# DATA STRUCTURES

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# DATA STRUCTURES

**TEXT BOOKS:**

1. Fundamentals of Data structures in C, 2nd Edition, E.Horowitz, S.Sahni and Susan Anderson-Freed, Universities Press.
2. Data structures A Programming Approach with C, D.S.Kushwaha and A.K.Misra, PHI.

# UNIT-I

**Topics:**

**Basic concepts-** Algorithm Specification-Introduction, Recursive algorithms, Data Abstraction Performance analysis- time complexity and space complexity, Asymptotic Notation-Big O, Omega and Theta notations, introduction to Linear and Non Linear data structures.

**Singly Linked Lists**-Operations-Insertion, Deletion, Concatenating singly linked lists, circularly linked lists-Operations for Circularly linked lists, Doubly Linked Lists-Operations- Insertion, Deletion. Representation of single, two dimensional arrays, sparse matrices-array and linked representations.

# ALGORITHM

An algorithm is a step by step representation or a procedure for solving a problem.

or

It is a method of finding a right solution to a problem or to a different problem or to a different problem breaking into simple cases.

# PROPERTIES OF AN ALGORITHM

Finitness:

An algorithm should terminate at finite number of steps.

Definiteness:

Each step of an algorithm must be precisely stated.

Effectiveness:

It consists of basic instructions that are realizable.

This means that the instructions can be performed by using the given inputs in a finite amount of time.

Input:

An algorithm accepts zero or more inputs.

Output:

It produces at least one output.

# PSEUDOCODE

It is a representation of algorithm in which instruction sequence can be given with the help of programming constructs.

or

Pseudo code, on the other hand, is not a programming language, but simply an informal way of describing a program.

Because it is not an actual programming language, pseudo code cannot be compiled into an executable program.

Therefore, pseudo code must be converted into a specific programming language if it is to become an usable application.

# PSEUDOCODE CONVENTIONS

1. Algorithm is a procedure consisting of heading and body. The heading consists of a name of the procedure and parameter list. The syntax is

   Algorithm name_of _procedure(paramater1,parameter2,…..parameter n).

2.Using assignment operator:=an assignment statement can be given. For instance: variable:=expression;

3. Boolean operators, logical operators, relational operators can be used in pseudo code.

4. All different types of arrays can be used and array indices stored in [ and ] brackets.

# PSEUDOCODE CONVENTIONS

5. The beginning and end of block should be indicted by { and} resp. the compound statements should be enclosed within { and } brackets.

6. The delimiters ; are used at the end of each statement.

7. Single line comments are written using // as beginning of comment.

8.The identifier should beginning by letter only.

9.No need to write data types explicitly for identifiers.

# PSEUDOCODE CONVENTIONS

10. The inputting and outputting can be done using read and write.

11. The conditional statements and the looping statements have the same syntax as in C language.

# EXAMPLES

1) write an algorithm to count the sum of n numbers

```
 Algorithm sum(1,n)
{
    Result:=0;
    for i:=1 to n do i:=i+1
   Result:=result+i;
}
```

# EXAMPLES

2) write an algorithm to check whether given number is even or odd

```
 Algorithm events (val)
{
     if  (val%2==0) then
       write(“given no is even”):
       else
       write(“given no is odd”);
    }
```

# EXAMPLES

3) write an algorithm to find factorial of n number.

```
Algorithm fact(n)
{
    if n:=1 then
            return  1;
    else
            return n*fact(n-1);
}
```

# EXAMPLES

4) write an algorithm to perform multiplication of two matrices.

```
Algorithm Mul(A,B,n)
{
    for i:=1 to n do
           for j:=1 to n do
                      c[i,j]:=0
           for k:=1 to n do
                       c[i,j]:=c[i,j]+a[i,k]*b[k,j];
}
```

# RECURSIVE ALGORITHM

A recursive routine is one whose design includes a call to itself.

Or

A function that calls itself is known as recursive function and this technique is known as recursion in C programming.

# EXAMPLES

Factorial of a number

```
Algorithm factorial(a)
int a;
        {
          int fact=1
              if(a>1)
              Fact = a* factorial(a-1);
              Return(fact);
        }
```

# DATA ABSTRACTION

Data abstraction refers to, providing only essential information to the outside world and hiding their background details, i.e., to represent the needed information in program without presenting the details.

Data abstraction is a programming (and design) technique that relies on the separation of interface and implementation.

# PERFORMANCE EVALUATION

The total effectiveness of a computer system, including throughput, individual response time, and availability.

Performance evaluation can be loosely divided into 2 phases

1) A priori estimates which is known as performance analysis

1) A posterior testing which is known as performance measurement.

# **PERFORMANCE ANALYS**IS

The efficiency of an algorithm can be decided by measuring the performance of an algorithm.

The performance of an algorithm by computing amount of time(time complexity )and storage requirement(space complexity).

# TIME COMPLEXITY

This is the amount of computing time required by an algorithm to run to completion.

There are 2 types of computing time- compile time and run time.

The time complexity is generally computed using run time or execution time.

The time complexity is given in terms of frequency count.

Frequency count is basically a count denoting number of times of execution of statement.

# TIME COMPLEXITY

The time complexity is computed using run time is calculated by input size and asymptotic notations .

Input size: the input size of any instance of a problem is defined as the number of words required to describe that instance of problem.

Asymptotic notations: This is the shorthand way to represent the time complexity.

Time complexity is given as fastest possible, slowest possible or average time.

Notations such as Ω,θ,O are the asymptotic notations.

# BIG OH NOTATION

It represents the upper bound of algorithms running time.

The longest amount of time taken by the algorithm to complete is calculated by big oh (O).

Def: let f(n) and g(n) are two non-negative functions. And is there exists an integer no and constant C such that C>0 and for all integers n>$n_0$ ,f(n)<=c*g(n), then f(n) is big oh of g(n). It is also denoted as “ f(n) = O(g(n))”.

# BIG OH NOTATION

Various meanings associated with big-oh are

O(1)- constant computing time

O(n)- linear

$O(n^2)$-quadratic

$O(n^3)$-cubic

O(2n)-exponential

O(log n)-logarithmic

# OMEGA NOTATION

It represents the lower bound of algorithms running time.

It is the shortest amount of time taken by algorithm to complete.

$F(n)>C*g(n)$.

This is denoted by $f(n)=\Omega g(n)$.

# THETA NOTATION

It represents the running time between upper bound and lower bound.

$c1g(n)<=f(n)<=c2g(n)$

It is denoted by $f(n)=\theta\ g(n)$.

# SPACE COMPLEXITY

This is the amount of memory required by an algorithm to run.

There are two factors to compute space complexity.

1) constant

2)instance

# SPACE COMPLEXITY

The space requirement S(p) can be given as

S(p)= C+Sp

C is the constant

Sp is a space dependent upon instance characteristics.

# DATA STRUCTURE

The data structure can be defined as the collection of elements and all the possible operations which are required for those set of elements.

Or

Data structure is a combination of a set of elements and corresponding set of operations.

The data structures can be implemented by building the suitable algorithms for them.

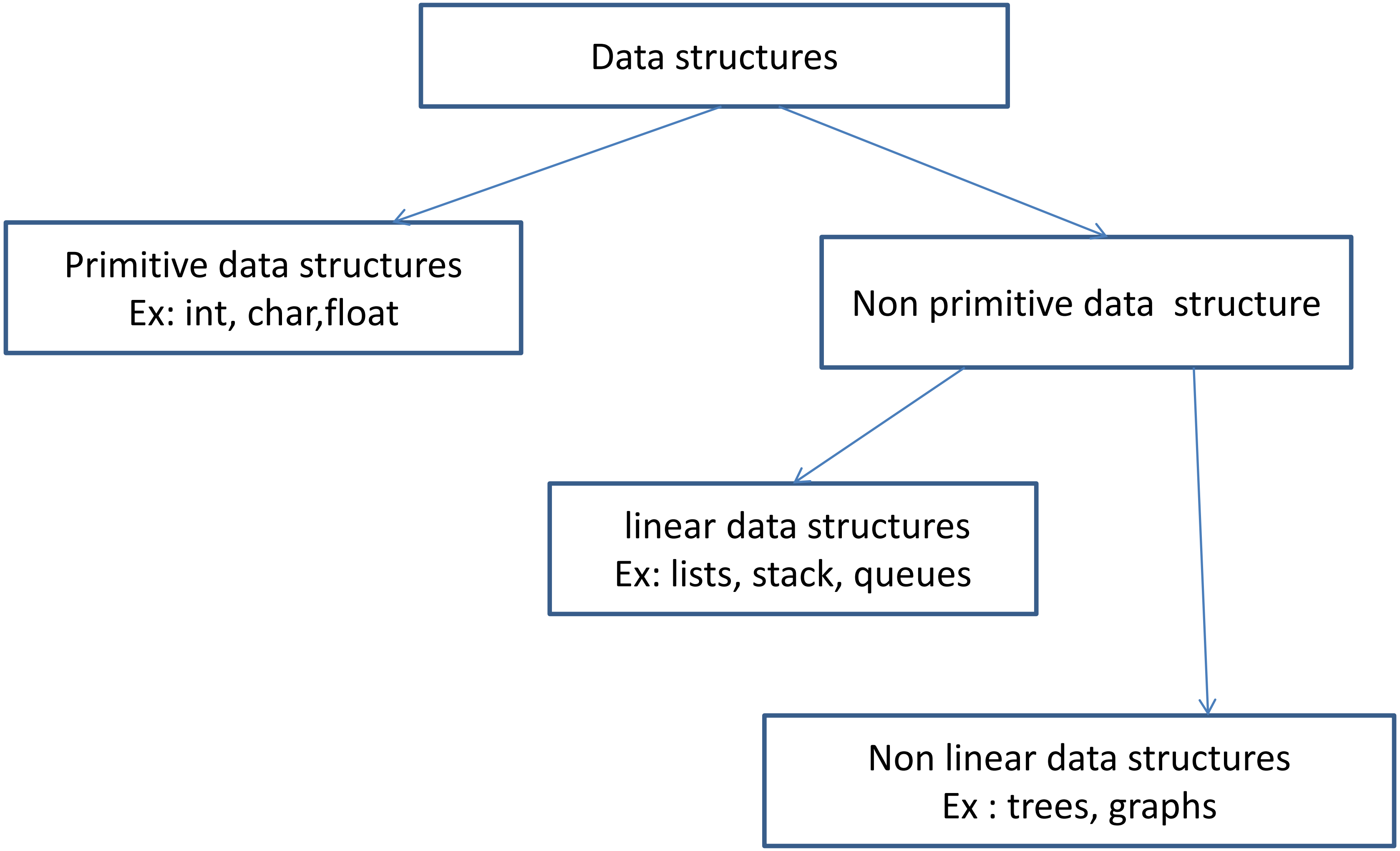
# TYPES OF DATA STRUCTURES

The data structure can be divided into two basic types.

1) Preliminary data structures

1) Secondary data structures

# TYPES OF DATA STRUCTURES



Data structures

Primitive data structures
Ex: int, char,float

Non primitive data  structure

linear data structures
Ex: lists, stack, queues

Non linear data structures
Ex : trees, graphs

# LIST

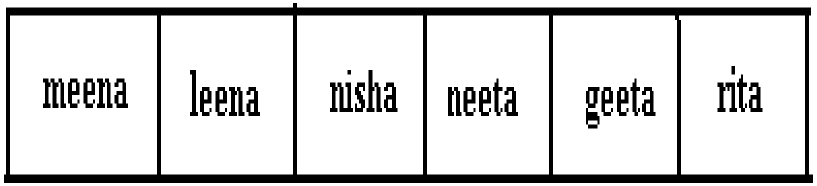
List is the collection of elements arranged in a sequential manner.

There are two representations

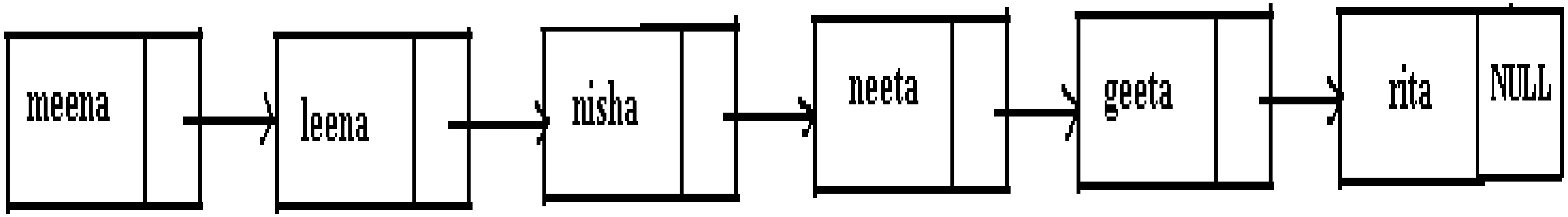
1) list of sequentially stored elements----using arrays

2) list of elements with associated pointers---using linked list.

# LIST REPRESENTATION



list of sequentially stored elements using arrays



# OPERATIONS ON AN ORDERED LIST

1)display of list.

2)search an element in the list.

3) insert an element into the list.

4) delete an element from the list.

# SINGLY LINKED LIST

In the single linked list, a node is connected to the next node by a single link.

In this list a node contains two types of fields-

data:

which holds a list element

next(pointer):

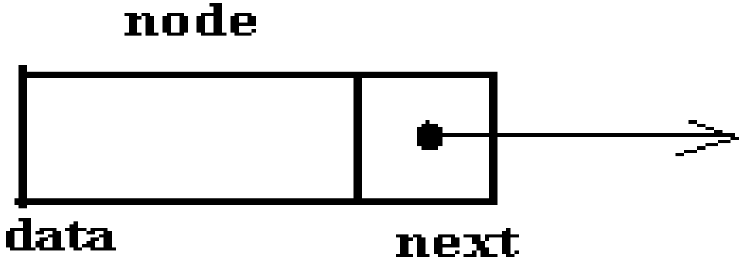
which holds a link to the next node in the list.

The head of the pointer is used to gain  access to the list and the end of the list is denoted by a NULL pointer

# STRUCTURE OF A SINGLE LINKED LIST

```
struct node
{
     int data;
            struct node * next;
}
```

The list holds two members ,an integer type variable “data” which holds the elements and another member of type “node”, which h



# SINGLE LINKED LIST OPERATIONS

Creating a linked list

Inserting in a linked list

Deleting a linked list

Searching an element in the linked list

Display the elements

Merging two linked list

Sorting a linked list

Reversing a list

# CREATING A LINKED LIST

List can be created by using pointers and dynamic memory allocation function such as **malloc**.

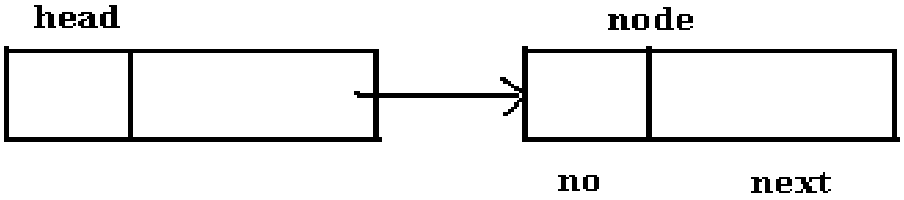
The head pointer is used to create and access unnamed nodes.

# CREATING A LINKED LIST

```
struct list
{
        int no;

        struct list *next;
};

typedef struct list node;

node *head;

head=(node*) malloc (size of(node));
```

# CREATING A LINKED LIST

The statement obtains memory to store a node and assigns its address to head which is a pointer variable.



To store values  in the member fields :

```
head→no=10;
head→next=NULL;
```

The second node can be added as:

```
head→next=(node*)malloc(size_of(node));
head→next→number=20;
head→next→next=NULL;
```

# INSERTING AN ELEMENT

Insertion is done in three ways:

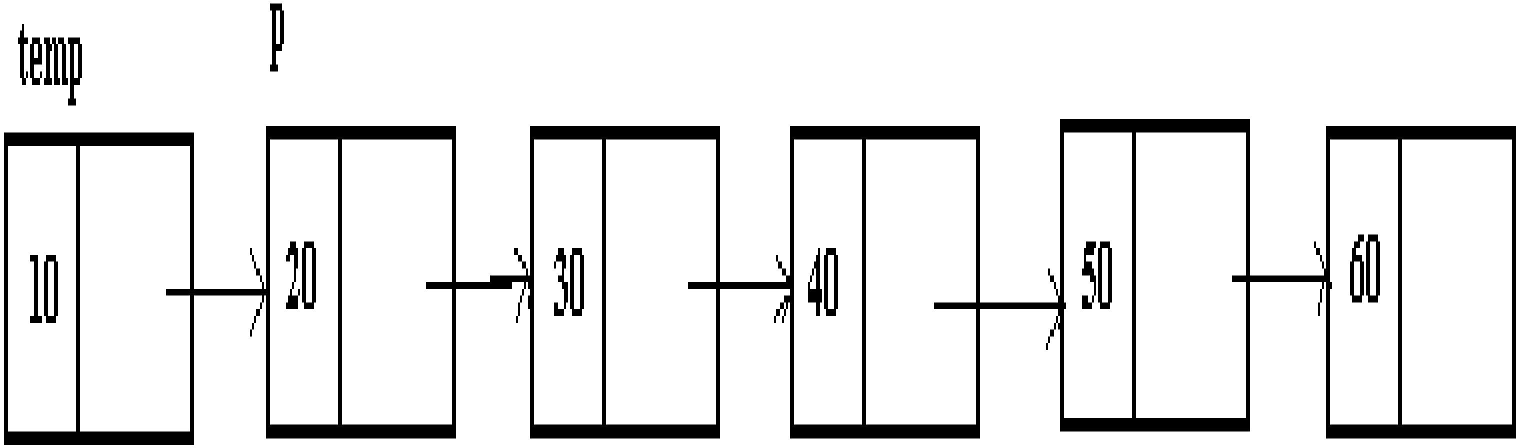
Insertion at the beginning of the list.

Insertion after any specified node.

Inserting node at the end of the list.

# INSERTING AN ELEMENT

Function to insert a node at the beginning of the list:



# INSERTING AN ELEMENT

Function to insert a node at the beginning of the list:

```
void add_beg(struct node **q, int no)
 {
        struct node *temp;        /*add new node*/

         temp→data=no;

         temp→next=*q;t

         *q=temp;

 }
```

here temp variable is take and space is allocated using “malloc” function.

# INSERTING AN ELEMENT

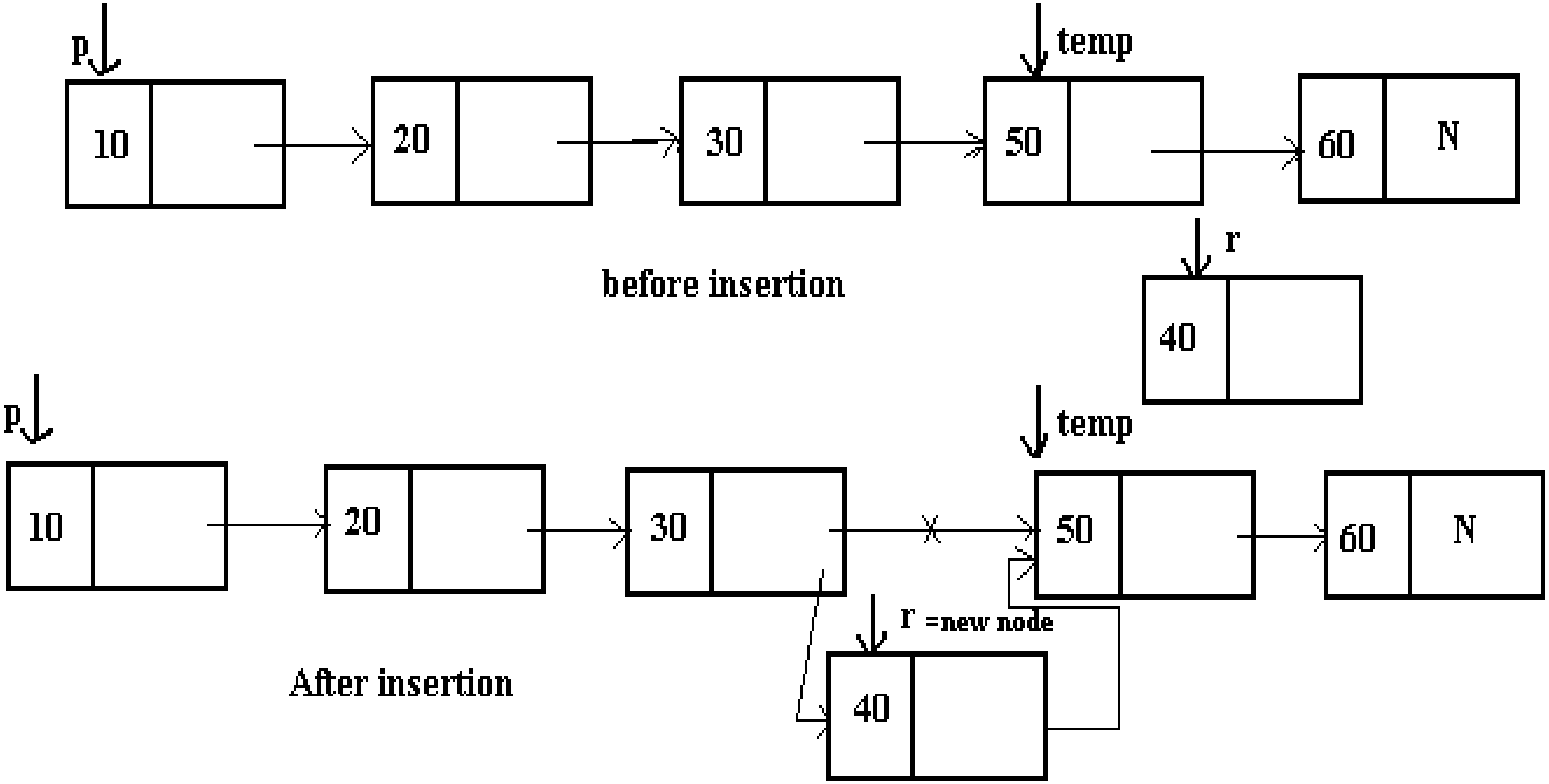
Insertion after any specified node:

Inserting a node in the middle of the list,

if you consider to insert a node after the element then the

process is as follows.

# INSERTING AN ELEMENT



# INSERTING AN ELEMENT

Function to insert a node at the middle of the list:

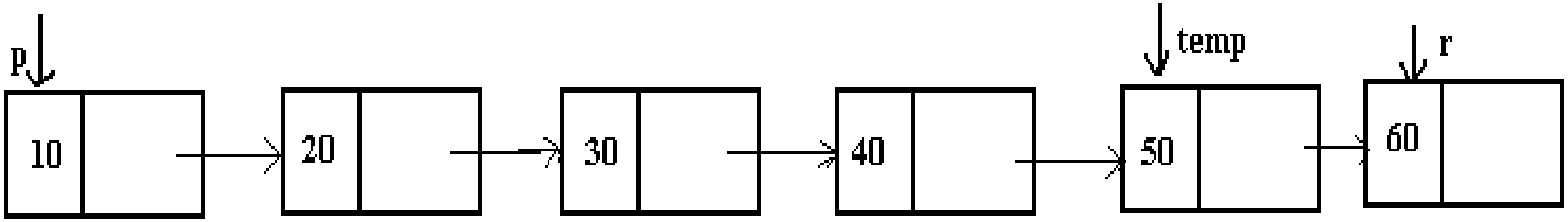
```
Void add_after(struct node *q, int loc, int no)

{
 struct node *temp, *r;

int l;

temp=q;/*skip to desire portion*/

for(i=0;i<loc;i++)

    {
          temp=temp→next;
```

# INSERTING AN ELEMENT

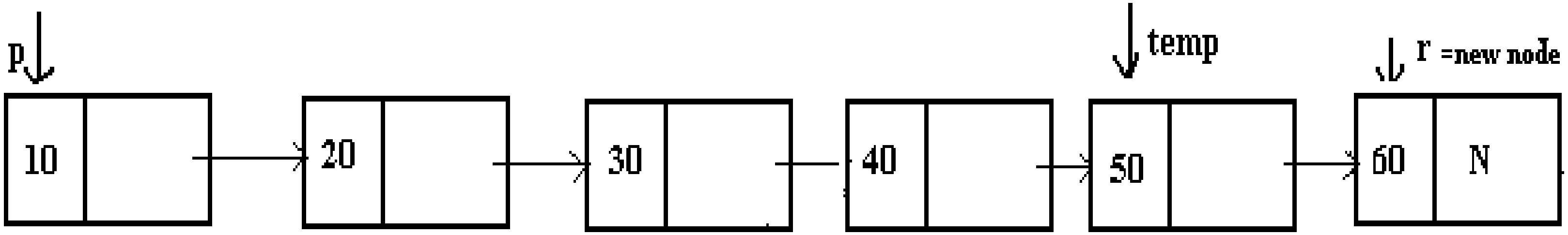
```
        if(temp==NULL)
        {
        printf(“\n there are less than %d elements in list”,loc);
         return;
        }
}                  ?/*insert new node*/
    r=malloc(sizeof(struct node));

    r→data=n0;

    r→next=temp→next;

    temp→next=r;
```

# INSERTING AN ELEMENT

Inserting node at the end of the list:



before insertion



After insertion

# INSERTING AN ELEMENT

Inserting node at the end of the list:

```
void create(struct node **q, int no)
{
 struct node *temp,*r;

    if(*q==NULL)       /*if the list is empty,create first node*/

    {
     temp=malloc(sizeof(struct node));

    temp→data=no;
```

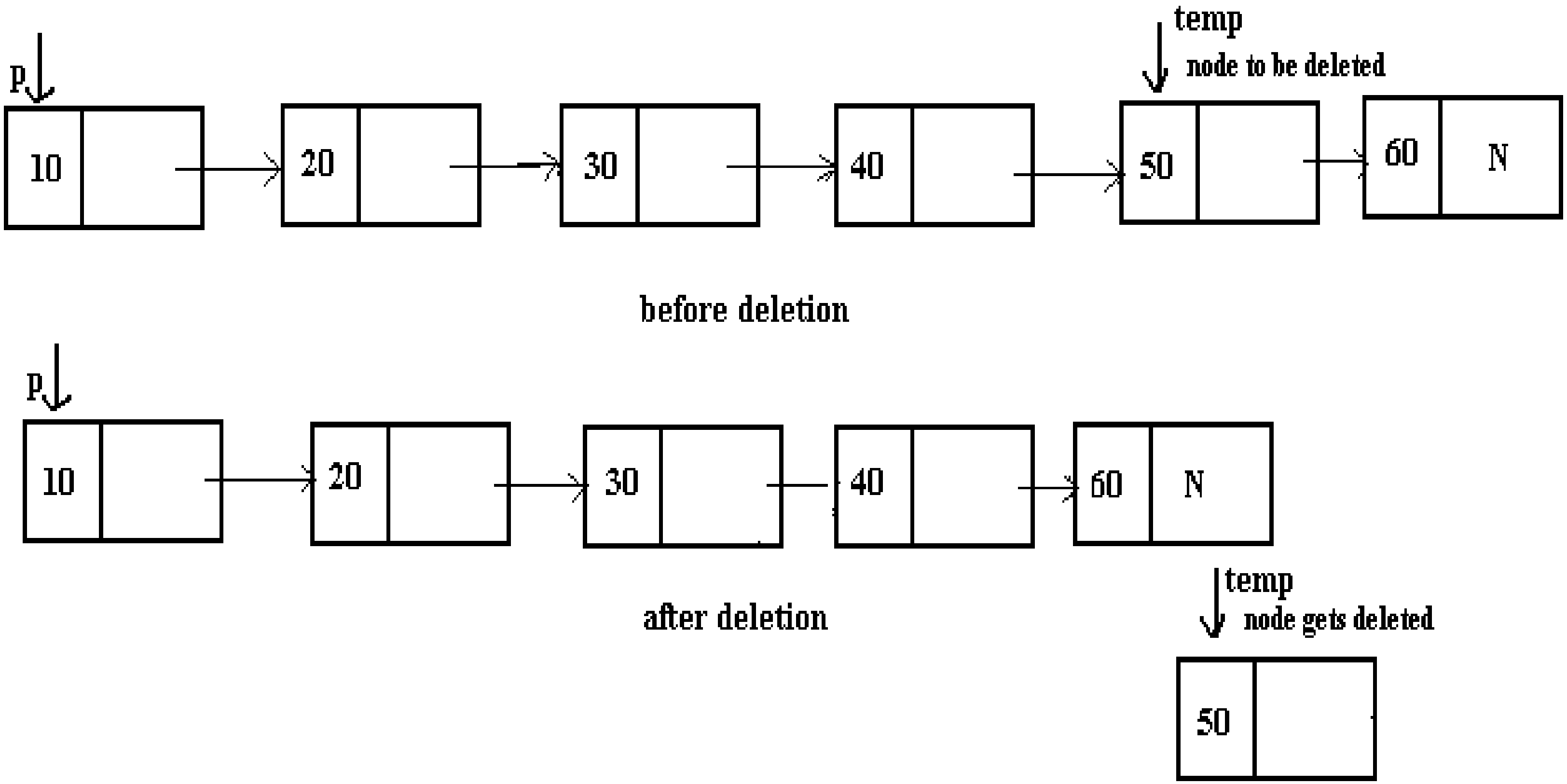
# INSERTING AN ELEMENT

```
  temp→next=NULL;

  *q=temp;

  }
else

   {

  temp=*q;    /* go to last node*/

  while(temp→next!=NULL)
```

# INSERTING AN ELEMENT

```
        temp=temp→next;
        r=malloc(sizeof(struct node));
        r→data=no;
        r→next=NULL;
        temp→next=r;
        }
}
```

# DELETING AN ELEMENT



# DELETING AN ELEMENT

We traverse through the entire linked list to check each node whether it has to be deleted.

if we want to delete the first node in the list then we shift the structure type pointer variable to the next node and then delete the entire node.

if the node is a intermediate node then the various pointers the linked list before and after deletion should be taken care of

# DISPLAYING THE CONTENTS OF THE LINKED LIST

Displays the elements of the linked list contained in the data part.

Function to display the contents of the linked list.

```
void display(struct node *start)
{
printf(“\n”);
```

# DISPLAYING THE CONTENTS OF THE LINKED LIST

```
    /*traverse the entire list*/

    while(start!=NULL)

    {
    printf(“%d”,start→data);

    start=start→next;

    }
}
```

# OTHER OPERATIONS OF SINGLY LINKED LIST

Searching the linked list:

Searching means finding information in a given linked list.

Reversing a linked list:

The reversing of the linked list that last node becomes the first node and first becomes the last.

# OTHER OPERATIONS OF SINGLY LINKED LIST

Sorting the list:

In sorting function the node containing the largest element is removed from the linked list and is appended to the new list in the ascending order.

CREATING A LINKED LIST

Merging the two linked list:

Merging two list pointed by two pointers into a third list.

While merging be ensure that the elements common to the lists appear only once in the third list.

# CIRCULAR LINKED LIST

A linked list in which last node points to the header node is called the circular linked list.

The list have neither a beginning nor an end.

In this list the last node contains a pointer back to the first node rather than the NULL pointer.

# CIRCULAR LINKED LIST

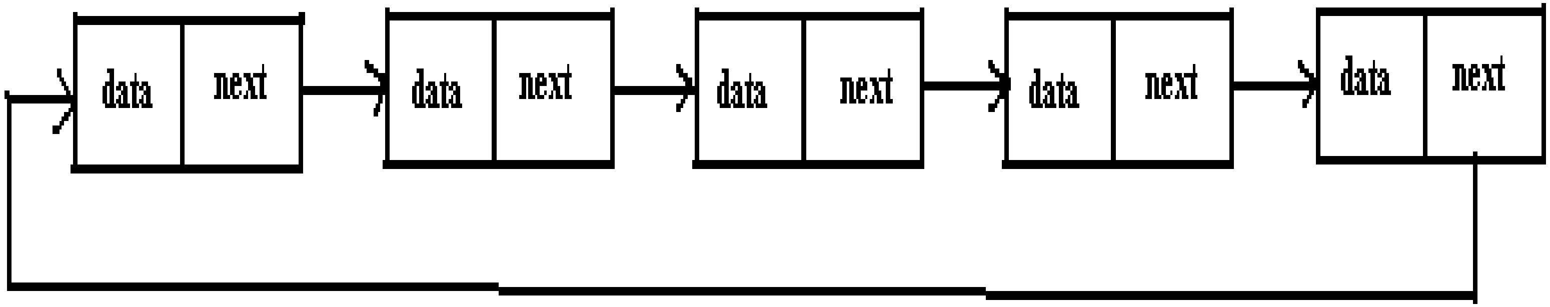
The structure defined for circular linked list

```
struct node

{
        int data;

        struct node *next;

   }
```

# CIRCULAR LINKED LIST

A circular linked list is represented as follows:

A circular linked list can be used to represent a stack and a queue.



# OPERATION OF CIRCULAR LINKED LIST

Adding elements in the circular linked list.

Deleting element from the circular list.

Displaying elements from the circular list.

# ADDING ELEMENTS IN THE CIRCULAR LINKED LIST

Ciradd():

this function accepts three parameters:

receives the address of the pointer to the first node.

receives the address of the pointer to the last node.

holds the data items that need to add in the list.

# DELETING ELEMENTS FROM THE CIRCULAR LINKED LIST

delcirq():

this function receives two parameters.

the pointer to the front .

the pointer to the rear .

# DELETING ELEMENTS FROM THE CIRCULAR LINKED LIST

The condition is checked for the empty list.
If the list is not empty,

then it is checked whether the front and rear
point to the same node or not.

If they point to the same node,

then the memory occupied by the node
is released and front and rear are both
assigned a NULL value.

# DISPLAYING THE CIRCULAR LIST

Cirq_disp():

the function receives the pointer to the first node in the list as a parameter.

The q is also made to point to the first node in the list.

The entire list is traversed using q.

Another pointer p is set to NULL initially.

The circular list is traversed through a loop till the time it reach the first node again.

It reach first node again when q equals p.

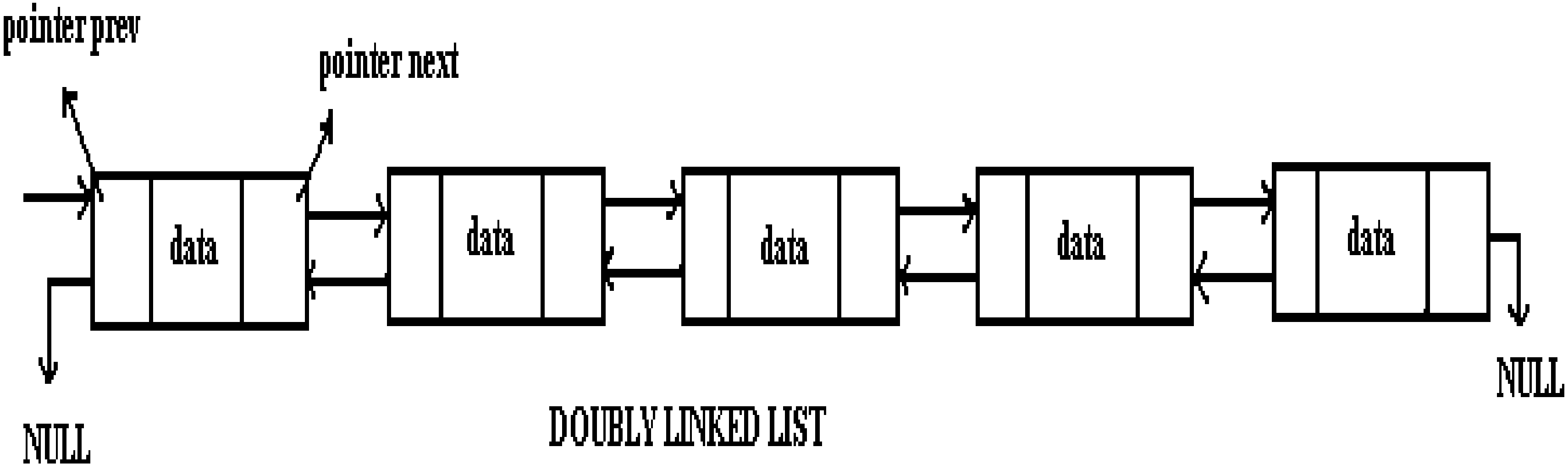
# DOUBLY LINKED LIST

The doubly linked list uses double set of pointer’s, one pointing to the next item and the other pointing to the preceding item.

It can traverse in two directions:

from the beginning of the list to the end
or
In the backward direction from the end of the list to the beginning.

# DOUBLY LINKED LIST



# DOUBLY LINKED LIST

Each node contains three parts:

An information field which contains the data.

A pointer field next which contains the location of the next node n the list.

A pointer field prev which contains the location of the preceding node in the list.

Structure to define DLL:

```
struct node
{  int data;
  struct node *next;
  struct node  *prev;
}
```

# CREATING A DLL

To create DLL at the nodes to the existing list:

To create the list the function d_create can be used before creating the list the function  checks if the list is empty.

Here the function accepts two parameters.

s of type struct dnode ** which contains the address of the pointer to the first node of the list.

parameter num is an integer which is to be added in the list.

# CREATING A DLL

To create DLL at the nodes to the existing list:

To create the list the function d_create can be used before creating the list the function  checks if the list is empty.

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s of type struct dnode ** which contains the address of the pointer to the first node of the list.

parameter num is an integer which is to be added in the list.

# OPERATIONS OF DLL

Adding a node in the beginning of DLL:

To add the node at the beginning of the list the function d_addatbeg() is used .

This function takes two parameters:
s of type dounode ** which contains the address of the pointer to the first node .

num is an integer to be added in the list.

# OPERATIONS OF DLL

The allocation of memory for the new node is done whose address is stored in q.

The num is the data part of the node.

A NULL value is stored in the prev part of new node a this is the first node in the list.

# OPERATIONS OF DLL

Function to add a node at the beginning of list.

```
Void d_addatbeg(struct dnode  **s,int num)
{
struct dnode *q;
     q=malloc(sizeof(struct dnode));
    q→prev=NULL;
    q→data=num;
    q→next=*s;
    (*s)→prev=q;
    *s=q;
    }
```

# OPERATIONS OF DLL

Adding a node in the middle of the list:

To add the node in the middle of the list we use the function d_addafter().

The function accepts three parameters.
q points to the first node of the list.
loc specifies the node number after which new node must be inserted.
num which is to be added to the list.

To reach to the position where node is to be inserted, a loop is executed.

# OPERATIONS OF DLL

Deleting a node from DLL:

This function deletes a node from the list if the data part matches a with num.

The function receives two parameters

the address of the pointer to the first node.

the number to be deleted.

To traverse the list ,a loop is run.

The data part of each node is compared with the num.

If the num value matches the data part, then the position of the node to be deleted is checked

# OPERATIONS OF DLL

Display the contents of DLL.

to display the contents of the doubly linked list, we follow the same algorithm that had used in the singly linked list.

Here q points to the first node in the list and the entire list is traversed .

Function to display the DLL.

```
void d_disp(struct dnode *q)
{  printf(“\n”);
while(q!=NULL)
{ printf(“%2d”,q→data);
 q=q→next;
}
}
```

# ARRAYS

A collection of objects of the *same type* stored contiguously in memory under one name.

May be type of any kind of variable

May even be collection of arrays!

The elements of the array are stored in consecutive memory locations and are referenced by an index (subscript).

# ARRAYS

To refer to an element, specify

—Array name

—Position number

Syntax:

array_name[ position number ]

# ARRAYS

Array Declaration

When declaring arrays

- Name
- Type of data elements
- Number of elements

Syntax

*Data_Type      array_Name[ Number_Of_Elements ];*

# ARRAYS

Examples:

```
int c[ 10 ];
float myArray[ 3284 ];
```

Declaring multiple arrays of same type

- Format similar to regular variables
- Example:

```
int b[ 100 ], x[ 27 ];
```

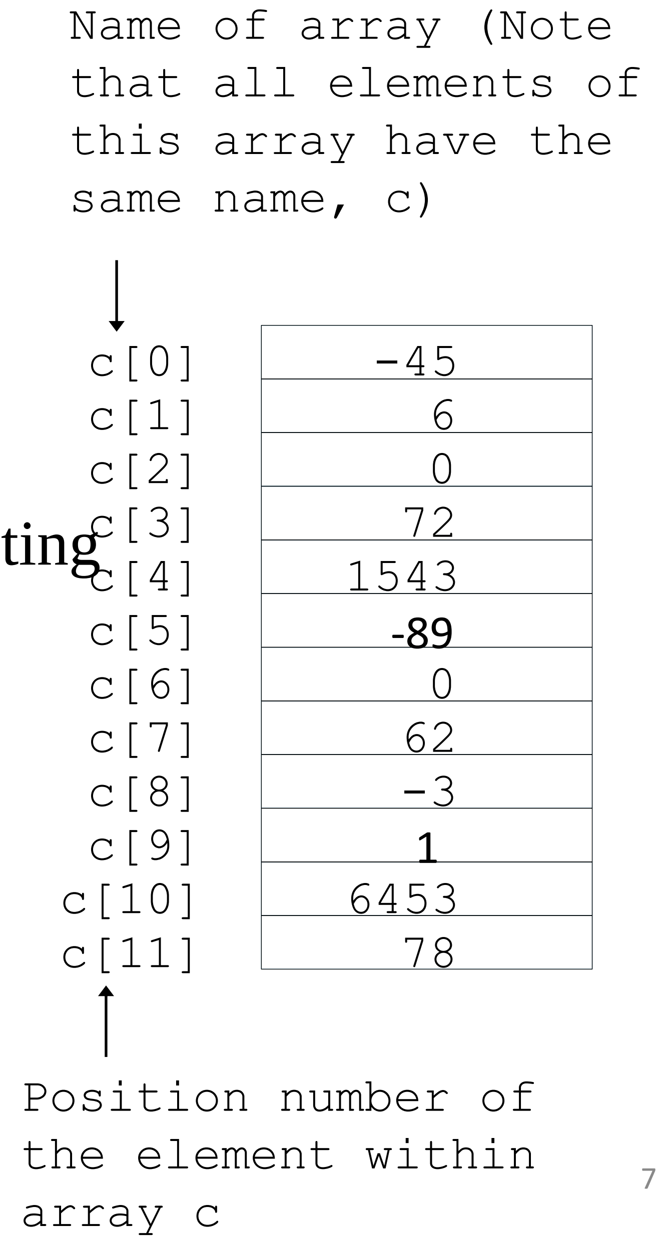
# ARRAYS

int c[12]

- An array of ten integers
- c[0], c[1], …, c[11]

double B[20]

- An array of twenty long floating point numbers
- B[0], B[1], …, B[19]



Name of array (Note that all elements of this array have the same name, c)

| | |
|---|---|
| c[0] | -45 |
| c[1] | 6 |
| c[2] | 0 |
| c[3] | 72 |
| c[4] | 1543 |
| c[5] | -89 |
| c[6] | 0 |
| c[7] | 62 |
| c[8] | -3 |
| c[9] | 1 |
| c[10] | 6453 |
| c[11] | 78 |

Position number of the element within array c

# ARRAYS

Arrays of structs, unions,

pointers, etc., are also allowed

Array indexes always

start at zero in *C*

# ARRAYS

Two Dimensional Array

- Syntax

  Data_Type array_Name[ Row_Elements][Column_Elements];

- Example

  int D[10][20]

  – An array of ten rows, each of which is an array of twenty integers

  – D[0][0], D[0][1], …, D[1][0], D[1][1], …, D[9][19]

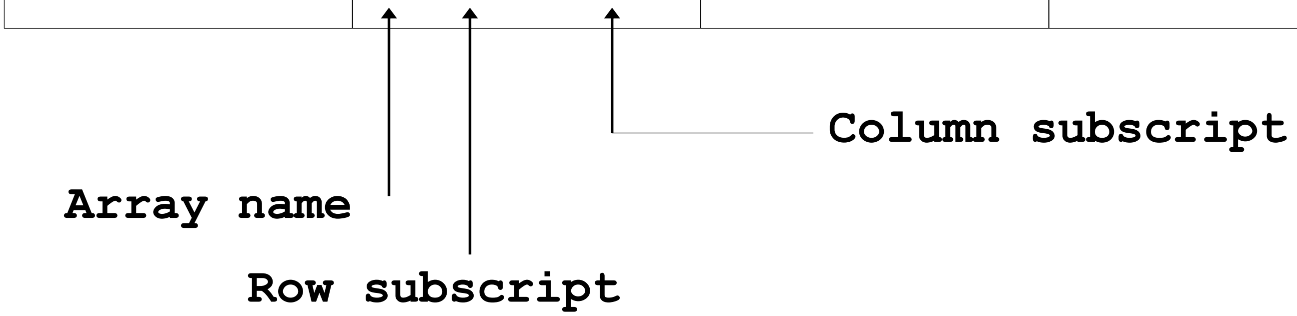
  – Not used so often as arrays of pointers

# ARRAYS

Two Dimensional Array

- Multiple subscripted arrays as
  - Tables with rows and columns (m×n array)
  - Like matrices: specify row, then column

| | Column 0 | Column 1 | Column 2 | Column 3 |
|---|---|---|---|---|
| Row 0 | a[0][0] | a[0][1] | a[0][2] | a[0][3] |
| Row 1 | a[1][0] | a[1][1] | a[1][2] | a[1][3] |
| Row 2 | a[2][0] | a[2][1] | a[2][2] | a[2][3] |



Column subscript

Array name

Row subscript

# ARRAYS

Multi Dimensional Arrays

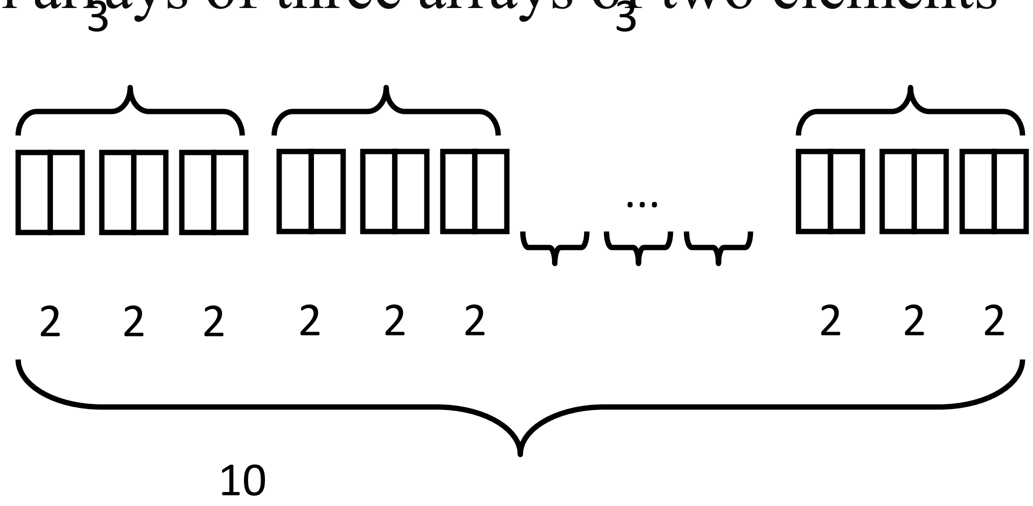
- Array declarations read right-to-left
- Syntax

  Data_Type  array_Name[ Size ][Size][Size] … Size];
- Example

        int a[10][3][2];

  "an array of ten arrays of three arrays of two elements"

  in memory



# SPARSE MATRIX

A sparse matrix is a matrix that allows special techniques to take advantage of the large number of zero elements.

Sparse matrix is very useful in engineering field, when solving the partial differentiation equations.

if there are maximum zeros then the matrix is known as sparse. matrix.

if there are few zeros then the matrix is dense matrix.

# CATEGORIES OF SPARSE MATRIX

Sparse matrix has

$N^2$ sparse matrix and

Triangular sparse matrix

A matrix with zero entries that form a square or a bar is $N^2$ sparse matrix.

A matrix with zero entries in its diagonal either in the upper or lower side is known as triangular sparse matrix.

# REPRESENTATION OF SPARSE MATRIX

Sparse matrix can be represented in

Tuple method

Array representation

Linked list representation

Only non zero elements are stored in any of the above representations.

# TUPLE METHOD

consider a matrix

[15 0 0 21<br>
22 11 0 0<br>
0 19 35 16]

This is $N^2$ sparse matrix

Tuple matrix is

| row | column | value |
|---|---|---|
| 1 | 1 | 15 |
| 1 | 4 | 21 |
| 2 | 1 | 22 |
| 2 | 2 | 11 |
| 3 | 2 | 19 |
| 3 | 3 | 35 |
| 3 | 4 | 16 |

# TUPLE METHOD

consider a matrix

$$\begin{bmatrix} 4 & 0 & 0 & 0 \\ 3 & 11 & 0 & 0 \\ 1 & 22 & 33 & 0 \\ 7 & 45 & 41 & 22 \end{bmatrix}$$

This is triangular matrix

Tuple matrix is

| row | column | value |
|---|---|---|
| 1 | 1 | 4 |
| 2 | 1 | 3 |
| 2 | 2 | 11 |
| 3 | 1 | 1 |
| 3 | 2 | 22 |
| 3 | 3 | 33 |
| 4 | 1 | 7 |
| 4 | 2 | 45 |
| 4 | 3 | 41 |
| 4 | 4 | 22 |

# ARRAY METHOD

consider a matrix

[15 0 0 21<br>
22 11 0 0<br>
0 19 35 15]

This is triangular matrix

The elements are represented as follows

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1,1,15 | 1,4,21 | 2,1,22 | 2,2,11 | 3,2,11 | 3,2,19 | 3,3,35 | 3,4,16 |

# SPARSE MATRIX OPERATIONS USING ARRAYS

Addition of two sparse matrix:

The function addmat() carries addition

The function display() displays the result.

Multiplication of two sparse matrix:

# SPARSE MATRIX OPERATIONS USING ARRAYS

This holds three functions

Sparseprod() stores the result.

Search_nonzero()checks whether an non zero element is present or not.

Searchinb()searches an element whose row number is equal to column number.

Transpose of a sparse matrix:

Transpose() is used to allocate memory to store the elements.

# REPRESENTATION OF SPARSE MATRIX THROUGH LINKED LIST

The elements of sparse matrix consist of three integers.

Its row number

Its column number

Its value

The head node consist of three parts.

Row number indicates the row to which the “head” node is pointing to the component element.

The head also points to another head the node for the next row.

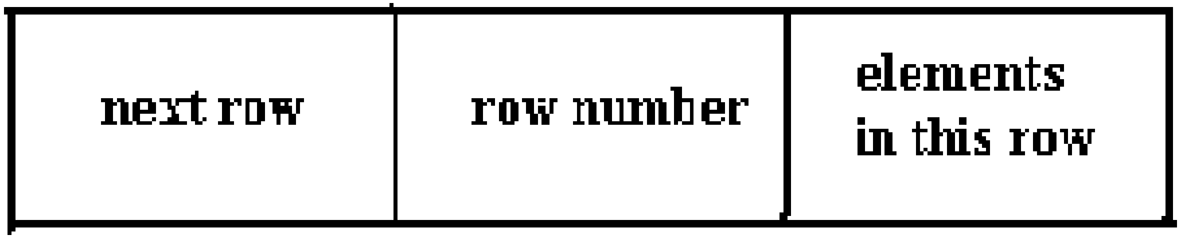
# REPRESENTATION OF SPARSE MATRIX THROUGH LINKED LIST

The create_list() function stores the information in the form of linked list.

Insert() accepts a pointer  to the special node .

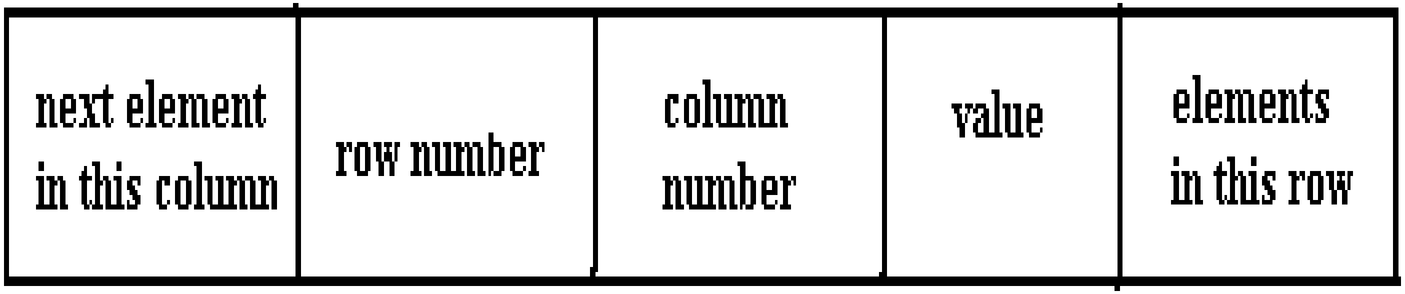
show_list() reads and displays the data stored in the linked list.

# REPRESENTATION OF SPARSE MATRIX THROUGH LINKED LIST



parts of head node for a row

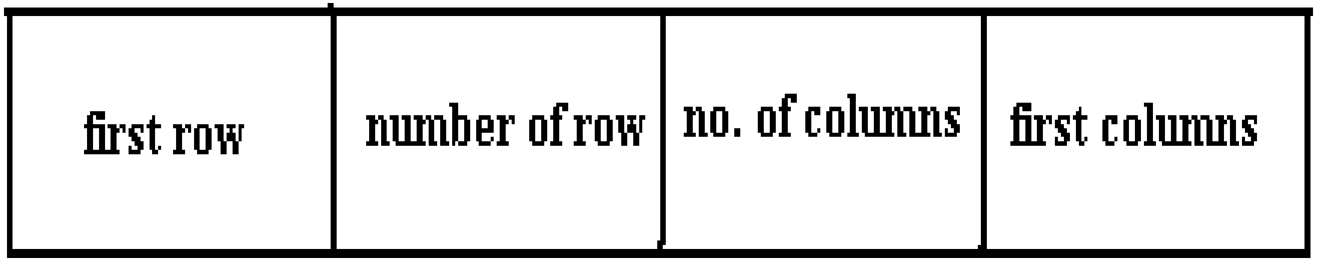
# REPRESENTATION OF SPARSE MATRIX THROUGH LINKED LIST



| | | | | |
|---|---|---|---|---|
| next element in this column | row number | column number | value | elements in this row |

structure for an element

# REPRESENTATION OF SPARSE MATRIX THROUGH LINKED LIST



| | | | |
|---|---|---|---|
| first row | number of row | no. of columns | first columns |

node

# UNIT-II

**Topics:**

Stack ADT, definition, operations, array and linked implementations in C, applications-infix to postfix conversion, Postfix expression evaluation, recursion implementation, Queue ADT, definition and operations ,array and linked Implementations in C, Circular queues-Insertion and deletion operations, Deque (Double ended queue) ADT, array and linked implementations in C.

# STACKS

A stack is a linear structures in which addition or deletion of elements takes place at the same end.

Or

The stack is an ordered list in which insertion and deletion is done at the same end.

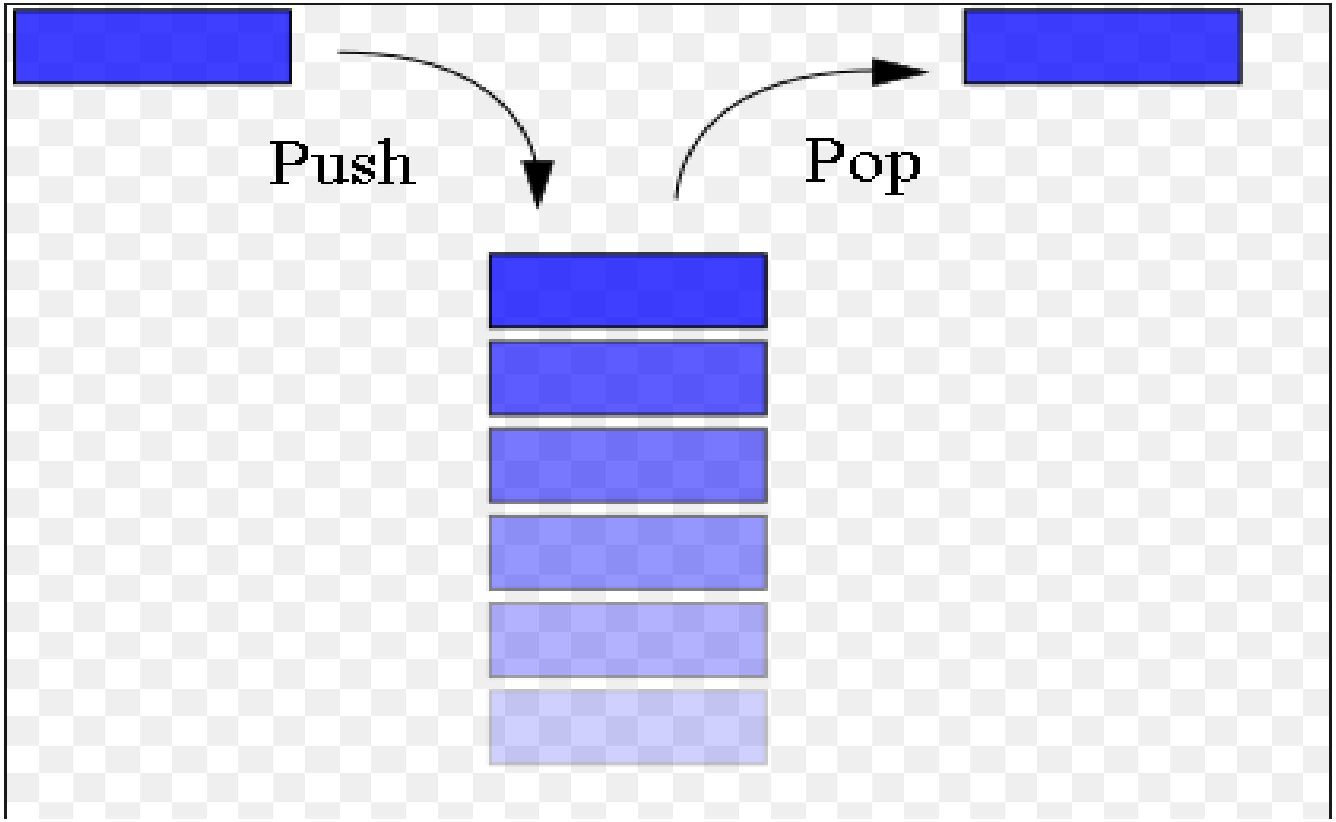
The end is called the top of stack.

Insertion and deletion cannot be done from the middle.

A technique of Last In First Out is followed.

Stack can be implemented by using both arrays and linked lists.

# STACKS



# STACK ADT

Stacks can also be defined as Abstract Data Types(ADT).

A stack of elements of any particular type is a finite sequence of elements of that type together with specific operations.

Therefore, stacks are called LIFO lists.

# STACK OPERATIONS

The primitive operations on stack are

To create a stack.

To insert an element on to the stack.

To delete an element from the stack.

To check which element is at the top of the stack.

To check whether a stack is empty or not.

# STACK OPERATIONS

If Stack is not full ,

then add a new node at one end of the stack

this operation is called PUSH.

If the stack is not empty

then delete the node at its top.

This operation is called POP.

PUSH and POP are functions of stack used to fulfill the stack operations.

TOP is the pointer locating the stack current position.

# ARRAY IMPLEMENTATION IN C

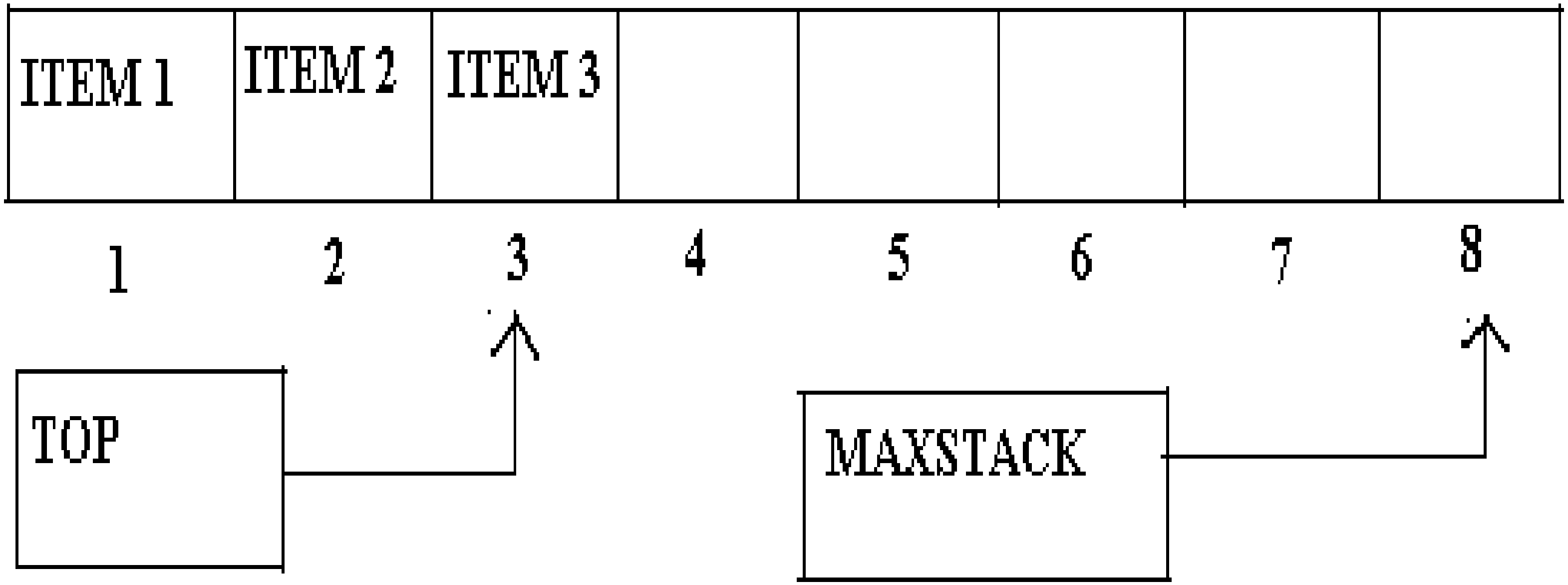
Stacks can be represented in the memory arrays by maintaining a linear array STACK and a pointer variable TOP which contains the location of top element.

The Variable MAXSTACK gives maximum number of elements held by the stack.

The TOP=NULL /0 will indicate that the stack is empty.

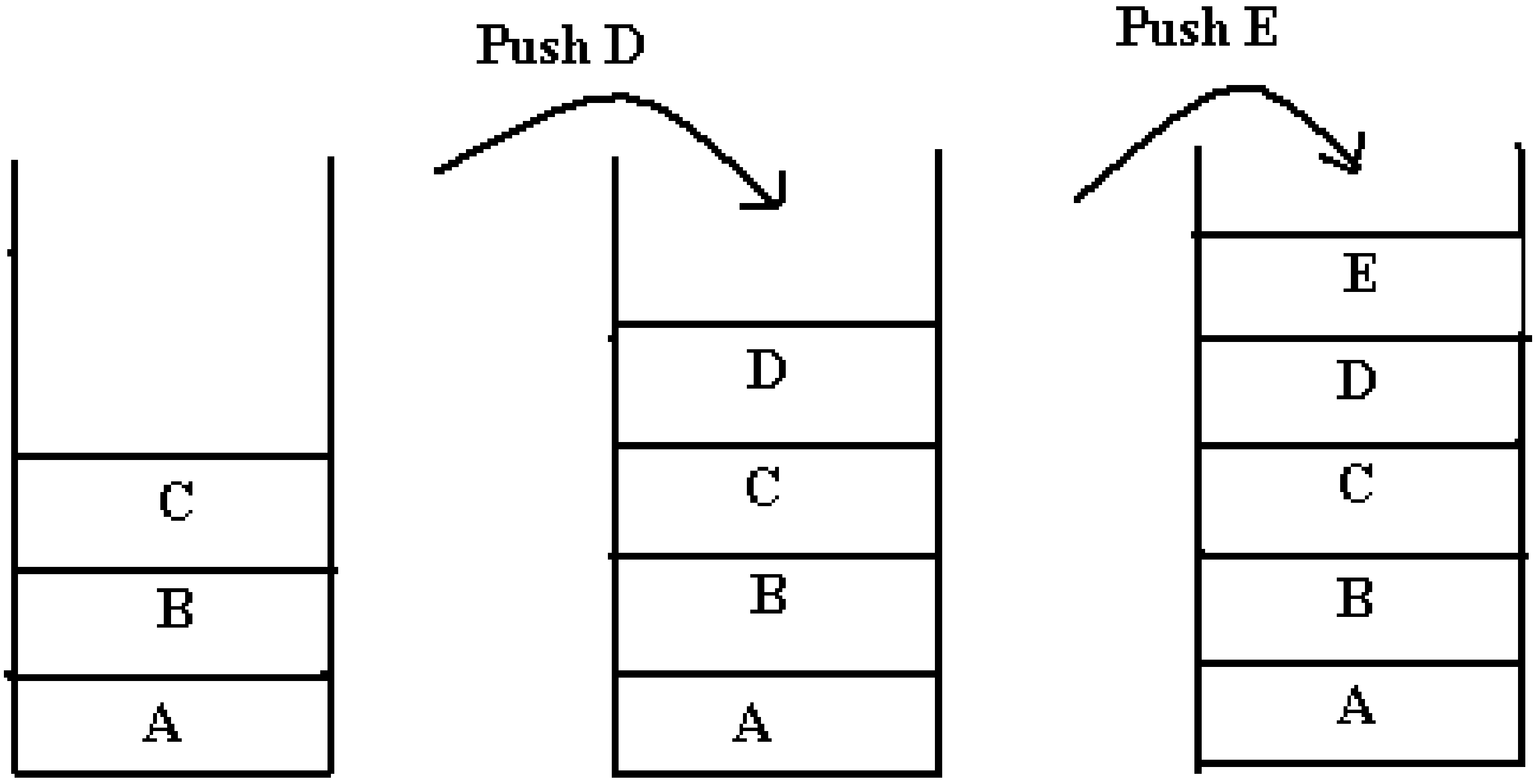
The operation of adding and removing an item in the stack can be implemented using the PUSH and POP functions.

# Figure shows the array representation



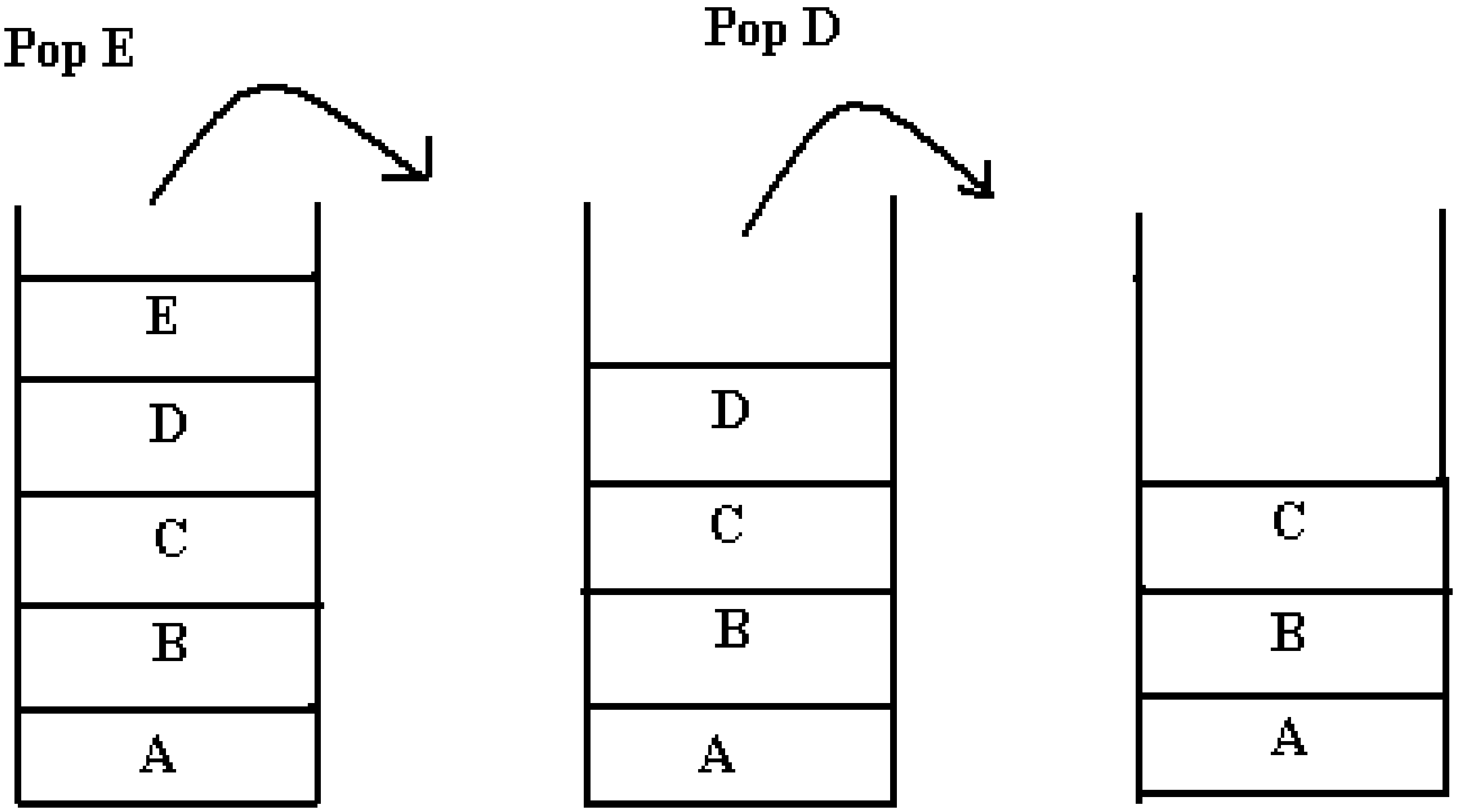
ARRAY REPRESENTATION OF A STACK

# Pictorial depiction of pushing elements in stack



Pushing Elements in Stack

# Pictorial depiction of popping elements in stack



Popping Elements from stack

# DISADVANTAGE OF STACK USING ARRAYS

The array representation of stack suffers from the drawbacks of the array's size, that cannot be increased or decreased once it is declared .

The space is wasted, if not used , or, there is shortage of space if needed.

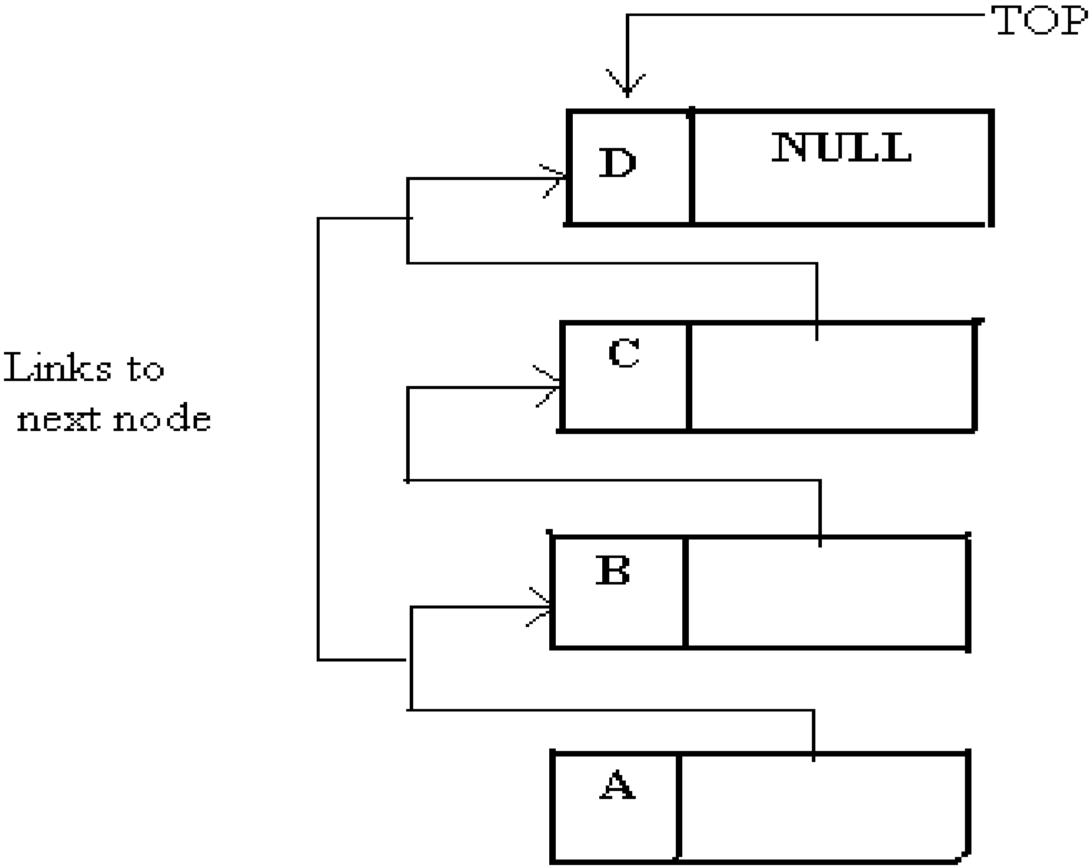
# LINKED IMPLEMENTATION IN C

The stack can be implemented using linked lists.

The stack as linked list is represented as a single linked list.

Each node in the list contains data and a pointer to the next node.

# Pictorial depiction of stack in linked list



# APPLICATION OF STACKS

Reversing a list.

Conversion of Infix to Postfix Expression.

Evaluation of Postfix Expression.

Conversion of Infix to Prefix Expression.

Evaluation of Prefix Expression.

# CONVERSION OF INFIX TO POSTFIX EXPRESSION

While evaluating an infix expression,

operations are executed according to the order as follows:

Brackets / Parentheses.

Exponentiation.

Multiplication / Division.

Addition / Subtraction.

the operators with the same priority(e.g. * and /) are evaluated from left to right.

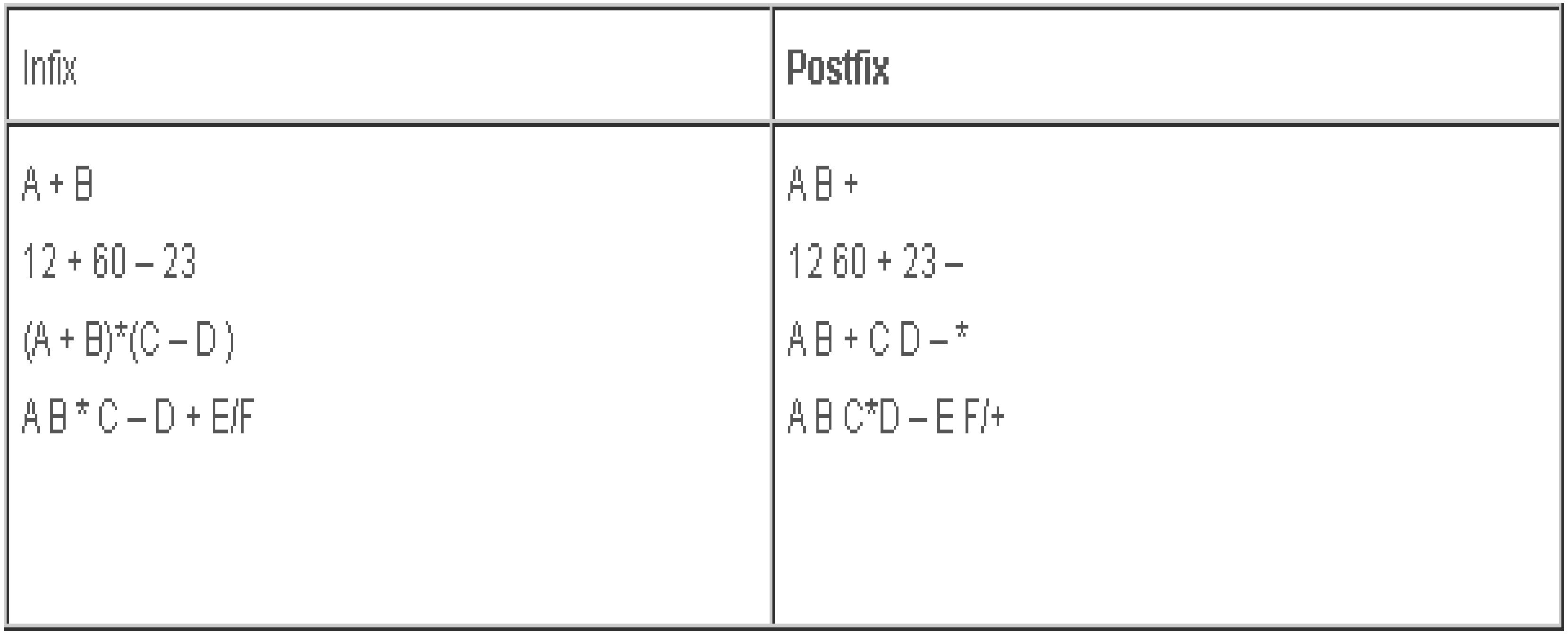
# STEPS TO CONVERT INFIX TO POSTFIX EXPRESSION

Step 1: The actual evaluation is determined by inserting braces.

Step 2: Convert the expression in the innermost braces into postfix notation by putting the operator after the operands.

Step 3: Repeat the above step (2) until the entire expression is converted into postfix notation.

# EXAMPLE OF INFIX TO POSTFIX CONVERSION



| Infix | Postfix |
|---|---|
| A + B | A B + |
| 12 + 60 – 23 | 12 60 + 23 – |
| (A + B)*(C – D ) | A B + C D – * |
| A B * C – D + E/F | A B C*D – E F/+ |

# RECURSION IMPLEMENTATION

If a procedure contains either a call statement to itself/to a second procedure that may eventually result in a cell statement back to the original procedure. Then such a procedure is called as recursive procedure.

Recursion may be useful in developing algorithms for specific problems. The stack may be used to implement recursive procedures.

# QUEUE

Queue is a linear list of elements in which deletion of an element can take place only at one end,

called the front

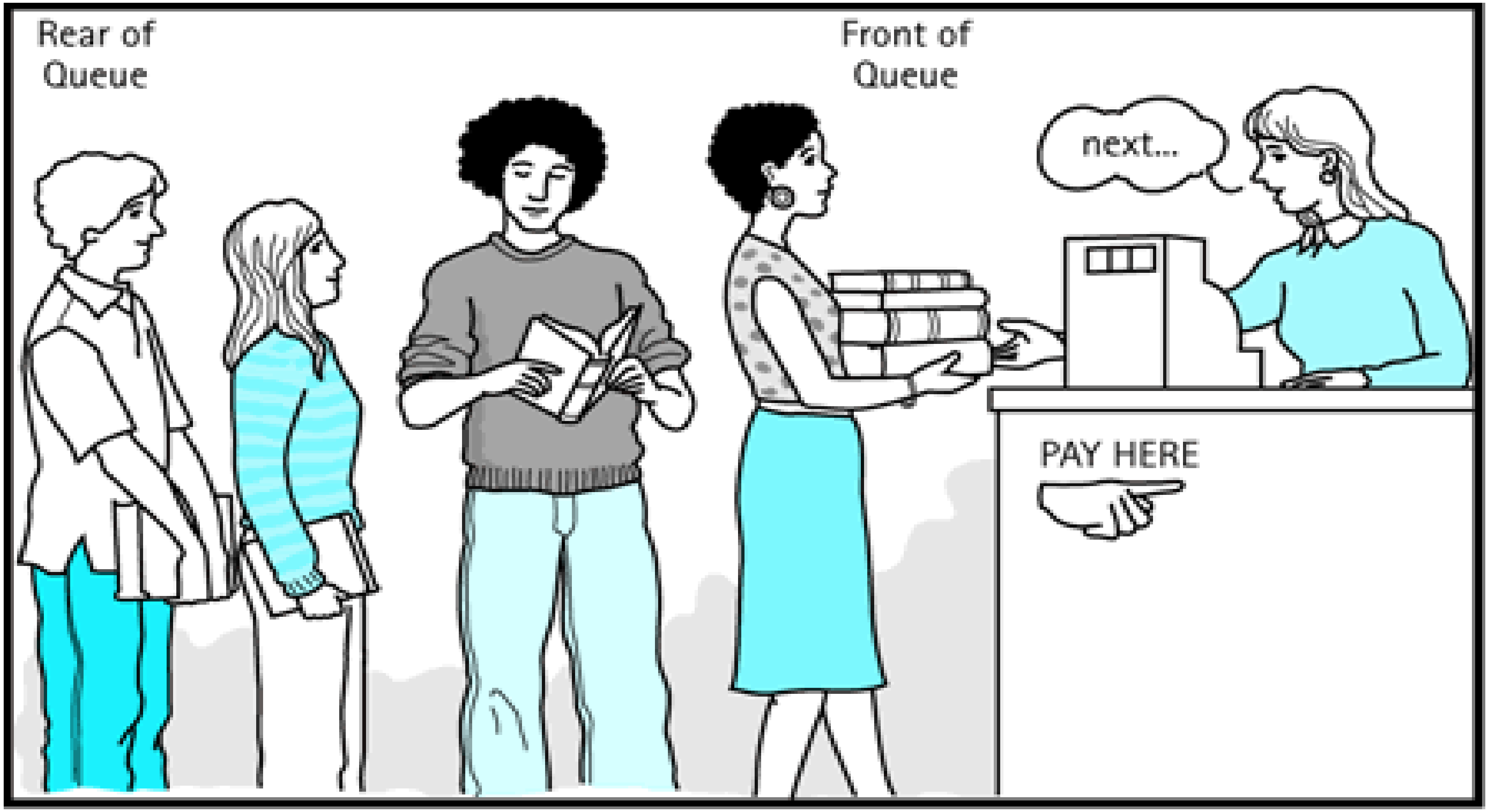
and insertion can take place only at the other end,

called the rear.

The first element in a queue will be the first one to be removed from the list.

Therefore, queues are called FIFO lists.

# QUEUE



# QUEUE ADT

The definition of an abstract data type clearly states that for a data structure to be abstract, it should have the two characteristics as follows.

There should be a particular way in which components are related to each other.

A statement of the operations that can be performed on element of the abstract data type should specified.

# QUEUE OPERATIONS

Queue overflow.

Insertion of the element into the queue.

Queue underflow.

Deletion of the element from the queue.

Display of the queue.

# ARRAY IMPLEMENTATION IN C

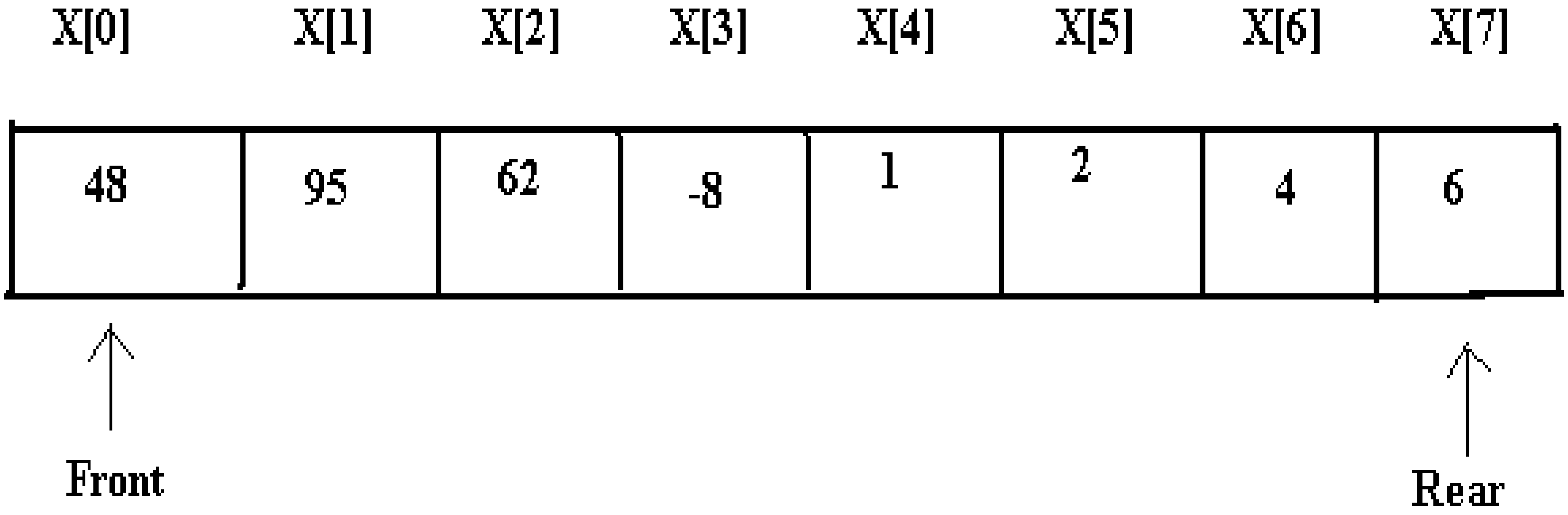
Array is a data structure that stores a fixed number of elements.

One of the major limitations of an array is that its size should be fixed prior to using it.

The size of the queue keeps on changing as the elements are either removed from the front end or added at the rear end.

The solution of this problem is to declare an array with a maximum size.

# FIGURE TO REPRESENT A QUEUE USING ARRAY



| X[0] | X[1] | X[2] | X[3] | X[4] | X[5] | X[6] | X[7] |
|---|---|---|---|---|---|---|---|
| 48 | 95 | 62 | -8 | 1 | 2 | 4 | 6 |

# INSERTION AND DELETION OPERATIONS IN QUEUE USING ARRAYS

We consider two variables front and rear which are declared to point to both the ends of the queue.

The array begins with index therefore , the maximum number of elements that can be stored can be consider as MAX-1(n-1).

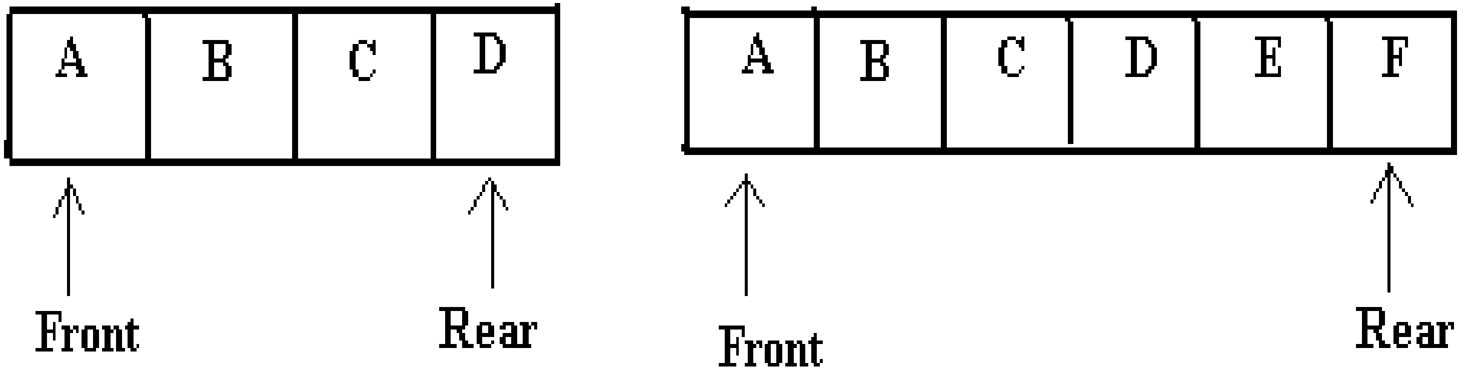
If the number of elements are already stored in the queue is reported to be full.

If the elements are added then the rear is incremented using the pointer and new item is stored in the array.

# ADDING ELEMENTS IN A QUEUE

The front and rear variables are initially set to -1, which denotes that the queue is empty.

If the item being added is the first  element then as the item is added, .the queue front is set to 0 indicating that the queue is now full.

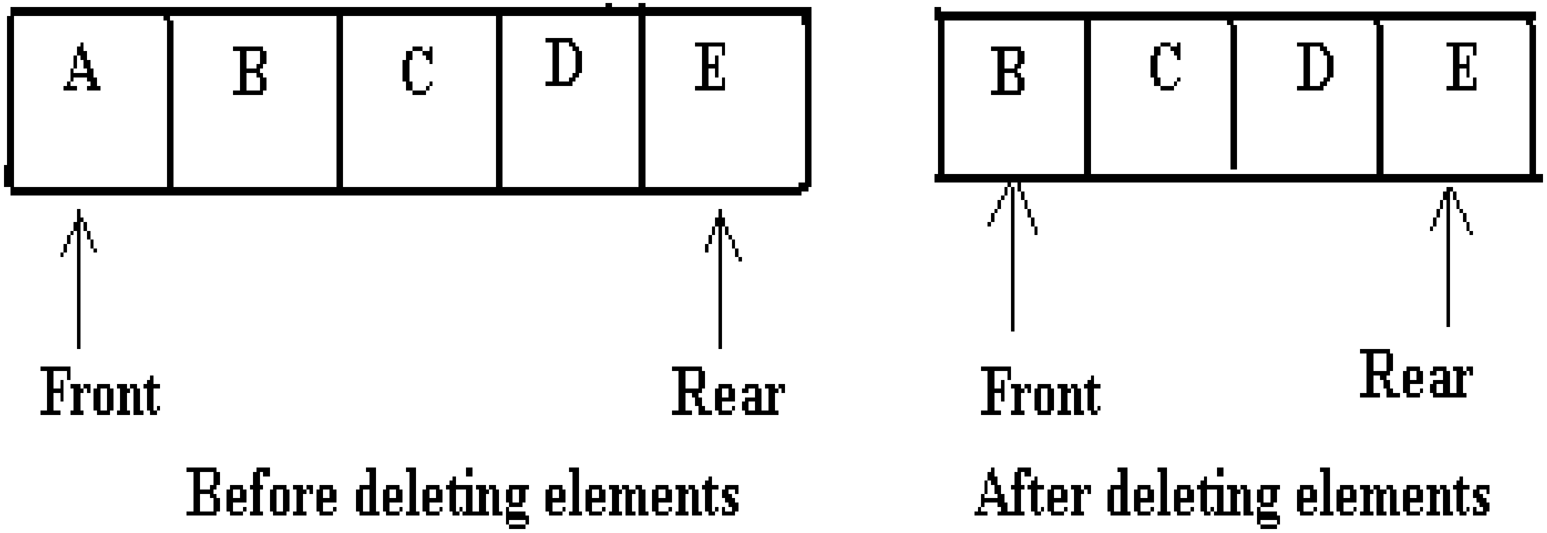


Before adding elements

After adding elements

# DELETING ELEMENTS IN A QUEUE

For deleting elements from the queue, the function first checks if there are any elements for deletion. If not , the queue is said to be empty otherwise an element is deleted.



# LINKED IMPLEMENTATION IN C

The linked list representation of a queue does not have any restrictions on the number of elements it can hold.

The elements are allocated dynamically , hence it can grow as long as there is sufficient memory available for dynamic allocation.

# APPLICATION OF QUEUE

Job scheduling.

Categorizing data.

Random number generation.

# TYPES OF QUEUES

Circular queue.

De queue (double ended queue).

Priority queue.

# CIRCULAR QUEUE

Circular queues are implemented in circular form rather than in a straight line.

This form over come the problem of unutilized space in linear queue implemented as an array.

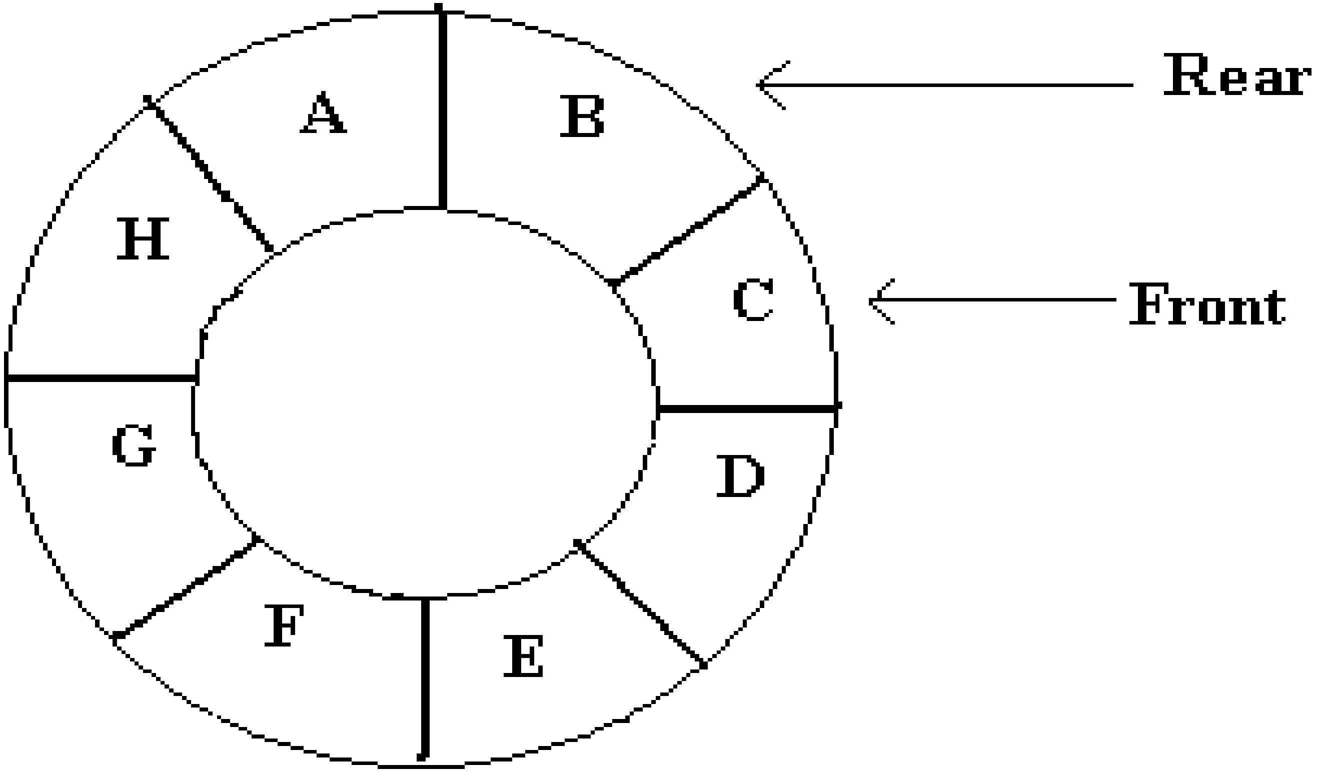
In the array implementation there is a possibility that the queue is reported full even though slots of the queue are empty.

# CIRCULAR QUEUE

Suppose an array x of n elements is used to implement a circular queue. If we go on adding elements to the queue we may reach x[n-1].

In a queue array if the elements reach the end then it reports the queue is full even some slots are empty but in circular queue ,it would not report as full until all the slots are occupied.

# REPRESENTATION OF CIRCULAR QUEUE



Circular queue

# ADDING ELEMENTS INTO CIRCULAR QUEUE

The conditions that are checked before inserting the elements :

If the front and rear are in adjacent locations(i.e. rare following front)the message ‘Queue is full’ is displayed.

If the value of front is -1 then it denotes that the queue is empty and that the element to be added would be the first element in the queue . The value of front and rear in such a case are set to 0 and new element gets placed at 0Th position.

# ADDING ELEMENTS INTO CIRCULAR QUEUE

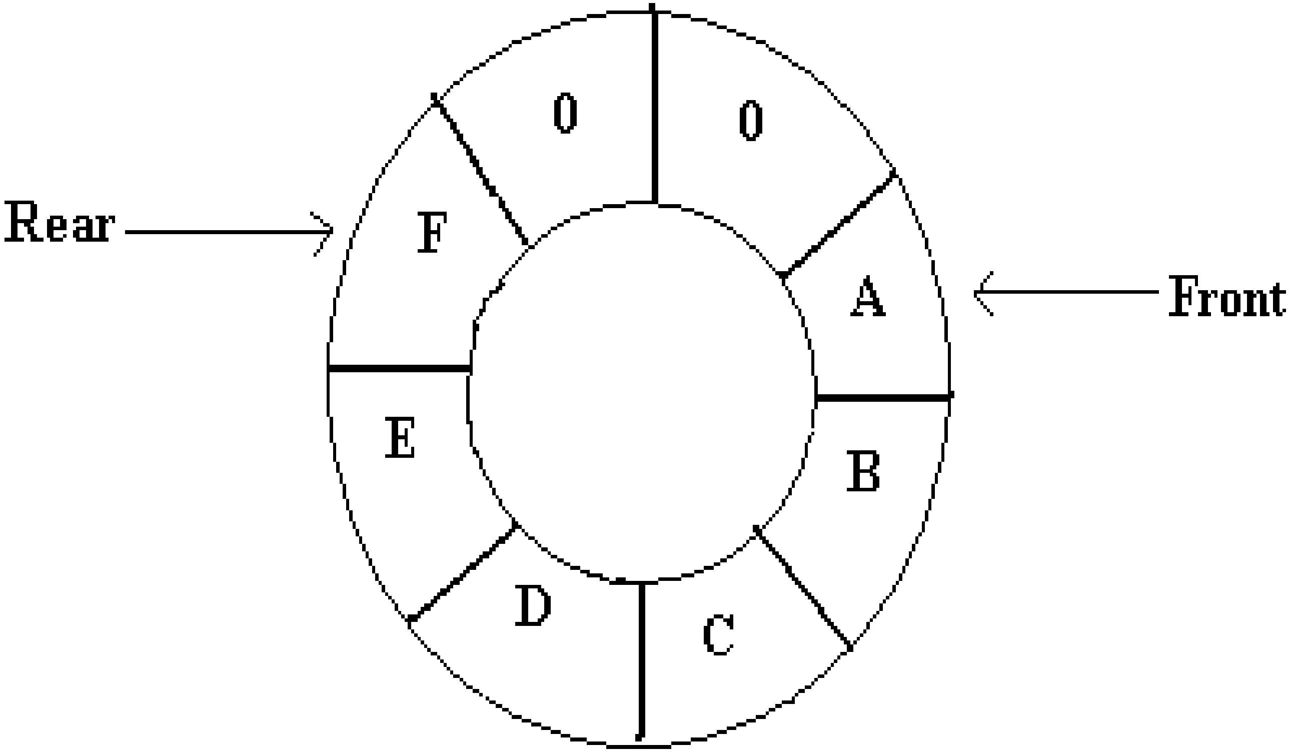
Some of the positions at the front end of the array might be empty .

This happens if we have deleted some elements from the queue , when the value of rear is MAX-1 and the value of front is greater than 0.

In such a case value of rear is set to 0 and the element to be added is added to this position.

The element is added at the rear position in case the value of front is either equal to or greater than 0 and the value of rear is less than MAX-1.

# ADDING ELEMENTS IN CIRCULAR QUEUE



Circular queue after adding 6 elements

# DELETING ELEMENTS INTO CIRCULAR QUEUE

The conditions that are checked before deleting the elements :

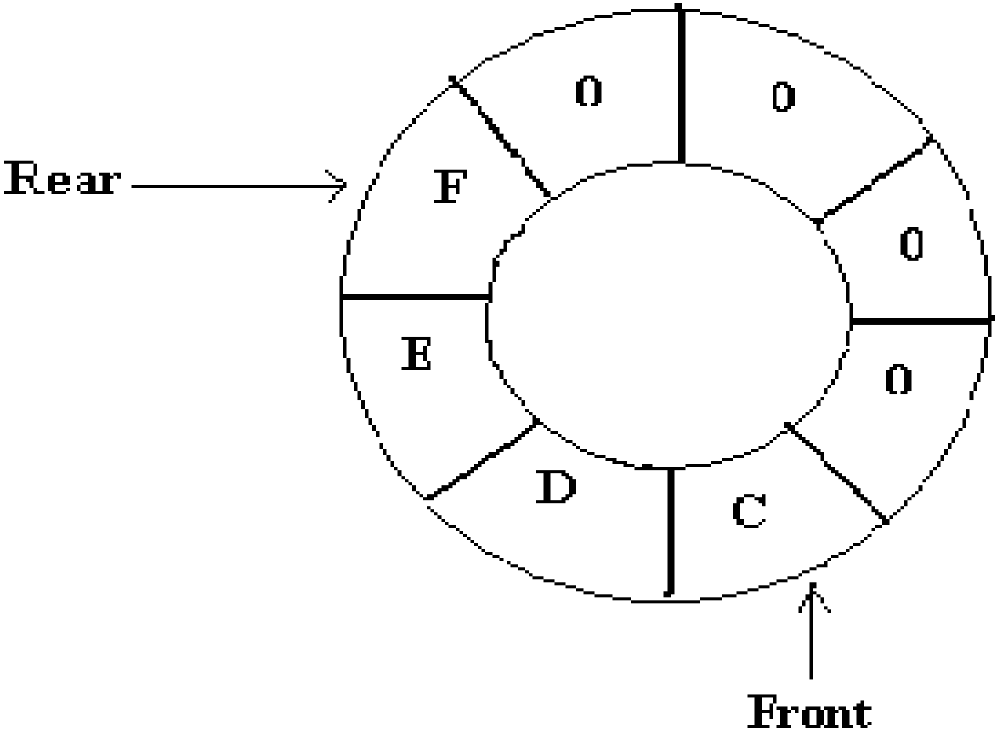
First it is checked whether the queue is empty or not . The elements at the front position will be deleted.

Now , it is checked if the value of front is equal to rear . If it is, then the element which will be deleted is the only element in the queue .

If the element is removes, the queue  will be empty and front and rear are set to -1.

# DELETING ELEMENTS IN CIRCULAR QUEUE

On Deleting an element from the queue the value of front is set to 0 if it is equal to MAX-1 otherwise front is simply incremented by 1.



Circular queue After deleting 2 elements

# DOUBLE ENDED QUEUE

A deque is a linear list in which elements can be added or removed at either end but not in the middle.

There are two variations of a deque an input restricted deque and an output restricted deque which are intermediate between deque and a regular queue.

An input  restricted deque is a deque which allows insertions at only one end of the list , but allows deletions at both ends of the list

# DOUBLE ENDED QUEUE

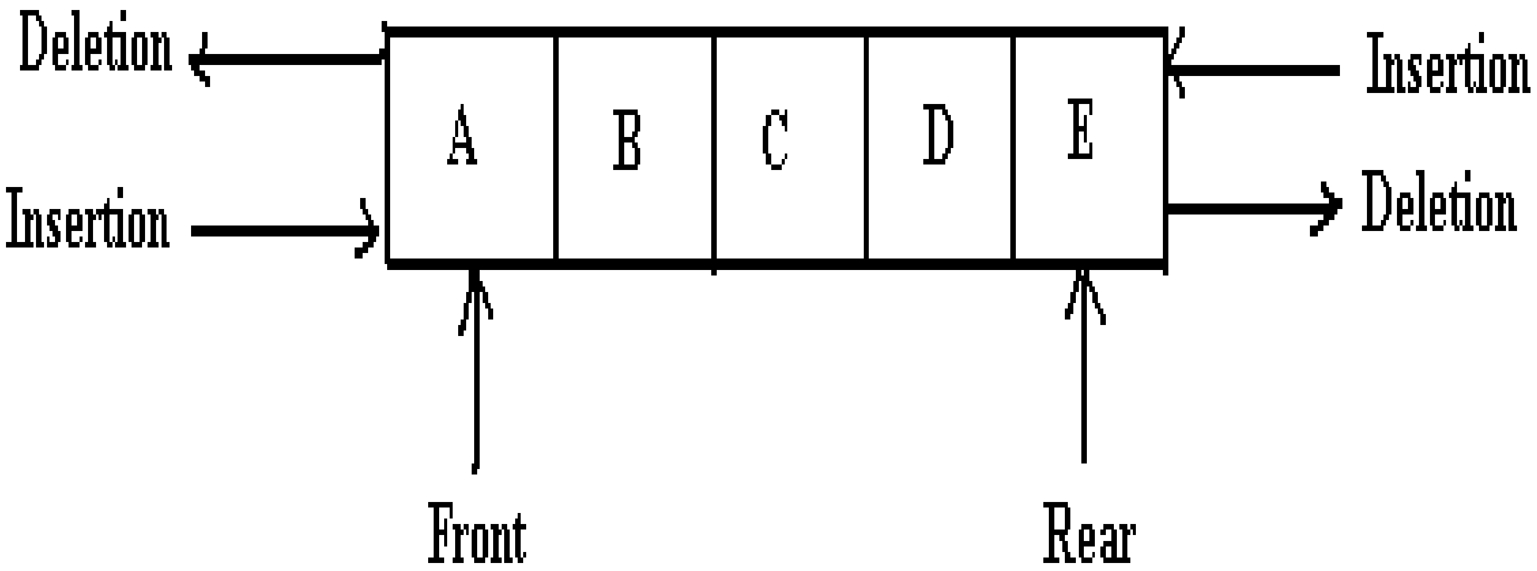
The output restricted deque is a deque which allows deletions at only one end of the list but allows insertions at both ends of the list.

The two possibilities that must consider while inserting /deleting elements into the queue are:

When an attempt is made to insert an element into a deque which is already full, an overflow occurs.

When an attempt is made to delete an element from a deque which is empty, underflow occurs.

# REPRESENTATION OF DEQUE



Represention of a deque

# UNIT-III

**Topics:**

**Trees** – Terminology, Representation of Trees, Binary tree ADT, Properties of Binary Trees, Binary Tree Representations-array and linked representations, Binary Tree traversals, threaded binary trees, Max Priority Queue ADT-implementation-Max Heap-Definition, Insertion into a Max Heap, Deletion from a Max Heap.

**Graphs** – Introduction, Definition, Terminology, Graph ADT, Graph Representations- Adjacency matrix, Adjacency lists, Graph traversals- DFS and BFS.

# Definition of Tree

- A tree is a finite set of one or more nodes such that:
- There is a specially designated node called the root.
- The remaining nodes are partitioned into n>=0 disjoint sets T1, ..., Tn, where each of these sets is a tree.
- We call T1, ..., Tn the subtrees of the root.

# Representation of Tree

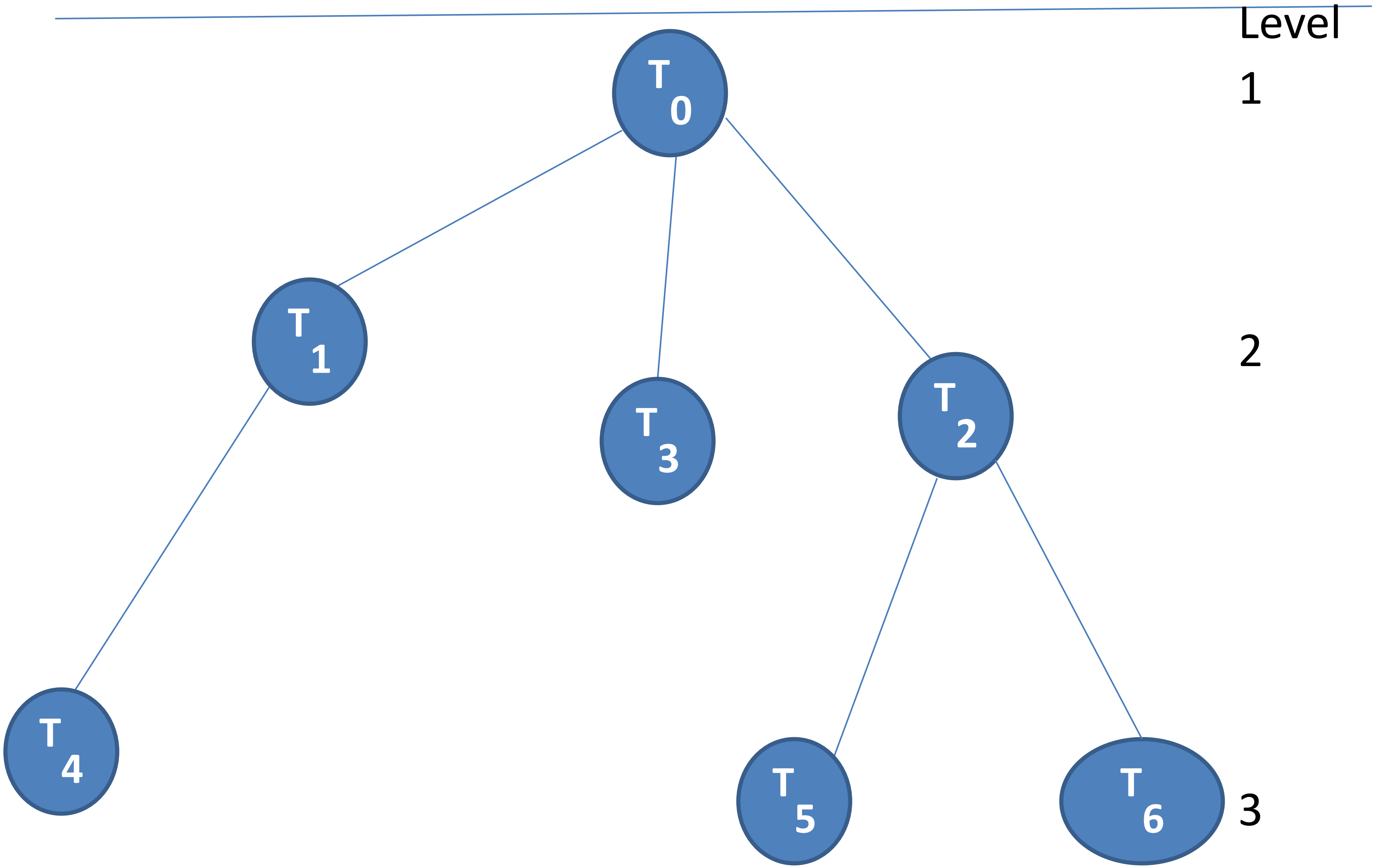


Fig.Tree 1

# Terminology

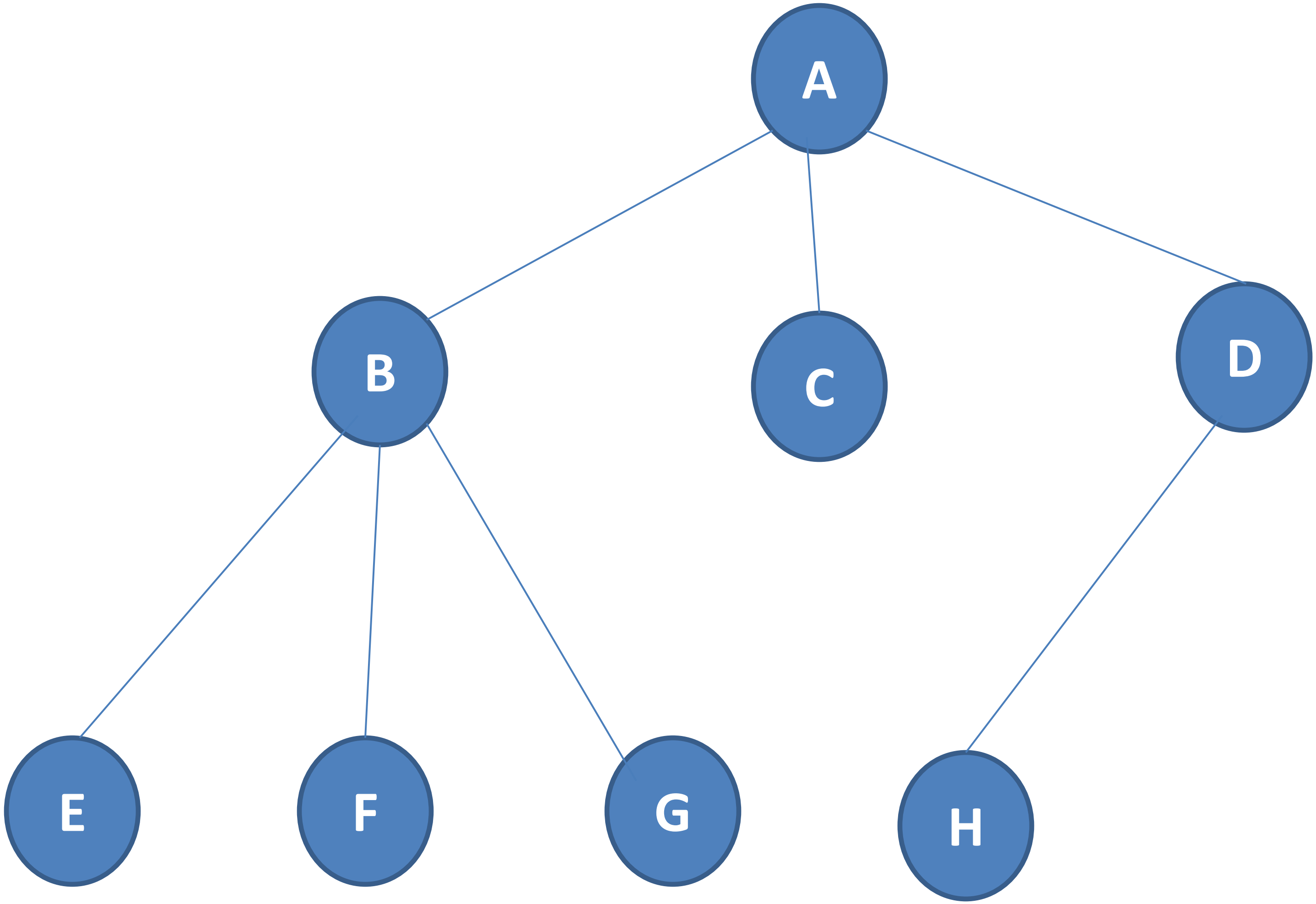


Fig.Tree 2

## ➢ROOT:

This is the unique node in the tree to which further subtrees are attached.in the above fig node A is a root node.

## ➢Degree of the node:

The total number of sub-trees attached to the node is called the degree of the node.

| Node | degree |
|---|---|
| A | 3 |
| E | 0 |

## ➢Leaves:

These are terminal nodes of the tree.The nodes with degree 0 are always the leaf nodes.In above given tree E,F,G,C and H are the leaf nodes.

## ➢Internal nodes:

The nodes other than the root node and the leaves are called the internal nodes.Here B and D are internal nodes.

## Parent nodes:

The node which is having further sub-trees(branches)is called the parent node of those sub-trees. In the given example node B is parent node of E,F and G nodes.

## Predecessor:

While displaying the tree ,if some particular node occurs previous to some other node then that node is called the predecessor of the other node.In above figure E is a predecessor of the node B.

## successor:

The node which occurs next to some other node is a successor node.In above figure B is successor of F and G.

## Level of the tree:

The root node is always considered at level 0,then its adjacent children are supposed to be at level 1 and so on.In above figure the node A is at level 0,the nodes B,C,D are at level 1,the nodes E,F,G,H are at level 2.

## Height of the tree:

The maximum level is the height of the tree.Here height of the tree is 3.The height of the tree is also called depth of the tree.

## Degree of tree:

The maximum degree of the node is called the degree of the tree.

The degree of a node is the number of subtrees of the node

- The degree of A is 3; the degree of C is 1.

- The node with degree 0 is a leaf or terminal node.
- A node that has subtrees is the *parent* of the roots of the subtrees.
- The roots of these subtrees are the *children* of the node.
- Children of the same parent are *siblings.*
- The *ancestors* of a node are all the nodes along the path from the root to the node.

# Binary Trees

- A binary tree is a finite set of nodes that is either empty or consists of a root and two disjoint binary trees called *the left subtree* and *the right subtree*.
- Any tree can be transformed into binary tree.
  - by left child-right sibling representation
- The left subtree and the right subtree are distinguished.

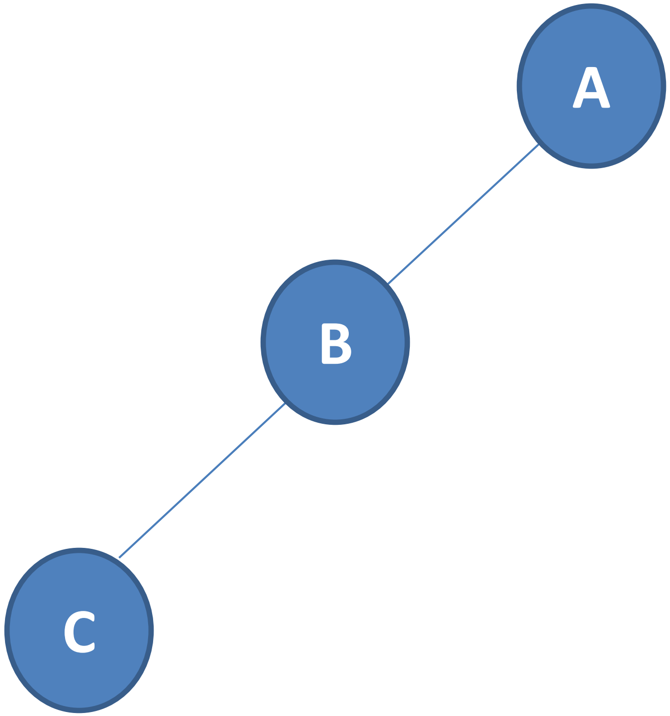
# Types Of Binary Trees

There are three types of binary trees

- Left skewed binary tree
- Right skewed binary tree
- Complete binary tree

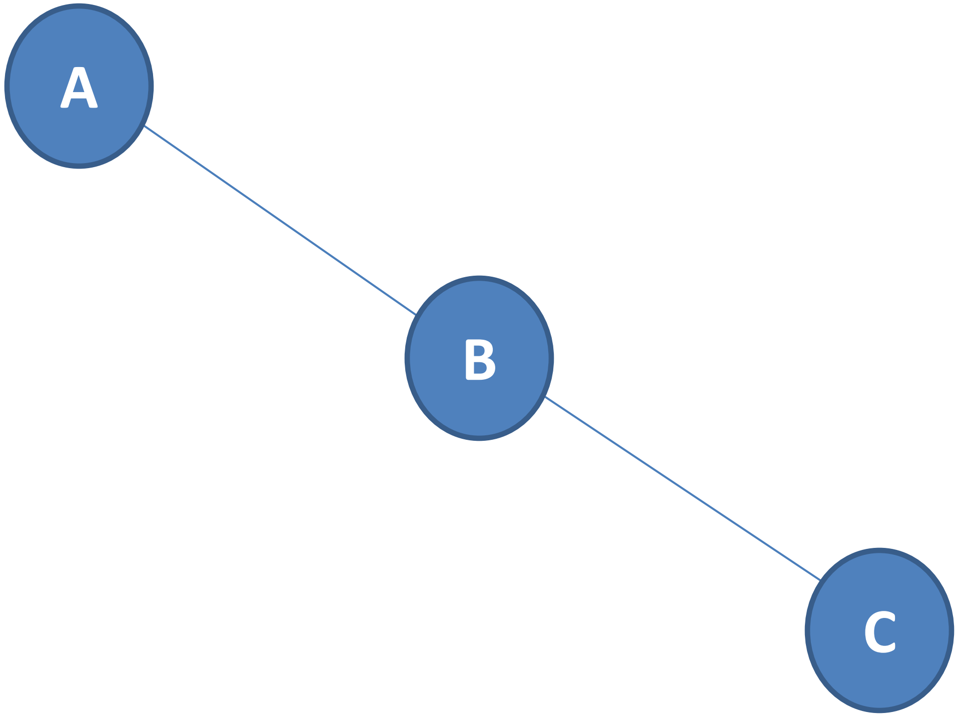
# Left skewed binary tree

- If the right subtree is missing in every node of a tree we cal it as left skewed tree.



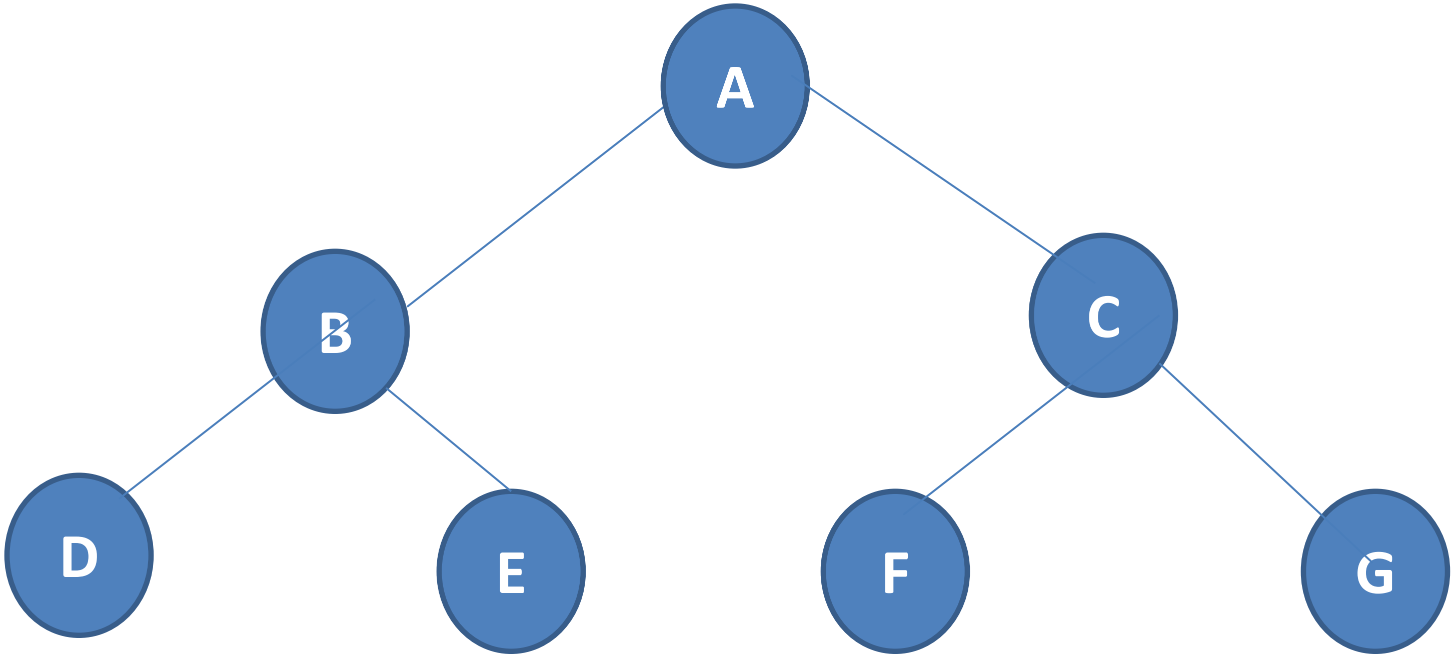
# Right skewed binary tree

- If the left subtree is missing in every node of a tree we call it as right subtree.



# Complete binary tree

- The tree in which degree of each node is at the most two is called a complete binary tree.In a complete binary tree there is exactly one node at level 0,twonodes at level 1 and four nodes at level 2 and so on.so we can say that a complete binary tree of depth d will contains exactly $2^l$ nodes at each level l,where l is from 0 to d.



# Abstract Data Type Binary_Tree

structure *Binary_Tree*(abbreviated *BinTree*) is

objects: a finite set of nodes either empty or consisting of a root node, left *Binary_Tree*, and right *Binary_Tree*.

functions:

for all *bt*, *bt1*, *bt2* ∈ *BinTree*, *item* ∈ *element*

*Bintree* Create()::= creates an empty binary tree

*Boolean* IsEmpty(*bt*)::= if (*bt*==empty binary tree) return *TRUE* else return *FALSE*

*BinTree* MakeBT(*bt1*, *item*, *bt2*)::= return a binary tree
whose left subtree is *bt1*, whose right subtree is *bt2*,
and whose root node contains the data *item*

*Bintree* Lchild(*bt*)::= if (IsEmpty(*bt*)) return error
else return the left subtree of *bt*

*element* Data(*bt*)::= if (IsEmpty(*bt*)) return error
else return the data in the root node of *bt*

*Bintree* Rchild(*bt*)::= if (IsEmpty(*bt*)) return error
else return the right subtree of *bt*

# Maximum Number of Nodes in BT

- The maximum number of nodes on level i of a binary tree is $2^{i-1}$, i>=1.
- The maximum nubmer of nodes in a binary tree of depth k is $2^k-1$, k>=1.

**Prove by induction.**

$$\sum_{i=1}^{k} 2^{i-1} = 2^k - 1$$

# Binary Tree Representation

- Sequential(Arrays)  representation
- Linked  representation

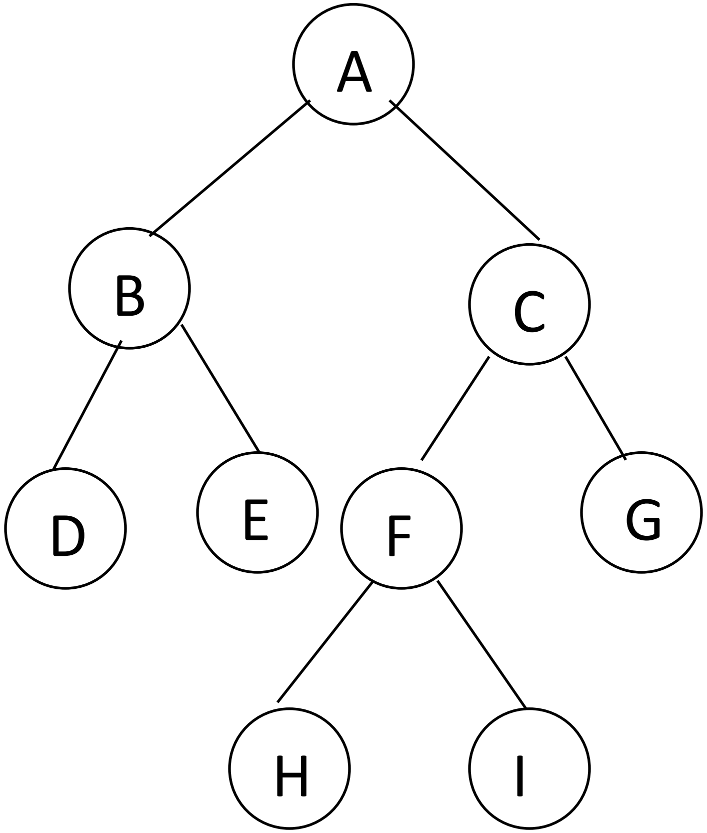
# Array Representation of Binary Tree

This representation uses only a single linear array tree as follows:

i)The root of the tree is stored in tree[0].

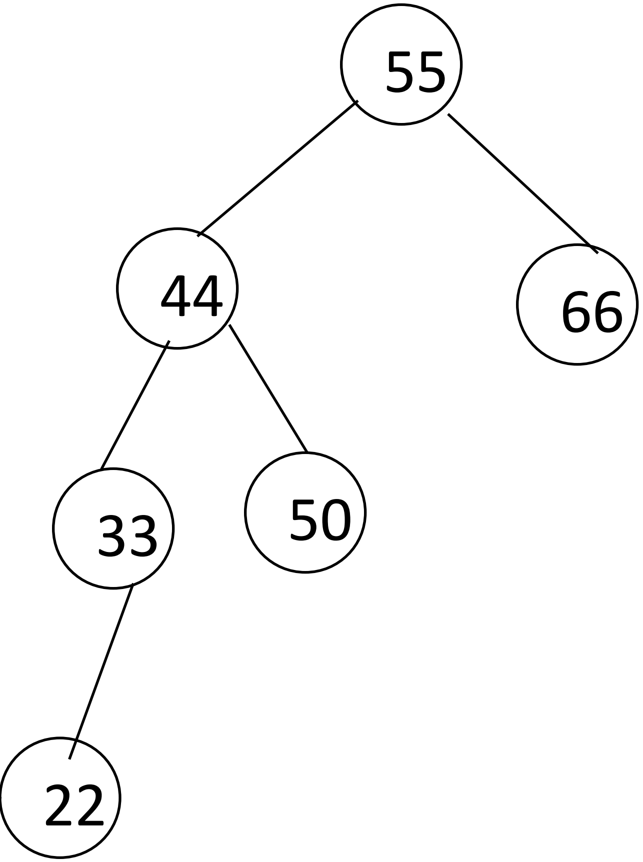
ii)if a node occupies tree[i],then its left child is stored in tree[2*i+1],its right child is stored in tree[2*i+2],and the parent is stored in tree[(i-1)/2].

# Sequential Representation



| | |
|---|---|
| [1] | A |
| [2] | B |
| [3] | C |
| [4] | D |
| [5] | E |
| [6] | F |
| [7] | G |
| [8] | H |
| [9] | I |
| . | . |
| . | . |
| . | . |
| . | . |

# Sequential Representation



| | |
|---|---|
| [0] | 55 |
| [1] | 44 |
| [2] | 66 |
| [3] | 33 |
| [4] | 50 |
| [5] | |
| [6] | |
| [7] | 22 |
| [8] | |

# Advantages of sequential representation

The only advantage with this type of representation is that the direct access to any node can be possible and finding the parent or left right children of any particular node is fast because of the random access.

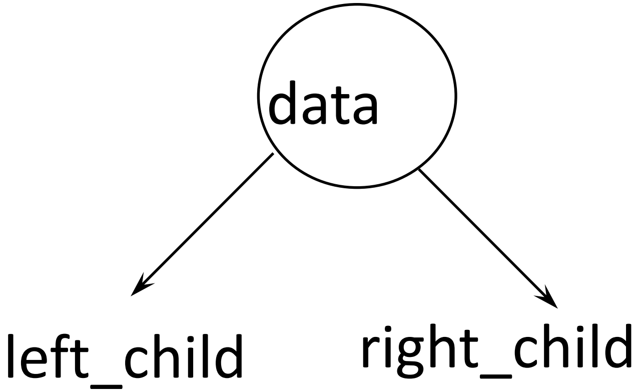
# Disadvantages of sequential representation

- The major disadvantage with this type of representation is wastage of memory.
- The maximum depth of the tree has to be fixed.
- The insertions and deletion of any node in the tree will be costlier as other nodes has to be adjusted at appropraite positions so that the meaning of binary tree can be preserved.

# Linked Representation

```
struct node
 {
 int data;
struct node * left_child, *right_child;
};
```

| | | |
|---|---|---|
| left_child | data | right_child |

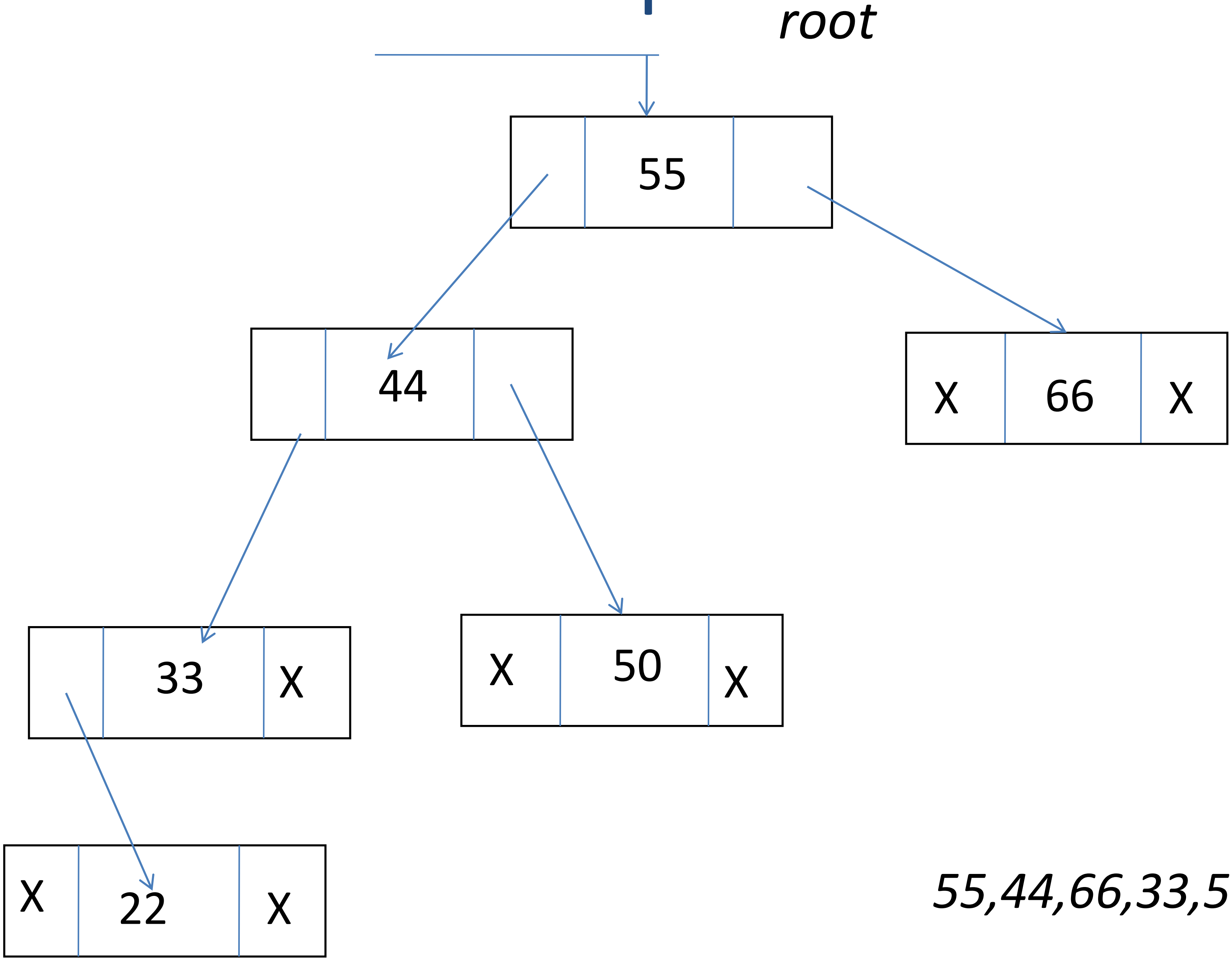


data

left_child

right_child

# Linked Representation



*55,44,66,33,50,22*

# Advantages of Linked representation

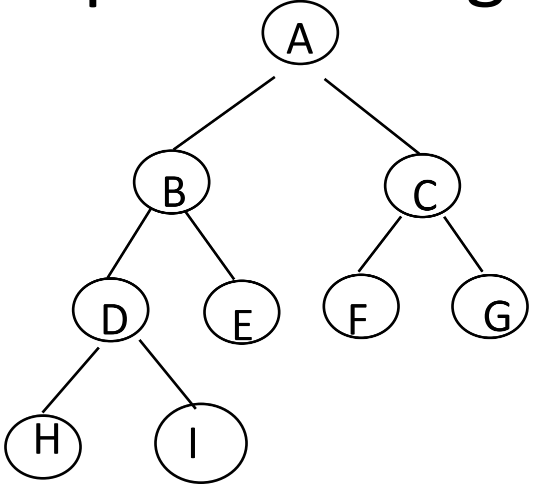
- This representation is superior to our representation as there is no wastage of memory.
- Insertions and deletions which are the most common operations can be done without moving the other nodes.

# Disadvantages of linked representation

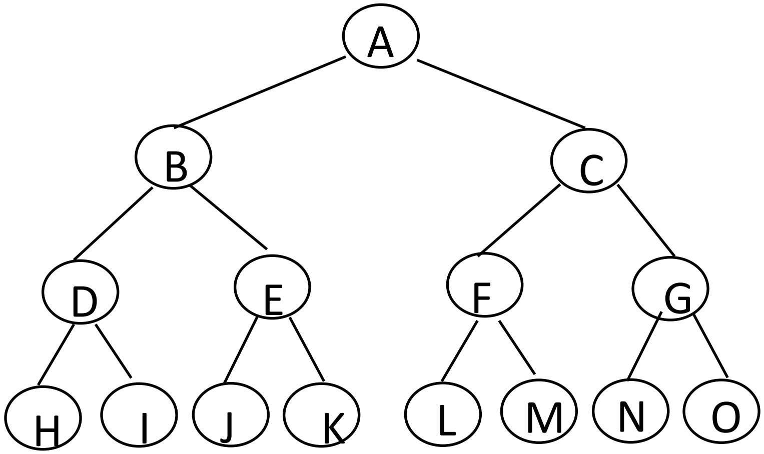
- This representation does not provide direct access to a node and special algorithms are required.
- This representation needs additional space in each node for storing the left and right sub-trees.

# Full BT VS Complete BT

- A binary tree with *n* nodes and depth *k* is complete *iff* its $\overset{k}{\text{nodes}}$ correspond to the nodes numbered from 1 to *n* in the full binary tree of depth *k*.
- A full binary tree of depth *k* is a binary tree of depth *k* having 2 -1 nodes, *k*>=0.



Complete binary tree



Full binary tree of depth 4

# Binary Tree Traversals

The process of going through a tree in such a way that each node is visted once is tree traversal.several method are used for tree traversal.the traversal in a binary tree involves three kinds of basic activities such as:

*Visiting the root*

*Traverse left subtree*

*Traverse right subtree*

We will use some notations to traverse a given binary tree as follows:

L means move to the Left child.

R means move to the Right child.

D means the root/parent node.

The only difference among the methods is the order in which these three operations are performed.

There are three standard ways of traversing a non empty binary tree namely :

*Preorder*

*Inorder*

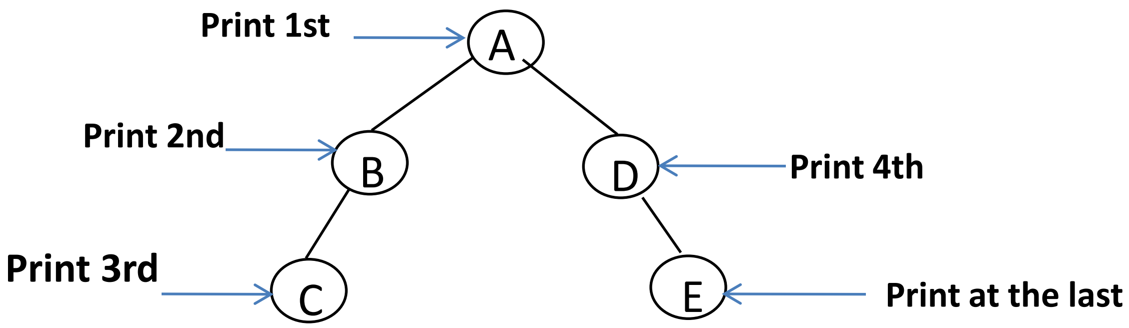
*Postorder*

# Preorder(also known as depth-first order)

1.Visit the root(D)

2.Traverse the left subtree in preorder(L)

3.Traverse the right subtree in preorder(R)



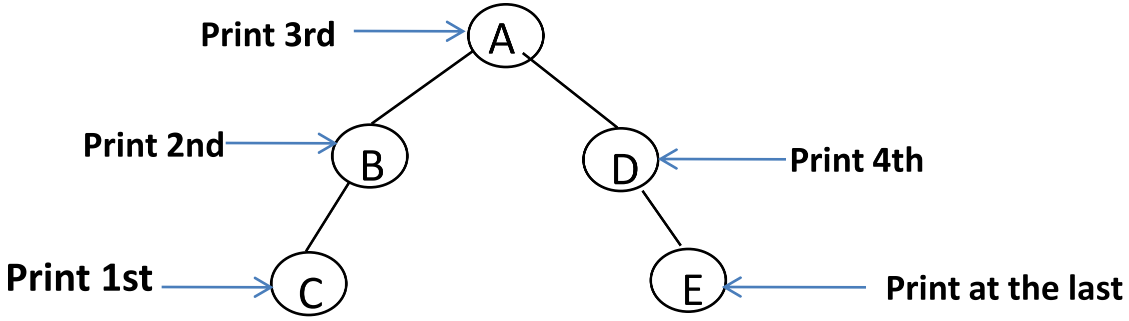
A-B-C-D-E is the preorder traversal of the above figure.

# Inorder(also known as symmetric order)

1.Traverse the left subtree in Inorder(L)

2.Visit the root(D)

3.Traverse the right subtree in Inorder(R)



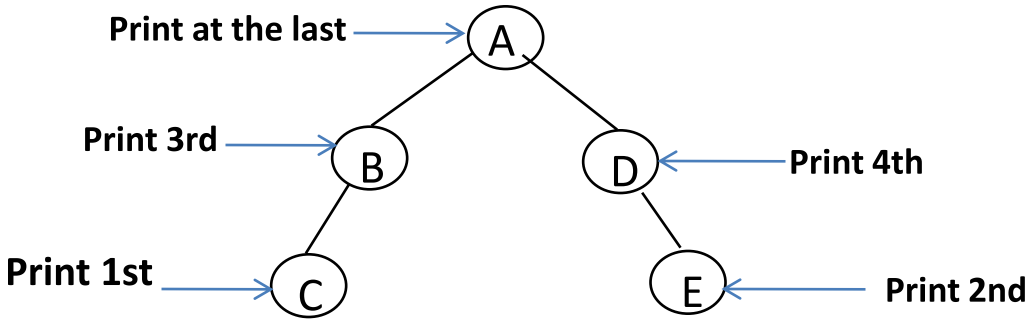
C-B-A-D-E is the Inorder traversal of the above figure.

# Postorder

1.Traverse the left subtree in postorder(L)

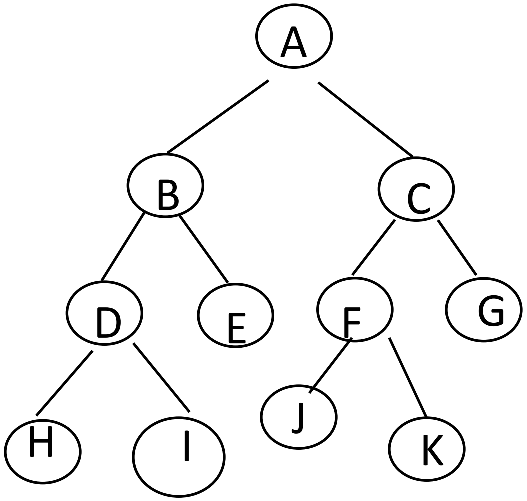
2.Traverse the right subtree in postorder(R)

3.Visit the root(D)

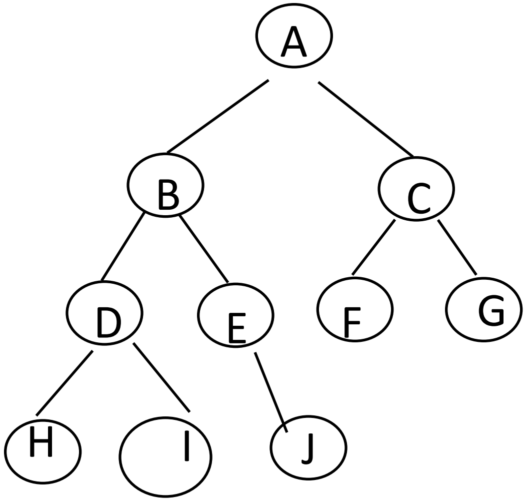


C-D-B-E-A is the postorder traversal of the above figure.

# Binary tree traversals



FIG(a)

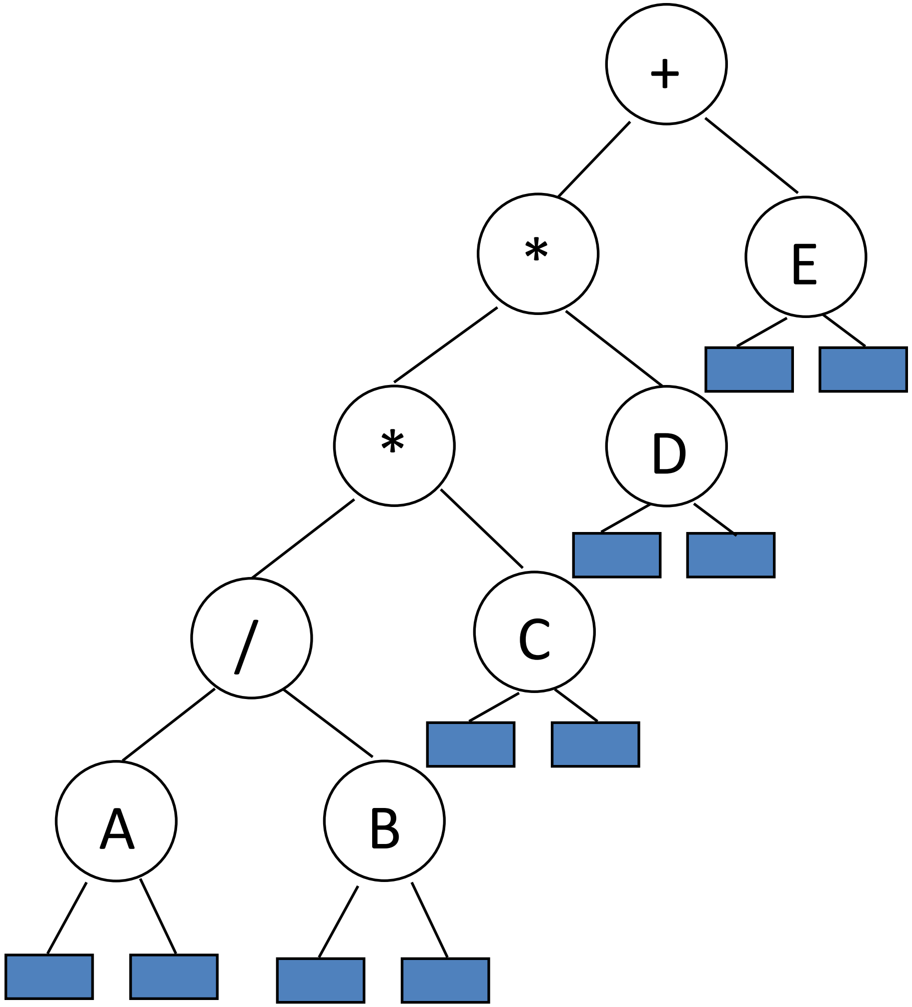


FIG(b)

Preorder:ABDHIECFJKG
Inorder:HDIBEAJFKCG
Postorder:HIDEBJKFGCA

preorder :ABDHIEJCFG
inorder: HDIBJEAFCG
postorder:HIDJEBFGCA

# Arithmetic Expression Using BT



inorder traversal

A / B * C * D + E

infix expression

preorder traversal

+ * * / A B C D E

prefix expression

postorder traversal

A B / C * D * E +

postfix expression

level order traversal

+ * E * D / C A B

# Inorder Traversal (recursive version)

```
void inorder(tree_pointer ptr)
/* inorder tree traversal */
{
    if (ptr) {
        inorder(ptr->left_child);
        printf(“%d”, ptr->data);
        indorder(ptr->right_child);
    }
}
```

A / B * C * D + E

# Preorder Traversal (recursive version)

```
void preorder(tree_pointer ptr)
/* preorder tree traversal */
{
    if (ptr) {
        printf(“%d”, ptr->data);
        preorder(ptr->left_child);
        predorder(ptr->right_child);
    }
}
```

+ * * / A B C D E

# Postorder Traversal (recursive version)

```
void postorder(tree_pointer ptr)
/* postorder tree traversal */
{
   if (ptr) {
      postorder(ptr->left_child);
      postdorder(ptr->right_child);
      printf(“%d”, ptr->data);
   }
}
```

A B / C * D * E +

# Iterative Inorder Traversal

(using stack)

```
void iter_inorder(tree_pointer node)
{
  int top= -1; /* initialize stack */
  tree_pointer stack[MAX_STACK_SIZE];
  for (;;) {
   for (; node; node=node->left_child)
     add(&top, node);/* add to stack */
   node= delete(&top);
                /* delete from stack */
   if (!node) break; /* empty stack */
   printf(“%D”, node->data);
   node = node->right_child;
 }
}
```

O(n)

# Trace Operations of Inorder Traversal

| Call of inorder | Value in root | Action | Call of inorder | Value in root | Action |
|---|---|---|---|---|---|
| 1 | + |  | 11 | C |  |
| 2 | * |  | 12 | NULL |  |
| 3 | * |  | 11 | C | printf |
| 4 | / |  | 13 | NULL |  |
| 5 | A |  | 2 | * | printf |
| 6 | NULL |  | 14 | D |  |
| 5 | A | printf | 15 | NULL |  |
| 7 | NULL |  | 14 | D | printf |
| 4 | / | printf | 16 | NULL |  |
| 8 | B |  | 1 | + | printf |
| 9 | NULL |  | 17 | E |  |
| 8 | B | printf | 18 | NULL |  |
| 10 | NULL |  | 17 | E | printf |
| 3 | * | printf | 19 | NULL |  |

# Level Order Traversal

(using queue)

```
void level_order(tree_pointer ptr)
/* level order tree traversal */
{
  int front = rear = 0;
  tree_pointer queue[MAX_QUEUE_SIZE];
  if (!ptr) return; /* empty queue */
  addq(front, &rear, ptr);
  for (;;) {
    ptr = deleteq(&front, rear);
    if (ptr) {
      printf("%d", ptr->data);
      if (ptr->left_child)
        addq(front, &rear,
                    ptr->left_child);
      if (ptr->right_child)
        addq(front, &rear,
                    ptr->right_child);
    }
    else break;
  }
}
```

+ * E * D / C A B

# Copying Binary Trees

```
tree_poointer copy(tree_pointer original)
{
tree_pointer temp;
if (original) {
 temp=(tree_pointer) malloc(sizeof(node));
 if (IS_FULL(temp)) {
   fprintf(stderr, “the memory is full\n”);
   exit(1);
 }
 temp->left_child=copy(original->left_child);
 temp->right_child=copy(original->right_child)
 temp->data=original->data;
 return temp;
}
return NULL;
}
```

postorder

# Post-order-eval function

```
void post_order_eval(tree_pointer node)
{
/* modified post order traversal to evaluate a propositional
calculus tree */
if (node) {
post_order_eval(node->left_child);
post_order_eval(node->right_child);
switch(node->data) {
case not:  node->value =
!node->right_child->value;
break;
case and:    node->value =
          node->right_child->value &&
          node->left_child->value;
          break;
     case or:      node->value =
          node->right_child->value | |
          node->left_child->value;
          break;
      case true:    node->value = TRUE;
          break;
      case false:  node->value = FALSE;
     }
  }
}
```

# Threaded Binary Trees

- Two many null pointers in current representatio of binary trees

  n: number of nodes

  number of non-null links: n-1

  total links: 2n

  null links: 2n-(n-1)=n+1

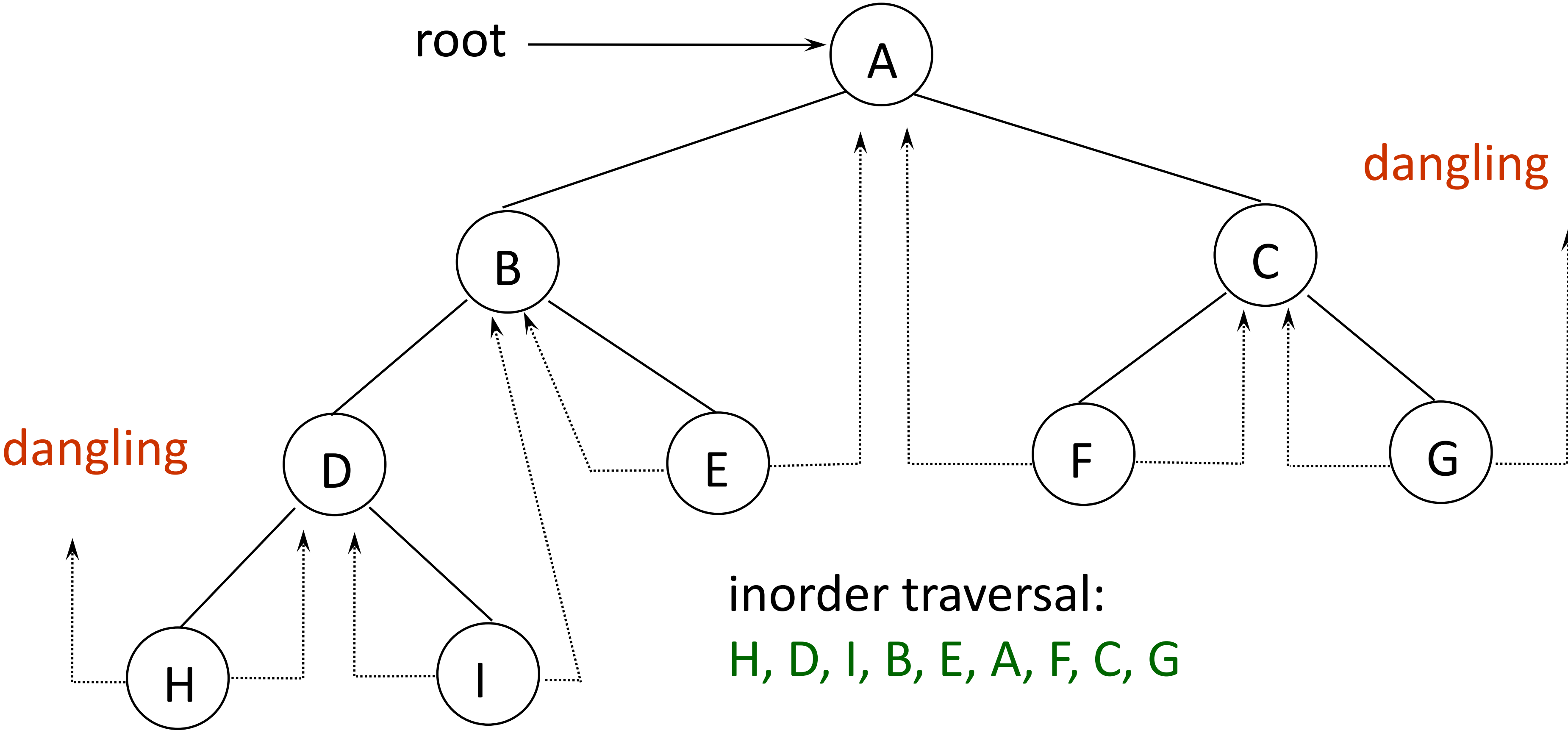
- Replace these null pointers with some useful "threads".

# Threaded Binary Trees (*Continued*)

If `ptr->left_child` is null,
replace it with a pointer to the node that would be visited *before* `ptr` in an *inorder traversal*

If `ptr->right_child` is null,
replace it with a pointer to the node that would be visited *after* `ptr` in an *inorder traversal*

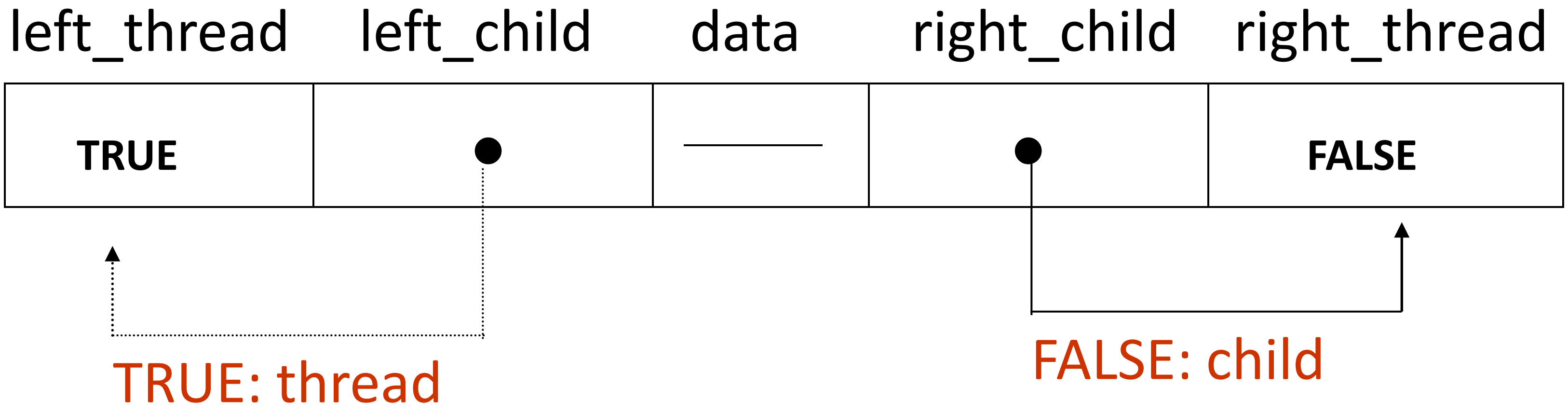
# A Threaded Binary Tree



inorder traversal:

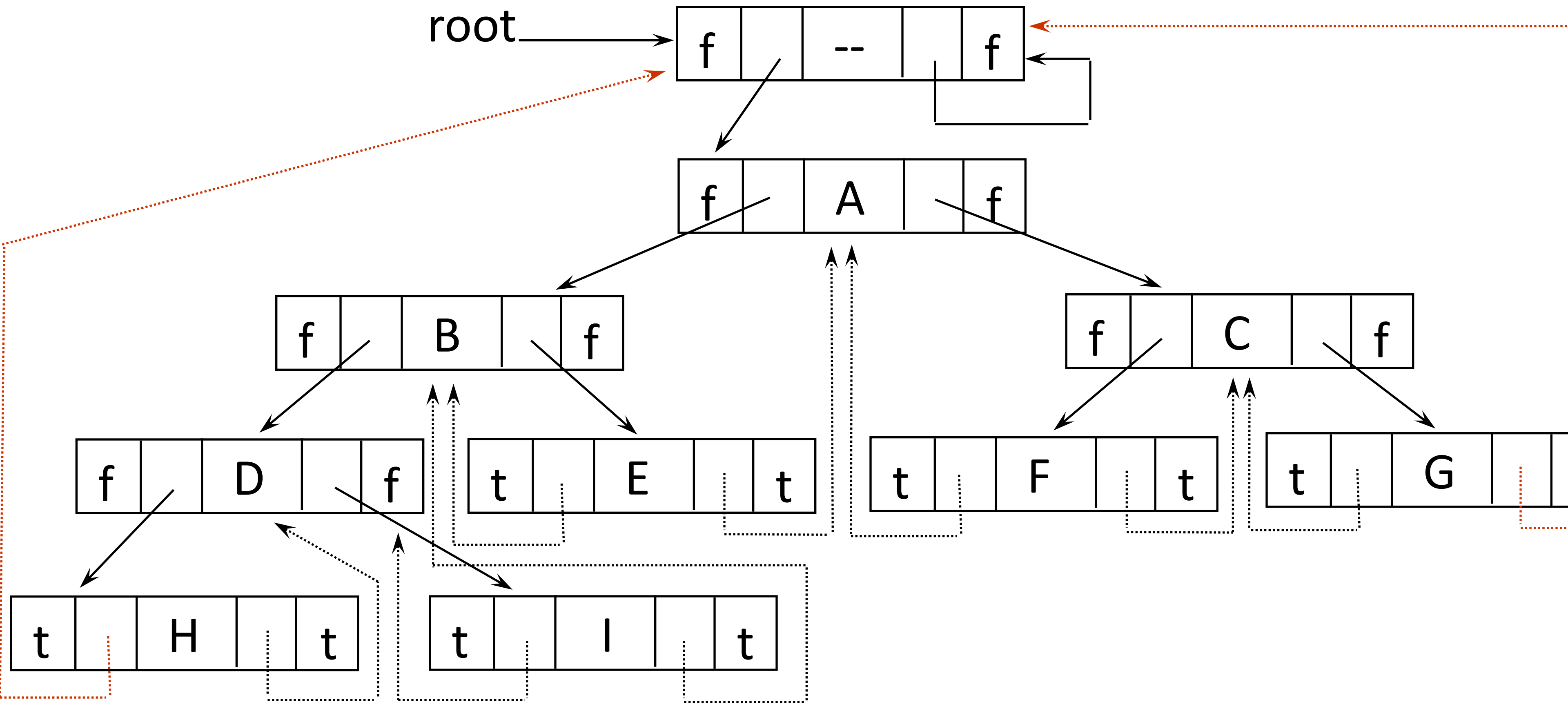
H, D, I, B, E, A, F, C, G

# Data Structures for Threaded BT



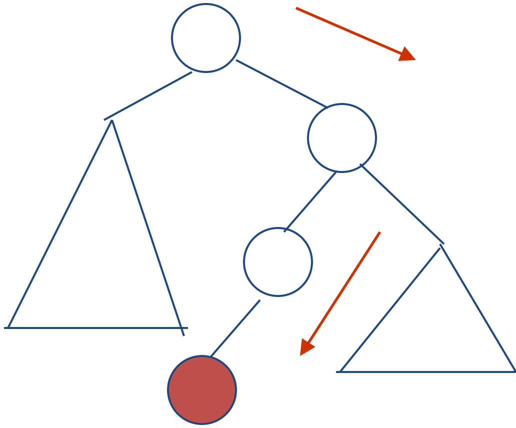
```
typedef struct threaded_tree *threaded_pointer;
typedef struct threaded_tree {
   short int left_thread;
   threaded_pointer left_child;
   char data;
   threaded_pointer right_child;
   short int right_thread;  };
```

# Memory Representation of A Threaded BT



# Next Node in Threaded BT

```
threaded_pointer insucc(threaded_pointer
  tree)
{
   threaded_pointer temp;
   temp = tree->right_child;
   if (!tree->right_thread)
      while (!temp->left_thread)
         temp = temp->left_child;
   return temp;
}
```



# Inorder Traversal of Threaded BT

```
void tinorder(threaded_pointer tree)
{
/* traverse the threaded binary tree
  inorder */
    threaded_pointer temp = tree;
    for (;;) {
        temp = insucc(temp);
        if (temp==tree) break;
        printf(“%3c”, temp->data);
    }
}
```

O(n)(timecomplexity)

# Inserting Nodes into Threaded BTs

- Insert `child` as the right child of node `parent`
  - change `parent->right_thread` to FALSE
  - set `child->left_thread` and `child->right_thread` to TRUE
  - set `child->left_child` to point to `parent`
  - set `child->right_child` to `parent->right_child`
  - change `parent->right_child` to point to `child`

# Examples

Insert a node D as a right child of B.

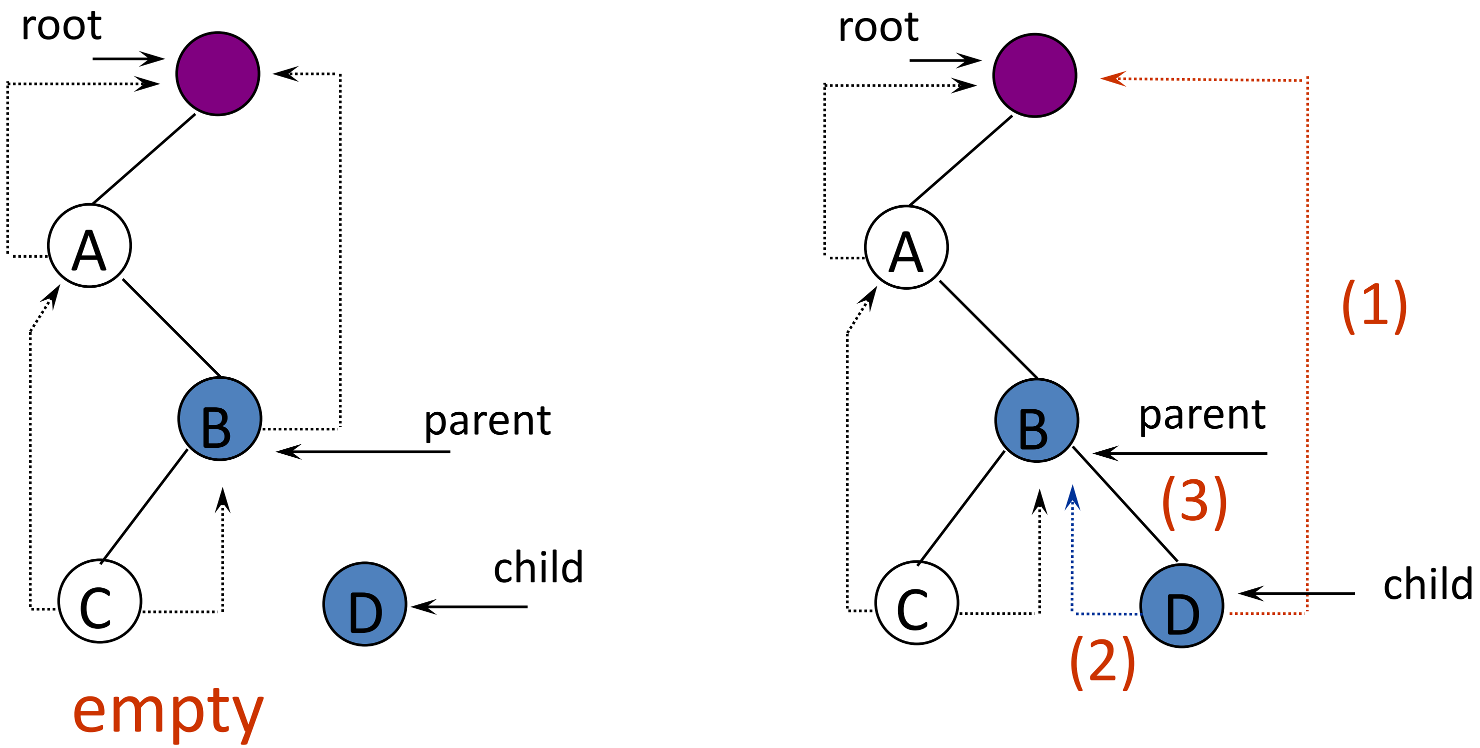
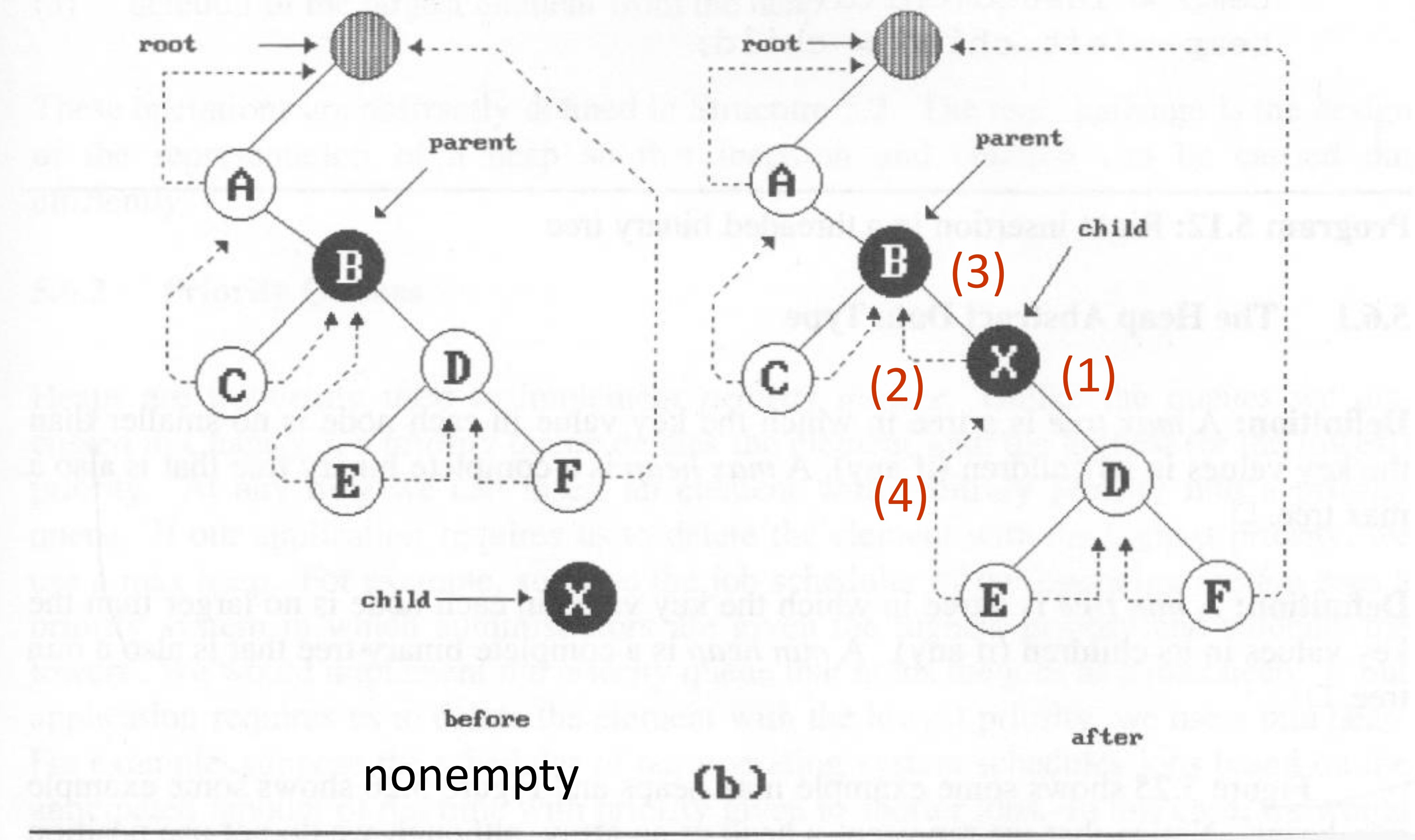


***Figure 5.24:** Insertion of child as a right child of parent in a threaded binary tree (p.217)



nonempty

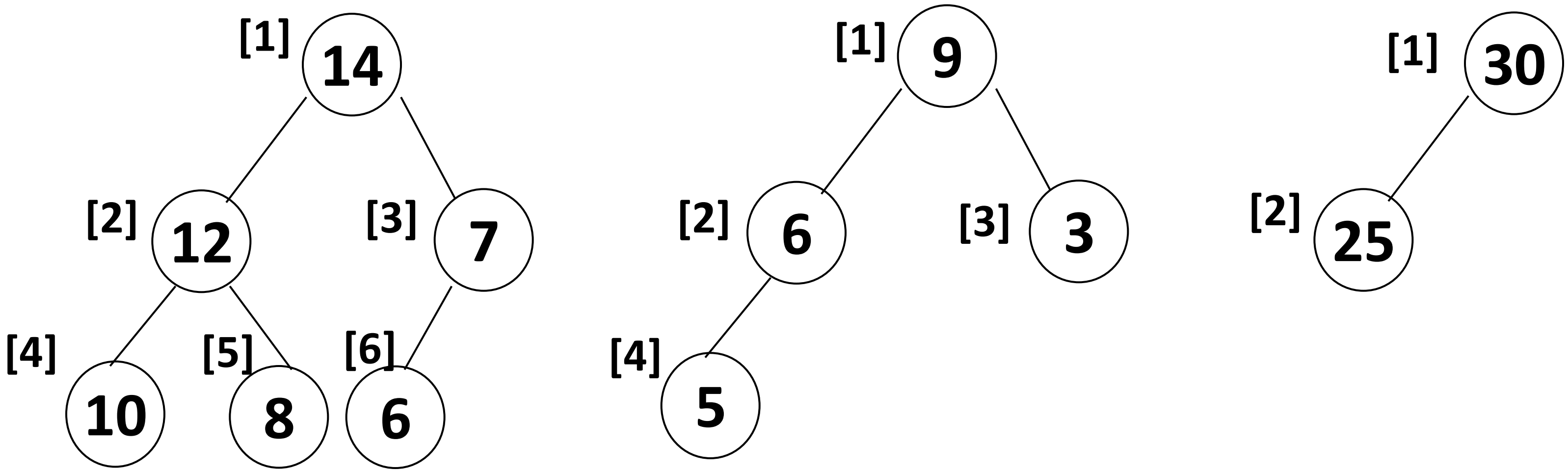
# Right Insertion in Threaded BTs

```
void insert_right(threaded_pointer parent,
                  threaded_pointer child)
{
    threaded_pointer temp;
(1) child->right_child = parent->right_child;
    child->right_thread = parent->right_thread;
(2) child->left_child = parent;  case (a)
    child->left_thread = TRUE;
(3) parent->right_child = child;
    parent->right_thread = FALSE;
    if (!child->right_thread) { case (b)
(4)     temp = insucc(child);
        temp->left_child = child;
    }
}
```

# Heap

- A *max tree* is a tree in which the key value in each node is no smaller than the key values in its children. A *max heap* is a complete binary tree that is also a max tree.
- A *min tree* is a tree in which the key value in each node is no larger than the key values in its children. A *min heap* is a complete binary tree that is also a min tree.
- Operations on heaps
  - creation of an empty heap
  - insertion of a new element into the heap;
  - deletion of the largest element from the heap

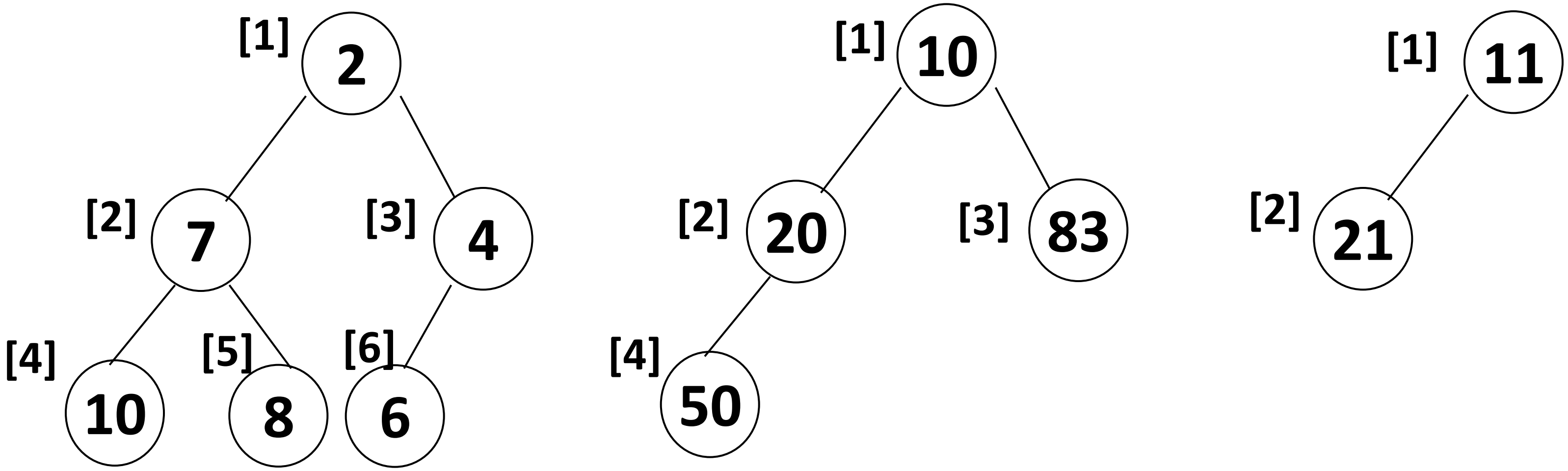
# Sample max heaps



Property:

The root of max heap contains the largest .

## Sample min heaps



Property:

The root of min heap contains the smallest.

# ADT for Max Heap

structure MaxHeap

objects: a complete binary tree of n > 0 elements organized so that the value in each node is at least as large as those in its children

functions:

for all *heap* belong to *MaxHeap*, *item* belong to *Element*, *n*, *max_size* belong to integer

MaxHeap Create(max_size)::= create an empty heap that can
hold a maximum of max_size elements

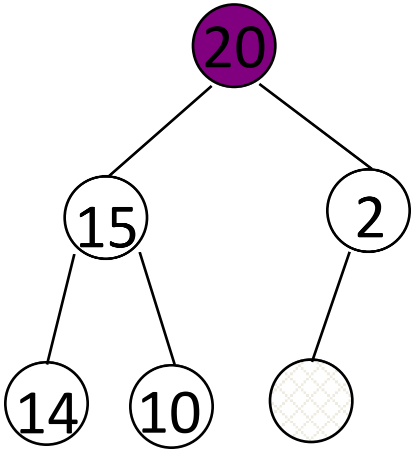
Boolean HeapFull(heap, n)::= if (n==max_size) return TRUE
else return FALSE

MaxHeap Insert(heap, item, n)::= if (!HeapFull(heap,n)) insert
item into heap and return the resulting heap
else return error

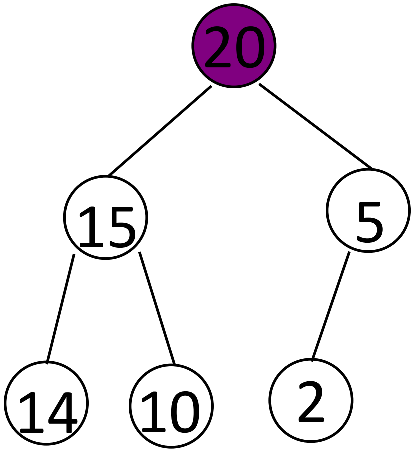
Boolean HeapEmpty(heap, n)::= if (n>0) return FALSE
else return TRUE

Element Delete(heap,n)::= if (!HeapEmpty(heap,n)) return one
instance of the largest element in the heap
and remove it from the heap
else return error

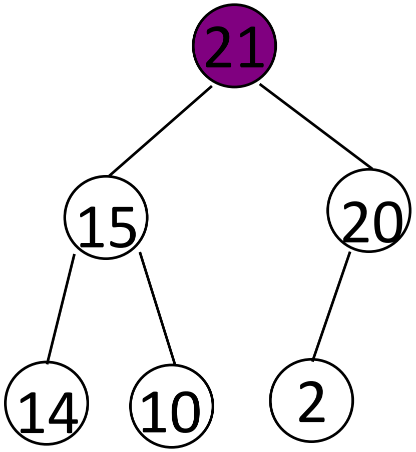
# Example of Insertion to Max Heap



initial location of new node



insert 5 into heap



insert 21 into heap

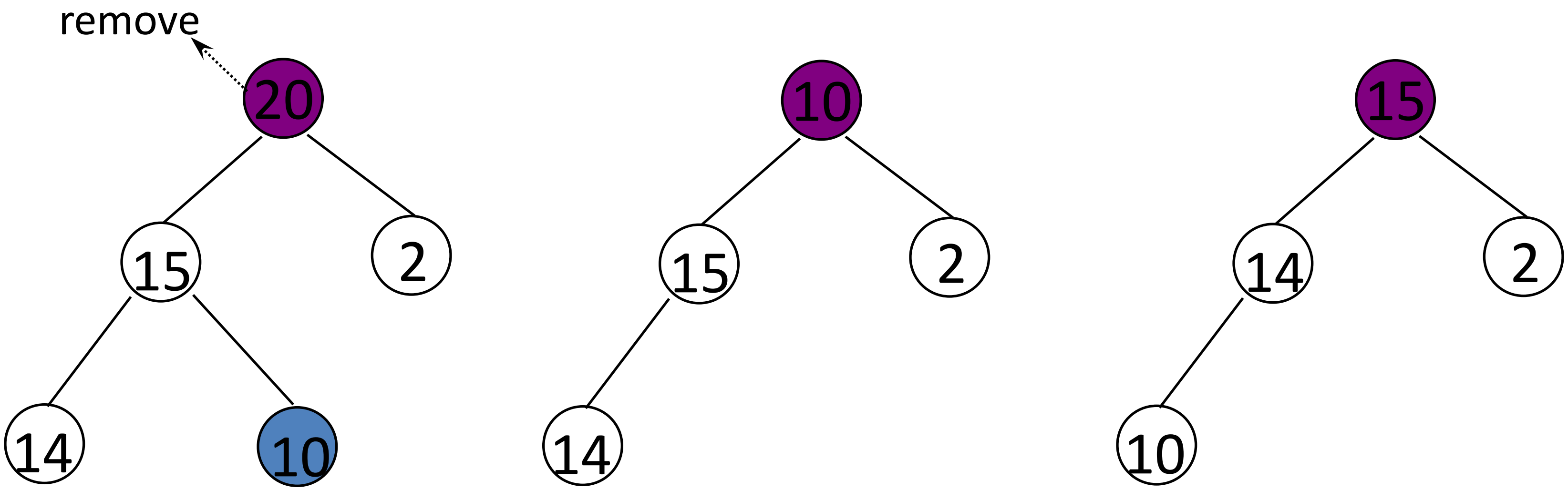
# Insertion into a Max Heap

```
void insert_max_heap(element item, int *n)
{
  int i;
  if (HEAP_FULL(*n)) {
    fprintf(stderr, “the heap is full.\n”);
    exit(1);
  }
  i = ++(*n);
  while ((i!=1)&&(item.key>heap[i/2].key)) {
    heap[i] = heap[i/2];
    i /= 2;
  }
  heap[i]= item;
}
```

$2^k-1=n \implies k=\lceil \log_2(n+1) \rceil$

$O(\log_2 n)$

# Example of Deletion from Max Heap



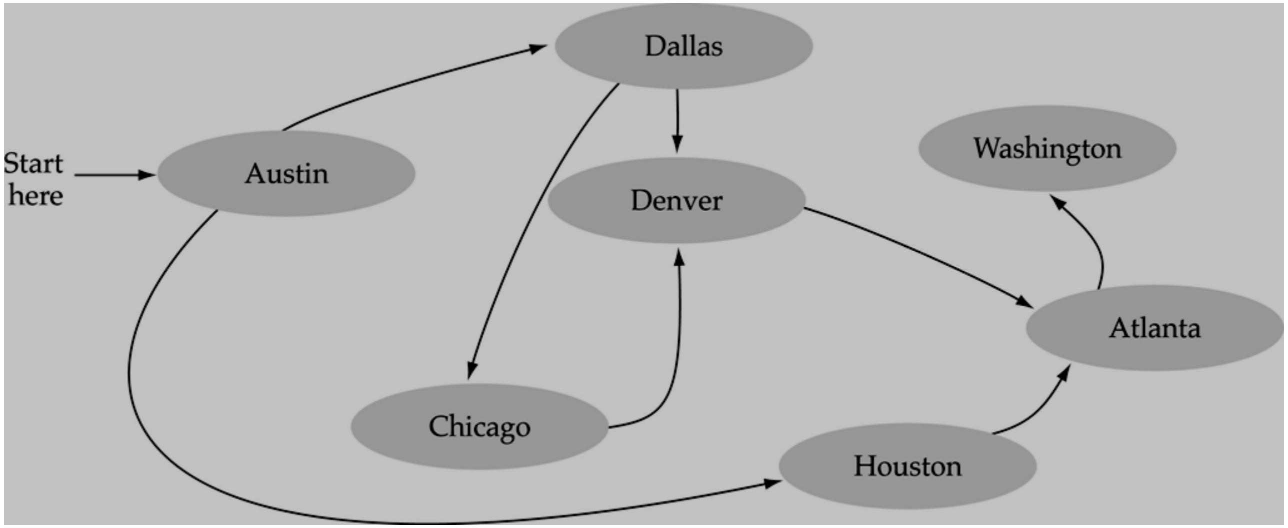
# Deletion from a Max Heap

```
element delete_max_heap(int *n)
{
  int parent, child;
  element item, temp;
  if (HEAP_EMPTY(*n)) {
    fprintf(stderr, “The heap is empty\n”);
    exit(1);
  }
  /* save value of the element with the
     highest key */
  item = heap[1];
  /* use last element in heap to adjust heap
  temp = heap[(*n)--];
  parent = 1;
  child = 2;
while (child <= *n) {
    /* find the larger child of the current
       parent */
    if ((child < *n)&&
        (heap[child].key<heap[child+1].key))
      child++;
    if (temp.key >= heap[child].key) break;
    /* move to the next lower level */
    heap[parent] = heap[child];
    child *= 2;
  }
  heap[parent] = temp;
  return item;
}
```

# Graphs

# What is a graph?

- A data structure that consists of a set of nodes (*vertices*) and a set of edges that relate the nodes to each other
- The set of edges describes relationships among the vertices



# Formal definition of graphs

- A graph *G* is defined as follows:

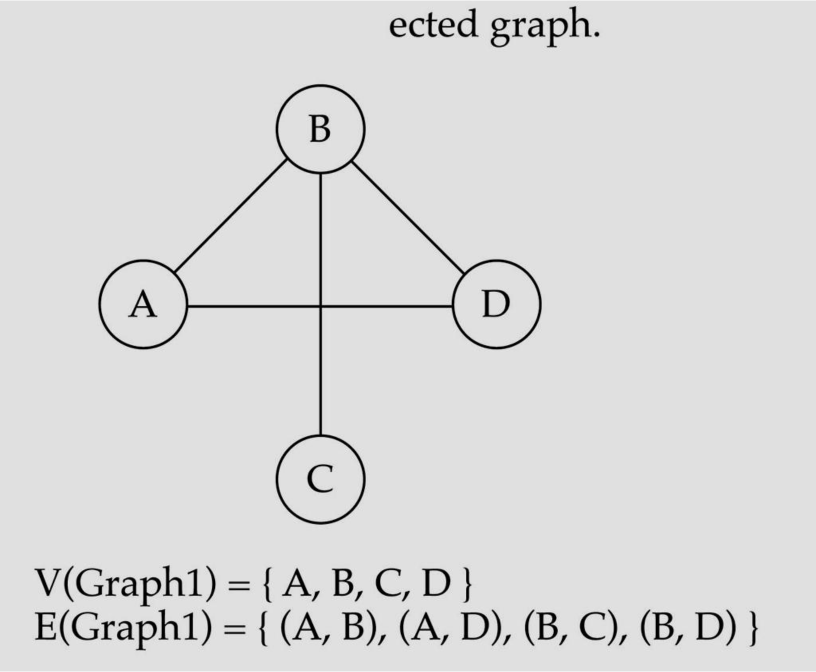
  *G=(V,E)*

  *V(G):* a finite, nonempty set of vertices

  *E(G):* a set of edges (pairs of vertices)

# Directed vs. undirected graphs

- When the edges in a graph have no direction, the graph is called *undirected*

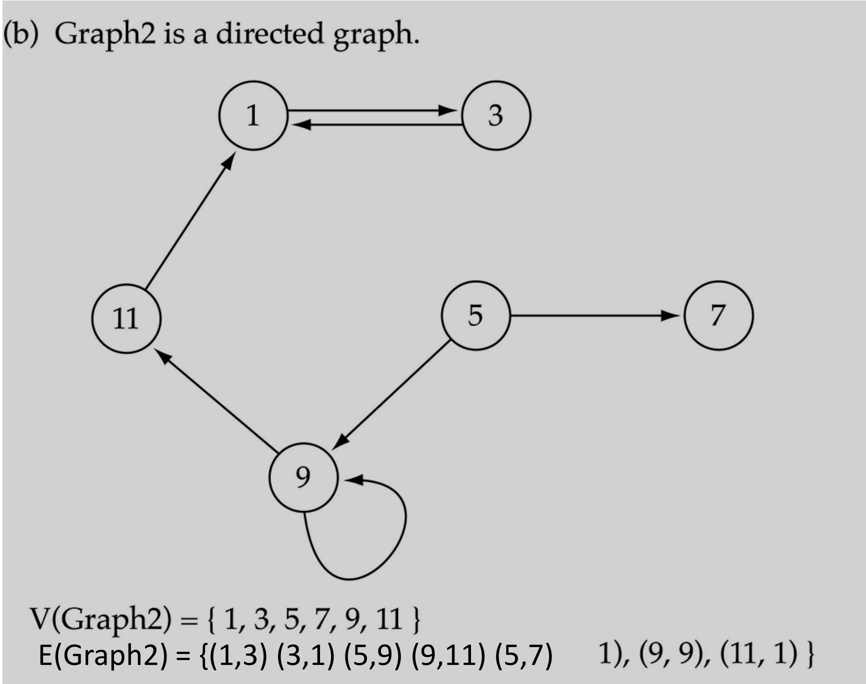


ected graph.

V(Graph1) = { A, B, C, D }
E(Graph1) = { (A, B), (A, D), (B, C), (B, D) }

# Directed vs. undirected graphs (cont.)

- When the edges in a graph have a direction, the graph is called *directed* (or *digraph*)



(b) Graph2 is a directed graph.

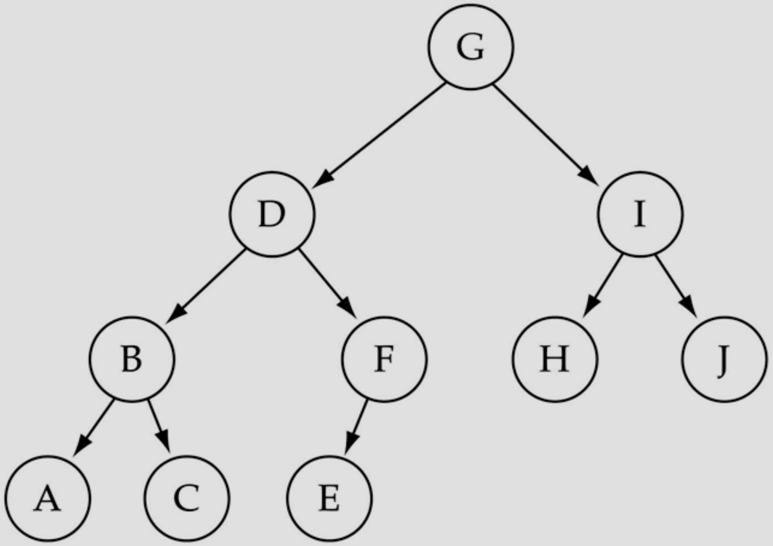
V(Graph2) = { 1, 3, 5, 7, 9, 11 }

E(Graph2) = {(1,3) (3,1) (5,9) (9,11) (5,7) 1), (9, 9), (11, 1) }

# Trees vs graphs

- Trees are special cases of graphs!!

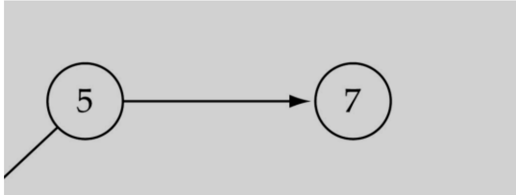
(c) Graph3 is a directed graph.



V(Graph3) = { A, B, C, D, E, F, G, H, I, J }
E(Graph3) = { (G, D), (G, J), (D, B), (D, F) (I, H), (I, J), (B, A), (B, C), (F, E) }

# Graph terminology

- Adjacent nodes: two nodes are adjacent if they are connected by an edge



5 is adjacent to 7
7 is adjacent from 5

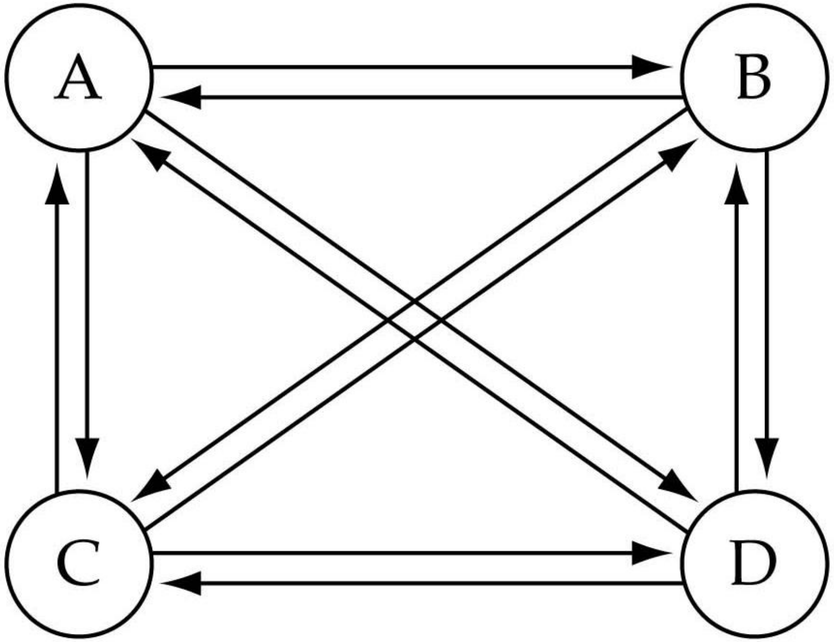
- Path: a sequence of vertices that connect two nodes in a graph
- Complete graph: a graph in which every vertex is directly connected to every other vertex

# Graph terminology (cont.)

- What is the number of edges in a complete directed graph with N vertices?

*N * (N-1)*

$O(N^2)$



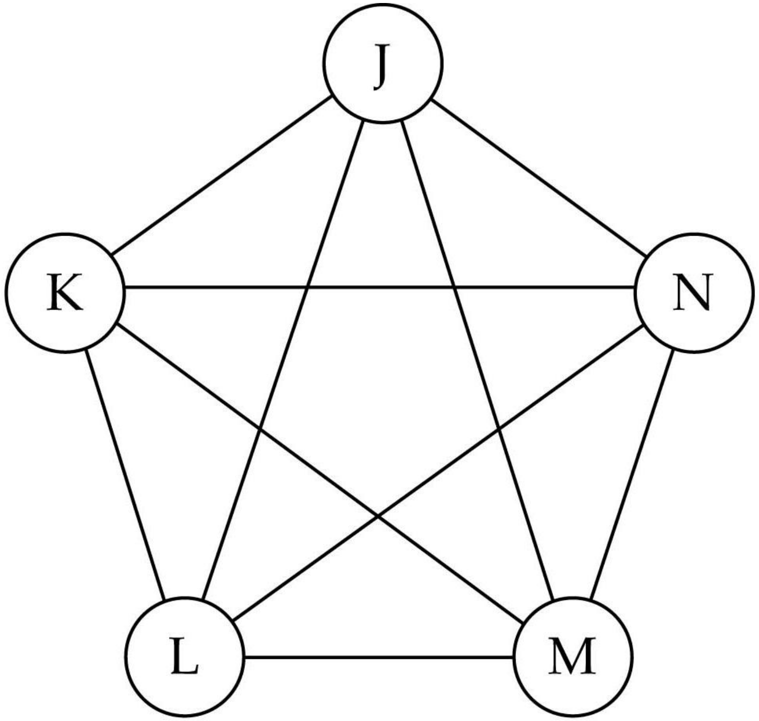
(a) Complete directed graph.

# Graph terminology (cont.)

- What is the number of edges in a complete undirected graph with N vertices?

*N * (N-1) / 2*

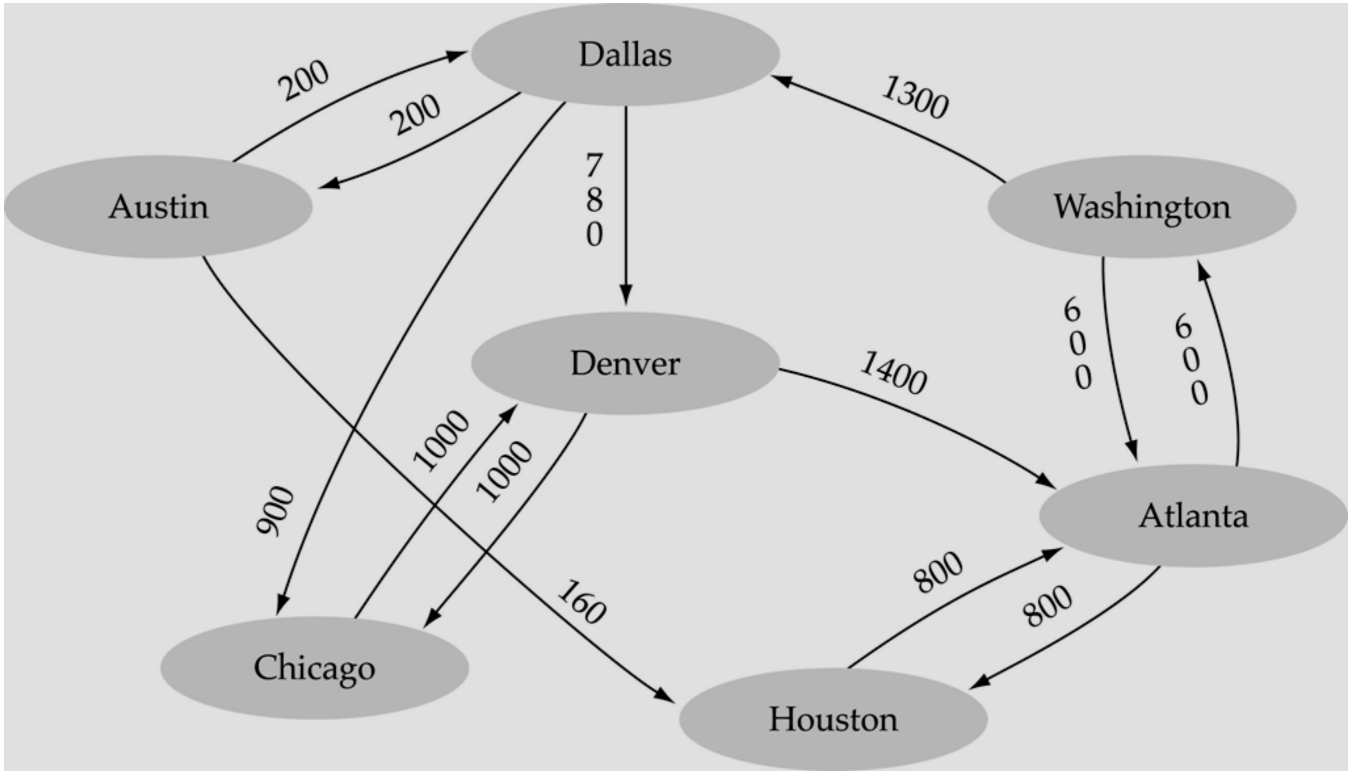
$O(N^2)$



(b) Complete undirected graph.

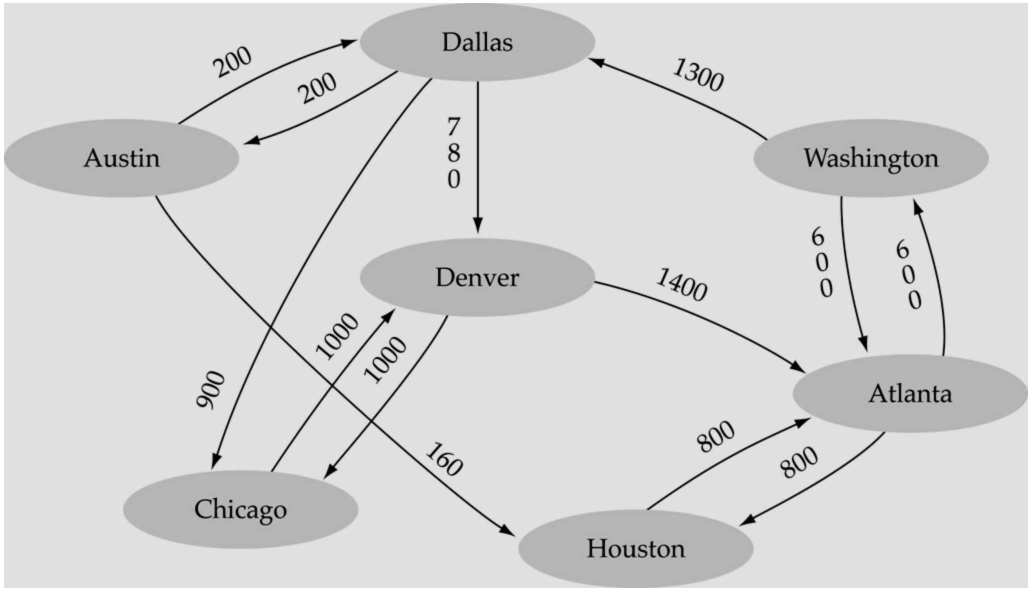
# Graph terminology (cont.)

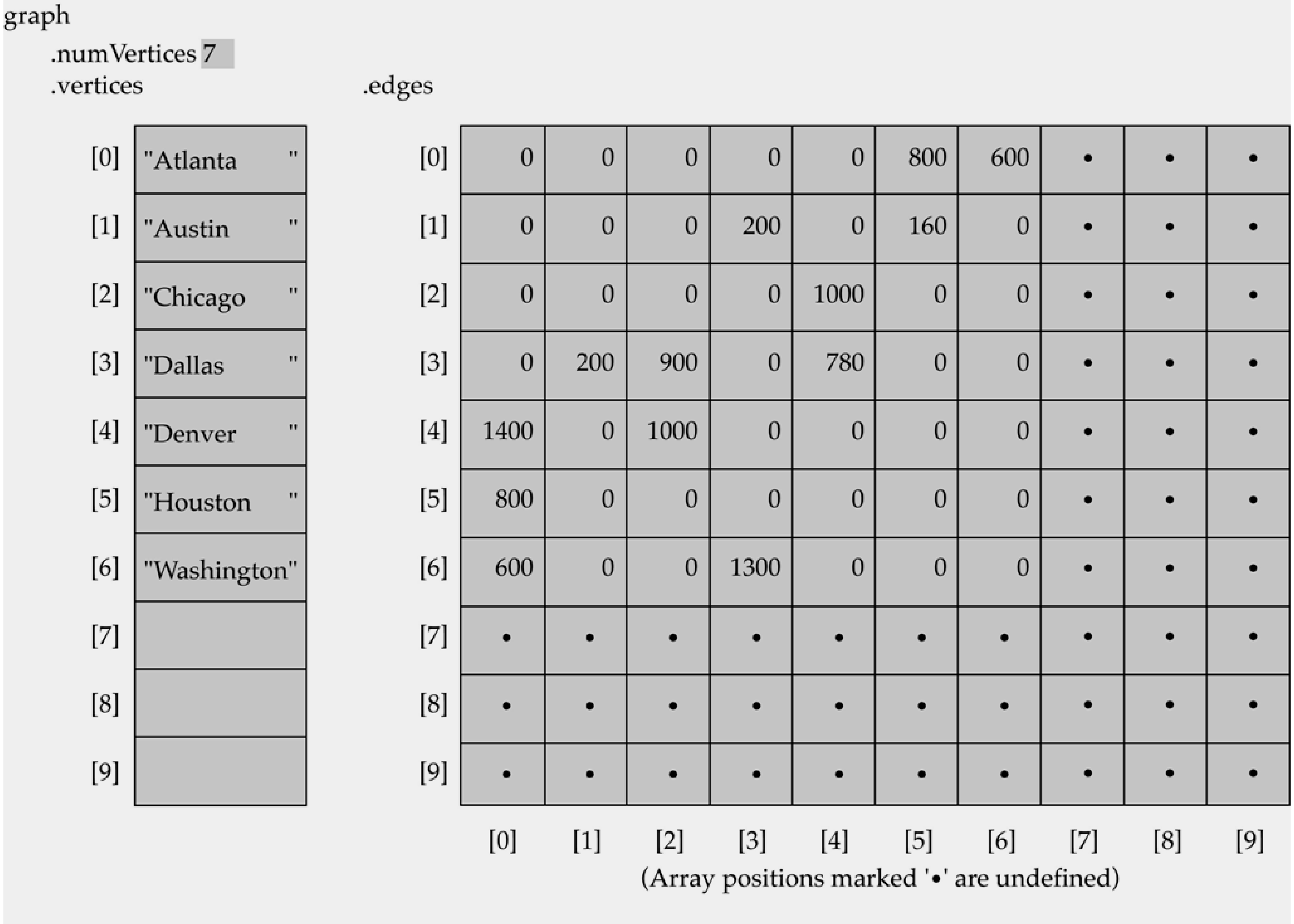
- Weighted graph: a graph in which each edge carries a value



# Graph implementation

- Array-based implementation
  - A 1D array is used to represent the vertices
  - A 2D array (adjacency matrix) is used to represent the edges





graph

.numVertices 7

.vertices

| | |
|---|---|
| [0] | "Atlanta " |
| [1] | "Austin " |
| [2] | "Chicago " |
| [3] | "Dallas " |
| [4] | "Denver " |
| [5] | "Houston " |
| [6] | "Washington" |
| [7] | |
| [8] | |
| [9] | |

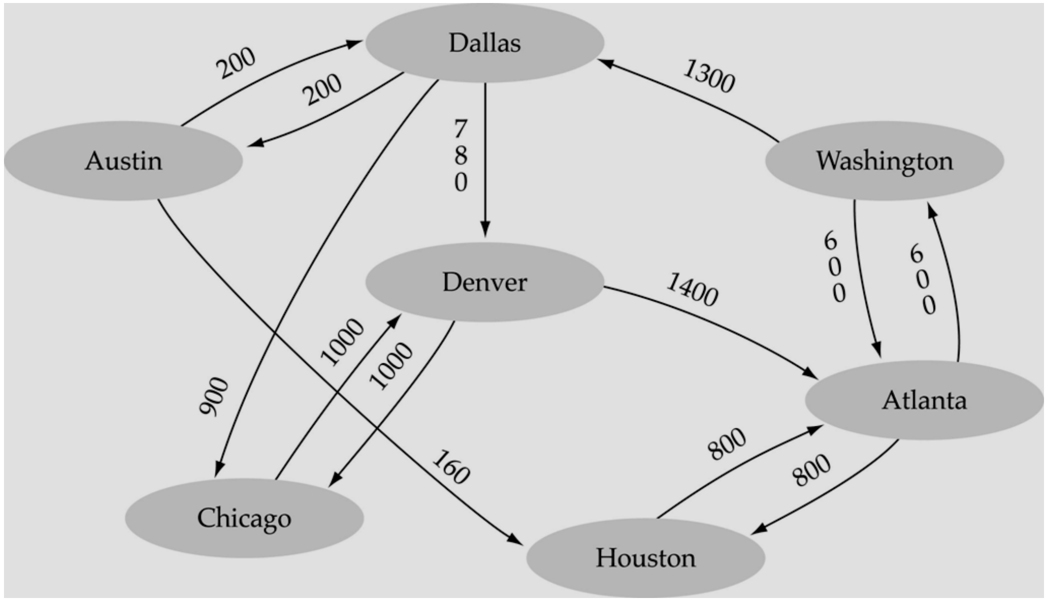
.edges

| | [0] | [1] | [2] | [3] | [4] | [5] | [6] | [7] | [8] | [9] |
|---|---|---|---|---|---|---|---|---|---|---|
| [0] | 0 | 0 | 0 | 0 | 0 | 800 | 600 | • | • | • |
| [1] | 0 | 0 | 0 | 200 | 0 | 160 | 0 | • | • | • |
| [2] | 0 | 0 | 0 | 0 | 1000 | 0 | 0 | • | • | • |
| [3] | 0 | 200 | 900 | 0 | 780 | 0 | 0 | • | • | • |
| [4] | 1400 | 0 | 1000 | 0 | 0 | 0 | 0 | • | • | • |
| [5] | 800 | 0 | 0 | 0 | 0 | 0 | 0 | • | • | • |
| [6] | 600 | 0 | 0 | 1300 | 0 | 0 | 0 | • | • | • |
| [7] | • | • | • | • | • | • | • | • | • | • |
| [8] | • | • | • | • | • | • | • | • | • | • |
| [9] | • | • | • | • | • | • | • | • | • | • |

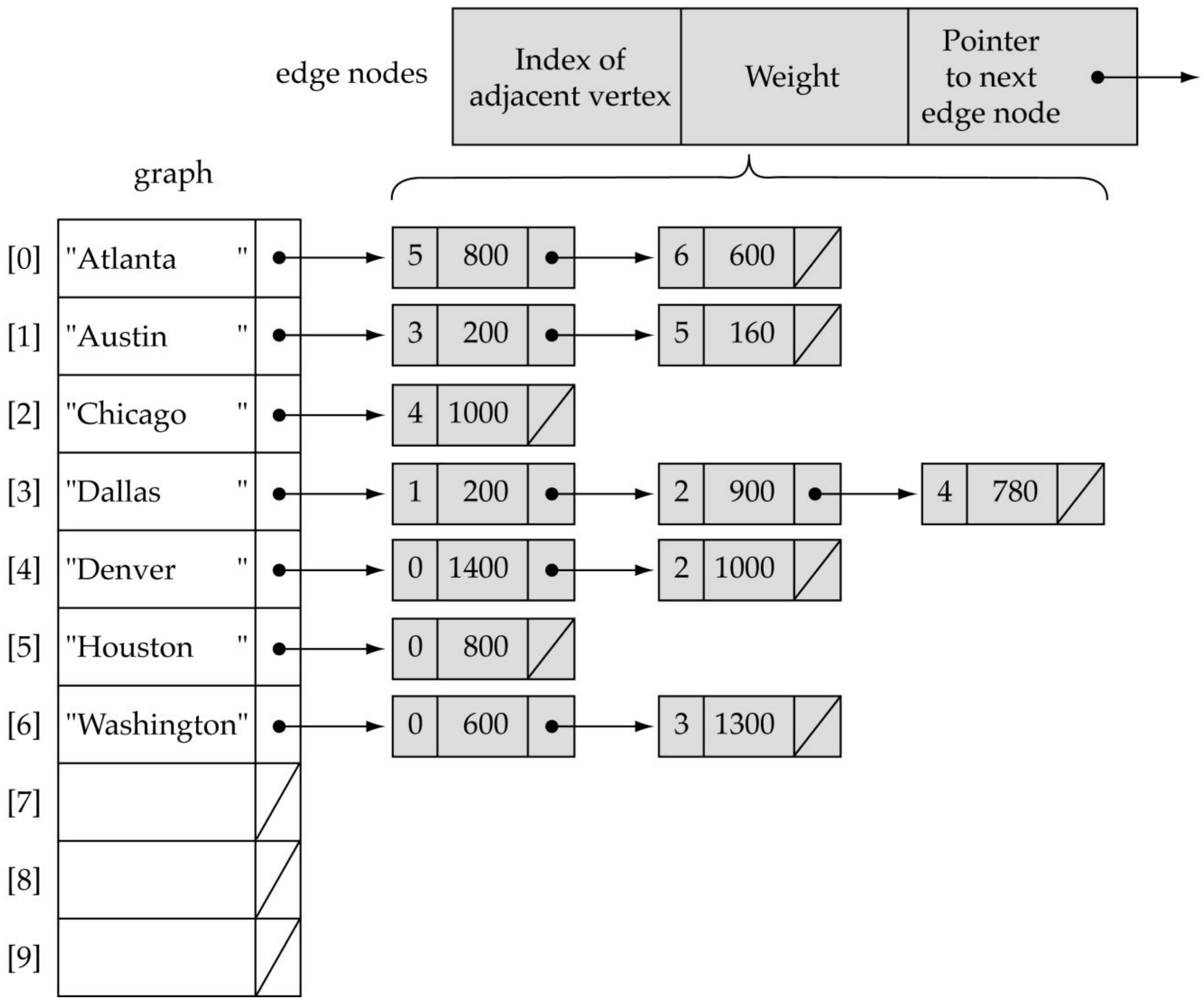
(Array positions marked '•' are undefined)

# Graph implementation (cont.)

- Linked-list implementation
  - A 1D array is used to represent the vertices
  - A list is used for each vertex *v* which contains the vertices which are adjacent from *v* (adjacency list)



(a)



edge nodes

Index of adjacent vertex | Weight | Pointer to next edge node

graph

[0] "Atlanta " → 5 800 → 6 600

[1] "Austin " → 3 200 → 5 160

[2] "Chicago " → 4 1000

[3] "Dallas " → 1 200 → 2 900 → 4 780

[4] "Denver " → 0 1400 → 2 1000

[5] "Houston " → 0 800

[6] "Washington" → 0 600 → 3 1300

[7]

[8]

[9]

# Adjacency matrix vs. adjacency list representation

- **Adjacency matrix**
  - Good for dense graphs --$|E| \sim O(|V|^2)$
  - Memory requirements: $O(|V| + |E|) = O(|V|^2)$
  - Connectivity between two vertices can be tested quickly
- **Adjacency list**
  - Good for sparse graphs -- $|E| \sim O(|V|)$
  - Memory requirements: $O(|V| + |E|) = O(|V|)$
  - Vertices adjacent to another vertex can be found quickly

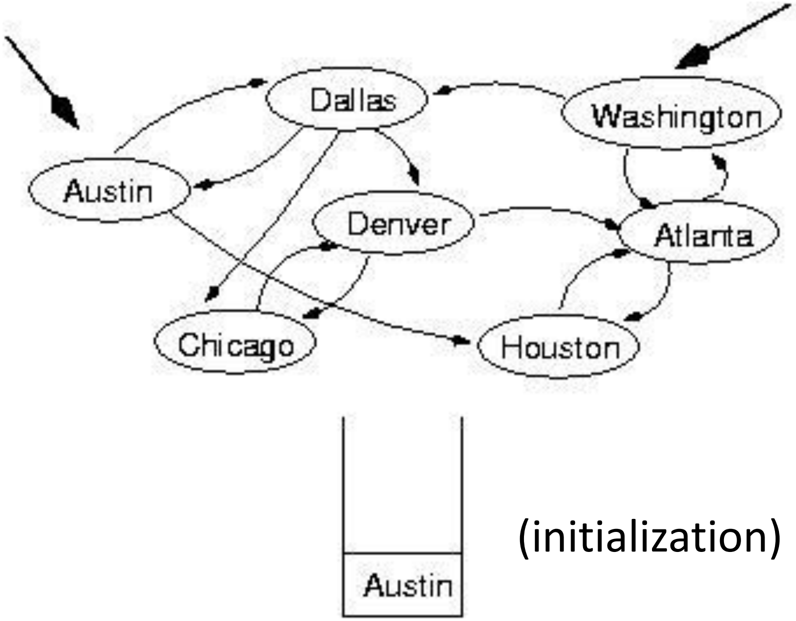
# Depth-First-Search (DFS)

- What is the idea behind DFS?
  - Travel as far as you can down a path
  - Back up *as little as possible* when you reach a "dead end" (i.e., next vertex has been "marked" or there is no next vertex)
- DFS can be implemented efficiently using a *stack*

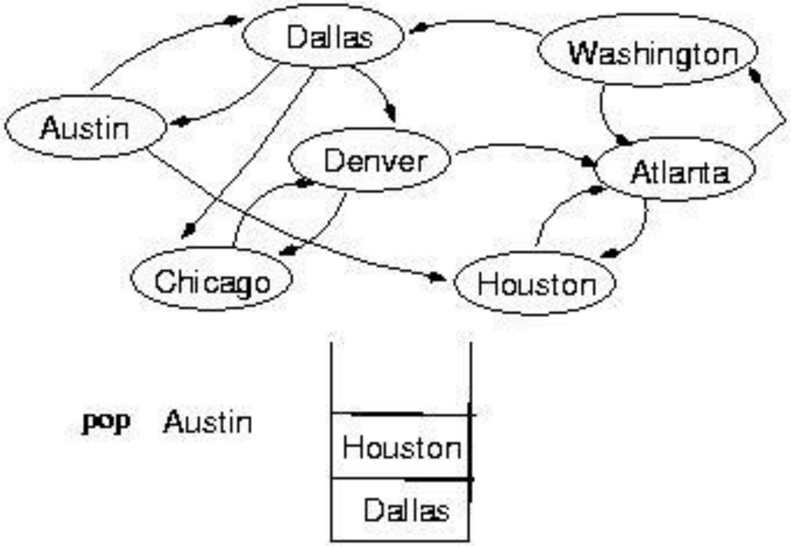
# Depth-First-Search (DFS) *(cont.)*

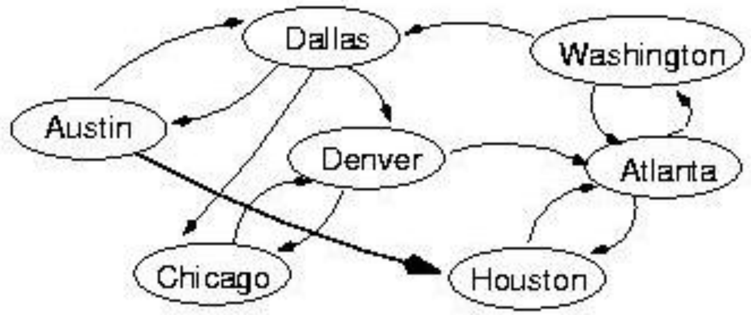
```
Set found to false
stack.Push(startVertex)
DO
  stack.Pop(vertex)
  IF vertex == endVertex
    Set found to true
  ELSE
    Push all adjacent vertices onto stack
WHILE !stack.IsEmpty() AND !found

IF(!found)
  Write "Path does not exist"
```



(initialization)

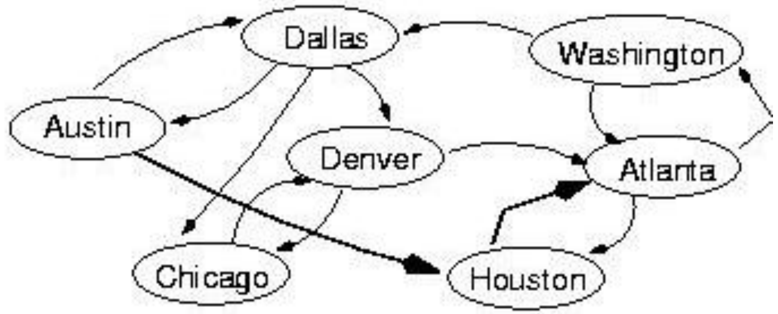




**pop** Houston

Atlanta

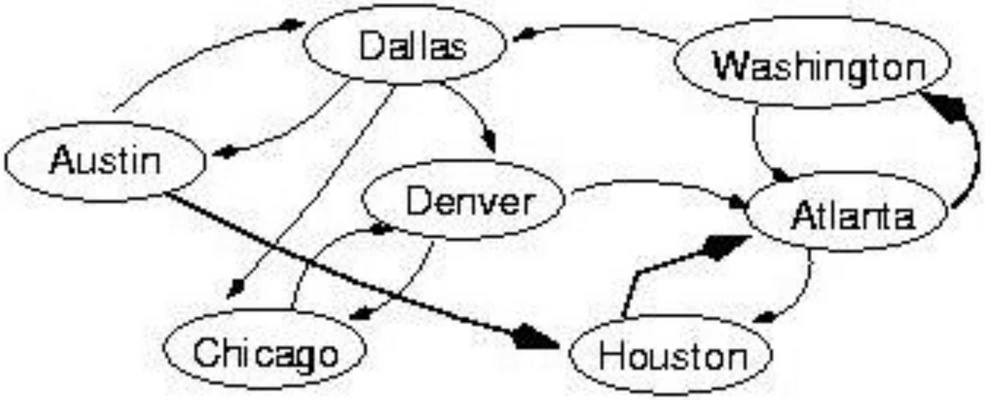
Dallas



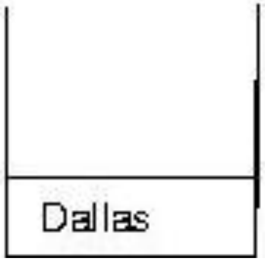
**pop** Atlanta

Washington

Dallas



**pop** Washington



Dallas

```
template <class ItemType>
void DepthFirstSearch(GraphType<VertexType> graph, VertexType
    startVertex, VertexType endVertex)
{
 StackType<VertexType> stack;
 QueType<VertexType> vertexQ;

 bool found = false;
 VertexType vertex;
 VertexType item;

 graph.ClearMarks();
 stack.Push(startVertex);
 do {
   stack.Pop(vertex);
   if(vertex == endVertex)
     found = true;
```

(continues)

```
 else {
     if(!graph.IsMarked(vertex)) {
       graph.MarkVertex(vertex);
       graph.GetToVertices(vertex, vertexQ);

       while(!vertexQ.IsEmpty()) {
         vertexQ.Dequeue(item);
         if(!graph.IsMarked(item))
           stack.Push(item);
       }
     }
   } while(!stack.IsEmpty() && !found);

   if(!found)
     cout << "Path not found" << endl;
}
```

(continues)

```
template<class VertexType>
void GraphType<VertexType>::GetToVertices(VertexType vertex,
                               QueTye<VertexType>& adjvertexQ)
{
 int fromIndex;
 int toIndex;

 fromIndex = IndexIs(vertices, vertex);
 for(toIndex = 0; toIndex < numVertices; toIndex++)
   if(edges[fromIndex][toIndex] != NULL_EDGE)
     adjvertexQ.Enqueue(vertices[toIndex]);
}
```

# Breadth-First-Searching (BFS)

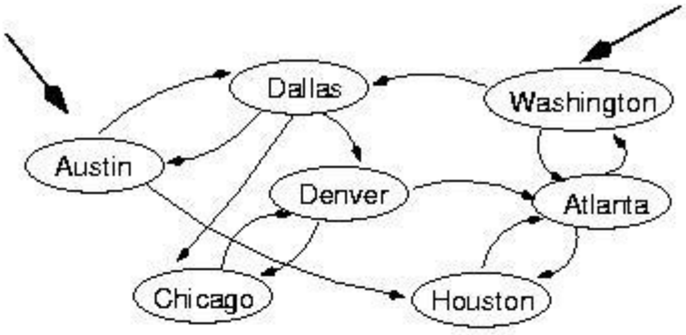
- What is the idea behind BFS?
  - Look at all possible paths at the same depth before you go at a deeper level
  - Back up *as far as possible* when you reach a "dead end" (i.e.,  next vertex has been "marked" or there is no next vertex)

# Breadth-First-Searching (BFS) (cont.)

- BFS can be implemented efficiently using a *queue*

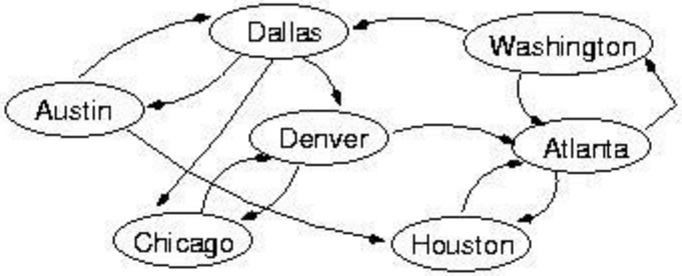
```
Set found to false
queue.Enqueue(startVertex)
DO
  queue.Dequeue(vertex)
  IF vertex == endVertex
    Set found to true
  ELSE
    Enqueue all adjacent vertices onto queue
WHILE !queue.IsEmpty() AND !found
```

- Should we mark a vertex when it is enqueued or when it is dequeued ?



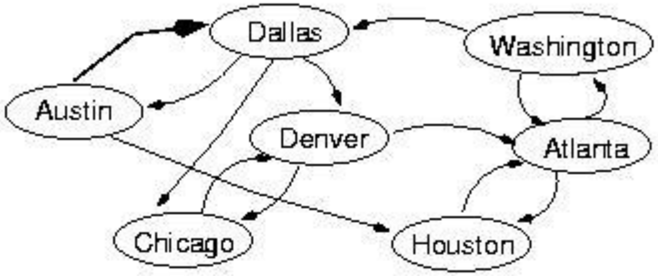
(initialization)

| | | | |
|---|---|---|---|
| | | | Austin |



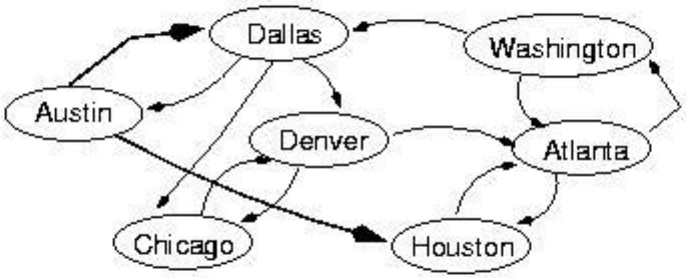
**dequeue** Austin

| | | | |
|---|---|---|---|
| | | Dallas | Houston |



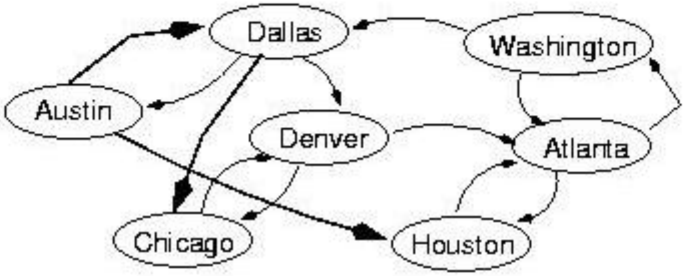
**dequeue** Dallas

| | | | |
|---|---|---|---|
| | Houston | Chicago | Denver |



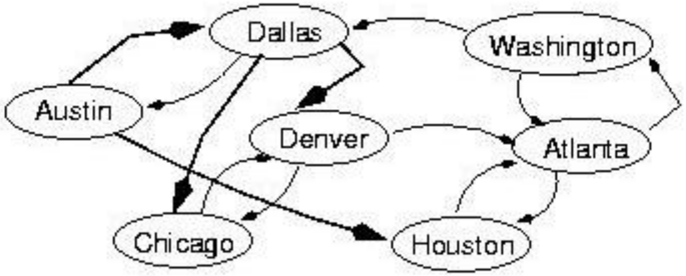
**dequeue** Houston

| | | | |
|---|---|---|---|
| | Chicago | Denver | Atlanta |



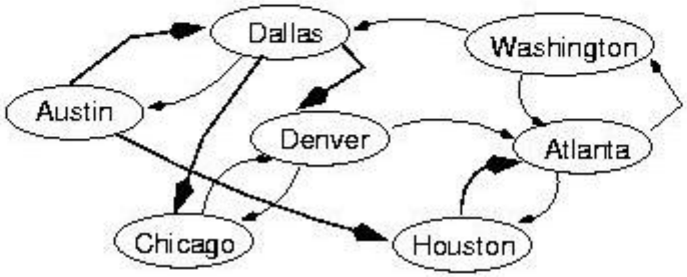
**dequeue** Chicago

| | | | |
|---|---|---|---|
| | Denver | Atlanta | Denver |



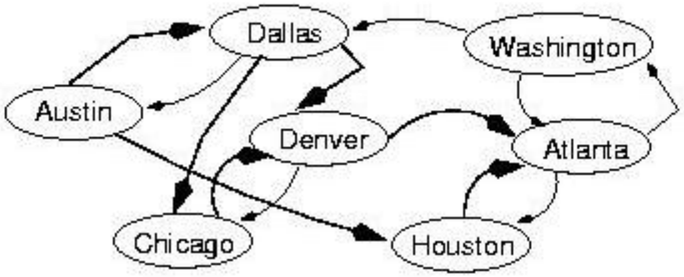
**dequeue** Denver

| | | | |
|---|---|---|---|
| | Atlanta | Denver | Atlanta |



**dequeue** Atlanta

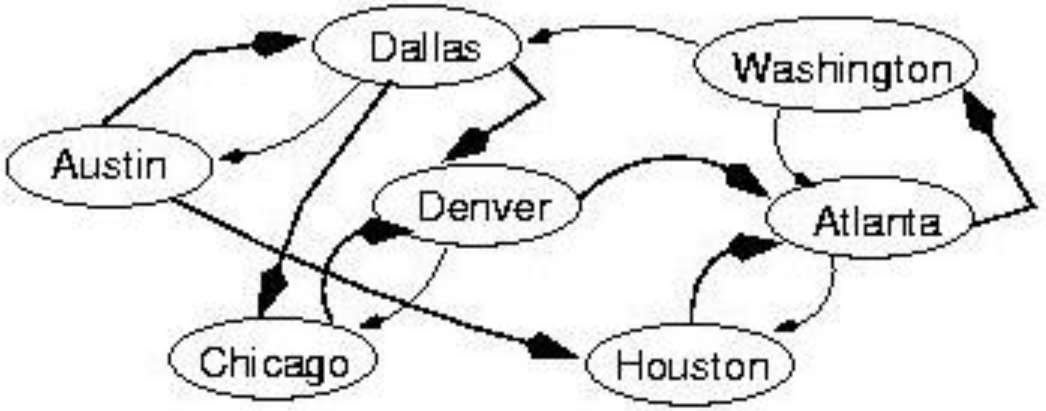
| | | | |
|---|---|---|---|
| | Denver | Atlanta | Washington |



**dequeue** Denver,

next: Atlanta

| | | |
|---|---|---|
| | Washington | Washington |



**dequeue** Washington

| | | |
|---|---|---|
| | | Washington |

```
template<class VertexType>
void BreadthFirtsSearch(GraphType<VertexType> graph,
     VertexType startVertex, VertexType endVertex);
{
 QueType<VertexType> queue;
 QueType<VertexType> vertexQ;//

 bool found = false;
 VertexType vertex;
 VertexType item;

 graph.ClearMarks();
 queue.Enqueue(startVertex);
 do {
   queue.Dequeue(vertex);
   if(vertex == endVertex)
     found = true;
```

(continues)

```
 else {
   if(!graph.IsMarked(vertex)) {
    graph.MarkVertex(vertex);
    graph.GetToVertices(vertex, vertexQ);

    while(!vertxQ.IsEmpty()) {
     vertexQ.Dequeue(item);
     if(!graph.IsMarked(item))
      queue.Enqueue(item);
    }
   }
  }
 } while (!queue.IsEmpty() && !found);

 if(!found)
  cout << "Path not found" << endl;
}
```

# Single-source shortest-path problem

- There are multiple paths from a source vertex to a destination vertex
- *Shortest path*: the path whose total weight (i.e., sum of edge weights) is minimum
- Examples:
  - Austin->Houston->Atlanta->Washington: 1560 miles
  - Austin->Dallas->Denver->Atlanta->Washington: 2980 miles

# Single-source shortest-path problem (cont.)

- Common algorithms: *Dijkstra's* algorithm, *Bellman-Ford* algorithm
- BFS can be used to solve the shortest graph problem when the graph is **weightless** or all the weights are the same

(mark vertices before Enqueue)

# UNIT-IV

**Topics:**

**Searching**- Linear Search, Binary Search, Static Hashing-Introduction, hash tables, hash functions, Overflow Handling.

**Sorting**-Insertion Sort, Selection Sort, Radix Sort, Quick sort, Heap Sort, Comparison of Sorting methods.

# Sequential Search
O (n)

- A **sequential search** of a list/array begins at the beginning of the list/array and continues until the item is found or the entire list/array has been searched

# Sequential Search

```
bool LinSearch(double x[ ], int n, double item){

        for(int i=0;i<n;i++){
                if(x[i]==item) return true;
                else return false;
        }
        return false;
   }
```

# Linear Search - Example

- Array `numlist` contains:

| | | | | | | |
|---|---|---|---|---|---|---|
| 17 | 23 | 5 | 11 | 2 | 29 | 3 |

- Searching for the the value `11`, linear search examines `17, 23, 5,` and `11`
- Searching for the the value `7`, linear search examines `17, 23, 5, 11, 2, 29,` and `3`

# Search Algorithms

Suppose that there are *n* elements in the array. The following expression gives the average number of comparisons:

$$\frac{1+2+\ldots+n}{n}$$

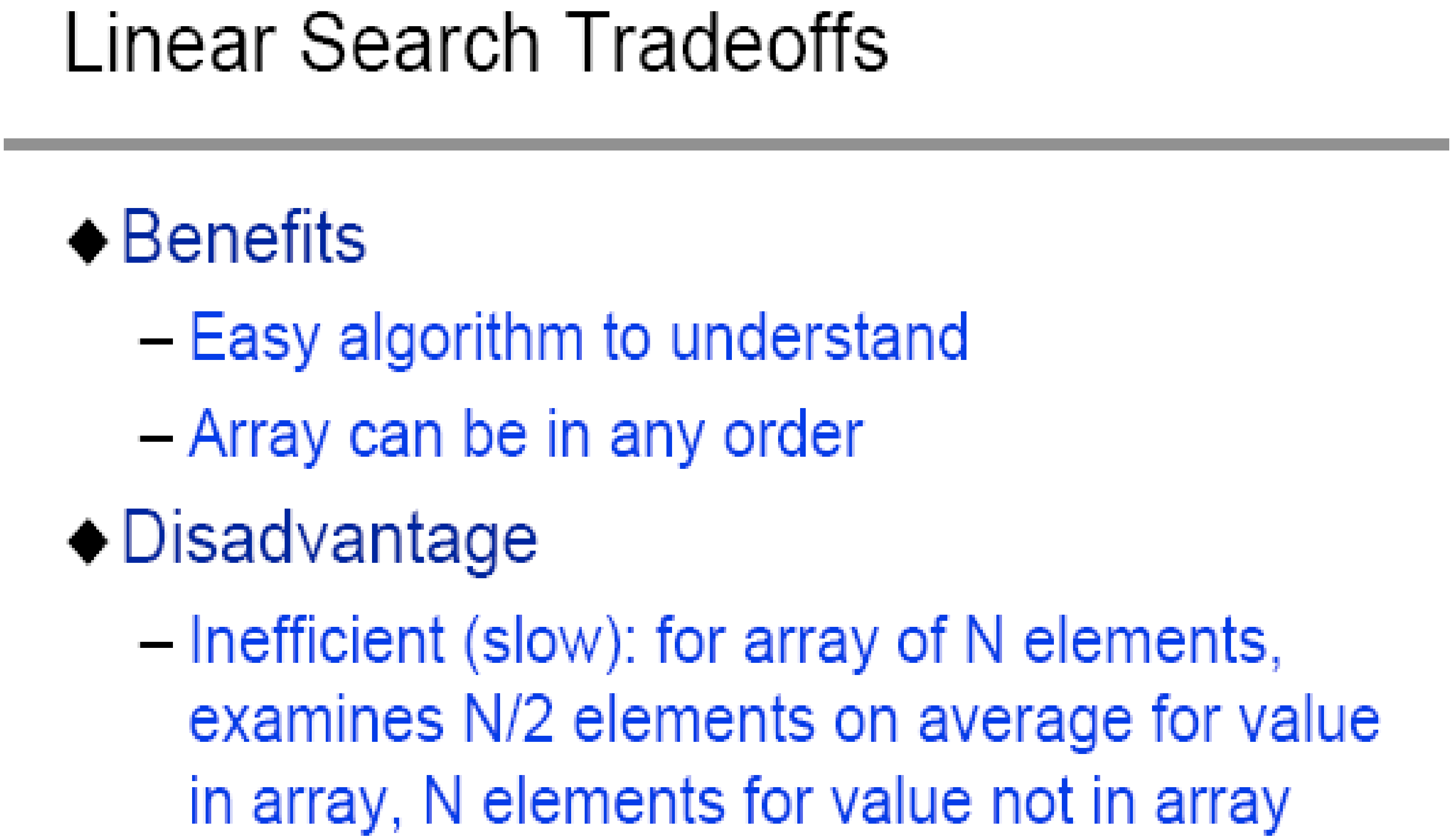
It is known that

$$1+2+\ldots+n=\frac{n(n+1)}{2}$$

Therefore, the following expression gives the average number of comparisons made by the sequential search in the successful case:

$$\frac{1+2+\ldots+n}{n}=\frac{1}{n}\frac{n(n+1)}{2}=\frac{n+1}{2}$$

# Search Algorithms



## Linear Search Tradeoffs

- Benefits
  - Easy algorithm to understand
  - Array can be in any order
- Disadvantage
  - Inefficient (slow): for array of N elements, examines N/2 elements on average for value in array, N elements for value not in array

# Binary Search
# O(log2 n)

- A **binary search** looks for an item in a list using a divide-and-conquer strategy

# Binary Search

Requires array elements to be in order

1. Divides the array into three sections:
   - middle element
   - elements on one side of the middle element
   - elements on the other side of the middle element
2. If the middle element is the correct value, done. Otherwise, go to step 1. using only the half of the array that may contain the correct value.
3. Continue steps 1. and 2. until either the value is found or there are no more elements to examine

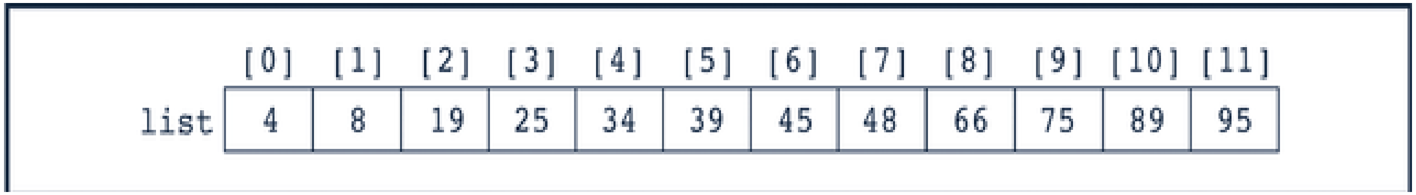
# Binary Search: middle element

$$\texttt{mid} = \frac{\texttt{left + right}}{\texttt{2}}$$

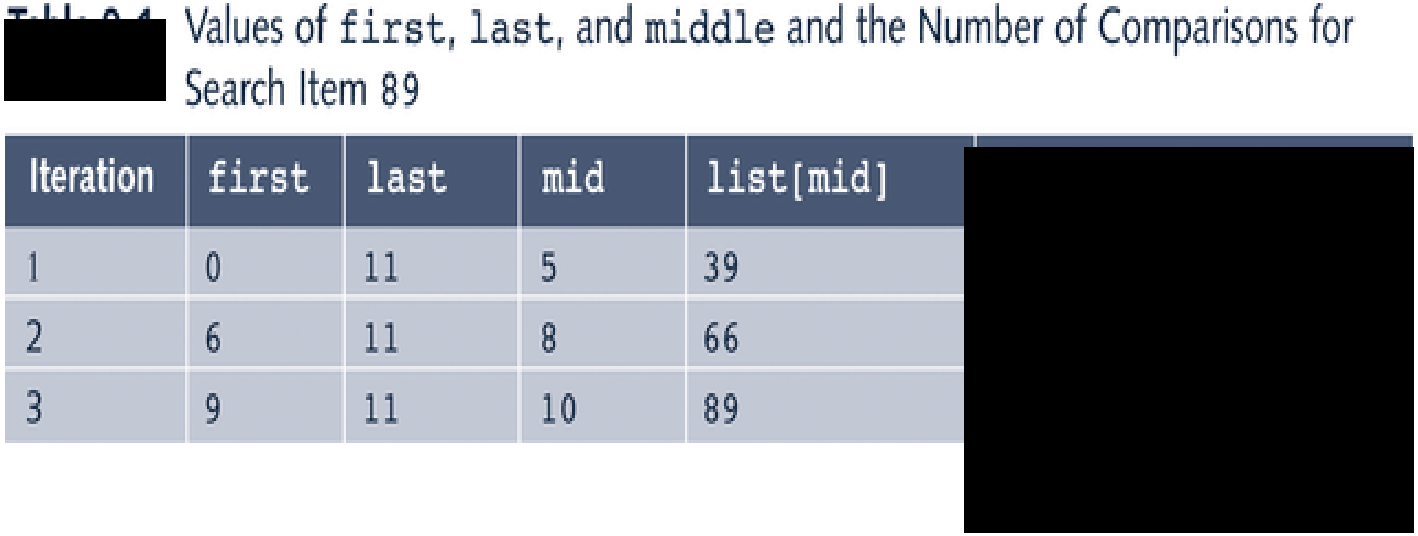
# Binary Search

```
bool BinSearch(double list[ ], int n, double
  item, int&index){
        int left=0;
        int right=n-1;
        int mid;
        while(left<=right){
         mid=(left+right)/2;
if(item> list [mid]){ left=mid+1; }
        else if(item< list [mid]){right=mid-1;}
        else{
        item= list [mid];
        index=mid;
        return true; }
      }// while
    return false;
  }
```

# Binary Search: Example



|  | [0] | [1] | [2] | [3] | [4] | [5] | [6] | [7] | [8] | [9] | [10] | [11] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| list | 4 | 8 | 19 | 25 | 34 | 39 | 45 | 48 | 66 | 75 | 89 | 95 |



[redacted] Values of first, last, and middle and the Number of Comparisons for Search Item 89

| Iteration | first | last | mid | list[mid] | [redacted] |
|---|---|---|---|---|---|
| 1 | 0 | 11 | 5 | 39 | [redacted] |
| 2 | 6 | 11 | 8 | 66 | [redacted] |
| 3 | 9 | 11 | 10 | 89 | [redacted] |

# Binary Search - Example

- Array numlist2 contains:

| | | | | | | |
|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 17 | 23 | 29 |

- Searching for the the value 11, binary search examines 11 and stops
- Searching for the the value 7, binary search examines 11, 3, 5, and stops

# Binary Search - Tradeoffs

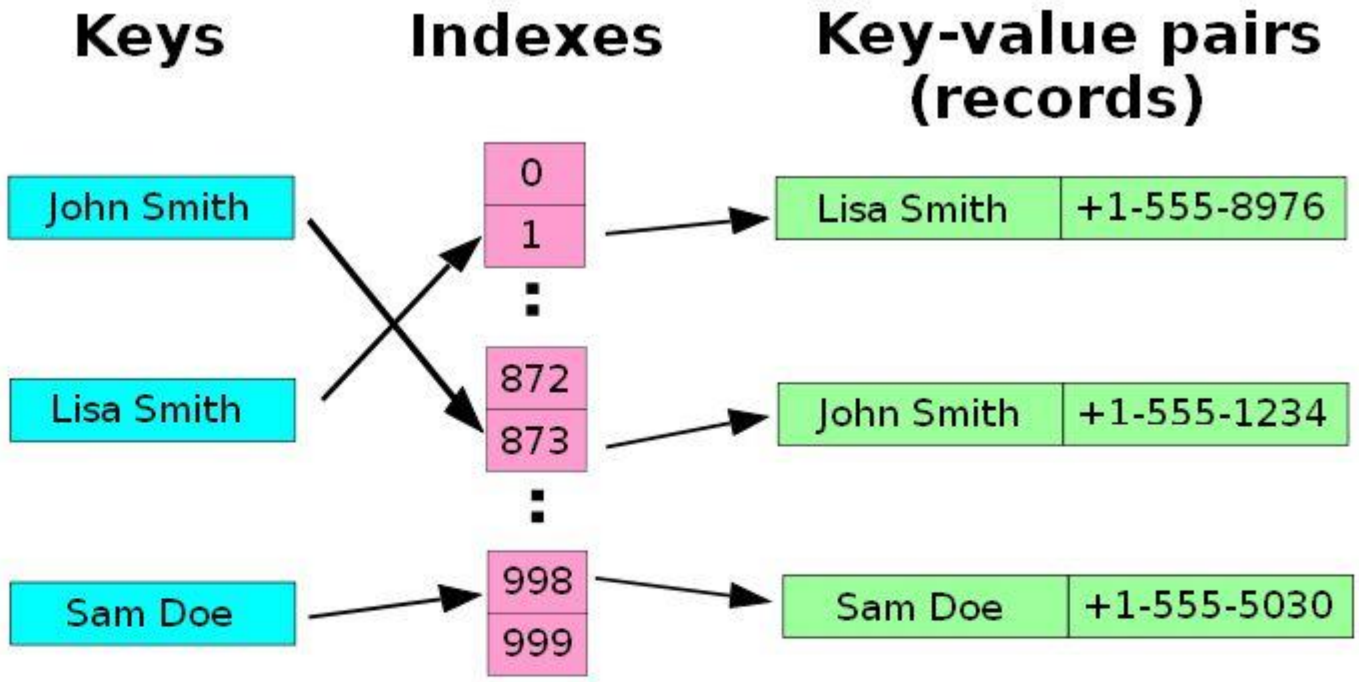
- Benefits:
  - Much more efficient than linear search. For array of N elements, performs at most $\boldsymbol{log_2 N}$ comparisons

- Disadvantages:
  - Requires that array elements be sorted

# Concept of Hashing

- In CS, a **hash table**, or a **hash map**, is a data structure that associates keys (names) with values (attributes).
  - Look-Up Table
  - Dictionary
  - Cache
  - Extended Array

# Example



A small phone book as a hash table.

(Figure is from Wikipedia)

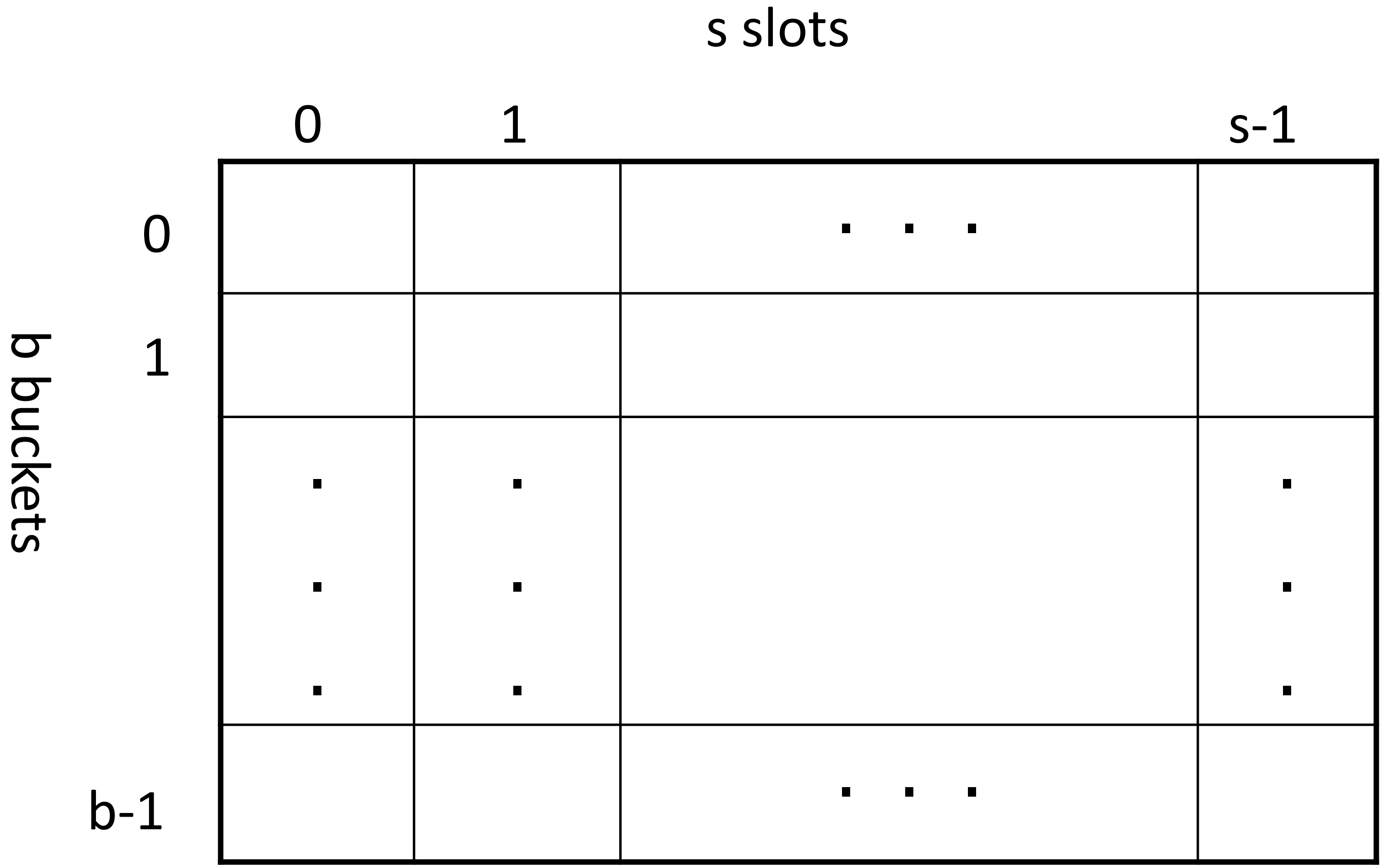
# Search vs. Hashing

- Search tree methods: key comparisons
  - Time complexity: O(size) or O(log n)
- Hashing methods: hash functions
  - Expected time: O(1)
- Types
  - Static hashing (section 8.2)
  - Dynamic hashing (section 8.3)

# Static Hashing

- Key-value pairs are stored in a fixed size table called a *hash table*.
  - A hash table is partitioned into many ***buckets***.
  - Each bucket has many ***slots***.
  - Each slot holds one record.
  - A hash function f(x) transforms the identifier (key) into an address in the hash table

# Hash table



# Data Structure for Hash Table

```
#define MAX_CHAR  10
#define TABLE_SIZE  13
typedef struct {
    char key[MAX_CHAR];
    /* other fields */
} element;
element hash_table[TABLE_SIZE];
```

# Some Issues

- **Choice of hash function.**
  - ***Really tricky!***
  - To avoid collision (two different pairs are in the same the same bucket.)
  - Size (number of buckets) of hash table.
- **Overflow handling method.**
  - Overflow: there is no space in the bucket for the new pair.

# Example (fig 8.1)

synonyms:
char, ceil,
clock, ctime

↑

overflow

| | Slot 0 | Slot 1 |
|---|---|---|
| 0 | acos | atan |
| 1 | | |
| 2 | char | ceil |
| 3 | define | |
| 4 | exp | |
| 5 | float | floor |
| 6 | | |
| … | | |
| 25 | | |

synonyms

synonyms

# Choice of Hash Function

- Requirements
  - easy to compute
  - minimal number of collisions
- If a hashing function groups key values together, this is called clustering of the keys.
- A good hashing function distributes the key values uniformly throughout the range.

# Some hash functions

- Middle of square
  - H(x):= return middle digits of x^2
- Division
  - H(x):= return x % k
- Multiplicative:
  - H(x):= return the first few digits of the fractional part of x*k, where k is a fraction.

# Some hash functions II

- Folding:
  - Partition the identifier x into several parts, and add the parts together to obtain the hash address
  - e.g. x=12320324111220; partition x into 123,203,241,112,20; then return the address 123+203+241+112+20=699
  - Shift folding vs. folding at the boundaries
- Digit analysis:
  - If all the keys have been known in advance, then we could delete the digits of keys having the most skewed distributions, and use the rest digits as hash address.

# Overflow Handling

- An overflow occurs when the home bucket for a new pair (key, element) is full.
- We may handle overflows by:
  - Search the hash table in some systematic fashion for a bucket that is not full.
    - .

    - Linear probing (linear open addressing).
    - Quadratic probing.
    - Random probing.
  - Eliminate overflows by permitting each bucket to keep a list of all pairs for which it is the home bucket.
    - Array linear list.
    - Chain

# Linear probing (linear open addressing)

- **Open addressing** ensures that all elements are stored directly into the hash table, thus it attempts to resolve collisions using various methods.

- **Linear Probing** resolves collisions by placing the data into the next open slot in the table.

# Linear Probing – Get And Insert

- divisor = b (number of buckets) = 17.
- Home bucket = key % 17.

| 0 | | | | 4 | | | | 8 | | | | 12 | | | | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 34 | 0 | 45 | | | | 6 | 23 | 7 | | | 28 | 12 | 29 | 11 | 30 | 33 |

- Insert pairs whose keys are 6, 12, 34, 29, 28, 11, 23, 7, 0, 33, 30, 45

# Linear Probing – Delete

| 0 | | | | 4 | | | | 8 | | | | 12 | | | | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 34 | 0 | 45 | | | | 6 | 23 | 7 | | | 28 | 12 | 29 | 11 | 30 | 33 |

- Delete(0)

| 0 | | | | 4 | | | | 8 | | | | 12 | | | | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 34 | 🟨 | 45 | | | | 6 | 23 | 7 | | | 28 | 12 | 29 | 11 | 30 | 33 |

- Search cluster for pair (if any) to fill vacated bucket.

| 0 | | | | 4 | | | | 8 | | | | 12 | | | | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 34 | 45 | 🟨 | | | | 6 | 23 | 7 | | | 28 | 12 | 29 | 11 | 30 | 33 |

# Linear Probing – Delete(34)

| 0 |  |  |  | 4 |  |  |  | 8 |  |  |  | 12 |  |  |  | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 34 | 0 | 45 |  |  |  | 6 | 23 | 7 |  |  | 28 | 12 | 29 | 11 | 30 | 33 |

| 0 |  |  |  | 4 |  |  |  | 8 |  |  |  | 12 |  |  |  | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 🟨 | 0 | 45 |  |  |  | 6 | 23 | 7 |  |  | 28 | 12 | 29 | 11 | 30 | 33 |

- Search cluster for pair (if any) to fill vacated bucket.

| 0 |  |  |  | 4 |  |  |  | 8 |  |  |  | 12 |  |  |  | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 🟨 | 45 |  |  |  | 6 | 23 | 7 |  |  | 28 | 12 | 29 | 11 | 30 | 33 |

| 0 |  |  |  | 4 |  |  |  | 8 |  |  |  | 12 |  |  |  | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 45 | 🟨 |  |  |  | 6 | 23 | 7 |  |  | 28 | 12 | 29 | 11 | 30 | 33 |

# Linear Probing – Delete(29)

| 0 | | | | 4 | | | | 8 | | | | 12 | | | | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 34 | 0 | 45 | | | | 6 | 23 | 7 | | | 28 | 12 | 29 | 11 | 30 | 33 |

| 0 | | | | 4 | | | | 8 | | | | 12 | | | | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 34 | 0 | 45 | | | | 6 | 23 | 7 | | | 28 | 12 | 🟨 | 11 | 30 | 33 |

- Search cluster for pair (if any) to fill vacated bucket.

| 0 | | | | 4 | | | | 8 | | | | 12 | | | | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 34 | 0 | 45 | | | | 6 | 23 | 7 | | | 28 | 12 | 11 | 🟨 | 30 | 33 |

| 0 | | | | 4 | | | | 8 | | | | 12 | | | | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 34 | 0 | 45 | | | | 6 | 23 | 7 | | | 28 | 12 | 11 | 30 | 🟨 | 33 |

| 0 | | | | 4 | | | | 8 | | | | 12 | | | | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 34 | 0 | 🟨 | | | | 6 | 23 | 7 | | | 28 | 12 | 11 | 30 | 45 | 33 |

# Performance Of Linear Probing

| 0 |  |  |  | 4 |  |  |  | 8 |  |  |  | 12 |  |  |  | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 34 | 0 | 45 |  |  |  | 6 | 23 | 7 |  |  | 28 | 12 | 29 | 11 | 30 | 33 |

- Worst-case find/insert/erase time is $\Theta(n)$, where n is the number of pairs in the table.
- This happens when all pairs are in the same cluster.

# Problem of Linear Probing

- Identifiers tend to cluster together
- Adjacent cluster tend to coalesce
- Increase the search time

# Quadratic Probing

- Linear probing searches buckets $(H(x)+i^2)\%b$
- Quadratic probing uses a quadratic function of *i* as the increment
- Examine buckets $H(x)$, $(H(x)+i^2)\%b$, $(H(x)-i^2)\%b$, for $1<=i<=(b-1)/2$
- b is a prime number of the form 4j+3, j is an integer

# Random Probing

- Random Probing works incorporating with random numbers.
  - H(x):= (H’(x) + S[i]) % b
  - S[i] is a table with size b-1
  - S[i] is a random permuation of integers [1,b-1].

# Some Applications of Hash Tables

- **Database systems**: Specifically, those that require efficient random access. Generally, database systems try to optimize between two types of access methods: sequential and random. Hash tables are an important part of efficient random access because they provide a way to locate data in a constant amount of time.

- **Symbol tables**: The tables used by compilers to maintain information about symbols from a program. Compilers access information about symbols frequently. Therefore, it is important that symbol tables be implemented very efficiently.

- **Data dictionaries**: Data structures that support adding, deleting, and searching for data. Although the operations of a hash table and a data dictionary are similar, other data structures may be used to implement data dictionaries. Using a hash table is particularly efficient.

- **Network processing algorithms**: Hash tables are fundamental components of several network processing algorithms and applications, including route lookup, packet classification, and network monitoring.

- **Browser Cashes**: Hash tables are used to implement browser cashes.

# Problems for Which Hash Tables are not Suitable

1. **Problems for which data ordering is required.** Because a hash table is an unordered data structure, certain operations are difficult and expensive. Range queries, proximity queries, selection, and sorted traversals are possible only if the keys are copied into a sorted data structure. There are hash table implementations that keep the keys in order, but they are far from efficient.

- 2. Problems having **multidimensional data**.
- 3. **Prefix searching** especially if the keys are long and of variable-lengths.

- 4. **Problems that have dynamic data**:
- Open-addressed hash tables are based on 1D-arrays, which are difficult to resize
- once they have been allocated. Unless you want to implement the table as a
- dynamic array and rehash all of the keys whenever the size changes. This is an
- incredibly expensive operation. An alternative is use a separate-chained hash tables or dynamic hashing.

- 5. **Problems in which the data does not have unique keys.**
- Open-addressed hash tables cannot be used if the data does not have unique keys. An alternative is use separate-chained hash tables.

# SORTING

# Sorting

- To arrange a set of items in sequence.
- It is estimated that 25~50% of all computing power is used for sorting activities.
- Possible reasons:
  - Many applications require sorting;
  - Many applications perform sorting when they don't have to;
  - Many applications use inefficient sorting algorithms.

# Sorting: Definition

***Sorting: an operation that segregates items into groups according to specified criterion.***

***A = { 3 1 6 2 1 3 4 5 9 0 }***

***A = { 0 1 1 2 3 3 4 5 6 9 }***

# Some Definitions

- Internal Sort
  - The data to be sorted is all stored in the computer’s main memory.
- External Sort
  - Some of the data to be sorted might be stored in some external, slower, device.
- In Place Sort
  - The amount of extra space required to sort the data is constant with the input size.

# Types of Sorting Algorithms

There are many, many different types of sorting algorithms, but the primary ones are:

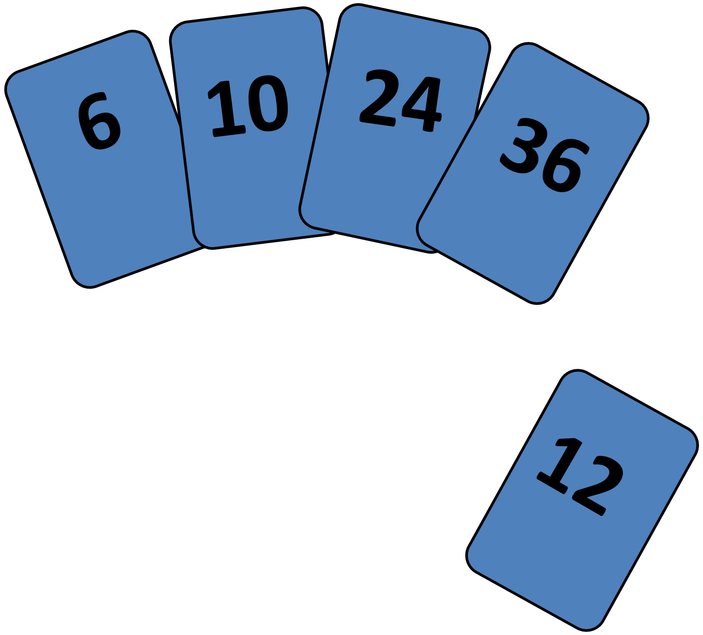
- Bubble Sort
- Selection Sort
- Insertion Sort
- Merge Sort
- Quick Sort
- Shell Sort
- Radix Sort
- Swap Sort
- Heap Sort

# Insertion Sort

- Idea: like sorting a hand of playing cards
  - Start with an empty left hand and the cards facing down on the table.
  - Remove one card at a time from the table, and insert it into the correct position in the left hand

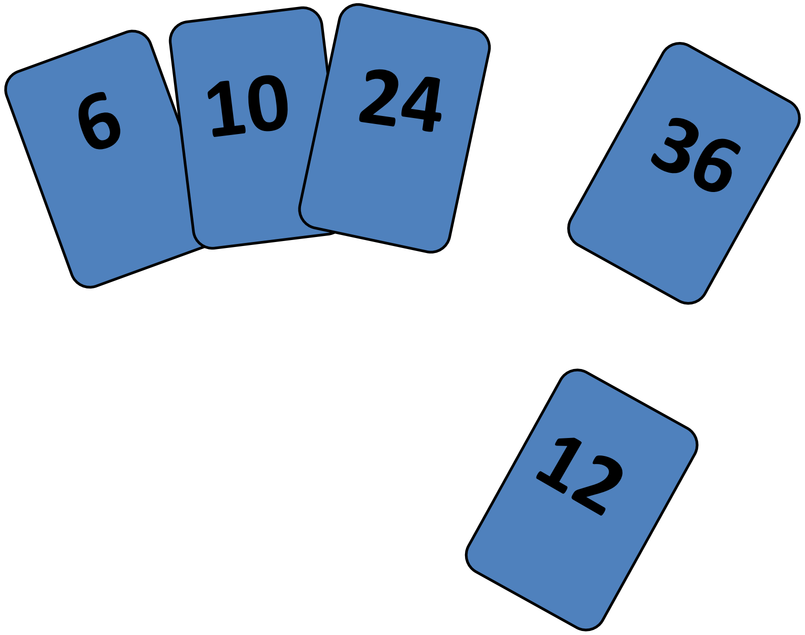
    - compare it with each of the cards already in the hand, from right to left
- The cards held in the left hand are sorted
    - these cards were originally the top cards of the pile on the table

# Insertion Sort

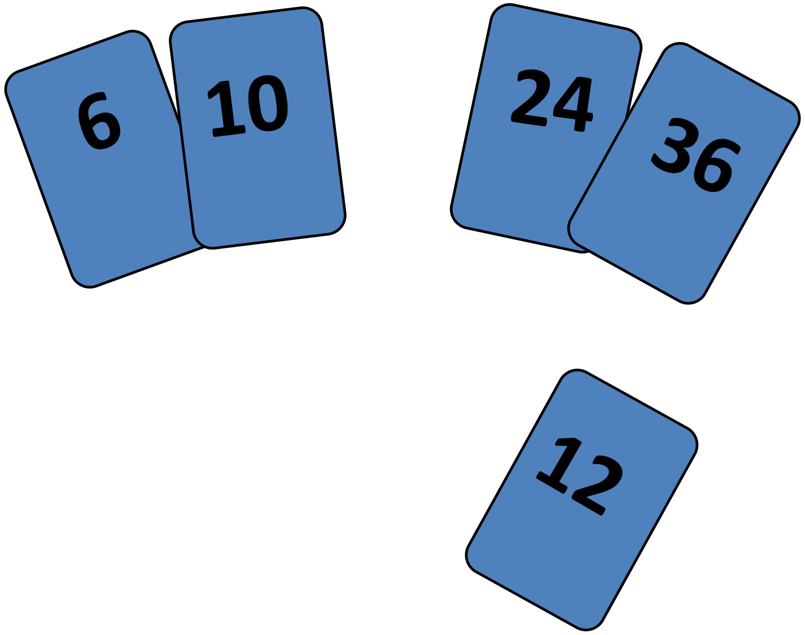


**To insert 12, we need to make room for it by moving first 36 and then 24.**

# Insertion Sort



# Insertion Sort



# Insertion Sort

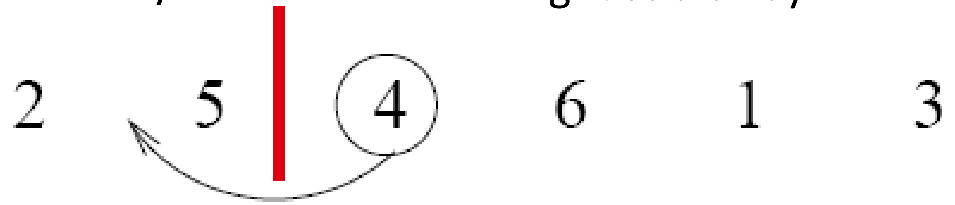
input array

5 2 4 6 1 3

at each iteration, the array is divided in two sub-arrays:

left sub-array

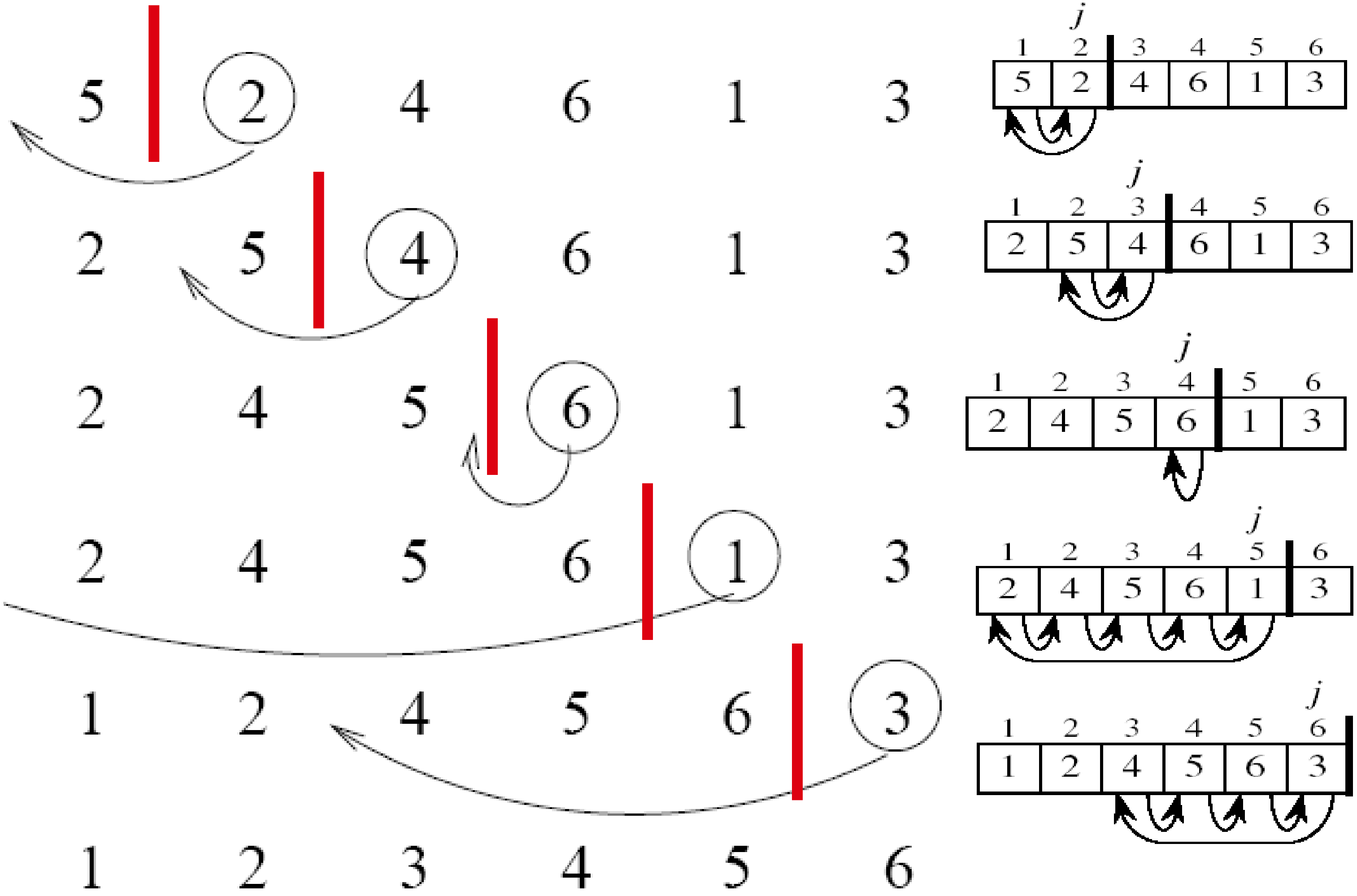
right sub-array



sorted

unsorted

# Insertion Sort



# Insertion Sort: Analysis

- Running time analysis:
  - Worst case: O(N$^2$)
  - Best case: O(N)

# Selection Sort: Idea

1. We have two group of items:
   - sorted group, and
   - unsorted group
2. Initially, all items are in the unsorted group. The sorted group is empty.
   - We assume that items in the unsorted group unsorted.
   - We have to keep items in the sorted group sorted.

# Selection Sort: Cont’d

1. Select the “best” (eg. smallest) item from the unsorted group, then put the “best” item at the end of the sorted group.
2. Repeat the process until the unsorted group becomes empty.

# Selection Sort

|  |  |  |  |  |  |
|---|---|---|---|---|---|
| 5 | 1 | 3 | 4 | 6 | 2 |

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort

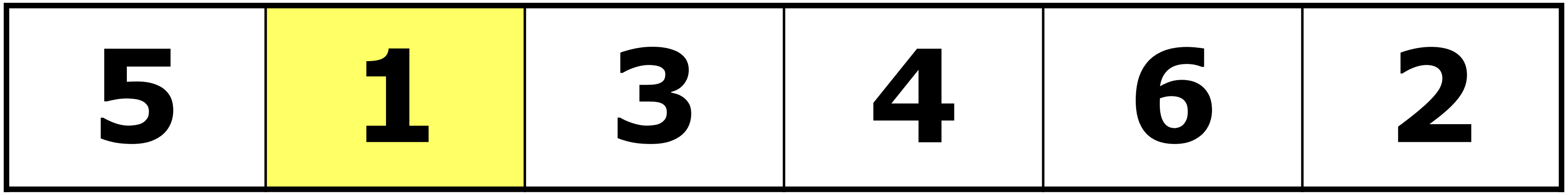
| | | | | | |
|---|---|---|---|---|---|
| 5 | 1 | 3 | 4 | 6 | 2 |

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



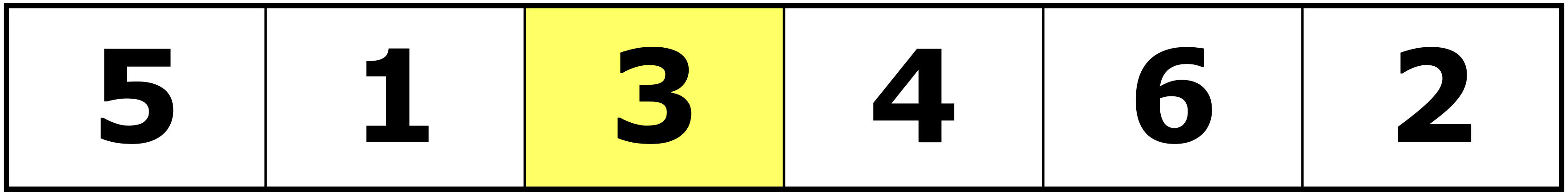
| | | | | | |
|---|---|---|---|---|---|
| 5 | 1 | 3 | 4 | 6 | 2 |

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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| 5 | 1 | 3 | 4 | 6 | 2 |

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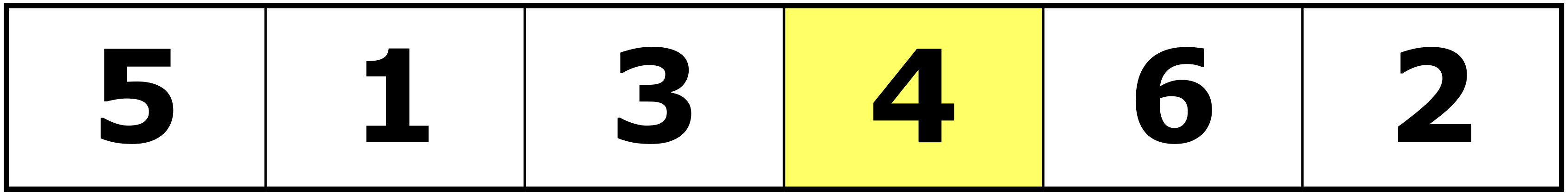
| 🟨 | Comparison |
| 🟩 | Data Movement |
| 🟦 | Sorted |

# Selection Sort



| 5 | 1 | 3 | 4 | 6 | 2 |
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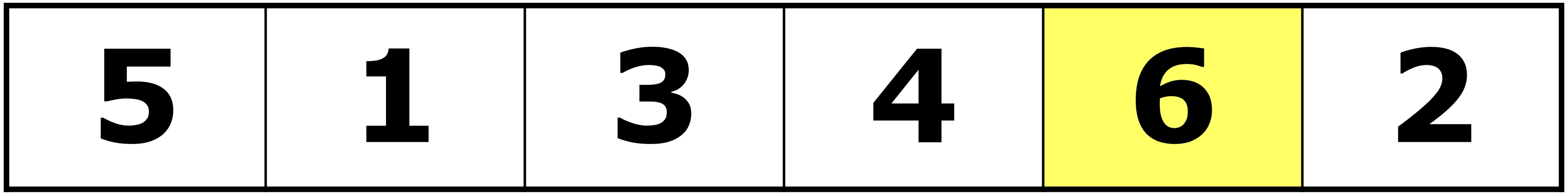
🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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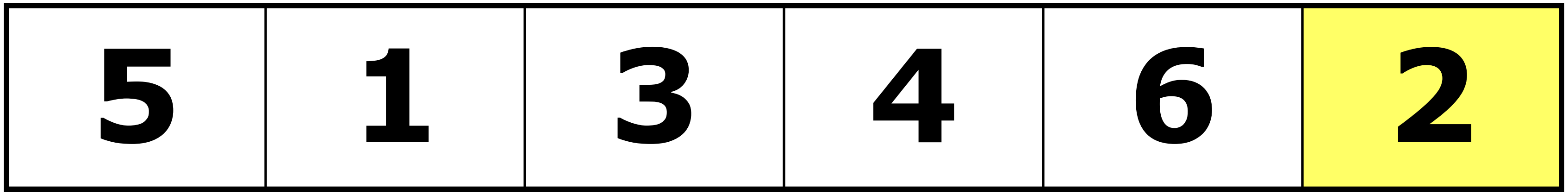
| 5 | 1 | 3 | 4 | 6 | 2 |

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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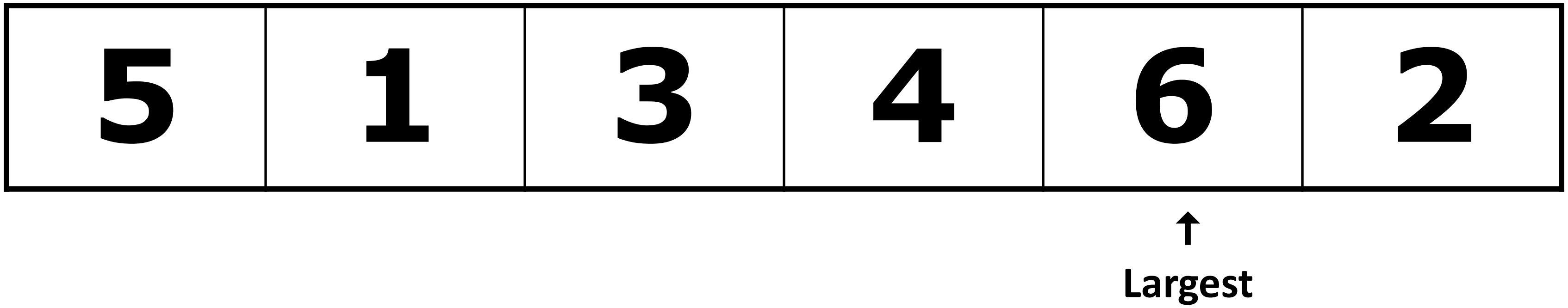
🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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| 5 | 1 | 3 | 4 | 6 | 2 |

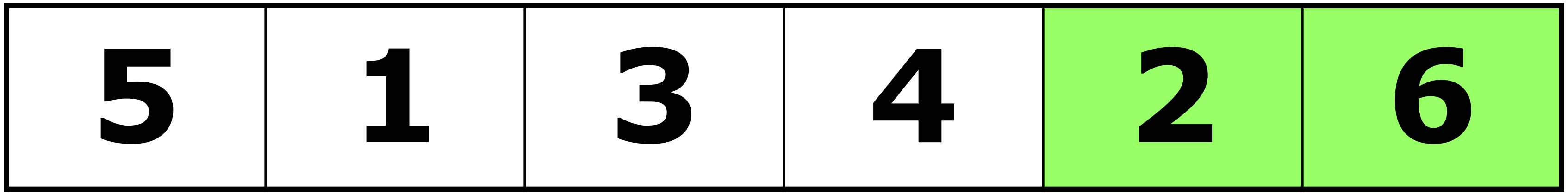
Largest

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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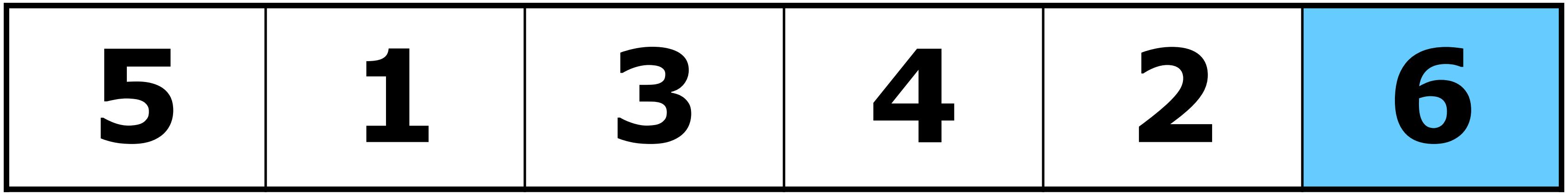
| 5 | 1 | 3 | 4 | 2 | 6 |

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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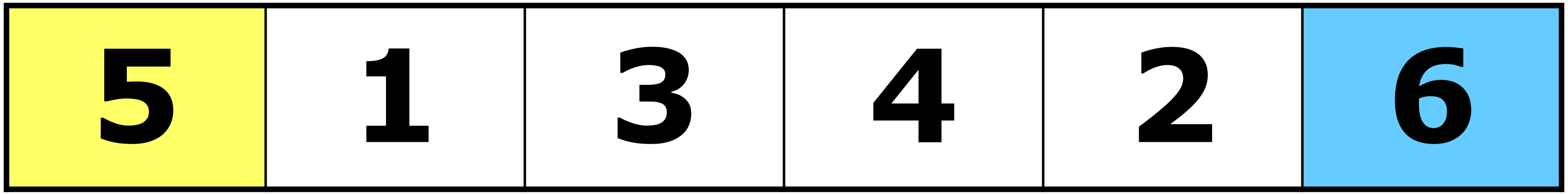
🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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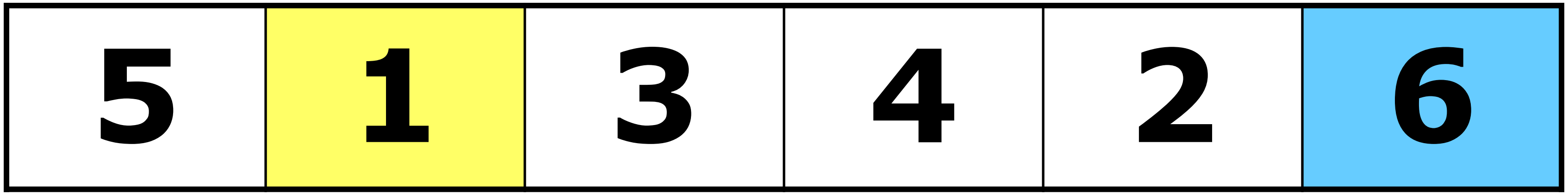
| 5 | 1 | 3 | 4 | 2 | 6 |

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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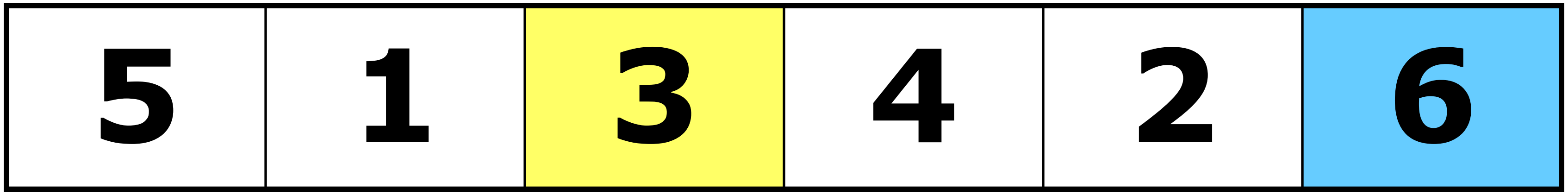
🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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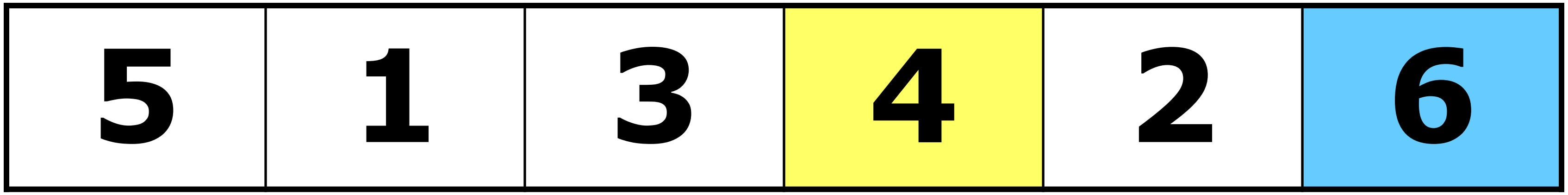
- 🟨 Comparison
- 🟩 Data Movement
- 🟦 Sorted

# Selection Sort



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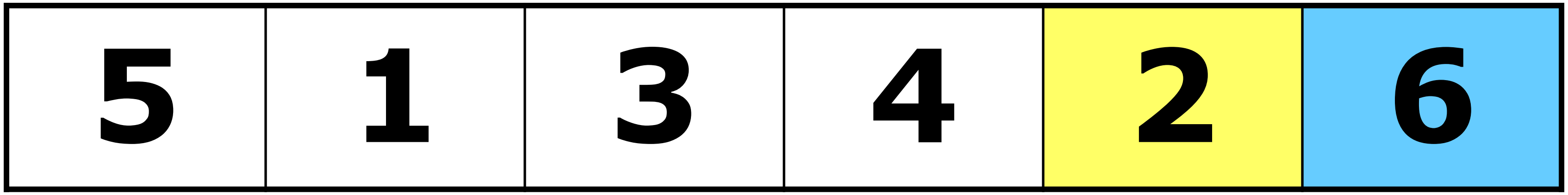
🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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🟨 Comparison

🟩 Data Movement

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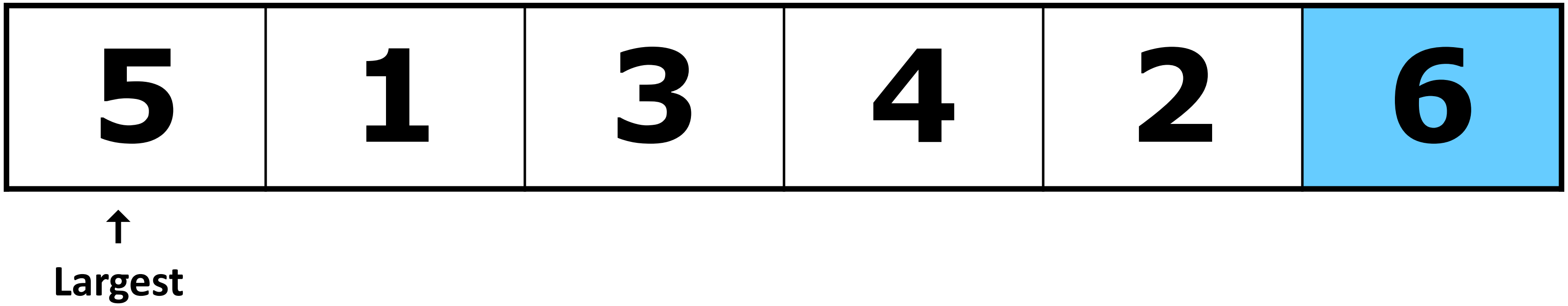
# Selection Sort



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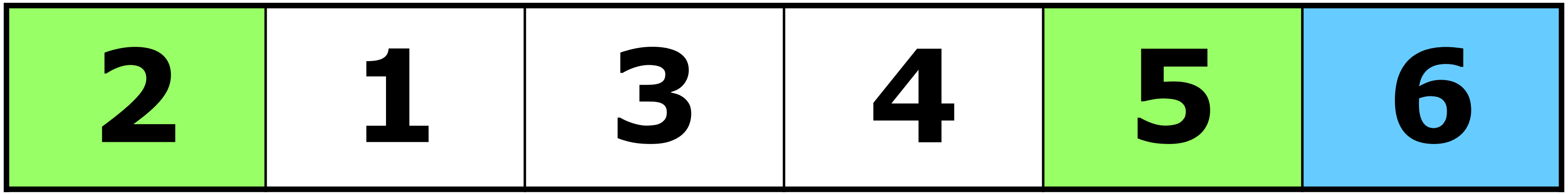
| 5 | 1 | 3 | 4 | 2 | 6 |

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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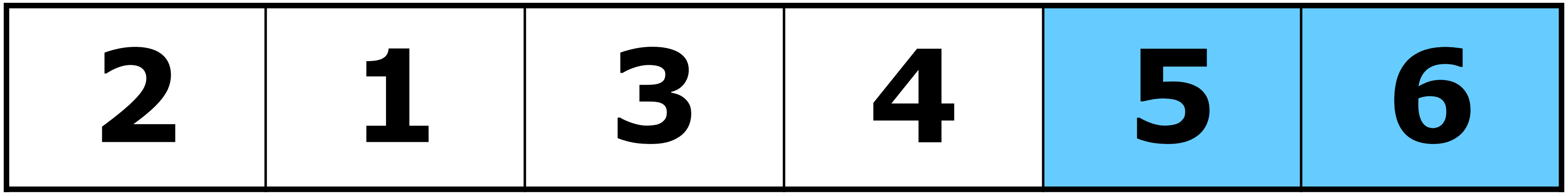
| 🟩 2 | 1 | 3 | 4 | 🟩 5 | 🟦 6 |

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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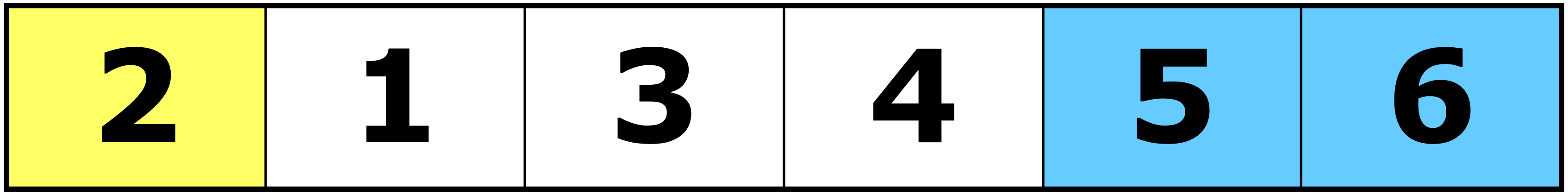
| 2 | 1 | 3 | 4 | 5 | 6 |

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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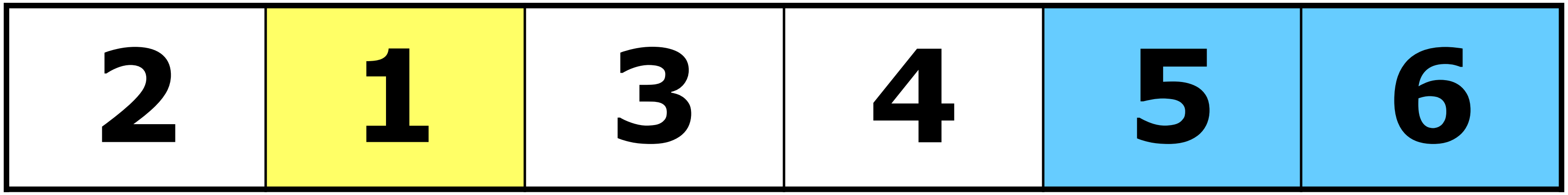
| 2 | 1 | 3 | 4 | 5 | 6 |

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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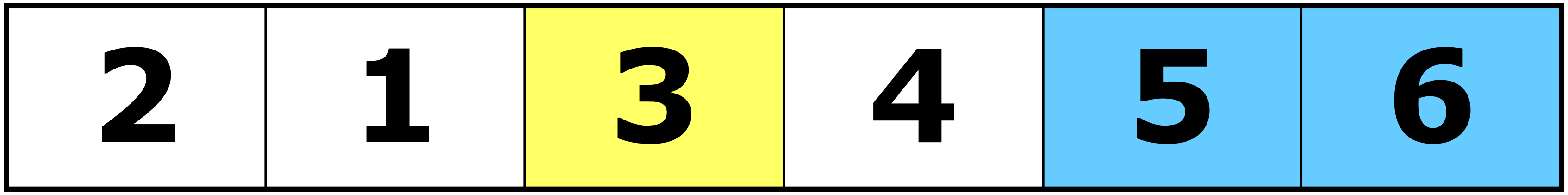
| 2 | 1 | 3 | 4 | 5 | 6 |

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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🟨 Comparison

🟩 Data Movement

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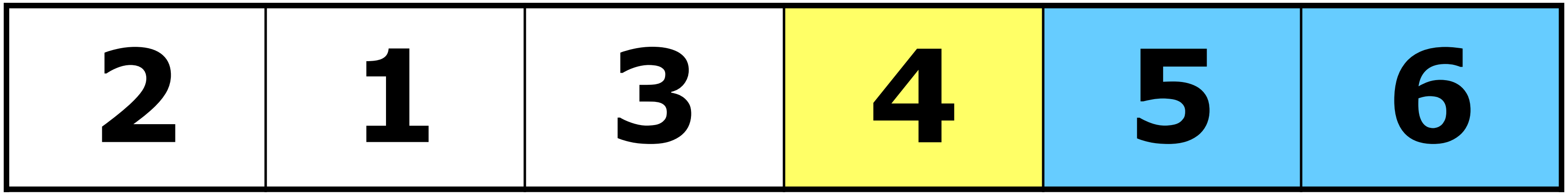
# Selection Sort

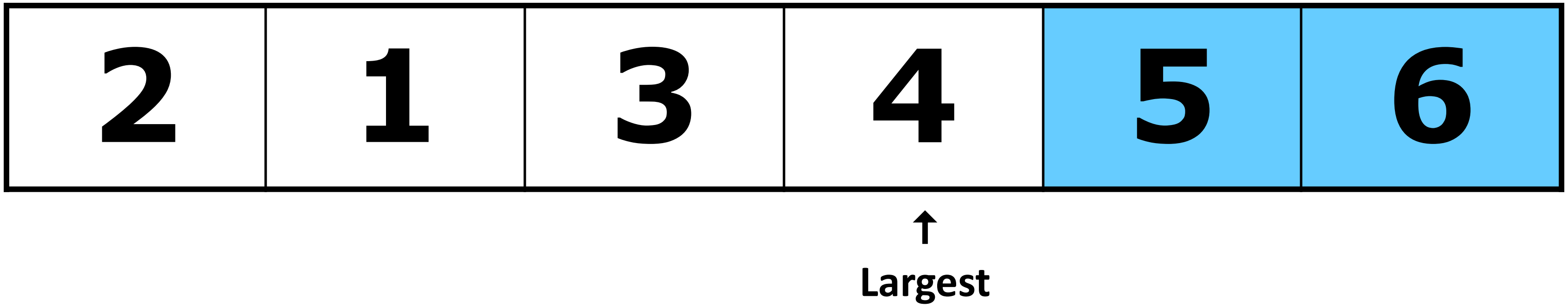


🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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| 2 | 1 | 3 | 4 | 5 | 6 |

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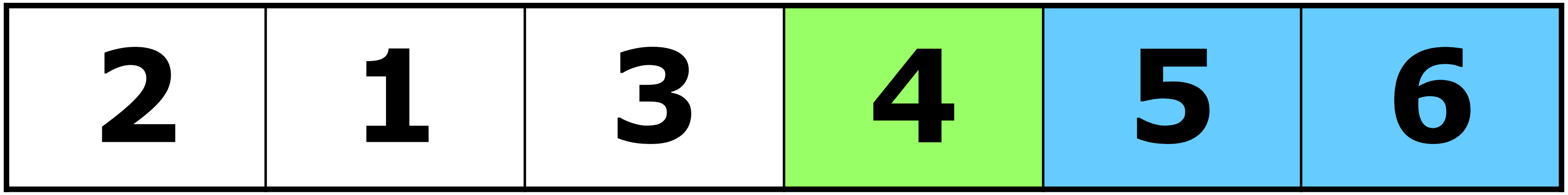
**Largest**

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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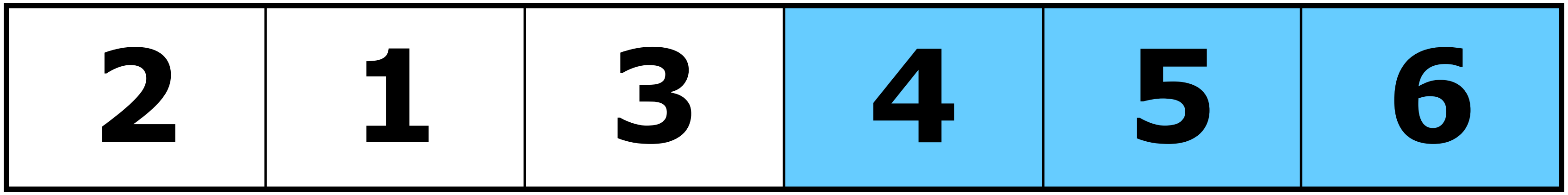
| 2 | 1 | 3 | 4 | 5 | 6 |

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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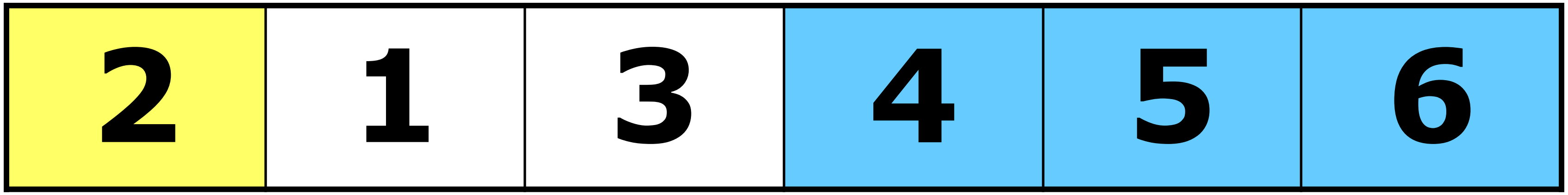
| 2 | 1 | 3 | 4 | 5 | 6 |

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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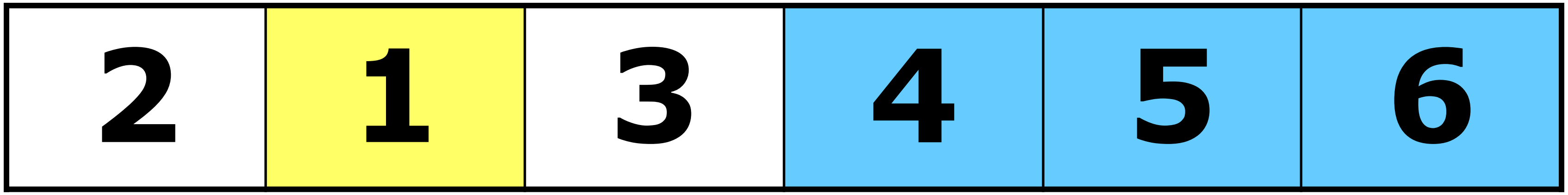
🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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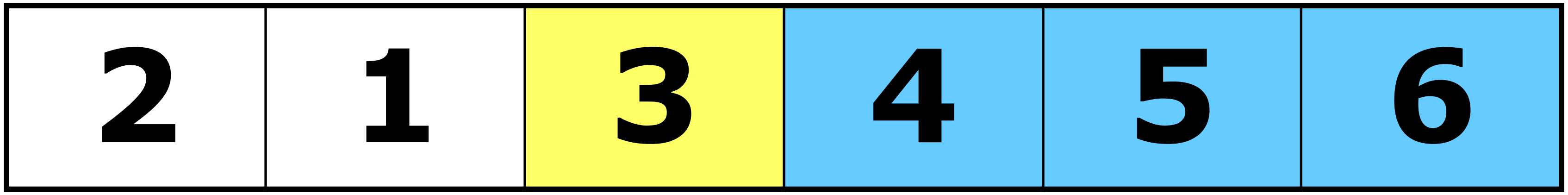
🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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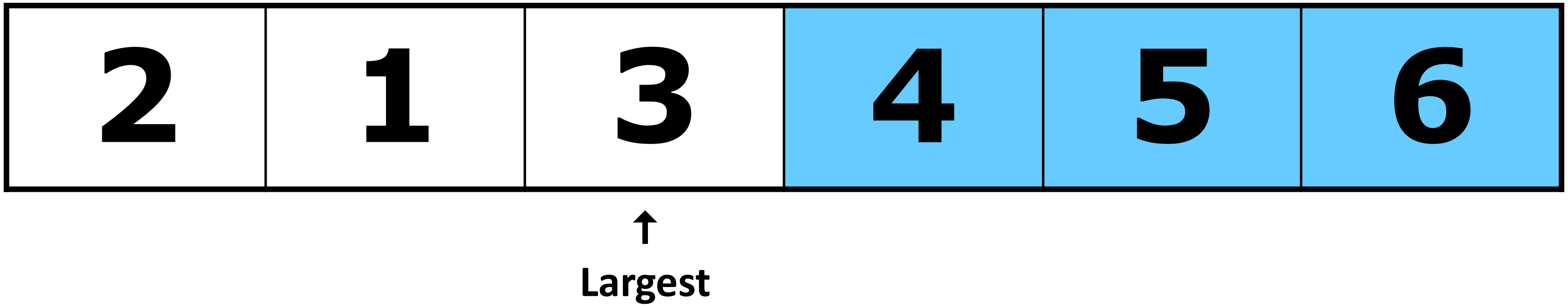
| 2 | 1 | 3 | 4 | 5 | 6 |

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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| 2 | 1 | 3 | 4 | 5 | 6 |

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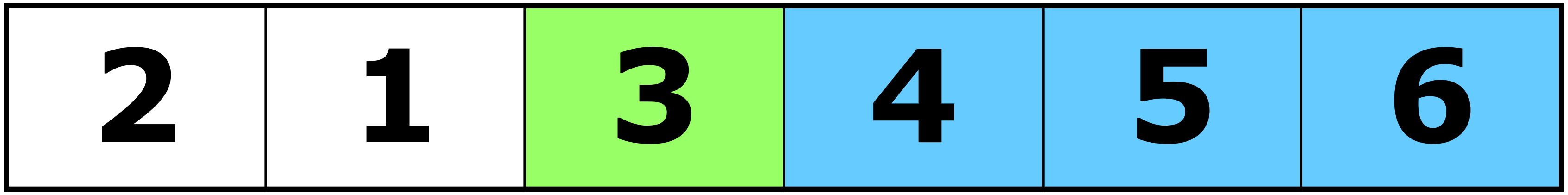
Largest

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



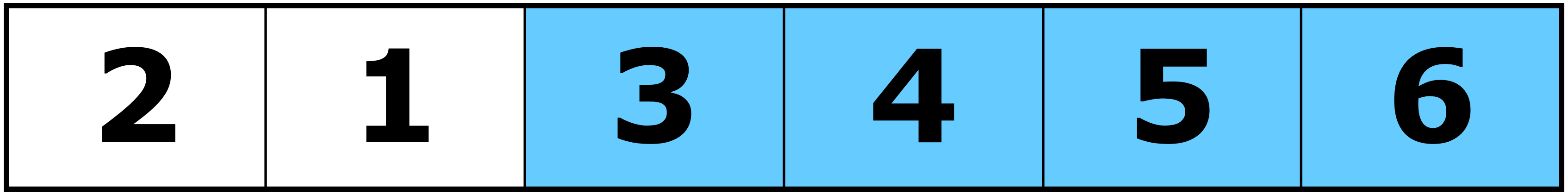
2 1 3 4 5 6

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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| 2 | 1 | 3 | 4 | 5 | 6 |

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| 🟨 | Comparison |
| 🟩 | Data Movement |
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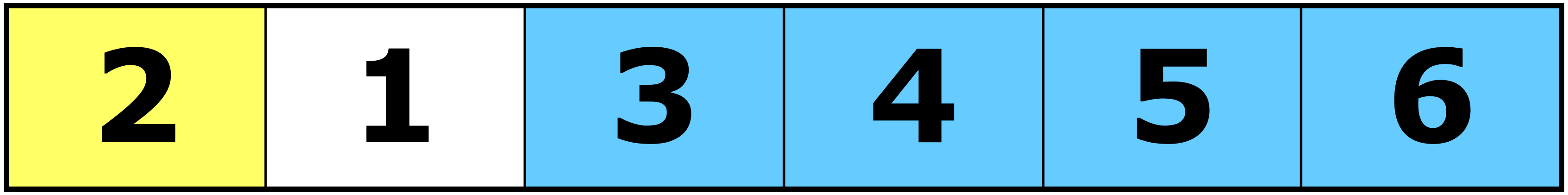
# Selection Sort



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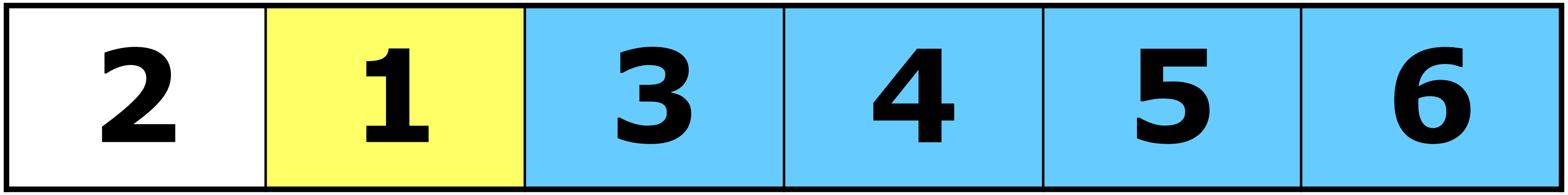
🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



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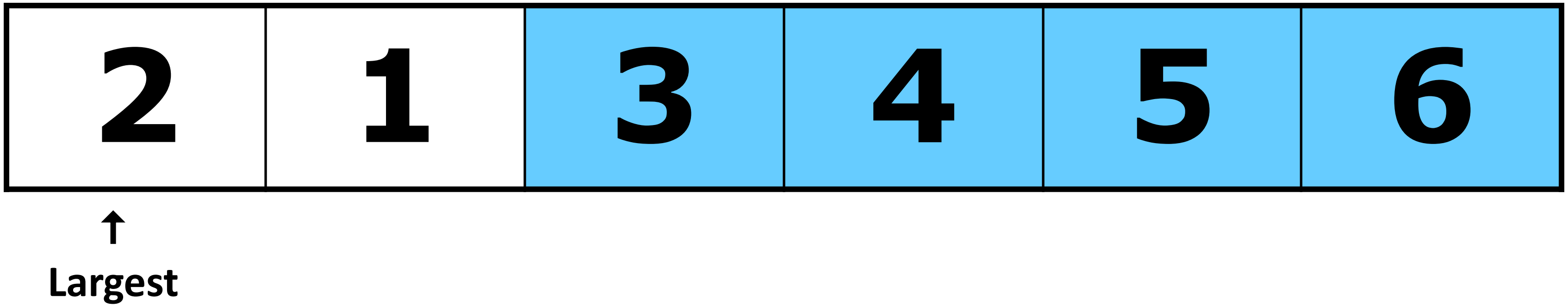
|---|---|---|---|---|---|
| 2 | 1 | 3 | 4 | 5 | 6 |

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort



🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort

| | | | | | |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort

| | | | | | |
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| 1 | 2 | 3 | 4 | 5 | 6 |

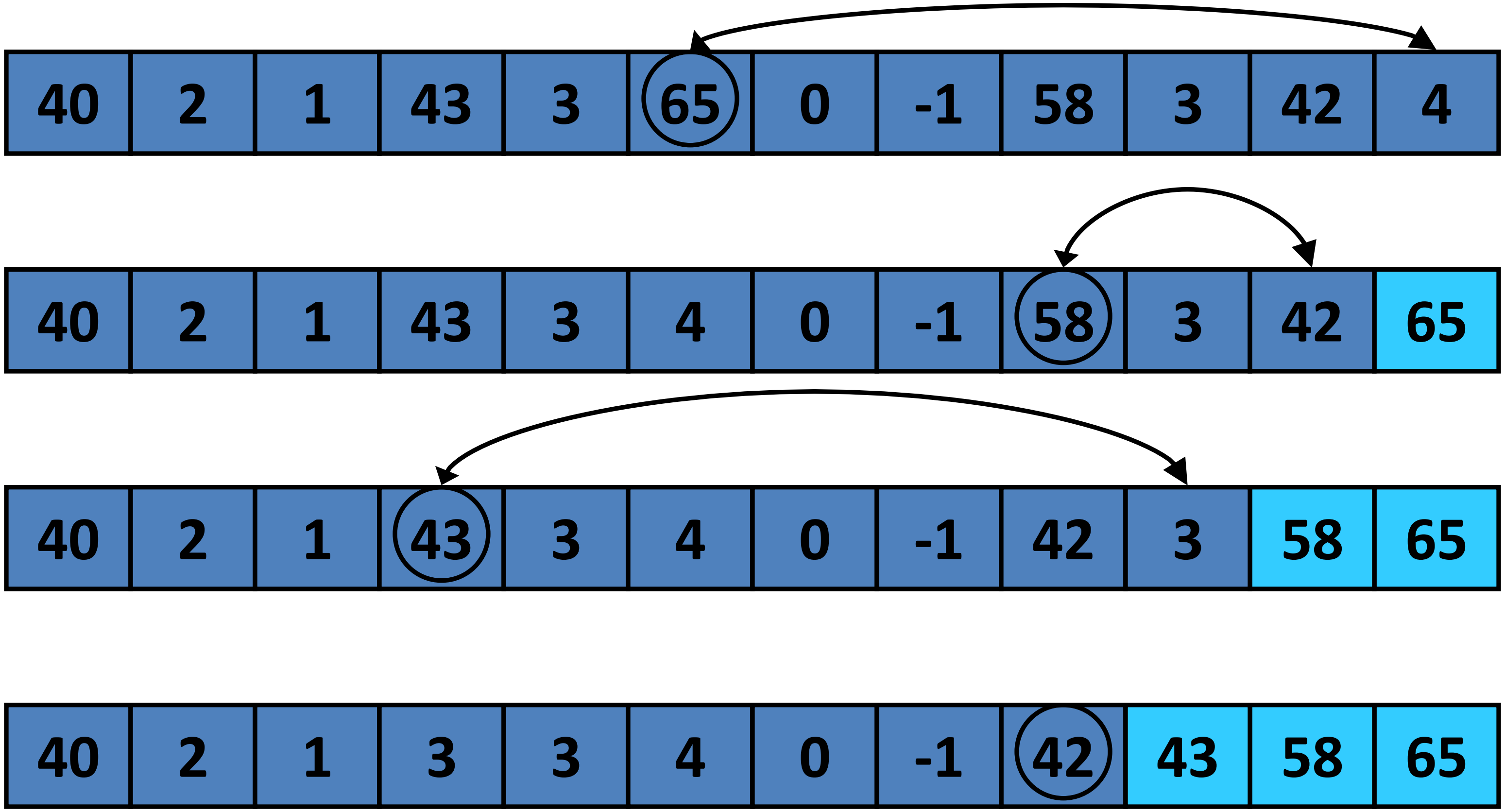
**DONE!**

🟨 Comparison

🟩 Data Movement

🟦 Sorted

# Selection Sort: Example



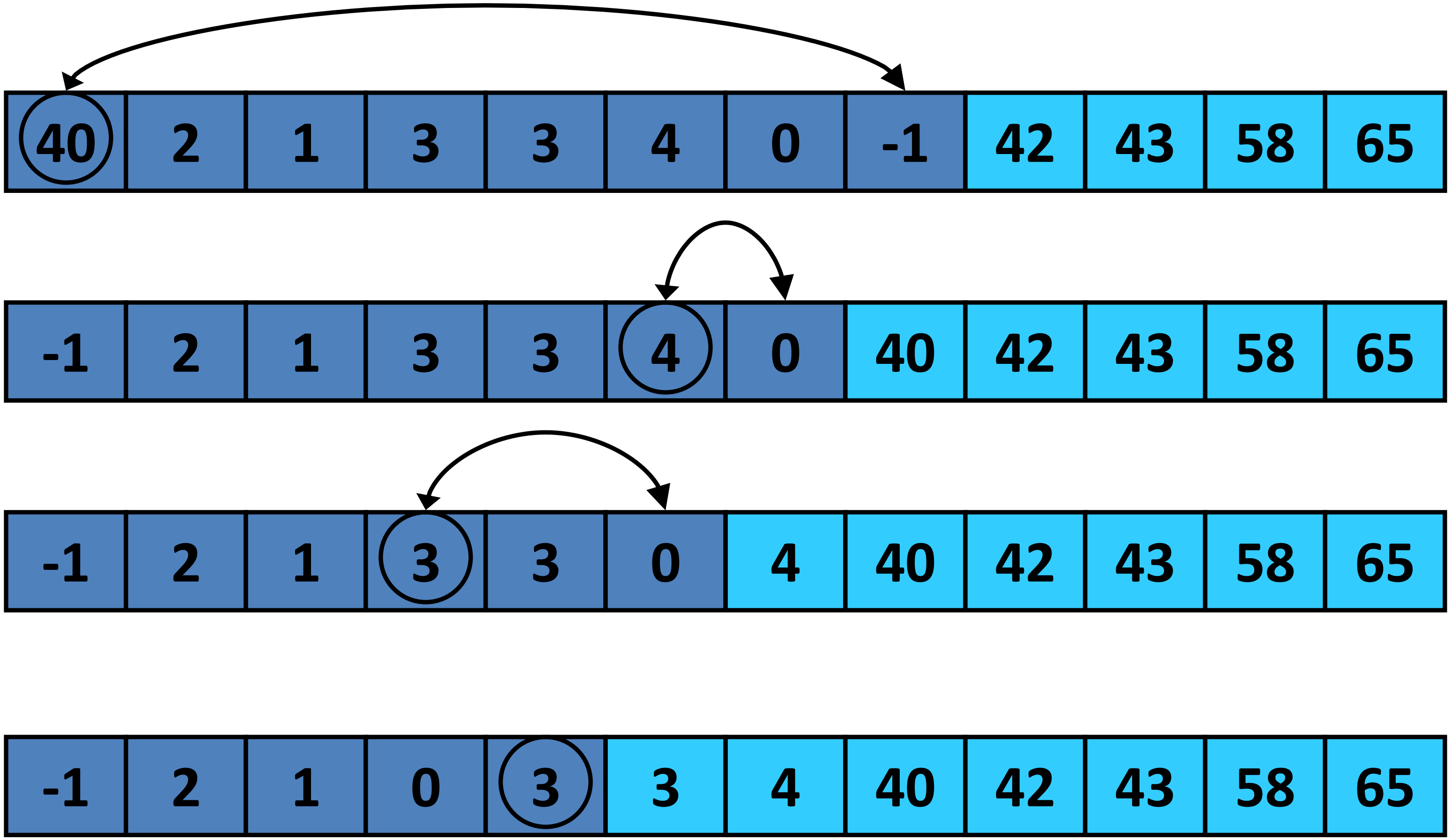
| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 40 | 2 | 1 | 43 | 3 | 65 | 0 | -1 | 58 | 3 | 42 | 4 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 40 | 2 | 1 | 43 | 3 | 4 | 0 | -1 | 58 | 3 | 42 | 65 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 40 | 2 | 1 | 43 | 3 | 4 | 0 | -1 | 42 | 3 | 58 | 65 |

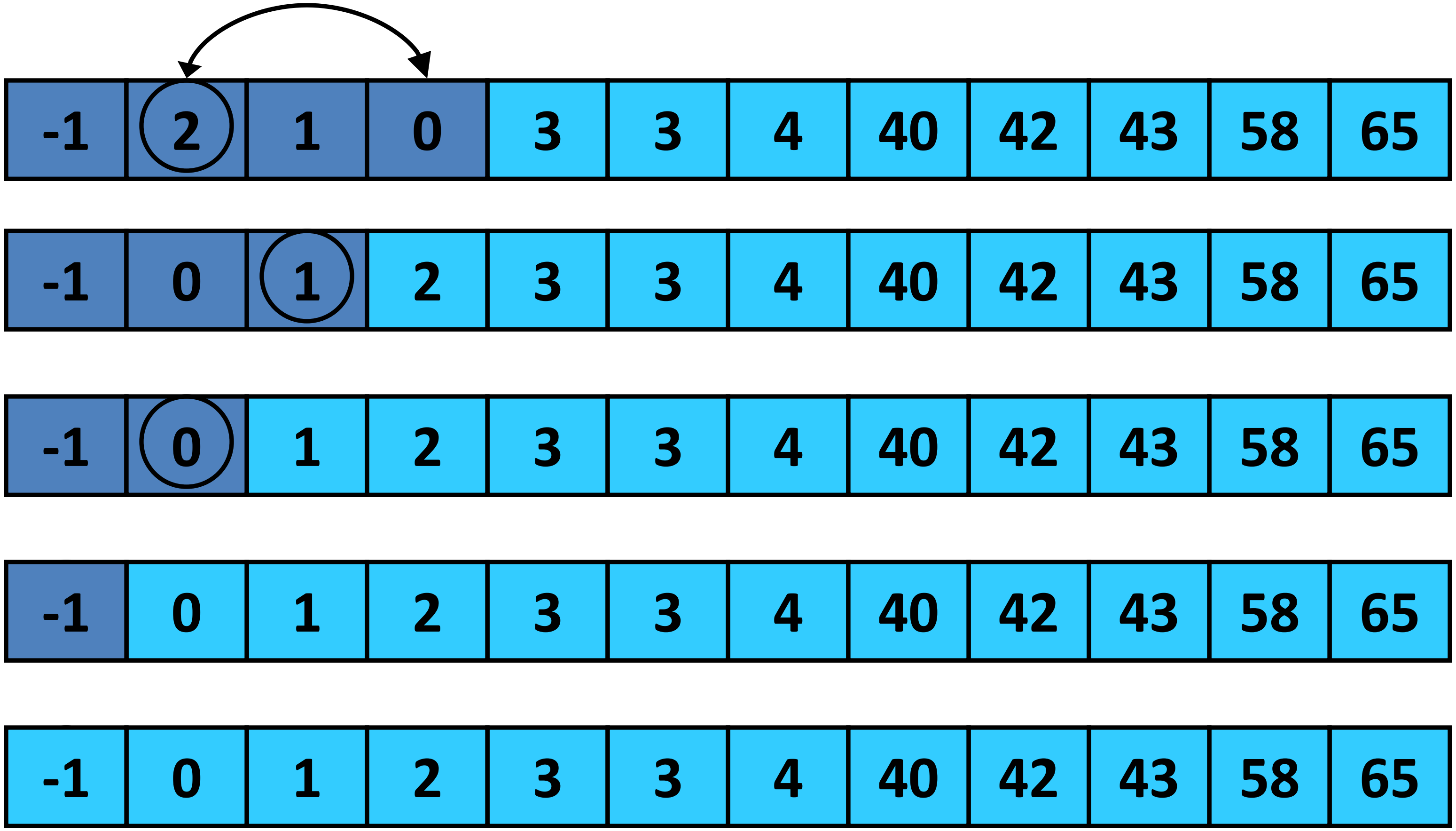
| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 40 | 2 | 1 | 3 | 3 | 4 | 0 | -1 | 42 | 43 | 58 | 65 |

# Selection Sort: Example



| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 40 | 2 | 1 | 3 | 3 | 4 | 0 | -1 | 42 | 43 | 58 | 65 |
| -1 | 2 | 1 | 3 | 3 | 4 | 0 | 40 | 42 | 43 | 58 | 65 |
| -1 | 2 | 1 | 3 | 3 | 0 | 4 | 40 | 42 | 43 | 58 | 65 |
| -1 | 2 | 1 | 0 | 3 | 3 | 4 | 40 | 42 | 43 | 58 | 65 |

# Selection Sort: Example



| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| -1 | 2 | 1 | 0 | 3 | 3 | 4 | 40 | 42 | 43 | 58 | 65 |
| -1 | 0 | 1 | 2 | 3 | 3 | 4 | 40 | 42 | 43 | 58 | 65 |
| -1 | 0 | 1 | 2 | 3 | 3 | 4 | 40 | 42 | 43 | 58 | 65 |
| -1 | 0 | 1 | 2 | 3 | 3 | 4 | 40 | 42 | 43 | 58 | 65 |
| -1 | 0 | 1 | 2 | 3 | 3 | 4 | 40 | 42 | 43 | 58 | 65 |

# Selection Sort: Analysis

- Running time:
  - Worst case: $O(N^2)$
  - Best case: $O(N^2)$

# Radix Sort

- This sort is unusual because it does not directly compare any of the elements

- We instead create a set of buckets and repeatedly separate the elements into the buckets

- On each pass, we look at a different part of the elements

# Radix Sort

- Assuming decimal elements and 10 buckets, we would put the elements into the bucket associated with its units digit

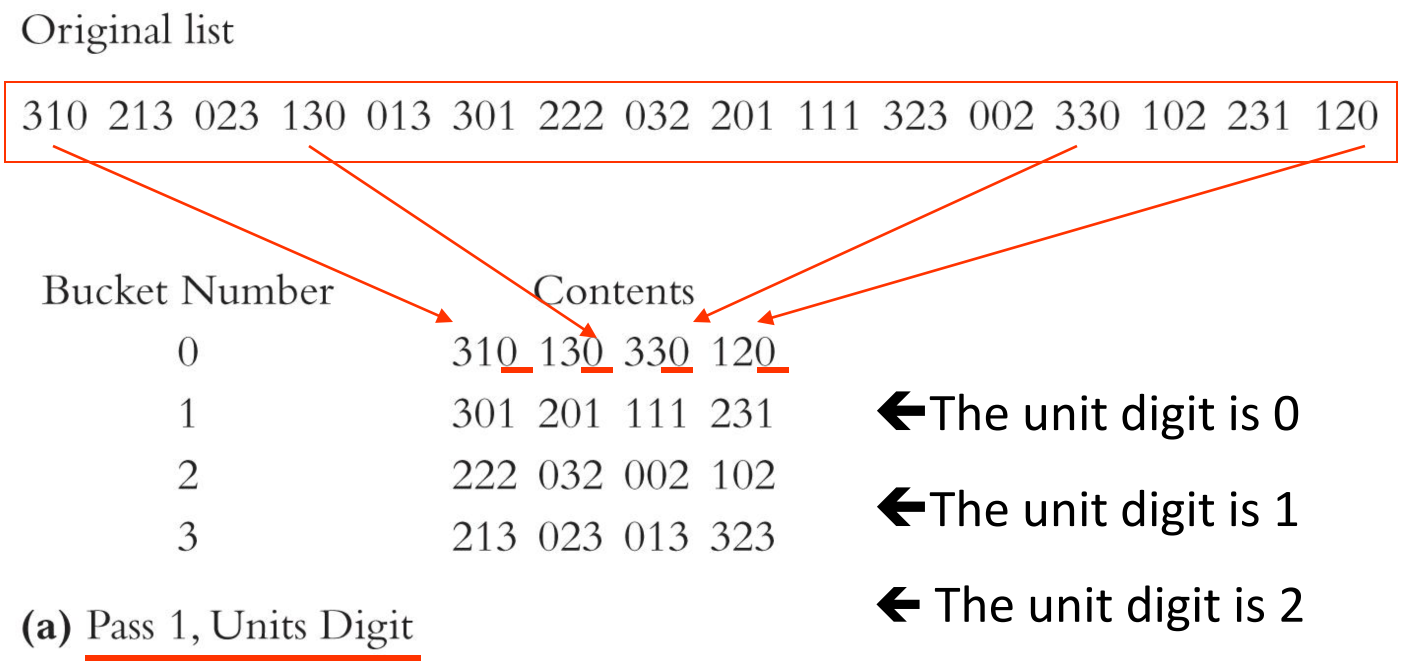
- The buckets are actually queues so the elements are added at the end of the bucket
- At the end of the pass, the buckets are combined in increasing order

# Radix Sort

- On the second pass, we separate the elements based on the “tens” digit, and on the third pass we separate them based on the “hundreds” digit

- Each pass must make sure to process the elements in order and to put the buckets back together in the correct order

# Radix Sort Example



Original list

310 213 023 130 013 301 222 032 201 111 323 002 330 102 231 120

| Bucket Number | Contents |
|---|---|
| 0 | 310 130 330 120 |
| 1 | 301 201 111 231 |
| 2 | 222 032 002 102 |
| 3 | 213 023 013 323 |

(a) Pass 1, Units Digit

FIGURE 4.4 The three passes of a radix sort

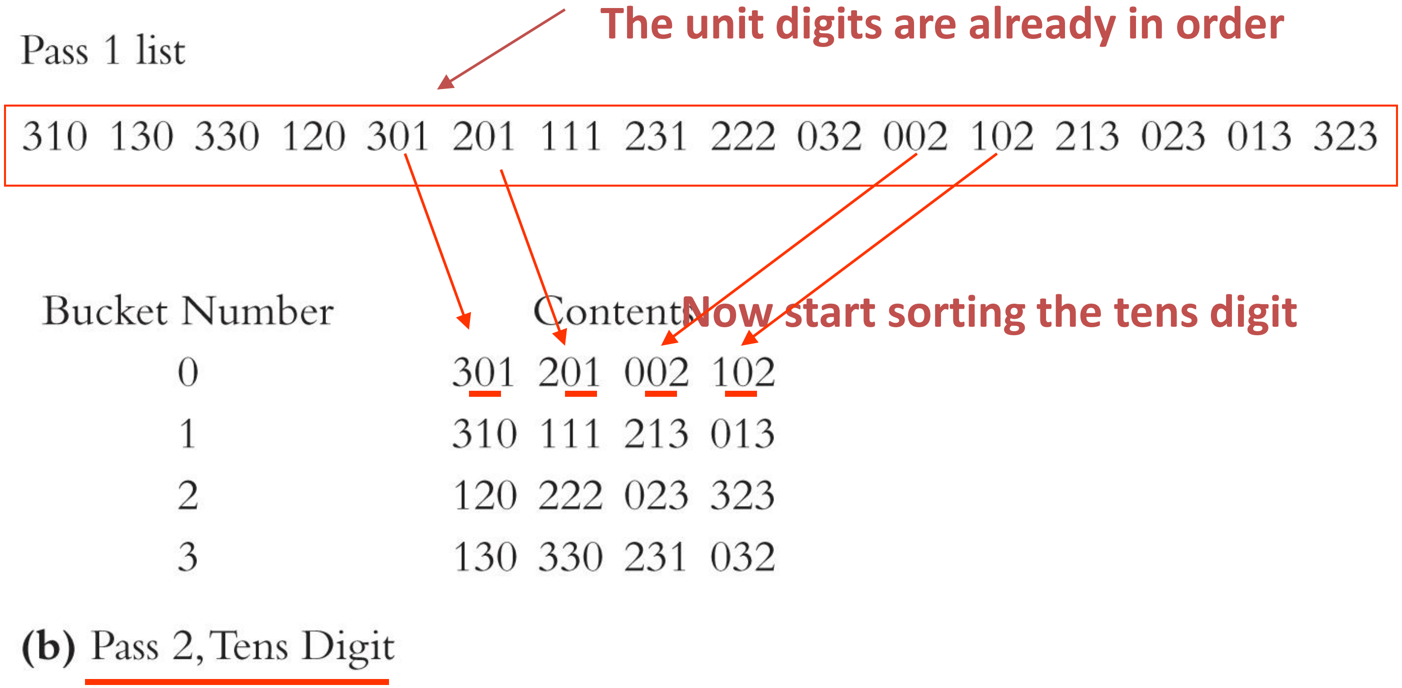
← The unit digit is 0

← The unit digit is 1

← The unit digit is 2

← The unit digit is 3

# Radix Sort Example (continued)



The unit digits are already in order

Pass 1 list

310 130 330 120 301 201 111 231 222 032 002 102 213 023 013 323

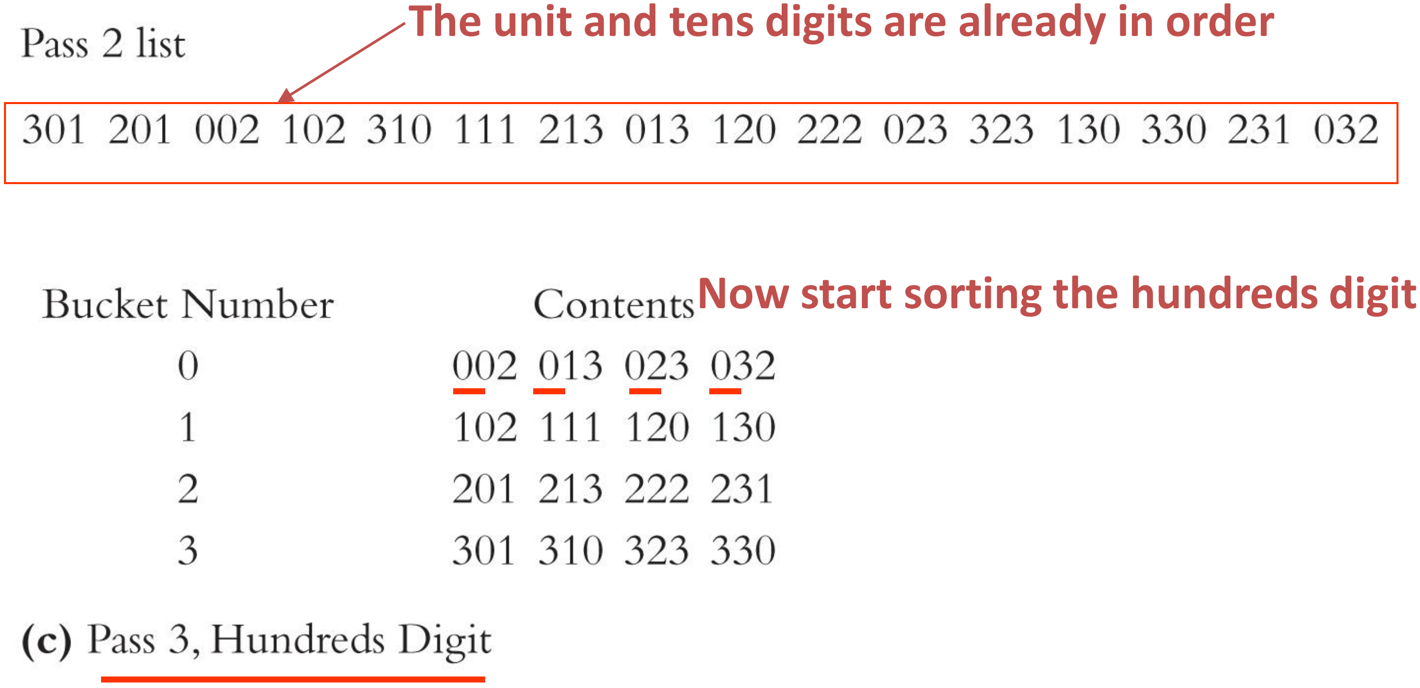
Now start sorting the tens digit

| Bucket Number | Contents |
|---|---|
| 0 | 301 201 002 102 |
| 1 | 310 111 213 013 |
| 2 | 120 222 023 323 |
| 3 | 130 330 231 032 |

**(b)** Pass 2, Tens Digit

**FIGURE 4.4**
The three passes of a radix sort

# Radix Sort Example (continued)



Pass 2 list

The unit and tens digits are already in order

301 201 002 102 310 111 213 013 120 222 023 323 130 330 231 032

Now start sorting the hundreds digit

| Bucket Number | Contents |
|---|---|
| 0 | 002 013 023 032 |
| 1 | 102 111 120 130 |
| 2 | 201 213 222 231 |
| 3 | 301 310 323 330 |

(c) Pass 3, Hundreds Digit

FIGURE 4.4
The three passes of a radix sort

Values in the buckets are now in order

# The Algorithm to sort a set of numeric keys

# of digits of the longest key

# of elemnts in the list

```
shift = 1
for pass = 1 to keySize do
    for entry = 1 to N do
        bucketNumber = (list[entry] / shift) mod 10
        Append( bucket[bucketNumber], list[entry] )
    end for
    list = CombineBuckets()
    shift = shift * 10
end for
```

quotient

remainder

bucketNumber: lies between 0 and 9

# Radix Sort Analysis

- Each element is examined once for each of the digits it contains, so if the elements have at most *M* digits and there are *N* elements this algorithm has order O(*M***N*)
- This means that sorting is linear based on the number of elements
- Why then isn’t this the only sorting algorithm used?

# Radix Sort Analysis

- Though this is a very time efficient algorithm it is not space efficient
- If an array is used for the buckets and we have ***B* buckets**, we would need ***N**B* extra memory locations** because it's possible for all of the elements to wind up in one bucket
- If linked lists are used for the buckets you have the overhead of pointers

# Radix Sort

- **Radix** is the base of a number system or logarithm.

- Radix sort is a multiple pass distribution sort.
  - It distributes each item to a bucket according to part of the item's key.

  - After each pass, items are collected from the buckets, keeping the items in order, then redistributed according to the next most significant part of the key.

- This sorts keys digit-by-digit (hence referred to as digital sort), or, if the keys are strings that we want to sort alphabetically, it sorts character-by-character.
- It was used in card-sorting machines.

- Radix sort uses bucket or count sort as the stable sorting algorithm, where the initial relative order of equal keys is unchanged.

Integer representations can be used to represent strings of characters as well as integers.
So, anything that can be represented by integers can be rearranged to be in order by a radix sort.

Execution of Radix sort is in $\Theta(d(n + k))$, where n is instance size or number of elements that need to be sorted. k is the number of buckets that can be generated and d is the number of digits in the element, or length of the keys.

# Radix sort

- There's also a bottom-up version of bucket sort called radix sort, which is easiest to state for character strings of the same length p:
  - for i from p down to 1
  - for each string s, assign s to the bucket corresponding to its ith character
  - concatenate the buckets into an output list
  - clear each bucket
- For b buckets, the time is Q(b+n) per iteration

# Radix sort details

- Concatenation is easiest if linked lists are used for the individual buckets.
- It is important that distribution into buckets be stable – elements should appear in the buckets in the order of the original input.
- If strings have different lengths, they can be padded (explicitly or implicitly) with nulls on the right

# Radix sort analysis

- Note that if p and b are independent of n, then radix sort has $\Theta(n)$ time complexity
- However if p is independent of n, then there can be at most $\Theta(b^p)$ distinct strings.
- So if all strings are distinct, then n is $O(b^p)$, so p is $\Omega(\log n)$.
- And thus the time complexity is $\Omega(n \log n)$

# Selection using bucket sort

- Top-down bucket sort can easily be converted to a selection algorithm
- To find the kth smallest item, distribute the items into buckets, counting the number of buckets
- Then select recursively from the appropriate bucket, replacing k by a value that depends on the counts of the preceding buckets

# Radix sort example

- To sort:
  - 123, 12, 313, 321, 212, 112, 221, 132, 131
- Pass 1 assignment to buckets:
  - 0:
  - 1: 321, 221, 131
  - 2: 12, 212, 112, 132
  - 3: 123, 313
- Concatenated result
  - 321, 221, 131, 12, 212, 112, 132, 123, 313

# Pass 2

- From previous pass
  - 321, 221, 131, 212, 112, 132, 123, 313
- Pass 2 assignment to buckets:
  - 0:
  - 1: 12, 212, 112, 313
  - 2: 321, 221, 123
  - 3: 131, 132
- Concatenated result
  - 12, 212, 112, 313, 321, 221, 123, 131, 132

# Pass 3

- From previous pass
  - 12, 212, 112, 313, 321, 221, 123, 131, 132
- Pass 3 assignment to buckets:
  - 0: 12
  - 1: 112, 123, 131, 132
  - 2: 212, 221
  - 3: 313, 321
- Concatenated result
  - 12, 112, 123, 131, 132, 212, 221, 313, 321

# Classification of Radix Sort

Radix sort is classified based on how it works internally:

- **least significant digit (LSD) radix sort:** processing starts from the least significant digit and moves towards the most significant digit.
- **most significant digit (MSD) radix sort:** processing starts from the most significant digit and moves towards the least significant digit. This is recursive. It works in the following way:

- If we are sorting strings, we would create a bucket for ‘a’,’b’,’c’ upto ‘z’.

- After the first pass, strings are roughly sorted in that any two strings that begin with different letters are in the correct order.
- If a bucket has more than one string, its elements are recursively sorted (sorting into buckets by the next most significant character).
- Contents of buckets are concatenated.

- The differences between LSD and MSD radix sorts are
  - In MSD, if we know the minimum number of characters needed to distinguish all the strings, we can only sort these number of characters. So, if the strings are long, but we can distinguish them all by just looking at the first three characters, then we can sort 3 instead of the length of the keys.

# Classification of Radix Sort…contd

- LSD approach requires padding short keys if key length is variable, and guarantees that all digits will be examined even if the first 3-4 digits contain all the information needed to achieve sorted order.
- MSD is recursive. LSD is non-recursive.
- MSD radix sort requires much more memory to sort elements. LSD radix sort is the preferred implementation between the two.

- MSD recursive radix sorting has applications to parallel computing, as each of the sub-buckets can be sorted independently of the rest.
- Each recursion can be passed to the next available processor.

The Postman's sort is a variant of MSD radix sort where attributes of the key are described so the algorithm can allocate buckets efficiently. This is the algorithm used by letter-sorting machines in the post office: first states, then post offices, then routes, etc. The smaller buckets are then recursively sorted.

# Example of LSD-Radix Sort

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 12 | 34 | 42 | 32 | 44 | 41 | 34 | 11 | 32 | 23 | 87 | 50 | 77 | 58 | 08 |

Input is an array of 15 integers. For integers, the number of buckets is 10, from 0 to 9. The first pass distributes the keys into buckets by the least significant digit (LSD). When the first pass is done, we have the following.

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 50 | 41 | 12 | 23 | 34 |  |  | 87 | 58 |  |
|  | 11 | 42 |  | 44 |  |  | 77 | 08 |  |
|  |  | 32 |  | 34 |  |  |  |  |  |
|  |  | 32 |  |  |  |  |  |  |  |

# Example of LSD-Radix Sort…contd

We collect these, keeping their relative order:

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 41 | 11 | 12 | 42 | 32 | 32 | 23 | 34 | 44 | 34 | 87 | 77 | 58 | 08 |

Now we distribute by the next most significant digit, which is the highest digit in our example, and we get the following.

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 08 | 11 | 23 | 32 | 41 | 50 | | 77 | 87 | |
| | 12 | | 32 | 42 | 58 | | | | |
| | | | 34 | 44 | | | | | |
| | | | 34 | | | | | | |

When we collect them, they are in order.

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 08 | 11 | 12 | 23 | 32 | 32 | 34 | 34 | 41 | 42 | 44 | 50 | 58 | 77 | 87 |

# Advantages and Disadvantages

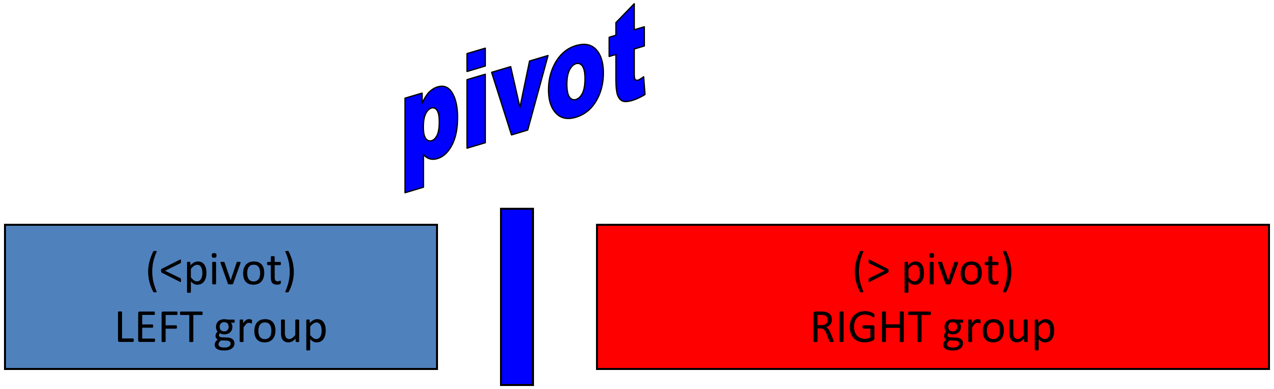
- **Advantages**
  - Radix and bucket sorts are stable, preserving existing order of equal keys.
  - They work in linear time, unlike most other sorts. In other words, they do not bog down when large numbers of items need to be sorted. Most sorts run in O(n log n) or O(n^2) time.
  - The time to sort per item is constant, as no comparisons among items are made. With other sorts, the time to sort per time increases with the number of items.

  - Radix sort is particularly efficient when you have large numbers of records to sort with short keys.

- **Drawbacks**
  - Radix and bucket sorts do not work well when keys are very long, as the total sorting time is proportional to key length and to the number of items to sort.
  - They are not “in-place”, using more working memory than a traditional sort.

# Quicksort Concept

- Basic Concept: divide and conquer
- Select a pivot and split the data into two groups: (< pivot) and (> pivot):



- Recursively apply Quicksort to the subgroups

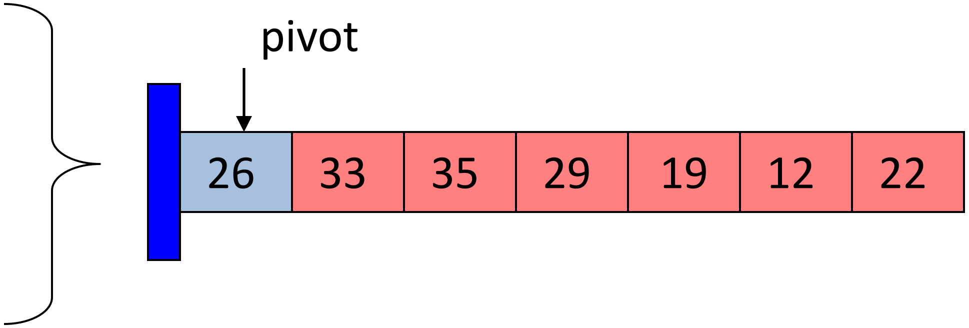
# Quicksort Start

Start with all data in an array, and consider it unsorted

Unsorted Array

# Quicksort Step 1

Step 1, select a pivot
(it is arbitrary)



pivot

|  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|
| 26 | 33 | 35 | 29 | 19 | 12 | 22 |

We will select the first element, as presented in the original algorithm by C.A.R. Hoare in 1962.

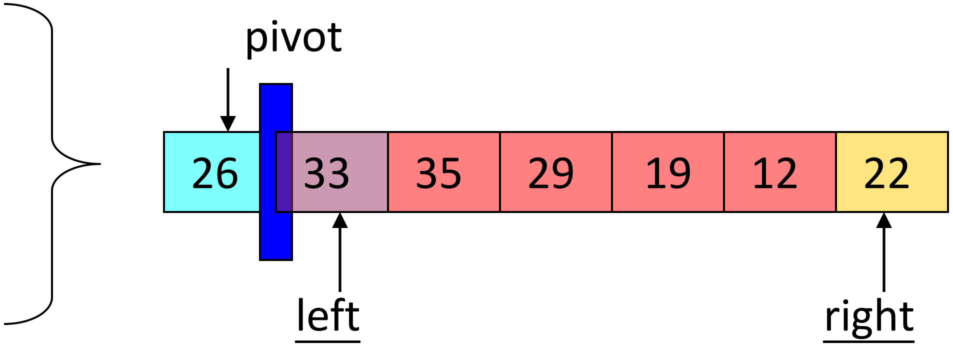
# Quicksort Step 2

Step 2, start process of dividing data into LEFT and RIGHT groups:

The LEFT group will have elements less than the pivot.

The RIGHT group will have elements greater that the pivot.

Use markers left and right

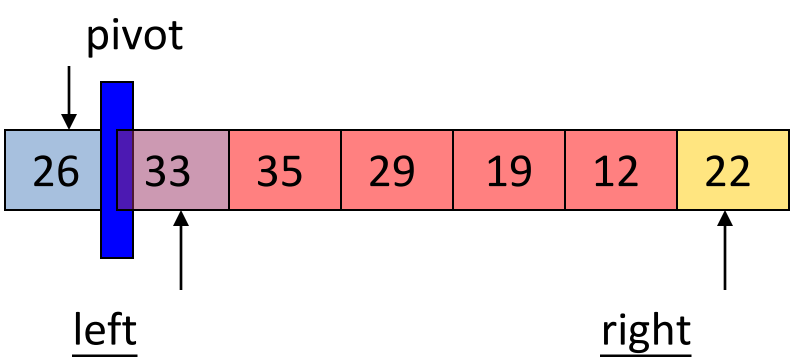


# Quicksort Step 3

Step 3,
If left element belongs
to LEFT group, then increment
left index.

If right index element belongs
to RIGHT, then decrement right.

Exchange when you find
elements that belong to the other
group.



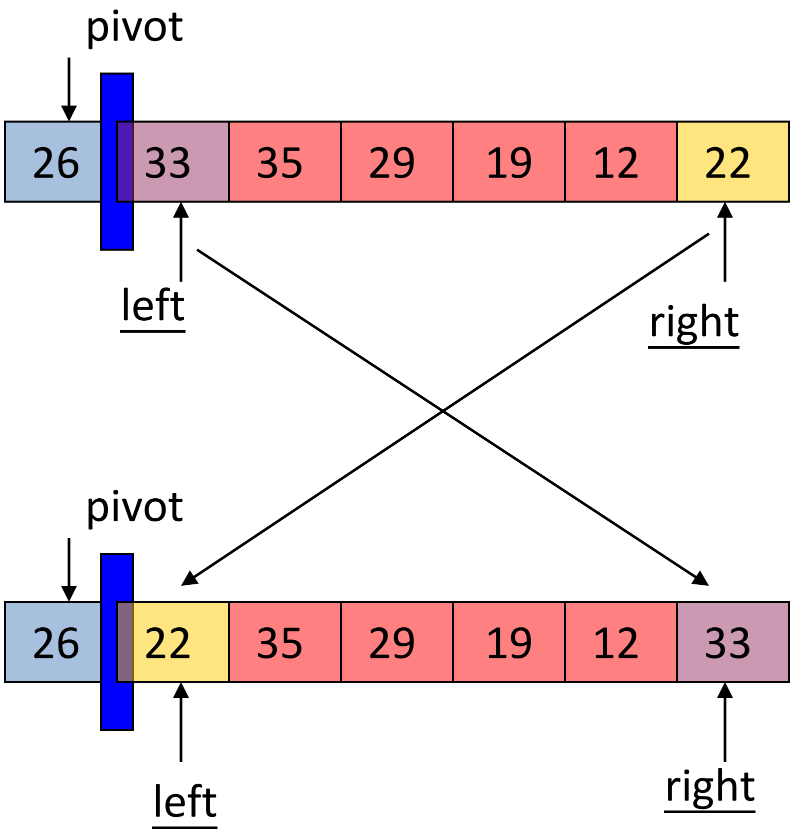
# Quicksort Step 4

Step 4:

Element 33 belongs to RIGHT group.

Element 22 belongs to LEFT group.

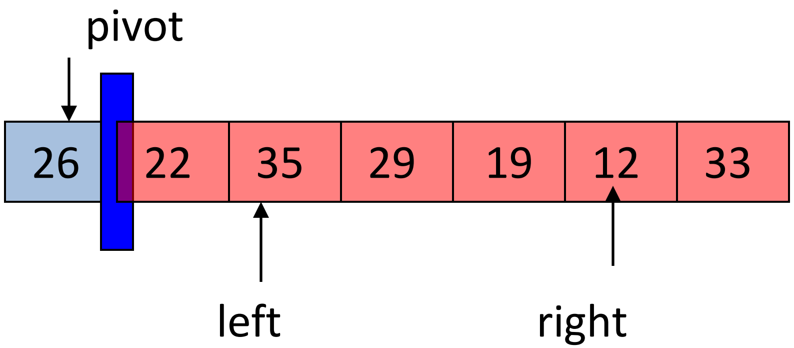
Exchange the two elements.



# Quicksort Step 5

Step 5:

After the exchange, increment left marker, decrement right marker.



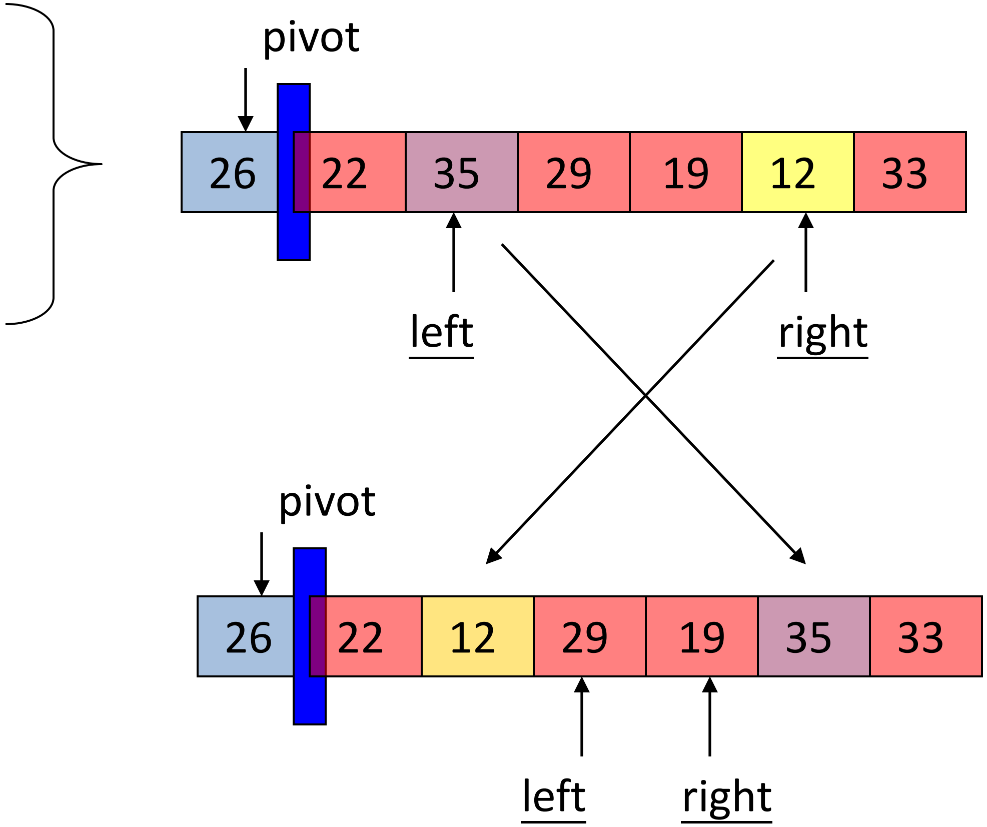
# Quicksort Step 6

Step 6:

Element 35 belongs to RIGHT group.

Element 12 belongs to LEFT group.

Exchange, increment left, and decrement right.



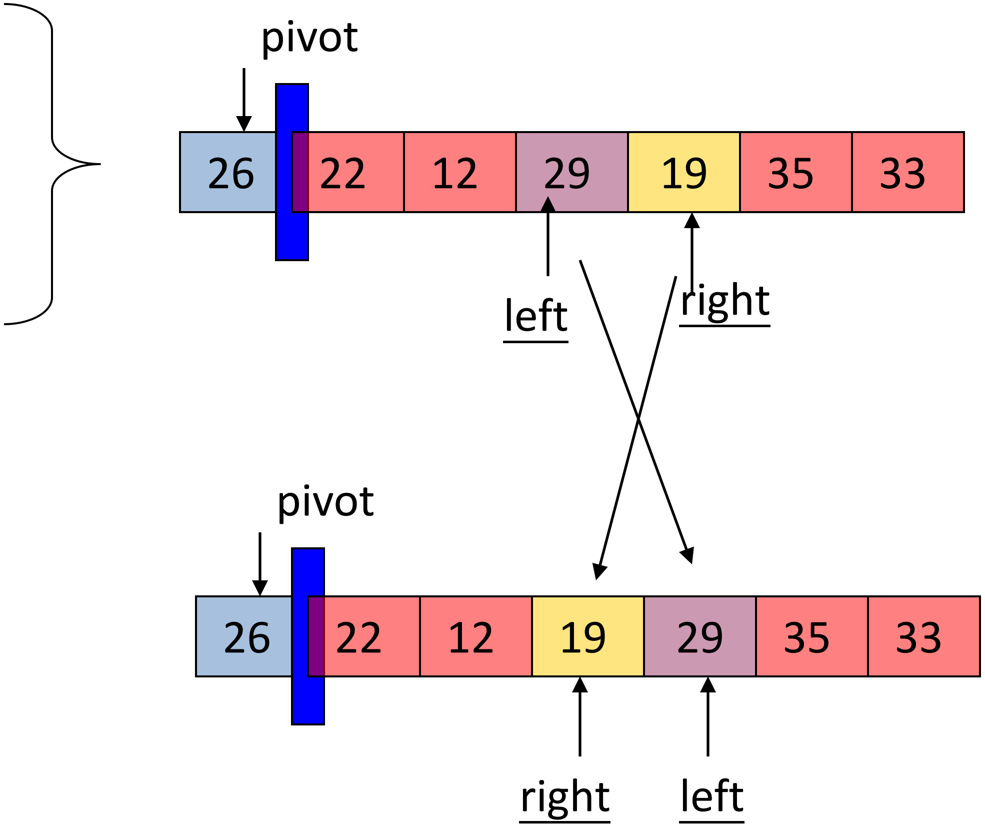
# Quicksort Step 7

Step 7:

Element 29 belongs to RIGHT.

Element 19 belongs to LEFT.

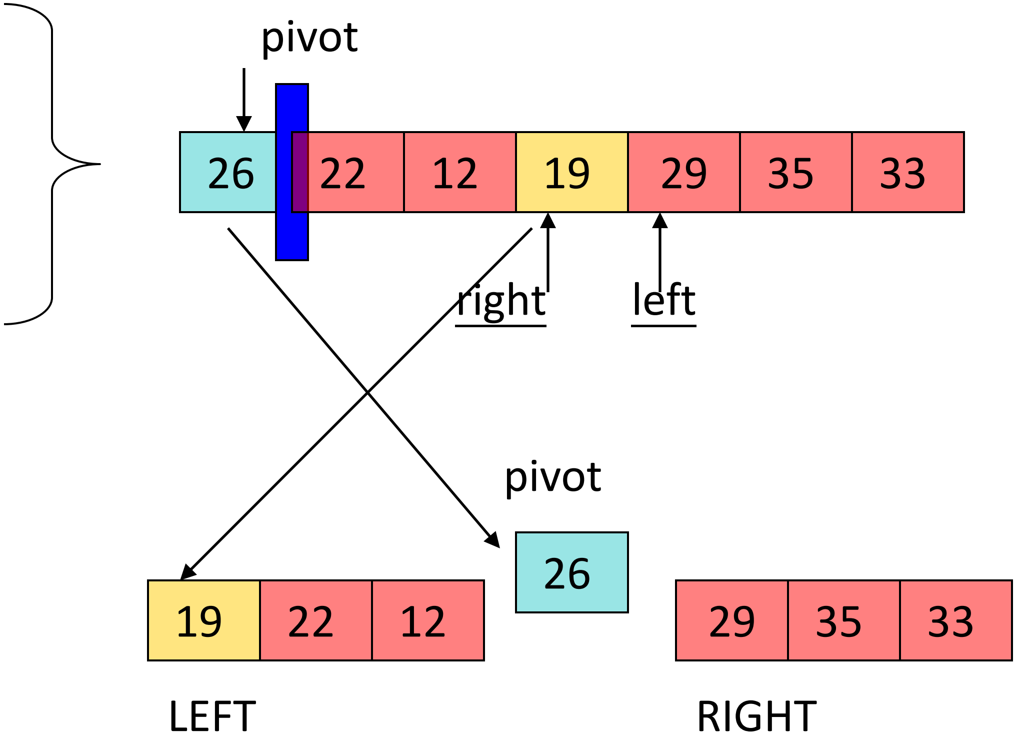
Exchange, increment left, decrement right.



# Quicksort Step 8

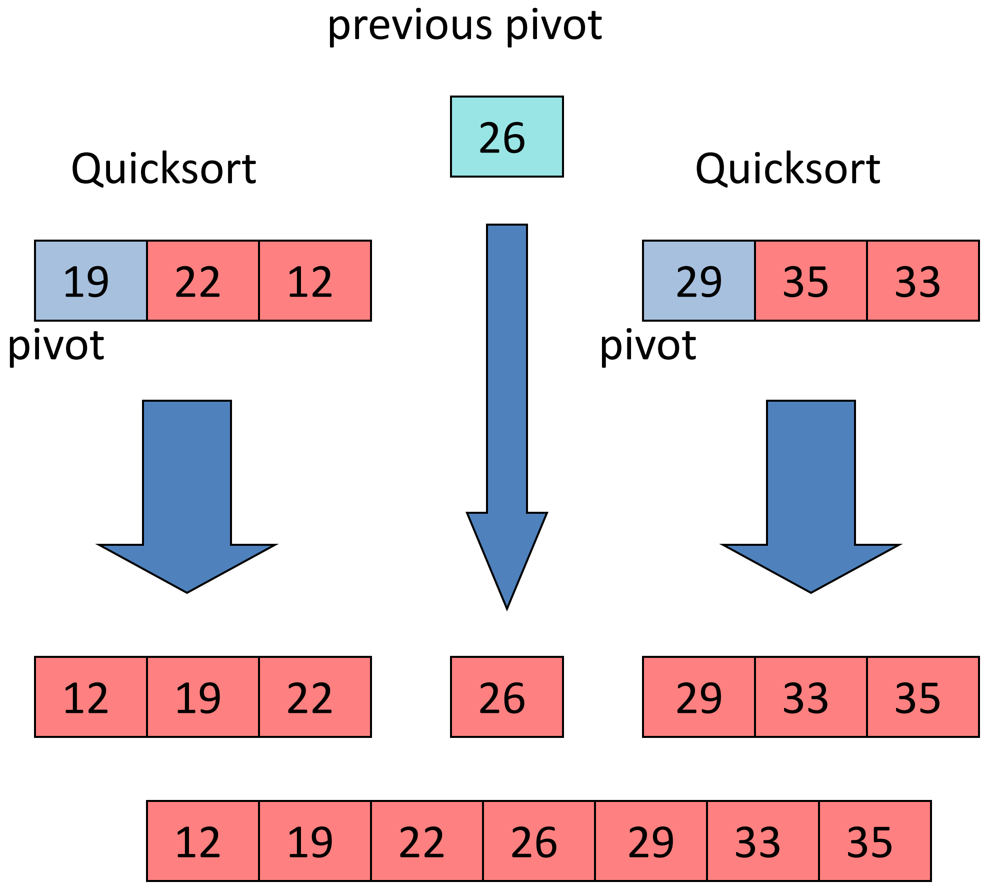
Step 8:
When the left and right markers pass each other, we are done with the partition task.

Swap the right with pivot.



# Quicksort Step 9

Step 9:
Apply Quicksort
to the LEFT and
RIGHT groups,
recursively.



previous pivot

26

Quicksort

19 | 22 | 12

pivot

Quicksort

29 | 35 | 33

pivot

Assemble parts when done

12 | 19 | 22

26

29 | 33 | 35

12 | 19 | 22 | 26 | 29 | 33 | 35

# Quicksort Efficiency

The partitioning of an array into two parts is O(n)

The number of recursive calls to Quicksort depends on how many times we can split the array into two groups.
On average this is O ($\log_2$ n)

The overall Quicksort efficiency is O(n) = n $\log_2$n

What is the worst-case efficiency?
Compare this to the worst case for the heapsort.

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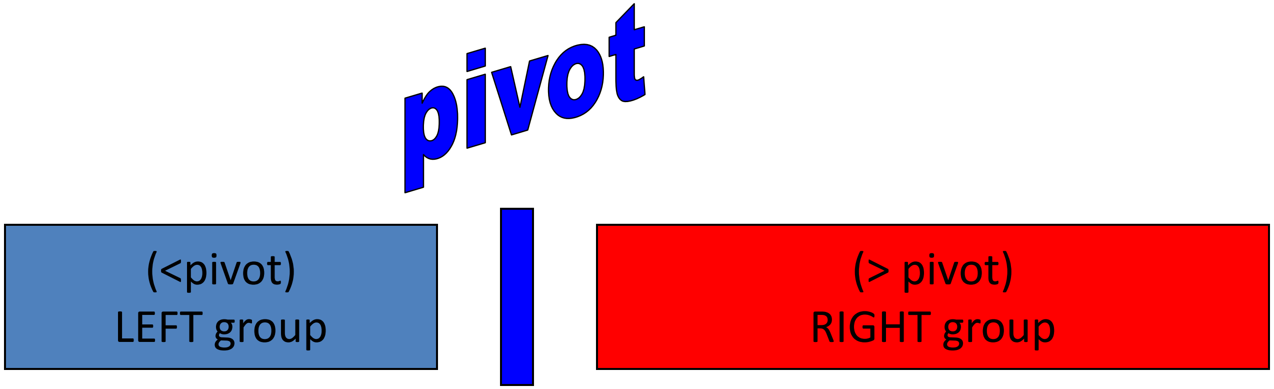

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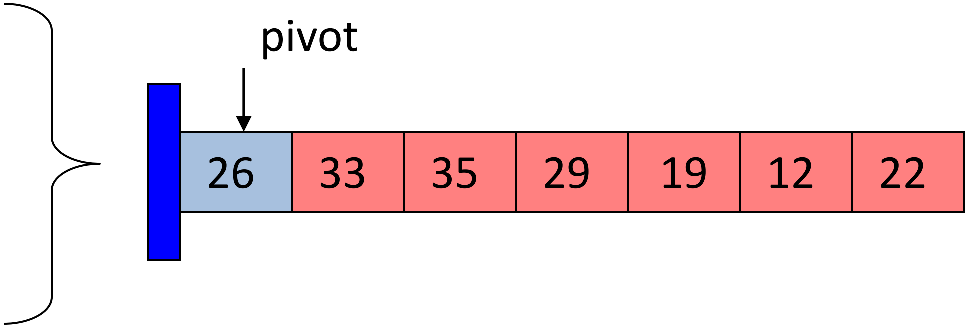

| | | | | | | |
|---|---|---|---|---|---|---|
| 26 | 33 | 35 | 29 | 19 | 12 | 22 |

We will select the first element, as presented in the original algorithm by C.A.R. Hoare in 1962.

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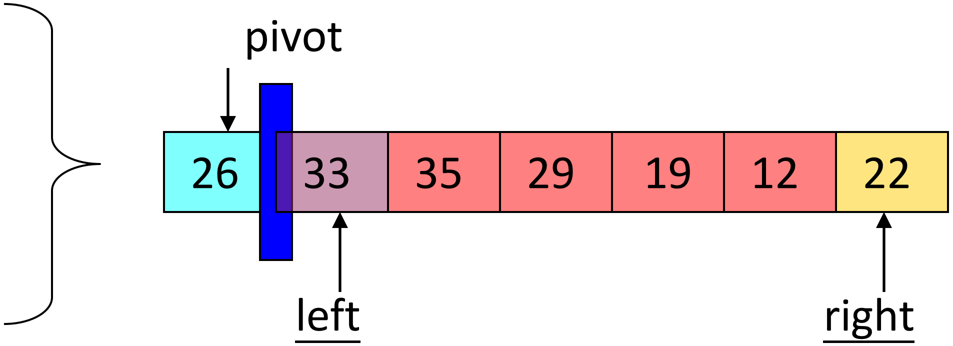

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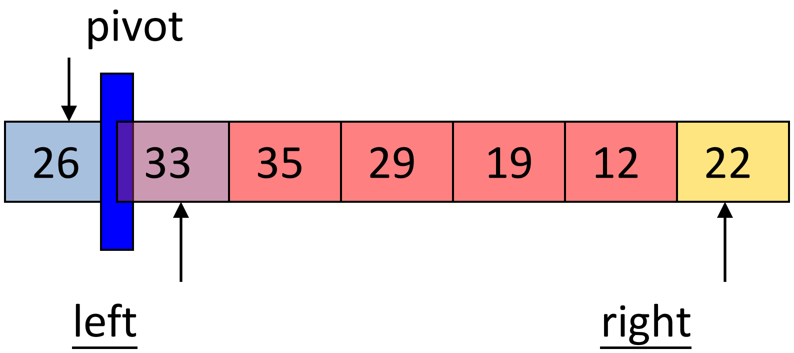

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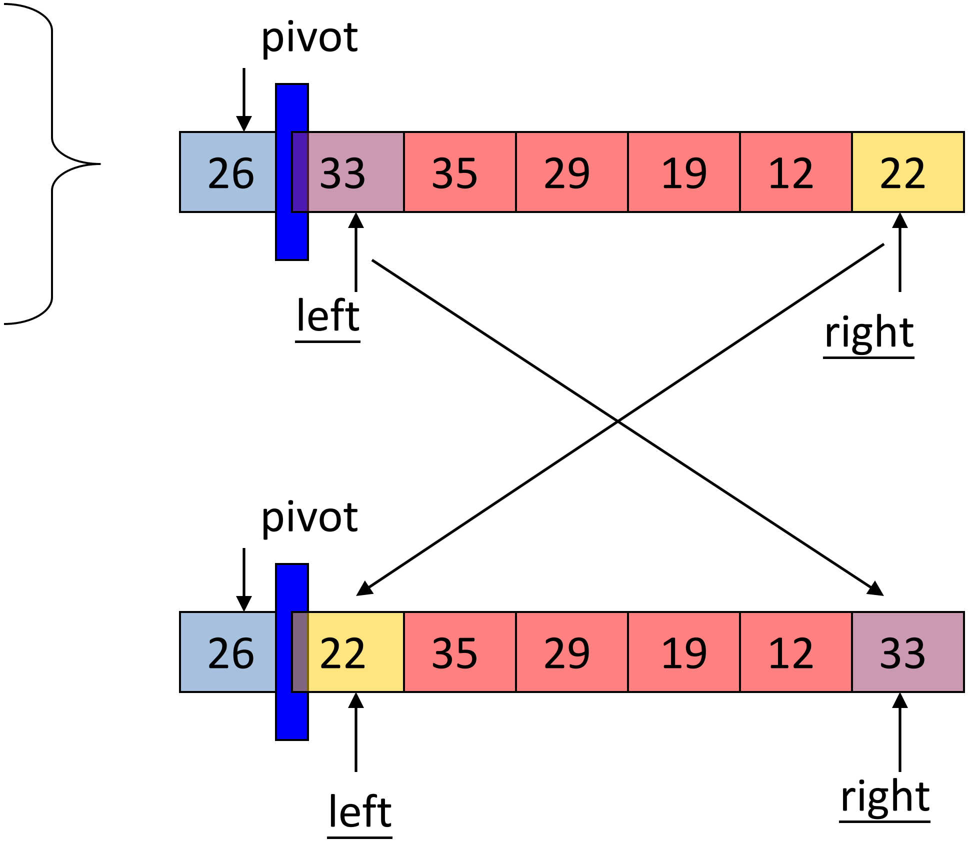
Element 22 belongs to LEFT group.

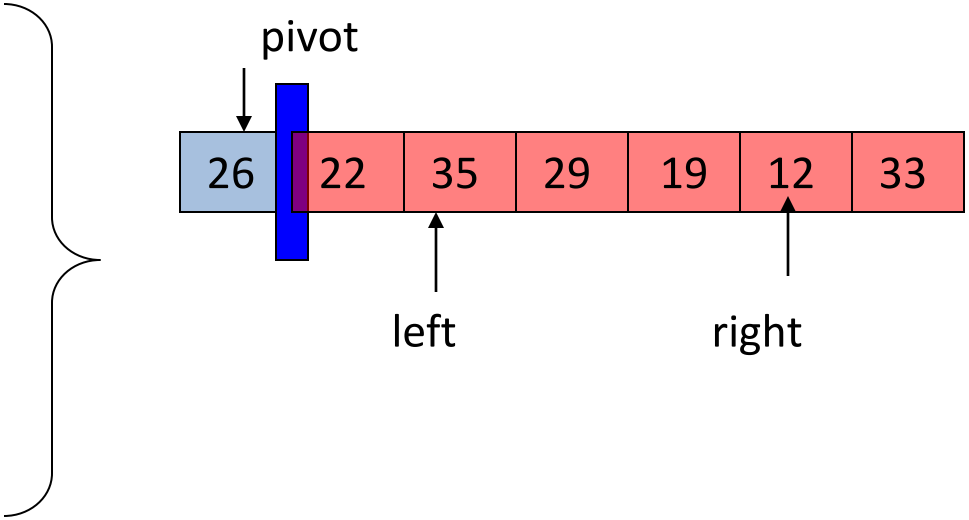
Exchange the two elements.



# Quicksort Step 5

Step 5:

After the exchange, increment left marker, decrement right marker.



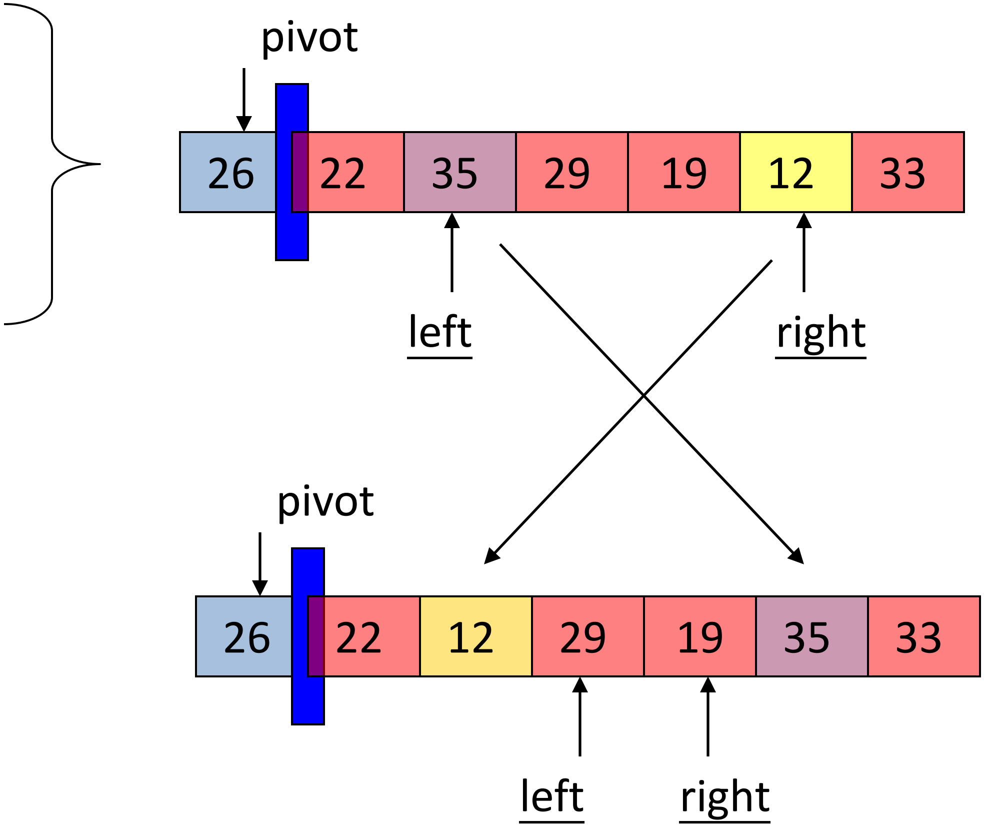
# Quicksort Step 6

Step 6:

Element 35 belongs to RIGHT group.

Element 12 belongs to LEFT group.

Exchange, increment left, and decrement right.



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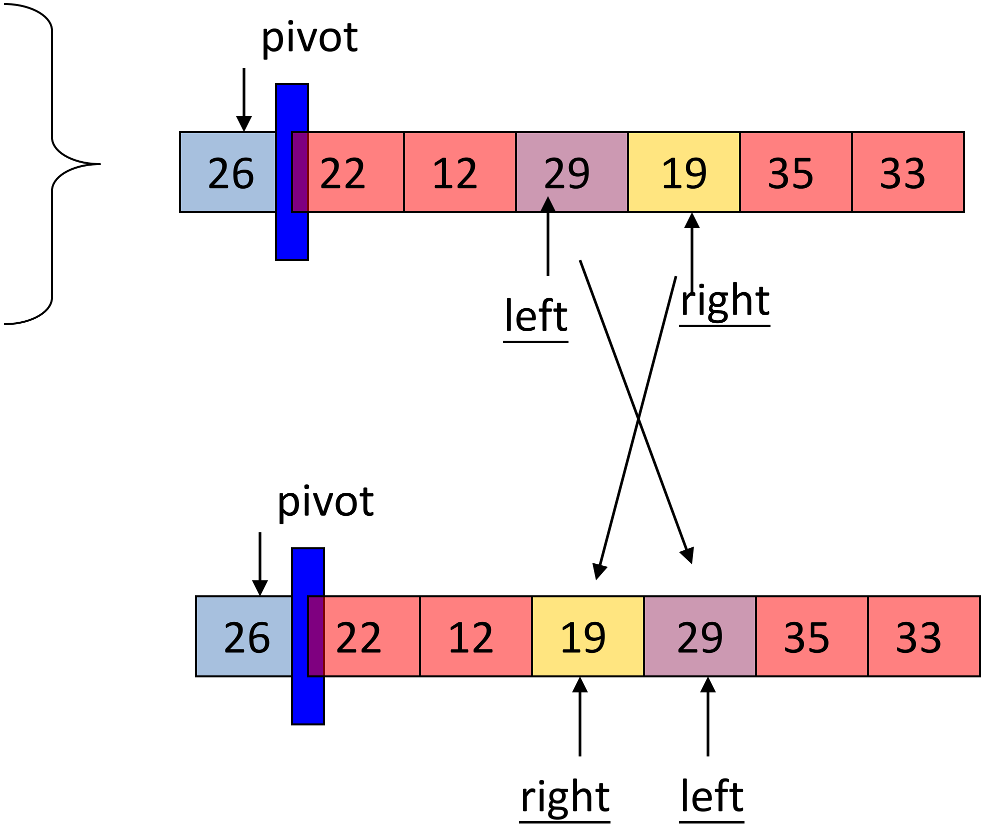
Step 7:

Element 29 belongs to RIGHT.

Element 19 belongs to LEFT.

Exchange, increment left, decrement right.



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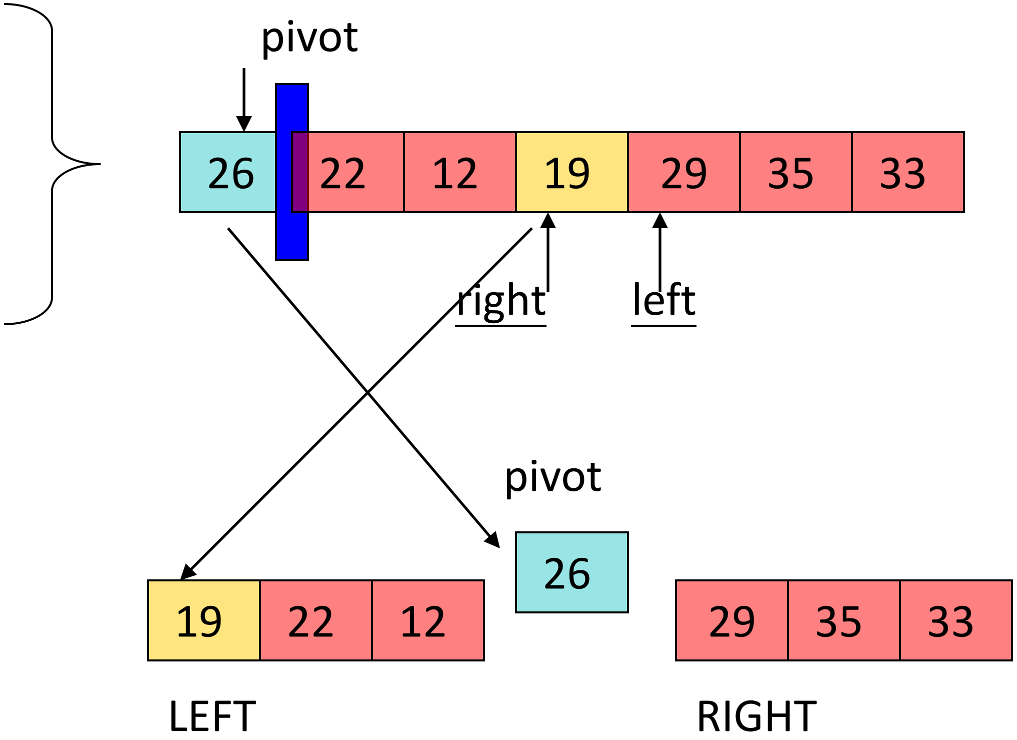
When the left and right markers pass each other, we are done with the partition task.

Swap the right with pivot.



# Quicksort Step 9

Step 9:
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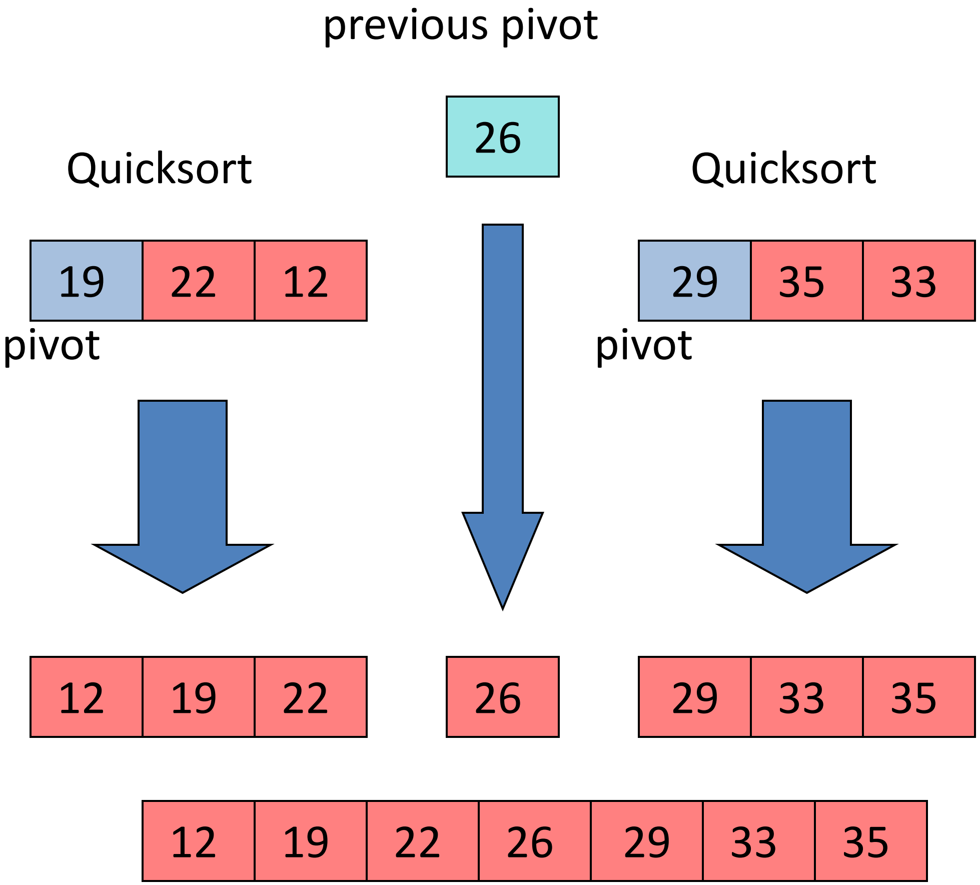
RIGHT groups,
recursively.

Assemble parts when done



previous pivot

26

Quicksort

19 | 22 | 12

pivot

Quicksort

29 | 35 | 33

pivot

12 | 19 | 22

26

29 | 33 | 35

12 | 19 | 22 | 26 | 29 | 33 | 35

# Quicksort Efficiency

The partitioning of an array into two parts is O(n)

The number of recursive calls to Quicksort depends on how many times we can split the array into two groups.

On average this is O (log$_2$ n)

The overall Quicksort efficiency is O(n) = n log$_2$n

What is the worst-case efficiency?

Compare this to the worst case for the heapsort.

# Heap

- The root of the tree A[1] and given index *i* of a node, the indices of its parent, left child and right child can be computed

```
PARENT ($i$)
    return floor($i/2$)
LEFT ($i$)
    return $2i$
RIGHT ($i$)
    return $2i + 1$
```

# Heap order property

- For every node *v*, other than the root, the key stored in *v* is greater or equal (smaller or equal for max heap) than the key stored in the parent of *v*.
- In this case the maximum value is stored in the root

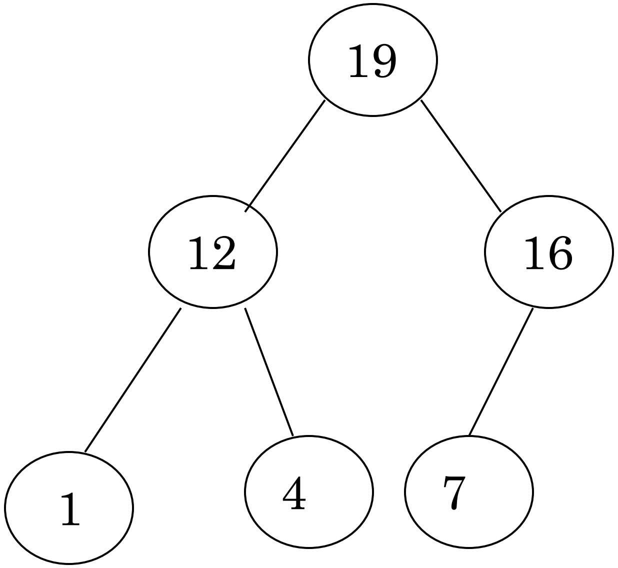
# Definition

- Max Heap
  - Store data in ascending order
  - Has property of

    $A[\text{Parent}(i)] \geq A[i]$
- Min Heap
  - Store data in descending order
  - Has property of

    $A[\text{Parent}(i)] \leq A[i]$

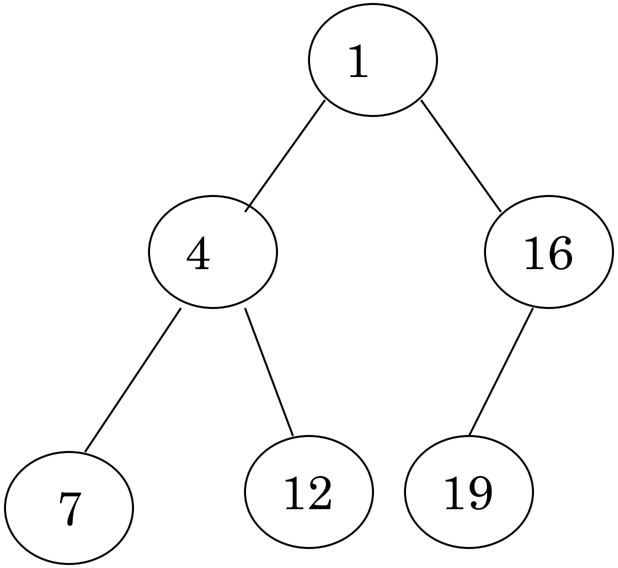
# Max Heap Example



| | | | | | |
|---|---|---|---|---|---|
| 19 | 12 | 16 | 1 | 4 | 7 |

Array A

# Min heap example



| | | | | | |
|---|---|---|---|---|---|
| 1 | 4 | 16 | 7 | 12 | 19 |

Array A

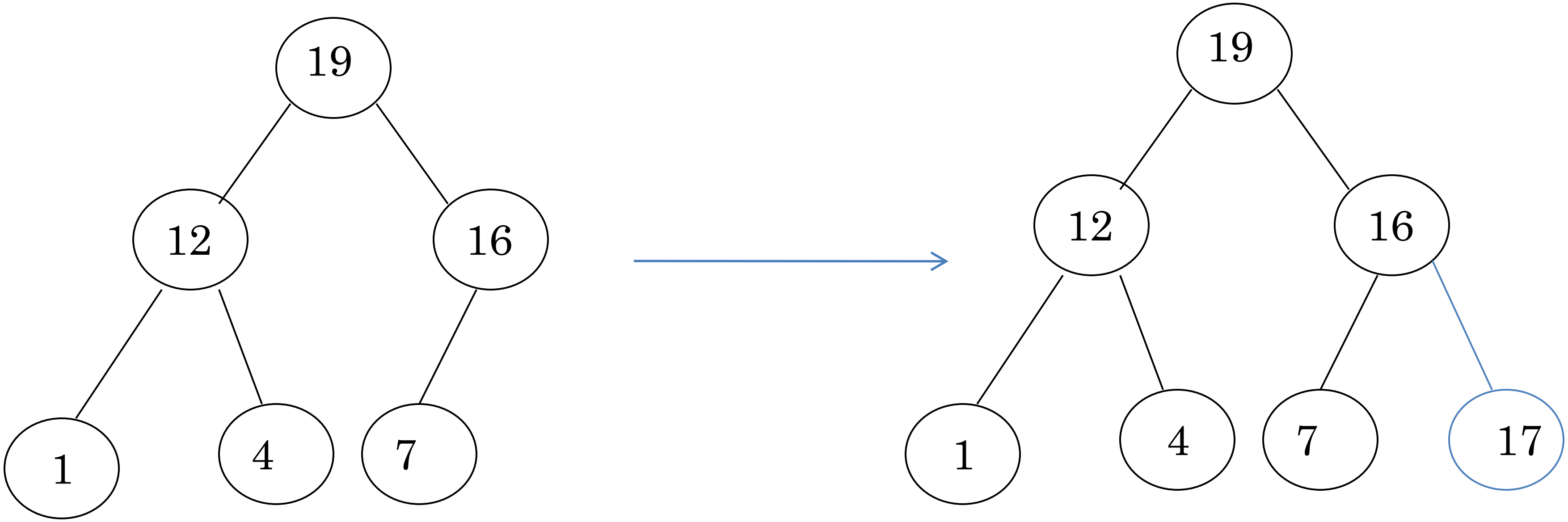
# Insertion

- Algorithm
  1. Add the new element to the next available position at the lowest level
  2. Restore the max-heap property if violated
     - General strategy is percolate up (or bubble up): if the parent of the element is smaller than the element, then interchange the parent and child.

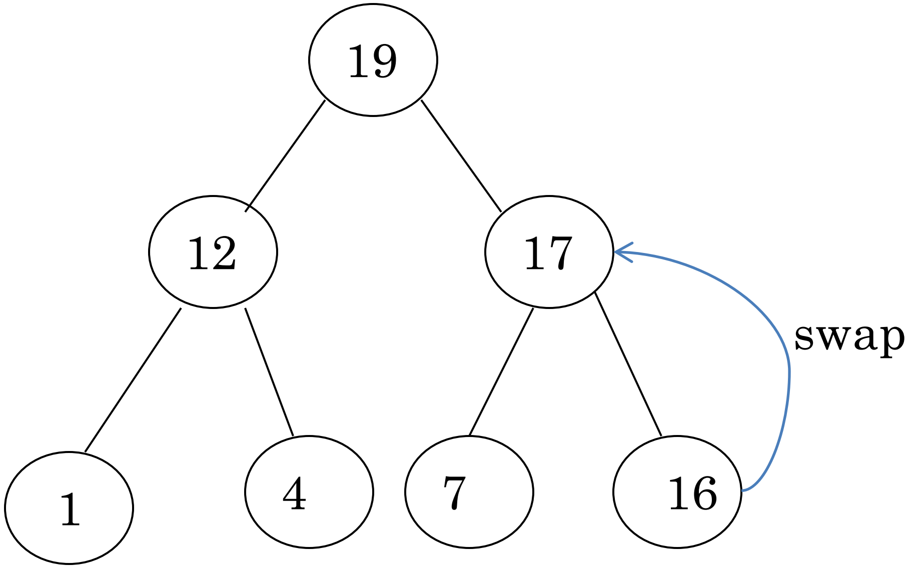
OR

Restore the min-heap property if violated

- General strategy is percolate up (or bubble up): if the parent of the element is larger than the element, then interchange the parent and child.



Insert 17



Percolate up to maintain the heap property

# Deletion

- Delete max
  - Copy the last number to the root ( overwrite the maximum element stored there ).
  - Restore the max heap property by percolate down.

- Delete min
  - Copy the last number to the root ( overwrite the minimum element stored there ).
  - Restore the min heap property by percolate down.

# Heap Sort

A sorting algorithm that works by first organizing the data to be sorted into a special type of binary tree called a heap

# Procedures on Heap

- Heapify
- Build Heap
- Heap Sort

# Heapify

- Heapify picks the largest child key and compare it to the parent key. If parent key is larger than heapify quits, otherwise it swaps the parent key with the largest child key. So that the parent is now becomes larger than its children.

```
Heapify(A, i)
{
    l ← left(i)
    r ← right(i)
    if l <= heapsize[A] and A[l] > A[i]
       then largest ←l
       else largest ← i
    if r <= heapsize[A] and A[r] > A[largest]
       then largest ← r
    if largest != i
       then swap A[i] ←→ A[largest]
          Heapify(A, largest)
}
```

# BUILD HEAP

- We can use the procedure 'Heapify' in a bottom-up fashion to convert an array A[1 . . $n$] into a heap. Since the elements in the subarray A[$n$/2 +1 . . $n$] are all leaves, the procedure BUILD_HEAP goes through the remaining nodes of the tree and runs 'Heapify' on each one. The bottom-up order of processing node guarantees that the subtree rooted at children are heap before 'Heapify' is run at their parent.

```
 Buildheap(A)
{
       heapsize[A] ←length[A]
       for i ←|length[A]/2  //down to 1
           do Heapify(A, i)
 }
```

# Heap Sort Algorithm

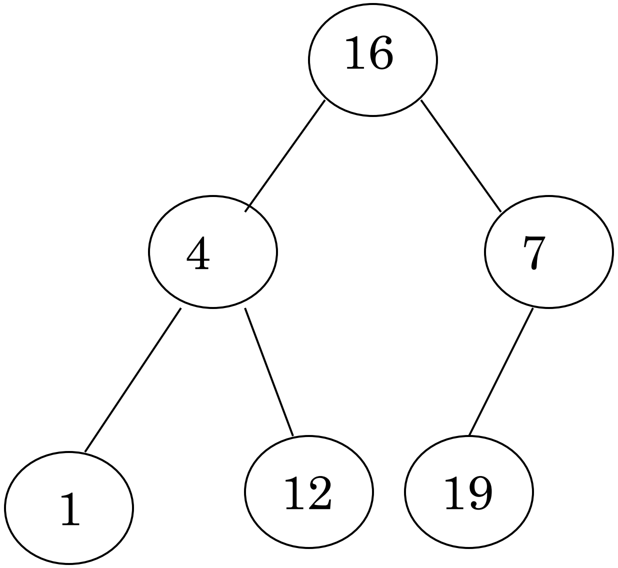
- The heap sort algorithm starts by using procedure BUILD-HEAP to build a heap on the input array A[1 . . *n*]. Since the maximum element of the array stored at the root *A*[1], it can be put into its correct final position by exchanging it with *A*[*n*] (the last element in *A*). If we now discard node n from the heap than the remaining elements can be made into heap. Note that the new element at the root may violate the heap property. All that is needed to restore the heap property.

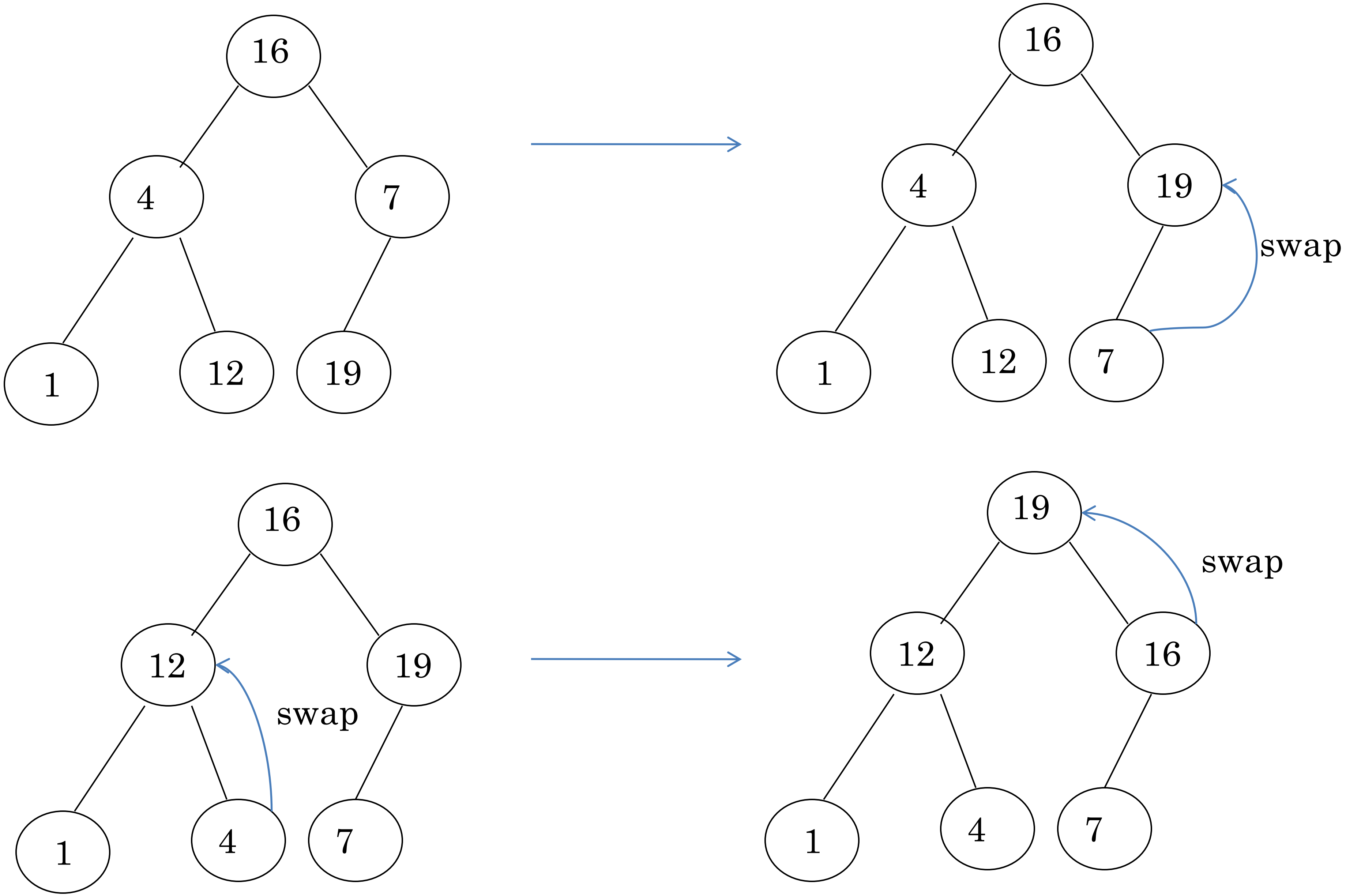
```
Heapsort(A)
{
        Buildheap(A)
        for i ← length[A] //down to 2
            do swap A[1] ←→ A[i]
            heapsize[A] ← heapsize[A] - 1
            Heapify(A, 1)
}
```

**Example:** Convert the following array to a heap

| | | | | | |
|---|---|---|---|---|---|
| 16 | 4 | 7 | 1 | 12 | 19 |

Picture **the array as a complete binary tree:**

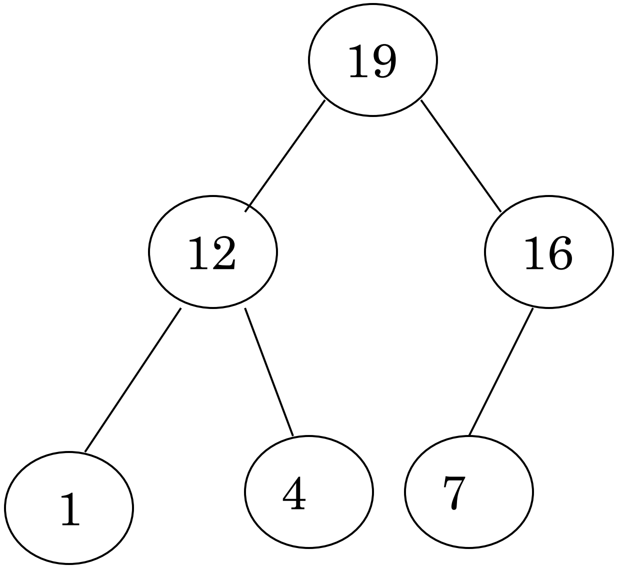



16
4
7
1
12
19
16
4
19
1
12
7
swap
16
12
19
1
4
7
swap
19
12
16
1
4
7
swap

# Heap Sort

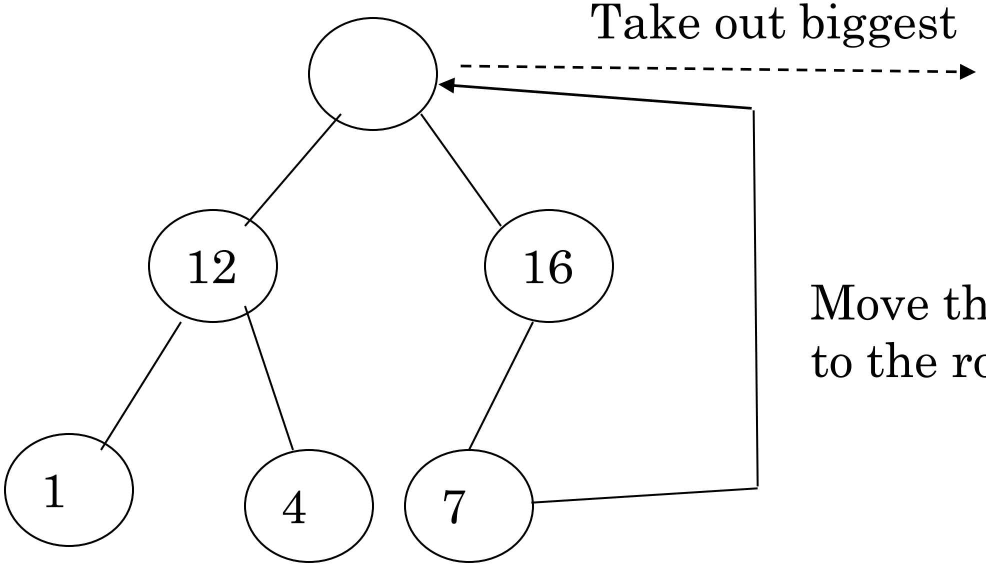
- The heapsort algorithm consists of two phases:
  - build a heap from an arbitrary array
  - use the heap to sort the data

- To sort the elements in the decreasing order, use a min heap
- To sort the elements in the increasing order, use a max heap



# Example of Heap Sort

Take out biggest



19

Move the last element to the root

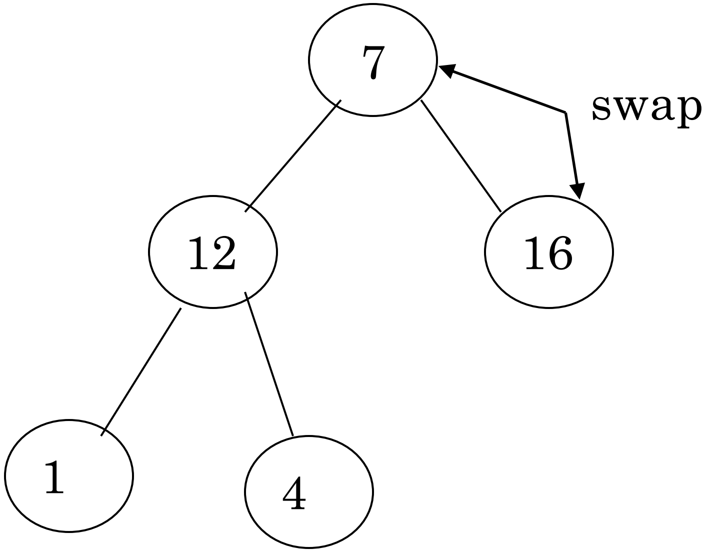
Array A

|    |    |   |   |   |
|----|----|---|---|---|
| 12 | 16 | 1 | 4 | 7 |

Sorted:

| |
|----|
| 19 |

HEAPIFY()

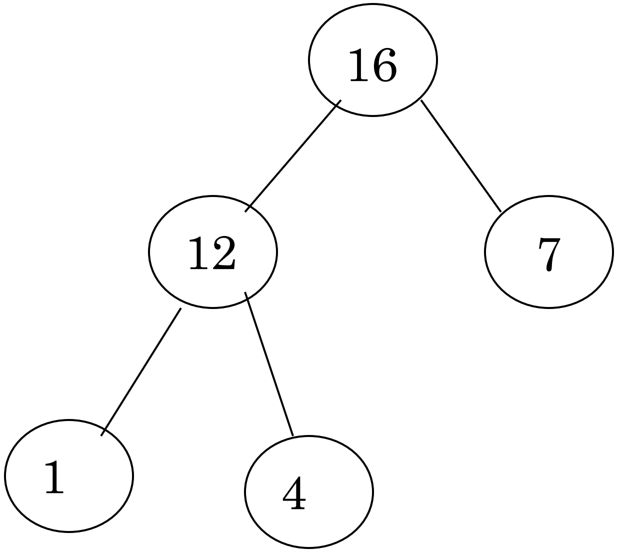


Sorted:

Array A

| | | | | |
|---|---|---|---|---|
| 7 | 12 | 16 | 1 | 4 |

| |
|---|
| 19 |

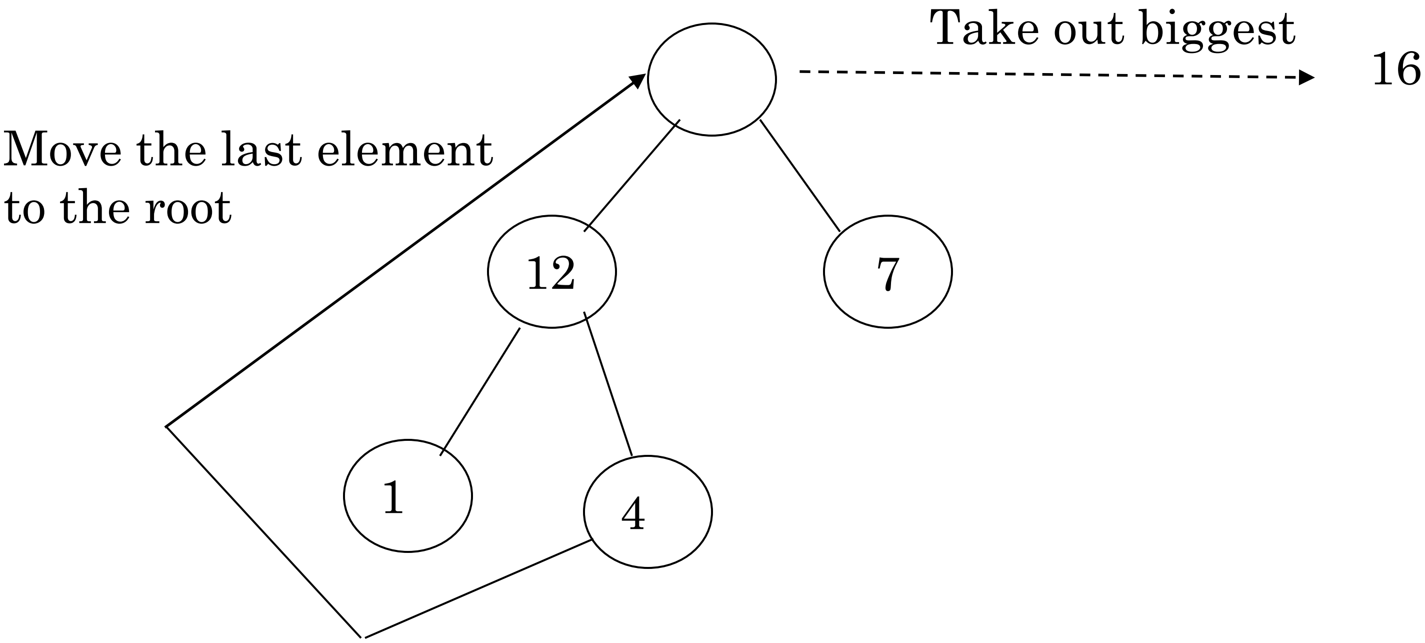


Array A

| | | | | |
|---|---|---|---|---|
| 16 | 12 | 7 | 1 | 4 |

Sorted:

19

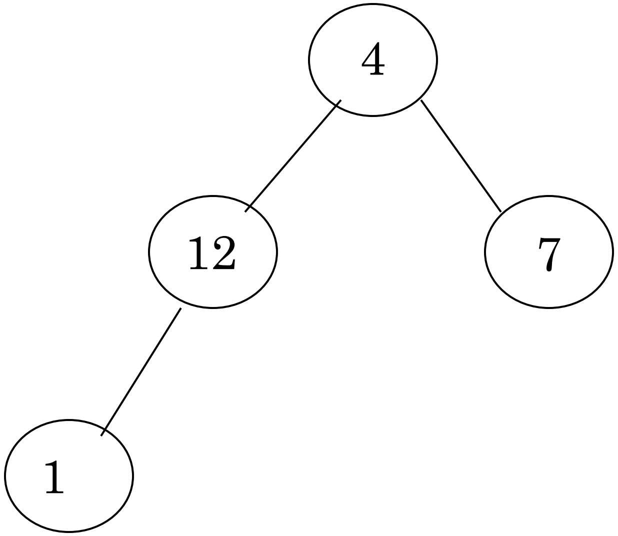


Array A

|    |   |   |   |
|----|---|---|---|
| 12 | 7 | 1 | 4 |

Sorted:

|    |    |
|----|----|
| 16 | 19 |



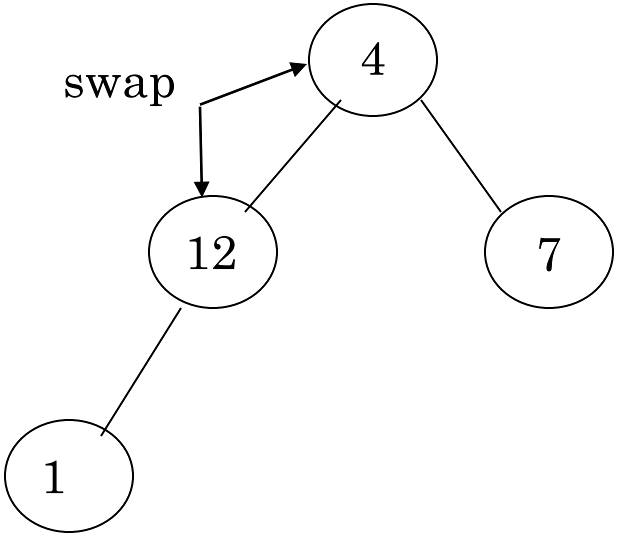
Sorted:

Array A

| | | | |
|---|---|---|---|
| 4 | 12 | 7 | 1 |

| | |
|---|---|
| 16 | 19 |

HEAPIFY()

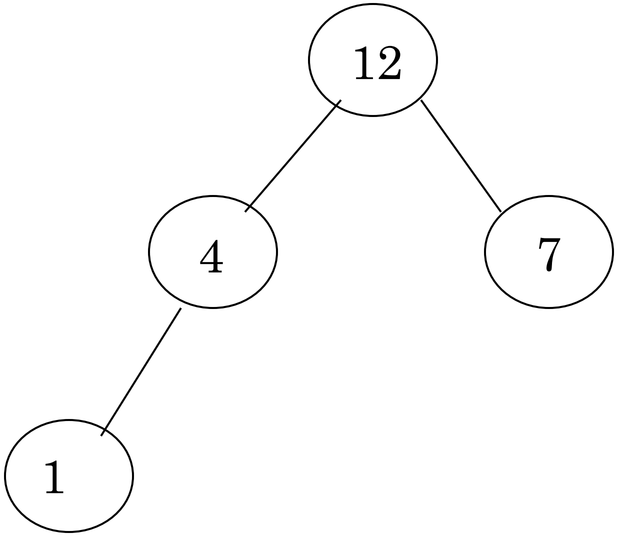


Array A

|  |  |  |  |
|---|---|---|---|
| 4 | 12 | 7 | 1 |

Sorted:

|  |  |
|---|---|
| 16 | 19 |

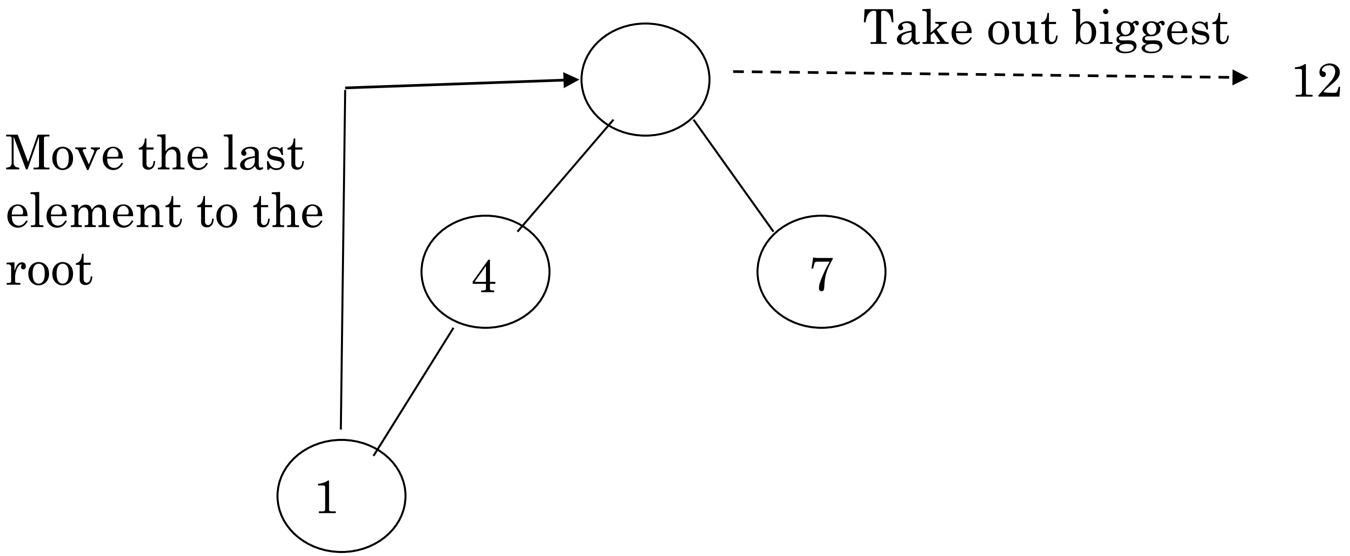


Array A

| | | | |
|---|---|---|---|
| 12 | 4 | 7 | 1 |

Sorted:

| | |
|---|---|
| 16 | 19 |



Take out biggest

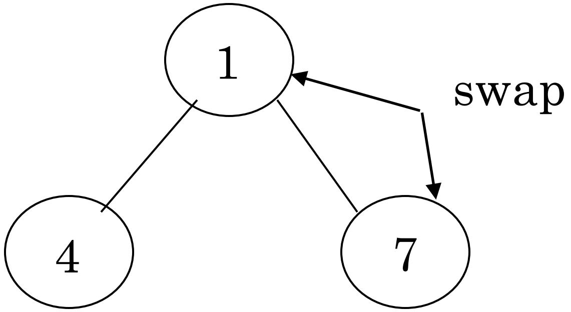
Move the last element to the root

Array A

|   |   |   |
|---|---|---|
| 4 | 7 | 1 |

Sorted:

|    |    |    |
|----|----|----|
| 12 | 16 | 19 |

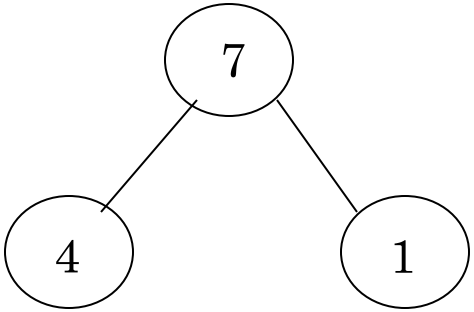


Array A

| | | |
|---|---|---|
| 1 | 4 | 7 |

Sorted:

| | | |
|---|---|---|
| 12 | 16 | 19 |

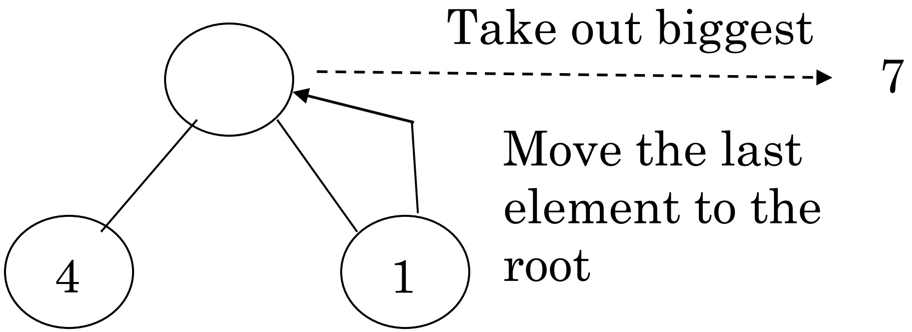


Array A

| | | |
|---|---|---|
| 7 | 4 | 1 |

Sorted:

| | | |
|---|---|---|
| 12 | 16 | 19 |



Take out biggest

7

Move the last element to the root

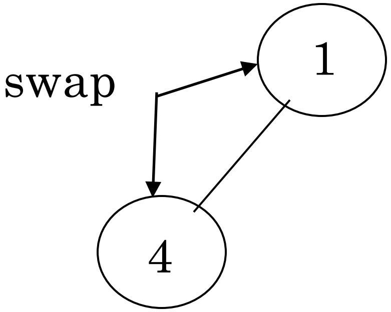
Array A

|   |   |
|---|---|
| 1 | 4 |

Sorted:

|   |    |    |    |
|---|----|----|----|
| 7 | 12 | 16 | 19 |

HEAPIFY()



swap

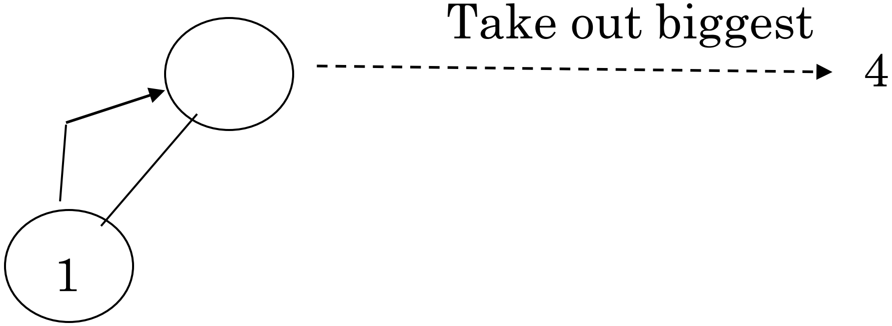
Array A

|   |   |
|---|---|
| 4 | 1 |

Sorted:

|   |    |    |    |
|---|----|----|----|
| 7 | 12 | 16 | 19 |

Move the last element to the root



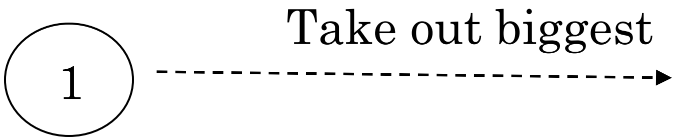
Take out biggest

Array A

| |
|---|
| 1 |

Sorted:

| | | | | |
|---|---|---|---|---|
| 4 | 7 | 12 | 16 | 19 |



Take out biggest

Array A

Sorted:

| | | | | | |
|---|---|---|---|---|---|
| 1 | 4 | 7 | 12 | 16 | 19 |

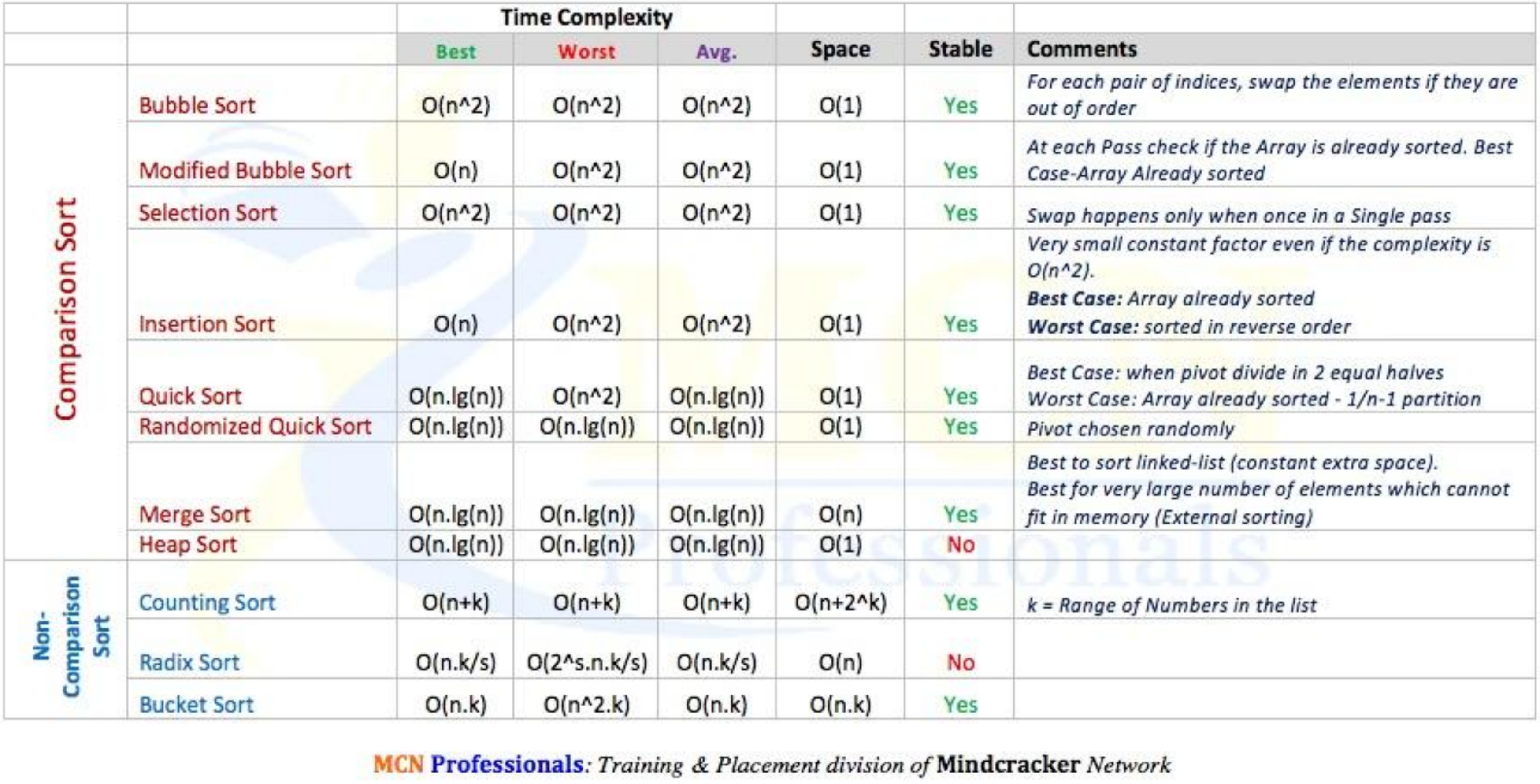
Sorted:

|  |  |  |  |  |  |
|---|---|---|---|---|---|
| 1 | 4 | 7 | 12 | 16 | 19 |

# Time Analysis

- Build Heap Algorithm will run in O(n) time
- There are *n*-1 calls to Heapify each call requires O(log *n*) time
- Heap sort program combine Build Heap program and Heapify, therefore it has the running time of O(n log n) time
- Total time complexity: O(n log n)

# Comparison of Sorting Methods



| | | Time Complexity | | | | | |
|---|---|---|---|---|---|---|---|
| | | Best | Worst | Avg. | Space | Stable | Comments |
| Comparison Sort | Bubble Sort | O(n^2) | O(n^2) | O(n^2) | O(1) | Yes | *For each pair of indices, swap the elements if they are out of order* |
| | Modified Bubble Sort | O(n) | O(n^2) | O(n^2) | O(1) | Yes | *At each Pass check if the Array is already sorted. Best Case-Array Already sorted* |
| | Selection Sort | O(n^2) | O(n^2) | O(n^2) | O(1) | Yes | *Swap happens only when once in a Single pass* |
| | Insertion Sort | O(n) | O(n^2) | O(n^2) | O(1) | Yes | *Very small constant factor even if the complexity is O(n^2).*<br>***Best Case:** Array already sorted*<br>***Worst Case:** sorted in reverse order* |
| | Quick Sort | O(n.lg(n)) | O(n^2) | O(n.lg(n)) | O(1) | Yes | *Best Case: when pivot divide in 2 equal halves*<br>*Worst Case: Array already sorted - 1/n-1 partition* |
| | Randomized Quick Sort | O(n.lg(n)) | O(n.lg(n)) | O(n.lg(n)) | O(1) | Yes | *Pivot chosen randomly* |
| | Merge Sort | O(n.lg(n)) | O(n.lg(n)) | O(n.lg(n)) | O(n) | Yes | *Best to sort linked-list (constant extra space).*<br>*Best for very large number of elements which cannot fit in memory (External sorting)* |
| | Heap Sort | O(n.lg(n)) | O(n.lg(n)) | O(n.lg(n)) | O(1) | No | |
| Non-Comparison Sort | Counting Sort | O(n+k) | O(n+k) | O(n+k) | O(n+2^k) | Yes | *k = Range of Numbers in the list* |
| | Radix Sort | O(n.k/s) | O(2^s.n.k/s) | O(n.k/s) | O(n) | No | |
| | Bucket Sort | O(n.k) | O(n^2.k) | O(n.k) | O(n.k) | Yes | |

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# UNIT-V

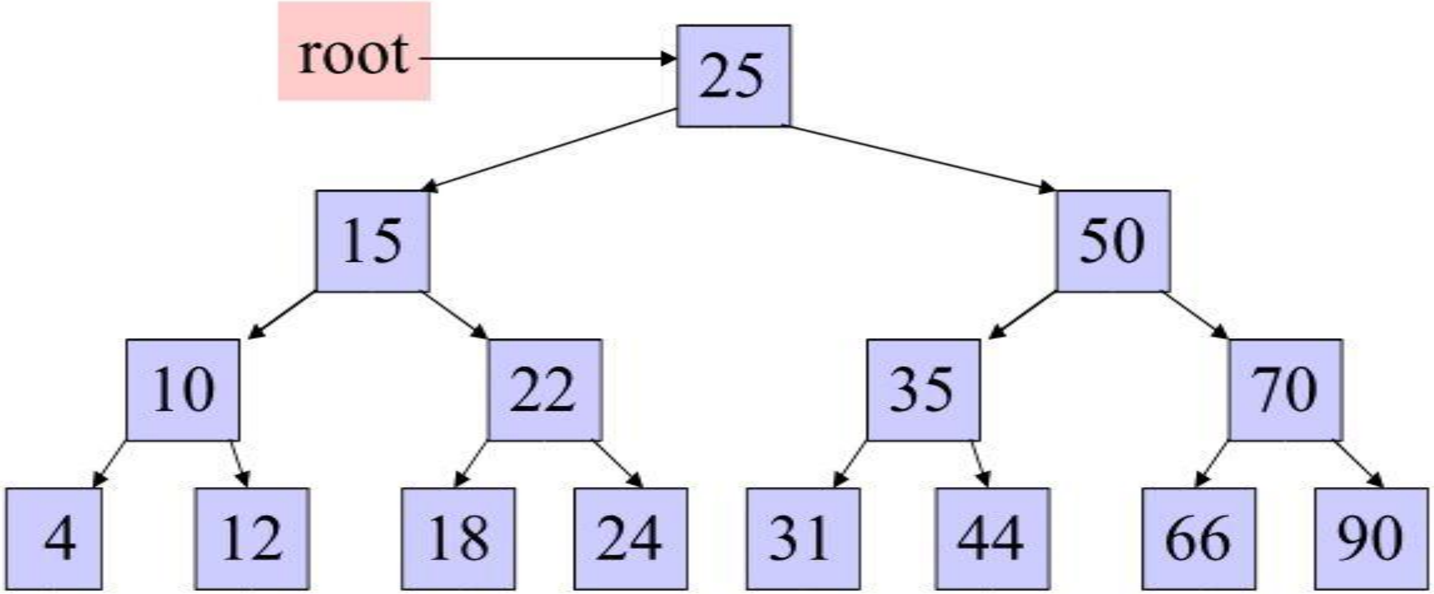
**Topics:**

- Search Trees-Binary Search Trees, Definition, Operations- Searching, Insertion and Deletion, AVL Trees-Definition and Examples, Insertion into an AVL Tree ,B-Trees, Definition, B-Tree of order m, operations-Insertion and Searching, Introduction to Red-Black and Splay Trees(Elementary treatment-only Definitions and Examples),Comparison of Search Trees.

  Pattern matching algorithm- The Knuth-Morris-Pratt algorithm, Tries (examples only).

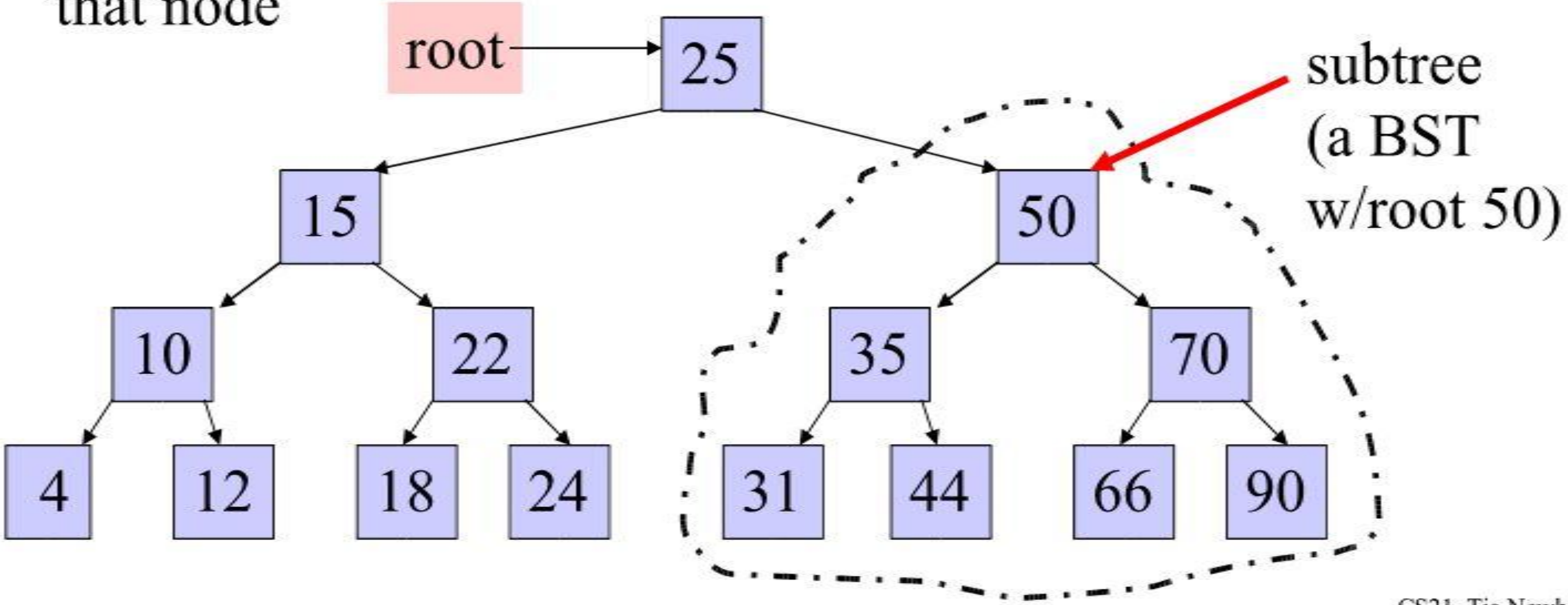
# Binary Search Trees (BST)

1. Hierarchical data structure with a single pointer to root node
2. Each node has at most two child nodes (a left and a right child)
3. Nodes are organized by the Binary Search property:
   - Every node is ordered by some key data field(s)
   - For every node in the tree, its key is greater than its left child's key and less than its right child's key



# Some BST Terminology

1. The Root node is the top node in the hierarchy
2. A Child node has exactly one Parent node, a Parent node has at most two child nodes, Sibling nodes share the same Parent node (ex. node 22 is a child of node 15)
3. A Leaf node has no child nodes, an Interior node has at least one child node (ex. 18 is a leaf node)
4. Every node in the BST is a Subtree of the BST rooted at that node



CS21, Tia Newhall

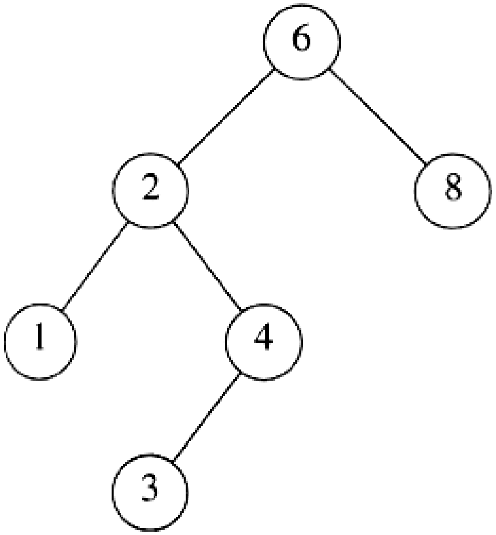
# Comparision Between Binary Tree & Binary Search Tree

- A binary search tree is a binary tree in which it has atmost two children, the key values in the left node is less than the root and the key values in the right node is greater than the root.
- It doesn't have any order.

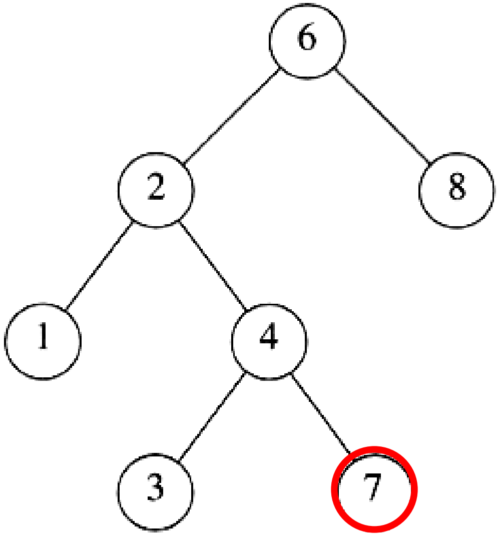
Note : * Every binary search tree is a binary tree.

- All binary trees need not be a binary search tree

# Example of Binary Search Tree



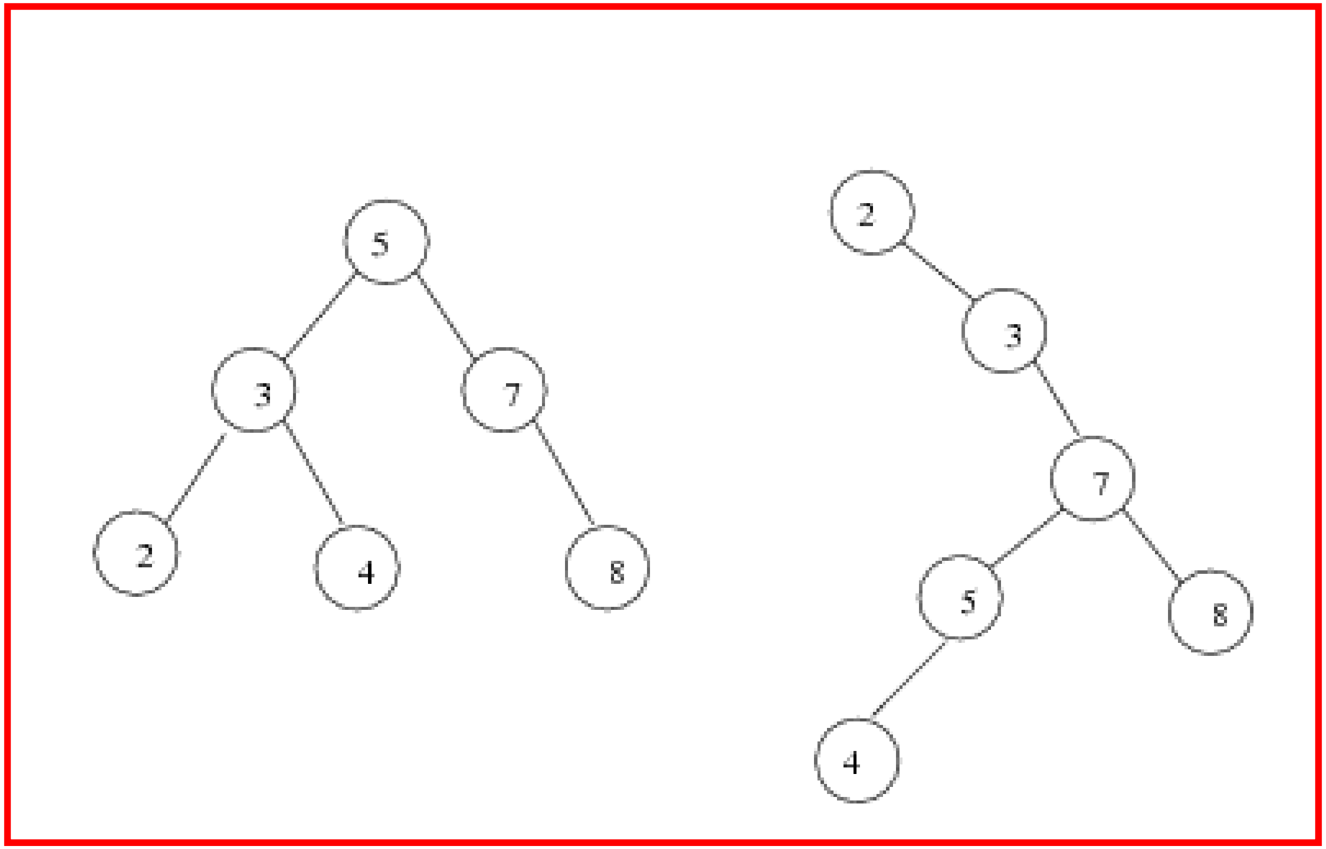
A binary search tree



Not a binary search tree

# Binary Search Trees

**The same set of keys may have different BSTs**



# DECLARATION ROUTINE FOR BINARY SEARCH TREE

```
Struct TreeNode
{
int Element ;
SearchTree Left;
SearchTree Right;
};
```

# BST Operations

- **The 3 basic BST operations are: search, insert, and delete; and develop algorithms for searches, insertion, and deletion.**
- **Searches**
- **Insertion**
- **Deletion**

# Three BST search algorithms:

- Find the smallest node
- Find the largest node
- Find a requested node

# Find : -

- Check whether the root is NULL if so then return NULL.
- Otherwise, Check the value X with the root node value (i.e. T data)
- (1) If X is equal to T data, return T.
- (2) If X is less than T data, Traverse the left of T recursively.
- (3) If X is greater than T data, traverse the right of T recursively.

# ROUTINE FOR FIND OPERATION

```
Int Find (int X, SearchTree T)
{
If T = = NULL)
Return NULL ;
If (X < T Element)
return Find (X, T →left);
else
If (X > T→ Element)
return Find (X, T →Right);
else
return T; // returns the position of
  the search element.
}
```

## ALGORITHM Search BST

```
Algorithm searchBST (root, targetKey)
Search a binary search tree for a given value.
  Pre    root is the root to a binary tree or
         targetKey is the key value requested
  Return the node address if the value is foun
         null if the node is not in the tree
1 if (empty tree)
     Not found
  1  return null
2 end if
3 if (targetKey < root)
  1  return searchBST (left subtree, targetKey
4 else if (targetKey > root)
  1  return searchBST (right subtree, targetKe
5 else
     Found target key
  1  return root
6 end if
end searchBST
```

# Find Min :

- **This operation returns the position of the smallest element in the tree.**
- **To perform FindMin, start at the root and go left as long as there is a left child. The stopping point is the smallest element.**

# RECURISVE ROUTINE FOR FINDMIN

```
int FindMin (SearchTree T)
{
if (T = = NULL);
return NULL ;
else if (T →left = = NULL)
return T;
else
return FindMin (T → left);
```

Example : - Root T

(a) T! = NULL and T→left!=NULL,

(b) (b) T! = NULL and T→left!=NULL,

Traverse left Traverse left until Min T

(c) Since Tleft is Null, return T as a minimum element.

# NON - RECURSIVE ROUTINE FOR FINDMIN

```
int FindMin (SearchTree T)
{
if (T! = NULL)
while (T →Left ! = NULL)
T = T →Left ;
return T;
}
```

# RECURSIVE ROUTINE FOR FINDMAX

```
int FindMax (SearchTree T)
{
if (T = = NULL)
return NULL ;
else if (T →Right = = NULL)
return T;
else FindMax (T →Right);
}
```

**Example :- Root T**

**(a) T! = NULL and T→Right!=NULL, (b) T! = NULL and T→Right!=NULL,**

**Traverse Right Traverse Right**

**Max**

**(c) Since T Right is NULL, return T as a Maximum element.**

# FindMax

- FindMax routine return the position of largest elements in the tree.
- To perform a FindMax, start at the root and go right as long as there is a right child.
- The stopping point is the largest element.

# RECURSIVE ROUTINE FOR FINDMAX

```
int FindMax (SearchTree T)
{
if (T = = NULL)
return NULL ;
else if (T →Right = = NULL)
return T;
else FindMax (T →Right);
}
```

**Example :- Root T**

**(a) T! = NULL and T→Right!=NULL, (b) T! = NULL and T→Right!=NULL,**

**Traverse Right Traverse Right Max**

**(c) Since T Right is NULL, return T as a Maximum element.**

# NON - RECURSIVE ROUTINE FOR FINDMAX

```
int FindMax (SearchTree T)
{
if (T! = NULL)
while (T Right ! = NULL)
T = T →Right ;
return T ;
}
```

# Find a Node into the BST

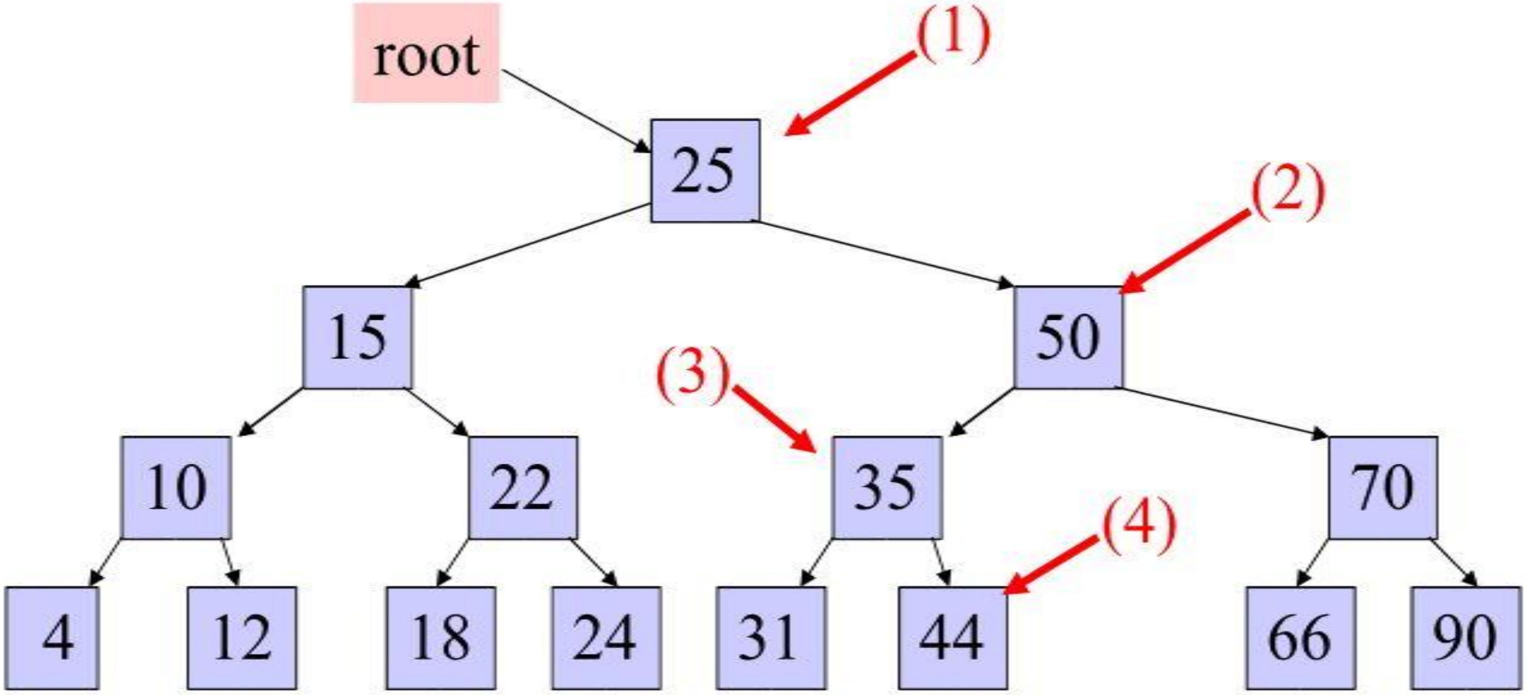
- Use the search key to direct a recursive binary

  search for a matching node

  1. Start at the root node as current node
  2. If the search key’s value matches the current node’s key then found a match
  3. If search key’s value is greater than current node’s
     1. If the current node has a right child, search right
     2. Else, no matching node in the tree
  4. If search key is less than the current node’s
     1. If the current node has a left child, search left
     2. Else, no matching node in the tree

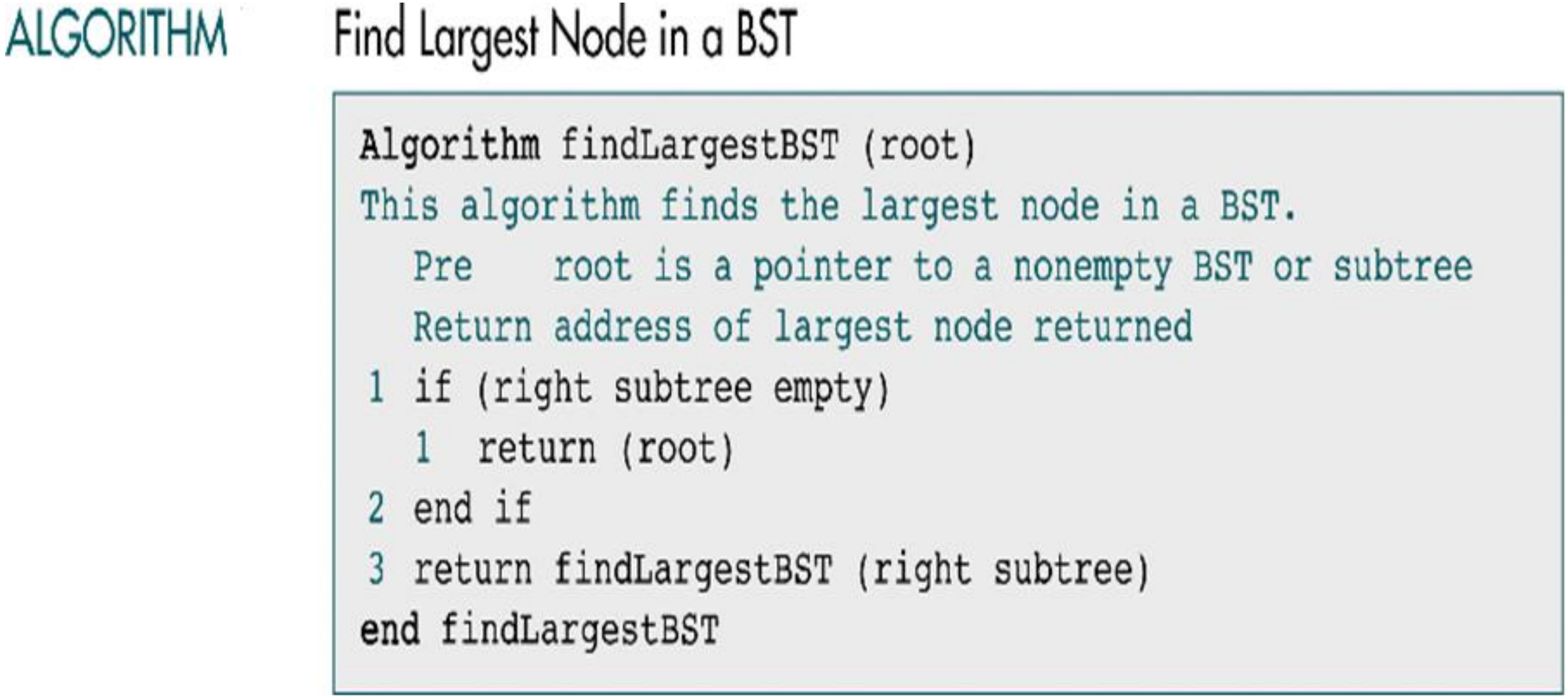
# Example: search for 45 in the tree:

1. start at the root, 45 is greater than 25, search in right subtree
2. 45 is less than 50, search in 50's left subtree
3. 45 is greater than 35, search in 35's right subtree
4. 45 is greater than 44, but 44 has no right subtree so 45 is not
   in the BST



## ALGORITHM Find Smallest Node in a BST

```
Algorithm findSmallestBST (root)
This algorithm finds the smallest node in a BST.
   Pre    root is a pointer to a nonempty BST or subtree
   Return address of smallest node
1  if (left subtree empty)
   1  return (root)
2  end if
3  return findSmallestBST (left subtree)
end findSmallestBST
```



ALGORITHM

## Find Largest Node in a BST

```
Algorithm findLargestBST (root)
This algorithm finds the largest node in a BST.
   Pre    root is a pointer to a nonempty BST or subtree
   Return address of largest node returned
1 if (right subtree empty)
   1 return (root)
2 end if
3 return findLargestBST (right subtree)
end findLargestBST
```

## Make Empty :-

**This operation is mainly for initialization when the programmer prefer to initialize the first element as a one - node tree.**

## ROUTINE TO MAKE AN EMPTY TREE :-

```
SearchTree MakeEmpty (SearchTree T)
{
if (T! = NULL)
{
MakeEmpty (T left);
MakeEmpty (T Right);
free (T);
}
return NULL ;
}
```

# Insert operation: -

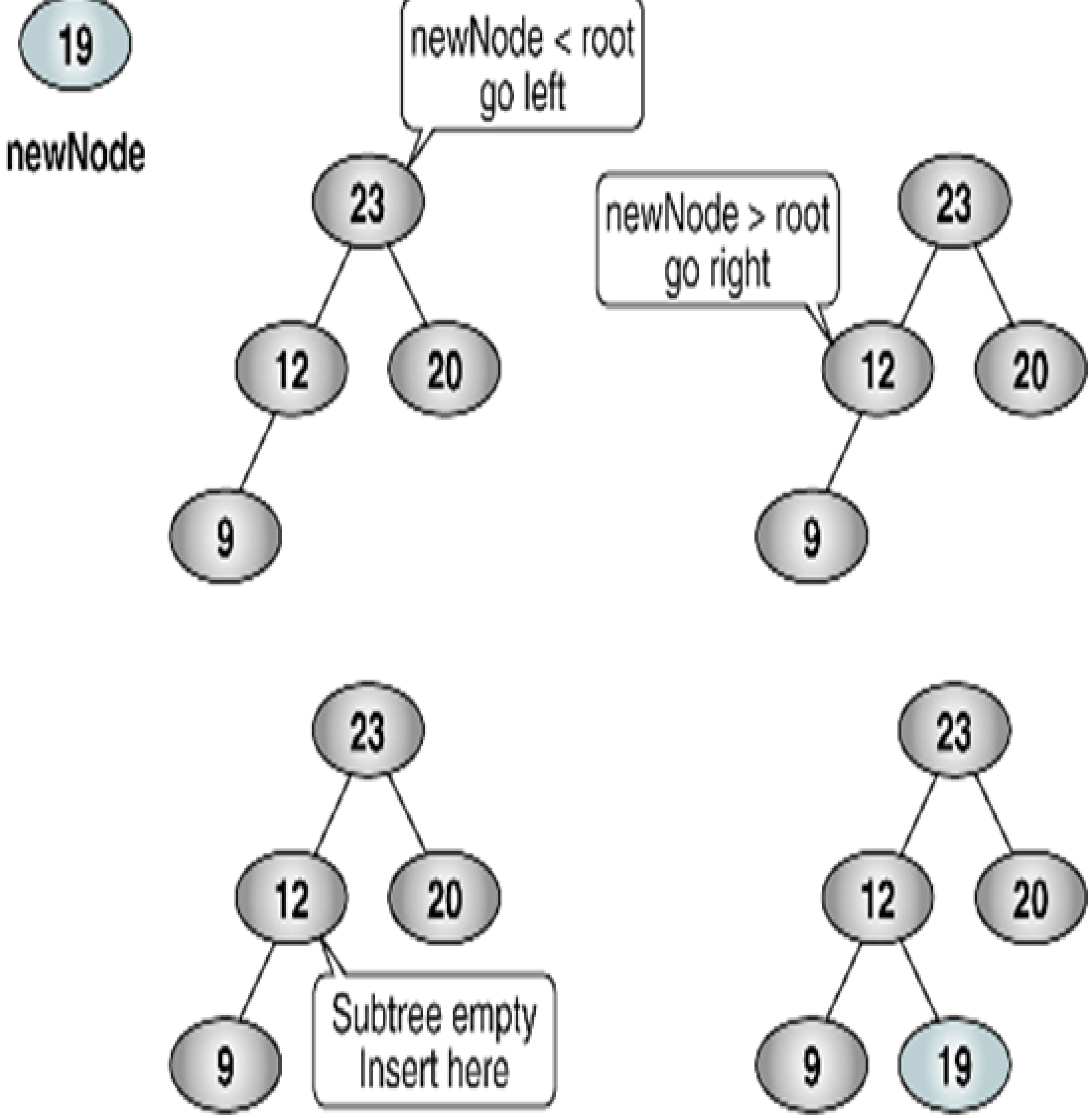
**To insert the element X into the tree,**

- **Check with the root node T**
- **If it is less than the root,**
- **Traverse the left subtree recursively until it reaches the T left equals to NULL. Then X is placed in T left.**
- **If X is greater than the root.**
- **Traverse the right subtree recursively until it reaches the T right equals to NULL. Then x is placed in TRight.**

# ROUTINE TO INSERT INTO A BINARY SEARCH TREE

```
SearchTree Insert (int X, searchTree T)
{
if (T = = NULL)
{
T = malloc (size of (Struct TreeNode));
if(T! = NULL)// First element is placed in
   the root.
{
T →Element = X;
T→ left = NULL;
T →Right = NULL;
 }} }
```

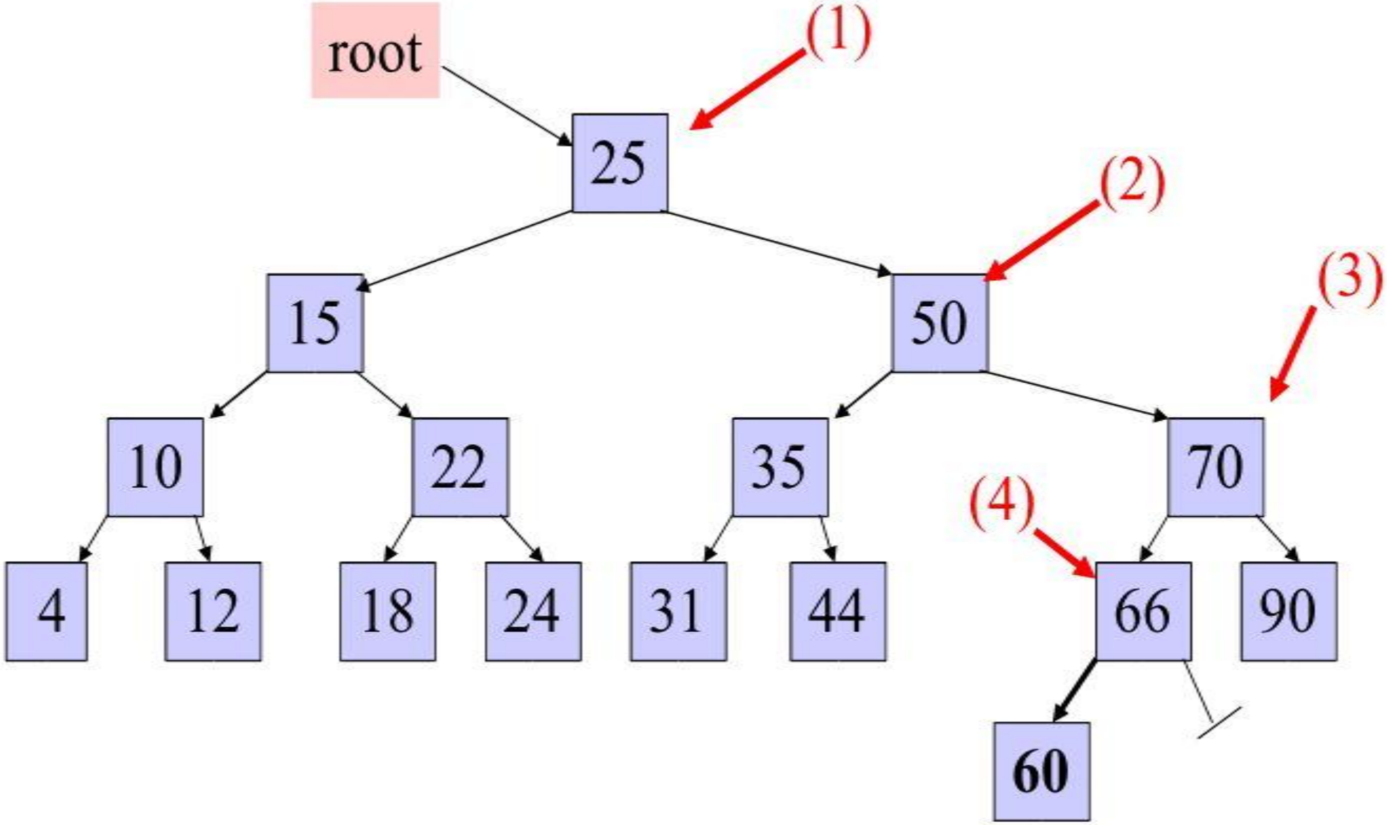
```
else
if (X < T →Element)
T left = Insert (X, T →left);
else
if (X > T →Element)
T Right = Insert (X, T →Right);
// Else X is in the tree already.
return T;
```

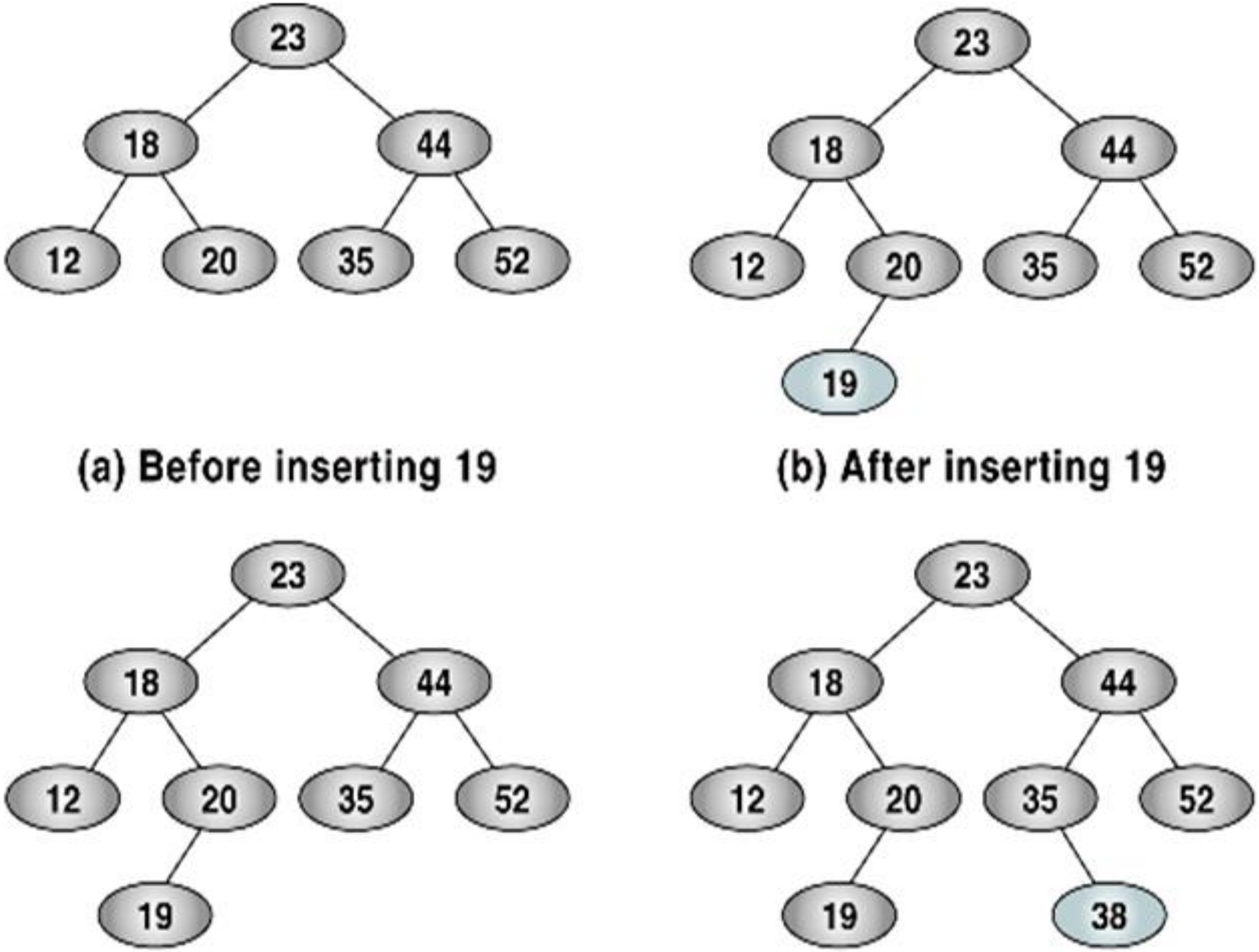


Trace of Recursive BST Insert

# Example: insert 60 in the tree:

1. start at the root, 60 is greater than 25, search in right subtree
2. 60 is greater than 50, search in 50's right subtree
3. 60 is less than 70, search in 70's left subtree
4. 60 is less than 66, add 60 as 66's left child





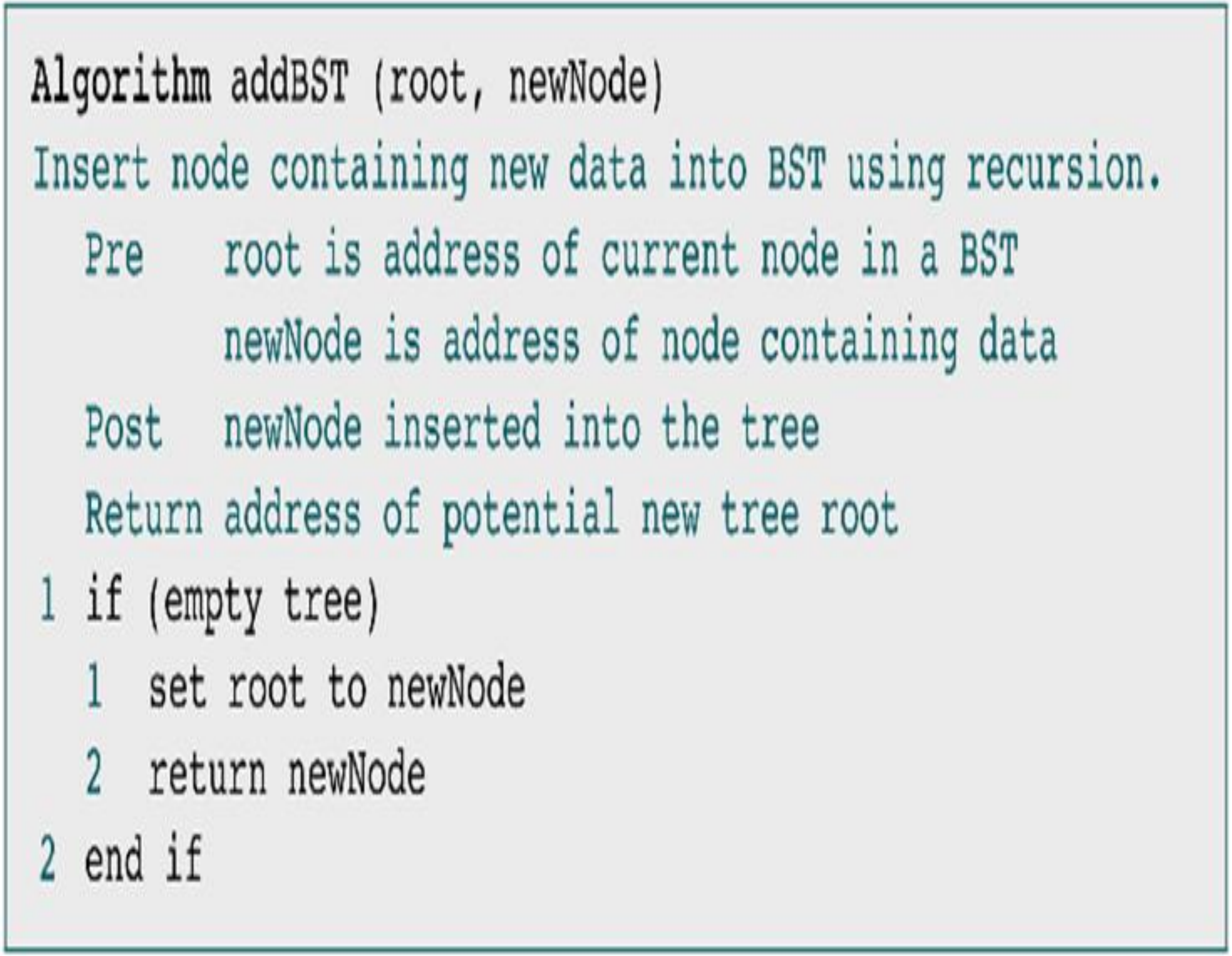
(a) Before inserting 19

(b) After inserting 19

(c) Before inserting 38

(d) After inserting 38

# Add Node to BST



```
Algorithm addBST (root, newNode)
Insert node containing new data into BST using recursion.
  Pre    root is address of current node in a BST
         newNode is address of node containing data
  Post   newNode inserted into the tree
  Return address of potential new tree root
1 if (empty tree)
  1 set root to newNode
  2 return newNode
2 end if
```

continued

ALGORITHM

## Add Node to BST *(continued)*

```
  Locate null subtree for insertion
3 if (newNode < root)
  1  return addBST (left subtree, newNode)
4 else
  1  return addBST (right subtree, newNode)
5 end if
end addBST
```

# Delete operation:

- **Deletion operation is the complex operation in the Binary search tree. To delete an element, consider the following three possibilities.**
- **CASE 1 Node with no children (Leaf node)**

**If the node is a leaf node, it can be deleted immediately.**

- **CASE 2 : - Node with one child**

**If the node has one child, it can be deleted by adjustingits parent pointer that points to its child node**

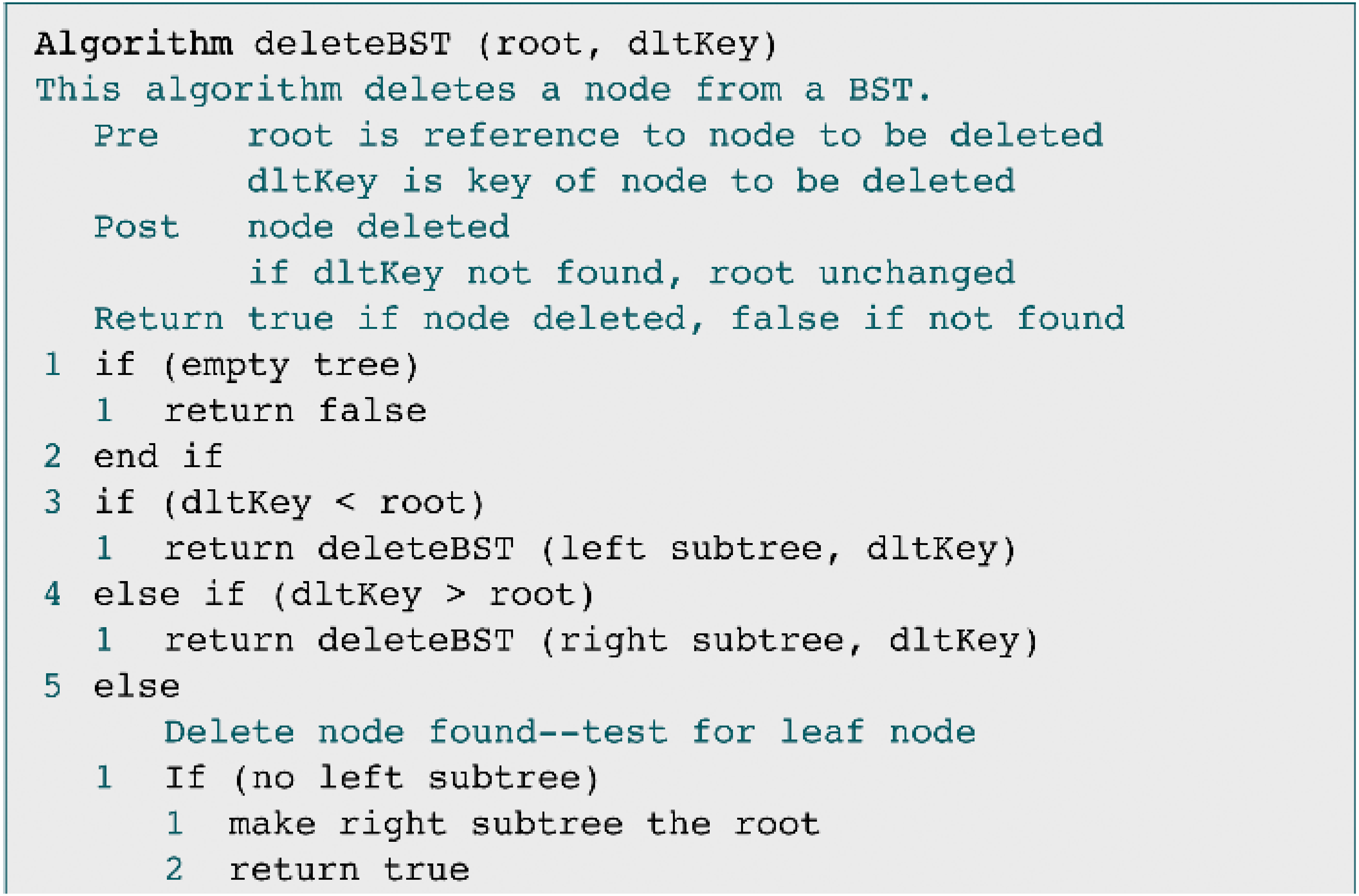
- **Case 3 : Node with two children**

**It is difficult to delete a node which has two children. The general strategy is to replace the data of the node to be deleted with its smallest data of the right subtree and recursively delete that node.**

# DELETION ROUTINE FOR BINARY SEARCH TREES

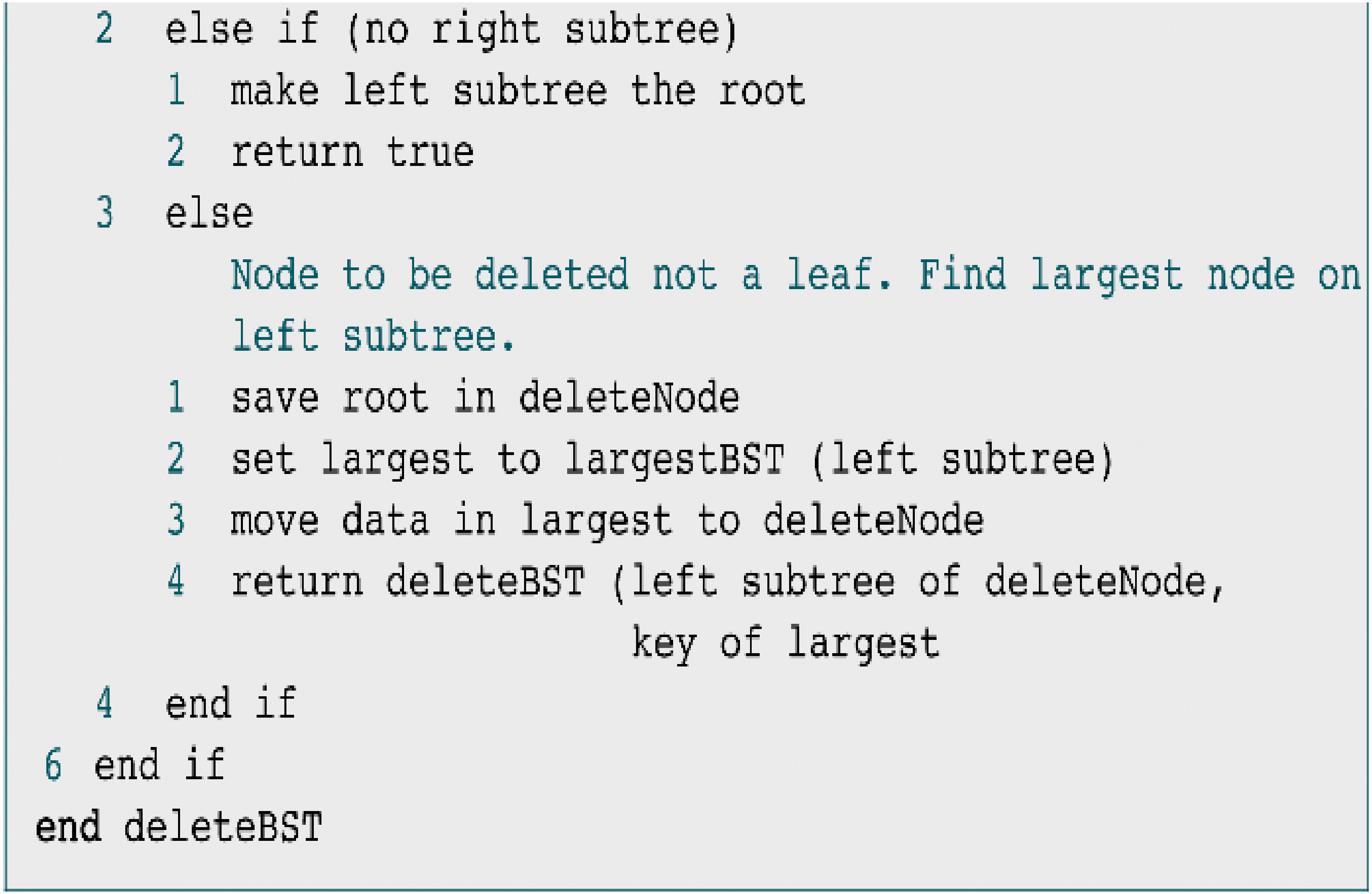
```
SearchTree Delete (int X, searchTree T)
{
int Tmpcell ;
if (T = = NULL)
Error ("Element not found");
else
if (X < T →Element) // Traverse towards left
T →Left = Delete (X, T Left);
else
if (X > T Element) // Traverse towards right
T →Right = Delete (X, T →Right);
// Found Element tobe deleted
else                                          // Two children
if (T→ Left && T→ Right)
{ // Replace with smallest data in right subtree
Tmpcell = FindMin (T→ Right);
T →Element = Tmpcell Element ;
T →Right = Delete (T →Element; T →Right);
}
else {// one or zero children
Tmpcell = T;
if (T →Left = = NULL)
T = T→ Right;
else if (T→ Right = = NULL)
T = T →Left ;
free (TmpCell);
}
return T; }
```

# Delete node from BST

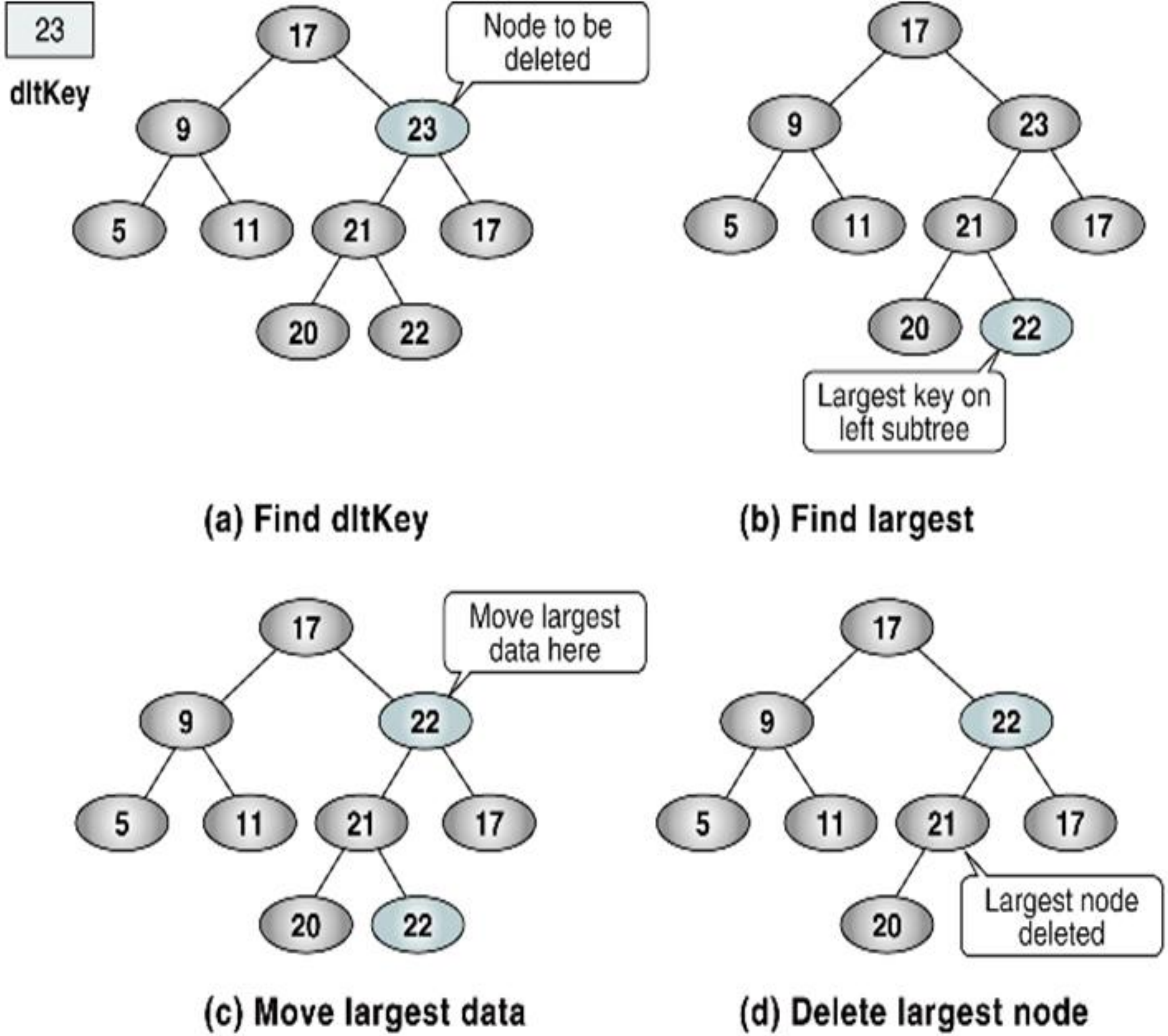


```
Algorithm deleteBST (root, dltKey)
This algorithm deletes a node from a BST.
   Pre     root is reference to node to be deleted
           dltKey is key of node to be deleted
   Post    node deleted
           if dltKey not found, root unchanged
   Return true if node deleted, false if not found
1  if (empty tree)
   1  return false
2  end if
3  if (dltKey < root)
   1  return deleteBST (left subtree, dltKey)
4  else if (dltKey > root)
   1  return deleteBST (right subtree, dltKey)
5  else
      Delete node found--test for leaf node
   1  If (no left subtree)
      1  make right subtree the root
      2  return true
```

# Delete node from BST (continued)



```
   2  else if (no right subtree)
      1  make left subtree the root
      2  return true
   3  else
         Node to be deleted not a leaf. Find largest node on
         left subtree.
      1  save root in deleteNode
      2  set largest to largestBST (left subtree)
      3  move data in largest to deleteNode
      4  return deleteBST (left subtree of deleteNode,
                                  key of largest
   4  end if
6  end if
end deleteBST
```



(a) Find dltKey

(b) Find largest

(c) Move largest data

(d) Delete largest node

# Delete BST Test Cases

# AVL Trees

These are self-adjusting, height-balanced binary search trees and are named after the inventors: Adelson-Velskii and Landis.

Definition:

The height of a binary tree is the maximum path length from the root to a leaf. A single-node binary tree has height 0, and an empty binary tree has height -1

- An **AVL tree** is a binary search tree in which every node is **height balanced**, that is, the difference in the heights of its two subtrees is at most 1.
- The **balance factor** of a node is the height of its right subtree minus the height of its left subtree. An equivalent definition, then, for an AVL tree is that it is a binary search tree in which each node has a balance factor of -1, 0, or +1.
- Note :balance factor of -1 means that the subtree is left-heavy, and
- a balance factor of +1 means that the subtree is right-heavy.

# AVL Trees

# AVL Trees

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Definition:

The height of a binary tree is the maximum path length from the root to a leaf. A single-node AVLtree has height 0, and an empty AVL tree has height -1

# AVL Tree

**Definition**

- Binary Search tree.
- If T is a nonempty binary Search tree with $T_L$ and $T_R$ as its left and right subtrees, then T is an AVL tree iff
  1. $T_L$ and $T_R$ are AVL trees, and
  2. **$|h_L - h_R| \leq 1$** where $h_L$ and $h_R$ are the heights of $T_L$ and $T_R$, respectively

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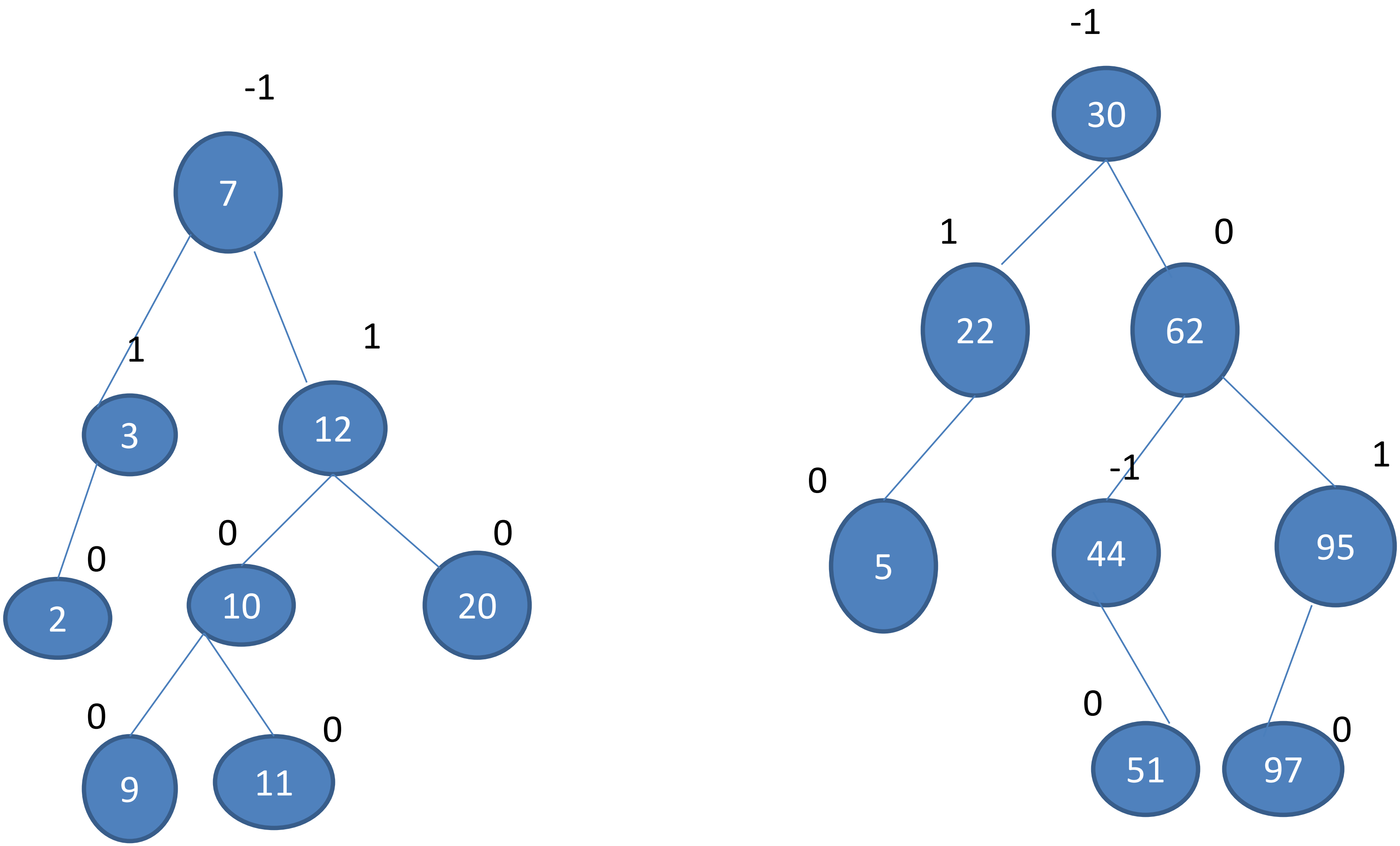
  2. **$|h_L - h_R| \leq 1$** where $h_L$ and $h_R$ are the heights of $T_L$ and $T_R$, respectively

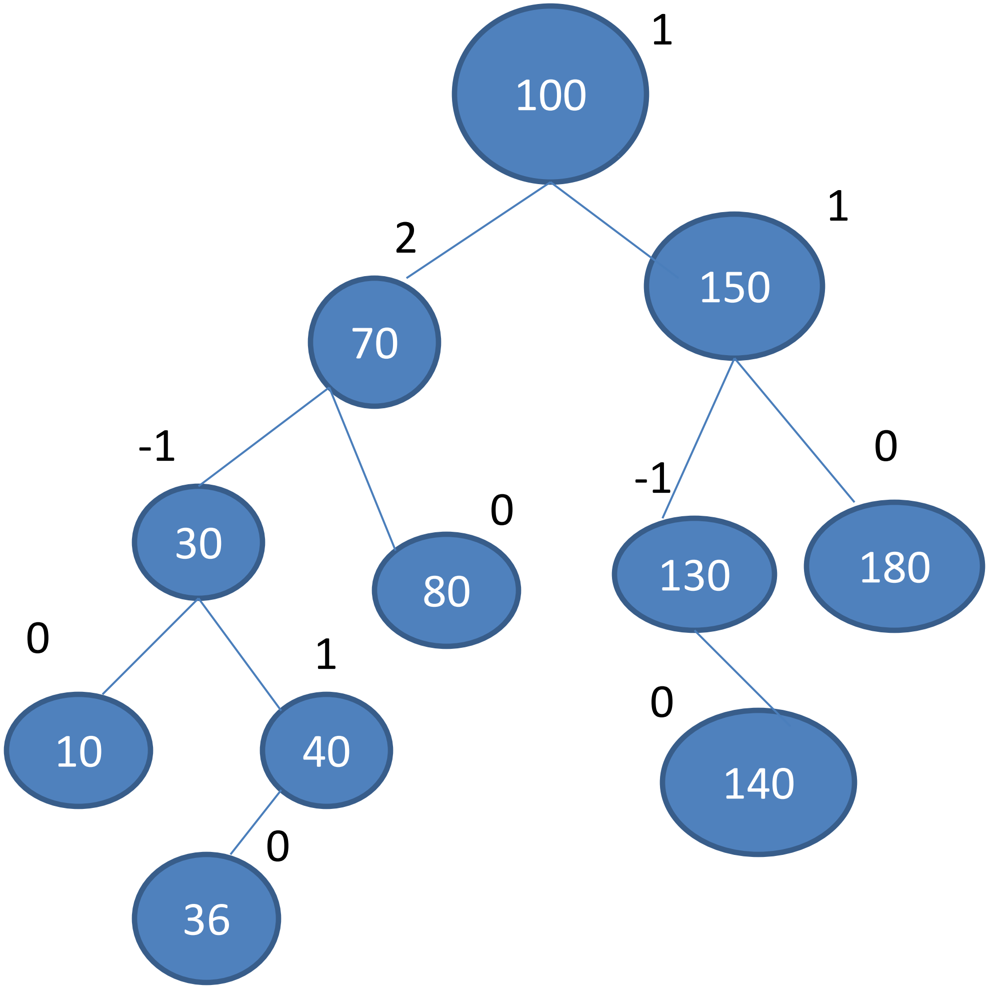
# Balance Factor

- **AVL trees are normally represented using the linked representation**
- **To facilitate insertion and deletion, a balance factor (bf) is associated with each node.**
- **The balance factor bf(x) of a node x is defined as**
  **height(x→leftChild) – height(x→rightChild)**
- **Balance factor of each node in an AVL tree must be –1, 0, or 1**

# Eg with balance factors



# Not an AVL TREE



# AVL tree structure in C

**For each node, the difference of height between left and right are no more than 1.**

```
struct AVLnode_s {
 Datatype element;
 struct AVLnode *left;
 struct AVLnode *right;
};
typedef struct AVLnode_s AVLnode;
```

# Inserting into an AVL Search Trees

- **If we insert an element into an AVL search tree, the result may not be an AVL tree**
- **That is, the tree may become unbalanced**
- **If the tree becomes unbalanced, we must adjust the tree to restore balance - this adjustment is called rotation.**
- **There are Four Models of rotations:**
- **There are four models about the operation of AVL Tree:**

1. **LL: new node is in the left subtree of the left subtree of A**
2. **LR: new node is in the right subtree of the left subtree of A**
3. **RR: new node is in the right subtree of the right subtree of A**
4. **RL: new node is in the left subtree of the right subtree of A**

# Rotation

**Definition**

- **To switch *children* and *parents* among two or three adjacent nodes to restore balance of a tree.**
- **A rotation may change the depth of some nodes, but does not change their relative ordering.**

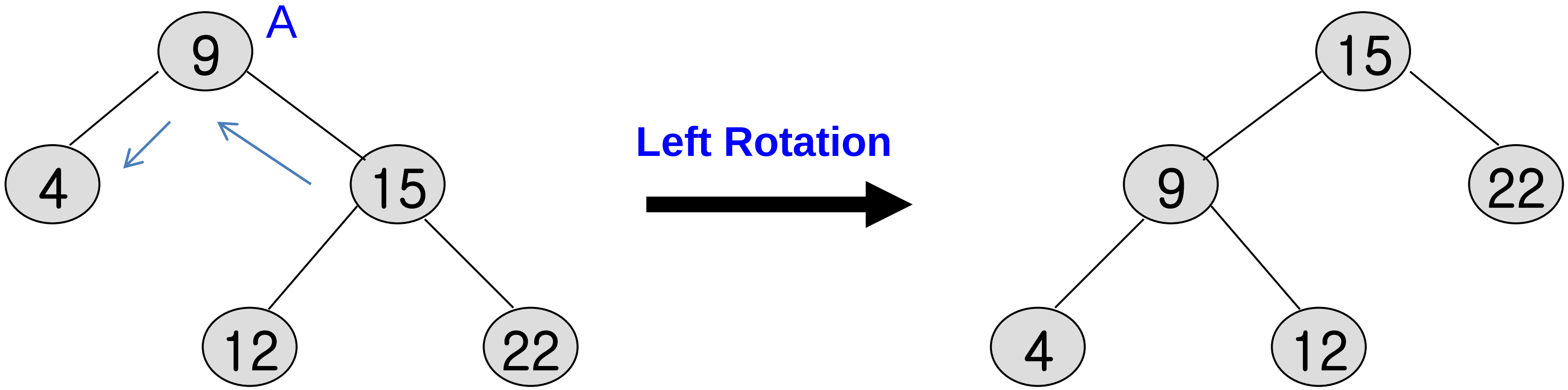
# Single and Double Rotations

- **Single rotations: the transformations done to correct LL and RR imbalances**
- **Double rotations: the transformations done to correct LR and RL imbalances**
- **The transformation to correct LR imbalance can be achieved by an RR rotation followed by an LL rotation**
- **The transformation to correct RL imbalance can be achieved by an LL rotation followed by an RR rotation**

# Left Rotation

**Definition**

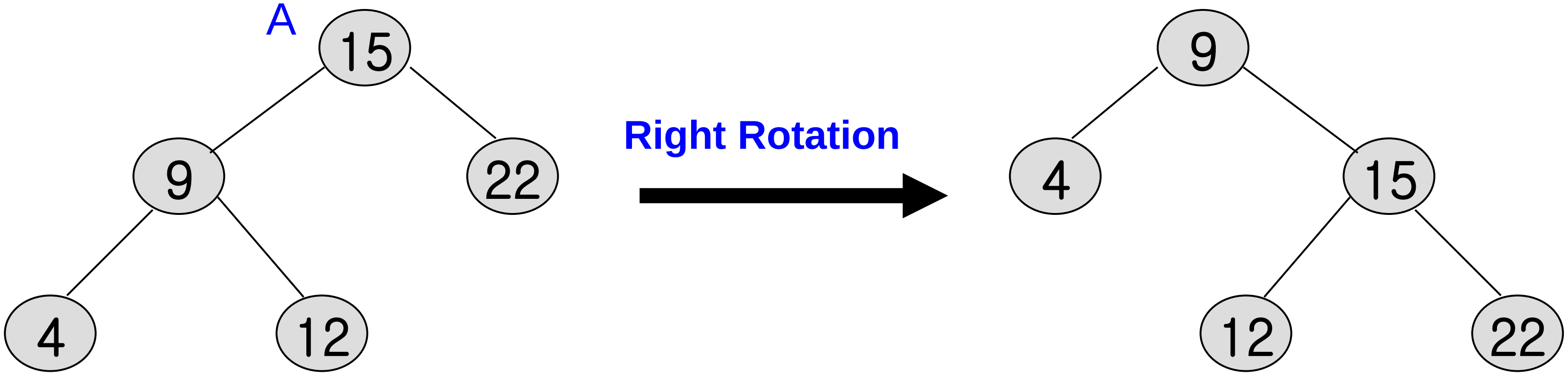
- **In a binary search tree, pushing a node A down and to the left to balance the tree.**
- **A's right child replaces A, and the right child's left child becomes A's right child.**



# Right Rotation

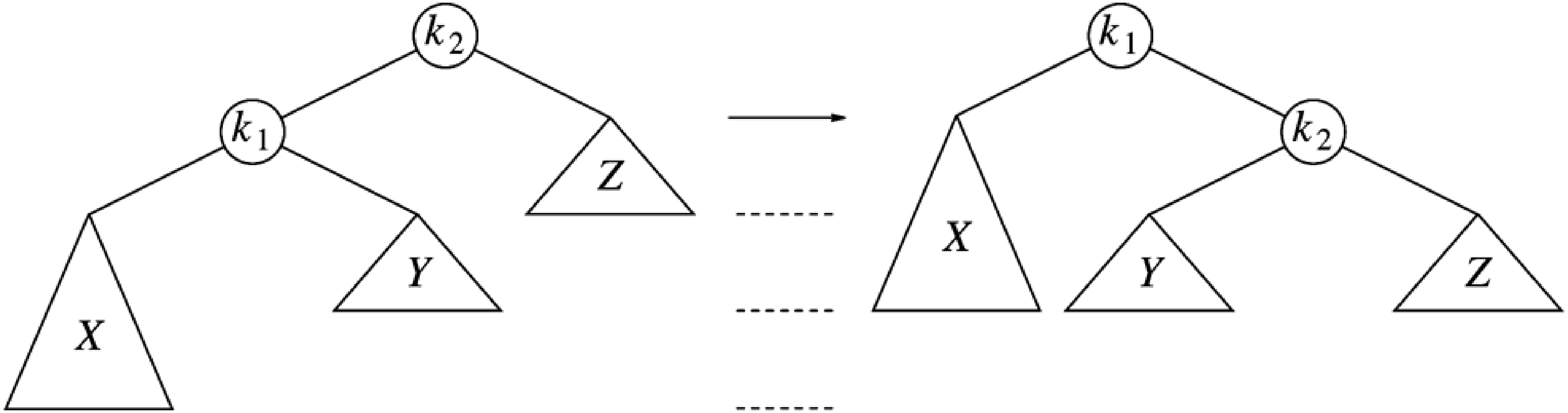
**Definition**

- **In a binary search tree, pushing a node A down and to the right to balance the tree.**
- **A's left child replaces A, and the left child's right child becomes A's left child.**



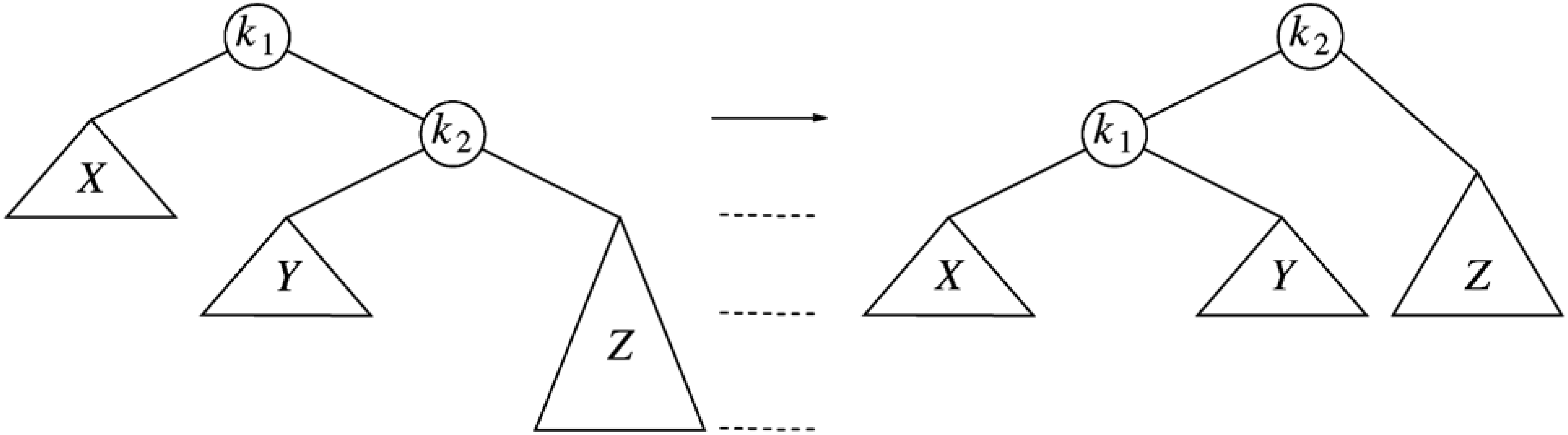
# Single Rotation (LL)

- **Let *$k_2$* be the first node on the path <u>up</u> violating AVL balance property. <u>Figure below is the only possible scenario</u> that allows *$k_2$* to satisfy the AVL property before the insertion but violate it afterwards. Subtree *X* has grown an extra level (2 levels deeper than *Z* now). *Y* cannot be at the same level as *X* (*$k_2$* then out of balance before insertion) and *Y* cannot be at the same level as *Z* (then *$k_1$* would be the first to violate).**



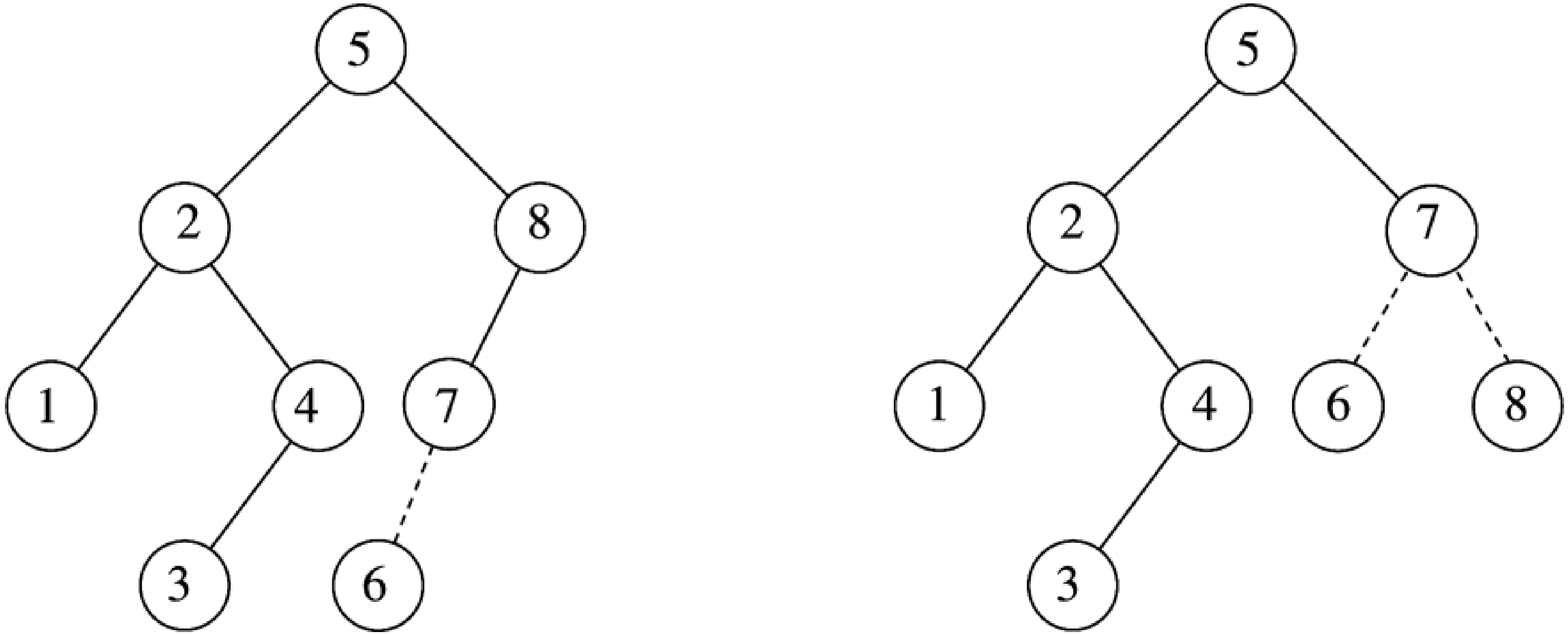
# Single Rotation (RR)

- **Note that in single rotation inorder traversal orders of the nodes are preserved.**
- **The new height of the subtree is exactly the same as before. Thus no further updating of the nodes on the path to the root is needed.**



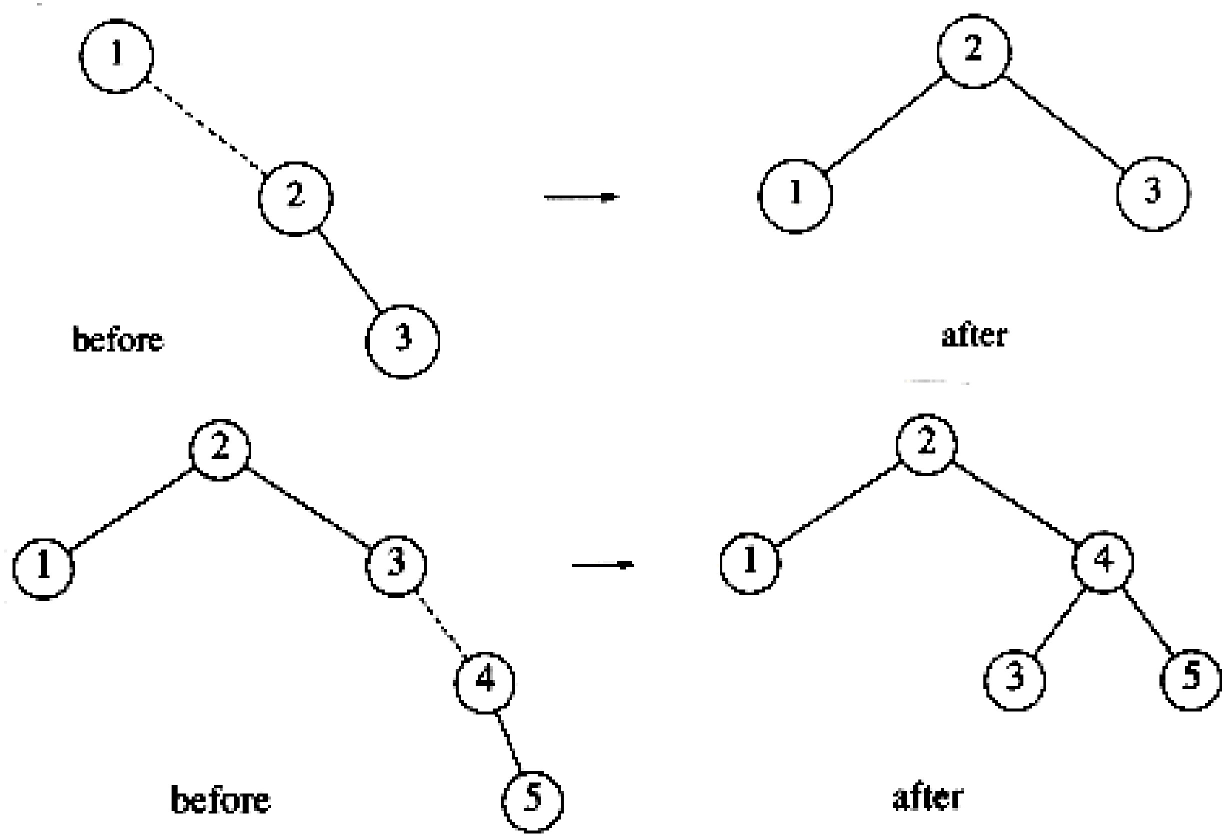
# Single Rotation-Example I

- AVL property destroyed by insertion of 6, then fixed by a single rotation.
- BST node structure needs an additional field for height.



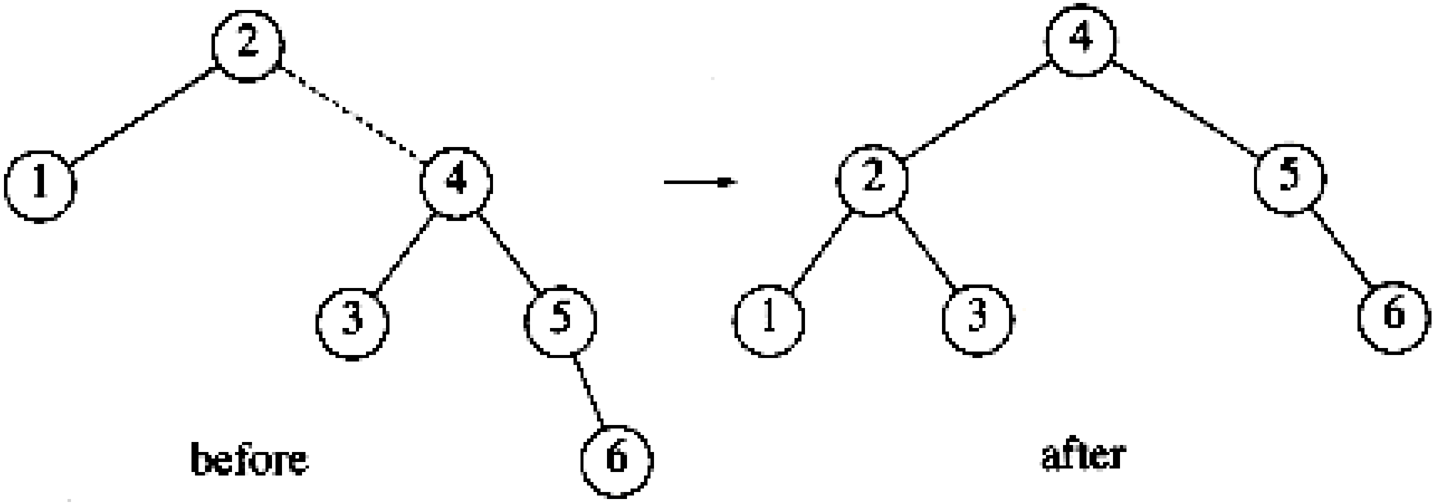
# Single Rotation-Example II

- Start with an initially empty tree and insert items 1 through 7 sequentially. Dashed line joins the two nodes that are the subject of the rotation.

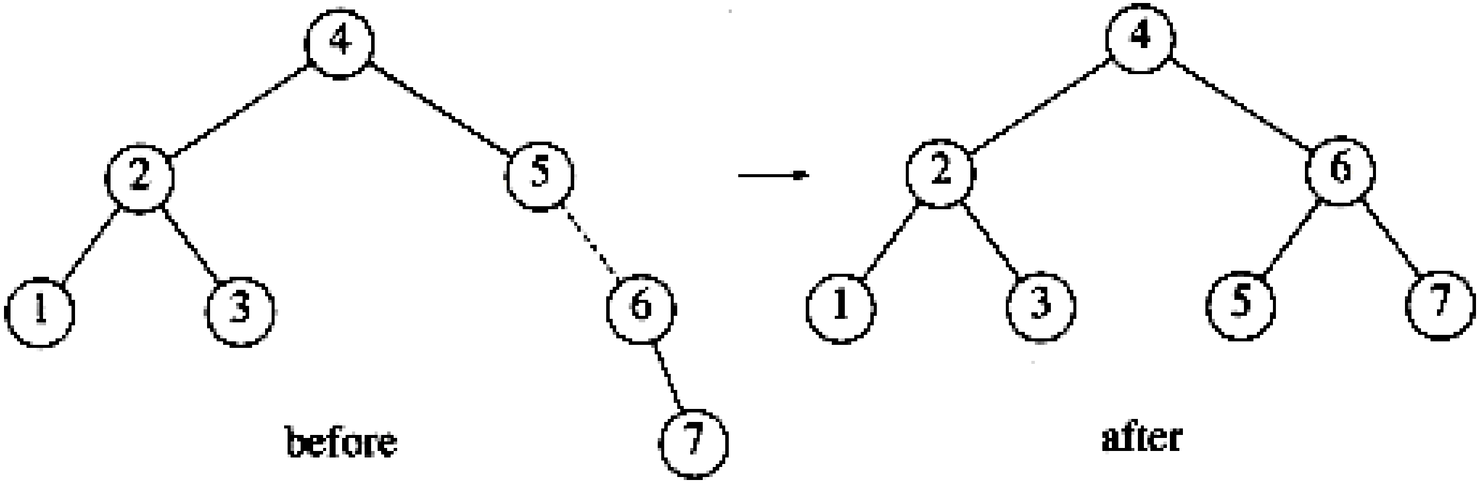


# Single Rotation-Example III

Insert 6. Balance problem at th root. So a single rotatio is performed.

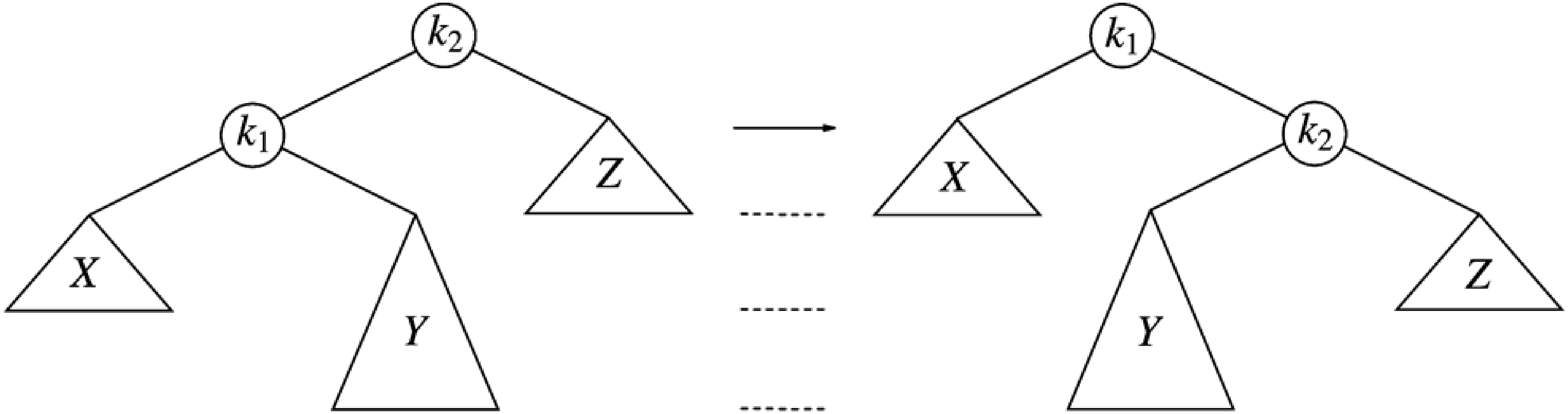


Finally, Insert 7 causing another rotation.



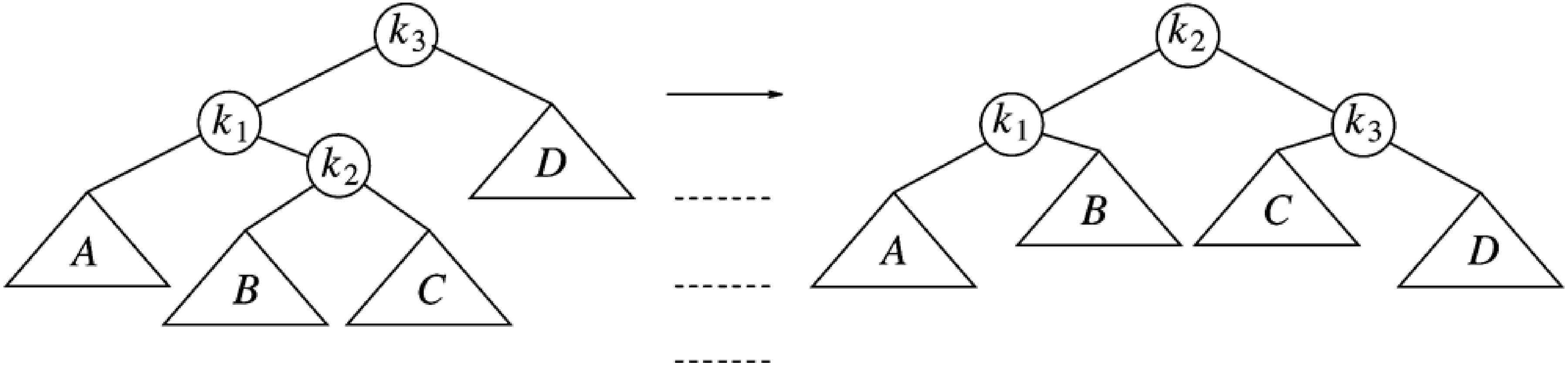
# Double Rotation (LR, RL) - I

- The algorithm that works for cases 1 and 4 (LL, RR) does not work for cases 2 and 3 (LR, RL). The problem is that subtree *Y* is too deep, and a single rotation does not make it any less deep.
- The fact that subtree *Y* has had an item inserted into it guarantees that it is nonempty. Assume it has a root and two subtrees.



# Double Rotation (LR) - II

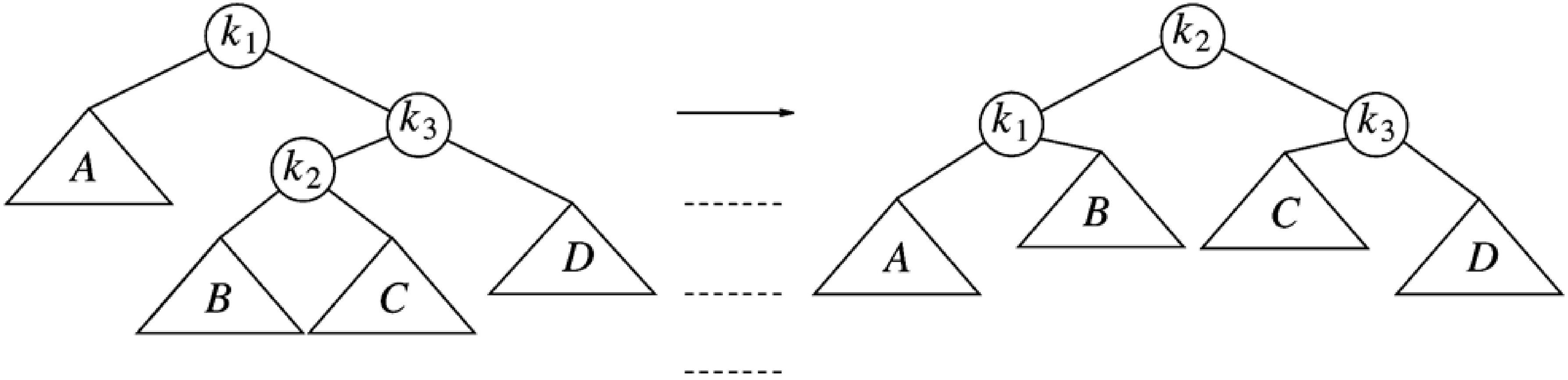
**Below are 4 subtrees connected by 3 nodes. Note that exactly one of tree *B* or *C* is 2 levels deeper than *D* (unless all empty). To rebalance, $k_3$ cannot be root and a rotation between $k_1$ and $k_3$ was shown not to work. So the only alternative is to place $k_2$ as the new root. This forces $k_1$ to be $k_2$'s left child and $k_3$ to be its right child. It also completely determines the locations of all 4 subtrees. AVL balance property is now satisfied. Old height of the tree is restored; so, all the balancing and and height updating is complete.**



# Double Rotation (RL) - III

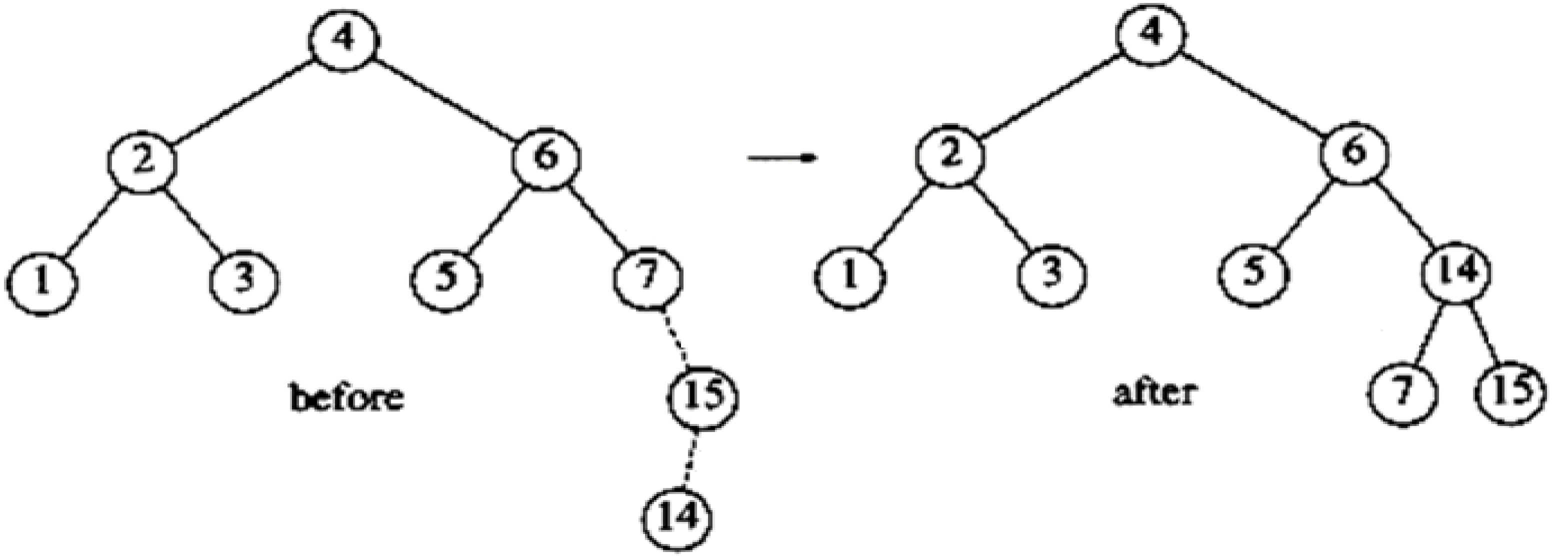
**In both cases (LR and RL), the effect is the same as rotating between *α*'s child and grandchild and then between *α* and its new child. Every double rotation can be modelled in terms of 2 single rotations. Inorder traversal orders are always preserved between $k_1$, $k_2$, and $k_3$.**

**Double RL = Single LL (*α*->right)+ Single RR (*α*)**
**Double LR = Single RR (*α*->left)+ Single LL (*α* )**



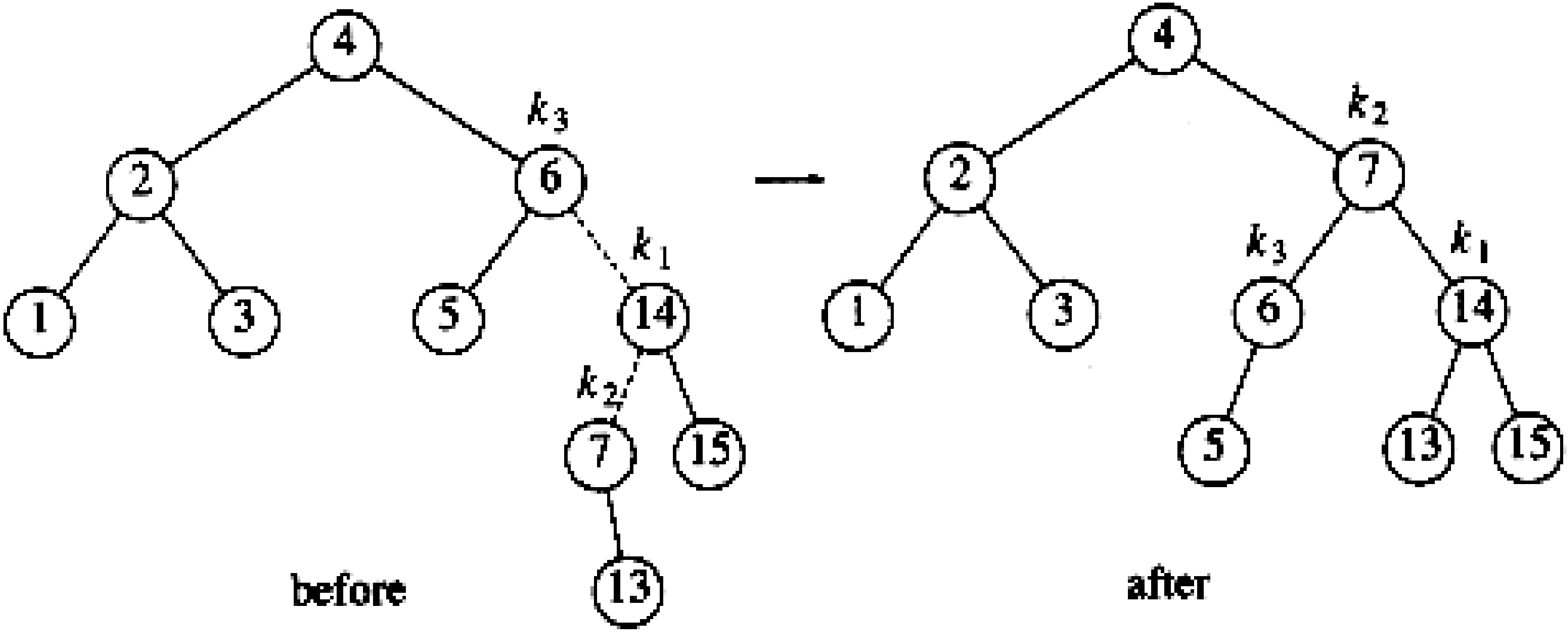
# Double Rotation Example - I

- Continuing our example, suppose keys 8 through 15 are inserted in reverse order. Inserting 15 is easy but inserting 14 causes a height imbalance at node 7. The double rotation is an RL type and involves 7, 15, and 14.



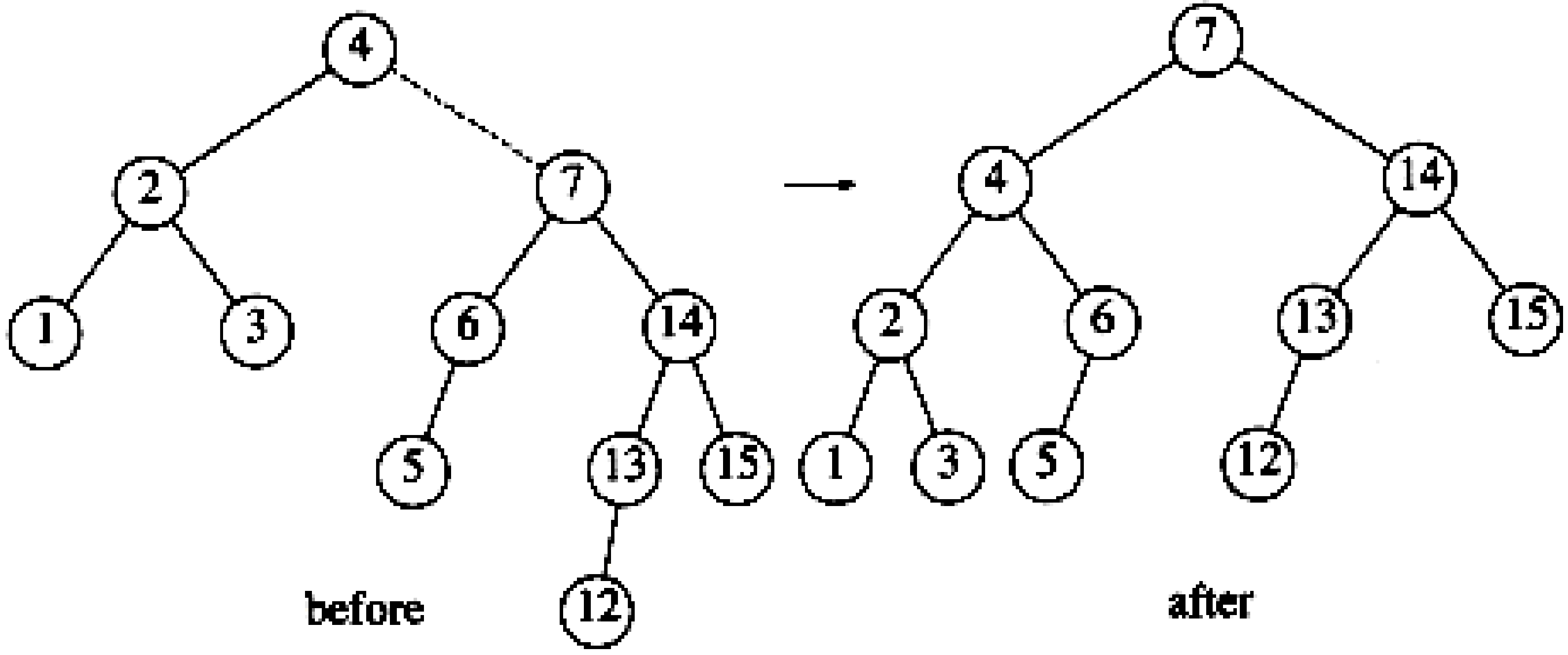
# Double Rotation Example - II

- **insert 13: double rotation is RL that will involve 6, 14, and 7 and will restore the tree.**



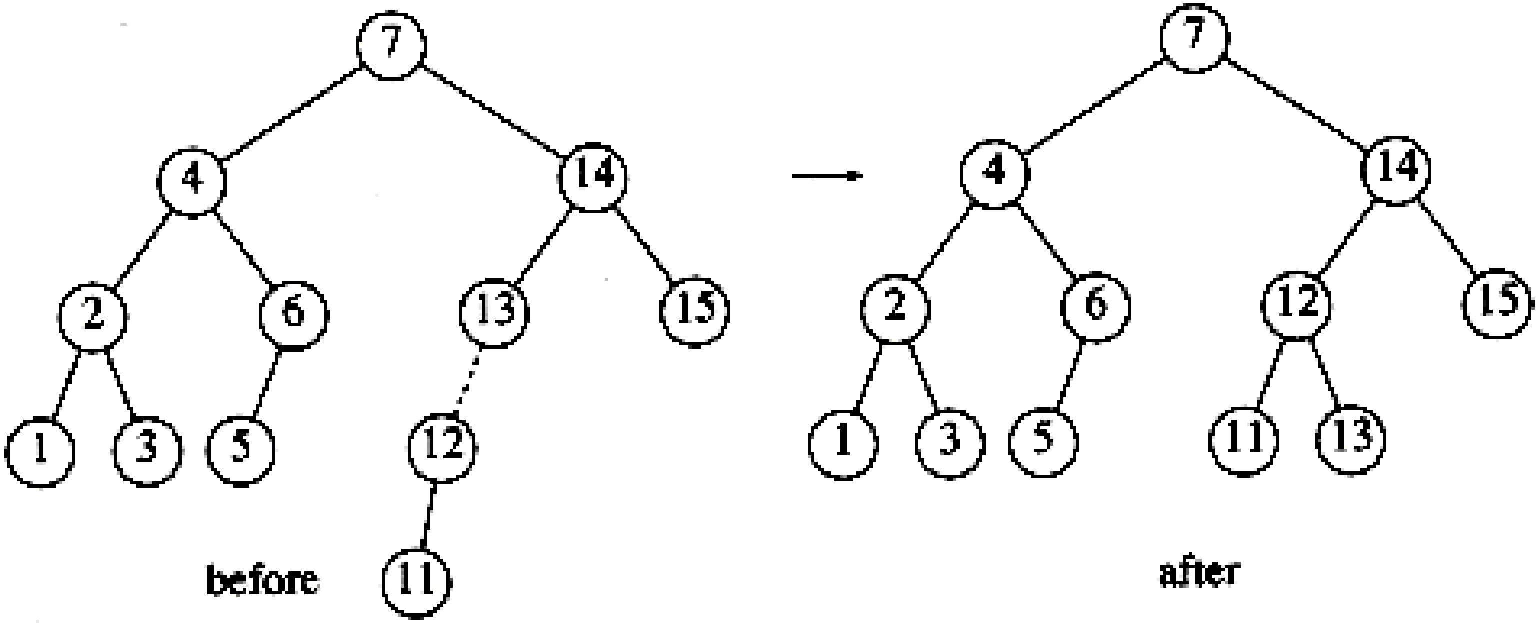
# Double Rotation Example - III

- **If 12 is now inserted, there is an imbalance at the root. Since 12 is not between 4 and 7, we know that the single rotation RR will work.**



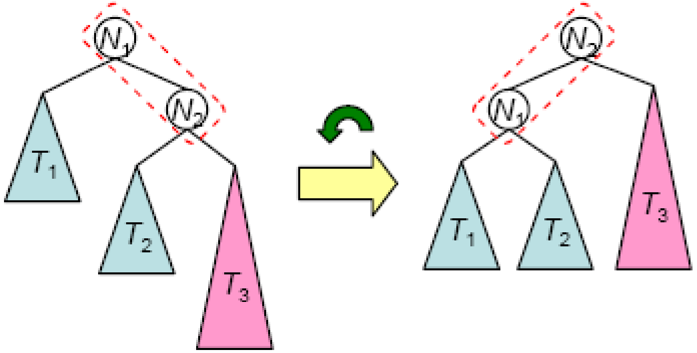
# Double Rotation Example - IV

- **Insert 11: single rotation LL; insert 10: single rotation LL; insert 9: single rotation LL; insert 8: without a rotation.**



Case 1: insertion to *right* subtree of *right* child

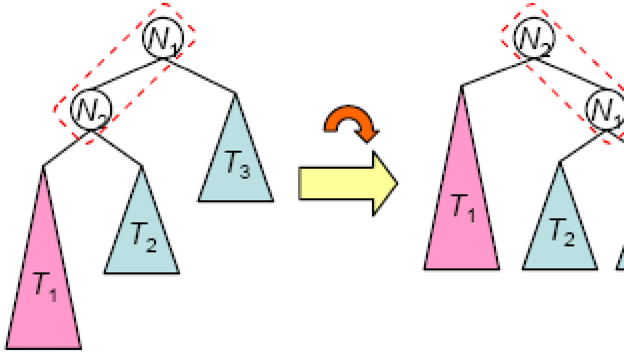
Solution: *Left* rotation



Case 2: insertion to *le*

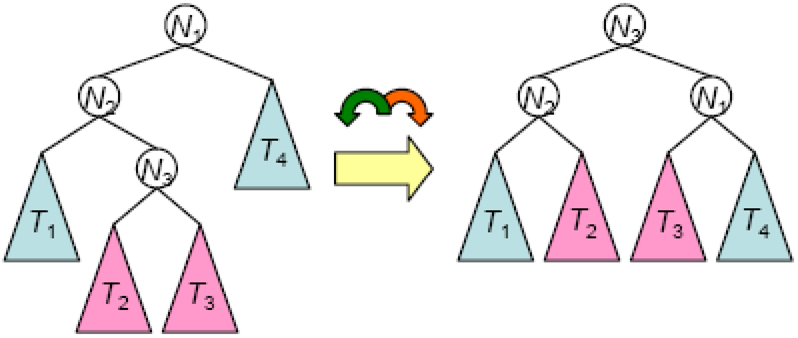
subtree of *left* child

Solution: *Right* rotatio



Case 3: insertion to *right* subtree of *left* child

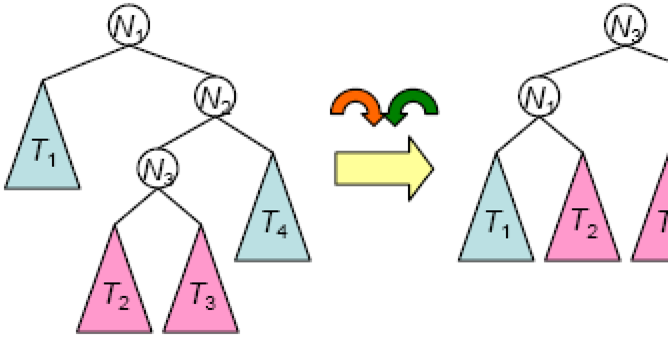
Solution: *Left*-*right* rotation



Case 4: insertion to *le*

subtree of *right* child

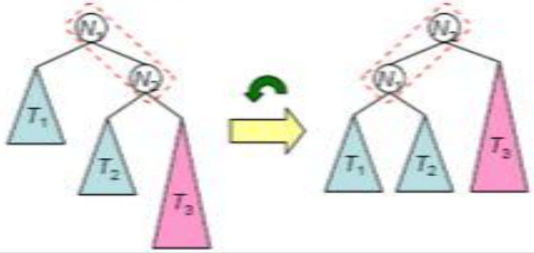
Solution: *Right*-*left* rota

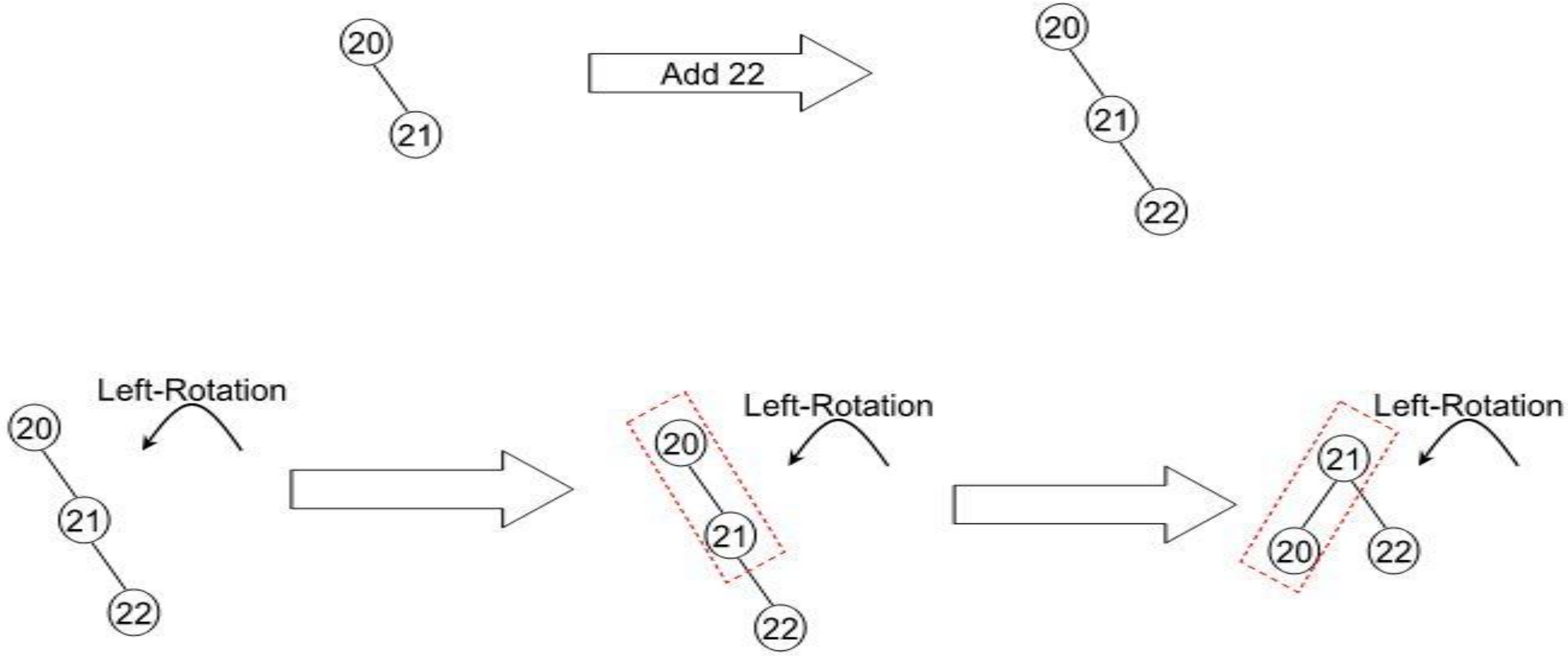


# examples
# Left-Rotation

Case 1: insertion to *right* subtree of *right* child

Solution: *Left* rotation

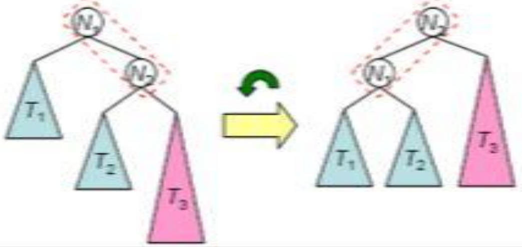


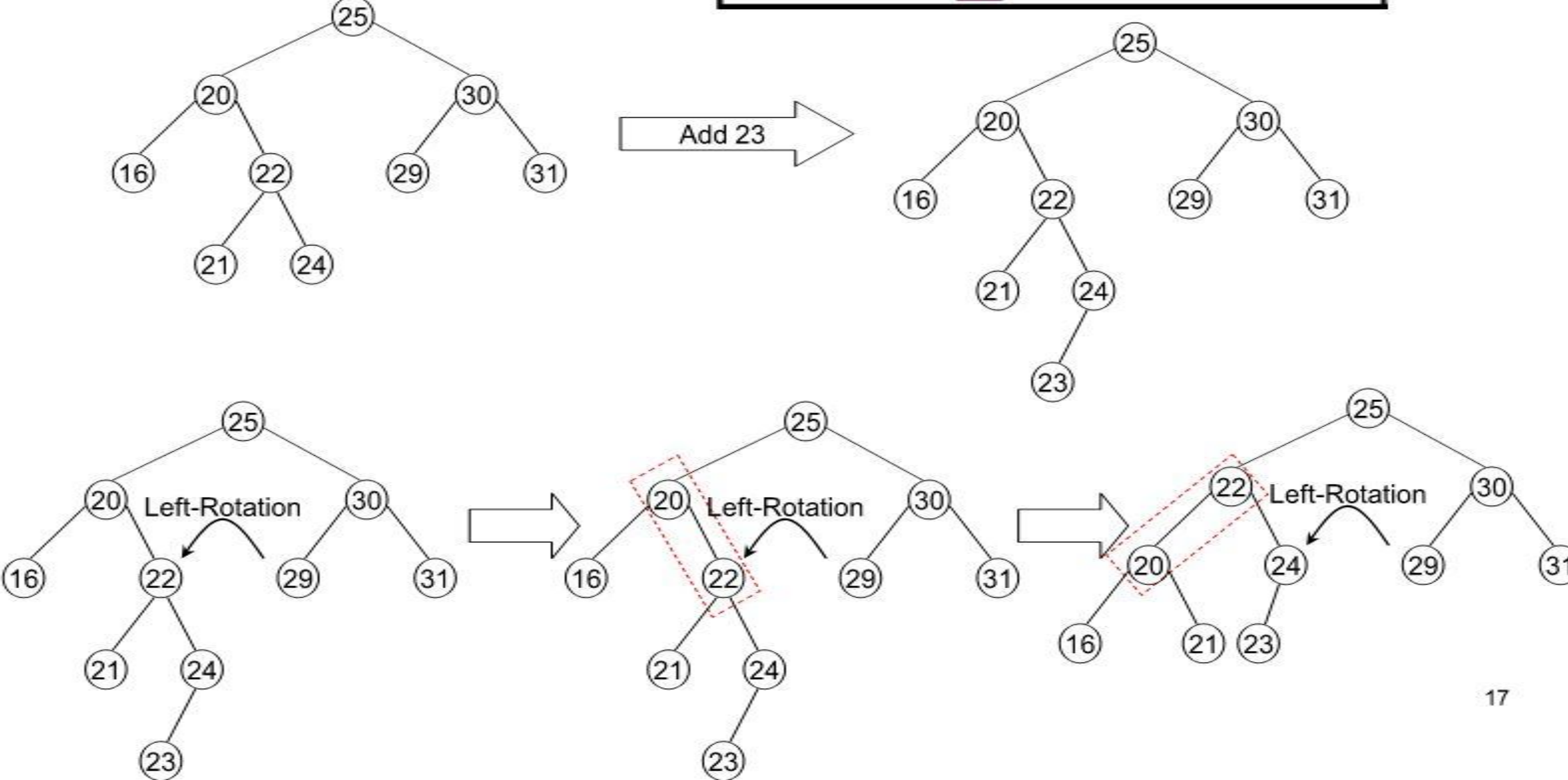


# Left-Rotation

Case 1: insertion to *right* subtree of *right* child

Solution: *Left* rotation

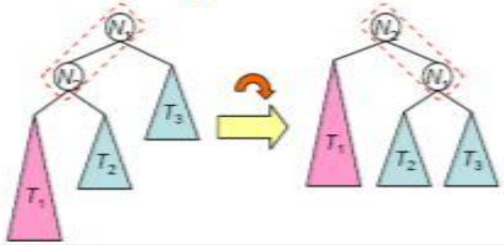


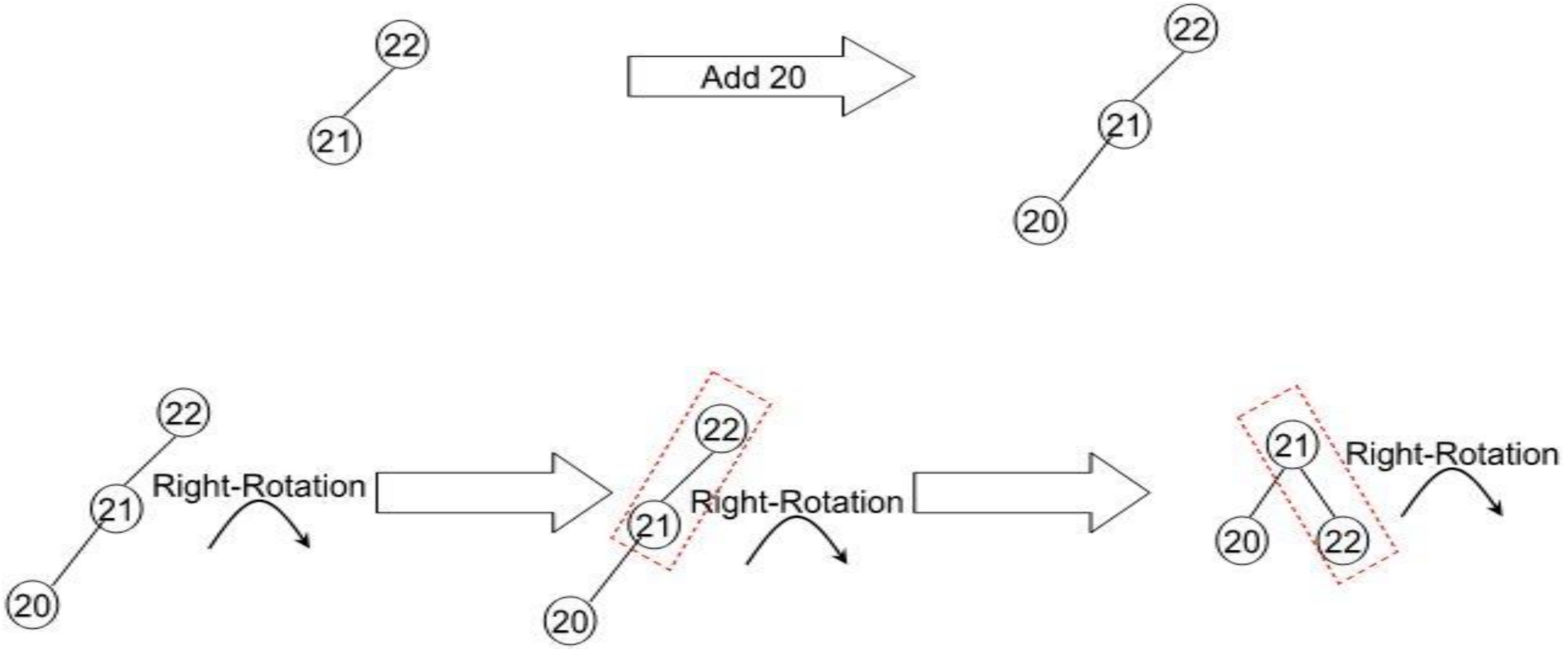


# Right-Rotation

Case 2: insertion to *left* subtree of *left* child

Solution: *Right* rotation




Add 20

Right-Rotation

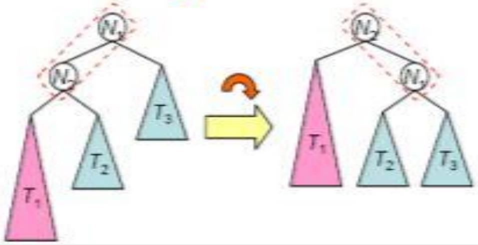
Right-Rotation

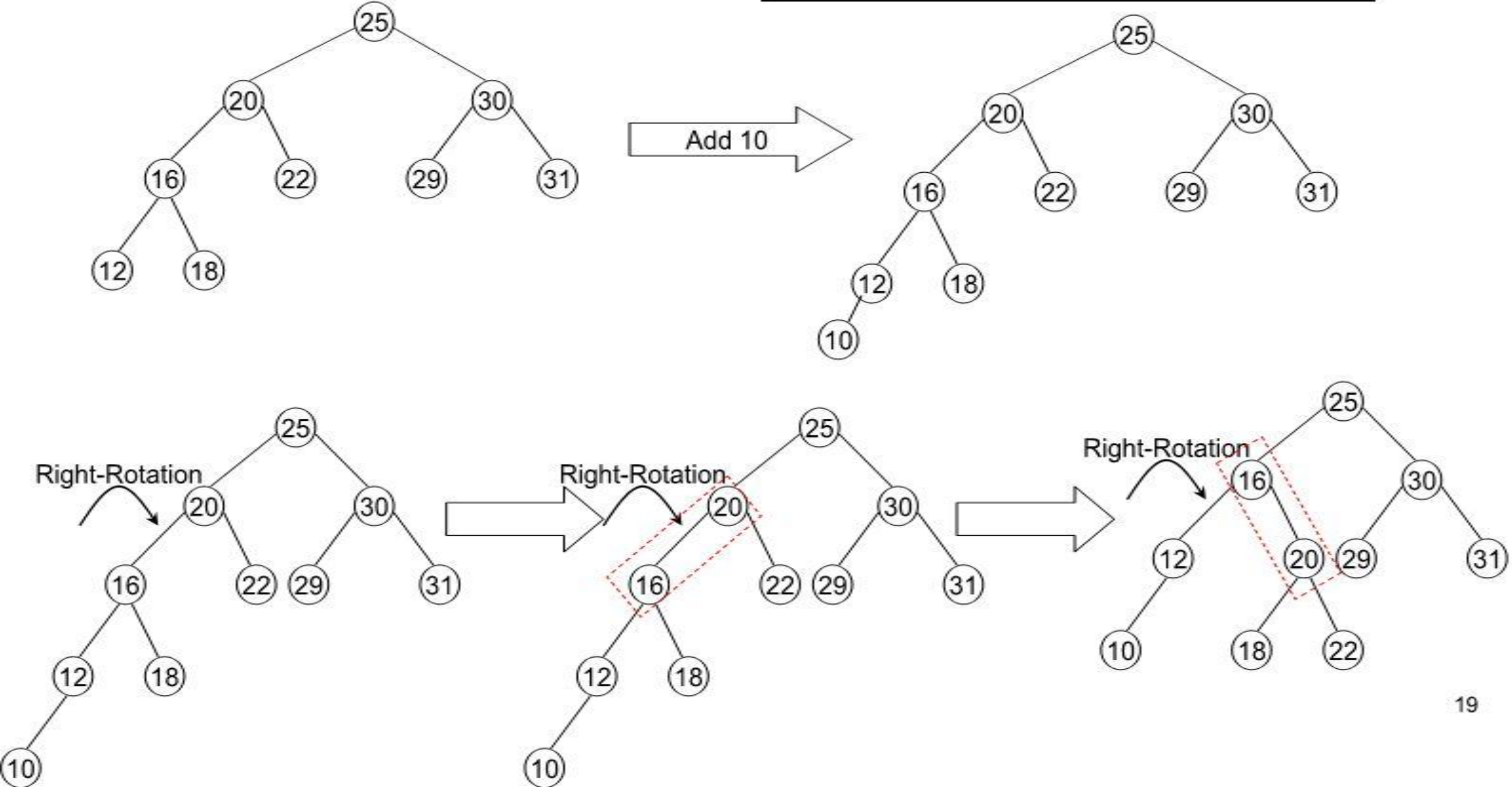
Right-Rotation

# Right-Rotation

Case 2: insertion to *left* subtree of *left* child

Solution: *Right* rotation

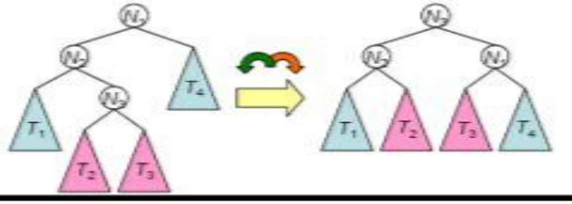


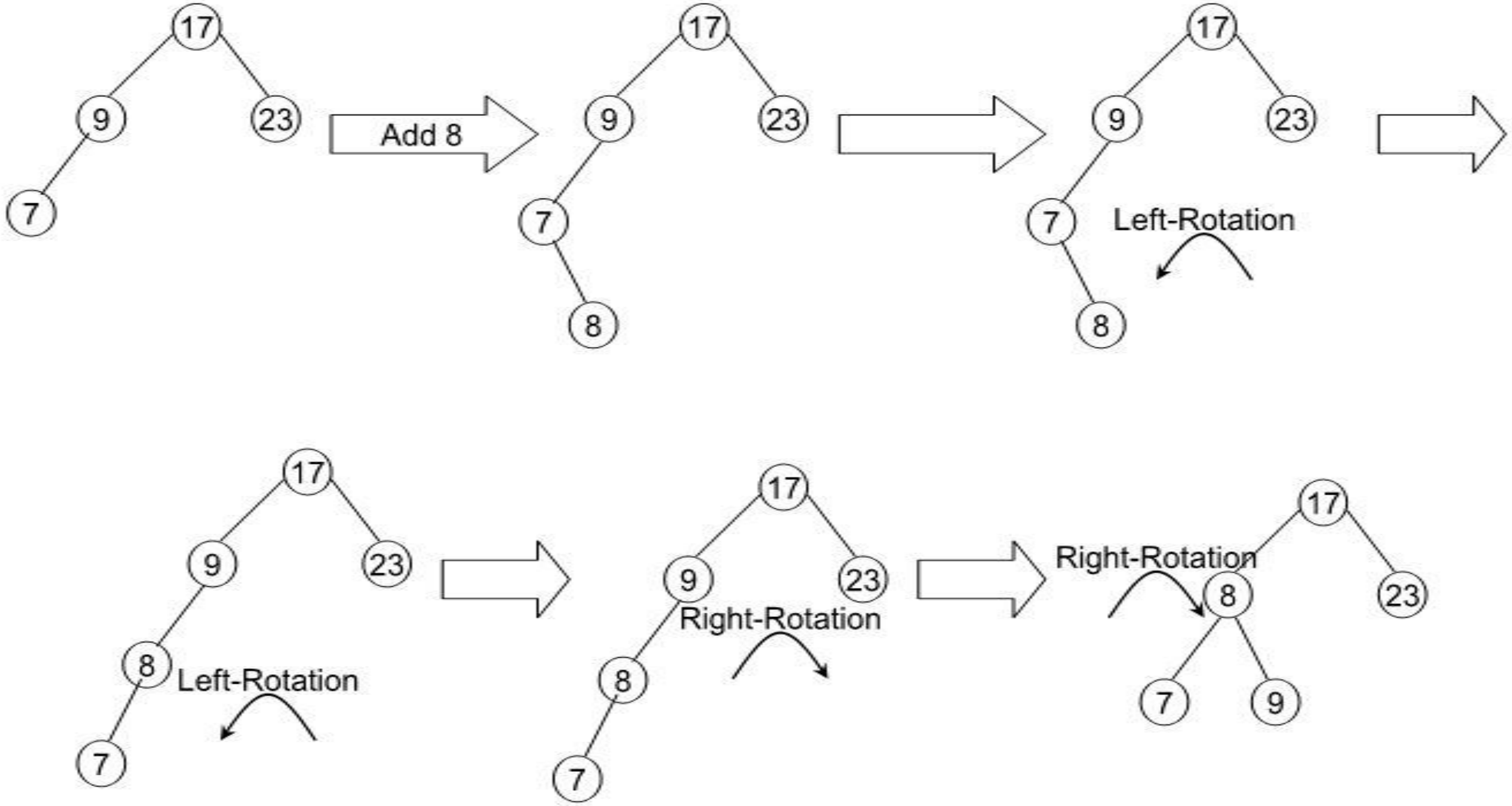


# Left-Right Rotation

Case 3: insertion to *right* subtree of *left* child

Solution: *Left*-*right* rotation

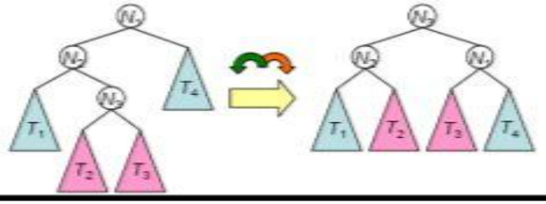


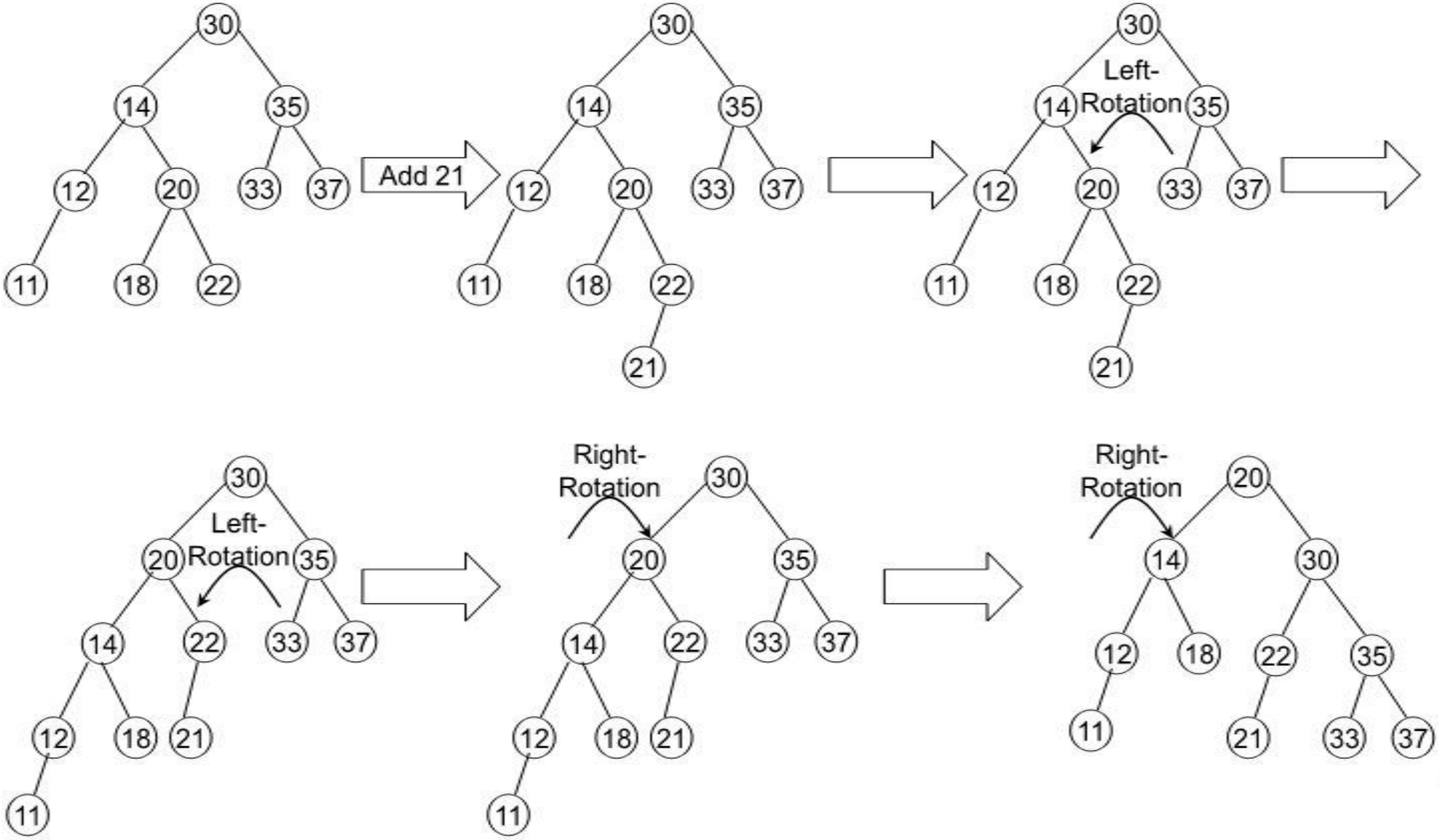


# Left-Right Rotation

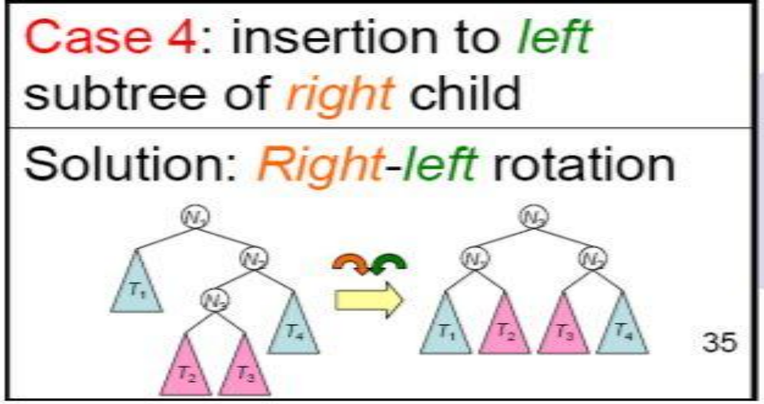
Case 3: insertion to *right* subtree of *left* child

Solution: *Left-right* rotation





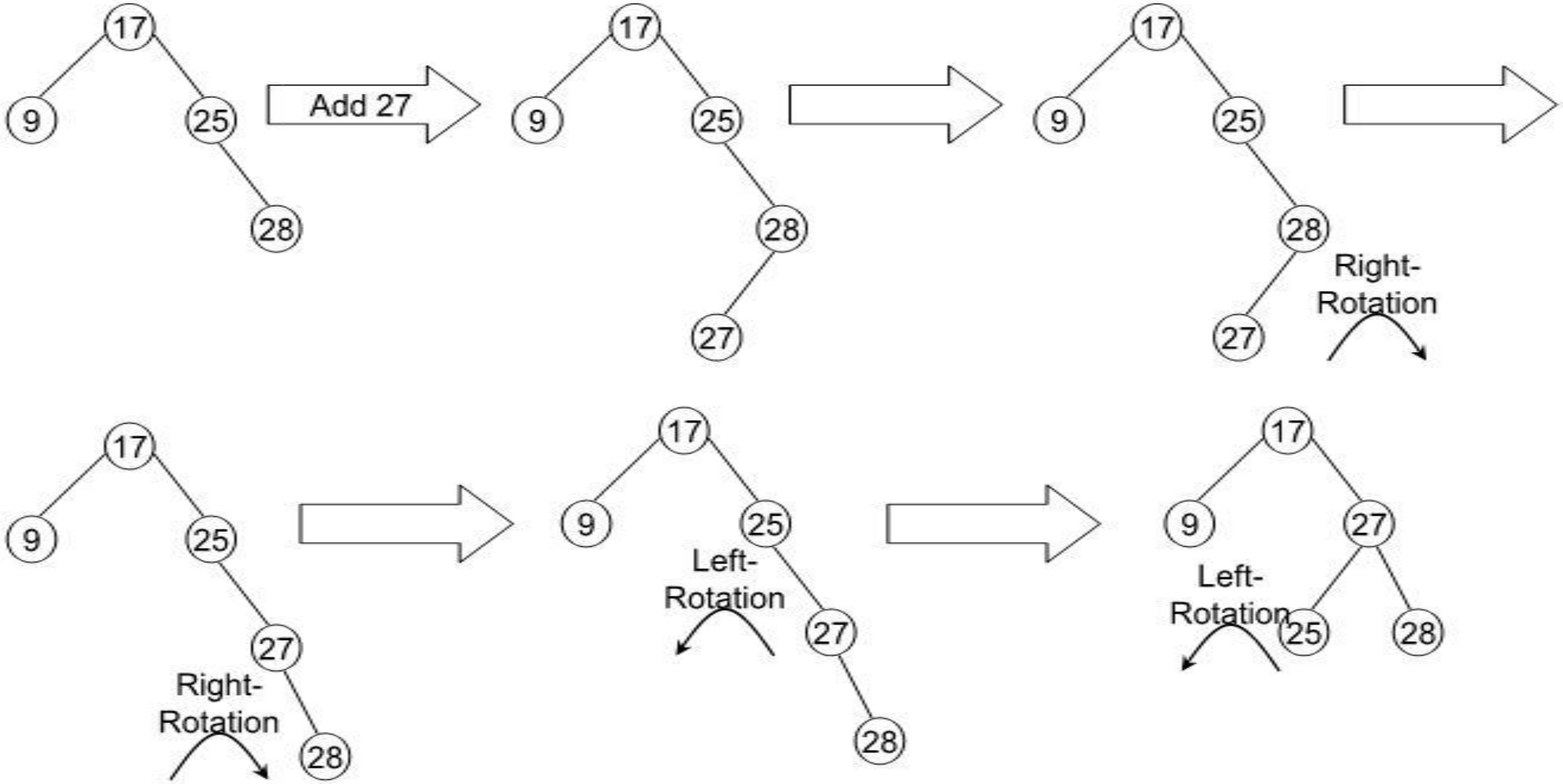
# Right-Left Rotation



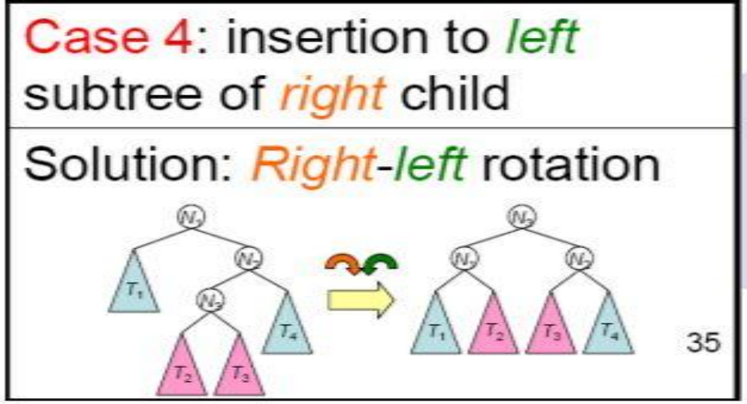
Case 4: insertion to *left* subtree of *right* child

Solution: *Right*-*left* rotation

N1
N2
N3
T1
T2
T3
T4
35



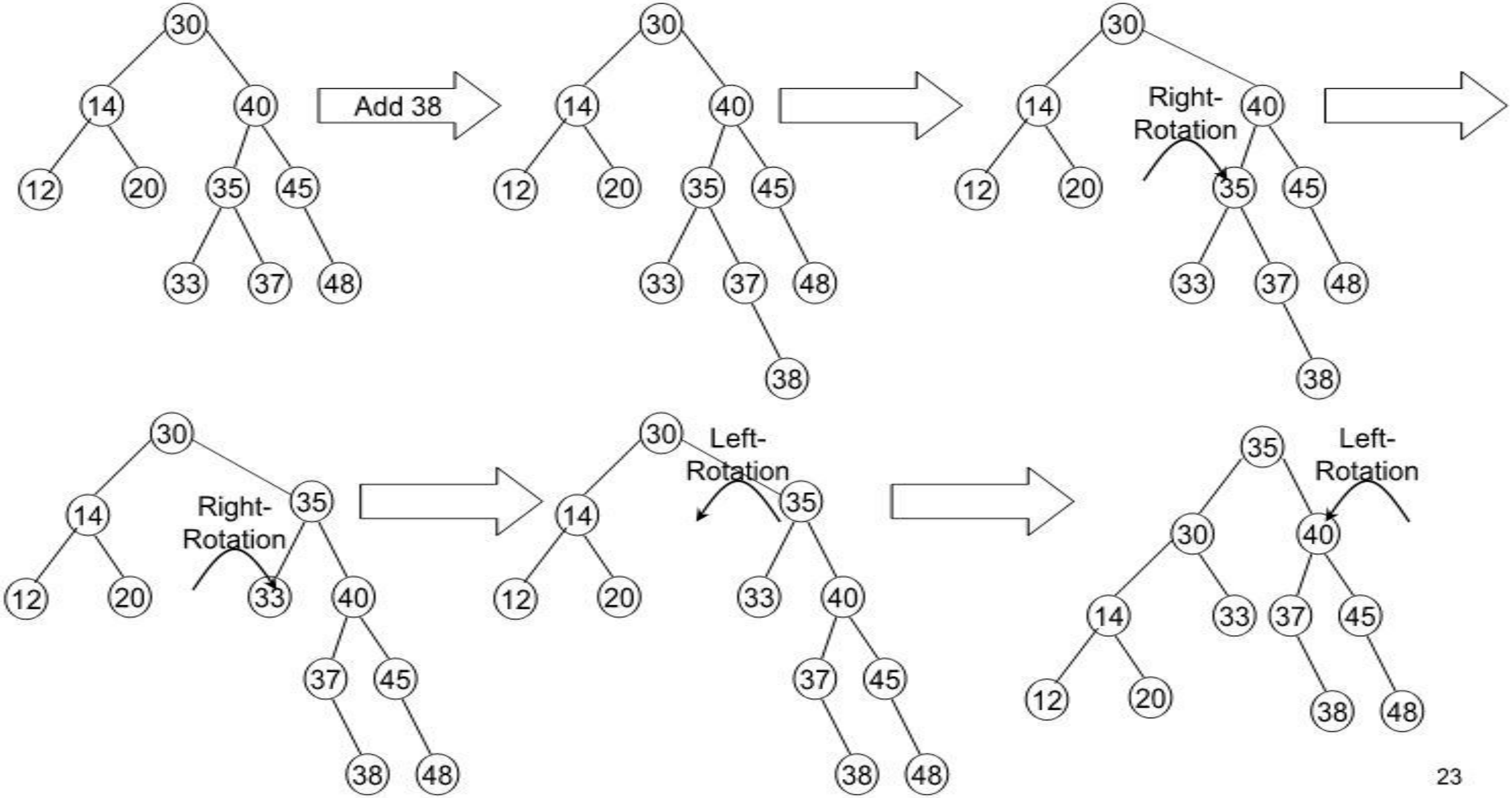
# Right-Left Rotation



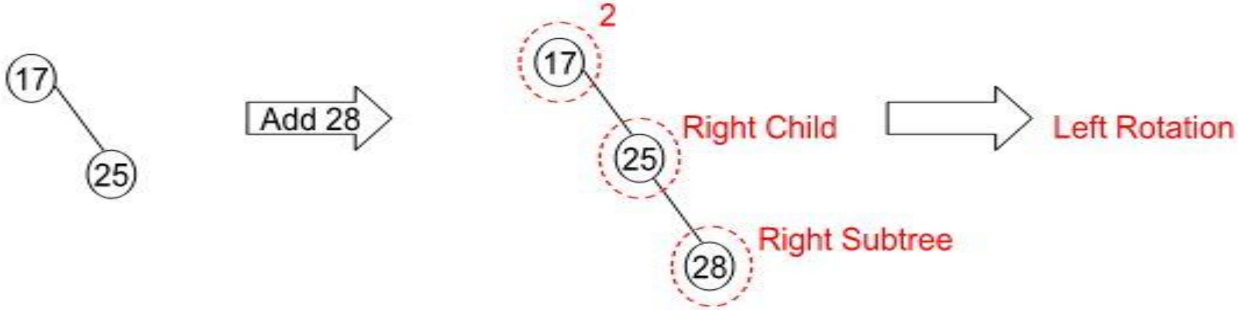
Case 4: insertion to *left* subtree of *right* child

Solution: *Right*-*left* rotation

35

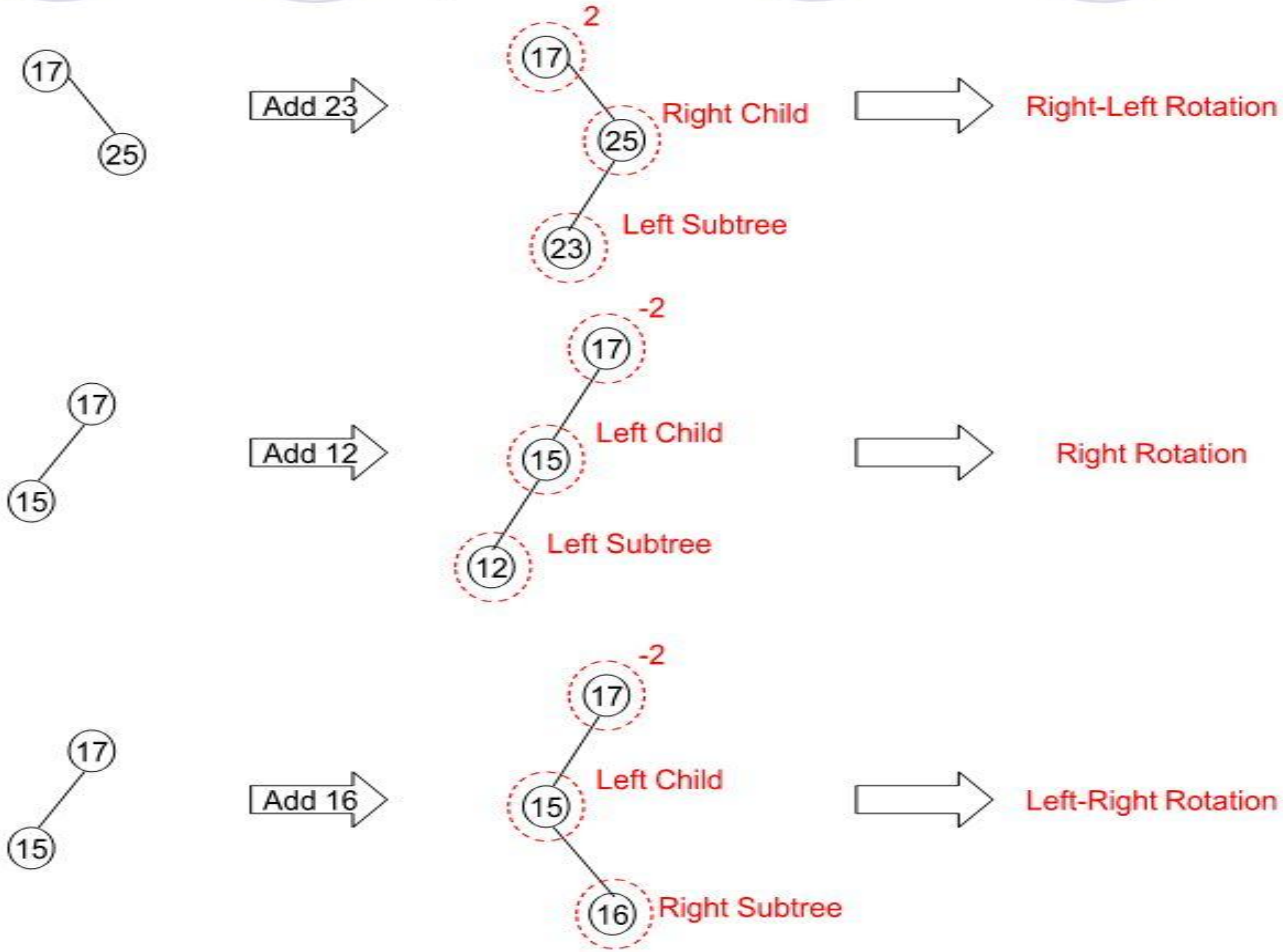


# How to identify rotations?



- First find the node that cause the imbalance (balance factor)
- Then find the corresponding child of the imbalanced node (left node or right node)
- Finally find the corresponding subtree of that child (left or right)

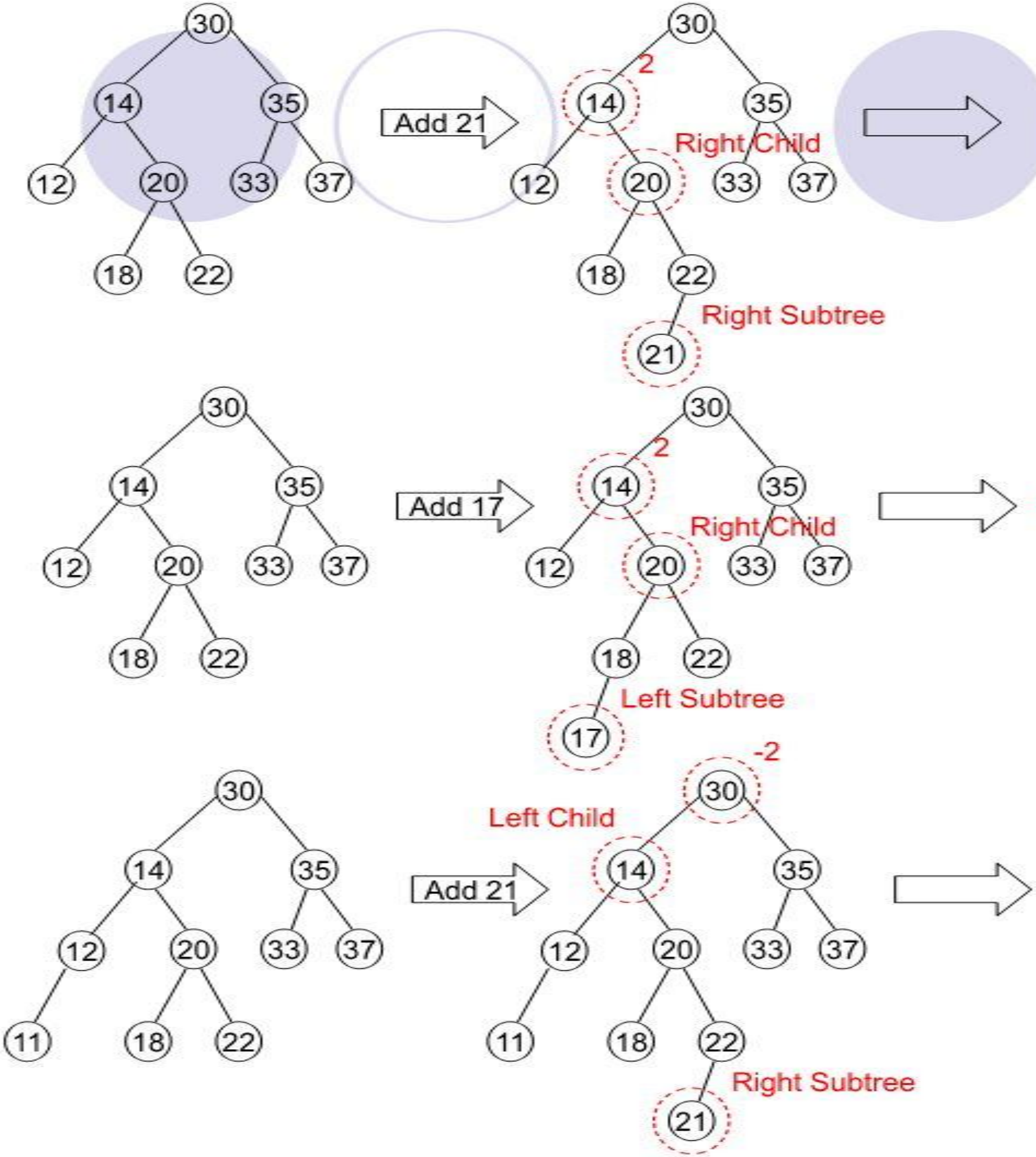
# How to identify rotations?



Add 23 → Right Child, Left Subtree → Right-Left Rotation

Add 12 → Left Child, Left Subtree → Right Rotation

Add 16 → Left Child, Right Subtree → Left-Right Rotation



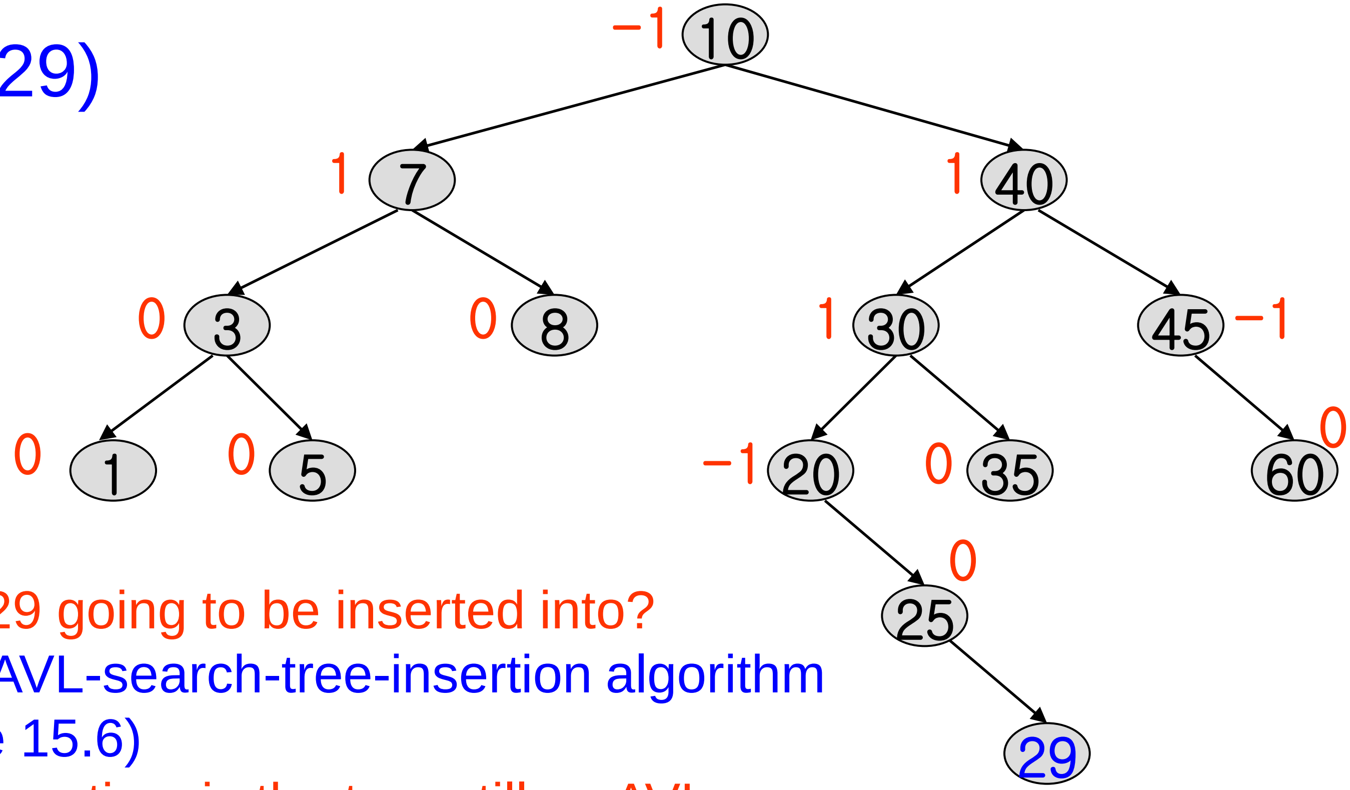
Left Rotation

Right-Left Rotation

Left-Right Rotation

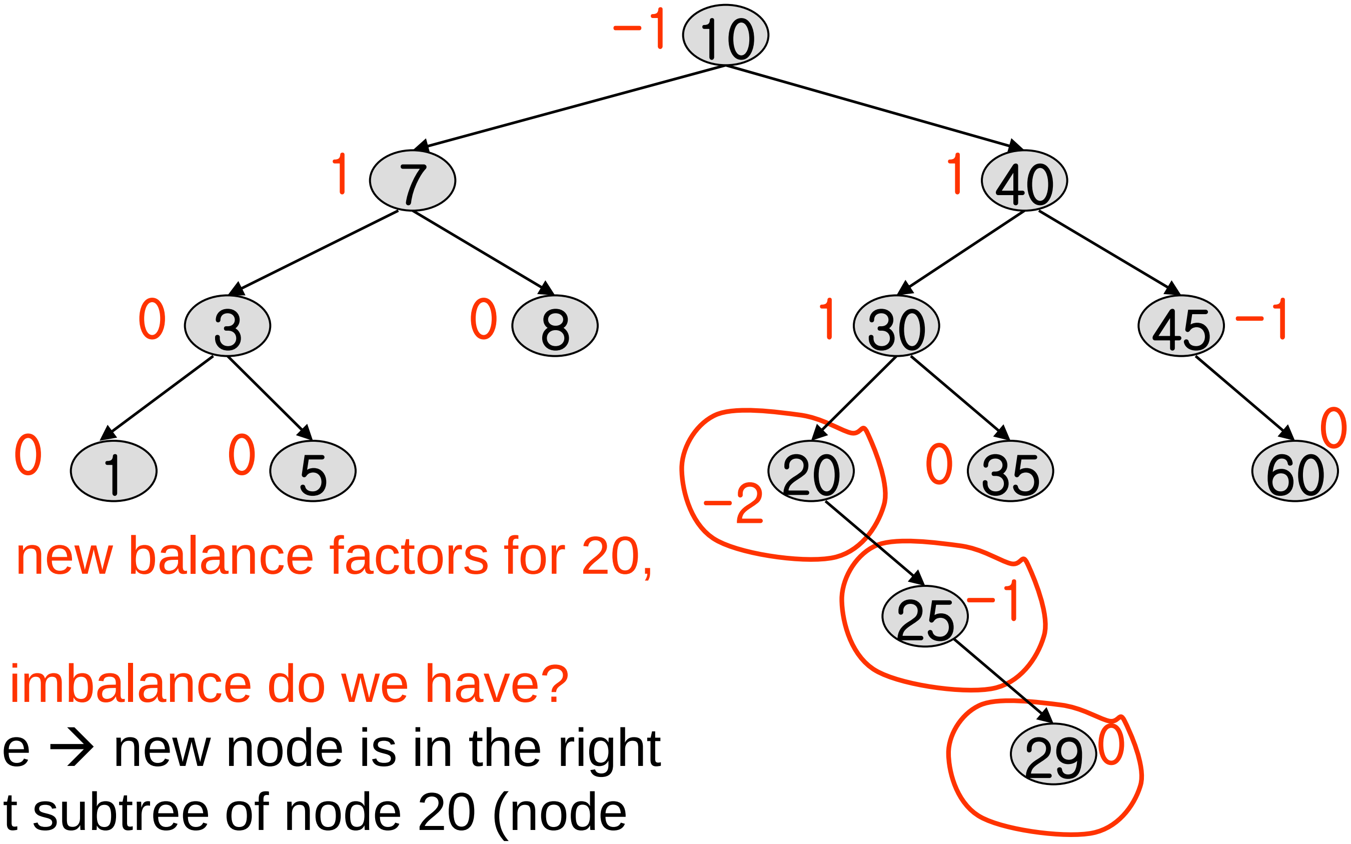
# Inserting into an AVL Search Tree

Insert(29)



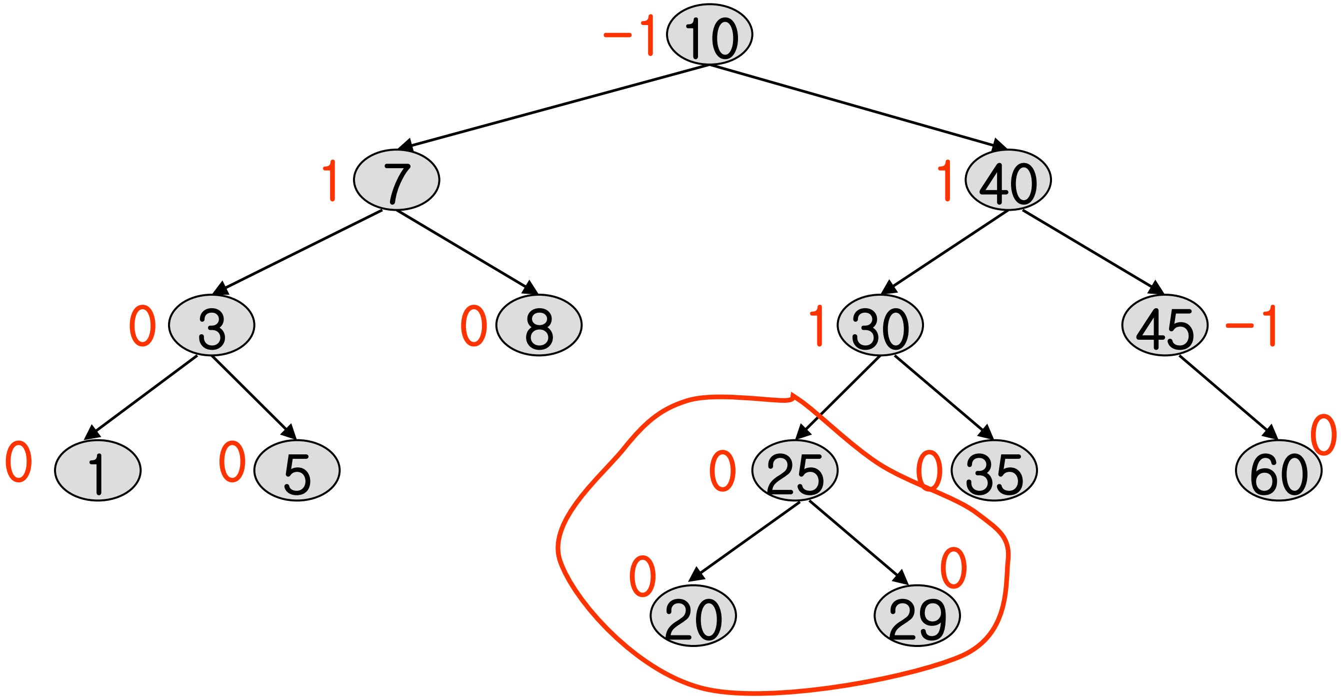
- Where is 29 going to be inserted into?
  - use the AVL-search-tree-insertion algorithm in Figure 15.6)
- After the insertion, is the tree still an AVL search tree? (i.e., still balanced?)

# Inserting into an AVL Search Tree



- What are the new balance factors for 20, 25, 29?
- What type of imbalance do we have?
- RR imbalance → new node is in the right subtree of right subtree of node 20 (node with bf = -2) → what rotation do we need?
- What would the left subtree of 30 look like after RR rotation?

# After RR Rotation



- After the RR rotation, is the resulting tree an AVL search tree?

# Balancing an AVL tree after an insertion

- Begin at the node containing the item which was just inserted and move back along the access path toward the root.{
  - For each node determine its height and check the balance condition. {
    - If the tree is AVL balanced and no further nodes need be considered.
    - else If the node has become unbalanced, a rotation is needed to balance it.

    }

} we proceed to the next node on the access path.

```
AVLnode *insert(Datatype x, AVLnode *t) {
  if (t == NULL) {
        /* CreateNewNode */
  }
  else if (x < t->element) {
        t->left = insert(x, t->left);
        /* DoLeft */
  }
  else if (x > t->element) {
        t->right = insert(x, t->right);
        /* DoRight */
  }
}
```

# AVL tree

- **CreateNewNode**

```
t = malloc(sizeof(struct AVLnode);
t->element = x;
t->left = NULL;
t->right = NULL;
```

# AVL tree

- **DoLeft**

```
if (height(t->left) - height(t->right) == 2)
  if (x < t->left->element)
    t = singleRotateWithLeft(t); // LL
  else
    t = doubleRotateWithLeft(t); // LR
```

# AVL tree

- **DoRight**

```
if (height(t->right) - height(t->left) == 2)
  if (x > t->right->element)
   t = singleRotateWithRight(t); // RR
  else
   t = doubleRotateWithRight(t); // RL
```

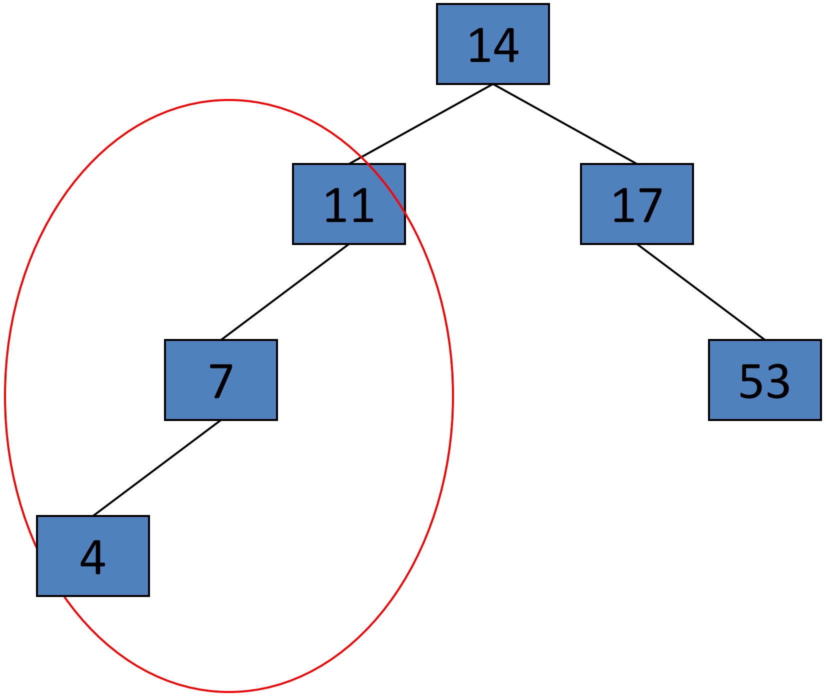
# Deletion from an AVL Search Tree

**Deletion procedure is more complex than insertion in 2 ways:**

- **1)More number of cases for rebalancing may arise in deletion;**
- **2)In insertion there is only one reblancing,but in deletion there can be as many rebalancing as the length of the path from the deleted node to the root.**

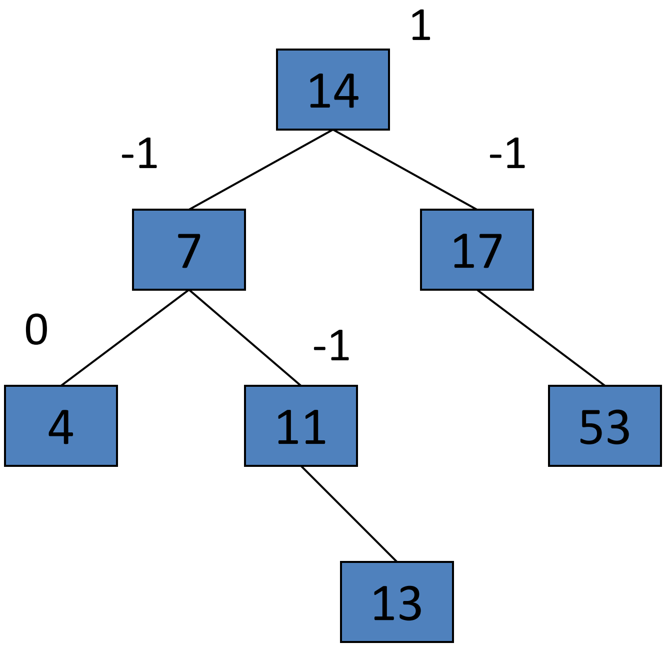
## AVL Tree Example:

- Insert 14, 17, 11, 7, 53, 4, 13 into an empty AVL tree



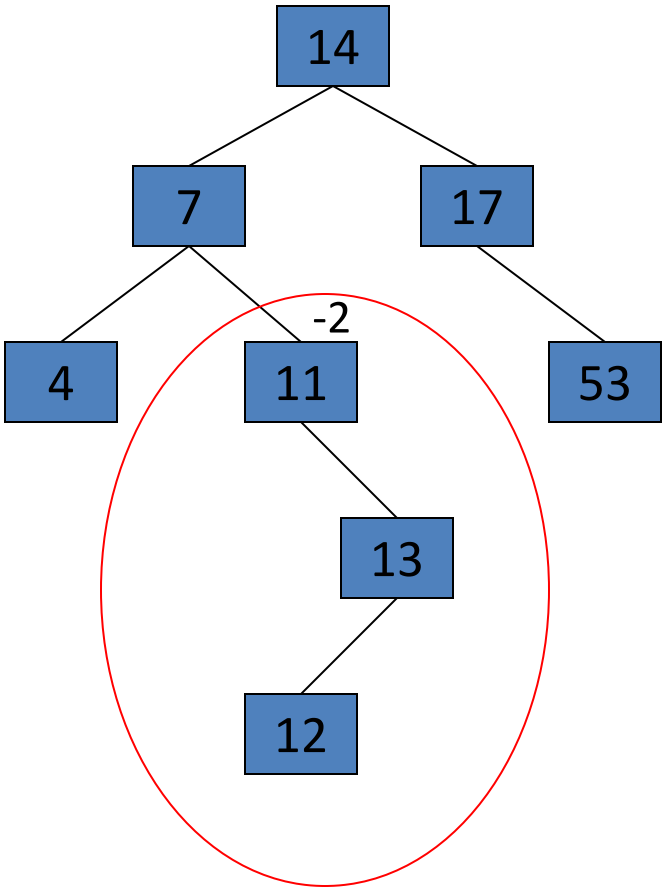
**AVL Tree Example:**

- **Insert 14, 17, 11, 7, 53, 4, 13 into an empty AVL tree**



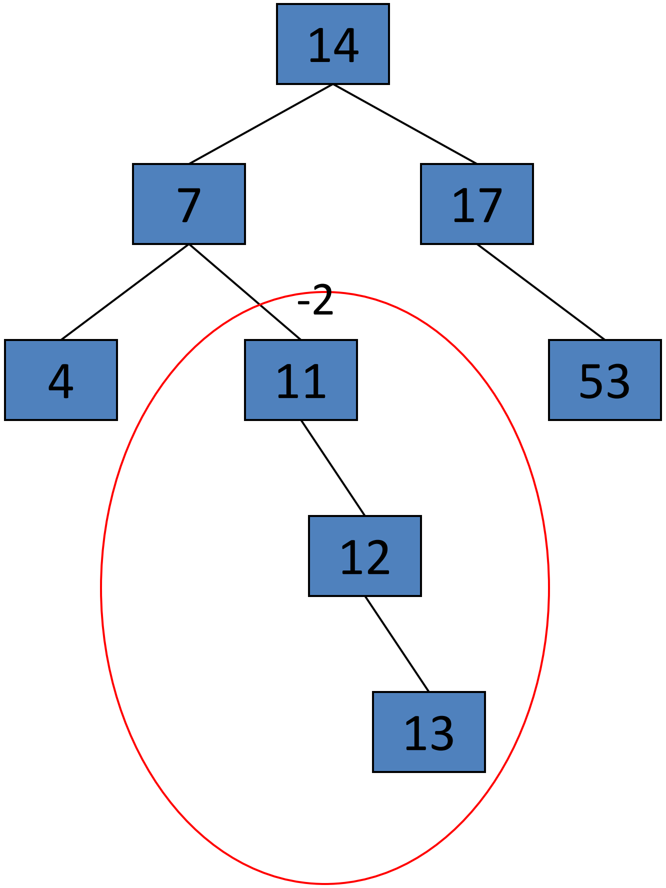
## AVL Tree Example:

- **Now insert 12**



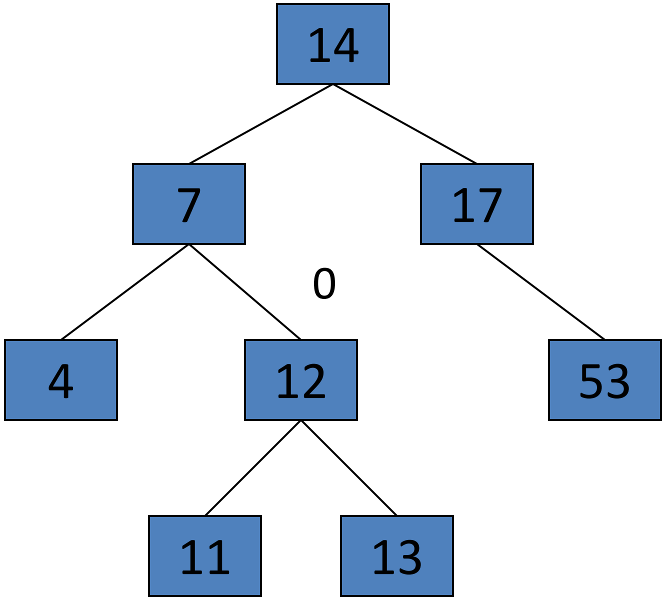
## AVL Tree Example:

- **Now insert 12**



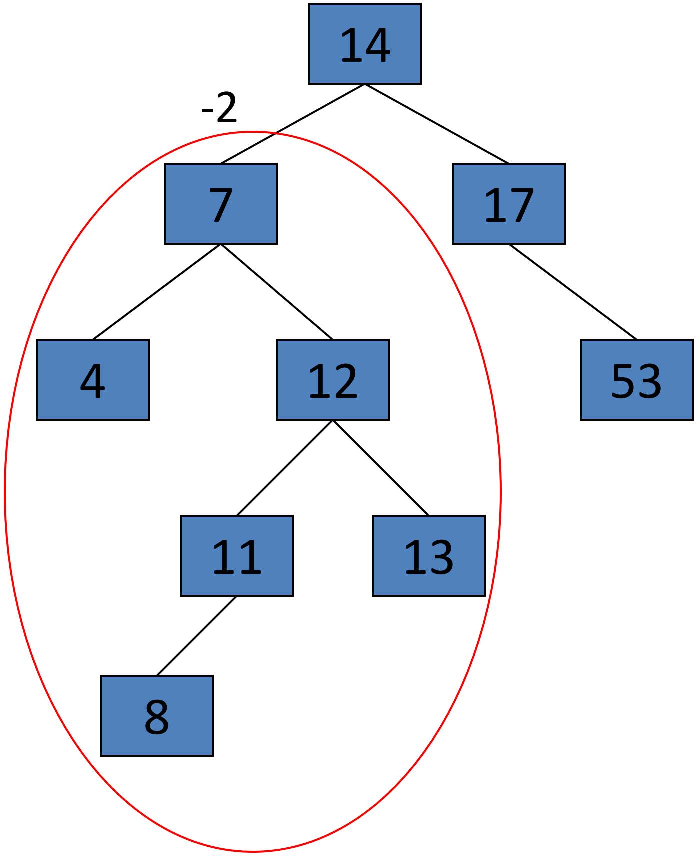
## AVL Tree Example:

- **Now the AVL tree is balanced.**



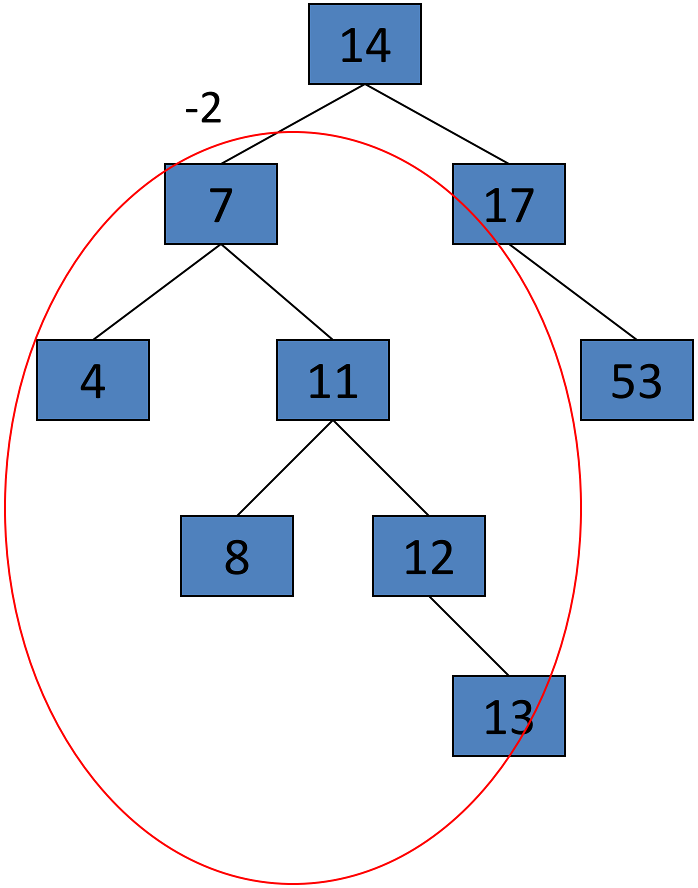
## AVL Tree Example:

- **Now insert 8**



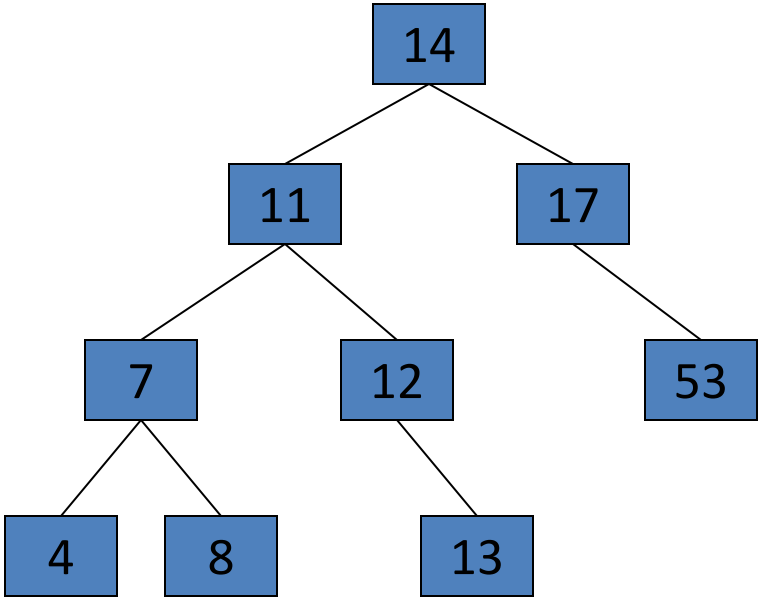
## AVL Tree Example:

- **Now insert 8**



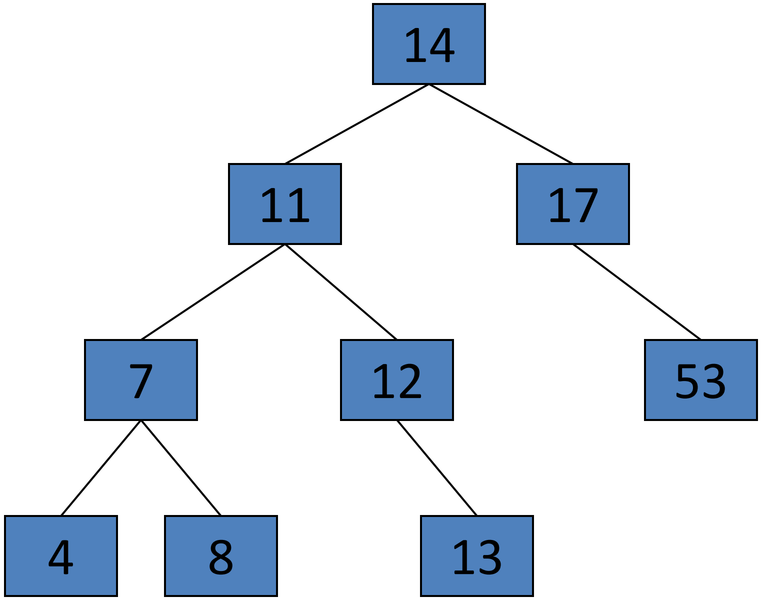
## AVL Tree Example:

- **Now the AVL tree is balanced.**



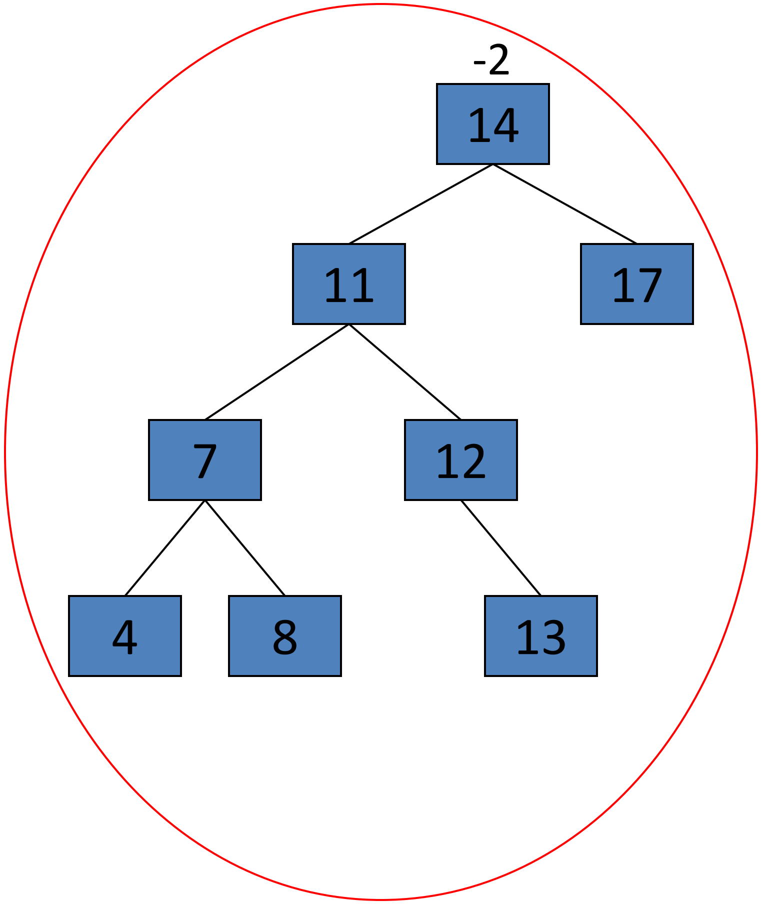
## AVL Tree Example:

- **Now remove 53**



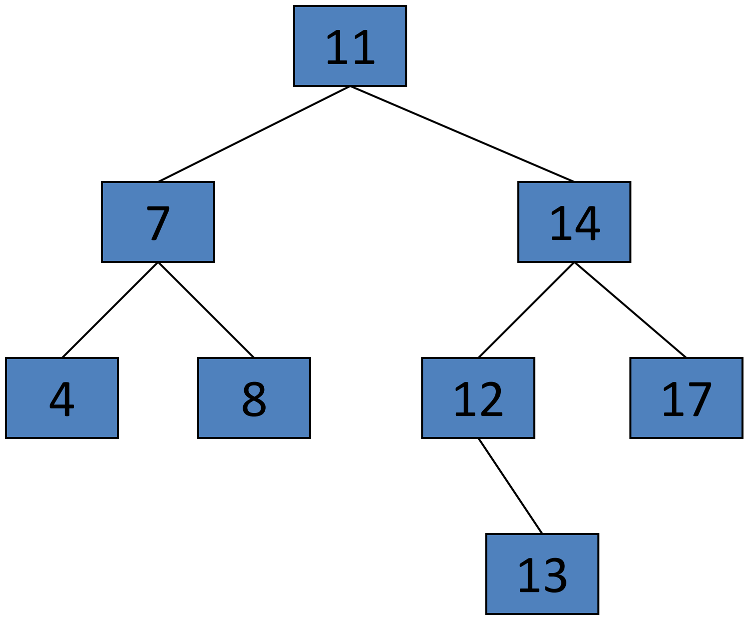
## AVL Tree Example:

- Now remove 53, unbalanced



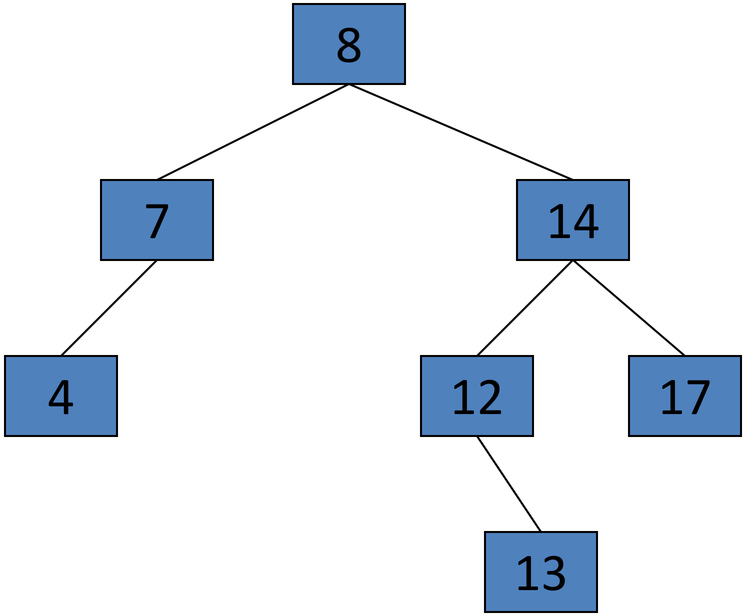
## AVL Tree Example:

- **Balanced! Remove 11**



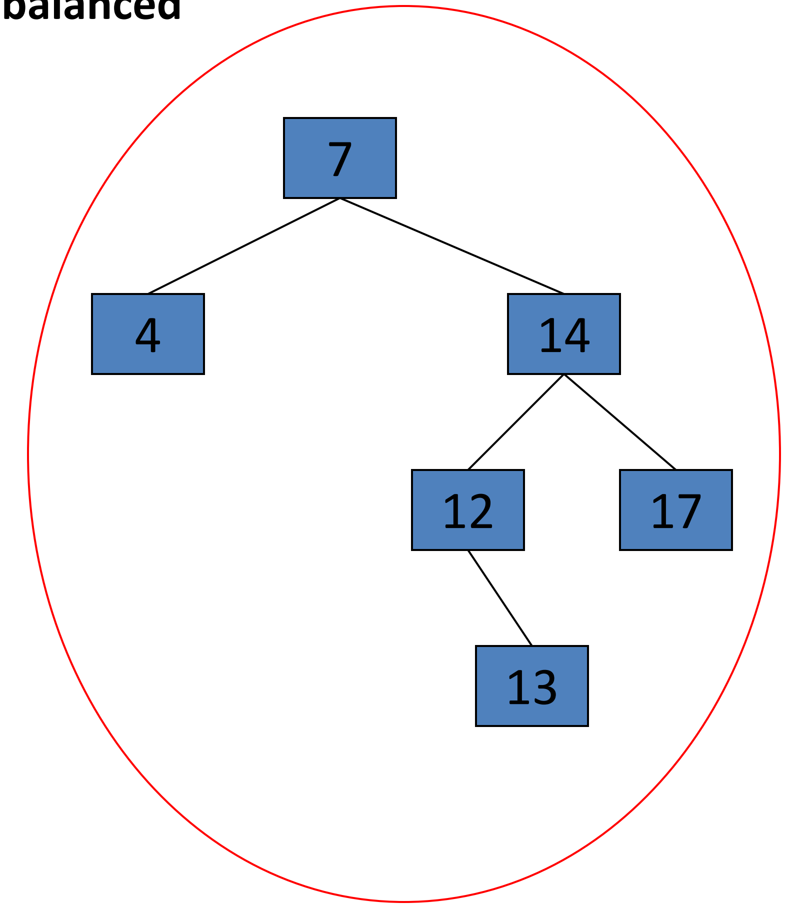
## AVL Tree Example:

- Remove 11, replace it with the largest in its left branch



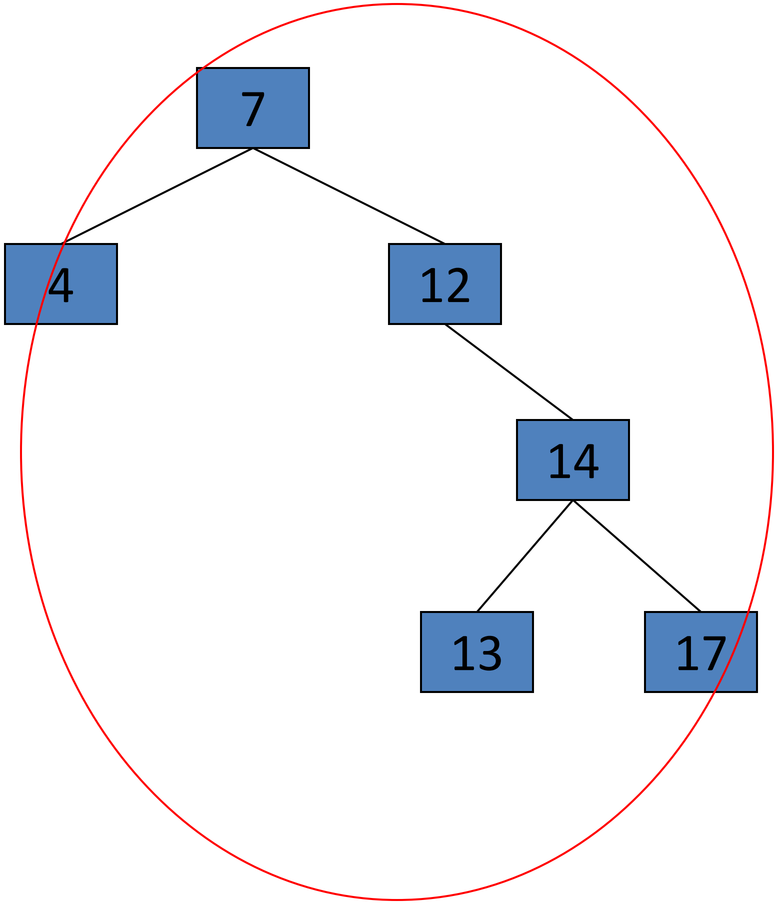
## AVL Tree Example:

- **Remove 8, unbalanced**



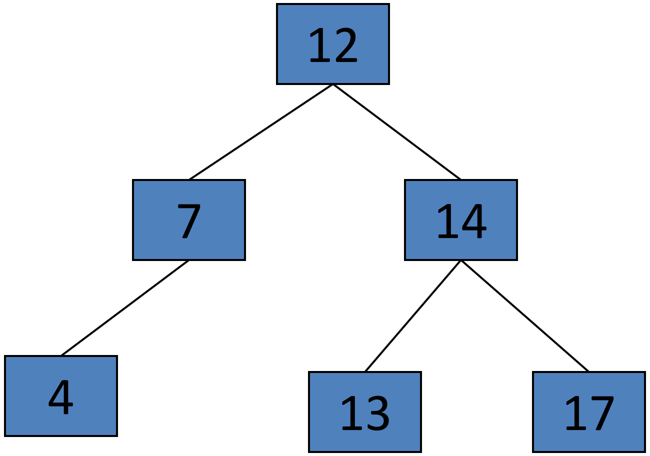
## AVL Tree Example:

- **Remove 8, unbalanced**



## AVL Tree Example:

- **Balanced!!**

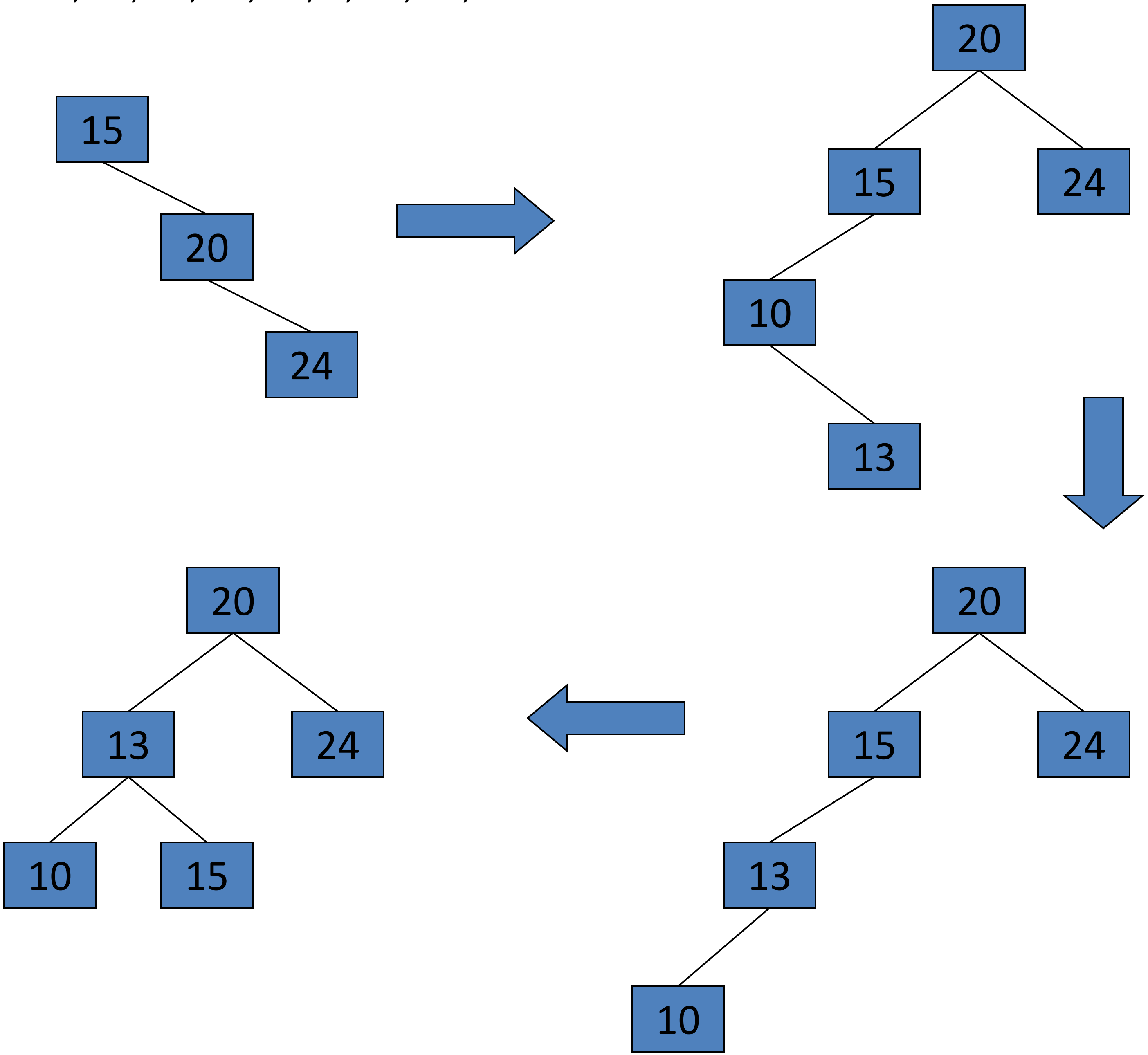


# Exercise

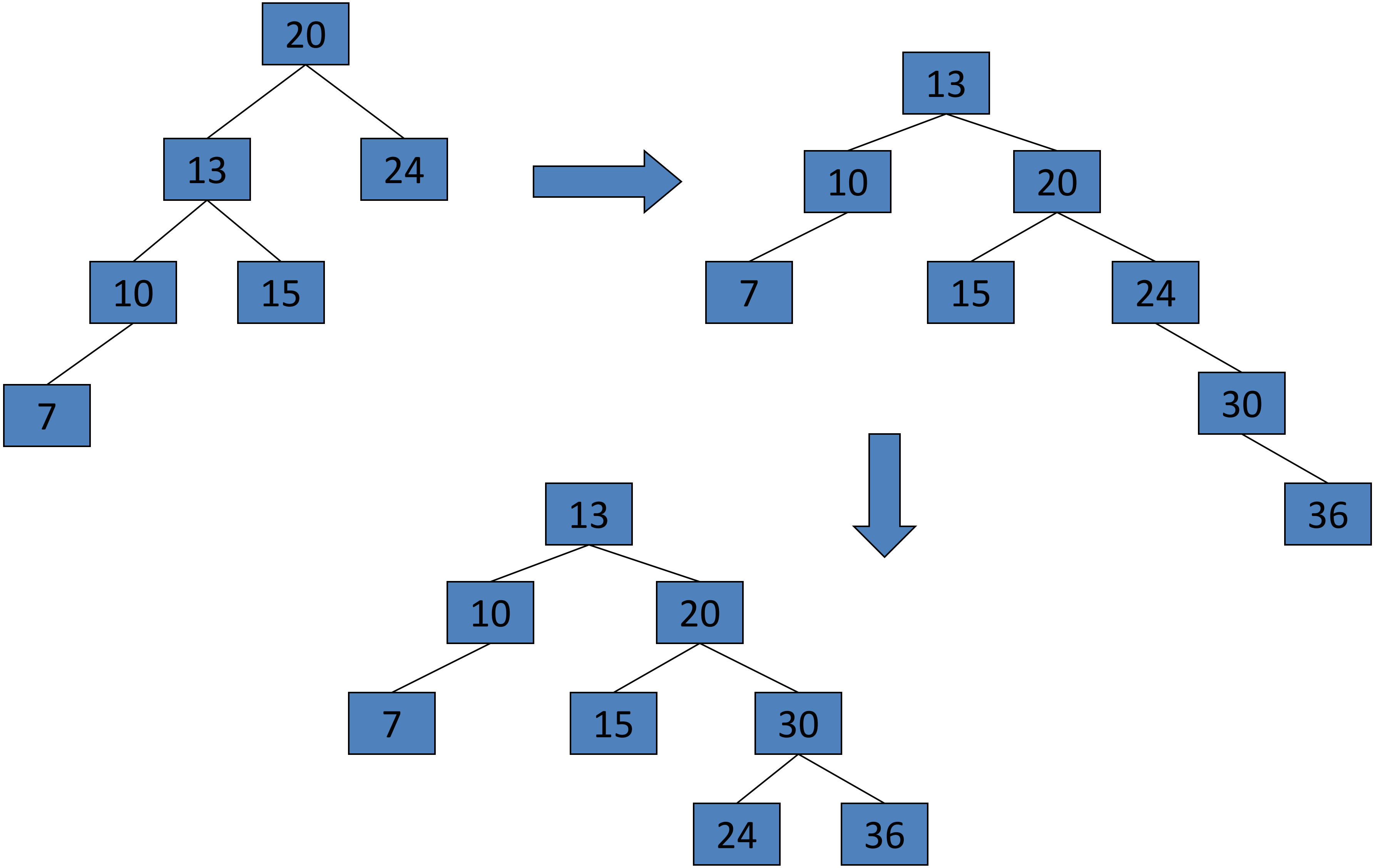
- Build an AVL tree with the following values:

  15, 20, 24, 10, 13, 7, 30, 36, 25

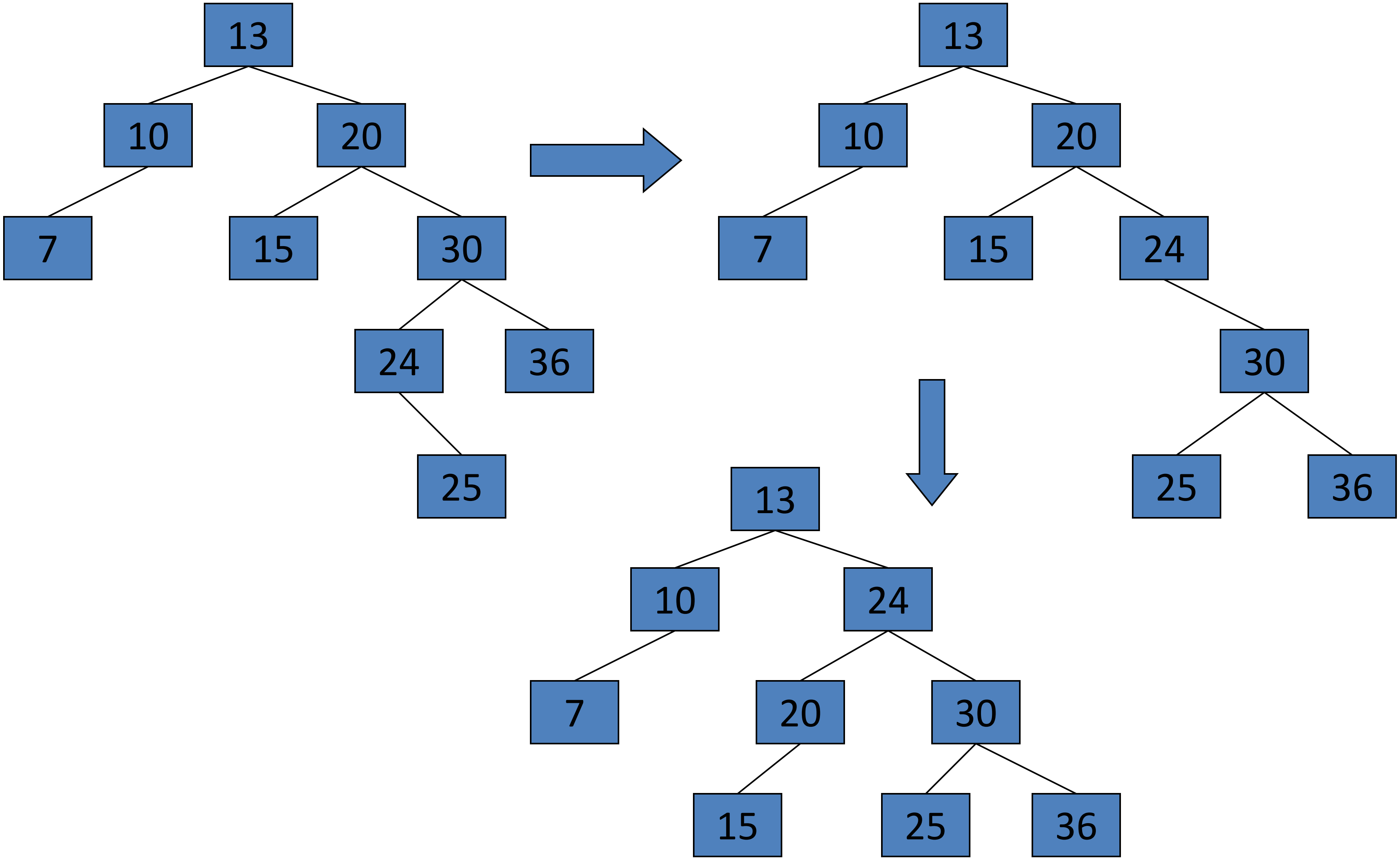
15, 20, 24, 10, 13, 7, 30, 36, 25



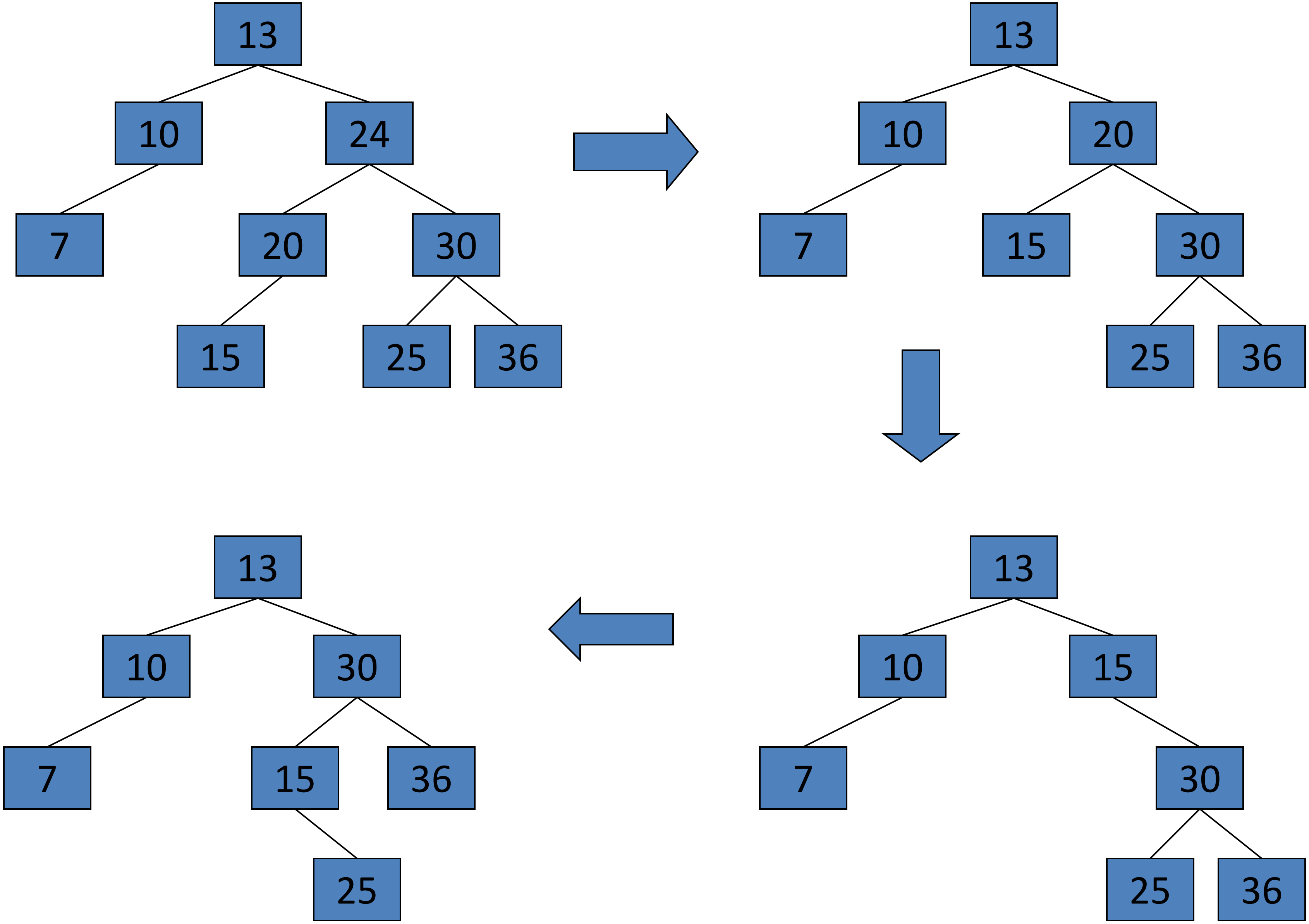
15, 20, 24, 10, 13, 7, 30, 36, 25



15, 20, 24, 10, 13, 7, 30, 36, 25



Remove 24 and 20 from the AVL tree.



# B -TREE

# B-tree of order n

- **Every B-tree is of some "order n", meaning nodes contain from n to 2n keys (so nodes are always at least half full of keys), and n+1 to 2n+1 pointers, and n can be any number.**
- **Keys are kept in sorted order within each node. A corresponding list of pointers are effectively interspersed between keys to indicate where to search for a key if it isn't in the current node.**

- **A B-tree of order n is a multi-way search tree with two properties:**
- **1.All leaves are at the same level**
- **2.The number of keys in any node lies between n and 2n, with the possible exception of the root which may have fewer keys.**

# Other definition

**A B-tree of order m is a m-way tree that satisfies the following conditions.**

- **Every node has ≤ m children.**
- **Every internal node (except the root) has ≤m/2 children.**
- **The root has ≥2 children.**
- **An internal node with k children contains (k-1) ordered keys. The leftmost child contains keys less than or equal to the first key in the node. The second child contains keys greater than the first keys but less than or equal to the second key, and so on.**

# B-tree of order n

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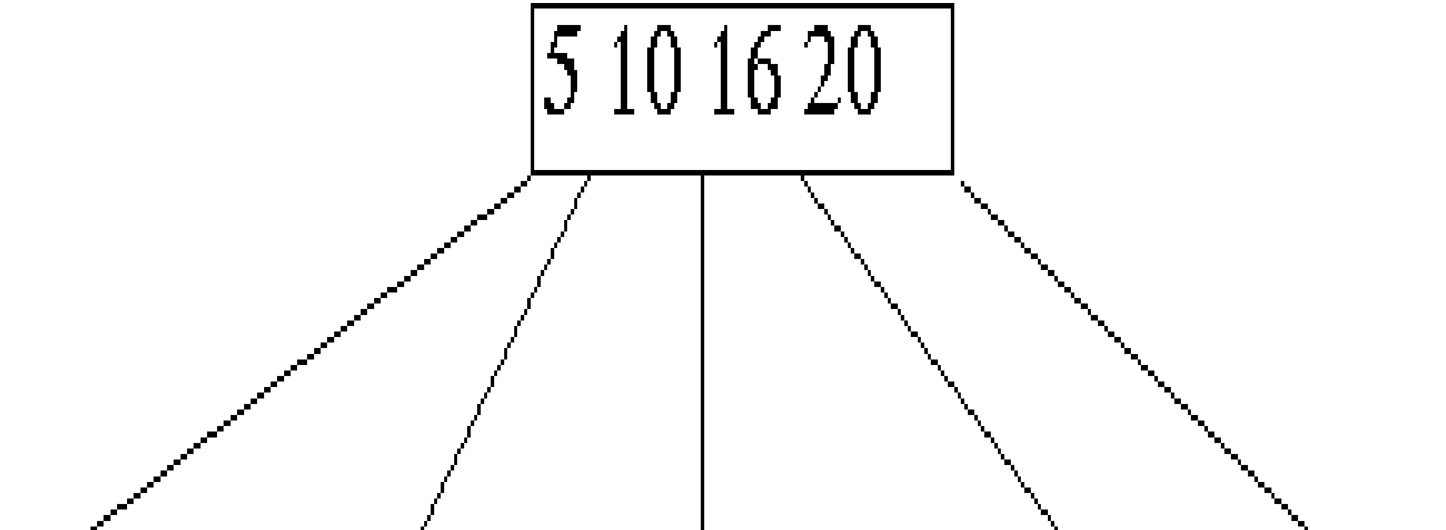
# A B-tree of order 2



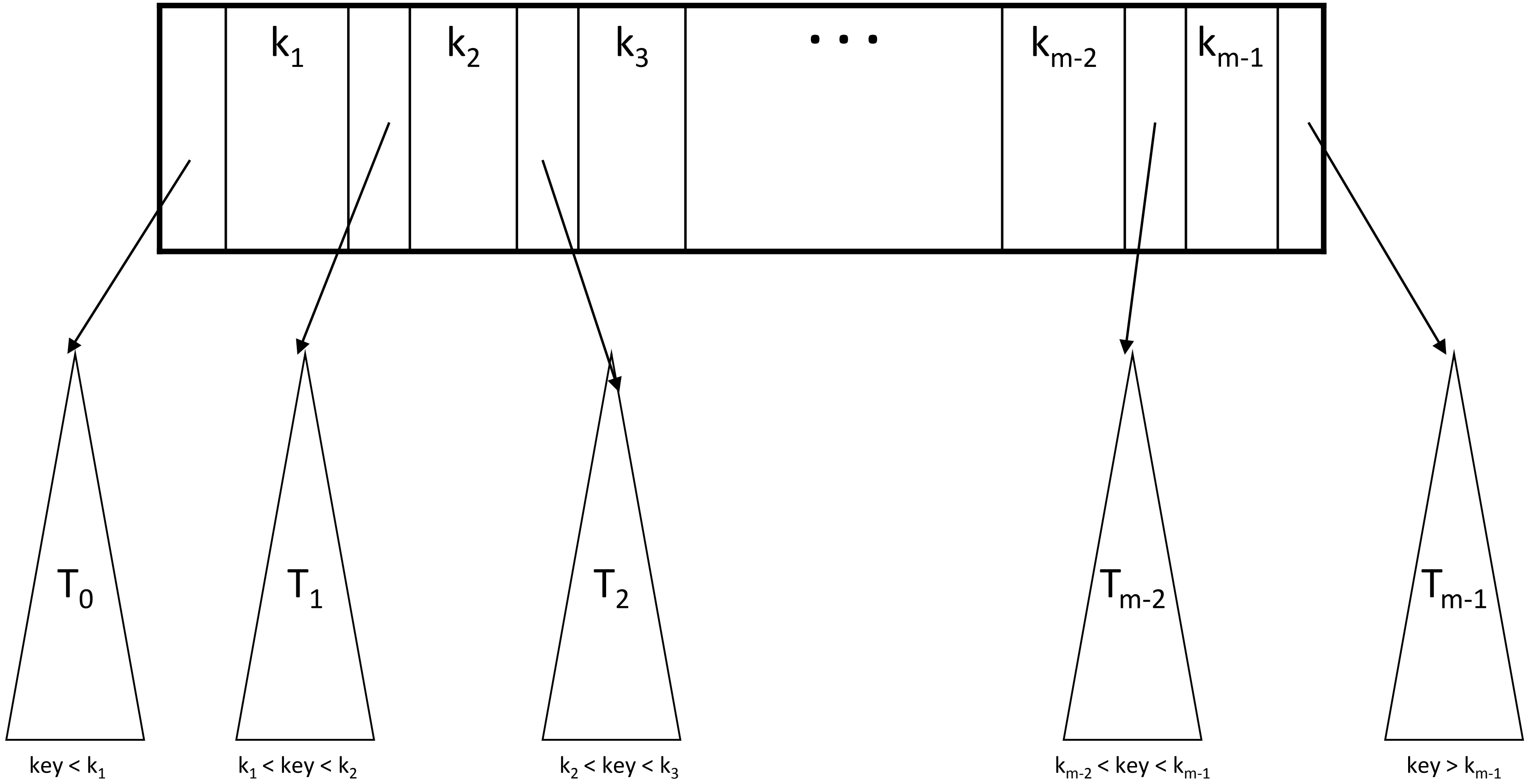
- **A multi-way (or m-way) search tree of order m is a tree in which**
  - **Each node has at-most m subtrees, where the subtrees <u>may be empty</u>.**
  - **Each node consists of at least 1 and at most m-1 distinct keys**
  - **The keys in each node are sorted.**

- **The keys and subtrees of a non-leaf node are ordered as:**

  **$T_0$, $k_1$, $T_1$, $k_2$, $T_2$, . . . , $k_{m-1}$, $T_{m-1}$ such that:**

  - **All keys in subtree T0 are less than k1.**
  - **All keys in subtree $T_i$ , 1 <= i <= m - 2, are greater than $k_i$ but less than $k_{i+1}$.**
  - **All keys in subtree $T_{m-1}$ are greater than $k_{m-1}$**

# Multi-way tree

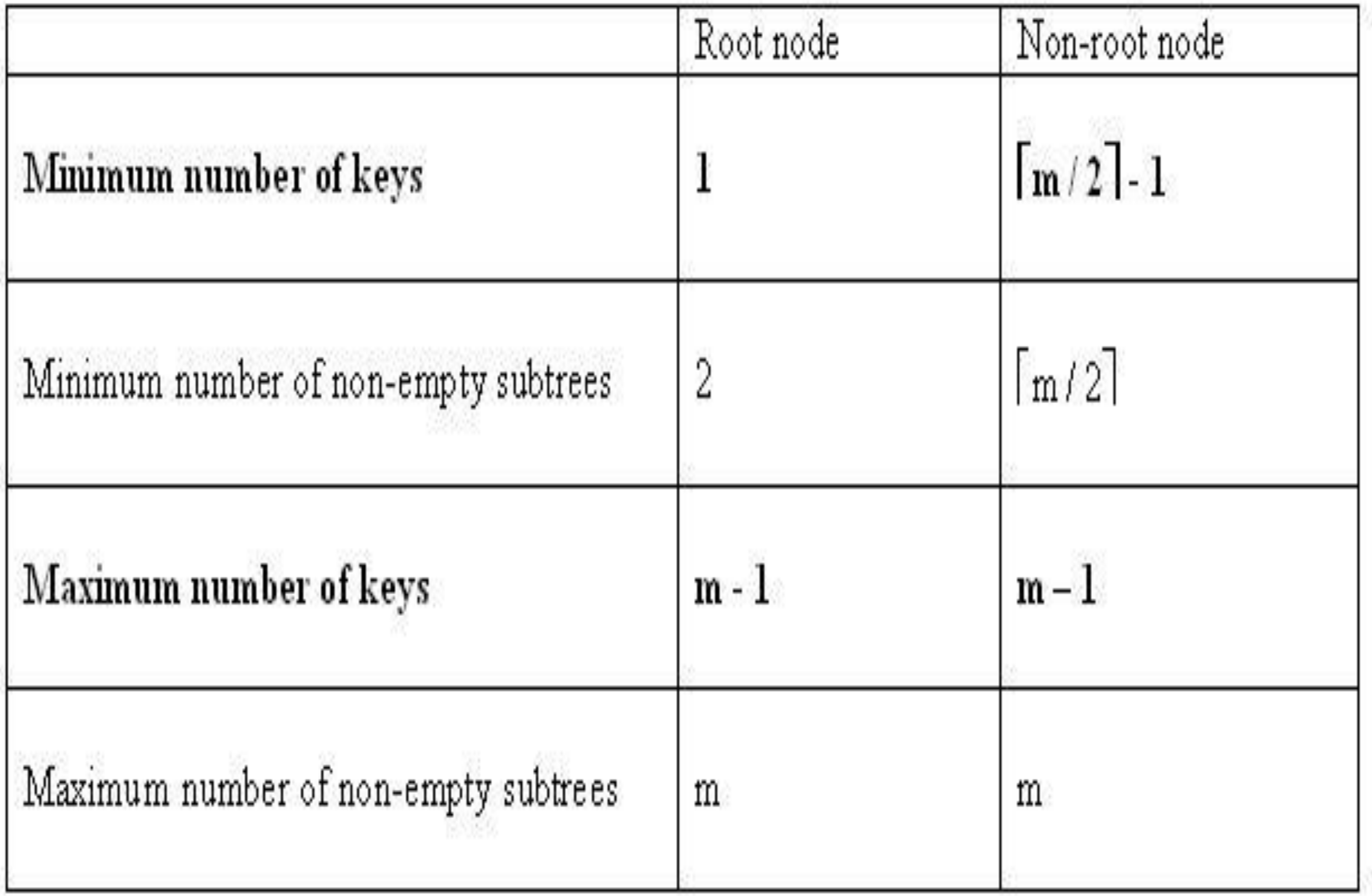


# What is B-tree?

- **B-tree of order m (or branching factor m), where m > 2, is either an empty tree or a multiway search tree with the following properties:**
  - –The root is either a leaf or it has at least two non-empty subtrees and at most m non-empty subtrees.
  - –Each non-leaf node, other than the root, has at least $\lceil m/2 \rceil$ non-empty subtrees and at most m non-empty subtrees. (Note: $\lceil x \rceil$ is the lowest integer > x ).
  - –The number of keys in each non-leaf node is one less than the number of non-empty subtrees for that node.
  - –All leaf nodes are at the same level; that is the tree is perfectly balanced

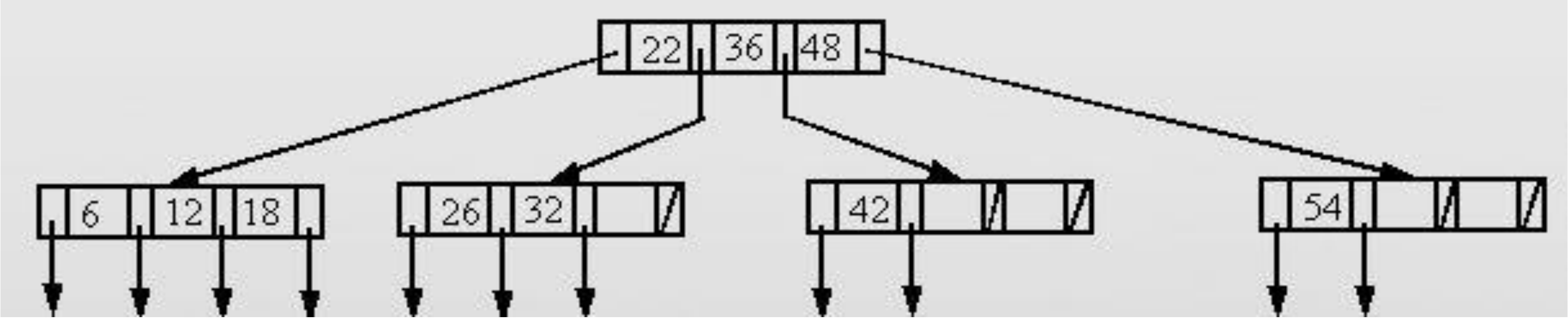
# What is a B-tree?

- **For a non-empty B-tree of order m:**

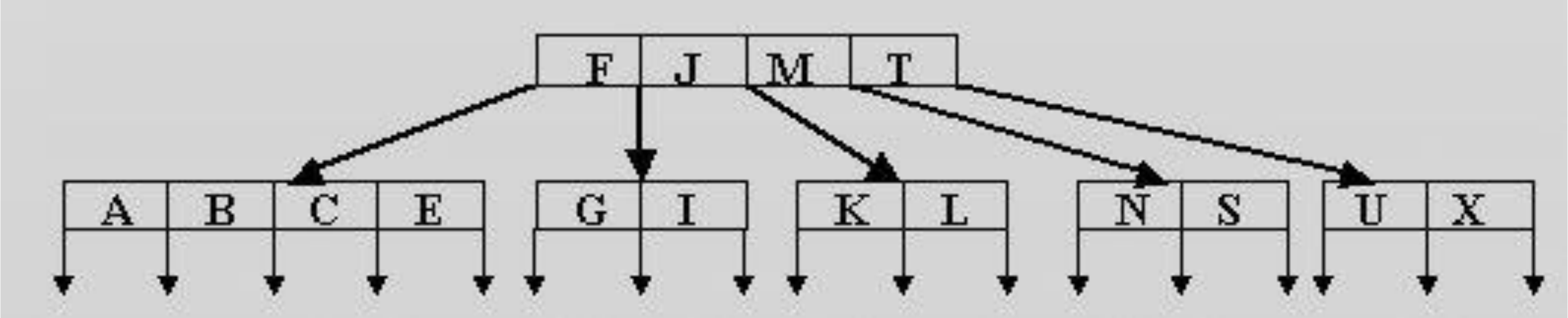


| | Root node | Non-root node |
|---|---|---|
| **Minimum number of keys** | **1** | **$\lceil m / 2 \rceil$ - 1** |
| Minimum number of non-empty subtrees | 2 | $\lceil m / 2 \rceil$ |
| **Maximum number of keys** | **m - 1** | **m – 1** |
| Maximum number of non-empty subtrees | m | m |

## Example: A B-tree of order 4



### Example: A B-tree of order 5



**Note:**

- The data references are not shown.
- The leaf references are to empty subtrees

# Height of B-Trees

- For *n* greater than or equal to one, the height of an *n*-key b-tree T of height *h* with a minimum degree *t* greater than or equal to 2

$$h \le \log_t \frac{n+1}{2}$$

# Operations of B-Trees

- **B-Tree-Search(x, k)**
  - The search operation on a b-tree is similar to a search on a binary tree. The *B-Tree-search* runs in time $O(\log_t n)$.
- **B-Tree-Create(T)**
  - The *B-Tree-Create* operation creates an empty b-tree by allocating a new root node that has no keys and is a leaf node. Only the root node is permitted to have these properties; all other nodes must meet the criteria outlined previously. The *B-Tree-Create* operation runs in time *O(1)*.

# Operations of B-Trees

- **B-Tree-Split-Child(x, i, y)**
  - If is node becomes "too full," it is necessary to perform a split operation. The split operation moves the median key of node x into its parent y where x is the ith child of y. A new node, z, is allocated, and all keys in x right of the median key are moved to z. The keys left of the median key remain in the original node x. The new node, z, becomes the child immediately to the right of the median key that was moved to the parent y, and the original node, x, becomes the child immediately to the left of the median key that was moved into the parent. The B-Tree-Split-Child algorithm will run in time O(t) , T is constrain

# Operations of B-Trees

- **B-Tree-Insert(T, k)**
- **B-Tree-Insert-Nonfull(x, k)**

**To perform an insertion on a b-tree, the appropriate node for the key must be located using an algorithm similiar to *B-Tree-Search*. Next, the key must be inserted into the node.**

- **If the node is not full prior to the insertion, no special action is required; however, if the node is full, the node must be split to make room for the new key. Since splitting the node results in moving one key to the parent node, the parent node must not be full or another split operation is required. This process may repeat all the way up to the root and may require splitting the root node.**
- **This approach requires two passes. The first pass locates the node where the key should be inserted; the second pass performs any required splits on the ancestor nodes. runs in time $O(t \log_t n)$**

- **OVERFLOW CONDITION:**

**A root-node or a non-root node of a B-tree of order m overflows if, after a key insertion, it contains m keys.**

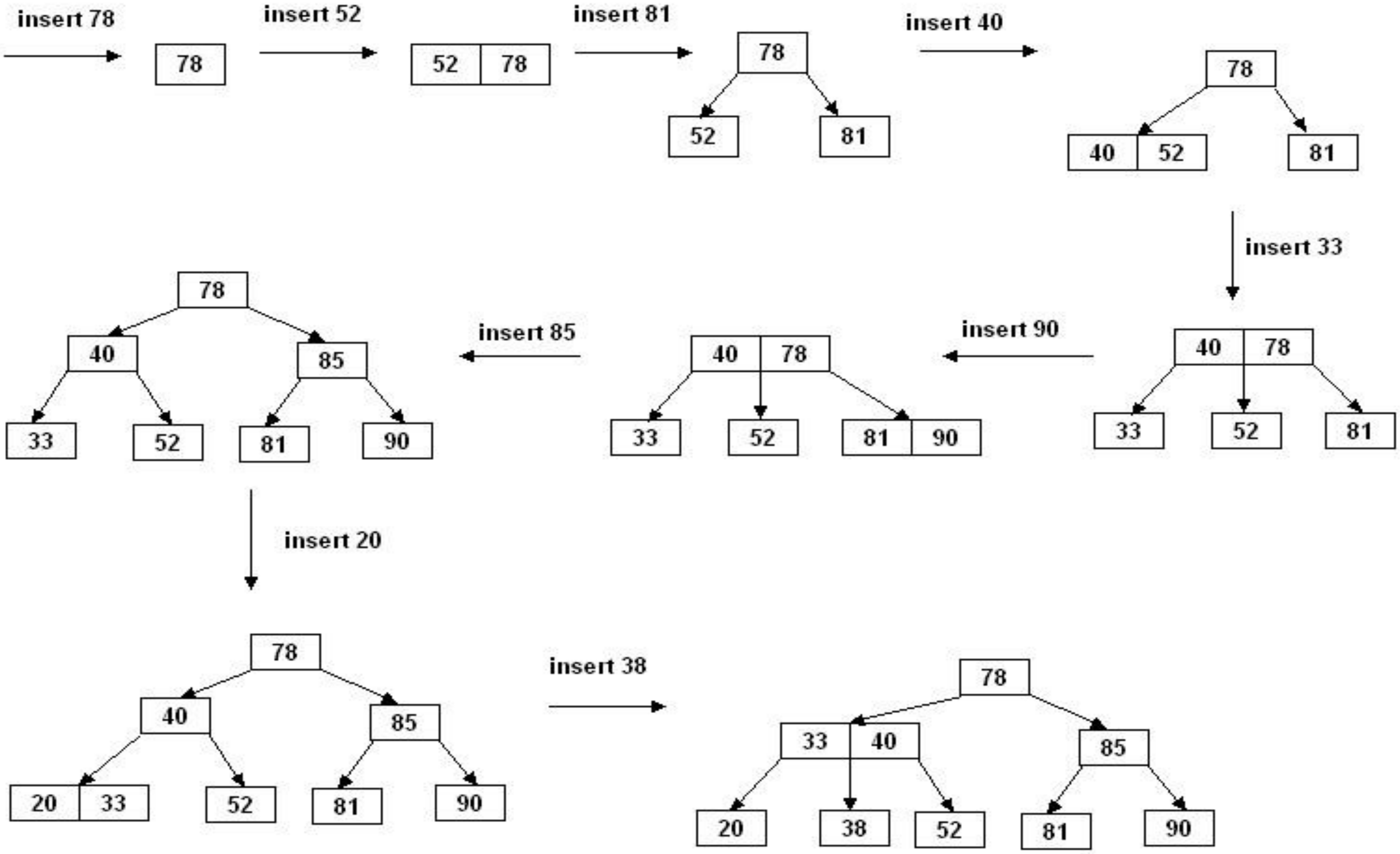
- **Insertion algorithm:**

**If a node overflows, split it into two, propagate the "middle" key to the parent of the node. If the parent overflows the process propagates upward. If the node has no parent, create a new root node.**

- **Note: Insertion of a key always <u>starts</u> at a leaf node.**

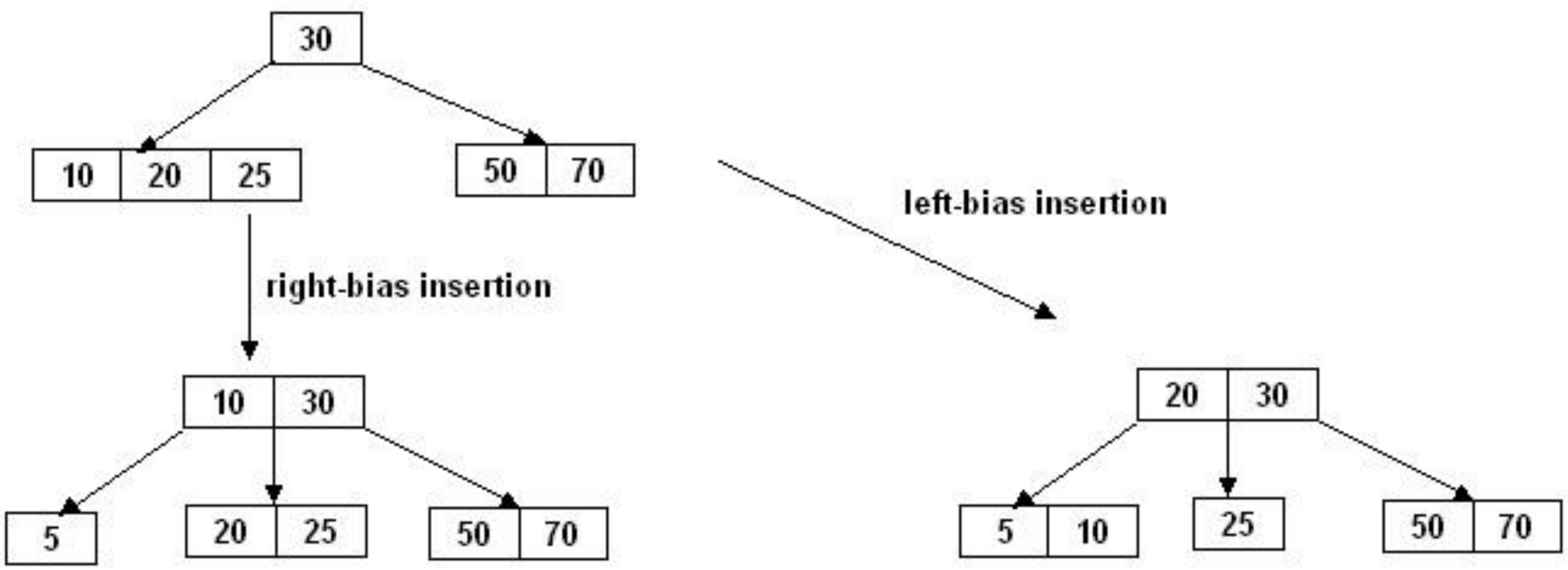
Insertion

- **Insertion in a B-tree of odd order**
- **Example: Insert the keys 78, 52, 81, 40, 33, 90, 85, 20, and 38 in this order in an initially empty B-tree of order 3**



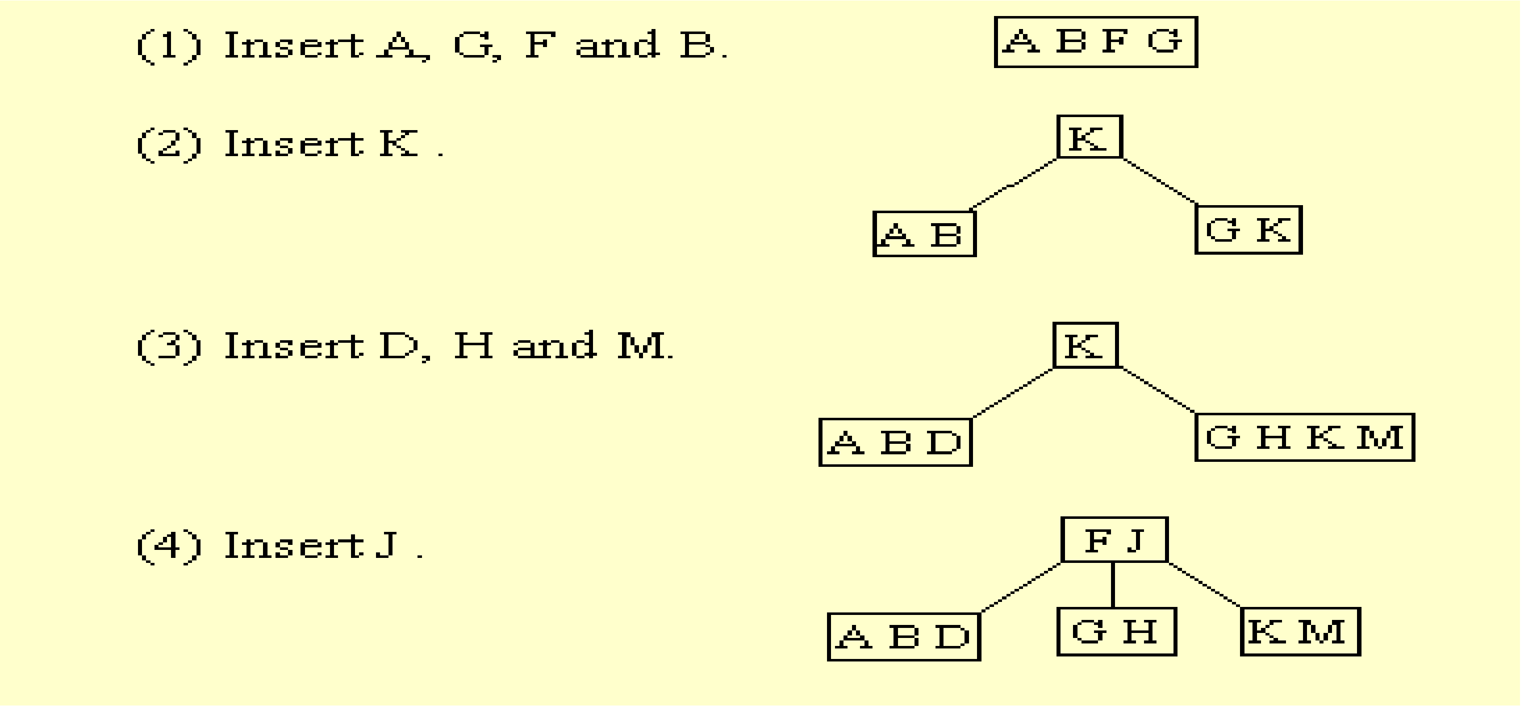
# Insertion in B-Trees

- **Insertion in a B-tree of <u>even</u> order**
- **right-bias: The node is split such that its right subtree has more keys than the left subtree.**
- **left-bias: The node is split such that its left subtree has more keys than the right subtree.**
- **Example: Insert the key 5 in the following B-tree of order 4:**



# Insertion

- **Insert the keys in the folowing order into a B-tree of order 5.**
- **A, G, F, B, K, D, H, M, J, E, S, I, R, X, C, L, N, T, U, P.**



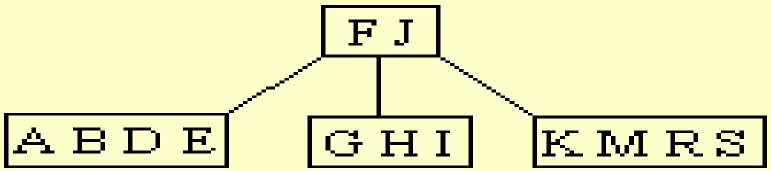
(1) Insert A, G, F and B.

(2) Insert K .

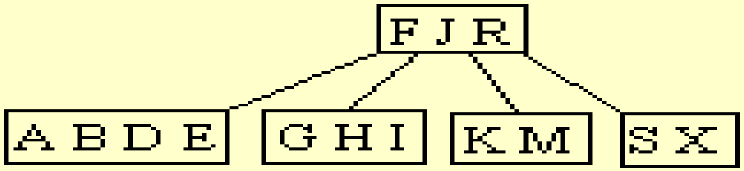
(3) Insert D, H and M.

(4) Insert J .

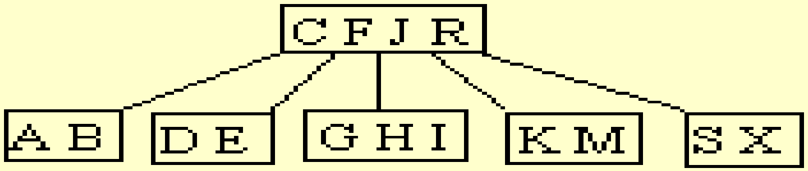
(5) Insert E, S, I and R.



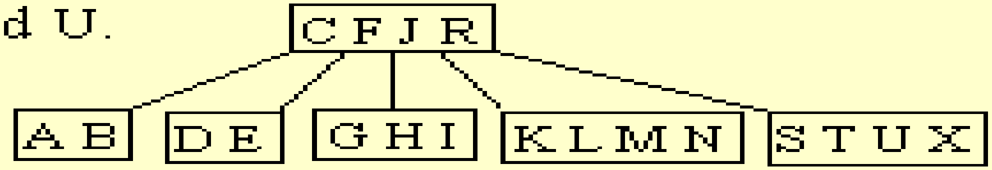
(6) Insert X.



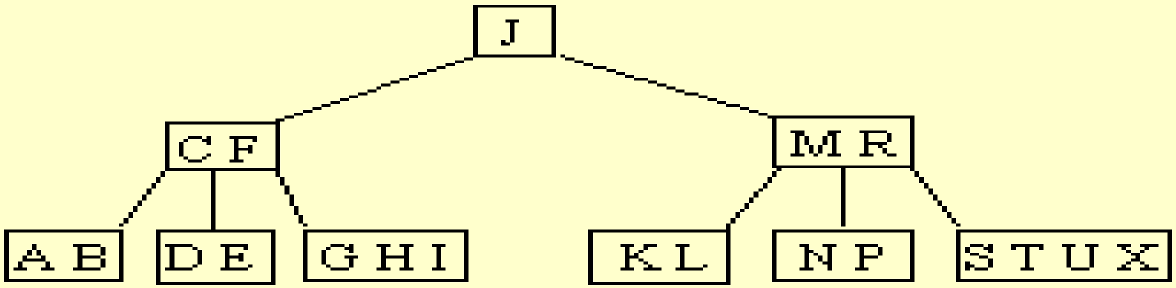
(7) Insert C.



(8) Insert L, N, T and U.



(9) Insert P.



# Searching

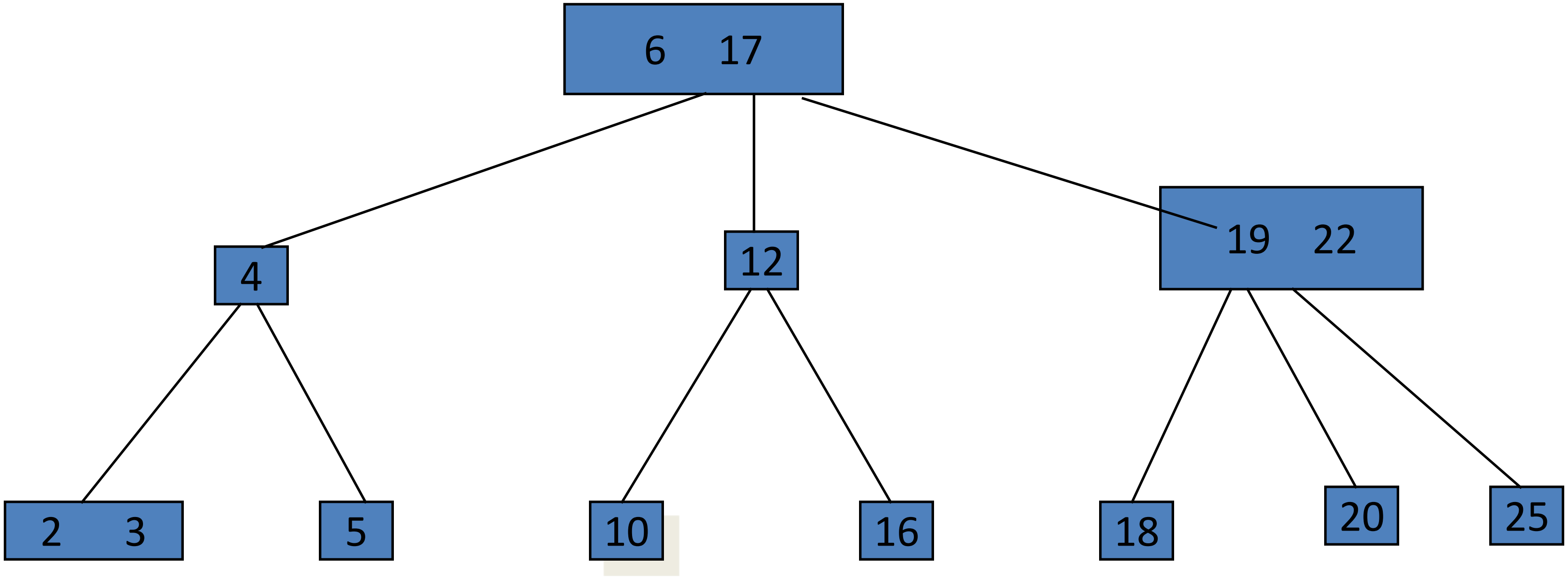
Searching for an Item in a B-Tree:

1. Make a local variable, i, equal to the first index such that data[i] >= target. If there is no such index, then set i equal to data_count, indicating that none of the entries is greater than or equal to the target.

```
2. if (we found the target at data[i])
        return true;
   else if (the root has no children)
        return false;
   else
        return subset[i]->contains (target);
```

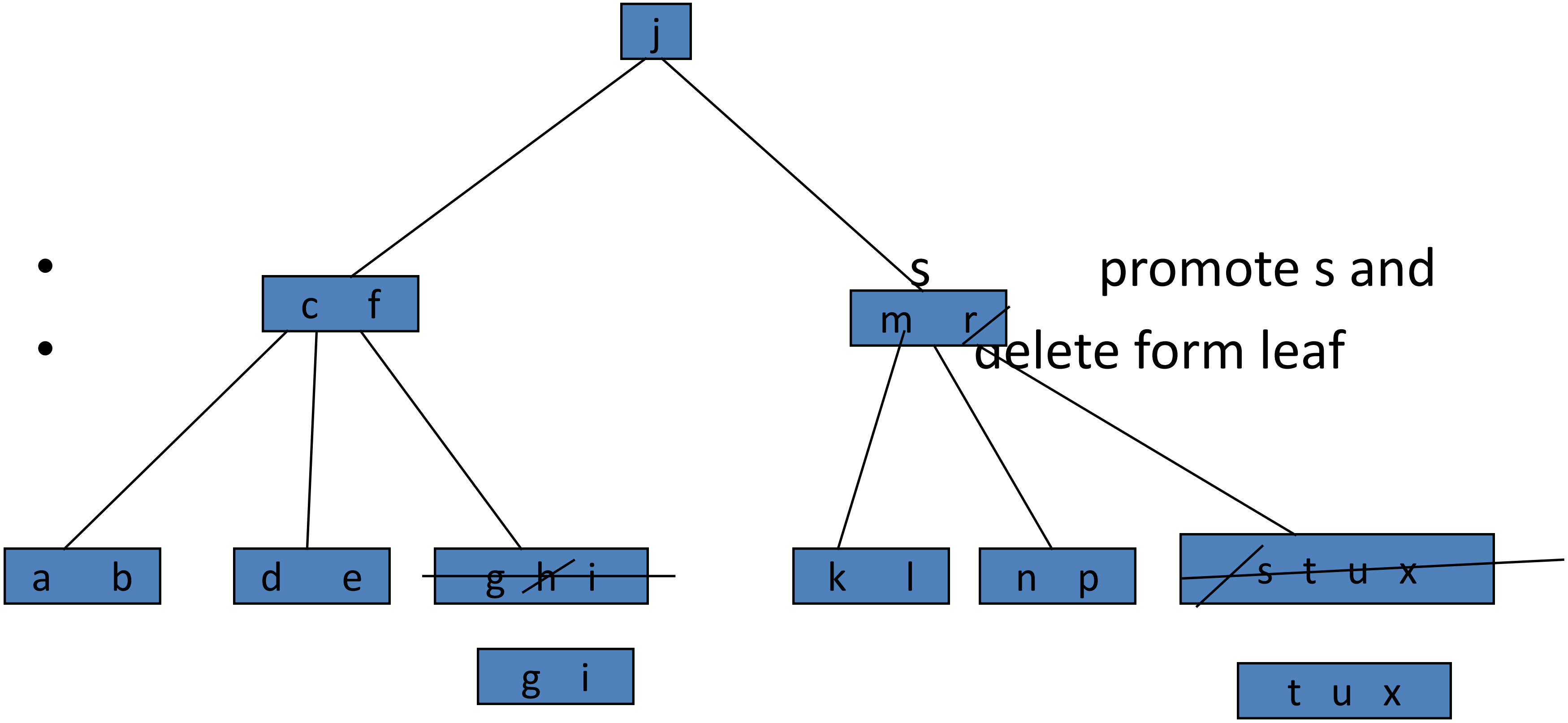
# Searching (cont.)

- Example: target = 10



# Deletion form a B-Tree

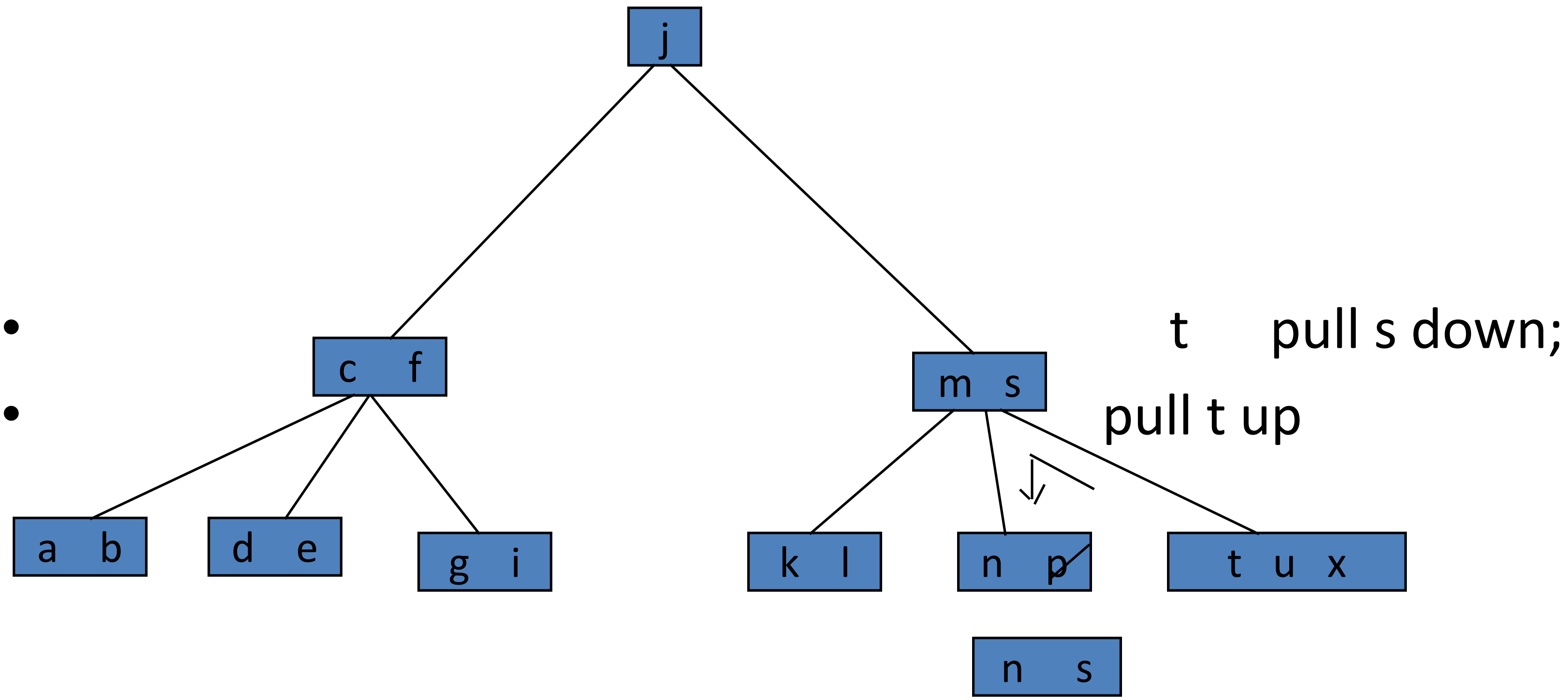
- 1. detete h, r :



promote s and
delete form leaf

# Deletion (cont.)

- 2. delete p :
-

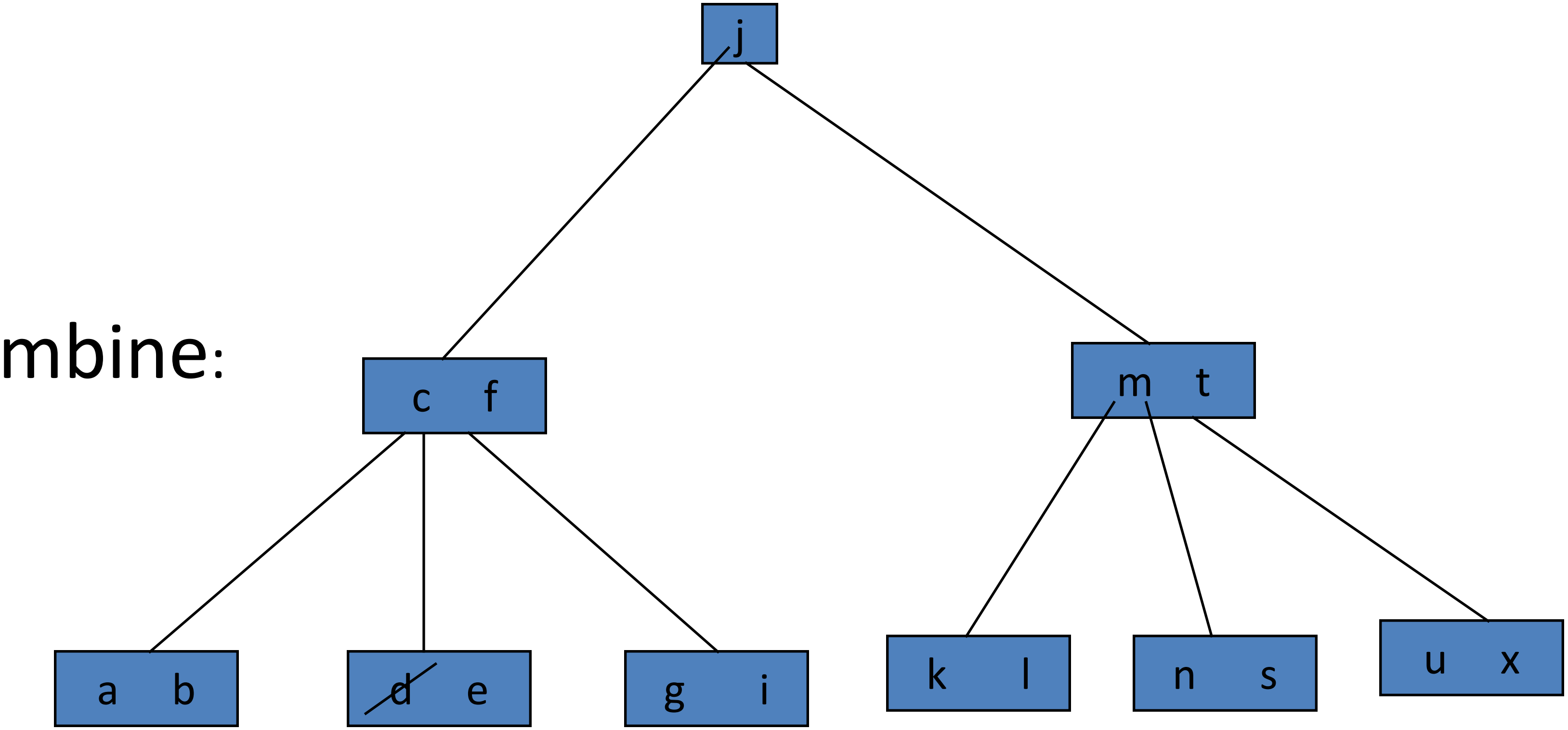


- t pull s down;
- pull t up

# Deletion (cont.)

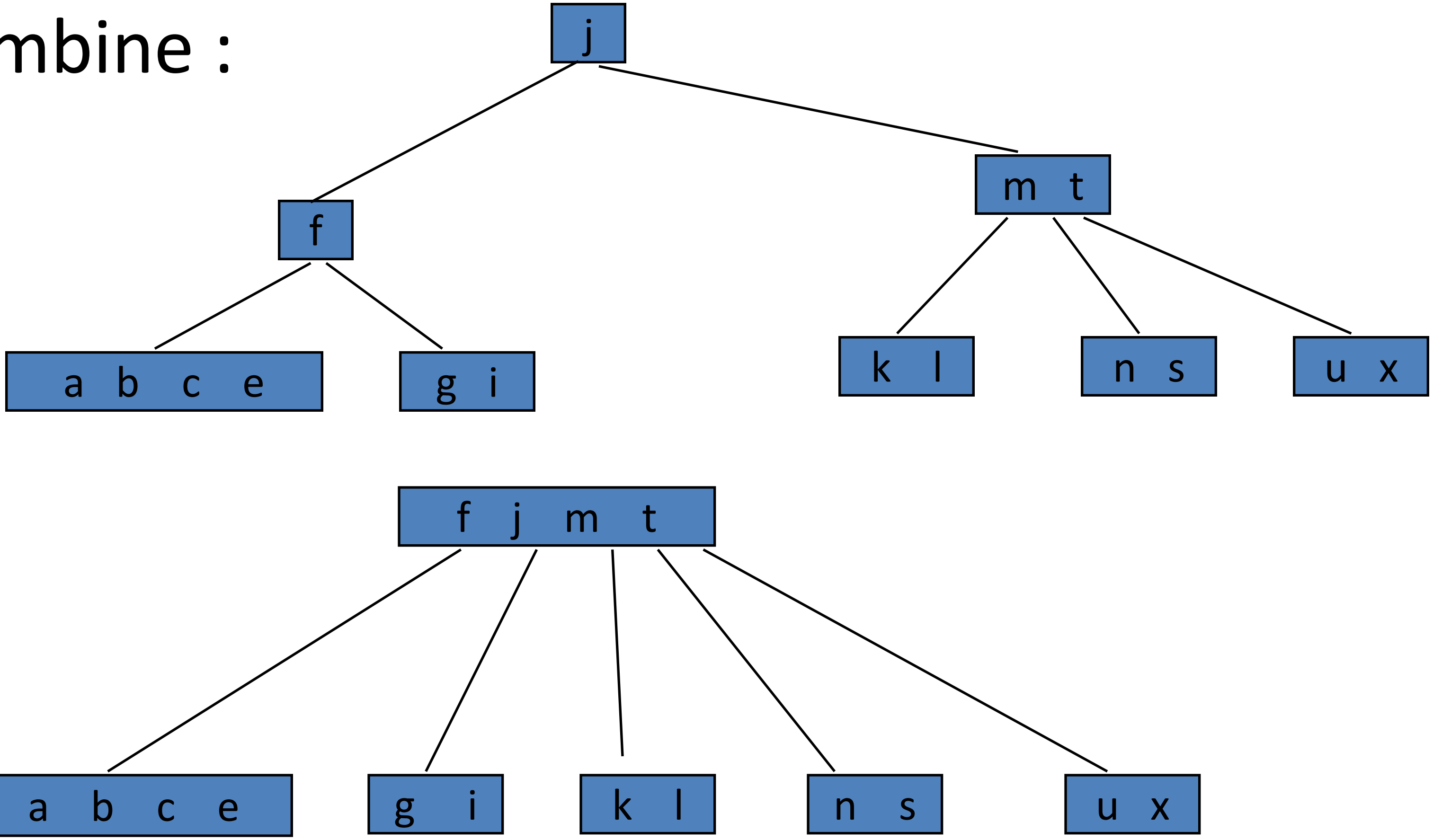
- 3. delete d:

- Combine:



# Deletion (cont.)

- combine :



# Deleting from a B-Tree

- To delete a key value x from a B-tree, first search to determine the leaf node that contains x.
- If removing x leaves that leaf node with fewer than the minimum number of keys, try to adopt a key from a neighboring node. If that's possible, then you're finished.

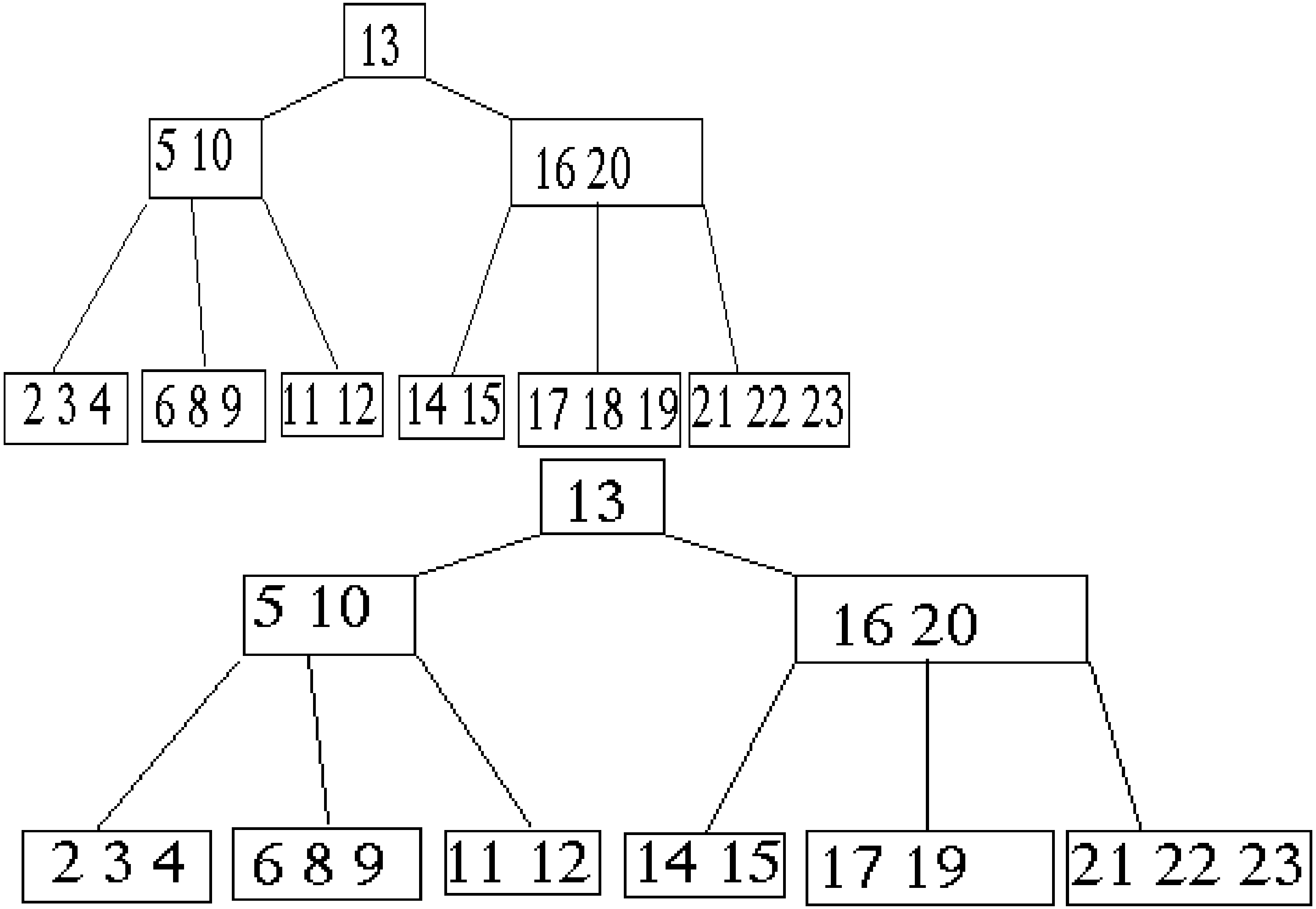
# Deleting from a B-Tree (continued)

- If the neighboring node is already at its minimum, combine the leaf node with its neighboring node, resulting in one full leaf node.
- This will require restructuring the parent node since it has lost a child
- If the parent now has fewer than the minimum keys, adopt a key from one of its neighbors. If that’s not possible, combine the parent with its neighbor.

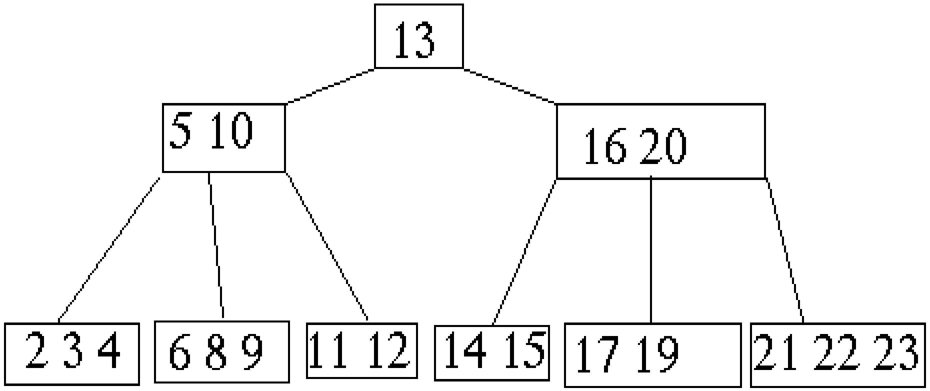
# Deleting from a B-Tree (continued)

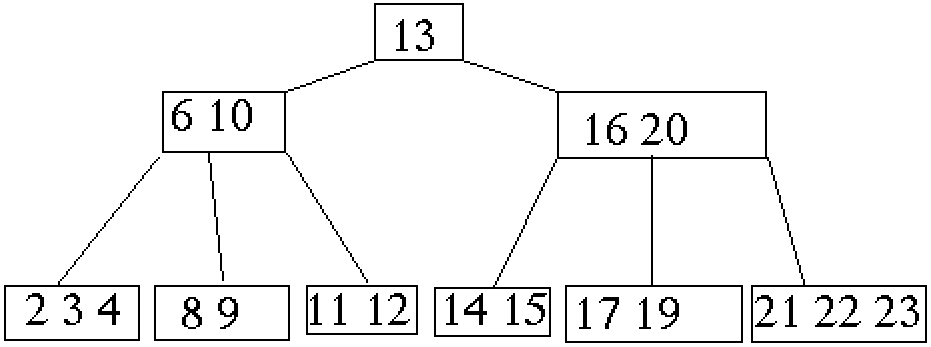
- This process may percolate all the way to the root.
- If the root is left with only one child, then remove the root node and make its child the new root.
- Both insertion and deletion are O(h), where h is the height of the tree.

# Delete 18

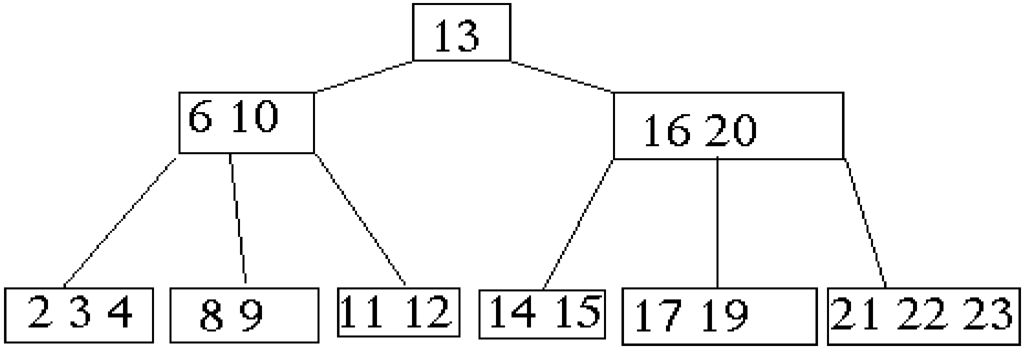

13
5 10
16 20
2 3 4
6 8 9
11 12
14 15
17 18 19
21 22 23
13
5 10
16 20
2 3 4
6 8 9
11 12
14 15
17 19
21 22 23

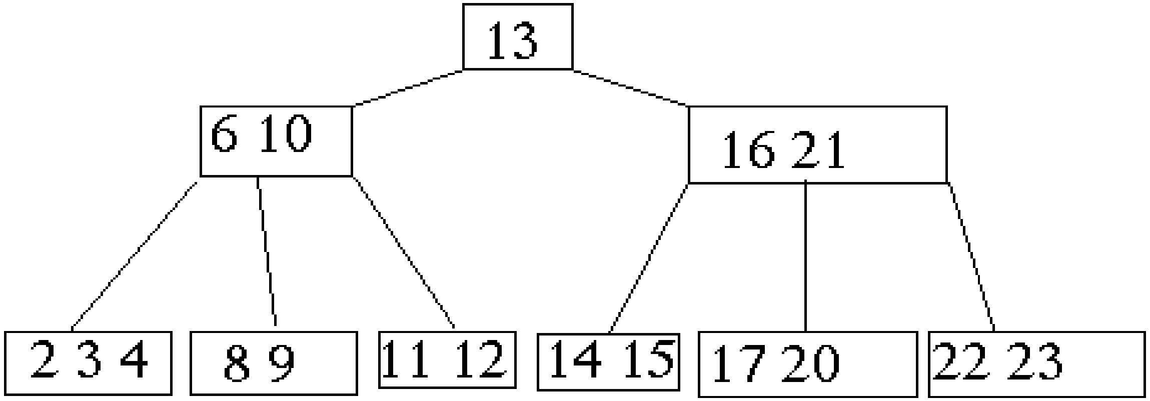
# Delete 5


13
5 10
16 20
2 3 4
6 8 9
11 12
14 15
17 19
21 22 23

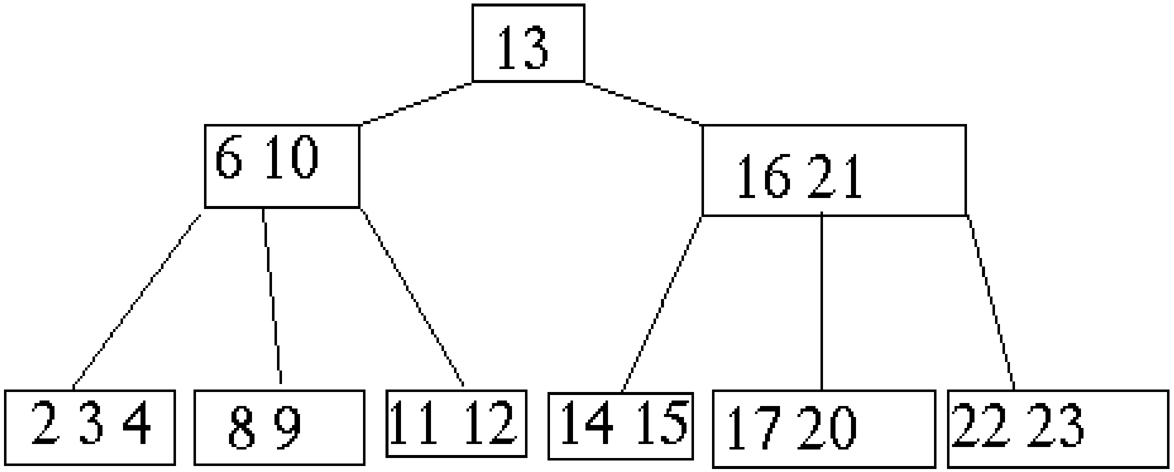

13
6 10
16 20
2 3 4
8 9
11 12
14 15
17 19
21 22 23

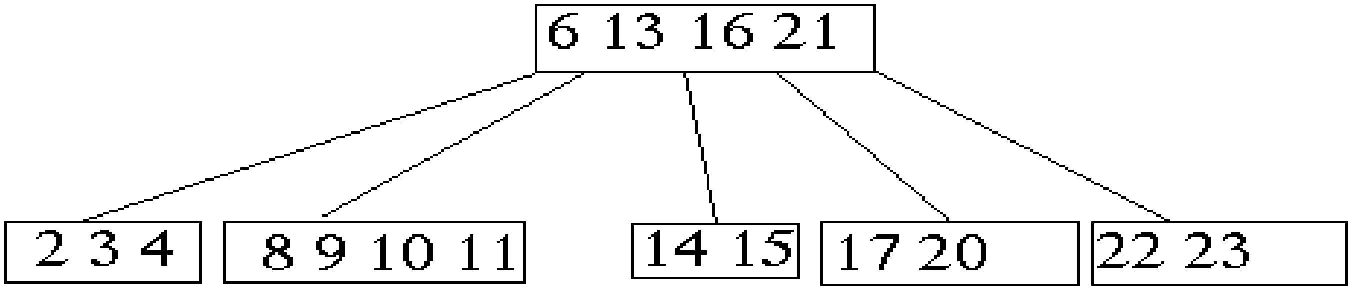
# Delete 19


13
6 10
16 20
2 3 4
8 9
11 12
14 15
17 19
21 22 23


13
6 10
16 21
2 3 4
8 9
11 12
14 15
17 20
22 23

# Delete 12


13
6 10
16 21
2 3 4
8 9
11 12
14 15
17 20
22 23


6 13 16 21
2 3 4
8 9 10 11
14 15
17 20
22 23

# Deletion in B-Tree

- **B-Tree-Delete**
  - UNDERFLOW CONDITION
  - A non-root node of a B-tree of order m underflows if, after a key deletion, it contains $\lceil$**m / 2**$\rceil$ **- 2** keys
  - The root node does not underflow. If it contains only one key and this key is deleted, the tree becomes empty.

# Deletion in B-Tree

- There are **five** deletion cases:

1. **The leaf does not underflow.**

**2. The leaf underflows and the adjacent right sibling has at least $\lceil m / 2 \rceil$ keys.**

**perform a left key-rotation**

**3. The leaf underflows and the adjacent left sibling has at least $\lceil m / 2 \rceil$ keys.**

**perform a right key-rotation**

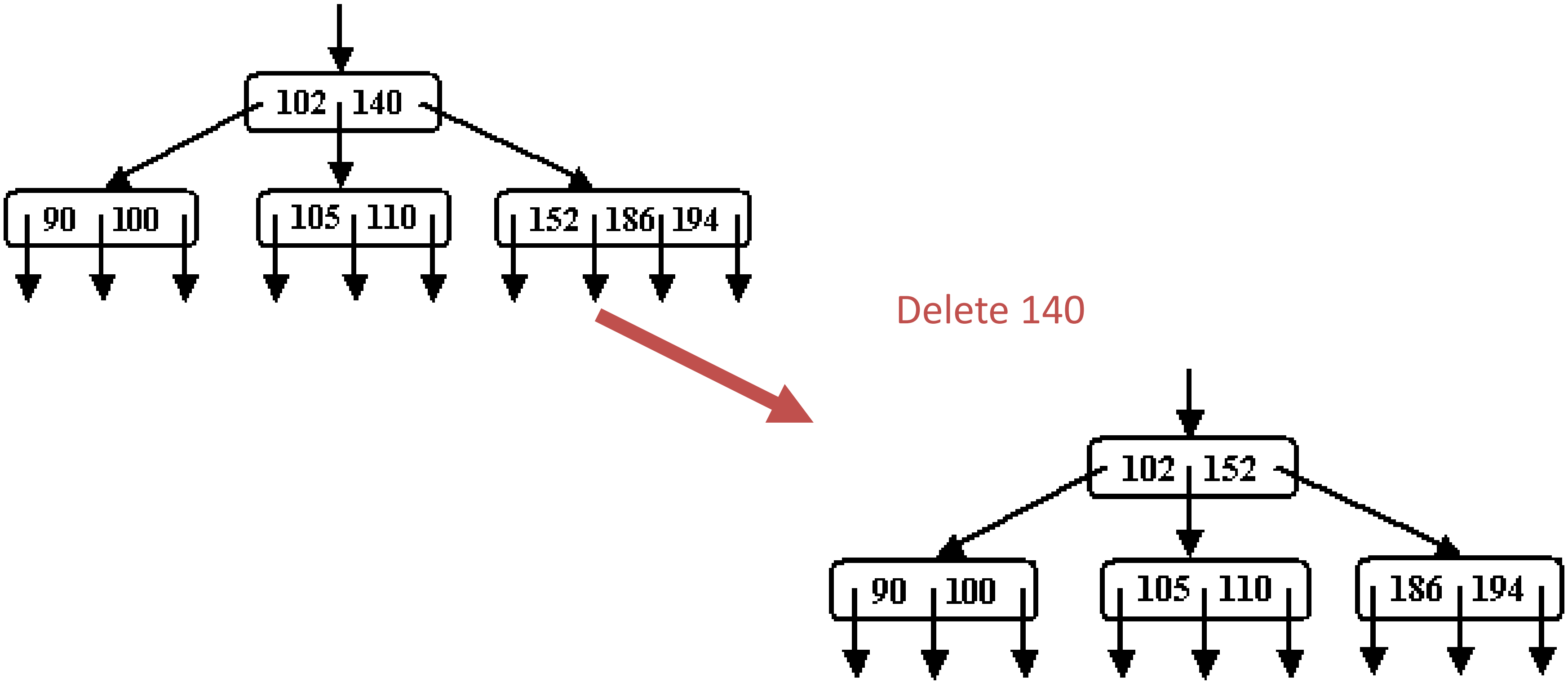
**4. The leaf underflows and each of the adjacent right sibling and the adjacent left sibling has at least $\lceil m / 2 \rceil$ keys.**

**perform either a left or a right key-rotation& perform a merging**

**5. The leaf underflows and each adjacent sibling has $\lceil m / 2 \rceil$ - 1 keys.**

# Deletion in B-Tree

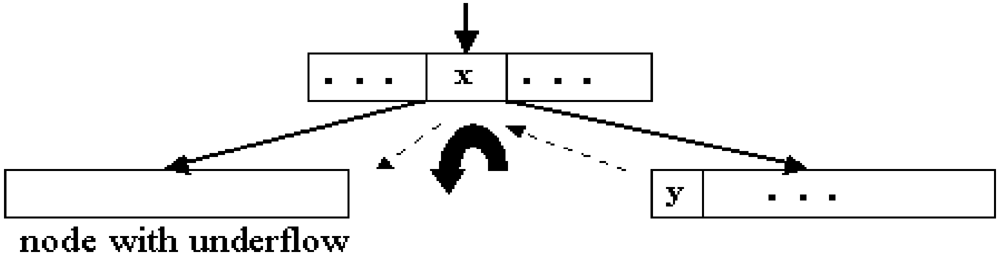
- Case1: The leaf does not underflow.
- **Example : B-tree of order 4**

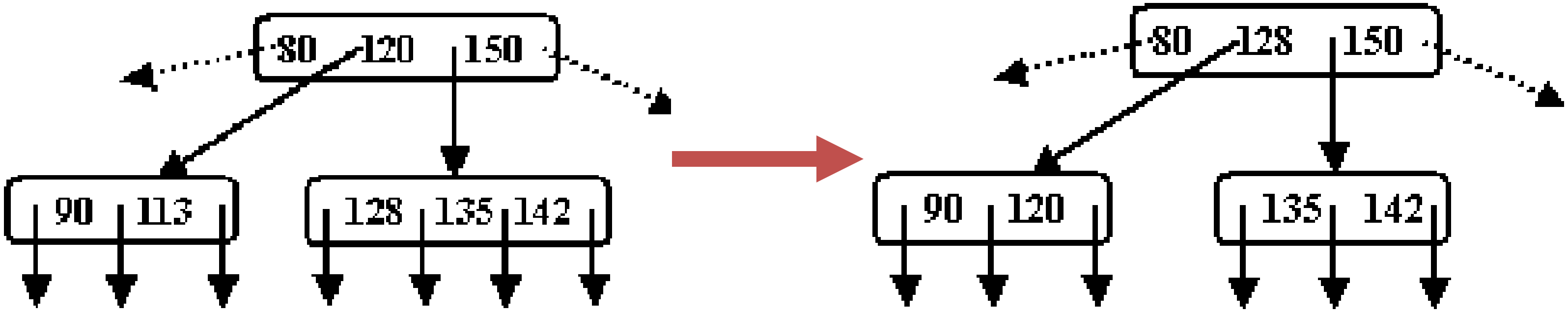


Delete 140

# Deletion in B-Tree

- Case2: The leaf underflows and the adjacent right sibling has at least ⌈m/2⌉ keys.
- **Example : B-tree of order 5**

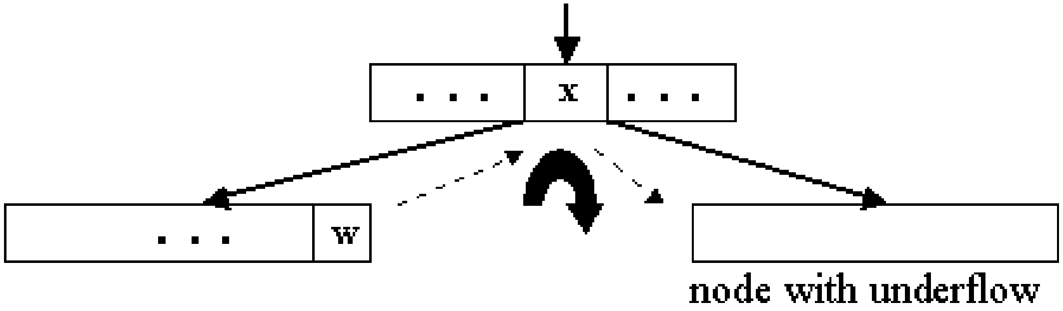




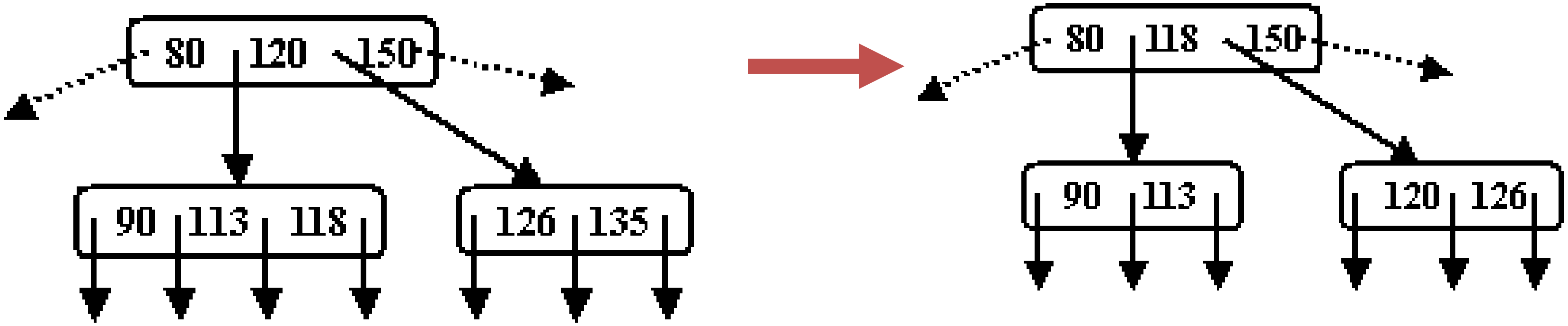
Delete 113

# Deletion in B-Tree

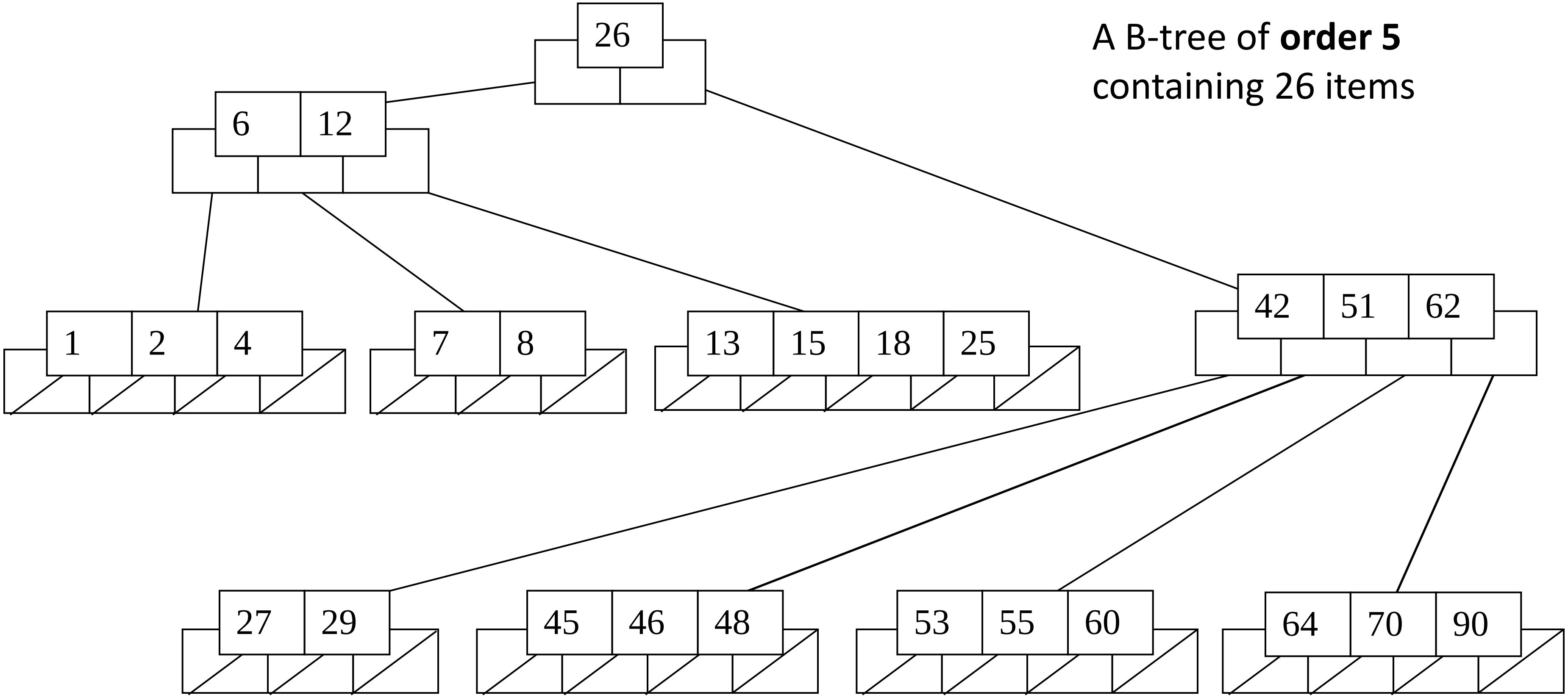
- Case 3: The leaf underflows and the adjacent left sibling has at least $\lceil m / 2 \rceil$ keys.
- **Example : B-tree of order 5**



Delete 135



# An example B-Tree

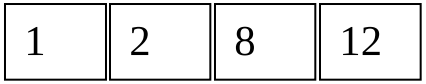


A B-tree of **order 5** containing 26 items

*Note that all the leaves are at the same level*

# Constructing a B-tree

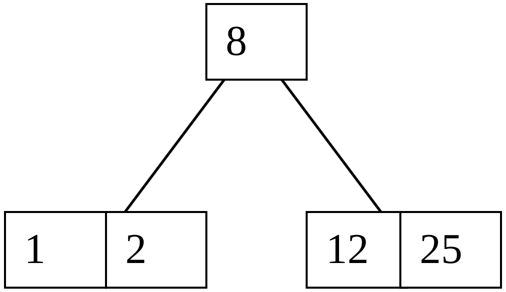
- Suppose we start with an empty B-tree and keys arrive in the following order:1  12  8  2  25  5  14  28  17  7  52  16  48  68  3  26  29  53  55  45
- We want to construct a B-tree of order 5
- The first four items go into the root:



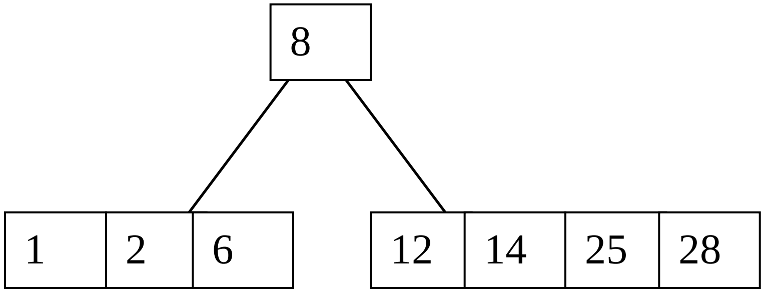
1 2 8 12

- To put the fifth item in the root would violate condition 5
- Therefore, when 25 arrives, pick the middle key to make a new root

# Constructing a B-tree (contd.)

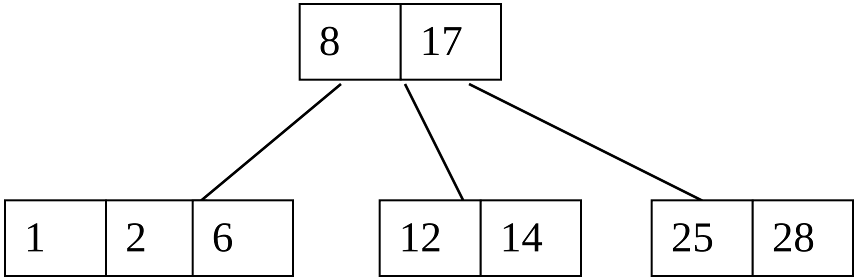


6, 14, 28 get added to the leaf nodes:

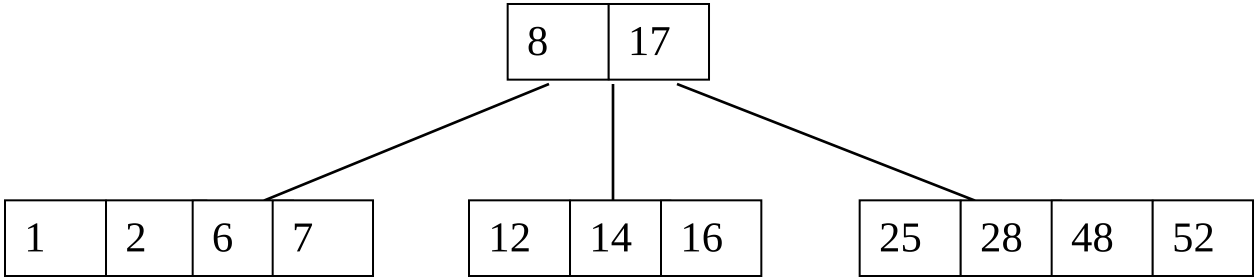


# Constructing a B-tree (contd.)

Adding 17 to the right leaf node would over-fill it, so we take the middle key, promote it (to the root) and split the leaf

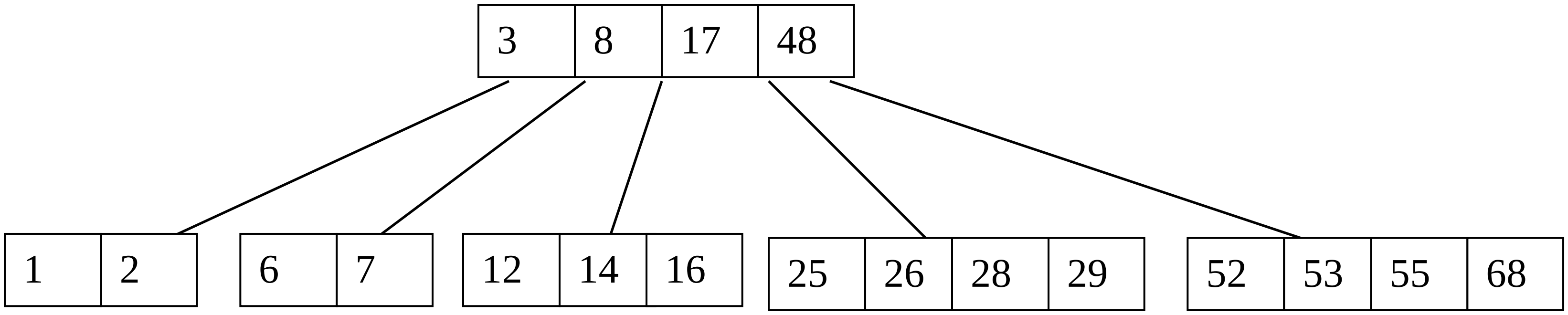


7, 52, 16, 48 get added to the leaf nodes



# Constructing a B-tree (contd.)

Adding 68 causes us to split the right most leaf, promoting 48 to the root, and adding 3 causes us to split the left most leaf, promoting 3 to the root; 26, 29, 53, 55 then go into the leaves



3 | 8 | 17 | 48

1 | 2

6 | 7

12 | 14 | 16

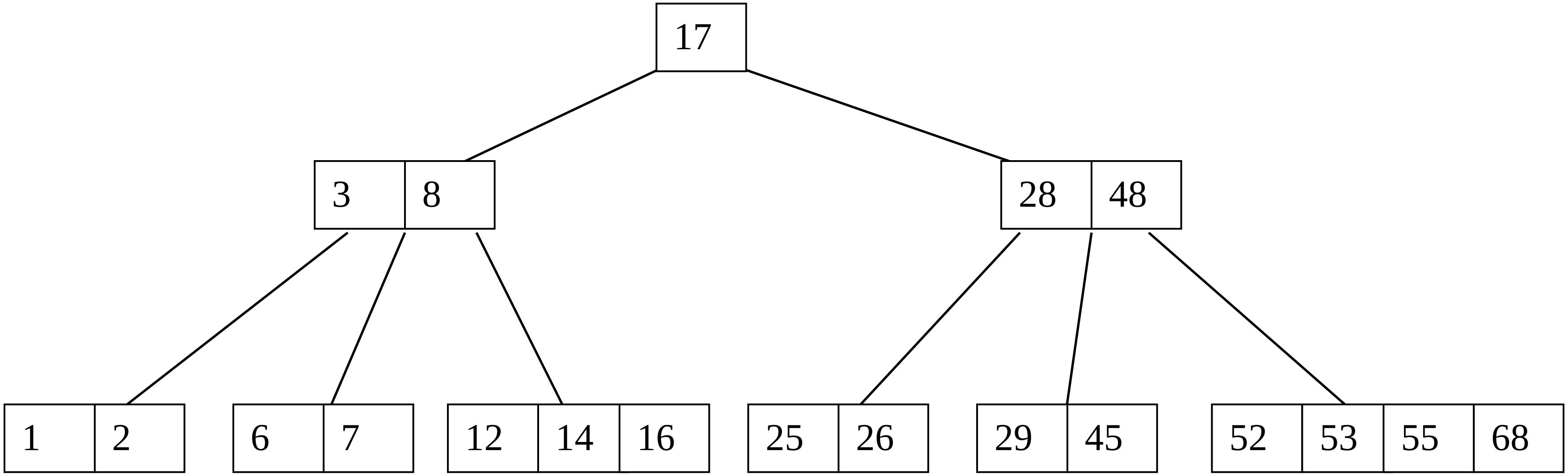
25 | 26 | 28 | 29

52 | 53 | 55 | 68

Adding 45 causes a split of 25 | 26 | 28 | 29

and promoting 28 to the root then causes the root to split

# Constructing a B-tree (contd.)



# Inserting into a B-Tree

- Attempt to insert the new key into a leaf
- If this would result in that leaf becoming too big, split the leaf into two, promoting the middle key to the leaf’s parent
- If this would result in the parent becoming too big, split the parent into two, promoting the middle key
- This strategy might have to be repeated all the way to the top
- If necessary, the root is split in two and the middle key is promoted to a new root, making the tree one level higher

# Exercise in Inserting a B-Tree

- Insert the following keys to a 5-way B-tree:
- 3, 7, 9, 23, 45, 1, 5, 14, 25, 24, 13, 11, 8, 19, 4, 31, 35, 56

# Removal from a B-tree

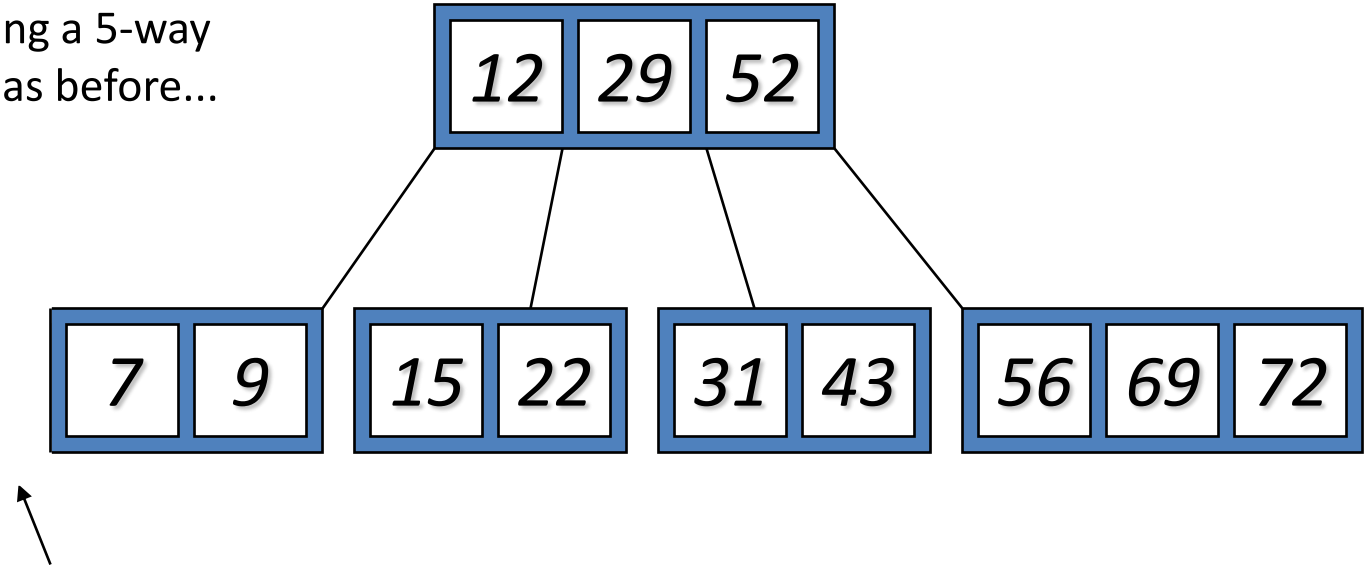
- During insertion, the key always goes *into* a *leaf*. For deletion we wish to remove *from* a leaf. There are three possible ways we can do this:
- 1 - If the key is already in a leaf node, and removing it doesn’t cause that leaf node to have too few keys, then simply remove the key to be deleted.
- 2 - If the key is *not* in a leaf then it is guaranteed (by the nature of a B-tree) that its predecessor or successor will be in a leaf -- in this case we can delete the key and promote the predecessor or successor key to the non-leaf deleted key’s position.

# Removal from a B-tree (2)

- If (1) or (2) lead to a leaf node containing less than the minimum number of keys then we have to look at the siblings immediately adjacent to the leaf in question:
  - 3: if one of them has more than the min. number of keys then we can promote one of its keys to the parent and take the parent key into our lacking leaf
  - 4: if neither of them has more than the min. number of keys then the lacking leaf and one of its neighbours can be combined with their shared parent (the opposite of promoting a key) and the new leaf will have the correct number of keys; if this step leave the parent with too few keys then we repeat the process up to the root itself, if required

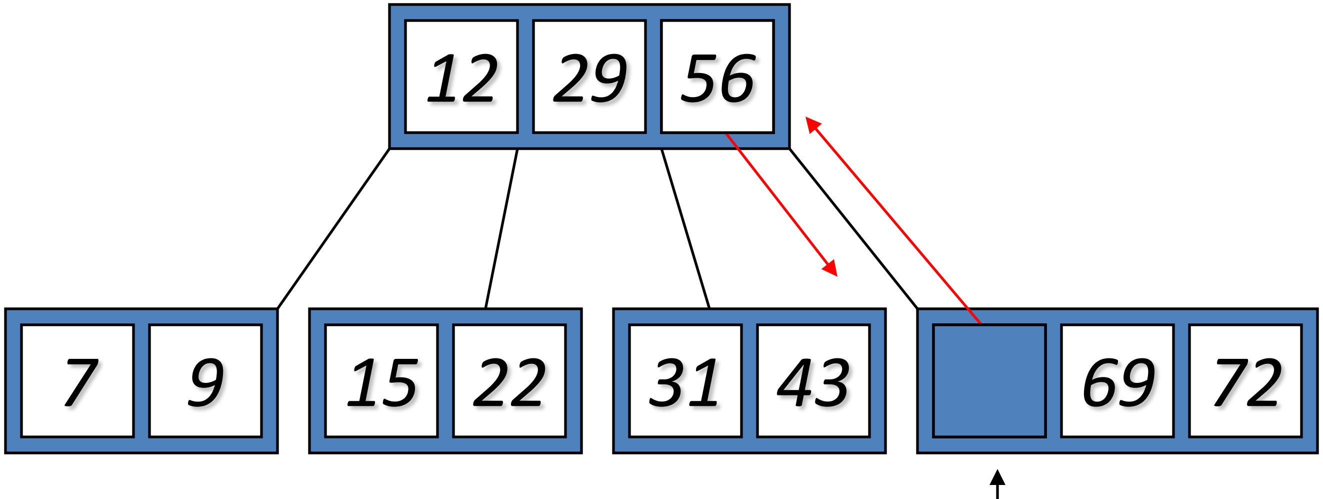
# Type #1: Simple leaf deletion

Assuming a 5-way
B-Tree, as before...



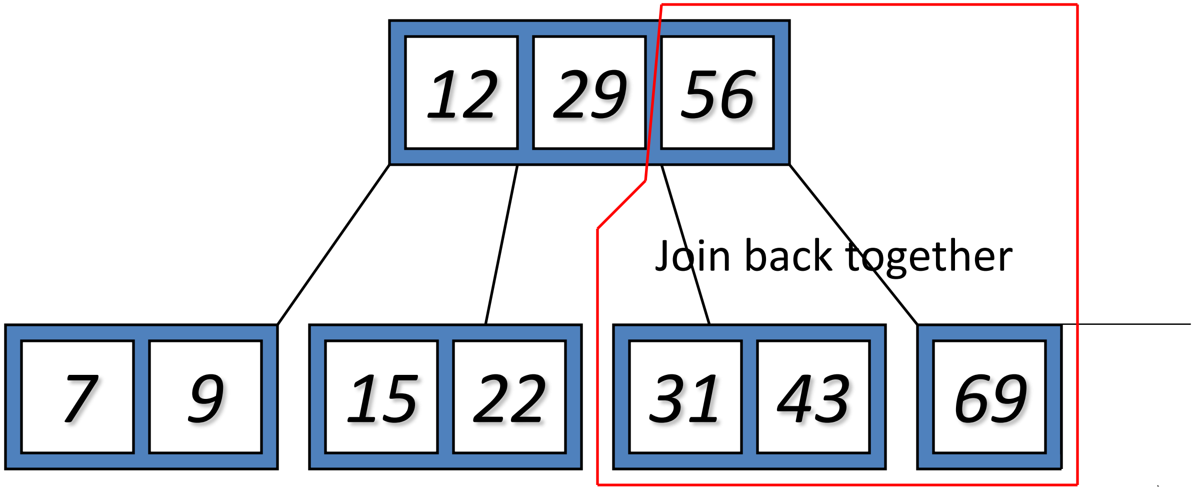
Delete 2: Since there are enough keys in the node, just delete it

# Type #2: Simple non-leaf deletion



Borrow the predecessor
or (in this case) successor

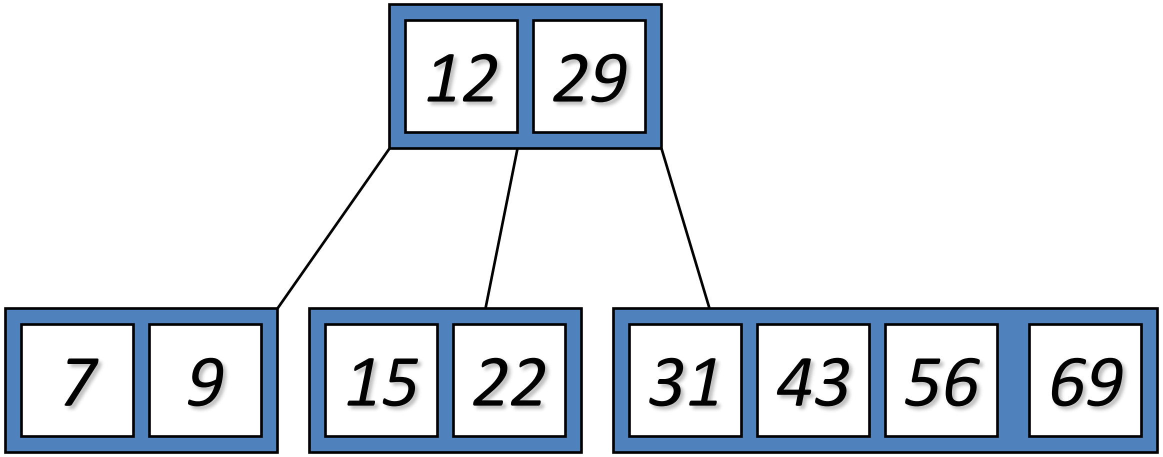
# Type #4: Too few keys in node and its siblings



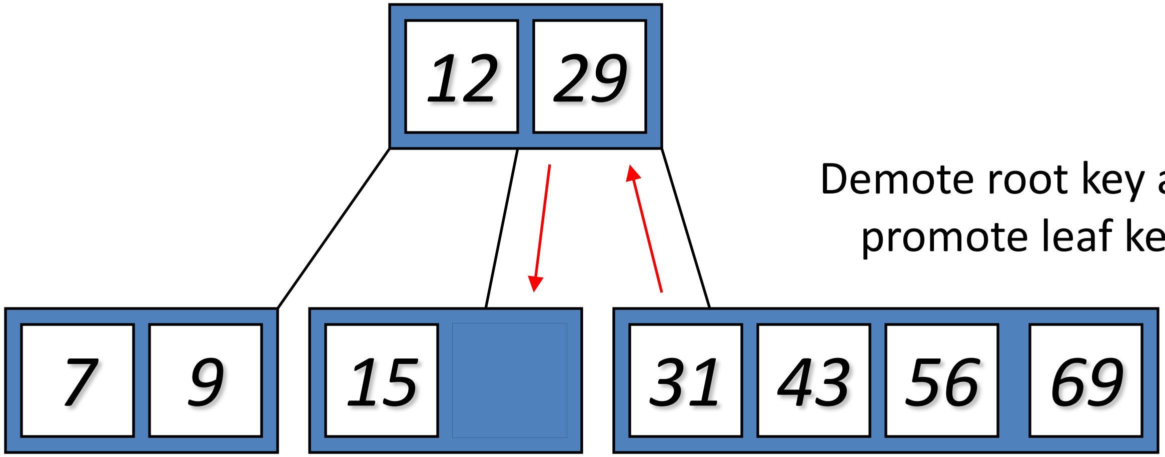
Join back together

Too few keys!

# Type #4: Too few keys in node and its siblings

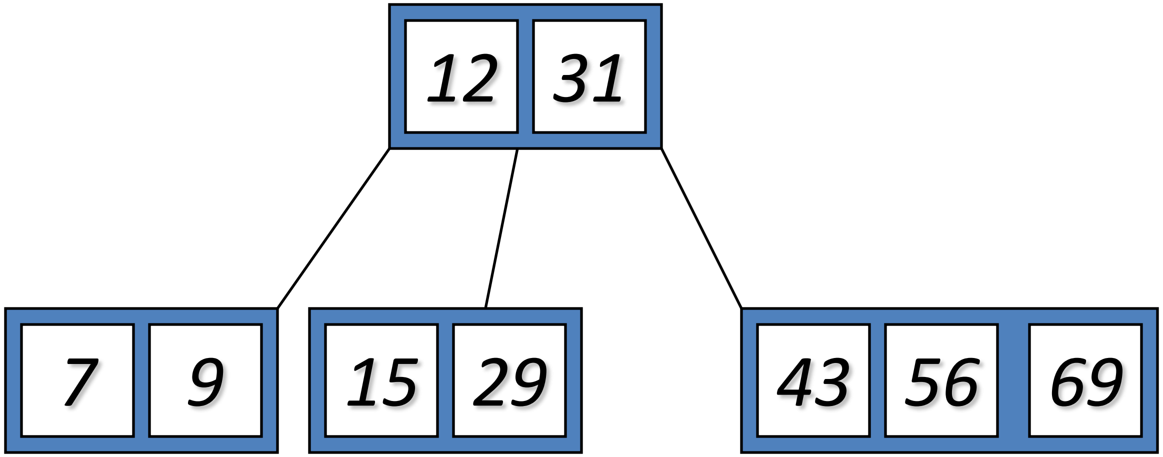


# Type #3: Enough siblings



Demote root key and promote leaf key

# Type #3: Enough siblings

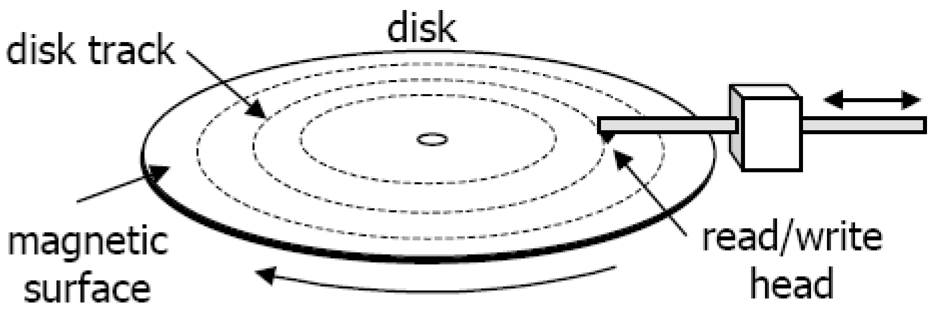


# Summary

- The B-tree is a tree-like structure that helps us to organize data in an efficient way.
- The B-tree index is a technique used to minimize the disk I/Os needed for the purpose of locating a row with a given index key value.
- Because of its advantages, the B-tree and the B-tree index structure are widely used in databases nowadays.
- In addition to its use in databases, the B-tree is also used in file systems to allow quick random access to an arbitrary block in a particular file. The basic problem is turning the file block *i* address into a disk block.

MS/Dos - FAT (File allocation table)

- entry for each disk block
- entry identifies whether its block is used by a file
- which block (if any) is the next disk block of the same file
- allocation of each file is represented as a linked list in the table



# Secondary Storages

# Red-Black Trees

# Red-Black Properties

The *red-black properties*:

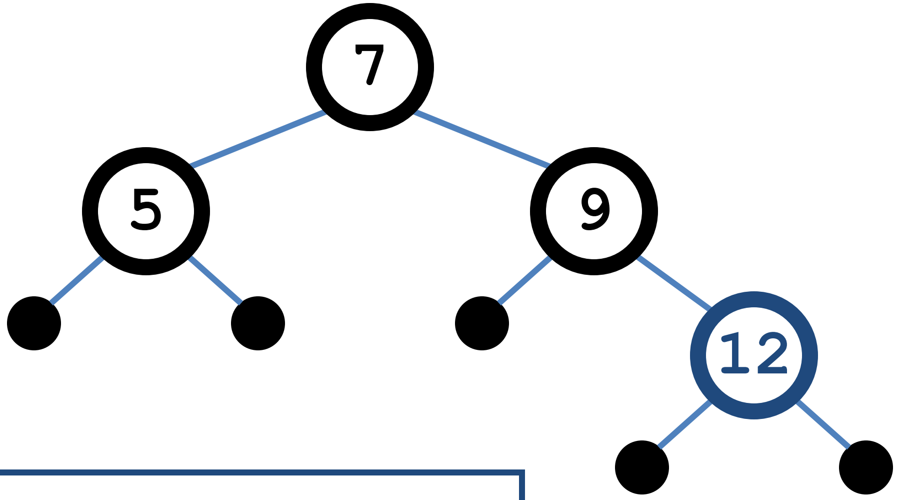
1. **Every node is either red or black**
2. **Every leaf (NULL pointer) is black**

   **Note: this means every "real" node has 2 children**
3. **If a node is red, both children are black**

   **Note: can't have 2 consecutive reds on a path**
4. **Every path from node to descendent leaf contains the same number of black nodes**
5. **The root is always black**

# Red-Black Trees: An Example

- Color this tree:

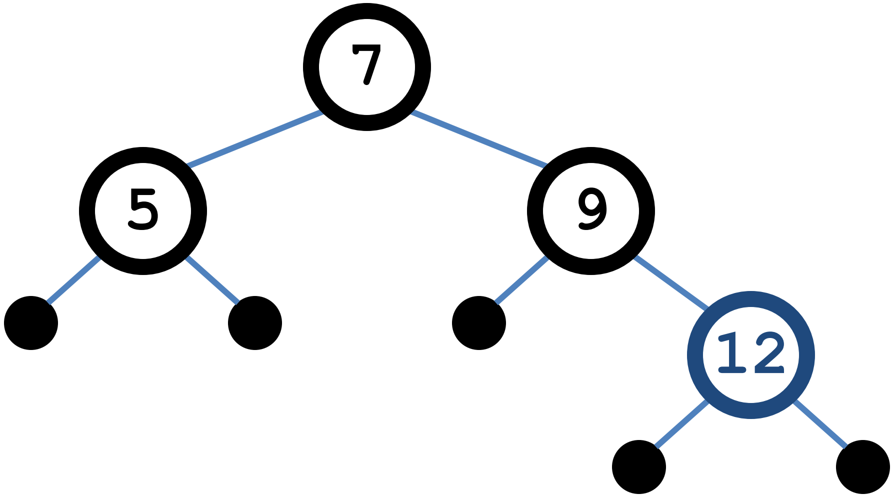


Red-black properties:

1. Every node is either red or black
2. Every leaf (NULL pointer) is black
3. If a node is red, both children are black
4. Every path from node to descendent leaf contains the same number of black nodes
5. The root is always black

# Red-Black Trees: The Problem With Insertion

- Insert 8
    - Where does it go?



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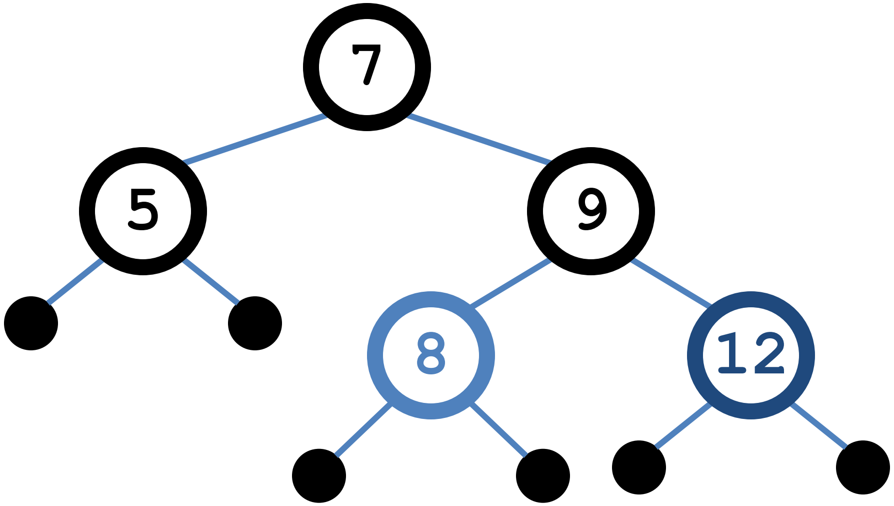
# Red-Black Trees: The Problem With Insertion

- Insert 8
  - Where does it go?
  - What color should it be?



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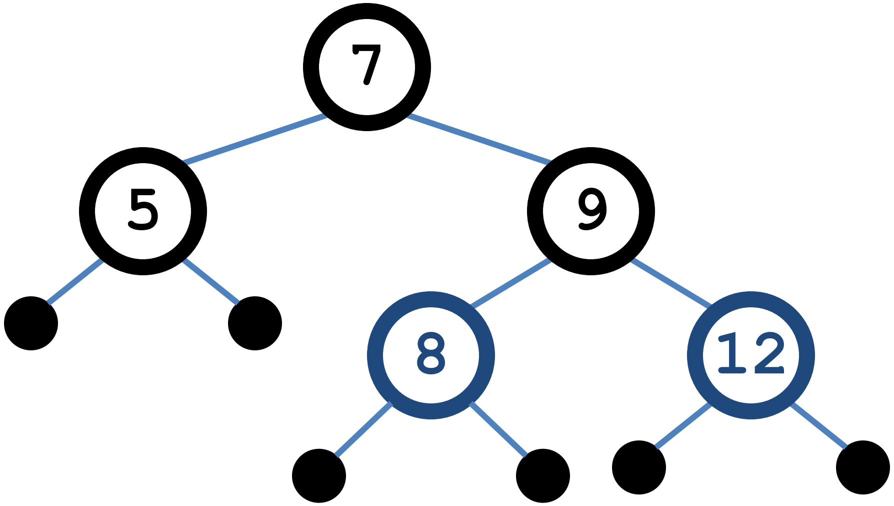

1. Every node is either red or black
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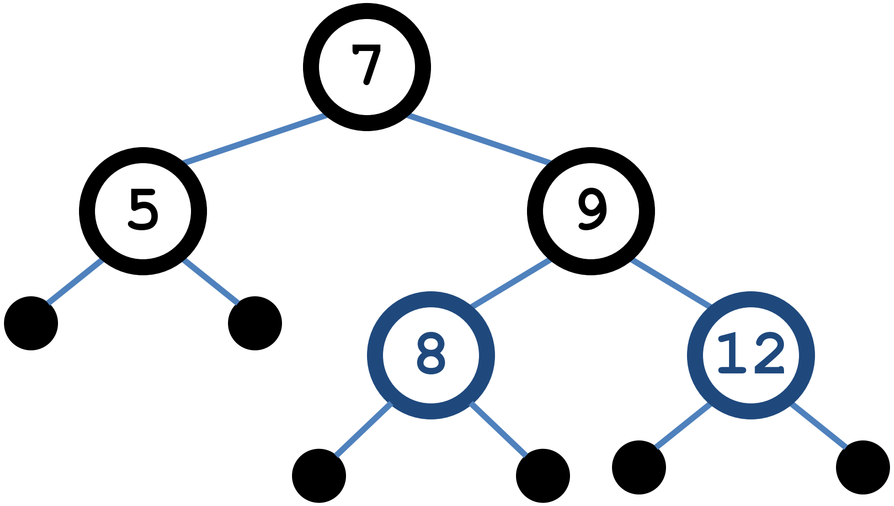
# Red-Black Trees: The Problem With Insertion

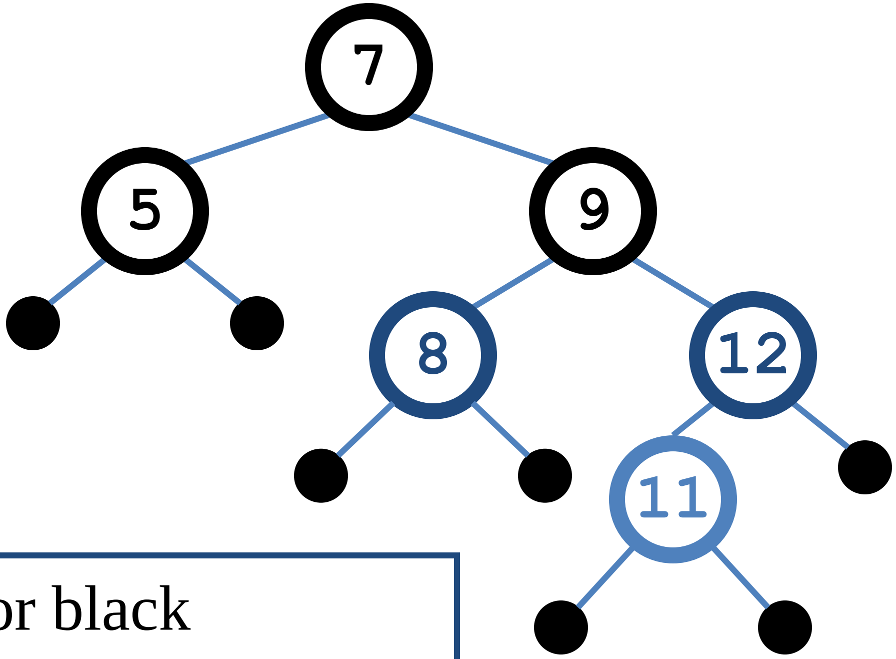
- Insert 11
  - Where does it go?



1. Every node is either red or black
2. Every leaf (NULL pointer) is black
3. If a node is red, both children are black
4. Every path from node to descendent leaf contains the same number of black nodes
5. The root is always black

# Red-Black Trees:
# The Problem With Insertion

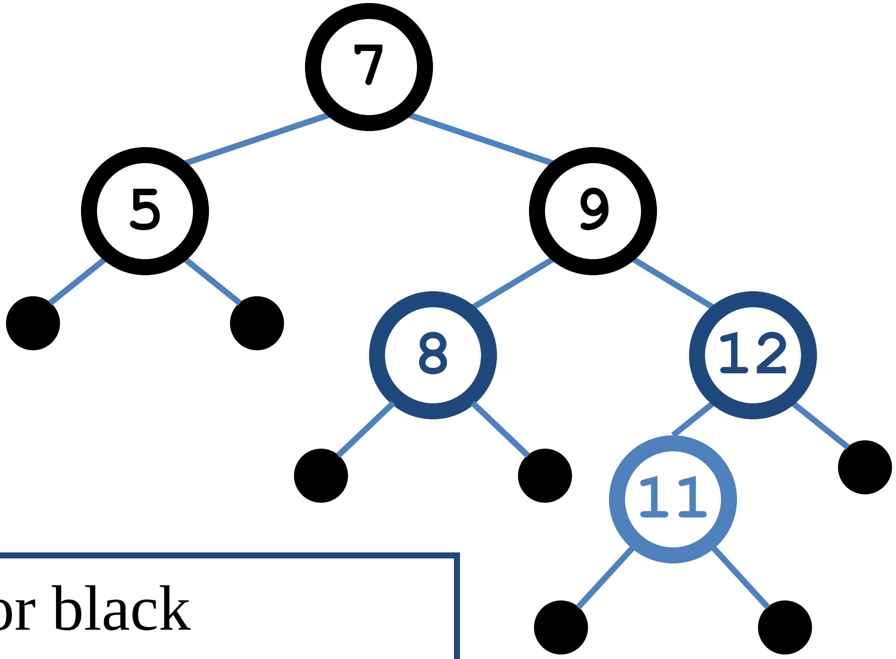
- Insert 11
  - Where does it go?
  - What color?



1. Every node is either red or black
2. Every leaf (NULL pointer) is black
3. If a node is red, both children are black
4. Every path from node to descendent leaf contains the same number of black nodes
5. The root is always black

# Red-Black Trees: The Problem With Insertion

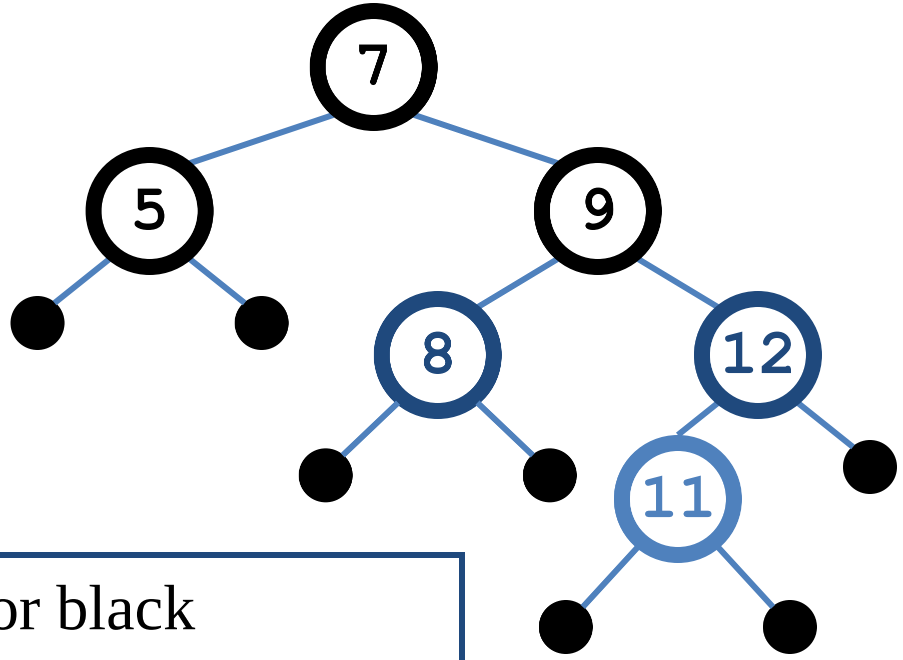
- Insert 11
  - Where does it go?
  - What color?
    - Can’t be red! (#3)



1. Every node is either red or black
2. Every leaf (NULL pointer) is black
3. If a node is red, both children are black
4. Every path from node to descendent leaf contains the same number of black nodes
5. The root is always black

# Red-Black Trees: The Problem With Insertion

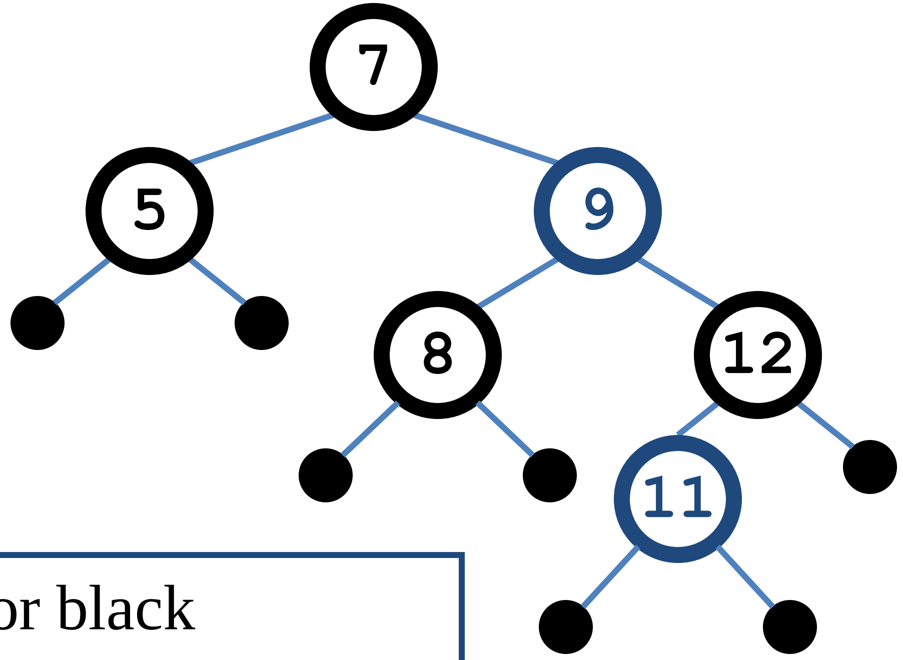
- Insert 11
  - Where does it go?
  - What color?
    - Can’t be red! (#3)
    - Can’t be black! (#4)



1. Every node is either red or black
2. Every leaf (NULL pointer) is black
3. If a node is red, both children are black
4. Every path from node to descendent leaf contains the same number of black nodes
5. The root is always black

# Red-Black Trees: The Problem With Insertion

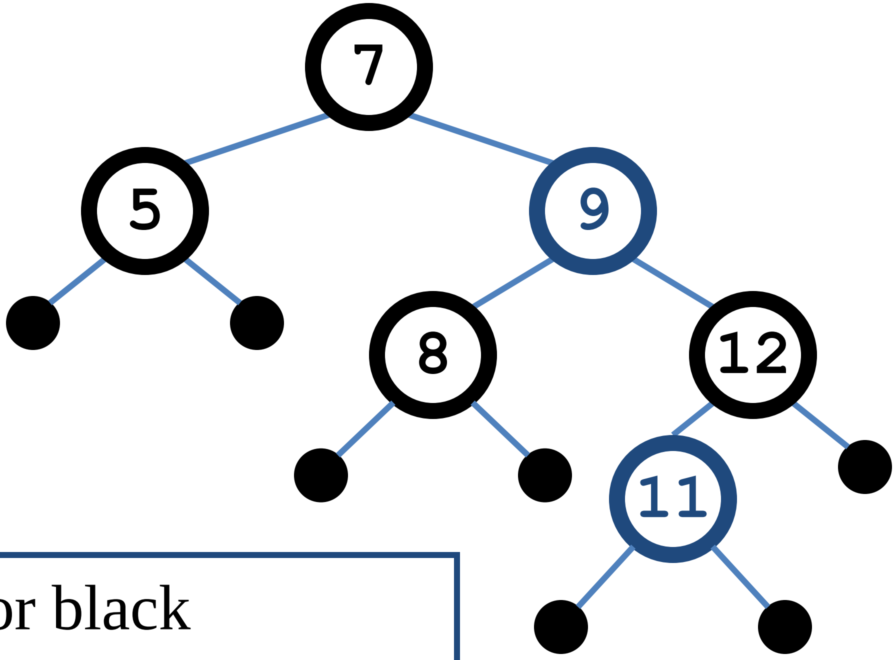
- Insert 11
  - Where does it go?
  - What color?
    - Solution: recolor the tree



1. Every node is either red or black
2. Every leaf (NULL pointer) is black
3. If a node is red, both children are black
4. Every path from node to descendent leaf contains the same number of black nodes
5. The root is always black

# Red-Black Trees:
# The Problem With Insertion

- Insert 10
  - Where does it go?



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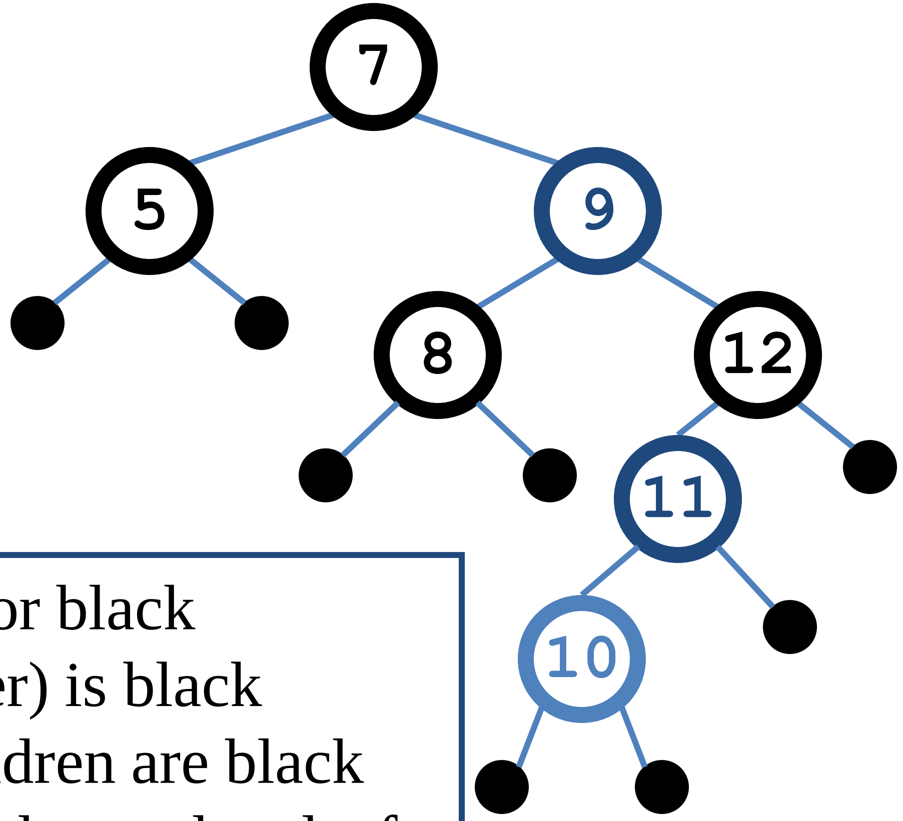
# Red-Black Trees:
# The Problem With Insertion

- Insert 10
  - Where does it go?
  - What color?



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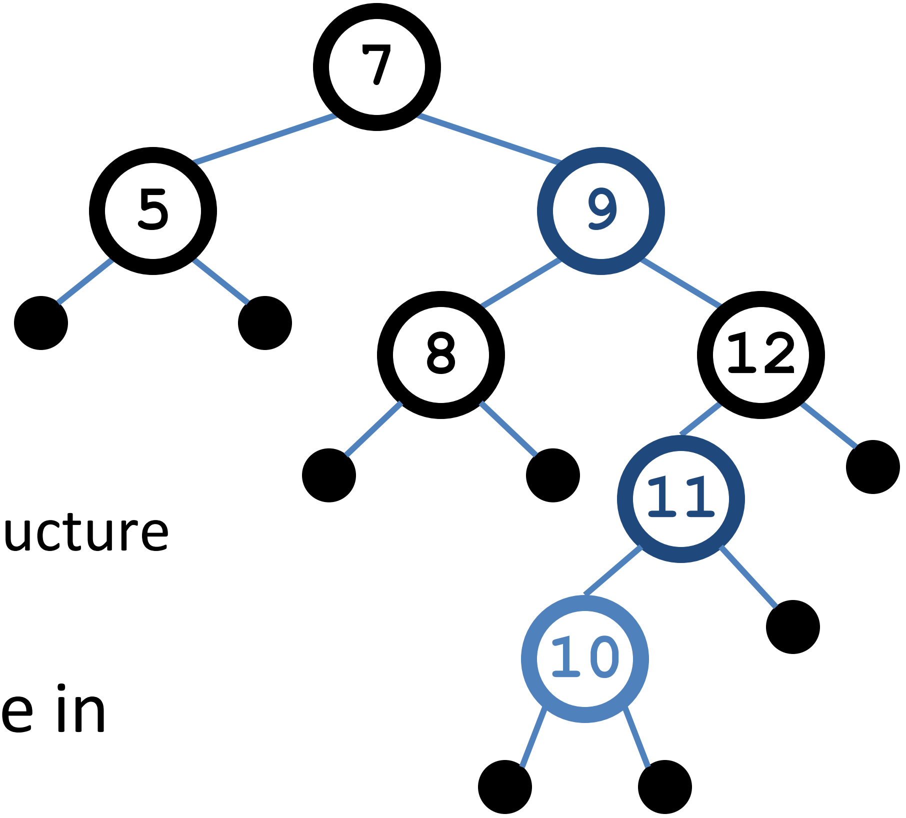
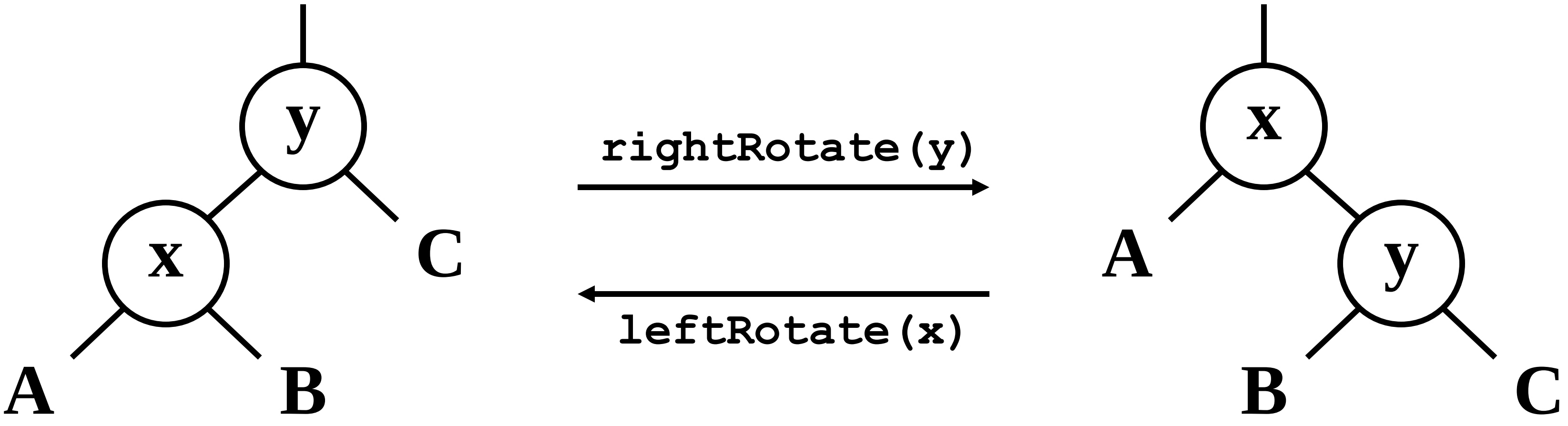
# Red-Black Trees: The Problem With Insertion

- Insert 10
  - Where does it go?
  - What color?
    - A: no color! Tree is too imbalanced
    - Must change tree structure to allow recoloring
  - Goal: restructure tree in O(lg n) time



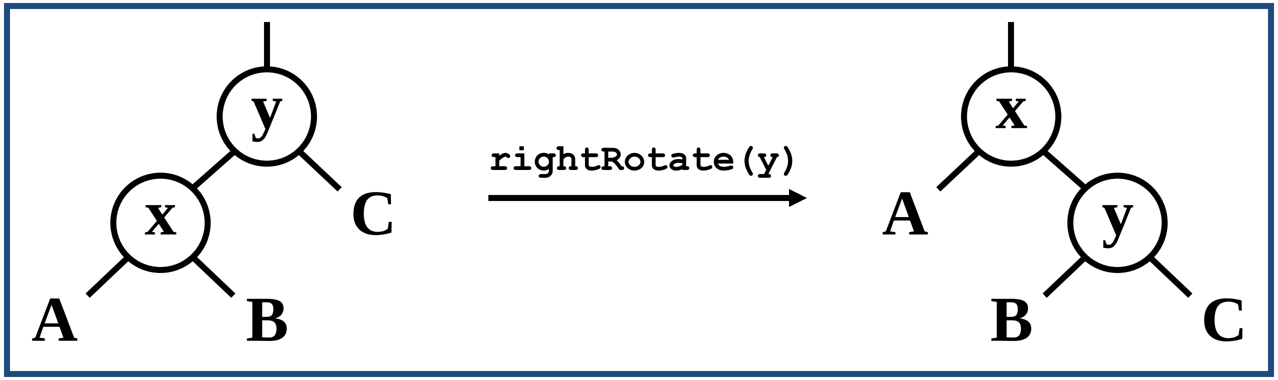
# RB Trees: Rotation

- Our basic operation for changing tree structure is called rotation:



- Does rotation preserve inorder key ordering?
- What would the code for `rightRotate()` actually do?

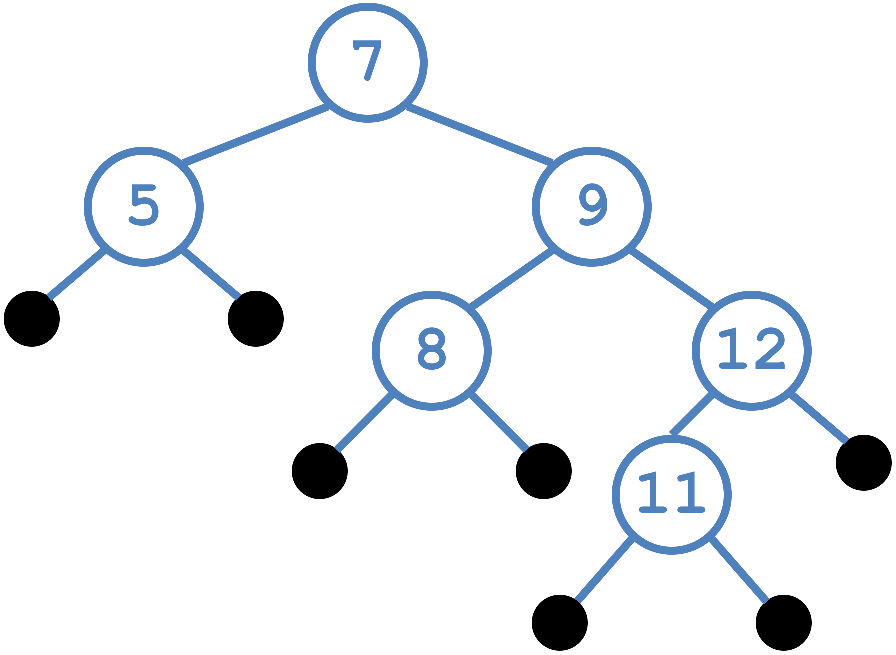
# RB Trees: Rotation



- Answer: A lot of pointer manipulation
  - x keeps its left child
  - y keeps its right child
  - x's right child becomes y's left child
  - x's and y's parents change
- What is the running time?

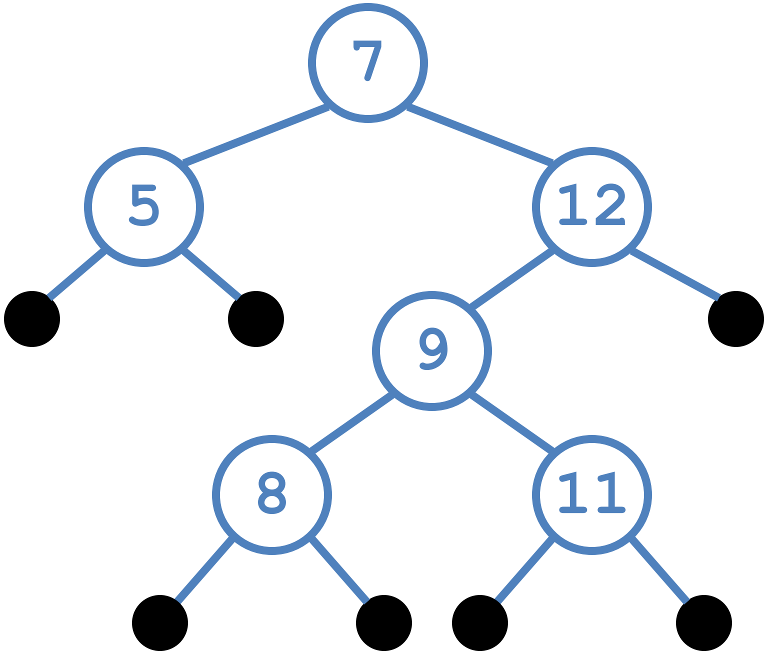
# Rotation Example

- Rotate left about 9:

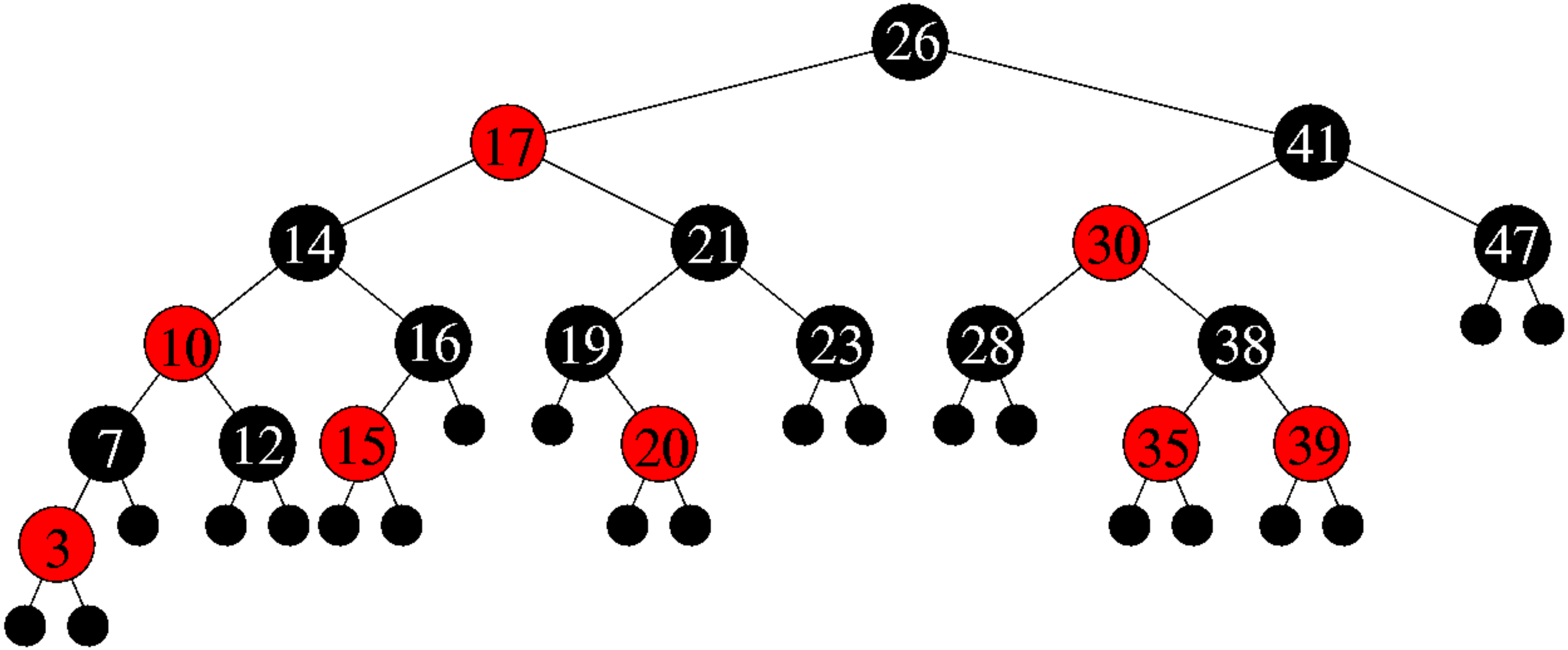


# Rotation Example

- Rotate left about 9:



# Example Red-Black Tree



# Splay Trees

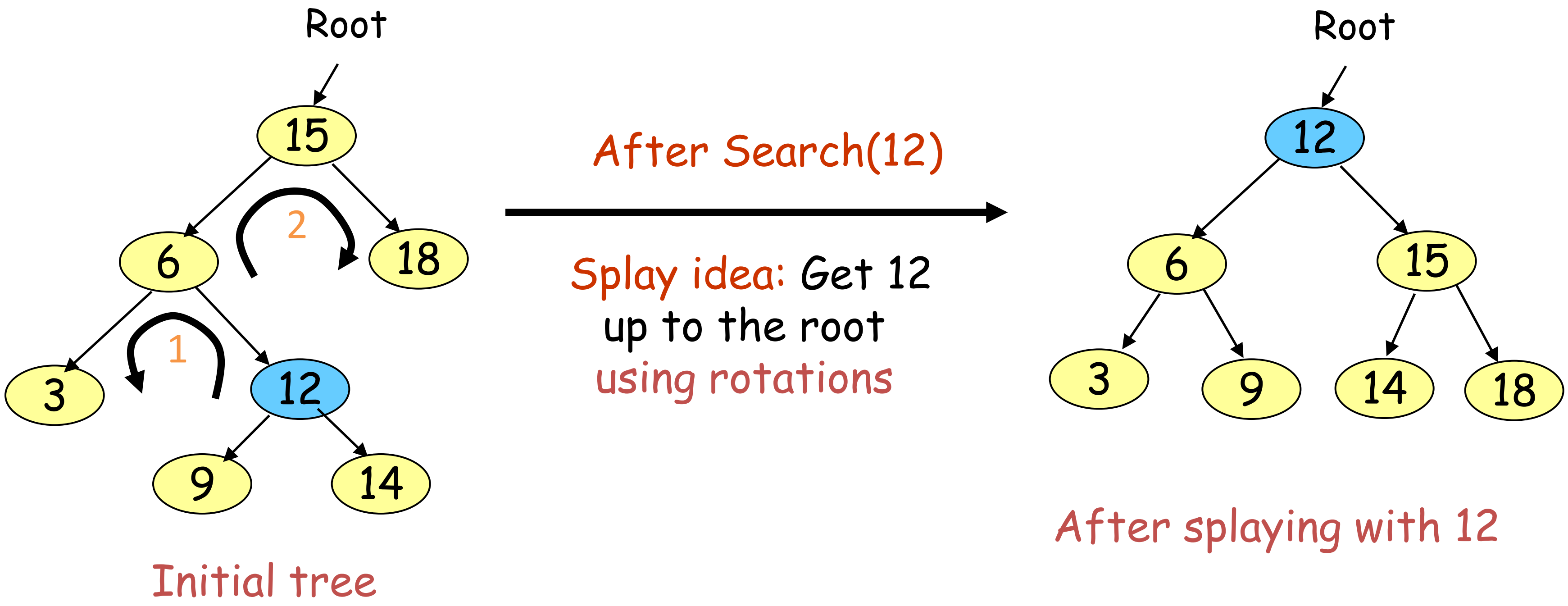
**Splay trees are binary search trees (BSTs) that:**

- **Are not perfectly balanced all the time**
- **Allow search and insertion operations to try to balance the tree so that future operations may run faster**

**Based on the heuristic:**

- **If X is accessed once, it is likely to be accessed again.**
- **After node X is accessed, perform "splaying" operations to bring X up to the root of the tree.**
- **Do this in a way that leaves the tree more or less balanced as a whole.**

# Example



After Search(12)

Splay idea: Get 12 up to the root using rotations

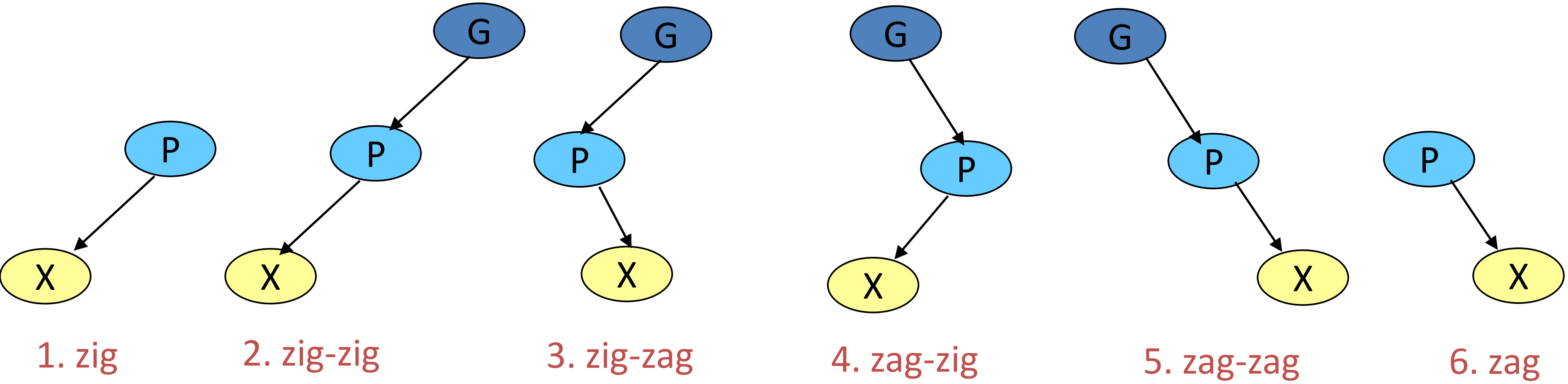
Initial tree

After splaying with 12

- Not only splaying with 12 makes the tree balanced, subsequent accesses for 12 will take O(1) time.
- Active (recently accessed) nodes will move towards the root and inactive nodes will slowly move further from the root

# Splay Tree Terminology

- Let X be a **non-root** node, i.e., has at least 1 ancestor.
- Let P be its **parent** node.
- Let G be its **grandparent** node (if it exists)
- Consider a path from **G to X**:
  - Each time we go **left**, we say that we "**zig**"
  - Each time we go **right**, we say that we "**zag**"
- There are 6 possible cases:

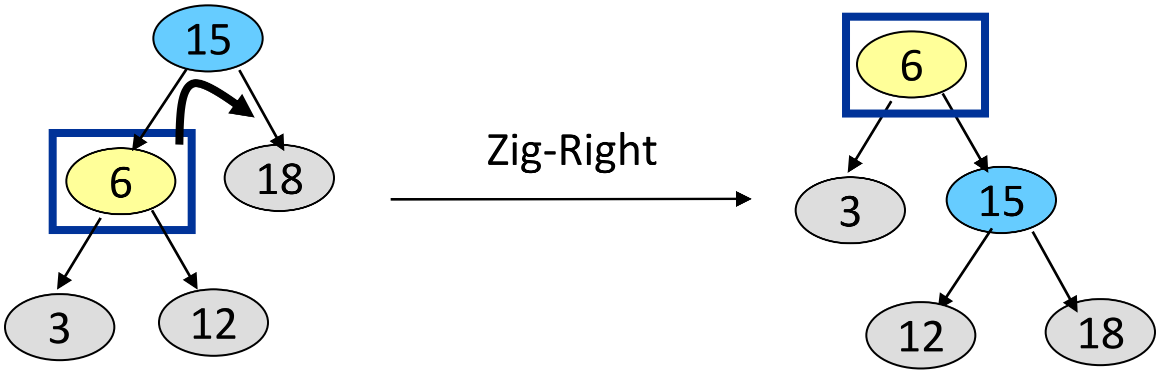


# Splay Tree Operations

- **When node X is accessed, apply one of six rotation operations:**
  - **Single Rotations (X has a P but no G)**
    - **zig, zag**
  - **Double Rotations (X has both a P and a G)**
    - **zig-zig, zig-zag**
    - **zag-zig, zag-zag**

# Splay Trees: Zig Operation

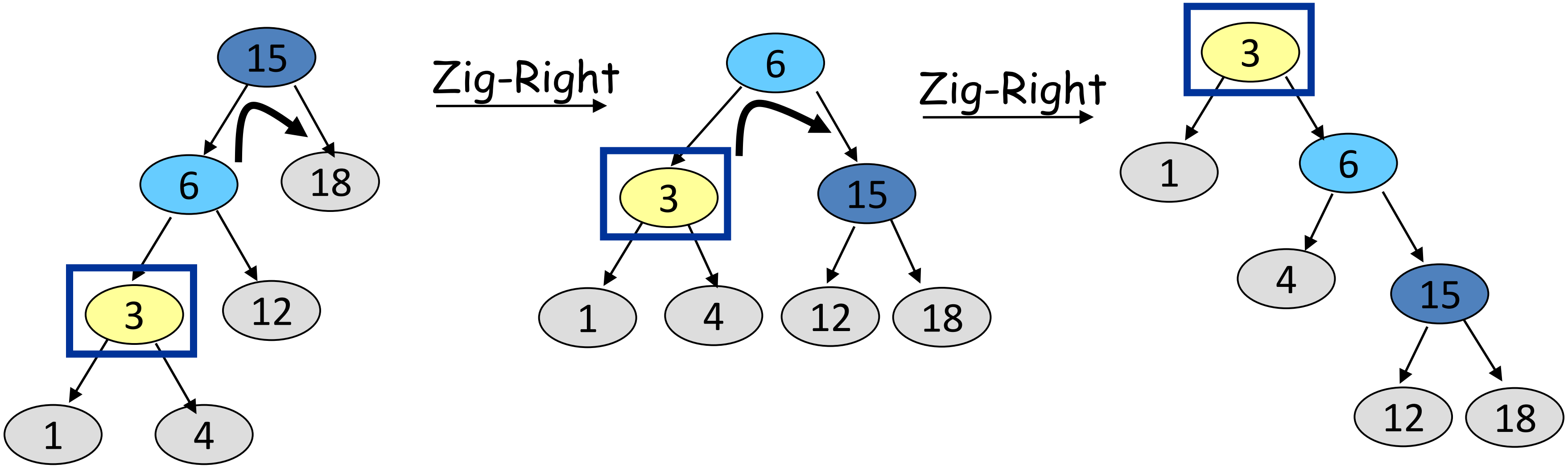
- **"Zig" is just a single rotation, as in an AVL tree**
- **Suppose 6 was the node that was accessed (e.g. using Search)**



- "Zig-Right" moves 6 to the root.
- Can access 6 faster next time: O(1)
- Notice that this is simply a right rotation in AVL tree terminology.

# Splay Trees: Zig-Zig Operation

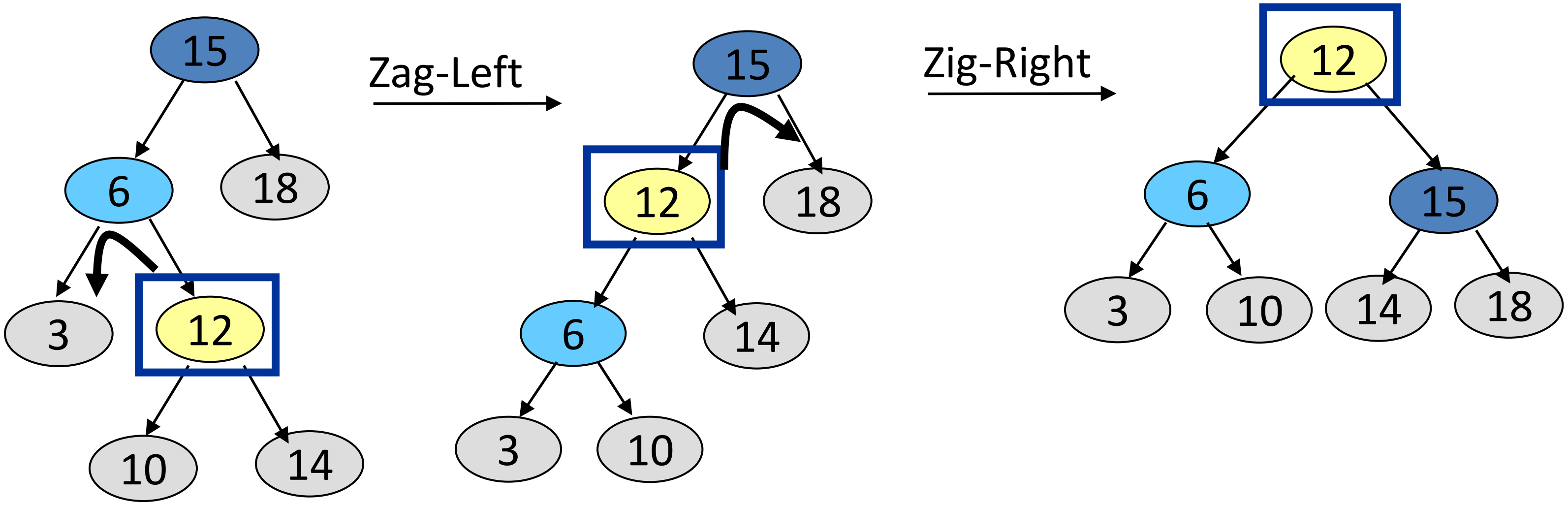
- **“Zig-Zig” consists of two single rotations of the same type**
- **Suppose 3 was the node that was accessed (e.g., using Search)**



- Due to “zig-zig“ splaying, 3 has bubbled to the top!
- Note: Parent-Grandparent is rotated first.

# Splay Trees: Zig-Zag Operation

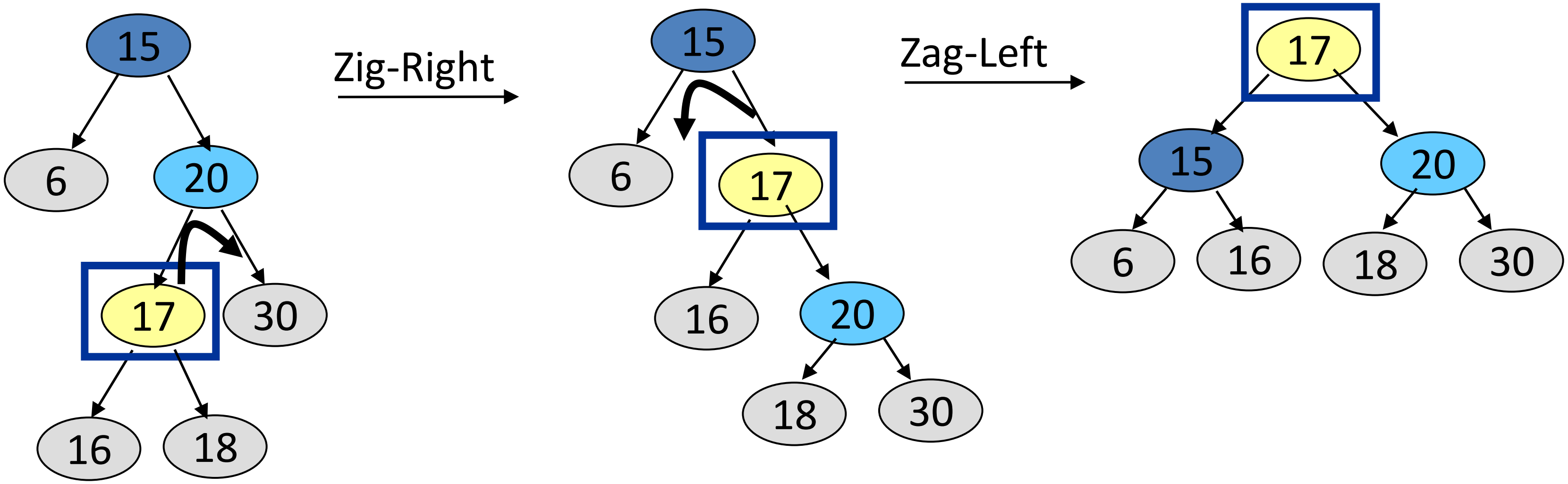
- **"Zig-Zag" consists of two rotations of the opposite type**
- **Suppose 12 was the node that was accessed (e.g., using Search)**



- Due to "zig-zag" splaying, 12 has bubbled to the top!
- Notice that this is simply an LR imbalance correction in AVL tree terminology (first a left rotation, then a right rotation)

# Splay Trees: Zag-Zig Operation

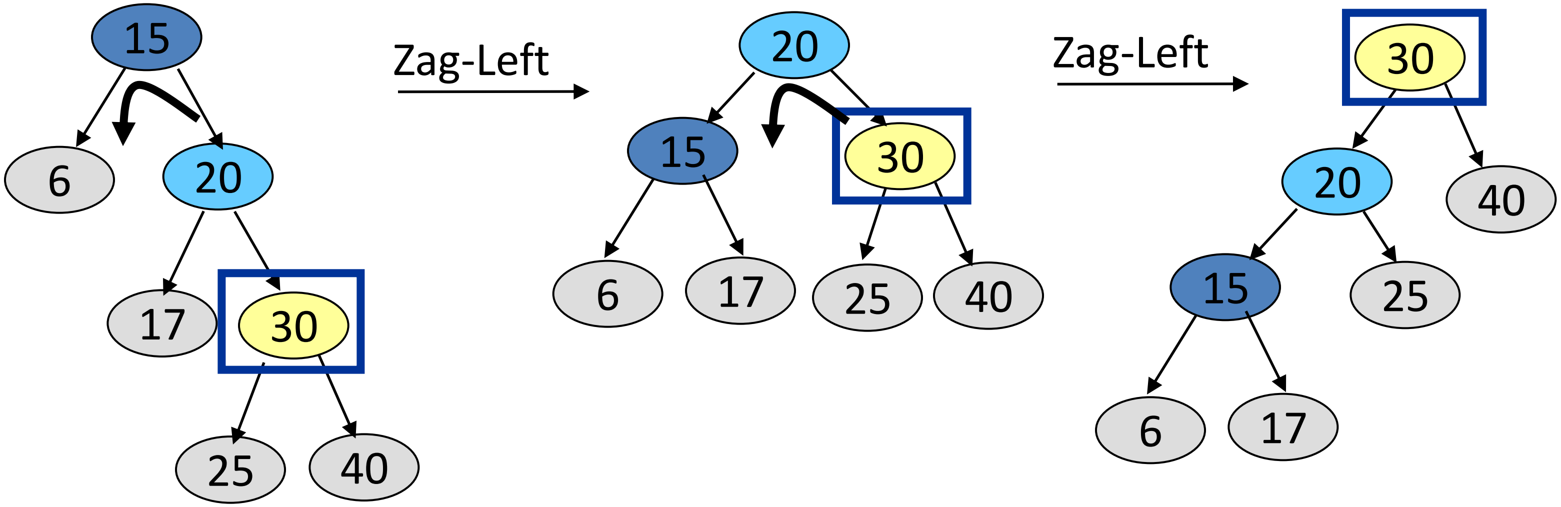
- **"Zag-Zig" consists of two rotations of the opposite type**
- **Suppose 17 was the node that was accessed (e.g., using Search)**



- Due to "zag-zig" splaying, 17 has bubbled to the top!
- Notice that this is simply an RL imbalance correction in AVL tree terminology (first a right rotation, then a left rotation)

# Splay Trees: Zag-Zag Operation

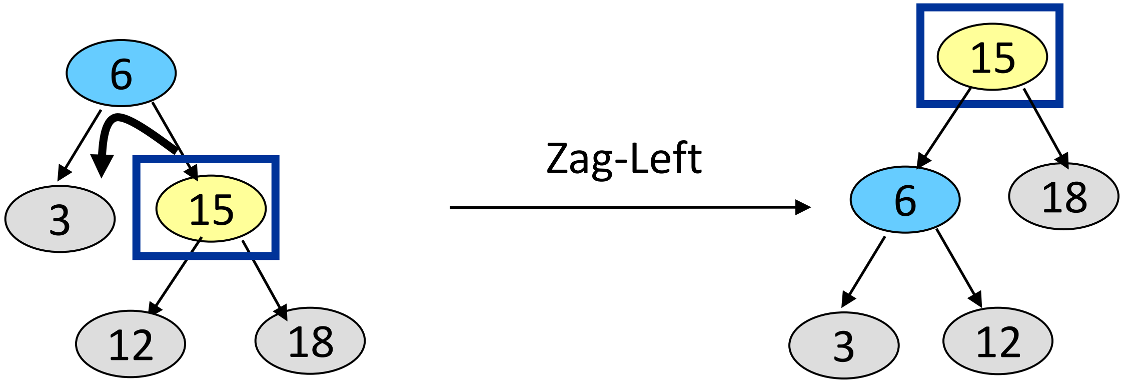
- **"Zag-Zag" consists of two single rotations of the same type**
- **Suppose 30 was the node that was accessed (e.g., using Search)**



- Due to "zag-zag" splaying, 30 has bubbled to the top!
- Note: Parent-Grandparent is rotated first.

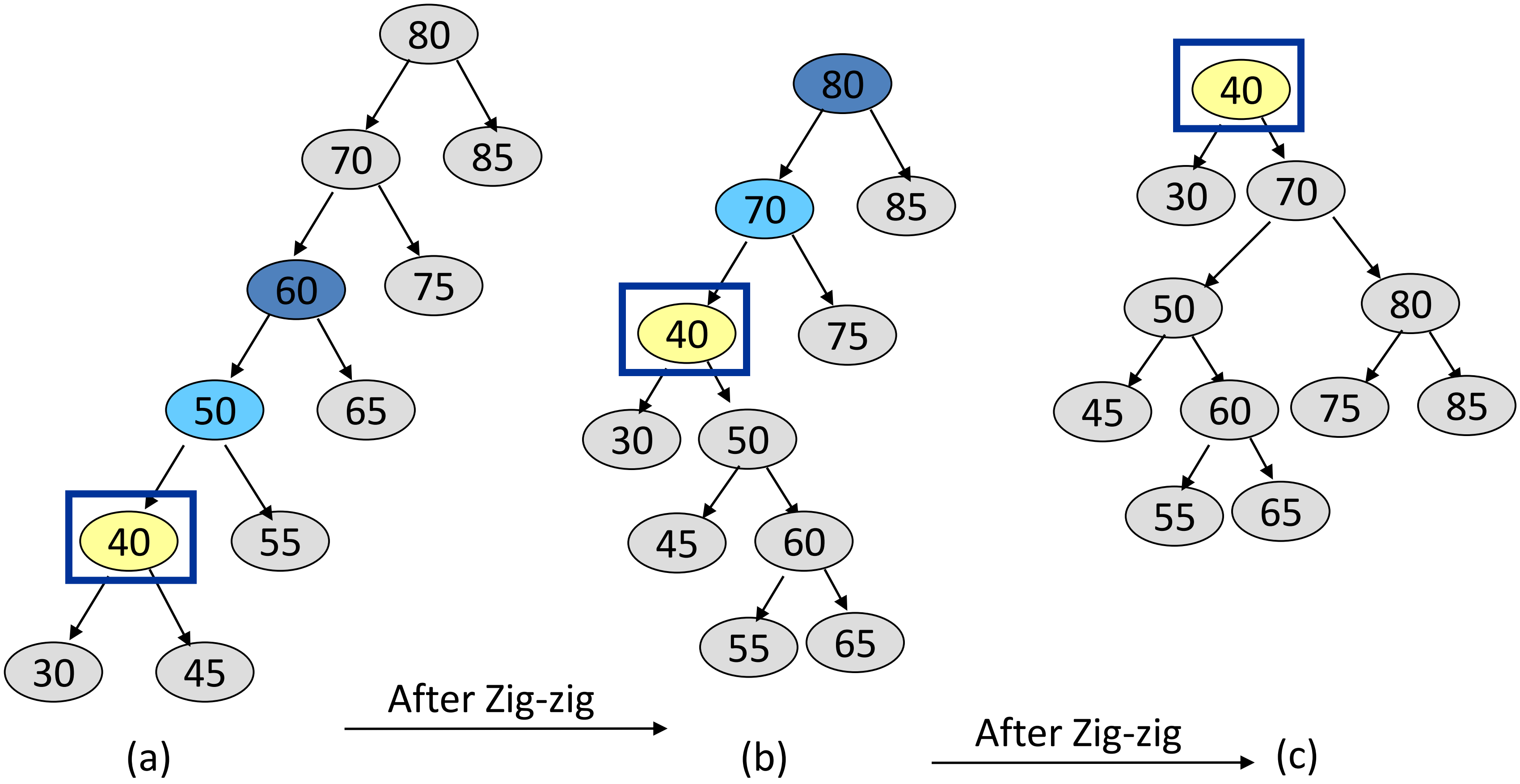
# Splay Trees: Zag Operation

- **"Zag" is just a single rotation, as in an AVL tree**
- **Suppose 15 was the node that was accessed (e.g., using Search)**

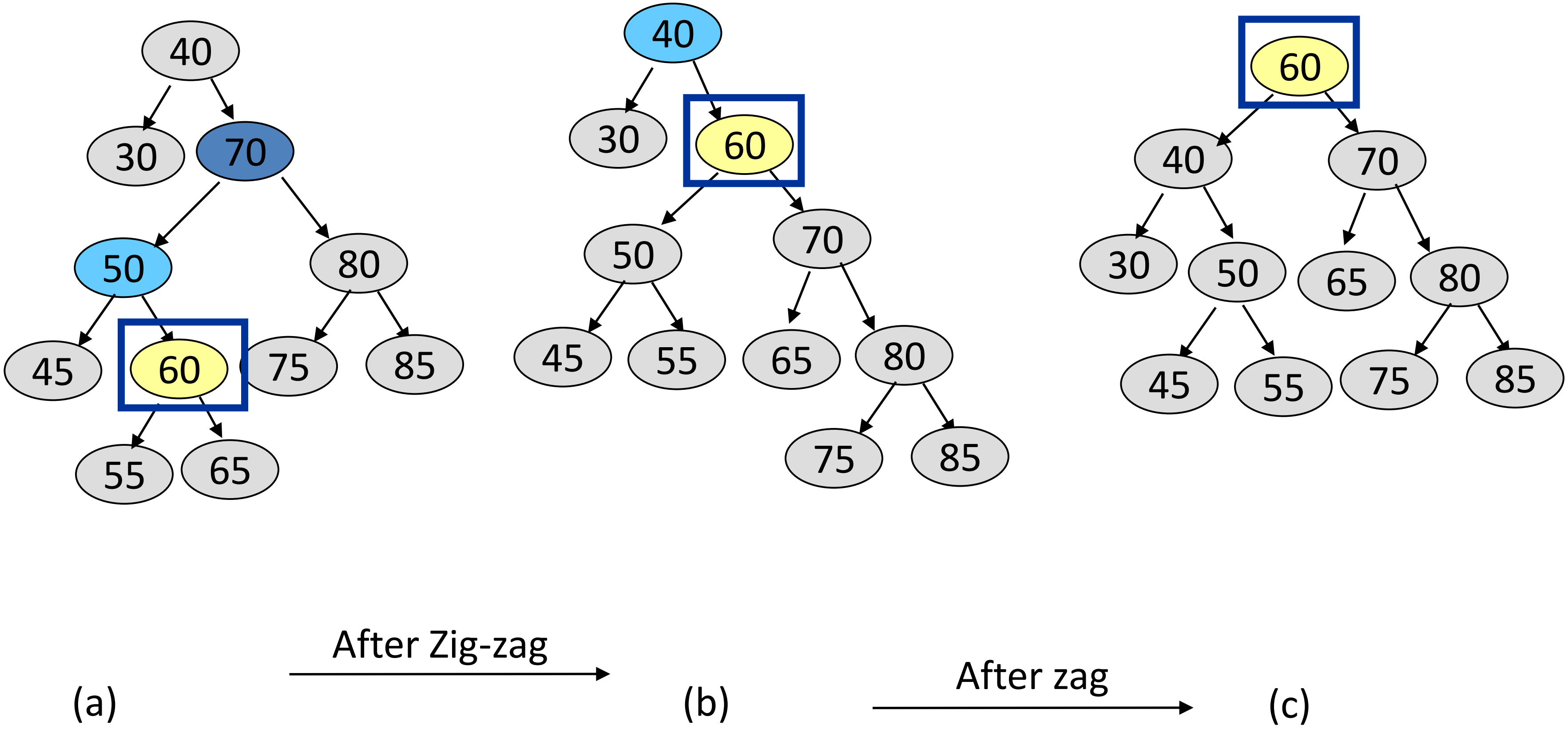


- "Zag-Left"moves 15 to the root.
- *Can access 15 faster next time: O(1)*
- Notice that this is simply a left rotation in *AVL* tree terminology

# Splay Trees: Example – 40 is accessed



# Splay Trees: Example – 60 is accessed



# Splaying during other operations

- **Splaying can be done not just after Search, but also after other operations such as Insert/Delete.**

- **Insert X: After inserting X at a leaf node (as in a regular BST), splay X up to the root**

- **Delete X: Do a Search on X and get X up to the root. Delete X at the root and move the largest item in its left sub-tree, i.e, its predecessor, to the root using splaying.**

- **Note on Search X: If X was not found, splay the leaf node that the Search ended up with to the root.**

# Summary of Splay Trees

- **Examples suggest that splaying causes tree to get balanced.**

- **The actual analysis is rather advanced and is in Chapter 11. Such analysis is called "amortized analysis"**

- **Result of Analysis: Any sequence of M operations on a splay tree of size N takes O(M log N) time. So, the amortized running time for one operation is O(log N).**

- **This guarantees that even if the depths of some nodes get very large, you cannot get a long sequence of O(N) searches because each search operation causes a rebalance.**

- **Without splaying, total time could be O(MN).**

# Comparison of  Search Trees

| Tree | Worst Case | | | Expected | | |
|---|---|---|---|---|---|---|
| | Search | Insert | Remove | Search | Insert | Remove |
| BST | n | n | n | log n | log n | log n |
| AVL tree | log n | log n | log n | log n | log n | log n |
| red-black tree | log n | log n | log n | log n | log n | log n |
| splay tree | n | n | n | log n | log n | log n |
| B-trees | log n | log n | log n | log n | log n | log n |

# Knuth-Morris-Pratt Algorithm

# The problem of String Matching

Given a string ‘S’, the problem of string matching deals with finding whether a pattern ‘p’ occurs in ‘S’ and if ‘p’ does occur then returning position in ‘S’ where ‘p’ occurs.

# …. a O(mn) approach

One of the most obvious approach towards the string matching problem would be to compare the first element of the pattern to be searched ‘p’, with the first element of the string ‘S’ in which to locate ‘p’. If the first element of ‘p’ matches the first element of ‘S’, compare the second element of ‘p’ with second element of ‘S’. If match found proceed likewise until entire ‘p’ is found. If a mismatch is found at any position, shift ‘p’ one position to the right and repeat comparison beginning from first element of ‘p’.

# How does the O(mn) approach work

Below is an illustration of how the previously described O(mn) approach works.

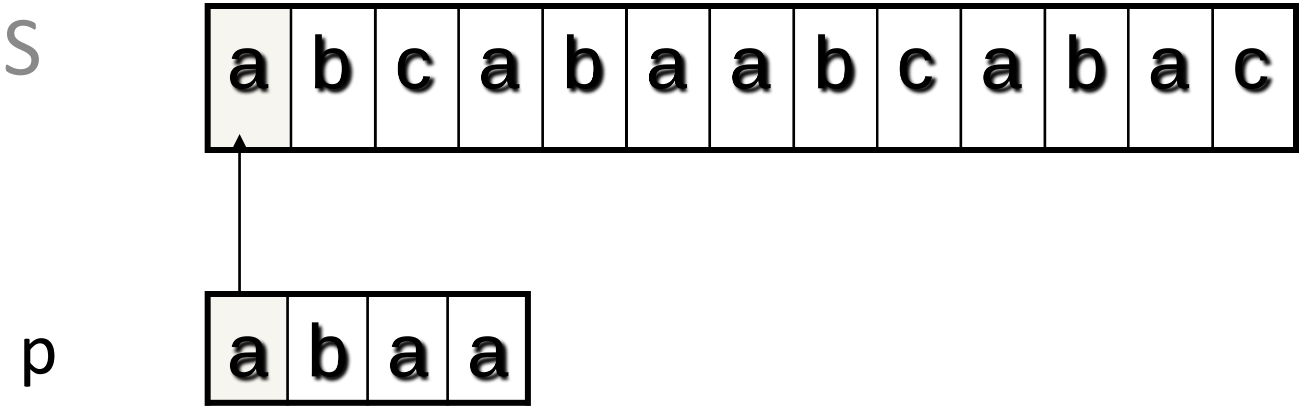
String S

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| a | b | c | a | b | a | a | b | c | a | b | a | c |

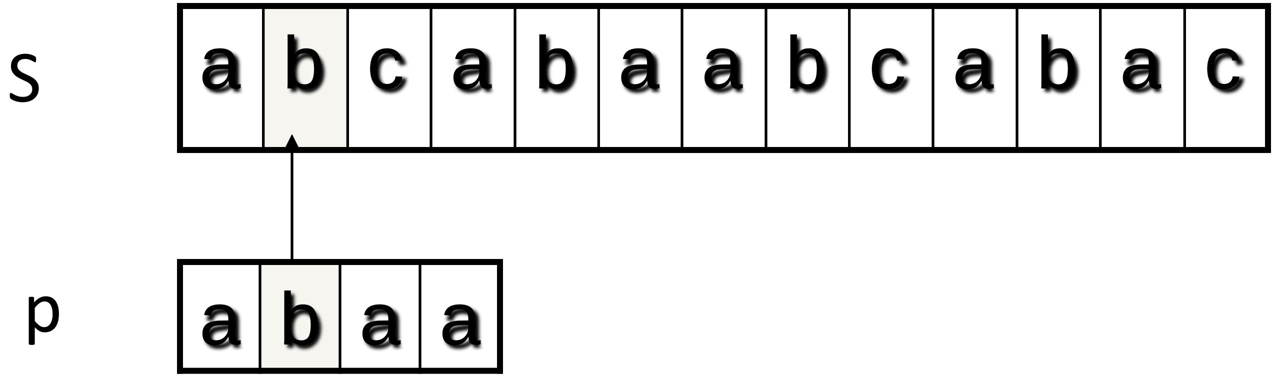
Pattern p

| | | | |
|---|---|---|---|
| a | b | a | a |

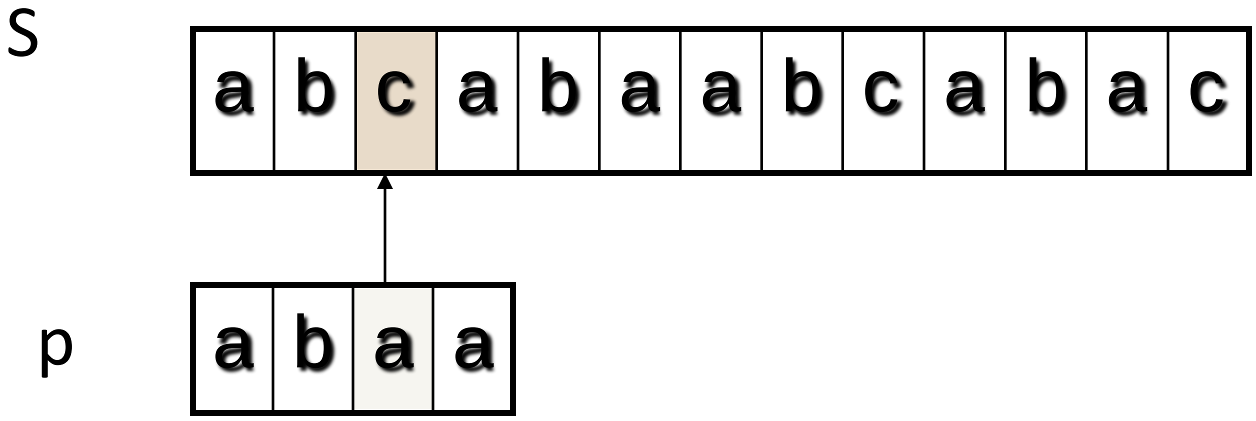
Step 1:compare p[1] with S[1]



Step 2: compare p[2] with S[2]



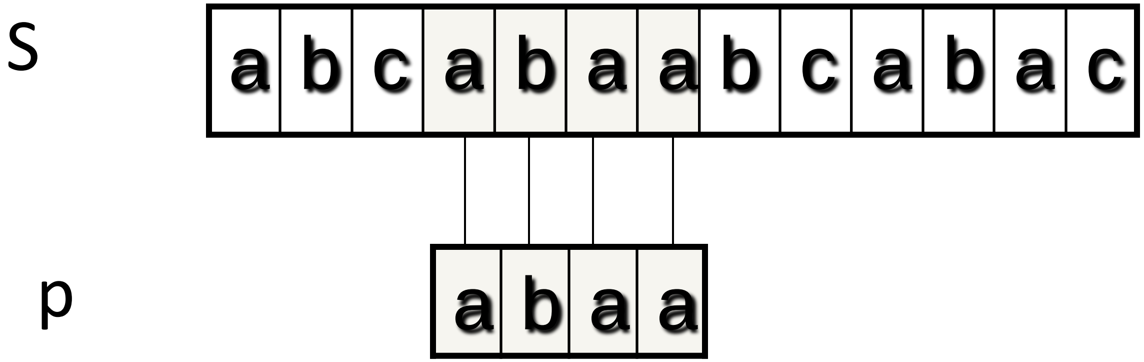
# Step 3: compare p[3] with S[3]



| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S | a | b | c | a | b | a | a | b | c | a | b | a | c |
| p | a | b | a | a | | | | | | | | | |

*Mismatch occurs here..*

Since mismatch is detected, shift ‘p’ one position to the right and repeat matching procedure.



S a b c a b a a b c a b a c

p a b a a

**Finally, a match would be found after shifting ‘p’ three times to the right side.**

**<u>Drawbacks of this approach:</u> if ‘m’ is the length of pattern ‘p’ and ‘n’ the length of string ‘S’, the matching time is of the order O(mn). This is a certainly a very slow running algorithm.**
**What makes this approach so slow is the fact that elements of ‘S’ with which comparisons had been performed earlier are involved again and again in comparisons in some future iterations. For example: when mismatch is detected for the first time in comparison of p[3] with S[3], pattern ‘p’ would be moved one position to the right and matching procedure would resume from here. Here the first comparison that would take place would be between p[0]=‘a’ and S[1]=‘b’. It should be noted here that S[1]=‘b’ had been previously involved in a comparison in step 2. this is a repetitive use of S[1] in another comparison.**
**It is these repetitive comparisons that lead to the runtime of O(mn).**

# The Knuth-Morris-Pratt Algorithm

**Knuth, Morris and Pratt proposed a linear time algorithm for the string matching problem.**

**A matching time of O(n) is achieved by avoiding comparisons with elements of ‘S’ that have previously been involved in comparison with some element of the pattern ‘p’ to be matched. i.e., backtracking on the string ‘S’ never occurs**

# Components of KMP algorithm

- **The prefix function, Π**

**The prefix function,Π for a pattern encapsulates knowledge about how the pattern matches against shifts of itself. This information can be used to avoid useless shifts of the pattern ‘p’. In other words, this enables avoiding backtracking on the string ‘S’.**

- **The KMP Matcher**

**With string ‘S’, pattern ‘p’ and prefix function ‘Π’ as inputs, finds the occurrence of ‘p’ in ‘S’ and returns the number of shifts of ‘p’ after which occurrence is found.**

# The prefix function, Π

**Following pseudocode computes the prefix fucnction, Π:**

```
Compute-Prefix-Function (p)
m ← length[p]              //'p' pattern to be matched
Π[1] ← 0
k ← 0
    for q ← 2 to m
        do while k > 0 and p[k+1] != p[q]
            do k ← Π[k]
            If p[k+1] = p[q]
              then k ← k +1
            Π[q] ← k
    return Π
```

# Example: compute Π for the pattern ‘p’ below:

p

| | | | | | | |
|---|---|---|---|---|---|---|
| a | b | a | b | a | c | a |

Initially: m = length[p] = 7

Π[1] = 0

k = 0

Step 1: q = 2, k=0

Π[2] = 0

| q | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| p | a | b | a | b | a | c | a |
| Π | 0 | 0 | | | | | |

Step 2: q = 3, k = 0,

Π[3] = 1

| q | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| p | a | b | a | b | a | c | a |
| Π | 0 | 0 | 1 | | | | |

Step 3: q = 4, k = 1

Π[4] = 2

| q | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| p | a | b | a | b | a | c | A |
| Π | 0 | 0 | 1 | 2 | | | |

Step 4: q = 5, k =2

Π[5] = 3

| q | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| p | a | b | a | b | a | c | a |
| Π | 0 | 0 | 1 | 2 | 3 |  |  |

Step 5: q = 6, k = 3

Π[6] = 1

| q | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| p | a | b | a | b | a | c | a |
| Π | 0 | 0 | 1 | 2 | 3 | 1 |  |

Step 6: q = 7, k = 1

Π[7] = 1

| q | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| p | a | b | a | b | a | c | a |
| Π | 0 | 0 | 1 | 2 | 3 | 1 | 1 |

After iterating 6 times, the prefix function computation is complete: →

| q | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| p | a | b | A | b | a | c | a |
| Π | 0 | 0 | 1 | 2 | 3 | 1 | 1 |

# The KMP Matcher

**The KMP Matcher, with pattern ‘p’, string ‘S’ and prefix function ‘Π’ as input, finds a match of p in S.**

**Following pseudocode computes the matching component of KMP algorithm:**

**KMP-Matcher(S,p)**

```
1  n ← length[S]
2  m ← length[p]
3  Π ← Compute-Prefix-Function(p)
4  q ← 0                                      //number of characters matched
5  for i ← 1 to n                             //scan S from left to right
6      do while  q > 0 and p[q+1] != S[i]
7              do  q ← Π[q]                   //next character does not match
8          if p[q+1] = S[i]
9            then q ← q + 1                   //next character matches
10         if q = m                           //is all of p matched?
11             then print “Pattern occurs with shift” i – m
12                 q ← Π[ q]                  // look for the next match
```

***Note: KMP finds every occurrence of a ‘p’ in ‘S’. That is why KMP does not terminate in step 12, rather it searches remainder of ‘S’ for any more occurrences of ‘p’.***

## Illustration: given a String ‘S’ and pattern ‘p’ as follows:

S

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| b | a | c | b | a | b | a | b | a | b | a | c | a | c | a |

p

| | | | | | | |
|---|---|---|---|---|---|---|
| a | b | a | b | a | c | a |

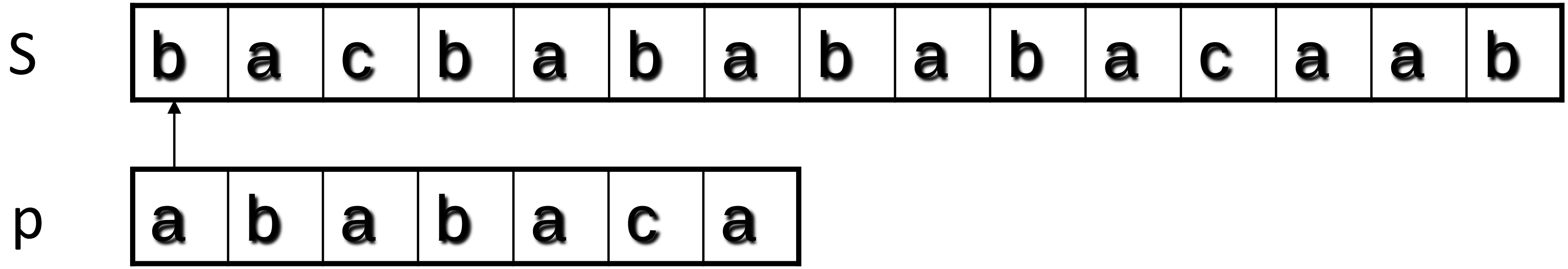
Let us execute the KMP algorithm to find whether ‘p’ occurs in ‘S’.

*For ‘p’ the prefix function, Π was computed previously and is as follows:*

| q | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| p | a | b | a | b | a | c | a |
| Π | 0 | 0 | 1 | 2 | 3 | 1 | 1 |

Initially: n = size of S = 15;
m = size of p = 7

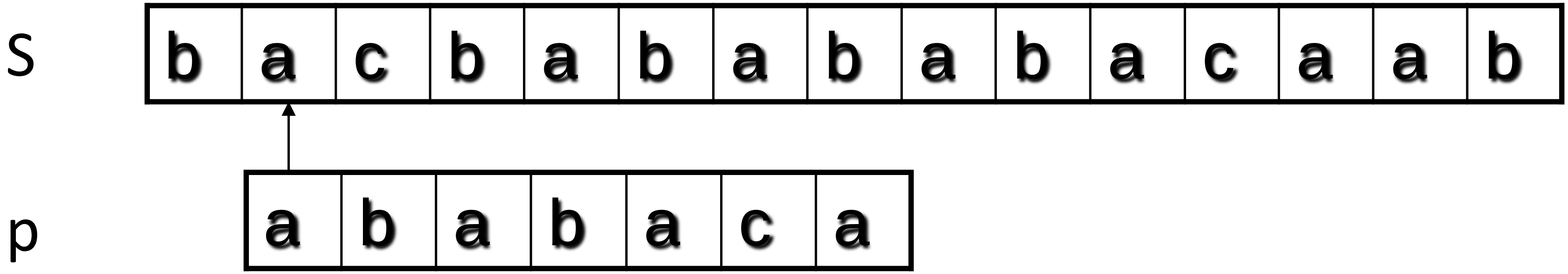
Step 1: i = 1, q = 0
comparing p[1] with S[1]



| S | b | a | c | b | a | b | a | b | a | b | a | c | a | a | b |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| p | a | b | a | b | a | c | a | | | | | | | | |

P[1] does not match with S[1]. ‘p’ will be shifted one position to the right.

Step 2: i = 2, q = 0
comparing p[1] with S[2]

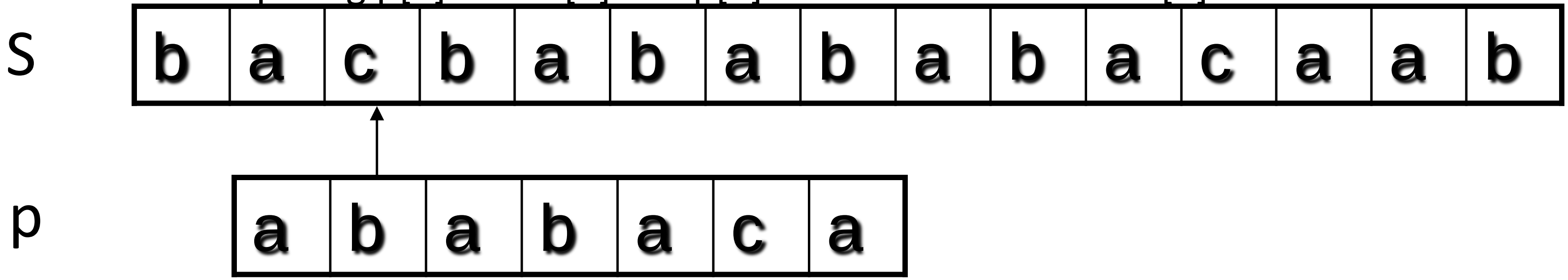


| S | b | a | c | b | a | b | a | b | a | b | a | c | a | a | b |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| p | | a | b | a | b | a | c | a | | | | | | | |

P[1] matches S[2]. Since there is a match, p is not shifted.

Step 3: i = 3, q = 1

Comparing p[2] with S[3]  p[2] does not match with S[3]

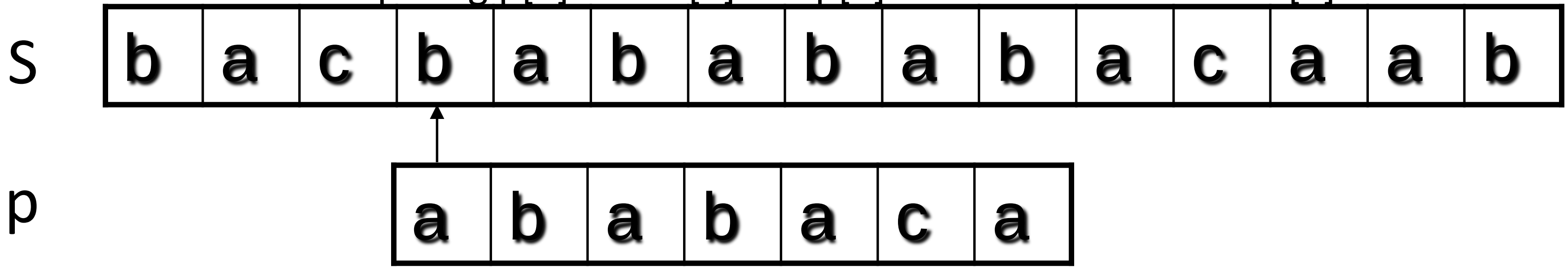


| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S | b | a | c | b | a | b | a | b | a | b | a | c | a | a | b |
| p | | a | b | a | b | a | c | a | | | | | | | |

Backtracking on p, comparing p[1] and S[3]

Step 4: i = 4, q = 0

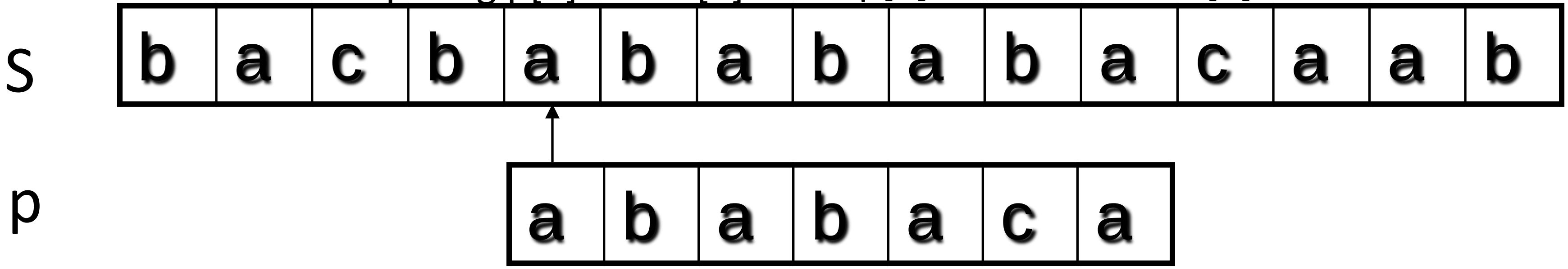
comparing p[1] with S[4]  p[1] does not match with S[4]



| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S | b | a | c | b | a | b | a | b | a | b | a | c | a | a | b |
| p | | | | a | b | a | b | a | c | a | | | | | |

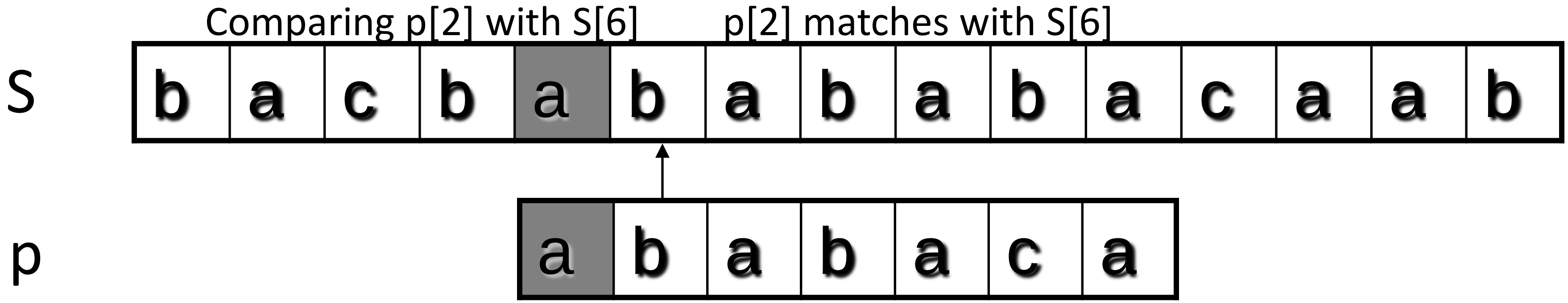
Step 5: i = 5, q = 0

comparing p[1] with S[5]  p[1] matches with S[5]



| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S | b | a | c | b | a | b | a | b | a | b | a | c | a | a | b |
| p | | | | | a | b | a | b | a | c | a | | | | |

**Step 6: i = 6, q = 1**



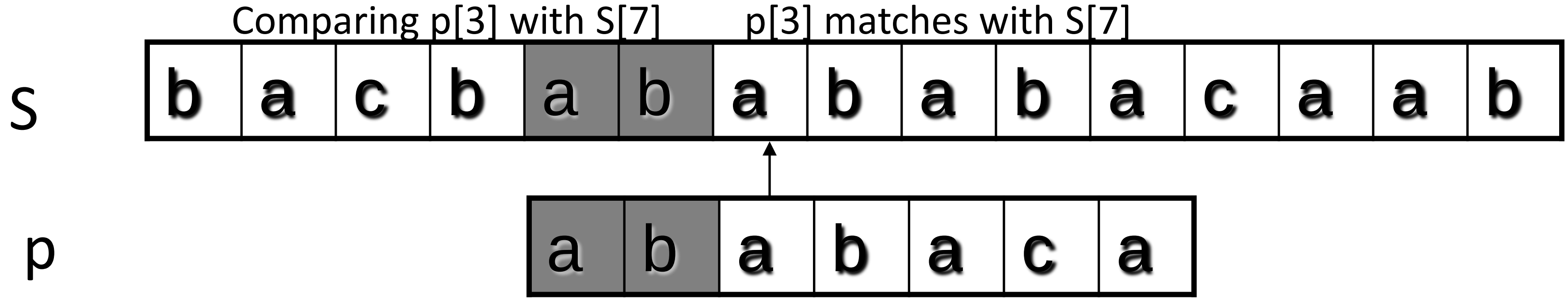
Comparing p[2] with S[6]

p[2] matches with S[6]

S b a c b a b a b a b a c a a b

p a b a b a c a

**Step 7: i = 7, q = 2**



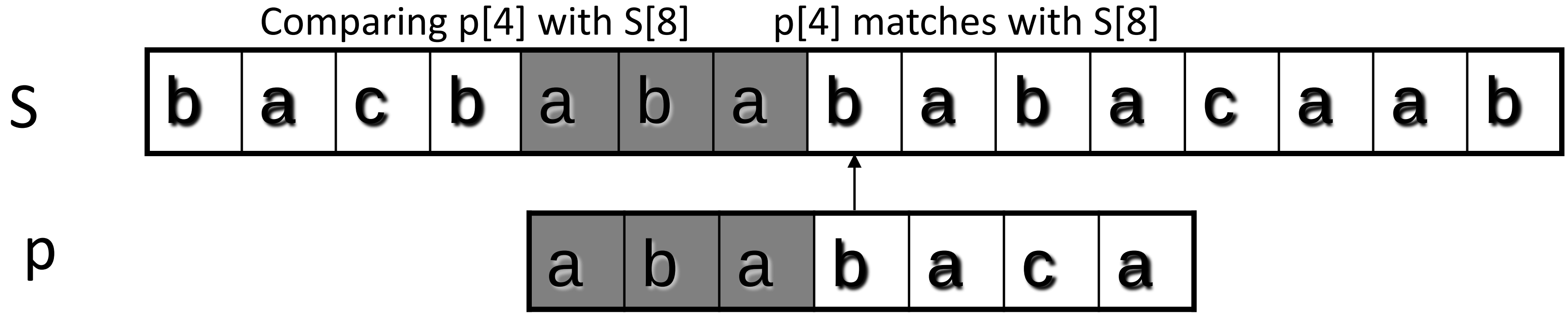
Comparing p[3] with S[7]

p[3] matches with S[7]

S b a c b a b a b a b a c a a b

p a b a b a c a

**Step 8: i = 8, q = 3**



Comparing p[4] with S[8]

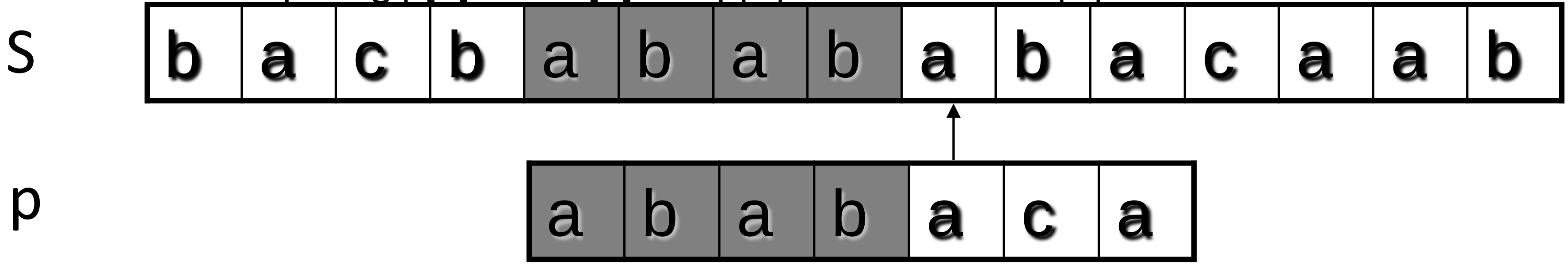
p[4] matches with S[8]

S b a c b a b a b a b a c a a b

p a b a b a c a

**Step 9: i = 9, q = 4**

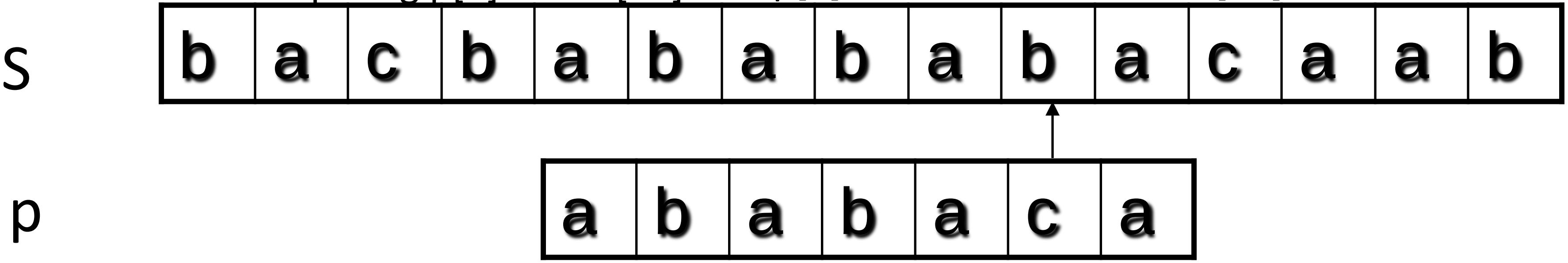
Comparing p[5] with S[9]     p[5] matches with S[9]



| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S | b | a | c | b | a | b | a | b | a | b | a | c | a | a | b |
| p | | | | | a | b | a | b | a | c | a | | | | |

**Step 10: i = 10, q = 5**

Comparing p[6] with S[10]     p[6] doesn’t match with S[10]

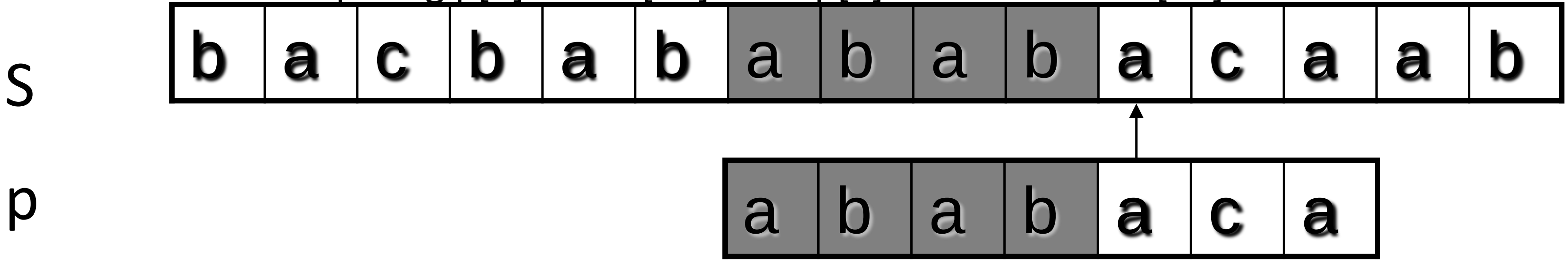


| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S | b | a | c | b | a | b | a | b | a | b | a | c | a | a | b |
| p | | | | | a | b | a | b | a | c | a | | | | |

Backtracking on p, comparing p[4] with S[10] because after mismatch q = Π[5] = 3

**Step 11: i = 11, q = 4**

Comparing p[5] with S[11]     p[5] matches with S[11]

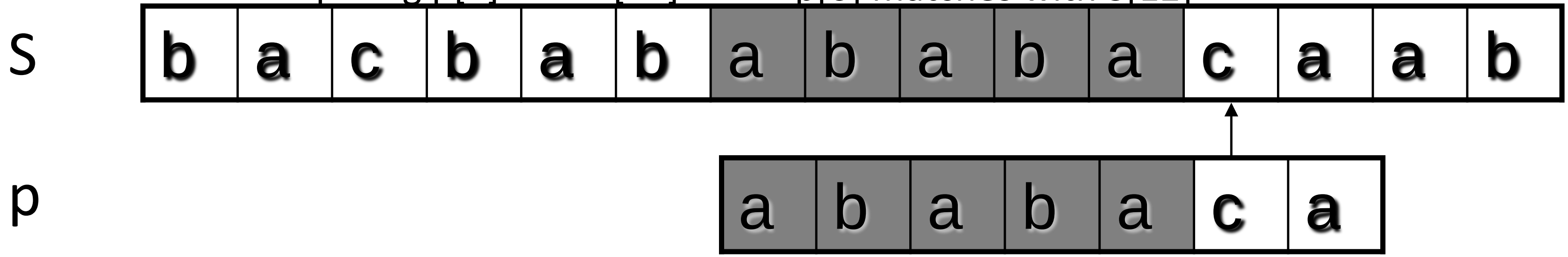


| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S | b | a | c | b | a | b | a | b | a | b | a | c | a | a | b |
| p | | | | | | | a | b | a | b | a | c | a | | |

**Step 12: i = 12, q = 5**

Comparing p[6] with S[12]

p[6] matches with S[12]

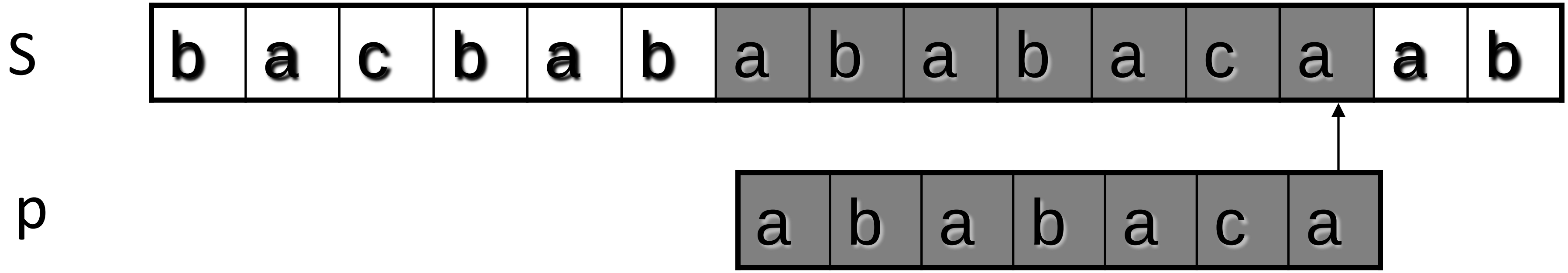


| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S | b | a | c | b | a | b | a | b | a | b | a | c | a | a | b |
| p | | | | | | | a | b | a | b | a | c | a | | |

**Step 13: i = 13, q = 6**

Comparing p[7] with S[13]

p[7] matches with S[13]



| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S | b | a | c | b | a | b | a | b | a | b | a | c | a | a | b |
| p | | | | | | | a | b | a | b | a | c | a | | |

Pattern ‘p’ has been found to completely occur in string ‘S’. The total number of shifts that took place for the match to be found are: i – m = 13 – 7 = 6 shifts.

# Running - time analysis

- **Compute-Prefix-Function (Π)**

```
1  m ← length[p]          //'p' pattern to be matched
2  Π[1] ← 0
3  k ← 0
4      for q ← 2 to m
5          do while k > 0 and p[k+1] != p[q]
6             do k ← Π[k]
7              If p[k+1] = p[q]
8                 then k ← k +1
9              Π[q] ← k
10     return Π
```

In the above pseudocode for computing the prefix function, the for loop from step 4 to step 10 runs 'm' times. Step 1 to step 3 take constant time. Hence the running time of compute prefix function is $\Theta(m)$.

- **KMP Matcher**

```
1 n ← length[S]
2 m ← length[p]
3 Π ← Compute-Prefix-Function(p)
4 q ← 0
5 for i ← 1 to n
6   do while  q > 0 and p[q+1] != S[i]
7           do  q ← Π[q]
8      if p[q+1] = S[i]
9         then q ← q + 1
10     if q = m
11       then print "Pattern occurs with shift" i – m
12              q ← Π[ q]
```

The for loop beginning in step 5 runs 'n' times, i.e., as long as the length of the string 'S'. Since step 1 to step 4 take constant time, the running time is dominated by this for loop. Thus running time of matching function is $\Theta(n)$.

# Tries

- ***Trie*** is a special structure to represent sets of character strings.
- Can also be used to represent data types that are objects of any type e.g. strings of integers.
- The word ***“trie”*** is derived from the middle letters of the word “***retrieval***”.

# Tries: Example

One way to implement a spelling checker is

- Read a text file.
- Break it into words( character strings separated by blanks and new lines).
- Find those words not in a standard dictionary of words.
- Words in the text but not in the dictionary are printed out as possible misspellings.

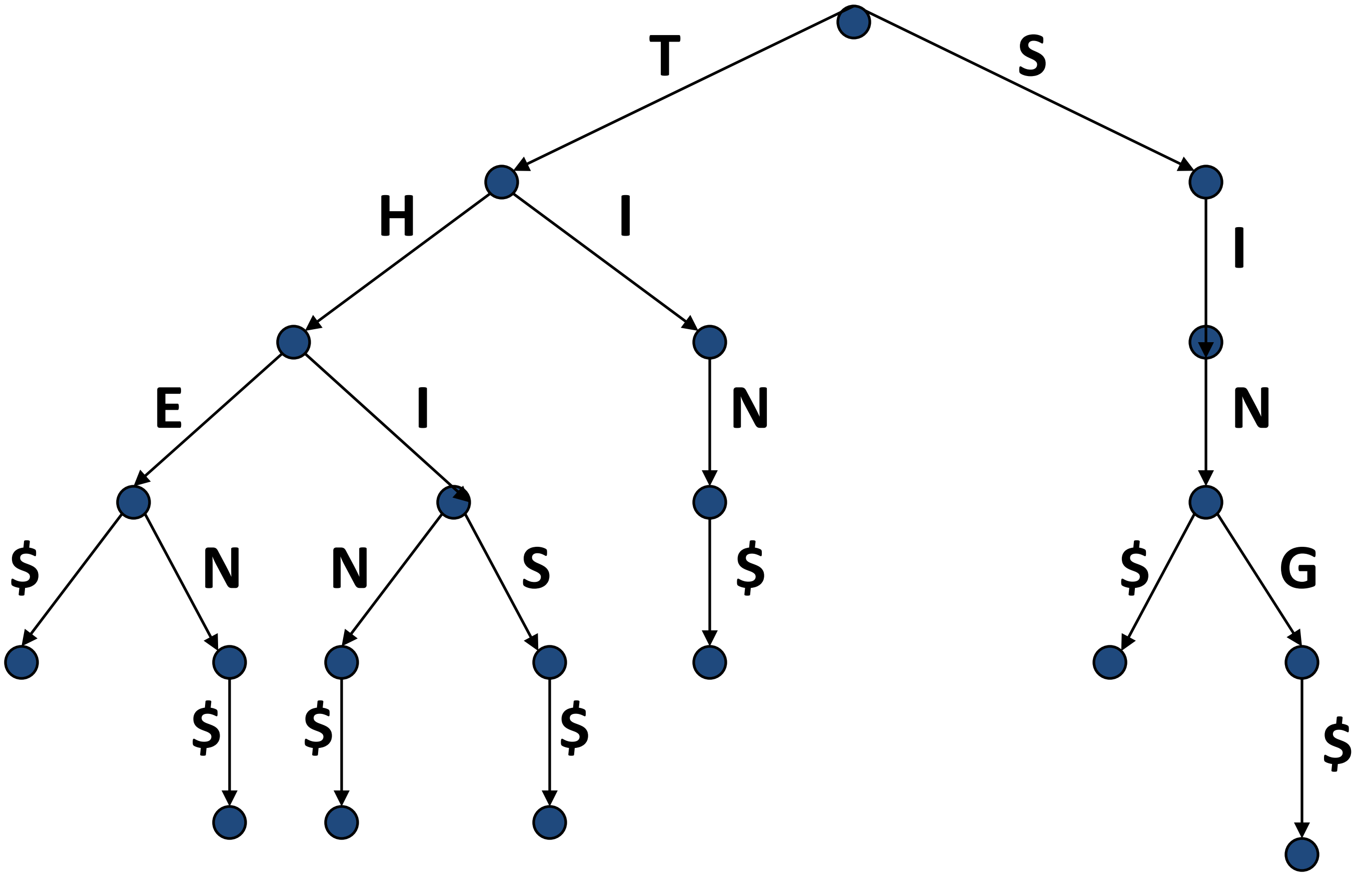
# Tries: Example

It can be implemented by a set having operations of :

- INSERT
- DELETE
- MAKENULL
- PRINT

A Trie structure supports these set operations when the element of the set are words.

# Tries: Example



# Tries: Example

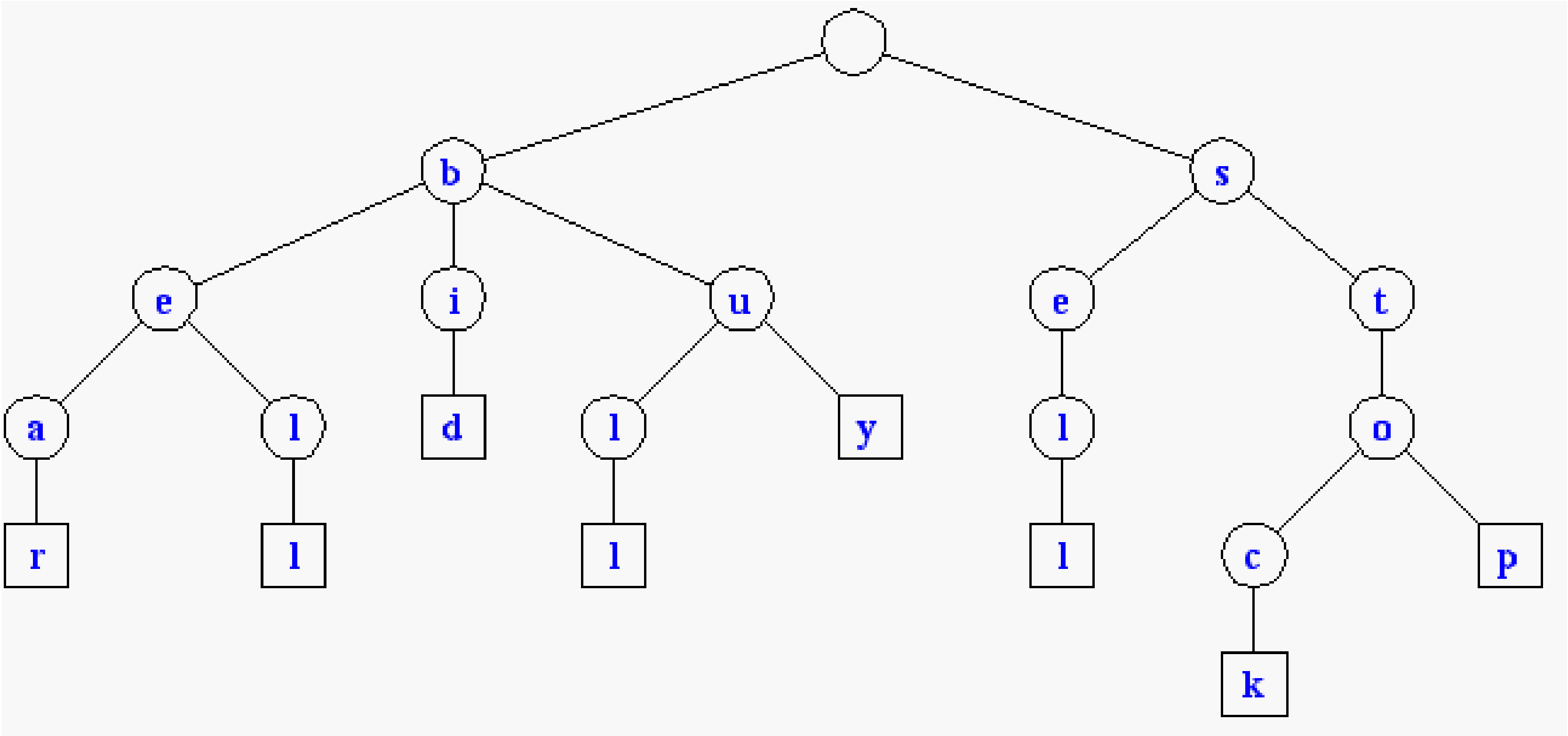
- Tries are appropriate when many words begin with the same sequence of letters.
- i.e; when the number of distinct prefixes among all words in the set is much less than the total length of all the words.
- Each path from the root to the leaf corresponds to one word in the represented set.
- Nodes of the trie correspond to the prefixes of words in the set.

# Tries: Example

- The symbol $ is added at the end of each word so that no prefix of a word can be a word itself.
- The Trie corresponds to the set {THE,THEN THIN, TIN, SIN, SING}
- Each node has at most 27 children, one for each letter and $
- Most nodes will have many fewer than 27 children.
- A leaf reached by an edge labeled $ cannot have any children.

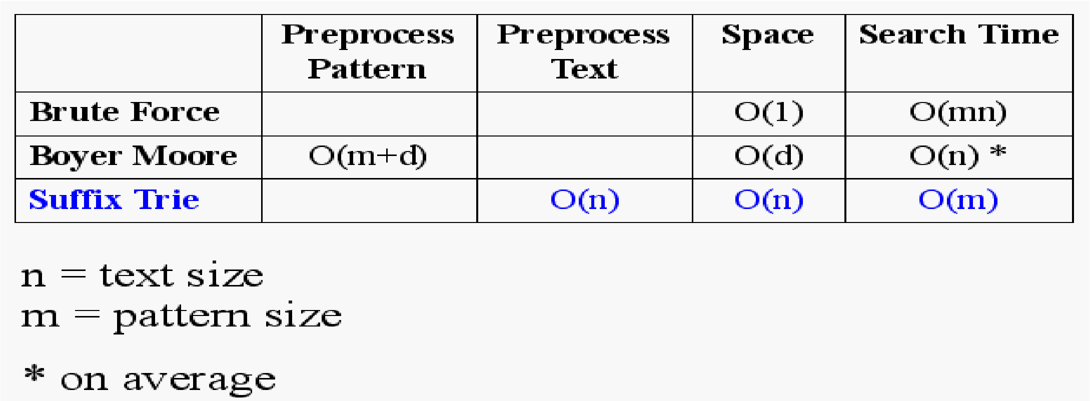
# Tries

- **Standard Tries**
- **Compressed Tries**
- **Suffix Tries**



# Text Processing

- We have seen that preprocessing the pattern speeds up pattern matching queries
- After preprocessing the pattern in time proportional to the pattern length, the Boyer-Moore algorithm searches an arbitrary English text in (average) time proportional to the text length
- If the text is large, immutable and searched for often (e.g., works by Shakespeare), we may want to preprocess the text instead of the pattern in order to perform pattern matching queries in time *proportional to the pattern length*.
- Tradeoffs in text searching



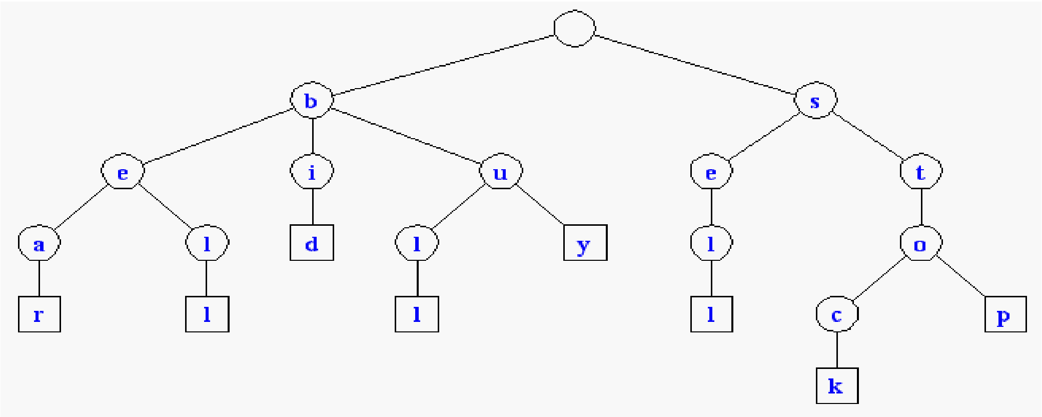
| | Preprocess Pattern | Preprocess Text | Space | Search Time |
|---|---|---|---|---|
| **Brute Force** | | | O(1) | O(mn) |
| **Boyer Moore** | O(m+d) | | O(d) | O(n) * |
| **Suffix Trie** | | O(n) | O(n) | O(m) |

n = text size
m = pattern size

* on average

# Standard Tries

- The ***standard trie*** for a set of strings S is an ordered tree such that:
  - **each node but the root is labeled with a character**
  - **the children of a node are alphabetically ordered**
  - **the paths from the external nodes to the root yield the strings of S**
- Example: standard trie for the set of strings S = { bear, bell, bid, bull, buy, sell, stock, stop }



•A standard trie uses O(n) space. Operations (find, insert, remove) take time O(dm) each, where:

-n = total size of the strings in S,

-m =size of the string parameter of the operation

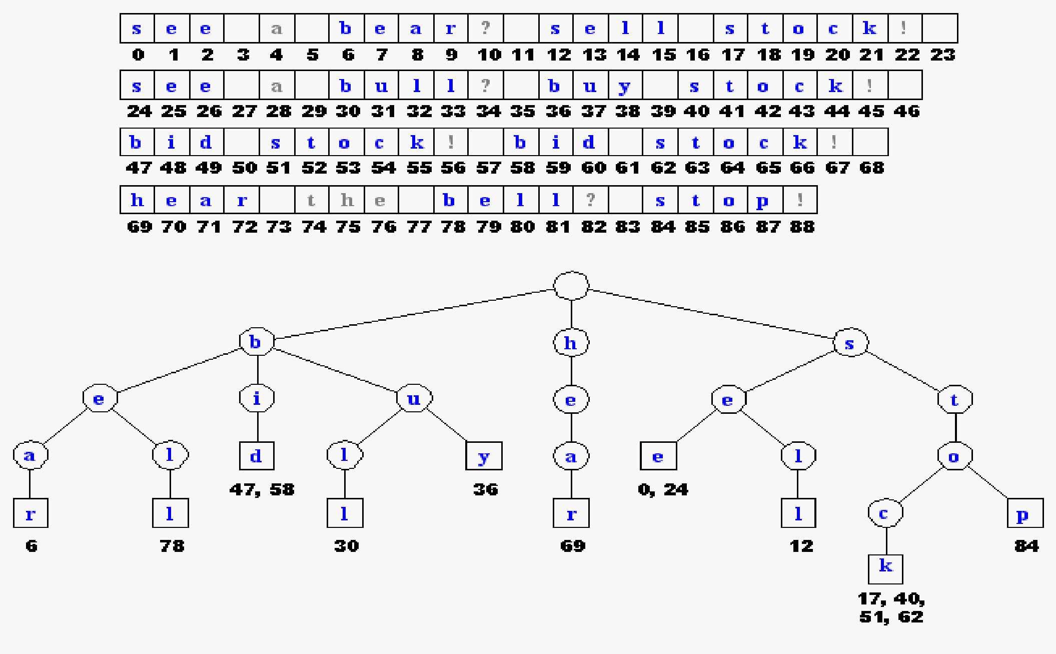
-d =alphabet size,

# Applications of Tries

- A standard trie supports the following operations on a preprocessed text in time O(m), where m = |X|

  -***word matching***: find the first occurence of word X in the text

  -***prefix matching***: find the first occurrence of the longest prefix of word X in the text
- Each operation is performed by tracing a path in the trie starting at the root



| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| s | e | e | | a | | b | e | a | r | ? | | s | e | l | l | | s | t | o | c | k | ! | |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |

| | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| s | e | e | | a | | b | u | l | l | ? | | b | u | y | | s | t | o | c | k | ! | |
| 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 |

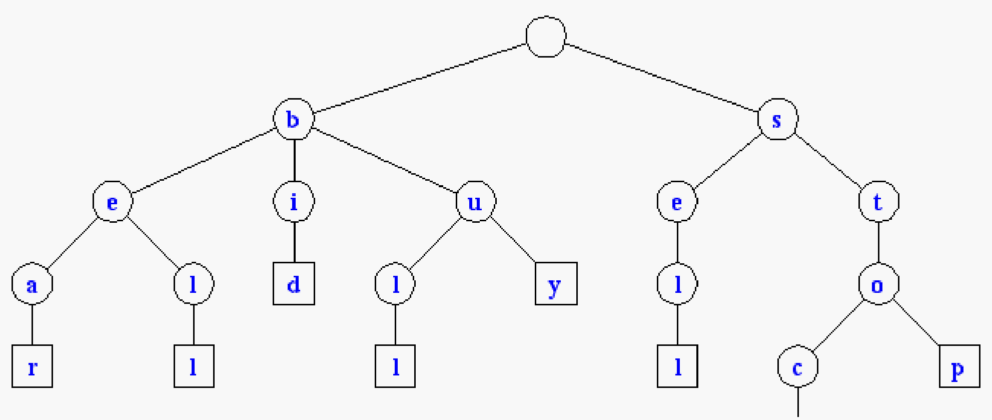
| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| b | i | d | | s | t | o | c | k | ! | | b | i | d | | s | t | o | c | k | ! | |
| 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 |

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| h | e | a | r | | t | h | e | | b | e | l | l | ? | | s | t | o | p | ! |
| 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 |

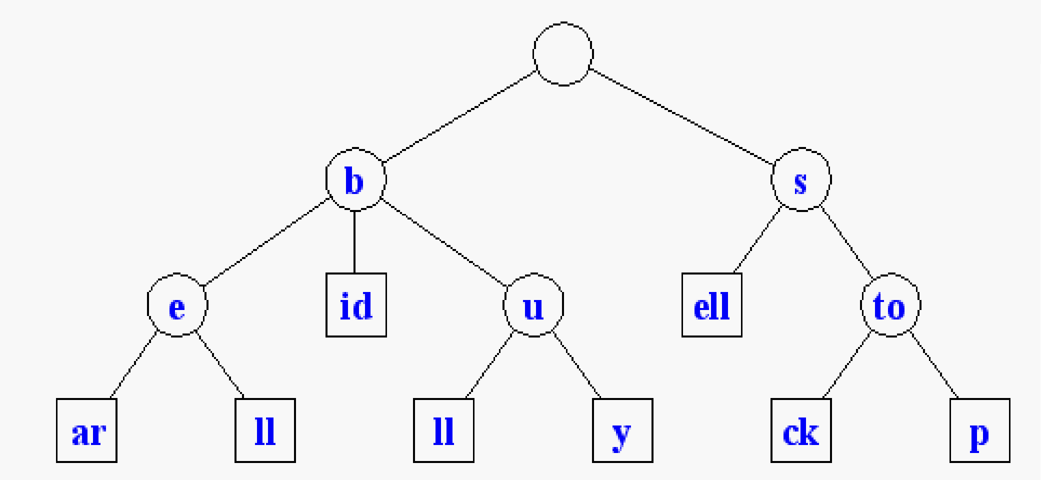
# Compressed Tries

- Trie with nodes of degree at least 2
- Obtained from standard trie by compressing chains of ***redundant nodes***

Standard Trie:

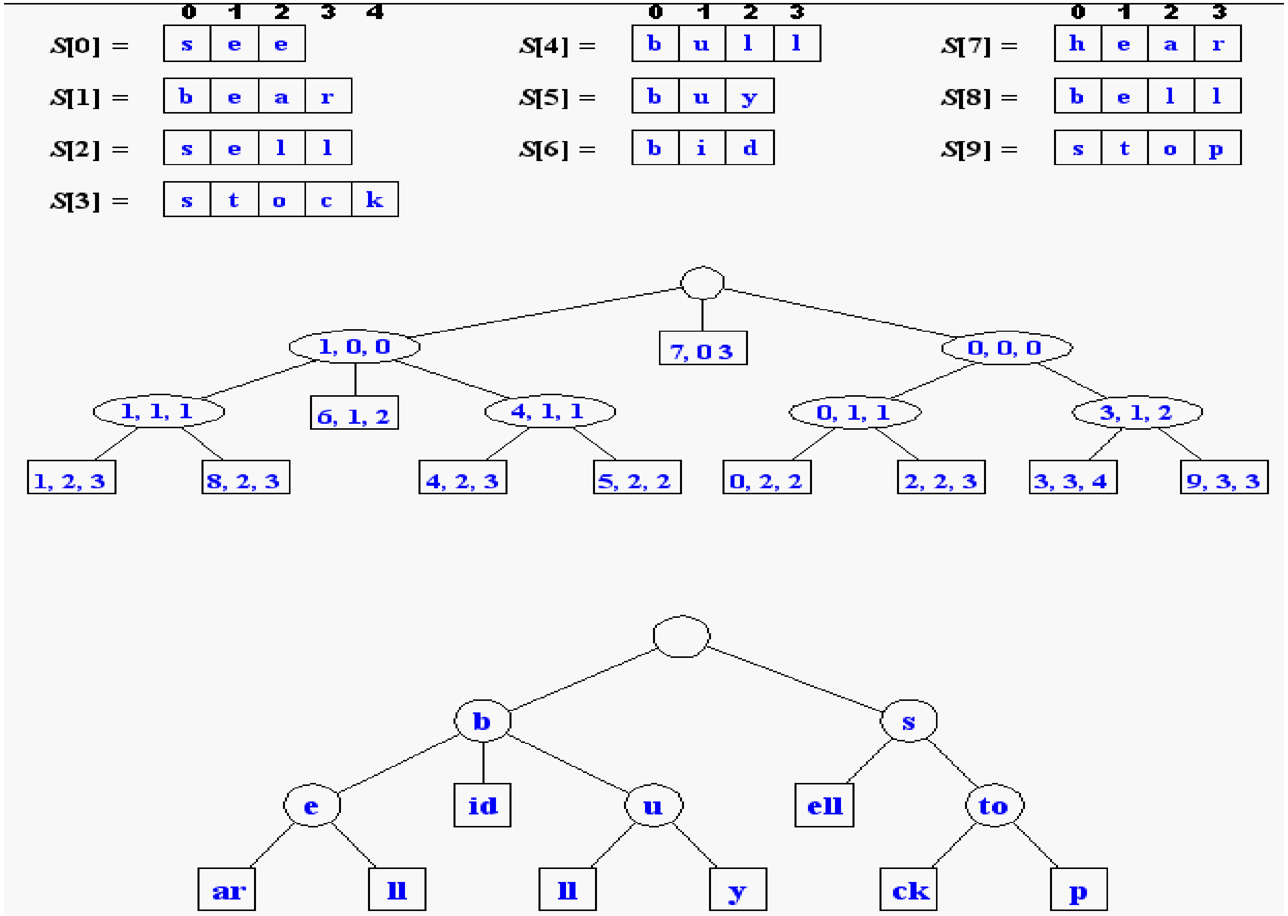


Compressed Trie:



# Compact Storage of Compressed Tries

- A compressed trie can be stored in space O(s), where s = |S|, by using O(1) space ***index ranges*** at the nodes

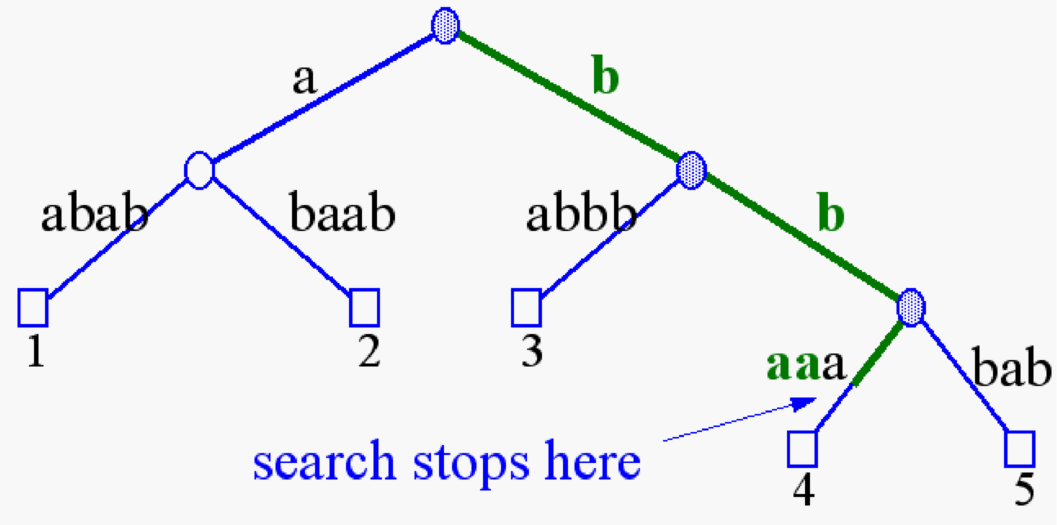


| | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| S[0] = | s | e | e | | |
| S[1] = | b | e | a | r | |
| S[2] = | s | e | l | l | |
| S[3] = | s | t | o | c | k |

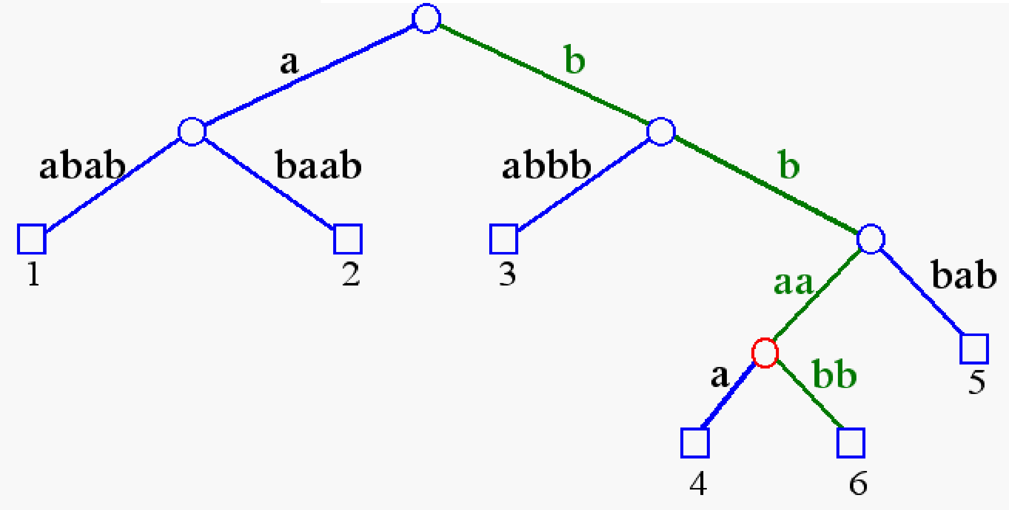
| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| S[4] = | b | u | l | l |
| S[5] = | b | u | y | |
| S[6] = | b | i | d | |

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| S[7] = | h | e | a | r |
| S[8] = | b | e | l | l |
| S[9] = | s | t | o | p |

# Insertion and Deletion into/from a Compressed Trie



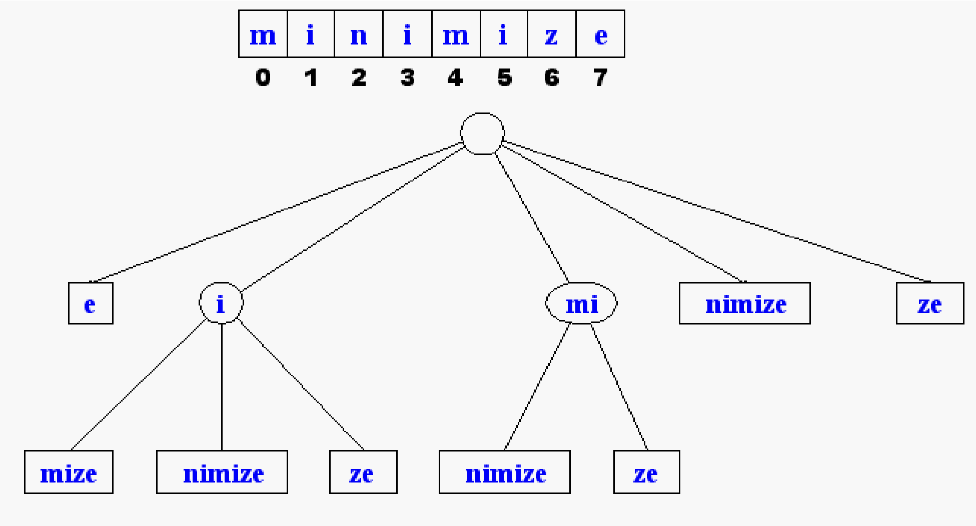
insert(bbaabb)



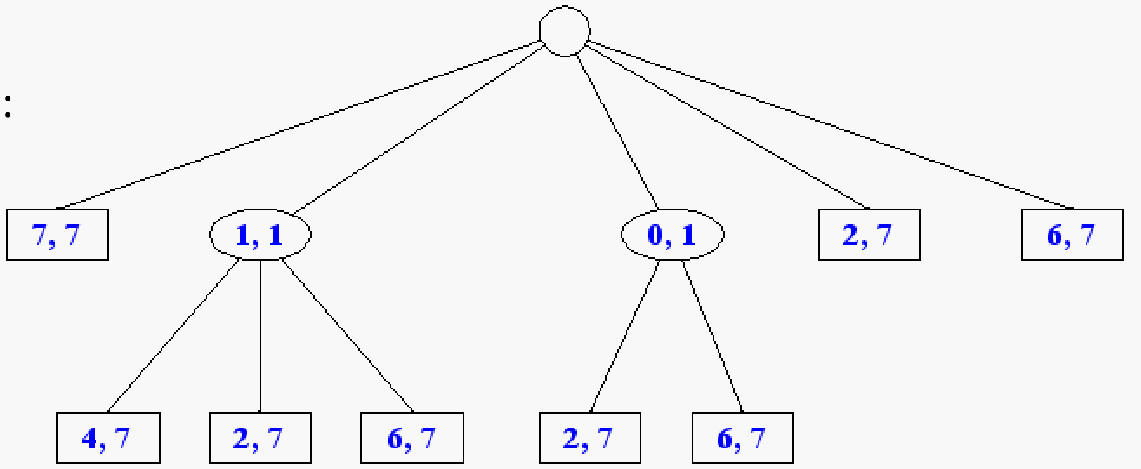
# Suffix Tries

- A ***suffix trie*** is a compressed trie for all the suffixes of a text

Example:



Compact representation:



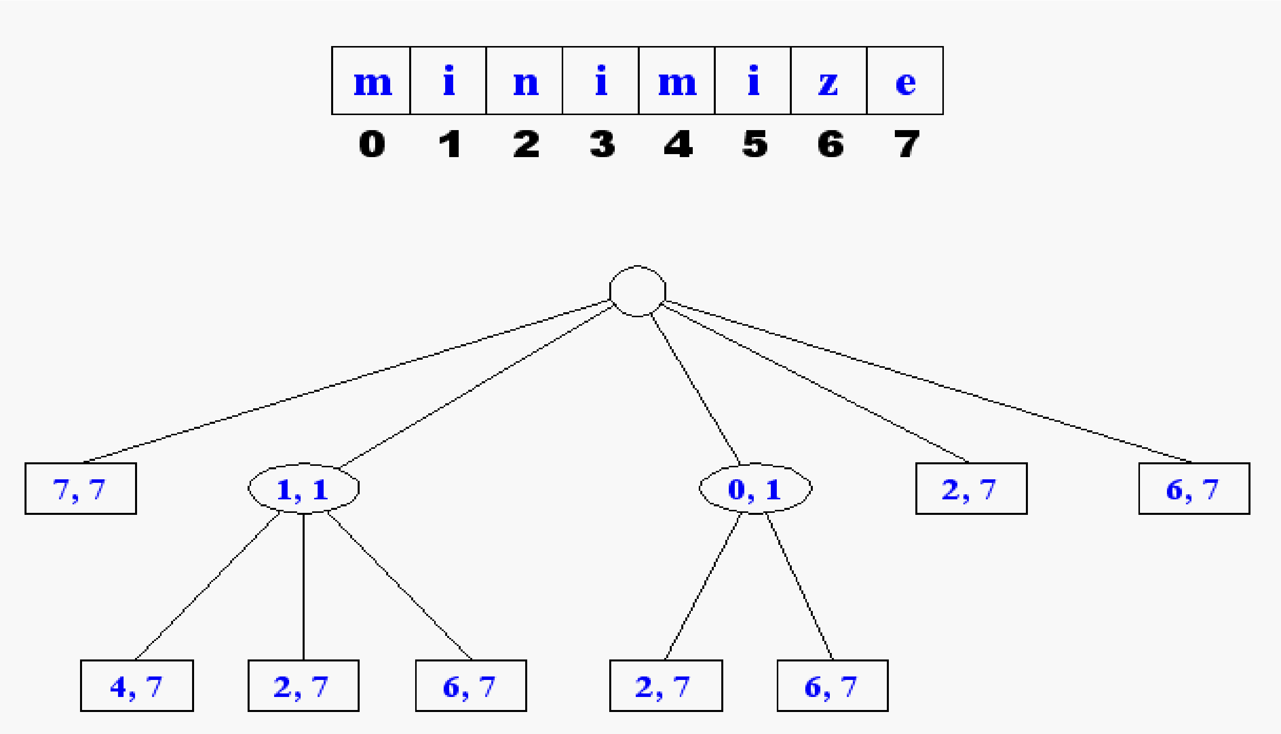
# Properties of Suffix Tries

- The ***suffix trie*** for a text X of size ***n*** from an alphabet of size ***d***

  -stores all the ***n(n-1)/2 suffixes*** of X in O(n) space

  -supports arbitrary ***pattern matching*** and prefix matching queries in ***O(dm) time***, where m is the length of the pattern

  -can be constructed in ***O(dn) time***



# Tries and Web Search Engines

- The ***index of a search engine*** (collection of all searchable words) is stored into a compressed trie
- Each leaf of the trie is associated with a word and has a list of pages (URLs) containing that word, called ***occurrence list***
- The trie is kept in internal memory
- The occurrence lists are kept in external memory and are ranked by relevance
- Boolean queries for sets of words (e.g., Java and coffee) correspond to set operations (e.g., intersection) on the occurrence lists
- Additional ***information retrieval*** techniques are used, such as
  - stopword elimination (e.g., ignore "the" "a" "is")
  - stemming (e.g., identify "add" "adding" "added")
  - link analysis (recognize authoritative pages)

# Tries and Internet Routers

- Computers on the internet (hosts) are identified by a unique 32-bit IP (***internet protocol***) addres, usually written in "dotted-quad-decimal" notation
- E.g., www.cs.brown.edu is 128.148.32.110
- Use nslookup on Unix to find out IP addresses
- An organization uses a subset of IP addresses with the same prefix, e.g., Brown uses 128.148.*.*, Yale uses 130.132.*.*
- Data is sent to a host by fragmenting it into packets. Each packet carries the IP address of its destination.
- The internet whose nodes are ***routers***, and whose edges are communication links.
- A router forwards packets to its neighbors using IP ***prefix matching*** rules. E.g., a packet with IP prefix 128.148. should be forwarded to the Brown gateway router.
- Routers use tries on the alphabet 0,1 to do prefix matching.