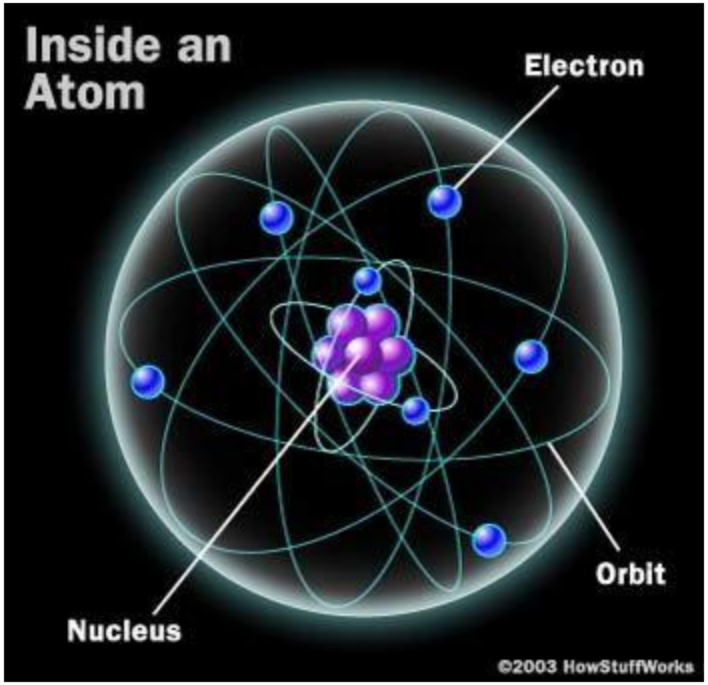
ELECTRICAL CIRCUITS

Prepared By:
G Hari Krishna, Assistant Professor,
M Diva Kumar, Assistant Professor,
N Sindhu, Assistant Professor

How electricity flows?

Electricity Basics

Electricity starts with **electrons.** Every atom contains one or more electrons. Electrons have a **negative charge**.

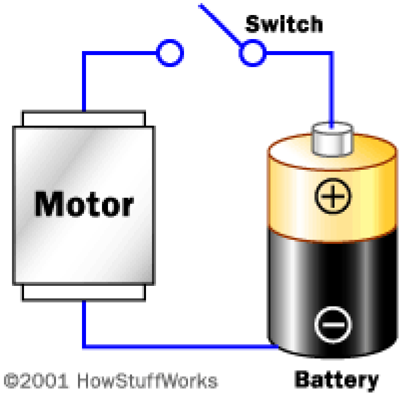


Simplest model of an atom

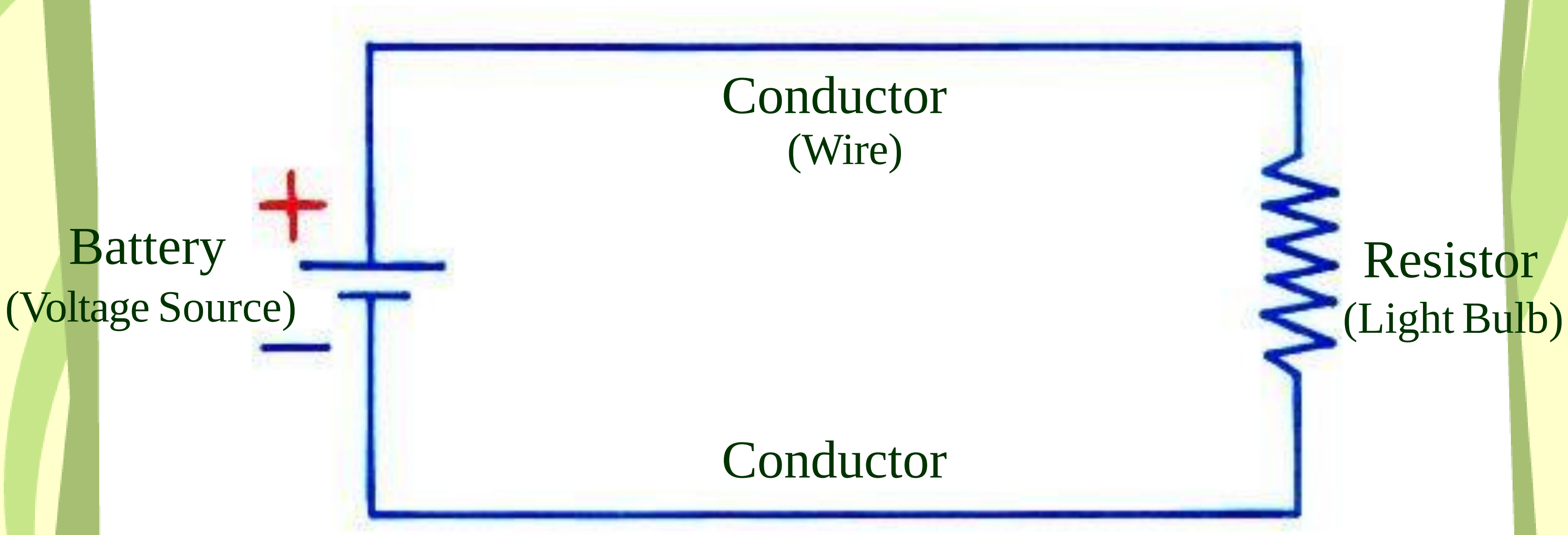
Electrical Circuits

Whether you are using a battery, a fuel cell or a solar cell to produce electricity, there are three things that are always the same:

- The source of electricity will have **two terminals**: a positive terminal and a negative terminal.



Basic Electrical Circuits



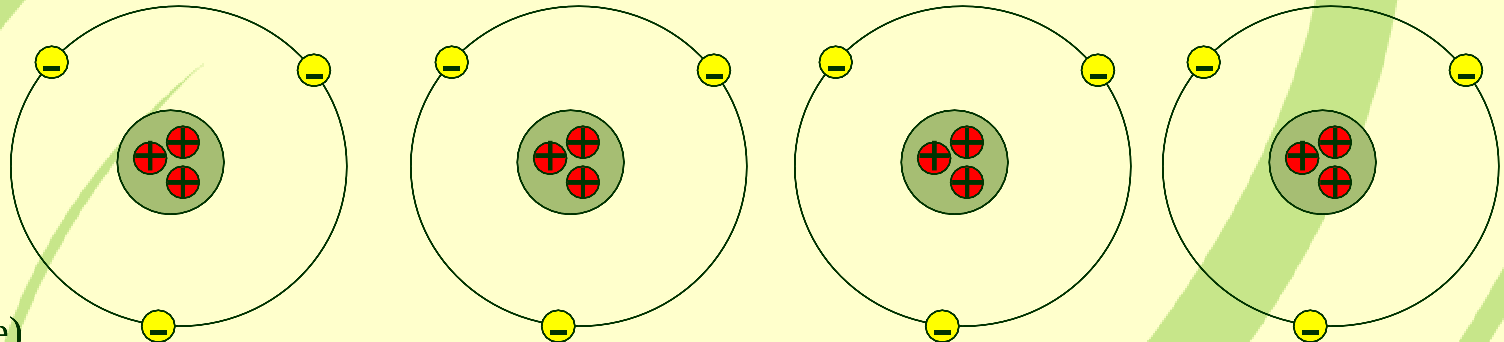
How does electricity flow?

What causes electrons to move from atom to atom?

Voltage

(Pressure)

(Electromotive Force)

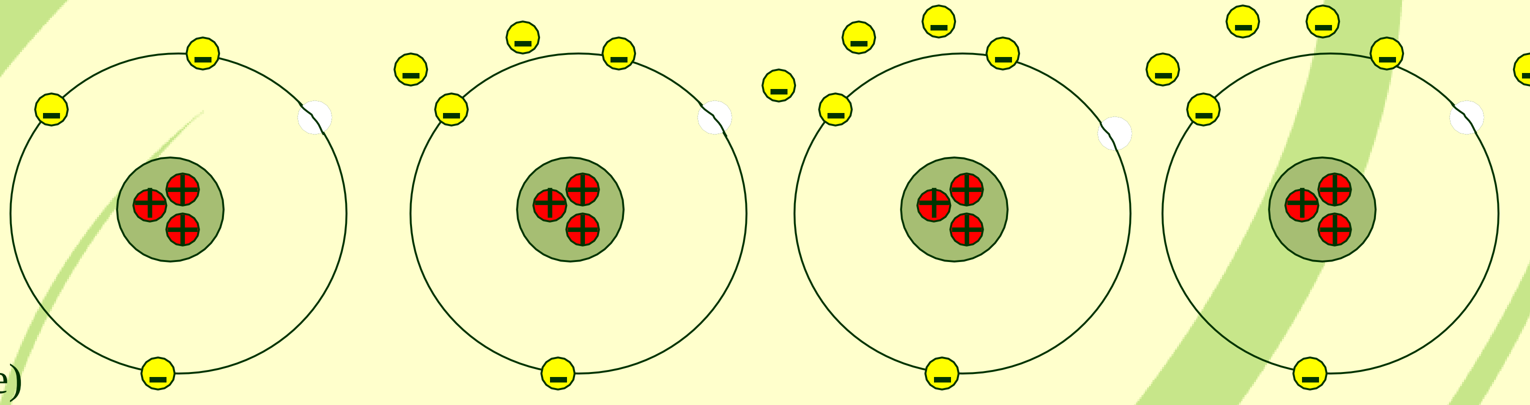


How does electricity flow?
What causes electrons to move from atom to atom?

Voltage

(Pressure)

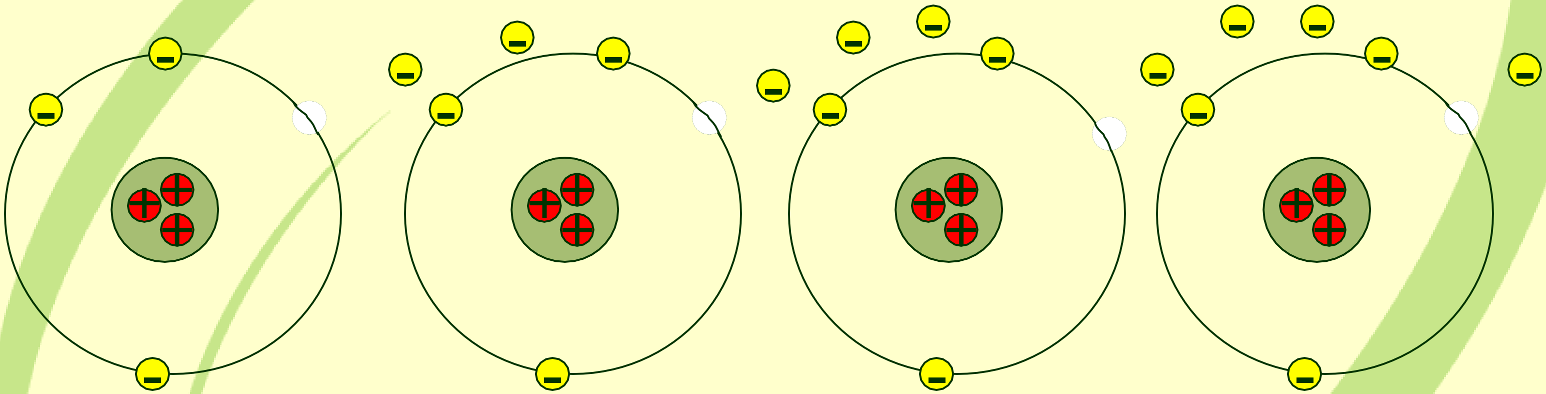
(Electromotive Force)



Voltage Pushes the electrons

How does electricity flow?

What causes electrons to move from atom to atom?



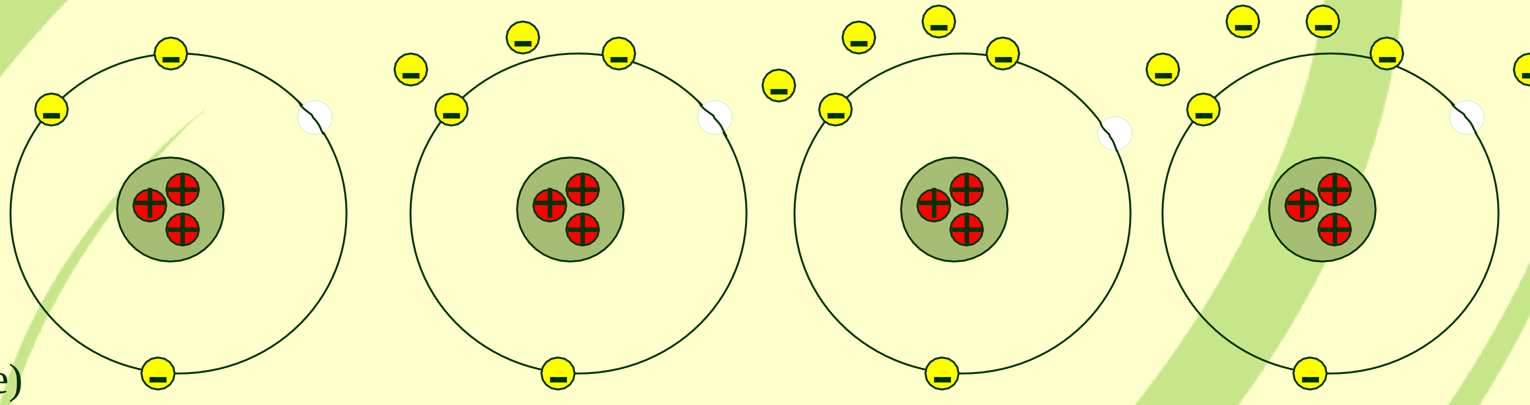
Voltage Pushes the electrons

How does electricity flow?
What causes electrons to move from atom to atom?

Voltage

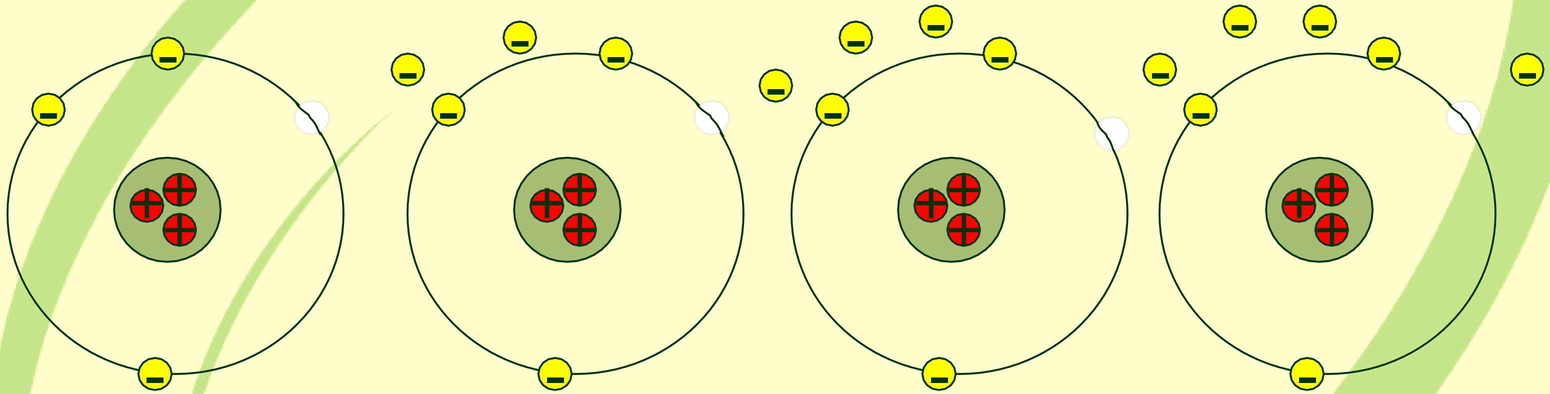
(Pressure)

(Electromotive Force)



The flow of the electrons is referred to as Current

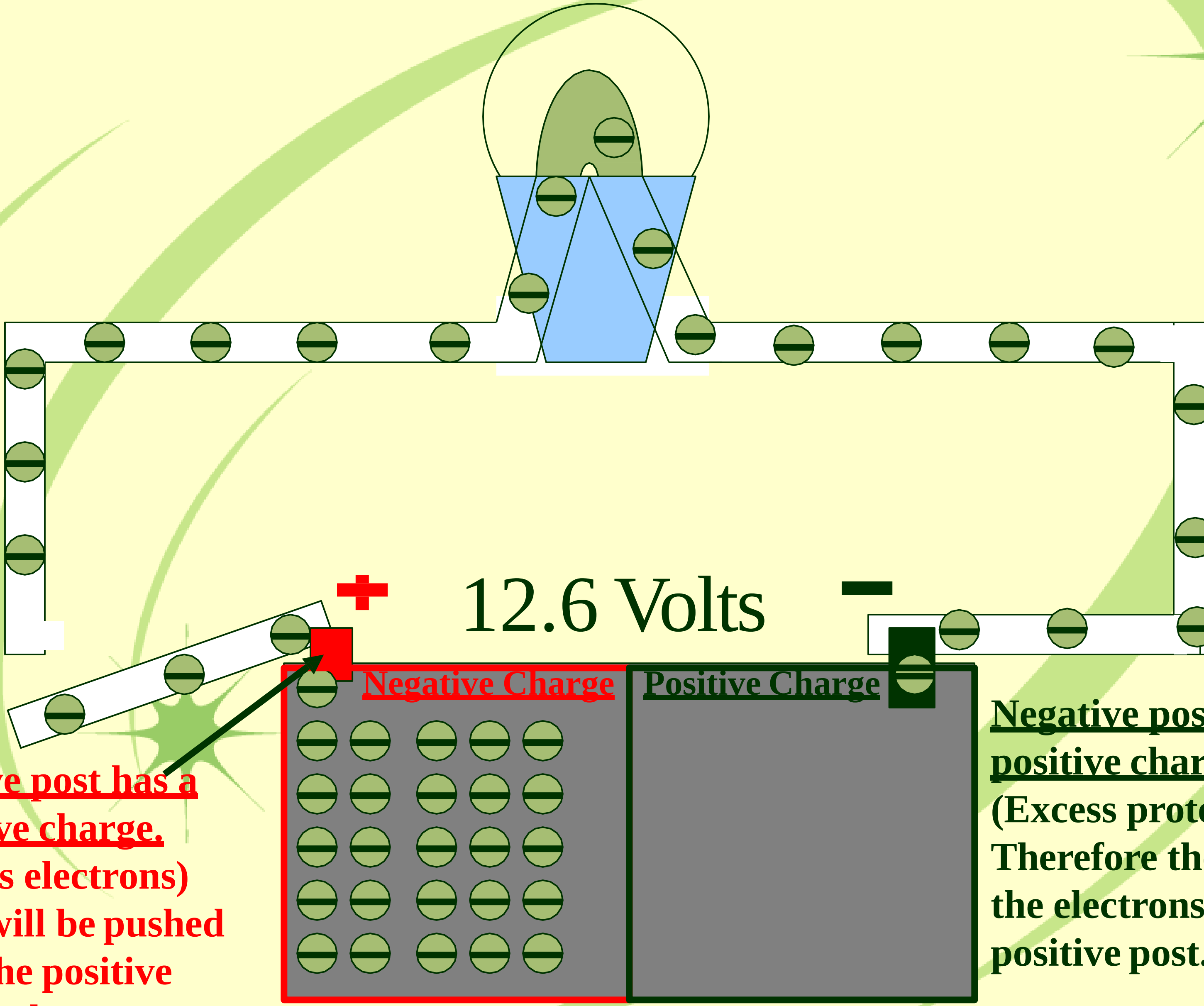
How does electricity flow?
What causes electrons to move from atom to atom?



Electron Flow is measured in Amps

The flow of the electrons is referred to as Current

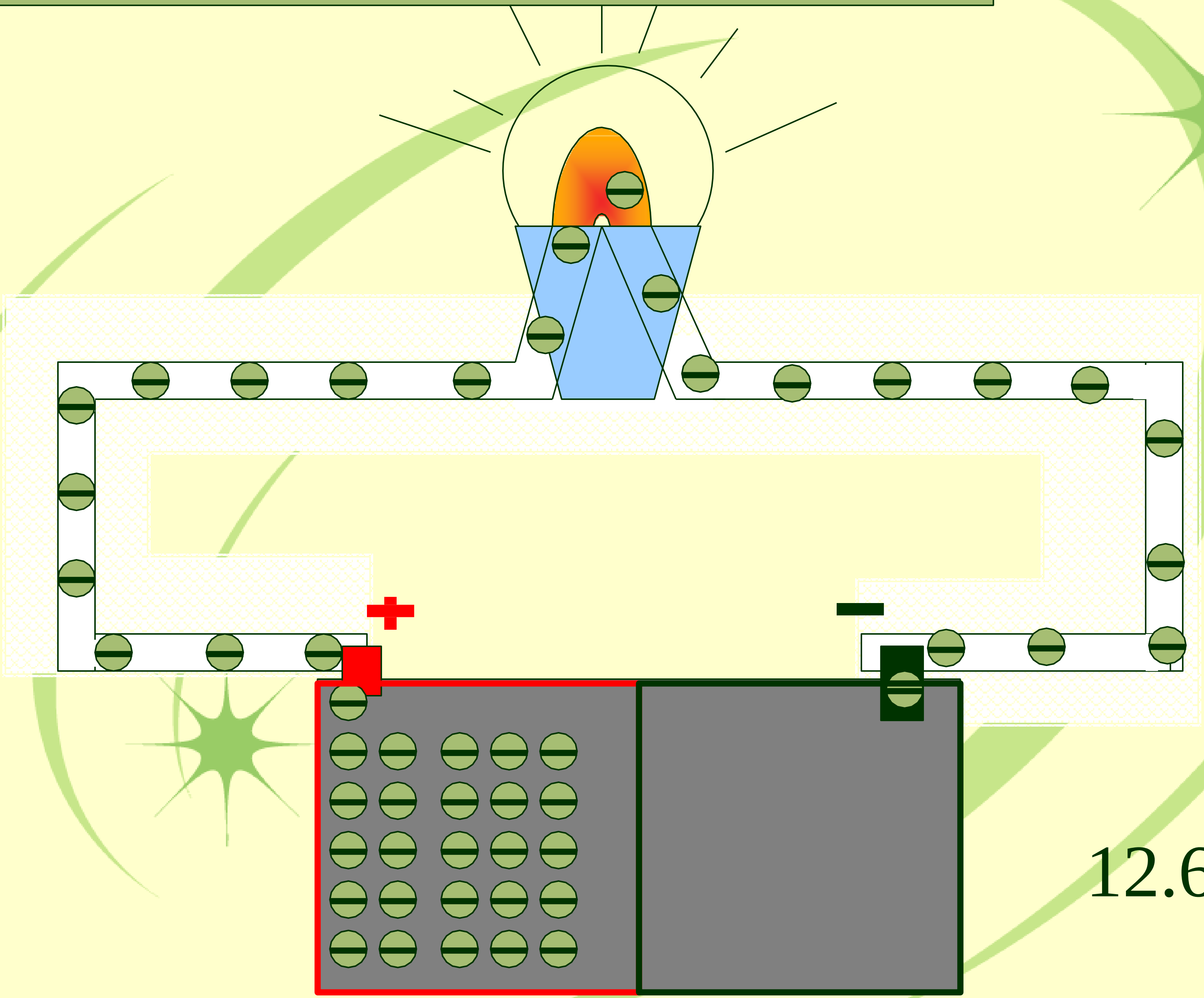
Basics Concept of Battery Discharging And Charging



Positive post has a negative charge.
(Excess electrons)
They will be pushed from the positive Terminal.

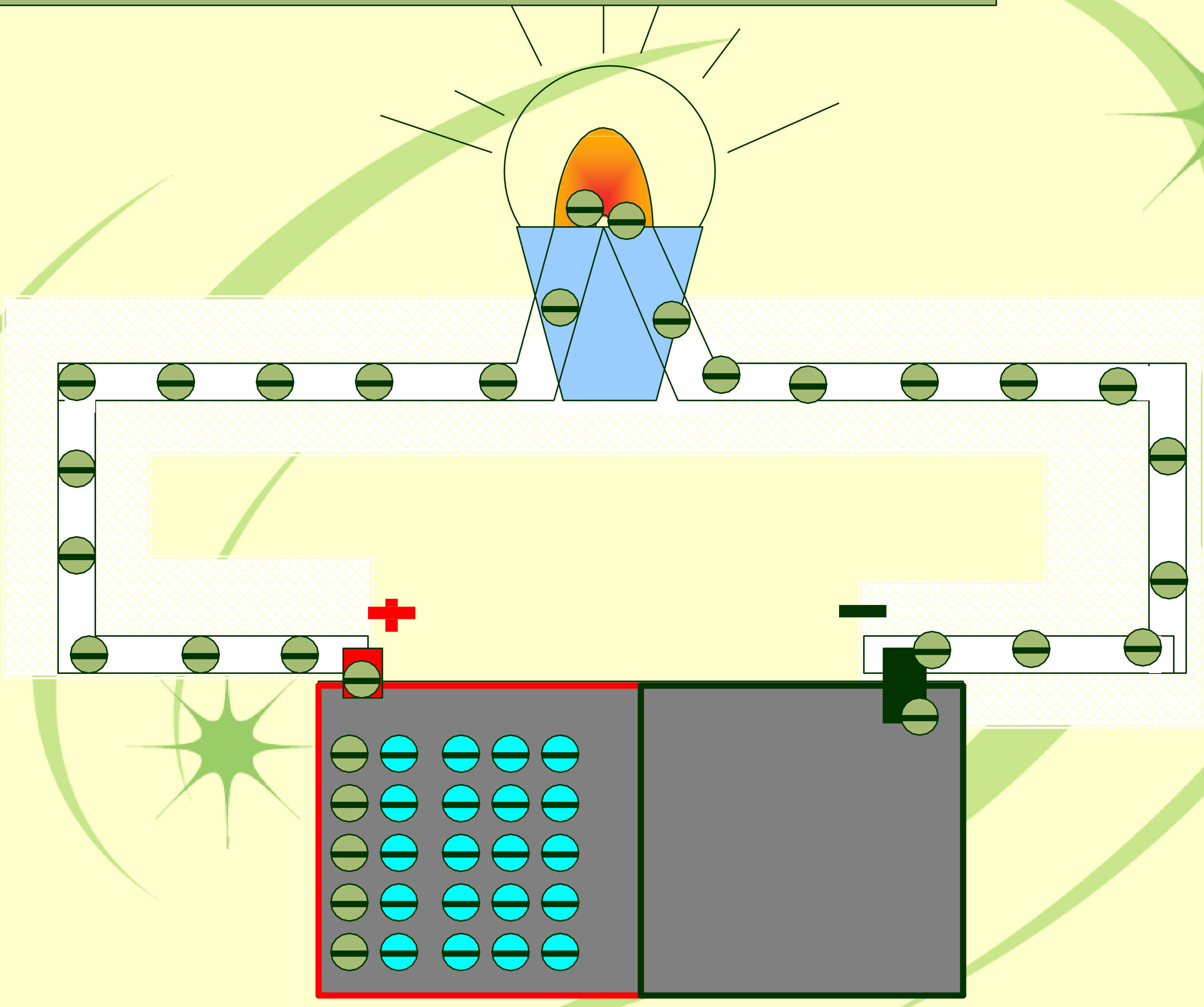
Negative post has a positive charge.
(Excess protons)
Therefore they will pull the electrons from the positive post.

Basics Concept of Battery Discharging And Charging

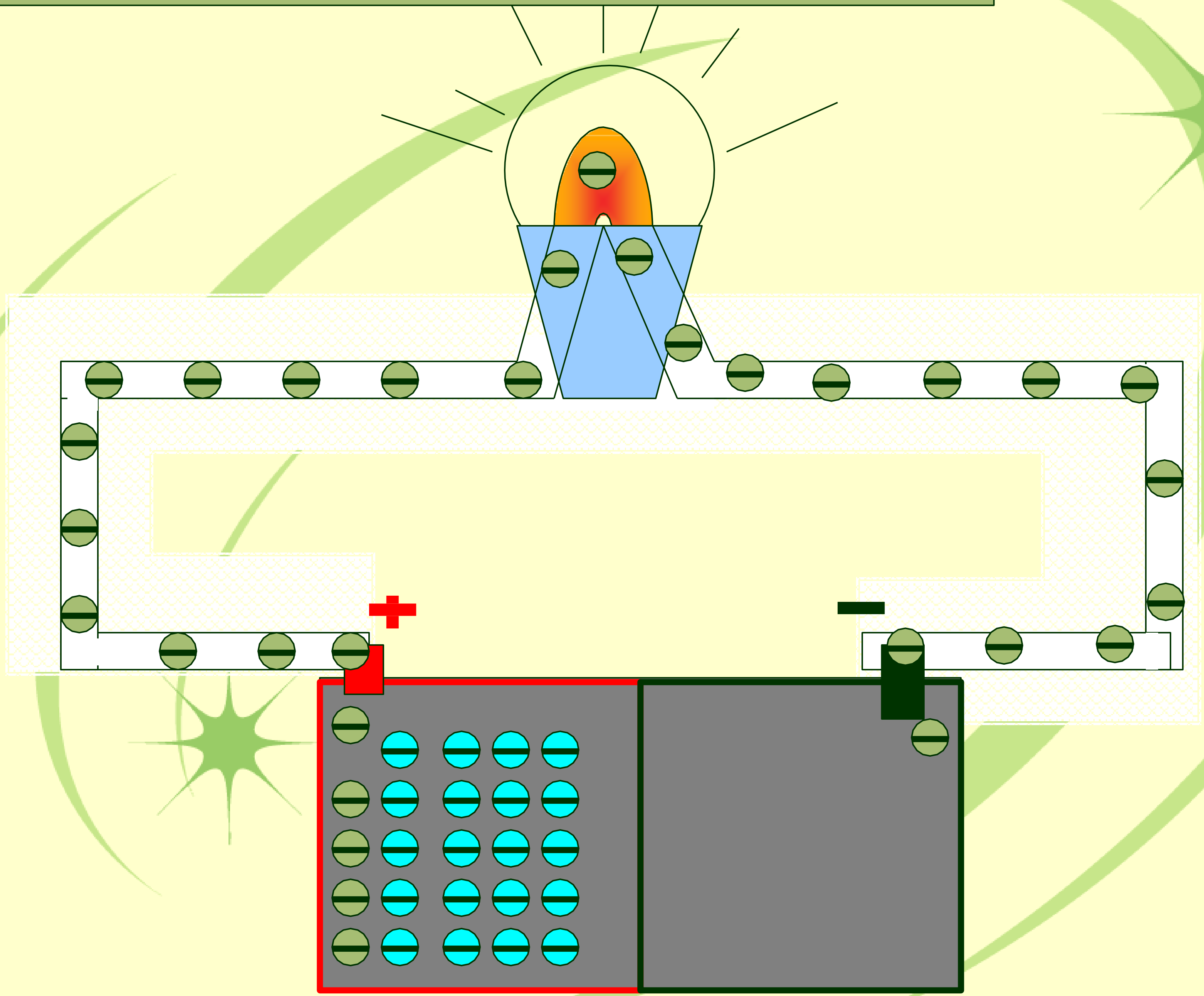


12.6 Volt

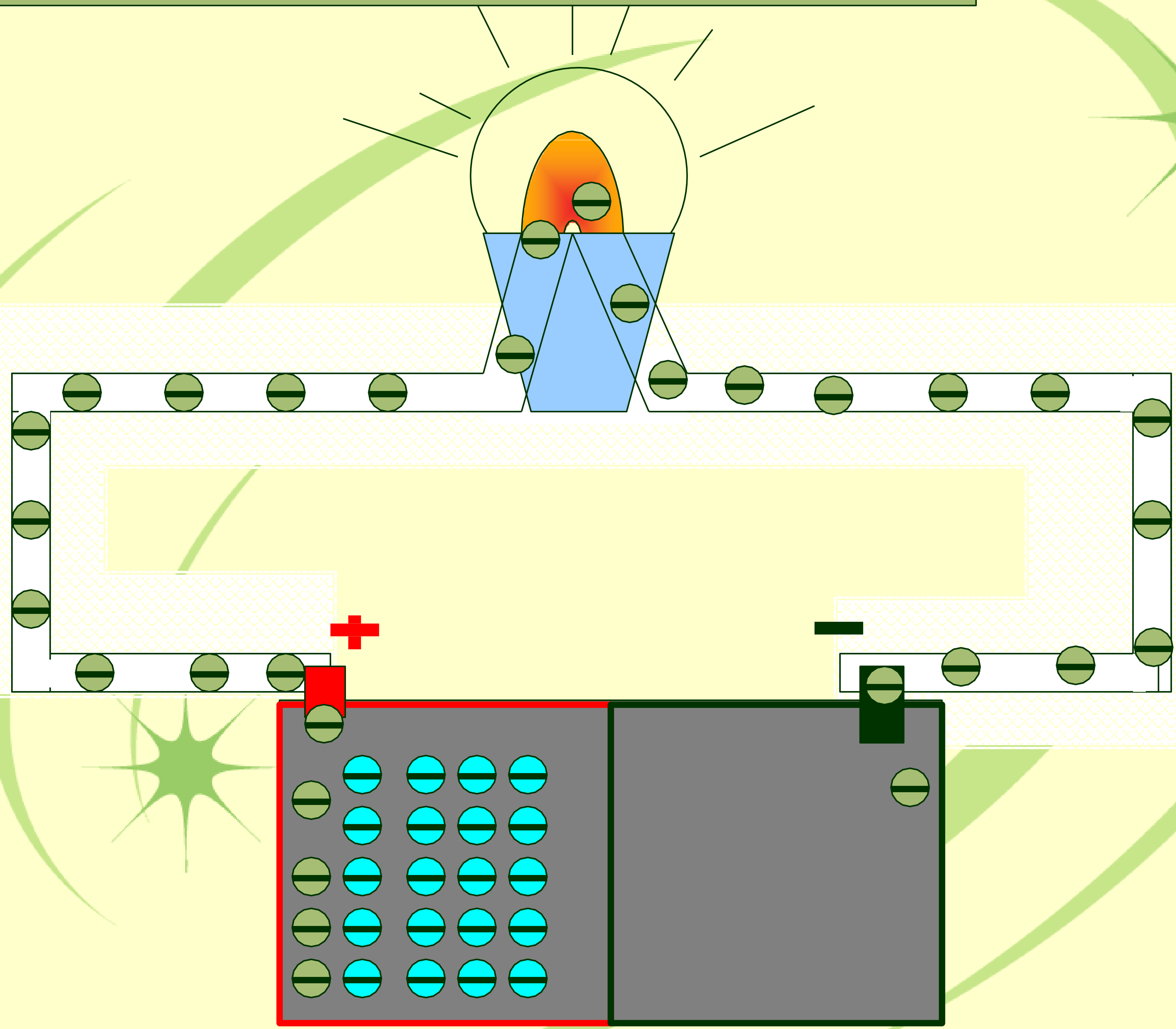
Basics Concept of Battery Discharging And Charging



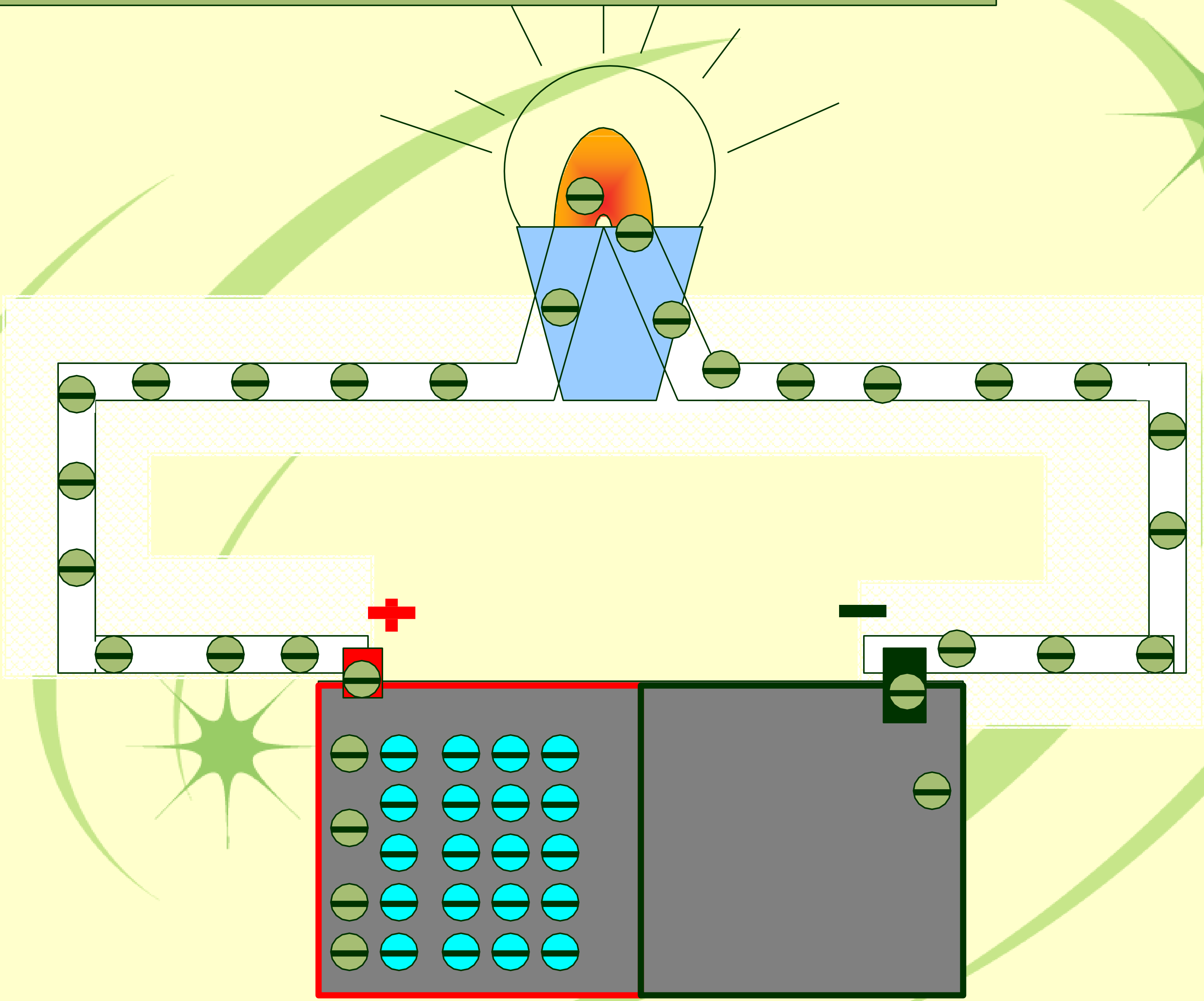
Basics Concept of Battery Discharging And Charging



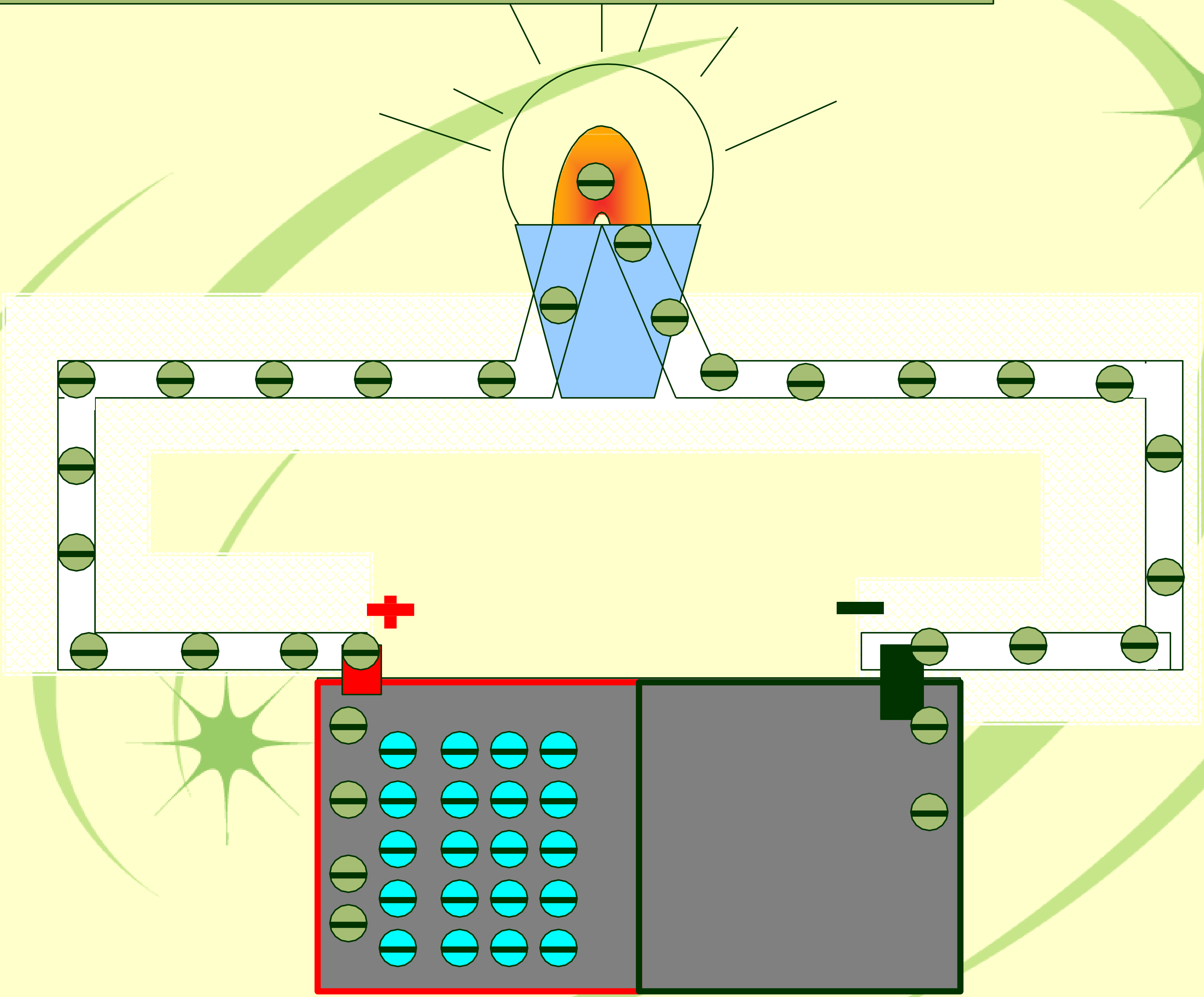
Basics Concept of Battery Discharging And Charging



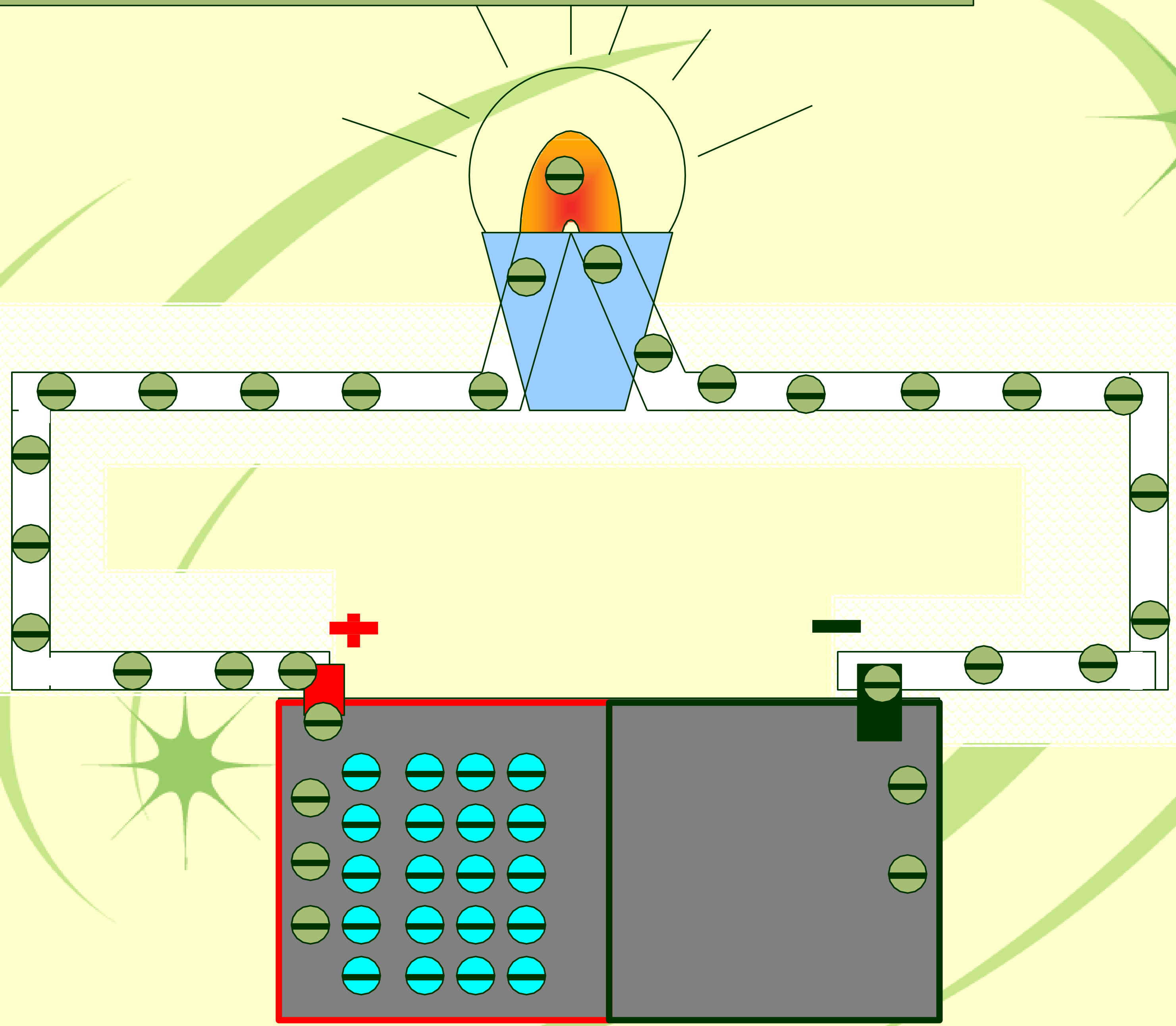
Basics Concept of Battery Discharging And Charging



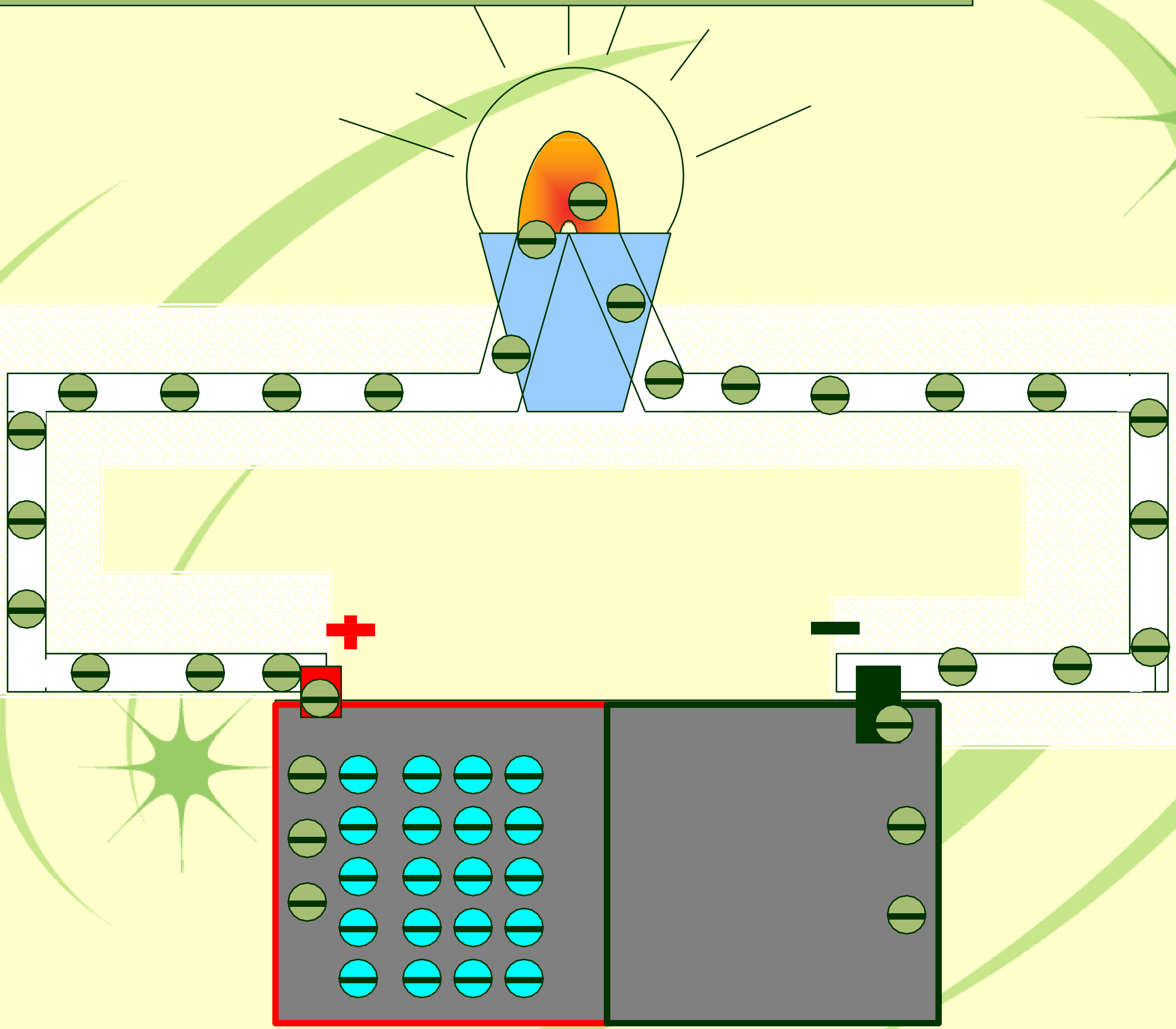
Basics Concept of Battery Discharging And Charging



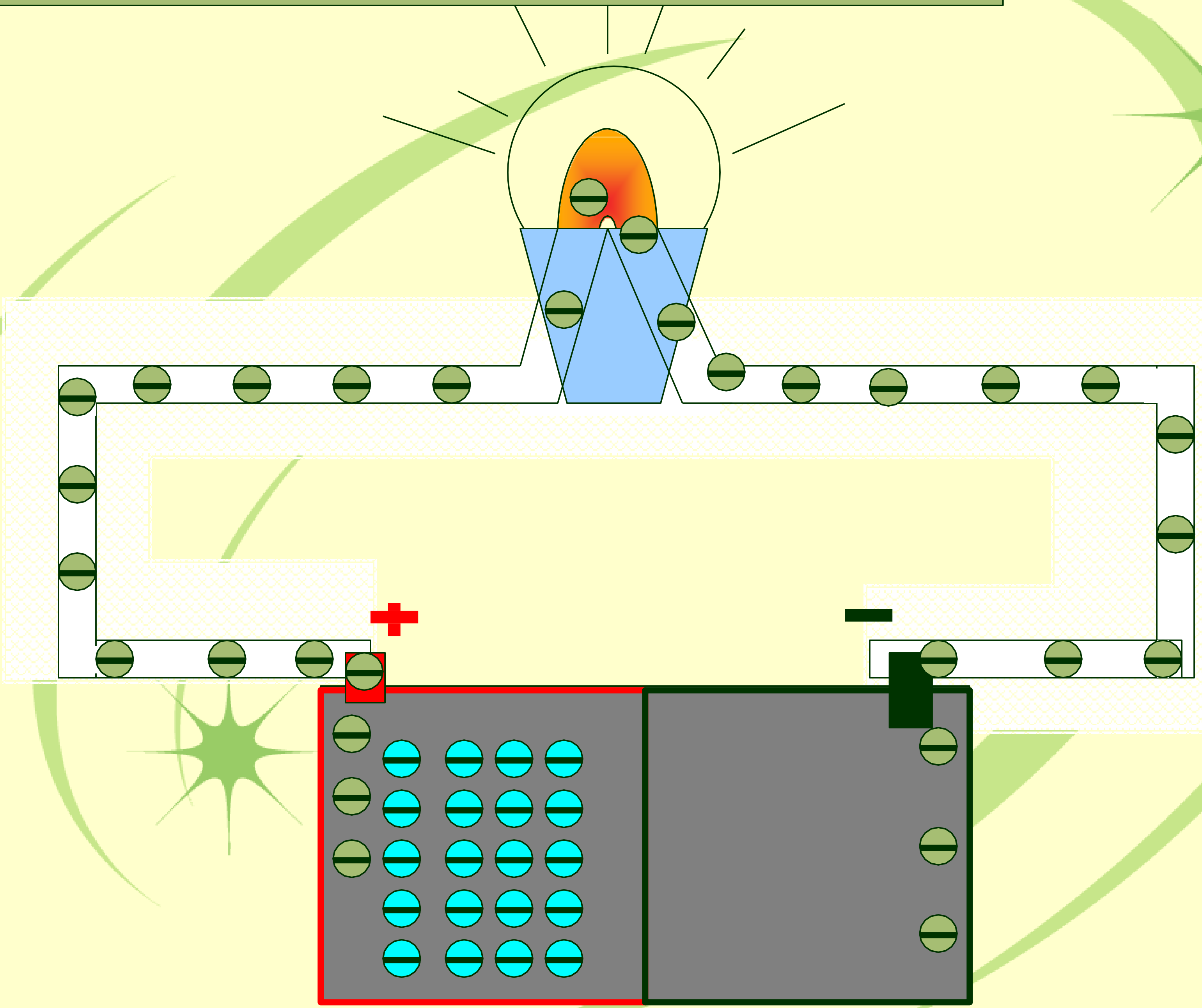
Basics Concept of Battery Discharging And Charging



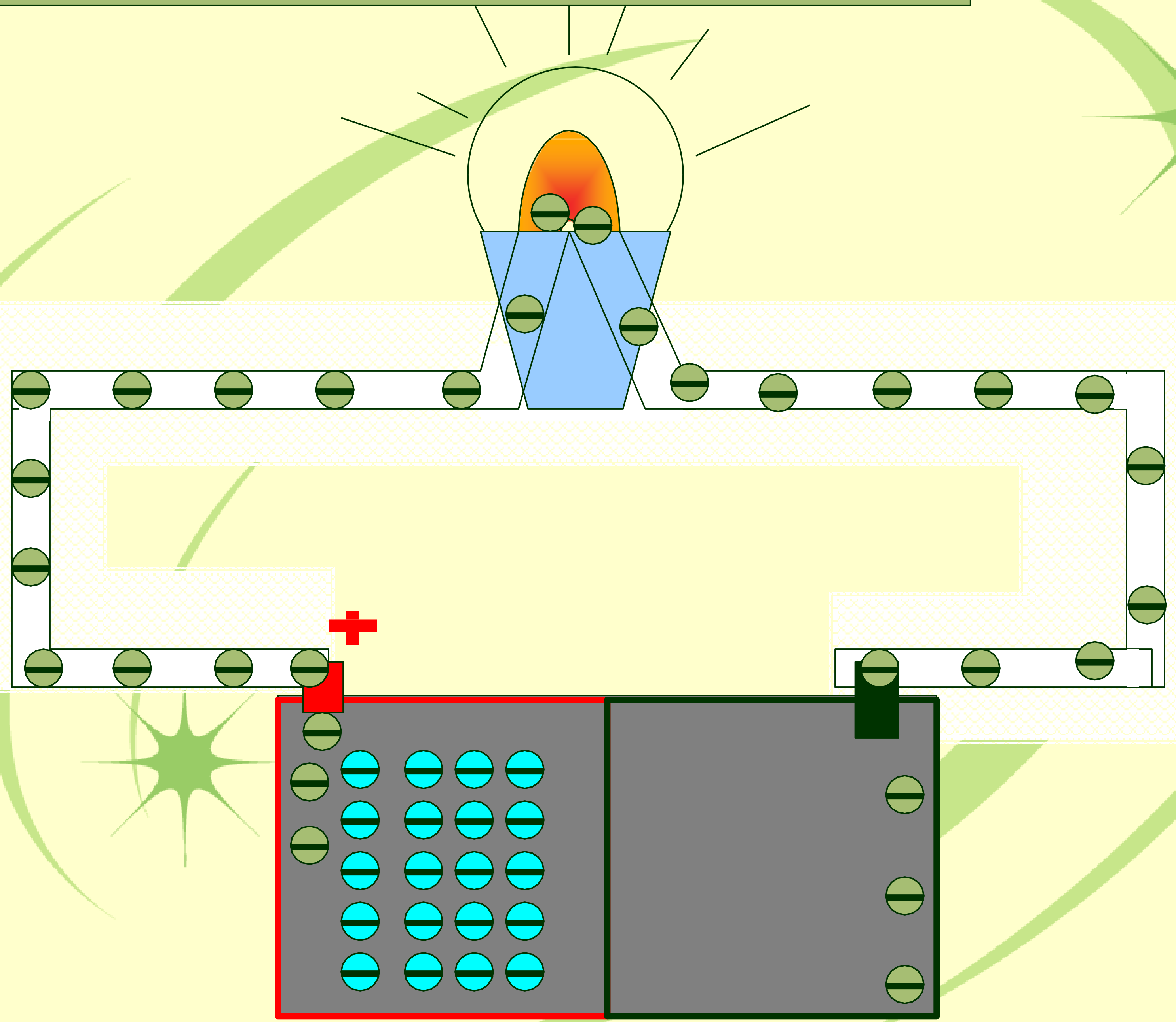
Basics Concept of Battery Discharging And Charging



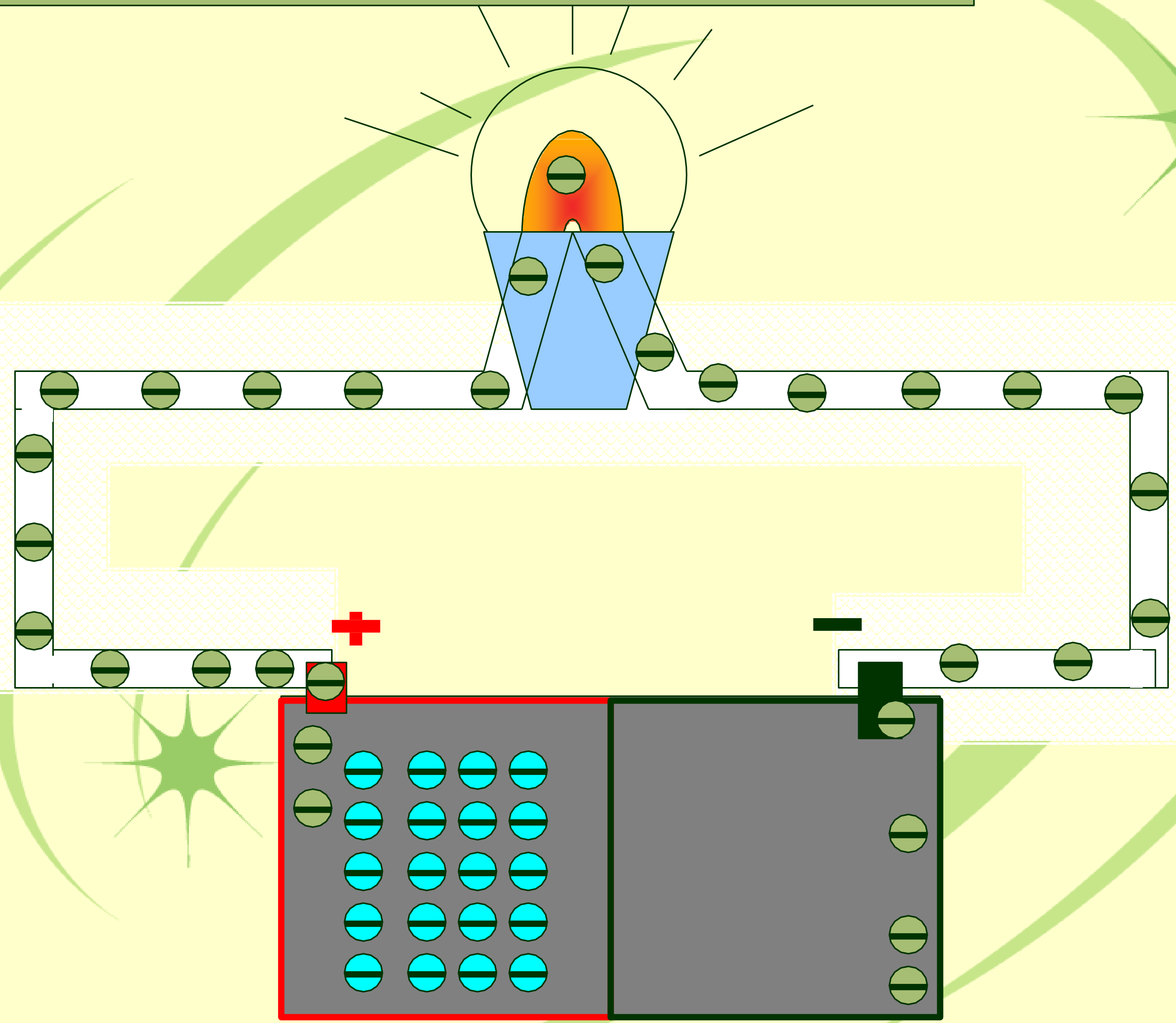
Basics Concept of Battery Discharging And Charging



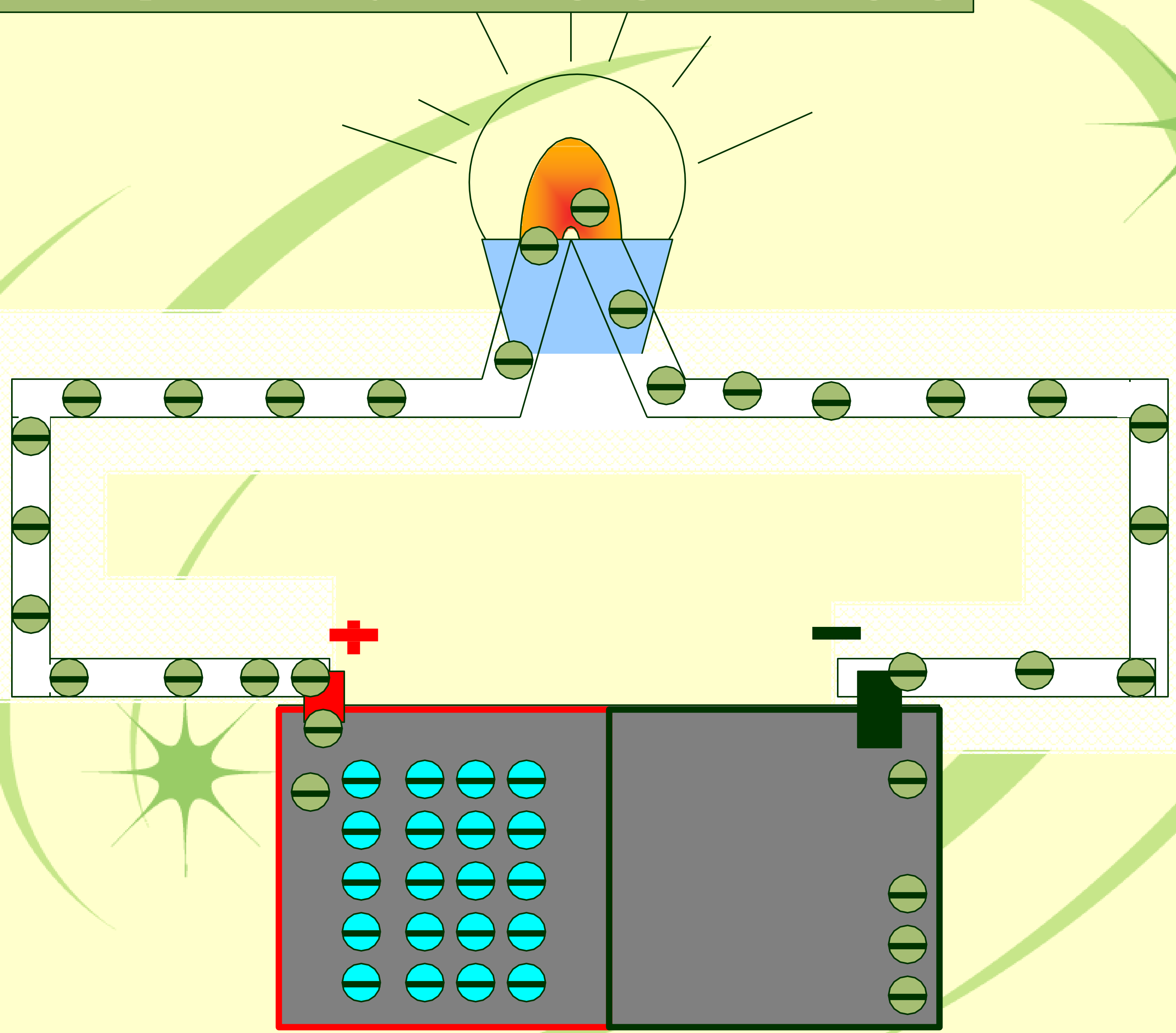
Basics Concept of Battery Discharging And Charging



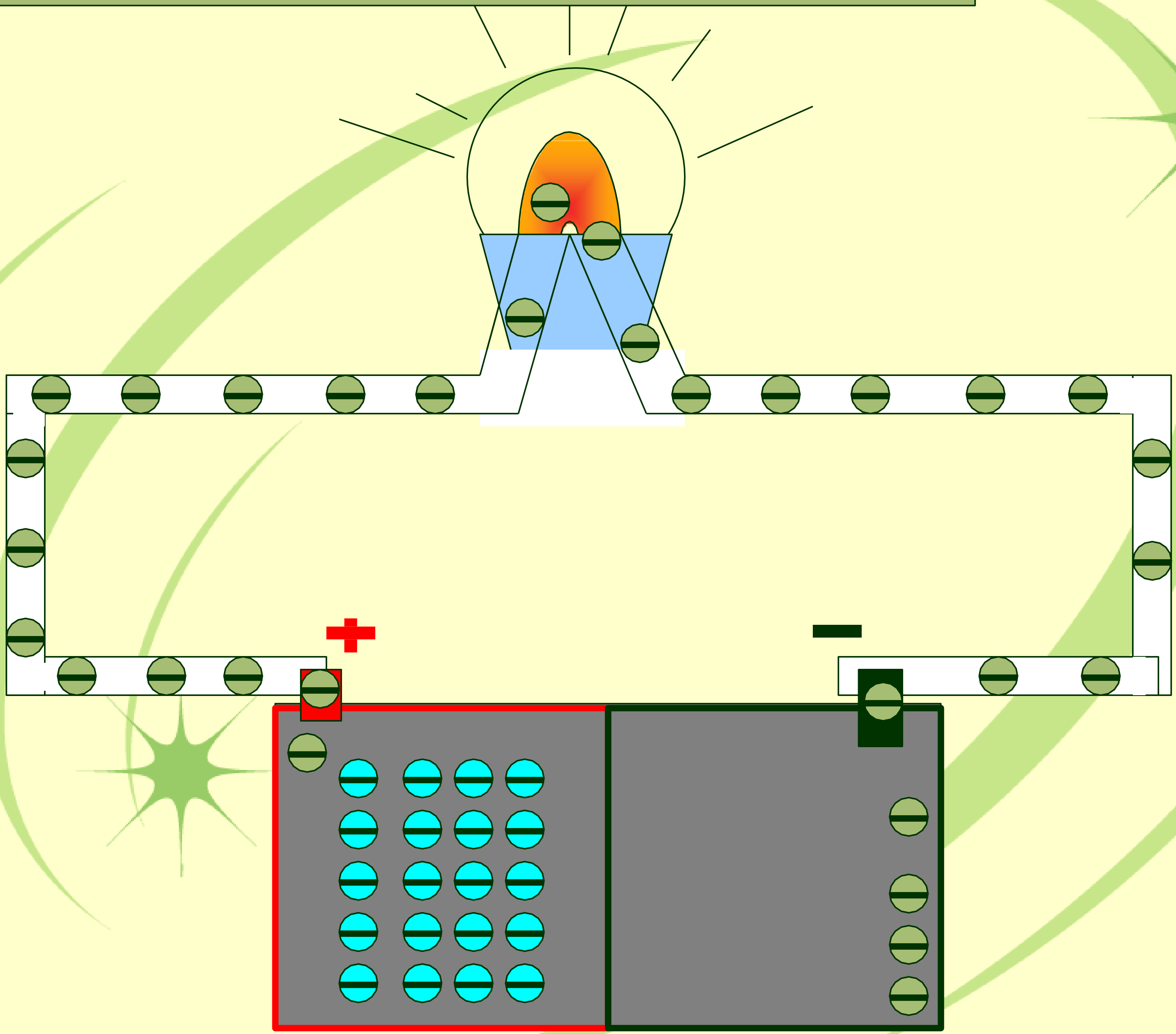
Basics Concept of Battery Discharging And Charging



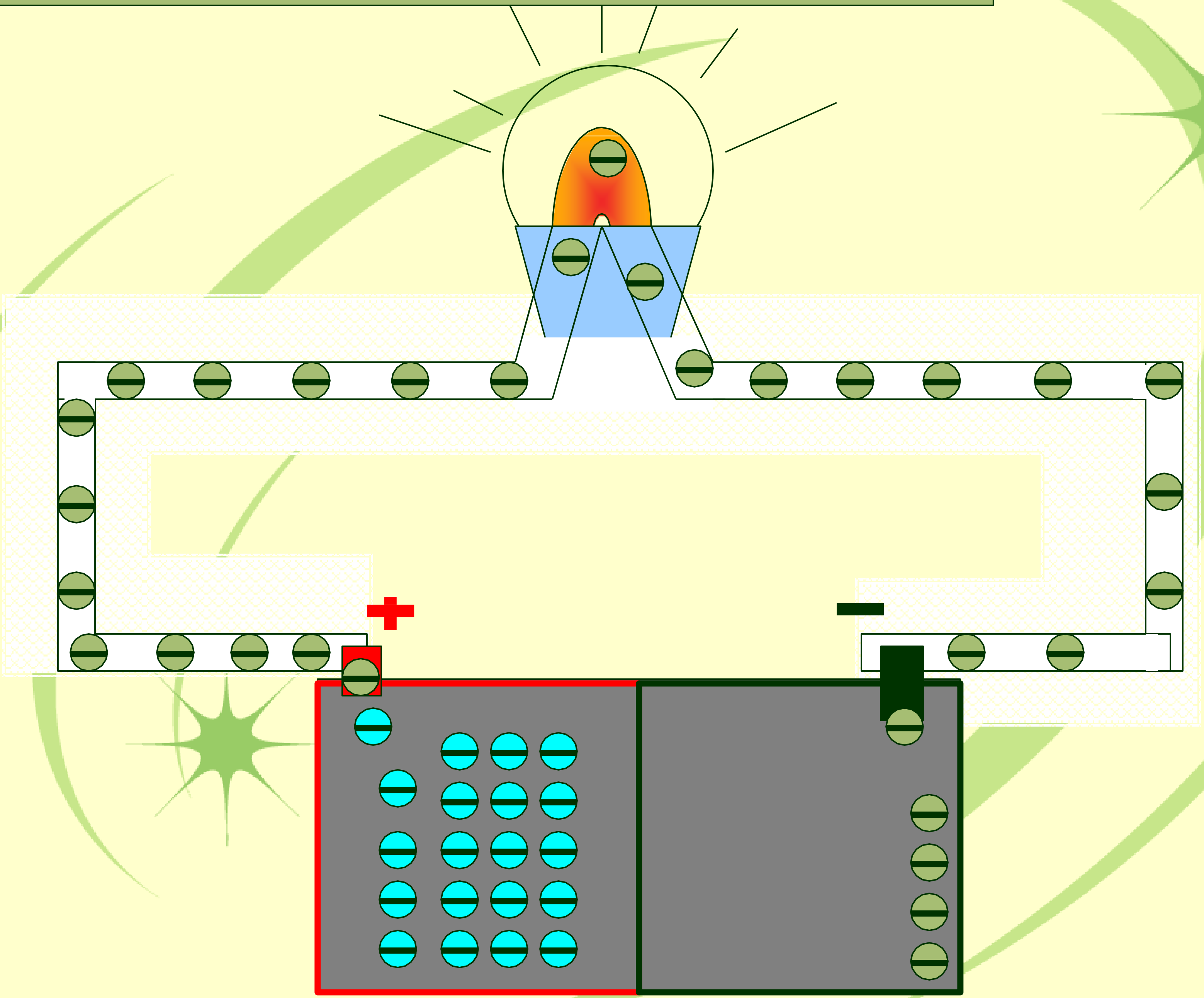
Basics Concept of Battery Discharging And Charging



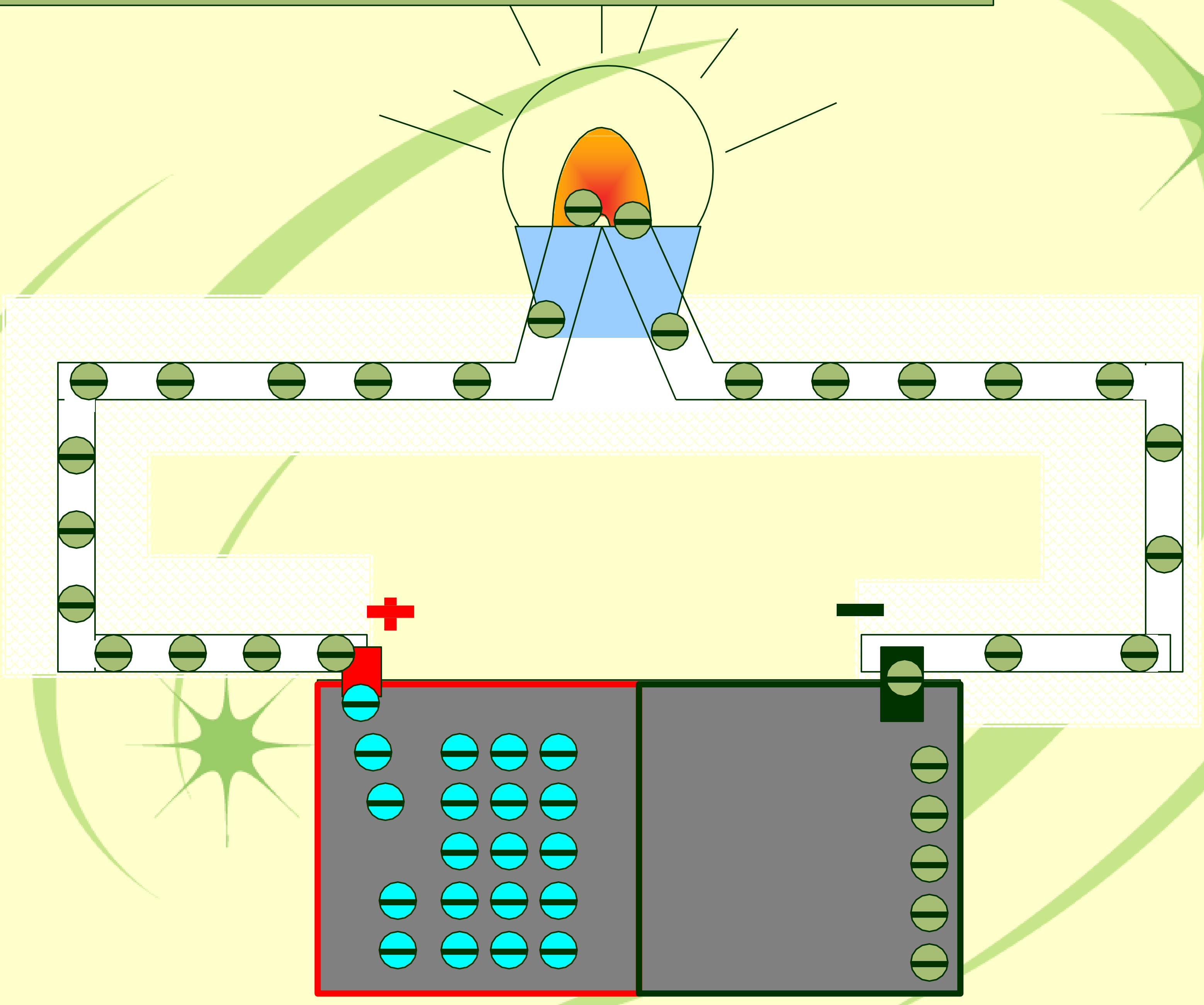
Basics Concept of Battery Discharging And Charging



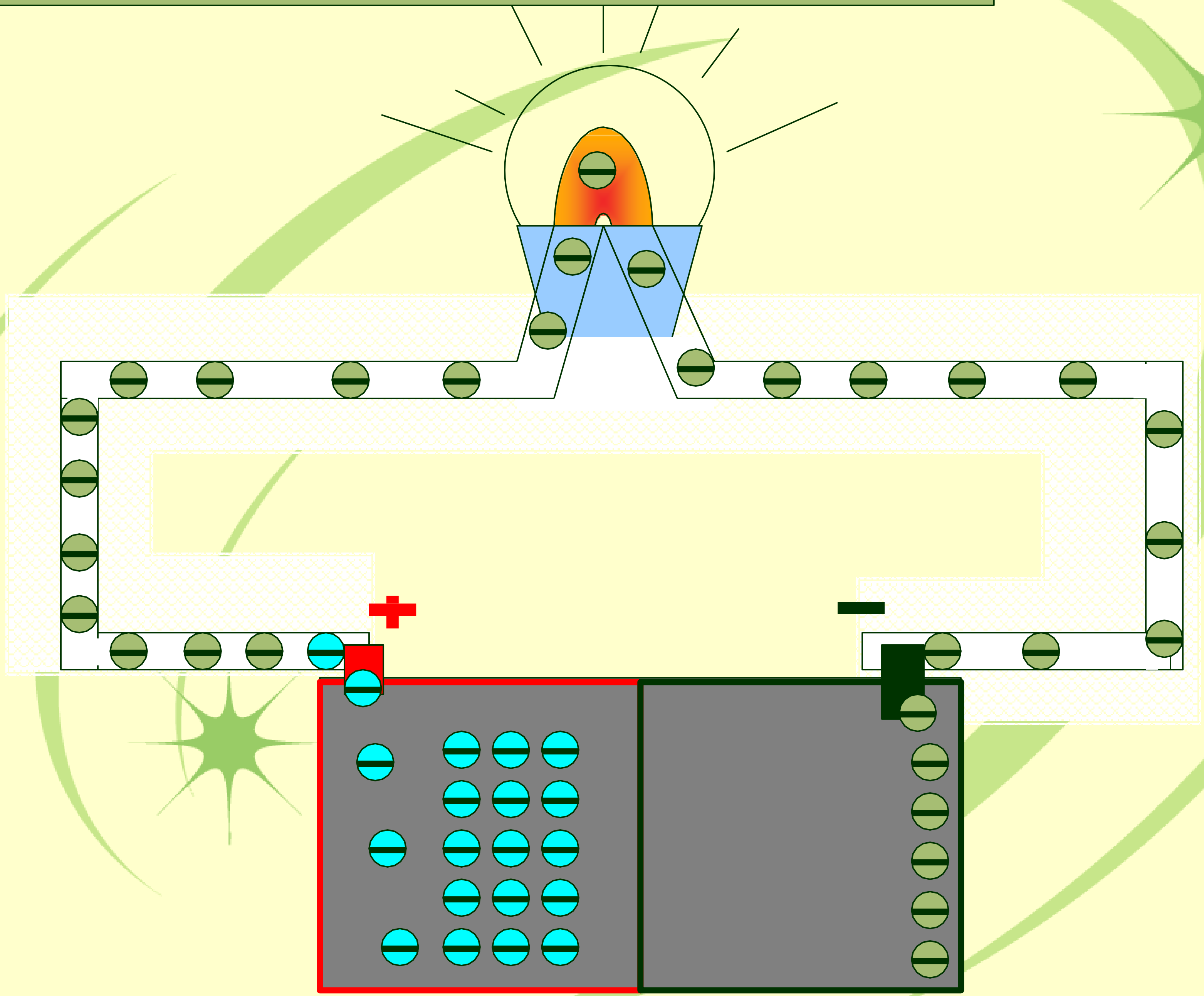
Basics Concept of Battery Discharging And Charging



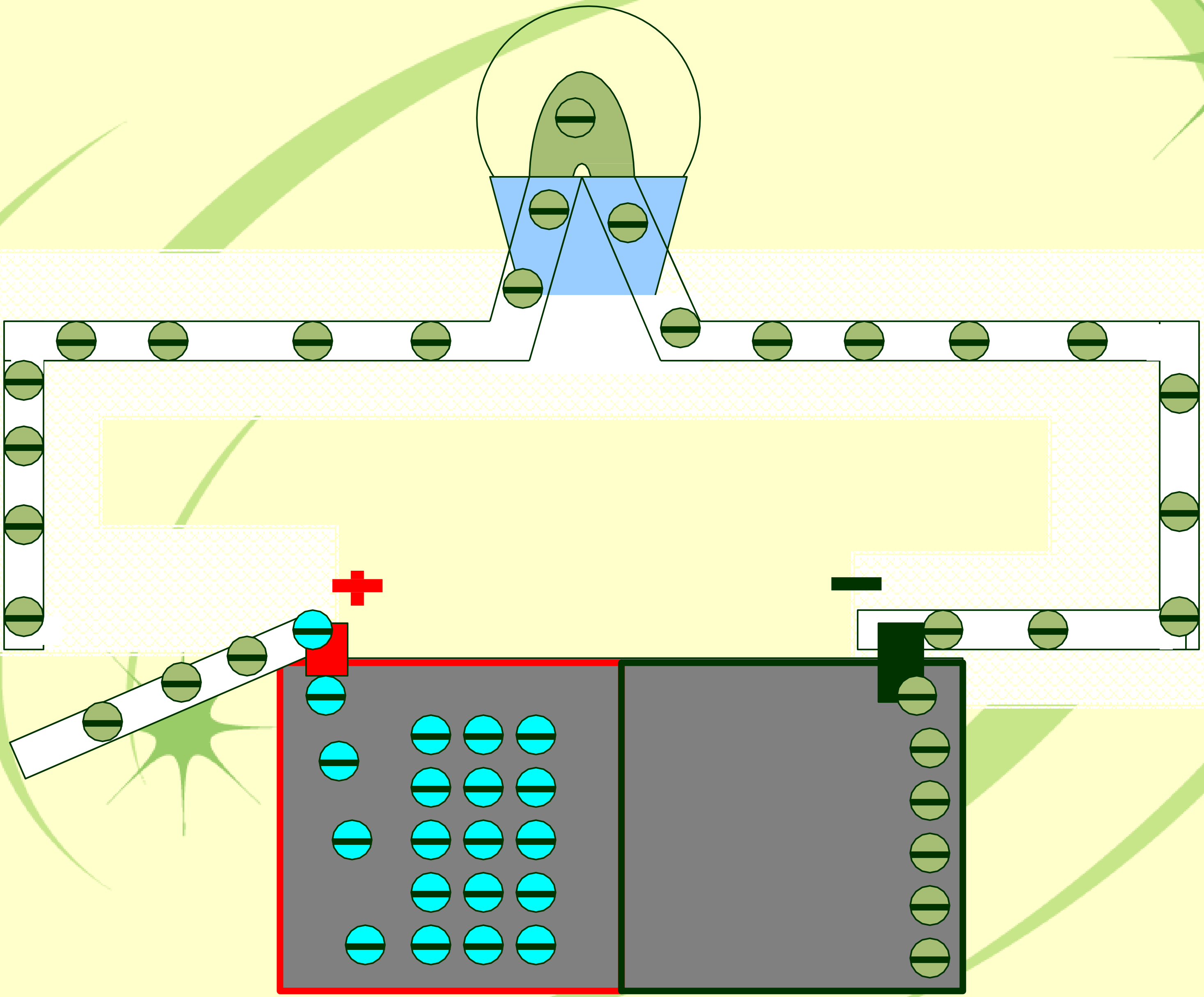
Basics Concept of Battery Discharging And Charging



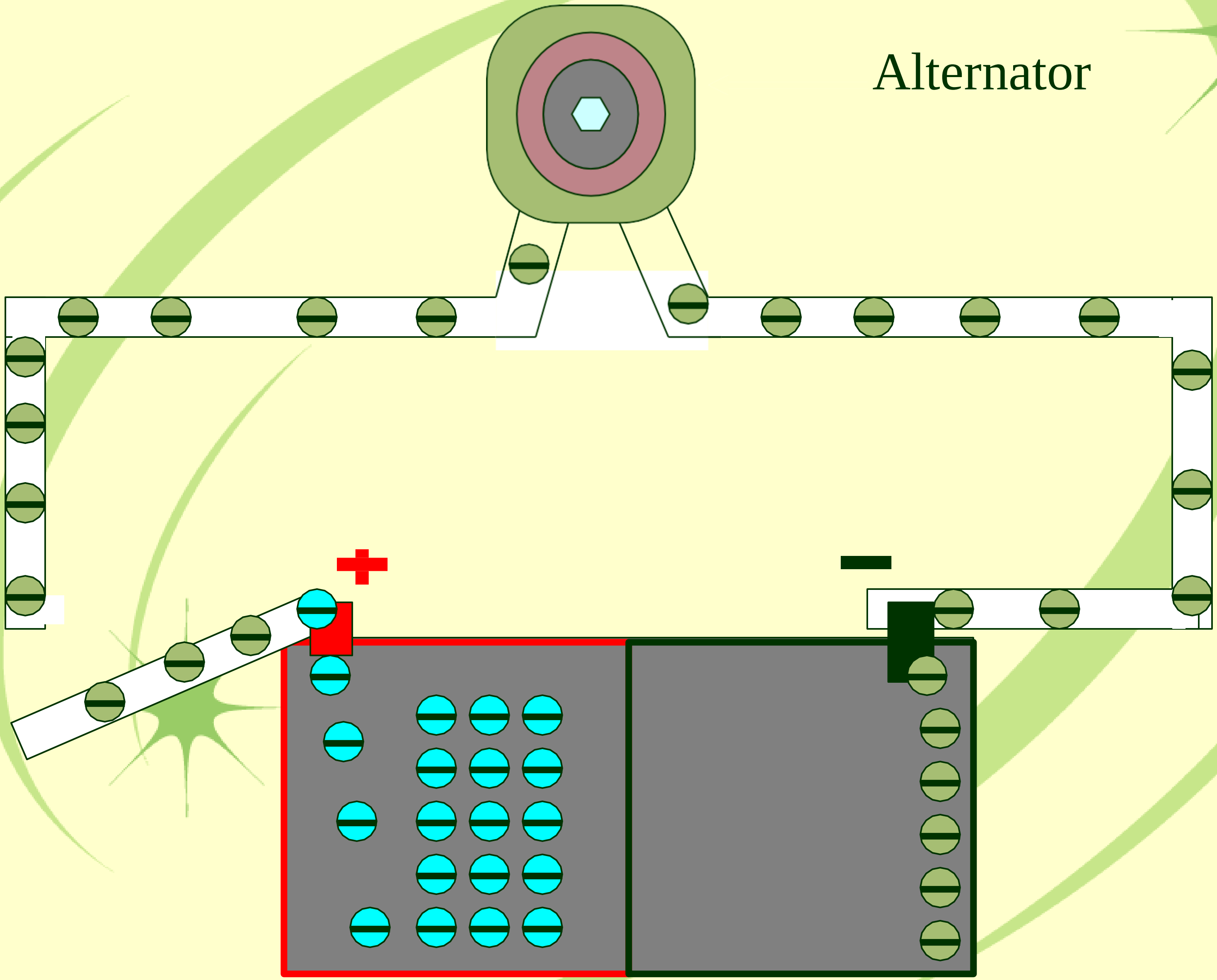
Basics Concept of Battery Discharging And Charging



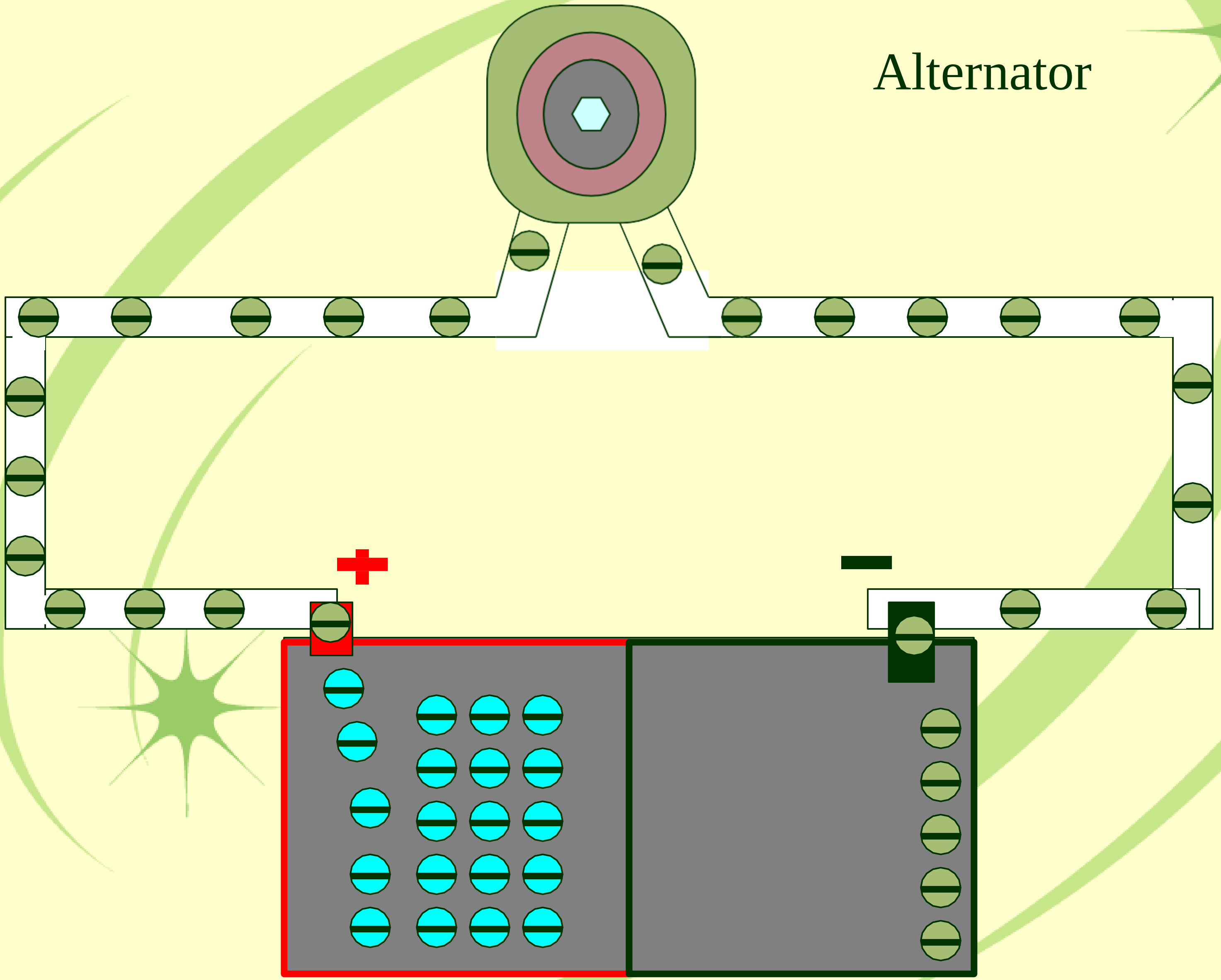
Basics Concept of Battery Discharging And Charging



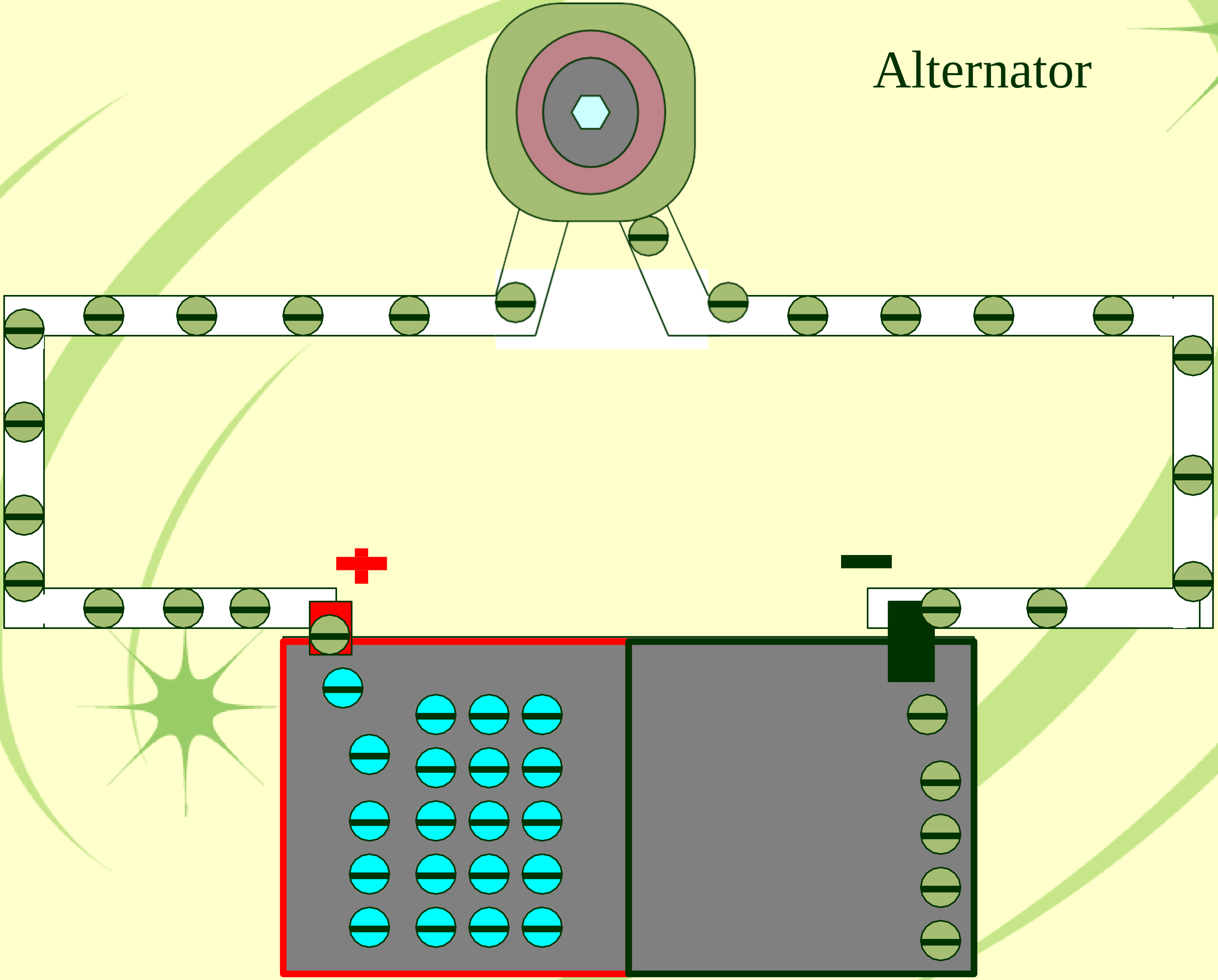
Basics Concept of Battery Discharging And Charging



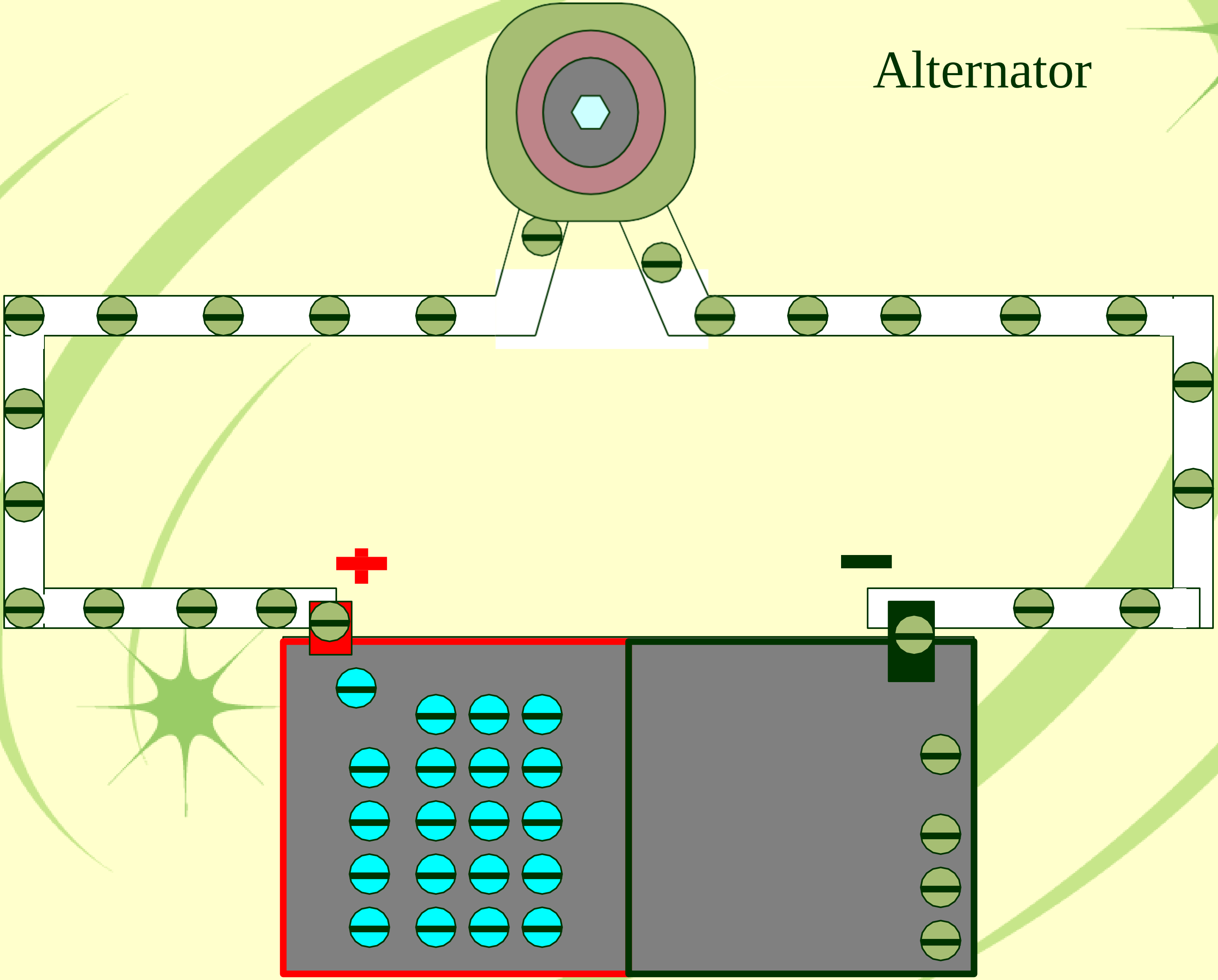
Basics Concept of Battery Discharging And Charging



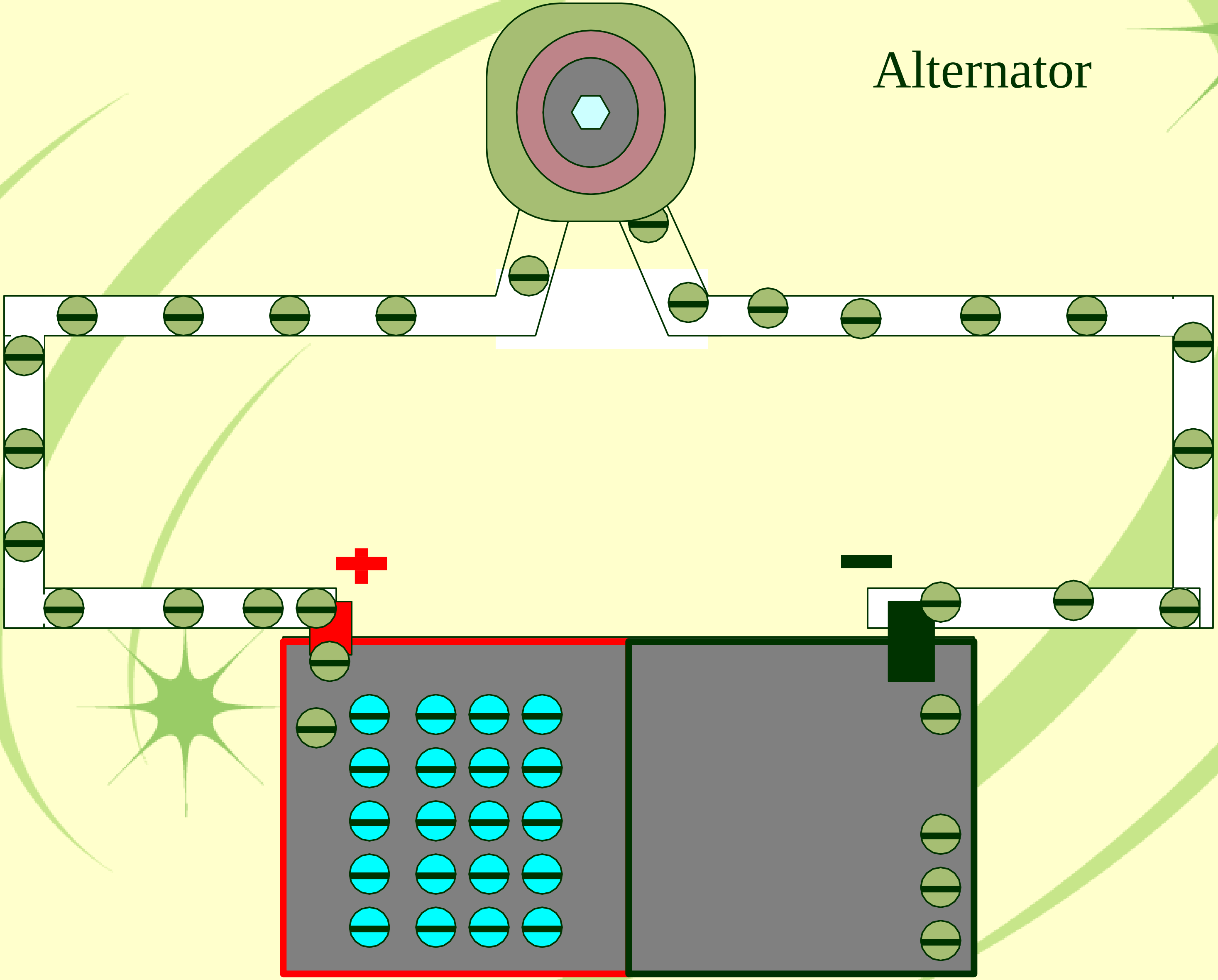
Basics Concept of Battery Discharging And Charging



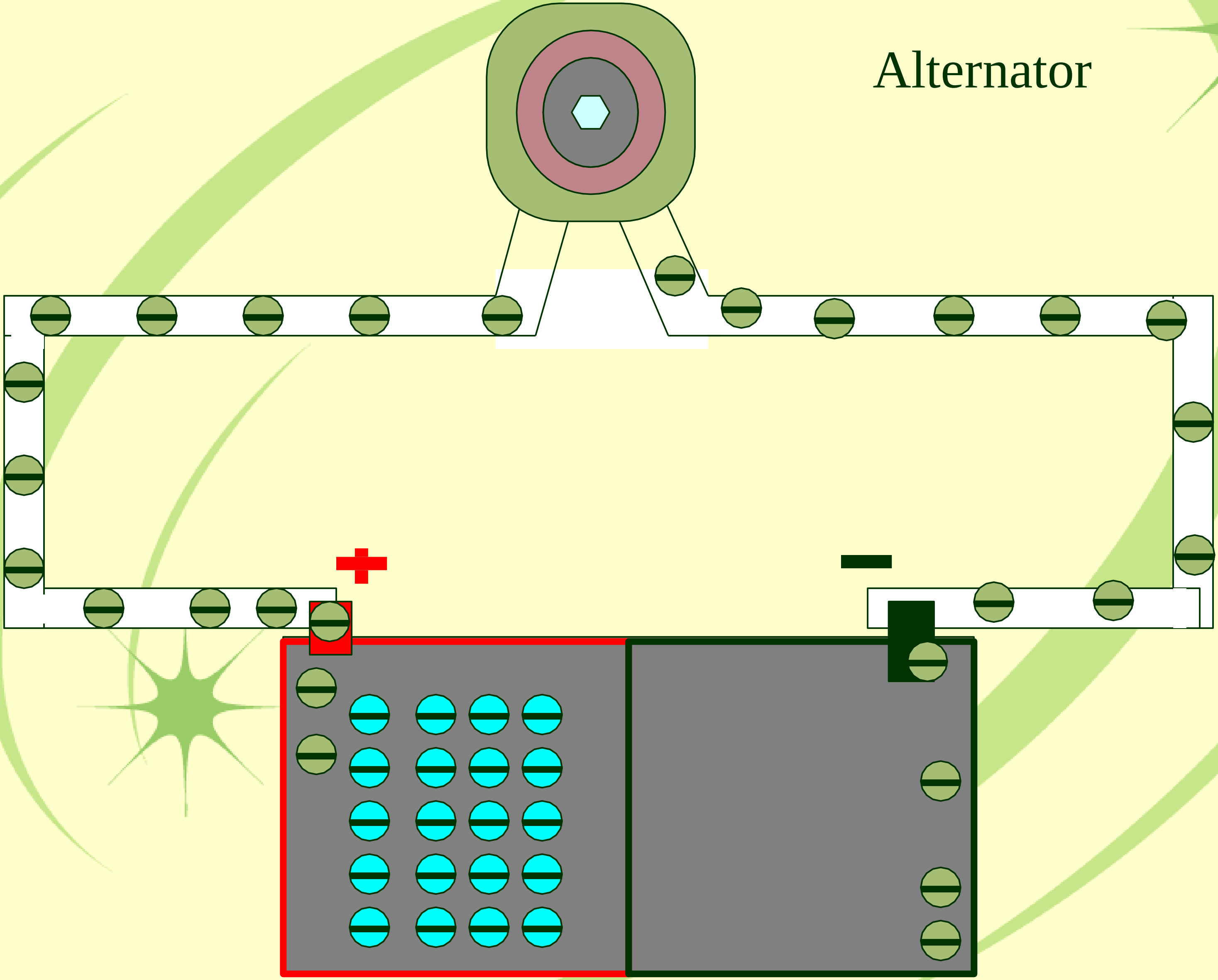
Basics Concept of Battery Discharging And Charging



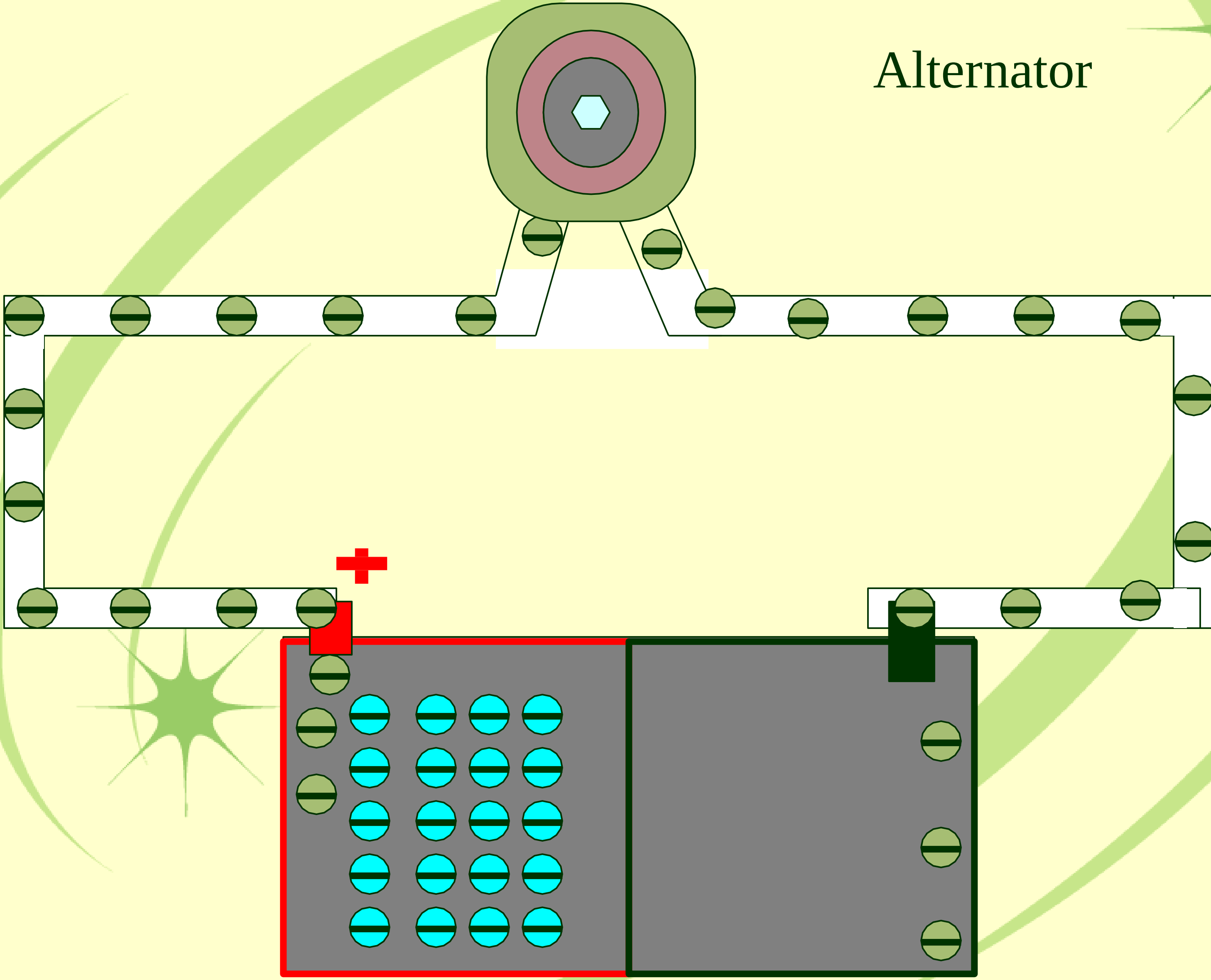
Basics Concept of Battery Discharging And Charging



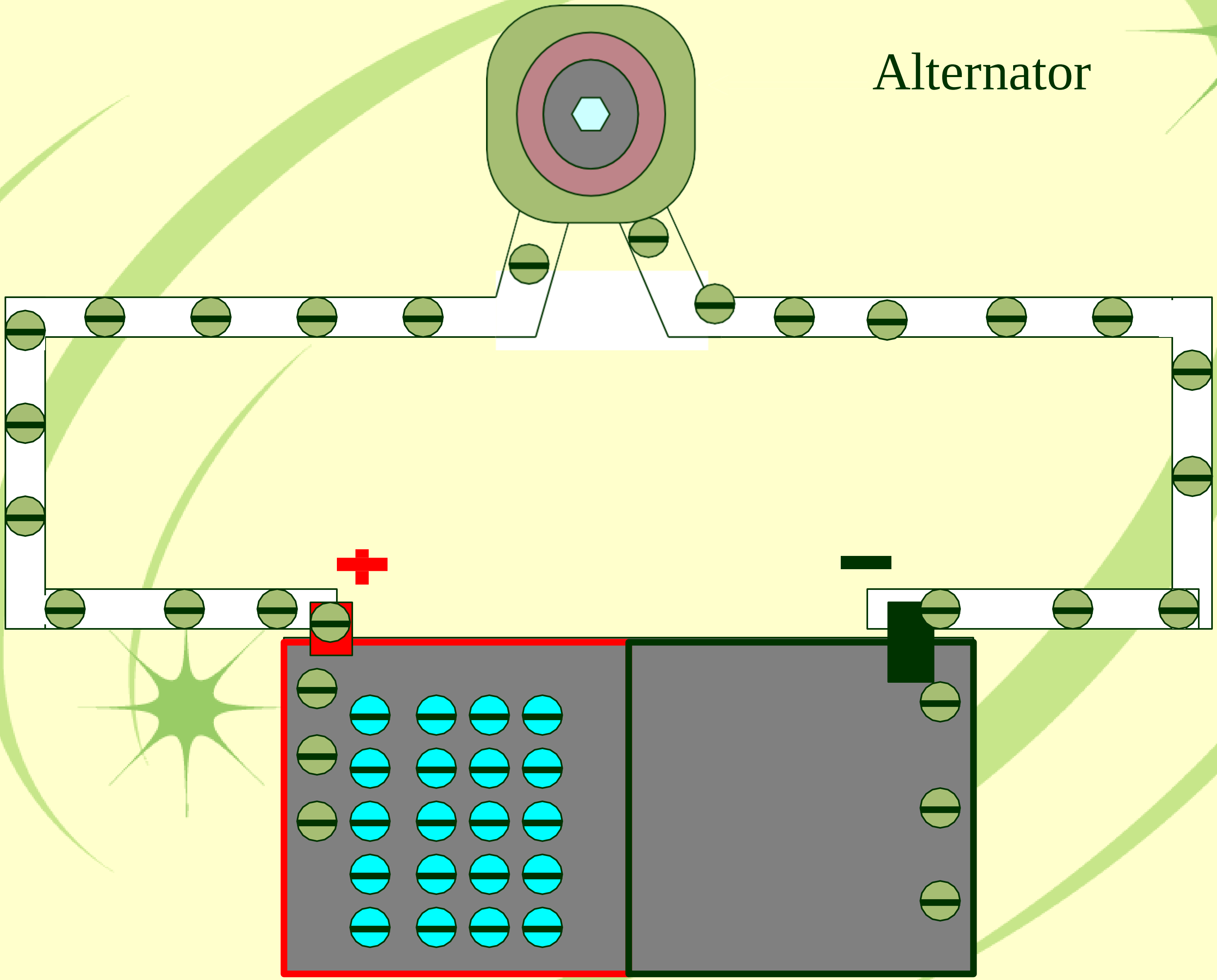
Basics Concept of Battery Discharging And Charging



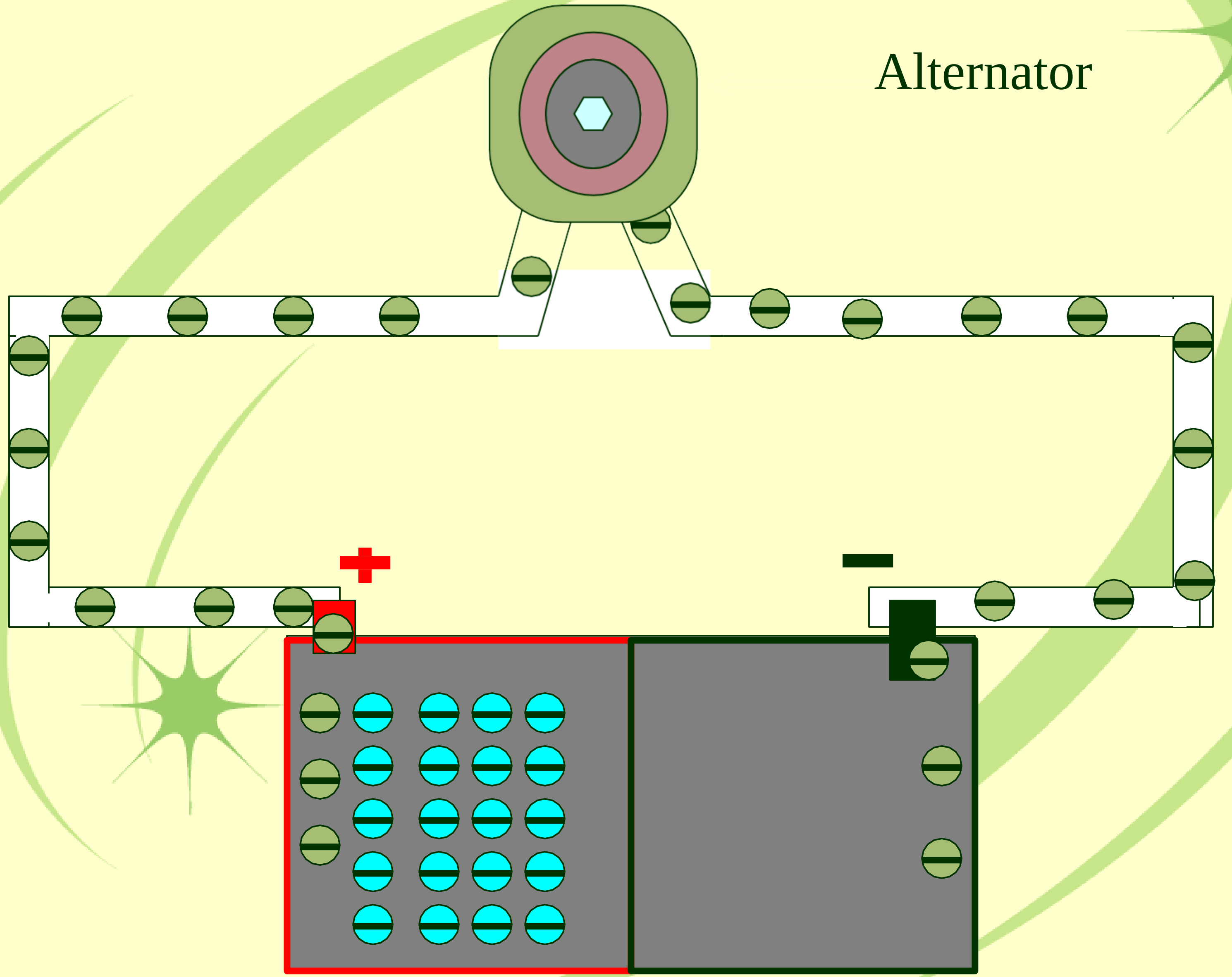
Basics Concept of Battery Discharging And Charging



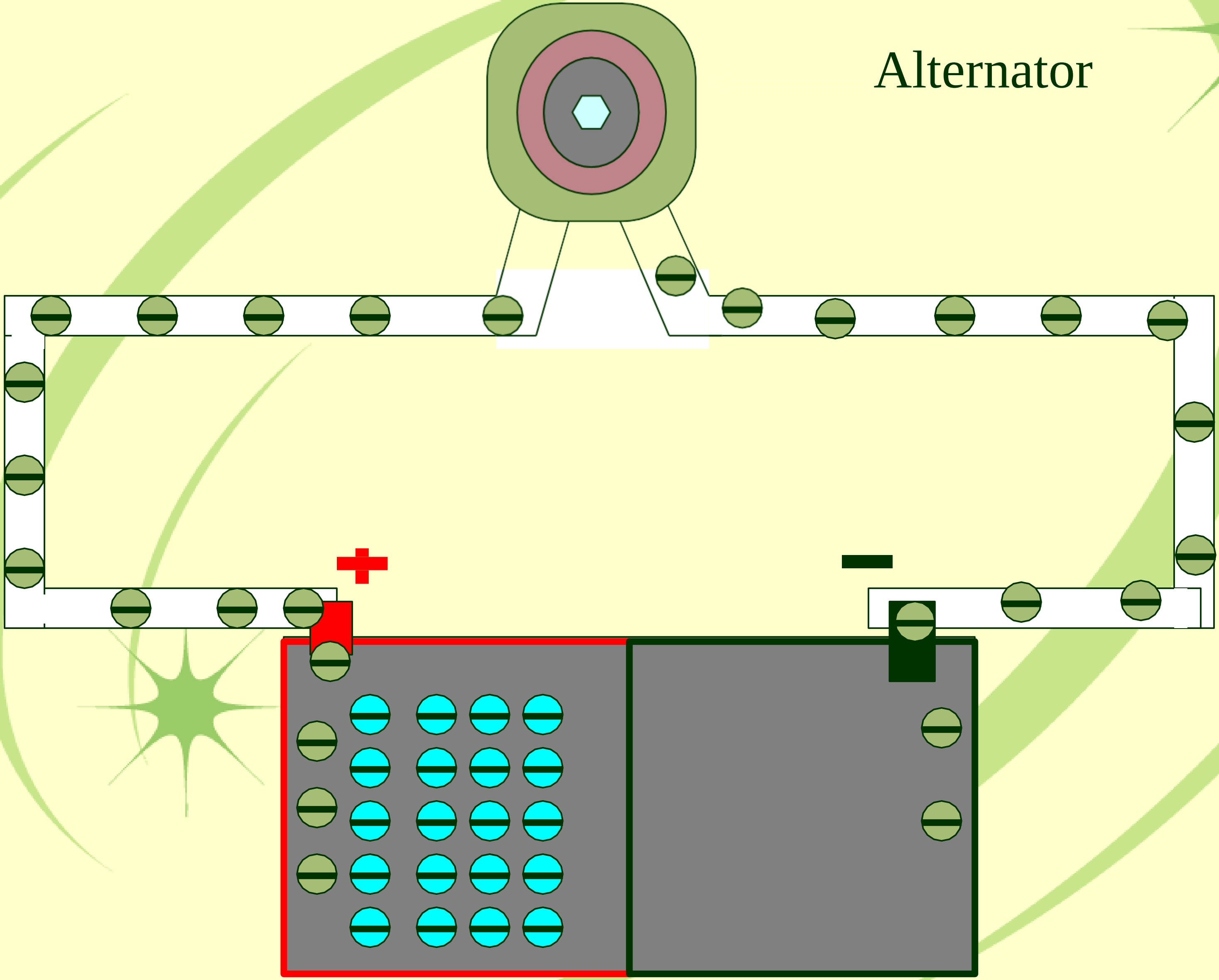
Basics Concept of Battery Discharging And Charging



Basics Concept of Battery Discharging And Charging

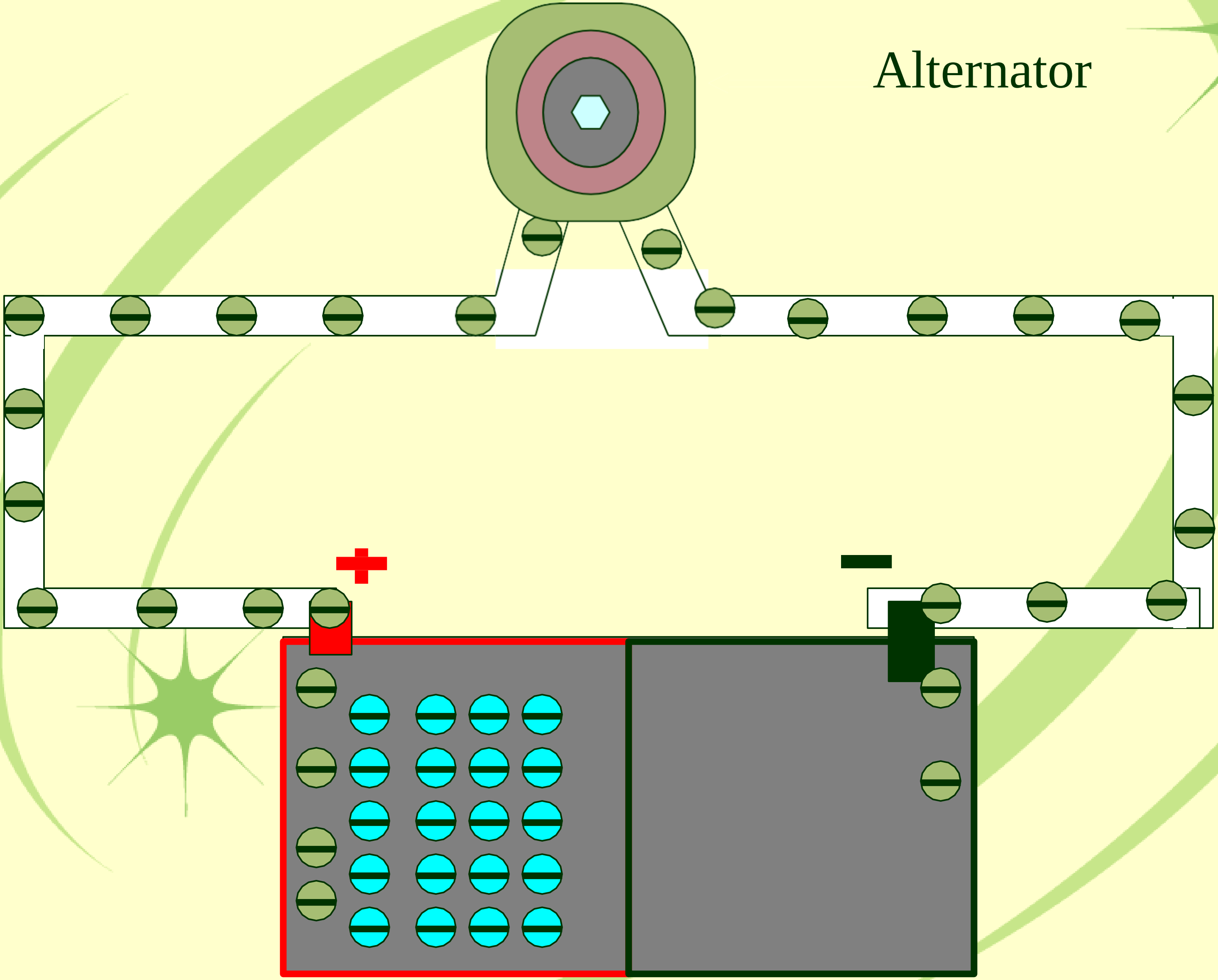


Basics Concept of Battery Discharging And Charging

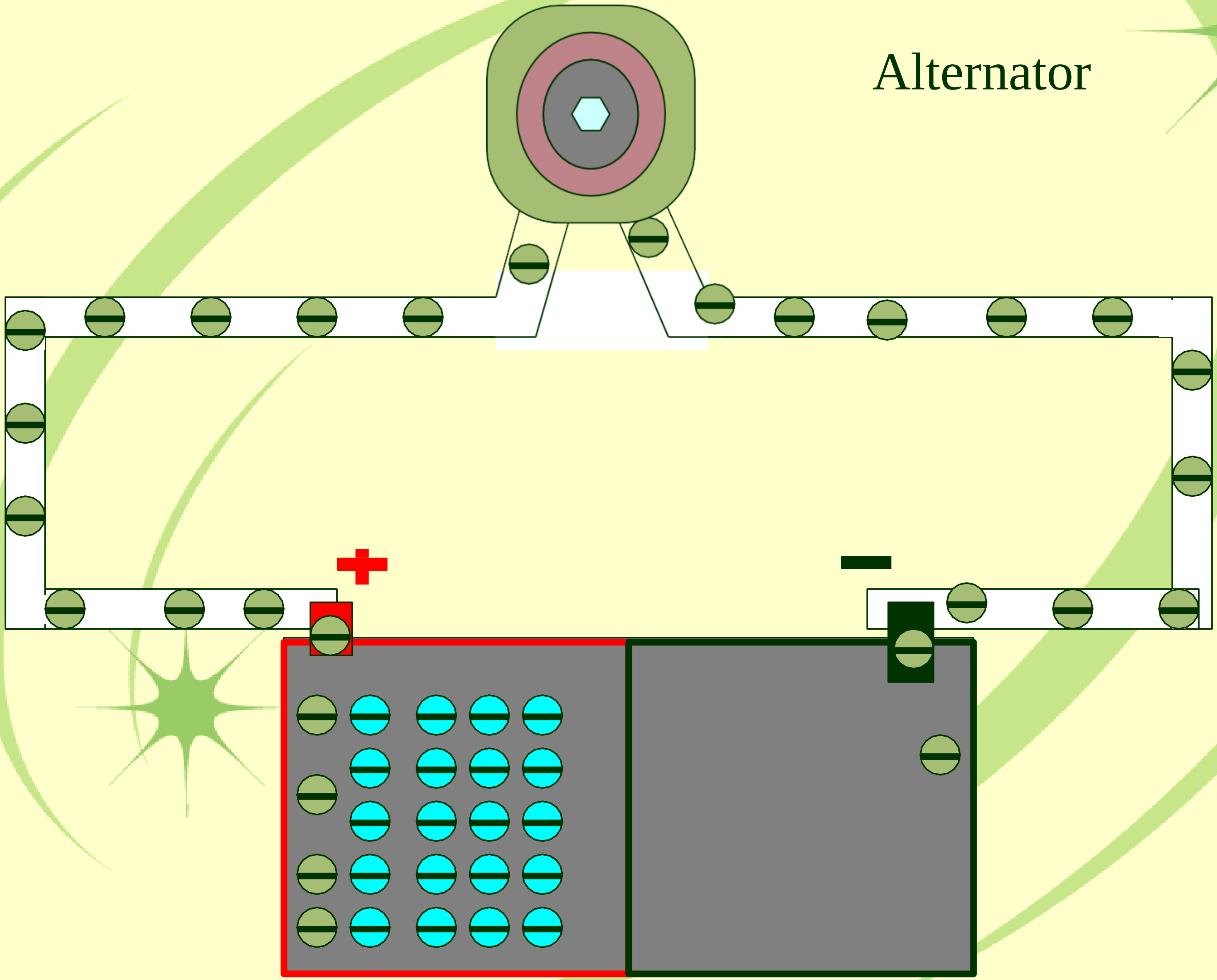


Basics Concept of Battery Discharging And Charging

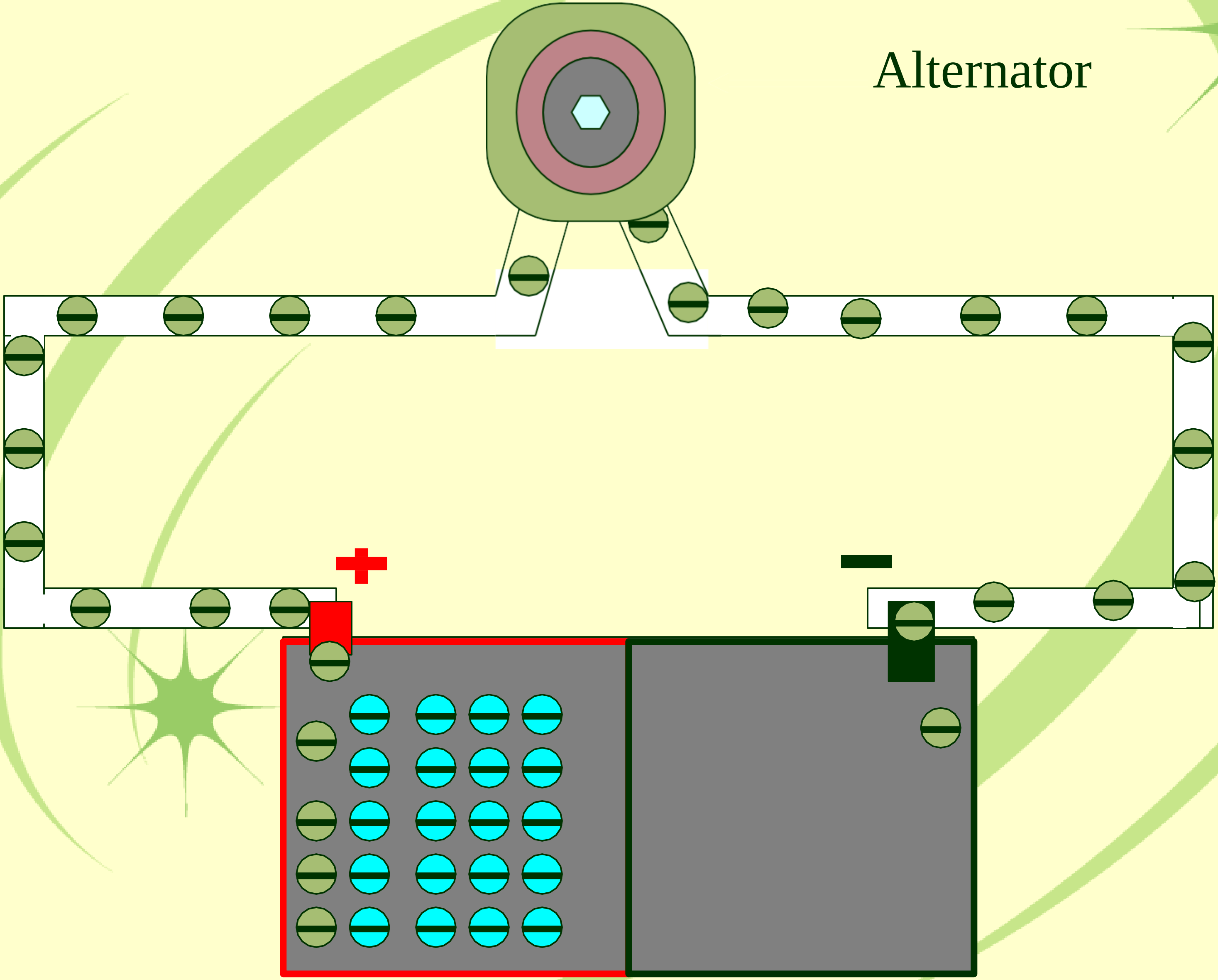
Alternator



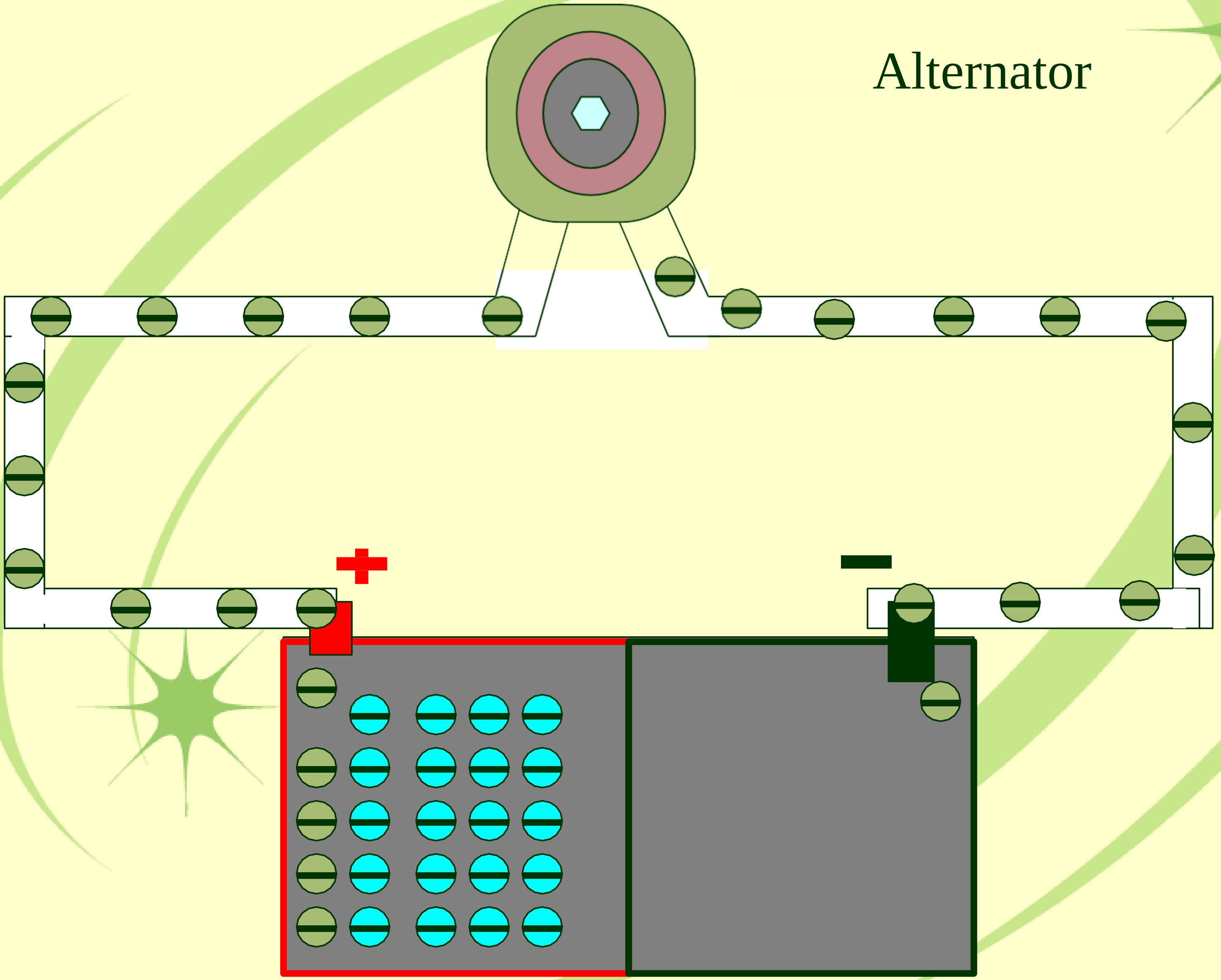
Basics Concept of Battery Discharging And Charging



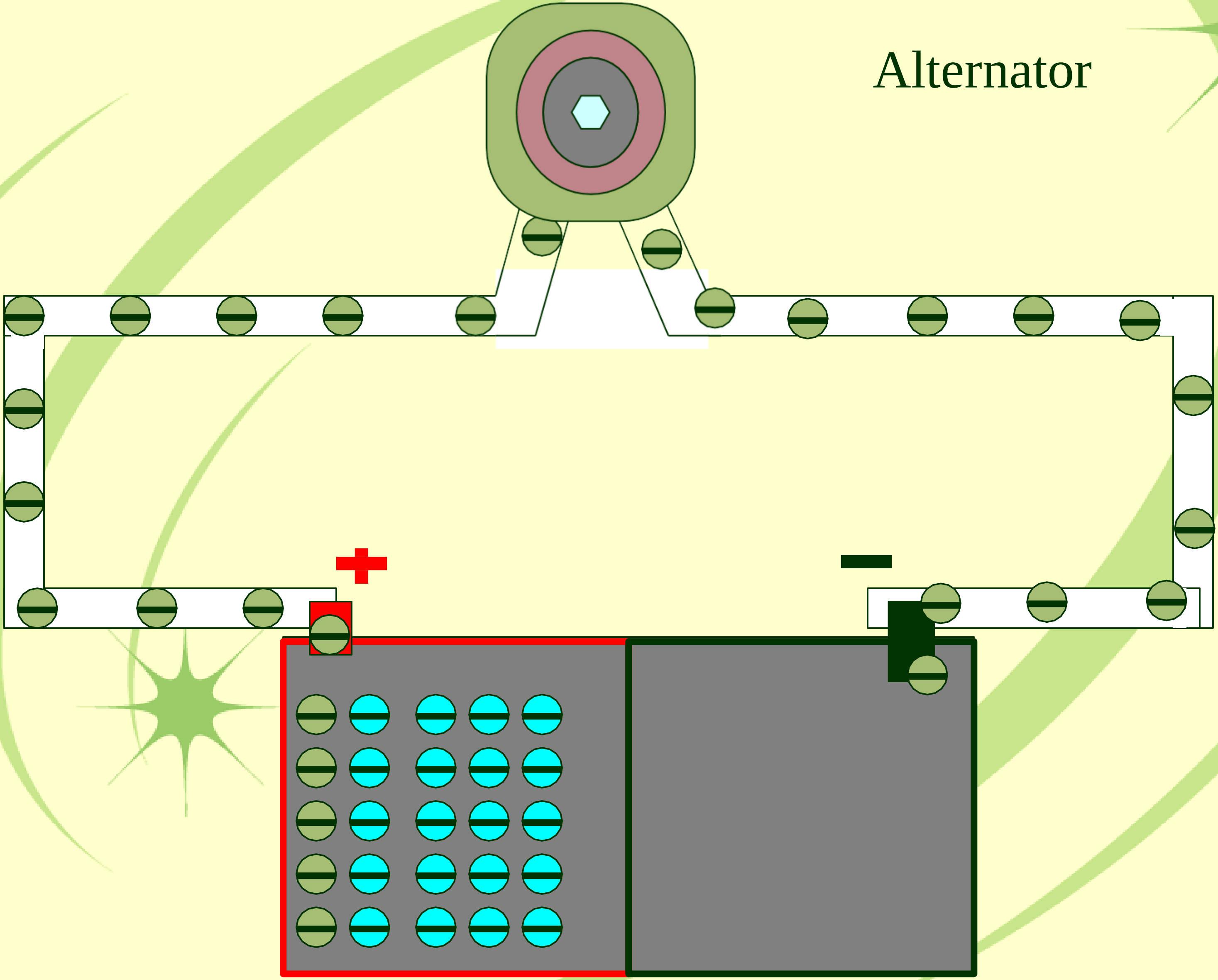
Basics Concept of Battery Discharging And Charging



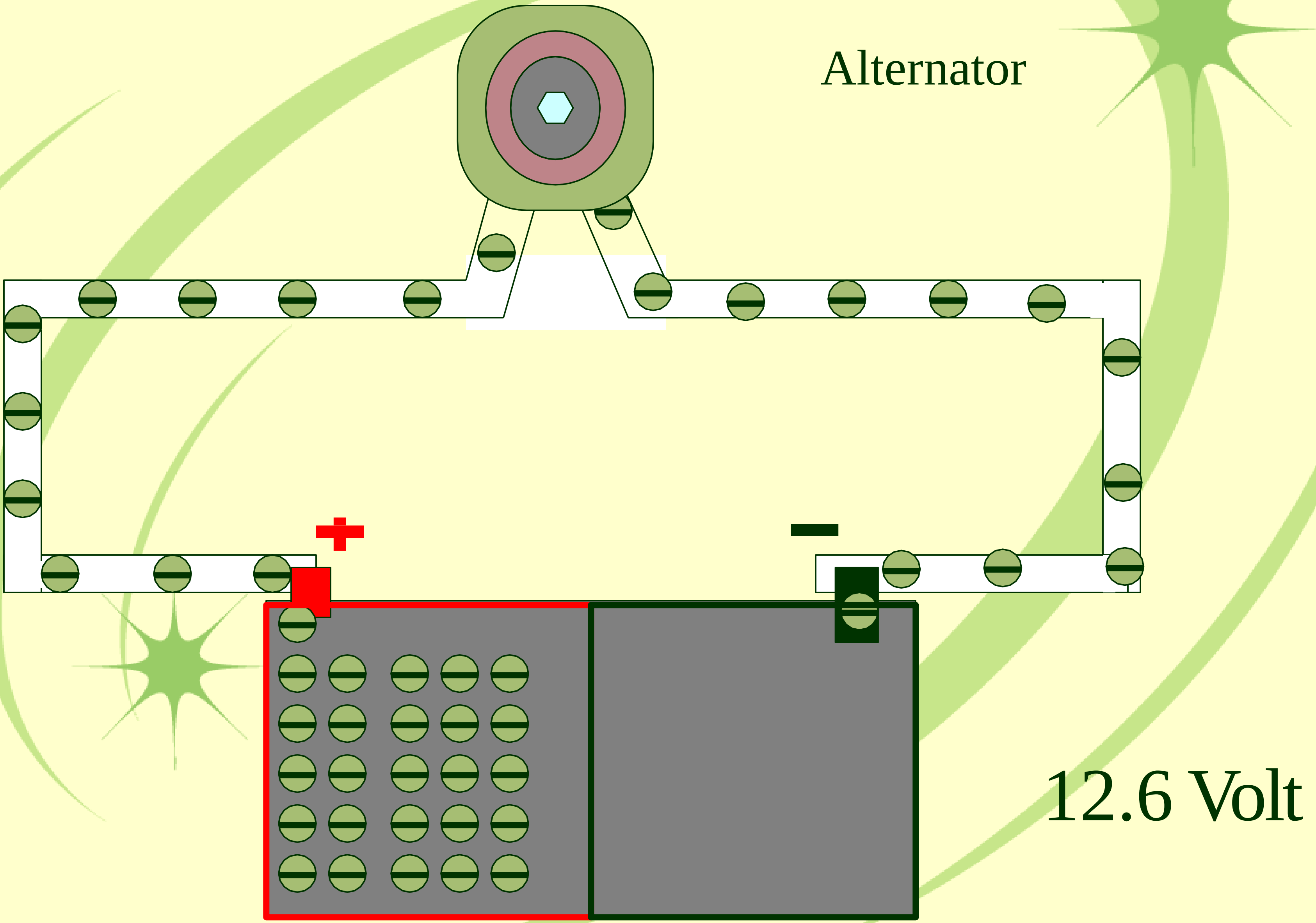
Basics Concept of Battery Discharging And Charging



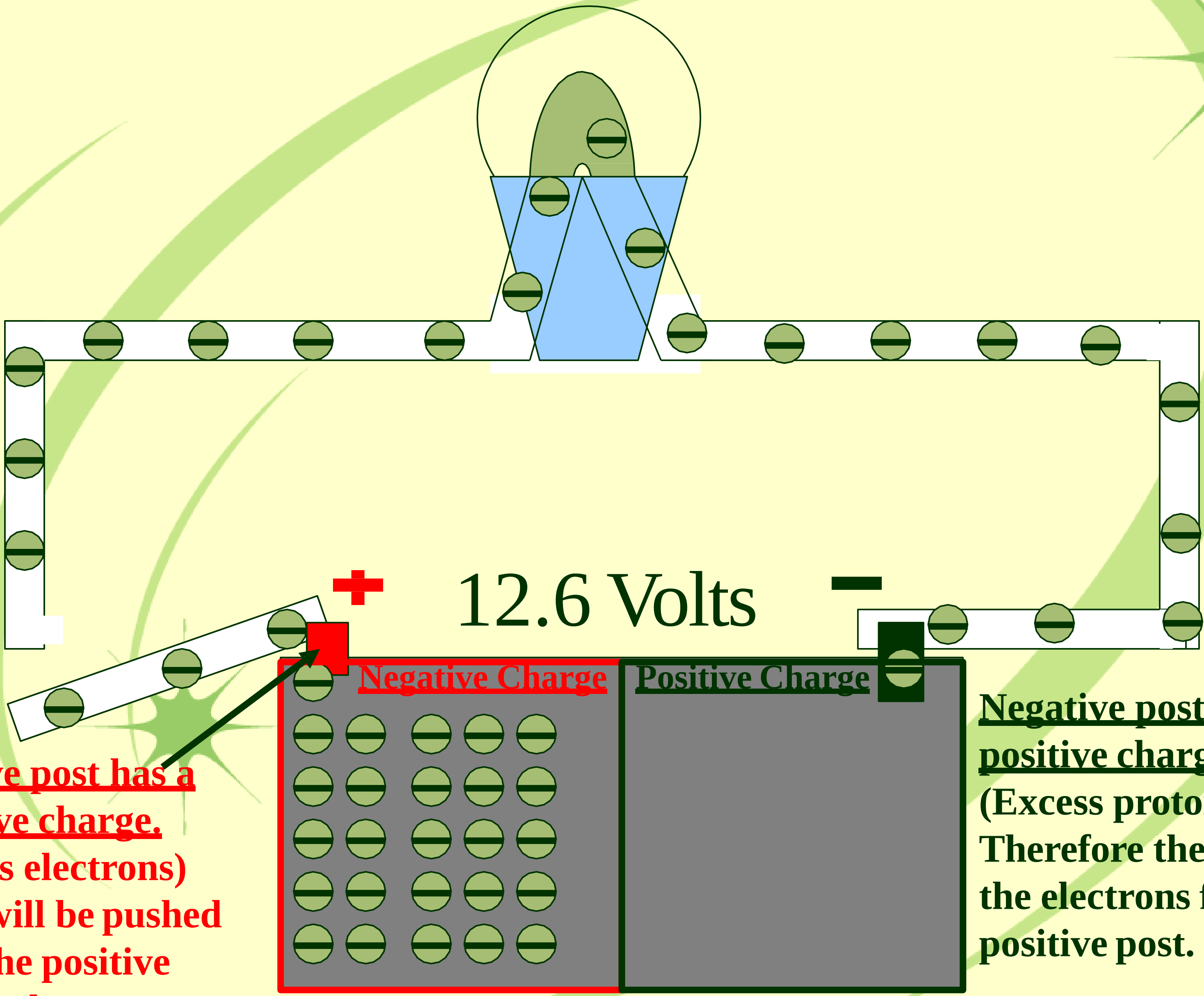
Basics Concept of Battery Discharging And Charging



Basics Concept of Battery Discharging And Charging



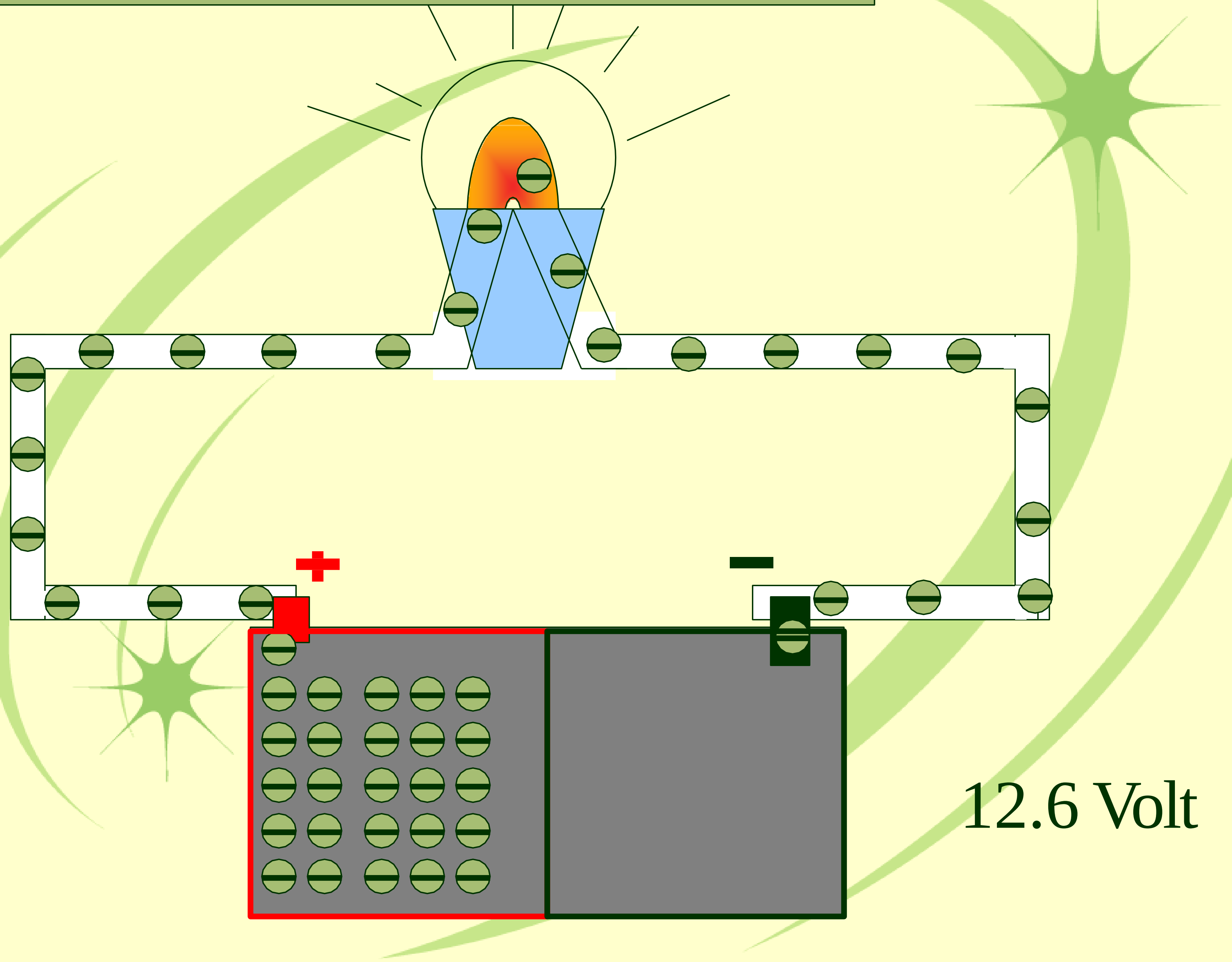
Basics Concept of Battery Discharging And Charging



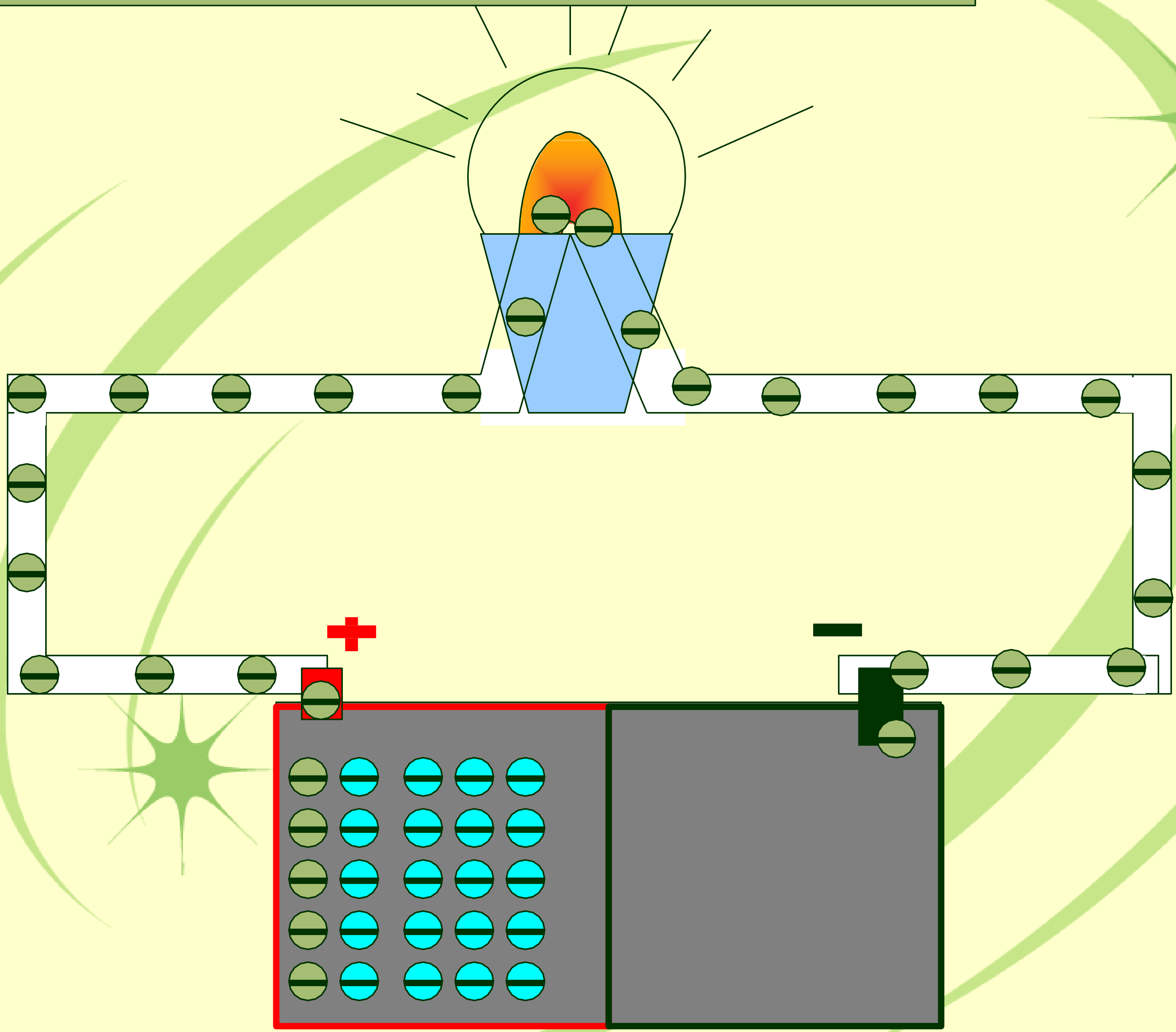
<u>**Positive post has a negative charge.**</u>
(Excess electrons)
They will be pushed from the positive Terminal.

<u>**Negative post has a positive charge.**</u>
(Excess protons)
Therefore they will pull the electrons from the positive post.

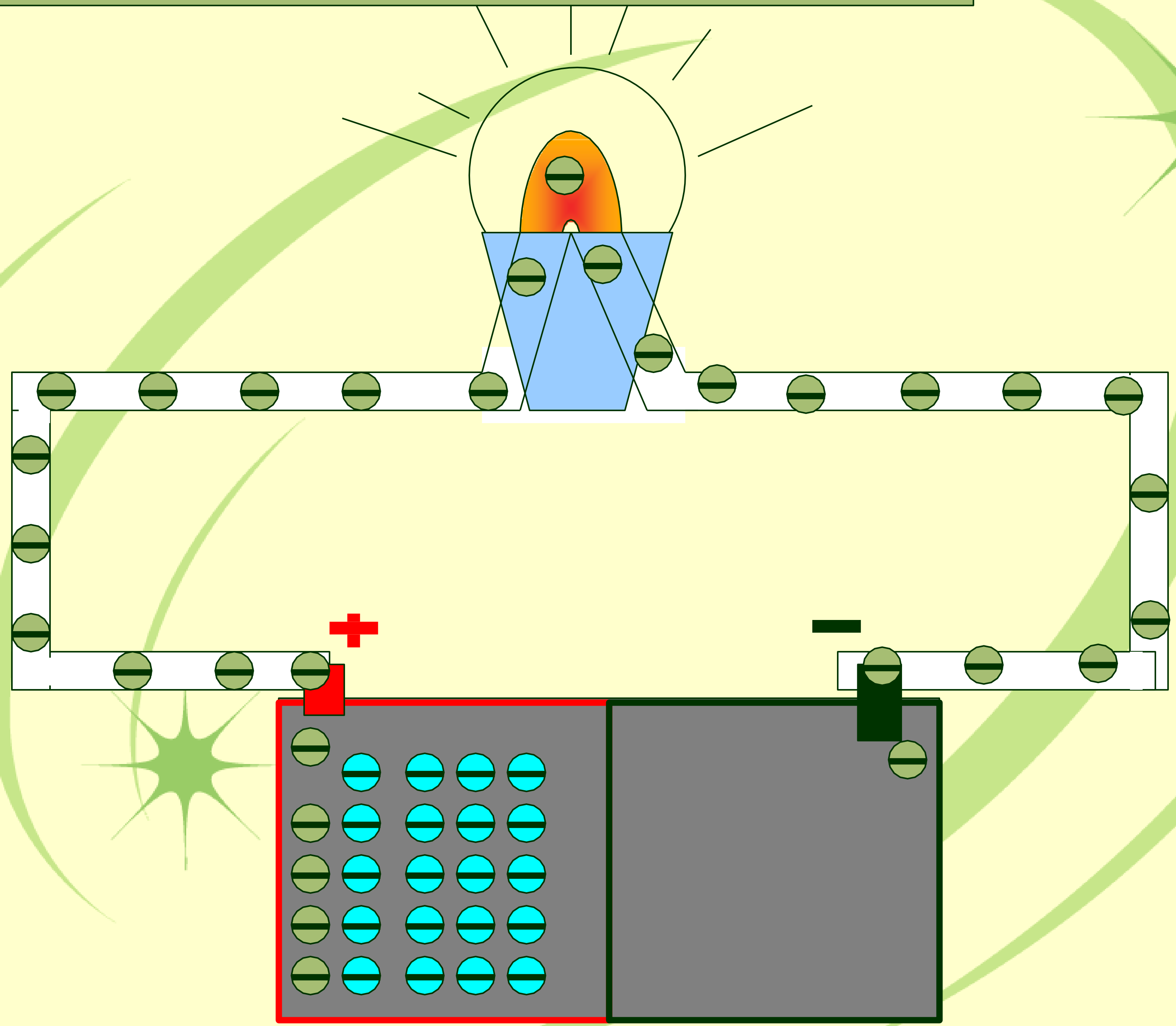
Basics Concept of Battery Discharging And Charging



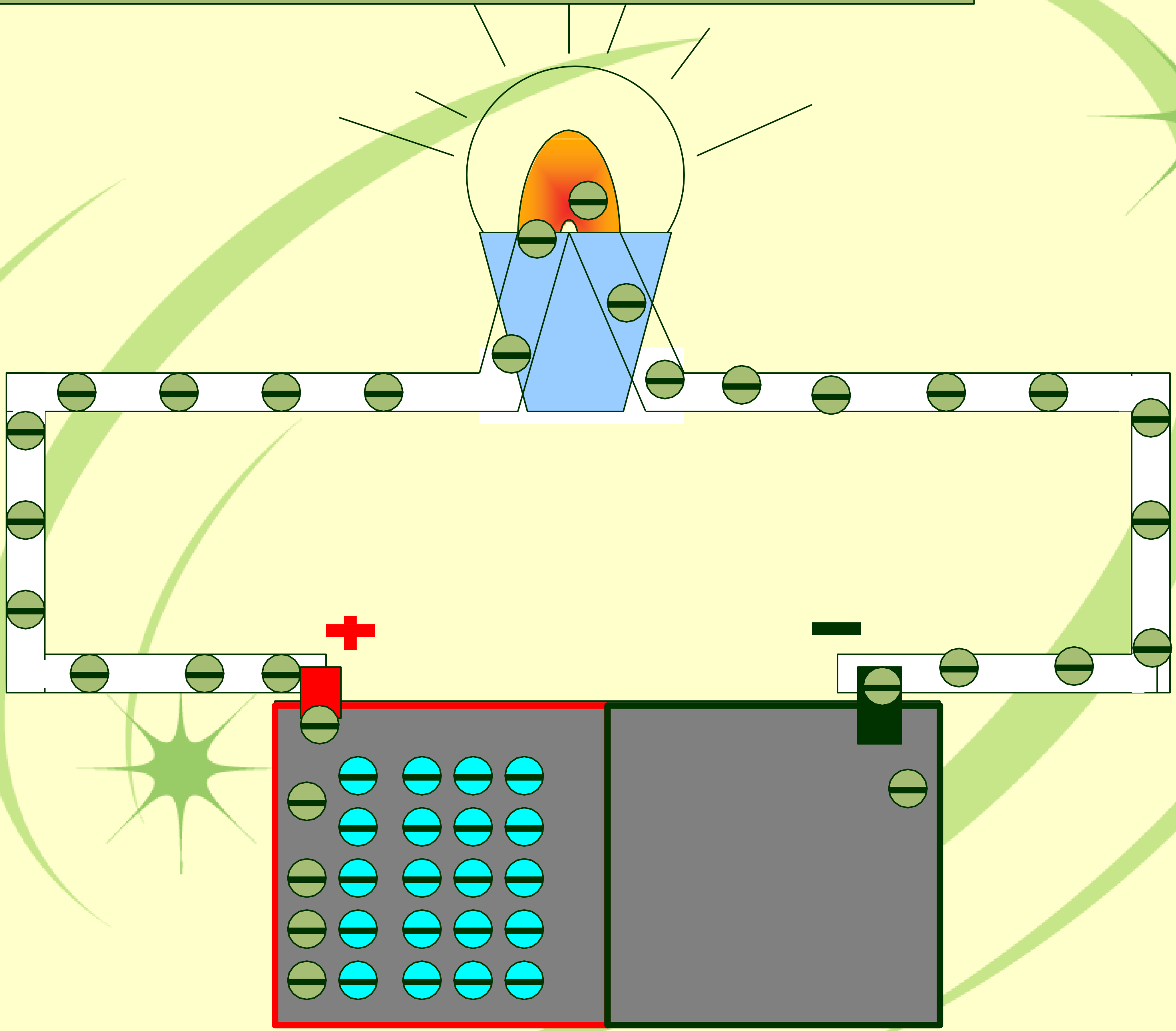
Basics Concept of Battery Discharging And Charging



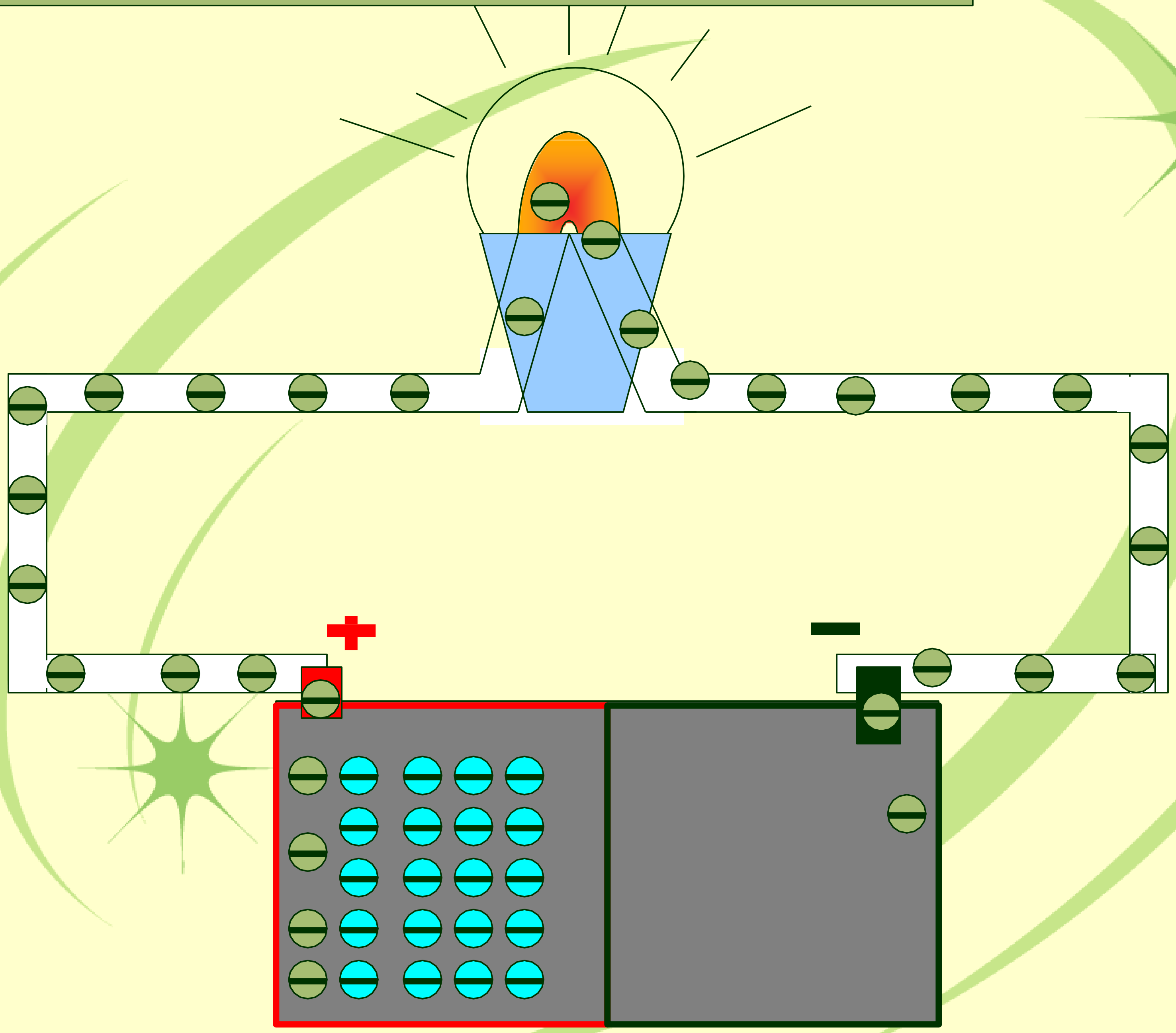
Basics Concept of Battery Discharging And Charging



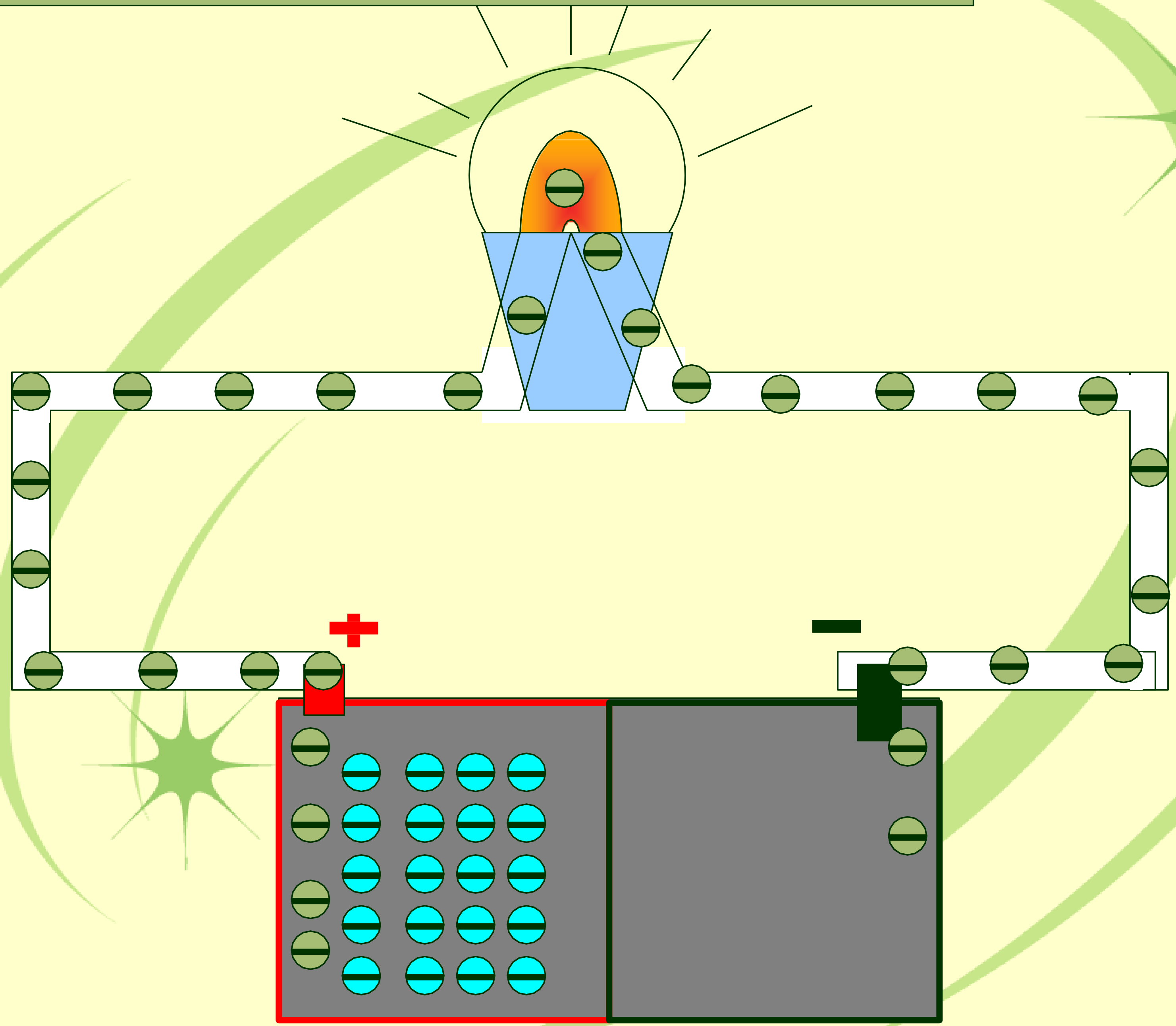
Basics Concept of Battery Discharging And Charging



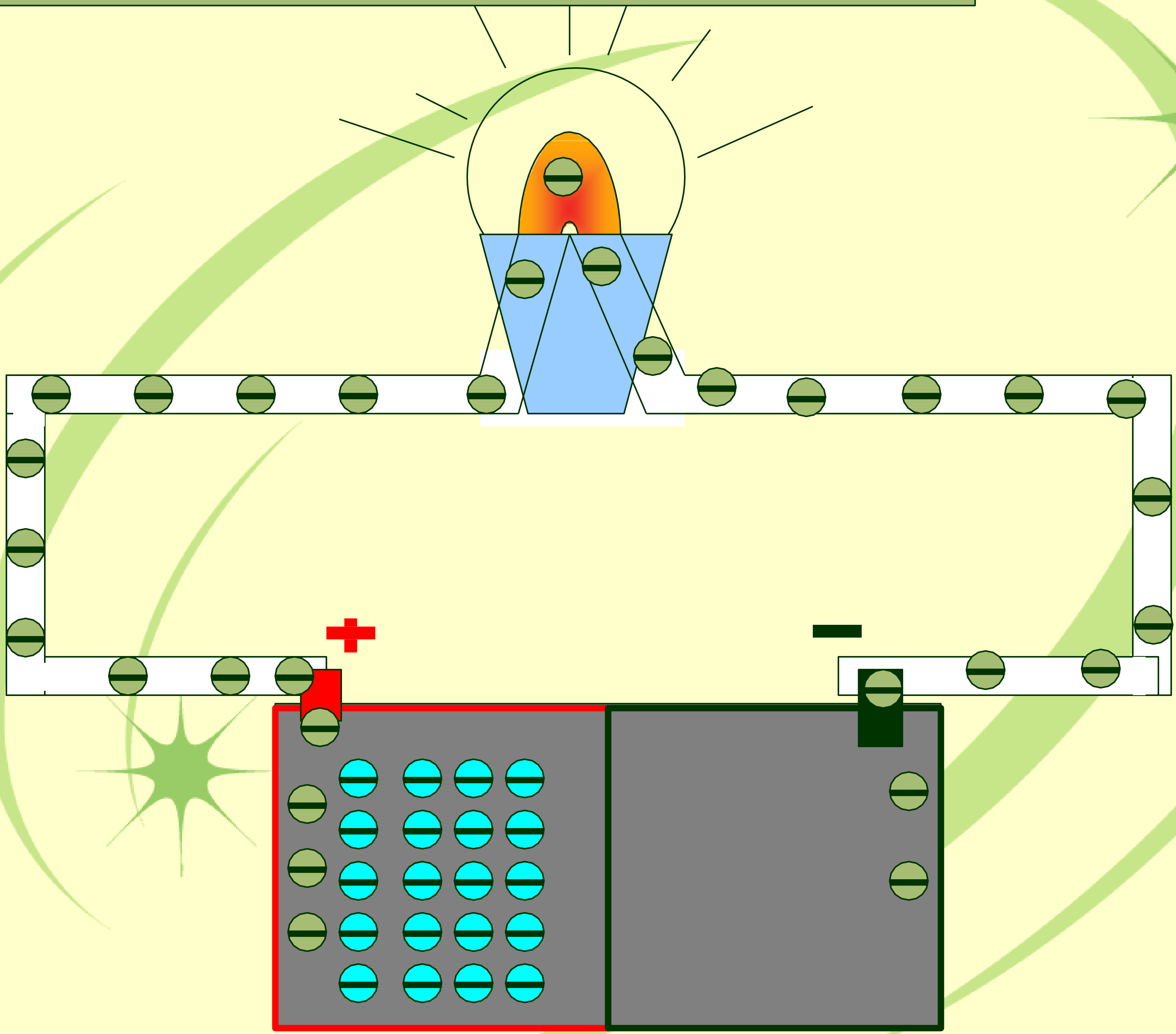
Basics Concept of Battery Discharging And Charging



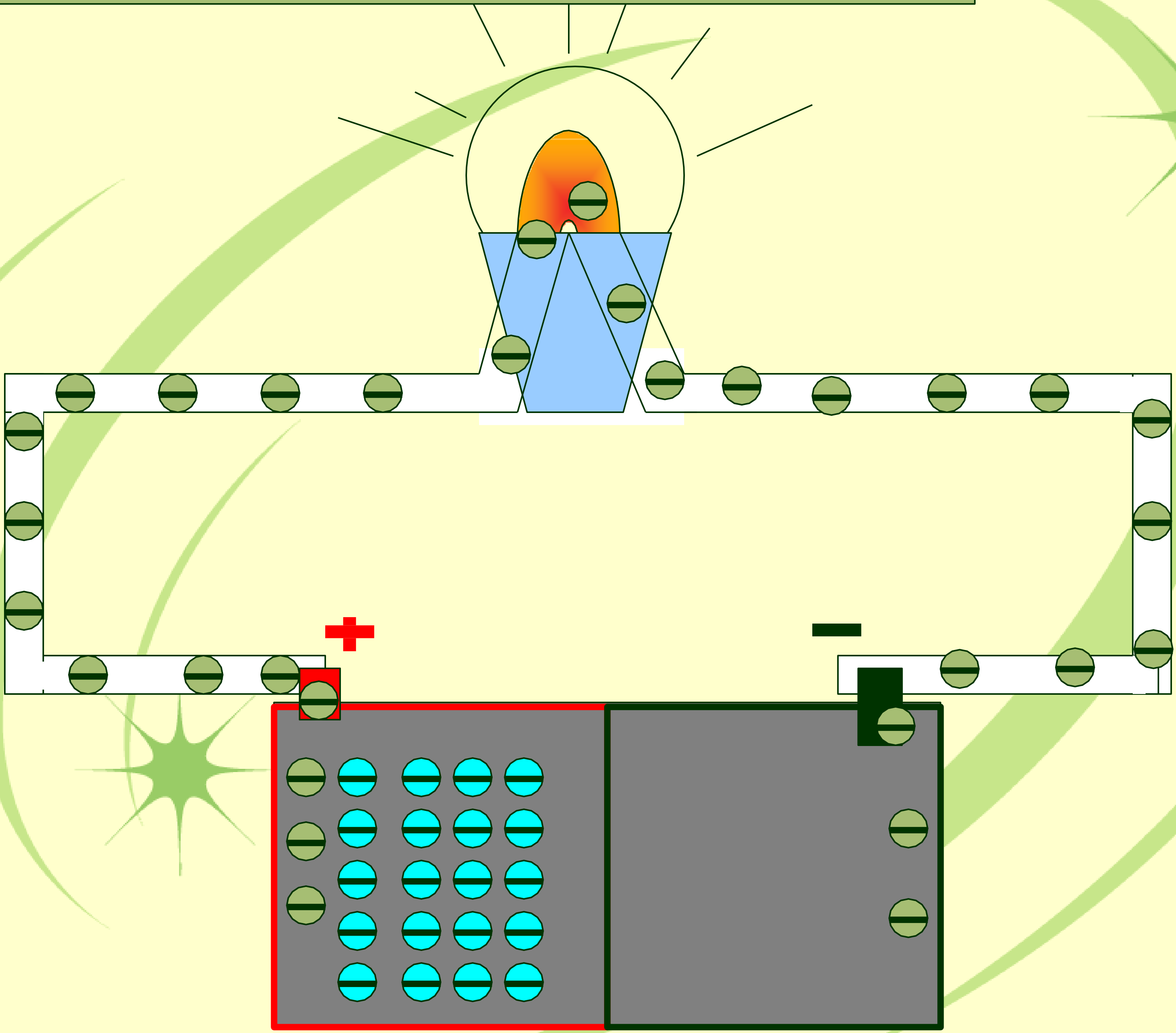
Basics Concept of Battery Discharging And Charging



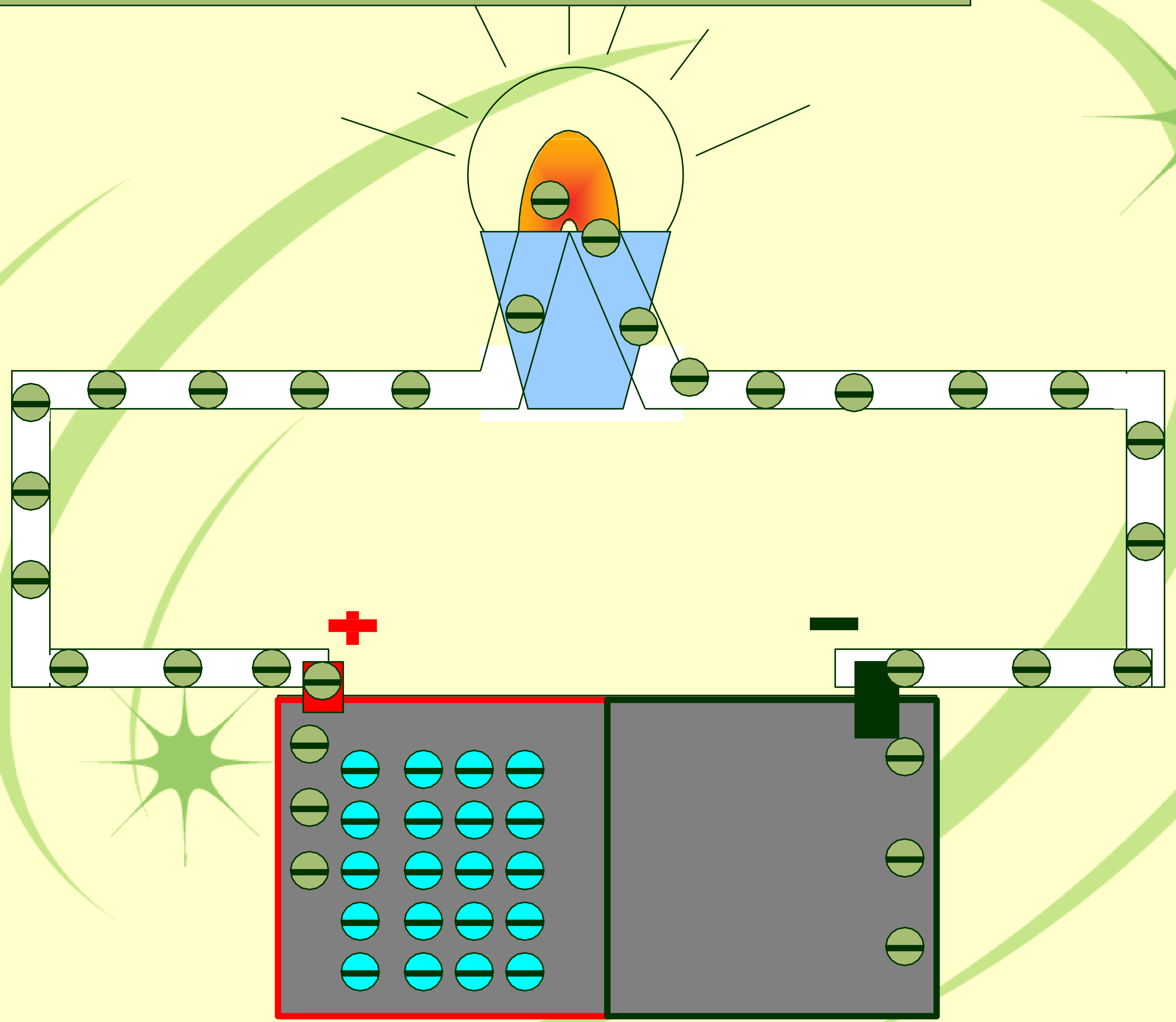
Basics Concept of Battery Discharging And Charging



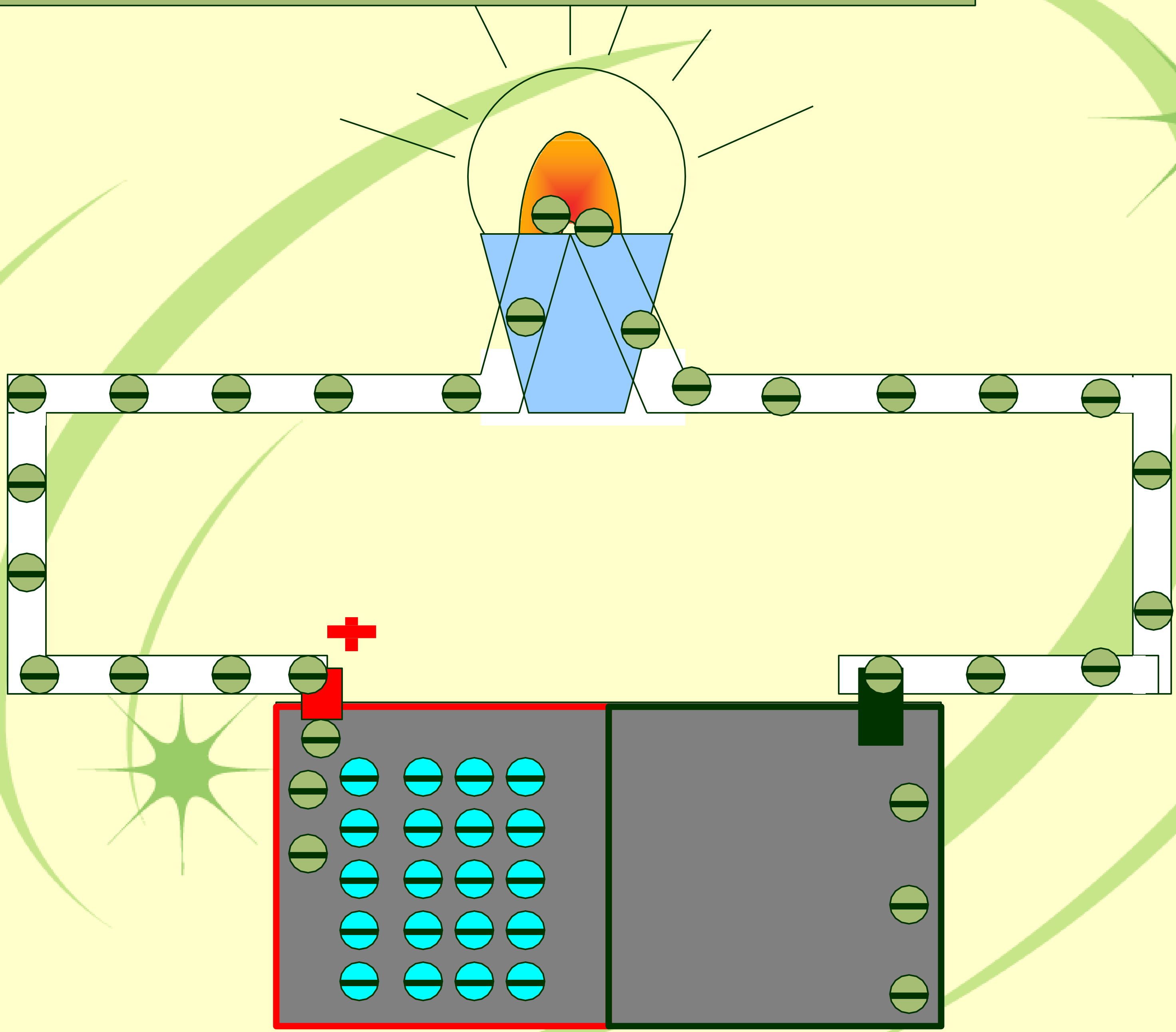
Basics Concept of Battery Discharging And Charging



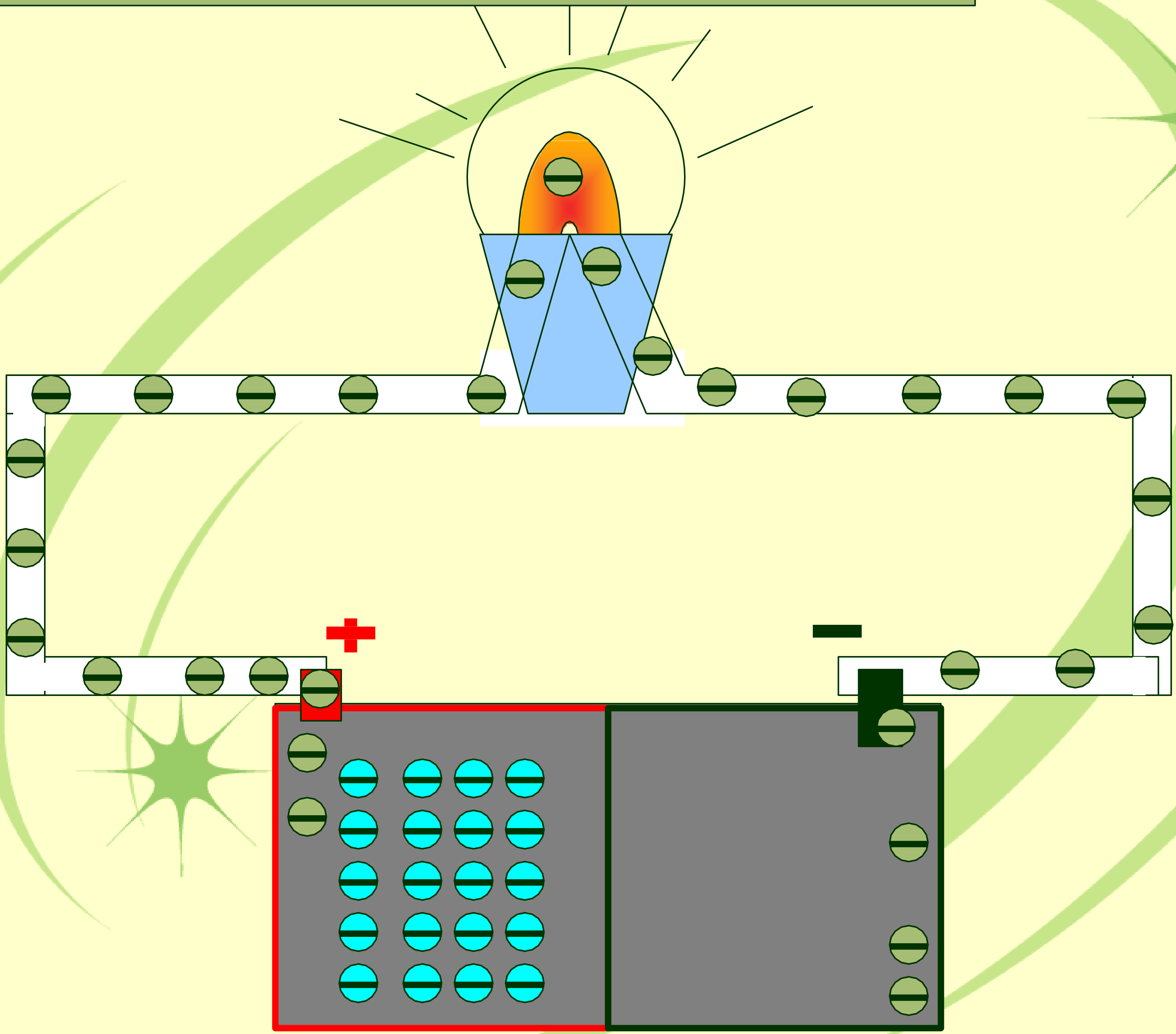
Basics Concept of Battery Discharging And Charging



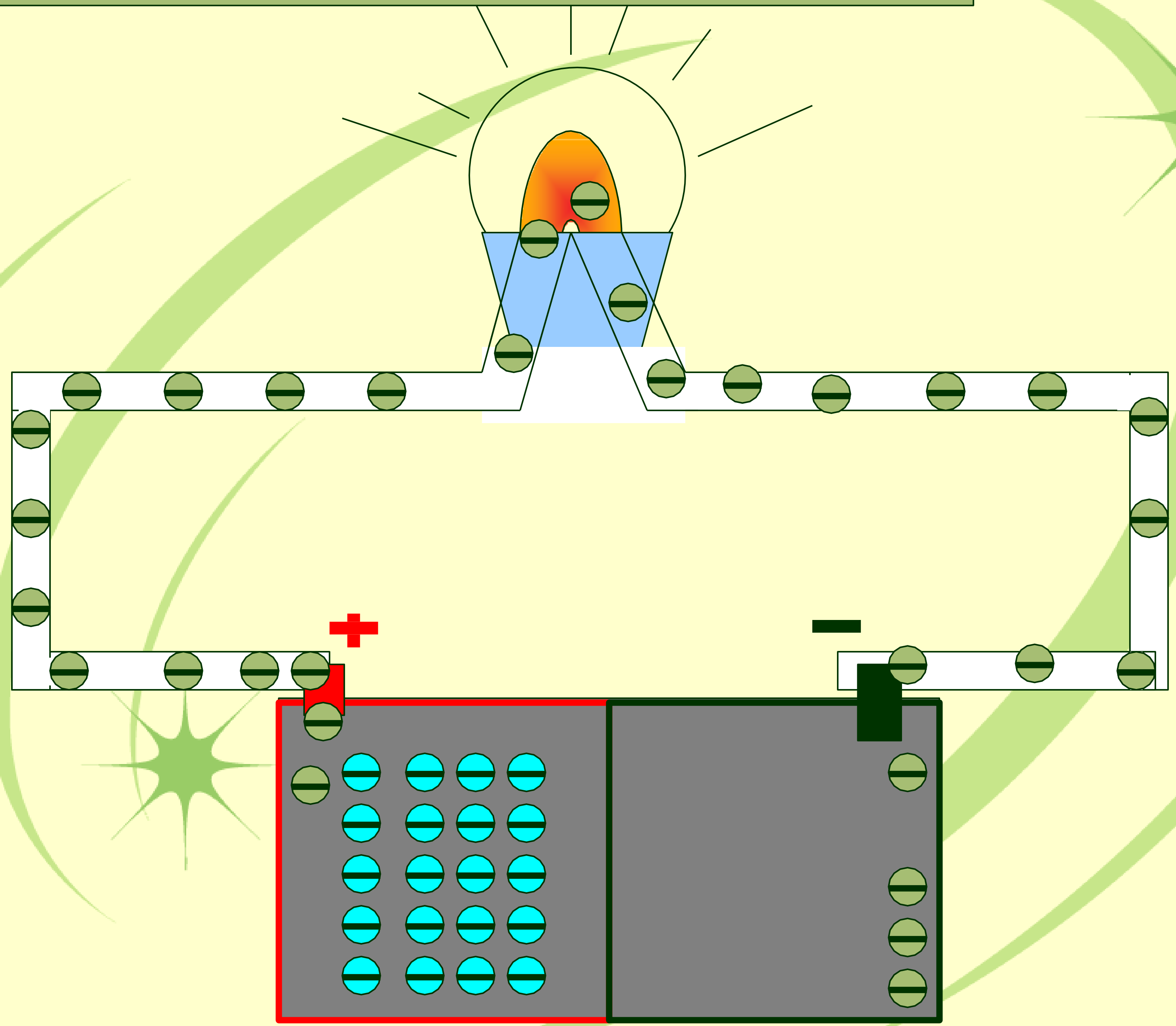
Basics Concept of Battery Discharging And Charging



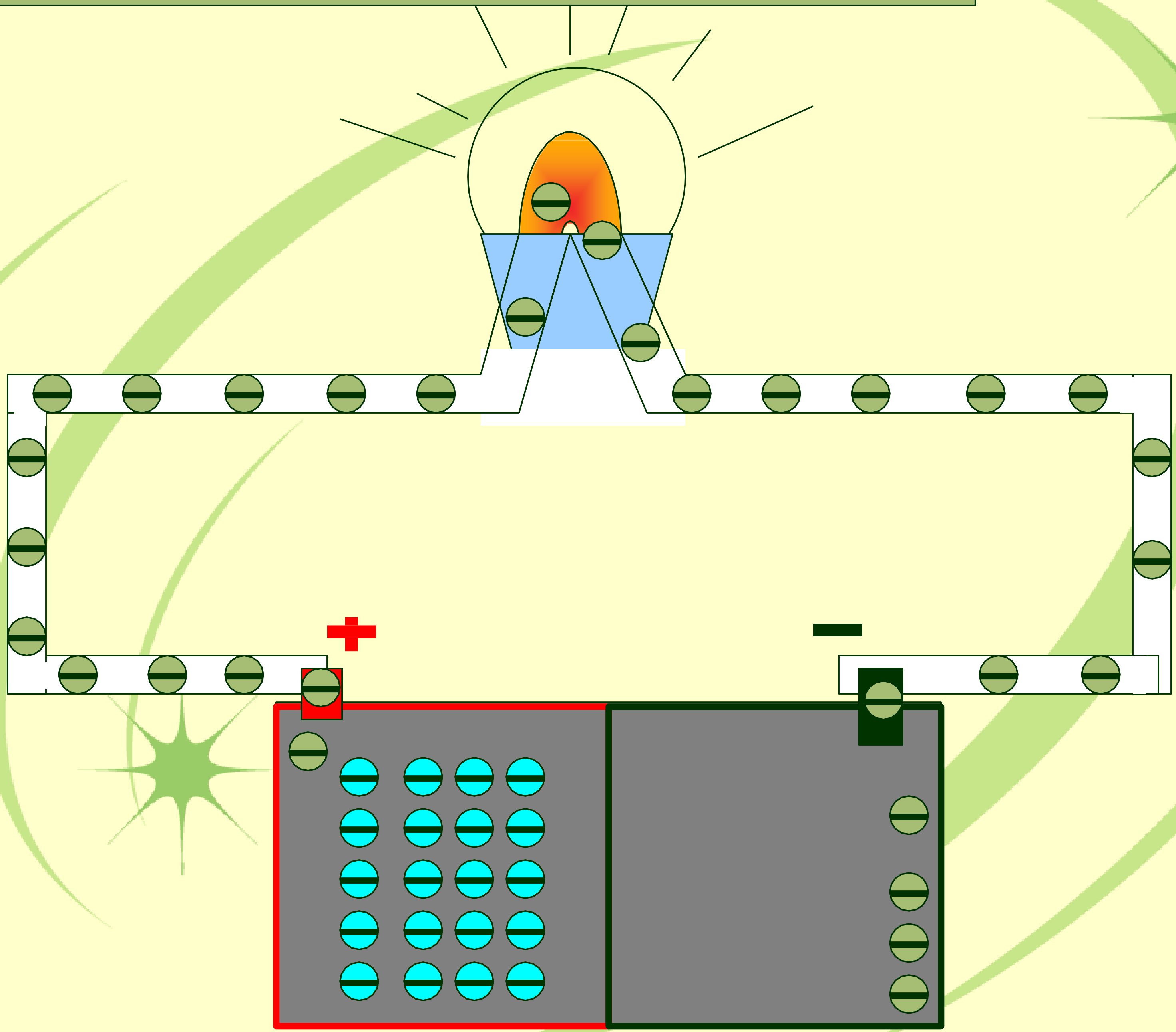
Basics Concept of Battery Discharging And Charging



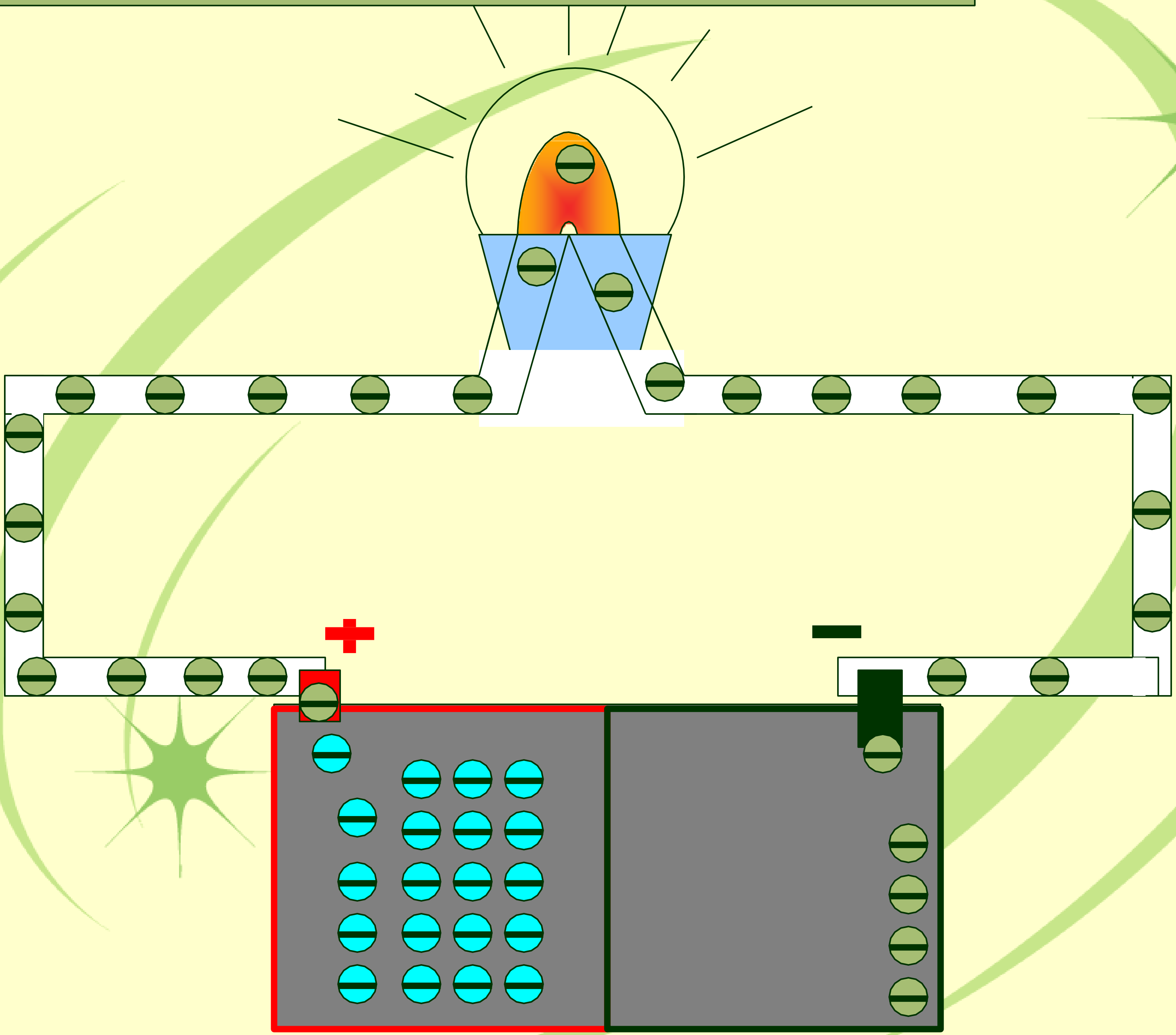
Basics Concept of Battery Discharging And Charging



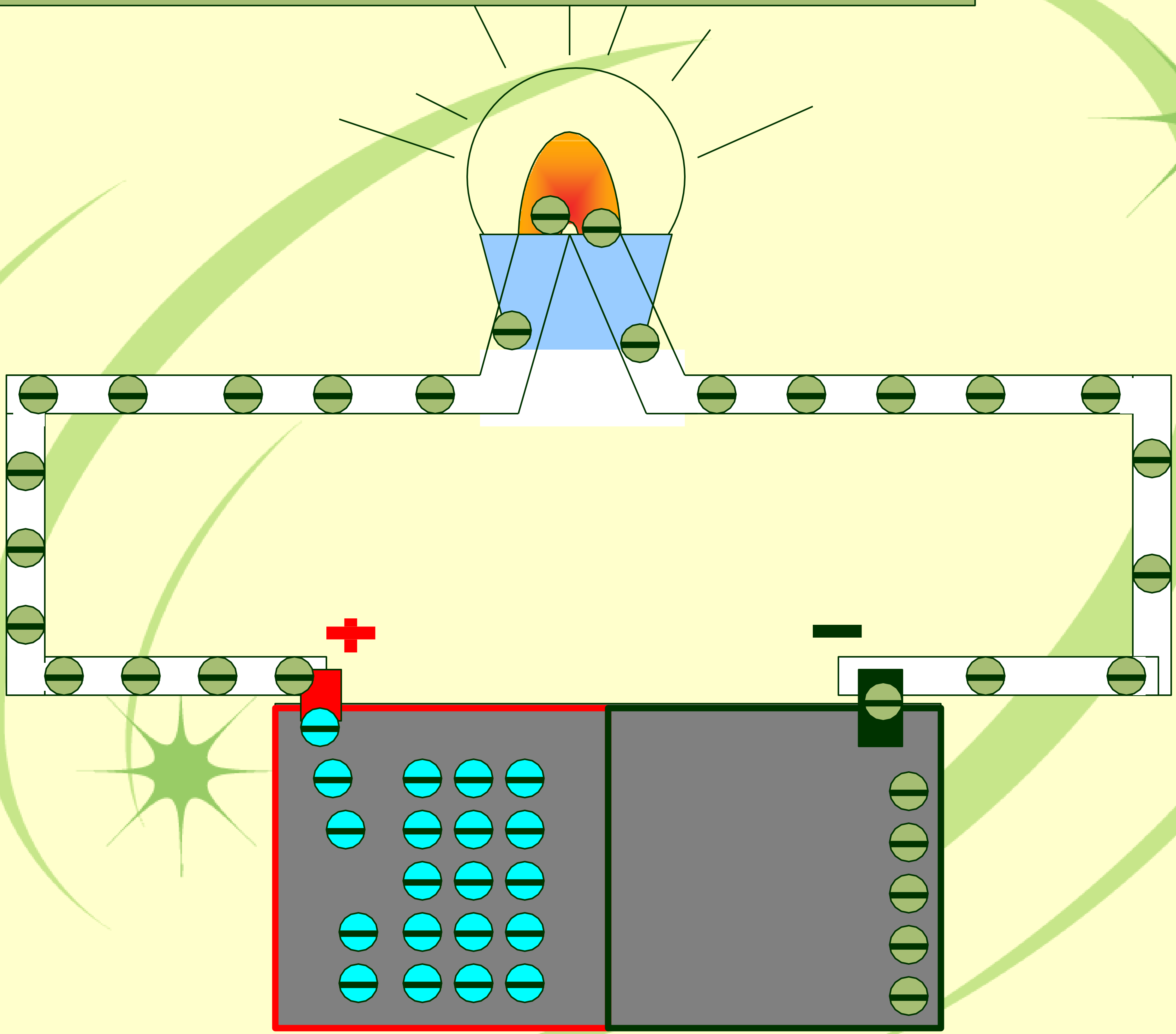
Basics Concept of Battery Discharging And Charging



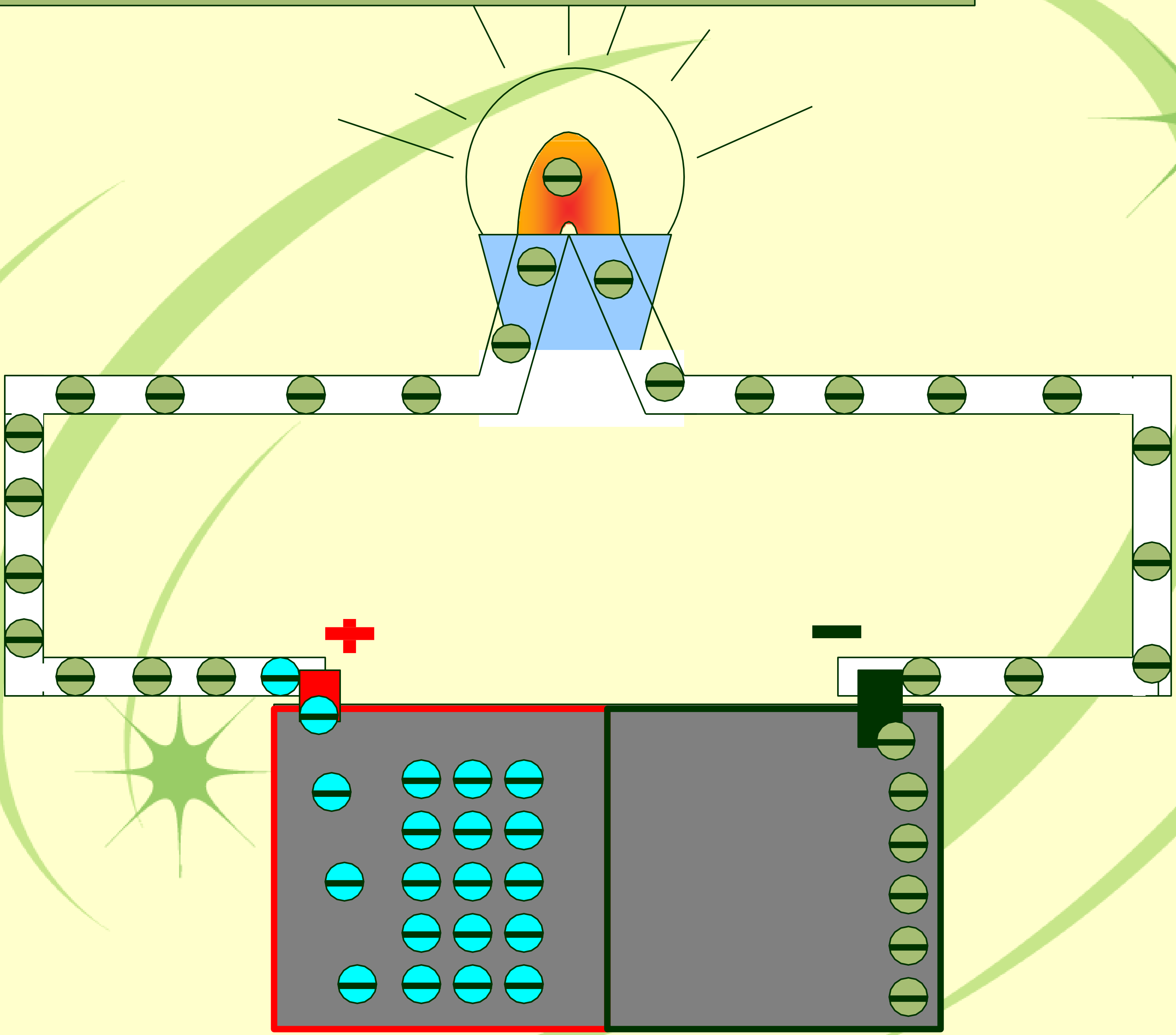
Basics Concept of Battery Discharging And Charging



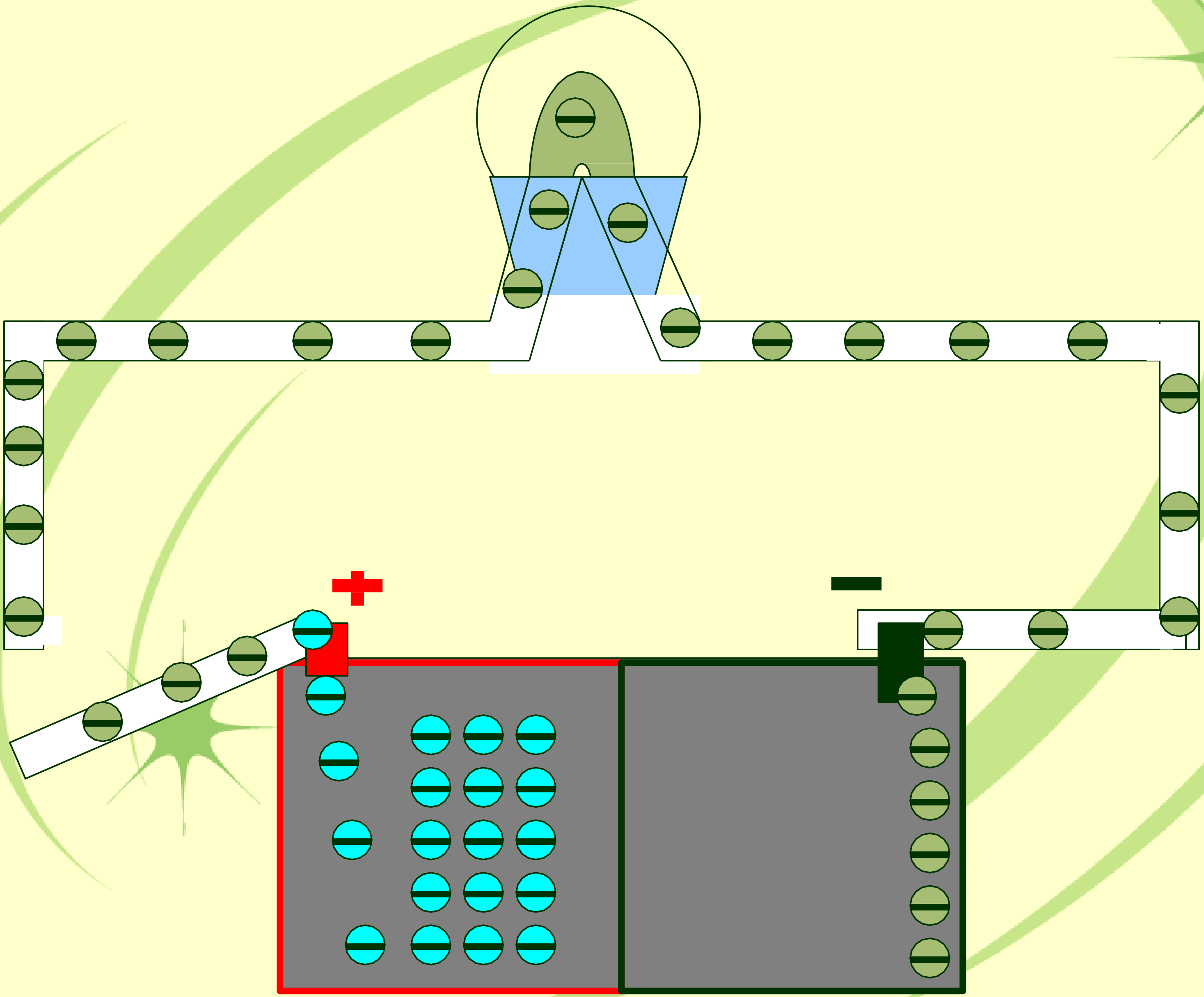
Basics Concept of Battery Discharging And Charging



Basics Concept of Battery Discharging And Charging



Basics Concept of Battery Discharging And Charging



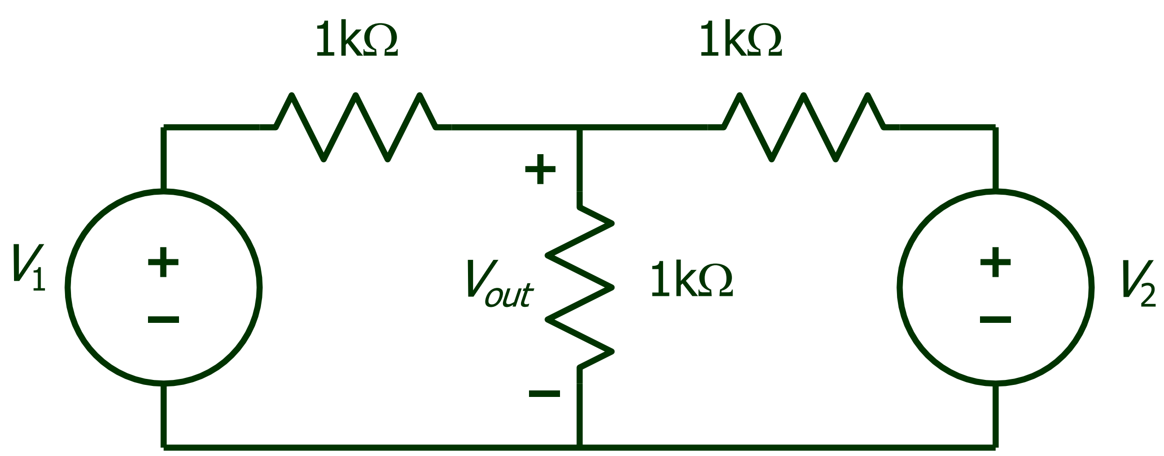
Loop (Mesh) Analysis

Loop Analysis

- Nodal analysis was developed by applying KCL at each non-reference node.
- Loop analysis is developed by applying KVL around loops in the circuit.
- Loop (mesh) analysis results in a system of linear equations which must be solved for unknown currents.

Another Summing Circuit

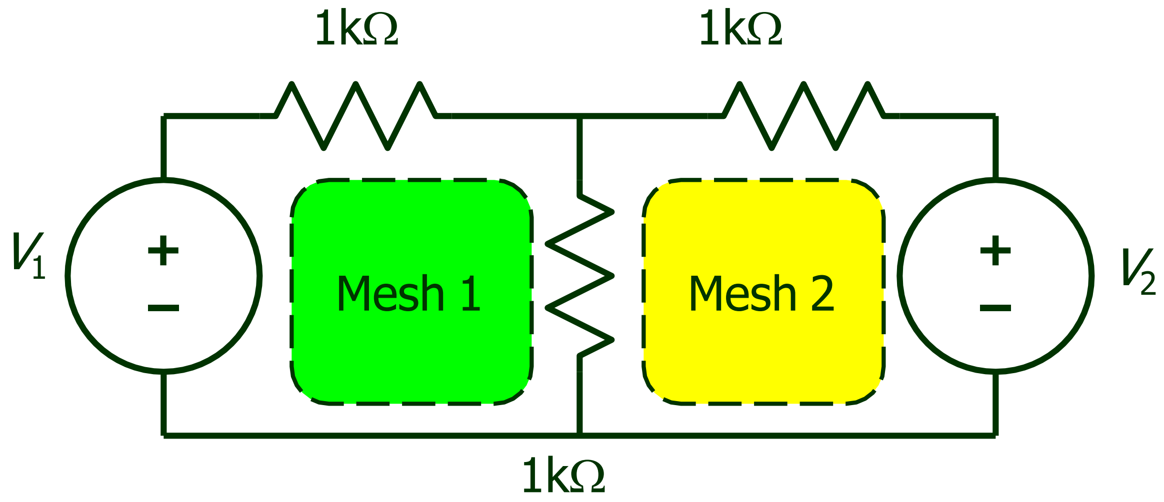
- The output voltage V of this circuit is proportional to the sum of the two input voltages V_1 and V_2.



Steps of Mesh Analysis

1. **Identify mesh (loops).**
2. Assign a current to each mesh.
3. Apply KVL around each loop to get an equation in terms of the loop currents.
4. Solve the resulting system of linear equations.

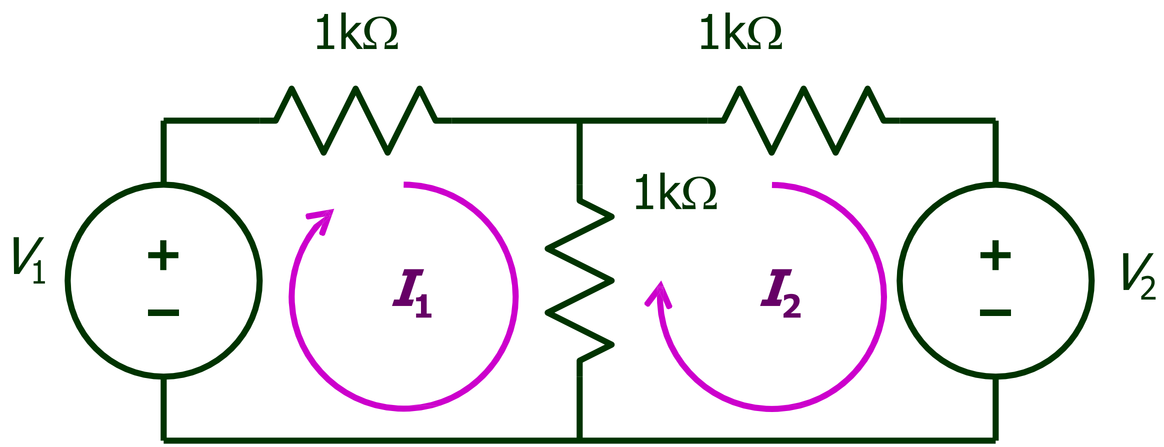
1. Identifying the Meshes



Steps of Mesh Analysis

1. Identify mesh (loops).
2. **Assign a current to each mesh.**
3. Apply KVL around each loop to get an equation in terms of the loop currents.
4. Solve the resulting system of linear equations.

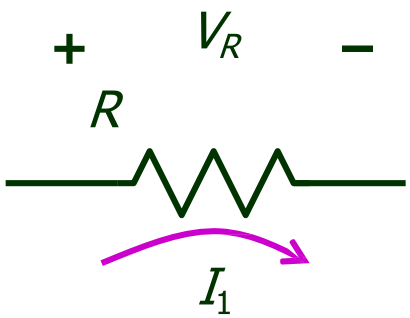
2. Assigning Mesh Currents

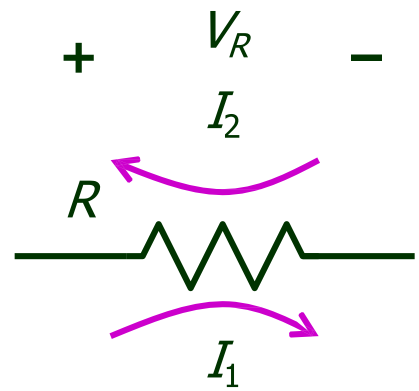


Steps of Mesh Analysis

1. Identify mesh (loops).
2. Assign a current to each mesh.
3. **Apply KVL around each loop to get an equation in terms of the loop currents.**
4. Solve the resulting system of linear equations.

Voltages from Mesh Currents

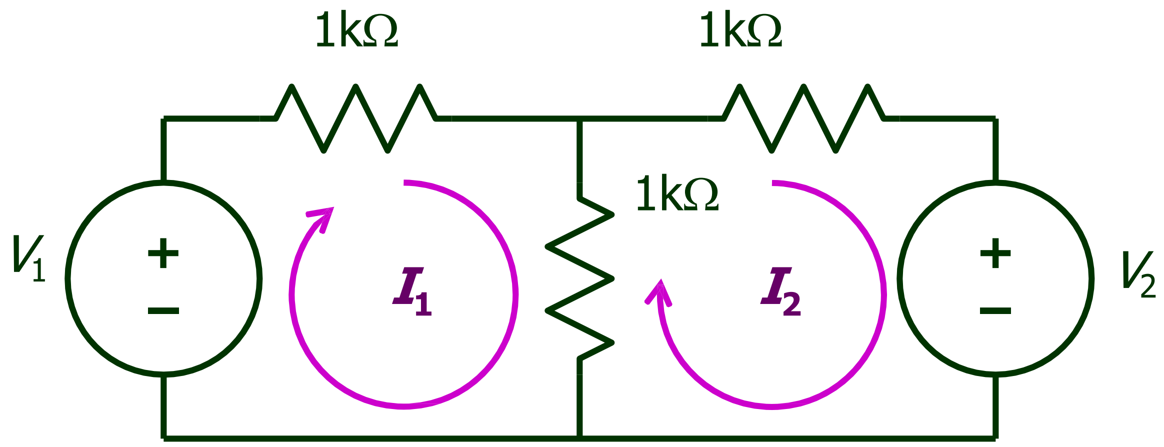




$$V_R = I_1 R$$

$$V_R = (I_1 - I_2) R$$

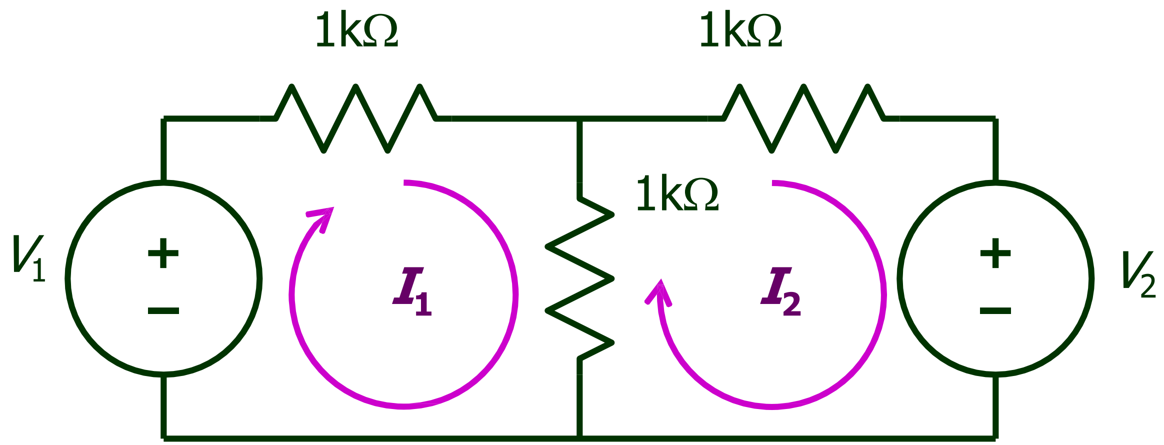
3. KVL Around Mesh 1



$$-V_1 + I_1\ 1\text{k}\Omega + (I_1 - I_2)\ 1\text{k}\Omega = 0$$

$$I_1\ 1\text{k}\Omega + (I_1 - I_2)\ 1\text{k}\Omega = V_1$$

3. KVL Around Mesh 2



$$(I_2 - I_1)\,1\text{k}\Omega + I_2\,1\text{k}\Omega + V_2 = 0$$

$$(I_2 - I_1)\,1\text{k}\Omega + I_2\,1\text{k}\Omega = -V_2$$

Steps of Mesh Analysis

1. Identify mesh (loops).
2. Assign a current to each mesh.
3. Apply KVL around each loop to get an equation in terms of the loop currents.
4. **Solve the resulting system of linear equations.**

Matrix Notation

- The two equations can be combined into a single matrix/vector equation.

$$\begin{bmatrix} 1\mathrm{k}\Omega + 1\mathrm{k}\Omega & -1\mathrm{k}\Omega \\ -1\mathrm{k}\Omega & 1\mathrm{k}\Omega + 1\mathrm{k}\Omega \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

Cramer's Rule

$$\begin{bmatrix} E_1 \\ -E_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \begin{bmatrix} (R_1 + R_2) & (-R_3) \\ (-R_3) & (R_2 + R_3) \end{bmatrix}$$

$$I_1 = \frac{\begin{bmatrix} E_1 & (-R_3) \\ -E_2 & (R_2 + R_3) \end{bmatrix}}{\begin{bmatrix} (R_1 + R_2) & (-R_3) \\ (-R_3) & (R_2 + R_3) \end{bmatrix}}$$

$$I_2 = \frac{\begin{bmatrix} (R_1 + R_2) & E_1 \\ (-R_3) & -E_2 \end{bmatrix}}{\begin{bmatrix} (R_1 + R_2) & (-R_3) \\ (-R_3) & (R_2 + R_3) \end{bmatrix}}$$

4. Solving the Equations

Let: $V_1 = 7V$ and $V_2 = 4V$

Results:

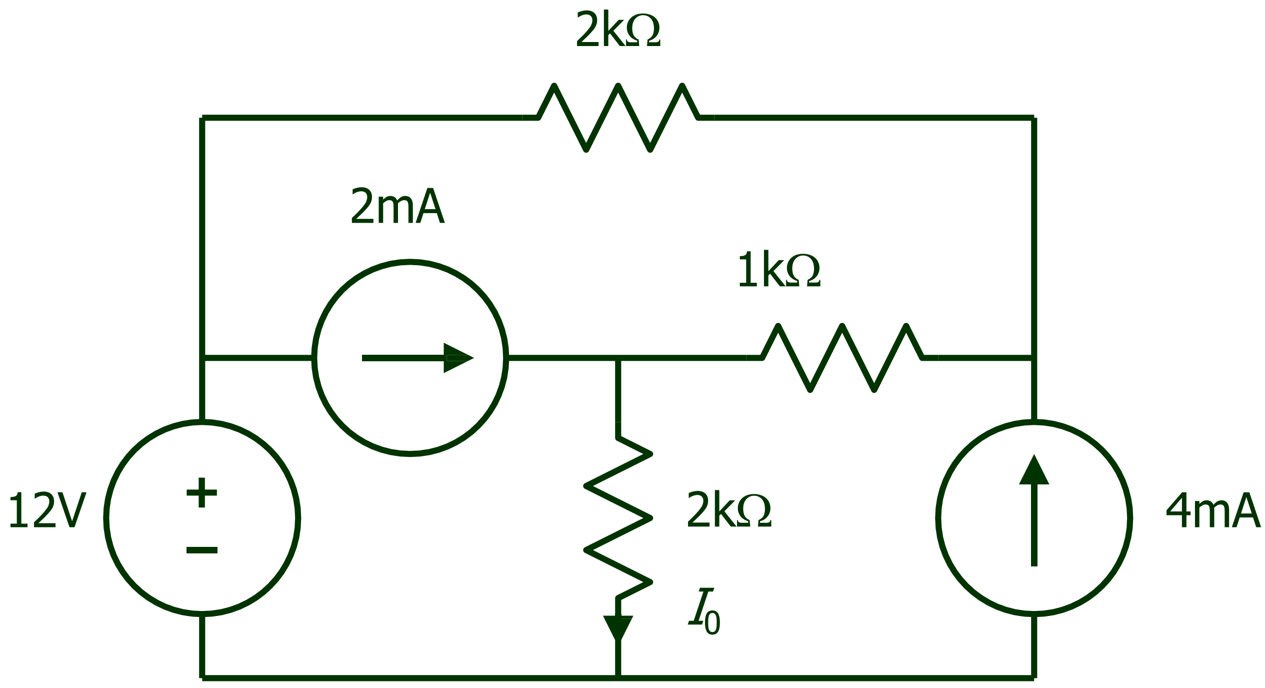
$$I_1 = 3.33\,\text{mA}$$

$$I_2 = -0.33\,\text{mA}$$

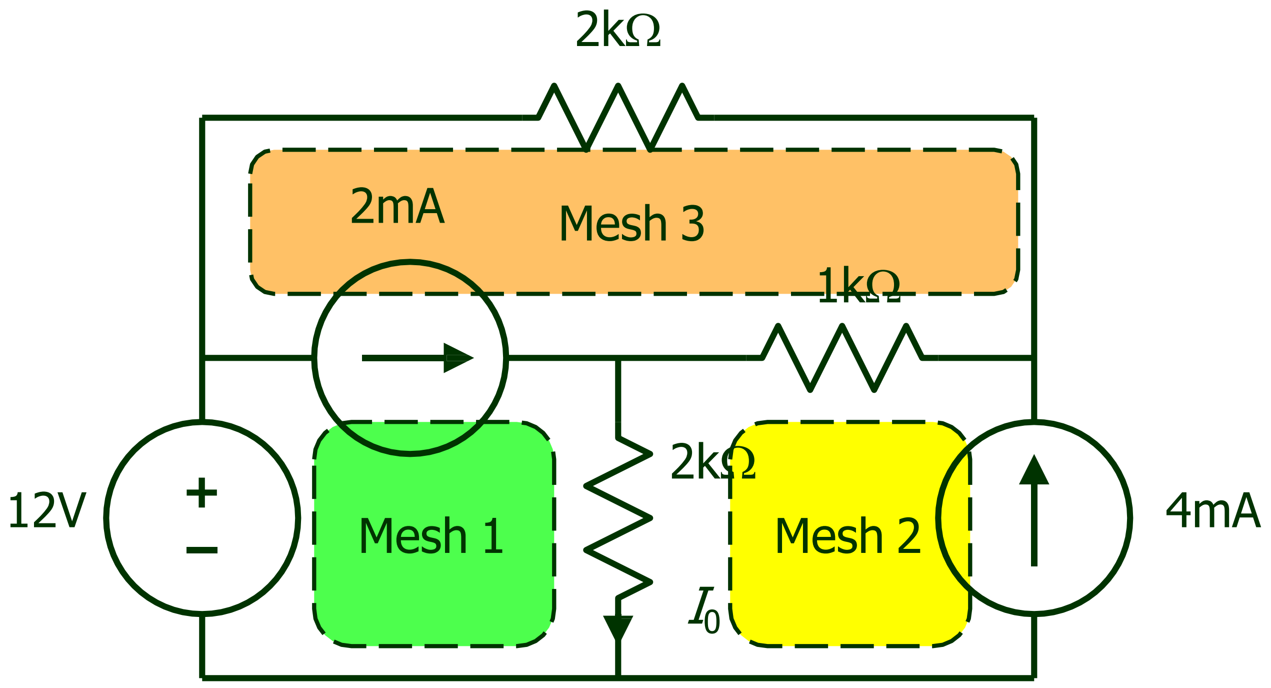
Finally

$$V_{out} = (I_1 - I_2)\,1\text{k}\Omega = 3.66\text{V}$$

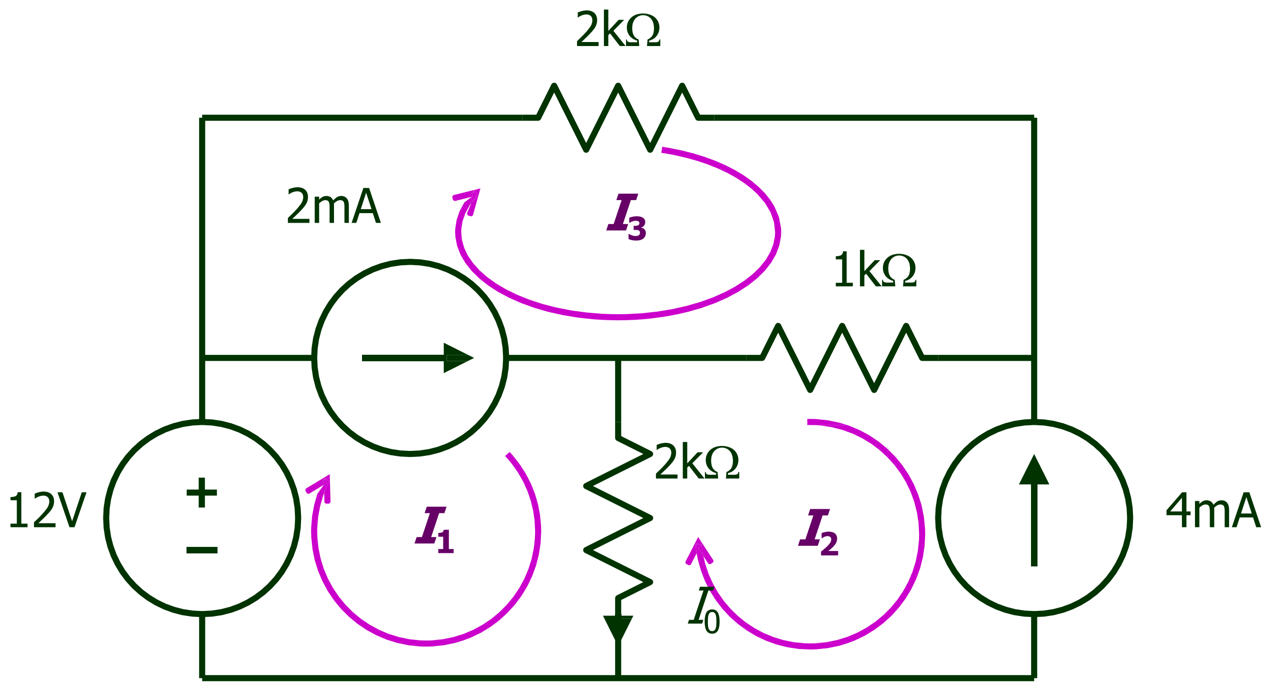
Another Example



1. Identify Meshes



2. Assign Mesh Currents



Current Sources

- The current sources in this circuit will have whatever voltage is necessary to make the current correct.
- We can't use KVL around any mesh because we don't know the voltage for the current sources.
- What to do?

Current Sources

- The 4mA current source sets I_2:

$$I_2 = -4\,\text{mA}$$

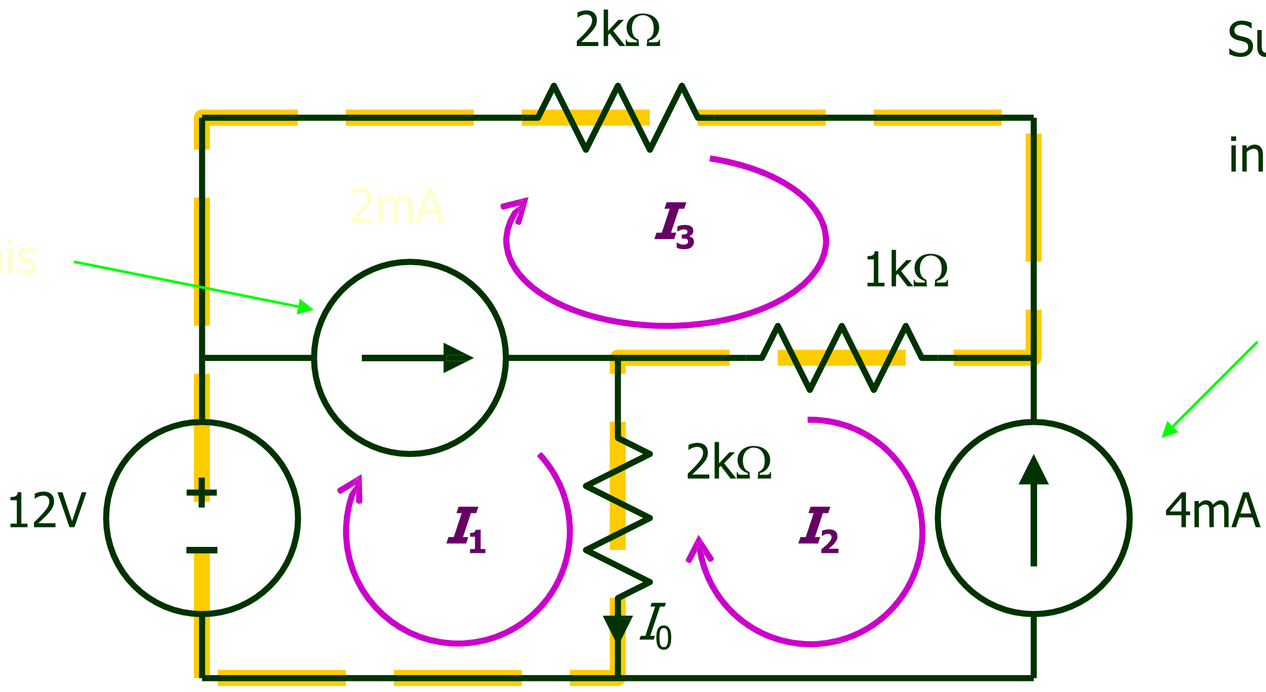
- The 2mA current source sets a *constraint* on I_1 and I_3:

$$I_1 - I_3 = 2\,\text{mA}$$

- We have two equations and three unknowns. Where is the third equation?

Supermesh

The Supermesh surrounds this source!



The Supermesh does not include this source!

3. KVL Around the Supermesh

$$-12V + I_3\, 2k\Omega + (I_3 - I_2)1k\Omega + (I_1 - I_2)2k\Omega = 0$$

$$I_3\, 2k\Omega + (I_3 - I_2)1k\Omega + (I_1 - I_2)2k\Omega = 12V$$

Matrix Notation

- The three equations can be combined into a single matrix/vector equation.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 2\text{k}\Omega & -1\text{k}\Omega - 2\text{k}\Omega & 2\text{k}\Omega + 1\text{k}\Omega \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -4\text{mA} \\ 2\text{mA} \\ 12\text{V} \end{bmatrix}$$

Advantages of Nodal Analysis

- Solves directly for node voltages.
- Current sources are easy.
- Voltage sources are either very easy or somewhat difficult.
- Works best for circuits with few nodes.
- Works for any circuit.

Advantages of Loop Analysis

- Solves directly for some currents.
- Voltage sources are easy.
- Current sources are either very easy or somewhat difficult.
- Works best for circuits with few loops.

Disadvantages of Loop Analysis

- Some currents must be computed from loop currents.
- Does not work with non-planar circuits.
- Choosing the supermesh may be difficult.

- FYI: Spice uses a nodal analysis approach

Magnetically Coupled Networks

- A new four-terminal element, the *transformer*, is introduced in this chapter
- A transformer is composed of two closely spaced inductors, that is, two or more magnetically coupled coils
  - *primary side* is connected to the source
  - *secondary side* is connected to the load

Dot Convention

- *dot convention*: dots are placed beside each coil (inductor) so that if the currents are entering (or leaving) both dotted terminals, then the fluxes add
- *right hand rule* says that curling the fingers (of the right hand) around the coil in the direction of the current gives the direction of the magnetic flux based on the direction of the thumb
- We need dots on the schematic to know how the coils are physically oriented wrt one

Mutually Coupled Coils

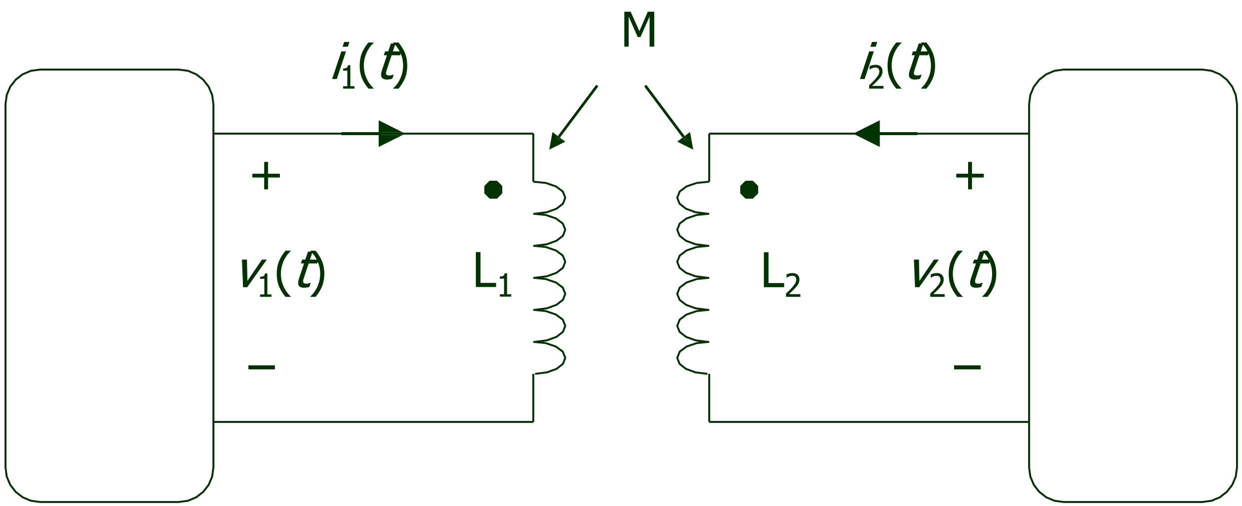
The following equations define the coupling between the two inductors assuming that each respective current enters the dot side which is also the positive voltage side

$$v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2(t) = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

where L_1 and L_2 are the *self-inductances* of the coils (inductors), and M is the *mutual inductance* between the two coils

Mutually Coupled Coils



$$v_1(t) = L_1 \frac{d i_1}{dt} + M \frac{d i_2}{dt}$$

$$v_2(t) = M \frac{d i_1}{dt} + L_2 \frac{d i_2}{dt}$$

Class Example

- Extension Exercise E11.1

Mutually Coupled Coils (AC)

- The frequency domain model of the coupled circuit is essentially identical to that of the time domain

$$\mathbf{V}_1 = j\omega L_1 \mathbf{I}_1 + j\omega M \mathbf{I}_2$$
$$\mathbf{V}_2 = j\omega M \mathbf{I}_1 + j\omega L_2 \mathbf{I}_2$$

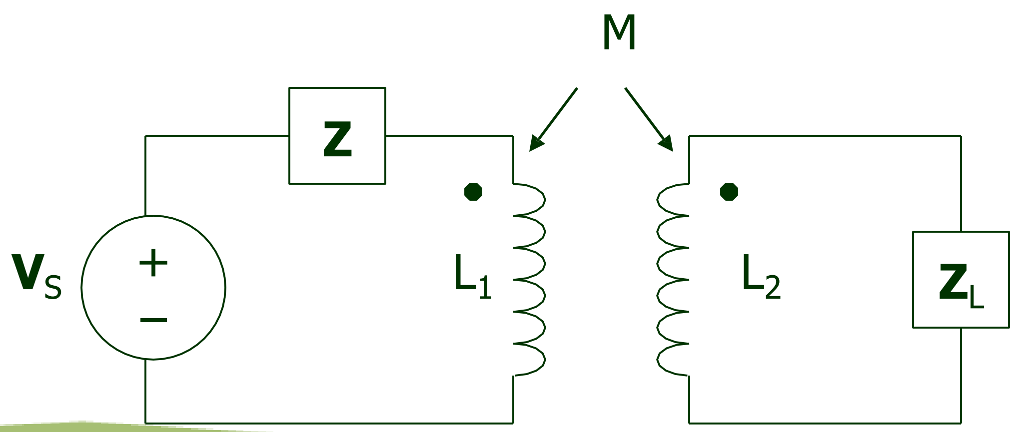
Class Examples

- Extension Exercise E11.2
- Extension Exercise E11.3

Source Input Impedance

The source sees an *input impedance*, $\mathbf{Z}_i$, that is the sum of the *primary impedance*, and a *reflected impedance*, $\mathbf{Z}_R$, due to the secondary (load) side

$$\mathbf{Z}_i \equiv \frac{\mathbf{V}_S}{\mathbf{I}_P} = \mathbf{Z}_P + \mathbf{Z}_R = \mathbf{Z}_P + f\left(\mathbf{Z}_L\right)$$



Class Example

- Extension Exercise E11.4

Energy Analysis

- An energy analysis of the mutually coupled inductors provides an expression for the instantaneous stored energy

$$w(t) = \tfrac{1}{2} L_1 \left[i_1(t)\right]^2 + \tfrac{1}{2} L_2 \left[i_2(t)\right]^2 \pm M\, i_1(t)\, i_2(t)$$

- The sign is positive (+) if currents are both entering (or leaving) the dots; sign is negative (-) if currents are otherwise

Quantifying the Coupling

- The *mutual inductance, M*, is in the range

$$0 \le M \le \sqrt{L_1 L_2}$$

- The *coefficient of coupling* (*k*) between two inductors is defined as

$$0 \le \left[k \equiv \frac{M}{\sqrt{L_1 L_2}} \right] \le 1$$

  – for k > 0.5, inductors are said to be tightly coupled
  – for k ≤ 0.5, coils are considered to be loosely coupled

Class Example

- Extension Exercise E11.5

Electromagnetic Induction

Chapter 31

Faraday's Law
Induced currents
Lenz's Law
Induced EMF
Magnetic Flux
Induced Electric fields

Electromagnetic Induction

**A changing magnetic field (intensity, movement)
will induce an electromotive force (emf)**

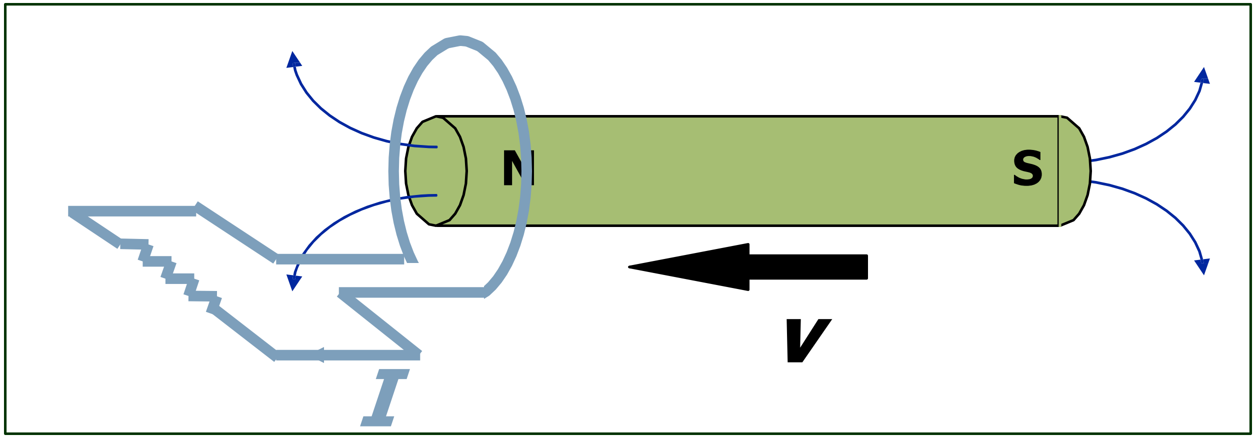
**In a closed electric circuit,
a changing magnetic field
will produce an electric current**

Electromagnetic Induction
Faraday's Law

The induced emf in a circuit is proportional to the rate of change of magnetic flux, through any surface bounded by that circuit.

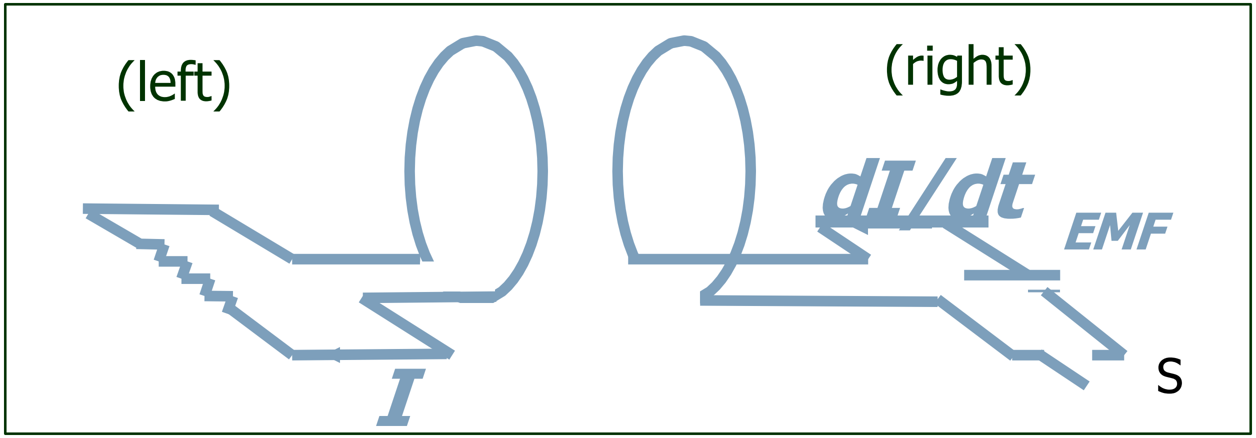
$$\mathcal{E} = - d\Phi_B / dt$$

Faraday's Experiments



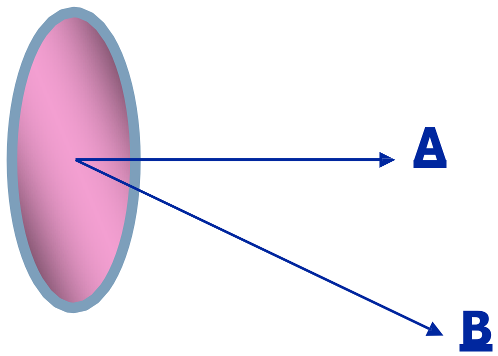
- Michael Faraday discovered induction in 1831.
- Moving the magnet induces a current I.
- Reversing the direction reverses the current.
- Moving the loop induces a current.
- The induced current is set up by an *induced EMF.*

Faraday's Experiments



- Changing the current in the right-hand coil induces
  a current in the left-hand coil.
- The induced current does not depend on the size of
  the current in the right-hand coil.
- The induced current depends on ***dI/dt***.

Magnetic Flux

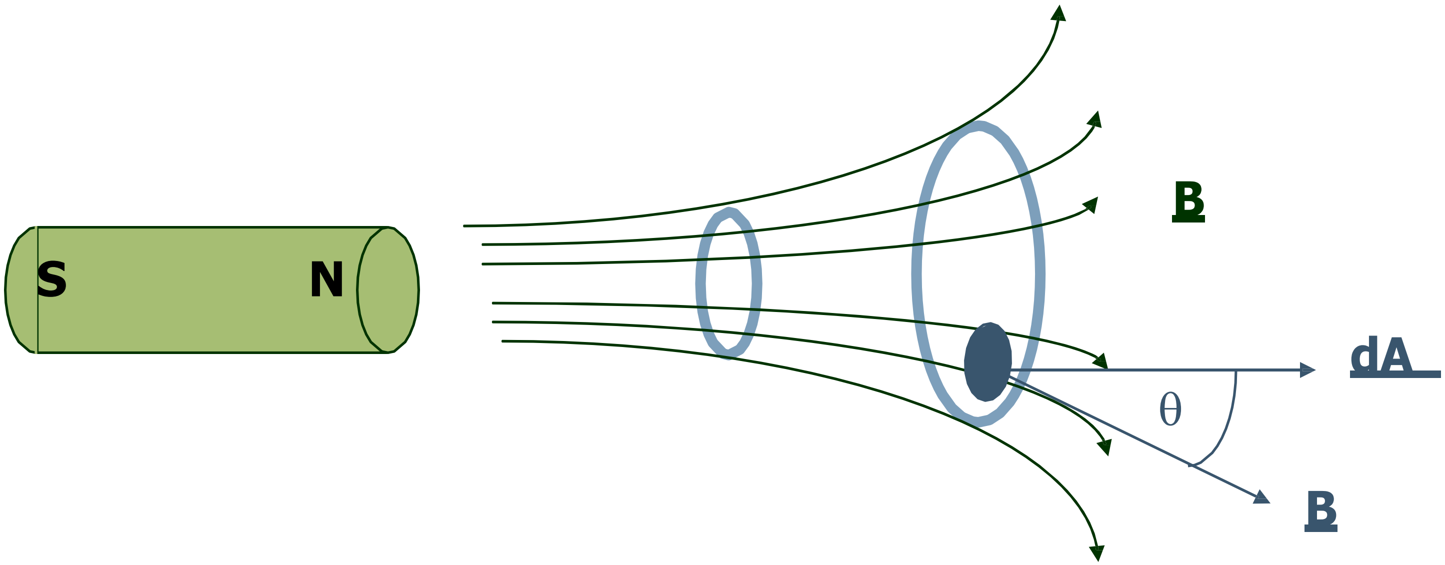


- In the easiest case, with a constant magnetic field **B**, and a flat surface of area **A**, the magnetic flux is

$$\boxed{\Phi_B = \mathbf{B} \cdot \mathbf{A}}$$

- Units : 1 tesla x m^2 = 1 weber

Magnetic Flux



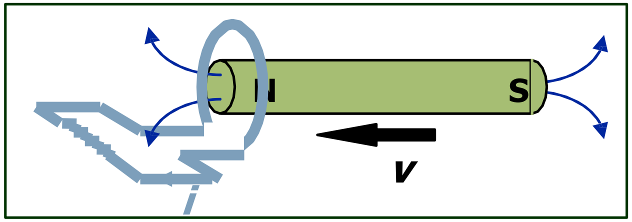
- When B is not constant, or the surface is not flat, one must

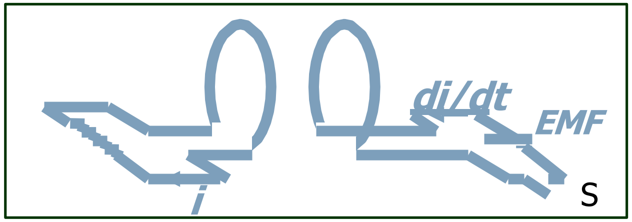
  do an integral.
- Break the surface into bits **dA**. The flux through one bit is

  $d\Phi_B = \mathbf{B} \cdot d\mathbf{A} = B\,dA\cos\theta$
- Add the bits:

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B\cos\theta\, dA$$

Faraday's Law

1) 

2) 

- Moving the magnet changes the flux Φ_B (1).
- Changing the current changes the flux Φ_B (2).
- ***Faraday:*** changing the flux induces an emf.

$$\boxed{\mathcal{E} = -\, d\Phi_B / dt}$$

Faraday's law

The emf induced around a loop

equals the rate of change of the flux through that loop

Lenz’s Law

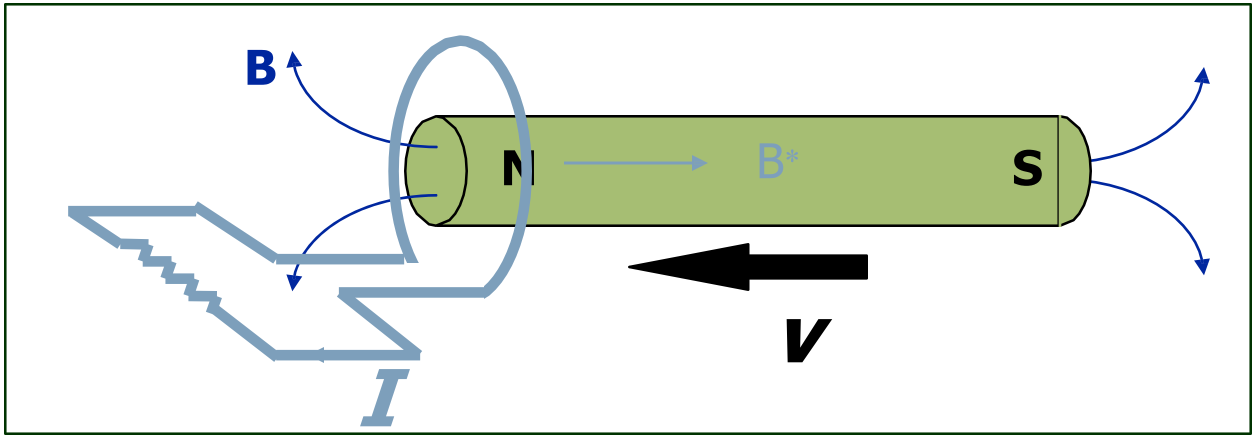
- Faraday’s law gives the direction of the induced emf and therefore the direction of any induced current.
- Lenz’s law is a simple way to get the directions straight, with less effort.
- **Lenz’s Law:**

  The induced emf is directed so that any induced current flow

  will *oppose* the *change* in magnetic flux (which causes the induced emf).
- This is easier to use than to say ...

  Decreasing magnetic flux $\Rightarrow$ emf creates additional magnetic field

Lenz's Law



If we move the magnet towards the loop

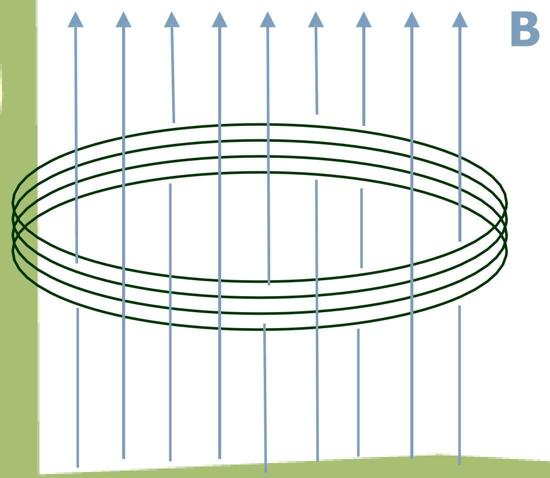
the flux of B will increase.

Lenz's Law $\Rightarrow$ the current induced in the

loop will generate a field B* opposed

Example of Faraday's Law

Consider a coil of radius 5 cm with N = 250 turns.
A magnetic field B, passing through it,
changes at the rate of $dB/dt = 0.6$ T/s.
The total resistance of the coil is 8 Ω.
What is the induced current ?

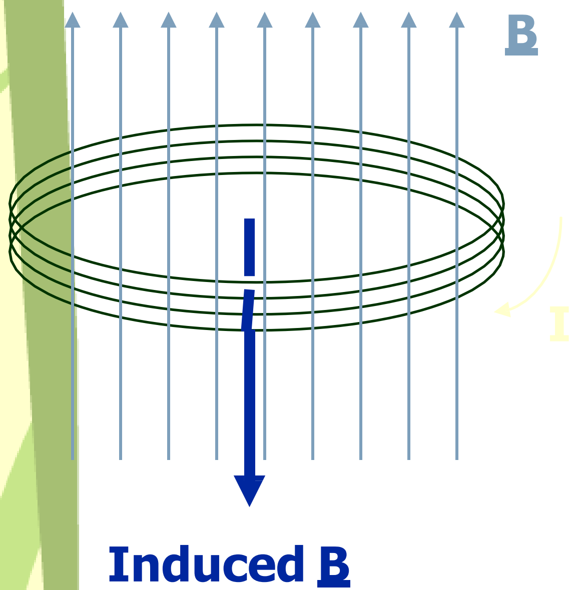


$B = 0.6\,t$ [T] (t = time in seconds)

Use Lenz's law to determine the direction of the induced current.

Apply Faraday's law to find the emf and then the current.

Example of Faraday's Law



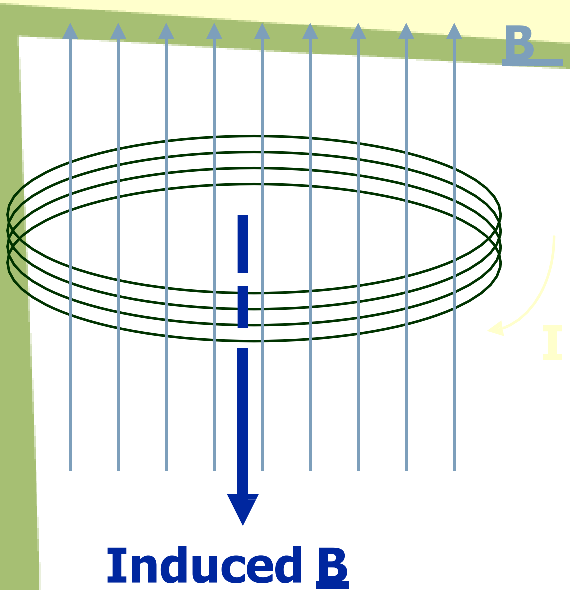
Lenz's law: dB/dt > 0

The change in B is increasing the
upward flux through the coil.

So the induced current will have a magnetic field whose flux (and therefore field) is ***down***.

Hence the induced current must be *clockwise* when looked at from above.

Use Faraday's law to get the magnitude of the induced emf and current.



$$\Phi_{\mathbf{B}} = \int \underline{B} \, . \, d\underline{A}$$

$$\mathcal{E} = - d\Phi_B / dt$$

The induced EMF is $\mathcal{E} = - d\Phi_B / dt$

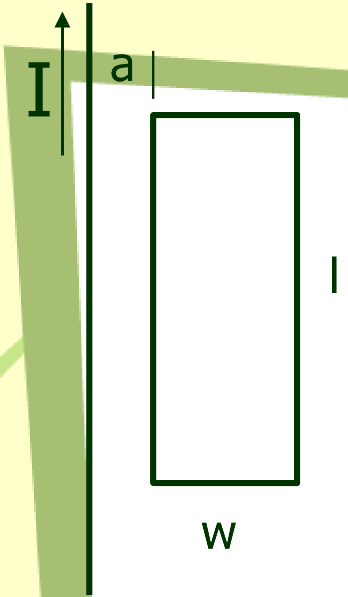
In terms of B: $\Phi_B = N(BA) = NB(\pi r^2)$

Therefore $\mathcal{E} = - N (\pi r^2) \, dB / dt$

$\mathcal{E} = - (250) (\pi \, 0.005^2)(0.6 T/s) = -1.18 \, V$

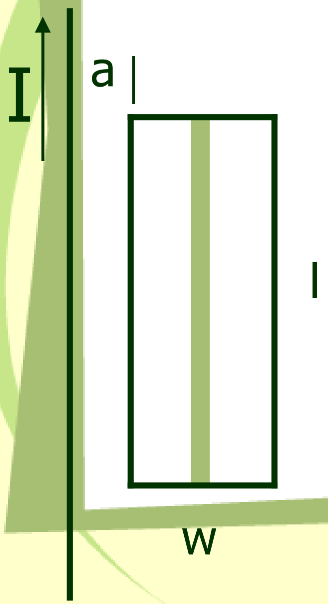
$(1V = 1Tm^2/s)$

Current I $= \mathcal{E} / R = (-1.18V) / (8 \, \Omega) = - 0.147$ A

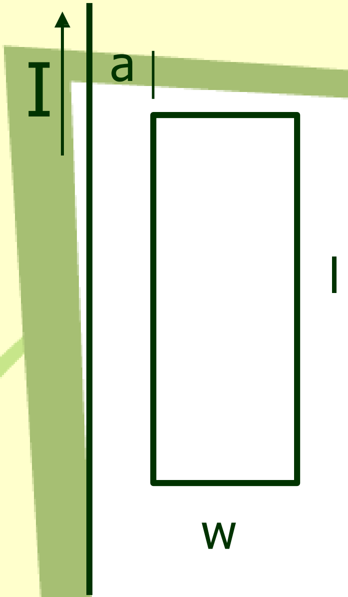


Magnetic Flux in a Nonuniform Field

A long, straight wire carries a current I. A rectangular loop (w by l) lies at a distance a, as shown in the figure. What is the magnetic flux through the loop?.



Induced emf Due to Changing Curren



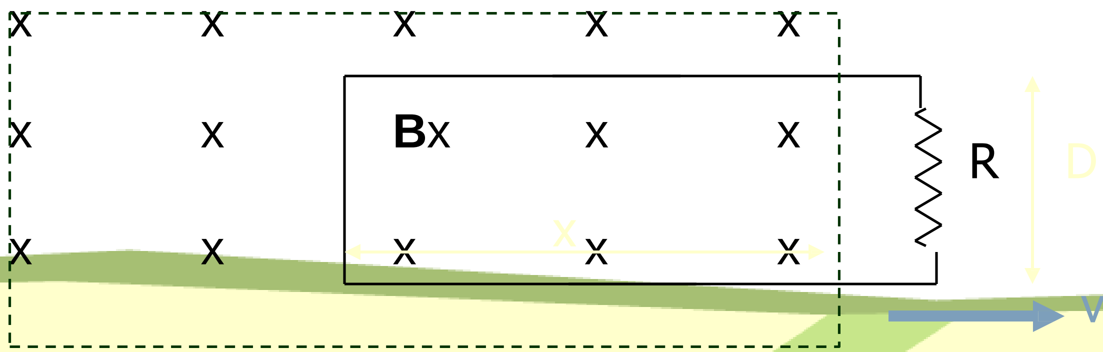
A long, straight wire carries a current $I = I_0 + i\,t$.
A rectangular loop (w by l) lies at a distance a,
as shown in the figure.
What is the induced emf in the loop?.
What is the direction of the induced current and field?

Motional EMF

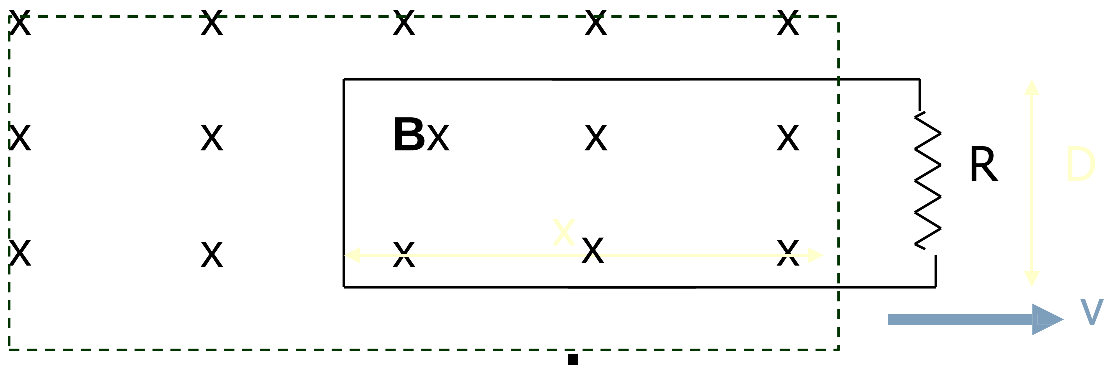
Up until now we have considered fixed loops. The flux through them changed because the magnetic field changed with time.

Now try moving the loop in a uniform and constant magnetic field. This changes the flux, too.

<u>B </u>points into screen



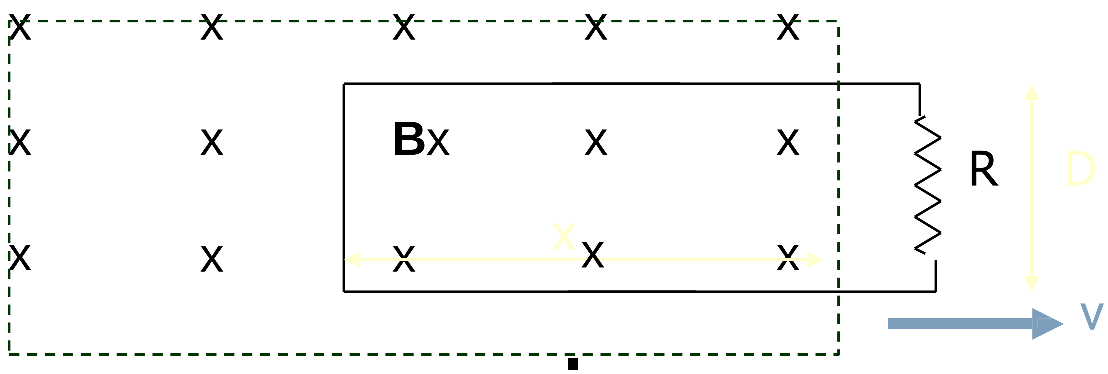
Motional EMF - Use Faraday's Law



The flux is $\Phi_B = \underline{B \cdot A} \quad = \text{BDx}$

This changes in time:

Motional EMF - Use Faraday's Law



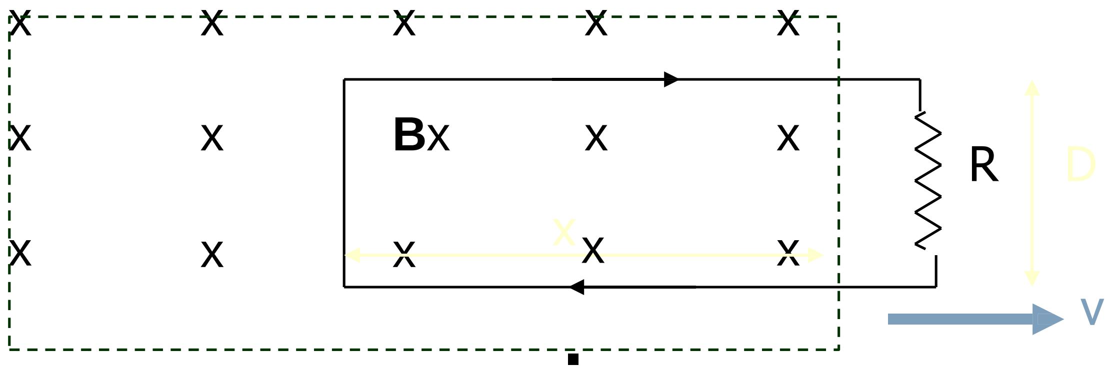
The flux is $\Phi_B = \underline{B \cdot A} \quad = \; BDx$

This changes in time:

$$d\Phi_B / dt = d(BDx)/dt = BDdx/dt = -BDv$$

Hence by Faraday's law there is an induced emf and current. What is the direction of the current?

Motional EMF - Use Faraday's Law



The flux is $\Phi_B = \underline{B \cdot A} \quad = \; BDx$

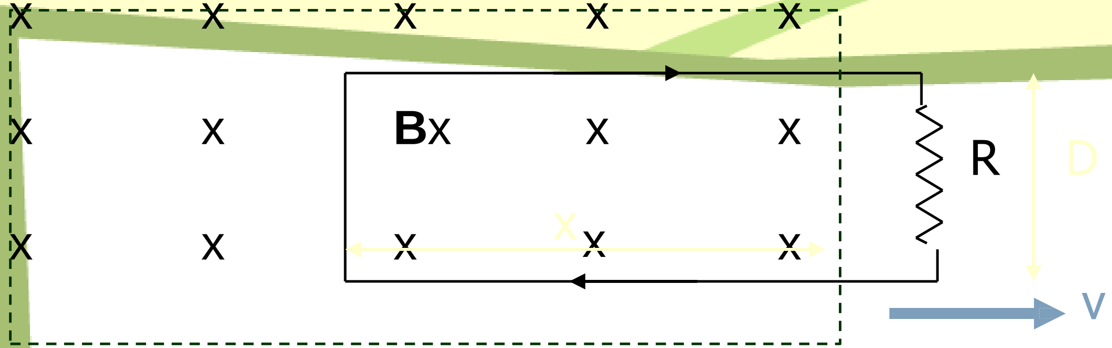
This changes in time:

$$d\Phi_B / dt = d(BDx)/dt \; = BDdx/dt \; = -BDv$$

Hence by Faraday's law there is an induced emf and current. What is the direction of the current?

Lenz's law: there is less inward flux through the loop. Hence the induced current gives inward flux.

$\Rightarrow$ So the induced current is clockwise.



Motional EMF
Faraday's Law

Now Faraday's Law $d\Phi_B/dt = -\mathcal{E}$

gives the EMF $\Rightarrow \mathcal{E} = BDv$

In a circuit with a resistor, this gives

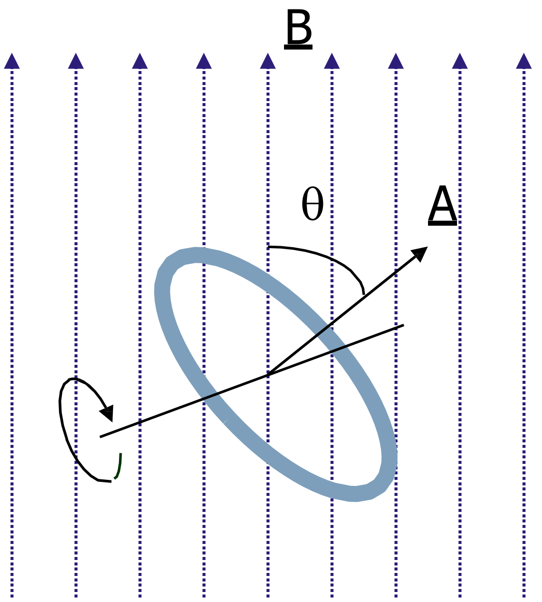
$\mathcal{E} = BDv = IR \Rightarrow I = BDv/R$

Thus moving a circuit in a magnetic field produces an emf exactly like a battery. This is the principle of an electric generator.

Rotating Loop - The Electricity Generator

Consider a loop of area A in a region of space in which there is a uniform magnetic field B. Rotate the loop with an angular frequency ω .

The flux changes because angle θ changes with time: $\theta = \omega t$.



Hence:

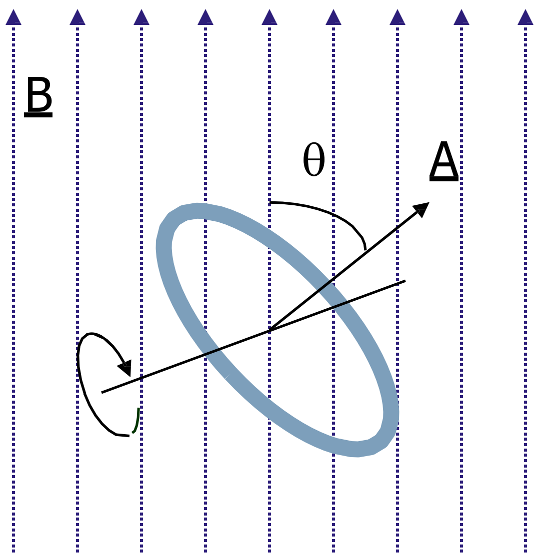
$d\Phi_B/dt = d(\underline{B} . \underline{A})/dt$

$= d(BA\cos\theta)/dt$

$= B\,A\,d(\cos(\omega t))/dt$

$= -BA\omega \sin(\omega t)$

Rotating Loop - The Electricity Genera



$$d\Phi_B/dt = - BA\omega \sin(\omega t)$$

- Then by Faraday's Law this motion causes an emf

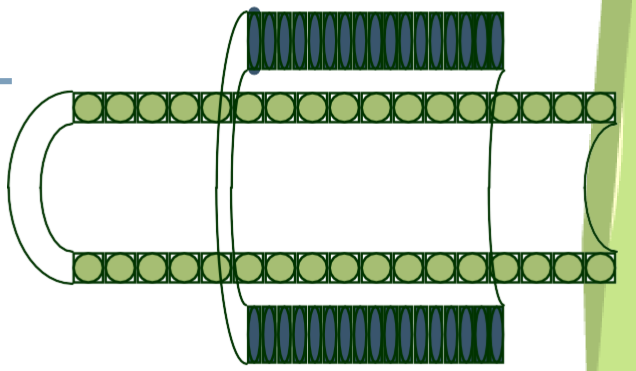
$$\mathcal{E} = - d\Phi_B /dt = BA\omega \sin(\omega t)$$

- This is an **AC (alternating current)**

A New Source of EMF

- If we have a conducting loop in a magnetic field, we can create an EMF (like a battery) by changing the value of B . A.
- This can be done by changing the area, by changing the magnetic field, or the angle between them.
- We can use this source of EMF in electrical circuits in the same way we used batteries.
- Remember we have to do work to move the loop or to change B, to generate the EMF (Nothing is for free!).

Example: a 120 turn coil (r= 1.8 cm, R = 5.3Ω) is placed outside a solenoid (r=1.6cm, n=220/cm, i=1.5A). The current in the solenoid is reduced to 0 in 0.16s. What current appears in the coil ?



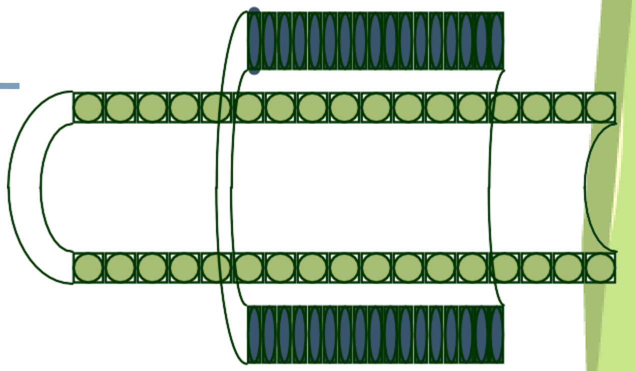
Current induced in coil:

$$i_C = \frac{EMF}{R} = \left(\frac{N}{R}\right)\frac{d\Phi_B}{dt}$$

$$\Phi_B = \vec{B} \cdot \vec{A} = (\mu_0 n i_s) A_s$$

Only field in coil is inside solenoid

Example: a 120 turn coil ($r = 1.8$ cm, $R = 5.3\Omega$) is placed outside a solenoid ($r=1.6$cm, $n=220$/cm, $i=1.5$A). The current in the solenoid is reduced to 0 in 0.16s. What current appears in the coil ?



Current induced in coil:

$$i_C = \frac{EMF}{R} = \left(\frac{N}{R}\right)\frac{d\Phi_B}{dt}$$

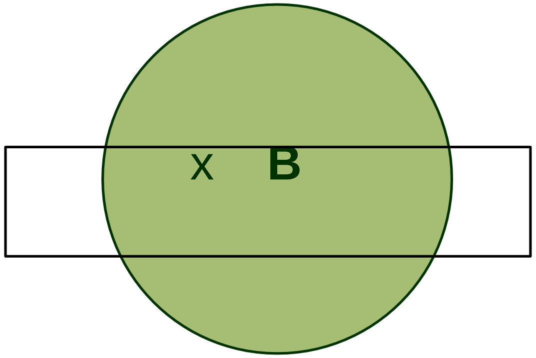
$$\Phi_B = \vec{B}\cdot\vec{A} = (\mu_0 n i_s)A_s$$

Only field in coil is inside solenoid

$$i_c = \left(\frac{N}{R}\right)\frac{d(\mu_0 n i_s A_s)}{dt} = \left(\frac{N}{R}\right)\mu_0 n A_s \frac{di_s}{dt}$$

$$= \left(\frac{N}{R}\right)\mu_0 n A \frac{i_0}{t} = 4.72\,mA$$

Induced Electric Fields



Consider a stationary conductor
in a time-varying magnetic field.
A current starts to flow.

So the electrons must feel a force $\underline{F}$.

It is not $\underline{F} = q\underline{v}\mathrm{x}\underline{B}$, because the wire is stationary.

Instead: we know that $\varepsilon = -\ d\Phi_B/dt$

This is equivalent to an induced electric field E, such that:

$$\underline{F} = q\underline{E} \quad \text{and} \quad \varepsilon = \oint \underline{E}.\underline{dl}$$

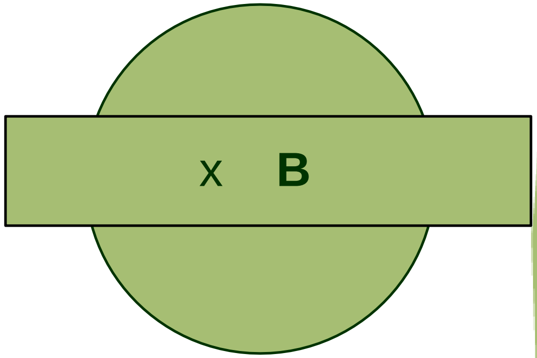
$\Rightarrow$ a time-varying magnetic field B causes an electric field E to appear in the conductor!

Induced Electric Fields

$$\mathcal{E} = \oint \underline{E}.\underline{dl}$$

and

$$\mathcal{E} = - d\Phi_B/dt$$



Then: $\boxed{\oint \underline{E}.\,\underline{dl} = - d\Phi_B/dt}$ **Faraday's Law**

The induced electric field E is NOT a conservative field

We can NOT write E = - dV/dl or $\underline{E} = -\underline{\nabla V}$

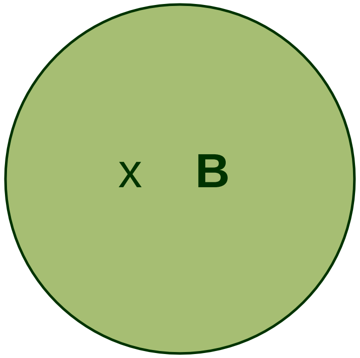
The electrostatic field E_e is conservative $\oint E_e.dl = 0$

Electrostatic Field	Induced Electric Field
$\underline{F} = q\,\underline{E}_e$	$\underline{F} = q\,\underline{E}$
$\Delta V_{ab} = -\int \underline{E}_e . \underline{dl}$	$\int \underline{E} . \underline{dl} = -d\Phi_B/dt$
$\int \underline{E}_e . \underline{dl} = 0$ and $\underline{E}_e = \underline{\nabla V}$	$\int \underline{E} . \underline{dl} \neq 0$
Conservative	Nonconservative
Work or energy difference does NOT depend on path	**Work or energy difference DOES depend on path**
Caused by stationary charges or emf sources	**Caused by changing magnetic fields**

Induced Electric Fields

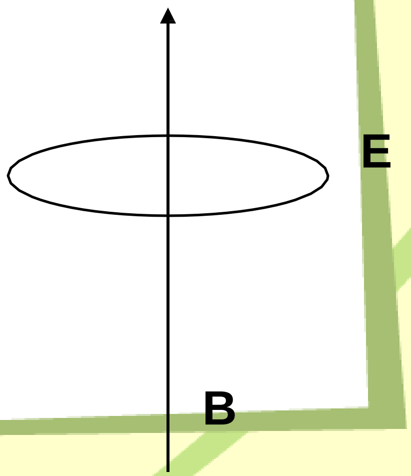
$$\oint \underline{E} \, . \, \underline{dl} = - d\Phi_B / dt$$

Faraday's Law



Now suppose there is no conductor:
Is there still an electric field?

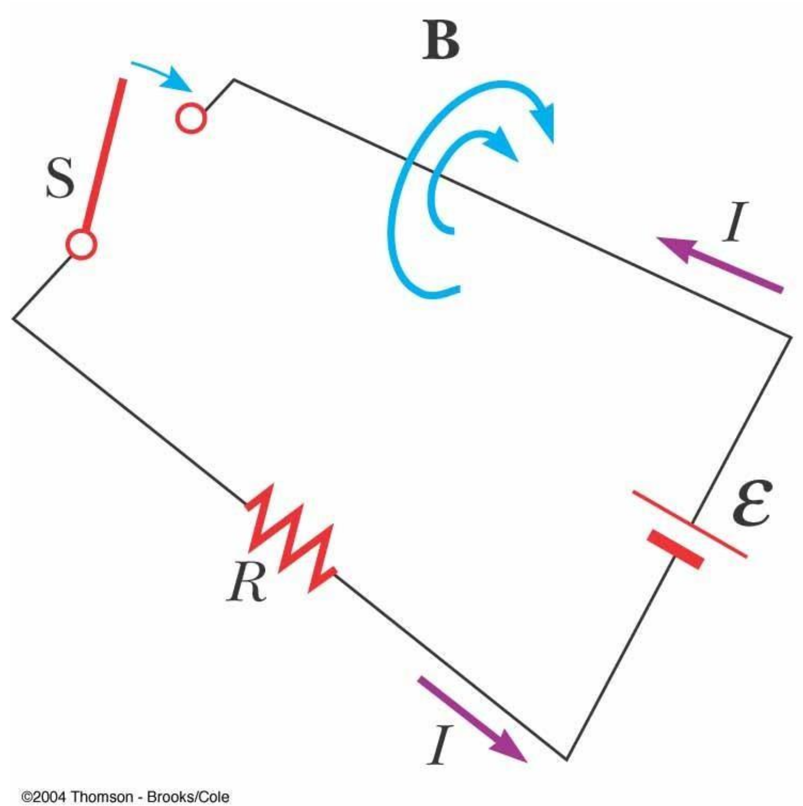
YES!, the field does not depend on the presence of the conductor.



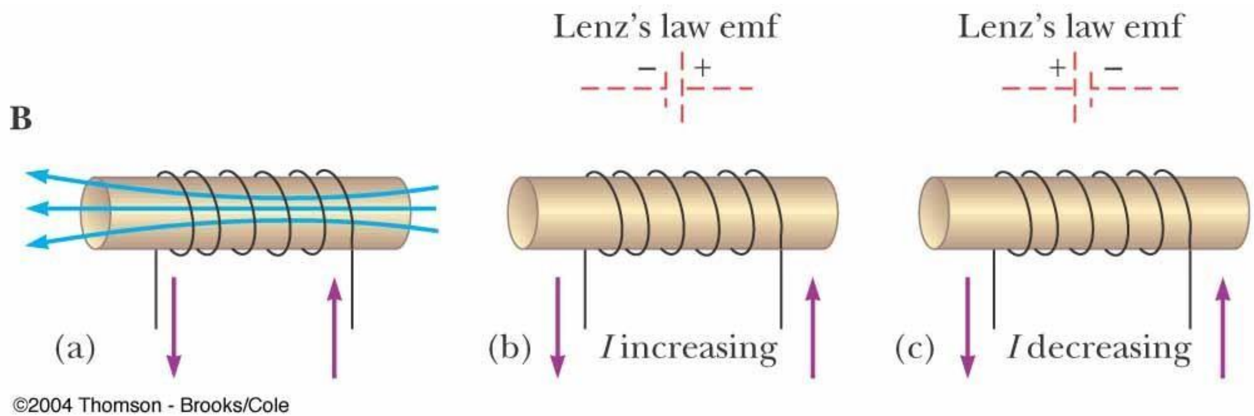
For a magnetic field with axial or cylindrical symmetry, the field lines of E are circles.

Self-Inductance

- When the switch is closed, the current does not immediately reach its maximum value
- Faraday's law can be used to describe the effect



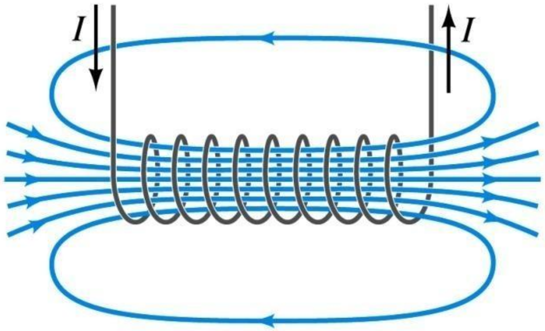
Self-induced emf



- A current in the coil produces a magnetic field directed toward the left (a)
- If the current increases, the increasing flux creates an induced emf of the polarity shown (b)
- The polarity of the induced emf reverses if the current decreases (c)

Self Inductance

$$B = \mu_o n I$$



$$\varepsilon = -N\frac{d\Phi_B}{dt} = -N\frac{d\left(\mu_o n I A\right)}{dt} = -N\mu_o nA\frac{dI}{dt} = -\frac{NBA}{I}\frac{dI}{dt}$$

$$\varepsilon = -\frac{N\Phi_B}{I}\frac{dI}{dt} = -L\frac{dI}{dt}$$

Define: Self Inductance

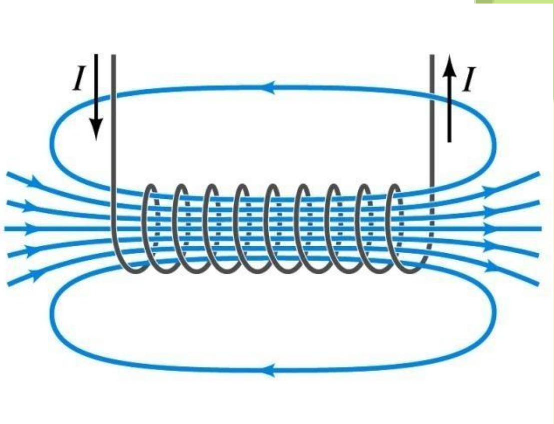
$$L = \frac{N\Phi_B}{I}$$

Inductance of a Solenoid

- The magnetic flux through each turn is

$$\Phi_B = BA = \left(\mu_o \frac{N}{\ell} I \right) A$$

- Therefore, the inductance is



$$L = \frac{N\Phi_B}{I} = \frac{\mu_o N^2 A}{\ell}$$

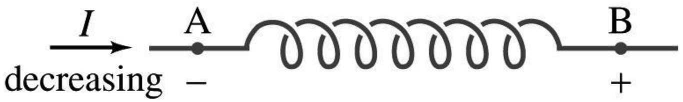
- This shows that *L* depends on the geometry of the object

Inductance Units

$$\varepsilon = -L\frac{dI}{dt} \qquad L = \frac{N\Phi_B}{I}$$

$$L = \left[\frac{V}{A/s}\right] = [\Omega - s] = [\text{Henry}] = [H]$$

(a)



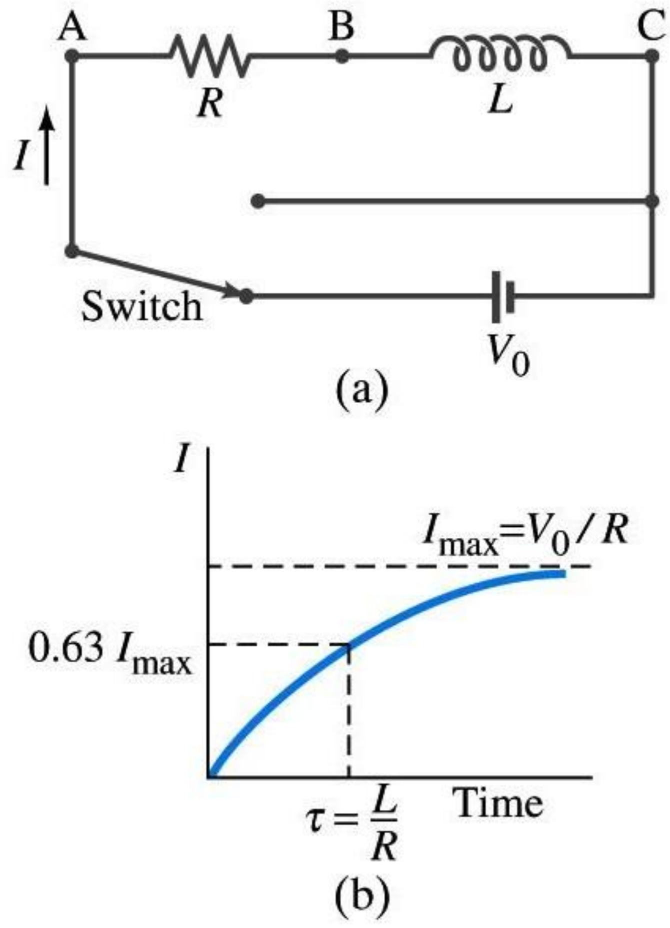
(b)

3. A 2.00-H inductor carries a steady current of 0.500 A. When the switch in the circuit is opened, the current is effectively zero after 10.0 ms. What is the average induced emf in the inductor during this time?

5. A 10.0-mH inductor carries a current $I = I_{max} \sin \omega t$, with I_{max} = 5.00 A and $\omega/2\pi$ = 60.0 Hz. What is the back emf as a function of time?

7. An inductor in the form of a solenoid contains 420 turns, is 16.0 cm in length, and has a cross-sectional area of 3.00 cm2. What uniform rate of decrease of current through the inductor induces an emf of 175 μV?

LR Circuits



Charging

Kirchhoff Loop Equation:

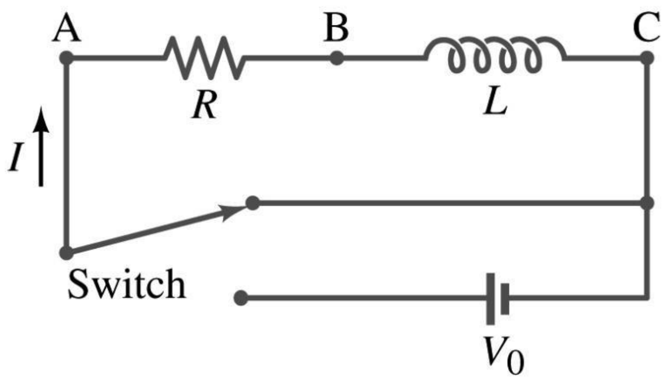
$$V_o - RI - L\frac{dI}{dt} = 0$$

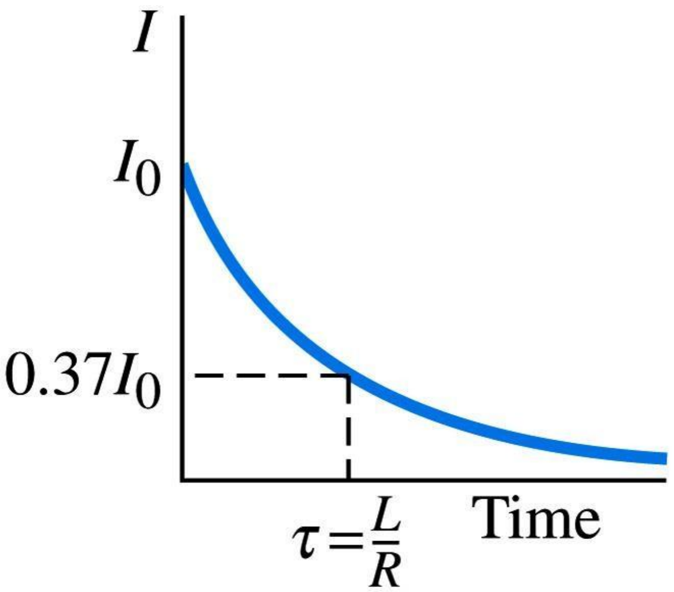
Solution:

$$I = \frac{V_o}{R}\left(1 - e^{-t/\tau}\right)$$

$$\tau = \frac{L}{R}$$

LR Circuits





Discharging

Kirchhoff Loop Equation:

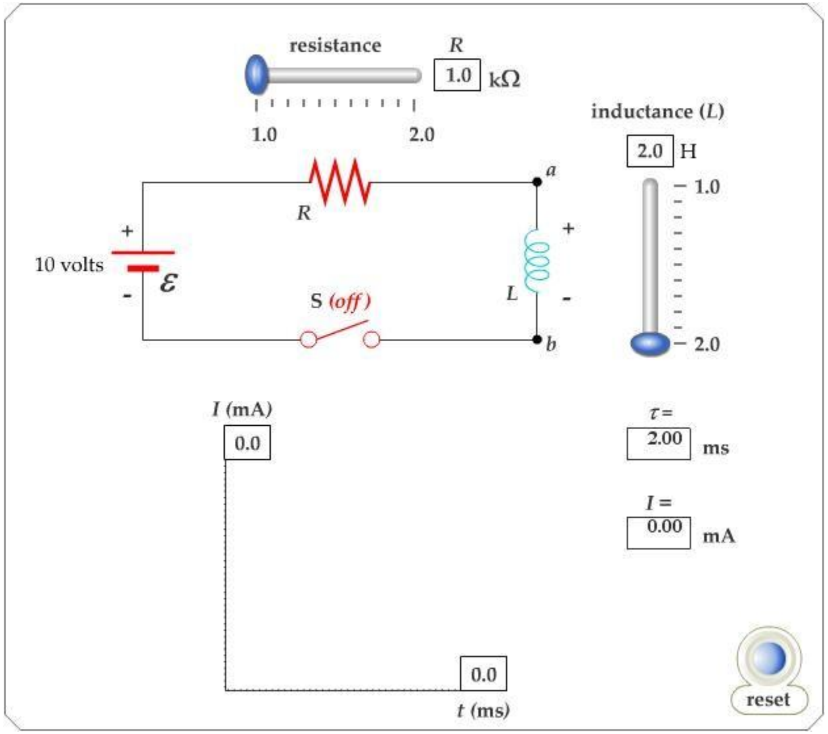
$$RI + L\frac{dI}{dt} = 0$$

Solution:

$$I = I_o e^{-t/\tau}$$

$$\tau = \frac{L}{R}$$

Active Figure 32.3



PLAY ANIMATION

(SLIDESHOW MODE ONLY)

14. Calculate the resistance in an *RL* circuit in which $L = 2.50$ H and the current increases to 90.0% of its final value in 3.00 s.

20. A 12.0-V battery is connected in series with a resistor and an inductor. The circuit has a time constant of 500 μs, and the maximum current is 200 mA. What is the value of the inductance?

24. A series *RL* circuit with $L = 3.00$ H and a series *RC* circuit with $C = 3.00\ \mu$F have equal time constants. If the two circuits contain the same resistance *R*, (a) what is the value of *R* and (b) what is the time constant?

Energy in a coil

$$P = VI = \left(L \frac{dI}{dt} \right) I$$

PE in an Inductor $$U = \frac{1}{2} L I^2$$

PE in an Capacitor $$U = \frac{1}{2} C V^2$$

Energy Density in a coil

PE in an Inductor

$$U = \frac{1}{2} L I^2$$

$$L = \frac{N\Phi_B}{I} = \frac{NBA}{I} = \frac{N\left(\mu_o N I / \ell\right) A}{I}$$

$$I = \frac{B\ell}{\mu_o N}$$

$$U = \frac{1}{2}\left(\frac{N\left(\mu_o N\right)A}{\ell}\right)\left(\frac{B\ell}{\mu_o N}\right)^2 = \frac{1}{2\mu_o} B^2 \left(A\ell\right)$$

$$u = \frac{1}{2\mu_o} B^2$$

$$u = \frac{1}{2} \varepsilon_o E^2$$

31. An air-core solenoid with 68 turns is 8.00 cm long and has a diameter of 1.20 cm. How much energy is stored in its magnetic field when it carries a current of 0.770 A?

33. On a clear day at a certain location, a 100-V/m vertical electric field exists near the Earth's surface. At the same place, the Earth's magnetic field has a magnitude of 0.500×10^{-4} T. Compute the energy densities of the two fields.

36. A 10.0-V battery, a 5.00-**Ω** resistor, and a 10.0-H inductor are connected in series. After the current in the circuit has reached its maximum value, calculate (a) the power being supplied by the battery, (b) the power being delivered to the resistor, (c) the power being delivered to the inductor, and (d) the energy stored in the magnetic field of the inductor.

Example 32-5: The Coaxial Cable

- Calculate L for the cable
- The total flux is

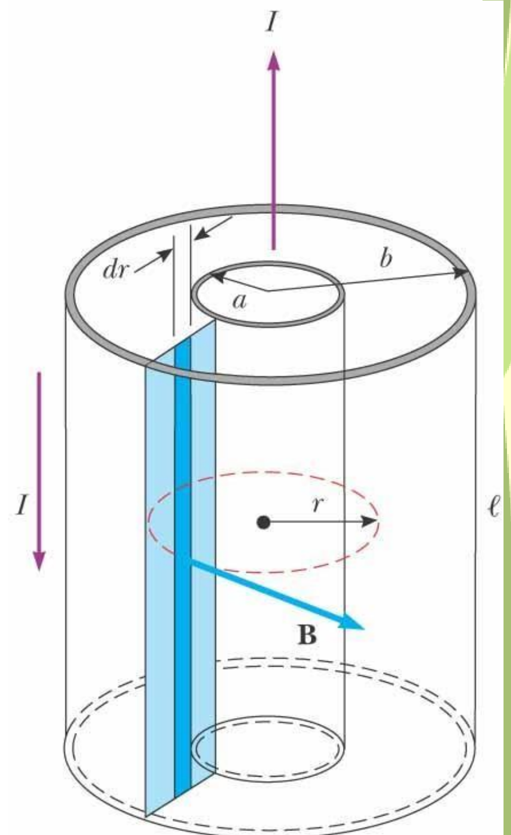
$$\Phi_B = \int B\, dA = \int_a^b \frac{\mu_o I}{2\pi r} \ell\, dr = \frac{\mu_o I \ell}{2\pi} \ln\left(\frac{b}{a}\right)$$

- Therefore, L is

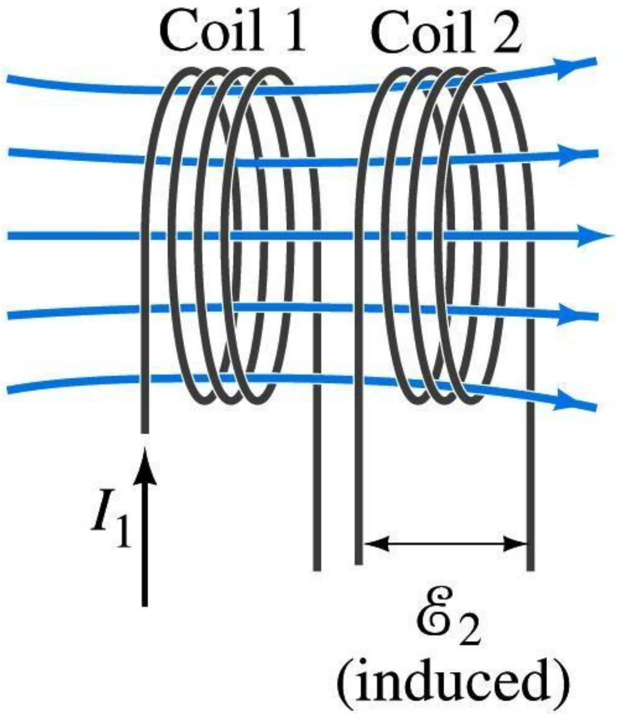
$$L = \frac{\Phi_B}{I} = \frac{\mu_o \ell}{2\pi} \ln\left(\frac{b}{a}\right)$$

- The total energy is

$$U = \frac{1}{2} L I^2 = \frac{\mu_o \ell I^2}{4\pi} \ln\left(\frac{b}{a}\right)$$



Mutual Inductance

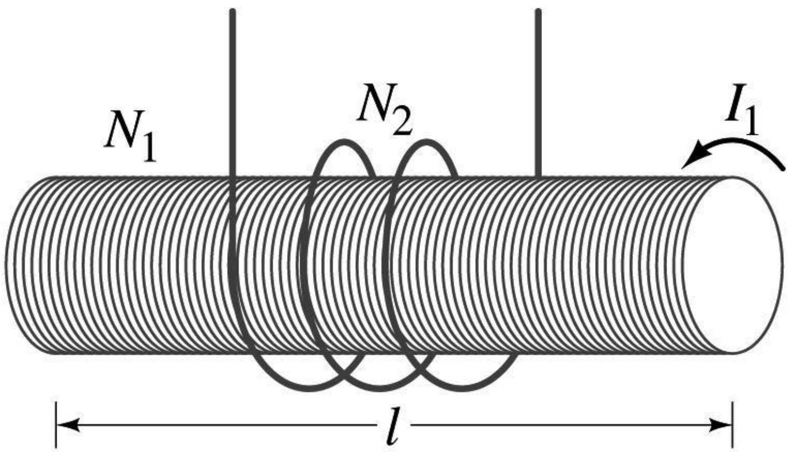


$$N_2\Phi_{12} \propto I_1$$

$$N_2\Phi_{12} = M_{12}I_1$$

$$\varepsilon_2 = -N_2\frac{d\Phi_{12}}{dt} = -N_2\frac{d\left(\dfrac{M_{12}I_1}{N_2}\right)}{dt} = -M_{12}\frac{dI_1}{dt} \qquad \varepsilon_1 = -M_{21}\frac{dI_2}{dt}$$

Mutual Inductance example



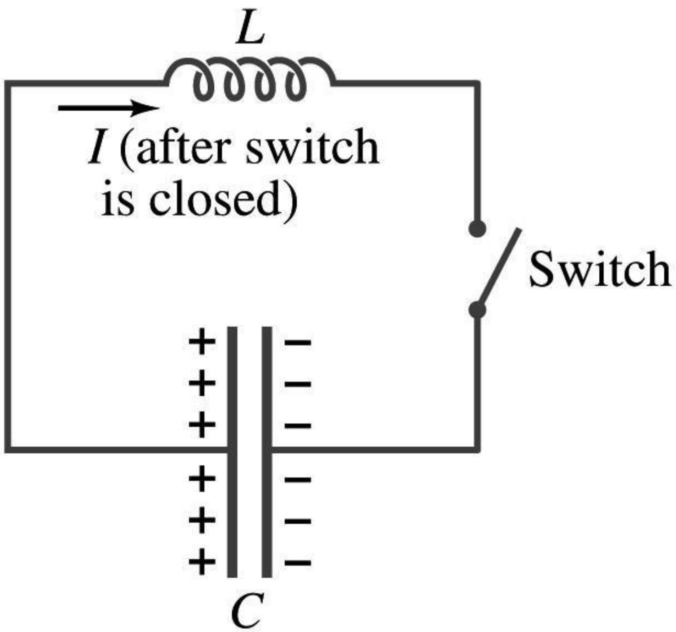
$$B_1 = \mu_0 \frac{N_1}{\ell} I_1$$

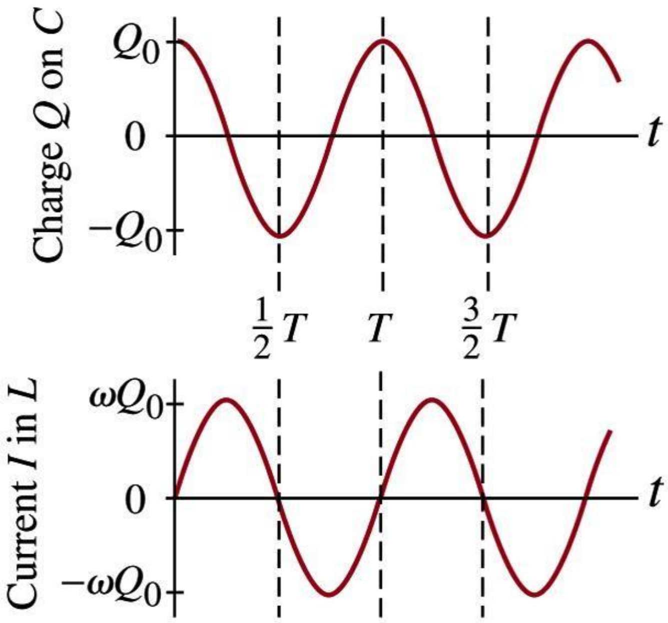
$$\Phi_{12} = B_1 A = \mu_0 \frac{N_1}{\ell} I_1 A$$

$$\varepsilon_2 = -N_2 \frac{d\Phi_{21}}{dt} = -N_2 \frac{d\left(\mu_0 \frac{N_1}{\ell} I_1 A\right)}{dt} = -\mu_0 \frac{N_1 N_2}{\ell} A \frac{dI}{dt}$$

$$M_{12} = \mu_0 \frac{N_1 N_2}{\ell} A = M$$

LC Circuits





Kirchhoff Loop Equation:

$$\frac{Q}{C} - L\frac{dI}{dt} = 0$$

$$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = 0$$

Solution:

$$Q = Q_{max}\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{1}{LC}}$$

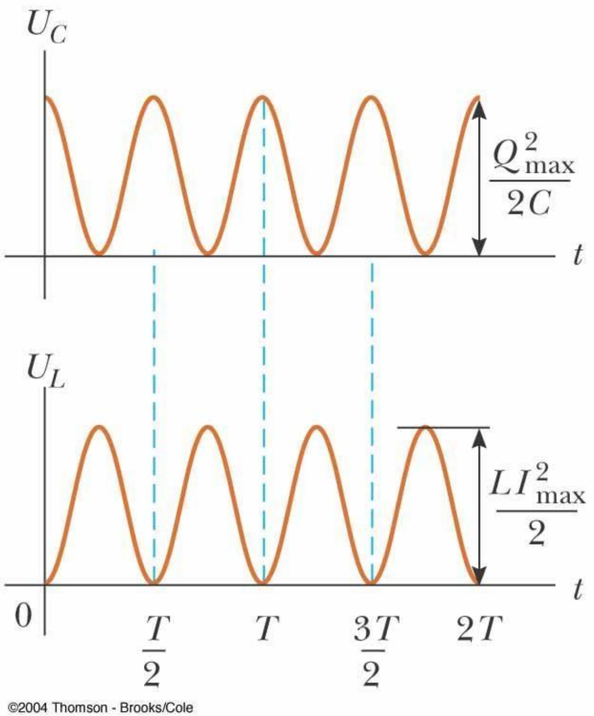
$$I(t=0) = 0 \qquad Q(t=0) = Q_{max}$$

Energy in an LC circuit

$$Q = Q_{max}\cos(\omega t)$$

$$\omega = \sqrt{\frac{1}{LC}}$$

$$I = -\frac{dQ}{dt} = Q_{max}\,\omega\sin(\omega t)$$

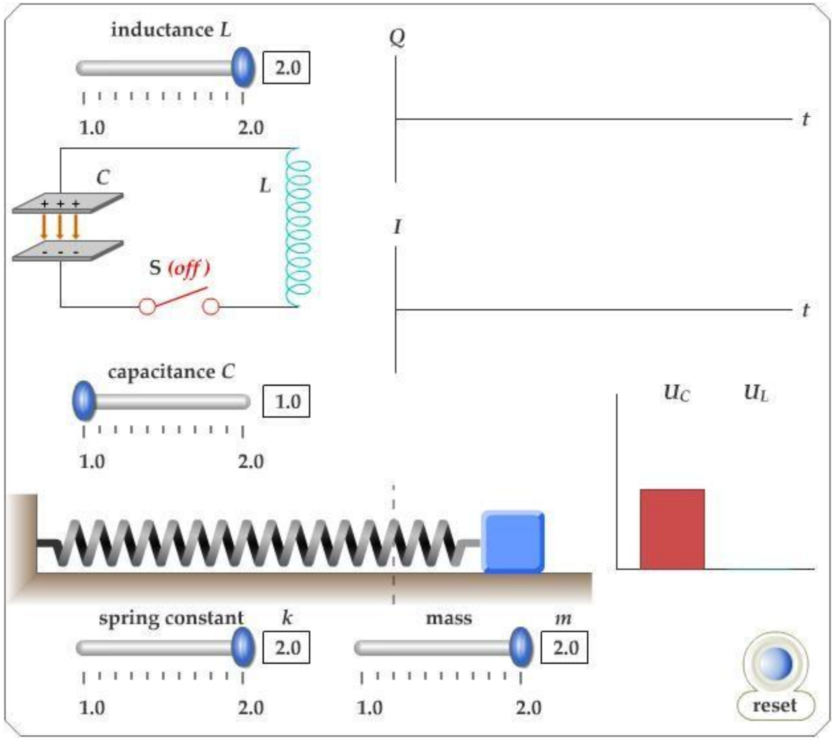


$$U_E = \frac{1}{2}\frac{Q^2}{C} = \frac{Q_{max}^2}{2C}\cos^2(\omega t + \phi)$$

$$U_B = \frac{1}{2}LI^2 = \frac{L\omega^2 Q_{max}^2}{2}\sin^2(\omega t) = \frac{Q_{max}^2}{2C}\sin^2(\omega t)$$

$$U_E + U_B = \frac{Q_{max}^2}{2C}\cos^2(\omega t) + \frac{Q_{max}^2}{2C}\sin^2(\omega t) = \frac{Q_{max}^2}{2C}$$

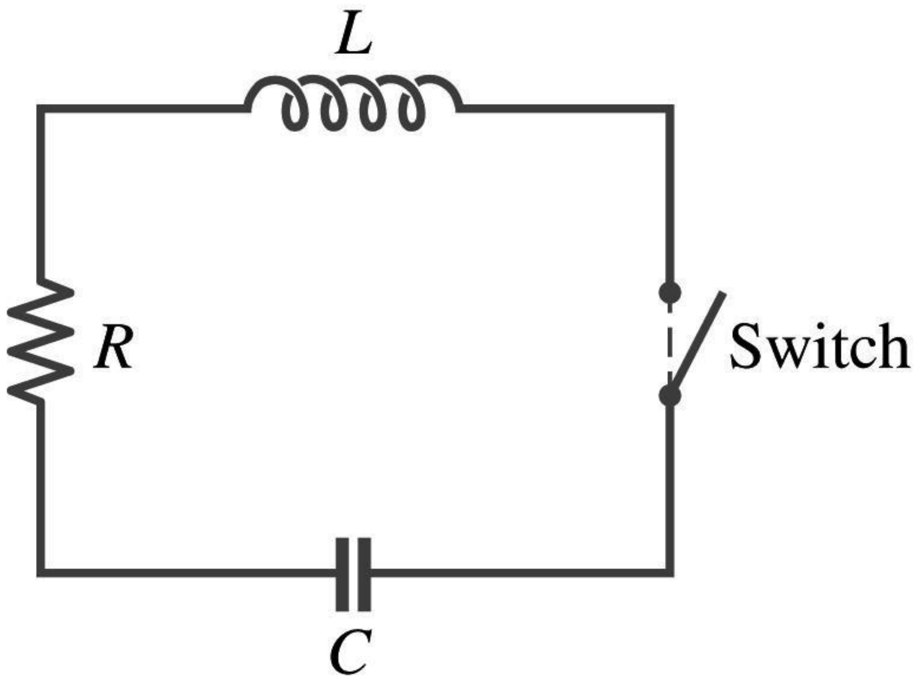
Active Figure 32.17



PLAY ANIMATION

(SLIDESHOW MODE ONLY)

LRC Circuits



Kirchhoff Loop Equation:

$$\frac{Q}{C} - RI - L\frac{dI}{dt} = 0$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

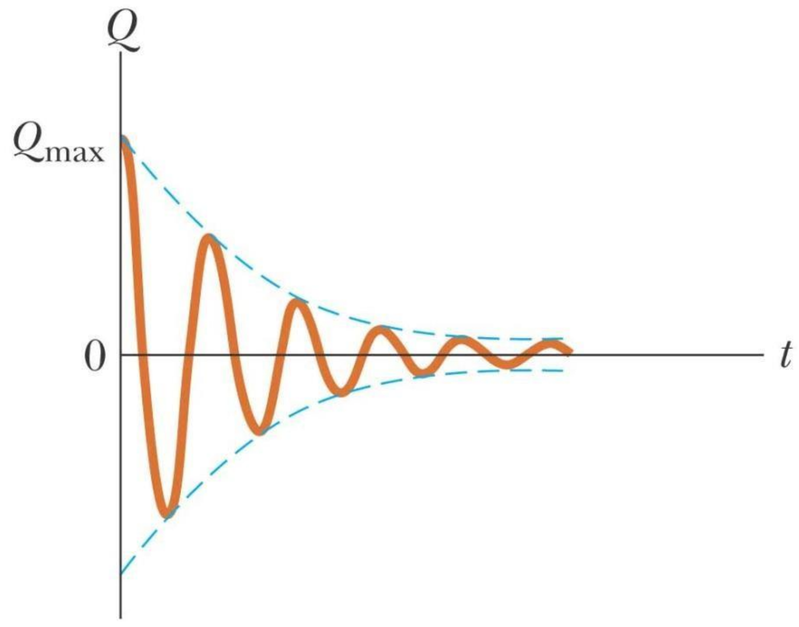
Solution:

$$Q = Q_{max} e^{-\alpha t} \cos\left(\omega_d t + \phi\right)$$

$$\alpha = \frac{R}{2L} \qquad \omega_d = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

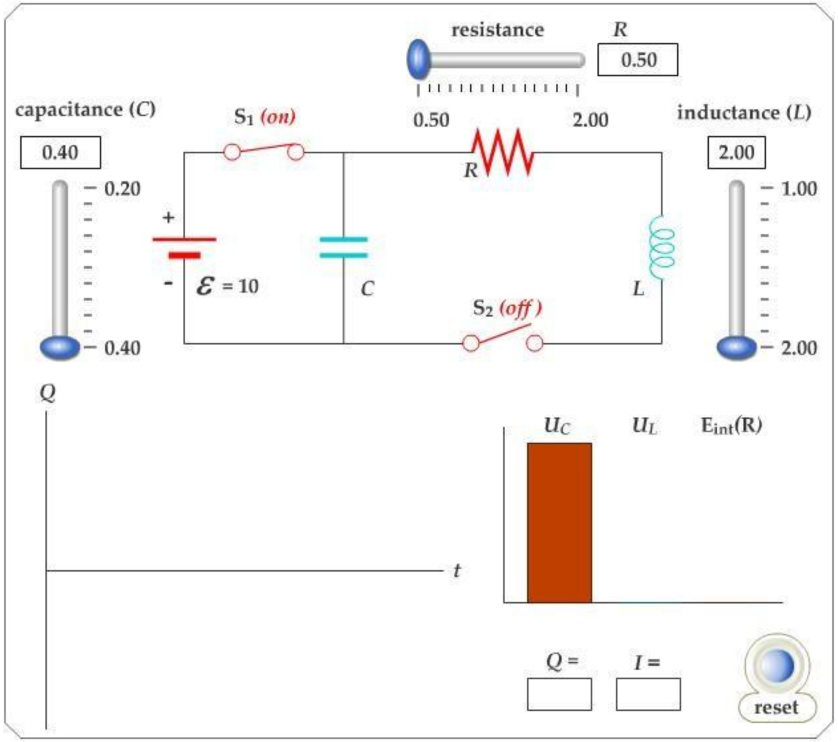
Damped *RLC* Circuit

- The maximum value of *Q* decreases after each oscillation
  - $R < R_C$
- This is analogous to the amplitude of a damped spring-mass system



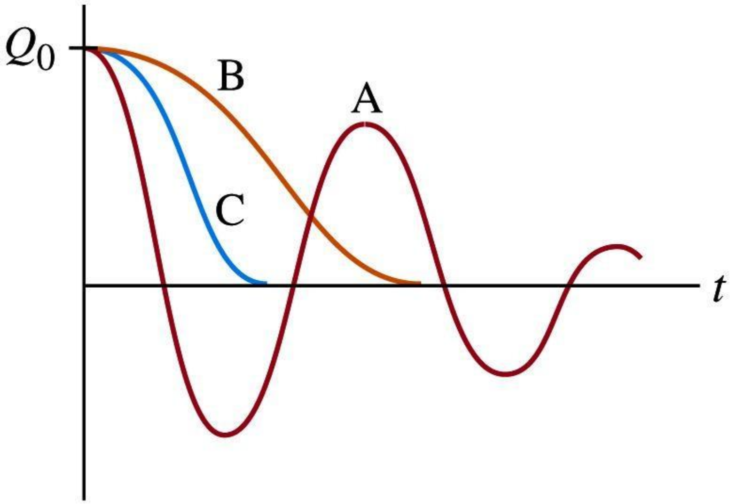
(a)

Active Figure 32.21



(SLIDESHOW MODE ONLY)

LRC Circuits



$$Q = Q_o e^{-\frac{R}{2L}t}\cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

- Underdamped
- Critically Damped
- Overdamped

$$\frac{1}{LC} > \frac{R^2}{4L^2} \qquad \frac{4L}{C} > R^2$$

$$\frac{1}{LC} = \frac{R^2}{4L^2} \qquad \frac{4L}{C} = R^2$$

$$\frac{1}{LC} < \frac{R^2}{4L^2} \qquad \frac{4L}{C} < R^2$$

Table 32.1

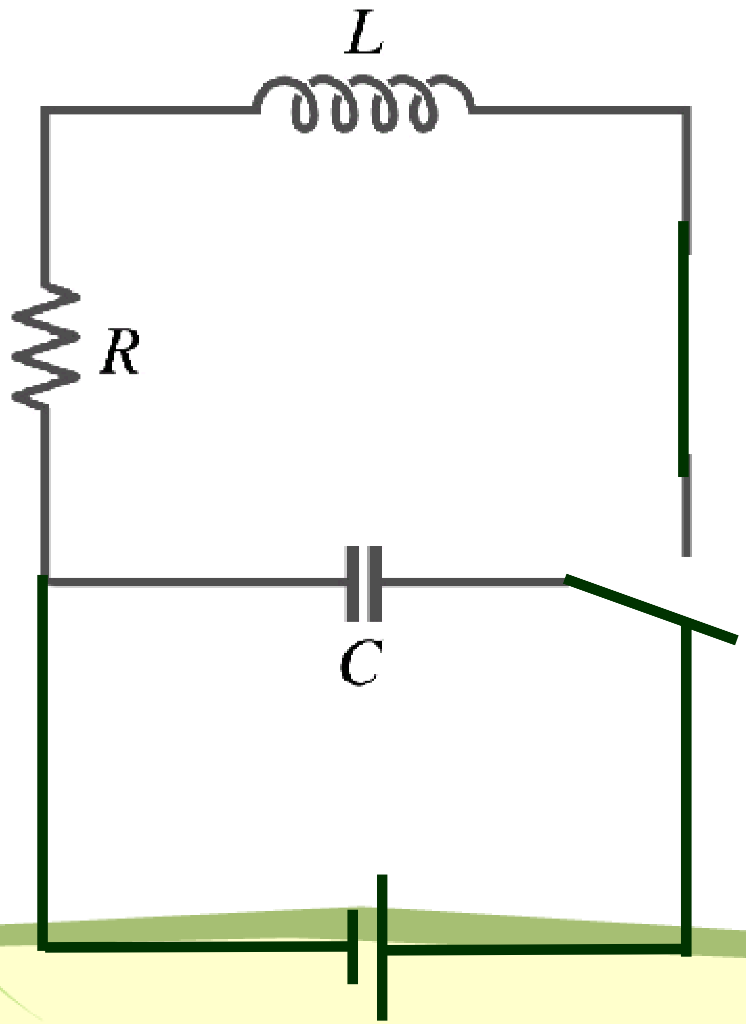
Analogies Between Electrical and Mechanical Systems		
Electric Circuit		**One-Dimensional Mechanical System**
Charge	$Q \leftrightarrow x$	Position
Current	$I \leftrightarrow v_x$	Velocity
Potential difference	$\Delta V \leftrightarrow F_x$	Force
Resistance	$R \leftrightarrow b$	Viscous damping coefficient
Capacitance	$C \leftrightarrow 1/k$	(k = spring constant)
Inductance	$L \leftrightarrow m$	Mass
Current = time derivative of charge	$I = \frac{dQ}{dt} \leftrightarrow v_x = \frac{dx}{dt}$	Velocity = time derivative of position
Rate of change of current = second time derivative of charge	$\frac{dI}{dt} = \frac{d^2Q}{dt^2} \leftrightarrow a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$	Acceleration = second time derivative of position
Energy in inductor	$U_L = \frac{1}{2}LI^2 \leftrightarrow K = \frac{1}{2}mv^2$	Kinetic energy of moving object
Energy in capacitor	$U_C = \frac{1}{2}\frac{Q^2}{C} \leftrightarrow U = \frac{1}{2}kx^2$	Potential energy stored in a spring
Rate of energy loss due to resistance	$I^2R \leftrightarrow bv^2$	Rate of energy loss due to friction
RLC circuit	$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0 \leftrightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$	Damped object on a spring

41. An emf of 96.0 mV is induced in the windings of a coil when the current in a nearby coil is increasing at the rate of 1.20 A/s. What is the mutual inductance of the two coils?

49. A fixed inductance L = 1.05 μH is used in series with a variable capacitor in the tuning section of a radiotelephone on a ship. What capacitance tunes the circuit to the signal from a transmitter broadcasting at 6.30 MHz?

55. Consider an LC circuit in which L = 500 mH and C = 0.100 μF. (a) What is the resonance frequency ω_0? (b) If a resistance of 1.00 k**Ω** is introduced into this circuit, what is the frequency of the (damped) oscillations? (c) What is the percent difference between the two frequencies?

LC Demo



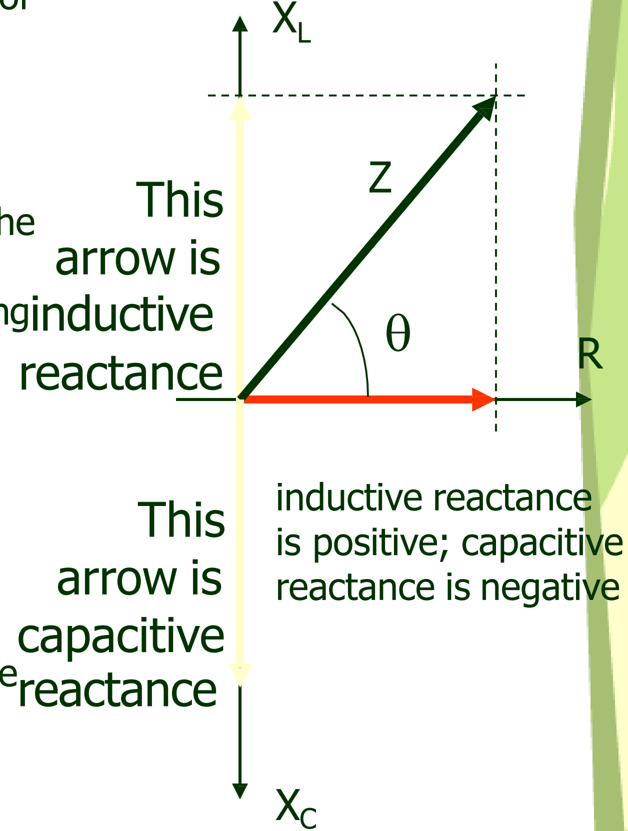
R = 10 W
C = 2.5 mF
L = 850 mH

1. Calculate period
2. What if we change C = 10 mF
3. Underdamped?
4. How can we change damping?

R and X in Series

Inductors and capacitors resist the flow of AC. This property is called reactance. Resistance also impedes the flow of AC. The combination of reactance and resistance in a circuit is called ***impedance***.

Reactance and resistance which are in series cannot simply be added because they differ in the phase relationship between current and voltage. With reactance, current and voltage are out of phase ("ELI the ICE man") while with resistance they are in phase. A method called triangulation is used to combine the two and to determine the resulting phase relationship.



The impedance of the reactance and resistance in series is found by completing the triangle whose legs are R and X:

$$Z = \sqrt{R^2 + X^2}$$

The phase angle between current and voltage (the angle by which the voltage leads the current is given by:

$$\theta = \arctan \frac{X}{R}$$

Make sure that X is given the correct sign (positive or negative) when finding the phase angle.

R and X in Parallel

For a parallel combination of resistance and reactance, we use a different formula for finding the resulting impedance:

$$Z = \frac{RX}{\sqrt{R^2 + X^2}}$$

In this case, the phase angle by which voltage leads current is given by

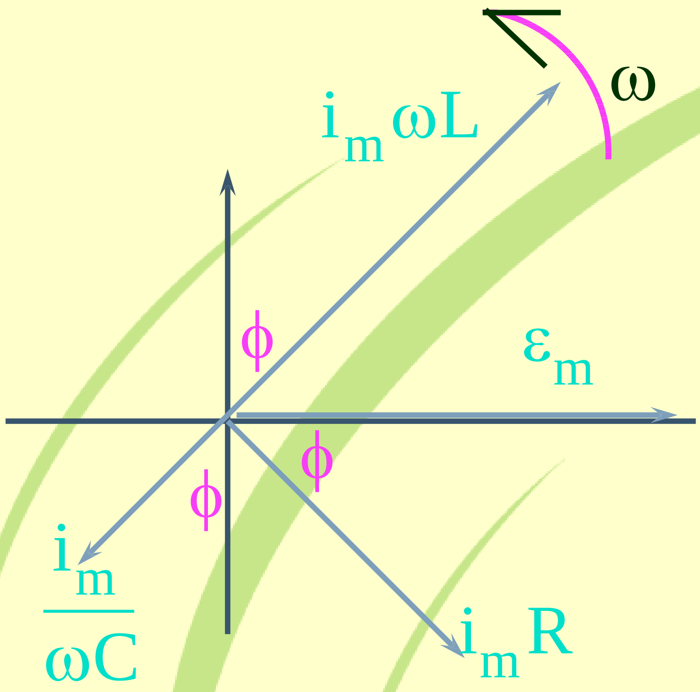
$$\theta = \arctan \frac{R}{X}$$

If the reactance is capacitive, the phase angle is negative. If the reactance is inductive, the phase angle is positive.

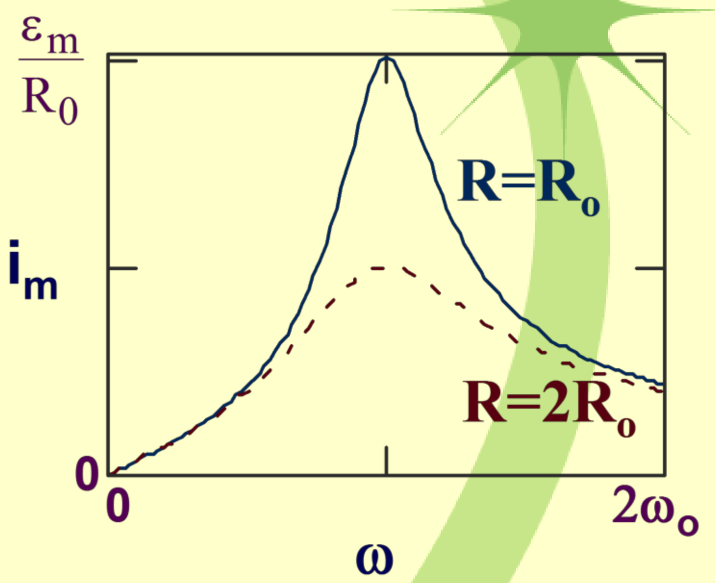
Ohm's Law for Impedance

$$E = IZ \qquad I = \frac{E}{Z} \qquad Z = \frac{E}{I}$$

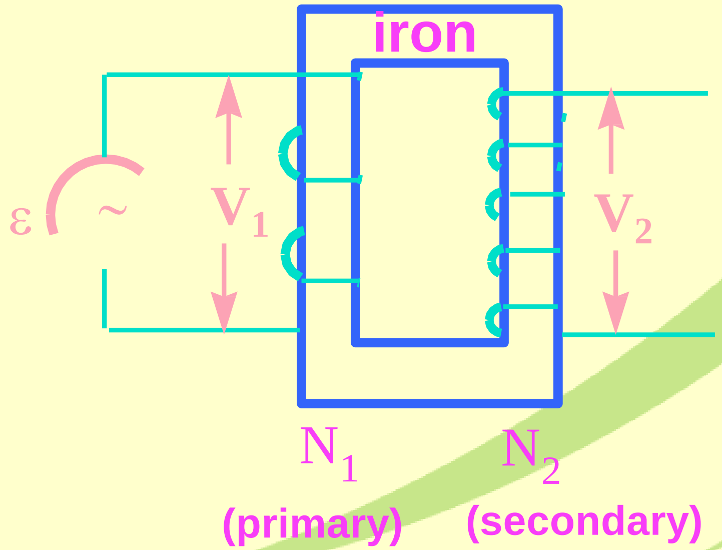
AC Circuits



ω
$i_m \omega L$
ϕ
ε_m
ϕ
ϕ
$\frac{i_m}{\omega C}$
$i_m R$



$\frac{\varepsilon_m}{R_0}$
i_m
R=R$_o$
R=2R$_o$
0
0
2ω$_o$
ω



iron
ε
~
V_1
V_2
N_1
(primary)
N_2
(secondary)

Lecture Outline

- Driven Series LCR Circuit:
  - General solution
  - Resonance condition
    - Resonant frequency
    - “Sharpness of resonance” = Q
  - Power considerations
    - Power factor depends on impedance
- Transformers
  - Voltage changes
  - Faraday’s Law in action gives induced primary current.
  - Power considerations

Text Reference: Chapter 33.4-6

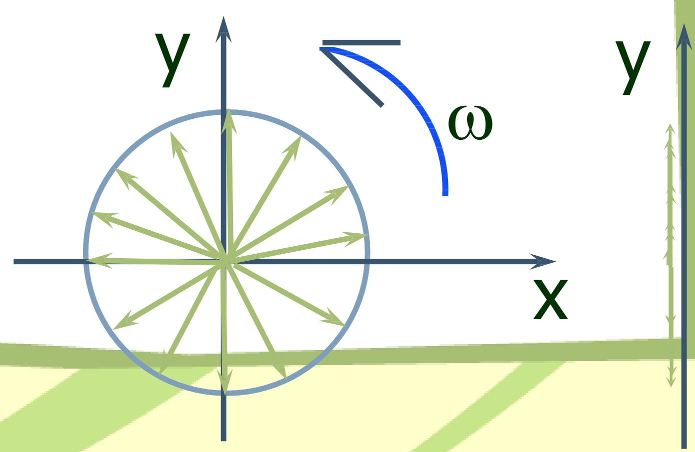
Phasors

- R: V in phase with i $\quad V_R = Ri_R = \varepsilon_m \sin\omega t \;\Rightarrow\; i_R = \frac{\varepsilon_m}{R}\sin\omega t$
- C: V lags i by 90° $\quad V_C = \frac{Q}{C} = \varepsilon_m \sin\omega t \;\Rightarrow\; i_C = \omega C \varepsilon_m \cos\omega t$
- L: V leads i by 90° $\quad V_L = L\frac{di_L}{dt} = \varepsilon_m \sin\omega t \;\Rightarrow\; i_L = -\frac{\varepsilon_m}{\omega L}\cos\omega t$

- A phasor is a vector whose magnitude is the maximum value of a quantity (eg V or I) and which rotates counterclockwise in a 2-d plane with angular velocity ω. Recall uniform circular motion:

$$x = r\cos\omega t$$
$$y = r\sin\omega t$$

The projections of r (on the vertical y axis) execute sinusoidal oscillation.



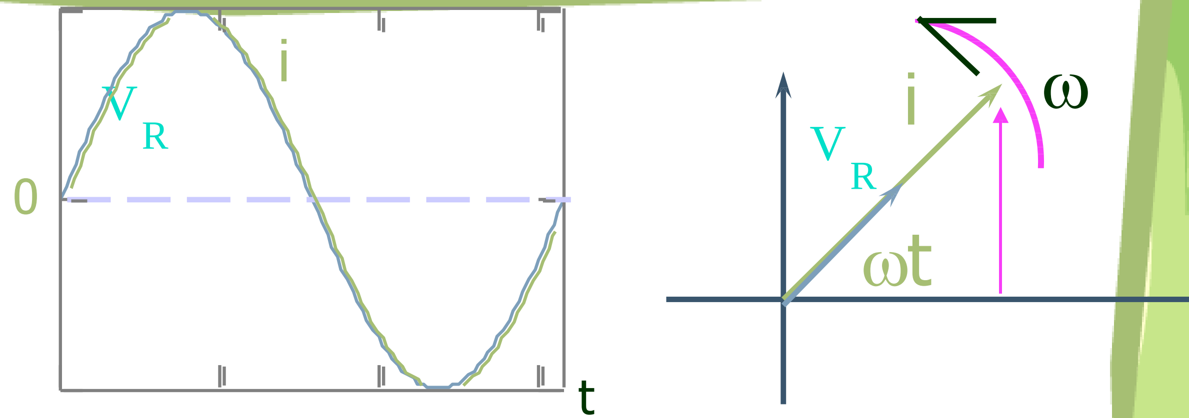
Phasors for L,C,R

Suppose:

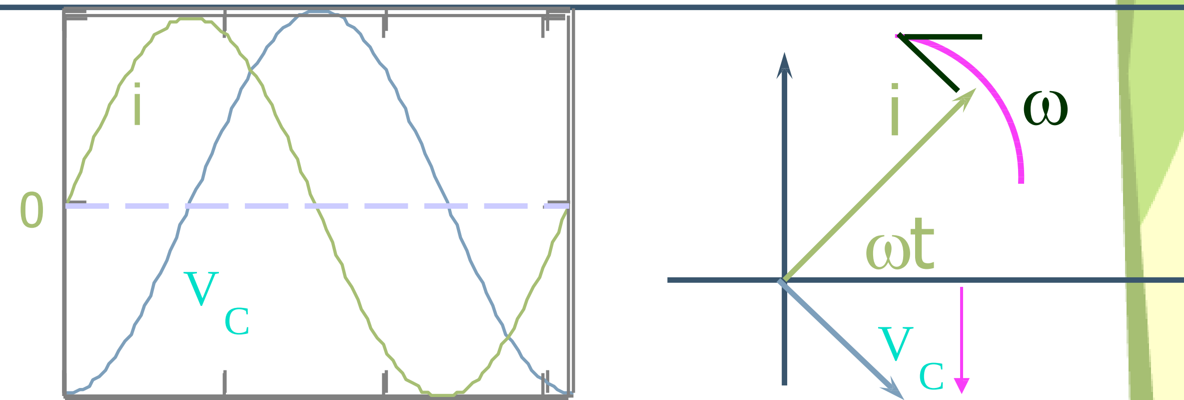
$i = i_m \sin \omega t$

$\Downarrow$

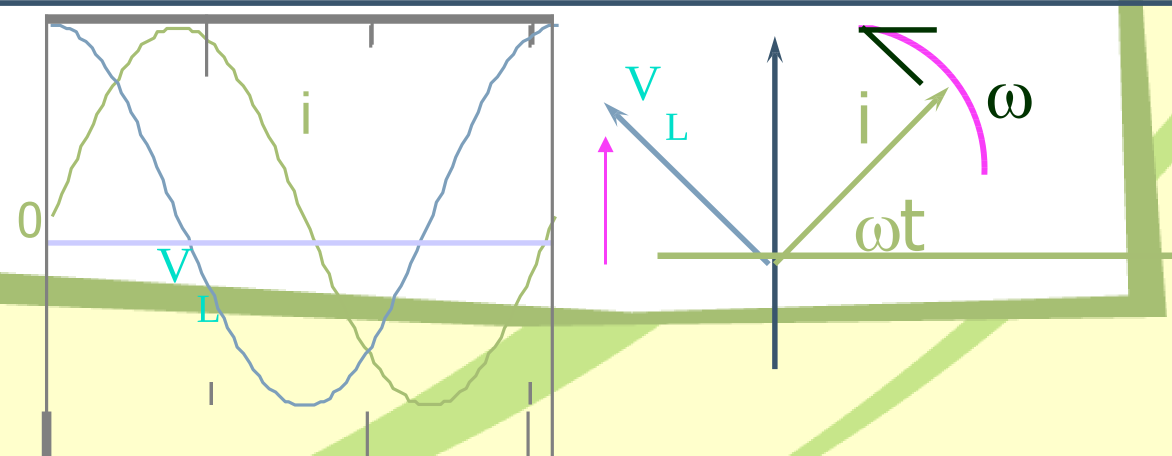
$V_R = R i_m \sin\omega t$



$V_C = -\dfrac{1}{\omega C} i_m \cos\omega t$

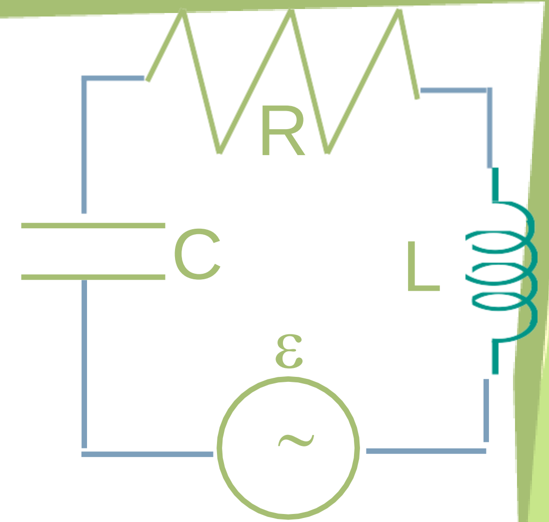


$V_L = \omega L i_m \cos\omega t$



Series LCR

AC Circuit



- Back to the original problem: the loop equation gives:

$$L\frac{d^2Q}{dt^2}+\frac{Q}{C}+R\frac{dQ}{dt}=\varepsilon_m \sin\omega t$$

- Assume a solution of the form:

$$i = i_m \sin(\omega t - \phi)$$

- Here all unknowns, (i_m,ϕ) , must be found from the loop eqn; the initial conditions have been taken care of by taking the emf to be: $\varepsilon = \varepsilon_m \sin\omega t$.

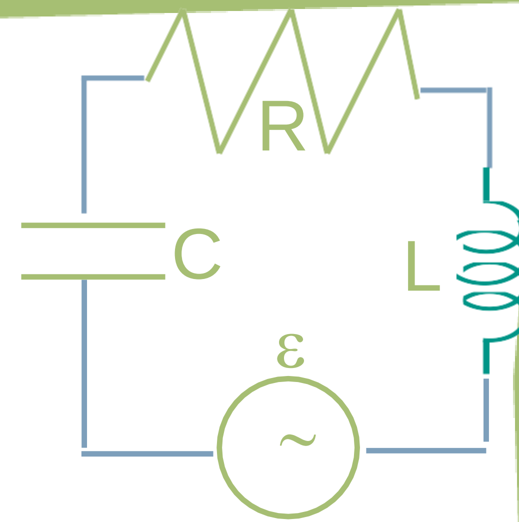
- To solve this problem graphically, first write down expressions for the voltages across R,C, and L and then plot the appropriate phasor diagram.

Phasors: LCR

- Given: $\varepsilon = \varepsilon_m \sin\omega t$
- Assume:

$$i = i_m \sin(\omega t - \phi) \quad \Rightarrow \quad Q = -\frac{i_m}{\omega}\cos(\omega t - \phi)$$

$$\frac{di}{dt} = i_m \omega \cos(\omega t - \phi)$$

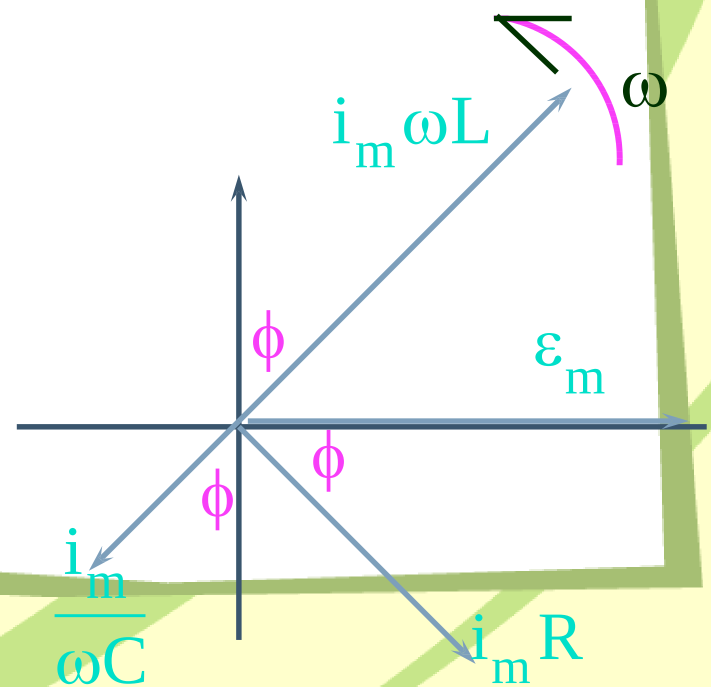


$$\Rightarrow \quad V_R = Ri = Ri_m \sin(\omega t - \phi)$$

$$V_C = \frac{Q}{C} = -\frac{1}{\omega C} i_m \cos(\omega t - \phi)$$

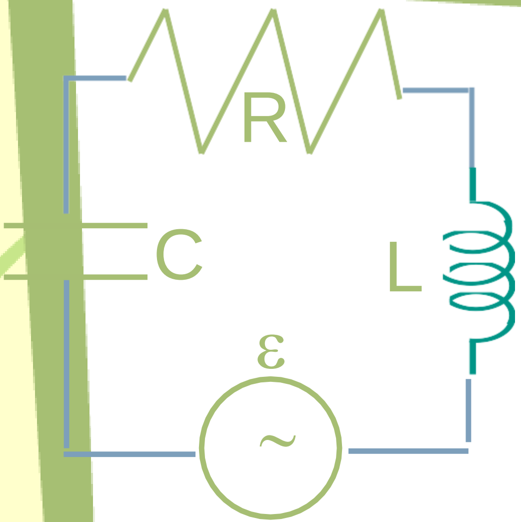
$$V_L = L\frac{di}{dt} = \omega L i_m \cos(\omega t - \phi)$$

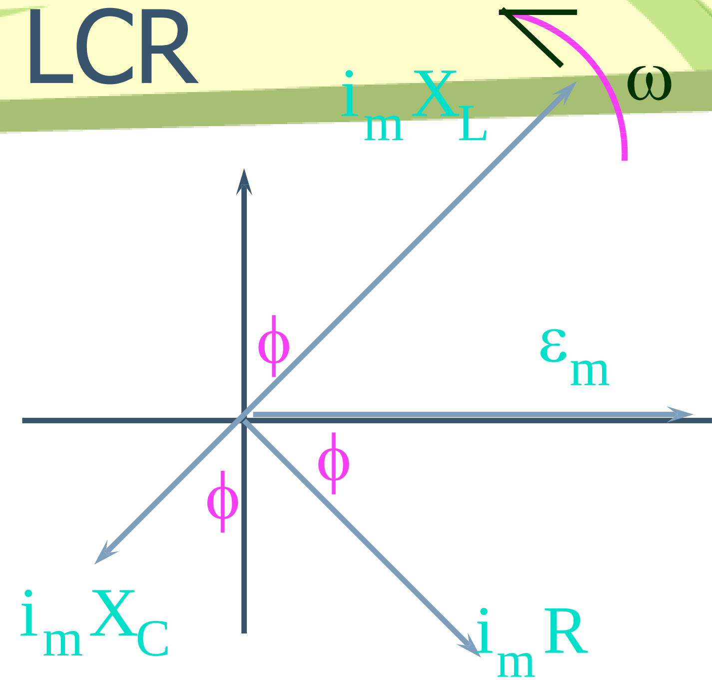
- From these equations, we can draw the phasor diagram to the right.



- This picture corresponds to a snapshot at t=0. The projections of these phasors along the vertical axis are the actual values of the voltages at the given time.

Phasors: LCR





- The phasor diagram has been relabeled in terms of the reactances defined from:

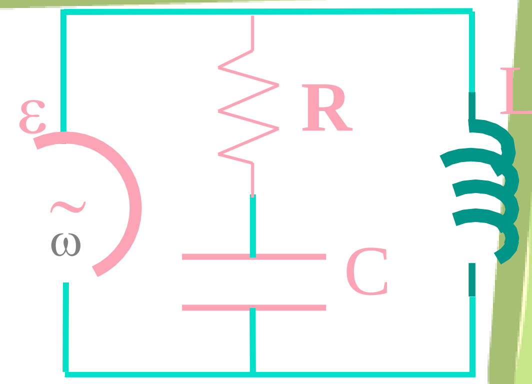
Ohms ->

$$X_L \equiv \omega L \qquad X_C \equiv \frac{1}{\omega C}$$

The unknowns (i_m, ϕ) can now be solved for graphically since the vector sum of the voltages $V_L + V_C + V_R$ must sum to the driving emf ε.

Lecture 20, ACT 3

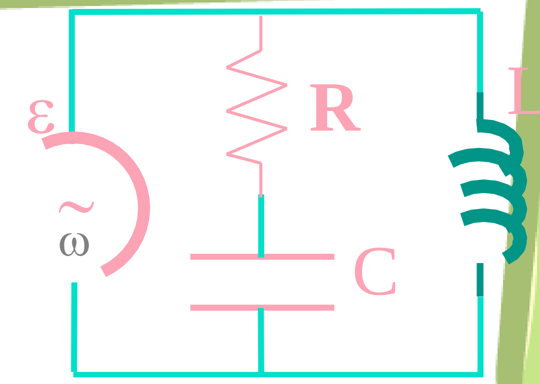
- A driven RLC circuit is connected as shown.
  - For what frequencies ω of the voltage source is the current through the resistor largest?



(a) ω small (b) ω large (c) $\omega = \frac{1}{\sqrt{LC}}$

Conceptual Question

- A driven RLC circuit is connected as shown.
  - For what frequencies ω of the voltage source is the current through the resistor largest?



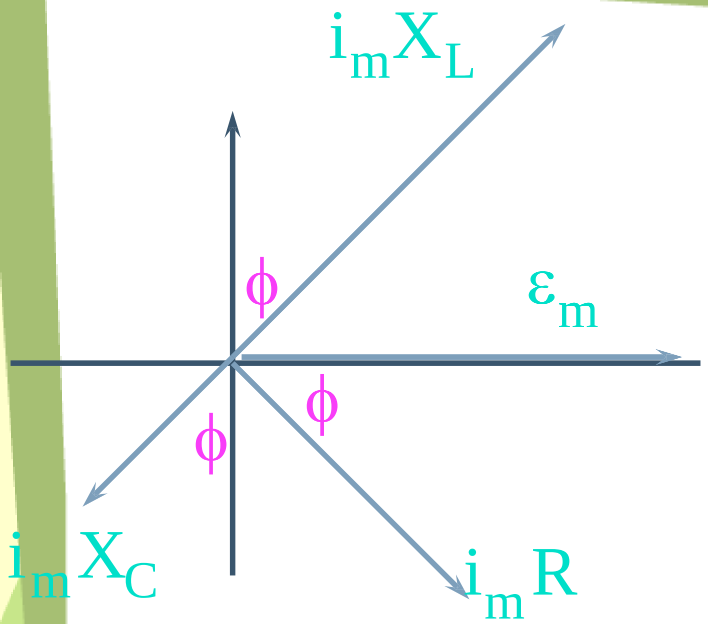
(a) ω small $\boxed{\text{(b) } \omega \text{ large} \quad \text{(c) } \omega =} \dfrac{1}{\sqrt{LC}}$

•This is NOT a series RLC circuit. We cannot blindly apply our techniques for solving the circuit. We must think a little bit.

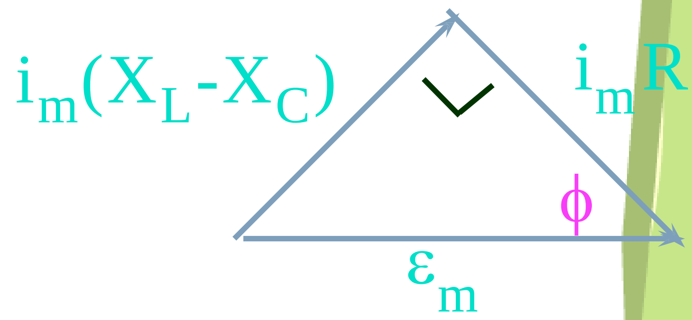
•However, we can use the frequency dependence of the impedances (reactances) to answer this question.

- The reactance of an inductor = $X_L = \omega L$.
- The reactance of a capacitor = $X_C = 1/(\omega C)$.
- Therefore,
  - in the low frequency limit, $X_L \rightarrow 0$ and $X_C \rightarrow \infty$.
    - •Therefore, as $\omega \rightarrow 0$, the current will flow mostly through the inductor; the current through the capacitor approaches 0.
  - in the high frequency limit, $X_L \rightarrow \infty$ and $X_C \rightarrow 0$.
    - •Therefore, as $\omega \rightarrow \infty$, the current will flow mostly through the capacitor, approaching a maximum $i_{max} = \varepsilon/R$.

Phasors:LCR



$\Rightarrow$



$$\tan\phi = \frac{X_L - X_C}{R}$$

$$X_L \equiv \omega L$$

$$X_C \equiv \frac{1}{\omega C}$$

$$\varepsilon_m^2 = i_m^2\left(R^2 + (X_L - X_C)^2\right)$$

$$\Downarrow$$

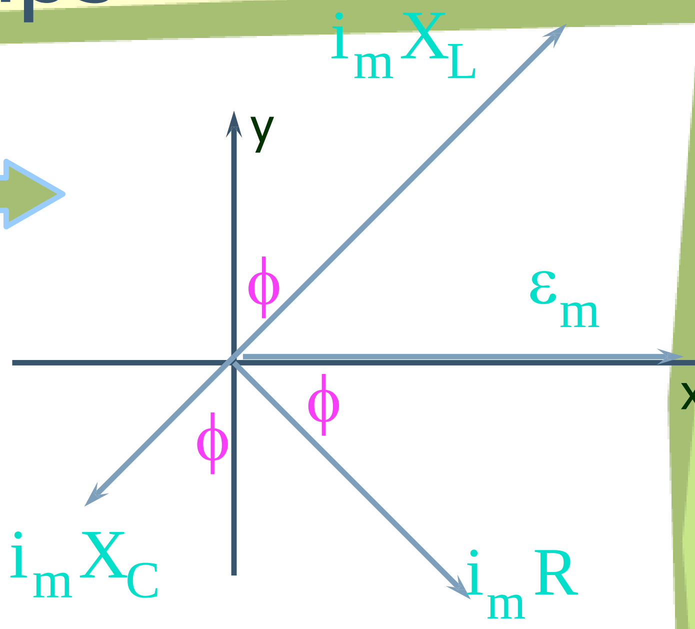
$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

$$i_m = \frac{\varepsilon_m}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\varepsilon_m}{Z}$$

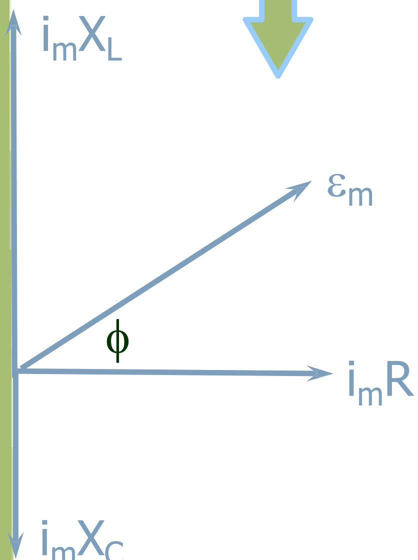
Impedance

Phasors:Tips

•This phasor diagram was drawn as a snapshot of time t=0 with the voltages being given as the projections along the y-axis.

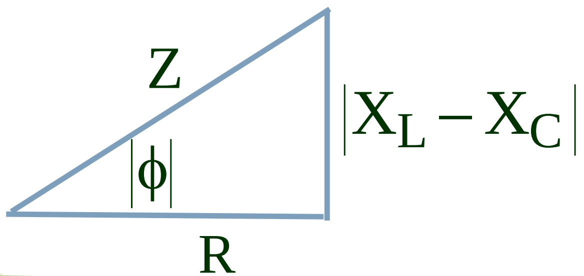


•Sometimes, in working problems, it is easier to draw the diagram at a time when the current is along the x-axis (when i=0).



"Full Phasor Diagram"

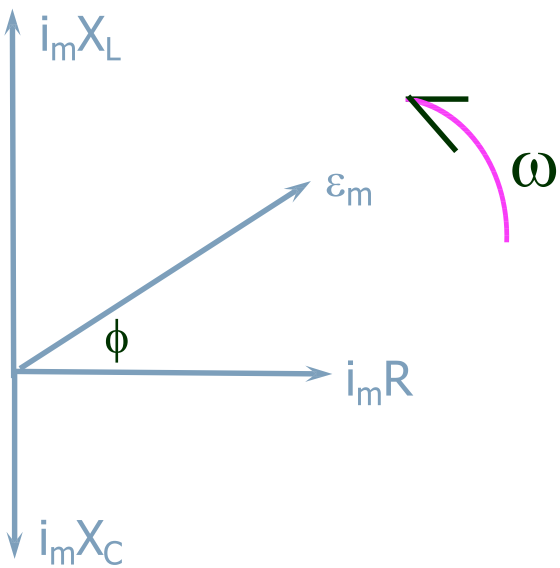
From this diagram, we can also create a triangle which allows us to calculate the impedance Z:



" Impedance Triangle"

Phasors:LCR

We have found the general solution for the driven LCR circuit:



the loop eqn

$$\vec{\varepsilon} = i_m \vec{Z}$$

$$i = i_m \sin(\omega t - \phi)$$

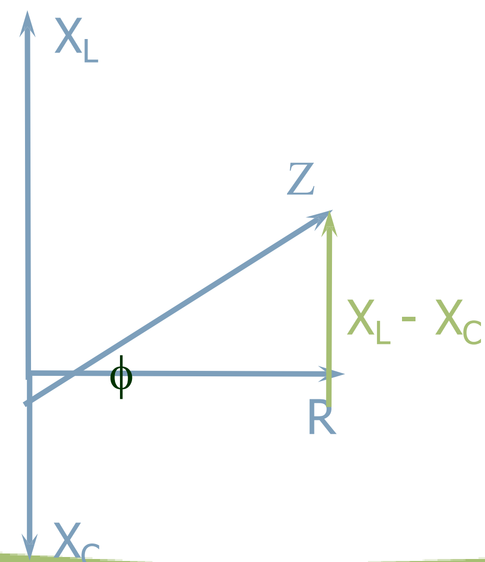
$$\tan\phi = \frac{X_L - X_C}{R}$$

$$i_m = \frac{\varepsilon_m}{Z}$$

$$X_L \equiv \omega L$$

$$X_C \equiv \frac{1}{\omega C}$$

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$



Lagging & Leading

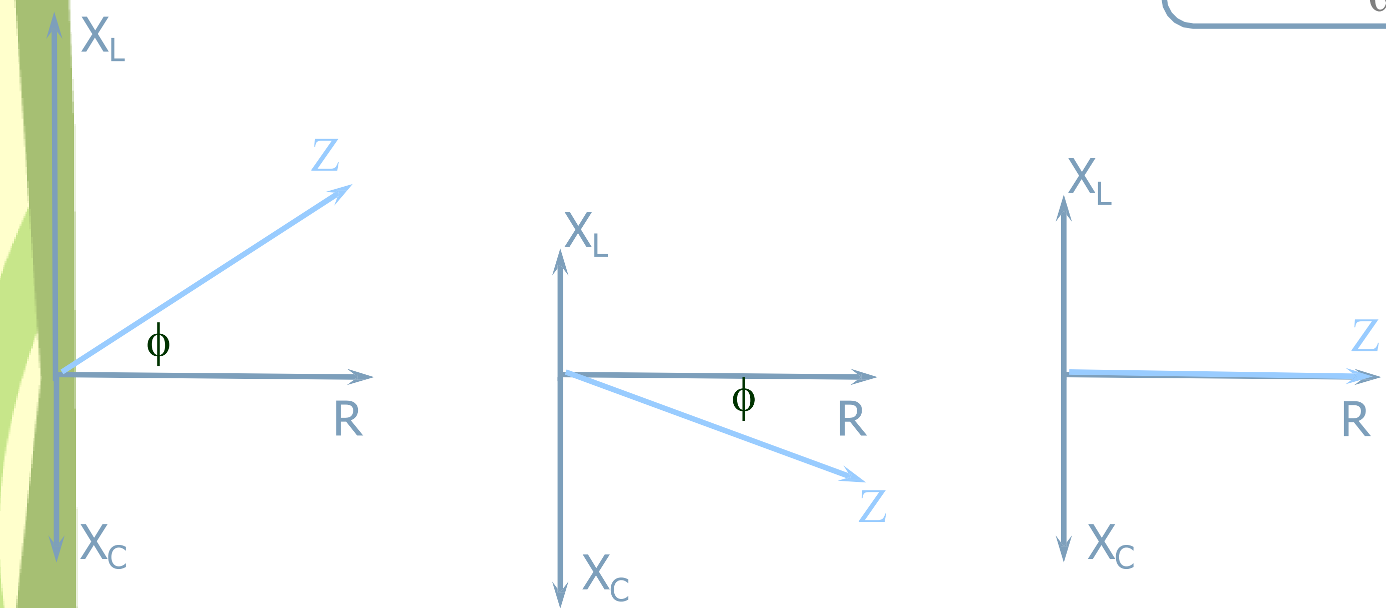
The phase ϕ between the current and the driving emf depends on the relative magnitudes of the inductive and capacitive reactances.

$$i_m = \frac{\varepsilon_m}{Z}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$X_L \equiv \omega L$$

$$X_C \equiv \frac{1}{\omega C}$$



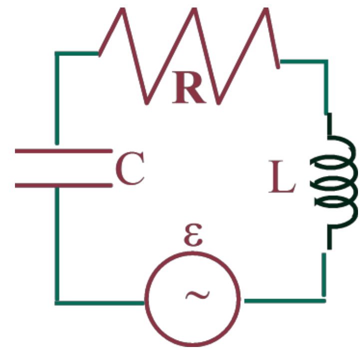
$X_L > X_C$
$\phi > 0$
current
LAGS
applied voltage

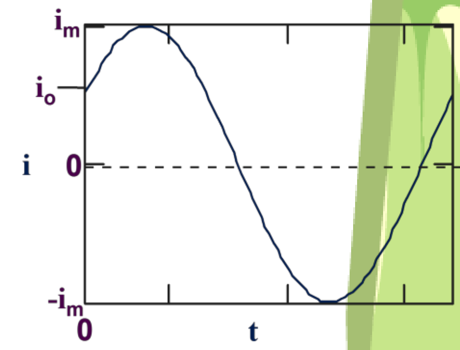
$X_L < X_C$
$\phi < 0$
current
LEADS
applied voltage

$X_L = X_C$
$\phi = 0$
current
IN PHASE
applied voltage

Conceptual Question

- The series LCR circuit shown is driven by a generator with voltage $\varepsilon = \varepsilon_m \sin\omega t$. The time dependence of the current i which flows in the circuit is shown in the plot.





1A

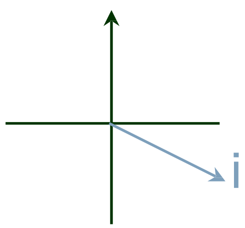
– How should ω be changed to bring the current and driving voltage into phase?

(a) increase ω (b) decrease ω (c) impossible

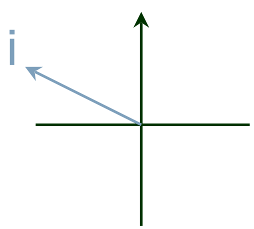
1B

- Which of the following phasors represents the current i at t=0?

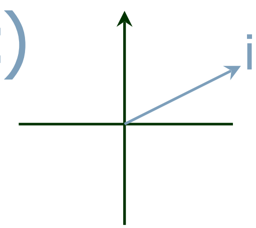
(a)



(b)

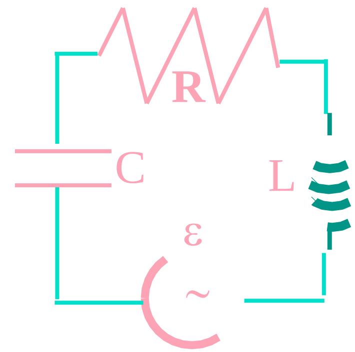


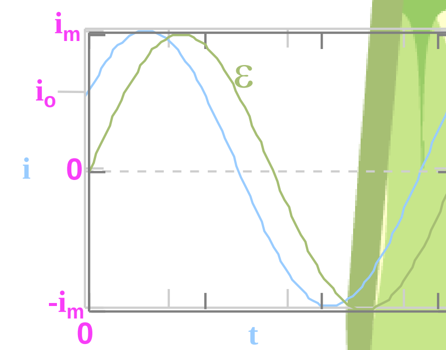
(c)



Lecture 21, ACT 1

- The series LCR circuit shown is driven by a generator with voltage $\varepsilon = \varepsilon_m \sin\omega t$. The time dependence of the current i which flows in the circuit is shown in the plot.
  - How should ω be changed to bring the current and driving voltage into phase?



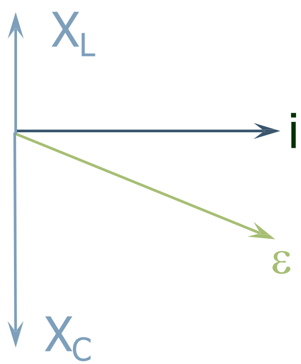


1A

(a) increase ω (b) decrease ω (c) impossible

- From the plot, it is clear that the current is LEADING the applied voltage.

Therefore, the phasor diagram must look like this:

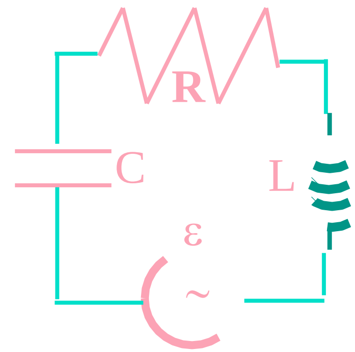


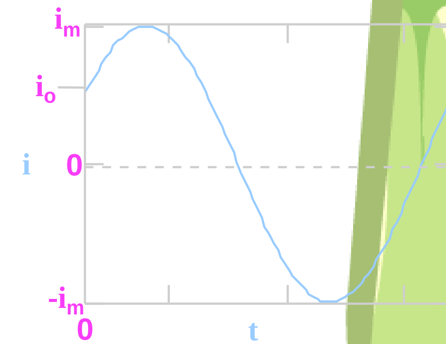
Therefore, $X_C > X_L$

- To bring the current into phase with the applied voltage, we need to increase X_L and decrease X_C.
- Increasing ω will do both!!

Lecture 21, ACT 1

- The series LCR circuit shown is driven by a generator with voltage $\varepsilon = \varepsilon_m \sin\omega t$. The time dependence of the current i which flows in the circuit is shown in the plot.
  - How should ω be changed to bring the current and driving voltage into phase?





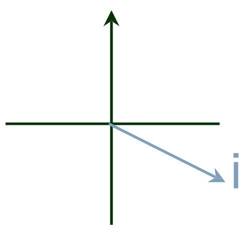
1A

(a) increase ω (b) decrease ω (c) impossible

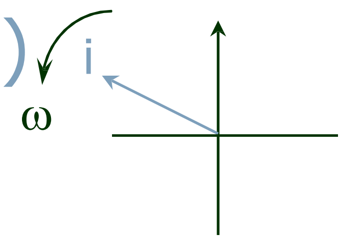
1B

- Which of the following phasors represents the current i at t=0?

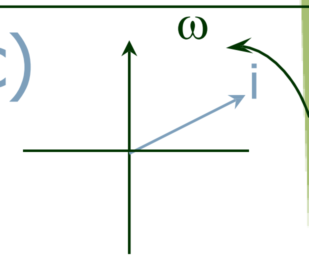
(a)



(b)



(c)



- The projection of i along the vertical axis is negative here.
- no way jose

- The sign of i is correct at t=0.
- However, it soon will become negative!
- nope

- This one looks just right!!
- $\phi = -30^\circ$

Resonance

- For fixed R,C,L the current i_m will be a maximum at the resonant frequency ω_0 which makes the impedance Z purely resistive.

ie: $$i_m = \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

reaches a maximum when: $$\boxed{X_L = X_C}$$

the frequency at which this condition is obtained is given from:

$$\omega_o L = \frac{1}{\omega_o C} \quad \Rightarrow \quad \boxed{\omega_o = \frac{1}{\sqrt{LC}}}$$

- Note that this resonant frequency is identical to the natural frequency of the LC circuit by itself!
- At this frequency, the current and the driving voltage are in phase! $\tan\phi = \dfrac{X_L - X_C}{R} = 0$

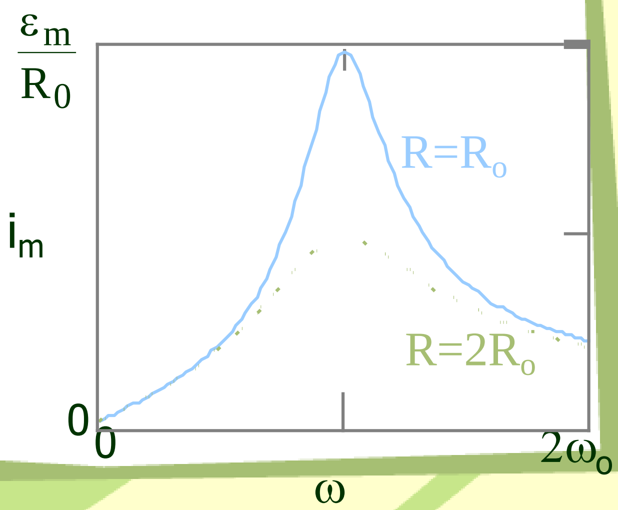
Resonance

The current in an LCR circuit depends on the values of the elements and on the driving frequency through the relation

$$i_m = \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\boxed{i_m = \frac{\varepsilon_m}{R}\frac{1}{\sqrt{1 + \tan^2\phi}} = \frac{\varepsilon_m}{R}\cos\phi}$$

Suppose you plot the current versus ω, the source voltage frequency, you would get:



Power in LCR Circuit

- The power supplied by the *emf* in a series LCR circuit depends on the frequency ω. It will turn out that the maximum power is supplied at the resonant frequency ω_0.

- The instantaneous power (for some frequency, ω) delivered at time t is given by:

$$P(t) = \varepsilon(t)i(t) = \left(\varepsilon_m \sin\omega t\right)\left(i_m \sin(\omega t - \phi)\right)$$

Remember what this stands for

- The most useful quantity to consider here is not the instantaneous power but rather the average power delivered in a cycle.

$$\langle P(t)\rangle = \varepsilon_m i_m \langle \sin\omega t \sin(\omega t - \phi)\rangle$$

- To evaluate the average on the right, we first expand the $\sin(\omega t-\phi)$ term.

Power in LCR Circuit

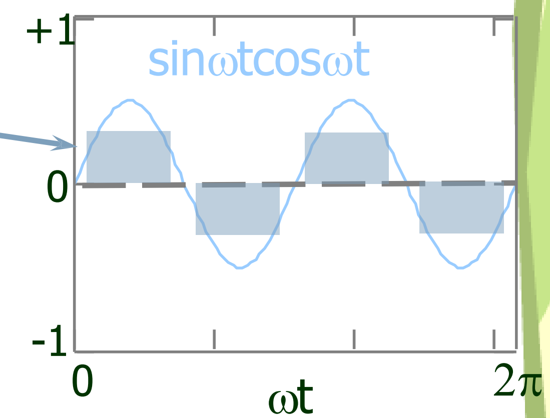
- Expanding,

$$\sin\omega t\sin(\omega t-\phi)=\sin\omega t(\sin\omega t\cos\phi-\cos\omega t\sin\phi)$$

- Taking the averages,

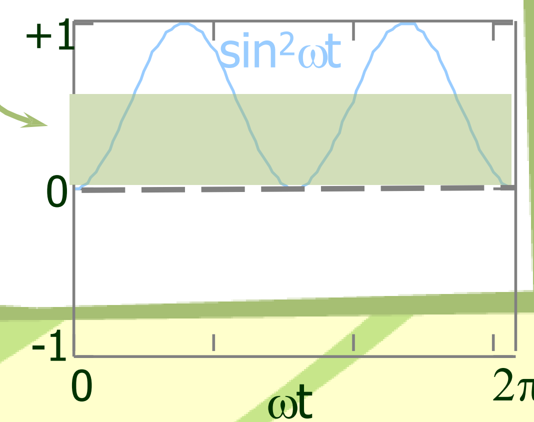
$$\langle\sin\omega t\cos\omega t\rangle=0$$

(Product of even and odd function = 0)



- Generally:

$$\langle\sin^2 x\rangle=\frac{1}{2\pi}\int_0^{2\pi}\sin^2 x\,dx=\frac{1}{2}$$



- Putting it all back together again,

$$\langle P(t)\rangle=\varepsilon_m i_m\left\{\cos\phi\cancelto{1/2}{\langle\sin^2\omega t\rangle}-\sin\phi\cancelto{0}{\langle\sin\omega t\cos\omega t\rangle}\right\}$$

$$\boxed{\langle P(t)\rangle=\frac{1}{2}\varepsilon_m i_m\cos\phi}$$

Power in LCR Circuit

- This result is often rewritten in terms of rms values:

$$\boxed{\varepsilon_{rms} \equiv \frac{1}{\sqrt{2}}\varepsilon_m} \qquad \boxed{i_{rms} \equiv \frac{1}{\sqrt{2}} i_m} \quad \Rightarrow \quad \boxed{\langle P(t) \rangle = \varepsilon_{rms} i_{rms} \cos\phi}$$

- Power delivered depends on the phase, ϕ, the "power factor"
- phase depends on the values of L, C, R, and ω
- therefore...

Power in RLC

$$\langle P(t)\rangle = \varepsilon_{rms} i_{rms} \cos\phi$$

- Power, as well as current, peaks at $\omega = \omega_0$. The *sharpness* of the resonance depends on the values of the components.
- Recall:

$$i_m = \frac{\varepsilon_m}{R}\frac{1}{\sqrt{1+\tan^2\phi}} = \frac{\varepsilon_m}{R}\cos\phi$$

- Therefore,

$$\langle P(t)\rangle = \frac{\varepsilon^2{}_{rms}}{R}\cos^2\phi$$

We can write this in the following manner (you can do the algebra):

$$\langle P(t)\rangle = \frac{\varepsilon^2{}_{rms}}{R}\frac{x^2}{x^2+Q^2(x^2-1)^2} \qquad x = \frac{\omega}{\omega_o}$$

...introducing the curious factors Q and x...

$$Q = \frac{L\omega_o}{R}$$

The Q factor

A parameter "Q" is often defined to describe the *sharpness* of resonance peaks in both mechanical and electrical oscillating systems. "Q" is defined as

$$Q \equiv 2\pi \frac{U_{max}}{\Delta U}$$

where U_{max} is max energy stored in the system and ΔU is the energy dissipated in one cycle

period

For RLC circuit, U_{max} is (e.g.)

$$U_{max} = \frac{1}{2} L I^2_{max}$$

And losses only in R, namely

$$\Delta U = \frac{1}{2} I^2_{max} R T = \frac{1}{2} I^2_{max} R \left(\frac{2\pi}{\omega_{res}} \right)$$

This gives

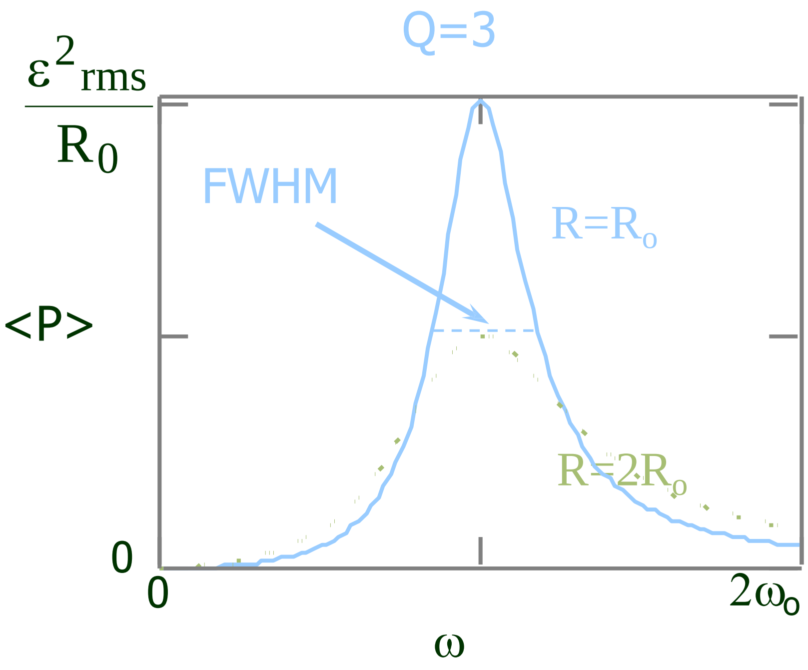
$$\boxed{Q \equiv \frac{\omega_{res} L}{R}}$$

And for completeness, note

$$\boxed{x \equiv \frac{\omega}{\omega_{res}}}$$

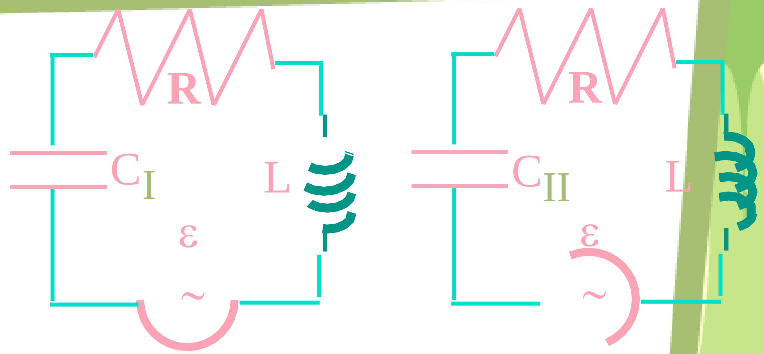
Power in RLC

For Q > few, $Q \approx \frac{\omega_{res}}{fwhm}$



Conceptual Question 2

- Consider the two circuits shown where $C_{II} = 2\ C_I$.
  - What is the relation between the quality factors, Q_I and Q_{II}, of the two circuits?



2A

(a) $Q_{II} < Q_I$ (b) $Q_{II} = Q_I$ (c) $Q_{II} > Q_I$

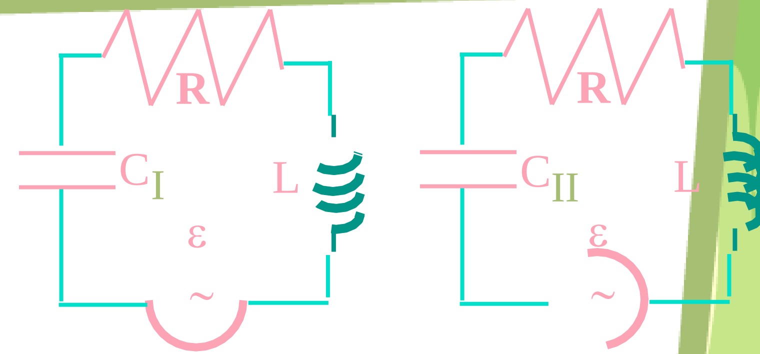
2B

- What is the relation between P_I and P_{II}, the power delivered by the generator to the circuit when each circuit is operated at its resonant frequency?

(a) $P_{II} < P_I$ (b) $P_{II} = P_I$ (c) $P_{II} > P_I$

Lecture 21, ACT 2

- Consider the two circuits shown where $C_{II} = 2\,C_I$.
  - What is the relation between the quality factors, Q_I and Q_{II}, of the two circuits?



2A

$\boxed{\text{(a) } Q_{II} < Q_I}$ (b) $Q_{II} = Q_I$ (c) $Q_{II} > Q_I$

- We know the definition of Q: $$Q \equiv \frac{\omega_0 L}{R}$$

- At first glance, it looks like Q is independent of C.
- At second glance, we see this cannot be true, since the resonant frequency ω_o depends on C!

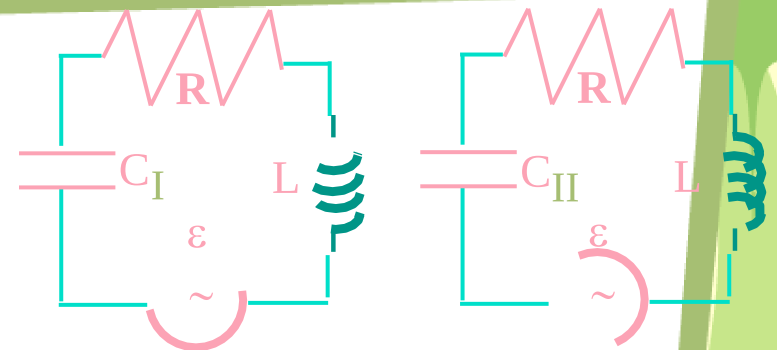
$$\omega_0 = \frac{1}{\sqrt{LC}}$$ ⇒ Doubling C decreases ω_o by sqrt(2)!

$\therefore$ Doubling C decreases Q by sqrt(2)!

Doubling C increases the width of the resonance!

Lecture 21, ACT 2

- Consider the two circuits shown where $C_{II} = 2\,C_I$.



2A

– What is the relation between the quality factors, Q_I and Q_{II}, of the two circuits?

$\boxed{\text{(a) } Q_{II} < Q_I}$ (b) $Q_{II} = Q_I$ (c) $Q_{II} > Q_I$

2B

- What is the relation between P_I and P_{II}, the power delivered by the generator to the circuit when each circuit is operated at its resonant frequency?

(a) $P_{II} < P_I$ (b) $P_{II} = P_I$ (c) $P_{II} > P_I$

- At the resonant frequency, the impedance of the circuit is purely resistive.
- Since the resistances in each circuit are the same, the impedances at the resonant frequency for each circuit are equal.
- Therefore, The power delivered by the generator to each circuit is identical.

Power Transmission

How can we transport power from power stations to homes? Why do we use "high tension" lin...

- At home, the AC voltage obtained from outlets in this country is 120V at 60Hz.
- Transmission of power is typically at very high voltages (eg ~500 kV)
- Transformers are used to raise the voltage for transmission and lower the voltage for use. We'll describe these next.

But why?

- Calculate ohmic losses in the transmission lines:
- Define *efficiency* of transmission:

$$\boxed{\varepsilon \equiv \frac{P_{out}}{P_{in}} = \frac{iV_{in} - i^2R}{iV_{in}} = 1 - \frac{iR}{V_{in}}\left(\frac{V_{in}}{V_{in}}\right) = 1 - \frac{P_{in}R}{V^2_{in}}}$$

Keep R small

Make V_{in} big

– Note for fixed input power and line resistance, the inefficiency $\propto 1/V^2$

Example: Quebec to Montreal

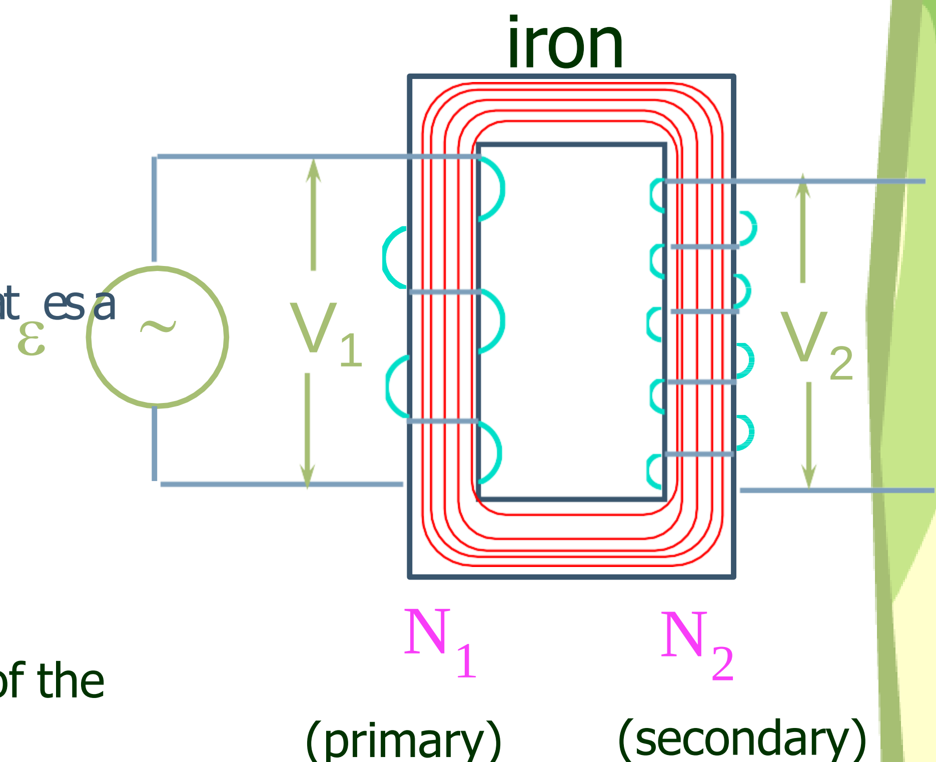
1000 km $\Rightarrow$ R= 220Ω

With V_{in}=735kV, ε = 80%.

The efficiency goes to zero quickly if V_{in} were lowered!

Transformers

- AC voltages can be *stepped up* or *stepped down* by the use of transformers.



- The AC current in the primary circuit creates a time-varying magnetic field in the iron

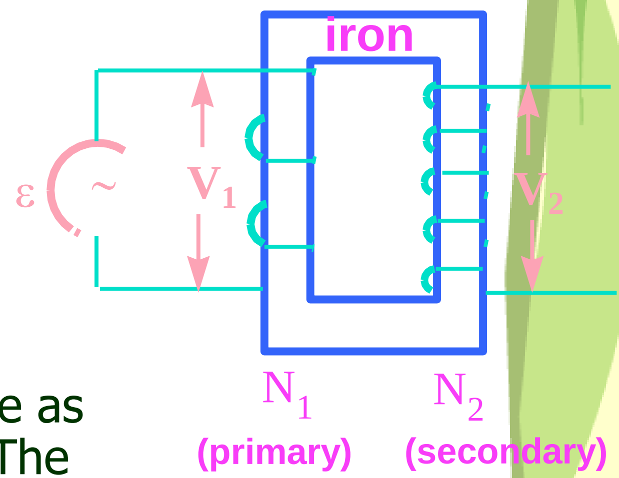
- This induces an *emf* on the secondary windings due to the mutual inductance of the two sets of coils.

- The iron is used to maximize the mutual inductance. We assume that the entire flux produced by each turn of the primary is trapped in the iron.

Ideal Transformers (no load)

resistance losses All flux contained in iron Nothing connected on secondary

- The primary circuit is just an AC voltage source in series with an inductor. The change in flux produced in each turn is given by:



$$\frac{d\phi_{turn}}{dt} = \frac{V_1}{N_1}$$

•The change in flux per turn in the secondary coil is the same as the change in flux per turn in the primary coil (ideal case). The induced voltage appearing across the secondary coil is given by:

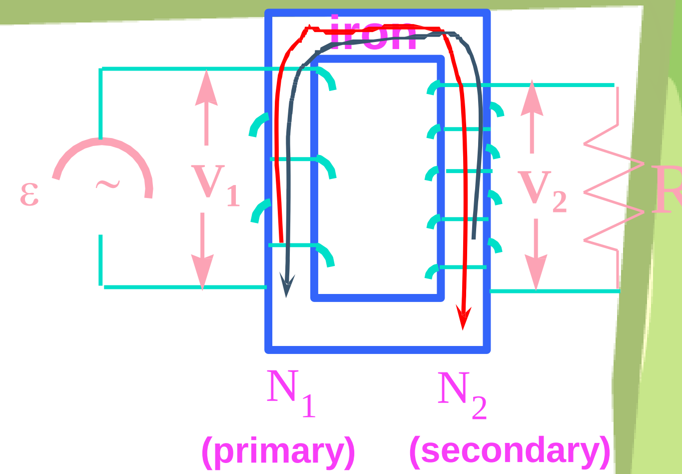
$$\boxed{V_2 = N_2 \frac{d\phi_{turn}}{dt} = \frac{N_2}{N_1} V_1}$$

- Therefore,
  - $N_2 > N_1 \Rightarrow$ secondary V_2 is larger than primary V_1 (step-up)
  - $N_1 > N_2 \Rightarrow$ secondary V_2 is smaller than primary V_1 (step-down)

•Note: "no load" means no current in secondary. The primary current, termed "the magnetizing current" is small!

Ideal Transformers with a Load

- What happens when we connect a resistive load to the secondary coil?
  - Flux produced by primary coil induces an *emf* in secondary



ç *emf* in secondary produces current i_2

$$i_2 = \frac{V_2}{R}$$

ç This current produces a flux in the secondary coil $\propto N_2 i_2$, which opposes the original flux -- Lenz's law

ç This changing flux appears in the primary circuit as well; the sense of it is to reduce the *emf* in the primary...

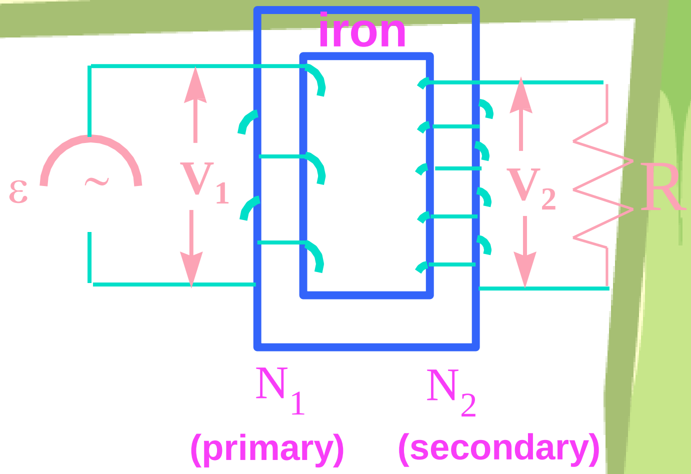
ç However, V_1 is a voltage source.

ç Therefore, there must be an increased current i_1 (supplied by the voltage source) in the primary which produces a flux $\propto N_1 i_1$ which exactly cancels the flux produced by i_2.

$$\boxed{i_1 = \frac{N_2}{N_1} i_2}$$

Transformers with a Load

- With a resistive load in the secondary, the primary current is given by:



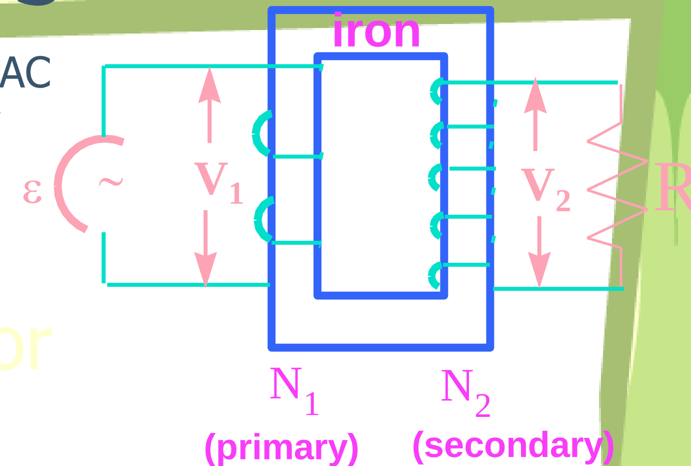
$$i_1 = \frac{N_2}{N_1} i_2 = \frac{N_2}{N_1}\frac{V_2}{R} = \frac{V_1}{(N_1/N_2)^2 R}$$

$$\boxed{\frac{V_1}{i_1} \equiv R_{eq} = R\frac{N_1^2}{N_2^2}}$$

This is the equivalent resistance seen by the source.

Lecture 21, ACT 3

- The primary coil of an ideal transformer is connected to an AC voltage source as shown. There are 50 turns in the primary and 200 turns in the secondary.



3A

– If V_1 = 120 V, what is the potential drop across the resistor R ?

(a) 30 V (b) 120 V (c) 480 V

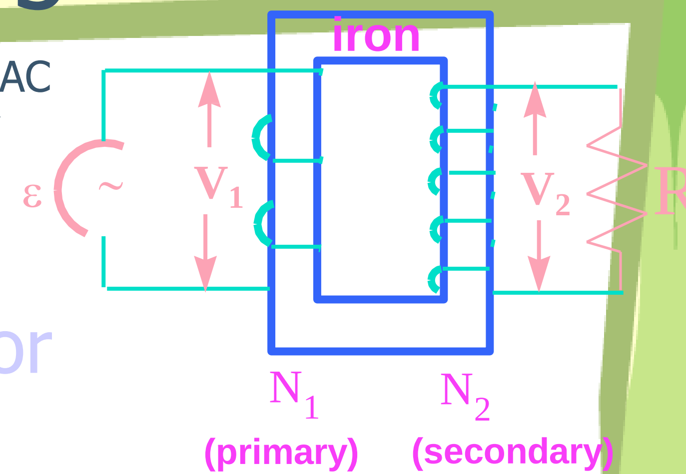
3B

If 960 W are dissipated in the resistor R, what is the current in the primary ?

(a) 8 A (b) 16 A (c) 32 A

Lecture 21, ACT 3

- The primary coil of an ideal transformer is connected to an AC voltage source as shown. There are 50 turns in the primary and 200 turns in the secondary.
  - If V_1 = 120 V, what is the potential drop across the resistor R ?



3A

(a) 30 V (b) 120 V (c) 480 V

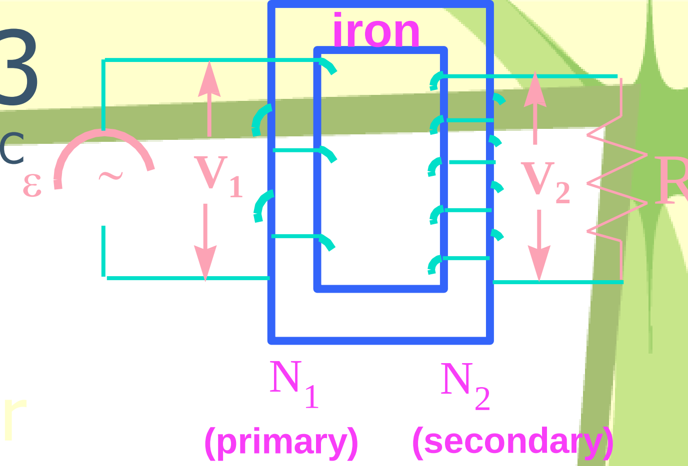
The ratio of turns, $(N_2/N_1) = (200/50) = 4$

The ratio of secondary voltage to primary voltage is equal to the ratio of turns, $(V_2/V_1) = (N_2/N_1)$

Therefore, (V_2/V_1) = 480 V

Lecture 21, ACT 3

- The primary coil of an ideal transformer is connected to an AC voltage source as shown. There are 50 turns in the primary and 200 turns in the secondary.



3A

– If V_1 = 120 V, what is the potential drop across the resistor R_2?

(a) 30 V (b) 120 V $\boxed{\text{(c) 480 V}}$

The ratio of turns, $(N_2/N_1) = (200/50) = 4$
The ratio $(V_2/V_1) = (N_2/N_1)$. Therefore, $(V_2/V_1) = 480$ V

3B

ç If 960 W are dissipated in the resistor R, what is the current in the primary ?

(a) 8 A (b) 16 A (c) 32 A

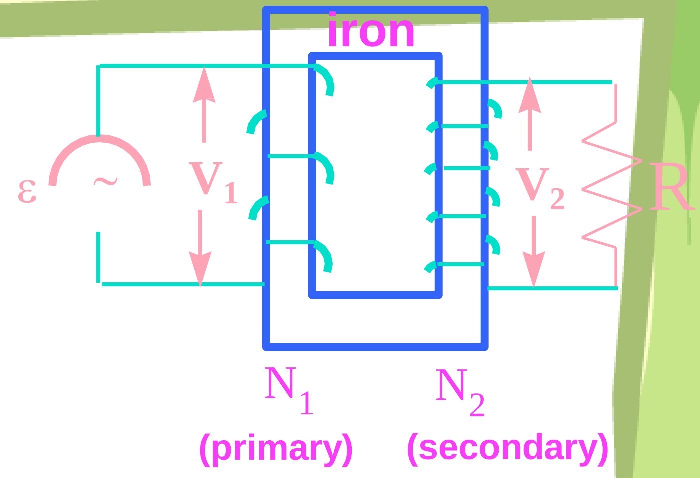
Gee, we didn't talk about power yet….

But, let's assume energy is conserved…since it usually is around here

Therefore, 960 W should be produced in the primary

$P_1 = V_1 I_1$ implies that 960W/120V = 8 A

Transformers with a Load



- To get that last ACT, you had to use a general philosophy -- energy conservation.
- An expression for the RMS power flow looks like this:

$$\boxed{P_{rms} = V_{1rms} i_{1rms} = \frac{N_1}{N_2} V_{2rms} \frac{N_2}{N_1} i_{2rms} = V_{2rms} i_{2rms}}$$

Note: This equation simply says that all power delivered by the generator is dissipated in the resistor ! Energy conservation!!

Chap. 4 Circuit Theorems

- Introduction
- Linearity property
- Superposition
- Source transformations
- Thevenin’s theorem
- Norton’s theorem
- Maximum power transfer

4.1 Introduction

A large complex circuits → **Simplify circuit analysis**

↑

Circuit Theorems

- **Thevenin's theorem**
- **Circuit linearity**
- **source transformation**
- **Norton theorem**
- **Superposition**
- **max. power transfer**

4.2 Linearity Property

Homogeneity property (Scaling)

$$i \rightarrow v = iR$$

$$ki \rightarrow kv = kiR$$

Additivity property

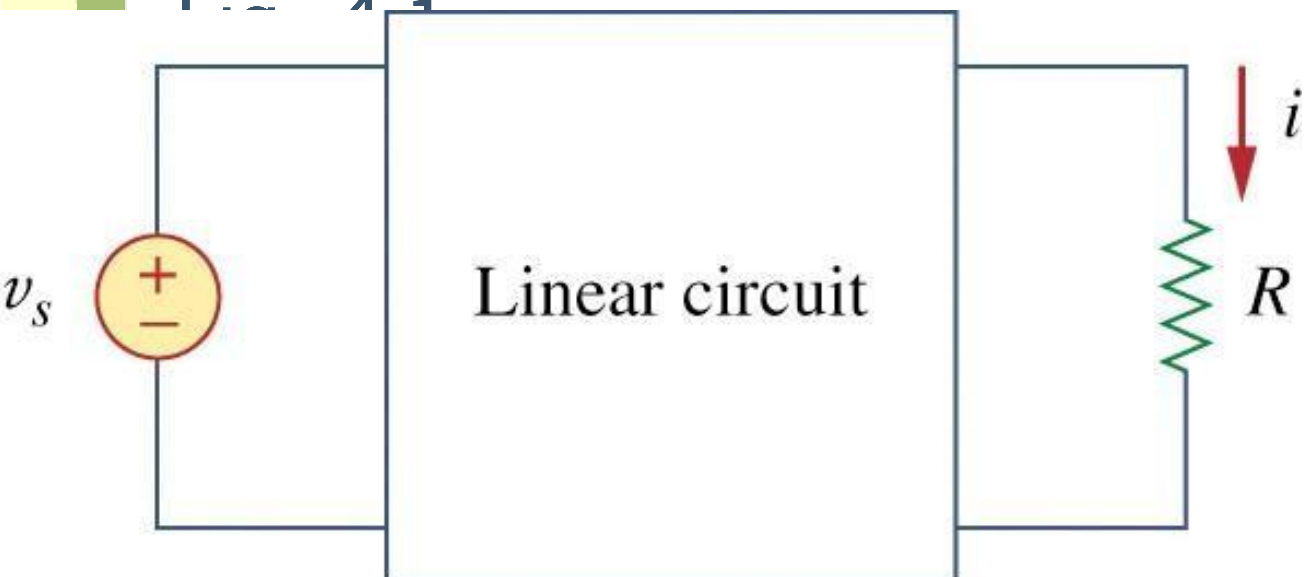
$$i_1 \rightarrow \quad v_1 = i_1 R$$

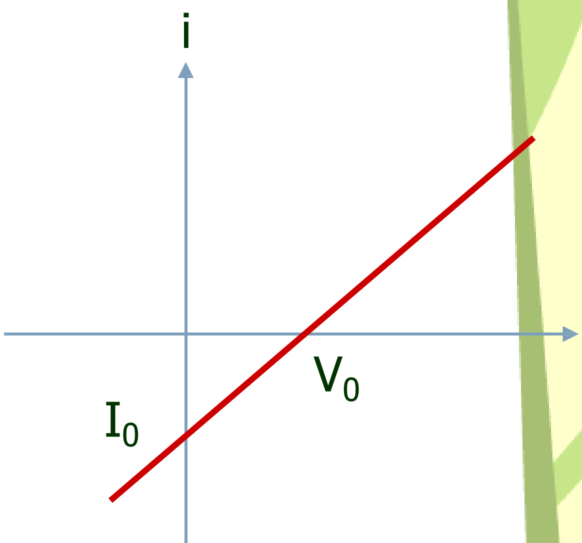
$$i_2 \rightarrow v_2 = i_2 R$$

$$i_1 + i_2 \rightarrow (i_1 + i_2)R = i_1 R + i_2 R = v_1 + v_2$$

- A linear circuit is one whose output is linearly related (or directly proportional) to its input

Fig. 4.1





- Linear circuit consist of
  - linear elements
  - linear dependent sources
  - independent sources
- $p = i^2 R = \frac{v^2}{R}$: nonlinear

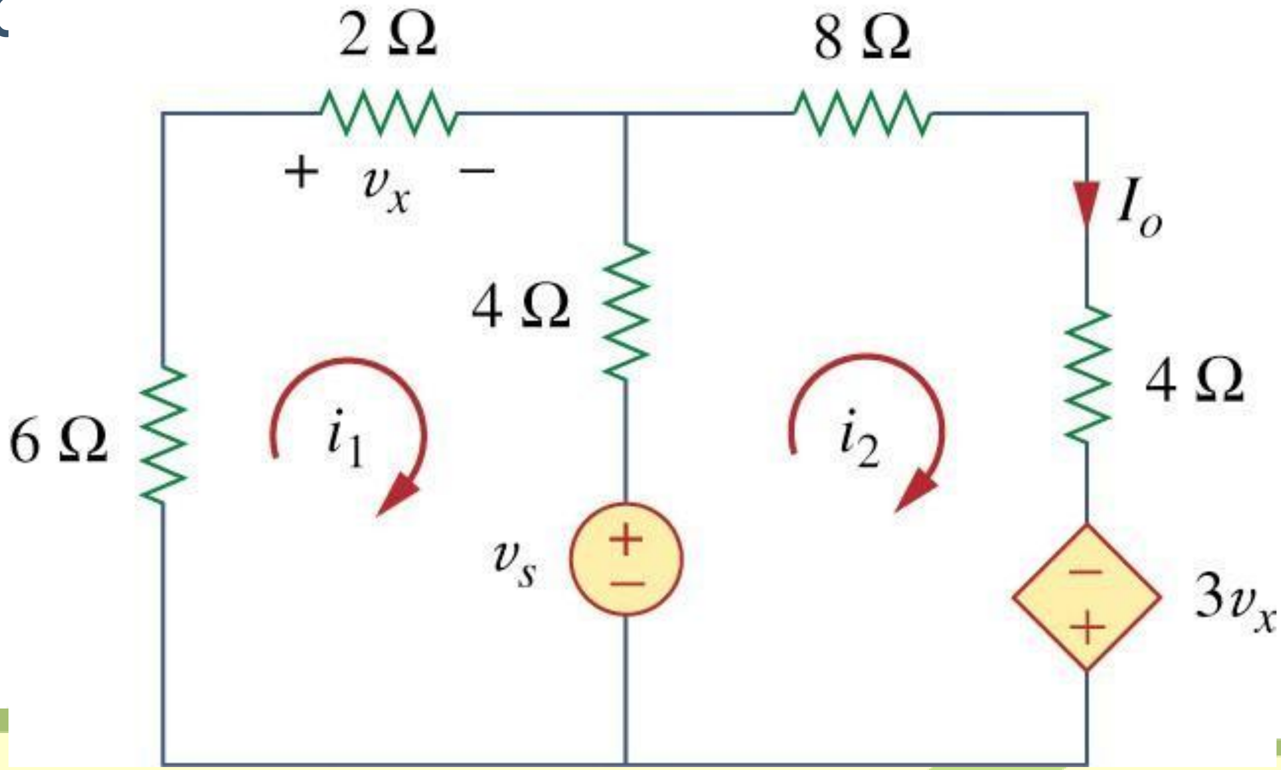
$$v_s = 10\text{V} \rightarrow i = 2\text{A}$$

$$v_s = 1\text{V} \rightarrow i = 0.2\text{A}$$

$$v_s = 5\text{mV} \leftarrow i = 1\text{mA}$$

Example 4.1

- For the circuit in fig 4.2 find I_0 when v_s=12V and



Example 4.1

- KVL $12i_1 - 4i_2 + v_s = 0$ (4.1.1)

$$-4i_1 + 16i_2 - 3v_x - v_s = 0 \qquad (4.1.2)$$

$$v_x = 2i_1$$

(4.1.2) becomes

$$-10i_1 + 16i_2 - v_s = 0 \qquad (4.1.3)$$

Eqs(4.1.1) and (4.1.3) we get

$$2i_1 + 12i_2 = 0 \rightarrow i_1 = -6i_2$$

Example 4.1

Eq(4.1.1), we get

$$-76 i_2 = v_s = 0 \quad \Rightarrow \quad i_2 = \frac{v_s}{76}$$

When $v_s = 12\mathrm{V}$

$$I_0 = i_2 = \frac{12}{76}\mathrm{A}$$

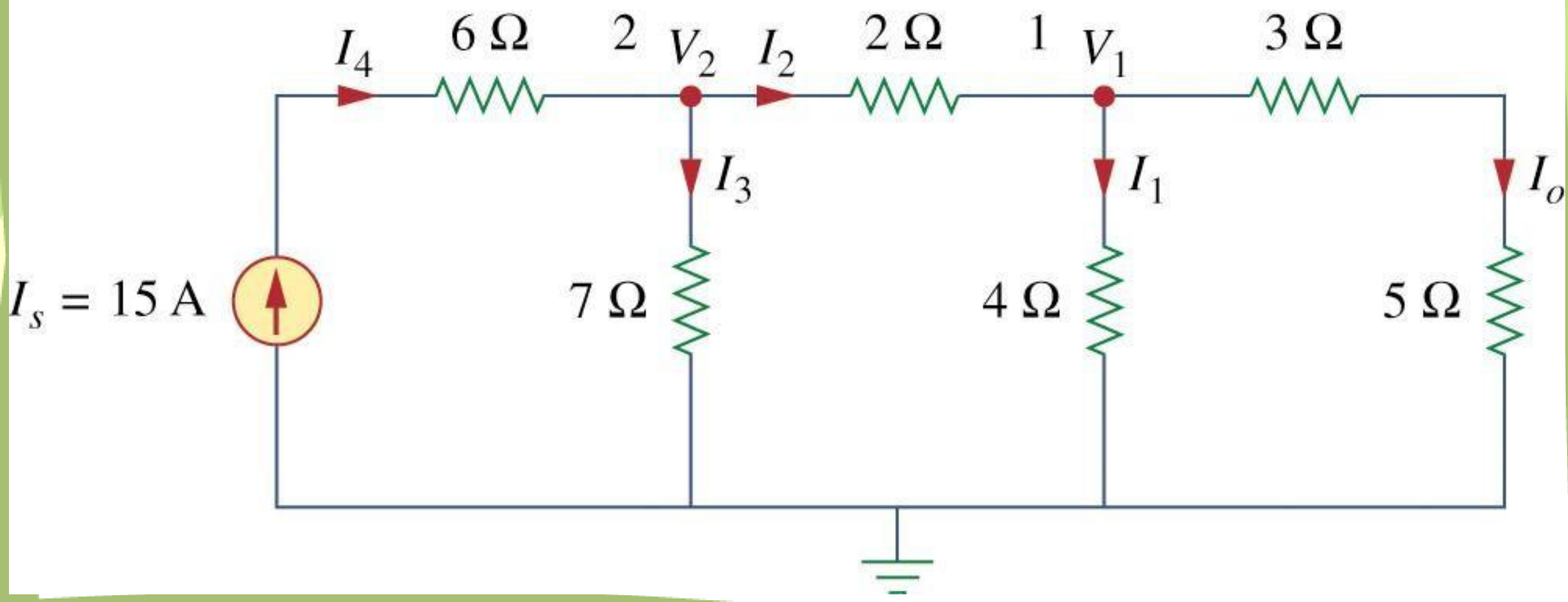
When $v_s = 24\mathrm{V}$

$$I_0 = i_2 = \frac{24}{76}\mathrm{A}$$

Showing that when the source value is doubled, I_0 doubles.

Example 4.2

- Assume $I_0 = 1$ A and use linearity to find the actual value of I_0 in the circuit in fig 4.4.



Example 4.2

$$\text{If } I_0 = 1\text{A, then } v_1 = (3+5)I_0 = 8\text{V}$$

$$I_1 = v_1 / 4 = 2\text{A}, \quad I_2 = I_1 + I_0 = 3\text{A}$$

$$V_2 = V_1 + 2I_2 = 8 + 6 = 14\text{V}, I_3 = \frac{V_2}{7} = 2\text{A}$$

$$I_4 = I_3 + I_2 = 5\text{A} \Rightarrow \quad I_S = 5\text{A}$$

$$I_0 = 1A \rightarrow I_S = 5\text{A}$$

$$I_0 = 3\text{A} \leftarrow I_S = 15\text{A}$$

4.3 Superposition

- The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.
- Turn off, killed, inactive source:
  - independent voltage source: 0 V (short circuit)
  - independent current source: 0 A (open circuit)
- Dependent sources are left intact.

- Steps to apply superposition principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

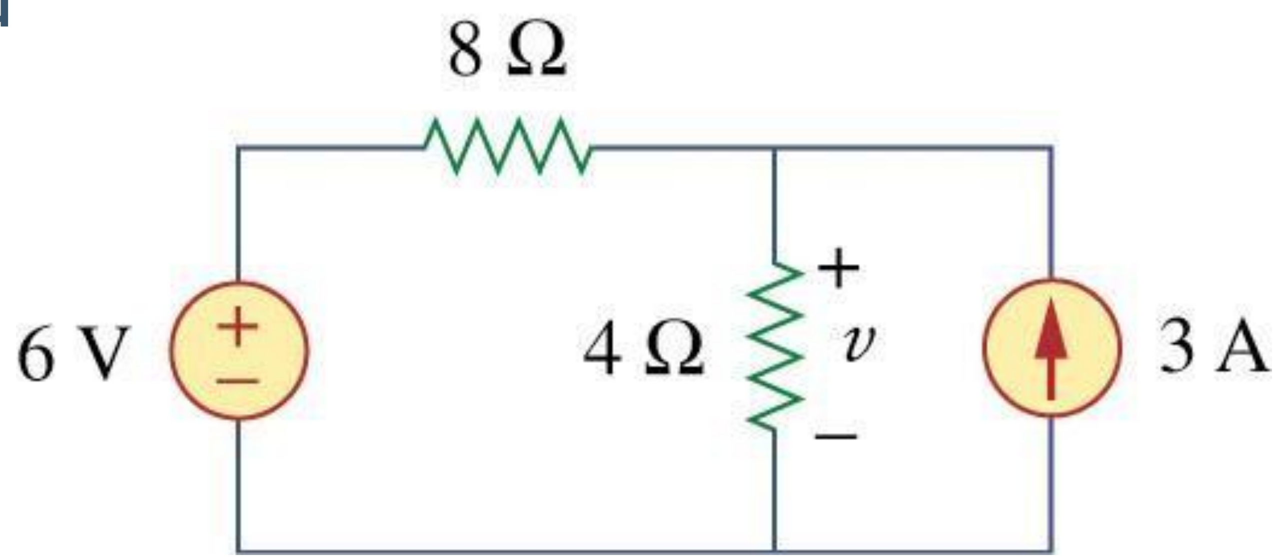
How to turn off independent sources

- Turn off voltages sources = short voltage sources; make it equal to zero voltage
- Turn off current sources = open current sources; make it equal to zero current

- Superposition involves more work but simpler circuits.
- Superposition is not applicable to the effect on power.

Example 4.3

- Use the superposition theorem to find in the circuit in Fig 4.6.



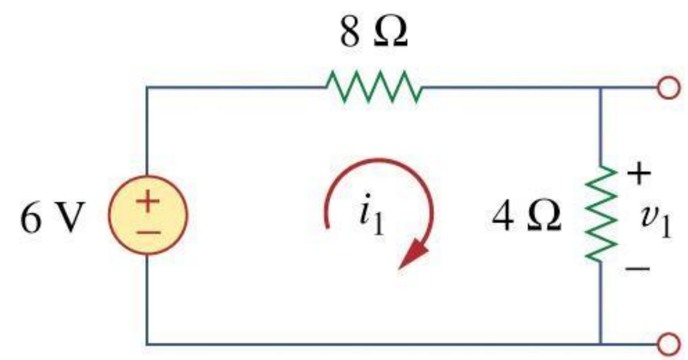
Example 4.3

Since there are two sources,

let $V = V_1 + V_2$

Voltage division to get

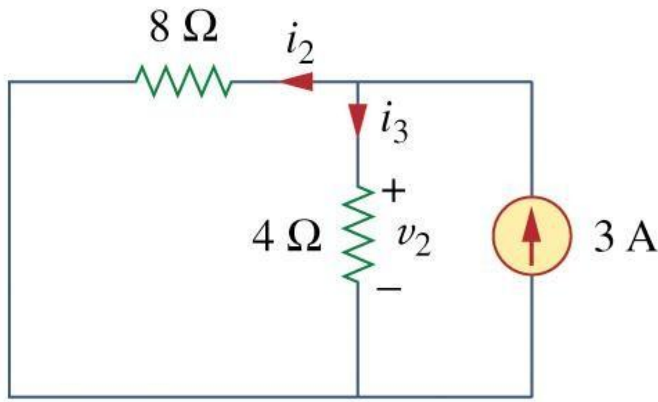
$$V_1 = \frac{4}{4+8}(6) = 2\text{V}$$



(a)

Current division, to get

$$i_3 = \frac{8}{4+8}(3) = 2\text{A}$$



(b)

Hence

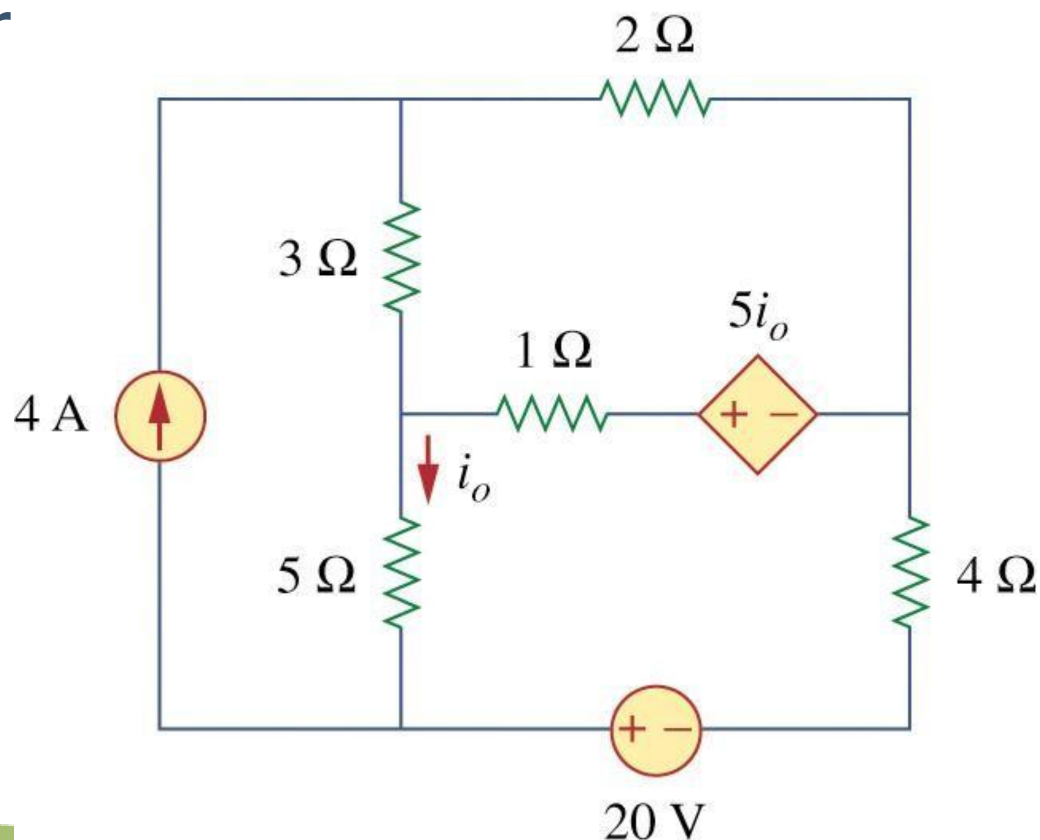
$$v_2 = 4i_3 = 8\text{V}$$

And we find

$$v = v_1 + v_2 = 2 + 8 = 10\text{V}$$

Example 4.4

- Find I_0 in the cir superposition.



Example 4.4

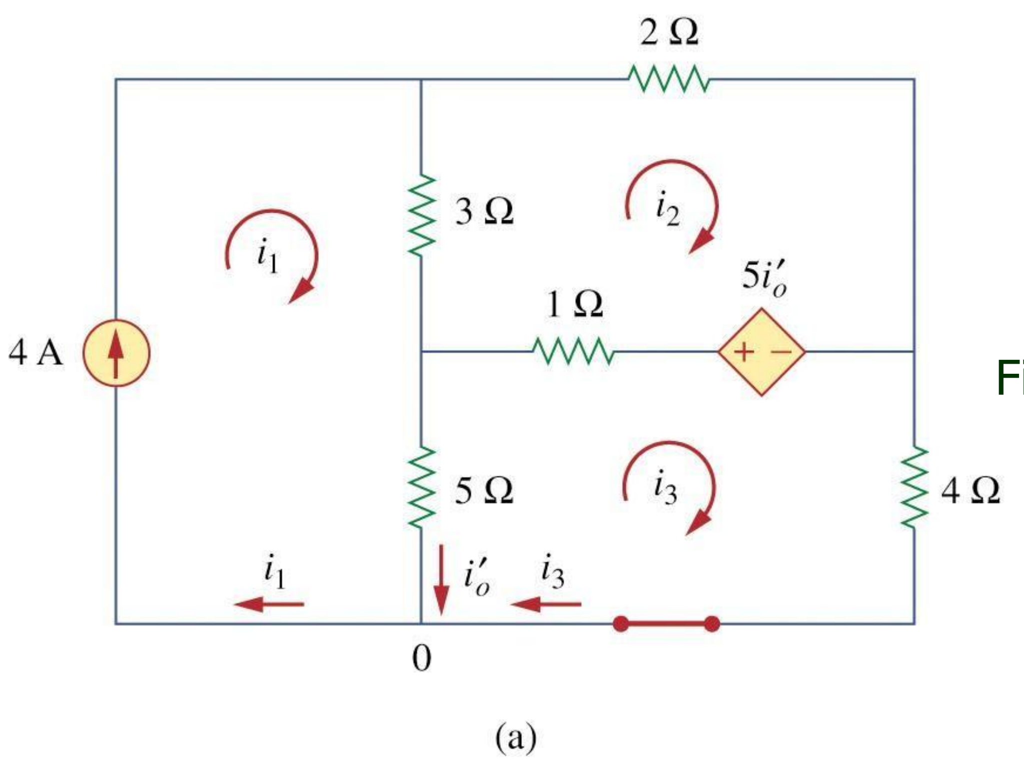


Fig. 4.10

Example 4.4

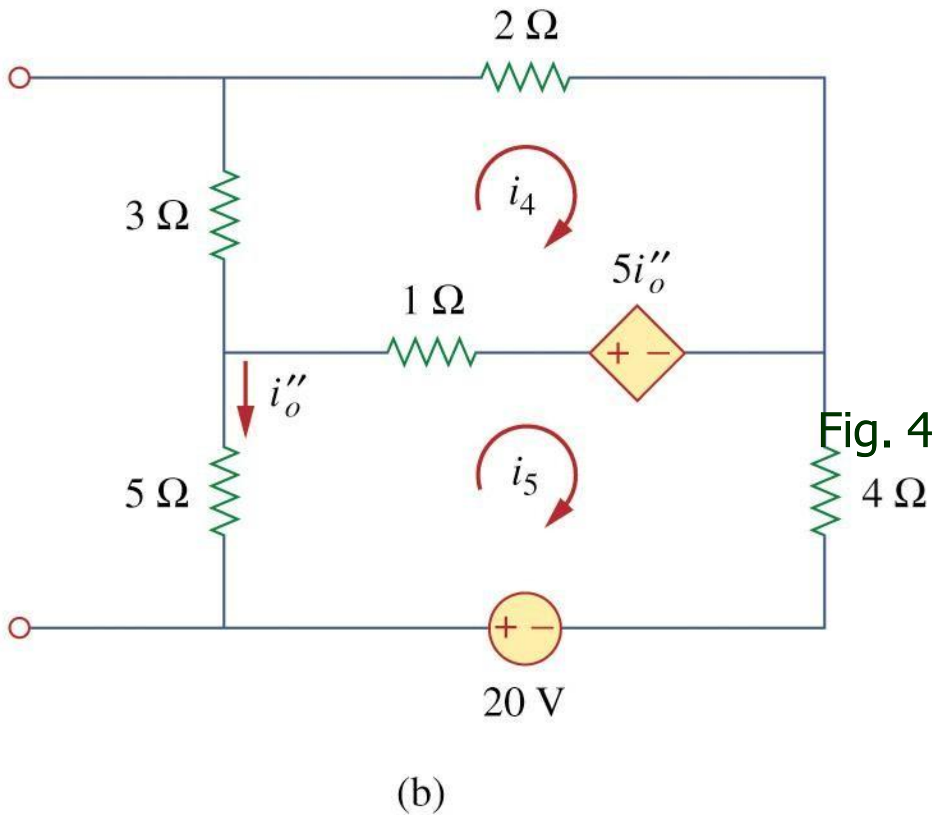
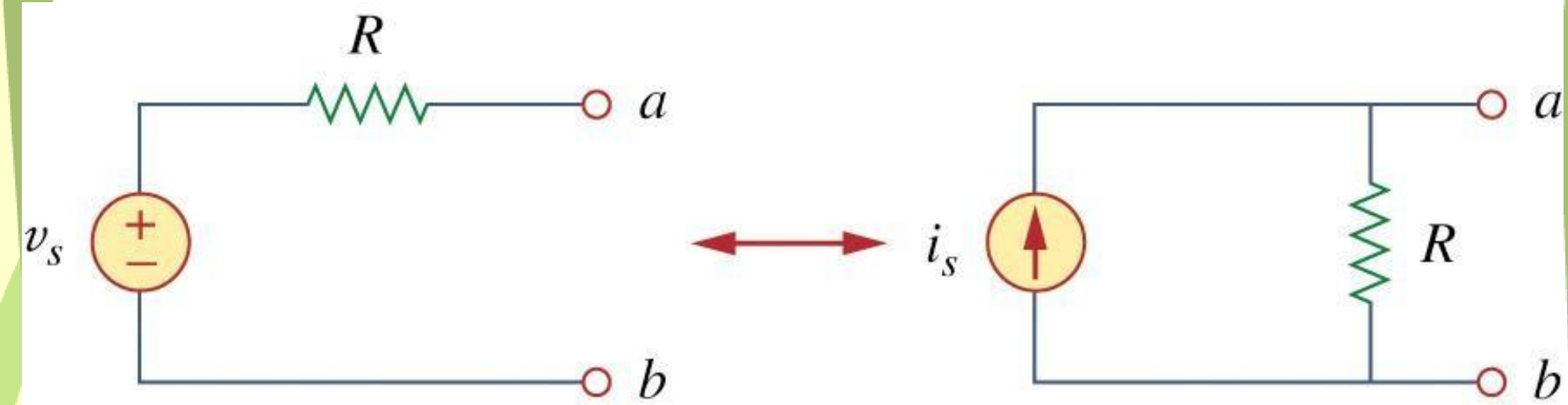


Fig. 4.10

4.5 Source Transformation

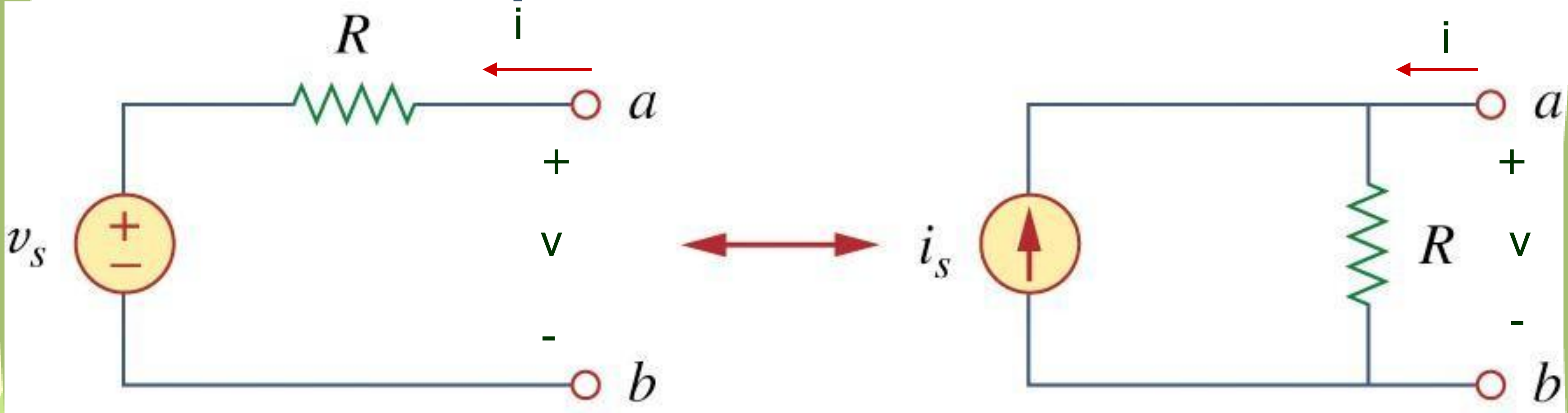
- A source transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa

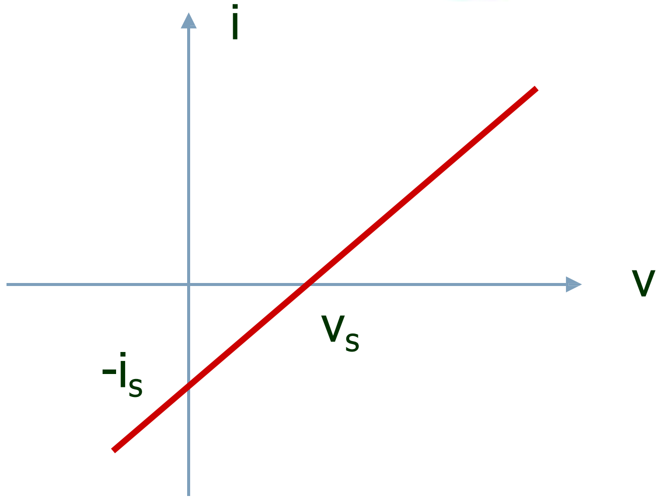
Fig. 4.15 & 4.16



$$v_s = i_s R \quad \text{or} \quad i_s = \frac{v_s}{R}$$

Equivalent Circuits

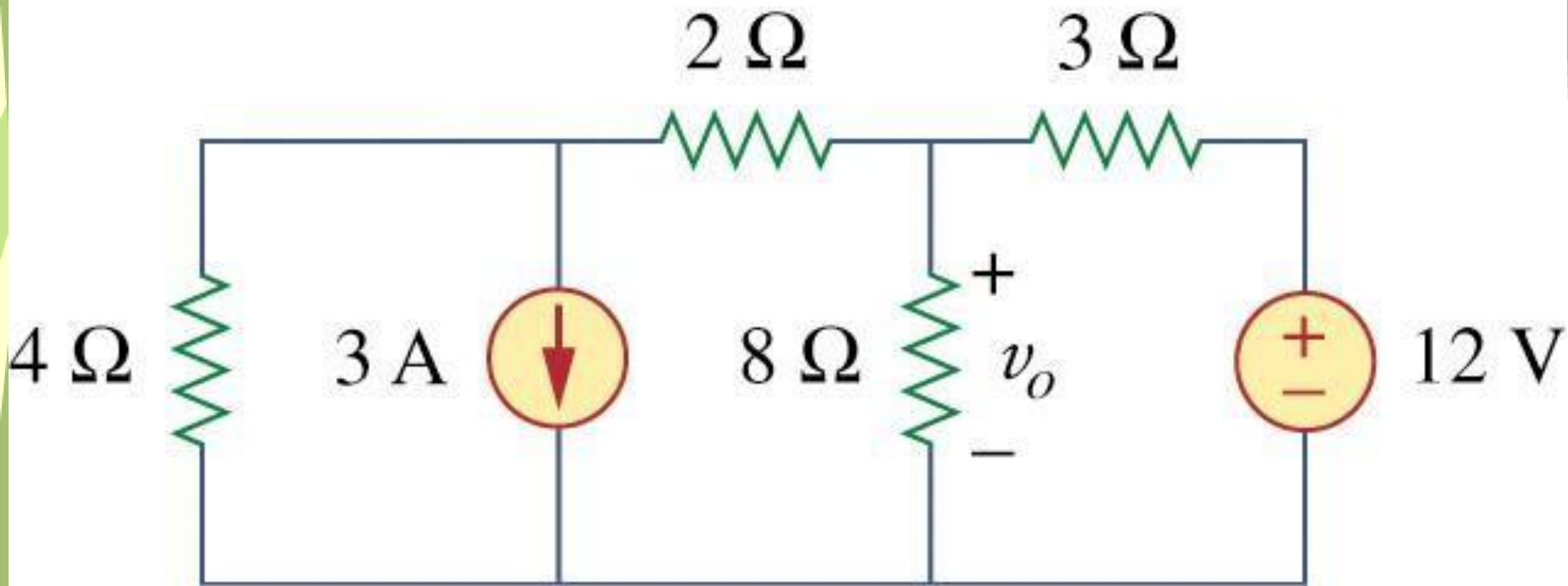




- Arrow of the current source
  positive terminal of voltage source
- Impossible source Transformation
  - ideal voltage source ($R = 0$)
  - ideal current source ($R=\infty$)

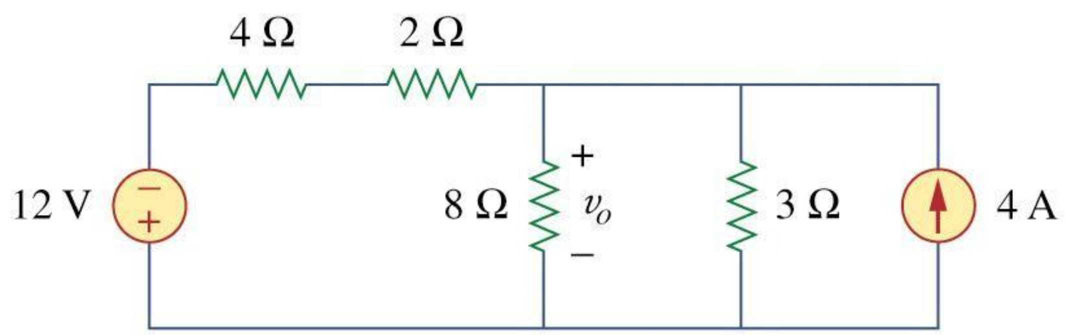
Example 4.6

- Use source transformation to find v_o in the circuit in Fig 4.17.

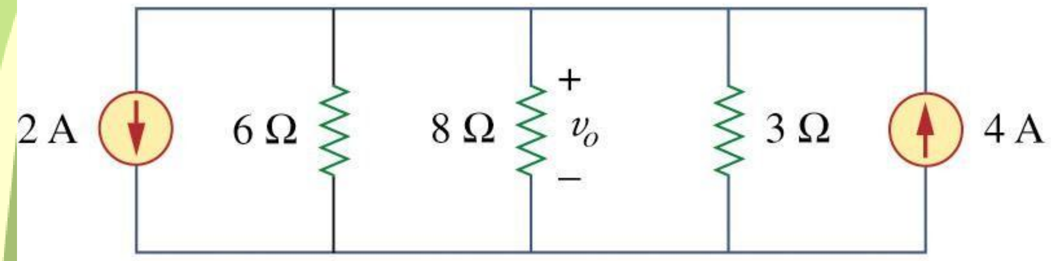


Example 4.6

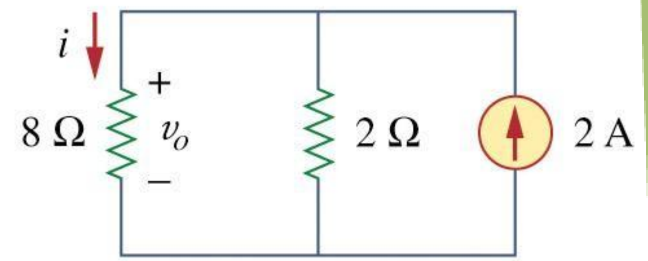
Fig 4.18



(a)



(b)



(c)

Example 4.6

we use current division in Fig.4.18(c) to get

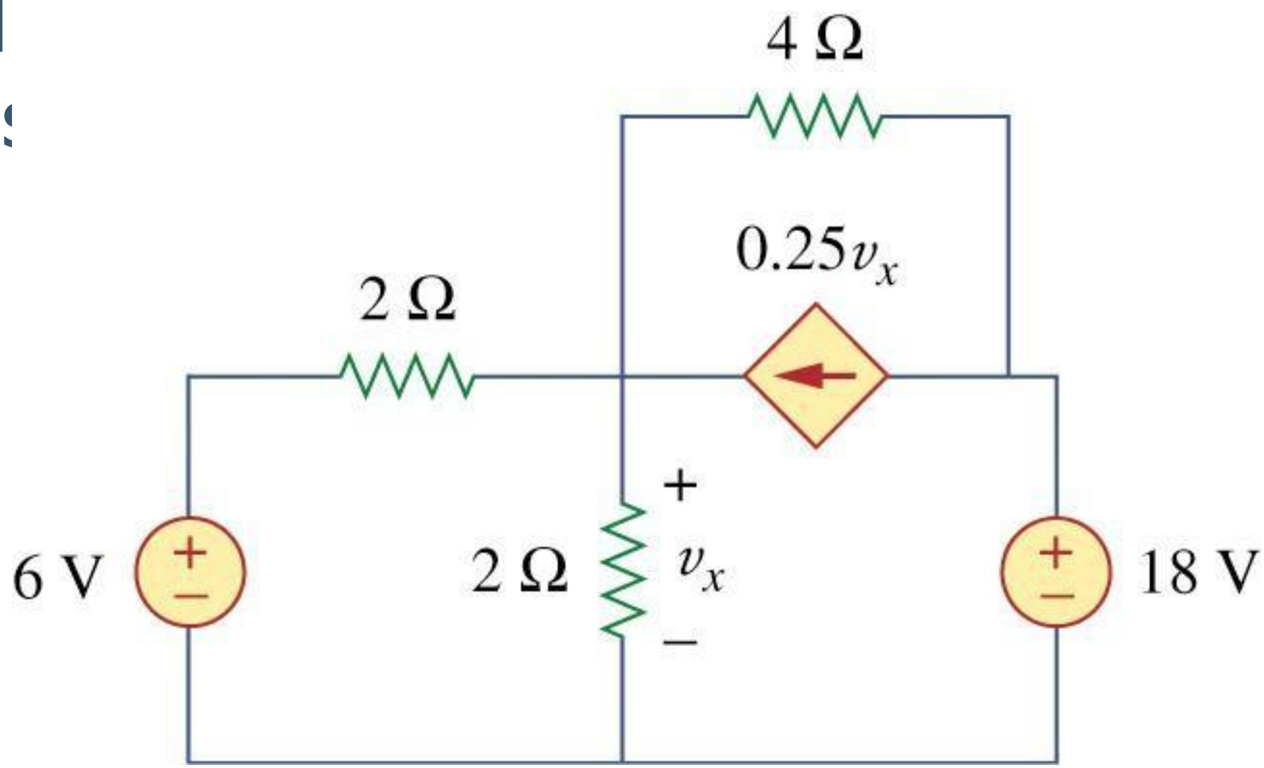
$$i = \frac{2}{2+8}(2) = 0.4\text{A}$$

and

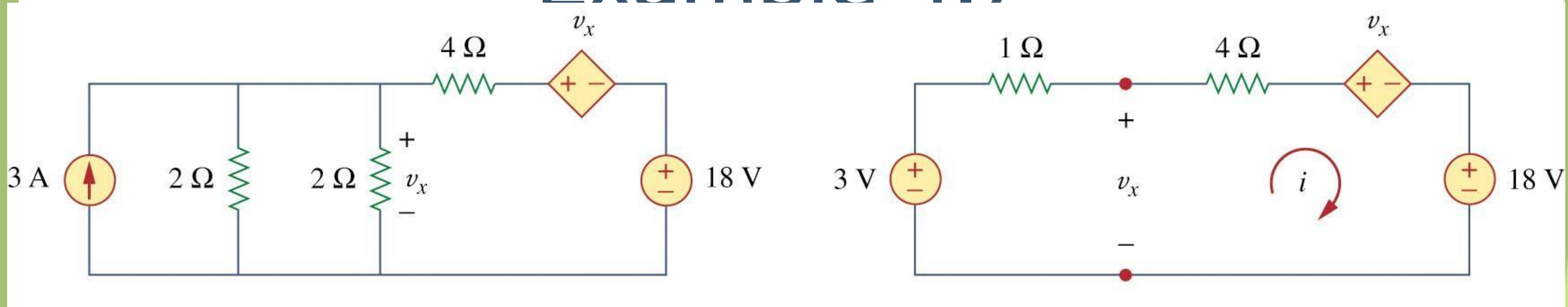
$$v_o = 8i = 8(0.4) = 3.2\text{V}$$

Example 4.7

- Find
  tran



Example 4.7



(a) (b)

Applying KVL around the loop in Fig 4.21(b) gives

$$-3 + 5i + v_x + 18 = 0$$

(4.7.1)

Appling KVL to the loop containing only the 3V voltage source, the 1Ω resistor and v_x yields

$$-3 + 1i + v_x = 0 \Rightarrow v_x = 3 - i$$

Example 4.7

Substituting this into Eq. (4.7.1), we obtain

$$15 + 5i + 3 = 0 \Rightarrow i = -4.5 \text{ A}$$

Alternatively

$$-v_x + 4i + v_x + 18 = 0 \Rightarrow i = -4.5\text{A}$$

thus

$$v_x = 3 - i = 7.5\text{V}$$

4.5 Thevenin's Theorem

- Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th} where V_{Th} is the open circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent source are turn off.

Property of Linear Circuits

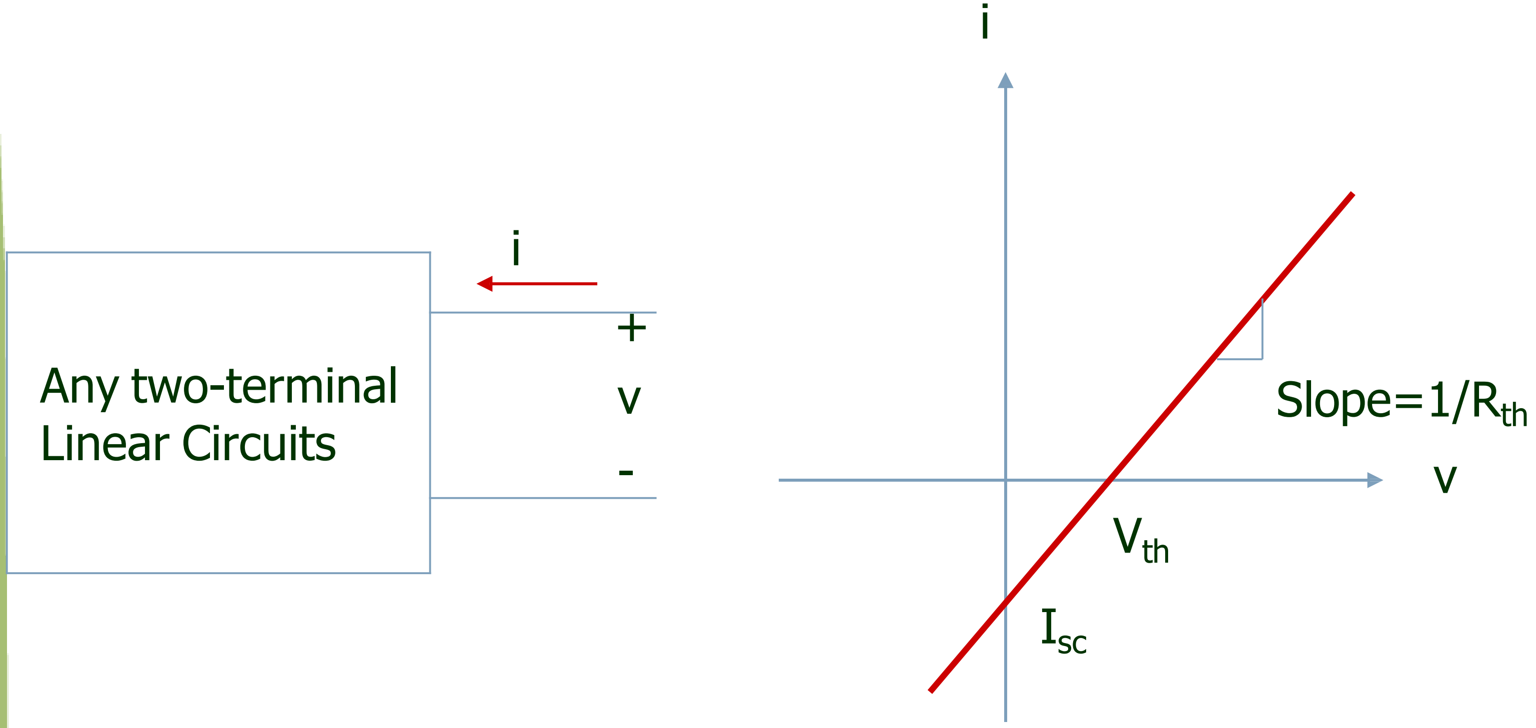
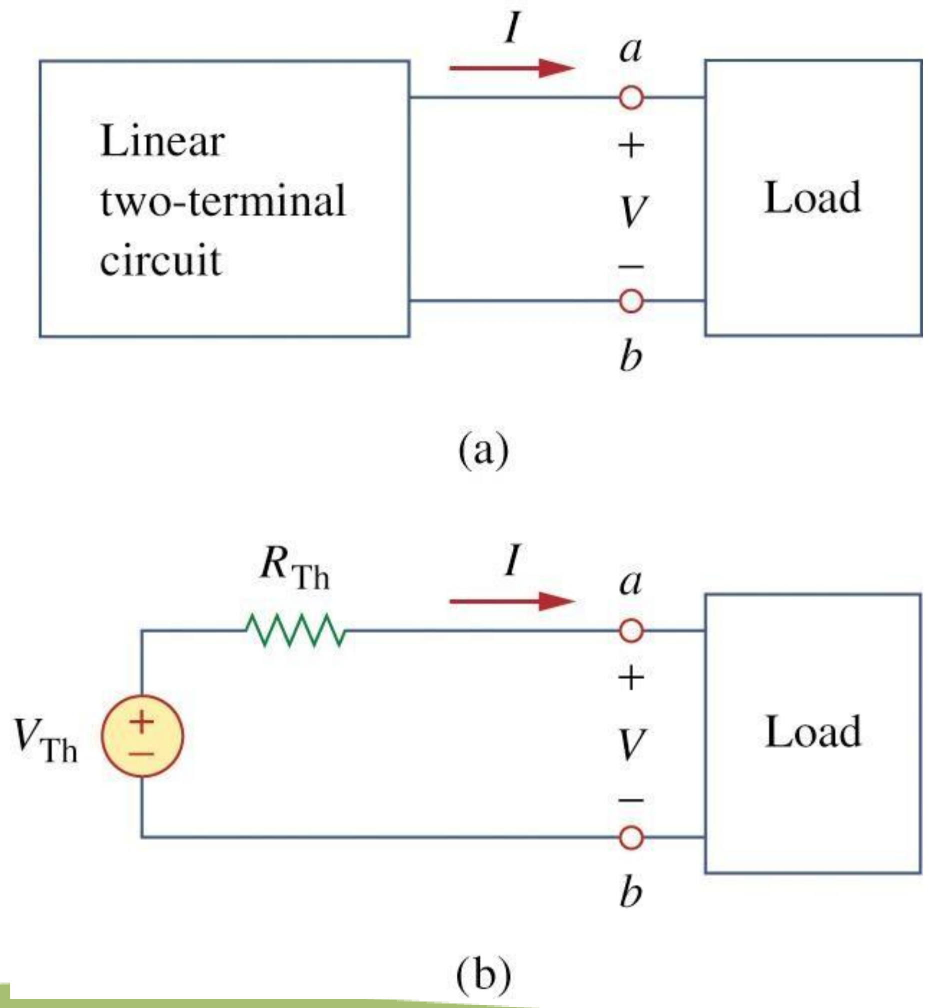


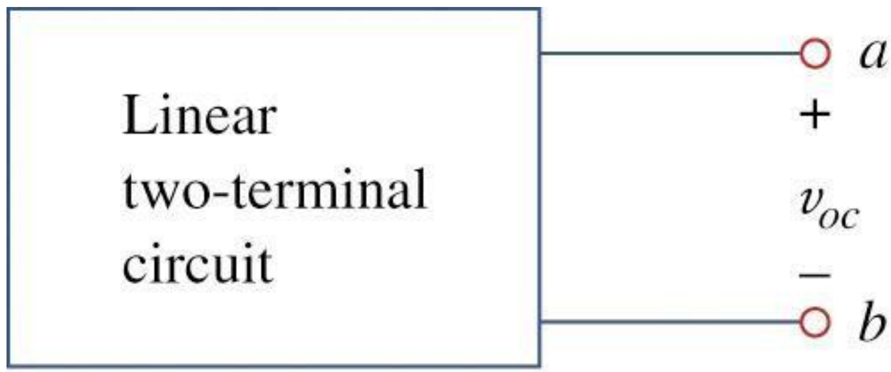
Fig. 4.23



How to Find Thevenin's Voltage

- Equivalent circuit: same voltage-current relation at the terminals.

$$V_{Th} = v_{oc}: \text{ open circuit voltage at } a - b$$

- 

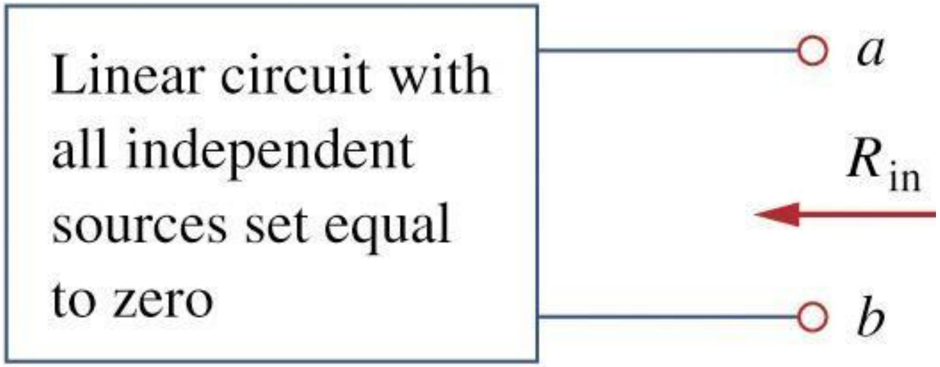
$V_{Th} = v_{oc}$

(a)

How to Find Thevenin's Resistance

$$R_{\text{Th}} = R_{\text{in}} :$$

- $\text{input} - \text{resistance of the dead circuit at } a - b.$
  - $a - b$ open circuited
  - Turn off all independent sources



$$R_{\text{Th}} = R_{\text{in}}$$

(b)

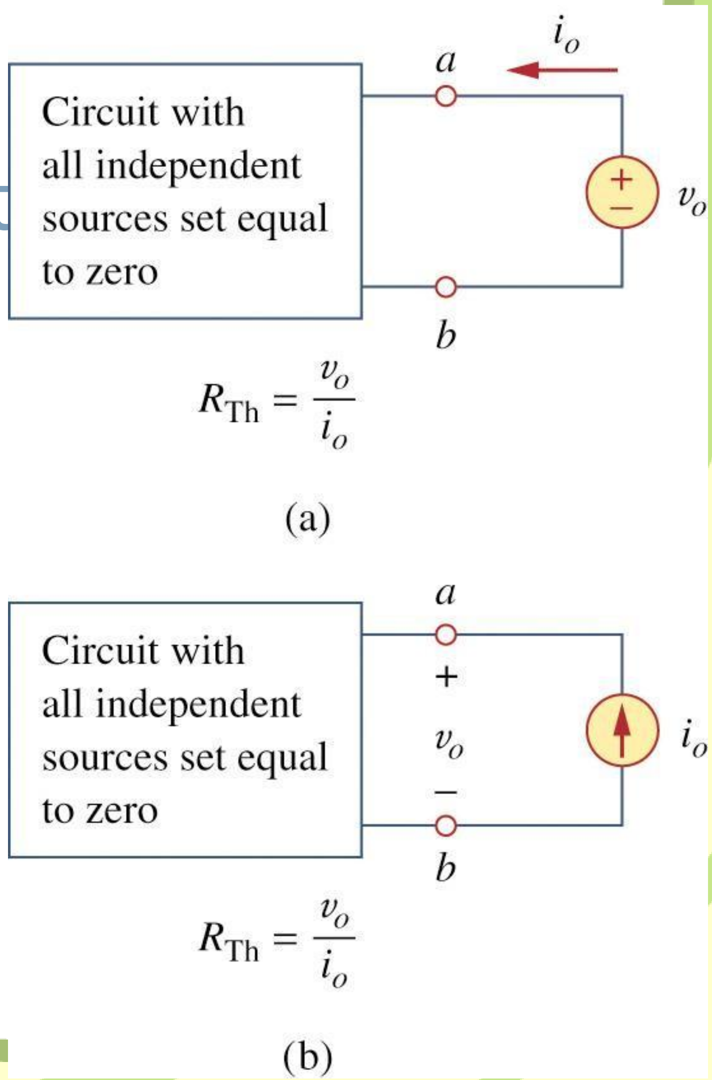
CASE 1

- If the network has no dependent sources:
  - Turn off all independent source.
  - R_{TH}: can be obtained via simplification of either parallel or series connection seen from a-b

Fig. 4.25

CASE 2

- If the network has dependent sources
  - Turn off all independent sources.
  - Apply a voltage source v_o at a-b
    $$R_{Th} = \frac{v_o}{i_o}$$
  - Alternatively, apply a current source i_o at a-b
    $$R_{Th} = \frac{v_o}{i_o}$$

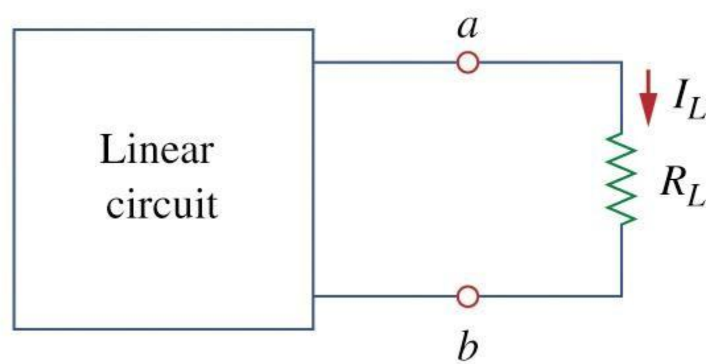


- The Thevenin’s resistance may be negative, indicating that the circuit has ability providing power

Fig. 4.26

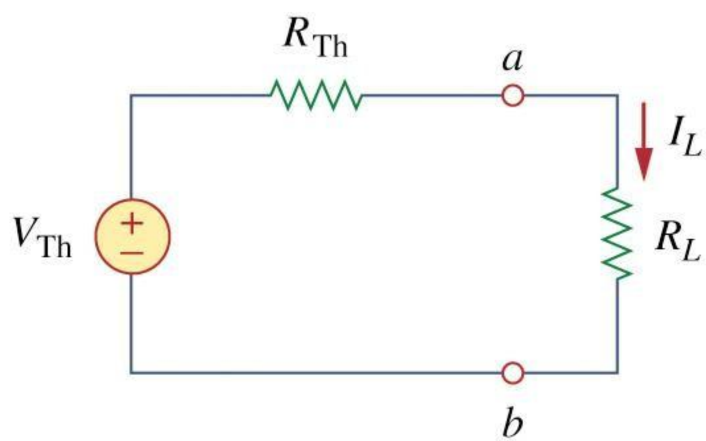
Simplified circuit

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$



(a)

$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

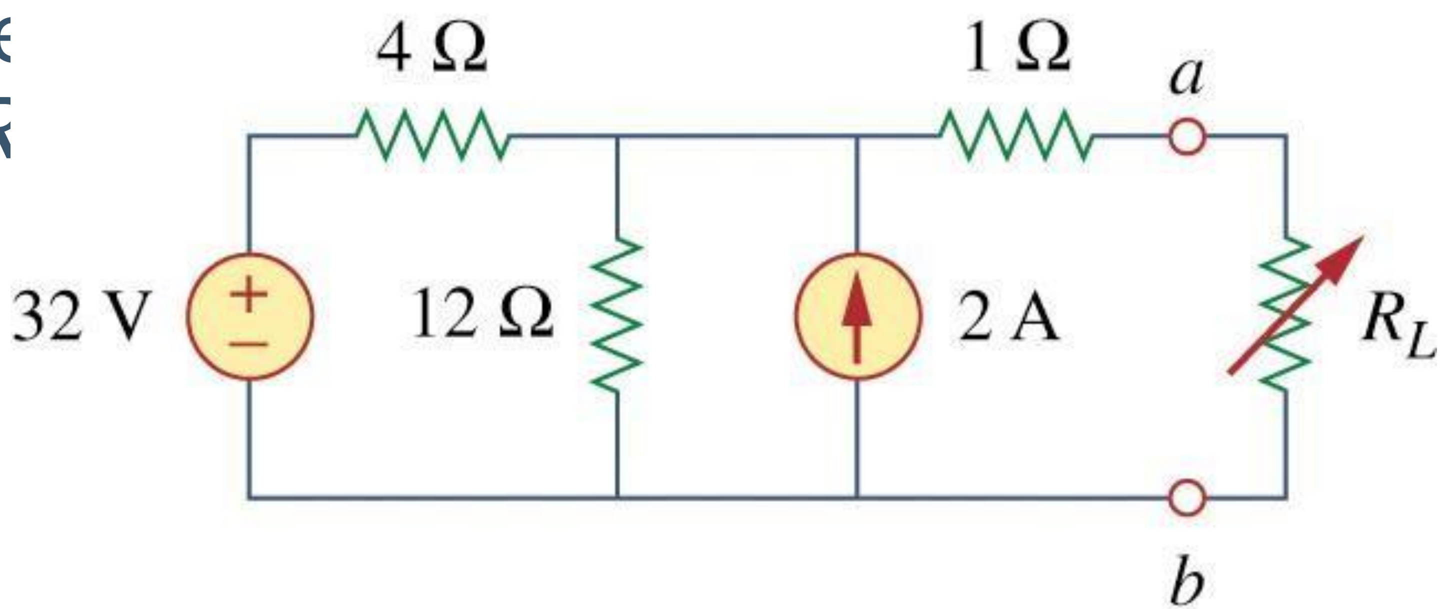


(b)

Voltage divider

Example 4.8

- Find the Thevenin's equivalent circuit of the circuit shown in Fig 4.27, to the left of the te gh
  R

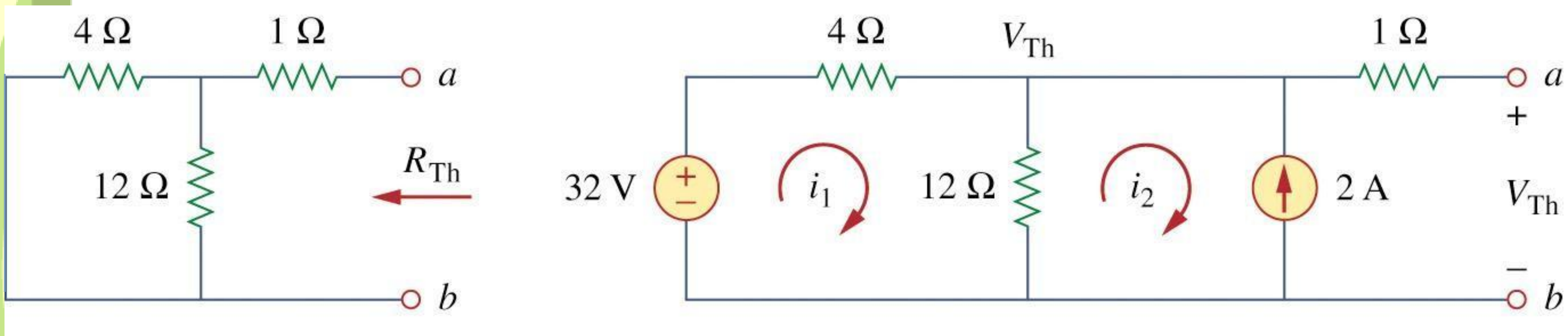


Find R_{th}

R_{Th} : 32V voltage source $\rightarrow$ short

2A current source $\rightarrow$ open

$$R_{Th} = 4 \| 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\Omega$$



(a) (b)

Find V_{th}

V_{Th} :

(1) Mesh analysis

$$-32 + 4i_1 + 12(i_1 - i_2) = 0\ ,\ \ i_2 = -2\mathrm{A}$$

$$\therefore i_1 = 0.5\mathrm{A}$$

$$V_{\mathrm{Th}} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30\mathrm{V}$$

(2) Alternatively, Nodal Analysis

$$(32 - V_{\mathrm{Th}})/4 + 2 = V_{\mathrm{Th}}/12$$

$$\therefore V_{\mathrm{Th}} = 30\mathrm{V}$$

Example 4.8

(3) Alternatively, sourcetransform

$$\frac{32 - V_{\mathrm{TH}}}{4} + 2 = \frac{V_{\mathrm{TH}}}{12}$$

$$96 - 3V_{\mathrm{TH}} + 24 = V_{\mathrm{TH}} \Rightarrow V_{\mathrm{TH}} = 30\mathrm{V}$$

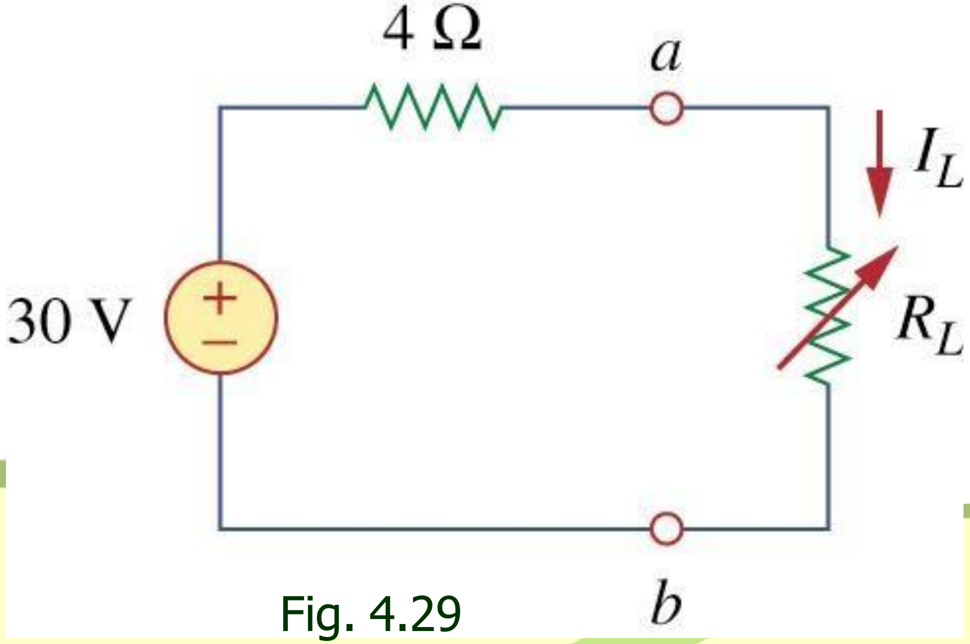


Fig. 4.29

Example 4.8

To get i_L:

$$i_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{30}{4 + R_L}$$

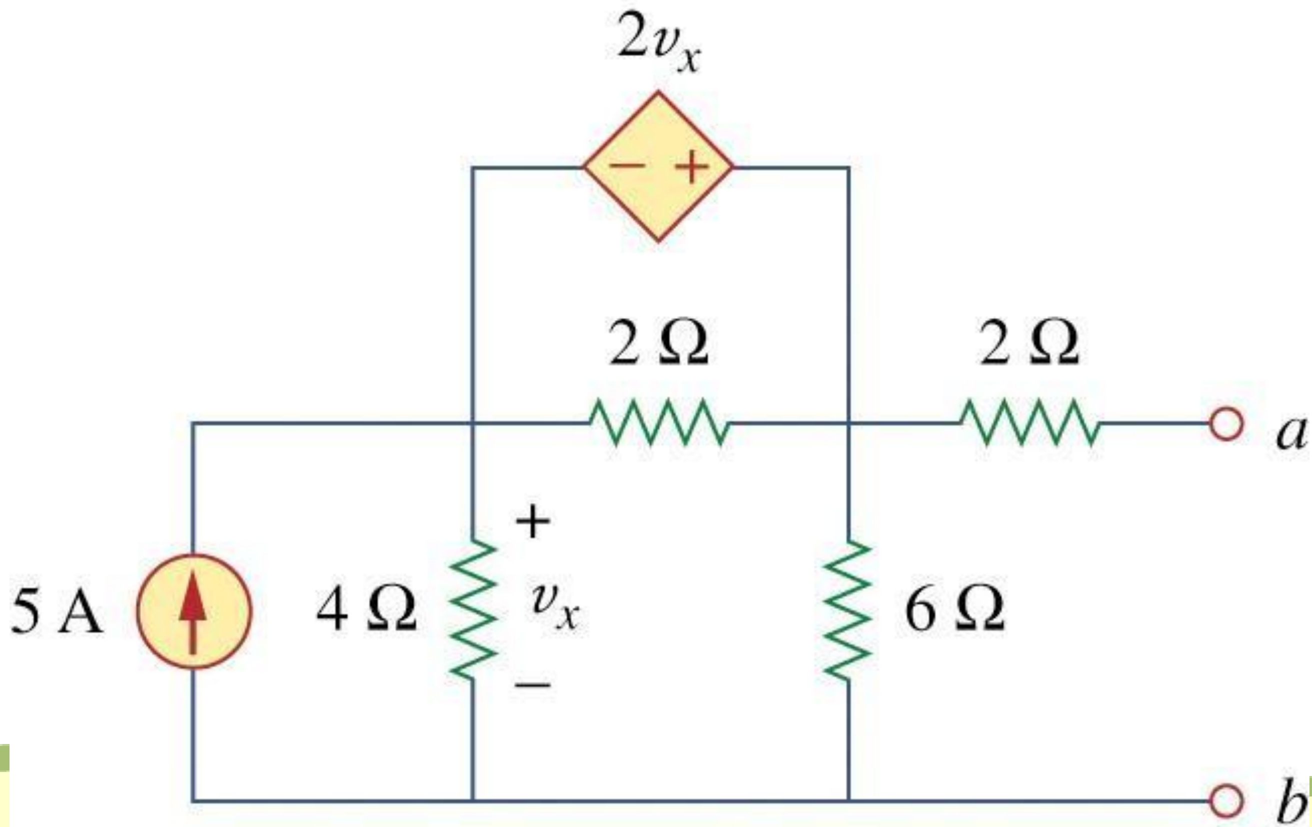
$$R_L = 6 \rightarrow I_L = 30/10 = 3\text{A}$$

$$R_L = 16 \rightarrow I_L = 30/20 = 1.5\text{A}$$

$$R_L = 36 \rightarrow I_L = 30/40 = 0.75\text{A}$$

Example 4.9

- Find the Thevenin's equivalent of the circuit in Fig.



Example 4.9

- To find R_{Th} : Fig(a)ependent source case)

$$\text{independent source} \rightarrow 0$$

$$\text{dependent source} \rightarrow \text{intact}$$

$$v_o = 1\text{V}, \quad R_{\text{Th}} = \frac{v_o}{i_o} = \frac{1}{i_o}$$

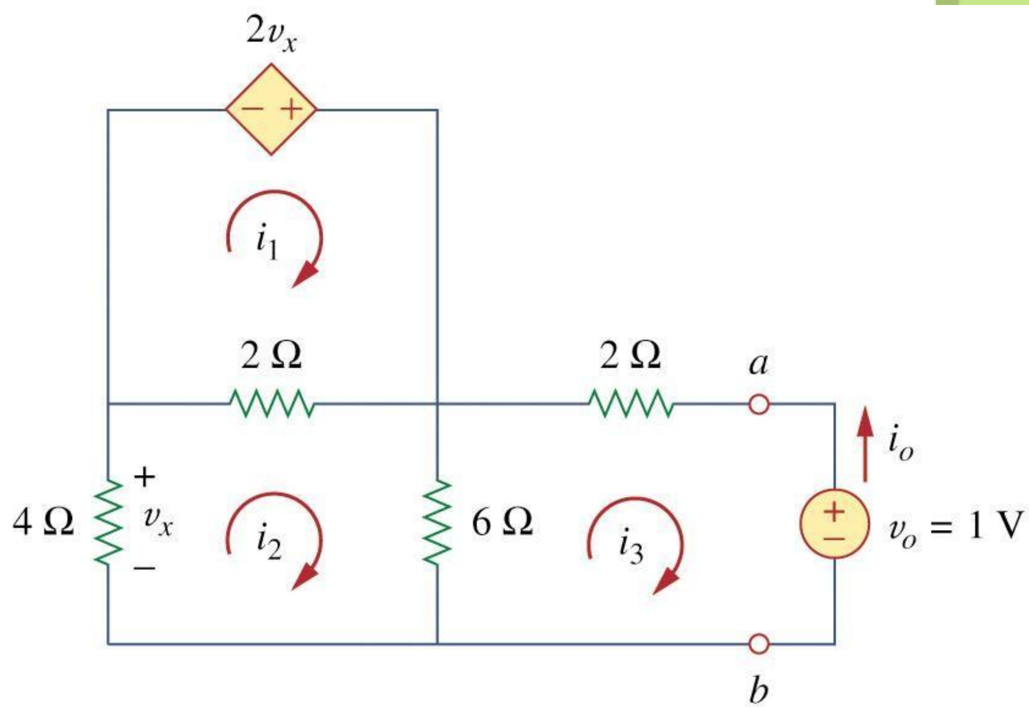
Example 4.9

- For loop 1,

$$-2v_x + 2(i_1 - i_2) = 0 \quad \text{or} \quad v_x = i_1 - i_2$$

$$\text{But} - 4i = v_x = i_1 - i_2$$

$$\therefore\ i_1 = -3i_2$$



(a)

Example 4.9

$$\text{Loop 2 and 3}:$$

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$$

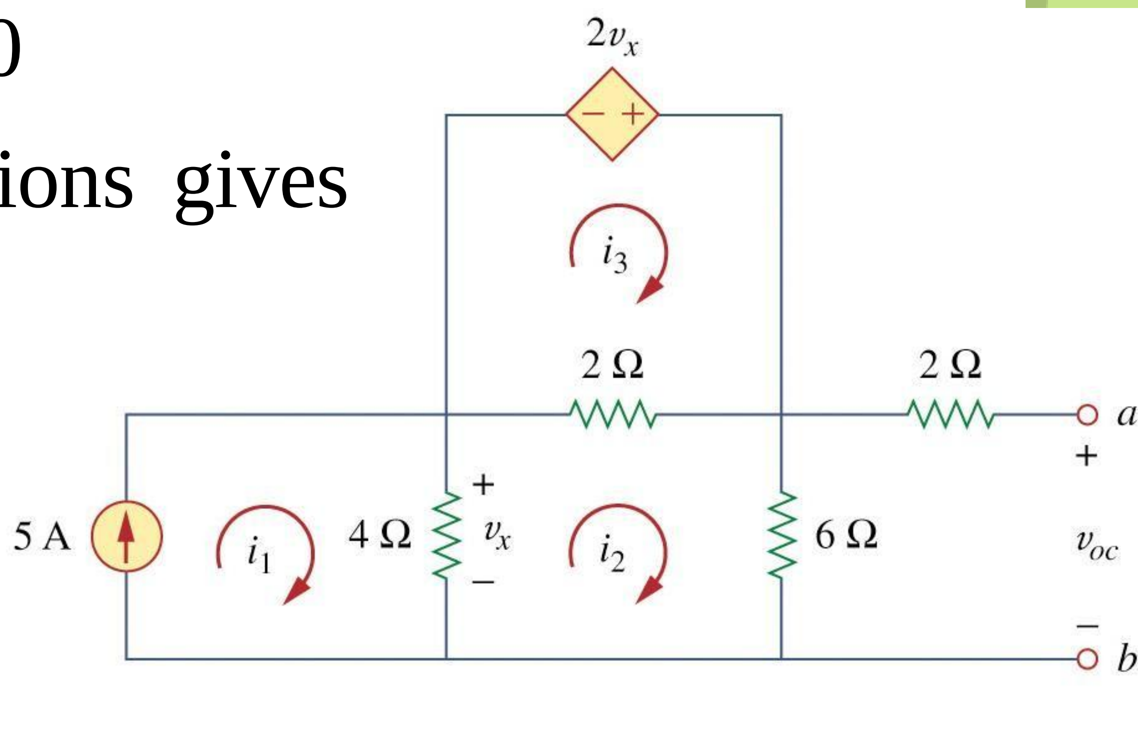
$$6(i_3 - i_2) + 2i_3 + 1 = 0$$

Solving these equations gives

$$i_3 = -1/6\text{A}.$$

$$\text{But } i_o = -i_3 = \frac{1}{6}\text{A}$$

$$\therefore R_{\text{Th}} = \frac{1V}{i_o} = 6\Omega$$



(b)

Example 4.9

To get V_{Th} :Fig(b) Mesh analysis

$$i_1 = 5$$

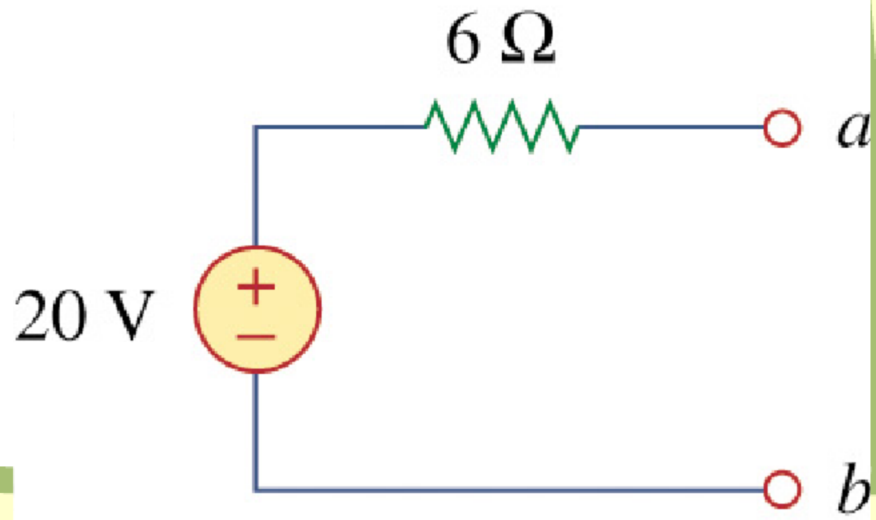
$$-2v_x + 2(i_3 - i_2) = 0 \Rightarrow v_x = i_3 - i_2$$

$$4(i_2 - i_1) + 2(i_2 - i_1) + 6i_2 = 0 \Rightarrow 12i_2 - 4i_1 - 2i_3 = 0$$

$$\text{But } 4(i_1 - i_2) = v_x$$

$$\therefore i_2 = 10/3.$$

$$V_{Th} = v_{oc} = 6i_2 = 20\text{V}$$



Example 4.10

- Determine the Thevenin equivalent circuit in Fig.4.35(a).

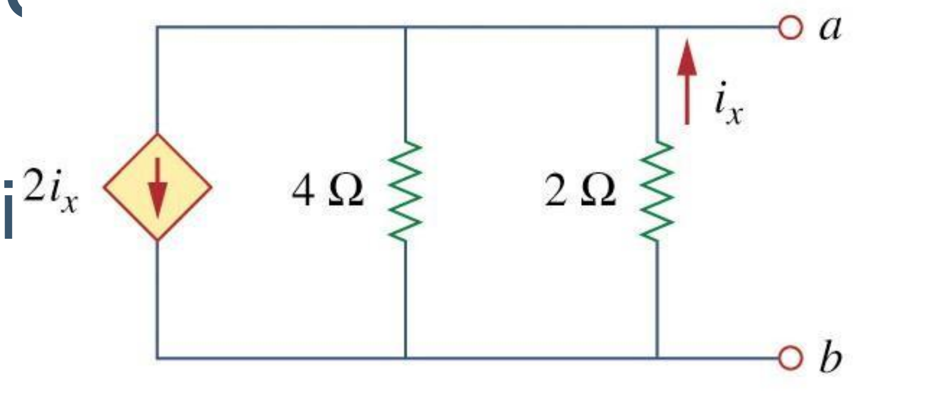
(dependent source only case)

- Solution

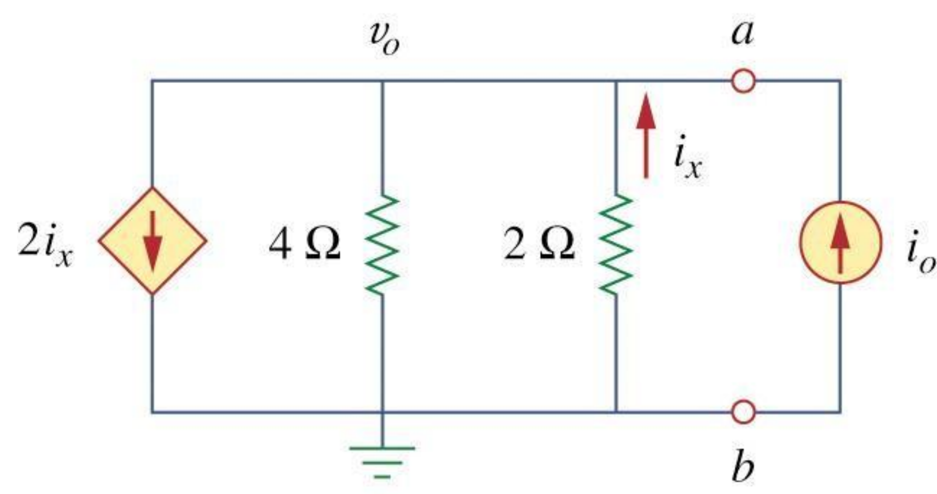
$$V_{\text{Th}} = 0 \qquad R_{\text{Th}} = \frac{v_o}{i_o}$$

Nodal anaysis:

$$i_o + i_x = 2i_x + v_o/4$$



(a)



(b)

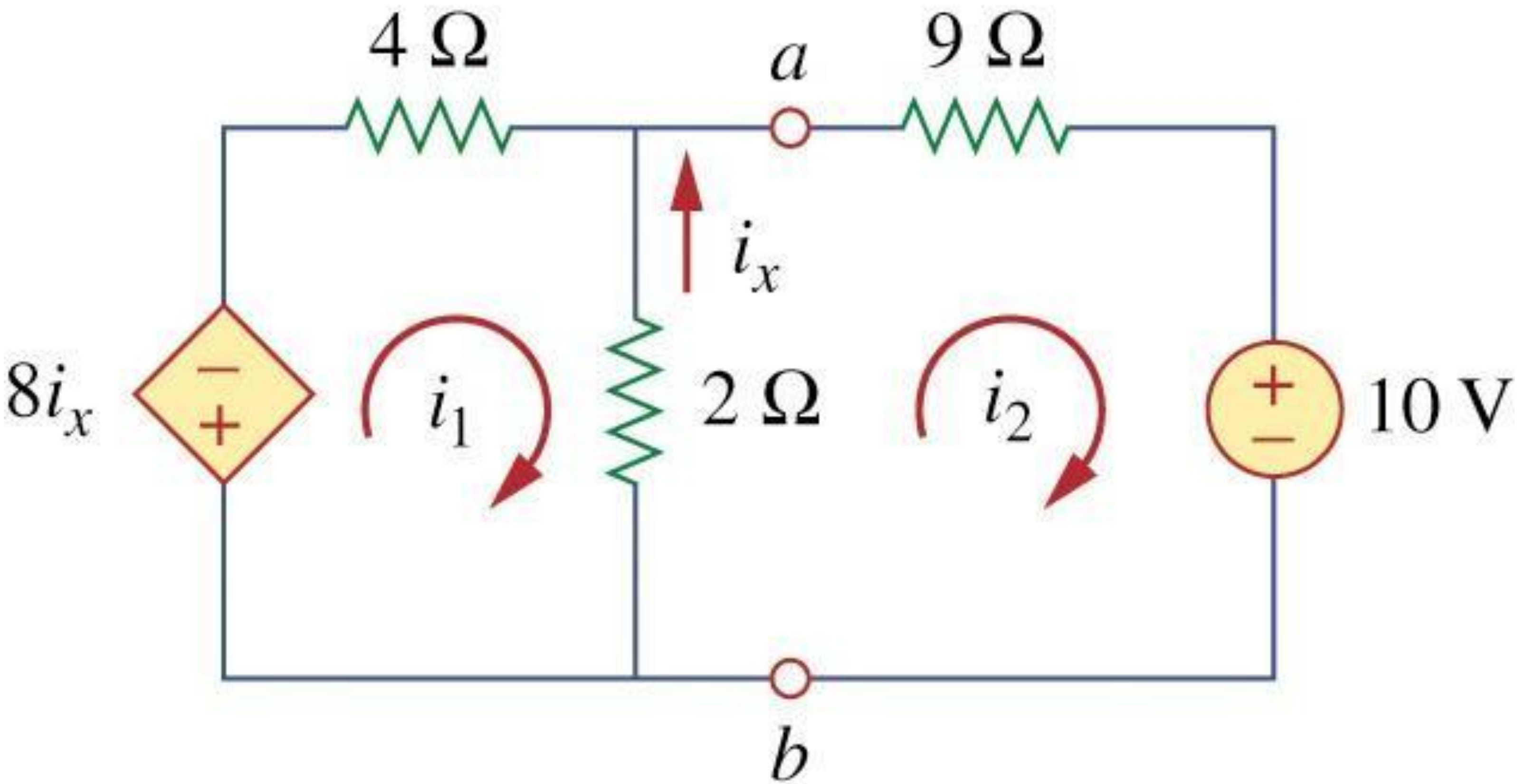
Example 4.10

But $$i_x = \frac{0 - v_o}{2} = -\frac{v_o}{2}$$

$$i_o = i_x + \frac{v_o}{4} = -\frac{v_o}{2} + \frac{v_o}{4} = -\frac{v_o}{4} \quad \text{or } v_o = -4i_o$$

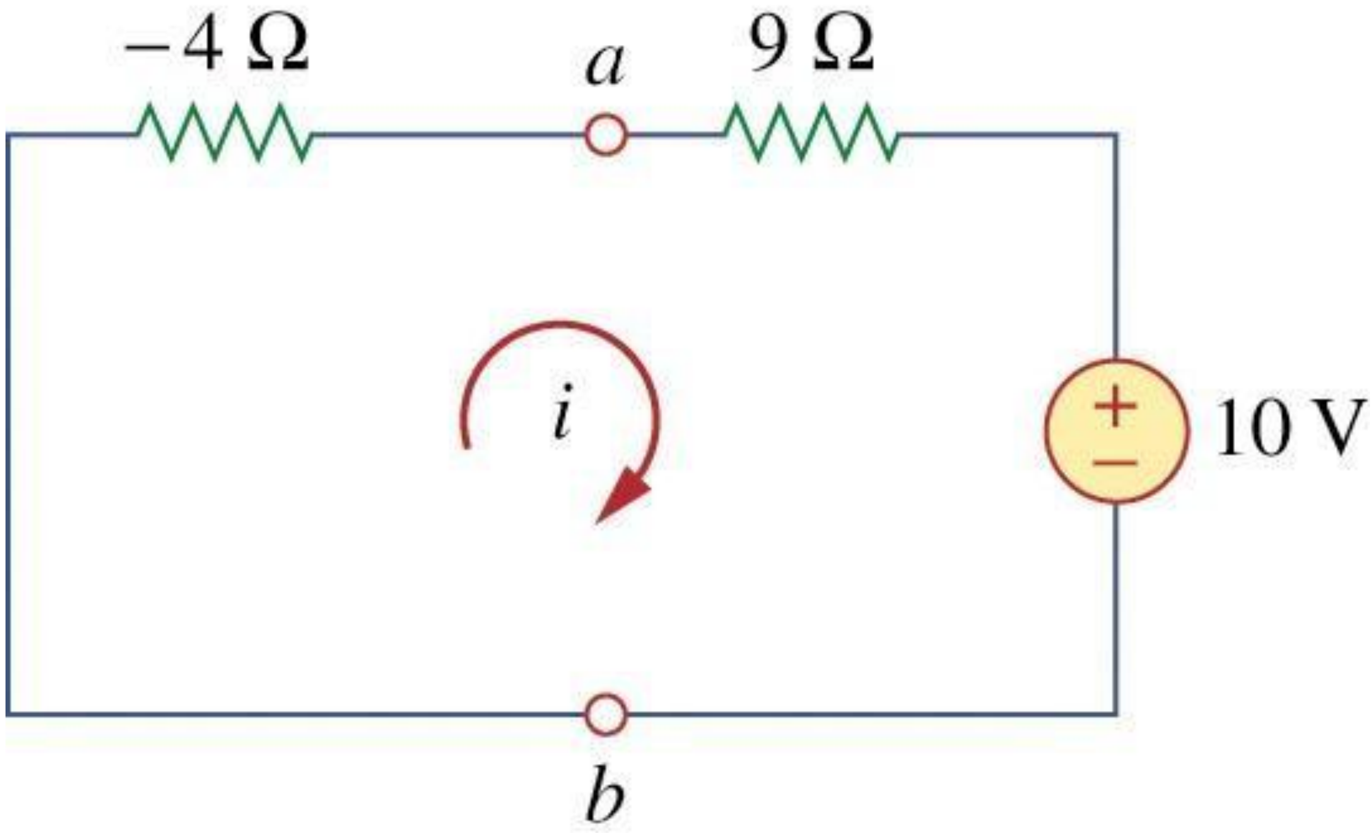
Thus $$R_{\text{Th}} = \frac{v_o}{i_o} = -4\Omega: \text{ Supplying power}$$

Example 4.10



(c)

Example 4.10

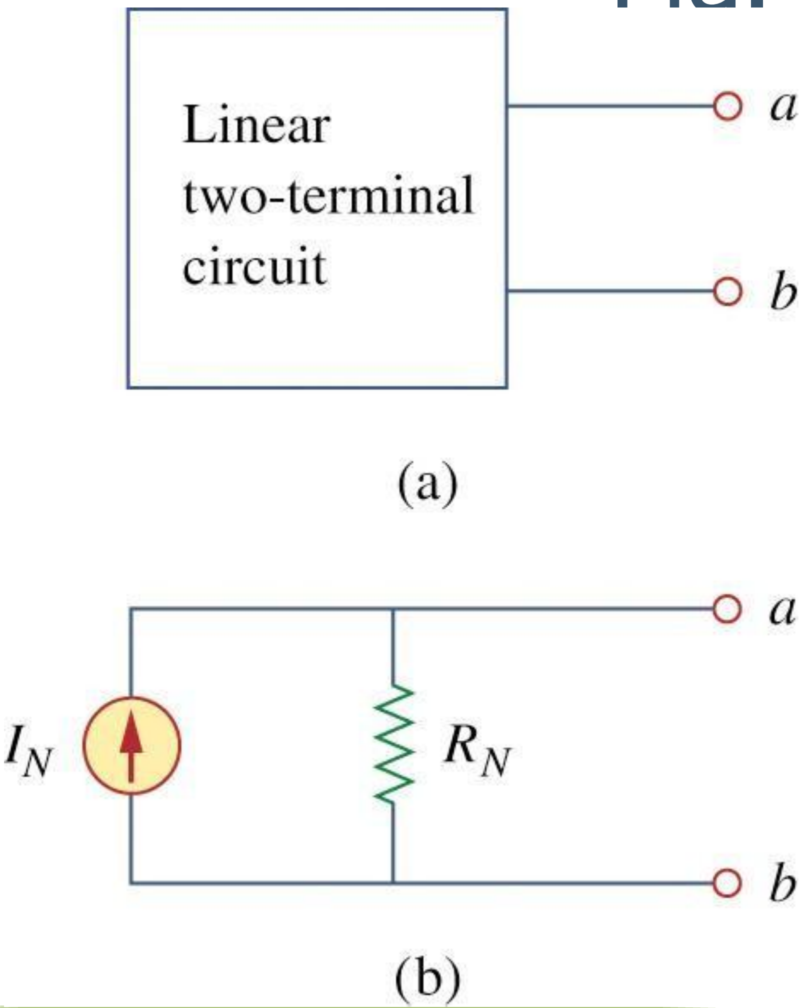


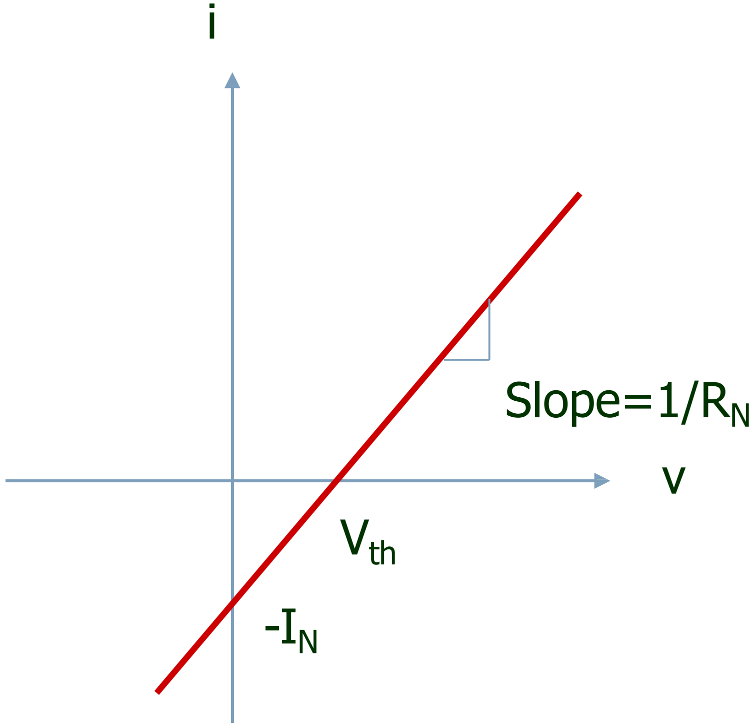
(d)

4.6 Norton's Theorem

- Norton's theorem states that a linear two-terminal circuit can be replaced by equivalent circuit consisting of a current source I_N in parallel with a resistor R_N where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent source are turn off.

Fig. 4.37

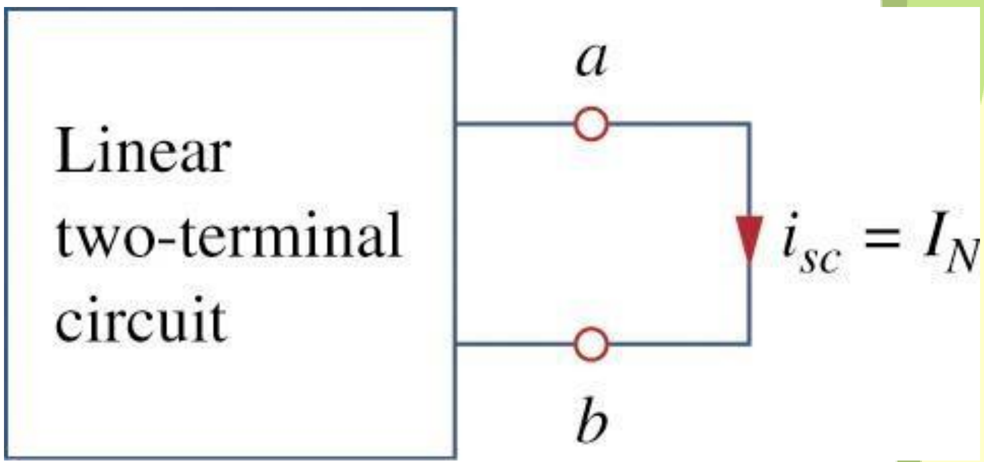




How to Find Norton Current

- Thevenin and Norton resistances are equal: $R_N = R_{\mathrm{Th}}$

- Short circuit current from *a* to *b* : $I_N = i_{sc} = \dfrac{V_{\mathrm{Th}}}{R_{\mathrm{Th}}}$



Thevenin or Norton equivalent circuit :

- The open circuit voltage v_{oc} across terminals a and b
- The short circuit current i_{sc} at terminals a and b
- The equivalent or input resistance R_{in} at terminals a and b when all independent source are turn off.

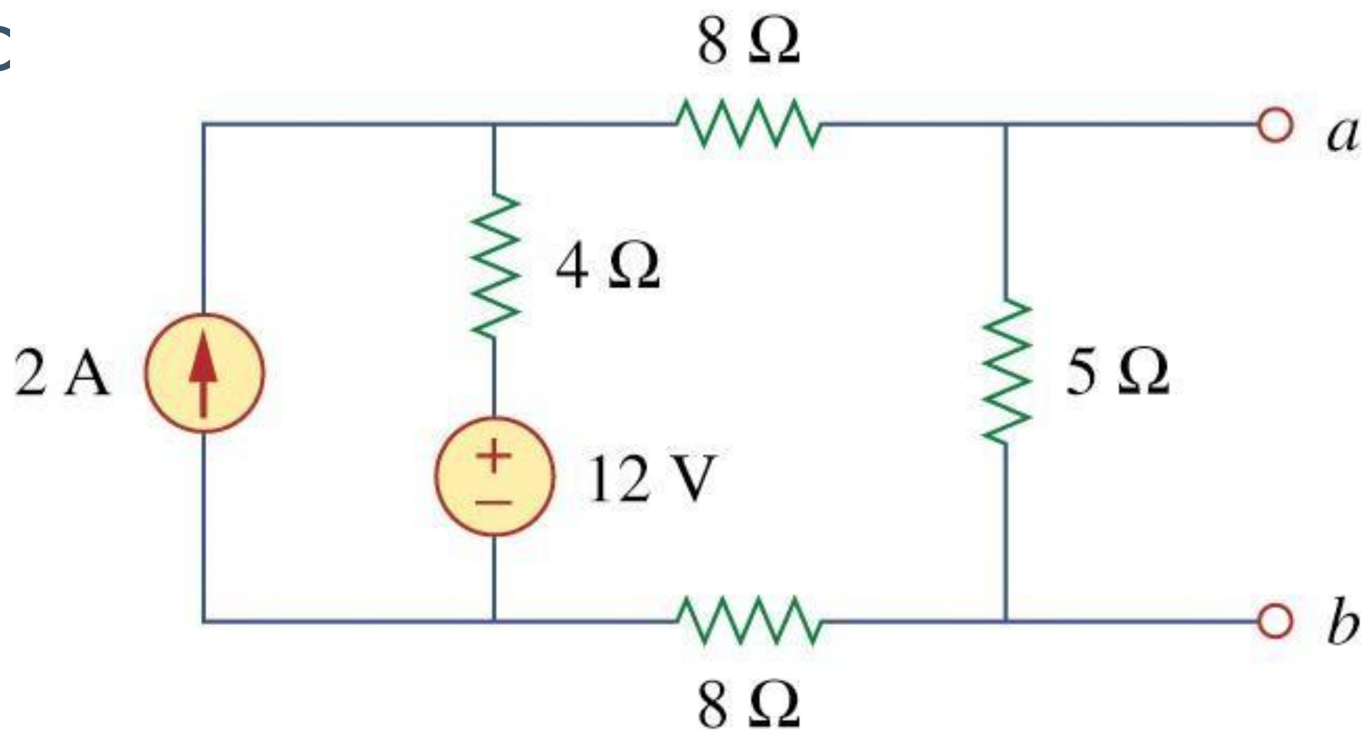
$$V_{Th} = v_{oc}$$

$$I_N = i_{sc}$$

$$R_{Th} = \frac{V_{Th}}{R_{Th}} = R_N$$

Example 4.11

- Find the Norton equivalent circuit of the circ

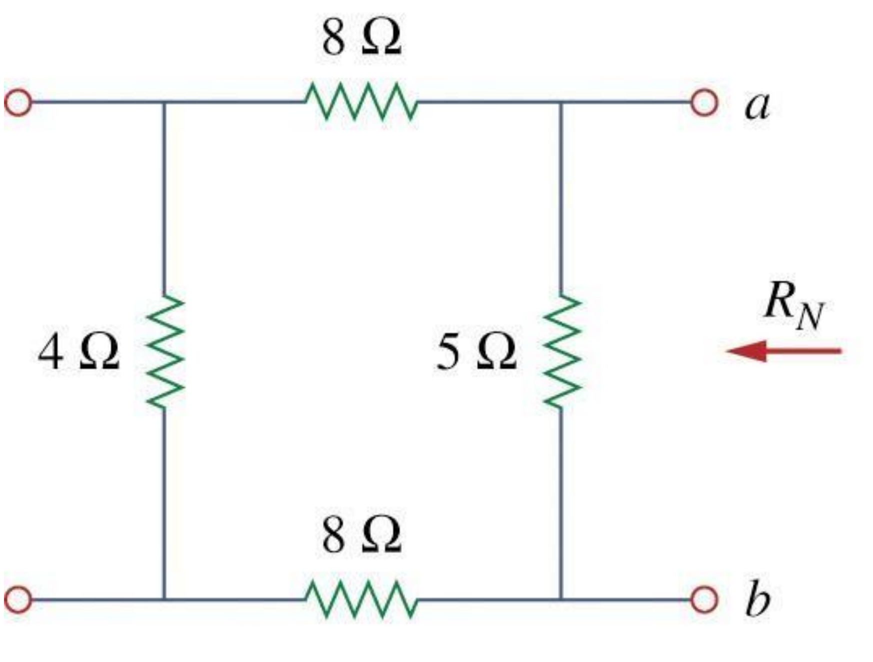


Example 4.11

$\boxed{\text{To find } R_N}$ $\text{Fig4.40}(a):$

$$R_N = 5 \| (8+4+8)$$

$$= 5 \| 20 = \frac{20 \times 5}{25} = 4\Omega$$



(a)

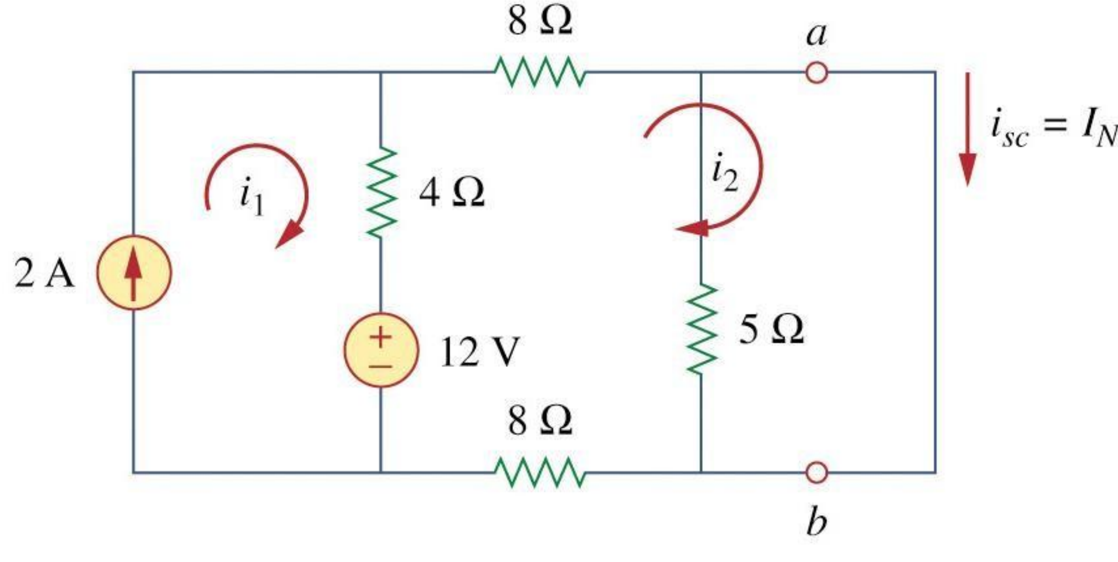
Example 4.11

$\boxed{\text{To find } i_N}$ (Fig.4.40(*b*))

short – circuit terminals a and b.

Mesh : $i_1 = 2\text{A}, \quad 20i_2 - 4i_1 - i_2 = 0$

$i_2 = 1\text{A} = i_{sc} = I_N$



(b)

Example 4.11

Alternative method for I_N

$$I_N = \frac{V_{Th}}{R_{Th}}$$

V_{Th}: open – circuit voltage across terminals a and b

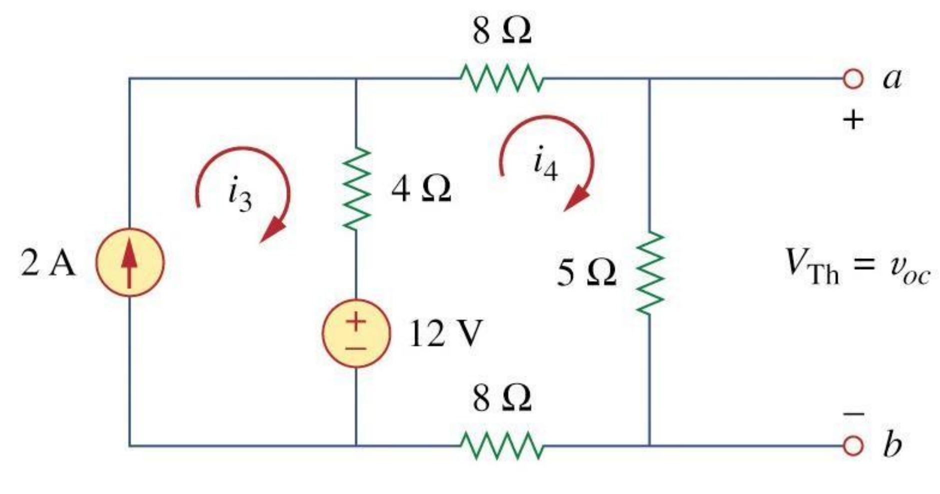
$(Fig\,4.40(c))$:

Mesh analysis :

$i_3 = 2A,\ \ 25i_4 - 4i_3 - 12 = 0$

$\therefore\ i_4 = 0.8\text{A}$

$\therefore v_{oc} = V_{Th} = 5i_4 = 4\text{V}$

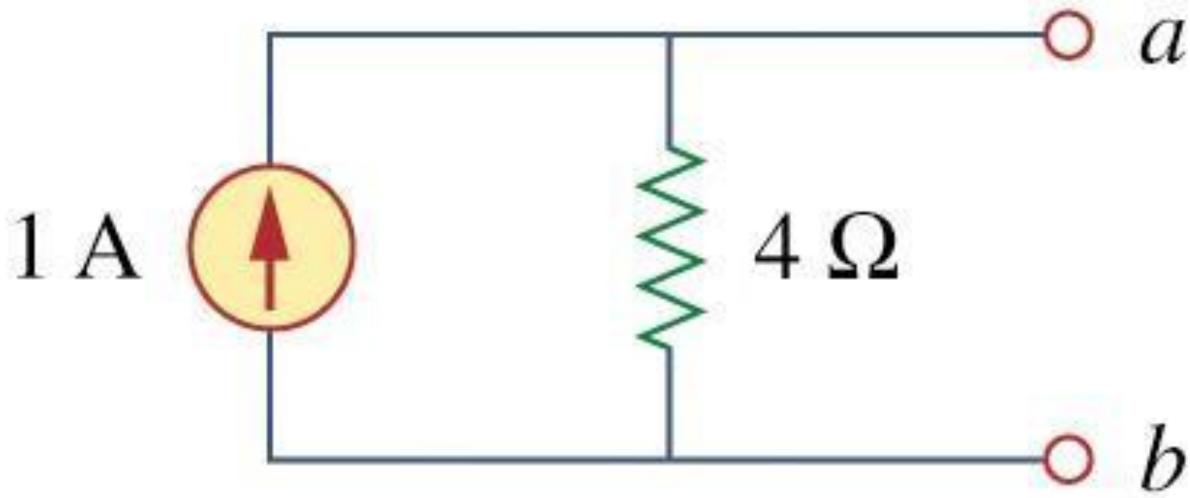


(c)

Example 4.11

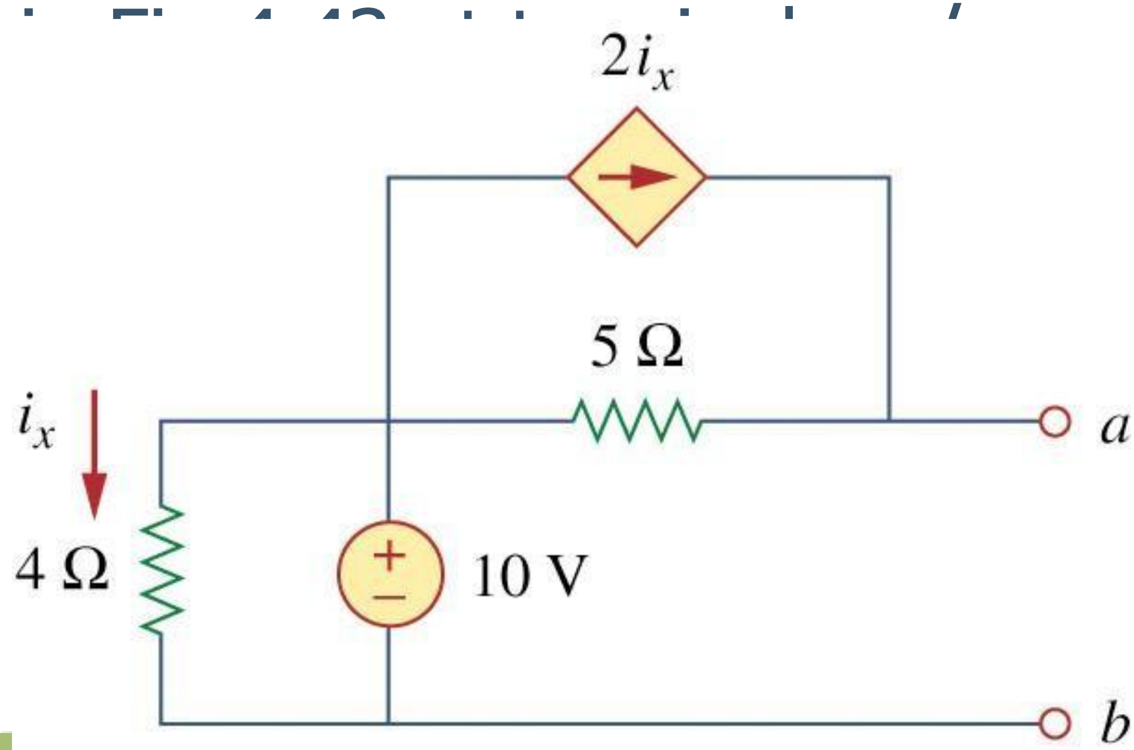
Hence,

$$I_N = \frac{V_{Th}}{R_{Th}} = 4/4 = 1\text{A}$$



Example 4.12

- Using Norton's theorem, find R_N and I_N of the circuit



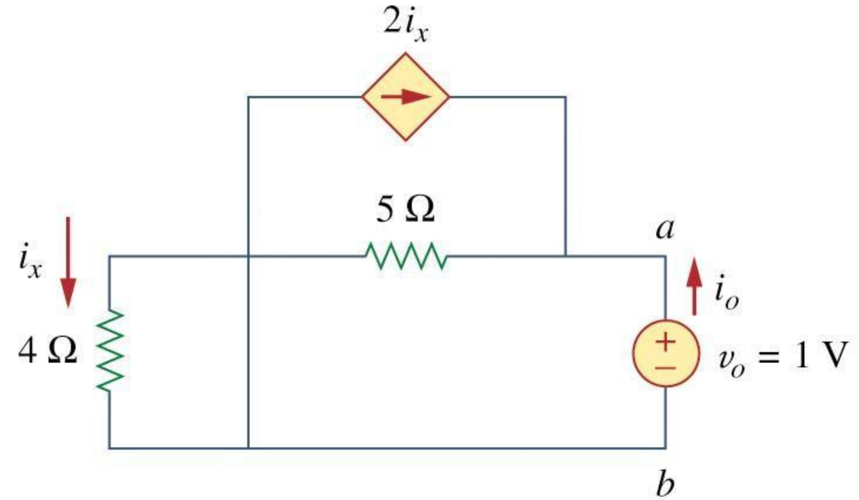
Example 4.12

To find R_N $\quad Fig.4.44(a)$

- 4Ω resistor shorted
- $5\Omega \| v_o \| 2i_x$: Parallel

Hence, $i_x = v_o / 5 = 1/5 = 0.2$

$$\therefore R_N = \frac{v_o}{i_o} = \frac{1}{0.2} = 5\Omega$$



(a)

Example 4.12

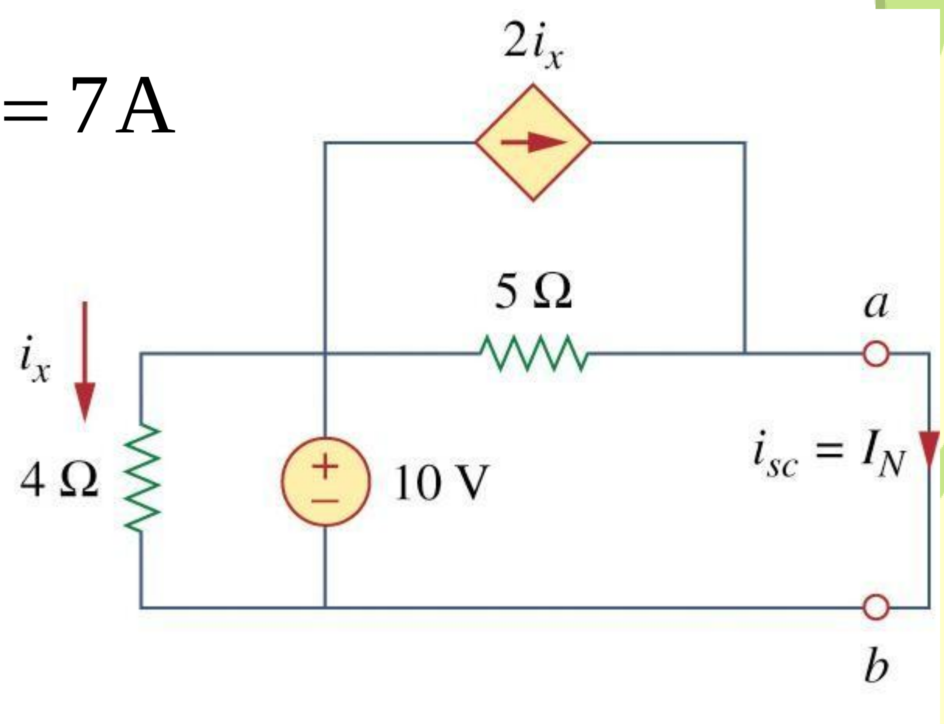
To find I_N $Fig.4.44(b)$

- $4\Omega \,||\, 10v \,||\, 5\Omega \,||\, 2i_x$:Parallel

$$i_x = \frac{10-0}{4} = 2.5\text{A},$$

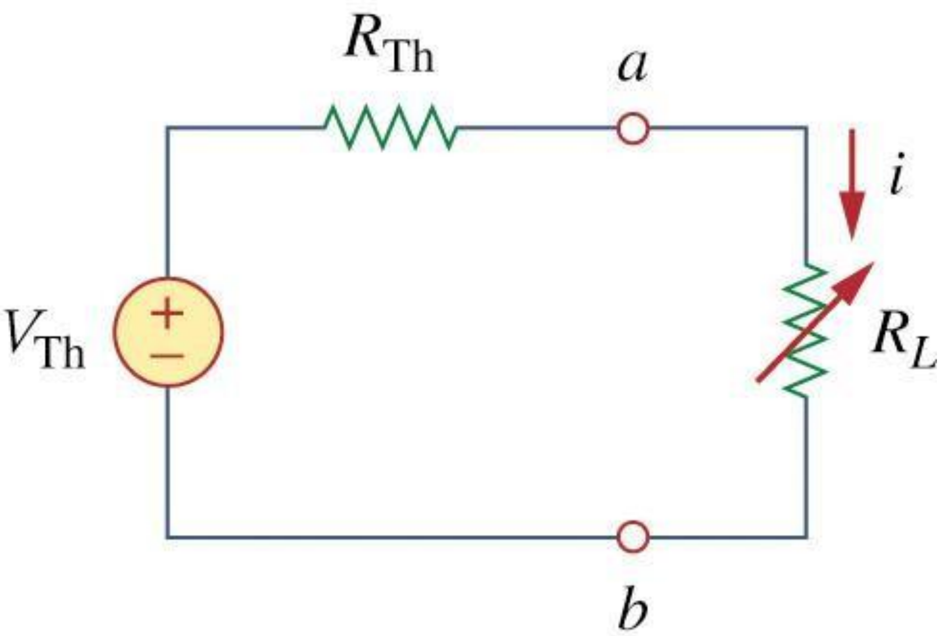
$$i_{sc} = i_x + 2i_x = \frac{10}{5} + 2(2.5) = 7\text{A}$$

$$\therefore I_N = 7\text{A}$$



(b)

4.8 Maximum Power Trandfer

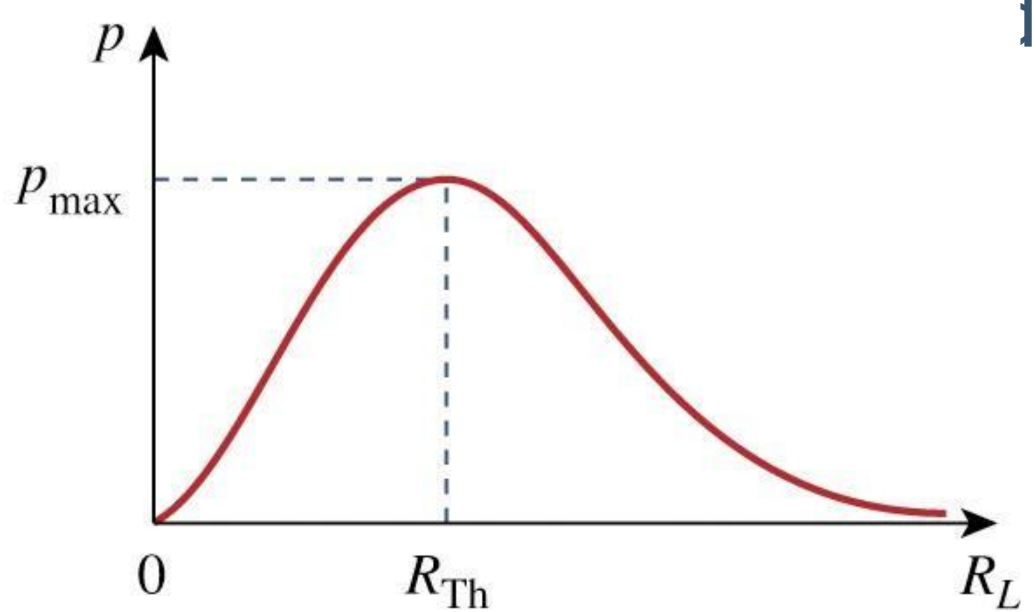


$$p = i^2 R_L = \left(\frac{V_{TH}}{R_{TH} + R_L} \right)^2 R_L$$

Fig 4.48

Fig. 4.49

- Maximum power is transferred to the load when the load resistance equals the Thever … d (R_L = R_{TH}).



$$\frac{dp}{dR_L} = V_{TH}^2 \left[\frac{(R_{TH} + R_L)^2 - 2R_L(R_{TH} + R_L)}{(R_{TH} + R_L)^4} \right]$$

$$= V_{TH}^2 \left[\frac{(R_{TH} + R_L - 2R_L)}{(R_{TH} + R_L)^3} \right] = 0$$

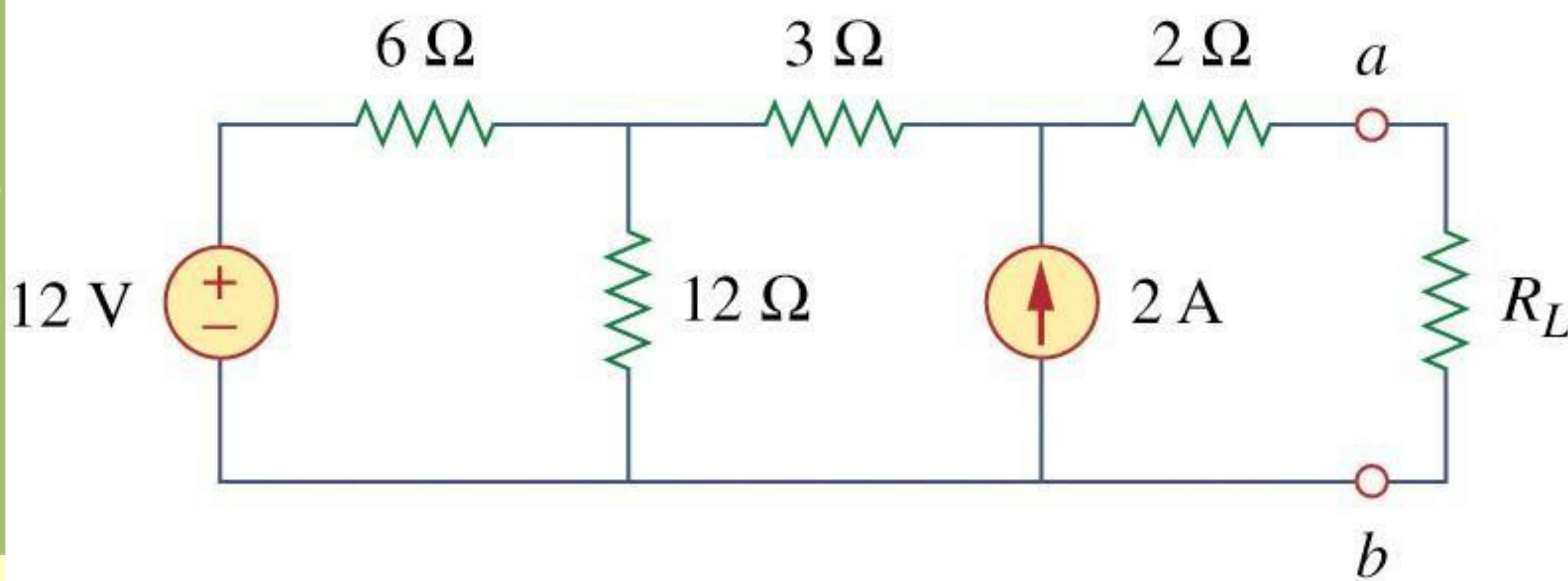
$$0 = (R_{TH} + R_L - 2R_L) = (R_{TH} - R_L)$$

$$\boxed{R_L = R_{TH}}$$

$$\boxed{p_{\max} = \frac{V_{TH}^2}{4R_{TH}}}$$

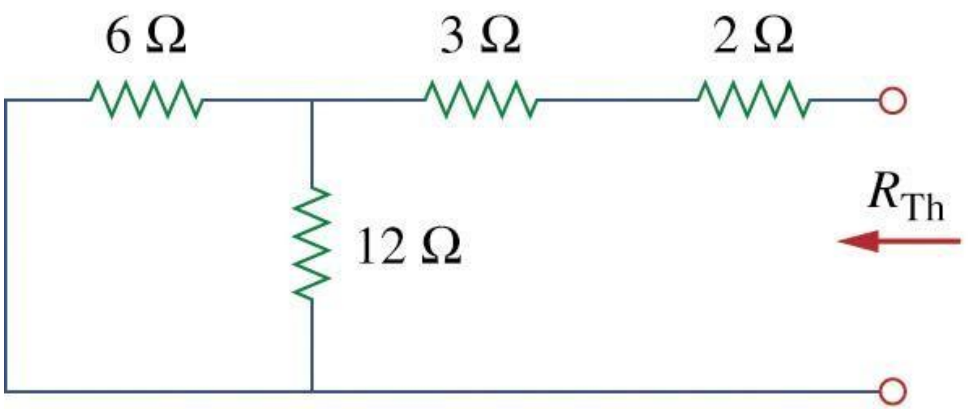
Example 4.13

- Find the value of R_L for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.



Example 4.13

$$R_{TH} = 2 + 3 + 6 \| 12 = 5 + \frac{6 \times 12}{18} = 9\Omega$$



(a)

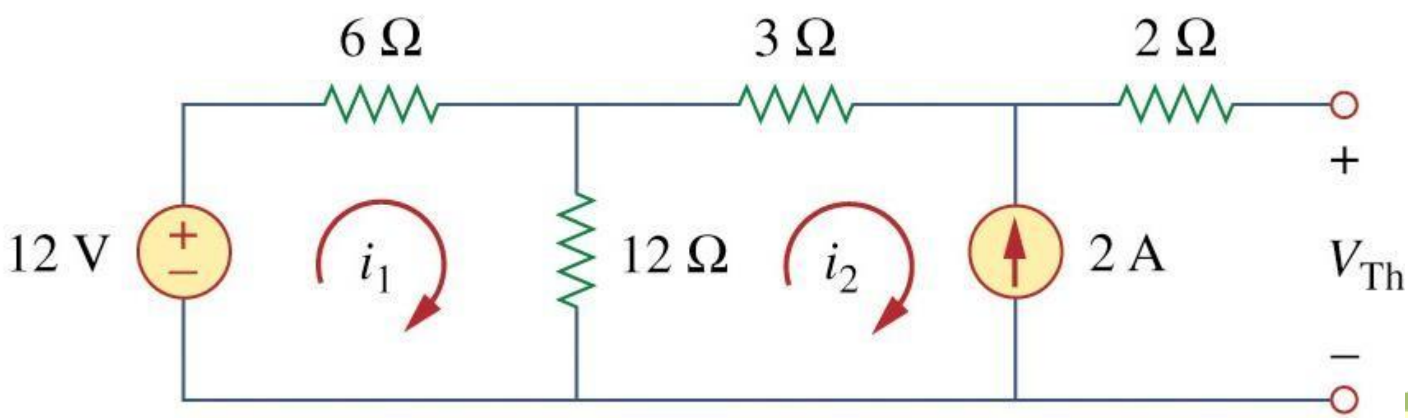
Example 4.13

$$-12 + 18i_1 - 12 i_2, \quad i_2 = -2A$$

$$-12 + 6i_i 1 + 3i_2 + 2(0) + V_{TH} = 0 \Rightarrow V_{TH} = 22V$$

$$R_L = R_{TH} = 9\Omega$$

$$p_{\max} = \frac{V_{TH}^2}{4R_L} = \frac{22^2}{4\times 9} = 13.44W$$



(b)

Extension of Resonant Circuits

What can be learned from this example?

- w_r does not seem to have much meaning in this problem. What is w_r if R = 3.99 ohms?
- Just because a circuit is operated at the resonant frequency does not mean it will have a peak in the response at the frequency.
- For circuits that are fairly complicated and can resonant, It is probably easier to use a simulation program similar to Matlab to find out what is going on in the circuit.