POWER POINT PRESENTATION
ON
ELECTRICAL CIRCUITS

II B. Tech I semester (JNTUH-R15)

Prepared
By

Mr. T Anil Kumar Associate Professor

ELECTRICAL AND ELECTRONICS ENGINEERING
INSTITUTE OF AERONAUTICAL ENGINEERING
(AUTONOMOUS)
DUNDIGAL, HYDERABAD - 500043

ELECTRICAL CIRCUITS

All you need to be an inventor is a good imagination and a pile of junk.

-Thomas Edison

Ohm's Law

$$I = V / R$$

Georg Simon Ohm (1787-1854)

I	= Current (Amperes) (amps)
V	= Voltage (Volts)
R	= Resistance (ohms)

How you should be thinking about electric circuits:

<u>Voltage</u>: a force that pushes the current through the circuit (in this picture it would be equivalent to gravity)

How you should be thinking about electric circuits:

Resistance: friction that impedes flow of current through the circuit (rocks in the river)

How you should be thinking about electric circuits:

Current: the actual “substance” that is flowing through the wires of the circuit (electrons!)

Would This Work?

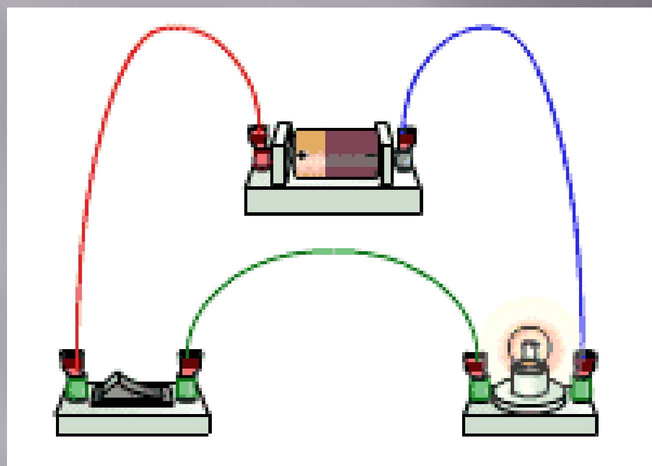
Would This Work?

Would This Work?

The Central Concept: Closed Circuit

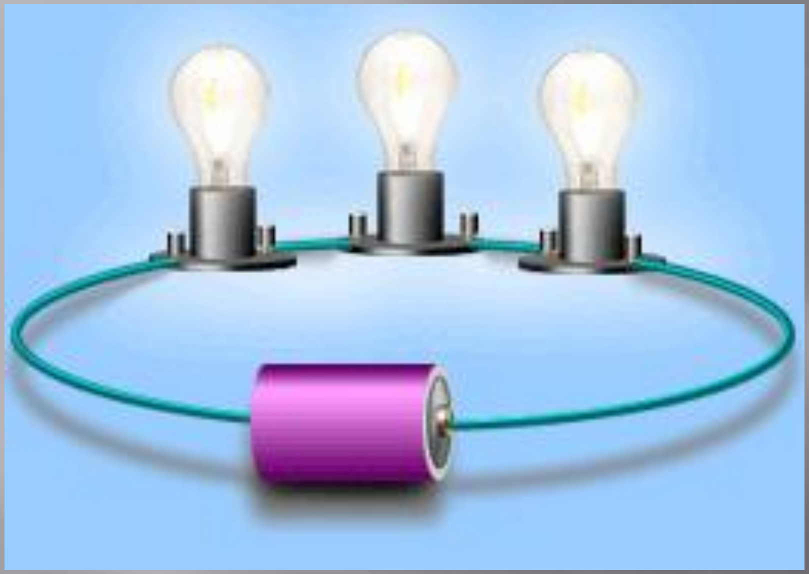
circuit diagram

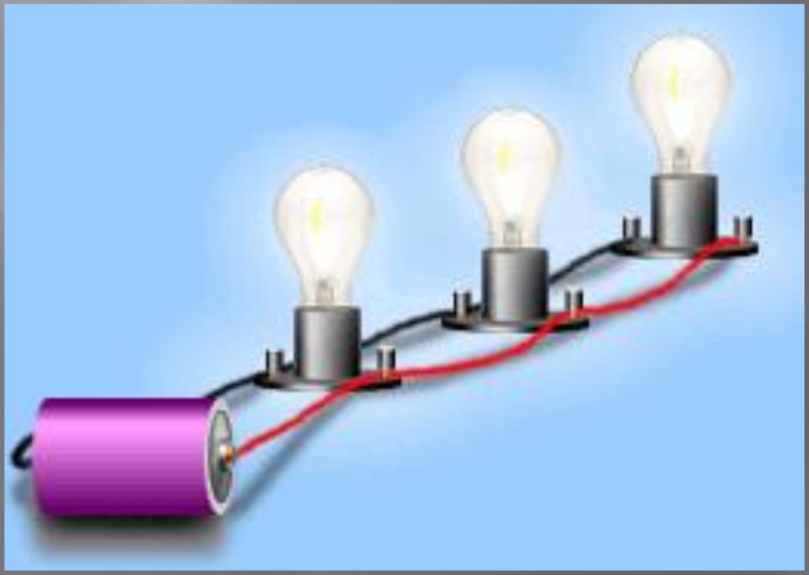
Scientists usually draw electric circuits using symbols;



cell lamp switch wires

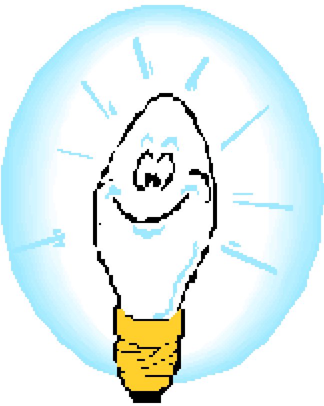
Simple Circuits

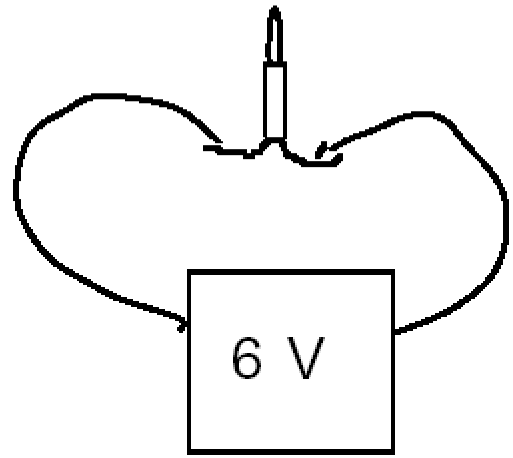




- Series circuit
  - All in a row
  - 1 path for electricity
  - 1 light goes out and the circuit is broken
- Parallel circuit
  - Many paths for electricity
  - 1 light goes out and the others stay on

SERIES CIRCUITS





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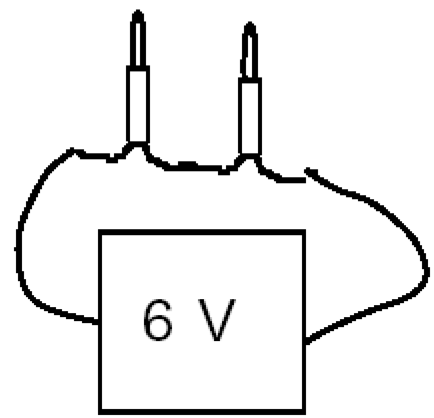
Connect one bulb to the battery.



Connect 2 bulbs and the battery to form a series circuit.

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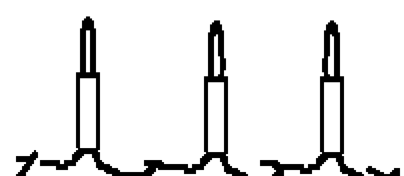
Connect 3 bulbs and the battery to form a series circuit.

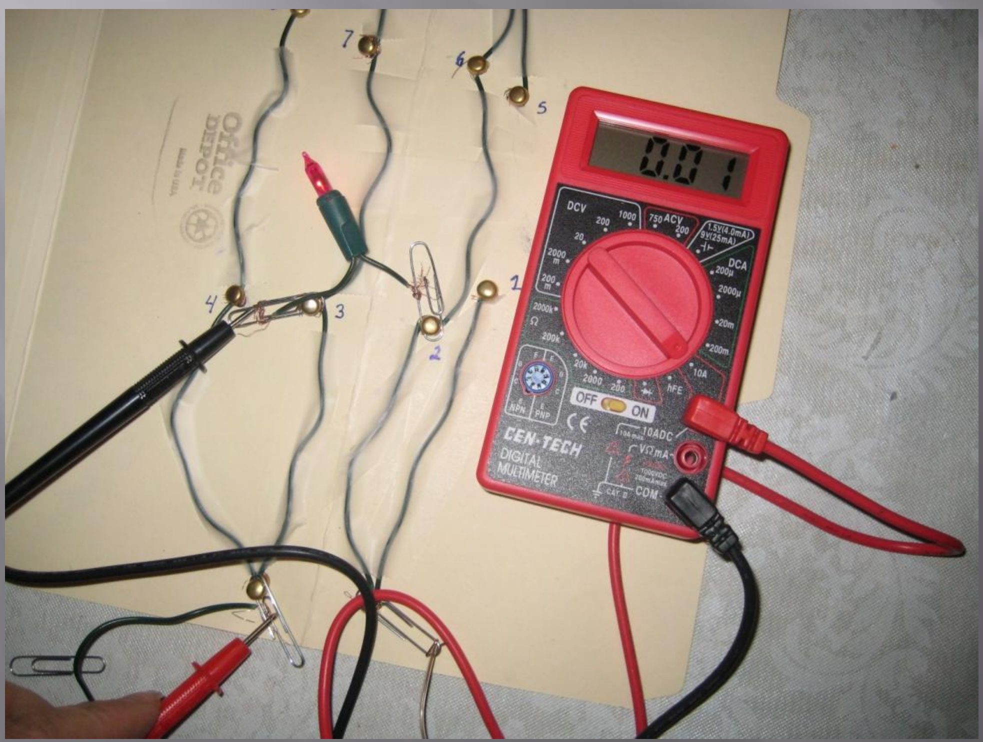


PARALLEL CIRCUIT

- Place two bulbs in parallel. What do you notice about the brightness of the bulbs?
- Add a third light bulb in the circuit. What do you notice about the brightness of the bulbs?
- Remove the middle bulb from the circuit. What happened?

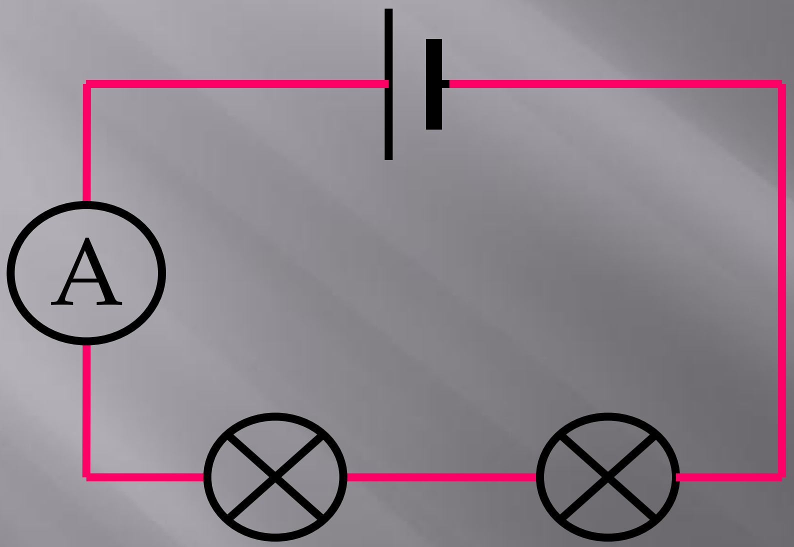
measuring current

Electric current is measured in **amps** (A) using an ammeter connected in series in the circuit.

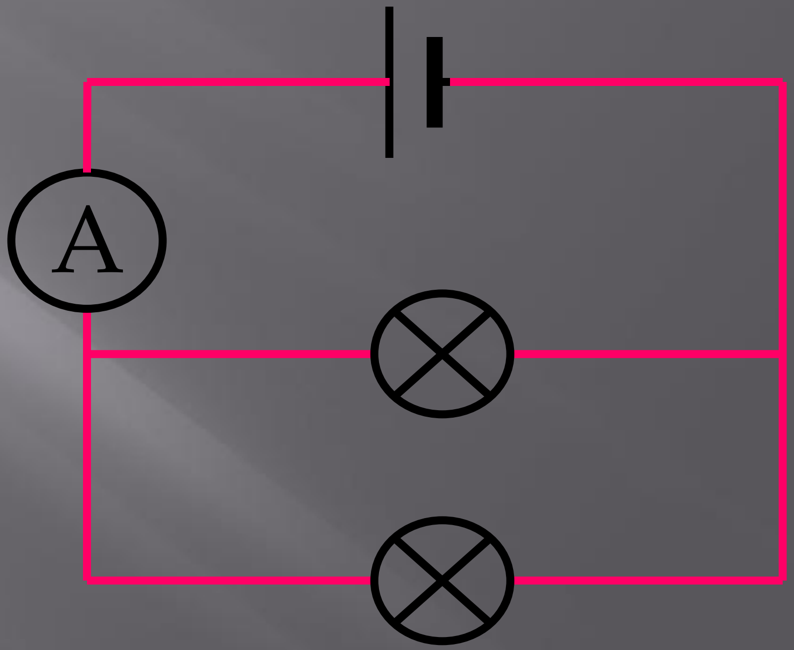


measuring current

This is how we draw an ammeter in a circuit.



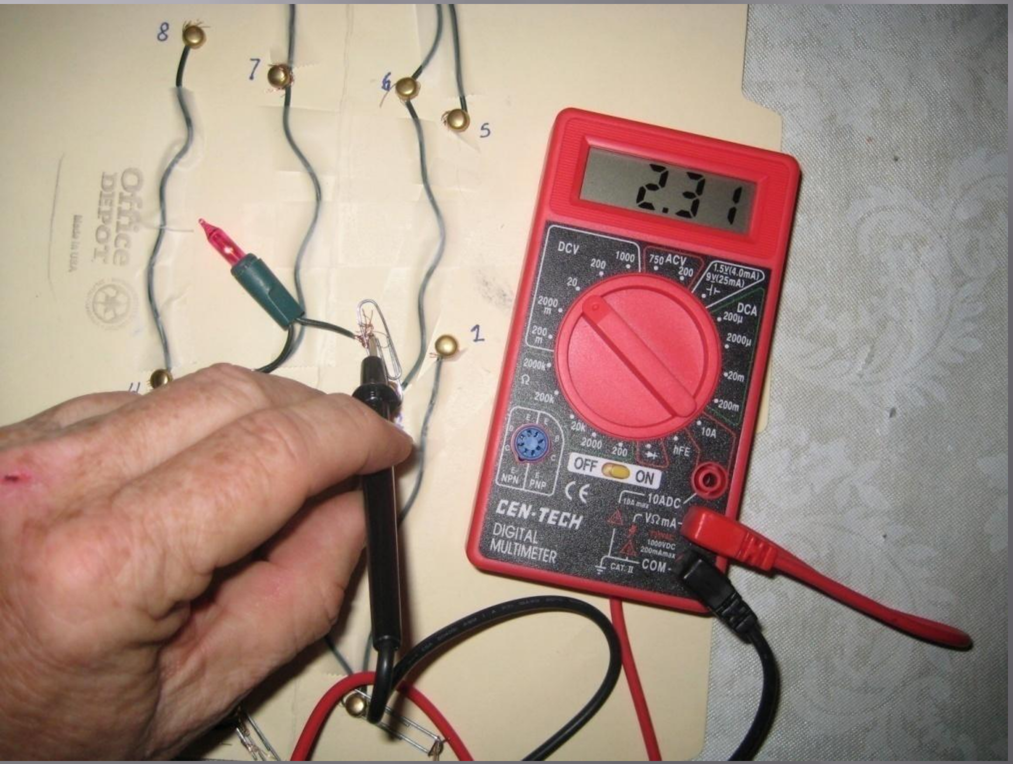
SERIES CIRCUIT



PARALLEL CIRCUIT

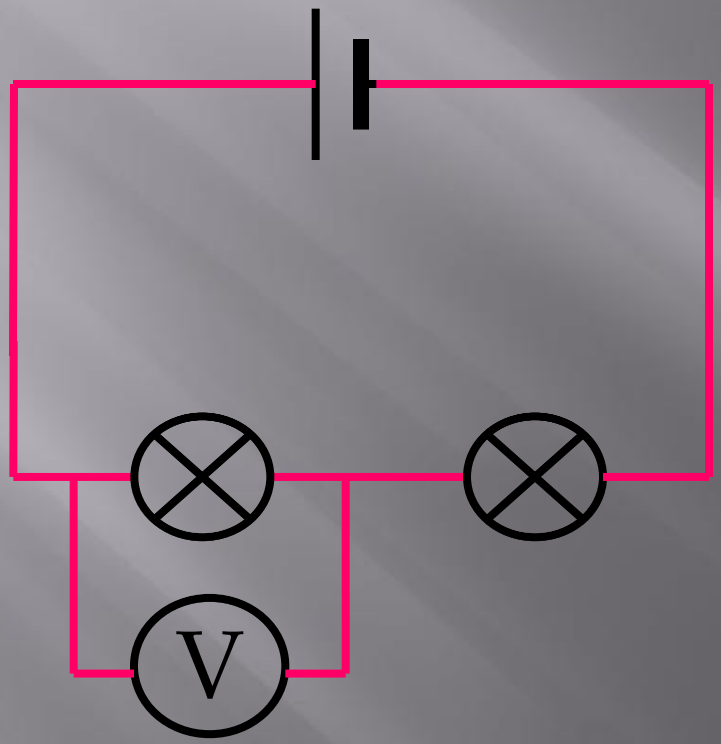
measuring voltage

The 'electrical push' which the cell gives to the current is called the voltage. It is measured in **volts** (V) on a **voltmeter**

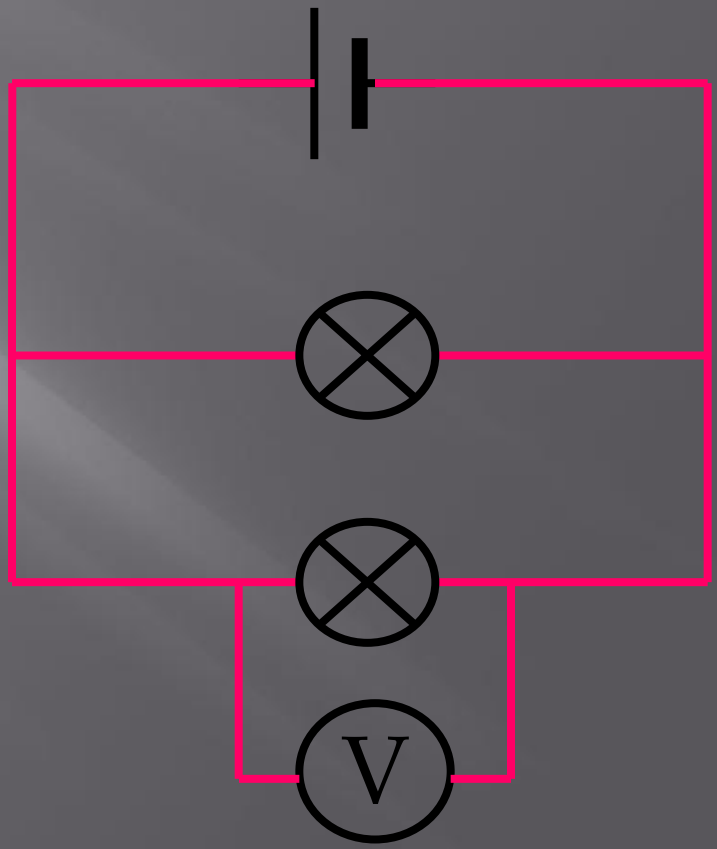


measuring voltage

This is how we draw a voltmeter in a circuit.



SERIES CIRCUIT



PARALLEL CIRCUIT

OHM's LAW

- Measure the current and voltage across each circuit.
- Use Ohm's Law to compute resistance

Series Circuit

Voltage	Current	Resistance

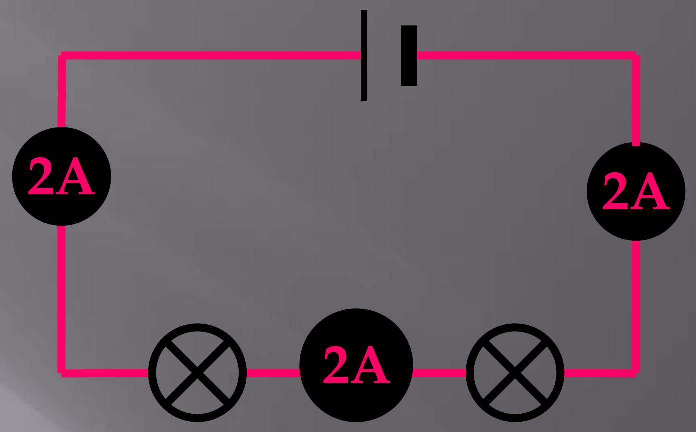
Parallel Circuit

Voltage	Current	Resistance

measuring current

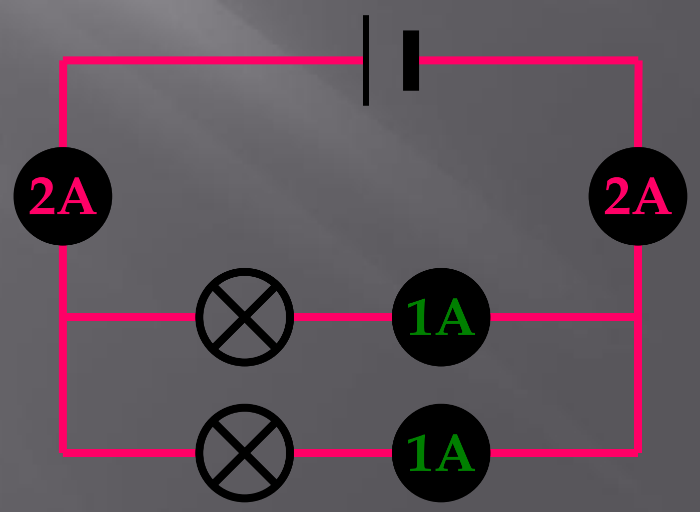
SERIES CIRCUIT

- current is the **same** at all points in the circuit.

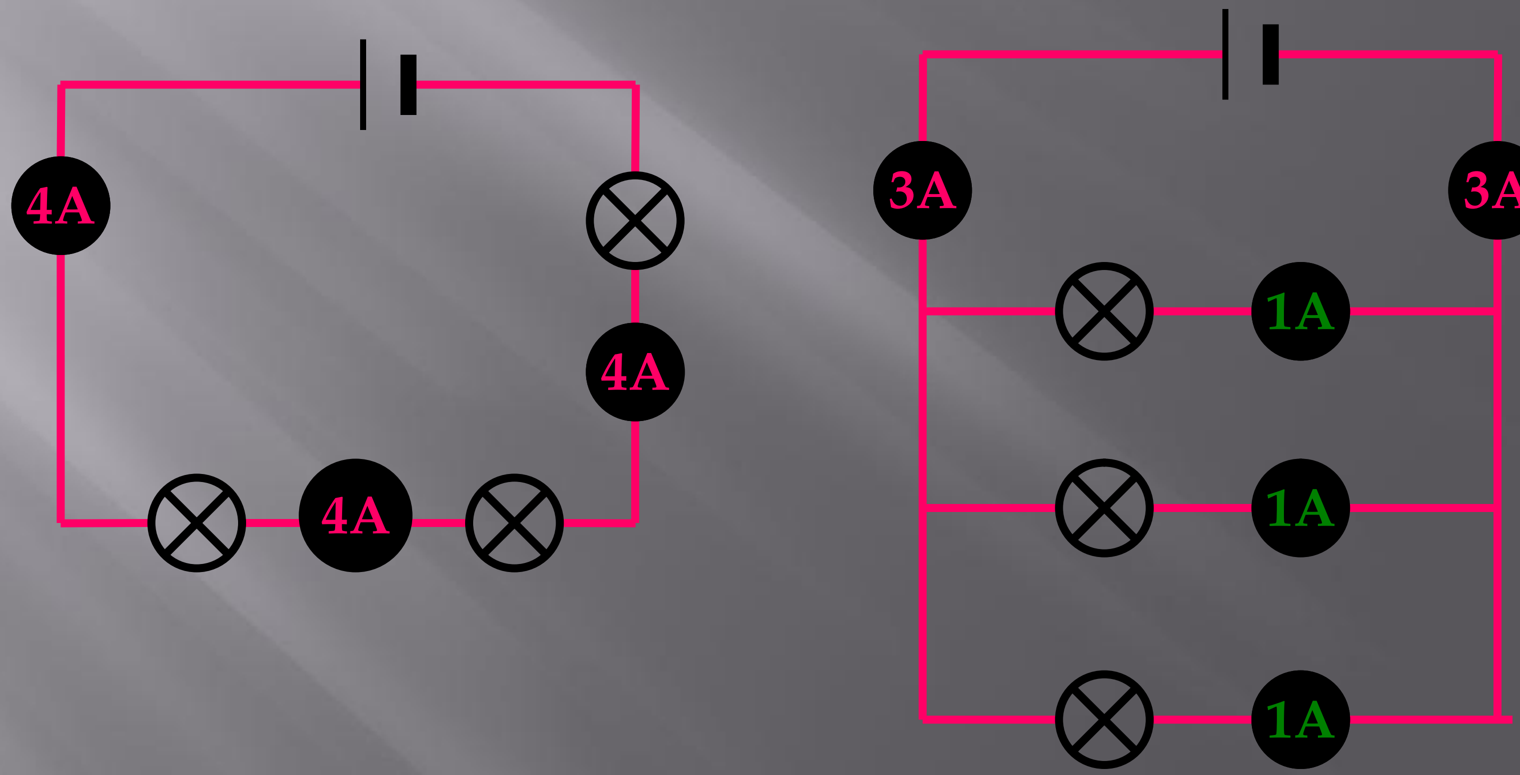


PARALLEL CIRCUIT

- current is **shared** between the components



fill in the missing ammeter readings.



SERIES CIRCUITS

Explain what happens to the current in a series circuit when there is a break in the circuit.

The circuit is no longer complete, therefore current can not flow

Explain what happens to the voltage across each bulb as more bulbs are added to the circuit.

The voltage decreases because the current is decreased and the resistance increases.

PARALLEL CIRCUITS

Explain what happens to the current in each bulb as more bulbs are added to the circuit.

The current remains the same. The total resistance drops in a parallel circuit as more bulbs are added

Explain what happens to the total current provided by the battery as more bulbs are added to the circuit.

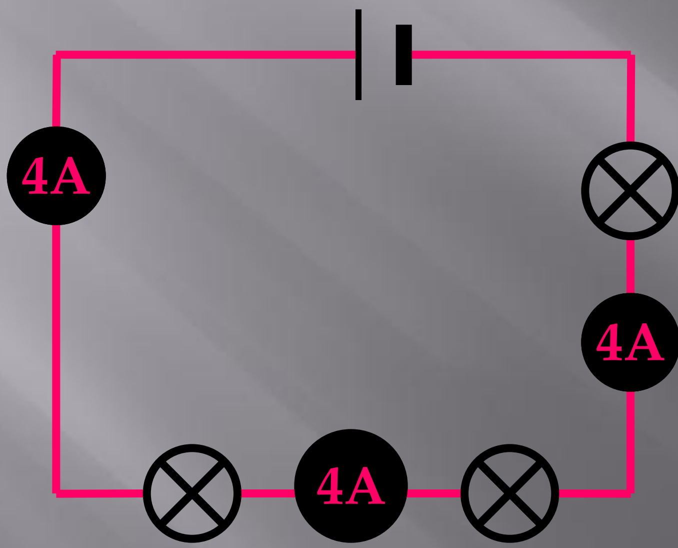
The current increases.

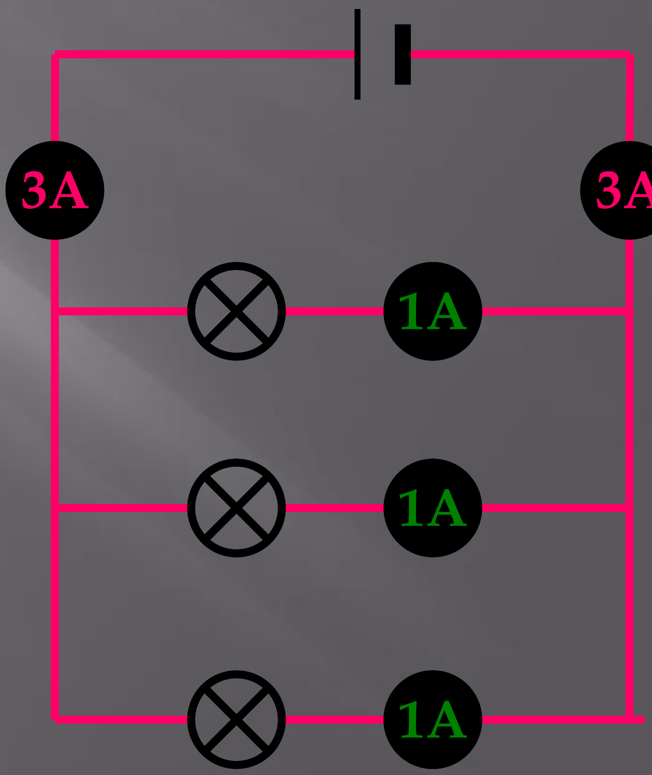
Series and Parallel Circuits

- Series Circuits
  - only one end of each component is connected
  - e.g. Christmas tree lights

- Parallel Circuits
  - both ends of a component are connected
  - e.g. household lighting

copy the following circuits and fill in the missing ammeter readings.





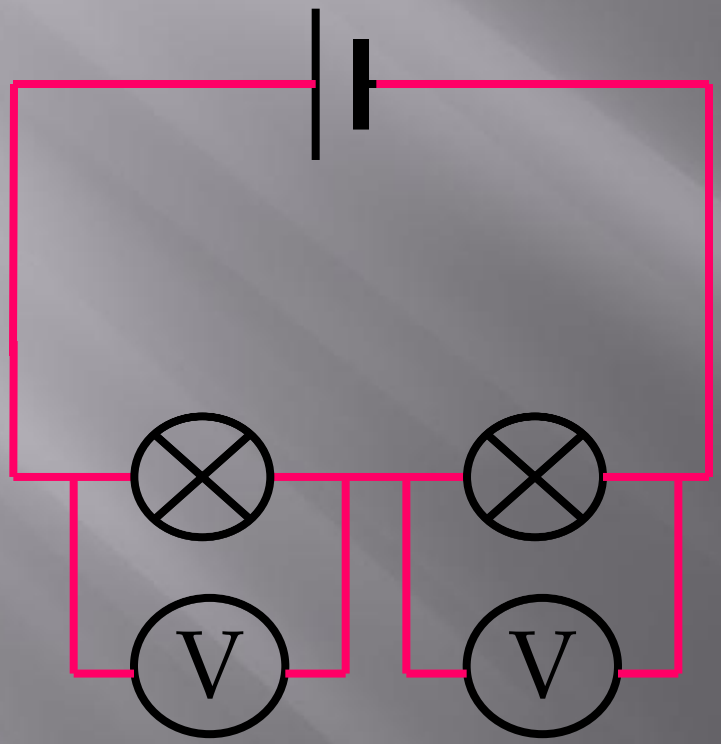
measuring voltage

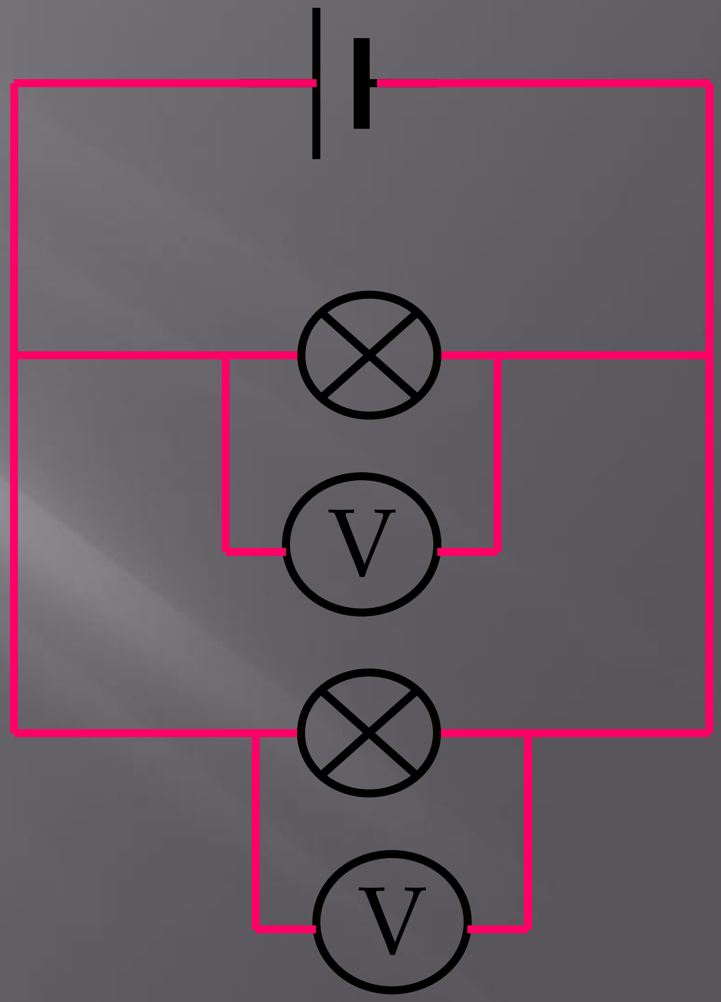
Different cells produce different voltages. The bigger the voltage supplied by the cell, the bigger the current.

Unlike an ammeter, a voltmeter is connected across the components

Scientist usually use the term **Potential Difference** (pd) when they talk about voltage.

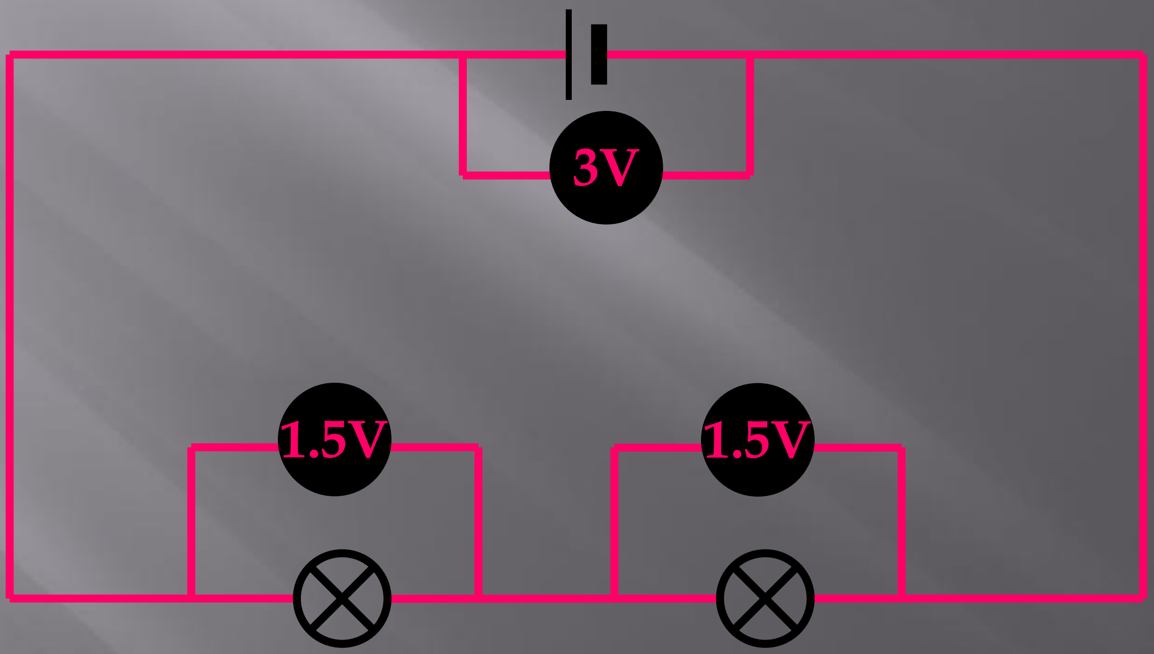
measuring voltage





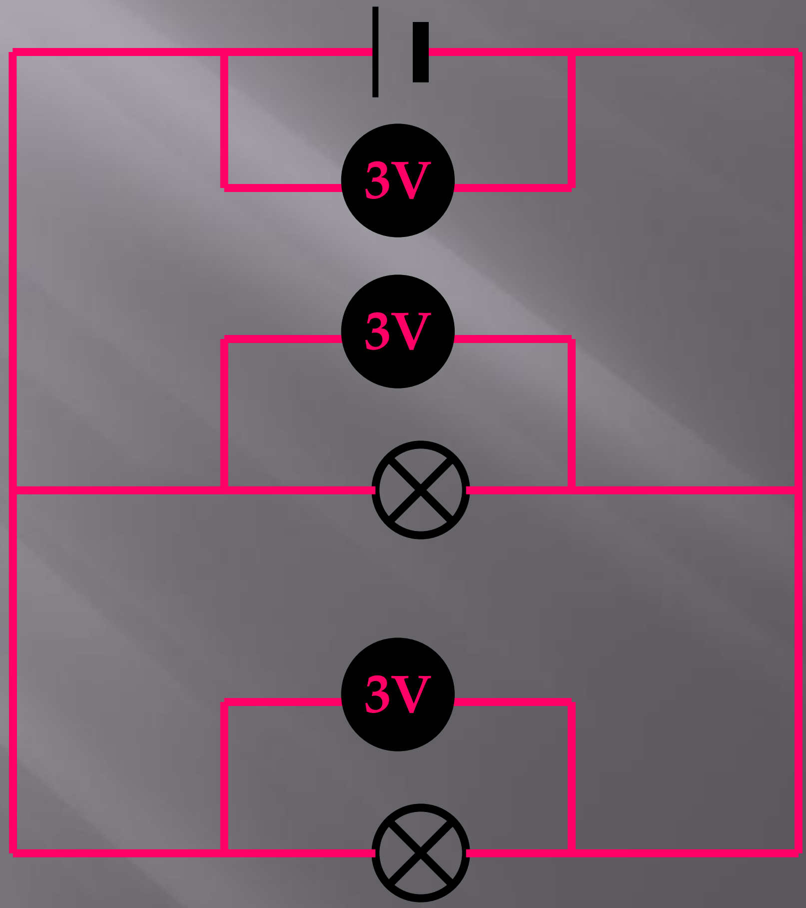
series circuit

- voltage is **shared** between the components



parallel circuit

- voltage is the **same** in all parts of the circuit.



measuring current & voltage

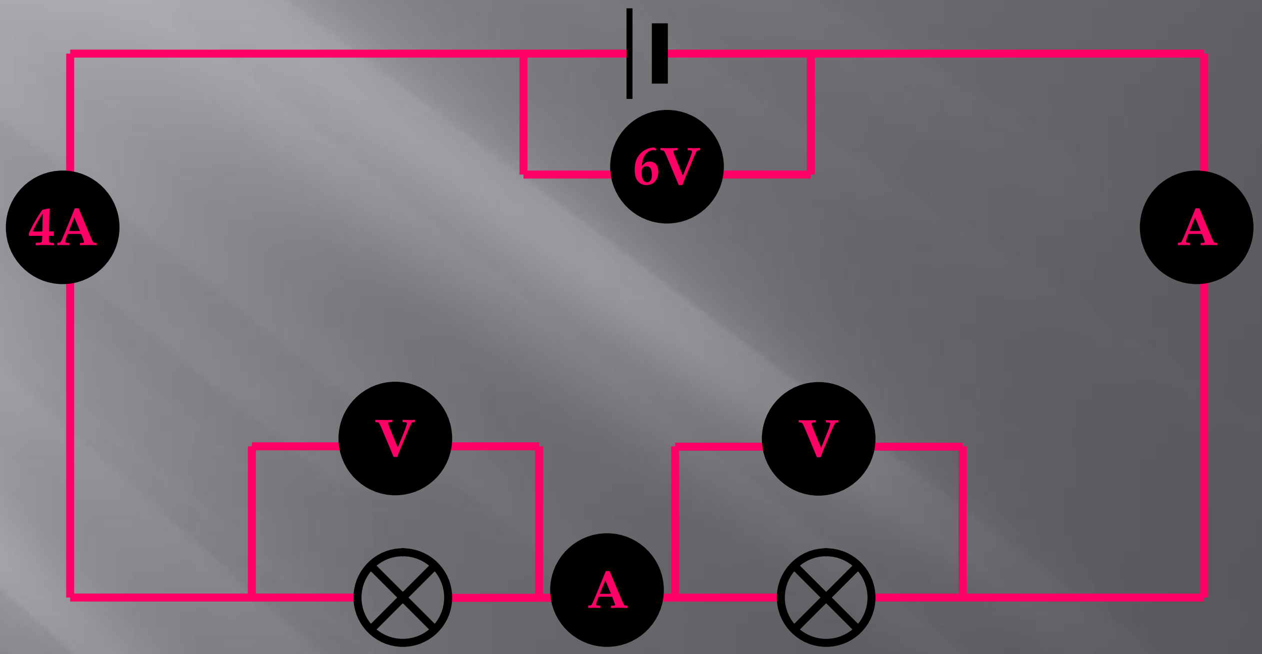
copy the following circuits on the next two slides.

complete the missing current and voltage readings.

remember the rules for current and voltage in series and parallel circuits.

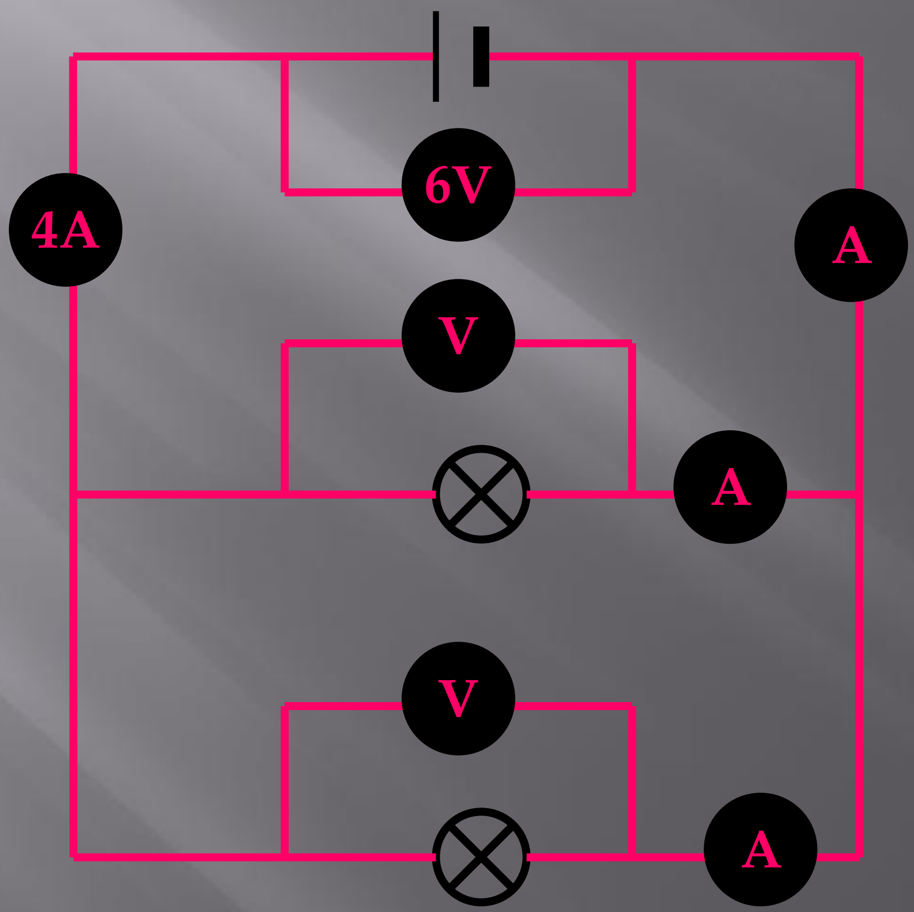
measuring current & voltage

a)



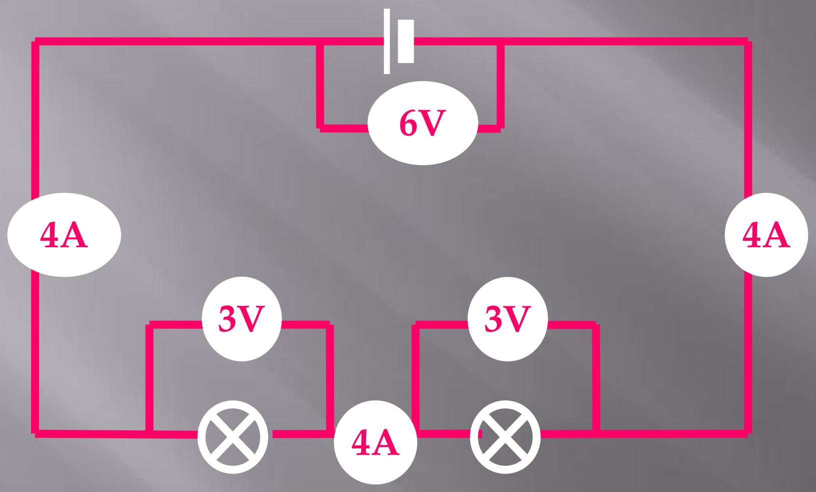
measuring current & voltage

b)

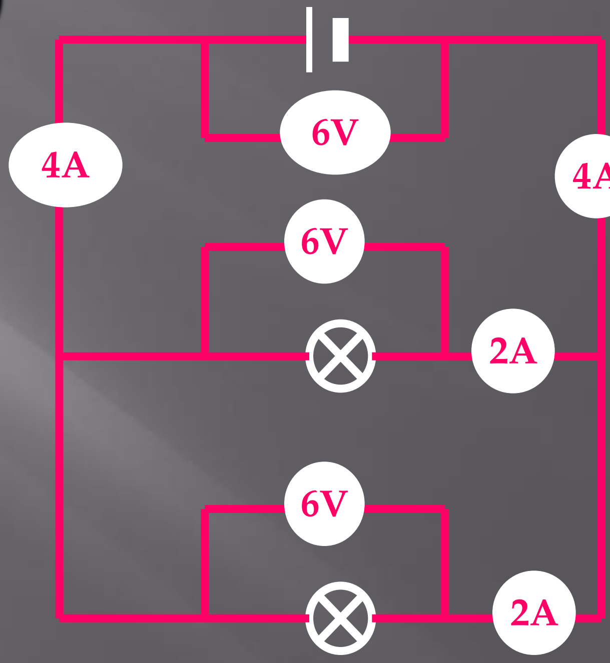


answers

a)



b)



Voltage, Current, and Power

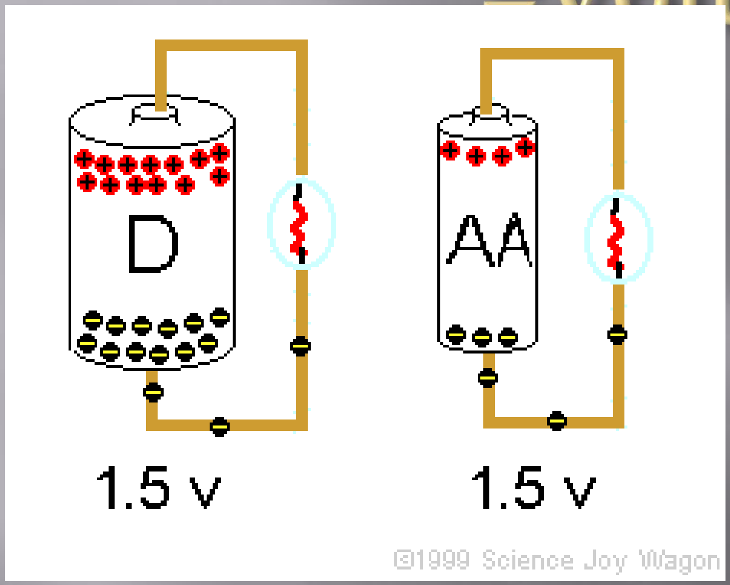
- One Volt is a Joule per Coulomb (J/C)
- One Amp of current is one Coulomb per second (6.24 x10^18 electrons/second).
- If I have one volt (J/C) and one amp (C/s), then multiplying gives Joules per second (J/s)
  - this is power: J/s = Watts
- So the formula for electrical power is just:
- More work is done per unit time the higher the voltage and/or the higher the current

$P = VI$: power = voltage × current

Electric Circuits

AP Physics B

Potential Difference =Voltage=EMF



Voltage = Potential Difference = Emf

$V = \Delta V = \varepsilon$

In a battery, a series of chemical reactions occur in which electrons are transferred from one terminal to another. There is a **potential difference (voltage)** between these poles.

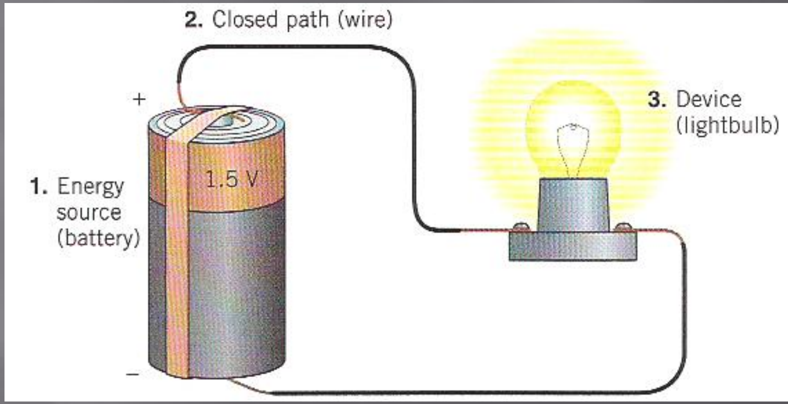
The **maximum potential difference** a power source can have is called the **electromotive force or (EMF),** ε. The term isn't actually a force, simply the amount of energy per charge (J/C or V)

A Basic Circuit

All electric circuits have three main parts

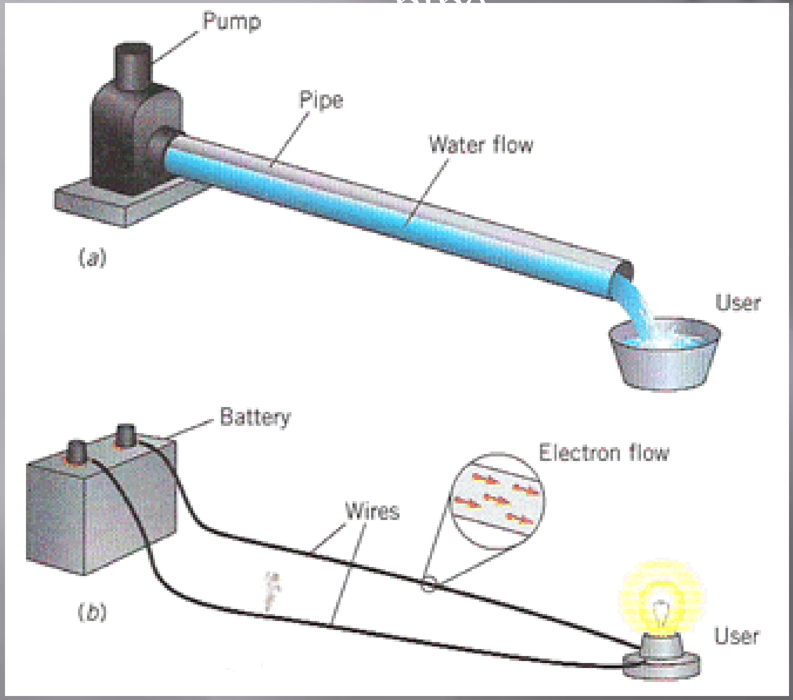
1. A source of energy
2. A closed path
3. A device which uses the energy

If ANY part of the circuit is open the device will not work!



Electricity can be symbolic of Fluids

Circuits are very similar to water flowing through a pipe



A pump basically works on TWO IMPORTANT PRINCIPLES concerning its flow

- There is a **PRESSURE DIFFERENCE** where the flow begins and ends
- A certain **AMOUNT** of flow passes each **SECOND**.

A circuit basically works on TWO IMPORTANT PRINCIPLES

- There is a "**POTENTIAL DIFFERENCE aka VOLTAGE**" from where the charge begins to where it ends
- The **AMOUNT of CHARGE** that flows **PER SECOND** is called **CURRENT.**

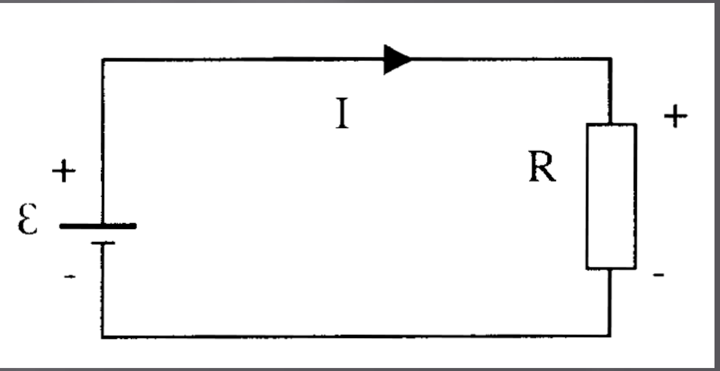
Current

Current is defined as the rate at which charge flows through a surface.

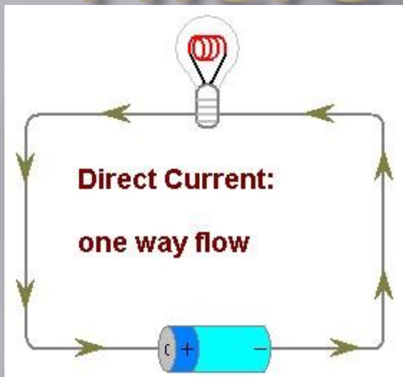
$$I = \frac{q}{t} = \frac{Coloumbs(C)}{Second(s)} = Amperes = Amps = A$$

The current is in the same direction as the flow of positive charge (for this course)

Note: The "I" stands for *intensity*

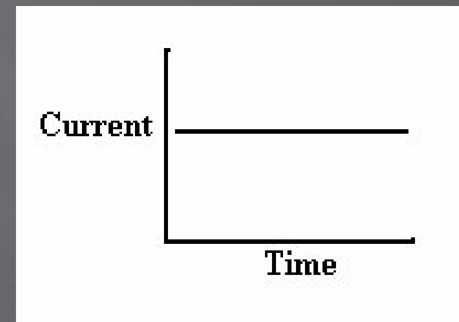


There are 2 types of Current

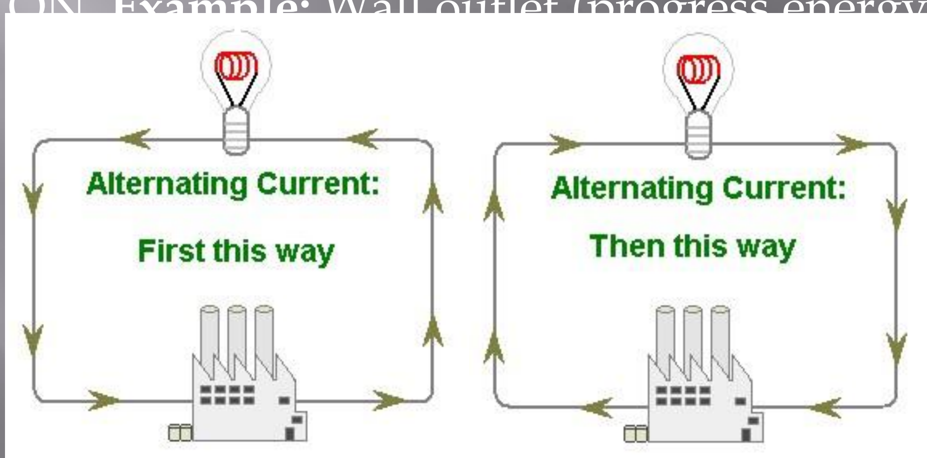


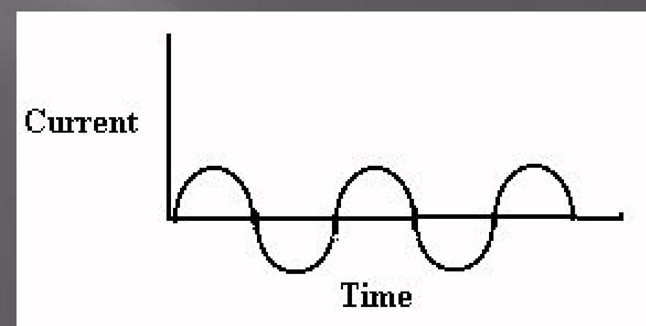
DC = Direct Current - current flows in one direction

Example: Battery



AC = Alternating Current- current reverses direction many times per second. This suggests that AC devices turn OFF and ON. **Example:** Wall outlet (progress energy)





Ohm's Law

"The voltage (potential difference, emf) is directly related to the current, when the resistance is constant"

$\Delta V \alpha I$

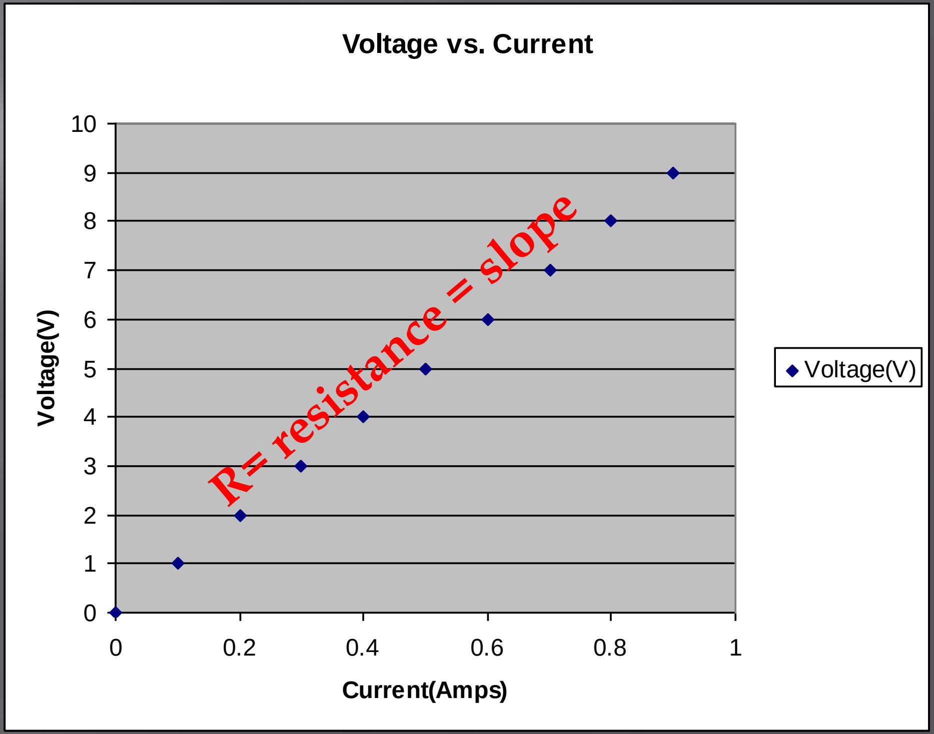
$R = \text{constant of proportionality}$

$R = \text{Resistance}$

$\Delta V = IR$

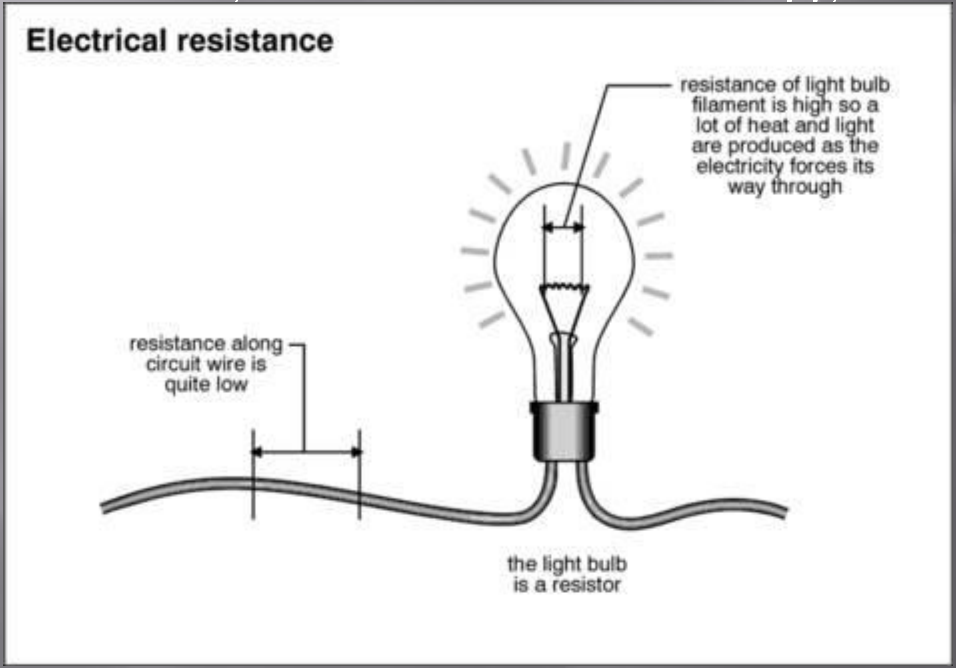
$\varepsilon = IR$

Since R=ΔV/I, the resistance is the SLOPE of a ΔV vs. I graph



Resistance

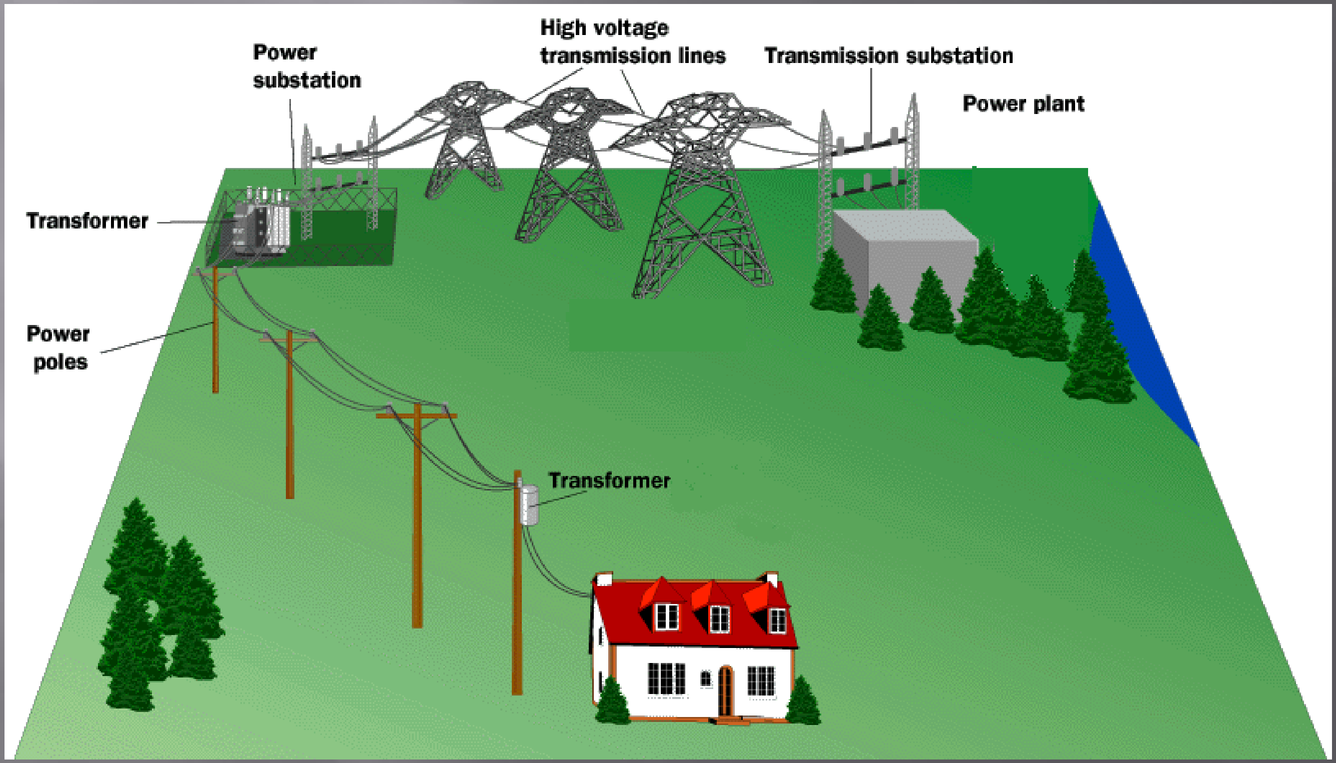
Resistance (R) – is defined as the restriction of electron flow. It is due to interactions that occur at the atomic scale. For example, as electron move through a conductor they are attracted to the protons on the nucleus of the conductor itself. This attraction doesn't stop the electrons, just slow them down a bit and cause the system to waste energy.



The unit for resistance is the OHM, Ω

Electrical POWER

We have already learned that POWER is the rate at which work (energy) is done. Circuits that are a prime example of this as batteries only last for a certain amount of time AND we get charged an energy bill each month based on the amount of energy we used over the course of a month…aka POWER.



POWER

It is interesting to see how certain electrical variables can be used to get POWER. Let's take Voltage and Current for example.

$$V = \frac{Joules}{Coloumb}$$

$$I = \frac{Coloumbs}{Second}$$

$$VxI = \frac{\textbf{Joules} \bullet \textbf{Coloumbs}}{\textbf{Coloumbs} \bullet \textbf{seconds}} = \frac{Joules}{Second} = WATT!$$

$$Power = P = VI$$

Other useful power formulas

$$P = VI$$

$$V = IR$$

$$P = (IR)I = I^2 R$$

$$I = \frac{V}{R}$$

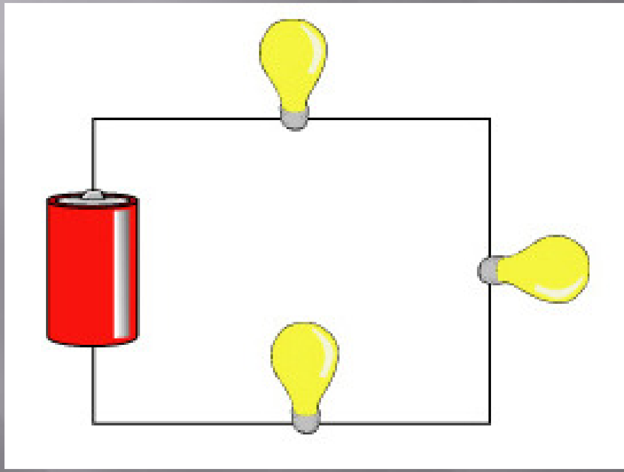
$$P = V\left(\frac{V}{R}\right) = \frac{V^2}{R}$$

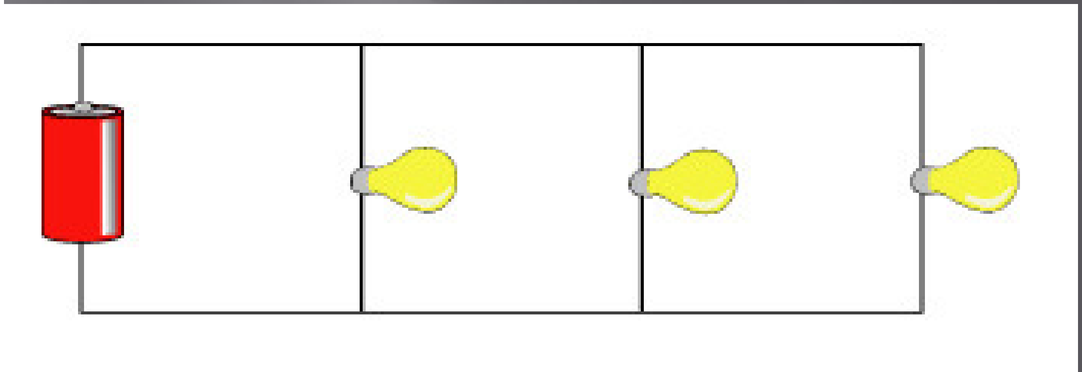
These formulas can also be used! They are simply derivations of the POWER formula with different versions of Ohm's law substituted in.

Ways to Wire Circuits

There are 2 basic ways to wire a circuit. Keep in mind that a resistor could be ANYTHING (bulb, toaster, ceramic material…etc)

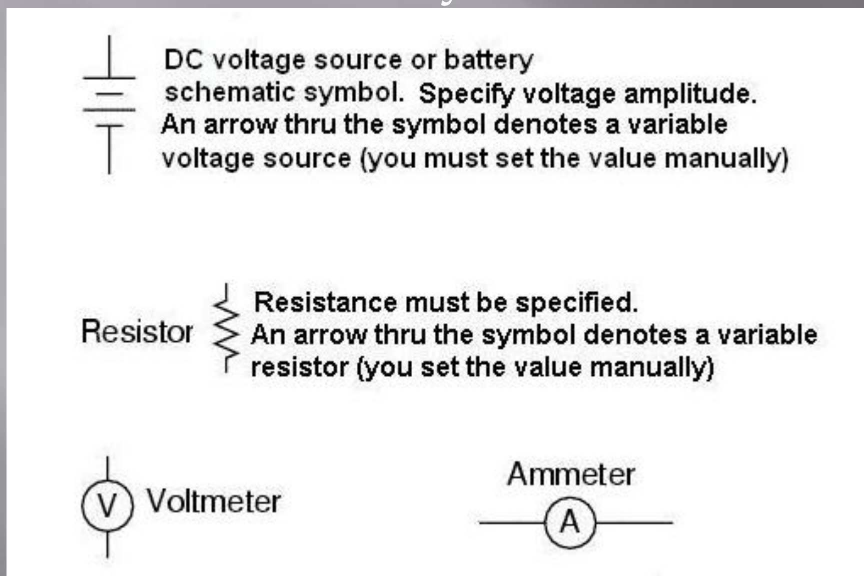
Series – One after another
Parallel – between a set of junctions and parallel to each other

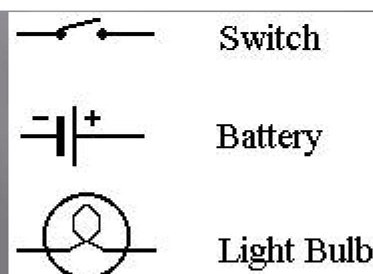




Schematic Symbols

Before you begin to understand circuits you need to be able to draw what they look like using a set of standard symbols understood anywhere in the world

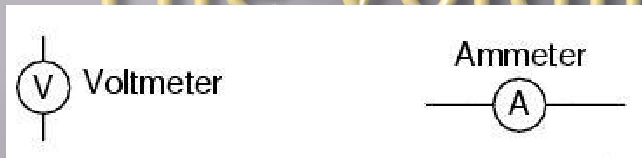




For the battery symbol, the LONG line is considered to be the POSITIVE terminal and the SHORT line , NEGATIVE.

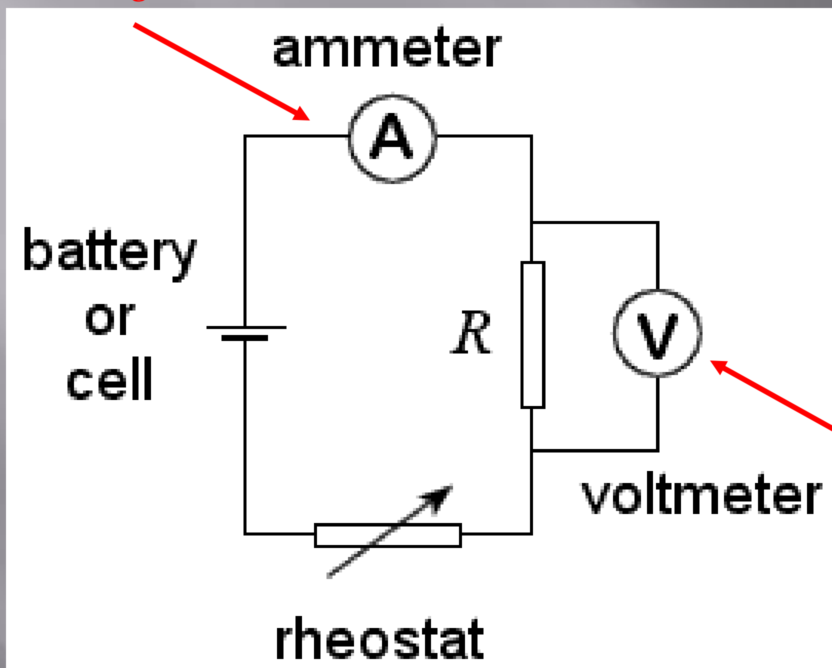
The **VOLTMETER** and **AMMETER** are special devices you place **IN** or **AROUND** the circuit to measure the **VOLTAGE** and **CURRENT**.

The Voltmeter and Ammeter



The voltmeter and ammeter cannot be just placed anywhere in the circuit. They must be used according to their DEFINITION.

Current goes THROUGH the ammeter

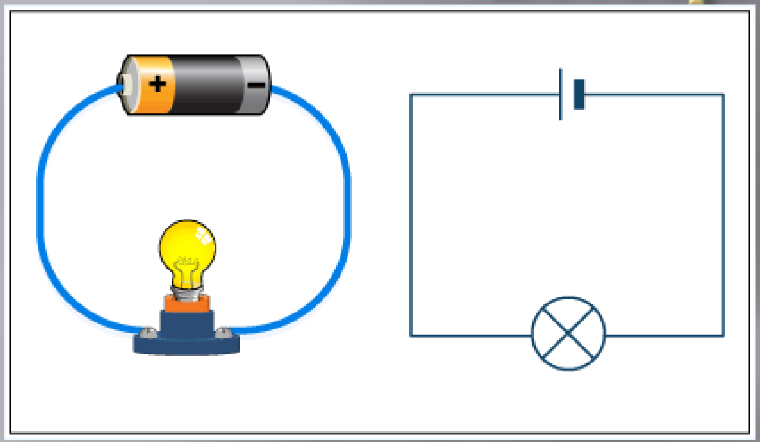


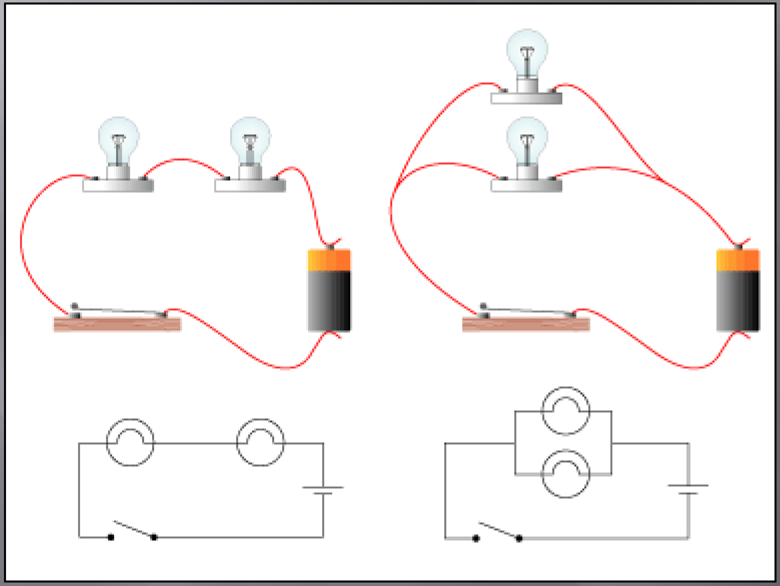
Since a voltmeter measures voltage or POTENTIAL DIFFERENCE it must be placed **ACROSS** the device you want to measure. That way you can measure the CHANGE on either side of the device.

Voltmeter is drawn ACROSS the resistor

…mmeter measures the current or FLOW it must be placed in such a way as the charges go **THROUGH** the device.

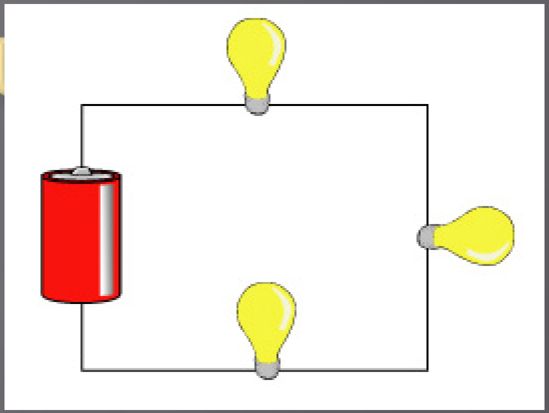
Simple Circuit





When you are drawing a circuit it may be a wise thing to start by drawing the battery first, then follow along the loop (closed) starting with positive and drawing what you see.

Series Circu



In in series circuit, the resistors are wired one after another. Since they are all part of the SAME LOOP they each experience the SAME AMOUNT of current. In figure, however, you see that they all exist BETWEEN the terminals of the battery, meaning they SHARE the potential (voltage).

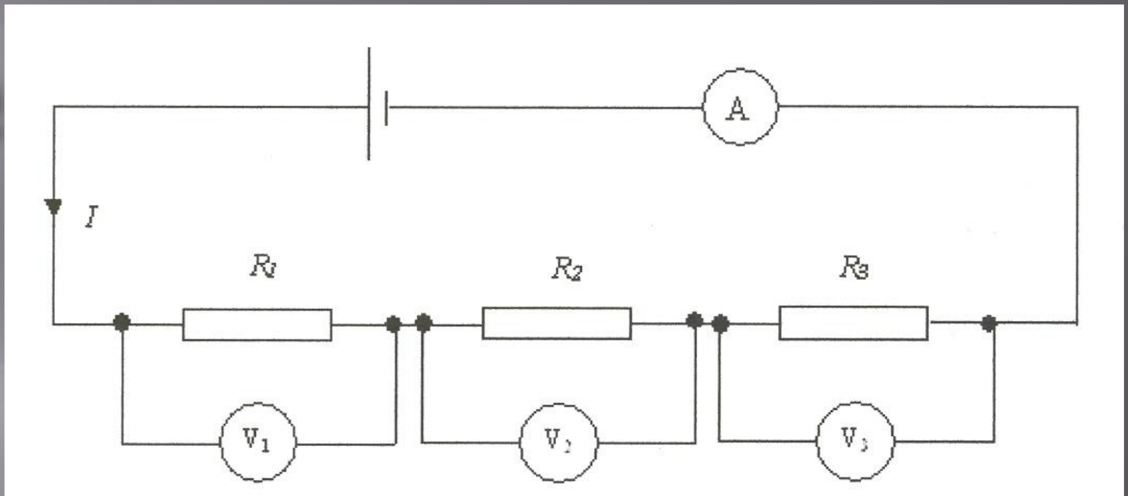
$$I_{(series)Total} = I_1 = I_2 = I_3$$

$$V_{(series)Total} = V_1 + V_2 + V_3$$

Se

$$I_{(series)Total} = I_1 = I_2 = I_3$$

$$V_{(series)Total} = V_1 + V_2 + V_3$$



As the current goes through the circuit, the charges must USE ENERGY to get through the resistor. So each individual resistor will get its own individual potential voltage). We call this VOLTAGE DROP.

$$V_{(series)Total} = V_1 + V_2 + V_3; \quad \Delta V = IR$$

$$(I_T R_T)_{series} = I_1 R_1 + I_2 R_2 + I_3 R_3$$

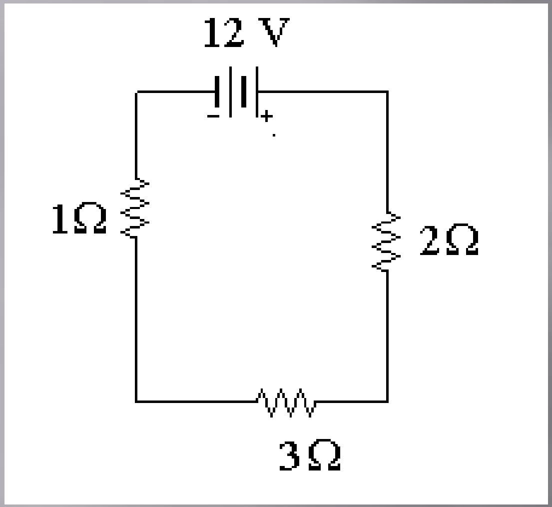
$$R_{series} = R_1 + R_2 + R_3$$

$$R_s = \sum R_i$$

Note: They may use the terms "effective" or "equivalent" to mean TOTAL!

Example

A series circuit is shown to the left.



a) What is the total resistance?

R(series) = 1 + 2 + 3 = 6Ω

b) What is the total current?

$\Delta V = IR$ $12 = I(6)$ $I = 2A$

c) What is the current across EACH resistor?

They EACH get 2 amps!

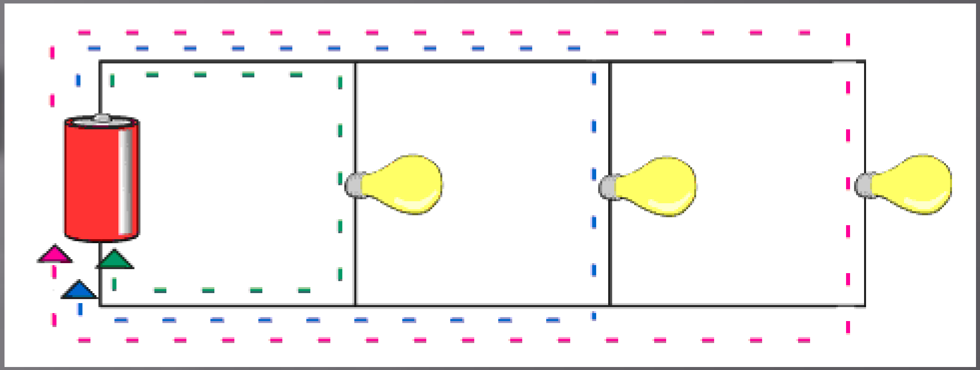
d) What is the voltage drop across each resistor?(Apply Ohm's law to each resistor separately)

$V_{1\Omega} = (2)(1) = 2\ V$ $V_{3\Omega} = (2)(3) = 6V$ $V_{2\Omega} = (2)(2) = 4V$

Notice that the individual VOLTAGE DROPS add up to the TOTAL!!

Parall

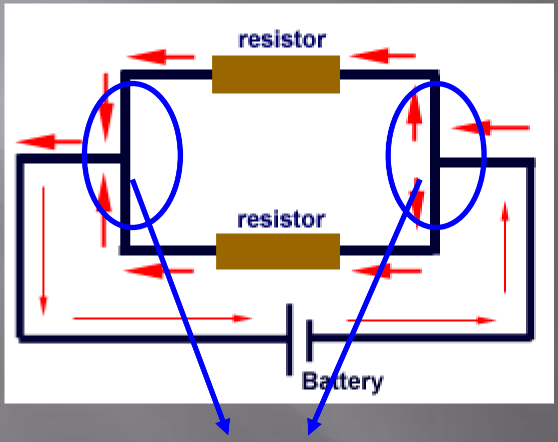
In a parallel circuit, we have multiple loops. So the current splits up among the loops with the individual loop currents **adding** to the total current



It is important to understand that parallel circuits will all have some position where the current splits and comes back together. We call these **JUNCTIONS**.

The current going IN to a junction will always equal the current going OUT of a junction.

$$I_{(parallel)Total} = I_1 + I_2 + I_3$$



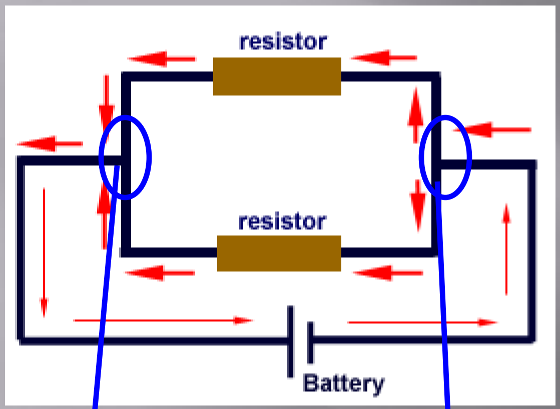
Junctions

$$\text{Regarding Junctions:}$$

$$I_{IN} = I_{OUT}$$

Parallel Circuit

Notice that the JUNCTIONS both touch the POSTIVE and NEGATIVE terminals of the battery. That means you have the SAME potential difference down EACH individual branch of the parallel circuit. This means that the individual voltages drops are equal.



This junction touches the **POSITIVE** terminal

This junction touches the **NEGATIVE** terminal

$$V_{(parallel)Total} = V_1 = V_2 = V_3$$

$$I_{(parallel)Total} = I_1 + I_2 + I_3; \;\; \Delta V = IR$$

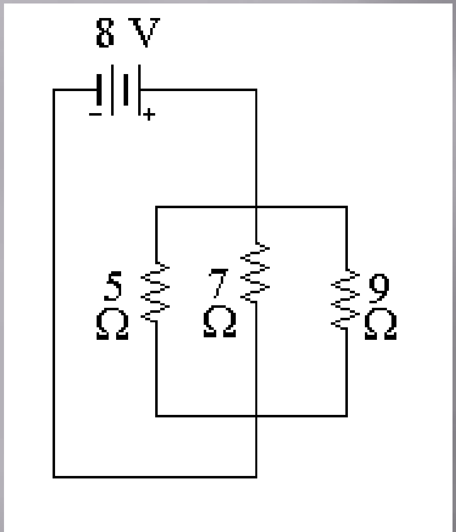
$$\left(\frac{V_T}{R_T}\right)_{Parallel} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_P} = \sum \frac{1}{R_i}$$

Example

To the left is an example of a parallel circuit.
a) What is the total resistance?



$$\frac{1}{R_P} = \frac{1}{5} + \frac{1}{7} + \frac{1}{9}$$

$$\frac{1}{R_p} = 0.454 \rightarrow R_P = \frac{1}{0.454} = \textbf{2.20 Ω}$$

b) What is the total current? $\Delta V = IR$

$$8 = I(R) = \textbf{3.64 A}$$

c) What is the voltage across EACH resistor?

8 V each!

d) What is the current drop across each resistor? (Apply Ohm's law to each resistor separately)

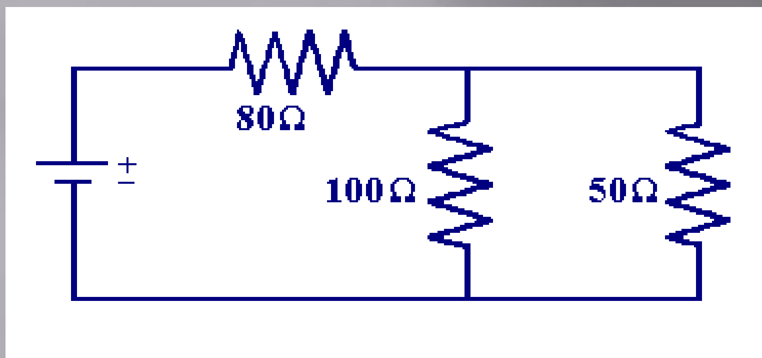
$$\Delta V = IR$$

$$I_{5\Omega} = \frac{8}{5} = \textbf{1.6 A} \quad I_{7\Omega} = \frac{8}{7} = \textbf{1.14 A} \quad I_{9\Omega} = \frac{8}{9} = \textbf{0.90 A}$$

Notice that the individual currents ADD to the total.

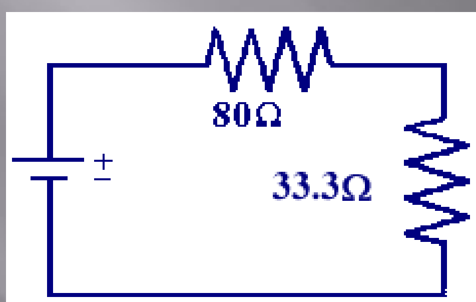
Compound (Complex) Circuits

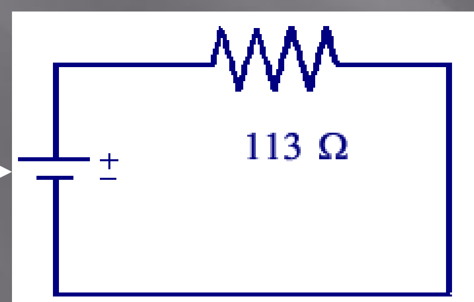
Many times you will have series and parallel in the SAME circuit.



Solve this type of circuit from the inside out.

WHAT IS THE TOTAL RESISTANCE?

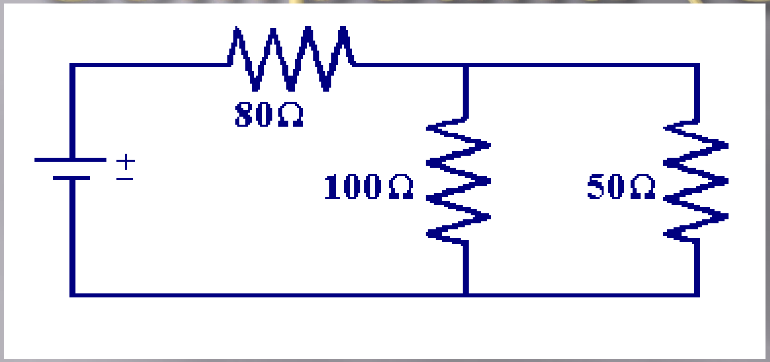




$$\frac{1}{R_P} = \frac{1}{100} + \frac{1}{50}; \quad R_P = 33.3\Omega$$

$$R_s = 80 + 33.3 = 113.3\Omega$$

Compound (Complex) Circuits



$$\frac{1}{R_P} = \frac{1}{100} + \frac{1}{50}; \quad R_P = 33.3\Omega$$

$$R_s = 80 + 33.3 = 113.3\Omega$$

Suppose the potential difference (voltage) is equal to **120V**. What is the total current?

$$\Delta V_T = I_T R_T$$

$$120 = I_T (113.3)$$

$$I_T = \ \mathbf{1.06\ A}$$

What is the VOLTAGE DROP across the 80Ω resistor?

$$\Delta V_{80\Omega} = I_{80\Omega} R_{80\Omega}$$

$$V_{80\Omega} = (1.06)(80)$$

$$V_{80\Omega} = \ \mathbf{84.8\ V}$$

Compound (Complex) Circuits

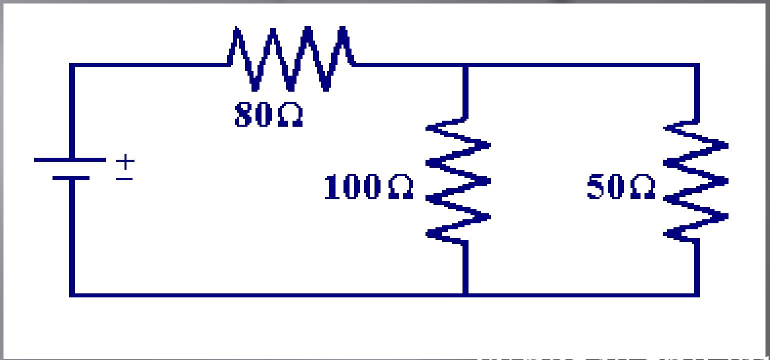
$$R_T = 113.3\Omega$$

$$V_T = 120V$$

$$I_T = 1.06A$$

$$V_{80\Omega} = 84.8V$$

$$I_{80\Omega} = 1.06A$$



What is the VOLTAGE DROP across the 100Ω and 50Ω resistor?

$$V_{T(parallel)} = V_2 = V_3$$

$$V_{T(series)} = V_1 + V_{2\&3}$$

$$120 = 84.8 + V_{2\&3}$$

$V_{2\&3} =$ **35.2 V Each!**

What is the current across the 100Ω and 50Ω resistor?

$$I_{T(parallel)} = I_2 + I_3$$

$$I_{T(series)} = I_1 = I_{2\&3}$$

$I_{100\Omega} = \frac{35.2}{100} =$ **0.352 A**

$I_{50\Omega} = \frac{35.2}{50} =$ **0.704 A**

Add to 1.06A

Objectives: After completing this module, you should be able to:

- Describe the sinusoidal variation in ac current and voltage, and calculate their effective values.
- Write and apply equations for calculating the inductive and capacitive reactances for inductors and capacitors in an ac circuit.
- Describe, with diagrams and equations, the phase relationships for circuits containing resistance, capacitance, and inductance.

Objectives (Cont.)

- Write and apply equations for calculating the impedance, the phase angle, the effective current, the average power, and the resonant frequency for a series ac circuit.
- Describe the basic operation of a step-up and a step-down transformer.
- Write and apply the transformer equation and determine the efficiency of a transformer.

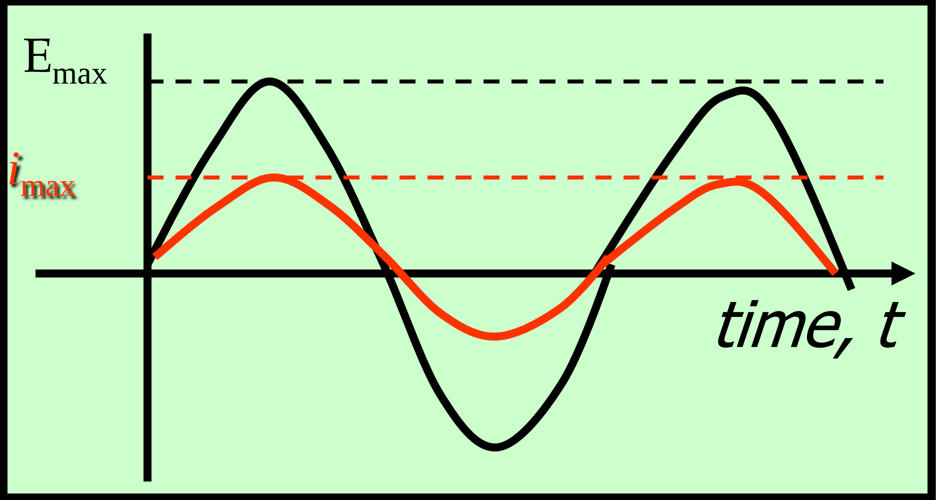
Alternating Currents

An alternating current such as that produced by a generator has no direction in the sense that direct current has. The magnitudes vary sinusoidally with time as given by:

AC-voltage and current

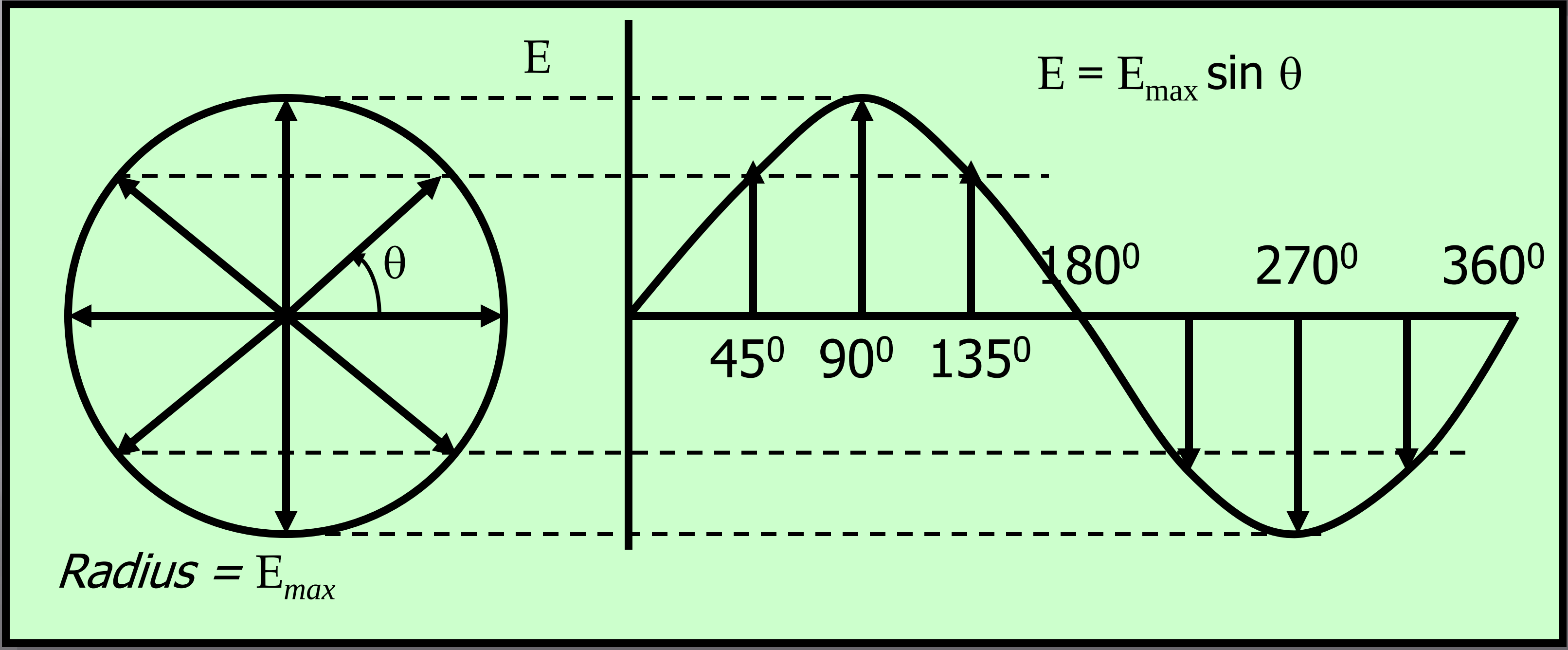
$E = E_{max} \sin \theta$

$i = i_{max} \sin \theta$



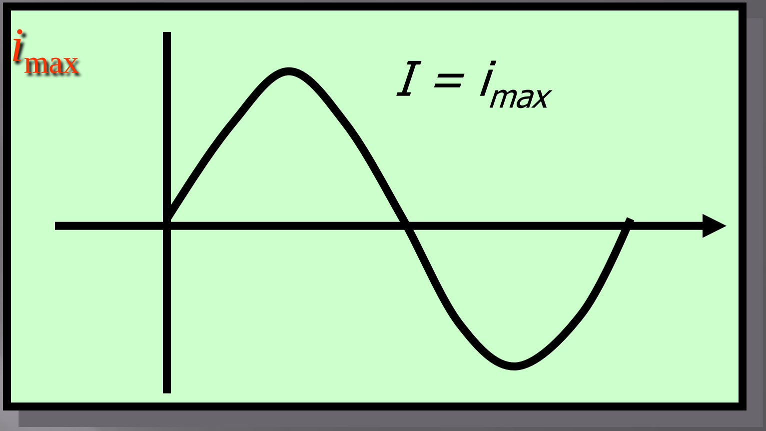
Rotating Vector Description

The coordinate of the emf at any instant is the value of E_{max} sin θ. Observe for incremental angles in steps of 45^0. Same is true for *i*.



Effective AC Current

The average current in a cycle is zero—half + and half -.



But energy is expended, regardless of direction. So the "root-mean-square" value is useful.

$$I_{rms} = \sqrt{\frac{I^2}{2}} = \frac{I}{0.707}$$

The rms value I_{rms} is sometimes called the effective current I_{eff}:

The effective ac current:

$$i_{eff} = 0.707\ i_{max}$$

AC Definitions

One effective ampere is that ac current for which the power is the same as for one ampere of dc current.

Effective current: $i_{eff} = 0.707\ i_{max}$

One effective volt is that ac voltage that gives an effective ampere through a resistance of one ohm.

Effective voltage: $V_{eff} = 0.707\ V_{max}$

Example 1: For a particular device, the house ac voltage is 120-V and the ac current is 10 A. What are their maximum values?

$i_{eff} = 0.707\ i_{max}$

$V_{eff} = 0.707\ V_{max}$

$$i_{max} = \frac{i_{eff}}{0.707} = \frac{10\ \text{A}}{0.707}$$

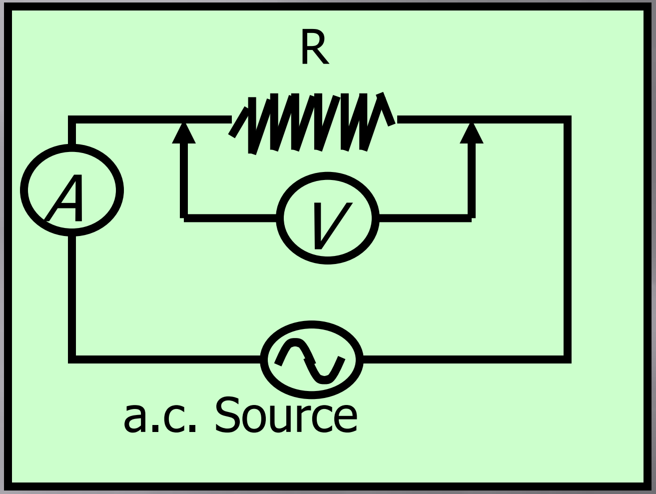
$$V_{max} = \frac{V_{eff}}{0.707} = \frac{120\text{V}}{0.707}$$

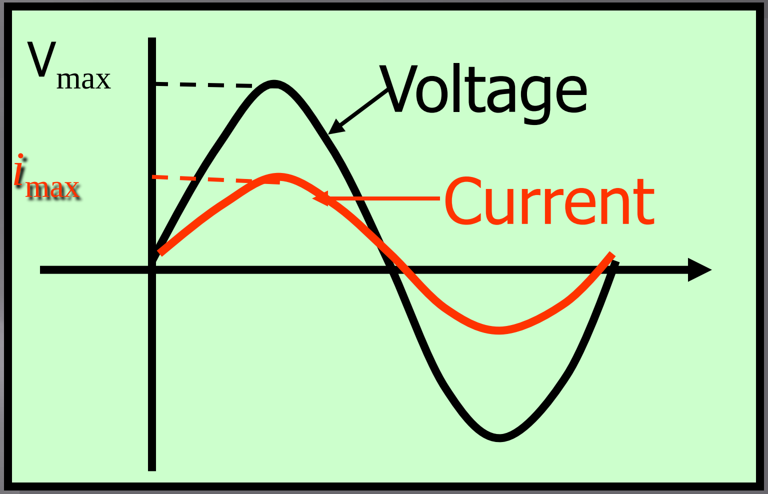
$i_{max} = 14.14$ A

$V_{max} = 170$ V

The ac voltage actually varies from +170 V to -170 V and the current from 14.1 A to –14.1 A.

Pure Resistance in AC Circuits

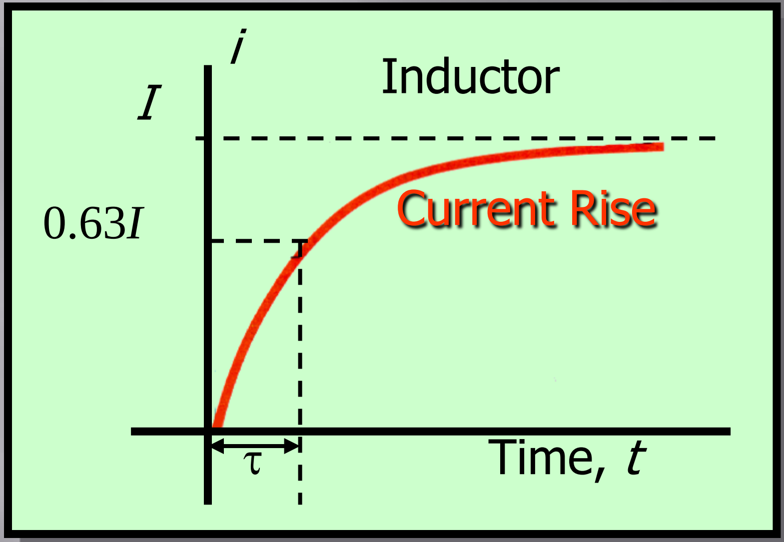


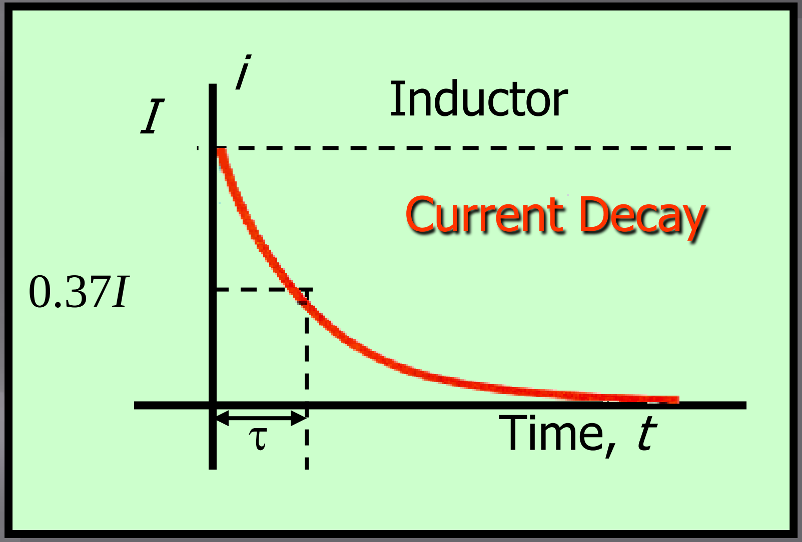


Voltage and current are in phase, and Ohm's law applies for effective currents and voltages.

Ohm's law: $V_{eff} = i_{eff}R$

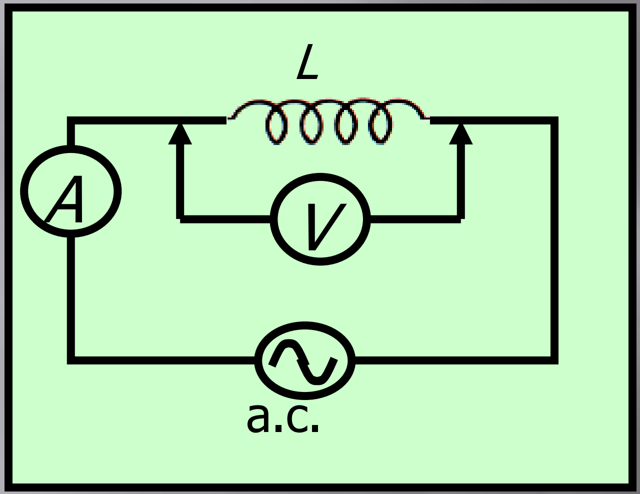
AC and Inductors

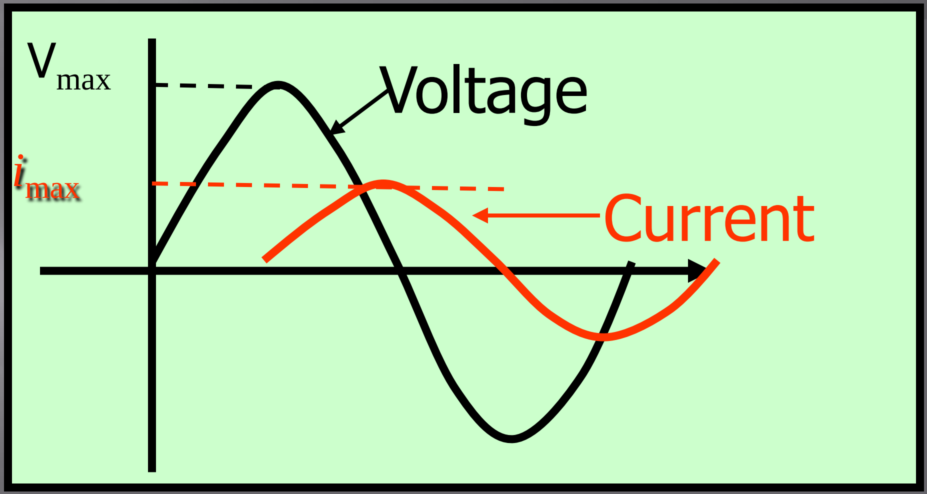




The voltage *V* peaks first, causing rapid rise in *i* current which then peaks as the emf goes to zero. Voltage leads (peaks before) the current by 90^0. Voltage and current are out of phase.

A Pure Inductor in AC Circuit



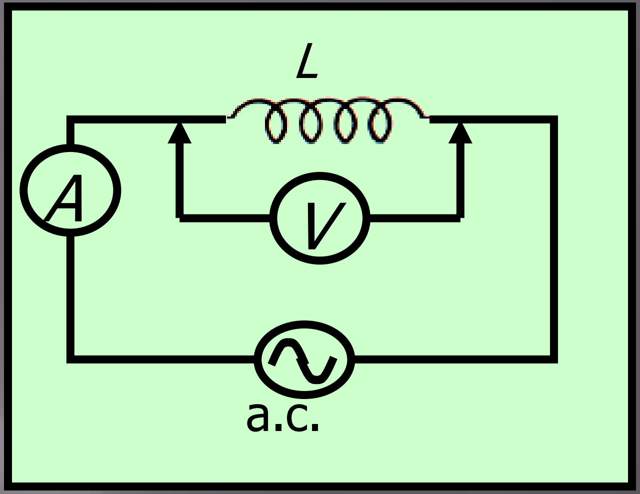


The voltage peaks 90^0 before the current peaks. One builds as the other falls and vice versa.

The reactance may be defined as the nonresistive opposition to the flow of ac current.

Inductive Reactance

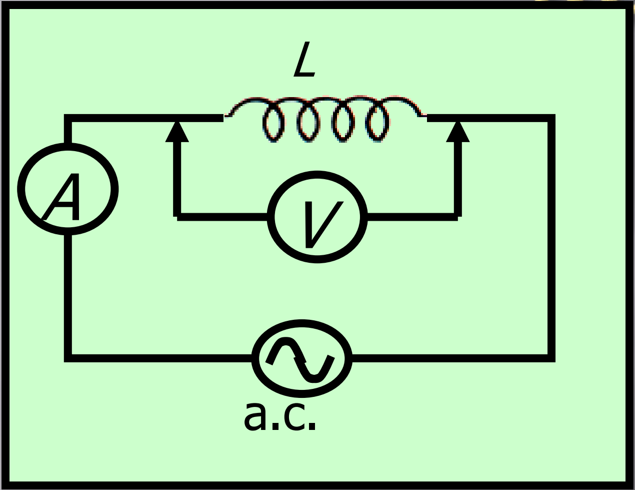
The back emf induced by a changing current provides opposition to current, called inductive reactance X_L.



Such losses are temporary, however, since the current changes direction, periodically re-supplying energy so that no net power is lost in one cycle.

Inductive reactance X_L is a function of both the inductance and the frequency of the ac current.

Calculating Inductive Reactance



Inductive Reactance:

$X_L = 2\pi fL$ Unit is the Ω

Ohm's law: $V_L = iX_L$

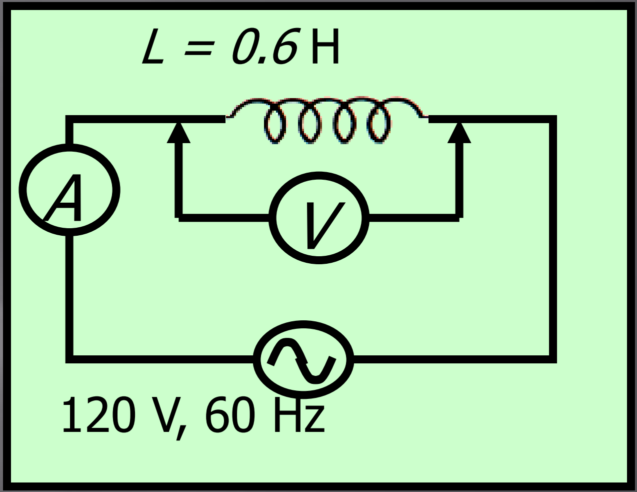
The voltage reading V in the above circuit at the instant the ac current is i can be found from the inductance in H and the frequency in Hz.

$$V_L = i(2\pi fL)$$

Ohm's law: $V_L = i_{eff}X_L$

Example 2: A coil having an inductance of 0.6 H is connected to a 120-V, 60 Hz ac source. Neglecting resistance, what is the effective current through the coil?

Reactance: $X_L = 2\pi fL$



$X_L = 2\pi(60\text{ Hz})(0.6\text{ H})$

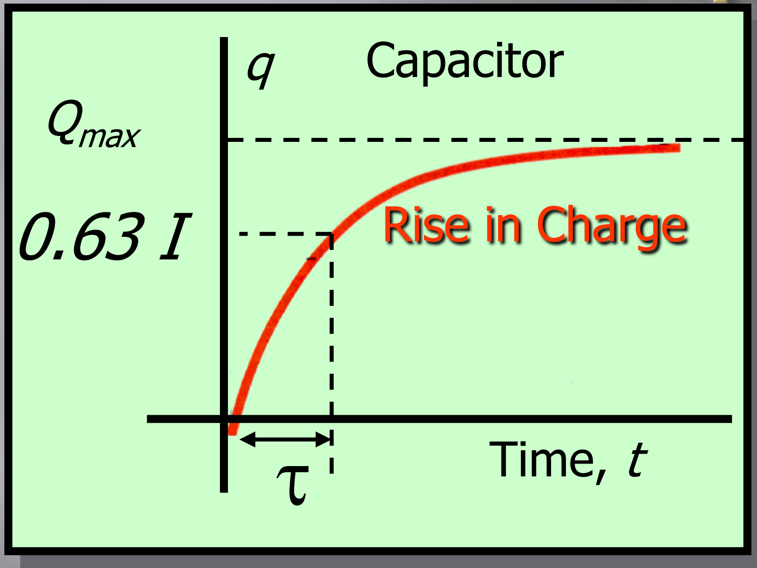
$X_L = 226\ \Omega$

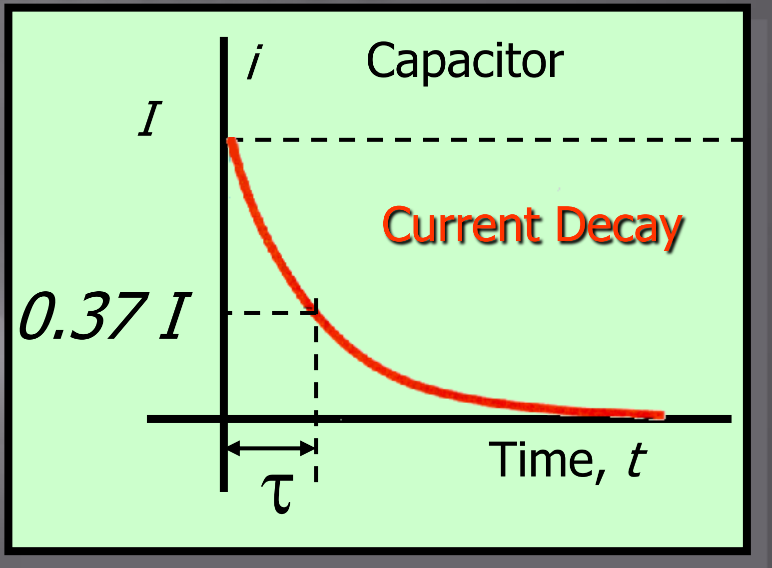
$$i_{eff} = \frac{V_{eff}}{X_L} = \frac{120\text{V}}{226\ \Omega}$$

$i_{eff} = 0.531$ A

Show that the peak current is $I_{max} = 0.750$ A

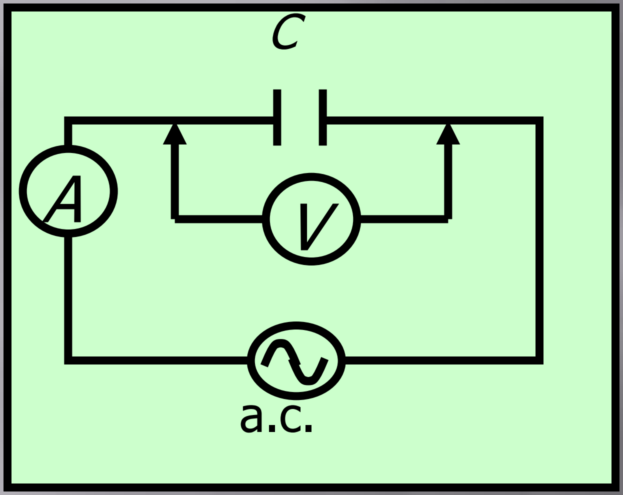
AC and Capacitance

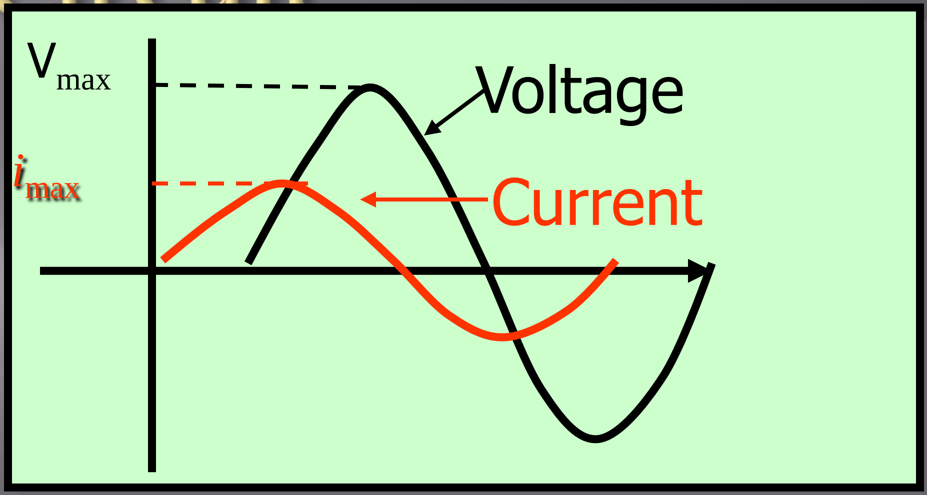




The voltage V peaks ¼ of a cycle after the current *i* reaches its maximum. The voltage lags the current. Current *i* and V out of phase.

A Pure Capacitor in AC Circuit



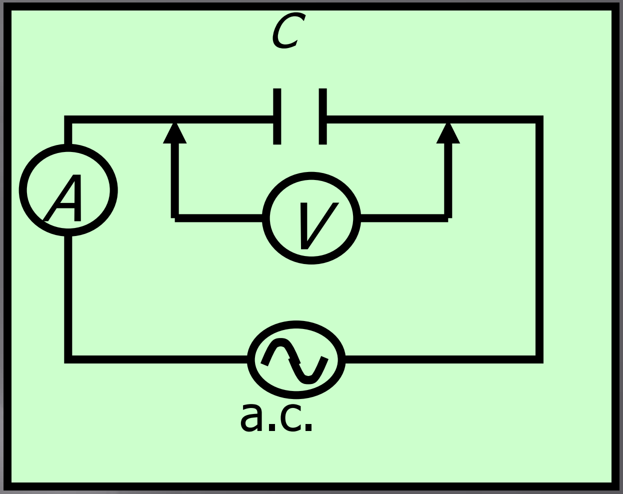


The voltage peaks 90^0 after the current peaks. One builds as the other falls and vice versa.

The diminishing current *i* builds charge on C which increases the back emf of V_C.

Capacitive Reactance

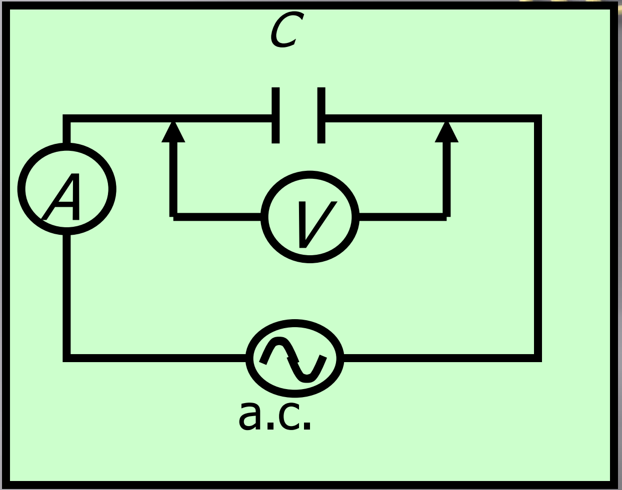
Energy gains and losses are also temporary for capacitors due to the constantly changing ac current.



No net power is lost in a complete cycle, even though the capacitor does provide nonresistive opposition (reactance) to the flow of ac current.

Capacitive reactance X_C is affected by both the capacitance and the frequency of the ac current.

Calculating Inductive Reactance



<u>Capacitive Reactance:</u>

$$X_C = \frac{1}{2\pi fC}$$ Unit is the Ω

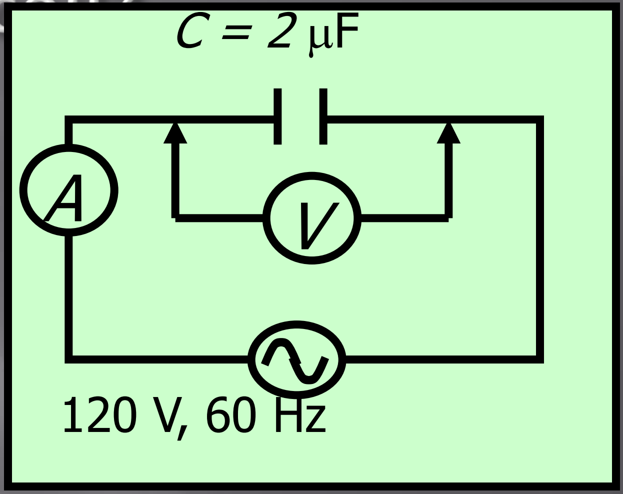
Ohm's law: $V_C = iX_C$

The voltage reading *V* in the above circuit at the instant the ac current is *i* can be found from the inductance in F and the frequency in Hz.

$$V_L = \frac{i}{2\pi fL}$$

Ohm's law: $V_C = i_{eff}X_C$

Example 3: A 2-μF capacitor is connected to a 120-V, 60 Hz ac source. Neglecting resistance, what is the effective current through the coil?



Reactance:

$$X_C = \frac{1}{2\pi fC}$$

$$X_C = \frac{1}{2\pi(60\,\text{Hz})(2 \text{ x } 10^{-6}\text{F})}$$

X_C = 1330 Ω

$$i_{eff} = \frac{V_{eff}}{X_C} = \frac{120\text{V}}{1330\ \Omega}$$

i_{eff} = 90.5 mA

Show that the peak current is i_{max} = 128 mA

Frequency and AC Circuits

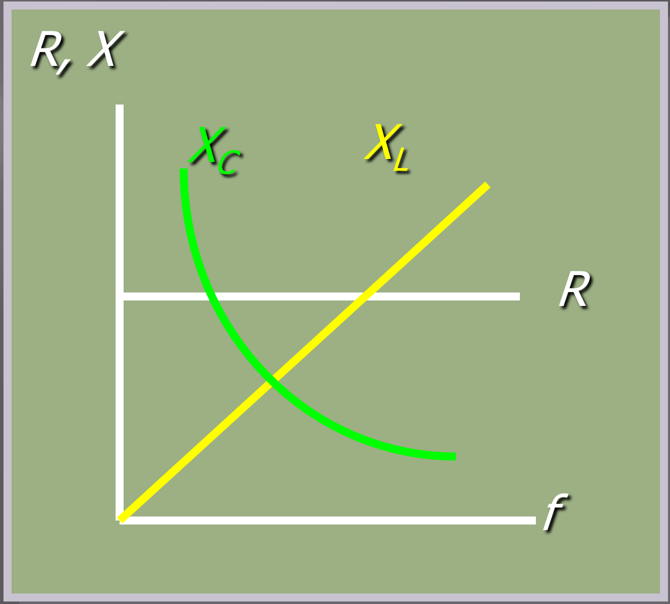
Resistance R is constant and not affected by f.

Inductive reactance X_L varies directly with frequency as expected since $E \propto \Delta i/\Delta t$.

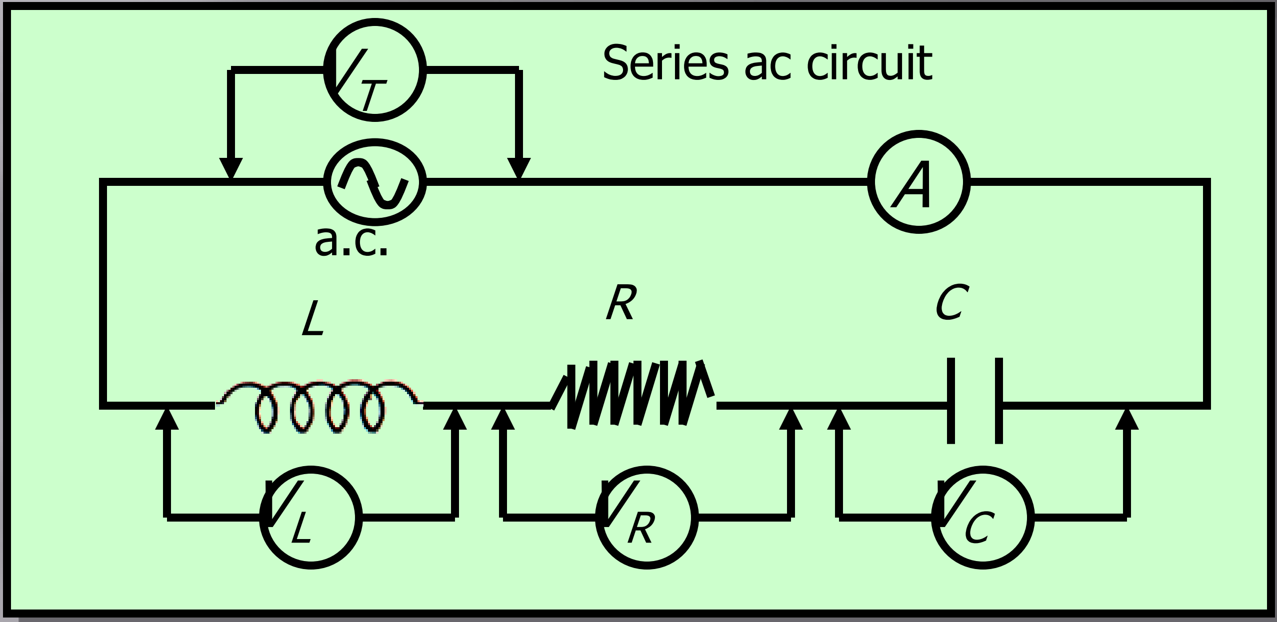
$$X_L = 2\pi f L$$

$$X_C = \frac{1}{2\pi f C}$$

Capacitive reactance X_C varies inversely with f since rapid ac allows little time for charge to build up on capacitors.



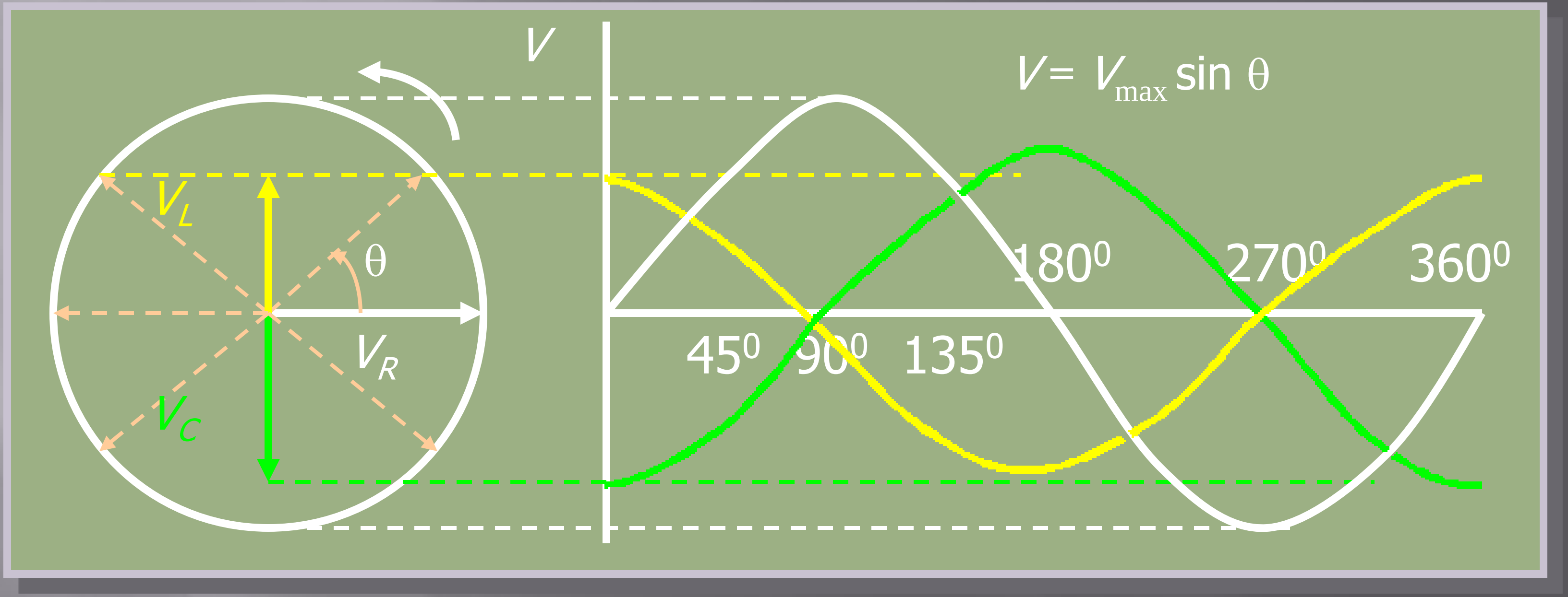
Series LRC Circuits



Consider an inductor *L*, a capacitor *C*, and a resistor *R* all connected in series with an ac source. The instantaneous current and voltages can be measured with meters.

Phase in a Series AC Circuit

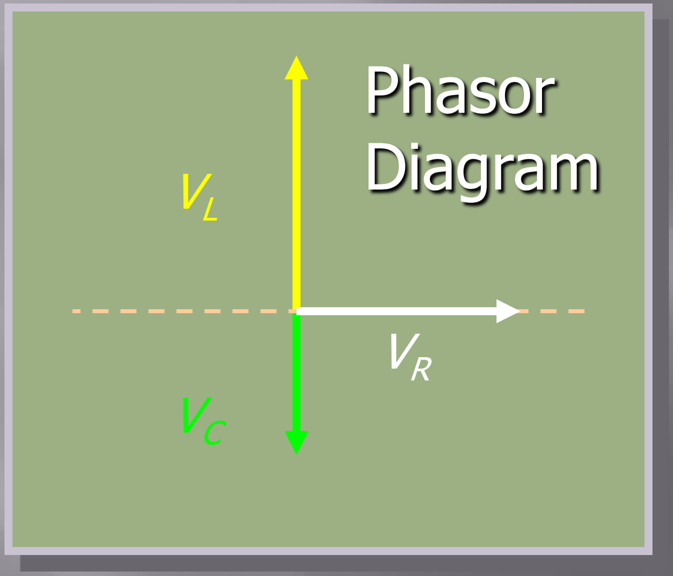
The voltage leads current in an inductor and lags current in a capacitor. In phase for resistance *R*.

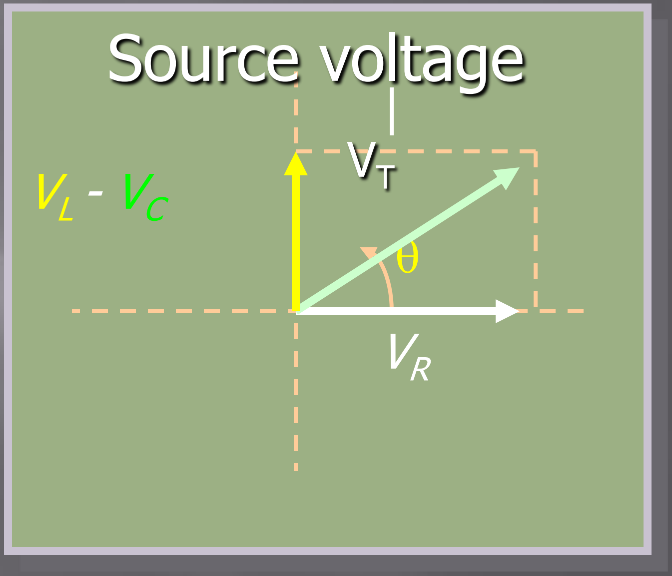


Rotating phasor diagram generates voltage waves for each element *R*, *L*, and *C* showing phase relations. Current *i* is always in phase with V_R.

Phasors and Voltage

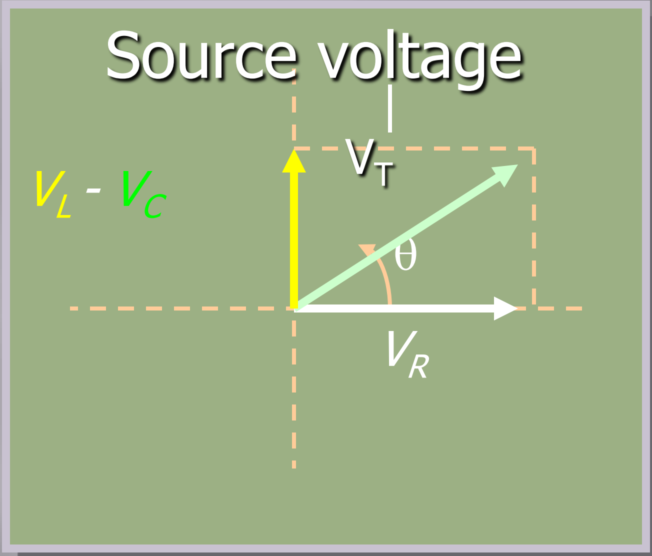
At time t = 0, suppose we read V_L, V_R and V_C for an ac series circuit. What is the source voltage V_T?





We handle phase differences by finding the vector sum of these readings. $V_T = \Sigma \mathbf{V_i}$. The angle θ is the phase angle for the ac circuit.

Calculating Total Source Voltage



Treating as vectors, we find:

$$V_T = \sqrt{V_R^2 + (V_L - V_C)^2}$$

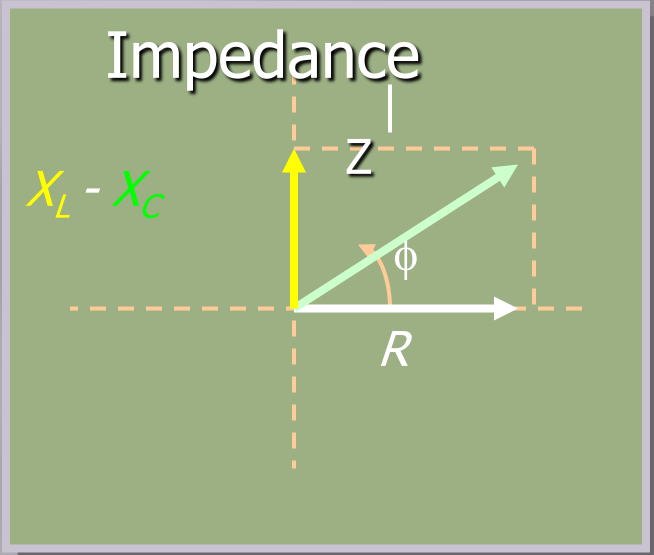
$$\tan\phi = \frac{V_L - V_C}{V_R}$$

Now recall that: $V_R = iR$; $V_L = iX_L$; and $V_C = iV_C$

Substitution into the above voltage equation gives:

$$V_T = i\sqrt{R^2 + (X_L - X_C)^2}$$

Impedance in an AC Circuit



$$V_T = i\sqrt{R^2 + (X_L - X_C)^2}$$

Impedance Z is defined:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

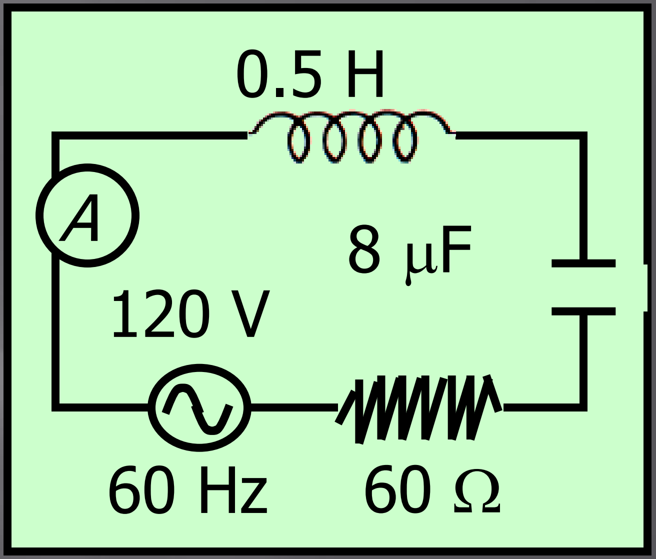
Ohm's law for ac current and impedance:

$$V_T = iZ \quad \text{or} \quad i = \frac{V_T}{Z}$$

The impedance is the combined opposition to ac current consisting of both resistance and reactance.

Example 3: A 60-Ω resistor, a 0.5 H inductor, and an 8-μF capacitor are connected in series with a 120-V, 60 Hz ac source. Calculate the impedance for this circuit.

$$X_L = 2\pi fL \quad \text{and} \quad X_C = \frac{1}{2\pi fC}$$



$$X_L = 2\pi(60\text{Hz})(0.6\text{ H}) = 226\Omega$$

$$X_C = \frac{1}{2\pi(60\text{Hz})(8 \text{ x } 10^{-6}\text{F})} = 332\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(60\Omega)^2 + (226\Omega - 332\Omega)^2}$$

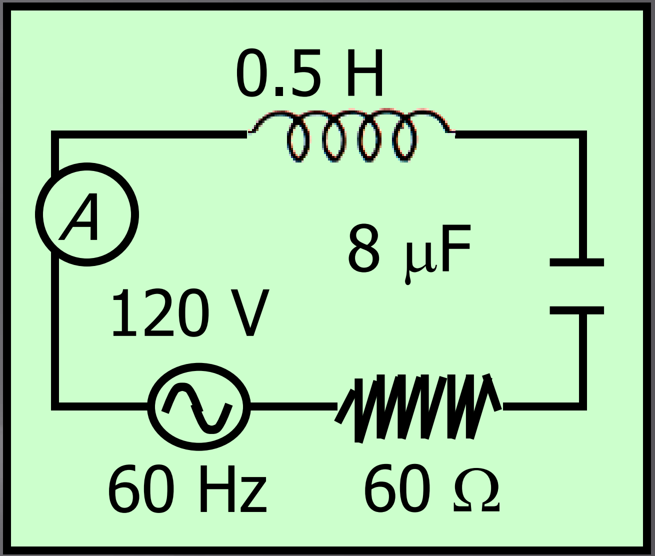
Thus, the impedance is:

$Z = 122\ \Omega$

Example 4: Find the effective current and the phase angle for the previous example.

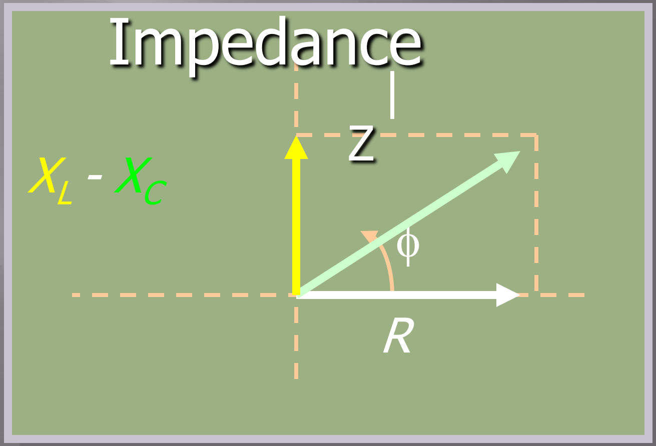
$X_L = 226\ \Omega;\ X_C = 332\ \Omega;$ $R = 60\ \Omega;\ Z = 122\ \Omega$

$$i_{eff} = \frac{V_T}{Z} = \frac{120\text{ V}}{122\ \Omega}$$



$i_{eff} = 0.985$ A

Next we find the phase angle:



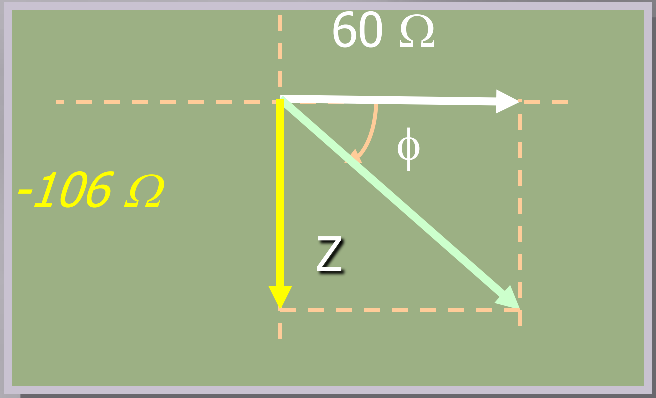
$X_L - X_C = 226 - 332 = -106\ \Omega$

$R = 60\ \Omega$

$$\tan\phi = \frac{X_L - X_C}{R}$$

Continued . . .

Example 4 (Cont.): Find the phase angle ϕ for the previous example.



$X_L - X_C = 226 - 332 = -106\ \Omega$

$R = 60\ \Omega$

$$\tan\phi = \frac{X_L - X_C}{R}$$

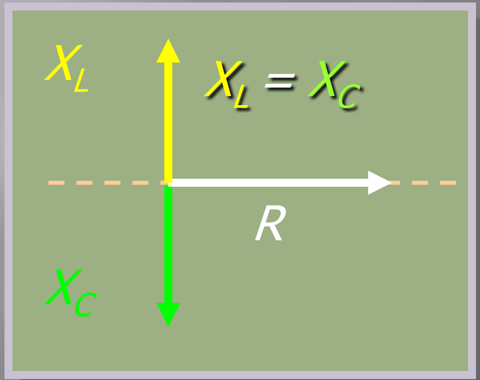
$$\tan\phi = \frac{-106\,\Omega}{60\,\Omega}$$

$\phi = -60.5^0$

The negative phase angle means that the ac voltage lags the current by 60.5^0. This is known as a capacitive circuit.

Resonant Frequency

Because inductance causes the voltage to lead the current and capacitance causes it to lag the current, they tend to cancel each other out.



Resonance (Maximum Power) occurs when $X_L = X_C$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = R$$

Resonant f_r *:* $X_L = X_C$

$$2\pi fL = \frac{1}{2\pi fC}$$

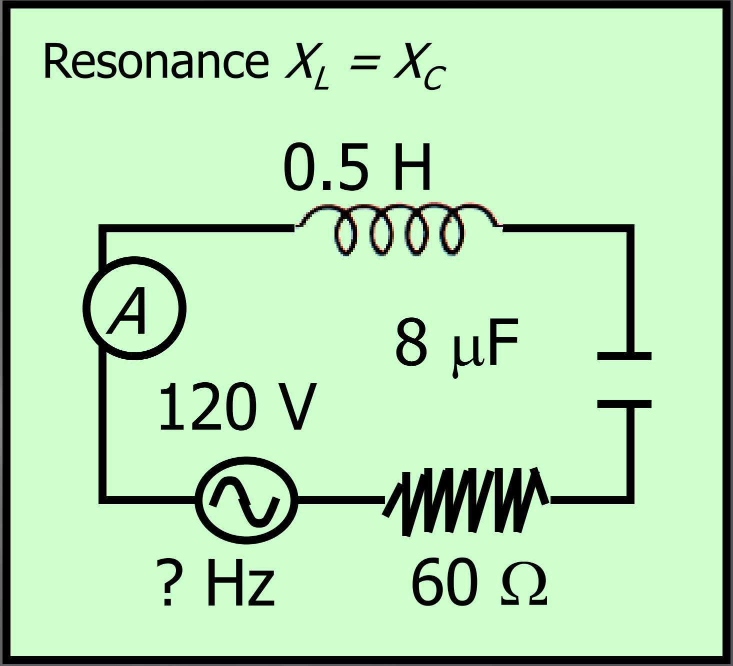
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Example 5: Find the resonant frequency for the previous circuit example: L = .5 H, C = 8 μF

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{(0.5\,\mathrm{H})(8 \times 10^{-6}\,\mathrm{F})}}$$

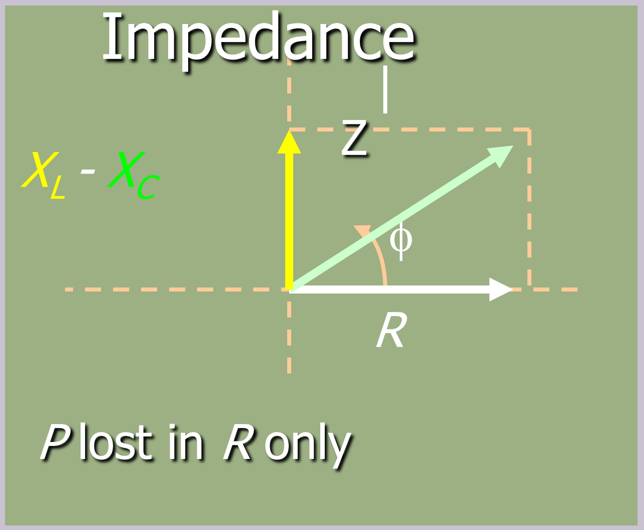
Resonant f_r = 79.6 Hz



At resonant frequency, there is zero reactance (only resistance) and the circuit has a phase angle of zero.

Power in an AC Circuit

No power is consumed by inductance or capacitance. Thus power is a function of the component of the impedance along resistance:



In terms of ac voltage:

$P = iV \cos \phi$

In terms of the resistance R:

$P = i^2 R$

The fraction Cos φ is known as the power factor.

Example 6: What is the average power loss for the previous example: V = 120 V, ϕ = -60.5^0, i = 90.5 A, and R = 60Ω .

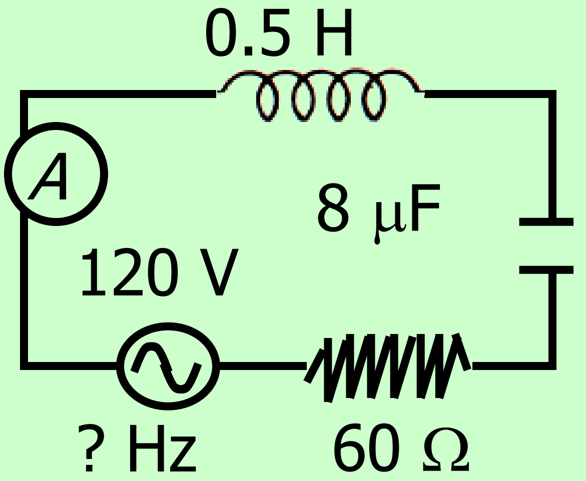
$P = i^2R$ = (0.0905 A)^2(60 Ω)

Average P = 0.491 W

The power factor is: Cos 60.5^0

Cos ϕ = 0.492 or 49.2%

Resonance $X_L = X_C$

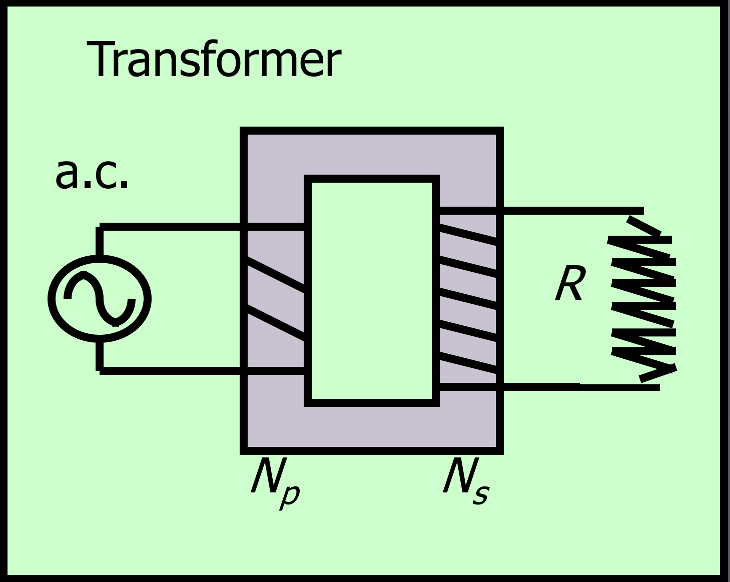


The higher the power factor, the more efficient is the circuit in its use of ac power.

The Transformer

A transformer is a device that uses induction and ac current to step voltages up or down.

An ac source of emf $\mathcal{E}_p$ is connected to primary coil with N_p turns. Secondary has N_s turns and emf of $\mathcal{E}_s$.

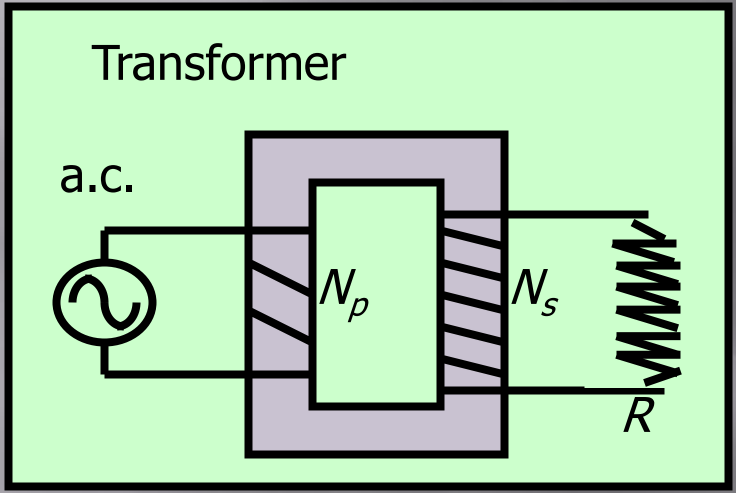


Induced emf's are:

$$\mathcal{E}_P = -N_P \frac{\Delta\Phi}{\Delta t}$$

$$\mathcal{E}_S = -N_S \frac{\Delta\Phi}{\Delta t}$$

Transformers (Continued):



$$\mathcal{E}_P = -N_P \frac{\Delta\Phi}{\Delta t}$$

$$\mathcal{E}_S = -N_S \frac{\Delta\Phi}{\Delta t}$$

Recognizing that $\Delta\phi/\Delta t$ is the same in each coil, we divide first relation by second and obtain:

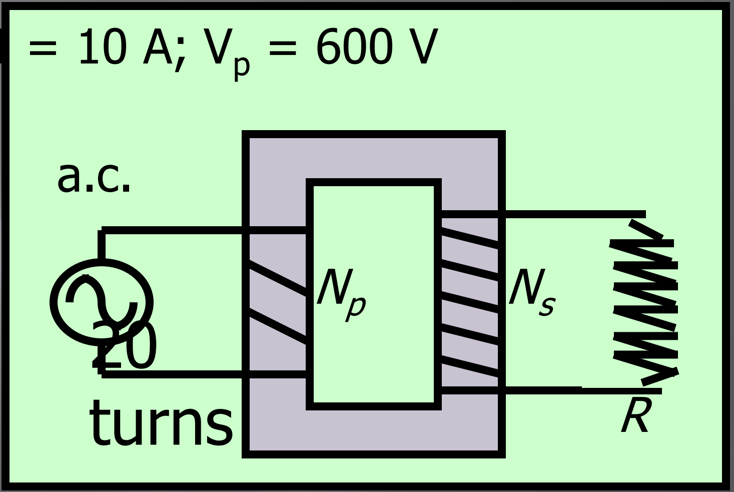
The transformer equation:

$$\frac{\mathcal{E}_P}{\mathcal{E}_S} = \frac{N_P}{N_S}$$

Example 7: A generator produces 10 A at 600 V. The primary coil in a transformer has 20 turns. How many secondary turns are needed to step up the voltage to 2400 V?

Applying the transformer equation:

$$\frac{V_P}{V_S} = \frac{N_P}{N_S}$$



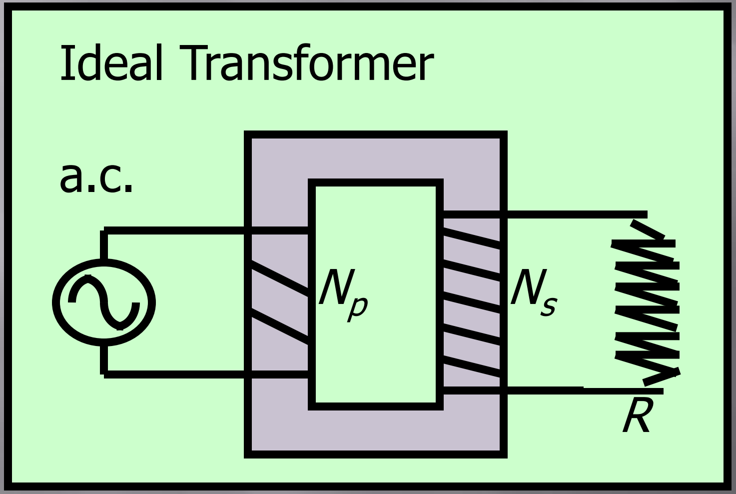
$$N_S = \frac{N_P V_S}{V_P} = \frac{(20)(2400\,\text{V})}{600\,\text{V}}$$

N_S = 80 turns

This is a step-up transformer; reversing coils will make it a step-down transformer.

Transformer Efficiency

There is no power gain in stepping up the voltage since voltage is increased by reducing current. In an ideal transformer with no internal losses:



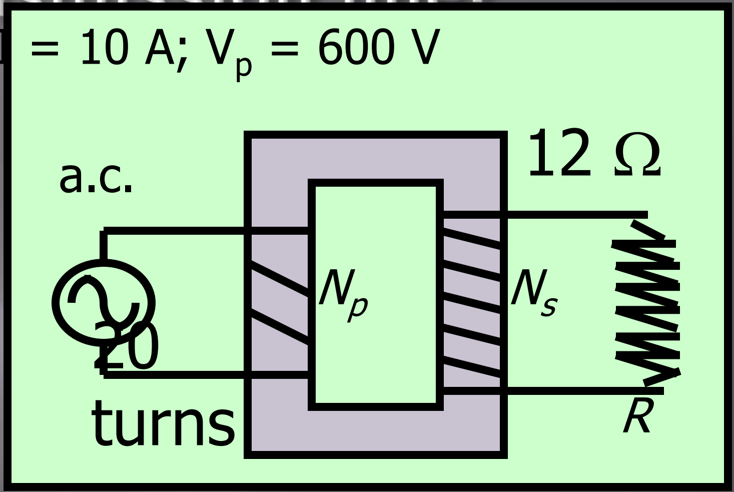
An ideal transformer:

$$\mathcal{E}_P i_P = \mathcal{E}_S i_S \quad \text{or} \quad \frac{i_P}{i_s} = \frac{\mathcal{E}_S}{\mathcal{E}_P}$$

The above equation assumes no internal energy losses due to heat or flux changes. Actual efficiencies are usually between 90 and 100%.

Example 7: The transformer in Ex. 6 is connected to a power line whose resistance is 12 Ω. How much of the power is lost in the transmission line?

$V_S = 2400\ V$



$$\mathcal{E}_P i_P = \mathcal{E}_S i_S \qquad i_S = \frac{\mathcal{E}_P i_P}{\mathcal{E}_S}$$

$$i_S = \frac{(600\text{V})(10\text{ A})}{2400\text{ V}} = 2.50\text{ A}$$

$P_{lost} = i^2R = (2.50\text{ A})^2(12\ \Omega)$ $\qquad P_{lost} = 75.0\text{ W}$

$P_{in} = (600\text{ V})(10\text{ A}) = 6000\text{ W}$

%Power Lost = (75 W/6000 W)(100%) = 1.25%

Summary

Effective current: $i_{eff} = 0.707\ i_{max}$

Effective voltage: $V_{eff} = 0.707\ V_{max}$

Inductive Reactance:

$$X_L = 2\pi fL \quad \text{Unit is the } \Omega$$

$$\text{Ohm's law: } V_L = iX_L$$

Capacitive Reactance:

$$X_C = \frac{1}{2\pi fC} \quad \text{Unit is the } \Omega$$

$$\text{Ohm's law: } V_C = iX_C$$

Summary (Cont.)

$$V_T = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\tan\phi = \frac{V_L - V_C}{V_R}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$V_T = iZ \quad \text{or} \quad i = \frac{V_T}{Z}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Summary (Cont.)

Power in AC Circuits:

In terms of ac voltage:

$P = iV \cos \phi$

In terms of the resistance R:

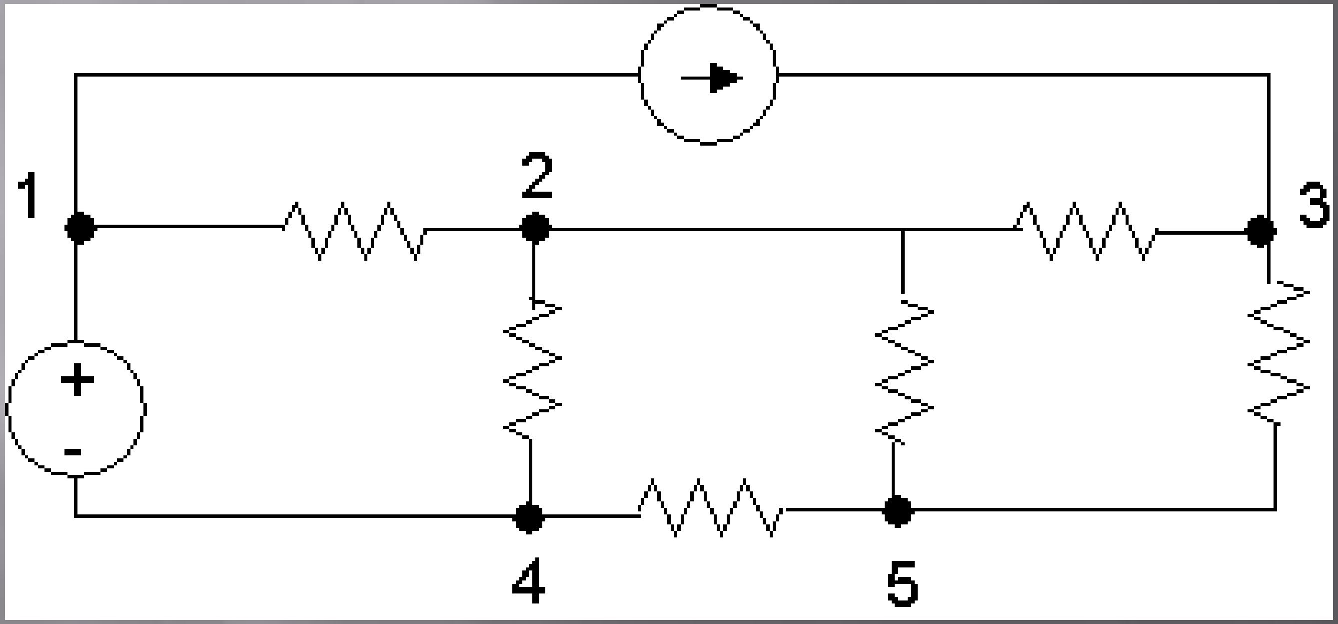
$P = i^2 R$

Transformers:

$$\frac{\mathcal{E}_P}{\mathcal{E}_S} = \frac{N_P}{N_S} \qquad \mathcal{E}_P i_P = \mathcal{E}_S i_S$$

Graph Theory in Circuit Analysis

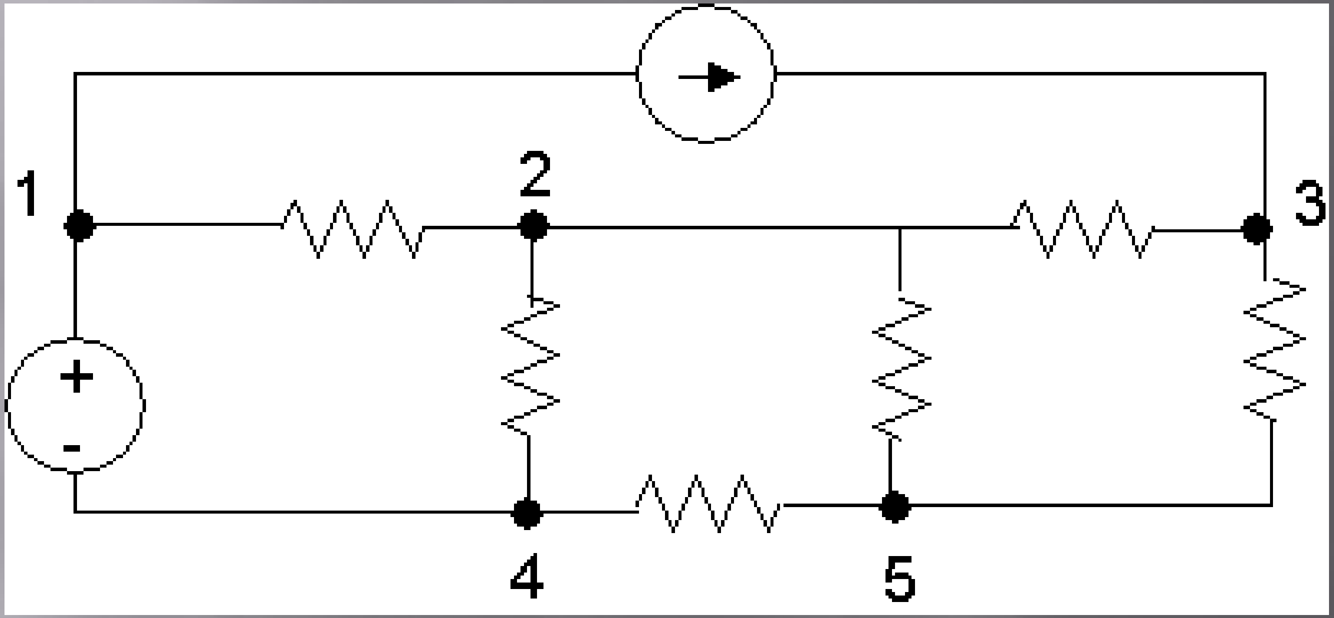
Suppose we wish to find the node voltages of the circuit below.



We know how to do this by hand.

For large-scale circuits, we may wish to do this via a computer simulation (i.e. PSpice). We will need to express this circuit in a standard form for input to the program.

Graph Theory in Circuit Analysis

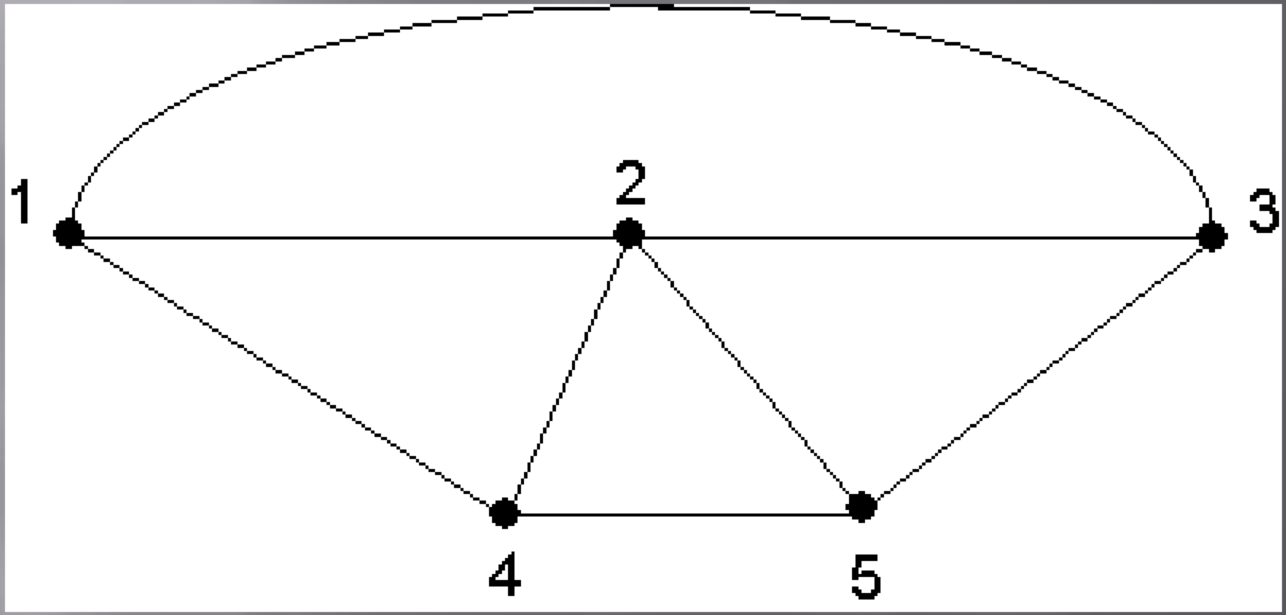


Whether the circuit is input via a GUI or as a text file, at some level the circuit will be represented as a graph, with elements as edges and nodes as nodes.

For example, when entering a circuit into PSpice via a text file, we number each node, and specify each element (edge) in the circuit with its value and endpoints.

Graph of a Circuit

a

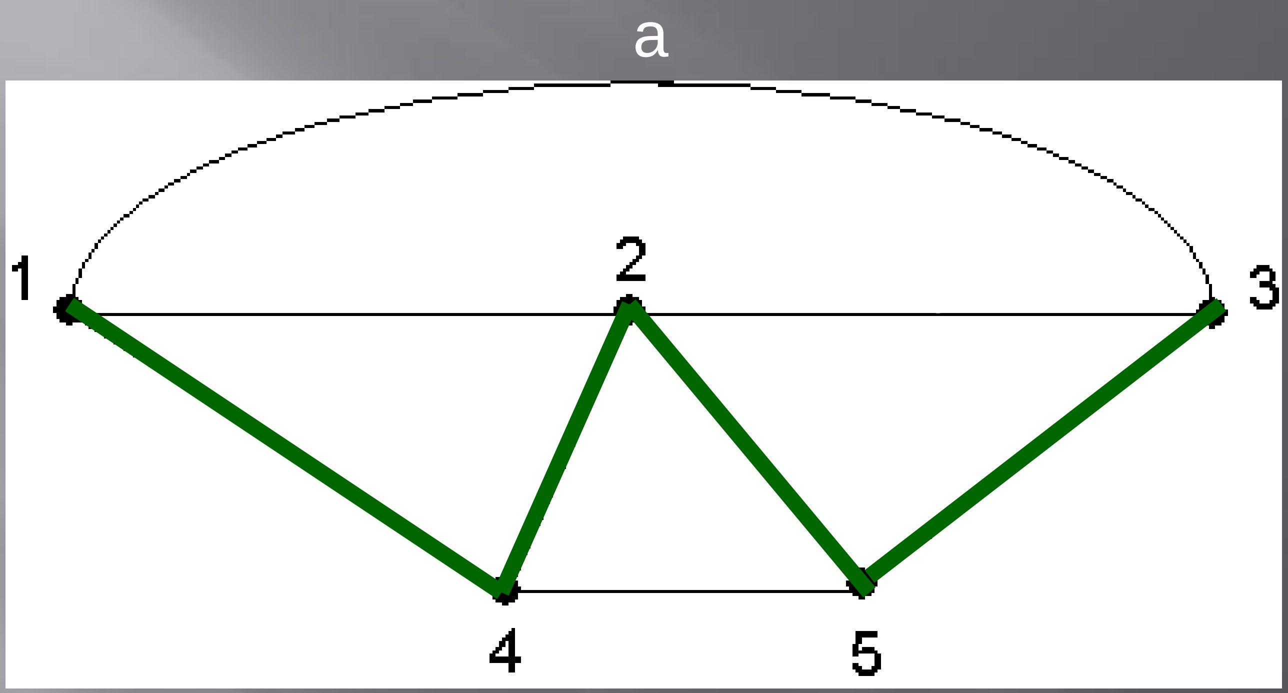


Here is a **graph** of the circuit. It is simply the circuit without elements.

We refer to the lines above as **edges** (and the nodes are **nodes**).

The graph provides **connectivity** information. To actually solve the circuit using this graph, the types of elements forming the edges would need to be provided.

Trees and Co-Trees



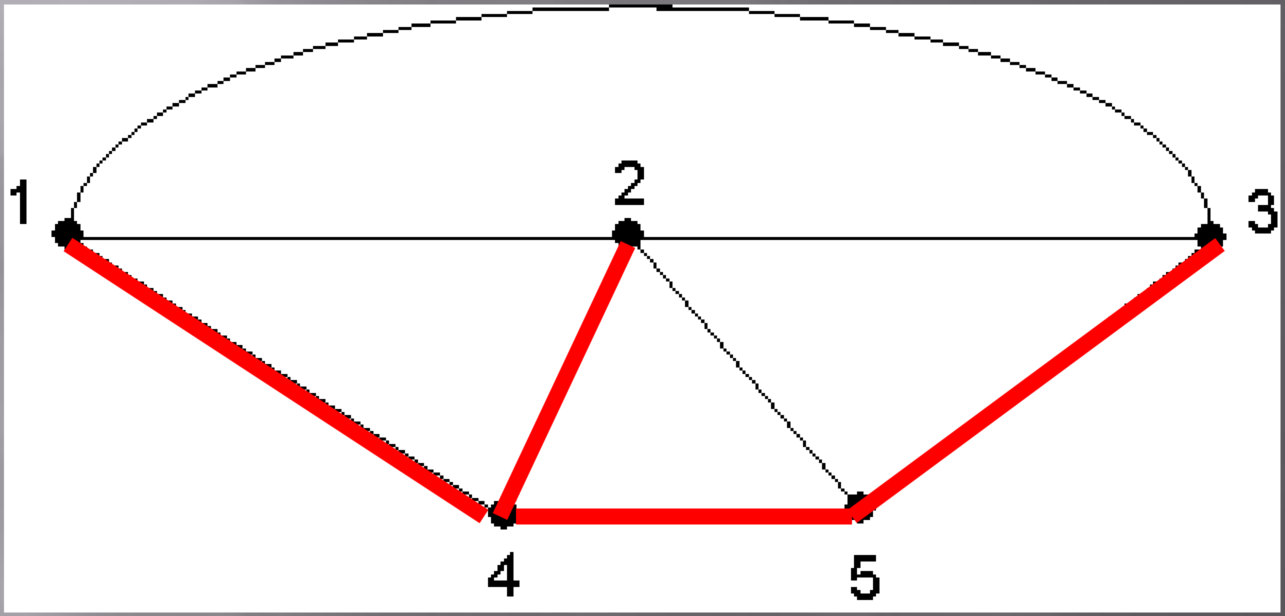
A **tree** is defined as any set of edges in a graph that touches every node *without forming any closed paths.*

Also known as Hamiltonian path!

Each tree has a **co-tree**, which is the set of edges not in the tree.

Cut Set

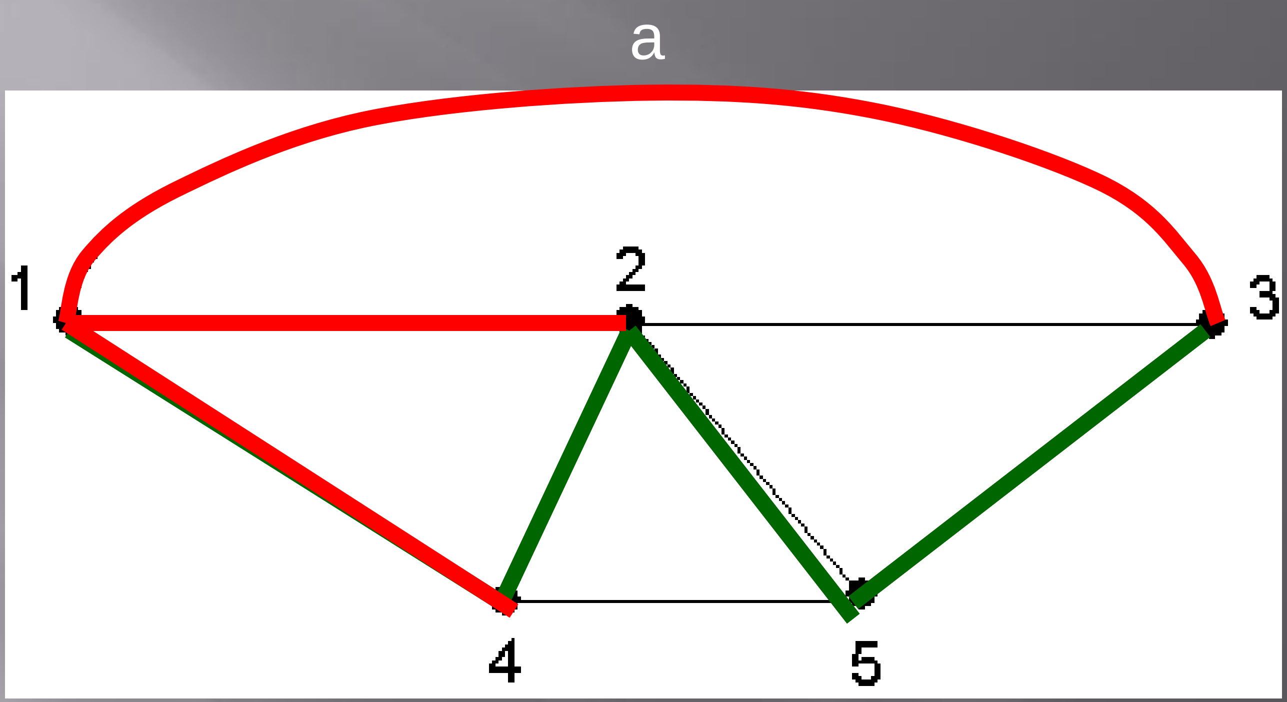
a



A **cut set** is a *minimal* set of edges that, when broken, breaks the graph into two completely separate parts (two groups of nodes.

Minimal means that a cut set cannot contain another smaller cut set that would break the graph into the same two parts.

Fundamental Cut Set



Suppose I am given a tree.

A **fundamental cut set** w.r.t. that tree is a cut set that only contains one branch of the tree.

There may be many fundamental cut sets w.r.t. a given tree.

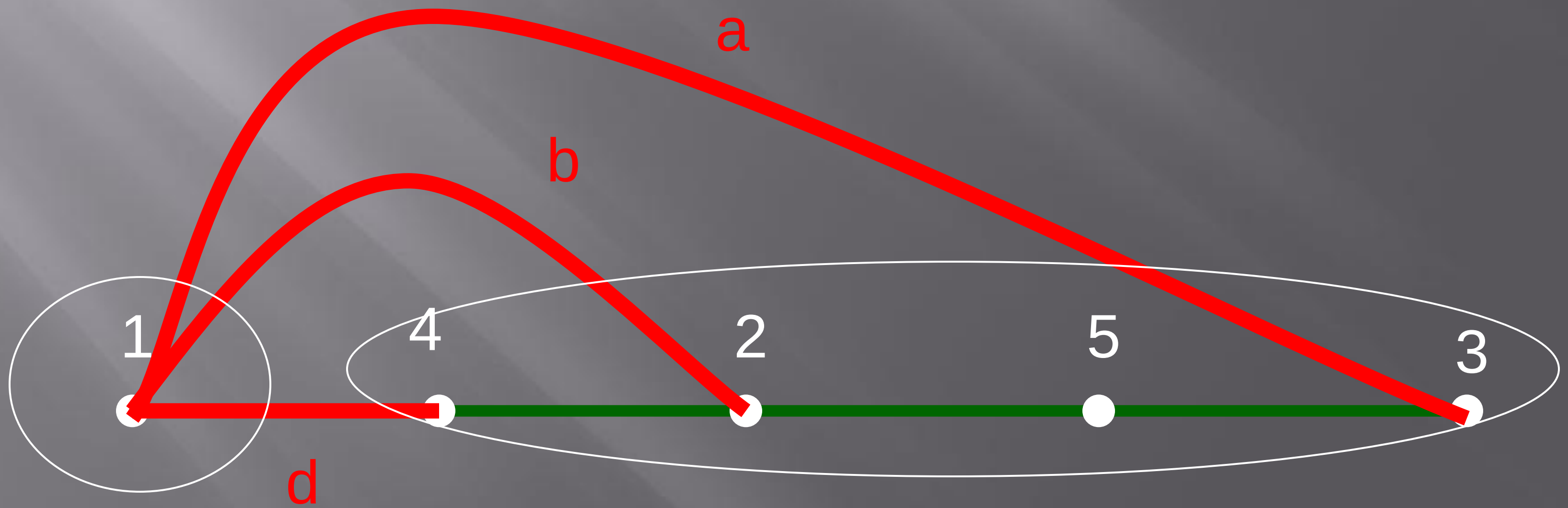
Finding Fundamental Cut Sets Systematically

1. Redraw the graph with the tree in a straight line.
2. For each tree edge, form its fundamental cut set as follows:

   2a) that tree edge is a member of this fundamental cut set

   2b) cut that edge…what two groups of nodes are separated?

   2c) the fundamental cut set also contains all edges in the co-tree that connect these two groups.



How is this used in circuit analysis?

- Identifying a tree for a circuit, and all of the fundamental cut sets that go with it, can be used in nodal analysis.
- Here are the steps simulation software may take to perform nodal analysis:

1. From user input, make a connectivity matrix (graph) and record the circuit element on each edge.
2. Choose a tree using the following guidelines:
   a) Place an edge in the **tree** if it contains a voltage source, or if the voltage over the edge controls a dependent source.
   b) Place an edge in the **co-tree** if it contains a current source, or if the current in the edge controls a dependent source.
3. Find all of the fundamental cut sets for this tree.
   - *n* nodes yields *n-1* fundamental cut sets

How is this used in circuit analysis?

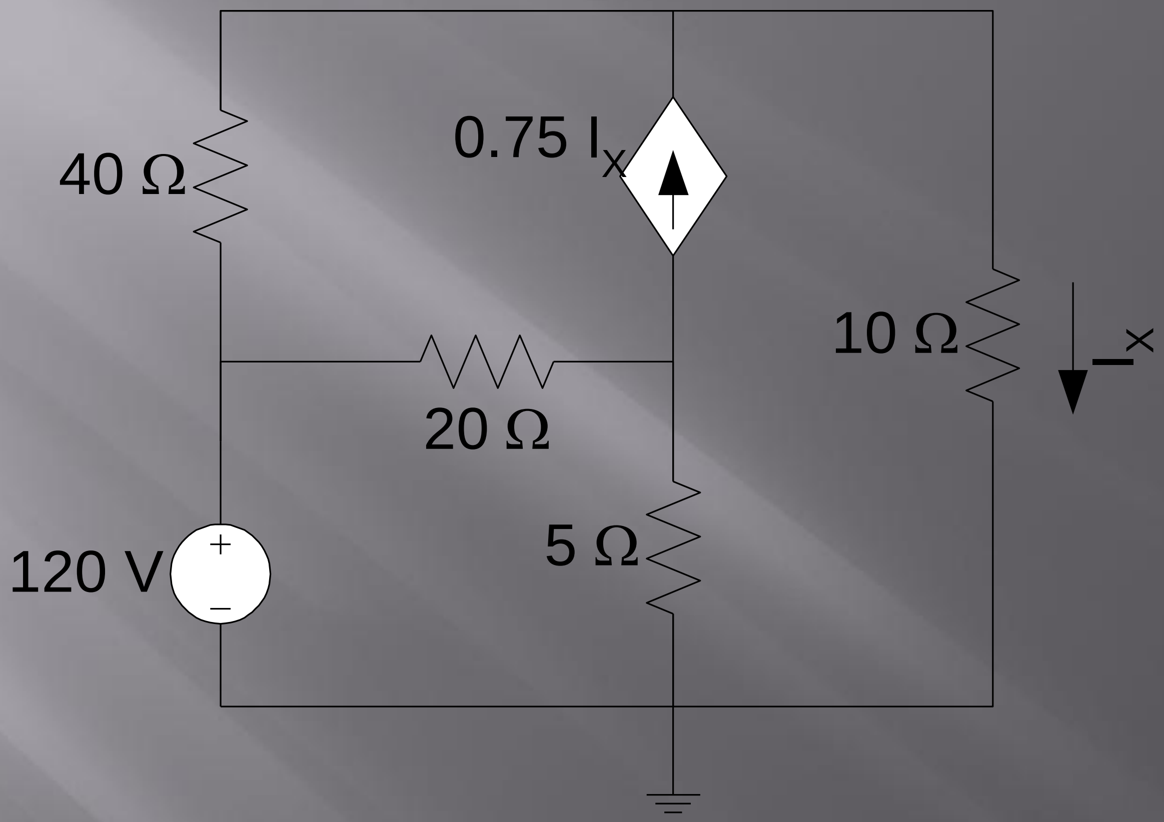
4. Each fundamental cut set breaks the circuit into two pieces: **two supernodes**. Write a KCL equation for one supernode in each fundamental cut set (in terms of node voltages).
   - The KCL equations for the two supernodes formed by a fundamental cut set will be the same.
   - This is where the circuit element info comes into play.
   - This yields *n-1* equations in *n* node voltage variables.
5. Set one node voltage to zero volts (ground) and solve

   .

Notes

- All of this can be done computationally.
  - Graph algorithms
  - Linear equation solution
- This algorithm shows why nodal analysis always works: you get *n-1* independent linear equations in *n-1* unknowns.
- The fundamental cut sets ensure independence of the equations—unless the circuit has impossible elements.
  - Each fundamental cut set contains a unique element (edge) from the tree. So each KCL equation provides new info.
  - The elements themselves could destroy the independence (redundant dependent source, shorted voltage source…) but this won't happen in real life circuits.

Example



Find the node voltages using the graph method.

Circuit from Nilsson's *Electric Circuits*, Addison-Wesley, 1993.

Magnetic and Electromagnetic Fields

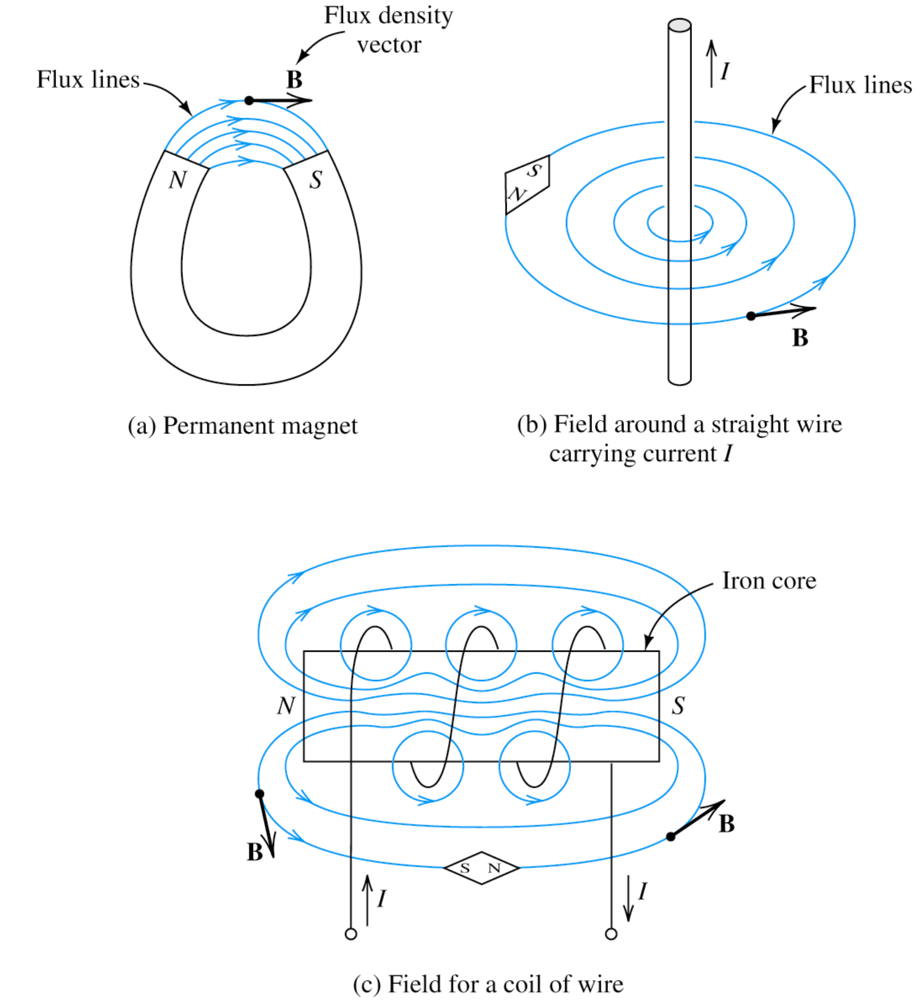
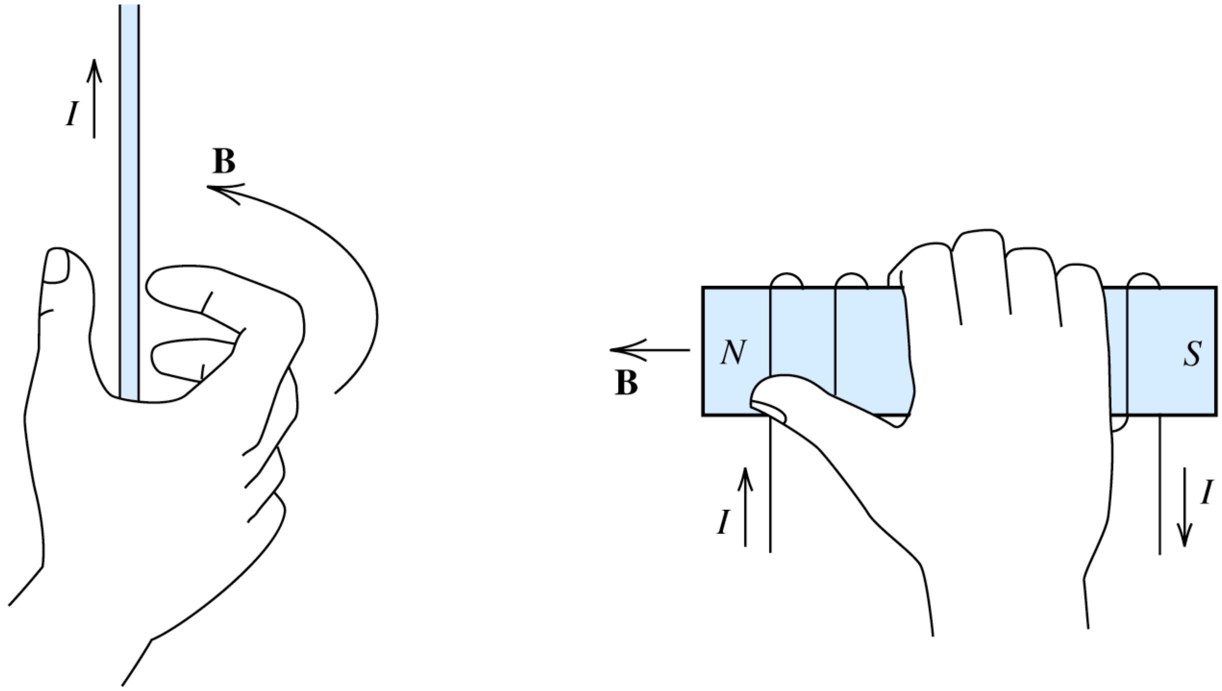


Figure 15.1 Magnetic fields can be visualized as lines of flux that form closed paths. Using a compass, we can determine the direction of the flux lines at any point. Note that the flux density vector **B** is tangent to the lines of flux.

Magnetic Materials

Iron, Cobalt and Nickel and various other alloys and compounds made using these three basic elements

Electric Current and Magnetic Field



(a) If a wire is grasped with the thumb pointing in the current direction, the fingers encircle the wire in the direction of the magnetic field

(b) If a coil is grasped with the fingers pointing in the current direction, the thumb points in the direction of the magnetic field inside the coil

Figure 15.2 Illustrations of the right-hand rule.

A Few Definitions Related to Electromagnetic Field

Φ (Unit is Weber (Wb)) = Magnetic Flux Crossing a Surface of Area 'A' in m^2.

B (Unit is Tesla (T)) = Magnetic Flux Density = Φ/A

H (Unit is Amp/m) = Magnetic Field Intensity = $\frac{B}{\mu}$

μ = permeability = $\mu_o \mu_r$

$\mu_o = 4\pi * 10^{-7}$ H/m (H ⇒Henry) = Permeability of free space (air)

μ_r = Relative Permeability

$\mu_r >> 1$ for Magnetic Material

Ampére's Law

The line integral of the magnetic field intensity around a closed path is equal to the sum of the currents flowing through the area enclosed by the path.

$$\oint \vec{H}.\vec{dl} = \sum i$$

$$\vec{H}.\vec{dl} = \left|\vec{H}\right|\left|\vec{dl}\right| \cos\theta$$

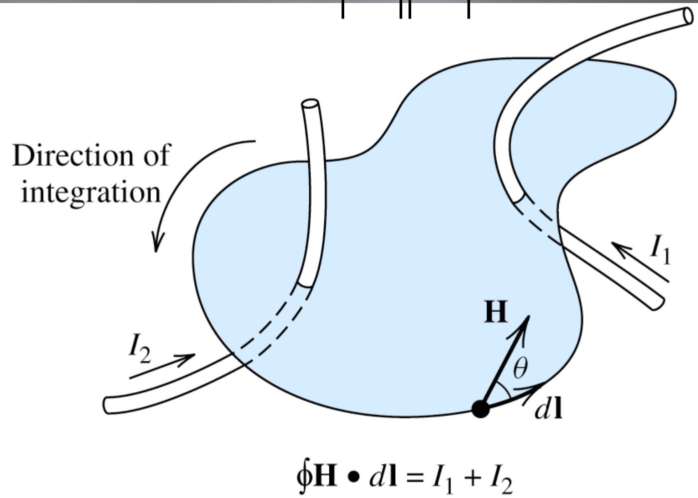


Figure 15.6 Ampère's law states that the line integral of magnetic field intensity around a closed path is equal to the sum of the currents flowing through the surface bounded by the path.

Example of Ampére's Law

Find the magnetic field along a circular path around an infinitely long Conductor carrying 'I' ampere of current.

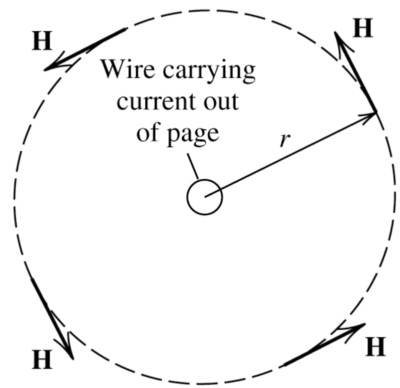
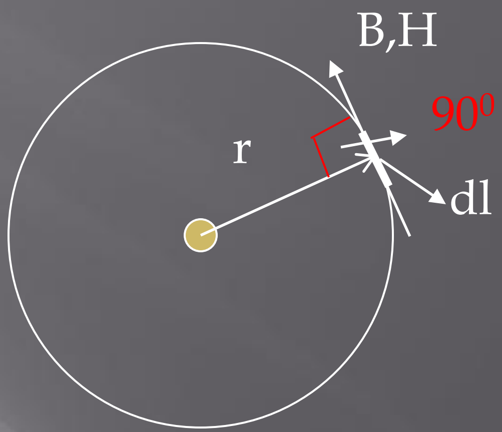


Figure 15.7 The magnetic field around a long straight wire carrying a current can be determined with Ampère's law aided by considerations of symmetry.



Since both $\vec{dl}$ and $\vec{H}$ are perpendicular to radius 'r' at any point 'A' on the circular path, the angle θ is zero between them at all points. Also since all the points on the circular path are equidistant from the current carrying conductor $\vec{H}$ is constant at all points on the circle

$$\oint \vec{H}.\vec{dl} = \left|\vec{H}\right| \oint \vec{dl} = \left|\vec{H}\right| 2\pi r = I \quad \text{or} \quad \left|\vec{H}\right| = \frac{I}{2\pi r}$$

Magnetic Circuits

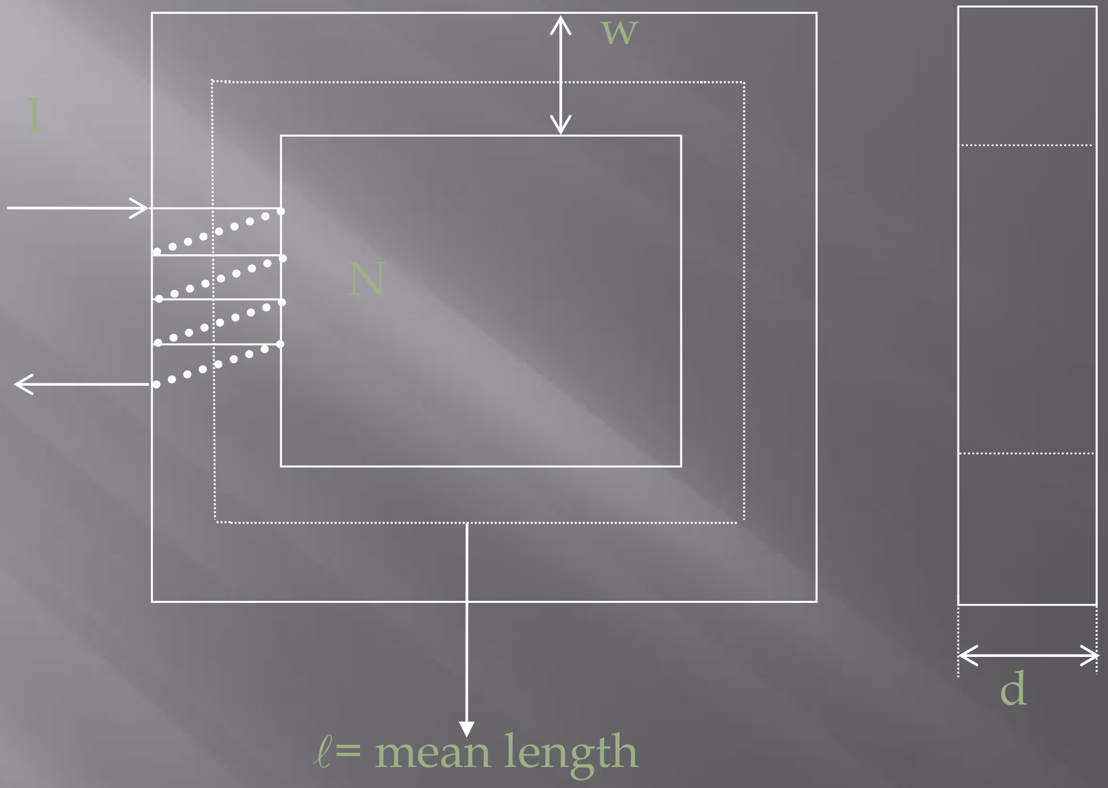
- They are basically ferromagnetic structures(mostly Iron, Cobalt, Nickel alloys and compounds) with coils wound around them

- Because of high permeability most of the magnetic flux is confined within the magnetic circuit

- Thus $\vec{H}$ is always aligned with $\vec{dl}(\theta = 0)$

- Examples: Transformers,Actuators, Electromagnets, Electric Machines

Magnetic Circuits (1)



Magnetic Circuits (2)

F =NI= Magneto Motive Force or MMF = # of turns * Current passing through it

F = NI = Hℓ (why!)

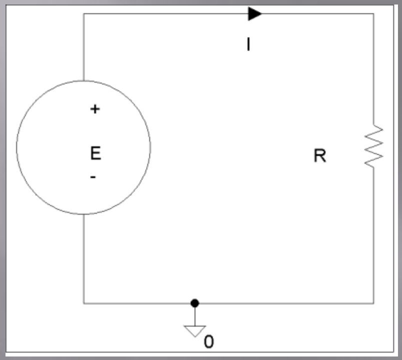
or $\dfrac{B}{\mu}\ell = NI$ or $\dfrac{\Phi}{\mu A}\ell = NI$

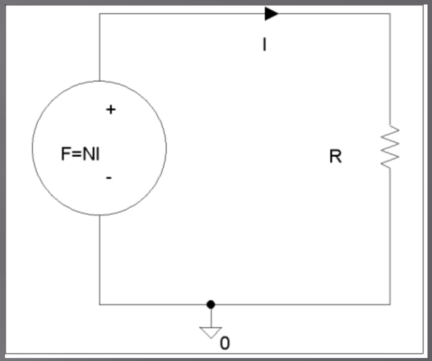
or $\Phi = \dfrac{NI}{\ell/(\mu A)}$

or $\Phi = \dfrac{NI}{\Re}$

$\Re$ = ***Reluctance*** of magnetic path

Analogy Between Magnetic and Electric Circuits





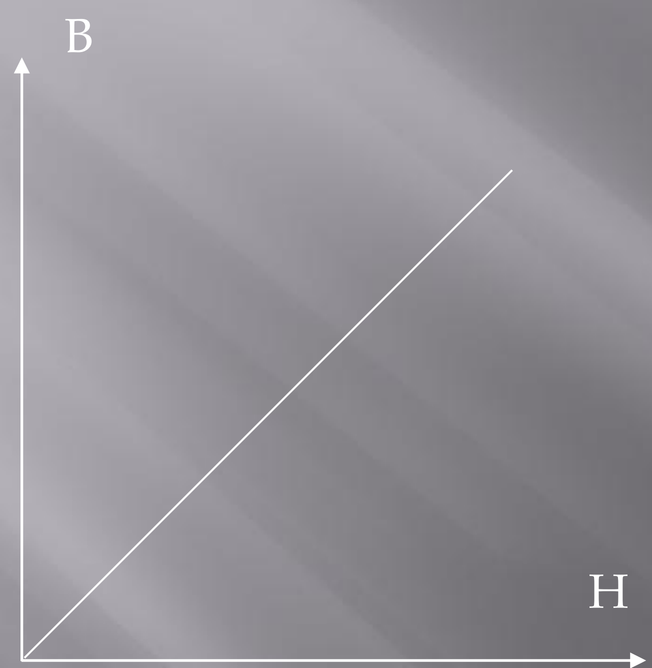
F =MMF is analogous to Electromotive force (EMF) =E

Φ = Flux is analogous to I = Current

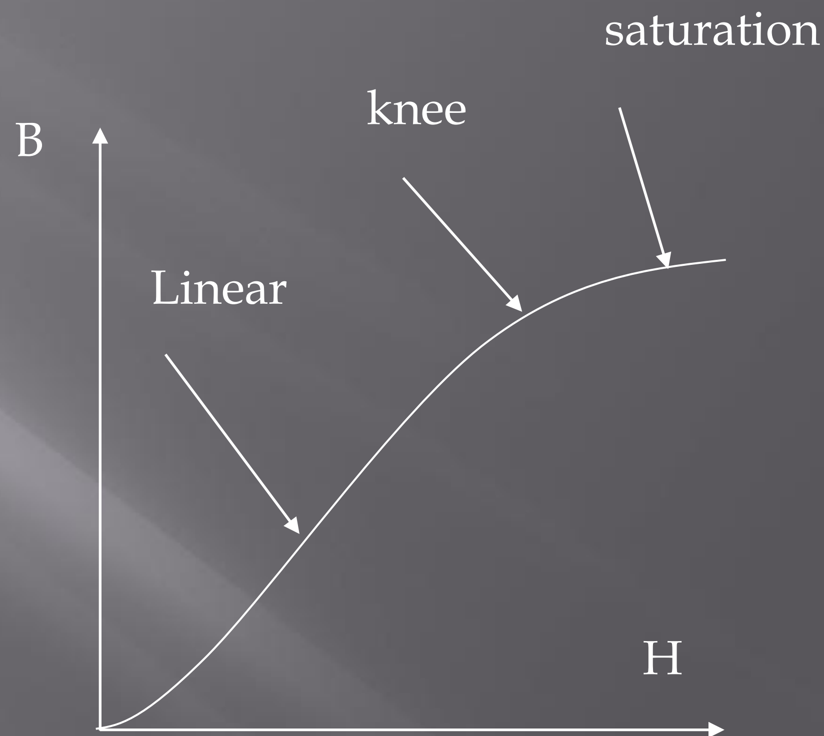
$\Re$ = Reluctance is analogous to R = Resistance

P = Permeance $= \frac{1}{\Re}$ = Analogous to conductance $G = \frac{1}{R}$

Magnetization Curves



Magnetization curve
(linear) (Ideal)



Magnetization curve
(non-linear) (Actual)
(see also Fig. 1.6 in the text)

Magnetization Curves(2)

- One can linearize magnetic circuits by including air-gaps

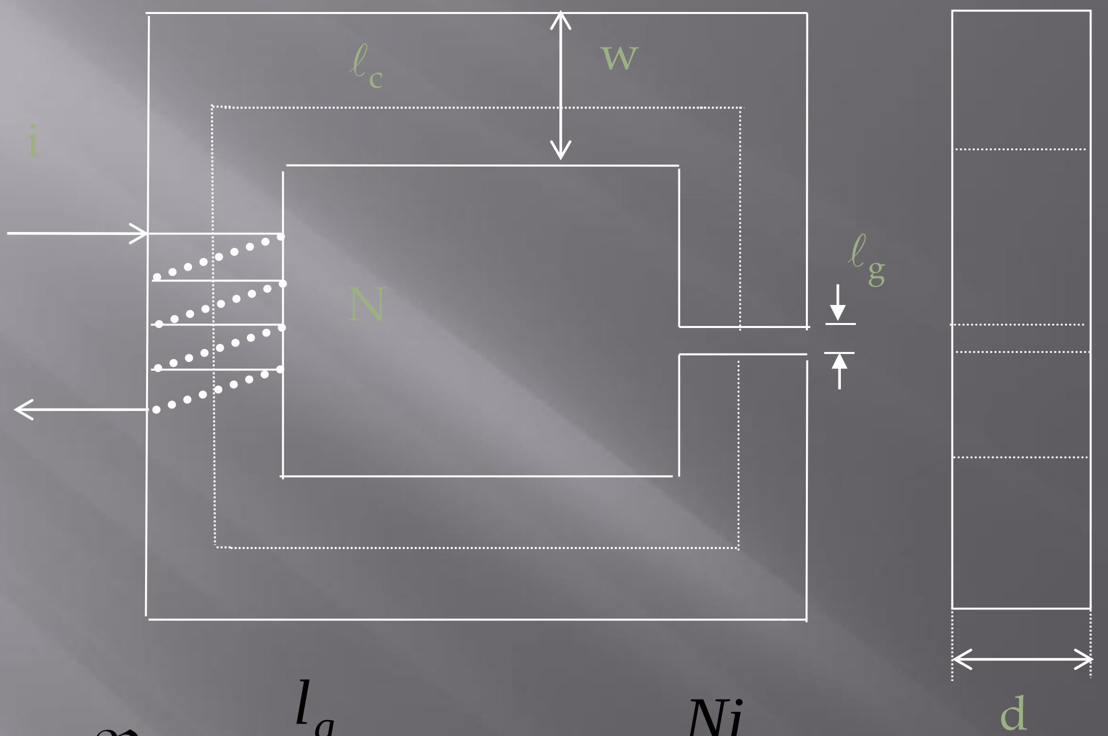
- However that would cause a large increase in ampere-turn requirements.

Ex: Transformers don’t have air-gaps. They have very little magnetizing current (5% of full load)

Induction motors have air-gaps. They have large magnetizing current (30-50%)

Question: why induction motors have air –gap and transformers don’t?

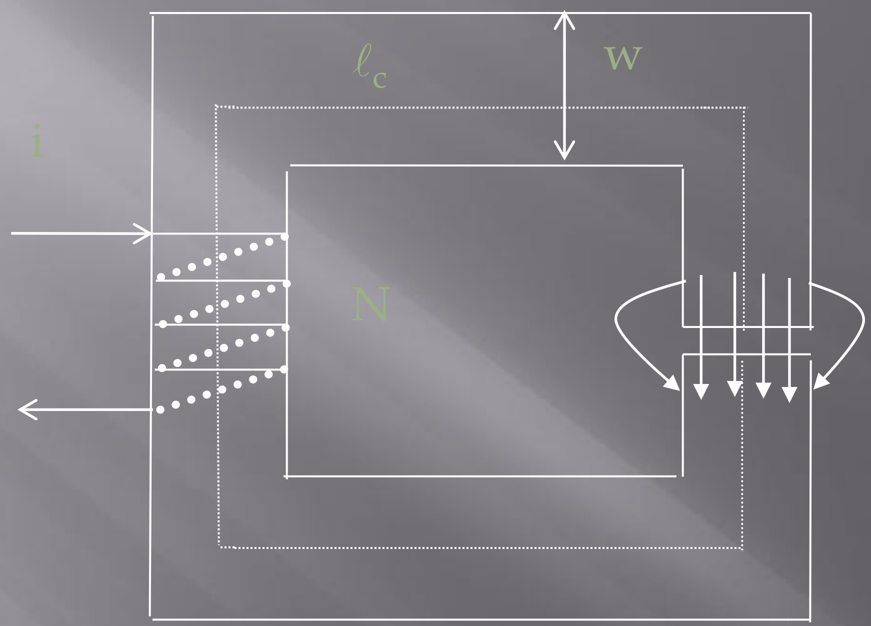
Magnetization Circuits with Air-gap



$$\Re_c = \frac{l_c}{\mu_c A_c} \qquad \Re_g = \frac{l_g}{\mu_g A_g} \qquad \Phi = \frac{Ni}{\Re_c + \Re_g}$$

$$Ni = H_c l_c + H_g l_g \qquad A_c = A_g = wd \; (Neglecting \;\; fringing)$$

Fringing

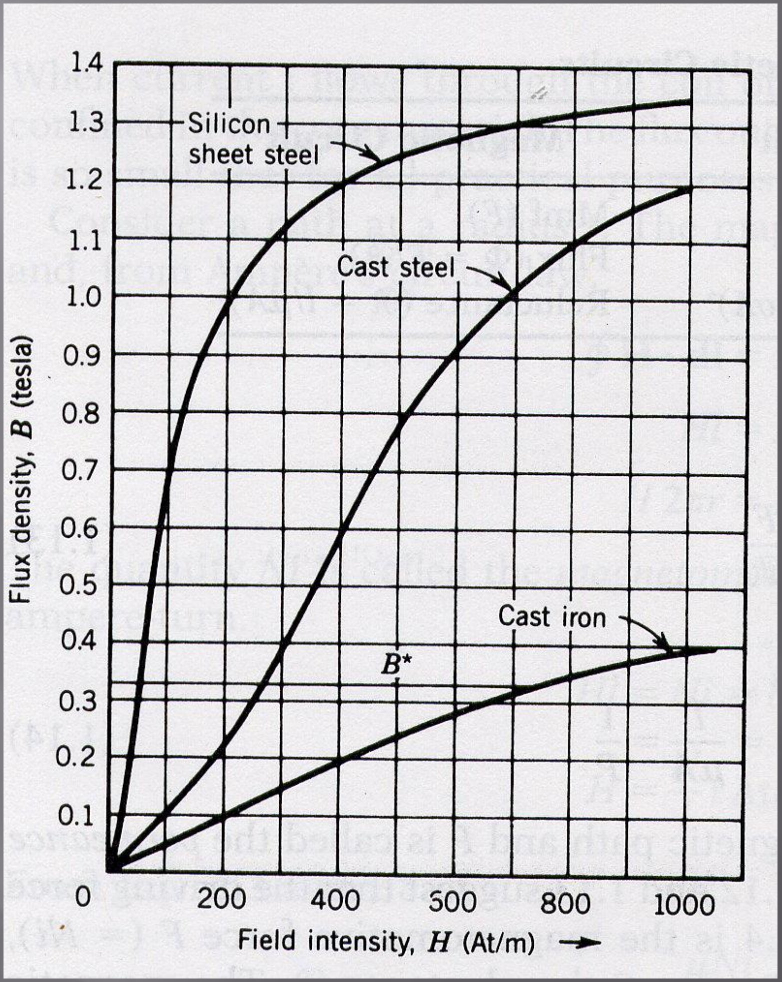


With large air-gaps, flux tends to leak outside the air –gap. This is called fringing which increases the effective flux area. One way to approximate this increase is:

$$w_n = w + l_g ; d_n = d + l_g ; A_{gn} = w_n d_n$$

Example of Magnetic Circuits On Greenboard

Magnetization Curves (for examples)



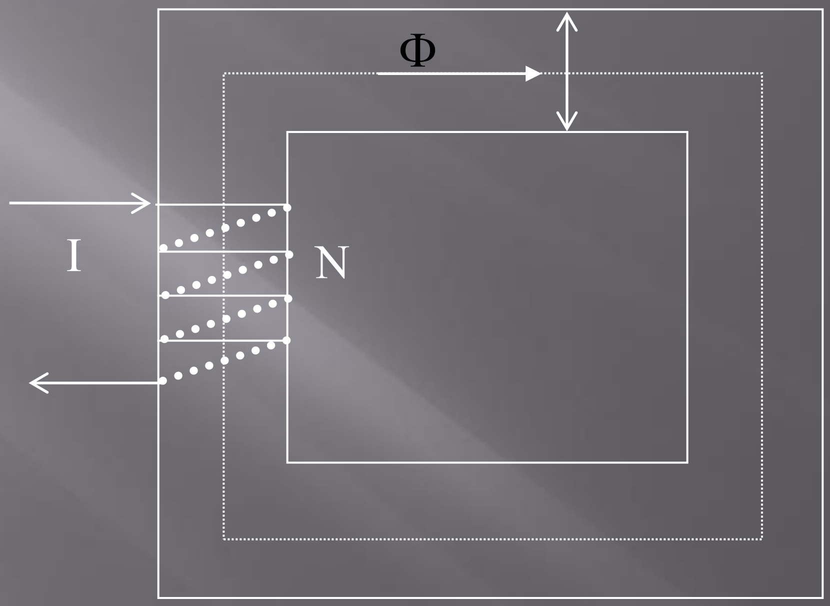
Inductance(L)

Definition: Flux Linkage(λ) per unit of current(I) in a magnetic circuit

$$L = \frac{\lambda}{I} = \frac{N\Phi}{I}$$

$$\Phi = \frac{NI}{\Re}$$

$$\therefore L = \frac{N^2}{\Re}$$



Thus inductance depends on the geometry of construction

Example of Inductances On Greenboard

How to find exact Inductances with magnetic circuit with finite thickness (say a torroid with finite thickness)

see problem 1.16

Faraday's law of Electromagnetic Induction

The EMF (Electromotive Force) induced in a magnetic circuit is
Equal to the rate of change of flux linked with the circuit

$$e = \frac{d\lambda}{dt} = \frac{d(N\Phi)}{dt} = N\frac{d\Phi}{dt}$$

$$\because Li = N\Phi$$

$$\therefore e = \frac{dLi}{dt} = L\frac{di}{dt}$$

Lenz's Law

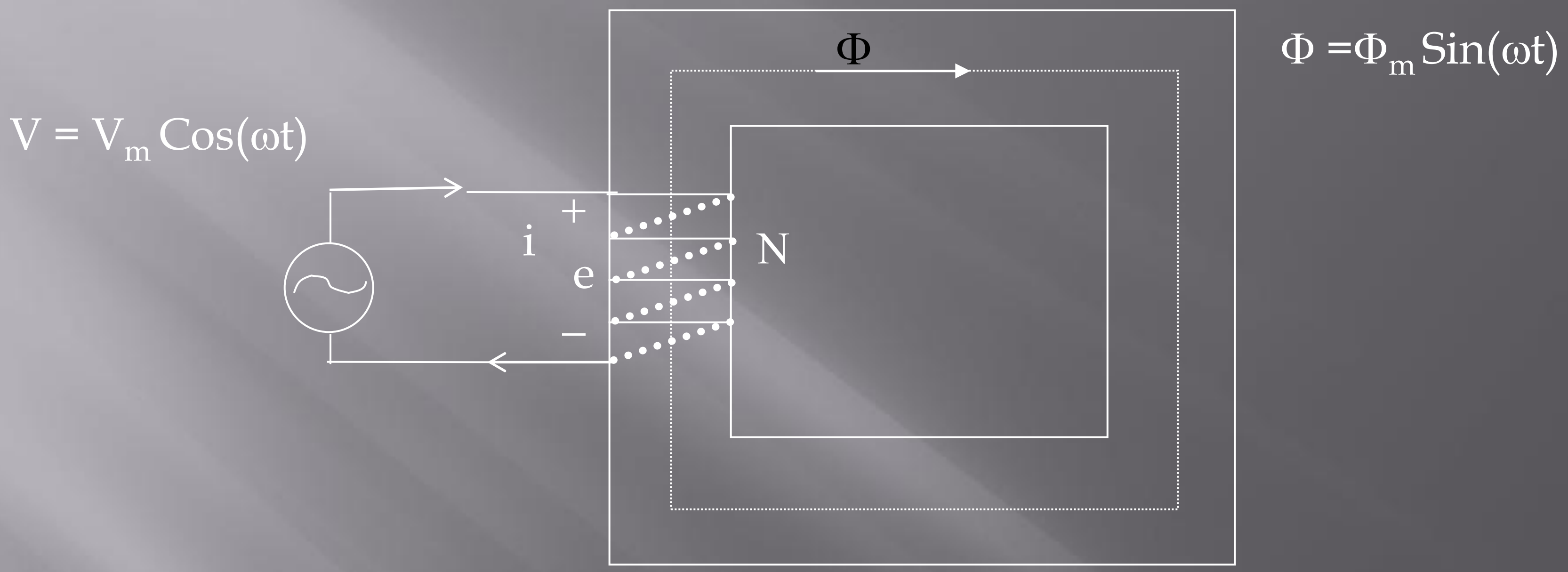
The polarity of the induced voltage is given by Lenz's law

The polarity of the induced voltage will be such as to oppose the very cause to which it is due

Thus *sometimes* we write

$$\therefore e = -\frac{dLi}{dt} = -L\frac{di}{dt}$$

A precursor to Transformer

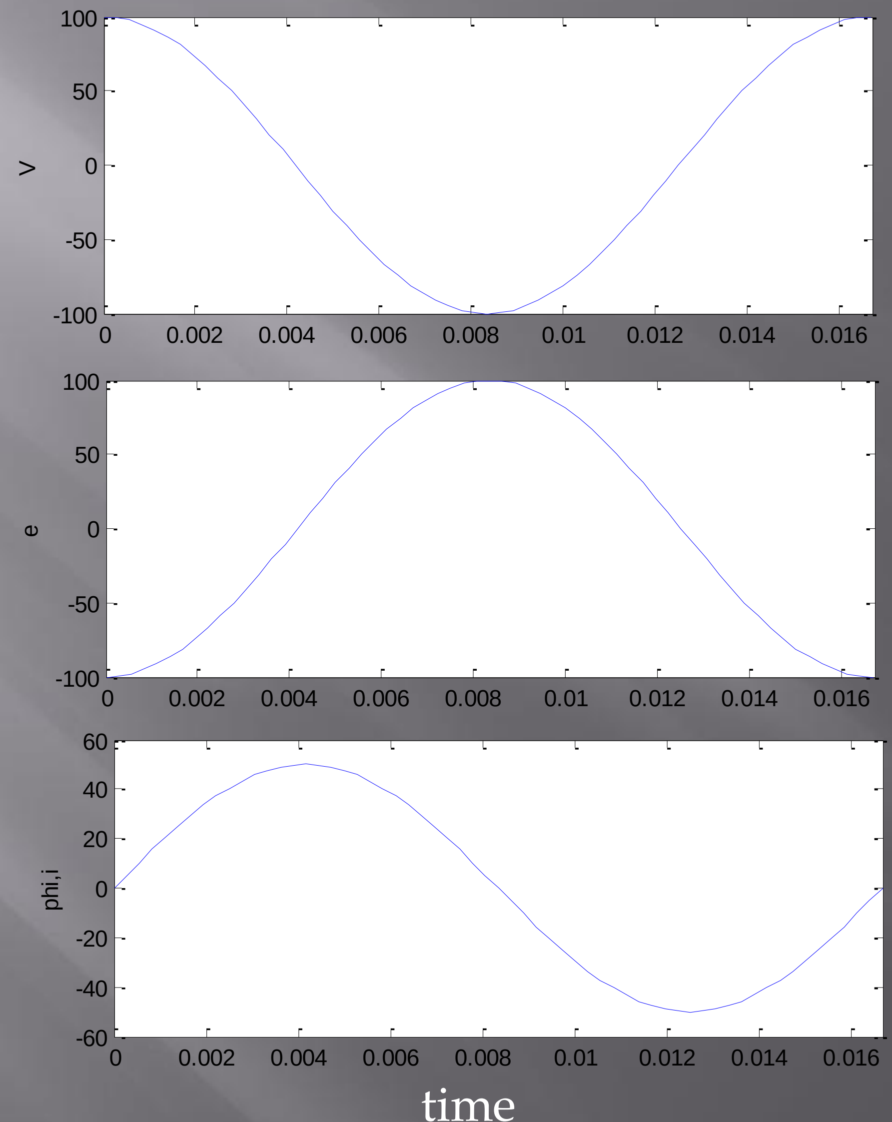


Ideally

$$e = -N\frac{d\Phi}{dt} = -N\Phi_m \omega Cos(\omega t) = -E_m Cos(\omega t) = -V_m Cos(\omega t)$$

$$i = \frac{N\Phi}{L} = \frac{N\Phi_m Sin(\omega t)}{L} = I_m Sin(\omega t)$$

A Precursor to Transformer(2)



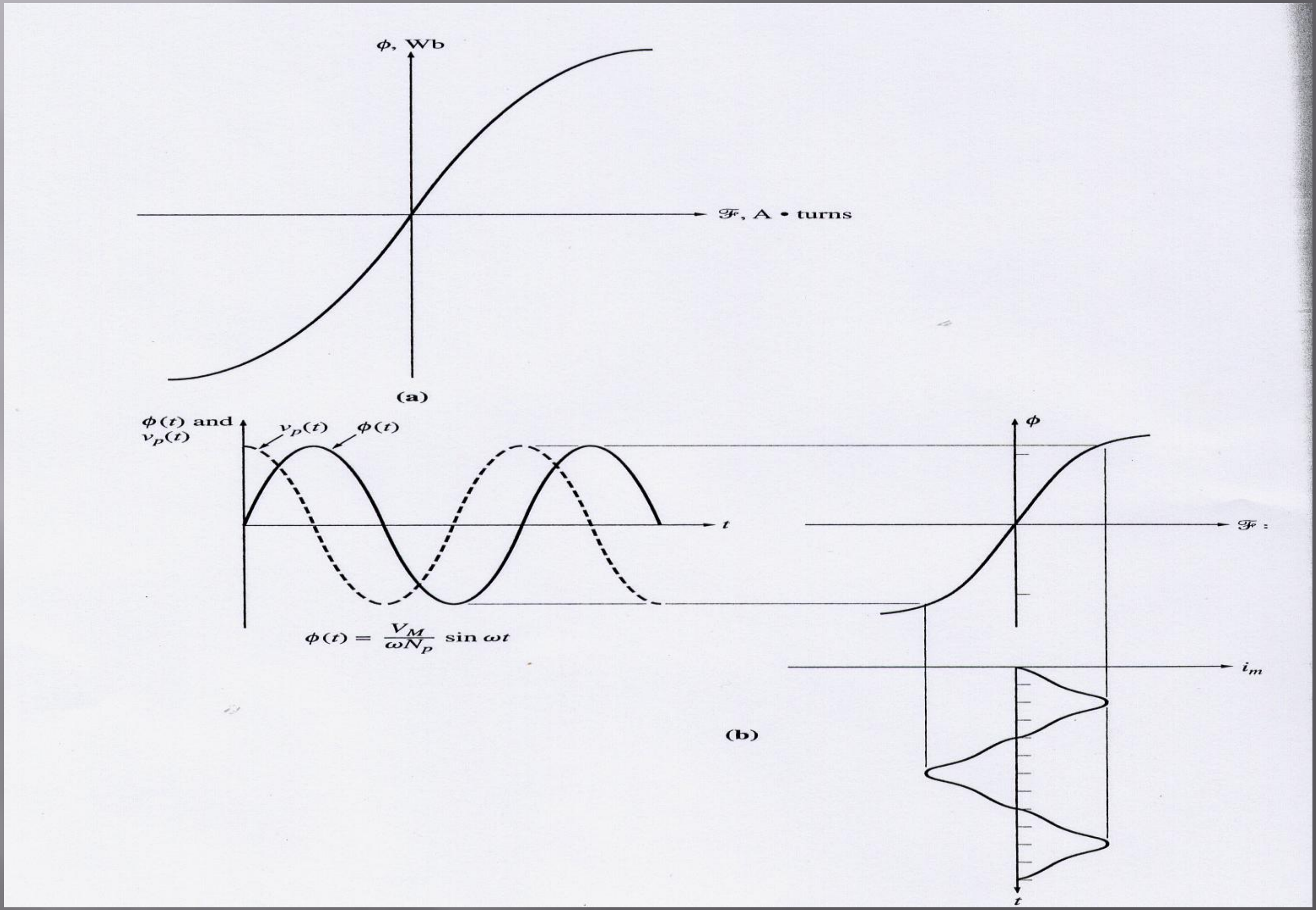
Example on excitation of magnetic circuit with sinusoidal flux On greenboard

Example on excitation of magnetic circuit with square flux on greenboar (Important for Switched Mode Powe Supplies)

What will non-linearity in magnetic circuit lead to?

- It would cause distortion in current waveforms since by Faraday's and Lenz's law the induced voltage always has to balance out the applied voltage that happens to be sinusoidal

Sinusoidal voltage non-sinusoidal current



Iron Losses in Magnetic Circuit

There are two types of iron losses

a) Hysteresis losses

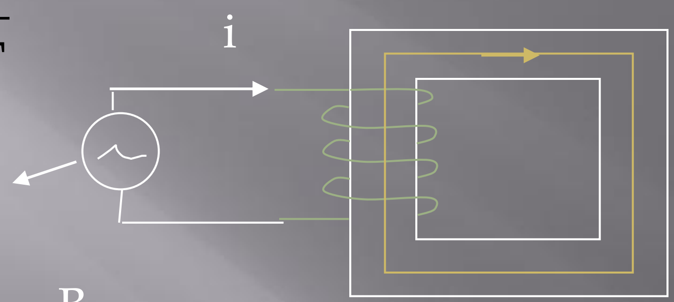
b) Eddy Current Losses

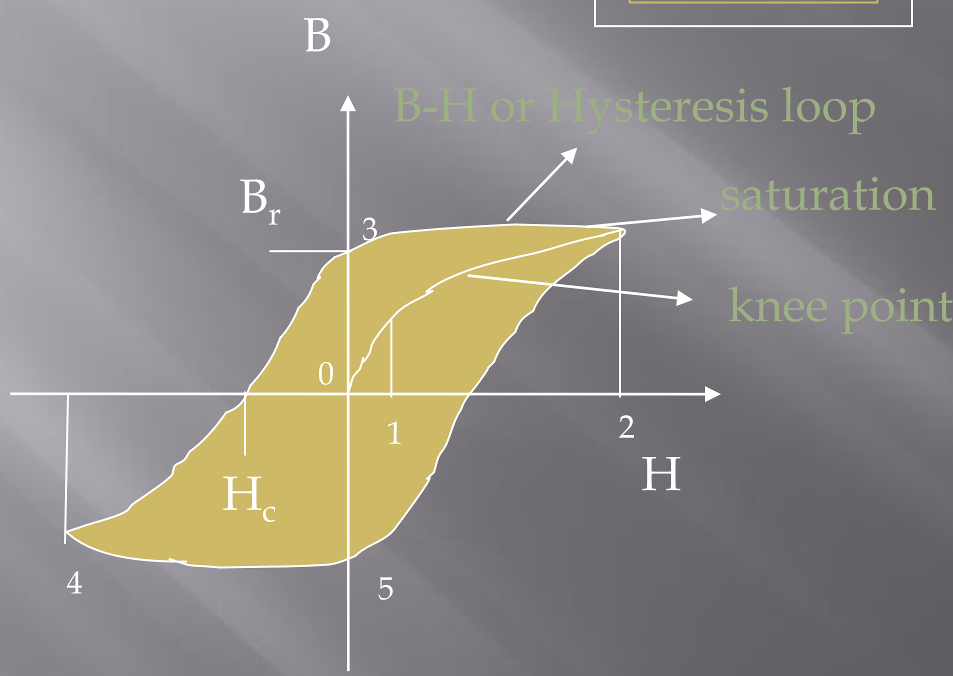
Total iron loss is the sum of these two losses

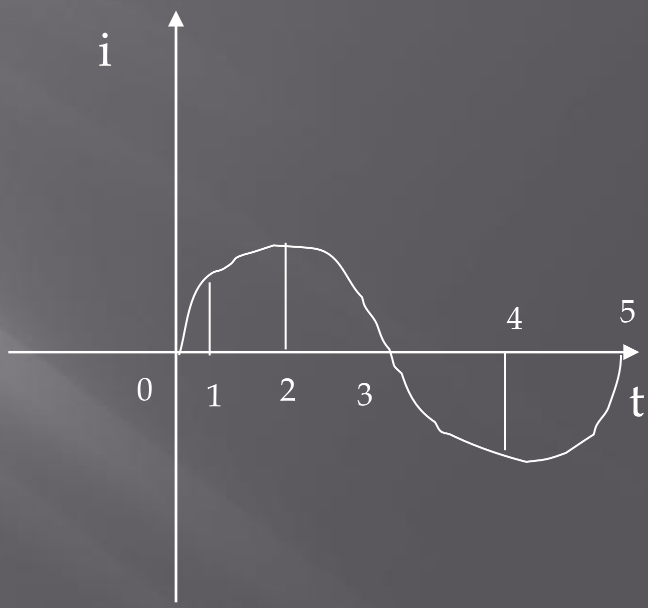
Hysteresis losses

$$f = \frac{1}{T}$$

f =frequency
of sine source







B_r = Retentive flux density (due to property of retentivity)
H_c= Coercive field intensity (due to property of coercivity)

Hysteresis losses (2)

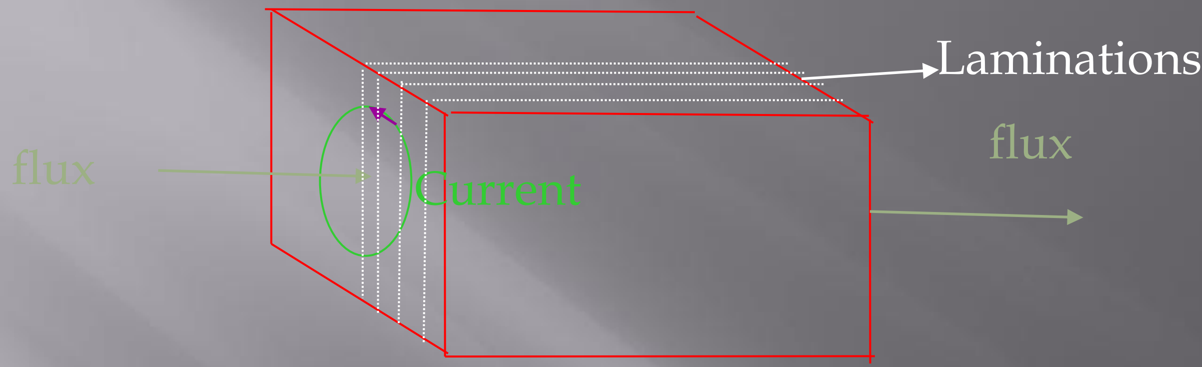
- The lagging phenomenon of B behind H is called hysteresis
- The tip of hysteresis loops can be joined to obtain the magnetization characteristics
- In each of the current cycle the energy lost in the core is proportional to the area of the B-H loop
- Energy lost/cycle = V_{core}

$$\oint HdB$$

- P_h = Hysteresis loss = $f\, V_{core} \oint HdB = k_h B^n_{max} f$

k_h = Constant, n = 1.5-2.5, B_{max} = Peak flux density

Eddy current loss



Because of time variation of flux flowing through the magnetic material as shown, current is induced in the magnetic material, following Faraday's law. This current is called eddy current. The direction of the current is determined by Lenz's law. This current can be reduced by using laminated (thin sheet) iron structure, with Insulation between the laminations.

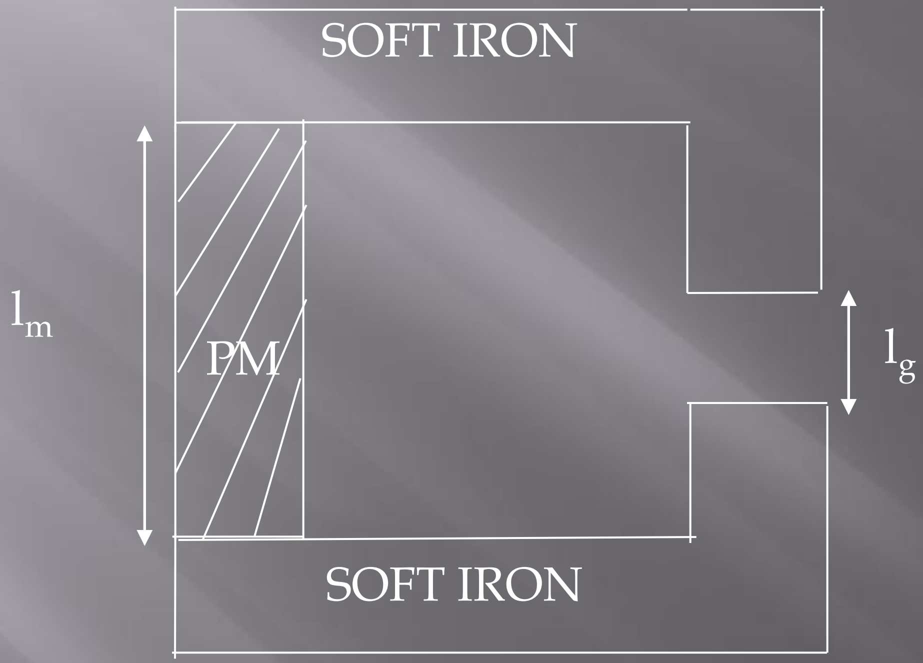
- P_e = Eddy current loss $= k_e B^2_{max} f$

k_e = Constant , B_{max} = Peak flux density

Permanent Magnets

- Alloys of Iron, Cobalt and Nickle
- Have large B-H loops, with large B_r and $-H_c$
- Due to heat treatment becomes mechanically hard and are thus called HARD IRON
- Field intensity is determined by the coercive field required to demagnetize it
- Operating points defined by B_m,H_m in the second quadrant of the B-H loop

Using Permanent Magnets for providing magnetic field



Designing Permanent Magnets

- The key issue here is to minimize the volume V_m of material required for setting up a required B_g in a given air gap

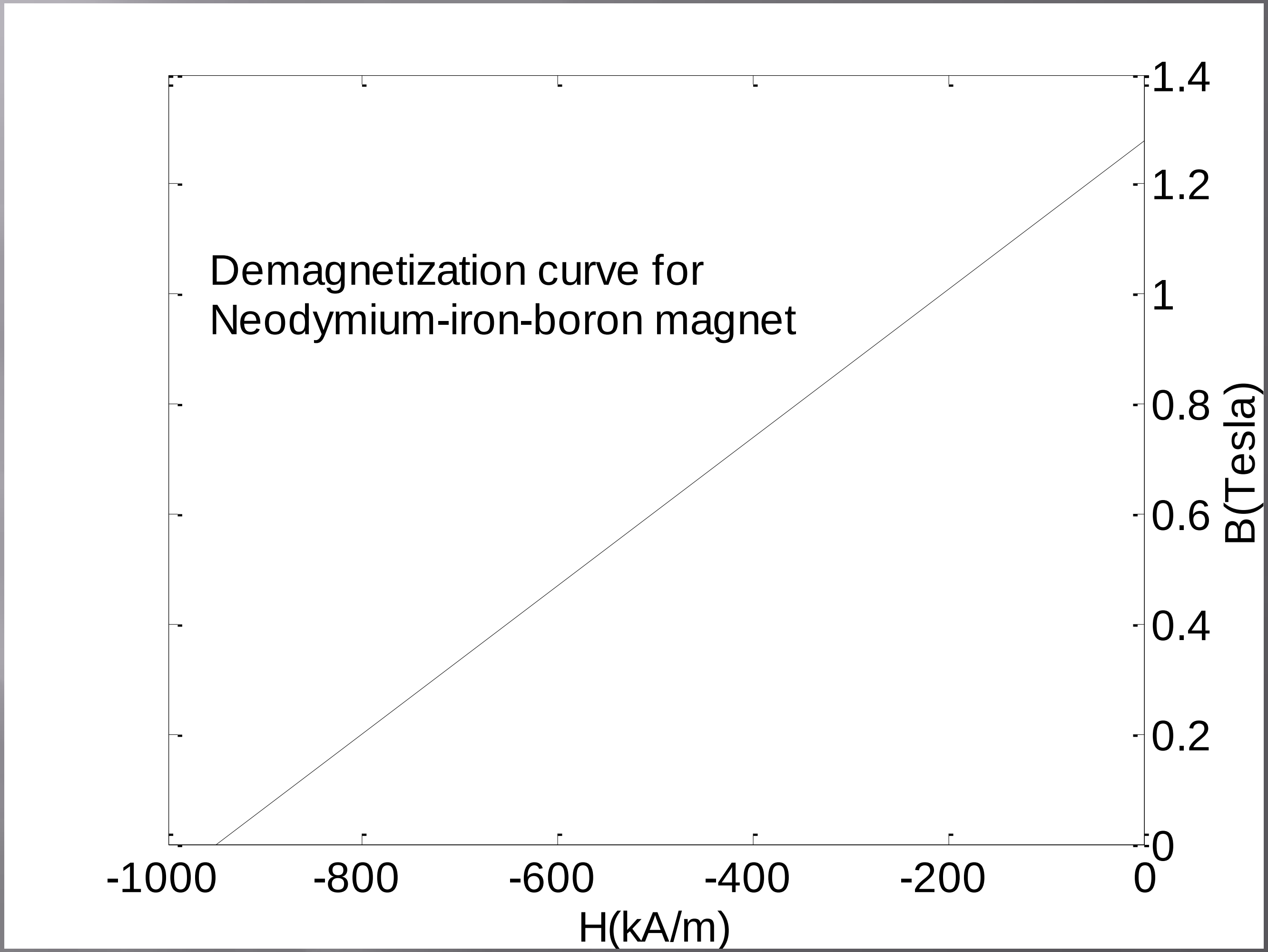
- It can be shown that $V_m = B_g^2 V_g / \mu_o B_m H_m$ (see derivation in text) where $V_g = A_g l_g$ Volume of air-gap, l_g = length of air-gap, A_g =area of air-gap

- Thus by maximizing B_m, H_m product V_m can be minimized

- Once B_m, H_m at the maximum B_m, H_m product point are known, l_m =length of permanent magnet, A_m =area of permanent magnet can be found as

- $l_m = -l_g H_g / H_m$ (applying ampère's law),
- $A_m = B_g A_g / B_m$ (same flux flows through PM as well as air-gap)

Finding the maximum product point



Finding the maximum product point

$B = mH + c$, m and c are constants.

To find maximum BH product, we need to differentiate

$$BH = mH^2 + cH;$$

and set it equal to 0. Thus we get

$H_m = -c/2m$. and $B_m = c/2$

Finding the maximum product point

Answer:

B_m=0.64 T, H_m = -475 kA/m

Introduction

- This chapter introduces important fundamental theorems of network analysis. They are the
  - **Superposition theorem**
  - **Thévenin's theorem**
  - **Norton's theorem**
  - **Maximum power transfer theorem**
  - **Substitution Theorem**
  - **Millman's theorem**
  - **Reciprocity theorem**

Superposition Theorem

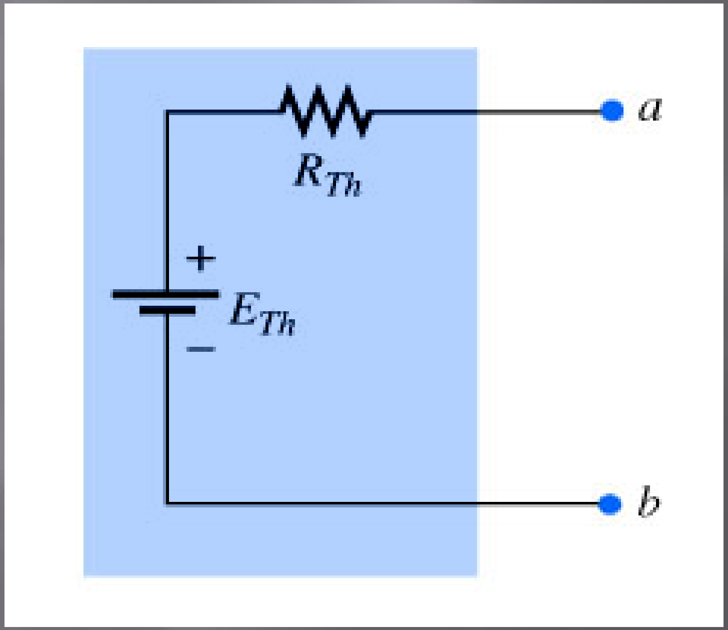
- Used to find the solution to networks with two or more sources that are not in series or parallel.
- The current through, or voltage across, an element in a network is equal to the algebraic sum of the currents or voltages produced independently by each source.
- Since the effect of each source will be determined independently, the number of networks to be analyzed will equal the number of sources.

Superposition Theorem

- The total power delivered to a resistive element must be determined using the total current through or the total voltage across the element and cannot be determined by a simple sum of the power levels established by each source.

Thévenin's Theorem

- Any two-terminal dc network can be replaced by an equivalent circuit consisting of a voltage source and a series resistor.



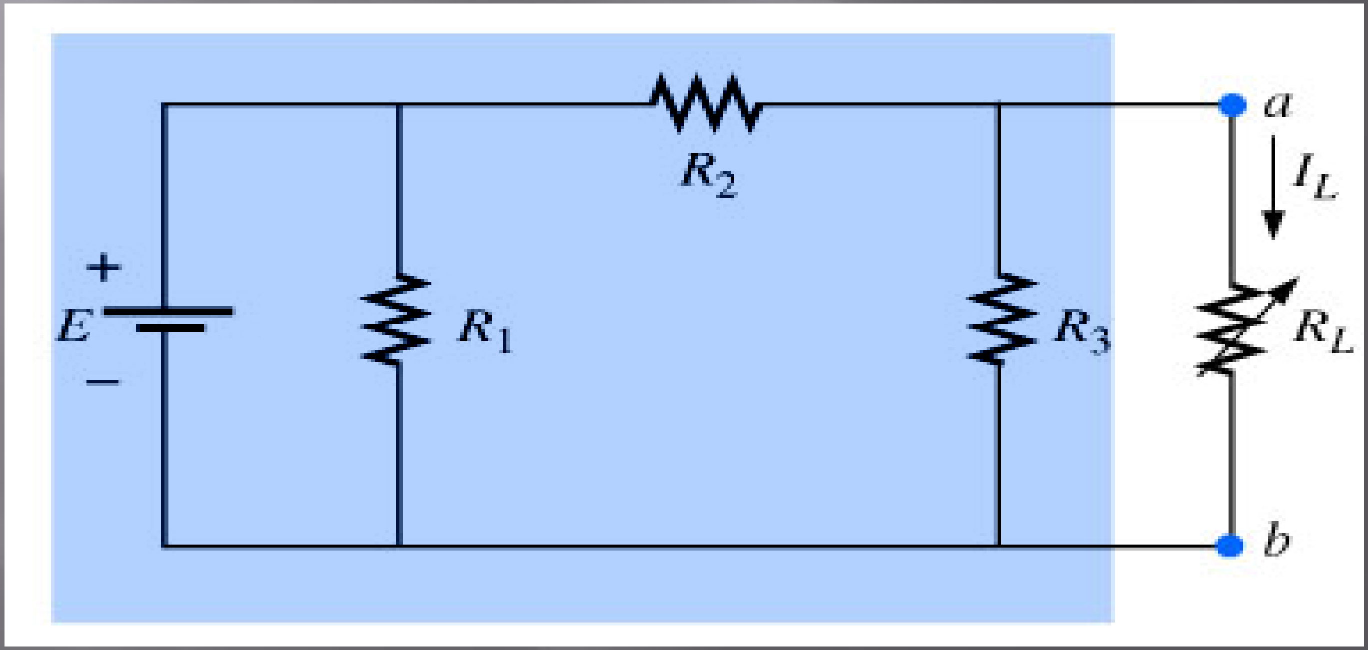
Thévenin's Theorem

- Thévenin's theorem can be used to:
  - Analyze networks with sources that are not in series or parallel.
  - Reduce the number of components required to establish the same characteristics at the output terminals.
  - Investigate the effect of changing a particular component on the behavior of a network without having to analyze the entire network after each change.

Thévenin's Theorem

- Procedure to determine the proper values of R_{Th} and E_{Th}
- Preliminary

1. **Remove that portion of the network across which the Thévenin equation circuit is to be found. In the figure below, this requires that the load resistor R_L be temporarily removed from the network.**



Thévenin's Theorem

2. Mark the terminals of the remaining two-terminal network. (The importance of this step will become obvious as we progress through some complex networks.)

R_{Th}:

3. Calculate R_{Th} by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)

Thévenin's Theorem

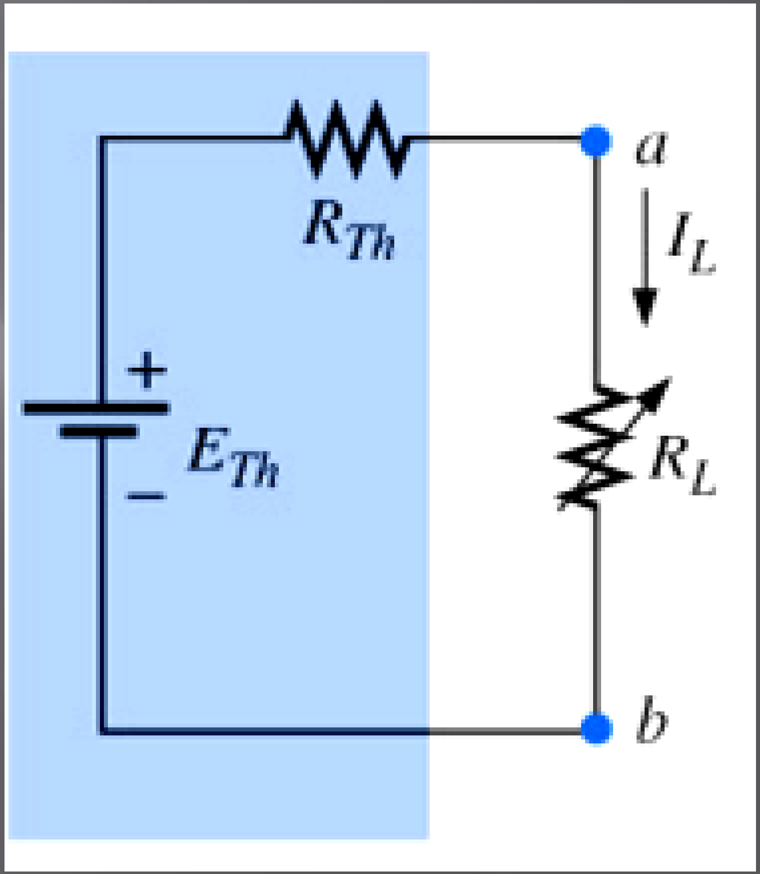
E_{Th}:

4. Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals. (This step is invariably the one that will lead to the most confusion and errors. In all cases, keep in mind that it is the open-circuit potential between the two terminals marked in step 2.)

Thévenin's Theorem

- Conclusion:

5. Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit. This step is indicated by the placement of the resistor R_L between the terminals of the Thévenin equivalent circuit.



Thévenin's Theorem

Experimental Procedures

- Two popular experimental procedures for determining the parameters of the Thévenin equivalent network:
  - Direct Measurement of E_{Th} and R_{Th}
    - For any physical network, the value of E_{Th} can be determined experimentally by measuring the open-circuit voltage across the load terminals.
    - The value of R_{Th} can then be determined by completing the network with a variable resistance R_L.

Thévenin's Theorem

- Measuring V_{OC} and I_{SC}
  - The Thévenin voltage is again determined by measuring the open-circuit voltage across the terminals of interest; that is, $E_{Th} = V_{OC}$. To determine R_{Th}, a short-circuit condition is established across the terminals of interest and the current through the short circuit (I_{sc}) is measured with an ammeter.
  - Using Ohm's law:

$$R_{Th} = V_{oc} / I_{sc}$$

Norton's Theorem

- Norton's theorem states the following:
  - Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current and a parallel resistor.
- The steps leading to the proper values of I_N and R_N.
- Preliminary steps:
  1. Remove that portion of the network across which the Norton equivalent circuit is found.
  2. Mark the terminals of the remaining two-terminal network.

Norton's Theorem

- Finding R_N:

3. Calculate R_N by first setting all sources to zero (voltage sources are replaced with short circuits, and current sources with open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.) Since $R_N = R_{Th}$ the procedure and value obtained using the approach described for Thévenin's theorem will determine the proper value of R_N.

Norton's Theorem

- Finding I_N :
  4. Calculate I_N by first returning all the sources to their original position and then finding the short-circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.
- Conclusion:
  5. Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

Maximum Power Transfer Theorem

- The maximum power transfer theorem states the following:

  A load will receive maximum power from a network when its total resistive value is exactly equal to the Thévenin resistance of the network applied to the load. That is,

$$R_L = R_{Th}$$

Maximum Power Transfer Theorem

- For loads connected directly to a dc voltage supply, maximum power will be delivered to the load when the load resistance is equal to the internal resistance of the source; that is, when:

$$R_L = R_{int}$$

Millman's Theorem

- Any number of parallel voltage sources can be reduced to one.
  - This permits finding the current through or voltage across R_L without having to apply a method such as mesh analysis, nodal analysis, superposition and so on.
    1. Convert all voltage sources to current sources.
    2. Combine parallel current sources.
    3. Convert the resulting current source to a voltage source and the desired single-source network is obtained.

Substitution Theorem

- The substitution theorem states:
  - If the voltage across and the current through any branch of a dc bilateral network is known, this branch can be replaced by any combination of elements that will maintain the same voltage across and current through the chosen branch.
  - Simply, for a branch equivalence, the terminal voltage and current must be the same.

Reciprocity Theorem

- The reciprocity theorem is applicable only to single-source networks and states the following:
  - The current *I* in any branch of a network, due to a single voltage source *E* anywhere in the network, will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current *I* was originally measured.
    - The location of the voltage source and the resulting current may be interchanged without a change in current

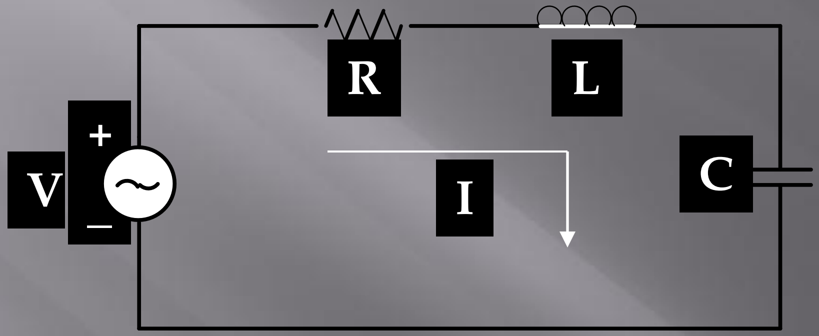
Resonance In Electric Circuits

- Any passive electric circuit will resonate if it has an inductor and capacitor.
- Resonance is characterized by the input voltage and current being in phase. The driving point impedance (or admittance) is completely real when this condition exists.
- In this presentation we will consider (a) series resonance, and (b) parallel resonance.

Series Resonance

Consider the series RLC circuit shown below.

$V = V_M \angle 0$



The input impedance is given by:

$$Z = R + j(wL - \frac{1}{wC})$$

The magnitude of the circuit current is;

$$I = |\bar{I}| = \frac{V_m}{\sqrt{R^2 + (wL - \frac{1}{wC})^2}}$$

Series Resonance

<u>**Resonance occurs when,**</u>

$$wL = \frac{1}{wC}$$

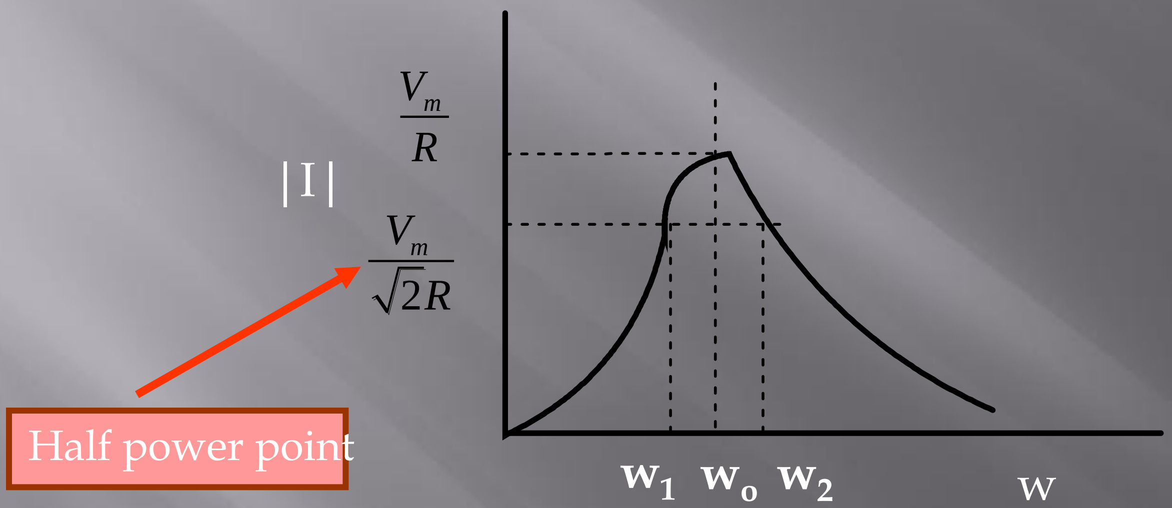
At resonance we designate w as w_o and write;

$$w_o = \frac{1}{\sqrt{LC}}$$

This is *<u>an important equation</u>* to remember. It applies to both series And parallel resonant circuits.

Series Resonance

The magnitude of the current response for the series resonance circuit is as shown below.



Bandwidth:

$$BW = w_{BW} = w_2 - w_1$$

Series Resonance

The peak power delivered to the circuit is;

$$P = \frac{V_m^{\,2}}{R}$$

The so-called half-power is given when $I = \frac{V_m}{\sqrt{2}R}$.

We find the frequencies, w_1 and w_2, at which this half-power occurs by using;

$$\sqrt{2}R = \sqrt{R^2 + (wL - \frac{1}{wC})^2}$$

Series Resonance

After some insightful algebra one will find two frequencies at which the previous equation is satisfied, they are:

$$w_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

an
d

$$w_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

The two half-power frequencies are related to the resonant frequency by

$$w_o = \sqrt{w_1 w_2}$$

Series Resonance

The bandwidth of the series resonant circuit is given by;

$$BW = w_b = w_2 - w_1 = \frac{R}{L}$$

We define the Q (quality factor) of the circuit as;

$$Q = \frac{w_o L}{R} = \frac{1}{w_o RC} = \frac{1}{R}\sqrt{\left(\frac{L}{C}\right)}$$

Using Q, we can write the bandwidth as;

$$BW = \frac{w_o}{Q}$$

These are all important relationships.

Series Resonance

An Observation:

If Q > 10, one can safely use the approximation;

$$w_1 = w_o - \frac{BW}{2} \qquad and \qquad w_2 = w_o + \frac{BW}{2}$$

These are useful approximations.

Series Resonance

An Observation:

By using $Q = w_o L/R$ in the equations for w_1 and w_2 we have;

$$w_1 = w_o\left[\frac{-1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]$$

and

$$w_2 = w_o\left[\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]$$

Series Resonance

In order to get some feel for how the numerical value of Q influences the resonant and also get a better appreciation of the s-plane, we consider the following example.

It is easy to show the following for the series RLC circuit.

$$\frac{I(s)}{V(s)} = \frac{1}{Z(s)} = \frac{\frac{1}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

In the following example, three cases for the about transfer function will be considered. We will keep w_o the same for all three cases. The numerator gain,k, will (a) first be set k to 2 for the three cases, then (b) the value of k will be set so that each response is 1 at resonance.

Series Resonance

An Example Illustrating Resonance:

The 3 transfer functions considered are:

Case 1:

$$\frac{ks}{s^2 + 2s + 400}$$

Case 2:

$$\frac{ks}{s^2 + 5s + 400}$$

Case 3:

$$\frac{ks}{s^2 + 10s + 400}$$

Series Resonance

An Example Illustrating Resonance:

The poles for the three cases are given below.

Case 1:

$$s^2 + 2s + 400 = (s + 1 + j19.97)(s + 1 - j19.97)$$

Case 2:

$$s^2 + 5s + 400 = (s + 2.5 + j19.84)(s + 2.5 - j19.84)$$

Case 3:

$$s^2 + 10s + 400 = (s + 5 + j19.36)(s + 5 - j19.36)$$

Series Resonance

Comments:

Observe the denominator of the CE equation.

$$s^2 + \frac{R}{L}s + \frac{1}{LC}$$

Compare to actual characteristic equation for *Case 1*:

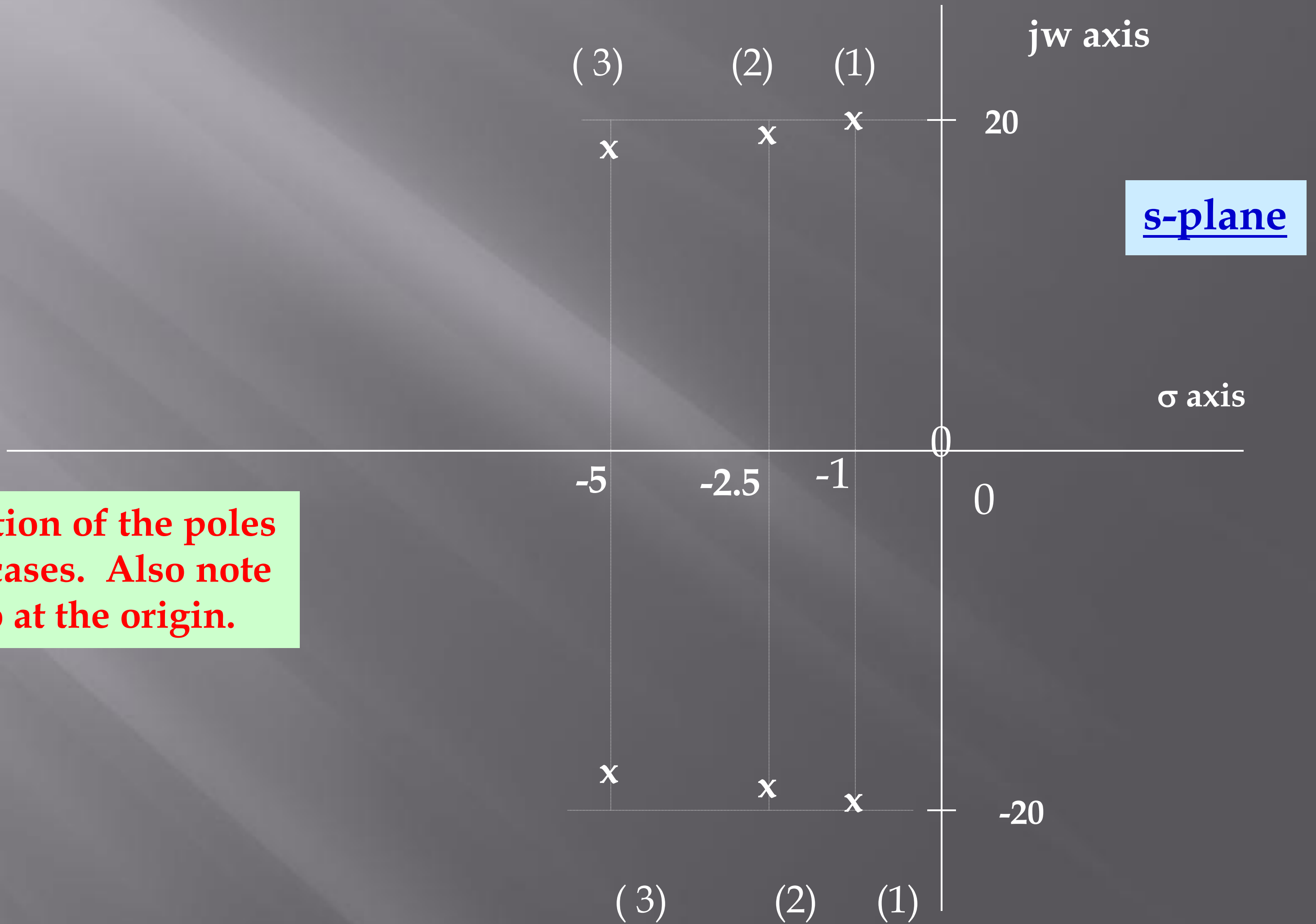
$$s^2 + 2s + 400$$

$$w_o^{\ 2} = 400 \rightarrow w = 20$$ **rad/sec**

$$BW = \frac{R}{L} = 2$$ **rad/sec** $$\rightarrow Q = \frac{w_o}{BW} = 10$$

Series Resonance

Poles and Zeros In the s-plane:



Note the location of the poles for the three cases. Also note there is a zero at the origin.

Series Resonance

Comments:

The frequency response starts at the origin in the s-plane. At the origin the transfer function is zero because there is a zero at the origin.

As you get closer and closer to the complex pole, which has a j parts in the neighborhood of 20, the response starts to increase.

The response continues to increase until we reach w = 20. From there on the response decreases.

We should be able to reason through why the response has the above characteristics, using a graphical approach.

Series Resonance

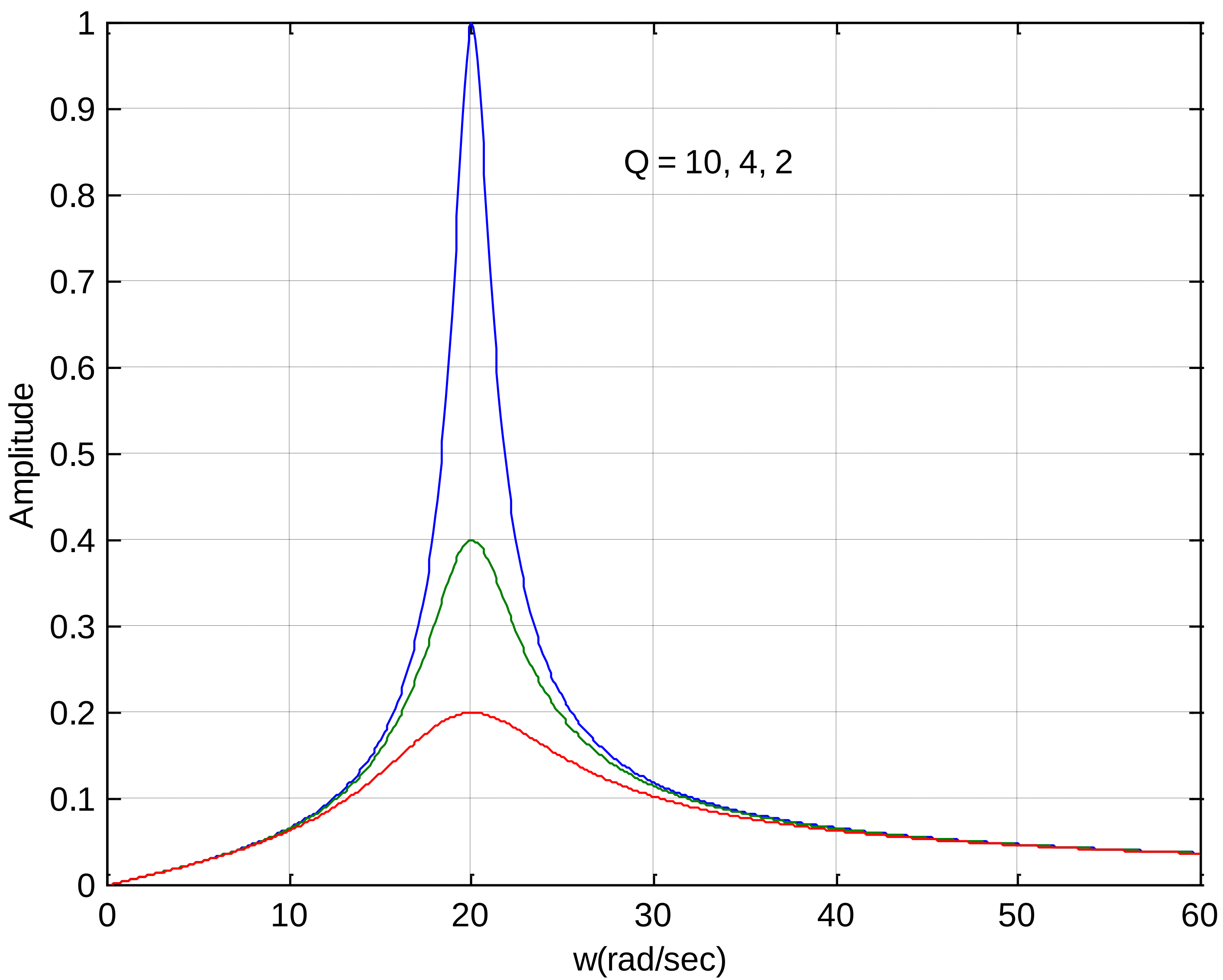
Matlab Program For The Study:

```
% name of program is freqtest.m
% written for 202 S2002, wlg
%CASE ONE DATA:
K = 2;
num1 = [K 0];
den1 = [1 2  400];

num2 = [K 0];
den2 = [1 5 400];

num3 = [K 0];
den3 = [1 10 400];

w = .1:.1:60;
```

```
grid
H1 = bode(num1,den1,w);
magH1=abs(H1);

H2 = bode(num2,den2,w);
magH2=abs(H2);

H3 = bode(num3,den3,w);
magH3=abs(H3);

plot(w,magH1, w, magH2, w,magH3)
grid
xlabel('w(rad/sec)')
ylabel('Amplitude')
gtext('Q = 10, 4, 2')
```

Series Resonance

Program Output



Series Resonance

Comments: cont.

From earlier work:

$$w_1, w_2 = w_o \left[\frac{\pm 1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right]$$

With Q = 10, this gives;

w_1= 19.51 rad/sec, w_2 = 20.51 rad/sec

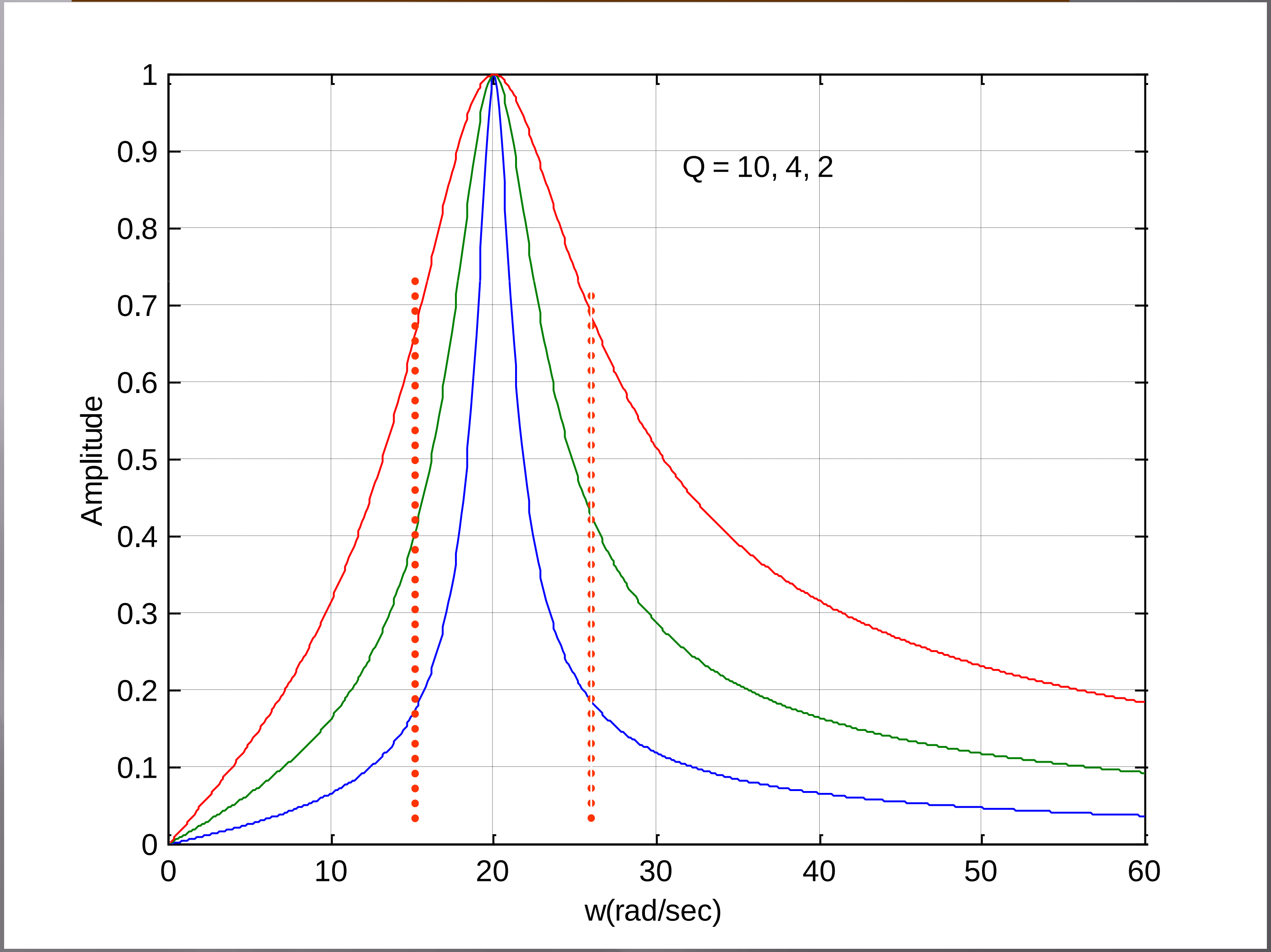
Compare this to the approximation:

$w_1 = w_0$ – BW = 20 – 1 = 19 rad/sec, w_2 = 21 rad/sec

So basically we can find all the series resonant parameters if we are given the numerical form of the CE of the transfer function.

Series Resonance

Next Case: Normalize all responses to 1 at w_o



Series Resonance

Three dB Calculations:

Now we use the analytical expressions to calculate w_1 and w_2. We will then compare these values to what we find from the Matlab simulation.

Using the following equations with Q = 2,

$$w_1, w_2 = w_o \left[\frac{\mp 1}{2Q} + w_o \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right]$$

we find,

w_1 = 15.62 rad/sec

w_2 = 21.62 rad/sec

Series Resonance

Checking w_1 and w_2

(cut-outs from the simulation)

	15.3000	0.6779
	15.4000	0.6871
	15.5000	0.6964
w_1 →	15.6000	0.7057
	15.7000	0.7150
	15.8000	0.7244

	25.3000	0.7254
	25.4000	0.7195
	25.5000	0.7137
w_2 →	25.6000	0.7080
	25.7000	0.7023
	25.8000	0.6967
	25.9000	0.6912

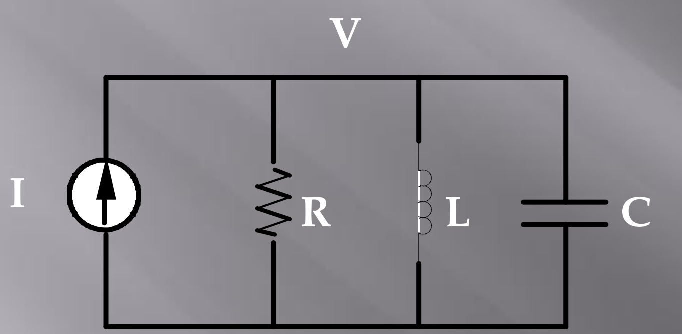
This verifies the previous calculations.

Now we shall look at Parallel Resonance.

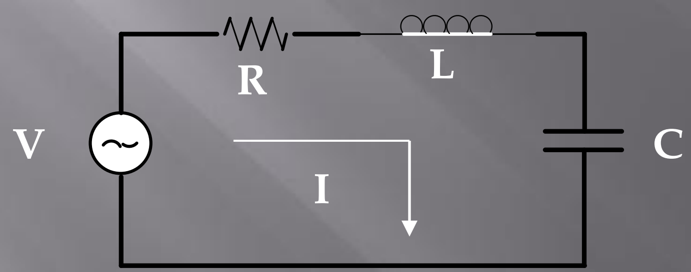
Parallel Resonance

Background

Consider the circuits shown below:



$$I = V\left[\frac{1}{R} + jwC + \frac{1}{jwL}\right]$$



$$V = I\left[R + jwL + \frac{1}{jwC}\right]$$

Series Resonance

Duality

$$I = V\left[\frac{1}{R} + jwC + \frac{1}{jwL}\right] \qquad V = I\left[R + jwL + \frac{1}{jwC}\right]$$

We notice the above equations are the same provided:

$$I \longleftrightarrow V$$

$$R \longleftrightarrow \frac{1}{R}$$

$$L \longleftrightarrow C$$

If we make the inner-change, then one equation becomes the same as the other.

For such case, we say the one circuit is the dual of the other.

Parallel Resonance

Background

What this means is that for all the equations we have derived for the parallel resonant circuit, we can use for the series resonant circuit provided we make the substitutions:

$$R \quad \text{replaced be} \quad \frac{1}{R}$$

$$L \quad \text{replaced by} \quad C$$

$$C \quad \text{replaced by} \quad L$$

Parallel Resonance

Parallel Resonance

$$w_o = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{w_o L}{R}$$

$$BW = (w_2 - w_1) = w_{BW} = \frac{R}{L}$$

$$w_1, w_2 = \left[\frac{\mp R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right]$$

$$w_1, w_2 = w_o \left[\frac{\mp 1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right]$$

$w_1, w w_2$

Series Resonance

$$w_o = \frac{1}{\sqrt{LC}}$$

$$Q = w_o RC$$

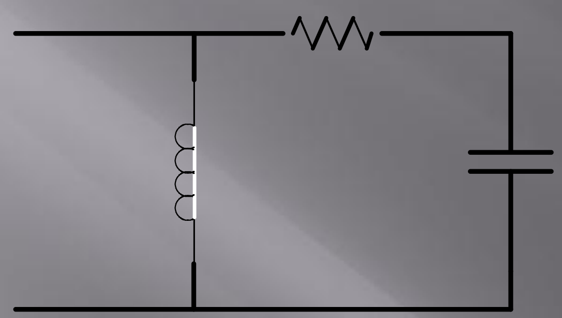
$$BW = w_{BW} = \frac{1}{RC}$$

$$w_1, w_2 = \left[\frac{\mp 1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \right]$$

$$w_1, w_2 = w_o \left[\frac{\mp 1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right]$$

Resonance

Example 1: **Determine the resonant frequency for the circuit below.**



$$Z_{IN} = \frac{jwL\left(R + \dfrac{1}{jwC}\right)}{R + jwL + \dfrac{1}{jwC}} = \frac{(-w^2 LRC + jwL)}{(1 - w^2 LC) + jwRC}$$

At resonance, the phase angle of Z must be equal to zero.

Resonance

Analysis

$$\frac{(-w^2LRC + jwL)}{(1 - w^2LC) + jwRC}$$

For zero phase;

$$\frac{wL}{(-w^2LCR)} = \frac{wRC}{(1 - w^2LC}$$

This gives;

$$w^2LC - w^2R^2C^2 = 1$$

or

$$w_o = \frac{1}{\sqrt{(LC - R^2C^2)}}$$

Parallel Resonance

Example 2:

A series RLC resonant circuit has a resonant frequency admittance of $2x10^{-2}$ S(mohs). The Q of the circuit is 50, and the resonant frequency is 10,000 rad/sec. Calculate the values of R, L, and C. Find the half-power frequencies and the bandwidth.

First, R = 1/G = 1/(0.02) = 50 ohms.

Second, from $Q = \frac{w_o L}{R}$, we solve for L, knowing Q, R, and w_o to find L = 0.25 H.

Third, we can use $C = \frac{Q}{w_o R} = \frac{50}{10{,}000 x 50} = 100 \ \mu F$

Parallel Resonance

Example 2: (continued)

Fourth: We can use
$$w_{BW} = \frac{w_o}{Q} = \frac{1x10^4}{50} = 200\,rad/\sec$$

and

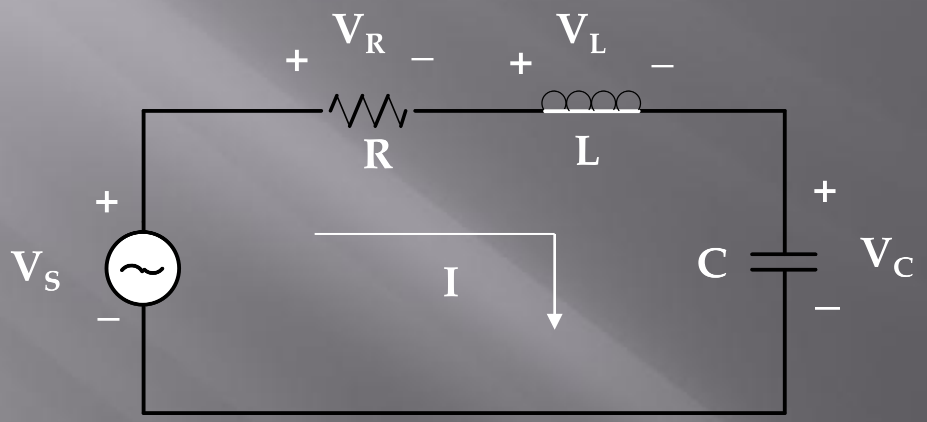
Fifth: Use the approximations;

$$w_1 = w_o - 0.5w_{BW} = 10{,}000 - 100 = 9{,}900 \text{ rad/sec}$$

$$w_2 = w_o - 0.5w_{BW} = 10{,}000 + 100 = 10{,}100 \text{ rad/sec}$$

Extension of Series Resonance

Peak Voltages and Resonance:



We know the following:

- When $w = w_o = \dfrac{1}{\sqrt{LC}}$, V_S and I are in phase, the driving point impedance is purely real and equal to R.
- A plot of $|I|$ shows that it is maximum at $w = w_o$. We know the standard equations for series resonance applies: Q, w_{BW}, etc.

Extension of Series Resonance

Reflection:

- A question that arises is what is the nature of V_R, V_L, and V_C? A little reflection shows that V_R is a peak value at w_o. But we are not sure about the other two voltages. We know that at resonance they are equal and they have a magnitude of QxV_S.

- Irwin shows that the frequency at which the voltage across the capacitor is a maximum is given by;

$$w_{\max} = w_o \sqrt{1 - \frac{1}{2Q^2}}$$

- The above being true, we might ask, what is the frequency at which the voltage across the inductor is a maximum?

We answer this question by simulation

Extension of Series Resonance

Series RLC Transfer Functions:

The following transfer functions apply to the series RLC circuit.

$$\frac{V_C(s)}{V_S(s)} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\frac{V_L(s)}{V_S(s)} = \frac{s^2}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\frac{V_R(s)}{V_S(s)} = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Extension of Series Resonance

Parameter Selection:

We select values of R, L. and C for this first case so that Q = 2 and w_o = 2000 rad/sec. Appropriate values are; R = 50 ohms, L = .05 H, C = 5µF. The transfer functions become as follows:

$$\frac{V_C}{V_S} = \frac{4x10^6}{s^2 + 1000s + 4x10^6}$$

$$\frac{V_L}{V_S} = \frac{s^2}{s^2 + 1000s + 4x10^6}$$

$$\frac{V_R}{V_S} = \frac{1000s}{s^2 + 1000s + 4x10^6}$$

Extension of Series Resonance

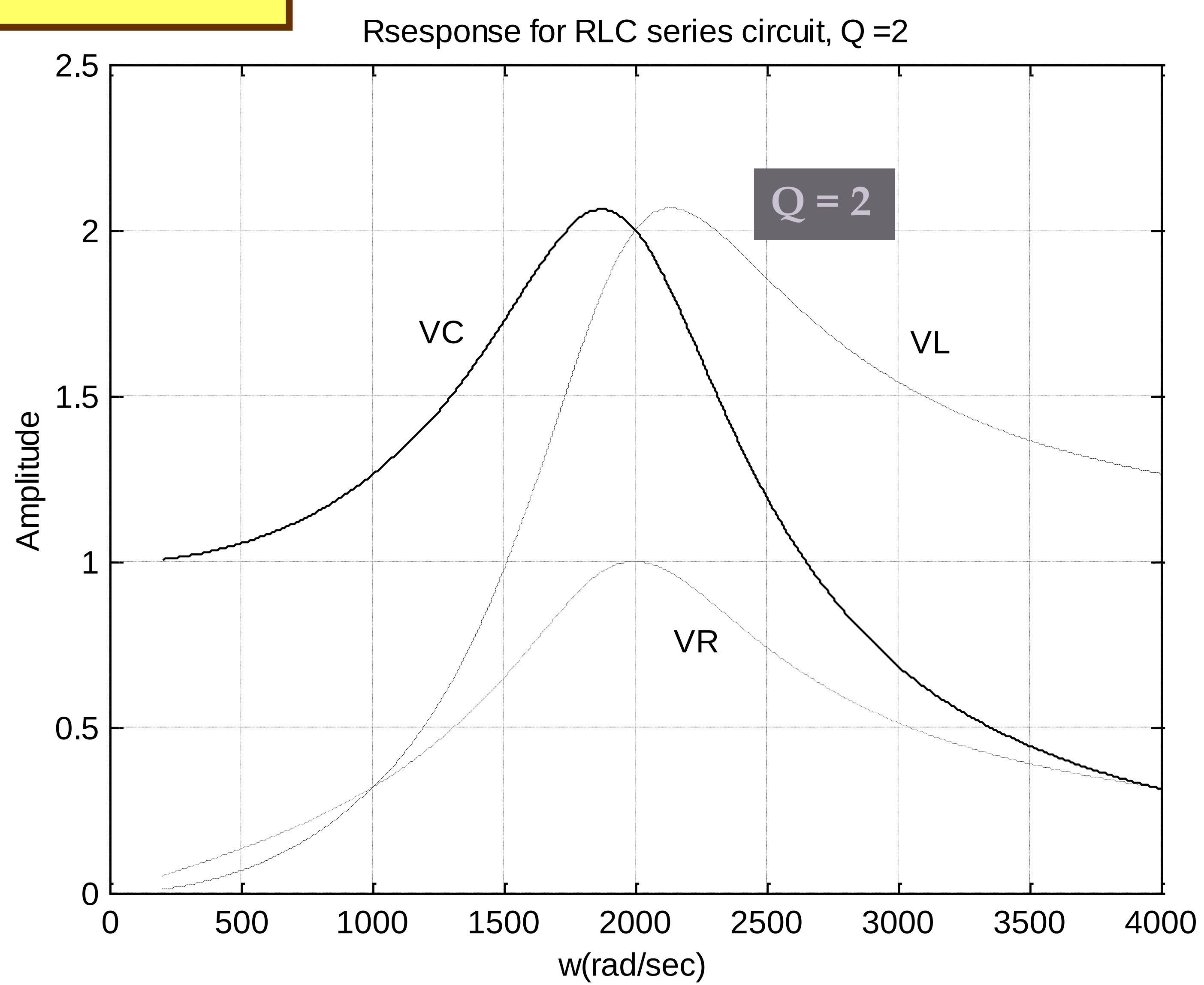
Matlab Simulation:

```
% program is freqcompare.m
% written for 202 S2002, wlg

numC = 4e+6;
denC = [1 1000 4e+6];

numL = [1 0 0];
denL = [1 1000 4e+6];

numR = [1000 0];
denR = [1 1000 4e+6];

w = 200:1:4000;

grid
HC = bode(numC,denC,w);
magHC = abs(HC);
```

```
grid
HC = bode(numC,denC,w);
magHC = abs(HC);

HL = bode(numL,denL,w);
magHL = abs(HL);

HR = bode(numR,denR,w);
magHR = abs(HR);

plot(w,magHC,'k-', w, magHL,'k--', w, magHR, 'k:')
grid

xlabel('w(rad/sec)')
ylabel('Amplitude')
title(' Rsesponse for RLC series circuit, Q =2')

gtext('VC')
gtext('VL')
gtext(' VR')
```

Exnsion of Series Resonance

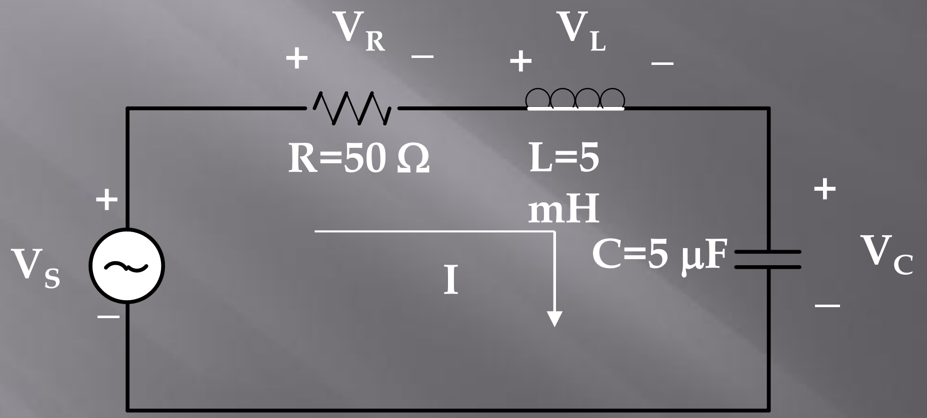
Simulation Results



Exnsion of Series Resonance

Analysis of the problem:

Given the previous circuit. Find Q, w_0, w_{max}, $|V_c|$ at w_o, and $|V_c|$ at w_{max}



Solution:

$$w_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50x10^{-2}x5x10^{-6}}} = 2000\, rad/\sec$$

$$Q = \frac{w_o L}{R} = \frac{2x10^{3}x5x10^{-2}}{50} = 2$$

Exnsion of Series Resonance

Problem Solution:

$$w_{MAX} = w_O \sqrt{1 - \frac{1}{2Q^2}} = 0.9354 w_o$$

$$|V_R| \, at \, w_O = Q|V_S| = 2x1 = 2 \, volts \, (peak)$$

$$|V_C| \, at \, w_{MAX} = \frac{Qx|V_S|}{\sqrt{1 - \frac{1}{4Q^2}}} = \frac{2}{0.968} = 2.066 \, volts \, (peak))$$

Now check the computer printout.

Exnsion of Series Resonance

Problem Solution (Simulation):

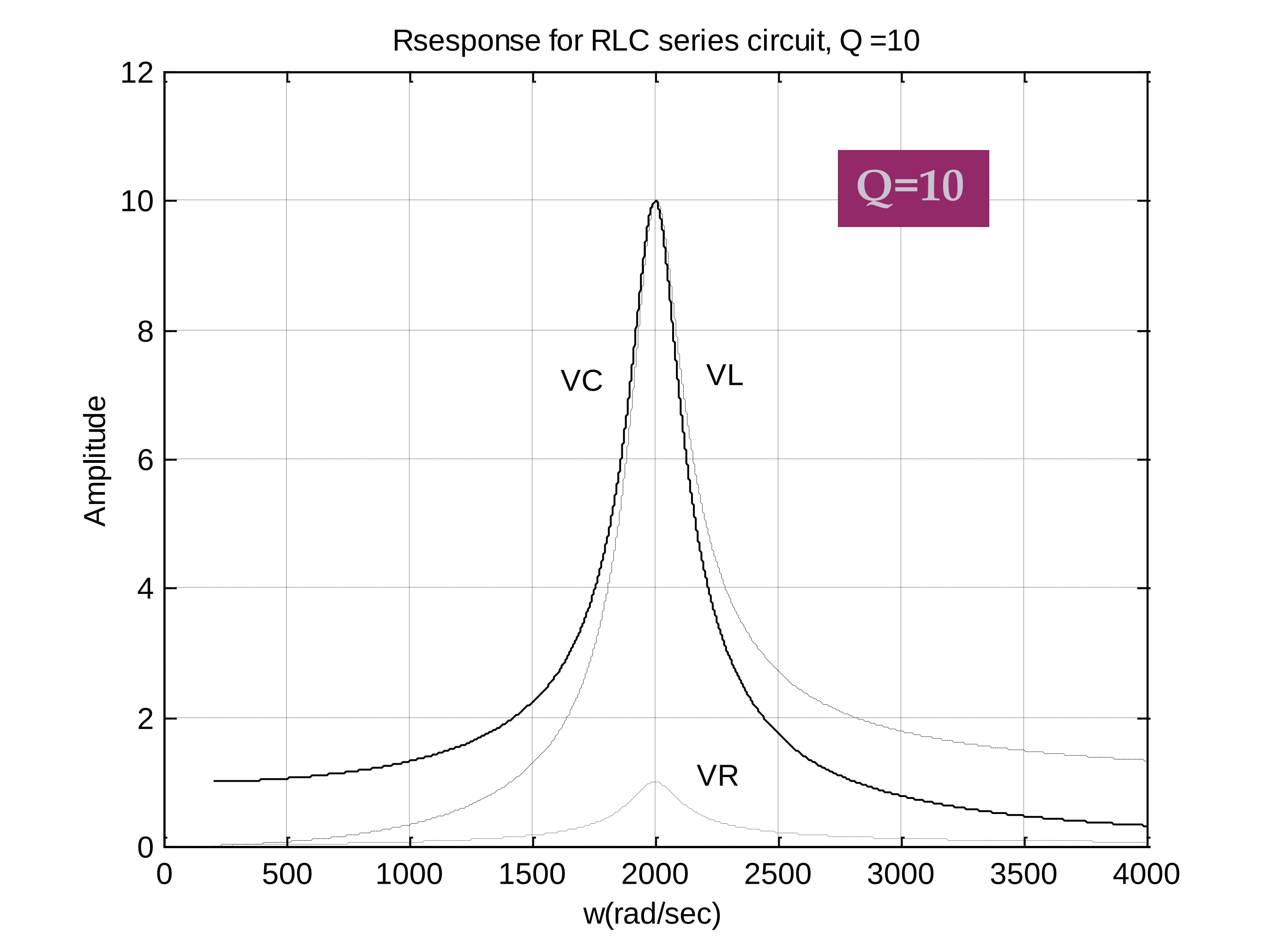
1.0e+003 *

1.8600000	0.002065141
1.8620000	0.002065292
1.8640000	0.002065411
1.8660000	0.002065501
1.8680000	0.002065560
1.8700000	0.002065588
1.8720000	0.002065585
1.8740000	0.002065552
1.8760000	0.002065487
1.8780000	0.002065392
1.8800000	0.002065265
1.8820000	0.002065107
1.8840000	0.002064917

Maximum

Extension of Series Resonance

Simulation Results:



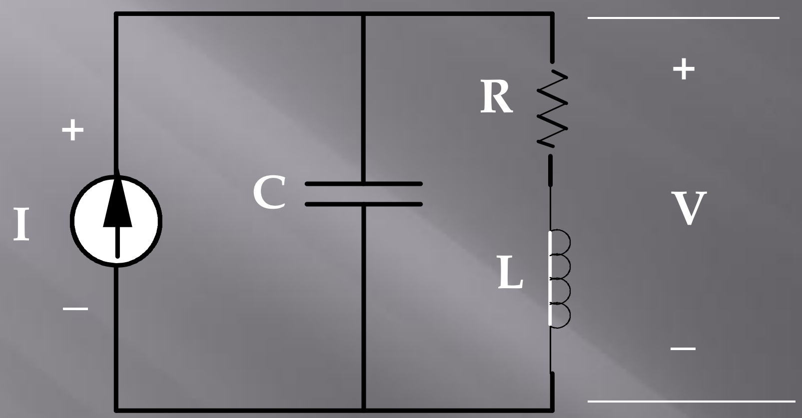
Exnsion of Series Resonance

Observations From The Study:

- The voltage across the capacitor and inductor for a series RLC circuit is not at peak values at resonance for small Q (Q <3).
- Even for Q<3, the voltages across the capacitor and inductor are equal at resonance and their values will be QxV_S.
- For Q>10, the voltages across the capacitors are for all practical purposes at their peak values and will be QxV_S.
- Regardless of the value of Q, the voltage across the resistor reaches its peak value at $w = w_o$.
- For high Q, the equations discussed for series RLC resonance can be applied to any voltage in the RLC circuit. For Q<3, this is not true.

Extension of Resonant Circuits

Given the following circuit:



- We want to find the frequency, w_r, at which the transfer function for V/I will resonate.
- The transfer function will exhibit resonance when the phase angle between V and I are zero.

Extension of Resonant Circuits

The desired transfer functions is;

$$\frac{V}{I} = \frac{(1/sC)(R+sL)}{R+sL+1/sC}$$

This equation can be simplified to;

$$\frac{V}{I} = \frac{R+sL}{LCs^2+RCs+1}$$

With s → jw

$$\frac{V}{I} = \frac{R+jwL}{(1-w^2LC)+jwR}$$

Extension of Resonant Circuits

Resonant Condition:

For the previous transfer function to be at a resonant point, the phase angle of the numerator must be equal to the phase angle of the denominator.

$$\angle\theta_{num} = \angle\theta_{dem}$$

or,

$$\theta_{num} = \tan^{-1}\left(\frac{wL}{R}\right), \qquad \theta_{den} = \tan^{-1}\left(\frac{wRC}{(1-w^2LC)}\right).$$

Therefore;

$$\frac{wL}{R} = \frac{wRC}{(1-w^2LC)}$$

Extension of Resonant Circuits

Resonant Condition Analysis:

Canceling the w's in the numerator and cross multiplying gives,

$$L(1 - w^2 LC) = R^2 C \quad or \quad w^2 L^2 C = L - R^2 C$$

This gives,

$$w_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Notice that if the ratio of R/L is small compared to 1/LC, we have

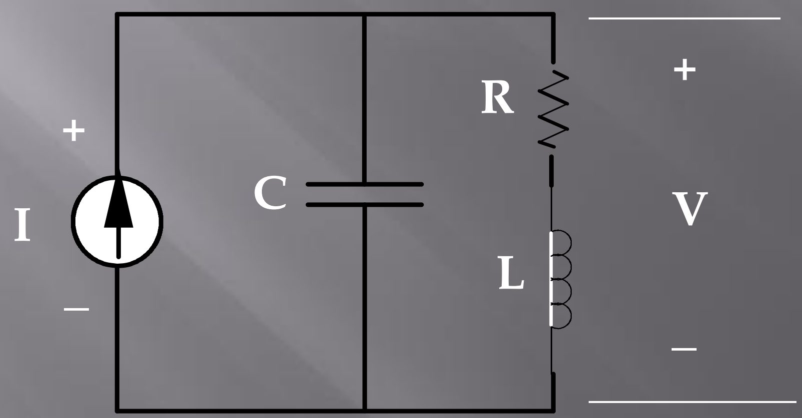
$$w_r = w_o = \frac{1}{\sqrt{LC}}$$

Extension of Resonant Circuits

Resonant Condition Analysis:

What is the significance of w_r and w_o in the previous two equations? Clearly w_r is a lower frequency of the two. To answer this question, consider the following example.

Given the following circuit with the indicated parameters. Write a Matlab program that will determine the frequency response of the transfer function of the voltage to the current as indicated.



Extension of Resonant Circuits

Resonant Condition Analysis: Matlab Simulation:

We consider two cases:

Case 1:

R = 3 ohms
C = 6.25×10^{-5} F
L = 0.01 H

$w_r = 2646$ rad/sec

Case 2:

R = 1 ohms
C = 6.25×10^{-5} F
L = 0.01 H

$w_r = 3873$ rad/sec

For both cases,

$w_o = 4000$ rad/sec

Extension of Resonant Circuits

Resonant Condition Analysis: Matlab Simulation:

The transfer functions to be simulated are given below.

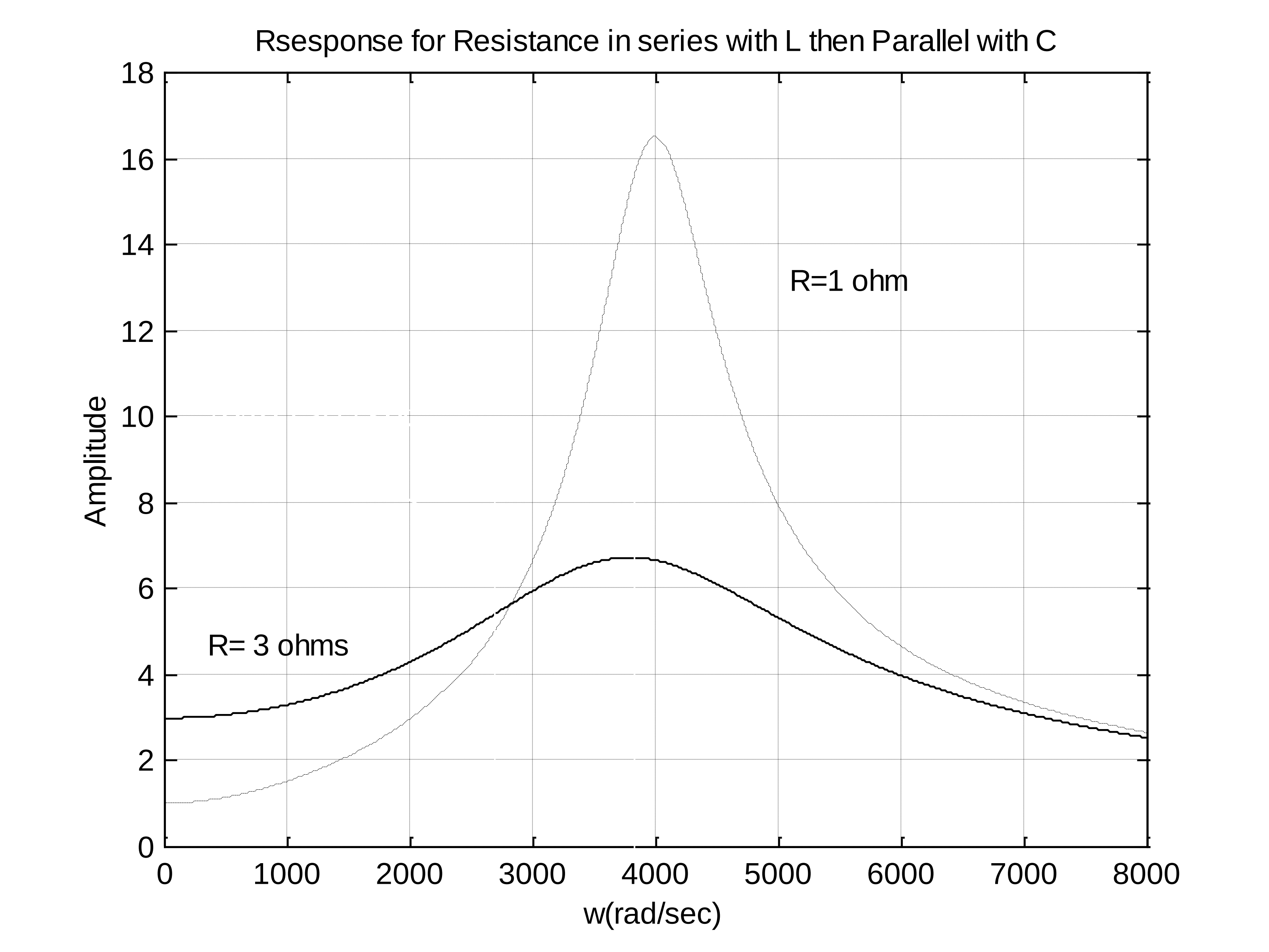
Case 1:

$$\frac{V}{I} = \frac{0.001s + 3}{6.25x10^{-8}s^2 + 1.875x10^{-7} + 1}$$

Case 2:

$$\frac{V}{I} = \frac{0.001s + 1}{6.25x10^{-8}s^2 + 6.25x10^{-5} + 1}$$

Extension of Resonant Circuits



Extension of Resonant Circuits

What can be learned from this example?

- w_r does not seem to have much meaning in this problem. What is w_r if R = 3.99 ohms?

- Just because a circuit is operated at the resonant frequency does not mean it will have a peak in the response at the frequency.

- For circuits that are fairly complicated and can resonant, It is probably easier to use a simulation program similar to Matlab to find out what is going on in the circuit.

Basic Laws of Circuits

End of Lesson

Resonant Circuits