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PPT ON ELECTROMAGNETIC FIELDS

II B.Tech I semester (JNTUH-R13)

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Introduction

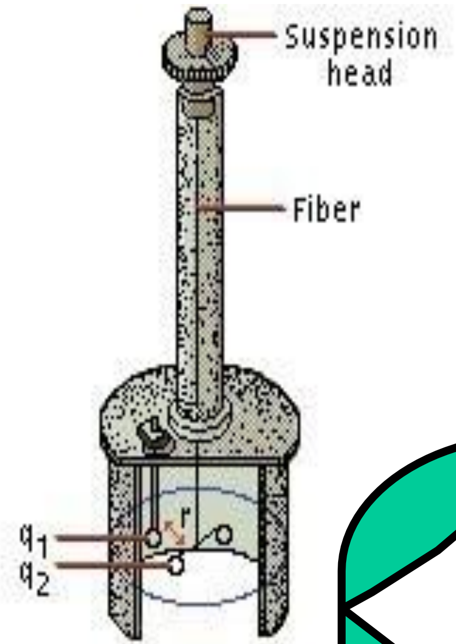
Electrostatics can be defined as the study of electric charges at rest. Electric fields have their sources in electric charges.

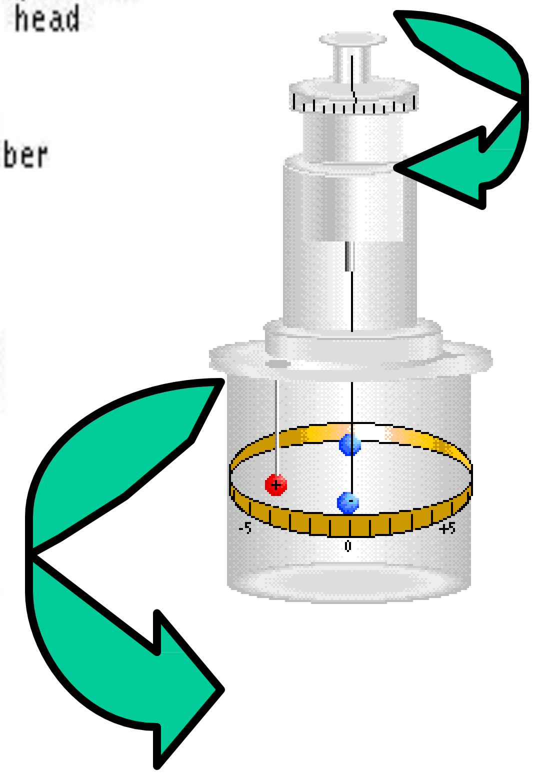
(Note: Almost all real electric fields vary to some extent with time. However, for many problems, the field variation is slow and the field may be considered as static. For some other cases spatial distribution is nearly same as for the static case even though the actual field may vary with time. Such cases are termed as quasi-static.)

In this chapter we first study two fundamental laws governing the electrostatic fields, viz, (1) Coulomb's Law and (2) Gauss's Law. Both these law have experimental basis. Coulomb's law is applicable in finding electric field due to any charge distribution, Gauss's law is easier to use when the distribution is symmetrical.

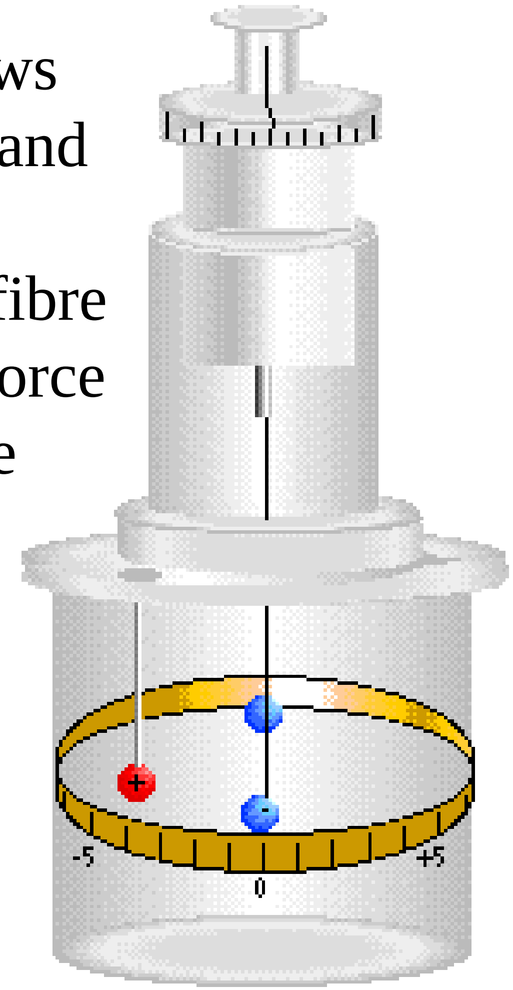
Unit I
Coulomb's Law

Coulomb's Torsion Balance



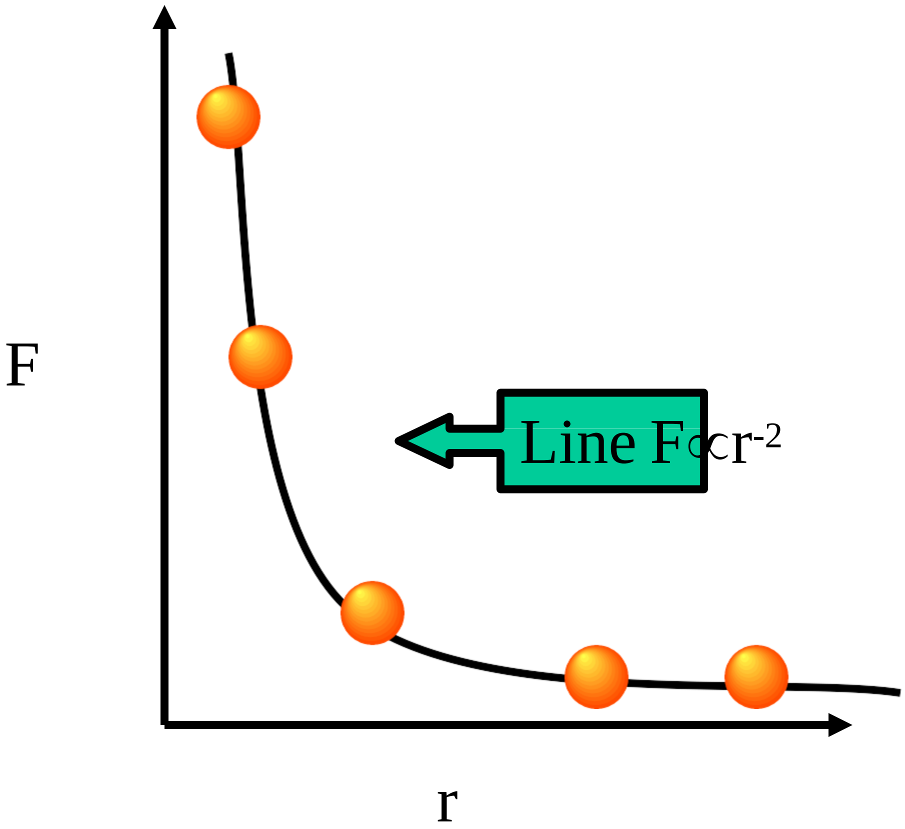


This dial allows you to adjust and measure the torque in the fibre and thus the force restraining the charge



This scale allows you to read the separation of the charges

Coulomb's Experiments



Coulomb's Law

- Coulomb determined
  - Force is attractive if charges are opposite sign
  - Force proportional to the product of the charges q_1 and q_2 along the lines joining them
  - Force inversly proportional square of the distance
- I.e.
  - $|F_{12}| \propto |Q_1|\ |Q_2| / r_{12}^2$
  - or
  - $|F_{12}| = k\ |Q_1|\ |Q_2| / r_{12\,2}$

Coulomb's Law

- Units of constant can be determined from Coulomb's Law
- Colomb (and others since) have determined this constant which (in a vacuum) in SI units is
  - k = 8.987.5x10^{9} Nm^{2}C^{-2}
- k is normally expressed as $k = 1/4\pi\varepsilon_0$
  - where is the permittivity of free space

Coulomb's Law

The equation for the magnitude of the Coulomb force between two point charges Q_1 and Q_2 in a vacuum is given by

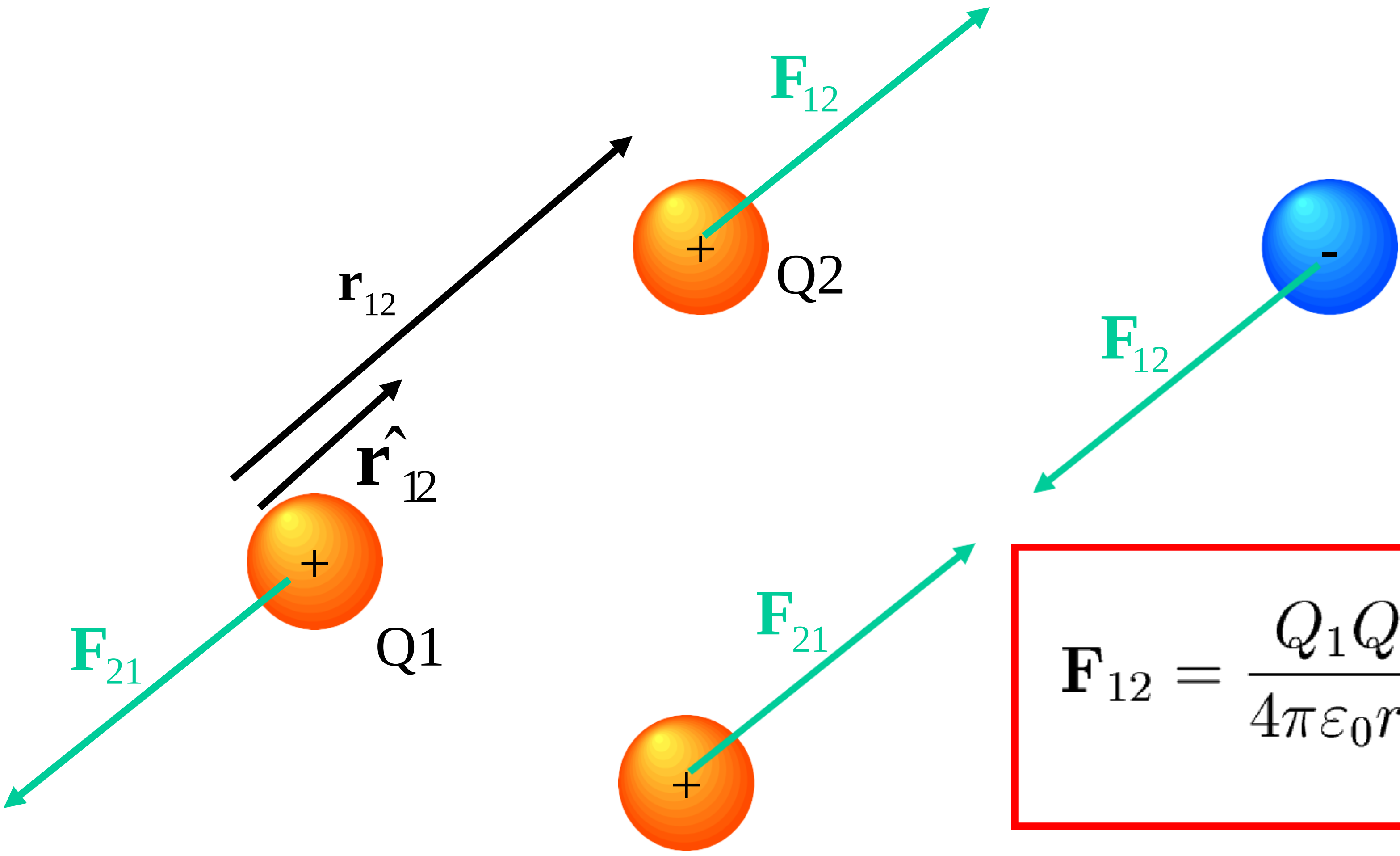
$$|\mathbf{F}_{12}| = \frac{|Q_1 Q_2|}{4\pi\varepsilon_0 r_{12}^2}$$

where

$\lvert Q_1 \rvert$	is	the magnitude of the charge Q_1 in coulombs (C)
$\lvert Q_2 \rvert$	is	the magnitude of the charge Q_2 in coulombs (C)
$\mathbf{F}_{12}$	is	the electrical force acting on the charge Q_1 due to charge Q_2 in newtons (N)
r_{12}	is	the distance between the point charges Q_1 and Q_2 in metres (m)
ε_0	is	the permittivity of free space in $C^2\ N^{-1}\ m^{-2}$
$\frac{1}{4\pi\varepsilon_0}$	is	the Coulomb constant in $N\ m^2\ C^{-2}$.

The direction of the force $\mathbf{F}_{12}$ is determined by the sign of the charges; the force is attractive if the charges have opposite signs, and repulsive if the charges have the same sign.

Vector form of Coulomb's Law

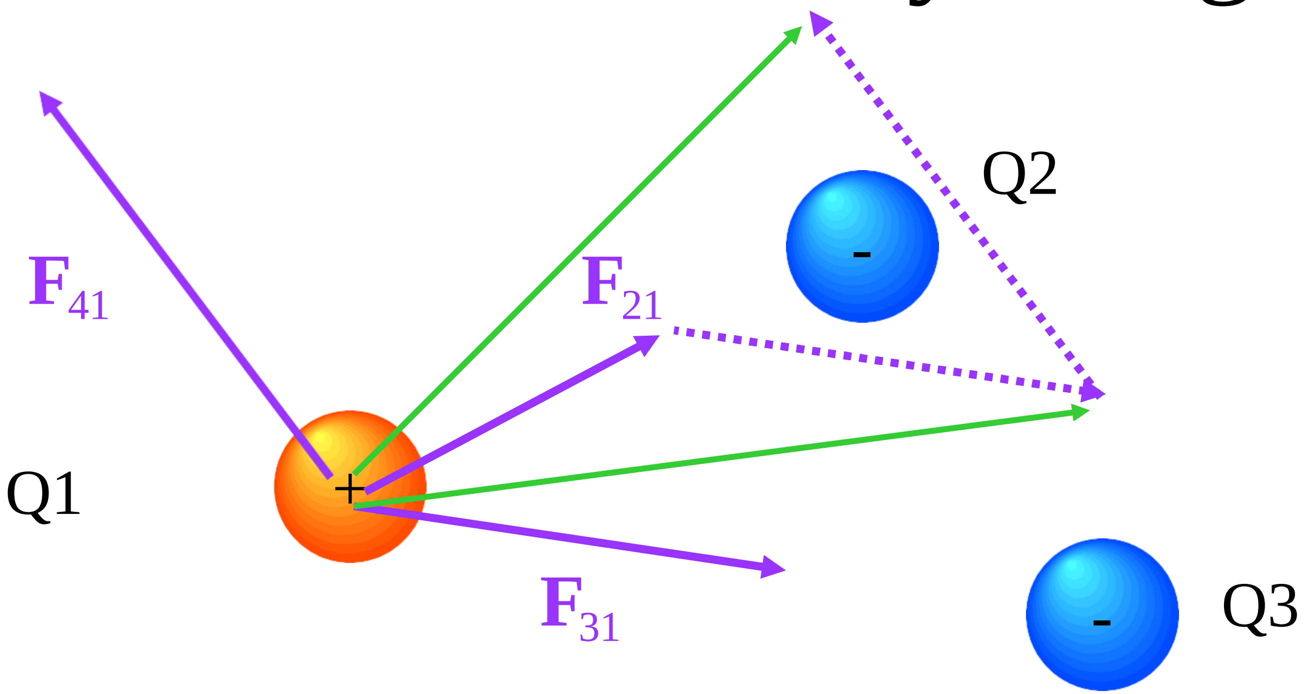


$$\mathbf{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r_{12}^2}\hat{\mathbf{r}}_{12},$$

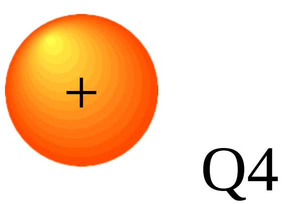
Force from many charges

Superposition

Force from many charges



Principle of superposition



Force on charge is vector sum of forces from all charges

$$\mathbf{F}_1 = \mathbf{F}_{21} + \mathbf{F}_{31} + \mathbf{F}_{41}$$

Coulomb's Law vs Newton's Law of Gravity

$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{|\mathbf{r}_{12}|^2}\hat{\mathbf{r}}_{12}$$

- Attractive or repulsive
- *1/r²*
- very strong
- only relevant relatively local scales

$$\mathbf{F}_{12} = -G\frac{m_1 m_2}{|\mathbf{r}_{12}|^2}\hat{\mathbf{r}}_{12}$$

- Always attractive
- *1/r²*
- very weak $\frac{e^2}{4\pi\varepsilon_0} >> -Gm^2$
- important on very large scales, planets, the Universe

Two spheres

The Electric Field

Van de Graaf Generator and thread

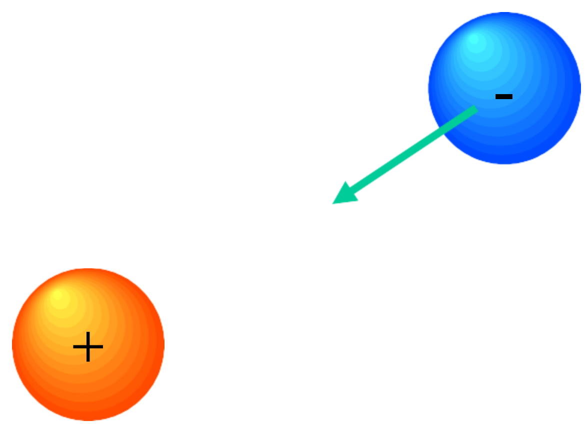
Van de Graaf Generator and many threads

Electric Field

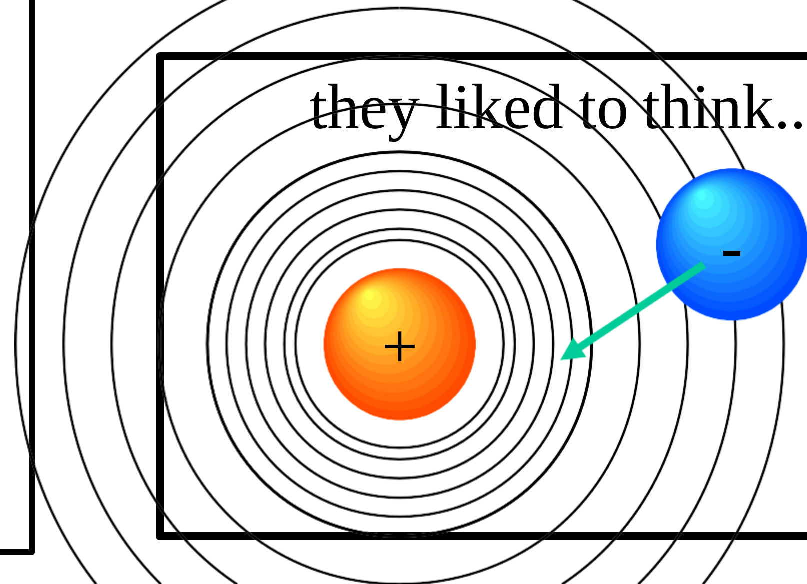
Physicists did not like the concept of "action at a distance" i.e. a force that was "caused" by an object a long distance away

They preferred to think of an object producing a "field" and other objects interacting with that field

Thus rather than ...



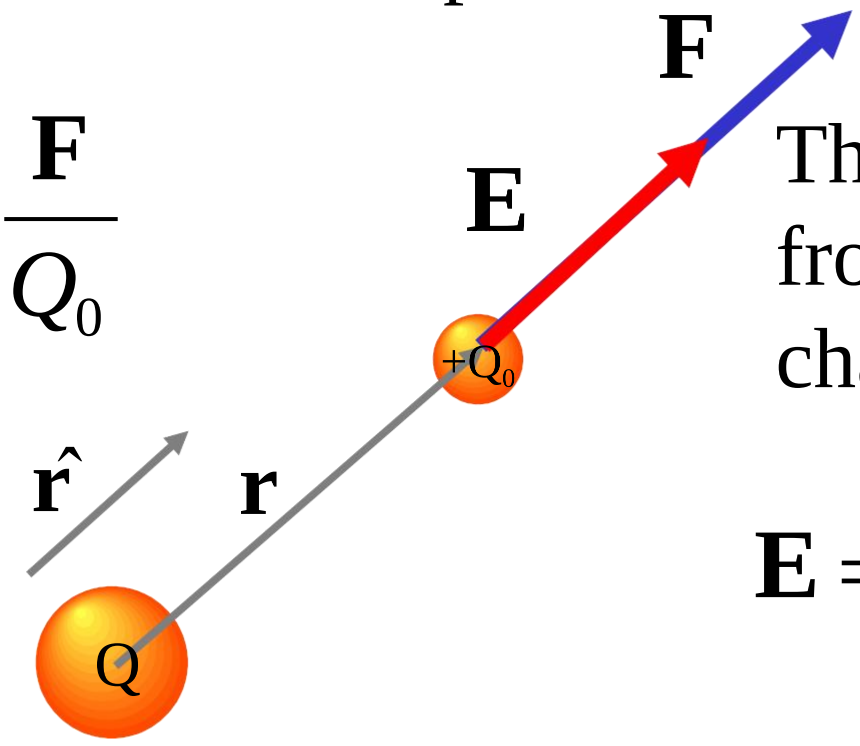
they liked to think...



Electric Field

Electric Field E is defined as the force acting on a test particle divided by the charge of that test particle

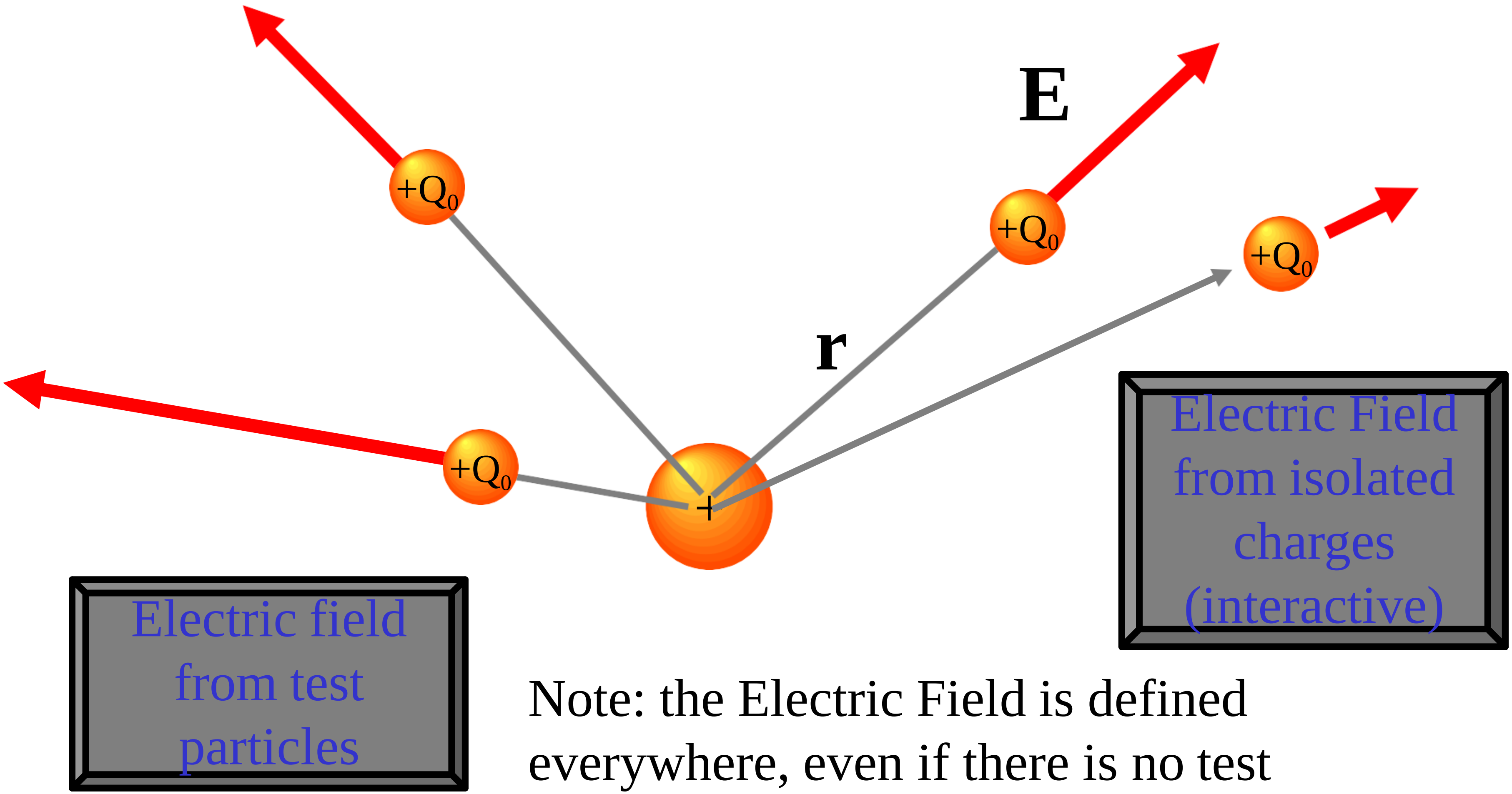
$$\mathbf{E} = \frac{\mathbf{F}}{Q_0}$$



Thus Electric Field from a single charge is

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\mathbf{r}|^2}\hat{\mathbf{r}}$$

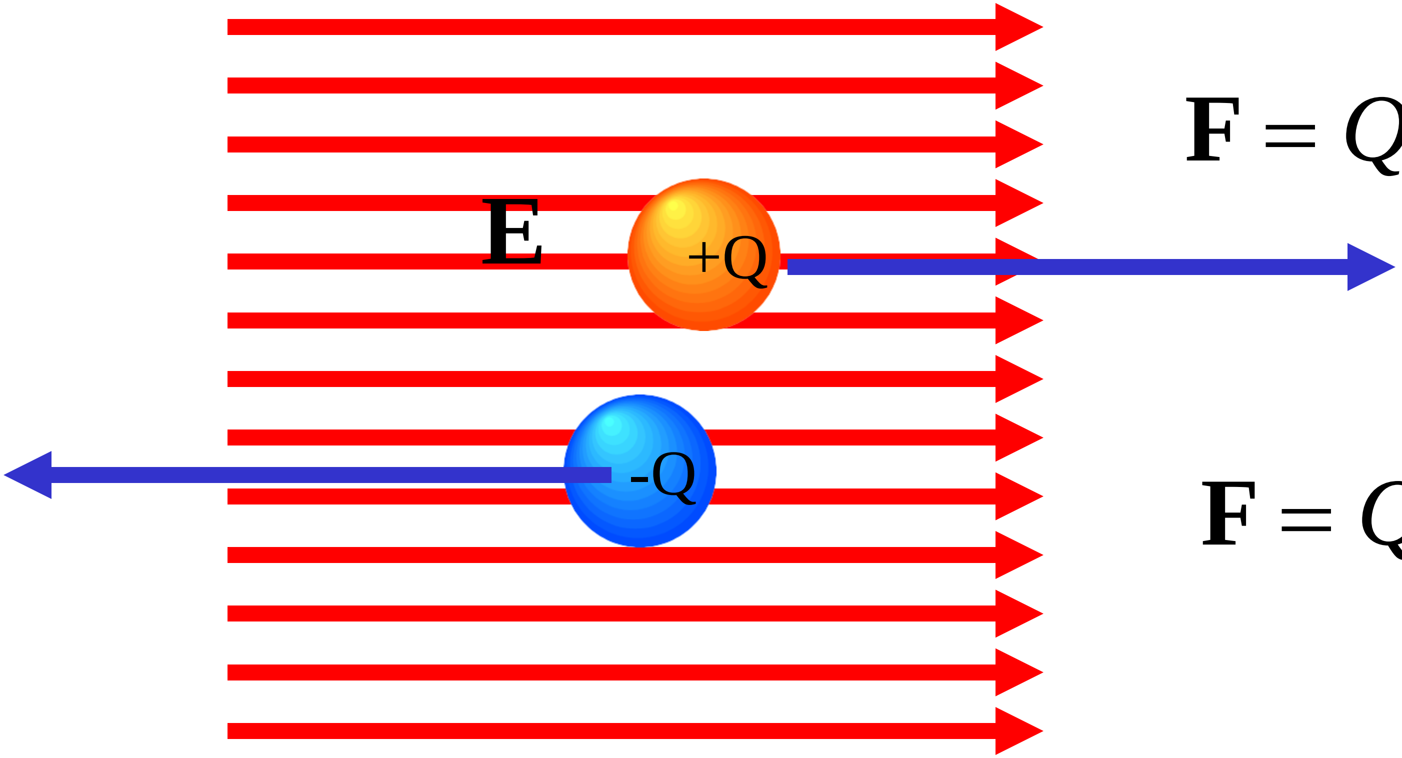
Electric Field of a single charge



Note: the Electric Field is defined everywhere, even if there is no test charge is not there.

Charged particles in electric field

Using the Field to determine the force



$$\mathbf{F} = Q\mathbf{E}$$

$$\mathbf{F} = Q\mathbf{E}$$

Vector & Scalar Fields

The Electric Field

Electric Field as a vector field

The Electric Field is one example of a Vector Field

A "field" (vector or scalar) is defined everywhere

A vector field has direction as well as size

The Electric Field has units of N/C

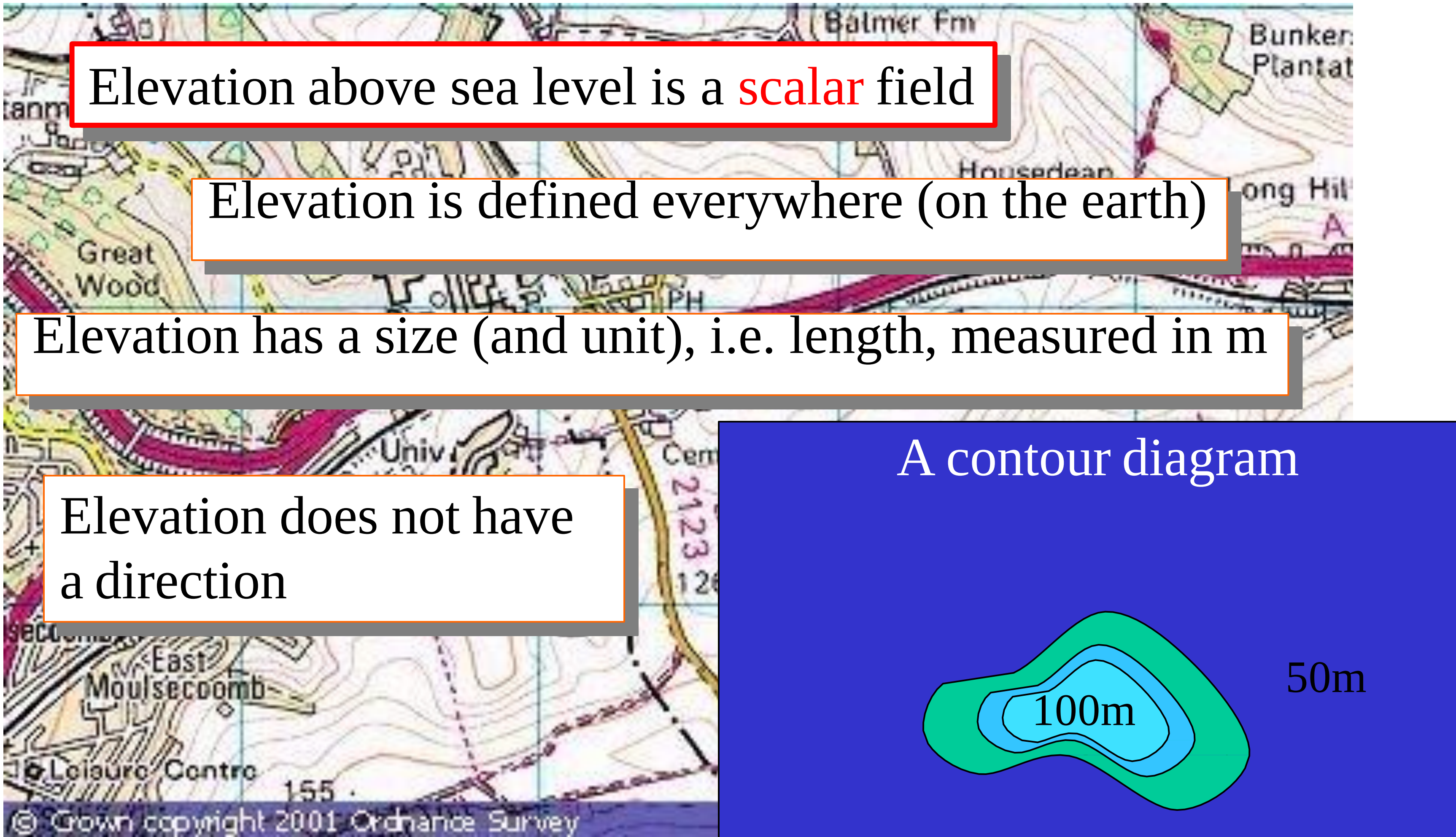
Other examples of fields: Elevation

Elevation above sea level is a scalar field

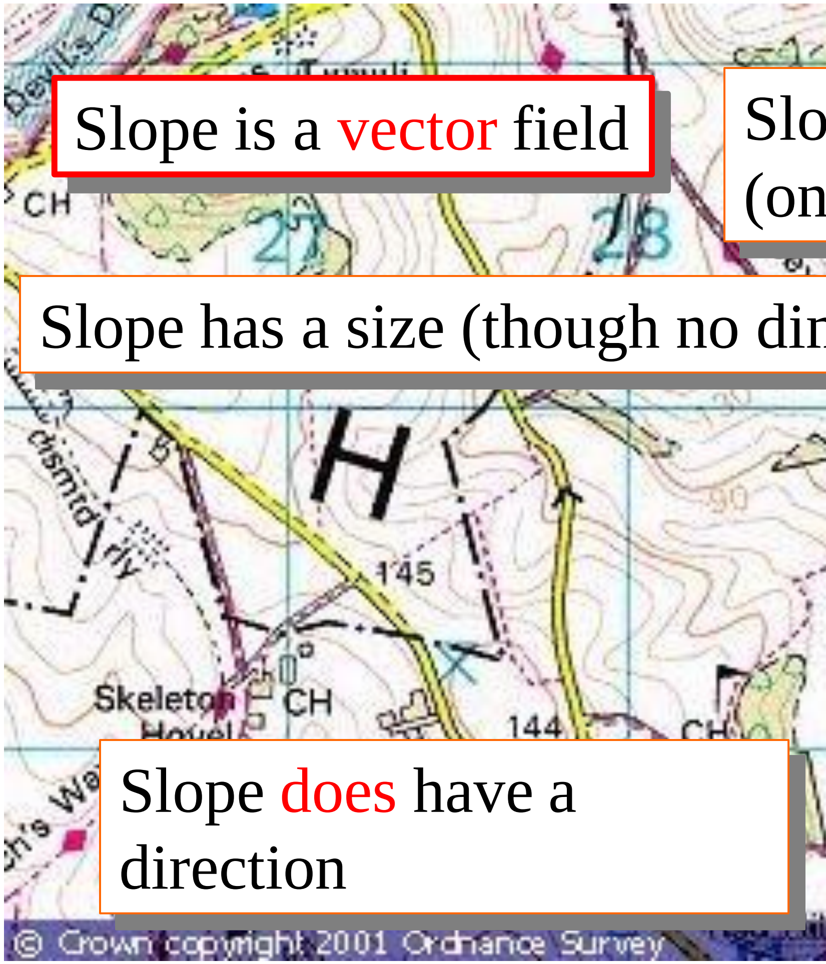
Elevation is defined everywhere (on the earth)

Elevation has a size (and unit), i.e. length, measured in m

Elevation does not have a direction



Other examples of fields: Slope

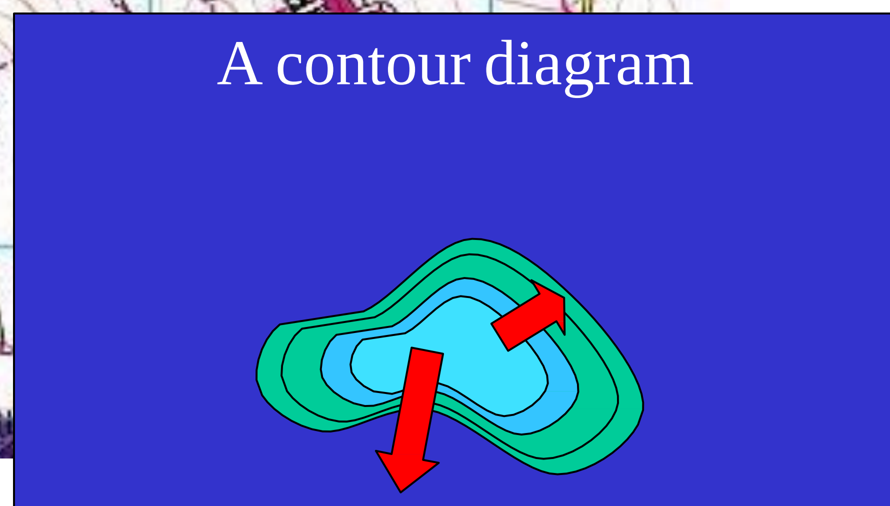


Slope is a vector field

Slope is defined everywhere (on the earth)

Slope has a size (though no dimension), i.e. 10%, 1 in 10, 2º

Slope does have a direction

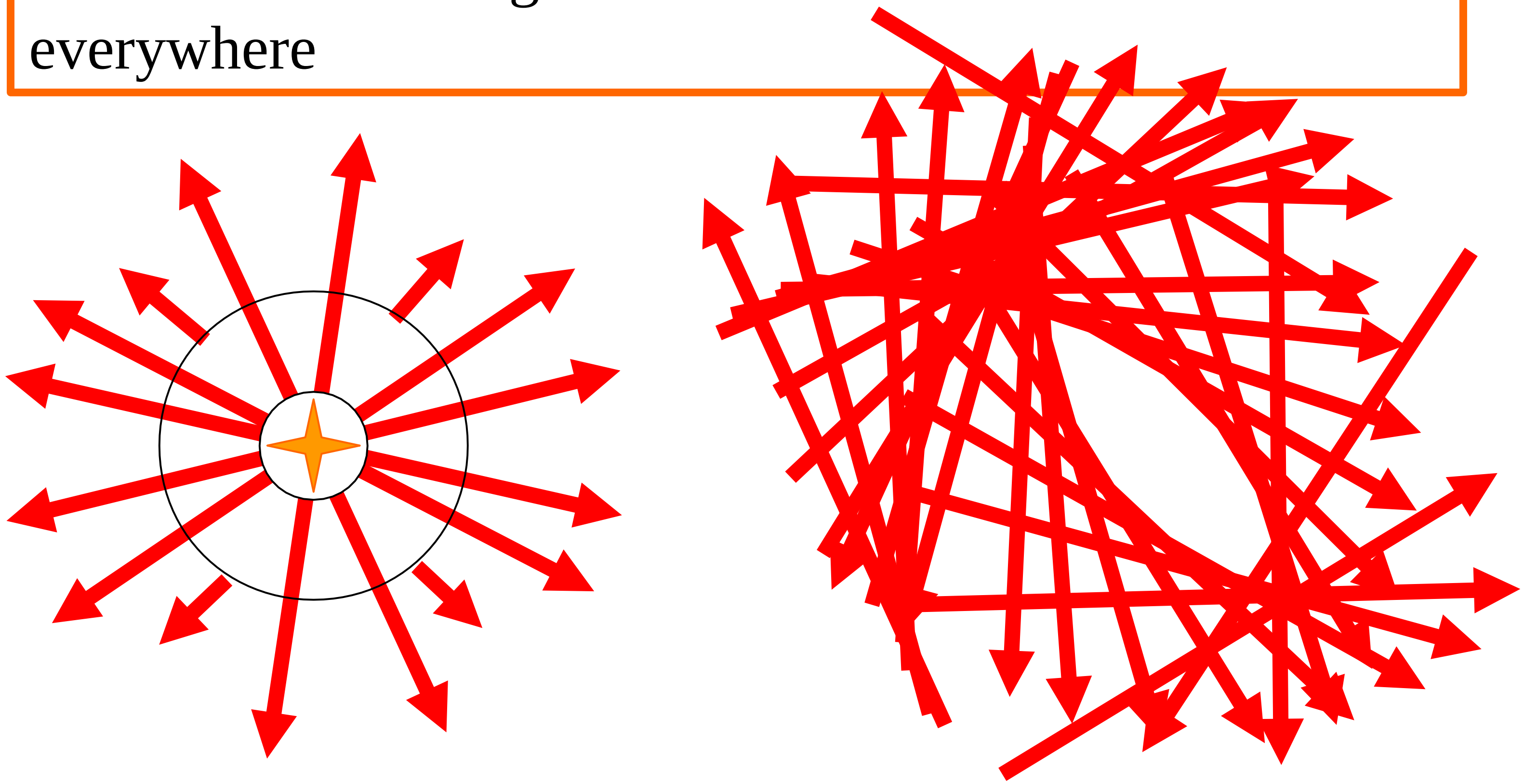


Representation of the Electric Field

Electric Field Lines

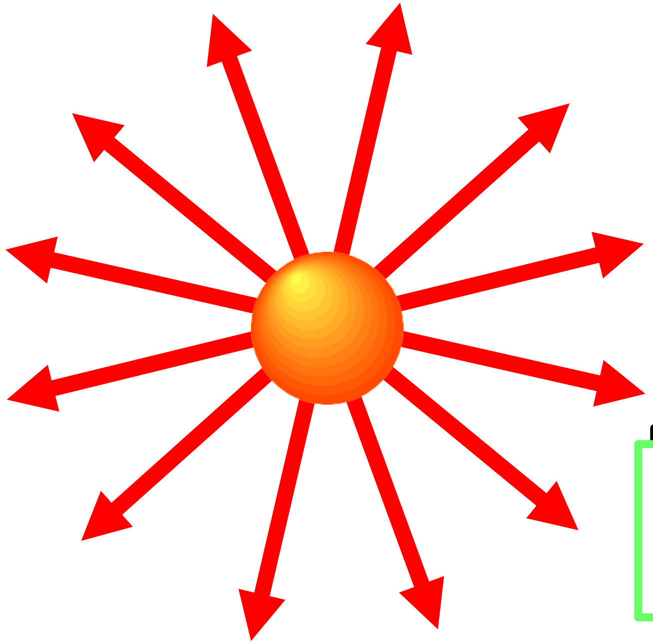
Representation of the Electric Field

It would be difficult to represent the electric field by drawing vectors whose direction was the direction of the field and whose length was the size of the field everywhere



Representation of the Electric Field

Instead we choose to represent the electric field with lines whose direction indicates the direction of the field



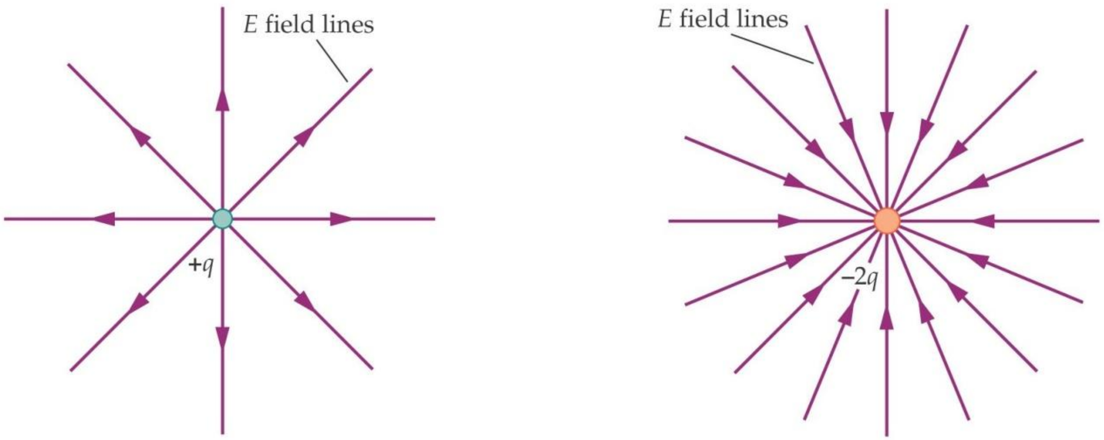
Notice that as we move away from the charge, the density of lines decreases

These are called Electric Field Lines

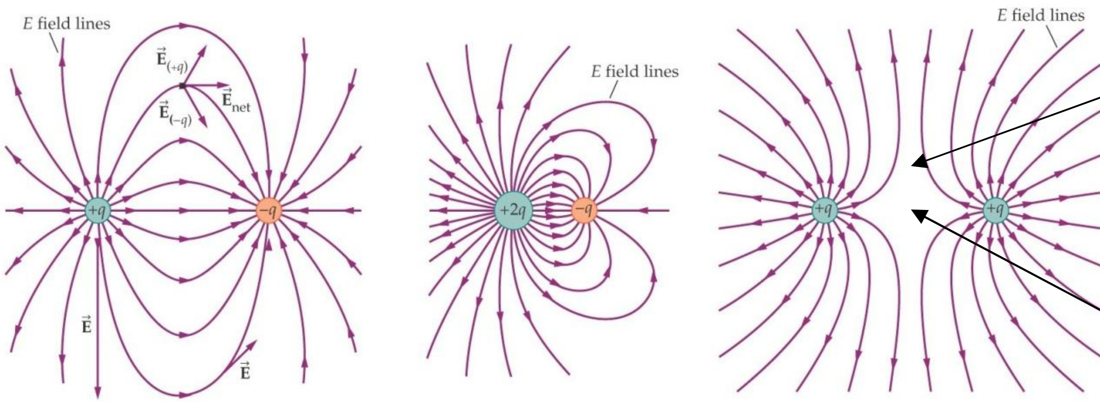
Drawing Electric Field Lines

- The lines must begin on positive charges (or infinity)
- The lines must end on negative charges (or infinity)
- The number of lines leaving a +ve charge (or approaching a -ve charge) is proportional to the magnitude of the charge
- electric field lines cannot cross

Electric Field Lines



(a) (b)

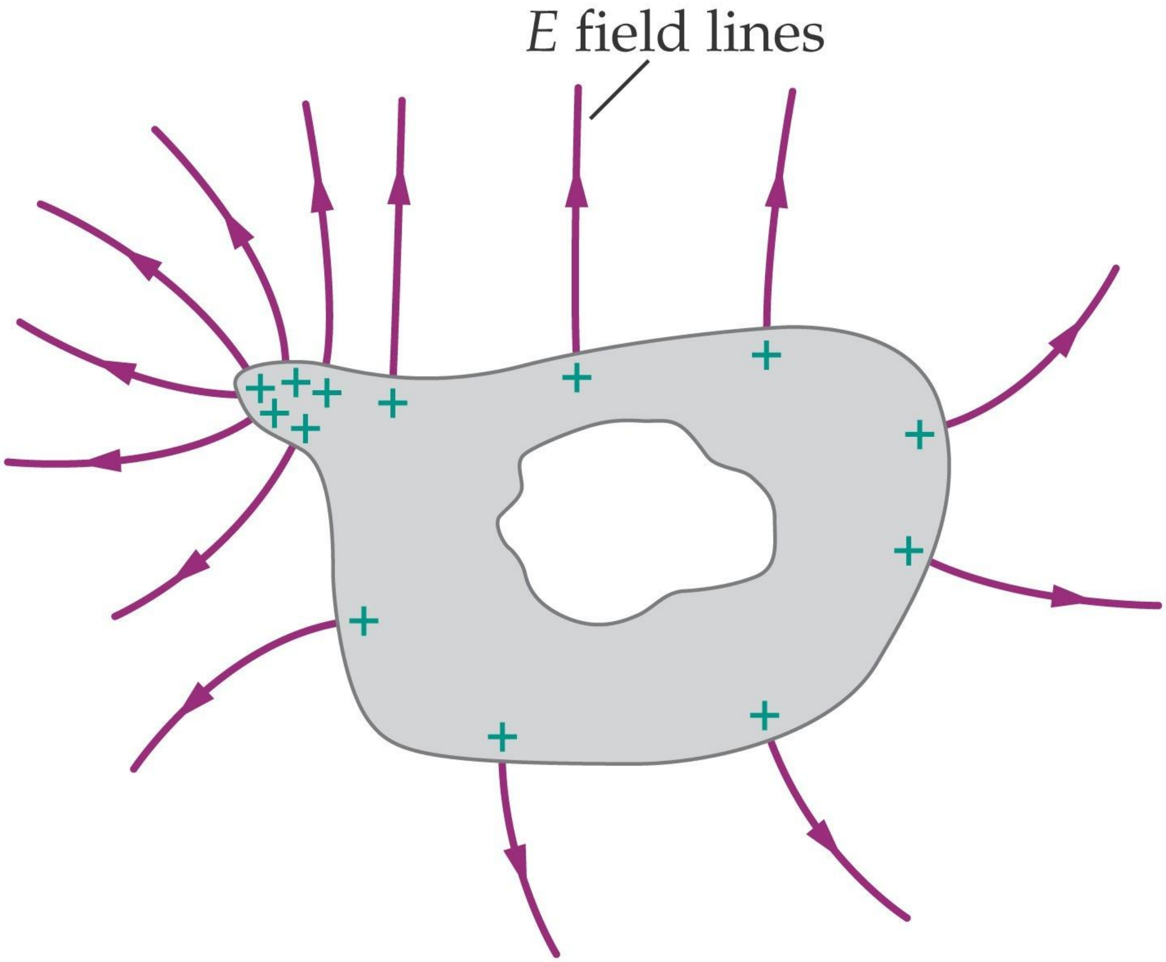


(a) (b) (c)

Field is *not* zero here

Field is zero at midpoint

Field lines for a conductor

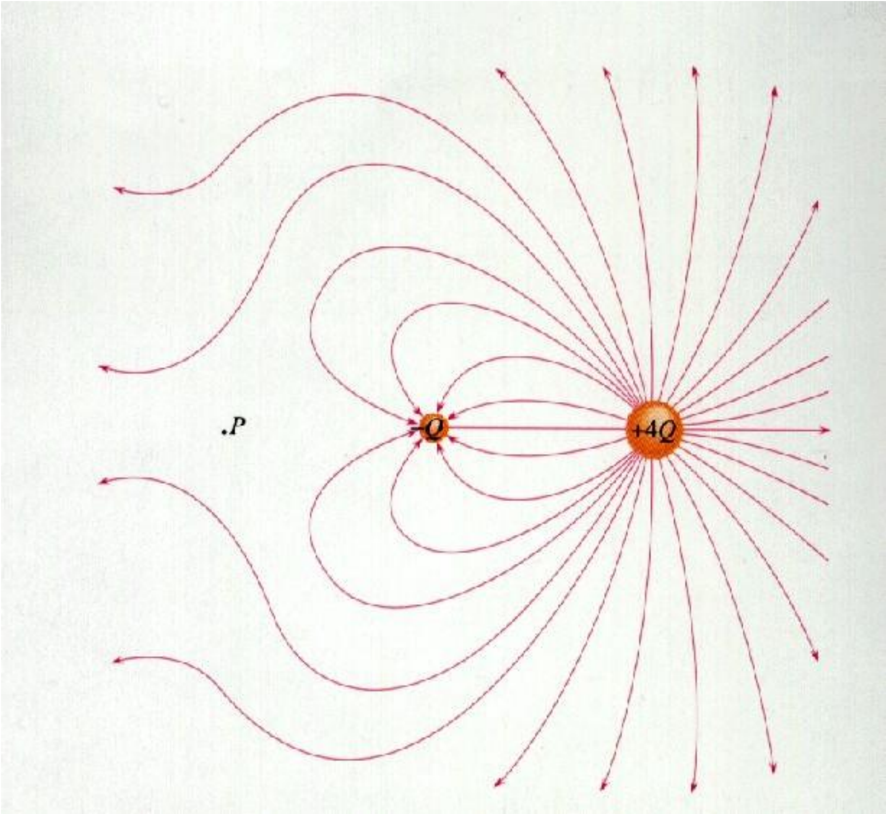


Drawing Electric Field Lines: Examples

From Electric field vectors to field lines

Field lines from all angles

Field lines representation



Electric Field Lines

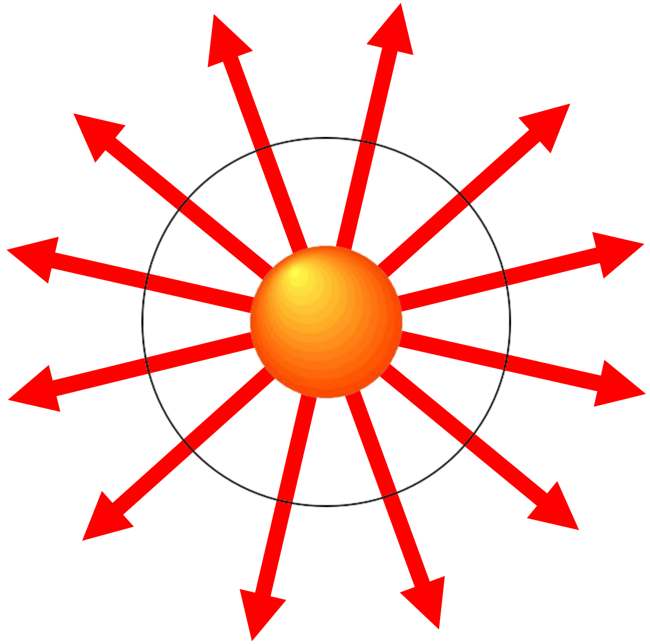
Define $$\rho \equiv \frac{N_{\text{lines}}}{A}$$ $$\rho = \frac{N}{4\pi r^2}$$

since $N_{\text{lines}} \propto Q$

$$\rho \propto \frac{Q}{4\pi r^2}$$

we know

$$|\mathbf{E}| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\mathbf{r}|^2}$$



The number density of field lines is

$$|\mathbf{E}| \propto \rho$$

Interpreting Electric Field Lines

- The electric field vector, **E**, is at a tangent to the electric field lines at each point along the lines
- The number of lines per unit area through a surface perpendicular to the field is proportional to the strength of the electric field in that region

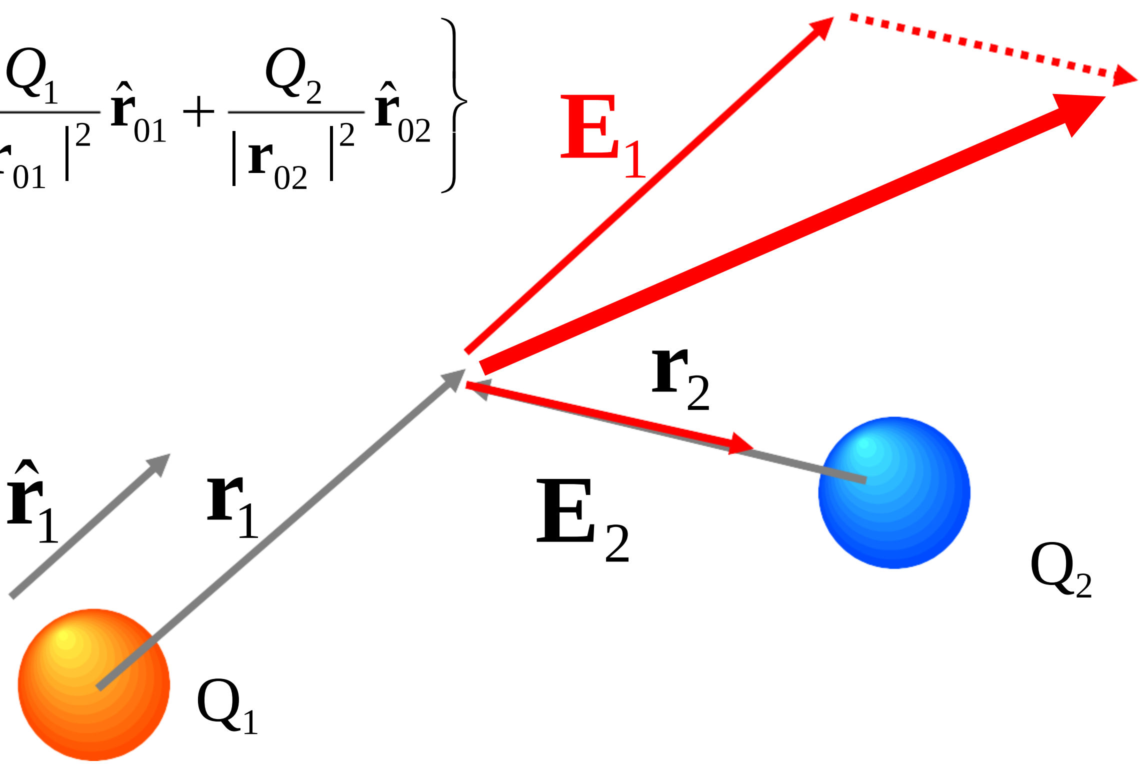
Superposition & Electric Field

Superposition & Electric Field

$$\mathbf{F}_0 = \frac{1}{4\pi\varepsilon_0}\left\{\frac{Q_0 Q_1}{|\mathbf{r}_{01}|^2}\hat{\mathbf{r}}_{01} + \frac{Q_0 Q_2}{|\mathbf{r}_{02}|^2}\hat{\mathbf{r}}_{02}\right\}$$

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{Q_i}{|\mathbf{r}_i|^2}\hat{\mathbf{r}}_i$$

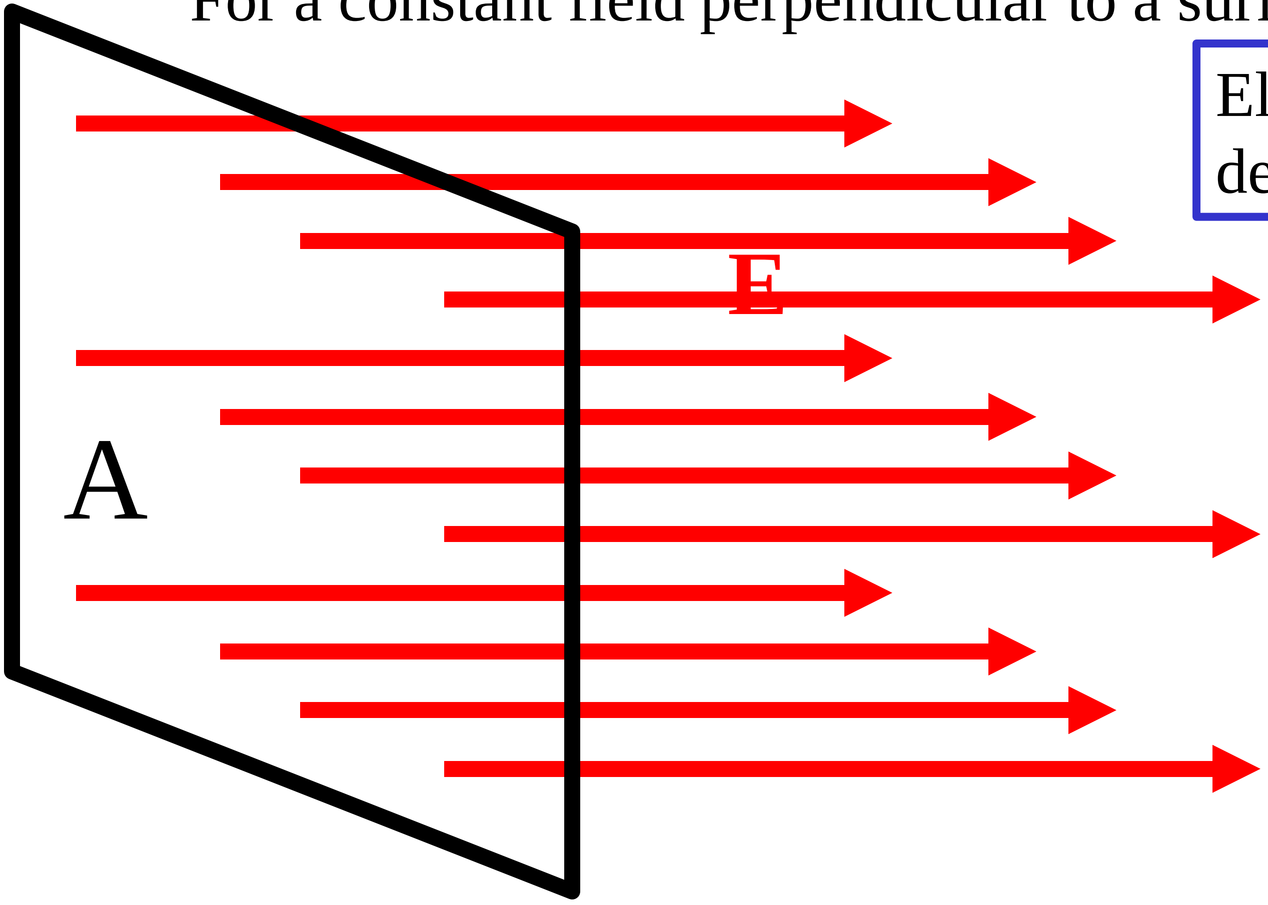
$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\left\{\frac{Q_1}{|\mathbf{r}_{01}|^2}\hat{\mathbf{r}}_{01} + \frac{Q_2}{|\mathbf{r}_{02}|^2}\hat{\mathbf{r}}_{02}\right\}$$



Electric Flux

Electric Flux: Field Perpendicular

For a constant field perpendicular to a surface A

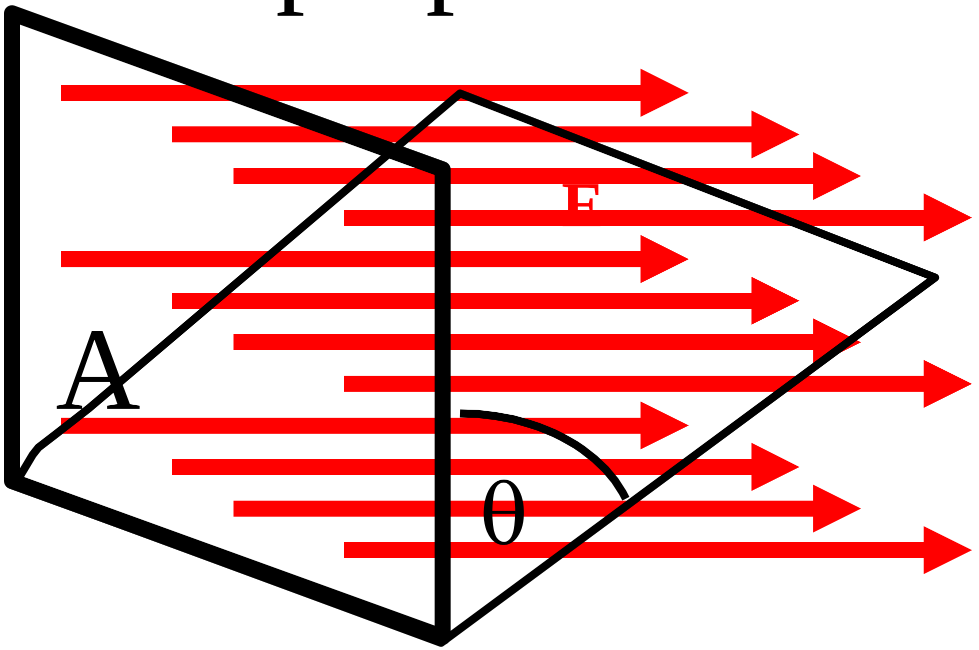


Electric Flux is defined as

$$\Phi = |\mathbf{E}| A$$

Electric Flux: Non perpendicular

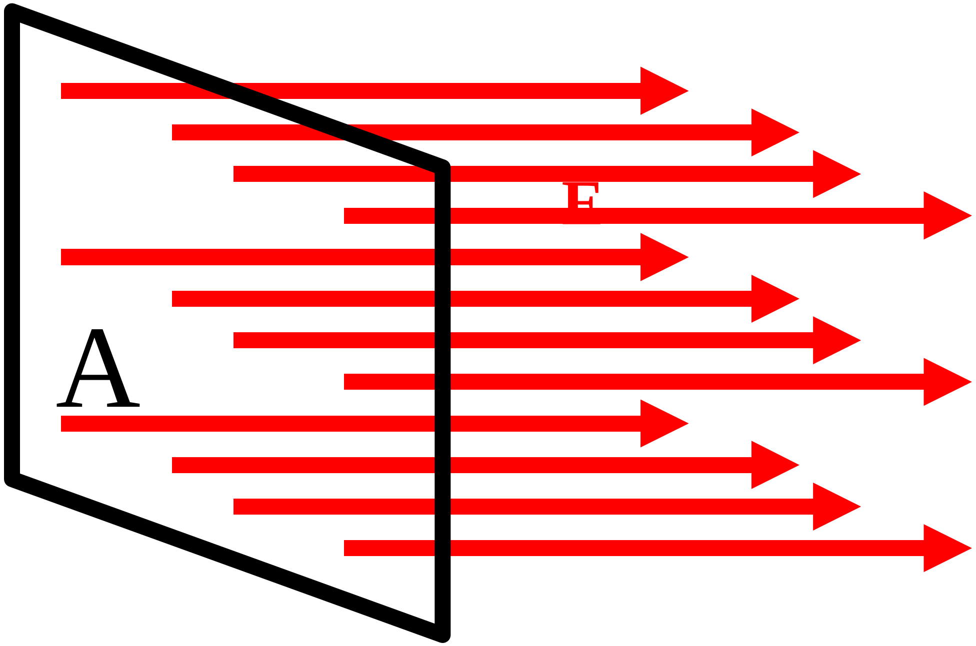
For a constant field NOT perpendicular to a surface A



Electric Flux is defined as

$$\Phi = |\mathbf{E}|\, A\cos\theta$$

Electric Flux:
Relation to field lines



$$\Phi = |\mathbf{E}|A$$

Field line density $\qquad \rho \propto |\mathbf{E}|$

Field line density × Area $\qquad \rho A \propto |\mathbf{E}|\, A$

FLUX

Number of flux lines $\qquad N \propto \Phi$

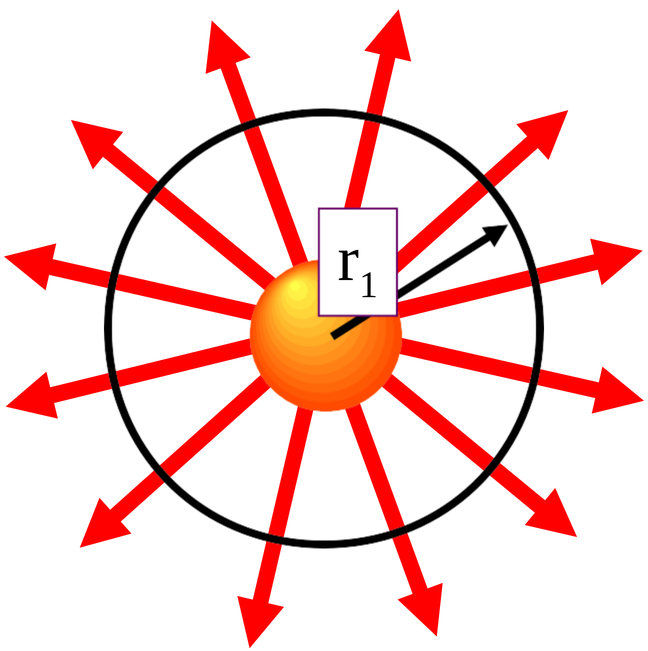
Gauss’s Law

Relates flux through a closed surface
to
charge within that surface

Flux through a sphere from a point charge

The electric field around a point charge

$$|\mathbf{E}| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\mathbf{r}_1|^2}$$



Thus the flux on a sphere is E × Area

E Area

$$\Phi = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\mathbf{r}_1|^2} \times 4\pi\,|\mathbf{r}_1|^2$$

Cancelling we get

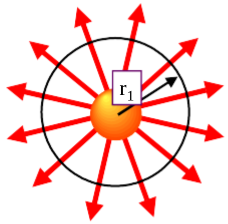
$$\Phi = \frac{Q}{\varepsilon_0}$$

Now we change the radius of sphere

Flux through a sphere from a point charge

The electric field around a point charge

$$|\mathbf{E}| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\mathbf{r}_1|^2}$$



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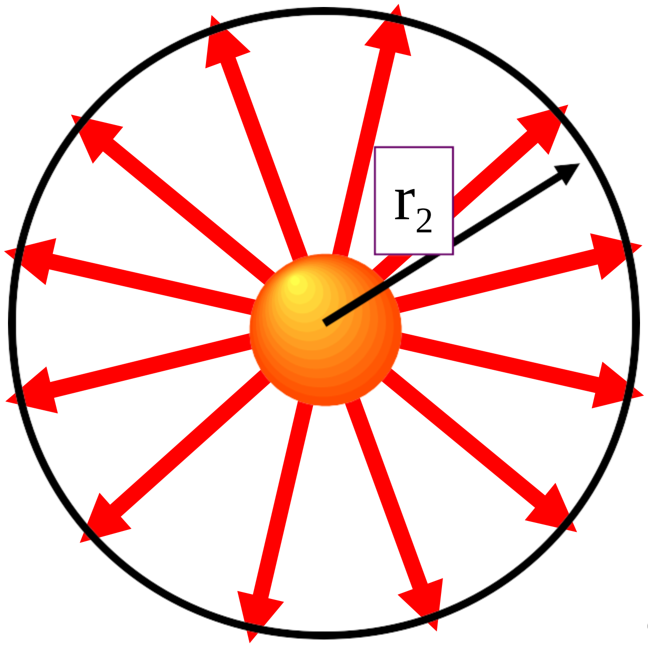
E Area

$$\Phi = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\mathbf{r}_1|^2} \times 4\pi\, |\mathbf{r}_1|^2$$

Cancelling we get

$$\Phi = \frac{Q}{\varepsilon_0}$$



$$|\mathbf{E}| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\mathbf{r}_2|^2}$$

$$\Phi_2 = \frac{1}{|4\pi\varepsilon_0|}\frac{Q}{|\mathbf{r}_2|^2} \times 4\pi\, |\mathbf{r}_2|^2$$

$$\Phi_2 = \frac{Q}{\varepsilon_0}$$

The flux is the same as before

$$\Phi_2 = \Phi_1 = \frac{Q}{\varepsilon_0}$$

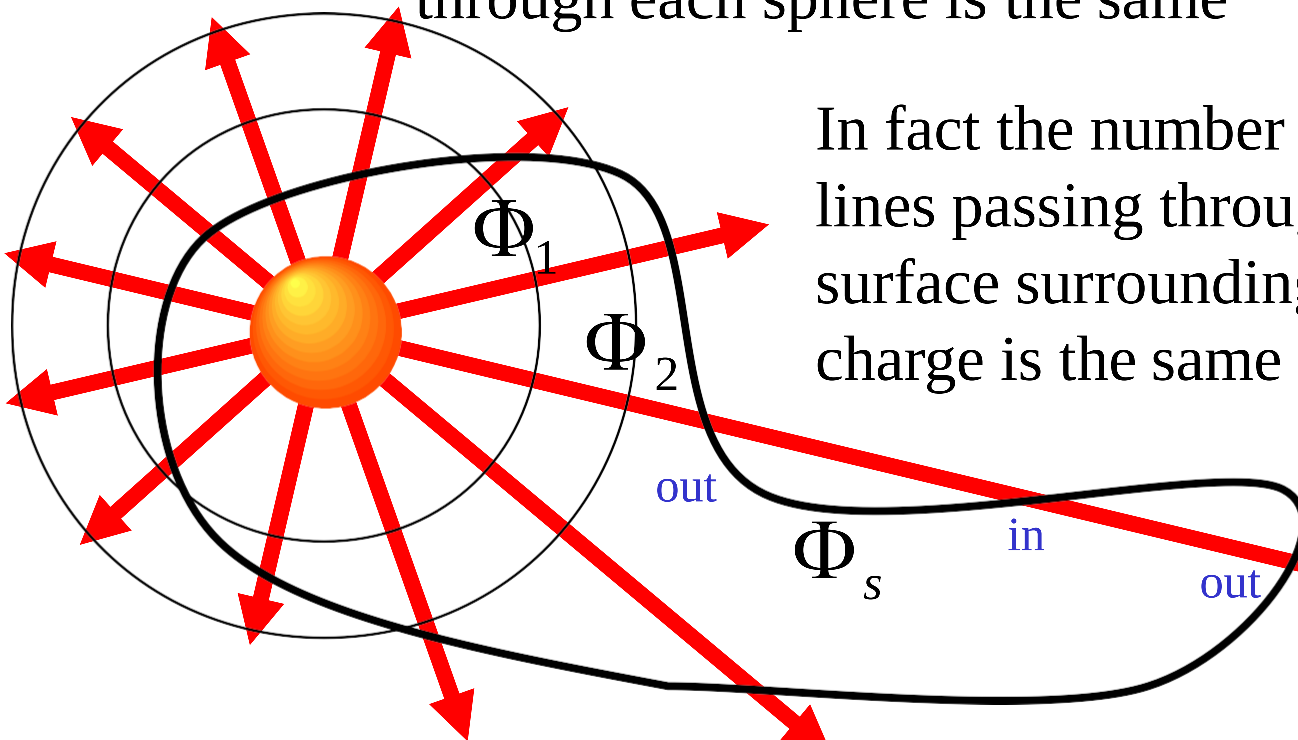
Flux lines & Flux

Just what we would expect because the number of field lines passing through each sphere is the same

$$N \propto \Phi \qquad \Phi \propto N$$

and number of lines passing through each sphere is the same

$$\Phi_S = \Phi_2 = \Phi_1 = \frac{Q}{\varepsilon_0}$$



In fact the number of flux lines passing through any surface surrounding this charge is the same

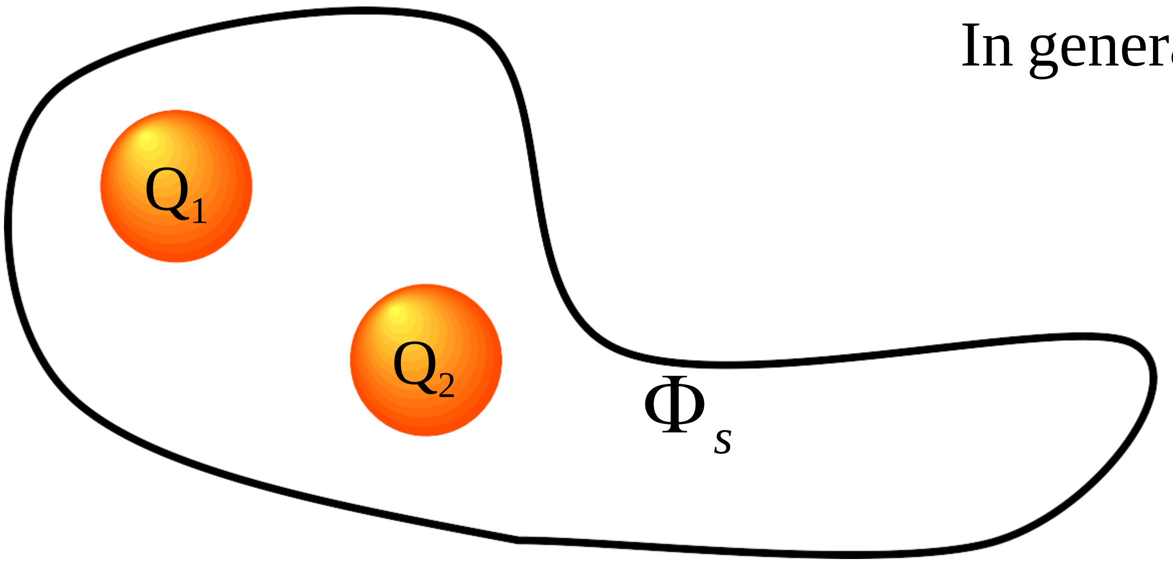
even when a line passes in and out of the surface it crosses out once more than in

Principle of superposition:

What is the flux from two charges?

Since the flux is related to the number of field lines passing through a surface the total flux is the total from each charge

$$\Phi_S = \frac{Q_1}{\varepsilon_0} + \frac{Q_2}{\varepsilon_0}$$



In general

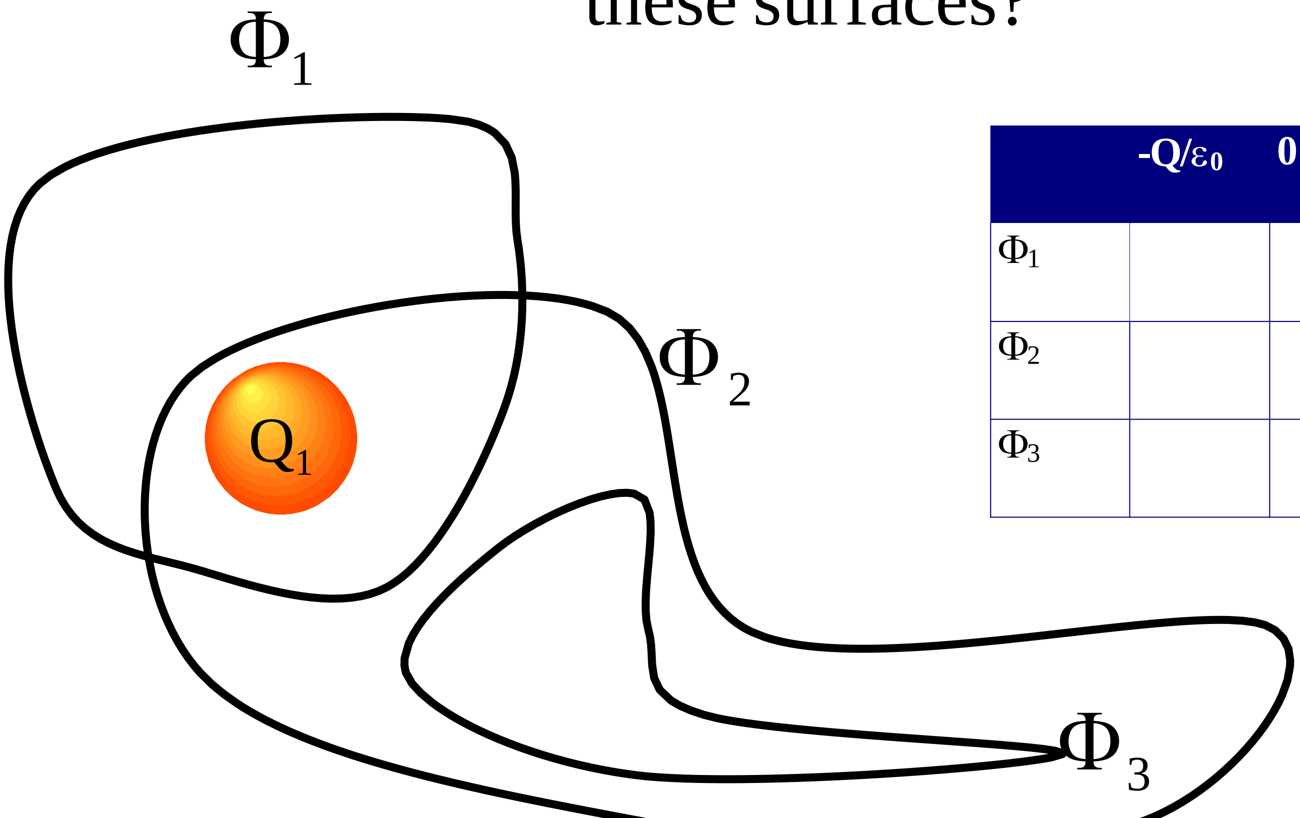
$$\Phi_S = \sum \frac{Q_i}{\varepsilon_0}$$

For any surface

Gauss's Law

Quiz

What flux is passing through each of these surfaces?



	$-Q/\varepsilon_0$	0	$+Q/\varepsilon_0$	$+2Q/\varepsilon_0$
Φ_1				
Φ_2				
Φ_3				

What is Gauss's Law?

Gauss's Law does not tell us anything new, it is NOT a new law of physics, but another way of expressing Coulomb's Law

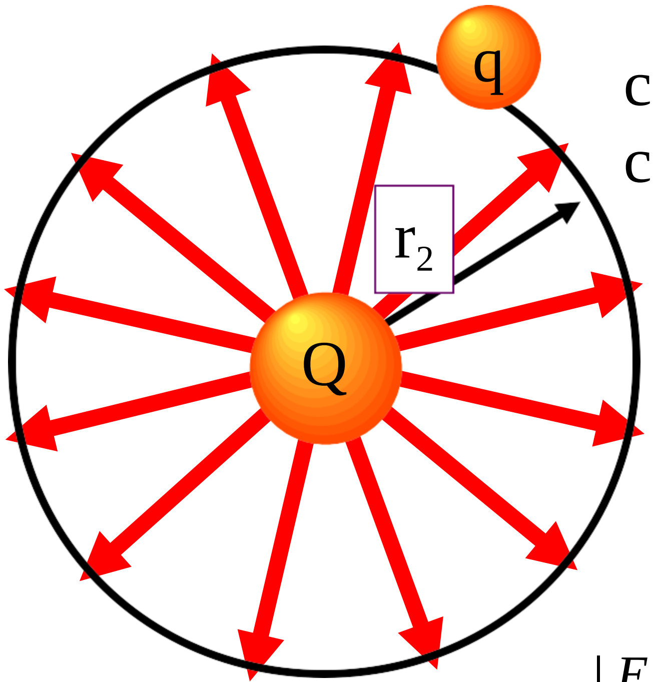
Gauss's Law is sometimes easier to use than Coulomb's Law, especially if there is lots of symmetry in the problem

Examples of using Gauss's Law

Example of using Gauss's Law 1

oh no! I've just forgotten Coulomb's Law!

Not to worry I remember Gauss's Law



consider spherical surface centred on charge

$$\Phi = \frac{Q}{\varepsilon_0}$$

By symmetry **E** is ⊥ to surface

$$\Phi = |E|\,A = \frac{Q}{\varepsilon_0} \qquad = |E|\,4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$|E| = \frac{1}{4\pi r^2}\frac{Q}{\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

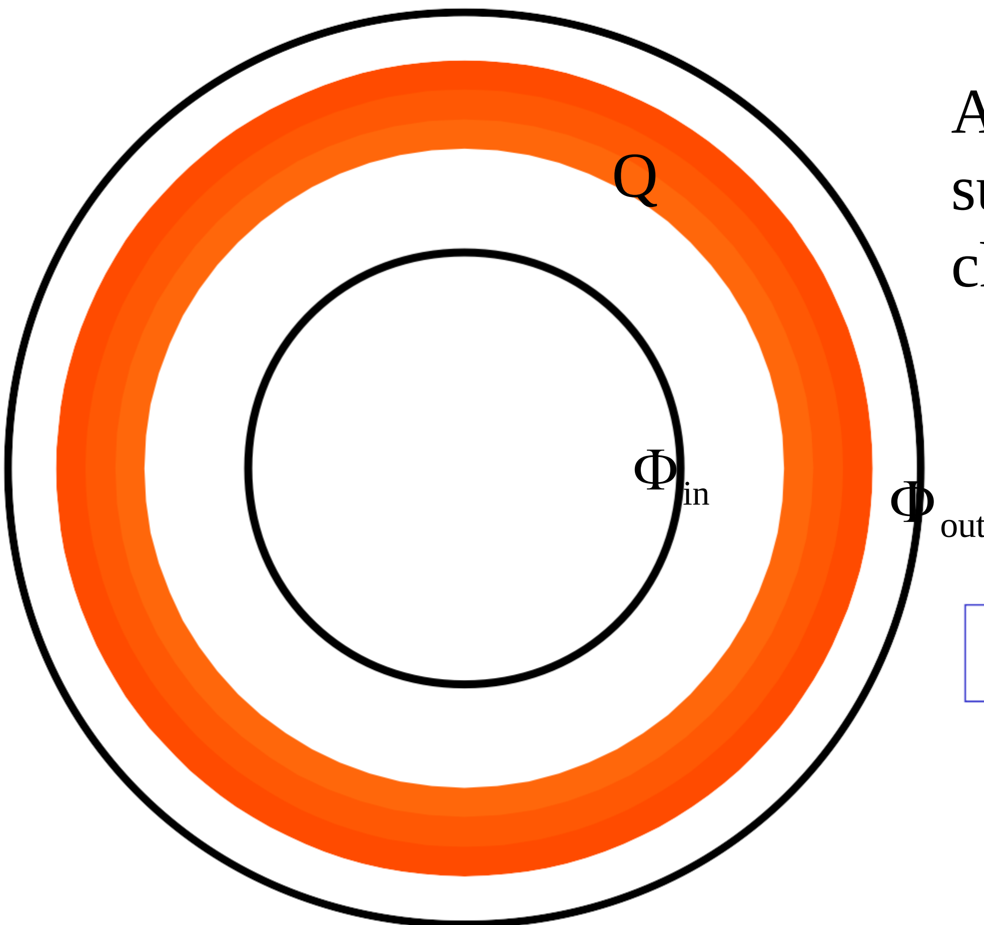
F=qE

$$F = \frac{1}{4\pi r^2}\frac{qQ}{\varepsilon_0}$$

Phew!

Example of using Gauss's Law 2

What's the field around a charged spherical shell?



Again consider spherical surface centred on charged shell

Outside

$$\Phi_{out} = \frac{Q}{\varepsilon_0}$$

So as e.g. 1

$$|E| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

Inside

charge within surface = 0

$$\Phi_{in} = 0 \qquad E = 0$$

Examples

Gauss's Law and a line of charge

Gauss's Law and a uniform sphere

Gauss's Law around a point charge

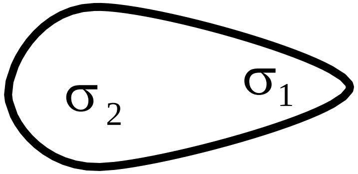
Properties of Conductors

Using Gauss's Law

Properties of Conductors

For a conductor in electrostatic equilibrium

1. E is zero within the conductor
2. Any net charge, Q, is distributed on surface (surface charge density $\sigma = Q/A$)
3. E immediately outside is $\perp$ to surface
4. σ is greatest where the radius of curvature is smaller



$\sigma_1 >> \sigma_{21}$

1. E is zero within conductor

If there is a field in the conductor, then the free electrons would feel a force and be accelerated. They would then move and since there are charges moving the conductor would not be in electrostatic equilibrium

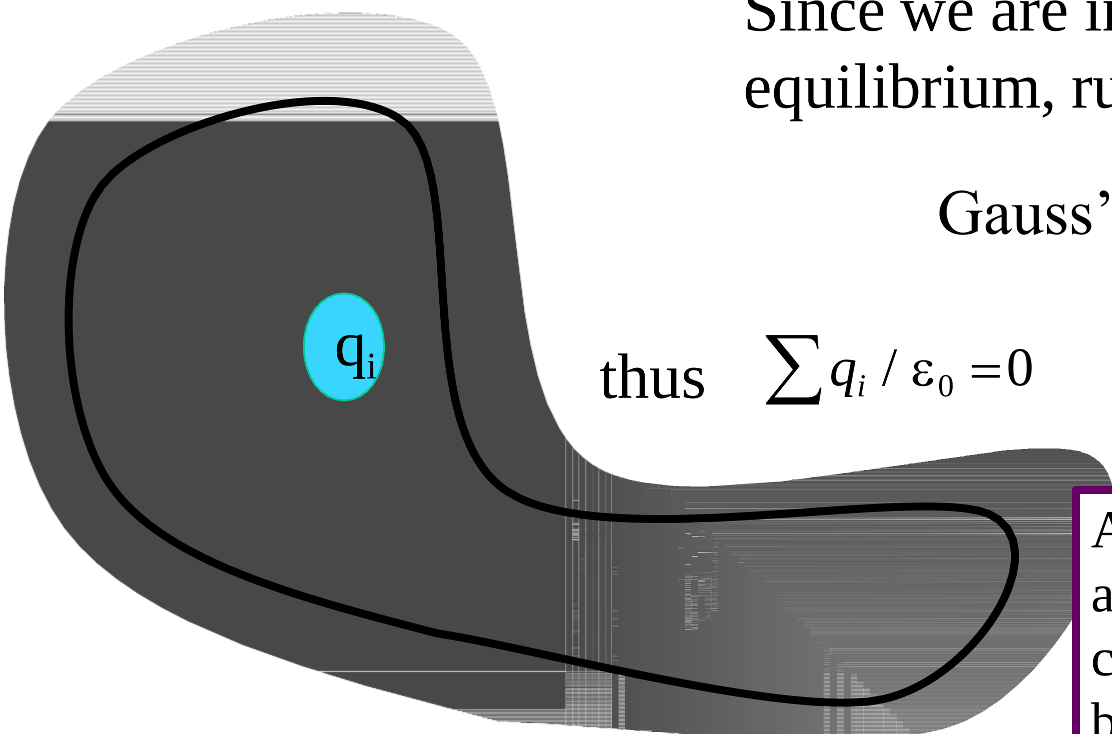
Thus E=0

2. Any net charge, Q, is distributed on surface

Consider surface S below surface of conductor

Since we are in a conductor in equilibrium, rule 1 says E=0, thus Φ=0

Gauss's Law $$\Phi = EA = \sum q / \varepsilon_0$$

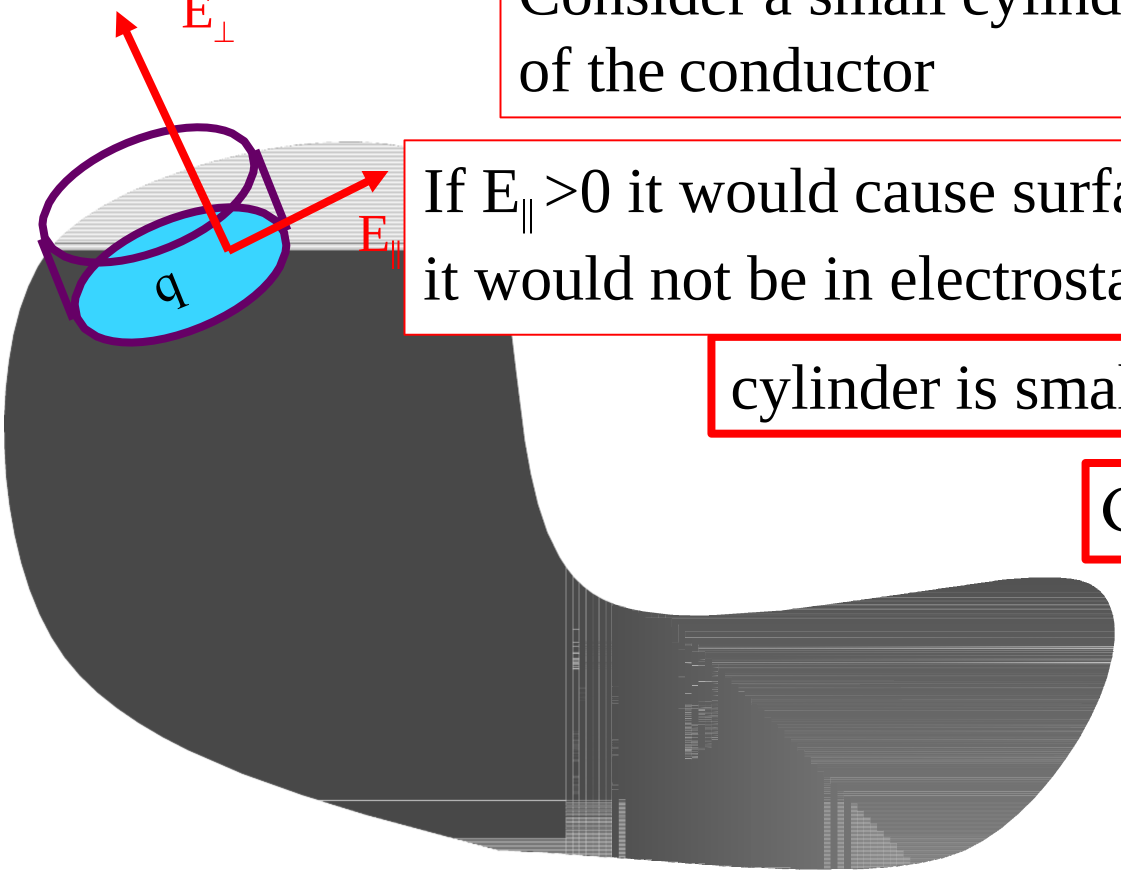


thus $$\sum q_i / \varepsilon_0 = 0$$

So, net charge within the surface is zero

As surface can be drawn arbitrarily close to surface of conductor, all net charge must be distributed on surface

3. E immediately outside is ⊥ to surface



Consider a small cylindrical surface at the surface of the conductor

If $E_\parallel > 0$ it would cause surface charge q to move thus it would not be in electrostatic equilibrium, thus $E_\parallel = 0$

cylinder is small enough that E is constant

Gauss's Law $\Phi = EA = q / \varepsilon$

thus $E = q / A\varepsilon$

$$E_\perp = \sigma / \varepsilon$$

Electromagnetic Fields

UNIT-II

- Electric Potential

Contents

ELECTRIC POTENTIAL

ELECTRIC POTENTIAL

- Suppose we wish to move a point charge *Q* from point *A* to point *B* in an electric field E some *work is done* in displacing the charge by *dl Given by*

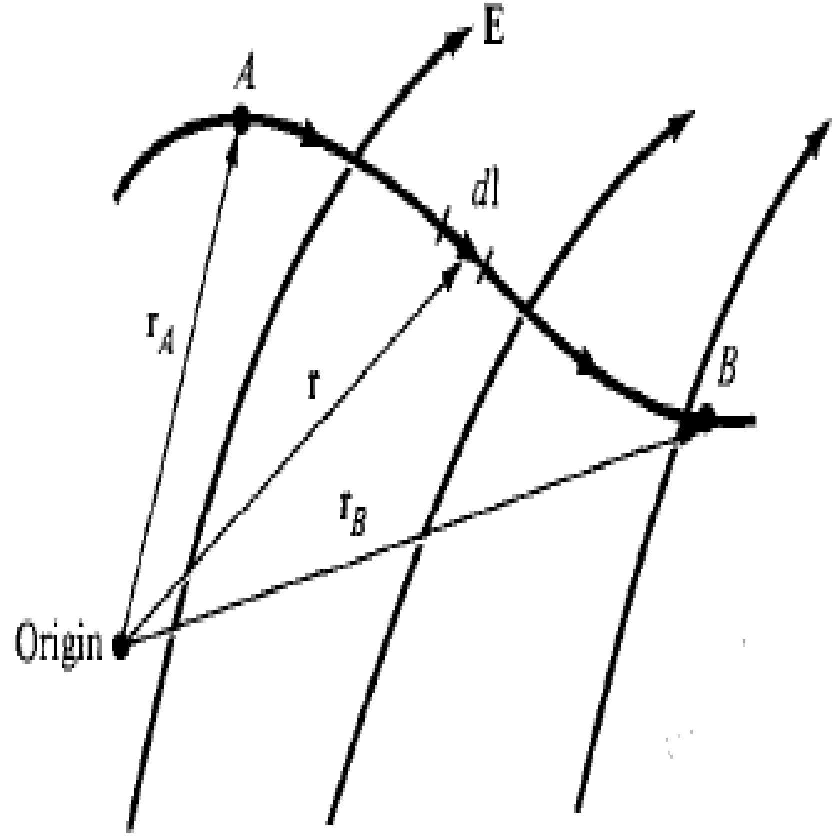
$$dW = -\mathbf{F} \cdot dl = -QE \cdot dl$$

Where E is the Electric field intensity.

The negative sign indicates that the work is being done by an external agent.

- Thus the total work done, or the potential energy required, in moving *Q* from A to *B* is

$$W = -Q \int_A^B \mathbf{E} \cdot d\mathbf{l}$$



Potential Difference

- The *potential difference* between points *A* and *B.*

$$V_{AB} = \frac{W}{Q} = -\int_A^B \mathbf{E} \cdot d\mathbf{l}$$

Note

- In determining *VAB, A* is the initial point while *B* is the final point.
- If V_{AB} is negative, there is a loss in potential energy in moving *Q* from *A* to *B;* this implies that the work is being done by the field. However, if V_{AB} is positive, there is a gain in potential energy in the movement; an external agent performs the work.
- V_{AB} is independent of the path taken (to be shown a little later).
- V_{AB} is measured in joules per coulomb, commonly referred to as volts (V).

Electric Potential Due to Point Charge

- Suppose a point charge *Q* located at the origin, Then

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_o r^2}\,\mathbf{a}_r$$

$$V_{AB} = -\int_{r_A}^{r_B} \frac{Q}{4\pi\varepsilon_o r^2}\,\mathbf{a}_r \cdot dr\,\mathbf{a}_r$$
$$= \frac{Q}{4\pi\varepsilon_o}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$V_{AB} = V_B - V_A$$

- Thus if *VA* = 0 as r_A —> ∞, the potential
- at any point r_B —> r due to a point charge *Q* located at the origin is

$$V = \frac{Q}{4\pi\varepsilon_o r}$$

Super position Principle

- For n point charges Qu Q2,. • • ,Qn located at points with position vectors r_1, r_2,. . ., r_n, the potential at r is

$$V(\mathbf{r}) = \frac{Q_1}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_1|} + \frac{Q_2}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_2|} + \cdots + \frac{Q_n}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_n|}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_o} \sum_{k=1}^{n} \frac{Q_k}{|\mathbf{r} - \mathbf{r}_k|} \qquad \text{(point charges)}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_o} \int_L \frac{\rho_L(\mathbf{r}')dl'}{|\mathbf{r} - \mathbf{r}'|} \qquad \text{(line charge)}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_o} \int_S \frac{\rho_S(\mathbf{r}')dS'}{|\mathbf{r} - \mathbf{r}'|} \qquad \text{(surface charge)}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_o} \int_v \frac{\rho_v(\mathbf{r}')dv'}{|\mathbf{r} - \mathbf{r}'|} \qquad \text{(volume charge)}$$

$$\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z \qquad \text{(cartesian)}$$

$$\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z \qquad \text{(cylindrical)}$$

$$\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi \qquad \text{(spherical)}$$

Relation Between E & V

- The potential difference between points A and B is independent of the path taken is given by

$$V_{BA} = -V_{AB}$$

- i.e

$$V_{BA} + V_{AB} = \oint \mathbf{E} \cdot d\mathbf{l} = 0$$

- Therefore

$$\boxed{\oint \mathbf{E} \cdot d\mathbf{l} = 0}$$

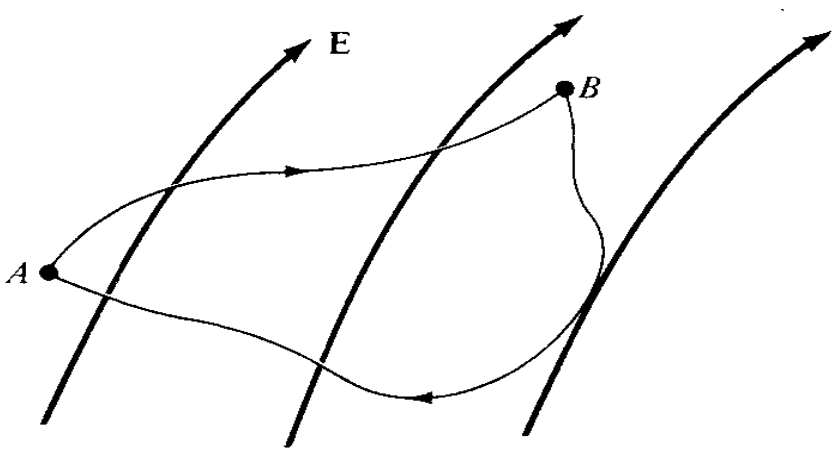
Stroke's Theorem

$$\oint \mathbf{E} \cdot d\mathbf{l} = \int (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = 0$$

$$\boxed{\nabla \times \mathbf{E} = 0}$$

E = -Grad V

$$dV = -\mathbf{E} \cdot d\mathbf{l} = -E_x\,dx - E_y\,dy - E_z\,dz$$

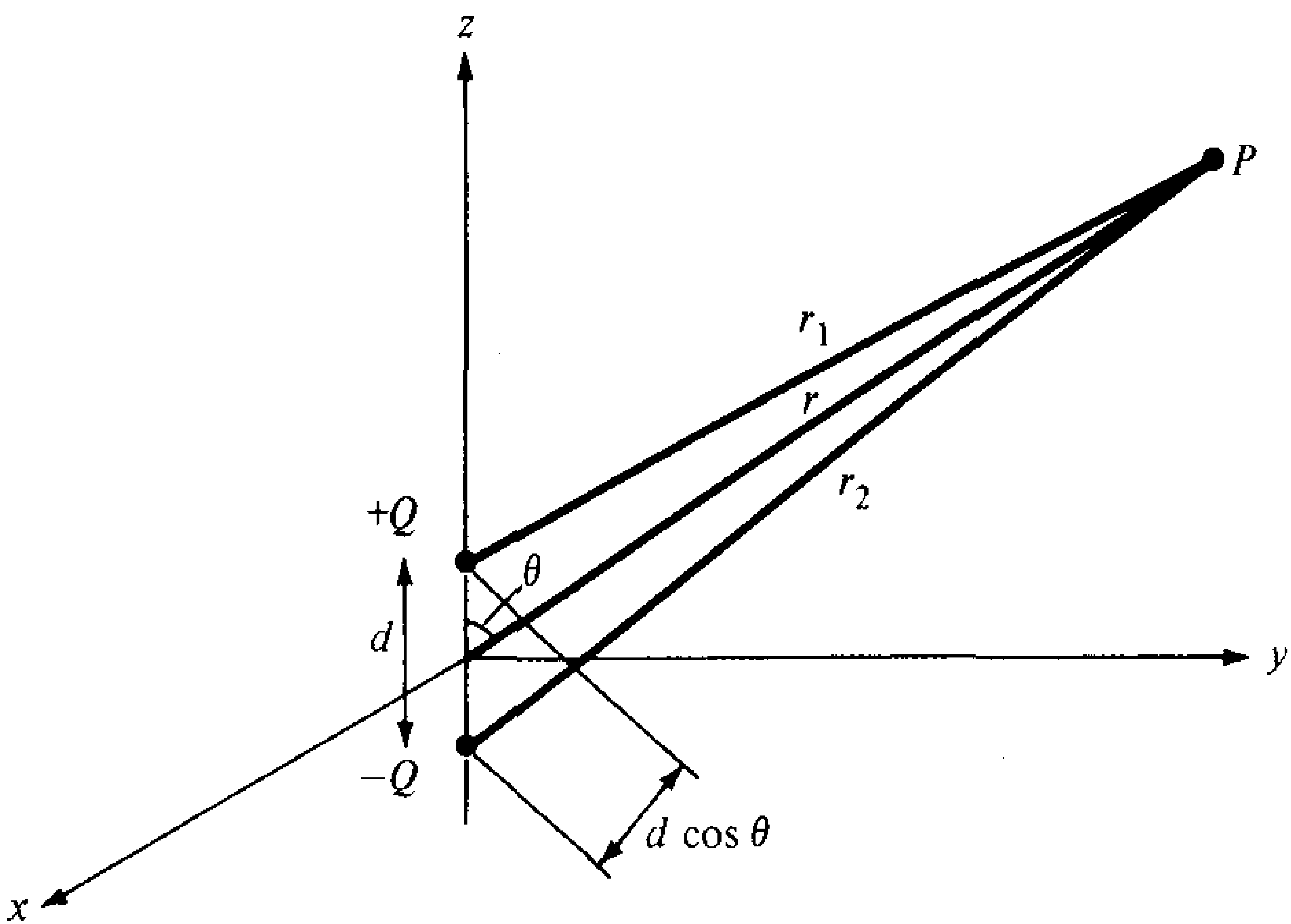


$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

$$E_x = -\frac{\partial V}{\partial x}, \qquad E_y = -\frac{\partial V}{\partial y}, \qquad E_z = -\frac{\partial V}{\partial z}$$

Electric Dipole

- **An electric dipole** is formed when two point charges of equal magnitude but opposite sign are separated by a small distance.



z
P
r_1
r
r_2
+Q
θ
d
y
−Q
d cos θ
x

Electric Potential

- Consider an electric dipole, the potential at point p is given by

$$V = \frac{Q}{4\pi\varepsilon_o}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{Q}{4\pi\varepsilon_o}\left[\frac{r_2 - r_1}{r_1 r_2}\right]$$

$$r \gg d,\ r_2 - r_1 \simeq d\cos\theta,\ r_2 r_1 \simeq r^2$$

$$V = \frac{Q}{4\pi\varepsilon_o} \frac{d\cos\theta}{r^2}$$

Since $d\cos\theta = \mathbf{d} \cdot \mathbf{a}_r$, where $\mathbf{d} = d\mathbf{a}_z$, if we define

$$\mathbf{p} = Q\mathbf{d}$$

$$V = \frac{\mathbf{p} \cdot \mathbf{a}_r}{4\pi\varepsilon_o r^2}$$

$$\boxed{V(\mathbf{r}) = \frac{\mathbf{p} \cdot (\mathbf{r} - \mathbf{r}')}{4\pi\varepsilon_o |\mathbf{r} - \mathbf{r}'|^3}}$$

$$\mathbf{E} = -\nabla V = -\left[\frac{\partial V}{\partial r}\,\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\,\mathbf{a}_\theta\right]$$

$$= \frac{Qd\cos\theta}{2\pi\varepsilon_o r^3}\,\mathbf{a}_r + \frac{Qd\sin\theta}{4\pi\varepsilon_o r^3}\,\mathbf{a}_\theta$$

$$\boxed{\mathbf{E} = \frac{p}{4\pi\varepsilon_o r^3}\left(2\cos\theta\,\mathbf{a}_r + \sin\theta\,\mathbf{a}_\theta\right)}$$

Current & Current Density

- Electric Charges in motion Constitute electric current.

$$I = \frac{dQ}{dt}$$

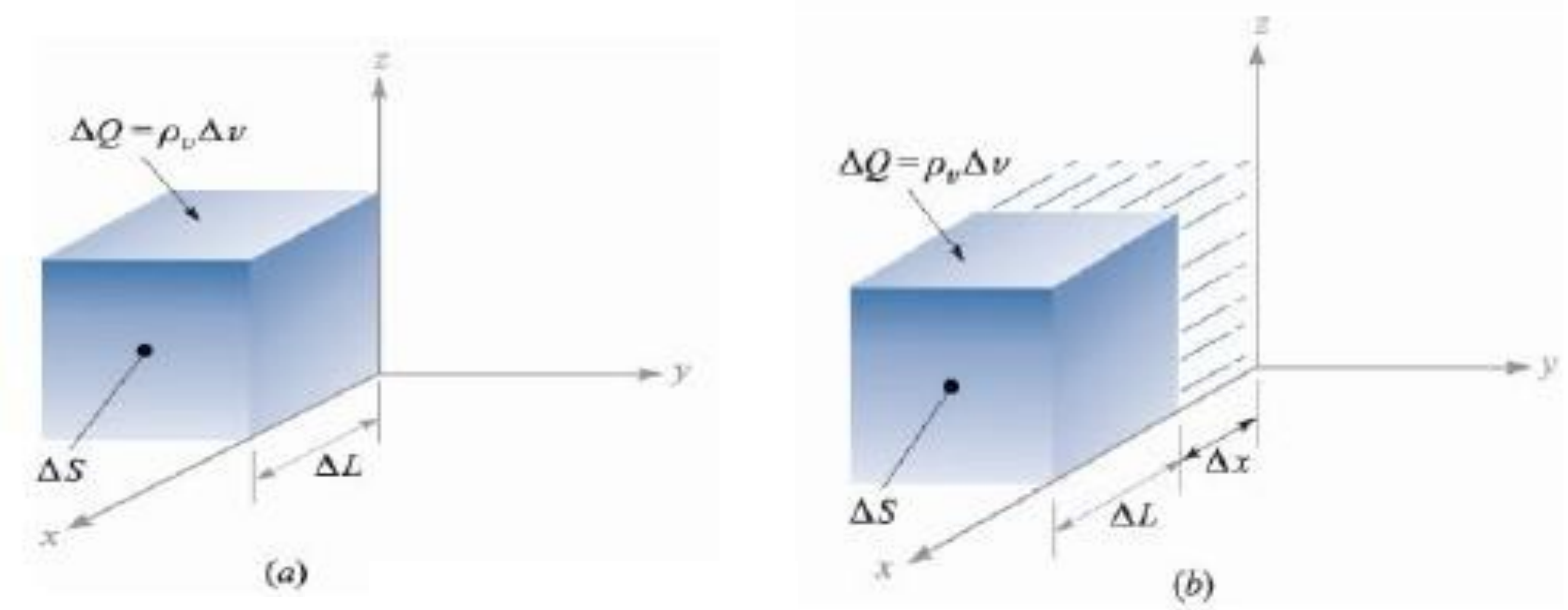
The increment of current ΔI crossing an incremental surface ΔS normal to the current density is

$$\Delta I = J_N \Delta S$$

$$\Delta I = \mathbf{J} \cdot \Delta \mathbf{S}$$

Total current is obtained by integrating,

$$I = \int_S \mathbf{J} \cdot d\mathbf{S}$$

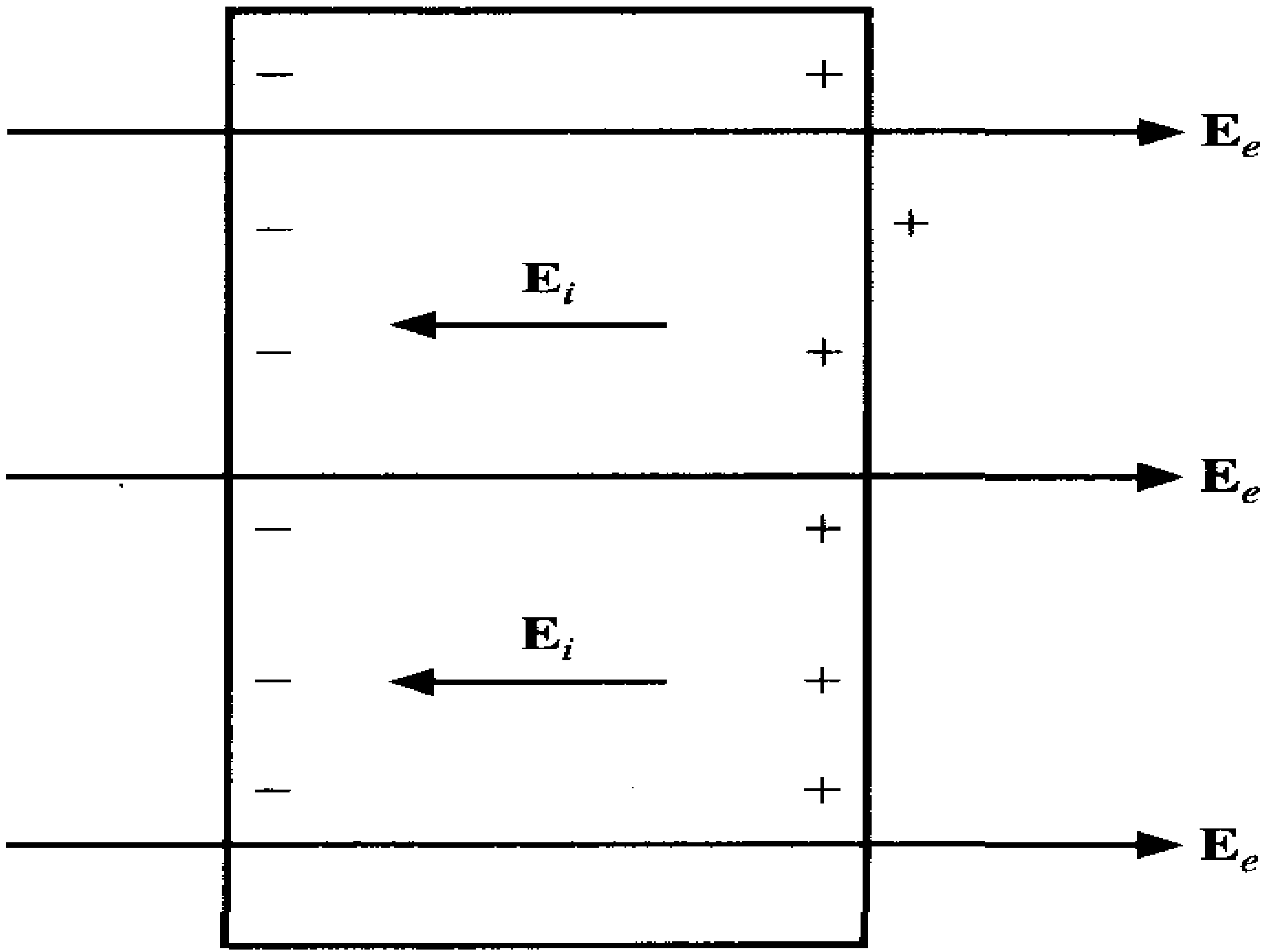


$$\mathbf{J} = \rho_v \mathbf{v}$$

CONDUCTORS

- A conductor has abundance of charge that is free to move.
- A perfect conductor cannot contain an electrostatic field within it.
- A conductor is called an equipotential body, implying that the potential is the same everywhere in the conductor.

$$\mathbf{E} = 0, \qquad \rho_v = 0, \qquad V_{ab} = 0 \qquad \text{inside a conductor}$$



$\mathbf{E}_e$
$\mathbf{E}_i$
$\mathbf{E}_e$
$\mathbf{E}_i$
$\mathbf{E}_e$

- When an external electric field Ee is applied, the positive free charges are pushed along the same direction as the applied field, while the negative free charges move in the opposite direction.
- This charge migration takes place very quickly. The free charges do two things.
- First, they accumulate on the surface of the conductor and form an *induced surface charge.*
- Second, the induced charges set up an internal induced field E,, which cancels the externally applied field Ee.

UNIT - III

- Static Magnetic Fields, Biot-Savart's Law
- Oesterd's Experiment
- Magnetic Field Intensity
- MFI due to a straight current carrying conductor
- MFI due to square and solenoid currents
- Div B = 0

Oesterd's Experiment

- An electrostatic field is produced by static or stationary charges.
- If the charges are moving with constant velocity, a static magnetic (or magnetostatic) field is produced.
- A magnetostatic field is produced by a constant current flow (or direct current).

Biot-Savart's Law

Two major laws governing magnetostatic fields:

(1) Biot-Savart's law and

(2) Ampere's circuit law.

Biot-Savart's Law - Definition

The magnetic field intensity dH , produced at a point P , as shown in Figure by the differential current clement Idl is proportional to the product Idl and the sine of the angle α between the clement and the line joining P to the element and is inversely proportional to the square of the distance R between P and the element.

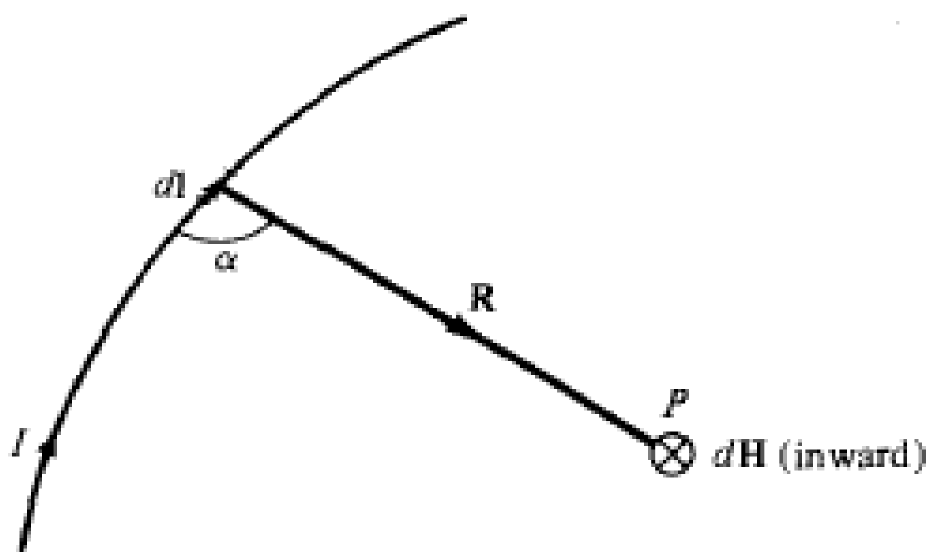
$$dH \propto \frac{I\, dl \sin \alpha}{R^2}$$

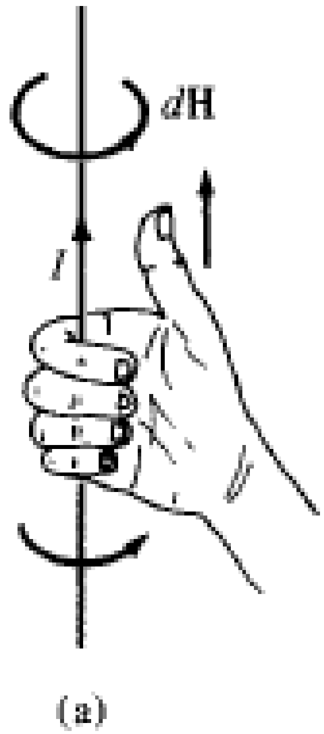
$$dH = \frac{kI\, dl \sin \alpha}{R^2}$$

, $k = 1/4\pi$,

$$dH = \frac{I\, dl \sin \alpha}{4\pi R^2}$$

$$d\mathbf{H} = \frac{I\, d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2} = \frac{I\, d\mathbf{l} \times \mathbf{R}}{4\pi R^3}$$





Determination of direction of dH

Different current distributions: line current, surface current, and volume current as shown in Figure .

K as the surface current density

and

J as the volume current density.

Then

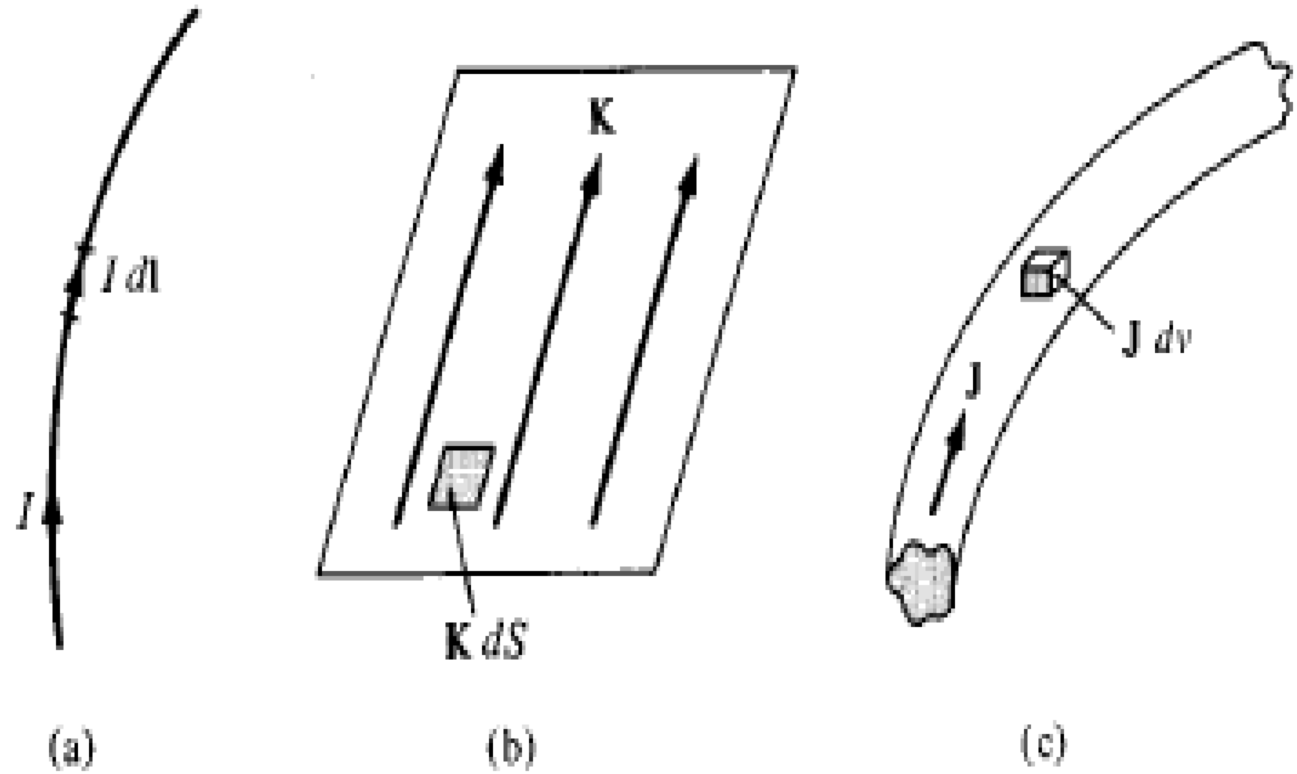
$$I\,d\mathbf{l} \equiv \mathbf{K}\,dS \equiv \mathbf{J}\,dv$$

The magnetic field produced by the current element I dl does not exert force on the element itself just as a point charge does not exert force on itself.

The B field that exerts force on I dl $_l$ must be due to another element.

If instead of the line current element I dl$_{l,}$ we have surface current elements K dS or a volume current element J dv, Then

$$dF = K\, dS\ X\ B \quad \text{Or} \quad dF = J\, dv\ X\ B$$



Current distributions:
(a) line current.
(b) surface current.
(c) volume current.

$$\mathbf{H} = \int_L \frac{I\, d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2} \qquad \text{(line current)}$$

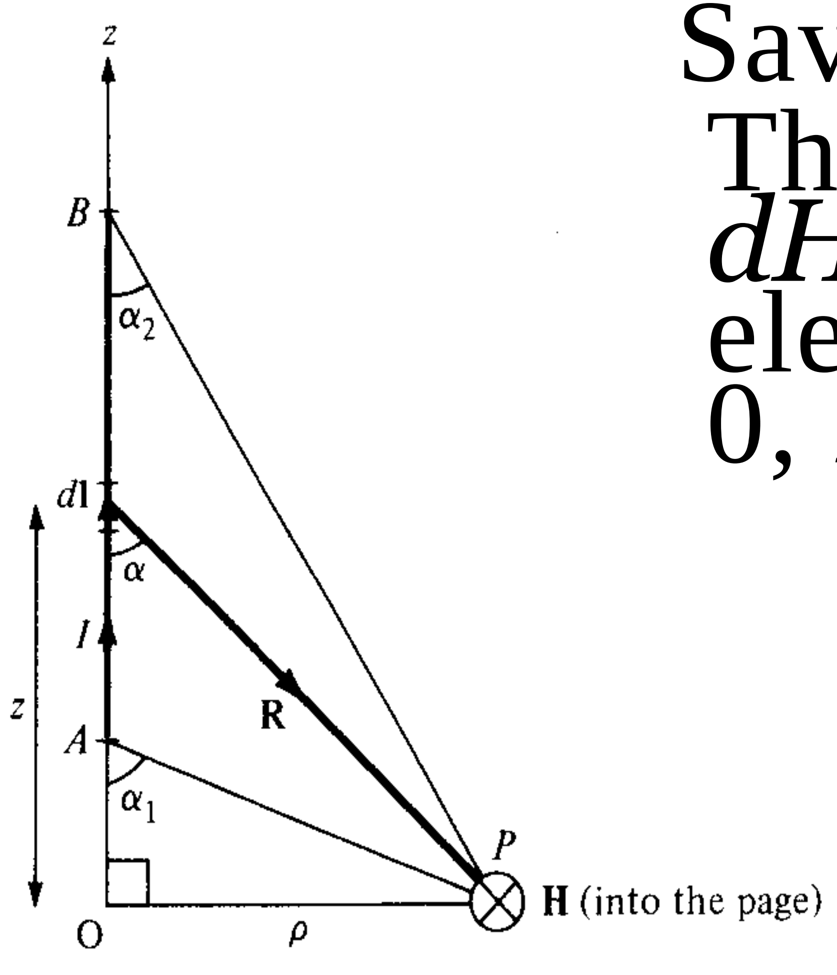
$$\mathbf{H} = \int_S \frac{\mathbf{K}\, dS \times \mathbf{a}_R}{4\pi R^2} \qquad \text{(surface current)}$$

$$\mathbf{H} = \int_v \frac{\mathbf{J}\, dv \times \mathbf{a}_R}{4\pi R^2} \qquad \text{(volume current)}$$

MFI due to a straight filamentary conductor.

According to Biot-Savart's Law

The contribution dH at P due to an element dl at (0, 0, z).



$$d\mathbf{H} = \frac{I\, d\mathbf{l} \times \mathbf{R}}{4\pi R^3}$$

But $d\mathbf{l} = dz\,\mathbf{a}_z$ and $\mathbf{R} = \rho\mathbf{a}_\rho - z\mathbf{a}_z$,

$$d\mathbf{l} \times \mathbf{R} = \rho\, dz\, \mathbf{a}_\phi$$

$$\mathbf{H} = \int \frac{I\rho\, dz}{4\pi[\rho^2 + z^2]^{3/2}}\, \mathbf{a}_\phi$$

Letting $z = \rho \cot \alpha$, $dz = -\rho\, \text{cosec}^2\, \alpha\, d\alpha$,

$$\mathbf{H} = -\frac{1}{4\pi} \int_{\alpha_1}^{\alpha_2} \frac{\rho^2\, \text{cosec}^2\, \alpha\, \text{d}\alpha}{\rho^3\, \text{cosec}^3\, \alpha}\, \mathbf{a}_\phi$$

$$= -\frac{I}{4\pi\rho}\, \mathbf{a}_\phi \int_{\alpha_1}^{\alpha_2} \sin \alpha\, d\alpha$$

$$\mathbf{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\mathbf{a}_\phi$$

When the conductor is semiinfinite (with respect to *P)* so that point *A* is now at O(0, 0, 0) while *B* is at (0, 0, ά); $\alpha_1 = 90°$ $\alpha_2 = 0°$.

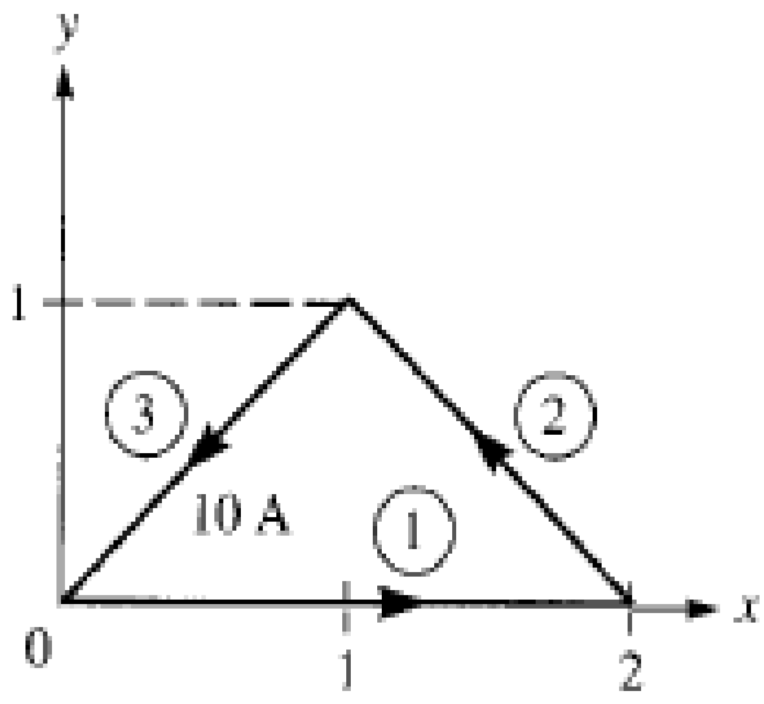
$$\mathbf{H} = \frac{I}{4\pi\rho}\mathbf{a}_\phi$$

When the conductor is *infinite* in length. point A is at (0, 0, -ά) while B is at (0, 0,ά); α1= 180°, α2= 0°

$$\mathbf{H} = \frac{I}{2\pi\rho}\mathbf{a}_\phi$$

$$\mathbf{a}_\phi = \mathbf{a}_\ell \times \mathbf{a}_\rho$$

MFI Due to a Triangular loop



$$\cos\alpha_1 = \cos 90^\circ = 0,$$

$$\cos\alpha_2 = \frac{2}{\sqrt{29}},$$

$$\rho = 5$$

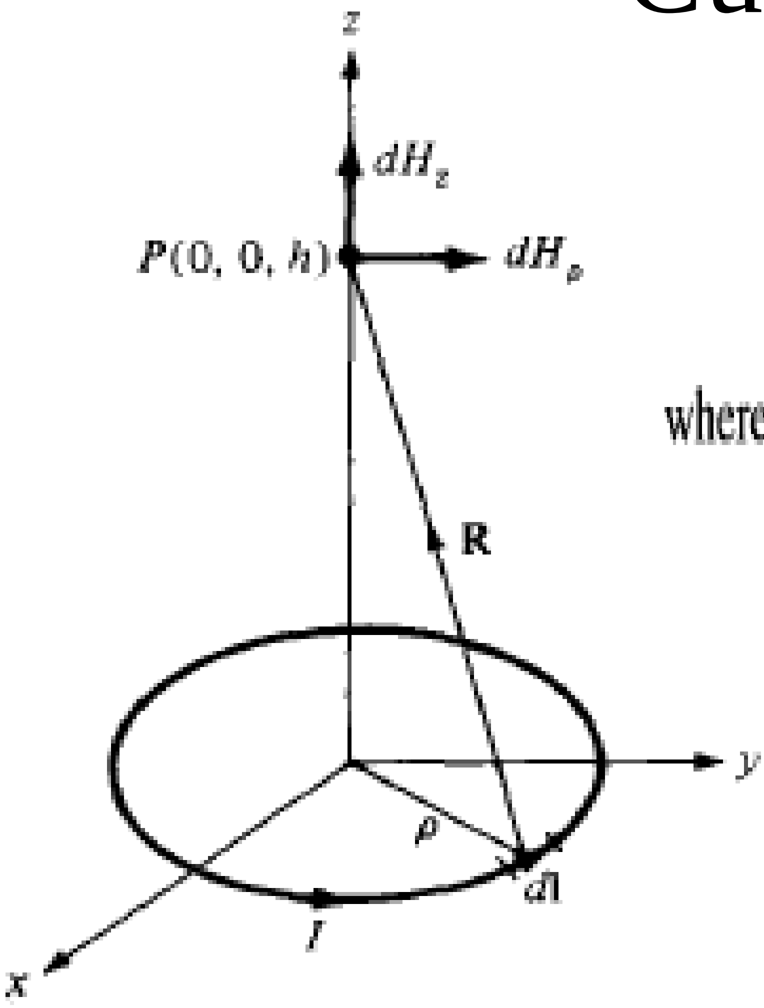
$$\mathbf{a}_\ell = \mathbf{a}_x \text{ and } \mathbf{a}_\rho = \mathbf{a}_z,$$

$$\mathbf{a}_\phi = \mathbf{a}_x \times \mathbf{a}_z = -\mathbf{a}_y$$

$$\mathbf{H}_1 = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\mathbf{a}_\phi = \frac{10}{4\pi(5)}\left(\frac{2}{\sqrt{29}} - 0\right)(-\mathbf{a}_y)$$

$$= -59.1\mathbf{a}_y \text{ mA/m}$$

MFI due to Circular Loop of Current



$$d\,\mathbf{H} = \frac{I\,d\mathbf{l} \times \mathbf{R}}{4\pi R^3}$$

where $d\mathbf{l} = \rho\, d\phi\, \mathbf{a}_\phi$, $\mathbf{R} = (0, 0, h) - (x, y, 0) = -\rho\mathbf{a}_\rho + h\mathbf{a}_z$,

$$d\mathbf{l} \times \mathbf{R} = \begin{vmatrix} \mathbf{a}_\rho & \mathbf{a}_\phi & \mathbf{a}_z \\ 0 & \rho\, d\phi & 0 \\ -\rho & 0 & h \end{vmatrix}$$

$$= \rho h\, d\phi\, \mathbf{a}_\rho + \rho^2\, d\phi\, \mathbf{a}_z$$

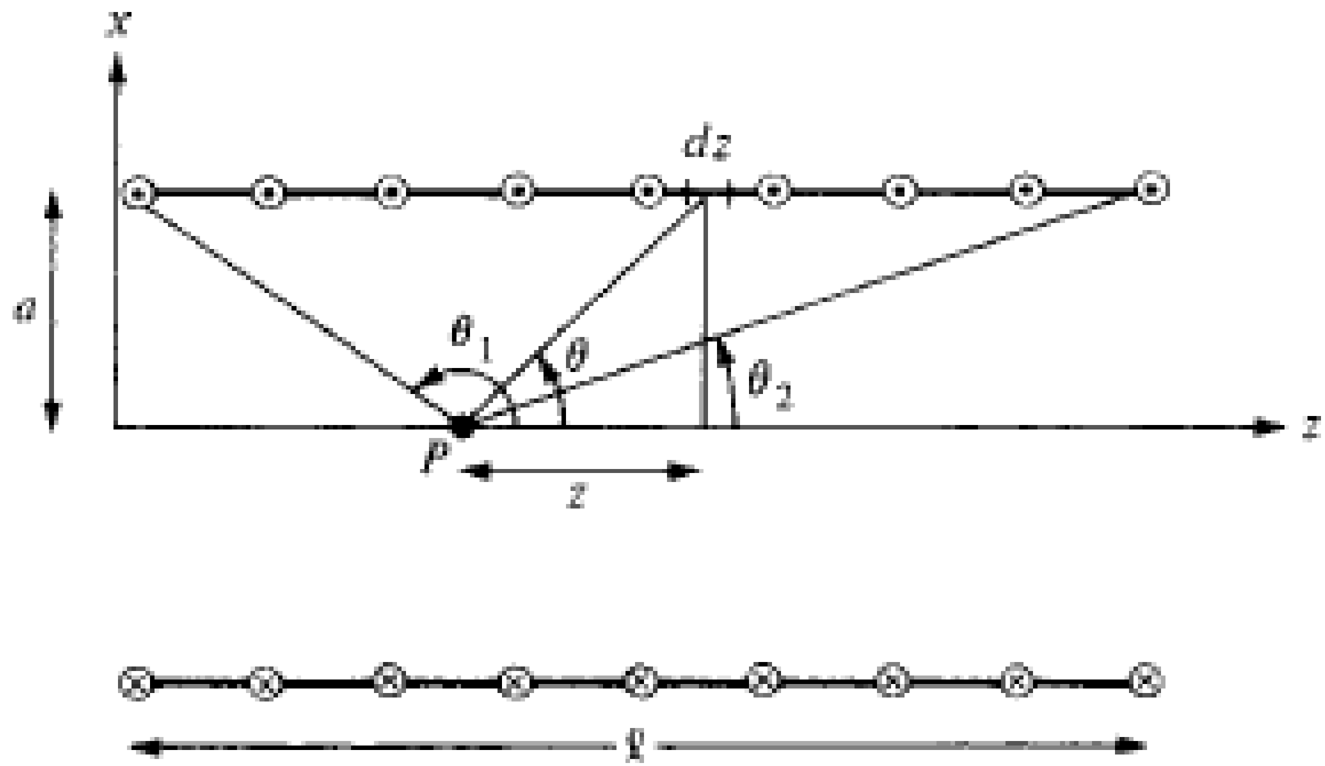
$$d\mathbf{H} = \frac{I}{4\pi[\rho^2 + h^2]^{3/2}} (\rho h \, d\phi \, \mathbf{a}_\rho + \rho^2 \, d\phi \, \mathbf{a}_z)$$

$$= dH_\rho \, \mathbf{a}_\rho + dH_z \, \mathbf{a}_z$$

$$\mathbf{H} = \int dH_z \, \mathbf{a}_z = \int_0^{2\pi} \frac{I\rho^2 \, d\phi \, \mathbf{a}_z}{4\pi[\rho^2 + h^2]^{3/2}} = \frac{I\rho^2 2\pi \mathbf{a}_z}{4\pi[\rho^2 + h^2]^{3/2}}$$

$$\mathbf{H} = \frac{I\rho^2 \mathbf{a}_z}{2[\rho^2 + h^2]^{3/2}}$$

MFI due to Solenoid Current



Cross sectional view of a solenoid.

z

The contribution to the magnetic field H at P by an element of the solenoid of length dz is

H

$$dH_z = \frac{I\,dl\,a^2}{2[a^2 + z^2]^{3/2}} = \frac{Ia^2 n\,dz}{2[a^2 + z^2]^{3/2}}$$

where $dl = n\,dz = (N/\ell)\,dz$.

$$\tan\theta = a/z$$

$$dz = -a\,\mathrm{cosec}^2\,\theta\,d\theta = -\frac{[z^2 + a^2]^{3/2}}{a^2}\sin\theta\,d\theta$$

$$dH_z = -\frac{nI}{2} \sin\theta \, d\theta$$

$$H_z = -\frac{nI}{2} \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$\mathbf{H} = \frac{nI}{2} (\cos\theta_2 - \cos\theta_1) \, \mathbf{a}_z$$

Substituting $n = N/\ell$ gives

$$\mathbf{H} = \frac{NI}{2\ell} (\cos\theta_2 - \cos\theta_1) \, \mathbf{a}_z$$

At the center of the solenoid,

$$\cos\theta_2 = \frac{\ell/2}{[a^2 + \ell^2/4]^{1/2}} = -\cos\theta_1$$

$$\mathbf{H} = \frac{In\ell}{2[a^2 + \ell^2/4]^{1/2}}\,\mathbf{a}_z$$

If $\ell \gg a$ or $\theta_2 \simeq 0°$, $\theta_1 \simeq 180°$,

$$\mathbf{H} = nI\mathbf{a}_z = \frac{NI}{\ell}\,\mathbf{a}_z$$

Ampere's Circuital Law

Ampere's circuit law states that the line integral of the tangential component of H around a closed path is the same as the net current I_{enc} enclosed by the path.

In other words, the circulation of **H** equals I_{enc}; that is,

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc}$$

$$I_{\text{enc}} = \oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{S}$$

$$I_{\text{enc}} = \int_S \mathbf{J} \cdot d\mathbf{S}$$

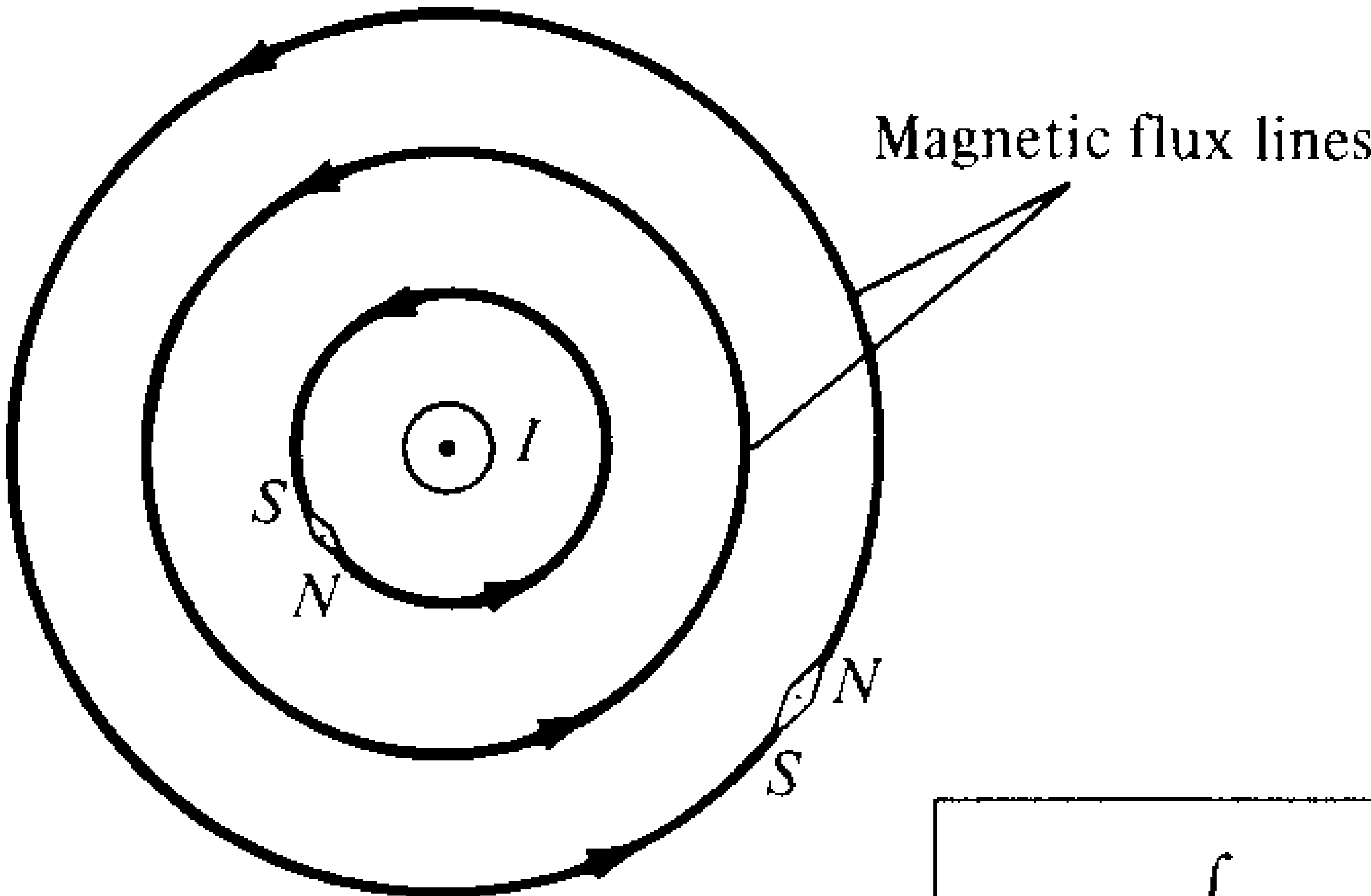
$$\boxed{\nabla \times \mathbf{H} = \mathbf{J}}$$

The magnetic flux density B is similar to the electric flux density D.
The magnetic flux density B is related to the magnetic field intensity H according to

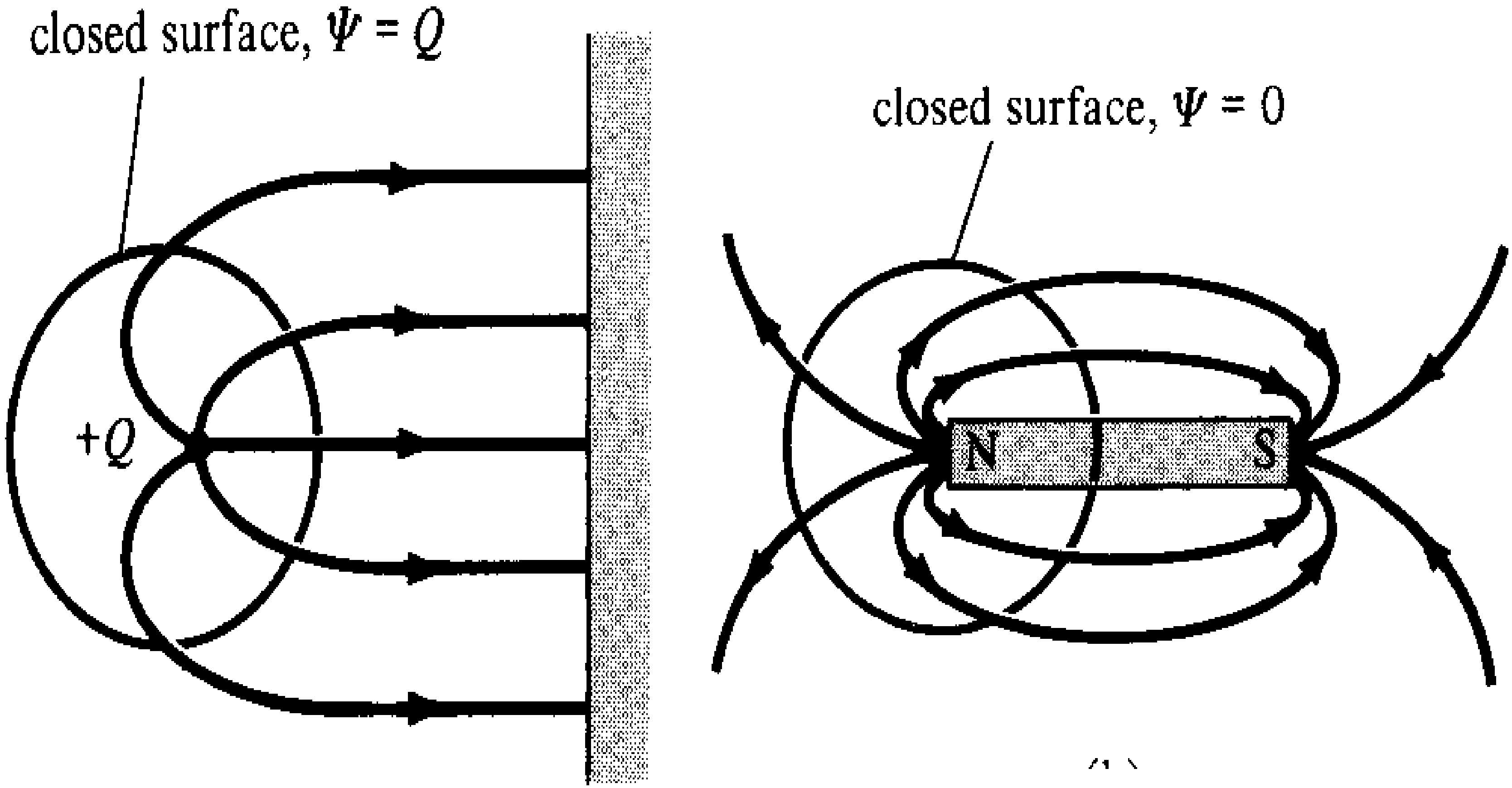
$$\mathbf{B} = \mu_o \mathbf{H}$$

Where μ_o is a constant known as the permeability of free space. The constant is in henrys/meter (H/m) and has the value of

$$\mu_o = 4\pi \times 10^{-7} \text{ H/m}$$



$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}$$


closed surface, Ψ = Q
+Q
closed surface, Ψ = 0
N
S

The Total Flux through a closed Surface in a magnetic field must be zero

$$\boxed{\oint \mathbf{B} \cdot d\mathbf{S} = 0}$$

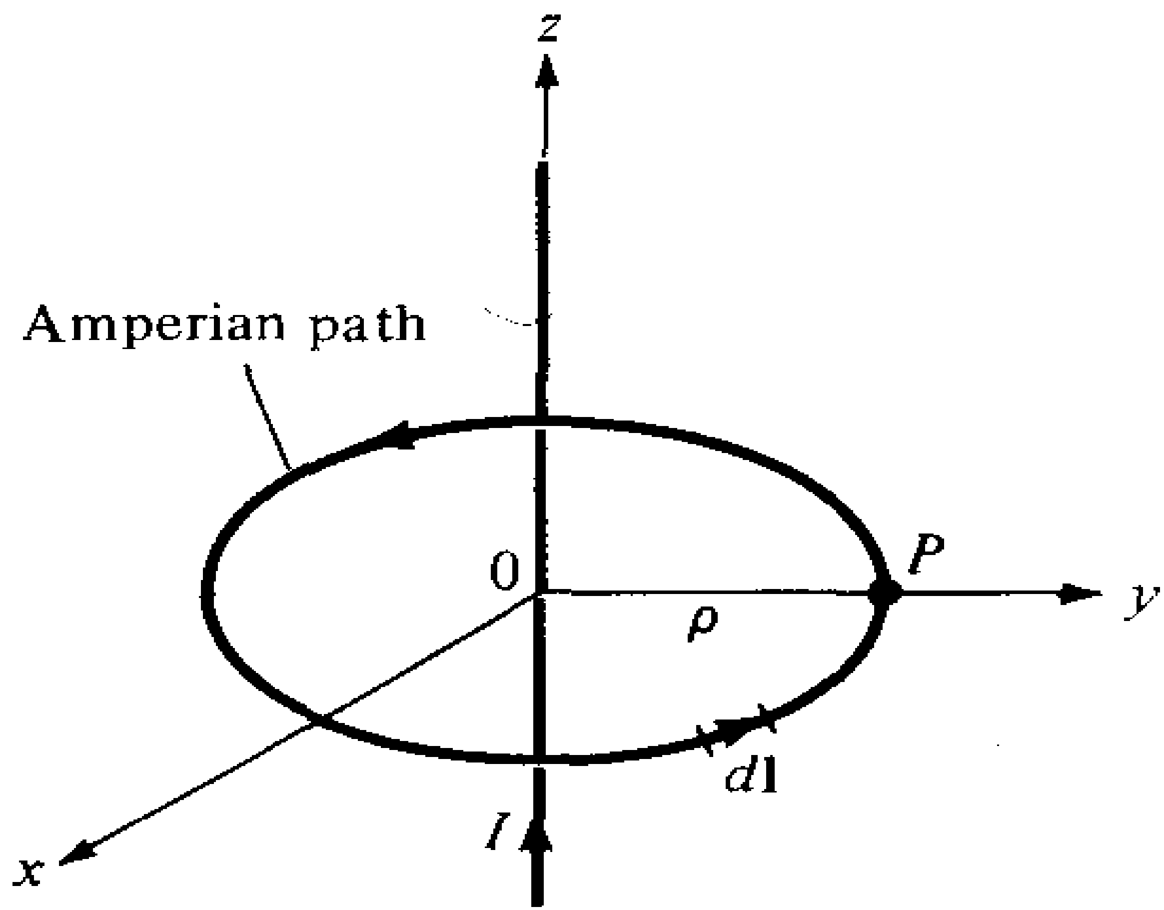
$$\oint_S \mathbf{B} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{B}\, dv = 0$$

$$\boxed{\nabla \cdot \mathbf{B} = 0}$$

- MFI due to infinite sheet of current and a long current carrying conductor
- Point form of Ampere's Law
- Field due to circular loop, rectangular and square loops.

MFI Due to Infinite line Current

- Consider an infinitely long filamentary current I along the z-axis as in Figure.
- We choose a concentric circle as the Amperian path in view of ampere's law, which shows that H is constant provided ρ is constant.



Ampere's law applied to an infinite filamentary line current.

Since this path encloses the whole current I, according to Ampere's law

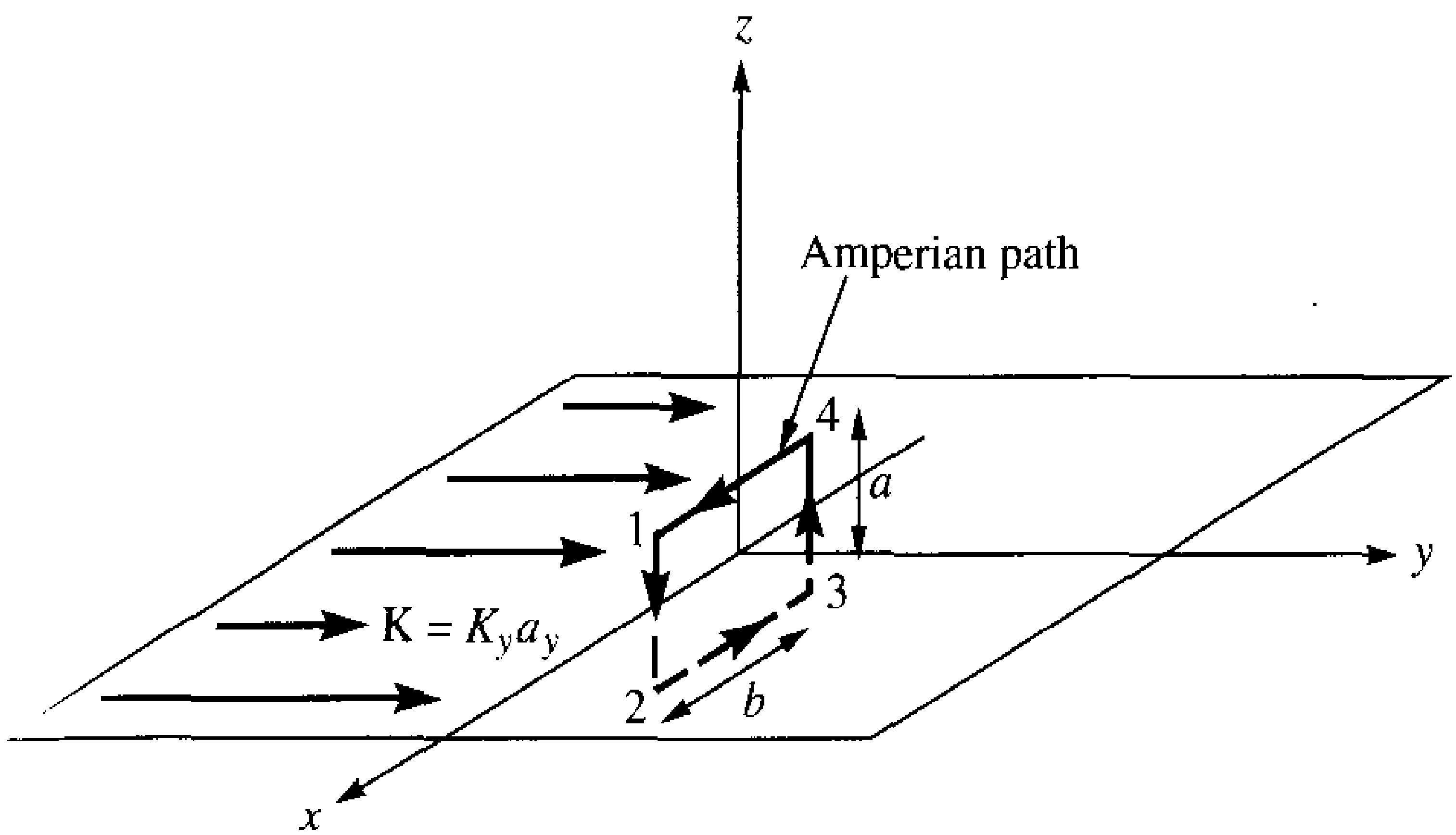
$$I = \int H_\phi \mathbf{a}_\phi \cdot \rho \, d\phi \, \mathbf{a}_\phi$$

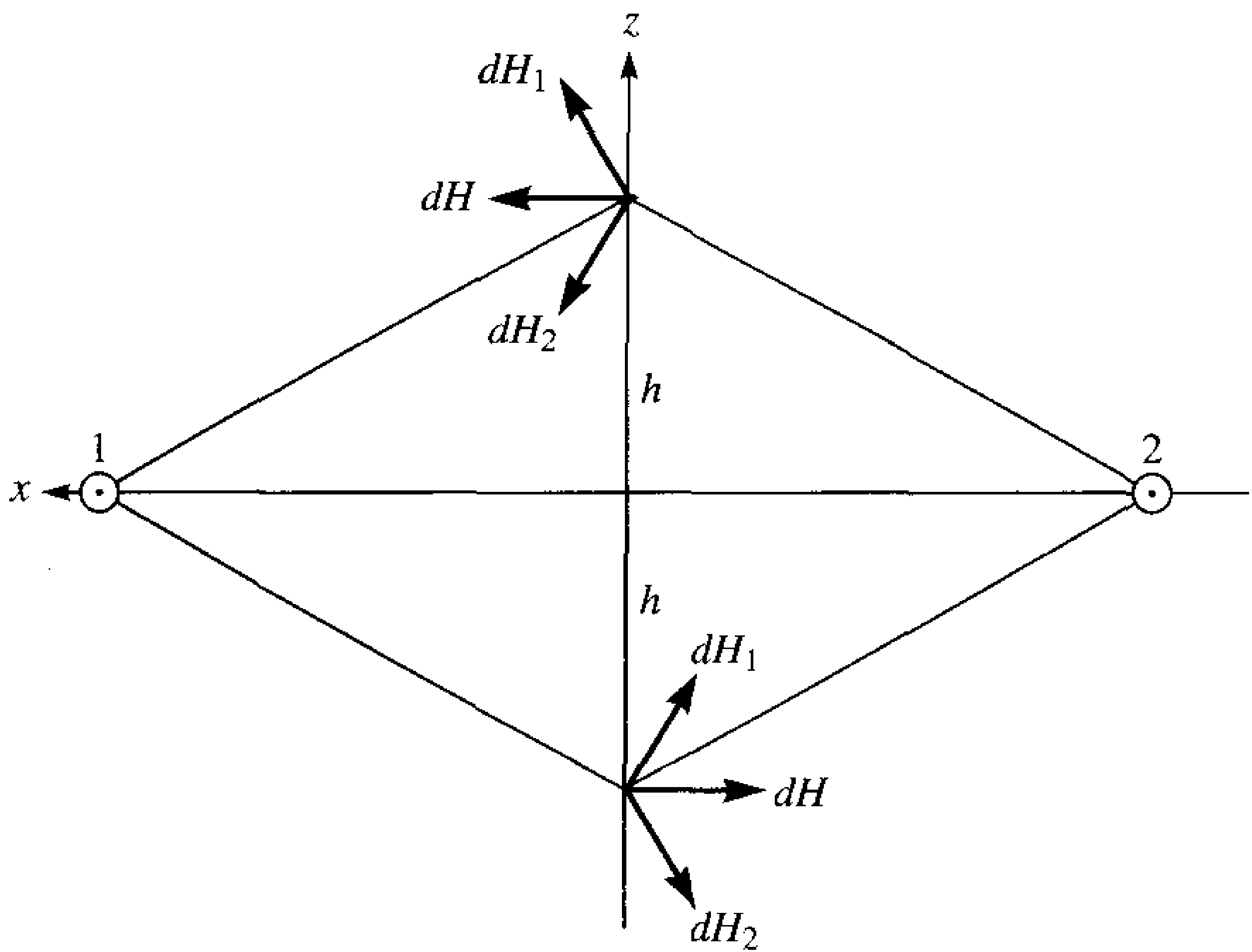
$$= H_\phi \int \rho \, d\phi$$

$$= H_\phi \cdot 2\pi\rho$$

$$\boxed{\mathbf{H} = \frac{I}{2\pi\rho} \mathbf{a}_\phi}$$

MFI Due to Infinite Sheet of Current





z
dH1
dH
dH2
h
1
2
x
h
dH1
dH
dH2

Consider an infinite current sheet in the $z = 0$ plane.
If the sheet has a uniform current density $\mathbf{K} = K_y \mathbf{a}_y$ A/m

Applying Ampere's law to the rectangular closed path gives

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc} = K_y b$$

the resultant dH has only an x-component.
Also, H on one side of the sheet is the negative of that on the other side.

Due to the infinite extent of the sheet, the sheet can be regarded as consisting of such filamentary pairs so that the characteristics of H for a pair are the same for the infinite current sheets, that is,

$$\mathbf{H} = \begin{cases} H_o \mathbf{a}_x & z > 0 \\ -H_o \mathbf{a}_x & z < 0 \end{cases}$$

Evaluating the line integral of H along the closed path in Figure gives

$$\oint \mathbf{H} \cdot d\mathbf{l} = \left(\int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 \right) \mathbf{H} \cdot d\mathbf{l}$$

$$= 0(-a) + (-H_o)(-b) + 0(a) + H_o(b)$$
$$= 2H_o b$$

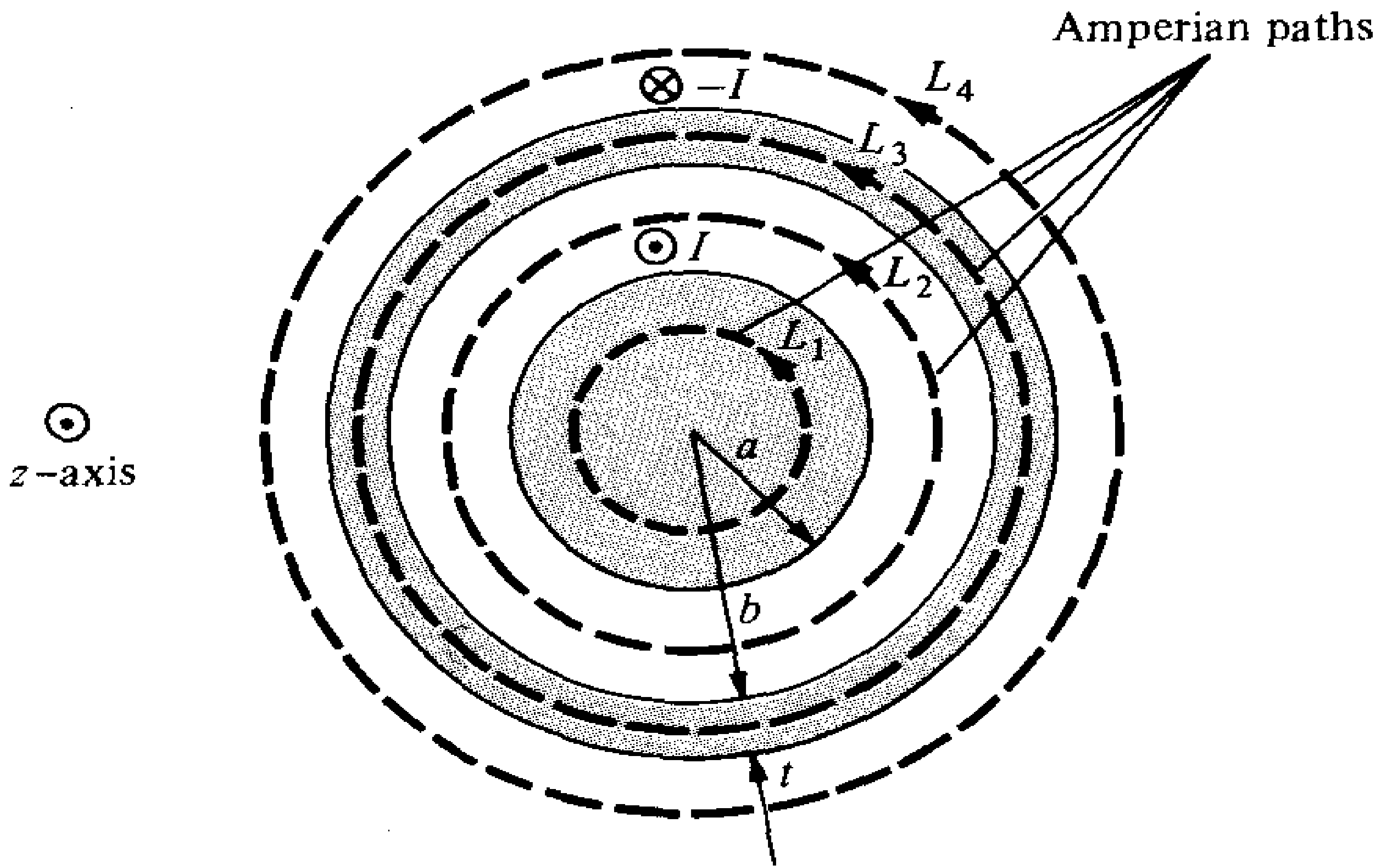
$$\mathbf{H} = \begin{cases} \frac{1}{2} K_y \mathbf{a}_x, & z > 0 \\ -\frac{1}{2} K_y \mathbf{a}_x, & z < 0 \end{cases}$$

$$\boxed{\mathbf{H} = \frac{1}{2} \mathbf{K} \times \mathbf{a}_n}$$

Infinitely Long Coaxial Transmission Line

- Consider an infinitely long transmission line consisting of two concentric cylinders having their axes along the z-axis.

- The cross section of the line is shown in Figure, where the z-axis is out of the page. The inner conductor has radius a and carries current I while the outer conductor has inner radius b and thickness t



Cross section of the transmission line, the positive - direction is out of the page.

$0 \leq \rho \leq a, a \leq \rho \leq b, b \leq \rho \leq b + t,$

and $\rho \geq b + t.$

For region $0 \leq \rho \leq a$, we apply Ampere's law to path L_1, giving

$$\oint_{L_1} \mathbf{H} \cdot d\mathbf{l} = I_{enc} = \int \mathbf{J} \cdot d\mathbf{S}$$

$$\mathbf{J} = \frac{I}{\pi a^2}\mathbf{a}_z, \qquad d\mathbf{S} = \rho\, d\phi\, d\rho\, \mathbf{a}_z$$

$$I_{enc} = \int \mathbf{J} \cdot d\mathbf{S} = \frac{I}{\pi a^2}\iint \rho\, d\phi\, d\rho = \frac{I}{\pi a^2}\pi\rho^2 = \frac{I\rho^2}{a^2}$$

$$H_\phi \int dl = H_\phi 2\pi\rho = \frac{I\rho^2}{a^2}$$

$$H_\phi = \frac{I\rho}{2\pi a^2}$$

$$\oint_{L_2} \mathbf{H} \cdot d\mathbf{l} = I_{\text{enc}} = I$$

$$H_\phi 2\pi\rho = I$$

$$H_\phi = \frac{I}{2\pi\rho}$$

For region $b \leq \rho \leq b + t$, we use path L_3, getting

$$\oint \mathbf{H} \cdot d\mathbf{l} = H_\phi \cdot 2\pi\phi = I_{\text{enc}}$$

$$I_{\text{enc}} = I + \int \mathbf{J} \cdot d\mathbf{S}$$

$$\mathbf{J} = -\frac{I}{\pi[(b + t)^2 - b^2]}\,\mathbf{a}_z$$

$$I_{\text{enc}} = I - \frac{I}{\pi[(b + t)^2 - t^2]} \int_{\phi=0}^{2\pi} \int_{\rho=b}^{\rho} \rho\, d\rho\, d\phi$$

$$= I\left[1 - \frac{\rho^2 - b^2}{t^2 + 2bt}\right]$$

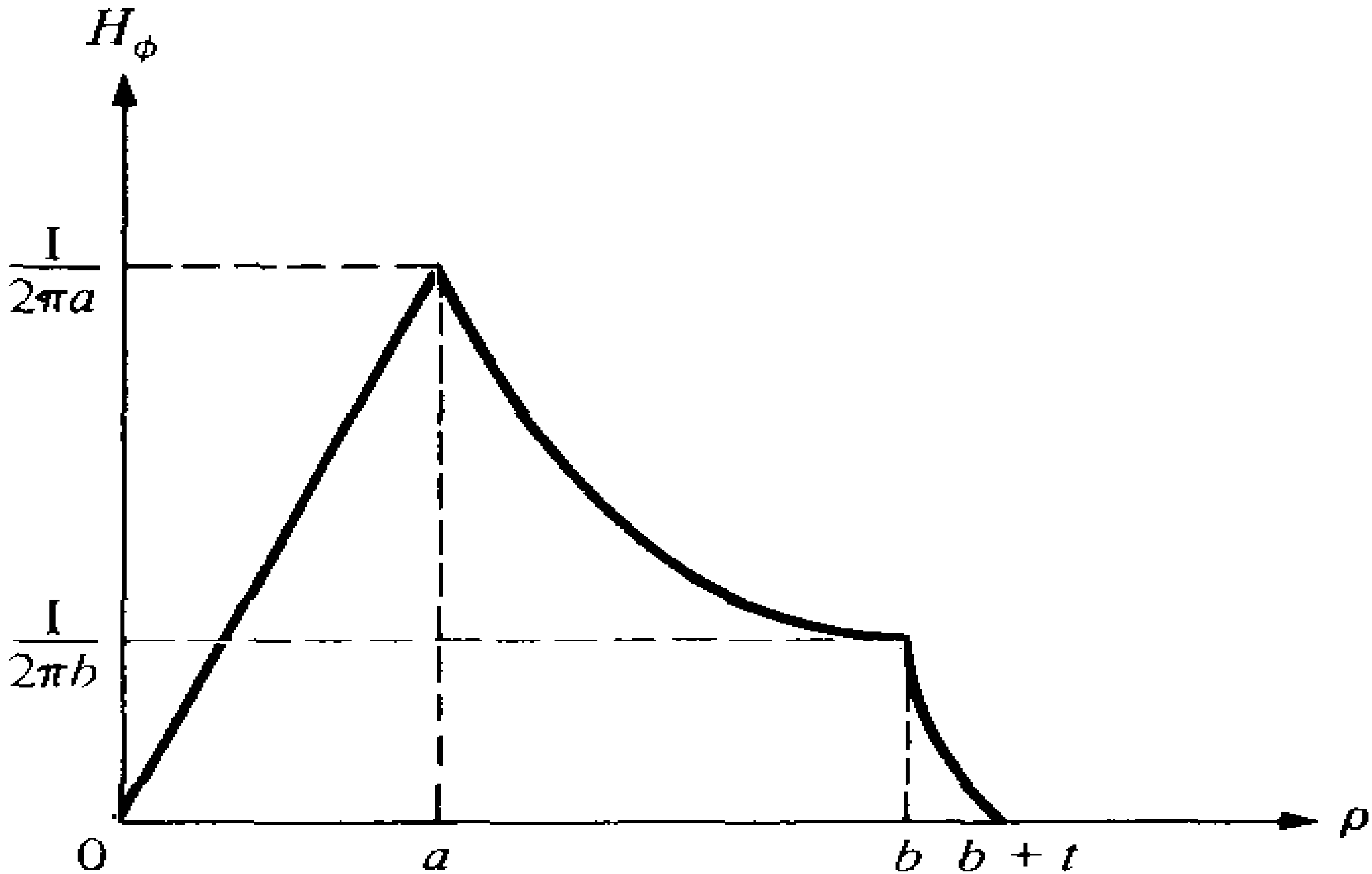
$$H_\phi = \frac{I}{2\pi\rho}\left[1 - \frac{\rho^2 - b^2}{t^2 + 2bt}\right]$$

For region $\rho \geq b + t$, we use path L_4, getting

$$\oint_{L_4} \mathbf{H} \cdot d\mathbf{l} = I - I = 0$$

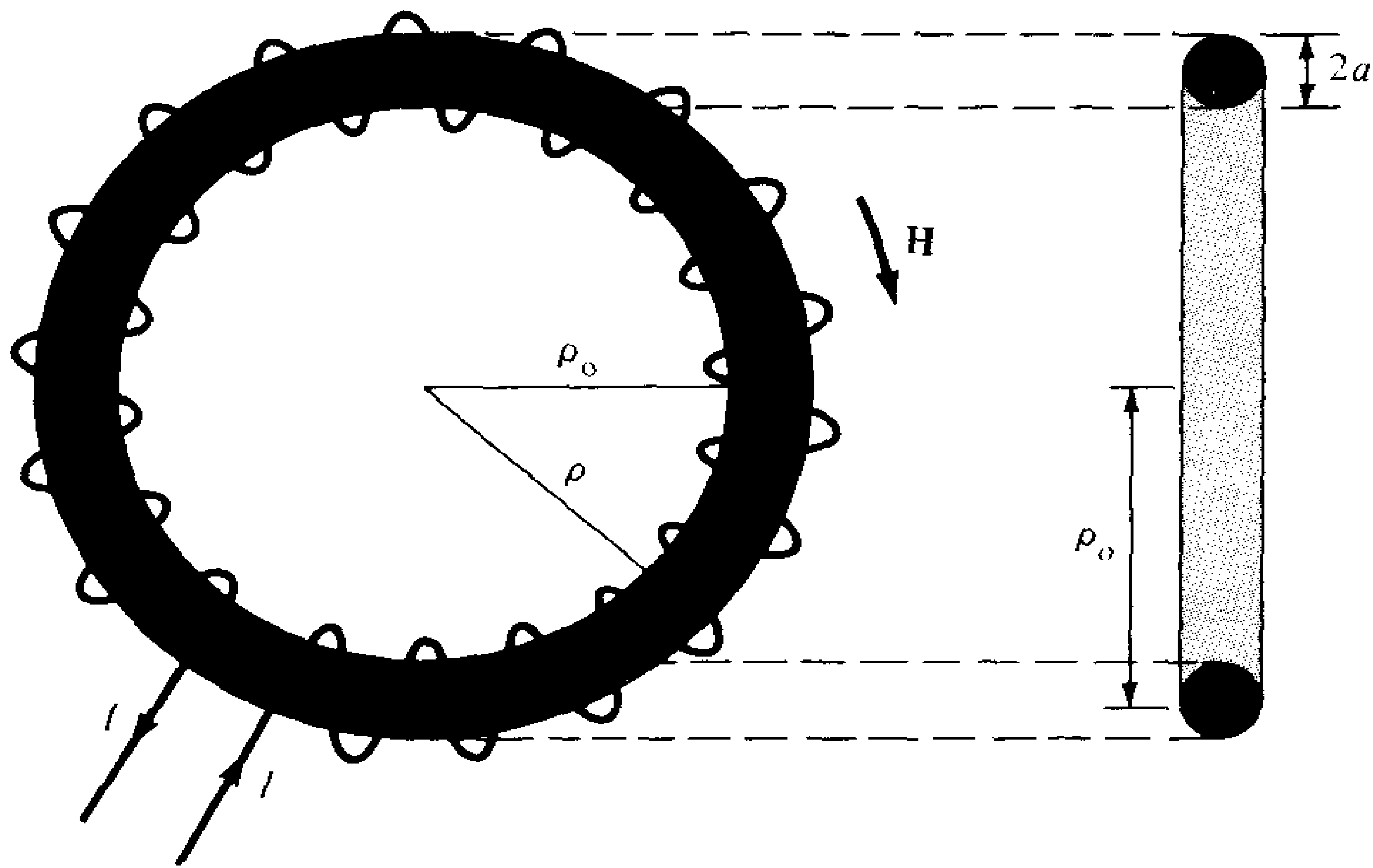
$$H_\phi = 0$$

$$
\mathbf{H} = \begin{cases} \dfrac{I\rho}{2\pi a^2}\,\mathbf{a}_\phi, & 0 \le \rho \le a \\ \dfrac{I}{2\pi\rho}\,\mathbf{a}_\phi, & a \le \rho \le b \\ \dfrac{I}{2\pi\rho}\left[1 - \dfrac{\rho^2 - b^2}{t^2 + 2bt}\right]\mathbf{a}_\phi, & b \le \rho \le b + t \\ 0, & \rho \ge b + t \end{cases}
$$



Plot of H_φ against ρ.

Toroid



$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{\text{enc}} \rightarrow H \cdot 2\pi\rho = NI$$

$$H = \frac{NI}{2\pi\rho}, \qquad \text{for } \rho_o - a < \rho < \rho_o + a$$

$$H_{\text{approx}} = \frac{NI}{2\pi\rho_o} = \frac{NI}{\ell}$$

Ampere' Circuital Law

- Ampere'scircuit law states that the line integral of the tangential component of H around a dosed path is the same as the net current I_{enc}.IK. enclosed by the path.

$$dH_z = -\frac{nI}{2}\sin\theta\, d\theta$$

$$H_z = -\frac{nI}{2}\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta$$

$$\mathbf{H} = \frac{nI}{2}(\cos\theta_2 - \cos\theta_1)\,\mathbf{a}_z$$

Substituting $n = N/\ell$ gives

$$\mathbf{H} = \frac{NI}{2\ell}(\cos\theta_2 - \cos\theta_1)\,\mathbf{a}_z$$

At the center of the solenoid,

$$\cos\theta_2 = \frac{\ell/2}{[a^2 + \ell^2/4]^{1/2}} = -\cos\theta_1$$

$$\mathbf{H} = \frac{In\ell}{2[a^2 + \ell^2/4]^{1/2}}\,\mathbf{a}_z$$

If $\ell \gg a$ or $\theta_2 \simeq 0°$, $\theta_1 \simeq 180°$,

$$\mathbf{H} = nI\mathbf{a}_z = \frac{NI}{\ell}\,\mathbf{a}_z$$

Ampere's Circuital Law

Ampere's circuit law states that the line integral of the tangential component of H around a closed path is the same as the net current I_{enc} enclosed by the path.

In other words, the circulation of **H** equals I_{enc}; that is,

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc}$$

$$I_{\text{enc}} = \oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{S}$$

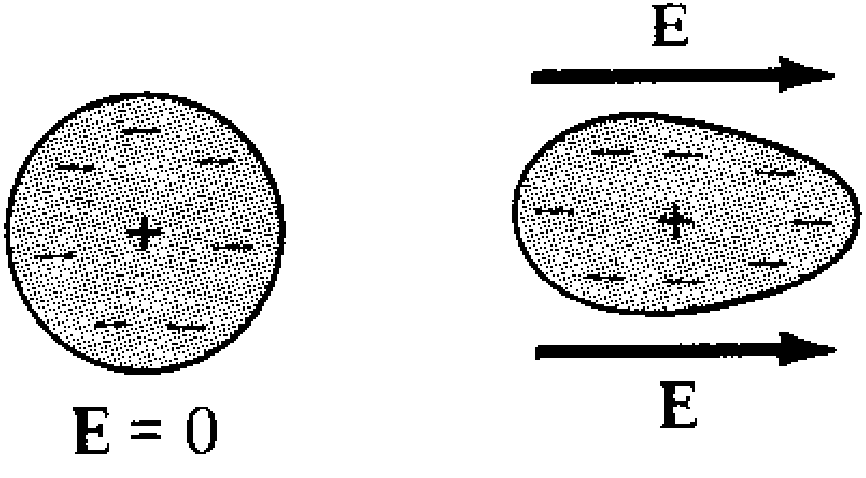
$$I_{\text{enc}} = \int_S \mathbf{J} \cdot d\mathbf{S}$$

$$\boxed{\nabla \times \mathbf{H} = \mathbf{J}}$$

- Electric Field inside a dielectric Material
- Dielectric- Conductor And Dielectric – Dielectric Boundary Conditions
- Capacitance
- Current Density
- Ohm's Law
- Equation of Continuity

Electric Field inside a Dielectric Material

consider an atom of the dielectric as consisting of a negative charge -Q (electron cloud) and a positive charge +Q(nucleus) as in Figure



- When an electric field E is applied, the positive charge is displaced from its equilibrium position in the direction of E by the force $F_+ = QE$ while the negative charge is displaced in the opposite direction by the force $F_- = -QE$

- A dipole results from the displacement of the charges and the dielectric is said to be polarized.

- $$\mathbf{p} = Q\mathbf{d}$$

$$Q_1\mathbf{d}_1 + Q_2\mathbf{d}_2 + \cdots + Q_N\mathbf{d}_N = \sum_{k=1}^{N} Q_k\mathbf{d}_k$$

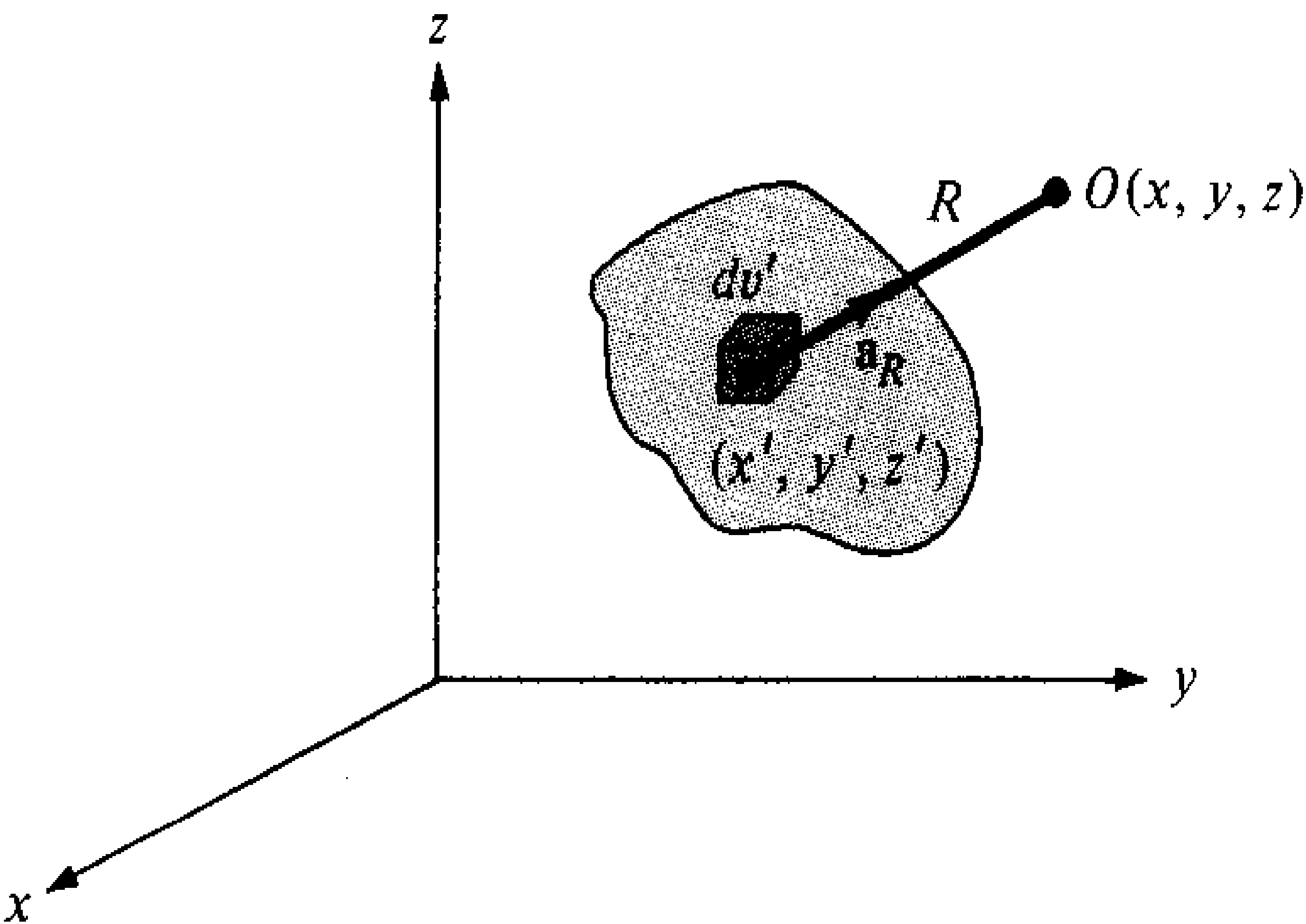
$$\boxed{\mathbf{P} = \frac{\lim\limits_{\Delta v \to 0} \sum\limits_{k=1}^{N} Q_k\mathbf{d}_k}{\Delta v}}$$

The major effect of the electric field E on a dielectric is the creation of dipole moments that align themselves in the direction of E.

$$dV = \frac{\mathbf{P} \cdot \mathbf{a}_R \, dv'}{4\pi\varepsilon_o R^2}$$

$$\nabla' = \frac{1}{R} = \frac{\mathbf{a}_R}{R^2}$$

$$\frac{\mathbf{P} \cdot \mathbf{a}_R}{R^2} = \mathbf{P} \cdot \nabla' \left(\frac{1}{R} \right)$$



z
R
O(x, y, z)
dv'
a_R
(x', y', z')
y
x

$$\nabla' \cdot f\mathbf{A} = f\nabla' \cdot \mathbf{A} + \mathbf{A} \cdot \nabla' f,$$

$$\frac{\mathbf{P} \cdot \mathbf{a}_R}{R^2} = \nabla' \cdot \frac{\mathbf{P}}{R} - \frac{\nabla' \cdot \mathbf{P}}{R}$$

$$V = \int_{v'} \frac{1}{4\pi\varepsilon_o}\left[\nabla' \cdot \frac{\mathbf{P}}{R} - \frac{1}{R}\nabla' \cdot \mathbf{P}\right]dv'$$

$$V = \int_{S'} \frac{\mathbf{P} \cdot \mathbf{a}'_n}{4\pi\varepsilon_o R}\,dS' + \int_{v'} \frac{-\nabla' \cdot \mathbf{P}}{4\pi\varepsilon_o R}\,dv'$$

$$\boxed{\begin{aligned} \rho_{ps} &= \mathbf{P} \cdot \mathbf{a}_n \\ \rho_{pv} &= -\nabla \cdot \mathbf{P} \end{aligned}}$$

The total positive bound charge on surface S bounding the dielectric is

$$Q_b = \oint \mathbf{P} \cdot d\mathbf{S} = \int \rho_{ps}\, dS$$

while the charge that remains inside S is

$$-Q_b = \int_v \rho_{pv}\, dv = -\int_v \nabla \cdot \mathbf{P}\, dv$$

Thus the total charge of the dielectric material remains zero, that is,

$$\text{Total charge} = \oint_S \rho_{ps}\, dS + \int_v \rho_{pv}\, dv = Q_b - Q_b = 0$$

We now consider the case in which the dielectric region contains free charge. If p_v is the free charge volume density, the total volume charge density p_t is given by

$$\rho_t = \rho_v + \rho_{pv} = \nabla \cdot \varepsilon_o \mathbf{E}$$

$$
\begin{aligned}
\rho_v &= \nabla \cdot \varepsilon_o \mathbf{E} - \rho_{pv} \\
&= \nabla \cdot (\varepsilon_o \mathbf{E} + \mathbf{P}) \\
&= \nabla \cdot \mathbf{D}
\end{aligned}
$$

$$
\boxed{\mathbf{D} = \varepsilon_o \mathbf{E} + \mathbf{P}}
$$

We would expect that the polarization P would vary directly as the applied electric field E. For some dielectrics, this is usually the case and we have

$$
\boxed{\mathbf{P} = \chi_e \varepsilon_o \mathbf{E}}
$$

DIELECTRIC CONSTANT AND STRENGTH

$$\mathbf{D} = \varepsilon_o(1 + \chi_e)\,\mathbf{E} = \varepsilon_o\varepsilon_r\mathbf{E}$$

$$\boxed{\mathbf{D} = \varepsilon\mathbf{E}}$$

$$\boxed{\varepsilon = \varepsilon_o\varepsilon_r}$$

$$\boxed{\varepsilon_r = 1 + \chi_e = \frac{\varepsilon}{\varepsilon_o}}$$

ε is called the *permittivity* of the dielectric, ε_o is the permittivity of free space, as approximately $10^{-9}/36\pi$ F/m, and ε_r is called the *dielectric constant* or *relative permittivity*.

The dielectric strength is the maximum electric field that a dielectric can tolerate or withstand without breakdown.

A dielectric material is linear if ε does not change with applied E field. homogeneous if ε does not change from point to point, and isotropic if ε does not change with direction.

$$\mathbf{F} = \frac{Q_1 Q_2}{4\pi\varepsilon_o\varepsilon_r R^2}\,\mathbf{a}_R$$

$$W = \frac{1}{2}\int \varepsilon_o\varepsilon_r E^2\, dv$$

BOUNDARY CONDITIONS

Dielectric-Dielectric Boundary Conditions

Consider the E field existing in a region consisting of two different dielectrics characterized by $\varepsilon_1 = \varepsilon_0 \varepsilon_{r1}$ and $\varepsilon_2 = \varepsilon_0 \varepsilon_{r2}$ as shown in Figure. E_1 and E_2 in media 1 and 2, respectively, can be decomposed as

$$\mathbf{E}_1 = \mathbf{E}_{1t} + \mathbf{E}_{1n}$$

$$\mathbf{E}_2 = \mathbf{E}_{2t} + \mathbf{E}_{2n}$$

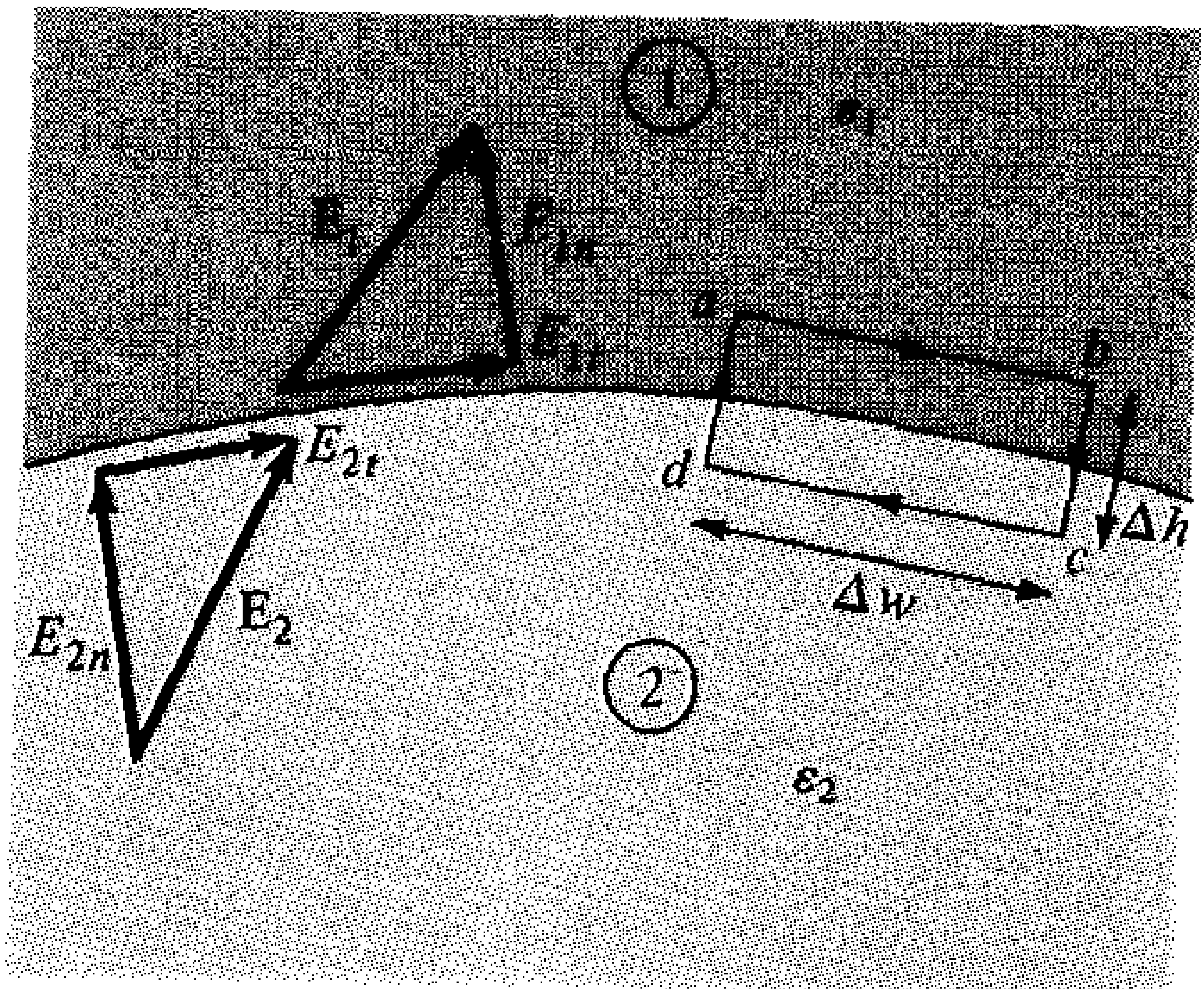
$$0 = E_{1t}\,\Delta w - E_{1n}\frac{\Delta h}{2} - E_{2n}\frac{\Delta h}{2} - E_{2t}\,\Delta w + E_{2n}\frac{\Delta h}{2} + E_{1n}\frac{\Delta h}{2}$$

$E_t = |\mathbf{E}_t|$ and $E_n = |\mathbf{E}_n|$. As $\Delta h \to 0$,

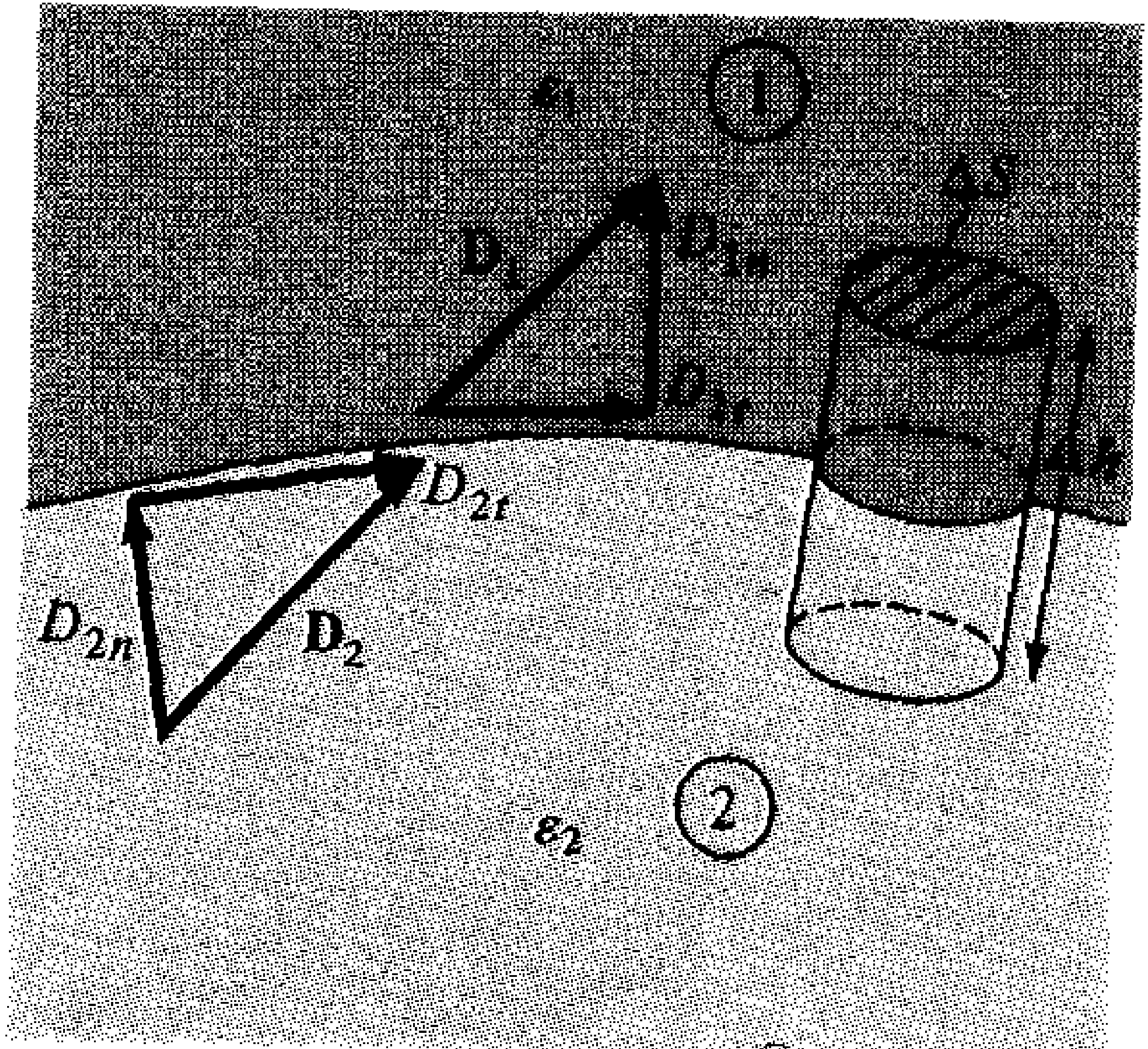
$$\boxed{E_{1t} = E_{2t}}$$

$$\frac{D_{1t}}{\varepsilon_1} = E_{1t} = E_{2t} = \frac{D_{2t}}{\varepsilon_2}$$

$$\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$$



1
ε_1
$\mathbf{E}_1$
E_{1n}
E_{1t}
a
b
E_{2t}
d
c
Δh
Δw
$\mathbf{E}_2$
E_{2n}
2
ε_2


1
D1
D2t
D2n
D2
ε2
2

$$\Delta Q = \rho_S \, \Delta S = D_{1n} \, \Delta S - D_{2n} \, \Delta S$$

$$\boxed{D_{1n} - D_{2n} = \rho_S}$$

$$\boxed{D_{1n} = D_{2n}}$$

$$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$$

$$E_1 \sin \theta_1 = E_{1t} = E_{2t} = E_2 \sin \theta_2$$

$$E_1 \sin\theta_1 = E_2 \sin\theta_2$$

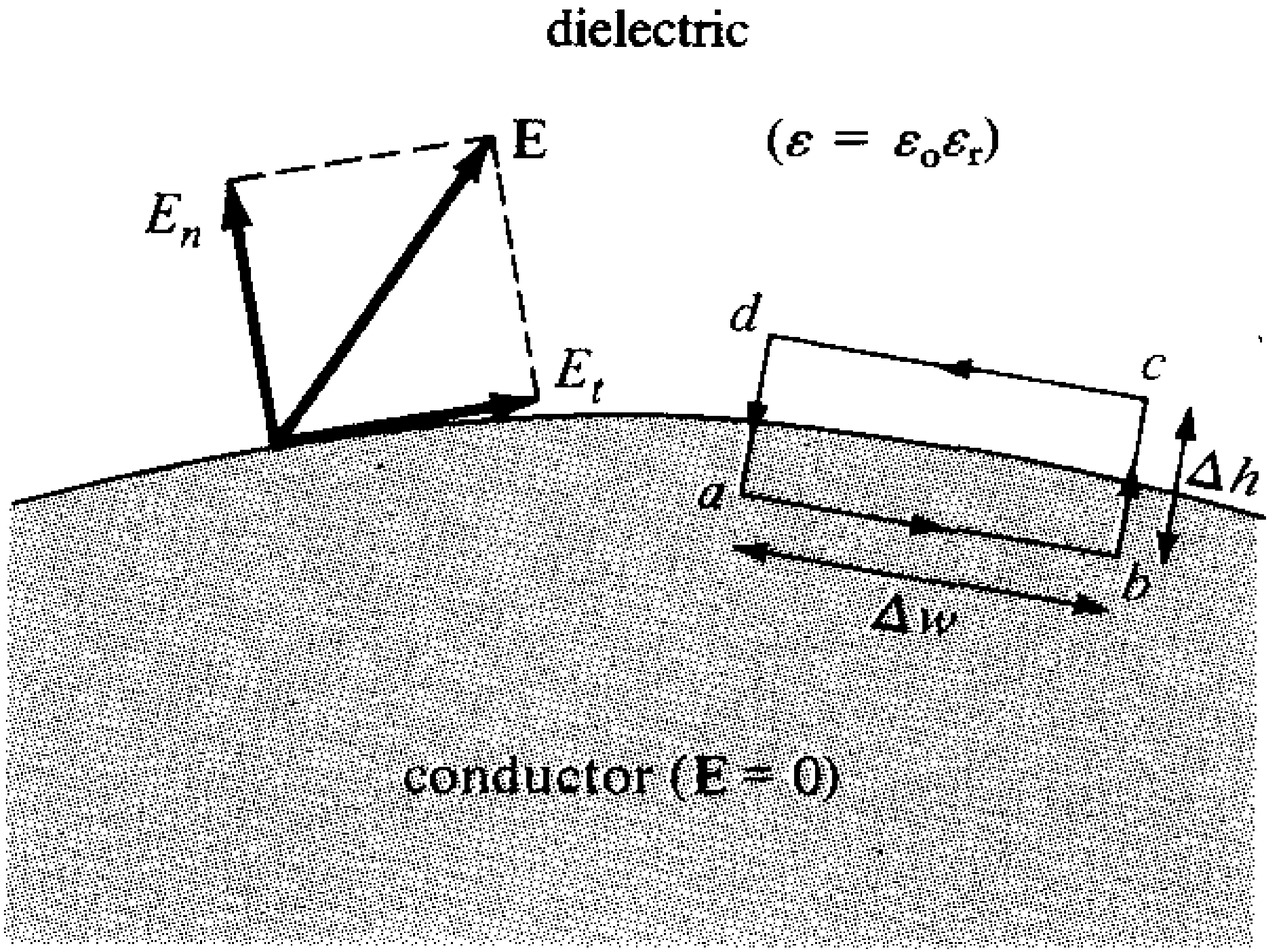
$$\varepsilon_1 E_1 \cos\theta_1 = D_{1n} = D_{2n} = \varepsilon_2 E_2 \cos\theta_2$$

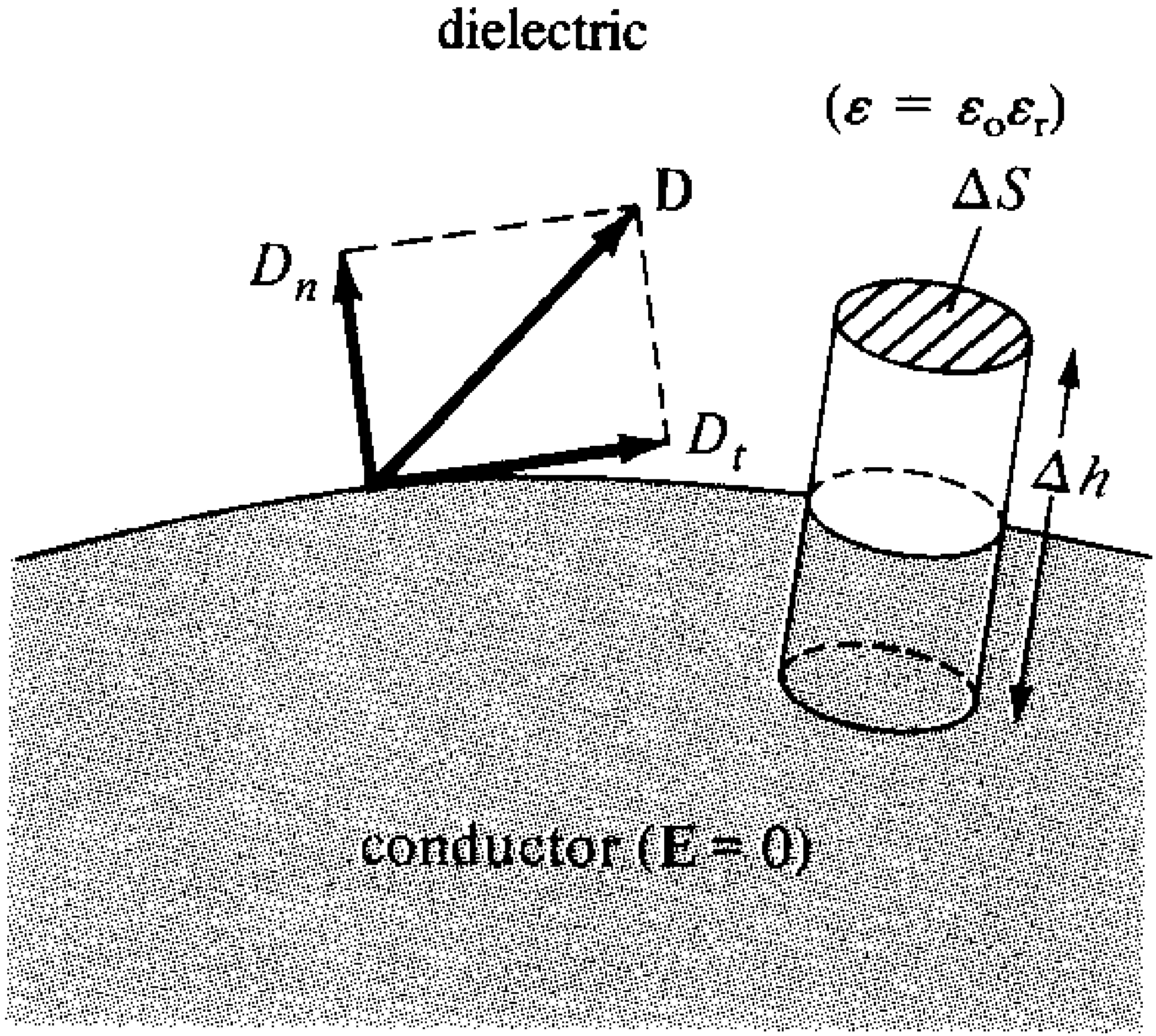
$$\varepsilon_1 E_1 \cos\theta_1 = \varepsilon_2 E_2 \cos\theta_2$$

$$\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}}$$

Conductor-Dielectric Boundary Conditions




dielectric
$(\varepsilon = \varepsilon_o \varepsilon_r)$
$\mathbf{D}$
D_n
D_t
ΔS
Δh
conductor ($\mathbf{E} = 0$)

$$0 = 0 \cdot \Delta w + 0 \cdot \frac{\Delta h}{2} + E_n \cdot \frac{\Delta h}{2} - E_t \cdot \Delta w - E_n \cdot \frac{\Delta h}{2} - 0 \cdot \frac{\Delta h}{2}$$

As $\Delta h \to 0$,

$$E_t = 0$$

$$\Delta Q = D_n \cdot \Delta S - 0 \cdot \Delta S$$

$$D_n = \frac{\Delta Q}{\Delta S} = \rho_S$$

$$D_n = \rho_S$$

1. No electric field may exist *within* a conductor; that is,

$$\boxed{\rho_v = 0, \qquad \mathbf{E} = 0} \tag{5.70}$$

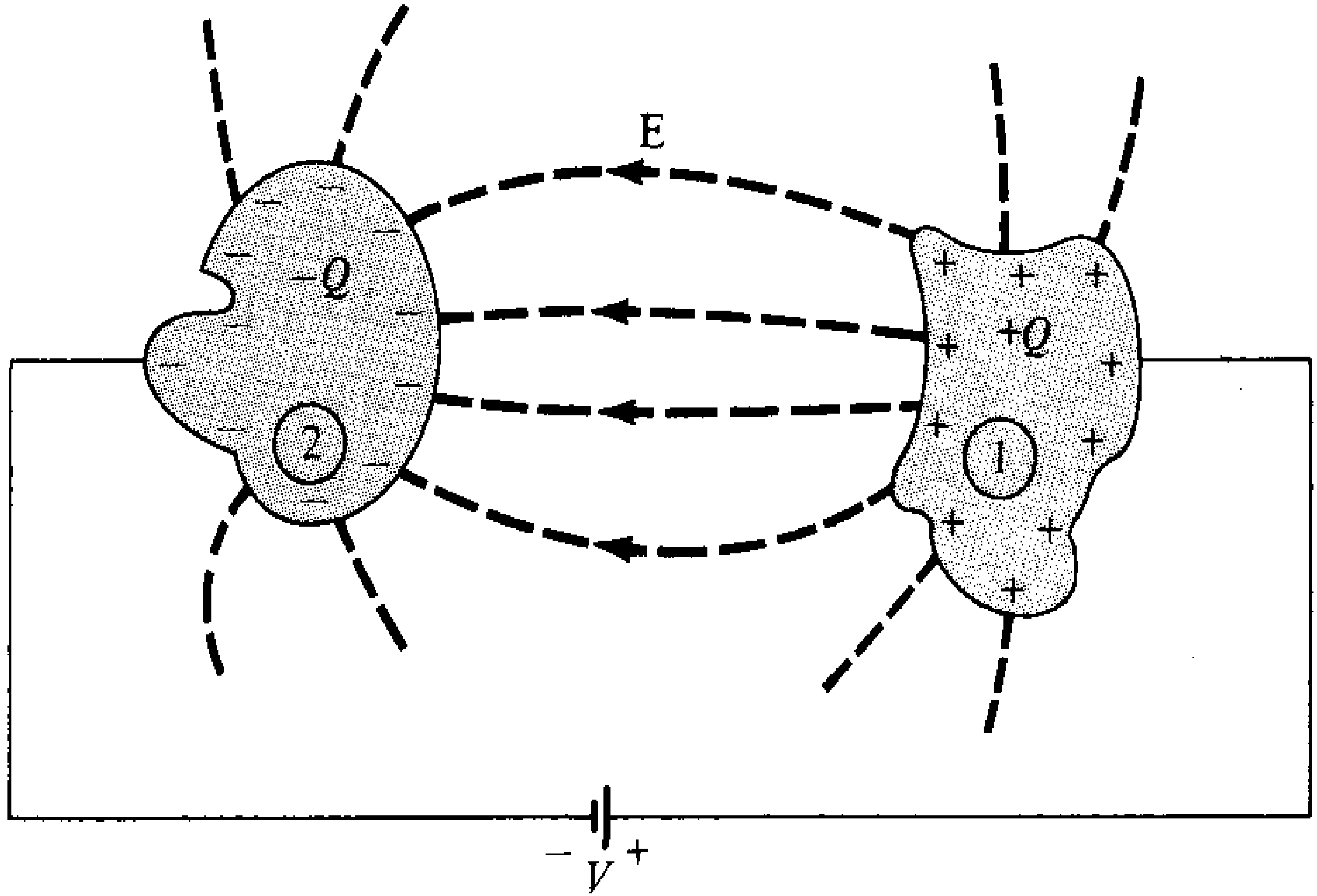
2. Since $\mathbf{E} = -\nabla V = 0$, there can be no potential difference between any two points in the conductor; that is, a conductor is an equipotential body.
3. The electric field **E** can be external to the conductor and *normal* to its surface; that is

$$\boxed{D_t = \varepsilon_0 \varepsilon_r E_t = 0, \qquad D_n = \varepsilon_0 \varepsilon_r E_n = \rho_S} \tag{5.71}$$

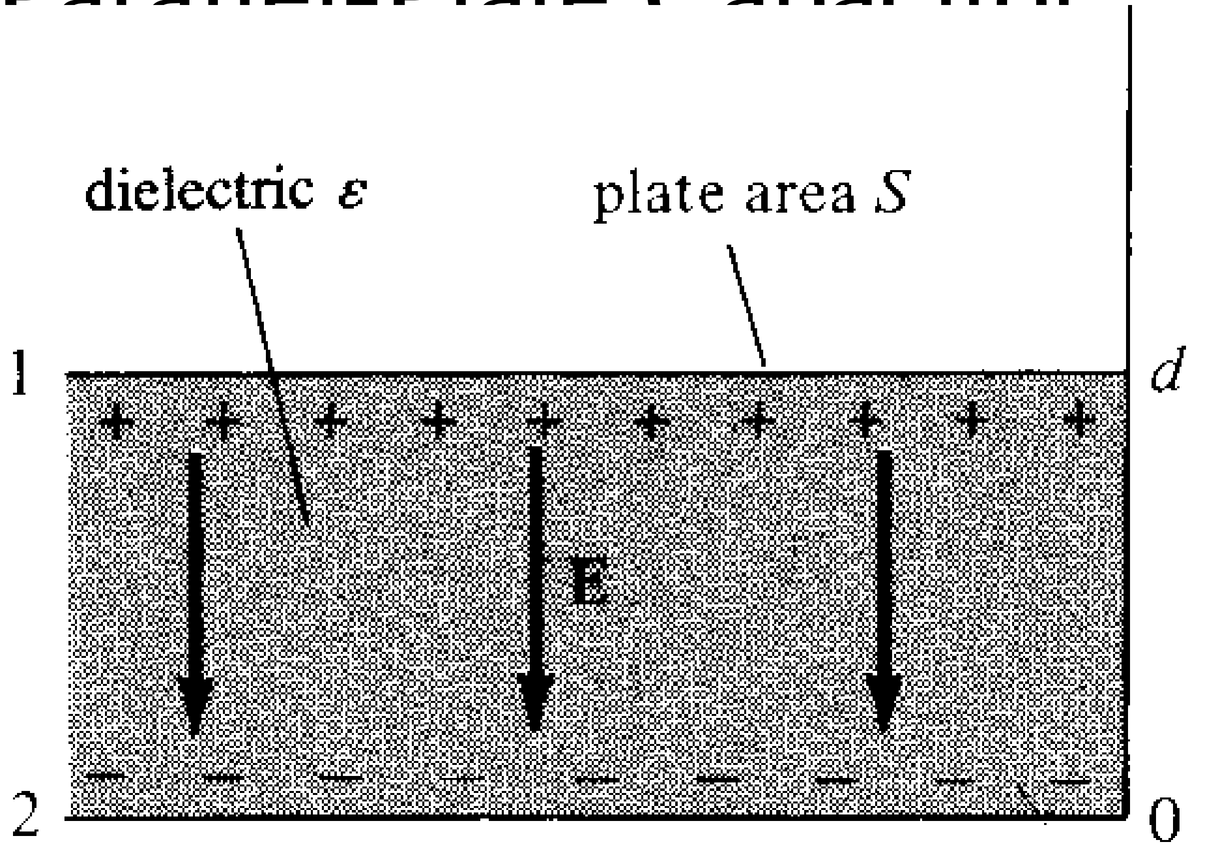
Capacitance

$$C = \frac{Q}{V} = \frac{\varepsilon \oint \mathbf{E} \cdot d\mathbf{S}}{\int \mathbf{E} \cdot d\mathbf{l}}$$

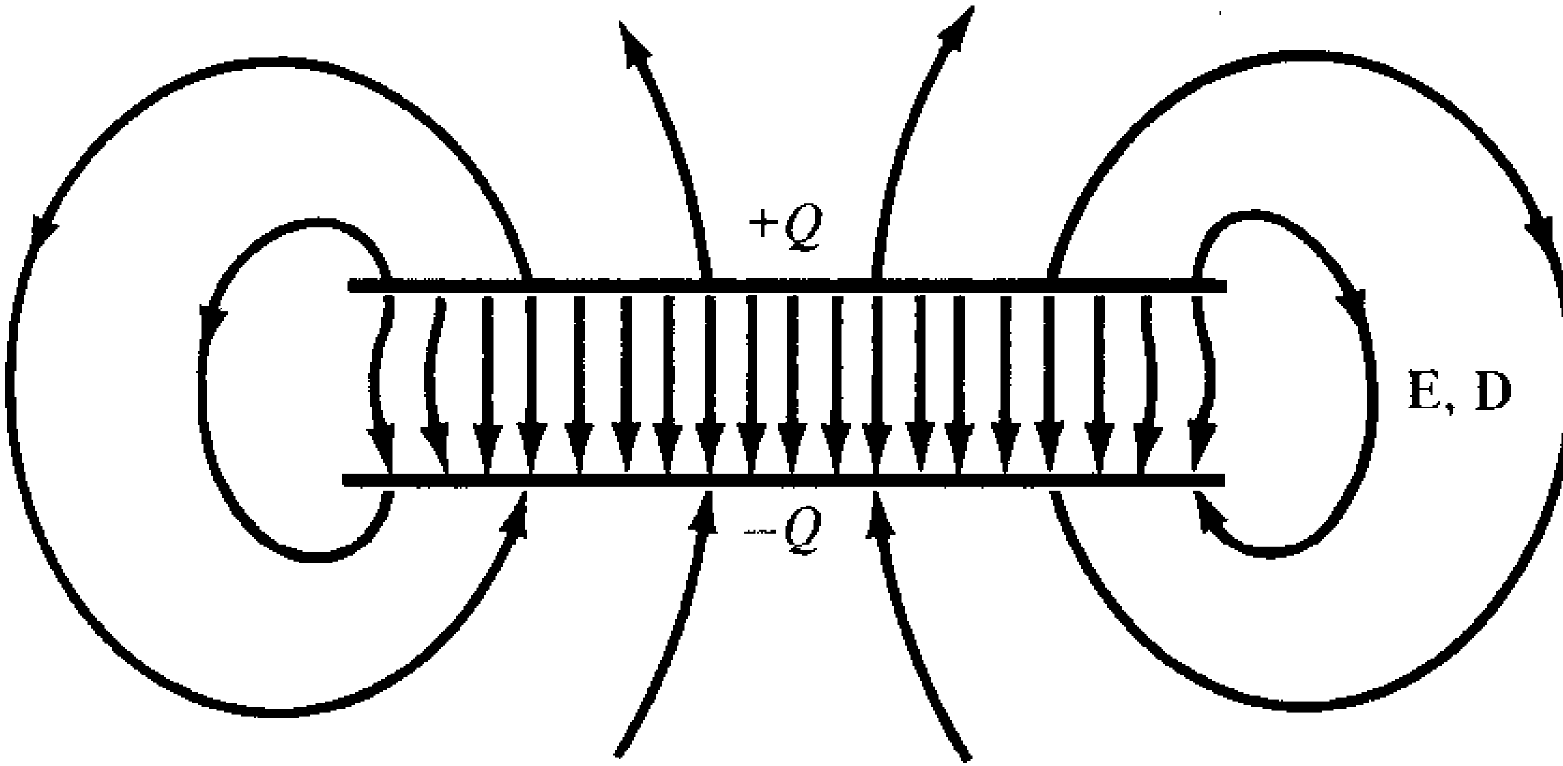
$$V = V_1 - V_2 = -\int_2^1 \mathbf{E} \cdot d\mathbf{l}$$


E
−Q
+Q
2
1
− V +

Parallel-Plate Capacitor



$$\rho_S = \frac{Q}{S}$$


+Q
−Q
E, D

$$\mathbf{E} = \frac{\rho_S}{\varepsilon}(-\mathbf{a}_x)$$

$$= -\frac{Q}{\varepsilon S}\mathbf{a}_x$$

$$V = -\int_2^1 \mathbf{E} \cdot d\mathbf{l} = -\int_0^d \left[-\frac{Q}{\varepsilon S}\mathbf{a}_x\right] \cdot dx\,\mathbf{a}_x = \frac{Qd}{\varepsilon S}$$

$$\boxed{C = \frac{Q}{V} = \frac{\varepsilon S}{d}}$$

$$\varepsilon_r = \frac{C}{C_o}$$

$$\boxed{W_E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}}$$

$$W_E = \frac{1}{2}\int_v \varepsilon \frac{Q^2}{\varepsilon^2 S^2}\,dv = \frac{\varepsilon Q^2 S d}{2\varepsilon^2 S^2}$$

$$= \frac{Q^2}{2}\left(\frac{d}{\varepsilon S}\right) = \frac{Q^2}{2C} = \frac{1}{2}QV$$

Coaxial Capacitor

$$Q = \varepsilon \oint \mathbf{E} \cdot d\mathbf{S} = \varepsilon E_\rho 2\pi\rho L$$

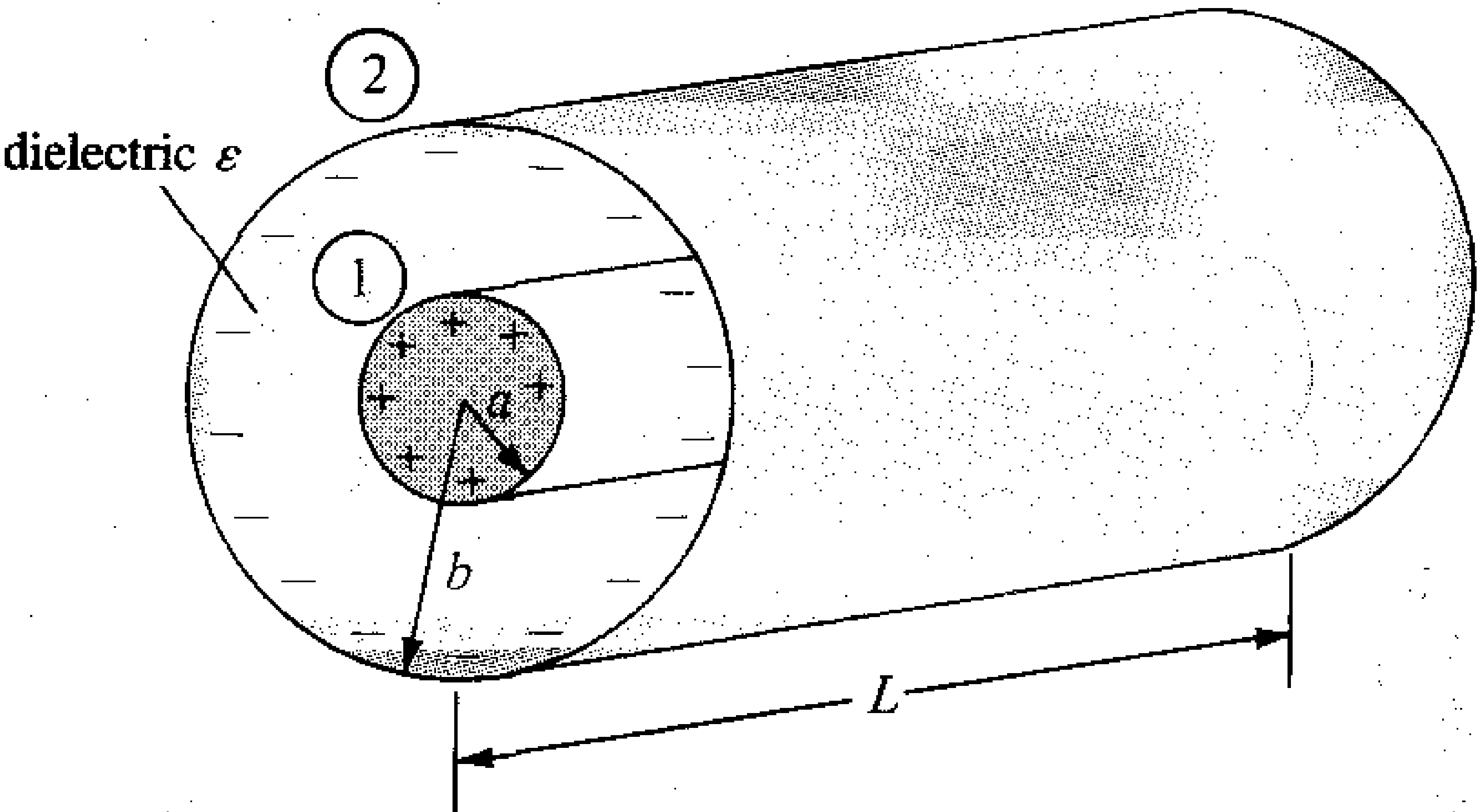
$$\mathbf{E} = \frac{Q}{2\pi\varepsilon\rho L}\,\mathbf{a}_\rho$$

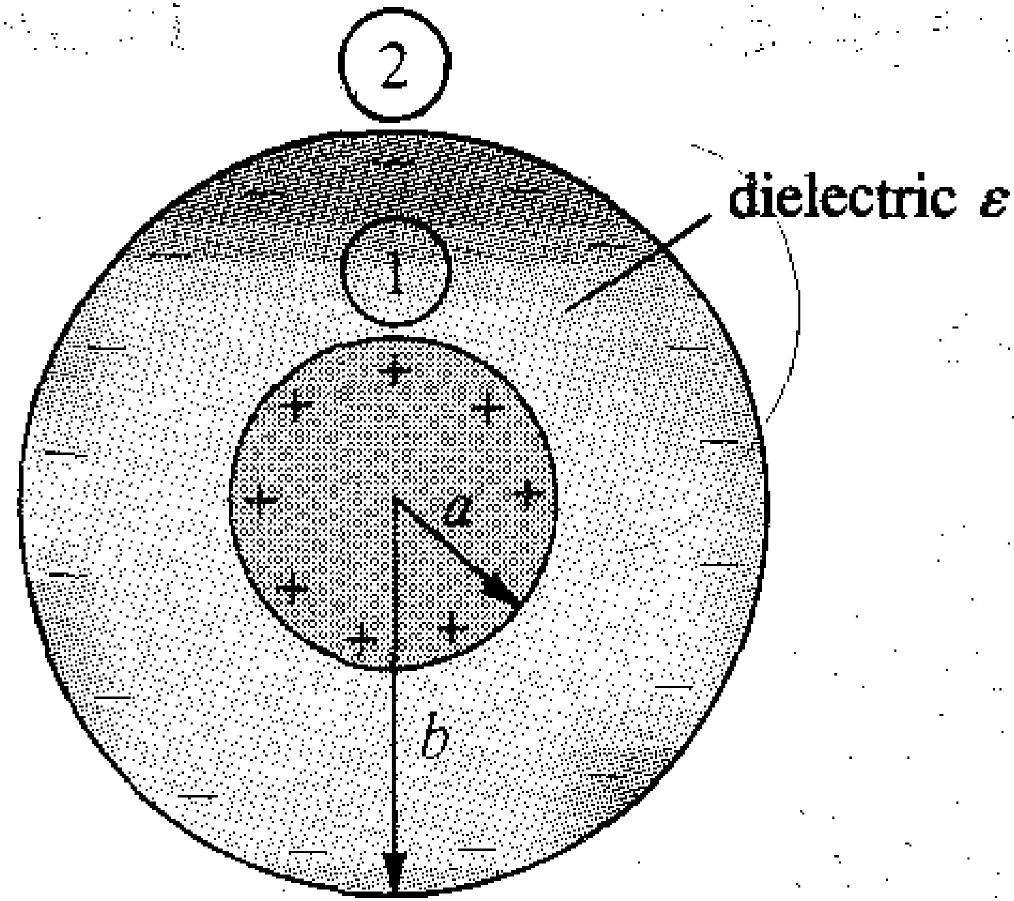
$$V = -\int_2^1 \mathbf{E} \cdot d\mathbf{l} = -\int_b^a \left[\frac{Q}{2\pi\varepsilon\rho L}\,\mathbf{a}_\rho\right] \cdot d\rho\,\mathbf{a}_\rho$$

$$= \frac{Q}{2\pi\varepsilon L} \ln\frac{b}{a}$$

$$C = \frac{Q}{V} = \frac{2\pi\varepsilon L}{\ln\frac{b}{a}}$$

Spherical Capacitor




2
dielectric ε
1
a
b

$$Q = \varepsilon \oint \mathbf{E} \cdot d\mathbf{S} = \varepsilon E_r 4\pi r^2$$

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon r^2}\,\mathbf{a}_r$$

$$V = -\int_2^1 \mathbf{E} \cdot d\mathbf{l} = -\int_b^a \left[\frac{Q}{4\pi\varepsilon r^2}\,\mathbf{a}_r\right] \cdot dr\,\mathbf{a}_r$$

$$= \frac{Q}{4\pi\varepsilon}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$C = \frac{Q}{V} = \frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{b}}$$

Current And Current Density

$$I = \frac{dQ}{dt}$$

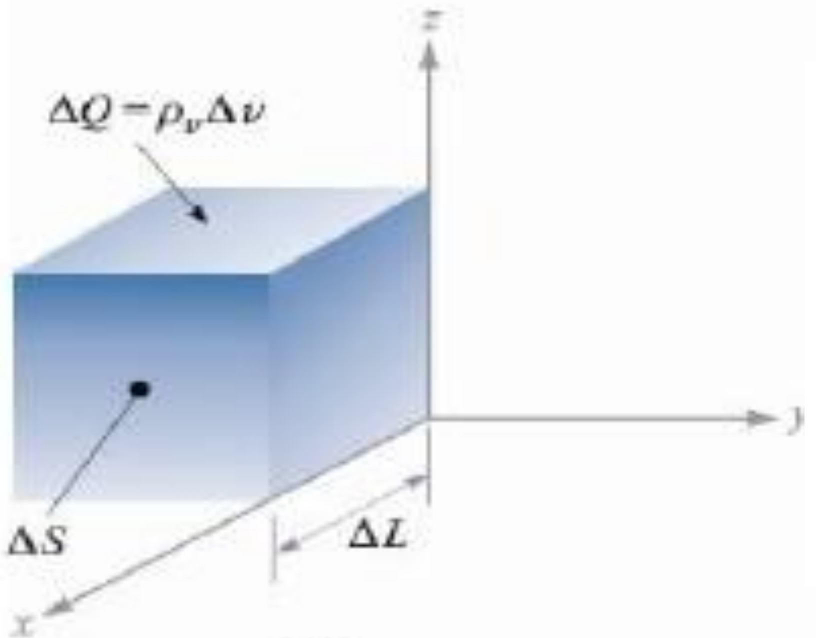
$$\Delta I = J_N \Delta S$$

$$\Delta I = \mathbf{J} \cdot \Delta \mathbf{S}$$

$$I = \int_S \mathbf{J} \cdot d\mathbf{S}$$

$$\Delta I = \frac{\Delta Q}{\Delta t} = \rho_v \, \Delta S \frac{\Delta x}{\Delta t}$$

$$\Delta I = \rho_v \, \Delta S \, v_x$$



(a)

$$J_x = \rho_v \, v_x$$

$$\mathbf{J} = \rho_v \mathbf{v}$$

Continuity of Current

$$I = \oint_S \mathbf{J} \cdot d\mathbf{S} = -\frac{dQ_i}{dt}$$

$$\oint_S \mathbf{J} \cdot d\mathbf{S} = \int_{\text{vol}} (\nabla \cdot \mathbf{J})\, dv$$

$$\int_{\text{vol}} (\nabla \cdot \mathbf{J})\, dv = -\frac{d}{dt}\int_{\text{vol}} \rho_v\, dv$$

$$\int_{\text{vol}} (\nabla \cdot \mathbf{J})\, dv = \int_{\text{vol}} -\frac{\partial \rho_v}{\partial t}\, dv$$

$$(\nabla \cdot \mathbf{J})\, \Delta v = -\frac{\partial \rho_v}{\partial t}\, \Delta v$$

$$\boxed{(\nabla \cdot \mathbf{J}) = -\frac{\partial \rho_v}{\partial t}}$$

Resistance & Ohm's Law

$$\mathbf{F} = -e\mathbf{E}$$

$$\mathbf{v}_d = -\mu_e \mathbf{E}$$

$$\boxed{\mathbf{J} = -\rho_e \mu_e \mathbf{E}}$$

$$\boxed{\mathbf{J} = \sigma \mathbf{E}}$$

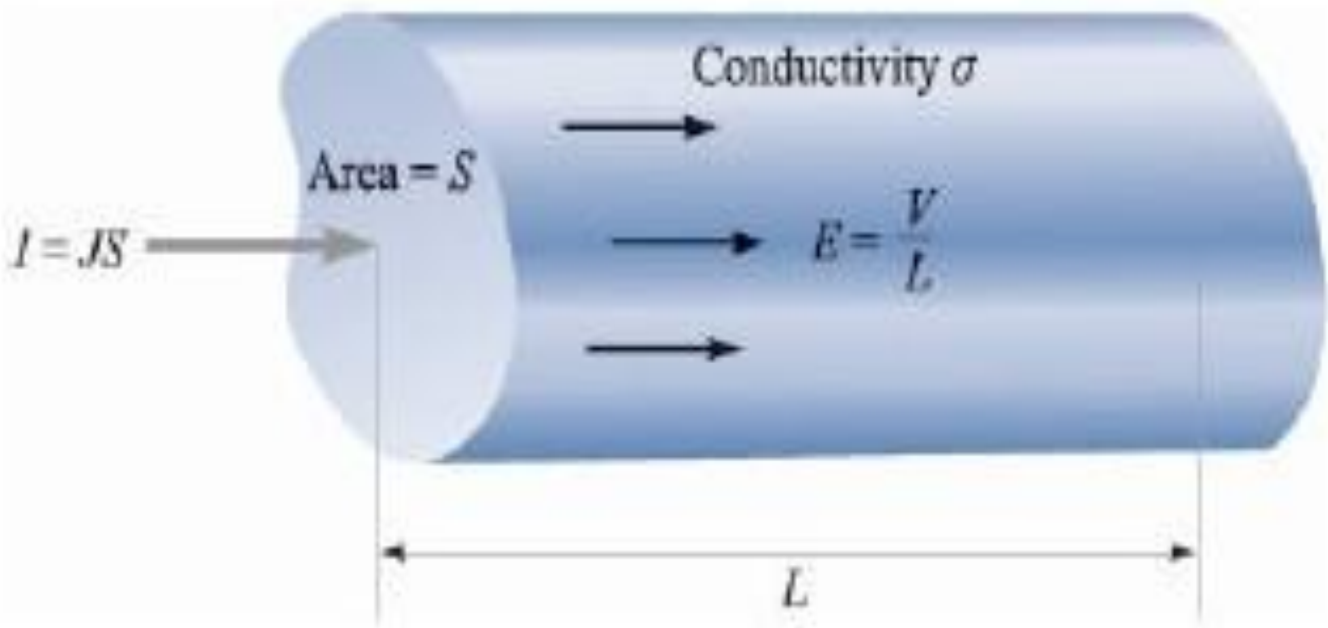
$$\boxed{\sigma = -\rho_e \mu_e}$$

$$I = \int_S \mathbf{J} \cdot d\mathbf{S} = J\,S$$

$$V_{ab} = -\int_b^a \mathbf{E} \cdot d\mathbf{L} = -\mathbf{E} \cdot \int_b^a d\mathbf{L} = -\mathbf{E} \cdot \mathbf{L}_{ba}$$

$$= \mathbf{E} \cdot \mathbf{L}_{ab}$$

$$V = EL$$



$$J = \frac{I}{S} = \sigma E = \sigma \frac{V}{L}$$

$$V = \frac{L}{\sigma S} I$$

$$\boxed{V = IR}$$

$$\boxed{R = \frac{L}{\sigma S}}$$

$$R = \frac{V_{ab}}{I} = \frac{-\int_b^a \mathbf{E} \cdot d\mathbf{L}}{\int_S \sigma \mathbf{E} \cdot d\mathbf{S}}$$

UNIT - VI

- Magnetic force
- Charges Movement in magnetic field
- Lorentz Force equation
- Force on current carrying elements
- Magnetic Dipoles
- Torque on a current carrying loop in magnetic field.

FORCES DUE TO MAGNETIC FIELDS

- There are at least three ways in which force due to magnetic fields can be experienced.

1. Due to a moving charged particle in a B field.
2. On a current element in an external B field.
3. Between two current elements.

Force on a Charged Particle

The electric force *Fe* on a stationary or moving electric charge *Q* in an electric field is given by Coulomb's experimental law and is related to the electric field intensity E as

$$F_e = QE$$

This shows that if F_Q is positive, F and E have the same direction.

A magnetic field can exert force only on a moving charge.

From experiments, it is found that the magnetic force Fm experienced by a charge Q moving with a velocity u in a magnetic field B is

$$F_m = Qu\ X\ B$$

F_m is perpendicular to both u and B.

Lorentz Force equation

For a moving charge Q in the presence of both electric and magnetic fields, the total force on the charge is given by

$$F = Fe + Fm$$

Or

$$F = Q(E + u \times B)$$

This equation is known as Lorentz Force equation.

It relates mechanical force to electrical force.

If the mass of the charged particle moving in E and B fields is *m*, by Newton's second law of motion.

$$F = m\frac{du}{dt} = Q\left(E + u X B\right)$$

The solution to this equation is important in determining the motion of charged particles in E and B fields.

Force on a Current Element

To determine the force on a current element I *dl* of a current-carrying conductor due to the magnetic field B

$$J = \rho_v u$$

We know that

$$I\,dl = K\,dS = J\,dv$$

Then

$$I\,dl = \rho_v u\,dv = dQ\,u$$

Hence

$$I\,dl = dQ\,u$$

The force acting on an elemental charge dQ moving with velocity u is equivalent to a conduction current element I dl in a magnetic field B.

$$dF = I\ \ dl\ X\,B$$

If the current is through a closed path L or circuit, the force on circuit is given by

$$F = \oint\ I\,dl\ X\,B$$

The magnetic field produced by the current element I *dl* does not exert force on the element itself just as a point charge does not exert force on itself.

The B field that exerts force on I dl must be due to another element.

If instead of the line current element I dl, we have surface current elements K dS or a volume current element J dv, Then

$$dF = K\,dS \times B \quad \text{Or} \quad dF = J\,dv \times B$$

Then

$$F = \oint K \, dS \, X \, B$$

and

$$F = \oint J \, dv \, X \, B$$

Force between Two Current Elements

Let us now consider the force between two elements $I_1\ dl_1$ and $I_2\ dl_2$.

According to Biot-Savart's law, both current elements produce magnetic fields.

So we may find the force $d(dF_1)$ on element $I_1\ dl_1$ due to the field $dB2$ produced by element $I_2\ dl_2$

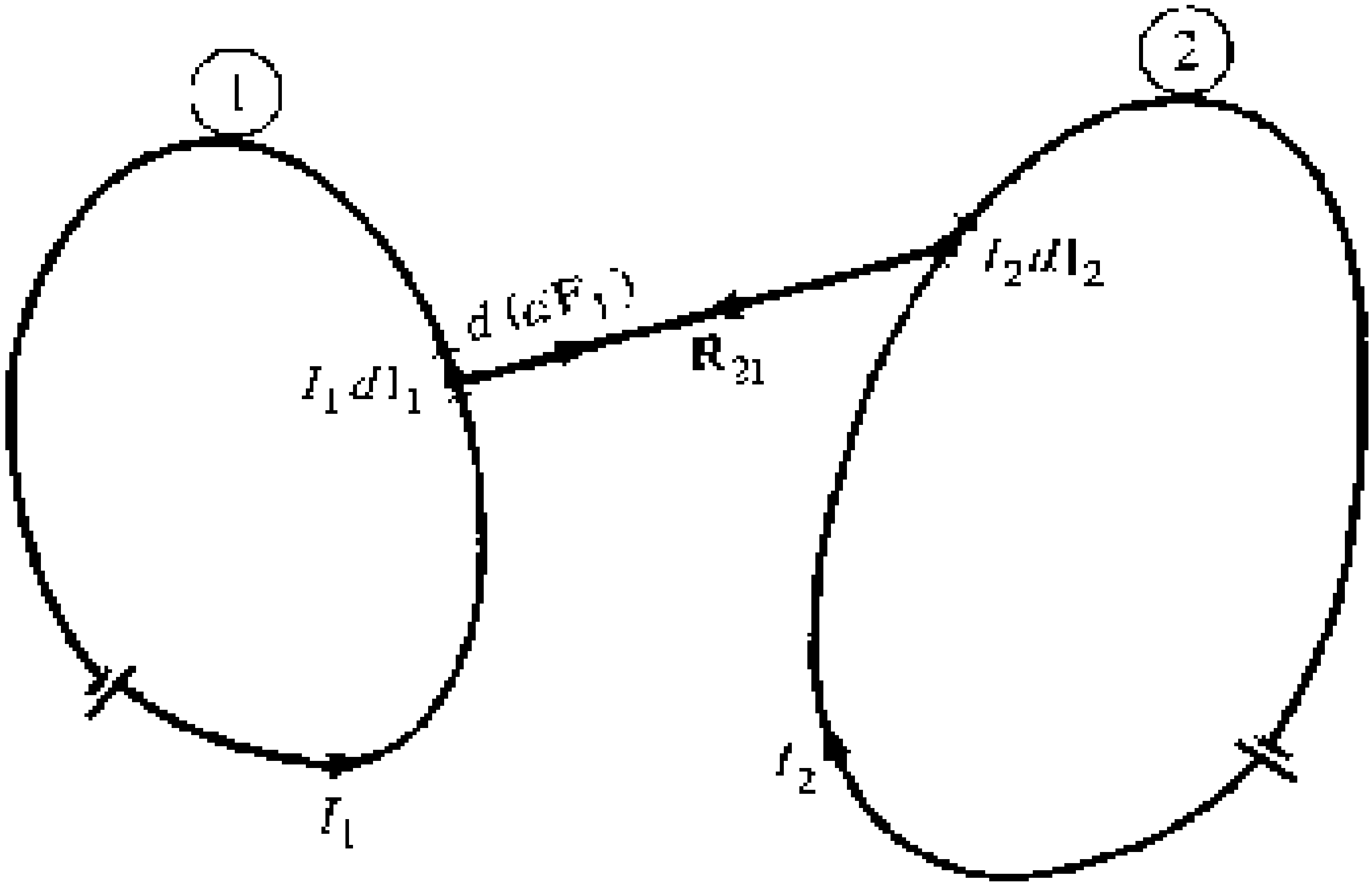
$$d(dF_1) = I_1\, dl_1\ X\ dB_2$$

From Biot-Savart's law

$$dB_2 = \frac{\square_0 I_2\, dI_2\, X\, a_{R_{21}}}{4\, R_{21}^{\;2}}$$

$$d\,\square dF_1\square = \frac{{}_0 I_1\, dI_1\, X\, \square I_2\, dI_2\, X\, a_{R_{21}} \square}{4\, R_{21}^{\;2}}$$

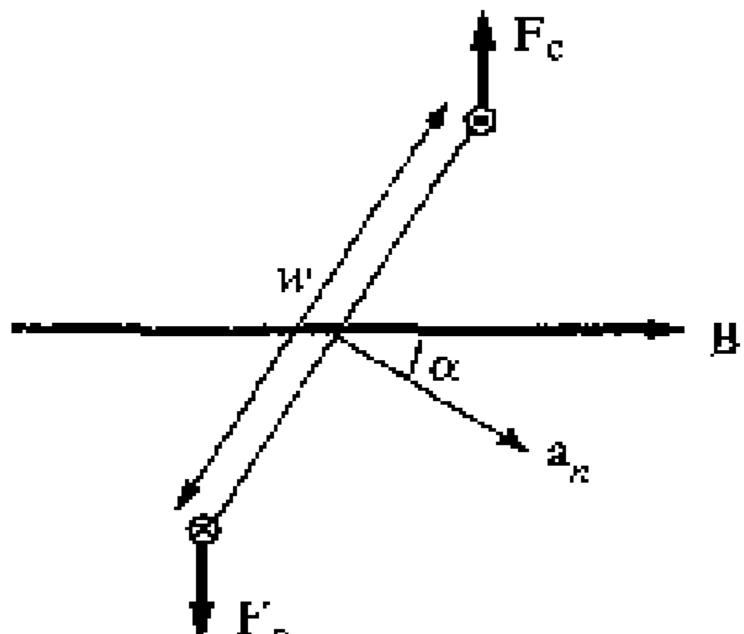
$$F_1 = \frac{{}_0 I_1 I_2}{4} \oint\!\!\oint \frac{dI_1\, X\, \square dI_2\, X\, a_{R_{21}} \square}{R_{21}^2}$$


1
2
d(dF1)
I1 dl1
R21
I2 dl2
I1
I2

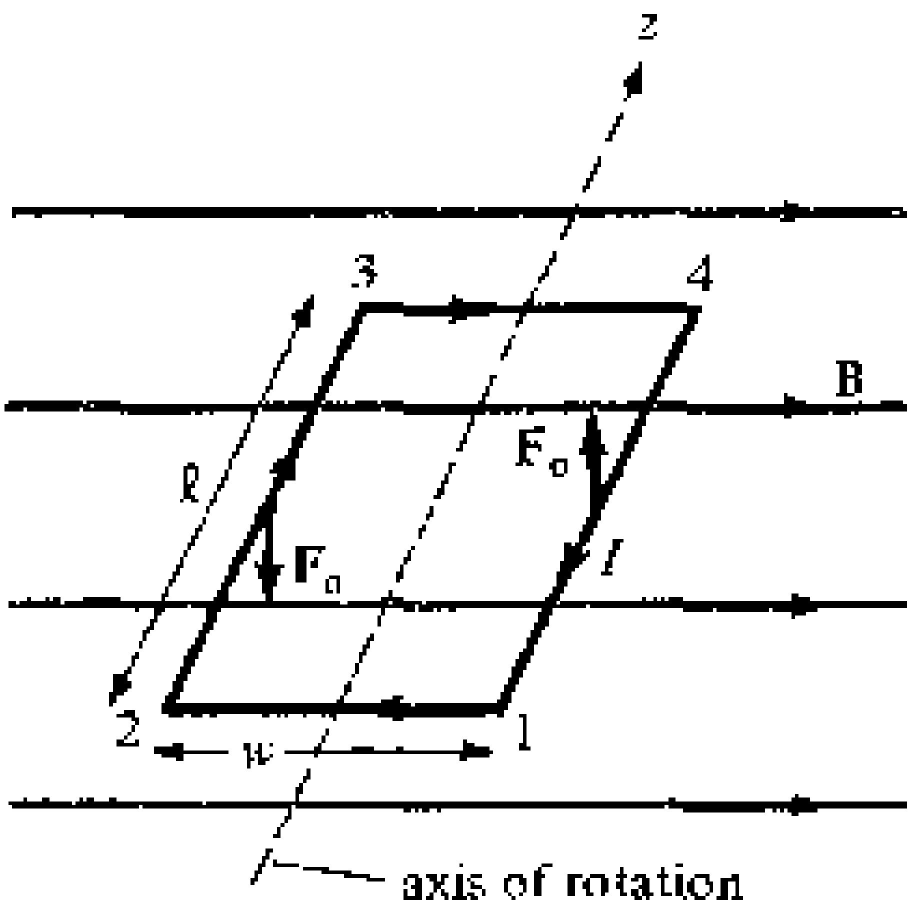
MAGNETIC TORQUE AND MOMENT

The **torque T** (or mechanical moincnl of force) on ihe loop is the vector product of the force F and the moment arm r.

That is T = r X F



Consider a rectangular loop of length l and width *w* placed in a uniform magnetic field B as shown in Figure



$$F = I \int_{2}^{3} dl \; X \; B \square I \int_{4}^{1} dl \; X \; B$$

$$F = I \int_{0}^{l} dz \, a_z \; X \; B \square I \int_{0}^{l} dz \, a_z \; X \; B$$

Or

$$F = F_0 - F_0 = 0$$

Thus F = Bil. Thus no force is exerted on the loop as a whole. However Fo and -Fo act at different points on the loop, there by creating a couple.

The torque on the loop is

$$|T| = |F_0| w \sin \square$$

Or

$$T = BIlw \sin \square$$

But lw = S, the area of the loop

$$T = BIS \sin \square$$

We define the quantity

$$m = IS\, a_n$$

m is defined as the magnetic dipole moment.

Units are A/m^2.

Magnetic Dipole Moment

The **magnetic dipole moment** is the product of current and area of the loop.

Its direction is normal to the loop.

Torque on a magnetic loop placed in Magnetic field is

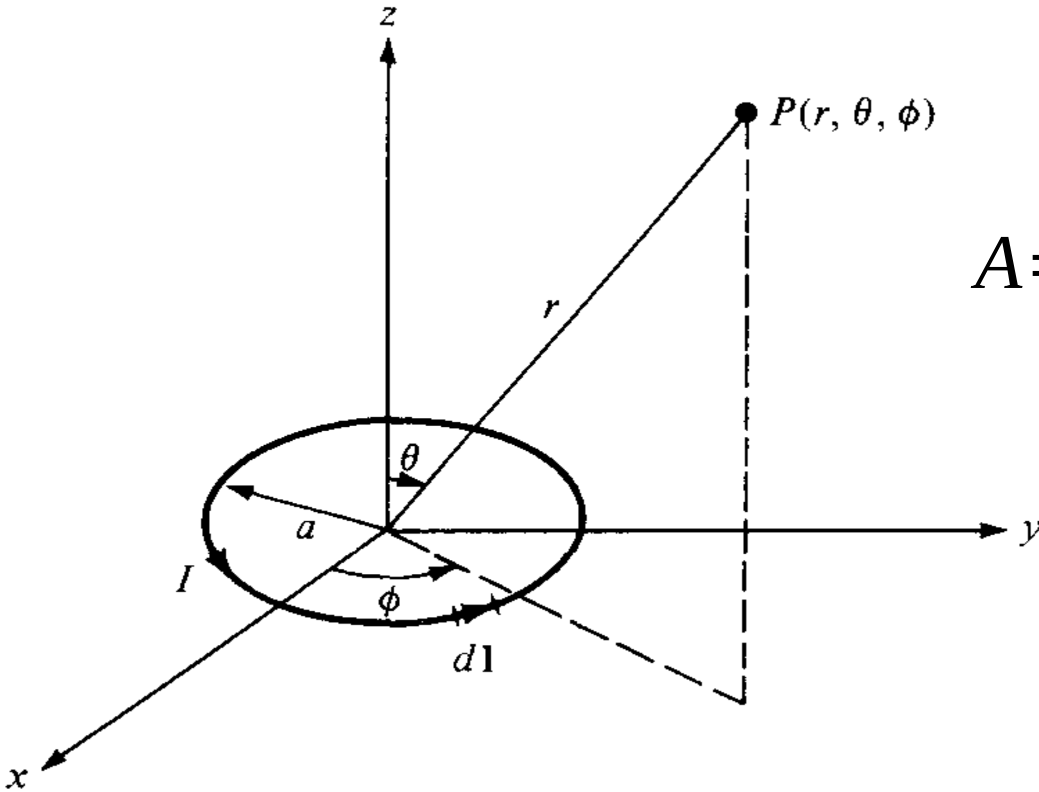
$$T = m \times B$$

This is applicable only when the magnetic field is uniform in nature.

Field due to a Magnetic Dipole

- A Bar magnet or a small filamentary current loop is usually referred to as a magnetic dipole.
- Consider a current carrying loop carrying a current of I amps, the the magnetic field due this at any arbitrary point P(r,θ,φ) due the loop is calculated as follows.

The magnetic Vector Potential at P is



$$A = \frac{\mu_0 I}{4\pi} \oint \frac{dl}{r}$$

$$A = \frac{\mu_0 I\, a^2 \sin\theta\, a_\phi}{4\pi r^2}$$

or

$$A = \frac{\mu_0 m \, X \, a_r}{4\pi r^2}$$

The magnetic Flux density B is determined as

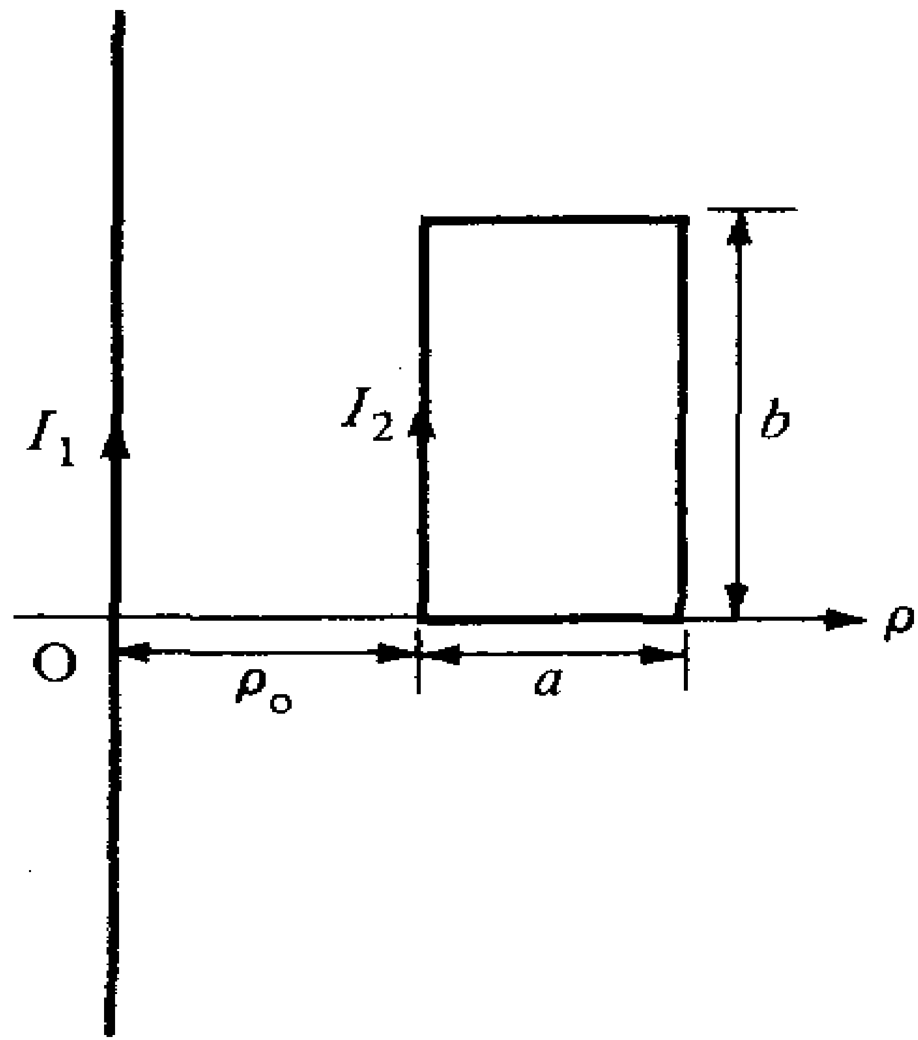
$$B = \nabla \ X \ A$$

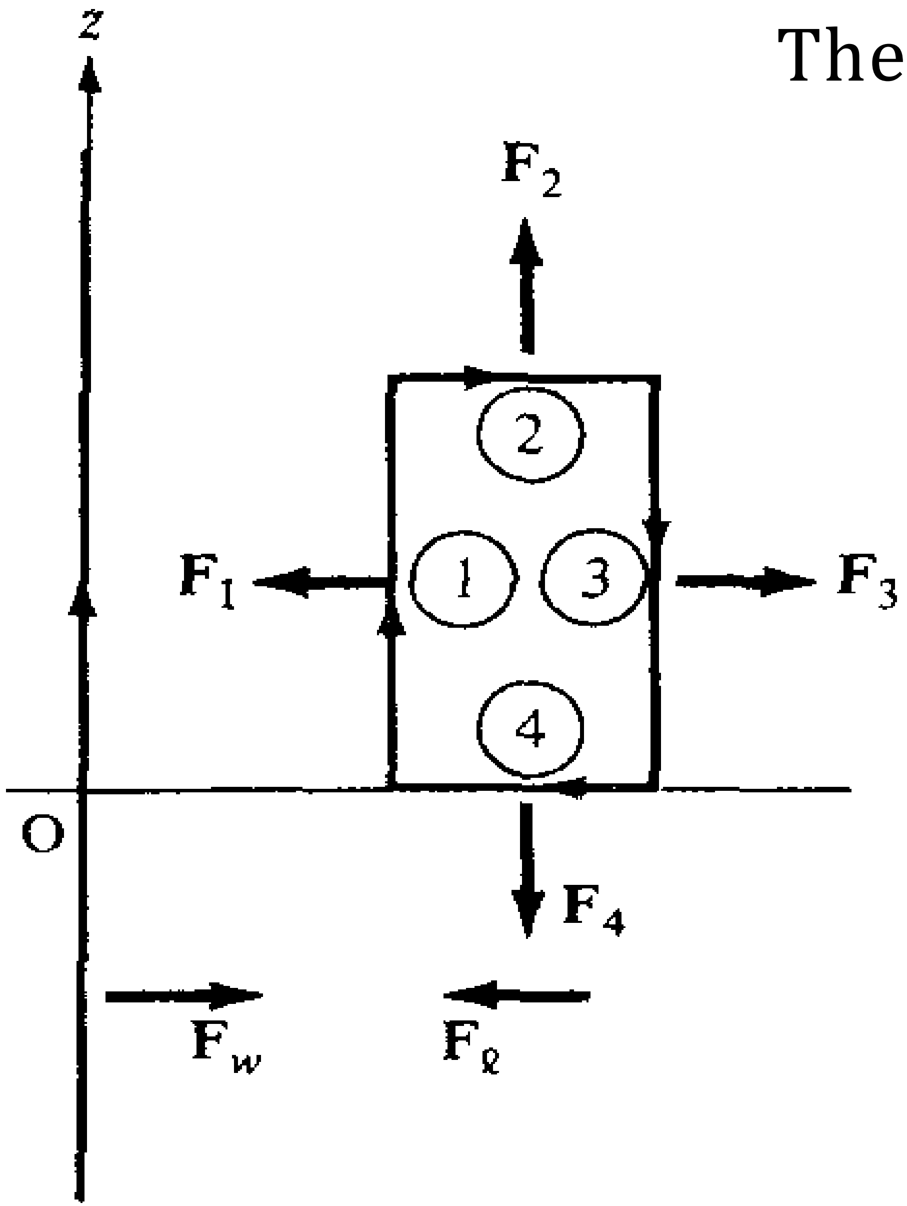
Therefore

$$B = \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta \, a_r + \sin\theta \, a_\theta\right)$$

Force Experienced by a

A rectangular loop carrying current I_2 is placed parallel to an infinitely long filamentary wire carrying current I_1 as shown in Figure





The force acting on loop is

$$F_l = F_1 + F_2 + F_3 + F_4$$

$$F_l = I_2 \oint dI_2 \; X \; B$$

where F_1, F_2, F_3, and F_4 are respectively, the forces exerted on sides of the loop labeled 1, 2, 3, and 4 in Figure

Due to the infinitely long wire

$$B_1 = \frac{\mu_0 I_1}{2\pi\rho_0} a_\phi$$

Then

$$F_1 = I_2 \oint dI_2 \, X \, B_1$$

$$F_1 = I_2 \oint_{z=0}^{b} dz \, a_z \, X \, \frac{\mu_0 I_1}{2\pi\rho_0} a_\phi$$

$$F_1 = -\frac{\mu_0 I_1 I_2 b}{2\pi\rho_0} a_\rho \qquad \text{(Attractive)}$$

$$F_3 = I_2 \oint dl_2 \; X \; B_1$$

$$F_3 = I_2 \oint_{z=0}^{b} dz\, a_z \; X \; \frac{\mu_0 I_1}{2\pi\rho_0} a_\phi$$

$$F_3 = \frac{\mu_0 I_1 I_2 b}{2\pi\rho_0} a_\rho \qquad \text{(Repulsive)}$$

$$F_2 = I_2 \oint_{\rho=\rho_0}^{\rho_0+a} d\rho\, a_\rho \; X \; \frac{\mu_0 I_1}{2\pi\rho_0} a_\phi$$

$$F_2 = \frac{\mu_0 I_1 I_2}{2\pi} \ln\left(\frac{\rho_0 + a}{\rho_0}\right) a_z \qquad \text{(Parallel)}$$

$$F_4 = I_2 \oint_{= {}_0 + a}^{{}_0} d \; a \quad X \frac{{}_0 I_1}{2 \, \square a \square \, {}_0} a_{\square}$$

$$F_4 = \frac{-\square I_1 I_2}{2} \ln \square \frac{{}_0 + a}{{}_0} \square a_z \quad \text{(Parallel)}$$

The Total Force

$$F_l = \frac{I_1 I_2 b}{2} \left[\frac{1}{{}_0} - \frac{1}{\square a \, {}_0} \right] \square - a \; \square$$

Magnetization in Materials

- The magnetization M (in amperes/meter) is the magnetic dipole moment per unit volume.
- If there are N atoms in a given volume Δv and the k^{th} atom has a magnetic moment m_K.

$$M = \lim_{\Delta v \to 0} \frac{\sum_{k=1}^{N} m_k}{\Delta v}$$

- A medium for which M is not zero everywhere is said to be magnetized.

$$dA = \frac{\square_0 M \quad X\, a_R}{4\ R^2}\ dv^1$$

$$dA = \frac{\square_0 M \quad X\, R}{4\ R^3}\ dv^1$$

$$\frac{R}{R^3} = \nabla^1 \ \frac{1}{R}$$

Hence

$$A = \frac{0}{4} \int M\ X\ \nabla^1\ \frac{1}{R}\, dv^1$$

But

$$M \; X \; \nabla^{1} \; \frac{1}{R} = \frac{1}{R} \nabla^{1} X \, M - \nabla^{1} X \frac{M}{R}$$

Substituting the above equation in A

$$A = \frac{0}{4} \int \frac{\nabla^{1} X \; M}{R} dv^{1} - \frac{0}{4} \int \nabla^{1} X \; \frac{M}{R} \; dv^{1}$$

Applying the Vector Identity

$$\int_{v^{1}} \nabla_{1} X \, F \, dv^{1} = -\int_{S^{1}} F \; X \; dS$$

$$A = \frac{\mu_0}{4\pi} \int_{v'} \frac{\nabla' \times M}{R}\, dv' + \frac{\mu_0}{4\pi} \oint_{S'} \frac{M \times a_n}{R}\, dS'$$

$$A = \frac{\mu_0}{4\pi} \int_{v'} \frac{\nabla' \times M}{R}\, dv' + \frac{\mu_0}{4\pi} \oint_{S'} \frac{M \times a_n}{R}\, dS'$$

$$A = \frac{\mu_0}{4\pi} \int_{v'} \frac{J_b}{R}\, dv' + \frac{\mu_0}{4\pi} \oint_{S'} \frac{K_b}{R}\, dS'$$

Comparing the equations

$$J_b = \nabla \times M$$

$$K_b = M \times a_n$$

J_b is the bound volume current density

or magnetizing volume current density.

K_b is the bound surface current density.

In free Space

$$\nabla X H = J_f \quad \text{Or} \quad \nabla X \left(\frac{B}{\mu_0}\right) = J_f$$

J_f is the free current volume density

In a medium where M is not equal to zero, then

$$\nabla X \left(\frac{B}{\mu_0}\right) = J_f + J_b = J = \nabla X B \square \nabla X M$$

or

$$B = \mu_0 (H + M)$$

For linear materials, M depends linearly on H such that

$$M = \chi_m H$$

χ_m is called Magnetic susceptibility of the medium.

$$B = \mu_0 (1 + \chi_m) H$$

$$B = \mu_0 \mu_r H$$

Where

$$\mu_r = 1 + \chi_m = \frac{\mu}{\mu_0}$$

μ is called as the permeability of the material

μ_r is called as the relative permeability of the material

- Scalar and Vector Magnetic Potentials
- Vector Potentials due to simple configurations
- Self and Mutual Inductances
- Determination of inductance
- Energy

Magnetic Potential

- we can define a potential associated with magnetostatic field B.
- The magnetic potential could be scalar or vector

Scalar Magnetic Potential

- We define the *magnetic scalar potential Vm*.

$$\boxed{\mathbf{H} = -\nabla V_m}$$

$$\mathbf{J} = \nabla \times \mathbf{H} = \nabla \times (-\nabla V_m) = 0$$

Thus the magnetic scalar potential V_m is only defined in a region where J = 0

Vector Magnetic Potential

$$\mathbf{A} = \int_L \frac{\mu_o I \, d\mathbf{l}}{4\pi R}$$

for line current

$$\mathbf{A} = \int_S \frac{\mu_o \mathbf{K} \, dS}{4\pi R}$$

for surface current

$$\boxed{\mathbf{A} = \int_v \frac{\mu_o \mathbf{J}\, dv}{4\pi R}}$$

for volume current

$$\boxed{\mathbf{B} = \nabla \times \mathbf{A}}$$

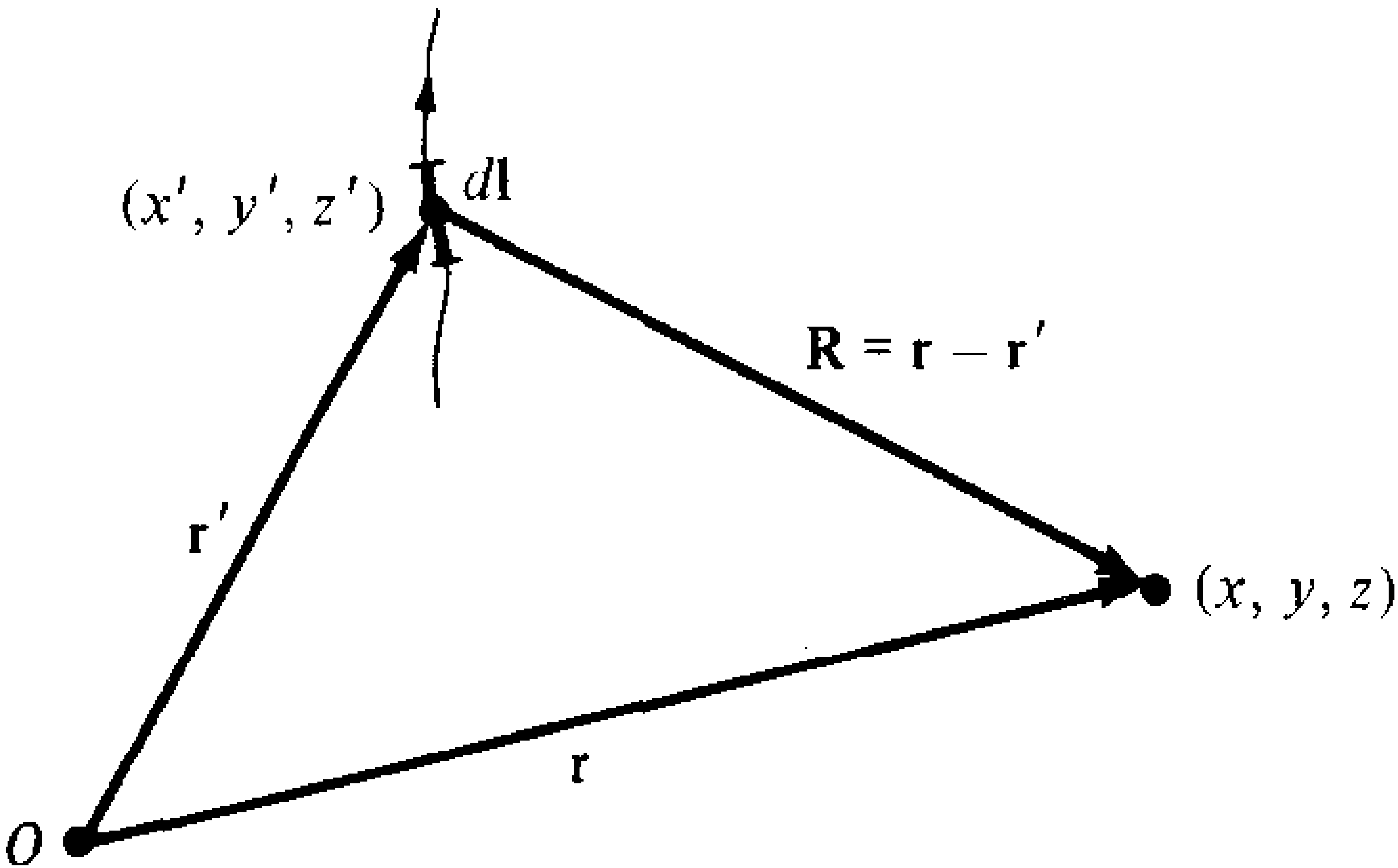
Proof

- We Know that

$$\mathbf{B} = \frac{\mu_o}{4\pi} \int_L \frac{I\, d\mathbf{l}' \times \mathbf{R}}{R^3}$$

$$R = |\mathbf{r} - \mathbf{r}'| = [(x - x')^2 + (y - y')^2 + (z - z')^2]^{1/2}$$

$$\nabla\left(\frac{1}{R}\right) = -\frac{(x - x')\mathbf{a}_x + (y - y')\mathbf{a}_y + (z - z')\mathbf{a}_z}{[(x - x')^2 + (y - y')^2 + (z - z')^2]^{3/2}} = -\frac{\mathbf{R}}{R^3}$$


(x′, y′, z′)
dl
R = r − r′
r′
(x, y, z)
r
O

$$\frac{\mathbf{R}}{R^3} = -\nabla\left(\frac{1}{R}\right) \qquad \left(= \frac{\mathbf{a}_R}{R^2}\right)$$

$$\mathbf{B} = -\frac{\mu_o}{4\pi}\int_L I\, d\mathbf{l}' \times \nabla\left(\frac{1}{R}\right)$$

$$\nabla \times (f\mathbf{F}) = f\nabla \times \mathbf{F} + (\nabla f) \times \mathbf{F}$$

$$d\mathbf{l}' \times \nabla\left(\frac{1}{R}\right) = \frac{1}{R}\nabla \times d\mathbf{l}' - \nabla \times \left(\frac{d\mathbf{l}'}{R}\right)$$

$$d\mathbf{l}' \times \nabla\left(\frac{1}{R}\right) = -\nabla \times \frac{d\mathbf{l}'}{R}$$

$$\mathbf{A} = \int_L \frac{\mu_o I \, d\mathbf{l}'}{4\pi R}$$

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S} = \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S} = \oint_L \mathbf{A} \cdot d\mathbf{l}$$

$$\boxed{\Psi = \oint_L \mathbf{A} \cdot d\mathbf{l}}$$

We Know that

$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

$$\mathbf{B} = \nabla \times \oint_L \frac{\mu_o I \, d\mathbf{l}'}{4\pi R} = \frac{\mu_o I}{4\pi} \oint_L \nabla \times \frac{1}{R} \, d\mathbf{l}',$$

$$\mathbf{B} = \frac{\mu_o I}{4\pi} \oint_L \left[\frac{1}{R} \nabla \times d\mathbf{l}' + \left(\nabla \frac{1}{R} \right) \times d\mathbf{l}' \right]$$

$$\frac{1}{R} = [(x - x')^2 + (y - y')^2 + (z - z')^2]^{-1/2}$$

$$\nabla\left[\frac{1}{R}\right] = -\frac{(x - x')\mathbf{a}_x + (y - y')\mathbf{a}_y + (z - z')\mathbf{a}_z}{[(x - x')^2 + (y - y')^2 + (z - z')^2]^{3/2}} = -\frac{\mathbf{a}_R}{R^2}$$

$$\mathbf{B} = \frac{\mu_o I}{4\pi} \oint_L \frac{d\mathbf{l} \times \mathbf{a}_R}{R^2}$$

$$\nabla \times \mathbf{B} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$$

$$\boxed{\nabla \cdot \mathbf{A} = 0}$$

$$\nabla^2 \mathbf{A} = -\mu_o \nabla \times \mathbf{H}$$

$$\boxed{\nabla^2 \mathbf{A} = -\mu_o \mathbf{J}}$$

This equation is called vector Poisson's equation. In Cartesian form these can be written as

$$\nabla^2 A_x = -\mu_o J_x$$

$$\nabla^2 A_y = -\mu_o J_y$$

$$\nabla^2 A_z = -\mu_o J_z$$

$$\oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S \nabla \times \mathbf{H} \cdot d\mathbf{S}$$

$$= \frac{1}{\mu_o} \int_S \nabla \times (\nabla \times \mathbf{A}) \cdot d\mathbf{S}$$

$$\nabla \times \nabla \times \mathbf{A} = -\nabla^2 \mathbf{A} = \mu_o \mathbf{J}$$

$$\oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S} = I$$

Flux Linkages

- A circuit (or closed conducting path) carrying current /produces a magnetic field B which causes a flux ψ = ∫ B • dS to pass through each turn of the circuit as shown in Figure.
- If the circuit has N identical turns, we define *the flux linkage* λ as

$$\lambda = N\psi$$

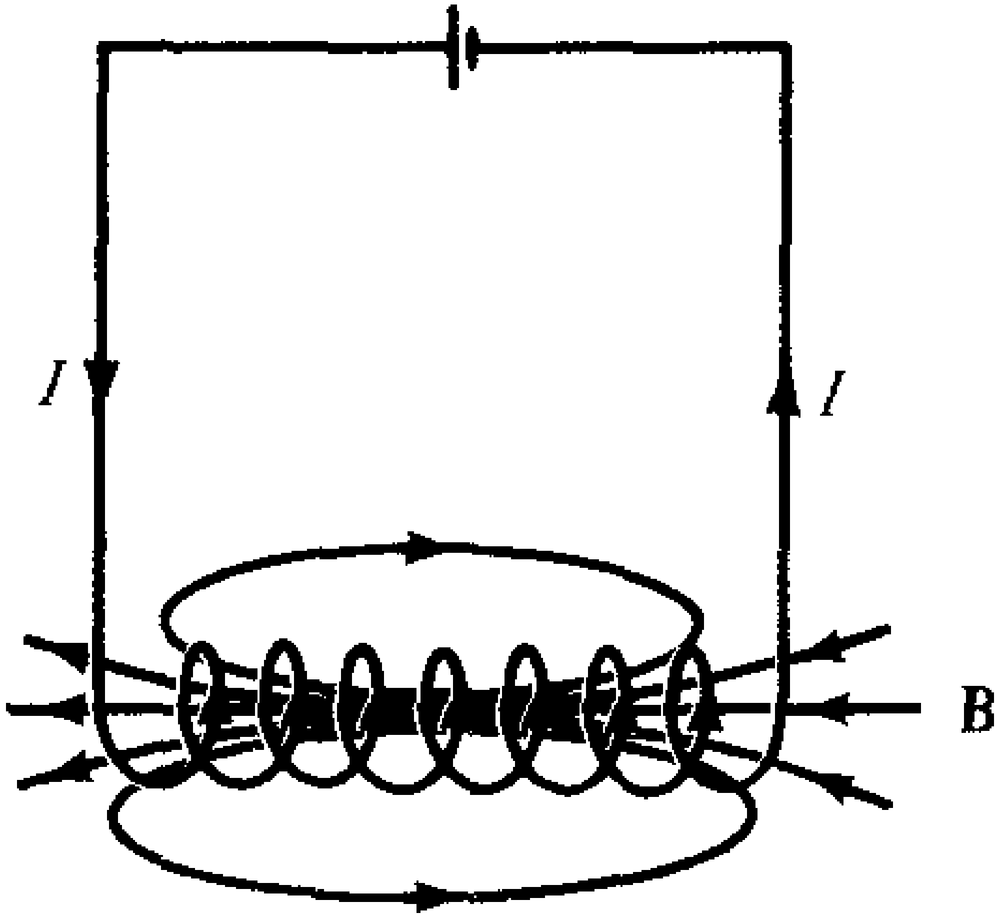
- If the medium surrounding the circuit is linear, the flux linkage X is proportional to the current I producing it.

Inductance

Inductance L of an inductor as the ratio of the magnetic flux linkage λ to the current I through the inductor

$$L = \frac{\lambda}{I} = \frac{N\Psi}{I}$$

The unit of inductance is the henry (H) which is the same as webers/ampere.



Inductance is a measure of how much magnetic energy is stored in an inductor.

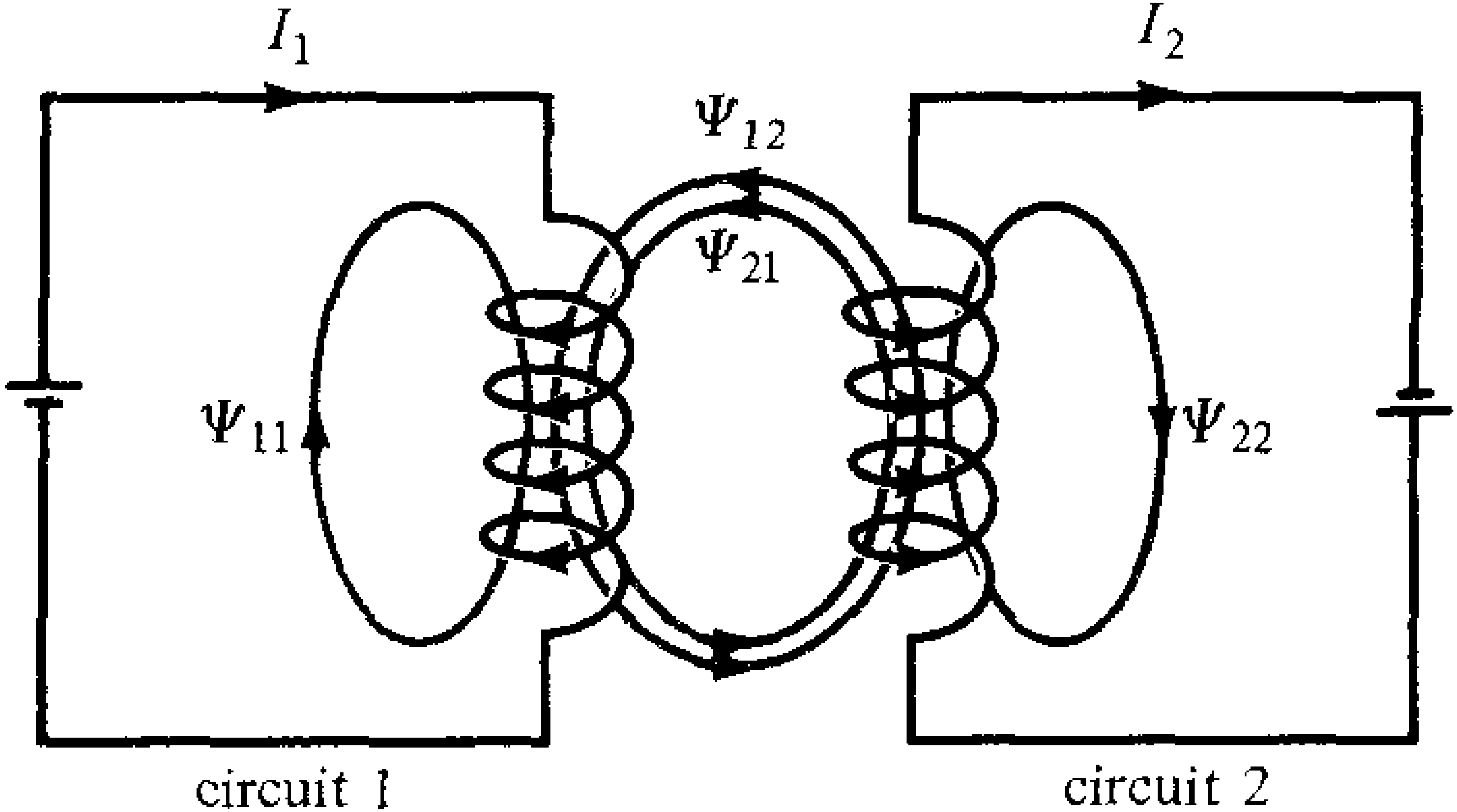
The magnetic energy (in joules) stored in an inductor is expressed as

$$W_m = \frac{1}{2}LI^2$$

$$\boxed{L = \frac{2W_m}{I^2}}$$

Mutual Inductance

- If instead of having a single circuit we have two circuits carrying current I_1 and I_2 as shown in Figure, a magnetic interaction exists between the circuits.
- Four component fluxes ψ_{11},ψ_{12},ψ_{21} and ψ_{22} are produced.
- The flux ψ_{12}, for example, is the flux passing through circuit 1 due to current I_2 in circuit 2. If B_2 in the field due to I_2 and S_1 is the area of circuit 1, then



I_1
I_2
Ψ_{12}
Ψ_{21}
Ψ_{11}
Ψ_{22}
circuit 1
circuit 2

$$\Psi_{12} = \int_{S_1} \mathbf{B}_2 \cdot d\mathbf{S}$$

We define the *mutual inductance* M_{12} as the ratio of the flux linkage $\lambda_{12} = N_1\psi_{12}$ on circuit 1 to current I_2, that is

$$M_{21} = \frac{\lambda_{21}}{I_1} = \frac{N_2\Psi_{21}}{I_1}$$

$$M_{12} = M_{21}$$

We define the self-inductance of circuits 1 and 2, respectively, as

$$L_1 = \frac{\lambda_{11}}{I_1} = \frac{N_1 \Psi_1}{I_1}$$

$$L_2 = \frac{\lambda_{22}}{I_2} = \frac{N_2 \Psi_2}{I_2}$$

$$\begin{aligned} W_m &= W_1 + W_2 + W_{12} \\ &= \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 \pm M_{12} I_1 I_2 \end{aligned}$$

Magnetic Energy

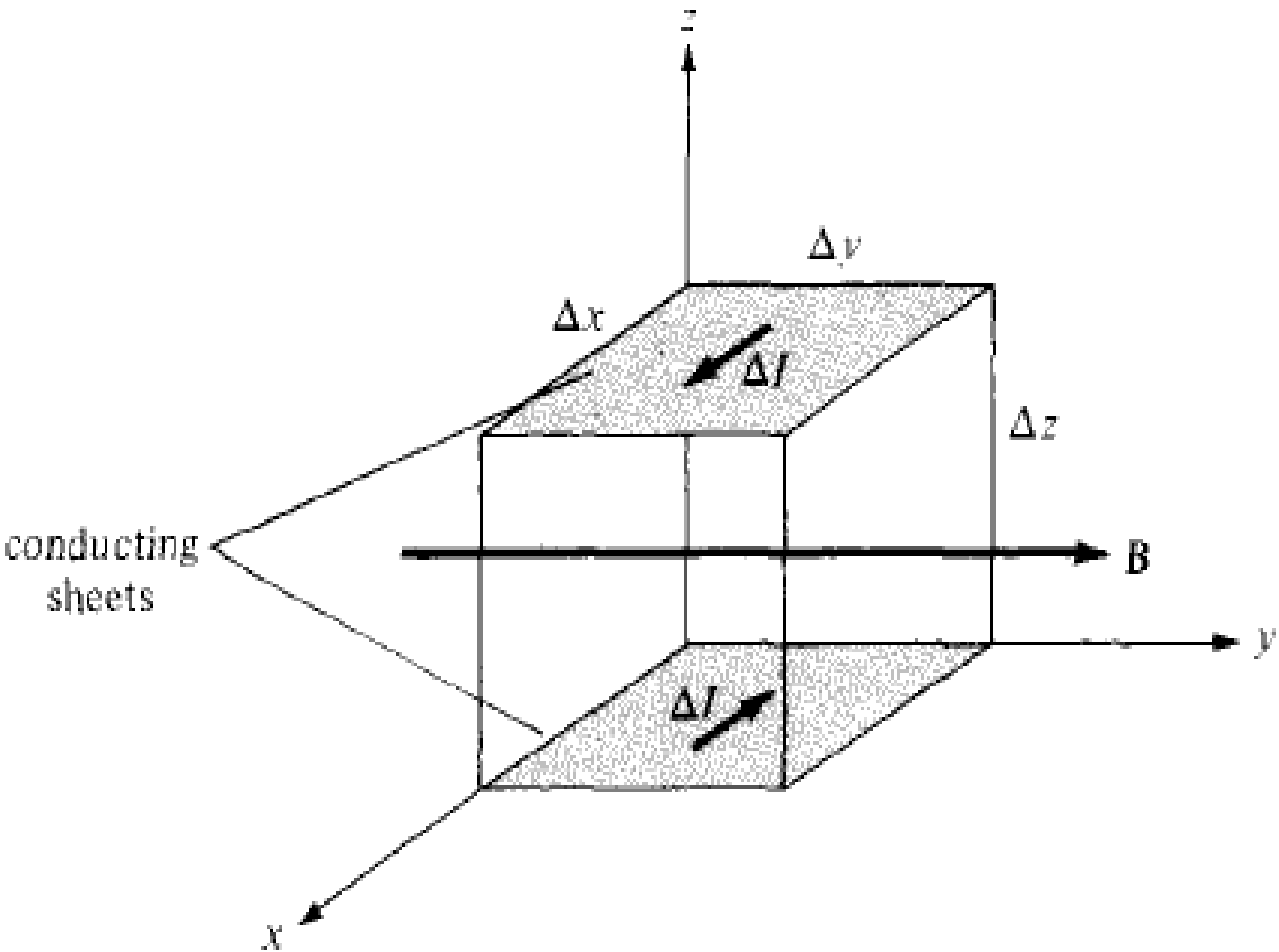
Just as the potential energy in an electrostatic field was derived as

$$W_E = \frac{1}{2}\int \mathbf{D}\cdot\mathbf{E}\,dv = \frac{1}{2}\int \epsilon E^2\,dv$$

$$\Delta L = \frac{\Delta\Psi}{\Delta I} = \frac{\mu H\,\Delta x\,\Delta z}{\Delta I}$$

$$\Delta W_m = \frac{1}{2}\Delta L\,\Delta I^2 = \frac{1}{2}\mu H^2\,\Delta x\,\Delta y\,\Delta z$$

$$\Delta W_m = \frac{1}{2}\mu H^2\,\Delta v$$


z
Δy
Δx
ΔI
Δz
conducting
sheets
B
y
ΔI
x

$$w_m = \lim_{\Delta v \to 0} \frac{\Delta W_m}{\Delta v} = \frac{1}{2} \mu H^2$$

$$w_m = \frac{1}{2} \mu H^2 = \frac{1}{2} B \cdot H = \frac{B^2}{2\mu}$$

$$W_m = \int w_m \, dv$$

$$\boxed{W_m = \frac{1}{2} \int \mathbf{B} \cdot \mathbf{H} \, dv = \frac{1}{2} \int \mu H^2 \, dv}$$

Inductance of a Solenoid

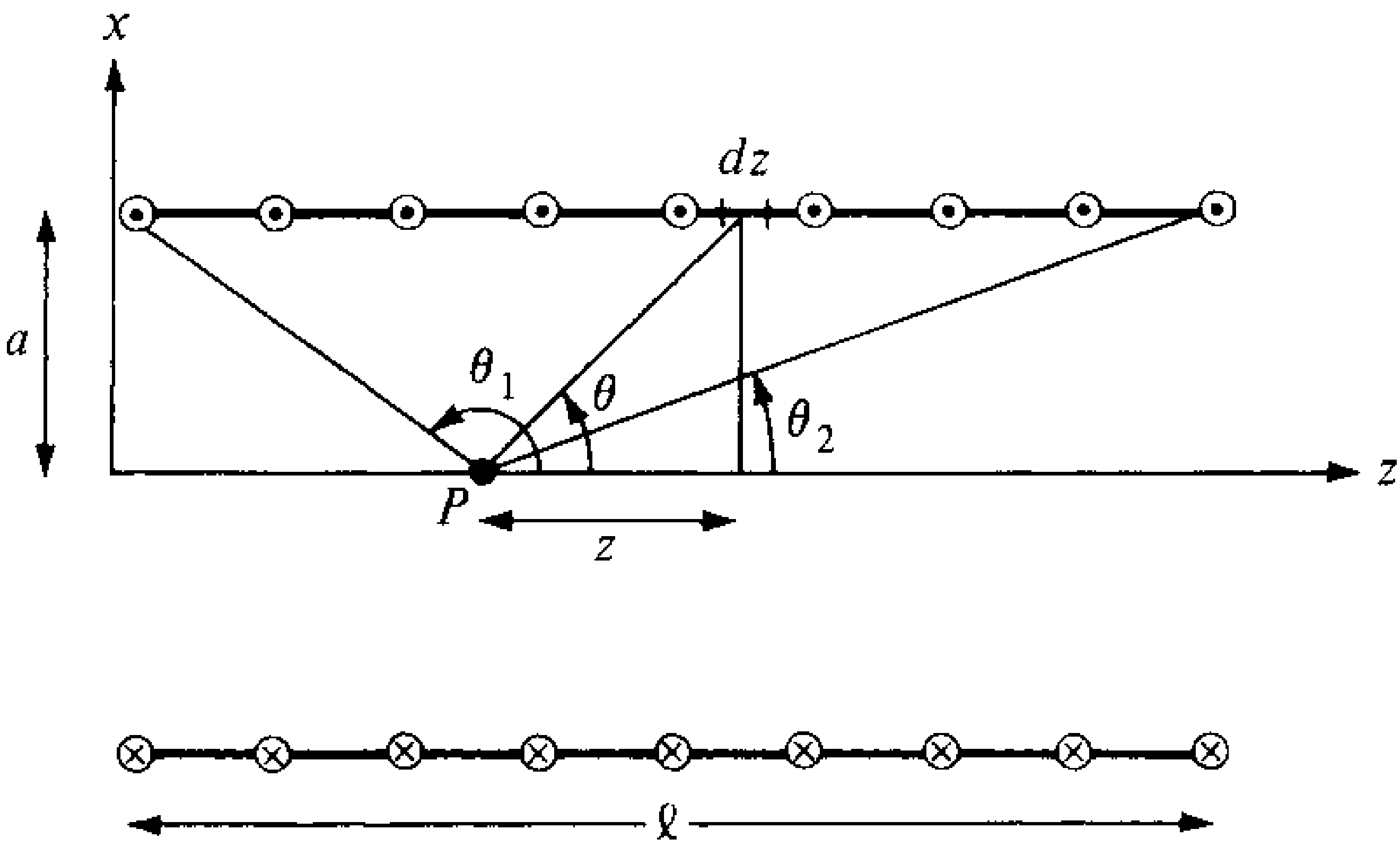
For an infinitely long solenoid, the magnetic flux inside the solenoid per unit length is

$$B = \mu H = \mu I n$$

$$\Psi = BS = \mu I n S$$

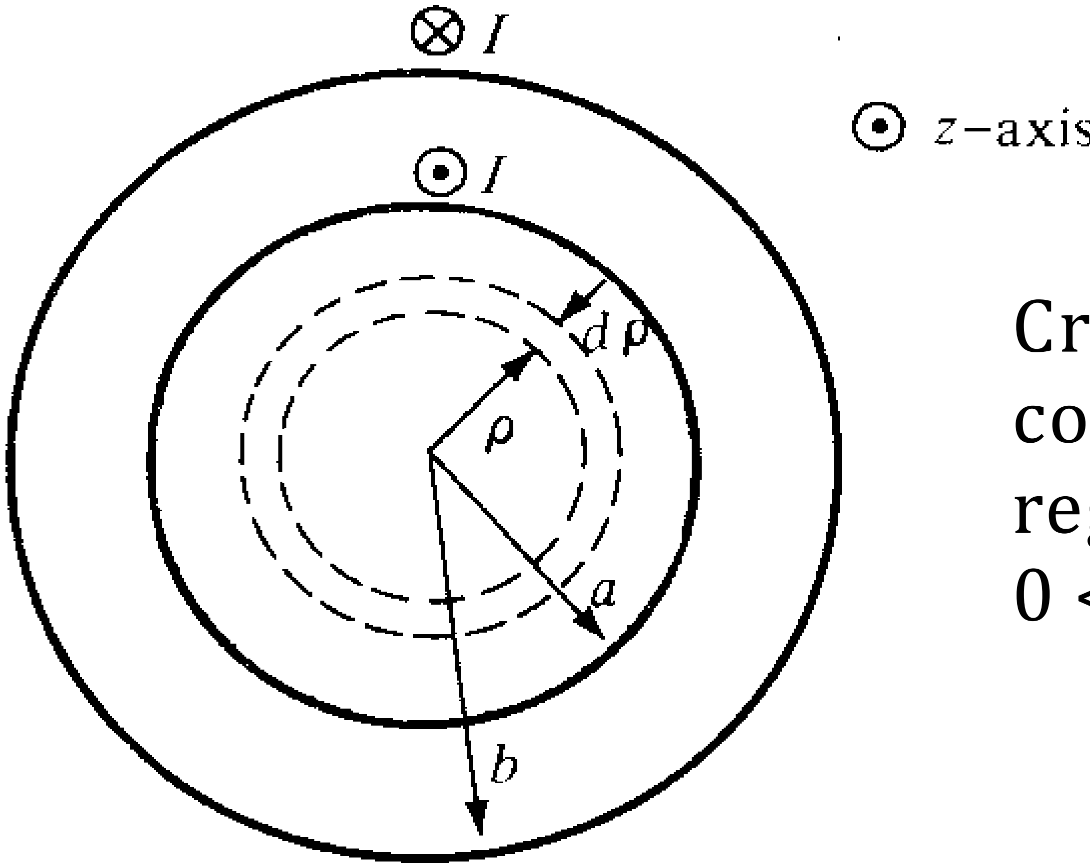
$$\lambda' = \frac{\lambda}{\ell} = n\Psi = \mu n^2 I S$$

$$L' = \frac{L}{\ell} = \frac{\lambda'}{I} = \mu n^2 S$$

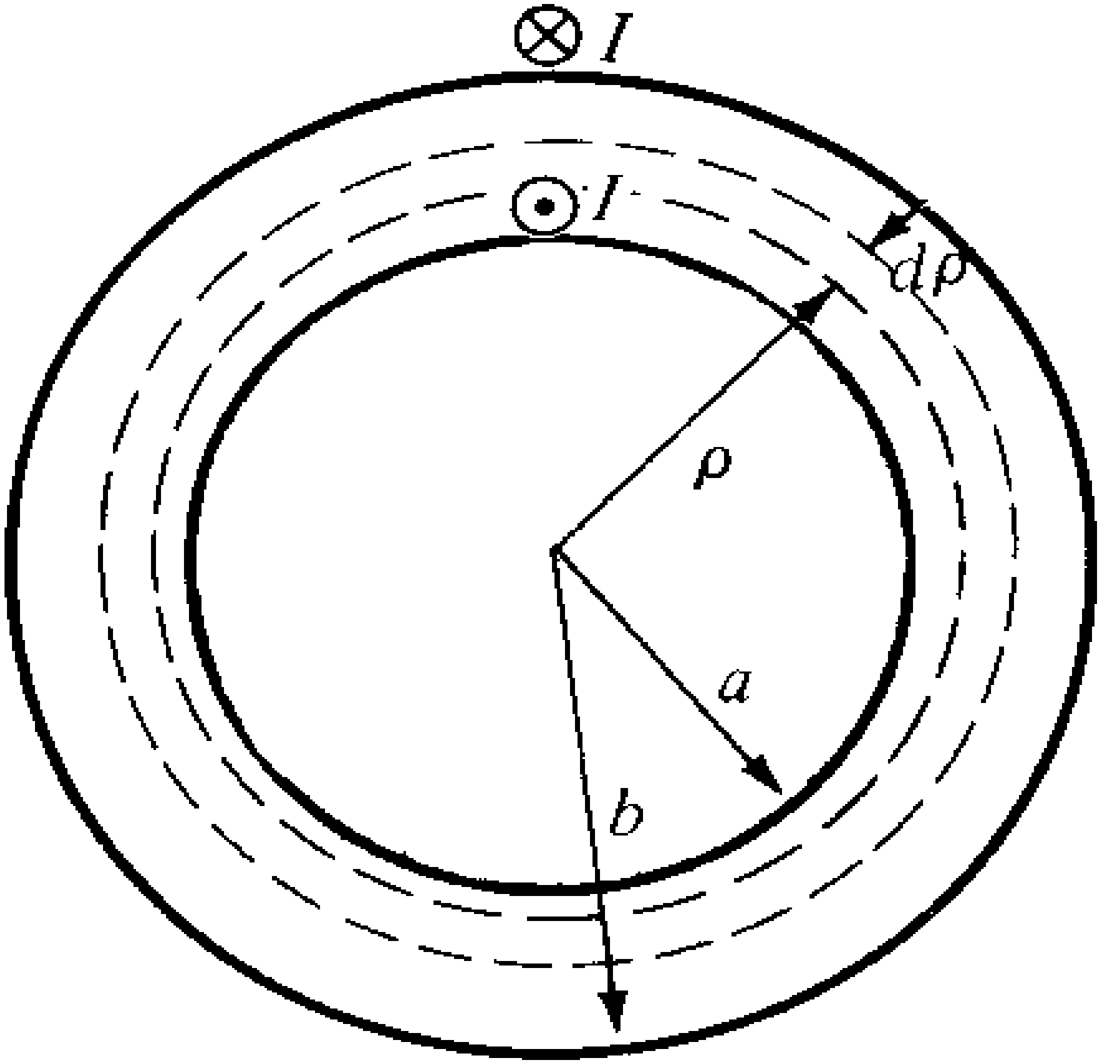

x
dz
a
θ_1
θ
θ_2
z
P
z
ℓ

$$\boxed{L' = \mu n^2 S} \qquad \text{H/m}$$

Co-Axial Cable



Cross section of the coaxial cable: for region 1,
$0 < p < a$,



(b) for region 2, $a < p < b$

We first find the internal inductance *Lin* by considering the flux linkages due to the inner conductor. From Figure, the flux leaving a differential shell of thickness *dp* is

$$d\Psi_1 = B_1\, d\rho\, dz = \frac{\mu I \rho}{2\pi a^2}\, d\rho\, dz$$

$$d\lambda_1 = d\Psi_1 \cdot \frac{I_{\text{enc}}}{I} = d\Psi_1 \cdot \frac{\pi\rho^2}{\pi a^2}$$

$$d\lambda_1 = \frac{\mu I \rho\, d\rho\, dz}{2\pi a^2} \cdot \frac{\rho^2}{a^2}$$

For length l of the cable,

$$\lambda_1 = \int_{\rho=0}^{a} \int_{z=0}^{\ell} \frac{\mu I \rho^3 \, d\rho \, dz}{2\pi a^4} = \frac{\mu I \ell}{8\pi}$$

$$L_{in} = \frac{\lambda_1}{I} = \frac{\mu \ell}{8\pi}$$

The internal inductance per unit length, given by

$$\boxed{L'_{in} = \frac{L_{in}}{\ell} = \frac{\mu}{8\pi}} \qquad \text{H/m}$$

the external inductance L_{ext} by considering the flux linkages between the inner and the outer conductor as in Figure. For a differential shell of thickness dρ.

$$d\Psi_2 = B_2\, d\rho\, dz = \frac{\mu I}{2\pi\rho}\, d\rho\, dz$$

$$\lambda_2 = \Psi_2 = \int_{\rho=a}^{b} \int_{z=0}^{\ell} \frac{\mu I\, d\rho\, dz}{2\pi\rho} = \frac{\mu I \ell}{2\pi} \ln \frac{b}{a}$$

$$L_{\text{ext}} = \frac{\lambda_2}{I} = \frac{\mu \ell}{2\pi} \ln \frac{b}{a}$$

$$L = L_{\text{in}} + L_{\text{ext}} = \frac{\mu \ell}{2\pi}\left[\frac{1}{4} + \ln\frac{b}{a}\right]$$

The inductance per length is

$$\boxed{L' = \frac{L}{\ell} = \frac{\mu}{2\pi}\left[\frac{1}{4} + \ln\frac{b}{a}\right]} \qquad \text{H/m}$$

Maxwell's Equations

- We have been examining a variety of electrical and magnetic phenomena
- James Clerk Maxwell summarized all of electricity and magnetism in just four equations
- Remarkably, the equations predict the existence of electromagnetic waves

Maxwell's Equations

Name	Partial differential form	Integral form
Gauss's law:	$\nabla \cdot \mathbf{D} = \rho$	$\oint_A \mathbf{D} \cdot d\mathbf{A} = Q_{encl}$
Gauss's law for magnetism:	$\nabla \cdot \mathbf{B} = 0$	$\oint_A \mathbf{B} \cdot d\mathbf{A} = 0$
Faraday's law of induction:	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_S \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$
Ampere's law + Maxwell's extension:	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$	$\oint_S \mathbf{H} \cdot d\mathbf{s} = I_{\text{enc}} + \frac{d\mathbf{\Phi_D}}{dt}$

Maxwell’s Equations

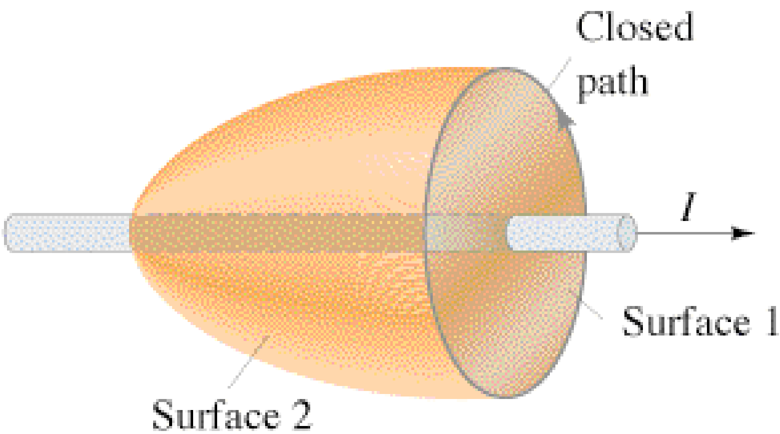
- The first is Gauss’s Law which is an extended form of Coulomb’s Law
- The second is the equivalent for magnetic fields, except that we know that magnetic poles always occur in pairs (north & south)

Maxwell’s Equations

- The third is Faraday’s Law that a changing magnetic field produces an electric field
- The fourth is that a changing electric field produces a magnetic field
- The latter is a bit of a stretch. We knew that a current produces a magnetic field

Maxwell's Equations

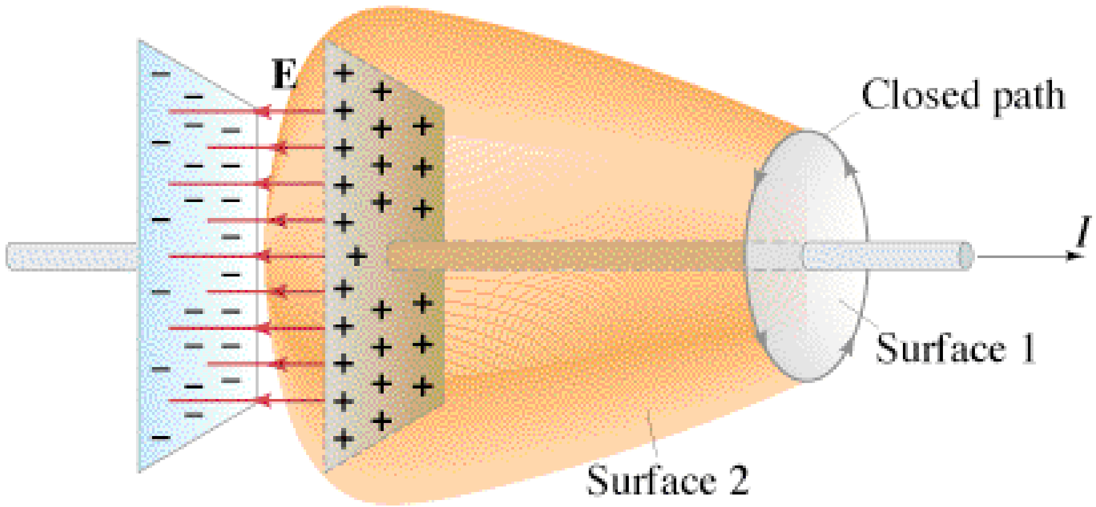
- Start with Ampere's Law $\sum B_{\parallel}\Delta l = \mu_0 I$



Earlier, we just went on a closed path enclosing surface 1. But according to Ampere's Law, we could have considered surface 2. The current enclosed is the same as for surface 1. We can say that the current flowing into any volume must equal that coming out.

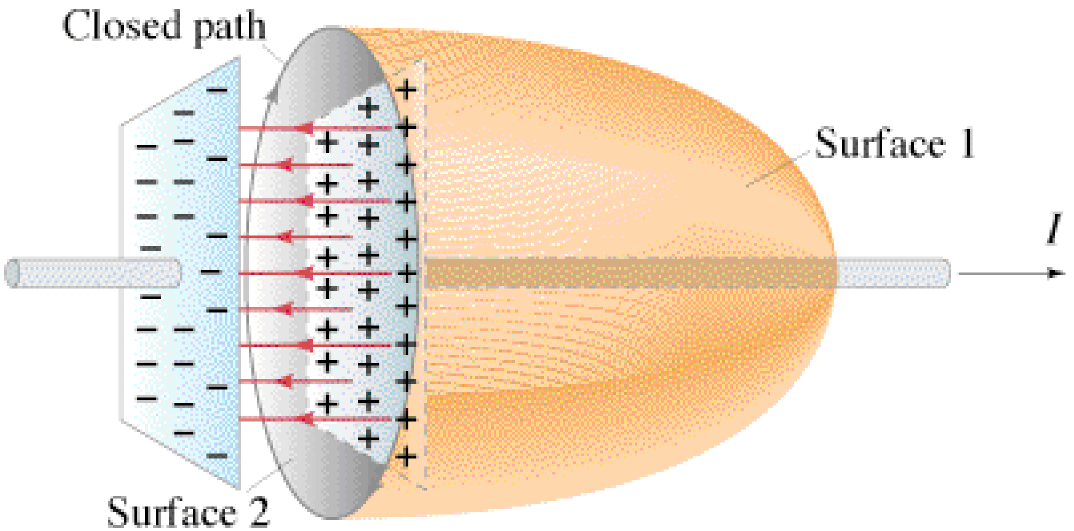
Maxwell’s Equations

- Suppose we have a charged capacitor and it begins to discharge



Surface 1 works but surface 2 has no current passing through the surface yet there is a magnetic field inside the surface.

Maxwell's Equations



Same problem here. Surface 1 works, but no current passes through surface two which encloses a magnetic field. What is happening???

Maxwell’s Equations

- While the capacitor is discharging, a current flows
- The electric field between the plates of the capacitor is decreasing as current flows
- Maxwell said the changing electric field is equivalent to a current
- He called it the **displacement current**

Maxwell's Equations

$$\sum B_{\parallel}\Delta l = \mu_0 \left(I_C + I_D\right)$$

$$Q = CV = \varepsilon_0 \left(\frac{A}{d}\right)\left(Ed\right) = \varepsilon_0 AE$$

$$\frac{\Delta Q}{\Delta t} = \varepsilon_0 A \frac{\Delta E}{\Delta t} = I_D$$

$$\Phi_E = AE$$

$$I_D = \varepsilon_0 \frac{\Delta \Phi_E}{\Delta t}$$

$$\sum B_{\parallel}\Delta l = \mu_0 I_C + \mu_0 \varepsilon_0 \frac{\Delta \Phi_E}{\Delta t}$$