POWER POINT PRESENTATION
ON
ELECTRICAL TECHNOLOGY

III Semester (IARE-R16)

Prepared

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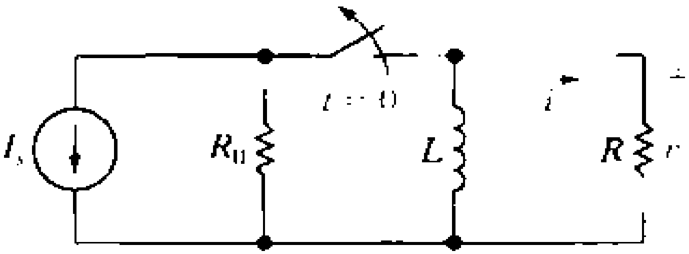
UNIT-I

TRANSIENT ANALYSIS

Natural Response of an RL circuit

- If the current is constant, the voltage across the inductor is zero.
- Thus the inductor behaves as a short circuit in the presence of a constant, or dc, current.
- Assume a constant current source.
- Assume that the switch has been closed for long time.



- So, no current passes through R_o or R and all source current I_s appears in the inductive branch.
- We find the natural response after the switch has been opened at t=0 ($i(0^-) = I_s$).

Time Constant, τ for RL circuit

- Time constant, τ determines the rate at which the current or voltage approaches zero.
- The time constant of a circuit is the time required for the response to decay to a factor of 1/e or 36.8% of its initial current
- Natural response of the RL circuit is an exponential decay of the initial current. The current response is shown in Fig. 5-1
- Time constant for RL circuit is

$$\tau = \frac{L}{R}$$

- And the unit is in seconds.

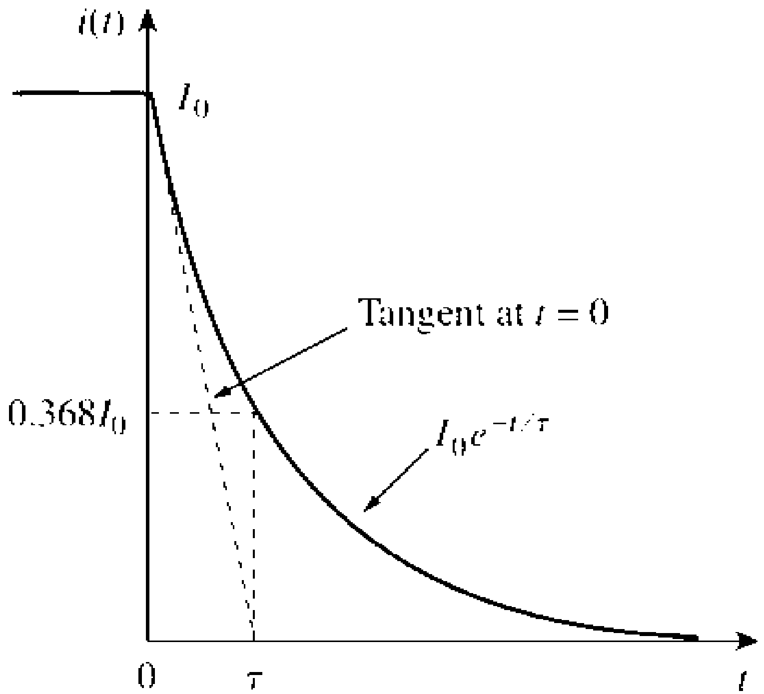


Figure 5-1

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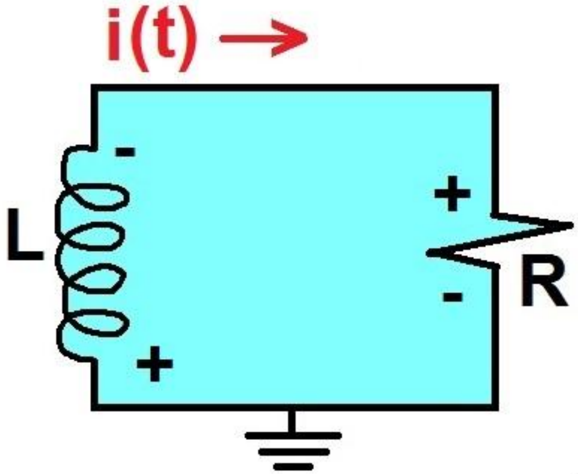
The RL Circuit without a Source



$i(t=0) = I_0$ energy stored:

$$u(t=0) = \frac{1}{2} L I_0^2$$

$v_L(t) = L\frac{di}{dt}$

using KVL:

$v_L + v_R = 0$

$$L\frac{di}{dt} + iR = 0$$

⋮

time constant: $\tau = \frac{L}{R}$

$$i(t) = I_0 e^{\frac{-t}{L/R}} \qquad i(t) = I_0 e^{\frac{-t}{\tau}}$$

TRANSIENT RESPONSE OF RC SERIES CIRCUIT

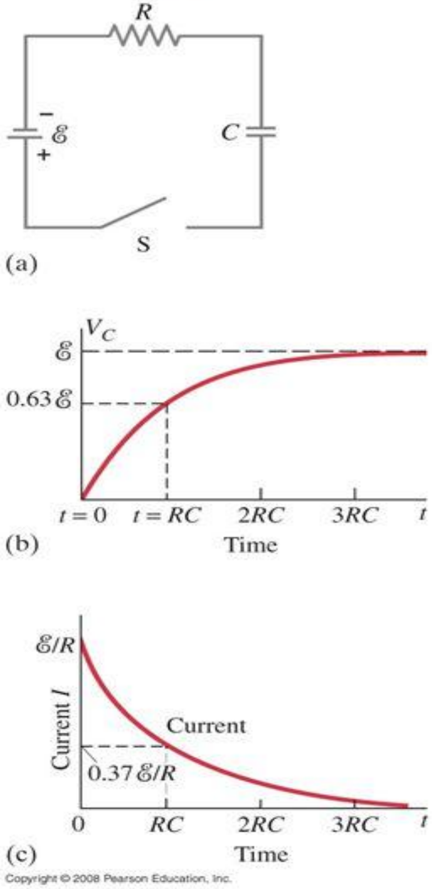
RC Circuit Transient Analysis

$$V_C = \xi\left(1 - e^{-t/RC}\right)$$

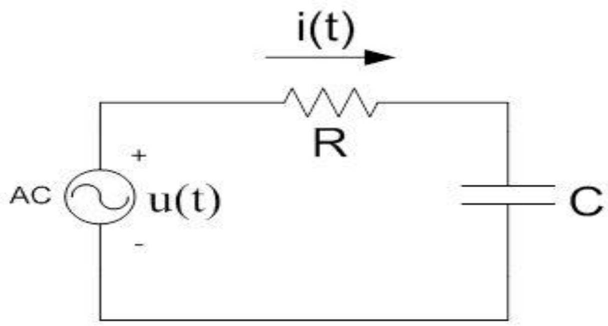
$$I = I_0 e^{-t/RC} \quad where \quad I_0 = \frac{\xi}{R}$$

$$and \quad let \quad \tau = RC$$

- τ is a time constant
- The voltage across the capacitor reaches 98% of the battery *EMF* in 4 τ
- The transient response of the circuit is over in approximately 4 - 5 τ



RC Circuits



The response of RC circuits can be categorized into two parts:

- **Transient Response**
- **Forced Response**

Transient response comes from the dynamic of R,C.
Forced response comes from the voltage source.

The complete response

- The combination of natural and step (or forced) responses
- For **RC** circuit, the complete response is:

$$v_c(t) = \underline{V_o e^{-\frac{t}{\tau}}} + \underline{V_s(1 - e^{-\frac{t}{\tau}})}$$

Natural response:

- Response due to initial energy stored in capacitor
- V_o is the <u>initial value,</u> i.e. $v_c(0)$

Forced response:

- Response due to the present of the source
- V_s is the <u>final value</u> i.e $v_c(\infty)$

Note: this is what we obtained when we solved the step response with initial energy (or initial voltage) at *t =0*

Step Response of A Series *RLC* Circuit

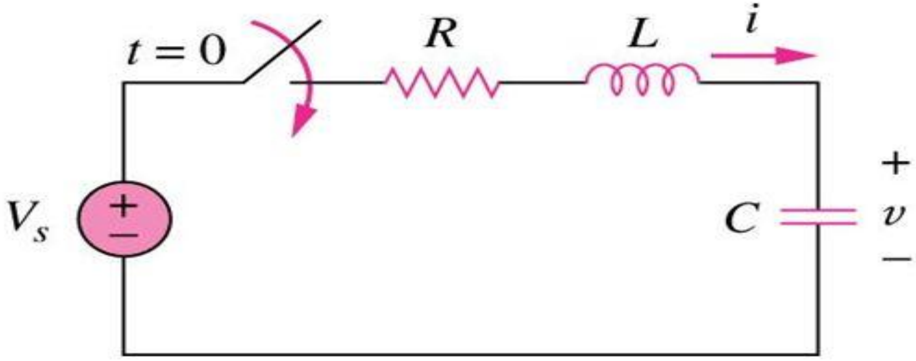
Applying KVL for $t > 0$,

$$Ri + L\frac{di}{dt} + v = V_S \qquad (1)$$

But $i = C\frac{dv}{dt}$

$$\Rightarrow \boxed{\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = \frac{V_S}{LC}} \quad (2)$$

(2) has the same form as in the source - free case.



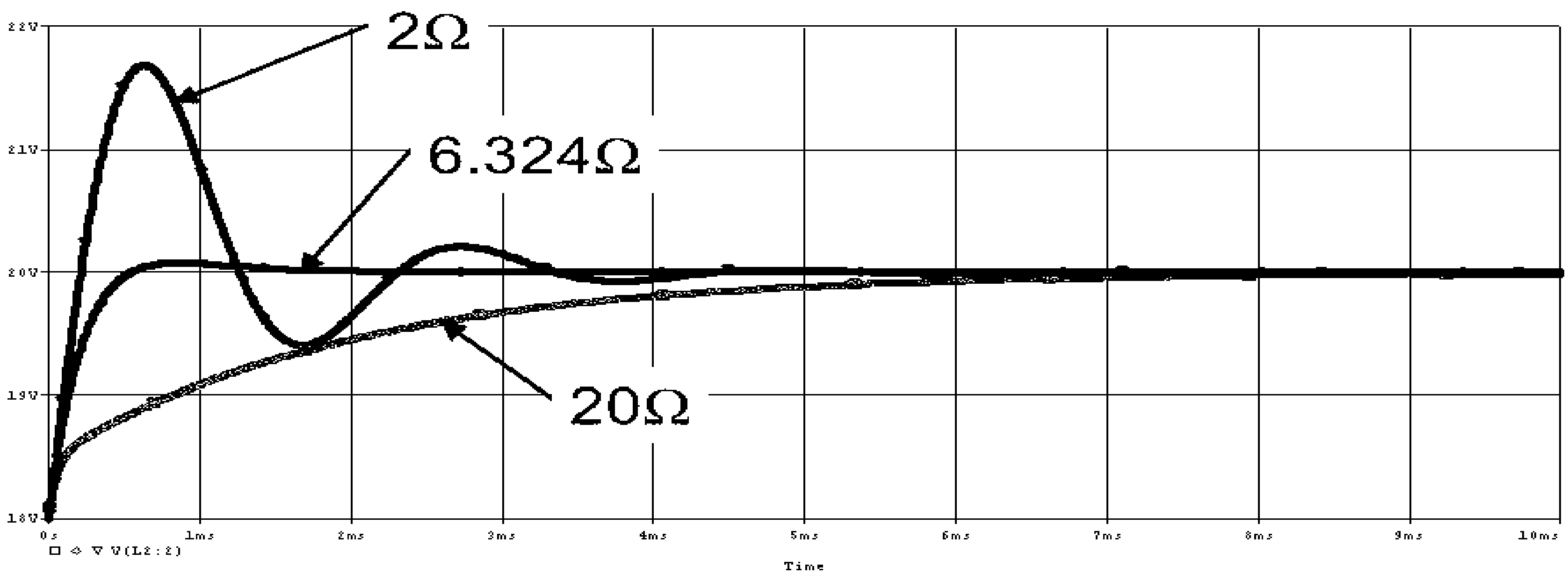
$$v(t) = v_t(t) + v_{ss}(t)$$

where

$$\begin{cases} v_t : \text{the transient response} \\ v_{ss} : \text{the steady - state response} \end{cases}$$

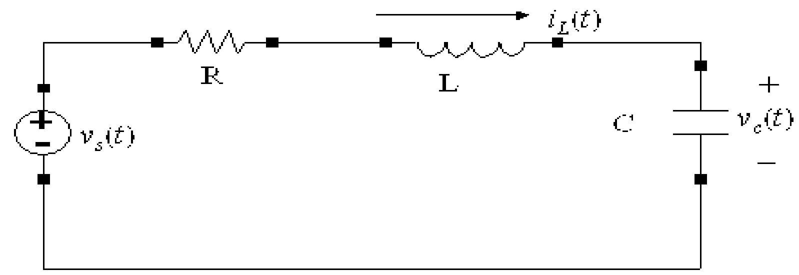
Step Response - Series RLC Circuit

- Comparison of the three responses with different R value.



Example

Find the differential equation for the circuit below in terms of v_c and also terms of i_L



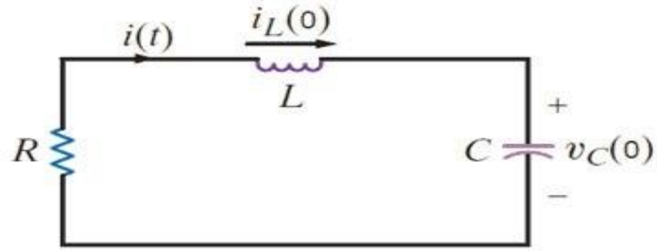
Show:

$$v_S(t) = LC\frac{d^2v_C}{dt^2} + RC\frac{dv_C}{dt} + v_C \quad \rightarrow \quad \frac{v_S(t)}{LC} = \frac{d^2v_C}{dt^2} + \frac{R}{L}\frac{dv_C}{dt} + \frac{1}{LC}v_C$$

$$v_S(t) = L\frac{di_L}{dt} + Ri_L + \frac{1}{C}\int_{-\infty}^{t} i_L(\tau)d\tau \quad \rightarrow \quad \frac{v_S(t)}{L} = \frac{di_L}{dt} + \frac{R}{L}i_L + \frac{1}{LC}\int_{-\infty}^{t} i_L(\tau)d\tau$$

Example

The series *RLC* circuit shown in Fig. 7.18 has the following parameters: C=0.04 F, L=1 H, R=6 , $i_L(0)$ = 4 A, and $v_C(0)$ = -4 V. Let us determine the expression for both the current and the capacitor voltage.



Applying Kirchhoff voltage law to the loop, $$iR + \frac{1}{C}\int_0^t i(x)dx + v_C(0) + L\frac{di}{dt} = 0$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0 \quad \Rightarrow \quad \frac{d^2 i}{dt^2} + 6\frac{di}{dt} + 25i = 0$$

Trial solution : $i(t) = Ke^{st}$

Characteristic equation : $s^2 + 6s + 25 = 0$

$$s = \frac{-6 \pm \sqrt{6^2 - 100}}{2} = \frac{-6 \pm 8j}{2} = -3 \pm 4j$$

First order transient circuits

Solution to 1st order differential equation :

$$\frac{dx(t)}{dt} + ax(t) = f(t)$$

$f(t) = 0 \rightarrow$ homogeneous equation
$f(t) \neq 0 \rightarrow$ inhomogeneous equation

$$\frac{dx_c(t)}{dt} + ax_c(t) = 0$$

$x_h(t)$ or $x_c(t) \rightarrow$ homogeneous or complementary solution
$x_p(t) \rightarrow$ inhomogeneous or particular solution

$$x(t) = x_p(t) + x_c(t)$$

Laplace Transform

- Applications of the Laplace transform
  - solve differential equations (both ordinary and partial)
  - application to RLC circuit analysis
- Laplace transform converts differential equations in the time domain to algebraic equations in the frequency domain, thus 3 important processes:

  (1) transformation from the time to frequency domain

  (2) manipulate the algebraic equations to form a solution

  (3) inverse transformation from the frequency to time domain

Transform Pairs

The Laplace transforms pairs

$f(t)$	$\mathbf{F}(s)$
$\delta(t)$	1
u(t) {a constant}	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$

Critically Damped Case (Series RLC)

- In the critically damped case, the two poles are real and equal. Assume that the poles are $s_1 = s_2 = -\alpha$, the forms for *I(s)* and $V_C(s)$ can be expressed as

$$I(s) = \frac{\frac{V_i}{L}}{(s+\alpha)^2}$$

The damping factor: $\alpha = \frac{R}{2L}$

$$V_C(s) = \frac{\frac{V_i}{LC}}{s(s+\alpha)^2}$$

- The inverse transforms are of the forms

$$i(t) = C_0 t e^{-\alpha t} = \frac{V_i t}{L} e^{-Rt/2L}$$

$C_0 = V_i / L$

$$v_C(t) = V_i + (C_1 t + C_2) e^{-Rt/2L}$$

The most significant aspect of the natural response function for the critically damped case is the $te^{-\alpha t}$ form. Although the t factor increases with increasing t, the $te^{-\alpha t}$ decreases at a faster rate, so the product eventually approaches zero.

UNIT-II

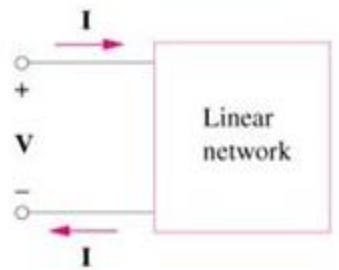
TWOPORT NETWORKS

Introduction to two port network

A port refers to a pair of terminals through which a current may enter or leave a network

We have focused only on one-port networks so far, where we consider the voltage across or current through a single pair of terminal

The rest of this unit deals with two-port networks

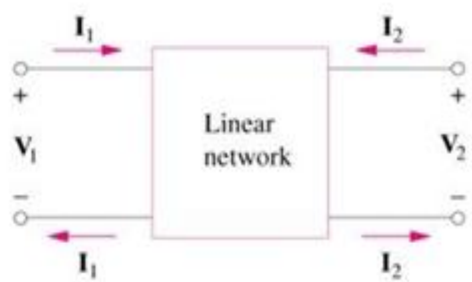


(a)

A two-port network is an electrical network with two separate ports for input and output

We will see examples of two-port networks (op amps and transistor circuits) later on in this course

Like for a one-port network, knowing the parameters of a two-port network enables use to treat it as a "black-box" placed within a larger network.



(b)

In a two-port network, we need to relate V_1, V_2, I_1, I_2

The terms relating these currents and voltages are known as **parameters**

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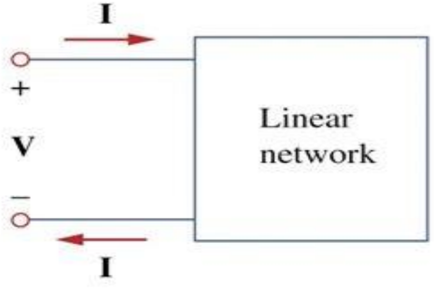
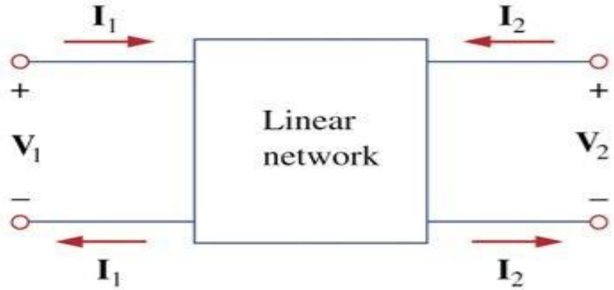
Cont..

- A **two-port Network** is an electrical network with two separate ports for input and output.



(a)



(b)

Impedance Parameters (Z)

Z parameters are also known as impedance parameters. When we use

Z parameter for analyzing two part network, the voltages are represented

as the function of currents.

Two Port Networks

Z parameters:

$$z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0}$$

z_{11} is the impedance seen looking into port 1 when port 2 is open.

$$z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$

z_{12} is a transfer impedance. It is the ratio of the voltage at port 1 to the current at port 2 when port 1 is open.

$$z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

z_{21} is a transfer impedance. It is the ratio of the voltage at port 2 to the current at port 1 when port 2 is open.

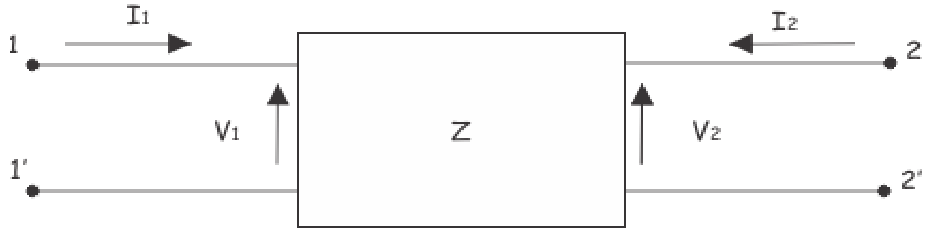
$$z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

z_{22} is the impedance seen looking into port 2 when port 1 is open.

Cont..

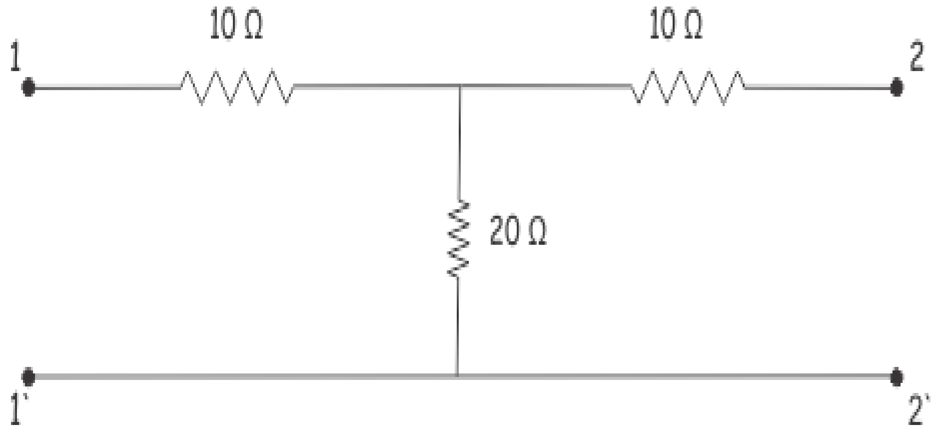
- The voltages are represented as

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \text{ and } V_2 = Z_{21}I_1 + Z_{22}I_2$$



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

- **Problem:** Find the z parameters for network shown in figure



Admittance Parameters (Y)

- We can represent current in terms of voltage for admittance parameters of a two port network. Then we will represent the current voltage relations as.

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

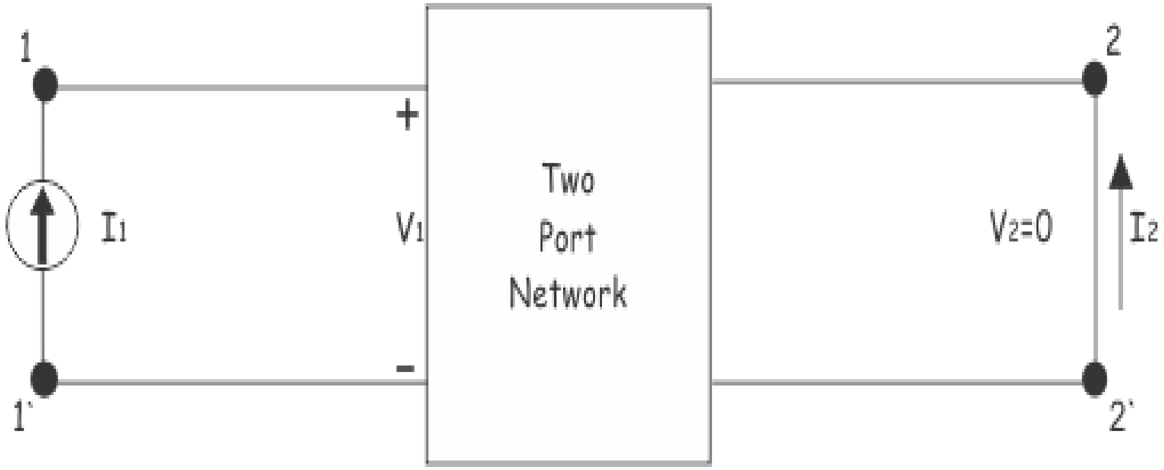
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

- This can also be represented in matrix form as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Cont..

- Here, Y11, Y12, Y21 and Y22 are admittance parameter. Sometimes these are called as Y parameters. We can determine the values of the parameters of a particular two port network by making short-circuited output port and input port alternatively as follows. First let us apply current source of I1 at input port keeping the output port short circuited as shown below.



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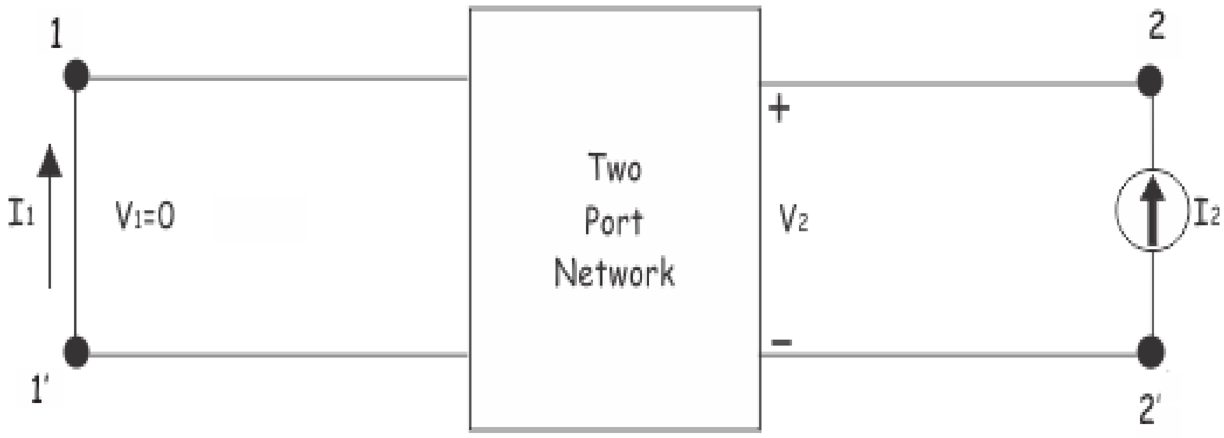
This is referred as short circuit transfer admittance from input port to output port. Now, let us short circuit the input port of the network and apply current I_2 at output port, as shown below



Cont..

$$y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$$

y_{11} is the admittance seen looking into port 1 when port 2 is shorted.

$$y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0}$$

y_{12} is a transfer admittance. It is the ratio of the current at port 1 to the voltage at port 2 when port 1 is shorted.

$$y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0}$$

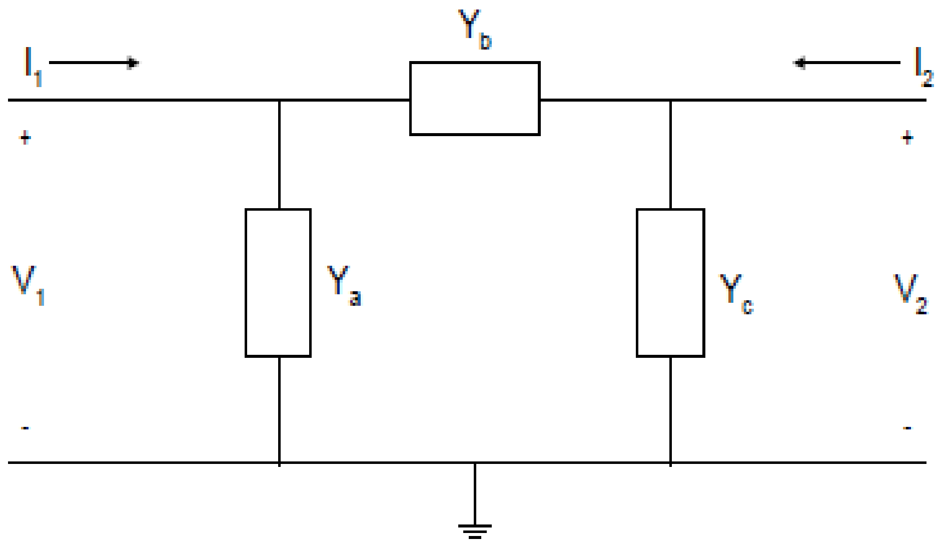
y_{21} is a transfer impedance. It is the ratio of the current at port 2 to the voltage at port 1 when port 2 is shorted.

$$y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0}$$

y_{22} is the admittance seen looking into port 2 when port 1 is shorted.

Numerical problems

1. Problem: Find the Y- Parameters for the given network shown in figure



Hybrid Parameters or h Parameters:

- Hybrid parameters are also referred as h parameters.
- These are referred as hybrid because, here Z parameters, Y parameters, voltage ratio, current ratio, all are used to represent the relation between voltage and current in a two port network.
- The relations of voltages and current in hybrid parameters are represented as

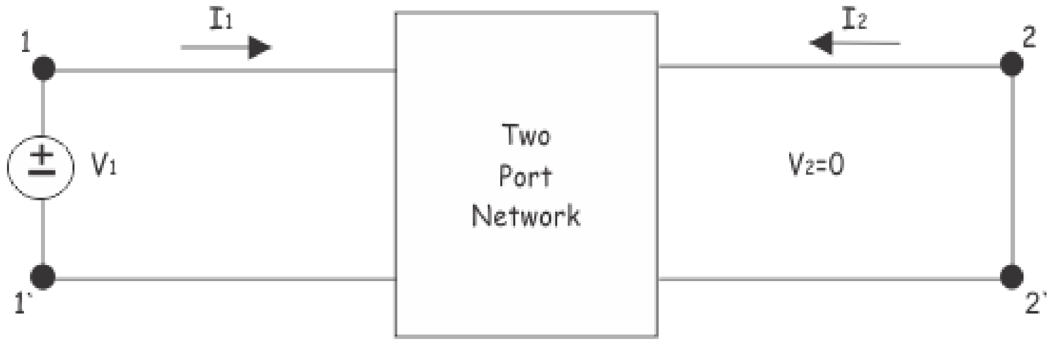
$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

- This can be represented in matrix form as,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Determining h Parameters

- Let us short circuit the output port of a two port network as shown below



- Now, ratio of input voltage to input current, at short circuited output port, is

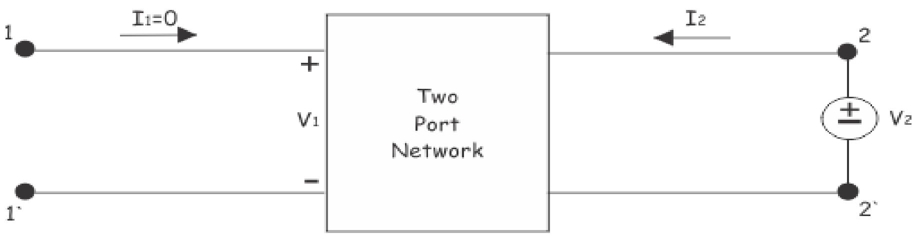
$$\left.\frac{V_1}{I_1}\right|_{V_2 = 0} = h_{11}$$

- This is referred as short circuit input impedance. Now, the ratio of the output current to input current at short circuited output port, is

$$\left.\frac{I_2}{I_1}\right|_{V_2 = 0} = h_{21}$$

Cont..

- This is called short circuit current gain of the network. Now, let us open circuit the port 1. At that condition, there will be no input current (I1=0) but open circuit voltage V1 appears across the port 1, as shown below



$$\left.\frac{V_1}{V_2}\right|_{I_1=0} = h_{12} = \textit{open circuit reverse voltage gain}$$

- This is referred as reverse voltage gain because, this is the ratio of input voltage to output voltage of the network, but voltage gain is defined as ratio of output voltage to input voltage of a network. Now

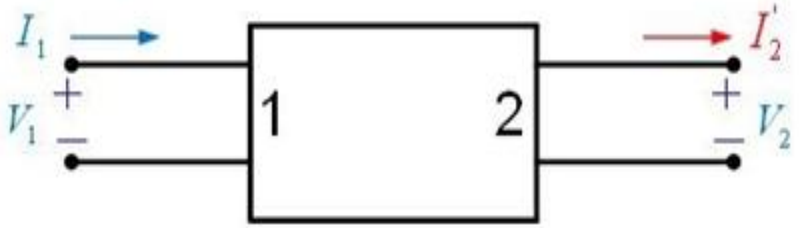
$$\left.\frac{I_2}{V_2}\right|_{I_1=0} = h_{21}$$

ABCD Parameters

- Concerning the equivalent port representations of networks we've seen in this course:
- 1. Z parameters are useful for series connected networks,
- 2. Y parameters are useful for parallel connected networks,
- 3. S parameters are useful for describing interactions of voltage and current waves with a network.
- There is another set of network parameters particularly suited for cascading two-port networks. This set is called the ABCD matrix or, equivalently, the transmission matrix

Cont..

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2^{'} \end{bmatrix}$$



$$I_2^{'} = -I_2$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2^{'}=0} \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2^{'}=0}$$

$$B = \left.\frac{V_1}{I_2^{'}}\right|_{V_2=0} \qquad D = \left.\frac{I_1}{I_2^{'}}\right|_{V_2=0}$$

Cont..

The ABCD parameters are defined as follows:

$$A = \frac{V_1}{V_2}\Big|_{I_2=0}$$ open circuit reverse voltage transfer ratio

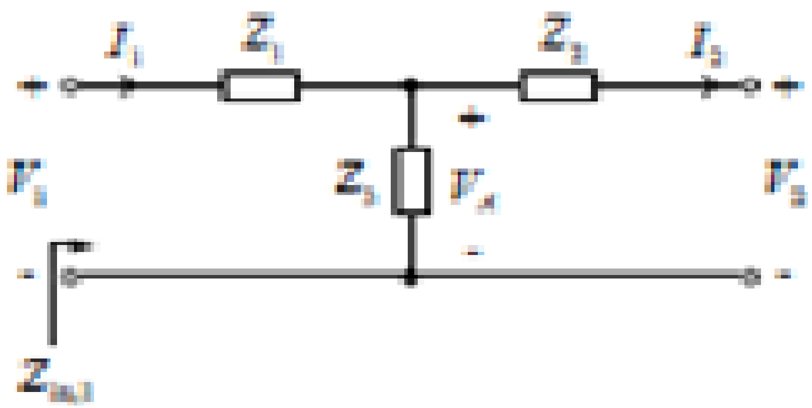
$$B = \frac{V_1}{-I_2}\Big|_{V_2=0}$$ = short circuit reverse transfer impedance

$$C = \frac{I_1}{V_2}\Big|_{I_2=0}$$ open circuit reverse transfer admittance

$$D = \frac{I_1}{-I_2}\Big|_{V_2=0}$$ short circuit reverse current transfer ratio.

Numerical Problems

Derive the ABCD Parameters for the T network



Condition for reciprocity

Properties:

I) Reciprocity

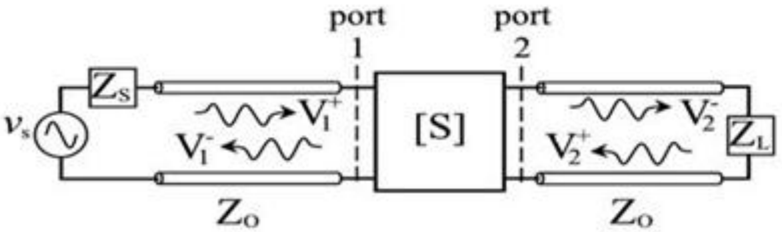
The two-port network is reciprocal if the transmission characteristics are the same in both directions (i.e. $S_{21} = S_{12}$).

It is a property of passive circuits (circuits with no active devices or ferrites) that they form reciprocal networks.

A network is reciprocal if it is equal to its transpose. Stated mathematically, for a reciprocal network

$$[S] = [S]^t ,$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^t = \begin{bmatrix} S_{11} & S_{21} \\ S_{12} & S_{22} \end{bmatrix}.$$



By inspection If the network is Symmetrical ⟹ Reciprocal

Condition for Reciprocity: $\mathbf{S_{12} = S_{21}}$

Condition for symmetry

A network is symmetrical if its input impedance is equal to its output impedance. Most often, but not necessarily, symmetrical networks are also physically symmetrical

Symmetry : For the network to be symmetrical, the voltage–to–current ratio at one port should be the same as the voltage–to–current ratio at the other port with one of the port short circuited.

Condition for symmetry

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

When the output port is short circuited, i.e., $V_2 = 0$. From the Y-parameter equation

$$I_1 = Y_{11} V_s$$
$$\frac{V_s}{I_1} = \frac{1}{Y_{11}}$$

When the input port is short circuited, i.e., $V_1 = 0$. From the Y-parameter equation,

$$I_2 = Y_{22} V_s$$
$$\frac{V_s}{I_2} = \frac{1}{Y_{22}}$$

Hence, for the network to be symmetrical,

$$\frac{V_s}{I_1} = \frac{V_s}{I_2}$$

$$\boxed{Y_{11} = Y_{22}}$$

Condition for symmetry

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{h_{11}I_1 + h_{12}V_2}{I_1}\right|_{I_2=0}$$
$$= h_{11} + h_{12}\frac{V_2}{I_1}$$

But with $I_2 = 0$, we have

$$0 = h_{21} I_1 + h_{22} V_2$$
$$\frac{V_2}{I_1} = \frac{-h_{21}}{h_{22}}$$
$$Z_{11} = h_{11} - \frac{h_{12}h_{21}}{h_{22}} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}}$$

where

$$\Delta h = h_{11} h_{22} - h_{12} h_{21}$$

Similarly,

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

With $I_1 = 0$, we have

$$I_2 = h_{22} V_2$$
$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{1}{h_{22}}$$

For a symmetrical network $Z_{11} = Z_{22}$

i.e.,
$$\frac{1}{h_{22}} = \frac{\Delta h}{h_{22}}$$

i.e.,
$$\Delta h = 1$$

$$h_{11} h_{22} - h_{12} h_{21} = 1$$

Cont...

Parameter	Symmetry	Reciprocity
Z	$Z_{11} = Z_{22}$	$Z_{12} = Z_{21}$
Y	$Y_{11} = Y_{22}$	$Y_{12} = Y_{21}$
h	$\Delta_h = 1$	$h_{12} = -h_{21}$
G	$\Delta_G = 1$	$G_{12} = -G_{21}$
T	$A = D$	$\Delta_T = 1$

Two port parameter conversions

- **Procedure of two port parameter conversions**
- convert one set of two-port network parameters into other set of two port network parameters. This conversion is known as two port network parameters conversion or simply, two-port parameters conversion.
- **Step 1** − Write the equations of a two port network in terms of desired parameters.
- **Step 2** − Write the equations of a two port network in terms of given parameters.
- **Step 3** − Re-arrange the equations of Step2 in such a way that they should be similar to the equations of Step1.
- **Step 4** − By equating the similar equations of Step1 and Step3, we will get the desired parameters in terms of given parameters. We can represent these parameters in matrix form.

Z parameters to T parameters

Here, we have to represent T parameters in terms of Z parameters. So, in this case T parameters are the desired parameters and Z parameters are the given parameters.

Step 1 − We know that, the following set of two equations, which represents a two port network in terms of **T parameters**.

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Step 2 − We know that the following set of two equations, which represents a two port network in terms of **Z parameters**.

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Step 3 − We can modify the above equation as

$$\Rightarrow V_2 - Z_{22}I_2 = Z_{21}I_1$$

$$\Rightarrow I_1 = \left(\frac{1}{Z_{21}}\right)V_2 - \left(\frac{Z_{22}}{Z_{21}}\right)I_2$$

Step 4 – The above equation is in the form of $I_1 = CV_2 - DI_2$. Here,

$$C = \frac{1}{Z_{21}}$$

$$D = \frac{Z_{22}}{Z_{21}}$$

Step 5 – Substitute I_1 value of Step 3 in V_1 equation of Step 2.

$$V_1 = Z_{11}\{(\frac{1}{Z_{12}})V_2 - (\frac{Z_{22}}{Z_{21}})I_2\} + Z_{12}I_2$$

$$\Rightarrow V_1 = (\frac{Z_{11}}{Z_{21}})V_2 - (\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}})I_2$$

Step 6 – The above equation is in the form of $V_1 = AV_2 - BI_2$. Here,

$$A = \frac{Z_{11}}{Z_{21}}$$

$$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$$

Two port network conversion

- **Admittance parameter to ABCD parameter conversion**

Y parameters to T parameters

Here, we have to represent T parameters in terms of Y parameters. So, in this case, T parameters are the desired parameters and Y parameters are the given parameters.

Step 1 − We know that, the following set of two equations, which represents a two port network in terms of **T parameters**.

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Step 2 − We know that the following set of two equations of two port network regarding Y parameters.

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Step 3 − We can modify the above equation as

$$\Rightarrow I_2 - Y_{22}V_2 = Y_{21}V_1$$

$$\Rightarrow V_1 = (\frac{-Y_{22}}{Y_{21}})V_2 - (\frac{-1}{Y_{21}})I_2$$

Step 4 − The above equation is in the form of $V_1 = AV_2 - BI_2$. Here,

$$A = \frac{-Y_{22}}{Y_{21}}$$

$$B = \frac{-1}{Y_{21}}$$

Step 5 − Substitute V_1 value of Step 3 in I_1 equation of Step 2.

$$I_1 = Y_{11}\{(\frac{-Y_{22}}{Y_{21}})V_2 - (\frac{-1}{Y_{21}})I_2\} + Y_{12}V_2$$

$$\Rightarrow I_1 = (\frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}})V_2 - (\frac{-Y_{11}}{Y_{21}})I_2$$

Step 6 − The above equation is in the form of $I_1 = CV_2 - DI_2$. Here,

$$C = \frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}}$$

$$D = \frac{-Y_{11}}{Y_{21}}$$

Cont...

Step 7 − Therefore, the **T parameters matrix** is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{-Y_{22}}{Y_{21}} & \frac{-1}{Y_{21}} \\ \frac{Y_{12}Y_{21}-Y_{11}Y_{22}}{Y_{21}} & \frac{-Y_{11}}{Y_{21}} \end{bmatrix}$$

- **ABCD Parameters to H- Parameters**

Here, we have to represent h-parameters in terms of T parameters. So, in this case hparameters are the desired parameters and T parameters are the given parameters.

Step 1 − We know that, the following **h-parameters** of a two port network.

$$h_{11} = \frac{V_1}{I_1},\ when\ V_2 = 0$$

$$h_{12} = \frac{V_1}{V_2},\ when\ I_1 = 0$$

$$h_{21} = \frac{I_2}{I_1},\ when\ V_2 = 0$$

$$h_{22} = \frac{I_2}{V_2},\ when\ I_1 = 0$$

Step 2 − We know that the following set of two equations of two port network regarding **T parameters**.

$$V_1 = AV_2 - BI_2 \qquad \textbf{Equation 5}$$

$$I_1 = CV_2 - DI_2 \qquad \textbf{Equation 6}$$

Step 3 − Substitute $V_2 = 0$ in the above equations in order to find the two h-parameters, h_{11} and h_{21}.

$$\Rightarrow V_1 = -BI_2$$

$$\Rightarrow I_1 = -DI_2$$

Substitute, V_1 and I_1 values in h-parameter, h_{11}.

$$h_{11} = \frac{-BI_2}{-DI_2}$$

Cont...

Substitute I_1 value in h-parameter h_{21}.

$$h_{21} = \frac{I_2}{-DI_2}$$

$$\Rightarrow h_{21} = -\frac{1}{D}$$

Step 4 − Substitute $I_1 = 0$ in the second equation of step 2 in order to find the h-parameter h_{22}.

$$0 = CV_2 - DI_2$$

$$\Rightarrow CV_2 = DI_2$$

$$\Rightarrow \frac{I_2}{V_2} = \frac{C}{D}$$

$$\Rightarrow h_{22} = \frac{C}{D}$$

Cont..

Step 5 – Substitute $I_2 = (\frac{C}{D})V_2$ in the first equation of step 2 in order to find the h-parameter, h_{12}.

$$V_1 = AV_2 - B(\frac{C}{D})V_2$$

$$\Rightarrow V_1 = (\frac{AD - BC}{D})V_2$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{AD - BC}{D}$$

$$\Rightarrow h_{12} = \frac{AD - BC}{D}$$

Step 6 – Therefore, the h-parameters matrix is

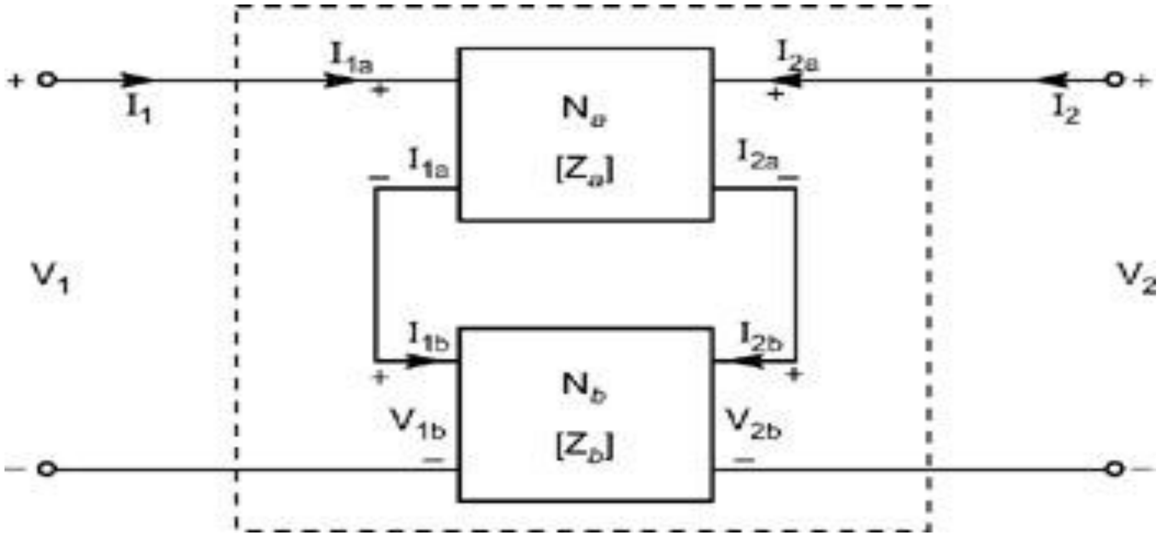
$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{B}{D} & \frac{AD-BC}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$$

Interconnections of two-port networks

- **Interconnections of two-port networks**

  Two-port networks may be interconnected in various configurations, such as series, parallel, cascade, series-parallel, and parallel-series connections. For each configuration a certain set of parameters may be more useful than others to describe the network.

- **Series connection**



Cont..

- Series connection of two two-port networks for network N $_a$

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}$$

$$(10.63) V_{2a} = Z_{21a} I_{1a} + Z_{22a} I_{2a}.$$

$$(10.63) V_{2a} = Z_{21a} I_{1a} + Z_{22a} I_{2a}.$$

$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22}b \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

$$(10.65) V_{2b} = Z_{21b} I_{1b} + Z_{22b} I_{2b}$$

Cont..

- The Z-parameters of the series-connected combined network can be written as

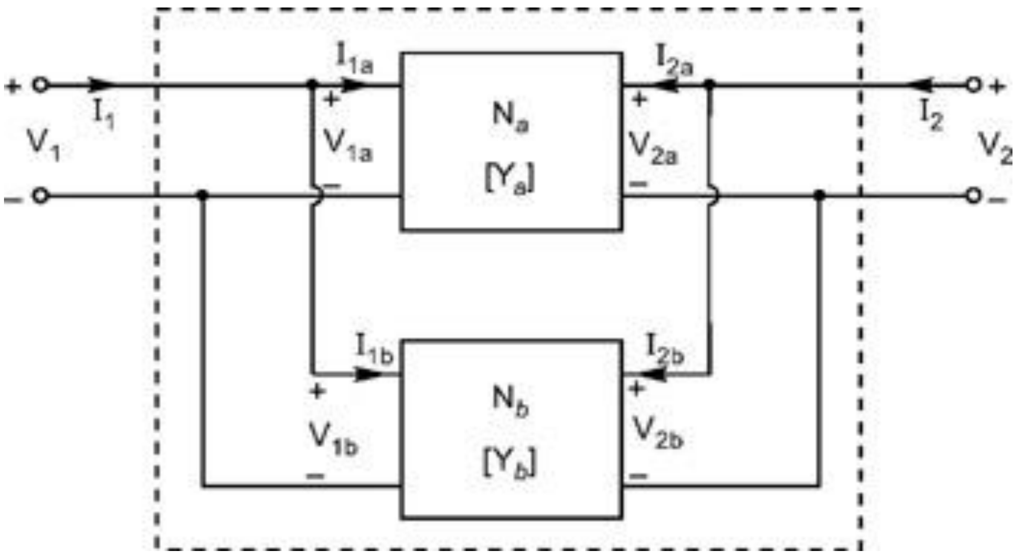
$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2,$$

$$Z_11 = Z_{11a} + Z_{11b}$$
$$Z_12 = Z_{12a} + Z_{12b}$$
$$Z_21 = Z_{21a} + Z_{21b}$$
$$Z_22 = Z_{22a} + Z_{22b}$$

$$[Z] = [Z_a] + [Z_b].$$

Parallel Connection

- Parallel connection of two two-port networks N a and N b. The resultant of two admittances connected in parallel is Y 1 + Y 2. So in parallel connection, the parameters are Y-parameters**.**

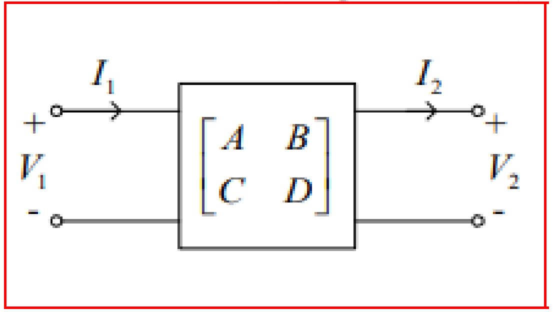


- Parallel connections for two two-port networks For network N $_a$

Cascade connection of two port networks

There is another set of network parameters particularly suited for cascading two-port networks. This set is called the *ABCD* matrix or, equivalently, the transmission matrix.

Consider this two-port network (Fig. 4.11a):



Unlike in the definition used for Z and Y parameters, notice that I_2 is directed away from the port. This is an important point and we'll discover the reason for it shortly.

It is easy to show that

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}, \quad B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}, \quad D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

Note that not all of these parameters have the same units.

To see this, consider the following two-port networks:

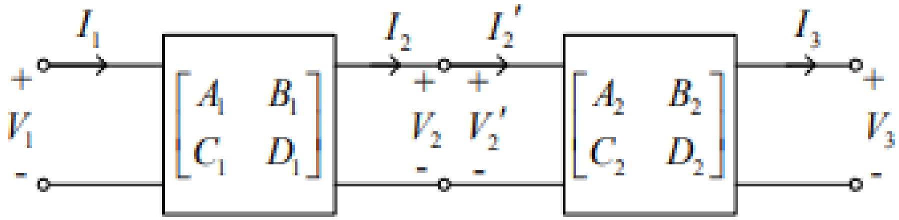
In matrix form

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad (2)$$

and

$$\begin{bmatrix} V_2' \\ I_2' \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdot \begin{bmatrix} V_3 \\ I_3 \end{bmatrix} \qquad (3)$$

When these two-ports are cascaded,



it is apparent that $V_2' = V_2$ and $I_2' = I_2$. (The latter is the reason for assuming I_2 out of the port.) Consequently, substituting (3) into (2) yields

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdot \begin{bmatrix} V_3 \\ I_3 \end{bmatrix} \quad (4)$$

We can consider the matrix-matrix product in this equation as describing the cascade of the two networks.

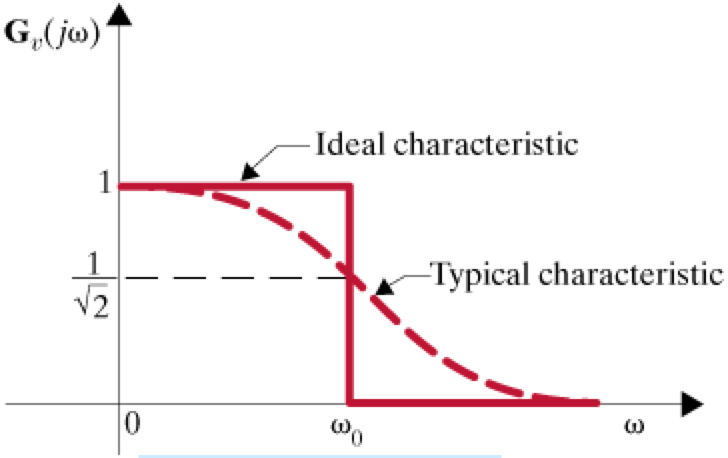
UNIT-III

FILTERS AND ATTENUATORS

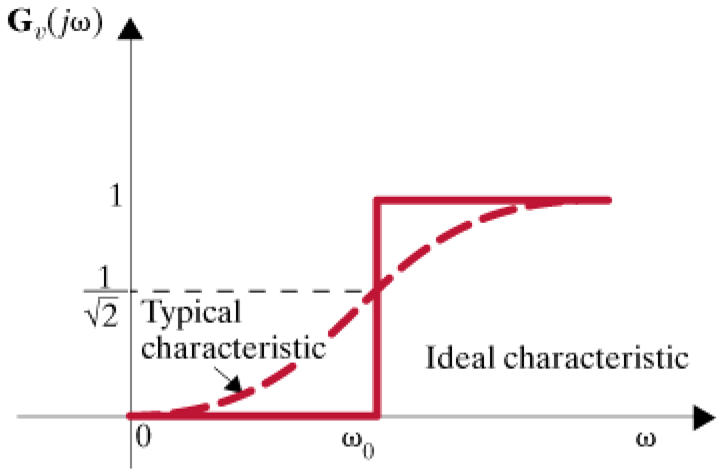
FILTER NETWORKS

Networks designed to have frequency selective behavior

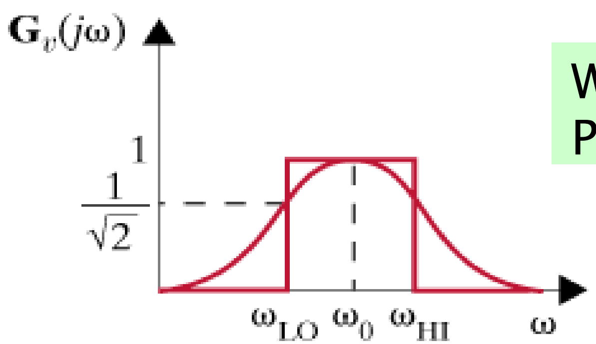
COMMON FILTERS



Low-pass filter

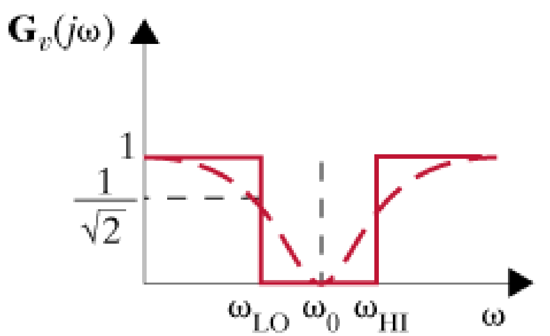


High-pass filter



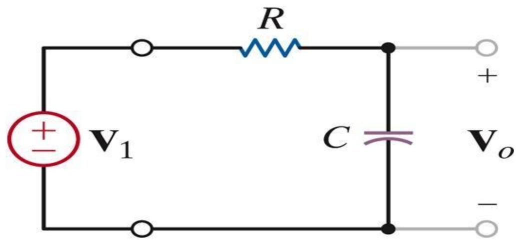
Band-pass filter

We focus first on PASSIVE filters



Band-reject filter

Simple Low pass filter



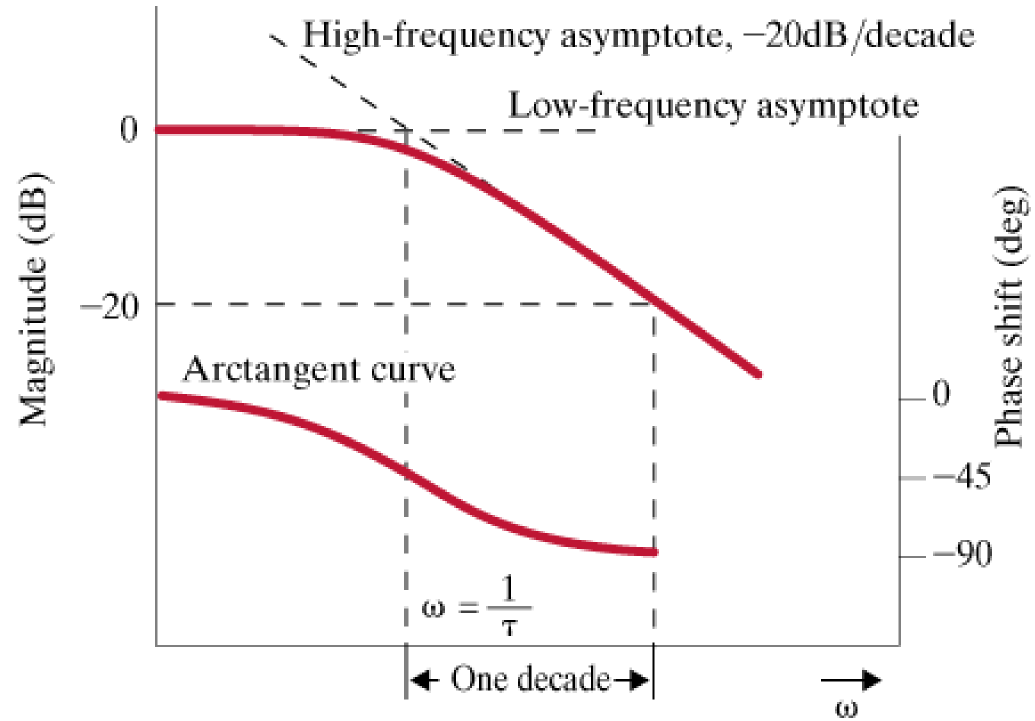
$$G_v = \frac{V_0}{V_1} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

$$G_v = \frac{1}{1 + j\omega\tau}; \quad \tau = RC$$

$$M(\omega) = |G_v| = \frac{1}{\sqrt{1 + (\omega\tau)^2}}$$

$$\angle G_v = \phi(\omega) = -\tan^{-1}\omega\tau$$

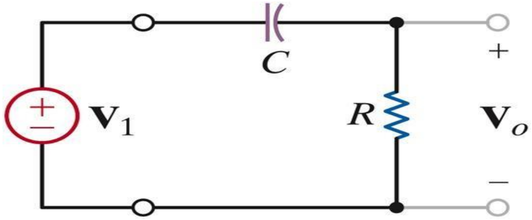
$$M_{max} = 1, \quad M\left(\omega = \frac{1}{\tau}\right) = \frac{1}{\sqrt{2}}$$



(c)

$$BW = \frac{1}{\tau}$$

High pass filter



$$G_v = \frac{V_0}{V_1} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega CR}{1 + j\omega CR}$$

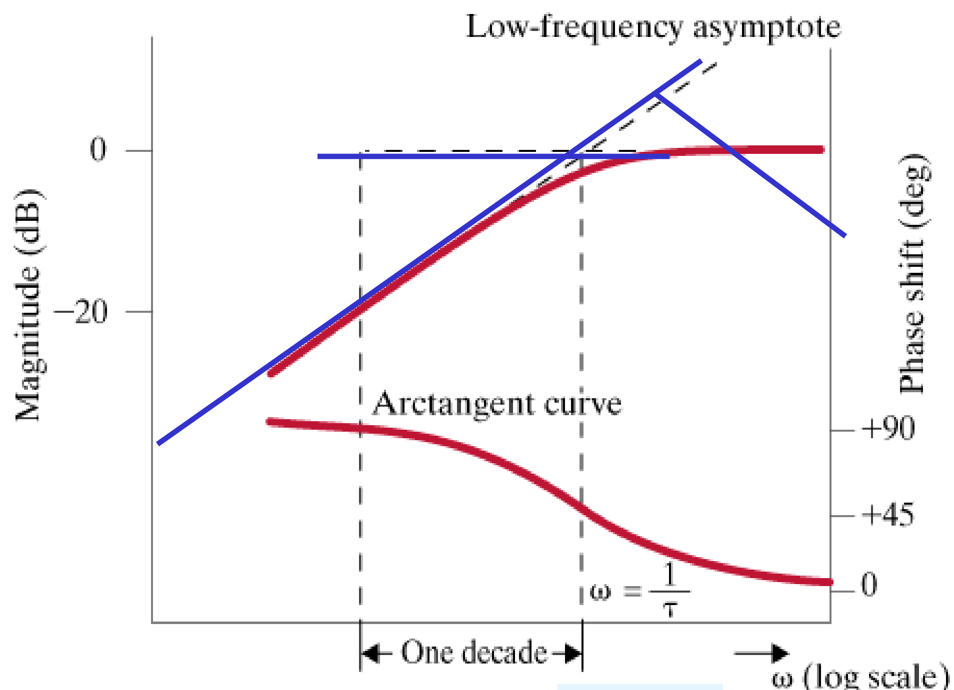
$$G_v = \frac{j\omega\tau}{1 + j\omega\tau}; \ \tau = RC$$

$$M(\omega) = |G_v| = \frac{\omega\tau}{\sqrt{1 + (\omega\tau)^2}}$$

$$\angle G_v = \phi(\omega) = \frac{\pi}{2} - \tan^{-1}\omega\tau$$

$$M_{max} = 1, M\left(\omega = \frac{1}{\tau}\right) = \frac{1}{\sqrt{2}}$$

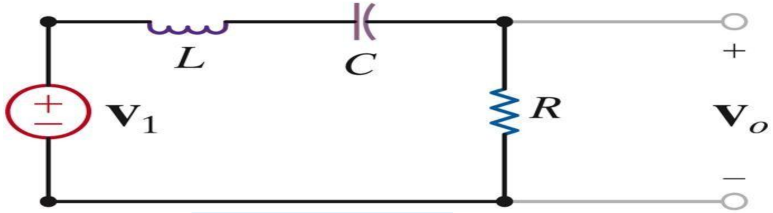
$\omega = \frac{1}{\tau} =$ half power frequency



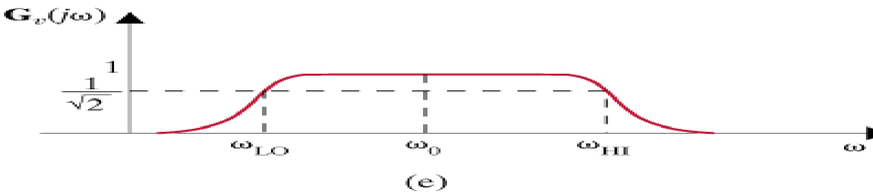
(c) $\omega_{LO} = \frac{1}{\tau}$

Simple band pass filter

r



Band-pass



$$G_v = \frac{V_0}{V_1} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$M(\omega) = \frac{\omega RC}{\sqrt{(\omega RC)^2 + (\omega^2 LC - 1)^2}}$$

$$M\left(\omega = \frac{1}{\sqrt{LC}}\right) = 1 \quad M(\omega = 0) = M(\omega = \infty) = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

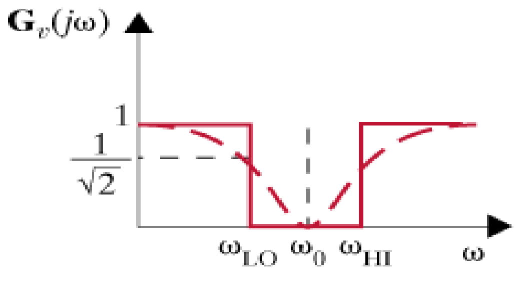
$$M(\omega_{LO}) = \frac{1}{\sqrt{2}} = M(\omega_{HI})$$

$$\omega_{LO} = \frac{-(R/L) + \sqrt{(R/L)^2 + 4\omega_0^2}}{2}$$

$$\omega_{HI} = \frac{(R/L) + \sqrt{(R/L)^2 + 4\omega_0^2}}{2}$$

$$BW = \omega_{HI} - \omega_{LO} = \frac{R}{L}$$

Simple Band reject filter



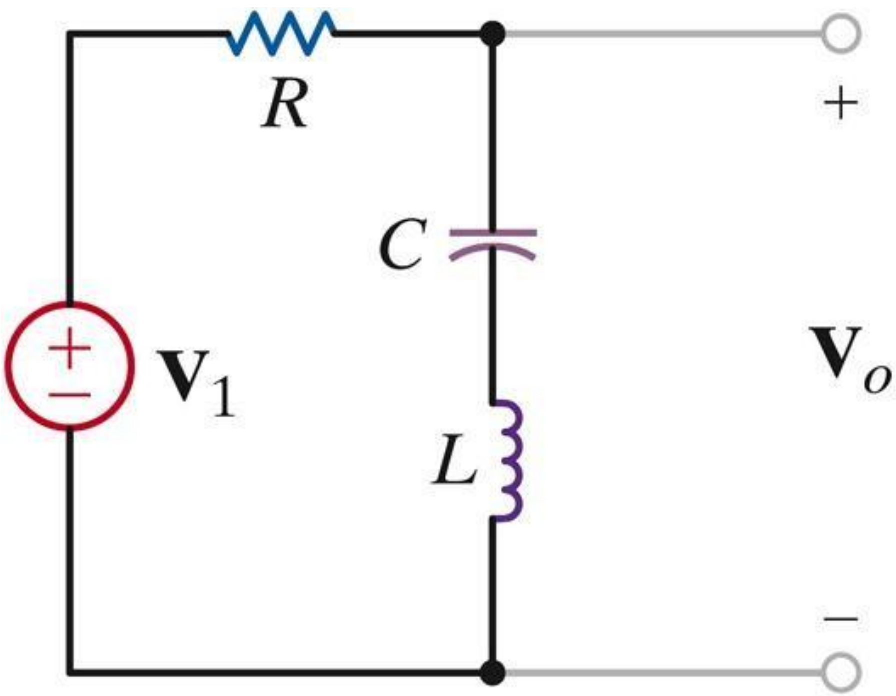
(b)

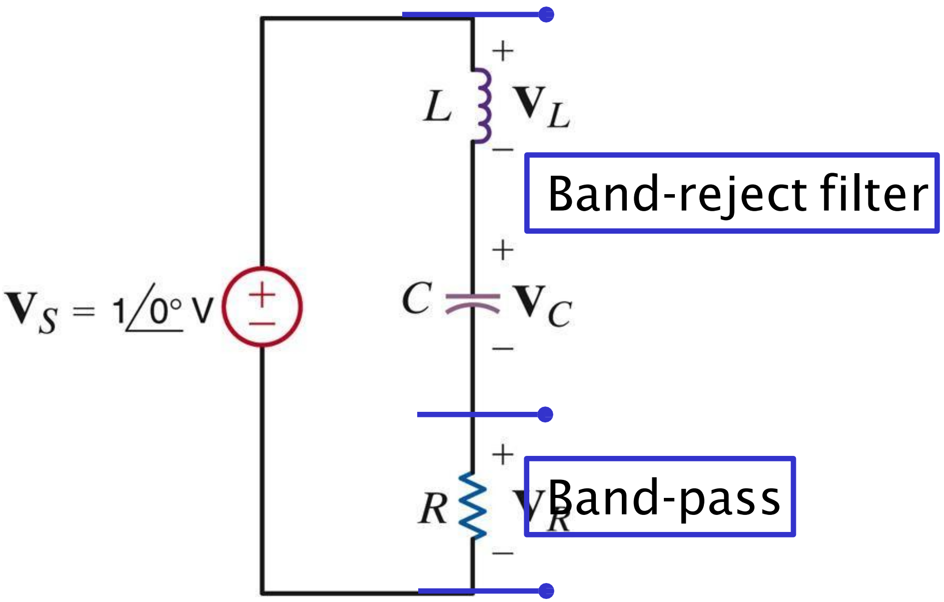
$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow j\left(\omega_0 L - \frac{1}{\omega_0 C}\right) = 0$$

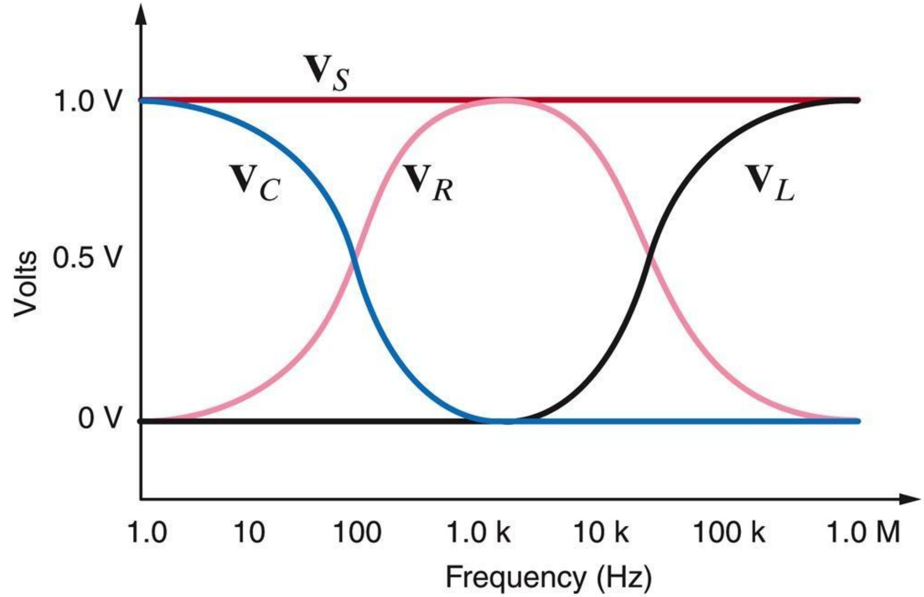
at $\omega = 0$ the capacitor acts as open circuit $\Rightarrow V_0 = V_1$

at $\omega = \infty$ the inductor acts as open circuit $\Rightarrow V_0 = V_1$

ω_{LO}, ω_{HI} are determined as in the band-pass filter







Bode plot for $R = 10\Omega,\ L = 159\,\mu H,\ C = 159\,\mu F$

$$\frac{V_L}{V_S} = \frac{j\omega L}{R + j\left(\omega L - \dfrac{1}{\omega C}\right)}$$

$$\frac{V_L}{V_S}(\omega = 0) = 0,\quad \frac{V_L}{V_S}(\omega = \infty) = 1$$

High-pass

$$\frac{V_C}{V_S} = \frac{\dfrac{1}{j\omega C}}{R + j\left(\omega L - \dfrac{1}{\omega C}\right)}$$

$$\frac{V_C}{V_S}(\omega = 0) = 1,\quad \frac{V_C}{V_S}(\omega = \infty) = 0$$

Low-pass

Active filters

Passive filters have several limitations

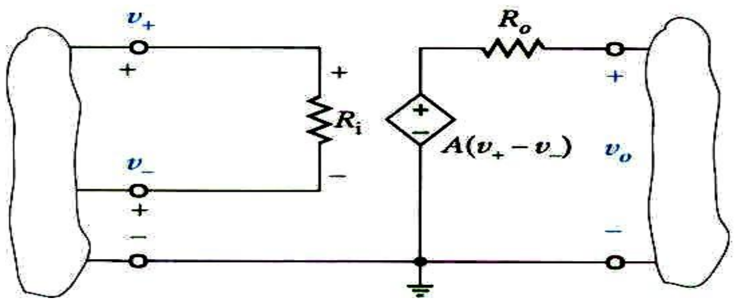
1. Cannot generate gains greater than one

2. Loading effect makes them difficult to interconnect

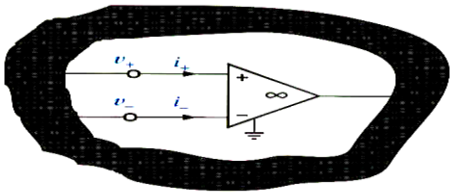
3. Use of inductance makes them difficult to handle

Using operational amplifiers one can design all basic filters, and more, with only resistors and capacitors

The linear models developed for operational amplifiers circuits are valid, in more general framework, if one replaces the resistors by impedances

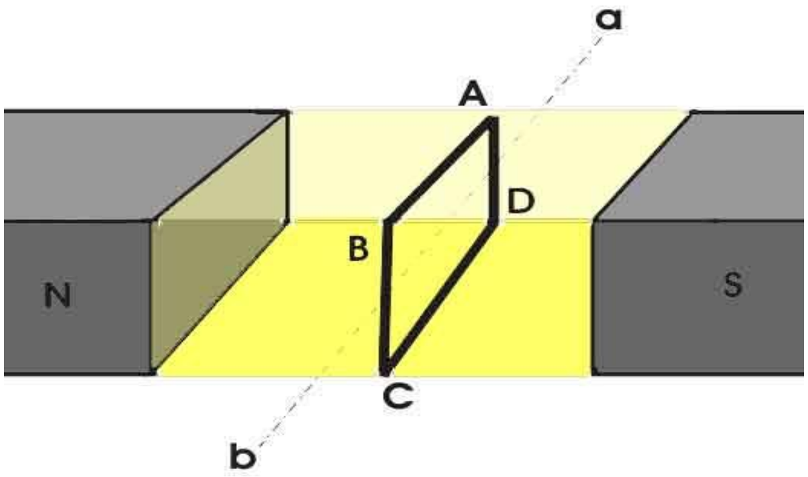


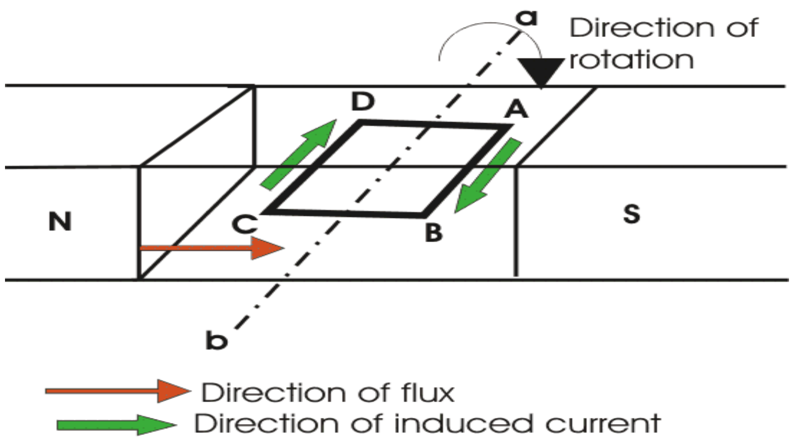
These currents are zero



UNIT-IV
DC MACHINES

Principle of DC Generator





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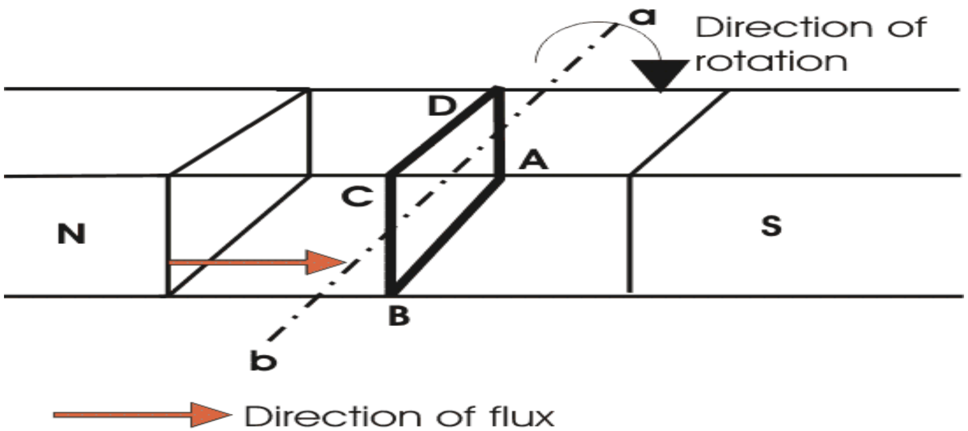
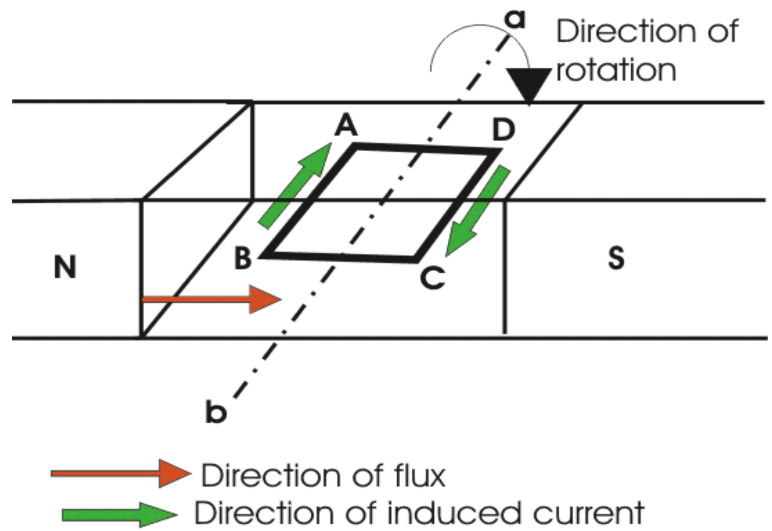

a
Direction of rotation
D
A
C
N
S
B
b
Direction of flux


a
Direction of rotation
A
D
N
B
C
S
b
Direction of flux
Direction of induced current

Construction of DC Generator

- During explaining working principle of DC Generator, we have used a single loop DC generator. But now we will discuss about practical construction of DC Generator**.**
- A DC generator has the following parts

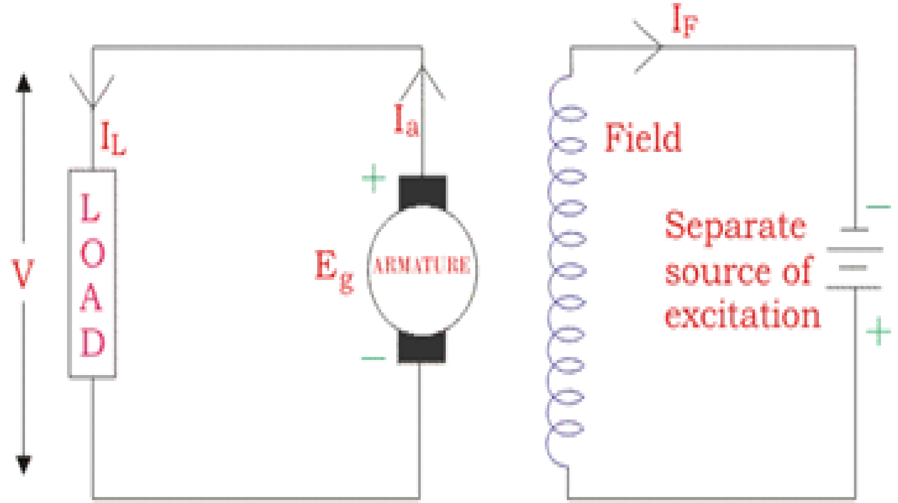
  1. Yoke
  2. Pole of generator
  3. Field winding
  4. Armature of DC generator
  5. Brushes of generator and Commutator
  6. Bearing

Types of DC Generators

- Generally DC generators are classified according to the ways of excitation of their fields. There are three methods of excitation.

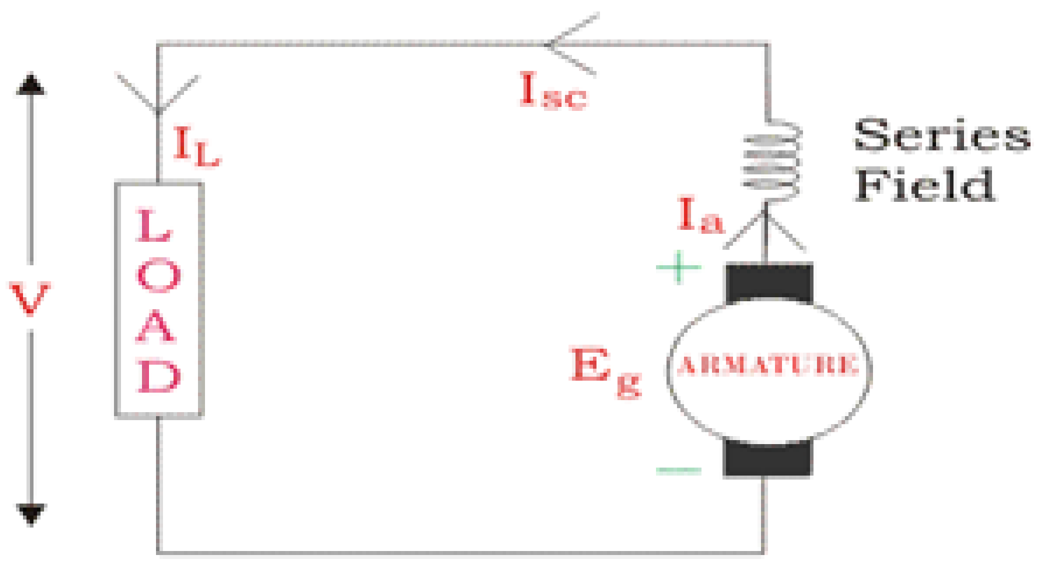
  1. Field coils excited by permanent magnets – Permanent magnet DC generators.
  2. Field coils excited by some external source – Separately excited DC generators.
  3. Field coils excited by the generator itself – Self excited DC generators.

Separately Excited DC Generator:

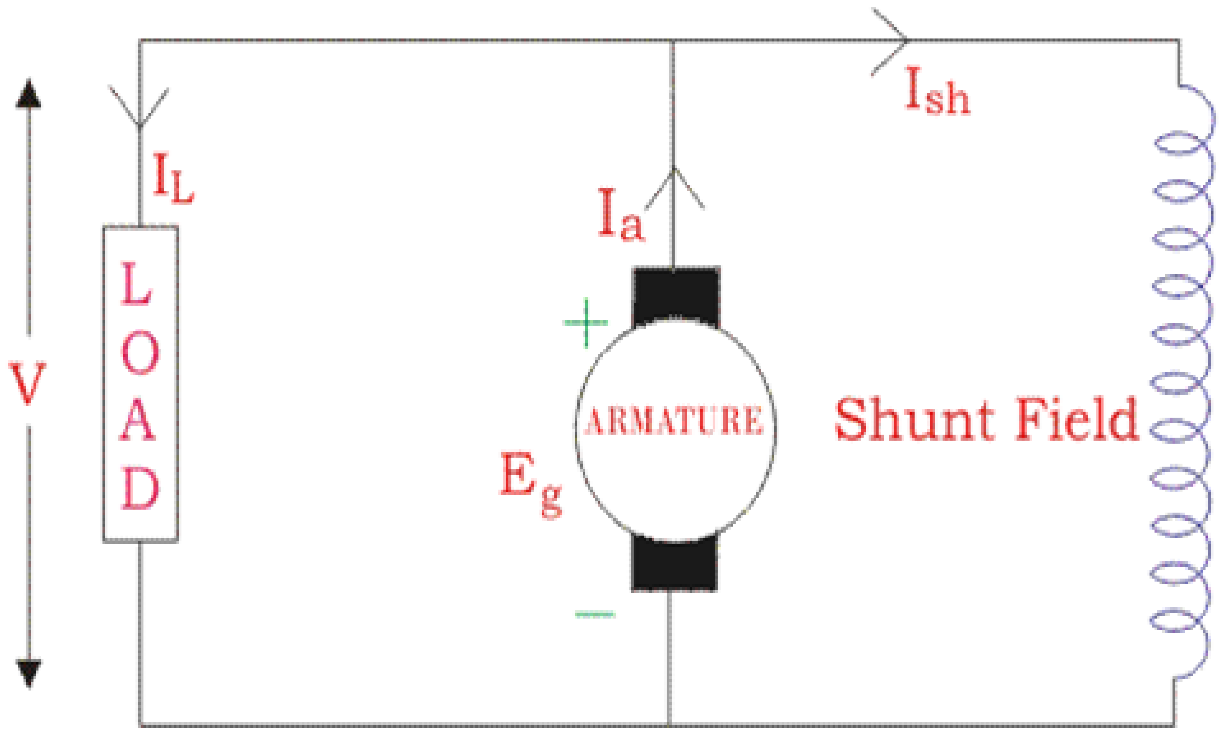


Self-excited DC Generators

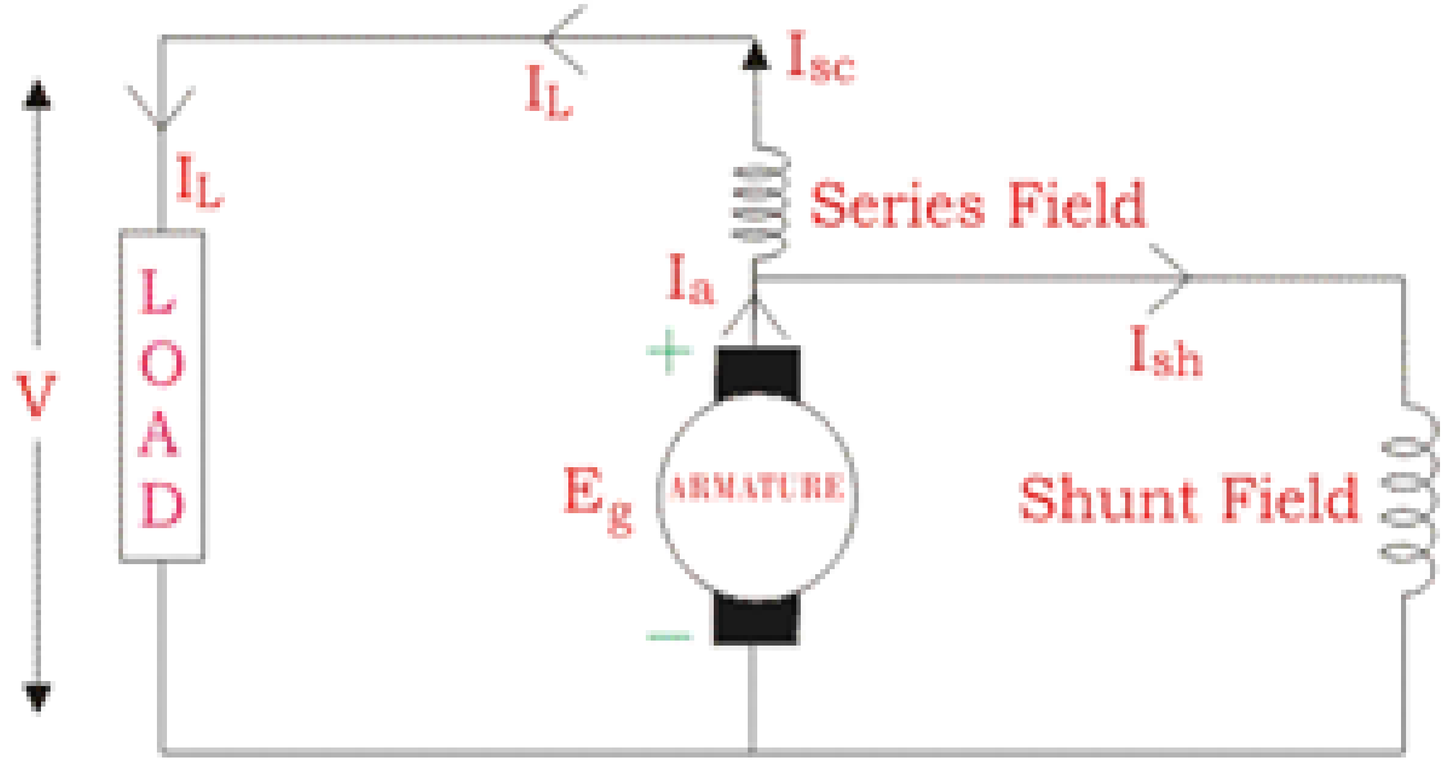
- According to the position of the field coils the self-excited DC generators may be classified as…
    1. Series wound generators
    2. Shunt wound generators
    3. Compound wound generators
- **Series Wound Generator:**



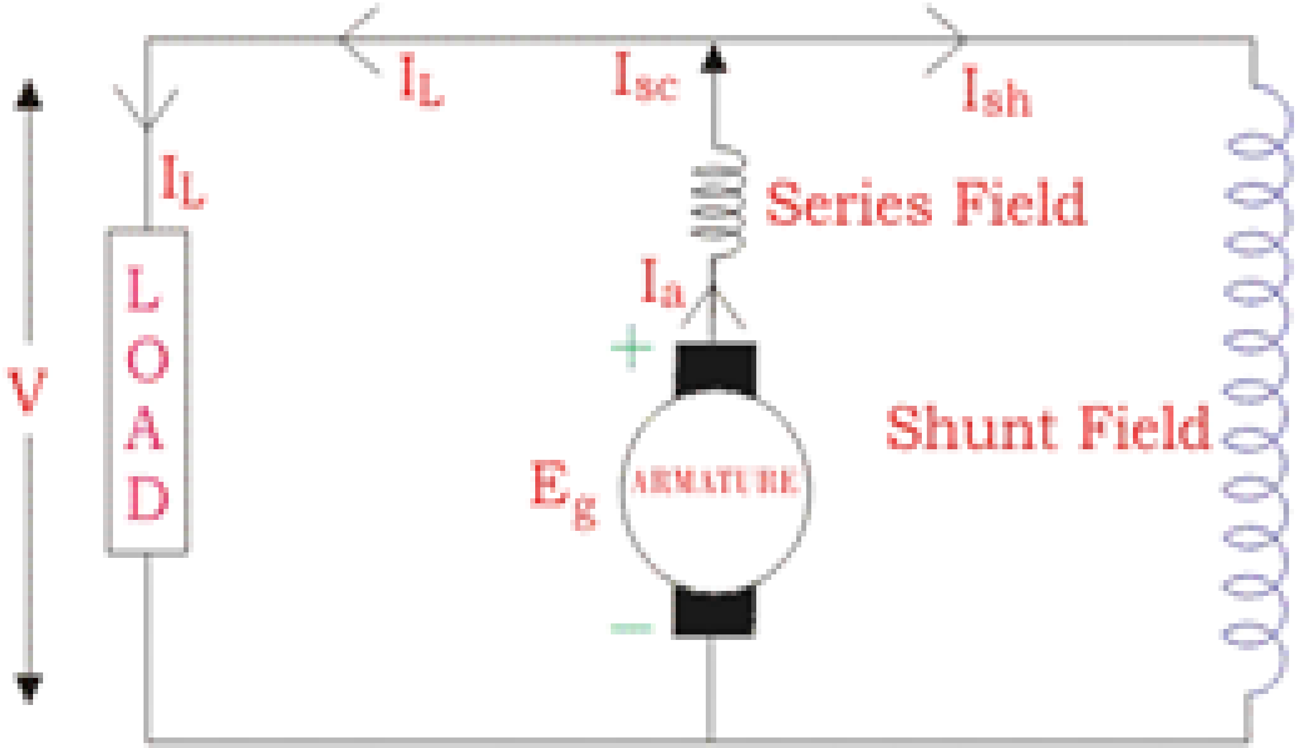
Shunt Wound DC Generators:



- **Compound Wound DC Generator:**
- **Short Shunt Compound Wound DC Generator:**



- **Long Shunt Compound Wound DC Generator:**



Induced EMF

- Let us suppose there are Z total numbers of conductor in a generator, and arranged in such a manner that all parallel paths are always in series.
- Here, Z = total numbers of conductor A = number of parallel paths
- Then, Z/A = number of conductors connected in series we know that induced EMF in each path is same across the line Therefore, Induced EMF of DC generator E = EMF of one conductor × number of conductor connected in series
- Induced EMF of DC generator is

$$e = \phi P \frac{N}{60} X \frac{Z}{A} volts$$

Cont..

- **Simple wave wound generator:**
- Numbers of parallel paths are only 2 = A Therefore, Induced EMF for wave type of winding generator is

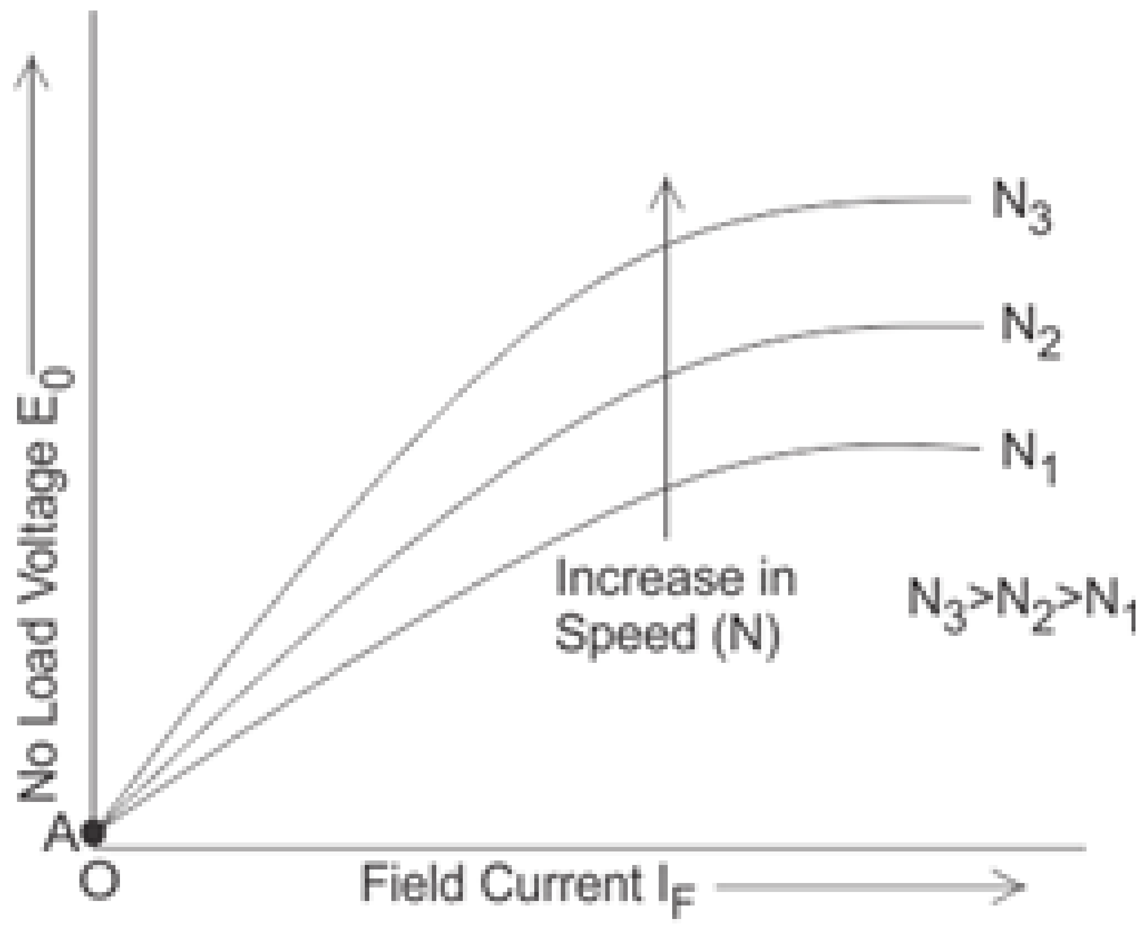
$$\frac{\phi PN}{60} X \frac{Z}{2} = \frac{\phi ZPN}{120} volts$$

- **Simple lap-wound generator:**
- Here, number of parallel paths is equal to number of conductors in one path i.e. P = A Therefore, Induced EMF for lap-wound generator is

$$E_g = \frac{\phi ZN}{60} X \frac{P}{A} volt$$

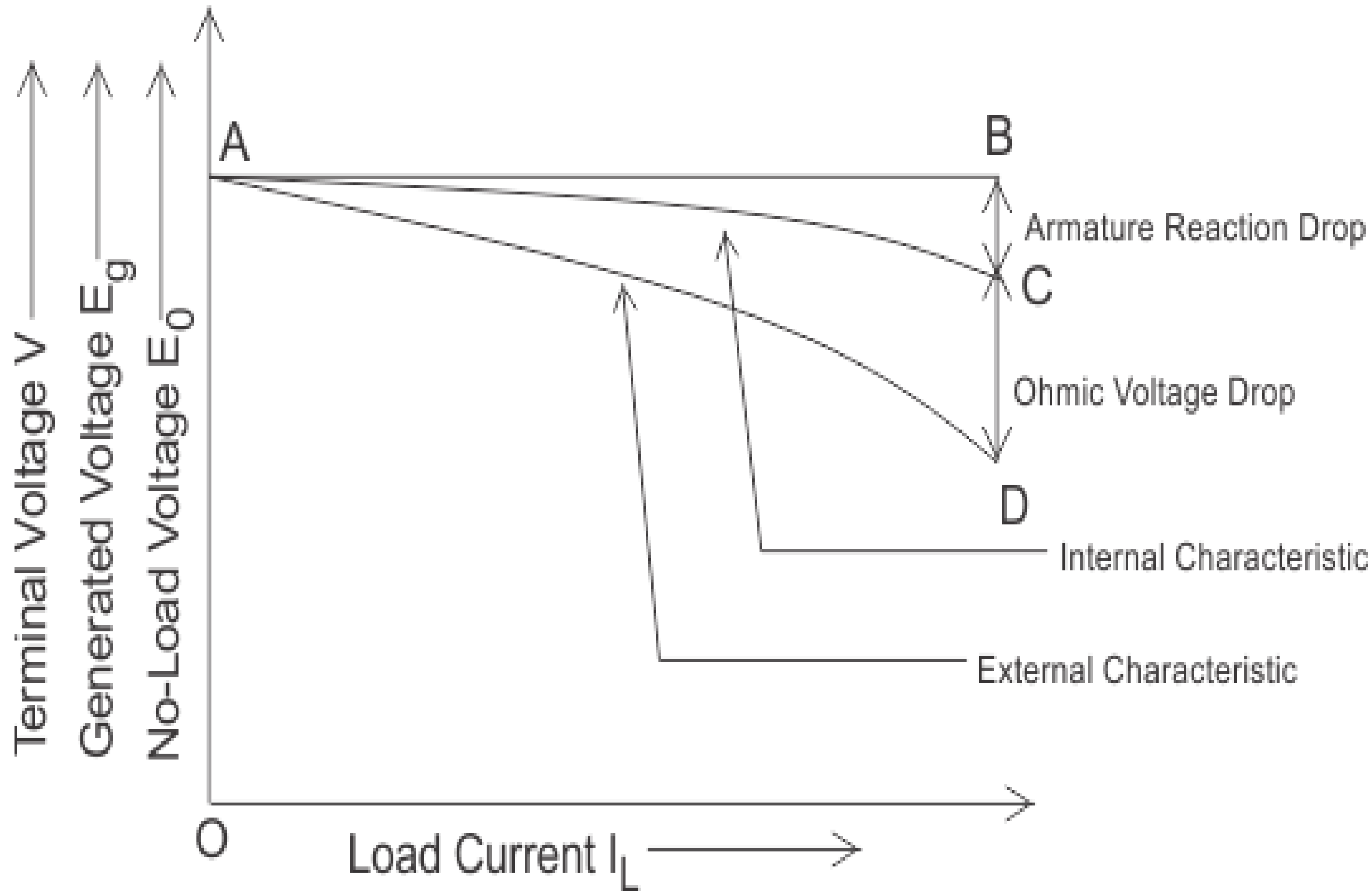
Characteristics of DC Generators

Magnetic or Open Circuit Characteristic of Separately Excited DC Generator:

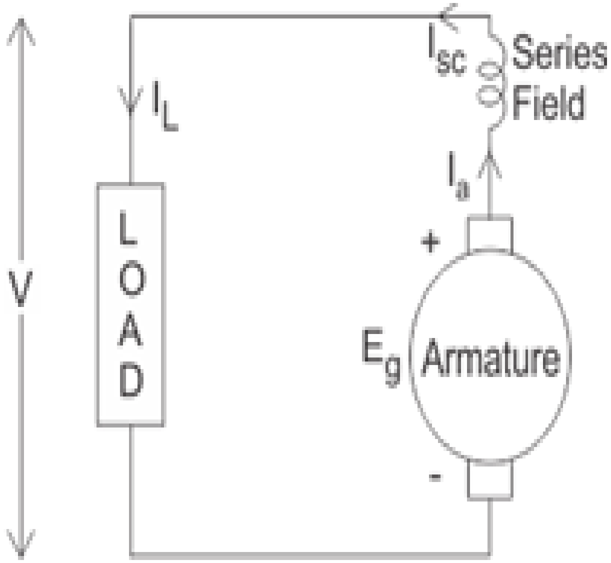


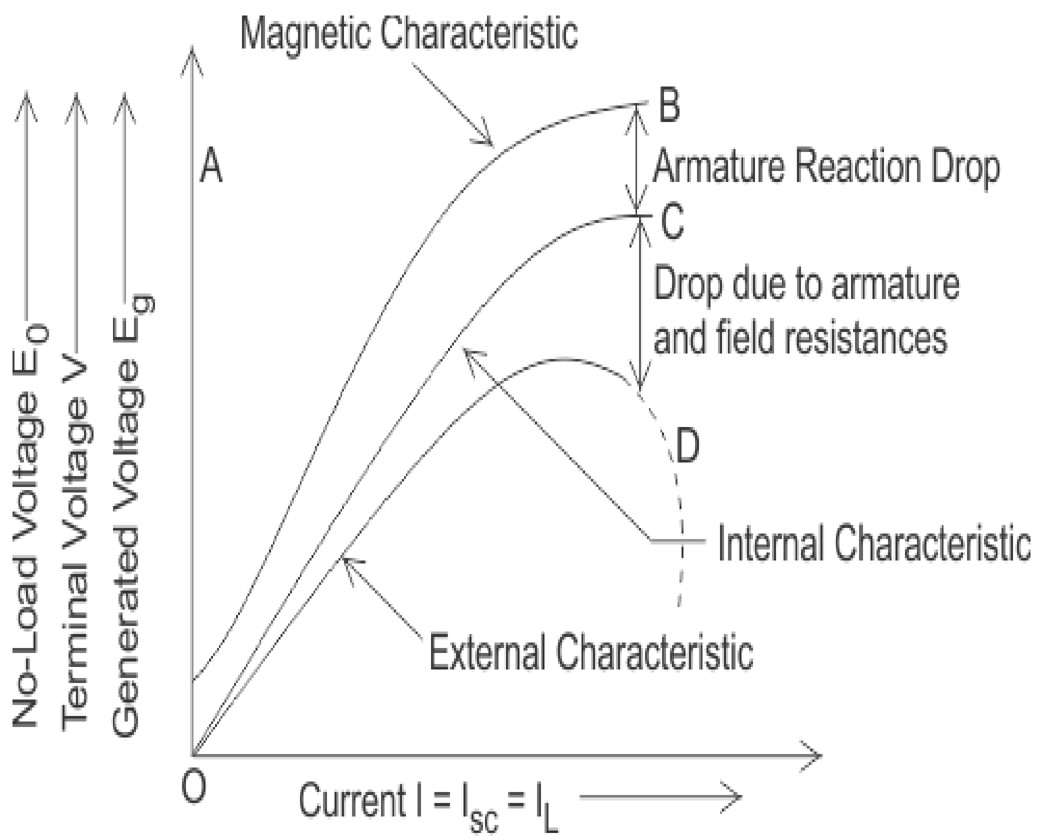
Cont..

- **Internal or Total Characteristic of Separately Excited DC Generator**



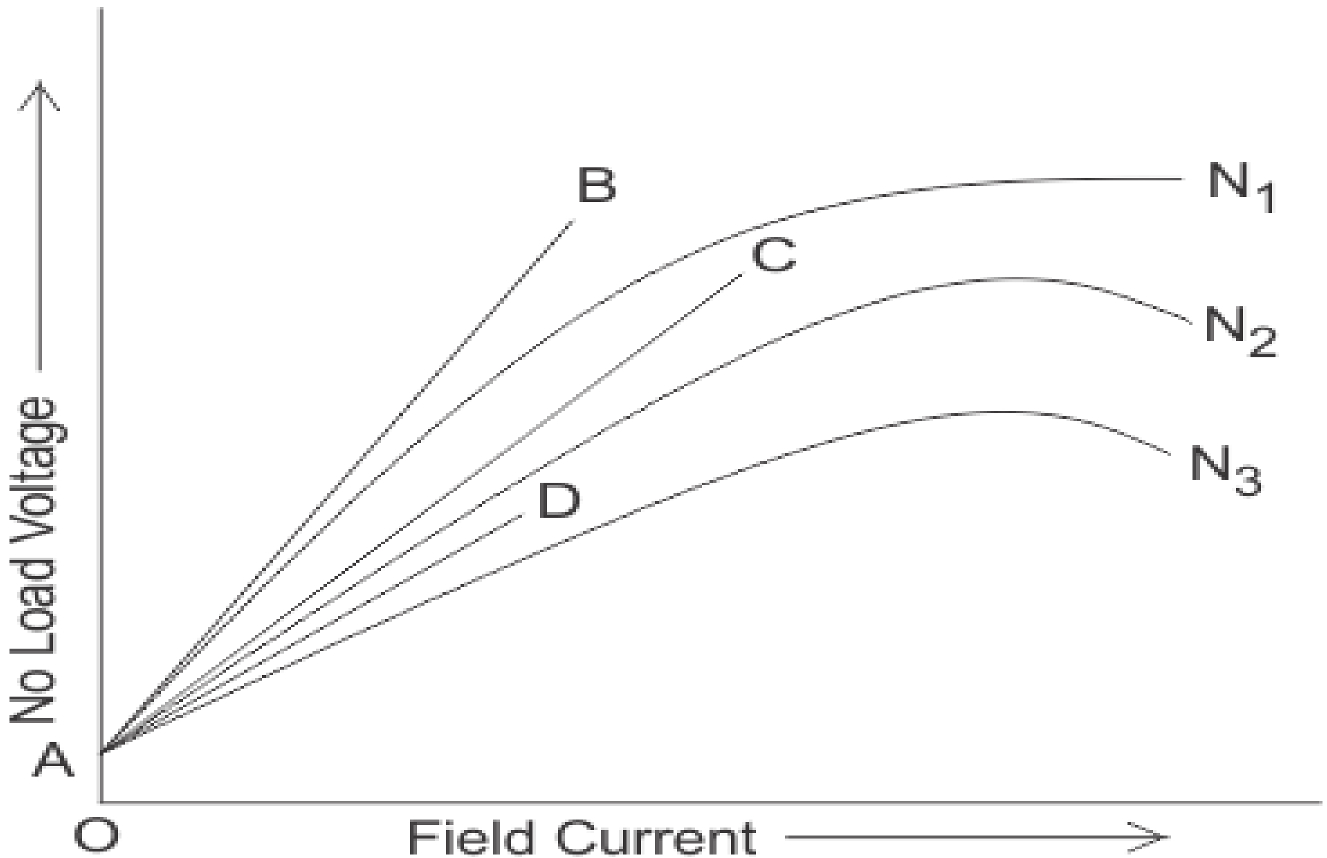
Characteristics of Series Wound DC Generator:



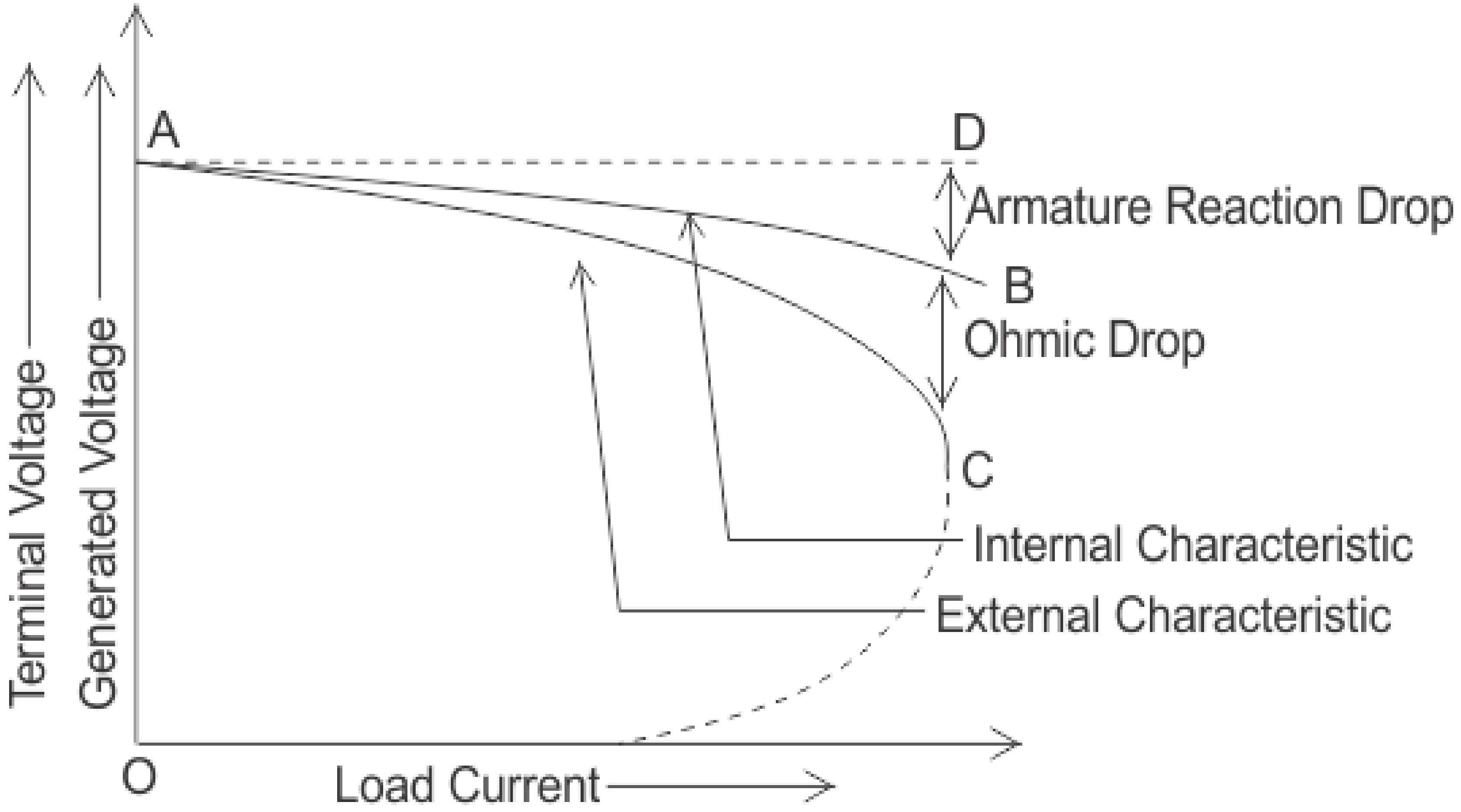


- **Characteristic of Shunt Wound DC Generator:**

  1. Open circuit characteristics

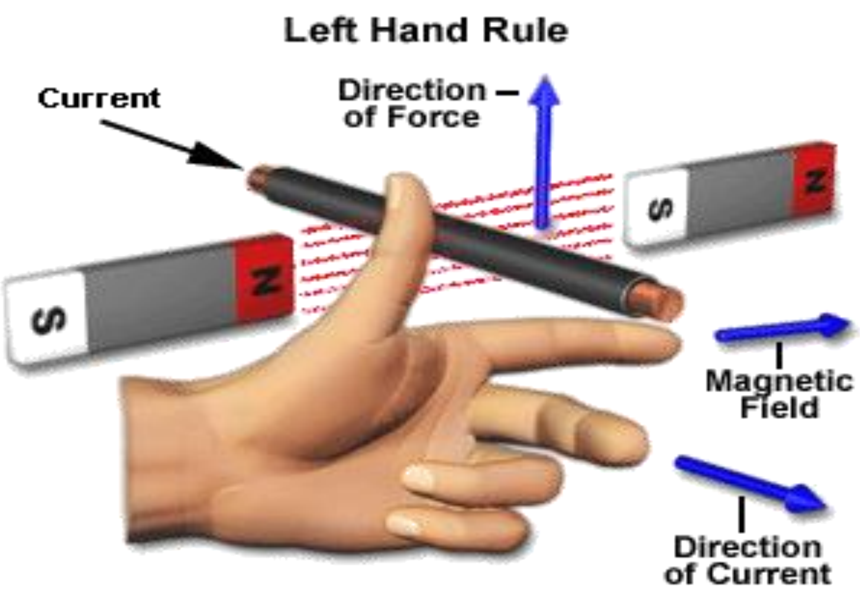


- **External Characteristic of Shunt Wound DC Generator:**



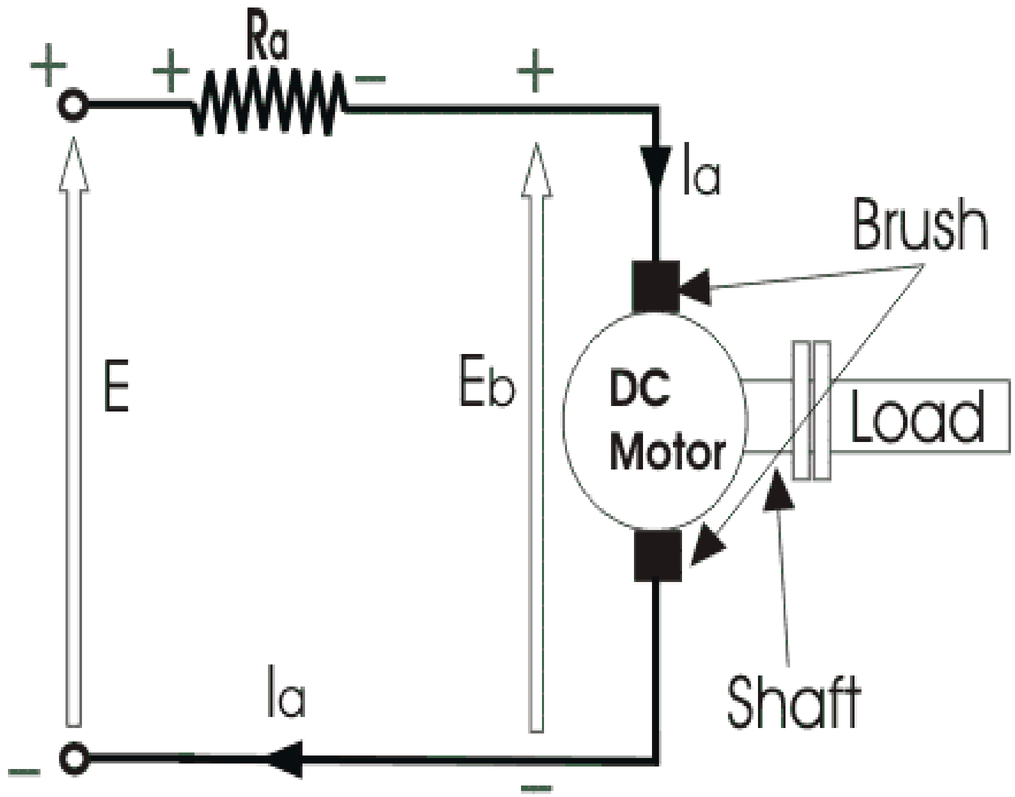
Working principle of DC motor

- This DC or direct current motor works on the principal, when a current carrying conductor is placed in a magnetic field; it experiences a torque and has a tendency to move. This is known as motoring action.
- If the direction of current in the wire is reversed, the direction of rotation also reverses.
- When magnetic field and electric field interact they produce a mechanical force, and based on that the working principle of DC motor is established



Detailed Description of a DC Motor:

- To understand the DC motor in details let's consider the diagram below,

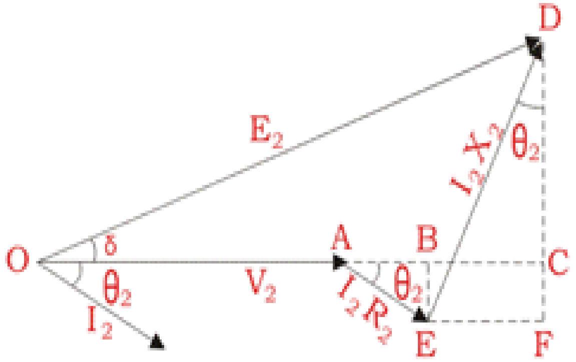


Voltage Regulation of Transformer

- The voltage regulation is the percentage of voltage difference between no load and full load voltages of a transformer with respect to its full load voltage
- Expression of Voltage Regulation of Transformer, represented in percentage is

$$Voltage\ regulation(\%) = \frac{E_2 - V_2}{V_2} \times 100\%$$

- **Voltage Regulation of Transformer for Lagging Power Factor:**
- Now we will derive the expression of voltage regulation in detail. Say lagging power factor of the load is cosθ_2, that means angle between secondary current and voltage is θ_2.



Cont..

- Here, from the above diagram

$$OC = OA + AB + BC$$

$$Here,\ OA = V_2$$

$$Here,\ AB = AE\cos\theta_2 = I_2R_2\cos\theta_2$$

$$and,\ BC = DE\sin\theta_2 = I_2X_2\sin\theta_2$$

- Angle between OC and OD may be very small, so it can be neglected and OD is
- Considered nearly equal to OC i.e,

$$E_2 = OC = OA + AB + BC$$

$$E_2 = OC = V_2 + I_2R_2\cos\theta_2 + I_2X_2\sin\theta_2$$

- Voltage regulation of transformer at lagging power factor,

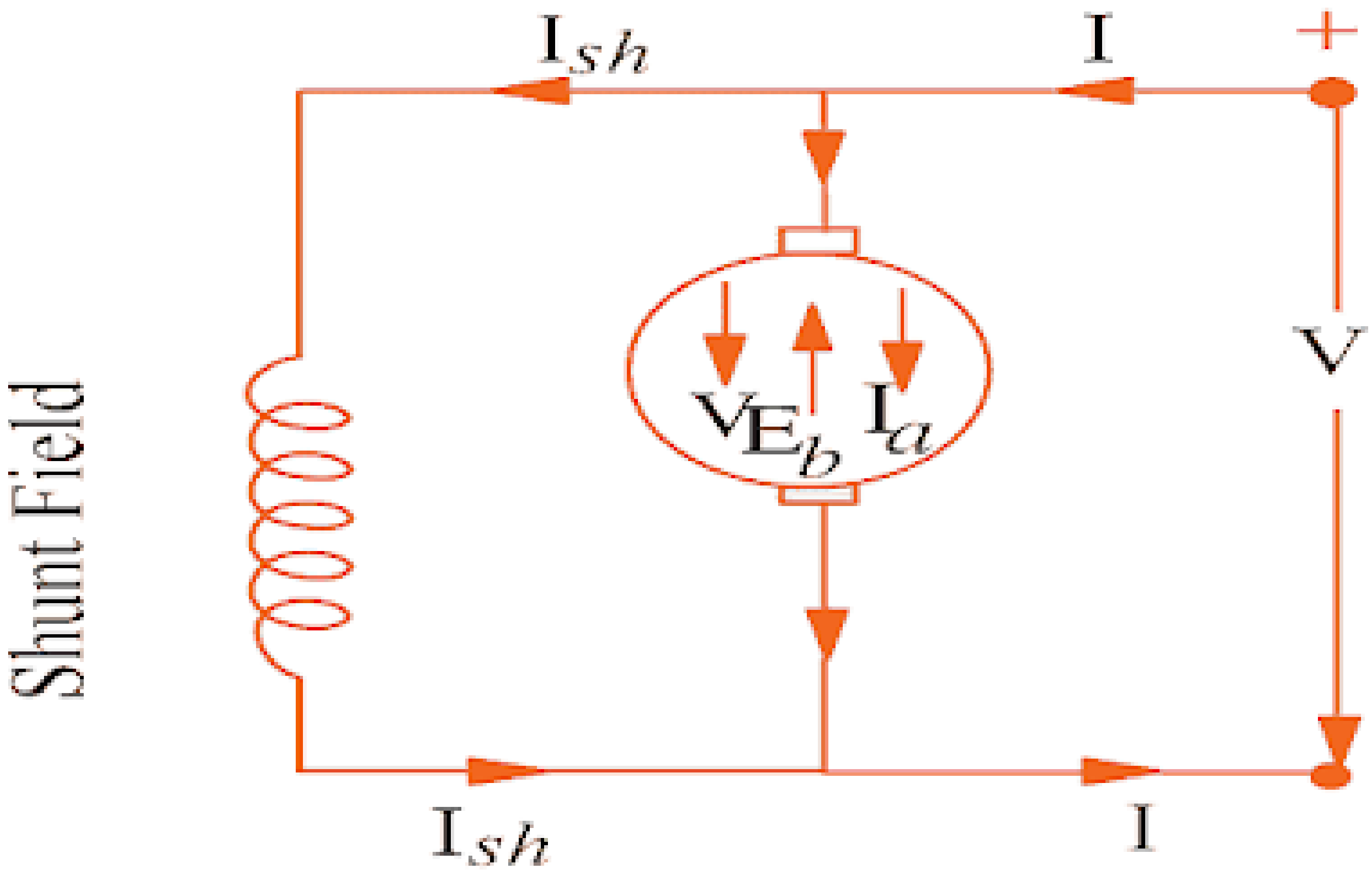
$$Voltage\ regulation\ (\%) = \frac{E_2 - V_2}{V_2} \times 100(\%)$$

$$= \frac{I_2R_2\cos\theta_2 + I_2X_2\sin\theta_2}{V_2} \times 100(\%)$$

Back EMF

- When the armature of a DC motor rotates under the influence of the driving torque, the armature conductors move through the magnetic field and hence EMF is induced in them as in a generator. The induced EMF acts in opposite direction to the applied voltage V (Lenz's law) and in known as back or counter EMF E_b

$$E_b = \frac{P\phi ZN}{60A}$$



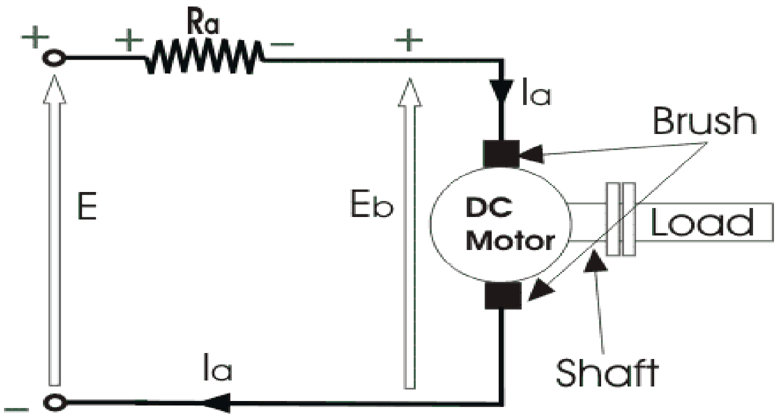
Shunt Field
I_{sh}
I
+
V
E_b
I_a
I_{sh}
I

Torque

- The term torque as best explained by Dr. Huge d Young is the quantitative measure of the tendency of a force to cause a rotational motion, or to bring about a change in rotational motion. It is in fact the moment of a force that produces or changes a rotational motion.
- The equation of torque is given by,

$$\tau = FR\sin\theta....................(1)$$

- To establish the torque equation, let us first consider the basic circuit diagram of a DC motor, and its voltage equation



Cont..

- if E is the supply voltage, E_b is the back EMF produced and I_a, R_a are the armature current and armature resistance respectively then the voltage equation is given by,

$$E = E_b + I_a R_a \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$Therefore,\ EI_a = E_b I_a + I_a^2 R_a \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

- Now $I_a^2.R_a$ is the power loss due to heating of the armature coil, and the true effective mechanical power that is required to produce the desired torque of DC machine is given by,

$$P_m = E_b I_a \ldots\ldots\ldots\ldots\ldots (4)$$

- The mechanical power P_m is related to the electromagnetic torque T_g as,

Cont..

- Where, ω is speed in rad/sec. Now equating equation (4) and (5) we get,

$$E_b I_a = T_g \omega$$

- Now for simplifying the torque equation of DC motor we substitute.

$$E_b = \frac{P\varphi ZN}{60A} \dots\dots\dots\dots\dots\dots (6)$$

- Where, P is no of poles, ϕ is flux per pole, Z is no. of conductors, A is no. of parallel paths, and N is the speed of the DC motor.

$$Hence,\ w = \frac{2\pi N}{60} \dots\dots\dots\dots\dots\dots (7)$$

- Substituting equation (6) and (7) in equation (4), we get

$$T_g = \frac{P.Z.\varphi.I_a}{2\pi A}$$

Losses in DC Machine

- In DC machine, mechanical energy is converted into the electrical energy. During this process, the total input power is not transformed into output power. Some part of input power gets wasted in various forms. The form of this loss may vary from one machine to another. These losses give in rise in temperature of machine and reduce the efficiency of the machine.
- In DC Machine, there are broadly four main categories of energy loss

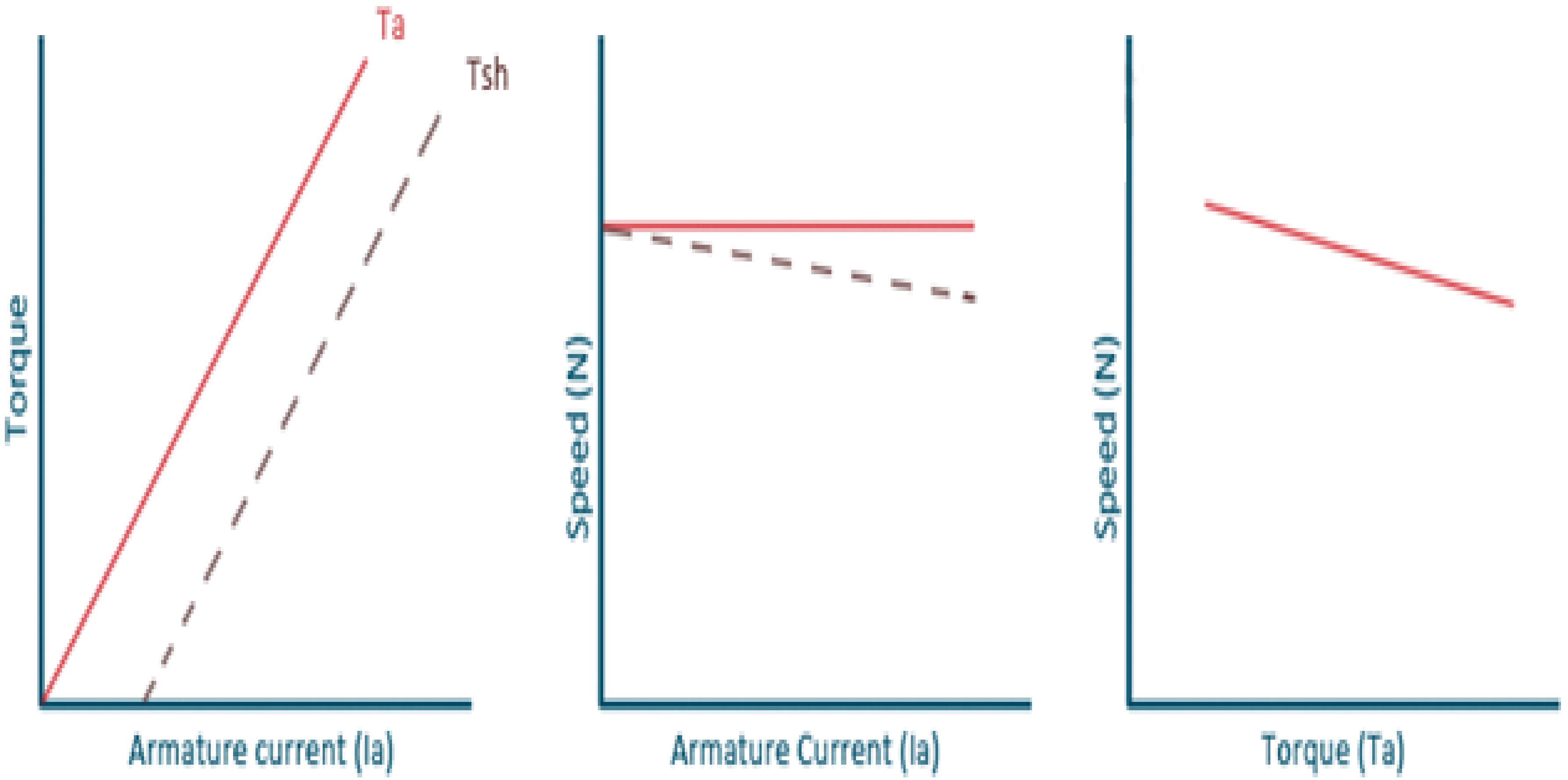
1. Copper Losses
   a) Armature Copper Loss
   b) Field Winding Copper Loss
   c) Brush Contact Resistance Loss
2. Core Losses or Iron Losses in DC Machine
   a) Hysteresis Loss
   b) Eddy Current Loss
3. Mechanical Losses in DC Machine
   a) Stray Load Losses

Characteristics of DC Motors

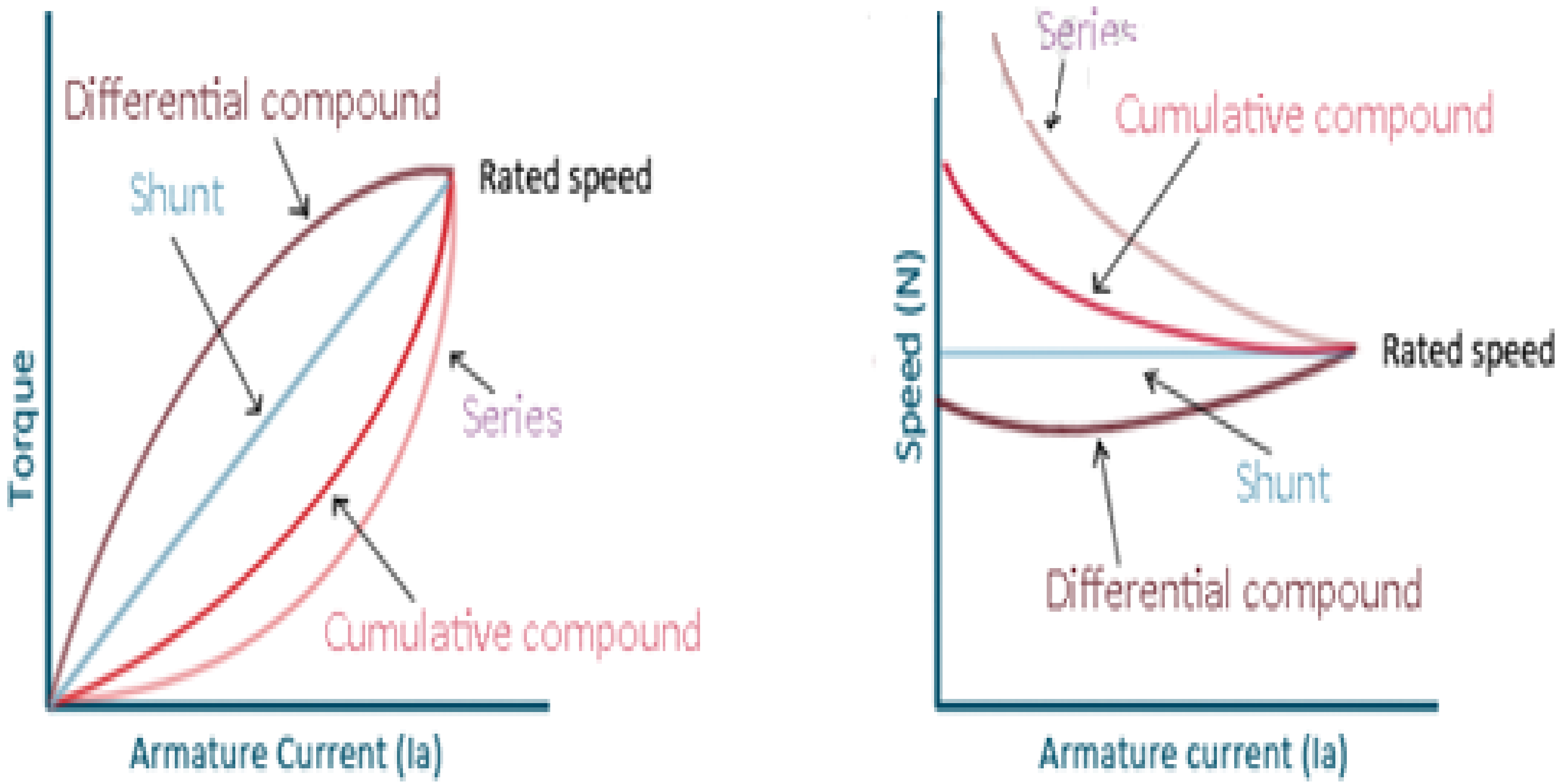
- Generally, three characteristic curves are considered important for DC motors which are

    1. Torque vs. armature
    2. Speed vs. armature current
    3. Speed vs. torque

Characteristics of DC Series Motors



Characteristics of DC Compound Motor



Testing of DC Machine

- The testing of dc machine is needed for proper fabrication and smooth trouble free operation. The tests which are mainly needed for these purposes are -

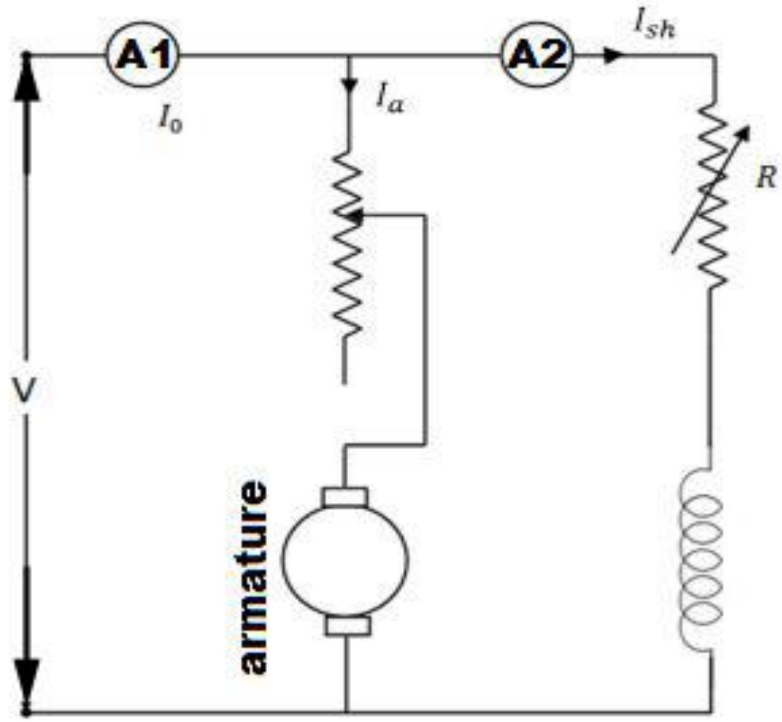
    1.Open circuit test

    2.Short circuit test

    3.Load test

    4.Determination of efficiency

Swinburne Test of DC Machine



Advantages of Swinburne's Test:

The main advantages of this test are:

- This test is very convenient and economical as it is required very less power from supply to perform the test.
- Since constant losses are known, efficiency of Swinburne's test can be pre-determined at any load.

Disadvantages of Swinburne's Test:

- Iron loss is neglected though there is change in iron loss from no load to full load due to armature reaction.
- We cannot be sure about the satisfactory commutation on loaded condition because the test is done on no-load.
- We can’t measure the temperature rise when the machine is loaded. Power losses can vary with the temperature.
- In DC series motors, the Swinburne’s test cannot be done to find its efficiency as it is a no load test

Speed Control of DC Motor

- Speed control means intentional change of the drive speed to a value required for performing the specific work process. Speed control is a different concept from speed regulation where there is natural change in speed due change in load on the shaft. Speed control is either done manually by the operator or by means of some automatic control device.
- Speed control of DC motor is classified as
  1. Armature control methods
  2. Field control methods.
- **Speed Control of DC Shunt Motor:**

  By this method speed control is obtained by any one of the following means

  1. Field Rheostat Control of DC Shunt Motor
  2. Field Voltage Control

Armature Control of DC Shunt Motor:

Speed control by this method involves two ways

1. Armature Resistance Control
2. Armature Voltage Control

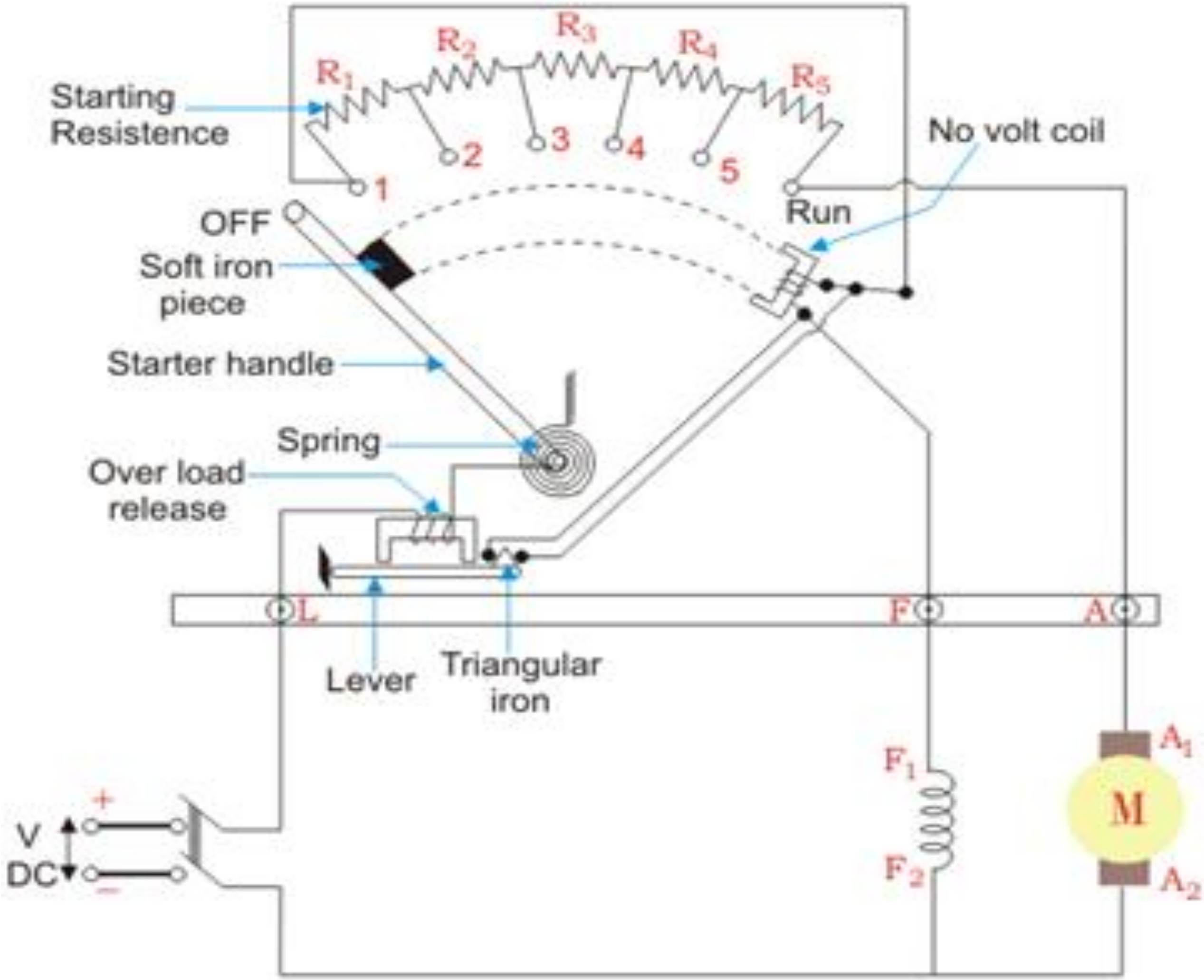
Advantages:

1. Very fine speed control over whole range in both directions
2. Uniform acceleration is obtained
3. Good speed regulation

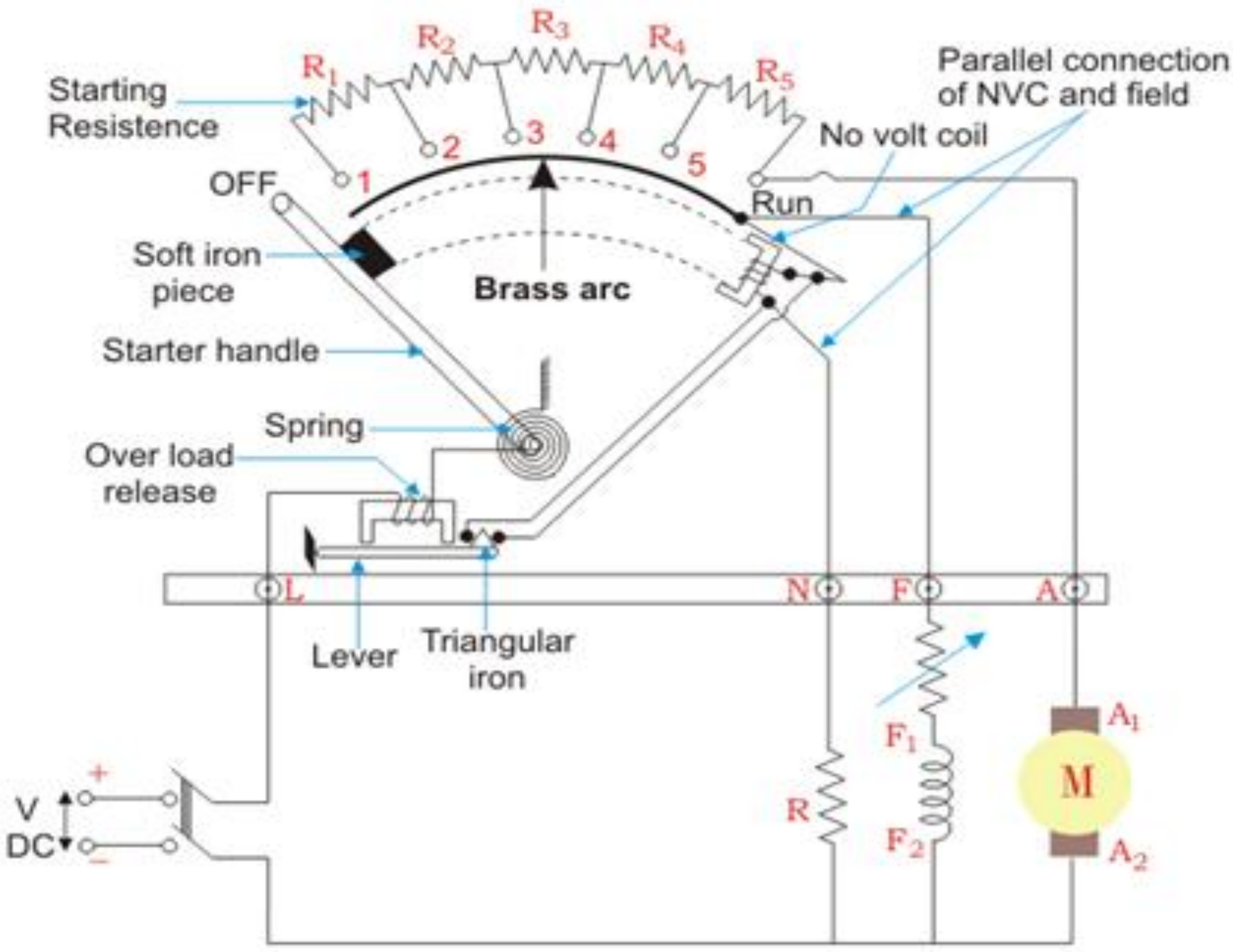
Disadvantages:

1. Low efficiency at light loads
2. Drive produced more noise.
3. Costly arrangement is needed, floor space required is more

Three Point Starter



Four Point Starter



UNIT-V

SINGLE PHASE TRANSFORMERS

SINGLE PHASE TRANSFORMERS

- The transformer is a device that transfers electrical energy from one electrical circuit to another electrical circuit.
- The two circuits may be operating at different voltage levels but always work at the same frequency. Basically transformer is an electro-magnetic energy conversion device.
- It is commonly used in electrical power system and distribution systems.

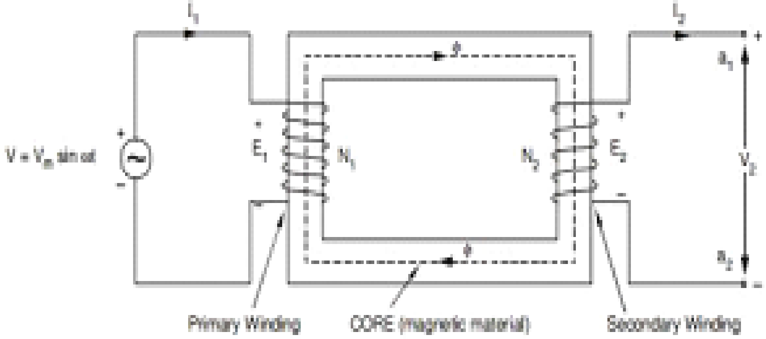


Fig.4.1 Basic arrangement of Transformer

PRINCIPLE OF OPERATION

- A single phase transformer works on the principle of mutual induction between two magnetically coupled coils. When the primary winding is connected to an alternating voltage of RMS value, V_1 volts, an alternating current flows through the primary winding and setup an alternating flux in the material of the core.
- This alternating flux φ, links not only the primary windings but also the secondary windings. Therefore, an EMF e_1 is induced in the primary winding and an EMF e_2 is induced in the secondary winding, e1 and e_2 are given

$$e1 = -N1\frac{d\varphi}{dt} \text{ ---------- (a)}$$

$$e2 = -N2\frac{d\varphi}{dt} \text{ ----------(b)}$$

Cont..

- If the induced EMF is e1 and e2 are represented by their RMS values E_1 and E_2 respectively, then

$$E1 = -N1\frac{d\varphi}{dt} \text{ --------- } (1)$$

$$E2 = -N2\frac{d\varphi}{dt} \text{ ---------- } (2)$$

$$\text{Therefore, } \frac{E2}{E1} = \frac{N2}{N1} = k \text{ ---------------- } (3)$$

- K is known as the transformation ratio of the transformer
- The directions of EMF's E_1 and E_2 induced in the primary and secondary windings are such that, they always oppose the primary applied voltage V_1.

$$E_1I_1 = E_2I_2$$

(Assuming that the power factor of the primary is equal to the secondary).

$$\text{Or, } \frac{E2}{E1} = \frac{I1}{I2} = k \text{ ------- } (4)$$

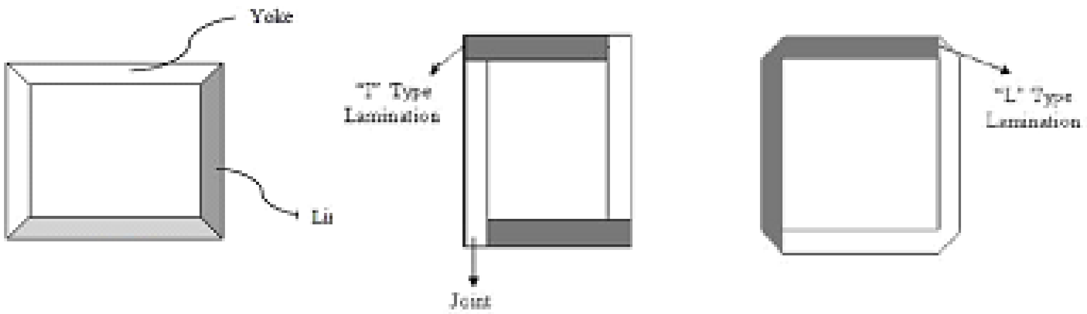
From eqn (3) and (4), we have

$$\frac{E2}{E1} = \frac{N2}{N1} = \frac{I1}{I2} = k \text{ -------------- } (5)$$

CONSTRUCTION OF A TRANSFORMER

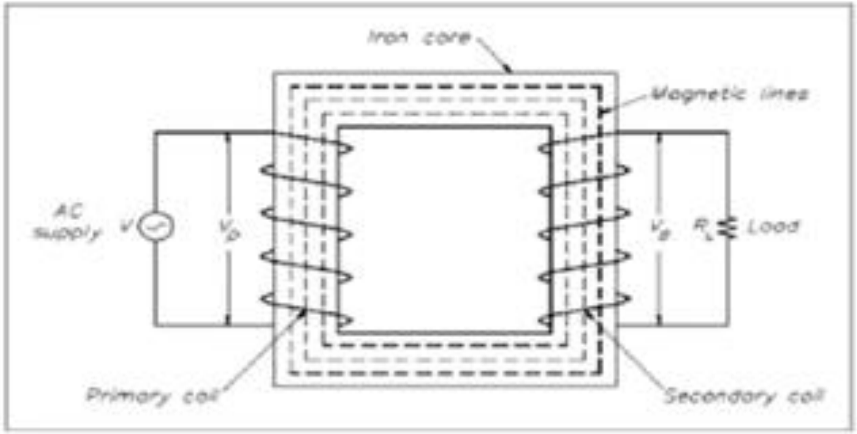
Core:

- The core of a transformer is either square or rectangular in size. It is further divided in two parts. The vertical portion on which the coils are bound is called limb, while the top and bottom horizontal portion is called yoke of the core
- Core is made up of laminations because of laminated type of construction, eddy current losses get minimized. Generally high grade silicon steel laminations (0.3 to 0.5 mm thick) are used



WINDING:

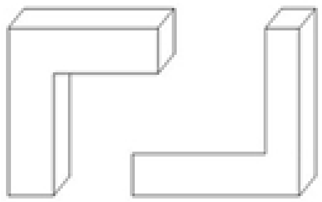
- There are two windings, which are wound on the two limbs of the core, which are insulated from each other and from the limbs
- The windings are made up of copper, so that, they possess a very small resistance



Types of Transformers

Core type Transformer:

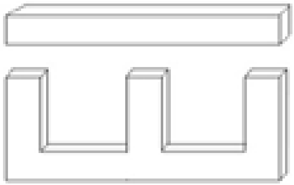
- The magnetic core of the transformer is made up of laminations to form a rectangular frame. The laminations are cut in the form of L-shape strips shown in the figure below.
- For avoiding the high reluctance at the joints where laminations are butted against each other, the alternate layer is stacked differently to eliminate continues joints



- The insulation layer is provided between the core and lower winding and between the primary and the secondary winding. For reducing the insulation, the low winding is always placed near to the core. The winding is cylindrical, and the lamination is inserted later on it.

Shell Type Transformer:

- The laminations are cut in the form of a long strip of E’s, and I’s as shown in the figure below. To reduce the high reluctance at the joints where the lamination is butted against each other, the alternate layers are stacked differently to eliminate continuous joint.



- The shell type transformer has three limbs or legs. The central limb carries the whole of the flux, and the side limb carries the half of the flux. Hence the width of the central limb is about to double to that of the outer limbs.
- The primary and secondary both the windings are placed on the central limbs

E.M.F. Equation

- As, shown in the fig., the flux rises sinusoidal to its maximum value Φ_m from 0. It reaches to the maximum value in one quarter of the cycle i.e. in T/4 sec (where, T is time period of the sin wave of the supply = 1/f).

  Therefore,

  average rate of change of flux = Φ_m /(T/4) = Φm/(1/4f)

  Therefore,

  average rate of change of flux = 4f Φ_m .... (Wb/s)

  Now,

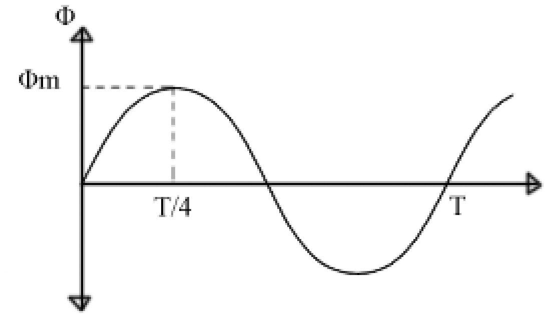
  Induced EMF per turn = rate of change of flux per turn

  Therefore, average EMF per turn = 4f Φ_m ....(Volts).

  Now, we know, Form factor = RMS value / average value

  Therefore, RMS value of EMF per turn = Form factor X average EMF per turn.

- As, the flux Φ varies sinusoidal, form factor of a sine wave is 1.11
- Therefore, RMS value of EMF per turn = 1.11 *4f Φ_m = 4.44f Φ_m



- RMS value of induced EMF in whole primary winding (E_1) = RMS value of EMF per turn X Number of turns in primary winding

  $E_1 = 4.44f\ N_1\ \Phi_m$ 1

  Similarly, RMS induced EMF in secondary winding (E_2) can be given as

  $E_2 = 4.44f\ N_2\ \Phi_m$ 2

  from the above equations 1 and 2,

- This is called the EMF equation of transformer, which shows, EMF / number of turns is same for both primary and secondary winding.

  For an ideal transformer on no load, $E_1 = V_1$ and $E_2 = V_2$

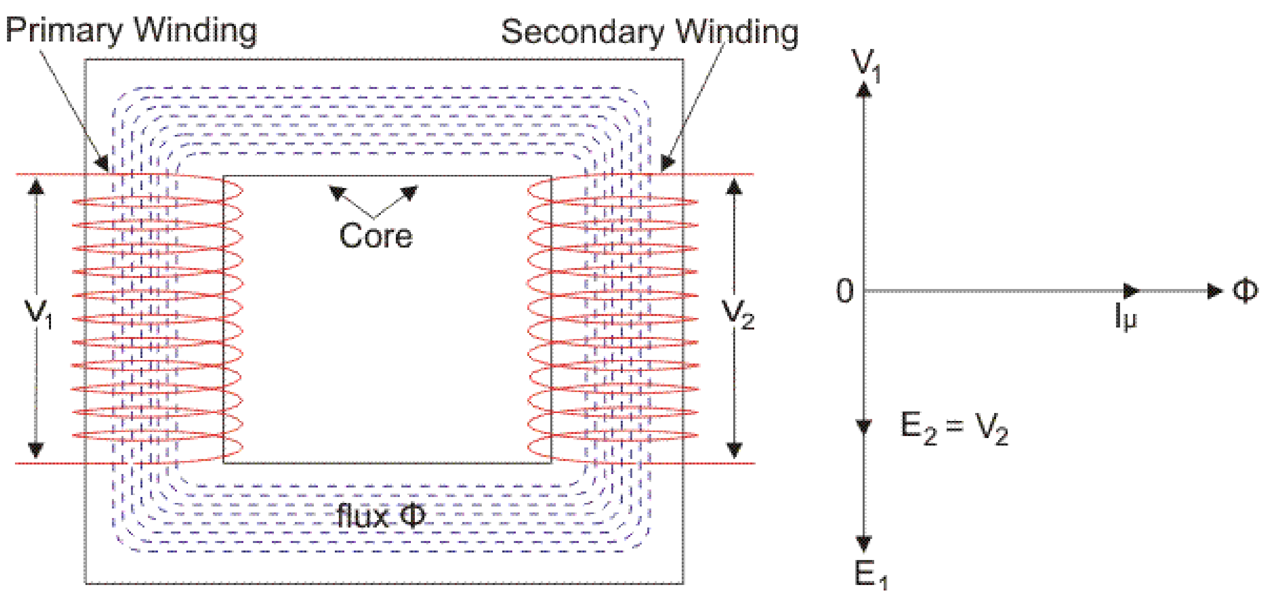
  where, V_1 = supply voltage of primary winding

  V_2 = terminal voltage of secondary winding

Ideal Transformer

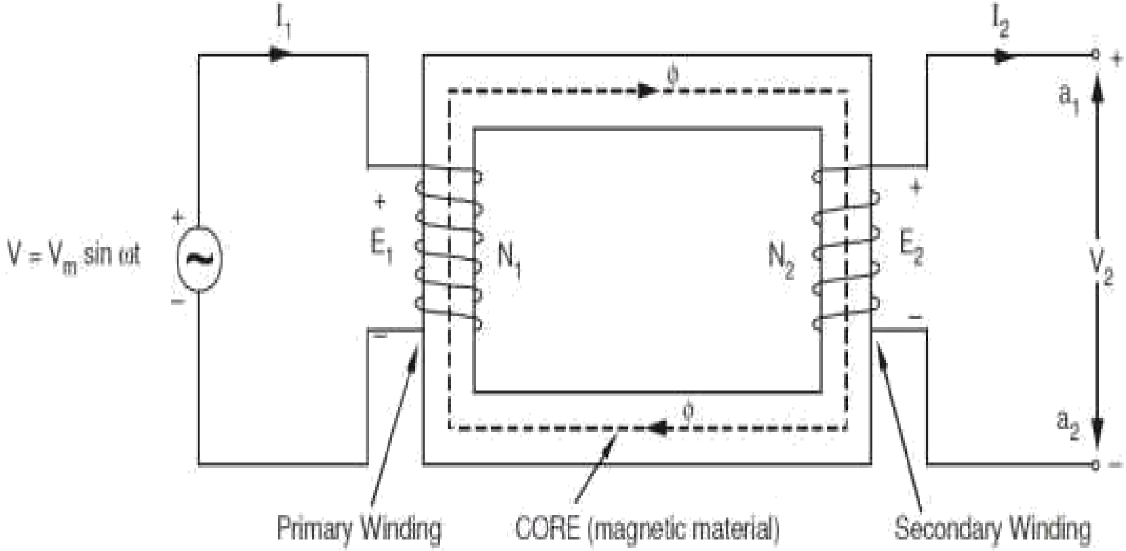
- **Definition of Ideal Transformer:**

An ideal transformer is an imaginary transformer which does not have any loss in it, means no core losses, copper losses and any other losses in transformer. Efficiency of this transformer is considered as 100%.



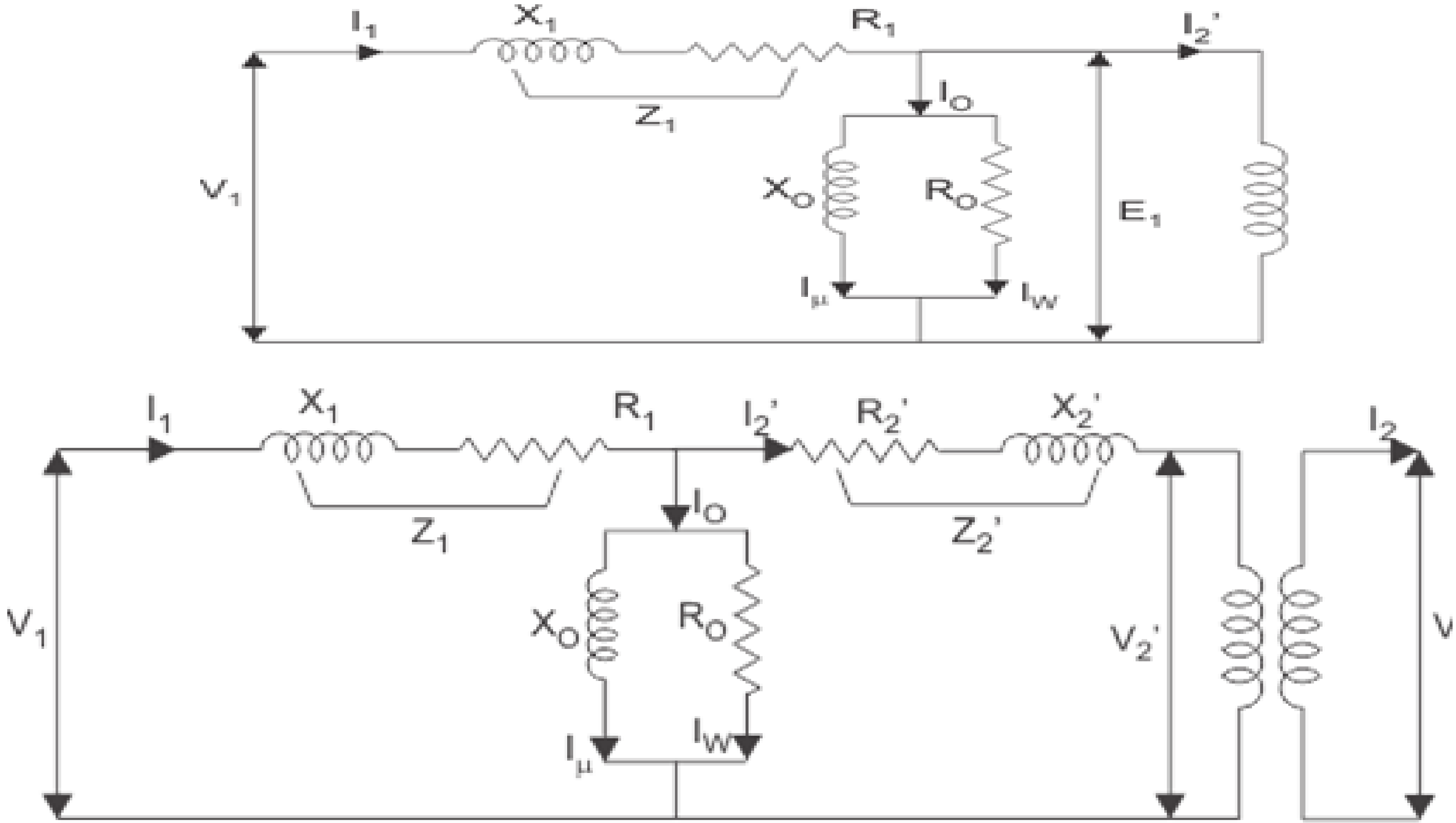
Practical Transformer

- As we said that, in an ideal transformer; there are no core losses in transformer i.e. loss free core of transformer. But in practical transformer, there are hysteresis and eddy current losses in transformer core.



Equivalent Circuit of Transformer

- **Equivalent Circuit of Transformer Referred to Primary:**



Cont..

- From Faraday's law of electromagnetic induction

$$emf,\ e = -T\frac{d\phi}{dt}$$

- Where ϕ is the instantaneous alternating flux and represented as

$$\phi = \phi_m \sin 2\pi ft$$

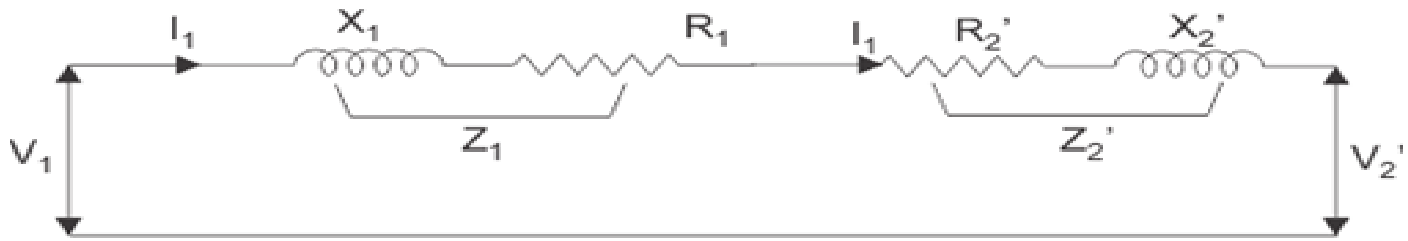
$$Hence,\ e = -T\frac{d(\phi_m \sin 2\pi ft)}{dt}$$

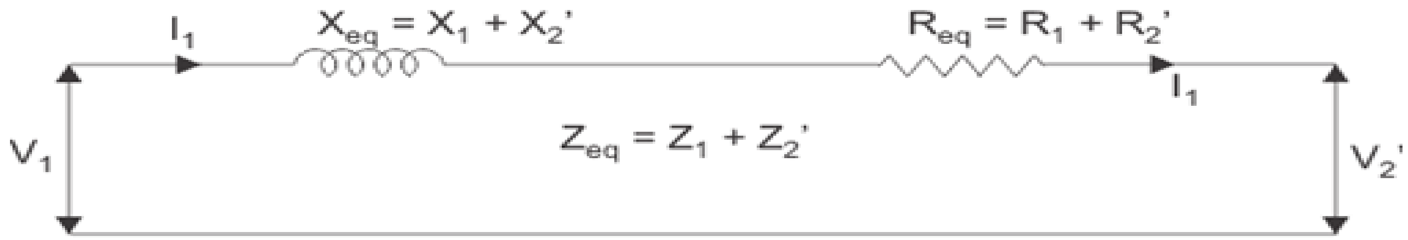
$$\Rightarrow e = -T\phi_m \cos(2\pi ft) \times 2\pi f$$

$$\Rightarrow e = -T\phi_m \times 2\pi f \cos(2\pi ft)$$

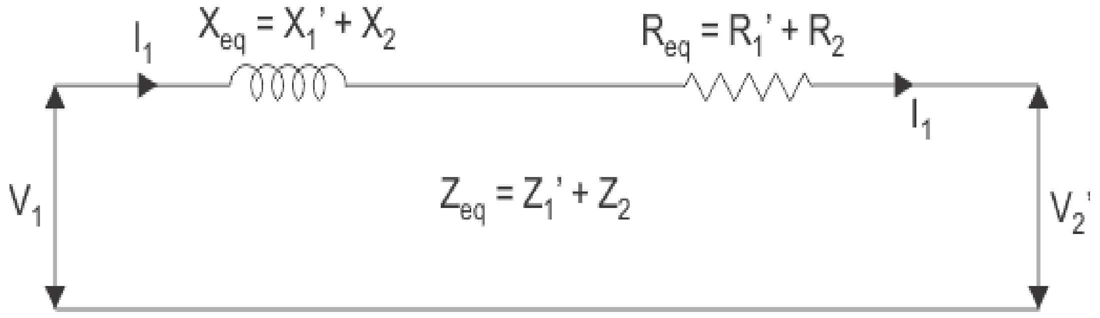
- As the maximum value of cos2πft is 1, the maximum value of induced EMF is

Approximate Equivalent Circuit of Transformer





•Equivalent Circuit of Transformer Referred to Secondary:



Losses in Transformer

- **Copper Loss in Transformer:**
- Copper loss is I^2R loss, in primary side it is $I_1^2R_1$ and in secondary side it is $I_2^2R_2$ loss, where I_1 and I_2 are primary and secondary current of transformer and R_1 and R_2 are resistances of primary and secondary winding. As the both primary & secondary currents depend upon load of transformer, copper loss in transformer vary with load.
- **Core Losses in Transformer:**
- Hysteresis loss and eddy current loss both depend upon magnetic properties of the materials used to construct the core of transformer and its design. So these losses in transformer are fixed and do not depend upon the load current. So core losses in transformer which is alternatively known as iron loss in transformer can be considered as constant for all range of loads

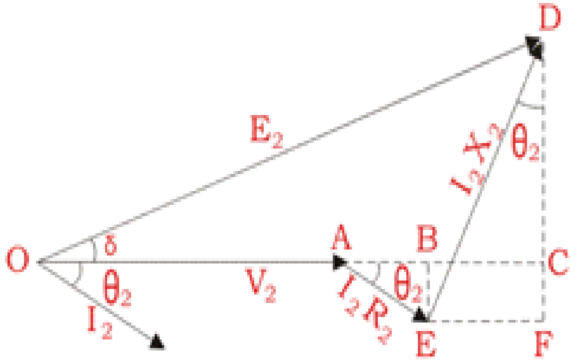
- **Hysteresis Loss in Transformer:**
- Hysteresis loss in transformer can be explained in different ways. We will discuss two of them, one is physical explanation and the other is mathematical explanation.
- **Eddy Current Loss:**
- In transformer, we supply alternating current in the primary, this alternating current produces alternating magnetizing flux in the core and as this flux links with secondary winding, there will be induced voltage in secondary, resulting current to flow through the load connected with it. Some of the alternating fluxes of transformer; may also link with other conducting parts like steel core or iron body of transformer etc. As alternating flux links with these parts of transformer, there would be a locally induced EMF. Due to these EMF's, there would be currents which will circulate locally at that parts of the transformer. These circulating current will not contribute in output of the transformer and dissipated as heat. This type of energy loss is called eddy current loss of transformer.

Voltage Regulation of Transformer

- The voltage regulation is the percentage of voltage difference between no load and full load voltages of a transformer with respect to its full load voltage
- Expression of Voltage Regulation of Transformer, represented in percentage is

$$Voltage\ regulation(\%) = \frac{E_2 - V_2}{V_2} \times 100\%$$

- **Voltage Regulation of Transformer for Lagging Power Factor:**
- Now we will derive the expression of voltage regulation in detail. Say lagging power factor of the load is $\cos\theta_2$, that means angle between secondary current and voltage is θ_2.



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$$OC = OA + AB + BC$$

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- Considered nearly equal to OC i.e,

$$E_2 = OC = OA + AB + BC$$

$$E_2 = OC = V_2 + I_2R_2\cos\theta_2 + I_2X_2\sin\theta_2$$

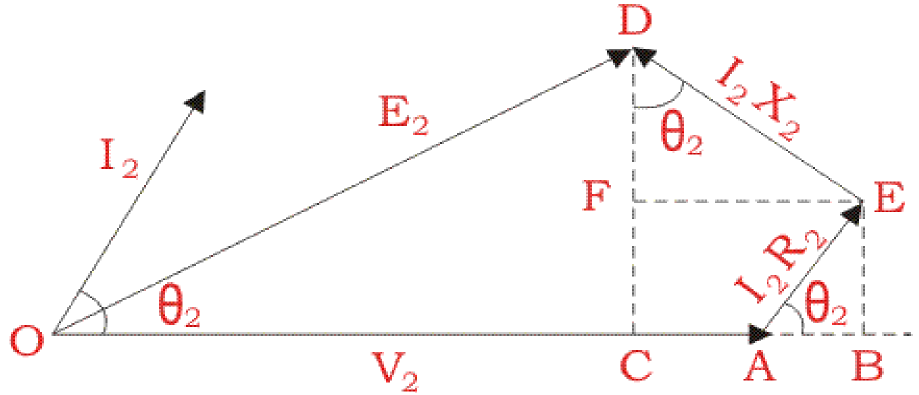
- Voltage regulation of transformer at lagging power factor,

$$Voltage\ regulation\ (\%) = \frac{E_2 - V_2}{V_2} \times 100(\%)$$

$$= \frac{I_2R_2\cos\theta_2 + I_2X_2\sin\theta_2}{V_2} \times 100(\%)$$

Cont..

- **Voltage Regulation of Transformer for Leading Power Factor:**
- Let's derive the expression of voltage regulation with leading current, say leading power factor of the load is $\cos\theta_2$, that means angle between secondary current and voltage is θ_2.



- Voltage regulation of transformer at leading power factor

$$Voltage\ regulation\ (\%) = \frac{E_2 - V_2}{V_2} \times 100(\%)$$

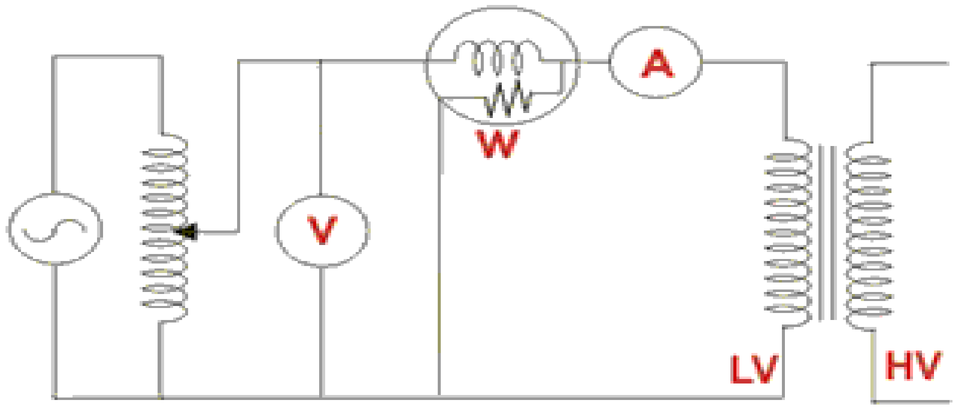
$$= \frac{I_2R_2\cos\theta_2 - I_2X_2\sin\theta_2}{V_2} \times 100(\%)$$

Open and Short Circuit Test on Transformer

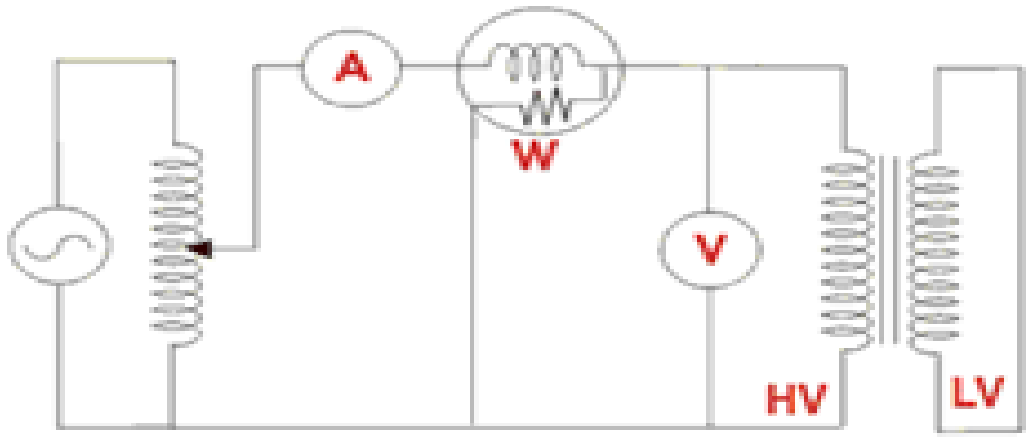
These two tests are performed on a transformer to determine

1. Equivalent circuit of transformer
2. Voltage regulation of transformer
3. Efficiency of transformer. The power required for these open circuit test and short circuit test on transformer is equal to the power loss occurring in the transformer.

Open Circuit Test on Transformer:



- **Short Circuit Test on Transformer:**



$$\therefore \quad \% \eta \text{ on full load} = \frac{V_2 (I_2) \text{F.L.} \cos\phi}{V_2 (I_2) \text{F.L.} \cos + W_o + W_{sc}} \times 100$$

$$\% \eta \text{ at any load} = \frac{n \times (\text{VA rating}) \times \cos\phi}{n \times (\text{VA rating}) \times \cos\phi + W_o + n^2 W_{sc}} \times 100$$

Cont..

- The rated voltages V_1, V_2 and rated currents (I_1) F.L. and (I_2) F.L. are known for the given transformer. Hence the regulation can be determined as,

$$\% R = \frac{I_2 R_{2e} \cos\phi \pm I_2 X_{2e} \sin\phi}{V_2} \times 100$$

$$= \frac{I_1 R_{1e} \cos\phi \pm I_1 X_{1e} \sin\phi}{V_1} \times 100$$