FLUID MECHANICS AND HYDRAULIC SYSTEMS

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Unit I

Fluid Properties and Fluid Statics

Syllabus

- Density, specific weight, specific gravity, surface tension and capillarity, Newton's law of viscosity, incompressible and compressible fluid, Hydrostatic forces on submerged bodies - Pressure at a point, Pascal's law, pressure variation with temperature and height, center of pressure plane, vertical and inclined surfaces; Manometers - simple and differential Manometers, inverted manometers, micro manometers, pressure gauges, Buoyancy - Archimedes principle, metacenter, Meta centric height calculations; Stability.

Introduction

- Fluid mechanics is a study of the behavior of fluids, either at rest (fluid statics) or in motion (fluid dynamics).
- The analysis is based on the fundamental laws of mechanics, which relate continuity of mass and energy with force and momentum.
- An understanding of the properties and behavior of fluids at rest and in motion is of great importance in engineering.

Fluid Mechanics

- **Fluid:** Fluids are substance which are capable of flowing and conforming the shapes of container.

  Fluids can be in gas or liquid states.

- **Mechanics:** Mechanics is the branch of science that deals with the state of rest or motion of body under the action of forces.

- **Fluid Mechanics:** Branch of mechanics that deals with the response or behavior of fluid either at rest or in motion.

Branches of Fluid Mechanics

- **Fluid Statics:** It is the branch of fluid mechanics which deals with the response/behavior of fluid when they are at rest.
- **Fluid kinematics:** It deals with the response of fluid when they are in motion without considering the energies and forces in them.
- **Hydrodynamics:** It deals with the behavior of fluids when they are in motion considering energies and forces in them.
- **Hydraulics:** It is the most important and **practical/experimental** branch of fluid mechanics which deals with the behavior of water and other fluid either at rest or in motion.

Significance of Fluid Mechanics

- Fluid is the most abundant available substance e.g., air, gases, ocean, river and canal etc.

To Design Dams, Spillways, Hydraulic Jumps

To Design water, oil and gas pipeline Networks

To study Oceanography and Coastal Engineering

To study Water and Air Pollution

To design slabs that resist to water and groundwater pressures

To Design bridges across rivers

To Design inshore Civil infrastructure

State of Matter

- 1. gas
- 2. Liquid
- 3. Solid

fluid

Solid Liquid Gas

Comparison Between Liquids and Gases

- Liquids have definite volume at any particular temperature
- Liquids have free level surface
- Molecules of liquid are close to each other
- Liquids have relatively more molecular attraction
- Liquids are slightly compressible
- Rate of diffusion of liquid is less

- Gases do not have any definite volume
- Gases do not have free level surface
- Molecules of gases are far apart
- Gases have less molecular attraction
- Gases are highly compressible
- Gases have higher rate of diffusion

Comparison Between Liquids and Solids

- Liquid conform the shape of any container
- Liquid can flow
- Molecules of liquid are distinctly apart
- Liquid have relatively less molecular attraction
- Liquid are slightly compressible
- Liquids cannot sustain shear forces

- Do not conform the shape of container
- Solids cannot flow
- Molecules of solids are very close to each other
- Solids have more molecular attraction
- Solids are highly incompressible
- Solids can sustain shear

Dimension and Units

- **System of Units**

System International (SI)

- Fundamental dimensions: length, mass and time
- Units: (meter, kilogram and second)

British Gravitation System (BG)

- Fundamental dimension: length, force and time
- Units: (ft, slug and second)

CGS System

- Fundamental dimensions: length, mass and time
- Units: (centimeter, gram and second)

Dimension and Units

- **Dimension**

Fundamental/Primary Dimension

length(L), mass (M) and time (T)

Derived/Secondary Dimensions

e.g., force, velocity, acceleration etc

Fundamental/Primary Dimension

Dimension	Symbol	Unit (SI)
Length	L	meter (m)
Mass	M	kilogram (kg)
Time	T	second (s)
Temperature	θ	kelvin (K)
Electric current	i	ampere (A)
Amount of light	C	candela (cd)
Amount of matter	N	mole (mol)

Units of Some Dimensions in Different Systems

Fundamental Units

- length(L), mass (M) and time (T)

Derived Units

- e.g., force(F), velocity(L/T), acceleration (L/T/T) etc

System	Length	Time	Force	Velocity	Acceleration	Energy	Power	Temperature
SI	m	s	N	m/s	m/s/s	N-m	kg-m/s	°C
BG	ft	s	lb	ft/s	ft/s/s	ft-lb	ft-lb/s	°F
CGS	cm	s	dyne	cm/s	cm/s/s	dyne-cm	dyne-cm/s	°C

Conversions

Length

- 1m=1000mm=100cm
- 1ft=12inch
- 1m=3.281ft
- 1Mile=5280ft=________km

Time

- 1day=24hours
- 1 hour=60min
- 1 min=60s

Mass

- 1kg=1000g
- 1kg=2.204lb
- 1kg=9.81N
- 1N=______lb ?

Volume

- $1m^3$=1000liters =________cm^3
  $1m^3=35.32ft^3$

Properties of Fluids- Mass Density, Specific Weight, Relative Density, Specific volume

Properties of Fluids

- The properties outlines the general properties of fluids which are of interest in engineering.
- The symbol usually used to represent the property is specified together with some typical values in SI units for common fluids.

Density

The density of a substance is the quantity of matter contained in a unit volume of the substance. It can be expressed in three different ways.

- **Mass Density**
- **Specific Weight**
- **Relative Density**

Mass Density

- **Mass Density:** Mass Density, ρ , is defined as the mass of substance per unit volume.

Units: Kilograms per cubic metre, (or)

Dimensions: ML^{-3}

Typical values:

Water = 1000 kgm^{-3}, Mercury = 13546 kgm^{-3}, Air = 1.23 kgm^{-3}, Paraffin Oil = 800 kgm^{-3}.

(at pressure =1.013x 10 $^{-5}$ Nm^{-2} and Temperature = 288.15 K.)

Specific Weight

- **Specific Weight:** Specific Weight ω, (sometimes , and sometimes known as *specific gravity*) is defined as the weight per unit volume.

or

- The force exerted by gravity, g, upon a unit volume of the substance.

  The Relationship between g and ω can be determined by Newton's 2nd Law, since

weight per unit volume = mass per unit volume g

$$\omega = \rho g$$

- Units: Newton's per cubic metre, N/m^3 (or) Nm^{-3}
- Dimensions: $ML^{-2}T^{-2}$

Typical values:

- Water =9814 Nm^{-3}
- Mercury = 132943Nm^{-3}
- Air =12.07 Nm^{-3}
- Paraffin Oil =7851 Nm^{-3}

Relative Density

- **Relative Density :** Relative Density,σ, is defined as the ratio of mass density of a substance to some standard mass density.
- For solids and liquids this standard mass density is the maximum mass density for water (which occurs at 4°C) at atmospheric pressure.

$$\sigma = \frac{\rho_{substance}}{\rho_{H_2O(at\,4^\circ c)}}$$

- Units: None, since a r umber.
- Dimensions: 1.
- Typical values: Water = 1, Mercury = 13.5, Paraffin Oil =0.8.

Specific Volume

- **Specific volume:** Specific volume is a property of materials, defined as the number of cubic meters occupied by one kilogram of a particular substance.
- The standard unit is the meter cubed per kilogram (m^3/kg or $m^3 \cdot kg^{-1}$).
- Specific volume is inversely proportional to density. If the density of a substance doubles, its specific volume, as expressed in the same base units, is cut in half. If the density drops to 1/10 its former value, the specific volume, as expressed in the same base units, increases by a factor of 10.

Dynamic viscosity, Kinematic viscosity, Newtonian and Non-Newtonian Fluids

Viscosity

- Viscosity, μ, is the property of a fluid, due to cohesion and interaction between molecules, which offers resistance to shear deformation. Different fluids deform at different rates under the same shear stress. Fluid with a high viscosity such as syrup, deforms more slowly than fluid with a low viscosity such as water.
- All fluids are viscous, "Newtonian Fluids" obey the linear relationship given by Newton's law of viscosity.

$$\tau = \mu \frac{du}{dy}$$

- where τ is the shear s
- Units Nm^{-2}; $Kgm^{-1}s^{-2}$
- Dimensions $ML^{-1}T^{-2}$.
- du/dy is the velocity gradient or rate of shear strain,
- μ is the "coefficient of dynamic viscosity"

Coefficient of Dynamic Viscosity

- **Coefficient of Dynamic Viscosity** The Coefficient of Dynamic Viscosity,μ, is defined as the shear force, per unit area, (or shear stress τ), required to drag one layer of fluid with unit velocity past another layer a unit distance away.

$$\mu = \tau \Big/ \frac{du}{dy} = \frac{\text{Force}}{\text{Area}} \Big/ \frac{\text{Velocity}}{\text{Distance}} = \frac{\text{Force} \times \text{Time}}{\text{Area}} = \frac{\text{Mass}}{\text{Length x Time}}$$

- Units: Newton seconds per square metre, or Kilograms per meter per second,.

- (Although note that is often expressed in Poise, P, where 10 P = 1kgm^{-1}s^{-1}.)

Typical values:

- Water =1.14x10^{-3} kgm^{-1}s^{-1},
- Air =1.78x 10^{-5}kgm^{-1}s^{-1},
- Mercury=1.552kgm^{-1}s^{-1} , Paraffin Oil =1.9kgm^{-1}s^{-1} .

Kinematic Viscosity

- **Kinematic Viscosity :**Kinematic Viscosity, υ , is defined as the ratio of dynamic viscosity to mass density.

- Units: square metres pe $\nu = \frac{\mu}{\rho}$ nd, m^2s^{-1}
- (Although note that υ is often expressed in Stokes, St, where $10\ ^4$St = $1m^2s^{-1}$.)
- Dimensions: L^2T^{-1}.

Typical values:

- Water =1.14 10^{-6} m^2s^{-1}
- Air =1.46 10^{-5} m^2s^{-1}
- Mercury =1.145 10^{-4} m^2s^{-1}
- Paraffin Oil =2.375 10^{-3} m^2s^{-1}

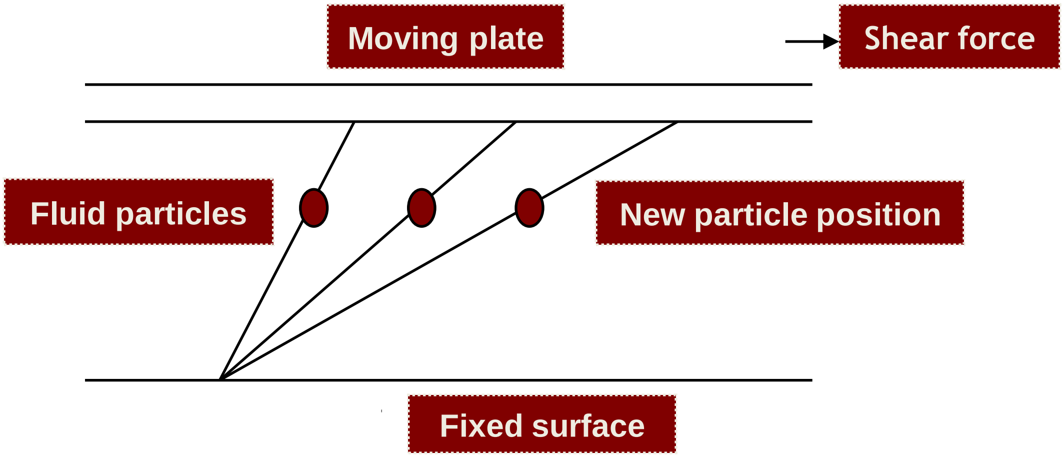
- A fluid is a substance, which deforms continuously, or flows, when subjected to shearing force
- In fact if a shear stress is acting on a fluid it will flow and if a fluid is at rest there is no shear stress acting on it.

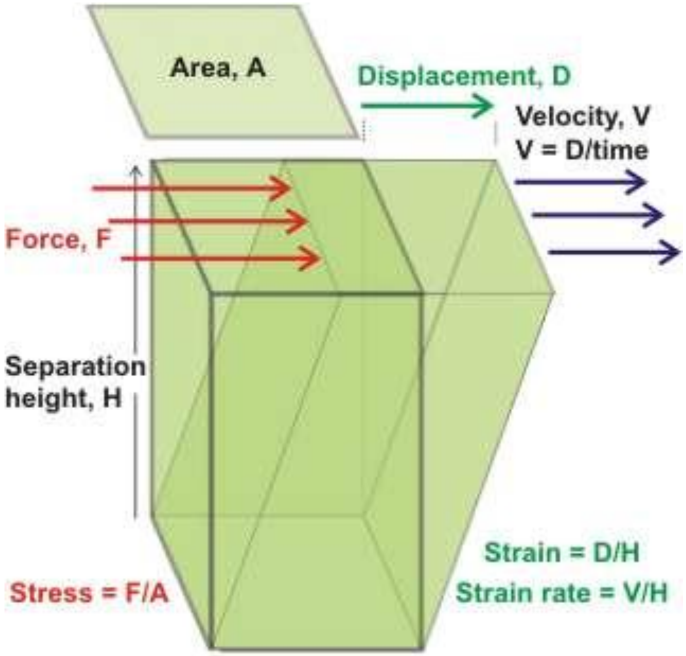
Fluid Flow → **Shear stress – Yes**

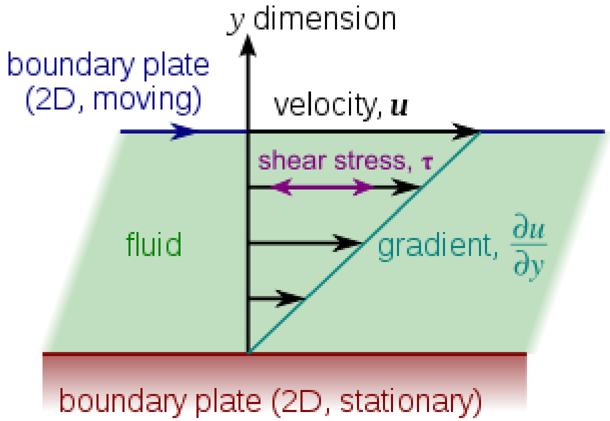
Fluid Rest → **Shear stress – No**

Shear stress in moving fluid

- If fluid is in motion, shear stress are developed if the particles of the fluid move relative to each other. Adjacent particles have different velocities, causing the shape of the fluid to become distorted.
- On the other hand, the velocity of the fluid is the same at every point, no shear stress will be produced, the fluid particles are at rest relative to each other.




Area, A
Displacement, D
Velocity, V
V = D/time
Force, F
Separation height, H
Strain = D/H
Stress = F/A
Strain rate = V/H


y dimension
boundary plate (2D, moving)
velocity, u
shear stress, τ
fluid
gradient, ∂u/∂y
boundary plate (2D, stationary)

Newtonian and Non-Newtonian Fluid

Fluid $\xrightarrow{\text{obey}}$ Newton's law of viscosity $\xrightarrow{\text{refer}}$ Newtonian fluids

Example:
Air
Water
Oil
Gasoline
Alcohol
Kerosene
Benzene
Glycerine

Newton's' law of viscosity is given by;

$$\tau = \mu \frac{du}{dy} \qquad (1.1)$$

τ = shear stress
μ = viscosity of fluid
du/dy = shear rate, rate of strain or velocity gradient

- The viscosity μ is a function only of the condition of the fluid, particularly its temperature.
- The magnitude of the velocity gradient (du/dy) has no effect on the magnitude of μ.

Newtonian and Non-Newtonian Fluid

Fluid $\xrightarrow{\text{Do not obey}}$ Newton's law of viscosity $\dashrightarrow$ Non- Newtonian fluids

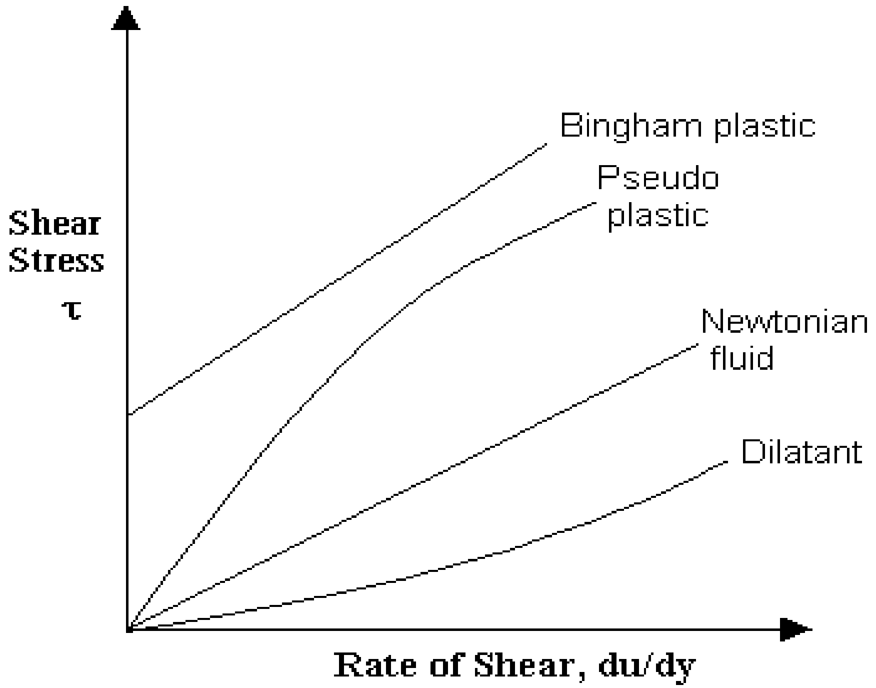
- The viscosity of the non-Newtonian fluid is dependent on the *velocity gradient* as well as the *condition of the fluid*.

Newtonian Fluids

- a linear relationship between shear stress and the velocity gradient (rate of shear),
- the slope is constant
- the viscosity is constant

Non-newtonian fluids

- slope of the curves for non-Newtonian fluids varies



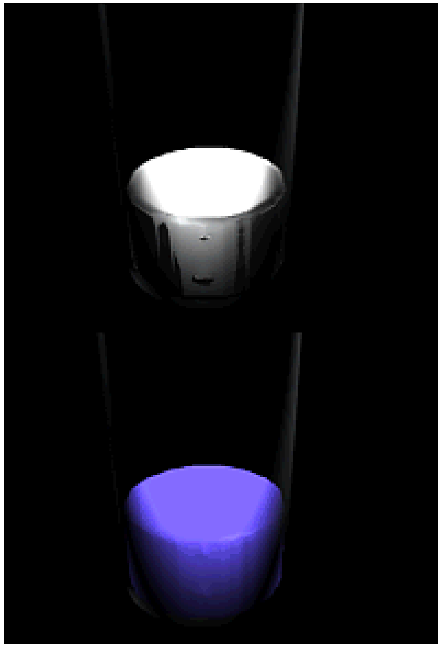
Bingham plastic : resist a small shear stress but flow easily under large shear stresses, e.g. sewage sludge, toothpaste, and jellies.

Pseudo plastic : most non-Newtonian fluids fall under this group. Viscosity decreases with increasing velocity gradient, e.g. colloidal substances like clay, milk, and cement.

Dilatants : viscosity decreases with increasing velocity gradient, e.g. quicksand.

Viscosity

Newtonian & Non Newtonian fluids



Numerical's

1. If $5.6 m^3$ of oil weighs 46800 N, what is the mass density in kg/m^3?
2. What is the relative density of the oil in question 1?
3. A fluid has absolute viscosity, μ, of 0.048 Pa s. If at point A, 75mm from the wall the velocity is measured as 1.125 m/s, calculate the intensity of shear stress at point B 50mm from the wall in N/m^2. Assume a linear (straight line) velocity distribution from the wall.
4. Calculate the specific weight, density and specific gravity of one liter of a liquid which weighs 7N.

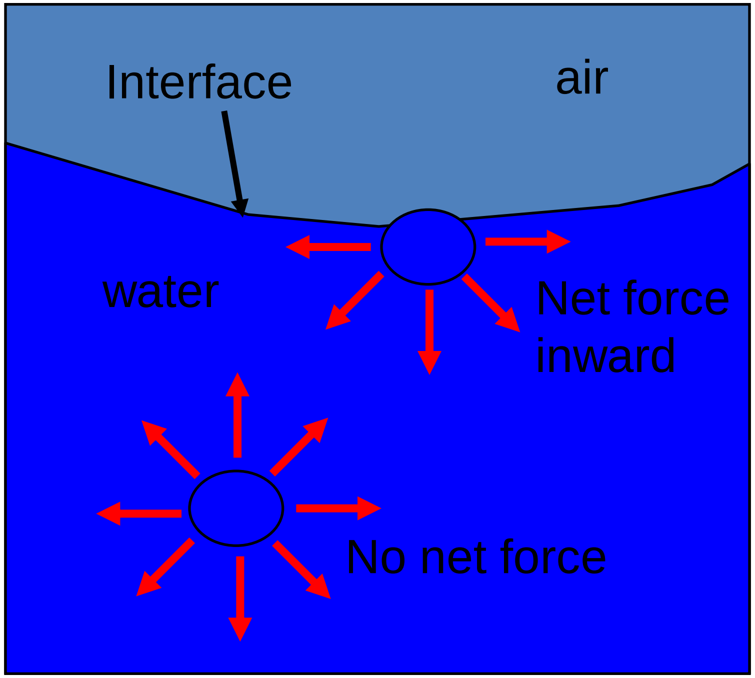
Answers

1. 852 kg/m^3
2. 0.852
3. 0.72 Pa
4. 7000N/m^3, 713.5kg/m^3,0.7135

Surface Tension, Capillarity, Bulk Modulus (Compressibility)

Surface Tension

- Below surface, forces act equally in all directions
- At surface, some forces are missing, pulls molecules down and together, like membrane exerting *tension* on the *surface*
- If interface is curved, higher pressure will exist on concave side
- Pressure increase is balanced by surface tension, σ
- σ = 0.073 N/m (@ 20^{o}C)



Surface Tension

● Molecular attraction forces in liquids:

– *Cohesion*: enables liquid to resist tensile stress

– *Adhesion*: enables liquid to adhere to another body

● Liquid-fluid interfaces:

– Liquid-gas interface: *free surface*

– Liquid-liquid (immiscible) interface

● At these interfaces, out-of-balance attraction forces forms imaginary surface film that exerts a tension force in the surface » *surface tension*

● Computed as a force per unit length

- Surface tension of various liquids

– Cover a wide range

– Decrease slightly with increasing temperature

- Values of surface tension for water between freezing and boiling points

– 0.00518 to 0.00404 lb/ft or 0.0756 to 0.0589 N/m

- Surface tension is responsible for the curved shapes of liquid drops and liquid sheets as in this example

Figure 1

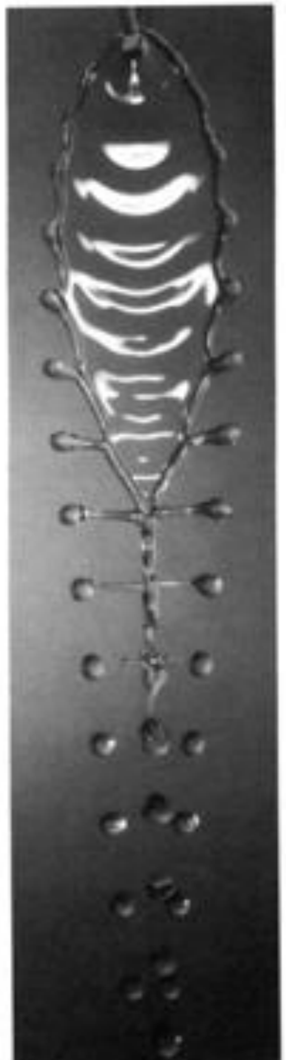


Figure 2

Figure 3

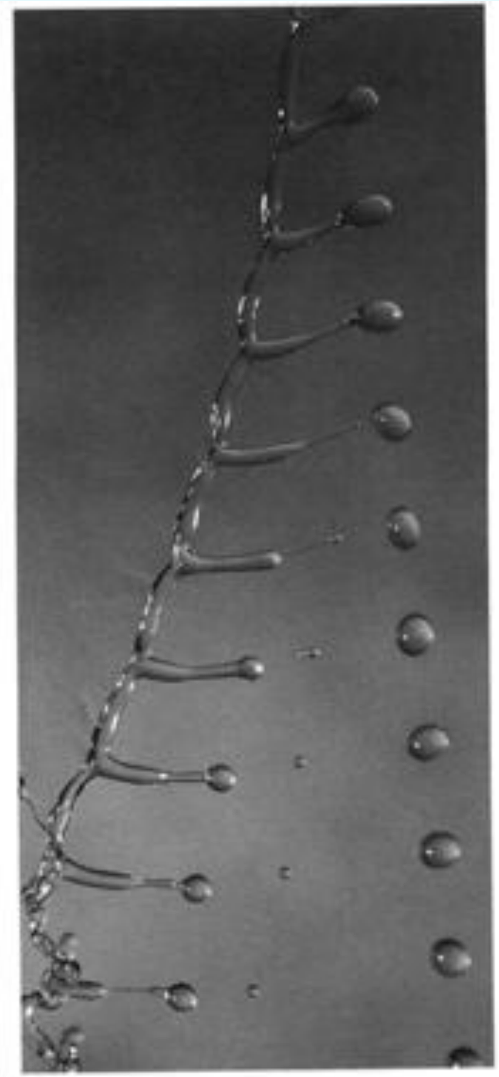
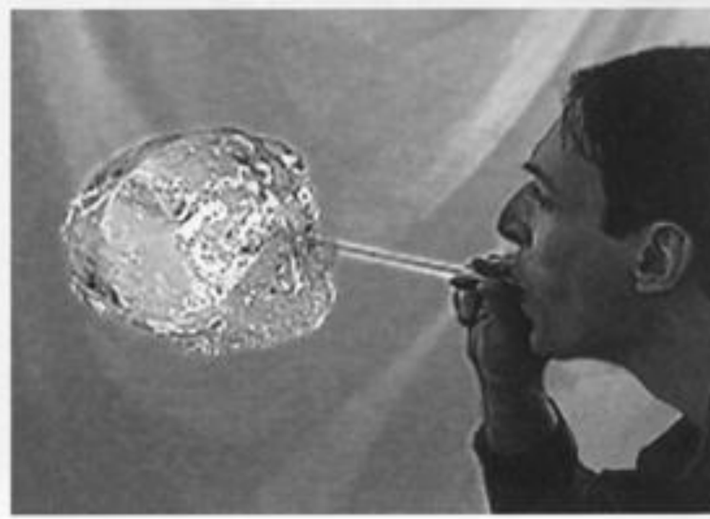


Figure 4

Figure 1

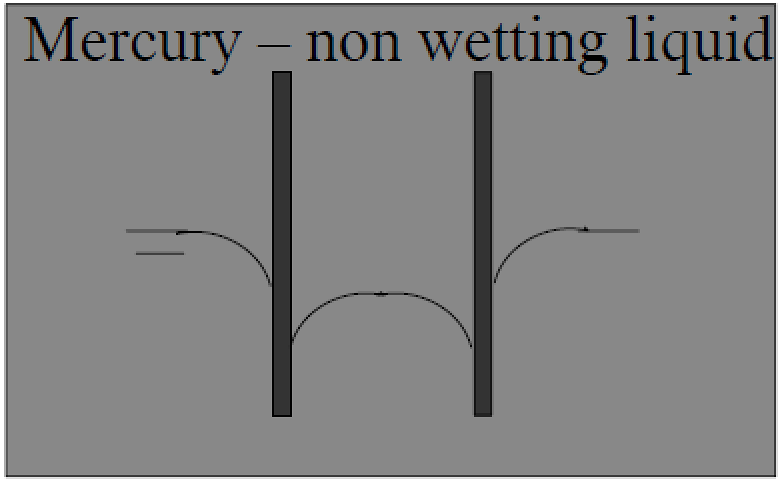
Figure 3

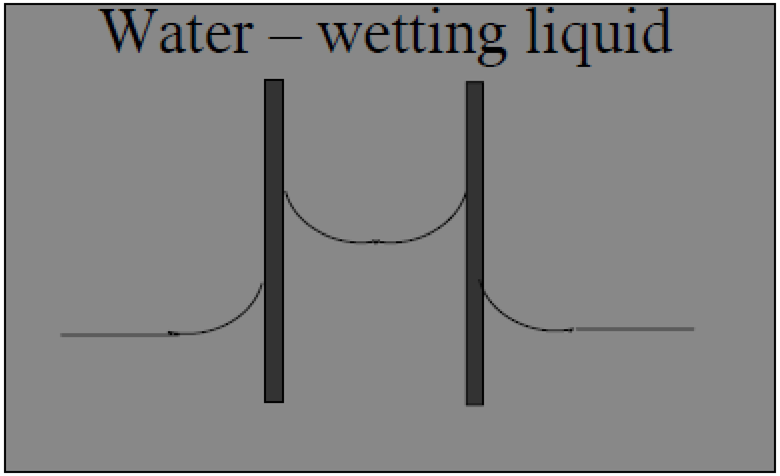


Surface Tension - Capillarity

- Property of exerting forces on fluids by fine tubes and porous media, due to both cohesion and adhesion
- Cohesion < adhesion, liquid wets solid, rises at point of contact
- Cohesion > adhesion, liquid surface depresses at point of contact
- Meniscus: curved liquid surface that develops in a tube

Surface Tension - Meniscus





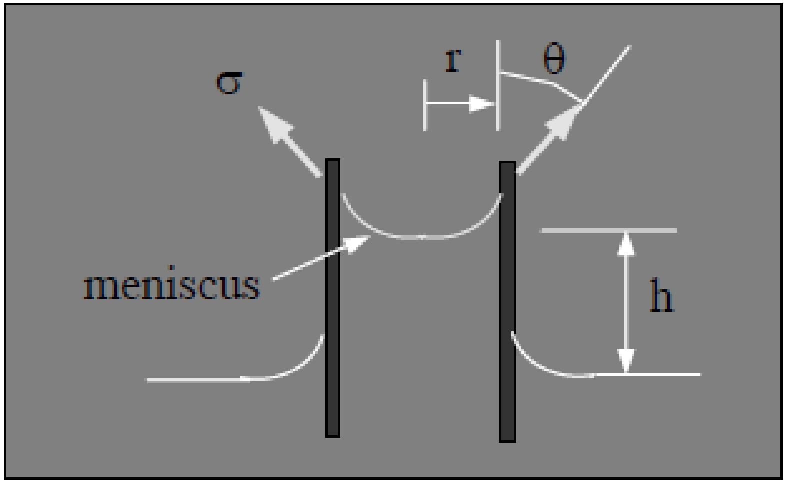
σ = surface tension,

θ = wetting angle,

γ = specific weight of liquid,

r = radius of tube,

h = capillary rise

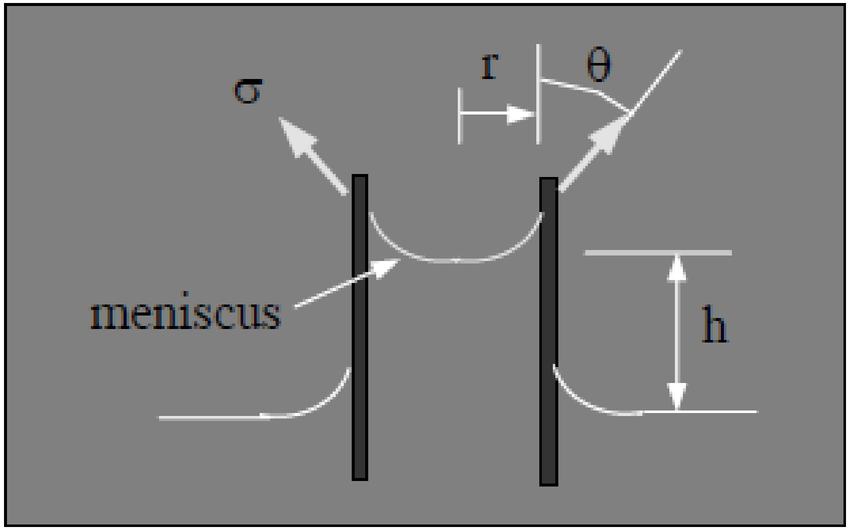


Surface Tension - Capillary Rise

- Equilibrium of surface tension force and gravitational pull on the water cylinder of height *h* produces:

$$2\pi r\sigma\cos\theta = \pi r^2 h\gamma$$

$$\boxed{h = 2\sigma\cos\theta / (\gamma r)}$$



Surface Tension

- Expression in previous slide calculates the *approximate* capillary rise in a small tube
- The meniscus lifts a small amount of liquid near the tube walls, as r increases this amount may become significant
- Thus, the equation developed *overestimates* the amount of capillary rise or depression, particularly for large r.
- For a clean tube, $\theta = 0^o$ for water, $\theta = 140^o$ for mercury
- For $r > ¼\ in$ ($6\ mm$), capillarity is negligible

Surface Tension - Applications

- Its effects are negligible in most engineering situations.
- Important in problems involving capillary rise, e.g., soil water zone, water supply to plants
- When small tubes are used for measuring properties, e.g., pressure, account must be made for capillarity
- Surface tension important in:

– Small models in hydraulic model studies

– Break up of liquid jets

– Formation of drops and bubbles

Elasticity (Compressibility)

- Deformation per unit of pressure change

$$E_v = -\frac{dp}{dV/V} = \frac{dp}{d\rho/\rho}$$

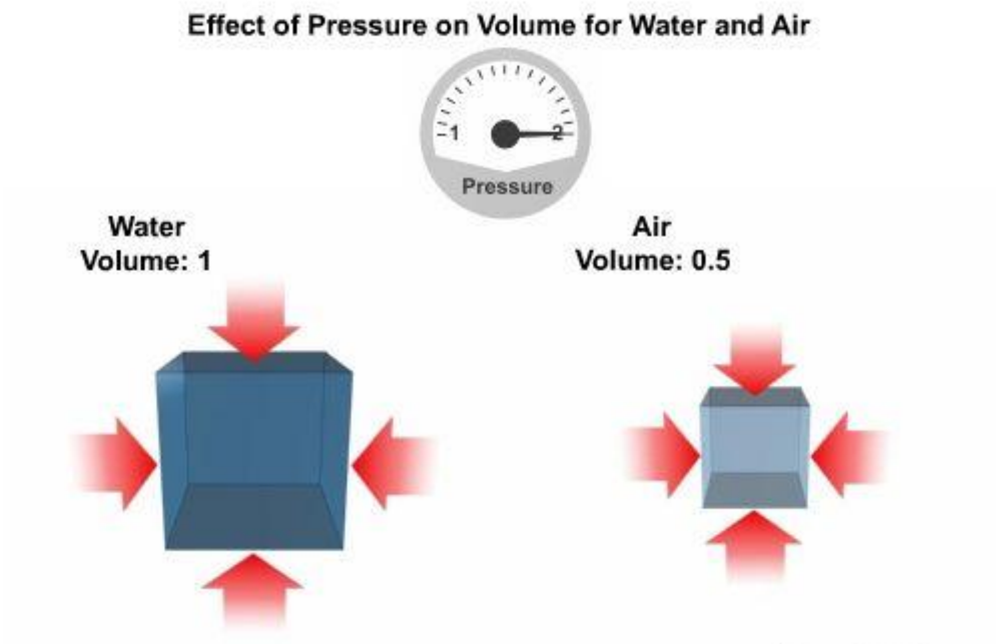
- For water E_v = 2.2 G Pa,

  1 M Pa pressure change = 0.05% volume change

  Water is relatively incompressible

Compressibility

- Compressibility is the fractional change in volume per unit increase in pressure. For each atmosphere increase in pressure, the volume of water would decrease 46.4 parts per million. The compressibility k is the reciprocal of the Bulk modulus, B.


Effect of Pressure on Volume for Water and Air
1
2
Pressure
Water
Volume: 1
Air
Volume: 0.5

Tutorials: Properties of Fluids

Numerical’s

- Determine the viscosity of a liquid having kinematic viscosity 6 stokes and specific gravity 1.9.
- The velocity distribution for flow over a flat plate is given by $u = 3/4y - y^2$ in which u is the velocity in metre per second at a distance y metre above the plate. Determine the shear stress at $y = 0.15$ m. Take Dynamic viscosity of fluid as 8.6 poise.
- If the velocity profile of a fluid over a plate is a parabolic with the vertex 20cm from the plate, where the velocity is 120cm/sec. Calculate the velocity gradients and shear stresses at a distance of 0, 10 and 20 cm from the plate, if the viscosity of the fluid is 8.5 poise.

Answers

- 11.40 poise
- 0.3825 N/m^2
- 10.2 N/m^2 ; 5.1 N/m^2, 0 N/m^2

Numerical

- Determine the bulk modulus of elasticity of a liquid, if pressure of the liquid is increased from 70 N/cm^2 to 130 N/cm^2. The volume of the liquid decreases by 0.15 percent.
- The surface tension of water in contact with air at 20^oC is 0.0725 N/m. The pressure inside a droplet of water is to be 0.02 N/cm^2 greater than the outside pressure. Calculate the diameter of the droplet of water.
- Find the surface tension in a soap bubble of 40mm diameter when the inside pressure is 2.5 N/m^2 above atmospheric pressure.

Answers

- $4x\ 10^4$ N/cm^2
- 1.45 mm
- 0.0125 N/m

Numerical's

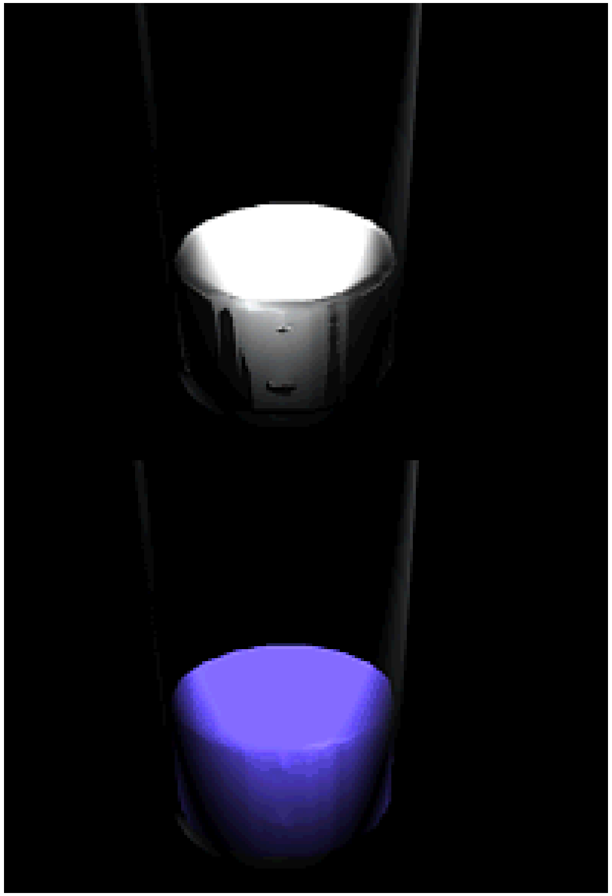
- Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in a (a) water and (b) mercury. Take surface tension $\sigma = 0.0725$ N/m for water and $\sigma = 0.52$ N/m for mercury in contact with air. The specific gravity for mercury is 13.6 and angle of contact = 130^o.
- Find out the minimum size of glass tube that can be used to measure water level if the capillarity rise in the tube is to be restricted to 2mm. Consider surface tension of water in contact with air as 0.073575 N/m.

Answers

- 1.18cm ; -0.4cm
- 1.5cm

Viscosity

- Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.
- Viscosity is also defined as the shear stress required to produce unit rate of shear strain.
- Units of viscosity are(SI) Ns/m^2.



Density and Buoyant Force

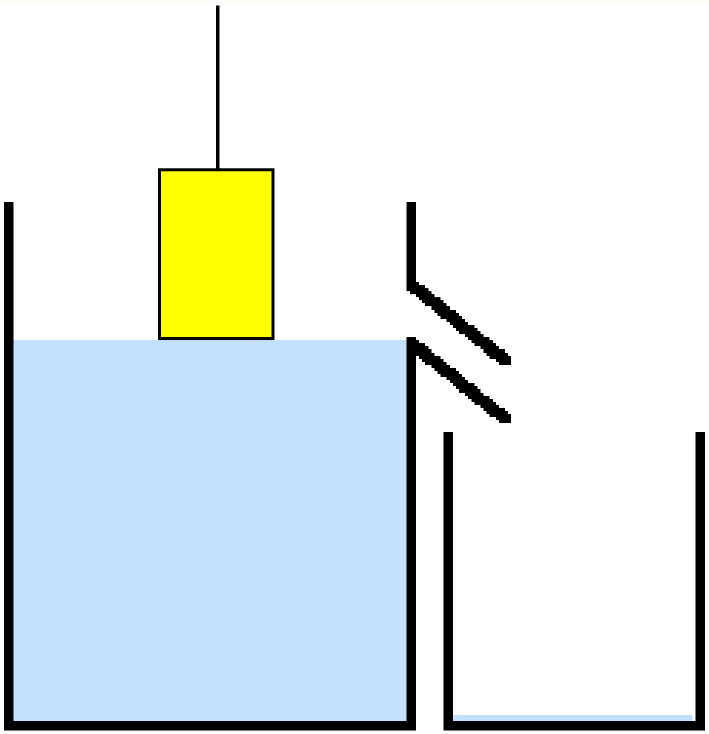
- The **buoyant force** is the upward force exerted by a liquid on an object immersed in or floating on the liquid.

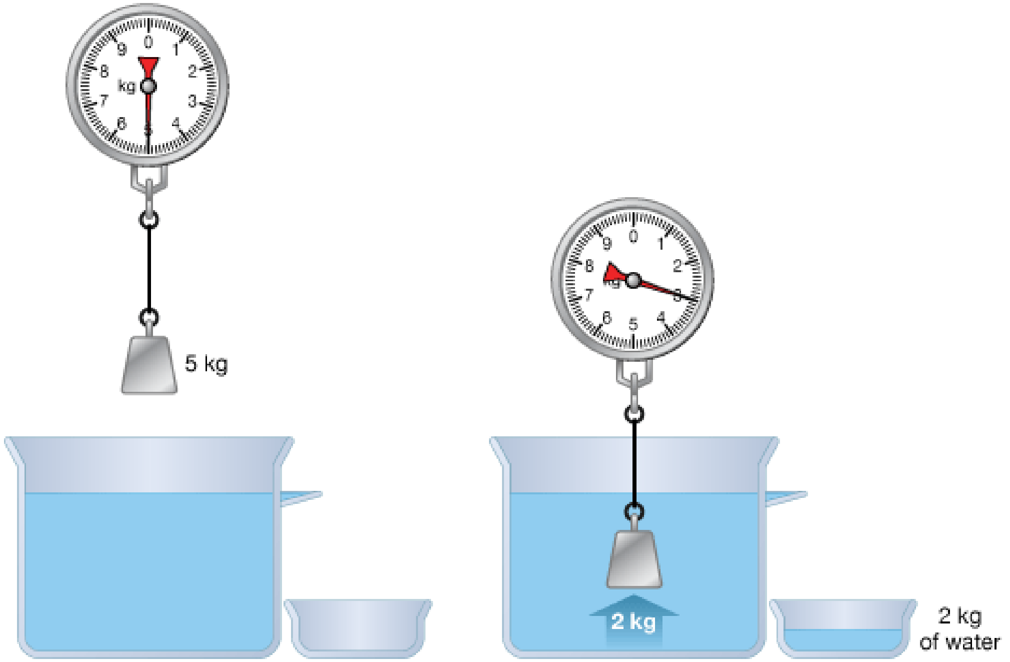
- Buoyant forces can keep objects afloat.

Buoyant Force

- For a floating object, the buoyant force equals the object's weight.
- The apparent weight of a submerged object depends on the density of the object.
- For an object with density ρ_O submerged in a fluid of density ρ_f, the buoyant force F_B obeys the following ratio:

$$\frac{F_g(object)}{F_B} = \frac{\rho_O}{\rho_f}$$





Archimedes' principle
5 kg
2 kg
2 kg
of water

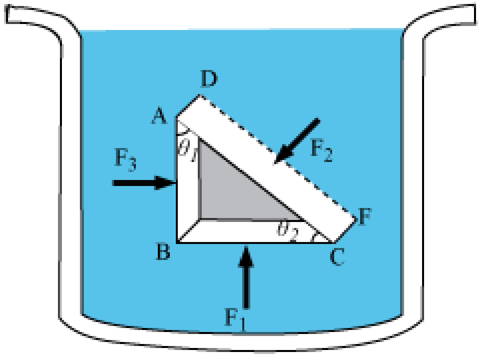
Fluids statics

- The general rules of statics (as applied in solid mechanics) apply to fluids at rest.

From earlier :

- a static fluid can have **no shearing force** acting on it, and that

- any force between the fluid and the boundary must be acting at right angles to the boundary.



Pressure force normal to the boundary

Pressure

- Deep sea divers wear atmospheric diving suits to resist the forces exerted by the water in the depths of the ocean.
- You experience this pressure when you dive to the bottom of a pool, drive up a mountain, or fly in a plane.

Pressure

- **Pressure** is the magnitude of the force on a surface per unit area.

$$P = \frac{F}{A}$$

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

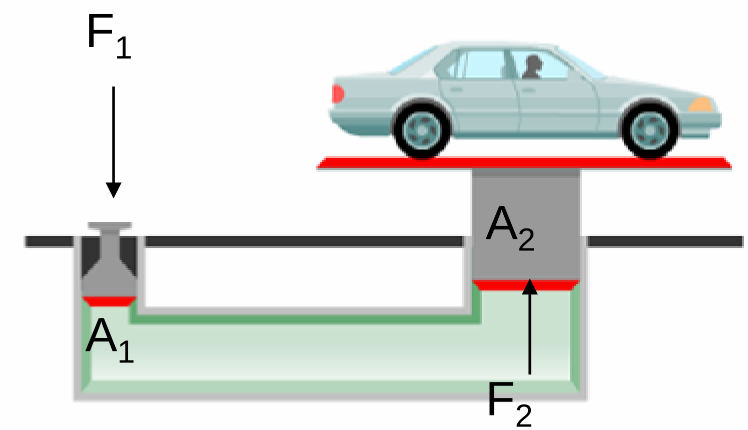
- Pascal's principle states that pressure applied to a fluid in a closed container is transmitted equally to every point of the fluid and to the walls of the container.

- The SI unit for pressure is the pascal, Pa.
- It is equal to 1 N/m^2.
- The pressure at sea level is about 1.01×10^5 Pa.
- This gives us another unit for pressure, the atmosphere, where 1 atm = 1.01×10^5 Pa

Pascal's Principle

- When you pump a bike tire, you apply force on the pump that in turn exerts a force on the air inside the tire.
- The air responds by pushing not only on the pump but also against the walls of the tire.
- As a result, the pressure increases by an equal amount throughout the tire.

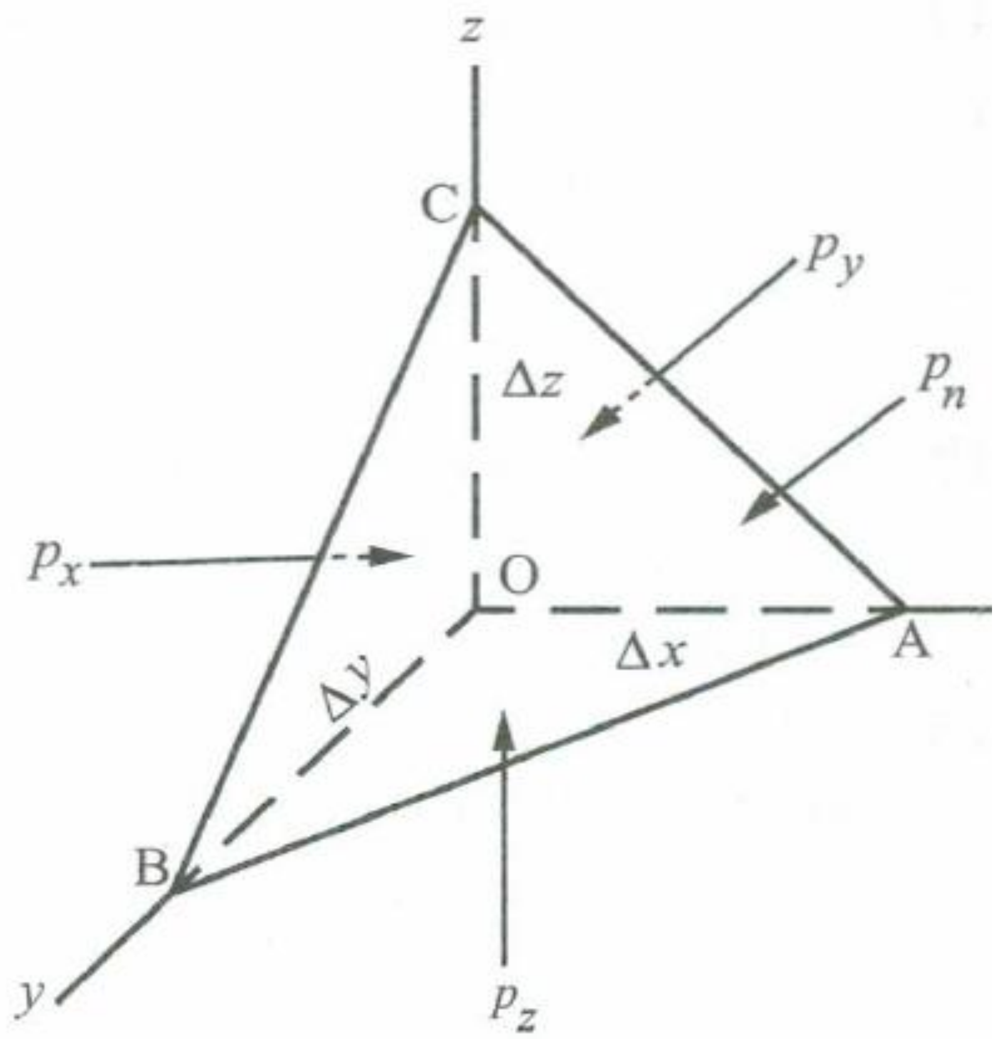
- A hydraulic lift uses Pascal's principle.
- A small force is applied (F_1) to a small piston of area (A_1) and cause a pressure increase on the fluid.
- This increase in pressure (P_{inc}) is transmitted to the larger piston of area (A_2) and the fluid exerts a force (F_2) on this piston.



$$P_{inc} = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = F_1 \frac{A_2}{A_1}$$

- **Pascal's Law:** Pascal's law states the pressure intensity at a point, in a static fluid, is equal in all directions. It can be proved in the following way.
- Consider a tetrahedron of sides Δx, Δy and Δz inside a shown in fig. Let 'O' a point inside the fluid be the origin.
- Let area ABC = ΔA and pressure on ΔA equal to p_n. The weight of a fluid element in the tetrahedron = $(\gamma.\ \Delta x.\ \Delta y.\ \Delta z)/6$.
- The weight is proportional to the third order of magnitude of very small quantities like Δx, Δy, Δz, whereas the pressure forces are proportional to the second order of magnitude.
- Hence the weight can be neglected in comparison to the pressures when Δx, Δy, Δz, tend to zero. Since the element is in static condition, the net forces in the x, y and z directions are zero.


z
C
p_y
p_n
Δz
p_x
O
Δx
A
Δy
B
y
p_z

- The component of p_n dA in the x direction = P_n. dA. cos(n,x), where cos(n,x) is the cosine of the angle between the normal to the surface and the x direction.
- Resolving forces in the x direction and equating the net force to zero, for static equilibrium conditions,

$$P_n.\ dA.\ \cos(n,x) = P_x 1/2\ dzdy.$$

But geometrically,

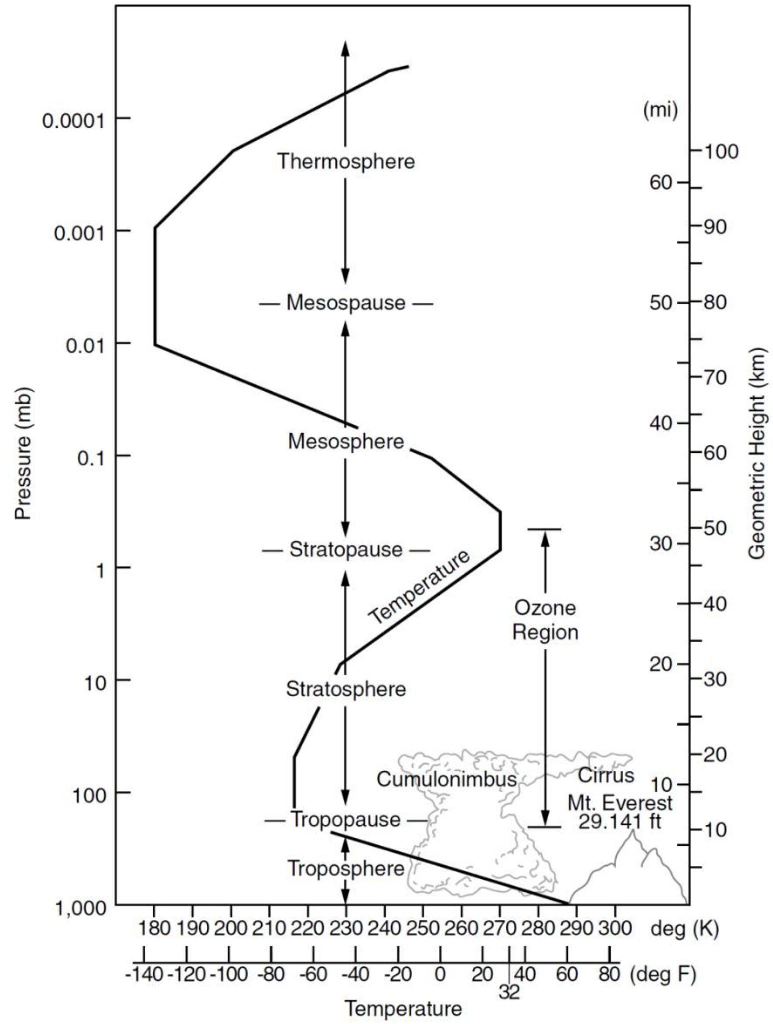
dAcos(n,x) = area OBC = 1/2dzdy.

So $P_n = P_x$.

Similarily, resolving forces in the y and z directions and equating the net forces to zero for the static condition, it can be proved that $P_n = P_y$ and $P_n = P_z$.

- Thus $P_n = P_x = P_y = P_z$, which proves pascal's law. Pascal's law does not hold good in the flows having shearing layers. In such cases, pressure p is defined as $p = \frac{p_x + p_z + p_y}{3}$, where P_x, P_y, P_z are pressure intensities in three mutually perpendicular directions.

Figure of standard atmosphere variation



Mathematical Model

– The standard atmosphere defines the temperature variation with altitude as shown

– Now need to find pressure and density as functions of either T or h

– Begin with the hydrostatic equation, divided by the equation of state for a perfect gas

$$dp / p = g_o dh/RT$$

– Can integrate this equation for pressure when we know the *P* relationship between T and h

Gradient Layers, T varies linearly with h

– Define a **lapse rate**, a, by: dT/dh = a

– Define the conditions at the layer base by h1, p1, ρ1, and T1

– In previous equation, replace dh with dT/a and integrate w.r.t. temperature to get:

$P/P_1 = (T/T_1)^{-g_o/aR}$

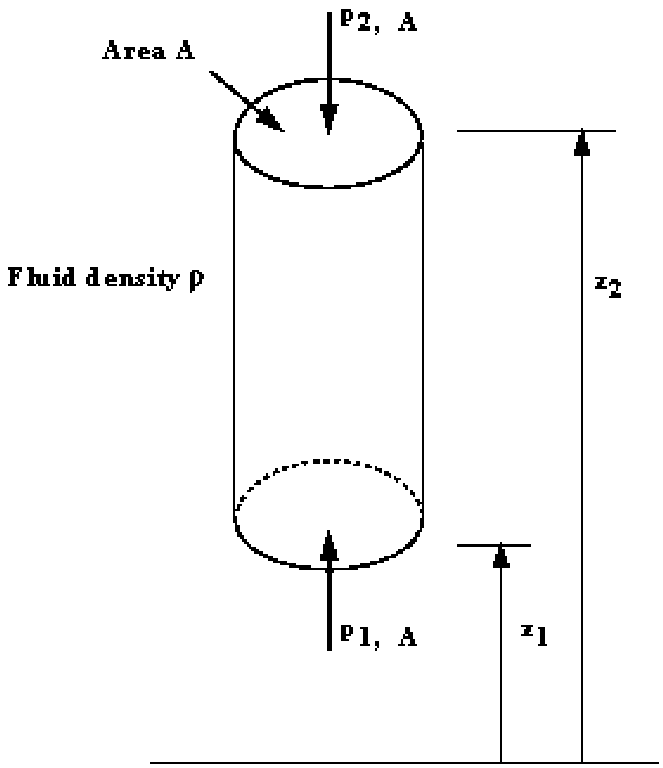
– And since $p/p_1 = (\rho T)/(\rho_1 T_1)$

$\rho/\rho_1 = (T/T_1)^{-[(g_o/aR)+1]}$

- Isothermal Layers, T is constant
  - Start at the base of the layer where we will define the conditions as h1, p1, ρ1, and T
  - Integrate the previous equation W.R.T. h holding T Constant

$$P/P_1 = e^{-(go/RT)(h-h1)} = \rho/\rho_1$$

Variation of Pressure Vertically In A Fluid Under Gravity



Vertical elemental cylinder of fluid

Variation of Pressure Vertically In A Fluid Under Gravity

- In the previous slide figure we can see an element of fluid which is a vertical column of constant cross sectional area, A, surrounded by the same fluid of mass density ρ.
- The pressure at the bottom of the cylinder is p_1 at level z_1 , and at the top is p_2 at level z_2.
- The fluid is at rest and in equilibrium so all the forces in the vertical direction sum to zero. i.e. we have
- Force due to p_1 on A (upward) $= p_1 A$
- Force due to p_2 on A (downward) $= p_2 A$
- Force due to weight of element (downward) mg

  $\rho g A(z_2 - z_1)$ mass density volume

- Taking upward as positive, in equilibrium we have

$$p_1 A - p_2 A = \rho g A(z_2 - z_1)$$

$$p_2 - p_1 = - \rho g A(z_2 - z_1)$$

- Thus in a fluid under gravity, pressure decreases with increase in height

$$z = (z_2 - z_1)$$

Pressure and Head

- In a static fluid of constant density we have the relationship

- This can be integrated to $\frac{dp}{dz} = -\rho g$ = -ρ*gz* + constant
- In a liquid with a free surface the pressure at any depth *z* measured from the free surface so that *z* = -*h*

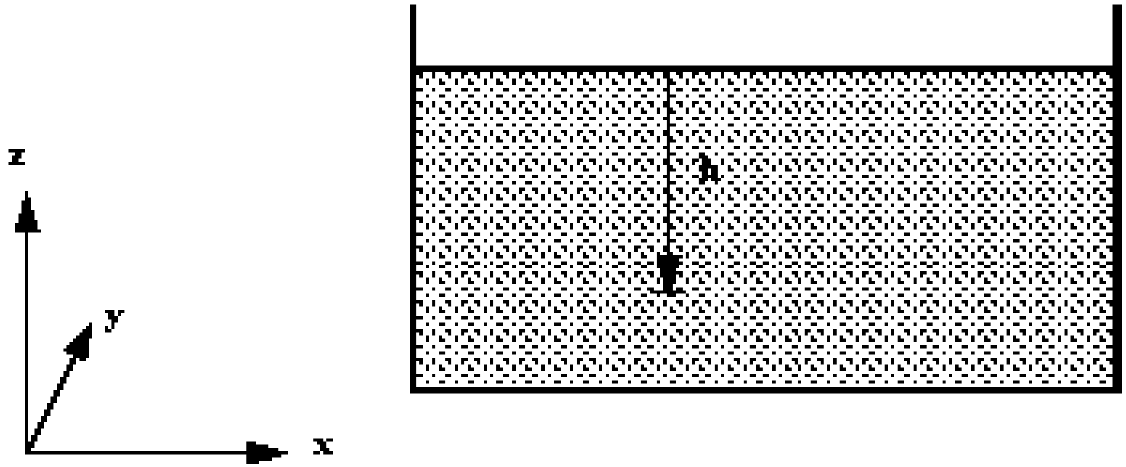


Fig. Fluid head measurement in a tank

This gives the pressure $p = \rho g h + \text{constant}$

- At the surface of fluids we are normally concerned with, the pressure is the atmospheric pressure, $p_{\text{atmospheric}}$. So

$$p = \rho g h + p_{\text{atmospheric}}$$

- As we live constantly under the pressure of the atmosphere, and everything else exists under this pressure, it is convenient (and often done) to take atmospheric pressure as the datum. So we quote pressure as above or below atmospheric.
- Pressure quoted in this way is known as gauge pressure i.e.
- **Gauge pressure** is $p = \rho g h$

- The lower limit of any pressure is zero - that is the pressure in a perfect vacuum. Pressure measured above this datum is known as absolute pressure i.e.
- **Absolute pressure** is $p = \rho g h + p_{\text{absolute atmospheric}}$
- Absolute pressure = Gauge pressure + Atmospheric pressure

- As g is (approximately) constant, the gauge pressure can be given by stating the vertical height of any fluid of density ρ which is equal to this pressure.

$$p = \rho g h$$

- This vertical height is known as **head** of fluid.

Note: If pressure is quoted in *head*, the density of the fluid *must* also be given.

Problem:

- We can quote a pressure of $500K\ N\ m$-2 in terms of the height of a column of water of density, ρ = 1000kgm-3.

Using $p = \rho gh$,

$$h = \frac{p}{\rho g} = \frac{500 \times 10^3}{1000 \times 9.81} = 50.95m \text{ of water}$$

And in terms of Mercury with density, ρ = 13.6x10³kgm-3.

$$h = \frac{500 \times 10^3}{13.6 \times 10^3 \times 9.81} = 3.75m \text{ of Mercury}$$

Simple and Differential Manometers

- Manometer: manometers are defined as the devices used for measuring the pressure at a point in the fluid by balancing the column of fluid by the same or another column of the fluid.
- They are classified as

  1.Simple manometer 2.Differential manometer

Simple manometer: simple manometer consist of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere.

Differential manometer: these are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes.

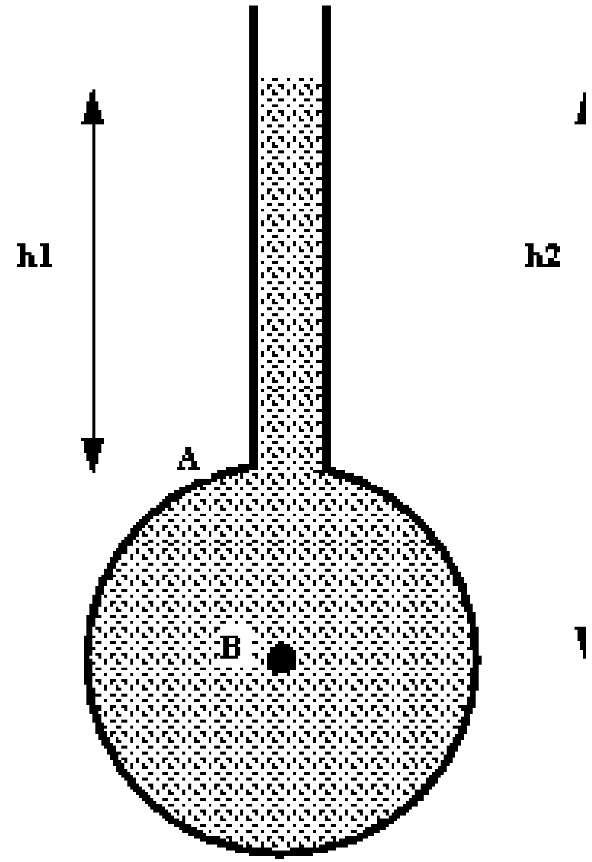
Pressure Measurement By Manometer

- The relationship between pressure and head is used to measure pressure with a manometer (also know as a liquid gauge).

1. Piezometer
2. U-tube Manometer
3. Single Column Manometer

The Piezometer Tube Manometer

- The simplest manometer is a tube, open at the top, which is attached to the top of a vessel containing liquid at a pressure (higher than atmospheric) to be measured.
- An example can be seen in the figure below. This simple device is known as a *Piezometer tube*.
- As the tube is open to the atmosphere the pressure measured is relative to atmospheric so is **gauge pressure**.



A simple piezometer tube manometer

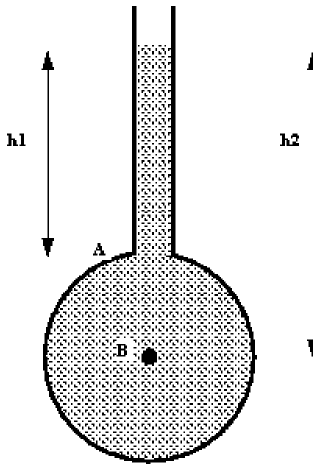
- Pressure at A = pressure due to column of liquid above A

$$p_A = \rho g h_1$$

- Pressure at B = pressure due to column of liquid above B

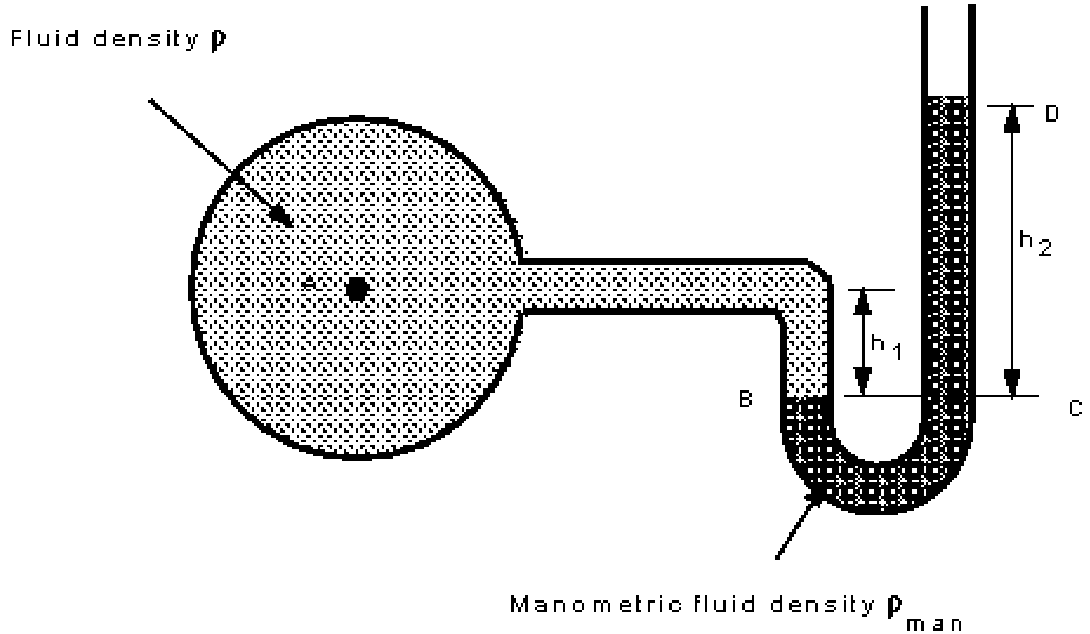
$$p_B = \rho g h_2$$

- This method can only be used for liquids (i.e. **not** for gases) and only when the liquid height is convenient to measure.



The “U”-Tube Manometer

- Using a “U”-Tube enables the pressure of both liquids and gases to be measured with the same instrument.
- The “U” is connected as in the figure and filled with a fluid called the *manometric fluid*.
- The fluid whose pressure is being measured should have a mass density less than that of the manometric fluid and the two fluids should not be able to mix readily - that is, they must be immiscible.



- A “U”-Tube manometer

Pressure in a continuous static fluid is the same at any horizontal level so,

pressure at B = pressure at C

$$p_B = p_C$$

For the **left hand** arm

pressure at B = pressure at A + pressure due to height h of fluid being measured

$$p_B = p_A + \rho g h_1$$

For the **right hand** arm

$$p_C = p_{\text{Amospheric}} + \rho g h_2$$

As we are measuring *gauge pressure* we can subtract $p_{\text{Atmospheric}}$ giving

$$p_B = p_C$$

$$p_A = \rho_{\text{man}}\, g h_2 - \rho g h_1$$

- If the fluid being measured is a gas, the density will probably be very low in comparison to the density of the manometric fluid i.e. $\rho_{man} >> \rho$.
- In this case the term $\rho g h_1$ can be neglected, and the gauge pressure given by

$$p_A = \rho_{man}\, gh_2$$

Single Column Manometer

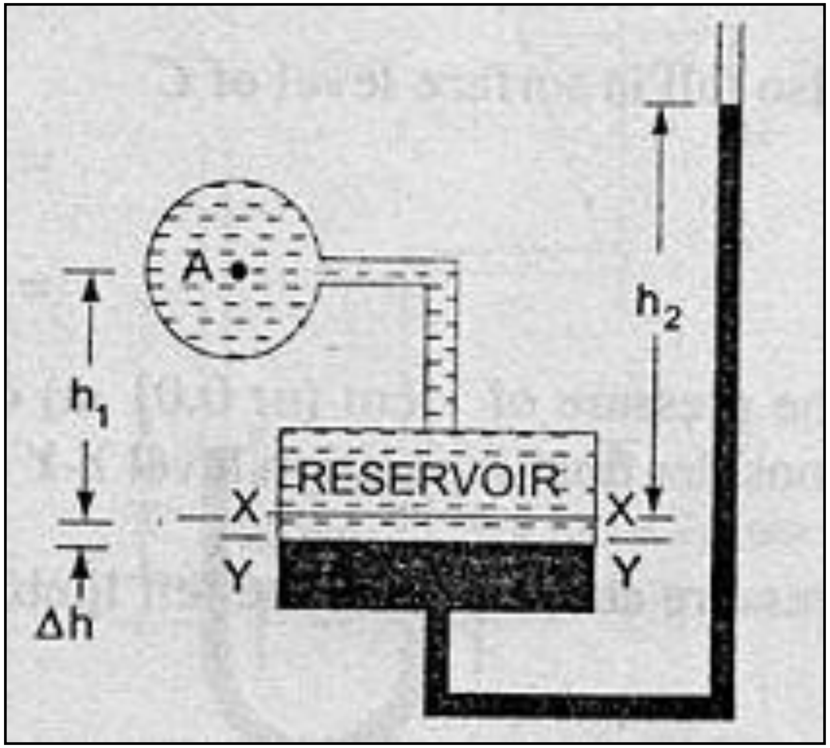
- Modified form of a U Tube manometer in which a reservoir, having a large cross-sectional area (about 100 times) as compared to tube. Due to large cross-sectional area of the reservoir, the change in the liquid level in this reservoir is negligible and the reading in the limb is taken as the pressure. The limb may be vertical or inclined
- When the fluid starts flowing in the pipe, the mercury level in the reservoir goes down by a very small amount which causes a large rise in the right limb.

Equating the pressure at this point about YY, on the right and left limb

$$P_A + \rho_1 g\,(\Delta h + h_1) = \rho_2 g(\Delta h + h_2)$$

$$P_A = \rho_2 g(\Delta h + h_2) - \rho_1 g\,(\Delta h + h_1)$$

$$= \Delta h\,(\rho_2 g - \rho_1 g\,) + h_2\,\rho_2 g - \rho_1 g h_1$$



As the volume of reservoir liquid remain same, the fall of liquid volume in the reservoir is equals to rise of liquid volume in the limb

$A\Delta h = a\,h_2$ So, $\Delta h = (a/A)\,h_2$

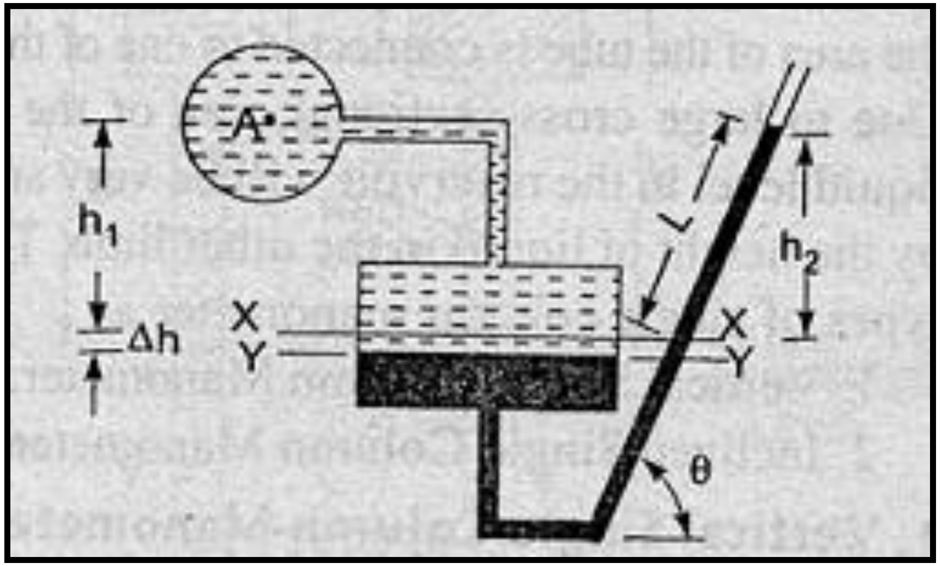
as A is very large and a is very small, a/A is very small and hence may be neglected

That means Δh term is neglected

So, $P_A = h_2\,\rho_2 g - h_1 \rho_1 g$

Inclined Manometers

Inclined manometers are more sensitive to pressure variations, rise of liquid in right limb will be more



If, L= length of liquid on right limb above X-X

ɵ= Inclination of right limb to horizontal

h_2= Vertical rise of liquid in right limb above X-X= L sinɵ

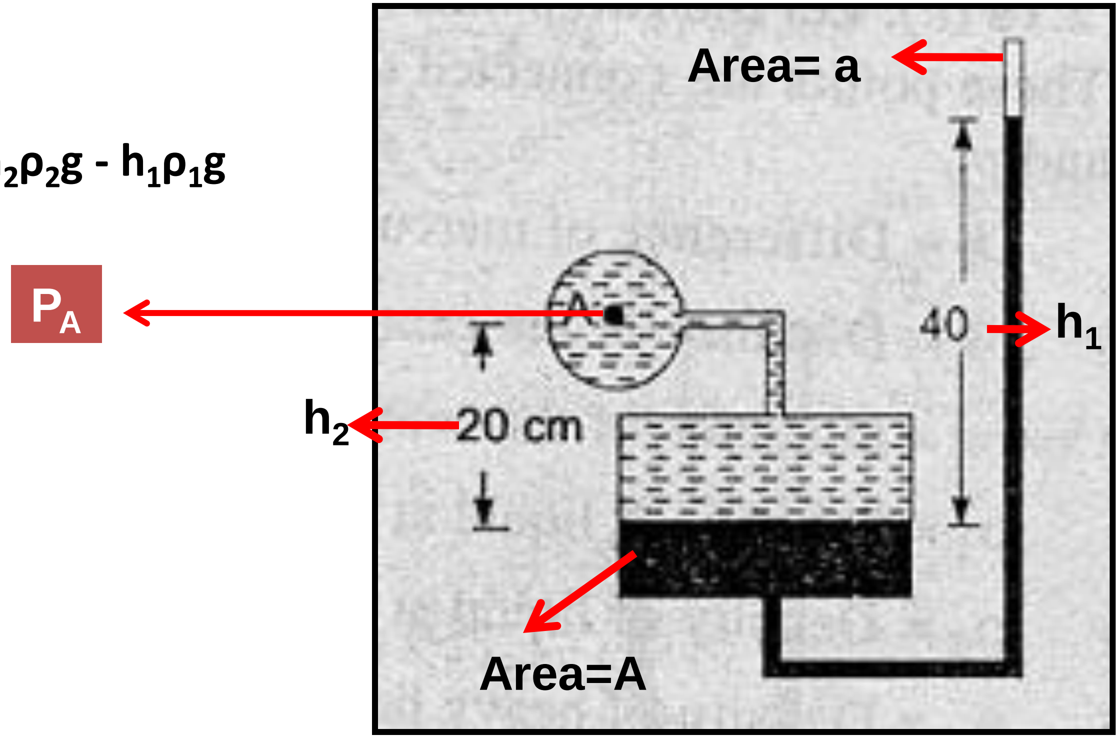
$P_A = h_2 \rho_2 g - h_1 \rho_1 g$ *become*

$P_A = L \sin\theta \rho_2 g - h_1 \rho_1 g$

A single column manometer is connected to a pipe containing a liquid of sp gravity 0.9, center of the pipe is 20cm from the surface of mercury in the reservoir, which has 100 times more area than that of tube. The mercury on the right limb is 40cm above the level of mercury in the reservoir . Find the pressure in the pipe.

$(A/a)=100$

We have, $P_A=(a/A)h_2(\rho_2 g - \rho_1 g) + h_2\rho_2 g - h_1\rho_1 g$



$P_A = (1/100)0.4[13.6X1000X9.81-900X9.81] + 0.4X13.6X1000X9.81 – 0.2X900X9.81$

$= 5.21 N/cm^2$

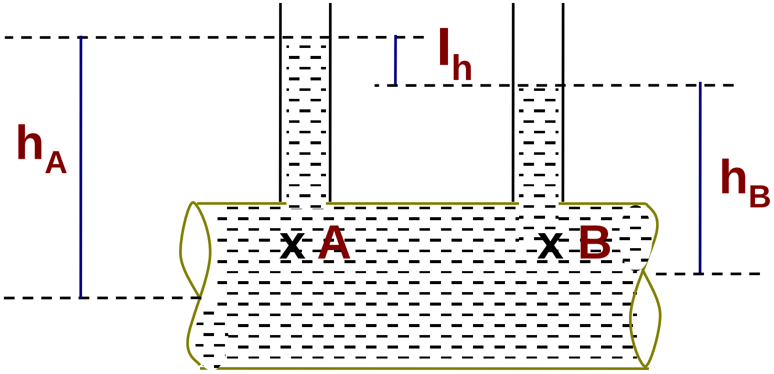
DIFFERENTIAL MANOMETERS

Differential manometers are used to measure pressure difference between any two points. Common varieties of differential manometers are:

- **Two piezometers.**
- **Inverted U-tube manometer.**
- **U-tube differential manometers.**
- **Micro manometers**

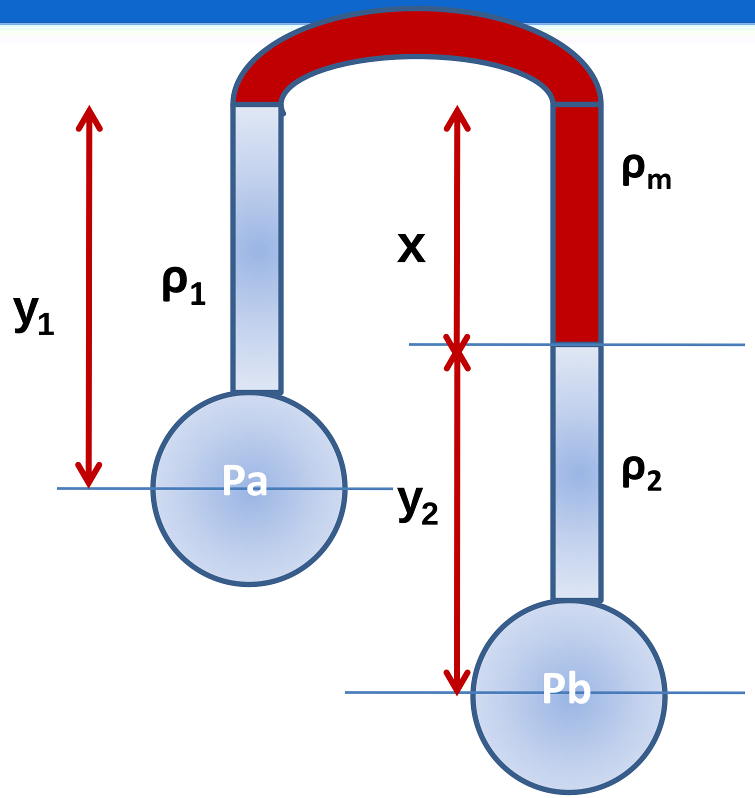
- **Two Piezometers**

The arrangement consists of two pizometers at the two points between which the pressure difference is required. The liquid will rise in both the piezometers. The difference in elevation of liquid levels can be recorded and the pressure difference can be calculated.



Inverted U-tube Manometers

Inverted U-tube manometer is used to measure small difference in pressure between any two points. It consists of an inverted U-tube connecting the two points between which the pressure difference is required. In between there will be a lighter sensitive manometric liquid. Pressure difference between the two points can be calculated by writing the gauge equations for the system.

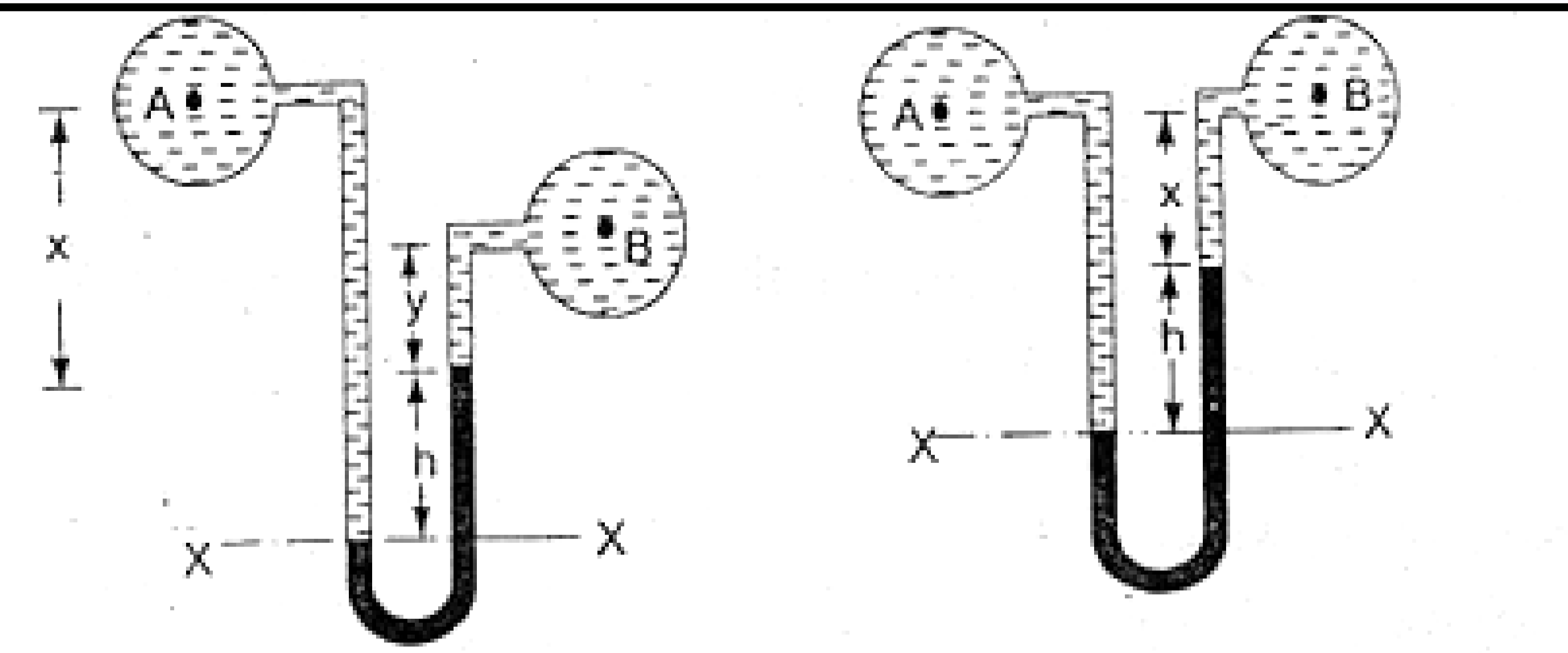


Let 'P_A' and 'P_B' be the pressure at 'A' and 'B'

$$P_a - [(y_1\rho_1) + (x\,\rho_m) + (y_2\rho_2)]g = P_b$$

$$P_a - P_b = [\,\rho_1\, y_1 - \rho_m\, x - \rho_2\, y_2]g$$

Differential U Tube Manometer



(a)Two pipes at different levels

(b) A and B are at the same level

Balancing the pressure on left and right limbs

$$\rho_1 g(h+x) + P_A = \rho_m g h + \rho_2 g y + P_B$$

$$(P_A - P_B) = \rho_m g h + \rho_2 g y - \rho_1 g(h+x)$$

$$(P_A - P_B) = hg(\rho_m - \rho_1) + \rho_2 g y - \rho_1 g x$$

$$\rho_m g h + \rho_1 g x + P_B = \rho_1 g(h+x) + P_A$$

$$(P_A - P_B) = \rho_m g h + \rho_1 g x - \rho_1 g(h+x)$$

$$(P_A - P_B) = hg(\rho_m - \rho_1)$$

A differential manometer is connected at the two points A and B of two pipes. Center of left pipe, A, is 3m above the center of right pipe B. The mercury level in the left limb is 2m below the center of right pipe. The height of mercury in the left limb is h m above the mercury surface in the right limb. s_A=1.5, s_B=0.9. P_A=1 bar and P_B=1.80bar. Find the difference in Hg level (h).

Balancing the pressures on left and right limbs

Pressure on left limb=

$$13.6X1000X9.81Xh + 1500X9.81X(2+3) + P_A$$

Pressure on right limb=

$$900X9.81X(h+2) + P_B$$

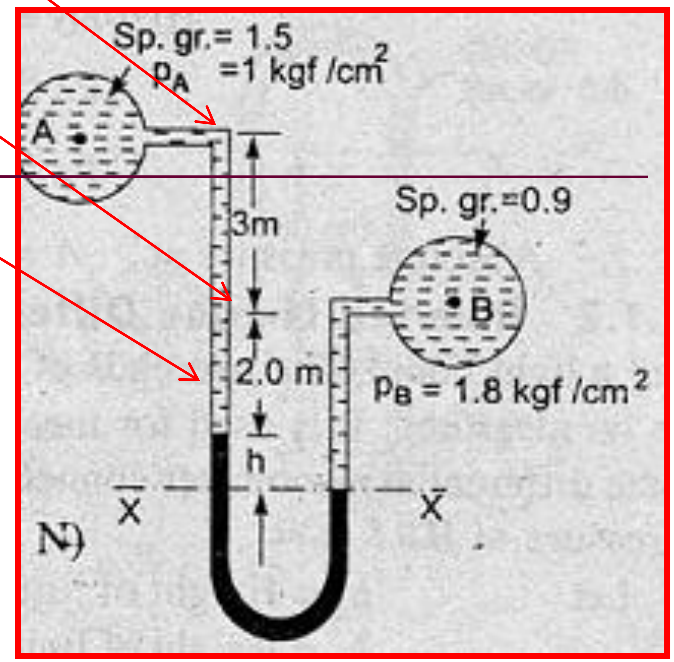
Pressure on left limb = Pressure on right limb

$$13.6X1000Xh + 7500 + 1X10^4 = 900X(h+2) + 1.8X10^4$$

$$13.6h + 7.5 + 1X10 = 0.9X(h+2) + 18$$

$$12.7h = 2.3$$

$$h = \frac{2.3}{12.7} = 18.1cm$$

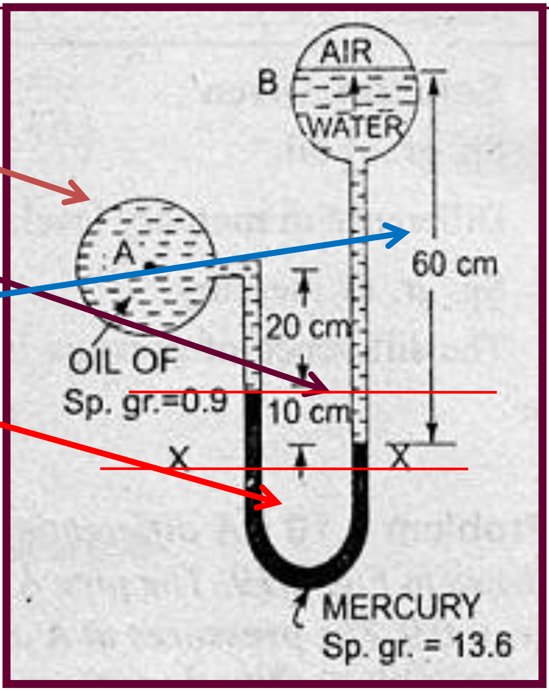


A differential manometer is connected at the two points A and B. P_B = 9.81 $N.cm^2$ (abs), find the absolute pressure at A. s_A=0.9 and s_B=1, s_M=13.6. Center points of pipe B is 30 cm above A and above this air is confined. Difference in mercury level is 10cm in left limb. Fluid in the left limb is 20cm below the center of left pipe.

Pressure above XX at LEFT Limb

= 13.6X1000X9.81X0.1 +900X9.81X0.2 +P_A

= 13341.6+1765.8+P_A



Pressure above XX at RIGHT Limb

=1000X9.81X0.6+P_B

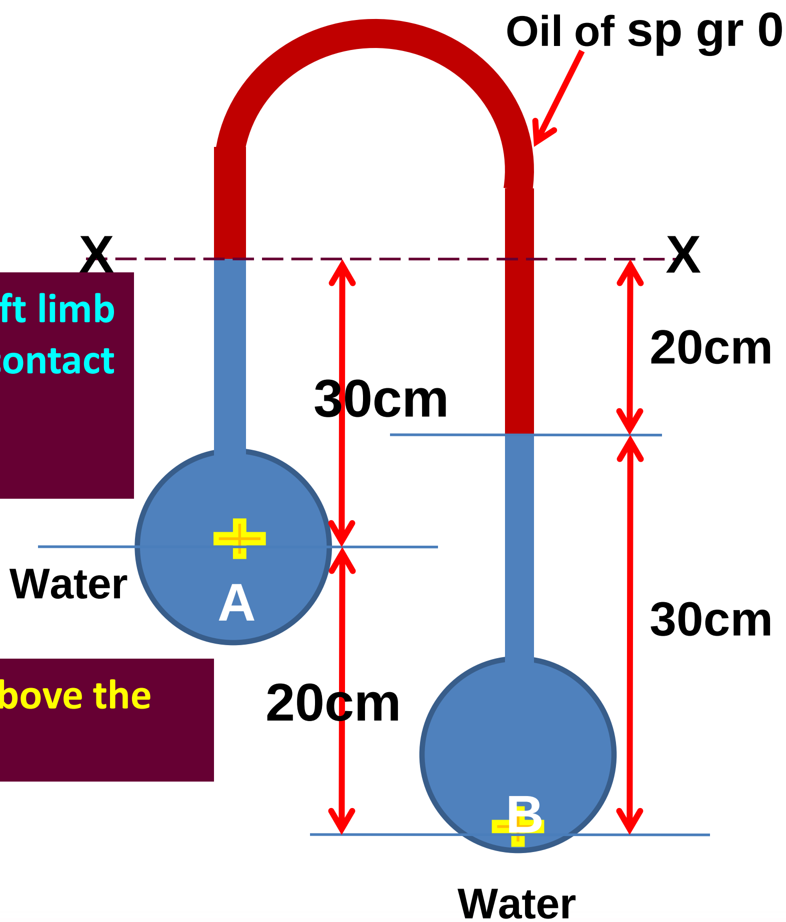
= 5886+98100=103986

EQUATING

103986=13341.6+1765.8+P_A

P_A=88876.8N/m2 = 8.88 N/cm2=absolute pressure at A

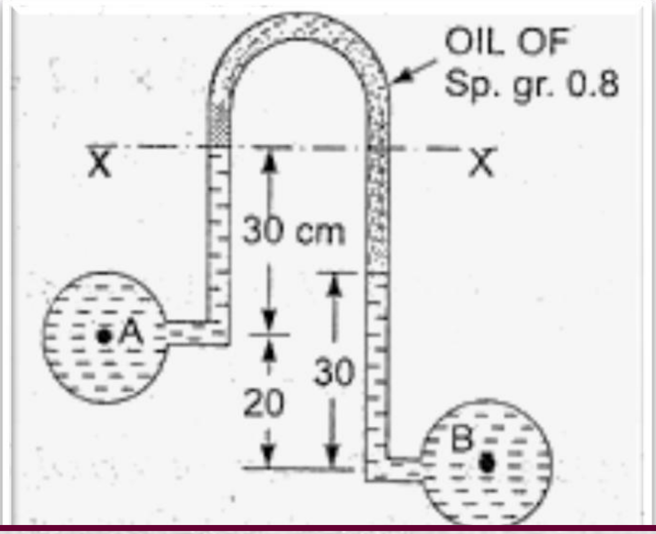
An inverted differential manometer is connected to two pipes A and B which convey water. The fluid in manometer is oil of specific gravity 0.8. Left side pipe center is 20cm above the right side pipe center. Water occupies 30cm in the left limb above its pipe center and in contact with manometer liquid. Difference in manometer liquid is 20cm. Find the differential pressure PA and PB



Water occupies 30cm in the left limb above its pipe center and in contact with manometer liquid

Difference in manometer liquid is 20cm

Left side pipe center is 20cm above the right side pipe center.



Pressure in the left limb below X-X

$$= p_A - 1000 \times 9.81 \times 0.3$$
$$= p_A - 2943$$

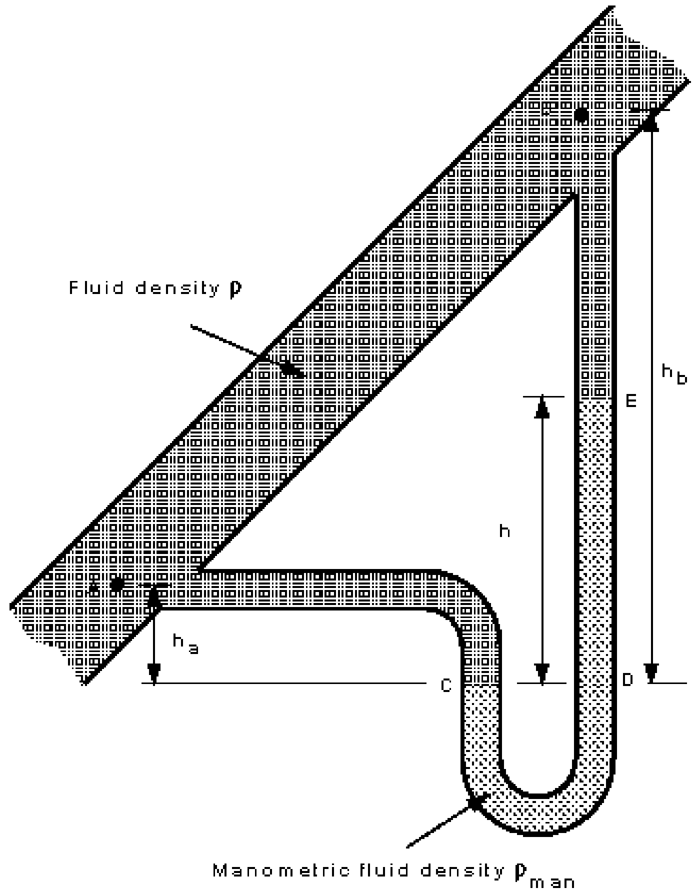
Pressure in the right limb below X-X

$$= p_B - 1000 \times 9.81 \times 0.3 - 800 \times 9.81 \times 0.2$$
$$= p_B - 2943 - 1569.6 = p_B - 4512.6$$

Equating the two pressure $p_A - 2943 = p_B - 4512.6$

$\therefore$ $p_B - p_A = 4512.6 - 2943 =$ **1569.6 N/m²**.

Measurement Of Pressure Difference Using a “U”-Tube manometer



- If the “U”-tube manometer is connected to a pressurized vessel at two points the *pressure difference* between these two points can be measured.
- If the manometer is arranged as in the figure above, then

Pressure at C = Pressure at D

$$p_C = p_D$$

$$p_C = p_A + \rho g h_a$$

$$p_D = p_B + \rho g(h_b - h) + \rho_{man} g h$$

$$p_A + \rho g h_a = p_B + \rho g(h_b - h) + \rho_{man} g h$$

Giving the pressure difference

$$p_A - p_B = \rho g(h_b - h_a) + (\rho_{man} - \rho) g h$$

- Again, if the fluid whose pressure difference is being measured is a gas and $\rho_{man} >> \rho$, then the terms involving ρ can be neglected, so

$$p_A - p_B = \rho_{man} g h$$

Total Pressure and Center Of Pressure

Total pressure:-It is defined as the force exerted by static fluid when the fluid comes in contact with the surface.

- This force always acts at right angle or normal to the surface.

Center of pressure:- it is defined as the point of application of the total pressure on the surface.

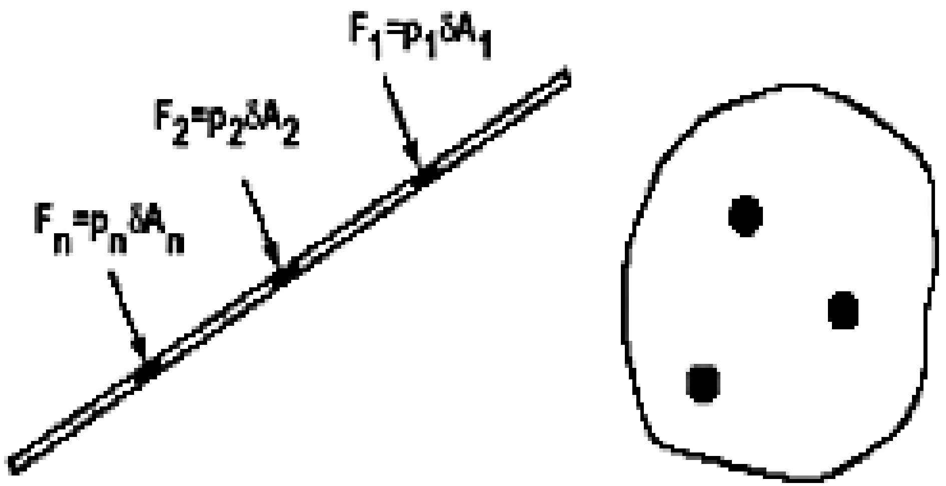
- The total pressure exerted by a liquid on the immersed the surfaces may be :

1. Horizontal plane surface
2. Vertical plane surface
3. Inclined plane surface
4. Curved surface

Pressure force on flat areas

- Pressure is force per unit area.
- Pressure p acting on a small area δA exerted force will be

$$F = p\delta A$$



Horizontally Immersed Surface

- The total pressure on the surface,

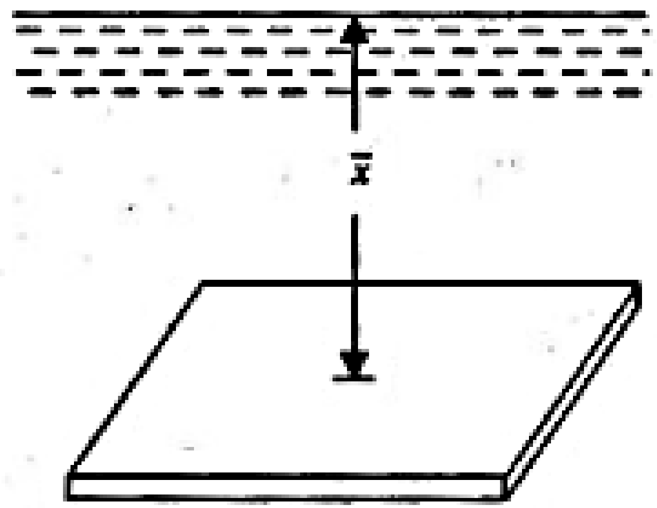
P = weight of the liquid above the immersed surface

= specific weight of liquid x Volume of fluid

= specific weight of liquid x area of surface x depth of liquid

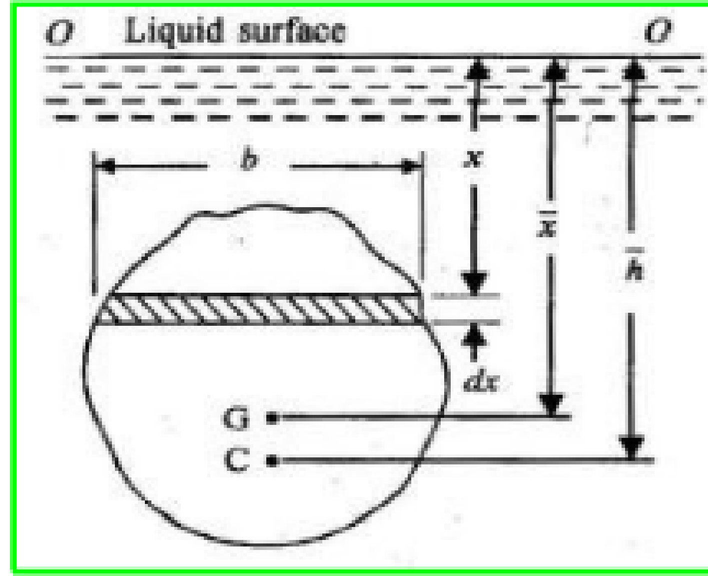
$= wA\bar{x}$

$= \rho gh*A$



Vertically immersed surface

Let A = total area of the surface,
G = centre of the area of surface,
$\bar{x}$ = depth of centre of area,
OO = free surface of liquid, and
$\bar{h}$ = distance of centre of pressure from free surface of liquid.



Pressure at any point along the strip $p = \rho g x$

Total pressure on the strip $= p \cdot dA = p.b.dx$

The Total pressure on the body $R = \int p.b.dx = \int \rho g x.b.dx$

$$R = \rho g \int bx.dx = \rho g (A\bar{x}) = \rho g \bar{x} A$$

$\int bdx \cdot x$ = moment of the surface area about the liquid level = $A\bar{x}$

Center of Pressure

Let $\bar{h}$ = depth of centre of pressure below free liquid surface,
I_0 = moment of inertia of the surface about OO.

Pressure at any point along the strip $\underline{p = \rho g x}$

Total pressure on the strip $= \underline{p.b.dx}$

The moment of the pressure about free surface o - o,

$$M = p.b.dx.(x)$$

M for all body $= \int p.b.dx.(x)$

$$= \int \rho g x^2 b\, dx = \rho g \int x^2 b.dx = \rho g I_o$$

But $\int x^2 b \cdot dx = I_0$ = moment of inertia of the surface about the free surface 00 (or second moment of area)

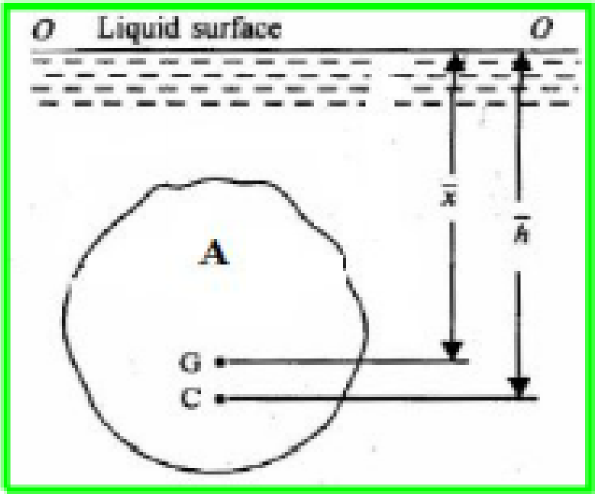
Center of Pressure

$$P \times \bar{h} = \rho g I_o$$

$$\bar{h} = \frac{\rho g I_o}{P} = \frac{\rho g I_o}{\rho g \bar{x} A} = \frac{I_o}{\bar{x} A} = \frac{I_G}{\bar{x} A} + \bar{x}$$

$$P = \rho g \bar{x} A$$

$$\bar{h} = \frac{I_G}{\bar{x} A} + \bar{x}$$



Inclined Immersed Surface

Total pressure (P):

The intensity of pressure on the strip

$= wl \sin\theta$

Area of the strip $= b \cdot dx$

Pressure on the strip

$=$ intensity of pressure $\times$ *area*

$= wl \sin\theta \cdot b \cdot dx$

Now total pressure on the surface,

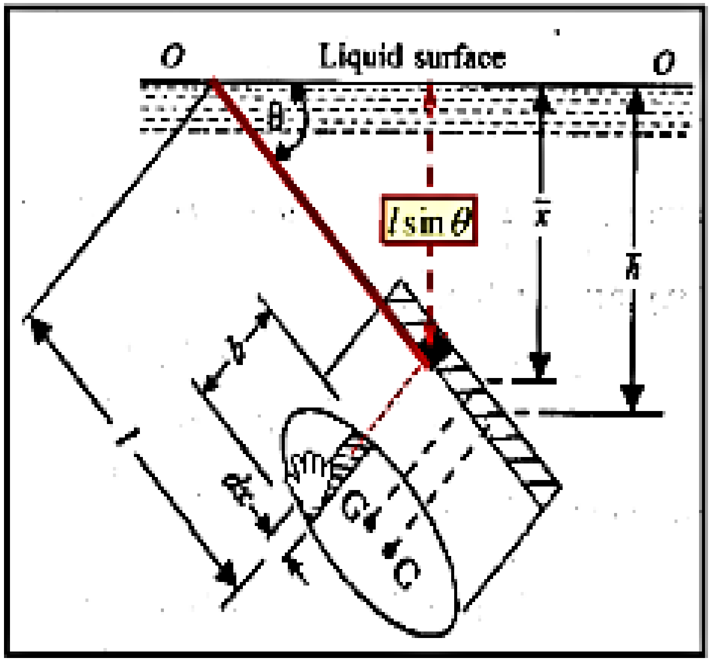
$$P = \int wl \sin\theta \cdot b \cdot dx = w \sin\theta \int l \cdot b \cdot dx$$

But $\int l \cdot b \cdot dx =$ moment of surface area about 00

$$= \frac{A\bar{x}}{\sin\theta},$$

$$\therefore \quad P = w \sin\theta \,.\, \frac{A\bar{x}}{\sin\theta} = wA\bar{x}$$

$$P = \rho g \bar{x} A$$



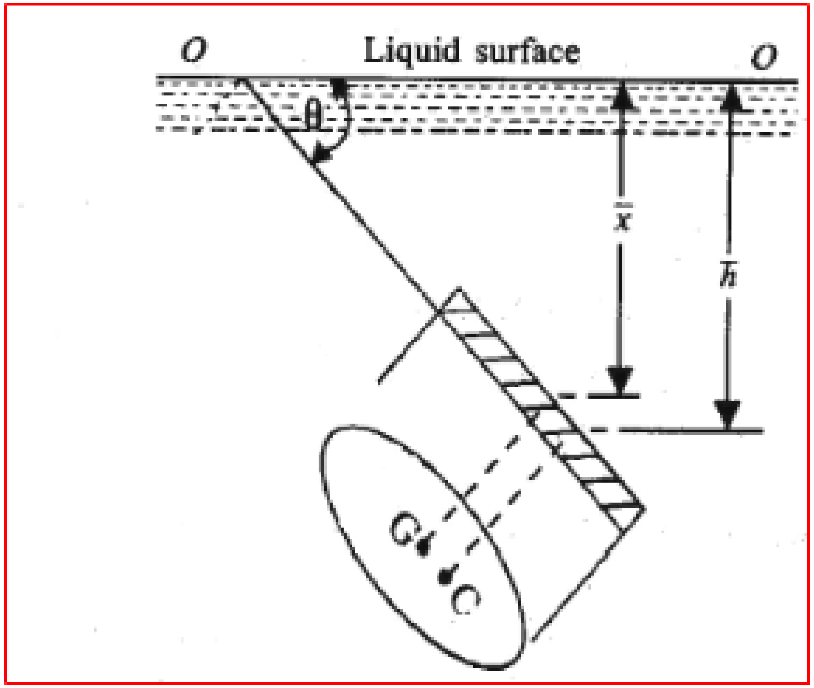
Centre of pressure ($\bar{h}$):

Using the same procedures as in Vertical surface

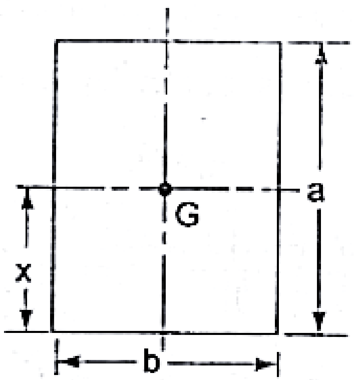
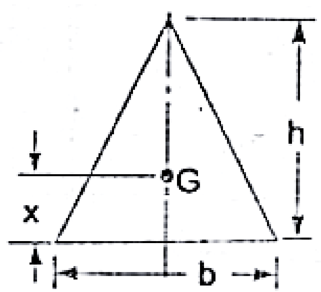
$$\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$$

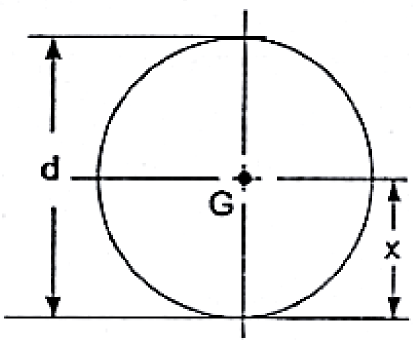
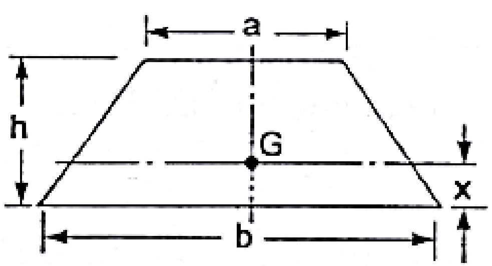
$$P = \rho g \bar{x} A$$

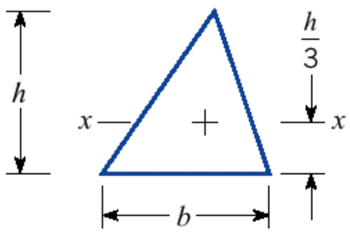
$$\bar{h} = \frac{I_G \sin^2 \theta}{\bar{x} A} + \bar{x}$$



The moments of inertia and other geometric properties of some plane surfaces

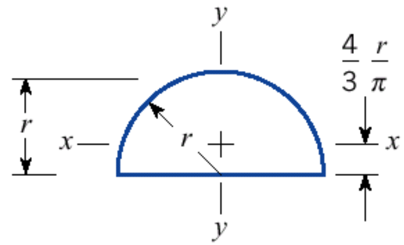
Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base (I_G)*	*Moment of inertia about base (I_0)*
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base (I_G)*	*Moment of inertia about base (I_0)*
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4. Trapezium 	$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{(a+b)}{2} \times h$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^3$	—



$$A = \frac{bh}{2}$$

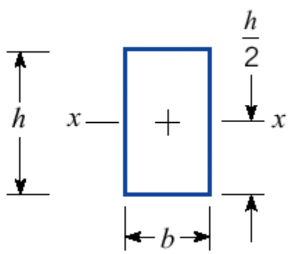
$$\bar{I}_{xx} = \frac{bh^3}{36}$$



$$A = \frac{\pi r^2}{2}$$

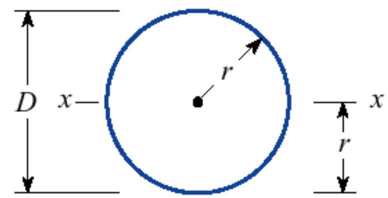
$$\bar{I}_{xx} = 0.110 r^4$$

$$\bar{I}_{xx} = \frac{\pi r^4}{8}$$



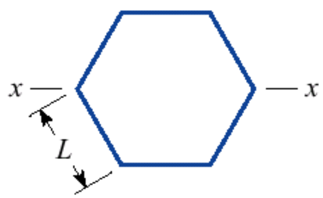
$$A = bh$$

$$\bar{I}_{xx} = \frac{bh^3}{12}$$



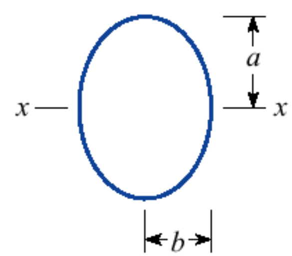
$$A = \pi r^2$$

$$\bar{I}_{xx} = \frac{\pi r^4}{4}$$



$$A = 2.5981 L^2$$

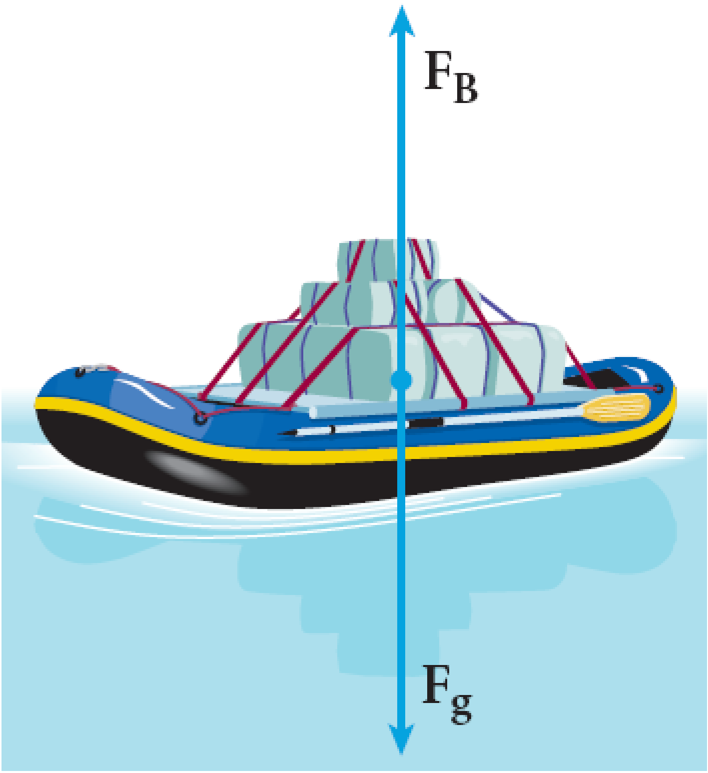
$$\bar{I}_x = 0.5127 L^4$$



$$A = \pi ab$$

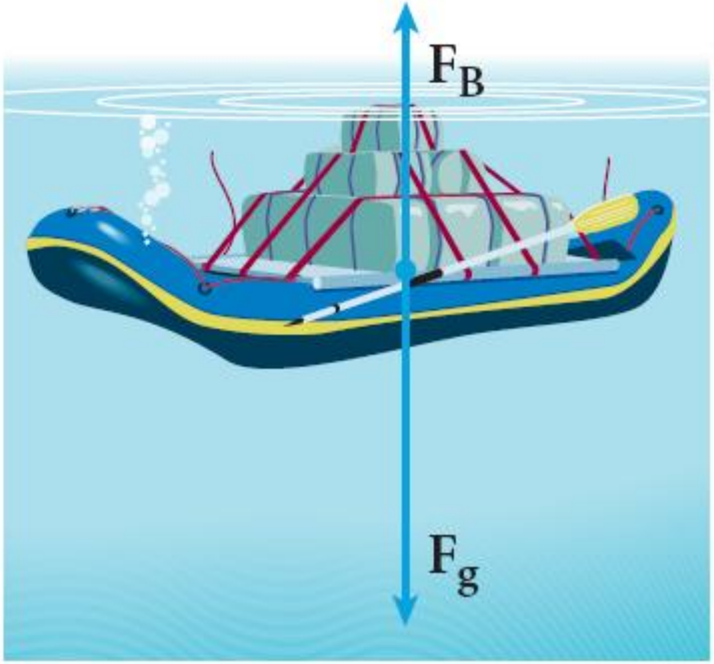
$$\bar{I}_{xx} = \frac{\pi a^4 b}{4}$$

Buoyant Force



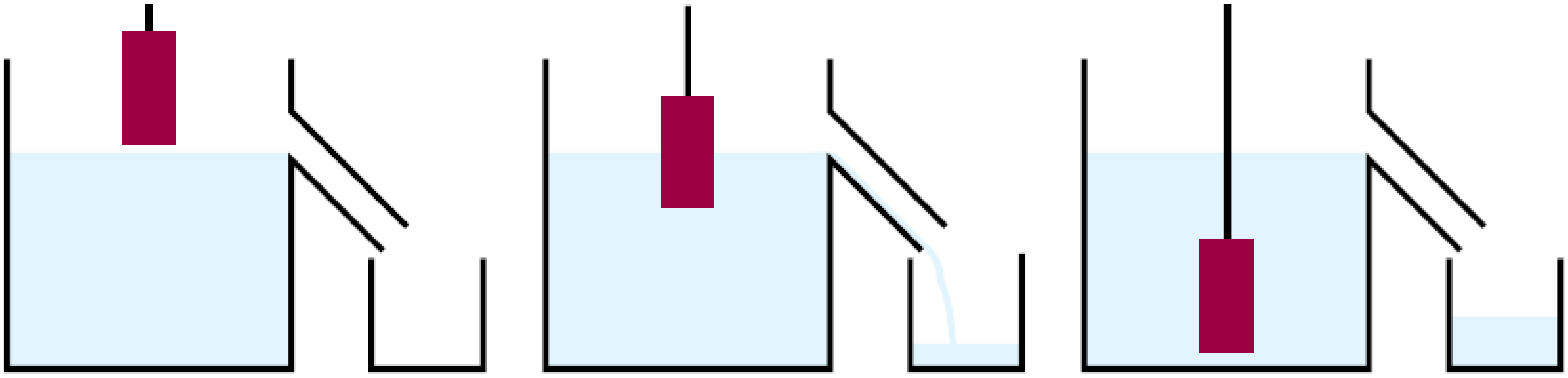
- The raft and cargo are floating because their weight and buoyant force are balanced.

- Now imagine a small hole is put in the raft.
- The raft and cargo sink because their density is greater than the density of the water.
- As the volume of the raft decreases, the volume of the water displaced by the raft and cargo also decreases, as does the magnitude of the buoyant force.



Buoyant Force and Archimedes' Principle

- The Brick, when added will cause the water to be displaced and fill the smaller container.
- What will the volume be inside the smaller container?
- The same volume as the brick!

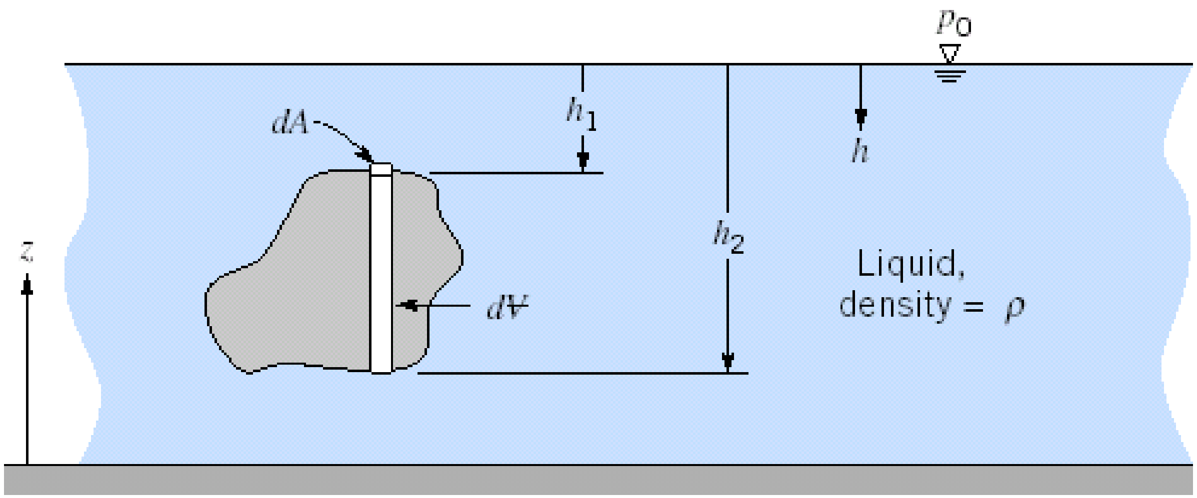


- Archimedes' principle describes the magnitude of a buoyant force.
- Archimedes' principle: *Any object completely or partially submerged in a fluid experiences an upward buoyant force equal in magnitude to the weight of the fluid displaced by the object.*

$$F_B = F_g \text{ (displaced fluid)} = m_f g$$

magnitude of buoyant force = weight of fluid displaced

Buoyancy



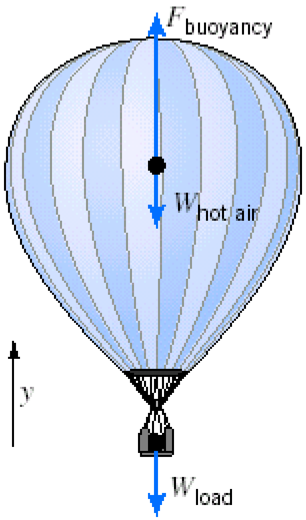
Immersed Body in Static Fluid

$$dF_z = (p_0 + \rho g h_2)\, dA - (p_0 + \rho g h_1)\, dA$$

$$dF_z = \rho g (h_2 - h_1)\, dA$$

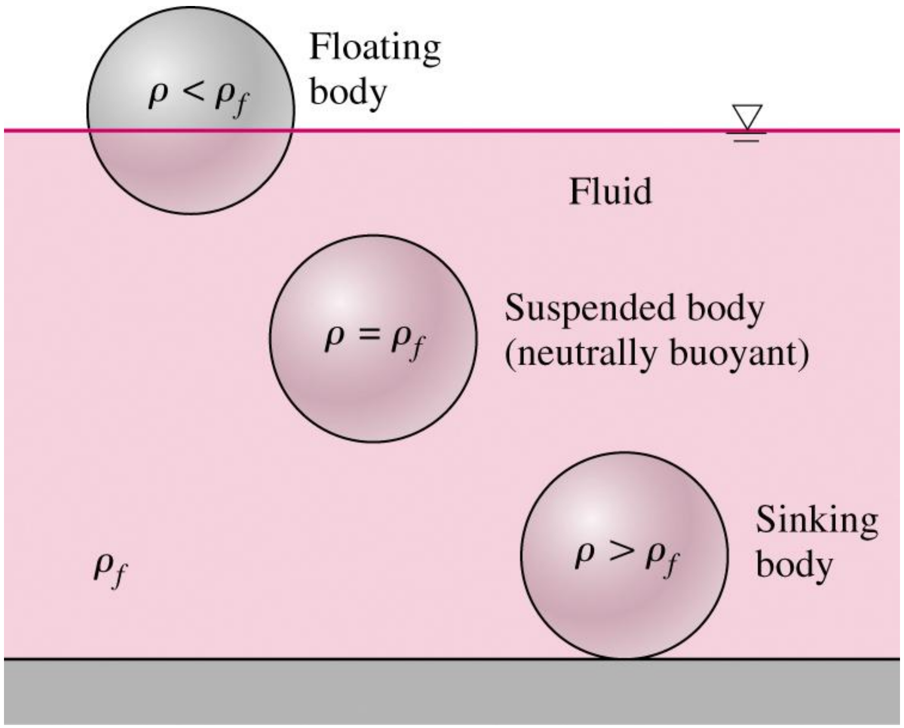
$$F_{\text{buoyancy}} = \rho g \forall$$

For example, for a hot air balloon



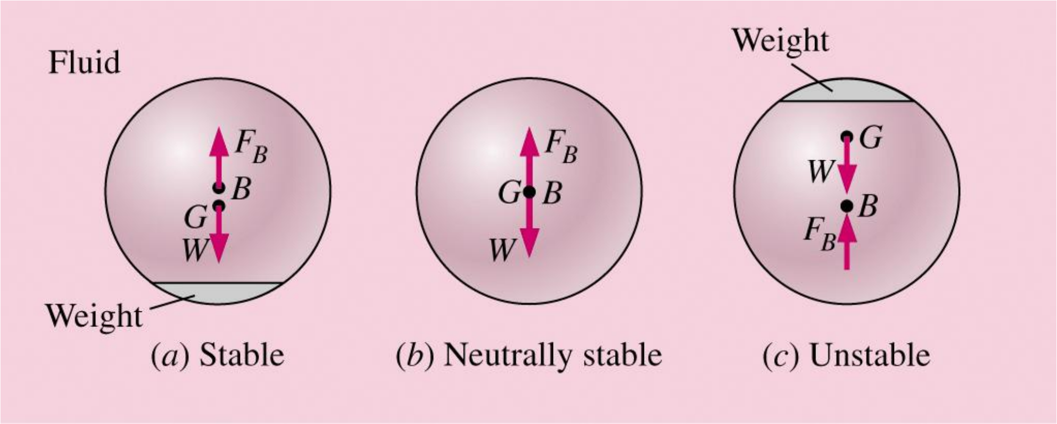
Buoyancy and Stability

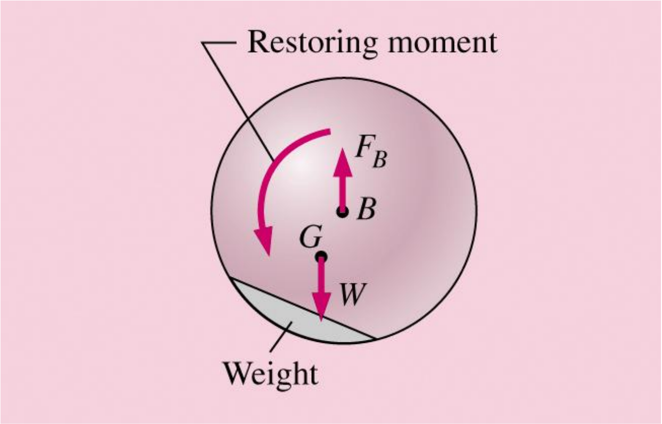
- Buoyancy is due to the fluid displaced by a body. $F_B=\rho_f gV$.
- Archimedes principle : The buoyant force = Weight of the fluid displaced by the body, and it acts through the centroid of the displaced volume.



- Buoyancy force F_B is equal only to the displaced volume $\rho_f g V_{displaced}$.
- Three scenarios possible
  1. $\rho_{body}<\rho_{fluid}$: Floating body
  2. $\rho_{body}=\rho_{fluid}$: Neutrally buoyant
  3. $\rho_{body}>\rho_{fluid}$: Sinking body

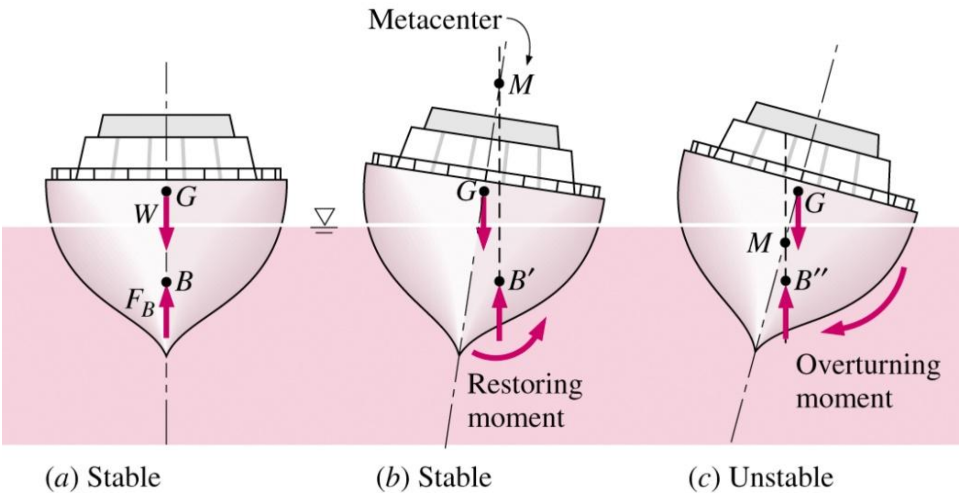
Stability of Immersed Bodies





- Rotational stability of immersed bodies depends upon relative location of *center of gravity G* and *center of buoyancy B*.
  - *G* below *B*: stable
  - *G* above *B*: unstable
  - *G* coincides with *B*: neutrally stable.

Stability of Floating Bodies

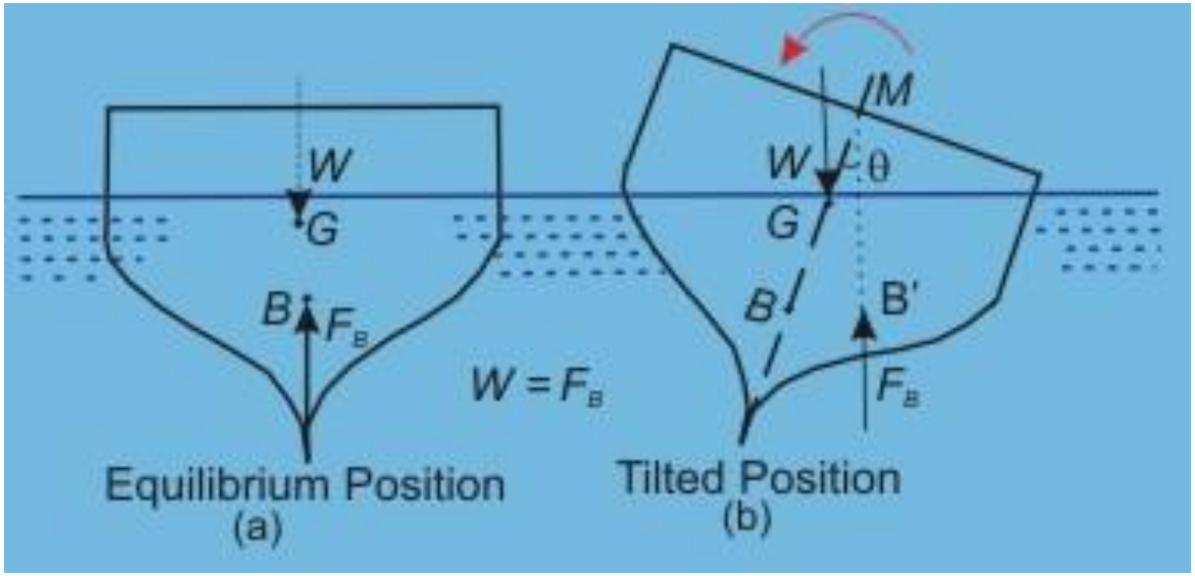


- If body is bottom heavy (*G* lower than *B*), it is always stable.
- Floating bodies can be stable when *G* is higher than *B* due to shift in location of center buoyancy and creation of restoring moment.
- Measure of stability is the metacentric height G*M*. If *GM*>1, ship is stable.

Stability of Floating Bodies in Fluid

- When the body undergoes an angular displacement about a horizontal axis, the shape of the immersed volume changes and so the centre of buoyancy moves relative to the body.
- As a result of above observation stable equilibrium can be achieved, under certain condition, even when G is above B. Figure illustrates a floating body -a boat, for example, in its equilibrium position.
- Let the new **line of action of the buoyant force** (which **is always vertical**) through B' intersects the axis BG (the old vertical line containing the centre of gravity G and the old centre of buoyancy B) at M. For small values of q the **point M is** practically constant in position and is **known as metacentre**

Floating body in Stable equilibrium



Important points

- The force of buoyancy F_B is equal to the weight of the body W
- Centre of gravity G is above the centre of buoyancy in the same vertical line.
  Figure b shows the situation after the body has undergone a small angular displacement q with respect to the vertical axis.
- The centre of gravity G remains unchanged relative to the body (This is not always true for ships where some of the cargo may shift during an angular displacement).
- During the movement, the volume immersed on the right hand side increases while that on the left hand side decreases. Therefore the centre of buoyancy moves towards the right to its new position B'.

- Hence the **condition of stable equilibrium for a floating body** can be expressed **in terms of metacentric height** as follows:

GM > 0 (M is above G)	**Stable equilibrium**
GM = 0 (M coinciding with G)	**Neutral equilibrium**
GM < 0 (M is below G)	**Unstable equilibrium**

- The angular displacement of a boat or ship about its longitudinal axis is known as 'rolling' while that about its transverse axis is known as "pitching".

Numericals

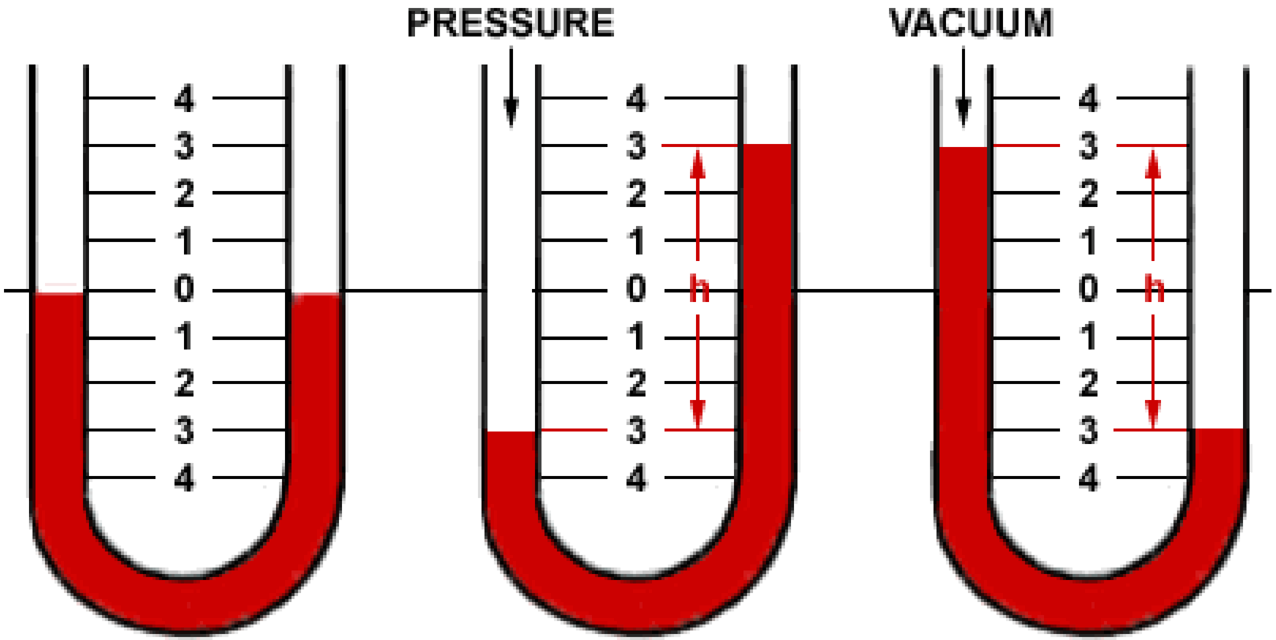
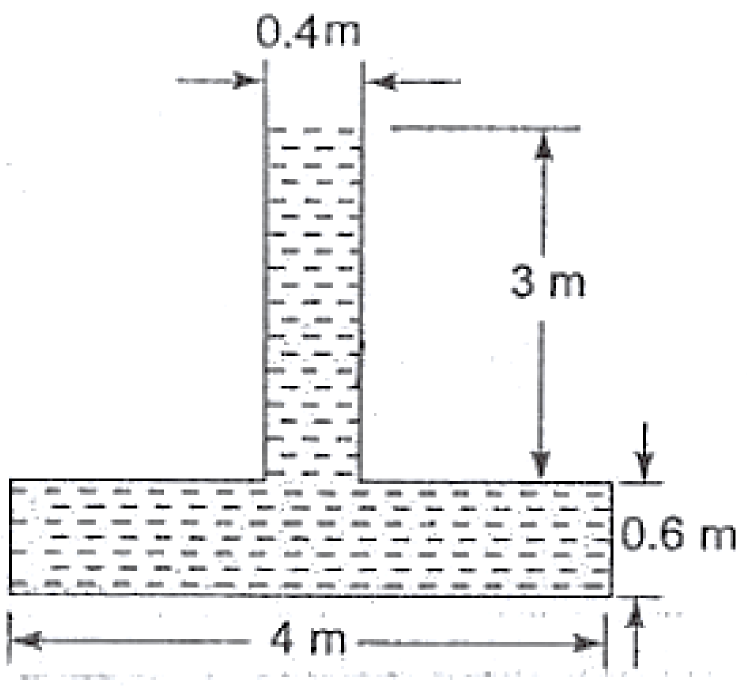


Figure shows a tank full of water. Find;

(i) *Total pressure on the bottom of tank.*

(ii) *Weight of water in the tank.*

(iii) *Hydrostatic paradox between the results of (i) and (ii). Width of tank is 2 m.*



Depth of water on bottom of tank

$$h_1 = 3 + 0.6 = 3.6 \text{ m}$$

Width of tank $= 2$ m

Length of tank at bottom $= 4$ m

$\therefore$ Area at the bottom, $A = 4 \times 2 = 8 \text{ m}^2$

(*i*) Total pressure F, on the bottom is

$$F = \rho g A \bar{h} = 1000 \times 9.81 \times 8 \times 3.6$$

$$= \mathbf{282528 \text{ N. Ans.}}$$

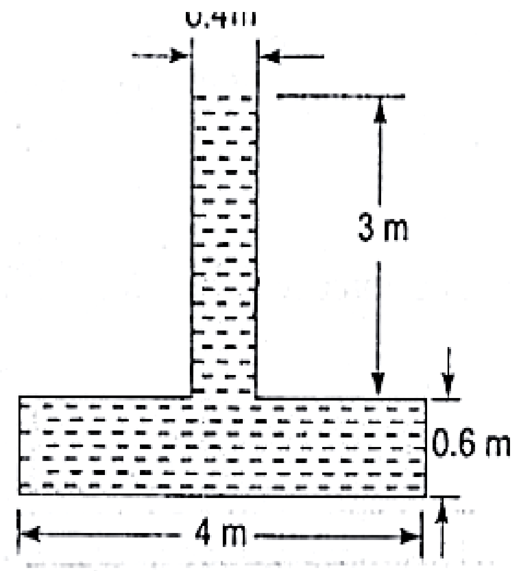


Fig. 3.17

(*ii*) Weight of water in tank $= \rho g \times$ Volume of tank

$$= 1000 \times 9.81 \times [3 \times 0.4 \times 2 + 4 \times .6 \times 2]$$

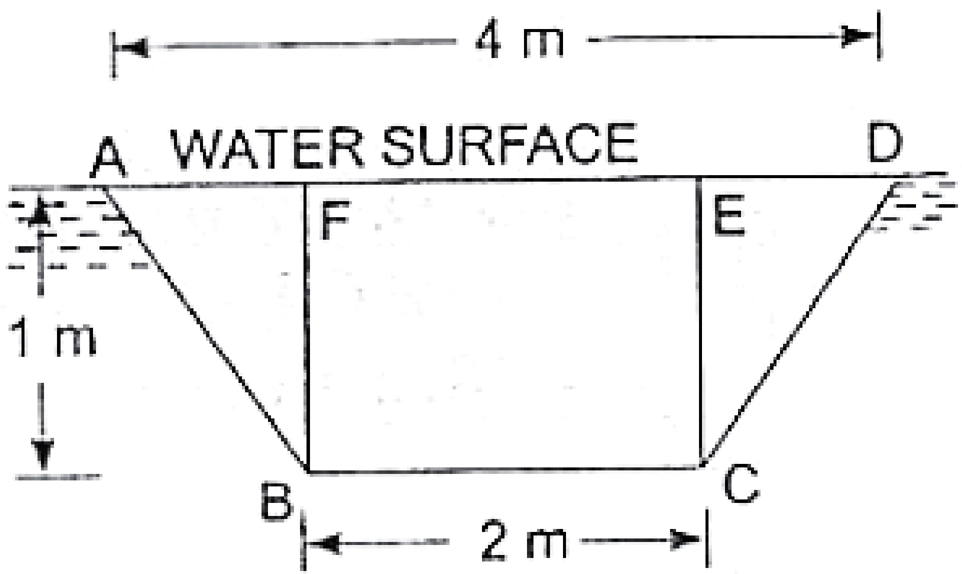
$$= 1000 \times 9.81 \, [2.4 + 4.8] = \mathbf{70632 \text{ N. Ans.}}$$

iii. From the results of i & ii it is observed that the total weight of water in the tank is much less than the total pressure at the bottom of the tank. This is known as Hydrostatic Paradox.

A trapezoidal channel 2m wide at the bottom and 1 m deep has slopes 1: 1. Determine:

i. The total Pressure

ii. The Centre of pressure on the vertical gate closing the channel when it is full of water.



$$x = \frac{(2a+b)}{(a+b)} \times \frac{h}{3} = \frac{(2\times2+4)}{(2+4)} \times \frac{1}{3} \qquad (\because \quad a = 2, b = 4 \text{ and } h = 1)$$

$$= 0.444 \text{ m}$$

$\therefore$ $$\bar{h} = x = 0.444 \text{ m}$$

$\therefore$ Total pressure, $$F = \rho g A\bar{h} = 1000 \times 9.81 \times 3.0 \times .444 \qquad (\because \quad A = 3.0)$$

$$= \mathbf{13079\ N.\ Ans.}$$

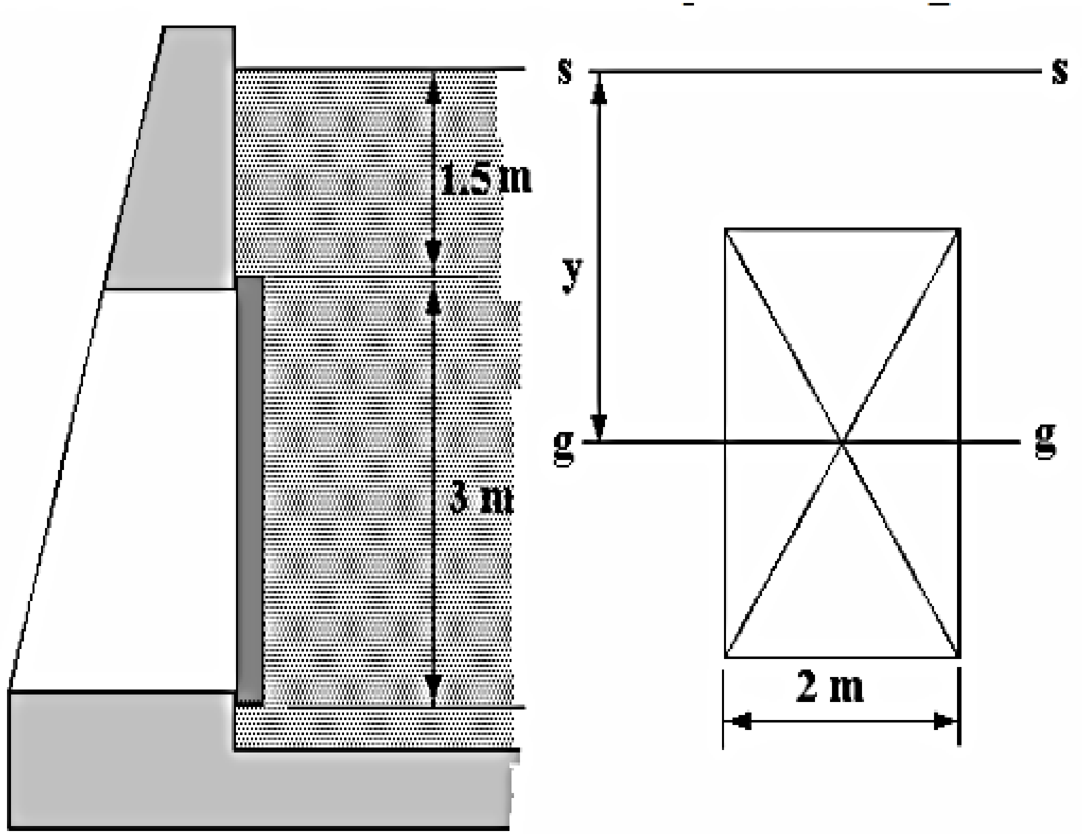
Centre of pressure, $$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

Where I_G is given by formula

$$I_G = \frac{(a^2+4ab+b^2)}{36(a+b)} \times h^3 = \frac{(2^2+4\times2\times4+4^2)}{36(2+4)} \times 1^3 = \frac{52}{36\times6} = 0.2407 \text{ m}^4$$

$\therefore$ $$h^* = \frac{0.2407}{3.0\times.444} + .444 = \mathbf{0.625\ m.\ Ans.}$$

A concrete wall retains water and has a hatch blocking off an outflow tunnel as shown. Find the total force on the hatch and position of the Centre of pressure. Calculate the total moment about the bottom edge of the hatch.



Total force = $P = \rho\, g\, A\, y$
For the rectangle shown $y = (1.5 + 3/2) = 3$ m. $A = 2 \times 3 = 6\ m^2$.

$P = 1000 \times 9.81 \times 6 \times 3 = 176580$ N or 176.58 kN

h = 2nd mom. of Area/ 1st mom. of Area

1^{st} moment of Area $= Ay = 6 \times 3 = 18\ m^3$.

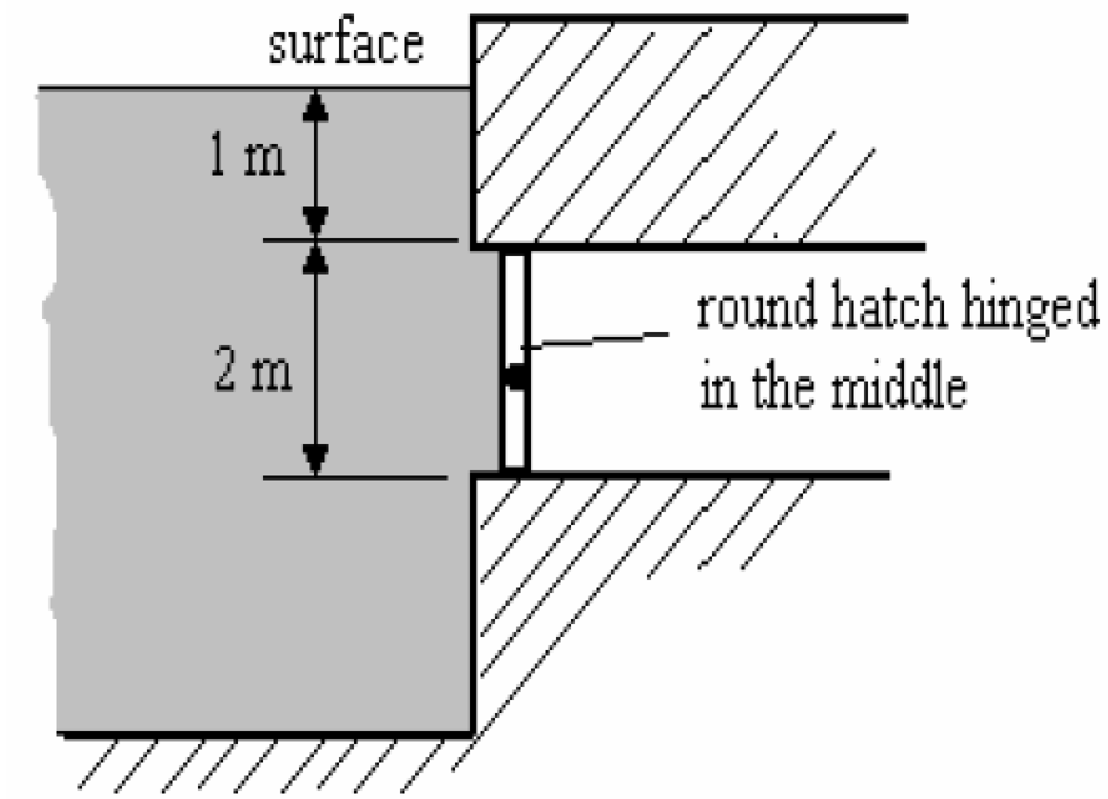
2^{nd} mom of area $= \mathbf{I}_{SS} = (BD^3/12) + Ay^2 = (2 \times 3^3/12) + (6 \times 3^2)$

$\mathbf{I}_{SS} = 4.5 + 54 = 58.5\ m^4$.

$h = 58.5/18 = 3.25$ m

The distance from the bottom edge is $x = 4.5 - 3.25 = 1.25$ m
Moment about the bottom edge is $= Rx = 176.58 \times 1.25 =$ ***220.725 kNm.***

Find the force required at the top of the circular hatch shown in order to keep it closed against the water pressure outside. The density of the water is 1030 kg/m^3.



y = 2 m from surface to middle of hatch.

Total Force = $P = \rho g A y = 1030 \times 9.81 \times (\pi \times 2^2/4) \times 2 = 63487$ N or 63.487 kN

Centre of Pressure $h = 2^{nd}$ moment/1^{st} moment

2^{nd} moment of area.

$I_{SS} = I_{gg} + Ay^2 = (\pi \times 2^4/64) + (\pi \times 2^2/4) \times 2^2$

$I_{SS} = 13.3518$ m^4.

1^{st} moment of area

$Ay = (\pi \times 2^2/4) \times 2 = 6.283$ m^3.

Centre of pressure.

$h = 13.3518/6.283 = 2.125$ m

This is the depth at which, the total force may be assumed to act. Take moments about the hinge.

This is the depth at which, the total force may be assumed to act. Take moments about the hinge.

F = force at top.

P = force at centre of pressure which is 0.125 m below the hinge.

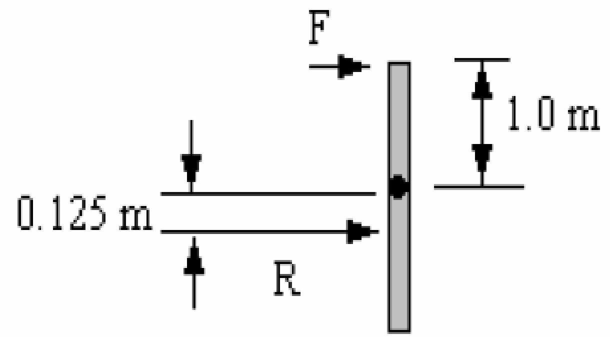


Fig. 9

For equilibrium F x 1 = 63.487 x 0.125

F = 7.936 kN

A circular plate of diameter 1.2m placed vertically in water in such a way that the center of the plate is 2.5m below the free surface of water. Determine : i) total pressure on the plate. ii) Position of center of pressure.

(i) Total pressure, P:

Using the relation

$$P = wA\bar{x} = 9{\cdot}81 \times 1{\cdot}13 \times 2{\cdot}5 = 27{\cdot}7 \text{ kN (Ans.)}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 1{\cdot}2^2 = 1{\cdot}13 \text{ m}^2$$

$$\bar{x} = 2{\cdot}5 \text{ m}$$

(ii) Position of centre of pressure, $\bar{h}$:

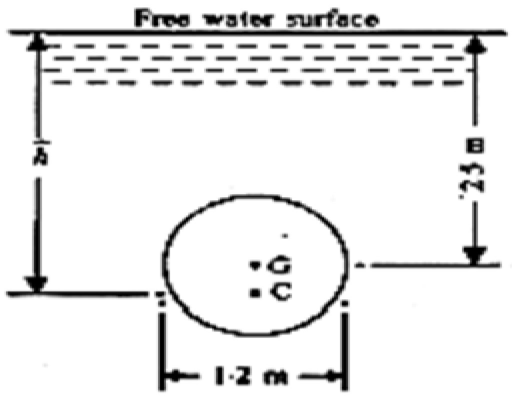
Using the relation,

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

where $$I_G = \frac{\pi}{64} d^4 = \frac{\pi}{64} \times 1{\cdot}2^4 = 0{\cdot}1018 \text{ m}^4$$

$$\bar{h} = \frac{0{\cdot}1018}{1{\cdot}13 \times 2{\cdot}5} + 2{\cdot}5 = 2{\cdot}536 \text{ m}$$

i.e. $$\bar{h} = 2{\cdot}536 \text{ m (Ans.)}$$



A rectangular plate 3 meters long and 1 m wide is immersed vertically in water in such a way that its 3 meters side is parallel to the water surface and is 1m below it. Find: i) Total pressure on the plate and ii) Position of center of pressure.

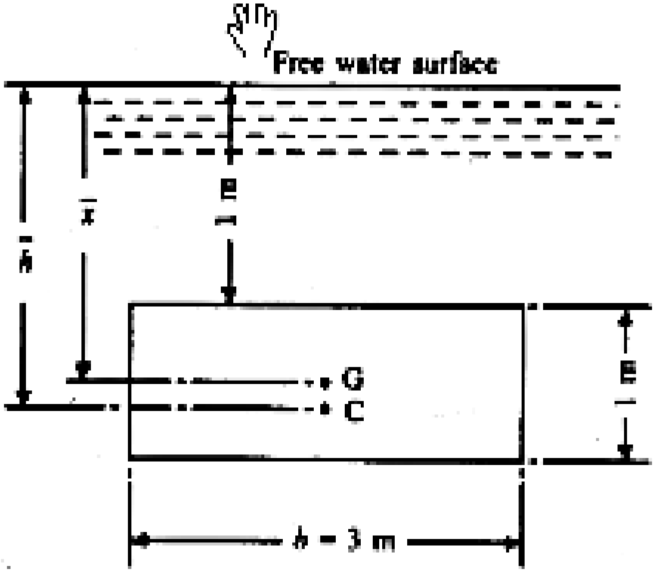
Sol. Width of the plane surface, $b = 3$ m

Depth of the plane surface, $d = 1$ m

Area of the plane surface,

$$A = b \times d = 3 \times 1 = 3 \text{ m}^2$$

$$\bar{x} = 1 + \frac{1}{2} = 1.5 \text{ m}$$



(*i*) **Total pressure P:**

Using the relation:

$$P = wA\bar{x} = 9.81 \times 3 \times 1.5$$
$$= 44.14 \text{ kN (Ans.)}$$

(*ii*) **Centre of pressure,** $\bar{h}$:

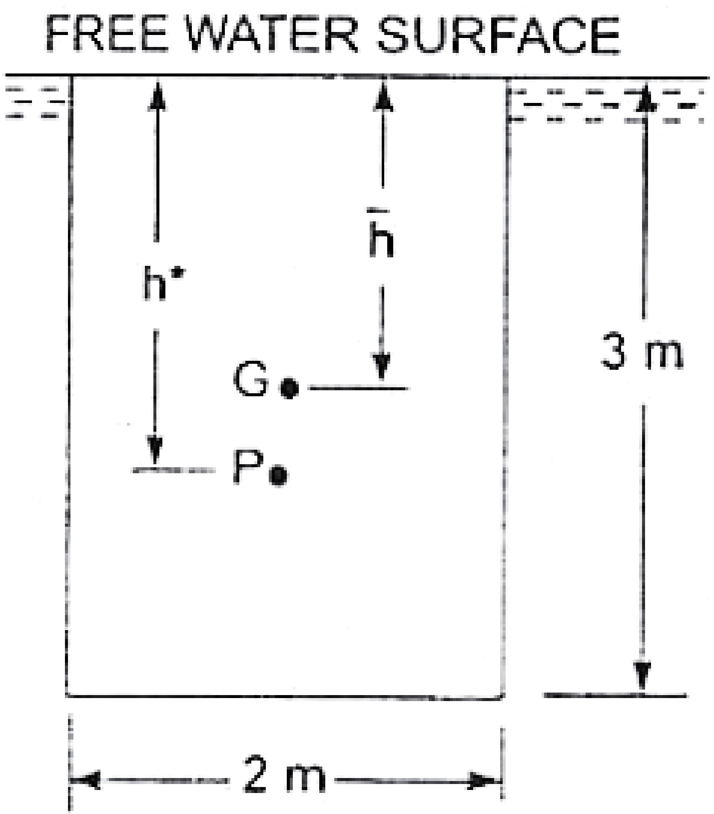
Using the relation: $\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$

$$I_G = \frac{bd^3}{12} = \frac{3 \times 1^3}{12} = 0.25 \text{ m}^4$$

$$\bar{h} = \frac{0.25}{3 \times 1.5} + 1.5 = 1.556 \text{ m}$$

$\bar{h} = 1.556$ m (Ans.)

A rectangular plane surface is 2 m wide and 3m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the plane surface when its upper edge is horizontal and a) coincides with water surface, b) 2.5 m below the free water surface.



$$F = \rho g A \bar{h}$$

where $\rho = 1000 \text{ kg/m}^3,\ g = 9.81 \text{ m/s}^2$

$$A = 3 \times 2 = 6 \text{ m}^2,\ \bar{h} = \frac{1}{2}(3) = 1.5 \text{ m.}$$

$\therefore$ $$F = 1000 \times 9.81 \times 6 \times 1.5$$

$$= \mathbf{88290 \text{ N. Ans.}}$$

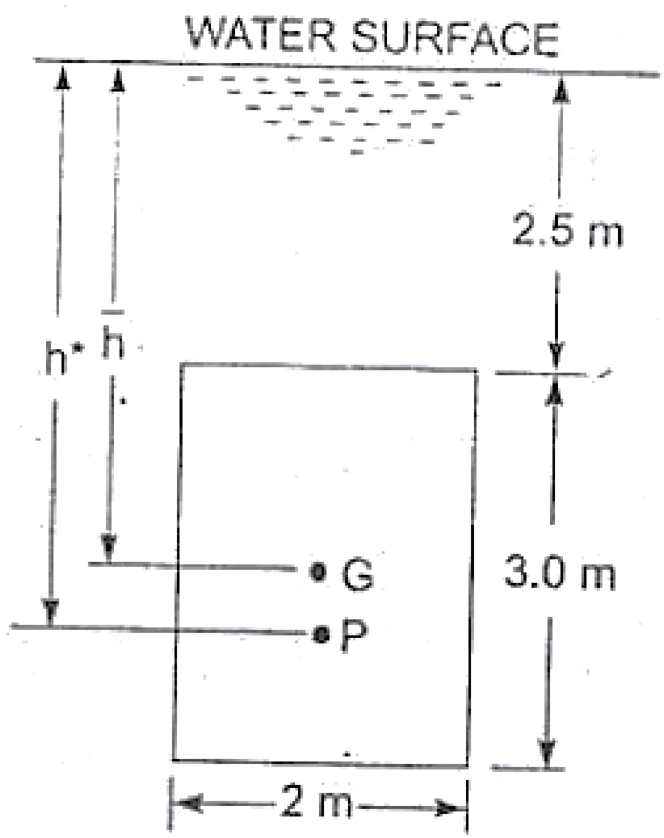
Depth of centre of pressure is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. about C.G. of the area of surface

$$= \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 \text{ m}^4$$

$$h^* = \frac{4.5}{6 \times 1.5} + 1.5 = 0.5 + 1.5 = \mathbf{2.0 \text{ m. Ans.}}$$


WATER SURFACE
2.5 m
h*
h
G
P
3.0 m
2 m

Case ii

$$F = \rho g A \bar{h}$$

where $\bar{h}$ = Distance of C.G. from free surface of water

$$= 2.5 + \frac{3}{2} = 4.0 \text{ m}$$

$\therefore$

$$F = 1000 \times 9.81 \times 6 \times 4.0$$

$$= \mathbf{235440 \text{ N. Ans.}}$$

Centre of pressure is given by $h^* = \frac{I_G}{A\bar{h}} + \bar{h}$

where $I_G = 4.5, A = 6.0, \bar{h} = 4.0$

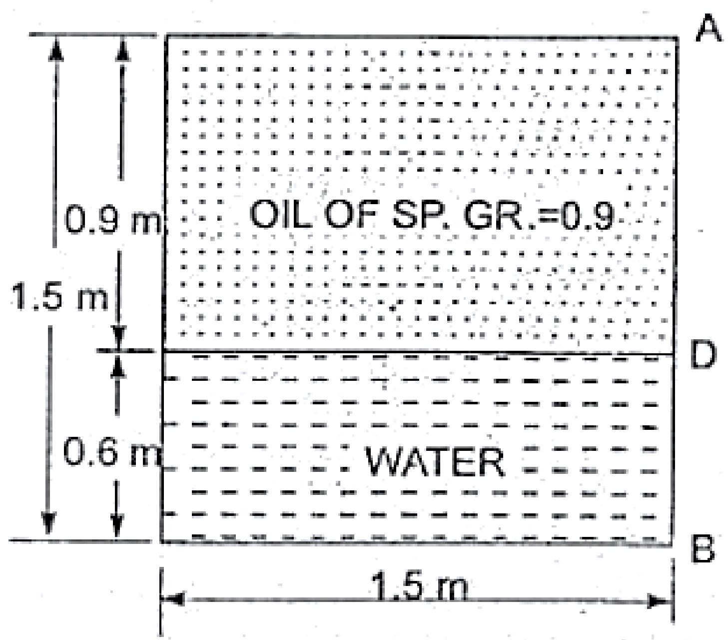
$$h^* = \frac{4.5}{6.0 \times 4.0} + 4.0$$

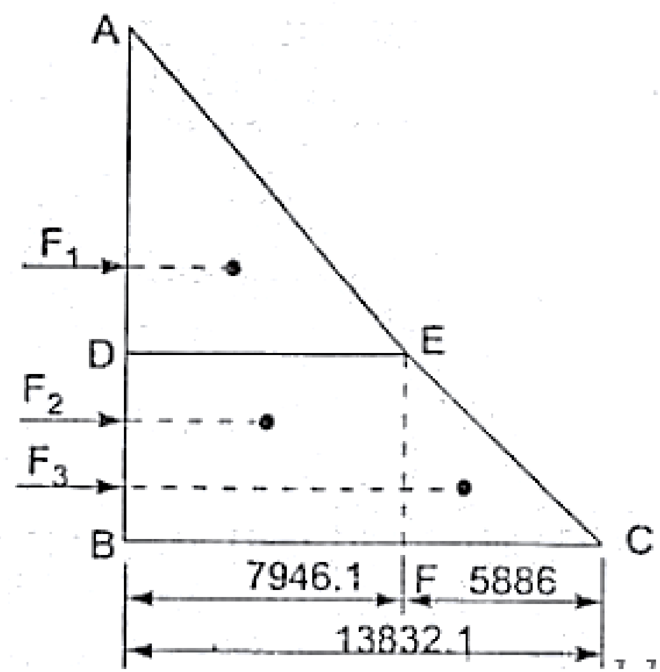
Fi

$$= 0.1875 + 4.0 = 4.1875 = \mathbf{4.1875 \text{ m. Ans.}}$$

A cubical Tank has sides of 1.5m. It contains water for the lower 0.6 m depth. The upper remaining part is filled with oil of specific gravity 0.9. calculate the one vertical side of the tank:

a. Total Pressure , and b. Position of Centre of pressure.

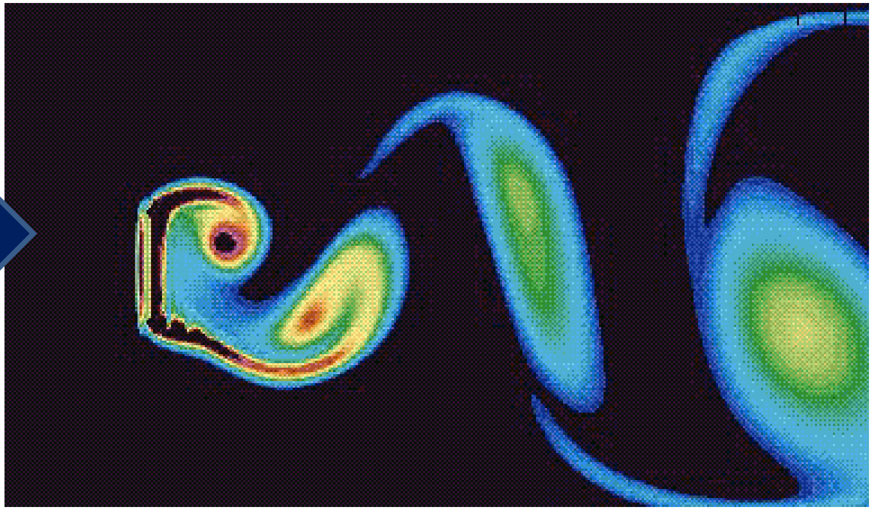


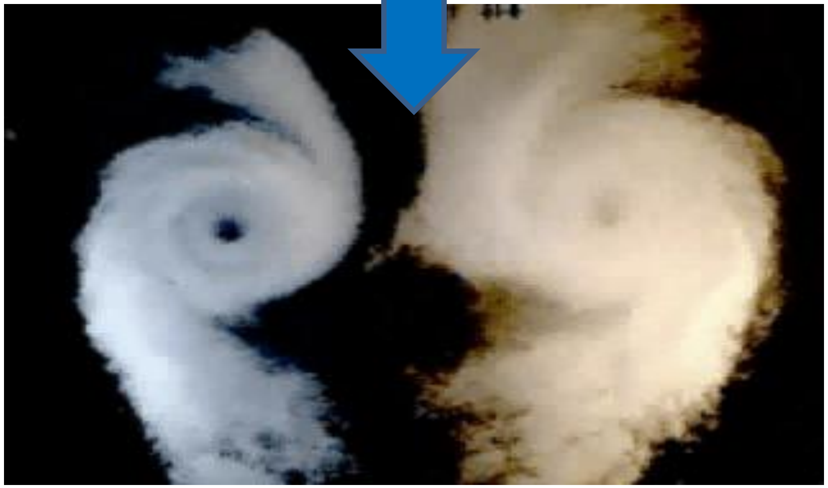


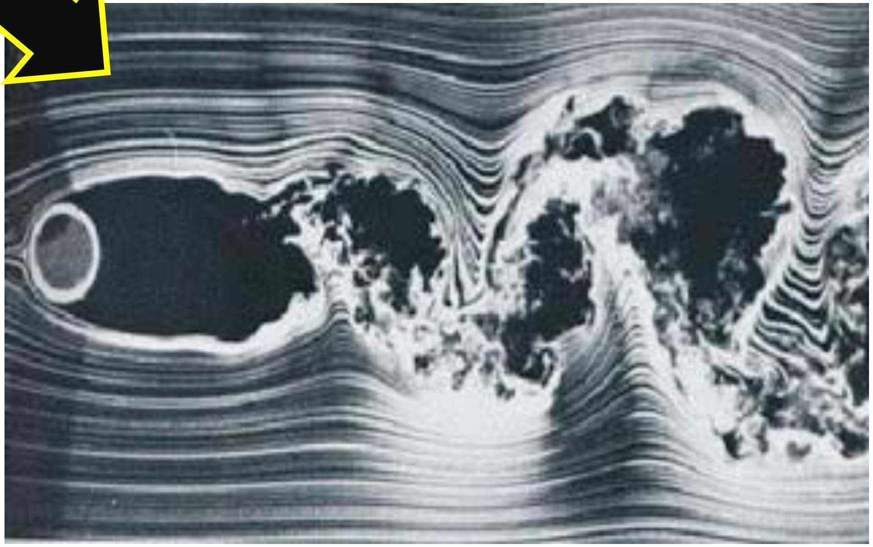
Thank You

UNIT 2

Kinematics of Fluids







Syllabus

Fluid Kinematics: Stream line, path line, streak line, stream tube, classification of flows, steady, unsteady, uniform, non-uniform, laminar, turbulent, rotational, irrotational flows, one, two and three dimensional flows, Continuity equation in 3D flow, stream function, velocity potential function.

What is fluid kinematics?

- Fluid kinematics is the study on fluid motion in space and time without considering the force which causes the fluid motion.
- According to the continuum hypothesis the local velocity of fluid is the velocity of an infinitesimally small fluid particle/element at a given instant ‘t’. It is generally a continuous function in space and time.

Three Aspects of Kinematics of Fluid

- Development of methods & techniques for describing and specifying the motions of fluids.
- Charecterization of different types of motion and associated deformation rates of any fluid element
- Determination of the conditions for the kinematic possibility of fluid motions.

Methods of Describing the Fluid Flow

- The fluid motion is described by two methods.

1. Lagrangian Method
2. Eulerian Method

Eularian and Lagrangian approaches

- Eularian and Lagrangian approaches are of the two methods to study fluid motion. The Eularian approach concentrates on fluid properties at a point P (x,y,z,t). Thus it is a field approach.
- In the Lagrangian approach one identifies a particle or a group of particles and follows them with time. This is bound to be a cumbersome method. But there may be situations where it is unavoidable. One such is the two phase flow involving particles.

Descriptions of Fluid Motion

Defined instantaneously

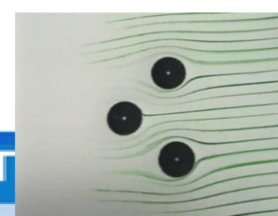
– streamline
  - has the direction of the velocity vector at each point
  - no flow across the streamline
  - steady flow streamlines are fixed in space
  - unsteady flow streamlines move

– pathline

Defined as particle moves (over time)

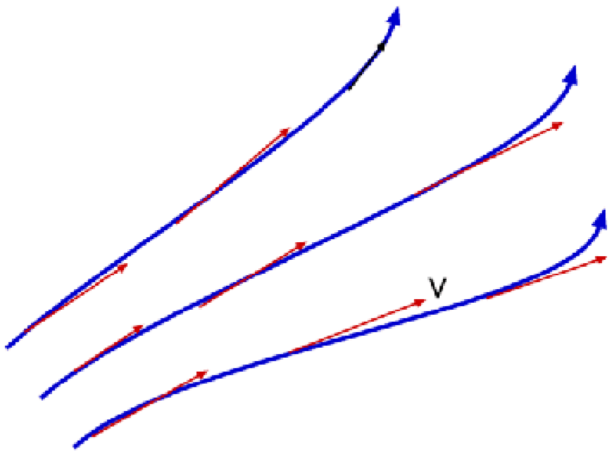
  - path of a particle
  - same as streamline for steady flow

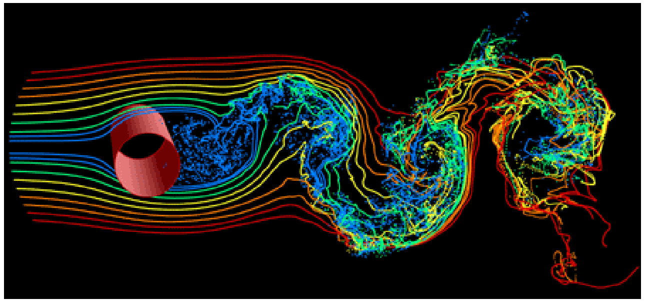
– streakline
  - tracer injected continuously into a flow
  - same as pathline and streamline for steady flow



Streamlines

Streamlines

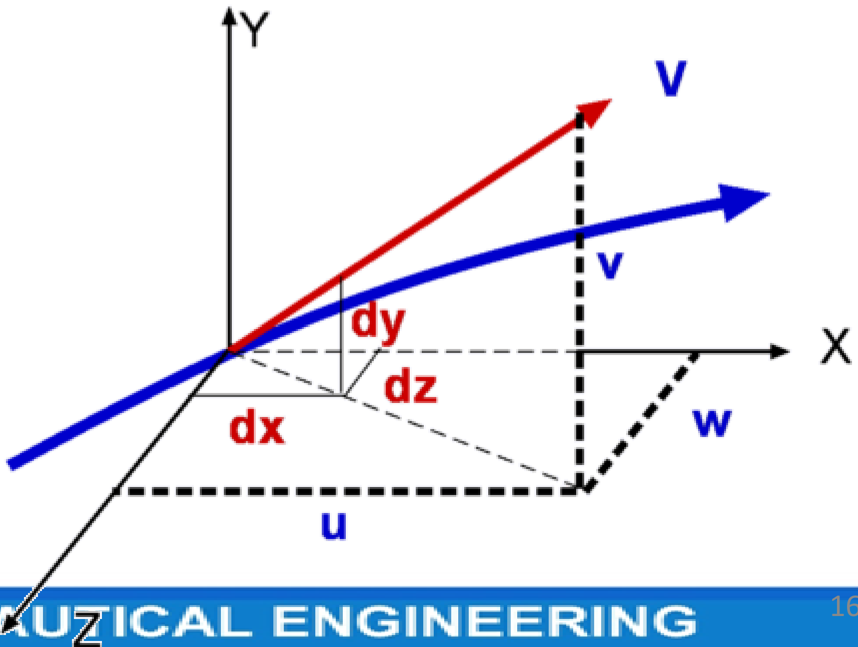




Streamline definition

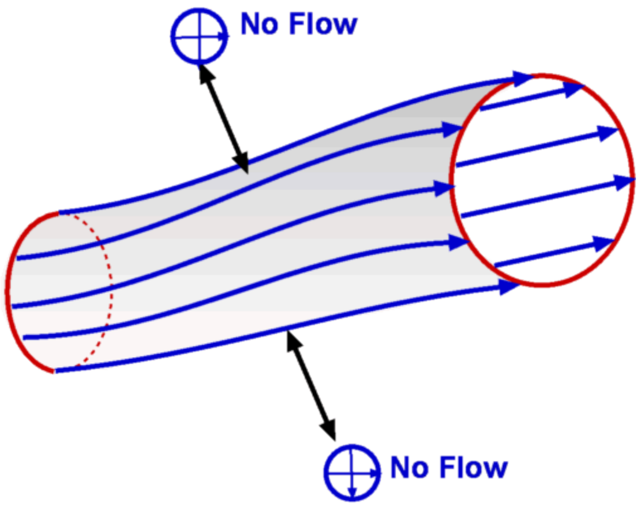
A streamline is one that drawn is tangential to the velocity vector at every point in the flow at a given instant and forms a powerful tool in understanding flows.

$$\frac{du}{u} = \frac{dv}{v} = \frac{dw}{w}$$

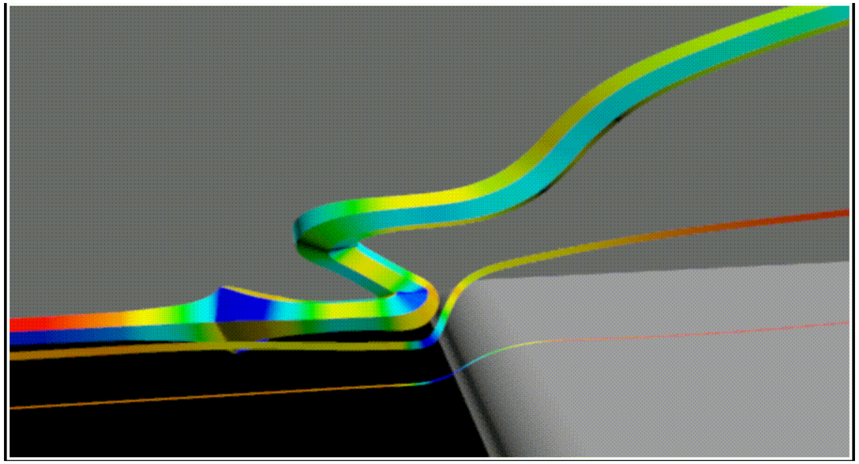


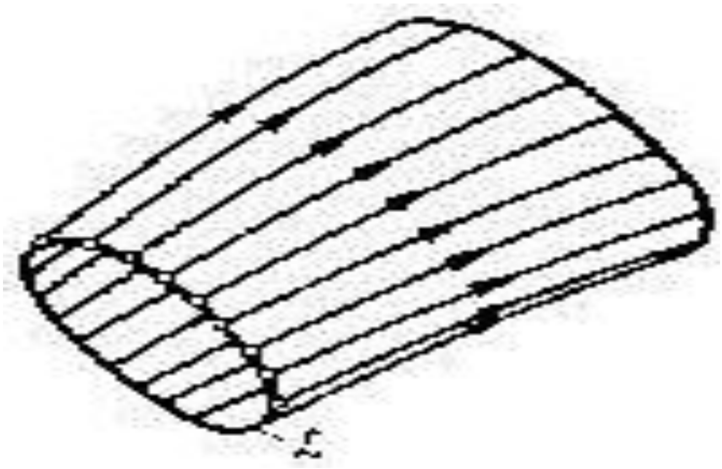
Stream-tube

- The definition of streamline is the fact that there cannot be a flow across it; i.e. there is no flow normal to it. Sometimes, as shown in Fig. we pull out a bundle of streamlines from inside of a general flow for analysis. Such a bundle is called stream tube and is very useful in analyzing flows. If one aligns a coordinate along the stream tube then the flow through it is one-dimensional.



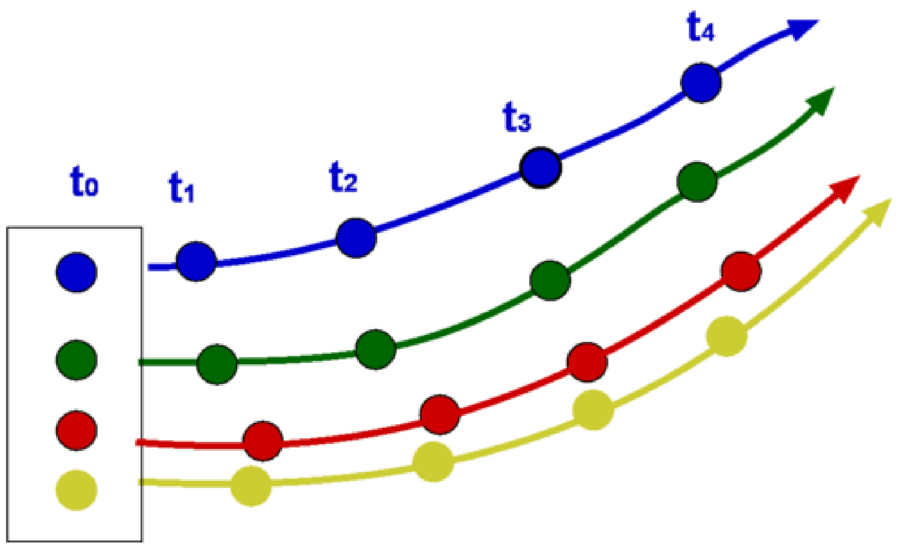
- Is the surface formed instantaneously by all the streamlines that pass through a given closed curve in the fluid.





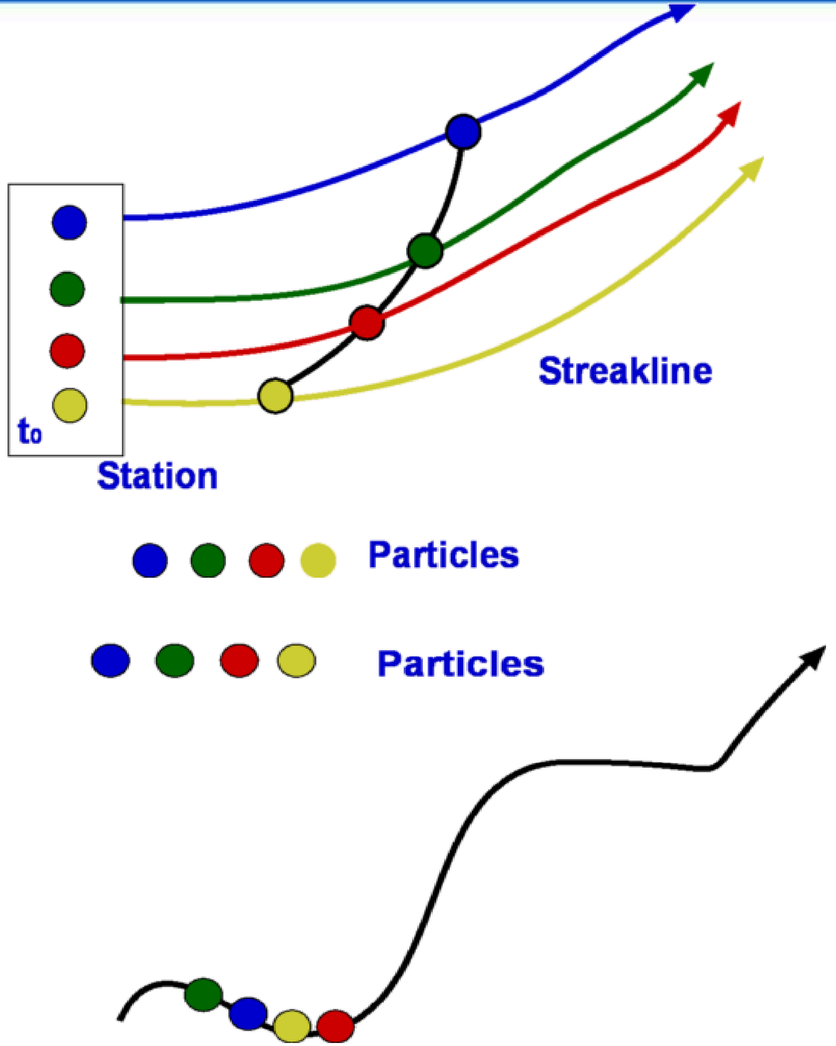
Pathlines

- Pathline is the line traced by a given particle. This is generated by injecting a dye into the fluid and following its path by photography or other means (Fig.)



Streakline

Streakline concentrates on fluid particles that have gone through a fixed station or point. At some instant of time the position of all these particles are marked and a line is drawn through them. Such a line is called a streakline (Fig.).



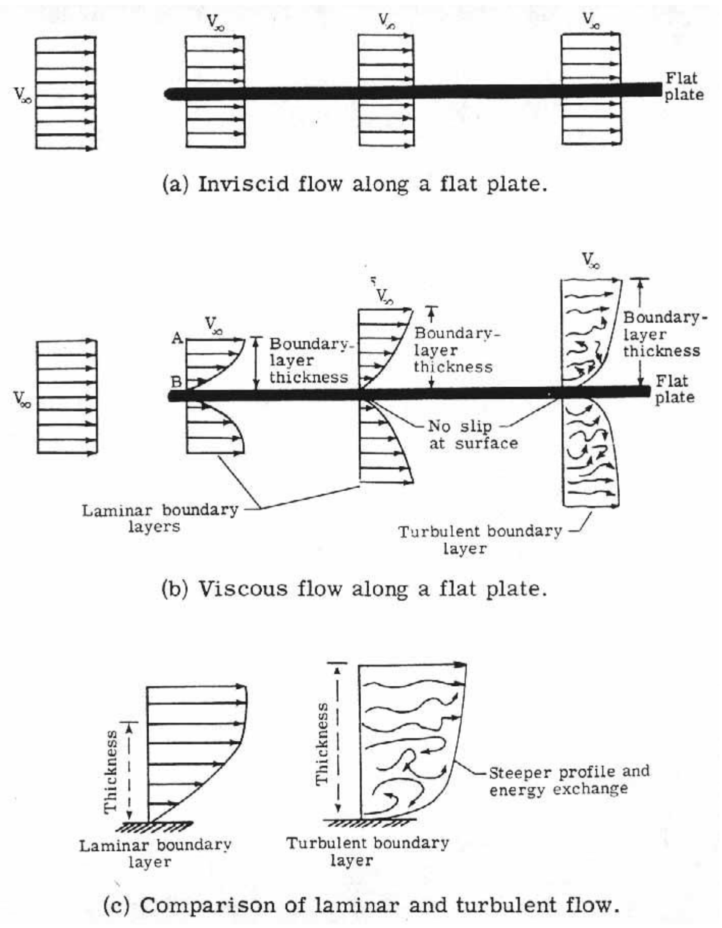
Timeline

- Timeline is generated by drawing a line through adjacent particles in flow at any instant of time.

- In a steady flow the streamline, pathline and streakline all coincide. In an unsteady flow they can be different. Streamlines are easily generated mathematically while pathline and streaklines are obtained through experiments.

Fluid Flow

- No real fluid has all the properties of an ideal fluid, it helps to explain the properties of real fluids.
- Viscosity refers to the amount of internal friction within a fluid. High viscosity equals a slow flow.
- Steady flow is when the pressure, viscosity, and density at each point in the fluid are constant.



Flow classifications

- Incompressible vs. compressible flow.
  - Incompressible flow: volume of a given fluid particle does not change.
    - Implies that density is constant everywhere.
    - Essentially valid for all liquid flows.
  - Compressible flow: volume of a given fluid particle can change with position.
    - Implies that density will vary throughout the flow field.
    - Compressible flows are further classified according to the value of the Mach number (M), where.
    - M < 1 - Subsonic.
    - M > 1 - Supersonic.

- Single phase vs. multiphase flow.
  - Single phase flow: fluid flows without phase change (either liquid or gas).
  - Multiphase flow: multiple phases are present in the flow field (e.g. liquid-gas, liquid-solid, gas-solid).
- Homogeneous vs. heterogeneous flow.
  - Homogeneous flow: only one fluid material exists in the flow field.
  - Heterogeneous flow: multiple fluid/solid materials are present in the flow field (multi-species flows).

Types of flows

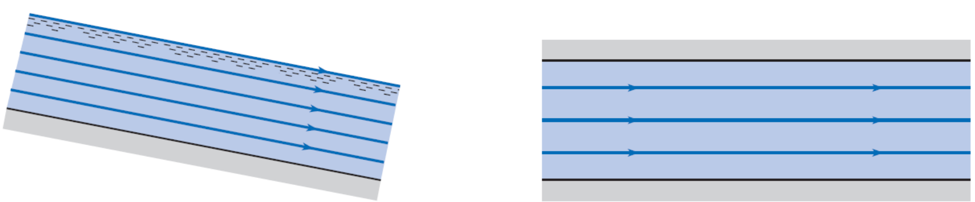
- Steady
- Unsteady
- Uniform
- Non-uniform
- Laminar
- Turbulent
- Rotational
- Irrotational
- One dimensional flows
- Two dimensional flows
- Three dimensional flows

Steady vs. Unsteady Flow

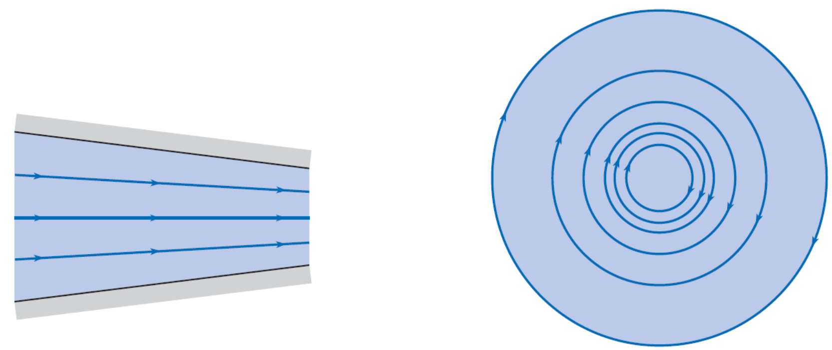
For steady flow, the velocity at a point or along a streamline does not change with time:

$$\frac{\partial \mathbf{V}}{\partial t} = 0$$

Any of the previous examples can be steady or unsteady, depending on whether or not the flow is accelerating:

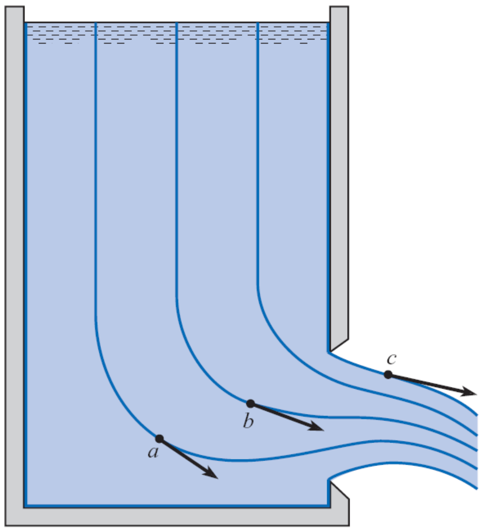


(a) (b)

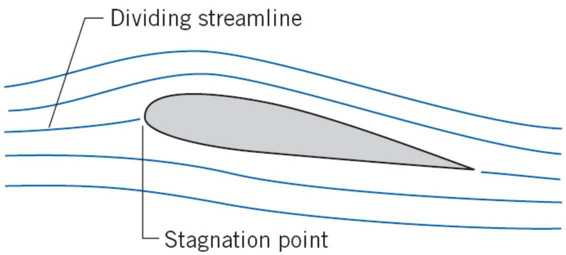


(a) (b)

Streamlines and Flow Patterns



(a)



(b)

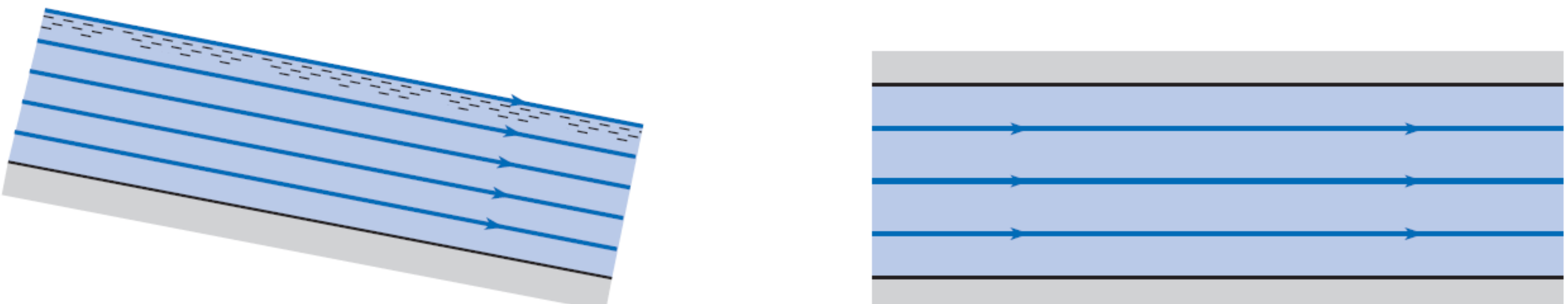
- Streamlines are used for visualizing the flow. Several streamlines make up a flow pattern.
- A streamline is a line drawn through the flow field such that the flow vector is tangent to it at every point at a given instant in time.

Uniform vs. Non-Uniform Flow

Using s as the spatial variable along the path (i.e., along a streamline):

Flow is uniform if $$\frac{\partial \mathbf{V}}{\partial s} = 0$$

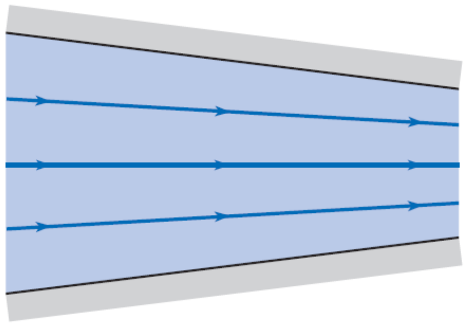
Examples of uniform flow:

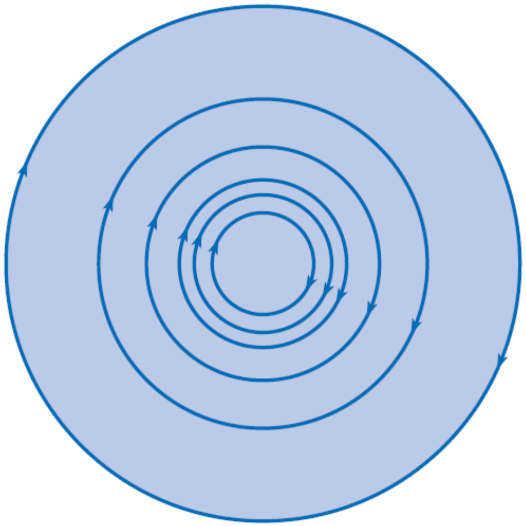


(a) (b)

Note that the velocity along different streamlines need not be the same! (in these cases it probably isn't).

Examples of non-uniform flow:


(a)


(b)

a) Converging flow: speed increases along each streamline.

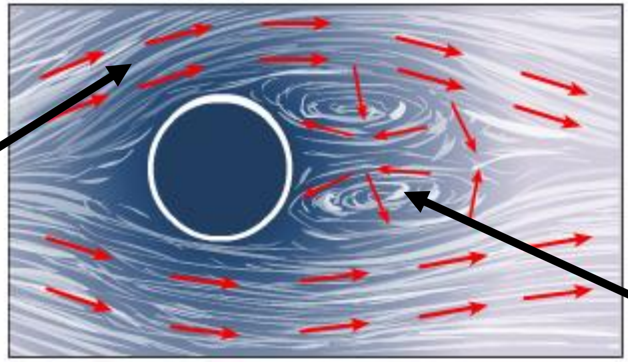
b) Vortex flow: Speed is constant along each streamline, but the direction of the velocity vector changes.


Uniform and Non-Uniform Flow
Uniform flow
Non-uniform flow
Flow direction
©The COMET Program

Laminar Turbulent Flow

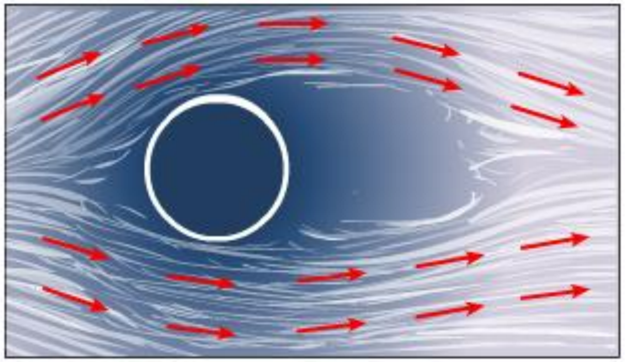
- Moving fluids can exhibit *laminar* (smooth) flow or *turbulent* (irregular) flow.

Laminar Flow



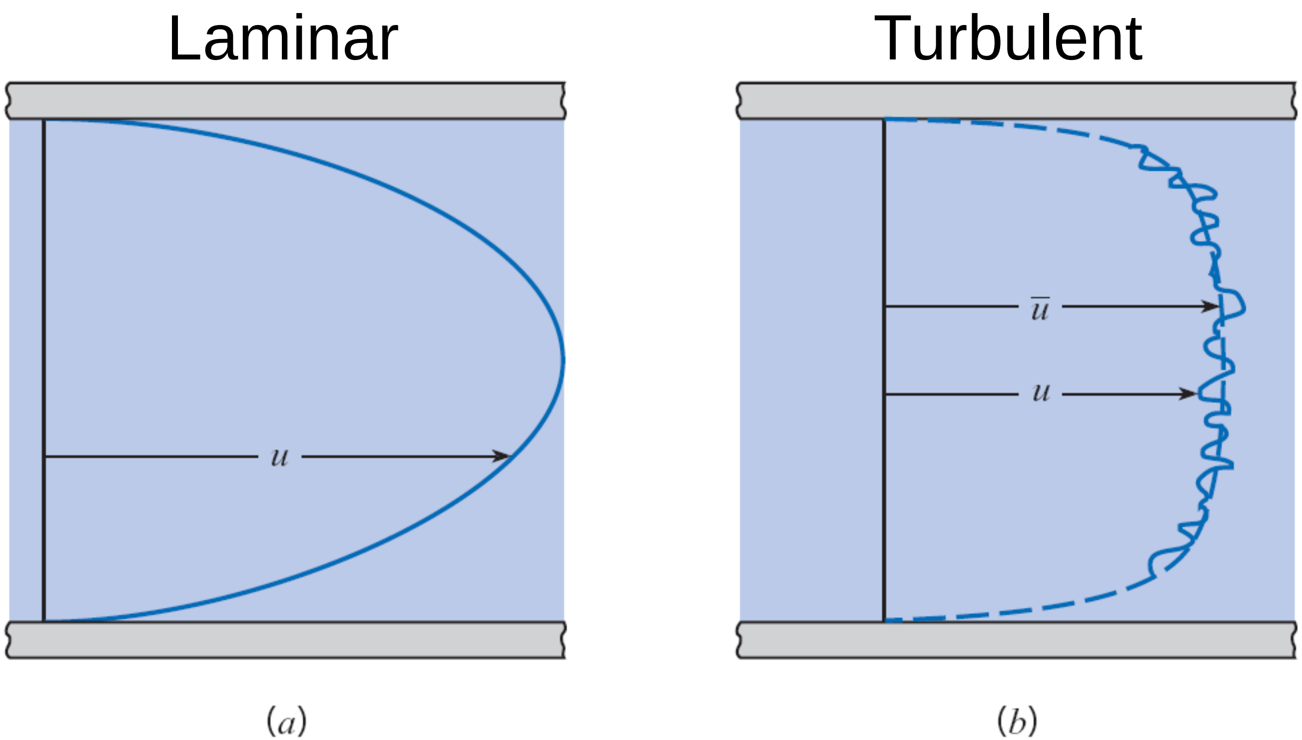
Turbulent Flow

Not an ideal fluid

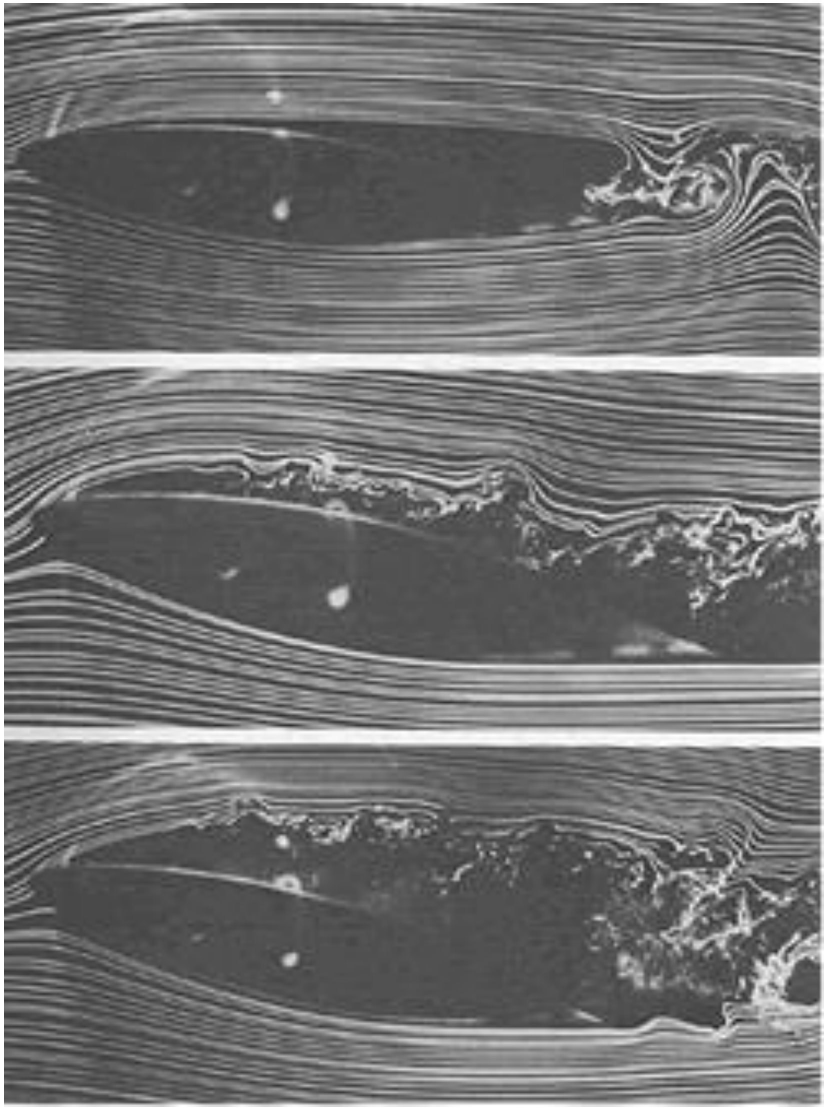


Ideal fluid

Flow inside a pipe:



- Turbulent flow is nearly constant across a pipe.
- Flow in a pipe becomes turbulent either because of high velocity, because of large pipe diameter, or because of low viscosity.



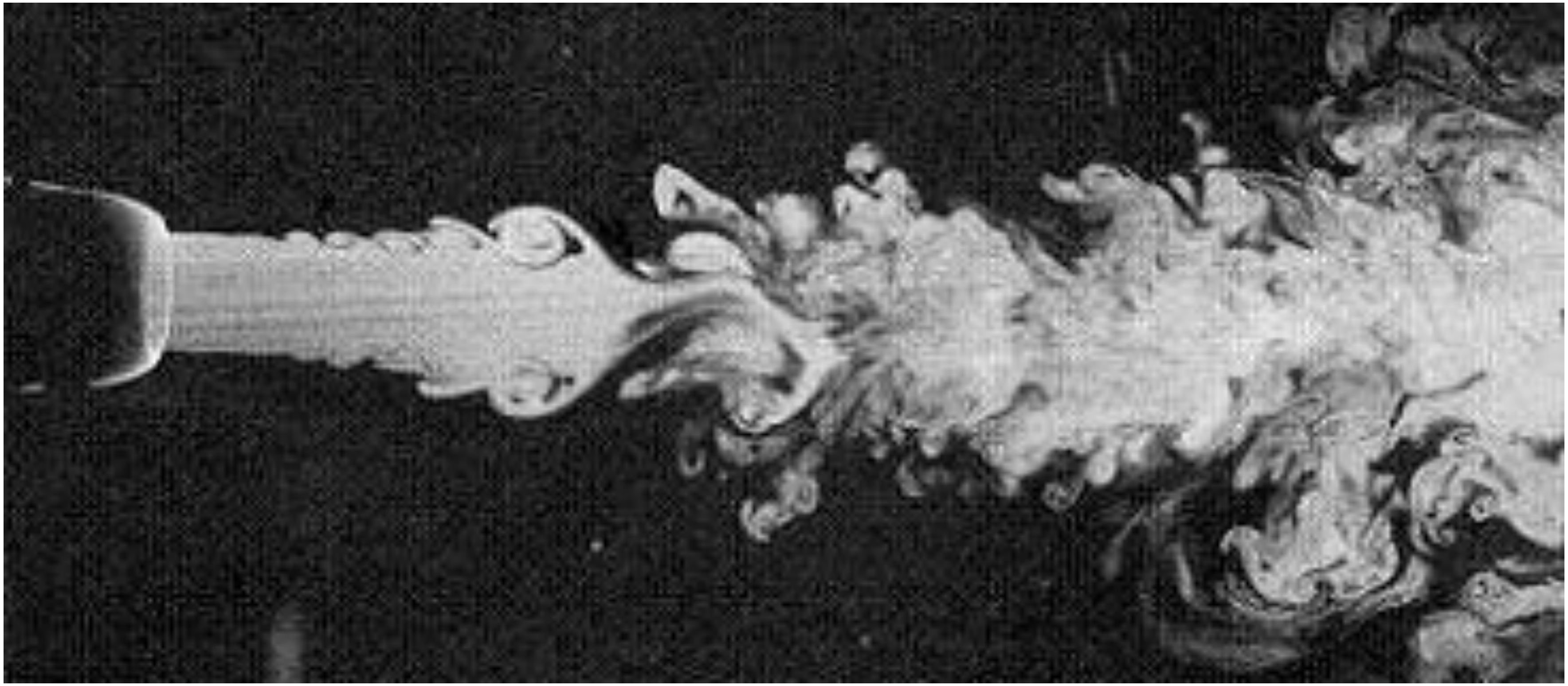
Flow around an airfoil

Partly laminar, i.e., flowing past the object in “layers” (laminar).

Turbulence forms mostly downstream from the airfoil.

(Flow becomes more turbulent with increased angle of attack.)

Turbulent flow in a jet

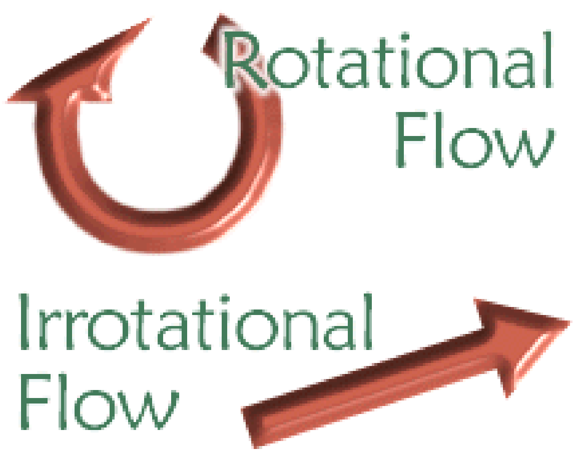


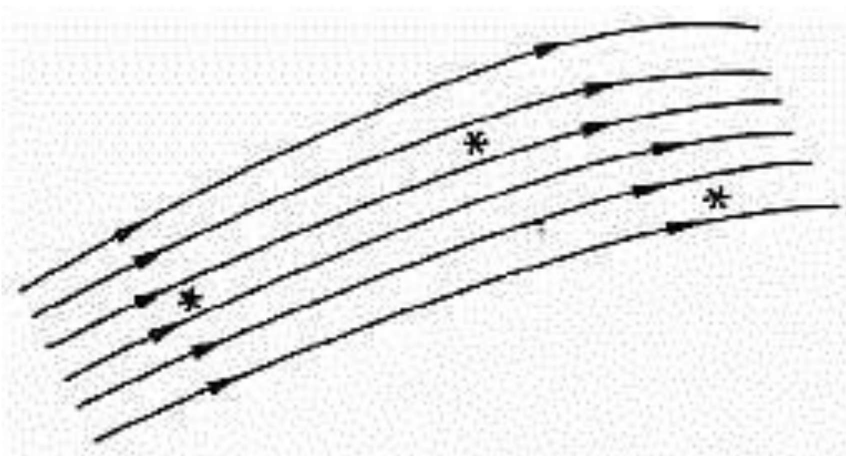
Turbulence is associated with intense mixing and unsteady flow.

Rotational Vs Irrotational Flows

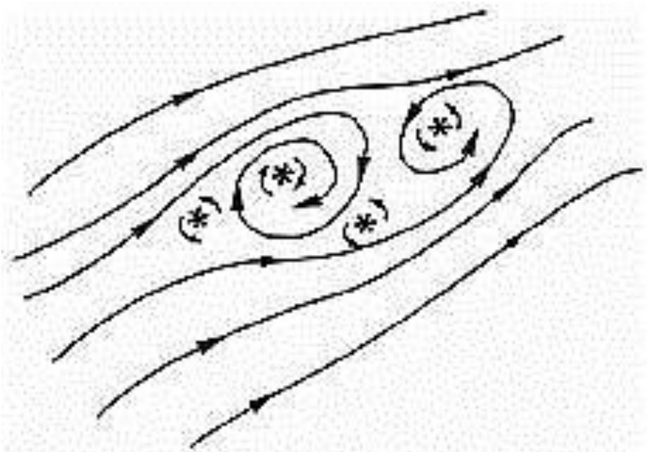
- Rotational flow is that type of flow in which the fluid particles while flowing along stream lines also rotate about their own axis.

- Irrotational flow is that type of flow in which the fluid particles while flowing along stream lines also do not rotate about their own axis.

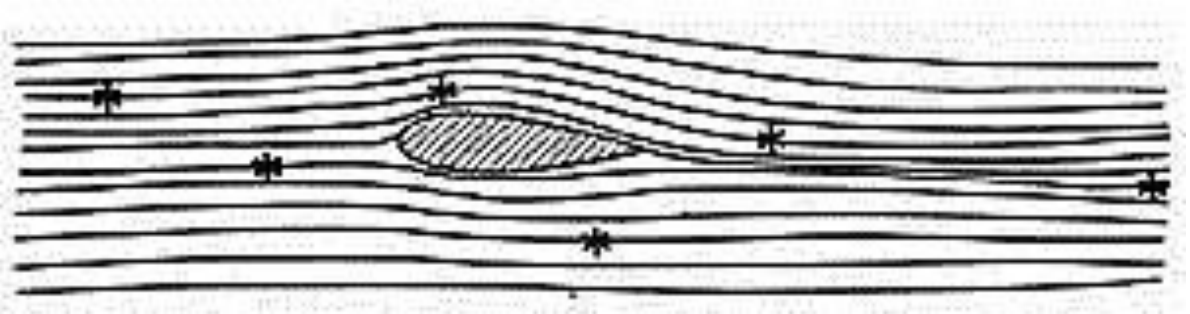




(a) Irrotational flow.



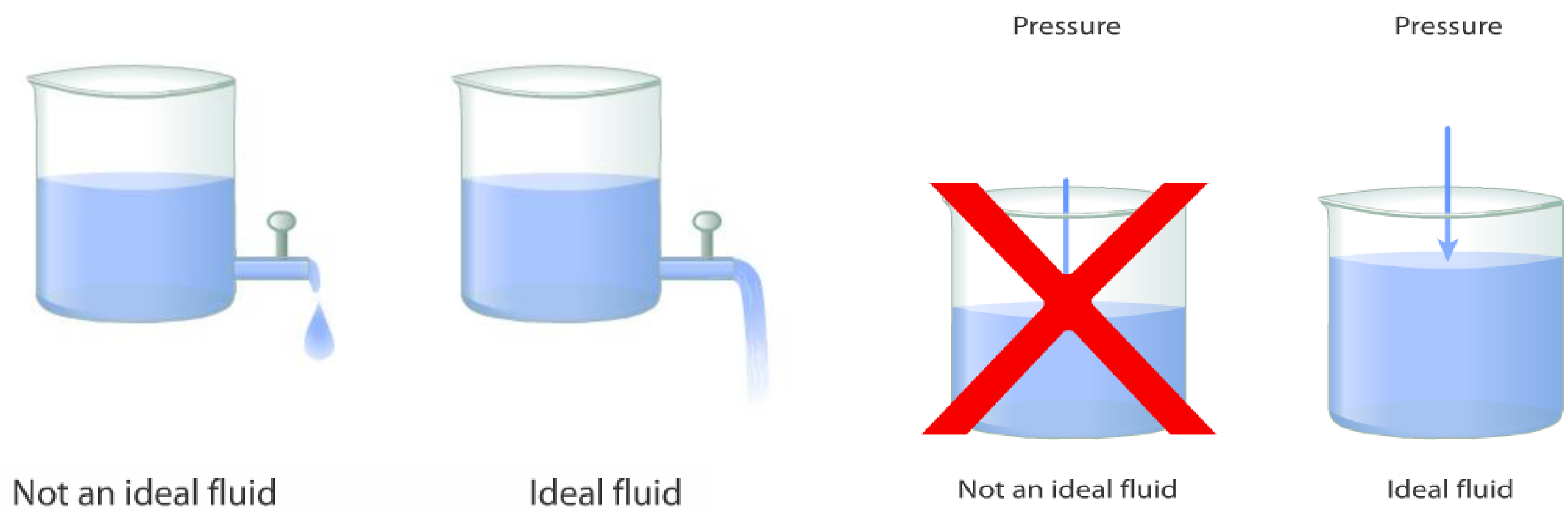
(b) Rotational flow.



(c) Inviscid, irrotational flow about an airfoil.

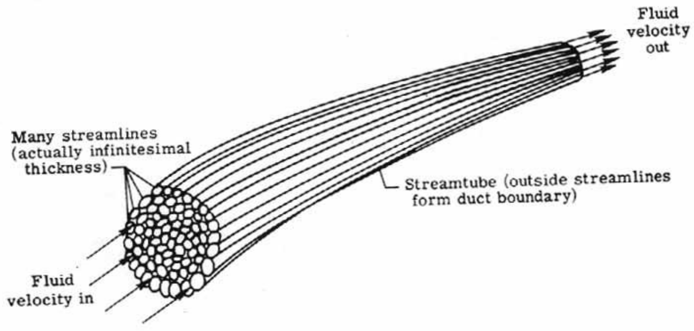
Fluid Flow

- An **ideal fluid** is a fluid that has no internal friction or viscosity and is incompressible.
- The ideal fluid model simplifies fluid-flow analysis

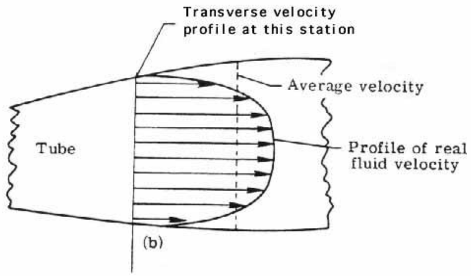


1D,2D,3D flows

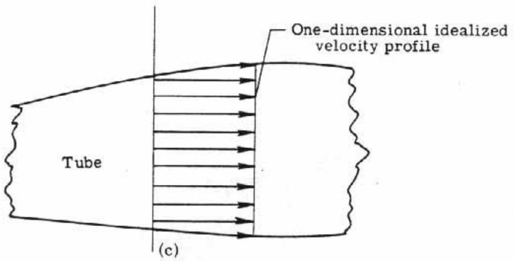
- <u>One dimensional flow: </u>Is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only.
- <u>Two Dimensional Flow: </u>Is that type flow in which the velocity is a function of time and two rectangular space co-ordinates.
- <u>Three Dimensional flow:</u> Is that type of flow in which the velocity is a function of time and three mutually perpendicular directions.



(a) Stream tubes.



(b) Real velocity flow profile.



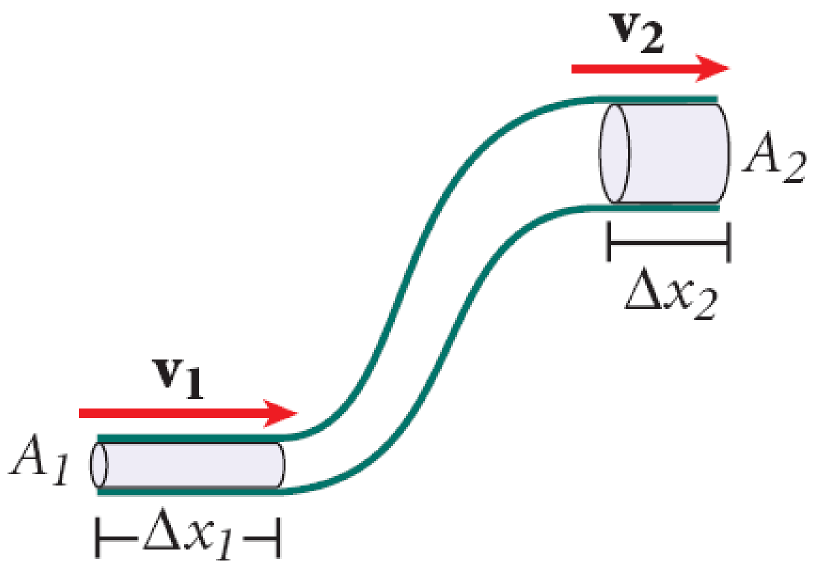
(c) One-dimensional flow profile.

Principles of Fluid Flow

- The continuity equation results from conservation of mass.
- Continuity equation:

$$A_1 v_1 = A_2 v_2$$

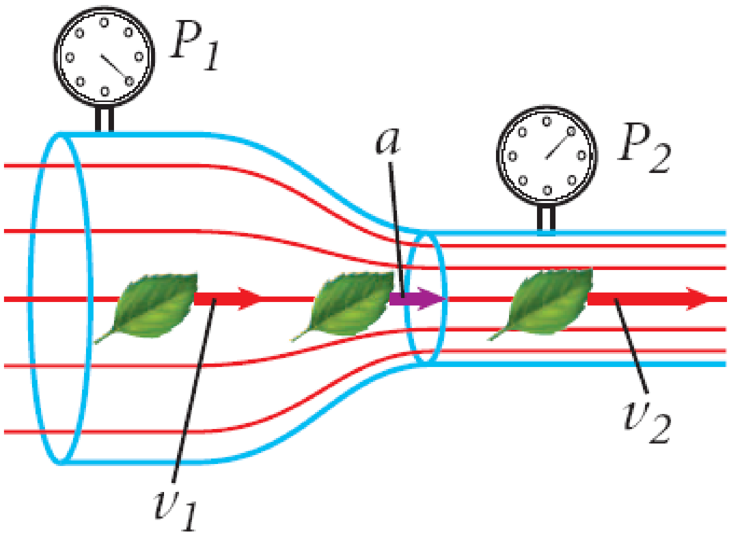
Area × speed in region 1 = area × speed in region 2



Principles of Fluid Flow

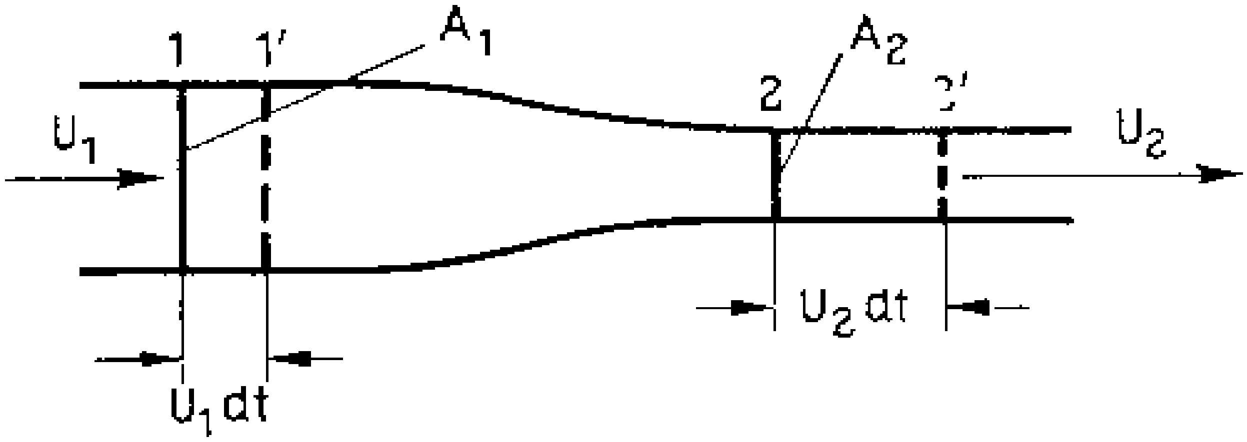
- The speed of fluid flow depends on cross-sectional area.

- Bernoulli's principle states that the pressure in a fluid decreases as the fluid's velocity increases.



Continuity Equation

- For a steady flow the stream-tube formed by a closed curved fixed in space is also fixed in space, and no fluid can penetrate through the stream-tube surface, like a duct wall.



Considering a stream-tube of cylindrical cross sections $A_1 \ and \ A_2$ with velocities $u_1 \ and \ u_2$ perpendicular to the cross sections $A_1 \ and \ A_2$ and densities $\rho_1 \ and \ \rho_2$ at the respective cross sections $A_1 \ and \ A_2$ and assuming the velocities and densities are constant across the whole cross section $A_1 \ and \ A_2$, a fluid mass closed between cross section 1 and 2 at an instant t will be moved after a time interval dt by $u_1 \cdot dt \ and \ u_2 \cdot dt$ to the cross section 1' and 2' respectively.

Because the closed mass between 1 and 2 must be the same $\rho_1 A_1 u_1 dt$ between 1' and 2', and the mass between 1' and 2 for a steady flow can not change from t and t+dt, the mass between 1 and 1' moved in dt, $\rho_2 A_2 u_2 dt$ must be the same as the mass between 2 and 2' moved in the same time dt, :

- Therefore the continuity equation of steady flow :

$$\rho_1 A_1 u_1 = \rho_2 A_2 u_2 \qquad \textbf{(4.1)}$$

Interpretation : The mass flow rate $\dot{m} = \rho A u = const$.

through a steady stream-tube or a duct.

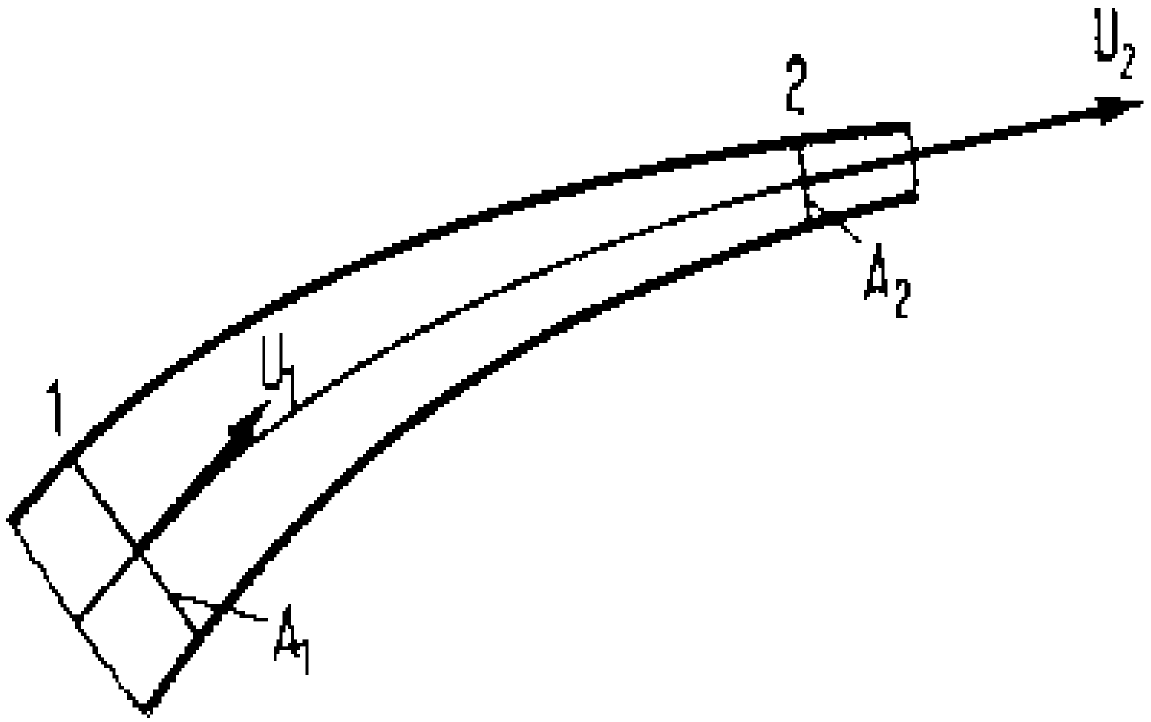
- For incompressible fluid with $\rho_1 = \rho_2$:

$$A_1 u_1 = A_2 u_2 \qquad \textbf{(4.2)}$$

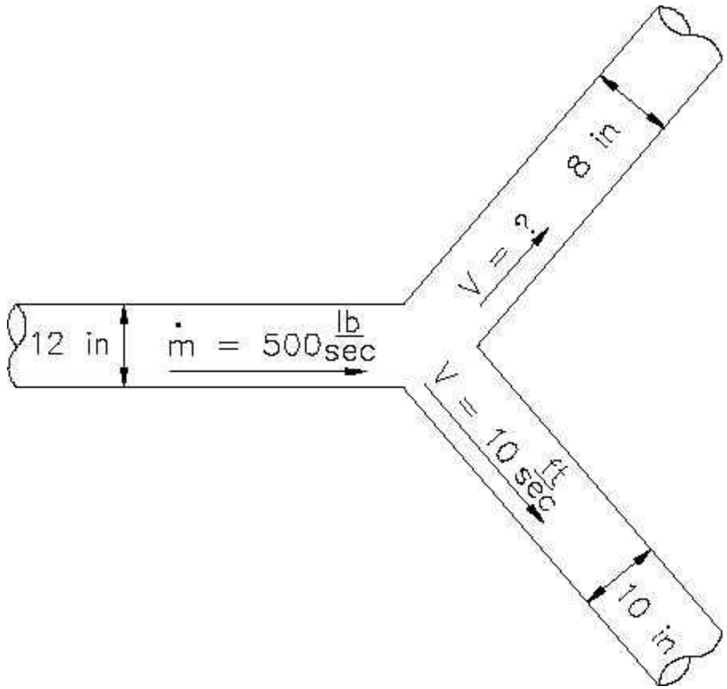
Interpretation : The volume flow rate $\dot{V} = A u = const$.

- From the continuity equation for incompressible fluid :

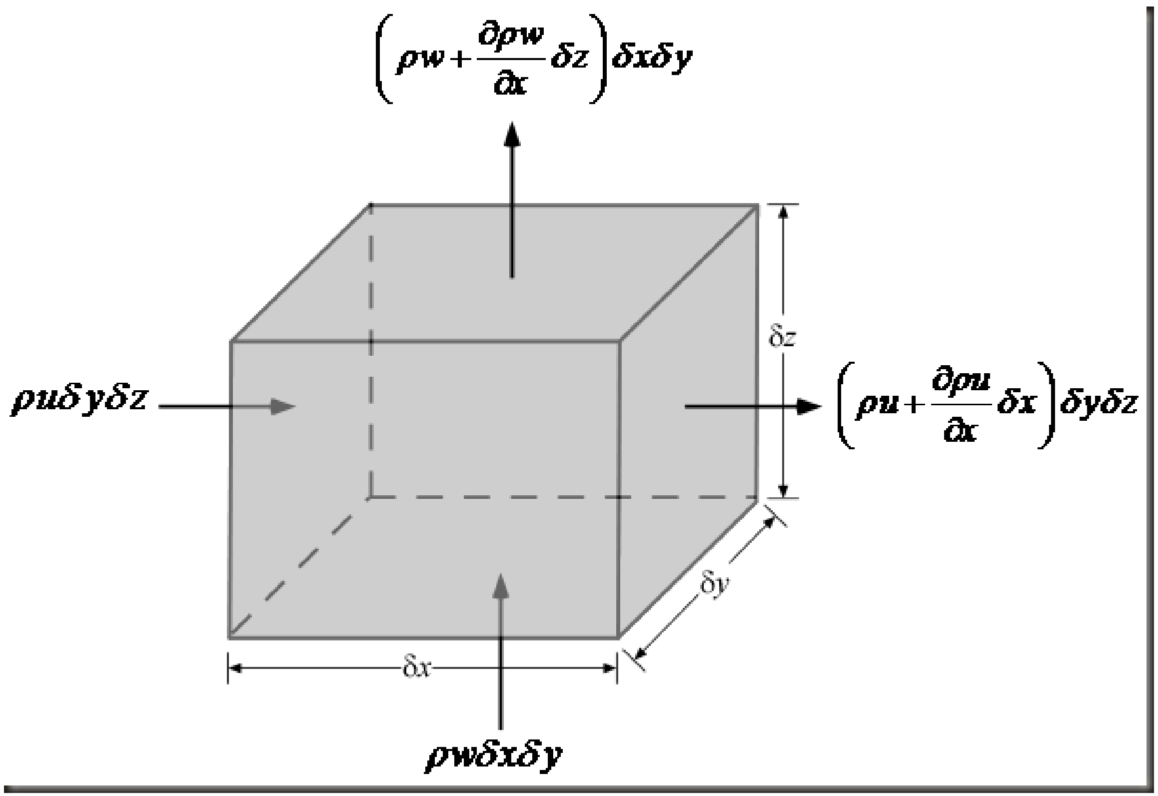
$$\frac{u_1}{u_2} = \frac{A_2}{A_1}$$ for a stream-tube.


U2
2
A2
U1
1
A1

Continuity Equation - Multiple Outlets A piping system has a "Y" configuration for separating the flow as shown in Figure. The diameter of the inlet leg is 12 in., and the diameters of the outlet legs are 8 and 10 in. The velocity in the 10 in. leg is 10 ft/sec. The flow through the main portion is 500 Kg.m/sec. The density of water is 62.4 kg/m^3. What is the velocity out of the 8 in. pipe section



Continuity Equation in three Dimensions



Stream Function & Velocity Potential

- Stream lines/ Stream Function (Y)
  - Rotation, vorticity
- Velocity Potential(f)
  - Relationship between stream function and velocity potential
  - Complex velocity potential

Velocity Potential vs Stream Function

	Stream Function (ψ)	Velocity Potential (ϕ)
Exists for	only 2D flow	all flows
	viscous or non-viscous flows	Irrotational (i.e. Inviscid or zero viscosity) flow
	Incompressible flow (steady or unsteady)	Incompressible flow (steady or unsteady state)
	compressible flow (steady state only)	compressible flow (steady or unsteady state)

- In 2D inviscid flow (incompressible flow or steady state compressible flow), both functions exist

Thank U

Fluid Dynamics

- Surface and Body forces – Euler's and Bernoulli's equation derivation, Navier-stokes equation (explanation only) Momentum equation - applications, vortex – Free and Forced. Forced vortex with free surface

Vortex flow

- Vortex flow is defined as the flow along a curved path or flow of the rotating mass of a fluid is known a vortex flow
- The vertex flow is of two types:

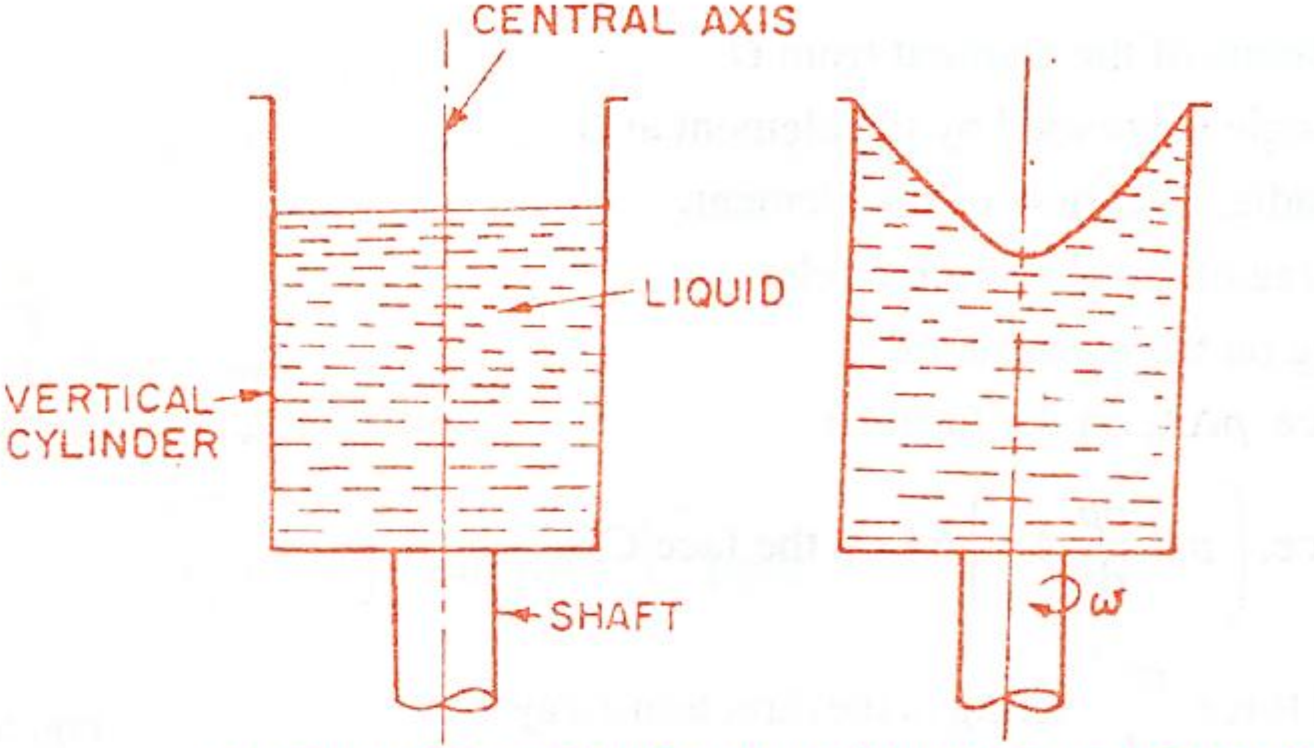
1. Forced vortex flow
2. Free vortex flow

Forced vertex flow

- Forced vertex flow is defined as that type of vertex flow, in which some external torque is required to rotate the fluid mass.
- The fluid mass in this type of flow, rotates at constant angular velocity, ω The tangential velocity of any fluid particle is given by $v=\omega r$
- Examples for forced vertex flow are:

1. A vertex flow containing liquid which is rotated about its central axis with a constant angular velocity ω.
2. Flow of liquid inside the impeller of a centrifugal pump.
3. Flow of water through the runner of a turbine.

Forced Vortex Flow



(a) CYLINDER STATIONARY (b) CYLINDER IS ROTATING

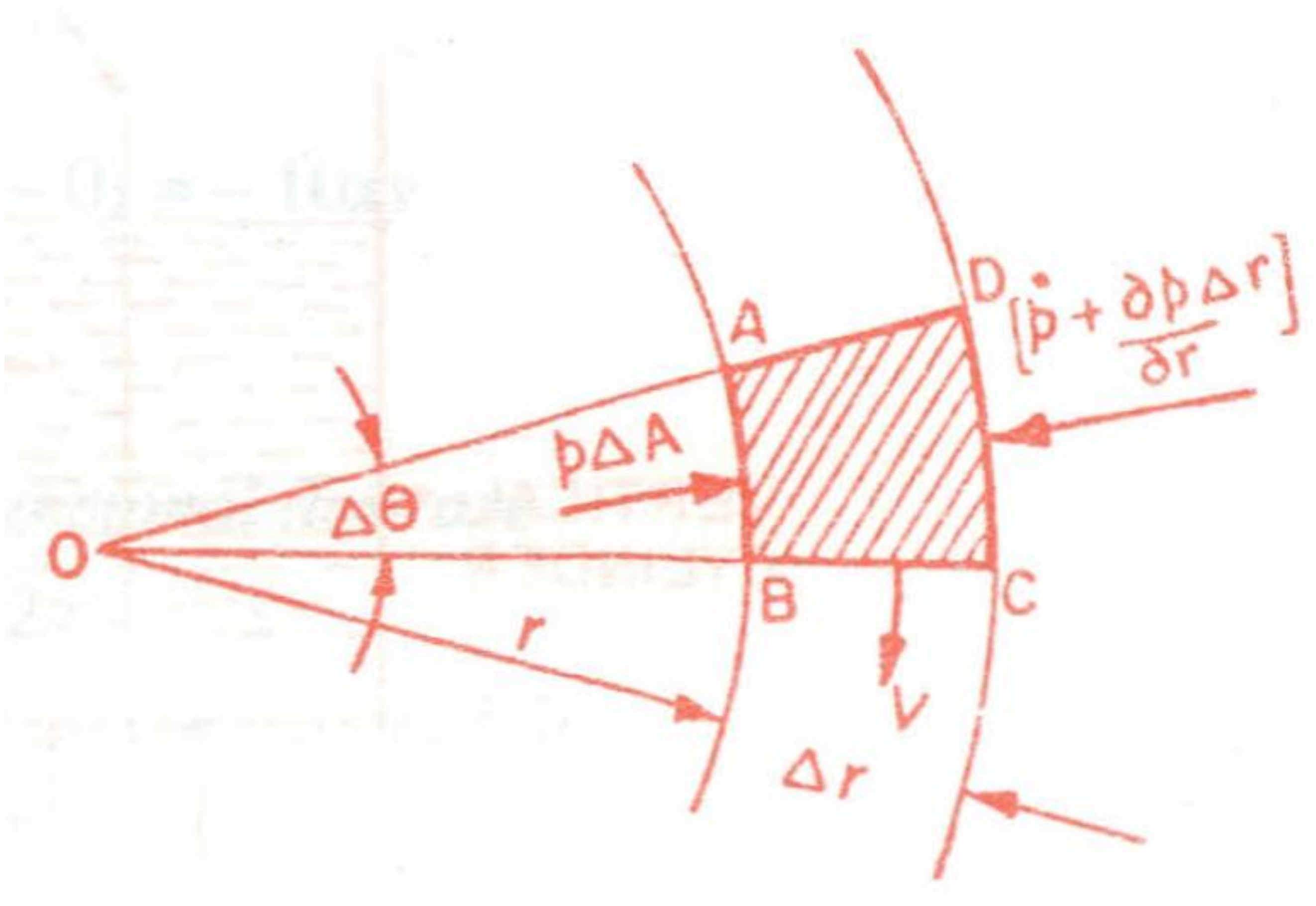
Free vortex flow

- When no external torque is required to rotate the fluid mass, that type of flow is called free vortex flow.
- Examples of free vertex flow are:

1. Flow of liquid through a hole provided at the bottom of a container.
2. Flow of liquid around a circular bend pipe.
3. A whirlpool in a river.
4. Flow of fluid in a centrifugal pump casing.

- The relation between velocity and radius, in free vortex is obtained by putting the value of external torque equal to zero, or, the time rate of change of angular momentum, *i.e.,* moment of momentum must be zero.
- Consider a fluid particle of mass 'm' at a radial distance r from the axis of rotation, having a tangential velocity
- Angular momentum = Mass x Velocity = $m \times v$
- Moment of momentum = Momentum x r = $m \times v \times r$
- Time rate of change of angular momentum : $\partial/\partial t\ (mvr)$
- For free vortex $\partial/\partial t\ (mvr) = 0$
- Integrating, we get mvr = Constant or vr = Constant

Equation of Motion for Vortex Flow



- Consider a fluid element *ABCD* (shown shaded) Fig. rotating at a uniform velocity in a horizontal plane about an axis perpendicular to the plane of paper and passing through *O.*
- Let r = Radius of the element from *O.*

$\Delta\theta$ = Angle subtended by the element at *O.*

Δr = Radial thickness of the element.

ΔA = Area of cross-section of element.

The forces acting on the element are :

(*i*) Pressure force, $p\Delta A$, on the face AB.

(*ii*) Pressure force, $\left(p + \frac{\partial p}{\partial r}\Delta r\right)\Delta A$ on the face CD.

(*iii*) Centrifugal force, $\frac{mv^2}{r}$ acting in the direction away

om the centre, O.

Now, the mass of the element $= \text{Mass density} \times \text{Volume}$

$$= \rho \times \Delta A \times \Delta r$$

$\therefore$ Centrifugal force $= \rho \Delta A \Delta r \dfrac{v^2}{r}$.

Equating the forces in the radial direction, we get

$$\left(p + \frac{\partial p}{\partial r} \Delta r\right) \Delta A - p \Delta A = \rho \Delta A \Delta r \frac{v^2}{r}$$

or

$$\frac{\partial p}{\partial r} \Delta r \, \Delta A = \rho \Delta A \Delta r \frac{v^2}{r}.$$

Cancelling $\Delta r \times \Delta A$ to both sides, we get $\dfrac{\partial p}{\partial r} = \rho \dfrac{v^2}{r}$

Equation (5.21) gives the pressure variation along the radical direction for a forced or free vortex f in a horizontal plane. The expression $\frac{\partial p}{\partial r}$ is called pressure gradient in the radial direction. As $\frac{\partial p}{\partial r}$ is +ve, he pressure increases with the increase of radius 'r'.

The pressure variation in the vertical plane is given by the hydrostatic law, *i.e.*,

$$\frac{\partial p}{\partial z} = -\rho g \quad \text{...(}$$

- In the equation above the pressure measured is in vertically upward direction.

The pressure, p varies with respect to r and z or p is a function of r and z and hence total derivative of p is

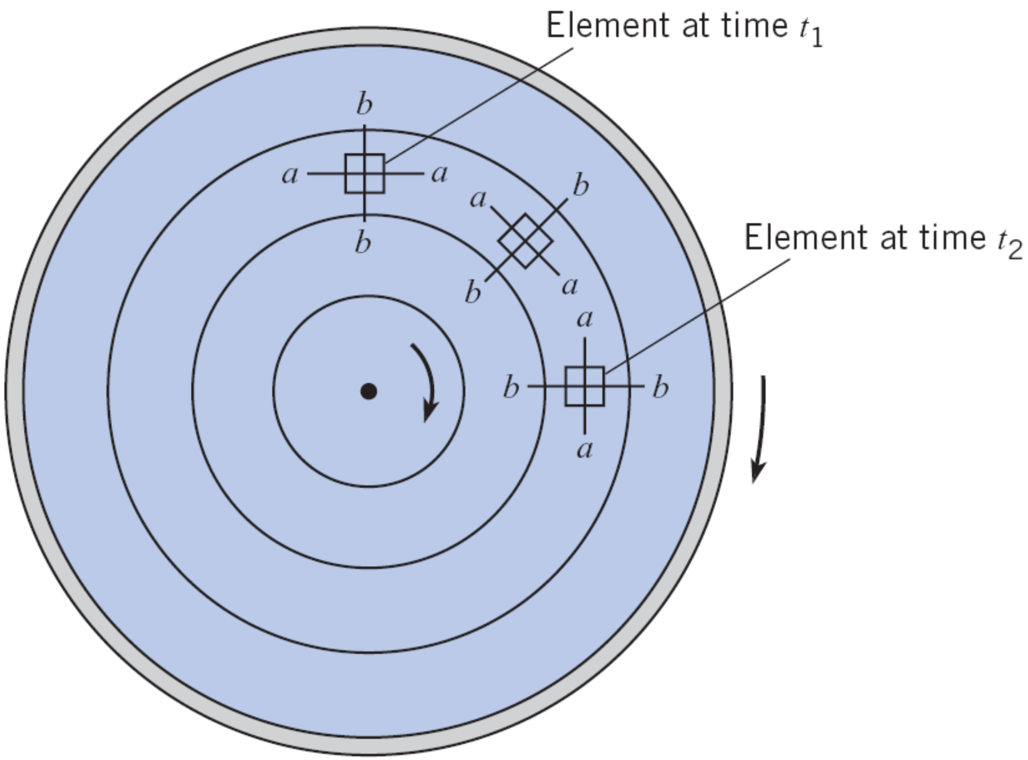
$$dp = \frac{\partial p}{\partial r} dr + \frac{\partial p}{\partial z} dz.$$

Substituting the values of $\frac{\partial p}{\partial r}$ from equation (5.21) and $\frac{\partial p}{\partial z}$ from equation (5.22), we get

$$dp = \rho \frac{V^2}{r} dr - \rho g dz \quad \text{...(5.23)}$$

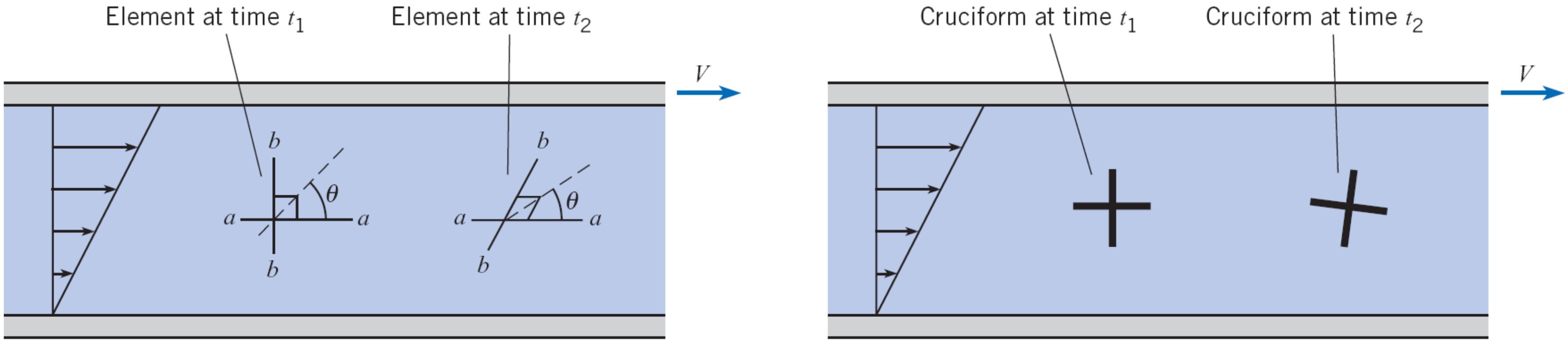
Equation (5.23) gives the variation of pressure of a rotating fluid in any plane.

Rotation and Vorticity , forced vertex with free surface



Rotation of a fluid element in a rotating tank of fluid (solid body rotation).

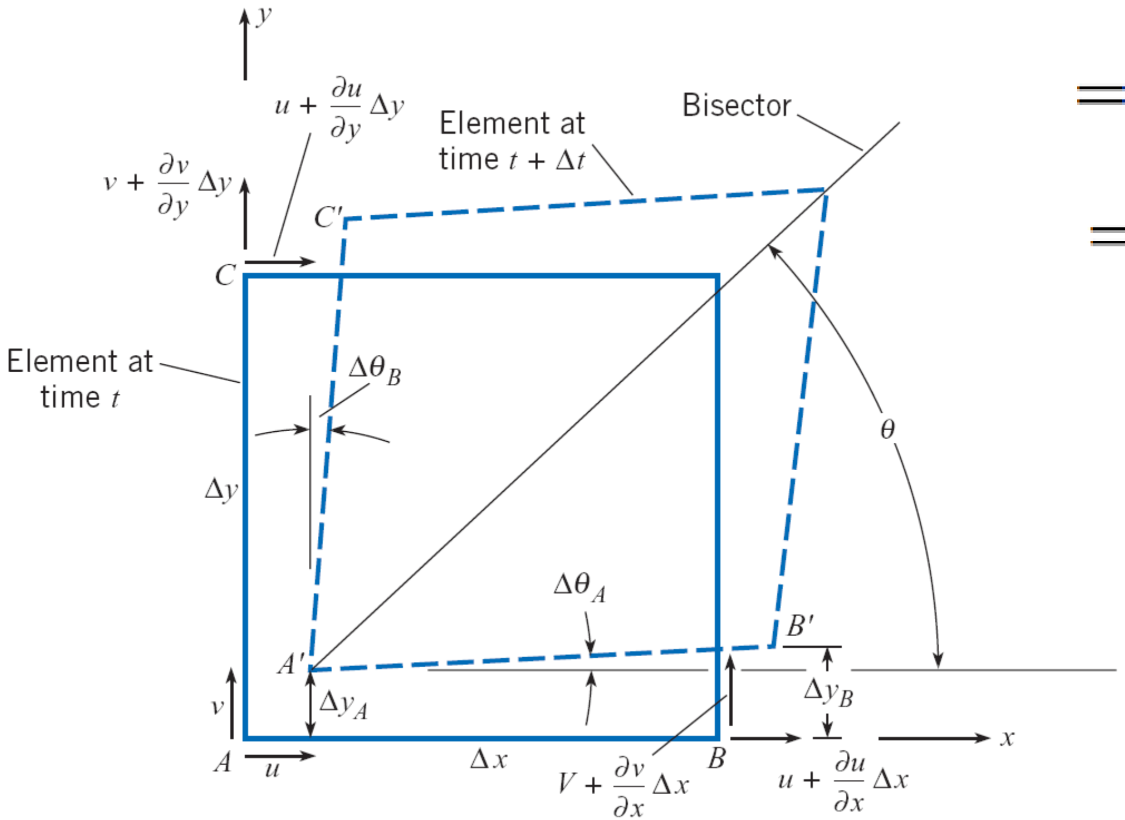
Rotation of fluid element in flow between moving and stationary parallel plates



(a) (b)

You can think of the “cruciforms” as small paddle wheels that are free to rotate about their center.

If the paddle wheel rotates, the flow is rotational at that point.



$$\Delta\theta_A = \frac{\Delta y_B - \Delta y_A}{\Delta x}$$

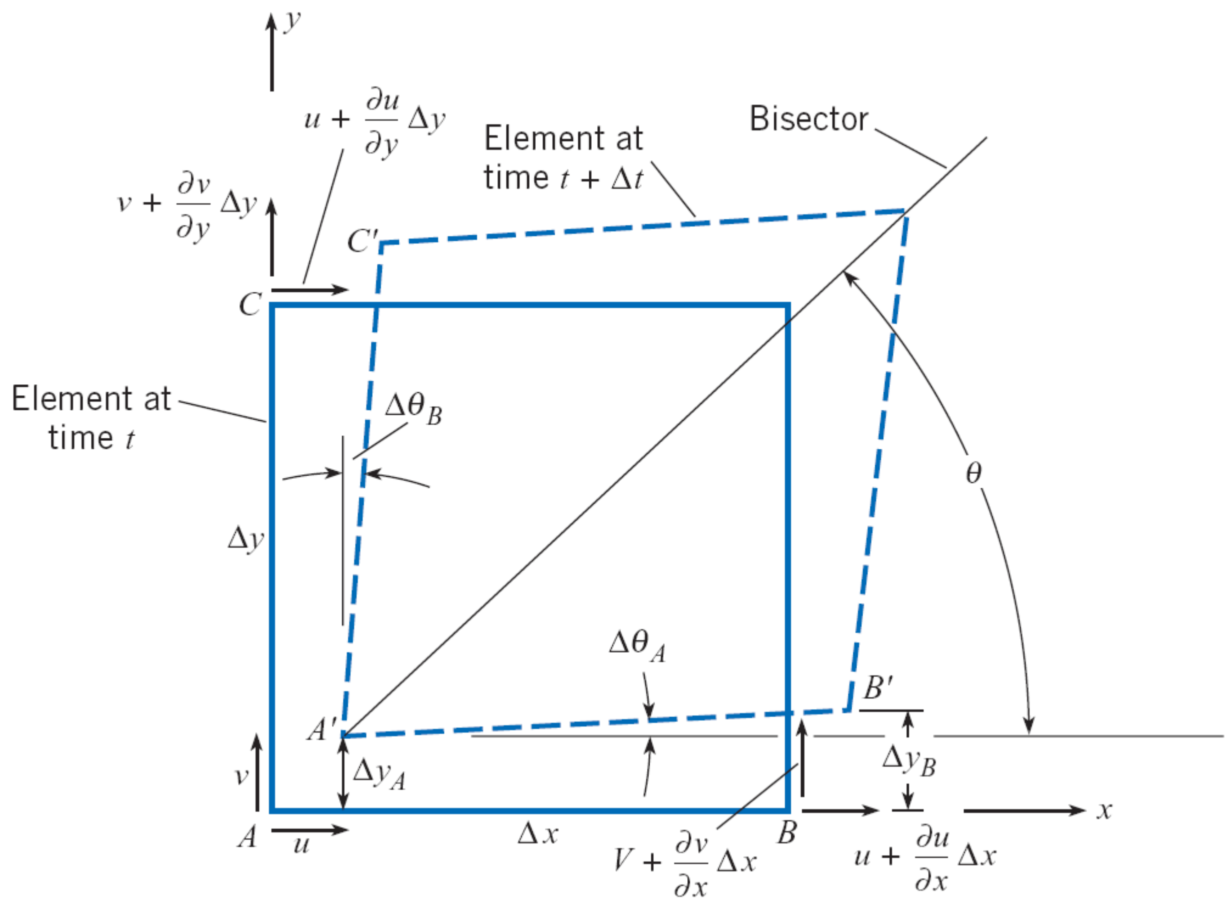
$$= \frac{\left(v + \frac{\partial v}{\partial x}\right)\Delta t - v\Delta t}{\Delta x}$$

$$= \frac{\partial v}{\partial x}\Delta t$$

As $\Delta t \rightarrow 0$

$$\dot{\theta}_A = \frac{\partial v}{\partial x}$$

And similarly $\dot{\theta}_B = \frac{\partial u}{\partial y}$



The net rate of rotation of the bisector is

$$\dot{\theta} = \frac{1}{2}\left(\dot{\theta}_A - \dot{\theta}_B\right)$$

$$= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

The rotation rate we just found was that about the z-axis; hence, we may call it

$$\Omega_z = \frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) \quad (= -\dot{\theta})$$

and similarly

$$\Omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\Omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

The rate-of-rotation vector is $\boldsymbol{\Omega} = \Omega_x \mathbf{i} + \Omega_y \mathbf{j} + \Omega_z \mathbf{k}$

Irrotational flow requires $\boldsymbol{\Omega} = 0$ (i.e., for all 3 components)

The property more frequently used is the **vorticity**

$$\boldsymbol{\omega} = 2\boldsymbol{\Omega}$$

$$= \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\mathbf{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\mathbf{j} + \left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)\mathbf{k}$$

$$= \boldsymbol{\nabla} \times \mathbf{V}$$

Vortices

A vortex is the motion of many fluid particles around a common center. The streamlines are concentric circles.

Choose coordinates such that z is perpendicular to flow.
In polar coordinates, the vorticity is

$$\Omega_z = \frac{1}{2}\left(\frac{dV}{dr} + \frac{V}{r}\right)$$ (V is function of r, only)

Solid body rotation <u>(forced vortex</u>):

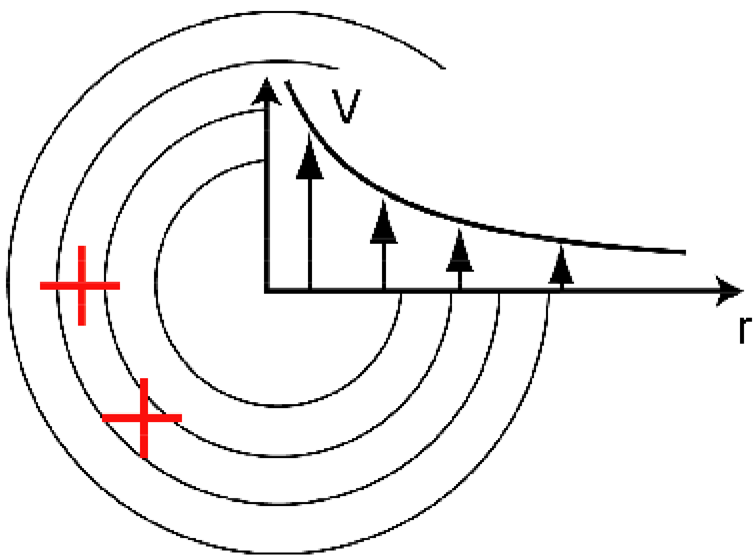
$V = \omega r$ or $\Omega_z = \omega$

Vortex with irrotational flow (<u>free vortex</u>):

$$\Omega_z = \frac{1}{2}\left(\frac{dV}{dr} + \frac{V}{r}\right) = 0$$

$$\Rightarrow \quad \frac{dV}{V} = -\frac{dr}{r}$$

$$\Rightarrow \quad V = \frac{C}{r}$$



A paddle wheel does not rotate in a free vortex!

Forced vertex with free surface

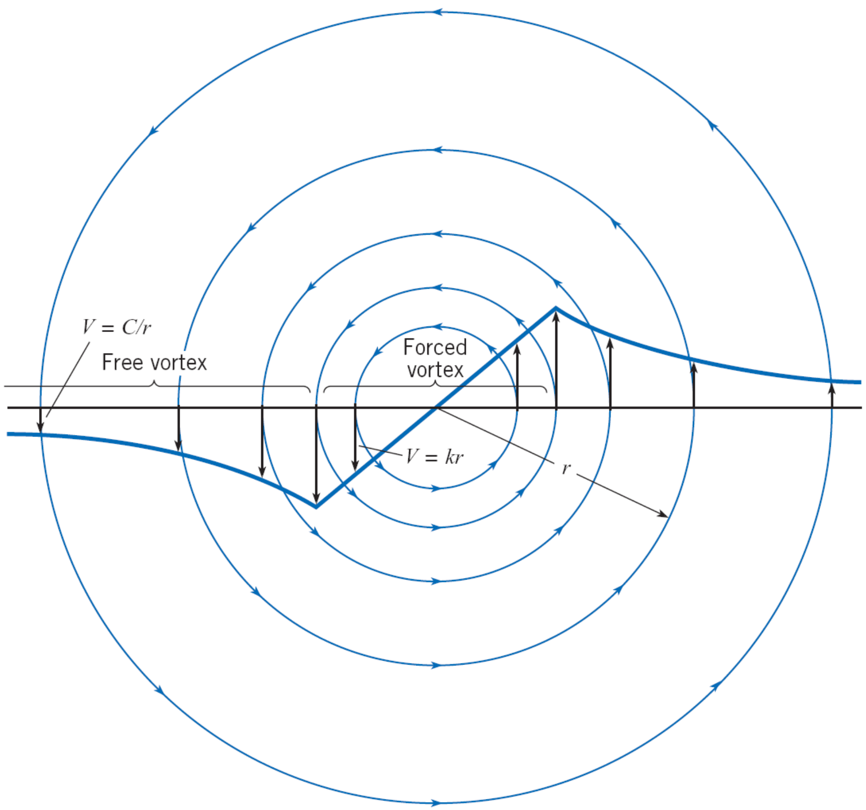
Forced vortex (interior) and free vortex (outside):

Good approximation to naturally occurring vortices such as tornadoes.

Euler's equation for any vortex:

$$-\frac{d}{dr}(p+\gamma z) = \rho a_r$$

$$a_r = -\frac{V^2}{r}$$



$$\Rightarrow \quad -\frac{d}{dr}(p+\gamma z) = -\rho\frac{V^2}{r}$$

We can find the pressure variation in different vortices (let's assume constant height z):

In general:

$$p = \int \rho \frac{V^2}{r}\, dr$$

$$V = \omega r$$

1) Solid body rotation:

$$\Rightarrow \quad p = \rho\omega \int \frac{r^2}{r}\, dr \qquad = \frac{1}{2}\omega r^2 + C_1$$

2) Free vortex (irrotational):

$$V = \frac{B}{r}$$

$$\Rightarrow \quad p = \rho B \int \frac{1}{r^3}\, dr \qquad = -\frac{\rho B}{2\, r^2} + C_2$$

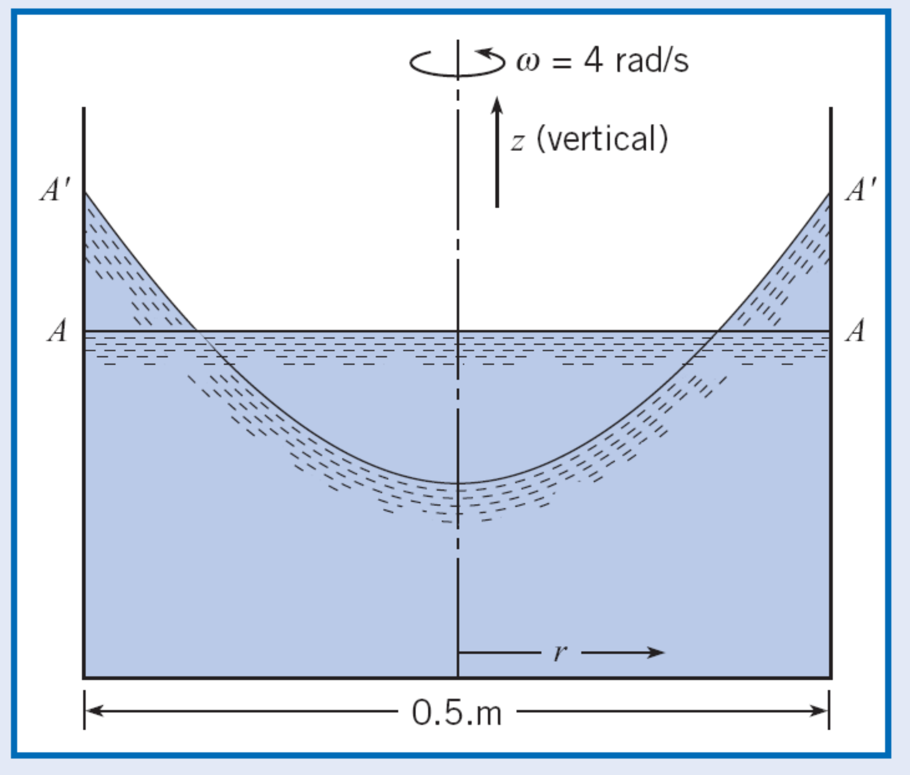
Application to forced vortex (solid body rotation):

$$-\frac{d}{dr}(p+\gamma z) = -\rho\frac{V^2}{r} \quad \text{with} \quad V = \omega r$$

$$\Rightarrow \frac{d}{dr}(p+\gamma z) = \rho r \omega^2$$

$$\Rightarrow \frac{p}{\gamma} + z - \frac{\omega^2 r^2}{2g} = C$$

Pressure as function of z and r



p = 0 gives free surface

Surface and Body Forces Euler Equation

- Fluid element accelerating in *l* direction & acted on by pressure and weight forces only (no friction)
- Newton's 2nd Law

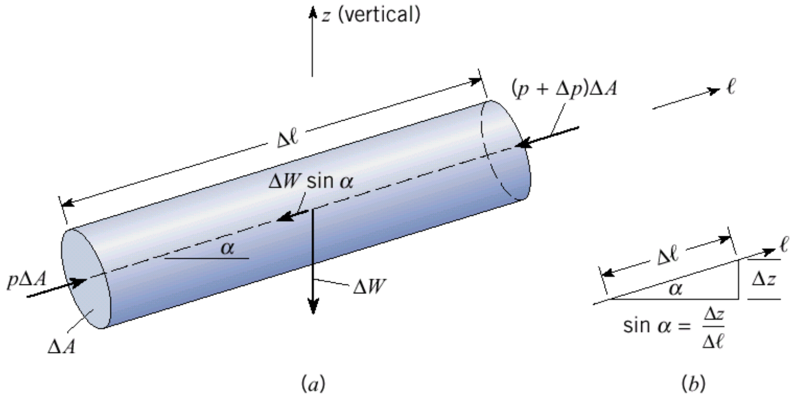
$$\sum F_l = Ma_l$$

$$p\Delta A - (p + \Delta p)\Delta A - \Delta W \sin\alpha = \rho \Delta l \Delta A a_l$$

$$p - (p + \Delta p) - \gamma \Delta l \sin\alpha = \rho \Delta l a_l$$

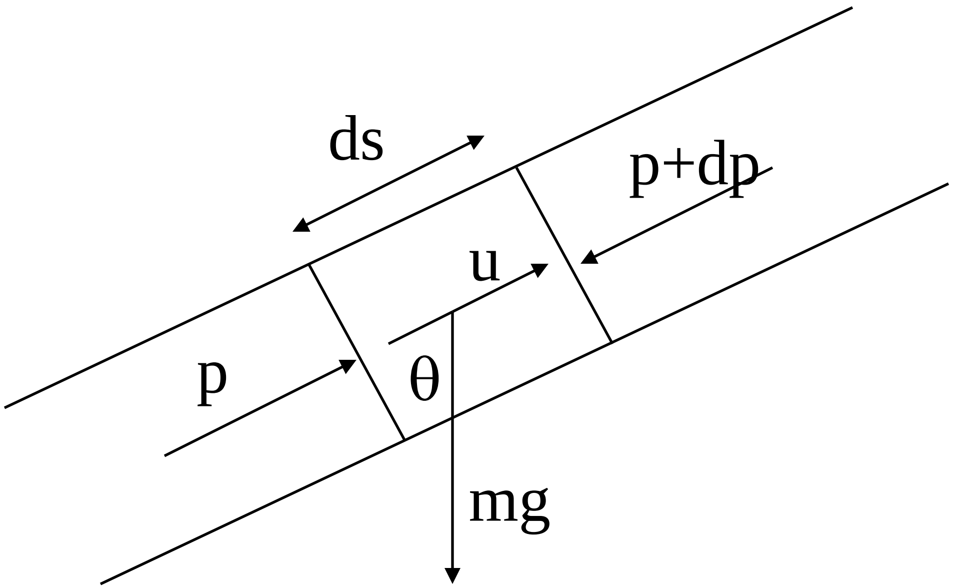
$$-\frac{dp}{dl} - \rho g \frac{dz}{dl} = \rho a_l$$

$$-\frac{d}{dl}\left(\frac{p}{\gamma} + z\right) = \frac{a_l}{g}$$



Euler's Equation

Flow along a streamtube of area A with no viscosity



Forces along streamline: $A\,dp + \rho g \cos(\theta) A\,ds + \rho A\,ds \dfrac{du}{dt} = 0$

pressure + gravity + inertia (or F=ma)

Euler's equation (cont)

Dividing through by ρAds:

$$\frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} + \frac{du}{dt} = 0$$

(We will come back to this equation later)

By the chain rule, the time derivative of u, which is a function of both s and t, may be expressed as:

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial t}$$

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial s}$$

Euler and Bernoulli

Euler's equation is independent of time, so for $\frac{\partial u}{\partial t} = 0$

$$\boxed{\frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} + u\frac{du}{ds} = 0}$$

Euler's equation

For an incompressible fluid, integrating along the streamline,

$$\boxed{\frac{p}{\rho g} + \frac{u^2}{2g} + z = \text{const}}$$

Bernoulli's equation

Navier-Stokes equations

- So far we have separately considered flow
  - in one dimension affected by pressure and gravity
  - in one dimension affected by pressure and viscosity
- Need three dimensions and all forces in order to provide a full solution for any general flow problem
- The following is not rigorous- see Bachelor for a rigorous derivation

Euler's equation (reminder)

$$\frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} + \frac{du}{dt} = 0$$

Navier-Stokes equations

Looking back to Euler's equation with unsteadiness, the gravity term is simply the component of gravity, g_s. Introducing viscosity as well gives 3 similar equations:

$$\rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \rho\frac{du}{dt}$$

$$\rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) = \rho\frac{dv}{dt}$$

$$\rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) = \rho\frac{dw}{dt}$$

Navier-Stokes equations

- There is no general solution to the N-S equations
- Some analytical solutions may be obtained by simplification
- The equations may be written in vector (div/grad) notation:

$$\boxed{\rho \underline{g} - \nabla p + \mu \nabla^2 \underline{u} = \rho \frac{d\underline{u}}{dt}}$$

Channel flow

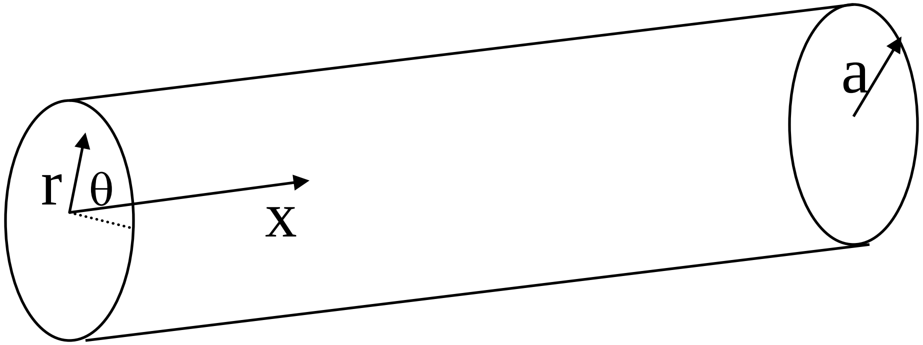
The n.s. equations therefore reduce to:

$$\mu \frac{d^2 u}{dy^2} = \frac{dp}{dx}$$

Which may be solved as before.

Applications

Consider steady laminar flow in a horizontal circular pipe of radius, a.



For flow in a pipe, cylindrical polar coordinates x, r, θ, are most useful.

The Navier-Stokes equation:

$$\rho \underline{g} - \nabla p + \mu \nabla^2 \underline{u} = \rho \frac{d\underline{u}}{dt}$$ (= 0 for steady flow in a straight pipe)

may be expressed as three components in x, r, θ, but for steady flow in the x-direction, we only need one component:

$$\rho g_x - \frac{\partial p}{\partial x} + \mu \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r} \right) = 0$$

And we will drop the gravitational term for a horizontal pipe.

$$\mu \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r} \right) = \frac{\partial p}{\partial x}$$

Integrating twice wrt r, gives:

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^{2} + A\, ln(r) + B$$

The constant A must be zero, because at the centre of the pipe *ln*(0) is infinite. The bc at the wall, u=0 at r=a gives a value for B.

Poisseuille equation

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x}\left(r^{2} - a^{2}\right)$$

Momentum Equation

Moment of momentum equation is derived from moment of momentum principle which states that the resulting torque acting on a rotating fluid is equal to the rate of change of moment of momentum.

Let V_1 = velocity of fluid at section 1

r_1 = radius of curvature at section 1,

Q = rate of flow of fluid,

ρ = density of fluid,

and V_2 and r_2 = velocity and radius of curvature at section 2

Momentum of fluid at section 1 = mass × velocity = $\rho Q \times V_1/s$

$\therefore$ Moment of momentum per second at section 1,

$$= \rho Q \times V_1 \times r_1$$

Similarly moment of momentum per second of fluid at section 2

$$= \rho Q \times V_2 \times r_2$$

$\therefore$ Rate of change of moment of momentum

$$= \rho Q V_2 r_2 - \rho Q V_1 r_1 = \rho Q [V_2 r_2 - V_1 r_1]$$

According to moment of momentum principle

Resultant torque = rate of change of moment of momentum

or $$T = \rho Q [V_2 r_2 - V_1 r_1] \quad \text{...(6.23)}$$

Equation (6.23) is known as moment of momentum equation. This equation is applied :

1. For analysis flow problems in turbines and centrifugal pumps.
2. For finding torque exerted by water on sprinkler.

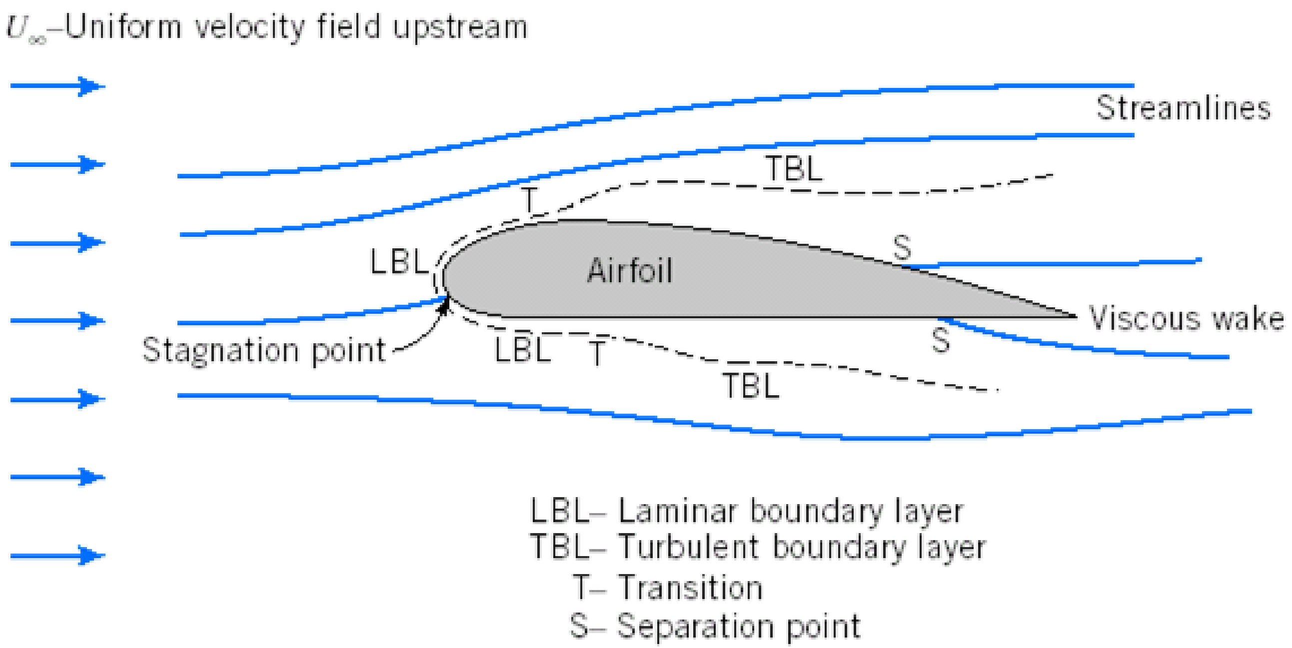
Thank U

UNIT - IV

Boundary Layer flows & Pipe Flows

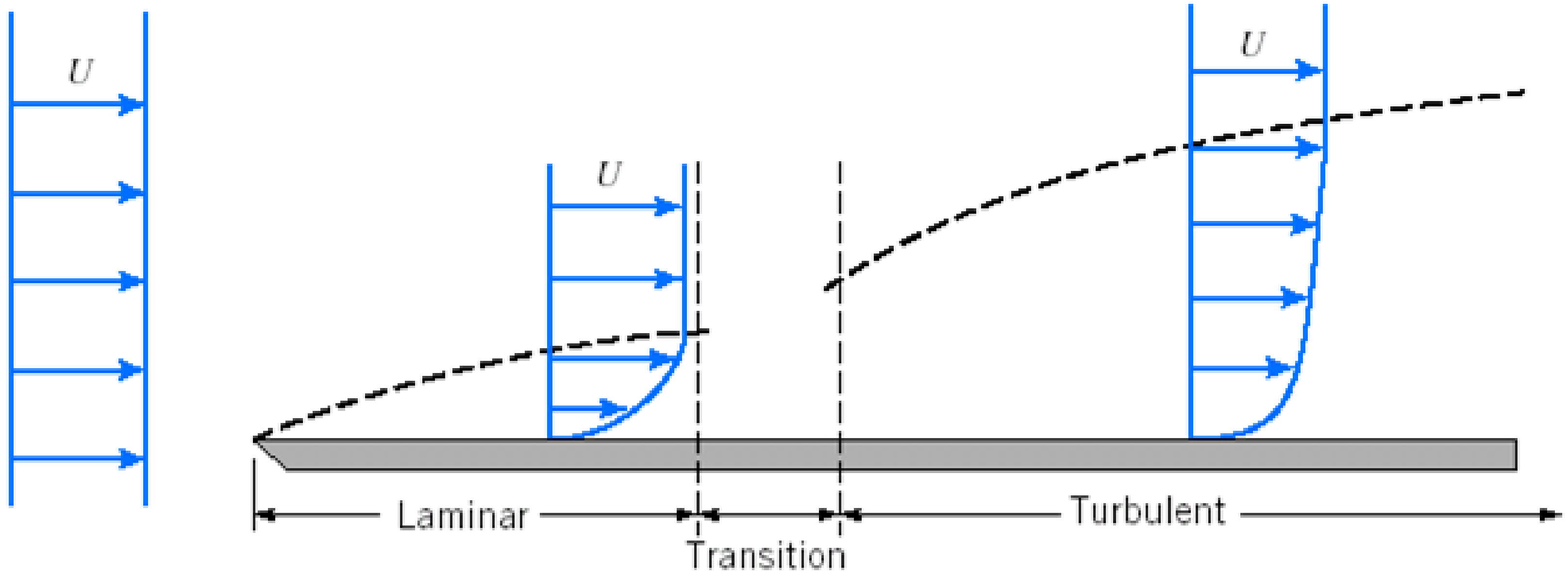
Boundary layer- concepts, Prandtl contribution, Characteristics of boundary layer along a thin flat plate, laminar and turbulent Boundary layers, BL in transition, separation of BL, control of BL separation, flow around submerged objects, Drag and lift types of drag magnus effect.

The Boundary-Layer Concept



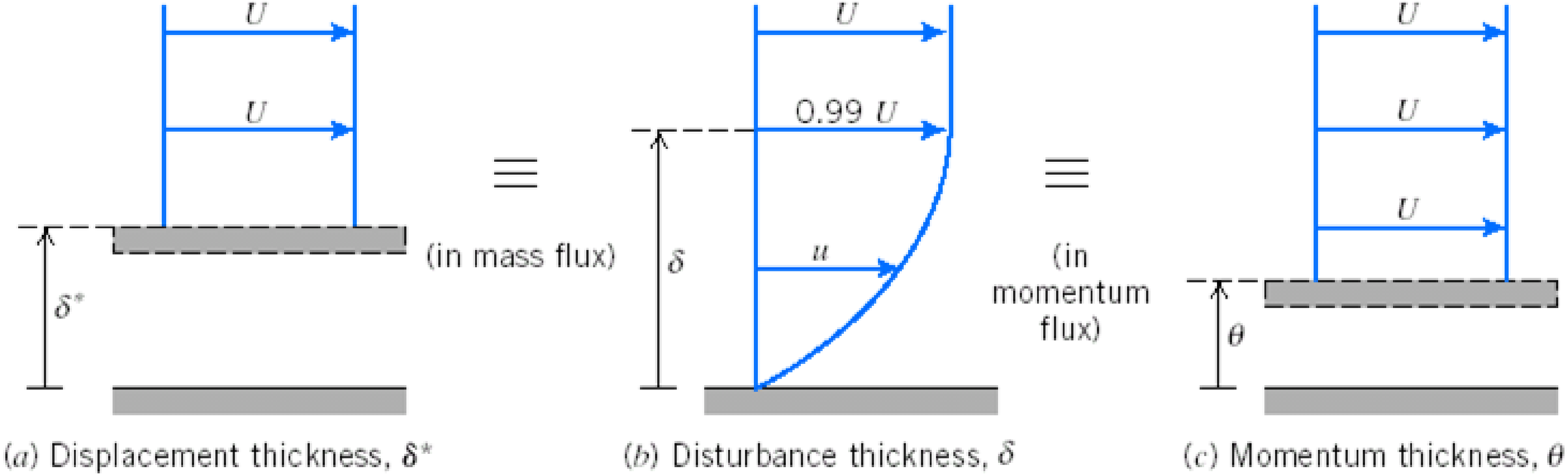
Details of Viscous flow Around an Airfoil.

The Boundary-Layer Concept



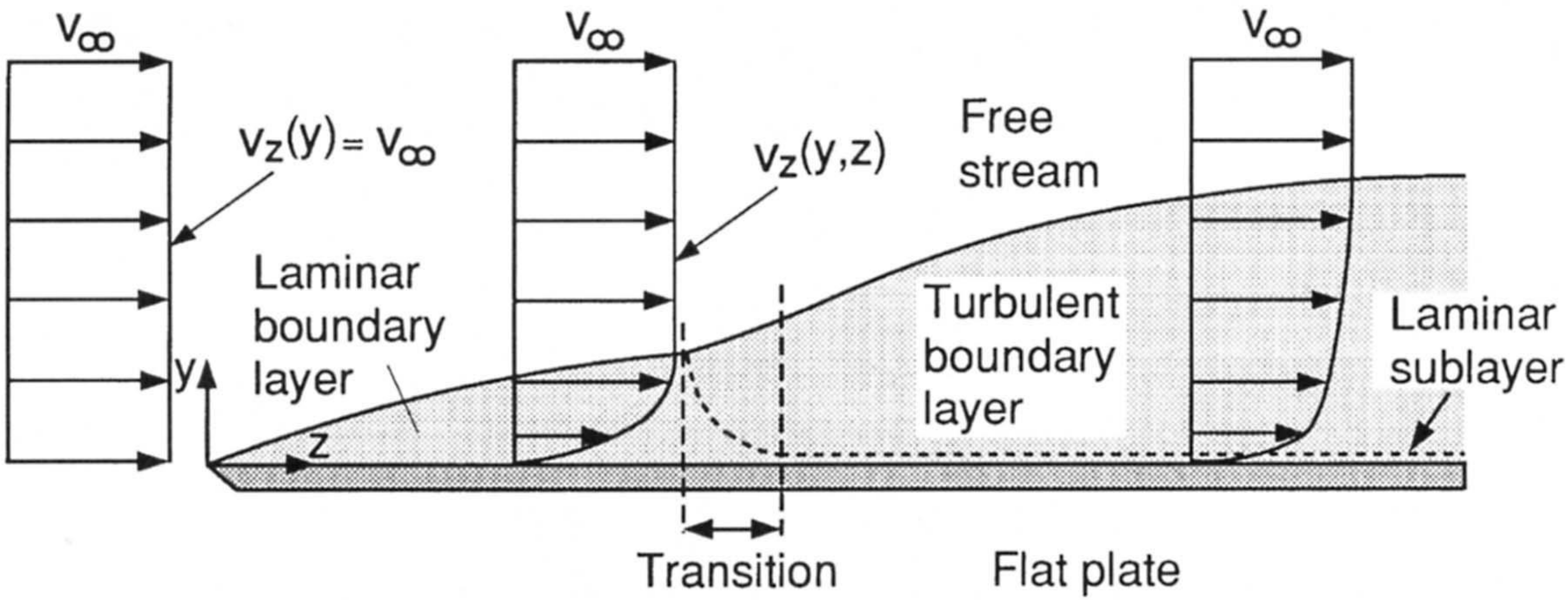
Boundary Layer on a flat Plate(Vertical thickness exaggerated greatly)

Boundary Layer Thicknesses



Boundary-Layer thickness Definitions

Boundary Layer Characteristics



Momentum boundary Layer over a flat plate: Laminar-to-Turbulent Transition

$$\frac{\partial v_z}{\partial y} = 0$$

Boundary Layer Thicknesses

- Disturbance Thickness, δ

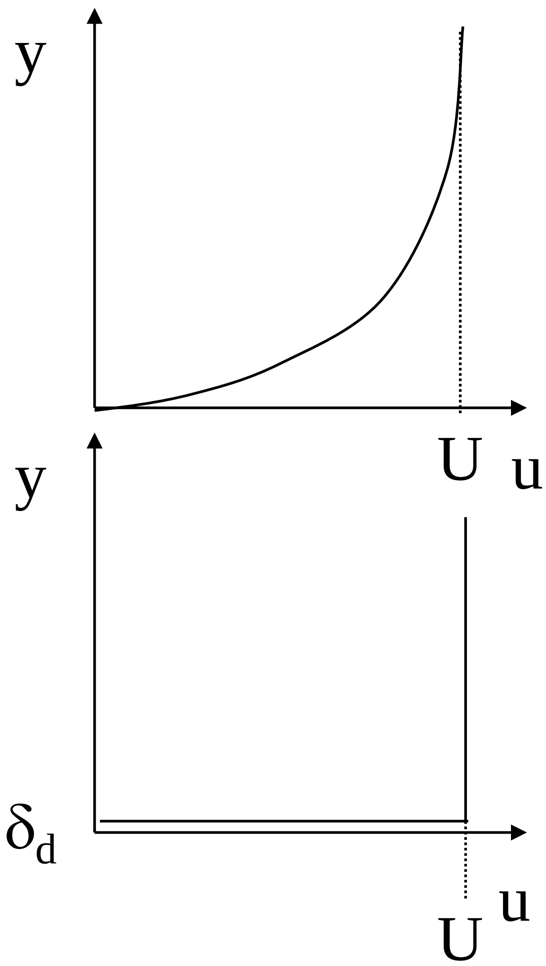
✓ **Displacement Thickness, δ^***

$$\delta^* \approx \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$$

✓ **Momentum Thickness, θ**

$$\theta \approx \int_0^{\delta} \frac{u}{U} \left(1 - \frac{u}{U}\right) dy$$

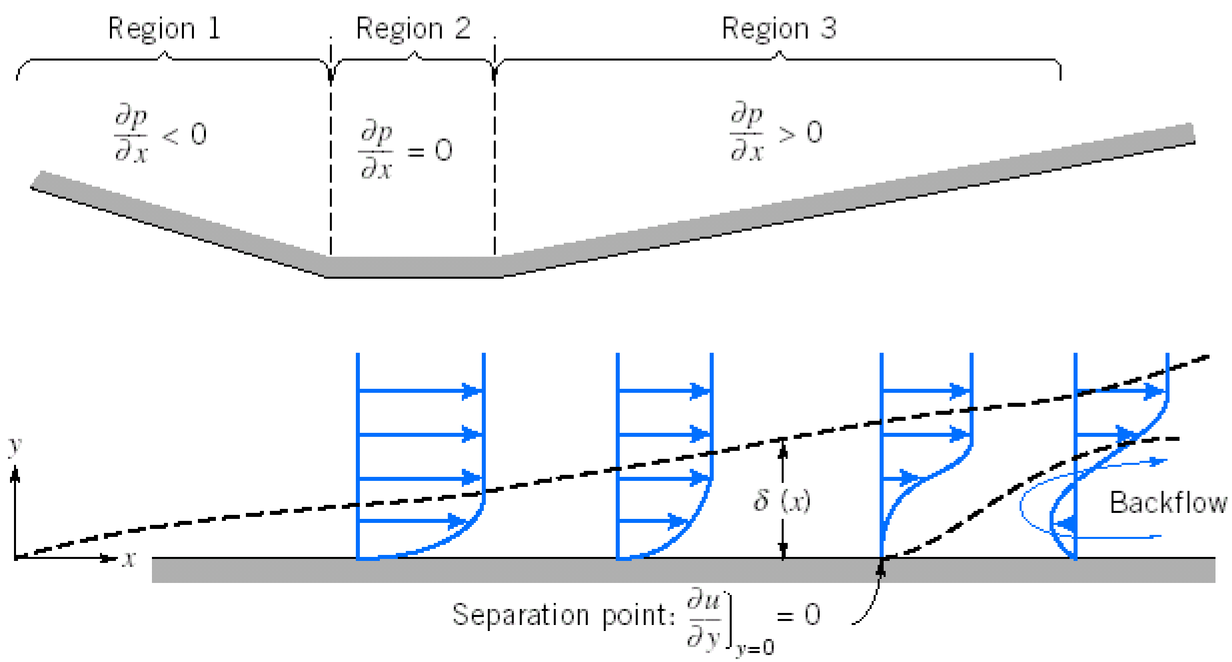
Displacement thickness



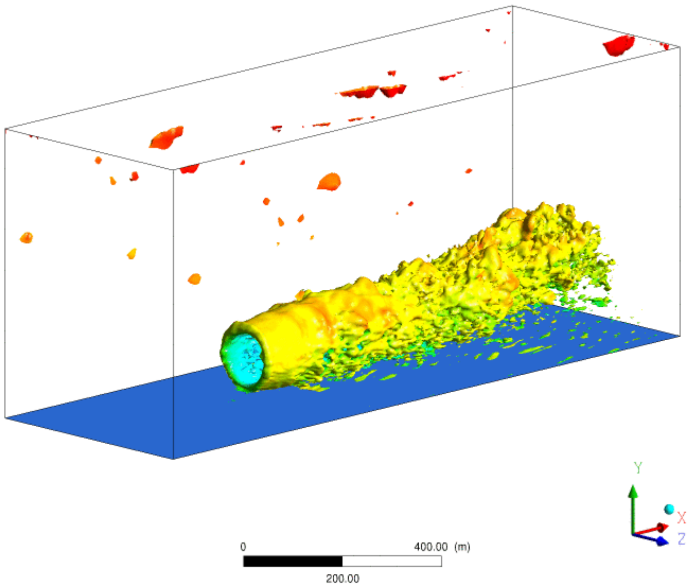
There is a reduction in the flow rate due to the presence of the boundary layer

This is equivalent to having a theoretical boundary layer with zero flow

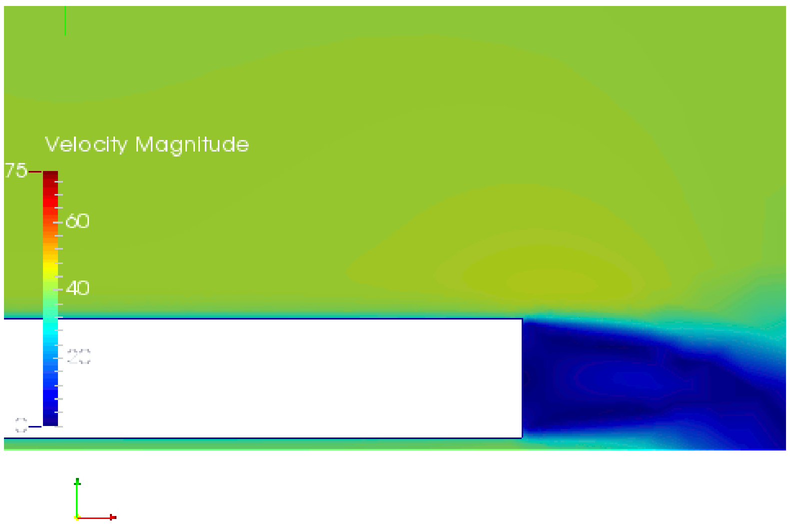
Pressure Gradients in Boundary-Layer Flow



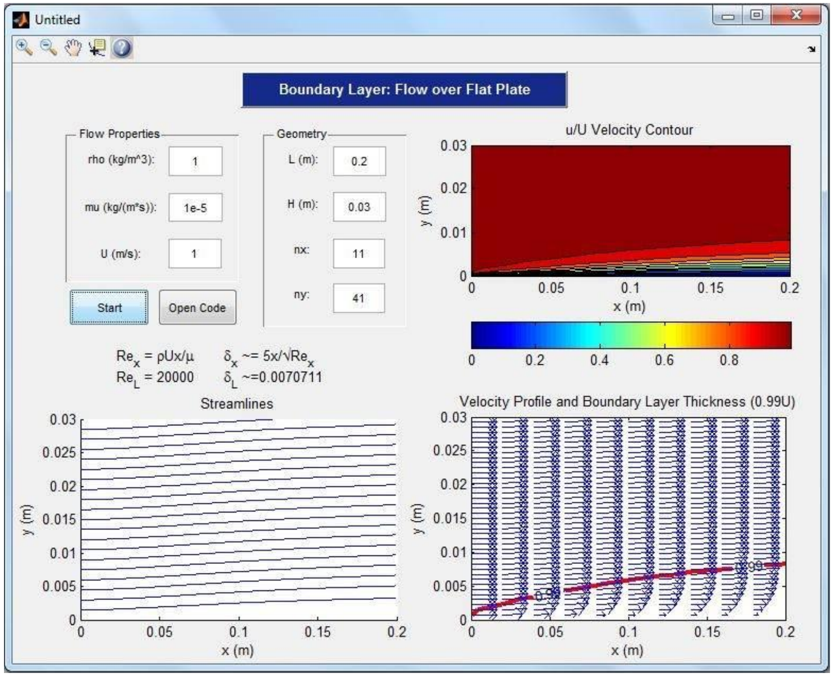
Boundary Layer flow with pressure gradient (Boundary layer thickness)

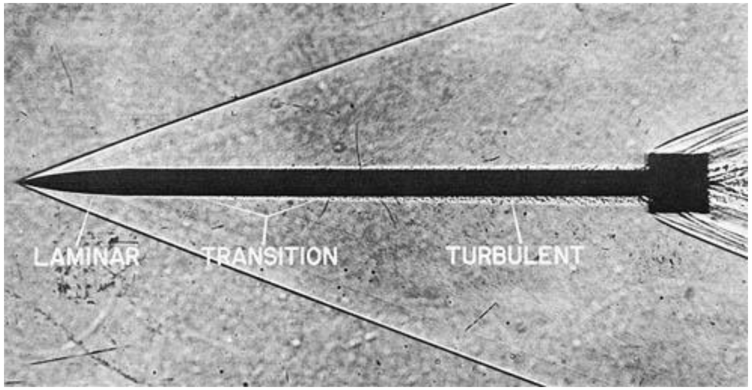


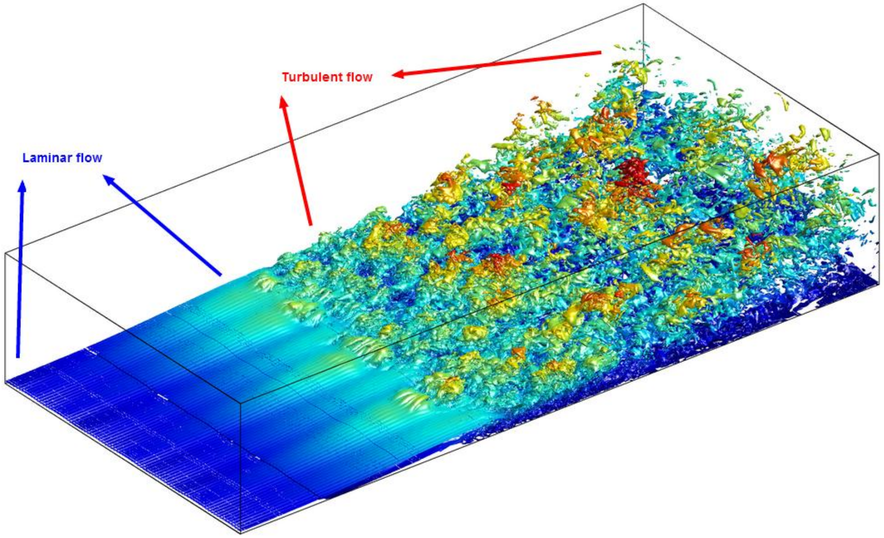
Y
X
Z
0
400.00 (m)
200.00



Velocity Magnitude
75
60
40


Untitled
Boundary Layer: Flow over Flat Plate
Flow Properties
rho (kg/m^3): 1
mu (kg/(m*s)): 1e-5
U (m/s): 1
Geometry
L (m): 0.2
H (m): 0.03
nx: 11
ny: 41
Start
Open Code
Re_x = ρUx/μ δ_x ~= 5x/√Re_x
Re_L = 20000 δ_L ~=0.0070711
u/U Velocity Contour
Streamlines
Velocity Profile and Boundary Layer Thickness (0.99U)
x (m)
y (m)


LAMINAR
TRANSITION
TURBULENT


Turbulent flow
Laminar flow

Methods to Prevent Separation of Boundary Layer

- Suction of the slow moving fluid by a suction slot
- Supplying additional energy from the blower
- Providing a bypass in the slotted wing
- Rotating boundary in the direction of flow
- Providing small divergence in a diffuser
- Providing guide-blades in a bend
- Providing a trip-wire ring in the laminar region for the flow a sphere.

Lift force

$F_L = \frac{1}{2} C_L \rho A_P v^2$

F_L is lift force,

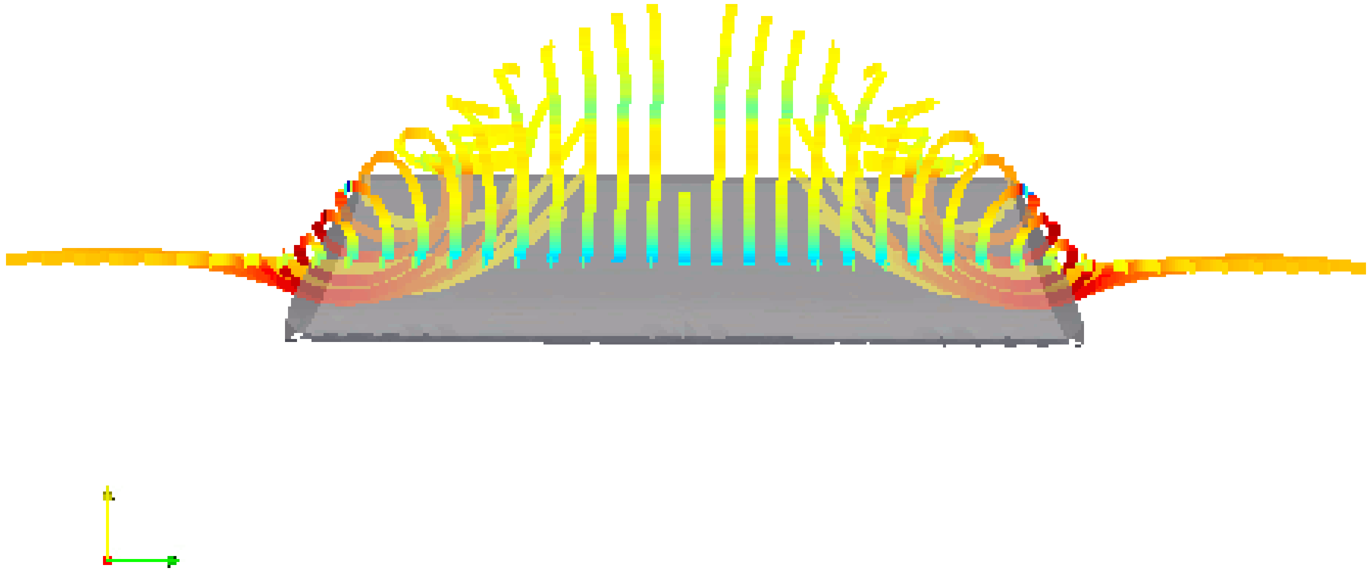
C_L is the coefficient of lift,

ρ is fluid density,

A_P is the projected area of the body or surface area orientated perpendicular to the fluid flow, and

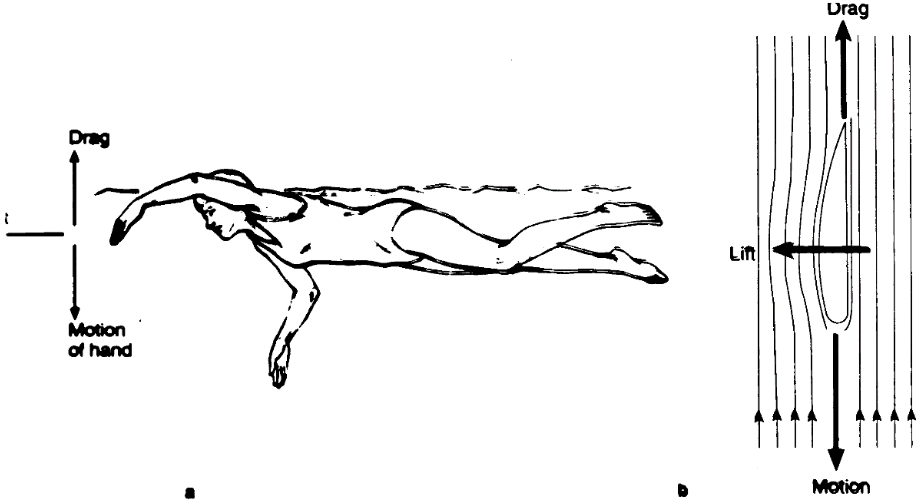
v is relative velocity of the body with respect to the fluid.

Note: The size, shape and orientation of the body (angle of attack) in the fluid are essential for generating lift force. The lift force increases with the square of the flow of velocity similar to drag force, but lift force increases are an advantage in sporting activities.

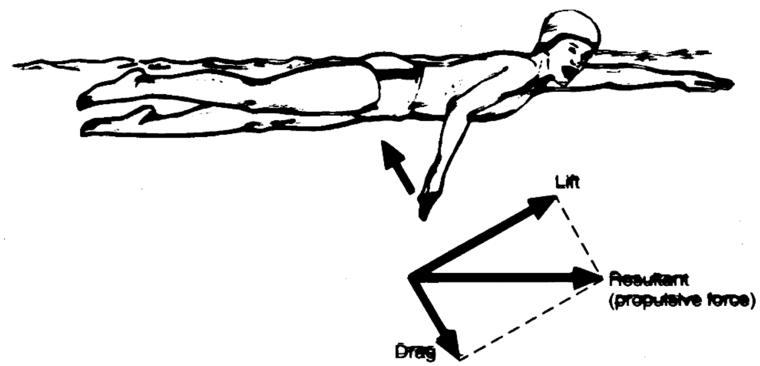


The lift/drag ratio

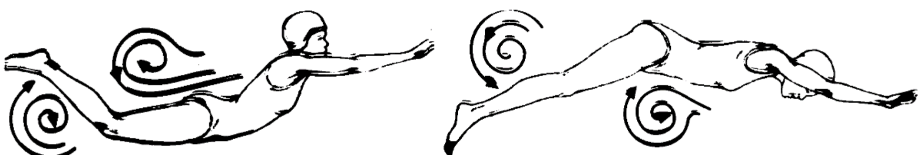
- The aim in sport is to maximize lift force while reducing drag force. The angle of attack of projected objects (a swimmer's hand) is constantly changing throughout the flight path, and therefore, the lift/drag ratio changes as well.



j. 6.22. The downward motion of the crawl swimmer's hand produces propulsive lift.



j. 6.23. In the final phase of the arm pull in the crawl stroke, the resultant of lift and drag is a propulsive force.



Drag Coefficient

$$C_D \equiv \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

with $$C_D = f(Re)$$

or $$C_D = f(Re, Fr, M)$$

Drag types:

- Pure Friction Drag: Flat Plate Parallel to the Flow
- Pure Pressure Drag: Flat Plate Perpendicular to the Flow
- Friction and Pressure Drag: Flow over a Sphere and Cylinder
- Streamlining

- Flow over a Flat Plate Parallel to the Flow: Friction Drag

$$F_D = \int_{\text{plate surface}} \tau_w \, dA$$

Boundary Layer can be 100% laminar, partly laminar and partly turbulent, or essentially 100% turbulent; hence several different drag coefficients are available

- Flow over a Flat Plate Parallel to the Flow: Friction Drag (Continued)

Laminar BL: $$C_D = \frac{1.33}{\sqrt{Re_L}}$$

Turbulent BL: $$C_D = \frac{0.0742}{Re_L^{1/5}}$$

… plus others for transitional flow

- Flow over a Flat Plate Perpendicular to the Flow: Pressure Drag

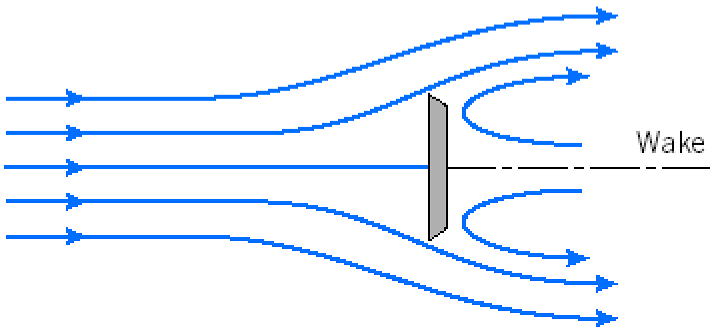
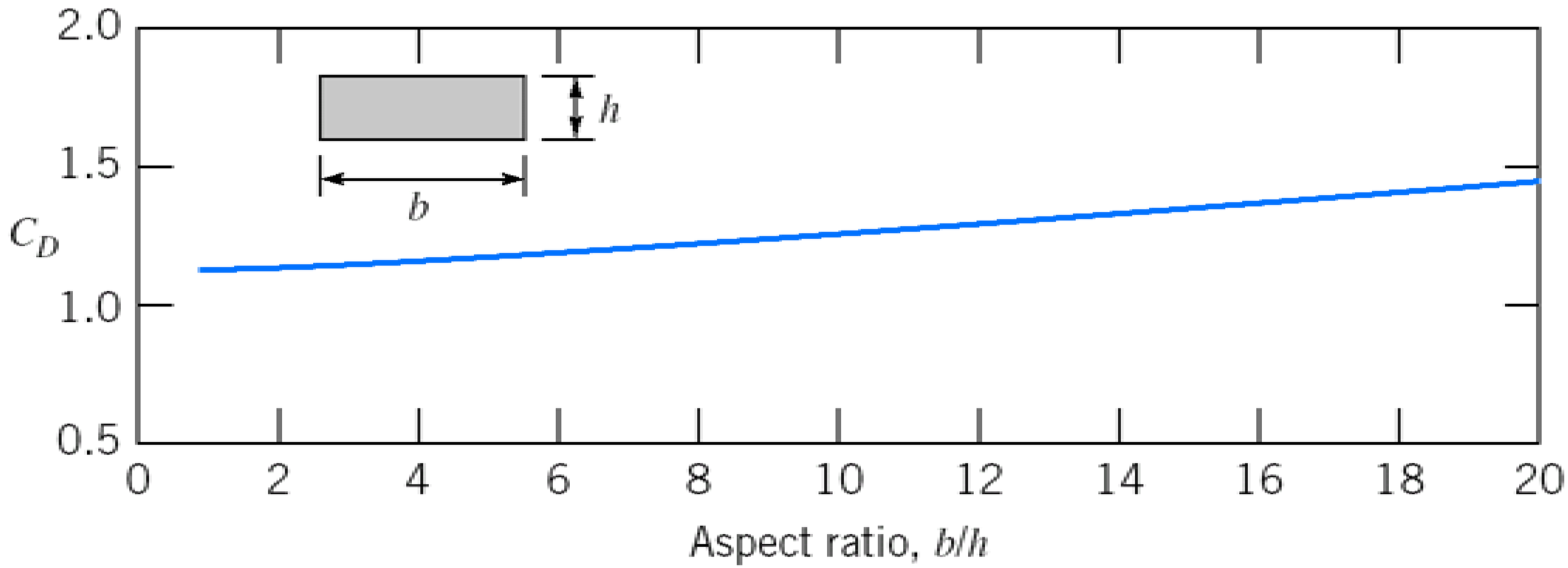


Fig. 9.9 Flow over a flat plate normal to the flow.

$$F_D = \int_{\text{surface}} p \, dA$$

Drag coefficients are usually obtained empirically

- Flow over a Flat Plate Perpendicular to the Flow: Pressure Drag (Continued)



Variation of drag coefficient with aspect ratio for a flat plate of finite width normal to the flow with $Re_h > 1000$ [16].

- Flow over a Sphere and Cylinder: Friction and Pressure Drag

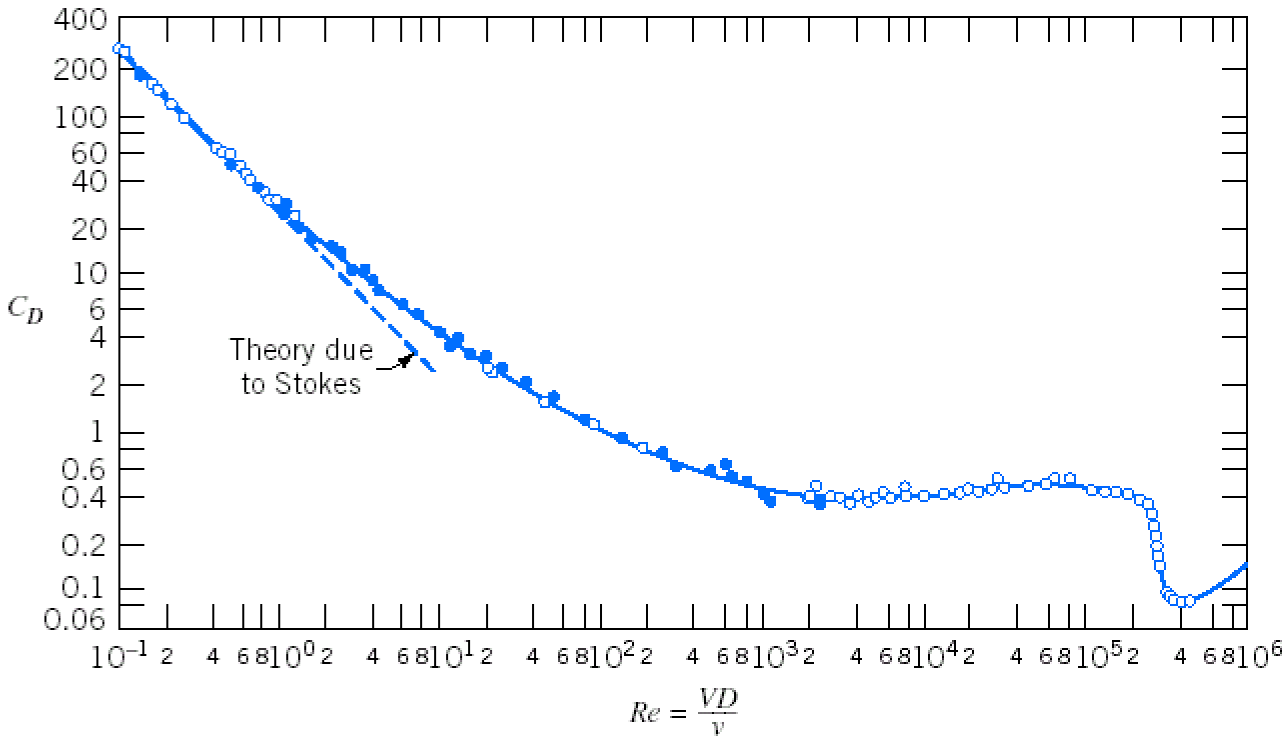
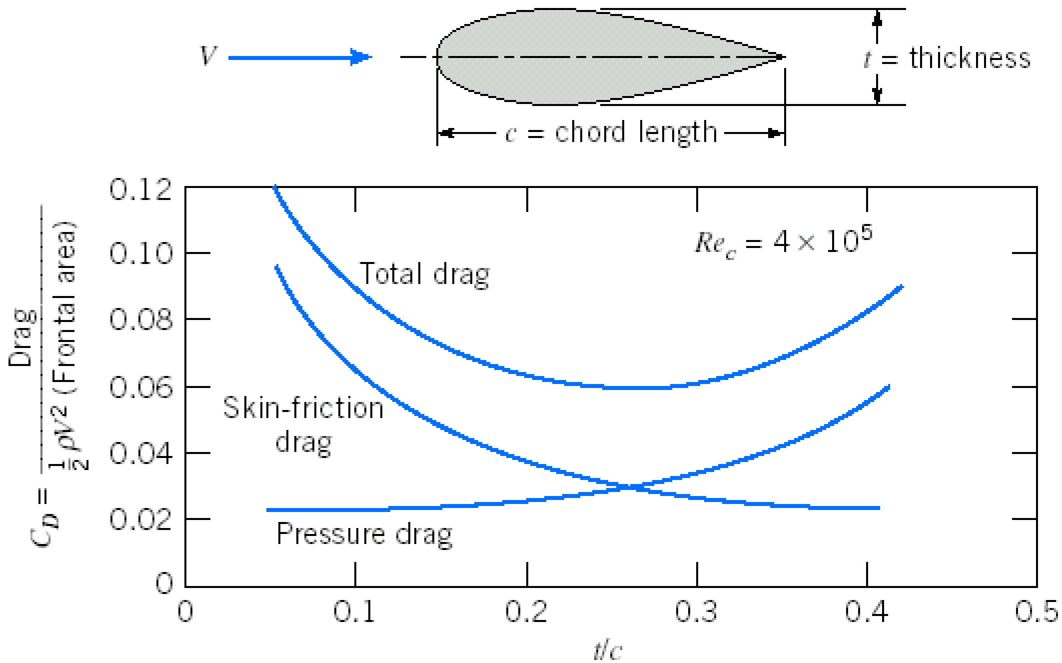


Fig. 9.11 Drag coefficient of a smooth sphere as a function of Reynolds number [3].

Streamlining(control of boundary layer separation)

- Used to Reduce Wake and hence Pressure Drag



Drag coefficient on a streamlined strut as a function of thickness ratio, showing contributions of skin friction and pressure to total drag [19].

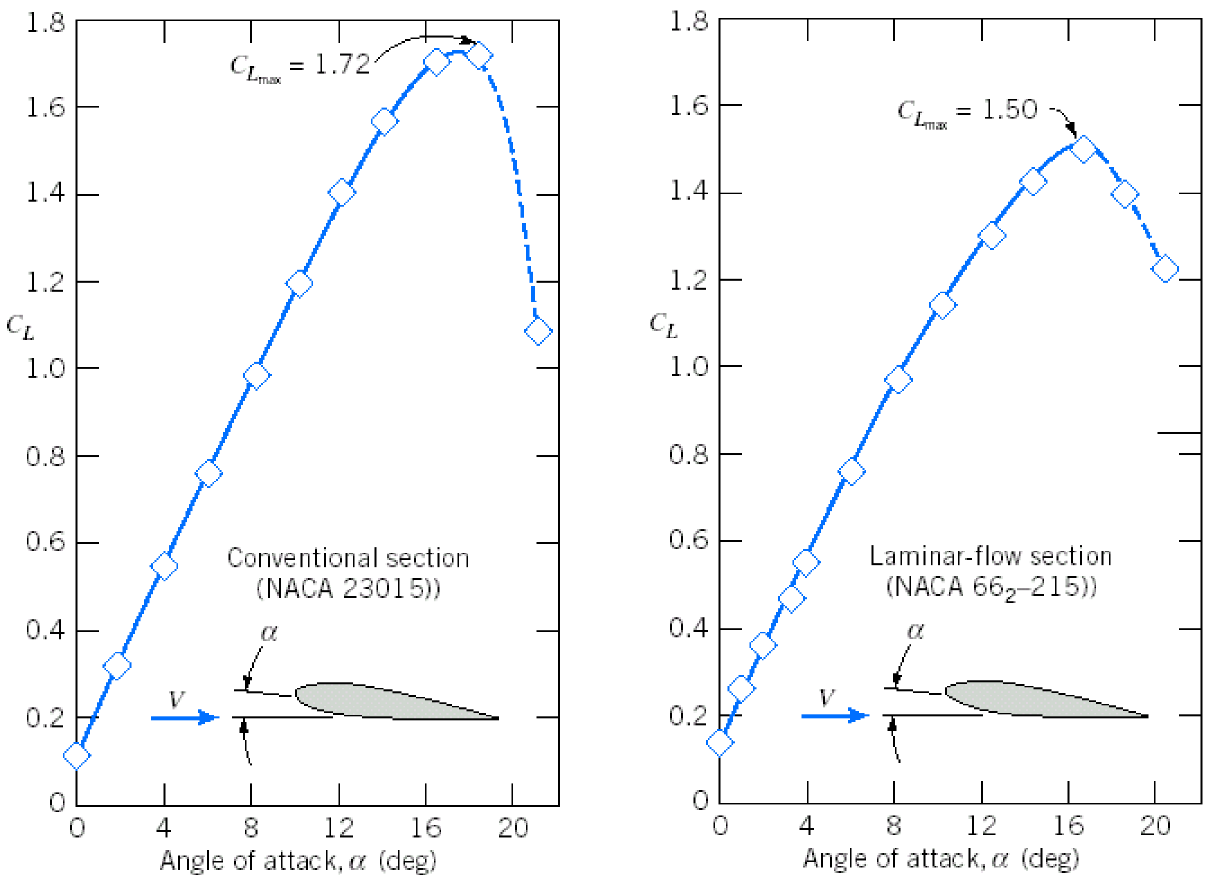
Lift

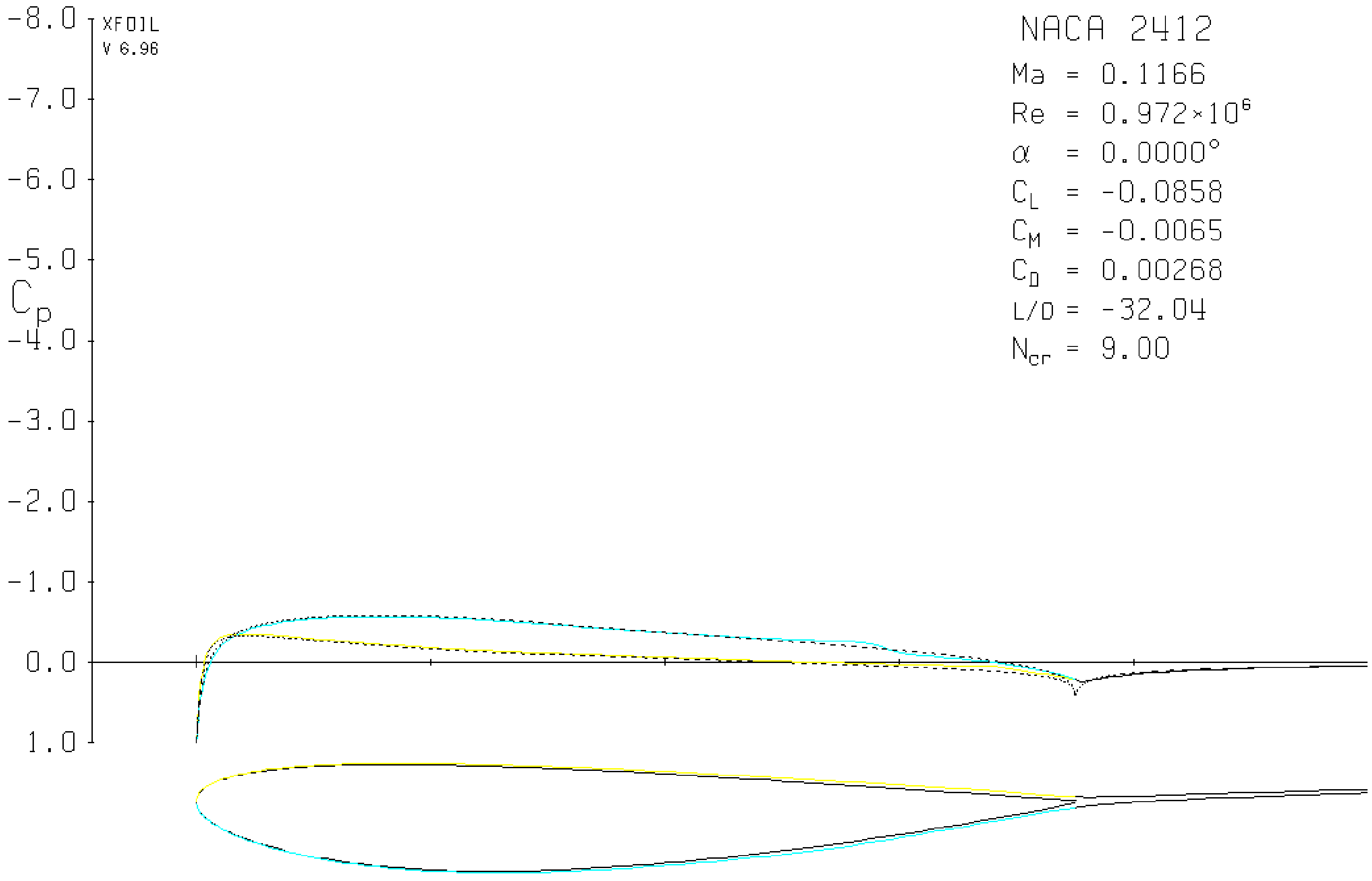
- Mostly applies to Airfoils

$$C_L \equiv \frac{F_L}{\frac{1}{2}\rho V^2 A_p}$$

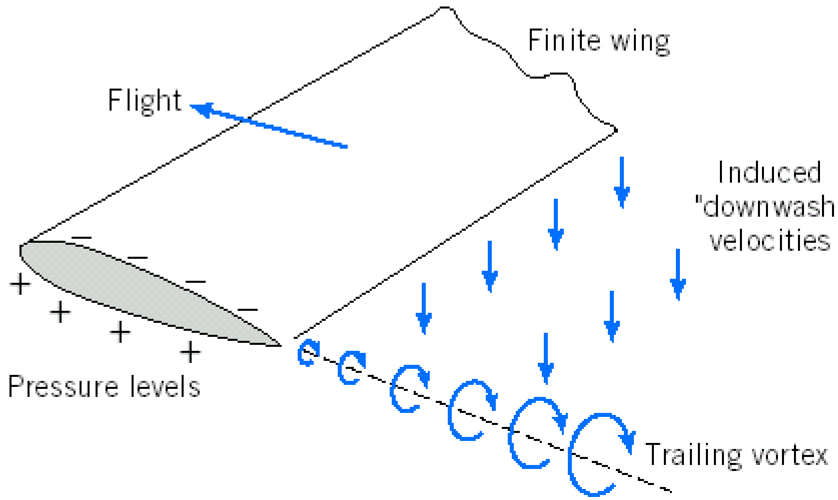
Note: Based on planform area *A_p*

- Examples: NACA 23015; NACA 66_2-215

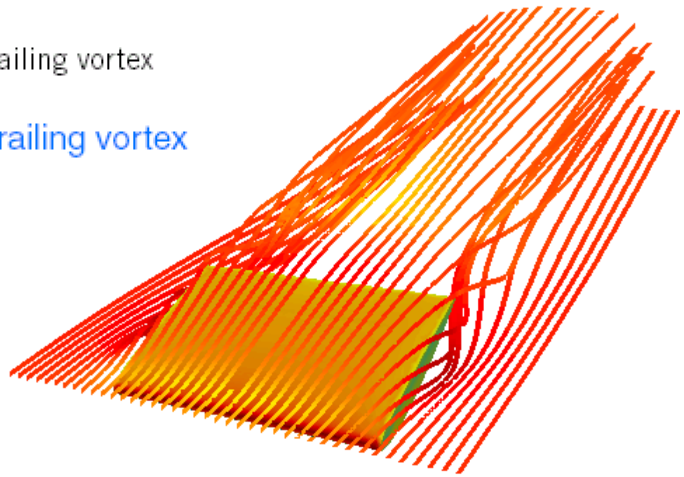



XFOIL
V 6.96
-8.0
-7.0
-6.0
-5.0
C_p
-4.0
-3.0
-2.0
-1.0
0.0
1.0
NACA 2412
Ma = 0.1166
Re = 0.972×10^6
α = 0.0000°
C_L = -0.0858
C_M = -0.0065
C_D = 0.00268
L/D = -32.04
N_{cr} = 9.00

- Induced Drag



Schematic representation of the trailing vortex system of a finite wing.



- Induced Drag (Continued)

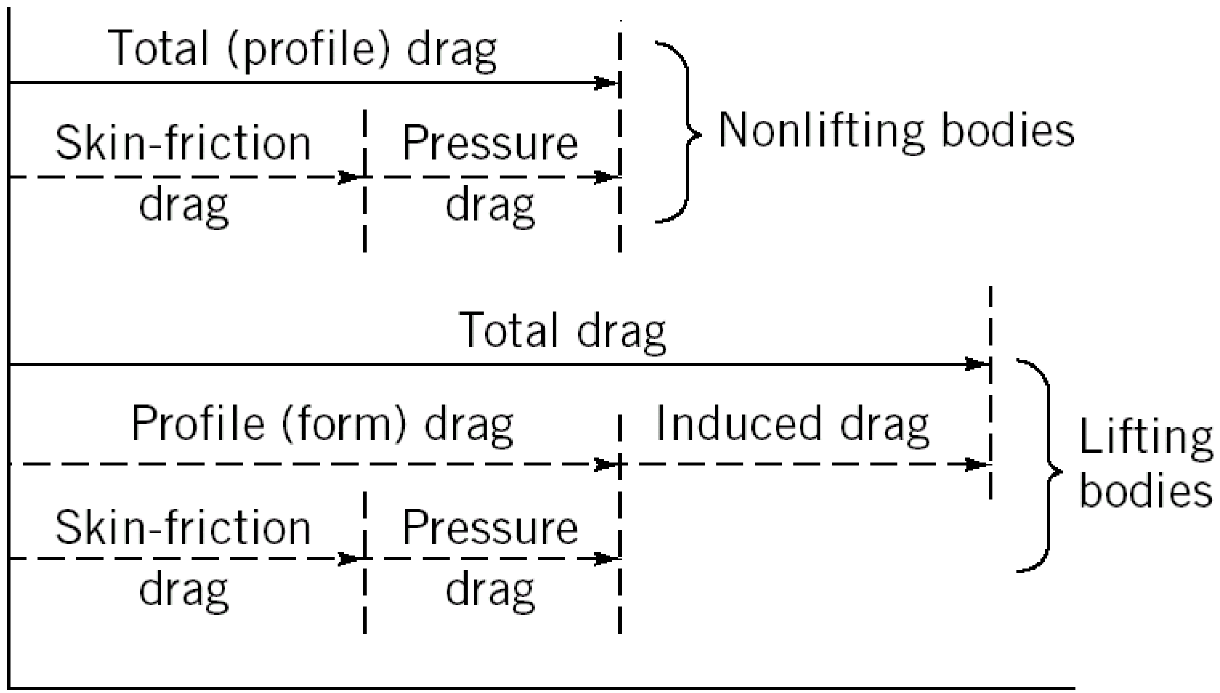
Reduction in Effective Angle of Attack:

$$\Delta\alpha \approx \frac{C_L}{\pi ar}$$

Finite Wing Drag Coefficient:

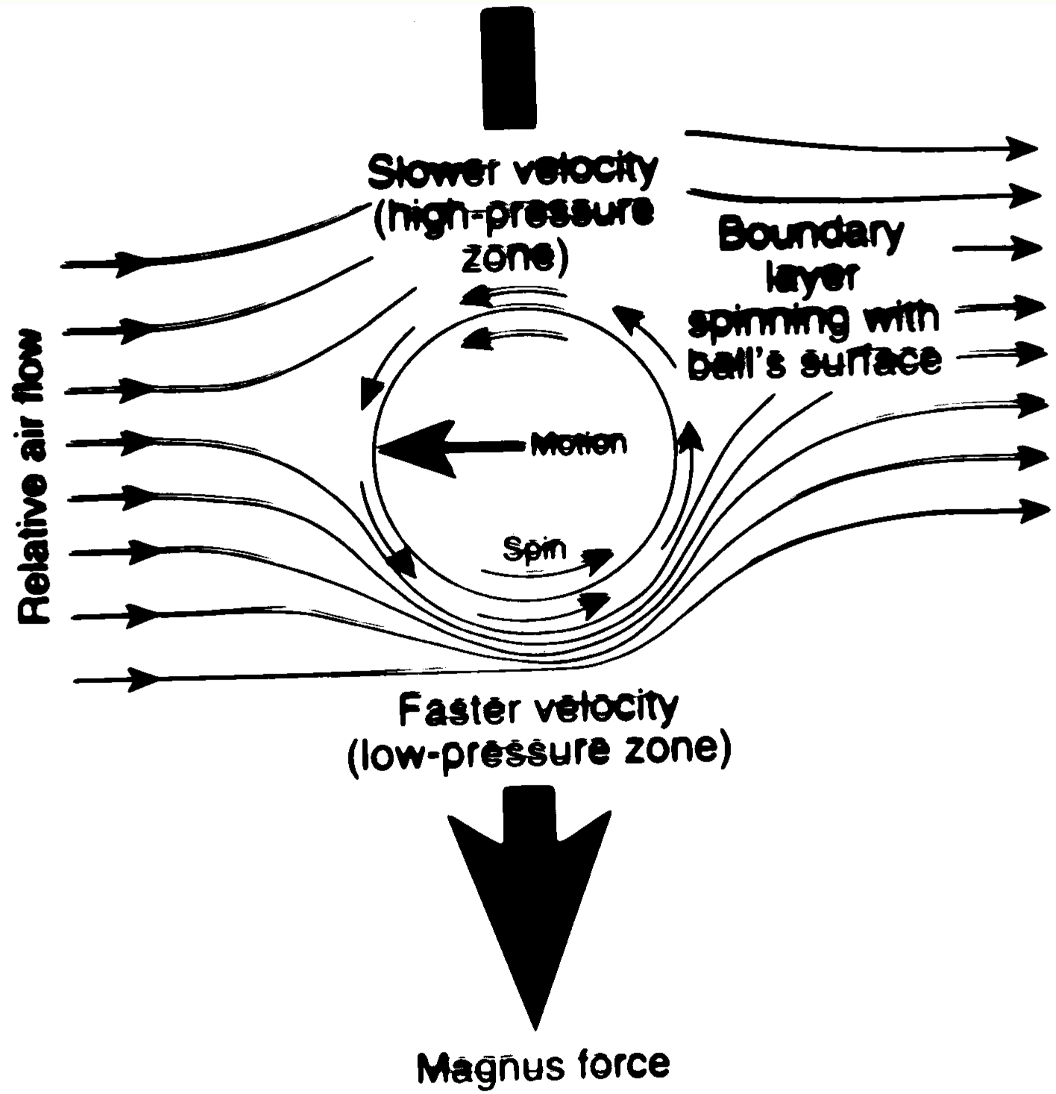
$$C_D = C_{D,\infty} + C_{D,i} = C_{D,\infty} + \frac{C_L^2}{\pi ar}$$

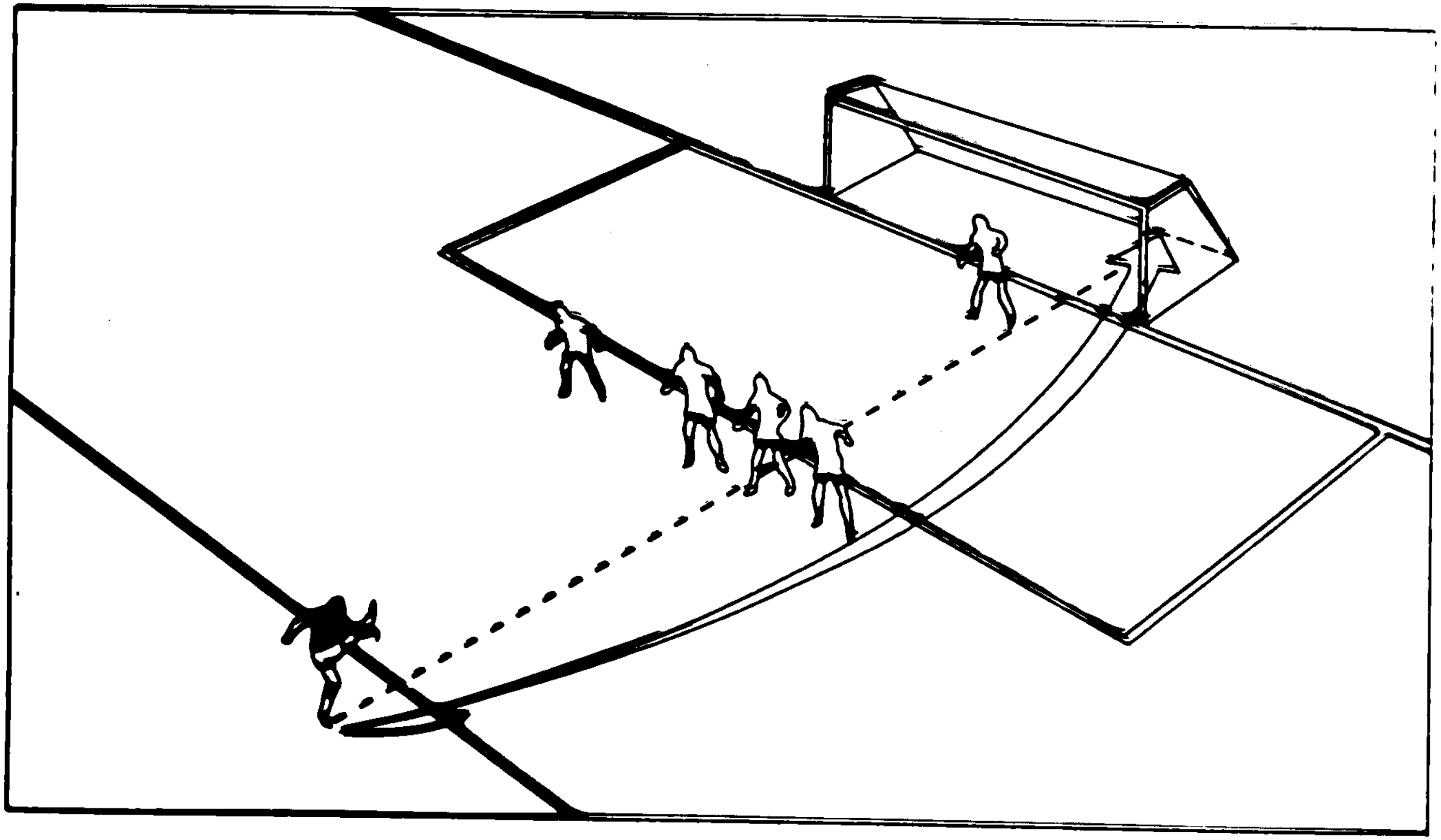
- Induced Drag (Continued)



Robbins – Magnus effect

- When a spherical body (such as a ball) rotates as it moves through the air, it carries with it a boundary layer of air.
- The object will seek the path of least resistance, (the side of the ball that is rotating the same direction as the oncoming air).
- The effect is to cause the ball to curve in the direction of this low pressure air.
- Examples are tennis, golf, soccer, volleyball and football.
- These are top spin or backward spin (around horizontal axis). Spin right or left known as a hook or slice is around the vertical axis.
- Spin about the axis of the line of flight is called gyroscopic action and acts to stabilize the object as it moves through the air.


Slower velocity
(high-pressure
zone)
Boundary
layer
spinning with
ball's surface
Relative air flow
Motion
Spin
Faster velocity
(low-pressure zone)
Magnus force



1. A flat plate 1.5m x 1.5m moves at 50 km/hr in stationary air of density 1.15kg/m3. If the coefficients of drag and lift are 0.15 and 0.75 respectively, determine

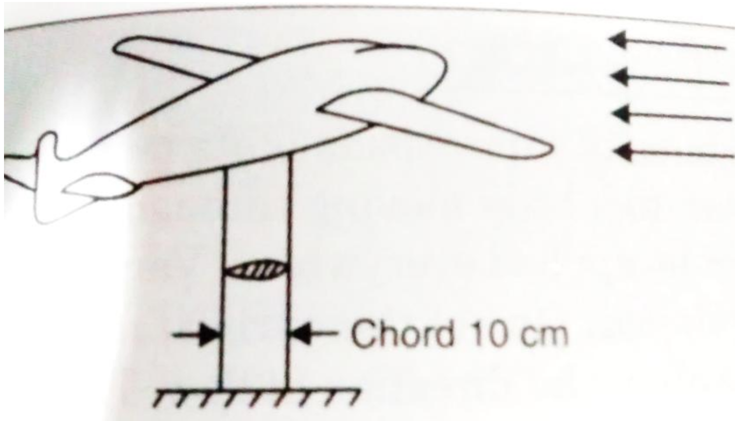
i. Lift force

ii. Drag force

iii. The resultant force and

iv. The power required to keep the plate in motion.

2. An aircraft model is mounted in a wind tunnel for drag determination. To decrease the drag contribution of the mounting system, the 1m long column is streamlined into an airfoil shape with a chord length of 10cm. At flow speed of 80m/s in the wind tunnel, estimate the error due to the drag on the mounting column, if the variation of C_D of the streamlined column with Re given in table.



C_D Vs Re for Mounting Column

Re	C_D
3×10^{5}	0.0045
6×10^{5}	0.0055
1×10^{6}	0.0060
3×10^{6}	0.0065

$$C_D = 0.072\, Re_L^{-1/5} - 1670/Re_L$$

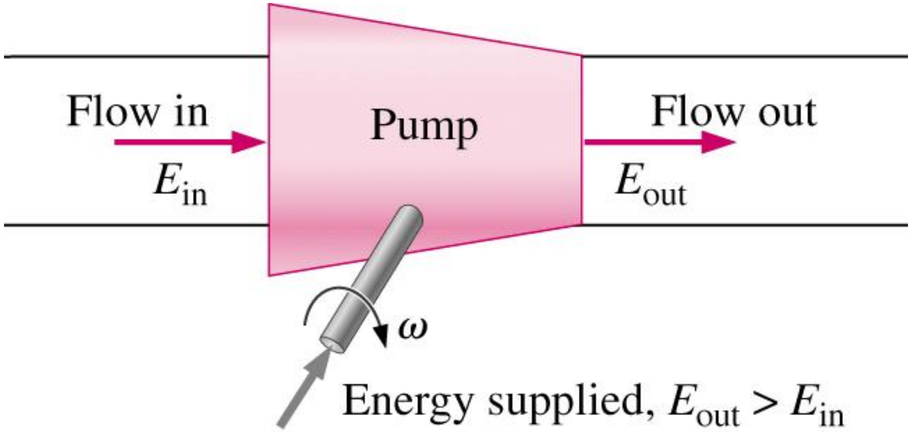
Thank You

Introduction to Turbo machinery

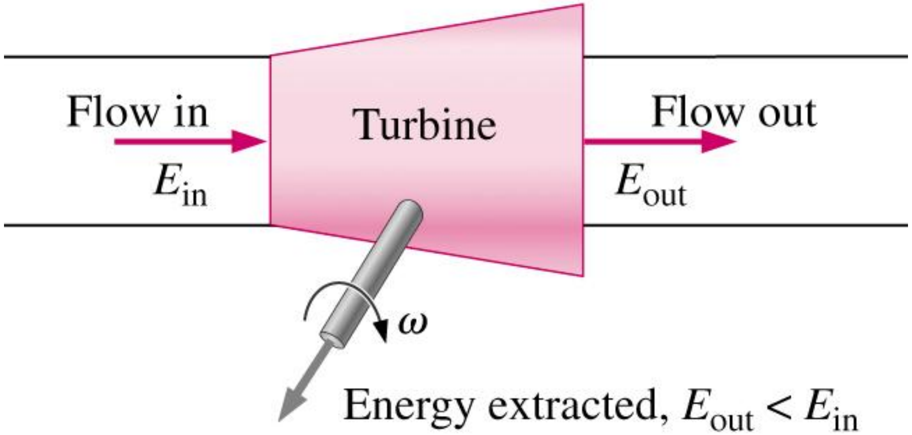
Objectives

- Identify various types of pumps and turbines, and understand how they work
- Apply dimensional analysis to design new pumps or turbines that are geometrically similar to existing pumps or turbines
- Perform basic vector analysis of the flow into and out of pumps and turbines
- Use specific speed for preliminary design and selection of pumps and turbines

Categories



(a)



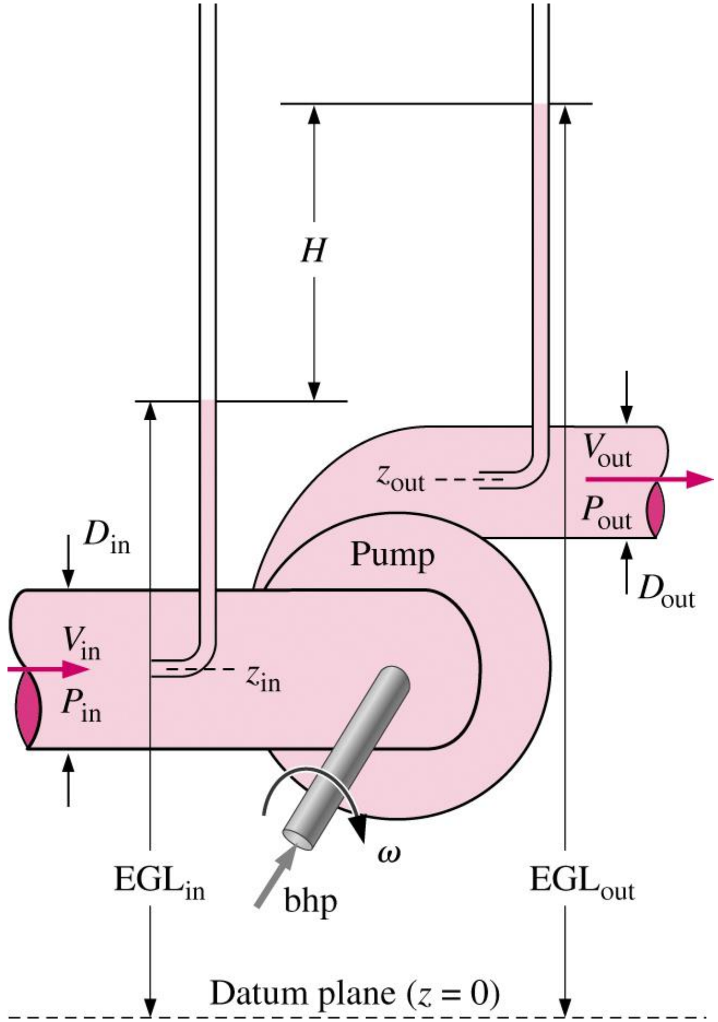
(b)

- Pump: adds energy to a fluid, resulting in an increase in pressure across the pump.
- Turbine: extracts energy from the fluid, resulting in a decrease in pressure across the turbine.

- For gases, pumps are further broken down into
  - **Fans**: Low *pressure gradient*, High *volume flow rate*. Examples include ceiling fans and propellers.
  - **Blower**: Medium *pressure gradient*, Medium *volume flow rate*. Examples include centrifugal and squirrel-cage blowers found in furnaces, leaf blowers, and hair dryers.
  - **Compressor**: High *pressure gradient*, Low *volume flow rate*. Examples include air compressors for air tools, refrigerant compressors for refrigerators and air conditioners.

- Positive-displacement machines
  - Closed volume is used to squeeze or suck fluid.
  - Pump: human heart
  - Turbine: home water meter
- Dynamic machines
  - No closed volume. Instead, rotating blades supply or extract energy.
  - Enclosed/Ducted Pumps: torpedo propulsor
  - Open Pumps: propeller or helicopter rotor
  - Enclosed Turbines: hydroturbine
  - Open Turbines: wind turbine

Pump Head



- Net Head

$$H = \left(\frac{P}{\rho g} + \frac{V^2}{2g} + z\right)_{out} - \left(\frac{P}{\rho g} + \frac{V^2}{2g} + z\right)_{in}$$

$$= EGL_{out} - EGL_{in}$$

- Water horsepower

$$\dot{W}_{water\ horsepower} = \dot{m}gH = \rho g\dot{V}H$$

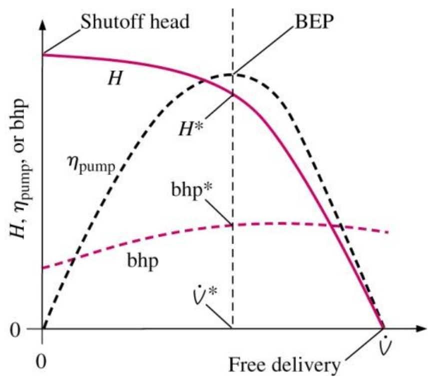
- Brake horsepower

$$bhp = \dot{W}_{shaft} = \omega T_{shaft}$$

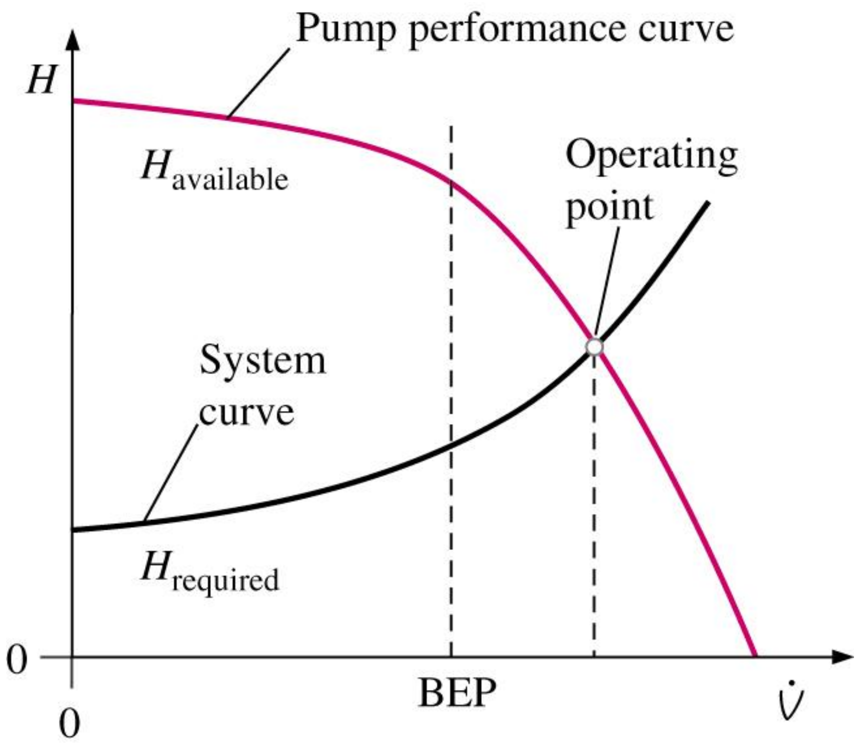
- Pump efficiency

$$\eta_{pump} = \frac{\rho g\dot{V}H}{\omega T_{shaft}}$$

Matching a Pump to a Piping System



- Pump-performance curves for a centrifugal pump
- BEP: best efficiency point
- *H*, bhp*, $\dot{V}$** correspond to BEP
- Shutoff head: achieved by closing outlet ($\dot{V}$=0)$
- Free delivery: no load on system ($H_{required}$ = 0)



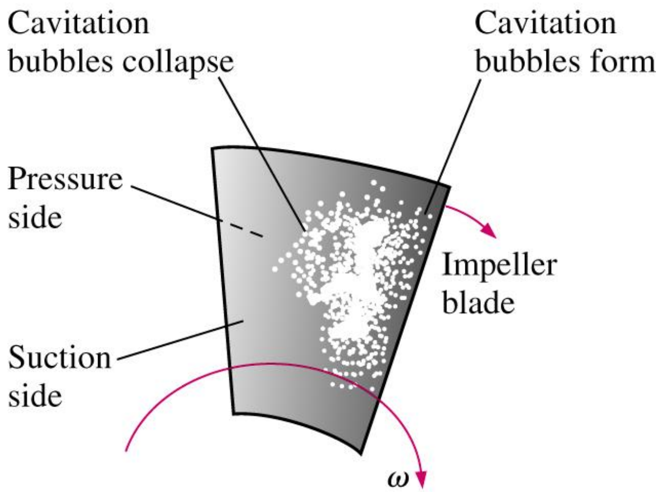
- Steady operating point:

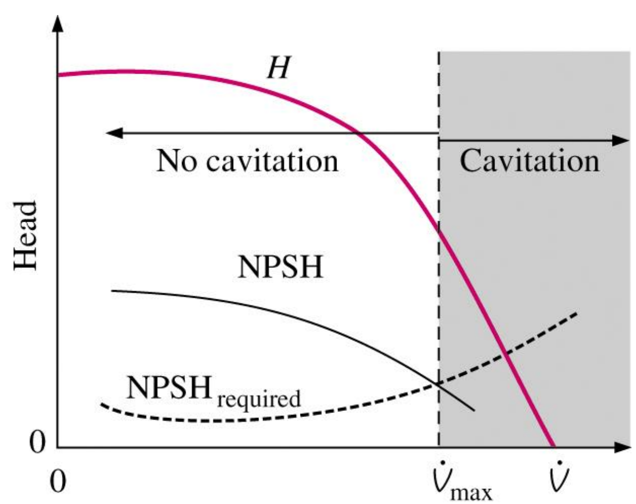
$$H_{required} = H_{available}$$

- Energy equation:

$$H_{required} = h_{pump,u} = \frac{P_2 - P_1}{\rho g} + \frac{\alpha_2 V_2^2 - \alpha_1 V_1^2}{2g} + z_2 - z_1 + h_{L,total}$$

Pump Cavitation and NPSH



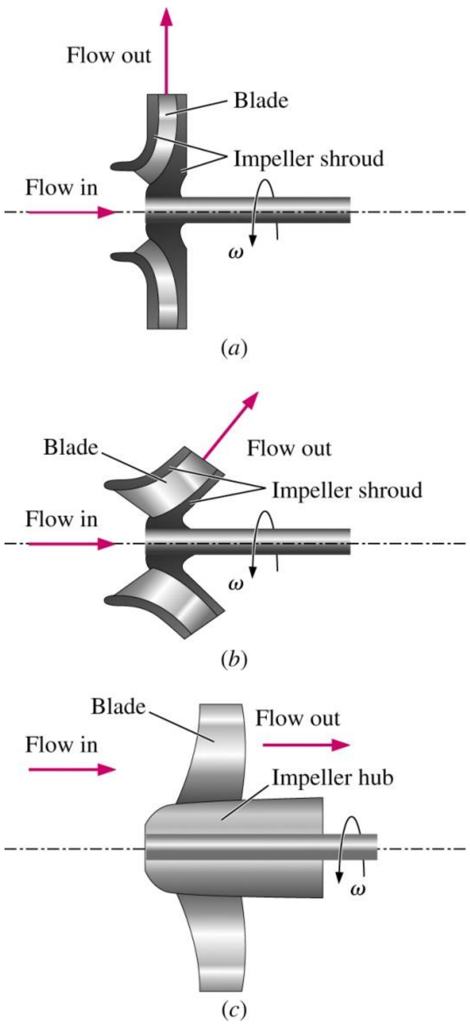


- Cavitation should be avoided due to erosion damage and noise.
- Cavitation occurs when $P < P_v$
- **Net positive suction head**

$$NPSH = \left(\frac{P}{\rho g} + \frac{V^2}{2g}\right)_{pump\ inlet} - \frac{P_v}{\rho g}$$

- $NPSH_{required}$ curves are created through systematic testing over a range of flow rates *V*.

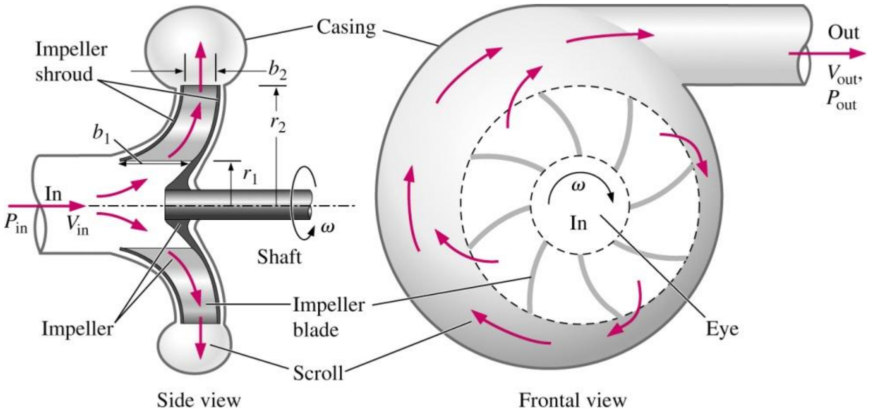
Dynamic Pumps



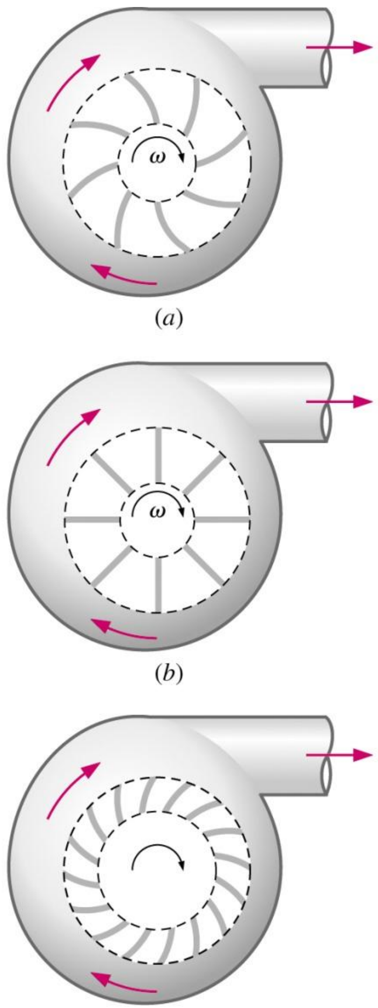
- Dynamic Pumps include
  - *centrifugal pumps*: fluid enters axially, and is discharged radially.
  - *mixed--flow pumps*: fluid enters axially, and leaves at an angle between radially and axially.
  - *axial pumps*: fluid enters and leaves axially.

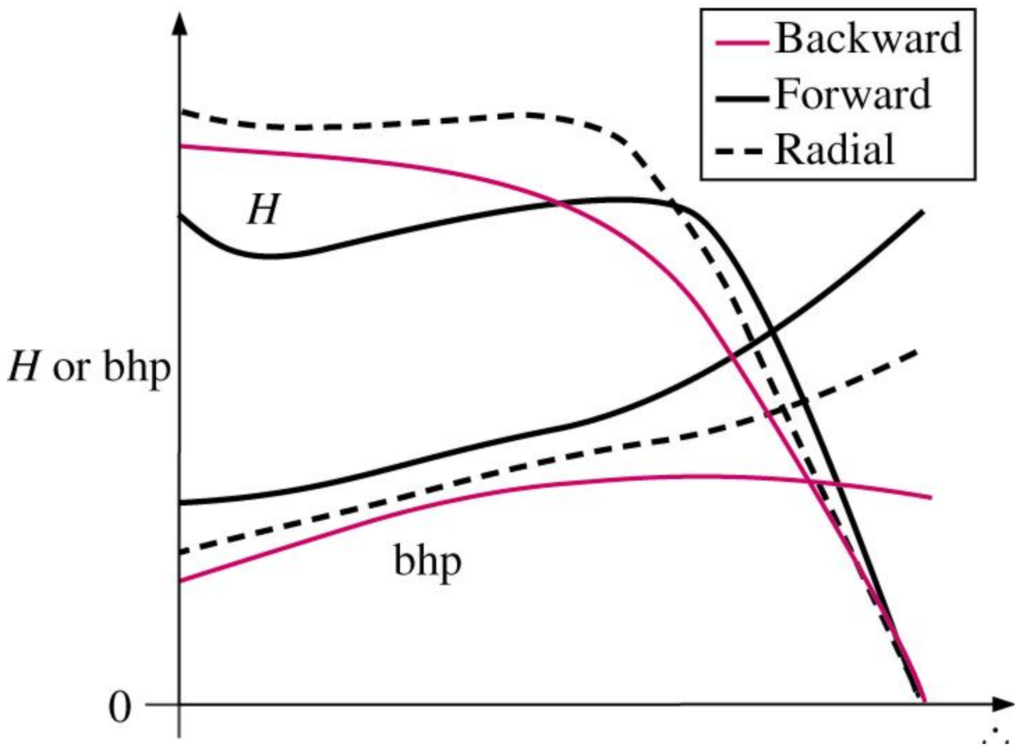
Centrifugal Pumps

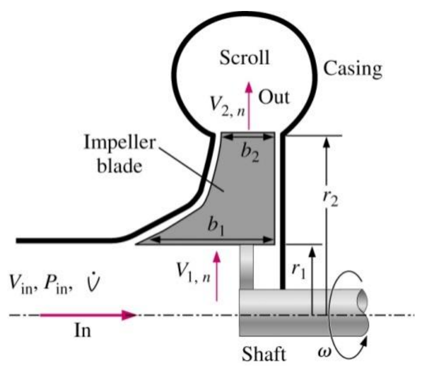
- Snail--shaped scroll
- Most common type of pump: homes, autos, industry.



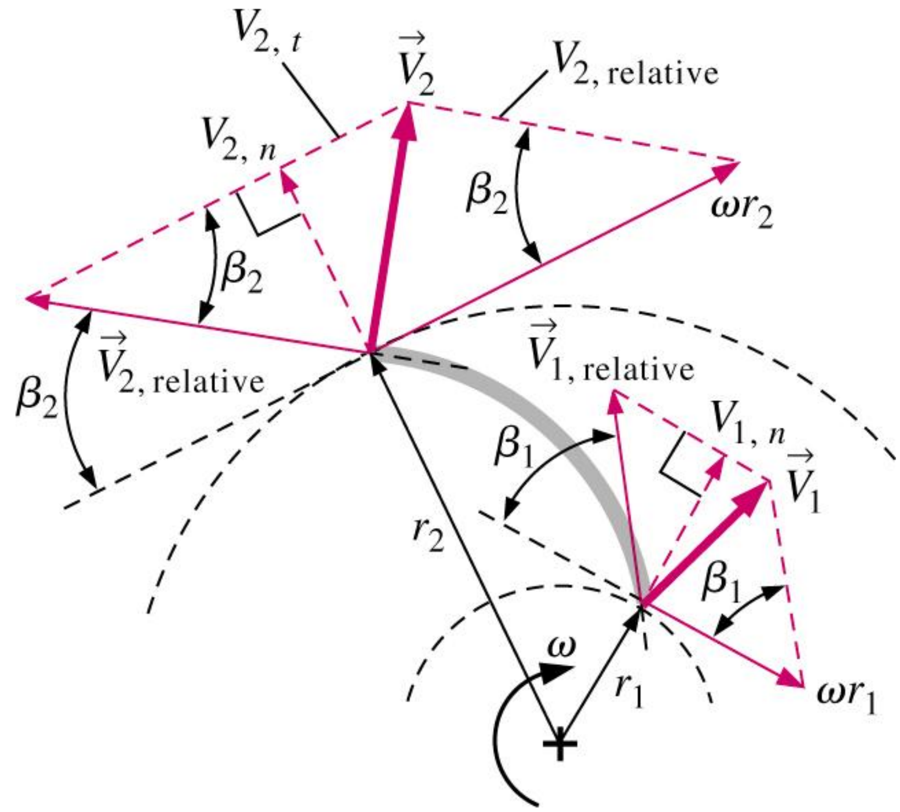
Centrifugal Pumps: Blade Design



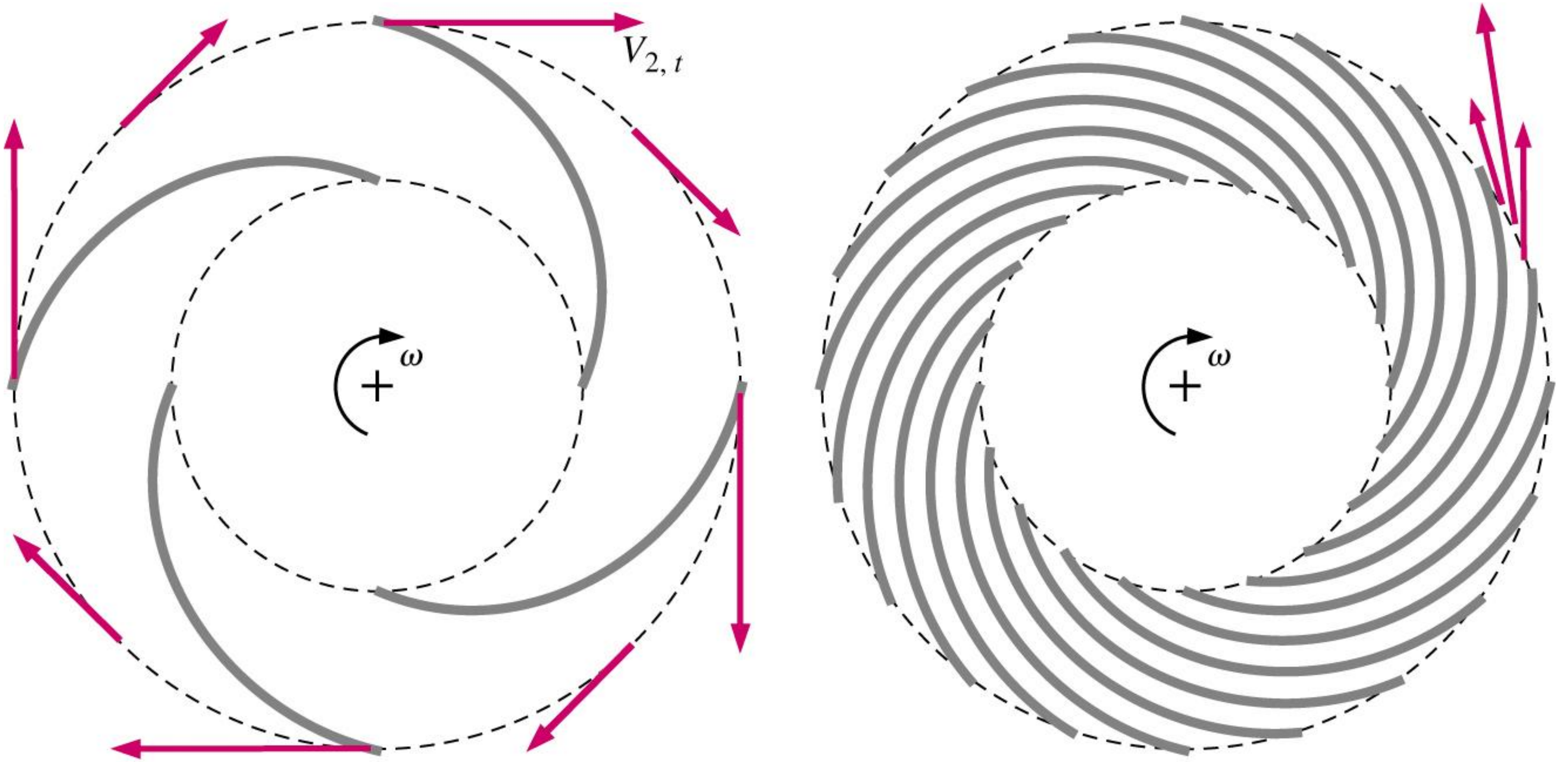




Side view of impeller blade.



Vector analysis of leading and trailing edges.



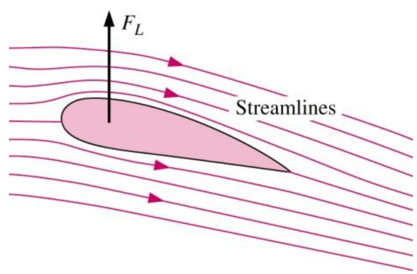
(a) (b)

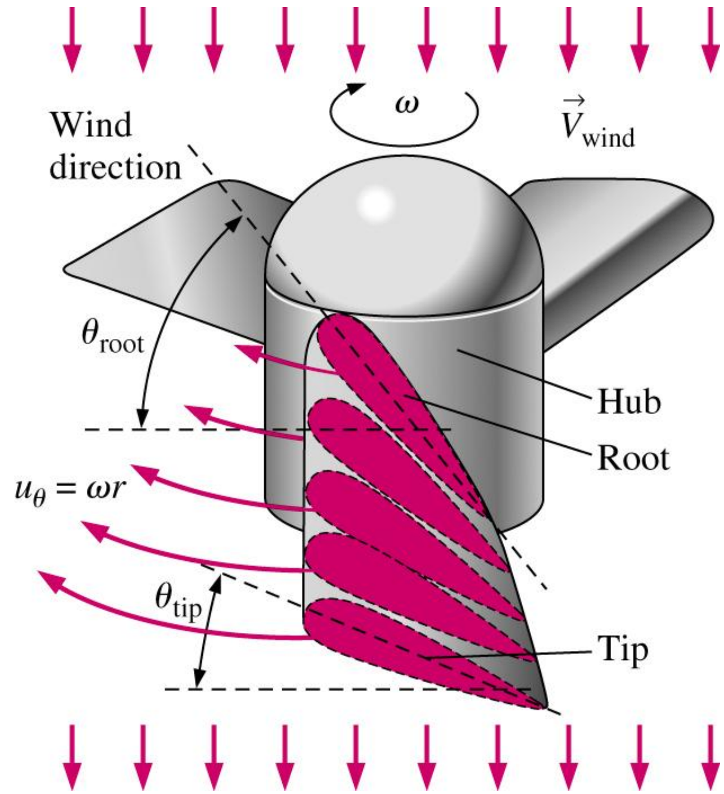
Blade number affects efficiency and introduces circulatory losses (too few blades) and passage losses (too many blades)

Axial Pumps

Open vs. Ducted Axial Pumps

Open Axial Pumps

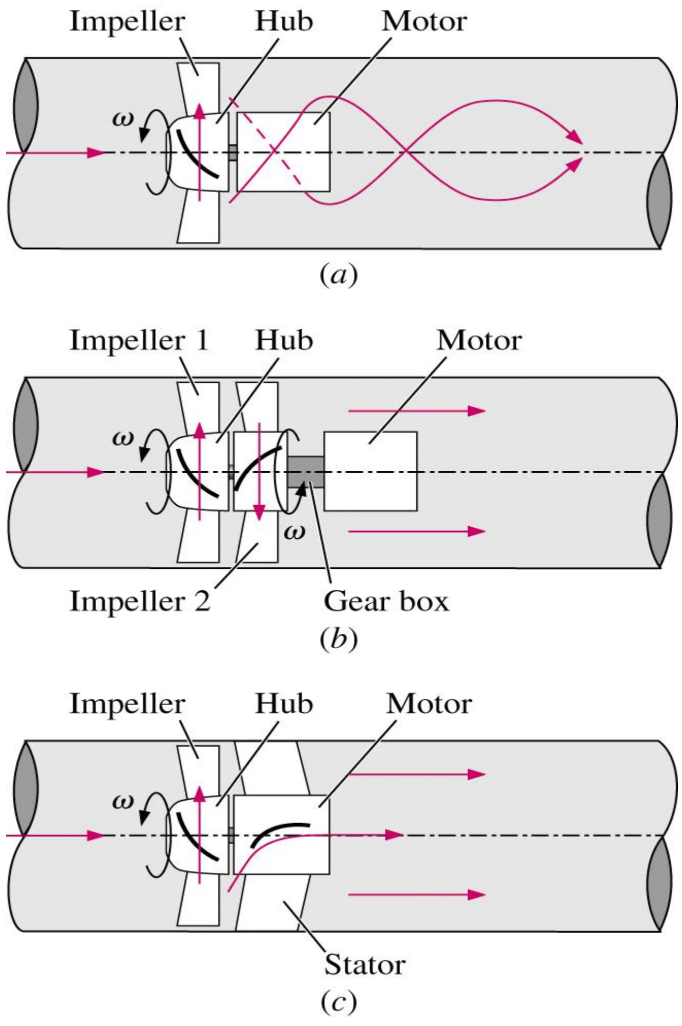




Blades generate thrust like wing generates lift.

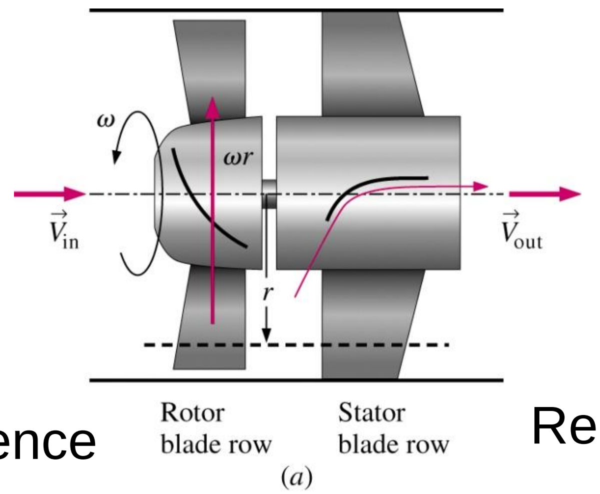
Propeller has radial twist to take into account for angular velocity (=ωr)

Ducted Axial Pumps



- Tube Axial Fan: Swirl downstream
- Counter-Rotating Axial-Flow Fan: swirl removed. Early torpedo designs
- Vane Axial-Flow Fan: swirl removed. Stators can be either pre-swirl or post-swirl.

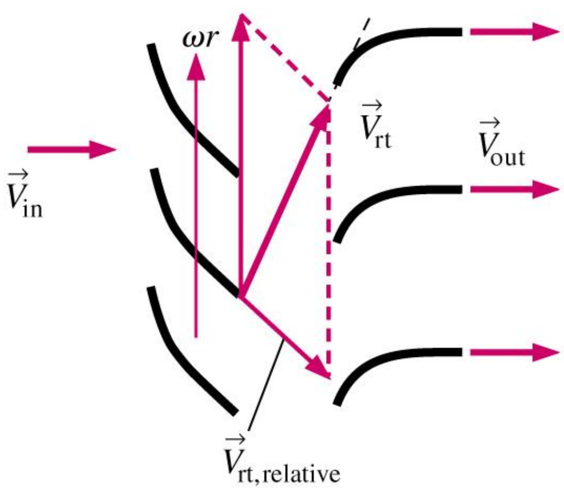
Ducted Axial Pumps: Blade Design

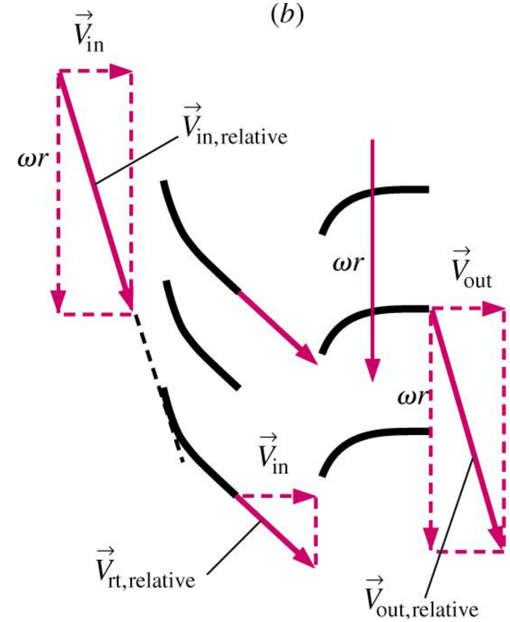


Absolute frame of reference

Relative frame of reference

(b)





Thank You