OPEN CHANNELS

(OPEN CHANNEL FLOW AND HYDRAULIC MACHINERY)

UNIT – I

IARE

Dr.G. Venkata Ramana Professor& HOD Civil Engineering

Learning Objectives

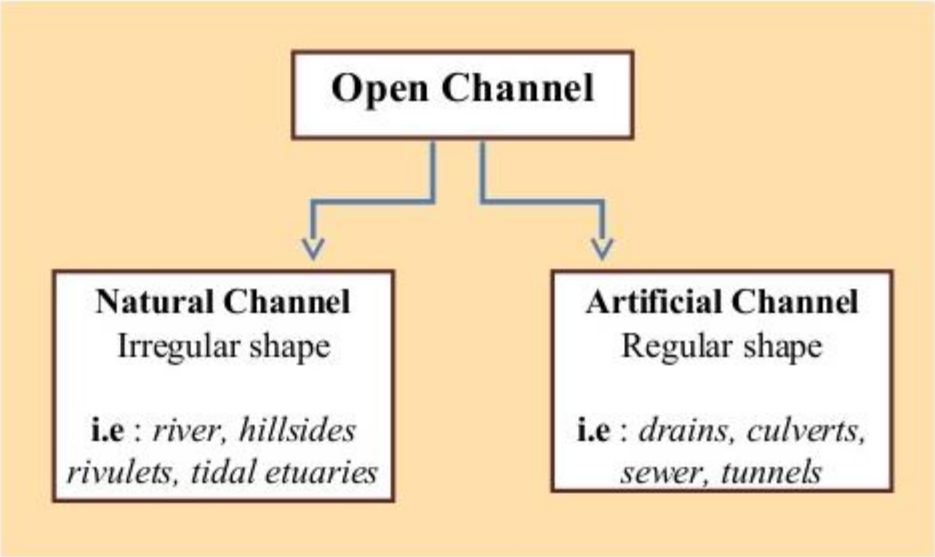
1. Types of Channels
2. Types of Flows
3. Velocity Distribution
4. Discharge through Open Channels
5. Most Economical Sections

Learning Objectives

6. Specific Energy and Specific Energy Curves
7. Hydraulic Jump (RVF)
8. Gradually Varied Flow (GVF)

Types of Channels

- Open channel flow is a flow which has a free surface and flows due to gravity.
- Pipes not flowing full also fall into the category of open channel flow
- In open channels, the flow is driven by the slope of the channel rather than the pressure


Open Channel
Natural Channel
Irregular shape
i.e : river, hillsides
rivulets, tidal etuaries
Artificial Channel
Regular shape
i.e : drains, culverts,
sewer, tunnels

Types of Flows

1. Steady and Unsteady Flow
2. Uniform and Non-uniform Flow
3. Laminar and Turbulent Flow
4. Sub-critical, Critical and Super-critical Flow

1. Steady and Unsteady Flow

- Steady flow happens if the conditions (flow rate, velocity, depth etc) do not change with time.
- The flow is unsteady if the depth is changes with time

2. Uniform and Non-uniform Flow

- If for a given length of channel, the velocity of flow, depth of flow, slope of the channel and cross section remain constant, the flow is said to be Uniform
- The flow is Non-uniform, if velocity, depth, slope and cross section is not constant

2. Non-uniform Flow

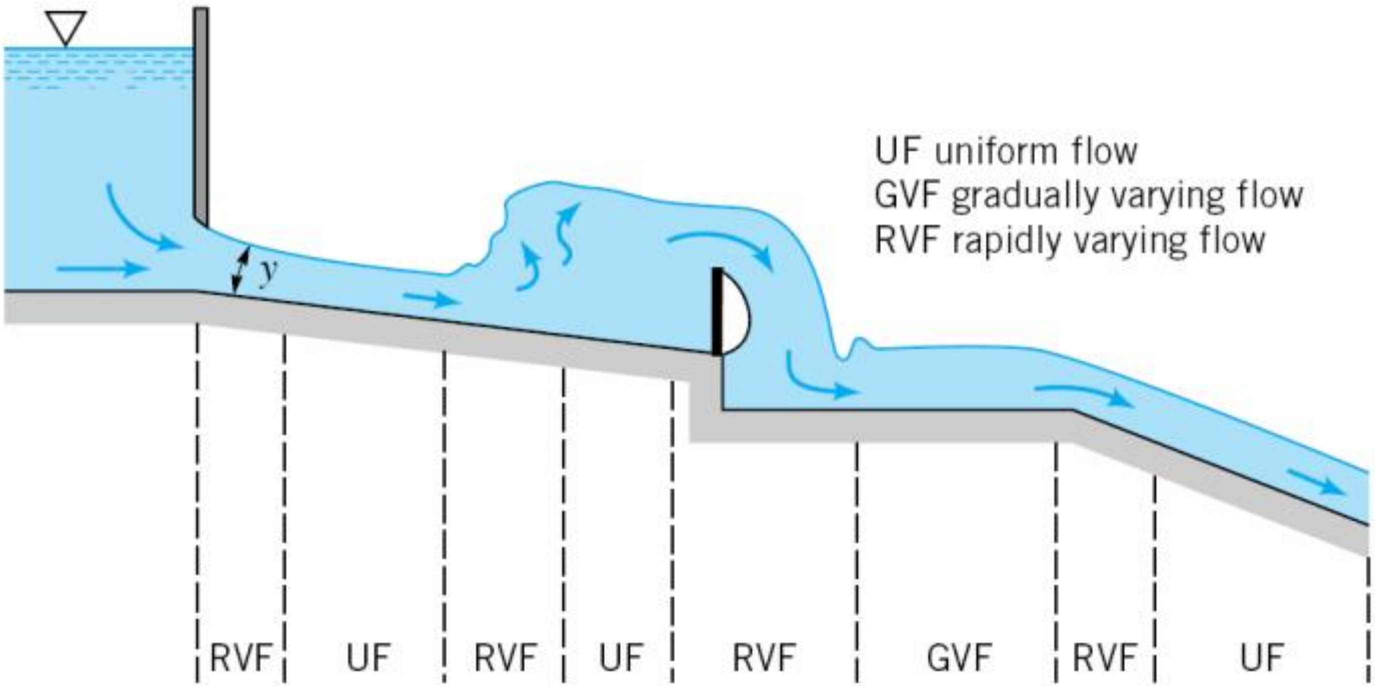
Types of Non-uniform Flow

1. Gradually Varied Flow (GVF)

   If the depth of the flow in a channel changes gradually over a length of the channel.

2. Rapidly Varied Flow (RVF)

   If the depth of the flow in a channel changes abruptly over a small length of channel



UF uniform flow
GVF gradually varying flow
RVF rapidly varying flow
y
RVF
UF
RVF
UF
RVF
GVF
RVF
UF

3. Laminar and Turbulent Flow

Both laminar and turbulent flow can occur in open channels depending on the Reynolds number (Re)

$$Re = \rho VR/\mu$$

Where,

ρ = density of water = 1000 kg/m^3

μ = dynamic viscosity

R = Hydraulic Mean Depth = Area / Wetted Perimeter

$$R_e = \rho V R / \mu$$

V is the average velocity of the fluid.
R is the hydraulic radius of the channel.

- Laminar flow: $Re < 500$
- Transitional flow: $Re > 500$ & $Re < 1000$
- Turbulent flow: $Re > 1000$

4. Sub-critical, Critical and Super-critical Flow

The flow in open channel is said to be sub-critical if the Froude number (F_e) is less than 1.0.

The Froude number is defined as : $F_e = \frac{V}{\sqrt{gD}}$

where V = Mean velocity of flow

D = Hydraulic depth of channel and is equal to the ratio of wetted area to the top width of channel

$= \frac{A}{T}$, where T = Top width of channel.

Sub-critical flow is also called tranquil or streaming flow. For sub-critical flow, $F_e < 1.0$.
The flow is called critical if $F_e = 1.0$. And if $F_e > 1.0$, the flow is called super critical or shooting or rapid or torrential.

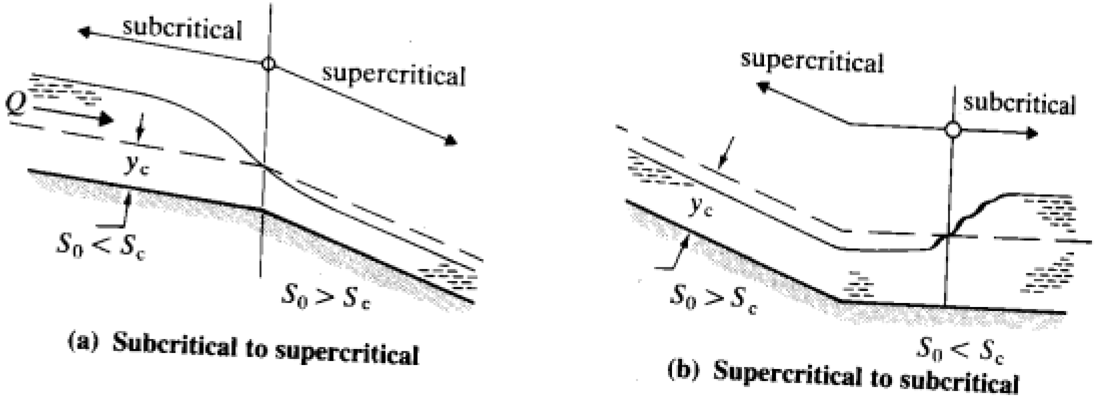
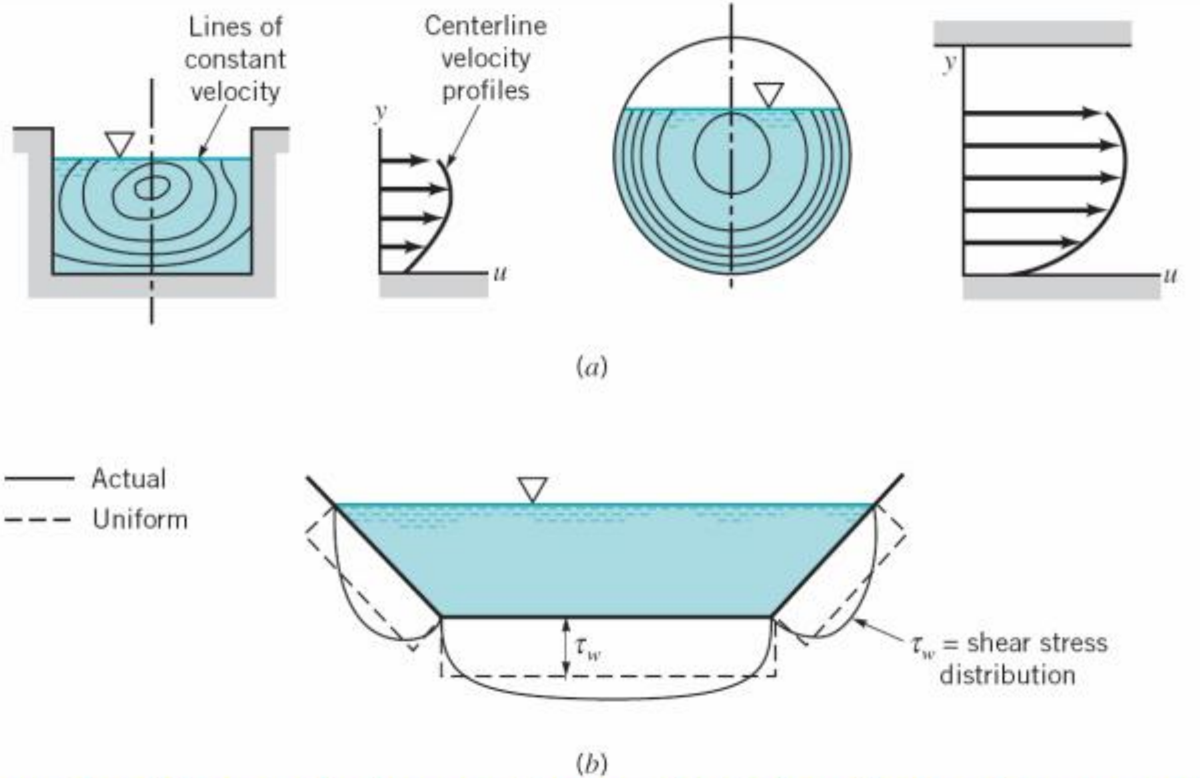


Figure of transition from sup to super-critical flow

Velocity Distribution

- Velocity is always vary across channel because of friction along the boundary
- The maximum velocity usually found just below the surface



Typical velocity and shear stress distributions in an open channel: (*a*) velocity distribution throughout the cross section. (*b*) shear stress distribution on the wetted perimeter.

Discharge through Open Channels

1. Chezy's C
2. Manning's N
3. Bazin's Formula
4. Kutter's Formula

▶ 16.3 DISCHARGE THROUGH OPEN CHANNEL BY CHEZY'S FORMULA

Forces acting on the water between sections 1-1 & 2-2

1. Component of weight of Water = W sin i →
2. Friction Resistance = f P L V^2 ←

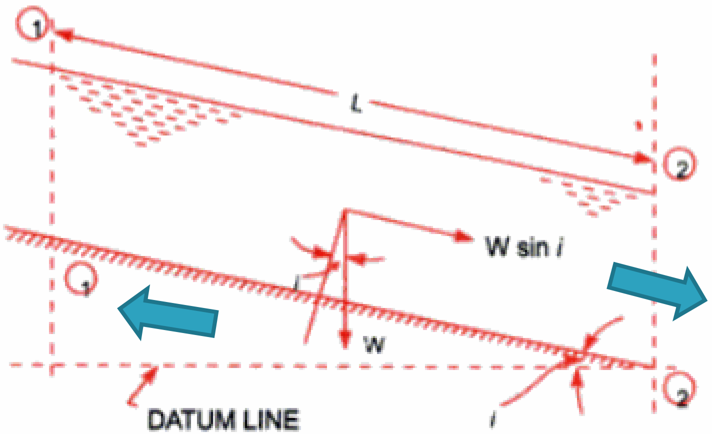


Fig. 16.2 *Uniform flow in open channel.*

where

W = density x volume

= w (AL) = wAL

Equate both Forces:

$f P L V^2 = wAL \sin i$

Chezy's Formula, $V = C\sqrt{mi}$

$$V = \sqrt{\frac{w}{f}}\sqrt{\frac{A}{P}\sin i} \rightarrow 1$$

$$\frac{A}{P} = m = \text{Hydraulic Radius} \rightarrow 2$$

$$\sqrt{\frac{w}{f}} = C = \text{Chezy's Constant} \rightarrow 3$$

Chezy's Formula, $V = C\sqrt{mi}$

$$\text{substitute Eqn. 2 \& 3 in Eqn. 1,}$$

$$V = C\sqrt{m.\sin i}$$

$$\text{for small values of } i, \ \sin i = \tan i = i$$

$$\therefore \boxed{V = C\sqrt{m.i}}$$

1. Manning's N

Chezy's formula can also be used with Manning's Roughness Coefficient

$$C = (1/n)\, R^{1/6}$$

where

R = Hydraulic Radius

n = Manning's Roughness Coefficient

2. Bazin's Formula

Chezy's formula can also be used with Bazins' Formula

$$C = \frac{157.6}{1.81 + \frac{k}{\sqrt{m}}}$$

where

k = Bazin's constant

m = Hydraulic Radius

Values of K in Bazin's Formula

No.	*Surface of channel*	*Bazin's constant (K)*
1.	Smooth cement plaster or planed wood	0·11
2.	Concrete, brick, or unplaned wood	0·21
3.	Smooth rubble masonry or poor brickwork	0·83
4.	Earth channels in very good condition	1·54
5.	Earth channels in rough condition	3·17
6.	Dredged earth channels, average condition	2·36

3. Kutter's Formula

Chezy's formula can also be used with Kutters' Formula

$$C = \frac{23 + 0.00155 + \dfrac{1}{N}}{1 + \left[23 + \dfrac{0.00155}{i}\right]\dfrac{N}{\sqrt{m}}}$$

where

N = Kutter's constant

m = Hydraulic Radius, i = Slope of the bed

Values of N in the Manning's & Kutter's Formula

.No.	*Surface of channel*	*N (Kutter's/Manning's constant)*
1.	Smooth cement plaster or planed wood	0·010
2.	Very smooth concrete and planed timber	0·011
3.	Smooth concrete	0·012
4.	Glazed brickwork	0·013
5.	Vitrified clay	0·014
6.	Brick surface lined with cement mortar	0·015
7.	Earth channels in best condition	0·017
8.	Straight unlined earth channels in good condition	0·020
9.	Rivers and earth channels in fair condition	0·025
10.	Canal and river of rough surface with weeds	0·030

Problems

1. Find the velocity of flow and rate of flow of water through a rectangular channel of 6 m wide and 3 m deep, when it is running full. The channel is having bed slope as 1 in 2000. Take Chezy's constant C = 55

2. Find slope of the bed of a rectangular channel of width 5m when depth of water is 2 m and rate of flow is given as 20 m^3/s. Take Chezy's constant, C = 50

Problems

3. Find the discharge through a trapezoidal channel of 8 m wide and side slopes of 1 horizontal to 3 vertical. The depth of flow is 2.4 m and Chezy's constant C = 55. The slope of bed of the channel is 1 in 4000

4. Find diameter of a circular sewer pipe which is laid at a slope of 1 in 8000 and carries a discharge of 800 litres/s when flowing half full. Take Manning's N = 0.020

Problems

5. Find the discharge through a channel show in fig. 16.5. Take the value of Chezy's constant C = 55. The slope of bed of the channel is 1 in 2000

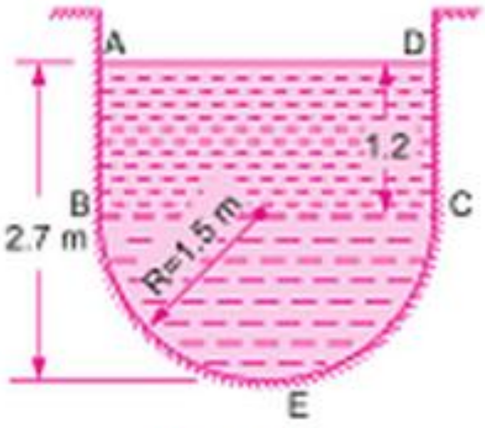


Fig. 16.5

Most Economical Sections

1. Cost of construction should be minimum
2. Discharge should be maximum

Types of channels based on shape:

1. Rectangular
2. Trapezoidal
3. Circular

Most Economical Sections

$$Q = A V = A C \sqrt{m i}$$

$$Q = K \frac{1}{\sqrt{P}} \quad \text{where } K = A C \sqrt{A i}$$

If P is minimum, Q will be maximum

Rectangular Section

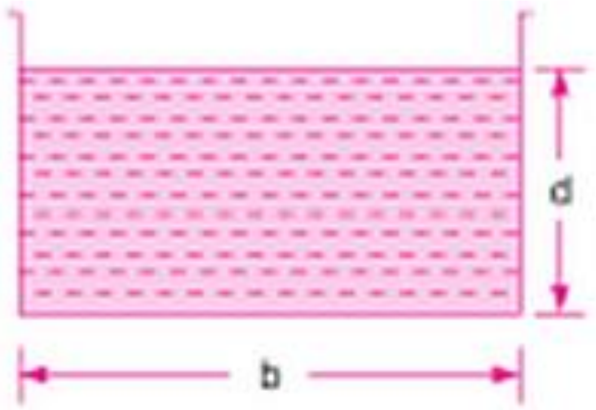


Fig. 16.9 *Rectangular channel.*

for most economical section,

P should be minimum

$$\frac{dP}{d(d)} = 0$$

$$A = bd \Rightarrow b = \frac{A}{d} \rightarrow 1$$

$$P = b + 2d = \frac{A}{d} + 2d \rightarrow 2$$

for most economical seciton, P should be minimum

$$\frac{dP}{d(d)} = 0 \Rightarrow \frac{d\left[\frac{A}{d} + 2d\right]}{d(d)} = 0 \Rightarrow \frac{-A}{d^2} + 2 = 0 \Rightarrow A = 2d^2 \Rightarrow bd = 2d^2$$

$$b = 2d \text{ or } d = b/2$$

$$m = \frac{A}{P} = \frac{bd}{b + 2d} = \frac{2d^2}{2d + 2d} = \frac{d}{2}$$

Trapezoidal Section

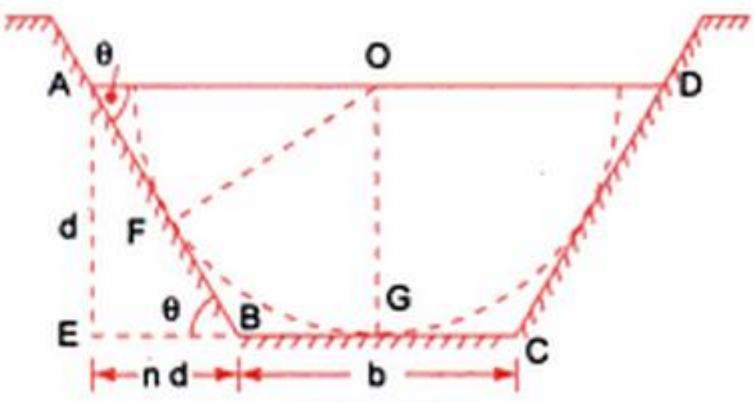


Fig. 16.11

for most economical section,

P should be minimum

$$\frac{dP}{d(d)} = 0$$

$$A=(b+nd)d \Rightarrow b=\frac{A}{d}-nd \rightarrow 1$$

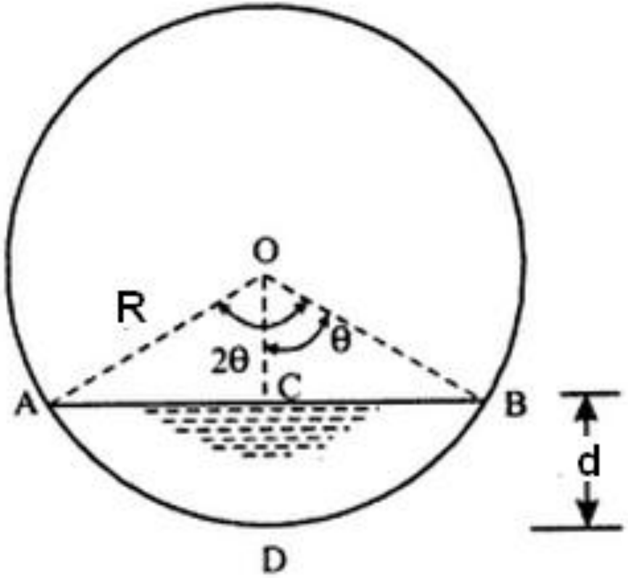
$$P=b+2d\sqrt{n^2+1}=\frac{A}{d}-nd+2d\sqrt{n^2+1} \rightarrow 2$$

for most economical seciton, P should be minimum

$$\frac{dP}{d(d)}=0 \Rightarrow \frac{d\left[\frac{A}{d}-nd+2d\sqrt{n^2+1}\right]}{d(d)}=0 \Rightarrow \frac{b+2nd}{2}=d\sqrt{n^2+1}$$

$m=\frac{d}{2}$ and $\theta=60^0$

Circular Section



Circular channel.

$$\text{for Max. Velocity,} \quad \frac{d\left[\frac{A}{P}\right]}{d\theta} = 0$$

$$\text{for Max. Discharge,} \quad \frac{d\left[\sqrt{\frac{A^3}{P}}\right]}{d\theta} = 0$$

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) \rightarrow 1$$

$$P = 2R\theta \rightarrow 2$$

$$m = \frac{A}{P} = \frac{R}{2\theta}\left(\theta - \frac{\sin 2\theta}{2}\right) \rightarrow 3$$

for max. velocity, $\frac{dm}{d\theta} = 0 \Rightarrow \theta = 128^0 45, d = 0.81D, m = 0.3D$

$Q = AC\sqrt{m\,i} = AC\sqrt{\frac{A}{P}i} = C\sqrt{\frac{A^3}{P}i}$, C and i are constants

for max. discharge, $\frac{d\left[\sqrt{\frac{A^3}{P}}\right]}{d\theta} = 0 \Rightarrow \theta = 154^0, d = 0.95D$

Problems

1. A trapezoidal channel has side slopes of 1 horizontal and 2 vertical and the slope of the bed is 1 in 1500. The area of cross section is $40m^2$. Find dimensions of the most economical section. Determine discharge if C=50

Hint:

- Equate Half of Top Width = Side Slope (condition 1) and find b in terms of d
- Substitute b value in Area and find d
- Find m = d/2 (condition 2)
- Find V and Q

Problem 16.16 *A trapezoidal channel has side slopes of 1 horizontal to 2 vertical and the slope of the bed is 1 in 1500. The area of the section is 40 m². Find the dimensions of the section if it is most economical. Determine the discharge of the most economical section if C = 50.*

Solution. Given :

Side slope, $n = \frac{\text{Horizontal}}{\text{Vertical}} = \frac{1}{2}$

Bed slope, $i = \frac{1}{1500}$

Area of section, $A = 40 \text{ m}^2$

Chezy's constant, $C = 50$

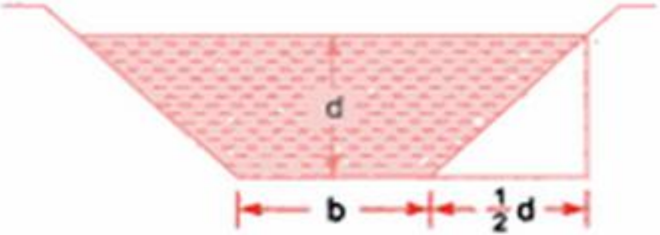


Fig. 16.12

For the most economical section, using equation (16.11)

$$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1} \quad \text{or} \quad \frac{b + 2 \times \frac{1}{2} \times d}{2} = d\sqrt{\left(\frac{1}{2}\right)^2 + 1}$$

or $$\frac{b + d}{2} = d\sqrt{\frac{1}{4} + 1} = 1.118\, d$$

or $$b = 2 \times 1.118d - d = 1.236\, d \quad \ldots(i)$$

But area of trapezoidal section, $$A = \frac{b + (b + 2nd)}{2} \times d = (b + nd)\, d$$

$$= (1.236\, d + \tfrac{1}{2}\, d)\, d \quad \left(\because \; b = 1.236\, d \text{ and } n = \tfrac{1}{2}\right)$$

$$= 1.736\, d^2$$

But $$A = 40 \text{ m}^2 \quad \text{(given)}$$

$\therefore$ $$40 = 1.736\, d^2$$

$\therefore$ $$d = \sqrt{\frac{40}{1.736}} = \textbf{4.80 m. Ans.}$$

Substituting the value of d in equation (i), we get

$$b = 1.236 \times 4.80 = \textbf{5.933 m. Ans.}$$

Discharge for most economical section. Hydraulic mean depth for most economical section is

$$m = \frac{d}{2} = \frac{4.80}{2} = 2.40 \text{ m}$$

$\therefore$ Discharge $$Q = AC\sqrt{mi} = 40 \times 50 \times \sqrt{2.40 \times \frac{1}{1500}}$$

$$= \textbf{80 m}^3\textbf{/s. Ans.}$$

Problems

2. A rectangular channel of width 4 m is having a bed slope of 1 in 1500. Find the maximum discharge through the channel. Take C=50

3. The rate of flow of water through a circular channel of diameter 0.6m is 150 litres/s. Find the slope of the bed of the channel for maximum velocity. Take C=50

Non-uniform Flow

In Non-uniform flow, velocity varies at each section of the channel and the Energy Line is not parallel to the bed of the channel.

This can be caused by

1. Differences in depth of channel and
2. Differences in width of channel.
3. Differences in the nature of bed
4. Differences in slope of channel and
5. Obstruction in the direction of flow

Specific Energy

Total Energy of flowing fluid, $E = z + h + \frac{v^2}{2g}$

where z = Height of bottom of channel above datus,

If the channel bottom is taken as datum,

$Es = h + \frac{v^2}{2g}$ which is called as Specific Energy

Specific Energy

$$Q = A\,V \Rightarrow V = \frac{Q}{A} = \frac{Q}{bh}$$

$$\text{If discharge per unit width, } q = \frac{Q}{b} = \text{constant}$$

$$V = \frac{Q}{bh} = \frac{q}{h}$$

$$\therefore Es = h + \frac{V^2}{2g} = h + \frac{q^2}{2g\,h^2}$$

Modified Equation
to plot Specific Energy Curve

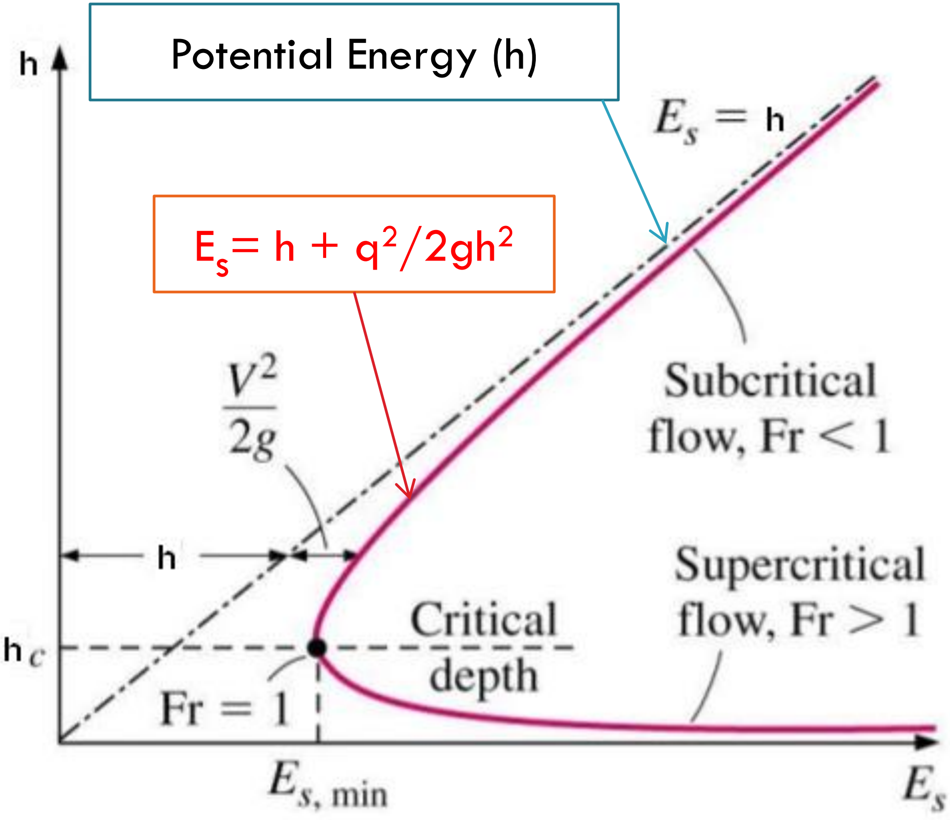
$$\text{for Critical Depth, } \frac{dE}{dh} = 0$$

$$\text{where, } E = h + \frac{q^2}{2g\,h^2}$$

$$h_c = \left[\frac{q^2}{g}\right]^{\frac{1}{3}} \Rightarrow h_c^{\,3} = \frac{q^2}{g} \Rightarrow h_c^{\,3}.g = q^2 \rightarrow 1$$

$$\text{subsitute value } q = \frac{Q}{b} = \frac{bh.\,v}{b} = h_c V_c \text{ in Eqn. 1}$$

$$\Rightarrow V_c = \sqrt{g\,h_c}$$



Specific Energy Curve

Minimum Specific Energy in terms of Critical Depth; $E = h + \frac{q^2}{2g\,h^2}$

when specific energy is minimum, Depth of flow is critical

$$E = h_c + \frac{q^2}{2g\,h_c^2} \text{ substitute } h_c = \left[\frac{q^2}{g}\right]^{\frac{1}{3}} \text{ or } h_c^3 = \frac{q^2}{g}$$

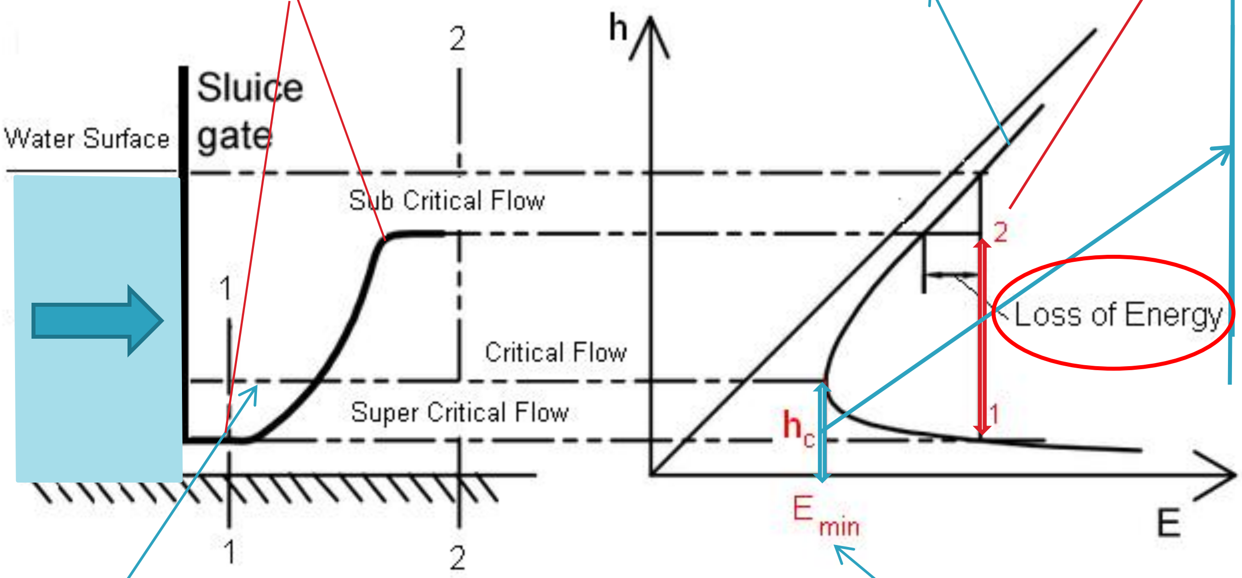
$$\boxed{E_{min} = h_c + \frac{h_c^3}{2g\,h_c^2} = h_c + \frac{h_c}{2} = \frac{3h_c}{2}}$$

$$\text{or } \boxed{h_c = \frac{2E_{min}}{3}}$$

$$E_s = h + \frac{V^2}{2g} = h + \frac{q^2}{2g\,h^2}$$

Hydraulic Jump

Alternate Depths 1 & 2



$$h_c = \frac{2E_{min}}{3}$$

$$h_c = \left[\frac{q^2}{g}\right]^{\frac{1}{3}}$$

$$V_c = \sqrt{g\,h_c}$$

Specific Energy Curve

$$E_{min} = \frac{3h_c}{2}$$

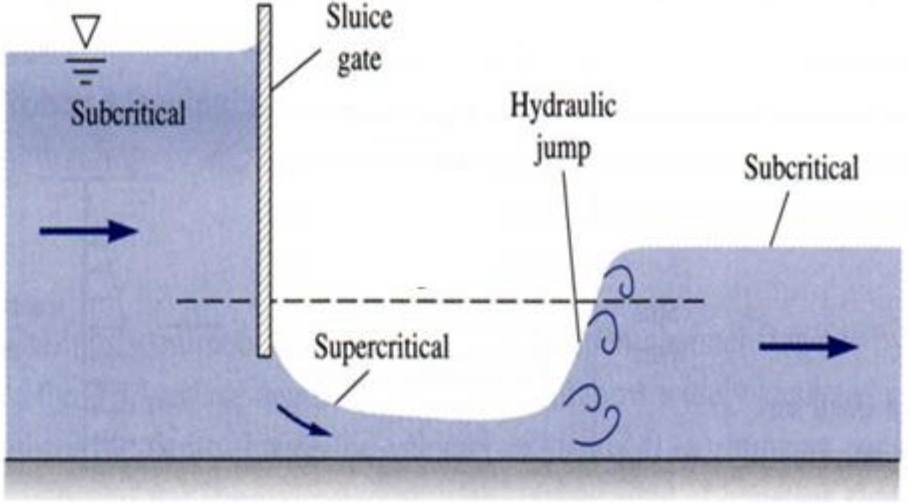
Problems

1. The specific energy for a 3 m wide channel is to be 3 kg-m/kg. What would be the max. possible discharge

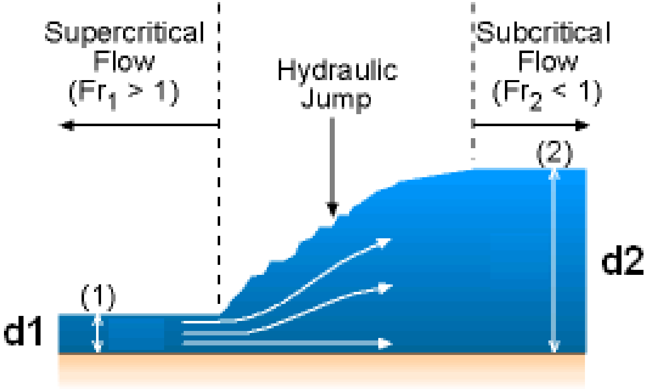
2. The discharge of water through a rectangular channel of width 6 m, is 18 m3/s when depth of flow of water is 2 m. Calculate: i) Specific Energy ii) Critical Depth iii) Critical Velocity iv) Minimum Energy

3. The specific energy for a 5 m wide rectangular channel is to be 4 Nm/N. If the rate of flow of water through the channel us 20 m^3/s, determine the alternate depths of flow.

Hydraulic Jump



Flow under a sluice gate accelerates from subcritical to critical to supercritical and then jumps back to subcritical flow

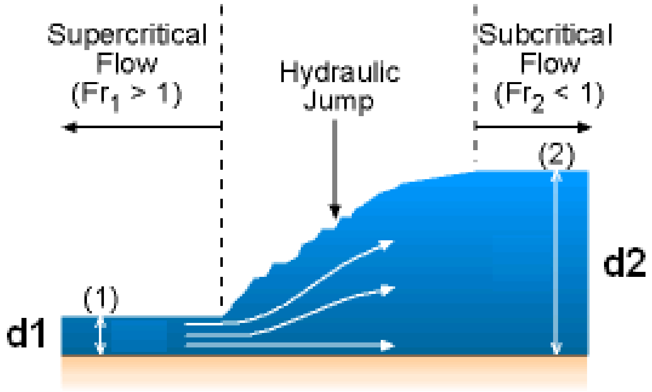


Froude Numbers and Fluid Depths across a Hydraulic Jump

Hydraulic Jump

The hydraulic jump is defined as the rise of water level, which takes place due to transformation of the unstable shooting flow (super-critical) to the stable streaming flow (sub-critical).

When hydraulic jump occurs, a loss of energy due to eddy formation and turbulence flow occurs.

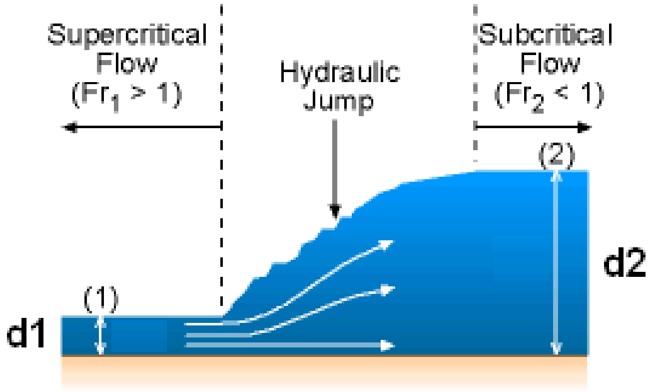


Froude Numbers and Fluid Depths across a Hydraulic Jump

Hydraulic Jump

The most typical cases for the location of hydraulic jump are:

1. Below control structures like weir, sluice are used in the channel
2. when any obstruction is found in the channel,
3. when a sharp change in the channel slope takes place.
4. At the toe of a spillway dam



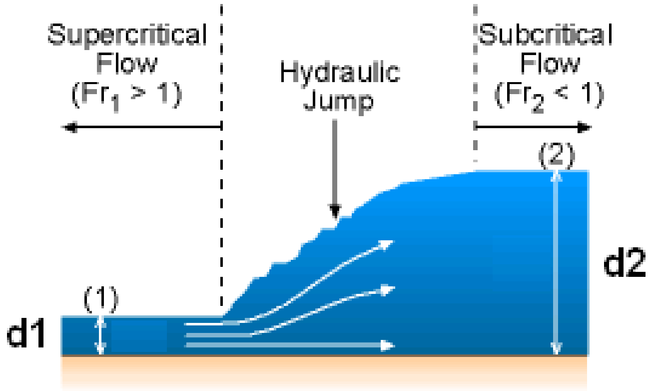
Froude Numbers and Fluid Depths across a Hydraulic Jump

Hydraulic Jump

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{g d_1}} \rightarrow \text{interms of } q$$

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2 v_1^2 d_1}{g_1}} \rightarrow \text{interms of } V_1$$

$$d_2 = \frac{d_1}{2}\left(\sqrt{1 + 8 F_e^2} - 1\right) \rightarrow \text{interms of } F_e$$



Froude Numbers and Fluid Depths across a Hydraulic Jump

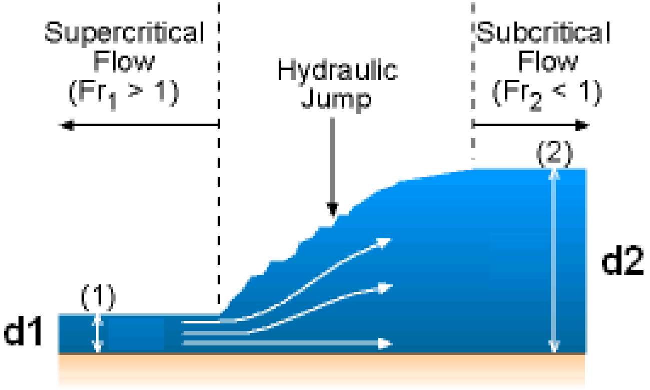
Hydraulic Jump

Loss of Energy :

$$h_L = E1 - E2 = \left[\frac{[d_2 - d_1]^3}{4 d_1 d_2} \right]$$

Length of jump $= 5$ to 7 times of $(d_2 - d_1)$

Hydrualic Jump $= d_2 - d_1$

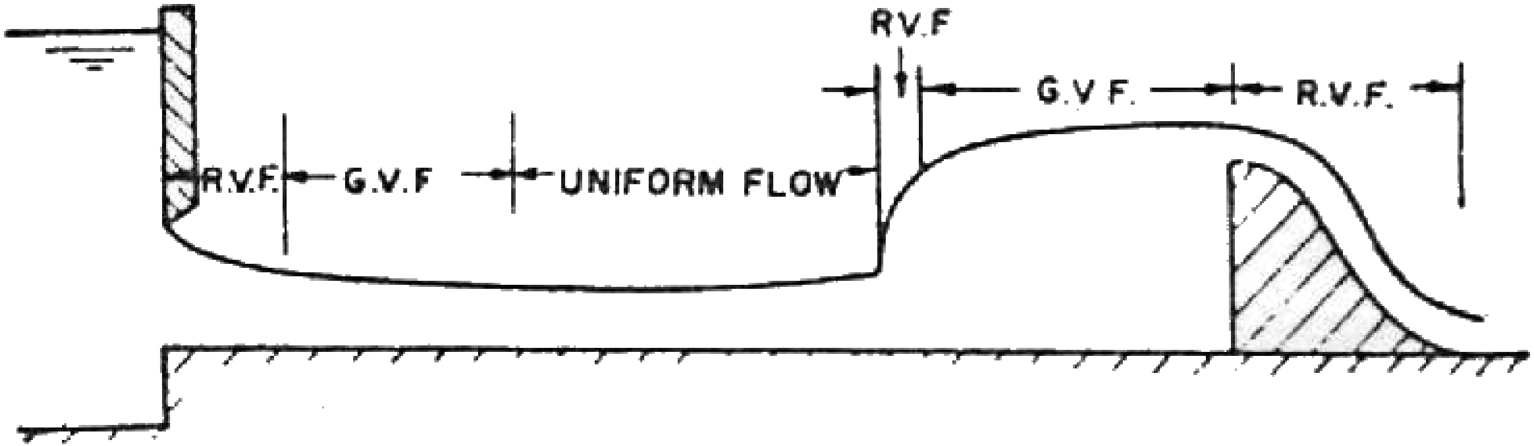


Froude Numbers and Fluid Depths across a Hydraulic Jump

Problems

1. The depth of flow of water, at a certain section of a rectangular channel of 2 m wide is 0.3 m. The discharge through the channel is 1.5 m^3/s. Determine whether a hydraulic jump will occur, and if so, find its height and loss of energy per kg of water.
2. A sluice gate discharges water into a horizontal rectangular channel with a velocity of 10 m/s and depth of flow of 1 m. Determine the depth of flow after jump and consequent loss in total head.

Gradually Varied Flow (GVF)



Gradually Varied Flow (GVF)

In GVF, depth and velocity vary slowly, and the free surface is stable

The GVF is classified based on the channel slope, and the magnitude of flow depth.

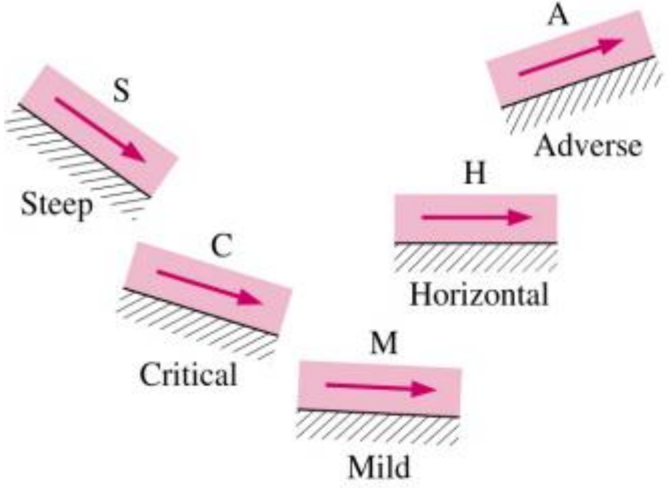
Steep Slope (S): $S_o > S_c$ or $h < h_c$

Critical Slope (C): $S_o = S_c$ or $h = h_c$

Mild Slope (M): $S_o < S_c$ or $h > h_c$

Horizontal Slope (H): $S_o = 0$

Adverse Slope(A): $S_o =$ Negative

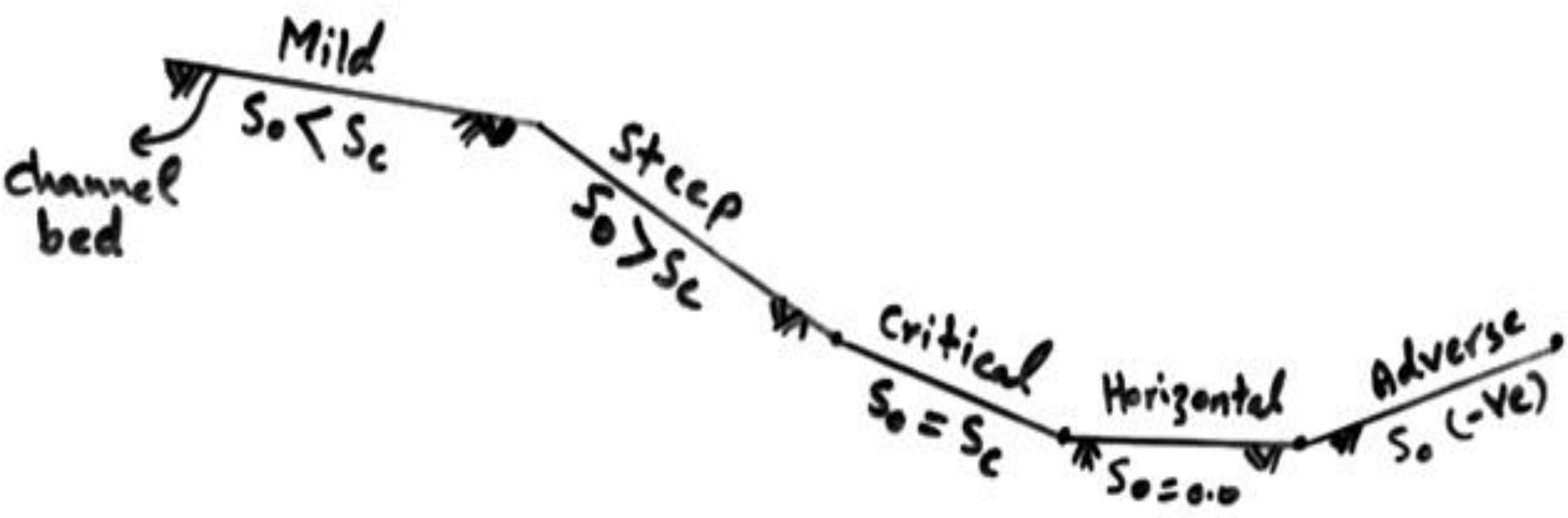


where

So : the slope of the channel bed,

Sc : the critical slope that sustains a given discharge as uniform flow at the critical depth (hc).

Gradually Varied Flow (GVF)



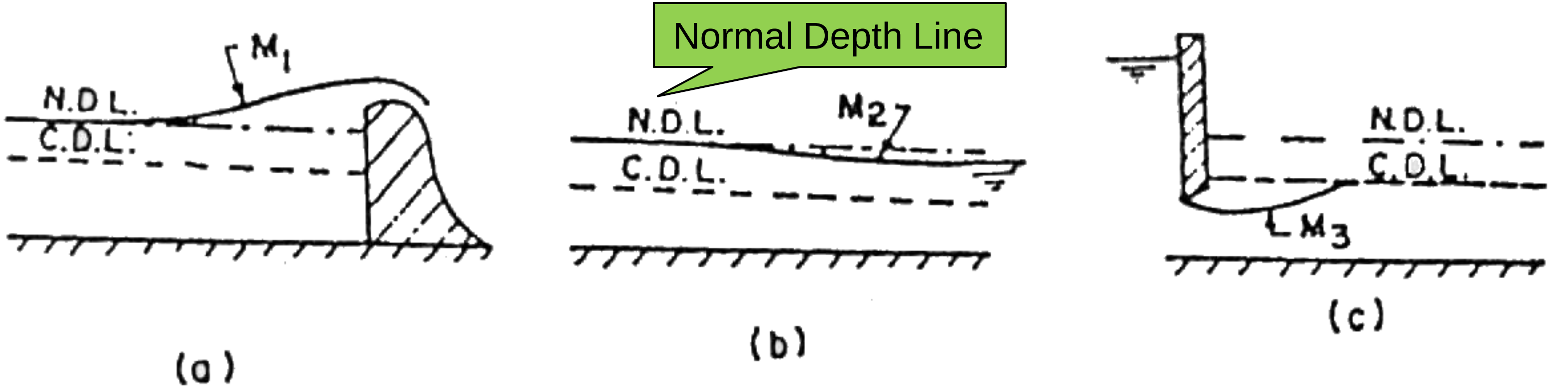
Flow Profiles

The surface curves of water are called flow profiles (or water surface profiles).

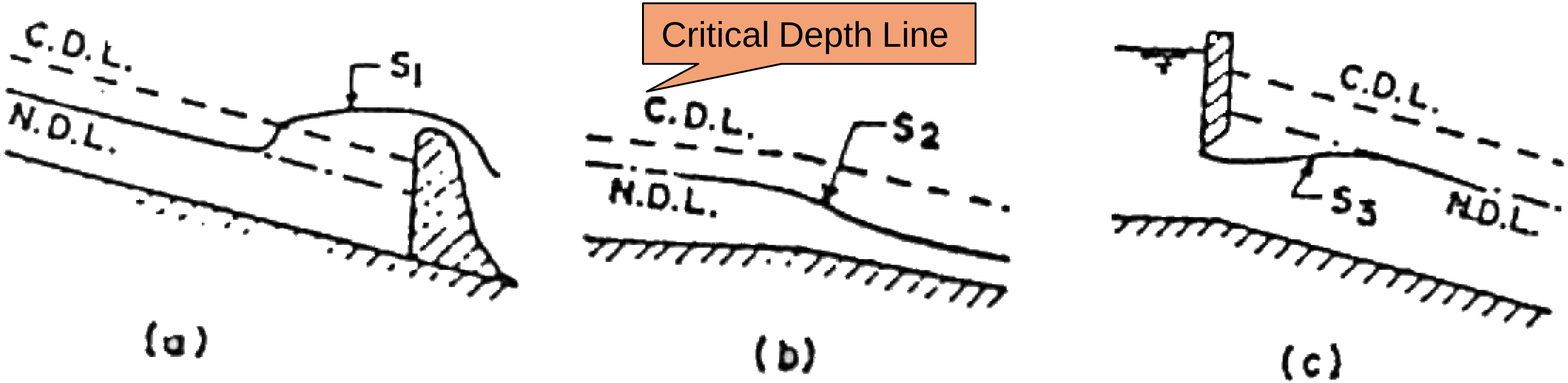
Depending upon the zone and the slope of the bed, the water profiles are classified into 13 types as follows:

1. Mild slope curves	M1, M2, M3
2. Steep slope curves	S1, S2, S3
3. Critical slope curves	C1, C2, C3
4. Horizontal slope curves	H2, H3
5. Averse slope curves	A2, A3

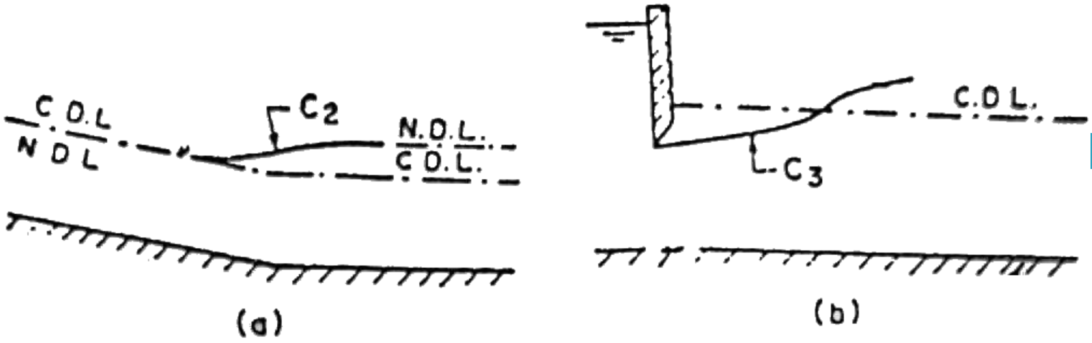
In all these curves, the letter indicates the slope type and the subscript indicates the zone. For example S2 curve occurs in the zone 2 of the steep slope



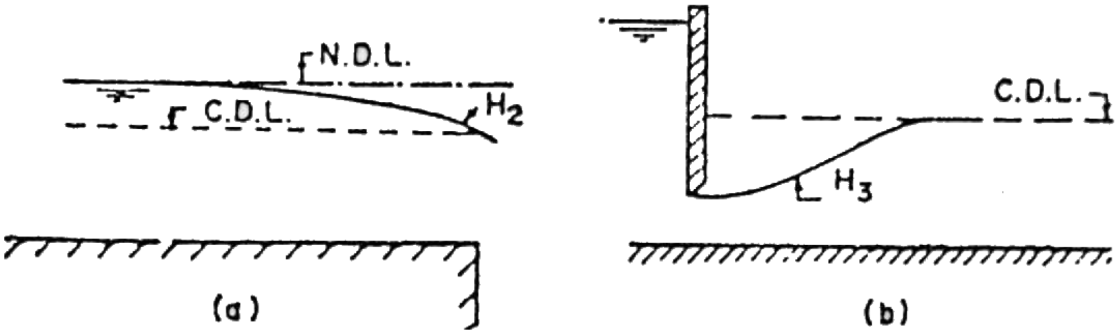
Flow Profiles in Mild slope



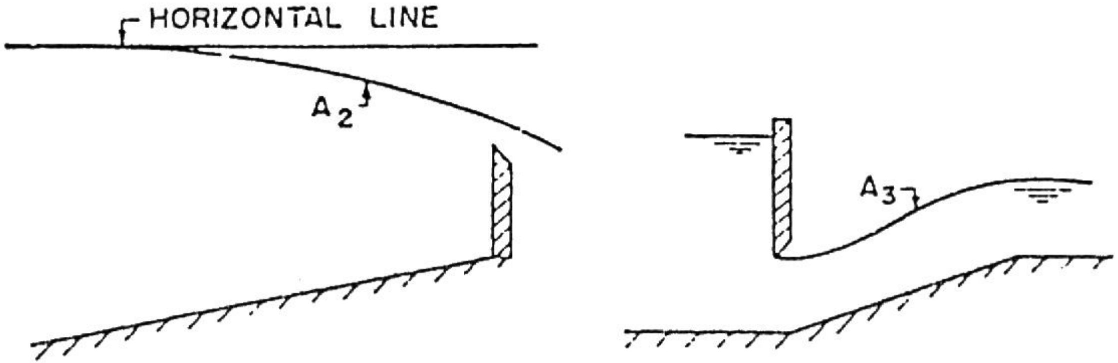
Flow Profiles in Steep slope



Flow Profiles in Critical slope



Flow Profiles in Horizontal slope



Flow Profiles in Adverse slope

Gradually Varied Flow (GVF)

Equation of GVF :

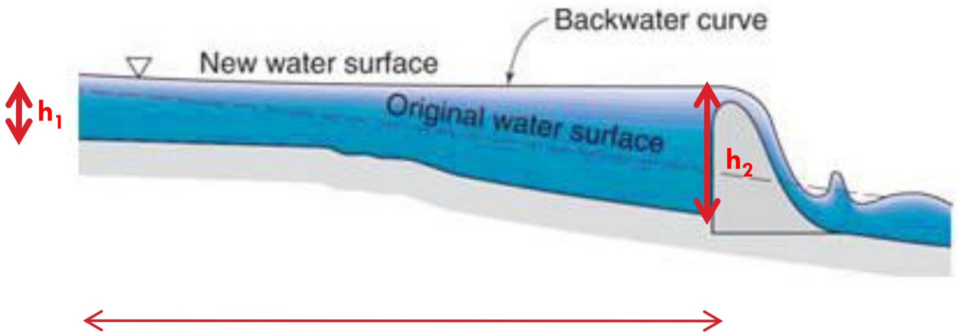
$$\frac{dh}{dx} = \frac{i_b - i_e}{\left[1 - \frac{V^2}{gh}\right]} \rightarrow \text{in terms of Velocity}$$

$$\frac{dh}{dx} = \frac{i_b - i_e}{\left[1 - (F_e)^2\right]} \rightarrow \text{in terms of Fe}$$

where $\frac{dh}{dx}$ represents the variation of water depth along the bottom of the channel

S_c or i_b Energy Line Slope

S_o or i_e Bed Slope



$$L = \frac{E_2 - E_1}{i_b - i_e}$$

Gradually Varied Flow (GVF)

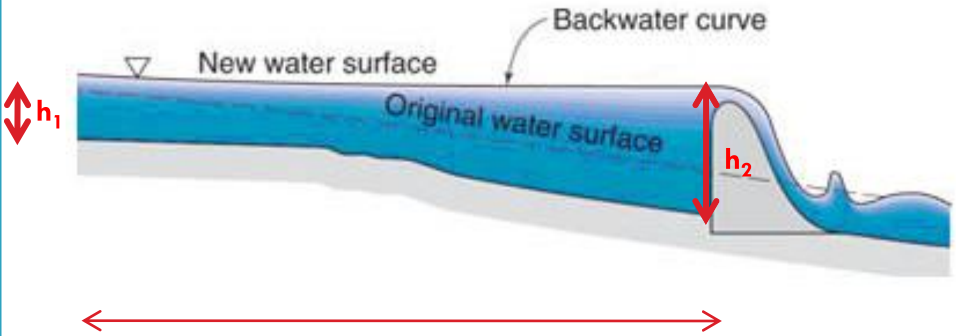
If $dh/dx = 0$, Free Surface of water is parallel to the bed of channel

If $dh/dx > 0$, Depth increases in the direction of water flow (Back Water Curve)

If $dh/dx < 0$, Depth of water decreases in the direction of flow (Dropdown Curve)

S_c or i_b Energy Line Slope

S_o or i_e Bed Slope



$$L = \frac{E_2 - E_1}{i_b - i_e}$$

where $\frac{dh}{dx}$ represents the variation of water depth along the bottom of the channel

Problems

1. Find the rate of change of depth of water in a rectangular channel of 10 m wide and 1.5 m deep, when water is flowing with a velocity of 1 m/s. The flow of water through the channel of bed slope in 1 in 4000, is regulated in such a way that energy line is having a slope of 0.00004

2. Find the slope of the free water surface in a rectangular channel of width 20 m, having depth of flow 5 m. The discharge through the channel is 50 m^3/s. The bed of channel is having a slope of 1 in 4000. Take C=60

DIMENSIONAL ANALYSIS

(OPEN CHANNEL FLOW AND HYDRAULIC MACHINERY)

UNIT – II

IARE

Dr.G. Venkata Ramana Professor& HOD Civil Engineering

Learning Objectives

1. Introduction to Dimensions & Units
2. Use of Dimensional Analysis
3. Dimensional Homogeneity
4. Methods of Dimensional Analysis
5. Rayleigh's Method

Learning Objectives

6. Buckingham’s Method

7. Model Analysis

8. Similitude

9. Model Laws or Similarity Laws

10. Model and Prototype Relations

Introduction

- Many practical real flow problems in fluid mechanics can be solved by using equations and analytical procedures. However, solutions of some real flow problems depend heavily on experimental data.

- Sometimes, the experimental work in the laboratory is not only time-consuming, but also expensive. So, the main goal is to extract maximum information from fewest experiments.

- In this regard, dimensional analysis is an important tool that helps in correlating analytical results with experimental data and to predict the prototype behavior from the measurements on the model.

Dimensions and Units

In dimensional analysis we are only concerned with the nature of the dimension i.e. its quality <u>not</u> its quantity.

- Dimensions are properties which can be measured.
  Ex.: Mass, Length, Time etc.,
- Units are the standard elements we use to quantify these dimensions.
  Ex.: Kg, Metre, Seconds etc.,

The following are the Fundamental Dimensions (MLT)

Mass	kg	M
Length	m	L
Time	s	T

Secondary or Derived Dimensions

Secondary dimensions are those quantities which posses more than one fundamental dimensions.

1. Geometric

a)	Area	m^2		L^2
b)	Volume	m^3		L^3

2. Kinematic

a)	Velocity	m/s	L/T	$L.T^{-1}$
b)	Acceleration	m/s^2	L/T^2	$L.T^{-2}$

3. Dynamic

a)	Force	N	ML/T	$M.L.T^{-1}$
b)	Density	kg/m^3	M/L^3	$M.L^{-3}$

Problems

Find Dimensions for the following:

1. Stress / Pressure
2. Work
3. Power
4. Kinetic Energy
5. Dynamic Viscosity
6. Kinematic Viscosity
7. Surface Tension
8. Angular Velocity
9. Momentum
10. Torque

Use of Dimensional Analysis

1. Conversion from one dimensional unit to another
2. Checking units of equations (Dimensional Homogeneity)
3. Defining dimensionless relationship using
   a) Rayleigh's Method
   b) Buckingham's π-Theorem
4. Model Analysis

Dimensional Homogeneity

Dimensional Homogeneity means the dimensions in each equation on both sides equal.

Let us consider the equation, $V = \sqrt{2gH}$

Dimension of L.H.S. $= V = \frac{L}{T} = LT^{-1}$

Dimension of R.H.S. $= \sqrt{2gH} = \sqrt{\frac{L}{T^2} \times L} = \sqrt{\frac{L^2}{T^2}} = \frac{L}{T} = LT^{-1}$

Dimension of L.H.S. $=$ Dimension of R.H.S. $= LT^{-1}$

$\therefore$ Equation $V = \sqrt{2gH}$ is dimensionally homogeneous. So it can be used in any system of units.

Problems

Check Dimensional Homogeneity of the following:

1. $Q = AV$
2. $E_K = v^2/2g$

Rayeligh's Method

To define relationship among variables

This method is used for determining the expression for a variable which depends upon maximum three or four variables only.

Rayeligh's Method

Methodology:

Let X is a function of X_1 ,X_2, X_3 and mathematically it can be written as

$X = f(X_1, X_2, X_3)$

This can be also written as

$X = K\ (X_1^a\ ,\ X_2^b\ ,\ X_3^c\ )$ where K is constant and a, b and c are arbitrarily powers

The values of a, b and c are obtained by comparing the powers of the fundamental dimension on both sides.

Rayeligh's Method

Problem: Find the expression for Discharge Q in a open channel flow when Q is depends on Area A and Velocity V.

Solution:

$Q = K.A^a.V^b \rightarrow 1$

where K is a Non-dimensional constant

Substitute the dimensions on both sides of equation 1

$M^0 L^3 T^{-1} = K.(L^2)^a.(LT^{-1})^b$

Equating powers of M, L, T on both sides,

Power of T, $-1 = -b \rightarrow b=1$

Power of L, $3= 2a+b \rightarrow 2a = 2-b = 2-1 = 1$

Substituting values of a, b, and c in Equation 1m

$Q = K.A^1.V^1 = V.A$

Problem : Find the equation for the power developed by a pump if it depends on head H discharge Q and specific weight γ of the fluid.

Solution:

Power	$= L^2MT^{-3}$
Head	$= LM^oT^o$
Discharge	$= L^3M^oT^{-1}$
Specific Weight	$= L^{-2}MT^{-2}$

$$P = f(H, Q, \gamma)$$

$$P = K \cdot H^a \cdot Q^b \cdot \gamma^c$$

$$[P] = [H]^a \cdot [Q]^b \cdot [\gamma]^c$$

$$[L^2MT^{-3}] = [LM^oT^o]^a \cdot [L^3M^oT^{-1}]^b \cdot [L^{-2}MT^{-2}]^c$$

Equating the powers of M, L and T on both sides,

Power of M, $1 = c$

Power of T, $-3 = -b - 2$ or $b = -2 + 3$ or $\mathbf{b = 1}$

Power of L, $2 = a + 3b - 2c$ or $2 = a + 3 - 2$ or $\mathbf{a = 1}$

Substituting the values of a, b and c

$$P = K \cdot H^1 \cdot Q^1 \cdot \gamma^1$$

$P = K \cdot H \cdot Q \cdot \gamma$ When, $K = 1$ $\mathbf{P = H \cdot Q \cdot \gamma}$

Problem 3: Find an expression for drag force R on a smooth sphere of diameter D moving with uniform velocity V in a fluid of density ρ and dynamic viscosity μ..

Solution:

Force	$= LMT^{-2}$
Diameter	$= LM^oT^o$
Velocity	$= LM^oT^{-1}$
Mass density	$= L^3MT^o$

$R = f(D, V, \rho, \mu)$

$R = K \cdot D^a \cdot V^b \cdot \rho^c, \mu^d$

$[R] = [D]^a \cdot [V]^b \cdot [\rho]^c \cdot [\mu]^d$

$[LMT^{-2}] = [LM^oT^o]^a \cdot [LM^oT^{-1}]^b \cdot [L^{-3}MT^o]^c \cdot [L^{-1}MT^{-1}]^d$

Equating the powers of M, L and T on both sides,

Power of M, $1 = c + d$ or $\mathbf{c = 1 - d}$

Power of T, $-2 = -b - d$ or $\mathbf{b = 2 - d}$

Power of L, $1 = a + b - 3c - d$ or $1 = a + 2 - d - 3(1 - d) - d$

$1 = a + 2 - d - 3 + 3d - d$ or $\mathbf{a = 2 - d}$

Substituting teh values of a, b, and c

$$R = K \cdot D^{2-d} \cdot V^{2-d} \cdot \rho^{1-d}, \mu^d = K \frac{D^2}{D^d} \cdot \frac{V^2}{V^d} \cdot \frac{\rho}{\rho^d} \cdot \mu^d$$

$$= K \cdot \rho V^2 D^2 \left[\frac{\mu}{\rho V D}\right]^d = \rho V^2 D^2 \phi \left[\frac{\mu}{\rho V D}\right] = \rho V^2 D^2 \phi \left[\frac{\rho V D}{\mu}\right]$$

Buckingham's π-Theorem

This method of analysis is used when number of variables are more.

Theorem:

If there are n variables in a physical phenomenon and those n variables contain m dimensions, then variables can be arranged into (n-m) dimensionless groups called Φ terms.

Explanation:

If $f(X_1, X_2, X_3, \ldots\ldots\ldots X_n) = 0$ and variables can be expressed using m dimensions then
$f(\pi_1, \pi_2, \pi_3, \ldots\ldots\ldots \pi_{n-m}) = 0$ where, $\pi_1, \pi_2, \pi_3, \ldots$ are dimensionless groups.
Each π term contains (m + 1) variables out of which m are of repeating type and one is of non-repeating type.
Each π term being dimensionless, the dimensional homogeneity can be used to get each π term.

π denotes a non-dimensional parameter

Buckingham's π-Theorem

Selecting Repeating Variables:

1. Avoid taking the quantity required as the repeating variable.
2. Repeating variables put together should not form dimensionless group.
3. No two repeating variables should have same dimensions.
4. Repeating variables can be selected from each of the following properties.
   - Geometric property → Length, height, width, area
   - Flow property → Velocity, Acceleration, Discharge
   - Fluid property → Mass density, Viscosity, Surface tension

Problem 12.11 *The pressure difference Δp in a pipe of diameter D and length l due to viscous flow depends on the velocity V, viscosity μ and density ρ. Using Buckingham's π-theorem, obtain an expression for Δp.*

Solution.

Δp is a function of D, l, V, μ, ρ or $\Delta p = f(D, l, V, \mu, \rho)$

or $$f_1(\Delta p, D, l, V, \mu, \rho) = 0 \qquad \ldots(i)$$

Total number of variables, $n = 6$

Number of fundamental dimension, $m = 3$

Number of π-terms $= n - 3 = 6 - 3 = 3$

Hence equation (i) is written as $f_1(\pi_1, \pi_2, \pi_3) = 0 \qquad \ldots(ii)$

Each π-term contains $m + 1$ variables, *i.e.*, $3 + 1 = 4$ variable. Out of four variables, three are repeating variables.

Choosing D, V, μ as repeating variables, we have π-terms as

$$\pi_1 = D^{a_1} \cdot V^{b_1} \cdot \mu^{c_1} \cdot \Delta p$$

$$\pi_2 = D^{a_2} \cdot V^{b_2} \cdot \mu^{c_2} \cdot l$$

$$\pi_3 = D^{a_3} \cdot V^{b_3} \cdot \mu^{c_3} \cdot \rho$$

First π-term $\pi_1 = D^{a_1} . V^{b_1} . \mu^{c_1} . \Delta p$

Substituting the dimensions on both sides,

$$M^0L^0T^0 = L^{a_1} . (LT^{-1})^{b_1} . (ML^{-1}T^{-1})^{c_1} . ML^{-1}T^{-2}$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_1 + 1$, $\therefore\ c_1 = -1$

Power of L, $0 = a_1 + b_1 - c_1 - 1$, $\therefore\ a_1 = -b_1 + c_1 + 1 = 1 - 1 + 1 = 1$

Power of T, $0 = -b_1 - c_1 - 2$, $\therefore\ b_1 = -c_1 - 2 = 1 - 2 = -1$

Substituting the values of a_1, b_1 and c_1 in π_1,

$$\pi_1 = D^1 . V^{-1} . \mu^{-1} . \Delta p = \frac{D\Delta p}{\mu V}.$$

Second π-term $\pi_2 = D^{a_2} . V^{b_2} . \mu^{c_2} . l$

Substituting the dimensions on both sides,

$$M^0L^0T^0 = L^{a_2} . (LT^{-1})^{b_2} . (ML^{-1}T^{-1})^{c_2} . L.$$

Equating the powers of M, L, T on both sides

Power of M, $0 = c_2$, $\therefore\ c_2 = 0$

Power of L, $0 = a_2 + b_2 - c_2 + 1$, $\therefore\ a_2 = -b_2 + c_2 - 1 = -1$

Power of T, $0 = -b_2 - c_2$, $\therefore\ b_2 = -c_2 = 0$

Substituting the values of a_2, b_2 and c_2 in π_2,

$$\pi_2 = D^{-1} . V^0 . \mu^0 . l = \frac{l}{D}.$$

Third π-term $\pi_3 = D^{a_3} . V^{b_3} . \mu^{c_3} . \rho$

Substituting the dimension on both sides,

$$M^0L^0T^0 = L^{a_3} . (LT^{-1})^{b_3} . (ML^{-1}T^{-1})^{c_3} . ML^{-3}.$$

Equating the powers of M, L, T on both sides

Power of M, $0 = c_3 + 1$, $\therefore\ c_3 = -1$

Power of L, $0 = a_3 + b_3 - c_3 - 3$, $\therefore\ a_3 = -b_3 + c_3 + 3 = -1 - 1 + 3 = 1$

Power of T, $0 = -b_3 - c_3$, $\therefore\ b_3 = -c_3 = -(-1) = 1$

Substituting the values of a_3, b_3 and c_3 in π_3,

$$\pi_3 = D^1 . V^1 . \mu^{-1} . \rho = \frac{\rho DV}{\mu}.$$

Substituting the values of π_1, π_2 and π_3 in equation (*ii*),

$$f_1\left(\frac{D\Delta p}{\mu V}, \frac{l}{D}, \frac{\rho DV}{\mu}\right) = 0 \quad \text{or} \quad \frac{D\Delta p}{\mu V} = \phi\left[\frac{l}{D}, \frac{\rho DV}{\mu}\right] \quad \text{or} \quad \Delta p = \frac{\mu V}{D}\phi\left[\frac{l}{D}, \frac{\rho DV}{\mu}\right].$$

Experiments show that the pressure difference Δp is a linear function $\frac{l}{D}$. Hence $\frac{l}{D}$ can be taken out of the functional as

$$\Delta p = \frac{\mu \mathbf{V}}{\mathbf{D}} \times \frac{\mathbf{L}}{\mathbf{D}}\phi\left[\frac{\rho \mathbf{DV}}{\mu}\right]. \textbf{ Ans.}$$

Expression for difference of pressure head for viscous flow

$$h_f = \frac{\Delta p}{\rho g} = \frac{\mu V}{D} \times \frac{l}{D} \times \frac{1}{\rho g}\phi\,[R_e] \qquad \left\{\because\ \frac{\rho DV}{\mu} = R_e\right\}$$

$$= \frac{\mu \mathbf{VL}}{\mathbf{wD^2}}\phi\,[\mathbf{R_e}]. \textbf{ Ans.}$$

Model Analysis

For predicting the performance of the hydraulic structures (such as dams, spillways etc.) or hydraulic machines (such as turbines, pumps etc.) before actually constructing or manufacturing, models of the structures or machines are made and tests are conducted on them to obtain the desired information.

Model is a small replica of the actual structure or machine

The actual structure or machine is called as **Prototype**

Models can be smaller or larger than the Prototype

Model Analysis is actually an experimental method of finding solutions of complex flow problems.

Similitude or Similarities

Similitude is defined as the similarity between the model and prototype in every aspect, which means that the model and prototype have similar properties.

Types of Similarities:

1. Geometric Similarity → Length, Breadth, Depth, Diameter, Area, Volume etc.,
2. Kinematic Similarity → Velocity, Acceleration etc.,
3. Dynamic Similarity → Time, Discharge, Force, Pressure Intensity, Torque, Power

Geometric Similarity

The geometric similarity is said to be exist between the model and prototype if the ratio of all corresponding linear dimensions in the model and prototype are equal.

$$\frac{L_P}{L_m} = \frac{B_P}{B_m} = \frac{D_P}{D_m} = L_r$$

$$\frac{A_P}{A_m} = L_r^2$$

$$\frac{V_P}{V_m} = L_r^3$$

where L_r is Scale Ratio

Kinematic Similarity

The kinematic similarity is said exist between model and prototype if the ratios of velocity and acceleration at corresponding points in the model and at the corresponding points in the prototype are the same.

$$\frac{V_P}{V_m} = V_r$$

$$\frac{a_P}{a_m} = a_r$$

where V_r is Velocity Ratio

where a_r is Acceleration Ratio

Also the directions of the velocities in the model and prototype should be same

Dynamic Similarity

The dynamic similarity is said exist between model and prototype if the ratios of corresponding **forces** acting at the corresponding points are equal

$$\frac{F_P}{F_m} = F_r$$

where F_r is Force Ratio

It means for dynamic similarity between the model and prototype, the dimensionless numbers should be same for model and prototype.

Types of Forces Acting on Moving Fluid

1. Inertia Force, F_i

- It is the product of mass and acceleration of the flowing fluid and acts in the direction opposite to the direction of acceleration.
- It always exists in the fluid flow problems

Types of Forces Acting on Moving Fluid

1. Inertia Force, F_i
2. Viscous Force, F_v

- It is equal to the product of shear stress due to viscosity and surface area of the flow.
- It is important in fluid flow problems where viscosity is having an important role to play

Types of Forces Acting on Moving Fluid

1. Inertia Force, F_i
2. Viscous Force, F_v
3. Gravity Force, F_g

- It is equal to the product of mass and acceleration due to gravity of the flowing fluid.
- It is present in case of open surface flow

Types of Forces Acting on Moving Fluid

1. Inertia Force, F_i
2. Viscous Force, F_v
3. Gravity Force, F_g
4. Pressure Force, F_p

- It is equal to the product of pressure intensity and cross sectional area of flowing fluid
- It is present in case of pipe-flow

Types of Forces Acting on Moving Fluid

1. Inertia Force, F_i
2. Viscous Force, F_v
3. Gravity Force, F_g
4. Pressure Force, F_p
5. Surface Tension Force, F_s

➢ It is equal to the product of surface tension and length of surface of the flowing fluid

Types of Forces Acting on Moving Fluid

1. Inertia Force, F_i
2. Viscous Force, F_v
3. Gravity Force, F_g
4. Pressure Force, F_p
5. Surface Tension Force, F_s
6. Elastic Force, F_e

- It is equal to the product of elastic stress and area of the flowing fluid

Dimensionless Numbers

Dimensionless numbers are obtained by dividing the inertia force by viscous force or gravity force or pressure force or surface tension force or elastic force.

1. Reynold's number, R_e = $\frac{\text{Inertia Force}}{\text{Viscous Force}} = \frac{\rho VL}{\mu} \; or \; \frac{\rho VD}{\mu}$
2. Froude's number, F_e = $\sqrt{\frac{\text{Inertia Force}}{\text{Gravity Force}}} = \frac{V}{\sqrt{Lg}}$
3. Euler's number, E_u = $\sqrt{\frac{\text{Inertia Force}}{\text{Pressure Force}}} = \frac{V}{\sqrt{p/\rho}}$
4. Weber's number, W_e = $\sqrt{\frac{\text{Inertia Force}}{\text{Surface Tension Force}}} = \frac{V}{\sqrt{\sigma/\rho L}}$
5. Mach's number, M = $\sqrt{\frac{\text{Inertia Force}}{\text{Elastic Force}}} = \frac{V}{C}$

12.8.1 Reynold's Number (R_e). It is defined as the ratio of inertia force of a flowing fluid and the viscous force of the fluid. The expression for Reynold's number is obtained as

$$\text{Inertia force } (F_i) = \text{Mass} \times \text{Acceleration of flowing fluid}$$

$$= \rho \times \text{Volume} \times \frac{\text{Velocity}}{\text{Time}} = \rho L^3 \frac{V}{T} = \rho L^2 \frac{L}{T} V = \rho L^2 V^2$$

$$= \rho A V^2 \qquad \text{...(12.11)}$$

$$\text{Viscous force } (F_v) = \text{Shear stress} \times \text{Area} \qquad \left\{\because \ \tau = \mu \frac{du}{dy} \ \therefore \ \text{Force} = \tau \times \text{Area}\right\}$$

$$= \tau \times A$$

$$= \left(\mu \frac{du}{dy}\right) \times A = \mu \cdot \frac{V}{L} \times A \qquad \left\{\because \ \frac{du}{dy} = \frac{V}{L}\right\}$$

By definition, Reynold's number,

$$R_e = \frac{F_i}{F_v} = \frac{\rho A V^2}{\mu \cdot \frac{V}{L} \times A} = \frac{\rho V L}{\mu}$$

$$= \frac{V \times L}{(\mu / \rho)} = \frac{V \times L}{\nu} \qquad \left\{\because \ \frac{\mu}{\rho} = \nu = \text{Kinematic viscosity}\right\}$$

In case of pipe flow, the linear dimension L is taken as diameter, d. Hence Reynold's number for pipe flow,

$$R_e = \frac{V \times d}{\nu} \quad \text{or} \quad \frac{\rho V d}{\mu}. \qquad \text{...(12.12)}$$

12.8.2 Froude's Number (F_e).

The Froude's number is defined as the square root of the ratio of inertia force of a flowing fluid to the gravity force. Mathematically, it is expressed as

$$F_e = \sqrt{\frac{F_i}{F_g}}$$

where F_i from equation (12.11) $= \rho A V^2$

and F_g = Force due to gravity

$= \text{Mass} \times \text{Acceleration due to gravity}$

$= \rho \times \text{Volume} \times g = \rho \times L^3 \times g$ $\{\because \text{Volume} = L^3\}$

$= \rho \times L^2 \times L \times g = \rho \times A \times L \times g$ $\{\because L^2 = A = \text{Area}\}$

$$\therefore \quad F_e = \sqrt{\frac{F_i}{F_g}} = \sqrt{\frac{\rho A V^2}{\rho A L g}} = \sqrt{\frac{V^2}{Lg}} = \frac{V}{\sqrt{Lg}} \quad \text{...(12.13)}$$

12.8.3 Euler's Number (E_u). It is defined as the square root of the ratio of the inertia force of a flowing fluid to the pressure force. Mathematically, it is expressed as

$$E_u = \sqrt{\frac{F_i}{F_p}}$$

where F_p = Intensity of pressure × Area = $p \times A$

and $F_i = \rho A V^2$

$$\therefore \quad E_u = \sqrt{\frac{\rho A V^2}{p \times A}} = \sqrt{\frac{V^2}{p/\rho}} = \frac{V}{\sqrt{p/\rho}} \quad \text{...(12.14)}$$

12.8.4 Weber's Number (W_e).

It is defined as the square root of the ratio of the inertia force of a flowing fluid to the surface tension force. Mathematically, it is expressed as

Weber's Number, $$W_e = \sqrt{\frac{F_i}{F_s}}$$

where F_i = Inertia force = $\rho A V^2$

and F_s = Surface tension force

= Surface tension per unit length × Length = $\sigma \times L$

$$\therefore \quad W_e = \sqrt{\frac{\rho A V^2}{\sigma \times L}} = \sqrt{\frac{\rho \times L^2 \times V^2}{\sigma \times L}} \qquad \{\because \ A = L^2\}$$

$$= \sqrt{\frac{\rho L \times V^2}{\sigma}} = \sqrt{\frac{V^2}{\sigma / \rho L}} = \frac{V}{\sqrt{\sigma / \rho L}}. \qquad \text{...(12.15)}$$

12.8.5 Mach's Number (M).

Mach's number is defined as the square root of the ratio of the inertia force of a flowing fluid to the elastic force. Mathematically, it is defined as

$$M = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} = \sqrt{\frac{F_i}{F_e}}$$

where $F_i = \rho A V^2$

and F_e = Elastic force = Elastic stress × Area

$= K \times A = K \times L^2$ $\qquad \{\because \ K = \text{Elastic stress}\}$

$$\therefore \quad M = \sqrt{\frac{\rho A V^2}{K \times L^2}} = \sqrt{\frac{\rho \times L^2 \times V^2}{K \times L^2}} = \sqrt{\frac{V^2}{K/\rho}} = \frac{V}{\sqrt{K/\rho}}$$

But $$\sqrt{\frac{K}{\rho}} = C = \text{Velocity of sound in the fluid}$$

$$\therefore \quad M = \frac{V}{C}. \qquad \text{...(12.16)}$$

Model Laws

The laws on which the models are designed for dynamic similarity are called model laws or laws of similarity.

1. Reynold's Model

Models based on Reynolds's Number includes:

a) Pipe Flow
b) Resistance experienced by Sub-marines, airplanes, fully immersed bodies etc.

Model Laws

The laws on which the models are designed for dynamic similarity are called model laws or laws of similarity.

1. Reynold's Model
2. Froude Model Law

Froude Model Law is applied in the following fluid flow problems:

a) Free Surface Flows such as Flow over spillways, Weirs, Sluices, Channels etc.,
b) Flow of jet from an orifice or nozzle
c) Where waves are likely to formed on surface
d) Where fluids of different densities flow over one another

Model Laws

The laws on which the models are designed for dynamic similarity are called model laws or laws of similarity.

1. Reynold’s Model
2. Froude Model Law
3. Euler Model Law

Euler Model Law is applied in the following cases:

a) Closed pipe in which case turbulence is fully developed so that viscous forces are negligible and gravity force and surface tension is absent
b) Where phenomenon of cavitations takes place

Model Laws

The laws on which the models are designed for dynamic similarity are called model laws or laws of similarity.

1. Reynold's Model
2. Froude Model Law
3. Euler Model Law
4. Weber Model Law

Weber Model Law is applied in the following cases:

a) Capillary rise in narrow passages
b) Capillary movement of water in soil
c) Capillary waves in channels
d) Flow over weirs for small heads

Model Laws

The laws on which the models are designed for dynamic similarity are called model laws or laws of similarity.

1. Reynold's Model
2. Froude Model Law
3. Euler Model Law
4. Weber Model Law
5. Mach Model Law

Mach Model Law is applied in the following cases:

a) Flow of aero plane and projectile through air at supersonic speed ie., velocity more than velocity of sound

b) Aero dynamic testing, c) Underwater testing of torpedoes, and

d) Water-hammer problems

Reynold’s Model Law

If the viscous forces are predominant, the models are designed for dynamic similarity based on Reynold’s number.

$$[\mathbf{Re}]_m = [\mathbf{Re}]_p$$

$$t_r = \text{Time Scale Ratio} = \frac{L_r}{V_r}$$

Velocity, V = Length/Time → T = L/V

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

$$a_r = \text{Acceleration Scale Ratio} = \frac{V_r}{t_r}$$

Acceleration, a = Velocity/Time → L = V/T

Problem 6.15 *A pipe of diameter 1.5 m is required to transport an oil of sp. gr. 0.90 and viscosity 3×10^{-2} poise at the rate of 3000 litre/s. Tests were conducted on a 15 cm diameter pipe using water at 20°C. Find the velocity and rate of flow in the model. Viscosity of water at 20°C = 0.01 poise.*

(Delhi University, 1992)

Solution. Given :

Dia. of prototype, $D_p = 1.5$ m

Viscosity of fluid, $\mu_p = 3 \times 10^{-2}$ poise

Q for prototype, $Q_p = 3000$ lit/s $= 3.0$ m^3/s

Sp. gr. of oil, $S_p = 0.9$

$\therefore$ Density of oil, $\rho_p = S_p \times 1000 = 0.9 \times 1000 = 900$ kg/m^3

Dia. of the model, $D_m = 15$ cm $= 0.15$ m

Viscosity of water at 20°C $= .01$ poise $= 1 \times 10^{-2}$ poise or $\mu_m = 1 \times 10^{-2}$ poise

Density of water or $\rho_m = 1000$ kg/m^3.

For pipe flow, the dynamic similarity will be obtained if the Reynold's number in the model and prototype are equal

Hence using equation (6.17), $\frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho_P V_P D_P}{\mu_P}$ {For pipe, linear dimension is D}

$$\therefore \quad \frac{V_m}{V_P} = \frac{\rho_P}{\rho_m} \cdot \frac{D_P}{D_m} \cdot \frac{\mu_P}{\mu_m}$$

$$= \frac{900}{1000} \times \frac{1.5}{0.15} \times \frac{1 \times 10^{-2}}{3 \times 10^{-2}} = \frac{900}{1000} \times 10 \times \frac{1}{3} = 3.0$$

But
$$V_P = \frac{\text{Rate of flow in prototype}}{\text{Area of prototype}} = \frac{3.0}{\frac{\pi}{4}(D_P)^2} = \frac{3.0}{\frac{\pi}{4}(1.5)^2}$$

$$= \frac{3.0 \times 4}{\pi \times 2.25} = 1.697 \text{ m/s}$$

$$\therefore \quad V_m = 3.0 \times V_P = 3.0 \times 1.697 = \mathbf{5.091 \text{ m/s. Ans.}}$$

Rate of flow through model, $Q_m = A_m \times V_m = \frac{\pi}{4} (D_m)^2 \times V_m = \frac{\pi}{4} (0.15)^2 \times 5.091 \text{ m}^3/\text{s}$

$$= 0.0899 \text{ m}^3/\text{s} = 0.0899 \times 1000 \text{ lit/s} = \mathbf{89.9 \text{ lit/s. Ans.}}$$

Problems

1. Water flowing through a pipe of diameter 30 cm at a velocity of 4 m/s. Find the velocity of oil flowing in another pipe of diameter 10cm, if the conditions of dynamic similarity is satisfied between two pipes. The viscosity of water and oil is given as 0.01 poise and 0.025 poise. The specific gravity of oil is 0.8.

Froude Model Law

If the gravity force is predominant, the models are designed for dynamic similarity based on Froude number.

$$[F_e]_m = [F_e]_p \rightarrow \frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_P}} \rightarrow V_r = \text{Velocity Scale Ratio} = \sqrt{L_r}$$

$$T_r = \text{Scale Ratio for Time} = \sqrt{L_r}$$

$$F_r = \text{Scale Ratio for Force} = L_r^{3}$$

$$T_r = \text{Scale Ratio for Acceleration} = 1$$

$$F_r = \text{Scale Ratio for Pressure Intensity} = L_r$$

$$Q_r = \text{Scale Ratio for Discharge} = L_r^{2.5}$$

$$P_r = \text{Scale Ratio for Power} = L_r^{3.5}$$

Velocity Ratio: According to Froude Model

$$(F_e)_{model} = (F_e)_{prototype} \text{ or } \frac{V_m}{\sqrt{g_m L_m}} = \frac{V_P}{\sqrt{g_P L_P}} \quad \ldots(6.18)$$

If the tests on the model are performed on the same place where prototype is to operate, then $g_m = g_P$ and equation (6.18) becomes as

$$\frac{V_m}{\sqrt{L_m}} = \frac{V_P}{\sqrt{L_P}} \quad \ldots(6.19)$$

or $$\frac{V_m}{V_P} \times \frac{1}{\sqrt{\frac{L_m}{L_P}}} = 1$$

$$\frac{V_P}{V_m} = \sqrt{\frac{L_P}{L_m}} = \sqrt{L_r} \qquad \left\{\because \frac{L_P}{L_m} = L_r\right\}$$

where L_r = Scale ratio for length

(*a*) **Scale ratio for time**

$$\text{As time} = \frac{\text{Length}}{\text{Velocity}},$$

then ratio of time for prototype and model is

$$T_r = \frac{T_P}{T_m} = \frac{\left(\frac{L}{V}\right)_P}{\left(\frac{L}{V}\right)_m} = \frac{\frac{L_P}{V_P}}{\frac{L_m}{V_m}} = \frac{L_P}{L_m} \times \frac{V_m}{V_P} = L_r \times \frac{1}{\sqrt{L_r}} \qquad \left\{\because \frac{V_P}{V_m} = \sqrt{L_r}\right\}$$

$$= \sqrt{L_r}. \qquad \text{...(6.21)}$$

(*b*) **Scale ratio for acceleration**

$$\text{Acceleration} = \frac{V}{T}$$

$$\therefore \quad a_r = \frac{a_P}{a_m} = \frac{\left(\frac{V}{T}\right)_P}{\left(\frac{V}{T}\right)_m} = \frac{V_P}{T_P} \times \frac{T_m}{V_m} = \frac{V_P}{V_m} \times \frac{T_m}{T_P}$$

$$= \sqrt{L_r} \times \frac{1}{\sqrt{L_r}} \qquad \left\{\because \; \frac{V_P}{V_m} = \sqrt{L_r}, \frac{T_P}{T_m} = \sqrt{L_r}\right\}$$

$$= 1. \qquad \text{...(6.22)}$$

(c) **Scale ratio for discharge**

$$Q = A \times V = L^2 \times \frac{L}{T} = \frac{L^3}{T}$$

$$\therefore \quad Q_r = \frac{Q_P}{Q_m} = \frac{\left(\frac{L^3}{T}\right)_P}{\left(\frac{L^3}{T}\right)_m} = \left(\frac{L_P}{L_m}\right)^3 \times \left(\frac{T_m}{T_P}\right) = L_r^{\ 3} \times \frac{1}{\sqrt{L_r}} = L_r^{\ 2.5} \quad \text{...(12.23)}$$

(d) **Scale ratio for force**

$$\text{As Force} = \text{Mass} \times \text{Acceleration} = \rho L^3 \times \frac{V}{T} = \rho L^2 \,.\, \frac{L}{T} \,.\, V = \rho L^2 V^2$$

$$\therefore \quad \text{Ratio for force,} \quad F_r = \frac{F_P}{F_m} = \frac{\rho_P L_P^2 V_P^2}{\rho_m L_m^2 V_m^2} = \frac{\rho_P}{\rho_m} \times \left(\frac{L_P}{L_m}\right)^2 \times \left(\frac{V_P}{V_m}\right)^2.$$

If the fluid used in model and prototype is same, then

$$\frac{\rho_P}{\rho_m} = 1 \quad \text{or} \quad \rho_P = \rho_m$$

and hence

$$F_r = \left(\frac{L_P}{L_m}\right)^2 \times \left(\frac{V_P}{V_m}\right)^2 = L_r^{\ 2} \times \left(\sqrt{L_r}\right)^2 = L_r^{\ 2} \,.\, L_r = L_r^{\ 3}. \quad \text{...(12.24)}$$

(*e*) **Scale ratio for pressure intensity**

As $$p = \frac{\text{Force}}{\text{Area}} = \frac{\rho L^2 V^2}{L^2} = \rho V^2$$

$\therefore$ Pressure ratio, $$p_r = \frac{p_P}{p_m} = \frac{\rho_P V_P^2}{\rho_m V_m^2}$$

If fluid is same, then $\rho_P = \rho_m$

$$\therefore \quad p_r = \frac{V_P^2}{V_m^2} = \left(\frac{V_P}{V_m}\right)^2 = L_r. \quad \text{...(12.25)}$$

(*f*) **Scale ratio for work, energy, torque, moment etc.**

$$\text{Torque} = \text{Force} \times \text{Distance} = F \times L$$

$\therefore$ Torque ratio, $$T_r^* = \frac{T_P^*}{T_m^*} = \frac{(F \times L)_P}{(F \times L)_m} = F_r \times L_r = L_r^3 \times L_r = L_r^4. \quad \text{...(12.26)}$$

(*g*) **Scale ratio for power**

As Power = Work per unit time

$$= \frac{F \times L}{T}$$

∴ Power ratio,

$$\rho_r = \frac{\rho_P}{\rho_m} = \frac{\dfrac{F_P \times L_P}{T_P}}{\dfrac{F_m \times L_m}{T_m}} = \frac{F_P}{F_m} \times \frac{L_P}{L_m} \times \frac{1}{\dfrac{T_P}{T_m}}$$

$$= F_r \,.\, L_r \,.\, \frac{1}{T_r} = L_r^{\,3} \,.\, L_r \,.\, \frac{1}{\sqrt{L_r}} = L^{3.5}. \qquad \text{...(12.27)}$$

Problems

1. In 1 in 40 model of a spillway, the velocity and discharge are 2 m/s and 2.5 m^3/s. Find corresponding velocity and discharge in the prototype

2. In a 1 in 20 model of stilling basin, the height of the jump in the model is observed to be 0.20m. What is height of hydraulic jump in the prototype? If energy dissipated in the model is 0.1kW, what is the corresponding value in prototype?

3. A 7.2 m height and 15 m long spillway discharges 94 m^3/s discharge under a head of 2m. If a 1:9 scale model of this spillway is to be constructed, determine the model dimensions, head over spillway model and the model discharge. If model is experiences a force of 7500 N, determine force on the prototype.

Problems

4. A Dam of 15 m long is to discharge water at the rate of 120 cumecs under a head of 3 m. Design a model, if supply available in the laboratory is 50 lps

5. A 1:50 spillway model has a discharge of 1.5 cumecs. What is the corresponding discharge in prototype?. If a flood phenomenon takes 6 hour to occur in the prototype, how long it should take in the model

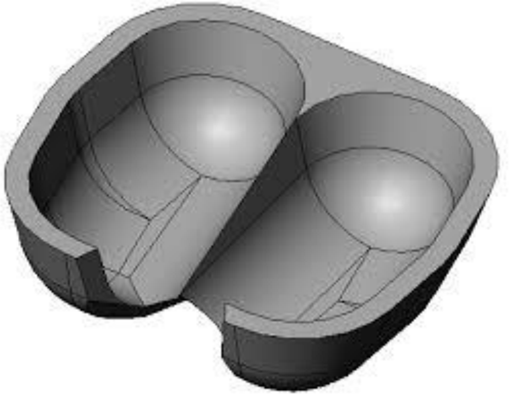
IMPACT OF FREE JETS

(OPEN CHANNEL FLOW AND HYDRAULIC MACHINERY)

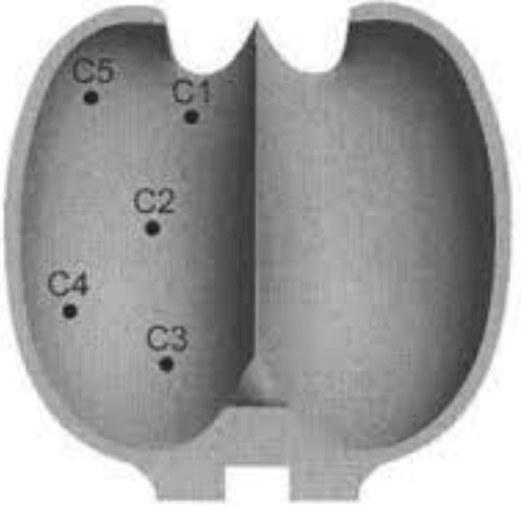
UNIT – III

IARE

Dr.G. Venkata Ramana Professor& HOD Civil Engineering



Vane or Bucket



Topics

1. Impulse-Momentum Principle
2. Hydrodynamic Force of Jets
3. Work done and Efficiency
4. Angular Momentum Principle
5. Applications to Radial Flow Turbines
6. Layout of Hydropower Installation
7. Heads and Efficiencies

Introduction

Analysis and Design of Hydraulic Machines (Turbines and Pumps) is essentially based on the knowledge of forces exerted on or by the moving fluids.

Learning Objective:

Evaluation of force, both in magnitude and direction, by free jets (constant pressure throughout) when they impinge upon stationary or moving objects such as flat plates and vanes of different shapes and orientation.

Force exerted by the jet on a stationary plate

Impact of Jets

The jet is a stream of liquid comes out from nozzle with a high velocity under constant pressure. When the jet impinges on plates or vanes, its momentum is changed and a hydrodynamic force is exerted. Vane is a flat or curved plate fixed to the rim of the wheel

1. **Force exerted by the jet on a stationary plate**
   a) Plate is vertical to the jet
   b) Plate is inclined to the jet
   c) Plate is curved
2. **Force exerted by the jet on a moving plate**
   a) Plate is vertical to the jet
   b) Plate is inclined to the jet
   c) Plate is curved

Impulse-Momentum Principle

From Newton's 2nd Law:

$$F = m\,a = m\,(V_1 - V_2)\,/\,t$$

Impulse of a force is given by the change in momentum caused by the force on the body.

$$Ft = mV_1 - mV_2 = \text{Initial Momentum} - \text{Final Momentum}$$

Force exerted by jet on the plate in the direction of jet, $F = m\,(V_1 - V_2)\,/\,t$

$= (\text{Mass} / \text{Time})\ (\text{Initial Velocity} - \text{Final Velocity})$

$= (\rho Q)\,(V_1 - V_2) =$ **$(\rho a V)\,(V_1 - V_2)$**

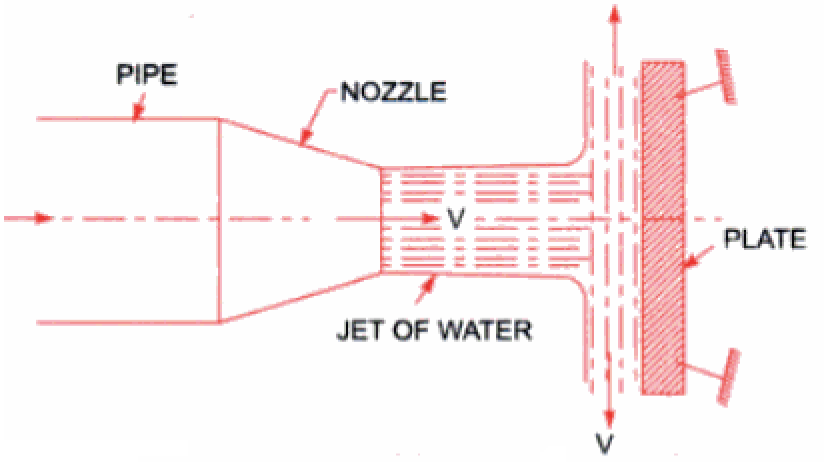
Force exerted by the jet on a stationary plate

Plate is vertical to the jet

$$F = \rho a V^2$$

If Plate is moving at a velocity of 'U' m/s,

$$F = \rho a (V-U)^2$$



Force exerted by jet on vertical plate.

Problems:

1. A jet of water 50 mm diameter strikes a flat plate held normal to the direction of jet. Estimate the force exerted and work done by the jet if

a. The plate is stationary

b. The plate is moving with a velocity of 1 m/s away from the jet along the line of jet.

The discharge through the nozzle is 76 lps.

2. A jet of water 50 mm diameter exerts a force of 3 kN on a flat vane held perpendicular to the direction of jet. Find the mass flow rate.

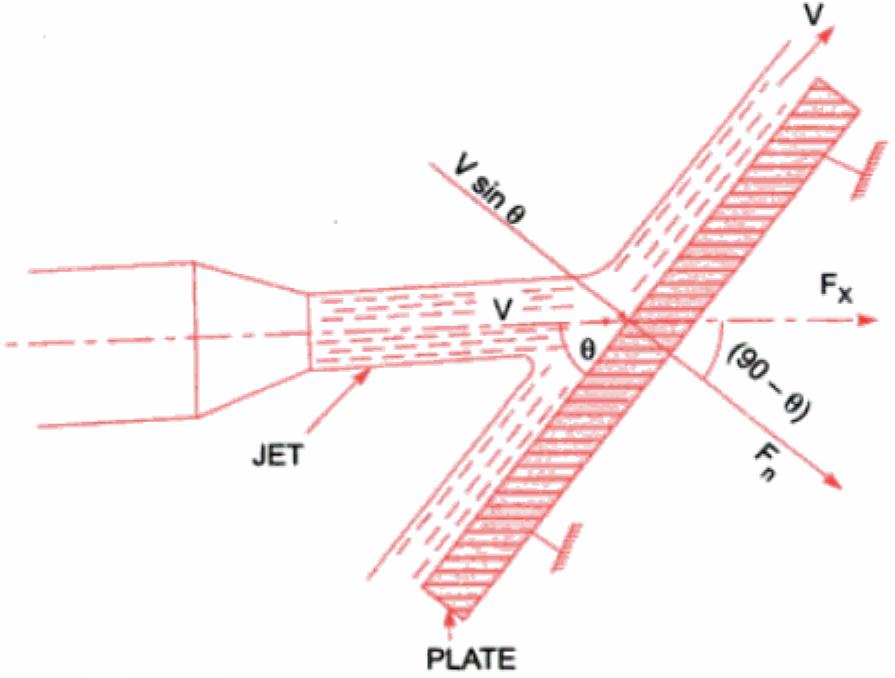
Force exerted by the jet on a stationary plate

Plate is inclined to the jet

$$F_N = \rho a V^2 \sin\theta$$

$$F_x = F_N \sin\theta$$

$$F_x = F_N \cos\theta$$



Jet striking stationary inclined plate.

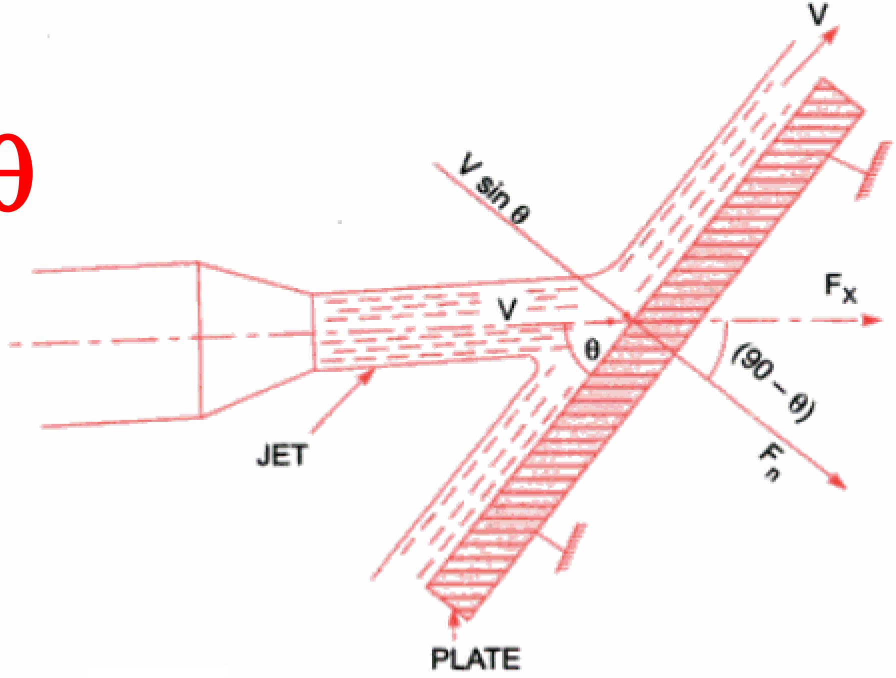
Force exerted by the jet on a moving plate

Plate is inclined to the jet

$$F_N = \rho a(V-U)^2 \sin\theta$$

$$F_x = F_N \sin\theta$$

$$F_x = F_N \cos\theta$$



Jet striking stationary inclined plate.

Problems:

1. A jet of data 75 mm diameter has a velocity of 30 m/s. It strikes a flat plate inclined at 45^0 to the axis of jet. Find the force on the plate when.

a. The plate is stationary

b. The plate is moving with a velocity of 15 m/s along and away from the jet.

Also find power and efficiency in case (b)

2. A 75 mm diameter jet having a velocity of 12 m/s impinges a smooth flat plate, the normal of which is inclined at 60^0 to the axis of jet. Find the impact of jet on the plate at right angles to the plate when the plate is stationery.

a. What will be the impact if the plate moves with a velocity of 6 m/s in the direction of jet and away from it.

b. What will be the force if the plate moves towards the plate.

Force exerted by the jet on a stationary plate

Plate is Curved and Jet strikes at Centre

$$F = \rho a V^2 (1 + \cos\theta)$$

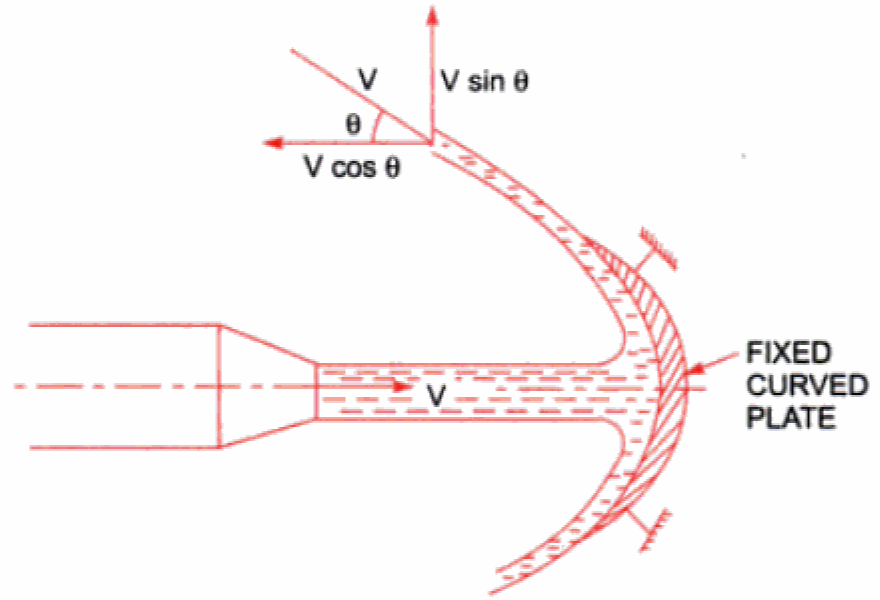
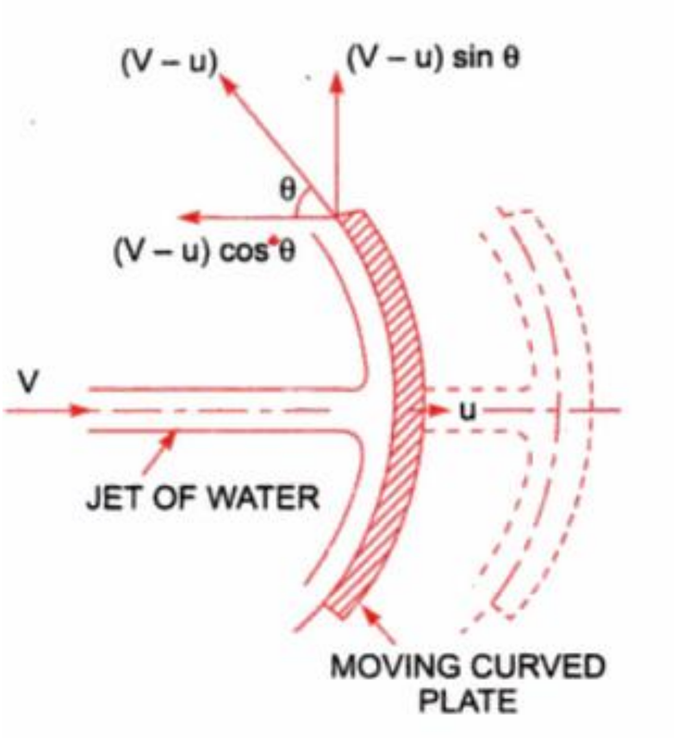


Fig. 17.3 *Jet striking a fixed curved plate at centre.*

Force exerted by the jet on a moving plate

Plate is Curved and Jet strikes at Centre

$$F = \rho a(V-U)^2 (1+ \cos \theta)$$



Problems:

1. A jet of water of diameter 50 mm strikes a stationary, symmetrical curved plate with a velocity of 40 m/s. Find the force extended by the jet at the centre of plate along its axis if the jet is deflected through 120^0 at the outlet of the curved plate

2. A jet of water from a nozzle is deflected through 60^0 from its direction by a curved plate to which water enters tangentially without shock with a velocity of 30m/s and leaver with a velocity of 25 m/s. If the discharge from the nozzle is 0.8 kg/s, calculate the magnitude and direction of resultant force on the vane.

Force exerted by the jet on a stationary plate (Symmetrical Plate)

Plate is Curved and Jet strikes at tip

$$F_x = 2\rho a V^2 \cos\theta$$

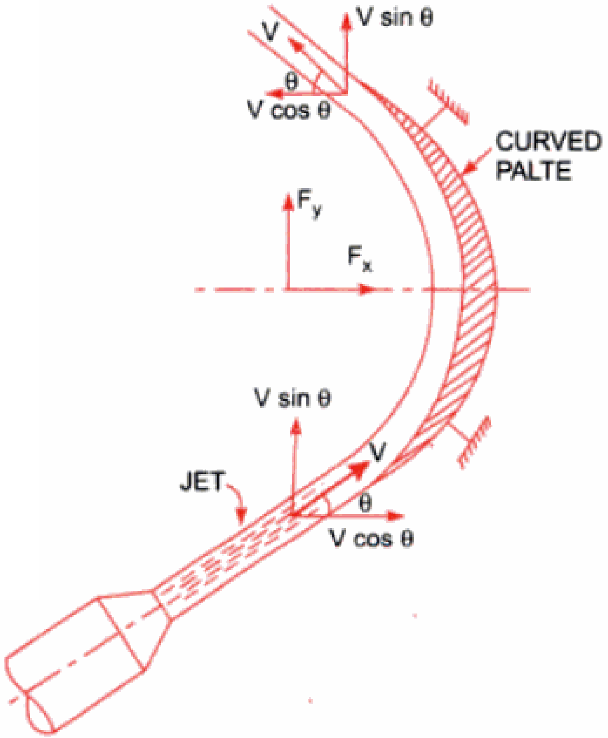
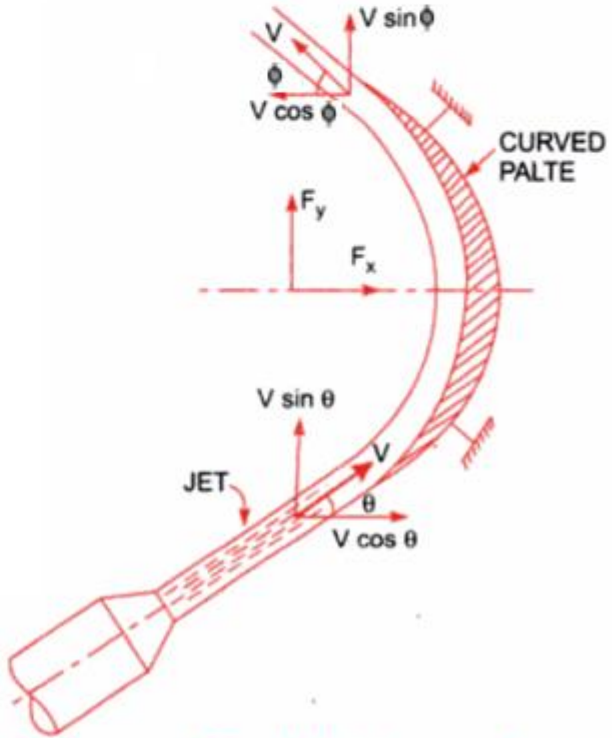


Fig. 17.4 Jet striking curved fixed plate at one end.

Force exerted by the jet on a stationary plate (Unsymmetrical Plate)

Plate is Curved and Jet strikes at tip

$$F_x = \rho a V^2 (\cos\theta + \cos\phi)$$



Jet striking curved fixed plate at one end.

Problems:

1. A jet of water strikes a stationery curved plate tangentially at one end at an angle of 30^0 . The jet of 75 mm diameter has a velocity of 30 m/s. The jet leaves at the other end at angle of 20^0 to the horizontal. Determine the magnitude of force exerted along ‘x’ and ‘y’ directions.

Force exerted by the jet on a moving plate

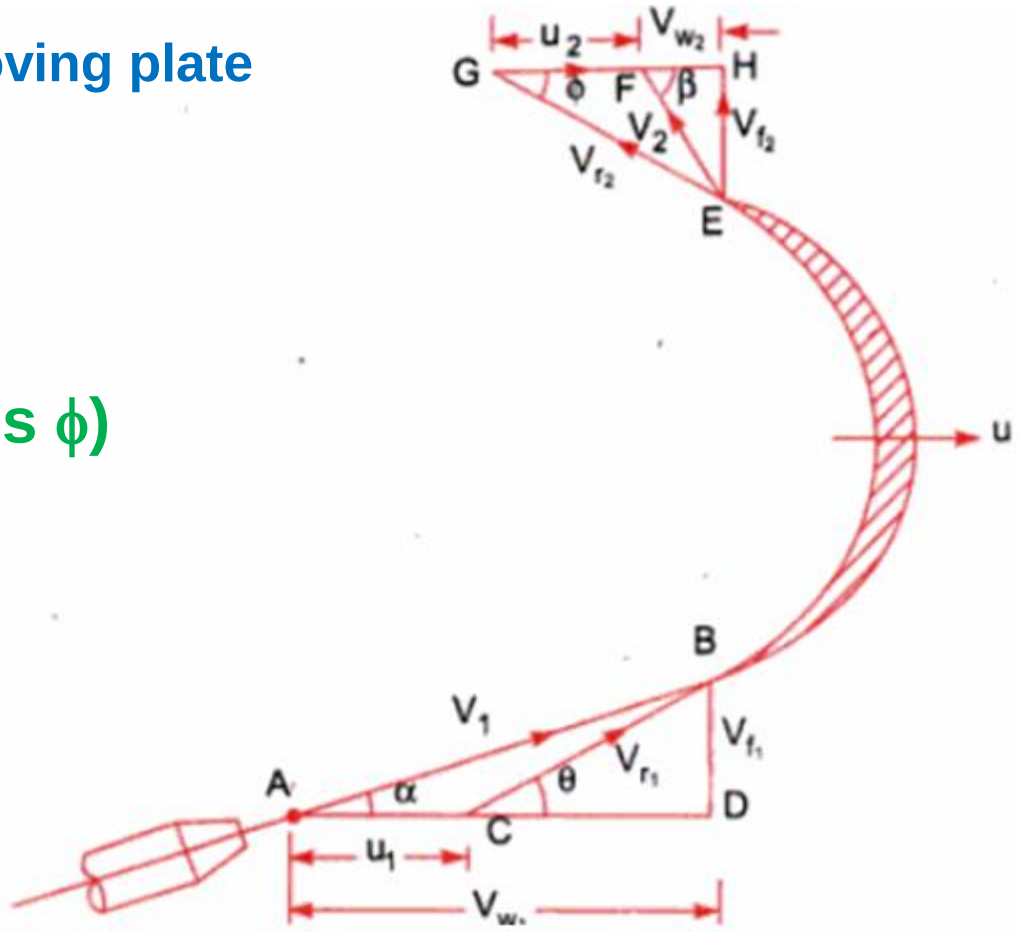
Considering Relative Velocity,

If $\beta < 90^0$

$$F_x = \rho a V_{r1} (V_{r1} \cos\theta + V_{r2} \cos\phi)$$

OR

$$F_x = \rho a V_{r1} (V_{W1} + V_{W2})$$



Jet striking a moving curved vane at one of the tips.

Force exerted by the jet on a moving plate

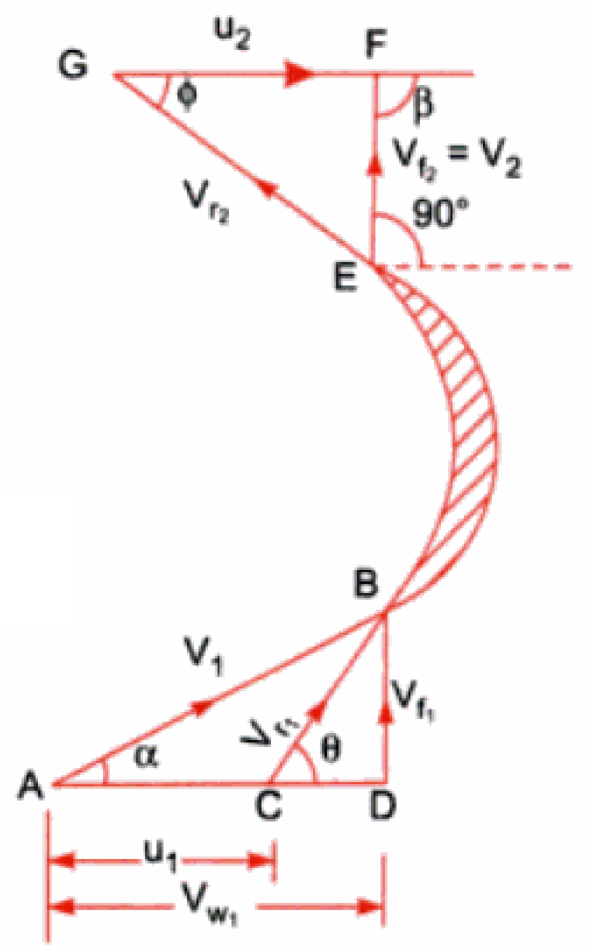
Considering Relative Velocity,

If $\beta = 90^0$

$$F_x = \rho a V_{r1} (V_{r1} \cos \theta - V_{r2} \cos \phi)$$

OR

$$F_x = \rho a V_{r1} (V_{W1})$$



Force exerted by the jet on a moving plate

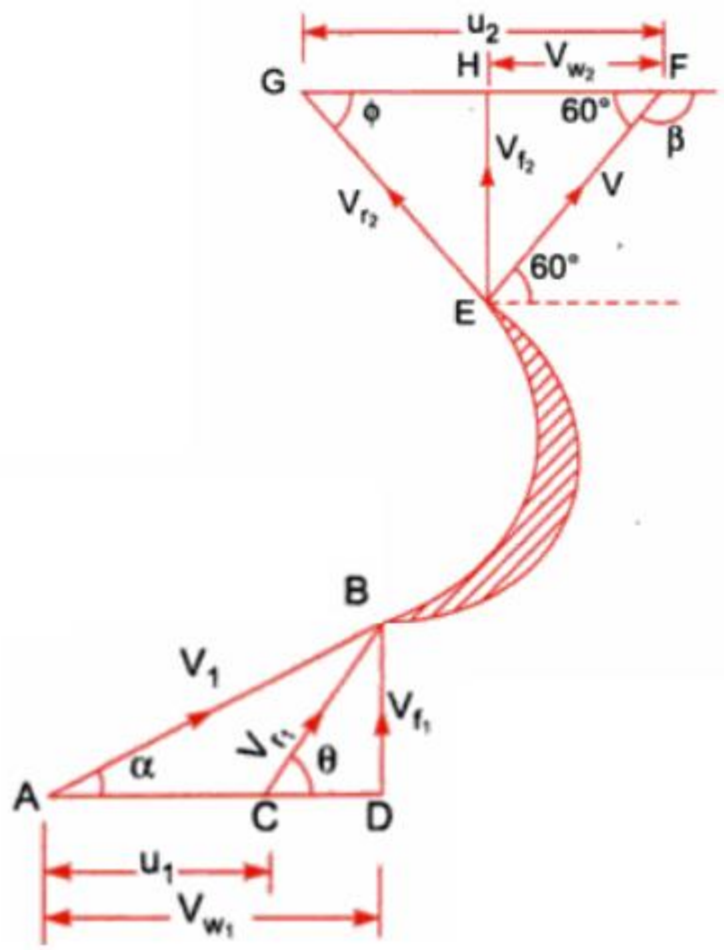
Considering Relative Velocity,

If $\beta = 90^0$

$$F_x = \rho a V_{r1} (V_{r1} \cos \theta - V_{r2} \cos \phi)$$

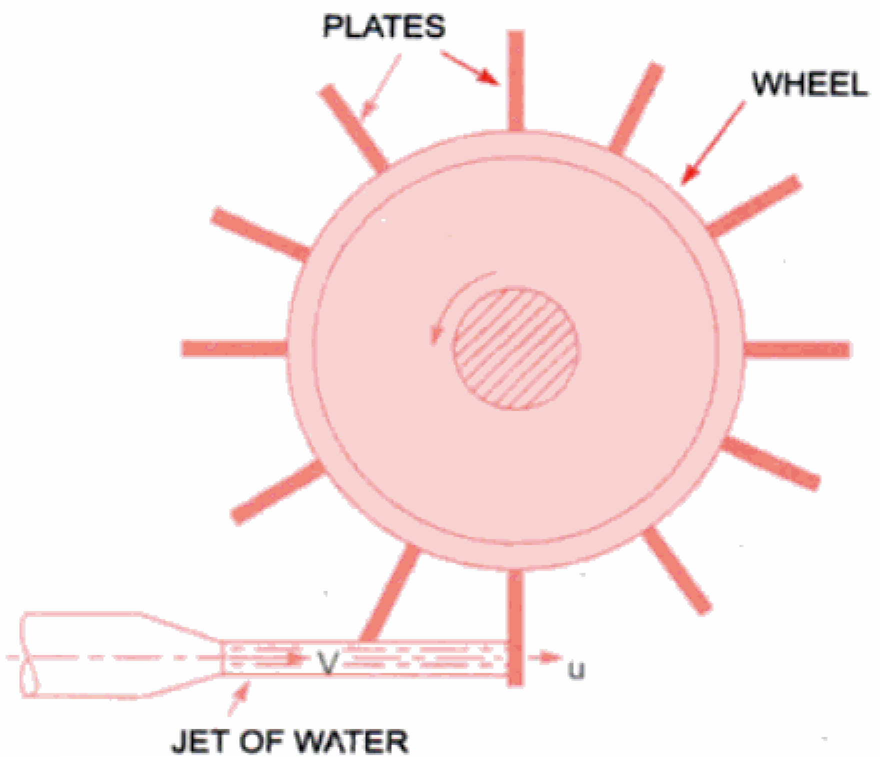
OR

$$F_x = \rho a V_{r1} (V_{W1} - V_{W2})$$



Impact of jet on a series of flat vanes mounted radially on the periphery of a circular wheel

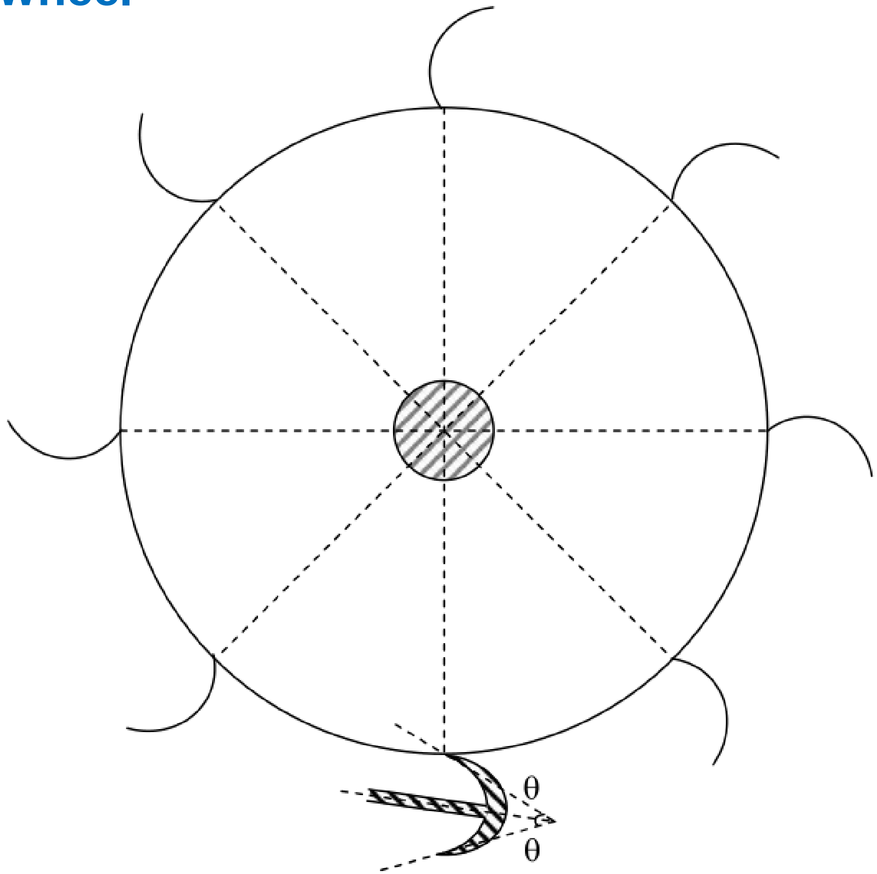
$$F = \rho a V (V - U)$$



Jet striking a series of vanes.

Impact of jet on a series of flat vanes mounted radially on the periphery of a circular wheel

$$F = \rho a V (V - U)(1 + \cos\theta)$$



Problems:

1. A jet of water of diameter 75 mm strikes a curved plate at its centre with a velocity of 25 m/s. The curved plate is moving with a velocity of 10 m/s along the direction of jet. If the jet gets deflected through 165^0 in the smooth vane, compute.

a) Force exerted by the jet.

b) Power of jet.

c) Efficiency of jet.

2. A jet of water impinges a curved plate with a velocity of 20 m/s making an angle of 20^0 with the direction of motion of vane at inlet and leaves at 130^0 to the direction of motion at outlet. The vane is moving with a velocity of 10 m/s. Compute.

i) Vane angles, so that water enters and leaves without shock.

ii) Work done per unit mass flow rate

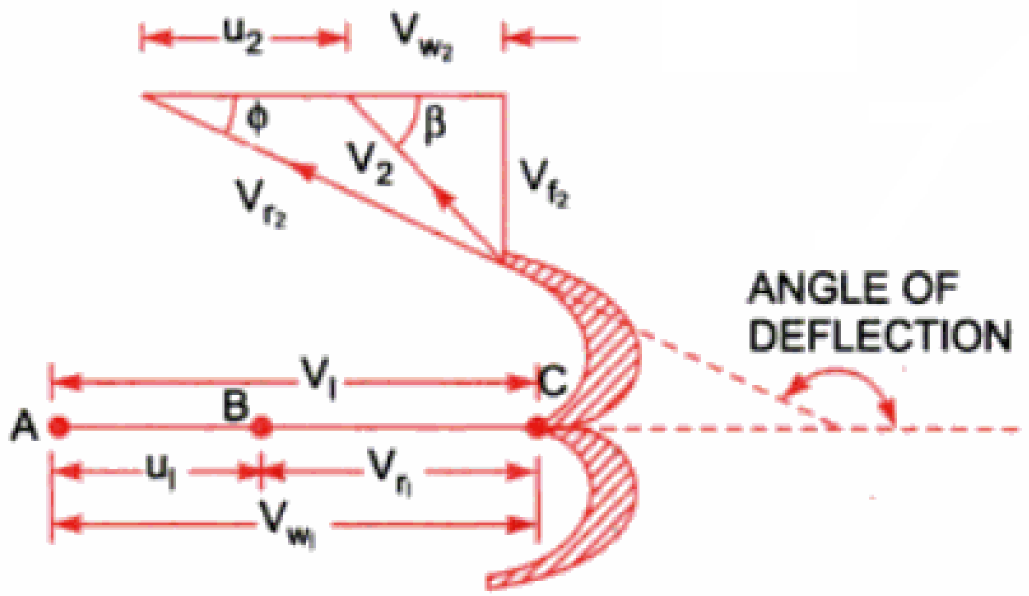
Force exerted by the jet on a moving plate (PELTON WHEEL)

Considering Relative Velocity,

$$F_x = \rho a V_{r1} (V_{r1} - V_{r2} \cos \phi)$$

OR

$$F_x = \rho a V_{r1} (V_{W1} - V_{W2})$$



Work done / sec = F.U

Power = F. U

$$\text{Efficiency} = \frac{F.U}{\frac{1}{2} m V^2}$$

Problems:

1. A jet of water having a velocity of 35 m/s strikes a series of radial curved vanes mounted on a wheel. The wheel has 200 rpm. The jet makes 20^0 with the tangent to wheel at inlet and leaves the wheel with a velocity of 5 m/s at 130^0 to tangent to the wheel at outlet. The diameters of wheel are 1 m and 0.5 m. Find

i) Vane angles at inlet and outlet for radially outward flow turbine.

ii) Work done

iii) Efficiency of the system

Applications to Radial Flow Turbines

$V_{W1} = V_{r1} \cos \theta$ & $V_{W2} = V_{r1} \cos \phi$

Considering Angular Momentum Principle,

Torque (T) = Rate of Change of Angular Momentum

$T = \rho Q\,(V_{W1} R_1 - V_{W1} R_2)$

Power (P) = Torque x Angular Velocity

$P = T.\omega$

If $\beta < 90^0$

$P = \rho Q\,[V_{W1} (R_1.\,\omega) - V_{W2} (R_2.\,\omega))$

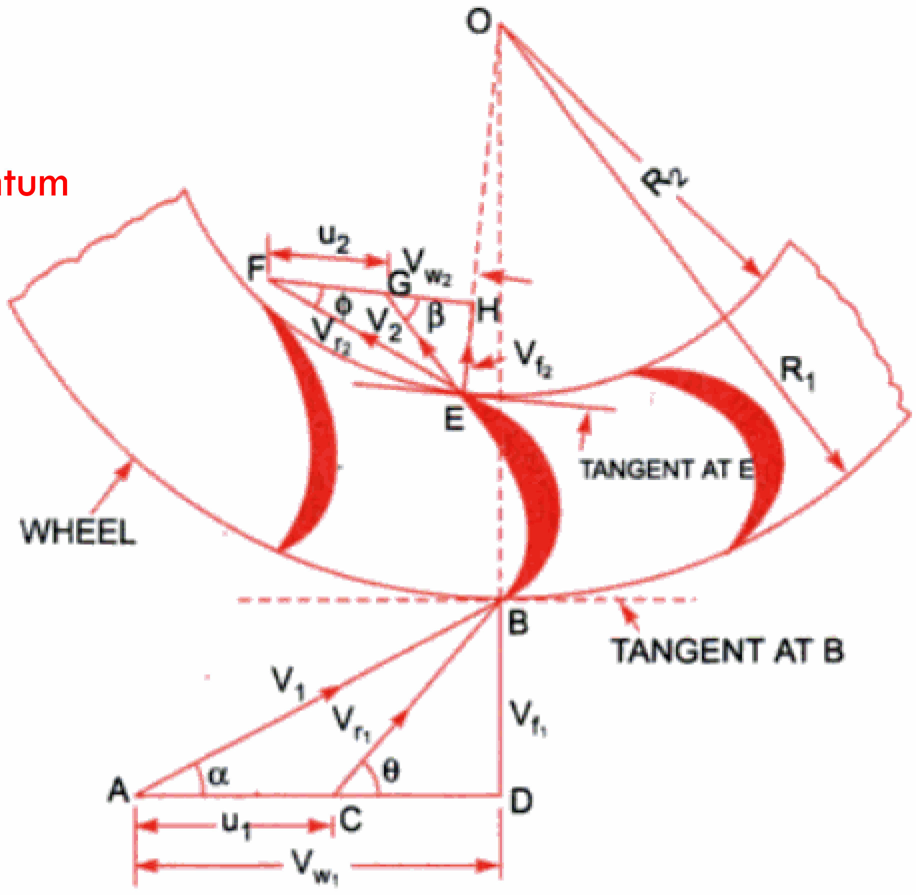
$P = \rho Q\,(V_{W1} U_1 - V_{W2} U_2)$

If $\beta = 90^0$

$P = \rho Q\,(V_{W1} U_1)$

If $\beta > 90^0$

$P = \rho Q\,(V_{W1} U_1 + V_{W2} U_2)$



Series of radial curved vanes mounted on a wheel.

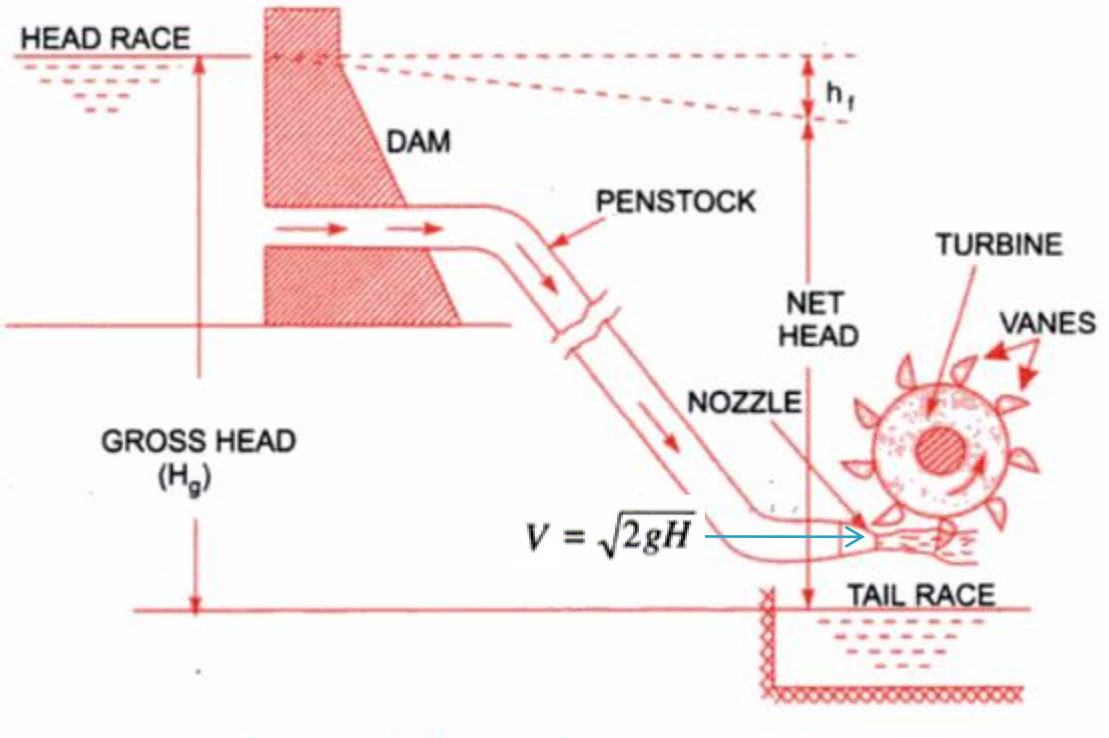
Layout of Hydropower Installation

H_g = Gross Head
h_f = Head Loss due to Friction

$$= \frac{4 \times f \times L \times V^2}{D \times 2g}$$

Where
V = Velocity of Flow in Penstock
L = Length of Penstock
D = Dia. of Penstock

H = Net Head
$= H_g - h_f$



Layout of a hydro-eletric power plant.

Efficiencies of Turbine

1. Hydraulic Efficiency

$$\eta_h = \frac{\text{Power delivered to runner}}{\text{Power supplied at inlet}} = \frac{\text{R.P.}}{\text{W.P.}}$$

2. Mechanical Efficiency

$$\eta_m = \frac{\text{Power at the shaft of the turbine}}{\text{Power delivered by water to the runner}} = \frac{\text{S.P.}}{\text{R.P.}}$$

3. Volumetric Efficiency

$$\eta_v = \frac{\text{Volume of water actually striking the runner}}{\text{Volume of water supplied to the turbine}}$$

4. Overall Efficiency

$$\eta_o = \frac{\text{Volume available at the shaft of the turbine}}{\text{Power supplied at the inlet of the turbine}} = \frac{\text{Shaft power}}{\text{Water power}}$$

$$= \frac{\text{S.P.}}{\text{W.P.}} = \frac{\text{S.P.}}{\text{W.P.}} \times \frac{\text{R.P.}}{\text{R.P.}} = \frac{\text{S.P.}}{\text{R.P.}} \times \frac{\text{R.P.}}{\text{W.P.}} \quad \left(\because \frac{\text{S.P.}}{\text{R.P.}} = \eta_m \text{ and } \frac{\text{R.P.}}{\text{W.P.}} = \eta_h \right)$$

$$= \eta_m \times \eta_h$$

HYDRAULIC TURBINES

(OPEN CHANNEL FLOW AND HYDRAULIC MACHINERY)

UNIT – IV

IARE

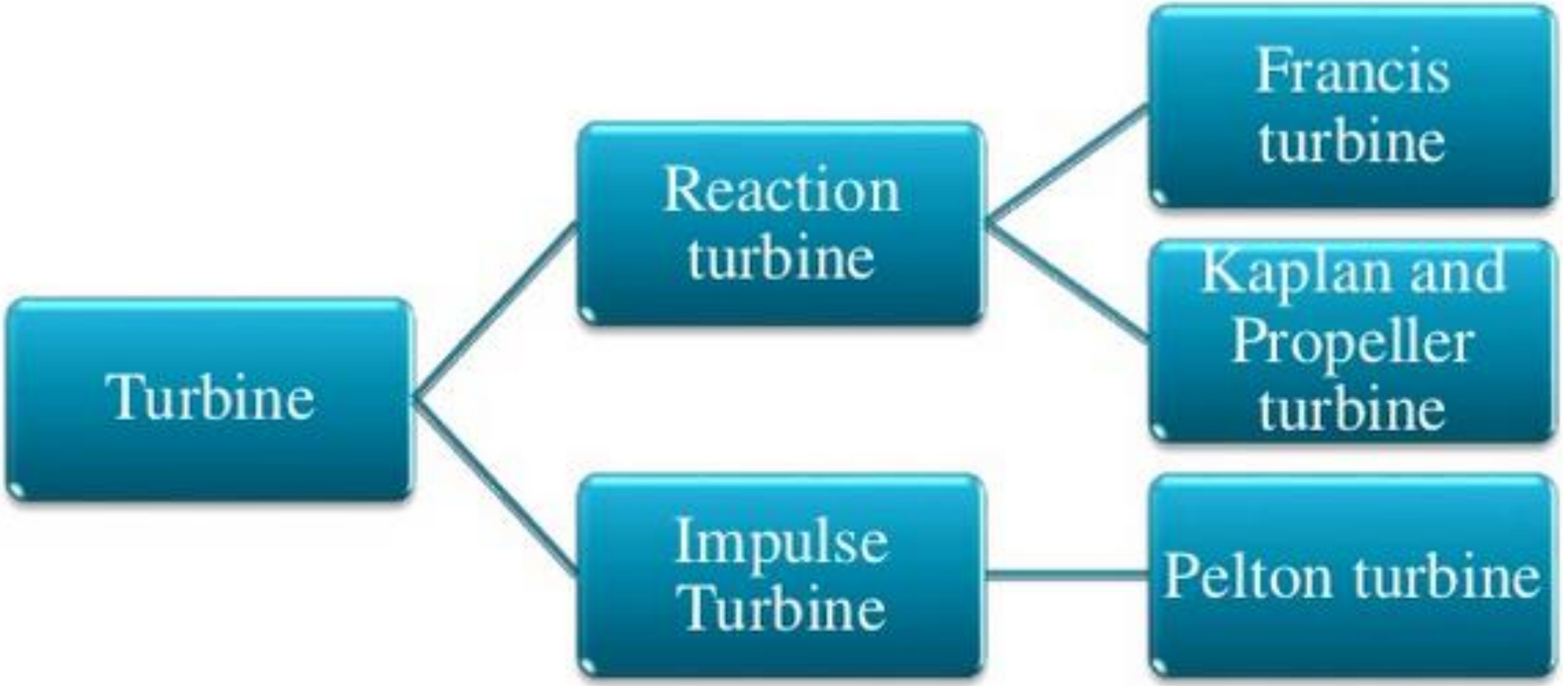
Dr.G. Venkata Ramana Professor& HOD Civil Engineering

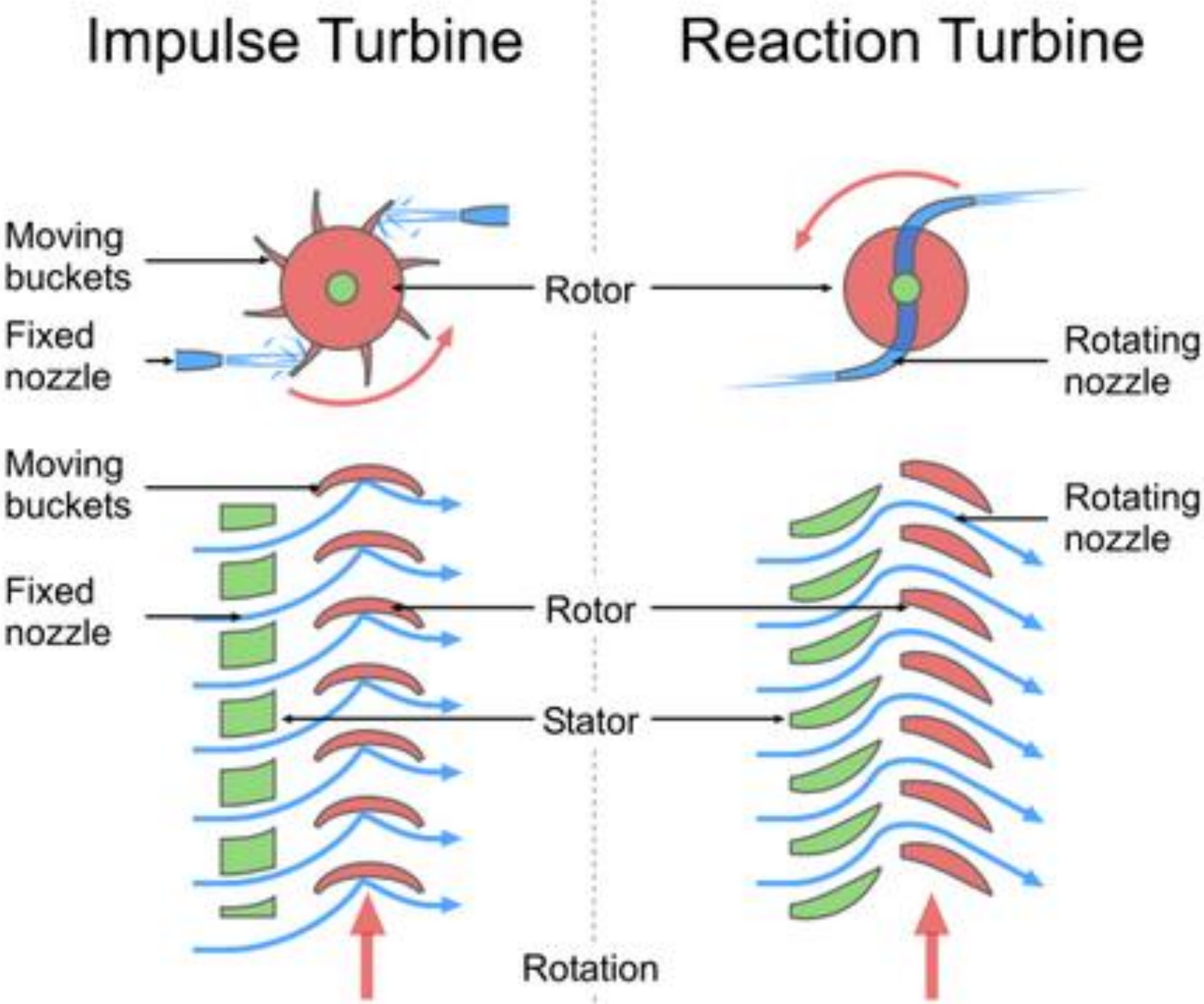
Topics

1. Classification of Turbines
2. Selection of Turbines
3. Design of Turbines - Pelton, Francis, Kaplan
4. Draft Tube
5. Surge Tanks
6. Governing of Turbines
7. Unit Speed, Unit Discharge, Unit Power
8. Characteristic Curves of Hydraulic Turbines
9. Similitude or Model Anlysis
10. Cavitations

Classification of Turbines

1. According to type of energy at Inlet
   a) Impulse Turbine - Pelton Wheel
      Requires High Head and Low Rate of Flow
   a) Reaction Turbine - Fancis, Kaplan
      Requires Low Head and High Rate of Flow
2. According to direction of flow through runner
   a) Tangential Flow Turbine - Pelton Wheel
   b) Radial Flow Turbine - Francis Turbine
   c) Axial Flow Turbine - Kaplan Turbine
   d) Mixed Flow Turbine - Modern Francis Turbine


Turbine
Reaction turbine
Impulse Turbine
Francis turbine
Kaplan and Propeller turbine
Pelton turbine


Impulse Turbine
Reaction Turbine
Moving buckets
Fixed nozzle
Rotor
Rotating nozzle
Moving buckets
Fixed nozzle
Rotor
Stator
Rotating nozzle
Rotation

Classification of Turbines

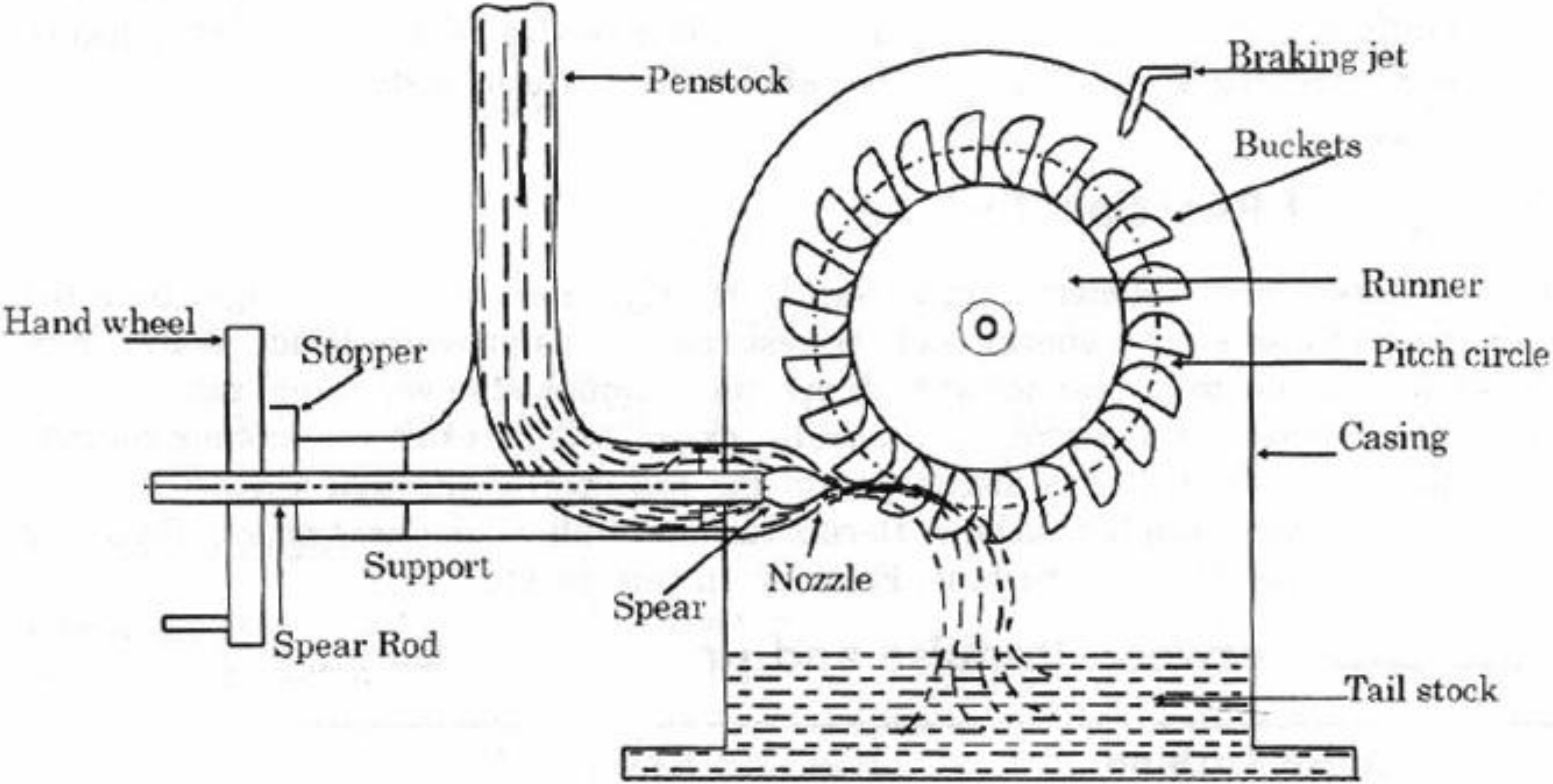
3. According to Head at Inlet of turbine
   a) High Head Turbine - Pelton Wheel
   b) Medium Head Turbine - Fancis Turbine
   c) Low Head Turbine - Kaplan Turbine
4. According to Specific Speed of Turbine
   a) Low Specific Speed Turbine - Pelton Wheel
   b) Medium Specific Speed Turbine - Fancis Turbine
   c) High Specific Speed Turbine - Kaplan Turbine

Classification according to Specific Speed of Turbines

Type of turbine	Type of runner	Specific speed
Pelton	Slow Normal Fast	10 to 20 20 to 28 28 to 35
Francis	Slow Normal Fast	60 to 120 120 to 180 180 to 300
Kaplan	–	300 to 1000

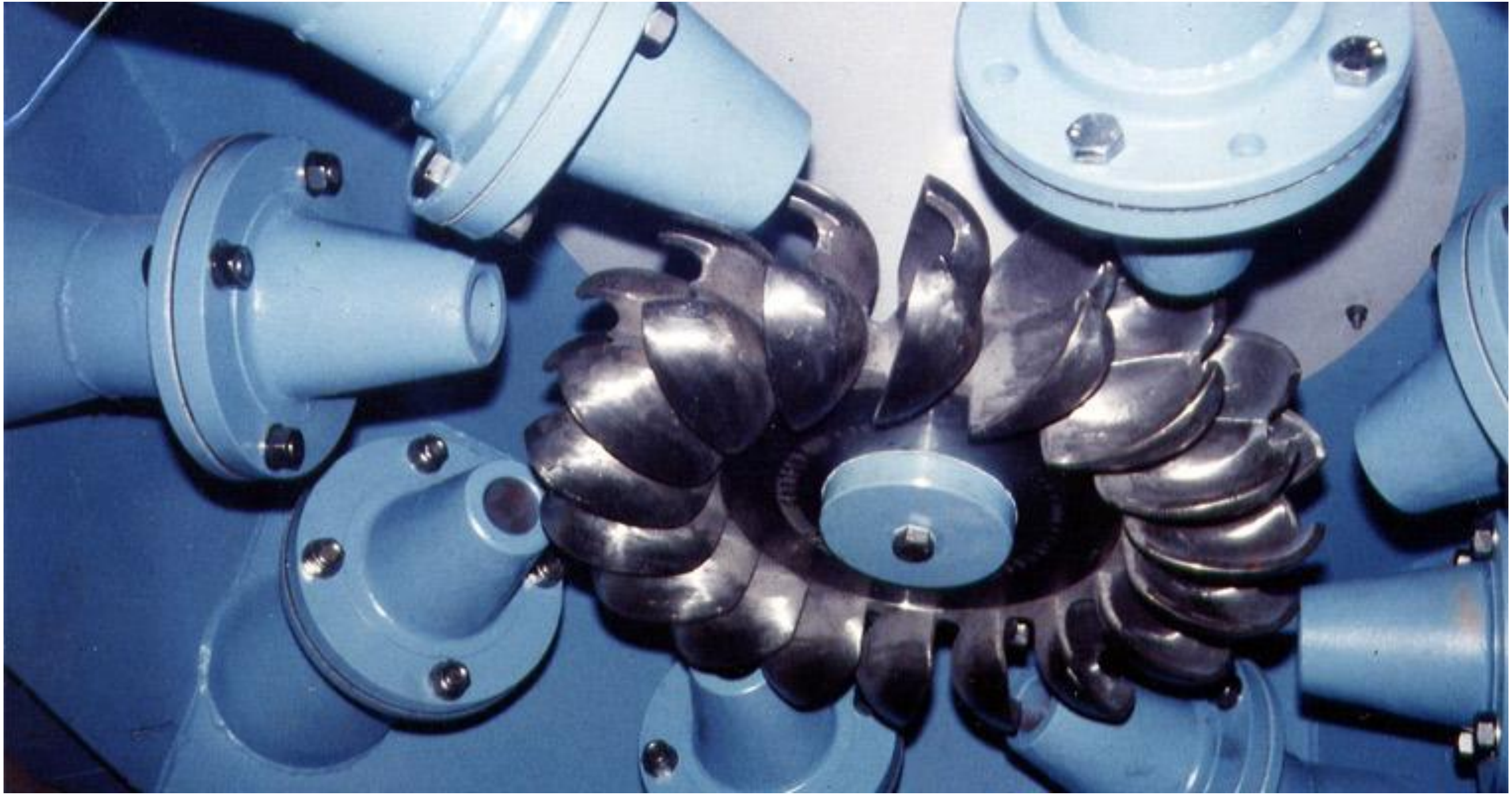
Classification of Turbines

5. According to Disposition of Turbine Shaft
   a) Horizontal Shaft - Pelton Wheel
   b) Vertical Shaft - Fancis & Kaplan Turbines



PELTON WHEEL

PELTON WHEEL



PELTON WHEEL WITH MULTILE JETS

Design of Pelton Wheel

Guidelines:

1. Jet Ratio = Pitch Diameter of wheel / Dia. of Jet = D/d
2. Speed Ratio = Velocity of Wheel / Velocity of Jet = u/V
3. Velocity of Wheel, $u = u_1 = u_2 = \frac{\pi D N}{60}$
4. Overall Efficiency , $\eta_0 = \eta_m \times \eta_h$ OR $\eta_o = \frac{\text{S.P.}}{\text{W.P.}}$
5. Water Power, W.P. = ½mV^2 = ρgQH
6. Shaft Power, S.P. = $\rho a V_1 [V_{w_1} + V_{w_2}] \times u = \rho Q [V_{w_1} + V_{w_2}] \times u$
7. No. of Buckets = (0.5 x Jet Ratio) + 15

Design of Pelton Wheel

Problems:

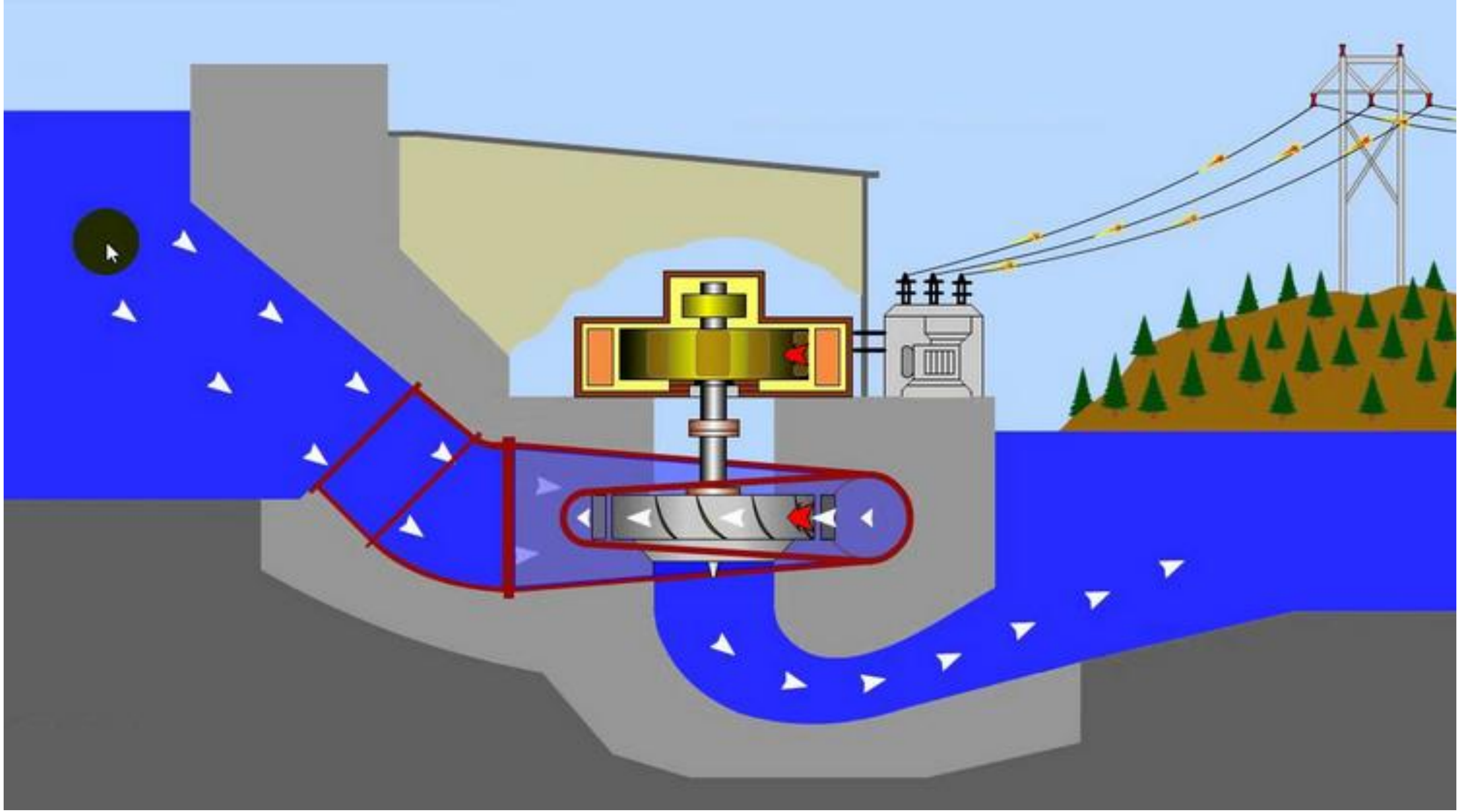
1. A Pelton wheel has a mean bucket speed of 10 m/s with a jet of water flowing at the rate of 700 lps under a head of 30 m. The buckets deflect the jet through an angle of 160°. Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume the coefficient of nozzle as 0.98.

2. A Pelton wheel has to develop 13230 kW under a net head of 800 m while running at a speed of 600 rpm. If the coefficient of Jet C y = 0.97, speed ratio is 0.46 and the ratio of the Jet diameter is 1 /16 of wheel diameter. Calculate
   i) Pitch circle diameter
   ii) the diameter of jet
   iii) the quantity of water supplied to the wheel

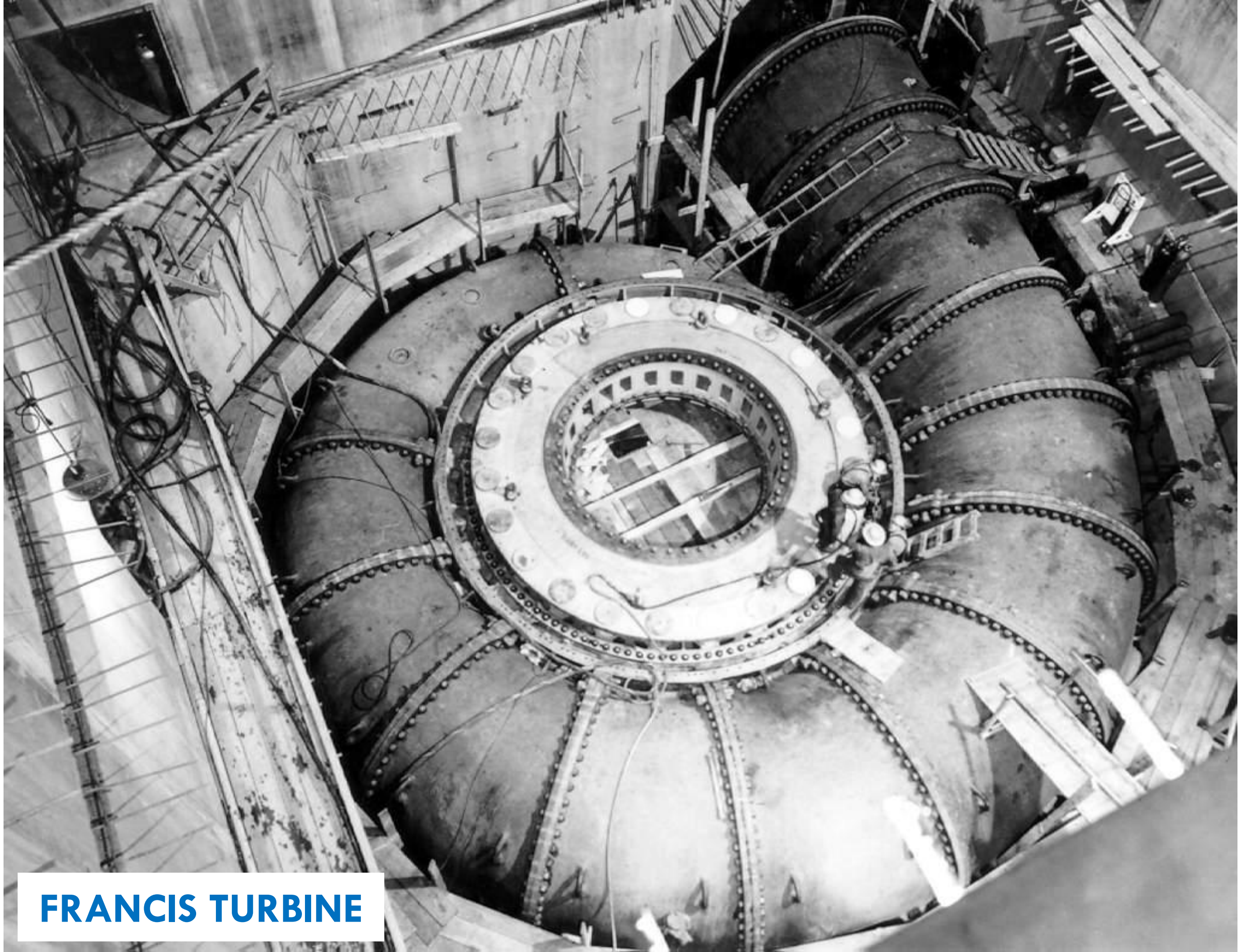
Design of Pelton Wheel

Problems:

3. Design a Pelton wheel for a head of 80m. and speed of 300 RPM. The Pelton wheel develops 110 kW. Take co-eficient of velocity= 0.98, speed ratio= 0.48 and overall efficiency = 80%.
4. A double jet Pelton wheel develops 895 MKW with an overall efficiency of 82% under a head of 60m. The speed ratio = 0.46, jet ratio = 12 and the nozzle coefficient = 0.97. Find the jet diameter, wheel diameter and wheel speed in RPM.



FRANCIS TURBINE



FRANCIS TURBINE

FRANCIS TURBINE

Design of Francis Turbine

Guidelines:

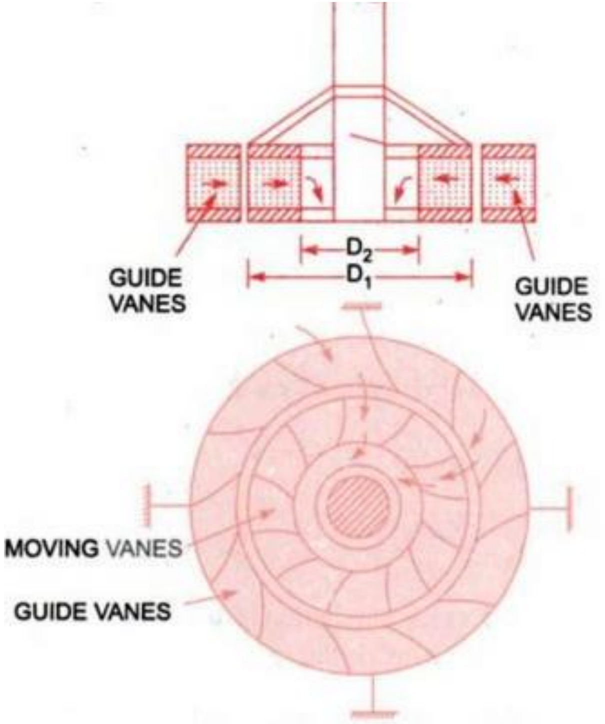
1. Velocity of Wheel, $u = u_1 = u_2 = \dfrac{\pi DN}{60}$

2. Work done per second or Power,

   $= \rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2] \quad = \rho Q [V_{w_1} u_1 \pm V_{w_2} u_2]$

3. Velocity of Wheel, $u_1 = \dfrac{\pi D_1 \times N}{60}, \quad u_2 = \dfrac{\pi D_2 \times N}{60}$

4. Discharge, $Q = \pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2}$

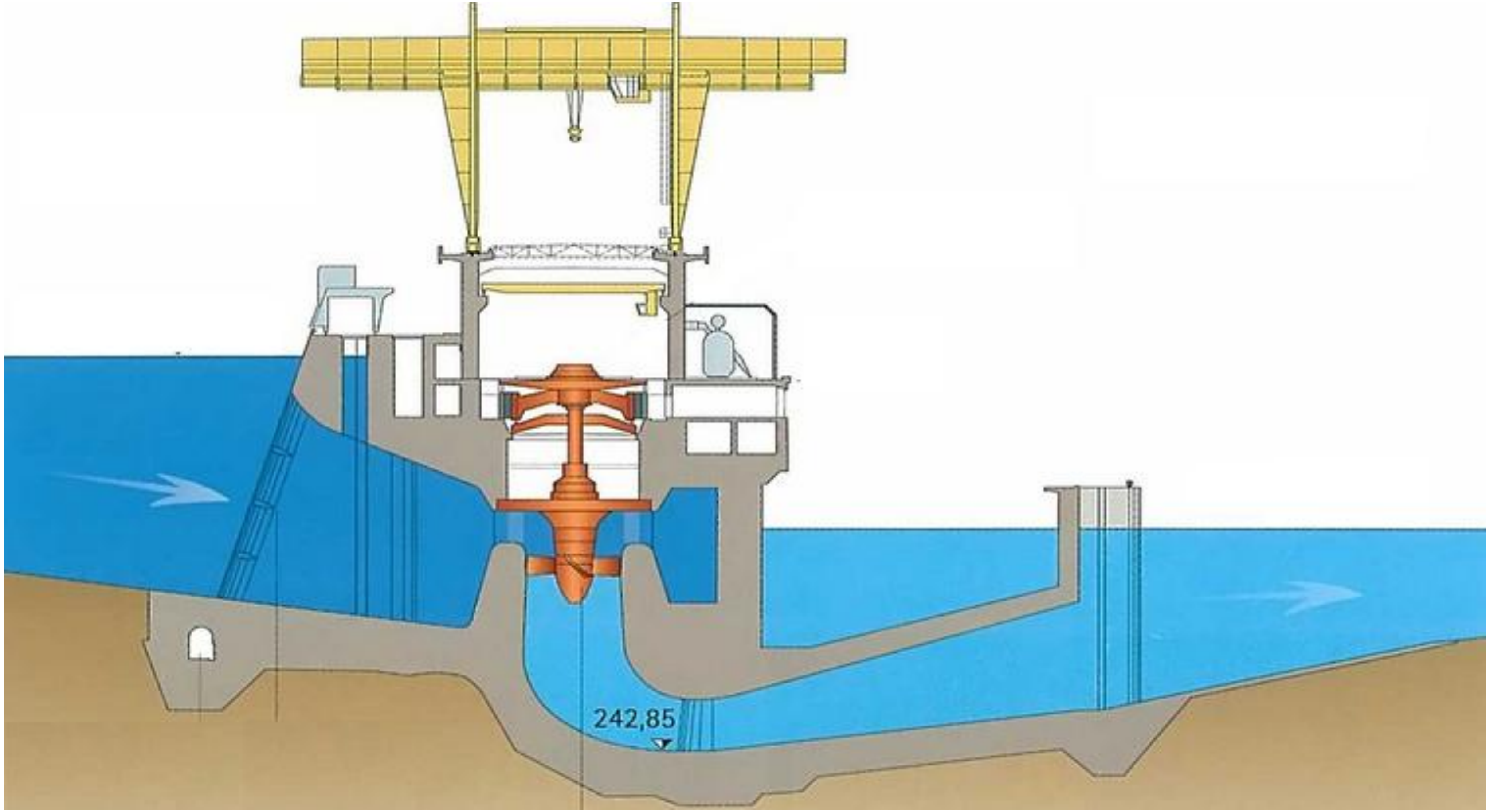


Inward radial flow turbine.

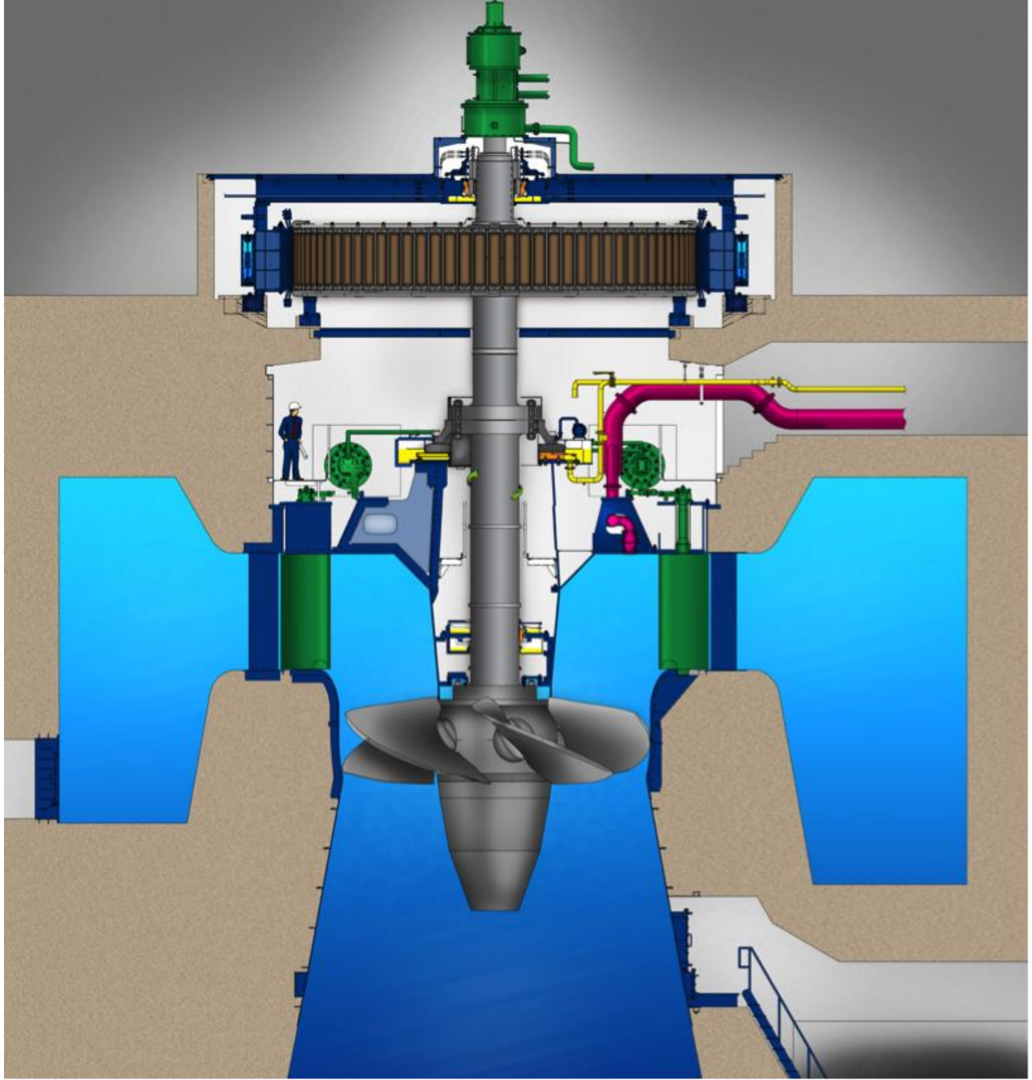
Design of Francis Turbine

Problems:

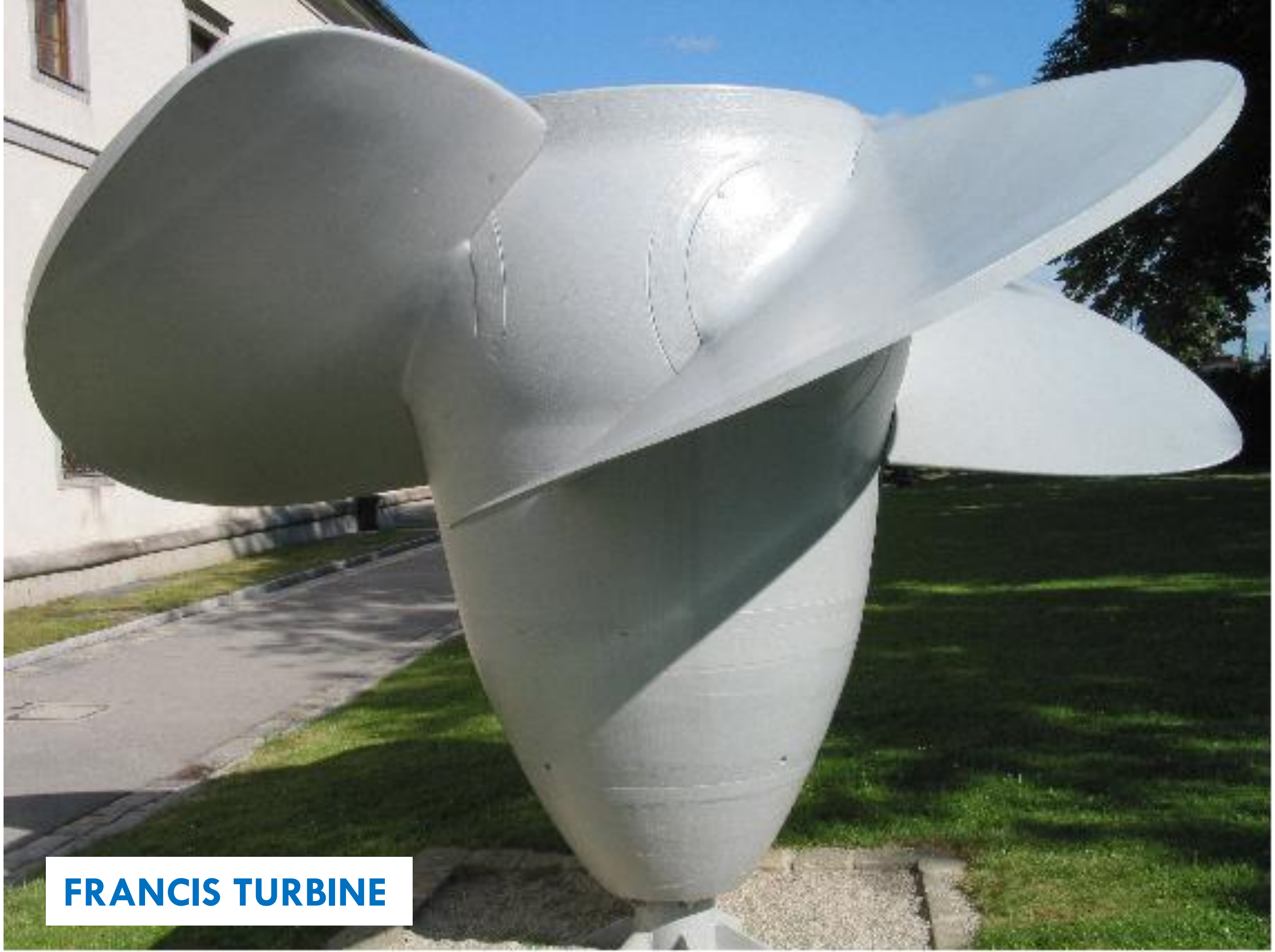
1. A reaction turbine works at 450 rpm under a head of 120 m. Its diameter at inlet is 1.2 m and the flow area is 0.4 m^2 . The angle made by the absolute and relative velocities at inlet are 20º and 60º respectively with the tangential velocity. Determine
   (i) the discharge through the turbine
   (ii) power developed   (iii) efficiency.
   Assume radial discharge at outlet.
2. A Francis turbine has inlet wheel diameter of 2 m and outlet diameter of 1.2 m. The runner runs at 250 rpm and water flows at 8 cumecs. The blades have a constant width of 200 mm. If the vanes are radial at inlet and the discharge is radially outwards at exit, make calculations for the angle of guide vane at inlet and blade angle at outlet



KAPLAN TURBINE



KAPLAN TURBINE

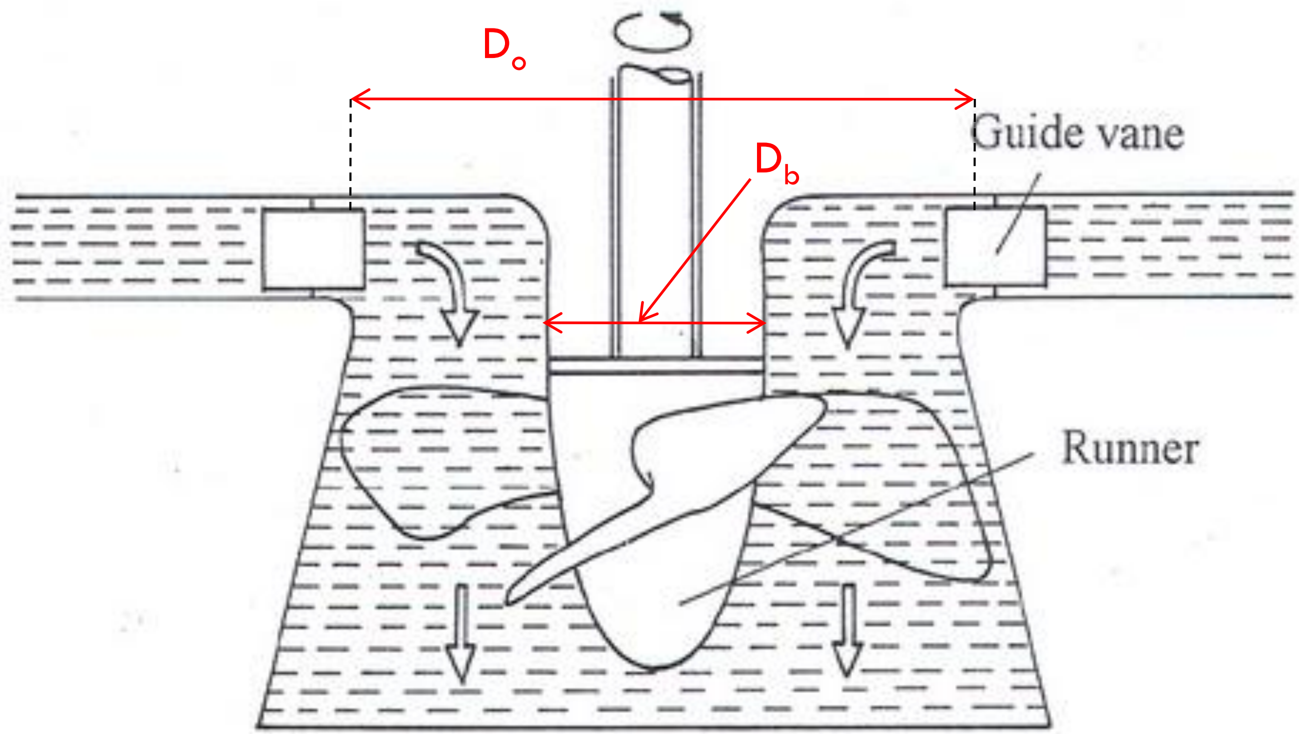


FRANCIS TURBINE

Design of Kaplan Turbine

Guidelines:

1. Velocity of Wheel, $u_1 = u_2 = \frac{\pi D_m \times N}{60}$ where Mean diameter, $D_m = \frac{D_o + D_b}{2}$
2. Work done per second = $\rho a V_1 [V_{w_1} + V_{w_2}] \times u = \rho Q [V_{w_1} + V_{w_2}] \times u$
3. Velocity of Flow at Inlet and Outlet are equal $V_{f_1} = V_{f_2}$
4. Discharge, $Q = \frac{\pi}{4}(D_o^2 - D_b^2) \times V_{f_1}$
5. Flow Ratio = $\frac{V_{f_1}}{\sqrt{2gH}}$



Kaplan Turbine

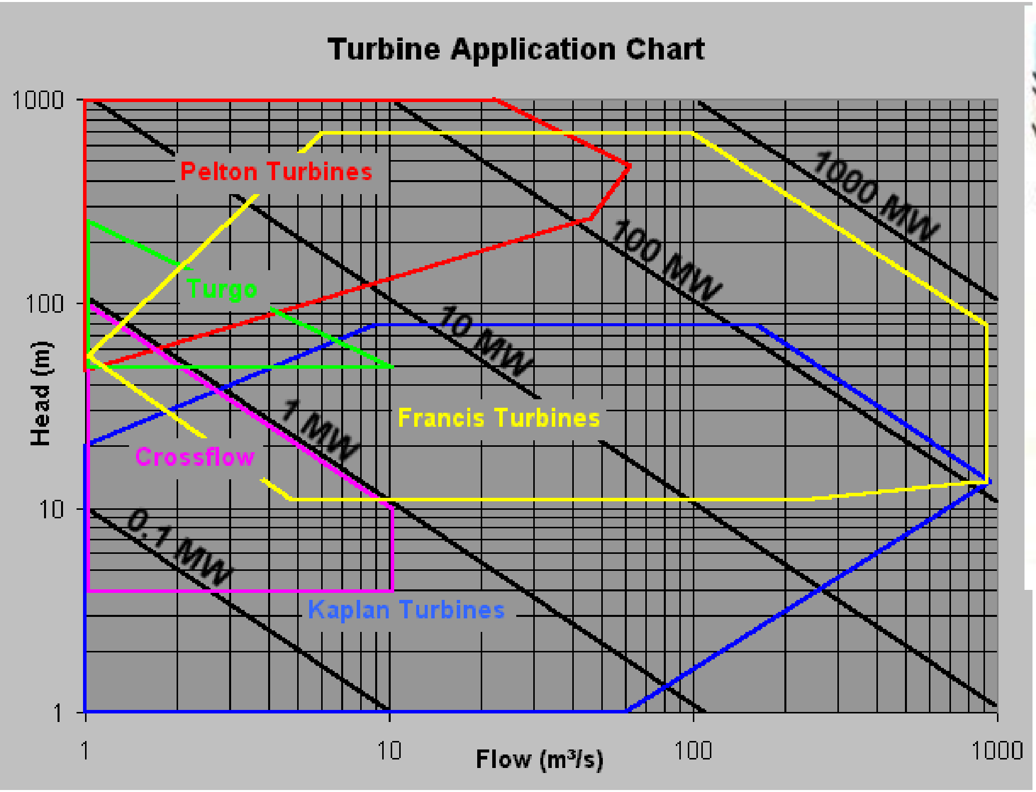
$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f_1}$$

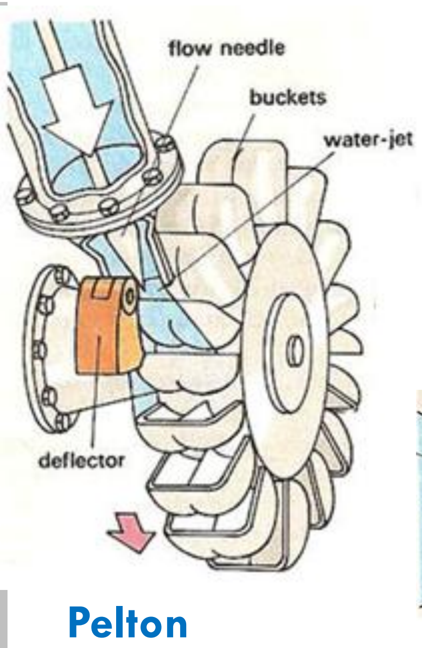
Design of Kaplan Turbine

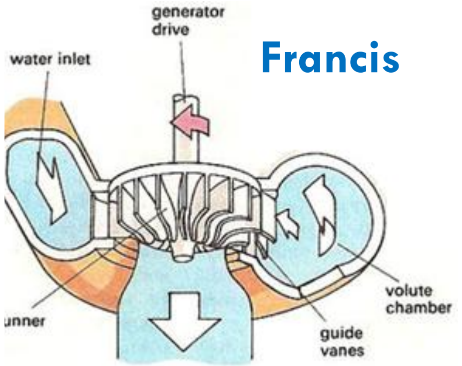
Problems:

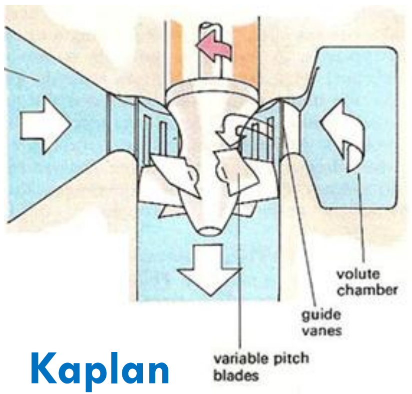
1. A Kaplan turbine develops 9000 kW under a net head of 7.5 m. Overall efficiency of the wheel is 86% The speed ratio based on outer diameter is 2.2 and the flow ratio is 0.66. Diameter of the boss is 0.35 times the external diameter of the wheel. Determine the diameter of the runner and the specific speed of the runner.
2. A Kaplan turbine working under a head of 25 m develops 16,000 kW shaft power. The outer diameter of the runner is 4 m and hub diameter is 2 m. The guide blade angle is 35˚. The hydraulic and overall efficiency are 90% and 85% respectively. If the velocity of whirl is zero at outlet, determine runner vane angles at inlet and outlet and speed of turbine.

Selection of Turbine







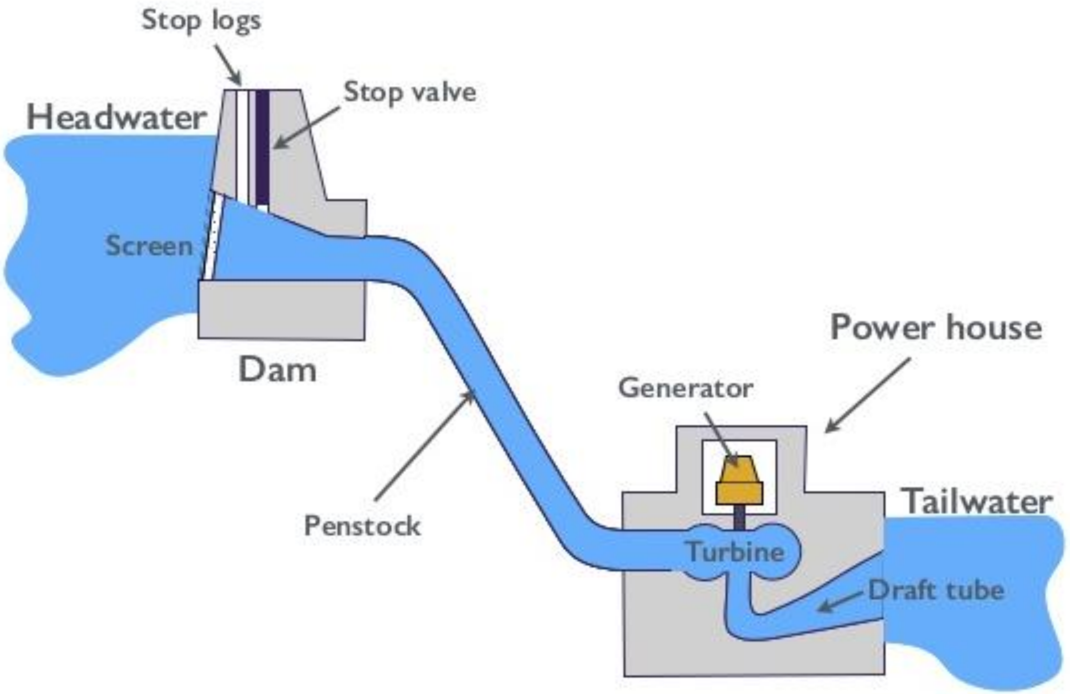


Draft Tube

The water after working on the turbine, imparts its energy to the vanes and runner, there by reducing its pressure less than that of atmospheric Pressure. As the water flows from higher pressure to lower Pressure, It can not come out of the turbine and hence a divergent tube is Connected to the end of the turbine.

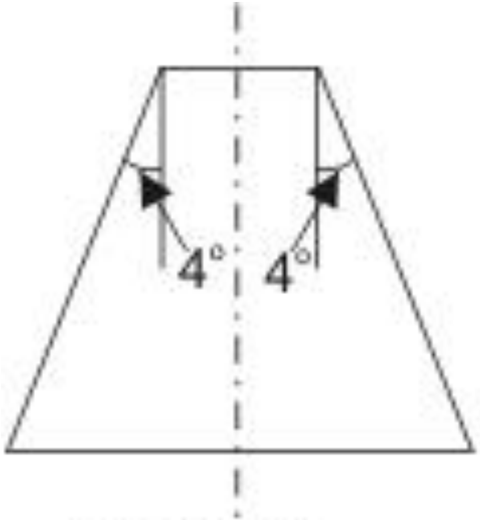
Draft tube is a divergent tube one end of which is connected to the outlet Of the turbine and other end is immersed well below the tailrace (Water level).

The major function of the draft tube is to increase the pressure from the inlet to outlet of the draft tube as it flows through it and hence increase it more than atmospheric pressure. The other function is to safely Discharge the water that has worked on the turbine to tailrace.

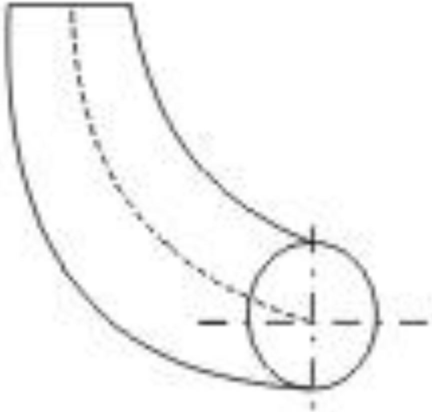

Stop logs
Stop valve
Headwater
Screen
Dam
Power house
Generator
Penstock
Tailwater
Turbine
Draft tube

Draft Tube

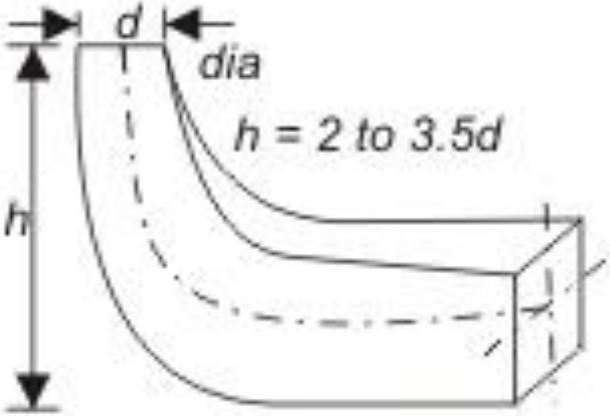
Types of Draft Tube



(a) Straight type



(b) Simple elbow type



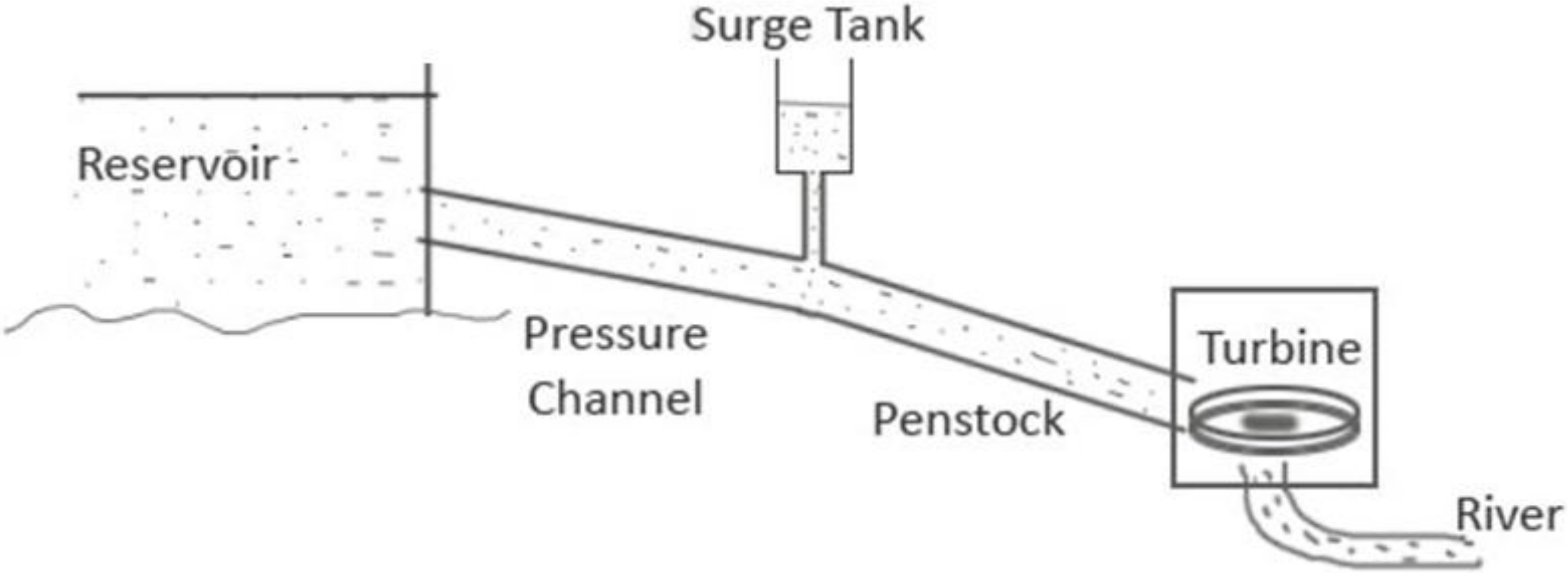
(c) Elbow type with varying cross-section

Surge Tanks

Surge tank (or surge chamber) is a device introduced within a hydropower water conveyance system having a rather long pressure conduit to absorb the excess pressure rise in case of a sudden valve closure. The surge tank is located between the almost horizontal or slightly inclined conduit and steeply sloping penstock and is designed as a chamber excavated in the mountain.

It also acts as a small storage from which water may be supplied in case of a sudden valve opening of the turbine.

In case of a sudden opening of turbine valve, there are chances of penstock collapse due to a negative pressure generation, if there is no surge tank.


Surge Tank
Reservoir
Pressure
Channel
Penstock
Turbine
River

Surge Tank

Governing of Turbines

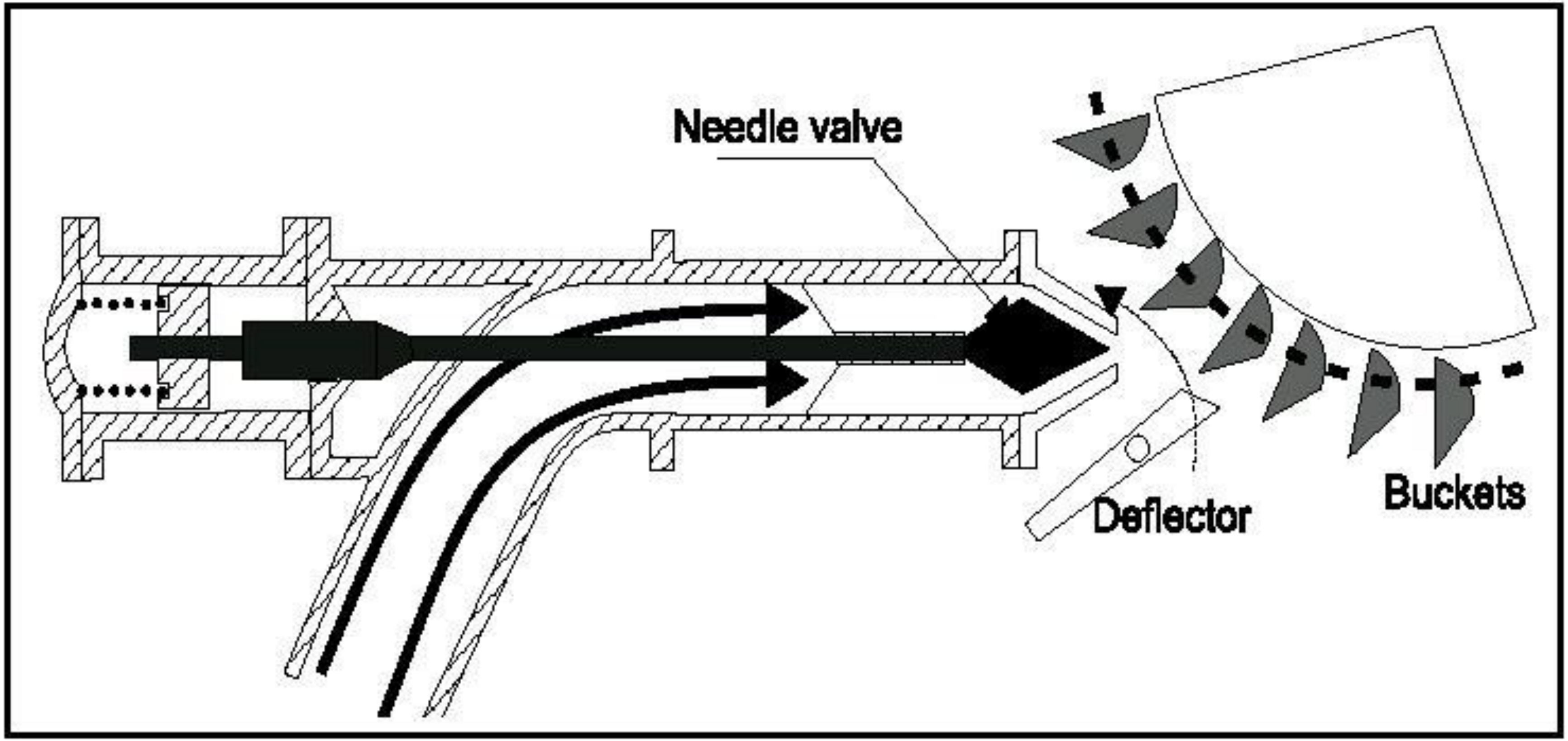
Governing means Speed Regulation.

Governing system or governor is the main controller of the hydraulic turbine. The governor varies the water flow through the turbine to control its speed or power output.

1. Impulse Turbine

a) Spear Regulation
b) Deflector Regulation
c) Combined

2. Reaction Turbine



Governor of Pelton Wheel

Performance of Turbines under unit quantities

The unit quantities give the speed, discharge and power for a particular turbine under a head of 1m assuming the same efficiency. Unit quantities are used to predict the performance of turbine.

1. Unit speed (N_u) - Speed of the turbine, working under unit head

$$Nu = \frac{N}{\sqrt{H}}$$

2. Unit power (P_u) - Power developed by a turbine, working under a unit head

$$Qu = \frac{Q}{\sqrt{H}}$$

3. Unit discharge (Q_u) - The discharge of the turbine working under a unit head

$$Pu = \frac{P}{H^{3/2}}$$

Unit Speed, Unit discharge and Unit Power is definite characteristics of a turbine.

If for a given turbine under heads $H_1, H_2, H_3, \ldots$. the corresponding speeds are $N_1, N_2, N_3, \ldots$, the corresponding discharges are $Q_1, Q_2, Q_3, \ldots$. and the powers developed are $P_1, P_2, P_3, \ldots$. Then

Unit speed = $N_u = \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}} = \frac{N_3}{\sqrt{H_3}}$

Unit Discharge = $Q_u = \frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}} = \frac{Q_3}{\sqrt{H_3}}$

Unit Power = $P_u = \frac{P_1}{H\sqrt{H_1}} = \frac{P_2}{H\sqrt{H_2}} = \frac{P_3}{H\sqrt{H_3}}$ or $P_u = \frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}} = \frac{P_3}{H_3^{3/2}}$

Thus if speed, discharge and power developed by a turbine under a certain head are known, the corresponding quantities for any other head can be determined.

Specific Speed of Turbine

Specific Speed of a Turbine (N_s)

The specific speed of a turbine is the speed at which the turbine will run when developing unit power under a unit head. This is the type characteristics of a turbine. For a set of geometrically similar turbines the specific speed will have the same value.

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

Unit Quantities & Specific Speed

Problems:

1. Suggest a suitable type of turbine to develop 7000 kW power under a head of 20m while operating at 220 rpm. What are the considerations for your suggestion.
2. A turbine is to operate under a head of 25m at 200 rpm. The discharge is 9 m^3/s. If the efficiency is 90%, determine:
   i) Power generated ii) Speed and Power at a head of 20m

Characteristics Curves of Turbine

These are curves which are characteristic of a particular turbine which helps in studying the performance of the turbine under various conditions. These curves pertaining to any turbine are supplied by its manufacturers based on actual tests.

The characteristic curves obtained are the following:

a) Constant head curves or main characteristic curves
b) Constant speed curves or operating characteristic curves
c) Constant efficiency curves or Muschel curves

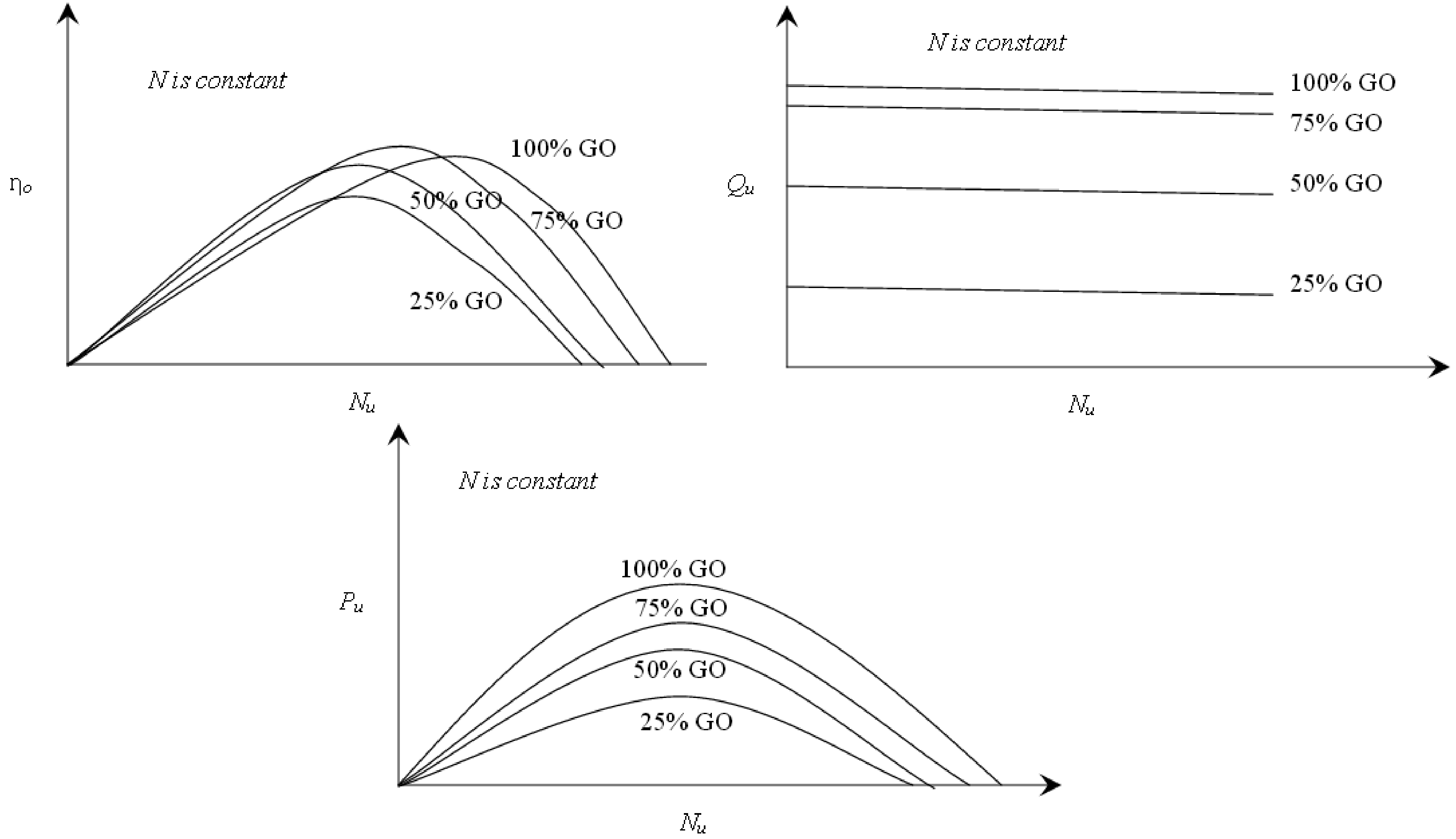
Constant head curves or main characteristic curves

Constant head curves:

Maintaining a constant head, the speed of the turbine is varied by admitting different rates of flow by adjusting the percentage of gate opening. The power P developed is measured mechanically. From each test the unit power Pu, the unit speed Nu, the unit discharge Qu and the overall efficiency are determined.

The characteristic curves drawn are

a) Unit discharge vs unit speed
b) Unit power vs unit speed
c) Overall efficiency vs unit speed



Main Characteristic curves of a Pelton turbine

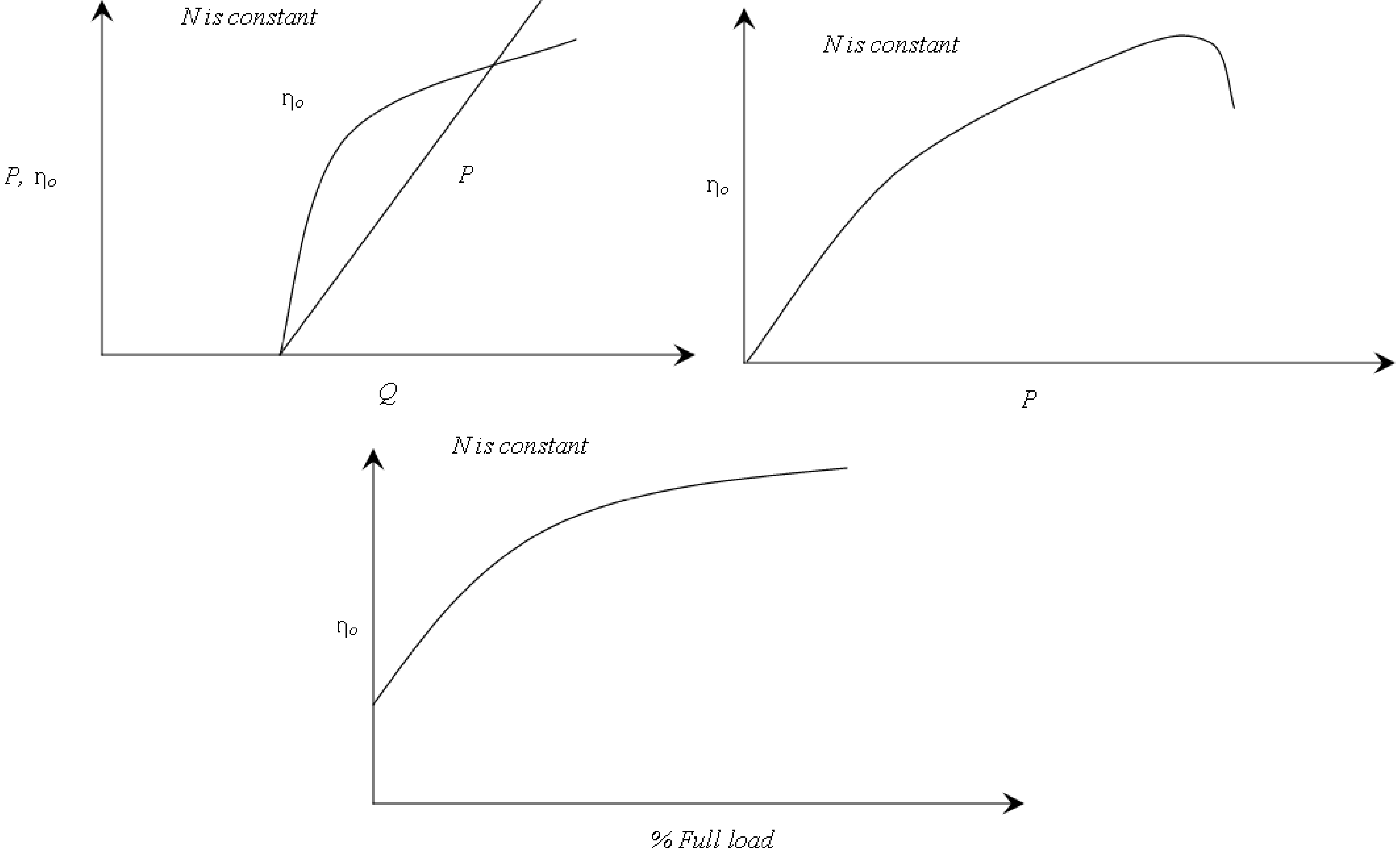
Constant speed curves or operating characteristic curves

Constant speed curves:

In this case tests are conducted at a constant speed varying the head H and suitably adjusting the discharge Q. The power developed P is measured mechanically. The overall efficiency is aimed at its maximum value.

The curves drawn are

P	vs	Q
η_o	vs	Q
η_o	vs	P_u
$\eta_{o\,max}$	vs	% Full load



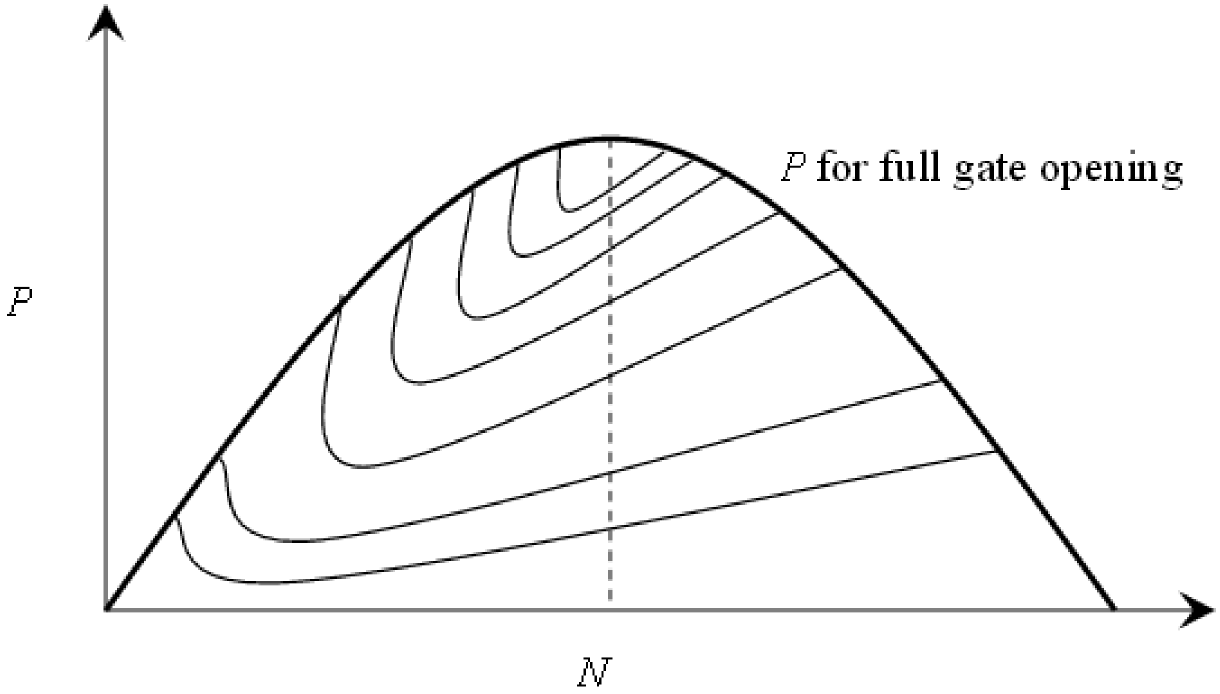
Operating Characteristic curves of a turbine

Constant efficiency curves or Muschel curves

Constant efficiency curves:

These curves are plotted from data which can be obtained from the constant head and constant speed curves. The object of obtaining this curve is to determine the zone of constant efficiency so that we can always run the turbine with maximum efficiency.

This curve also gives a good idea about the performance of the turbine at various efficiencies.



Constant Efficiency curves for Reaction turbine

Similitude of Turbines

Dimensionless Numbers:

$$\frac{Q}{ND^3}, \frac{gH}{N^2D^2}, \frac{P}{\rho N^3D^5}$$

Where

Q = Discharge

N = Speed of Wheel

D = Dia. of Wheel

H = Head

P = Shaft Power

Similitude of Turbines - Problems

Problems:

1. A hydraulic turbine develops 120 KW under a head of 10 m at a speed of 1200 rpm and gives an efficiency of 92%. Find the water consumption and the specific speed. If a model of scale 1: 30 is constructed to operate under a head of 8m what must be its speed, power and water consumption to run under the conditions similar to prototype.
2. A model turbine 1m in diameter acting under a head of 2m runs at 150 rpm. Estimate the scale ratio if the prototype develops 20 KW under a head of 225 m with a specific speed of 100.

Cavitations

If the pressure of a liquid in course of its flow becomes equal to its vapour pressure at the existing temperature, then the liquid starts boiling and the pockets of vapour are formed which create vapour locks to the flow and the flow is stopped. The phenomenon is known as **cavitation**.

To avoid cavitation, the minimum pressure in the passage of a liquid flow, should always be more than the vapour pressure of the liquid at the working temperature. In a reaction turbine, the point of minimum pressure is usually at the outlet end of the runner blades, i.e., at the inlet to the draft tube.

Methods to avoid Cavitations

(i) Runner/turbine may be kept under water.

(ii) Cavitation free runner may be designed.

(iii) By selecting materials that can resist better the cavitation effect.

(iv) By polishing the surfaces.

(v) By selecting a runner of proper specific speed for given load.

CENTRIFUGAL PUMPS

(OPEN CHANNEL FLOW AND HYDRAULIC MACHINERY)

UNIT – V

IARE

Dr.G. Venkata Ramana Professor& HOD Civil Engineering

Topics

1. Introduction
2. Classification of Pumps
3. Pump Installation Details
4. Work done by Pump – Velocity Triangles at Inlet & Outlet
5. Heads and Efficiencies
6. Minimum Starting Speed
7. Specific Speed of Pump
8. Model Analysis of Pumps
9. Cavitations in Pumps

Introduction

A pump is a hydraulic machine which converts mechanical energy into hydraulic energy or pressure energy.

A centrifugal pump works on the principle of centrifugal force.

In this type of pump the liquid is subjected to whirling motion by the rotating impeller which is made of a number of backward curved vanes. The liquid enters this impeller at its center or the eye and gets discharged into the casing enclosing the outer edge of the impeller.

Generally centrifugal pumps are made of the **radial flow** type only **($\alpha = 90^0$)**

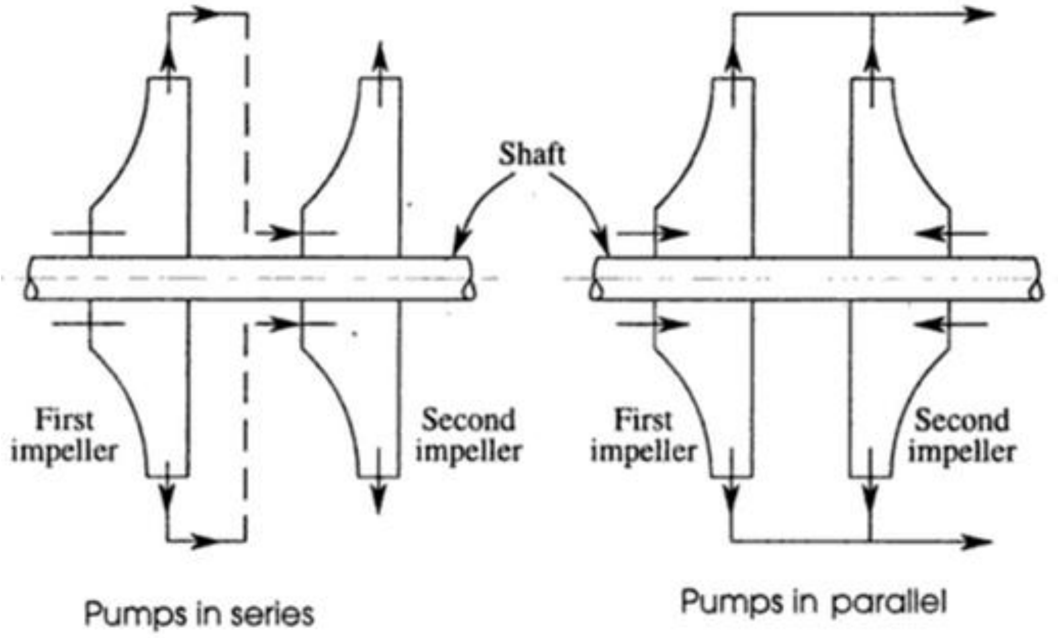
Classification of Pumps

1. According to No. of Impellers
   a) Single Stage Pump
   b) Multistage Pump
2. According to Disposition of Shaft
   a) Vertical Shaft Pump
   b) Horizontal Pump
3. According to Head
   a) Low Head Pump - $H < 15m$
   b) Medium Head Pump - $15m < H < 40m$
   c) High Specific Speed Turbine - $H > 40m$

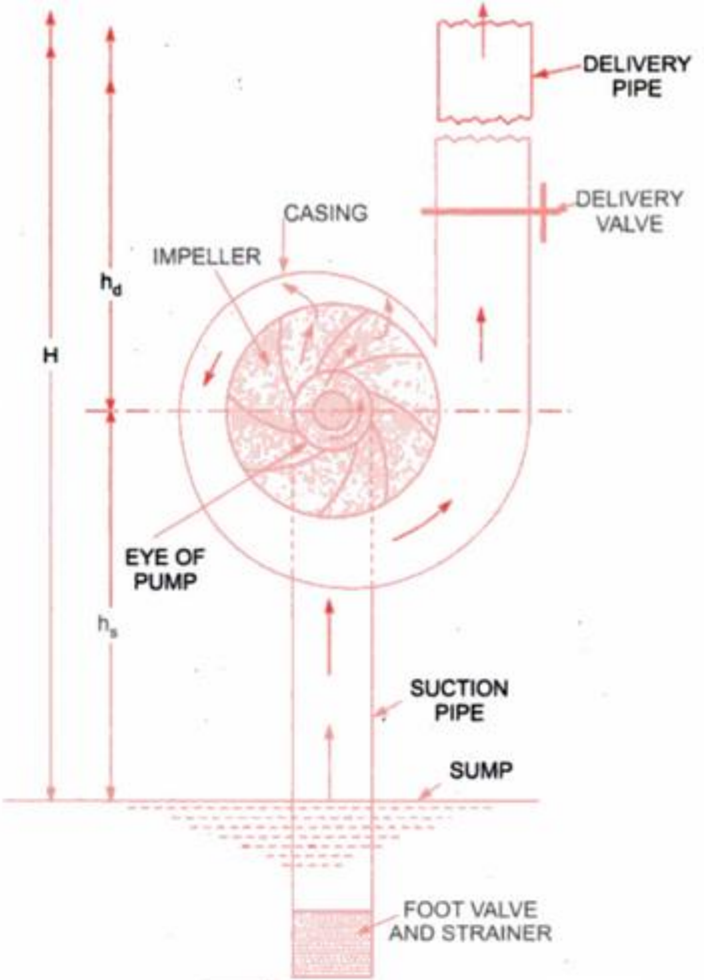
A centrifugal pump containing two or more impellers is called a multistage centrifugal pump.

a) For higher pressures at the outlet, impellers can be connected in series.

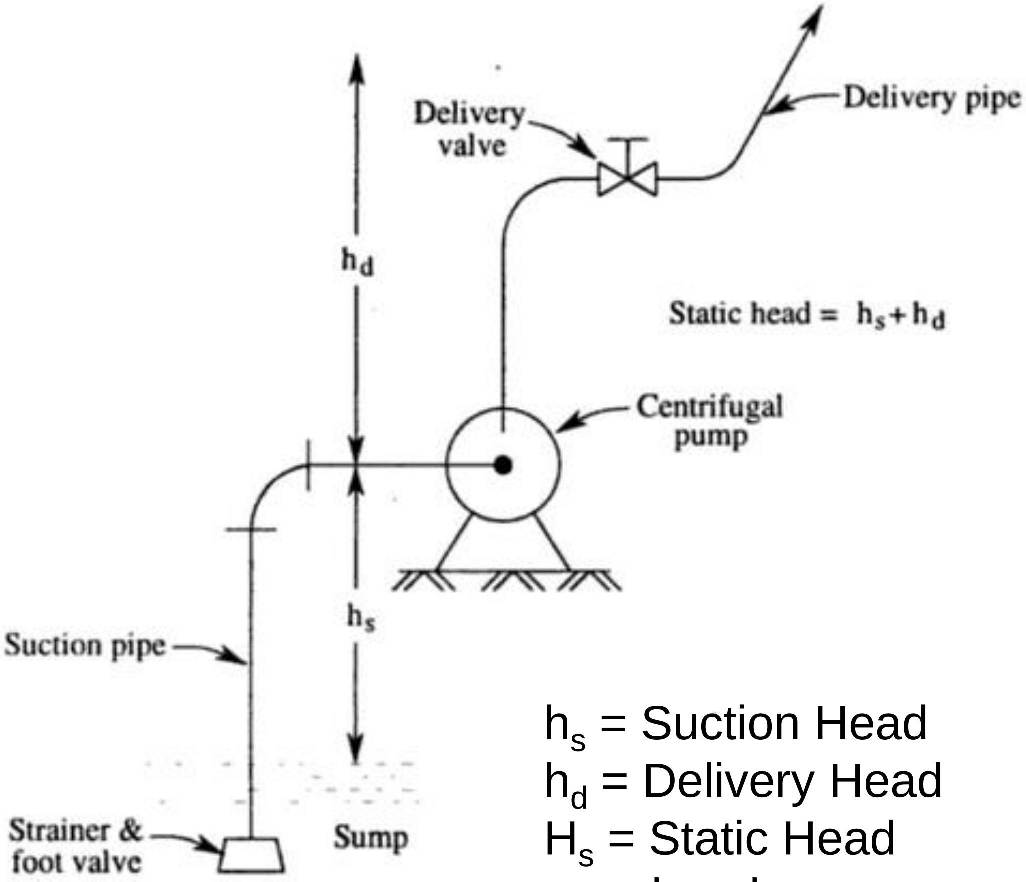
b) For higher flow output, impellers can be connected parallel.



MULTI-STAGE PUMPS



Main parts of a centrifugal pump.



h_s = Suction Head

h_d = Delivery Head

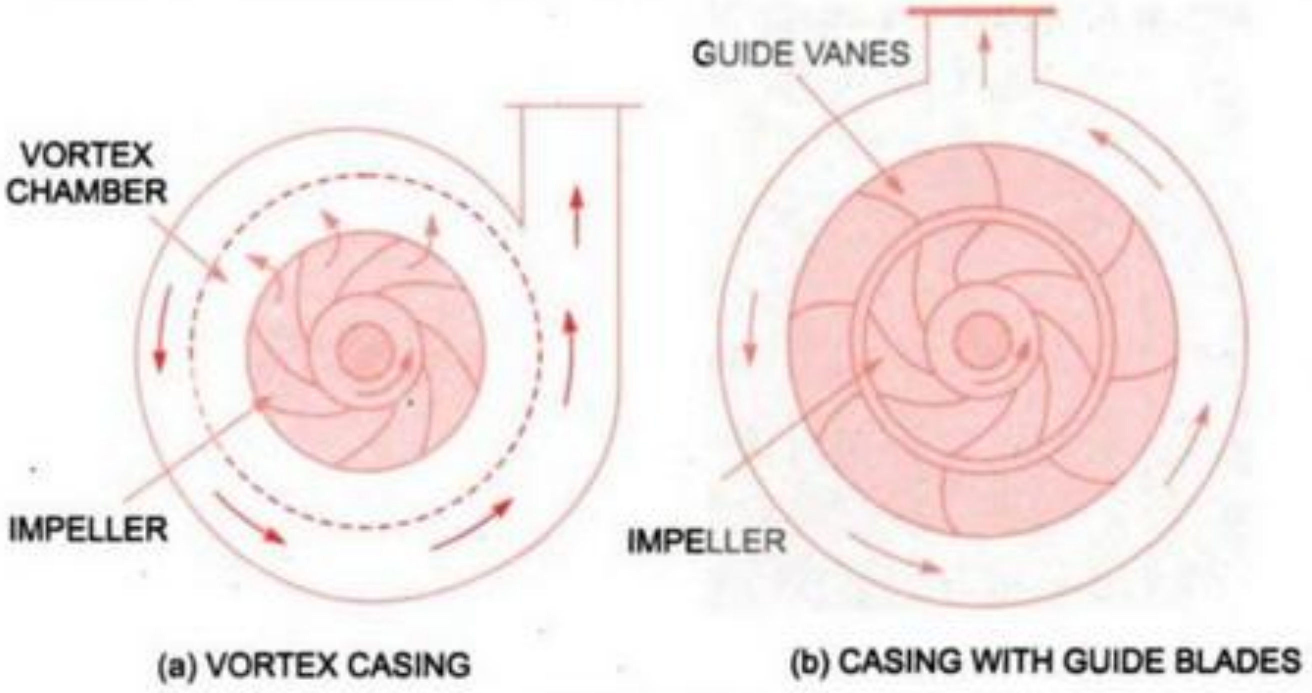
H_s = Static Head

= $h_s + h_s$

Components of Centrifugal Pump

Components of Pump

1. Strainer and Foot Valve
2. Suction Pipe and its fittings
3. Pump
4. Delivery Valve
5. Delivery Pipe and its fittings



Different types of casing.

Manometric Head

Manometric head (H_m):

It is the total head developed by the pump.

This head is slightly less than the head generated by the impeller due to some losses in the pump.

H_m = Suction Head + Delivery Head + Head Loss + Velocity Head in Delivery Pipe

$$= h_s + h_d + h_f + V_d^2/2g$$

Since $\alpha = 90^0$

$$Q = \text{Area} \times \text{Velocity of flow} = \pi D_1 B_1 \times V_{f_1} = \pi D_2 B_2 \times V_{f_2}$$

Head Imparted by Impeller to Water = Work done per Second

$= \rho Q(V_{W2} U_2)$

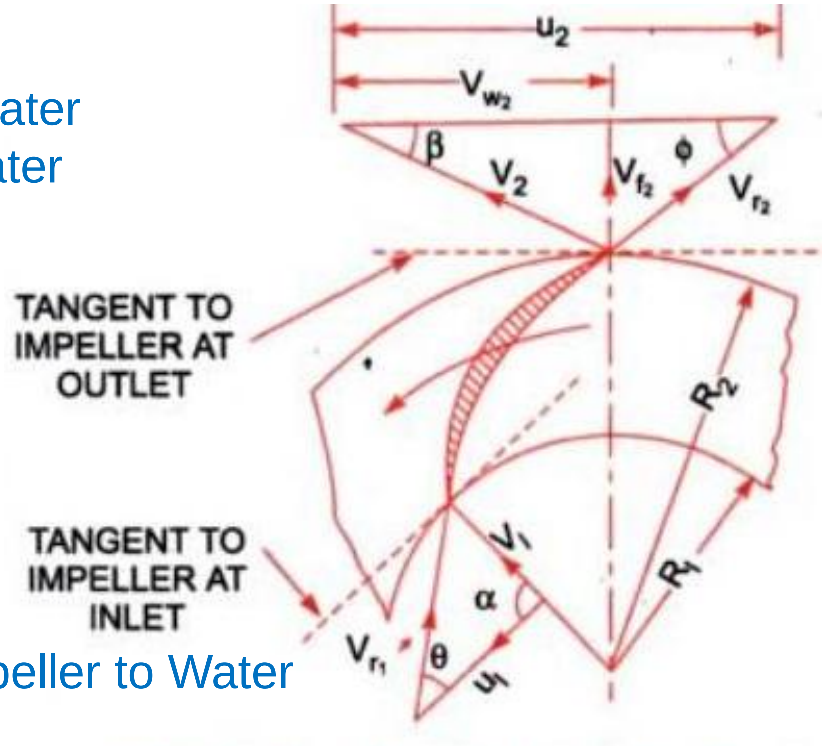
Head Imparted by Impeller to Unit Weight of Water

= Work done per Second per Unit Weight of Water

$= \rho Q(V_{W2} U_2) / mg$

$= \rho Q(V_{W2} U_2) / (\rho Q) g$

$= V_{W2} U_2 / g$



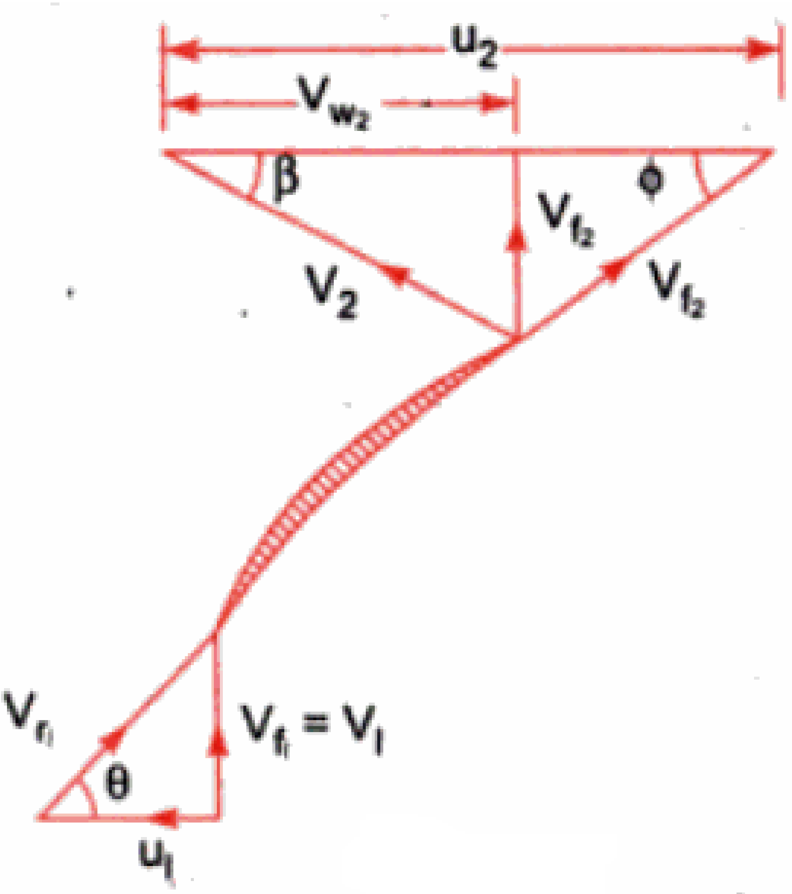
Velocity triangles at inlet and outlet.

Manometric Efficiency:

η_{man} = Manometric Head / Head Imparted by Impeller to Water

$= H_m / [(V_{W2} U_2) / g]$

$= g H_m / V_{W2} U_2$



Velocity Triangles at Inlet and Outlet

Minimum Starting Speed of Pump

A centrifugal pump will start delivering liquid only if the head developed by the impeller is more than the manometric head (H_m). If the head developed is less than H_m no discharge takes place although the impeller is rotating. When the impeller is rotating, the liquid in contact with the impeller is also rotating. This is a forced vertex, in which the increase in head in the impeller is given by

$$\text{Head rise in impeller} = \frac{u_2^2}{2g} - \frac{u_1^2}{2g}$$

Discharge takes place only when

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} \geq H_m$$

substituting for u_1, u_2 and H_m in Equation (10.13), we obtain

$$N = \frac{120\eta_m V_{w_2} D_2}{\pi(D_2^2 - D_1^2)}$$

which is the minimum speed for the pump to discharge liquid.

Specific Speed of Pump

The specific speed of a centrifugal pump is defined as the speed of a geometrically similar pump which would deliver *one cubic metre* of liquid per second against a head of *one metre*. It is denoted by 'N_s'.

$$N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}$$

Model Analysis of Pump

Before manufacturing the large sized pumps, their models which are in complete similarity with the actual pumps (also called prototypes) are made. Tests are conducted on the models and performance of the prototypes are predicted. The complete similarity between the model and actual pump (prototype) will exist if the following conditions are satisfied :

1. Specific speed of model = Specific speed of prototype

$$(N_s)_m = (N_s)_p \quad \text{or} \quad \left(\frac{N\sqrt{Q}}{H_m^{3/4}}\right)_m = \left(\frac{N\sqrt{Q}}{H_m^{3/4}}\right)_p$$

Cavitations in Pump

Cavitation is the formation of bubbles or cavities in liquid, developed in areas of relatively low pressure around an impeller. The imploding or collapsing of these bubbles trigger intense shockwaves inside the pump, causing significant damage to the impeller and/or the pump housing.

If left untreated, pump cavitations can cause:

a) Failure of pump housing
b) Destruction of impeller
c) Excessive vibration leading to premature seal and bearing failure
d) Higher than necessary power consumption

Precaution: NPSHA > NPSHR

Where NPSHA = Net Positive Suction Head Available
NPSHR = Net Positive Suction Head Required