POWER POINT PRESENTATION

ON

COMPUTER METHODS IN POWER SYSTEMS

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UNIT-I
POWER SYSTEM NETWORK MATRICES

Introduction

We should be able to analyze the performance of power systems both in normal operating conditions and under fault (short-circuit) condition. The analysis in normal steady-state operation is called a **power-flow study** (**load-flow study**) and it targets on determining the voltages, currents, and real and reactive power flows in a system under a given load conditions.

The purpose of power flow studies is to plan ahead and account for various hypothetical situations. For instance, what if a transmission line within the power system properly supplying loads must be taken off line for maintenance. Can the remaining lines in the system handle the required loads without exceeding their rated parameters?

Basic techniques for power-flow studies.

A **power-flow study** (**load-flow study**) is an analysis of the voltages, currents, and power flows in a power system under steady-state conditions. In such a study, we make an assumption about either a voltage at a bus or the power being supplied to the bus for each bus in the power system and then determine the magnitude and phase angles of the bus voltages, line currents, etc. that would result from the assumed combination of voltages and power flows.

The simplest way to perform power-flow calculations is by iteration:

1. Create a bus admittance matrix Y_{bus} for the power system;
2. Make an initial estimate for the voltages at each bus in the system;
3. Update the voltage estimate for each bus (one at a time), based on the estimates for the voltages and power flows at every other bus and the values of the bus admittance matrix: since the voltage at a given bus depends on the voltages at all of the other busses in the system (which are just estimates), the updated voltage will not be correct. However, it will usually be closer to the answer than the original guess.
4. Repeat this process to make the voltages at each bus approaching the correct answers closer and closer…

Basic techniques for power-flow studies.

The equations used to update the estimates differ for different types of busses. Each bus in a power system can be classified to one of three types:

1. Load bus (PQ bus) – a buss at which the real and reactive power are specified, and for which the bus voltage will be calculated. Real and reactive powers supplied to a power system are defined to be positive, while the powers consumed from the system are defined to be negative. All busses having no generators are load busses.

2. Generator bus (PV bus) – a bus at which the magnitude of the voltage is kept constant by adjusting the field current of a synchronous generator on the bus (as we learned, increasing the field current of the generator increases both the reactive power supplied by the generator and the terminal voltage of the system). We assume that the field current is adjusted to maintain a constant terminal voltage V_T. We also know that increasing the prime mover's governor set points increases the power that generator supplies to the power system. Therefore, we can *control and specify the magnitude of the bus voltage and real power supplied*.

Basic techniques for power-flow studies.

3. Slack bus (swing bus) – a special generator bus serving as the reference bus for the power system. Its voltage is assumed to be fixed in both magnitude and phase (for instance, $1\angle 0^\circ$ pu). The real and reactive powers are uncontrolled: the bus supplies whatever real or reactive power is necessary to make the power flows in the system balance.

In practice, a voltage on a load bus may change with changing loads. Therefore, load busses have specified values of P and Q, while V varies with load conditions.

Real generators work most efficiently when running at full load. Therefore, it is desirable to keep all but one (or a few) generators running at 100% capacity, while allowing the remaining (swing) generator to handle increases and decreases in load demand. Most busses with generators will supply a fixed amount of power and the magnitude of their voltages will be maintained constant by field circuits of generators. These busses have specific values of P and $|V_i|$.

The controls on the swing generator will be set up to maintain a constant voltage and frequency, allowing P and Q to increase or decrease as loads change.

Constructing Y_{bus} for power-flow analysis

The most common approach to power-flow analysis is based on the bus admittance matrix Y_{bus}. However, this matrix is slightly different from the one studied previously since the internal impedances of generators and loads connected to the system are not included in Y_{bus}. Instead, they are accounted for as specified real and reactive powers input and output from the busses.

Example 11.1: a simple power system has 4 busses, 5 transmission lines, 1 generator, and 3 loads. Series per-unit impedances are:

line #	Bus to bus	Series Z (pu)	Series Y (pu)
1	1-2	0.1+j0.4	0.5882-j2.3529
2	2-3	0.1+j0.5	0.3846-j1.9231
3	2-4	0.1+j0.4	0.5882-j2.3529
4	3-4	0.5+j0.2	1.1765-j4.7059
5	4-1	0.5+j0.2	1.1765-j4.7059

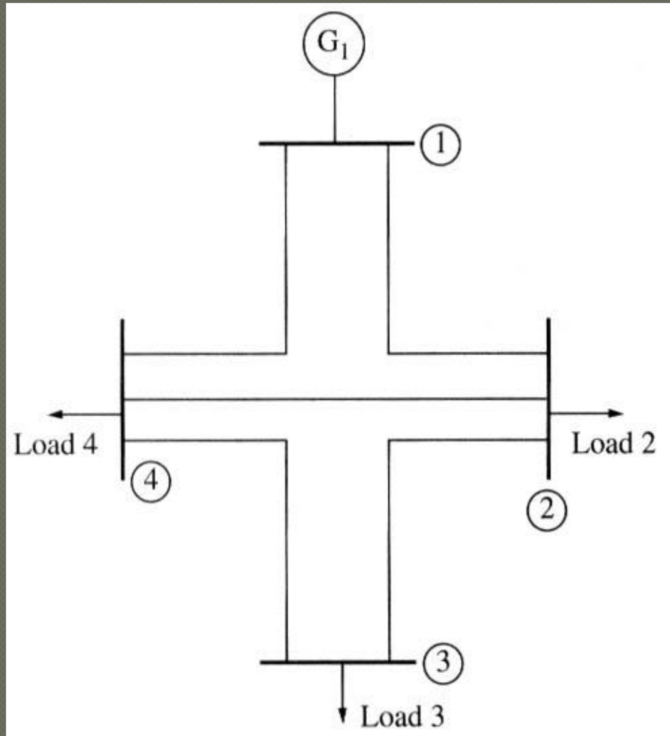


Table of Busses:

Bus 1	Slack bus
Bus 2	Load bus
Bus 3	Load bus
Bus 4	Load bus

Constructing Y_{bus} for power-flow analysis

The shunt admittances of the transmission lines are ignored. In this case, the Y_{ii} terms of the bus admittance matrix can be constructed by summing the admittances of all transmission lines connected to each bus, and the Y_{ij} ($i \neq j$) terms are just the negative of the line admittances stretching between busses i and j. Therefore, for instance, the term Y_{11} will be the sum of the admittances of all transmission lines connected to bus 1, which are the lines 1 and 5, so $Y_{11} = 1.7647 - j7.0588$ pu.

If the shunt admittances of the transmission lines are not ignored, the self admittance Y_{ii} at each bus would also include half of the shunt admittance of each transmission line connected to the bus.

The term Y_{12} will be the negative of all the admittances stretching between bus 1 and bus 2, which will be the negative of the admittance of transmission line 1, so $Y_{12} = -0.5882 + j2.3529$.

Constructing Y_{bus} for power-flow analysis

The complete bus admittance matrix can be obtained by repeating these calculations for every term in the matrix:

$$Y_{bus} = \begin{bmatrix} 1.7647 - j7.0588 & -0.5882 + j2.3529 & 0 & -1.1765 + j4.7059 \\ -0.5882 + j2.3529 & 1.5611 - j6.6290 & -0.3846 + j1.9231 & -0.5882 + j2.3529 \\ 0 & -0.3846 + j1.9231 & 1.5611 - j6.6290 & -1.1765 + j4.7059 \\ -1.1765 + j4.7059 & -0.5882 + j2.3529 & -1.1765 + j4.7059 & 2.9412 - j11.7647 \end{bmatrix}$$

Power-flow analysis equations

The basic equation for power-flow analysis is derived from the nodal analysis equations for the power system:

$$Y_{bus}V = I \tag{11.9.1}$$

For the four-bus power system shown above, (11.9.1) becomes

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} \tag{11.9.2}$$

where Y_{ij} are the elements of the bus admittance matrix, V_i are the bus voltages, and I_i are the currents injected at each node. For bus 2 in this system, this equation reduces to

$$Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4 = I_2 \tag{11.9.3}$$

Power-flow analysis equations

However, real loads are specified in terms of real and reactive powers, not as currents. The relationship between per-unit real and reactive power supplied to the system at a bus and the per-unit current injected into the system at that bus is:

$$S = VI^{*} = P + jQ \tag{11.10.1}$$

where *V* is the per-unit voltage at the bus; *I** - complex conjugate of the per-unit current injected at the bus; P and Q are per-unit real and reactive powers. Therefore, for instance, the current injected at bus 2 can be found as

$$V_2 I_2^{*} = P_2 + jQ_2 \quad \Rightarrow \quad I_2^{*} = \frac{P_2 + jQ_2}{V_2} \quad \Rightarrow \quad I_2 = \frac{P_2 + jQ_2}{V_2^{*}} \tag{11.10.2}$$

Substituting (11.10.2) into (11.9.3), we obtain

$$Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4 = \frac{P_2 + jQ_2}{V_2^{*}} \tag{11.10.3}$$

Power-flow analysis equations

Solving the last equation for V_2, yields

$$V_2 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - \left(Y_{21}V_1 + Y_{23}V_3 + Y_{24}V_4\right)\right] \qquad (11.11.1)$$

Similar equations can be created for each load bus in the power system.

(11.11.1) gives updated estimate for V_2 based on the specified values of real and reactive powers and the current estimates of all the bus voltages in the system. Note that the updated estimate for V_2 will not be the same as the original estimate of V_2^* used in (11.11.1) to derive it. We can repeatedly update the estimate wile substituting current estimate for V_2 back to the equation. The values of V_2 will converge; however, this would NOT be the correct bus voltage since voltages at the other nodes are also needed to be updated. Therefore, all voltages need to be updated during each iteration!

The iterations are repeated until voltage values no longer change much between iterations.

UNIT –II
POWER FLOW STUDIES

Power-flow analysis equations

This method is known as the Gauss-Siedel iterative method. Its basic procedure is:

1. Calculate the bus admittance matrix Y_{bus} including the admittances of all transmission lines, transformers, etc., between busses but exclude the admittances of the loads or generators themselves.
2. Select a slack bus: one of the busses in the power system, whose voltage will arbitrarily be assumed as $1.0\angle 0^{\circ}$.
3. Select initial estimates for all bus voltages: usually, the voltage at every load bus assumed as $1.0\angle 0^{\circ}$ (flat start) lead to good convergence.
4. Write voltage equations for every other bus in the system. The generic form is

$$V_i = \frac{1}{Y_{ii}}\left(\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{N} Y_{ik} V_k \right) \qquad (11.12.1)$$

Power-flow analysis equations

5. Calculate an updated estimate of the voltage at each load bus in succession using (11.12.1) except for the slack bus.

6. Compare the differences between the old and new voltage estimates: if the differences are less than some specified tolerance for all busses, stop. Otherwise, repeat step 5.

7. Confirm that the resulting solution is reasonable: a valid solution typically has bus voltages, whose phases range in less than 45°.

Example 11.2: in a 2-bus power system, a generator attached to bus 1 and loads attached to bus 2. the series impedance of a single transmission line connecting them is 0.1+j0.5 pu. The shunt admittance of the line may be neglected. Assume that bus 1 is the slack bus and that it has a voltage $V_1 = 1.0\angle 0^\circ$ pu. Real and reactive powers supplied to the loads from the system at bus 2 are $P_2 = 0.3$ pu, $Q_2 = 0.2$ pu (powers supplied to the system at each busses is negative of the above values). Determine voltages at each bus for the specified load conditions.

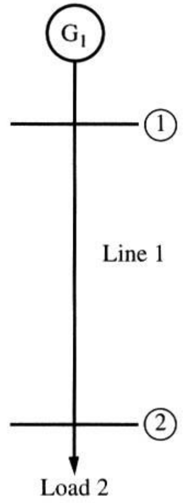


Table of Busses:

Bus 1	Slack bus
Bus 2	Load bus

Power-flow analysis equations

1. We start from calculating the bus admittance matrix Y_{bus}. The Y_{ii} terms can be constructed by summing the admittances of all transmission lines connected to each bus, and the Y_{ij} terms are the negative of the admittances of the line stretching between busses i and j. For instance, the term Y_{11} is the sum of the admittances of all transmission lines connected to bus 1 (a single line in our case). The series admittance of line 1 is

$$Y_{line\,1} = \frac{1}{Z_{line\,1}} = \frac{1}{0.1 + j0.5} = 0.3846 - j1.9231 = Y_{11} \tag{11.14.1}$$

Applying similar calculations to other terms, we complete the admittance matrix as

$$Y_{bus} = \begin{bmatrix} 0.3846 - j1.9231 & -0.3846 + j1.9231 \\ -0.3846 + j1.9231 & 0.3846 - j1.9231 \end{bmatrix} \tag{11.14.2}$$

2. Next, we select bus 1 as the slack bus since it is the only bus in the system connected to the generator. The voltage at bus 1 will be assumed $1.0\angle 0^{\circ}$.

Power-flow analysis equations

3. We select initial estimates for all bus voltages. Making a flat start, the initial voltage estimates at every bus are $1.0\angle 0^\circ$.

4. Next, we write voltage equations for every other bus in the system. For bus 2:

$$V_2 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_{2,old}^*} - Y_{21}V_1\right] \tag{11.15.1}$$

Since the real and reactive powers supplied to the system at bus 2 are P_2 = -0.3 pu and Q_2 = -0.2 pu and since Ys and V_1 are known, we may reduce the last equation:

$$V_2 = \frac{1}{0.3846 - j1.9231}\left[\frac{-0.3 - j0.2}{V_{2,old}^*} - \left(\left(-0.3846 + j1.9231\right)V_1\right)\right]$$

$$= \frac{1}{1.9612\angle -78.8^\circ}\left[\frac{0.3603\angle -146.3^\circ}{V_{2,old}^*} - \left(1.9612\angle 101.3^\circ\right)\left(1\angle 0^\circ\right)\right] \tag{11.15.2}$$

Power-flow analysis equations

5. Next, we calculate an updated estimate of the voltages at each load bus in succession. In this problem we only need to calculate updated voltages for bus 2, since the voltage at the slack bus (bus 1) is assumed constant. We repeat this calculation until the voltage converges to a constant value.

The initial estimate for the voltage is $V_{2,0} = 1\angle 0^\circ$. The next estimate for the voltage is

$$V_{2,1} = \frac{1}{1.9612\angle -78.8^\circ}\left[\frac{0.3603\angle -146.3^\circ}{V_{2,old}^{*}} - \left(1.9612\angle 101.3^\circ\right)\left(1\angle 0^\circ\right)\right]$$

$$= \frac{1}{1.9612\angle -78.8^\circ}\left[\frac{0.3603\angle -146.3^\circ}{1\angle 0^\circ} - \left(1.9612\angle 101.3^\circ\right)\right]$$

$$= 0.8797\angle -8.499^\circ \qquad (11.16.1)$$

This new estimate for V_2 substituted back to the equation will produce the second estimate:

Power-flow analysis equations

$$V_{2,2} = \frac{1}{1.9612\angle -78.8^\circ}\left[\frac{0.3603\angle -146.3^\circ}{0.8797\angle -8.499^\circ} - (1.9612\angle 101.3^\circ)(1\angle 0^\circ)\right]$$

$$= 0.8412\angle -8.499^\circ \qquad (11.17.1)$$

The third iteration will be

$$V_{2,3} = \frac{1}{1.9612\angle -78.8^\circ}\left[\frac{0.3603\angle -146.3^\circ}{0.8412\angle -8.499^\circ} - (1.9612\angle 101.3^\circ)(1\angle 0^\circ)\right]$$

$$= 0.8345\angle -8.962^\circ \qquad (11.17.2)$$

The fourth iteration will be

$$V_{2,4} = \frac{1}{1.9612\angle -78.8^\circ}\left[\frac{0.3603\angle -146.3^\circ}{0.8345\angle -8.962^\circ} - (1.9612\angle 101.3^\circ)(1\angle 0^\circ)\right]$$

$$= 0.8320\angle -8.962^\circ \qquad (11.17.3)$$

The fifth iteration will be

$$V_{2,5} = \frac{1}{1.9612\angle -78.8^\circ}\left[\frac{0.3603\angle -146.3^\circ}{0.8320\angle -8.962^\circ} - (1.9612\angle 101.3^\circ)(1\angle 0^\circ)\right]$$

$$= 0.8315\angle -8.994^\circ \qquad (11.17.4)$$

Power-flow analysis equations

6. We observe that the magnitude of the voltage is barely changing and may conclude that this value is close to the correct answer and, therefore, stop the iterations.

This power system converged to the answer in five iterations. The voltages at each bus in the power system are:

$$V_1 = 1.0\angle 0^\circ$$
$$V_2 = 0.8315\angle -8.994^\circ \qquad (11.18.1)$$

7. Finally, we need to confirm that the resulting solution is reasonable. The results seem reasonable since the phase angles of the voltages in the system differ by only 10°. The current flowing from bus 1 to bus 2 is

$$I_1 = \frac{V_1 - V_2}{Z_{line\,1}} = \frac{1\angle 0^\circ - 0.8315\angle -8.994^\circ}{0.1 + j0.5} = 0.4333\angle -42.65^\circ \qquad (11.18.2)$$

Power-flow analysis equations

The power supplied by the transmission line to bus 2 is

$$S = VI^* = \left(0.8315\angle -8.994^\circ\right)\left(0.4333\angle -42.65^\circ\right)^* = 0.2999 + j0.1997$$

This is the amount of power consumed by the loads; therefore, this solution appears to be correct.

Note that this example must be interpreted as follows: if the real and reactive power supplied by bus 2 is 0.3 + j0.2 pu and if the voltage on the slack bus is $1\angle 0^\circ$ pu, then the voltage at bus 2 will be $V_2 = 0.8315\angle -8.994^\circ$.

This voltage is correct only for the assumed conditions; another amount of power supplied by bus 2 will result in a different voltage V_2.

Therefore, we usually postulate some reasonable combination of powers supplied to loads, and determine the resulting voltages at all the busses in the power system. Once the voltages are known, currents through each line can be calculated.

The relationship between voltage and current at a load bus as given by (11.12.1) is fundamentally nonlinear! Therefore, solution greatly depends on the initial guess.

Adding generator busses to power-flow studies

At a generator bus, the real power P_i and the magnitude of the bus voltage $|V_i|$ are specified. Since the reactive power for that bus is usually unknown, we need to estimate it before applying (11.12.1) to get updated voltage estimates. The value of reactive power at the generator bus can be estimated by solving (11.12.1) for Q_i:

$$V_i = \frac{1}{Y_{ii}} \left(\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{N} Y_{ik} V_k \right) \Leftrightarrow P_i - jQ_i = V_i^* \left(Y_{ii} V_i - \sum_{\substack{k=1 \\ k \neq i}}^{N} Y_{ik} V_k \right) \quad (11.20.1)$$

Bringing the case $k = I$ into summation, we obtain

$$P_i - jQ_i = V_i^* \sum_{k=1}^{N} Y_{ik} V_k \quad \Rightarrow \quad Q_i = -\operatorname{Im} \left\{ V_i^* \sum_{k=1}^{N} Y_{ik} V_k \right\} \quad (11.20.2)$$

Once the reactive power at the bus is estimated, we can update the bus voltage at a generator bus using P_i and Q_i as we would at a load bus. However, the magnitude of the generator bus voltage is also forced to remain constant. Therefore, we must multiply the new voltage estimate by the ratio of magnitudes of old to new estimates.

Adding generator busses to power-flow studies

Therefore, the steps required to update the voltage at a generator bus are:

1. Estimate the reactive power Q_i according to (11.20.2);
2. Update the estimated voltage at the bus according to (11.12.1) as if the bus was a load bus;
3. Force the magnitude of the estimated voltage to be constant by multiplying the new voltage estimate by the ratio of the magnitude of the original estimate to the magnitude of the new estimate. This has the effect of updating the voltage phase estimate without changing the voltage amplitude.

Adding generator busses to power-flow studies

Example 11.3: a 4-bus power system with 5 transmission lines, 2 generators, and 2 loads. Since the system has generators connected to 2 busses, it will have one slack bus, one generator bus, and two load busses. Assume that bus 1 is the slack bus and that it has a voltage $V_1 = 1.0\angle 0^{\circ}$ pu. Bus 3 is a generator bus. The generator is supplying a real power $P_3 = 0.3$ pu to the system with a voltage magnitude 1 pu. The per-unit real and reactive power loads at busses 2 and 4 are $P_2 = 0.3$ pu, $Q_2 = 0.2$ pu, $P_4 = 0.2$ pu, $Q_4 = 0.15$ pu (powers supplied to the system at each busses are negative of the above values). The series impedances of each bus were evaluated in Example 11.1. Determine voltages at each bus for the specified load conditions.

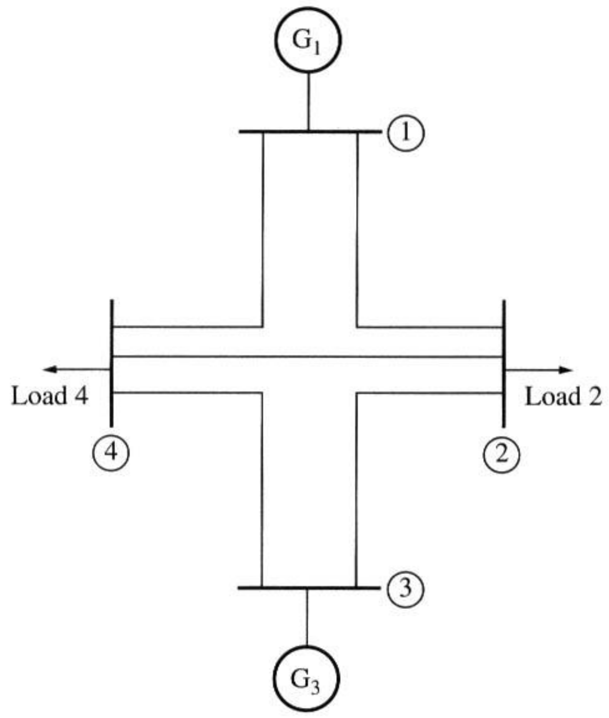


Table of Buses:

Bus 1	Slack bus
Bus 2	Load bus
Bus 3	Generator bus
Bus 4	Load bus

Adding generator busses to power-flow studies

The bus admittance matrix was calculated earlier as

$$Y_{bus} = \begin{bmatrix} 1.7647 - j7.0588 & -0.5882 + j2.3529 & 0 & -1.1765 + j4.7059 \\ -0.5882 + j2.3529 & 1.5611 - j6.6290 & -0.3846 + j1.9231 & -0.5882 + j2.3529 \\ 0 & -0.3846 + j1.9231 & 1.5611 - j6.6290 & -1.1765 + j4.7059 \\ -1.1765 + j4.7059 & -0.5882 + j2.3529 & -1.1765 + j4.7059 & 2.9412 - j11.7647 \end{bmatrix}$$

Since the bus 3 is a generator bus, we will have to estimate the reactive power at that bus before calculating the bus voltages, and then force the magnitude of the voltage to remain constant after computing the bus voltage. We will make a flat start assuming the initial voltage estimates at every bus to be $1.0\angle 0^{\circ}$.

Therefore, the sequence of voltage (and reactive power) equations for all busses is:

Adding generator busses to power-flow studies

$$V_2 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_{2,old}^*} - \left(Y_{21}V_1 + Y_{23}V_3 + Y_{24}V_4\right)\right] \quad (11.24.1)$$

$$Q_3 = -\operatorname{Im}\left\{V_3^* \sum_{k=1}^{N} Y_{ik}V_k\right\} \quad (11.24.2)$$

$$V_3 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_{3,old}^*} - \left(Y_{31}V_1 + Y_{32}V_2 + Y_{34}V_4\right)\right] \quad (11.24.3)$$

$$V_3 = V_3 \frac{\left|V_{3,old}\right|}{\left|V_3\right|} \quad (11.24.4)$$

$$V_4 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_{4,old}^*} - \left(Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3\right)\right] \quad (11.24.4)$$

Adding generator busses to power-flow studies

The voltages and the reactive power should be updated iteratively, for instance, using Matlab.

Computations converge to the following solution:

$$V_1 = 1.0\angle 0^\circ \ pu$$

$$V_2 = 0.964\angle -0.97^\circ \ pu$$

$$V_3 = 1.0\angle 1.84^\circ \ pu \qquad (11.25.1)$$

$$V_4 = 0.98\angle -0.27^\circ \ pu$$

The solution looks reasonable since the bus voltage phase angles is less than 45°.

The information derived from power-flow studies

After the bus voltages are calculated at all busses in a power system, a power-flow program can be set up to provide alerts if the voltage at any given bus exceeds, for instance, ±5% of the nominal value. This is important since the power needs to be supplied at a constant voltage level; therefore, such voltage variations may indicate problems…

Additionally, it is possible to determine the net real and reactive power either supplied or removed from the each bus by generators or loads connected to it. To calculate the real and reactive power at a bus, we first calculate the net current injected at the bus, which is the sum of all the currents leaving the bus through transmission lines.

The current leaving the bus on each transmission line can be found as:

$$I_i = \sum_{\substack{k=1 \\ k \neq i}}^{N} Y_{ik} \left(V_i - V_k \right) \tag{11.26.1}$$

The information derived from power-flow studies

The resulting real and reactive powers injected at the bus can be found from

$$S_i = -V_i I_i^* = P_i + jQ_i \qquad (11.27.1)$$

where the minus sign indicate that current is assumed to be injected instead of leaving the node.

Similarly, the power-flow study can show the real and reactive power flowing in every transmission line in the system. The current flow out of a node along a particular transmission line between bus *i* and bus *j* can be calculated as:

$$I_{ij} = Y_{ij}\left(V_i - V_j\right)$$

where Y_{ij} is the admittance of the transmission line between those two busses. The resulting real and reactive power can be calculated as:

$$S_{ij} = -V_i I_{ij}^* = P_{ij} + jQ_{ij}$$

The information derived from power-flow studies

Also, comparing the real and reactive power flows at either end of the transmission line, we can determine the real and reactive power losses on each line.

In modern power-flow programs, this information is displayed graphically. Colors are used to highlight the areas where the power system is overloaded, which aids "hot spot" localization.

Power-flow studies are usually started from analysis of the power system in its normal operating conditions, called the base case. Then, various (increased) load conditions may be projected to localize possible problem spots (overloads). By adding transmission lines to the system, a new configuration resolving the problem may be found. This estimated models can be used for planning.

Another reason for power-flow studies is modeling possible failures of particular lines and generators to see whether the remaining components can handle the loads.

Finally, it is possible to determine more efficient power utilization by redistributing generation from one locations to other. This variety of power-flow studies is called economic dispatch.

UNIT-III
SHORT CIRCUIT ANALYSIS

Modeling Voltage Dependent Load

So far we've assumed that the load is independent of the bus voltage (i.e., constant power). However, the power flow can be easily extended to include voltage depedence with both the real and reactive load. This is done by making P_{Di} and Q_{Di} a function of $|V_i|$:

$$\sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) - P_{Gi} + P_{Di}(|V_i|) = 0$$

$$\sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) - Q_{Gi} + Q_{Di}(|V_i|) = 0$$

Voltage Dependent Load Example

In previous two bus example now assume the load is constant impedance, so

$$P_2(\mathbf{x}) = |V_2|(10\sin\theta_2) + 2.0|V_2|^2 = 0$$

$$Q_2(\mathbf{x}) = |V_2|(-10\cos\theta_2) + |V_2|^2(10) + 1.0|V_2|^2 = 0$$

Now calculate the power flow Jacobian

$$J(\mathbf{x}) = \begin{bmatrix} 10|V_2|\cos\theta_2 & 10\sin\theta_2 + 4.0|V_2| \\ 10|V_2|\sin\theta_2 & -10\cos\theta_2 + 20|V_2| + 2.0|V_2| \end{bmatrix}$$

Voltage Dependent Load, cont'd

Again set $v = 0$, guess $\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Calculate

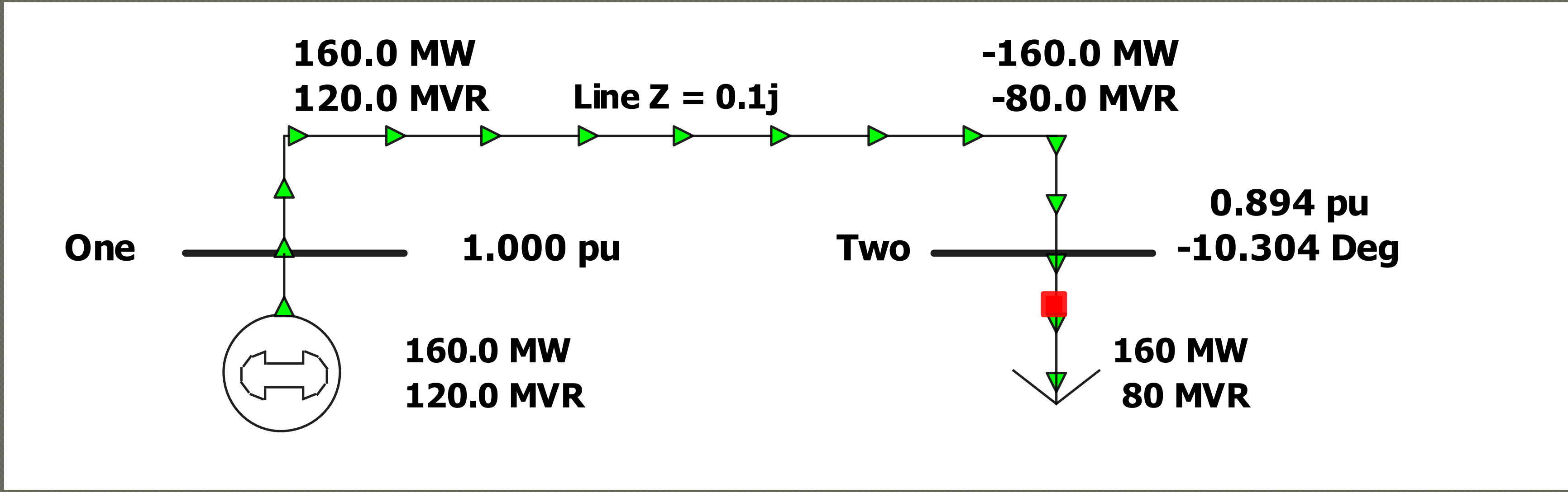
$$f(\mathbf{x}^{(0)}) = \begin{bmatrix} |V_2|(10 \sin\theta_2) + 2.0\,|V_2|^2 \\ |V_2|(-10\cos\theta_2) + |V_2|^2(10) + 1.0\,|V_2|^2 \end{bmatrix} = \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix}$$

$$\mathbf{J}(\mathbf{x}^{(0)}) = \begin{bmatrix} 10 & 4 \\ 0 & 12 \end{bmatrix}$$

Solve $\mathbf{x}^{(1)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 10 & 4 \\ 0 & 12 \end{bmatrix}^{-1} \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} -0.1667 \\ 0.9167 \end{bmatrix}$

Voltage Dependent Load, cont'd

With constant impedance load the MW/Mvar load at bus 2 varies with the square of the bus 2 voltage magnitude. This if the voltage level is less than 1.0, the load is lower than 200/100 MW/Mvar



Dishonest Newton-Raphson

- Since most of the time in the Newton-Raphson iteration is spend calculating the inverse of the Jacobian, one way to speed up the iterations is to only calculate/inverse the Jacobian occasionally
  - known as the "Dishonest" Newton-Raphson
  - an extreme example is to only calculate the Jacobian for the first iteration

Honest: $$\mathbf{x}^{(v+1)} = \mathbf{x}^{(v)} - \mathbf{J}(\mathbf{x}^{(v)})^{-1}\mathbf{f}(\mathbf{x}^{(v)})$$

Dishonest: $$\mathbf{x}^{(v+1)} = \mathbf{x}^{(v)} - \mathbf{J}(\mathbf{x}^{(0)})^{-1}\mathbf{f}(\mathbf{x}^{(v)})$$

Both require $\left\|\mathbf{f}(\mathbf{x}^{(v)})\right\| < \varepsilon$ for a solution

Dishonest Newton-Raphson Example

Use the Dishonest Newton-Raphson to solve

$$f(x) = x^2 - 2 = 0$$

$$\Delta x^{(v)} = -\left[\frac{df(x^{(0)})}{dx}\right]^{-1} f(x^{(v)})$$

$$\Delta x^{(v)} = -\left[\frac{1}{2x^{(0)}}\right]((x^{(v)})^2 - 2)$$

$$x^{(v+1)} = x^{(v)} - \left[\frac{1}{2x^{(0)}}\right]((x^{(v)})^2 - 2)$$

Dishonest N-R Example, cont'd

$$x^{(v+1)} = x^{(v)} - \left[\frac{1}{2x^{(0)}}\right]((x^{(v)})^2 - 2)$$

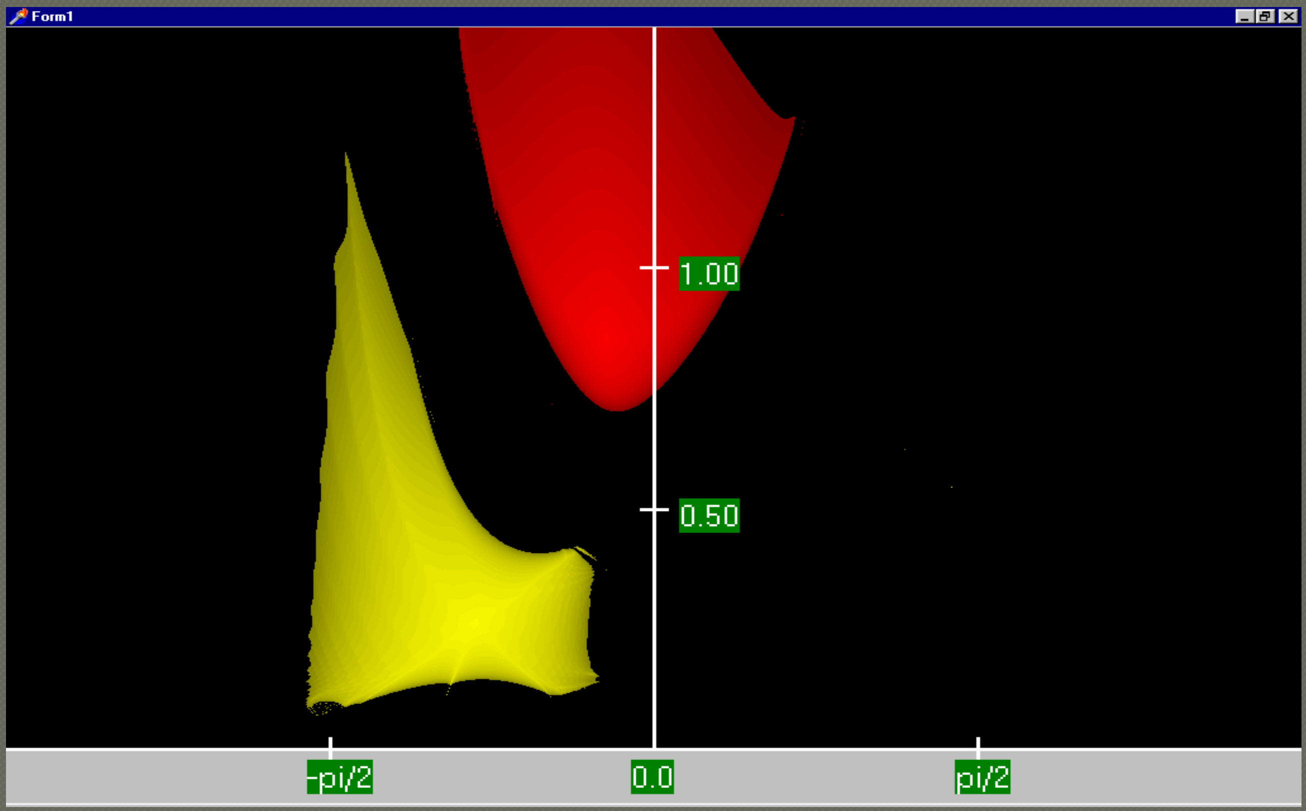
Guess $x^{(0)} = 1$. Iteratively solving we get

v	$x^{(v)}$ (honest)	$x^{(v)}$ (dishonest)
0	1	1
1	1.5	1.5
2	1.41667	1.375
3	1.41422	1.429
4	1.41422	1.408

We pay a price in increased iterations, but with decreased computation per iteration

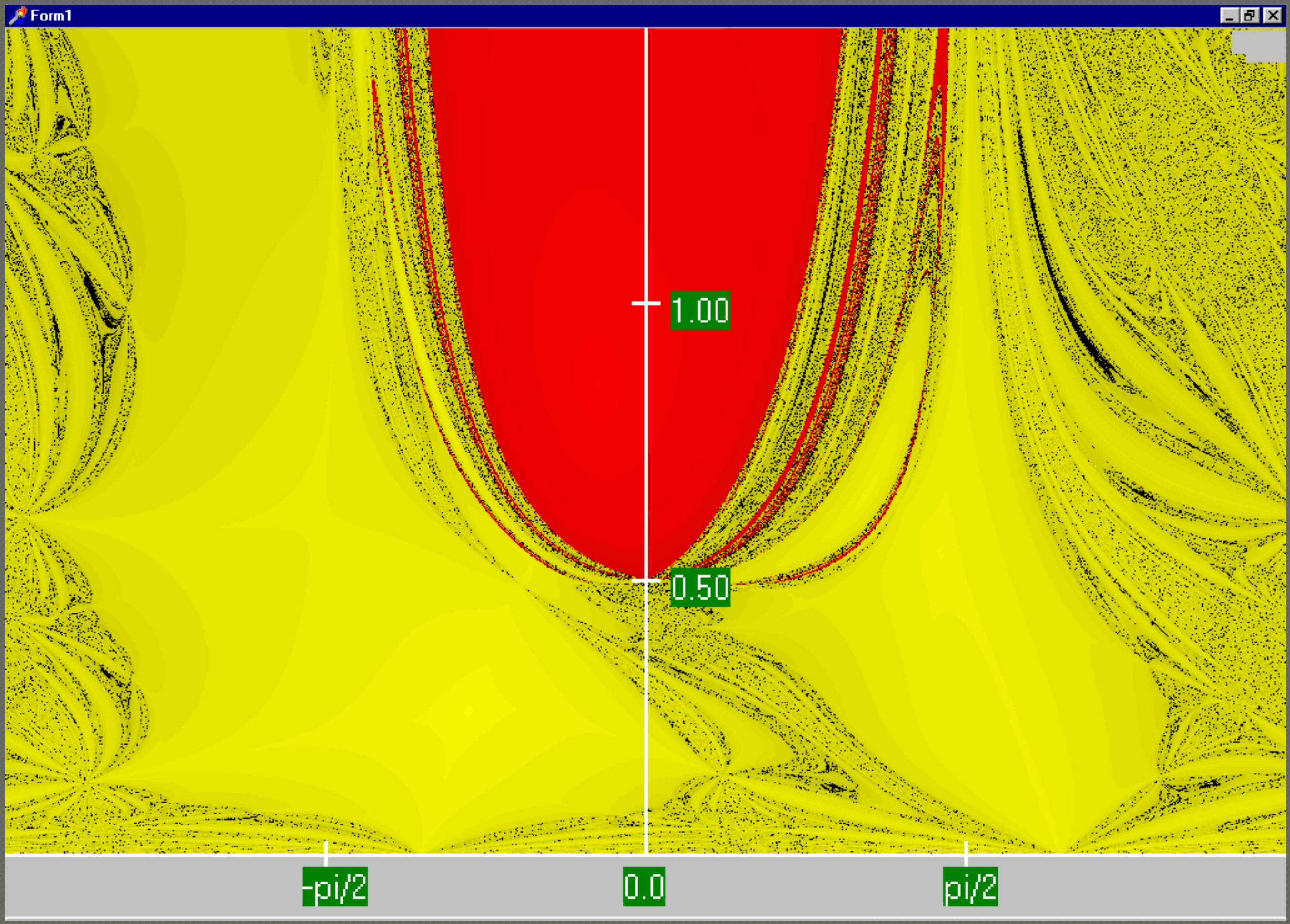
Two Bus Dishonest ROC

Slide shows the region of convergence for different initial
guesses for the 2 bus case using the Dishonest N-R



Red region converges to the high voltage solution, while the yellow region converges to the low voltage solution

Honest N-R Region of Convergence



Maximum of 15 iterations

Decoupled Power Flow

- The completely Dishonest Newton-Raphson is not used for power flow analysis. However several approximations of the Jacobian matrix are used.
- One common method is the decoupled power flow. In this approach approximations are used to decouple the real and reactive power equations.

Decoupled Power Flow Formulation

General form of the power flow problem

$$-\begin{bmatrix} \dfrac{\partial \mathbf{P}}{\partial \boldsymbol{\theta}}^{(v)} & \dfrac{\partial \mathbf{P}}{\partial |\mathbf{V}|}^{(v)} \\ \dfrac{\partial \mathbf{Q}}{\partial \boldsymbol{\theta}}^{(v)} & \dfrac{\partial \mathbf{Q}}{\partial |\mathbf{V}|}^{(v)} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\theta}^{(v)} \\ \Delta |\mathbf{V}|^{(v)} \end{bmatrix} = \begin{bmatrix} \Delta \mathbf{P}(\mathbf{x}^{(v)}) \\ \Delta \mathbf{Q}(\mathbf{x}^{(v)}) \end{bmatrix} = \mathbf{f}(\mathbf{x}^{(v)})$$

where

$$\Delta \mathbf{P}(\mathbf{x}^{(v)}) = \begin{bmatrix} P_2(\mathbf{x}^{(v)}) + P_{D2} - P_{G2} \\ \vdots \\ P_n(\mathbf{x}^{(v)}) + P_{Dn} - P_{Gn} \end{bmatrix}$$

Decoupling Approximation

Usually the off-diagonal matrices, $\dfrac{\partial \mathbf{P}}{\partial |\mathbf{V}|}^{(v)}$ and $\dfrac{\partial \mathbf{Q}}{\partial \boldsymbol{\theta}}^{(v)}$

are small. Therefore we approximate them as zero:

$$-\begin{bmatrix} \dfrac{\partial \mathbf{P}}{\partial \boldsymbol{\theta}}^{(v)} & \mathbf{0} \\ \mathbf{0} & \dfrac{\partial \mathbf{Q}}{\partial |\mathbf{V}|}^{(v)} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\theta}^{(v)} \\ \Delta |\mathbf{V}|^{(v)} \end{bmatrix} = \begin{bmatrix} \Delta \mathbf{P}(\mathbf{x}^{(v)}) \\ \Delta \mathbf{Q}(\mathbf{x}^{(v)}) \end{bmatrix} = \mathbf{f}(\mathbf{x}^{(v)})$$

Then the problem can be decoupled

$$\Delta \boldsymbol{\theta}^{(v)} = -\left[\dfrac{\partial \mathbf{P}}{\partial \boldsymbol{\theta}}^{(v)} \right]^{-1} \Delta \mathbf{P}(\mathbf{x}^{(v)}) \quad \Delta |\mathbf{V}|^{(v)} = -\left[\dfrac{\partial \mathbf{Q}}{\partial |\mathbf{V}|}^{(v)} \right]^{-1} \Delta \mathbf{Q}(\mathbf{x}^{(v)})$$

Off-diagonal Jacobian Terms

Justification for Jacobian approximations:

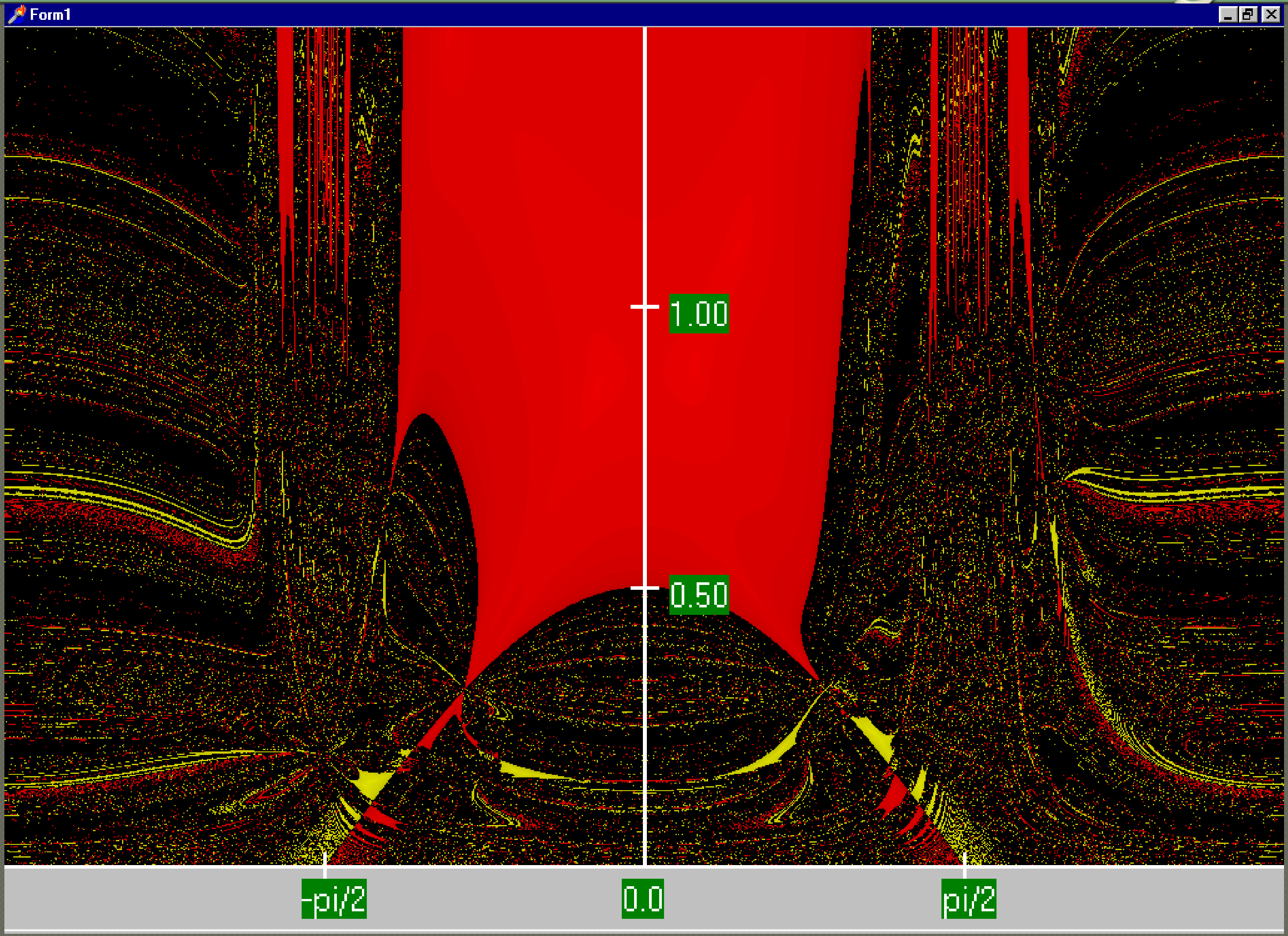
1. Usually r □ x, therefore $|G_{ij}|$ □ $|B_{ij}|$
2. Usually θ_{ij} is small so $\sin\theta_{ij} \approx 0$

Therefore

$$\frac{\partial \mathbf{P}_i}{\partial |\mathbf{V}_j|} = |V_i|\left(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}\right) \approx 0$$

$$\frac{\partial \mathbf{Q}_i}{\partial \boldsymbol{\theta}_j} = -|V_i||V_j|\left(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}\right) \approx 0$$

Decoupled N-R Region of Convergence



Fast Decoupled Power Flow

- By continuing with our Jacobian approximations we can actually obtain a reasonable approximation that is independent of the voltage magnitudes/angles.
- This means the Jacobian need only be built/inverted once.
- This approach is known as the fast decoupled power flow (FDPF)
- FDPF uses the same mismatch equations as standard power flow so it should have same solution
- The FDPF is widely used, particularly when we only need an approximate solution

FDPF Approximations

The FDPF makes the following approximations:

1. $\left| G_{ij} \right| = 0$
2. $\left| V_i \right| = 1$
3. $\sin\theta_{ij} = 0 \qquad \cos\theta_{ij} = 1$

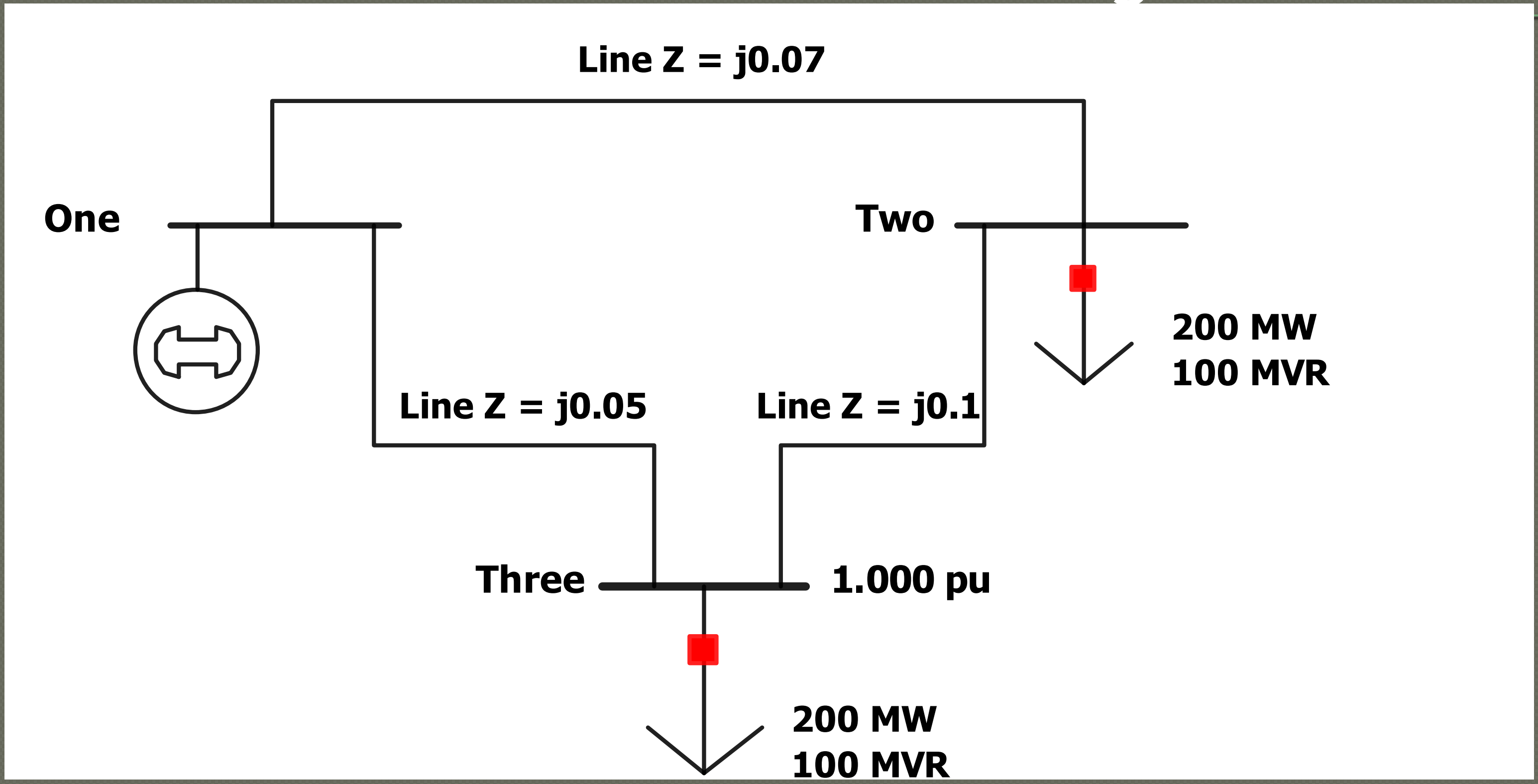
Then

$$\Delta\boldsymbol{\theta}^{(v)} = \mathbf{B}^{-1}\frac{\Delta\mathbf{P}(\mathbf{x}^{(v)})}{\mathbf{V}^{(v)}} \qquad \Delta\left|\mathbf{V}\right|^{(v)} = \mathbf{B}^{-1}\frac{\Delta\mathbf{Q}(\mathbf{x}^{(v)})}{\mathbf{V}^{(v)}}$$

Where $\mathbf{B}$ is just the imaginary part of the $\mathbf{Y}_{bus} = \mathbf{G} + j\mathbf{B}$, except the slack bus row/column are omitted

FDPF Three Bus Example

Use the FDPF to solve the following three bus system



$$\mathbf{Y}_{bus} = j\begin{bmatrix} -34.3 & 14.3 & 20 \\ 14.3 & -24.3 & 10 \\ 20 & 10 & -30 \end{bmatrix}$$

FDPF Three Bus Example, cont'd

$$\mathbf{Y}_{bus} = j\begin{bmatrix} -34.3 & 14.3 & 20 \\ 14.3 & -24.3 & 10 \\ 20 & 10 & -30 \end{bmatrix} \rightarrow \mathbf{B} = \begin{bmatrix} -24.3 & 10 \\ 10 & -30 \end{bmatrix}$$

$$\mathbf{B}^{-1} = \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix}$$

Iteratively solve, starting with an initial voltage guess

$$\begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix}^{(0)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} |V|_2 \\ |V|_3 \end{bmatrix}^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix}^{(1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -0.1272 \\ -0.1091 \end{bmatrix}$$

FDPF Three Bus Example, cont'd

$$\begin{bmatrix} |V|_2 \\ |V|_3 \end{bmatrix}^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.9364 \\ 0.9455 \end{bmatrix}$$

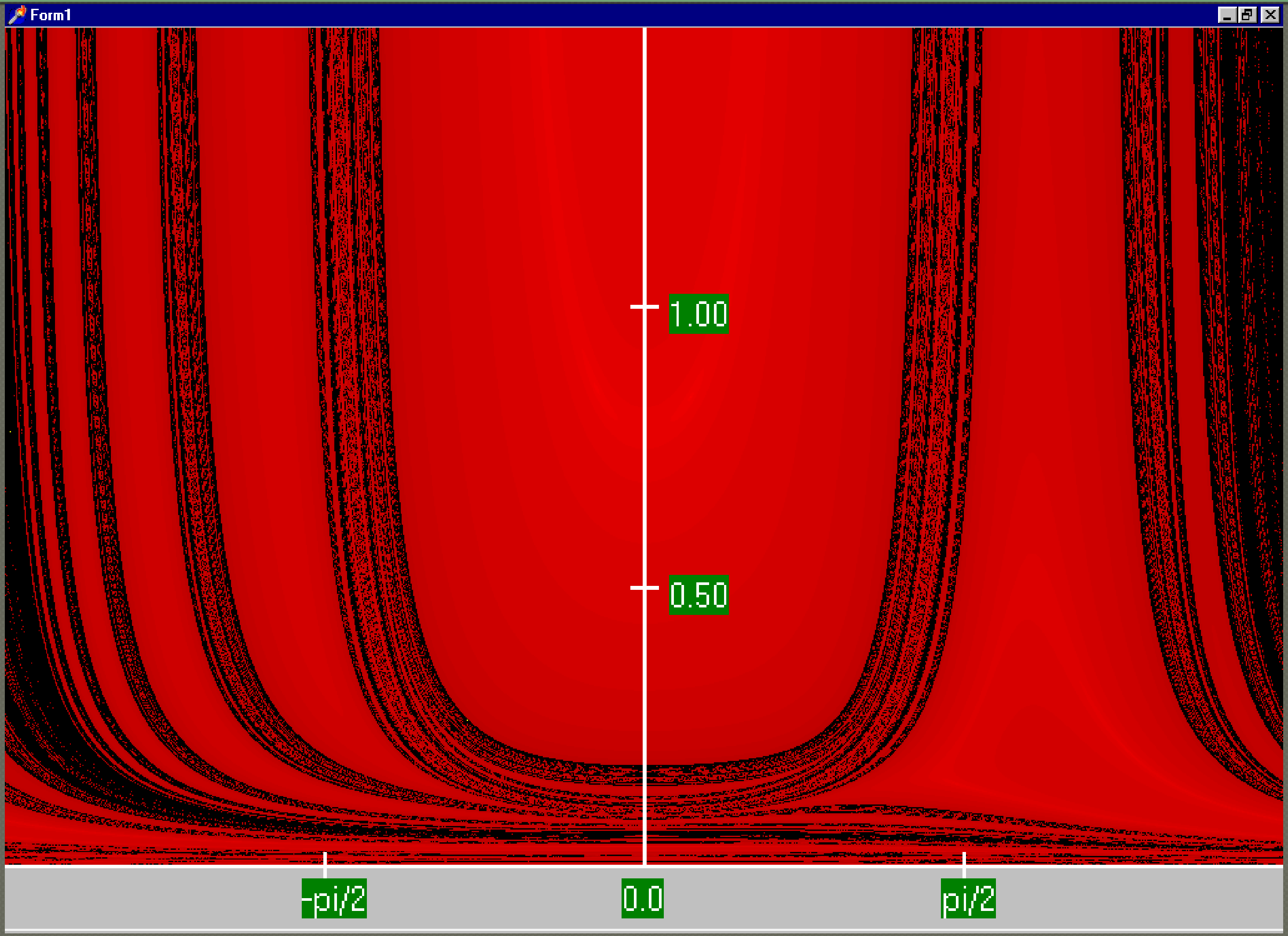
$$\frac{\Delta \mathrm{P_i}(\mathbf{x}\quad)}{|\mathrm{V_i}|} = \sum_{k=1}^{n} |V_k| (G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik}) + \frac{P_{Di} - P_{Gi}}{|\mathrm{V_i}|}$$

$$\begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix}^{(2)} = \begin{bmatrix} -0.1272 \\ -0.1091 \end{bmatrix} + \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix} \begin{bmatrix} 0.151 \\ 0.107 \end{bmatrix} = \begin{bmatrix} -0.1361 \\ -0.1156 \end{bmatrix}$$

$$\begin{bmatrix} |V|_2 \\ |V|_3 \end{bmatrix}^{(2)} = \begin{bmatrix} 0.924 \\ 0.936 \end{bmatrix}$$

Actual solution: $\boldsymbol{\theta} = \begin{bmatrix} -0.1384 \\ -0.1171 \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} 0.9224 \\ 0.9338 \end{bmatrix}$

FDPF Region of Convergence



“DC” Power Flow

- The “DC” power flow makes the most severe approximations:
  - completely ignore reactive power, assume all the voltages are always 1.0 per unit, ignore line conductance
- This makes the power flow a linear set of equations, which can be solved directly

$$\boldsymbol{\theta} = \mathbf{B}^{-1}\mathbf{P}$$

Power System Control

- A major problem with power system operation is the limited capacity of the transmission system
  - lines/transformers have limits (usually thermal)
  - no direct way of controlling flow down a transmission line (e.g., there are no valves to close to limit flow)
  - open transmission system access associated with industry restructuring is stressing the system in new ways
- We need to indirectly control transmission line flow by changing the generator outputs

DC Power Flow Example

EXAMPLE 6.17

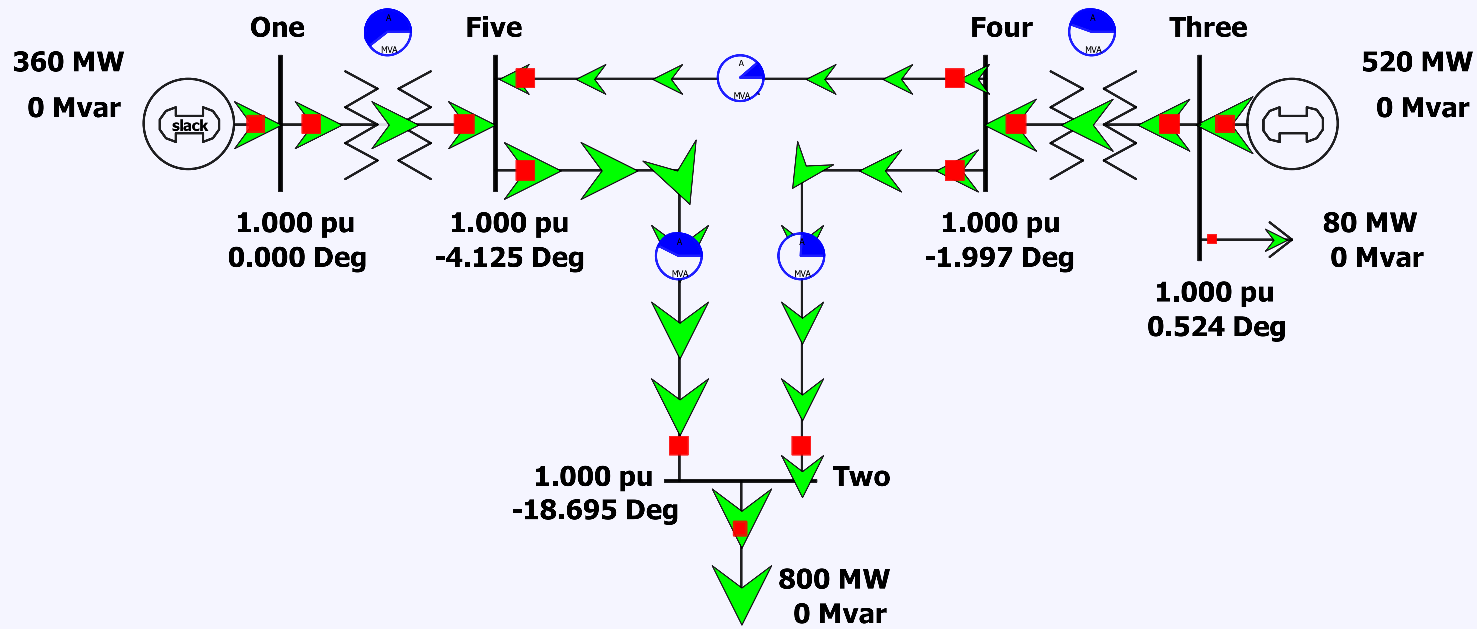
Determine the dc power flow solution for the five bus from Example 6.9.

SOLUTION With bus 1 as the system slack, the **B** matrix and **P** vector for this system are

$$\mathbf{B} = \begin{bmatrix} -30 & 0 & 10 & 20 \\ 0 & -100 & 100 & 0 \\ 10 & 100 & -150 & 40 \\ 20 & 0 & 40 & -110 \end{bmatrix} \quad \mathbf{P} = \begin{bmatrix} -8.0 \\ 4.4 \\ 0 \\ 0 \end{bmatrix}$$

$$\delta = -\mathbf{B}^{-1}\mathbf{P} = \begin{bmatrix} -0.3263 \\ 0.0091 \\ -0.0349 \\ -0.0720 \end{bmatrix} \text{radians} = \begin{bmatrix} -18.70 \\ 0.5214 \\ -2.000 \\ -4.125 \end{bmatrix} \text{degrees}$$

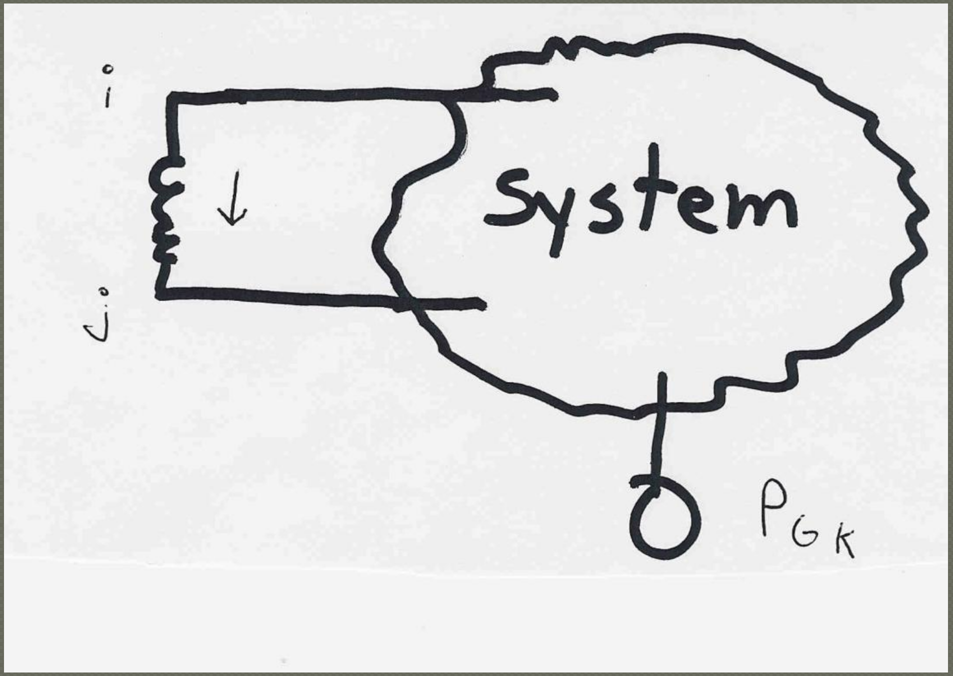
DC Power Flow 5 Bus Example



Notice with the dc power flow all of the voltage magnitudes are 1 per unit.

Indirect Transmission Line Control

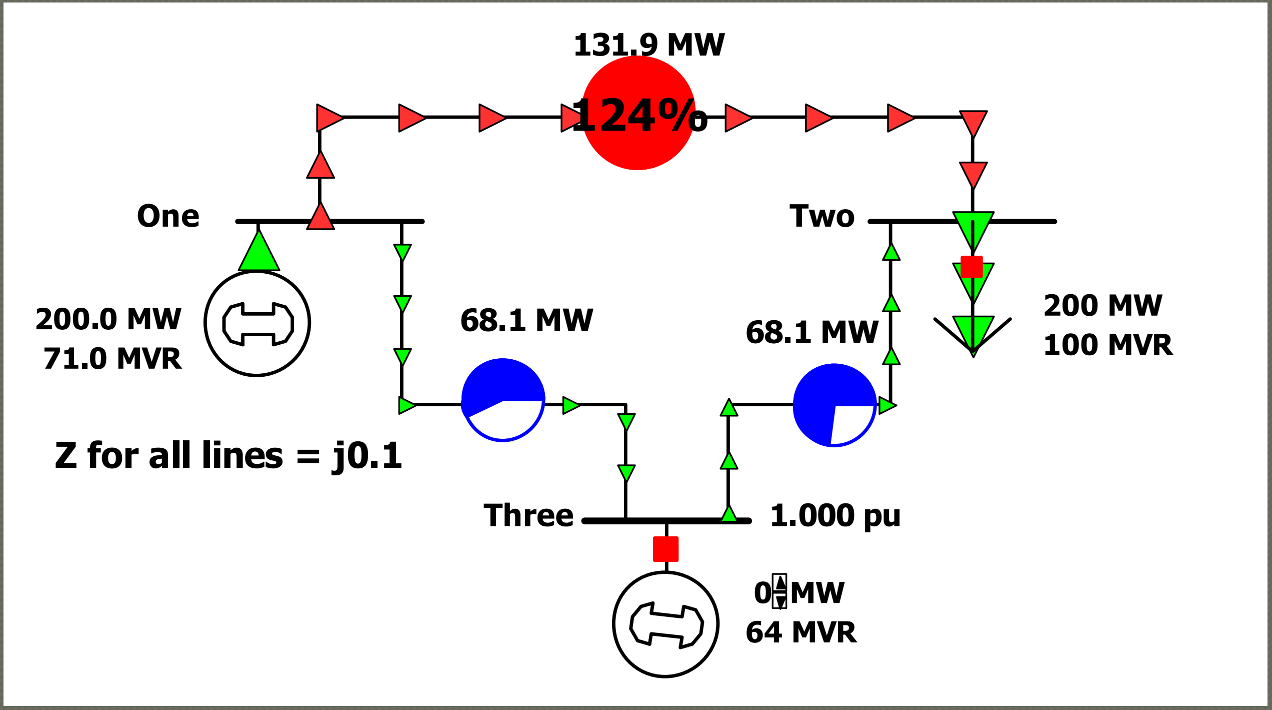
What we would like to determine is how a change generation at bus k affects the power flow on a line from bus i to bus j.



The assumption is that the change in generation is absorbed by the slack bus

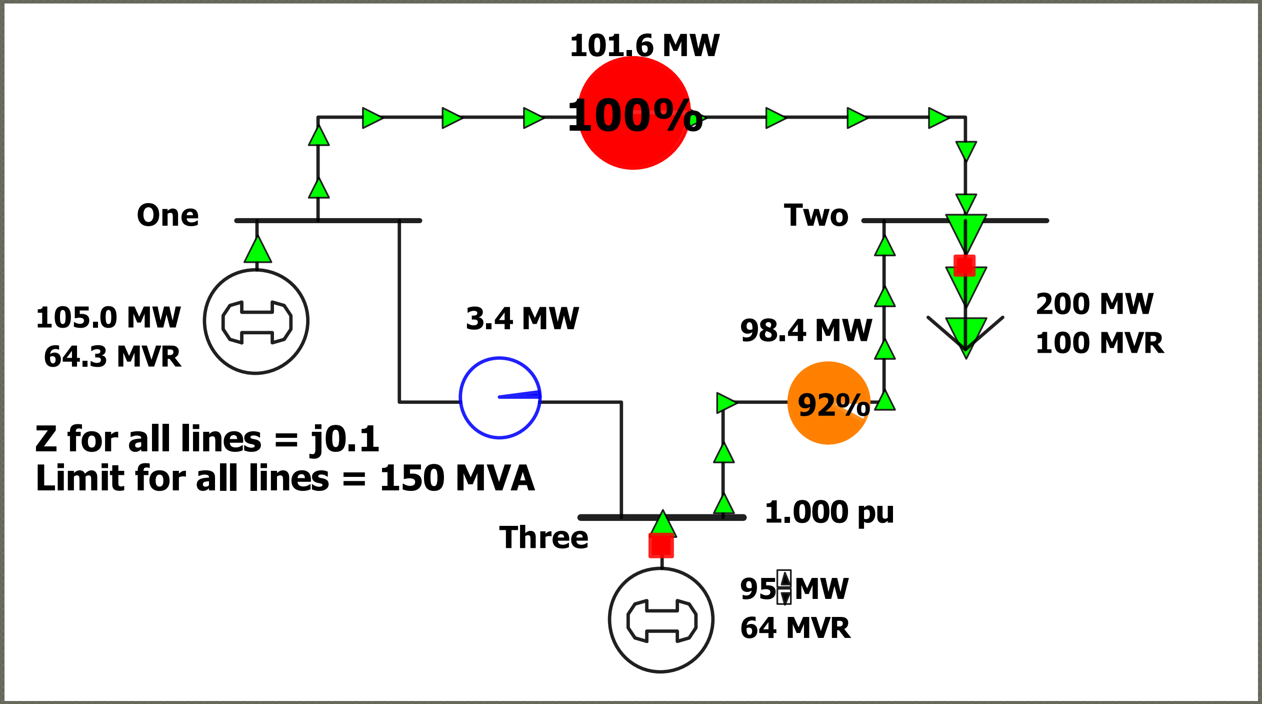
Power Flow Simulation - Before

One way to determine the impact of a generator change is to compare a before/after power flow.
For example below is a three bus case with an overload



Power Flow Simulation - After

Increasing the generation at bus 3 by 95 MW (and hence decreasing it at bus 1 by a corresponding amount), results in a 31.3 drop in the MW flow on the line from bus 1 to 2.



Analytic Calculation of Sensitivities

- Calculating control sensitivities by repeat power flow solutions is tedious and would require many power flow solutions. An alternative approach is to analytically calculate these values

The power flow from bus i to bus j is

$$P_{ij} \approx \frac{|V_i||V_j|}{X_{ij}} \sin(\theta_i - \theta_j) \approx \frac{\theta_i - \theta_j}{X_{ij}}$$

So $\Delta P_{ij} \approx \dfrac{\Delta\theta_i - \Delta\theta_j}{X_{ij}}$ We just need to get $\dfrac{\Delta\theta_{ij}}{\Delta P_{Gk}}$

Analytic Sensitivities

From the fast decoupled power flow we know

$$\Delta \boldsymbol{\theta} = \mathbf{B}^{-1} \Delta \mathbf{P}(\mathbf{x})$$

So to get the change in $\Delta \boldsymbol{\theta}$ due to a change of generation at bus k, just set $\Delta \mathbf{P}(\mathbf{x})$ equal to all zeros except a minus one at position k.

$$\Delta \mathbf{P} = \begin{bmatrix} 0 \\ \vdots \\ -1 \\ 0 \\ \vdots \end{bmatrix} \begin{matrix} \\ \\ \leftarrow \text{Bus k} \\ \\ \\ \end{matrix}$$

Per-Unit System

Per-Unit System

In the per-unit system, the voltages, currents, powers, impedances, and other electrical quantities are expressed on a per-unit basis by the equation:

$$\textbf{Quantity per unit} = \frac{\textbf{Actual value}}{\textbf{Base value of quantity}}$$

It is customary to select two base quantities to define a given per-unit system. The ones usually selected are voltage and power.

Per-Unit System

Assume:

$$V_b = V_{rated}$$

$$S_b = S_{rated}$$

Then compute base values for currents and impedances:

$$I_b = \frac{S_b}{V_b} \qquad Z_b = \frac{V_b}{I_b} = \frac{V_b^{\,2}}{S_b}$$

Per-Unit System

And the per-unit system is:

$$V_{p.u.} = \frac{V_{actual}}{V_b} \qquad I_{p.u.} = \frac{I_{actual}}{I_b}$$

$$S_{p.u.} = \frac{S_{actual}}{S_b} \qquad Z_{p.u.} = \frac{Z_{actual}}{Z_b}$$

$$Z\% = Z_{p.u.} \times 100 \ \%$$

Percent of base Z

Example 1

An electrical lamp is rated 120 volts, 500 watts. Compute the per-unit and percent impedance of the lamp. Give the p.u. equivalent circuit.

Solution:

(1) Compute lamp resistance

$$P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{(120)^2}{500} = 28.8\,\Omega$$

power factor = 1.0 $\quad Z = 28.8\angle 0\,\Omega$

Example 1

(2) Select base quantities

$$S_b = 500 \ VA$$

$$V_b = 120 \ V$$

(3) Compute base impedance

$$Z_b = \frac{V_b^2}{S_b} = \frac{(120)^2}{500} = 28.8\Omega$$

(4) The per-unit impedance is:

$$Z_{p.u.} = \frac{Z}{Z_b} = \frac{28.8\angle 0}{28.8} = 1\angle 0 \ p.u.$$

Example 1

(5) Percent impedance:

$$Z\% = 100\ \%$$

(6) Per-unit equivalent circuit:

Example 2

An electrical lamp is rated 120 volts, 500 watts. If the voltage applied across the lamp is twice the rated value, compute the current that flows through the lamp. Use the per-unit method.

Solution:

$$V_b = 120 \ V$$

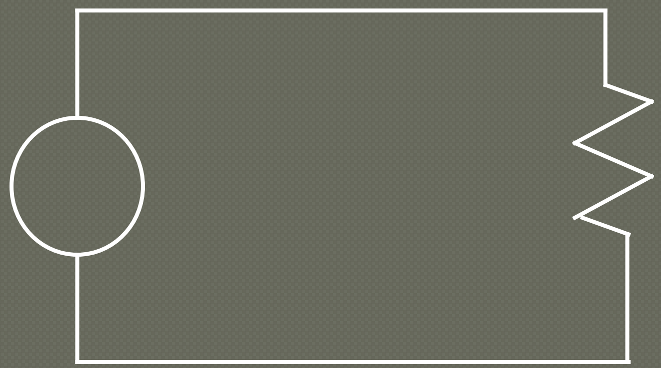
$$V_{p.u.} = \frac{V}{V_b} = \frac{240}{120} = 2\angle 0 \ p.u.$$

$$Z_{p.u.} = 1\angle 0 \ p.u.$$

Example 2

The per-unit equivalent circuit is as follows:

$V_S = 2\angle 0 \ p.u.$



$Z = 1\angle 0 \ p.u.$

$$I_{p.u.} = \frac{V_{p.u.}}{Z_{p.u.}} = \frac{2\angle 0}{1\angle 0} = 2\angle 0 \ p.u.$$

$$I_b = \frac{S_b}{V_b} = \frac{500}{120} = 4.167 \ A$$

$$I_{actual} = I_{p.u.} I_b = 2\angle 0 \times 4.167 = 8.334 \angle 0 \ A$$

Per-unit System for 1- ϕ Circuits

One-phase circuits

$$S_b = S_{1-\phi} = V_\phi I_\phi$$

where

$$V_\phi = V_{line-to-neutral}$$

$$I_\phi = I_{line-current}$$

$$V_{bLV} = V_{\phi LV} \qquad V_{bHV} = V_{\phi HV}$$

$$I_{bLV} = \frac{S_b}{V_{bLV}} \qquad I_{bHV} = \frac{S_b}{V_{bHV}}$$

Per-unit System for 1- ϕ Circuits

$$Z_{bLV} = \frac{V_{bLV}}{I_{bLV}} = \frac{(V_{bLV})^2}{S_b} \qquad Z_{bHV} = \frac{V_{bHV}}{I_{bHV}} = \frac{(V_{bHV})^2}{S_b}$$

$$S_{pu} = \frac{S}{S_b} = V_{pu} I_{pu}^*$$

$$P_{pu} = \frac{P}{S_b} = V_{pu} I_{pu} \cos\theta$$

$$Q_{pu} = \frac{Q}{S_b} = V_{pu} I_{pu} \sin\theta$$

Transformation Between Bases

Selection 1

$$S_{b1} = S_A \qquad V_{b1} = V_A$$

Then

$$Z_{b1} = \frac{V_{b1}^2}{S_{b1}} \qquad Z_{pu1} = \frac{Z_L}{Z_{b1}}$$

Selection 2

$$S_{b2} = S_B \qquad V_{b2} = V_B$$

Then

$$Z_{b2} = \frac{V_{b2}^2}{S_{b2}} \qquad Z_{pu2} = \frac{Z_L}{Z_{b2}}$$

Transformation Between Bases

$$\frac{Z_{pu\,2}}{Z_{pu\,1}} = \frac{Z_L}{Z_{b2}} \times \frac{Z_{b1}}{Z_L} = \frac{Z_{b1}}{Z_{b2}} = \frac{V_{b1}^2}{S_{b1}} \times \frac{S_{b2}}{V_{b2}^2}$$

$$Z_{pu\,2} = Z_{pu\,1}\left(\frac{V_{b1}}{V_{b2}}\right)^2 \times \left(\frac{S_{b2}}{S_{b1}}\right)$$

"1" – old
"2" - new

$$Z_{pu,new} = Z_{pu,old}\left(\frac{V_{b,old}}{V_{b,new}}\right)^2 \times \left(\frac{S_{b,new}}{S_{b,old}}\right)$$

Transformation Between Bases

Generally per-unit values given to another base can be converted to new base by by the equations:

$$(P,Q,S)_{pu\_on\_base\_2} = (P,Q,S)_{pu\_on\_base\_1} \frac{S_{base\ 1}}{S_{base\ 2}}$$

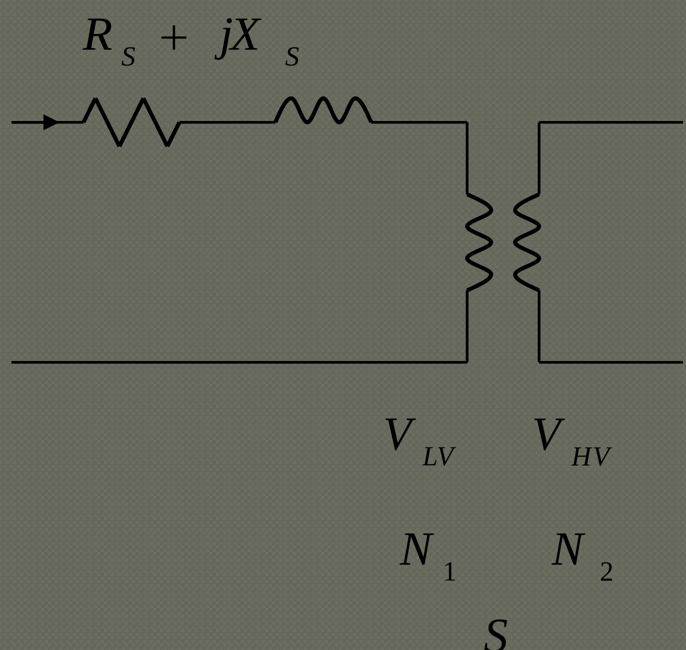
$$V_{pu\_on\_base\_2} = V_{pu\_on\_base\_1} \frac{V_{base\ 1}}{V_{base\ 2}}$$

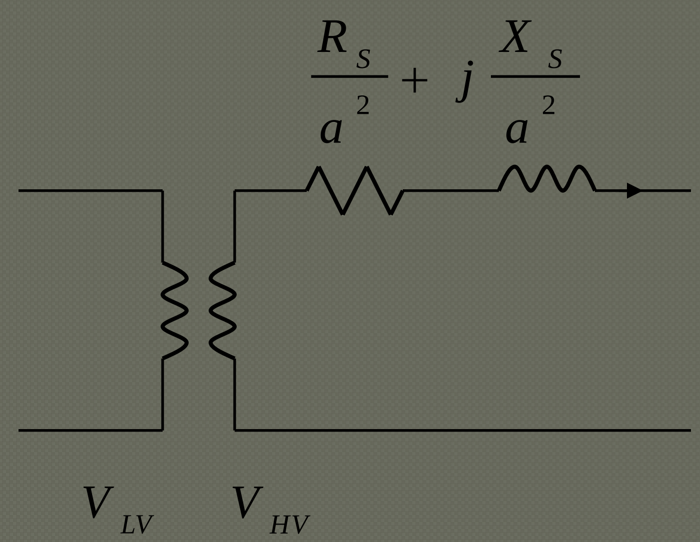
$$(R,X,Z)_{pu\_on\_base\_2} = (R,X,Z)_{pu\_on\_base\_1} \frac{(V_{base\ 1})^2 S_{base\ 2}}{(V_{base\ 2})^2 S_{base\ 1}}$$

When performing calculations in a power system, every per-unit value must be converted to the same base.

Per-unit System for 1-ϕ Transformer

Consider the equivalent circuit of transformer referred to LV side and HV side shown below:





Define $a = \dfrac{V_{LV}}{V_{HV}} = \dfrac{N_1}{N_2} < 1$

(1) Referred to LV side

(2) Referred to HV side

Per-unit System for 1-ϕ Transformer

Choose:

$$V_{b1} = V_{LV,rated}$$

$$S_b = S_{rated}$$

Normal choose rated values as base values

Compute:

$$V_{b2} = \frac{V_{HV}}{V_{LV}} V_{b1} = \frac{1}{a} V_{b1}$$

$$Z_{b1} = \frac{V_{b1}^2}{S_b} \qquad Z_{b2} = \frac{V_{b2}^2}{S_b}$$

$$\frac{Z_{b1}}{Z_{b2}} = \frac{V_{b1}^2}{V_{b2}^2} = \frac{V_{b1}^2}{(\frac{1}{a} V_{b1})^2} = a^2$$

Per-unit System for 1-ϕ Transformer

Per-unit impedances are:

$$Z_{p.u.1} = \frac{R_S + jX_S}{Z_{b1}}$$

$$Z_{p.u.2} = \frac{\frac{R_S}{a^2} + \frac{jX_S}{a^2}}{Z_{b2}} = \frac{\frac{R_S}{a^2} + \frac{jX_S}{a^2}}{\frac{Z_{b1}}{a^2}} = \frac{R_S + jX_S}{Z_{b1}}$$

So: $\boxed{Z_{p.u.1} = Z_{p.u.2}}$

Per-unit equivalent circuits of transformer referred to LV side and HV side are identical !!

Per-unit Eq. Ckt for 1-ϕ Transformer

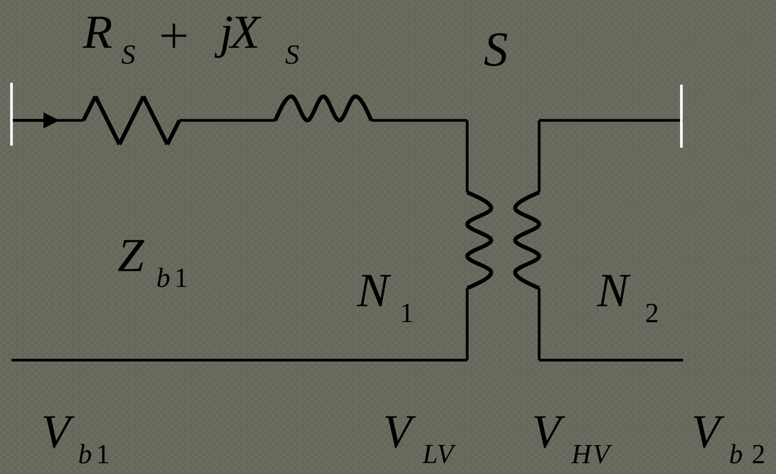


Fig 1. Eq Ckt referred to LV side

$$a = \frac{V_{LV}}{V_{HV}} = \frac{N_1}{N_2} < 1$$

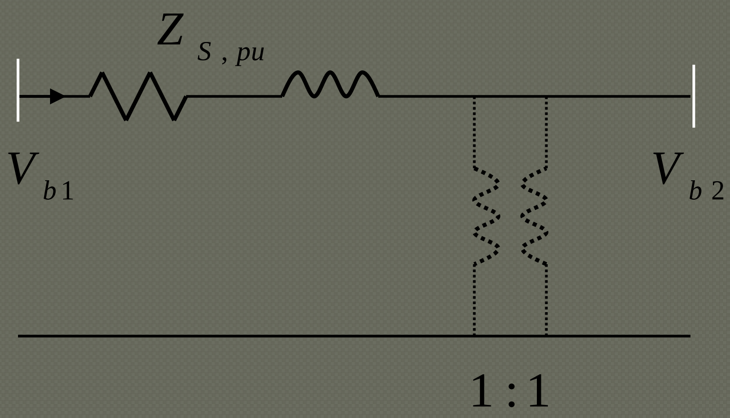


Fig 2. Per-unit Eq Ckt referred to LV side

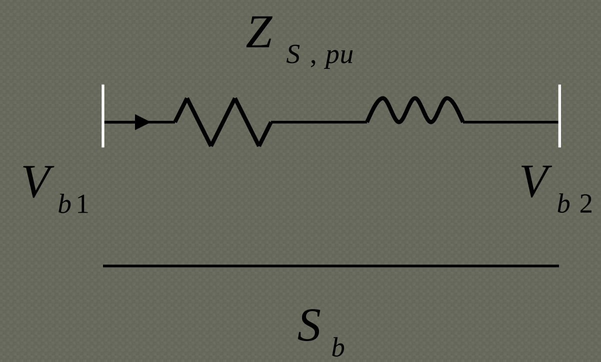


Fig 3.

Per-unit Eq. Ckt for 1-ϕ Transformer

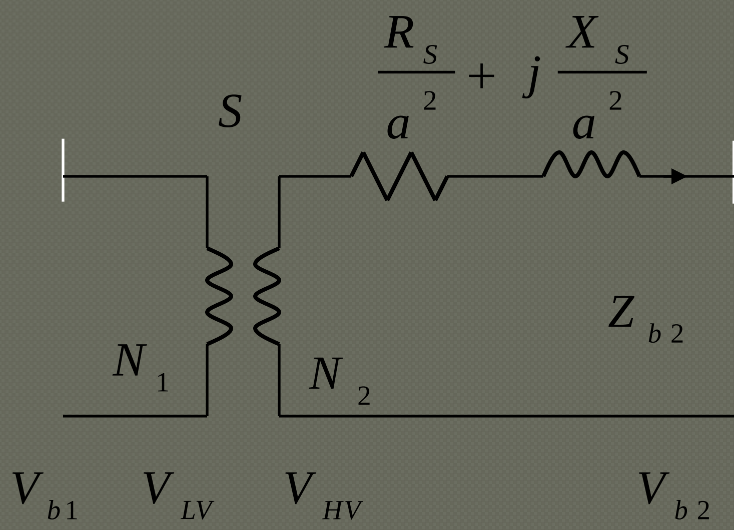


Fig 4. Eq Ckt referred to HV side

$$a = \frac{V_{LV}}{V_{HV}} = \frac{N_1}{N_2} < 1$$

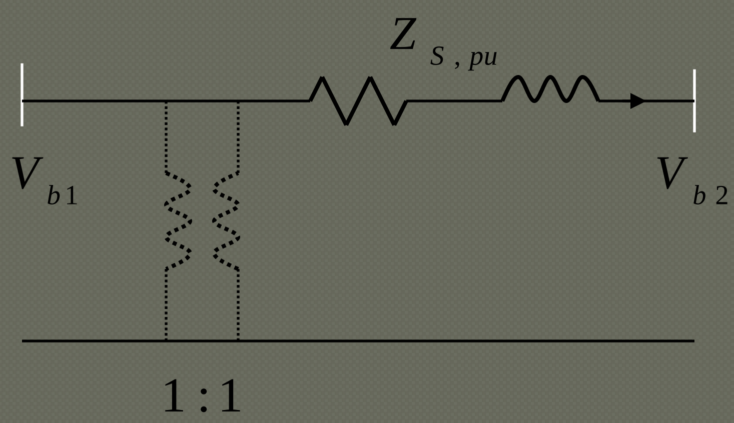


Fig 5. Per-unit Eq Ckt referred to HV side

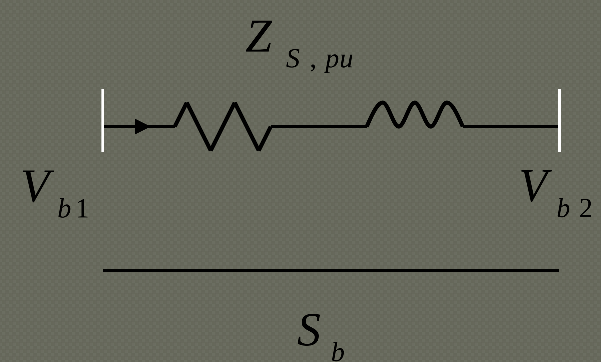


Fig 6.

Voltage Regulation

Voltage regulation is defined as:

$$VR = \frac{\left|V_{no-load}\right| - \left|V_{full-load}\right|}{\left|V_{full-load}\right|} \times 100\ \%$$

In per-unit system:

$$VR = \frac{\left|V_{pu,no-load}\right| - \left|V_{pu,full-load}\right|}{\left|V_{pu,full-load}\right|} \times 100\ \%$$

$V_{full\text{-}load}$: Desired load voltage at full load. It may be equal to, above, or below rated voltage

$V_{no\text{-}load}$: The no load voltage when the primary voltage is the desired voltage in order the secondary voltage be at its desired value at full load

Voltage Regulation Example

A single-phase transformer rated 200-kVA, 200/400-V, and 10% short circuit reactance. Compute the VR when the transformer is fully loaded at unity PF and rated voltage 400-V.

Solution:

$$V_{b2} = 400 \ V$$

$$S_b = 200 \ kVA$$

$$S_{load,pu} = 1\angle 0 \ pu$$

$$X_{S,pu} = j0.1 \, pu$$

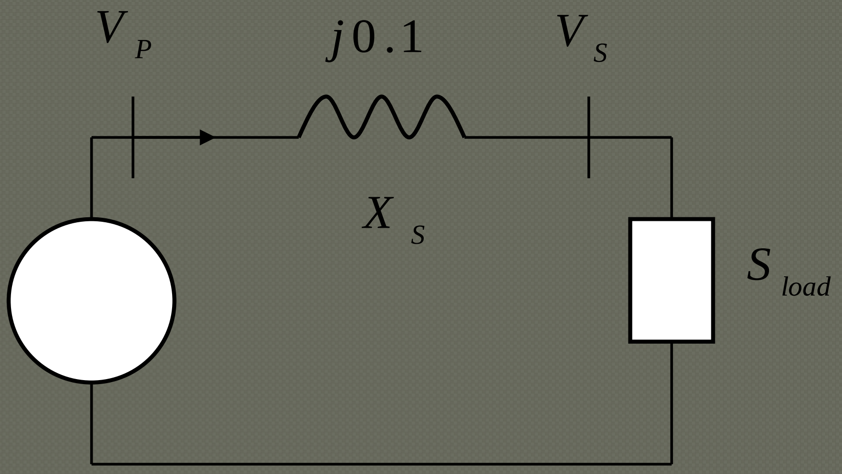


Fig 7. Per-unit equivalent circuit

Voltage Regulation Example

Rated voltage:

$$V_{S,pu} = 1.0\angle 0 \ pu$$

$$I_{load,pu} = \left(\frac{S_{load,pu}}{V_{S,pu}}\right)^* = \left(\frac{1.0\angle 0}{1.0\angle 0}\right)^* = 1.0\angle 0 \ pu$$

$$V_{P,pu} = V_{S,pu} + I_{pu} X_{S,pu}$$

$$= 1.0\angle 0 + 1.0\angle 0 \times j0.1 = 1 + j0.1$$

$$= \boxed{1.001\angle 5.7^o \ pu}$$

Voltage Regulation Example

Secondary side:

$$V_{pu,\ full-load} = V_{S,\ pu} = 1.0\angle 0\ pu$$

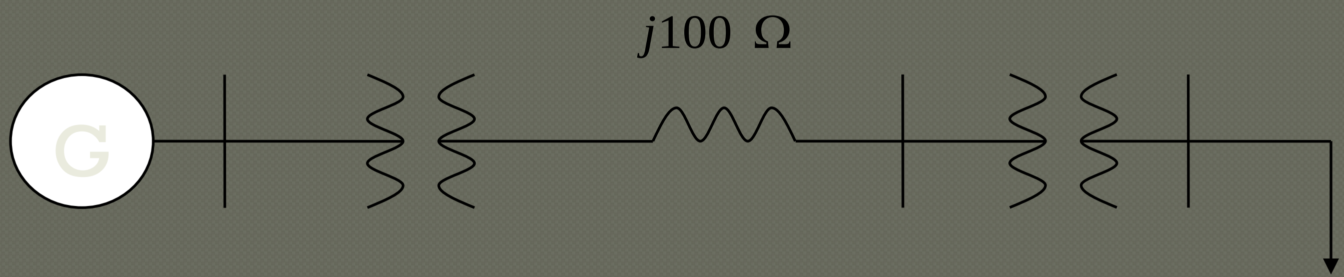
$$V_{pu,\ no-load} = V_{P,\ pu} = 1.001\angle 5.7^{o}\ pu$$

Voltage regulation:

$$VR = \frac{\left|V_{pu,\ no-load}\right| - \left|V_{pu,\ full-load}\right|}{\left|V_{pu,\ full-load}\right|} \times 100\ \%$$

$$= \frac{1.001 - 1.0}{1.0} \times 100\ \% = 0.1\%$$

Problem 1



20 kV	22kV/220kV	220kV/20kV	50MVA
	80MVA	50MVA	0.8 PF
	14%	10%	lagging

Select V_{base} in generator circuit and S_b=$100MVA$, compute p.u. equivalent circuit.

Per-unit System for 3-ϕ Circuits

Three-phase circuits

$$S_b = S_{3-\phi} = 3S_{1-\phi} = 3V_\phi I_\phi$$

where

$$V_\phi = V_{line-to-neutral} = V_{L(line)} / \sqrt{3}$$

$$I_\phi = I_{line-current} = I_L$$

$$S_b = \sqrt{3} V_L I_L$$

$$V_{bLV} = V_{L,LV} \qquad V_{bHV} = V_{L,HV}$$

$$S_b = \sqrt{3} V_{bLV} I_{bLV} = \sqrt{3} V_{bHV} I_{bHV}$$

Per-unit System for 3-ϕ Circuits

$$I_{bLV} = \frac{S_b}{\sqrt{3}V_{bLV}} \qquad I_{bHV} = \frac{S_b}{\sqrt{3}V_{bHV}}$$

$$Z_{bLV} = \frac{V_{\phi LV}}{I_{\phi LV}} = \frac{V_{bLV}}{\sqrt{3}} \frac{\sqrt{3}V_{bLV}}{S_b} = \frac{(V_{bLV})^2}{S_b}$$

$$Z_{bHV} = \frac{(V_{bHV})^2}{S_b}$$

$$S_{pu} = \frac{S_{3\phi}}{S_b} = \frac{\sqrt{3}V_L I_L^*}{\sqrt{3}V_b I_b} = V_{pu} I_{pu}^*$$

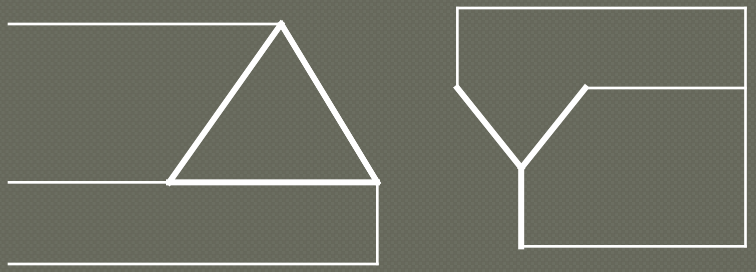
Per-unit System for 3-ϕ Transformer

Three 25-kVA, 34500/277-V transformers connected in Δ-Y. Short-circuit test on high voltage side:

$$V_{Line,SC} = 2010 \ V$$

$$I_{Line,SC} = 1.26 \ A$$

$$P_{3\phi,SC} = 912 \ W$$

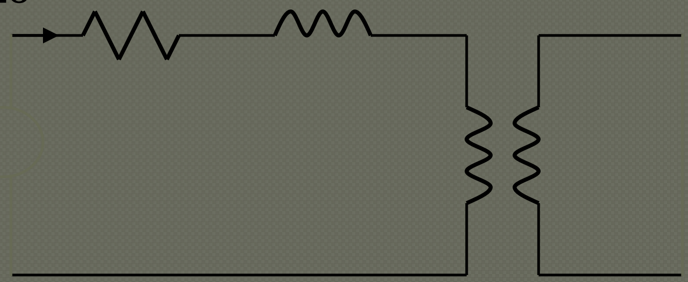


Determine the per-unit equivalent circuit of the transformer.

Per-unit System for 3-ϕ Transformer

(a) Using Y-equivalent

$$I_{SC} = 1.26 \qquad R_S + jX_S$$



$$V_{SC} = \frac{2010}{\sqrt{3}} \qquad S_b = 25000 \ VA$$

$$\frac{34500}{\sqrt{3}} \quad 277$$

$$\left|V_{SC}\right| = \frac{2010}{\sqrt{3}} = 1160.47 \ V$$

$$\left|Z_{SC}\right| = \left|\frac{1160.47}{1.26}\right| = 921.00 \ \Omega$$

Per-unit System for 3-ϕ Transformer

$$P_{\phi} = \frac{912}{3} = 304\ W \qquad R_S = \frac{P_{\phi}}{I_{SC}^2} = \frac{304}{1.26^2} = 191.48\ \Omega$$

$$X_S = \sqrt{\left|Z_{SC}\right|^2 - R_S^2} = \sqrt{921^2 - 191.48^2} = 900.86\ \Omega$$

So $Z_{SC} = 191.48 + j900.86\ \Omega$

$$S_b = 25000\ VA \qquad V_{b,HV} = \frac{34500}{\sqrt{3}} = 19918.58\ V$$

$$Z_{b,HV} = \frac{19918.58^2}{25000} = 15869.99\ \Omega$$

$$Z_{SC,pu,Y} = \frac{191.48 + j900.86}{15869.99} = 0.012 + j0.0568\ pu$$

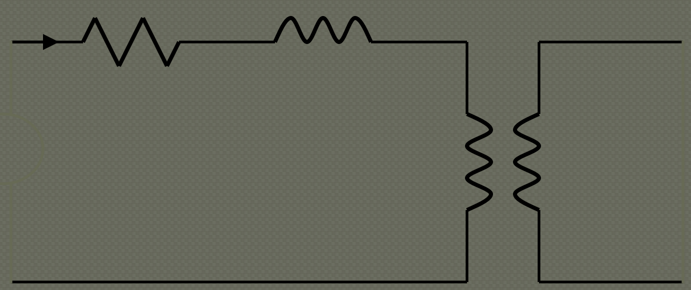
Per-unit System for 3-ϕ Transformer

(b) Using Δ-equivalent

$$I_{SC} = \frac{1.26}{\sqrt{3}}$$

$Z_{SC,\Delta}$

$V_{SC} = 2010$



$S_b = 25000 \quad VA$

34500 277

$$\left|V_{SC}\right| = 2010 \ V \qquad \left|I_{SC}\right| = \frac{1.26}{\sqrt{3}} = 0.727 \ A$$

$$\left|Z_{SC,\Delta}\right| = \left|\frac{2010}{0.727}\right| = 2764.79 \ \Omega$$

Per-unit System for 3-ϕ Transformer

$$P_{\phi} = \frac{912}{3} = 304\ W \qquad R_{S,\Delta} = \frac{P_{\phi}}{I_{SC}^{2}} = \frac{304}{0.727^{2}} = 575.18\,\Omega$$

$$X_{S,\Delta} = \sqrt{\left|Z_{SC,\Delta}\right|^{2} - R_{S,\Delta}^{2}} = \sqrt{2764.79^{2} - 575.18^{2}} = 2704.30\,\Omega$$

So $Z_{SC} = 191.48 + j900.86\ \Omega$

$$S_{b} = 25000\ VA \qquad V_{b,HV} = 34500\ V$$

$$Z_{b,HV} = \frac{34500^{2}}{25000} = 47610\ \Omega$$

$$Z_{SC,pu,\Delta} = \frac{575.18 + j1704.30}{47610} = 0.012 + j0.0568\ pu$$

UNIT-III
SHORT CIRCUIT ANALYSIS

Power System Equations

Start with Newton again ....

$T = I\,\alpha$

We want to describe the motion of the rotating masses of the generators in the system

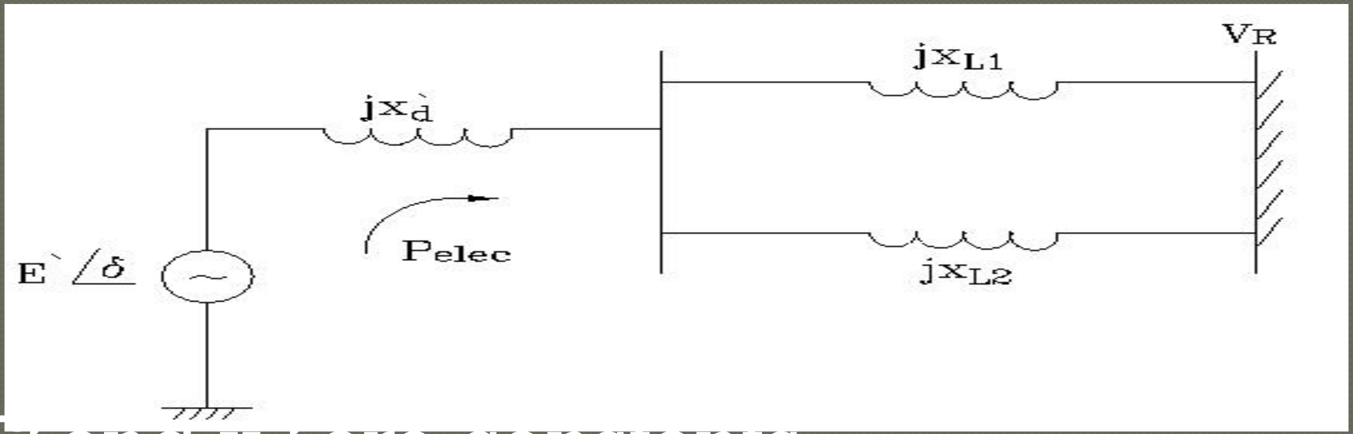
The swing equation

- $$\frac{2H}{\omega_o} \frac{d^2 \delta}{dt^2} = P_{acc}$$
- $P = T\,\omega$
- $\alpha = d^2\delta/dt^2$, acceleration is the second derivative of angular displacement w.r.t. time
- $\omega = d\delta/dt$, speed is the first derivative

- Accelerating Power, P_{acc}
- $P_{acc} = P_{mech} - P_{elec}$
- Steady State => No acceleration
- $P_{acc} = 0 \implies P_{mech} = P_{elec}$

Classical Generator Model

- Generator connected to Infinite bus through 2 lossless transmission lines

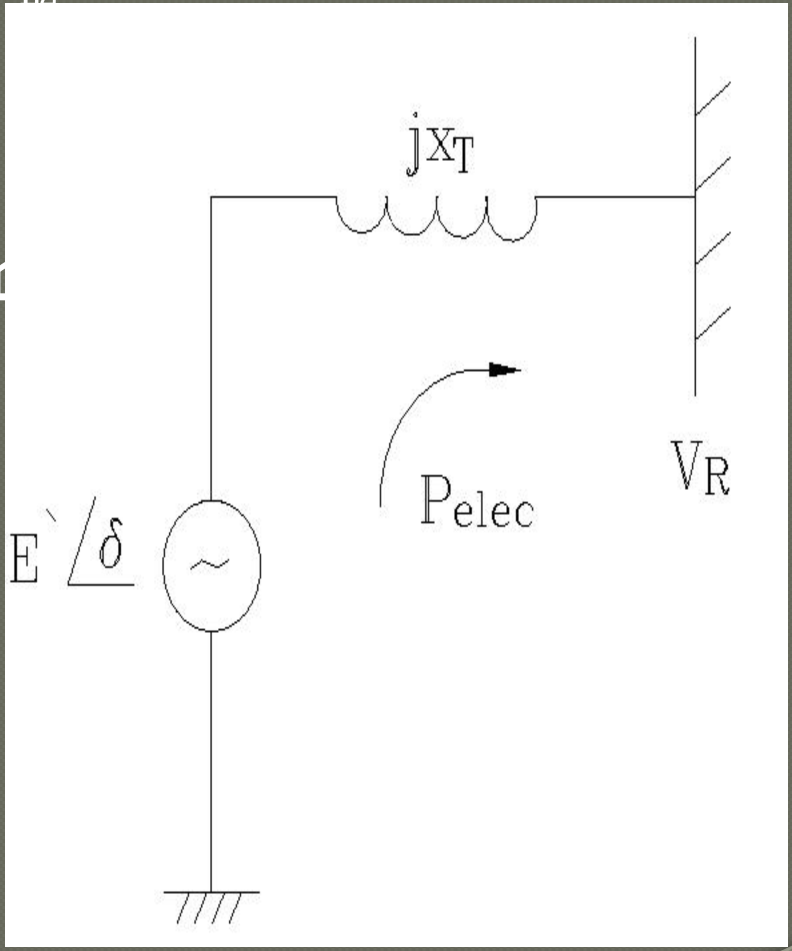


- E' and x_d' are constants
- δ is governed by the swing equation

Simplifying the system . . .

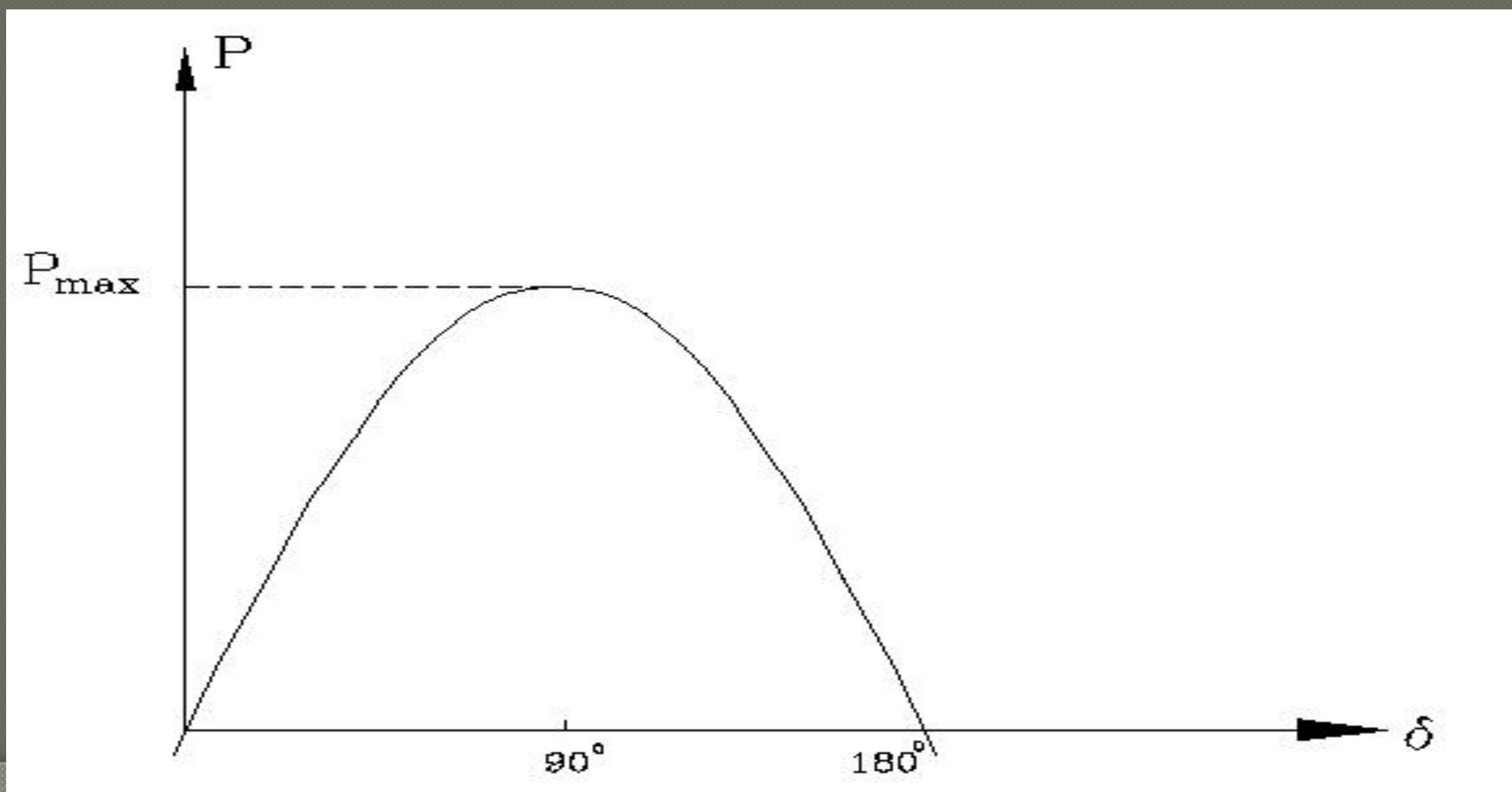
- Combine x_d' & X_{L1} & X_{L2}
- $jX_T = jx_d' + jX_{L1} \,||\, jX_{L2}$

- The simplified system



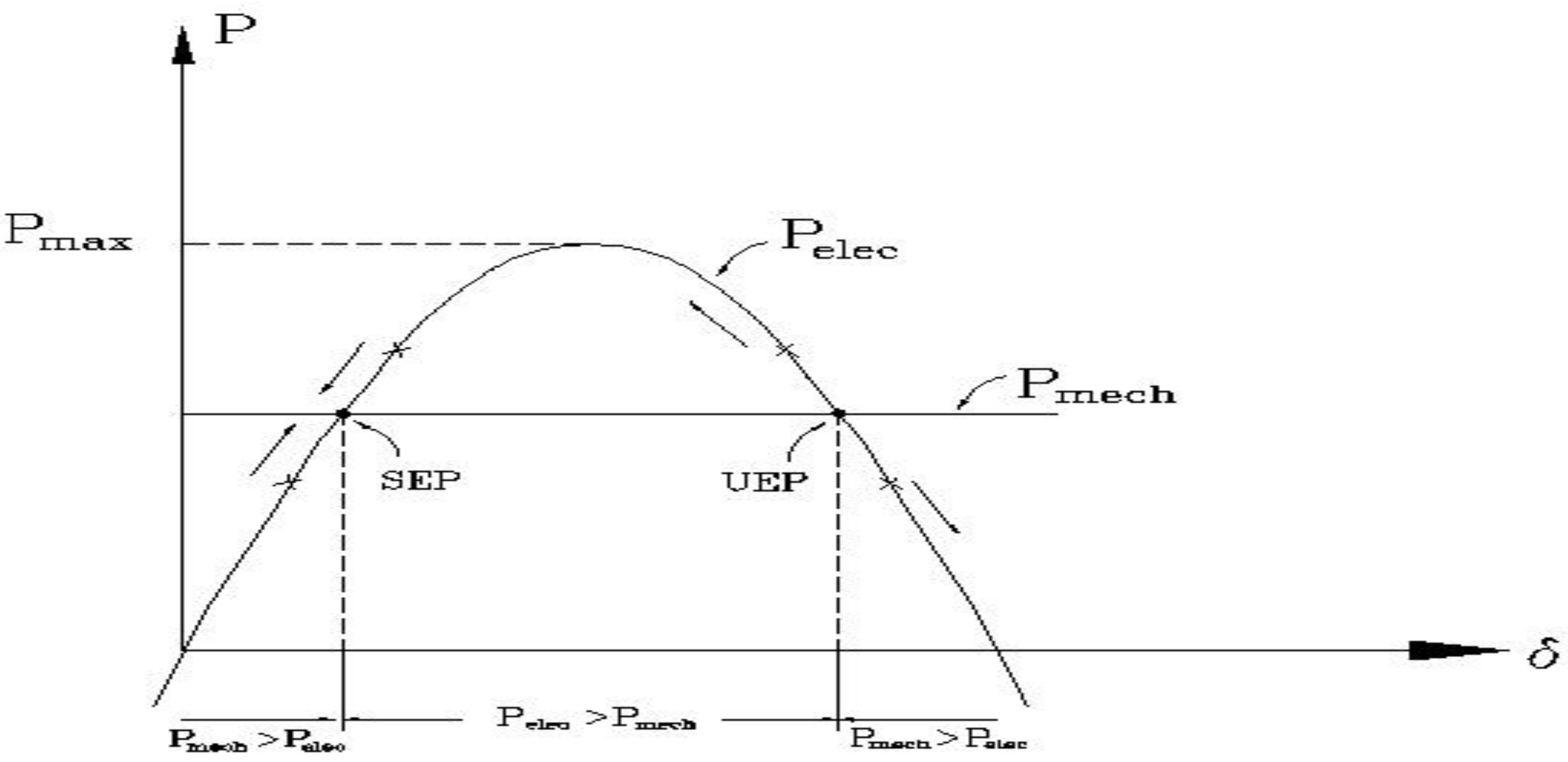
Recall the power-angle curve

- $P_{elec} = \frac{E' |V_R|}{X_T} \sin(\delta)$



Use power-angle curve

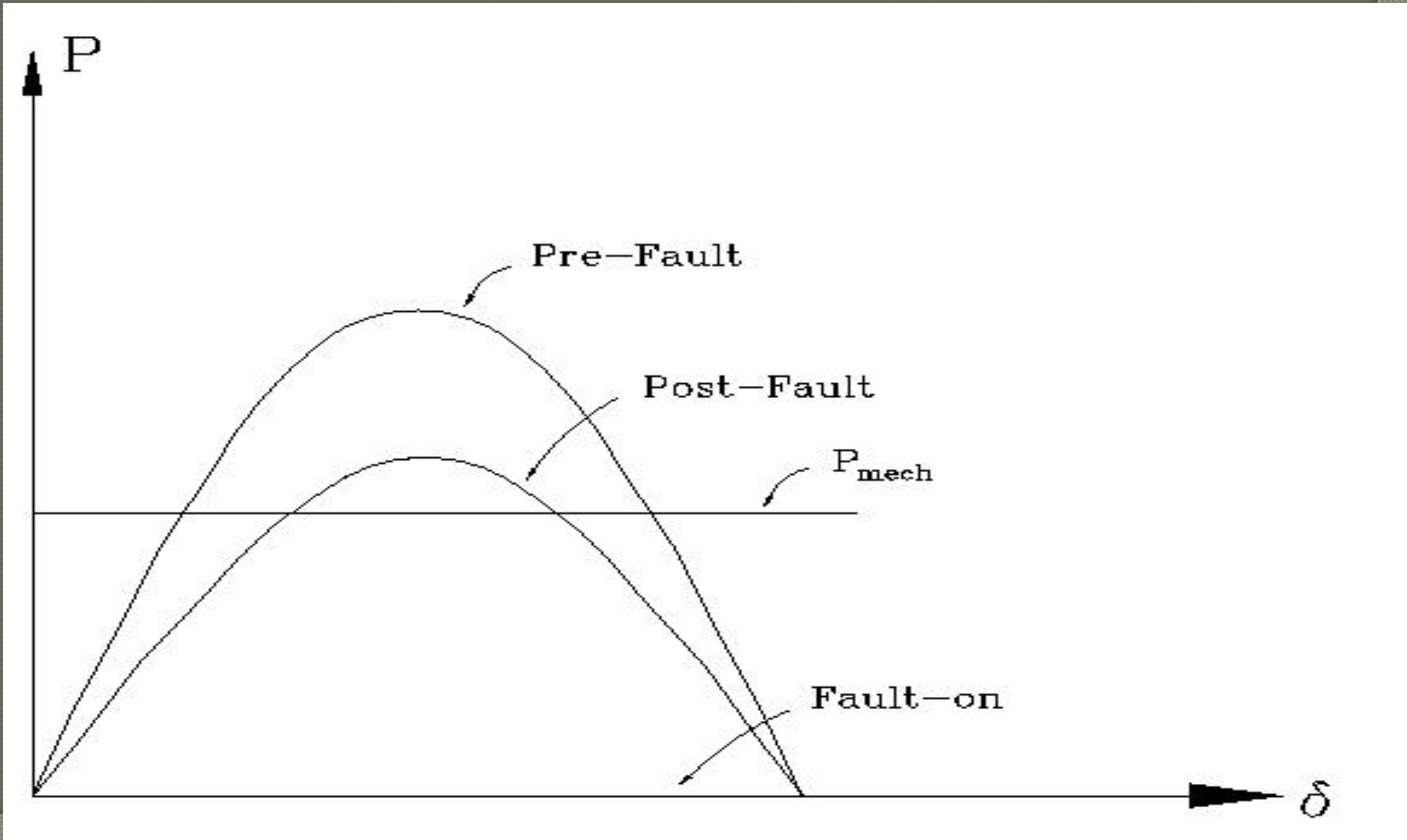
- Determine steady state (SEP)



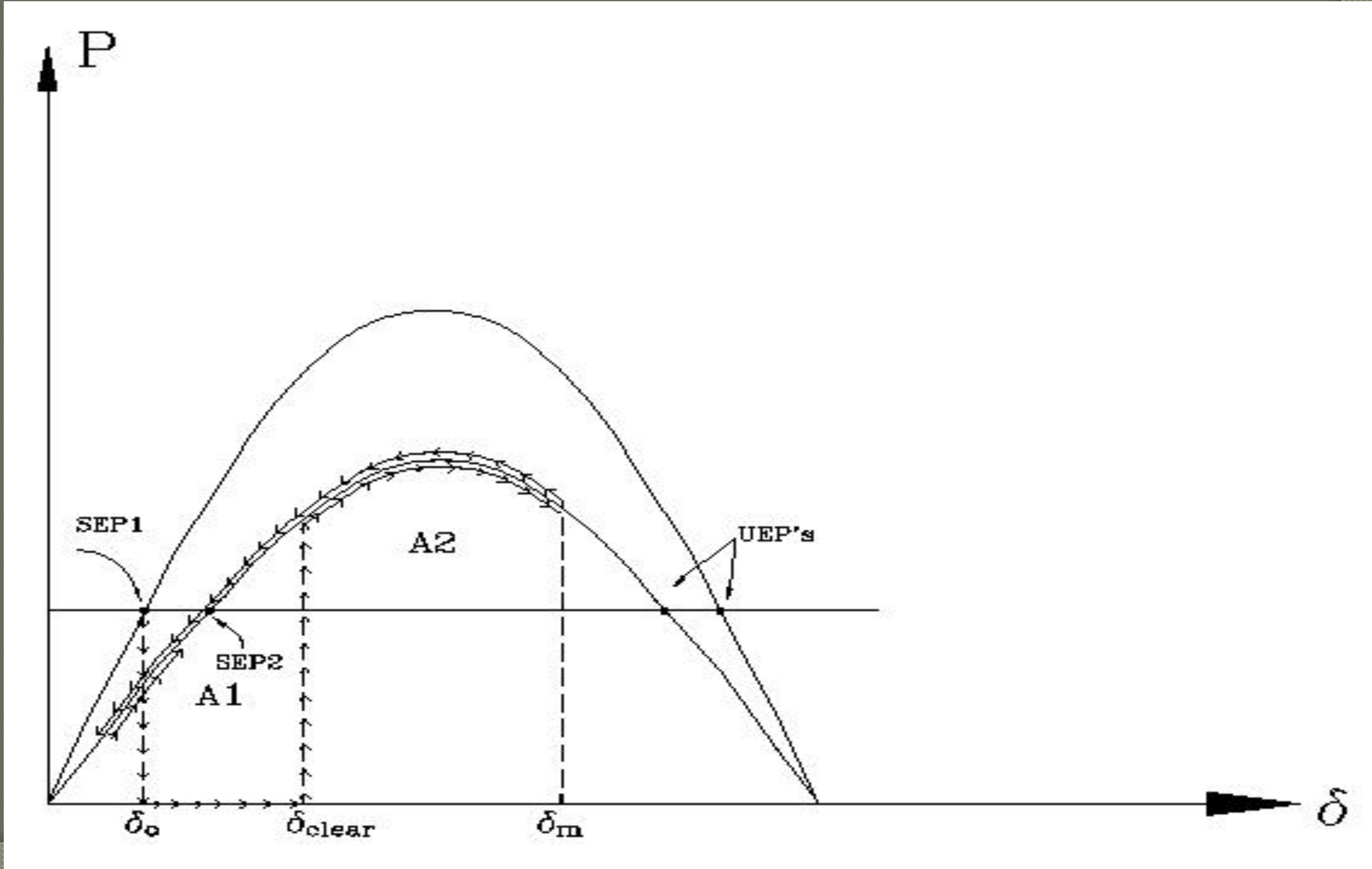
Fault study

- Pre-fault => system as given
- Fault => Short circuit at infinite bus
  - $P_{elec} = [E'(0)/\ jX_T]\sin(\delta) = 0$
- Post-Fault => Open one transmission line
  - $X_{T2} = x_d' + X_{L2} > X_T$

Power angle curves



Graphical illustration of the fault study



Equal Area Criterion

- $\frac{2H}{\omega_o} \frac{d^2 \delta}{dt^2} = P_{acc}$
- rearrange & multiply both sides by $2d\delta/dt$

- $2 \frac{d\delta}{dt} \frac{d^2\delta}{dt^2} = \frac{\omega_o}{H} P_{acc} \frac{d\delta}{dt}$

=>

$$\frac{d}{dt} \frac{\{d\delta\}^2}{\{dt\}} = \frac{\omega_o}{H} P_{acc} \frac{d\delta}{dt}$$

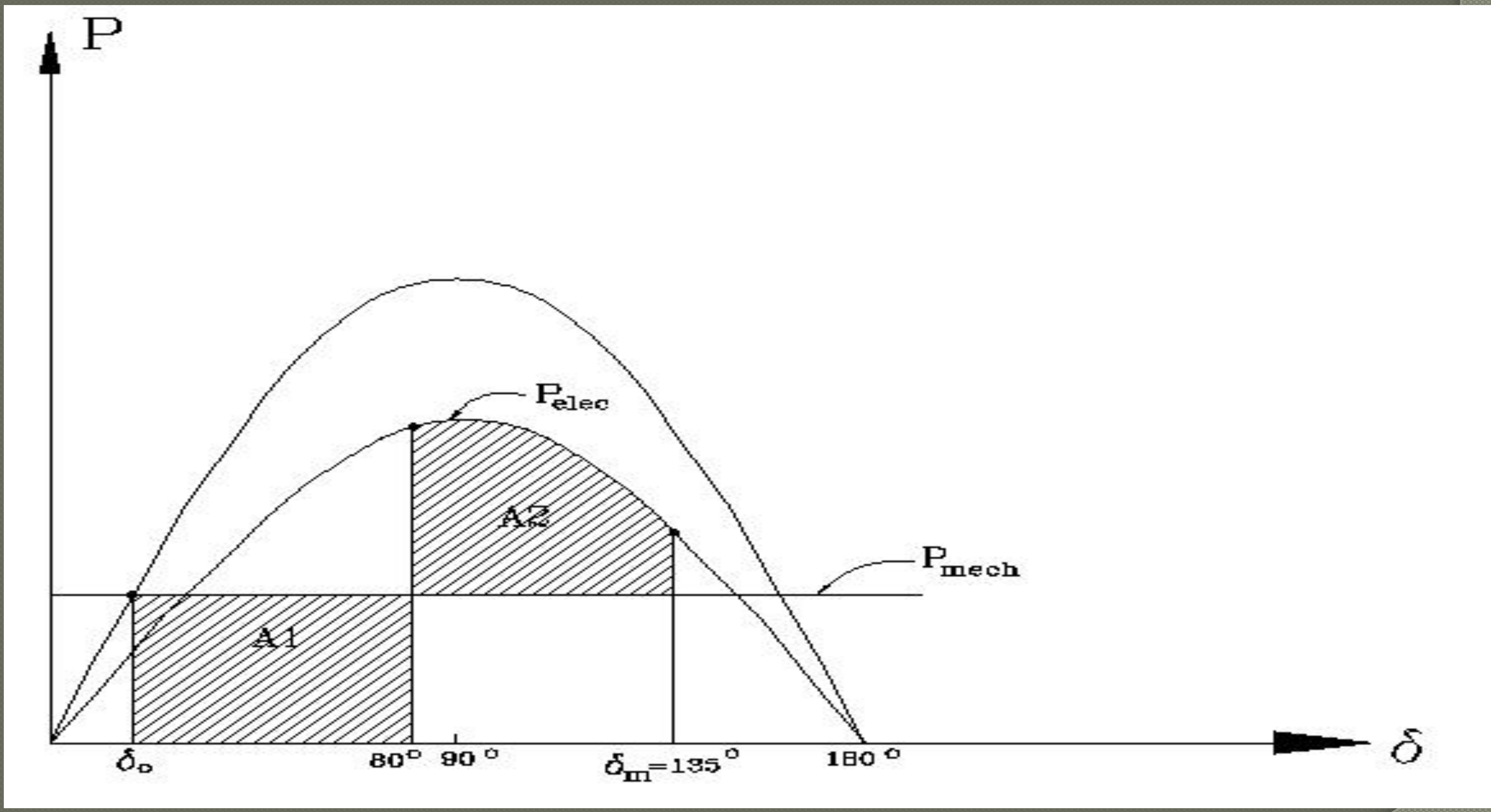
Integrating,

- $$\frac{\{d\delta\}^2}{\{dt\}} = \int \frac{\omega_o}{H} P_{acc} \frac{d\delta}{dt}$$
- For the system to be stable, δ must go through a maximum => dδ/dt must go through zero. Thus . . .
- $$\int_{\delta_o}^{\delta_m} \frac{\omega_o}{H} P_{acc} \; d\delta = 0 = \left\{\frac{d\delta}{dt}\right\}^2$$

The equal area criterion . . .

- For the total area to be zero, the positive part must equal the negative part. (A1 = A2)
- $\int_{\delta_0}^{\delta_{cl}} P_{acc}\, d\delta = A1$ <= "Positive" Area
- $\int_{\delta_{cl}}^{\delta_m} P_{acc}\, d\delta = A2$ <= "Negative" Area

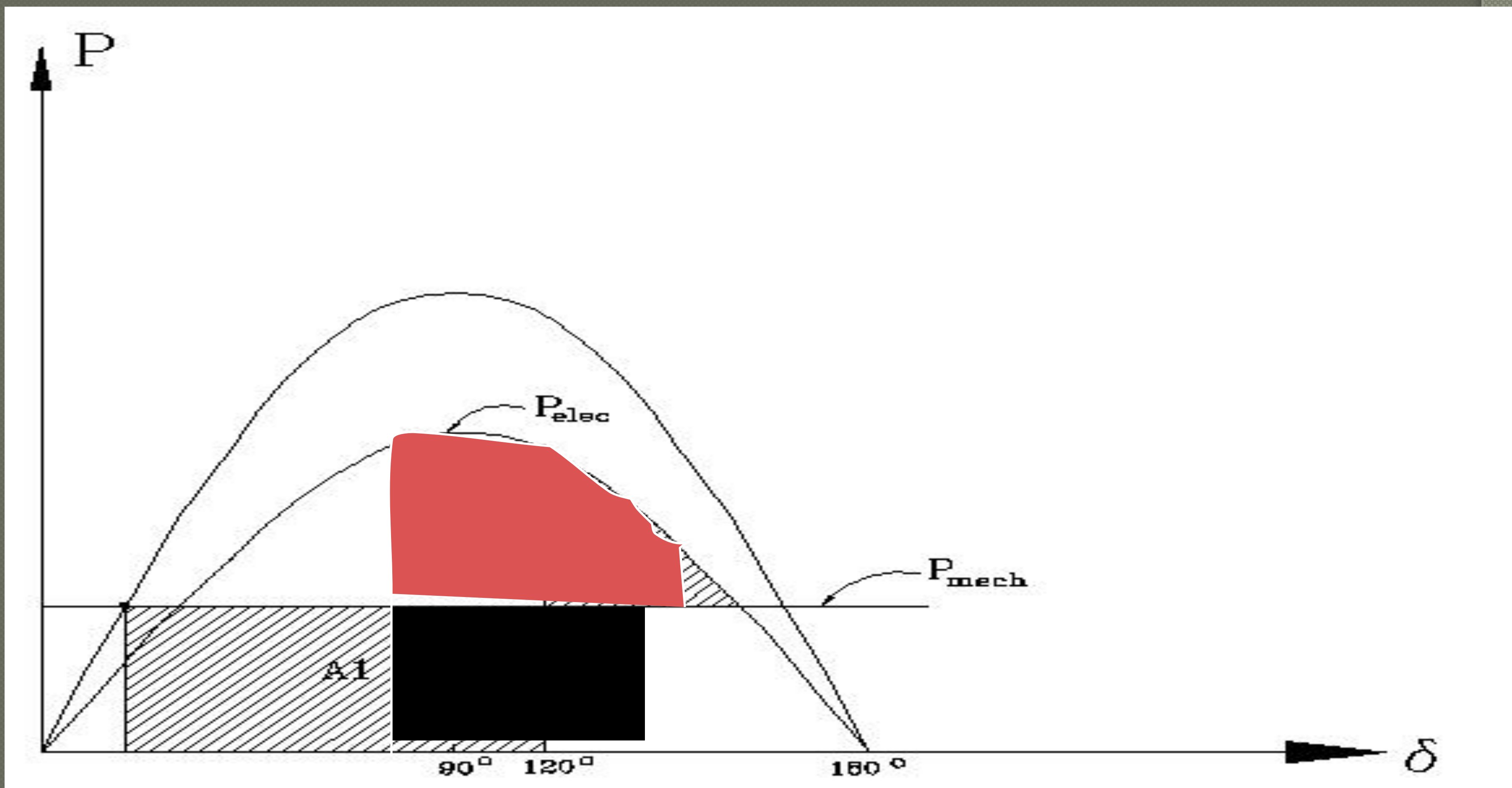
For the system to be stable for a given clearing angle δ, there must be sufficient area under the curve for A2 to "cover" A1.



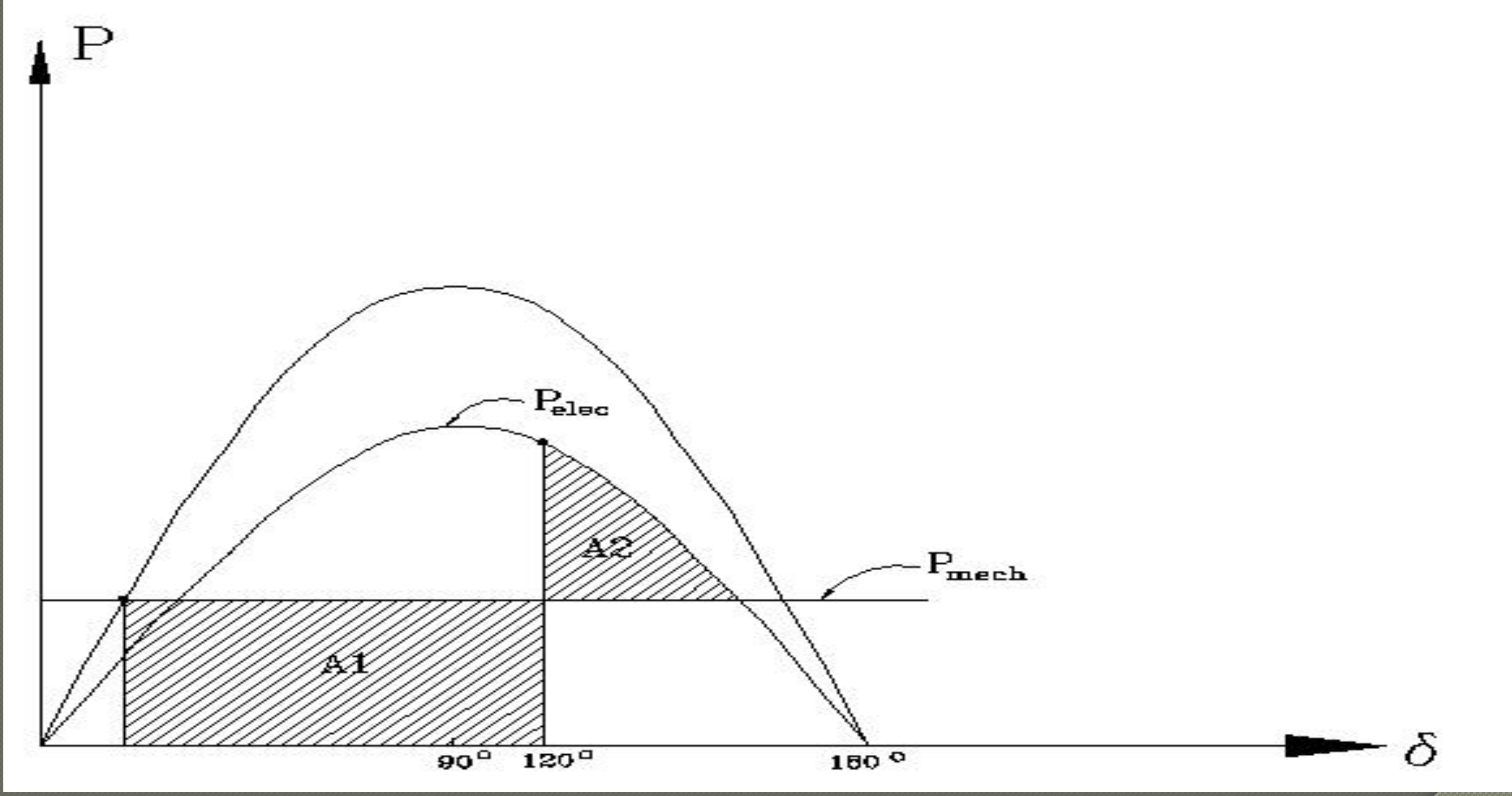
In-class Exercise . . .

- Draw a P-δ curve
- For a clearing angle of 80 degrees
  - is the system stable?
  - what is the maximum angle?
- For a clearing angle of 120 degrees
  - is the system stable?
  - what is the maximum angle?

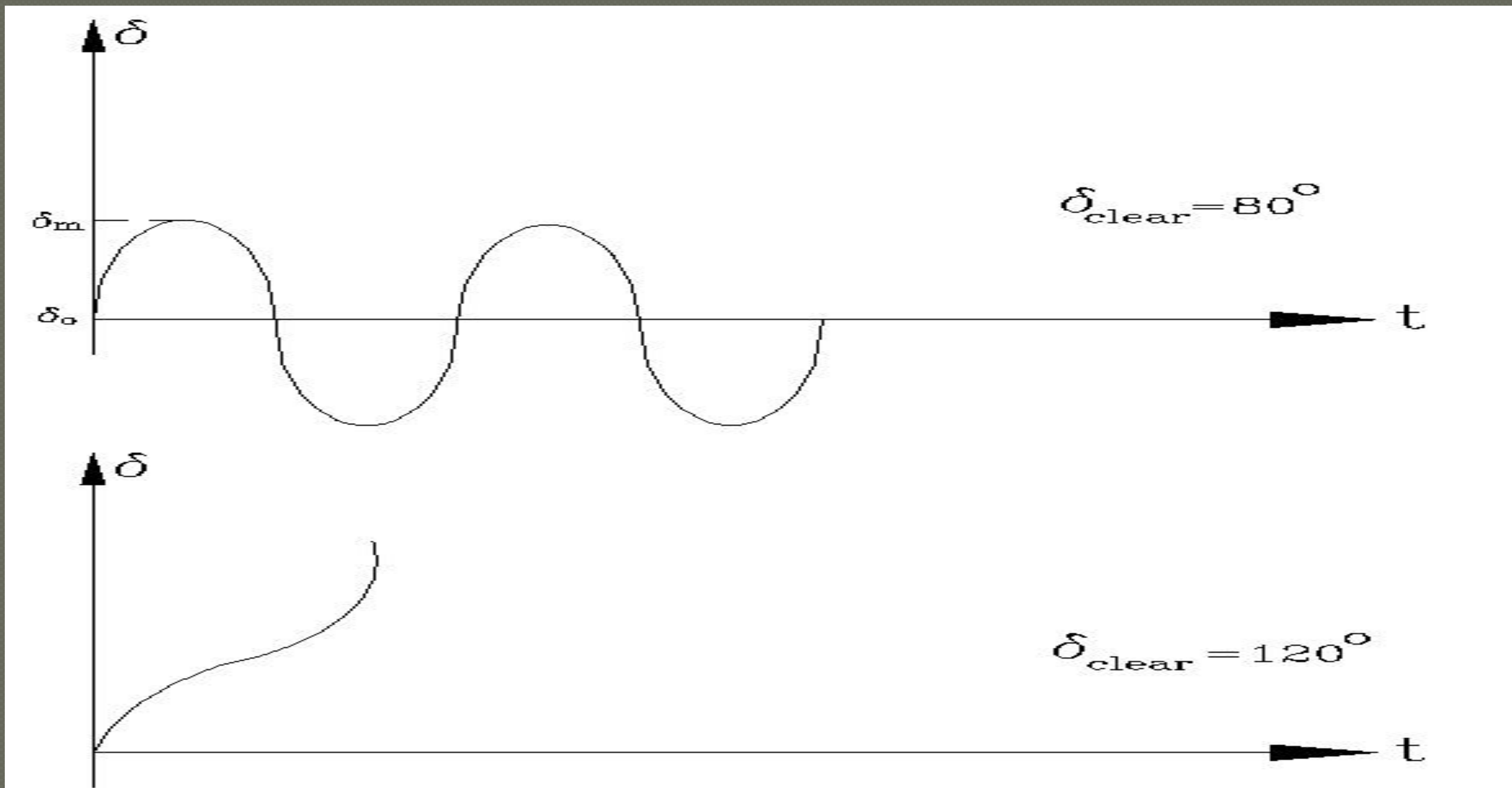
Clearing at 80 degrees



Clearing at 120 degrees



What would plots of δ vs. t look like for these 2 cases?



UNIT-V
POWER SYSTEM TRANSIENT STATE STABILITY

Introduction to Transient Stability

Steady-state = stable equilibrium

- things are not changing
- concerned with whether the system variables are within the correct limits

Transient Stability

- "Transient" means changing
- The state of the system is changing
- We are concerned with the transition from one equilibrium to another
- The change is a result of a "large" disturbance

Primary Questions

- 1. Does the system reach a new steady state that is acceptable?
- 2. Do the variables of the system remain within safe limits as the system moves from one state to the next?

Generally concerned with the synchronism of synchronous machines in the system

- Instability => at least one rotor angle becomes unbounded with respect to the rest of the system
- Also referred to as "going out of step" or "slipping a pole"

May also be concerned with other limits on other system variables

- Transient Voltage Dips
- Short-term current & power limits

Time Frame

- Typical time frame of concern
  - 1 - 30 seconds
- Model system components that are "active" in this time scale
- Faster changes -> assume instantaneous
- Slower changes -> assume constants

Primary components to be modeled

- Synchronous generators

Traditional control options

- **Generation based control**
  - **exciters, speed governors, voltage regulators, power system stabilizers**

Traditional Transmission Control Devices

- Slow changes
- modeled as a constant value

FACTS Devices

- May respond in the 1-30 second time frame
- modeled as active devices

Kundur's classification of methods for improving transient stability

- Minimization of disturbance severity and duration
- Increase in forces restoring synchronism
- Reduction of accelerating torque by reducing input mechanical power
- Reduction of accelerating torque by applying artificial load

improving transient stability

- High-speed fault clearing, reduction of transmission system impedance, shunt compensation, dynamic braking, reactor switching, independent and single-pole switching, fast-valving of steam systems, generator tripping, controlled separation, high-speed excitation systems, discontinuous excitation control, and control of HVDC links

FACTS devices = Exciting control opportunities!

- Deregulation & separation of transmission & generation functions of a utility
- FACTS devices can help to control transient problems from the transmission system

- THANK YOU