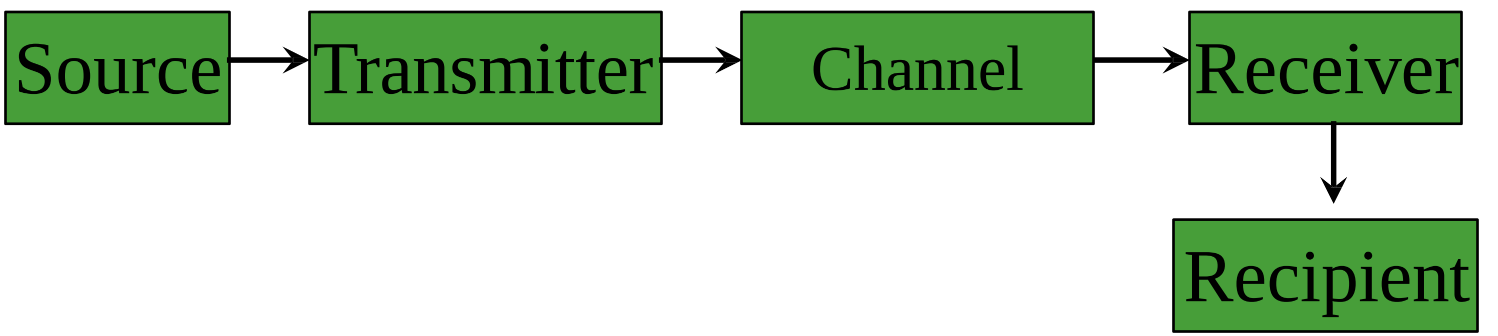
Digital Communications
ECE-III-II Sem
R15

Communication

- Main purpose of communication is to transfer information from a source to a recipient via a channel or medium.

- Basic block diagram of a communication system:



Brief Description

- **Source:** analog or digital
- **Transmitter:** transducer, amplifier, modulator, oscillator, power amp., antenna
- **Channel:** e.g. cable, optical fibre, free space
- **Receiver:** antenna, amplifier, demodulator, oscillator, power amplifier, transducer
- **Recipient:** e.g. person, (loud) speaker, computer

- **Types of information**
  Voice, data, video, music, email etc.

- **Types of communication systems**
  Public Switched Telephone Network (voice,fax,modem)
  Satellite systems
  Radio,TV broadcasting
  Cellular phones
  Computer networks (LANs, WANs, WLANs)

Information Representation

- Communication system converts information into electrical electromagnetic/optical signals appropriate for the transmission medium.
- Analog systems convert analog message into signals that can propagate through the channel.
- Digital systems convert bits(digits, symbols) into signals
  - Computers naturally generate information as characters/bits
  - Most information can be converted into bits
  - Analog signals converted to bits by sampling and quantizing (A/D conversion)

Why digital?

- Digital techniques need to distinguish between discrete symbols allowing regeneration versus amplification

- Good processing techniques are available for digital signals, such as medium.
  - Data compression (or source coding)
  - Error Correction (or channel coding)(A/D conversion)
  - Equalization
  - Security

- Easy to mix signals and data using digital techniques

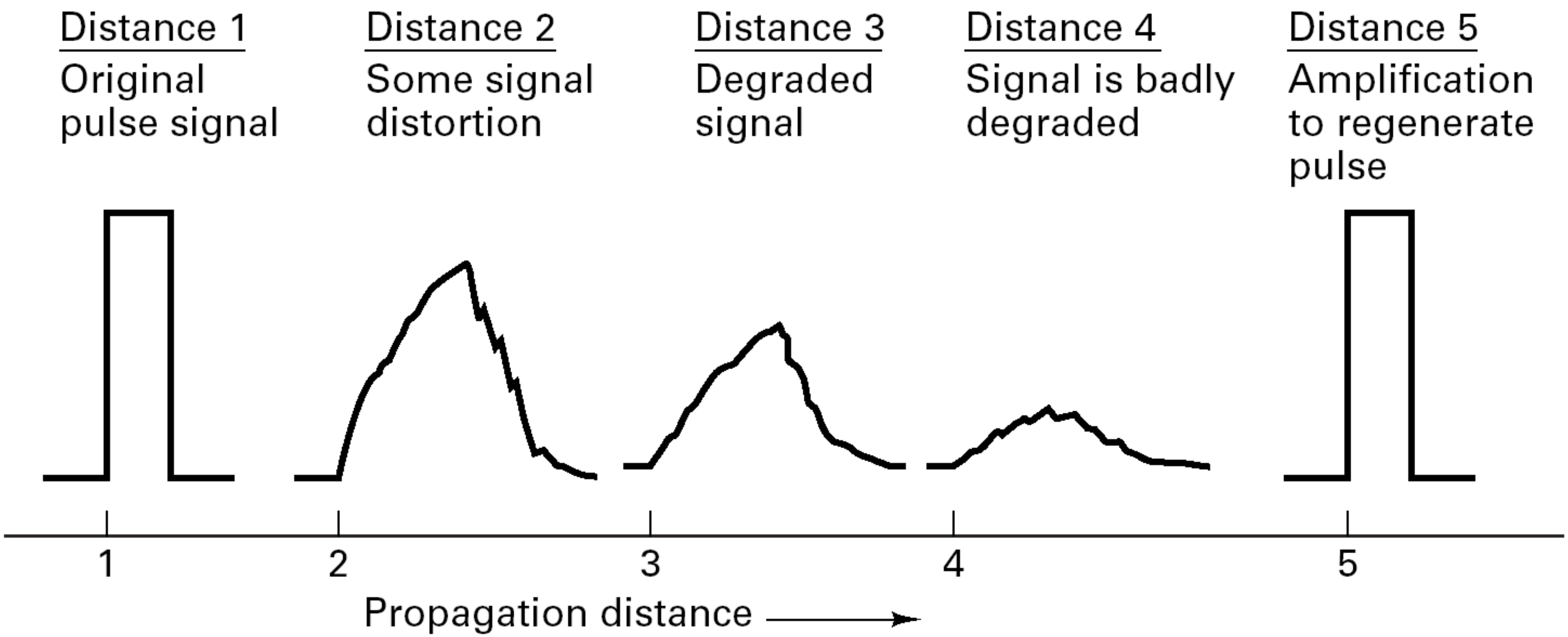


Figure 1.1 Pulse degradation and regeneration.

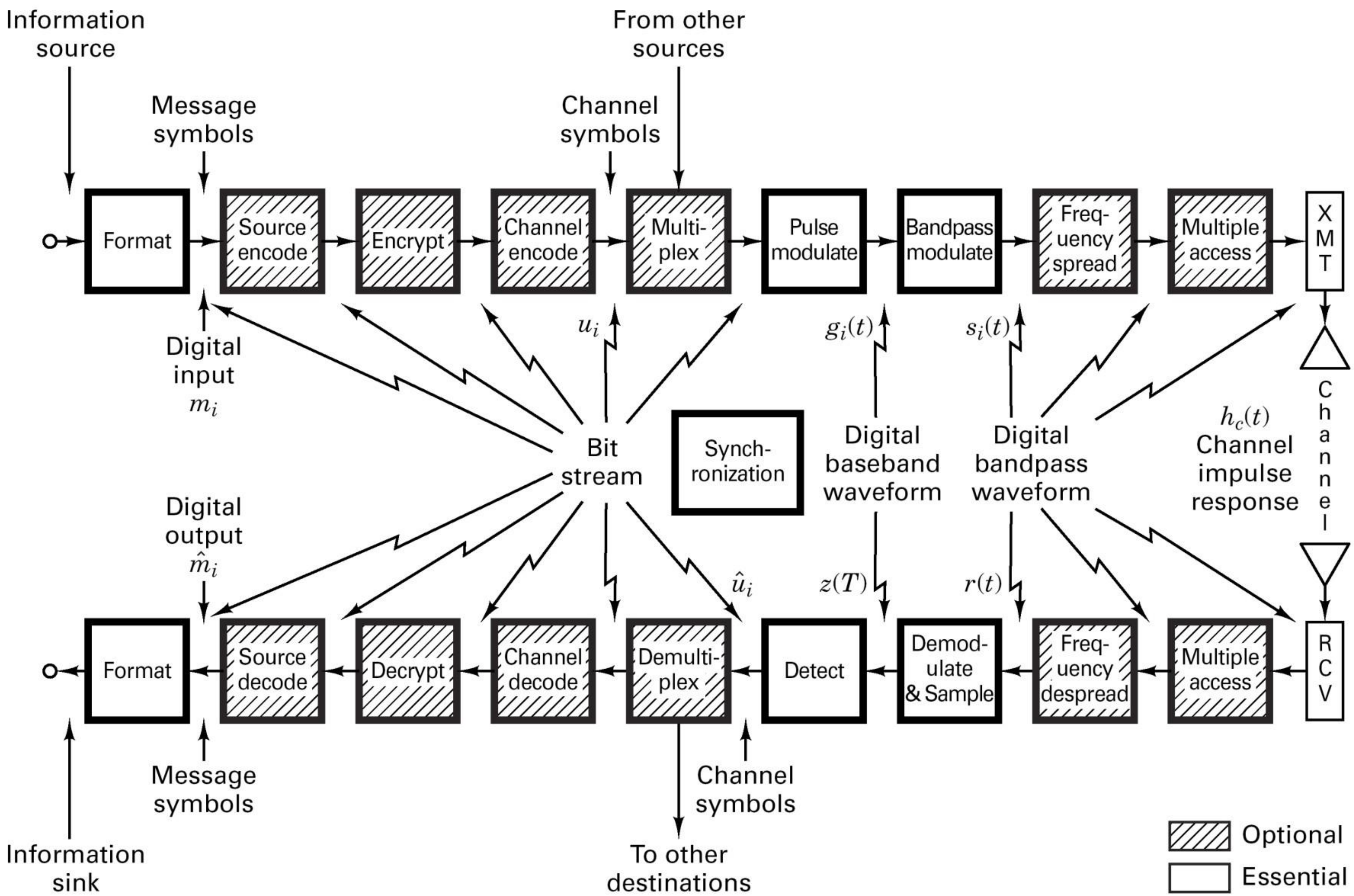


Figure 1.2 Block diagram of a typical digital communication system.

- Basic Digital Communication Transformations
  - Formatting/Source Coding
  - Transforms source info into digital symbols (digitization)
  - Selects compatible waveforms (matching function)
  - Introduces redundancy which facilitates accurate decoding despite errors
- *It is essential for reliable communication*
  - Modulation/Demodulation
  - Modulation is the process of modifying the info signal to facilitate transmission
  - Demodulation reverses the process of modulation. It involves the detection and retrieval of the info signal
    - Types
    - Coherent: Requires a reference info for detection
    - Noncoherent: Does not require reference phase information

Basic Digital Communication Transformations

- Coding/Decoding

  Translating info bits to transmitter data symbols

  Techniques used to enhance info signal so that they are less vulnerable to channel impairment (e.g. noise, fading, jamming, interference)

  - Two Categories
    - *Waveform Coding*
  - Produces new waveforms with better performance
    - *Structured Sequences*

- Involves the use of redundant bits to determine the occurrence of error (and sometimes correct it)
  - Multiplexing/Multiple Access Is synonymous with resource sharing with other users
  - Frequency Division Multiplexing/Multiple Access (FDM/FDMA

Formatting

- Character coding
- Sampling
- Quantization
- Pulse code modulation (PCM)

Source Coding

- Predictive coding
- Block coding
- Variable length coding
- Synthesis/analysis coding
- Lossless compression
- Lossy compression

Baseband Signaling

- PCM waveforms (line codes)
  - Nonreturn-to-zero (NRZ)
  - Return-to-zero (RZ)
  - Phase encoded
  - Multilevel binary
- *M*-ary pulse modulation
  - PAM, PPM, PDM

Equalization

- Maximum-likelihood sequence estimation (MLSE)
- Equalization with filters
  - Transversal or decision feedback
  - Preset or Adaptive
  - Symbol spaced or fractionally spaced

Bandpass Signaling

Coherent

- Phase shift keying (PSK)
- Frequency shift keying (FSK)
- Amplitude shift keying (ASK)
- Continuous phase modulation (CPM)
- Hybrids

Noncoherent

- Differential phase shift keying (DPSK)
- Frequency shift keying (FSK)
- Amplitude shift keying (ASK)
- Continuous phase modulation (CPM)
- Hybrids

Channel Coding

Waveform

- *M*-ary signaling
- Antipodal
- Orthogonal
- Trellis-coded modulation

Structured Sequences

- Block
- Convolutional
- Turbo

Synchronization

- Frequency synchronization
- Phase synchronization
- Symbol synchronization
- Frame synchronization
- Network synchronization

Multiplexing/Multiple Access

- Frequency division (FDM/FDMA)
- Time division (TDM/TDMA)
- Code division (CDM/CDMA)
- Space division (SDMA)
- Polarization division (PDMA)

Spreading

- Direct sequencing (DS)
- Frequency hopping (FH)
- Time hopping (TH)
- Hybrids

Encryption

- Block
- Data stream

Figure 1.3 Basic digital communication transformations.

Main Points

- Transmitters modulate analog messages or bits in case of a DCS for transmission over a channel.
- Receivers recreate signals or bits from received signal (mitigate channel effects)
- Performance metric for analog systems is fidelity, for digital it is the bit rate and error probability.

Why Digital Communications?

  - Easy to regenerate the distorted signal
  - Regenerative repeaters along the transmission path can detect a digital signal and retransmit a new, clean (noise free) signal
  - These repeaters prevent accumulation of noise along the path
- This is not possible with analog communication systems
  - Two-state signal representation
- The input to a digital system is in the form of a sequence of bits (binary or M_ary)
  - Immunity to distortion and interference
  - Digital communication is rugged in the sense that it is more immune to channel noise and distortion

Why Digital Communications?

  - Hardware is more flexible
  - Digital hardware implementation is flexible and permits the use of microprocessors, mini-processors, digital switching and VLSI
- Shorter design and production cycle
  - Low cost
- The use of LSI and VLSI in the design of components and systems have resulted in lower cost
  - Easier and more efficient to multiplex several digital signals
  - Digital multiplexing techniques – Time & Code Division Multiple Access - are easier to implement than analog techniques such as Frequency Division Multiple Access

Why Digital Communications?

- Can combine different signal types – data, voice, text, etc.
- Data communication in computers is digital in nature whereas voice communication between people is analog in nature
- The two types of communication are difficult to combine over the same medium in the analog domain.

- Using digital techniques, it is possible to combine both format for transmission through a common medium
- Encryption and privacy techniques are easier to implement
  - Better overall performance
  - Digital communication is inherently more efficient than analog in realizing the exchange of SNR for bandwidth
  - Digital signals can be coded to yield extremely low rates and high fidelity as well as privacy

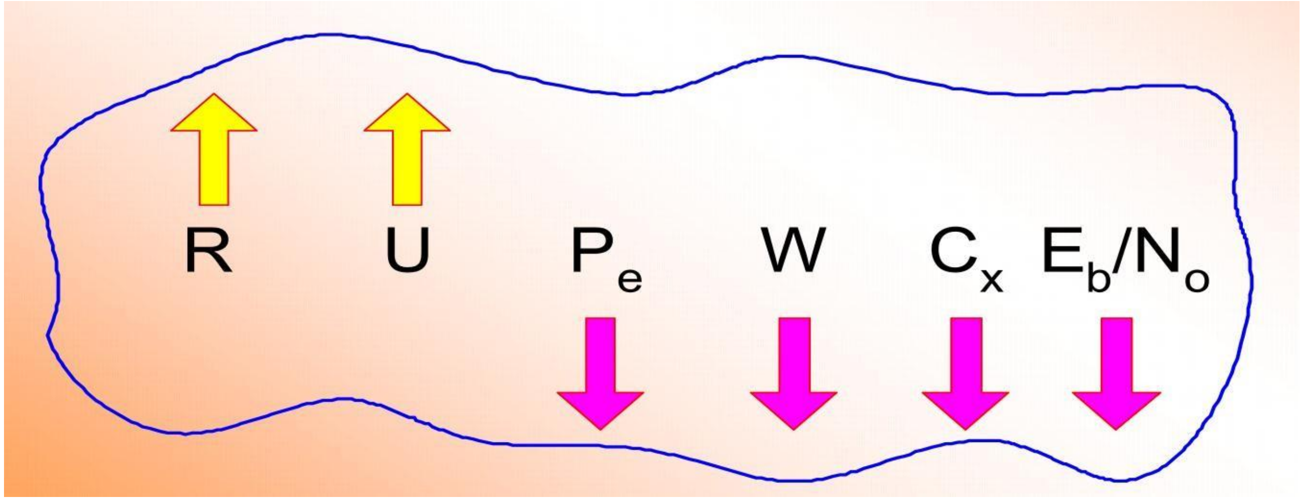
Why Digital Communications?

Disadvantages

- Requires reliable "synchronization"
- Requires A/D conversions at high rate
- Requires larger bandwidth
- Nongraceful degradation
- Performance Criteria
- Probability of error or Bit Error Rate

Goals in Communication System Design

- To maximize transmission rate, $\boldsymbol{R}$
- To maximize system utilization, $\boldsymbol{U}$
- To minimize bit error rate, $\boldsymbol{P_e}$
- To minimize required systems bandwidth, $\boldsymbol{W}$
- To minimize system complexity, $\boldsymbol{C_x}$
- To minimize required power, $\boldsymbol{E_b/N_o}$



Digital Signal Nomenclature

- **Information Source**
  - Discrete output values e.g. Keyboard
  - Analog signal source e.g. output of a microphone
- **Character**
  - Member of an alphanumeric/symbol (A to Z, 0 to 9)
  - Characters can be mapped into a sequence of binary digits using one of the standardized codes such as
    - **ASCII: American Standard Code for Information Interchange**
    - **EBCDIC: Extended Binary Coded Decimal Interchange Code**

Digital Signal Nomenclature

- **Digital Message**
  - Messages constructed from a finite number of symbols; e.g., printed language consists of 26 letters, 10 numbers, “space” and several punctuation marks. Hence a text is a digital message constructed from about 50 symbols
  - Morse-coded telegraph message is a digital message constructed from two symbols “**Mark**” and “**Space**”
- **M - ary**
  - A digital message constructed with *M* symbols
- **Digital Waveform**
  - Current or voltage waveform that represents a digital symbol
- **Bit Rate**
  - Actual rate at which information is transmitted per second

Digital Signal Nomenclature

- **Baud Rate**
  - Refers to the rate at which the signaling elements are transmitted, i.e. number of signaling elements per second.
- **Bit Error Rate**
  - The probability that one of the bits is in error or simply the probability of error

Digital Pulse Modulation

- The process of Sampling which we have already discussed in initial slides is also adopted in Digital pulse modulation.
- It is mainly of two types:

❑ Pulse Code Modulation(PCM)

❑ Delta Modulation(DM)

Pulse Code Modulation(PCM)

- Pulse-Code Modulation (PCM) is the most commonly used digital modulation scheme
- In PCM, the available range of signal voltages is divided into levels and each is assigned a binary number
- Each sample is represented by a binary number and transmitted serially
- The number of levels available depends upon the number of bits used to express the sample value
- The number of levels is given by: $N = 2^m$

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Pulse Code Modulation(PCM)

- PCM consists of three steps to digitize an analog signal:
  1. Sampling
  2. Quantization
  3. Binary encoding
- Before we sample, we have to filter the signal to limit the maximum frequency of the signal .Filtering should ensure that we do not distort the signal, ie remove high frequency components that affect the signal shape.

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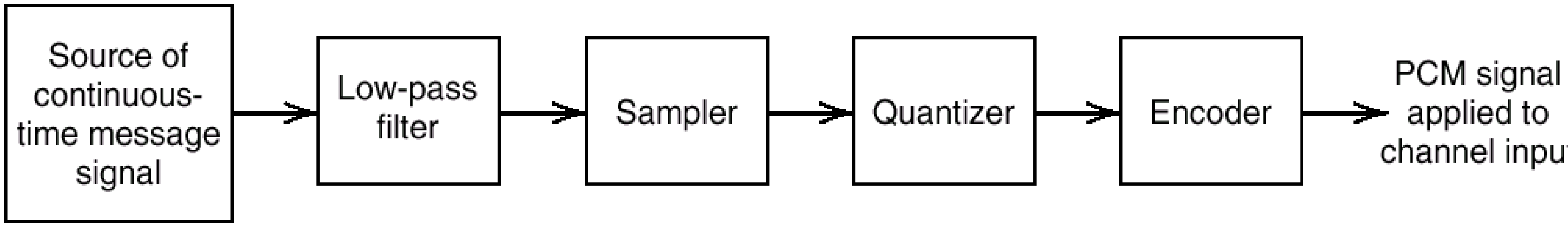
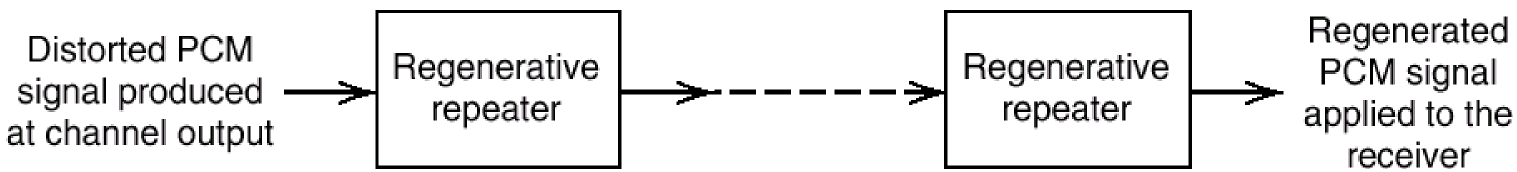
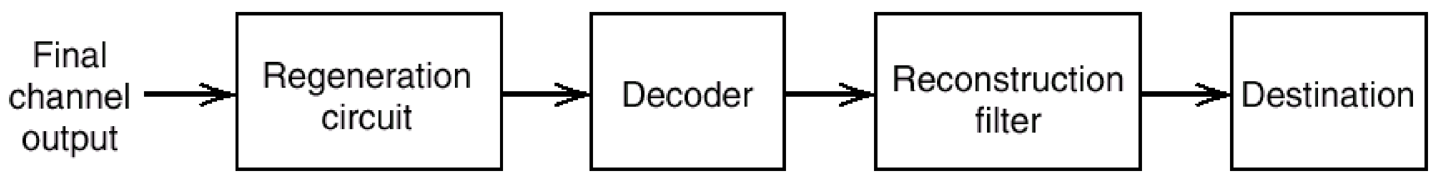
Pulse Code Modulation(PCM)



(*a*) Transmitter



(*b*) Transmission path



(*c*) Receiver

The basic elements of a PCM system.

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Pulse Code Modulation(PCM)

Analog to digital converter employs two techniques:

1. Sampling: The process of generating pulses of zero width and of amplitude equal to the instantaneous amplitude of the analog signal. The no. of pulses per second is called “sampling rate”.

Pulse Code Modulation(PCM)

- 2. Quantization: The process of dividing the maximum value of the analog signal into a fixed no. of levels in order to convert the PAM into a Binary Code.

  The levels obtained are called "quanization levels".

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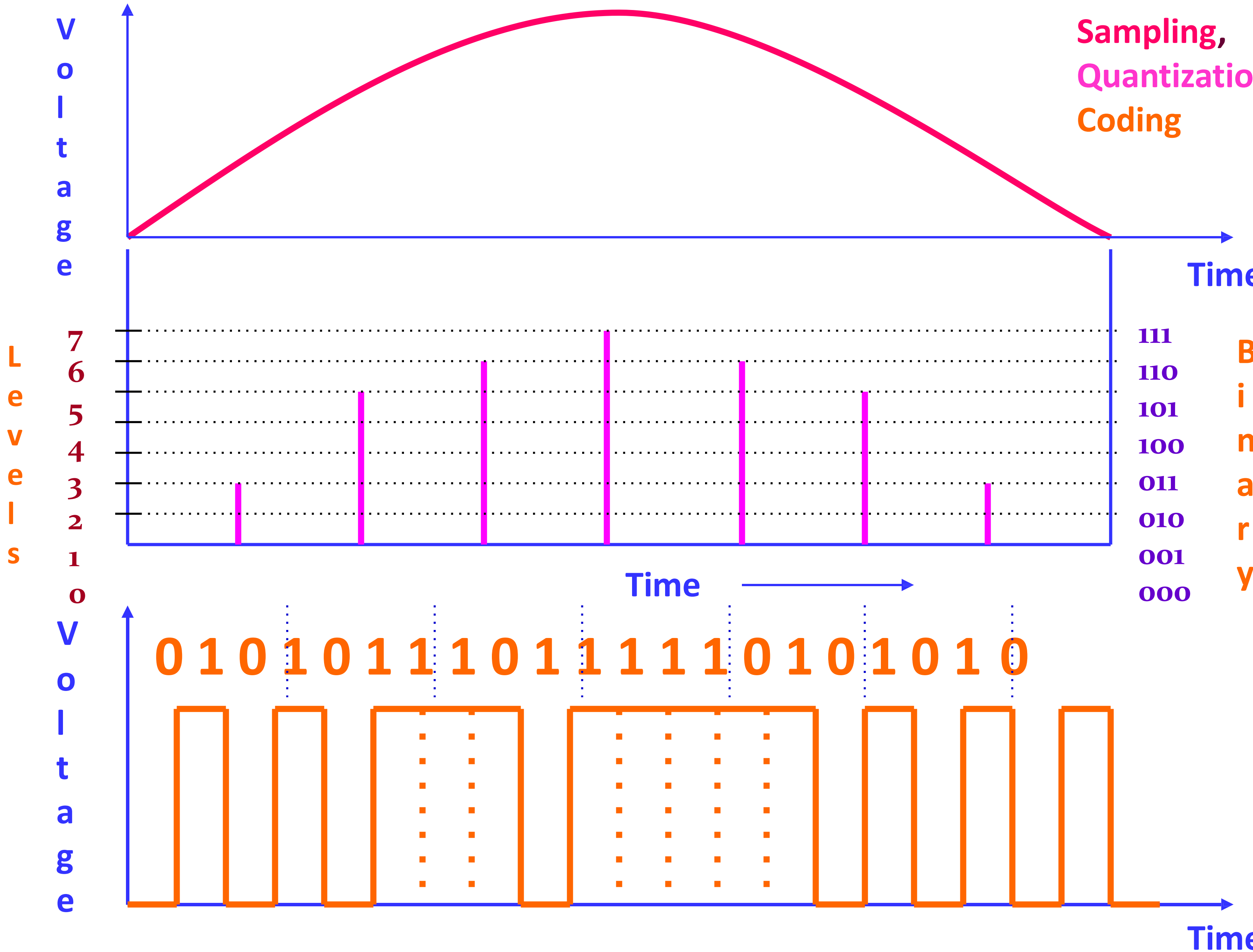
Sampling, Quantization and Coding



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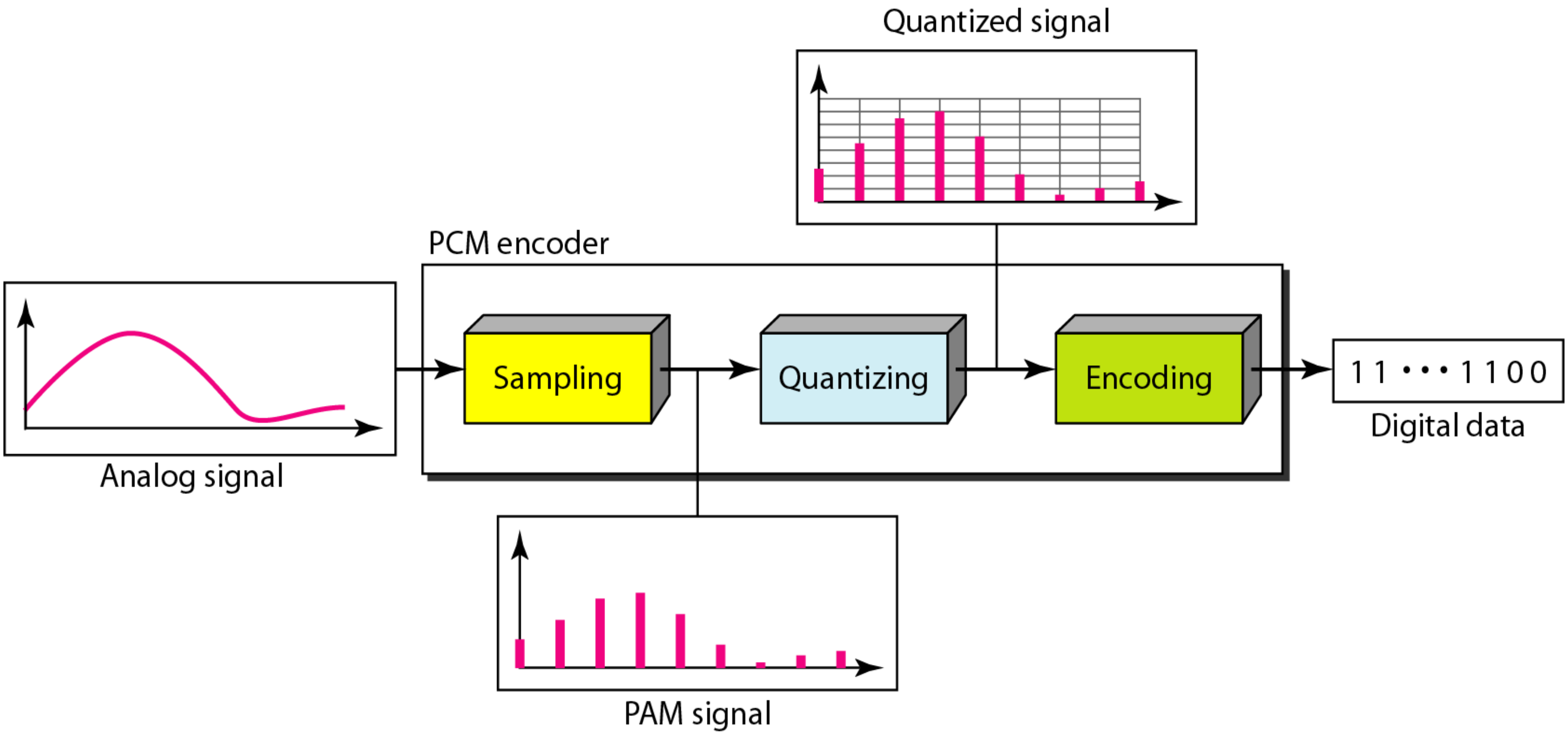
Pulse Code Modulation(PCM)



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Quantization

- By quantizing the PAM pulse, original signal is only approximated
- The process of converting analog signals to PCM is called *quantizing*
- Since the original signal can have an infinite number of signal levels, the quantizing process will produce errors called **quantizing errors** or **quantizing noise**

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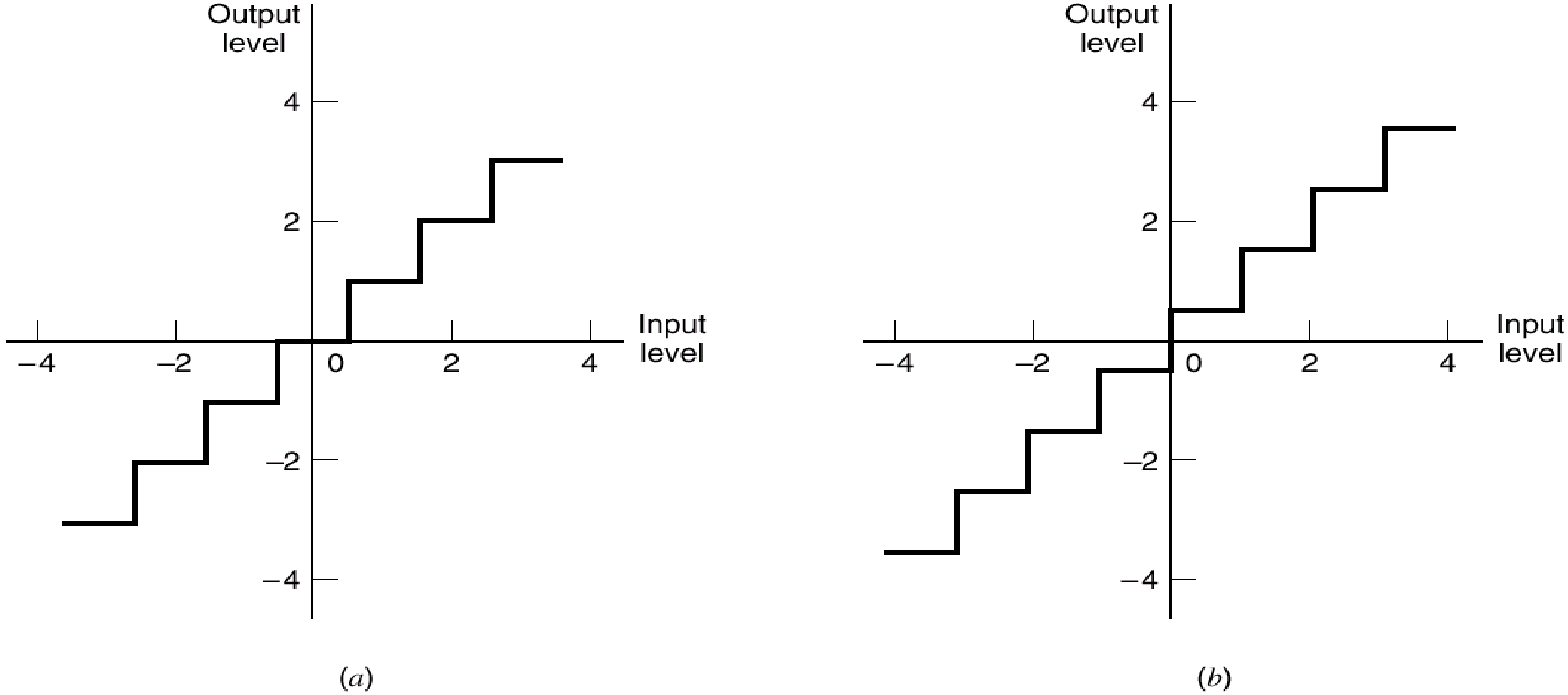
Quantization



Two types of quantization: (*a*) midtread and (*b*) midrise

Quantization

- Coding and Decoding
- The process of converting an analog signal into PCM is called coding, the inverse operation is called decoding
- Both procedures are accomplished in a **CODEC**

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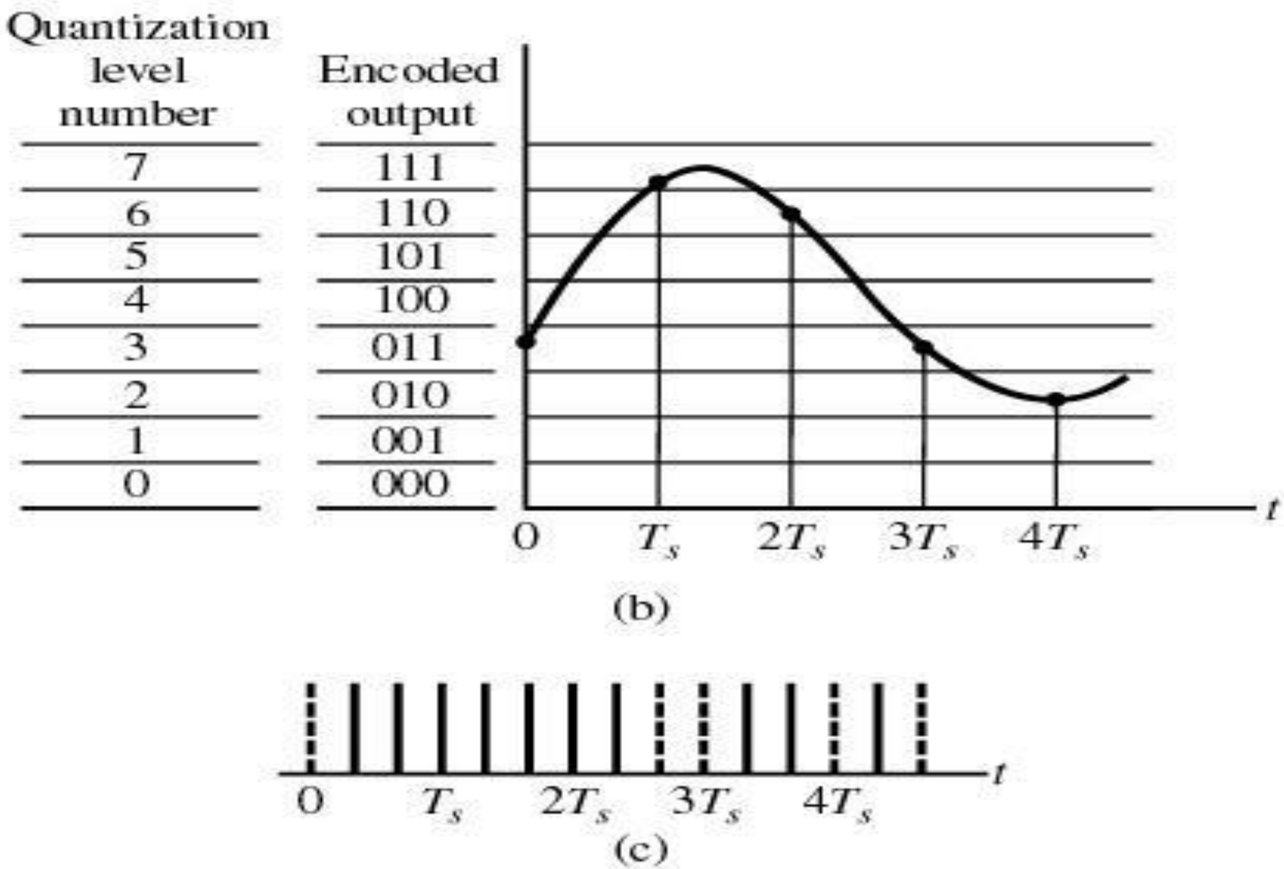
Quantization



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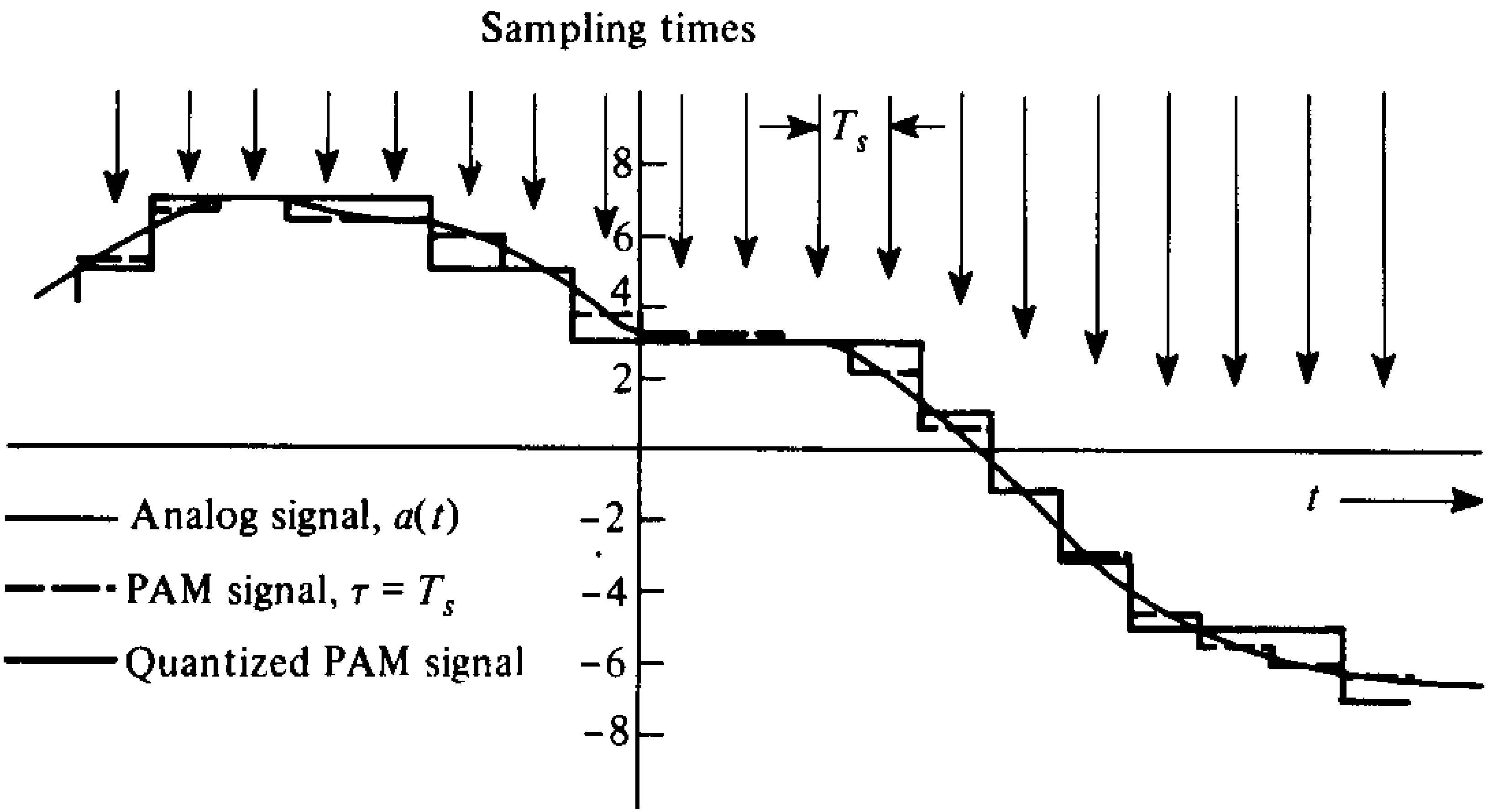
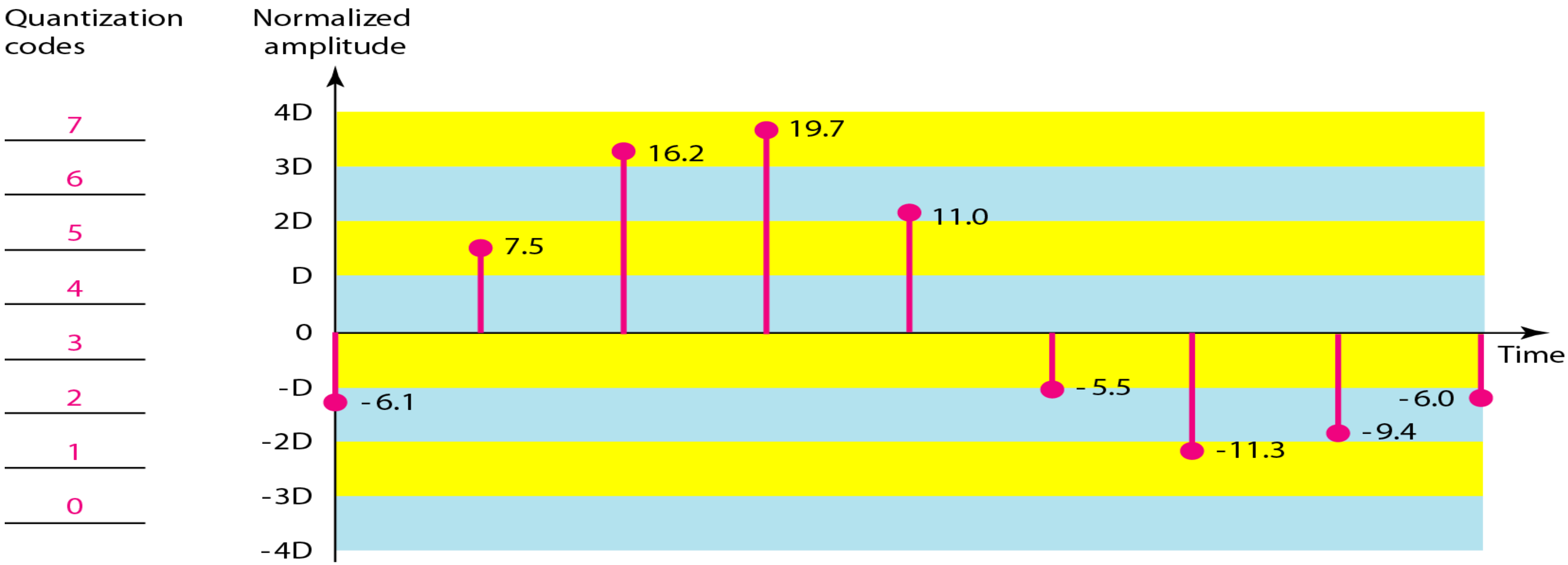
Quantization



(b) Analog Signal, Flat-top PAM Signal, and Quantized PAM Signal

Quantization and encoding of a sampled signal



Normalized PAM values	-1.22	1.50	3.24	3.94	2.20	-1.10	-2.26	-1.88	-1.20
Normalized quantized values	-1.50	1.50	3.50	3.50	2.50	-1.50	-2.50	-1.50	-1.50
Normalized error	-0.38	0	+0.26	-0.44	+0.30	-0.40	-0.24	+0.38	-0.30
Quantization code	2	5	7	7	6	2	1	2	2
Encoded words	010	101	111	111	110	010	001	010	010

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Quantization Error

- When a signal is quantized, we introduce an error - the coded signal is an approximation of the actual amplitude value.
- The difference between actual and coded value (midpoint) is referred to as the quantization error.
- The more zones, the smaller Δ which results in smaller errors.
- BUT, the more zones the more bits required to encode the samples -> higher bit rate

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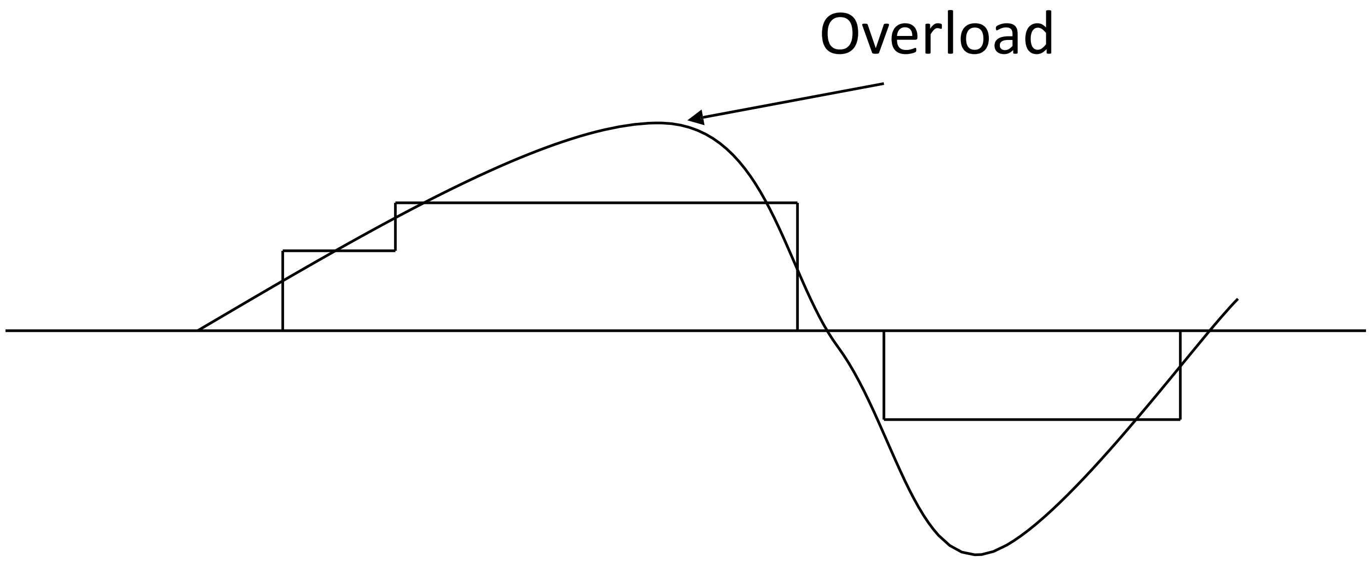
Quantization Error (cont.)

- Round-off error
- Overload error



Quantization Noise

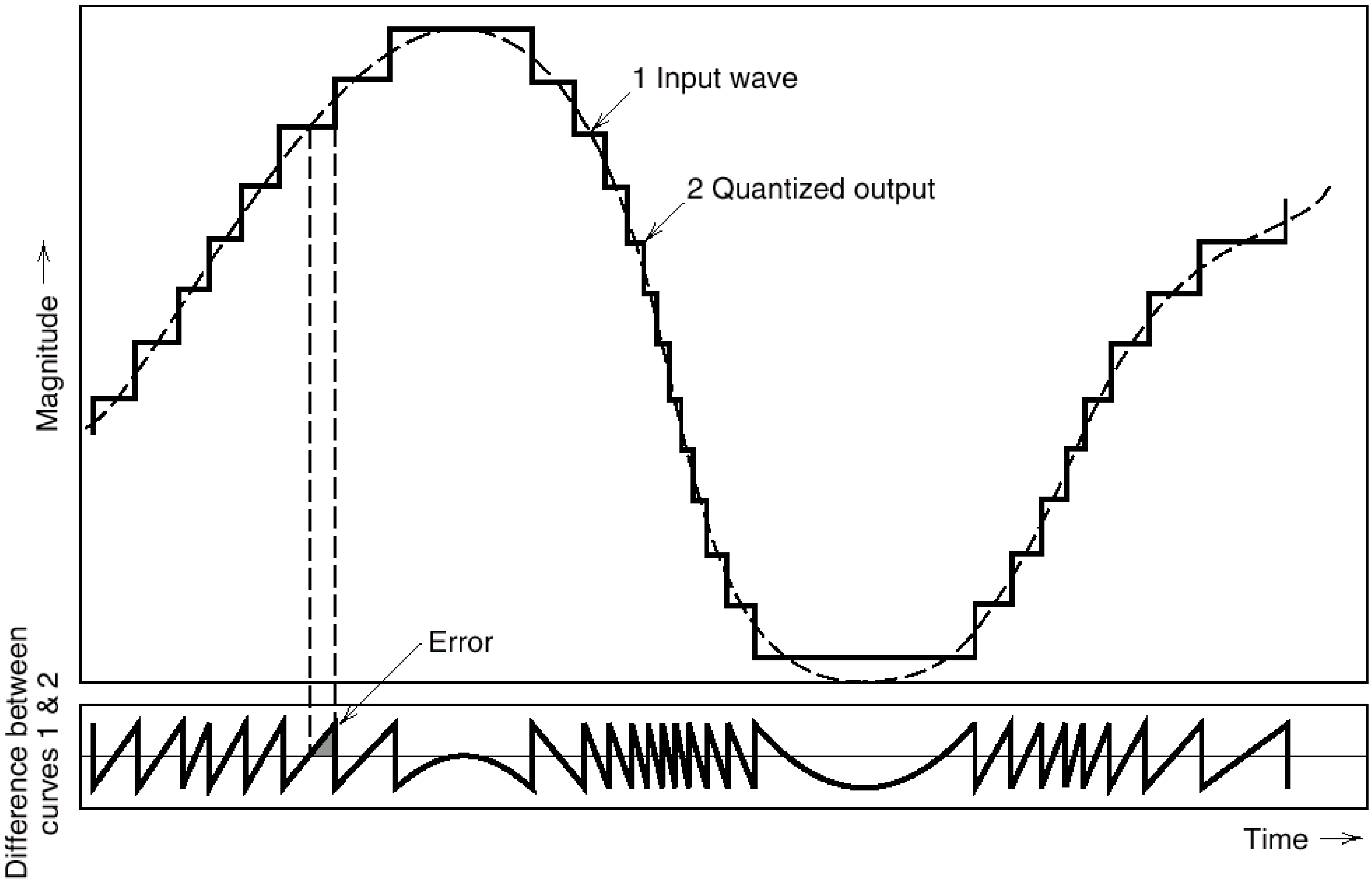


Illustration of the quantization process

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Pulse Code Modulation

- In PCM system,N number of binary digits are transmitted per sample.Hence the signaling rate and channel bandwidth of PCM are very large.
- Also encodind,decoding and quantizing circuitary of PCM is complex.

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Delta Modulation

- In Delta Modulation, only one bit is transmitted per sample
- That bit is a one if the current sample is more positive than the previous sample, and a zero if it is more negative
- Since so little information is transmitted, delta modulation requires higher sampling rates than PCM for equal quality of reproduction

Delta Modulation

- This scheme sends only the difference between pulses, if the pulse at time t_{n+1} is higher in amplitude value than the pulse at time t_n, then a single bit, say a "1", is used to indicate the positive value.
- If the pulse is lower in value, resulting in a negative value, a "0" is used.
- This scheme works well for small changes in signal values between samples.
- If changes in amplitude are large, this will result in large errors.

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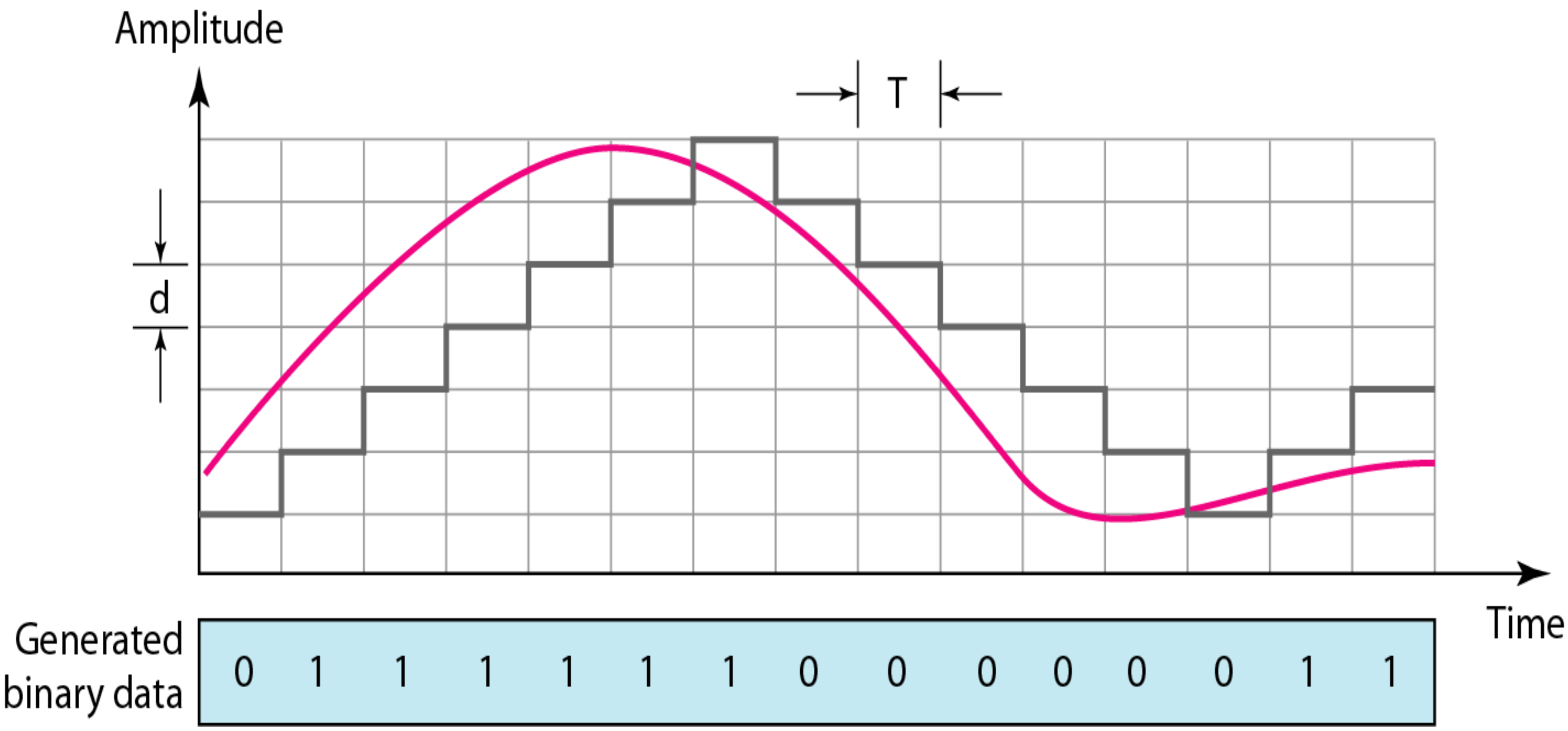
Delta Modulation



The process of delta modulation

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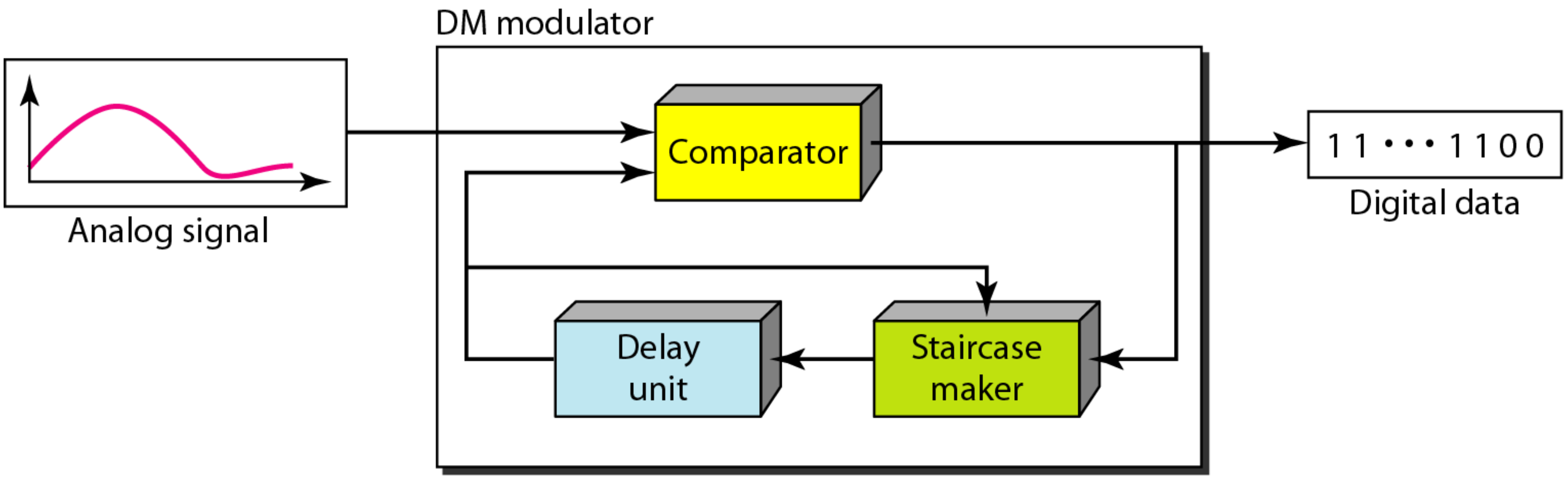
Delta Modulation

Components of Delta Modulation



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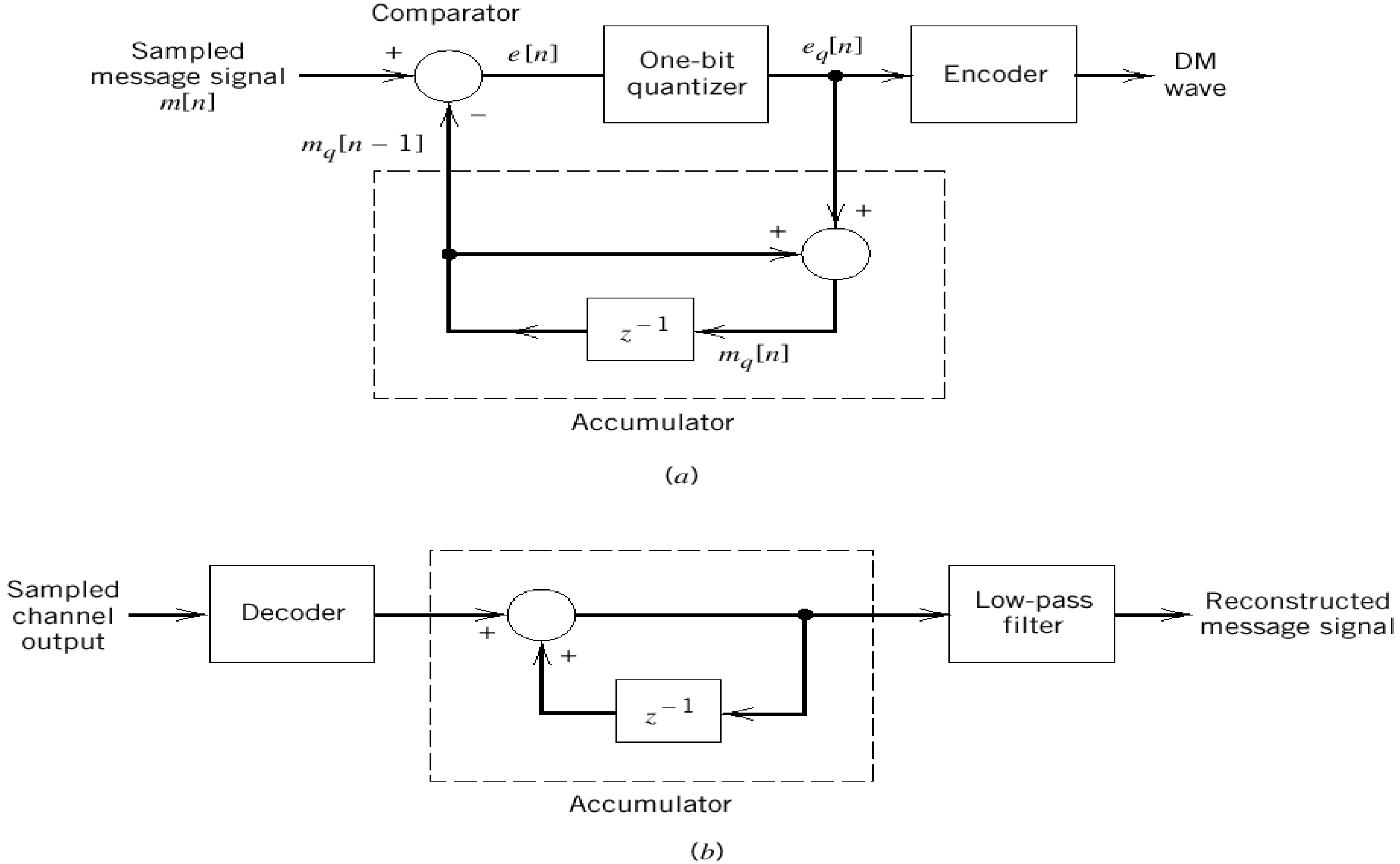
Delta Modulation



DM system. (*a*) Transmitter. (*b*) Receiver.

Delta Modulation

- Distortions in DM system

1. If the slope of analog signal is much higher than that of approximated digital signal over long duration,than this difference is called **Slope overload distortion**.
2. The difference between quantized signal and original signal is called as **Granular noise.** It is similar to quantisation noise.

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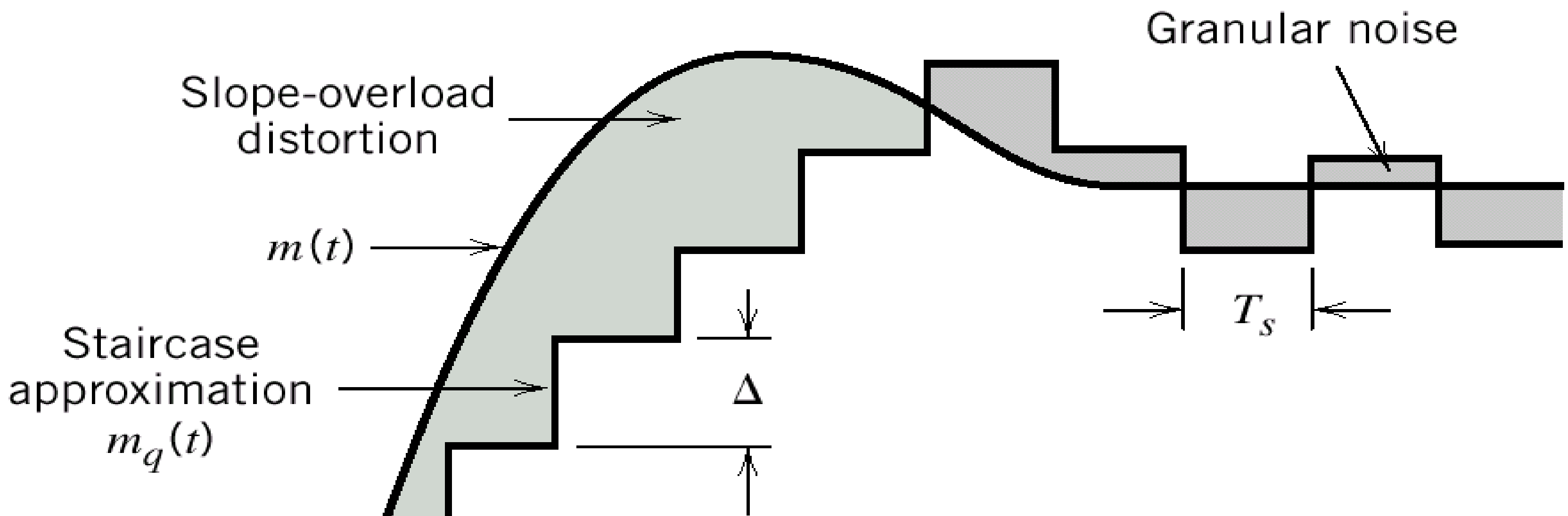
Delta Modulation



Two types of quantization errors :

Slope overload distortion and **granular noise**

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Delta Modulation

- Distortions in DM system

Granular noise occurs when step size ▲ is large relative to local slope m(t).

There is a further modification in this system,in which step size is not fixed.

That scheme is known as **Adaptive Delta Modulation**.

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Adaptive Delta Modulation

- A better performance can be achieved if the value of ▲ is not fixed.
- The value of ▲ changes according to the amplitude of the analog signal.
- It has wide dynamic range due to variable step size.
- Also better utilisation of bandwidth as compared to delta modulation.
- Improvement in signal to noise ratio.

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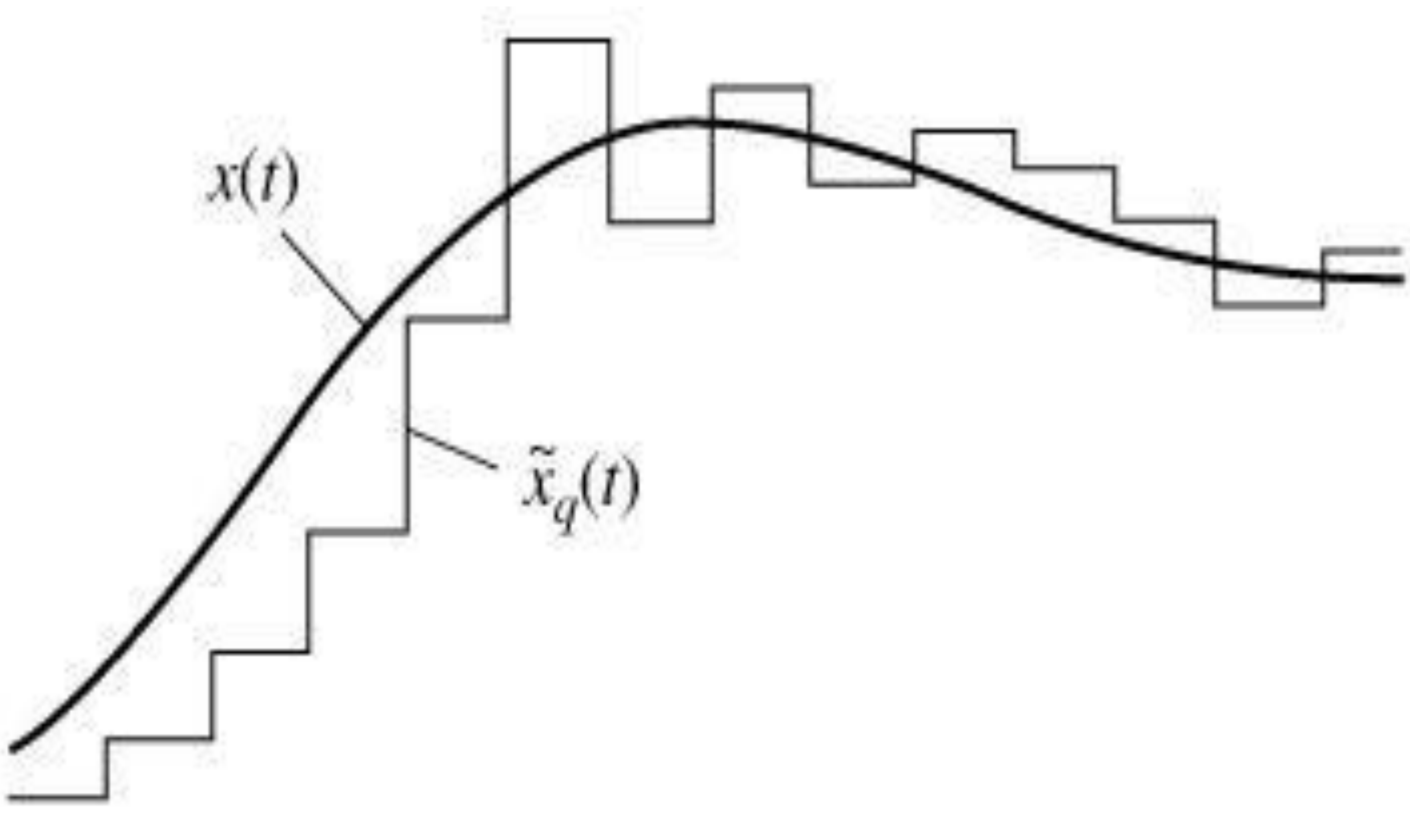
Adaptive Delta Modulation



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Unit 2
Digital Band-Pass Modulation Techniques

- Digital band-pass modulation techniques
  - Amplitude-shift keying
  - Phase-shift keying
  - Frequency-shift keying
- Receivers
  - Coherent detection
    - The receiver is synchronized to the transmitter with respect to carrier phases
  - Noncoherent detection
    - The practical advantage of reduced complexity but at the cost of degraded performance

Some Preliminaries

- Given a binary source
  - The modulation process involves switching ore keying the amplitude, phase, or frequency of a sinusoidal carrier wave between a pair of possible values in accordance with symbols 0 and 1.

$$c(t) = A_c \cos(2\pi f_c t + \varphi_c)$$

  - All three of them are examples of a band-pass process

1. Binary amplitude shift-keying (BASK)
   - The carrier amplitude is keyed between the two possible values used to represent symbols 0 and 1
2. Binary phase-shift keying (BPSK)
   - The carrier phase is keyed between the two possible values used to represent symbols 0 and 1.
3. Binary frequency-shift keying (BFSK)
   - The carrier frequency is keyed between the two possible values used to represent symbols 0 and 1.

– Decreasing the bit duration T_b has the effect of increasing the transmission bandwidth requirement of a binary modulated wave.

$$A_c = \sqrt{\frac{2}{T_b}}$$

$$c(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t + \phi_c) \qquad (7.3)$$

– Differences that distinguish digital modulation from analog modulation.
  - The transmission bandwidth requirement of BFSK is greater than that of BASK for a given binary source.
  - However, the same does not hold for BPSK.

- Band-Pass Assumption
  - The spectrum of a digital modulated wave is centered on the carrier frequency f_c
  - Under the assumption $f_c \gg W$,
    - There will be no spectral overlap in the generation of s(t)
    - The spectral content of the modulated wave for positive frequency is essentially separated from its spectral content for negative frequencies.

$$s(t) = b(t)c(t) \qquad (7.4)$$

$$s(t) = \sqrt{\frac{2}{T_b}}\, b(t)\cos(2\pi f_c t) \qquad (7.5)$$

  - The transmitted signal energy per bit

$$E_b = \int_0^{T_b} |s(t)|^2 \, dt$$

$$= \frac{2}{T_b}\int_0^{T_b} |b(t)|^2 \cos^2(2\pi f_c t)\, dt \qquad (7.6)$$

$$\cos^2(2\pi f_c t) = \frac{1}{2}[1 + \cos(4\pi f_c t)]$$

$$E_b = \frac{1}{T_b}\int_0^{T_b} |b(t)|^2\, dt + \frac{1}{T_b}\int_0^{T_b} |b(t)|^2 \cos^2(4\pi f_c t)\, dt \qquad (7.7)$$

– The band-pass assumption implies that $|b(t)|^2$ is essentially constant over one complete cycle of the sinusoidal wave $\cos(4\pi f_c t)$

$$\int_0^{T_b} |b(t)|^2 \cos^2(4\pi f_c t)\, dt \approx 0$$

$$E_b \approx \frac{1}{T_b}\int_0^{T_b} |b(t)|^2\, dt \qquad (7.8)$$

– For linear digital modulation schemes governed by Eq.(7.5), the transmitted signal energy is a scaled version of the energy in the incoming binary wave responsible for modulating the sinusoidal carrier.

7.2 Binary Amplitude-Shift Keying

– The ON-OFF signaling variety

$$b(t) = \begin{cases} \sqrt{E_b}, & \text{for binary symbol } 1 \\ 0, & \text{for binary symbol } 0 \end{cases} \qquad (7.9)$$

$$s(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t), & \text{for symbol } 1 \\ 0, & \text{for symbol } 0 \end{cases} \qquad (7.10)$$

– The average transmitted signal energy is (the two binary symbols must by equiprobable)

$$E_{\text{av}} = \frac{E_b}{2} \qquad (7.11)$$

- Generation and Detection of ASK Signals
  - Generation of ASK signal : by using a produce modulator with two inputs
    - The ON-OFF signal of Eq. (7.9)

$$b(t) = \begin{cases} \sqrt{E_b}, & \text{for binary symbol } 1 \\ 0, & \text{for binary symbol } 0 \end{cases} \qquad (7.9)$$

    - The sinusoidal carrier wave

$$c(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)$$

  - Detection of ASK signal
    - The simplest way is to use an envelope detector, exploiting the nonconstant-envelope property of the BASK signal

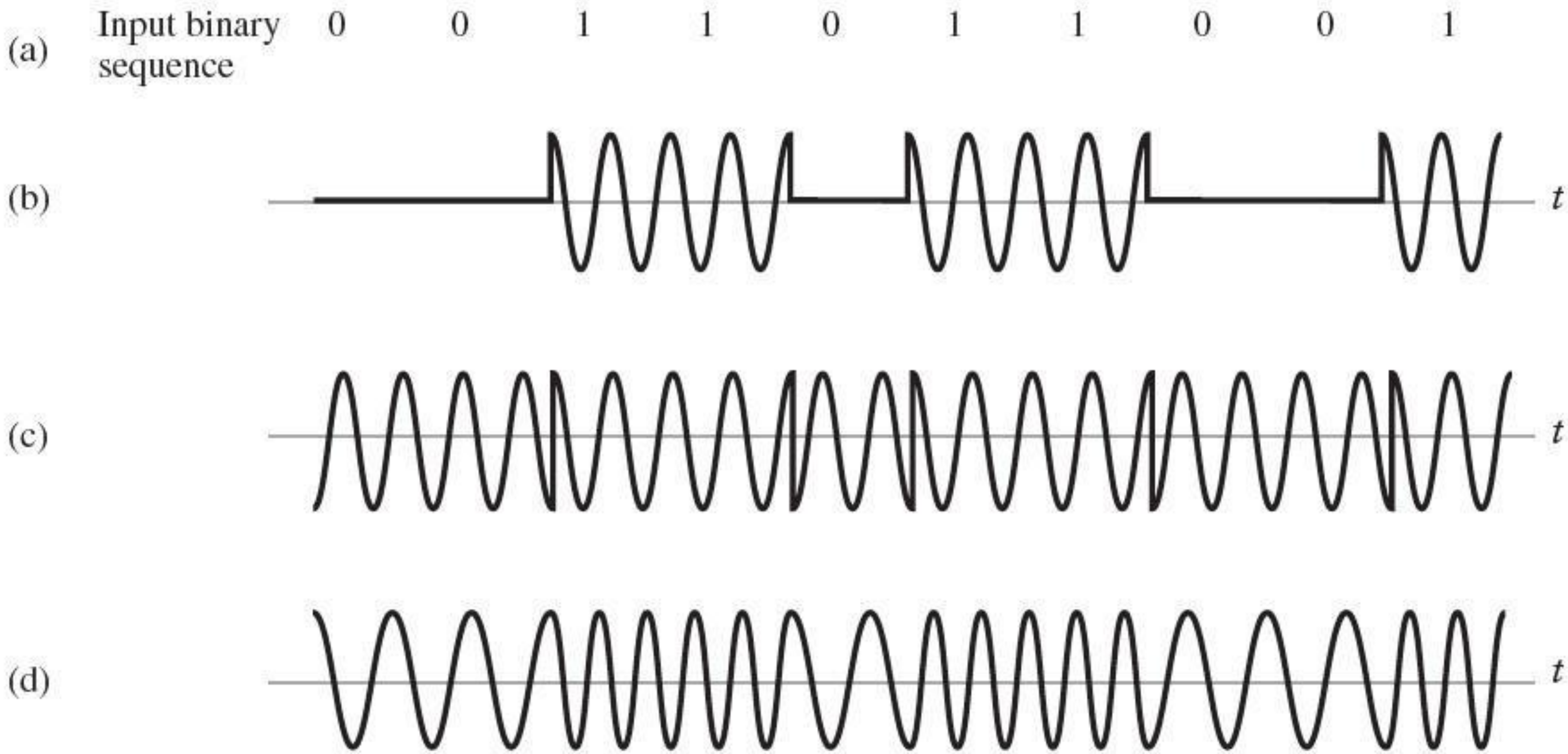


Figure 7.1 The three basic forms of signaling binary information. (*a*) Binary data stream. (*b*) Amplitude-shift keying. (*c*) Phase-shift keying. (*d*) Frequency-shift keying with continuous phase.

- Spectral Analysis of BASK
  - The objective
    1) To investigate the effect of varying the carrier frequency fc on the power spectrum of the BASK signal s(t), assuming that the wave is fixed.
    2) Recall that the power spectrum of a signal is defined as 10 times the logarithm of the squared magnitude spectrum of the signal
    3) To investigate the effect of varying the frequency of the square wave on the spectrum of the BASK signal, assuming that the sinusoidal carrier wave is fixed.

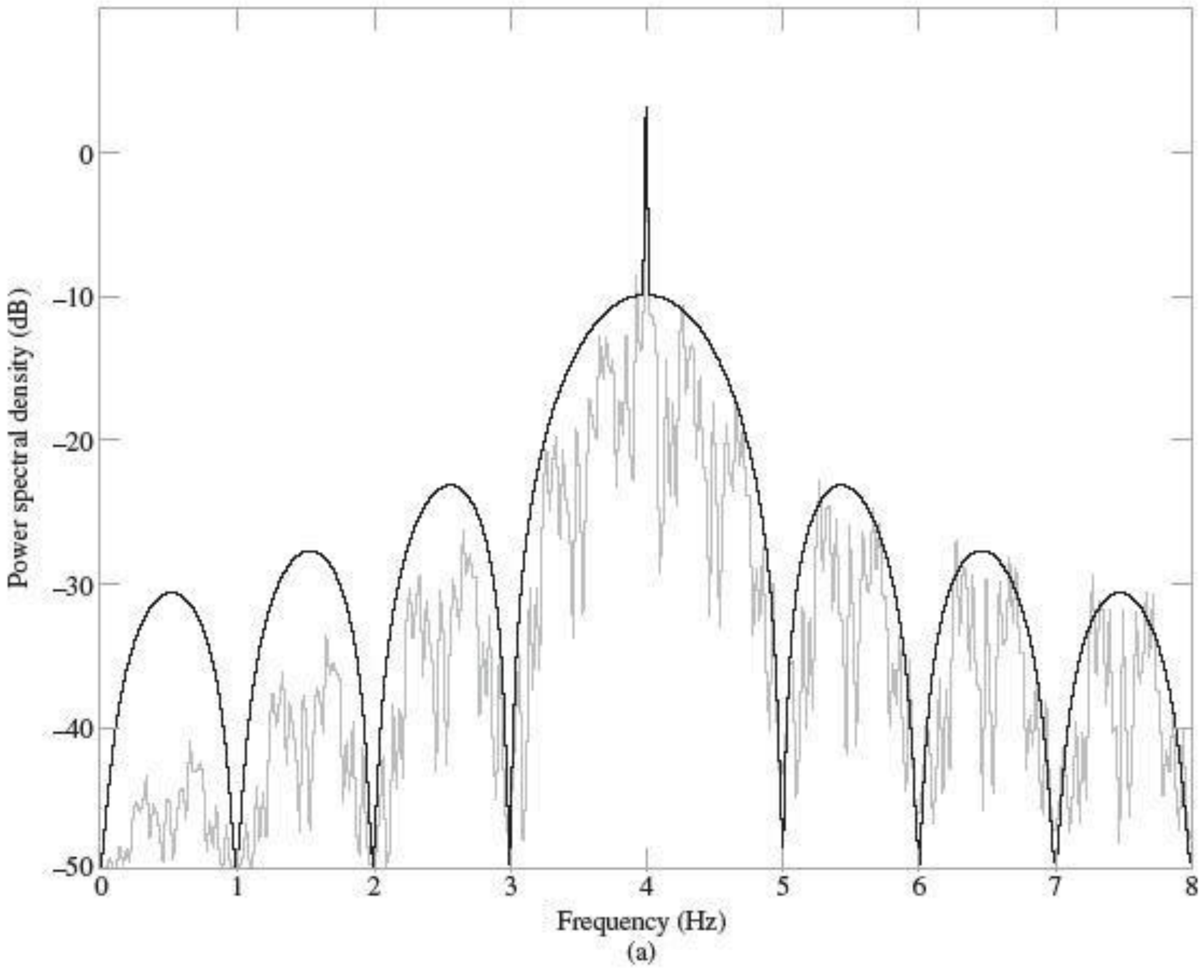


FIGURE 7.2 Power spectra of BASK signal produced by square wave as the modulating signal for varying modulation frequency: (*a*) $f_c = 4$ Hz and $T_b = 1$ s; (*b*) $f_c = 8$ Hz and $T_b = 1$ s.

1. The spectrum of the BASK signal contains a line component at $f=f_c$
2. When the square wave is fixed and the carrier frequency is doubled, the mid-band frequency of the BASK signal is likewise doubled.
3. When the carrier is fixed and the bit duration is halved, the width of the main lobe of the sinc function defining the envelope of the BASK spectrum is doubled, which, in turn, means that the transmission bandwidth of the BASK signal is doubled.
4. The transmission bandwidth of BASK, measured in terms of the width of the main lobe of its spectrum, is equal to $2/T_b$, where T_b is the bit duration.

Phase-Shift Keying

- Binary Phase-Shift Keying (BPSK)
  - The special case of double-sideband suppressed-carried (DSB-SC) modulation
  - The pair of signals used to represent symbols 1 and 0,

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t), & \text{for symbol 1 corresponding to } i = 1 \\ \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t + \pi) = -\sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t), & \text{for symbol 0 corresponding to } i = 2 \end{cases} \quad (7.12)$$

  - An antipodal signals
    - A pair of sinusoidal wave, which differ only in a relative phase-shift of π radians.

1. The transmitted energy per bit, E_b is constant, equivalently, the average transmitted power is constant.
2. Demodulation of BPSK cannot be performed using envelope detection, rather, we have to look to coherent detection as described next.

- Generation and Coherent Detection of BPSK Signals
  1. Generation
  - A product modulator consisting of two component
    1) Non-return-to-zero level encoder
       - The input binary data sequence is encoded in polar form with symbols 1 and 0 represented by the constant-amplitude levels ; $\sqrt{E_b}$ and - $\sqrt{E_b}$,
    2) Product modulator
       - Multiplies the level-encoded binary wave by the sinusoidal carrier c(t) of amplitude $\sqrt{2/T_b}$ to produce the BPSK signal

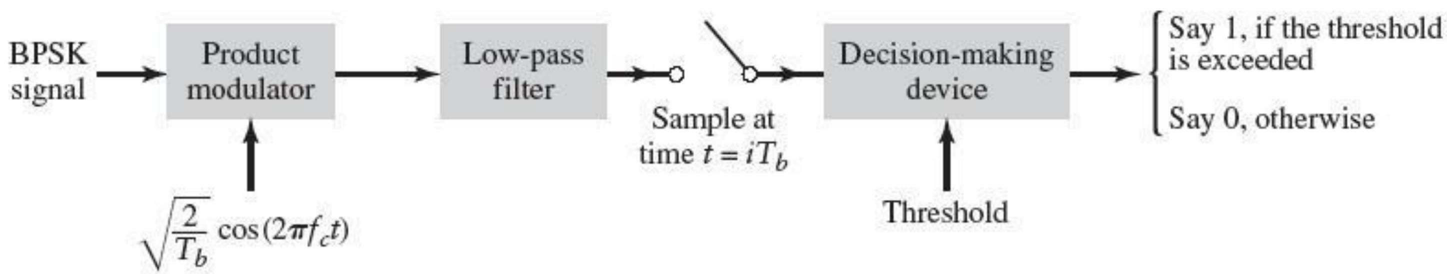
2. Detection

– A receiver that consists of four sections

1) Product modulator; supplied with a locally generated reference signal that is a replica of the carrier wave c(t)
2) Low-pass filter; designed to remove the double-frequency components of the product modulator output
3) Sampler ; uniformly samples the output of the low-pass filter, the local clock governing the operation of the sampler is synchronized with the clock responsible for bit-timing in the transmitter.
4) Decision-making device ; compares the sampled value of the low-pass filter's output to an externally supplied threshold. If the threshold is exceed, the device decides in favor of symbol 1, otherwise, it decides in favor of symbol 0.

– What should the bandwidth of the filter be ?

- The bandwidth of the low-pass filter in the coherent BPSK receiver has to be equal to or greater than the reciprocal of the bit duration T_b for satisfactory operation of the receiver.



(b)

FIGURE 7.4 (*a*) BPSK modulator. (*b*) Coherent detector for BPSK; for the sampler, integer $i = 0, \pm 1, \pm 2, \ldots$.

- Spectral Analysis of BPSK
  - The objectives
    1. To evaluate the effect of varying the carrier frequency fc on the power spectrum of the BPSK signal, for a fixed square modulating wave.
    2. To evaluate the effect of varying modulation frequency on the power spectrum of the BPSK signal, for a fixed carrier frequency.

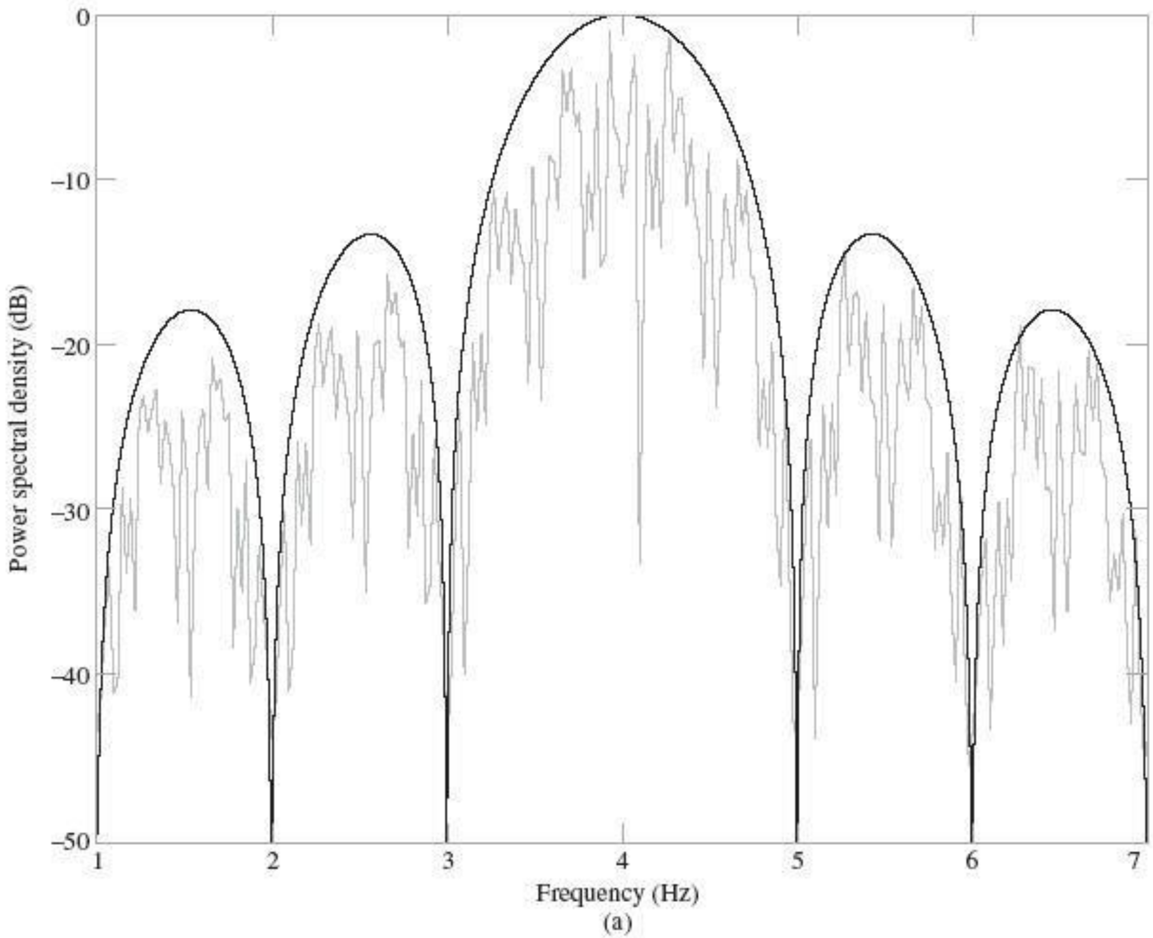


Figure 7.5 Power spectra of BPSK signal produced by square wave as the modulating signal for varying modulation frequency: (*a*) $f_c = 4$ Hz and $T_b = 1$ s; (*b*) $f_c = 8$ Hz and $T_b = 1$ s.

1. BASK and BPSK signals occupy the same transmission bandwidth, which defines the width of the main lobe of the sinc-shaped power spectra.
2. The BASK spectrum includes a carrier component, whereas this component is absent from the BPSK spectrum. With this observation we are merely restating the fact that BASK is an example of amplitude modulation, whereas BPSK is an example of double sideband-suppressed carrier modulation
   - The present of carrier in the BASK spectrum means that the binary data stream can be recovered by envelope detection of the BASK signal.
   - On the other hand, suppression of the carrier in the BPSK spectrum mandates the use of coherent detection for recovery of the binary data stream form the BASK signal

- Quadriphase-Shift Keying
  - An important goal of digital communication is the efficient utilization of channel bandwidth
  - In QPSK (Quadriphase-shift keying)
    - The phase of the sinusoidal carrier takes on one of the four equally spaced values, such as π/4, 3π/4, 5π/4, and 7π/4

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\dfrac{\pi}{4}\right], & 0 \le t \le T \\ 0, & \text{elsewhere} \end{cases} \qquad (7.13)$$

    - Each one of the four equally spaced phase values corresponds to a unique pair of bits called **dibit**

$$T = 2T_b \qquad (7.14)$$

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[(2i-1)\frac{\pi}{4}\right]\cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\left[(2i-1)\frac{\pi}{4}\right]\sin(2\pi f_c t) \qquad (7.15)$$

1. In reality, the QPSK signal consists of the sum of two BPSK signals
2. One BPSK signal, represented by the first term defined the product of modulating a binary wave by the sinusoidal carrier

   this binary wave has an amplitude equal to ±√E/2

$$\sqrt{2E/T}\cos\left[(2i-1)\frac{\pi}{4}\right]\cos(2\pi f_c t),$$

$$\sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right] = \begin{cases} \sqrt{E/2} & \text{for } i = 1, 4 \\ -\sqrt{E/2} & \text{for } i = 2, 3 \end{cases} \qquad (7.16)$$

3. The second binary wave also has an amplitude equal to ±√E/2

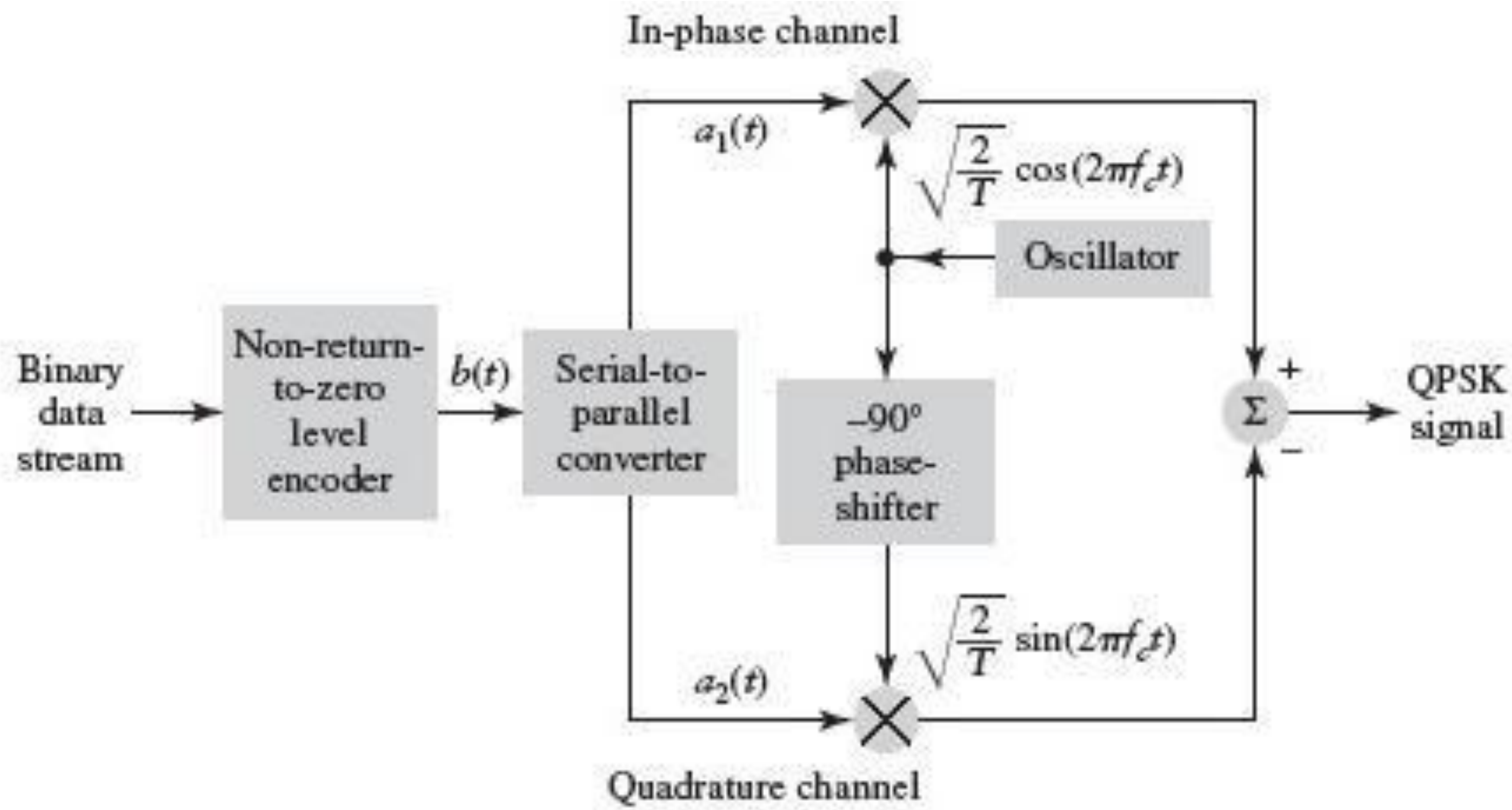
$$-\sqrt{2E/T}\sin\left[(2i-1)\frac{\pi}{4}\right]\sin(2\pi f_c t),$$

$$-\sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right] = \begin{cases} -\sqrt{E/2} & \text{for } i = 1, 2 \\ \sqrt{E/2} & \text{for } i = 3, 4 \end{cases} \qquad (7.17)$$

TABLE 7.1 *Relationship Between Index i And Identity of Corresponding Dibit, and Other Related Matters*

Index i	Phase of QPSK signal (radians)	Amplitudes of constituent binary waves: Binary wave 1 $a_1(t)$	Binary wave 2 $a_2(t)$	Input dibit $0 \le t \le T$
1	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$	10
2	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$	00
3	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$	01
4	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$	11

- Generation and Coherent Detection of QPSK Signals
  1. Generation
  - The incoming binary data stream is first converted into polar form by a non-return-to-zero level encoder
  - The resulting binary wave is next divided by means of a demultiplexer into two separate binary waves consisting of the odd- and even- mumbered input bits of *b(t)* – these are referred to as the demultiplexed components of the input binary wave.
  - The two BPSK signals are subtracted to produce the desired QPSK signals

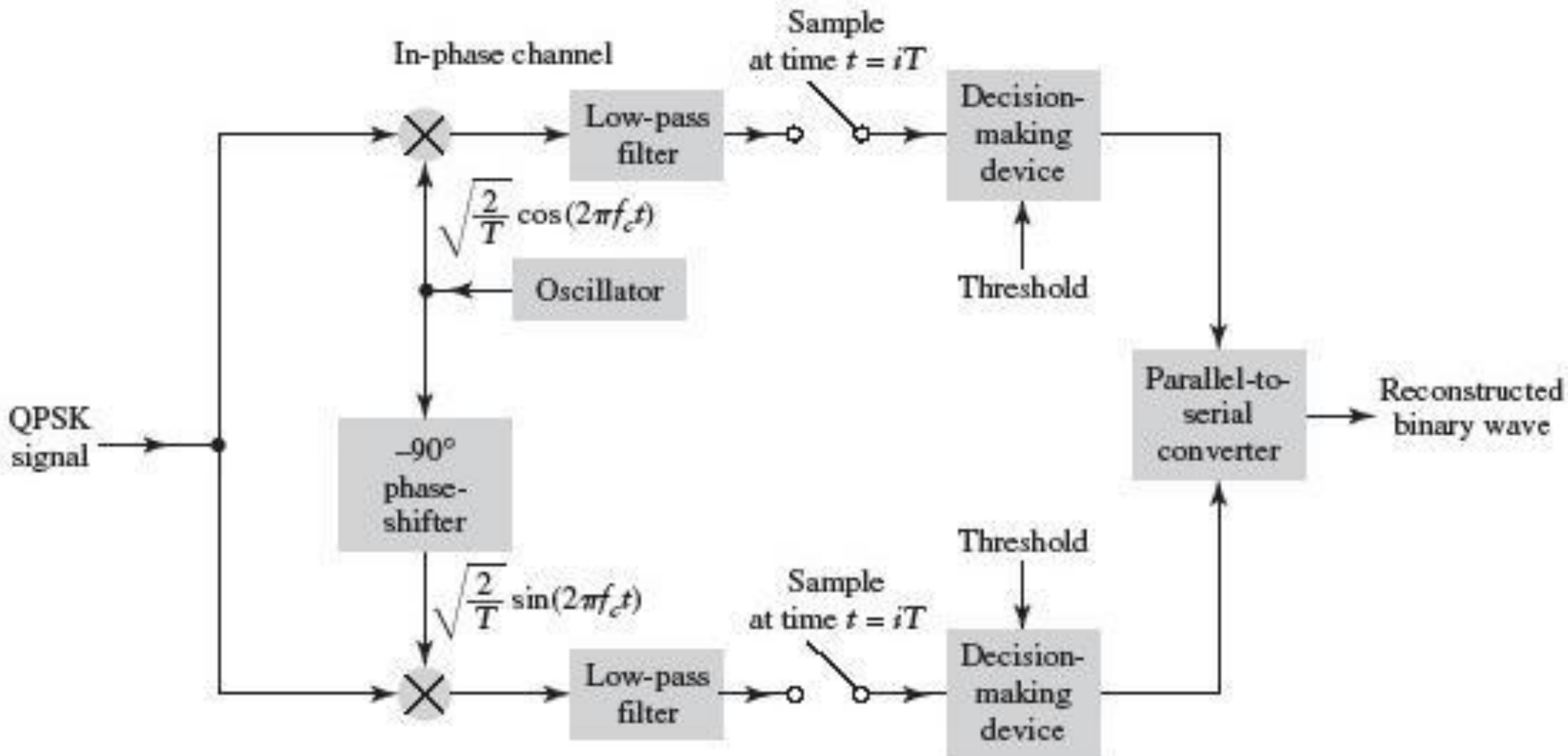

In-phase channel
$a_1(t)$
$\sqrt{\frac{2}{T}}\cos(2\pi f_c t)$
Oscillator
Binary data stream
Non-return-to-zero level encoder
$b(t)$
Serial-to-parallel converter
–90° phase-shifter
+
−
Σ
QPSK signal
$\sqrt{\frac{2}{T}}\sin(2\pi f_c t)$
$a_2(t)$
Quadrature channel

(a)

2. Detection

- The QPSK receiver consists of an In-phase and quadrature with a common input.
- Each channel is made up of a product modulator, low-pass filter, sampler, and decision-making device.
- The I- and Q-channnles of the receiver, recover the demultiplexed components $a_1(t)$ and $a_2(t)$
- By applying the outputs of these two channels to a multiplexer, the receiver recovers the original binary sequence

- Each of the two low-pass filters must be assigned a bandwidth equal to or greater than the reciprocal of the symbol duration T



(b)

FIGURE 7.7 Block diagrams of (*a*) QPSK transmitter and (*b*) coherent QPSK receiver; for the two synchronous samplers, integer $i = 0, \pm 1, \pm 2, \ldots$.

- Offset Quadriphase-Shift Keying (OQPSK)
  - The extent of amplitude fluctuations exhibited by QPSK signals may be reduced by using a variant of quadriphase-shift keying
  - The demultiplexed binary wave labeled $a_2(t)$ is delayed by one bit duration with respect to the other demultiplexed binary wave labled $a_1(t)$
  - $\pm 90^\circ$ phase transitions occur twice as frequency but with a reduced range of amplitude fluctuations.
  - Amplitude fluctuations in OQPSK due to filtering have a smaller amplitude than in QPSK.

EXAMPLE 7.1 Phase transitions

Parts (*a*) and (*b*) of Fig. 7.8 depict the waveforms of QPSK and OQPSK, both of which are produced by the binary data stream 0011011001 with the following composition over the interval $0 \le t \le 10T_b$:

- The input dibit (i.e., the pair of adjacent bits in the binary data stream) changes in going from the interval $0 \le t \le 2T_b$ to the next interval $2T_b \le t \le 4T_b$.
- The dibit changes again in going from the interval $2T_b \le t \le 4T_b$ to the next interval $4T_b \le t \le 6T_b$.
- The dibit changes yet again in going from the interval $4T_b \le t \le 6T_b$ to the next interval $6T_b \le t \le 8T_b$.

- Finally, the dibit is changed one last time in going from the interval $6T_b \le t \le 8T_b$ to the interval $8T_b \le t \le 10T_b$.

Examining the two waveforms of Fig. 7.8, we find the following:

(i) In QPSK, the carrier phase undergoes jumps of 0°, ±90°, or ±180° every $2T_b$ seconds.

(ii) In OQPSK, on the other hand, the carrier phase experiences only jumps of 0° or ±90° every T_b seconds.

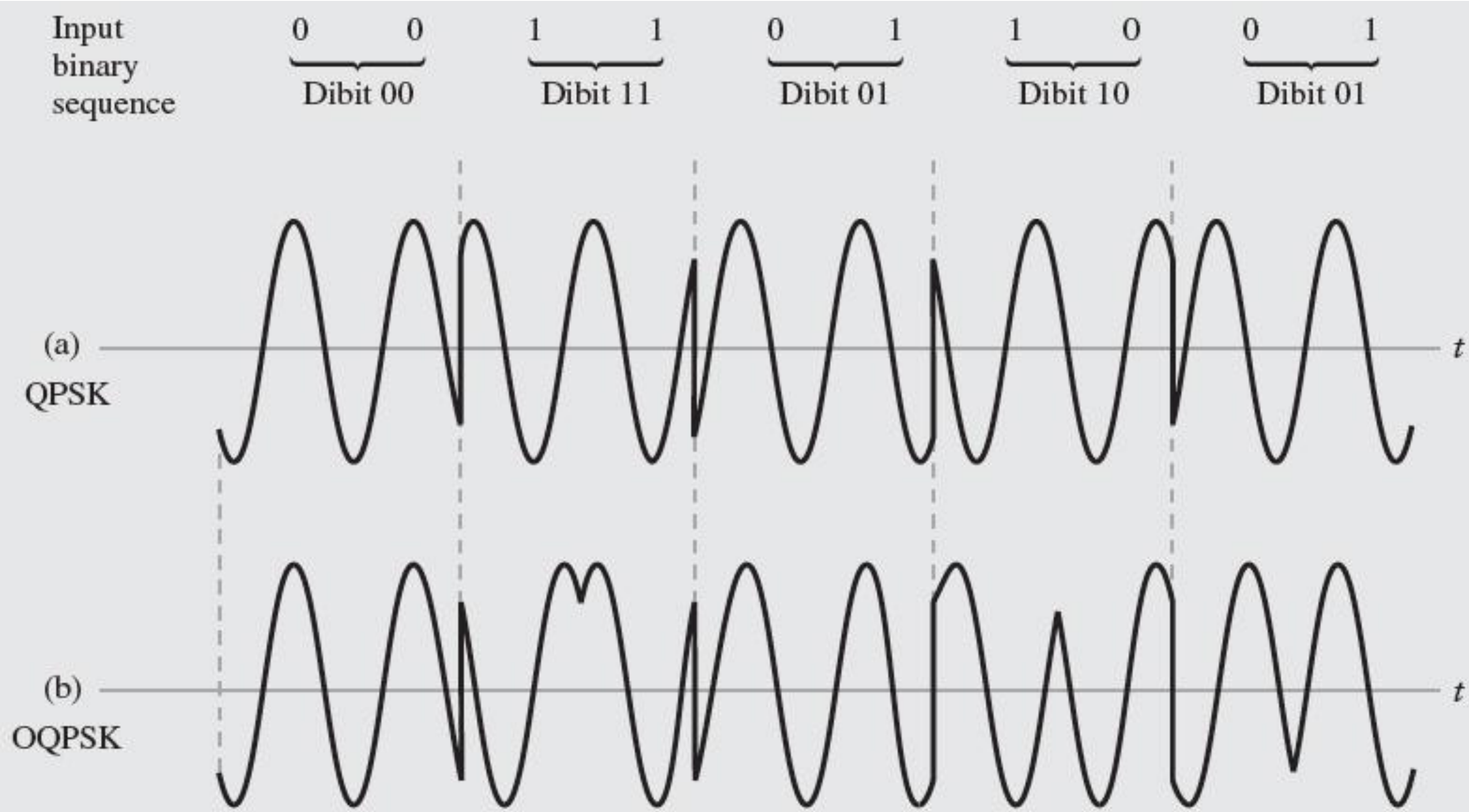


FIGURE 7.8 Graphical comparison of phase transitions in QPSK and OQPSK.

- Computer Experiment III : QPSK and OPQSK Spectra
  - QPSK Spectra

$$\text{Carrier Frequency,} \quad f_c = 8 \ Hz$$

$$\text{Bit duration,} \quad T_b = \begin{Bmatrix} 1s \text{ for part (a) of the figure} \\ 0\ s \text{ for part (b) of the figure} \end{Bmatrix}$$

  - OQPSK Spectra
    - For the same parameters used for QPSK
  - QPSK occupies a bandwidth equal to one half that of BPSK

Frequency-Shift Keying

- Binary Frequency-Shift Keying (BFSK)
  - Each symbols are distinguished from each other by transmitting one of two sinusoidal waves that differ in frequency by a fixed amount

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_1 t), & \text{for symbol 1 corresponding to } i = 1 \\ \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_2 t), & \text{for symbol 0 corresponding to } i = 2 \end{cases} \qquad (7.18)$$

  - Sunde's BFSK
    - When the frequencies f_1 and f_2 are chosen in such a way that they differ from each other by an amount equal to the reciprocal of the bit duration T_b

- Computer Experiment IV : Sunde's BFSK

1. Waveform

– Input binary sequence 0011011001 for a bit duration T_b=1s

– The latter part of the figure clearly displays the phase-continuous property of Sunde's BFSK

$$\text{Bit duration,} \quad T_b = 1\,s$$

$$\text{Carrier frequency,} \quad f_c = 8\ Hz$$

2. Spectrum

1. The spectrum contains two line components at the frequency $f=f_c \pm 1(2T_b)$; which equal 7.5Hz and 8.5Hz for f_c=8 Hz and T_b=1s
2. The main lobe occupies a band of width equal to $(3/T_b)$=3Hz, centered on the carrier frequency f_c=8 Hz
3. The largest sidelobe is about 21 dB below the main lobe.

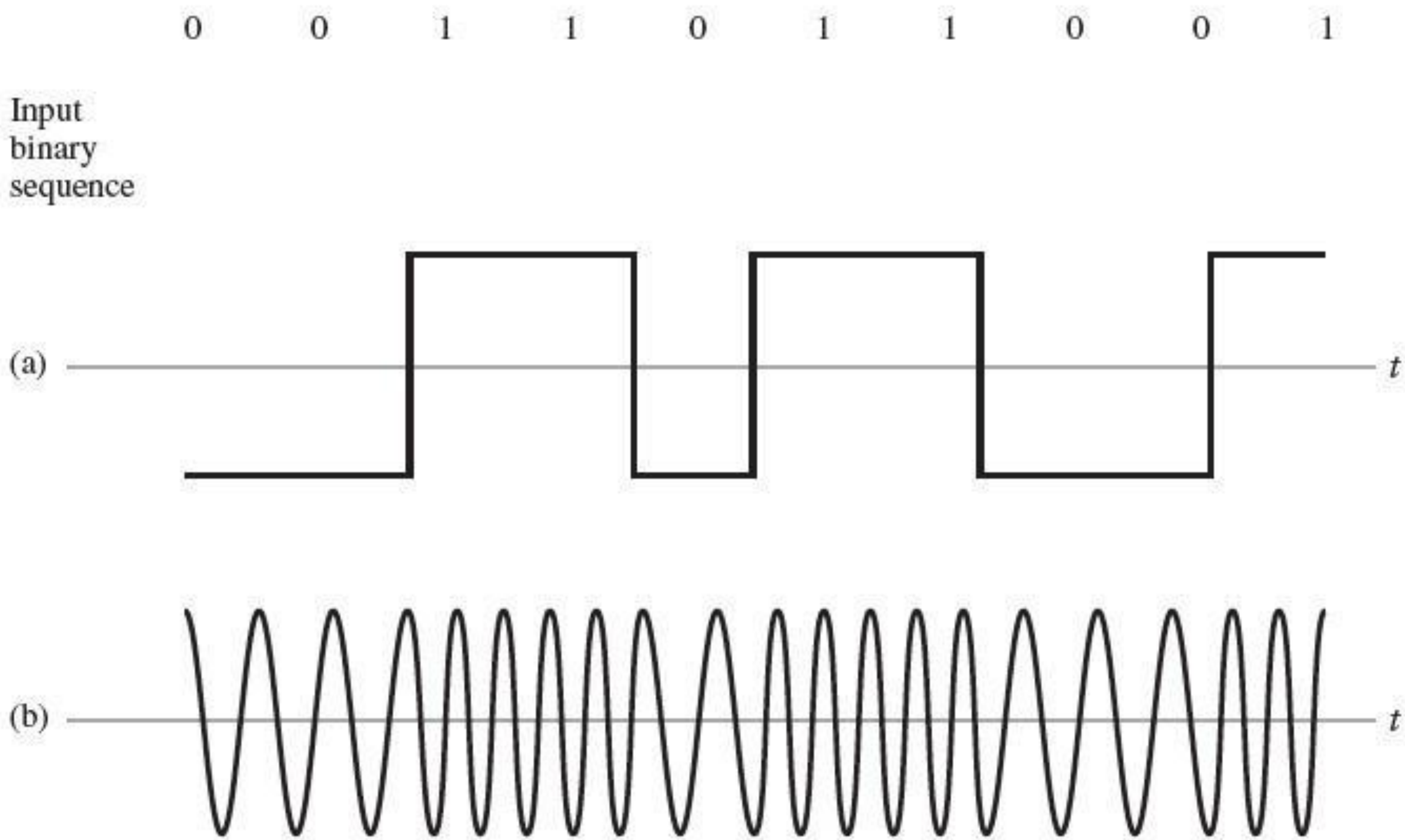


Figure 7.11 (*a*) Binary sequence and its non-return-to-zero level-encoded waveform. (*b*) Sunde's BFSK signal.

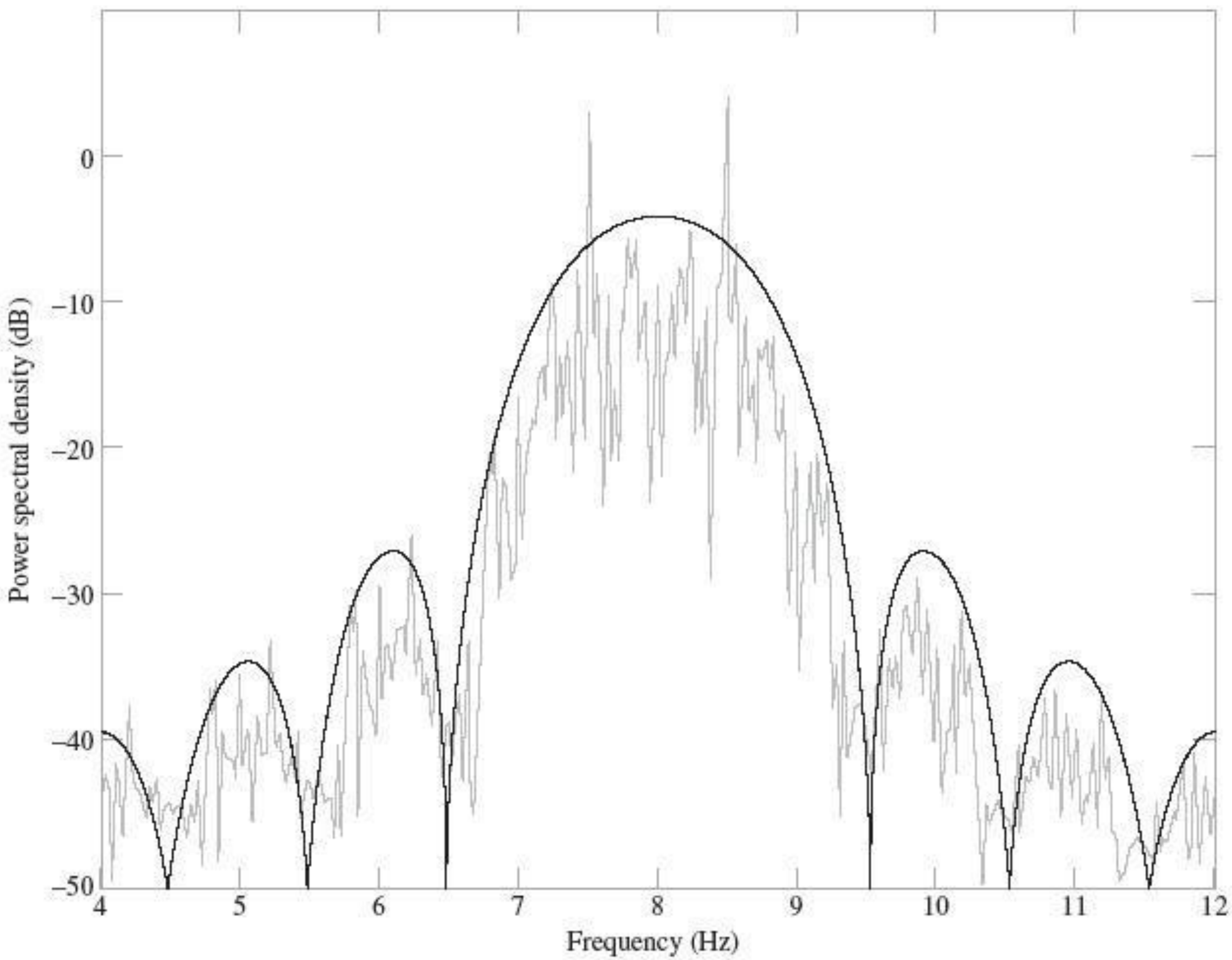


FIGURE 7.12 Power spectrum of Sunde's BFSK produced by square wave as the modulating signal for the following parameters: $f_c = 8$ Hz and $T_b = 1$ s.

- Continuous-phase Frequency-Shift Keying
  - The modulated wave maintains phase continuity at all transition points, even though at those points in time the incoming binary data stream switches back and forth
  - Sunde's BFSK, the overall excursion δf in the transmitted frequency from symbol 0 to symbol 1, is equal to the bit rate of the incoming data stream.
  - MSK (Minimum Shift Keying)
    - The special form of CPFSK
    - Uses a different value for the frequency excursion δf , with the result that this new modulated wave offers superior spectral properties to Sunde's BFSK.

- Minimum-Shift Keying
  - Overall frequency excursion δf from binary symbol 1 to symbol 0, is one half the bit rate

$$\delta f = f_1 - f_2$$

$$= \frac{1}{2T_b} \qquad (7.19)$$

$$f_c = \frac{1}{2}(f_1 + f_2) \qquad (7.20)$$

$$f_1 = f_c + \frac{\delta f}{2}, \quad \text{for symbol} \quad 1 \qquad (7.21)$$

$$f_2 = f_c - \frac{\delta f}{2}, \quad \text{for symbol} \quad 0 \qquad (7.22)$$

  - Define the MSK signal as the angle-modulated wave

$$s(t) = \sqrt{\frac{2E_b}{T_b}}\cos[2\pi f_c t + \theta(t)] \qquad (7.23)$$

– Sunde’s BFSK has no memory; in other words, knowing which particular change occurred in the previous bit interval provides no help in the current bit interval.

$$\theta(t) = 2\pi\left(\frac{\delta f}{2}t\right)$$

$$= \frac{\pi t}{2T_b}, \qquad \text{for symbol } 1 \qquad (7.24)$$

$$\theta(t) = 2\pi\left(-\frac{\delta f}{2}t\right)$$

$$= -\frac{\pi t}{2T_b}, \qquad \text{for symbol } 0 \qquad (7.25)$$

EXAMPLE 7.2: Relationship Between OQPSK and MSK Waveforms

The purpose of this example is to illustrate the relationship that exists between OQPSK and MSK waveforms. Figures 7.13 and 7.14 bear out this fundamental relationship:

- The five waveforms of Fig. 7.13 plot the components of the OQPSK signal for the input binary data stream 0011011001.
- The corresponding five waveforms of Fig. 7.14 plot the components of the MSK signal for the same input binary data stream 0011011001.

Comparing the results plotted in Figs. 7.13 and 7.14, we may make the following observation. Although the OQPSK and MSK are derived from different modulation principles, the MSK from frequency-shift keying and the OQPSK from phase-shift keying, these two digitally modulated waves are indeed closely related. The basic difference between them lies merely in the way in which the binary symbols in their in-phase and quadrature components are level-encoded. In OQPSK, the level-encoding is based on rectangular pulses, with one binary wave shifted from the other binary wave by one bit duration. On the other hand, in MSK, the level-encoding is based on the half cycle of a cosinusoid.

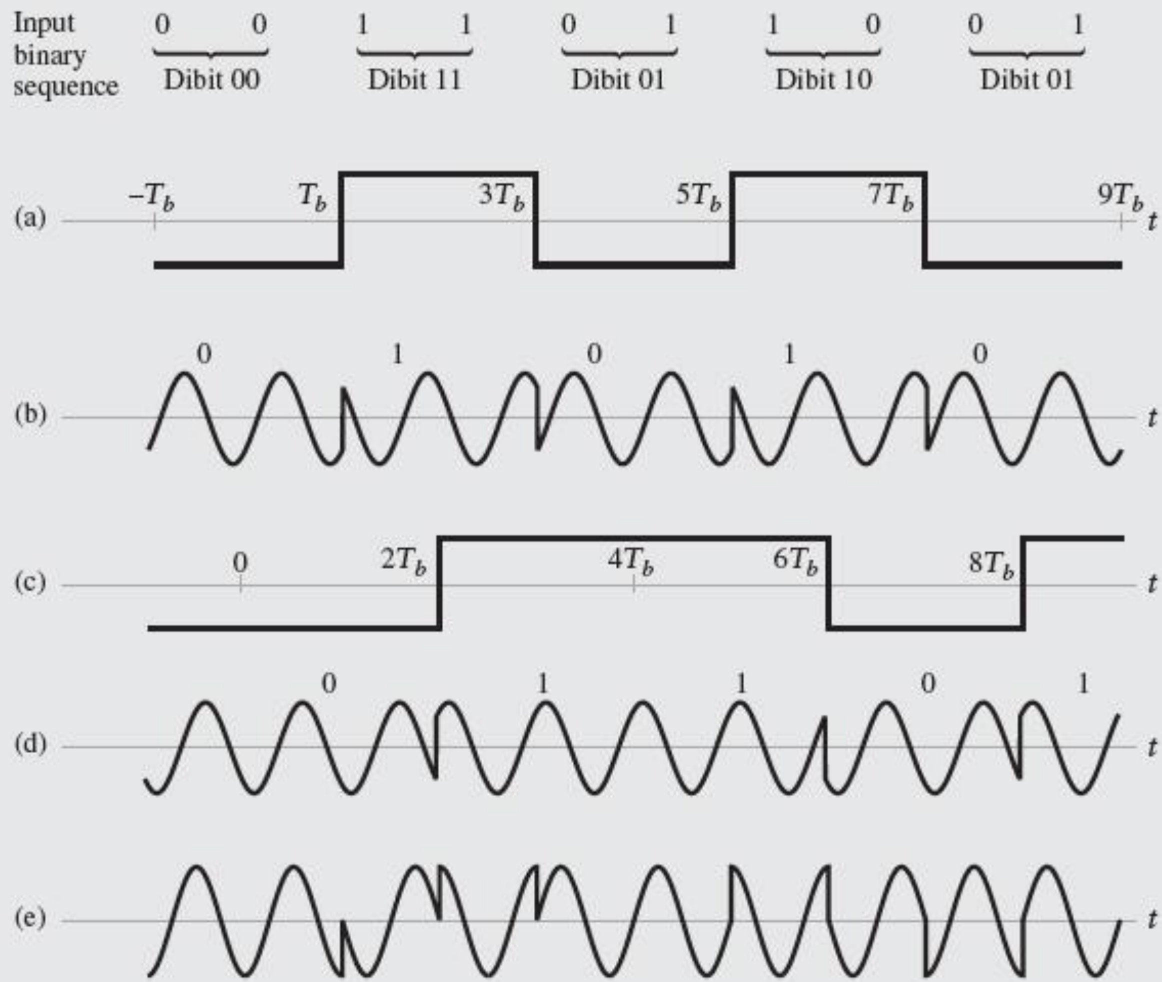


FIGURE 7.13 OQPSK signal components: (*a*) Modulating signal for in-phase component. (*b*) Modulated waveform of in-phase component. (*c*) Modulating signal for quadrature component. (*d*) Modulated waveform of quadrature component. (*e*) Waveform of OQPSK signal obtained by subtracting (*d*) from (*b*).

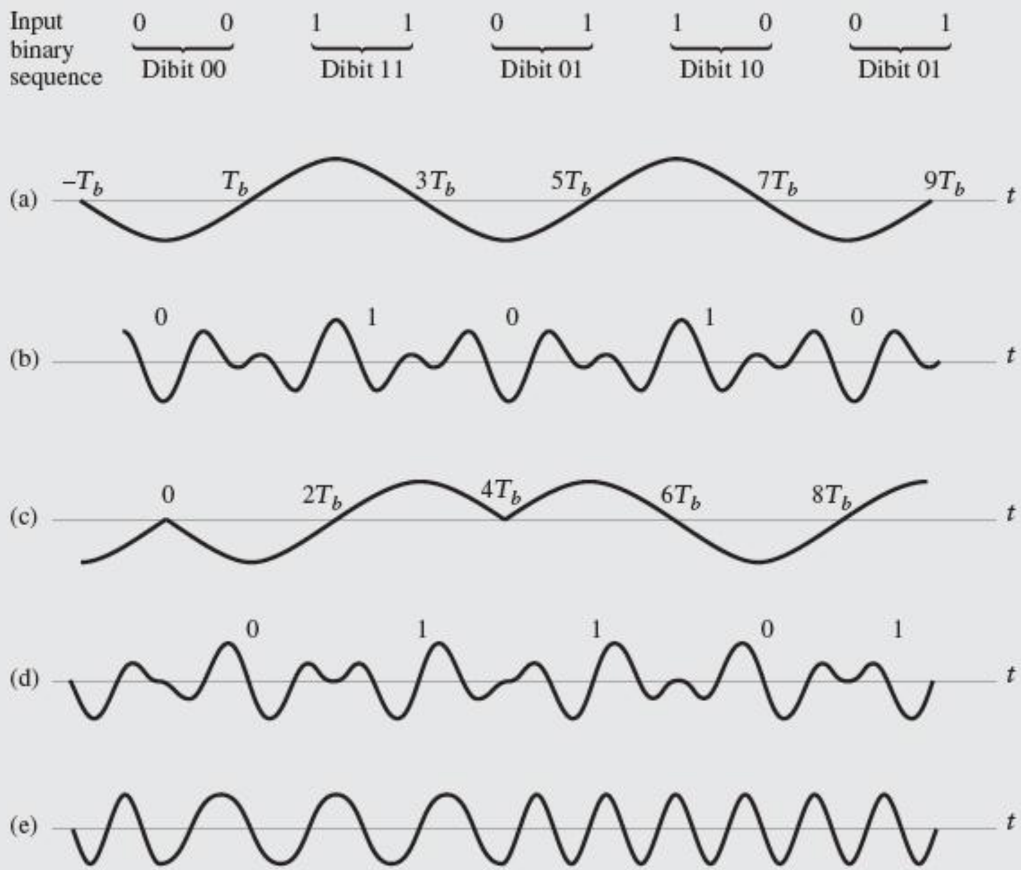


FIGURE 7.14 MSK signal components: (*a*) Modulating signal for in-phase component. (*b*) Modulated waveform of in-phase (*c*) Modulating signal for quadrature component (*d*) Modulated quadrature component. (*e*) Waveform of MSK signal obtained by subtracting (*d*) from (*b*).

- Formulation of Minimum-Shift Keying

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos(\theta(t)) \cos(2\pi f_c t) - \sqrt{\frac{2E_b}{T_b}} \sin(\theta(t)) \sin(2\pi f_c t) \qquad (7.26)$$

(i) $s_I(t) = \sqrt{E_b} \cos(\theta(t))$ is the in-phase (I) component associated with the carrier $\sqrt{2/T_b} \cos(2\pi f_c t)$. (7.27)

(ii) $s_Q(t) = \sqrt{E_b} \sin(\theta(t))$ is the quadrature (Q) component associated with the 90°-phase-shifted carrier. (7.28)

$$s_I(t) = a_1(t) \cos(2\pi f_0 t) \qquad (7.29)$$

$$s_Q(t) = a_2(t) \sin(2\pi f_0 t) \qquad (7.30)$$

$$\theta(t) = -\tan^{-1}\left[\frac{s_Q(t)}{s_I(t)}\right] = -\tan^{-1}\left[\frac{a_2(t)}{a_1(t)} \tan(2\pi f_0 t)\right] \qquad (7.31)$$

1. *$a_2(t)=a_1(t)$*

   This scenario arises when two successive binary symbols in the incoming data stream are the same

$$\theta(t) = -\tan^{-1}[\tan(2\pi f_0 t)]$$
$$= -2\pi f_0 t \qquad (7.32)$$

2. *$a_2(t)=-a_1(t)$*

   This second scenario arises when two successive binary symbols in the incoming data stream are different

$$\theta(t) = -\tan^{-1}[-\tan(2\pi f_0 t)]$$
$$= 2\pi f_0 t \qquad (7.33)$$

$$f_0 = \frac{1}{4T_b} \qquad (7.34)$$

– Given a non-return-to-zero level encoded binary wave *b(t)* of prescribed bit duration T_b and a sinusoidal carrier wave of frequency f_c, we may formulate the MSK signal by proceeding as follows

1. Use the given binary wave b(t) to construct the binary demultiplexed-offset waves $a_1(t)$ and $a_2(t)$
2. Use Eq. (7.34) to determine the frequency f_0
3. Use Eq. (7.29) and (7.30) to determine the in-phase component $s_I(t)$ and quadrature component $s_Q(t)$, respectively from which the MSK signal s(t) follows

- Computer Experinment V : MSK Spectrum
  - The parameters

$$\text{Bit duration,} \quad T_b = 1\,s$$

$$\text{Carrier frequency,} \quad f_c = 8\,Hz$$

1. MSK versus QPSK
   - The main lobe of MSK occupies a frequency band whose width is $1.5/T_b$=1.5Hz
   - The transmission bandwidth of MSK is 50 percent larger than that of QPSK
   - The sidelobes of MSK are considerably smaller than those of QPSK
2. MSK versus Sunde's BFSK
   - The transmission bandwidth of MSK is one half that of Sunde's BFSK
   - Sunde's BFSK exhibits two line components at $f=f_c \pm 1/(2T_b)$
   - The spectrum of MSK is continuous across the whole frequency band

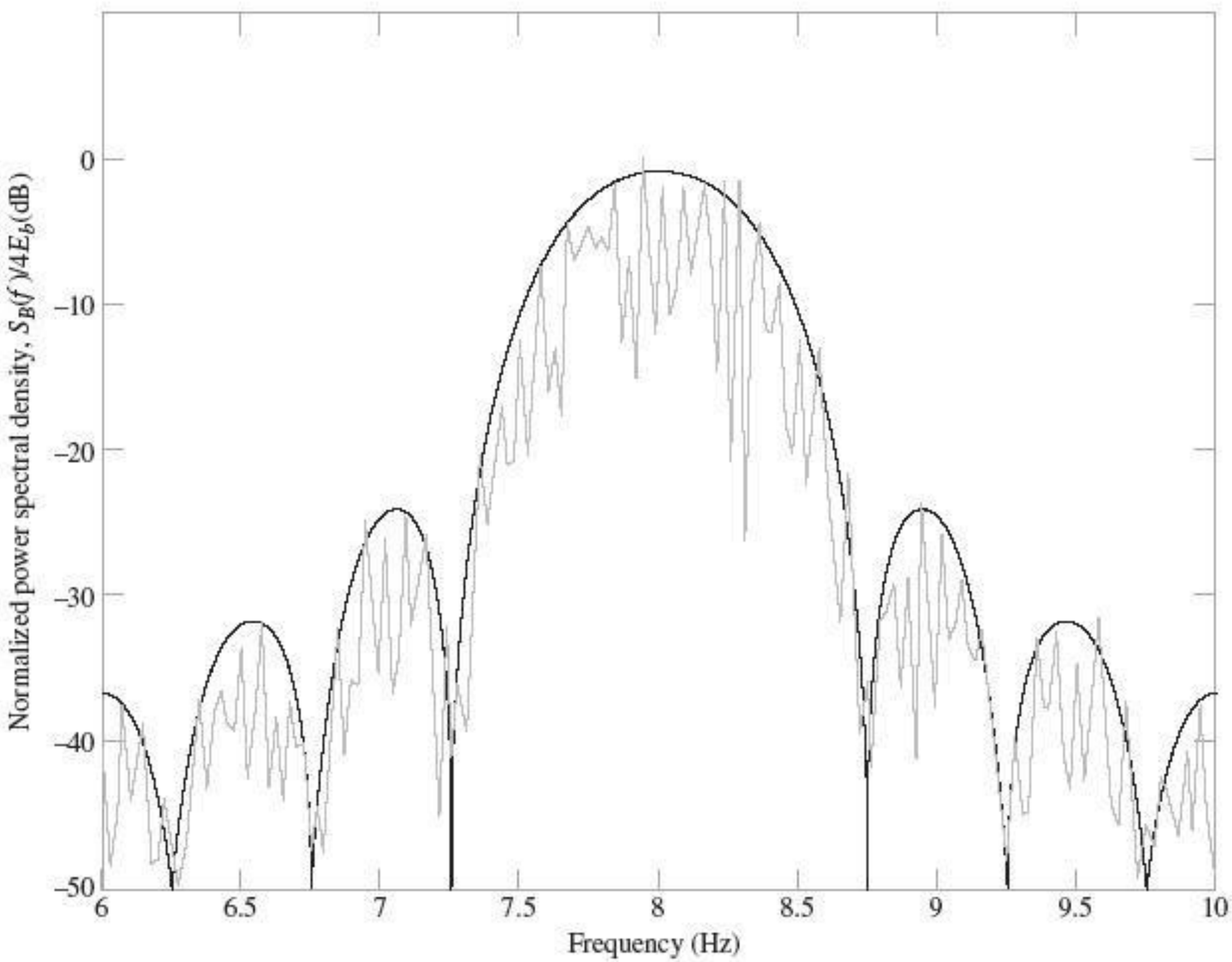


FIGURE 7.15 Power spectrum of MSK produced by square wave as the modulating signal for the following parameters: $f_c = 8$ Hz and $T_b = 1$ s.

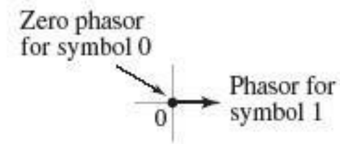
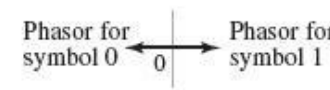
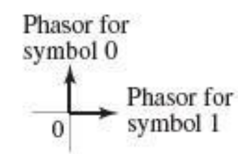
- Although the carrier frequency is not high enough to completely eliminate spectral overlap, the overlap is relatively small as evidenced by
  - The small value of the spectrum at zero frequency
  - The small degree of asymmetry about the carrier frequency f_c=8Hz

Summary of Three Binary Signaling Schemes

1. BASK, BPSK, and BFSK are the digital counterparts of amplitude modulation, phase modulation, and frequency modulation
2. Both BASK and BPSK exhibit discontinuity. It is possible to configure BFSK in such a way that phase continuity is maintained across the entire input binary data stream. The BFSK waveform plotted in part (d) of the figure is an example of minimum-shift keying

- Table 7.2 presents a summary of the three binary modulation schemes

TABLE 7.2 *Summary of Three Basic Binary Modulation Schemes*

Type of modulation scheme	*Variable parameter*	*Definition of modulated wave $s_1(t)$ or $s_2(t)$, for $0 \le t \le T_b$*	*Phasor representation of modulated wave*
1. Binary amplitude-shift keying (BASK)	$\begin{pmatrix}\text{Carrier amplitude}\\ A_c\end{pmatrix} = \begin{cases}\sqrt{\dfrac{2}{T_b}} & \text{for symbol 1}\\ 0 & \text{for symbol 0}\end{cases}$	$s_1(t) = \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t)$ for symbol 1 $s_2(t) = 0$ for symbol 0	
2. Binary phase-shift keying (BPSK)	$\begin{pmatrix}\text{Carrier phase}\\ \phi_c\end{pmatrix} = \begin{cases}0 & \text{for symbol 1}\\ \pi & \text{for symbol 0}\end{cases}$	$s_1(t) = \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t)$ for symbol 1 $s_2(t) = \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t + \pi)$ for symbol 0	
3. Binary frequency-shift keying (BFSK)	$\begin{pmatrix}\text{Carrier frequency}\\ f_c\end{pmatrix} = \begin{cases}f_1 & \text{for symbol 1}\\ f_2 & \text{for symbol 0}\end{cases}$	$s_1(t) = \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_1 t)$ for symbol 1 $s_2(t) = \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_2 t)$ for symbol 0	

Notations

T_b = bit duration

E_b = transmitted signal energy per bit

Carrier: $c(t) = A_c \cos(2\pi f_c t + \phi_c)$

The carrier phase ϕ_c is set equal to zero for both BASK and BFSK.

Non-coherent Digital Modulations Schemes

- Both BASK and BPSK are examples of linear modulation, with increasing complexity in going from BASK and BPSK.
- BFSK is in general an example of nonlinear modulation

- Noncoherent Detection of BASK Signal
  - The system designer would have knowledge of two system parameters
    - The carrier frequency f_c
    - The transmission bandwidth, which is determined by the bit duration T_b.
  - The band-pass filter is designed to have a mid-band frequency equal to the carrier frequency f_c and a bandwidth equal to the transmission bandwidth of the BASK signal.
  - The rise time and decay time of the response of the filter to a rectangular pulse be short compared to the bit duration T_b

1. Band-pass filter produces a pulsed sinusoid for symbol 1, no output for symbol 0.
2. Envelope detector traces the envelope of the filtered version of the BASK signal.
3. Decision-making device working in conjunction with the sampler, regenerates the original binary data stream

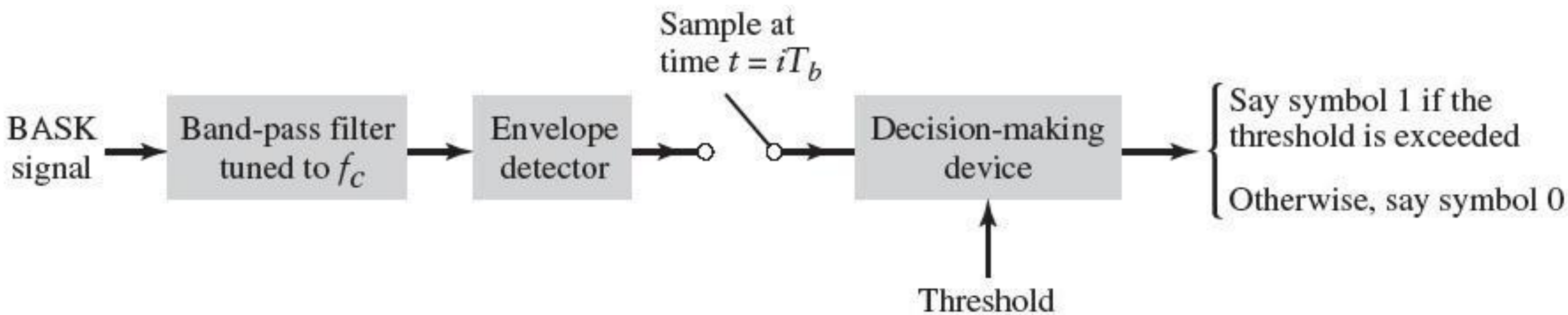


FIGURE 7.17 Noncoherent BASK receiver; the integer i for the sampler equals $0, \pm 1, \pm 2, \ldots$.

- Noncoherent Detection of BFSK Signals
  - The receiver consists of two paths
    - Path 1 : uses a band-pass filter of mid-band frequency f_1. produce the output v_1
    - Path 2 : uses a band-pass filter of mid-band frequency f_2. produce the output v_2
  - The output of the two paths are applied to a comparator

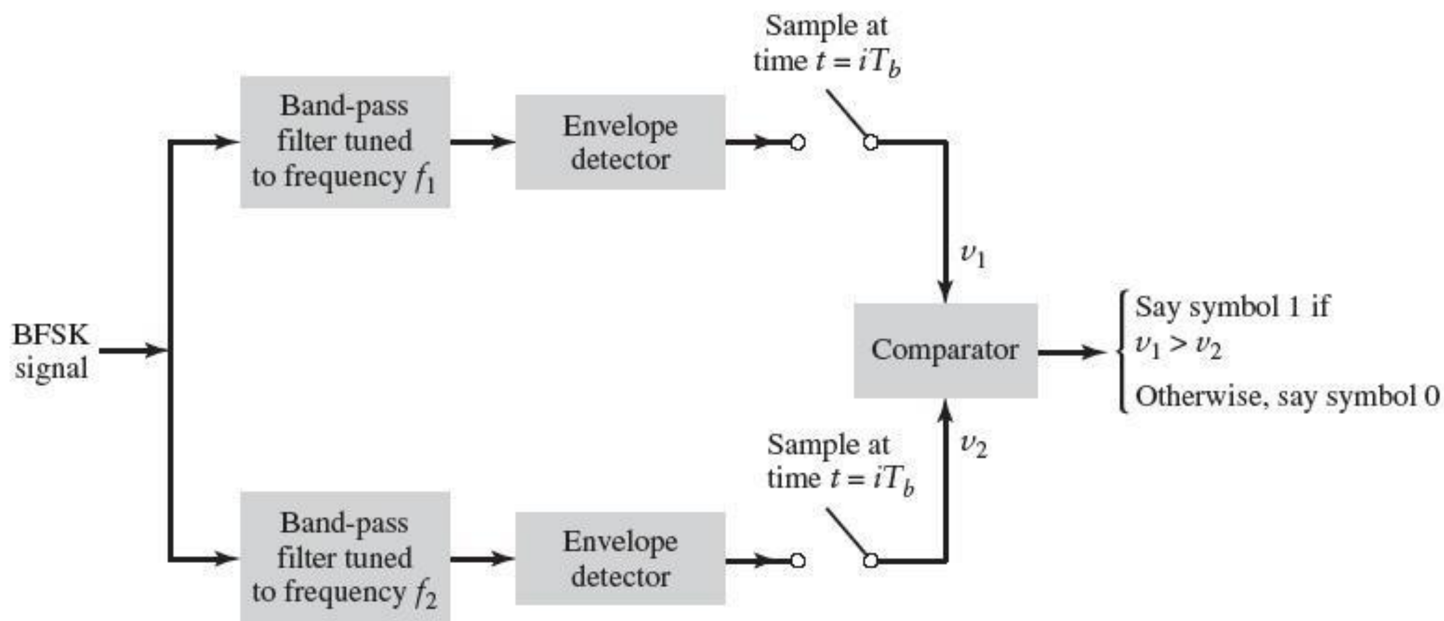


FIGURE 7.18 Noncoherent BFSK receiver; the two samplers operate synchronously, with $i = 0, \pm 1, \pm 2, \ldots$.

- Differential Phase-Shift Keying
  - In the case of phase-shift keying, we cannot have noncoherent detection in the traditional sense because the term “noncoherent” means having t to without carrier-phase information
  - We employ a “pseudo PSK” technique (differential phase-shift keying)
  - DPSK eliminates the need for a coherent reference signal at the receiver by combination two basic operations at the transmitter
    - Differential encoding of the input binary wave
    - Phase-shift keying
  - The receiver is equipped with a storage capability designed to measure the relative phase difference between the waveforms received during two successive bit intervals.
  - The phase difference between waveforms received in two succssive bit intervals will be essentially independent of ϑ.

1. Generation

– The differential encoding process at the transmitter input starts with an arbitrary first bit, serving merely as reference
    - If the incoming binary symbol b_k is 1, then the symbol d_k is unchanged with respect to the previous symbol d_{k-1}
    - If the incoming binary symbol b_k is 0, then the symbol d_k is changed with respect to the previous symbol d_{k-1}

2. Detection

– The phase-modulated pulses pertaining to two successive bits are identical except for a possible sign reversal
– The incoming pulse is multiplied by the preceding pulse
– The preceding pulse serves the purpose of a locally generated reference signal
– Applying the sampled output of the low-pass filter to a decision-making device supplied with a prescribed threshold, detection of the DPSK signal is accomplished.

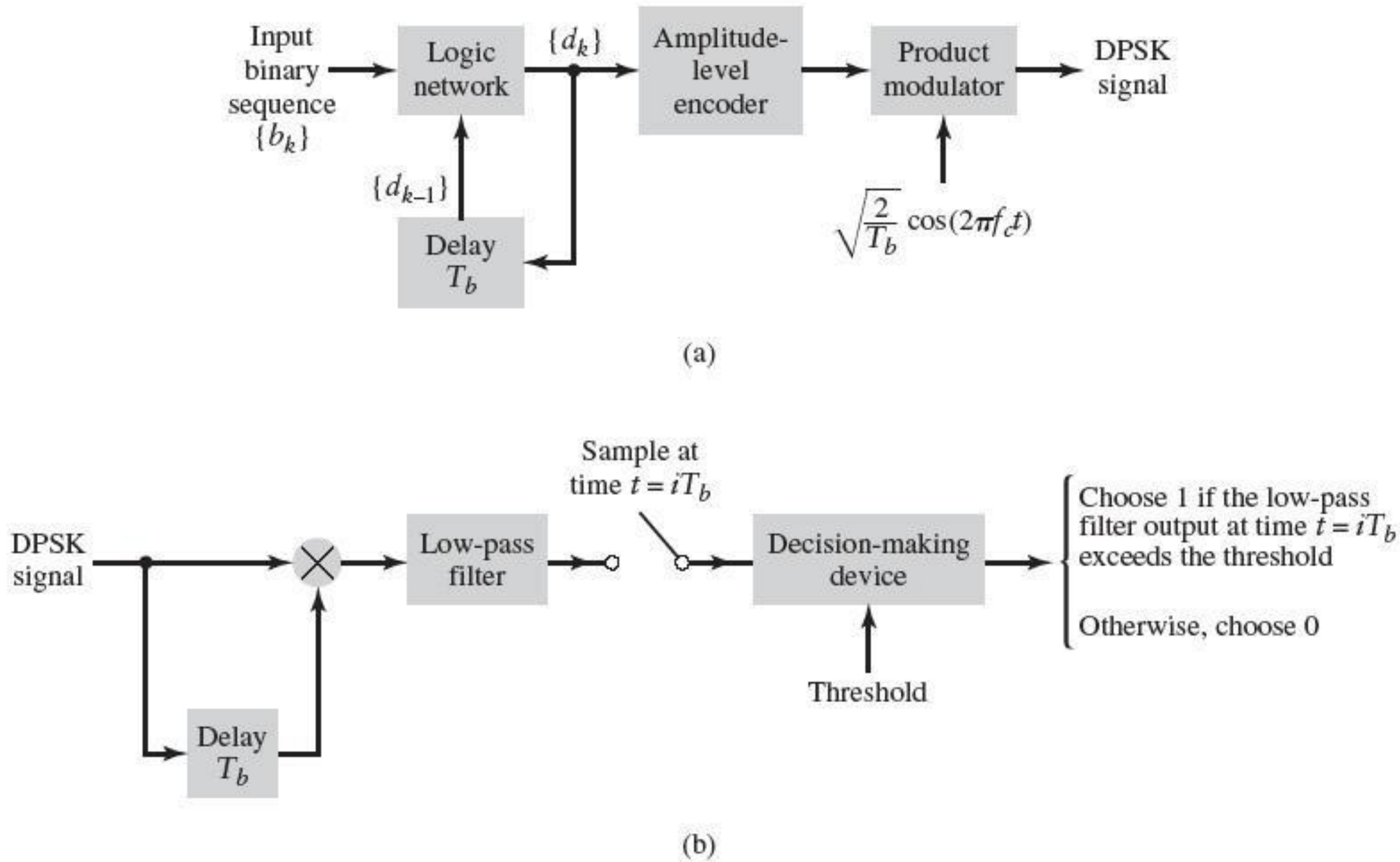


FIGURE 7.19 Block diagrams for (*a*) DPSK transmitter and (*b*) DPSK receiver; for the sampler, integer $i = 0, \pm 1, \pm 2, \ldots$.

TABLE 7.3 ***Illustration of the Generation and Detection of DPSK Signal***

$\{b_k\}$		1	0	0	1	0	0	1	1
$\{d_{k-1}\}$		1	1	0	1	1	0	1	1
Differentially encoded sequence $\{d_k\}$	1	1	0	1	1	0	1	1	1
Transmitted phase (radians)	0	0	π	0	0	π	0	0	0
Sampler's output (polarity)		+	−	−	+	−	−	+	+
Binary symbol at decision-maker's output		1	0	0	1	0	0	1	1

Note: The symbol 1 inserted at the beginning of the differentially encoded sequence d_k is the reference bit.

M-ary Digital Modulation Schemes

- we send any one of M possible signals during each signaling interval of duration T
- The requirement is to conserve bandwidth at the expense of both increased power and increased system complexity
- When the bandwidth of the channel is less than the required value, we resort to an M-ary modulation scheme for maximum bandwidth conservation

- M-ary Phase-Shift Keying
  - If we take blocks of m bits to produce a symbol and use an M-ary PSK scheme with $M=2^m$ and symbol duration $T=mT_b$
  - The bandwidth required is proportional to $1/(mT_b)$
  - The use of M-ary PSK provides a reduction in transmission bandwidth by a factor by a factor $m=log_2M$

– The discrete coefficients are respectively referred to as the in-phase and quadrature components of the M-ary PSK singal

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{2\pi}{M} i\right), \quad \begin{matrix} i = 0,1,\ldots, M-1 \\ 0 \le t \le T \end{matrix} \qquad (7.35)$$

$$s_i(t) = \left[\sqrt{E} \cos\left(\frac{2\pi}{M} i\right)\right]\left[\sqrt{\frac{2}{T}} \cos(2\pi f_c t)\right]$$

$$- \left[\sqrt{E} \sin\left(\frac{2\pi}{M} i\right)\right]\left[\sqrt{\frac{2}{T}} \sin(2\pi f_c t)\right], \quad \begin{matrix} i = 0,1,\ldots, M-1 \\ 0 \le t \le T \end{matrix} \qquad (7.36)$$

$$\left\{\left[\sqrt{E} \cos\left(\frac{2\pi}{M} i\right)\right]^2 + \left[\sqrt{E} \sin\left(\frac{2\pi}{M} i\right)\right]^2\right\}^{1/2} = \sqrt{E}, \quad \text{for all } i \qquad (7.37)$$

- Signal-Space Diagram
  - Pair of orthogonal functions

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t), \qquad 0 \le t \le T \qquad (7.38)$$

$$\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t), \qquad 0 \le t \le T \qquad (7.39)$$

1. M-ary PSK is described in geometric terms by a constellation of M signal points distributed uniformly on a circle of radius $\sqrt{E}$
2. Each signal point in the figure corresponds to the signal $s_i(t)$ of Eq. (7.35) for a particular value of the index i.
3. The squared length from the origin to each signal point is equal to the signal energy E.

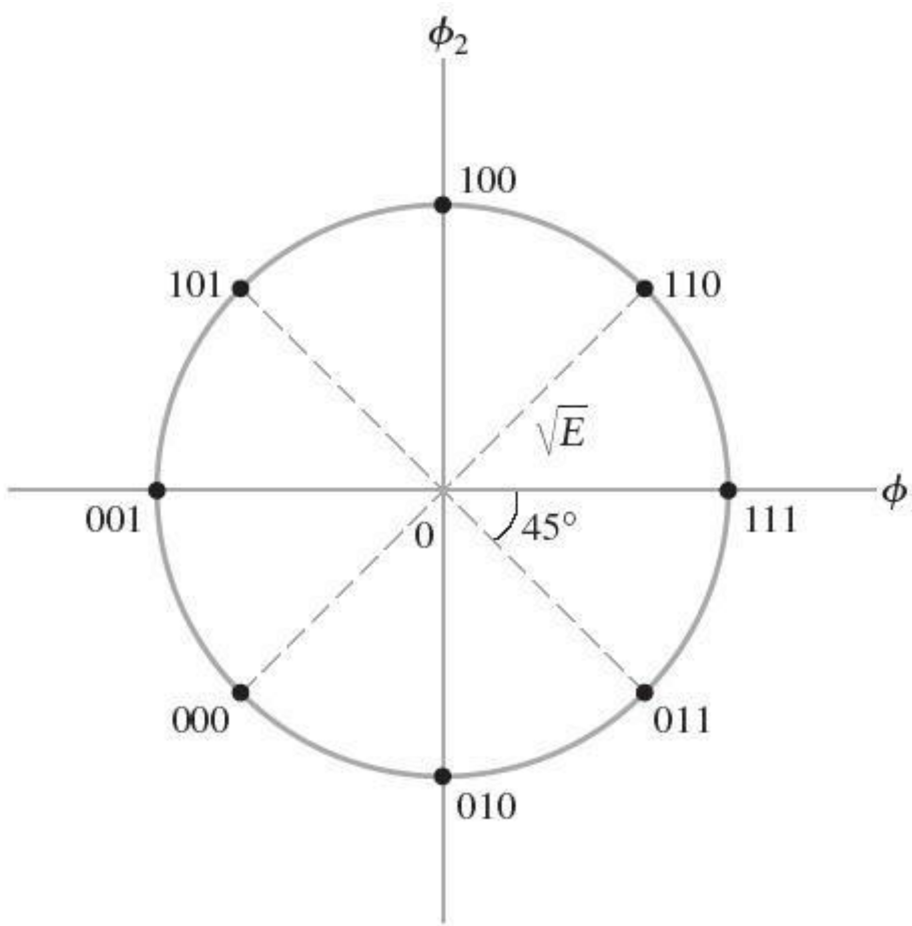


Figure 7.20 Signal-space diagram of 8-PSK.

- M-ary Quadrature Amplitude Modulation (QAM)
  - The mathematical description of the new modulated signal

$$s_i(t) = \sqrt{\frac{2E_0}{T}}a_i\cos(2\pi f_c t) - \sqrt{\frac{2E_0}{T}}b_i\sin(2\pi f_c t), \qquad \begin{array}{c} i = 0,1,\ldots,M-1 \\ 0 \le t \le T \end{array} \qquad (7.40)$$

  - The level parameter for in-phase component and quadrature component are independent of each other for all I
  - M-ary QAM is a hybrid form of M-ary modulation
  - M-ary amplitude-shift keying (M-ary ASK)
    - If b_i=0 for all i, the modulated signal $s_i(t)$ of Eq. (7.40) reduces to

$$s_i(t) = \sqrt{\frac{2E_0}{T}}a_i\cos(2\pi f_c t) \qquad i = 0,1,\ldots,M-1$$

  - M-ary PSK
    - If E0=E and the constraint is satisfied

$$(Ea_i^2 + Eb_i^2)^{1/2} = \sqrt{E}, \quad \text{for all } i$$

- Signal-Space Diagram
  - the signal-space representation of M-ary QAM for M=16
  - Unlike M-ary PSK, the different signal points of M-ary QAM are characterized by different energy levels
  - Each signal point in the constellation corresponds to a specific quadbit

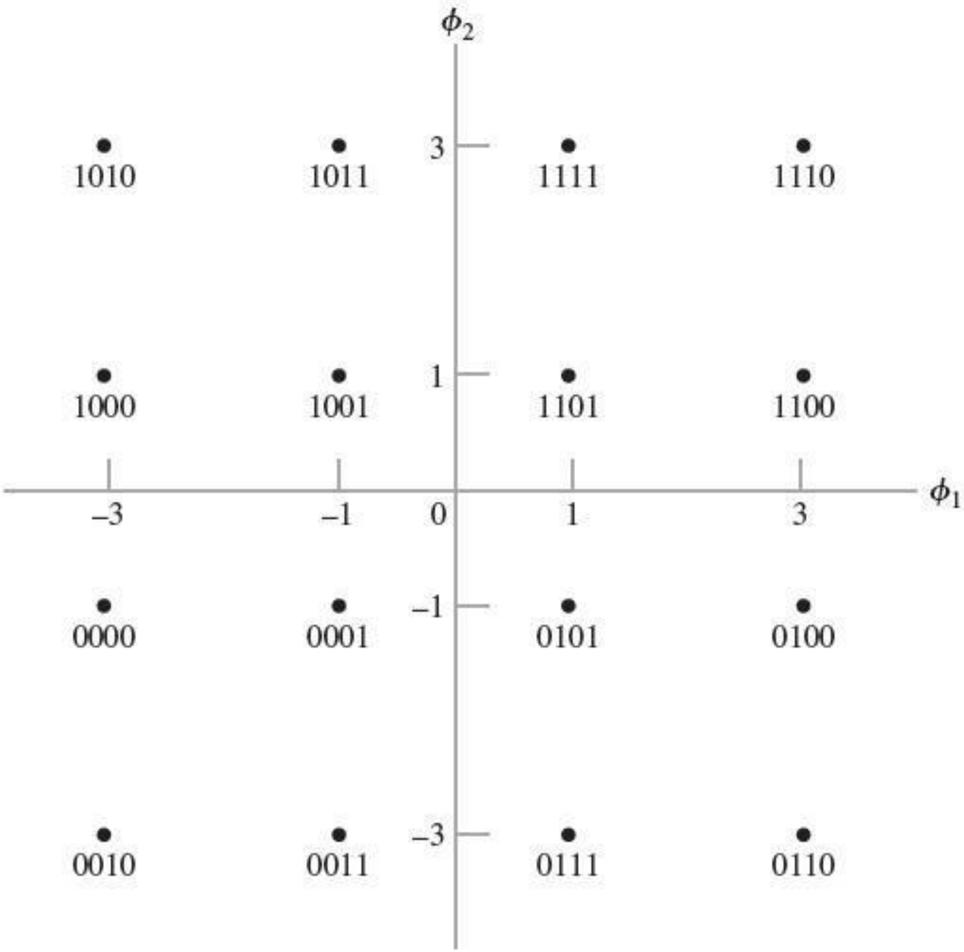


FIGURE 7.21 Signal-space diagram of Gray-encoded M-ary QAM for $M = 16$.

- M-ary Frequency-Shift Keying
  - In one form of M-ary FSK, the transmitted signals are defined for some fixed integer *n* as

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[\frac{\pi}{T}(n+i)t\right], \quad \begin{array}{l} i = 0,1,\ldots, M-1 \\ 0 \le t \le T \end{array} \qquad (7.41)$$

  - Like M-ary PSK, the envelope of M-ary FSK is constant for all M

$$\int_0^T s_i(t)\, s_j(t)\, dt = \begin{cases} E & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases} \qquad (7.42)$$

  - Signal-Space Diagram
    - Unlike M-ary PSK and M-ary QAM, M-ary FSK is described by an M-dimensional signal-space diagram

$$\phi_i(t) = \frac{1}{\sqrt{E}} s_i(t) \quad \begin{array}{l} i = 0,1,\ldots, M-1 \\ 0 \le t \le T \end{array} \qquad (7.43)$$

1. Correlating the signal

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t) \qquad \text{for symbol } 1 \qquad (7.45)$$

$$s_1 = \int_0^{T_b} \phi_1(t) s_1(t)\,dt$$

$$= \int_0^{T_b} \frac{2}{T_b}\sqrt{E_b}\cos^2(2\pi f_c t)\,dt \qquad (7.46)$$

$$s_1 = \sqrt{E_b} \qquad (7.47)$$

under the band-pass assumption,

$$s_2(t) = -\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t) \qquad \text{for symbol } 0 \qquad (7.48)$$

$$s_2 = -\sqrt{E_b} \qquad (7.49)$$

2. We may show that the signal

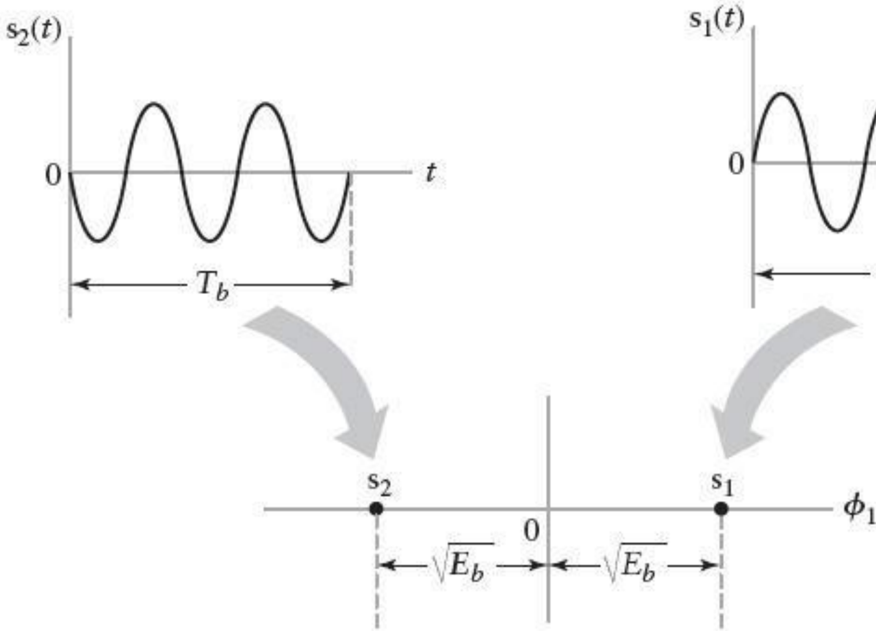


FIGURE 7.23 Mapping of BPSK signal onto a one-dimensional signal-space diagram.

- As with BPSK, the signal-space diagram consists of two transmitted signal points

$$s_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix} \quad (7.50) \qquad s_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix} \quad (7.51)$$

- Fig. 7.23 and 7.24 differ in one important respect : dimensionality

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_1 t) \quad (7.52) \qquad \phi_2(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_2 t) \quad (7.53)$$

1. The separation between the transmitted signal points for BPSK is √2 times that for BFSK
2. The received signal point lies inside a “cloud” centered on the transmitted signal point

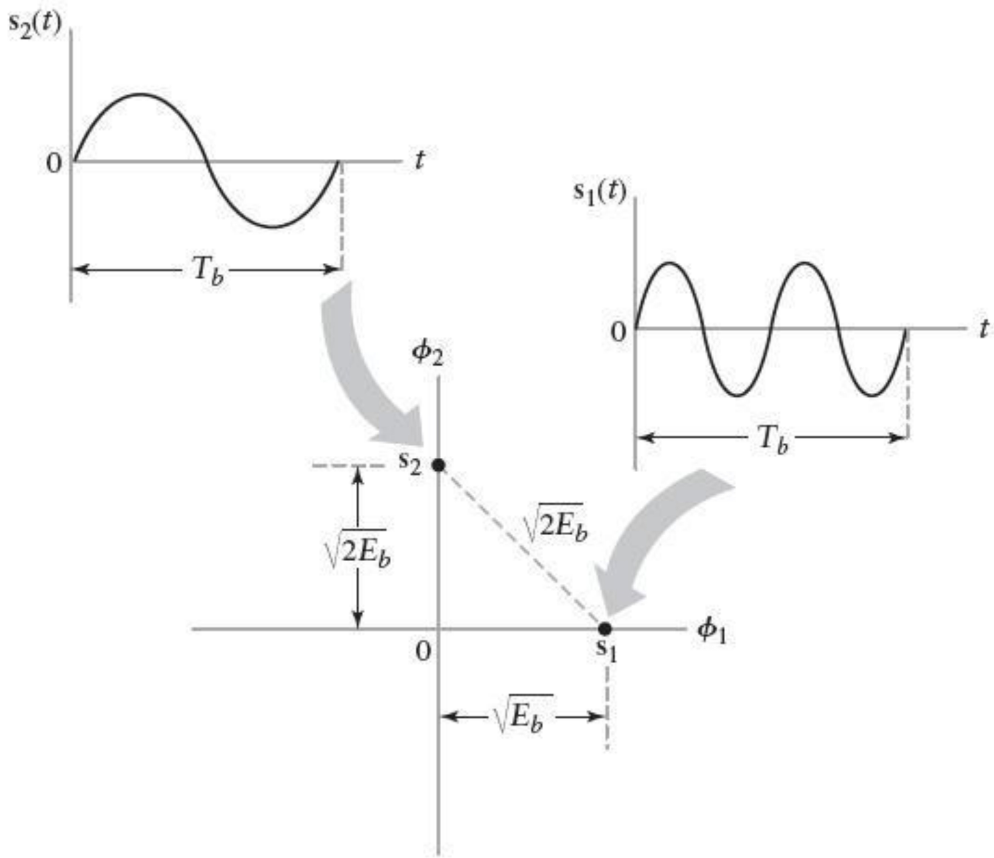


FIGURE 7.24 Mapping of BFSK onto two-dimensional signal-space diagram.

Digital Baseband Transmission

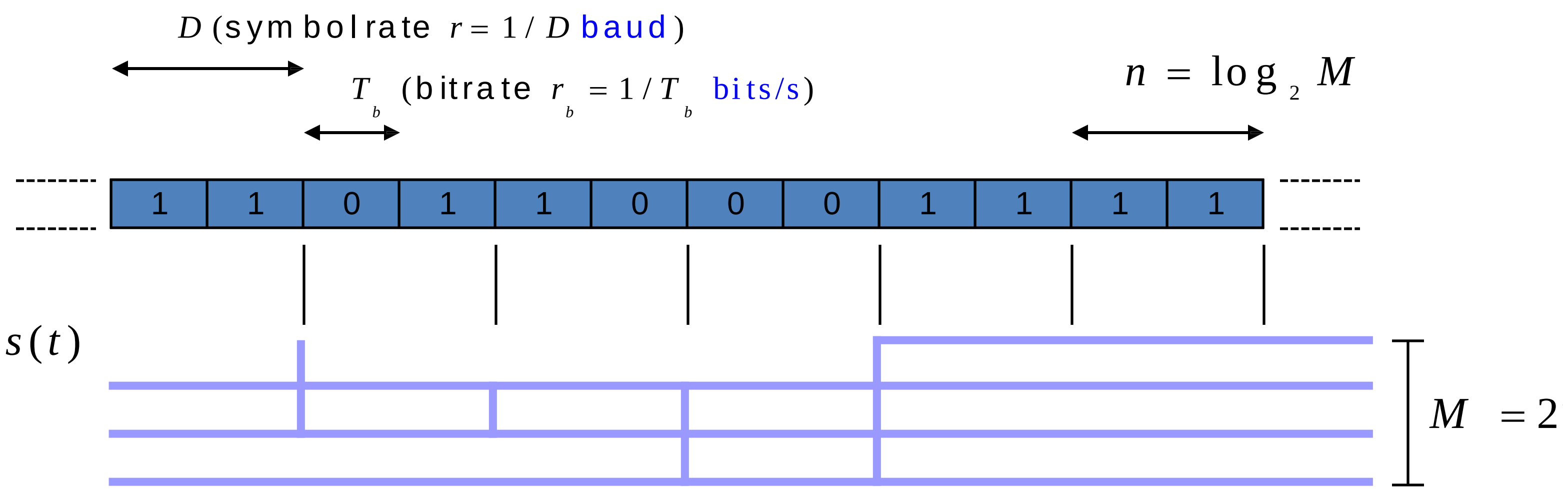
- Why to apply digital transmission?
- Symbols and bits
- Baseband transmission
  - Binary error probabilities in baseband transmission
- Pulse shaping
  - minimizing ISI and making bandwidth adaptation - cos roll-off signaling
  - maximizing SNR at the instant of sampling - matched filtering
  - optimal terminal filters
- Determination of transmission bandwidth as a function of pulse shape
  - Spectral density of Pulse Amplitude Modulation (PAM)
- Equalization - removing residual ISI - eye diagram

Why to Apply Digital Transmission?

- Digital communication withstands channel noise, interference and distortion better than analog system. For instance in PSTN inter-exchange STP*-links NEXT (Near-End Cross-Talk) produces several interference. For analog systems interference must be below 50 dB whereas in digital system 20 dB is enough. With this respect digital systems can utilize lower quality cabling than analog systems
- Regenerative repeaters are efficient. Note that cleaning of analog-signals by repeaters does not work as well
- Digital HW/SW implementation is straightforward
- Circuits can be easily reconfigured and preprogrammed by DSP techniques (an application: software radio)
- Digital signals can be coded to yield very low error rates
- Digital communication enables efficient exchanging of SNR to BW-> easy adaptation into different channels
- The cost of digital HW continues to halve every two or three years

STP: Shielded twisted pair

Symbols and Bits



Generally: $s(t) = \sum_k a_k \, p(t - kD)$ (a PAM* signal)

For *M*=2 (binary signalling): $s(t) = \sum_k a_k \, p(t - kT_b)$

$$\begin{cases} n: \text{number of bits} \\ M: \text{number of levels} \\ D: \text{Symbol duration} \\ T_b: \text{Bit duaration} \end{cases}$$

For non-Inter-Symbolic Interference (ISI), *p(t)* must satisfy:

$$p(t) = \begin{cases} 1, & t = 0 \\ 0, & t = \pm D, \pm 2D \ldots \end{cases}$$

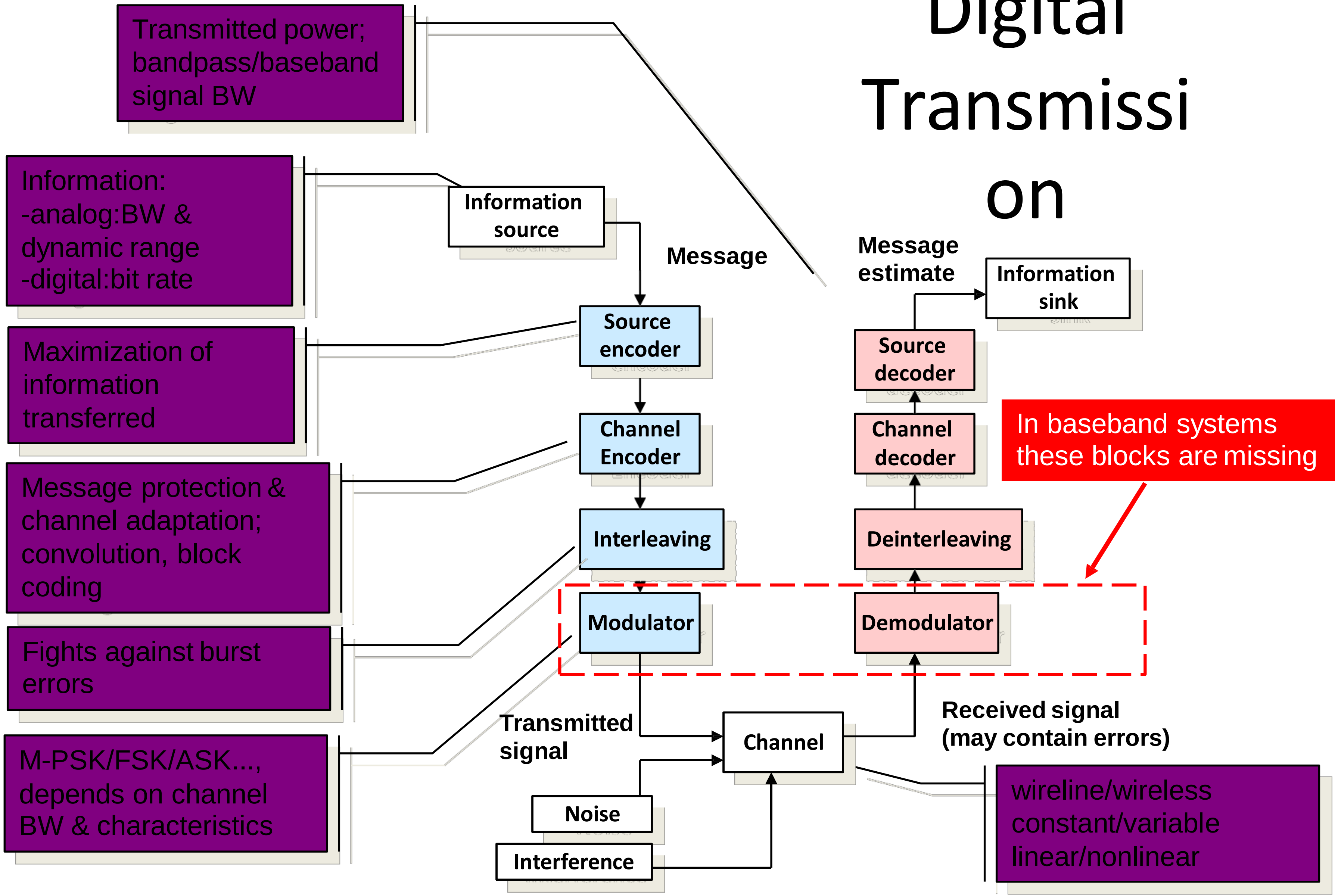
unipolar, 2-level pulses

This means that at the instant of decision

$$s(t) = \sum_k a_k \, p(t - kD) = a_k$$

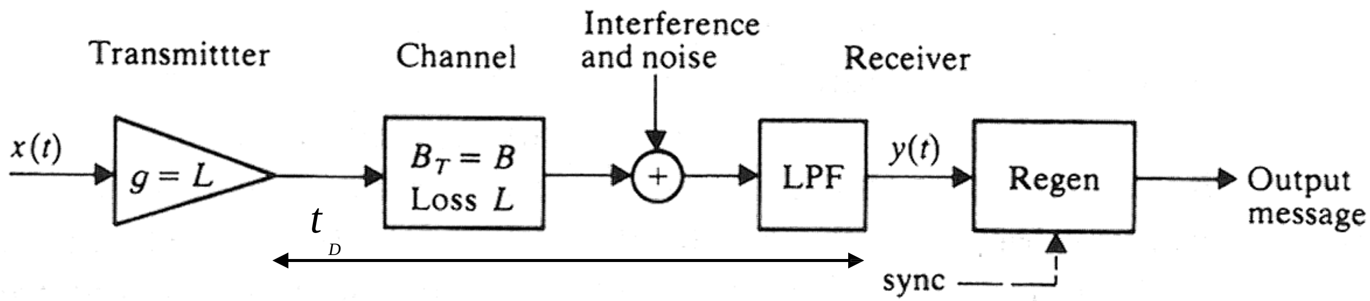
*Pulse Amplitude Modulation

Digital Transmission

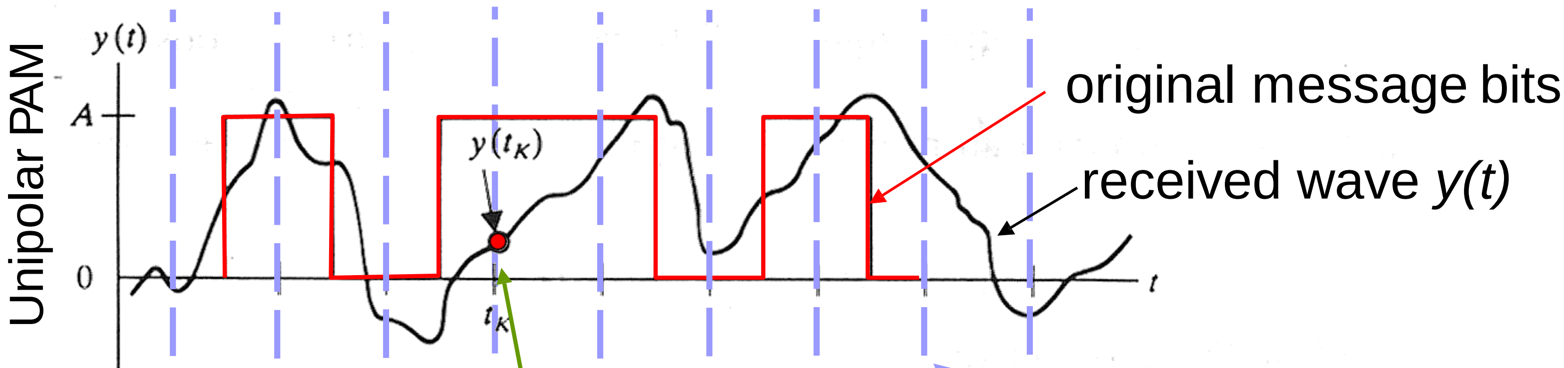


- ‘Baseband’ means that no carrier wave modulation is used for transmission

Baseband Digital Transmission Link



(a)



(b)

decision instances

$$y(t) = \sum_k a_k p(t - t_d - kD) + n(t)$$

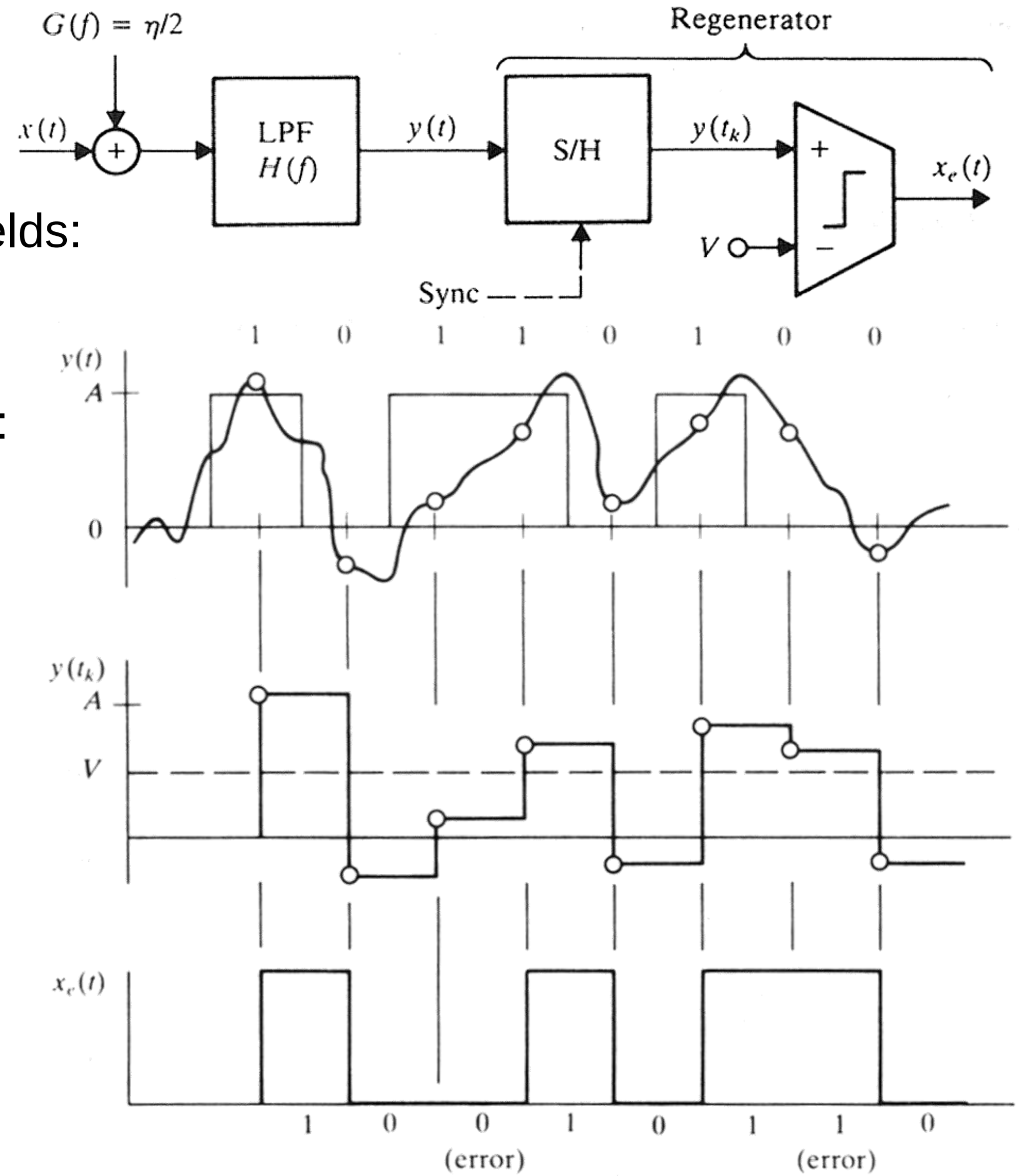
message reconstruction at $t_K = KD + t_d$ yields

$$y(t_K) = a_k + \sum_{k \neq K} a_k \tilde{p}(KD - kD) + n(t)$$

message ISI

Gaussian bandpass noise

Baseband Unipolar Binary Error Probability



Assume binary & unipolar *x(t)*

The sample-and-hold circuit yields:

$$\text{r.v. } Y : y(t_k) = a_k + n(t_k)$$

Establish H_0 and H_1 hypothesis:

$$H_0 : a_k = 0, Y = n$$

$$p_Y(y \mid H_0) = p_N(y)$$

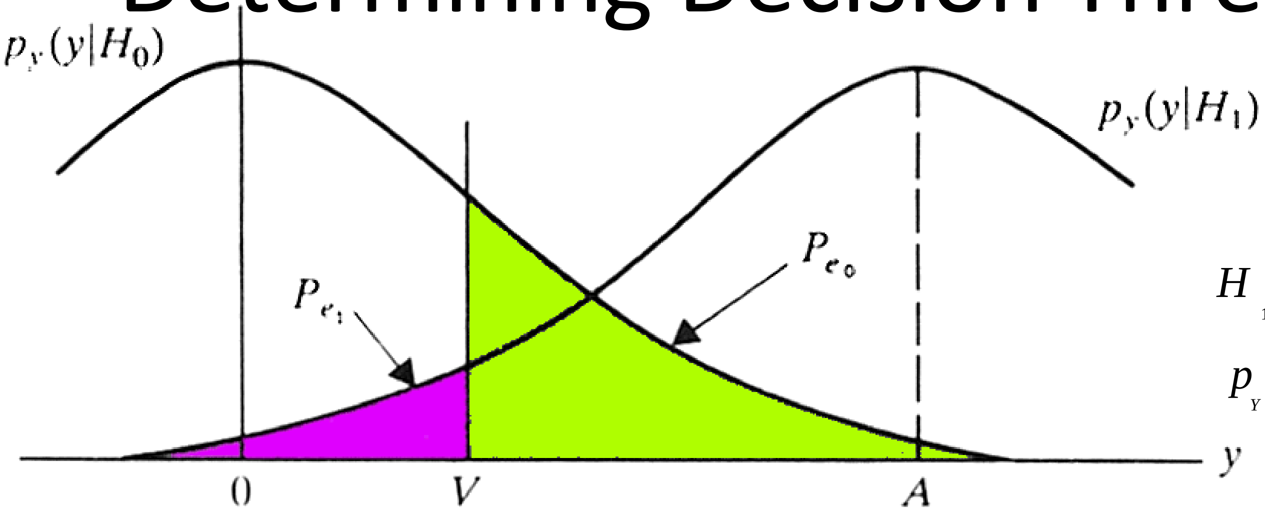
and

$$H_1 : a_k = 1, Y = A + n$$

$$p_Y(y \mid H_1) = p_N(y - A)$$

$p_N(y)$: Noise spectral density

Determining Decision Threshold



$H_0 : a_k = 0, Y = n$

$p_Y(y \mid H_0) = p_N(y)$

$H_1 : a_k = 1, Y = A + n$

$p_Y(y \mid H_1) = p_N(y - A)$

The comparator implements decision rule:

$$p_{e1} \equiv P(Y < V \mid H_1) = \int_{-\infty}^{V} p_Y(y|H_1)dy$$

$$p_{eo} \equiv P(Y > V \mid H_0) = \int_{V}^{\infty} p_Y(y|H_0)dy$$

Transmitted ‘0’ but detected as ‘1’

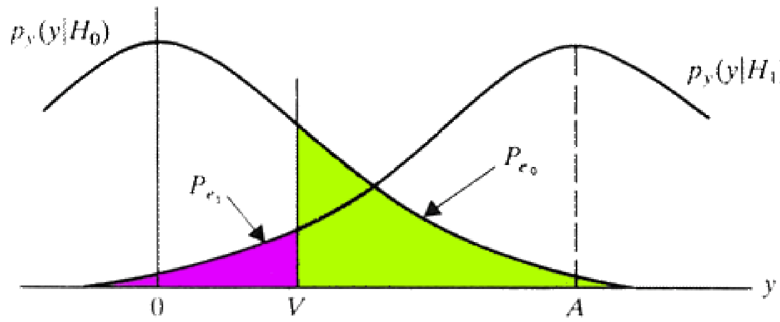
Average error error probability: $P_e = P_0 P_{e0} + P_1 P_{e1}$

$P_0 = P_1 = 1/2 \Rightarrow P_e = \frac{1}{2}(P_{e0} + P_{e1})$

Assume Gaussian noise:

$$p_N(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

Determining Error Rate



$$p_{e0} = \int_V^{\infty} p_N(y)dy$$

$$p_{e0} = \frac{1}{\sigma\sqrt{2\pi}}\int_V^{\infty} \exp\left(-\frac{x^2}{2\sigma^2}\right)dx$$

that can be expressed by using the Q-function, defined by

$$Q(k) = \frac{1}{\sqrt{2\pi}}\int_k^{\infty} \exp\left(-\frac{\lambda^2}{2}\right)d\lambda \Rightarrow$$

$$\sigma p_{e0} = \frac{1}{\sqrt{2\pi}}\int_V^{\infty} \exp\left(-\frac{x^2}{2\sigma^2}\right)dx = \sigma Q\left(\frac{V}{\sigma}\right)$$

and therefore

$$p_{e0} = Q\left(\frac{V}{\sigma}\right)$$ and also $$P_{e1} = \int_{-\infty}^{V} p_N(y-A)dy = Q\left(\frac{A-V}{\sigma}\right)$$

Baseband Binary Error Rate in Terms of Pulse Shape and γ

setting *V=A/2* yields then

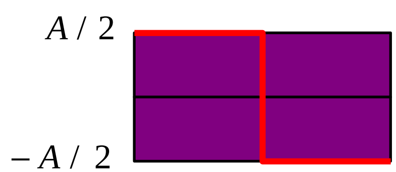
$$p_e = \tfrac{1}{2}(p_{e0} + p_{e1}) = p_{e0} = p_{e1} \Rightarrow \boxed{p_e = Q\left(\frac{A}{2\sigma}\right)}$$

for unipolar, **rectangular** NRZ [0,A] bits

$$S_R = \overline{x_{DC}^2} + \sigma^2 = A^2/2$$

for polar, **rectangular** NRZ [-A/2,A/2] bits

$$S_R = \underbrace{\overline{x_{DC}^2}}_{=0} + \sigma^2 = A^2/4$$



$$\overline{a_k^2} = \frac{2}{T}\int_0^{T/2}\left(\frac{A}{2}\right)^2 = \frac{\cancel{2T}}{\cancel{2T}}\left(\frac{A}{2}\right)^2 = \frac{A^2}{4}$$

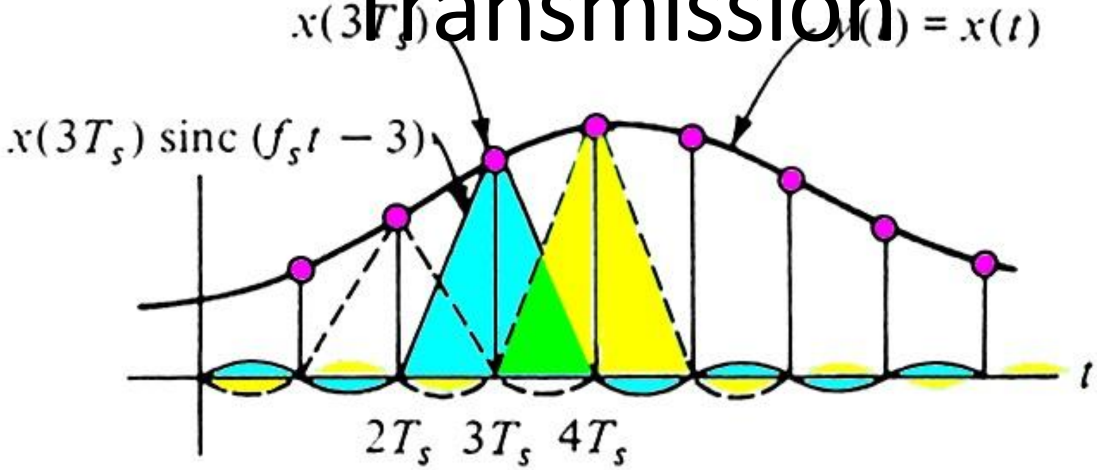
and hence

$$\left(\frac{A}{2\sigma}\right)^2 = \frac{A^2}{4N_R} = \begin{cases} S_R/(2N_R), \text{unipolar} \\ S_R/N_R, \text{polar} \end{cases}$$

$$\begin{cases} \gamma_b = E_b/N_0 = S_R/N_0 r_b \\ N_R = N_0 r_b/2 \end{cases} \Rightarrow \begin{cases} 2\gamma_b N_0 r_b/(2N_0 r_b) = \gamma_b, \text{unipolar} \\ 2\gamma_b N_0 r_b/N_0 r_b = 2\gamma_b, \text{polar} \end{cases}$$

Note that $N_R = N_0 B_N \geq N_0 r_b/2$ (lower limit with sinc-pulses (see later))

Pulse Shaping and Band-limited Transmission



- In digital transmission signaling pulse shape is chosen to satisfy the following requirements:
  – yields **maximum SNR at the time instance of decision** (matched filtering)
  – accommodates signal **to channel bandwidth:**
    - rapid **decrease of pulse energy** outside the main lobe in frequency domain alleviates filter design
    - **lowers cross-talk** in multiplexed systems

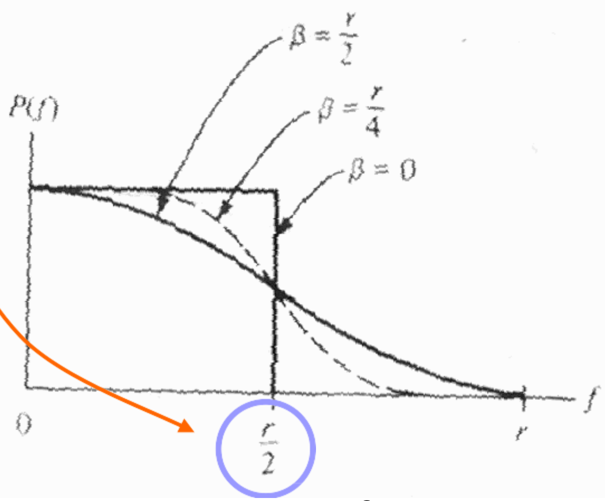
Signaling With Cosine Roll-off Signals

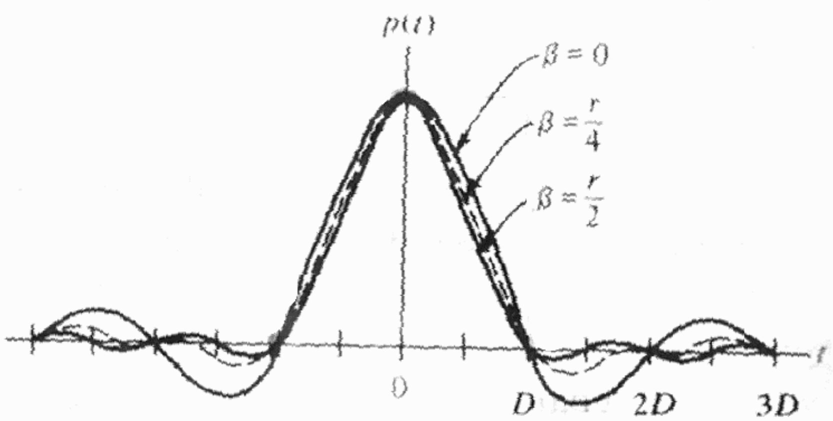
- **Maximum transmission rate** can be obtained with sinc-pulses

$$\begin{cases} p(t) = \mathrm{sinc}(rt) = \mathrm{sinc}(t/D) \\ P(f) = F[p(t)] = \dfrac{1}{r}\Pi\left(\dfrac{f}{r}\right) \end{cases}$$

- However, they are not time-limited. A more practical choice is the **cosine roll-off signaling**:

for **raised cos-pulses** $\beta=r/2$



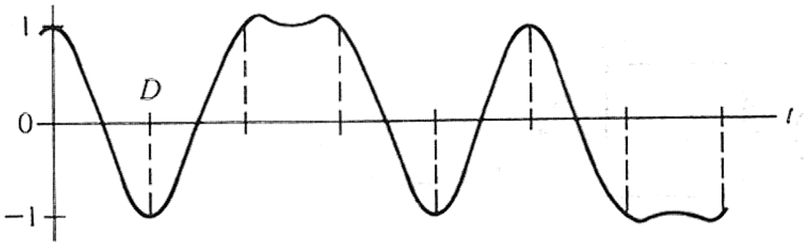


$$P(f)\Big|_{\beta=r/2} = \frac{1}{r}\cos^2\frac{\pi f}{2r}\Pi(f/2r)$$

$$p(t) = \frac{\cos 2\pi\beta t}{1-(4\beta t)^2}\mathrm{sinc}\, rt$$

Example

- By using $\beta = r/2$ and polar signaling, the following waveform is obtained:



- Note that the zero crossing are spaced by *D* at

$$t = \pm 0.5D, \pm 1.5D, \pm 2.5D, \ldots.$$

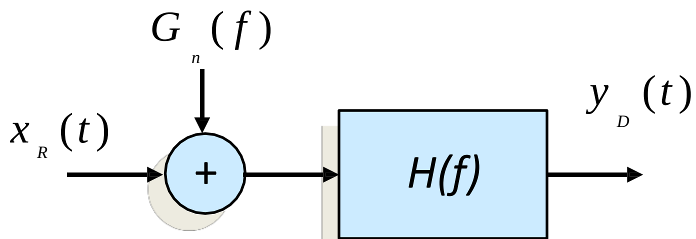
(this could be seen easily also in eye-diagram)

- The zero crossing are easy to detect for clock recovery. Note that unipolar baseband signaling involves performance penalty of 3 dB compared to polar signaling:

$$p_e = \begin{cases} Q(\sqrt{\gamma_b}), \text{ unipolar } [0/1] \\ Q(\sqrt{2\gamma_b}), \text{ polar}[\pm 1] \end{cases}$$

Matched Filtering

$$\begin{cases} x_R(t) = A_R\, p(t - t_0) \\ X_R(f) = A_R P_R(f) \exp(-j\omega t_0) \end{cases}$$



$$E_R = \int_{-\infty}^{\infty} \left| X_R(f) \right|^2 df = A_R^2 \int_{-\infty}^{\infty} \left| P(f) \right|^2 df$$

$$A = F^{-1}[H(f) X_R(f)] \Big|_{t = t_0 + t_d}$$

$$= A_R \int_{-\infty}^{\infty} H(f) P(f) \exp\left(j\omega t_d\right) df$$ Peak amplitude to be maximized

$$\sigma^2 = \int_{-\infty}^{\infty} \left| H(f) \right|^2 G_n(f)\, df = \frac{\eta}{2} \int_{-\infty}^{\infty} \left| H(f) \right|^2 df$$ Post filter noise

$$\left(\frac{A}{\sigma}\right)^2 = A_R^2 \frac{\left| \int_{-\infty}^{\infty} H(f) P(f) \exp\left(j\omega t_d\right) df \right|^2}{\frac{\eta}{2} \int_{-\infty}^{\infty} \left| H(f) \right|^2 df}$$ Should be maximized

Matched Filtering SNR and Transfer Function

$$\frac{\left| \int_{-\infty}^{\infty} V(f) W^*(f)\, df \right|^2}{\int_{-\infty}^{\infty} |V(f)|^2\, df} \le \int_{-\infty}^{\infty} |W(f)|^2\, df$$

Schwartz's inequality applies when

$$V(f) = K W^*(f)$$

$$\left(\frac{A}{\sigma}\right)^2 = A_R^2 \frac{\left| \int_{-\infty}^{\infty} H(f) P(f) \exp(j\omega t_d)\, df \right|^2}{\frac{\eta}{2} \int_{-\infty}^{\infty} |H(f)|^2\, df}$$

SNR at the moment of sampling

$$V(f) = H(f)$$
$$W^*(f) = P(f) \exp(j\omega t_d)$$

$$\Rightarrow H(f) = K P(f) \exp(j\omega t_d)$$

impulse response is: $\Rightarrow h(t) = K p(t_d - t)$

Considering right hand side yields max SNR

$$\left.\left(\frac{A}{\sigma}\right)^2\right|_{MAX} = \frac{2 A_R^2}{\eta} \int_{-\infty}^{\infty} |W(f)|^2\, df = \frac{2 A_R^2 \int_{-\infty}^{\infty} |P(f)|^2\, df}{\eta}$$

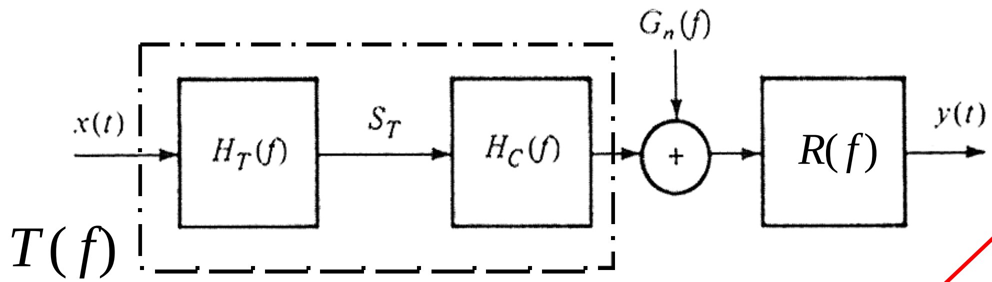
pulse energy

Optimum terminal filters

- Assume
  - arbitrary TX pulse shape *x(t)*
  - arbitrary channel response $h_c(t)$
  - multilevel PAM transmission

$$\begin{cases} P_x : \text{transmitting waveform} \\ H_T : \text{transmitter shaping filter} \\ H_C : \text{channel transfer function} \\ R : \text{receiver filter} \end{cases}$$

- What kind of filters are required for TX and RX to obtain matched filtered, non-ISI transmission?



Nyqvist shaped pulse in *y(t)*

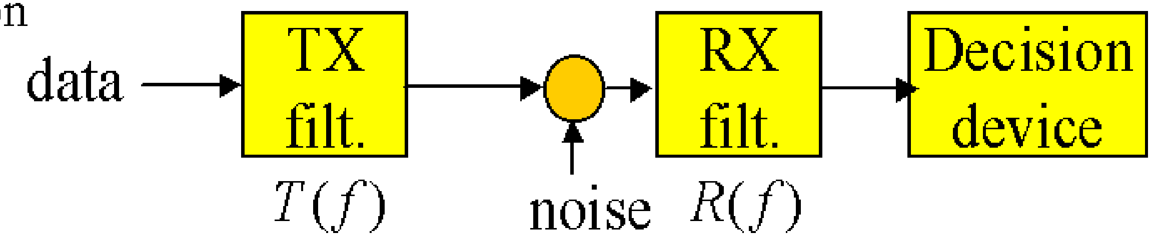
- The following condition must be fulfilled:

$$P_x(f)H_T(f)H_C(f)R(f) = P(f)\exp(-j\omega t_d)$$

that means that undistorted transmission is obtained

Avoiding ISI and enabling band-limiting in radio systems

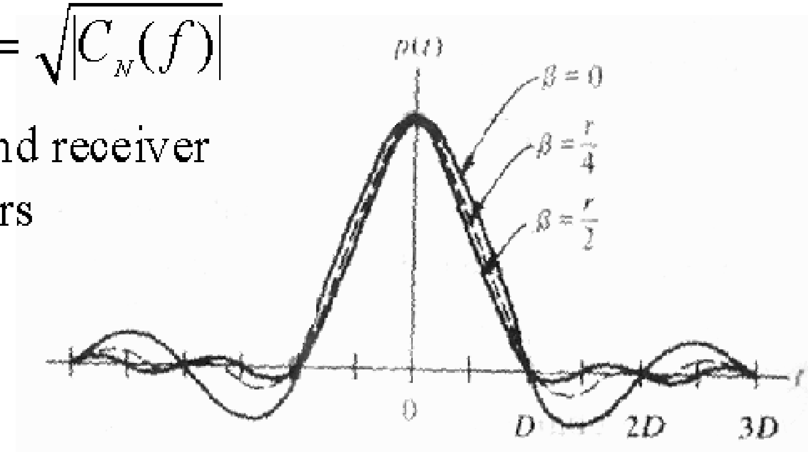
- Two goals to achieve: band limited transmission & matched filter reception



$$\begin{cases} T(f)R(f) \equiv C_N(f), & \text{raised-cos shaping} \\ T(f) = R^*(f), & \text{matched filtering} \end{cases}$$

$$\Rightarrow |R(f)| = |T(f)| = \sqrt{|C_N(f)|}$$

- Hence at the transmitter and receiver alike **root-raised** cos-filters must be applied



raised cos-spectra $C_N(f)$

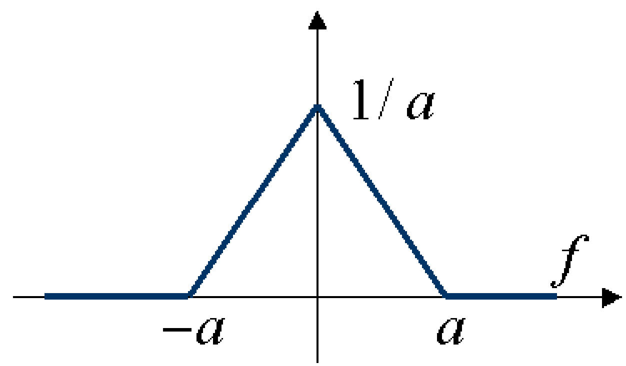
Determining Transmission Bandwidth for an Arbitrary Baseband Signaling Waveform

- Determine the relation between *r* and *B* when $p(t)=\mathrm{sinc}^2\, at$
- First note from time domain that

$$\mathrm{sinc}^2\, at = \begin{cases} 1, t = 0 \\ 0, t = \pm 1/a, \pm 2/a \ldots \end{cases} \Rightarrow r = a$$

  hence this waveform is suitable for signaling
- There exists a Fourier transform pair

$$\mathrm{sinc}^2\, at \leftrightarrow \frac{1}{a}\Lambda\left(\frac{f}{a}\right)$$



- From the spectra we note that $B_T \geq a$ and hence it must be that for baseband

$$\Rightarrow B_T \geq r$$

PAM Power Spectral Density (PSD)

- PSD for PAM can be determined by using a general expression

Amplitude autocorrelation

$$G_x(f) = \frac{1}{D}\left|P(f)\right|^2 \sum_{n=-\infty}^{\infty} R_a(n)\exp(-j2\pi nfD)$$

- For uncorrelated message bits

$$R_a(n) = \begin{cases} \sigma_a^2 + m_a^2, & n = 0 \\ m_a^2, & n \neq 0 \end{cases}$$

Total power

DC power

and therefore

$$\sum_{n=-\infty}^{\infty} R_a(n)\exp(-2\pi nfD) = \sigma_n^2 + m_a^2 \sum_{n=-\infty}^{\infty} \exp(-j2\pi nfD)$$

on the other hand $\sum_{n=-\infty}^{\infty} \exp(-j2\pi nfD) = \frac{1}{D}\sum_{n=-\infty}^{\infty}\delta\left(f-\frac{n}{D}\right)$ and $r = 1/D$

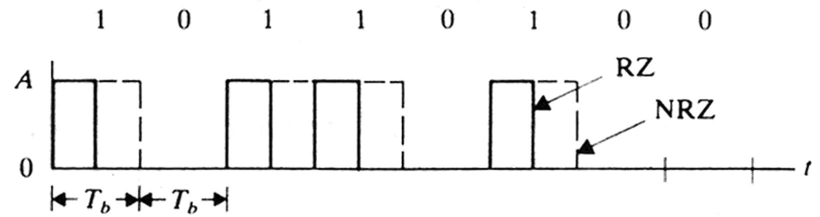
$$\boxed{G_x(f) = \sigma_a^2 r\left|P(f)\right|^2 + m_a^2 r^2 \sum_{n=-\infty}^{\infty}\left|P(nr)\right|^2 \delta(f - nr)}$$

Example

- For unipolar binary RZ signal:

$$P(f) = \frac{1}{2r_b}\operatorname{sinc}\frac{f}{2r_b}$$

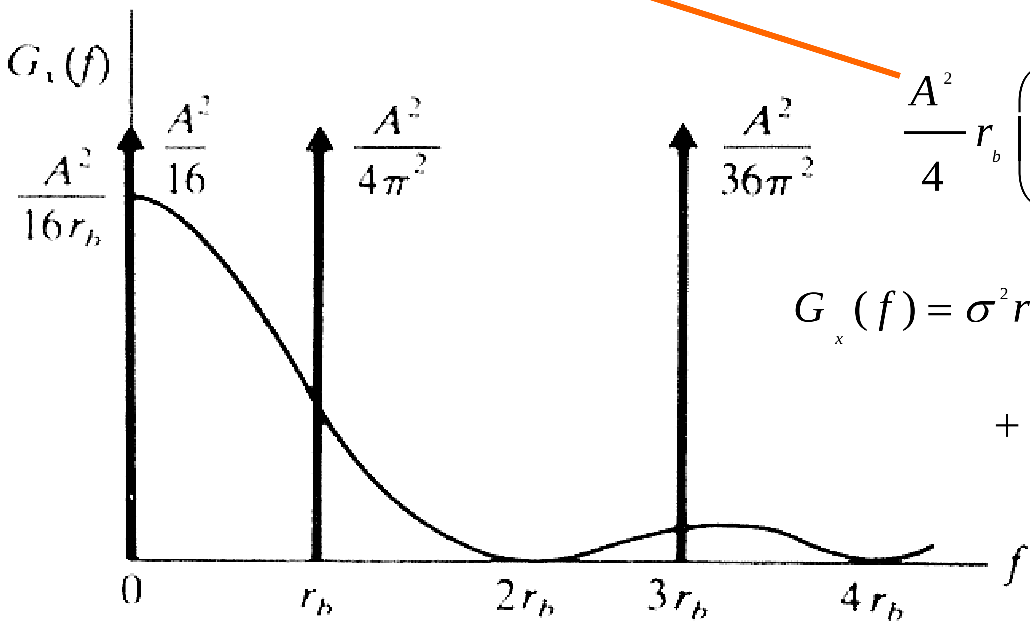
(a)



- Assume source bits are equally alike and independent, thus

$$\sigma_a^2 = (1/2T_b)\int_0^{T_b/2} A^2 dt = A^2/4,\; m_a^2 = \sigma_a^2$$

$$\Rightarrow G_x(f) = \frac{A^2}{16r_b}\operatorname{sinc}^2\frac{f}{2r_b} + \frac{A^2}{16}\sum_{n=-\infty}^{\infty}\delta(f-nr_b)\operatorname{sinc}^2\frac{n}{2}$$

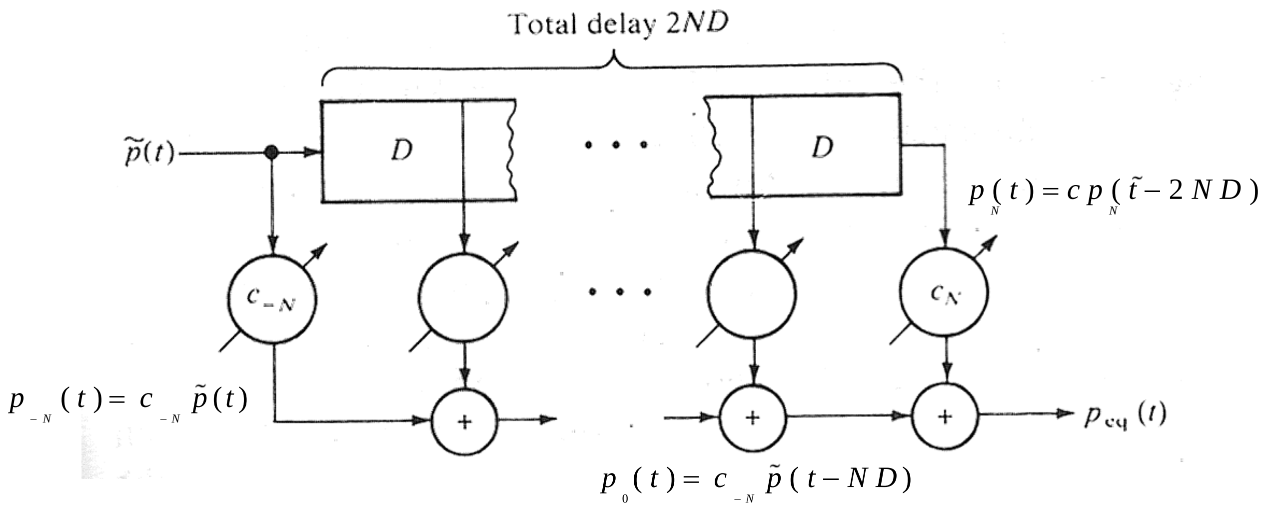


$$\frac{A^2}{4}r_b\left(\frac{1}{2r_b}\right)^2$$

$$G_x(f) = \sigma_a^2 r |P(f)|^2 + m_a^2 r^2 \sum_{n=-\infty}^{\infty} |P(nr)|^2 \delta(f-nr)$$

Equalization: Removing Residual ISI

- Consider a tapped delay line equalizer with



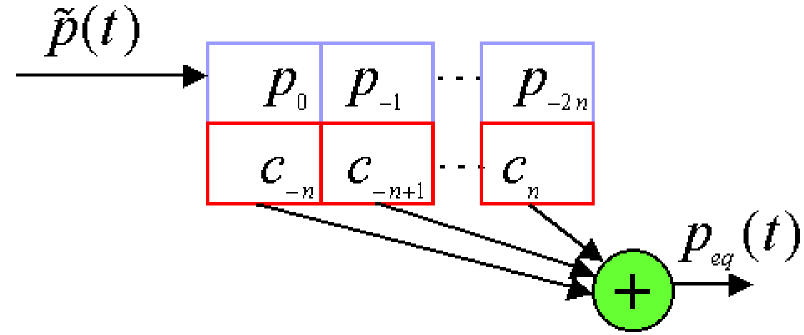
- Search for the tap gains c_N such that the output equals zero at sample intervals D except at the decision instant when it should be unity. The output is (think for instance paths c_{-N}, c_N or c_0)

$$p_{eq}(t) = \sum_{n=-N}^{N} c_n \tilde{p}(t - nD - ND)$$

that is sampled at $t_k = kD + ND$ yielding

$$p_{eq}(kD + ND) = \sum_{n=-N}^{N} c_n \tilde{p}(kD - nD) = \sum_{n=-N}^{N} c_n \tilde{p}\left[D(k-n)\right]$$

Tapped Delay Line: Matrix Representation



- At the instant of decision:

$$p_{eq}(t_k) = \sum_{n=-N}^{N} c_n \tilde{p}\left[D(k-n)\right] = \sum_{n=-N}^{N} c_n \tilde{p}_{k-n} = \begin{cases} 1, k = 0 \\ 0, k = \pm 1, \pm 2, \ldots, \pm N \end{cases}$$

- That leads into *(2N+1)x(2N+1)* matrix where *(2N+1)* tap coefficients can be solved:

$$\tilde{p}_0 c_{-n} + \tilde{p}_{-1} c_{-n+1} + \ldots + \tilde{p}_{-2n} c_n = 0$$
$$\tilde{p}_1 c_{-n} + \tilde{p}_0 c_{-n+1} + \ldots + \tilde{p}_{-2n+1} c_n = 0$$
$$\cdots$$
$$\tilde{p}_n c_{-n} + \tilde{p}_{n-1} c_{-n+1} + \ldots + \tilde{p}_{-n} c_n = 1$$
$$\cdots$$
$$\tilde{p}_{2n} c_{-n} + \tilde{p}_{2n-1} c_{-n+1} + \ldots + \tilde{p}_0 c_n = 0$$

$$\begin{bmatrix} \tilde{p}_0 & \cdots & \tilde{p}_{-2N} \\ \vdots & \cdots & \vdots \\ \tilde{p}_{N-1} & \cdots & \tilde{p}_{-N-1} \\ \tilde{p}_N & \cdots & \tilde{p}_{-N} \\ \tilde{p}_{N+1} & \cdots & \tilde{p}_{-N+1} \\ \vdots & & \vdots \\ \tilde{p}_{2N} & \cdots & \tilde{p}_0 \end{bmatrix} \begin{bmatrix} c_{-N} \\ \vdots \\ c_{-1} \\ c_0 \\ c_1 \\ \vdots \\ c_N \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

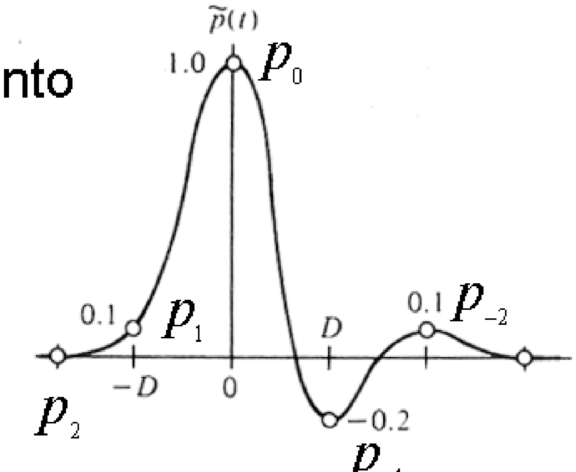
Example of Equalization

- Read the distorted pulse values into matrix from fig. (a)

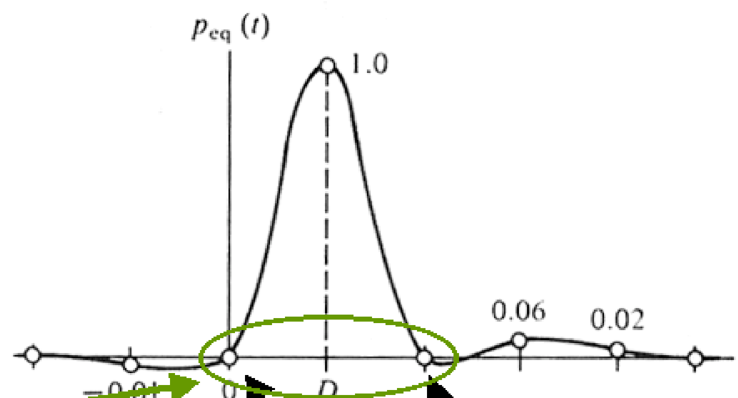
$$\begin{bmatrix} 1.0 & 0.1 & 0.0 \\ -0.2 & 1.0 & 0.1 \\ 0.1 & -0.2 & 1.0 \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

and the solution is

$$\begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} -0.096 \\ 0.96 \\ 0.2 \end{bmatrix}$$



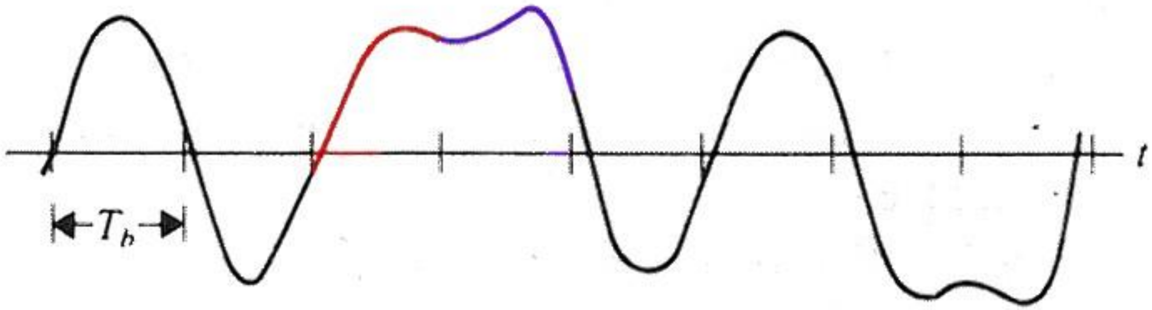
(a)

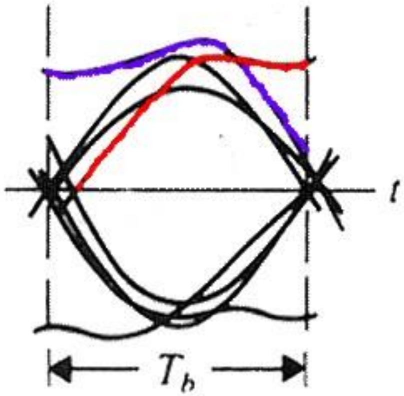


Question: what does these zeros help because they don't exist at the sampling instant?

Zero forced values

Monitoring Transmission Quality by Eye Diagram





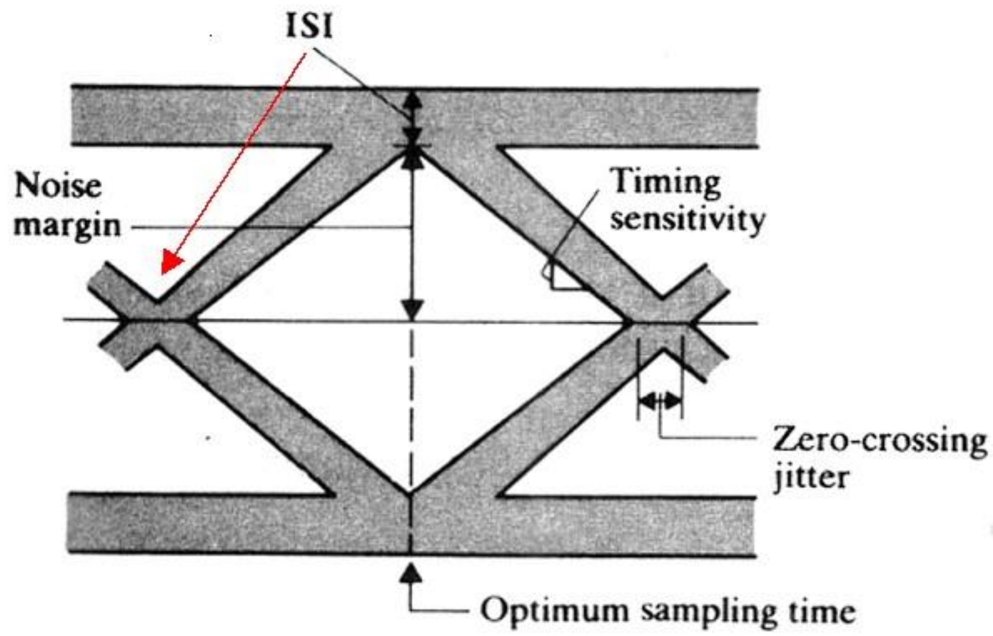
Required minimum bandwidth is

$$B_T \geq r/2$$

Nyqvist's sampling theorem:

Given an ideal LPF with the bandwidth B it is possible to transmit independent symbols at the rate:

$$B_T \geq r/2 = 1/(2T_b)$$



 Helsinki University of Technology, Communications Laboratory, Timo O. Korhonen

6.6 The Eye Pattern

- Eye Pattern
  - Be produced by the synchronized superposition of successive symbol intervals of the distorted waveform appearing at the output of the receive-filter prior to thresholding
  - From an experimental perspective, the eye pattern offers two compelling virtues
    - The simplicity of generation
    - The provision of a great deal of insightful information about the characteristics of the data transmission system, hence its wide use as a visual indicator of how well or poorly a data transmission system performs the task of transporting a data sequence across a physical channel.

- Timing Features
  - Three timing features pertaining to binary data transmission system,
    - Optimum sampling time : The width of the eye opening defines the time interval over the distorted binary waveform appearing at the output of the receive-filter
    - Zero-crossing jitter : in the receive-filter output, there will always be irregularities in the zero-crossings, which, give rise to jitter and therefore non-optimum sampling times
    - Timing sensitivity : This sensitivity is determined by the rate at which the eye pattern is closed as the sampling time is varied.

Fig. 6.5

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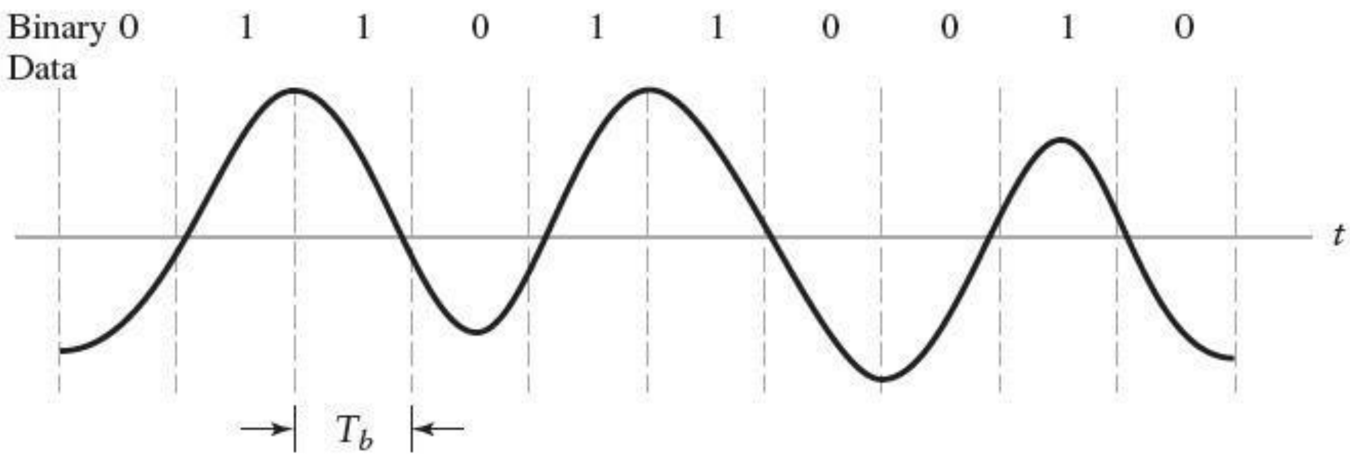
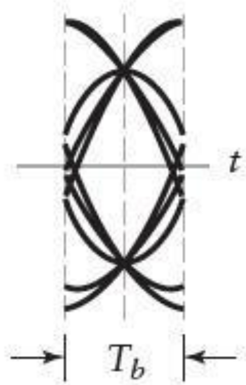
Fig.6.5



(a)



(b)

FIGURE 6.5 (*a*) Binary data sequence and its waveform. (*b*) Corresponding eye pattern.

- The Peak Distortion for Intersymbol Interference
  - In the absence of channel noise, the eye opening assumes two extreme values
    - An eye opening of unity, which corresponds to zero intersymbol interference
    - An eye opening of zero, which corresponds to a completely closed eye pattern; this second extreme case occurs when the effect of intersymbol interference is severe enough for some upper traces in the eye pattern to cross with its lower traces.

Fig. 6.6

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Fig.6.6

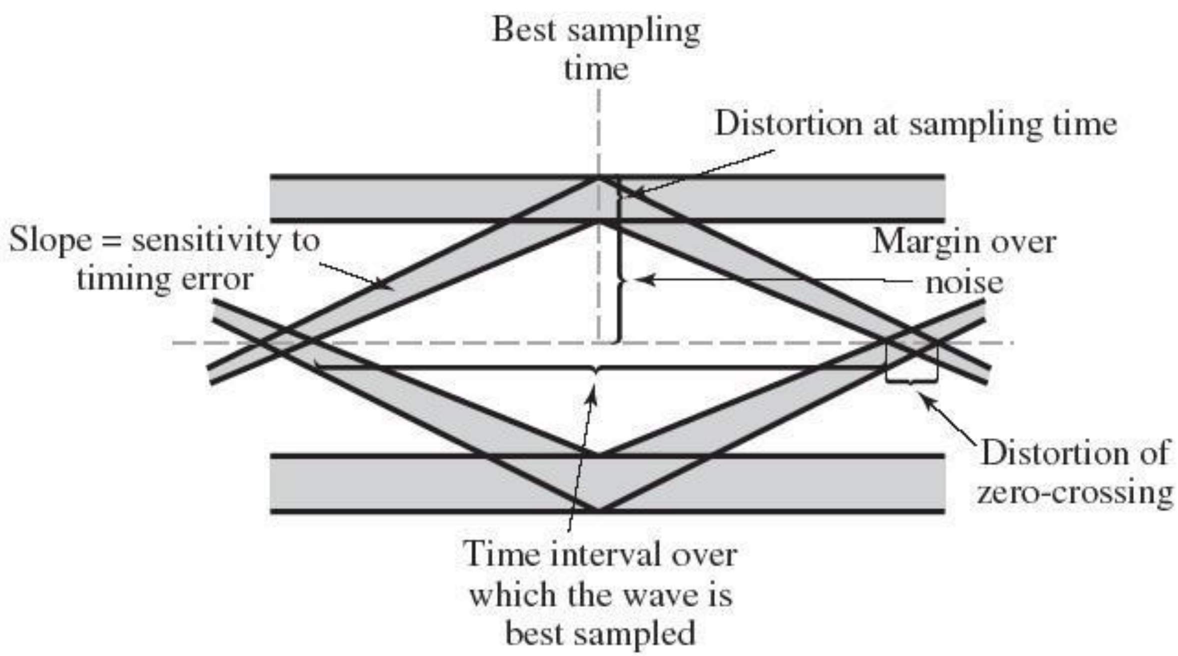

FIGURE 6.6 Interpretation of the eye pattern for a baseband binary data transmission system.

– Noise margin
  - In a noisy environment,
  - The extent of eye opening at the optimum sampling time provides a measure of the operating margin over additive channel noise

$$(\text{Eye opening}) = 1 - D_{\text{peak}} \qquad (6.34)$$

Fig. 6.7

– Eye opening
  - Plays an important role in assessing system performance
  - Specifies the smallest possible noise margin

- Zero peak distortion , which occurs when the eye opening is unity
- Unity peak distortion, which occurs when the eye pattern is completely closed.
- The idealized signal component of the receive-filter output is defined by the first term in Eq. (6.10)
- The intersymbol interference is defined by the second term

$$y_i = \sqrt{E}\,a_i + \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p_{i-k}, \quad i = 0, \pm 1, \pm 2, \ldots \qquad (6.10)$$

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Fig.6.7

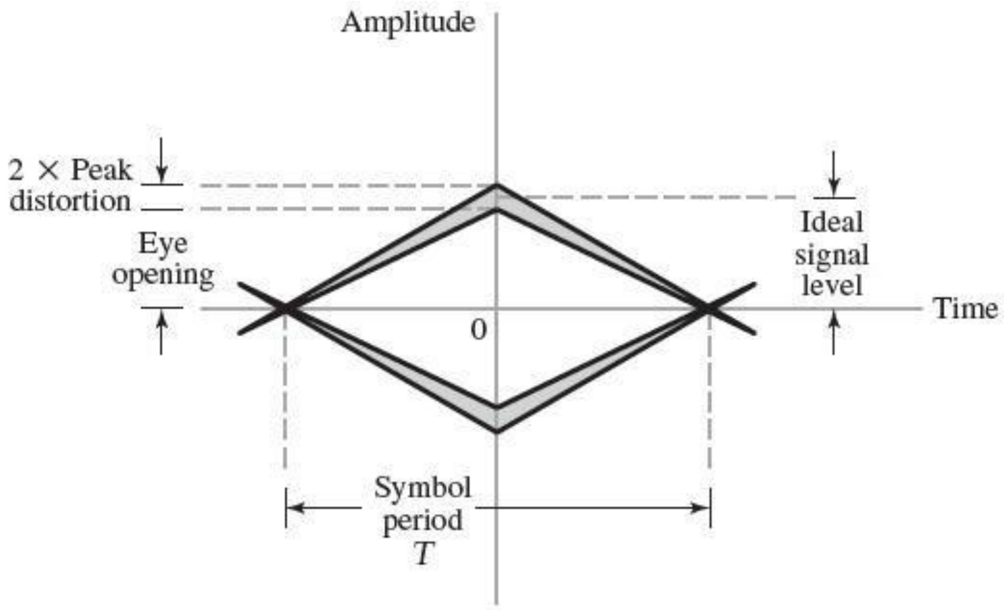


FIGURE 6.7 Illustrating the relationship between peak distortion and eye opening. *Note:* The ideal signal level is scaled to lie inside the range −1 to +1.

$$(\text{Maximum ISI}) = \sum_{\substack{m=-\infty \\ k \neq i}}^{\infty} \left| p_{i-k} \right|$$

$$D_{\text{peak}} = \sum_{k=-\infty}^{\infty} \left| p_{i-k} \right|$$

$$= \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} \left| p(i-k)T_b \right| \qquad (6.35)$$

- Eye pattern for M-ary Transmission
  - M-ary data transmission system uses M encoded symbols
  - The eye pattern for an M-ary data transmission system contains (M-1) eye openings stacked vertically one on top of the other.
  - It is often possible to find asymmetries in the eye pattern of an M-ary data-transmission system, which are caused by nonlinearities in the communication channel or other parts of the system.

6.7 Computer Experiment : Eye Diagrams for Binary and Quanternary Systems

- Fig. 6.8(a) and 6.8(b) show the eye diagrams for a baseband PAM transmission system using M=2 and M=4.

Fig. 6.8

- Fig. 6.9(a) and 6.9(b) show the eye diagrams for these two baseband-pulse transmission systems using the same system parameters as before, but this time under a bandwidth-limited condition.

$$|H(f)| = \frac{1}{1 + (f/f_0)^{2N}}$$

Fig. 6.9

$$1.\ N = 3,\ \text{and}\quad f_0 = 0.6\ Hz \quad \text{for binary PAM}$$

$$2.\ N = 3,\ \text{and}\quad f_0 = 0.3\ Hz \quad \text{for } 4-\text{PAM}$$

$$B_T = 0.5(1 + 0.5) = 0.75\ Hz$$

Fig.6.8

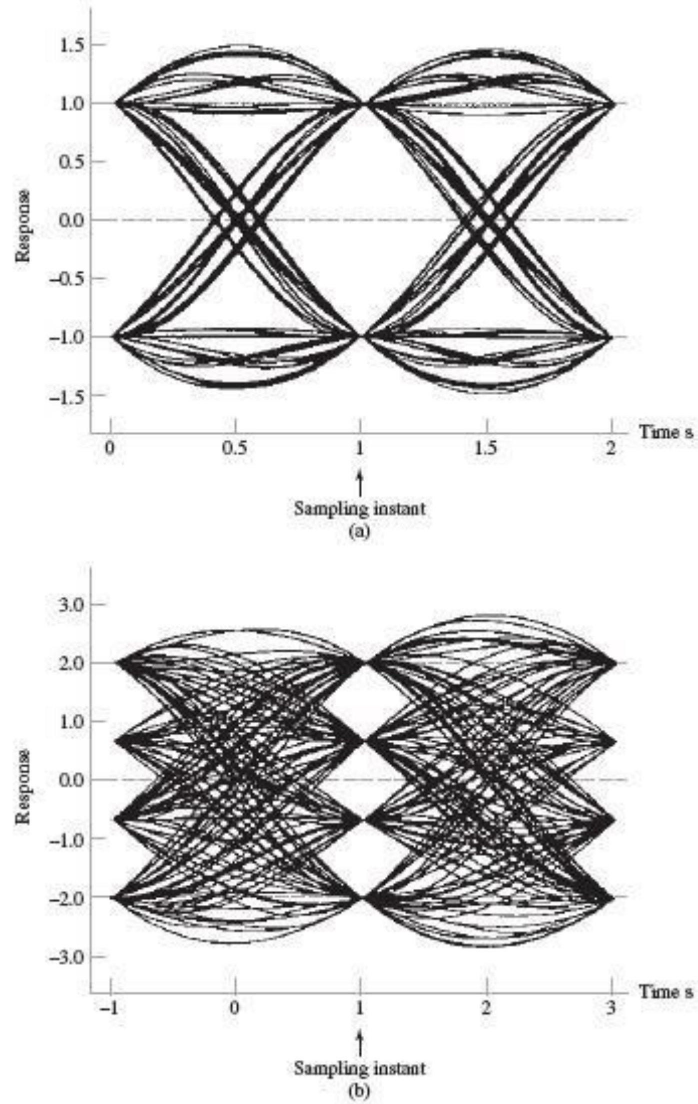


FIGURE 6.8 Eye diagram of received signal with no bandwidth limitation. (*a*) $M = 2$. (*b*) $M = 4$.

Fig.6.9

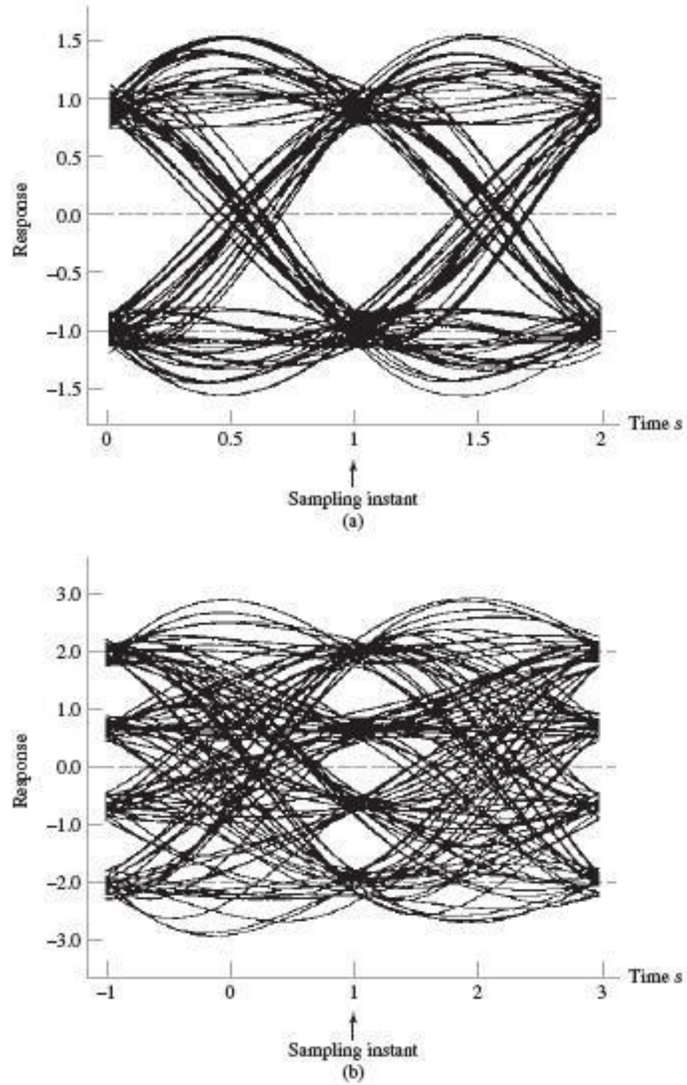


FIGURE 6.9 Eye diagram of received signal, using a bandwidth-limited channel. (*a*) $M = 2$. (*b*) $M = 4$.

Introduction

- What is *information theory* ?
  - *Information theory* is needed to enable the communication system to carry information (signals) from sender to receiver over a communication channel
    - it deals with mathematical modelling and analysis of a communication system
    - its major task is to answer to the questions of *signal compression* and *transfer rate*
  - Those answers can be found and solved by *entropy* and *channel capacity*

Entropy

- *Entropy* is defined in terms of probabilistic behaviour of a source of information
- In information theory the source output are discrete random variables that have a certain fixed finite alphabet with certain probabilities
  - *Entropy is an average information content for the given source symbol*

$$H(\wp) = \sum_{k=0}^{K-1} p_k \log_2\left(\frac{1}{p_k}\right)$$

Entropy (example)

- Entropy of Binary Memoryless Source
- Binary memoryless source has symbols 0 and 1 which have probabilities p0 and p1 (1-p0)
- Count the entropy as a function of p0

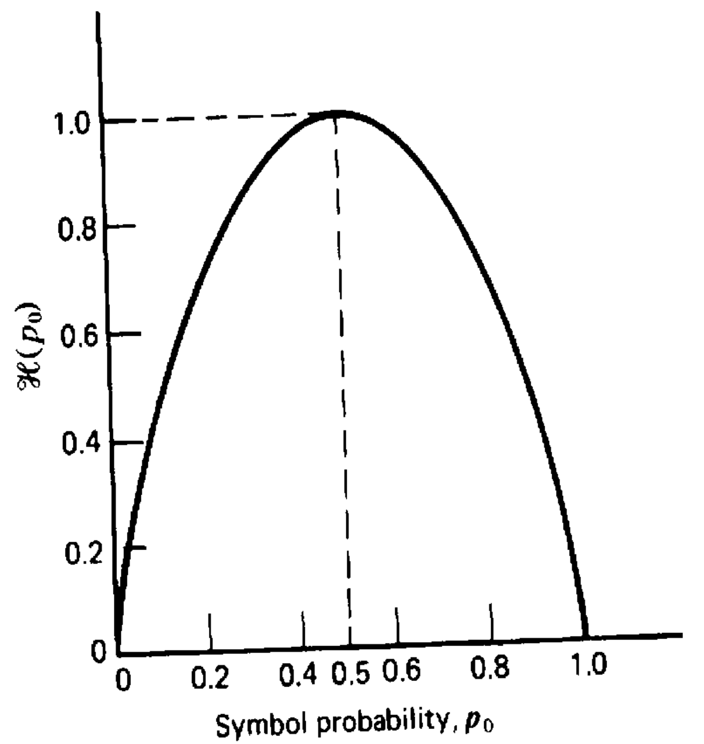


Figure 10.2 Entropy function $\mathcal{H}(p_0)$.

Source Coding Theorem

- *Source coding* means an effective representation of data generated by a discrete source
  - *representation by source encoder*
- statistics of the source must be known (e.g. if coding priorities exist)

Data Compaction

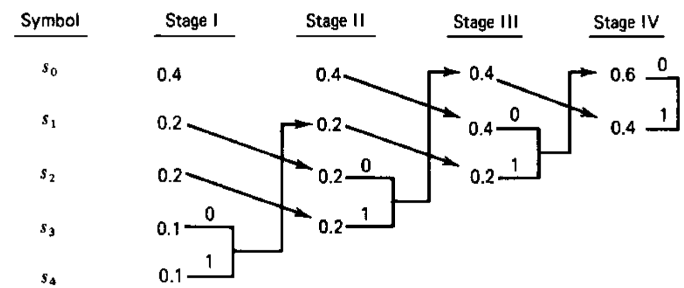
- *Data compaction* (a.k.a lossless data compression) means that we will remove redundant information from the signal prior the transmission
  - *basically this is achieved by assigning short descriptions to the most frequent outcomes of the source output and vice versa*
- Source-coding schemes that are used in data compaction are e.g. prefix coding, huffman coding, lempel-ziv

Data Compaction example

- *Prefix coding* has an important feature that it is always uniquely decodable and it also satisfies Kraft-McMillan (see formula 10.22 p. 624) inequality term
- Prefix codes can also be referred to as instantaneous codes, meaning that the decoding process is achieved immediately

Data Compaction example

- In *Huffman coding* to each symbol of a given alphabet is assigned a sequence of bits accor- ding to the symbol probability



(a)

Symbol	Probability	Code word
s_0	0.4	00
s_1	0.2	10
s_2	0.2	11
s_3	0.1	010
s_4	0.1	011

(b)

Figure 10.5 (*a*) Example of the Huffman encoding algorithm. (*b*) Source code.

Data Compaction example

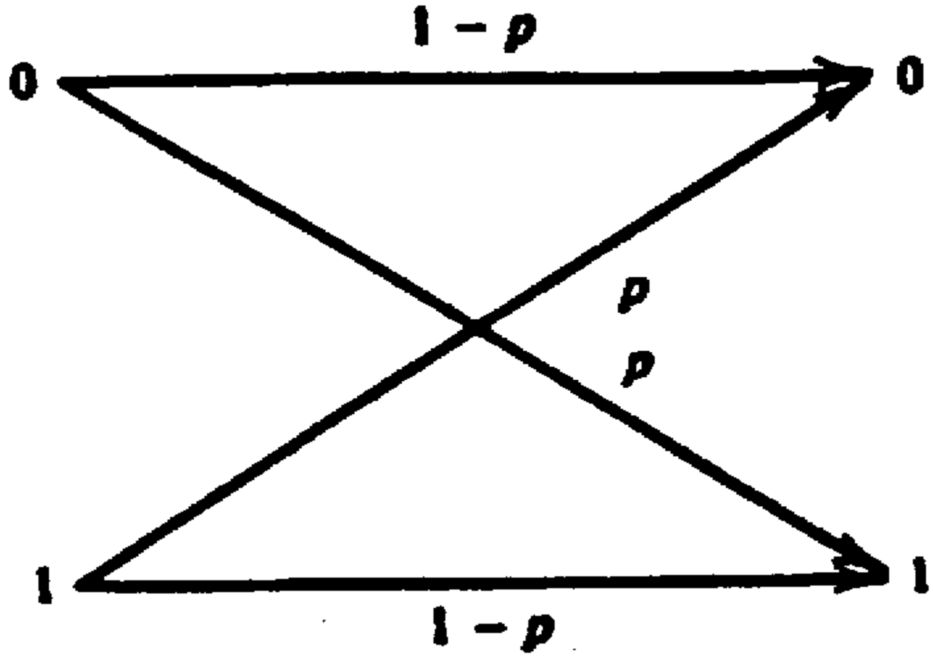
- In *Lempel-Ziv coding* no probabilities of the source symbols is needed, which is actually most often the case
- LZ algorithm parses the source data stream into segments that are the shortest subsequences not encountered previously

Discrete Memoryless Channels

- A *discrete memoryless channel* is a statistical model with an input of X and output of Y, which is a noisy version of X (here both are random variables)
- In each time slot the channel accepts an input symbol X selected from a given alphabet
- We can create channel matrix that corresponds fixed channel inputs and outputs and we can assume the probabilities of the symbols

Discrete Memoryless Channels

- Example : Binary symmetric channel



Mutual Information

- *Mutual information* uses conditional entropy of *X* selected from a known alphabet
  - *conditional entropy means the uncertainty remaining about the channel input after the channel output has been observed*
- Mutual information has several properties :
  - *symmetric channel*
  - *always nonnegative*
  - *relation to the joint entropy of a channel input and channel output*

Mutual Information

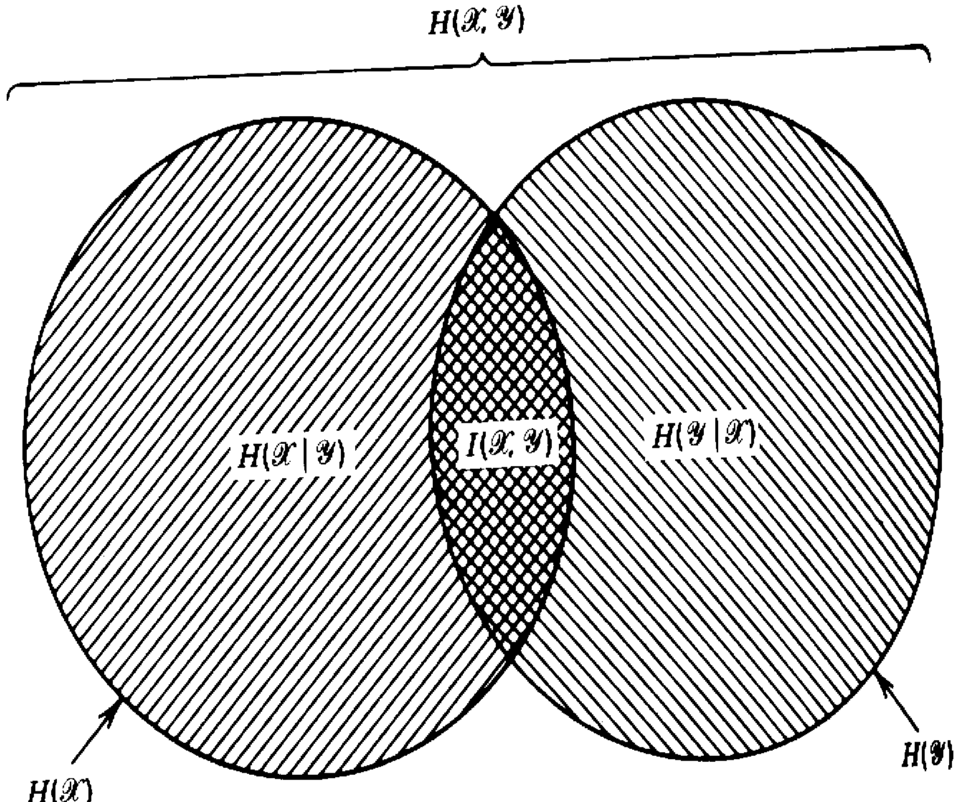


Figure 10.10 Illustrating the relations among various channel parameters.

Mutual Information

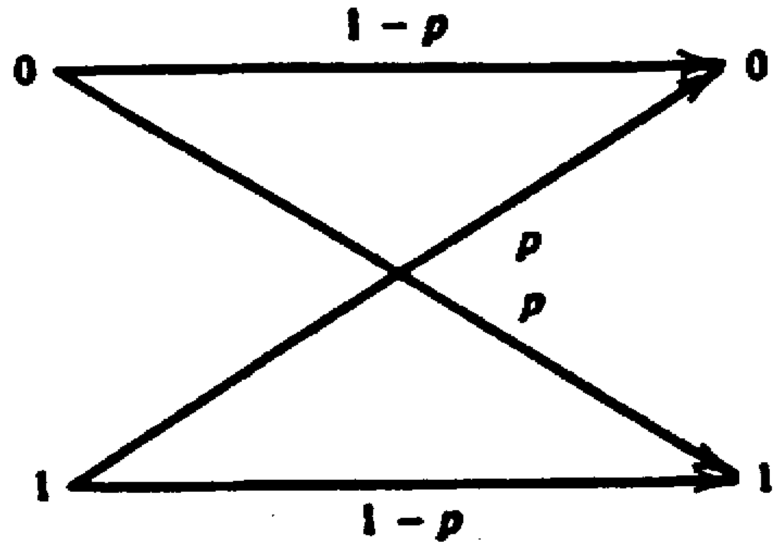
- Example : Imagine a village in a distance, in which the inhabitants are nerd-smurfs. One could divide these smurfs into two distinctive groups.
  - *In group A 50% always tells correctly the bitcode if asked, 30 % will lie if asked, 20 % go beserk*
  - *In group B the same percentages are 30 %, 50 % and 20%*
  - *calculate mutual information*

Channel Capacity

- *Capacity* in the channel is defined as a intrinsic ability of a channel to convey information
- Using mutual information the channel capacity of a discrete memoryless channel is a maximum average mutual information in any single use of channel over all possible probability distributions

Discrete Memoryless Channels

- Example : Binary symmetric channel *revisited*
  - *capacity of a binary symmetric channel with given input probabilities*
  - *variability with the error probability*



Channel Coding Theorem

- Channel coding consists of mapping the incoming data sequence into a channel input sequence and vice versa via inverse mapping
  - *mapping operations performed by encoders*

Information Capacity Theorem

- A channel with noise and the signal are received is described as discrete time, memoryless Gaussian channel (with power-limitation)
  - *example : Sphere Packing*

contents

Important classes of error-control coding:

- Linear block codes
- Cyclic codes
- Convolutional codes
- Turbo codes
- Low-density parity-check codes

Classification of Error-Control Coding:

Systematic Code Nonsystematic Code

Linear code
Nonlinear code

Random error-correcting code
Burst error-correcting code

Error-detecting code
Error-correcting code

Block code
Convolution code

Rule: the message sequence is subdivided into sequential blocks each *k* bits long, and each k-bit block is mapped into an *n*-bit block, where *n*>*k*. the number of redundant bits added by the channel encoder to each transmitted block is n-k bits.

Code Rate:

$$r = \frac{k}{n} \qquad r \leq 1$$

Hamming Distance and Hamming Weight

- Hamming weight $w(c)$: defined as the number of nonzero elements in the code vector c.

1 0 1 1 0 0 1 1 → w = 5

- Hamming Distance $d(c_1,c_2)$: defined as the number of locations in which their respective elements differ between two code words c_1 and c_2.

1 0 1 1 0 0 1 1

1 0 0 1 1 0 1 0

→ d = 3

Minimum Distance and Minimum Weight

- The minimum weight w_{min} is defined as the smallest weight of all nonzero code vectors in the code.

- The minimum distance d_{min} is defined as the smallest Hamming distance between any pair of code vectors in the code.

In linear block code

$$d_{\min} = w_{\min}$$

Relation between minimum distance and capability of error-detecting and error-correcting

$$t \le \left\lfloor \frac{1}{2}(d_{\min} - 1) \right\rfloor \qquad (10.25)$$

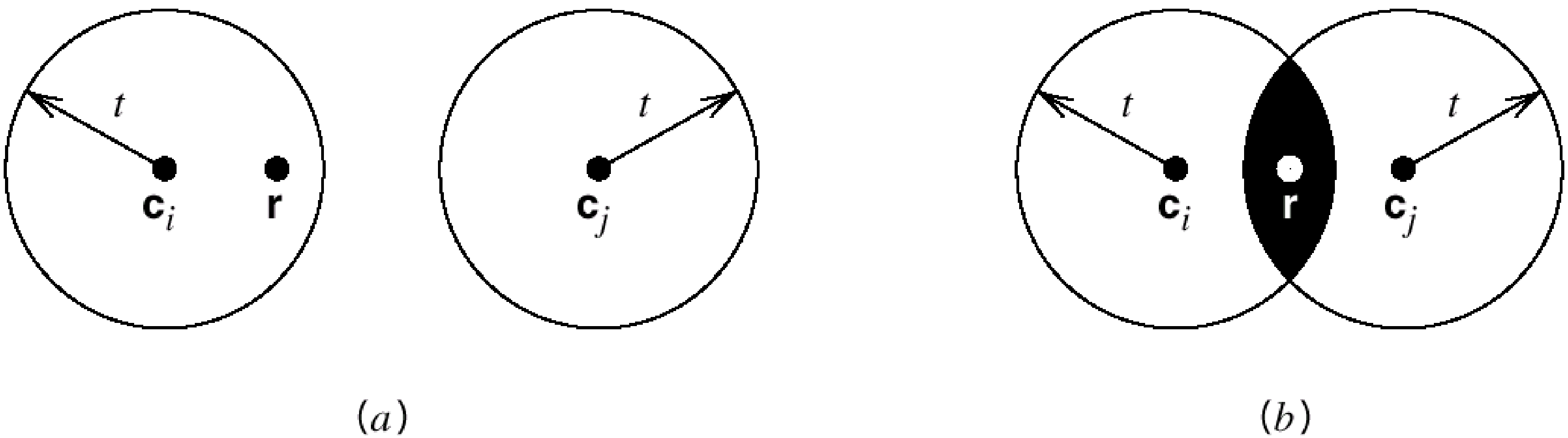


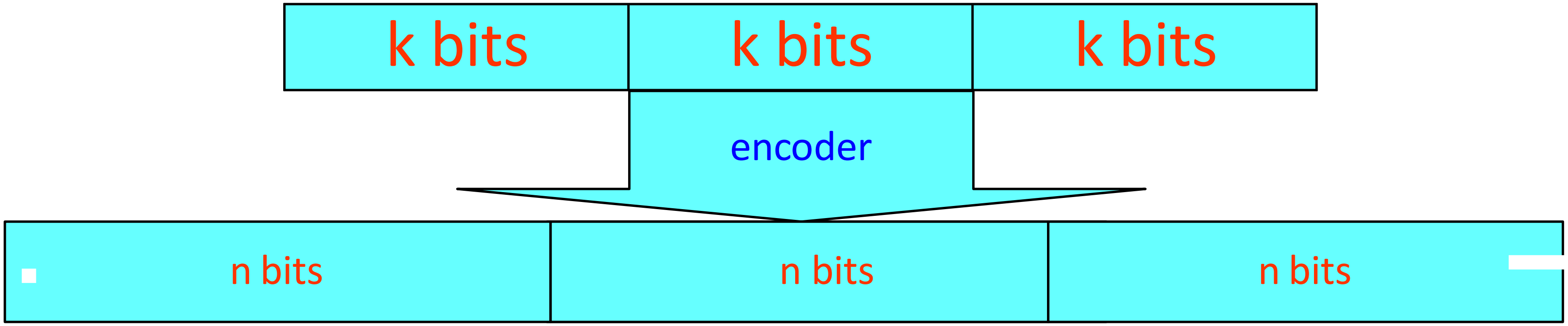
Figure 10.6

(*a*) Hamming distance $d(ci, cj) \geq 2t + 1$. (*b*) Hamming distance $d(ci, cj) < 2t$. The received vector is denoted by r.

10.3 Linear Block Codes

What is block code?

Message sequence



What is Systematic linear block codes?

The message bits are transmitted in unaltered form.

Figure 10.4 Structure of systematic code word

$$\underbrace{\boxed{b_0, b_1, \ldots, b_{n-k-1}}}_{\text{Parity bits}}\underbrace{\boxed{m_0, m_1, \ldots, m_{k-1}}}_{\text{Message bits}}$$

K bits: 1 1 0 1

N bits: 0 0 1 1 0 1
1 1 0 1 --→ systematic code

N bits: 1 0 1 0 0 1--→ non-systematic code k-bit message

$$m_0, m_1, \ldots, m_{k-1}$$

n-bit channel code word

$$c_0, c_1, \ldots c_{n-1} = \boxed{b_0, b_1, \ldots b_{n-k-1}}, m_0, m_1, \ldots, m_{k-1}$$

parity check bits

message bits

The n-k bits are computed from message bits in accordance with a given encoding rule.

Relation between parity bits and message bits

The (n-k) parity bits are linear sums of the k message bits, as shown by generalized relation

$$b_i = p_{0i} m_0 + p_{1i} m_1 + \cdots + p_{k-1,i} m_{k-1} \qquad (10.2)$$

Where, the coefficients are defined as follows:

$$p_{ij} = \begin{cases} 1 & \text{if } b_i \text{ depends on } m_j \\ 0 & \text{otherwise} \end{cases}$$

Matrix Notation

1-by-k message vector $m = [m_0, m_1, \cdots, m_{k-1}]$

1-by-(n-k) parity vector $b = [b_0, b_1, \cdots, b_{n-k-1}]$

1-by-n code vector $c = [c_0, c_1, \cdots, c_{n-1}]$

All of them are row vectors.

It is clear that *c* can be expressed in terms of *m* and *b* as follows

$$c = [b \vdots m]$$

$$b_i = p_{0i} m_0 + p_{1i} m_1 + \cdots + p_{k-1,i} m_{k-1} \qquad (10.2)$$

$$\boldsymbol{p_{00}} \qquad \boldsymbol{p_{10}} \qquad \boldsymbol{p_{k-1,0}}$$

k-by-(n-k)

$$P = \begin{bmatrix} p_{00} & p_{01} & \cdots & p_{0,n-k-1} \\ p_{10} & p_{11} & \cdots & p_{1,n-k-1} \\ \vdots & \vdots & \vdots & \vdots \\ p_{k-1,0} & p_{k-1,1} & \cdots & p_{k-1,n-k-1} \end{bmatrix}$$

$$\boxed{\boldsymbol{b} = \boldsymbol{m} \boldsymbol{P}}$$

Where p_{ij} is 0 or 1.

Because

therefore $$c = [mP \vdots m] = m[P \vdots I_k]$$

I_k: *k-by-k* identity matrix

$$I_k = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

k-by-n generator matrix

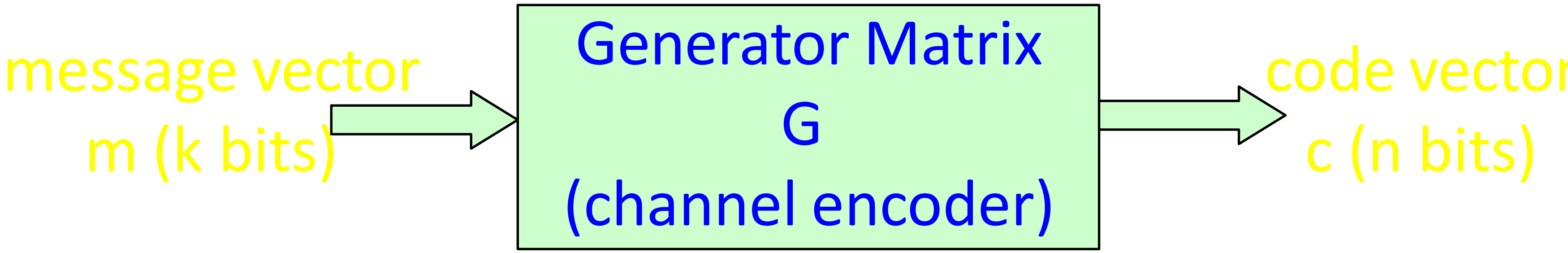
$$G = [P \vdots I_k] \qquad (10.12)$$

Hence , we get generation equation

$$\boldsymbol{c} = \boldsymbol{m}\boldsymbol{G} \qquad \mathbf{(10.13)}$$

Why is G called generator matrix?

$$\boldsymbol{c} = \boldsymbol{m}\,\boldsymbol{G}$$



How to get the full set of code words?

The full set of code words, referred to simply as code, is generated in accordance with *c=mG* by setting the message vector m range through the set of all 2^k binary k-tuples (1-by-k vectors) 2^k.

$$b_i = p_{0i} m_0 + p_{1i} m_1 + \cdots + p_{k-1,i} m_{k-1} \qquad (10.2)$$

Parity-Check Matrix H

Let H denote an *(n-k)-by-n* matrix, defined as

$$H = [I_{n-k} \vdots P^T]$$

P^T : *(n-k)-by-k* matrix, the transpose of P

since $$G = [P \vdots I_k]$$

Hence
$$HG^T = [I_{n-k} \vdots P^T]\begin{bmatrix} P^T \\ \cdots \\ I_k \end{bmatrix} = P^T + P^T = 0$$

Modulo-2 arithmetic

That is

$$HG^T = 0$$

We have known that

$$c = mG$$

Postmultiplying both sides by H^T, we get

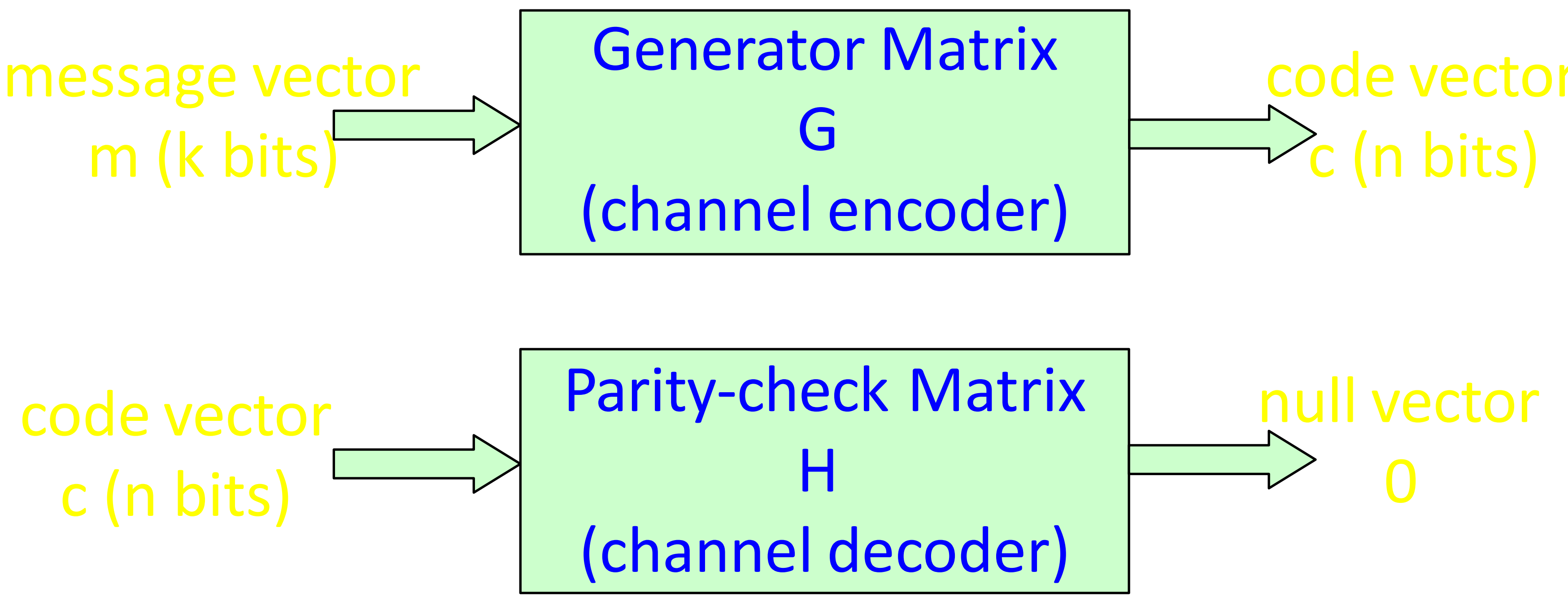
$$cH^T = mGH^T = 0 \qquad (10.16)$$

Equation (10.16) is called parity-check equation.

Generator equation $$c = mG \qquad (10.13)$$

Parity-check equation $$cH^T = 0 \qquad (10.16)$$

They are basic to the description and operation of a linear block code.



Repetition Codes

- It is the simplest type of linear block codes.
- (n,1) block code: only 1-bit message, (n-1) parity-check bits, which are the repetition of the message bit.

Example:

$$k = 1 \quad n = 5 \quad I_k = [1]$$

$$c_0, c_1, \ldots c_4 = b_0, b_1, b_2, b_3, m_0$$

$$\left.\begin{aligned} b_0 &= 1 \cdot m_0 \\ b_1 &= 1 \cdot m_0 \\ b_2 &= 1 \cdot m_0 \\ b_3 &= 1 \cdot m_0 \end{aligned}\right\} \Rightarrow P = [1 \;\; 1 \;\; 1 \;\; 1]$$

$$\Downarrow$$

$$G = [1 \;\; 1 \; 1 \;\; 1 \vdots 1]$$

$$H = [I_{n-k} \vdots P^T] = \begin{bmatrix} 1 & 0 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & 0 & \vdots & 1 \\ 0 & 0 & 1 & 0 & \vdots & 1 \\ 0 & 0 & 0 & 1 & \vdots & 1 \end{bmatrix}$$

Because: m=0 or 1

Thus, c=mG=00000 or 11111

Error Pattern

transmitted code： c (1×n vector)
received code： r (1×n vector)
error pattern： e (1×n error vector)

$$r = c + e$$

$$e = \{e_1, e_2, \ldots e_i, \ldots e_n\}$$

$$e_i = \begin{cases} 1, & \text{if an error has occurred in the } i^{\text{th}} \text{ location} \\ 0, & \text{otherwise} \end{cases}$$

If c= [0 0 1 1 0 1 0]
r= [0 1 1 1 0 0 0]

Then e= [0 1 0 0 0 1 0]

Our task now is to decode the code vector c from the received vector r. How to do it?

Definition and properties of Syndrome

Definition:

$$s = rH^T \qquad (10.19)$$

H^T is a $n \times (n\text{-}k)$ vector, so, s is a $1 \times (n\text{-}k)$ vector.

Property 1: it depends only on the error pattern, and not on the transmitted code word.

because:

$$s = rH^T = (c+e)H^T = cH^T + eH^T = eH^T$$

Syndrome Decoding

The decoding procedure for a linear block code

① For the received vector r, compute the syndrome $s=rH^T$

② Identify the error pattern e_{max} with the largest probability of occurrence.

③ Compute the code vector $c = r + e_{max}$ as the decoded version of the received vector r.

Likelihood decoding:

$$\boldsymbol{e} = \{\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots \boldsymbol{e}_i, \ldots \boldsymbol{e}_n\}$$

$$s = rH^T = (c + e)H^T = cH^T + eH^T = eH^T$$

$$s = eH^T = e_1 H_1^T + e_2 H_2^T + \cdots e_i H_i^T + \cdots + e_n H_n^T$$

$$s = eH^T = e_1 H_1^T + e_2 H_2^T + \cdots e_i H_i^T + \cdots + e_n H_n^T$$

If and only if e_i=1, then

$$\boldsymbol{s} = \boldsymbol{H}_i^T \quad \text{or} \quad \boldsymbol{s}^T = \boldsymbol{H}_i$$

Example 10.2 Hamming Codes

It is a special (n, k) linear block code that can correct 1-bit error. Speciality :

Block length:

$$n = 2^m - 1$$

Number of message bits:

$$k = 2^m - m - 1$$

Number of parity bits:

$$n - k = m$$

Such as (7, 4) Hamming code.

(7, 4) Hamming Code

Generator matrix

$$G = \begin{bmatrix} 1 & 1 & 0 & \vdots & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & \vdots & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & \vdots & 0 & 0 & 1 & 0 \\ \underbrace{1 \quad 0 \quad 1}_{p} & & & \vdots & \underbrace{0 \quad 0 \quad 0 \quad 1}_{I_k} \end{bmatrix}$$

Parity-check matrix

$$H = \begin{bmatrix} 1 & 0 & 0 & \vdots & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & \vdots & 1 & 1 & 1 & 0 \\ \underbrace{0 \quad 0 \quad 1}_{I_{n-k}} & & & \vdots & \underbrace{0 \quad 1 \quad 1 \quad 1}_{p^T} \end{bmatrix}$$

Suppose： received vector is [1100010]

Determine: is it correct?

Because

$$s = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{s} \neq \mathbf{0}$$

So, the received code vector is wrong.

Cyclic Codes

- Cyclic code is a kind of linear block codes.
- Any cyclic shift of a code word in the code is also a code word.
- Cyclic codes are easy to encode.
- Cyclic codes possess a well-define mathematical structure, which has led to the development of very efficient decoding schemes for them.

Cyclic Property

Assume that is a code word of an (n,k) linear block code.

Code words:

All of them are also code words in the code.

Code Polynomial

Code word:

Code polynomial:

$$c(X) = c_0 + c_1 X + c_2 X^2 + \cdots + c_{n-1} X^{n-1}$$

$$c^{(i)}(X) = c_{n-i} + \cdots + c_{n-1} X^{i-1} + c_0 X^i + c_1 X^{i+1} + \cdots + c_{n-i-1} X^{n-1} \quad (10.30)$$

$$X \; c(X) = c_0 X + c_1 X^2 + c_2 X^3 + \cdots + c_{n-1} X^n$$

$$X^i c(X)$$

$$= X^i (c_0 + c_1 X + \cdots + c_{n-i-1} X^{n-i-1} + c_{n-i} X^{n-i} + \cdots + c_{n-1} X^{n-1})$$

$$= c_0 X^i + c_1 X^{i+1} + \cdots + c_{n-i-1} X^{n-1} + c_{n-i} X^n + \cdots + c_{n-1} X^{n+i-1}$$

Because in modulo-2 addition,

$$X^i c(X) = c_{n-i} + \cdots + c_{n-1} X^{i-1} + c_0 X^i + c_1 X^{i+1} + \cdots + c_{n-i-1} X^{n-1}$$
$$+ c_{n-i} (X^n + 1) + \cdots + c_{n-1} X^{i-1} (X^n + 1)$$

$$c^{(i)}(X) = c_{n-i} + \cdots + c_{n-1} X^{i-1} + c_0 X^i + c_1 X^{i+1} + \cdots + c_{n-i-1} X^{n-1} \quad \mathbf{(10.30)}$$

Relation between $X^i c(X)$ and $c^{(i)}(X)$

$$\frac{X^i c(X)}{X^n + 1} = q(X) + \frac{c^{(i)}(X)}{X^n + 1}$$

remainder

Quotient

$$q(X) = c_{n-i} + c_{n-i+1} X + \cdots + c_{n-1} X^{i-1}$$

$$X^i c(X) = q(X)(X^n + 1) + c^{(i)}(X)$$

$$c^{(i)}(X) = X^i c(X) \bmod (X^n + 1) \qquad (10.33)$$

Generator Polynomial

Let *g(X)* be a polynomial of degree n-k that is a factor of X^n+1. It may be expanded as:

$$g(x) = 1 + \sum_{i=1}^{n-k-1} g_i X^i + X^{n-k}, \quad g_i = 0 \text{ or } 1$$

Generator polynomial

[redacted]

$$c(X) = a(X)g(X) \qquad (10.35)$$

Encoding Procedure

[redacted]

Message polynomial $m(X) = m_0 + m_1 X + \cdots + m_{k-1} X^{k-1}$

$$\boldsymbol{c_0, c_1, \ldots c_{n-1} = \boxed{b_0, b_1, \ldots b_{n-k-1}}, m_0, m_1, \ldots, m_{k-1}}$$

n-k parity check bits

k message bits

$$b(X) = b_0 + b_1 X + \cdots + b_{n-k-1} X^{n-k-1}$$

Therefore, code polynomial

$$c(X) = b(X) + X^{n-k} m(X) \qquad (10.38)$$

How to determine b(X)?

Because

$$c(X) = a(X)g(X) \qquad (10.35)$$

$$c(X) = b(X) + X^{n-k} m(X) \qquad (10.38)$$

Thus,

$$a(X)g(X) = b(X) + X^{n-k} m(X)$$

$$\frac{X^{n-k} m(X)}{g(X)} = a(X) + \frac{b(X)}{g(X)} \qquad (10.39)$$

It states that the polynomial *b(X)* is the remainder left over after dividing x^{n-k} *m(X)* by *g(X)*.

Steps for encoding an (n,k) systematic cyclic code

1. Multiply the message polynomial m(X) by X^{n-k};
2. Divide $X^{n-k}\,m(X)$ by the generator polynomial g(X), obtaining the remainder b(x);
3. Add b(x) to $X^{n-k}\,m(X)$, obtaining the code polynomial c(X).

How to select generator polynomial?

Rule: Any factor of X^n+1 can be used as a generator polynomial, the degree of the factor determines the number of parity bits.

It is difficult to select a polynomial factor to construct a GOOD cyclic code.

Example: (7, k) cyclic code

$$X^7 + 1 = (1 + X)(1 + X^2 + X^3)(1 + X + X^3)$$

(7,6) cyclic code

(7,4) cyclic code

(7,4) cyclic code

(7,3) cyclic code

(7,3) cyclic code

(7,1) cyclic code

Generator and Parity-Check Matrices

Generator matrix polynomial

$$G(X) = \begin{bmatrix} g(X) \\ X g(X) \\ \vdots \\ X^{k-1} g(X) \end{bmatrix} \xrightarrow{\text{coefficients}} G \longrightarrow H$$

$$\boldsymbol{G} = [\boldsymbol{P} \,\vdots\, \boldsymbol{I}_{\boldsymbol{k}}]$$

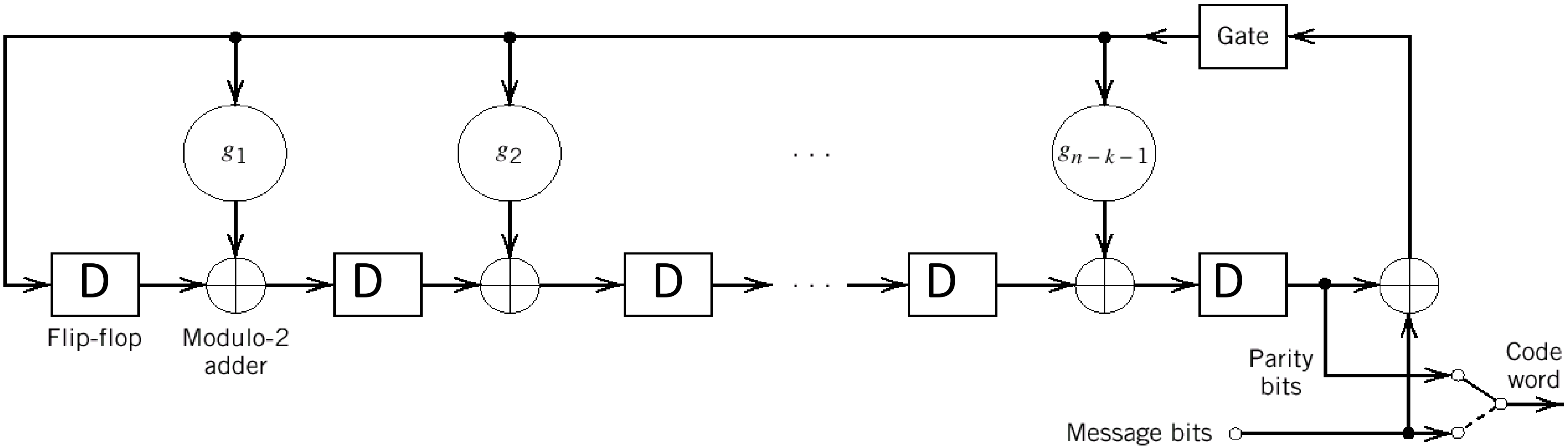
Note: G produced by this way may not be systematic .

Encoder for Cyclic Codes

How to design the cyclic encoder?

These encoding procedure can be implemented by means of encoder consisting of a linear feedback shift register with (n-k) stages.

Fig.10.8 Encoder for an (n, k) cyclic code



$$g(x) = 1 + \sum_{i=1}^{n-k-1} g_i X^i + X^{n-k}, \qquad g_i = 1 \text{ or } 0$$

$$\mathbf{m} = [m_0, m_1, \cdots, m_{k-1}]$$

$$c_0, c_1, \ldots c_{n-1} = \boxed{b_0, b_1, \ldots b_{n-k-1}}, m_0, m_1, \ldots, m_{k-1}$$

Operation of the encoder

1. The gate is switched on. The k message bits are shifted into the channel, and enter the shift registers.
2. The gate is switched off, thereby breaking the feedback connections.
3. The contents of the shift register are read out into the channel.

Calculation of the Syndrome

$$s = r H^T$$

- If the syndrome is zero, there are no transmission errors in the received word.
- If the syndrome is nonzero, the received word contains transmission errors that require correction.

In the case of a cyclic code in systematic form, the syndrome can be calculated easily.

Syndrome polynomial

Suppose: Code word

Received code

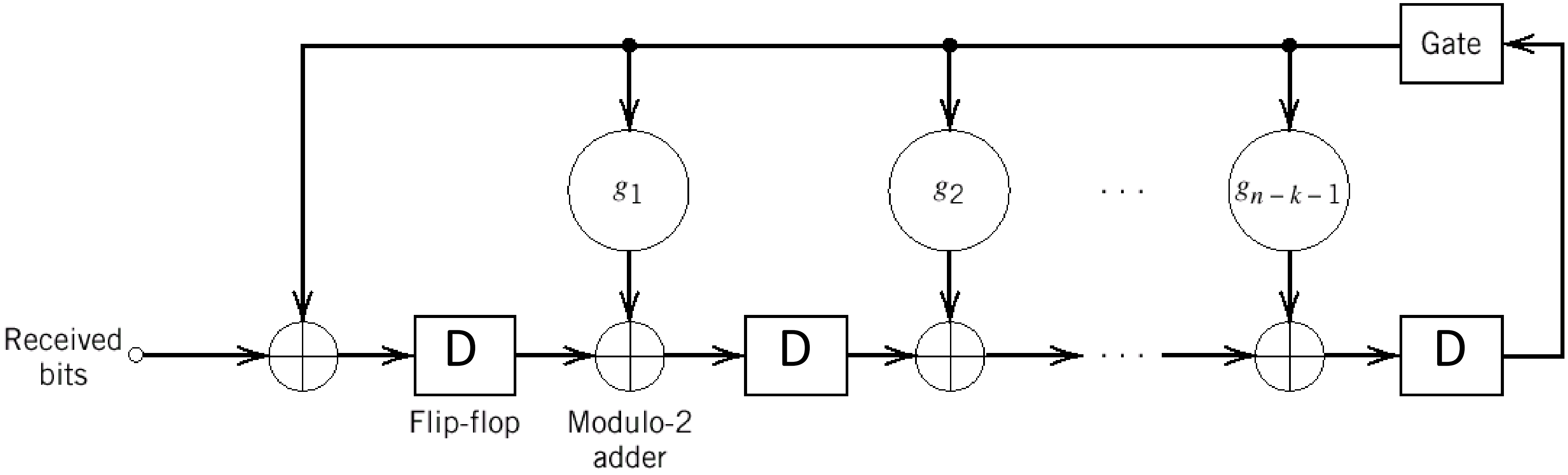
Then, the received code polynomial

$$\frac{r(X)}{g(X)} \longrightarrow$$

Quotient q(X)
Remainder s(X)

$$r(X) = q(X)g(X) + s(X) \qquad (10.47)$$

Syndrome Calculator for (n,k) Cyclic Code



- Syndrome calculator is identical to the encoder.
- As soon as all the received bits have been shifted into the shift register, its contents are the syndrome S.

In this example, we have two primitive polynomials:

$$1 + X^2 + X^3 \qquad 1 + X + X^3$$

Here, we take

$$g(X) = (1 + X + X^3)$$

Hence, parity-check polynomial

$$h(X) = \frac{X^7 + 1}{g(x)} = (1 + X)(1 + X^2 + X^3)$$

$$= 1 + X + X^2 + X^4$$

?

Suppose: message sequence 1001

Determine: the whole code word xxx1001

Message polynomial

$$m(X) = 1 + X^3 \Rightarrow X^{n-k} m(X) = X^3 + X^6$$

$$\frac{X^3 + X^6}{1 + X + X^3} = X + X^3 + \frac{X + X^2}{1 + X + X^3}$$

Quotient $a(X) = X + X^3$ remainder $b(X) = X + X^2$

Code polynomial

$$c(X) = b(X) + X^{n-k} m(X) = X + X^2 + X^3 + X^6$$

Code word 0111001

Suppose: $g(X)=1+X+X^3$ is given

Determine: G and H

$$G(X)=\begin{bmatrix} g(X) \\ Xg(X) \\ \vdots \\ X^{k-1}g(X) \end{bmatrix}$$

$$g(X)=1+X+X^3$$

$$Xg(X)=X+X^2+X^4$$

$$X^2g(X)=X^2+X^3+X^5$$

$$X^3g(X)=X^3+X^4+X^6$$

$$G(X)=\begin{bmatrix} 1+X+X^3 & & & \\ & X+X^2+X^4 & & \\ & & X^2+X^3+X^5 & \\ & & & X^3+X^4+X^6 \end{bmatrix}$$

$$G'=\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$G' = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Non-systematic form

$$G = \left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Systematic form

If m=(1001) $\longrightarrow$ *C=0111001*

$$G = [P \,\vdots\, I_k] \qquad (10.12) \qquad\qquad H = [I_{n-k} \,\vdots\, P^T] \qquad (10.14)$$

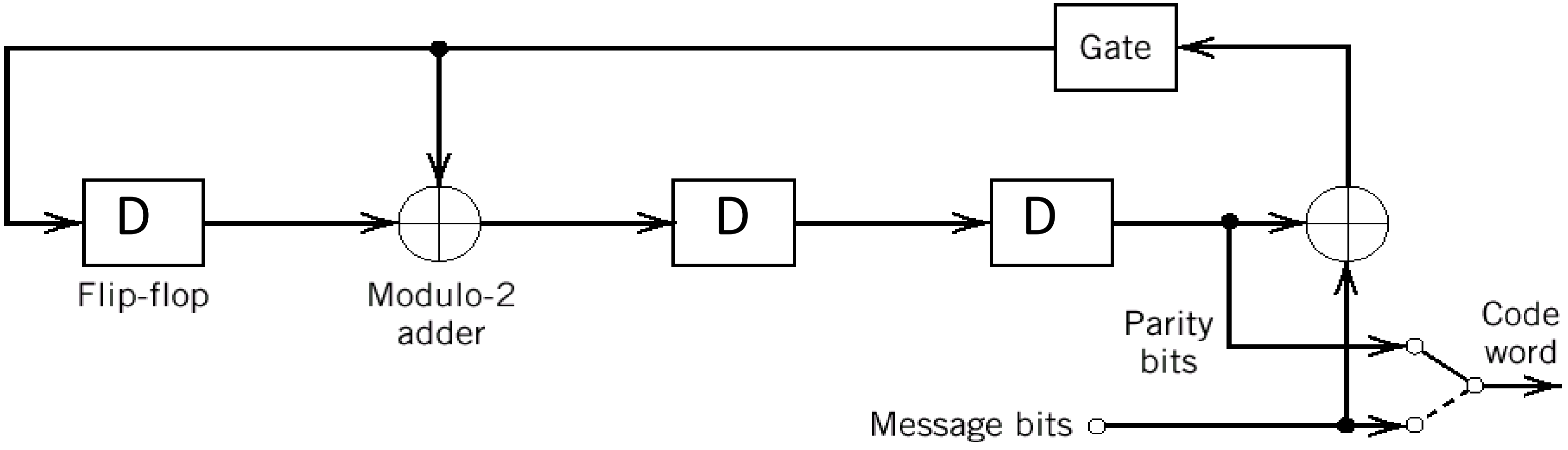
$$H = \left[\begin{array}{ccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right]$$

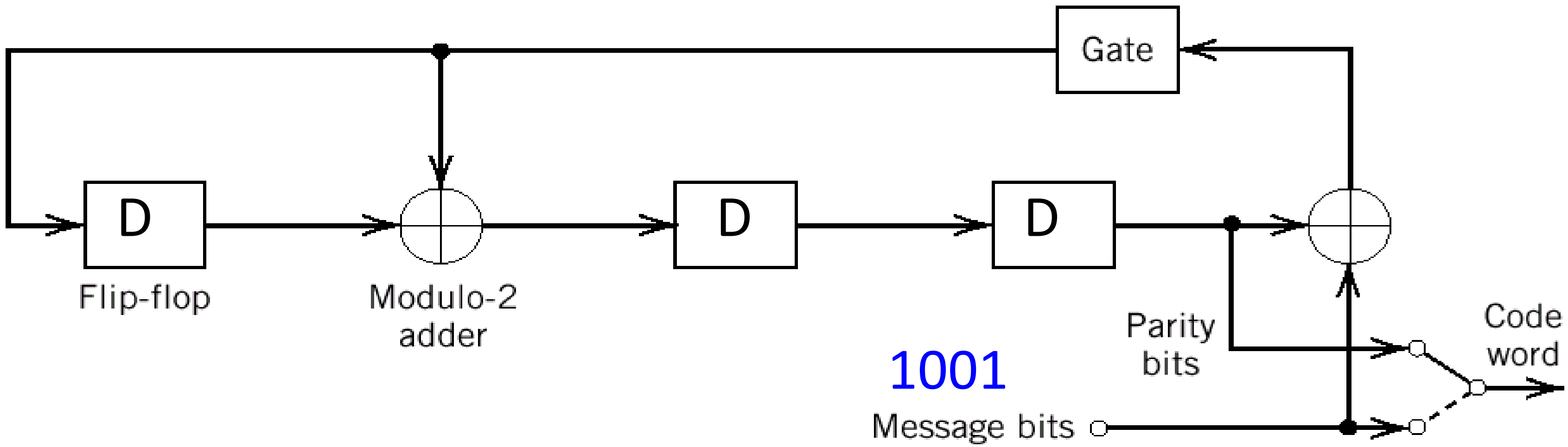
Suppose: g(X)=1+X+X^3 is given

Determine: encoder and syndrome calculator

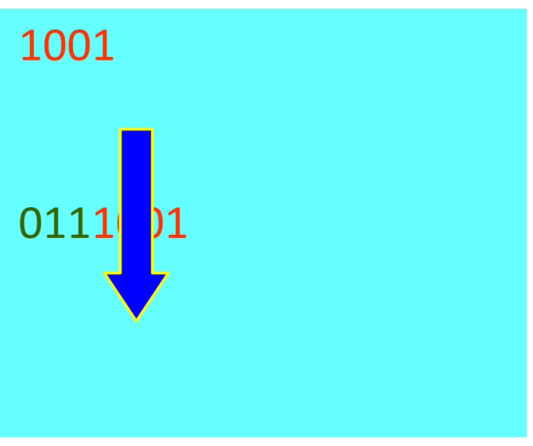
$$g(x) = 1 + \sum_{i=1}^{n-k-1} g_i X^i + X^{n-k}$$

g(X)=1+X+X^3 $\Rightarrow$ $g_1 = 1, \quad g_2 = 0$





shift	input	Register contents
		0 0 0 initial state
1	1	1 1 0
2	0	0 1 1
3	0	1 1 1
4	1	0 1 1 parity bits



Suppose: received code is 0110001

Determine: is it correct?

$$s = rH^T = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$$

$$s \neq 0$$

So, the received code vector is wrong.

Syndrome calculator

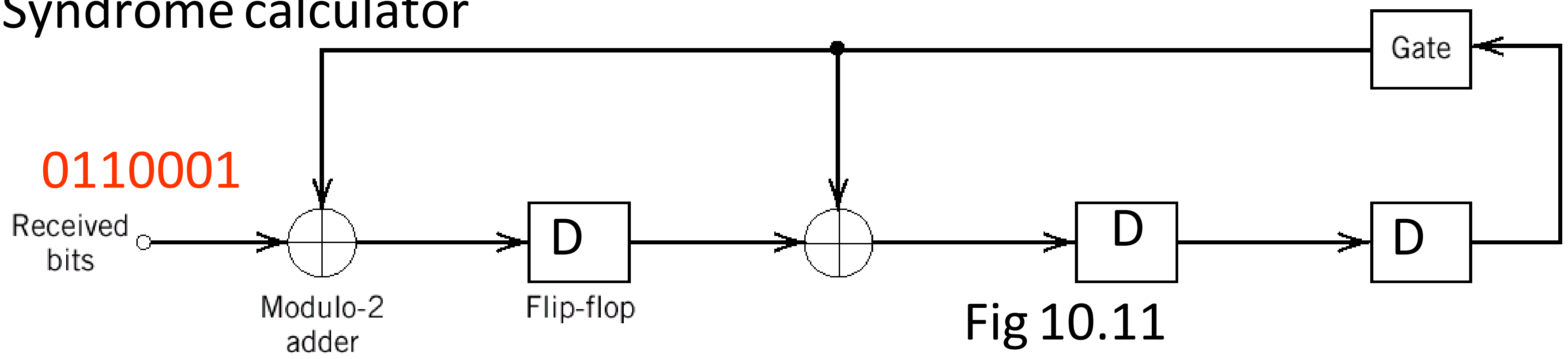


Fig 10.11

Shift	Input	Register contents
		0 0 0 (initial state)
1	1	1 0 0
2	0	0 1 0
3	0	0 0 1
4	0	1 1 0
5	1	1 1 1
6	1	0 0 1
7	0	1 1 0 syndrome

$$s \neq 0$$

So, the received code vector is wrong.

Convolutional codes

Introduction

- Convolutional codes map information to code bits sequentially by convolving a sequence of information bits with “generator” sequences
- A convolutional encoder encodes K information bits to $N>K$ code bits at one time step
- Convolutional codes can be regarded as block codes for which the encoder has a certain structure such that we can express the encoding operation as convolution

Properties of convolutional codes

Consider a convolutional encoder. Input to the encoder is a information bit sequence $\underline{u}$(partitioned into blocks of length *K*):

$$\underline{u} = (\underline{u}_0, \underline{u}_1, \ldots)$$

$$\underline{u_i} = (u_i^{(1)}, u_i^{(2)}, \ldots u_i^{(K)}),$$

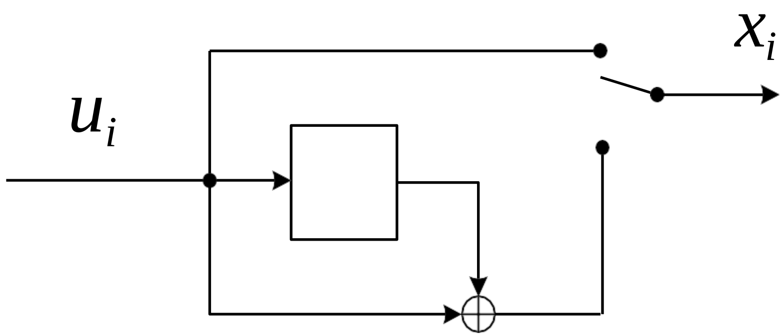
The encoder output the code bit sequence $\underline{x}$(partitioned into blocks of length *N*)

$$\underline{x} = (\underline{x}_0, \underline{x}_1, \ldots)$$

$$\underline{x}_i = (x_i^{(1)}, x_i^{(2)}, \ldots x_i^{(N)}),$$

$$R = \frac{K}{N}$$

Example: Consider a rate ½ convolutional code with K=1 and N=2 defined by the circuit:



The sequences $(x_0^{(1)}, x_1^{(1)}, \ldots)$, $(x_0^{(2)}, x_1^{(2)}, \ldots)$ are generated as follows:

$$x_i^{(1)} = u_i \quad \text{and} \quad x_i^{(2)} = u_i + u_{i-1}$$

Multiplexing between $x_i^{(1)}$ and $x_i^{(2)}$ gives the code bit sequence

$$\underline{x} = ((x_0^{(1)} x_0^{(2)}), (x_1^{(1)} x_1^{(2)}), \ldots) = (\underline{x}_0, \underline{x}_1, \ldots)$$

- The convolutional code is linear

- The encoding mapping is bijective

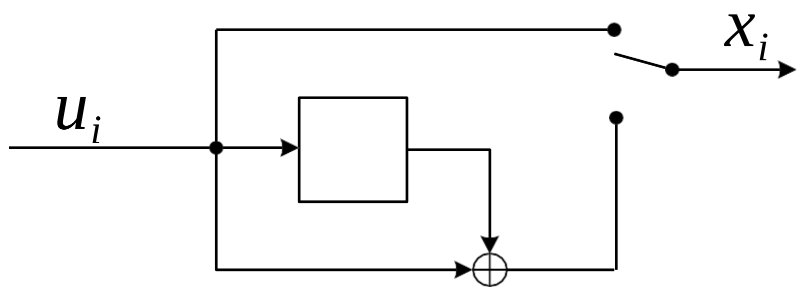
- Code bits generated at time step i are affected by information bits up to M time steps $i-1, i-2, \ldots, i-M$ back in time. M is the maximal delay of information bits in the encoder

- Code memory is the (minimal) number of registers to construct an encoding circuit for the code.

- Constraint length is the overall number of information bits affecting code bits generated at time step i: $= \text{code memory} + K = MK + K = (M+1)K$

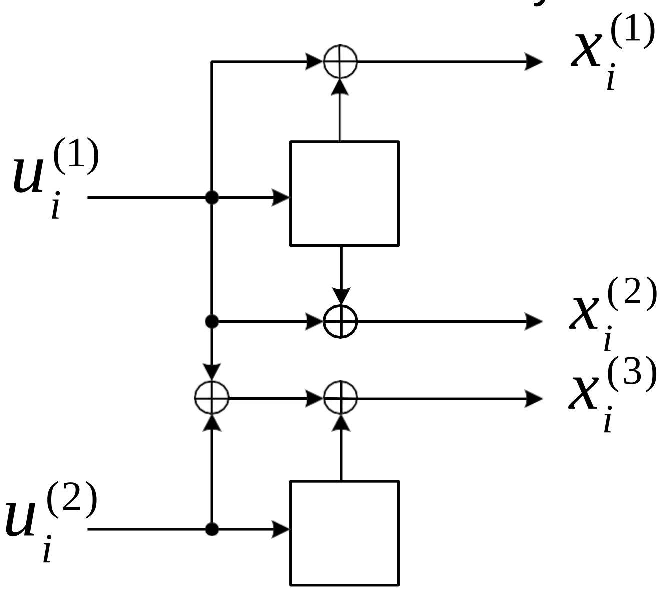
- A convolutional code is systematic if the N code bits generated at time step i contain the K information bits

Example: The rate ½ code defined by the circuit



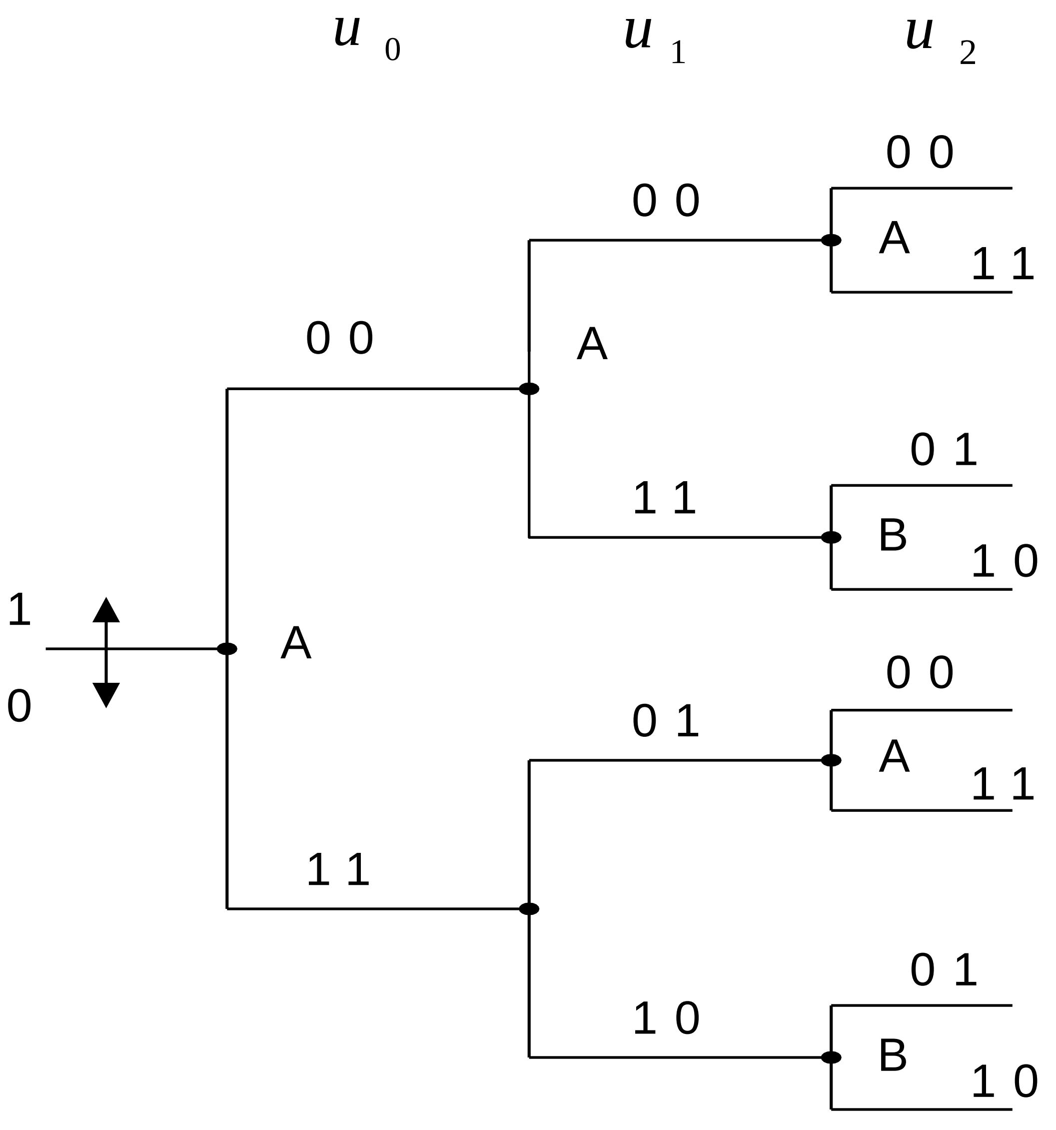
has delay M=1, memory 1, constraint length 2, and it is systematic

Example: the rate 2/3 code defined by the circuit



has delay M=1, memory 2, constraint length 4, and not systematic

Tree



Trellis

The tree graph can be contracted to a direct graph called trellis of the convolutional code having at most *S* nodes at distance *i*=0,1,… to the root

The contents of the (at most) *MK* encoder registers are assigned the variables $s_i^{(j)} \in GF(2), j = 0,1,\ldots,M \cdot K - 1$

The vector $\underline{s}_i = \left(s_i^{(0)}, s_i^{(1)}, \ldots s_i^{(M \cdot K - 1)}\right)$
combibing all register contents at time step *i* is called state of the encoder at time step *i*.

The code bit block $\underline{x}_i$ is clearly a function of $\underline{s}_i$ and $\underline{u}_i$, only

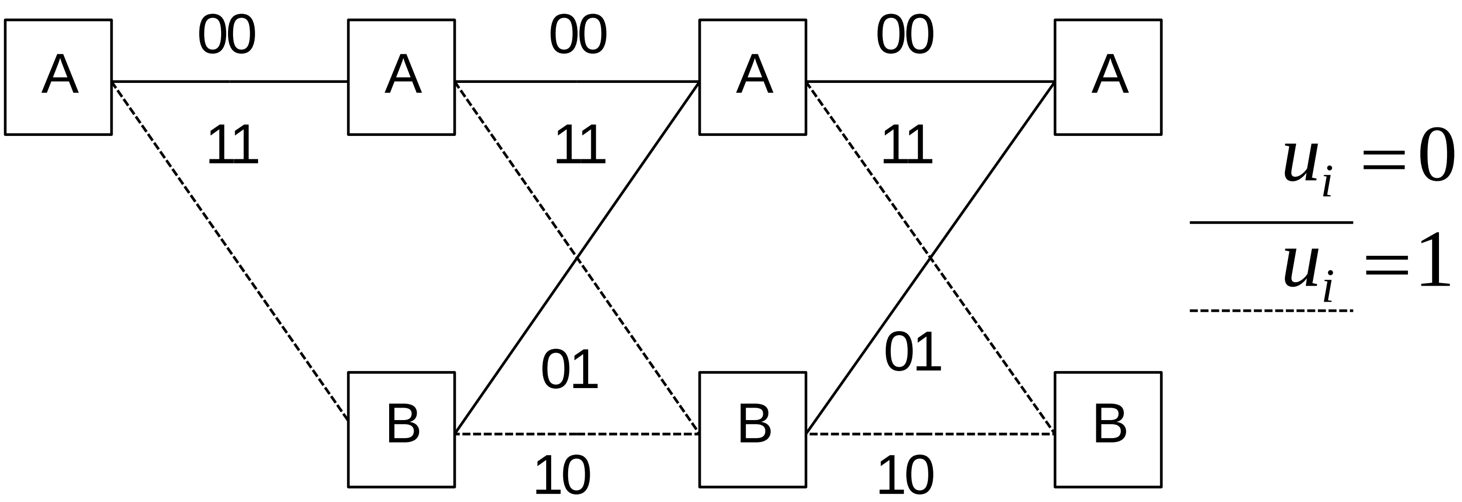
Example:

The encoder of the rate ½ convolutional code has $S = 2^1 = 2$ different states. The state is given by

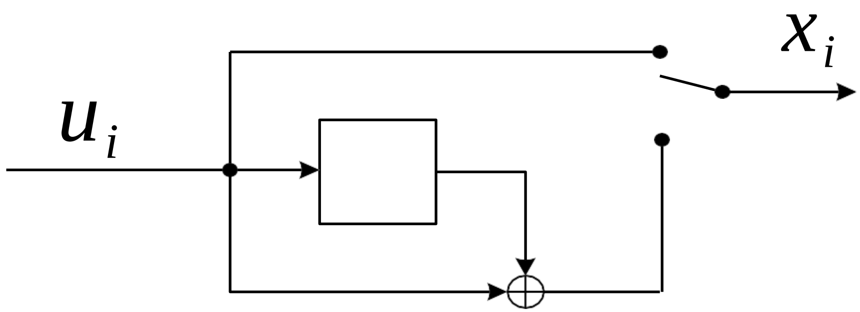
$$\underline{s} =$$

The code bit block $\underline{x}_i$ at time step *i* is computed from s_i and u_i by

$$x_i^{(1)} = u_i \text{ and } \quad x_i^{(2)} = u_i + s_i$$



Example: Constructing a trellis section



$$x_i^{(1)} = u_i \text{ and } x_i^{(2)} = u_i + s_i$$

Two equations are required:

(1) How does $\underline{s}_i$ depend on $\underline{u}_{i-m}$ and possibly $\underline{s}_{i-m}, m \rangle 0$

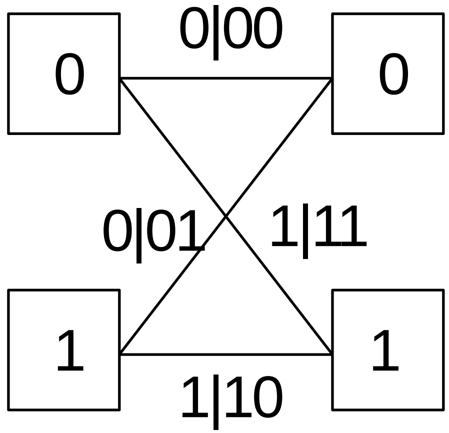
$$s_i = u_{i-1}$$

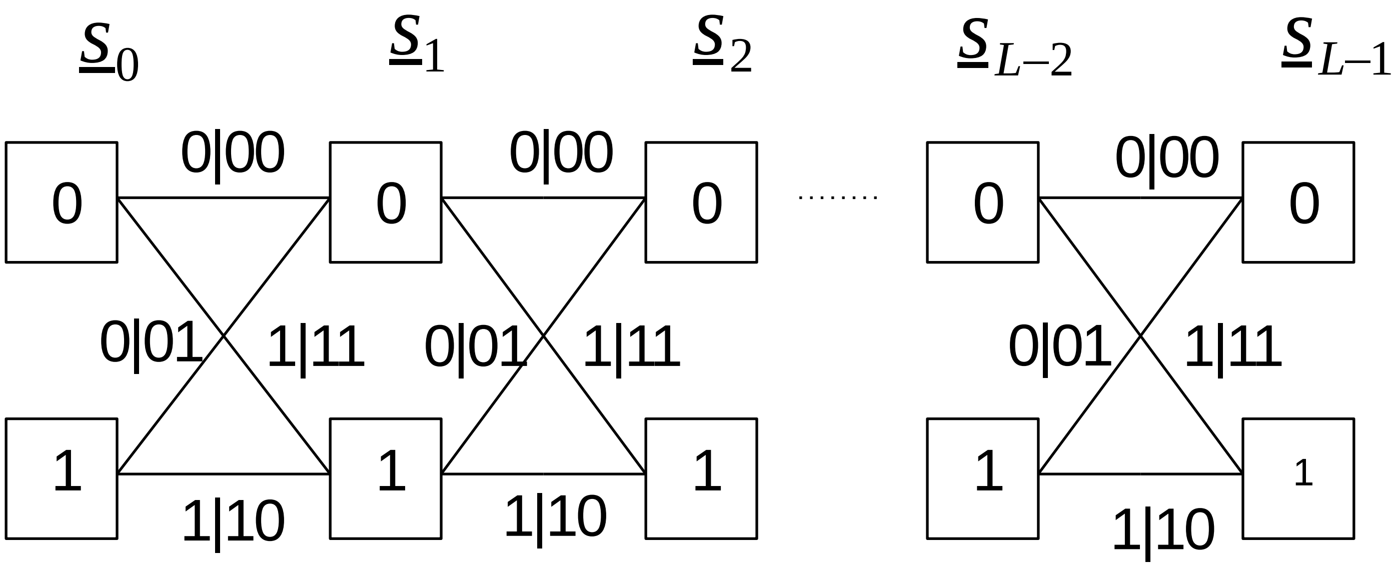
(2) How does $\underline{x}_i$ depend on $\underline{s}_i$ and $\underline{u}_i$

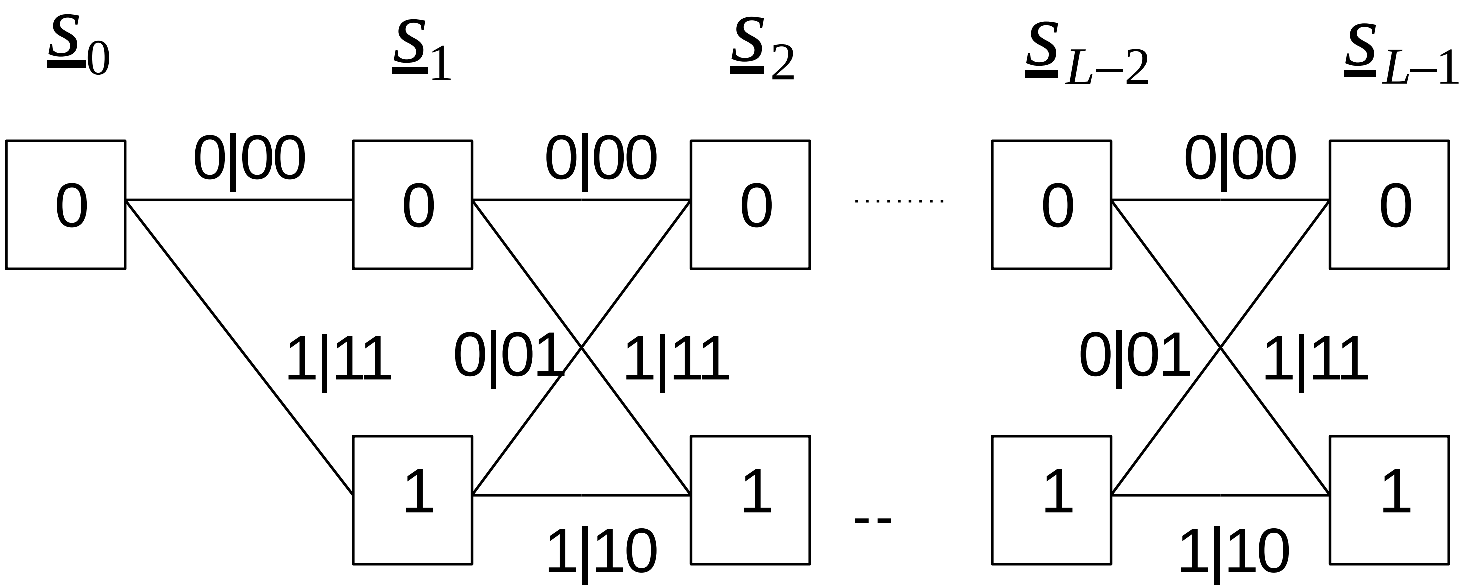
$$x_i^{(1)} = u_i \text{ and } x_i^{(2)} = u_i + s_i$$

The branches are labeled with $\underline{u}_i \mid \underline{x}_i$ called state transition leading from a state $\underline{s}_i$ to a new state $\underline{s}_{i+1}$

Trellis section:



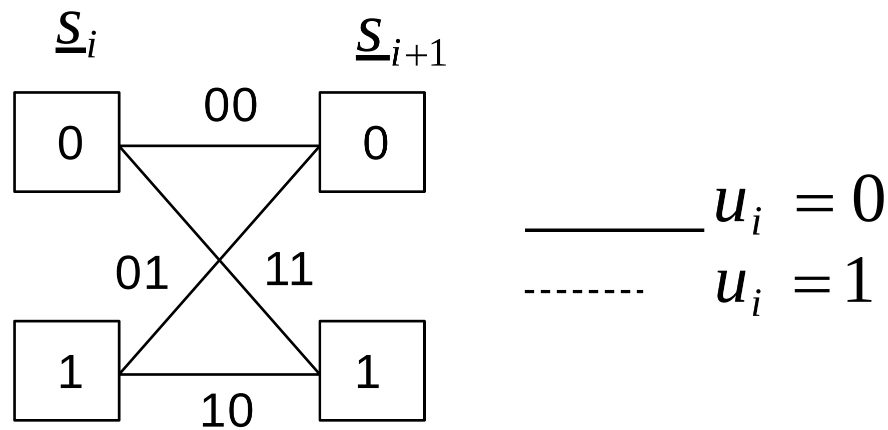



$\underline{s}_0$
$\underline{s}_1$
$\underline{s}_2$
$\underline{s}_{L-2}$
$\underline{s}_{L-1}$
0|00
0|00
0|00
0
0
0
0
0
1|11
0|01
1|11
0|01
1|11
1
1
1
1
1|10
1|10

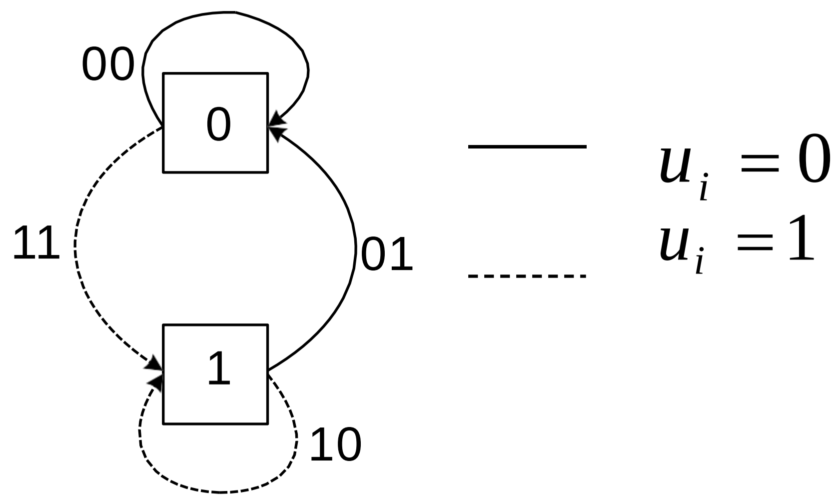
State diagram

Example: Trellis of the rate ½ convolutional code

$$x_i^{(1)} = u_i \text{ and } x_i^{(2)} = u_i + s_i$$



State diagram:



Description with submatrices

Definition: A convolutional code is a set *C* of code bit sequences

$$\underline{x} = (\underline{x}_0, \underline{x}_1, \ldots, \underline{x}_i, \ldots),$$

$$\underline{x}_i = (x_i^{(1)}, x_i^{(2)}, \ldots x_i^{(N)}),\; x_i^{(j)} \in GF(2)$$

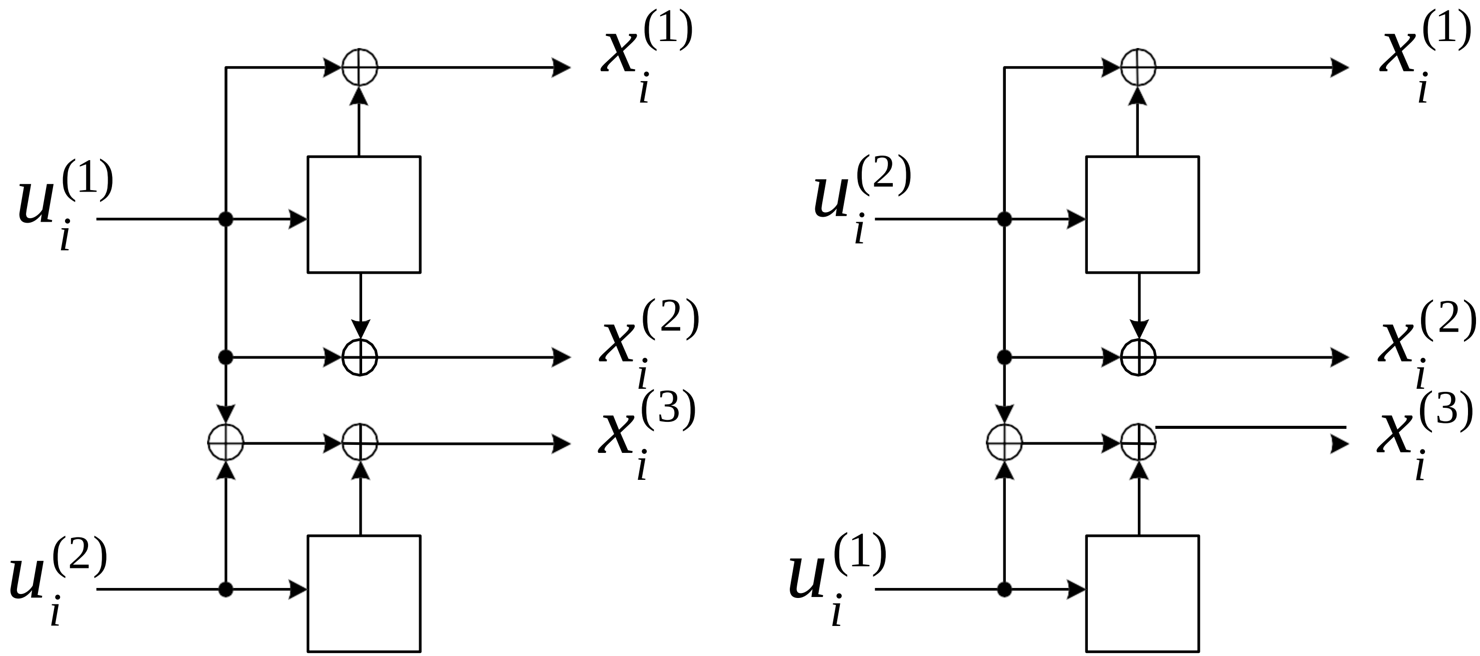
There exist many encoders mapping information bit sequences partitioned into lenth *N* blocks

$$\underline{u} = (\underline{u}_0, \underline{u}_1, \ldots)$$

$$\underline{u_i} = (u_i \quad , u_i \qquad\qquad u_i^{(j)} \in GF(2)$$

(partitioned into length *K*<*N* blocks) to code bit sequences $\underline{x}$ for the same code

Example: the following two encoding curcuits generate the same set of code word sequences



Generator matrix

$$\underline{x} = \underline{u} \cdot \underline{G},$$

where

$$\underline{G} = \begin{pmatrix} \underline{G}_0 & \underline{G}_1 & \underline{G}_2 & \cdots & \underline{G}_M & & & \\ & \underline{G}_0 & \underline{G}_1 & \underline{G}_2 & \cdots & \underline{G}_M & & \\ & & \underline{G}_0 & \underline{G}_1 & \underline{G}_2 & \cdots & \underline{G}_M & \\ & & & \ddots & \ddots & & & \ddots \end{pmatrix}$$

$$\underline{x}_i = \sum_{m=0}^{M} \underline{u}_{i-m} \underline{G}_m, \forall i$$

The generated convolutional code has rate *R=K/N*, memory *K*M*, and constraint length *K*(M+1)*

Example:

The rate ½ code is given by

$$x_i^{(1)} = u_i \quad \text{and} \quad x_i^{(2)} = u_i + s_i$$

$\underline{G}_0$ governs how $\underline{u}_i$ affects $\underline{x}_i = (x_i^{(1)} x_i^{(2)}) : \underline{G}_0 = \begin{pmatrix} 1 & 1 \end{pmatrix}$

$\underline{G}_1$ governs how $\underline{u}_{i-1}$ affects $\underline{x}_i : \underline{G}_1 = \begin{pmatrix} 0 & 1 \end{pmatrix}$

$$((x_0^{(1)} x_0^{(2)}), (x_1^{(1)} x_1^{(2)}), (x_2^{(1)} x_2^{(2)})) = (u_0, u_1, u_2) \cdot \underline{G},$$

where

$$\underline{G} = \begin{pmatrix} 11 & 01 & \\ & 11 & 01 \\ & & 11 \end{pmatrix}$$

Description with polynomials

$$\underline{G}(D)=\begin{pmatrix} g_1^{(1)}(D) & g_1^{(2)}(D) & \cdots & g_1^{(N)}(D) \\ g_2^{(1)}(D) & g_2^{(2)}(D) & \cdots & g_2^{(N)}(D) \\ \vdots & \vdots & & \vdots \\ g_K^{(1)}(D) & g_K^{(2)}(D) & \cdots & g_K^{(N)}(D) \end{pmatrix}$$

$$g_i^{(j)}(D)=g_{i,0}^{(j)}+g_{i,1}^{(j)}D^1+g_{i,2}^{(j)}D^2+\ldots+g_{i,M}^{(j)}D^M, g_{i,m}^{(j)}\in GF(2)$$

$$\underline{u}(D)=(u^{(1)}(D),u^{(2)}(D),\ldots u^{(K)}(D))\text{ where}$$

$$u^{(j)}(D)=u_0^{(j)}+u_1^{(j)}D+\ldots+u_i^{(j)}D^i+\ldots, j=1,2,\ldots,K,$$

$$\underline{x}(D)=(x^{(1)}(D),x^{(2)}(D),\ldots,x^{(N)}(D))\quad\text{where}$$

$$x^{(j)}(D)=x_0^{(j)}+x_1^{(j)}D+\ldots+x_i^{(j)}D^i+\ldots, j=1,2,\ldots,N$$

$$\underline{x}(D)=\underline{u}(D)\underline{G}(D)$$

$$g_{i,m}^{(j)}=\underline{G}_m(i,j), i=1,\ldots,K, j=1,\ldots,N, m=0,\ldots,M,$$

$$M=\max_{i,j}\deg(g_i^{(j)}(D)), i=1,\ldots,K, j=1,\ldots,N$$

Example:

The rate ½ code is given by

$$x_i^{(1)} = u_i \quad \text{and} \quad x_i^{(2)} = u_i + s_i$$

$$g_1^{(1)}(D) \text{ and } \quad g_1^{(2)}(D) \quad \underline{G}(D) = (g_1^{(1)}(D) g_1^{(2)}(D))$$

From M=1 follows that $\deg((g_i^{(j)}(D))) \leq 1$

The polynomial $g_1^{(1)}(D)$ governs how $u_{l-m}, m = 0,1,$ affects

$$x_l^{(1)} : g_1^{(1)}(D) = 1 + 0 \cdot D = 1$$

The polynomial $g_1^{(2)}(D)$ governs how $u_{l-m}, m = 0,1,$ affects

$$x_l^{(2)} : g_1^{(2)}(D) = 1 + 1 \cdot D = D + 1$$

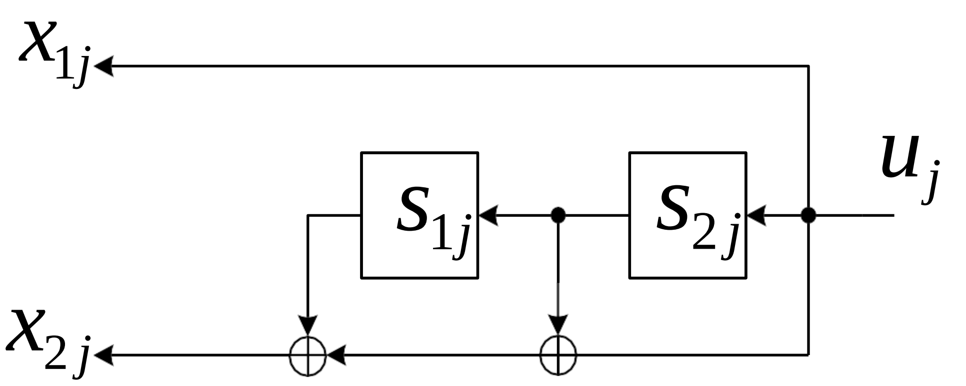
$$\underline{u} = (1,1,0,\ldots) \leftrightarrow u(D) = 1 + D$$

$\underline{x} = \underline{u} \cdot \underline{G}$, yielding $\underline{x} = (11,10,01,00,\ldots)$

$\underline{x}(D) = \underline{u}(D)\underline{G}(D)$ yielding $\underline{x}(D) = \begin{pmatrix} 1+D & 1+D^2 \end{pmatrix}$

Decoding of convolutional codes

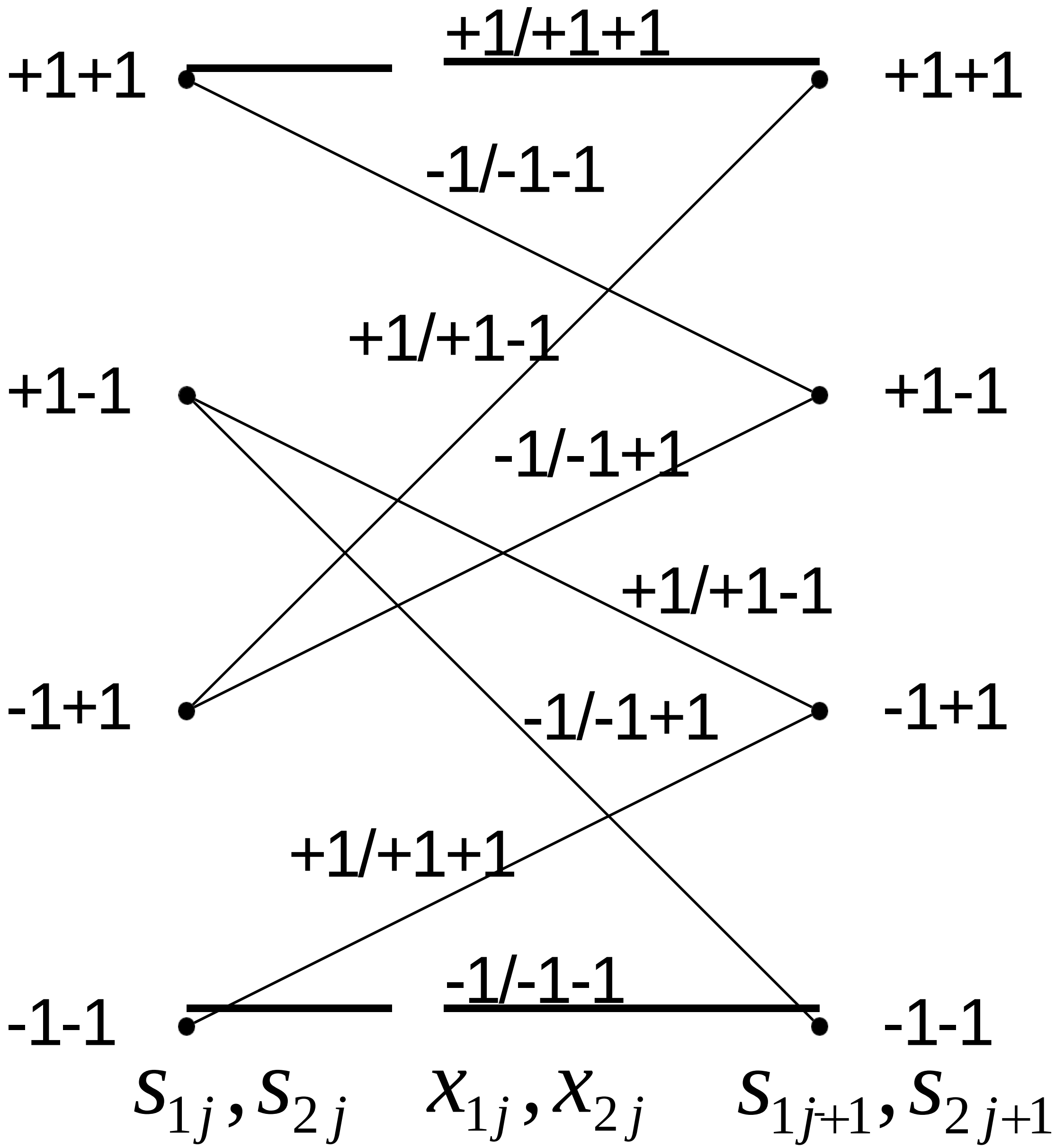
The Viterbi algorithm



$$\underline{u} = (+1,+1,+1,+1,+1,+1,+1)$$

$$(s_{10}, s_{20}) = (+1,+1)$$

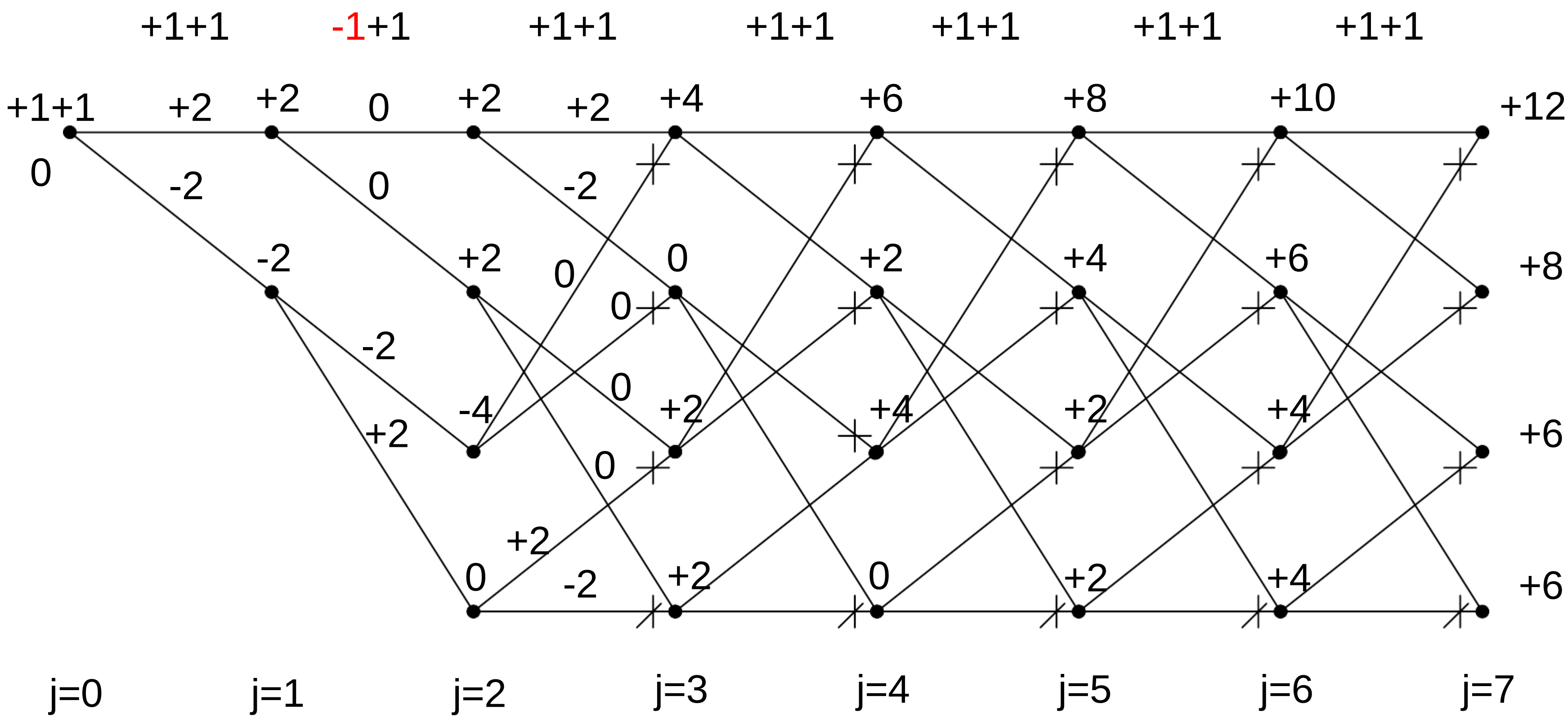
$$\underline{x} = (+1+1,+1+1,+1+1,+1+1,+1+1,+1+1,+1+1)$$

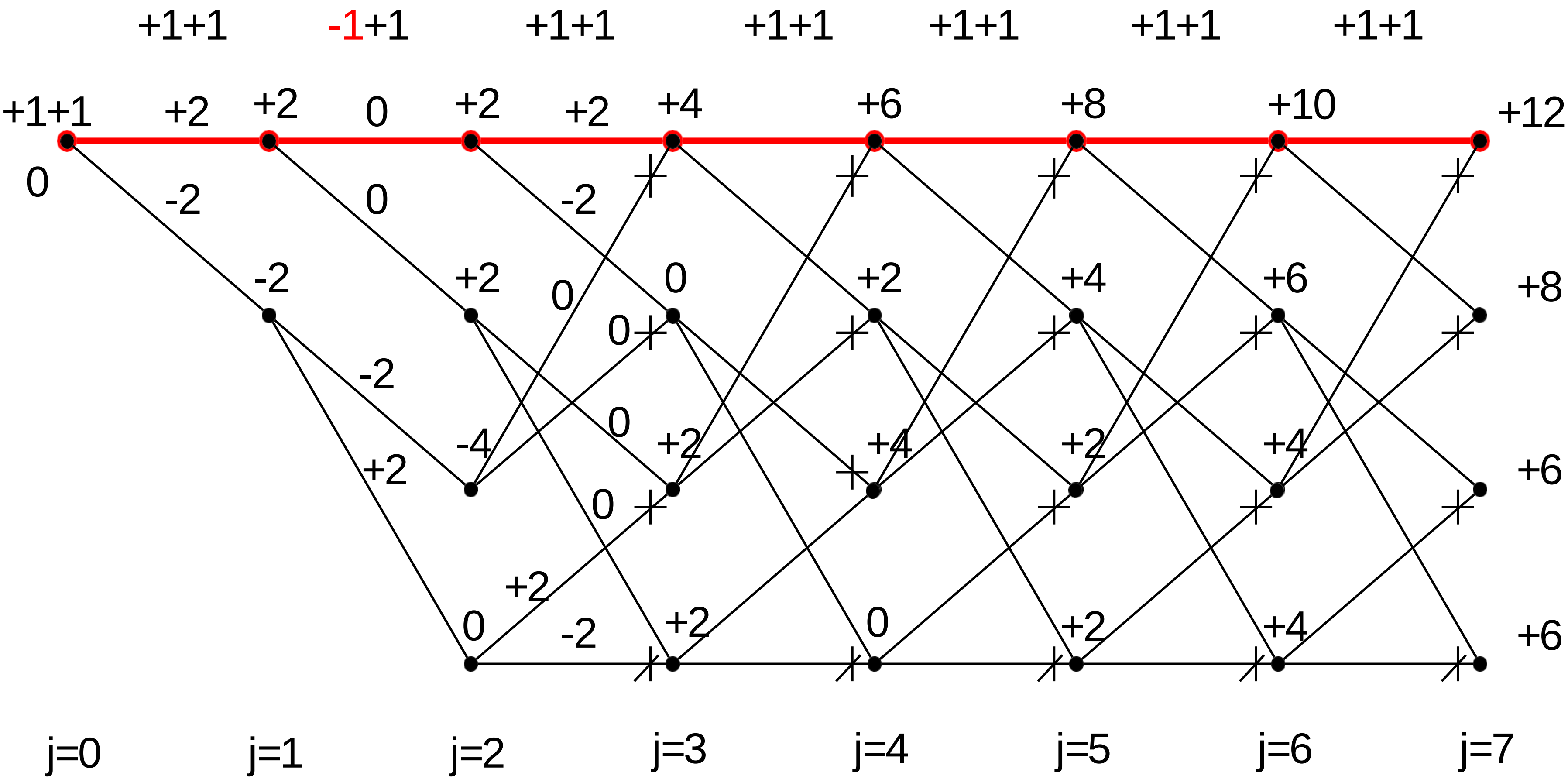

+1+1
+1-1
-1+1
-1-1
+1/+1+1
-1/-1-1
+1/+1-1
-1/-1+1
+1/+1-1
-1/-1+1
+1/+1+1
-1/-1-1
+1+1
+1-1
-1+1
-1-1
s_{1j}, s_{2j}
x_{1j}, x_{2j}
s_{1j+1}, s_{2j+1}

Hard decisions

$$\underline{y} = (+1+1,-1+1,+1+1,+1+1,+1+1,+1+1,+1+1)$$

$$\Lambda_j^{(m)} = x_{1j}^{(m)} \cdot y_{1j} + x_{2j}^{(m)} \cdot y_{2j}$$



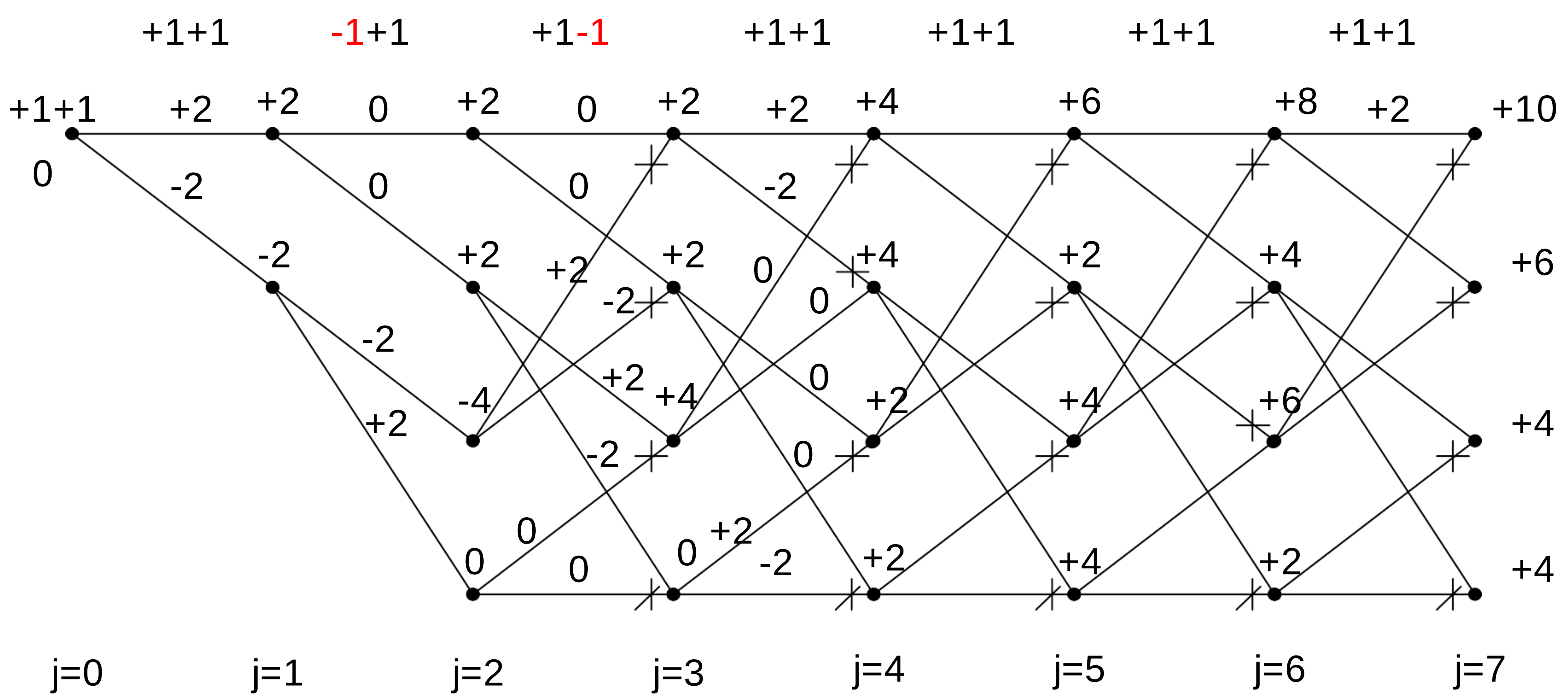


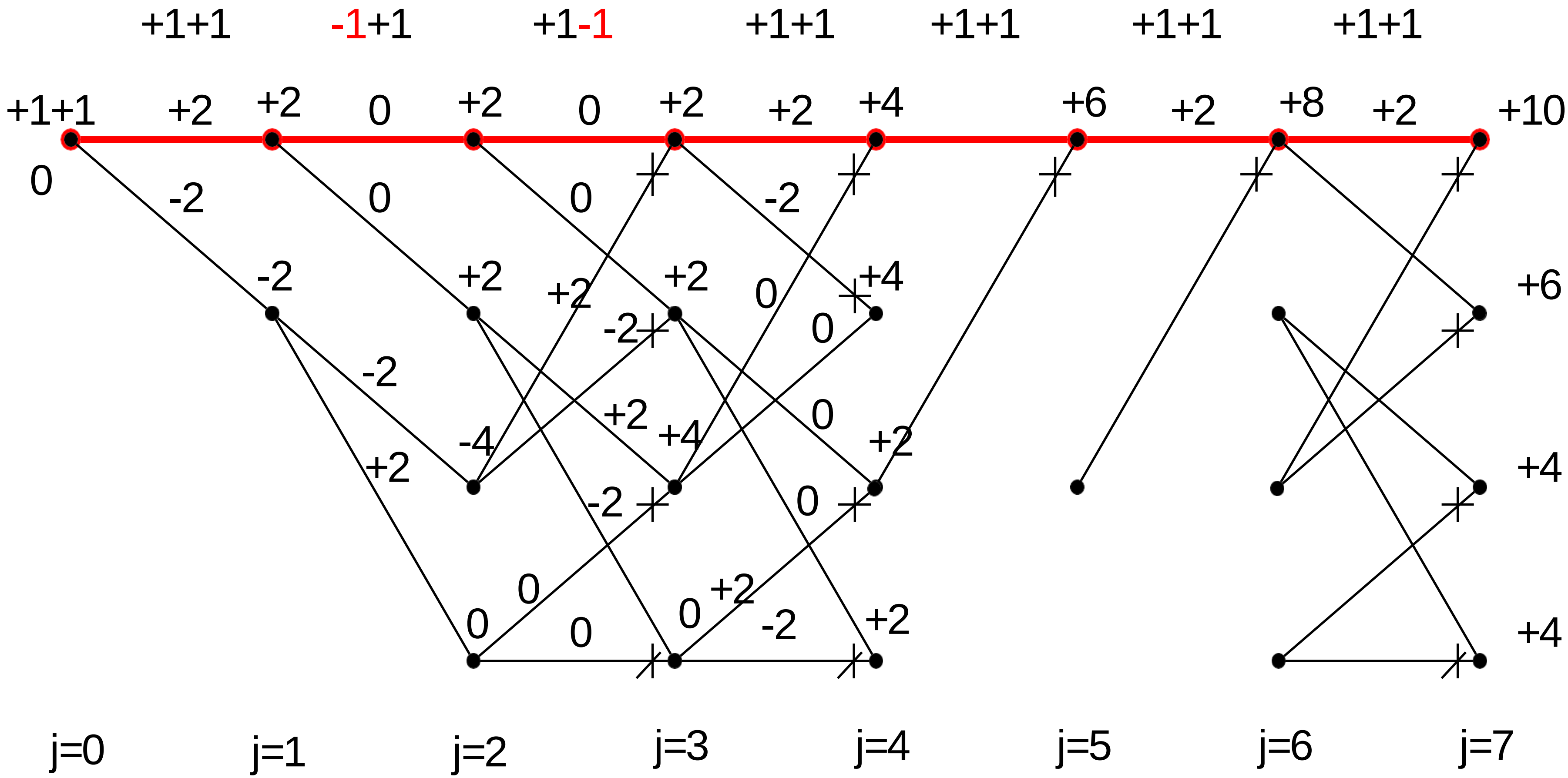
$$\hat{u}_j = +1\ +1\ +1\ +1\ +1\ +1\ +1$$ - No error

Hard decisions

$$\underline{y} = (+1+1, -1+1, +1-1, +1+1, +1+1, +1+1, +1+1)$$

$$\Lambda_j^{(m)} = x_{1j}^{(m)} \cdot y_{1j} + x_{2j}^{(m)} \cdot y_{2j}$$



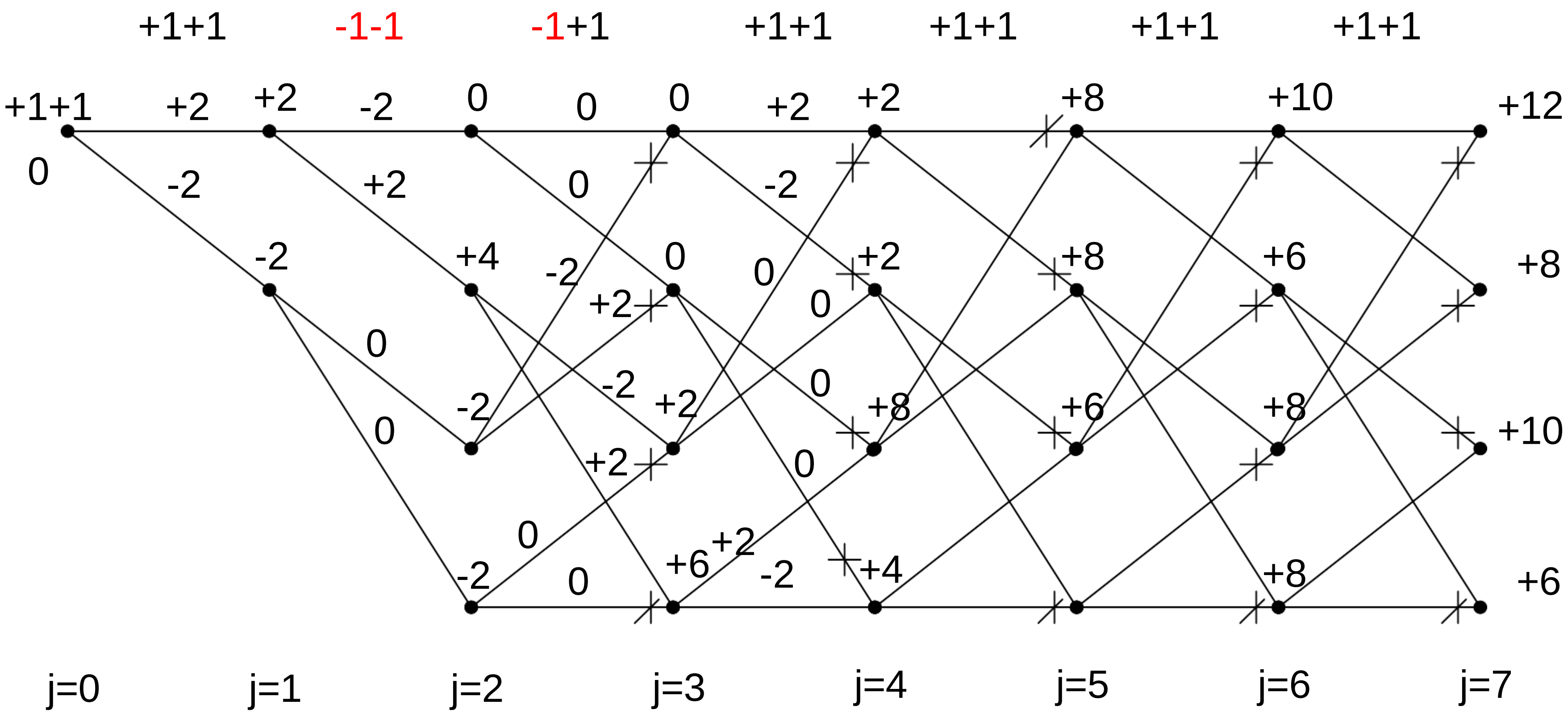


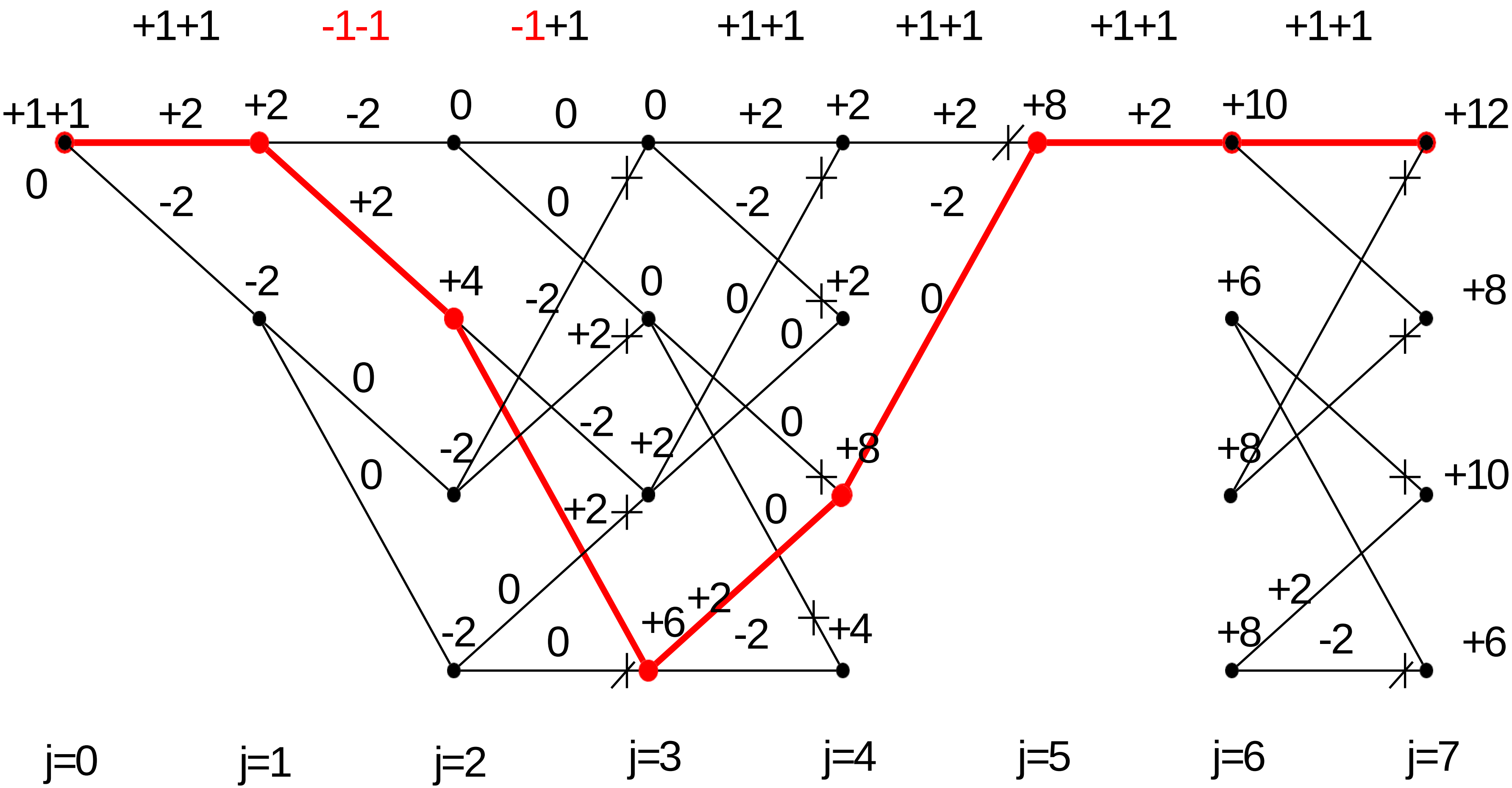
$$\hat{u}_j = +1\ +1\ +1\ +1\ +1\ +1\ +1$$ - No error

Hard decisions

$$\underline{y} = (+1+1,-1-1,-1+1,+1+1,+1+1,+1+1,+1+1)$$

$$\Lambda_j^{(m)} = x_{1j}^{(m)} \cdot y_{1j} + x_{2j}^{(m)} \cdot y_{2j}$$





$$\hat{u}_j = +1\ -1\ -1\ +1\ +1\ +1\ +1$$ - 2 decoding errors

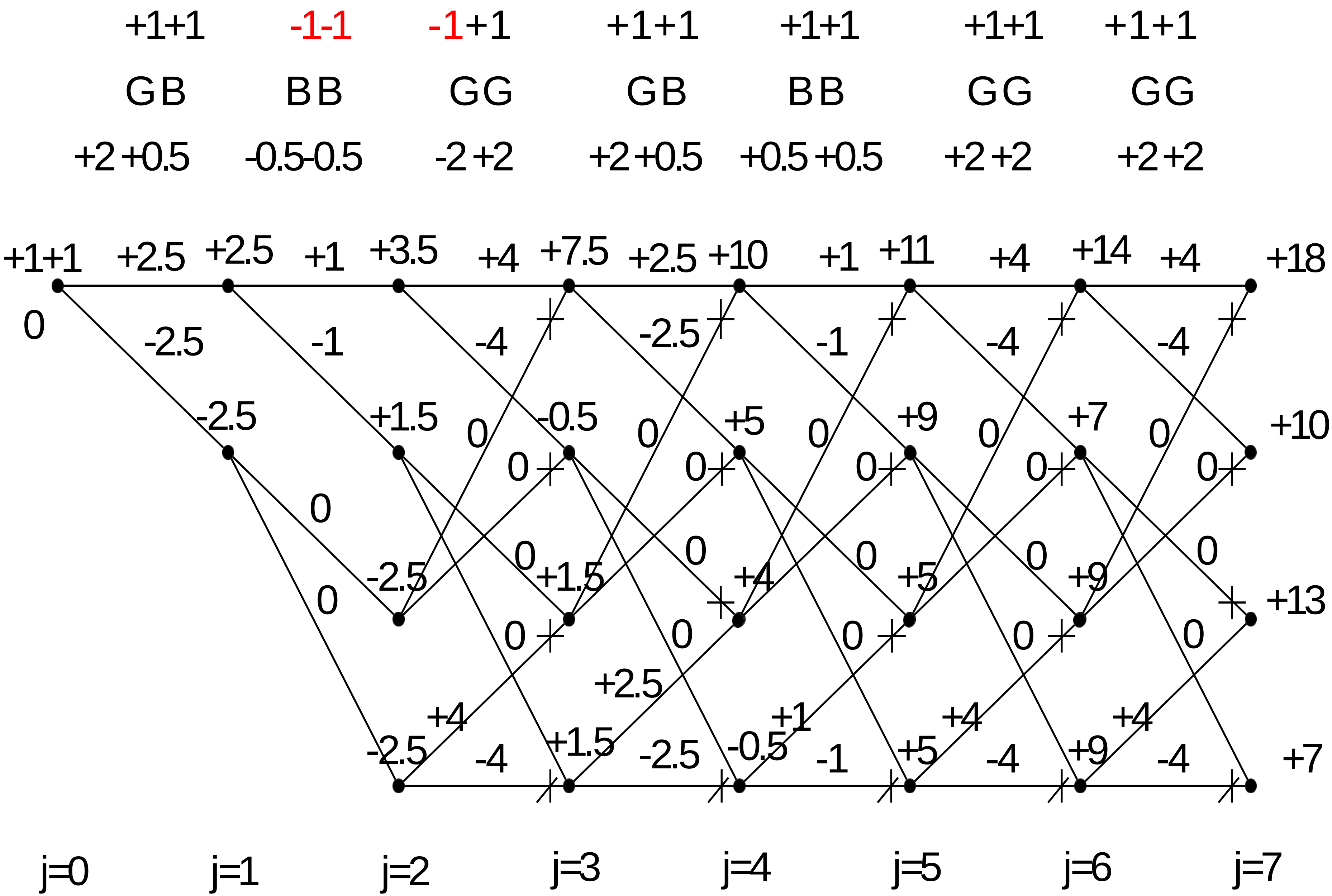
Soft decisions

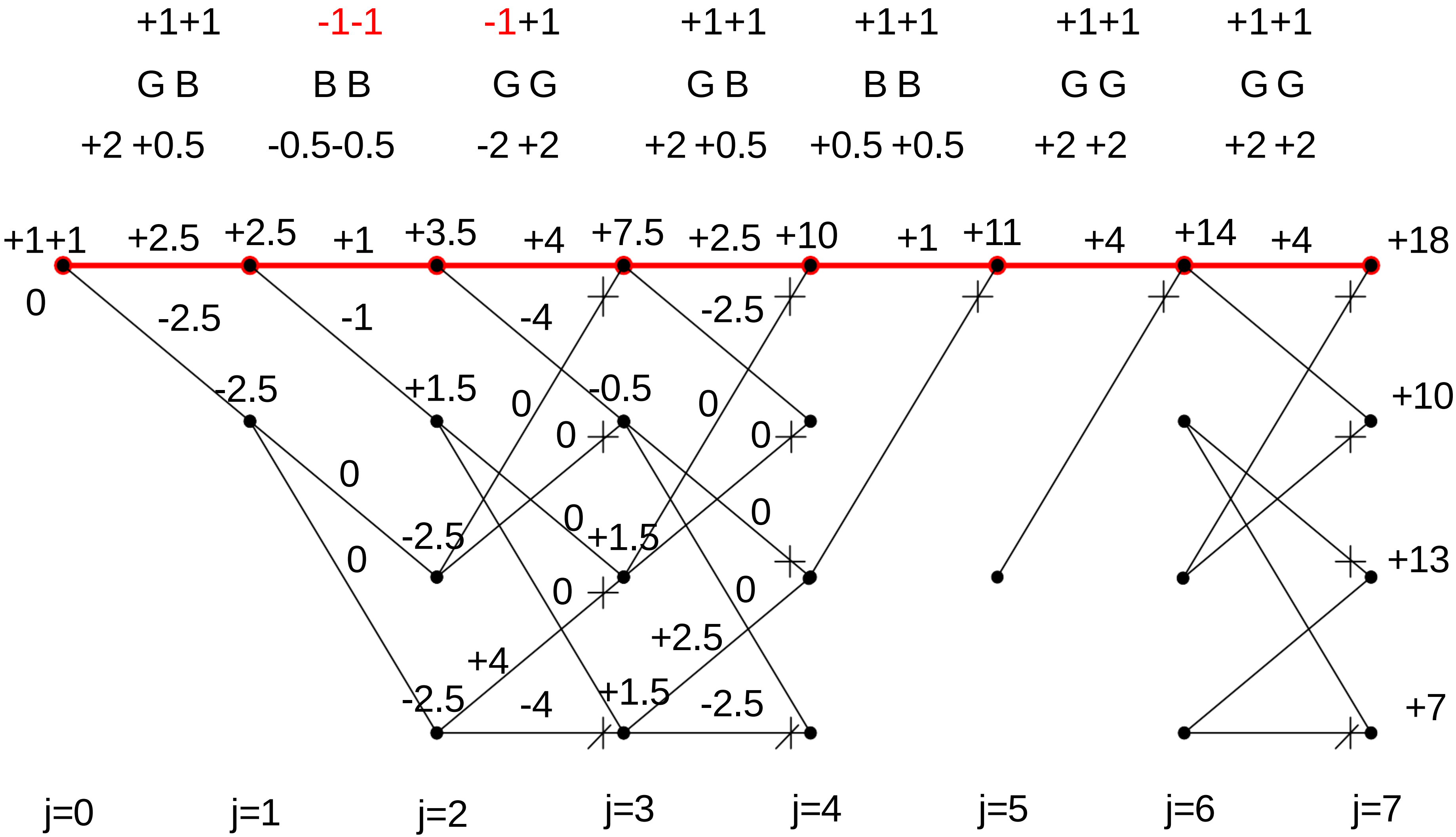
$$l_{ij} = \begin{cases} +2, x_{ij}^{(m)} = y_{ij}, & \text{GOOD channel} \\ +1/2, x_{ij}^{(m)} = y_{ij}, & \text{BAD channel} \\ -1/2, x_{ij}^{(m)} \neq y_{ij}, & \text{BAD channel} \\ -2, x_{ij}^{(m)} \neq y_{ij}, & \text{GOOD channel} \end{cases}$$

$$\lambda_j^{(m)} = x_{1j} l_{1j} y_{1j} + x_{2j} l_{2j} y_{2j}$$

CSI values ((G,B),(B,B),(G,G),(G,B),(B,B),(G,G),(G,G))

$$\underline{y} = (+1+1,-1-1,-1+1,+1+1,+1+1,+1+1,+1+1)$$


+1+1 -1-1 -1+1 +1+1 +1+1 +1+1 +1+1
GB BB GG GB BB GG GG
+2 +0.5 -0.5-0.5 -2 +2 +2 +0.5 +0.5 +0.5 +2 +2 +2 +2
+1+1
+2.5
+1
+4
+2.5
+1
+4
+4
+18
+10
+13
+7
j=0 j=1 j=2 j=3 j=4 j=5 j=6 j=7



$\hat{u}_j =$ +1 +1 +1 +1 +1 +1 +1 - No error

Spread Spectrum

Introduction to Spread Spectrum

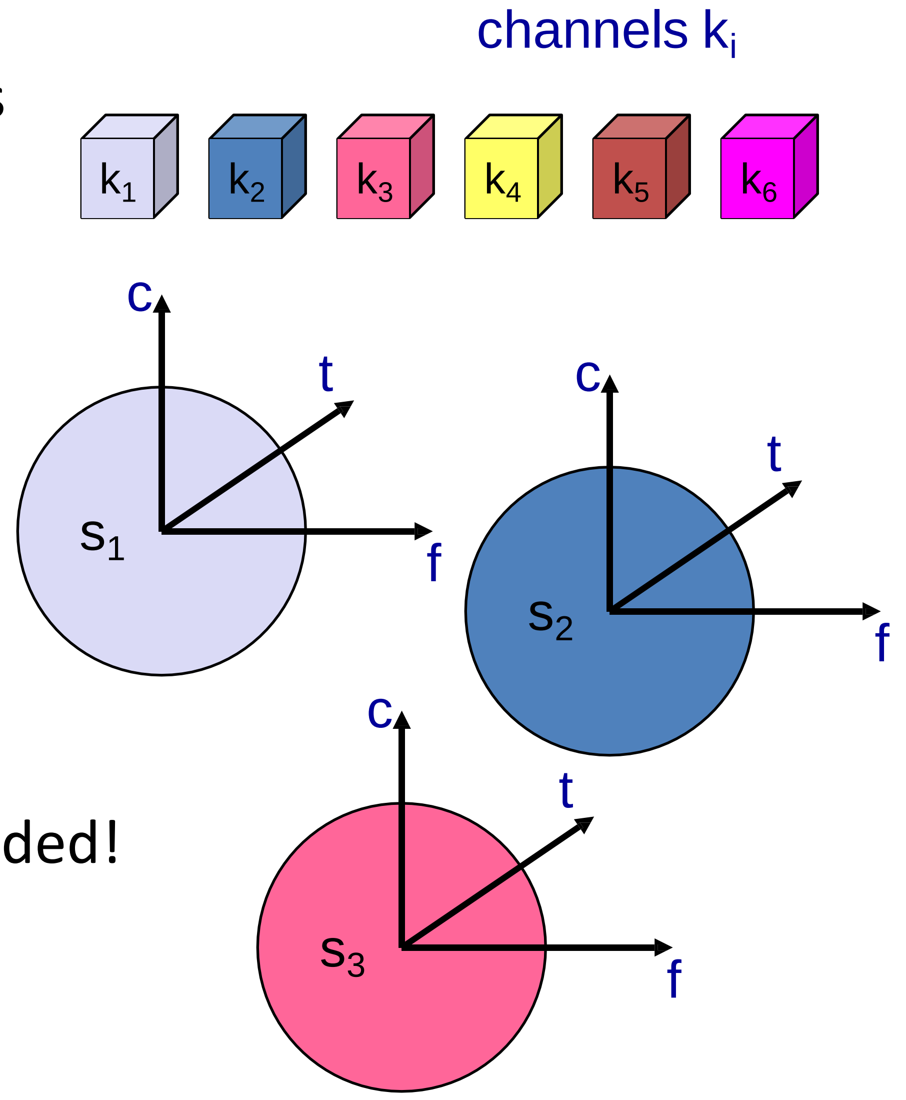
- Problems such as capacity limits, propagation effects, synchronization occur with wireless systems
- Spread spectrum modulation spreads out the modulated signal bandwidth so it is much greater than the message bandwidth
- Independent code spreads signal at transmitter and despreads signal at receiver

Multiplexing

- Multiplexing in 4 dimensions
  - space (s_i)
  - time (t)
  - frequency (f)
  - code (c)

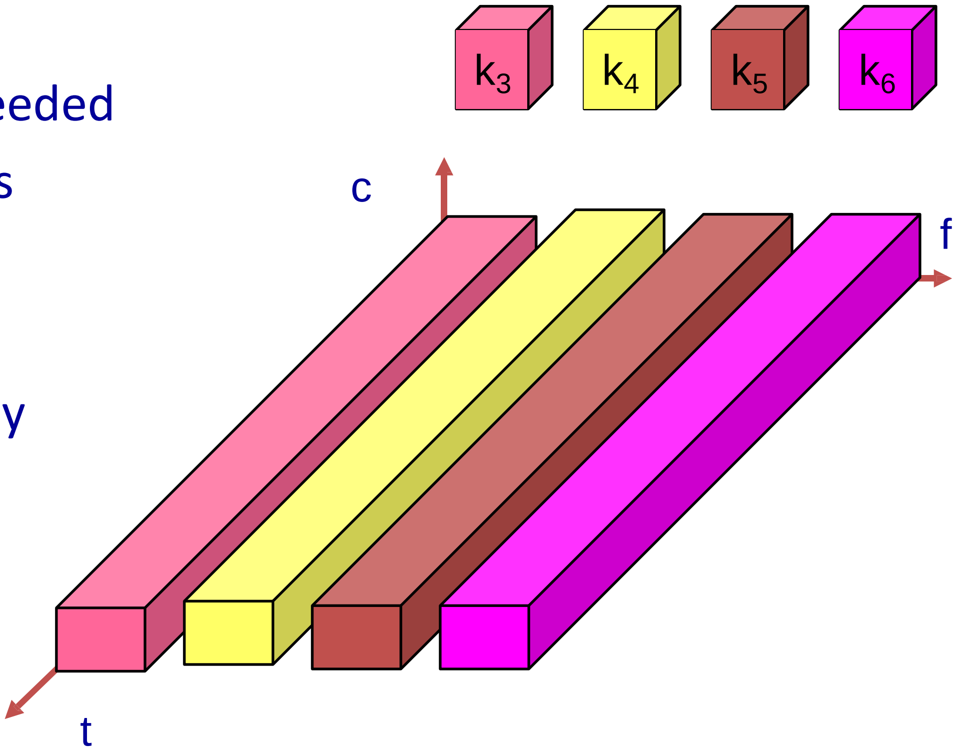
- Goal: multiple use of a shared medium

- Important: guard spaces needed!



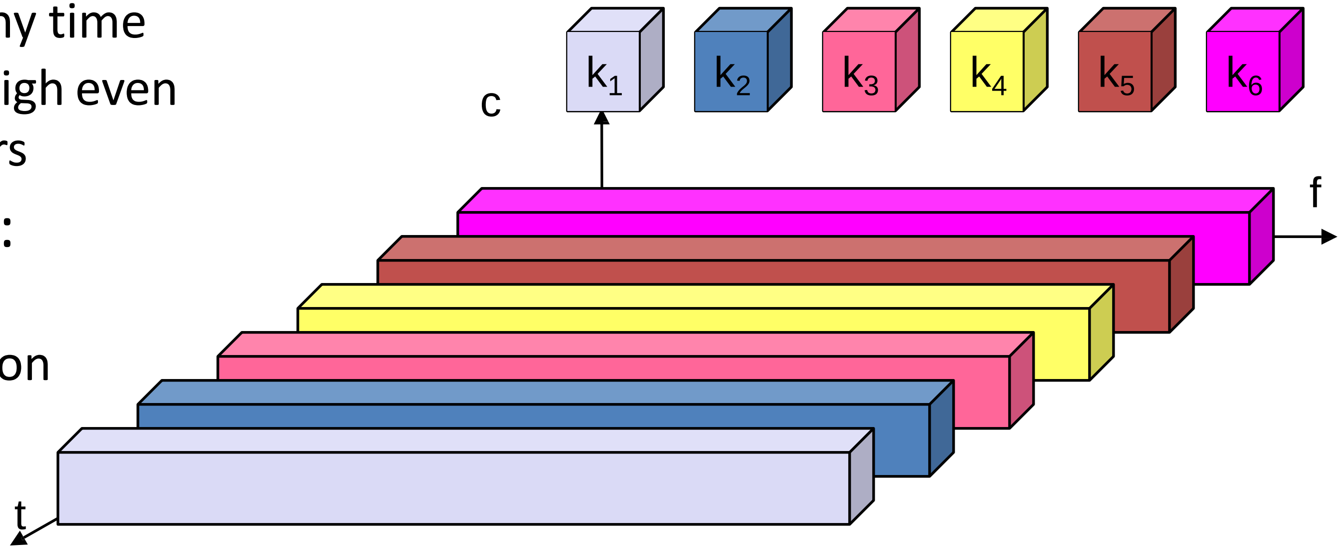
Frequency multiplex

- Separation of spectrum into smaller frequency bands
- Channel gets band of the spectrum for the whole time
- Advantages:
  - no dynamic coordination needed
  - works also for analog signals
- Disadvantages:
  - waste of bandwidth
    if traffic distributed unevenly
  - inflexible
  - guard spaces



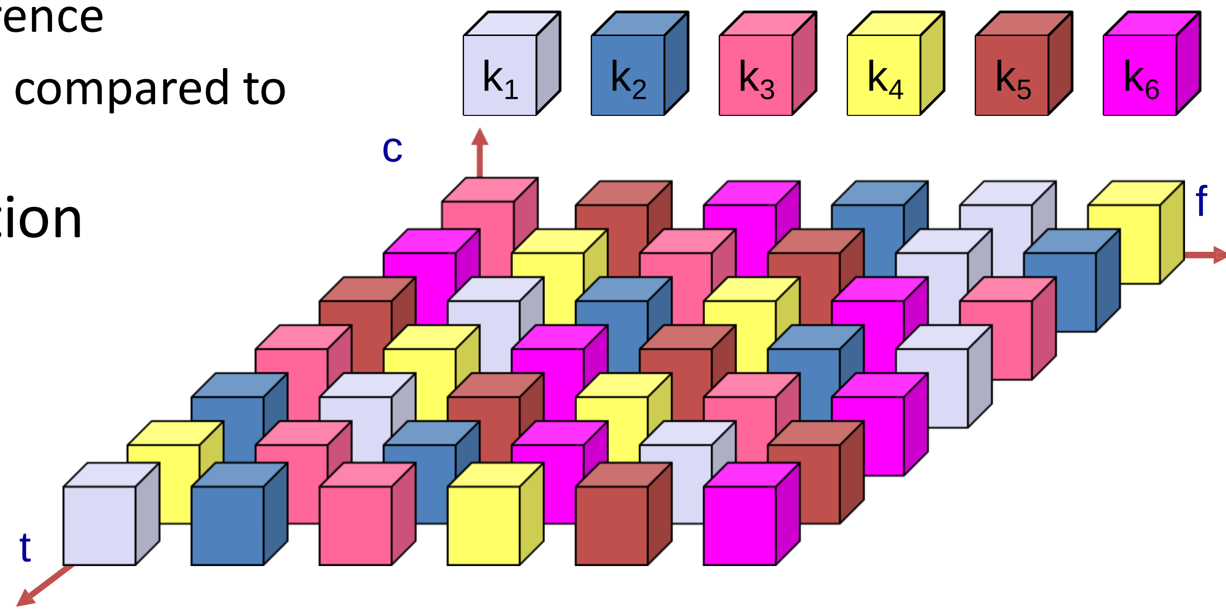
Time multiplex

- Channel gets the whole spectrum for a certain amount of time
- Advantages:
  - only one carrier in the medium at any time
  - throughput high even for many users
- Disadvantages:
  - precise synchronization necessary



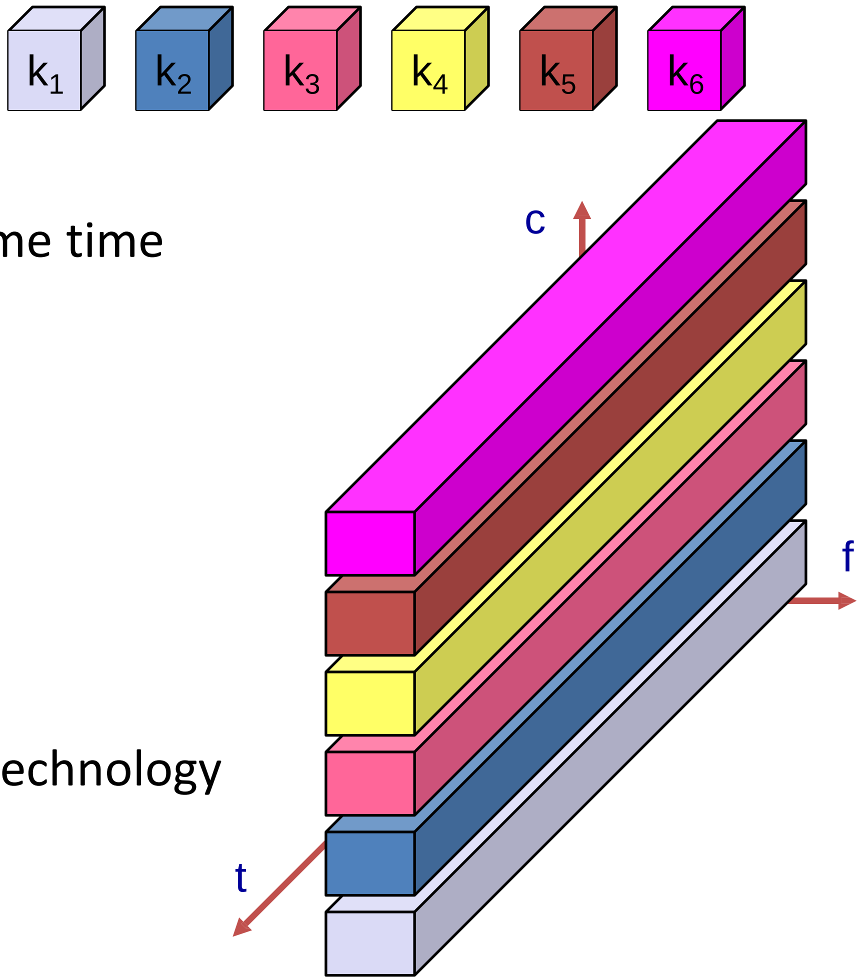
Time and frequency multiplex

- A channel gets a certain frequency band for a certain amount of time (e.g. GSM)
- Advantages:
  - better protection against tapping
  - protection against frequency selective interference
  - higher data rates compared to code multiplex
- Precise coordination required



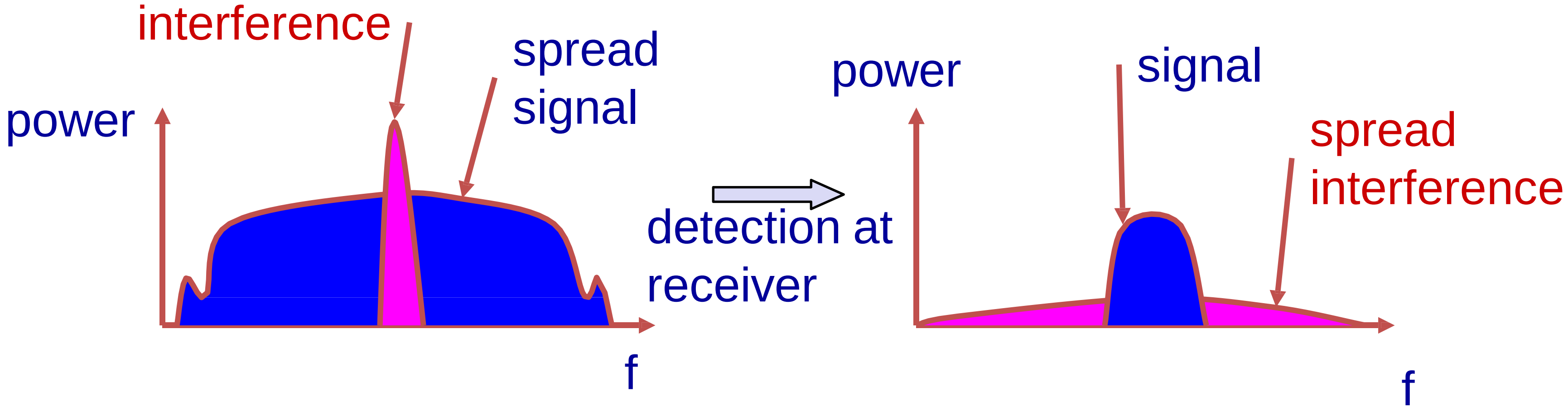
Code multiplex

- Each channel has unique code
- All channels use same spectrum at same time
- Advantages:
  - bandwidth efficient
  - no coordination and synchronization
  - good protection against interference
- Disadvantages:
  - lower user data rates
  - more complex signal regeneration
- Implemented using spread spectrum technology



Spread Spectrum Technology

- Problem of radio transmission: frequency dependent fading can wipe out narrow band signals for duration of the interference
- Solution: spread the narrow band signal into a broad band signal using a special code

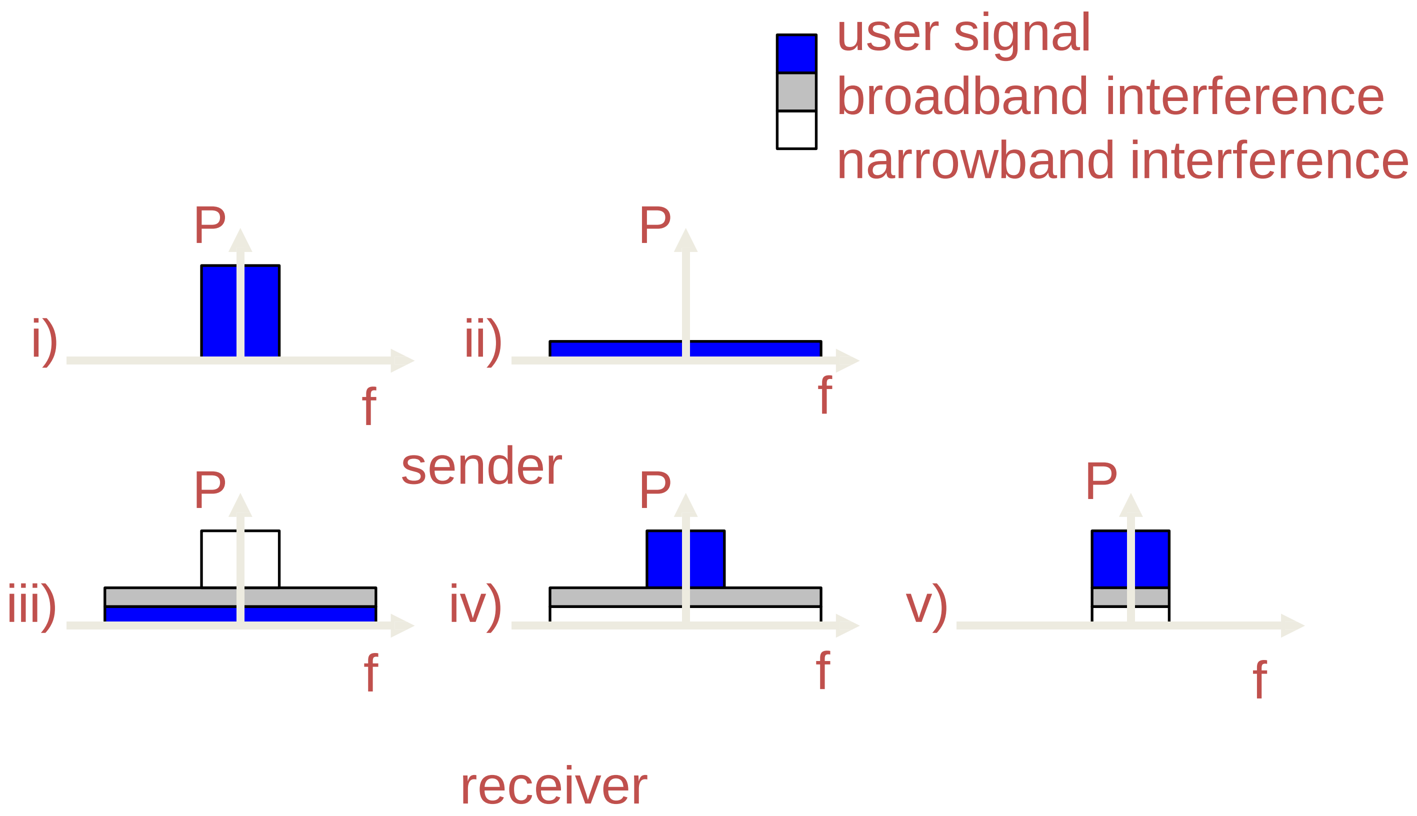


Spread Spectrum Technology

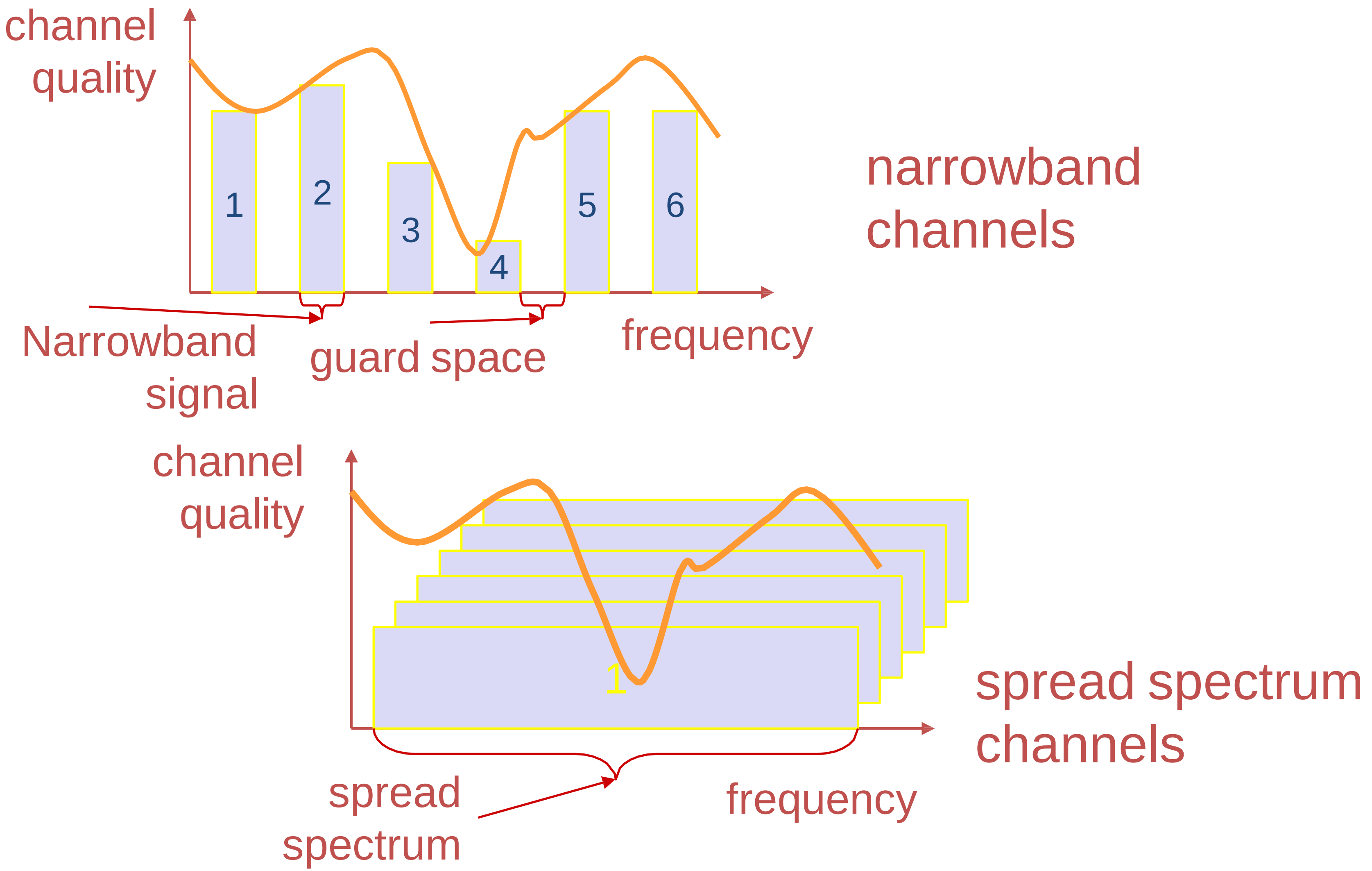
- Side effects:
  - coexistence of several signals without dynamic coordination
  - tap-proof
- Alternatives: Direct Sequence (DS/SS), Frequency Hopping (FH/SS)
- Spread spectrum increases BW of message signal by a factor *N*, Processing Gain

$$\text{Processing Gain } N = \frac{B_{ss}}{B} = 10\log_{10}\left(\frac{B_{ss}}{B}\right)$$

Effects of spreading and interference

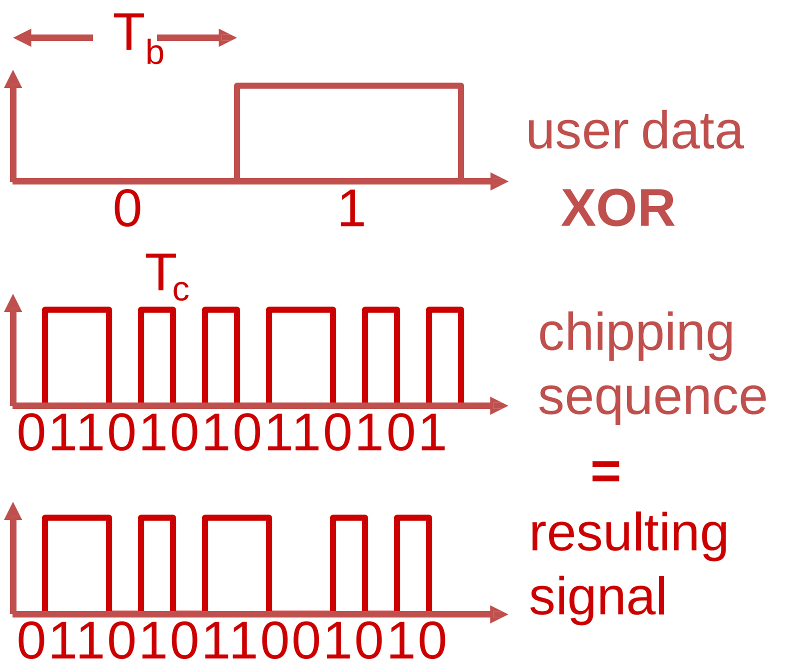


Spreading and frequency selective fading

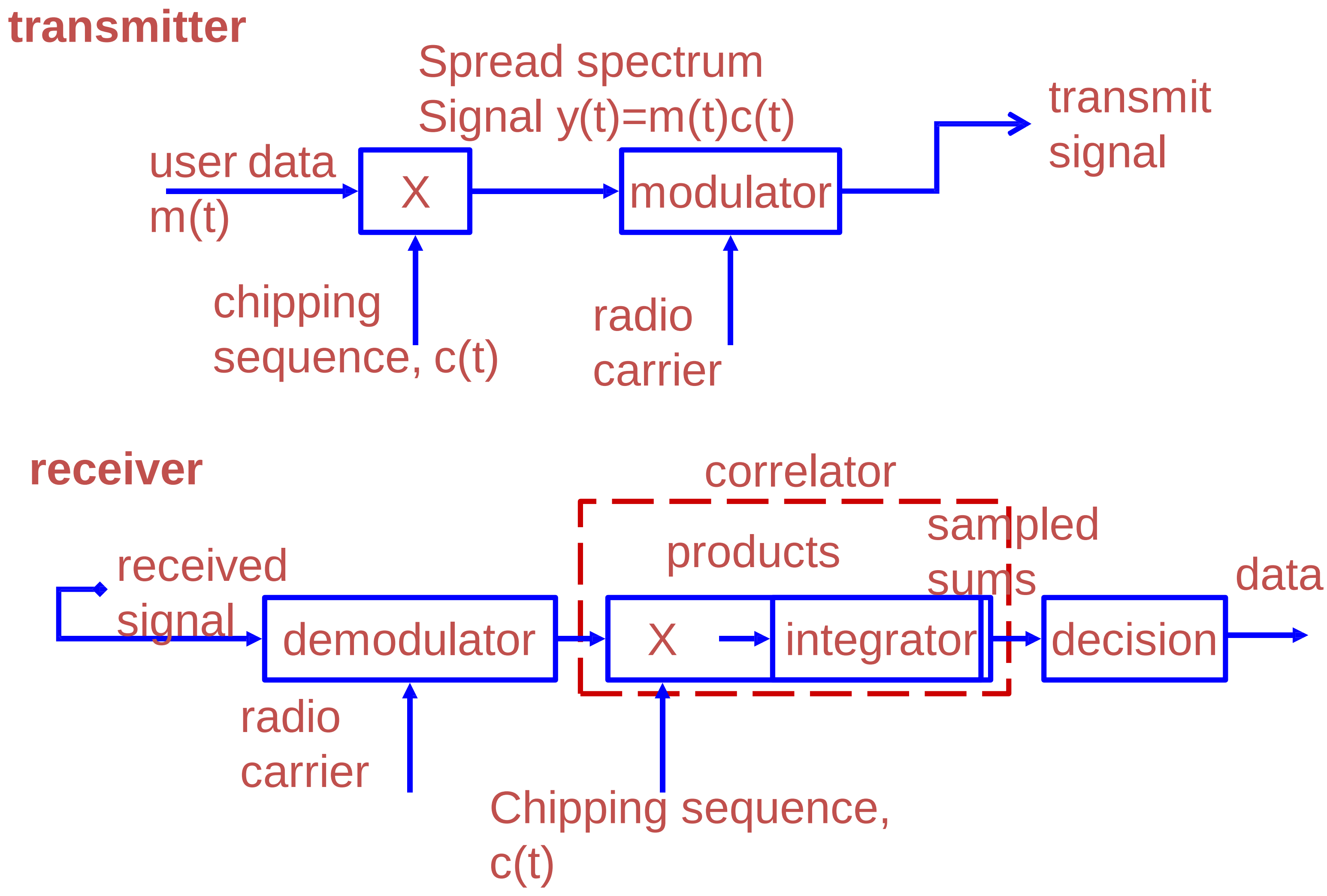


DSSS (Direct Sequence Spread Spectrum) I

- XOR the signal with pseudonoise (PN) sequence (chipping sequence)
- Advantages
  - reduces frequency selective fading
  - in cellular networks
    - base stations can use the same frequency range
    - several base stations can detect and recover the signal
- But, needs precise power control



DSSS (Direct Sequence Spread Spectrum) II



DS/SS Comments III

- Pseudonoise(PN) sequence chosen so that its autocorrelation is very narrow => PSD is very wide
  - Concentrated around $\tau \leq T_c$
  - Cross-correlation between two user's codes is very small

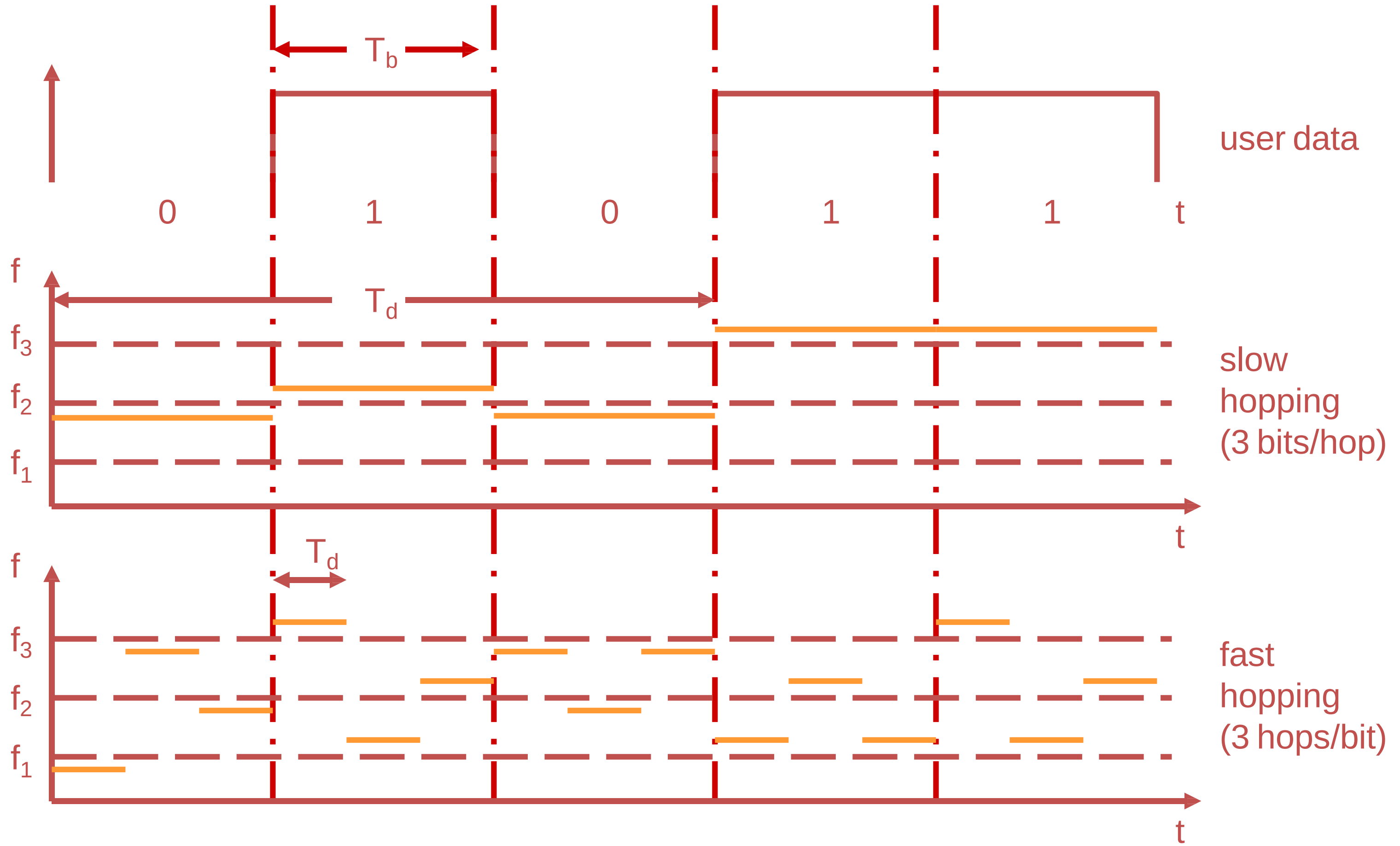
DS/SS Comments IV

- Secure and Jamming Resistant
  - Both receiver and transmitter must know c(t)
  - Since PSD is low, hard to tell if signal present
  - Since wide response, tough to jam everything
- Multiple access
  - If $c_i(t)$ is orthogonal to $c_j(t)$, then users do not interfere
- Near/Far problem
  - Users must be received with the same power

FH/SS (Frequency Hopping Spread Spectrum) I

- Discrete changes of carrier frequency
  - sequence of frequency changes determined via PN sequence
- Two versions
  - **Fast Hopping**: several frequencies per user bit (FFH)
  - **Slow Hopping**: several user bits per frequency (SFH)
- Advantages
  - frequency selective fading and interference limited to short period
  - uses only small portion of spectrum at any time
- Disadvantages
  - not as robust as DS/SS
  - simpler to detect

FHSS (Frequency Hopping Spread Spectrum) II

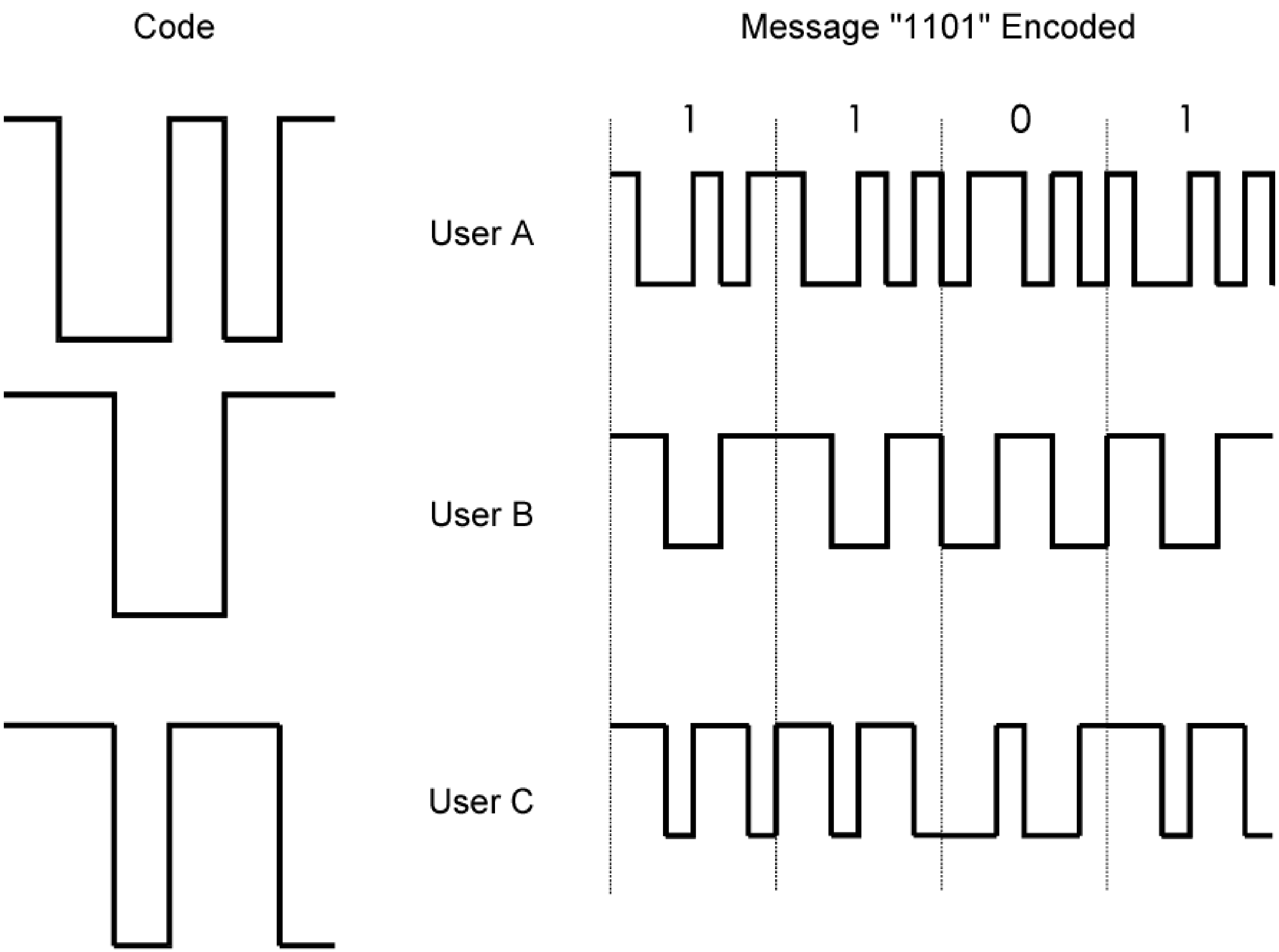


T_b: bit period T_d: dwell time

Code Division Multiple Access (CDMA)

- Multiplexing Technique used with spread spectrum
- Start with data signal rate *D*
  - Called bit data rate
- Break each bit into *k* chips according to fixed pattern specific to each user
  - User's code
- New channel has chip data rate *kD* chips per second
- E.g. *k*=6, three users (A,B,C) communicating with base receiver R
- Code for A = <1,-1,-1,1,-1,1>
- Code for B = <1,1,-1,-1,1,1>
- Code for C = <1,1,-1,1,1,-1>

CDMA Example



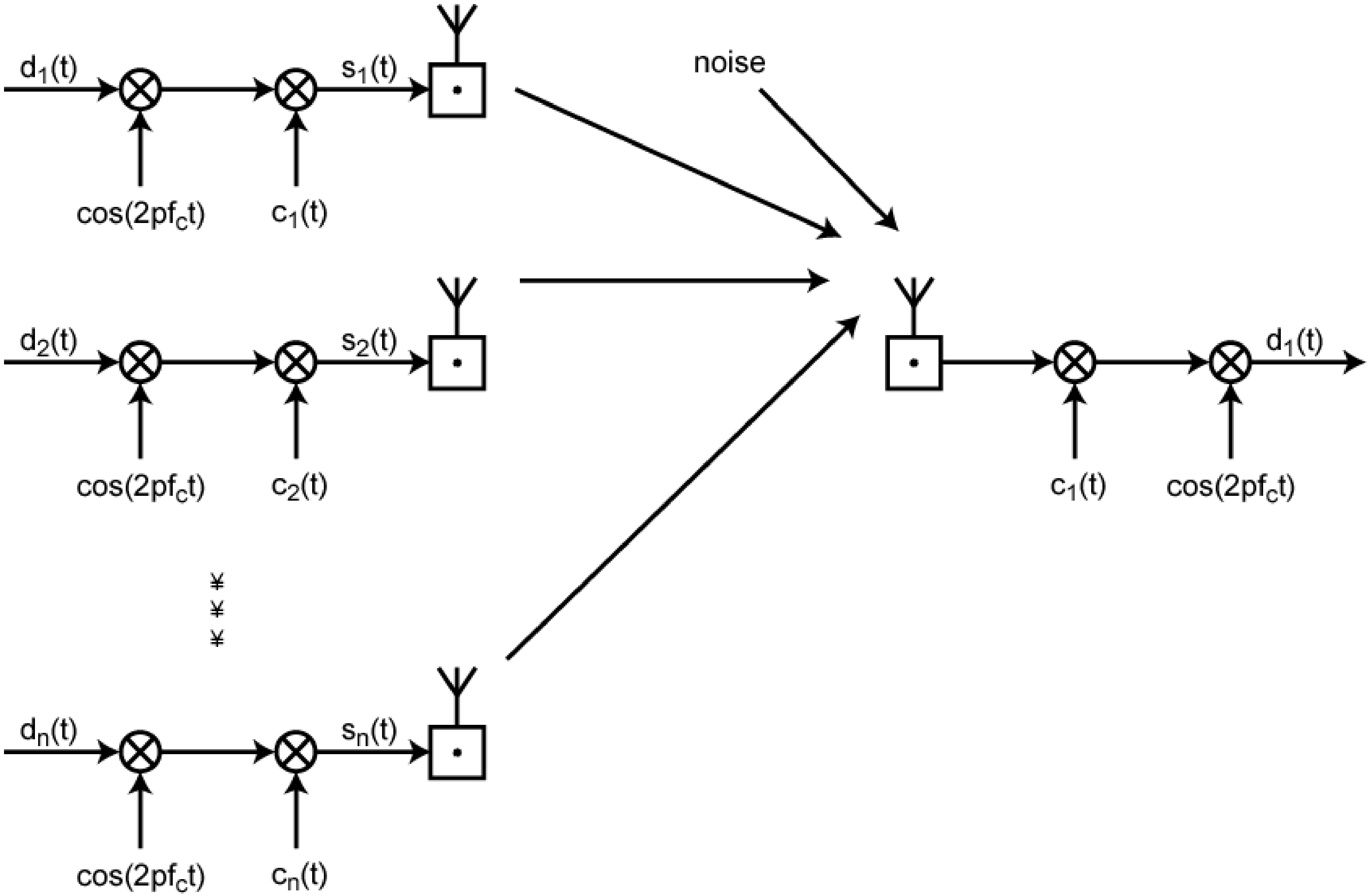
CDMA Explanation

- Consider A communicating with base
- Base knows A’s code
- Assume communication already synchronized
- A wants to send a 1
  - Send chip pattern <1,-1,-1,1,-1,1>
    - A’s code
- A wants to send 0
  - Send chip[pattern <-1,1,1,-1,1,-1>
    - Complement of A’s code
- Decoder ignores other sources when using A’s code to decode
  - Orthogonal codes

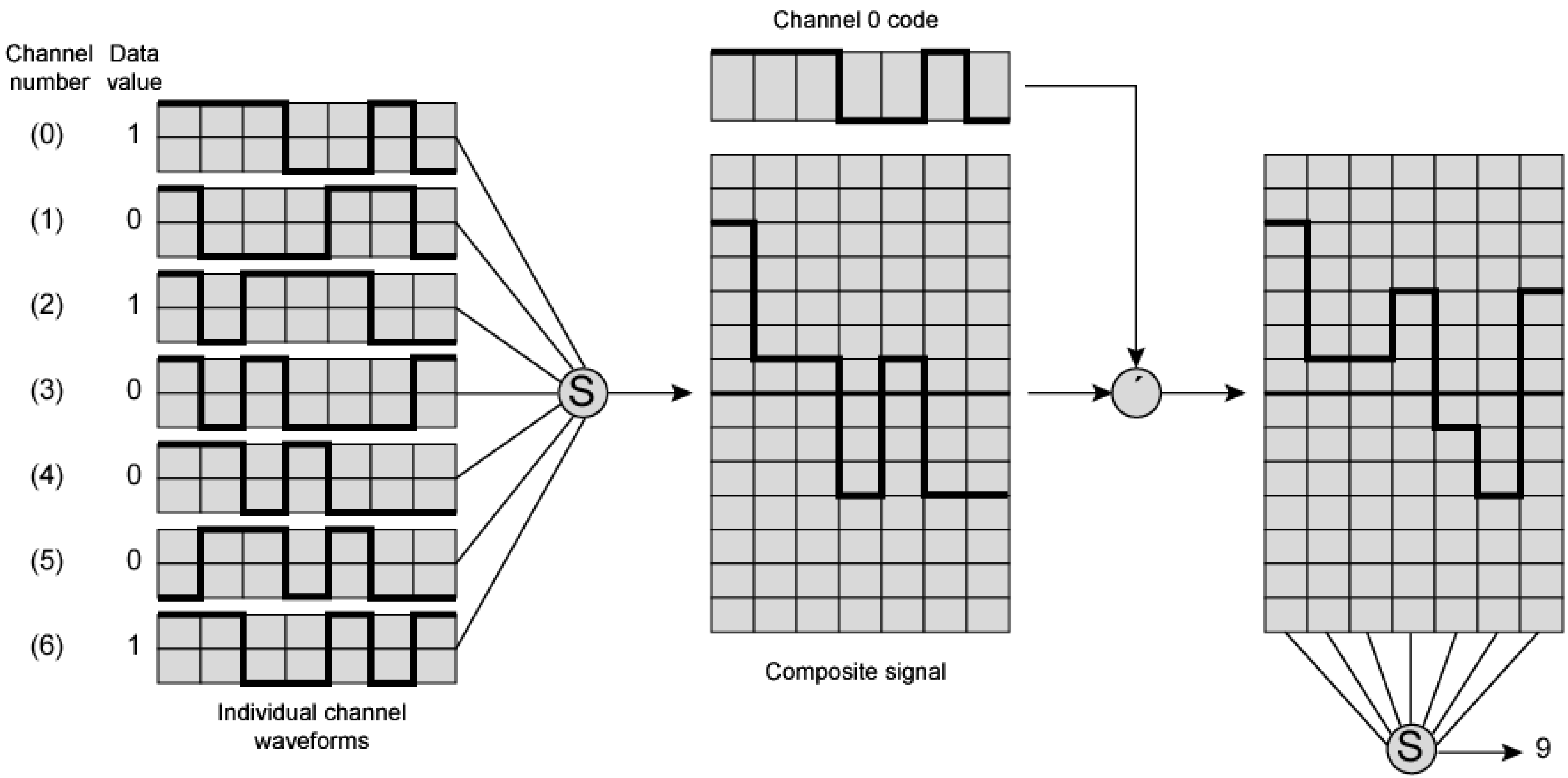
CDMA for DSSS

- n users each using different orthogonal PN sequence
- Modulate each users data stream
  - Using BPSK
- Multiply by spreading code of user

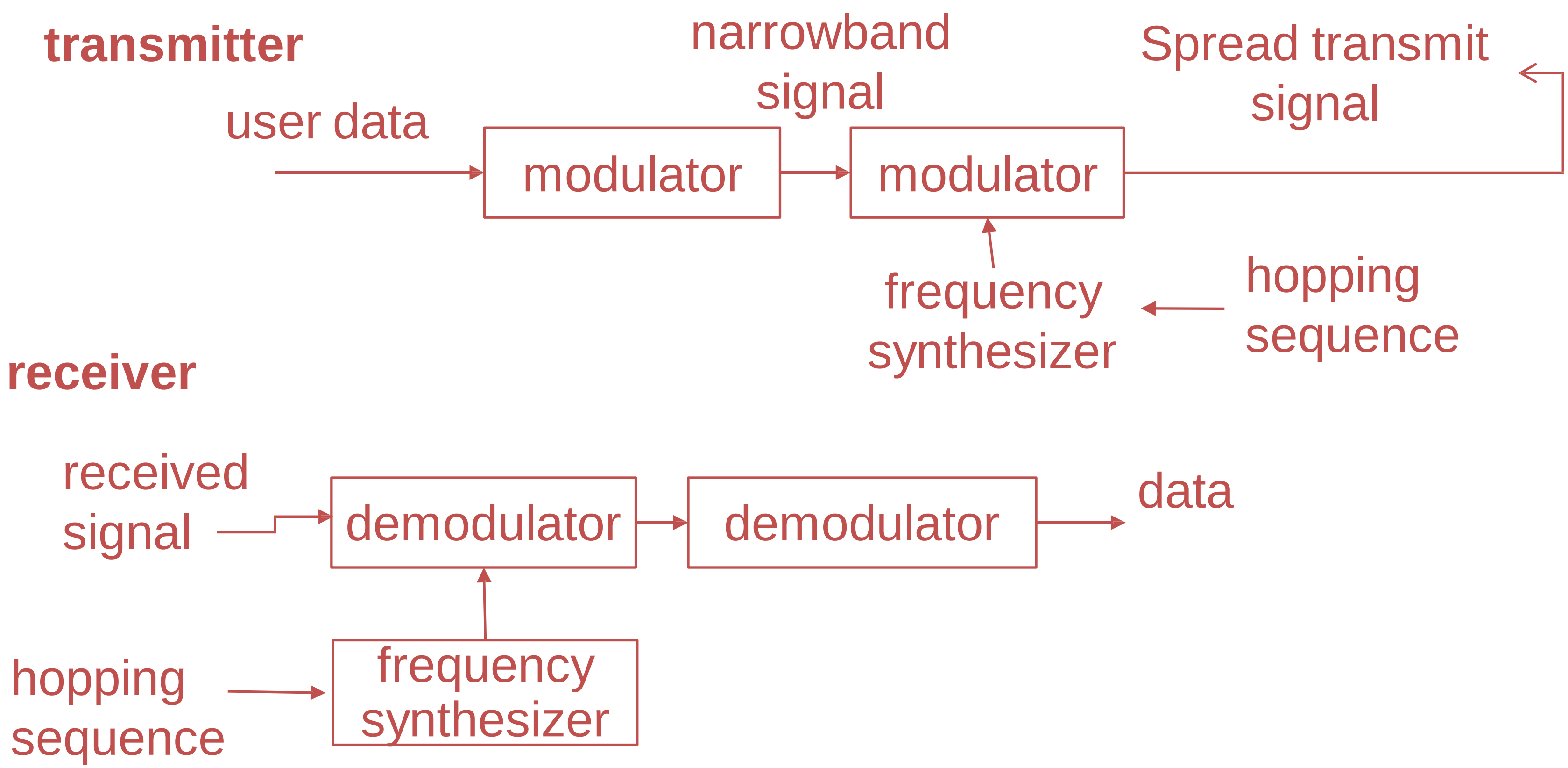
CDMA in a DSSS Environment



Seven Channel CDMA Encoding and Decoding



FHSS (Frequency Hopping Spread Spectrum) III



Applications of Spread Spectrum

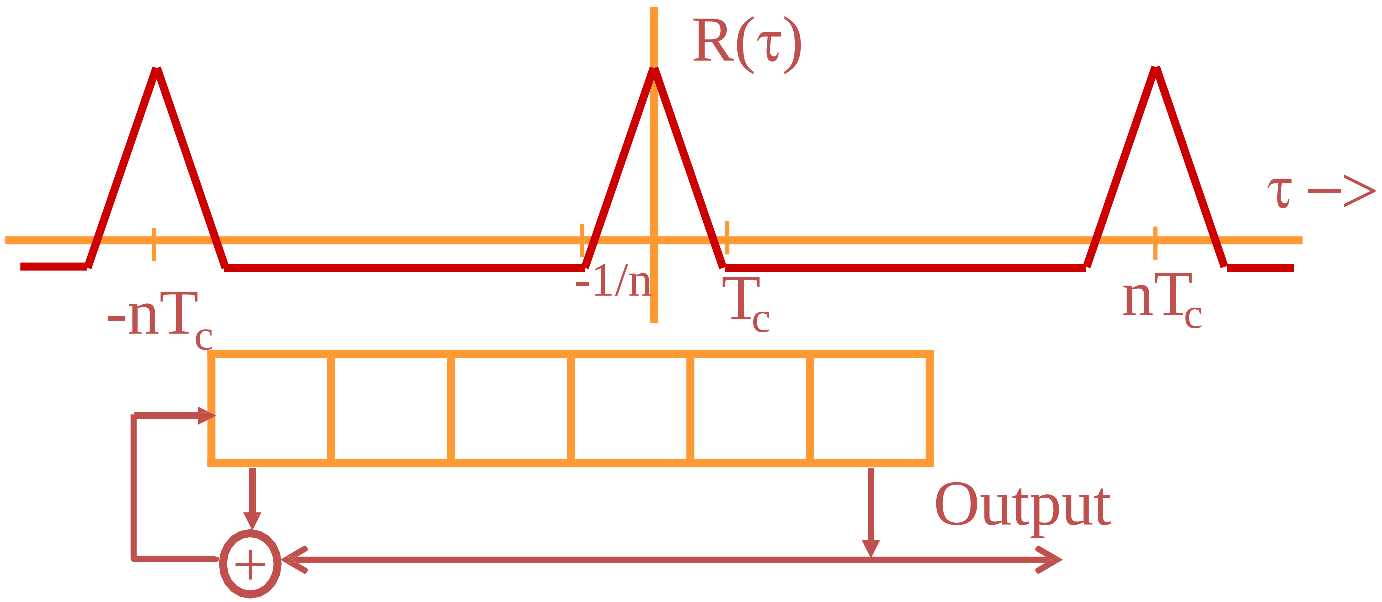
- Cell phones
  - IS-95 (DS/SS)
  - GSM
- Global Positioning System (GPS)
- Wireless LANs
  - 802.11b

Performance of DS/SS Systems

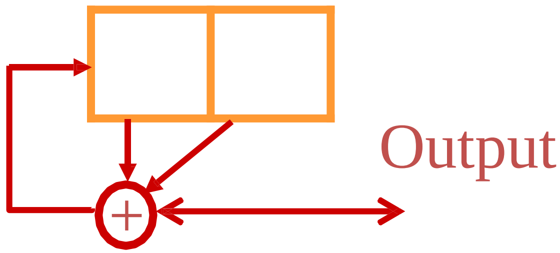
- Pseudonoise (PN) codes
  - Spread signal at the transmitter
  - Despread signal at the receiver
- Ideal PN sequences should be
  - Orthogonal (no interference)
  - Random (security)
  - Autocorrelation similar to white noise (high at τ=0 and low for τ not equal 0)

PN Sequence Generation

- Codes are periodic and generated by a shift register and XOR
- Maximum-length (ML) shift register sequences, *m*-stage shift register, length: $n = 2^m - 1$ bits



Generating PN Sequences



- Take m=2 =>L=3
- c_n=[1,1,0,1,1,0, . . .], usually written as bipolar c_n=[1,1,-1,1,1,-1, . . .]

$$R_c(m) = \frac{1}{L}\sum_{n=1}^{L} c_n c_{n+m}$$

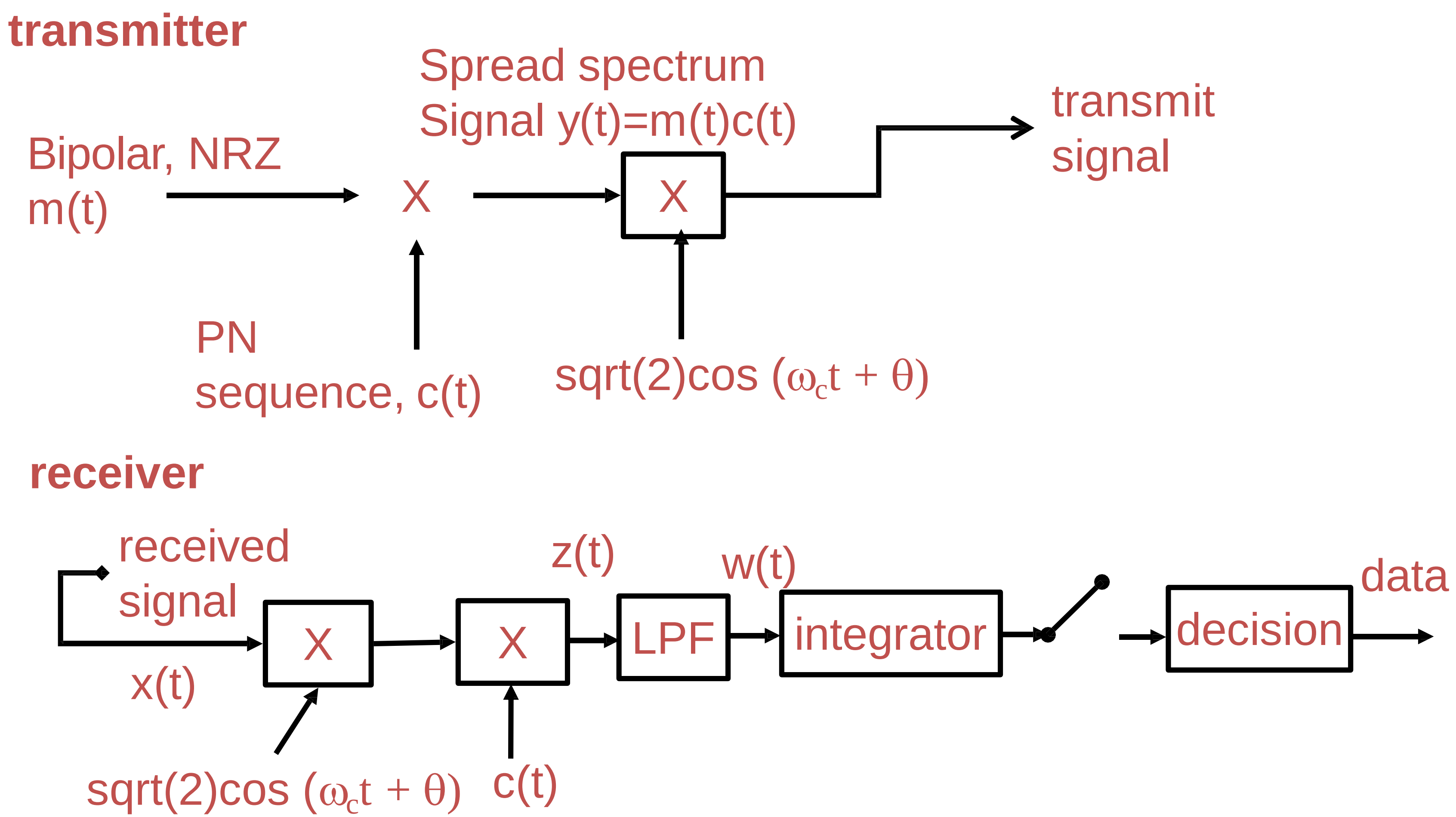
$$= \begin{cases} 1 & m = 0 \\ -1/L & 1 \le m \le L-1 \end{cases}$$

m	Stages connected to modulo-2 adder
2	1,2
3	1,3
4	1,4
5	1,4
6	1,6
8	1,5,6,7

Problems with *m*-sequences

- Cross-correlations with other *m*-sequences generated by different input sequences can be quite high
- Easy to guess connection setup in $2m$ samples so not too secure
- In practice, Gold codes or Kasami sequences which combine the output of m-sequences are used.

Detecting DS/SS PSK Signals



Optimum Detection of DS/SS PSK

- Recall, bipolar signaling (PSK) and white noise give the optimum error probability

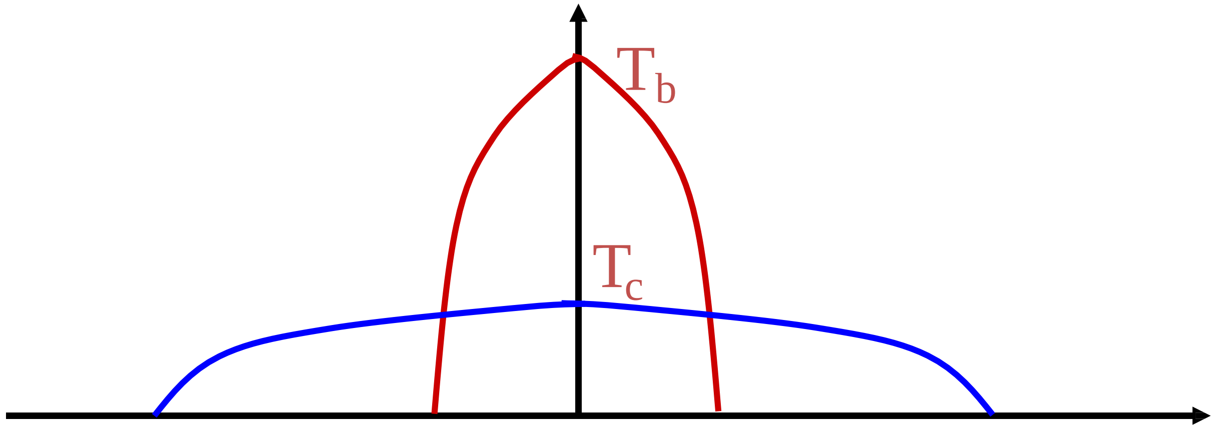
$$P_b = Q\left(\sqrt{\frac{2E_b}{\aleph}}\right)$$

- Not effected by spreading
  - Wideband noise not affected by spreading
  - Narrowband noise reduced by spreading

Signal Spectra

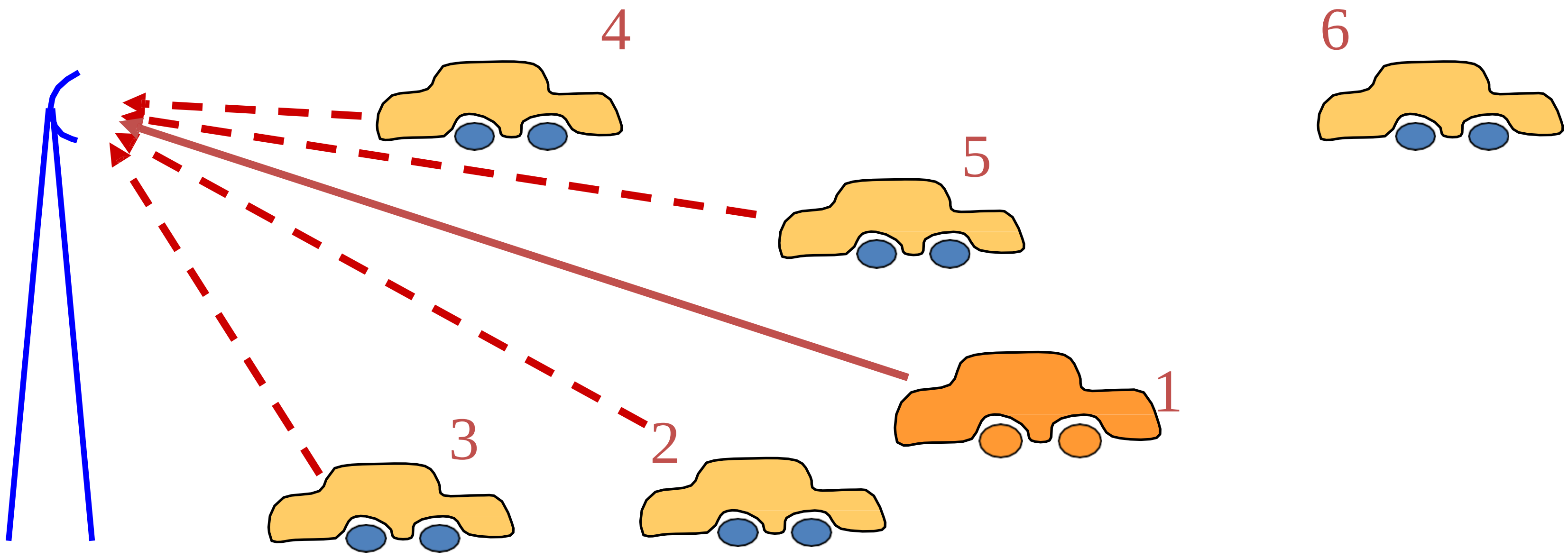
$$\text{Processing Gain } N = \frac{B_{ss}}{B} = 10\log_{10}\left(\frac{B_{ss}}{B}\right) = \frac{T_b}{T_c}$$

- Effective noise power is channel noise power plus jamming (NB) signal power divided by *N*



Multiple Access Performance

- Assume *K* users in the same frequency band,
- Interested in user 1, other users interfere



Signal Model

- Interested in signal 1, but we also get signals from other *K-1* users:

$$x_k(t) = \sqrt{2}\, m_k(t-\tau_k)\, c_k(t-\tau_k) \cos\left(\omega_c (t-\tau_k) + \theta_k\right)$$

$$= \sqrt{2}\, m_k(t-\tau_k)\, c_k(t-\tau_k) \cos\left(\omega_c t + \phi_k\right) \qquad \phi_k = \theta_k - \omega_c \tau_k$$

- At receiver,

$$x(t) = x_1(t) + \sum_{k=2}^{K} x_k(t)$$

Interfering Signal

- After mixing and despreading (assume τ_1=0)

$$z_k(t) = 2\, m_k(t-\tau_k)\, c_k(t-\tau_k)\, c_1(t)\cos(\omega_c t + \phi_k)\cos(\omega_c t + \theta_1)$$

- After LPF

$$w_k(t) = m_k(t-\tau_k)\, c_k(t-\tau_k)\, c_1(t)\cos(\phi_k - \theta_1)$$

- After the integrator-sampler

$$I_k = \cos(\phi_k - \theta_1)\int_0^{T_b} m_k(t-\tau_k)\, c_k(t-\tau_k)\, c_1(t)\, dt$$

At Receiver

- m(t) =+/-1 (PSK), bit duration Tb
- Interfering signal may change amplitude at τk

$$I_k = \cos(\phi_k - \theta_1)\left[b_{-1}\int_0^{\tau_k} c_k(t-\tau_k)c_1(t)\,dt + b_0\int_{\tau_k}^{T_b} c_k(t-\tau_k)c_1(t)\,dt\right]$$

- At User 1: $I_1 = \int_0^{T_b} m_1(t)\,c_1(t)c_1(t)\,dt$
- Ideally, spreading codes are Orthogonal:

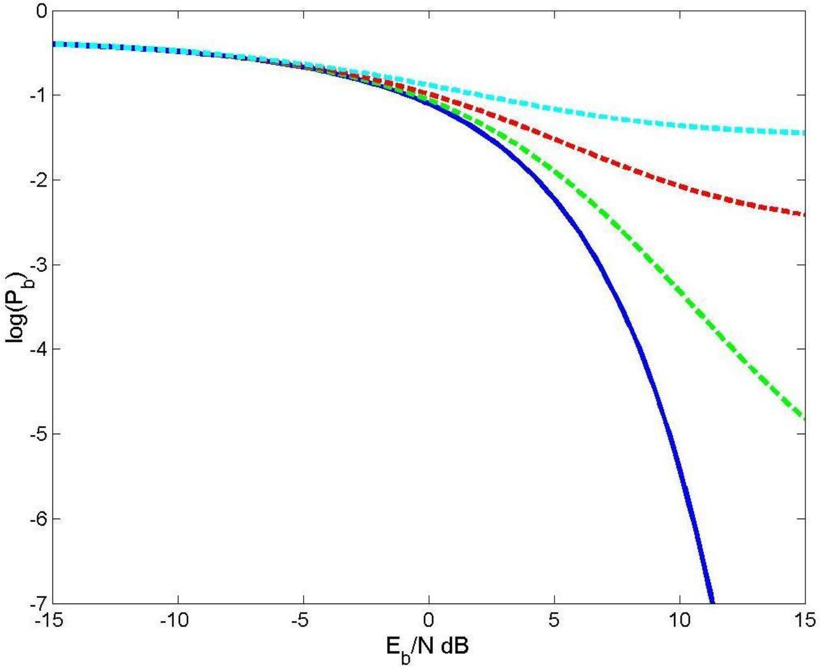
$$\int_0^{T_b} c_1(t)c_1(t)\,dt = A \qquad \int_0^{T_b} c_k(t-\tau_k)c_1(t)\,dt = 0$$

Multiple Access Interference (MAI)

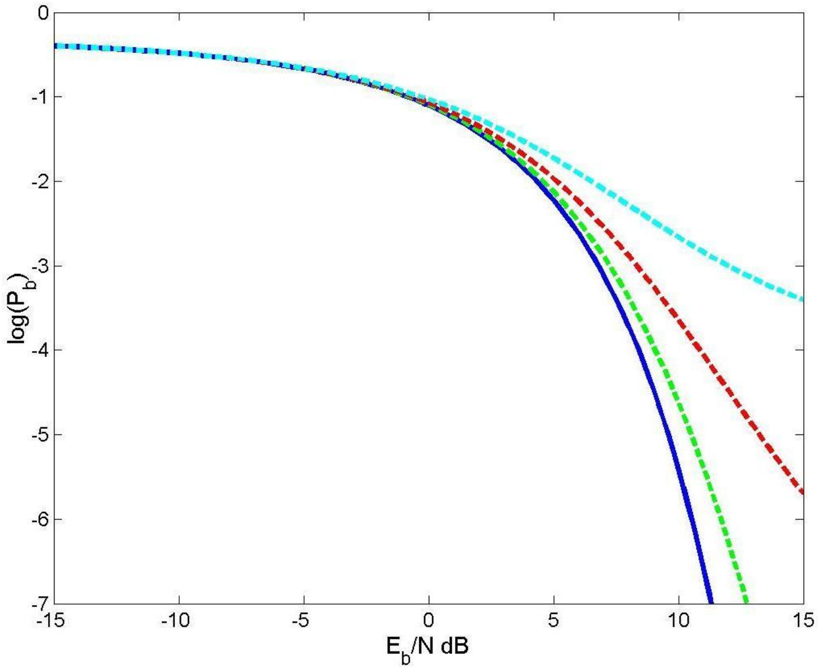
$$P_b = Q\left[\frac{1}{\sqrt{(K-1)/3N + \aleph/2E_b}}\right]$$

- If the users are assumed to be equal power interferers, can be analyzed using the central limit theorem (sum of IID RV’s)

Example of Performance Degradation



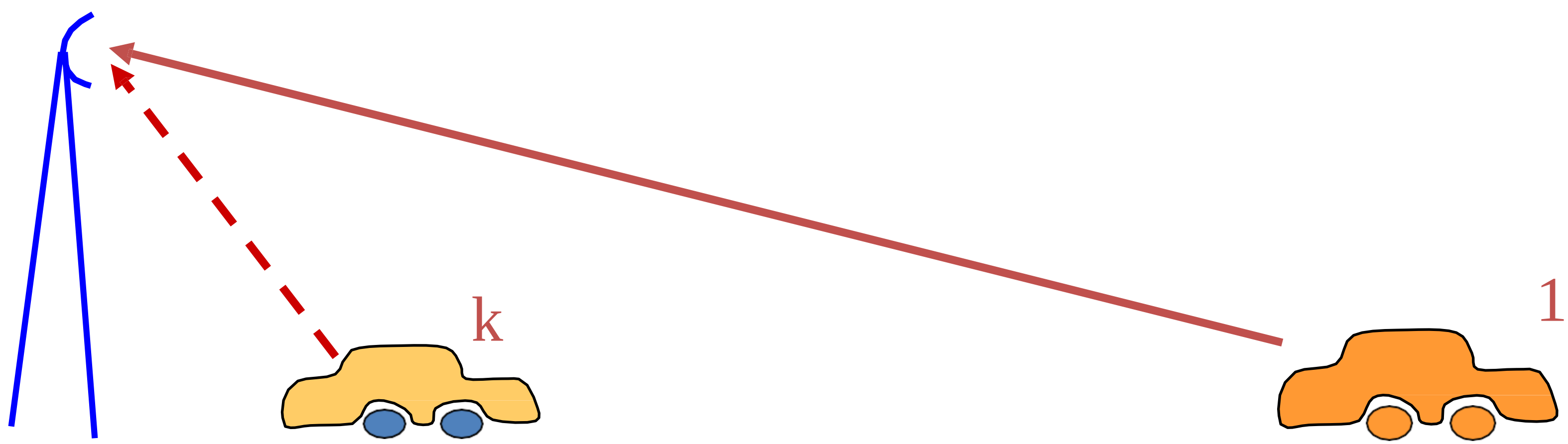
N=8



N=32

Near/Far Problem (I)

- Performance estimates derived using assumption that all users have same power level
- Reverse link (mobile to base) makes this unrealistic since mobiles are moving
- Adjust power levels constantly to keep equal

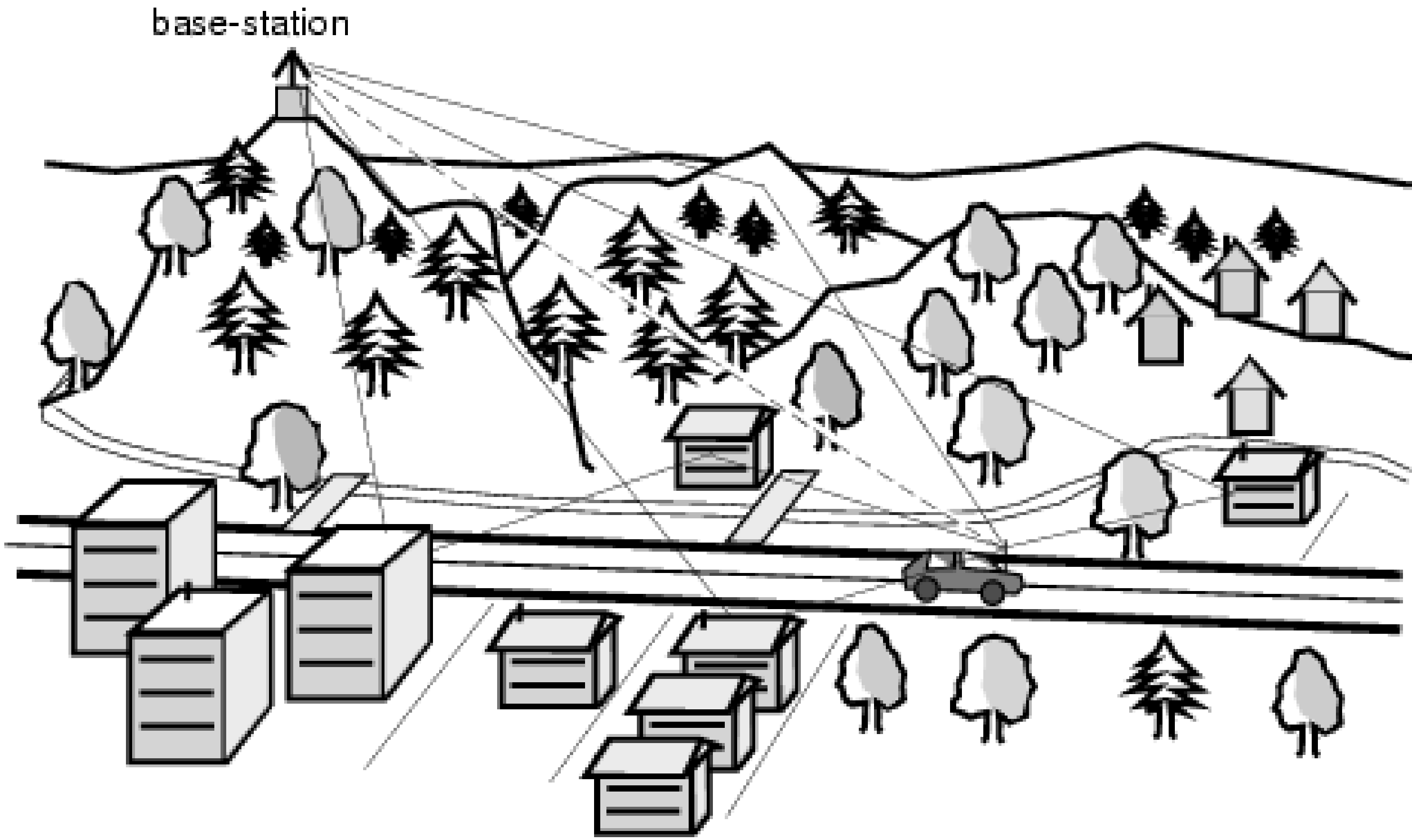


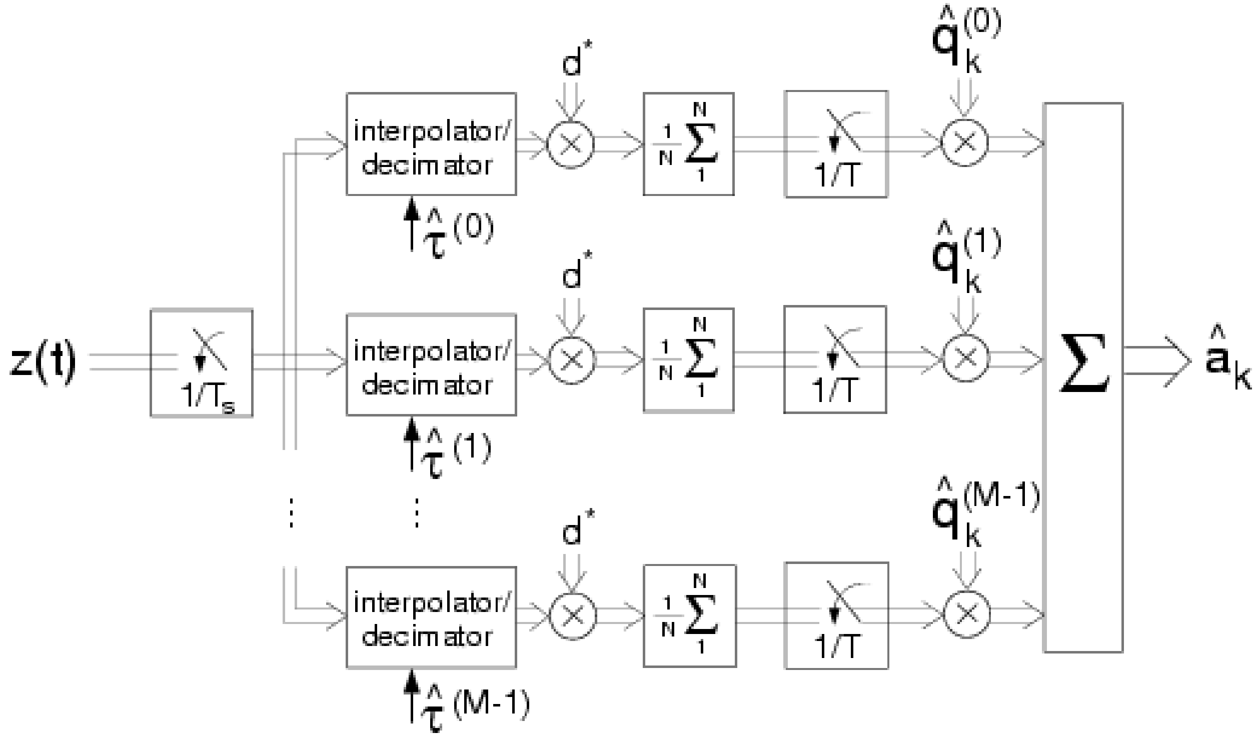
Near/Far Problem (II)

$$P_b^{(1)} = Q\left[\frac{1}{\sqrt{\sum_{k=2}^{K} E_b^{(k)} \Big/ 3E_b^{(1)}N + \aleph \Big/ 2E_b^{(1)}}}\right]$$

- *K* interferers, one strong interfering signal dominates performance
- Can result in capacity losses of 10-30%

Multipath Propagation





- Received signal sampled at the rate 1/Ts$\geq$ 2/Tc for detection and synchronization
- Fed to all *M* RAKE fingers. Interpolation/decimation unit provides a data stream on chiprate 1/Tc
- Correlation with the complex conjugate of the spreading sequence and weighted (maximum-ratio criterion)summation over one symbol