ANALOG COMMUNICATIONS (AEC005)

ELECTRONICS AND COMMUNICATION ENGINEERING

B.Tech IV Semester-IARE R16

Prepared by

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UNIT I

SIGNAL ANALYSIS AND LTI SYSTEMS

Classification of Signals

- Deterministic & Non Deterministic Signals
- Periodic & A periodic Signals
- Even & Odd Signals
- Energy & Power Signals

Classification of Signals

- Important Condition of Periodicity forDiscrete Time Signals

- A discrete time signal is periodic if $x(n) = x(n+N)$
- For satisfying the above condition the frequency of the discrete time signal should be ratio of two integers
- i.e. $f_o = k/N$

Classification of Signals

- Sum of periodic Signals

- X(t) = x1(t) + X2(t)
- X(t+T) = x1(t+m1T1) + X2(t+m2T2)
- m1T1=m2T2 = T_o = Fundamental period
- Example: cos(t/3)+sin(t/4)
- - T1=(2)/(/3)=6; T2 =(2)/(/4)=8;
- T1/T2=6/8 = ¾ = (rational number) = m2/m1
- m1T1=m2T2Find m1 and m2
- 6.4 = 3.8 = 24 = T_o

Classification of Signals

- Deterministic signals
- Behavior of these signals is predictable w.r.t time
- There is no uncertainty with respect to its value at any time.
- These signals can be expressed mathematically.
- For example $x(t) = \sin(3t)$ is deterministic signal.

Classification of Signals

- Non Deterministic or Random signals
- Behavior of these signals is random i.e. not predictable w.r.t time.
- There is an uncertainty with respect to its value at any time.
- These signals can't be expressed mathematically.
- For example Thermal Noise generated is non deterministic signal.

Periodic and Non-periodic Signals

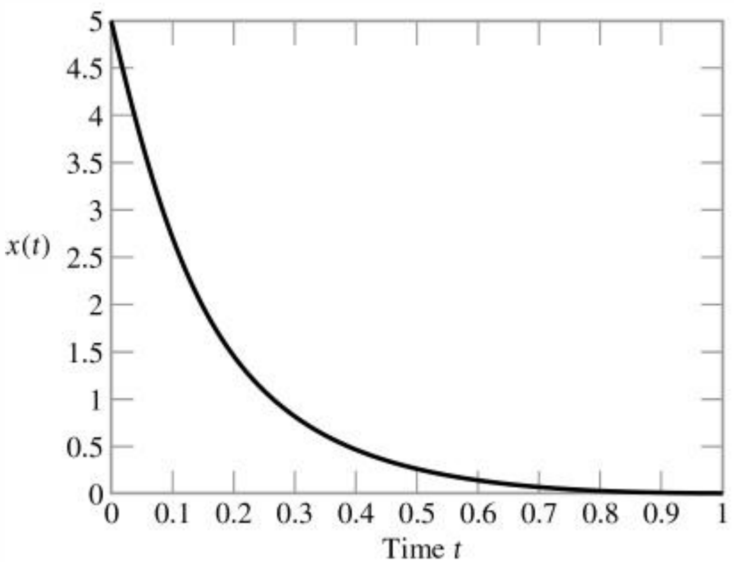
- Given x(t) is a continuous-time signal
- x (t) is periodic iff $x(t) = x(t+T_o)$ for any T and any integer n
- Example
-
  - $x(t) = A \cos(wt)$
  - $x(t+T_o) = A \cos[w\ \ t+T_o)] = A \cos(wt+wT_o) = A$
- $\cos(wt+2) = A \cos(wt)$

Classification of Signals

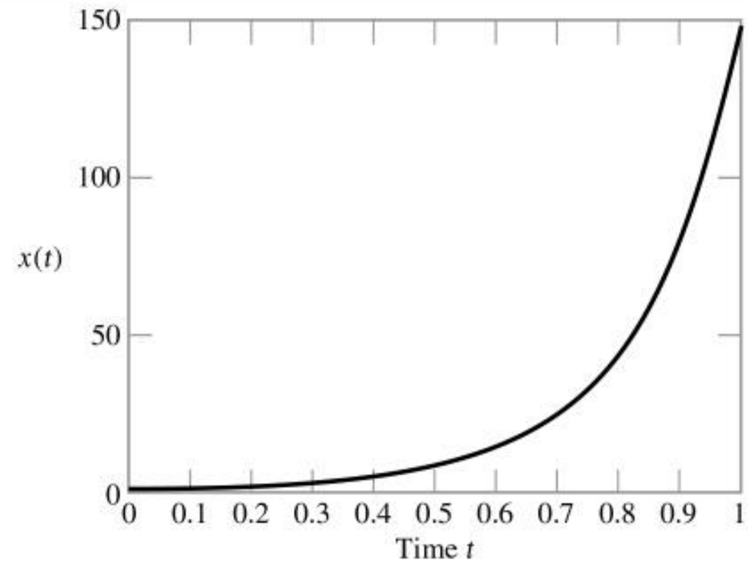
- Exponential signals
- Sinusoidal signals
- Step function
- Rectangular pulse
- Impulse function
- Derivatives of the impulse
- Ramp function

Classification of Signals

- Exponential signals



(a)



(b)

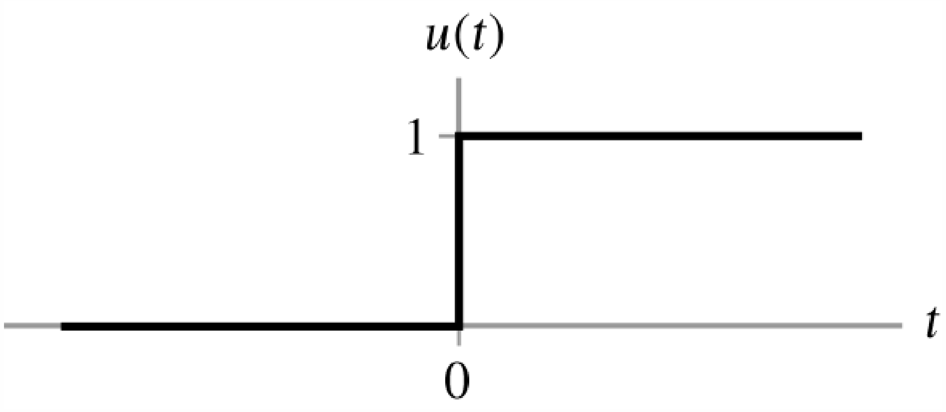
$$x(t) = Be^{at}, \quad a < 0$$

$$x(t) = Be^{at}, \quad a > 0$$

Classification of Signals

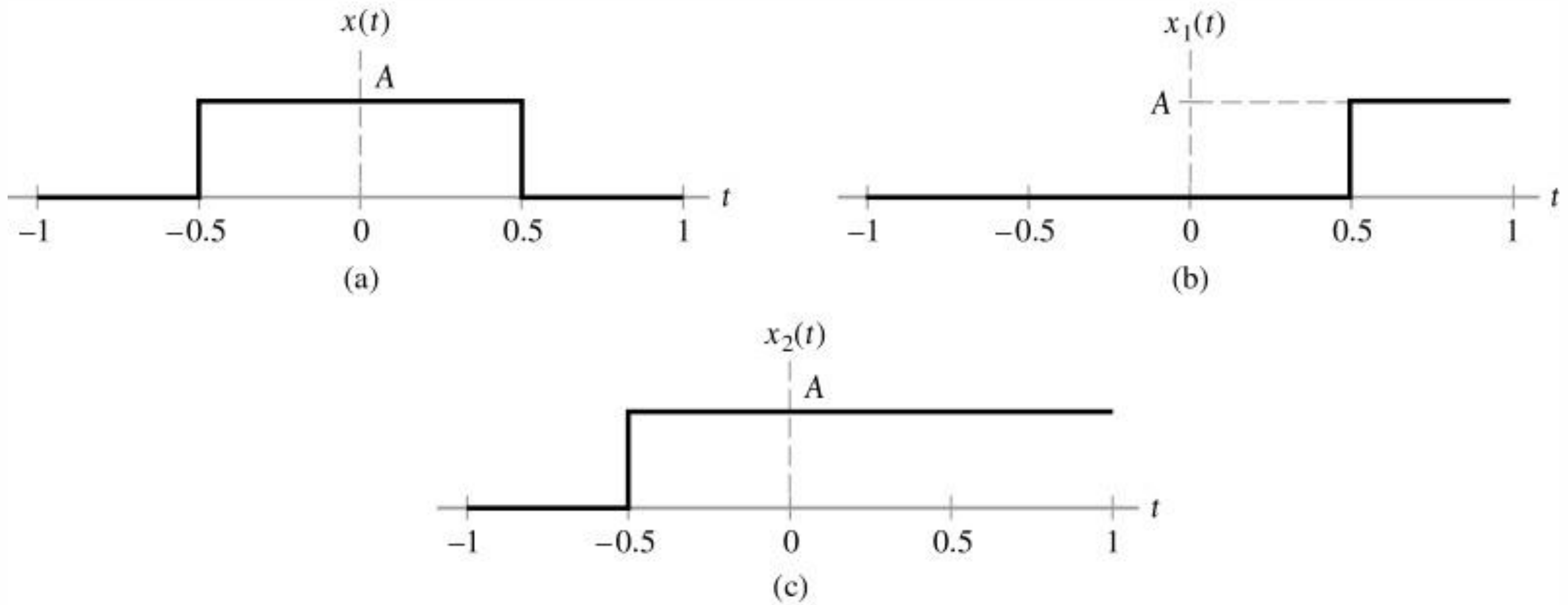
- Step function

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$



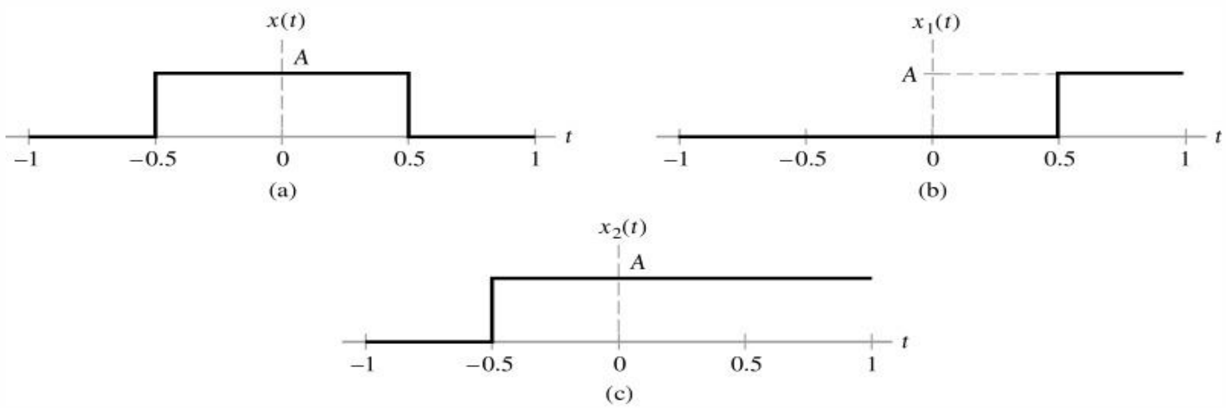
Elementary Signals

Rectangular pulse



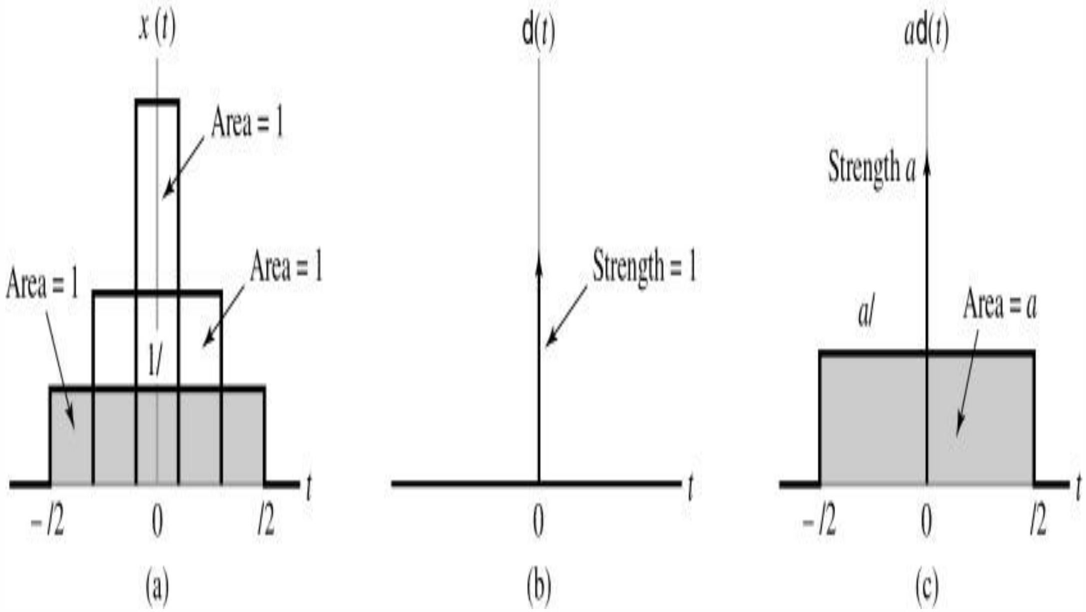
$$x(t) = \begin{cases} A, & -0.5 \le |t| \le 0.5 \\ 0, & otherwise \end{cases}$$

Rectangular pulse



$$x(t) = \begin{cases} A, & -0.5 \le |t| \le 0.5 \\ 0, & otherwise \end{cases}$$

Impulse function



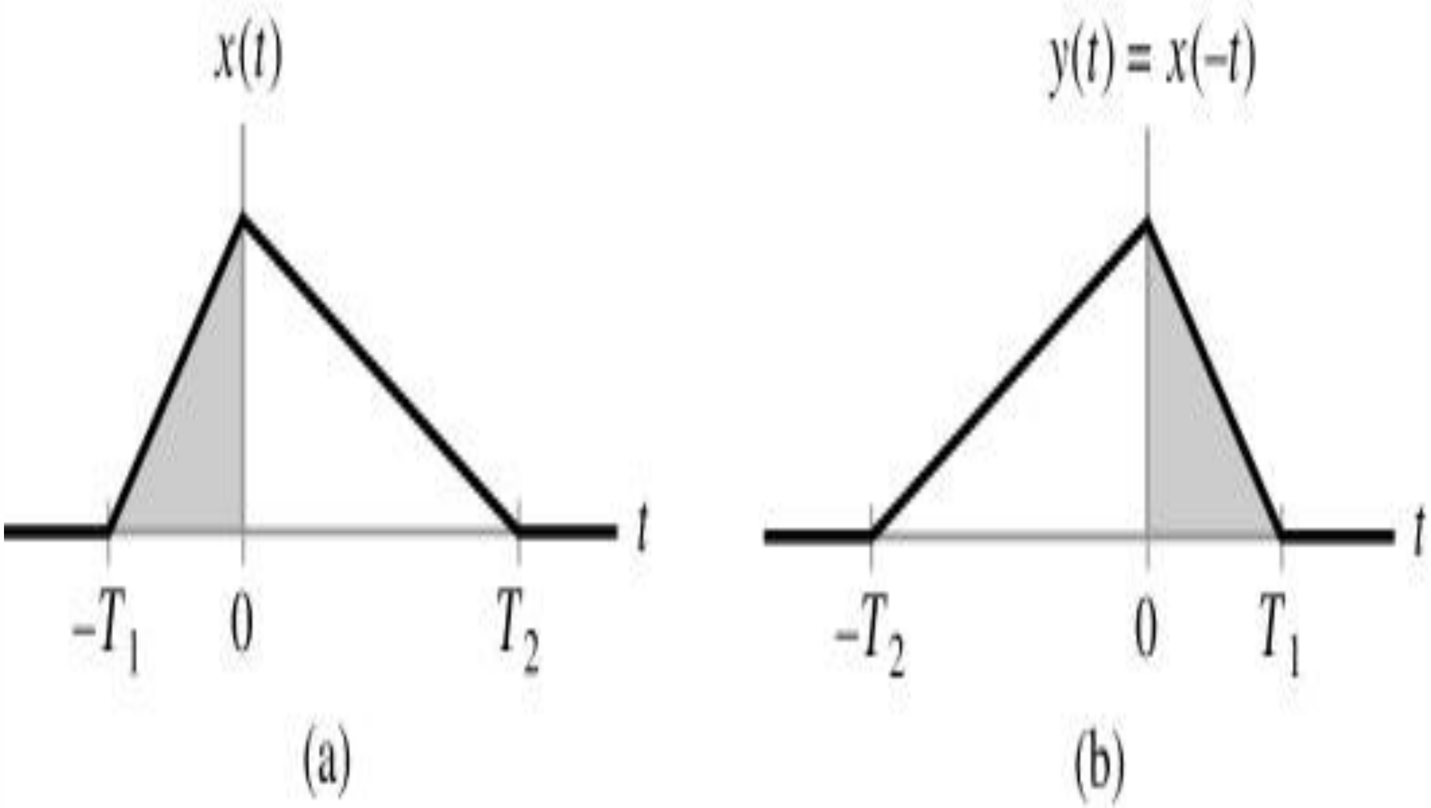
$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

$$\delta(t) = 0 \qquad for \qquad t \neq 0$$

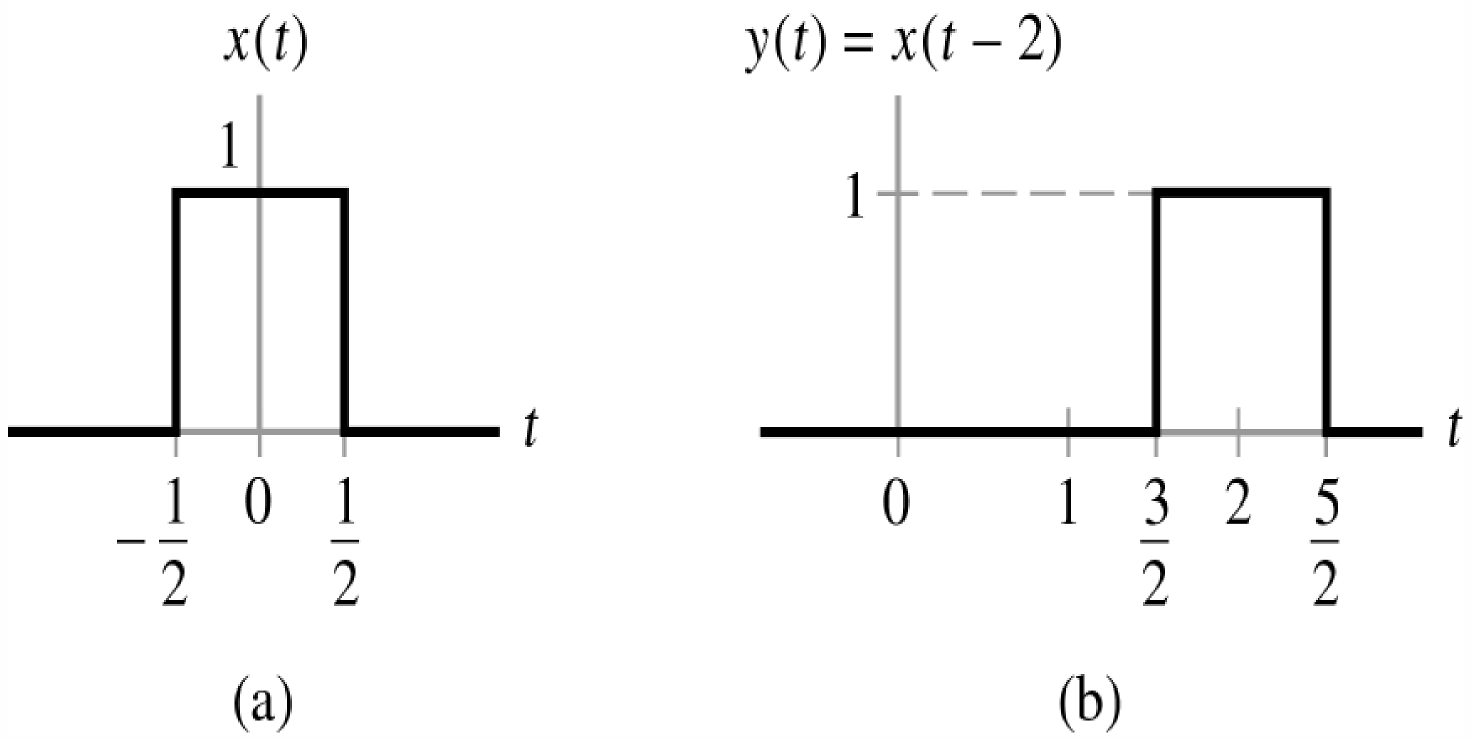
Operations On Dependent Variable

- Operations performed on dependent variables
  - Amplitude scaling, Addition, Multiplication, differentiation
- Operations performed on independent variables
  - Time scaling
  - Reflection
  - Time shifting

Reflection



Time shifting



classification of systems

- A system takes a signal as an input and transforms it into another signal

classification of systems

- Properties of a system
- Linear (time invariant) LTI systems

System with and without Memory

- A system is said to be memoryless if its output for each value of the independent variable at a given time is dependent on the output at only that same time (no system dynamics)

$$Y(n)=x(n)$$

System Causality

- A system is causal if the output at any time depends on values of the output at only the present and past times. Referred to as non-anticipative, as the system output does not anticipate future values of the input
- If two input signals are the same up to some point t_0/n_0, then the outputs from a causal system must be the same up to then.

System Causality

The accumulator system is causal

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

properties of a systems

- A system takes a signal as an input and transforms it into another signal

classification of systems

- Properties of a system
- Linear (time invariant) LTI systems

System Stability

- Informally, a stable system is one in which small input signals lead to responses that do not diverge
- If an input signal is bounded, then the output signal must also be bounded, if the system is stable

System stability

$$\forall x : |x| < U \;\rightarrow\; |y| < V$$

To show a system is stable we have to do it for **all** input signals. To show instability, we just have to find one counterexample

E.g. Consider the DT system of the bank account

$$y[n] = x[n] + 1.01\, y[n-1]$$

when $x[n] = \delta[n]$, $y[0] = 0$

This grows without bound, due to 1.01 multiplier. This system is unstable.

System Causality

- A system is causal if the output at any time depends on values of the output at only the present and past times. Referred to as non-anticipative, as the system output does not anticipate future values of the input
- If two input signals are the same up to some point t_0/n_0, then the outputs from a causal system must be the same up to then.

System Causality

The accumulator system is causal

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

Invertible and Inverse Systems

- A system is said to be **invertible** if distinct inputs lead to distinct outputs (similar to matrix invertibility)
- If a system is invertible, an inverse system exists which, when **cascaded** with the original system, yields an output equal to the input of the first signal
- E.g. the CT system is invertible:
  - $y(t) = 2x(t)$

Invertible and Inverse Systems

- E.g. the CT system is not-invertible
- $y(t) = x^2(t)$
- because distinct input signals lead to the same output signal
- Widely used as a design principle:
  - Encryption, decryption
  - System control, where the reference signal is input

Impulse Response of the System

- A system takes a signal as an input and transforms it into another signal
- The unit impulse function, $\delta(t)$, could have been used to directly derive the sifting function.

Impulse Response of the System

- Properties of a system
- Linear (time invariant) LTI systems

$$\delta(t-\tau) = 0 \qquad t \neq \tau$$

$$\int_{-\infty}^{\infty} \delta(t-\tau)\, d\tau = 1$$

Impulse Response of the System

$$x(\tau)\delta(t-\tau) = 0 \qquad t \neq \tau$$

$$\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau = \int_{-\infty}^{\infty} x(t)\delta(t-\tau)d\tau$$

$$= x(t)\int_{-\infty}^{\infty} \delta(t-\tau)d\tau$$

$$= x(t)$$

Impulse Response of the System

- The previous derivation strongly emphasises the close relationship between the structure for **both** discrete and continuous-time signals

Linear Time Invariant Convolution

- For a linear, time invariant system, all the impulse responses are simply time shifted versions:

$$H(t)=h(t-T)$$

- convolution for an LTI system is defined by

Linear Time Invariant Convolution

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

This is known as the **convolution integral** or the **superposition integral**

Algebraically, it can be written as:

$$y(t) = x(t) * h(t)$$

To evaluate the integral for a specific value of t, obtain the signal $h(t-\tau)$ and multiply it with $x(\tau)$ and the value $y(t)$ is obtained by integrating over τ from $-\infty$ to ∞.

Demonstrated in the following examples

Calculate the convolution of the following signals

$$x(t) = e^{2t}u(-t)$$

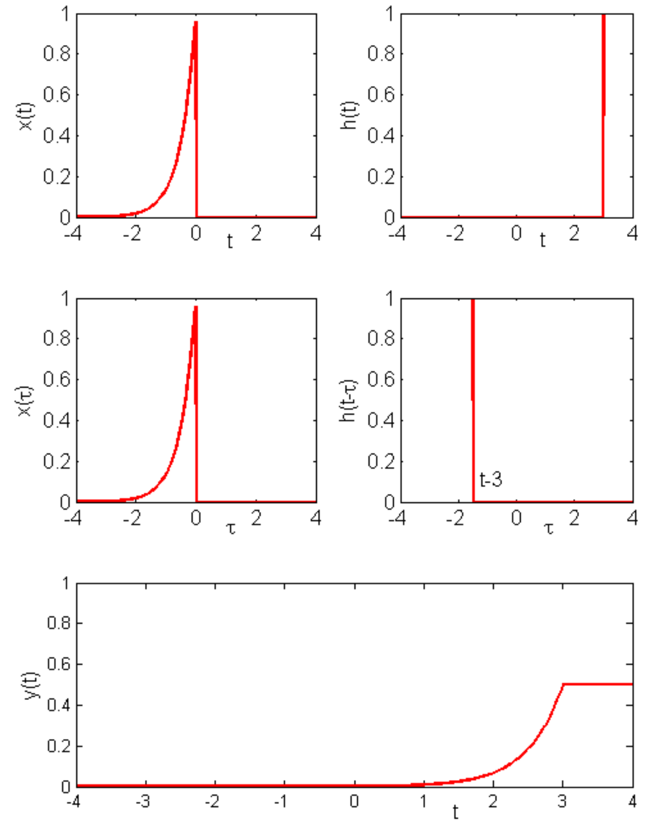
$$h(t) = u(t-3)$$

The convolution integral becomes:

$$y(t) = \int_{-\infty}^{t-3} e^{2\tau}\, d\tau = \tfrac{1}{2} e^{2(t-3)}$$

For t-3≥0, the product $x(\tau)h(t-\tau)$ is non-zero for -∞<τ<0, so the convolution integral becomes:

$$y(t) = \int_{-\infty}^{0} e^{2\tau}\, d\tau = \tfrac{1}{2}$$



Properties of Convolution

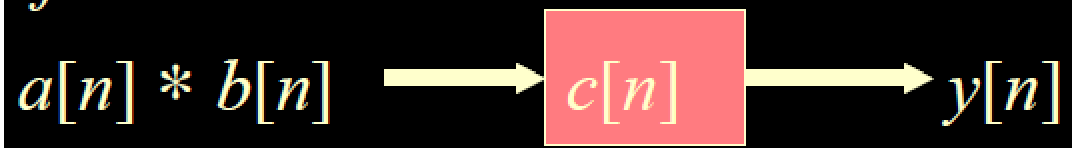
- A system takes a signal as an input and transforms it into another signal
- The unit impulse function, $\delta(t)$, could have been used to directly derive the sifting function.

Properties of convolution

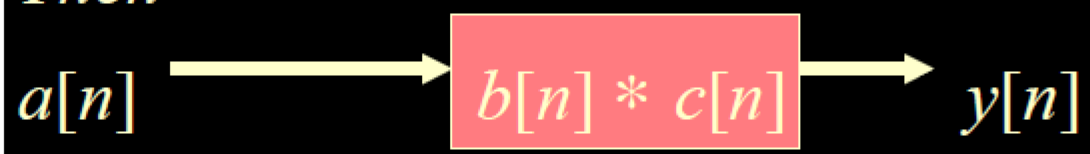
Associative:

$$\{a[n] * b[n]\} * c[n] = a[n] * \{b[n] * c[n]\}$$

If



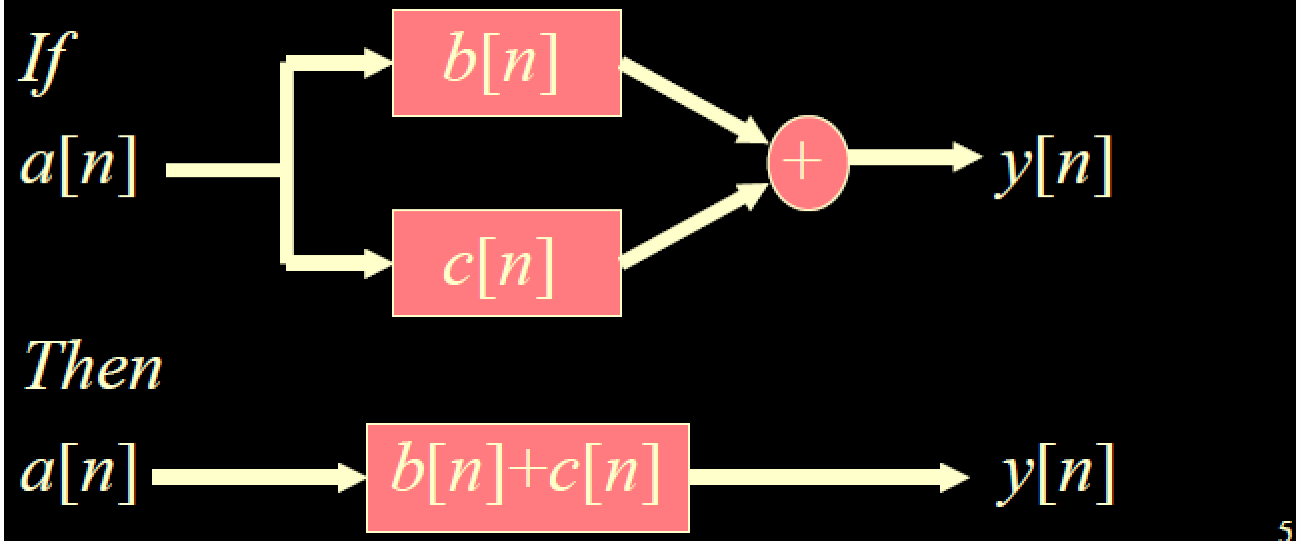
Then



Properties of convolution

Distributive

$$a[n]*b[n] + a[n]*c[n] = a[n]*\{b[n] + c[n]\}$$



Properties of convolution

Transference: between Input & Output

Suppose $x[n] * h[n] = y[n]$

If L is a linear system,

$x_1[n] = L\{x[n]\}$, $y_1[n] = L\{y[n]\}$

Then

$x_1[n] * h[n] = y_1[\text{n}]$

Static vs. Dynamic Systems

- A discrete-time system is called ***static*** or ***memoryless*** if its output at any time instant *n* depends on the input sample at the same time, but not on the past or future samples of the input. In the other case, the system is said to be ***dynamic*** or to have ***memory***.
- If the output of a system at time *n* is completly determined by the input samples in the interval from *n-N* to *n* (N>0), the system is said to have memory of ***duration N***.
- If , the system is ***static*** or ***memoryless.***
- If , the system is said to have ***finite memory***.
- If , the system is said to have ***infinite memory***

Transfer function

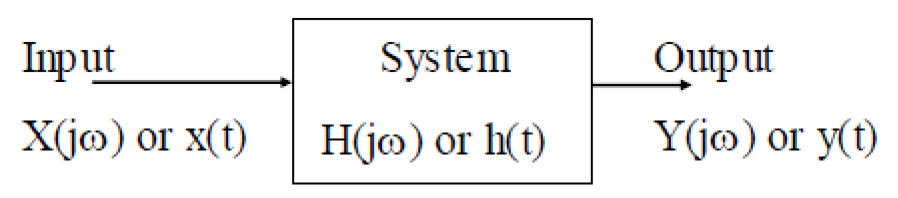
- Transfer Function & Sinusoidal Response of LTI Systems:

- a transfer function (also known as system function or network function) of an electronic or control system component is mathematical function giving the corresponding output value for each possible value of the input to the device.

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\mathcal{L}\{y(t)\}}{\mathcal{L}\{x(t)\}}$$

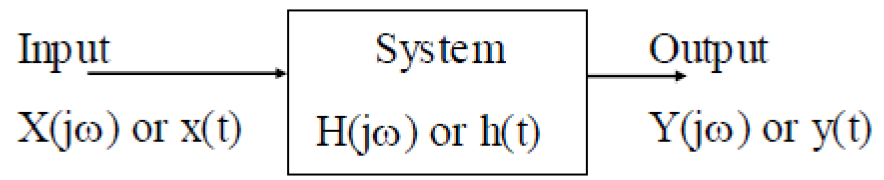
FREQUENCY RESPONSE OF CONTINUOUS LTI SYSTEMS

Most of the physical systems are considered as black boxes as indicated in Fig. 1 and modeled according to their responses to specific inputs. There are two basic tests that can be carried out in laboratories easily if the system is accessible.



Step Response of LTI Systems:

The **step response** of a system in a given initial state consists of the time evolution of its outputs when its control inputs are Heaviside step functions. In electronic engineering and control theory, step response is the time behaviour of the outputs of a general system when its inputs change from zero to one in a very short time. The concept can be extended to the abstract mathematical notion of a dynamical system using an evolution parameter.



Step Response of LTI Systems

We have:

$$H(s) = \frac{y(s)}{u(s)} = \frac{3}{2s+1}$$

Meaning that:

$$y(s) = \frac{3}{2s+1} u(s)$$

where $u(s) = \frac{1}{s}$

This means:

$$y(s) = \frac{3}{2s+1} \cdot \frac{1}{s} = \frac{3}{(2s+1)s}$$

Then we use the final value theorem (sluttverditeoremet):

$$y_s = \lim_{t\to\infty} y(t) = \lim_{s\to 0} s \cdot y(s) = \lim_{s\to 0} s \frac{3}{(2s+1)s} = \lim_{s\to 0} \frac{3}{2s+1} = \frac{3}{2\cdot 0+1} = \underline{3}$$

The Frequency Response

- All dynamic systems have some form of frequency selectivity in their characteristics. This is also called **filtering. A linear time-invariant continuous-time system translates a sinusoidal**
- signal into a sinusoidal output signal at steady state. The frequency of the signal is not affected by the translation, but the magnitude and phase of the input signal are modified by the system dynamics. Sinusoidal input signals can be expressed by complex exponentials in

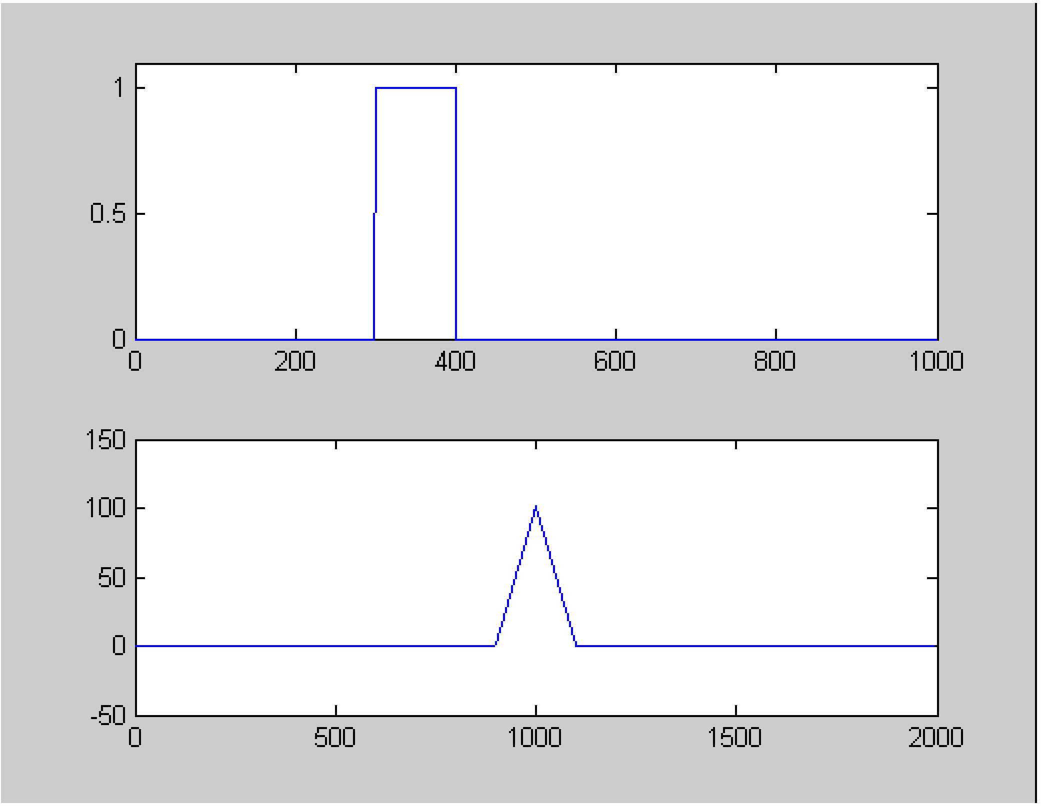
Cross Correlation of Aperiodic And Periodic Signals:

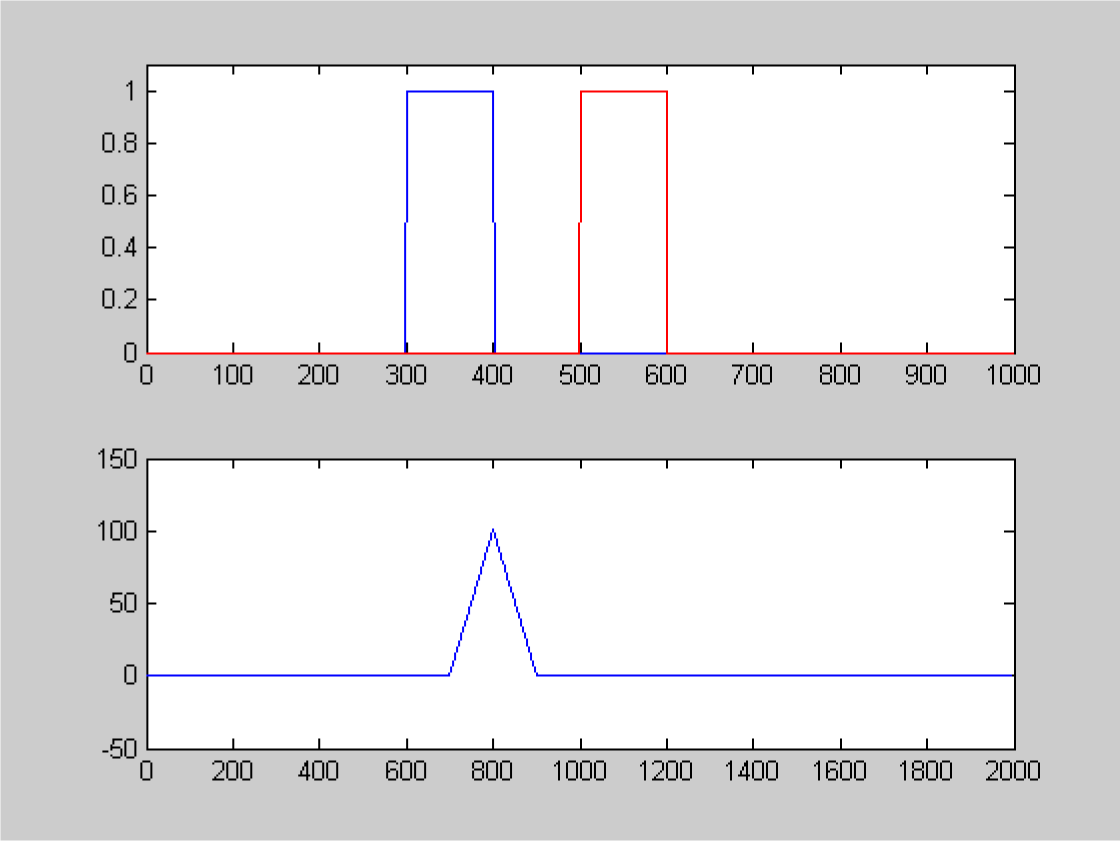
- Correlation of time series
  - Similarity
  - Time shitfs

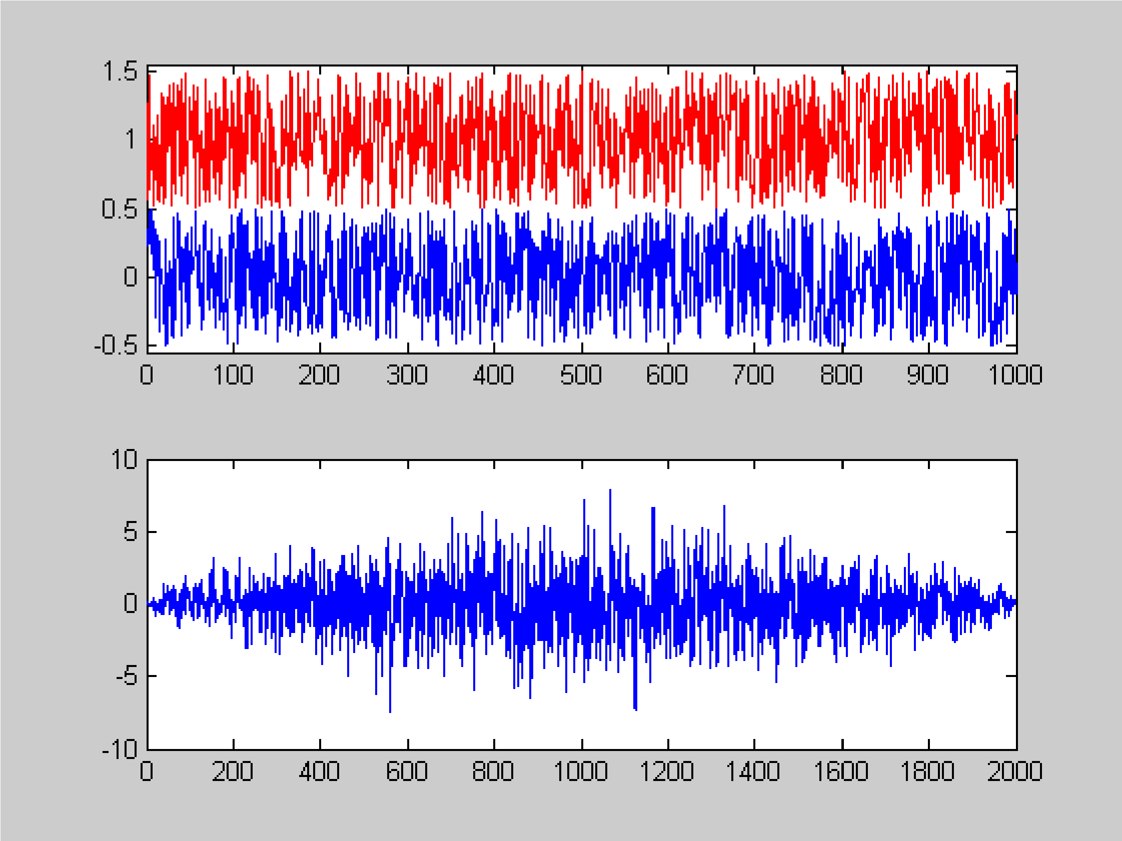
- Applications
  - Correlation of rotations/strains and translations
  - Ambient noise correlations
  - Coda correlations
  - Random media: correlation length

Scope: Appreciate that the use of noise (and coda) plus correlation techniques is one of the most innovative direction in data analysis at the moment: *passive imaging*

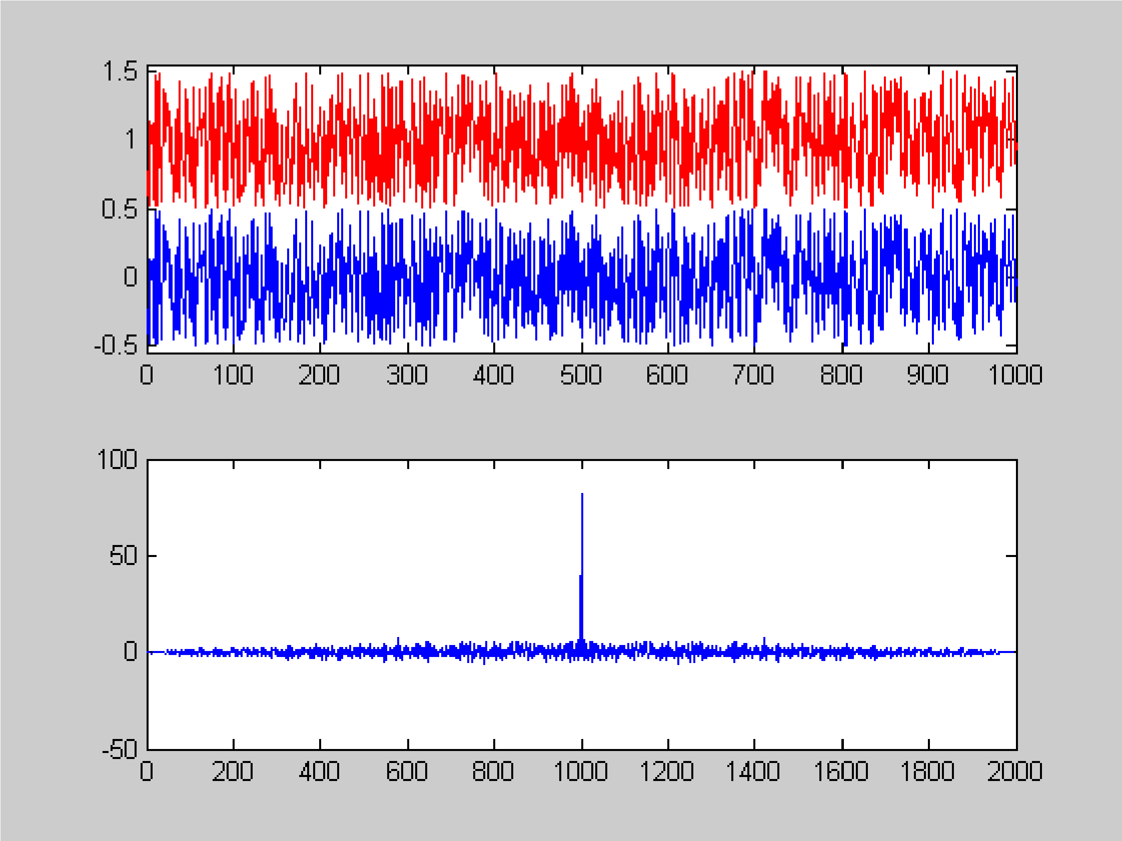
Step Response of LTI Systems




1
0.8
0.6
0.4
0.2
0
0 100 200 300 400 500 600 700 800 900 1000
150
100
50
0
-50
0 200 400 600 800 1000 1200 1400 1600 1800 2000



1.5
1
0.5
0
-0.5
0 100 200 300 400 500 600 700 800 900 1000
10
5
0
-5
-10
0 200 400 600 800 1000 1200 1400 1600 1800 2000


1.5
1
0.5
0
-0.5
0 100 200 300 400 500 600 700 800 900 1000
100
50
0
-50
0 200 400 600 800 1000 1200 1400 1600 1800 2000

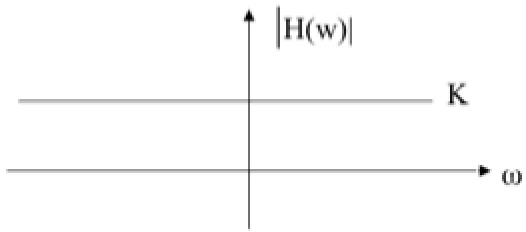
distortion-less transmission :

- Transmission is said to be distortion-less if the input and output have identical wave shapes. i.e., in distortion-less transmission, the input x(t) and output y(t) satisfy the condition:
- $y(t) = Kx(t - t_d)$
- Where t_d = delay time and k = constant.
- Take Fourier transform on both sides, $FT[y(t)] = FT[Kx(t - t_d)]$
- $= K\,FT[x(t - t_d)]$

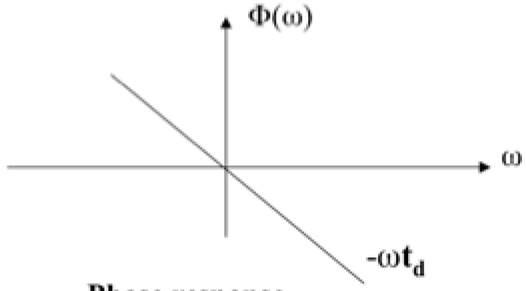
distortion-less transmission

- According to time shifting property,
- = KX(w) e−jωtde−jωtd
- ∴Y(w)=KX(w)e−jωtd
- ∴Y(w)=KX(w)e−jωtd
- Thus, distortion less transmission of a signal x(t) through a system with impulse response h(t) is achieved when
- |H(ω)|=K and|H(ω)|=Kand (amplitude response)
- Φ(ω)=−ωtd=−2πftd
- Φ(ω)=−ωtd=−2πftd (phase response)

distortion-less transmission

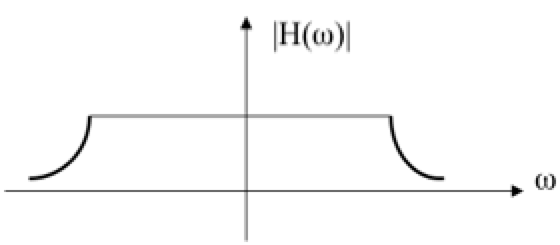


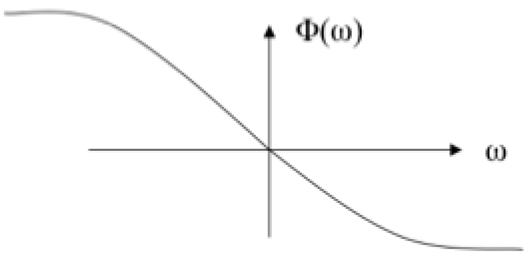
Amplitude response



Phase response

A physical transmission system may have amplitude and phase responses as shown below:



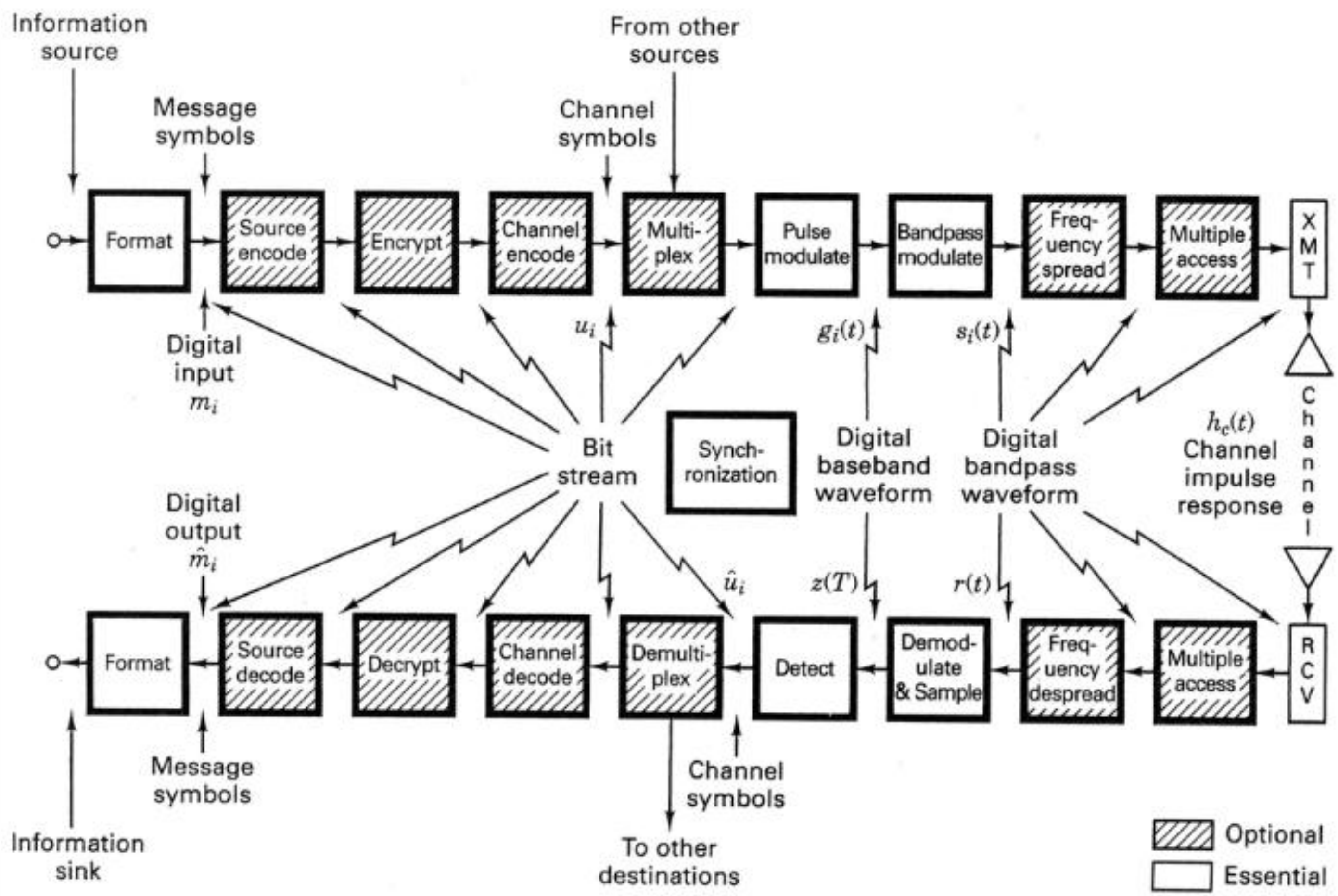


UNIT II

AMPLITUDE AND DOUBLE SIDE BAND SUPPRESSED CARRIER MODULATION

- Communication is the process of sending information from one point to another point. Technically speaking, the term communication refers to the sending, receiving and processing of information by electronic means.

Comm System Block Diagram



- Information Source Any communication system serves to communicate a message or information. This message originates in the information source.
- this electrical signal is processed to restrict its range of audio frequencies the signal or message in a radio transmitter is amplified in several stages of small signal amplifiers (voltage amplifiers) and large signal amplifiers (power amplifiers) and may be possibly encoded, to make it suitable for transmission and sub sequent reception

AM - Definition

- AM, modulation index can be defined as the measure of extent of amplitude variation about an un-modulated carrier.

$$\text{Modulation index } m = \frac{M}{A}$$

Calculation for transmitted power:

It is important to use as high percentage of modulation as possible (k=1) while ensuring that over modulation (k>1) does not occur.

The sidebands contain the information and have maximum power at 100% modulation.

Useful equation

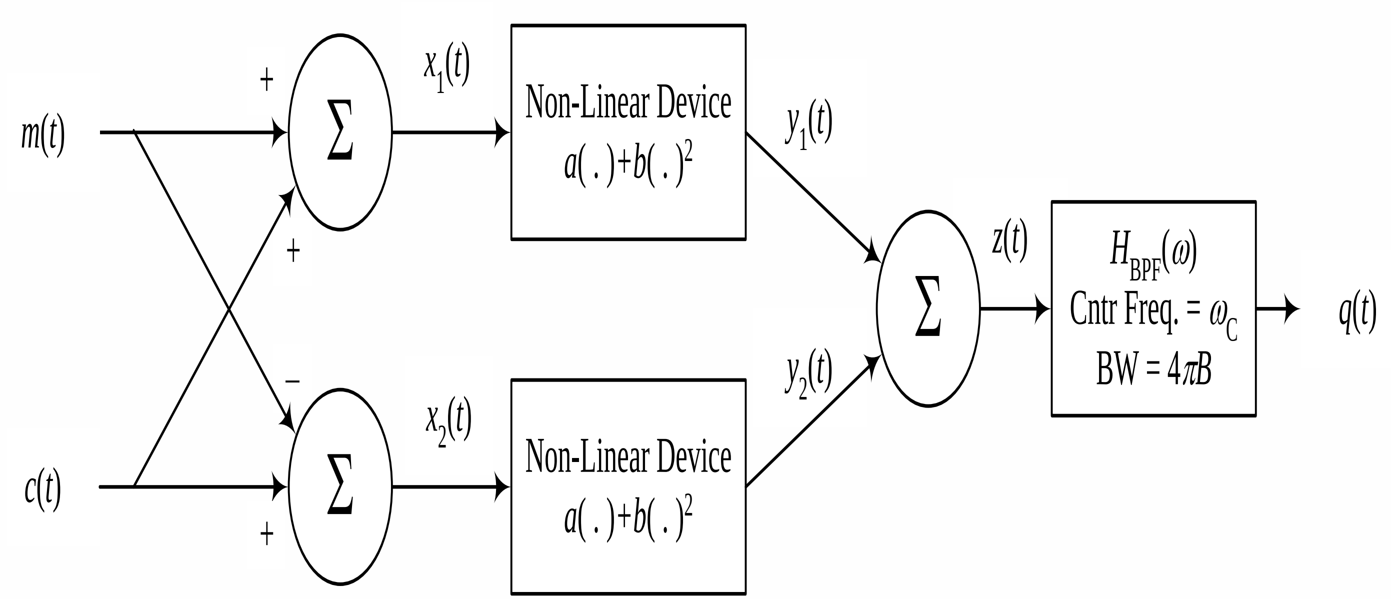
$$\mathbf{P_t = P_c(1 + k^2/2)}$$

P_t =Total transmitted power (sidebands and carrier)
P_c = Carrier power

AM Generation: Square Law & Switching Modulators

- Basically we are after multiplying a signal with a carrier.
- There are three realizations of this operation:
  - – Multiplier Circuits
  - – Non-Linear Circuits
  - – Switching Circuits

Square law Modulator



DSBSC modulation using non-linear device

Square law Modulator

$$y_1(t) = a\left[\cos(\omega_C t) + m(t)\right] + b\left[\cos(\omega_C t) + m(t)\right]^2$$

$$= a\cos(\omega_C t) + am(t) + bm^2(t) + 2bm(t)\cdot\cos(\omega_C t) + b\cos^2(\omega_C t)$$

$$= \underbrace{am(t)}_{Undesired} + \underbrace{bm^2(t)}_{Undesired} + \underbrace{2bm(t)\cdot\cos(\omega_C t)}_{Desired} + \underbrace{a\cos(\omega_C t)}_{Undesired} + \underbrace{\frac{b}{2}}_{Undesired} + \underbrace{\frac{b}{2}\cos(2\omega_C t)}_{Undesired}$$

$$y_2(t) = a\left[\cos(\omega_C t) - m(t)\right] + b\left[\cos(\omega_C t) - m(t)\right]^2$$

$$= a\cos(\omega_C t) - am(t) + bm^2(t) - 2bm(t)\cdot\cos(\omega_C t) + b\cos^2(\omega_C t)$$

$$= \underbrace{-am(t)}_{Undesired} + \underbrace{bm^2(t)}_{Undesired} - \underbrace{2bm(t)\cdot\cos(\omega_C t)}_{Desired} + \underbrace{a\cos(\omega_C t)}_{Undesired} + \underbrace{\frac{b}{2}}_{Undesired} + \underbrace{\frac{b}{2}\cos(2\omega_C t)}_{Undesired}$$

Switching Modulator

- Any periodic function can be expressed as a series of cosines (Fourier Series).
- The information signal, $m(t)$, can therefore be, equivalently, multiplied by any periodic function, and followed by BPF.
- Let this periodic function be a train of pulses.
- Multiplication by a train of pulses can be realized by simple *switching*.

Switching Modulator

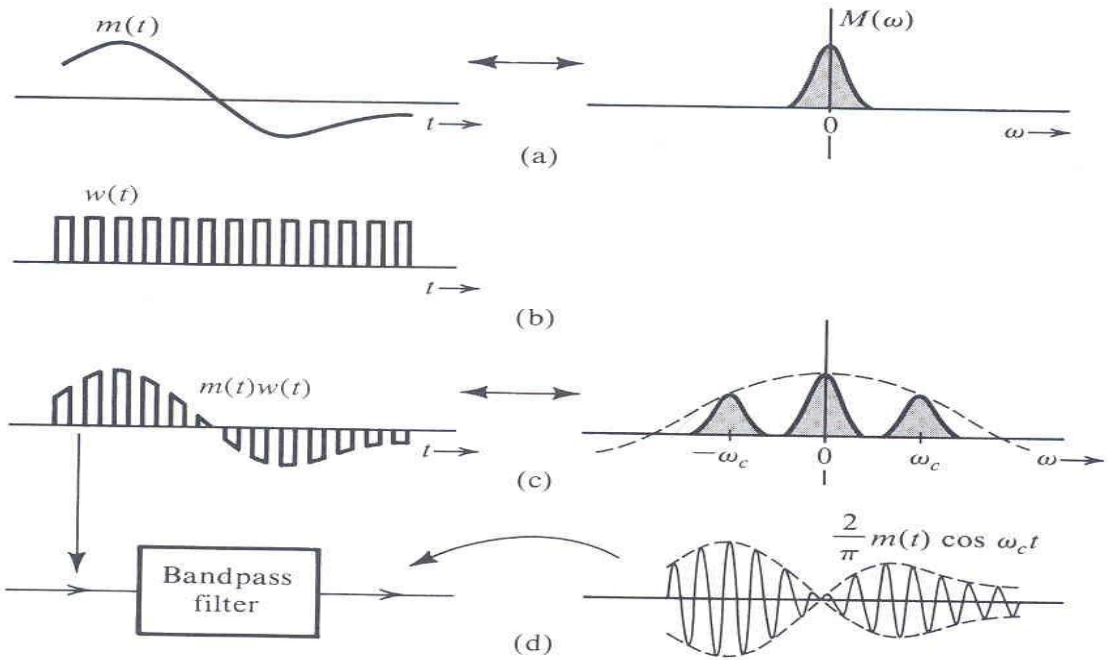


Figure 4.4 Switching modulator for DSB-SC.

Switching Modulator: Diode Bridge

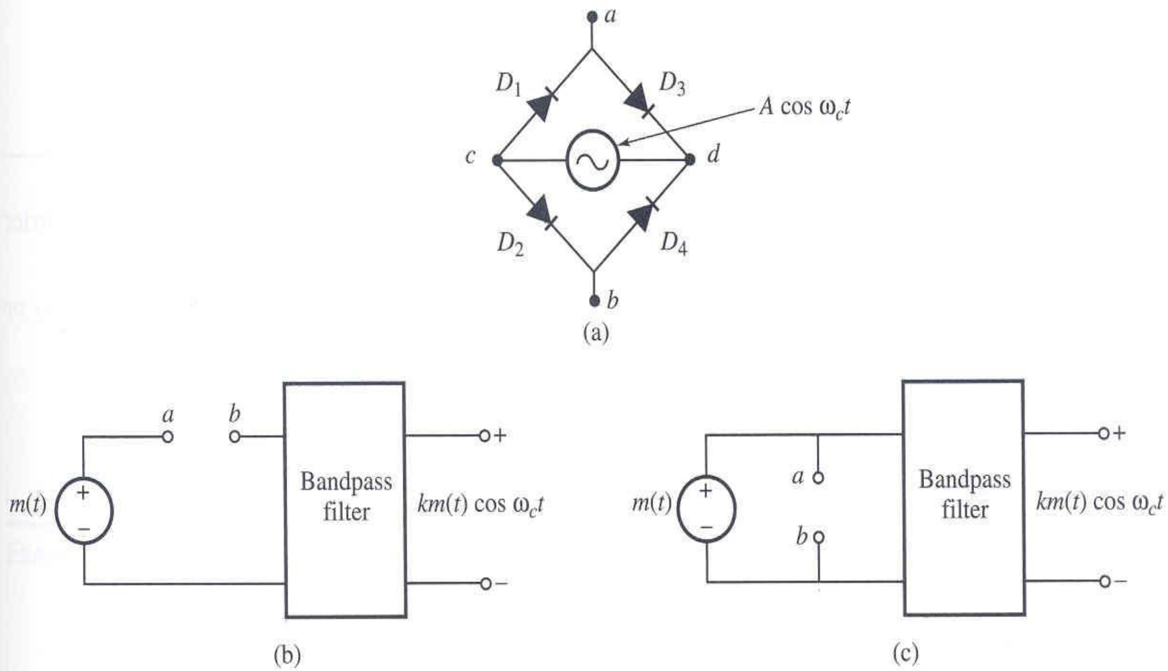


Figure 4.5 (a) Diode-bridge electronic switch. (b) Series-bridge diode modulator. (c) Shunt-bridge diode modulator.

Square-law modulator: -

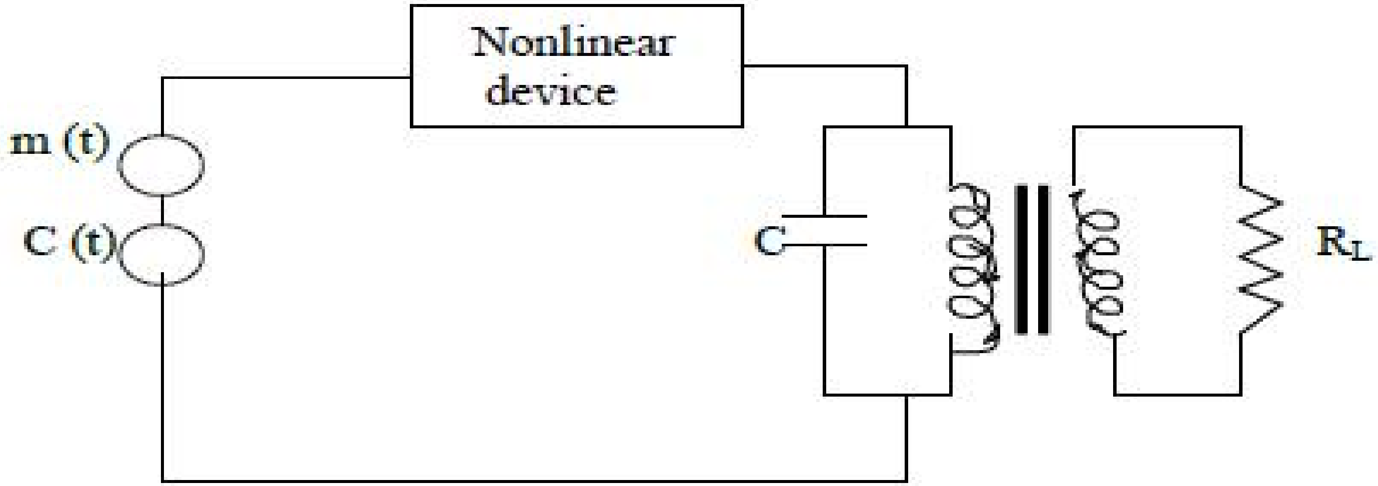


Fig. Square-law Modulator.

- A Square-law modulator requires three features: a means of summing the carrier and modulating waves, a nonlinear element, and a band pass filter for extracting the desired modulation products. Semi-conductor diodes and transistors are the most common nonlinear devices used for implementing square law modulators. The filtering requirement is usually satisfied by using a single or double tuned filters.

- When a nonlinear element such as a diode is suitably biased and operated in restricted portion of its characteristic curve, that is ,the signal applied to the diode is relatively weak, we find that transfer characteristic of diode-load resistor combination can be represented closely by a square law :

- $V_0 (t) = a_1 V_i (t) + a_2 V_i^2(t)$ ………………..(i)

- Where a1, a2 are constants

- Now, the input voltage Vi (t) is the sum of both carrier and message signals i.e.,

  $Vi\ (t) = A_c \cos 2\pi fct + m\ (t)$ …………… (ii)

- Substitute equation (ii) in equation (i) we get

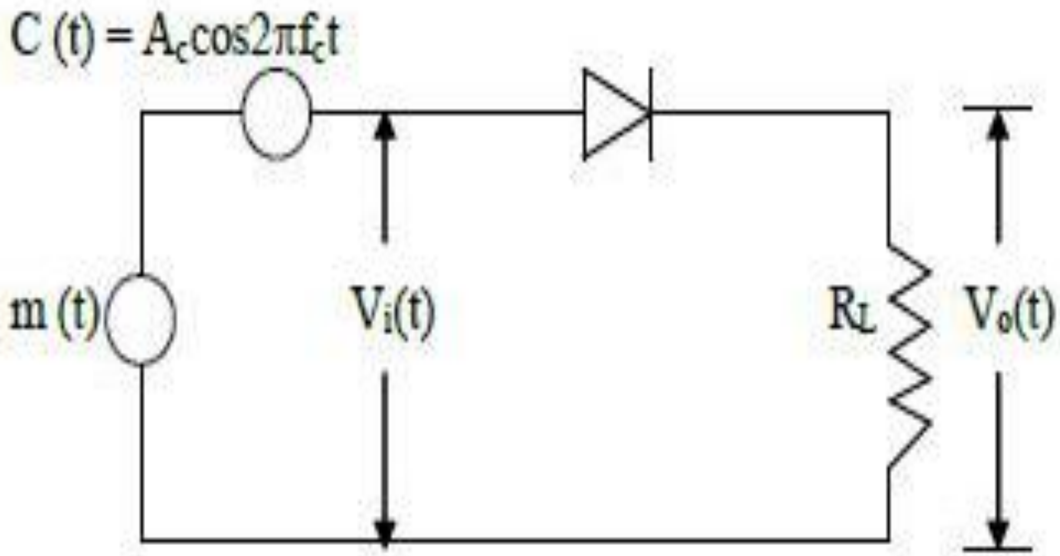
$V_0\ (t) = a_1 A_c\ [1+k_a m\ (t)]\ \cos 2\ f_c t + a_1 m\ (t) + a_2 A_c 2\cos 22\ fct + a_2 m^2(t)$……….(iii)

- Where $ka = 2a_2/a_1$

- Now design the tuned filter /Band pass filter with center frequency fc and pass band frequency width 2W.We can remove the unwanted terms by passing this output voltage V0(t) through the band pass filter and finally we will get required AM signal.

- V0 (t) =a1Ac [1+2a2/a1 m (t)] cos2 fct
- Assume the message signal m (t) is band limited to the interval –W f W

Switching Modulator: -



- Assume that carrier wave C (t) applied to the diode is large in amplitude, so that it swings right across the characteristic curve of the diode .we assume that the diode acts as an ideal switch, that is, it presents zero impedance when it is forward-biased and infinite impedance when it is reverse-biased. We may thus approximate the transfer characteristic of the diode-load resistor combination by a piecewise-linear characteristic.

- The input voltage applied Vi (t) applied to the diode is the sum of both carrier and message signals.

- $V_i(t) = A_c \cos 2\pi f_c t + m(t)$ ……………(i)

- During the positive half cycle of the carrier signal i.e. if C (t)>0, the diode is forward biased, and then the diode acts as a closed switch. Now the output voltage Vo (t) is same as the input voltage Vi (t) . During the negative half cycle of the carrier signal i.e. if C(t)<0, the diode is reverse biased, and then the diode acts as a open switch. Now the output voltage V_O (t) is zero i.e. the output voltage varies periodically between the values input voltage Vi (t) and zero at a rate equal to the carrier frequency fc.

- i.e., Vo (t) = [$A_c$cos 2π fct+m (t)] P(t)……….(ii)

- Where gp(t) is the periodic pulse train with duty cycle one-half and period Tc=1/fc and which is given by

gP(t)= ½+2/ ∑ [(-1)n-1/(2n-1)]cos [2 fct(2n-1)]………..(iii)

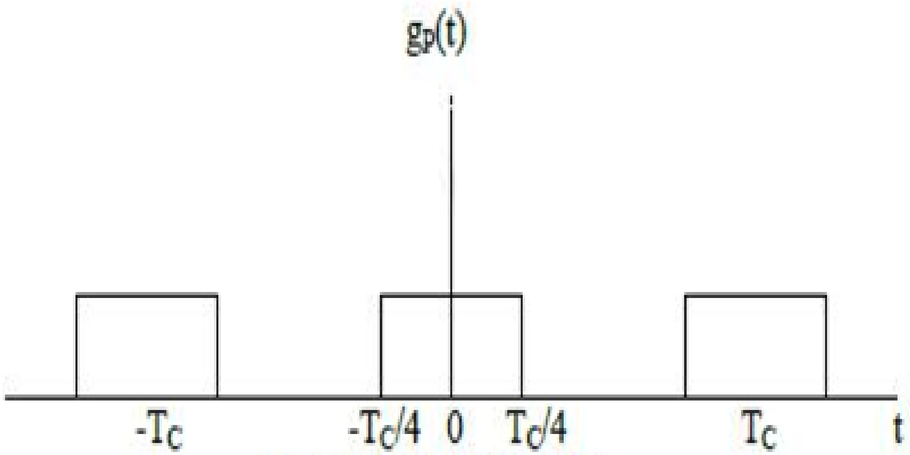


Fig. Periodic pulse train

- V0 (t) =Ac/2[1+kam (t)] cos2 fct +m (t)/2+2AC/cos 2πfct……….(iii)

Where ka = 4/ AC

- Now design the tuned filter /Band pass filter with center frequency fc and pass band frequency width 2W.We can remove the unwanted terms by passing this output voltage V0(t) through the band pass filter and finally we will get required AM signal.

- V0 (t) =Ac/2[1+kam (t)] cos2πfct

- A Square-law modulator requires nonlinear element and a low pass filter for extracting the desired message signal. Semi-conductor diodes and transistors are the most common nonlinear devices used for implementing square law modulators. The filtering requirement is usually satisfied by using a single or double tuned filters. When a nonlinear element such as a diode is suitably biased and we find that transfer characteristic of diode-load resistor combination can be represented closely by a square law :

- V0 (t) = a1Vi (t) + a2 Vi2 (t) ………………..(i)
- Where a1, a2 are constants

- Now, the input voltage Vi (t) is the sum of both carrier and message signals i.e., Vi (t) = Ac [1+kam (t)] cos2 fct……………(ii)

  Substitute equation (ii) in equation (i) we get

V0 (t) = a1Ac [1+kam (t)] cos2 fct + 1/2 a2Ac2 [1+2 kam (t) + ka2m2 (t)] [cos4 fct]………..(iii)

- Now design the low pass filter with cutoff frequency f is equal to the required message signal bandwidth. We can remove the unwanted terms by passing this output voltage V0 (t) through the low pass filter and finally we will get required message signal.

- V0 (t) = Ac2 a2 m (t)

- The Fourier transform of output voltage VO (t) is given by VO (f) = Ac2 a2 M (f)

Demodulation of AM waves

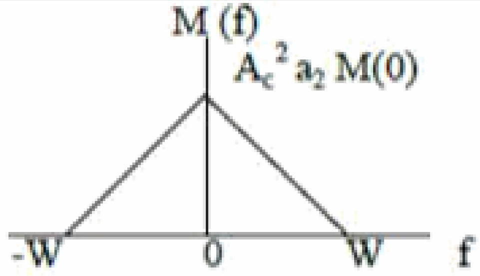


Fig: Spectrum of Output signal

Envelope Detector:

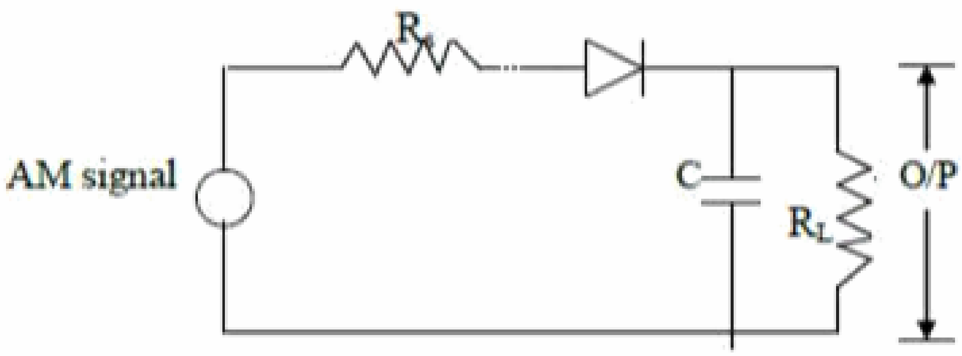


Fig: Envelope detector

- The charging time constant RsC is very small when compared to the carrier period 1/fc i.e., RsC << 1/fc
    - Where Rs = internal resistance of the voltage source.
        - C = capacitor
        - fc = carrier frequency
- i.e., the capacitor C charges rapidly to the peak value of the signal. The discharging time constant RlC is very large when compared to the
-
- charging time constant i.e.,
        - 1/fc << RlC << 1/W

Spectrum equation

a) $\cos(\omega_c t)\cos(\omega_1 t)$

from cosAcosB= 1/2[cos(A-B)+cos(A+B)]

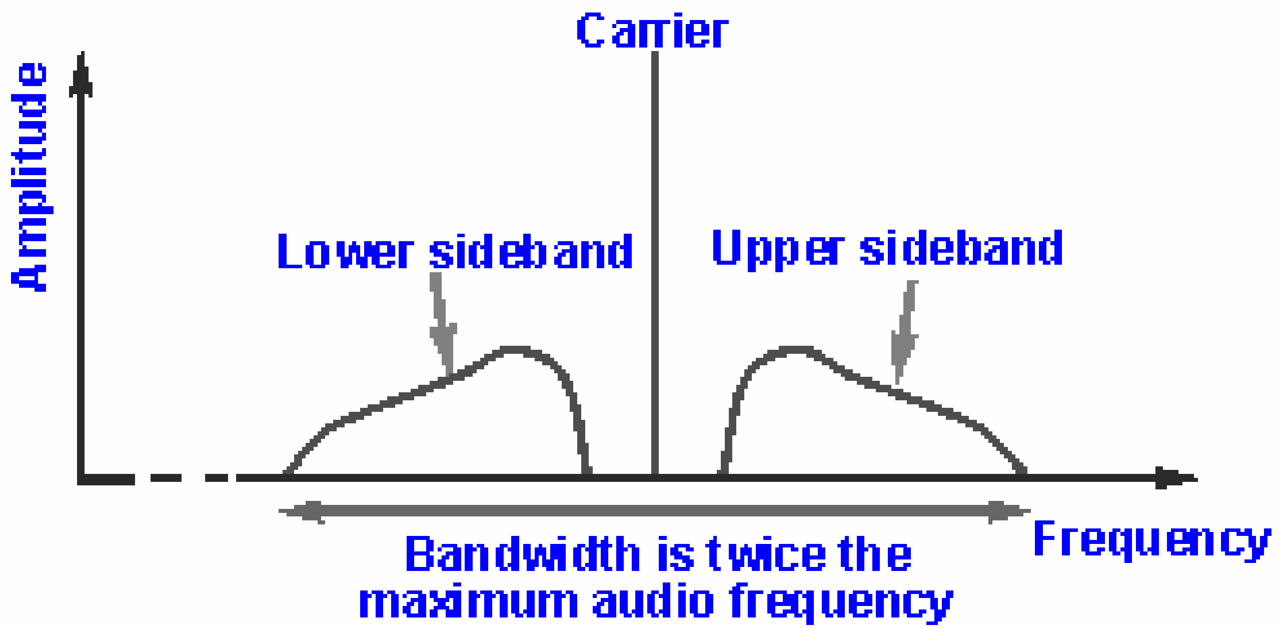
we get: $\cos(\omega_c t)\cos(\omega_1 t)=1/2[\cos(\omega_c-\omega_1)t + \cos(\omega_c+\omega_1)t]$

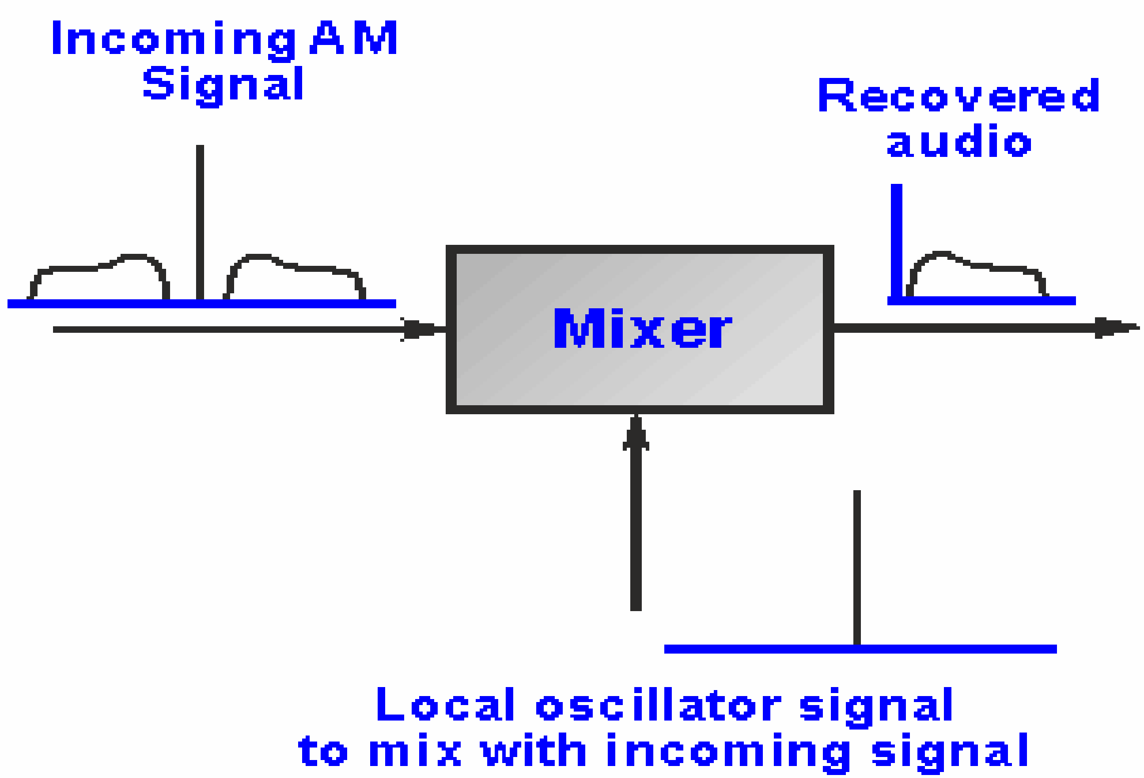
b) $\cos^2\omega t$

from $\cos^2 A=1/2[1+\cos 2A]$

we get: $\cos^2\omega t=1/2[1+\cos 2\omega t]$

- the simplest form of detection for an amplitude modulated signal utilises a simple diode rectifier. To achieve improved performance a form of demodulation known as synchronous demodulation can be used.
- When looking at the synchronous demodulation of an AM signal, it is first useful to look at the spectrum of an amplitude modulated signal. It can be seen that it comprises a carrier with the two sidebands carrying he audio or other information spreading out either side. These two sidebands are reflections of each other.


Carrier
Amplitude
Lower sideband
Upper sideband
Bandwidth is twice the
maximum audio frequency
Frequency


Incoming AM
Signal
Mixer
Recovered
audio
Local oscillator signal
to mix with incoming signal

- Double sideband-suppressed (DSB-SC) modulation, in which the transmitted wave consists of only the upper and lower sidebands. Transmitted power is saved through the suppression of the carrier wave, but the channel bandwidth requirement is same as in AM (i.e. twice the bandwidth of the message signal). Basically, double sideband-suppressed (DSB-SC) modulation consists of the product of both the message signal m (t) and the carrier signal c(t),as follows:
- $S(t) = c(t)\, m(t)$

The Fourier transform of s (t) is

$$S(f) = A_c A_m/4[\delta (f-f_c-f_m) + \delta (f+f_c+f_m)] + A_c A_m/4[\delta f_c+f_m) + \delta(f+f_c+f_m)]$$

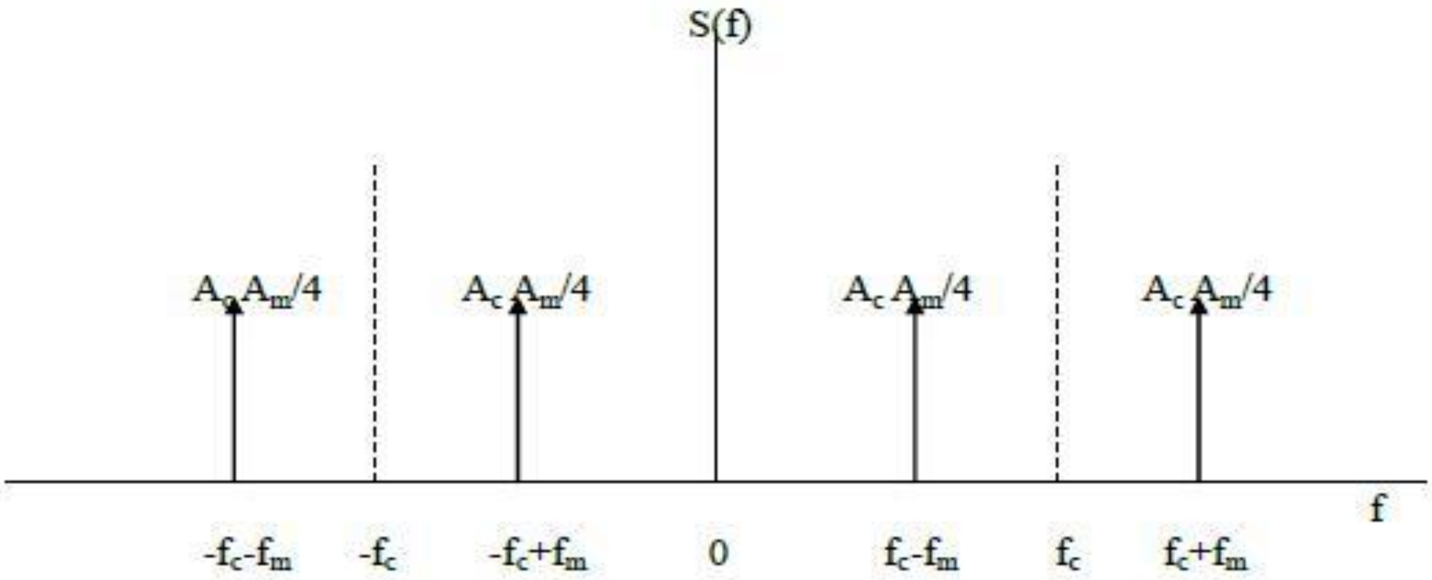


Fig. Spectrum of Single tone DSBSC wave

Power calculations of DSB-SC waves:-

- Total power PT = PLSB+PUSB
- Total power PT =Ac2Am2/8+Ac2Am2/8
- Total power PT =Ac2Am2/4

- Double sideband-suppressed (DSB-SC) modulation, in which the transmitted wave consists of only the upper and lower sidebands. Transmitted power is saved through the suppression of the carrier wave, but the channel bandwidth requirement is same as in AM (i.e. twice the bandwidth of the message signal). Basically, double sideband-suppressed (DSB-SC) modulation consists of the product of both the message signal m (t) and the carrier signal c(t),as follows:
- S (t) =c (t) m (t)

Balanced Modulator:-

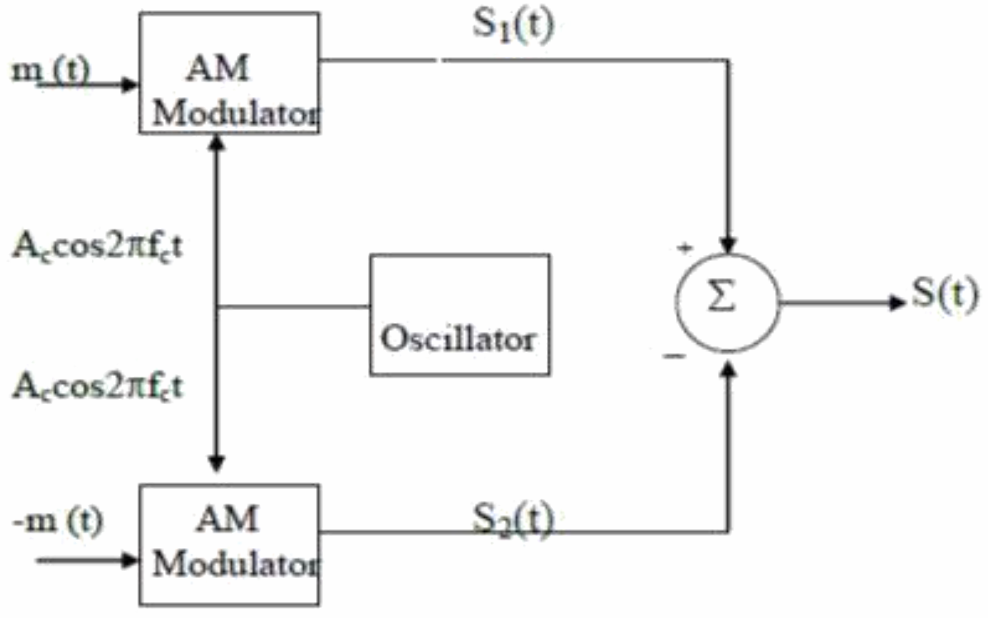


Fig. Balanced Modulator

$$S_1(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t$$

and

$$S_2(t) = A_c[1 - k_a m(t)] \cos 2\pi f_c t$$

Subtracting $S_2(t)$ from $S_1(t)$, we obtain

$$S(t) = S_1(t) - S_2(t)$$

$$S(t) = 2A_c k_a m(t) \cos 2\pi f_c t$$

Hence, except for the scaling factor $2k_a$ the balanced modulator output is equalto product of the modulating signal and the carrier signal

The Fourier transform of s (t) is

$$S(f) = k_a A_c [M(f - f_c) + M(f + f_c)]$$

Assume that the message signal is band-limited to the interval $-W \leq f \leq W$

- Double sideband-suppressed (DSB-SC) modulation, in which the transmitted wave consists of only the upper and lower sidebands. Transmitted power is saved through the suppression of the carrier wave, but the channel bandwidth requirement is same as in AM (i.e. twice the bandwidth of the message signal). Basically, double sideband-suppressed (DSB-SC) modulation consists of the product of both the message signal m (t) and the carrier signal c(t),as follows:
- S (t) =c (t) m (t)

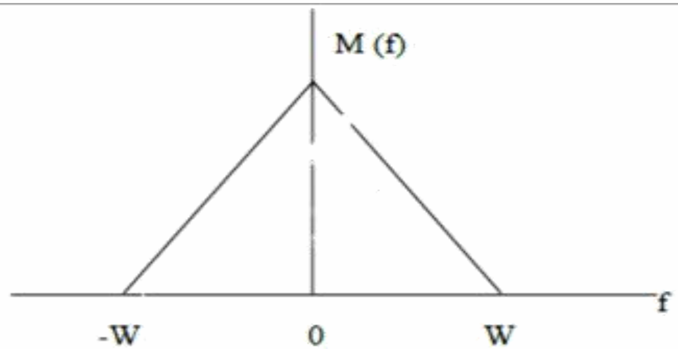
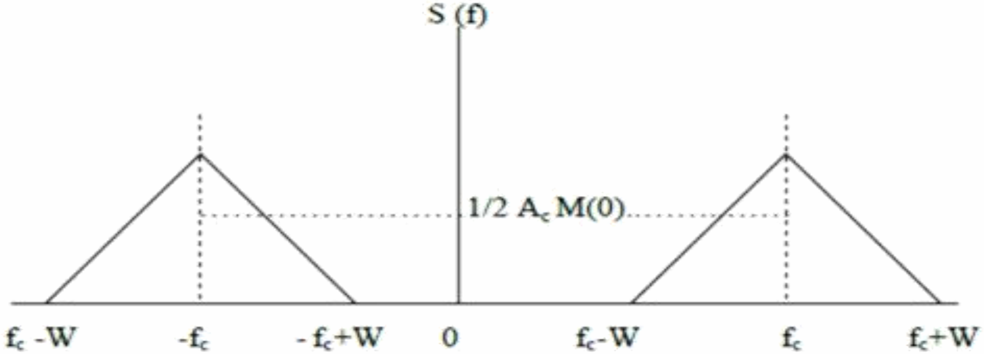
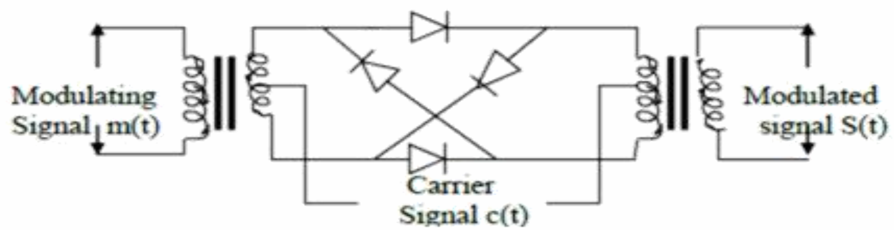


Fig. Spectrum of Baseband signal



Ring modulator:-



. The square wave carrier c (t) can be represented by a Fourier series as follows:

$$c(t) = 4/\pi \sum_{n=1}^{\infty} (-1)^{n-1}/(2n-1) \cos[2\pi f_c t(2n-1)]$$

When the carrier supply is positive, the outer diodes are switched ON
and the inner diodes are switched OFF, so that the modulator multiplies the message signal by +1
When the carrier supply is positive, the outer diodes are switched ON and the inner diodes are switched OFF, so that the modulator multiplies the message signal by +1.when the carrier supply is negative, the outer diodes are switched OFF and the inner diodes are switched ON, so that the modulator multiplies the message signal by -1. Now, the Ring modulator output is the product of both message signal m (t) and carrier signal c (t).

$$S(t) = c(t)\, m(t)$$

$$S(t) = 4/\pi \sum_{n=1}^{\infty} (-1)^{n-1}/(2n-1) \cos[2\pi f_c t(2n-1)]\, m(t)$$

For n=1

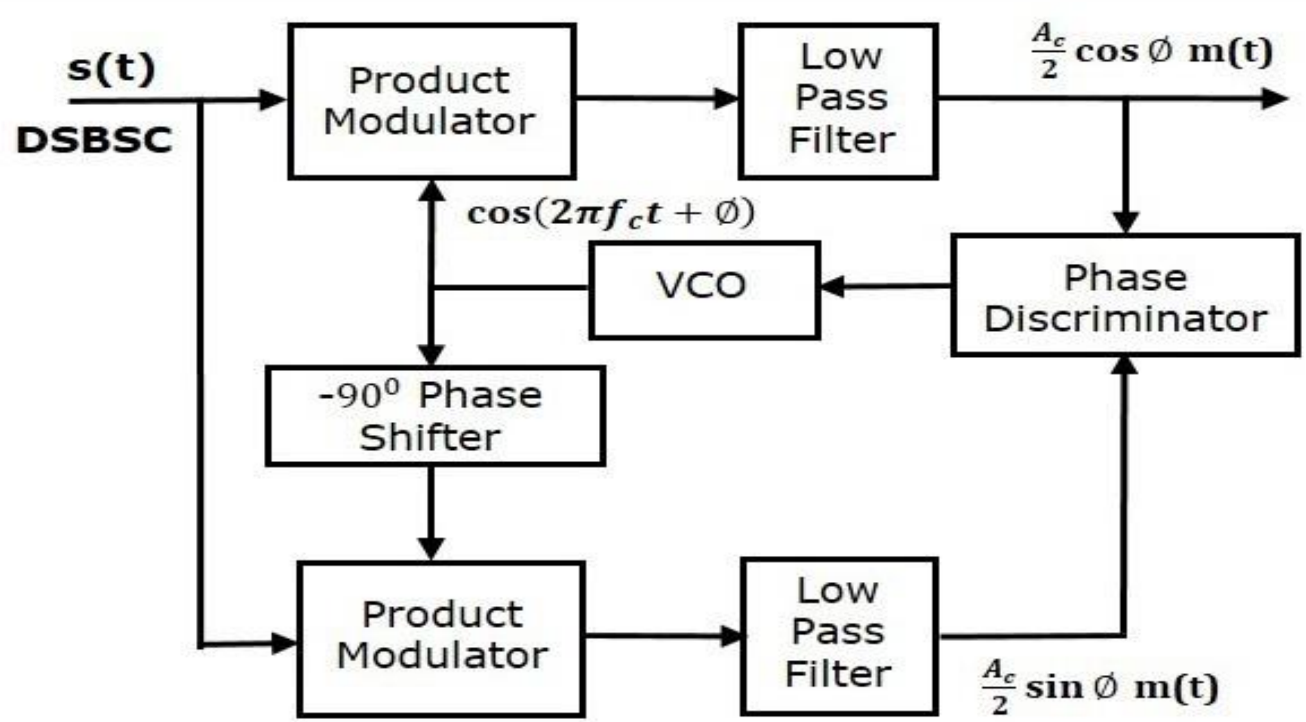
$$S(t) = 4/\pi \cos(2\pi f_c t)\, m(t)$$

There is no output from the modulator at the carrier frequency i.e the
modulator output consists of modulation products. The ring modulator is
sometimes referred to as a double-balanced modulator, because it is balanced with respect to both the message signal and the square wave carrier signal. The Fourier transform of s (t) is

$$S(f) = 2/\pi [M(f-f_c) + M(f+f_c)]$$

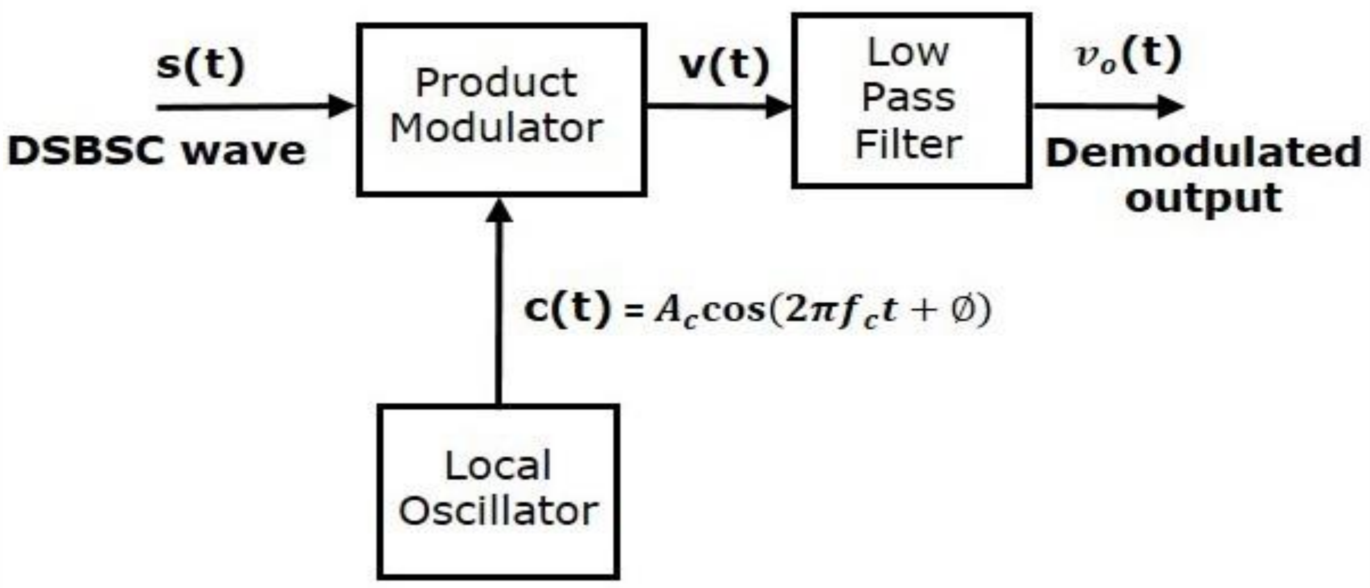
Assume that the message signal is band-limited to the interval $-W \leq f \leq W$

- COSTAS LOOP


s(t)
DSBSC
Product Modulator
Low Pass Filter
$\frac{A_c}{2}\cos\emptyset\ m(t)$
$\cos(2\pi f_c t + \emptyset)$
VCO
Phase Discriminator
-90^0 Phase Shifter
Product Modulator
Low Pass Filter
$\frac{A_c}{2}\sin\emptyset\ m(t)$

- **Costas loop** consists of two product modulators with common input $s\left (t \right)$, which is DSBSC wave. The other input for both product modulators is taken from **Voltage Controlled Oscillator** (VCO) with -90^0 phase shift to one of the product modulator as shown in figure.
- We know that the equation of DSBSC wave is
- $$s\left (t \right)=A_c \cos\left (2 \pi f_ct \right)m\left (t \right)$$

- Coherent detection



- In this process, the message signal can be extracted from DSBSC wave by multiplying it with a carrier, having the same frequency and the phase of the carrier used in DSBSC modulation. The resulting signal is then passed through a Low Pass Filter. Output of this filter is the desired message signal.

Noise in SSB Receivers

- The SSB signal is mathematically expressed as under

$$S(t) = \frac{1}{2} CV_c \cos(2*pi*f_c*t)\, m(t) + \frac{1}{2} CV_c \sin(2*pi*f_c*t)\, m\_(t)$$

- m_(t) represents the Hilbert transform of m(t). The Hilbert transform is obtained by passing the message through a 90 degree phase shifter. Hence, m (t) and m_(t) will always be orthogonal to each other.

- The general expression of SSB is given as

$S(t)= A_c m(t)/2 \cos(2*pi*f_c*t) \pm)= A_c m\_(t)/2 \sin(2*pi*f_c*t)$

Figure of Merit Calculation

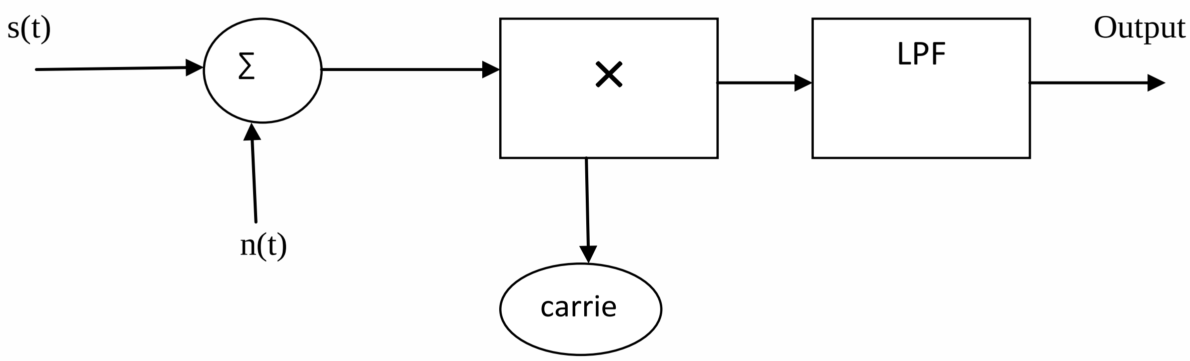
$$S_i = S_{dsb}/2$$

$$S_i = A_c^2P/4$$

Noise power affecting message signal is given as

$$N_i = N_0W \quad \text{watts}$$

$$S_i/N_i = A_c^2P/4\,N_0W$$



- $(Mul)_{o/p} = \{ s(t)+n(t) \} \cos(2*pi*f_c*t)$
- $(LPF)_{o/p} = A_c m(t)/4 + n_c(t)/2$
- S_0 = power $\{ A_c m(t)/4\}$

  $A_c^2 m(t)^2/16$

  $A_c^2 p/16$
- N_0 = power $\{ n_c(t)/2\}$

  $\frac{1}{4} n_c^2(t)$

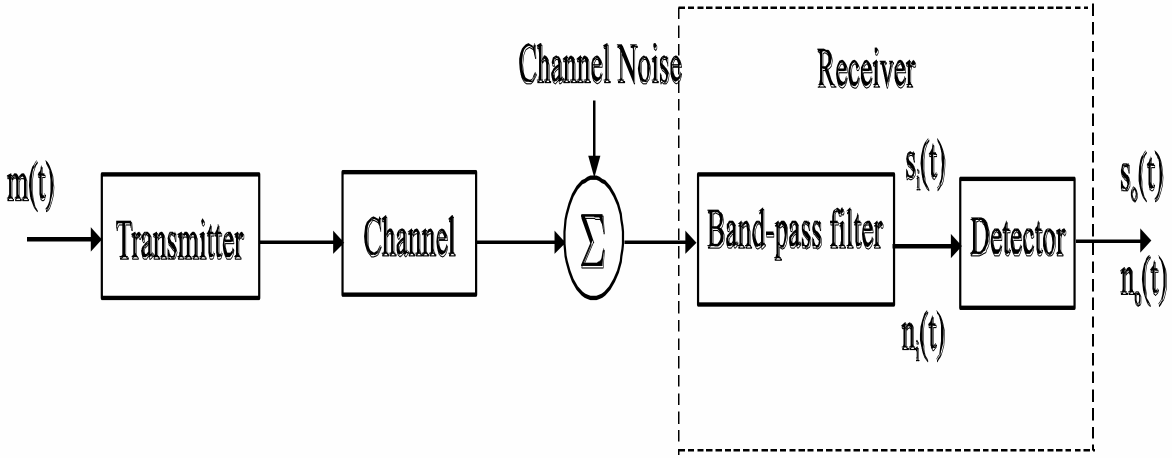
  $\frac{1}{4} N_0 W$
- $S_{0/} N_0 = A_c^2 p/16/\frac{1}{4} N_0 W$

  $A_c^2 p/ 4N_0 W$

- Figure of merit = $S_{0/} N_{0/} S_i/ N_i$

  1

In communication systems, message signal travels from the transmitter to the receiver via a channel. The channel introduces additive noise in the message and, hence, the message reaching the receiver becomes corrupted. The receiver detects both noise and message signals, it reproduces a noisy message at the output. It is thus necessary to discuss the noise effect in the communication system.



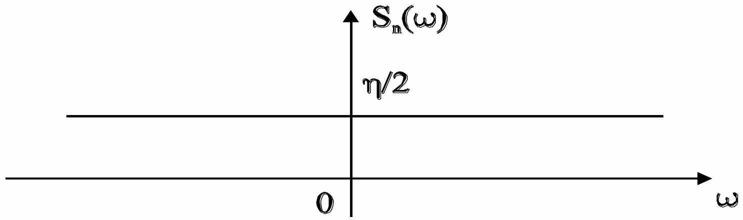
In a communication system, the noise is introduced by the transmission channel as well as the circuit components in the transmitter and/or receiver. Noise is random in nature and we can only model its statistical properties. The most common form of noise are known as *white noise*.

White noise

White noise is noise whose power spectral density is uniform over the entire the frequency range. The term *white* is used in analogy with white light, which is a superposition of all visible spectral components.

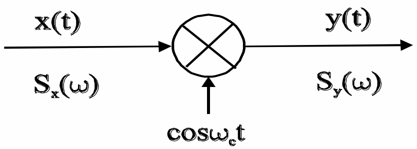
$$S_n(\omega) = \eta / 2$$

Where η is a constant.



Noise response of a product device

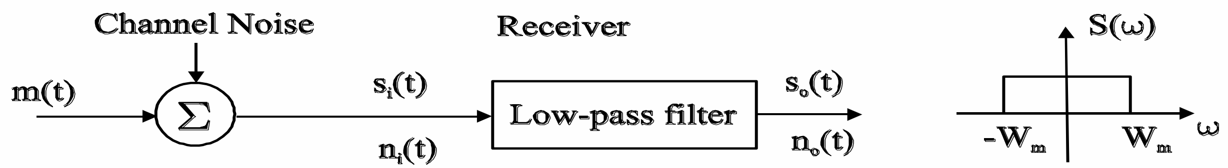
In coherent detection scheme, the input signal of the detector is sent to a product device in order to multiply the signal by a local carrier wave $\cos\omega_c t$ and then pass through a low-pass filter. Since noise travels along the signal, we have to study the noise response of a product device.



If the random signal x(t) has power spectral density $S_x(\omega)$, we have mentioned that the PSD of y(t) is given by

$$S_y(\omega) = \frac{1}{4}[S_x(\omega + \omega_c) + S_x(\omega - \omega_c)]$$

Noise performance of Baseband system



The baseband signal is assumed to be band limited and the channel be distortion-less, then
The output signal:

$$s_o(t) = s_i(t) = m(t)$$

The output signal power:

$$S_o = S_i = \overline{m^2(t)}$$

The output noise power:

$$N_o = N_i = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\eta}{2}\,d\omega = \frac{2}{2\pi}\int_{0}^{\infty}\frac{\eta}{2}\,d\omega = \frac{\eta}{2\pi}\int_{0}^{W_m} d\omega = \frac{\eta W_m}{2\pi}$$

The output signal-to-noise ratio:

$$\frac{S_o}{N_o} = \frac{2\pi\,\overline{m^2(t)}}{\eta W_m} = \frac{2\pi S_i}{\eta W_m}$$

In order to compare the noise performance of different amplitude modulation system, we define a parameter γ as

$$\gamma = \frac{2\pi S_i}{\eta W_m}$$

then

$$\gamma = \frac{S_o}{N_o}$$

UNIT III

SINGLE SIDE BAND MODULATION AND VESTIGIAL SIDE BAND MODULATION

Noise in SSB Receivers

- The SSB signal is mathematically expressed as under

$$S(t) = ½\, CV_c \cos(2*pi*f_c*t)\, m(t) + ½\, CV_c \sin(2*pi*f_c*t)\, m\_(t)$$

- m_(t) represents the Hilbert transform of m(t). The Hilbert transform is obtained by passing the message through a 90 degree phase shifter. Hence, m (t) and m_(t) will always be orthogonal to each other.

- The general expression of SSB is given as

$$S(t)= A_c m(t)/2 \cos(2*pi*f_c*t) \pm)= A_c m\_(t)/2 \sin(2*pi*f_c*t)$$

Figure of Merit Calculation

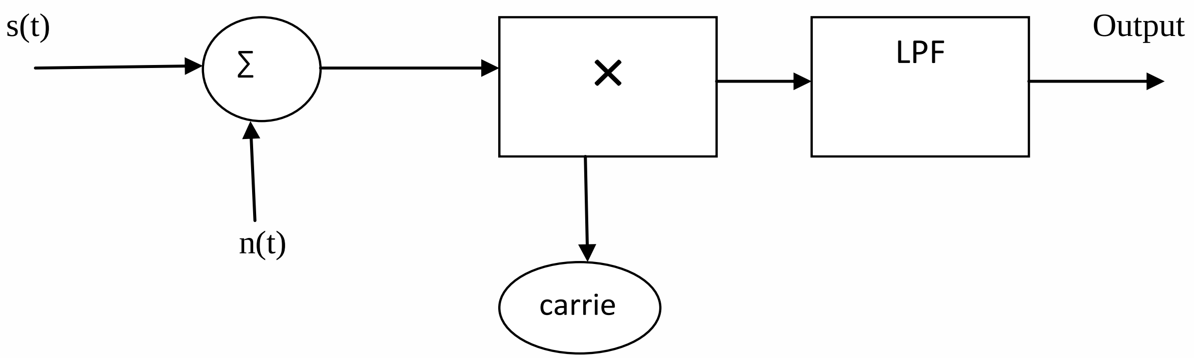
$$S_i = S_{dsb}/2$$

$$S_i = A_c^2 P/4$$

Noise power affecting message signal is given as

$$N_i = N_0 W \quad \text{watts}$$

$$S_i / N_i = A_c^2 P / 4 N_0 W$$



- $(Mul)_{o/p}$ = { s(t)+n(t) } cos (2*pi*f_c*t)
- $(LPF)_{o/p}$ = $A_c m(t)/4 + n_c(t)/2$
- S_0 = power { $A_c m(t)/4$}

  $A_c^2 m(t)^2/16$

  $A_c^2 p/16$
- N_0 = power { $n_c(t)/2$}

  $\frac{1}{4} n_c^2(t)$

  $\frac{1}{4} N_0 W$
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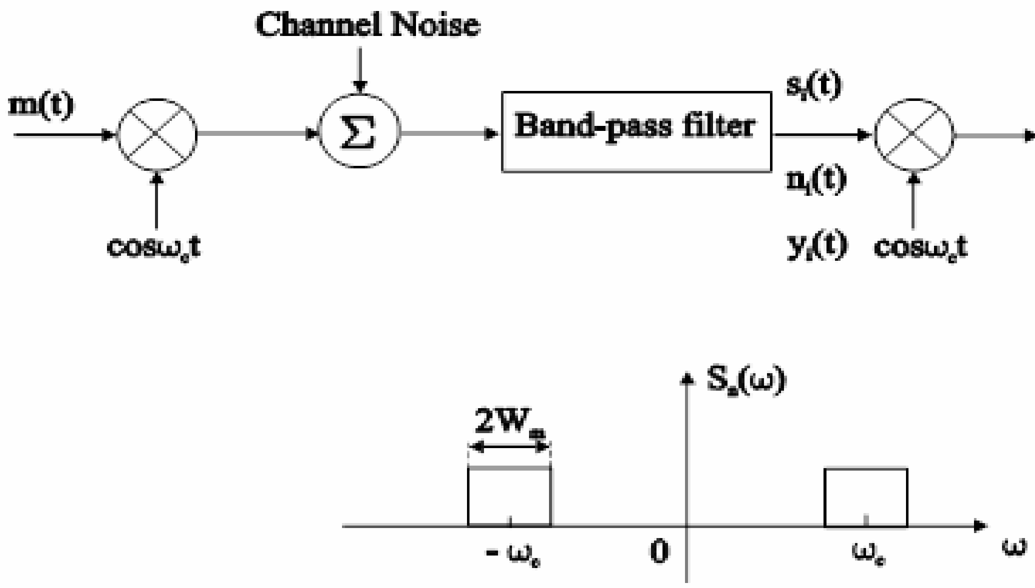
- Figure of merit = $S_{0/} N_{0/} S_i/ N_i$

  1

- Coherent detection



Input signal power:

$$S_i = \overline{[m(t)\cos\omega_c t]^2} = \frac{\overline{m^2(t)}}{2}$$

Input noise power:

$$N_i = 2\frac{2}{2\pi}\int_{\omega_c}^{\omega_c+W_m}\frac{\eta}{2}d\omega = \frac{\eta W_m}{\pi}$$

Input signal-to noise ratio:

$$\frac{S_i}{N_i} = \frac{\pi\overline{m^2(t)}}{2\eta W_m}$$

Output noise power:

$$N_o = \frac{1}{2\pi}\int_{-\infty}^{\infty}S_{n_o}(\omega)d\omega = \frac{1}{2\pi}\frac{1}{4}\int_{-\infty}^{\infty}[S_{n_i}(\omega+\omega_c)+S_{n_i}(\omega-\omega_c)]d\omega = \frac{1}{2\pi}\frac{2}{4}\int_{-\infty}^{\infty}S_{n_i}(\omega-\omega_c)d\omega$$

$$= \frac{1}{2\pi}\frac{1}{4}\int_{\omega_c-W_m}^{\omega_c+W_m}\eta\, d\omega = \frac{1}{2\pi}\frac{2}{4}\int_{\omega_c}^{\omega_c+W_m}\eta\, d\omega = \frac{\eta W_m}{4\pi} = \frac{N_i}{4}$$

Output signal-to-noise ratio:

$$\frac{S_o}{N_o} = \frac{\pi\overline{m^2(t)}}{\eta W_m} = \frac{2\pi S_i}{\eta W_m} = \gamma$$

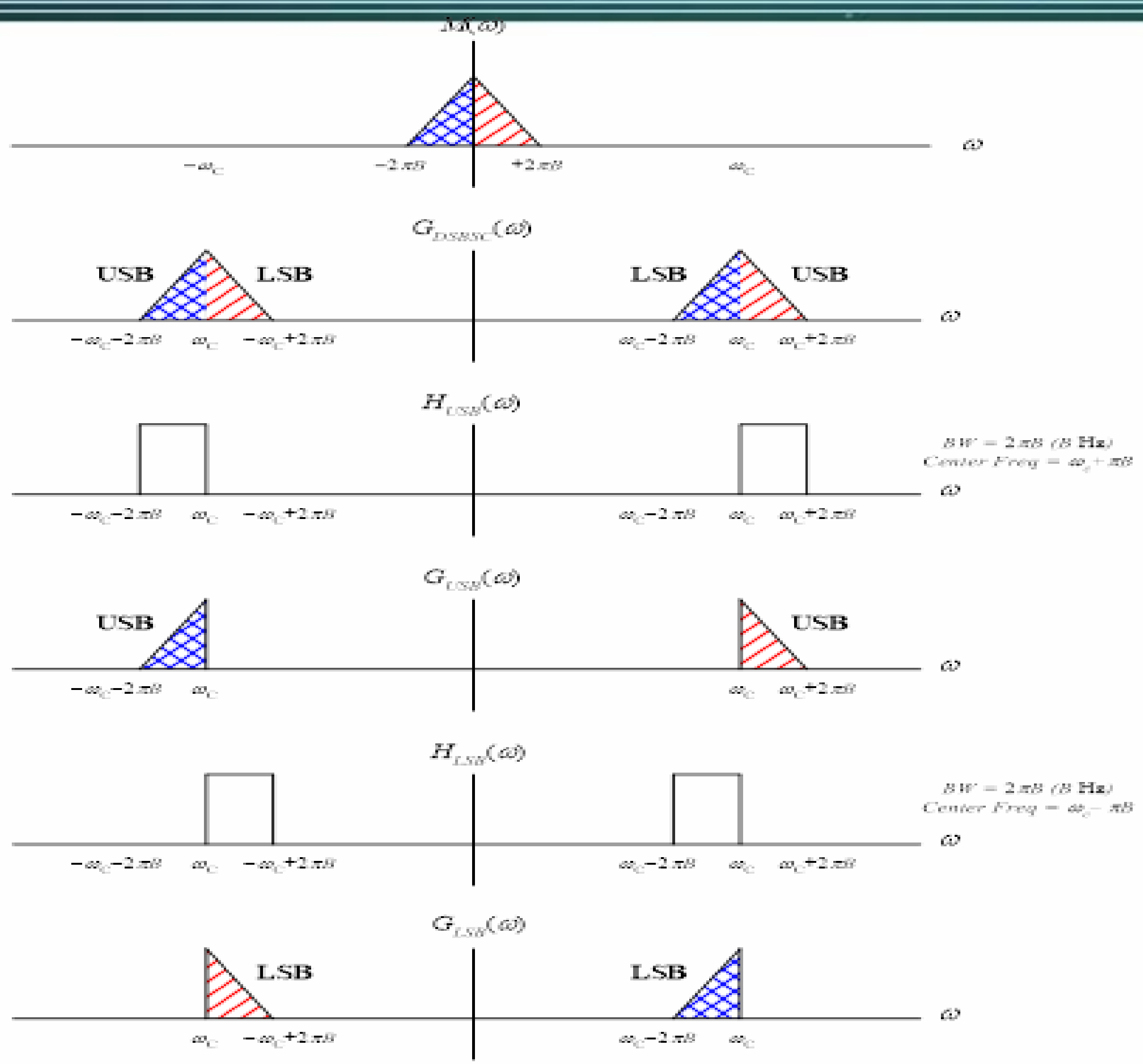
DSB-SC system has the same output signal-to-noise ratio as the baseband system. This shows that for a fixed input signal power, the output signal-to-noise ratio is the same for the baseband and the DSB-SC systems. Thus, theoretically, baseband and DSB-SC systems have identical capabilities.

*SSB system also has the same output signal-to-noise ratio as the baseband system.

$$\frac{S_o}{N_o} = \frac{2\pi S_i}{\eta W_m} = \gamma$$

- DSBSC Filtering Method: Since SSB modulation is the transmission of the upper or lower side bands, SSB modulation can be generated by filtering the undesired side band of a DSBSC signal and retaining the desired one using a bandpass filter with bandwidth equal that of the message signal (not twice its bandwidth) and a center frequency equal to the center frequency of the desired side band (not the carrier).

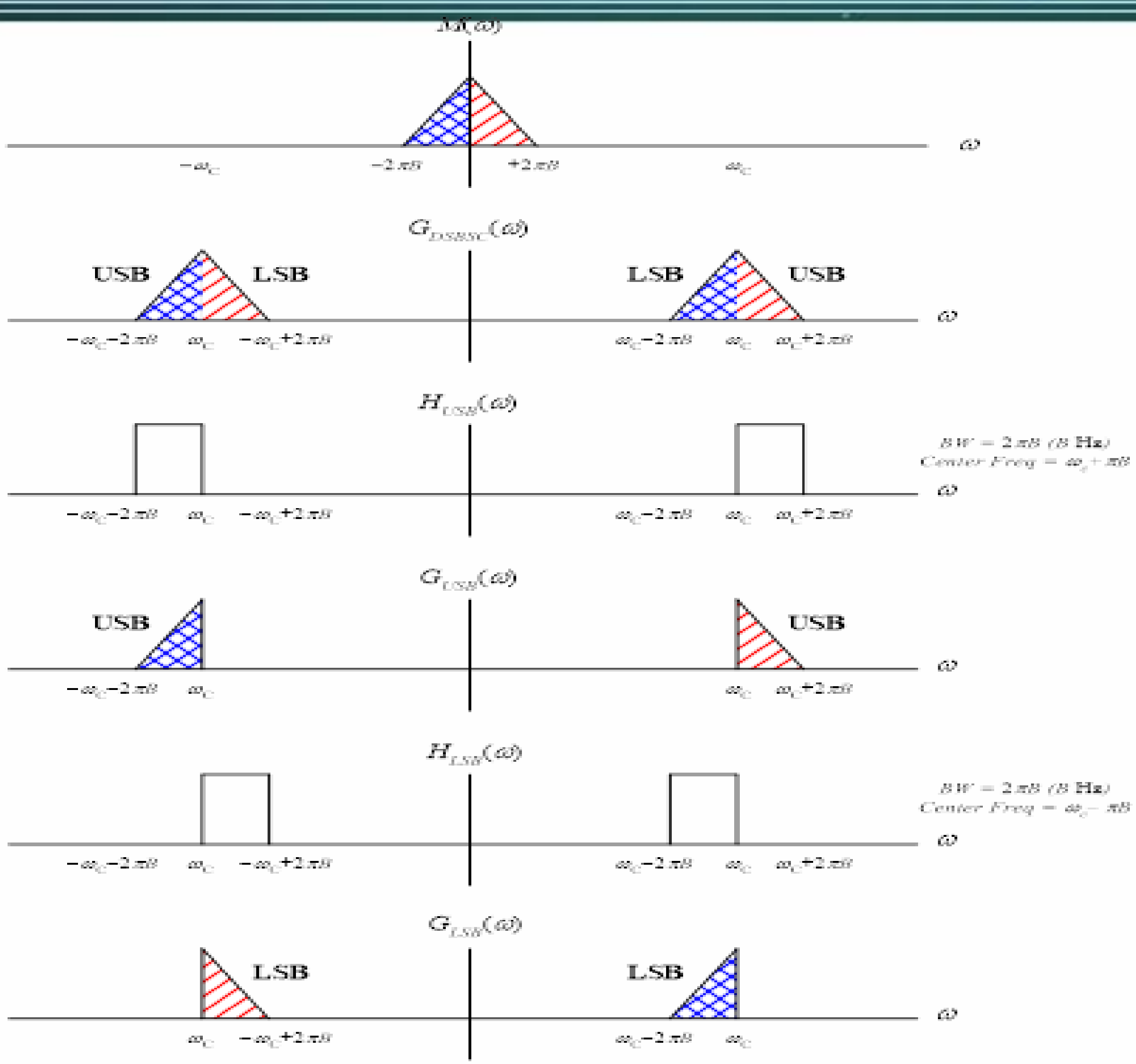
- The PROBLEM with this modulation method is that it is suitable only for message signals that have a small guard–band (no signal components) around zero frequency, as it is the case for voice signals. The important components of human voice start from a frequency around 300 Hz. The reason is that ideal filters with sharp edges do not exist and therefore, filters with non–sharp edges must be used. Any non–zero components of the message signals close to zero frequency may be lost or distorted because of the filtering process.


M(ω)
ω
G_DSBSC(ω)
USB
LSB
LSB
USB
H_USB(ω)
BW = 2πB (B Hz)
Center Freq = ω_c + πB
G_USB(ω)
USB
USB
H_LSB(ω)
BW = 2πB (B Hz)
Center Freq = ω_c − πB
G_LSB(ω)
LSB
LSB

Generation of SSB-SC

- DSBSC Filtering Method: Since SSB modulation is the transmission of the upper or lower side bands, SSB modulation can be generated by filtering the undesired side band of a DSBSC signal and retaining the desired one using a bandpass filter with bandwidth equal that of the message signal (not twice its bandwidth) and a center frequency equal to the center frequency of the desired side band (not the carrier).

- The PROBLEM with this modulation method is that it is suitable only for message signals that have a small guard–band (no signal components) around zero frequency, as it is the case for voice signals. The important components of human voice start from a frequency around 300 Hz. The reason is that ideal filters with sharp edges do not exist and therefore, filters with non–sharp edges must be used. Any non–zero components of the message signals close to zero frequency may be lost or distorted because of the filtering process.



M(ω)
ω
G_DSBSC(ω)
USB
LSB
LSB
USB
H_USB(ω)
BW = 2πB (B Hz)
Center Freq = ω_c + πB
G_USB(ω)
USB
USB
H_LSB(ω)
BW = 2πB (B Hz)
Center Freq = ω_c − πB
G_LSB(ω)
LSB
LSB

Noise in SSB Receivers

- The SSB signal is mathematically expressed as under

$$S(t) = ½ CV_c \cos(2*pi*f_c*t)\ m(t) + ½ CV_c \sin(2*pi*f_c*t)\ m\_(t)$$

- m_(t) represents the Hilbert transform of m(t). The Hilbert transform is obtained by passing the message through a 90 degree phase shifter. Hence, m (t) and m_(t) will always be orthogonal to each other.

- The general expression of SSB is given as

S(t)= A_cm(t)/2 cos(2*pi*f_c*t) ±)= A_cm_(t)/2 sin(2*pi*f_c*t)

Figure of Merit Calculation

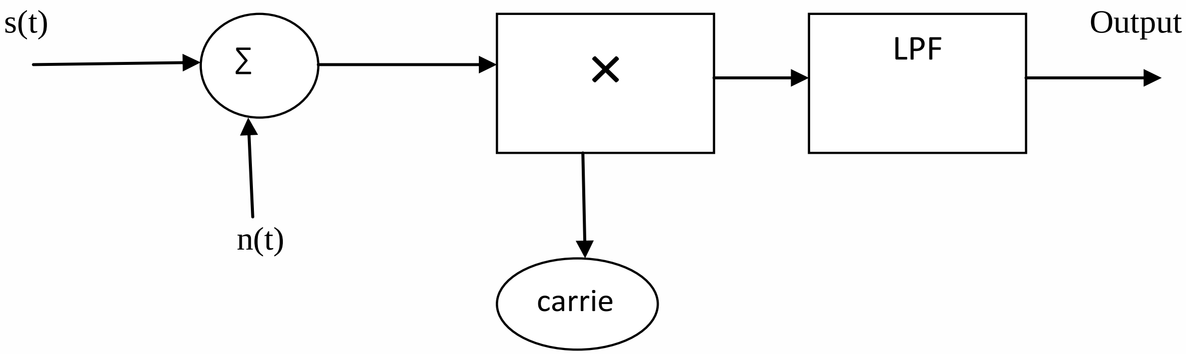
$$S_i = S_{dsb}/2$$

$$S_i = A_c^2 P/4$$

Noise power affecting message signal is given as

$$N_i = N_0 W \quad \text{watts}$$

$$S_i / N_i = A_c^2 P / 4 N_0 W$$



- $(Mul)_{o/p} = \{ s(t)+n(t) \} \cos (2*pi*f_c*t)$
- $(LPF)_{o/p} = A_c m(t)/4 + n_c(t)/2$
- S_0 = power $\{ A_c m(t)/4\}$

  $A_c^2 m(t)^2/16$

  $A_c^2 p/16$
- N_0 = power $\{ n_c(t)/2\}$

  $\frac{1}{4} n_c^2(t)$

  $\frac{1}{4} N_0 W$
- $S_{0/} N_0 = A_c^2 p/16/\frac{1}{4} N_0 W$

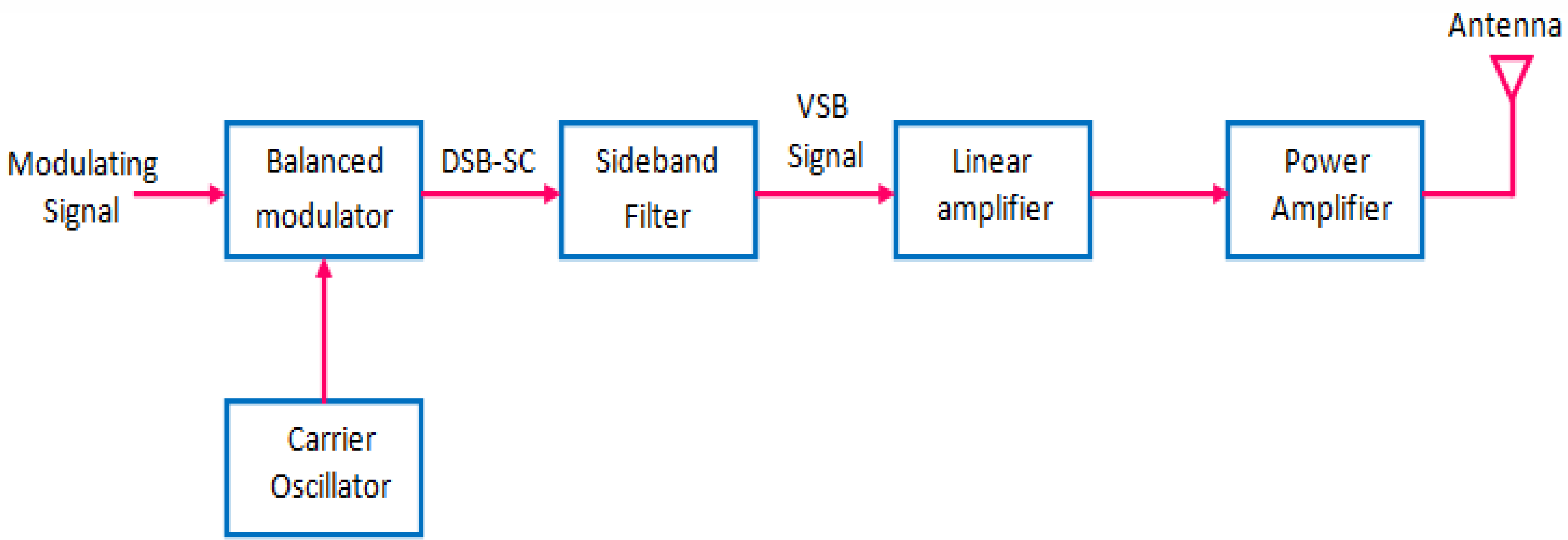
  $A_c^2 p/ 4N_0 W$

- Figure of merit = $S_{0/} N_{0/} S_i/ N_i$

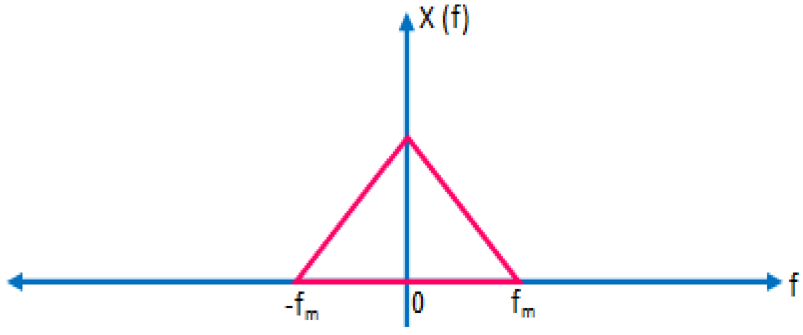
  1

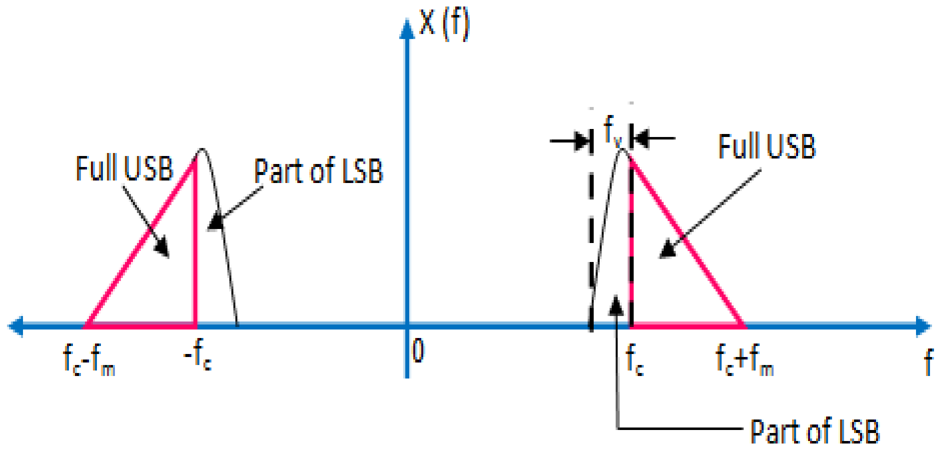
Introduction to VSB

- The stringent frequency response requirements on the sideband filter in SSB-SC system can be relaxed by allowing a part of the unwanted sideband called as Vestige to appear in the output of the modulator.


Antenna
VSB
Signal
Modulating
Signal
Balanced
modulator
DSB-SC
Sideband
Filter
Linear
amplifier
Power
Amplifier
Carrier
Oscillator

Frequency domain representation





Bandwidth and Advantages

- 1. Transmission bandwidth

  It is evident that the transmission of bandwidth of the VSB modulated wave is given by

  $$B = (fm+fv) \text{ Hz}$$

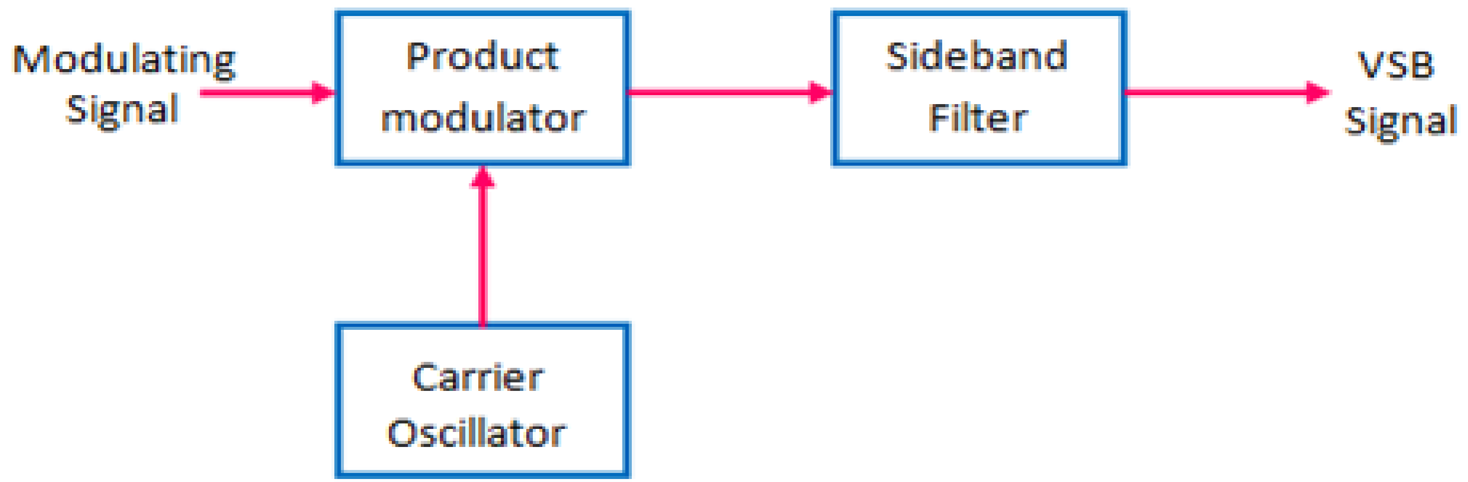
  Where fm is message bandwidth and fv is the width of the vestigial sideband.

- 2. Advantages of VSB

  1.The main advantage of VSB modulation is the reduction in bandwidth. It is almost as efficient as SSB.

  2.It possesses good phase characteristics and makes the transmission of low frequency components possible.

VSB Generation



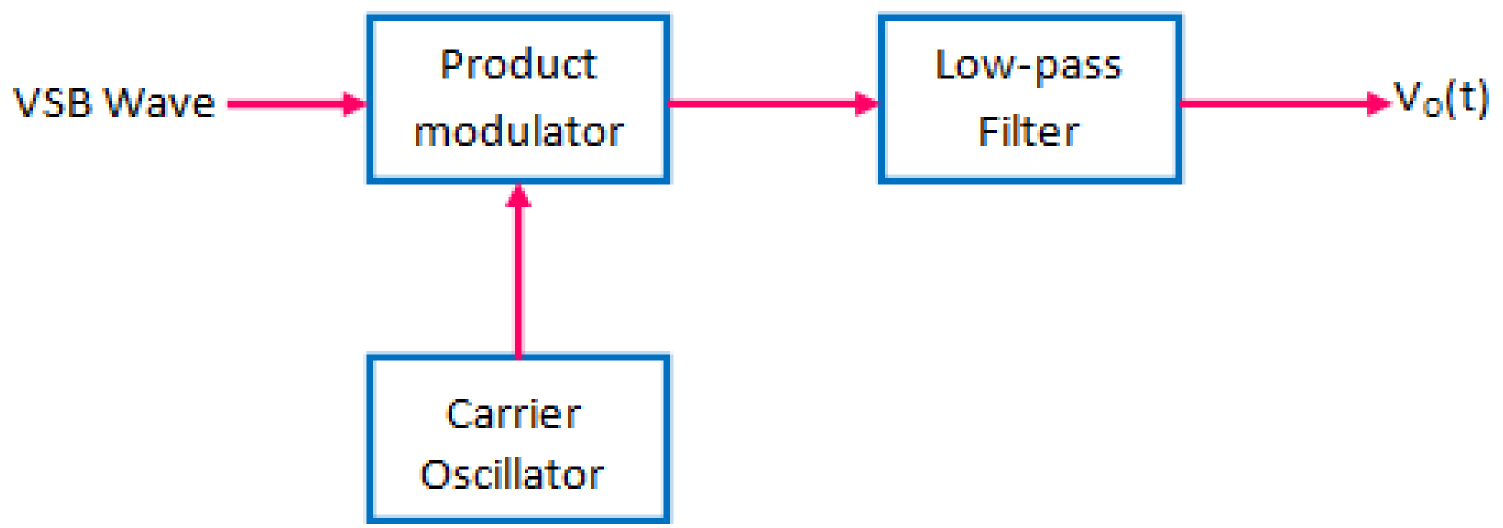
$$m(t) = x(t) \cdot c(t)$$

$$= x(t) \cdot V_c \cos(2\pi f_c t)$$

- This represents a DSB-SC modulated wave .This DSB-SC signal is then applied to a sideband shaping filter . The design of this filter depends on the desired spectrum of the VSB modulated

$$S(f) = V_c/2 \, [x(f-f_c)+x(f+f_c)] \, H(f)$$

Demodulation of VSB

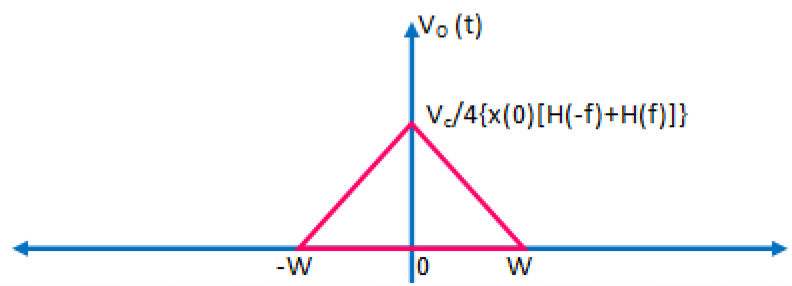


$$S(f) = \frac{V_c}{2}[X(f - f_c) + X(f + f_c)]H(f)$$

$$M(f) = \frac{V_c}{2}[X(f - 2f_c)H(f - f_c) + X((f + 2f_c)H(f + f_c)] + \frac{V_c}{4}[X(f)[H(f - f_c) + H(f + f_c)]$$

The first term in the above expression represents the VSB modulated wave, corresponding to a carrier frequency of $2f_c$. This term will be eliminated by the filter to produce output $v_o(t)$. The second term in the above expression for M(f) represents the spectrum of demodulated VSB output

$$V_O(f) = \frac{V_c}{4}[X(f)[H(f - f_c) + H(f + f_c)]$$



Time Domain representation of VSB

- $s(t)= V_c/2$ [$x(t) \cos(2*pi*f_c*t)-x_Q (t) \sin(2*pi* f_c*t)$]

    it represents the VSB wave with full USB and a vestige of LSB.

The time domain description for the VSB modulated wave with full LSB and vestige of USB is as follows:

- $s(t)= V_c/2$ [$x(t) \cos(2*pi*f_c*t)+x_Q (t) \sin(2*pi* f_c*t)$]

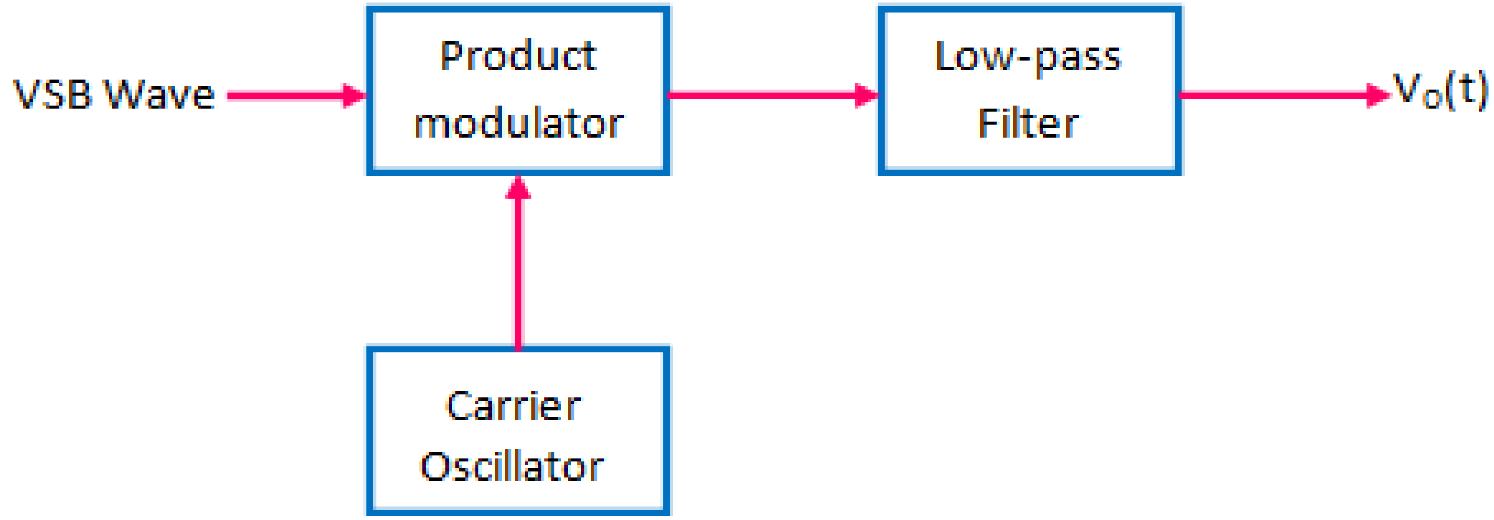
Envelope Detection of VSB

- The type of detection used for a VSB wave is envelope detection which we have discussed for the AM detection. VSB modulation is used in the commercial TV Broadcasting in which along with VSB transmission a carrier signal of substantial size is transmitted.
- s(t)= Vc/2 [x(t) cos(2*pi*fc*t)-xQ (t) sin(2*pi* fc*t)]+ Vc cos(2*pi*fc*t)

  or
- s(t)= Vc/2 [1+ m/2 x(t)] cos(2*pi*fc*t)- m/2 Vc xQ (t) sin(2*pi* fc*t)

  where m represents the percentage modulation.

Demodulation of VSB



- When we pass this VSB signal through an envelope detector, then its output is given by,

$$a(t) = V_c\{[1+1/2\ mx(t)]^2 + [1/2\ m.\ x_Q\ (t)\ ]^2\}^{1/2}$$

$$a(t) = V_c\{[1+1/2\ mx(t)]^2 + [1/2\ m.\ x_Q\ (t)/(1+(1/2)mx(t))\ ]^2\}^{1/2}$$

distortion in the detected signal can be reduced by taking the following correcting measures:

By reducing the percentage modulation to reduce m

By increasing the width of vestigal sideband to reduce the quadrature component x_Q (t)

UNIT IV
ANGLE MODULATION

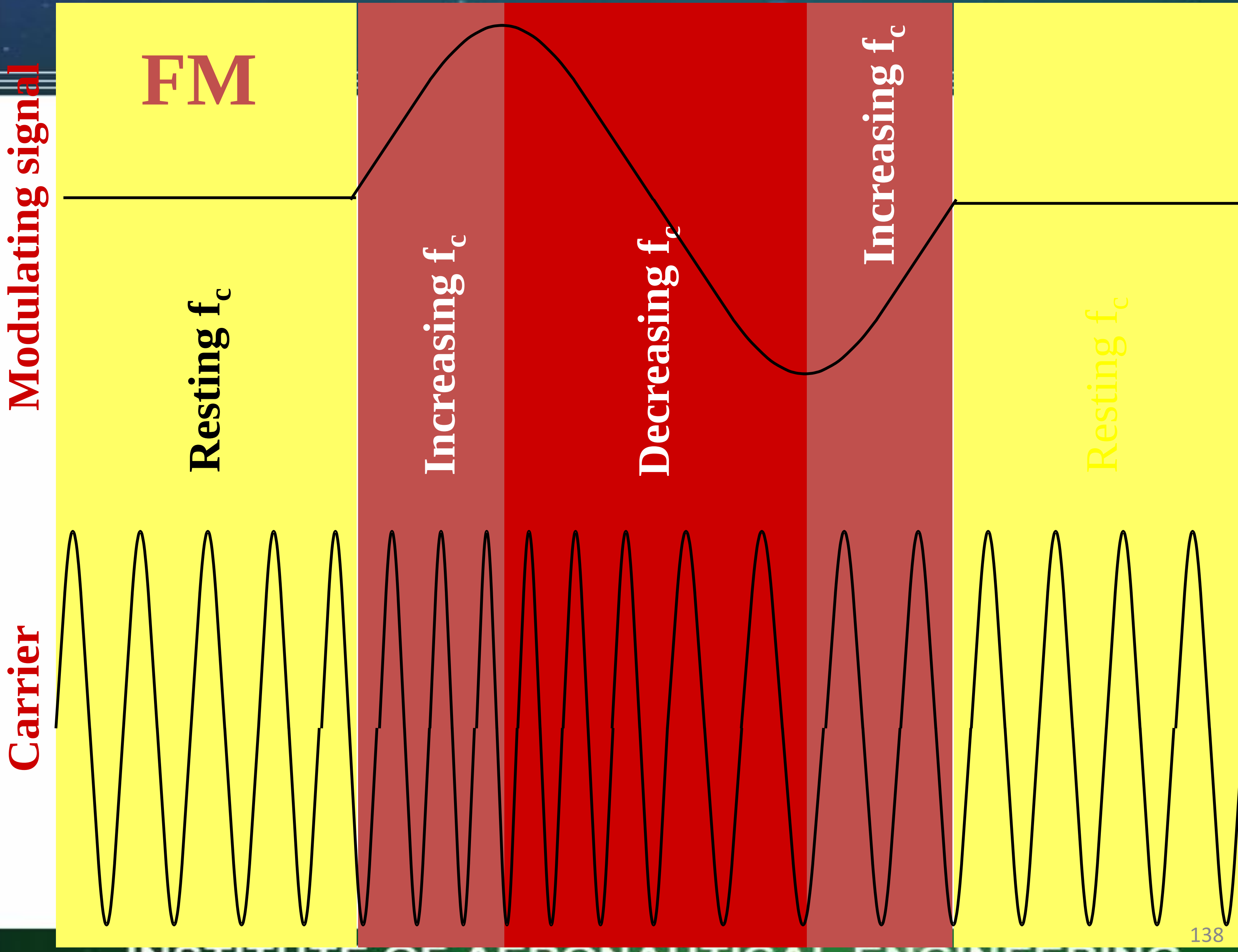
ANGLEMODULATION

- Angle modulation is the process by which the angle (frequency or phase) of the carrier signal is changed in accordance with the instantaneous amplitude of modulating or message signal.

- classified into two types such as
  - ***Frequency modulation (FM)***
  - ***Phase modulation (PM)***
- Used for :
  - Commercial radio broadcasting
  - Television sound transmission
  - Two way mobile radio
  - Cellular radio
  - Microwave and satellite communication system

FM MODULATION

- In FM the carrier amplitude remains constant, the carrier frequency varies with the amplitude of modulating signal.
- The amount of change in carrier frequency produced by the modulating signal is known as ***frequency deviation.***


FM
Modulating signal
Carrier
Resting f c
Increasing f c
Decreasing f c
Increasing f c
Resting f c

PHASE MODULATION(PM)

- The process by which changing the phase of carrier signal in accordance with the instantaneous of message signal. The amplitude remains constant after the modulation process.
- Mathematical analysis:

Let message signal:

$$v_m(t) = V_m \cos \varpi_m t$$

And carrier signal:

$$v_c(t) = V_c \cos[\varpi_c t + \theta]$$

- Where   = phase angle of carrier signal. It is changed in accordance with the amplitude of the message signal;

i.e. $$\theta = KV_m(t) = KV_m \cos \omega_m t$$

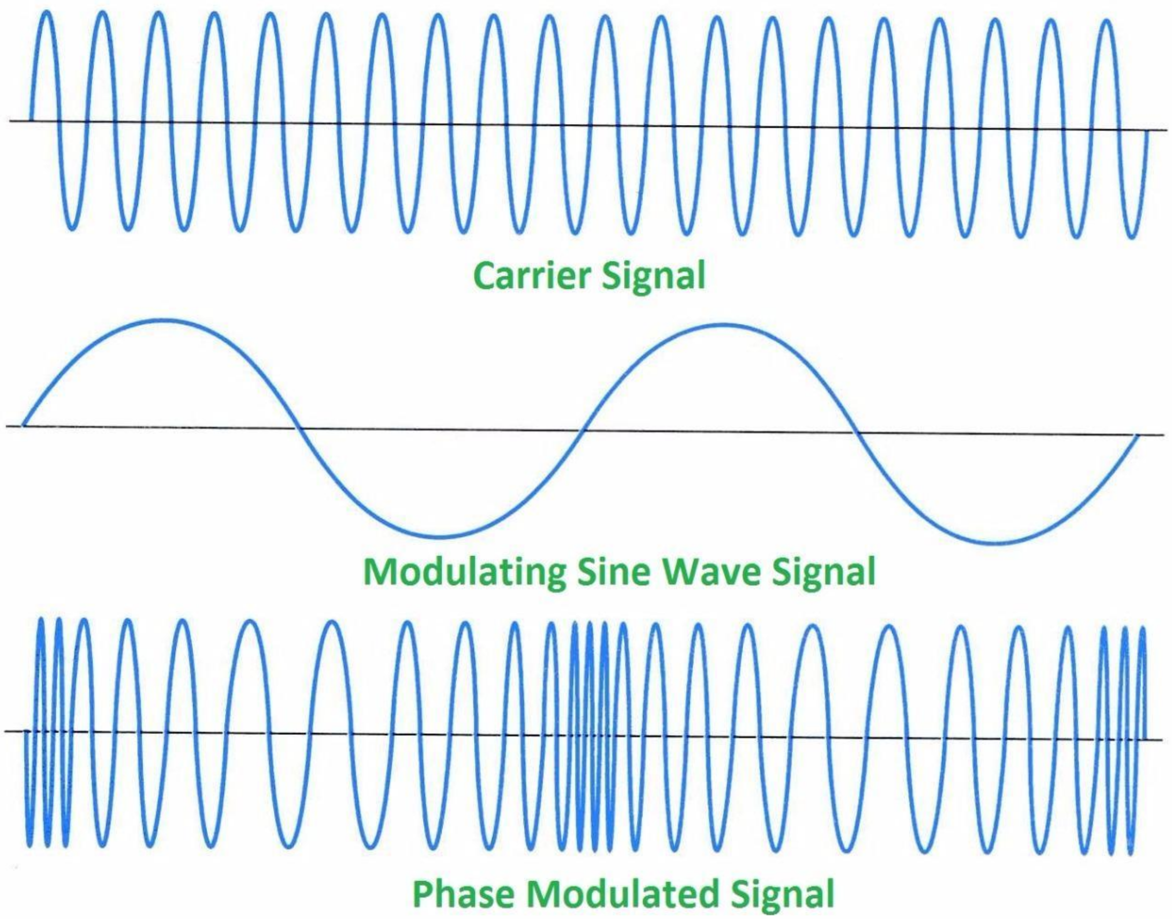
- After phase modulation the instantaneous voltage will be or

$$v_{pm}(t) = V_C \cos(\omega_C t + KV_m \cos \omega_m t)$$

$$v_{pm}(t) = V_C \cos(\omega_C t + m_p \cos \omega_m t)$$

- Where m_p = Modulation index of phase modulation
- K is a constant and called deviation sensitivities of the phase

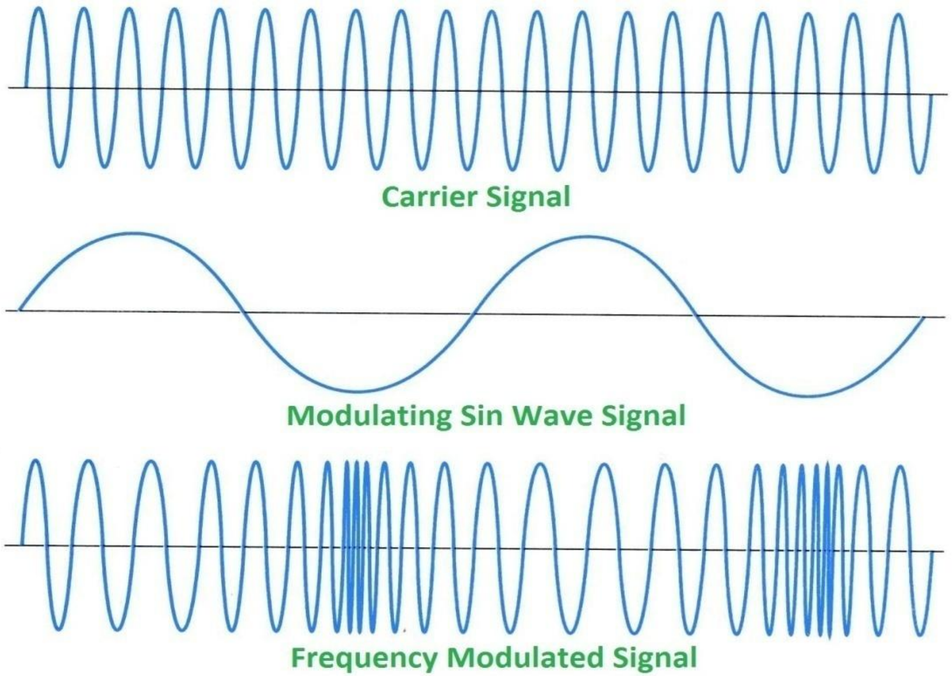
PM waveforms



FREQUENCY MODULATION (FM)

- A process where the frequency of the carrier wave varies with the magnitude variations of the modulating or audio signal.
- The amplitude of the carrier wave is kept constant.

FM waveforms



- Mathematical analysis:
- Let message signal:
- And carrier signal: $v_m(t) = V_m \cos \varpi_m t$

$$v_c(t) = V_c \cos[\varpi_c t + \theta]$$

- During the process of frequency modulations the frequency of carrier signal is changed in accordance with the instantaneous amplitude of message signal .Therefore the frequency of carrier after modulation is written as

$$\omega_i = \omega_c + K_1 v_m(t) = \omega_C + K_1 V_m \cos \omega_m t$$

- To find the instantaneous phase angle of modulated signal, integrate equation above w.r.t. ***t***

$$\phi_i = \int \omega_i dt = \int (\omega_C + K_1 V_m \cos \omega_m t) dt = \omega_C t + \frac{K_1 V_m}{\omega_m} \sin \omega_m t$$

- Thus, we get the FM wave as:

$$v_{FM}(t) = Vc \cos\phi_1 = V_C \cos(\omega_C t + \frac{K_1 V_m}{\omega_m} \sin\omega_m t)$$

$$v_{FM}(t) = V_C \cos(\omega_C t + m_f \sin\omega_m t)$$

- Where modulation index for FM is given by

$$m_f = \frac{K_1 V_m}{\omega_m}$$

- **Frequency deviation:** Δf is the relative placement of carrier frequency (Hz) w.r.t its unmodulated value. Given as:

$$\omega_{max} = \omega_C + K_1 V_m$$

$$\omega_{min} = \omega_C - K_1 V_m$$

$$\omega_d = \omega_{max} - \omega_C = \omega_C - \omega_{min} = K_1 V_m$$

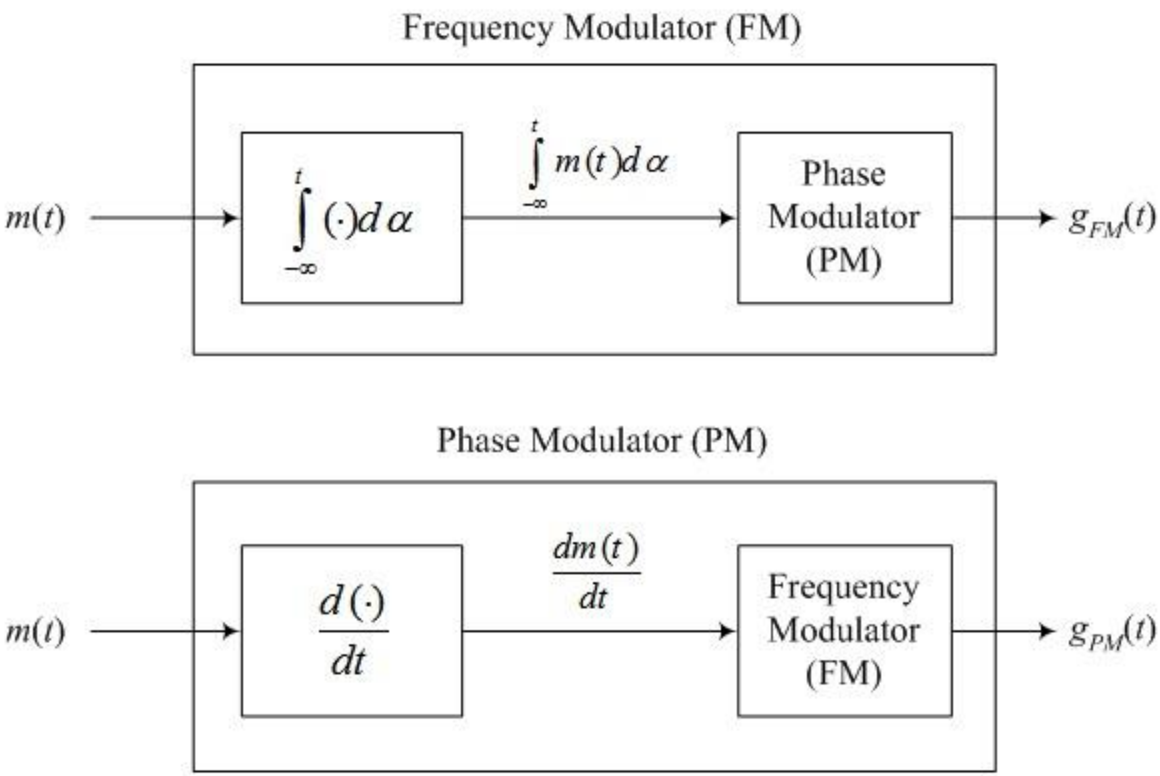
$$\Delta f = \frac{\omega_d}{2\pi} = \frac{K_1 V_m}{2\pi}$$

- Therefore:

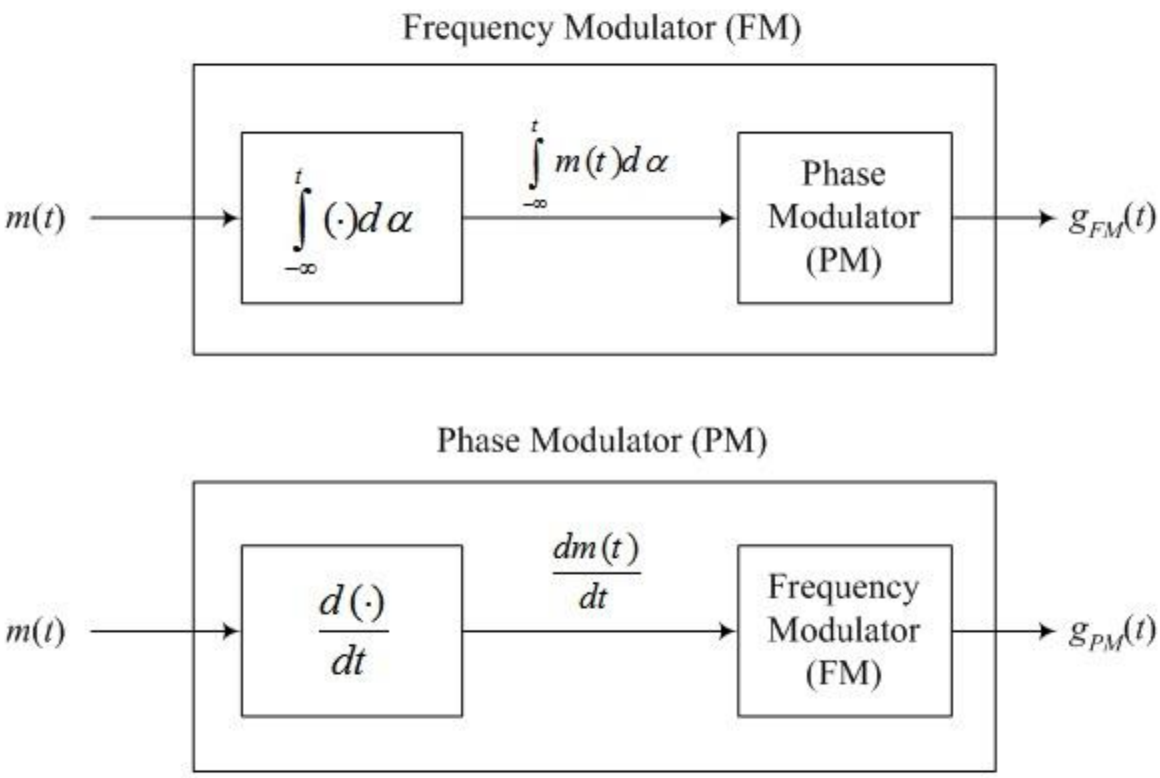
$$\Delta f = \frac{K_1 V_m}{2\pi};$$

$$m_f = \frac{\Delta f}{f_m}$$

Relation between FM and PM



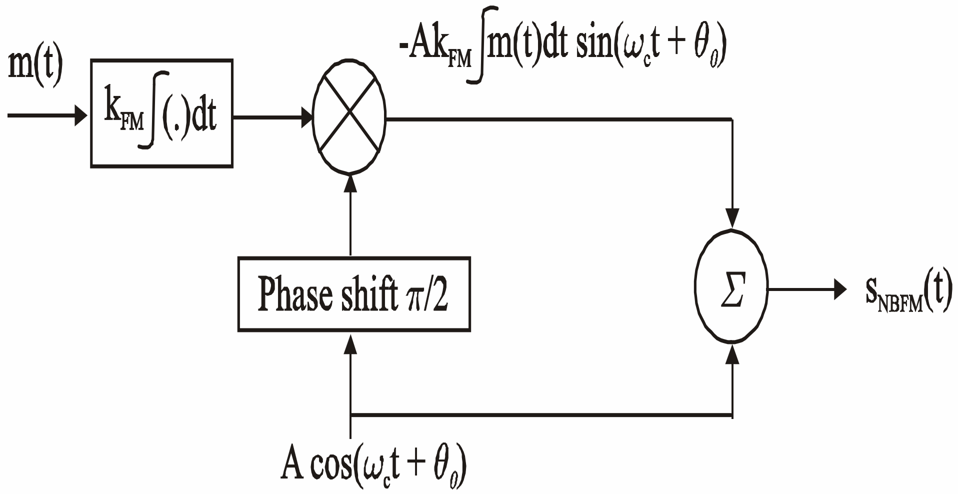
Relation between FM and PM



Equations for Phase- and Frequency-Modulated Carriers

Type of Modulation	Modulating Signal	Angle-Modulated Wave, $m(t)$
(a) Phase	$v_m(t)$	$V_c \cos[\omega_c t + K v_m(t)]$
(b) Frequency	$v_m(t)$	$V_c \cos[\omega_c t + K_1 \int v_m(t)\, dt]$
(c) Phase	$V_m \cos(\omega_m t)$	$V_c \cos[\omega_c t + K V_m \cos(\omega_m t)]$
(d) Frequency	$V_m \cos(\omega_m t)$	$V_c \cos\left[\omega_c t + \frac{K_1 V_m}{\omega_m} \sin(\omega_m t)\right]$

NARROW BAND FM



FM&PM (Bessel function)

- Thus, for general equation:

$$v_{FM}(t) = V_C \cos(\omega_C t + m_f \cos \omega_m t)$$

$$\cos(\alpha + m\cos\beta) = \sum_{n=-\infty}^{\infty} J_n(m)\cos\left(\alpha + n\beta + \frac{n\pi}{2}\right)$$

$$m(t) = V_C \sum_{n=-\infty}^{\infty} J_n(m)\cos\left(\omega_c t + n\omega_m t + \frac{n\pi}{2}\right)$$

Bessel Function

$$v(t)_{FM} = V_C\{J_0(m_f)\cos\omega_C t + J_1(m_f)\cos\left[(\omega_C + \omega_m)t + \frac{\pi}{2}\right] - J_1(m_f)\cos\left[(\omega_C - \omega_m)t - \frac{\pi}{2}\right]$$

$$+ J_2(m_f)\cos\left[(\omega_C + 2\omega_m)t\right] + J_2(m_f)\cos\left[(\omega_C - 2\omega_m)t\right] + \ldots J_n(m_f)\ldots\}$$

B.F. (cont'd)

- It is seen that each pair of side band is preceded by **J** coefficients. The order of the coefficient is denoted by subscript m. The Bessel function can be written as

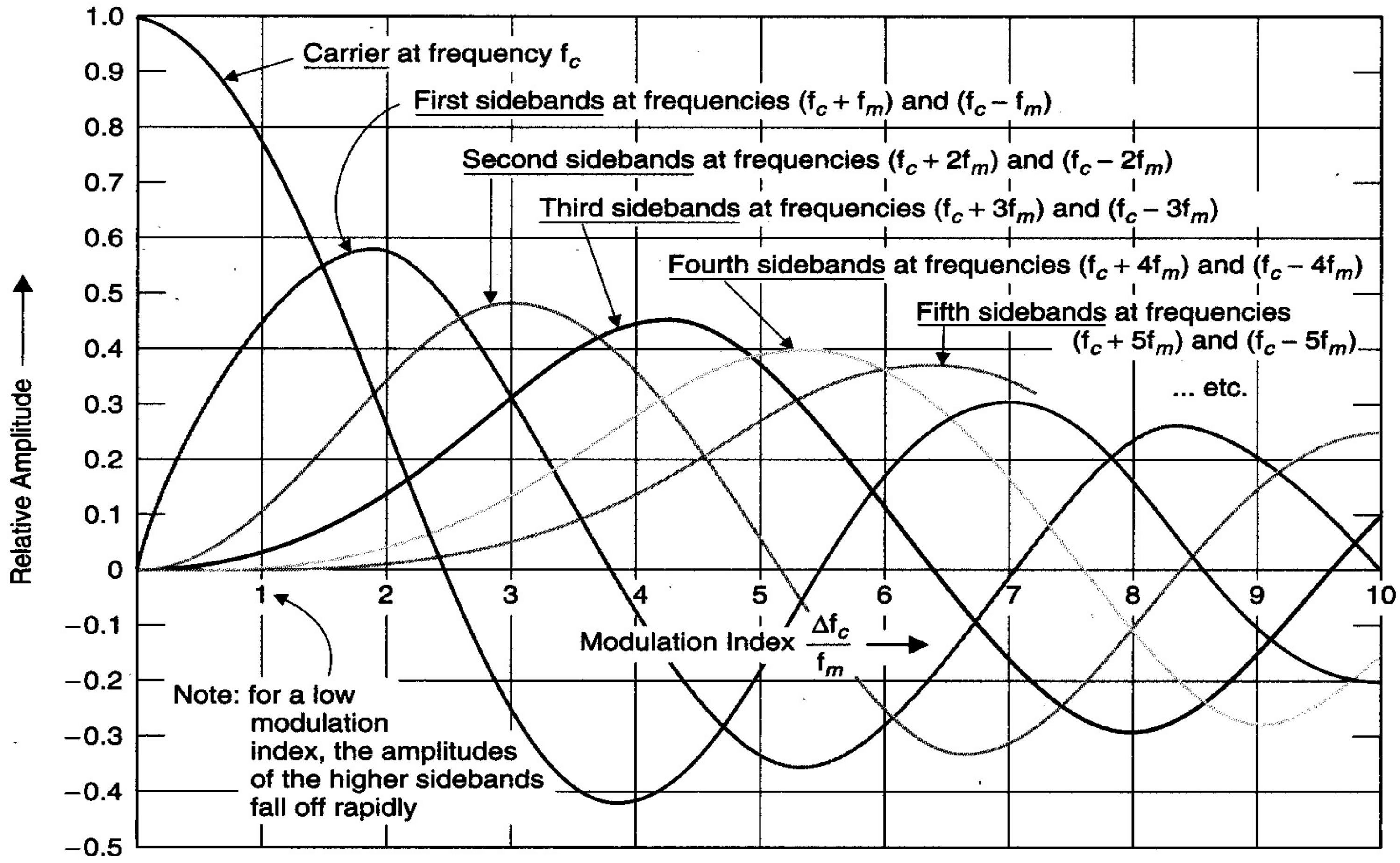
$$J_n(m_f) = \left(\frac{m_f}{2}\right)^n \left[\frac{1}{n} - \frac{(m_f/2)^2}{1!(n+1)!} + \frac{(m_f/2)^4}{2!(n+2)!} - ....\right]$$

- N = number of the side frequency
- M_f = modulation index

Bessel Functions of the First Kind, $J_n(m)$ for some value of modulation index

Modulation Index	Carrier	Side Frequency Pairs													
m	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—
2.4	0	0.52	0.43	0.20	0.06	0.02	—	—	—	—	—	—	—	—	—
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	0.01	—	—	—	—	—	—	—	—
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—
5.45	0	−0.34	−0.12	0.26	0.40	0.32	0.19	0.09	0.03	0.01	—	—	—	—	—
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	—	—	—
8.65	0	0.27	0.06	−0.24	−0.23	0.03	0.26	0.34	0.28	0.18	0.10	0.05	0.02	—	—
9.0	−0.09	0.25	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.31	0.21	0.12	0.06	0.03	0.01	—
10.0	−0.25	0.05	0.25	0.06	−0.22	−0.23	−0.01	0.22	0.32	0.29	0.21	0.12	0.06	0.03	0.01

B.F. (cont'd)


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
Relative Amplitude
Carrier at frequency f_c
First sidebands at frequencies $(f_c + f_m)$ and $(f_c - f_m)$
Second sidebands at frequencies $(f_c + 2f_m)$ and $(f_c - 2f_m)$
Third sidebands at frequencies $(f_c + 3f_m)$ and $(f_c - 3f_m)$
Fourth sidebands at frequencies $(f_c + 4f_m)$ and $(f_c - 4f_m)$
Fifth sidebands at frequencies $(f_c + 5f_m)$ and $(f_c - 5f_m)$
... etc.
1
2
3
4
5
6
7
8
9
10
Modulation Index $\frac{\Delta f_c}{f_m}$
Note: for a low modulation index, the amplitudes of the higher sidebands fall off rapidly

Representation of frequency spectrum

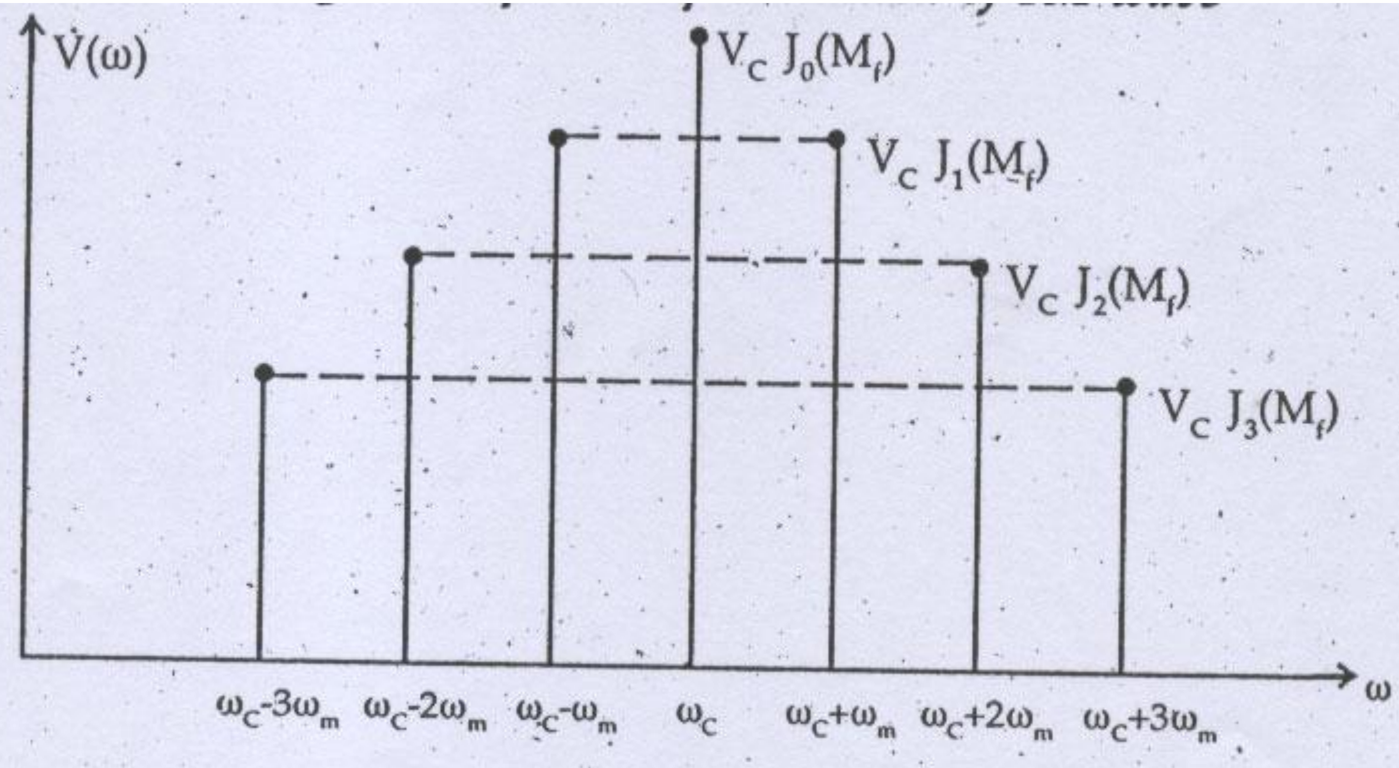


Fig. 2. Phasor representation of FM

FM Bandwidth

- Theoretically, the generation and transmission of FM requires infinite bandwidth. Practically, FM system have finite bandwidth and they perform well.
- The value of modulation index determine the number of sidebands that have the significant relative amplitudes
- If n is the number of sideband pairs, and line of frequency spectrum are spaced by fm, thus, the bandwidth is:

$$B_{fm} = 2nf_m$$

- For n≥1

FM Bandwidth (cont'd)

- Estimation of transmission b/w;
- Assume m_f is large and n is approximate $m_f + 2$; thus
- $B_{fm}=2(m_f + 2)f_m$

$$= 2\left(\frac{\Delta f}{f_m} + 2\right) f_m$$

$$B_{fm} = 2(\Delta f + f_m) \ldots\ldots\ldots (1)$$

(1) is called Carson's rule

Deviation Ratio (DR)

- The worse case modulation index which produces the widest output frequency spectrum.

$$DR = \frac{\Delta f_{(\max)}}{f_{m\,(\max)}}$$

- Where
  - $\Delta f_{(max)}$ = max. peak frequency deviation
  - $f_{m(max)}$ = max. modulating signal frequency

FM Power Distribution

- As seen in Bessel function table, it shows that as the sideband relative amplitude increases, the carrier amplitude,J_0 decreases.
- This is because, in FM, the total transmitted power is always constant and the total average power is equal to the unmodulated carrier power, that is the amplitude of the FM remains constant whether or not it is modulated.

FM Power Distribution (cont’d)

- In effect, in FM, the total power that is originally in the carrier is redistributed between all components of the spectrum, in an amount determined by the modulation index, m_f, and the corresponding Bessel functions.
- At certain value of modulation index, the carrier component goes to zero, where in this condition, the power is carried by the sidebands only.

Average Power

- The average power in unmodulated carrier

- The total instantaneous power in the angle modulated carrier.

$$P_t = \frac{m(t)^2}{R} = \frac{V_c^2}{R}\cos^2[\omega_c t + \theta(t)]$$

$$P_t = \frac{V_c^2}{R}\left\{\frac{1}{2} + \frac{1}{2}\cos[2\omega_c t + 2\theta(t)]\right\} = \frac{V_c^2}{2R}$$

- The total modulated power

$$P_t = P_0 + P_1 + P_2 + .. + P_n = \frac{V_c^2}{2R} + \frac{2(V_1)^2}{2R} + \frac{2(V_2)^2}{2R} + .. + \frac{2(V_n)^2}{2R}$$

Generation of FM

- Two major FM generation:

i) Direct method:

  i) straight forward, requires a VCO whose oscillation frequency has linear dependence on applied voltage.
  ii) Advantage: large frequency deviation
  iii) Disadvantage: the carrier frequency tends to drift and must be stabilized.

    i) Varactor diode

Indirect method:

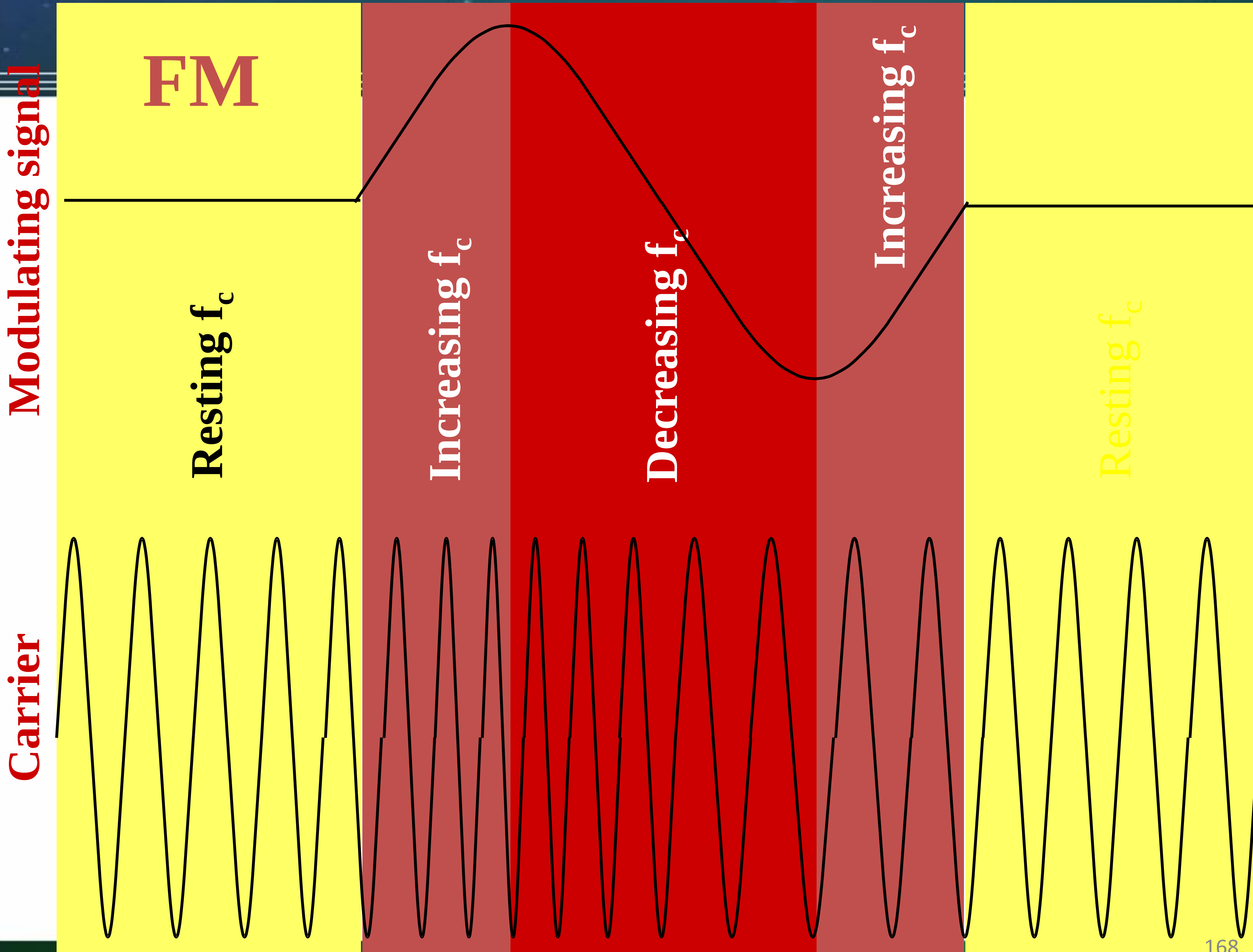
One most popular indirect method is the Armstrong modulator

Comparision of AM Techniques

Parameter of comparision	DSB-FC	DSB-SC	SSB	VSB
Carrier supression	N.A	Fully	Fully	N.A
Sideband Supression	N.A	N.A	One side band completely	One side band partially
Bandwidth	$2*f_m$	$2*f_m$	f_m	$f_m<B.W<2*f_m$
Transmission Efficiency	Minimum	Moderate	Maximum	Moderate
No of Modulating inputs	1	1	1	2
Applications	Radio Broadcasting	Radio Broadcasting	Point to point mobile Communications	T.V.

FM MODULATION

- In FM the carrier amplitude remains constant, the carrier frequency varies with the amplitude of modulating signal.
- The amount of change in carrier frequency produced by the modulating signal is known as ***frequency deviation.***


FM
Modulating signal
Carrier
Resting f_c
Increasing f_c
Decreasing f_c
Increasing f_c
Resting f_c

PHASE MODULATION(PM)

- The process by which changing the phase of carrier signal in accordance with the instantaneous of message signal. The amplitude remains constant after the modulation process.
- Mathematical analysis:

Let message signal:

$$v_m(t) = V_m \cos \varpi_m t$$

And carrier signal:

$$v_c(t) = V_c \cos[\varpi_c t + \theta]$$

- Where = phase angle of carrier signal. It is changed in accordance with the amplitude of the message signal;

i.e.

- After phase modulation the instantaneous voltage will be or

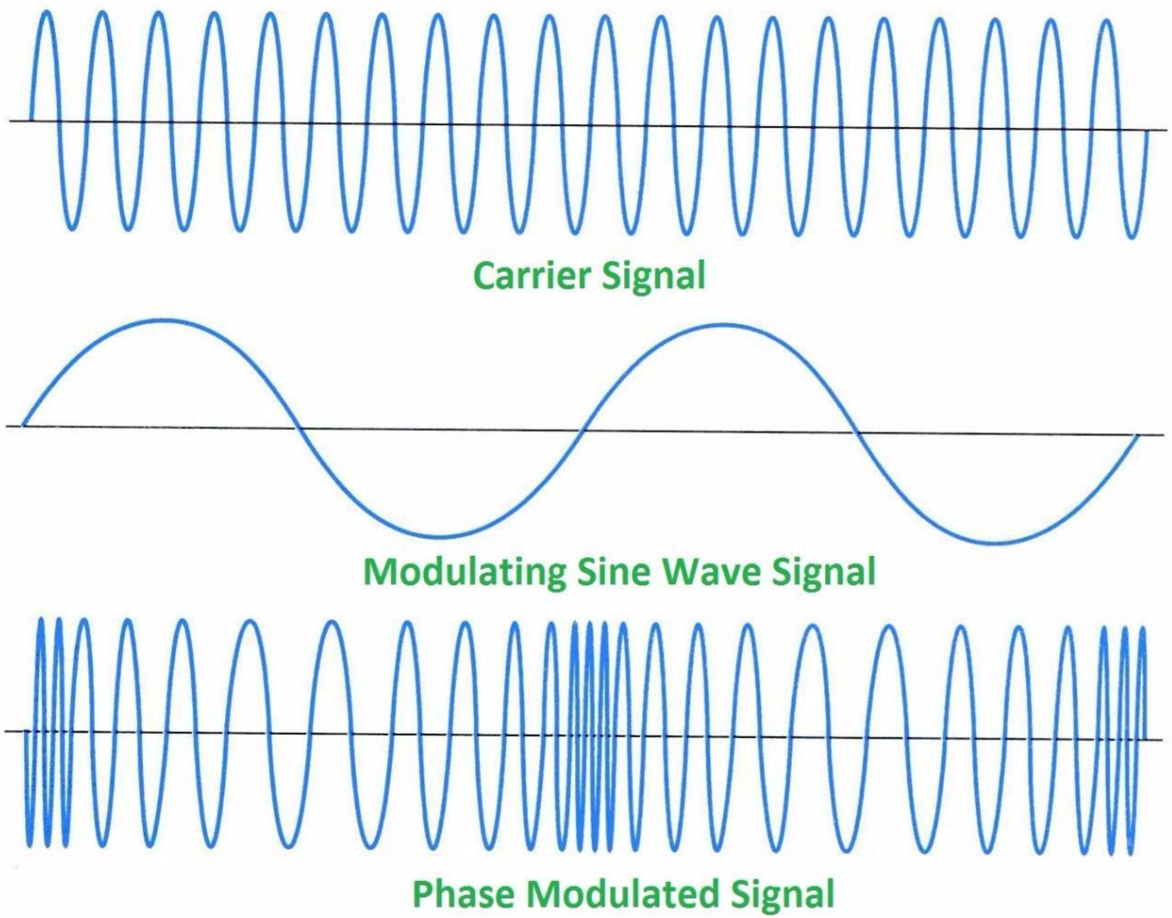
$$\theta = KV_m(t) = KV_m \cos \omega_m t$$

$$v_{pm}(t) = V_C \cos(\omega_C t + KV_m \cos \omega_m t)$$

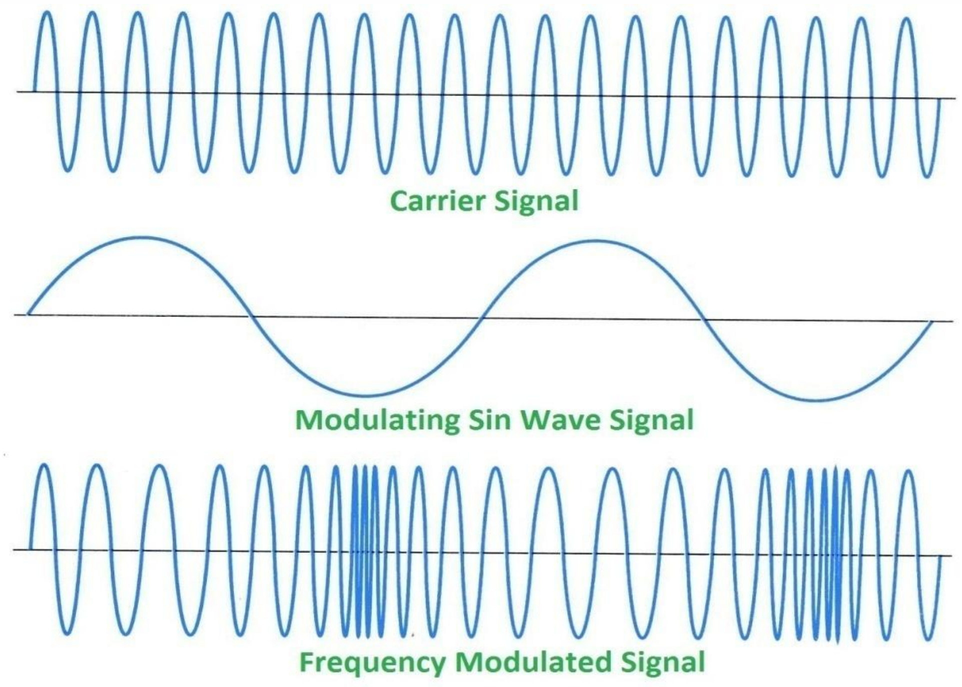
$$v_{pm}(t) = V_C \cos(\omega_C t + m_p \cos \omega_m t)$$

- Where m_p = Modulation index of phase modulation
- K is a constant and called deviation sensitivities of the phase

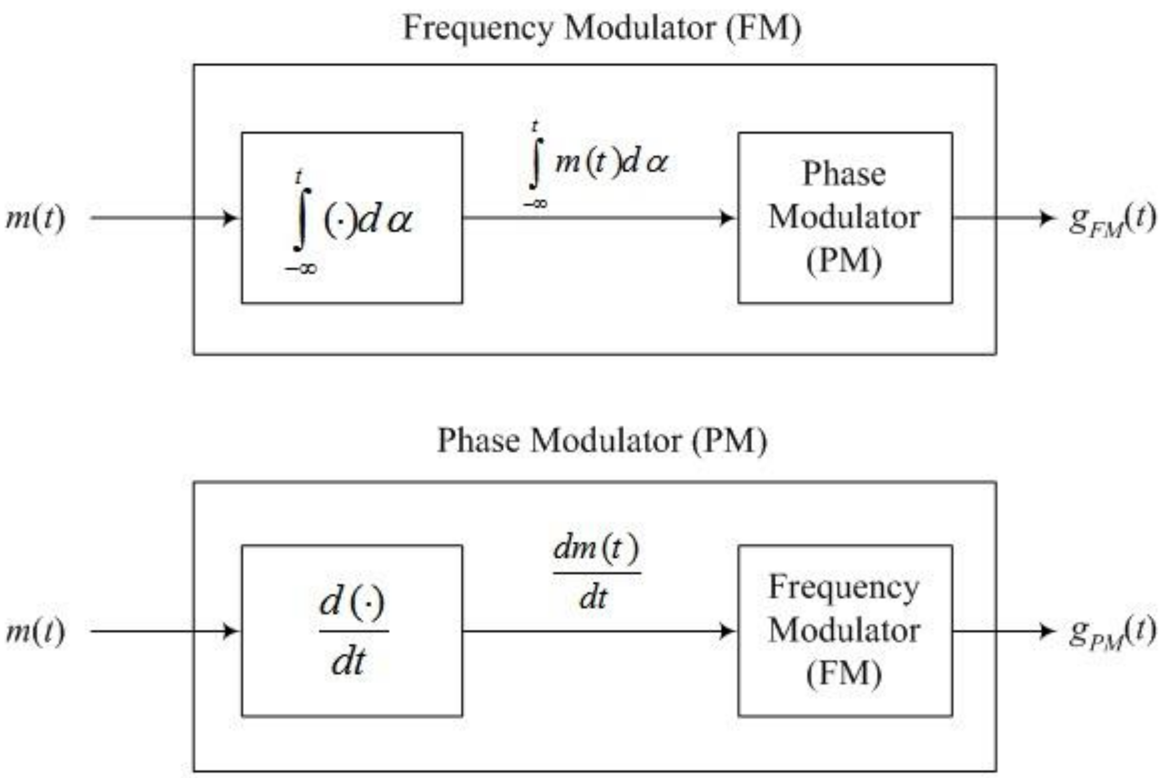
PM waveforms



FM waveforms



Relation between FM and PM



- **Frequency deviation:** Δf is the relative placement of carrier frequency (Hz) w.r.t its unmodulated value. Given as:

$$\omega_{max} = \omega_C + K_1 V_m$$

$$\omega_{min} = \omega_C - K_1 V_m$$

$$\omega_d = \omega_{max} - \omega_C = \omega_C - \omega_{min} = K_1 V_m$$

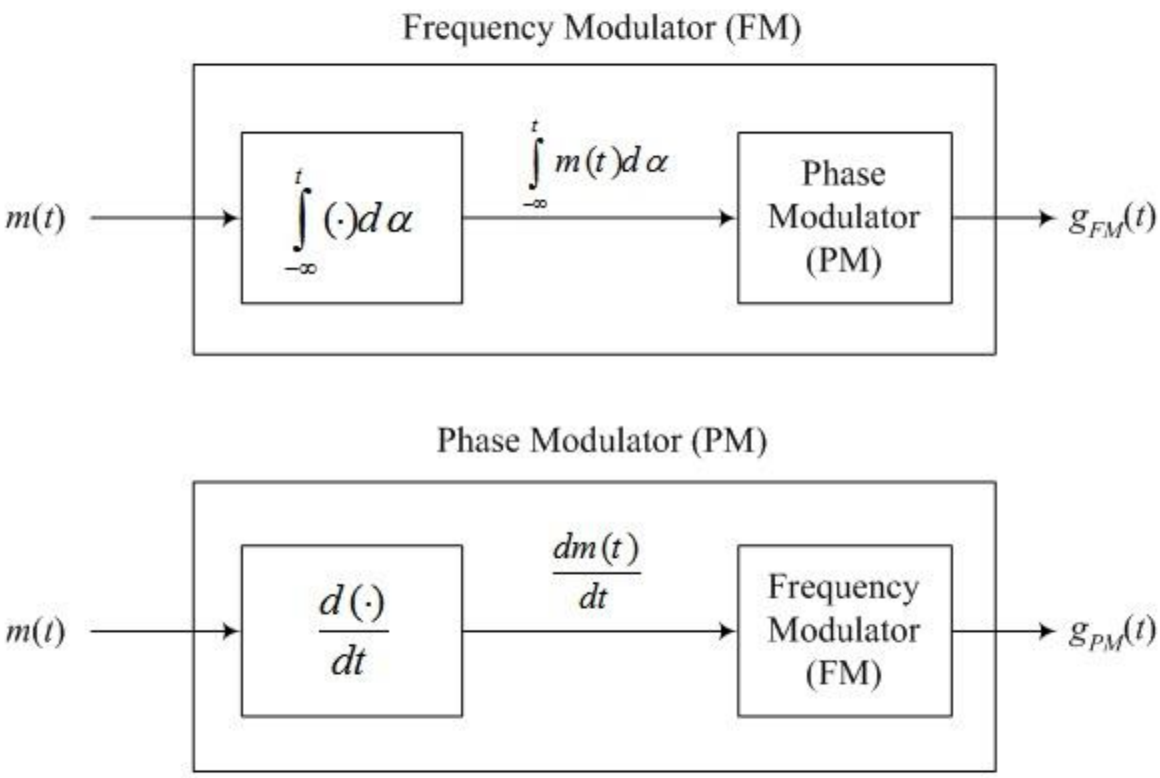
$$\Delta f = \frac{\omega_d}{2\pi} = \frac{K_1 V_m}{2\pi}$$

- Therefore:

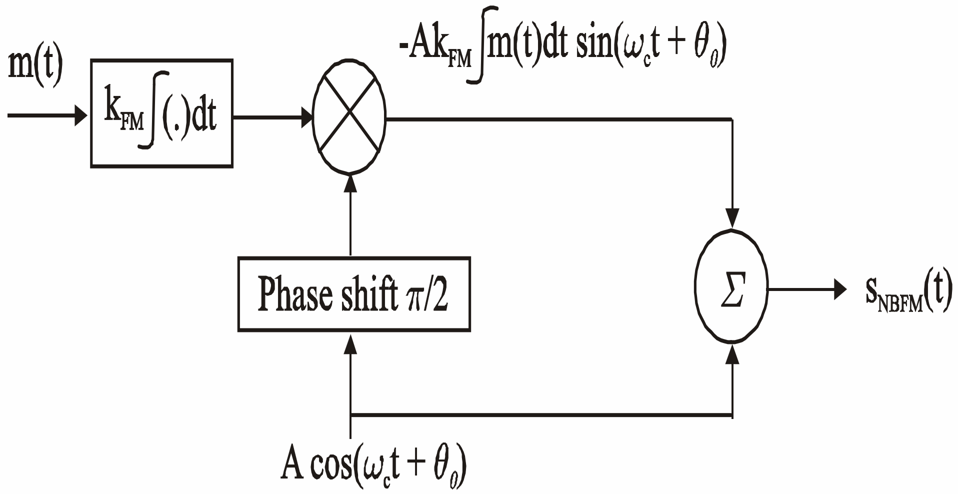
$$\Delta f = \frac{K_1 V_m}{2\pi};$$

$$m_f = \frac{\Delta f}{f_m}$$

Relation between FM and PM



NARROW BAND FM



FM&PM (Bessel function)

- Thus, for general equation:

$$v_{FM}(t) = V_C \cos(\omega_C t + m_f \cos \omega_m t)$$

$$\cos(\alpha + m \cos \beta) = \sum_{n=-\infty}^{\infty} J_n(m) \cos\left(\alpha + n\beta + \frac{n\pi}{2}\right)$$

$$m(t) = V_C \sum_{n=-\infty}^{\infty} J_n(m) \cos\left(\omega_c t + n\omega_m t + \frac{n\pi}{2}\right)$$

Bessel Function

$$v(t)_{FM} = V_C\{J_0(m_f)\cos\omega_C t + J_1(m_f)\cos\left[(\omega_C+\omega_m)t+\frac{\pi}{2}\right] - J_1(m_f)\cos\left[(\omega_C-\omega_m)t-\frac{\pi}{2}\right]$$

$$+ J_2(m_f)\cos\left[(\omega_C+2\omega_m)t\right] + J_2(m_f)\cos\left[(\omega_C-2\omega_m)t\right] + \ldots J_n(m_f)\ldots\}$$

B.F. (cont'd)

- It is seen that each pair of side band is preceded by **J** coefficients. The order of the coefficient is denoted by subscript m. The Bessel function can be written as

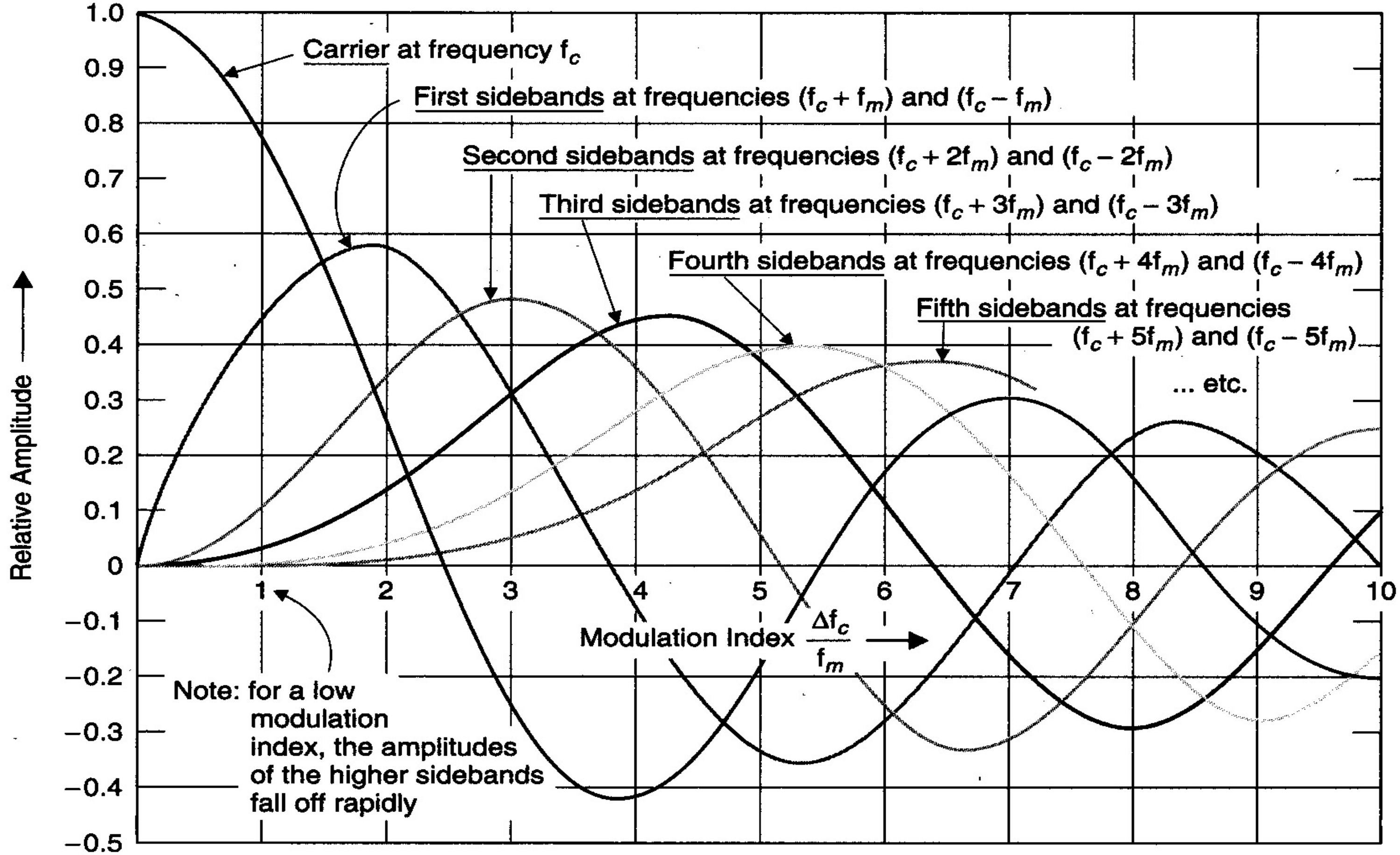
$$J_n(m_f) = \left(\frac{m_f}{2}\right)^n \left[\frac{1}{n} - \frac{(m_f/2)^2}{1!(n+1)!} + \frac{(m_f/2)^4}{2!(n+2)!} - \ldots\right]$$

- N = number of the side frequency
- M_f = modulation index

Bessel Functions of the First Kind, $J_n(m)$ for some value of modulation index

Modulation Index	Carrier							Side Frequency Pairs							
m	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—
2.4	0	0.52	0.43	0.20	0.06	0.02	—	—	—	—	—	—	—	—	—
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	0.01	—	—	—	—	—	—	—	—
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—
5.45	0	−0.34	−0.12	0.26	0.40	0.32	0.19	0.09	0.03	0.01	—	—	—	—	—
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	—	—	—
8.65	0	0.27	0.06	−0.24	−0.23	0.03	0.26	0.34	0.28	0.18	0.10	0.05	0.02	—	—
9.0	−0.09	0.25	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.31	0.21	0.12	0.06	0.03	0.01	—
10.0	−0.25	0.05	0.25	0.06	−0.22	−0.23	−0.01	0.22	0.32	0.29	0.21	0.12	0.06	0.03	0.01

B.F. (cont'd)



Carrier at frequency f_c
First sidebands at frequencies $(f_c + f_m)$ and $(f_c - f_m)$
Second sidebands at frequencies $(f_c + 2f_m)$ and $(f_c - 2f_m)$
Third sidebands at frequencies $(f_c + 3f_m)$ and $(f_c - 3f_m)$
Fourth sidebands at frequencies $(f_c + 4f_m)$ and $(f_c - 4f_m)$
Fifth sidebands at frequencies $(f_c + 5f_m)$ and $(f_c - 5f_m)$
... etc.
Relative Amplitude
Modulation Index $\frac{\Delta f_c}{f_m}$
Note: for a low modulation index, the amplitudes of the higher sidebands fall off rapidly
1.0
0.9
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Representation of frequency spectrum

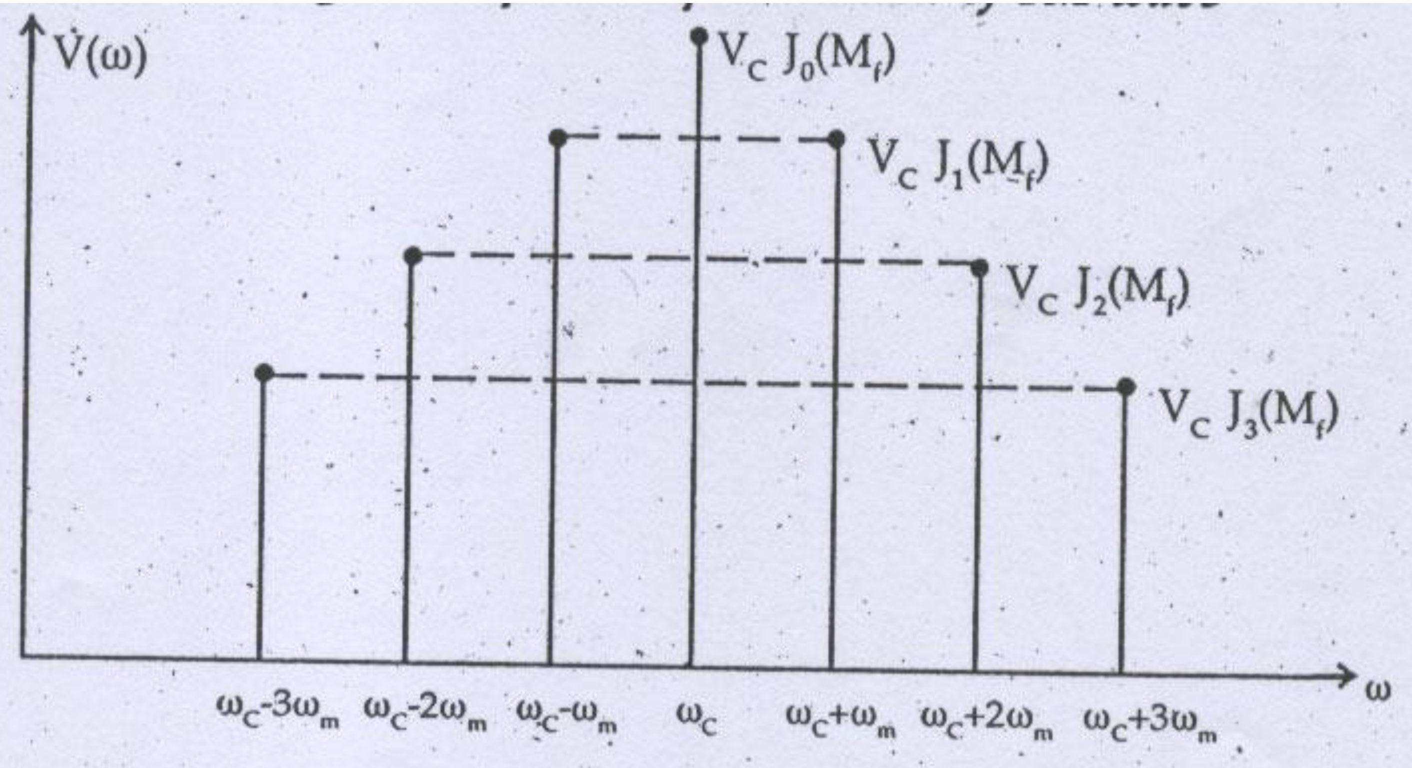

Fig. 2. Phasor representation of FM

FM Bandwidth

- Theoretically, the generation and transmission of FM requires infinite bandwidth. Practically, FM system have finite bandwidth and they perform well.
- The value of modulation index determine the number of sidebands that have the significant relative amplitudes
- If n is the number of sideband pairs, and line of frequency spectrum are spaced by fm, thus, the bandwidth is:

$$B_{fm} = 2nf_m$$

- For n≥1

FM Bandwidth (cont'd)

- Estimation of transmission b/w;
- Assume m_f is large and n is approximate $m_f + 2$; thus
- $B_{fm}=2(m_f + 2)f_m$

$$= 2\left(\frac{\Delta f}{f_m} + 2\right) f_m$$

$$B_{fm} = 2(\Delta f + f_m) \ldots\ldots\ldots (1)$$

(1) is called Carson's rule

Single tone FM

- **Frequency modulation (FM)** is that of angle modulation in which the instantaneous frequency fi(t) is varied linearly with the message signal m(t), as shown by

- fi(t) =fc+kfm(t)

  Where fc represents the frequency of the unmodulated carrier

- kf represents the frequency sensitivity of the modulator(Hz/volt) The frequency modulated wave
- s(t)=Accos[2πfct+2πkf otm(t)dt]

- FM wave can be generated by first integrating m(t) and then using the result as the input to a phase modulator

- PM wave can be generated by first differentiating m(t) and then using the result as the input to a frequency modulator. Frequency modulation is a Non-linear modulation process. Single tone FM:

  Consider m(t)=Amcos(2πfmt)

- The instantaneous frequency of the resulting FM wave
  fi(t) =fc+kf Amcos(2πfmt)

fc+ f cos(2πfmt)

wheref = kf Am is called as frequency deviation

theta(t) =2π fi(t)dt

=2πfct+ f/fm sin(2πfmt)

2πfct+β sin(2πfmt)

FM&PM (Bessel function)

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$$\cos(\alpha + m \cos \beta) = \sum_{n=-\infty}^{\infty} J_n(m) \cos\left(\alpha + n\beta + \frac{n\pi}{2}\right)$$

$$m(t) = V_C \sum_{n=-\infty}^{\infty} J_n(m) \cos\left(\omega_c t + n\omega_m t + \frac{n\pi}{2}\right)$$

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$$v(t)_{FM} = V_C\{J_0(m_f)\cos\ \omega_C t + J_1(m_f)\cos\left[(\omega_C + \omega_m)t + \frac{\pi}{2}\right] - J_1(m_f)\cos\left[(\omega_C - \omega_m)t - \frac{\pi}{2}\right]$$

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- N = number of the side frequency
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0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—
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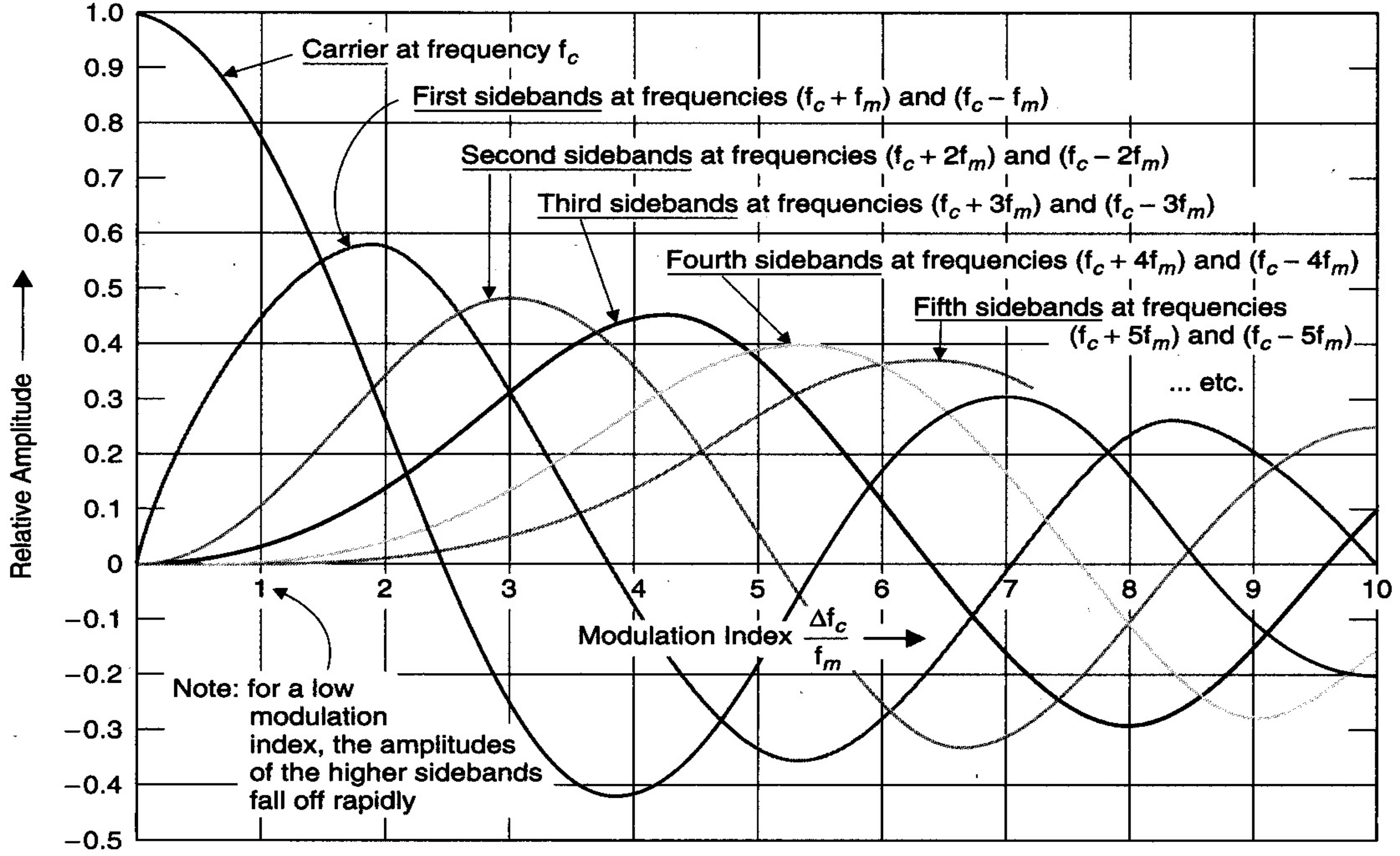

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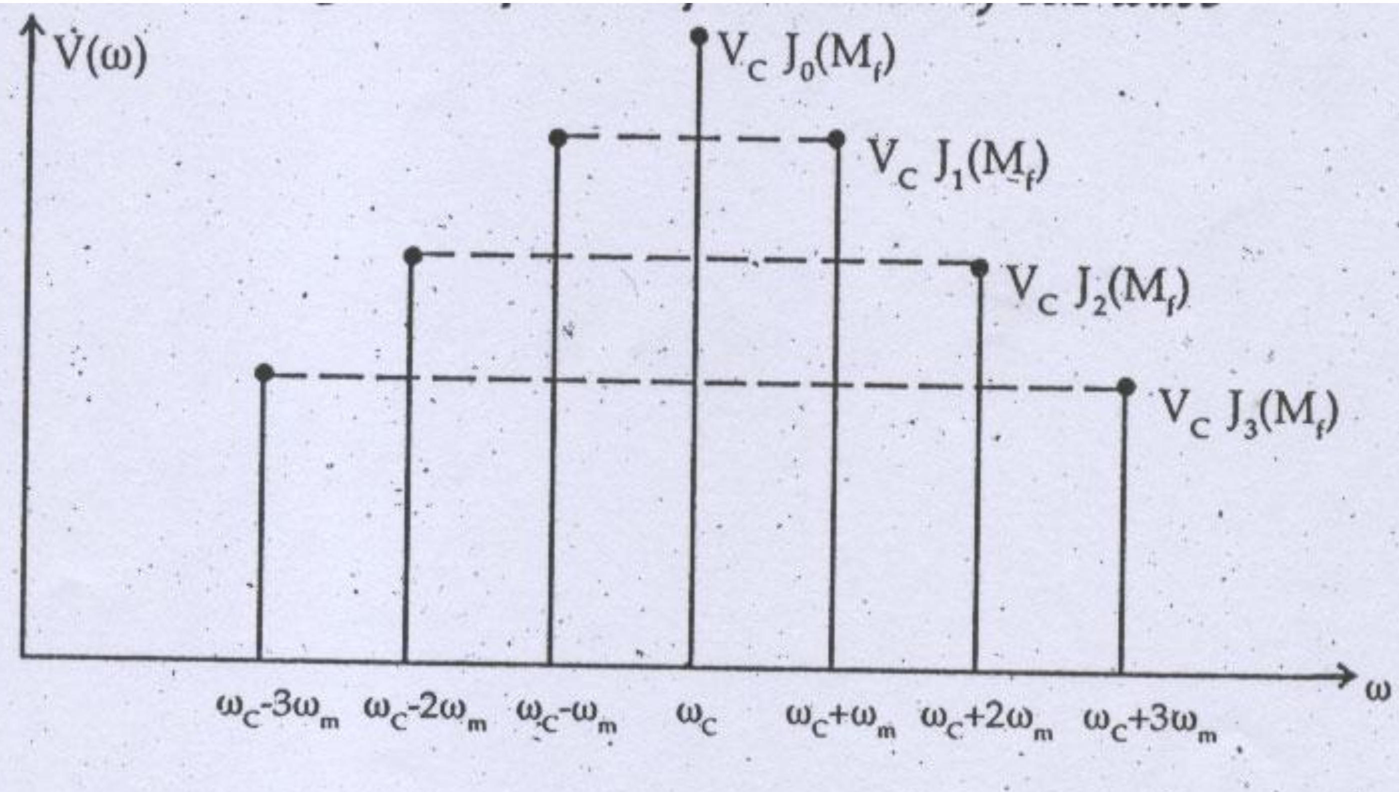
Representation of frequency spectrum



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$$= 2\left(\frac{\Delta f}{f_m} + 2\right) f_m$$

$$B_{fm} = 2(\Delta f + f_m) \text{........} (1)$$

(1) is called Carson's rule

Multi tone FM

- In FM modulation, the modulation index is not important in itself but the bandwidth of the broadcasted FM signal is.

  As you know, the FM modulation index for a sinusoidal modulating signal is defined as:
  $\beta = \Delta Fc/Fm$
  where:
  ΔFc is the carrier deviation
  Fm is the modulating frequency.

  For EACH sinusoidal modulating signal of freqency Fm, the bandwith of the FM signal could be given as:
  $BW = 2 * (\Delta Fc + Fm) = 2 * (\beta*Fm + Fm)$
  $BW = 2 * Fm * (\beta + 1)$

  For multi tone FM broadcasting, the FM bandwidth should not exceed usually a limit (as 200 KHz for the radio FM Band, if I remember well). Therefore for a constant BW, increasing Fm imposes on β to be lowered.

- A generic N+tone modulated FM signal can be represented as:

  y(t)=cos{ωc•t+K•[A1/ω1*sin(ω1•t)+A2/ω2*sin(ω2•t)+...+AN/ωN*sin(ωN•t)}

  where K is the constant in Hz/V to convert from amplitude to frequency and Ai is the amplitude of the i-th tone.

  each tone will have its own modulation index. they will be:

  index1=K•A1/ω1
  index2=K•A2/ω2
  .
  .
  indexN=K•AN/ωN

  (of course wi=2•Π•fi)

  To estimate the occupated bandwidth you can use the Carson's rule:

  first of all we need to calculate the maximum deviation frequency
  Δfmax=K•max(Ai)
  then we will need the deviation rate: DR=Δfmax/max(fi)

  BW≈2•Δfmax•(1+1/DR)

  Finally the spectrum will be expressed in terms of products of Jn bessel functions.

Bessel Function

$$v(t)_{FM} = V_C\{J_0(m_f)\cos\omega_C t + J_1(m_f)\cos\left[(\omega_C+\omega_m)t+\frac{\pi}{2}\right] - J_1(m_f)\cos\left[(\omega_C-\omega_m)t-\frac{\pi}{2}\right]$$

$$+ J_2(m_f)\cos\left[(\omega_C+2\omega_m)t\right] + J_2(m_f)\cos\left[(\omega_C-2\omega_m)t\right] + \ldots J_n(m_f)\ldots\}$$

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0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—
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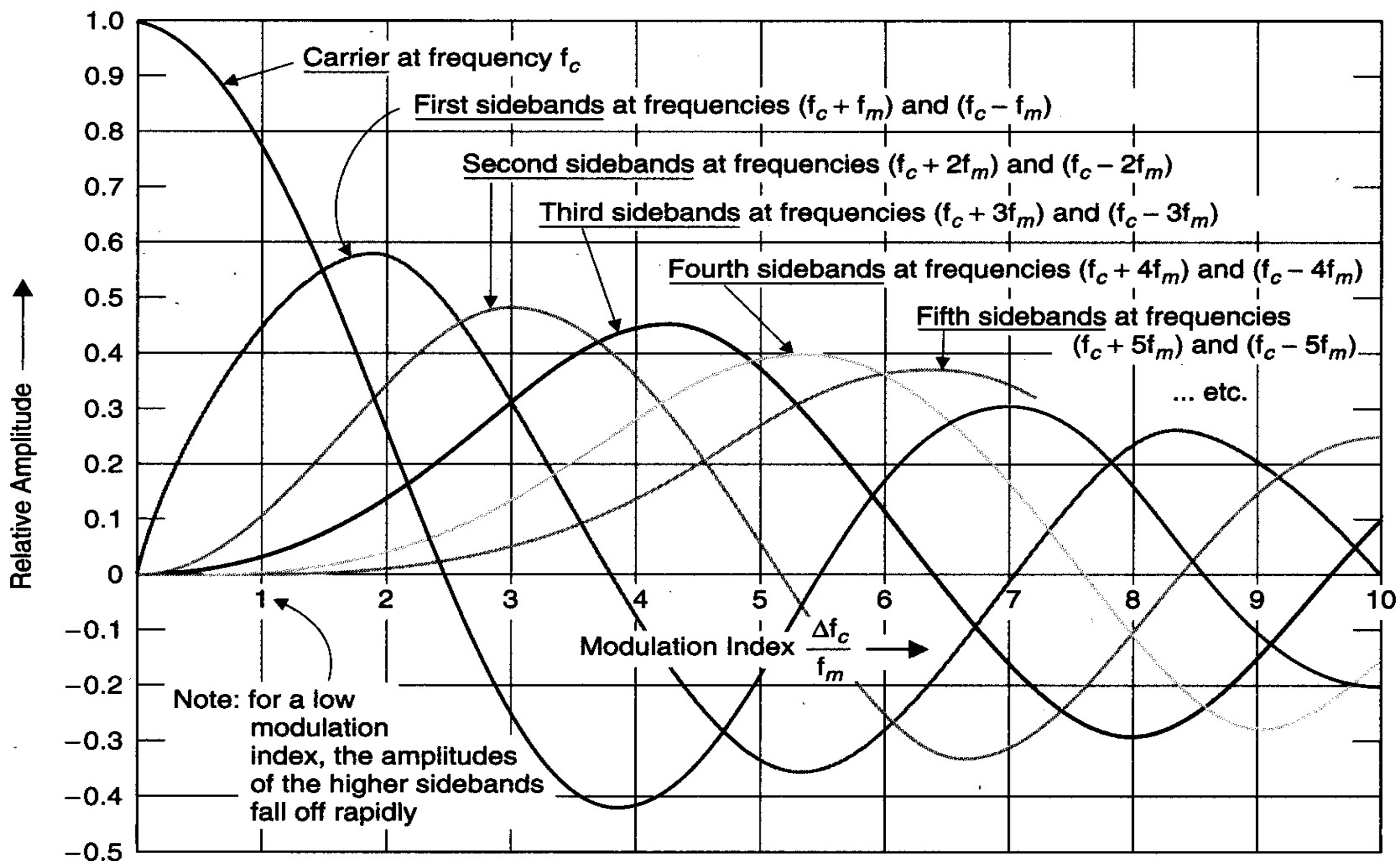

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Modulation Index $\frac{\Delta f_c}{f_m}$
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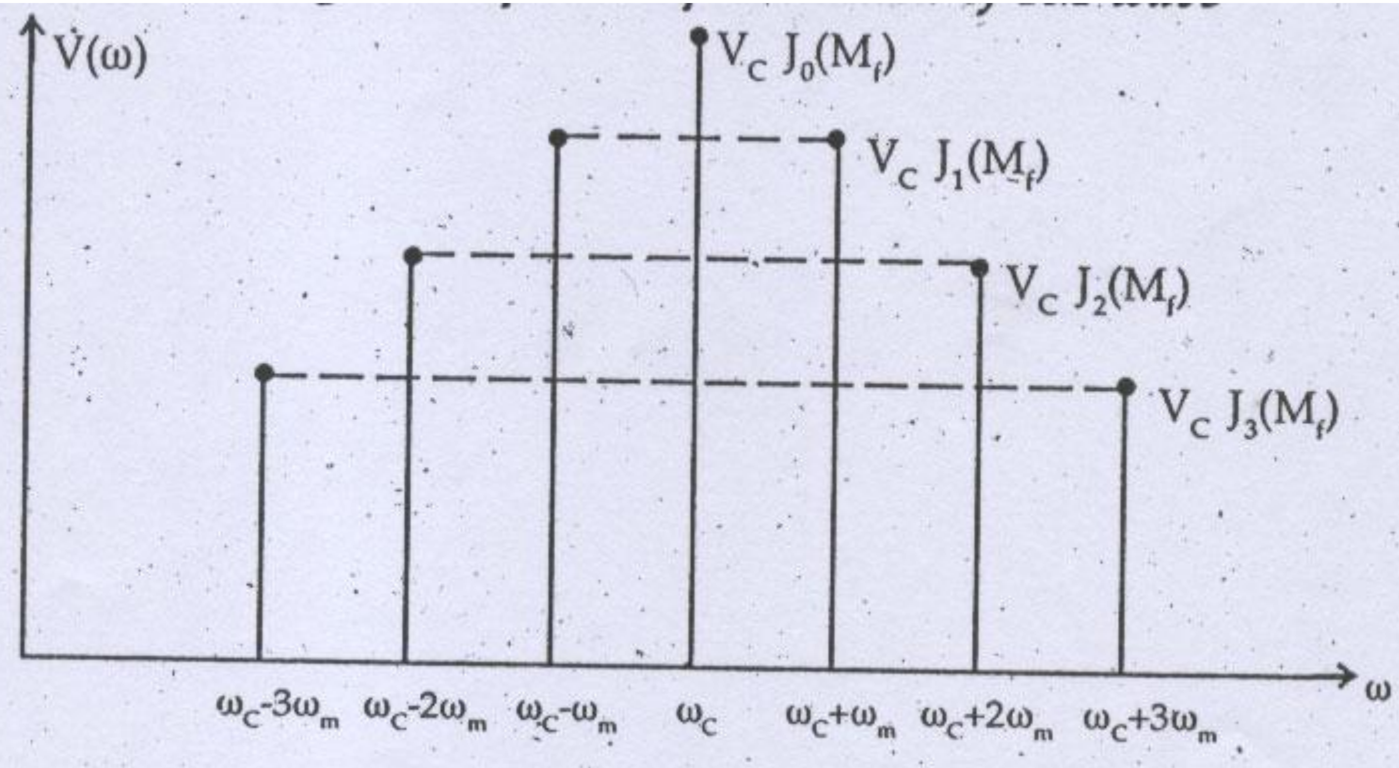
Representation of frequency spectrum



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Bessel Function

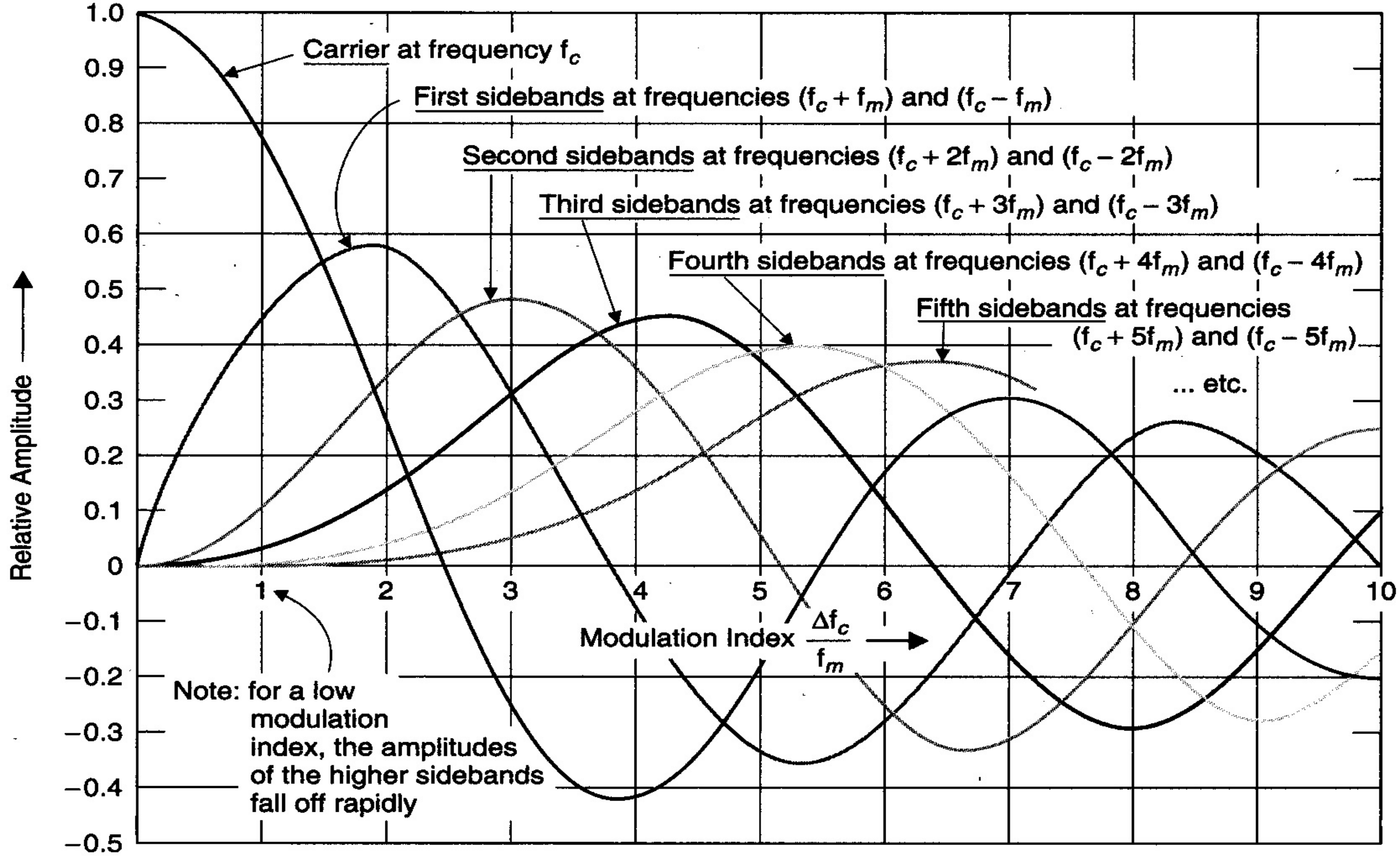
$$v(t)_{FM} = V_C\{J_0(m_f)\cos\,\omega_C t + J_1(m_f)\cos\left[(\omega_C + \omega_m)t + \frac{\pi}{2}\right] - J_1(m_f)\cos\left[(\omega_C - \omega_m)t - \frac{\pi}{2}\right]$$

$$+ J_2(m_f)\cos\left[(\omega_C + 2\omega_m)t\right] + J_2(m_f)\cos\left[(\omega_C - 2\omega_m)t\right] + \ldots J_n(m_f)\ldots\}$$

Bessel Functions of the First Kind, $J_n(m)$ for some value of modulation index

Modulation Index	Carrier	Side Frequency Pairs													
m	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—
2.4	0	0.52	0.43	0.20	0.06	0.02	—	—	—	—	—	—	—	—	—
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	0.01	—	—	—	—	—	—	—	—
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—
5.45	0	−0.34	−0.12	0.26	0.40	0.32	0.19	0.09	0.03	0.01	—	—	—	—	—
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	—	—	—
8.65	0	0.27	0.06	−0.24	−0.23	0.03	0.26	0.34	0.28	0.18	0.10	0.05	0.02	—	—
9.0	−0.09	0.25	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.31	0.21	0.12	0.06	0.03	0.01	—
10.0	−0.25	0.05	0.25	0.06	−0.22	−0.23	−0.01	0.22	0.32	0.29	0.21	0.12	0.06	0.03	0.01

B.F. (cont'd)



1.0
0.9
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0.5
0.4
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0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
Relative Amplitude
Carrier at frequency f_c
First sidebands at frequencies $(f_c + f_m)$ and $(f_c - f_m)$
Second sidebands at frequencies $(f_c + 2f_m)$ and $(f_c - 2f_m)$
Third sidebands at frequencies $(f_c + 3f_m)$ and $(f_c - 3f_m)$
Fourth sidebands at frequencies $(f_c + 4f_m)$ and $(f_c - 4f_m)$
Fifth sidebands at frequencies $(f_c + 5f_m)$ and $(f_c - 5f_m)$
... etc.
1
2
3
4
5
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10
Modulation Index $\frac{\Delta f_c}{f_m}$
Note: for a low modulation index, the amplitudes of the higher sidebands fall off rapidly

Representation of frequency spectrum

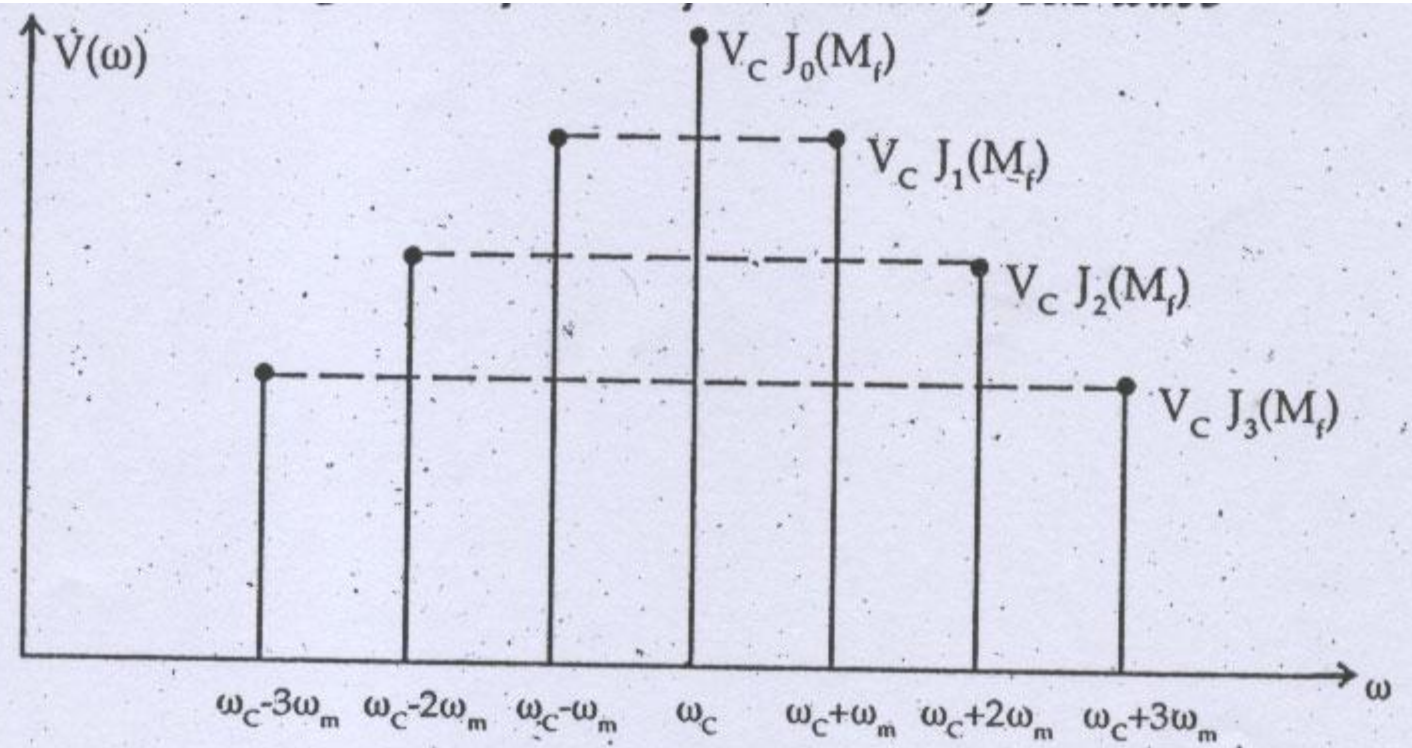

Fig. 2. Phasor representation of FM

Representation of frequency spectrum

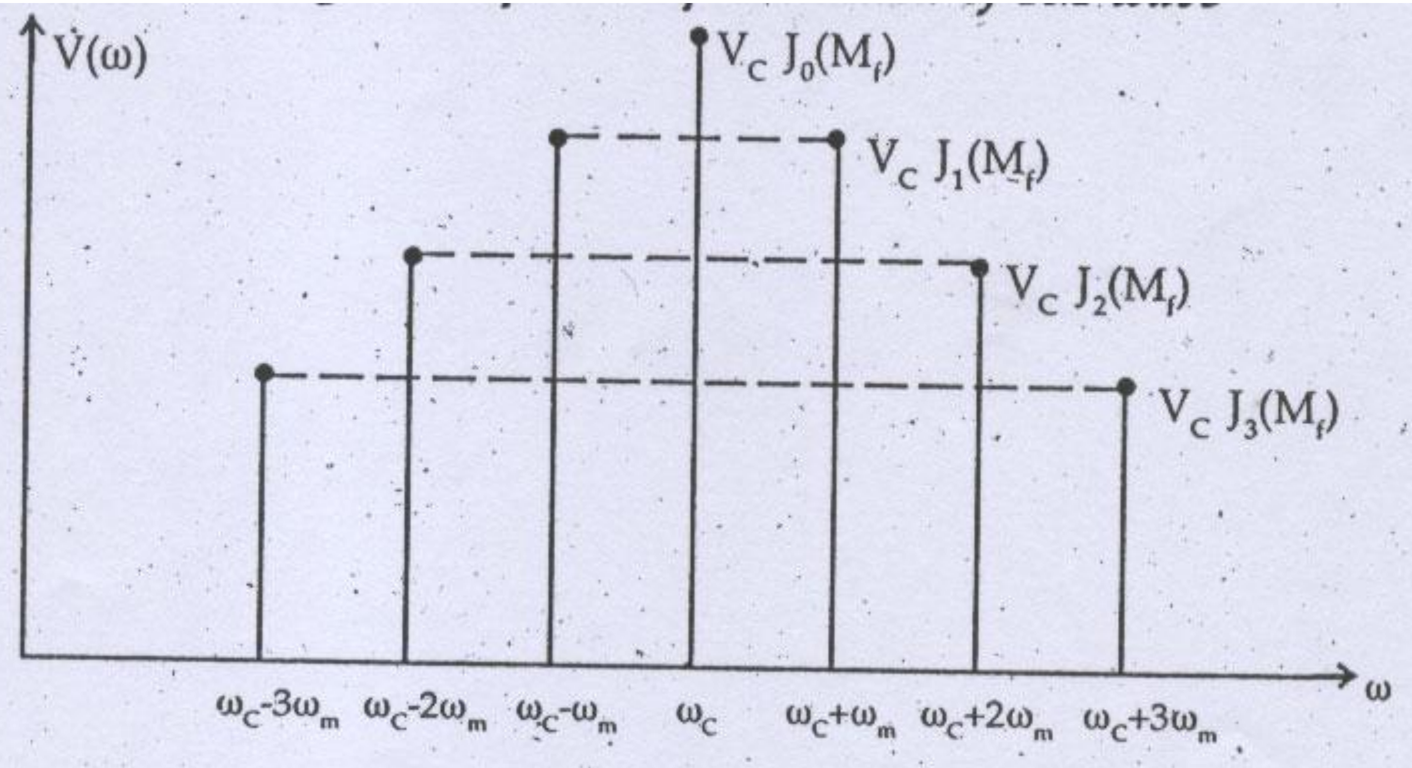

Fig. 2. Phasor representation of FM

FM Bandwidth

- Theoretically, the generation and transmission of FM requires infinite bandwidth. Practically, FM system have finite bandwidth and they perform well.
- The value of modulation index determine the number of sidebands that have the significant relative amplitudes
- If n is the number of sideband pairs, and line of frequency spectrum are spaced by fm, thus, the bandwidth is:

$$B_{fm} = 2nf_m$$

- For $n \geq 1$

FM Bandwidth (cont'd)

- Estimation of transmission b/w;
- Assume m_f is large and n is approximate $m_f + 2$; thus
- $B_{fm}=2(m_f + 2)f_m$

$$= 2\left(\frac{\Delta f}{f_m} + 2\right) f_m$$

$$B_{fm} = 2(\Delta f + f_m) \ldots\ldots\ldots (1)$$

(1) is called Carson's rule

Multi tone FM

- In FM modulation, the modulation index is not important in itself but the bandwidth of the broadcasted FM signal is.

  As you know, the FM modulation index for a sinusoidal modulating signal is defined as:
  $\beta = \Delta Fc/Fm$
  where:
  ΔFc is the carrier deviation
  Fm is the modulating frequency.

  For EACH sinusoidal modulating signal of freqency Fm, the bandwith of the FM signal could be given as:
  $BW = 2 * (\Delta Fc + Fm) = 2 * (\beta*Fm + Fm)$
  $BW = 2 * Fm * (\beta + 1)$

  For multi tone FM broadcasting, the FM bandwidth should not exceed usually a limit (as 200 KHz for the radio FM Band, if I remember well). Therefore for a constant BW, increasing Fm imposes on β to be lowered.

- A generic N+tone modulated FM signal can be represented as:

  $y(t)=\cos\{\omega c \bullet t+K \bullet [A1/\omega 1 * \sin(\omega 1 \bullet t)+A2/\omega 2 * \sin(\omega 2 \bullet t)+\ldots+AN/\omega N * \sin(\omega N \bullet t)\}$

  where K is the constant in Hz/V to convert from amplitude to frequency and Ai is the amplitude of the i-th tone.

  each tone will have its own modulation index. they will be:

  $index1=K \bullet A1/\omega 1$
  $index2=K \bullet A2/\omega 2$
  .
  .
  $indexN=K \bullet AN/\omega N$

  (of course $wi=2 \bullet \Pi \bullet fi$)

  To estimate the occupated bandwidth you can use the Carson's rule:

  first of all we need to calculate the maximum deviation frequency
  $\Delta fmax=K \bullet max(Ai)$
  then we will need the deviation rate: $DR=\Delta fmax/max(fi)$

  $BW \approx 2 \bullet \Delta fmax \bullet (1+1/DR)$

  Finally the spectrum will be expressed in terms of products of Jn bessel functions.

Bessel Function

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$$+ J_2(m_f)\cos\left[(\omega_C + 2\omega_m)t\right] + J_2(m_f)\cos\left[(\omega_C - 2\omega_m)t\right] + \ldots J_n(m_f)\ldots\}$$

Bessel Functions of the First Kind, $J_n(m)$

for some value of modulation index

Modulation Index	Carrier	Side Frequency Pairs													
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0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—
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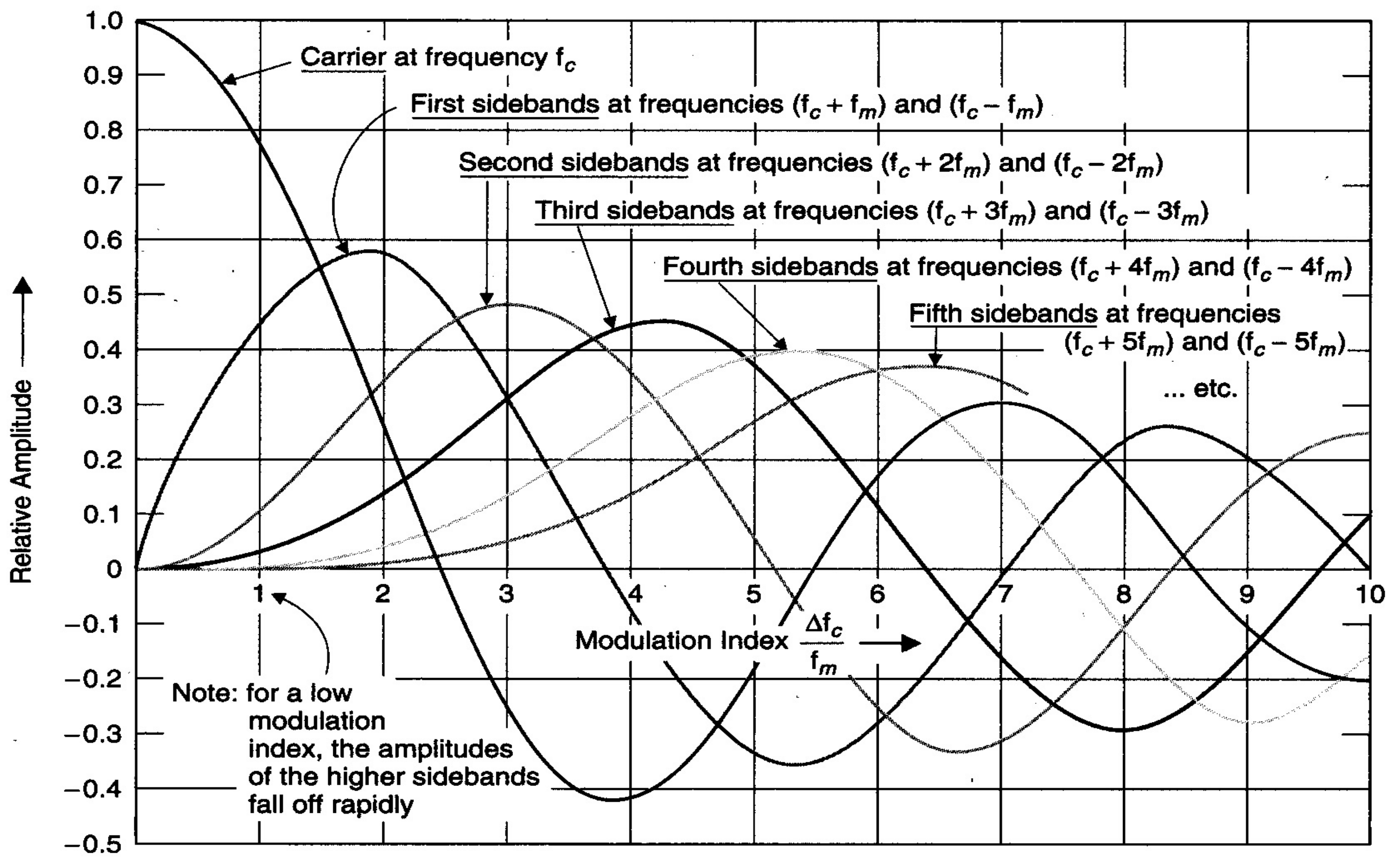
B.F. (cont'd)



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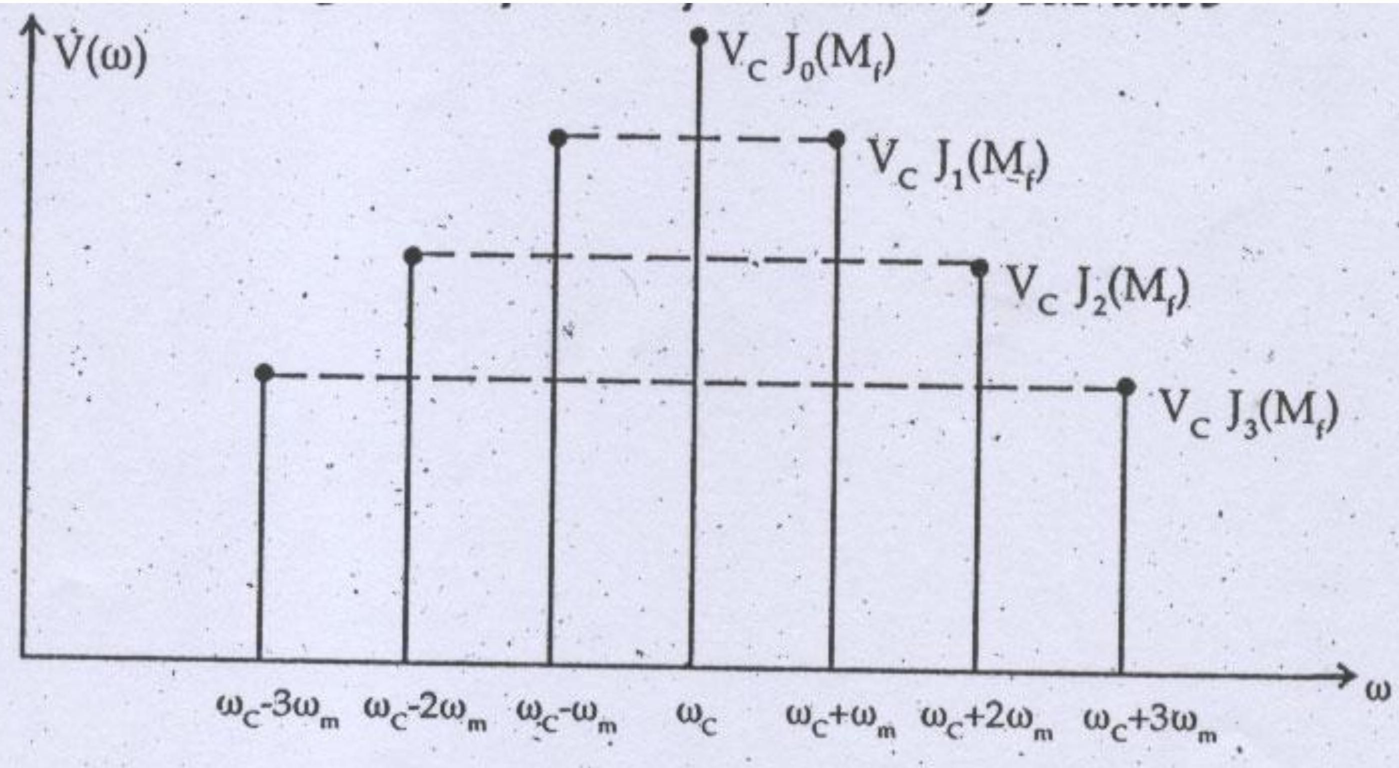
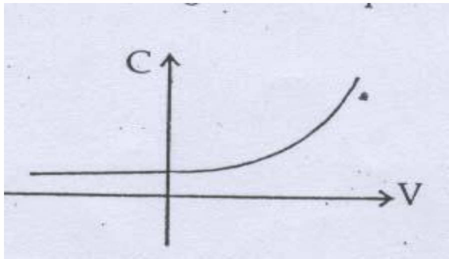


Fig. 2. Phasor representation of FM

Figure how the characteristics curve of a typical variable capacitance diode (varactor diode) displaying the capacitance as function of reverse bias.



Transfer characteristics of Varactor Diode

Increasing the bias increase the width of PN junction and reduces the capacitance .It can be mathematically written as

$$C \, \alpha \, \frac{1}{\sqrt{V}} \, where V \quad = \, reverse$$

Bias voltage

FM Generation

- The frequency modulated signals can be generated in 2 ways:
- Direct method of FM
- Indirect method of FM.

- The prime requirement of FM generation sis a viable output frequency. The frequency is directly propositional to the instantaneous amplitude of the modulating voltage.

- The subsidiary requirement of FM generation is that the frequency deviation is independent of modulating frequency. However if the system does not properly produce these charctictics, corrections can be introduced during the modulation process.

Generation of FM

- Two major FM generation:

i) Direct method:

  i) straight forward, requires a VCO whose oscillation frequency has linear dependence on applied voltage.
  ii) Advantage: large frequency deviation
  iii) Disadvantage: the carrier frequency tends to drift and must be stabilized.

    i) Varactor diode

Indirect method:

One most popular indirect method is the Armstrong modulator

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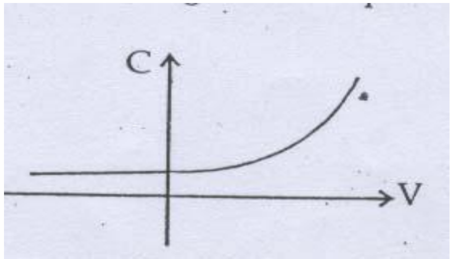

Transfer characteristics of Varactor Diode

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$$C \,\alpha\, \frac{1}{\sqrt{V}} \; where V = reverse$$

Bias voltage

- The D.C bias to the varactor diode is regulated in such a ways that the oscillator frequency is not affected by varactor supply fluctuations. The modulating signal is fed in series with this regulated supply and at any instant the effective bias to the varactor diode equals the algebraic sum of the d.c bias volt 'V' and the instantaneous values of the modulating signal.

- As a result, the capacitance changes with amplitude of the modulating signal resulting in frequency modulating of the oscillator output.

- The rate of change of carrier frequency depends on the information signal. Since the information signal directly controls the frequency of the oscillator the output is frequency modulated .The chief advanced for this circuit is the use of two terminal devices but makes its applications limited.

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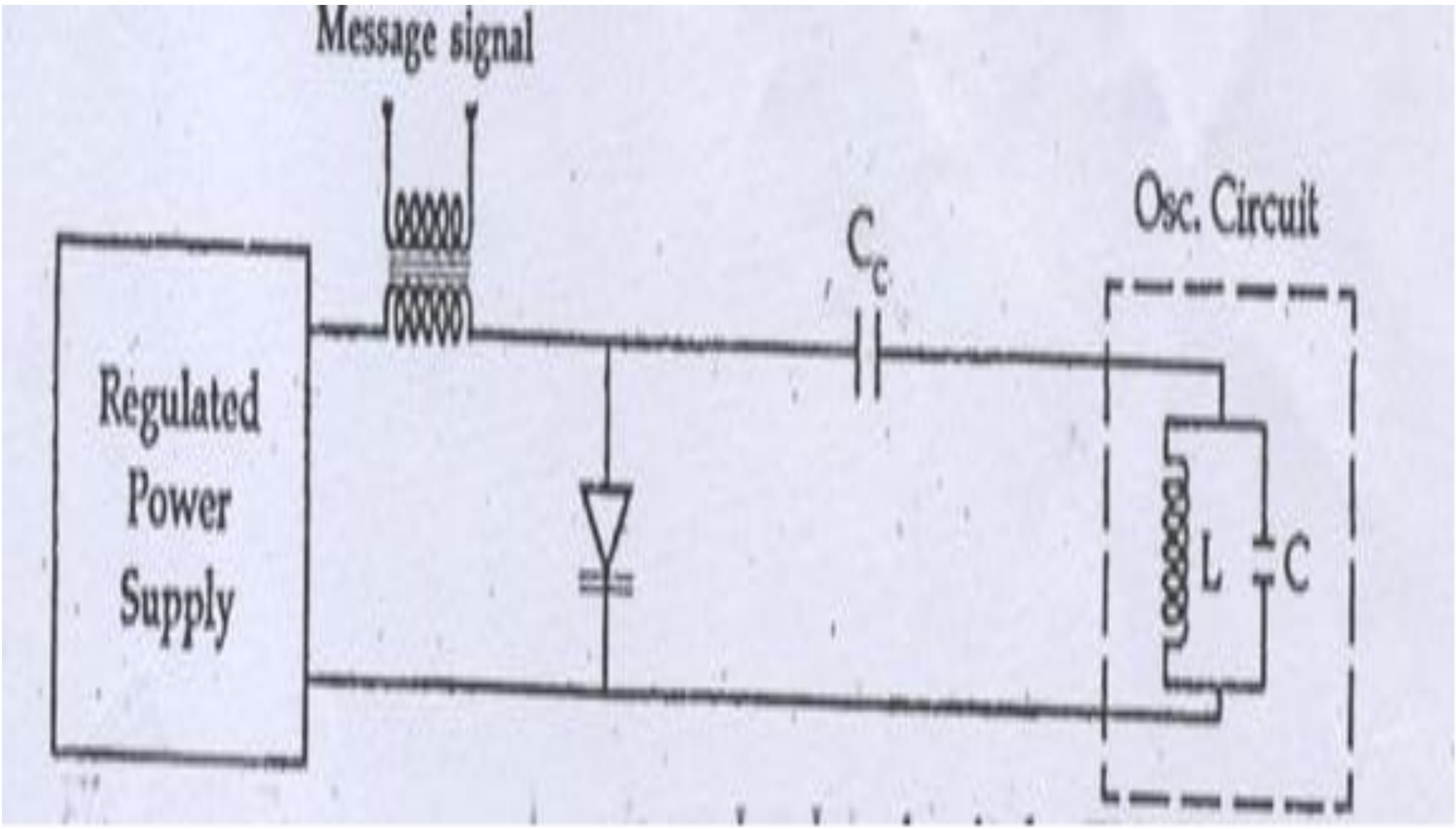
Message signal
Osc. Circuit
C_C
Regulated
Power
Supply
L
C

- The D.C bias to the varactor diode is regulated in such a ways that the oscillator frequency is not affected by varactor supply fluctuations. The modulating signal is fed in series with this regulated supply and at any instant the effective bias to the varactor diode equals the algebraic sum of the d.c bias volt 'V' and the instantaneous values of the modulating signal.

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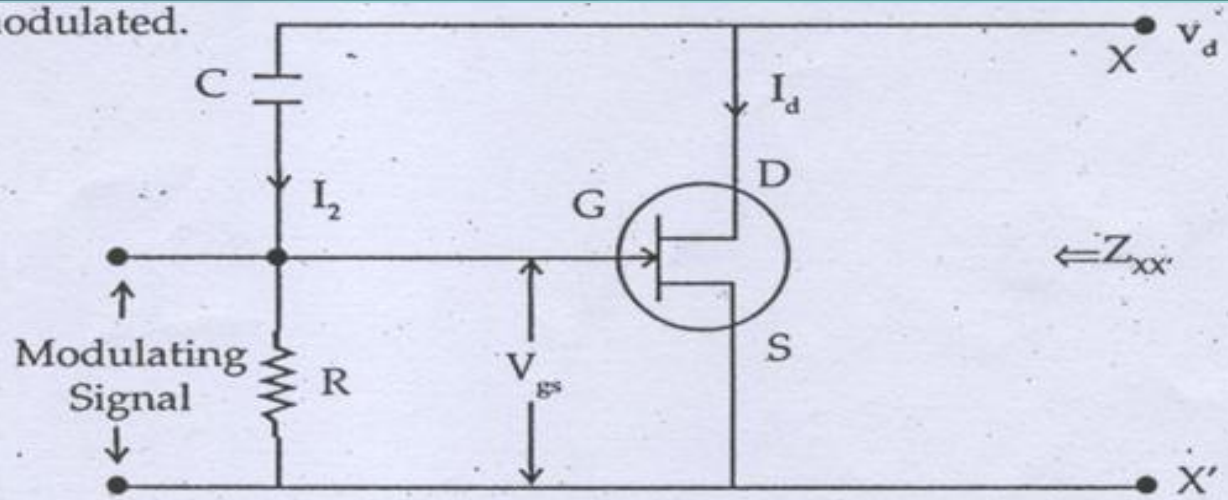
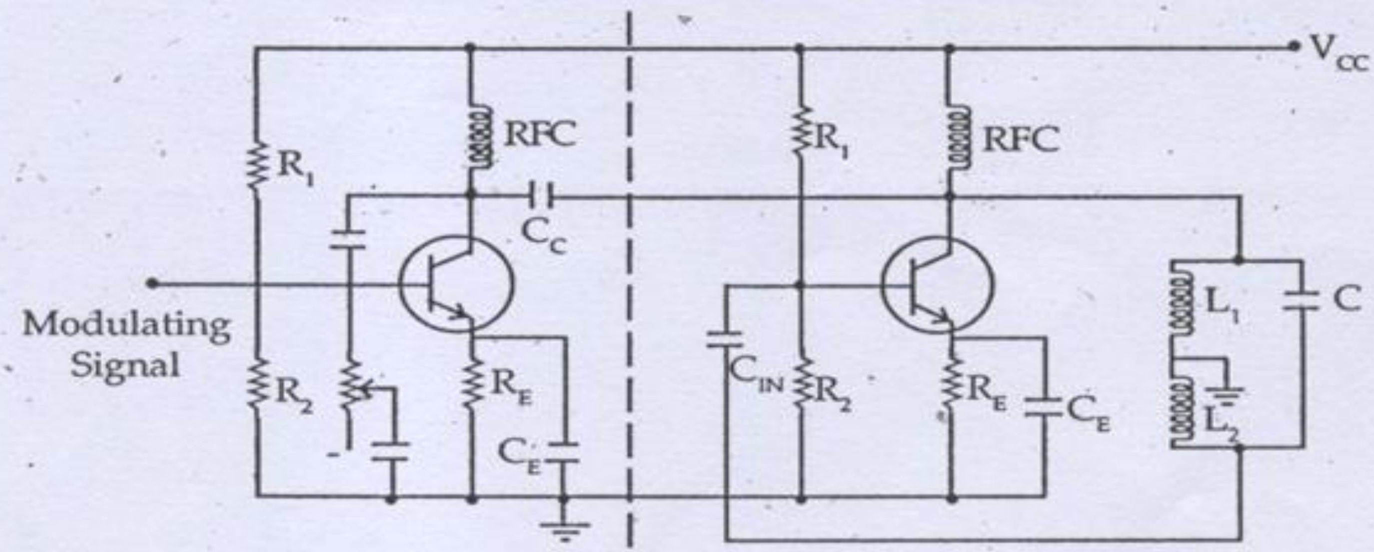
modulated.



Reactance Tube Modulator



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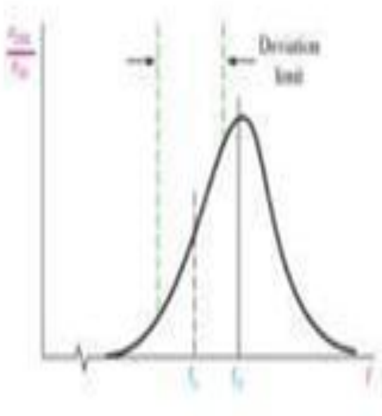
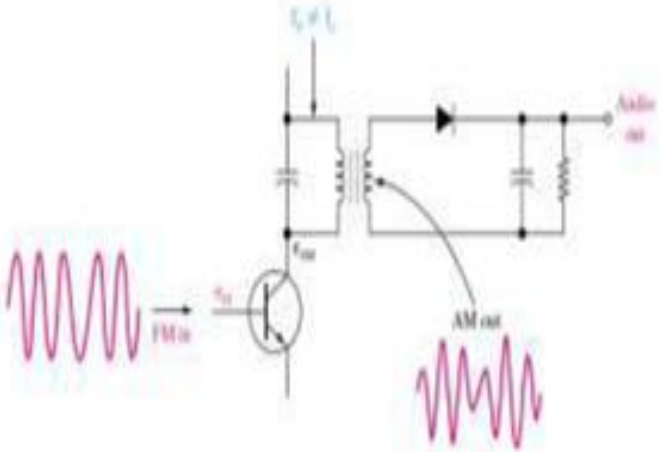
- The rate of change of carrier frequency depends on the information signal. Since the information signal directly controls the frequency of the oscillator the output is frequency modulated .The chief advanced for this circuit is the use of two terminal devices but makes its applications limited.

Slope detector

- The operation of the slope detector is very simple. The output network of an amplifier is tuned to a frequency that is slightly more than the carrier frequency + peak deviation. As the input signal varies in frequency, the output signal across the LC network will vary in amplitude because of the band pass properties of the tank circuit. The output of this amplifier is AM, which can be detected using a diode detector.

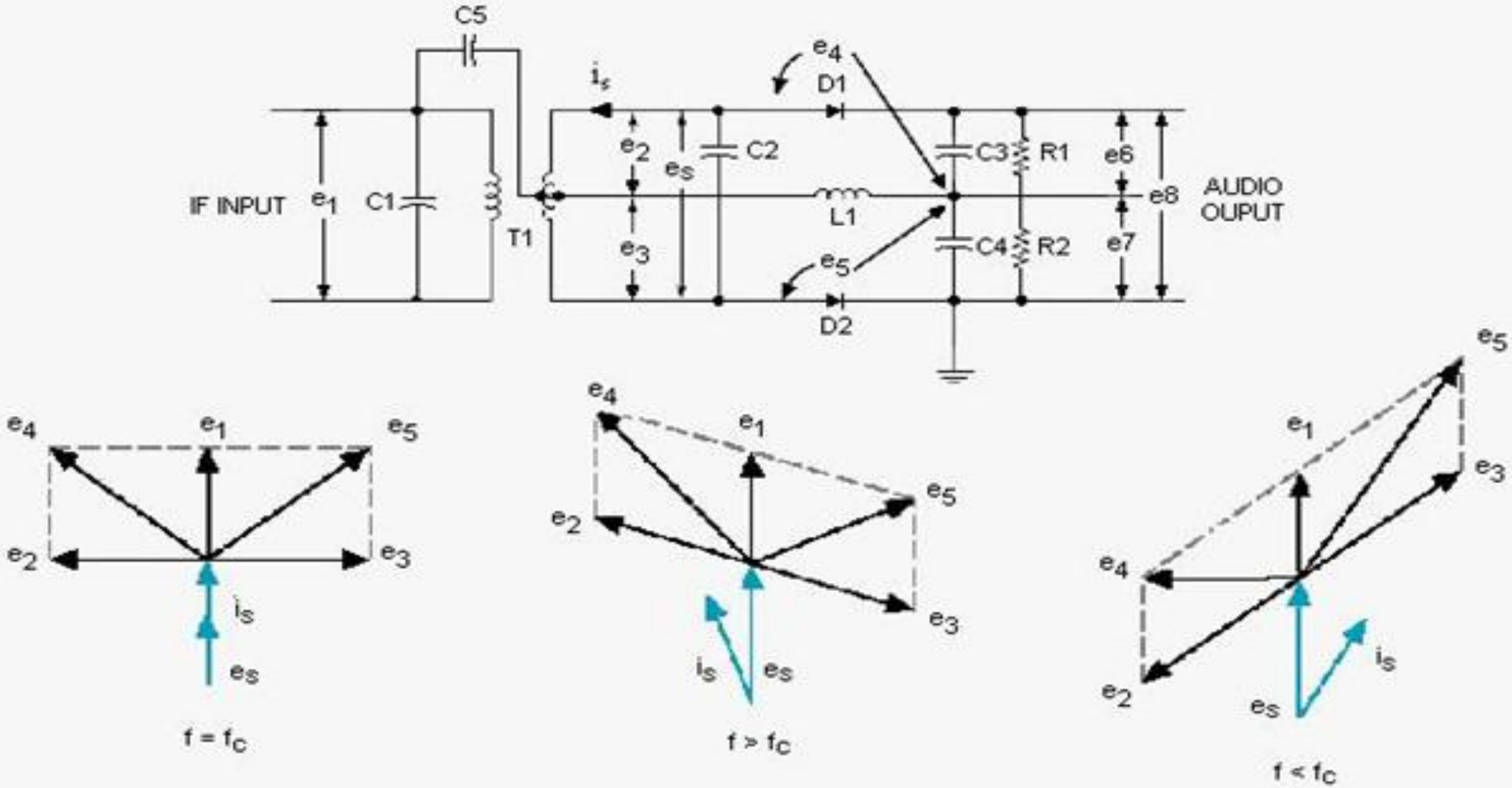

Deviation
limit


AM out

- The circuit shown in the diagram above looks very similar to the last IF amplifier and detector of an AM receiver, and it is possible to receive NBFM on an AM receiver by detuning the last IF transformer. If this transformer is tuned to a frequency of approximately 1 KHz above the IF frequency, the last IF amplifier will convert NBFM to AM.
- In spite of its simplicity, the slope detector is rarely used because it has poor linearity. To see why this is so, it is necessary to look at the expression for the voltage across the primary of the tuned transformer in the sloped detector

Fooster Seley Detector

- The Foster-Seely Discriminator is a widely used FM detector. The detector consists of a special center-tapped IF transformer feeding two diodes. The schematic looks very much like a full wave DC rectifier circuit. Because the input transformer is tuned to the IF frequency, the output of the discriminator is zero when there is no deviation of the carrier; both halves of the center tapped transformer are balanced. As the FM signal swings in frequency above and below the carrier frequency, the balance between the two halves of the center-tapped secondary are destroyed and there is an output voltage proportional to the frequency deviation.


C5
e_4
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D1
IF INPUT
e_1
C1
T1
e_2
e_s
C2
e_3
L1
C3
R1
C4
R2
e_5
D2
e6
e7
e8
AUDIO
OUPUT
e_4
e_1
e_5
e_2
e_3
i_s
e_s
$f = f_c$
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e_1
e_5
e_2
e_3
i_s
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$f > f_c$
e_5
e_1
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e_4
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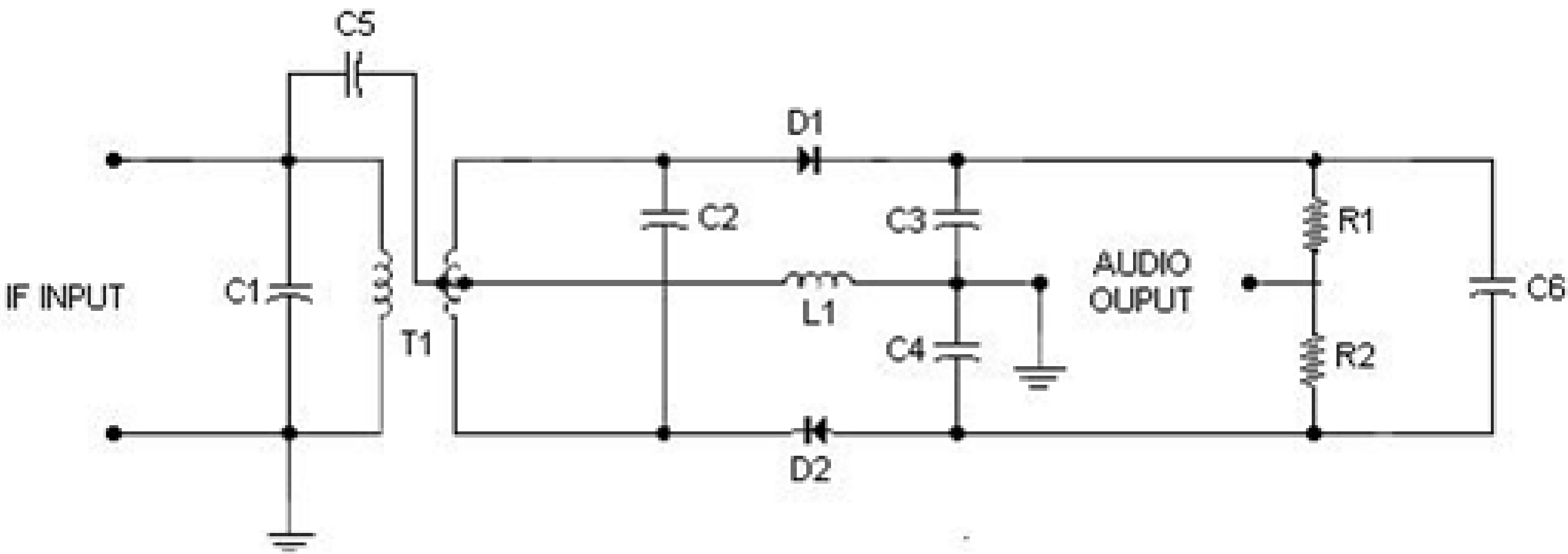
C5
D1
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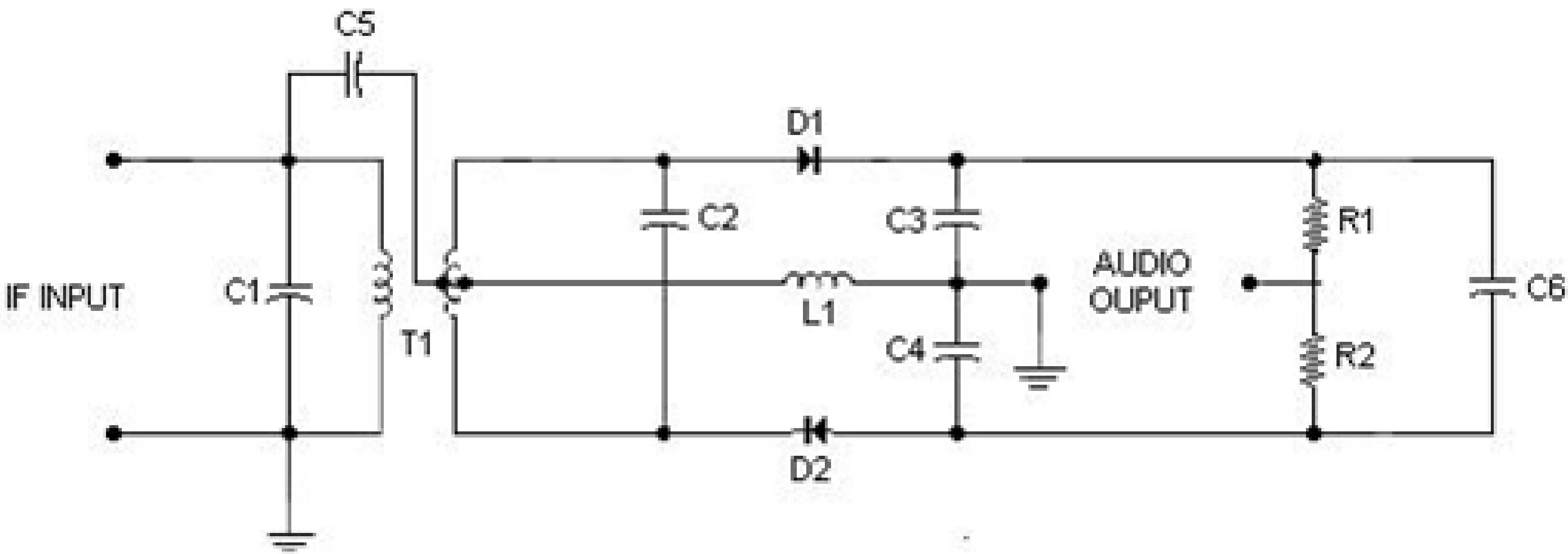
C3
R1
AUDIO
OUPUT
IF INPUT
C1
T1
L1
C4
C6
R2
D2

Ratio Detector

- The ratio detector is a variant of the discriminator. The circuit is similar to the discriminator, but in a ratio detector, the diodes conduct in opposite directions. Also, the output is not taken across the diodes, but between the sum of the diode voltages and the center tap. The output across the diodes is connected to a large capacitor, which eliminates AM noise in the ratio detector output. The operation of the ratio detector is very similar to the discriminator, but the output is only 50% of the output of a discriminator for the same input signal.


C5
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OUPUT
IF INPUT
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L1
C6
T1
C4
R2
D2

Lecture 7
AM and FM Signal Demodulation

- Introduction
- Demodulation of AM signals
- Demodulation of FM Signals
- Regeneration of Digital Signals and Bias Distortion
- Noise and Transmission Line Capacity
- Channel capacity
- Conclusion

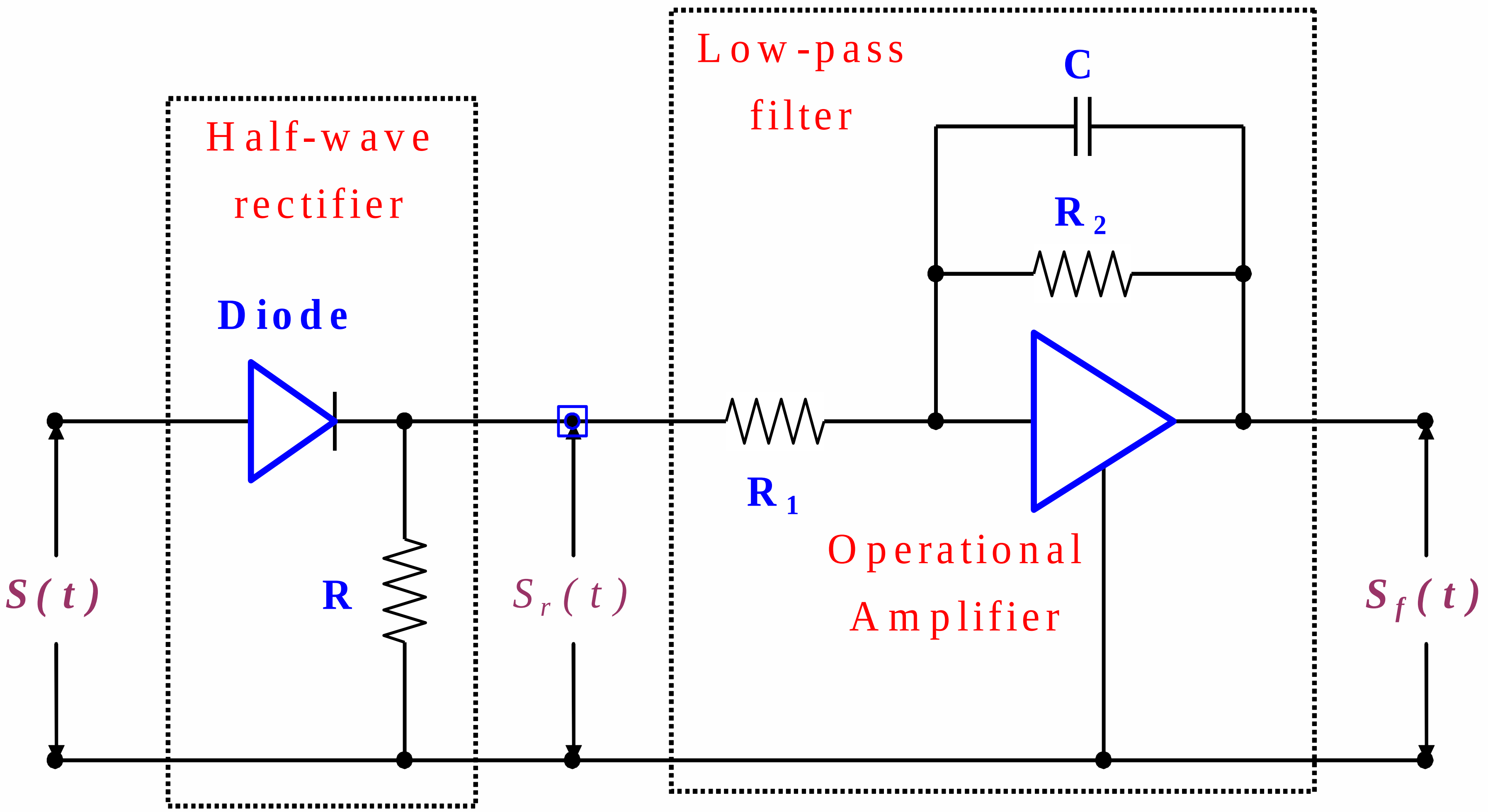
Introduction

- The goal of demodulation.
- **Demodulation**
- **Regeneration** can exactly reproduce the original digital signal.
- An AM signal preserves the frequency domain information of the baseband signal in each sideband,
- Two methods for demodulation of an AM signal:
- Envelope detection (for DSBTC AM signal)
- Synchronous detection (coherent or homodyne)

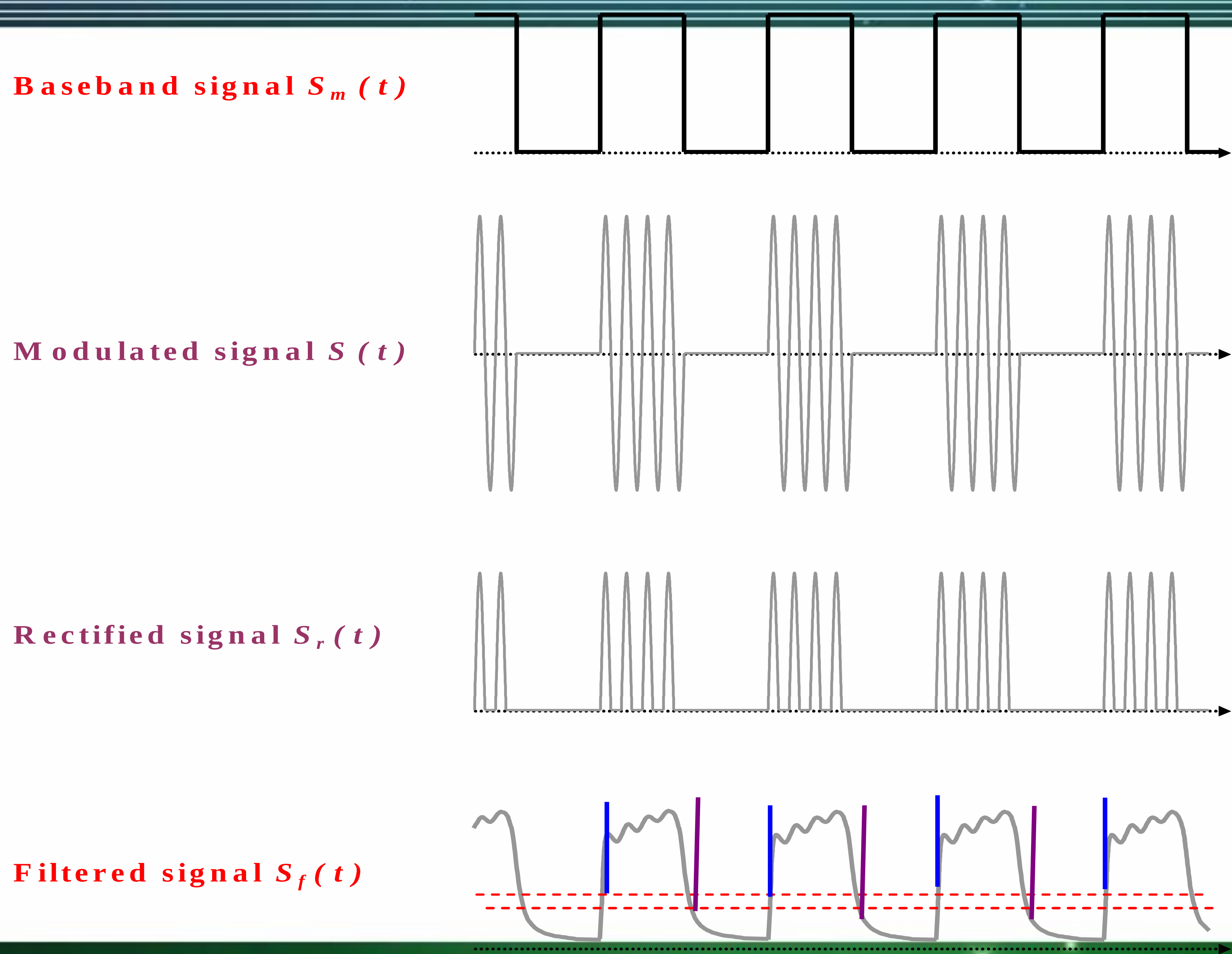
FM signal demodulation

- It is more resistant to noise than an AM signal.
- **filtering** and **Limiting** the transmitted signal.
- Differentiation to obtain the phase information in the modulated signal.
- There are four ways to implement differentiation:
- **Phase-Locked Loop**
- **Zero-Crossing Detection**

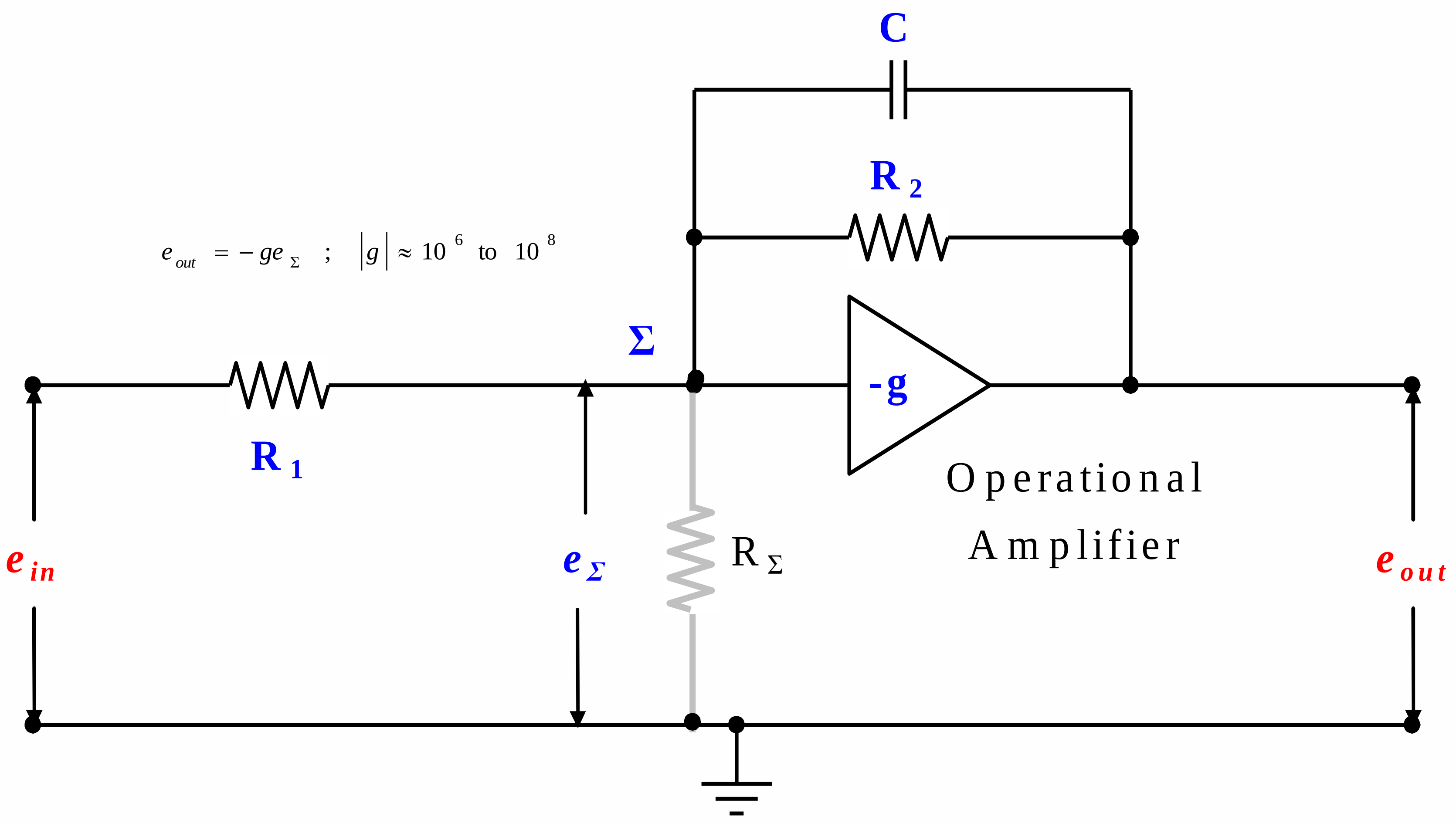
Envelope detection circuit.



Half-wave rectification and filtration of DSBTC AM signal.



Circuit diagram of the low-pass filter.



$$\frac{e_{in} - e_\Sigma}{R_1} = \frac{e_\Sigma - e_{out}}{R_2} + C \cdot \frac{d(e_\Sigma - e_{out})}{dt} + \frac{e_\Sigma - 0}{R_\Sigma}$$

In the limit as $|g| \to \infty$, the voltage, $e_\Sigma \to 0$ otherwise $e_{out} = -g$ $e_\Sigma \to \infty$

$$\frac{e_{in}}{R_1} = -\frac{e_{out}}{R_2} - C \cdot \frac{de_{out}}{dt} \quad \text{or} \quad -\frac{R_2}{R_1} \cdot e_{in} = e_{out} + R_2 C \cdot \frac{de_{out}}{dt}$$

$$F\left[\frac{R_2}{R_1} \cdot e_{in}\right] = \frac{R_2}{R_1} \cdot \overline{U}_{in} \quad ; \quad F[e_{out}] = \overline{U}_{out} \quad ;$$

$$F\left[R_2 C \cdot \frac{de_{out}}{dt}\right] = R_2 C \, j\omega \cdot \overline{U}_{out}(j\omega)$$

$$H(j\omega) = \frac{\overline{U}_{out}}{\overline{U}_{in}} = -\frac{R_2}{R_1} \cdot \frac{1}{1 + j\omega R_2 C}$$

$$|H(j\omega)| = \frac{R_2}{R_1} \cdot \frac{1}{\sqrt{1 + (\omega R_2 C)^2}}$$

$$\varphi(\omega) = \tan^{-1}(-\omega R_2 C)$$

$$20\cdot\log_{10}\left|H(j\omega)\right| = 20\cdot\log_{10}\left(\frac{R_2}{R_1}\cdot\frac{1}{\sqrt{1+(\omega R_2 C)^2}}\right)$$

$$= 20\cdot\log_{10}\left(\frac{R_2}{R_1}\right) - 10\cdot\log_{10}\left[1+(\omega R_2 C)^2\right]$$

$\omega \ll \frac{1}{R_2C}$:

$$20\cdot\log\left|H(j\omega)\right| \approx 20\cdot\log_{10}\left(\frac{R_2}{R_1}\right) - 10\cdot\log_{10}(1)$$

$$\approx 20\cdot\log_{10}\left(\frac{R_2}{R_1}\right)$$

$\omega = \frac{1}{R_2C}$:

$$20\cdot\log\left|H(j\omega)\right| = 20\cdot\log_{10}\left(\frac{R_2}{R_1}\right) - 10\cdot\log_{10}(2)$$

$$= 20\cdot\log_{10}\left(\frac{R_2}{R_1}\right) - 3.01\ \text{dB}$$

$\omega \gg \frac{1}{R_2C}$:

$$20\cdot\log\left|H(j\omega)\right| \approx 20\cdot\log_{10}\left(\frac{R_2}{R_1}\right) - 10\cdot\log_{10}\left[(\omega R_2 C)^2\right]$$

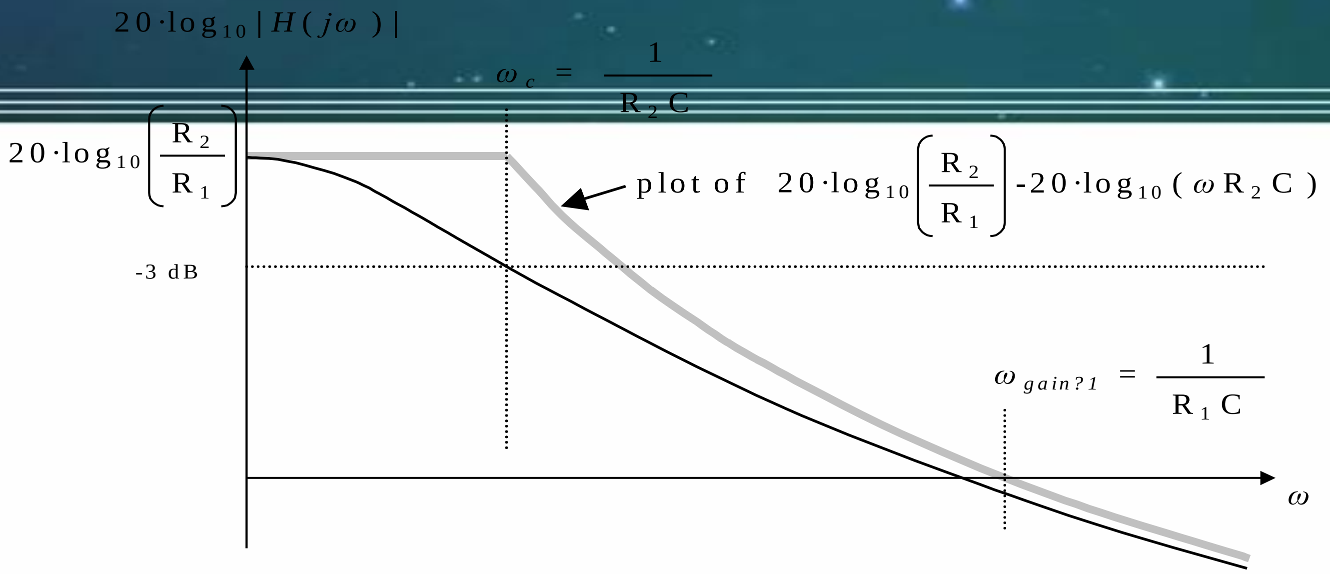
$$\approx 20\cdot\log_{10}\left(\frac{R_2}{R_1}\right) - 20\cdot\log_{10}(\omega R_2 C)$$

$$\varphi(\omega) = \tan^{-1}(-\omega R_2 C)$$

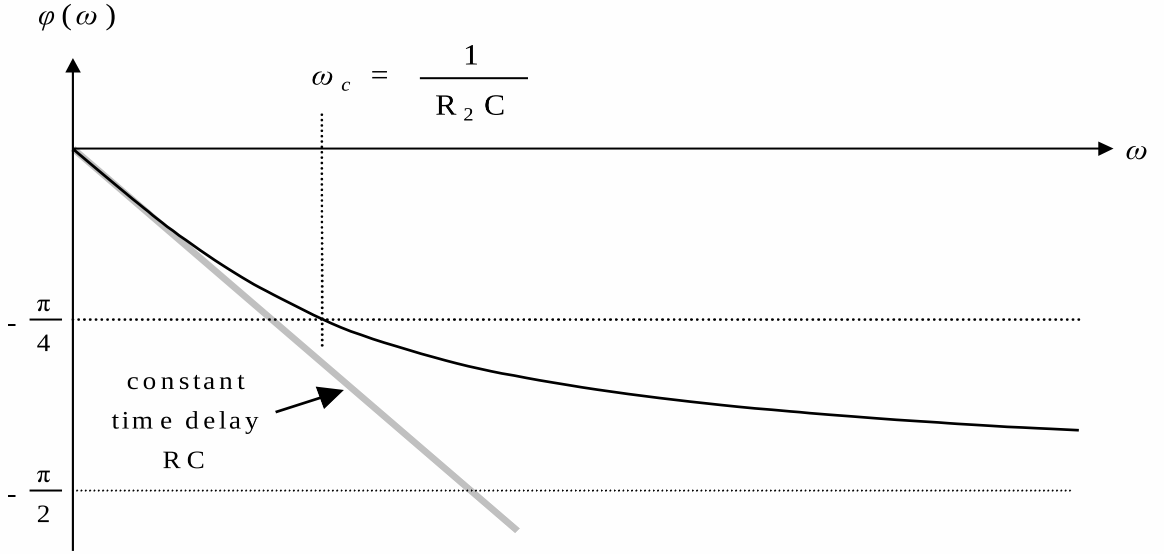
$\omega \ll \frac{1}{R_2C}$ $\quad \varphi(\omega) \approx -\omega R_2 C$

$\omega = \omega_c = \frac{1}{R_2C}$ $\quad \varphi(\omega) = -\frac{\pi}{4}$

$\omega = \infty$ $\quad \varphi(\omega) = -\frac{\pi}{2}$



(a) Amplitude Bode plot (in decibels)



(b) Phase Bode plot (in radians)

Synchronous Demodulation of AM signals

$$S(t) = A_c \cdot [A_c + S_m(t)] \times \cos(2\pi f_c t)$$

$$S_{demod}(t) = \tfrac{1}{k} A^2{}_c \cdot [A_c + S_m(t)] \times \cos(2\pi f_c t) \times \cos(2\pi f_c t)$$

$$= \tfrac{1}{k} A^2{}_c \cdot [A_c + S_m(t)] \times \cos^2(2\pi f_c t)$$

$$= \tfrac{1}{k} A^2{}_c \cdot [A_c + S_m(t)] \times \frac{1}{2} \cdot [1 + \cos(2 \cdot 2\pi f_c t)]$$

$$= \frac{A^3_c}{2k} + \frac{A^2{}_c}{2k} \cdot S_m(t) + \frac{1}{2k} A^2{}_c \cdot [A_c + S_m(t)] \cdot \cos(4\pi f_c t)$$

$$\cos^2(\alpha) = \frac{1}{2} \cdot [1 + \cos 2(\alpha)]$$

$$S_{demod}(t) = \frac{A^2{}_c}{2k} \cdot S_m(t)$$

Demodulation of FM Signal

1 - **filter** the signal in order to eliminate all noise outside of the signal band. Broadcast FM signals are filtered by a band-pass filter prior to transmitting.

2 - Modulated FM signal is to pass it through a **limiter**. This will restrict the signal amplitude to the range $-V_L$ to $+V_L$. The output is a series of nearly rectangular pulses.

3 - low-pass **filter** eliminates the higher frequency components from these pulses to obtain a signal which very closely resembles the transmitted FM signal:

$$S_f(t) \approx \frac{4}{\pi} g_{filter} V_L \cos[\omega_c t + \phi(t)]$$

g_{filter}: gain of low-pass filter (ratio of R_2 to R_1)

This amplitude variation in the received signal does not appear at the output of the low-pass filter, but the phase function $\phi(t)$ is preserved.

After the added noise is removed, the demodulator must restore the original signal $S_m(t)$. It is possible to accomplish this by differentiating the filtered output signal with respect to time:

(Af : amplitude of filter output, Af ≈ gfilter · VL)

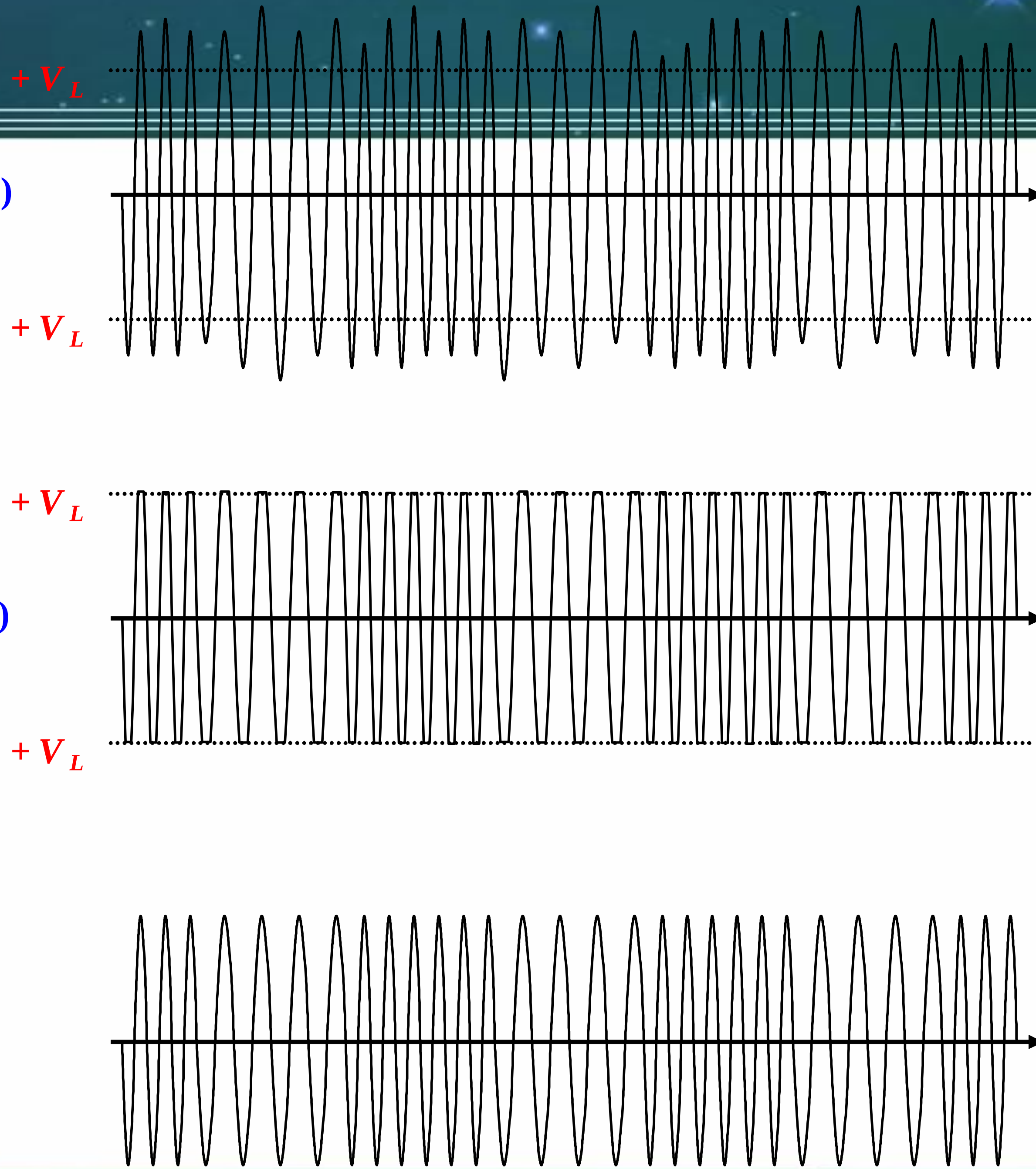
$$S(t) = A_c \cdot \cos\left(2\pi f_c t + \int_{-\infty}^{t} k \cdot S_m(\xi)\, d\xi\right)$$

$$\frac{d}{dt}\left[A_f \cos(\omega_c t + \phi(t))\right] = -A_f\left(\omega_c + \frac{d\phi(t)}{dt}\right)\sin(\omega_c t + \phi(t))$$

Received signal $S(t)$

Limited signal $S_L(t)$

Filtered signal $S_f(t)$



- The DC offset can be removed with a capacitor placed in series to the differentiator. The varying portion of the signal is proportional to the original signal:

- By passing the differentiated signal through an ideal envelope detector and low-pass filter, we can recover the original signal. The carrier frequency determines the DC offset of this signal, which will be much larger than the varying portion of the signal:

$$S_{env}(t) = \left| -A_f \left(\omega_c + \frac{d\phi(t)}{dt} \right) \right| = A_f \omega_c + A_f \frac{d\phi(t)}{dt}$$

$$\frac{d\phi}{dt} = K \cdot S_m(t) \ ; \quad S_{env}(t) = A_f \omega_c + \frac{A_f}{K} S_m(t)$$

- There are four ways to implement a differentiator:

A. **<u>Phase-Locked Loop (PLL)</u>**

B. **Zero-Crossing Detection**

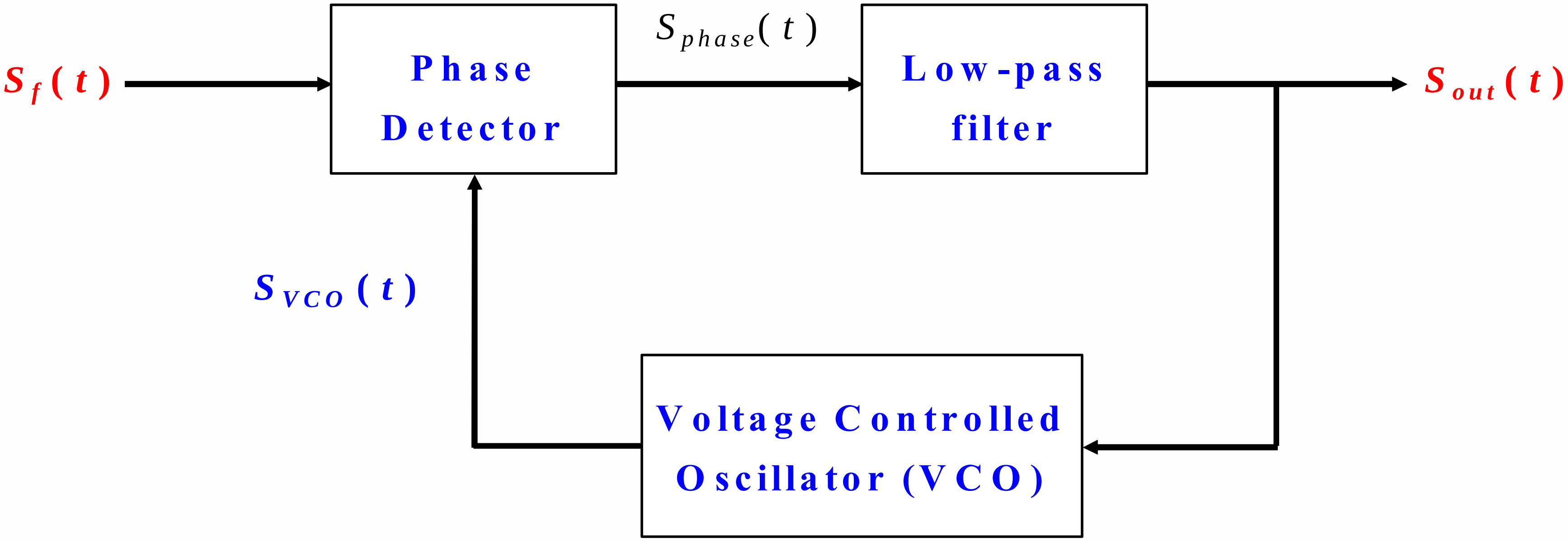
C. **FM-to-AM Conversion (also called a slope detector)**

D. **Phase Shift or Quadrature Detection**

Phase-Locked Loop (PLL) - negative feedback.

The PLL consists of three basic components:

A. Phase detector (PD)
B. Low-pass filter (LPF)
C. Voltage controlled oscillator (VCO)

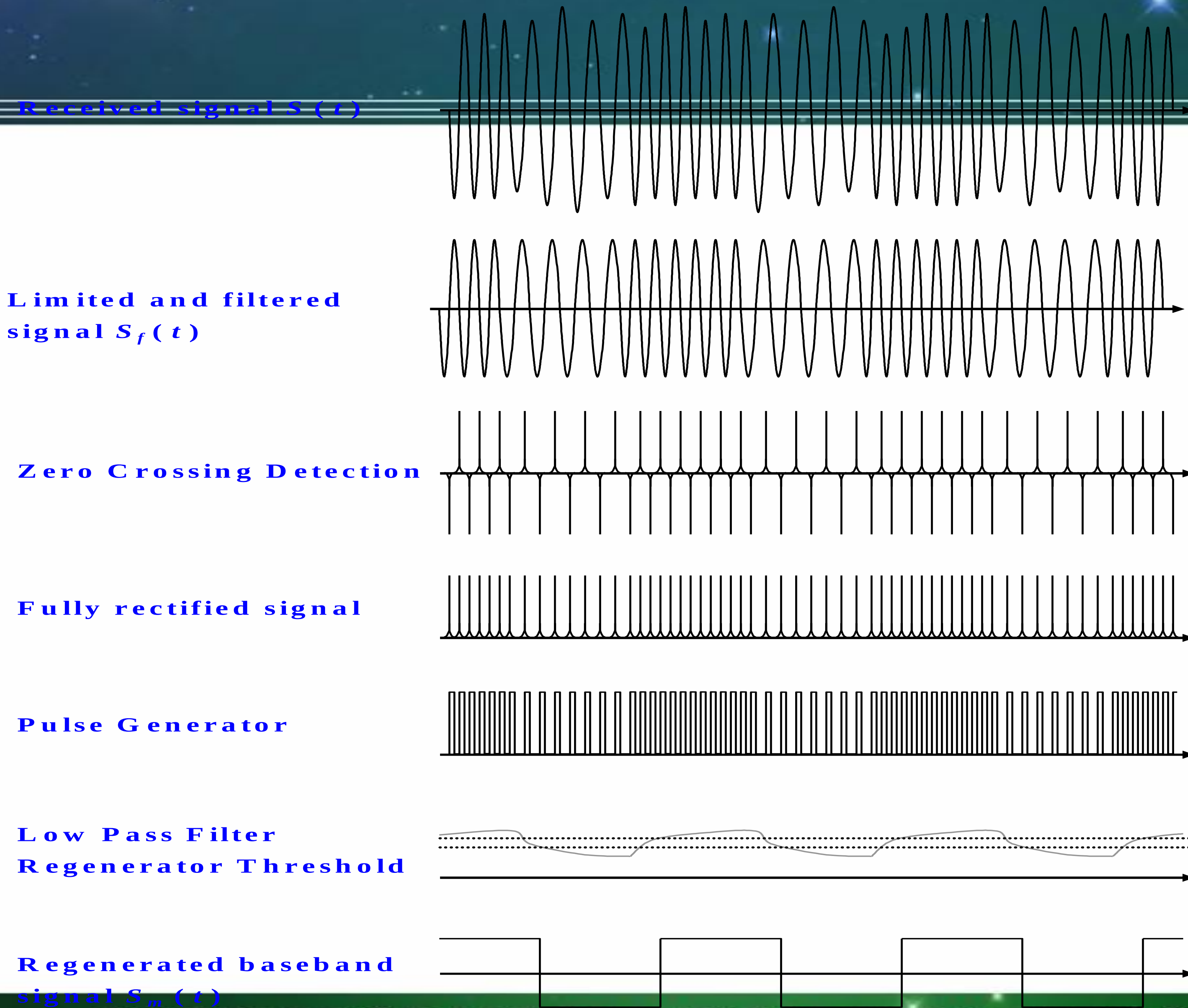


$$S_f(t) = A_f \cdot \cos[\omega_c t + \phi(t)]$$

$$S_{VCO}(t) = A_{VCO} \cdot \sin[\omega_0 t + \phi_0(t)]$$

Demodulation by Zero Crossing Detection

- **Zero crossing detector**
- Positive voltage.
- Negative voltage.
- Pulse generator.
- low-pass filter.
- The advantage of zero crossing detection (and FM-to-AM conversion) is that no source of the carrier frequency is required to demodulate the signal. A digital signal can easily be recovered from a FM signal in this manner.
- Decoding an analog signal may be difficult by this method, since the signal at the low-pass filter output does not closely resemble the baseband signal.


Received signal $S(t)$
Limited and filtered signal $S_f(t)$
Zero Crossing Detection
Fully rectified signal
Pulse Generator
Low Pass Filter
Regenerator Threshold
Regenerated baseband signal $S_m(t)$

Regeneration of Digital Signals and Bias Distortion

- To produce rectangular pulses, we send the demodulated signal to a regenerator, which detects whether the signal level is above a certain threshold.
- A poorly adjusted regenerator threshold can cause “bias distortion”, where the digital signal produced is not identical to the original signal.

$$BD = 1 - \frac{\tau_{reg}}{\tau_{orig}}$$

Original digital signal

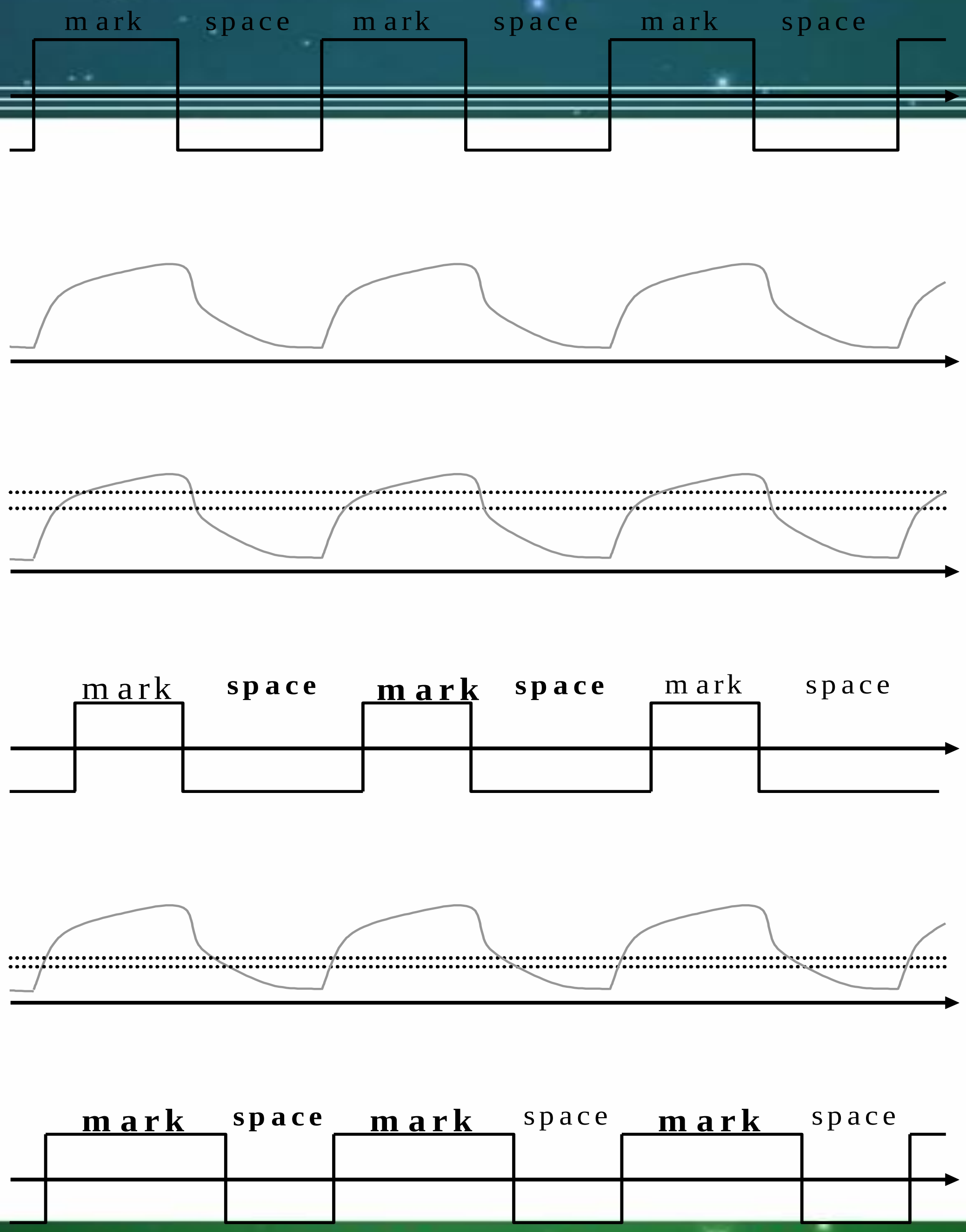
Demodulated signal

Regenerator threshold is too high

Regenerated signal with positive bias distortion

Regenerator threshold is too low

Regenerated signal with negative bias distortion



Noise is any signal that interferes with a transmitted signal. It can be another message signal, a random fluctuation in the amount of signal attenuation, environmental noise, or additional voltages introduced by the transmitting or receiving equipment.

$$N = k \cdot T \cdot W$$

k: the Boltzmann constant = 1.3710×10^{-23} Joules per degree Kelvin
T: temperature degrees Kelvin;
W: bandwidth in Hertz

- The channel capacity is the maximum rate at which data can be accurately transmitted over a given communication link (transmission line or radio link) under a given set of conditions.
- Shannon proved that if signals are sent with power S over a transmission line perturbed by AWGN of power *N*, the upper limit to the channel capacity in bits per second is:

$$C = W \cdot \log_2\left(1 + \frac{S}{N}\right)$$

- ***W*:** bandwidth of the channel in Hertz
- ***S*:** power of the signal in the transmission bandwidth
- ***N*:** power of the noise in the transmission bandwidth

Effect of Noise on Angle Modulation

- In this section, we study the performance of angle-modulated signals when contaminated by additive white Gaussian noise (AWGN)
- We will also compare this with the performance of AM signals.
- Recall that in AM, the message is contained in the amplitude of the modulated signal
- Since noise is additive, the noise is directly added to the signal.
- However, in a frequency-modulated signal, the noise is added to the amplitude and the message is contained in the frequency of the modulated signal.
- Therefore, the message is contaminated by the noise to the extent that the added noise changes the frequency of the modulated signal.
- The frequency of a signal can be described by its zero crossings.
- So the effect of additive noise on the demodulated FM signal can be described by the changes that it produces in the zero crossings of the modulated FM signal.

Effect of Noise on Angle Modulation

- A figure shown in below is the effect of additive noise on zero crossings of two FM signals, one with high power and the other with low power.
- From the previous discussion and also from the figure it should be clear that the effect of noise in an FM system is different from that for an AM system.
- We also observe that the effect of noise in a low-power FM system is more severe than in a high-power FM system.
- In a low power signal, noise causes more changes in the zero crossings.
- The analysis that we present in this chapter verifies our intuition based on these observations.

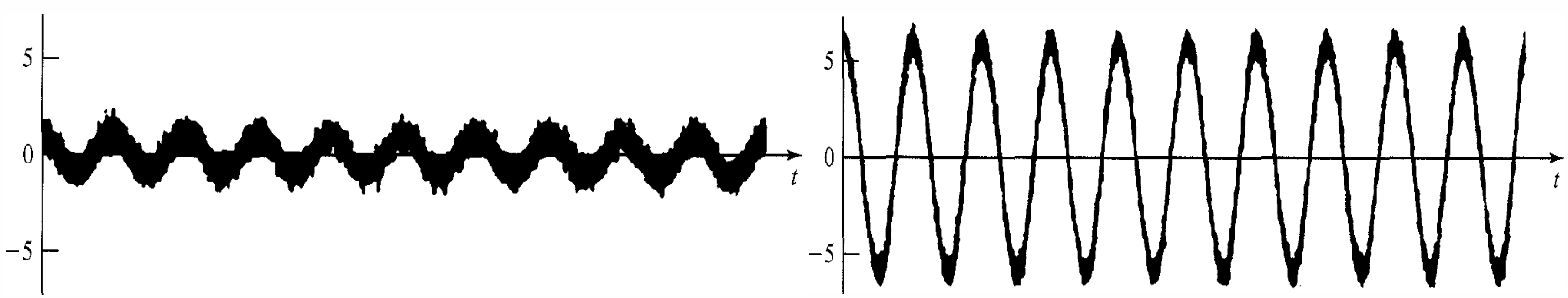


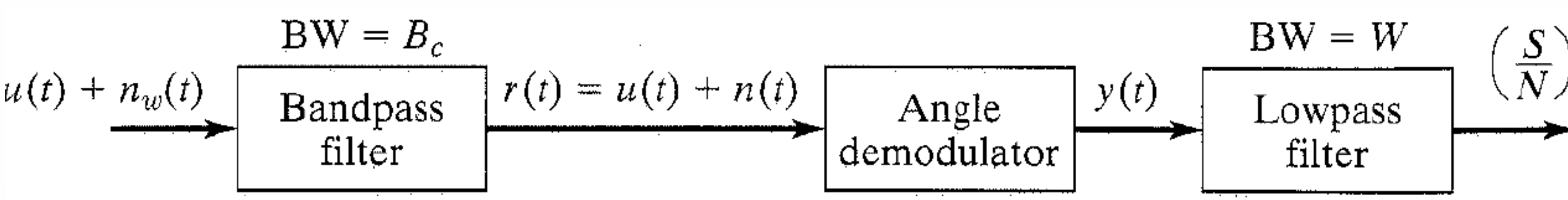
Fig. 6.1 Effect of noise in FM

Effect of Noise on Angle Modulation

- The receiver for a general angle-modulated signal is shown in below
- The angle-modulated signal is represented as

$$u(t) = A_c \cos\left(2\pi f_c t + \phi(t)\right) = \begin{cases} A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\,d\tau\right) & FM \\ A_c \cos\left(2\pi f_c t + k_p m(t)\right) & PM \end{cases}$$

- The AWGN $n_w(t)$ is added to $u(t)$, and the result is passed through a noise-limiting filter whose role is to remove the out-of-band noise.
- The bandwidth of this filter is equal to that of the modulated signal
- Therefore, it passes the modulated signal without distortion.
- However, it eliminates the out-of-band noise.
- Hence, the noise output of the filter is a filtered noise denoted by n(t).



Effect of Noise on Angle Modulation

- The output of this filter is

$$r(t) = u(t) + n(t) = u(t) + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

- A precise analysis is complicate due to the nonlinearity of demodulation .
- Let us assume that the signal power is much higher than the noise power.
- Then, the bandpass noise is represented as

$$n(t) = \sqrt{n_c^2(t) + n_s^2(t)}\cos\left(2\pi f_c t + \arctan\frac{n_s(t)}{n_c(t)}\right) = V_n(t)\cos(2\pi f_c t + \Phi_n(t))$$

  – where $V_n(t)$ and $\Phi_n(t)$ represent the envelope and the phase of the bandpass noise process, respectively.

Effect of Noise on Angle Modulation

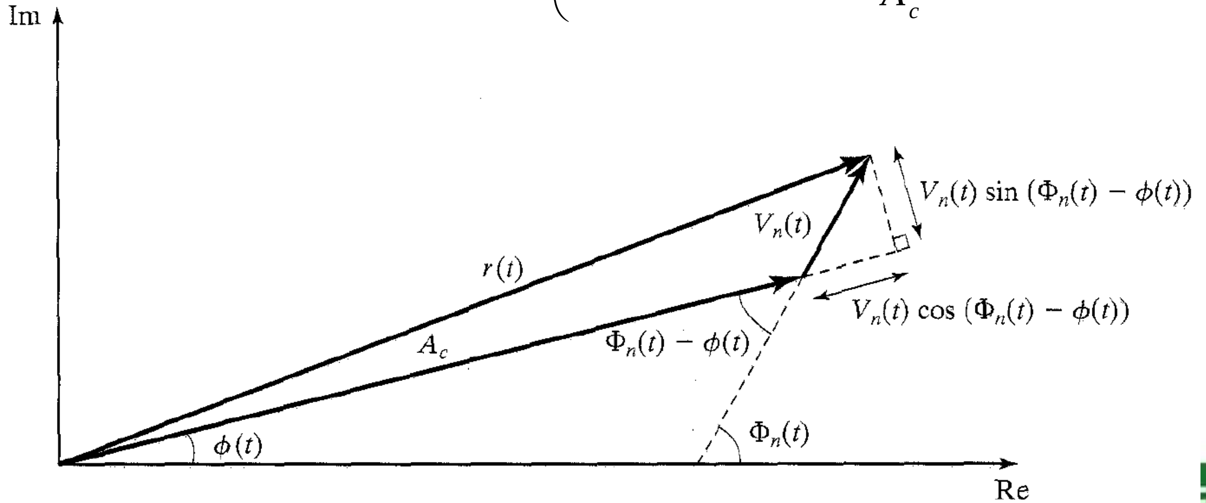
- Assume that the signal is much larger than the noise, that is,

$$P\left(V_n(t) \ll A_c\right) \approx 1$$

- The phasor diagram of signal and noise are shown in below.
- From this figure, it is obvious that we can write

$$r(t) \approx \left[A_c + V_n(t)\cos\left(\Phi_n(t) - \phi(t)\right)\right]\cos\left(2\pi f_c t + \phi(t) + \arctan\frac{V_n(t)\sin\left(\Phi_n(t) - \phi(t)\right)}{A_c + V_n(t)\cos\left(\Phi_n(t) - \phi(t)\right)}\right)$$

$$\approx \left[A_c + V_n(t)\cos\left(\Phi_n(t) - \phi(t)\right)\right]\cos\left(2\pi f_c t + \phi(t) + \frac{V_n(t)}{A_c}\sin\left(\Phi_n(t) - \phi(t)\right)\right)$$



Effect of Noise on Angle Modulation

- Noting that $$\phi(t) = \begin{cases} k_p m(t), & PM \\ 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau, & FM \end{cases}$$
- We see that the output of the demodulator is given by

$$y(t) = \begin{cases} \phi(t) + \frac{V_n(t)}{A_c} \sin\left(\Phi_n(t) - \phi(t)\right) & PM \\ \frac{1}{2\pi} \frac{d}{dt}\left[\phi(t) + \frac{V_n(t)}{A_c} \sin\left(\Phi_n(t) - \phi(t)\right)\right] & FM \end{cases}$$

$$= \begin{cases} k_p m(t) + \frac{V_n(t)}{A_c} \sin\left(\Phi_n(t) - \phi(t)\right) & PM \\ k_f m(t) + \frac{1}{2\pi} \frac{d}{dt} \frac{V_n(t)}{A_c} \sin\left(\Phi_n(t) - \phi(t)\right) & FM \end{cases} = \begin{cases} k_p m(t) + Y_n(t) & PM \\ k_f m(t) + \frac{1}{2\pi} \frac{d}{dt} Y_n(t) & FM \end{cases}$$

  – where we define $$Y_n(t) \overset{def}{=\!=} \frac{V_n(t)}{A_c} \sin\left(\Phi_n(t) - \phi(t)\right)$$

Effect of Noise on Angle Modulation

$$y(t) = \begin{cases} k_p m(t) + \dfrac{V_n(t)}{A_c} \sin\left(\Phi_n(t) - \phi(t)\right) & PM \\ k_f m(t) + \dfrac{1}{2\pi}\dfrac{d}{dt}\dfrac{V_n(t)}{A_c} \sin\left(\Phi_n(t) - \phi(t)\right) & FM \end{cases} = \begin{cases} k_p m(t) + Y_n(t) & PM \\ k_f m(t) + \dfrac{1}{2\pi}\dfrac{d}{dt} Y_n(t) & FM \end{cases}$$

(Eq. 6.2.7)

- The first term in above equation is the desired signal component.
- The second term is the noise component.
- The noise component is inversely proportional to the signal amplitude A_c.
- Hence, the higher the signal level, the lower the noise level.
- This is in agreement with the intuitive reasoning presented at the beginning of this section and based on Fig. 6.1.
- This is not the case with amplitude modulation.
- In AM systems, the noise component is independent of the signal component, and a scaling of the signal power does not affect the received noise power.

Effect of Noise on Angle Modulation

- The properties of the noise component

$$Y_n(t) = \frac{V_n(t)}{A_c}\sin\left(\Phi_n(t) - \phi(t)\right) = \frac{1}{A_c}\left[V_n(t)\sin\Phi_n(t)\cos\phi(t) - V_n(t)\cos\Phi_n(t)\sin\phi(t)\right]$$

$$= \frac{1}{A_c}\left[n_s(t)\cos\phi(t) - n_c(t)\sin\phi(t)\right]$$

- when we compare variations in $n_c(t)$ and $n_s(t)$, we can assume that $\phi(t)$ is almost constant, i.e., $\phi(t) \approx \phi$.

$$Y_n(t) = \frac{1}{A_c}\left[n_s(t)\cos\phi - n_c(t)\sin\phi\right]$$

$$= \frac{\cos\phi}{A_c} n_s(t) - \frac{\sin\phi}{A_c} n_c(t)$$

$$= a n_s(t) - b n_c(t),$$ where $a = \cos\phi/A_c$ and $b = -\sin\phi/A_c$

Effect of Noise on Angle Modulation

- By using the result of Exercise *5.3.3,* we have

$$S_{Y_n}(f) = \left(a^2 + b^2\right) S_{n_c}(f) = \frac{S_{n_c}(f)}{A_c^2}$$

- $S_{nc}(f)$ is the power spectral density (psd) of the in-phase component of the filtered noise given in (Eq. *5.3.10*).

$$S_{n_c}(f) = \begin{cases} N_0 & |f| < \frac{B_c}{2} \\ 0 & otherwise \end{cases}$$

  – Note that the bandwidth of the filtered noise extends from $f_c - B_c/2$ to $f_c + B_c/2$. Hence, the spectrum of $n_c(t)$ extends from $-B_c/2$ to $+B_c/2$.

- Therefore

$$S_{Y_n}(f) = \begin{cases} \frac{N_0}{A_c^2} & |f| < \frac{B_c}{2} \\ 0 & otherwise \end{cases} \qquad \text{(Eq. 6.2.13)}$$

Effect of Noise on Angle Modulation

- This equation provides an expression for the power spectral density of the filtered noise at *the front end of the receiver.*
- After demodulation, another filtering is applied; this reduces the noise bandwidth to W, which is the bandwidth of the message signal.
- Note that in the case of FM modulation, as seen in (Eq. *6.2.7),* the process $Y_n(t)$ is differentiated and scaled by $1/2\pi$.
- The PSD of the process $(1/2\pi)\,(dY_n(t)/dt)$ is given by (see Eq. *5.2.17)*

$$\frac{4\pi^2 f^2}{4\pi^2} S_{Y_n}(f) = f^2 S_{Y_n}(f) = \begin{cases} \frac{N_0}{A_c^2} f^2 & |f| < \frac{B_c}{2} \\ 0 & otherwise \end{cases} \quad \text{(Eq. 6.2.14)}$$

Effect of Noise on Angle Modulation

- In PM, the demodulated-noise PSD is given by (Eq. *6.2.13)*
- In FM, it is given by (Eq. *6.2.14).*
- In both cases, $B_c/2$ must be replaced by *W* after Lowpass filter.
- Hence, for *|f|<W*

$$S_{n_0}(f) = \begin{cases} \frac{N_0}{A_c^2} & PM \\ \frac{N_0}{A_c^2} f^2 & FM \end{cases}$$

- Fig. *6.4* shows the power spectrum of the noise component at the output of the demodulator, for PM and FM.

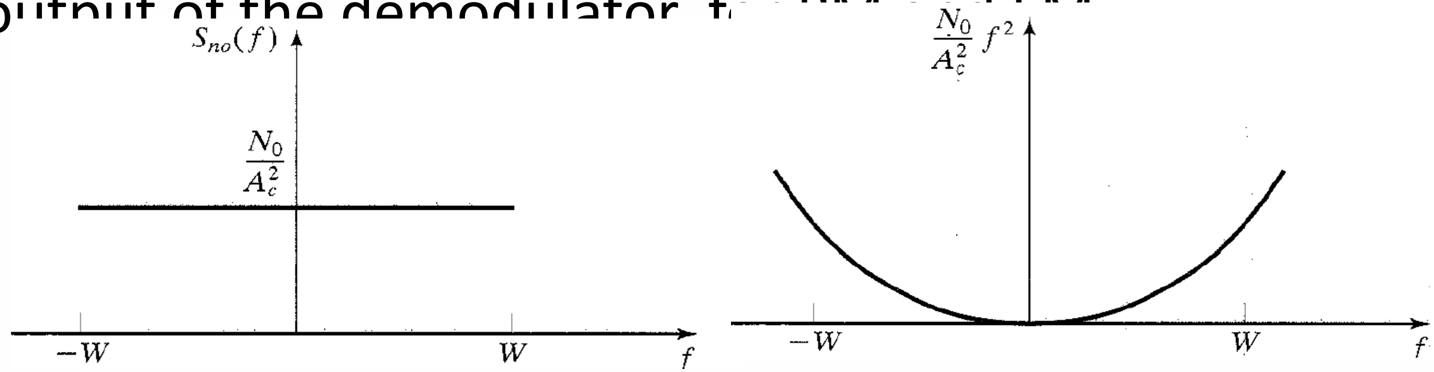


Fig. 6.4 Noise power spectrum at demodulator output for |*f*|<W in (a) PM (b) and (b) FM.

Effect of Noise on Angle Modulation

- It is interesting to note that **PM has a flat noise spectrum and FM has a parabolic noise spectrum**.
- Therefore, the effect of noise in FM for higher frequency components is much higher than the effect of noise on lower frequency components.
- The noise power at the output of the lowpass filter is the noise power in the frequency range [W, +W].
- Therefore, it is given by

$$P_{n_0} = \int_{-W}^{W} S_{n_0}(f)\,df = \begin{cases} \int_{-W}^{W} \frac{N_0}{A_c^2}\,df \\ \int_{-W}^{W} \frac{N_0}{A_c^2} f^2\,df \end{cases} = \begin{cases} \frac{2WN_0}{A_c^2} & PM \\ \frac{2N_0W^3}{3A_c^2} & FM \end{cases} \quad \text{(Eq. 6.2.16)}$$

Effect of Noise on Angle Modulation

- (Eq. 6.2.7) is used to determine the output SNR in angle modulation.
- First, we have the output signal power $P_{S_O} = \begin{cases} k_p^2 P_M & PM \\ k_f^2 P_M & FM \end{cases}$
- Then the SNR, which is defined as

$$\left(\frac{S}{N}\right)_O \stackrel{def}{=\!=} \frac{P_{S_O}}{P_{n_O}} \longrightarrow \left(\frac{S}{N}\right)_O = \begin{cases} \frac{k_p^2 A_c^2}{2} \frac{P_M}{N_0 W} & PM \\ \frac{3 k_f^2 A_c^2}{2W^2} \frac{P_M}{N_0 W} & FM \end{cases}$$

- Noting that $A_c^2/2$ is the received signal power, denoted by P_R, and

$$\begin{cases} \beta_p = k_p \max\left[\left|m(t)\right|\right] & PM \\ \beta_f = \frac{k_f \max\left[\left|m(t)\right|\right]}{W} & FM \end{cases} \longrightarrow \left(\frac{S}{N}\right)_O = \begin{cases} P_R \left(\frac{\beta_p}{\max\left[\left|m(t)\right|\right]}\right)^2 \frac{P_M}{N_0 W} & PM \\ 3 P_R \left(\frac{\beta_f}{\max\left[\left|m(t)\right|\right]}\right)^2 \frac{P_M}{N_0 W} & FM \end{cases}$$

Effect of Noise on Angle Modulation

- **In FM, the effect of noise is higher at higher frequencies**. This means that signal components at higher frequencies will suffer more from noise than signal components at lower frequencies.
- **To compensate for this effect, preemphasis and deemphasis filtering are used**.

Effect of Noise on Angle Modulation

- Denote $$\left(\frac{S}{N}\right)_b = \frac{P_M}{N_0 W}$$

- The SNR of a baseband system with the same received power, we obtain
$$\left(\frac{S}{N}\right)_O = \begin{cases} \dfrac{P_M \beta_p^2}{\left(\max |m(t)|\right)^2}\left(\dfrac{S}{N}\right)_b & PM \\ 3\dfrac{P_M \beta_f^2}{\left(\max |m(t)|\right)^2}\left(\dfrac{S}{N}\right)_b & FM \end{cases}$$

- $P_M / \left(\max |m(t)|\right)^2$: The average-to-peak-power-ratio of the message signal (or equivalently, the power content of the normalized message, P_{Mn}).
$$\left(\frac{S}{N}\right)_O = \begin{cases} \beta_p^2 P_{M_n}\left(\dfrac{S}{N}\right)_b & PM \\ 3\beta_f^2 P_{M_n}\left(\dfrac{S}{N}\right)_b & FM \end{cases}$$

Effect of Noise on Angle Modulation

- Now using Carson's rule B_c = 2(β + 1)W, we can express the output SNR in terms of the bandwidth expansion factor, which is defined as the ratio of the channel bandwidth to the message bandwidth and is denoted by Ω :

$$\Omega \overset{def}{=\!=} \frac{B_c}{W} = 2(\beta + 1)$$

- From this relationship, we have β =Ω/2 – 1.
- Therefore,

$$\left(\frac{S}{N}\right)_O = \begin{cases} P_M \left(\dfrac{\frac{\Omega}{2} - 1}{\max|m(t)|}\right)^2 \left(\dfrac{S}{N}\right)_b & PM \\ 3P_M \left(\dfrac{\frac{\Omega}{2} - 1}{\max|m(t)|}\right)^2 \left(\dfrac{S}{N}\right)_b & FM \end{cases}$$

Effect of Noise on Angle Modulation

$$\left(\frac{S}{N}\right)_O = \begin{cases} P_R\left(\dfrac{\beta_p}{\max\left[\left|m(t)\right|\right]}\right)^2 \dfrac{P_M}{N_0 W} & PM \\ 3P_R\left(\dfrac{\beta_f}{\max\left[\left|m(t)\right|\right]}\right)^2 \dfrac{P_M}{N_0 W} & FM \end{cases}$$

$$\left(\frac{S}{N}\right)_O = \begin{cases} P_M\left(\dfrac{\frac{\Omega}{2}-1}{\max\left|m(t)\right|}\right)^2 \left(\dfrac{S}{N}\right)_b & PM \\ 3P_M\left(\dfrac{\frac{\Omega}{2}-1}{\max\left|m(t)\right|}\right)^2 \left(\dfrac{S}{N}\right)_b & FM \end{cases}$$

- **Observations**
  - In both PM and FM, the output SNR is proportional to β^2. Therefore, increasing β increases the output SNR.
  - Increasing β increase the bandwidth (from Carson's rule).

  So **angle modulation provides a way to trade off bandwidth for transmitted power**.

Effect of Noise on Angle Modulation

- Although we can increase the output SNR by increasing β, having a large β means having a large B_c (by Carson's rule).
- Having a large B_c means having a large noise power at the input of the demodulator. This means that the approximation $P(V_n(t) << A_c) \approx 1$ will no longer apply and that the preceding analysis will not hold.
- In fact, if we increase β such that the preceding approximation does not hold, a phenomenon known as the *threshold* effect will occur and the signal will be lost in the noise.
- This means that **although increasing the modulation index, β, up to a certain value improves the performance of the system, this cannot continue indefinitely**.
- After a certain point, increasing β will be harmful and deteriorates the performance of the system.

Effect of Noise on Angle Modulation

- A comparison of the preceding result with the SNR in AM shows that, **in both cases (AM and angle modulation), increasing the transmitter power** (and consequently the received power) **will increase the output SNR**
- But the mechanisms are totally different. **In AM, any increase in the received power directly increases the signal power** at the output of the demodulator.
- This is basically because the message is in the amplitude of the transmitted signal and an increase in the transmitted power directly affects the demodulated signal power.
- However, in angle modulation, the message is in the phase of the modulated signal and increasing the transmitter power does not increase the demodulated message power.
- **In angle modulation, the output SNR is increased by a *decrease in the received noise power,*** as seen from Equation (6.2.16) and Fig. 6.1.

Effect of Noise on Angle Modulation

- **In FM, the effect of noise is higher at higher frequencies**. This means that signal components at higher frequencies will suffer more from noise than signal components at lower frequencies.
- **To compensate for this effect, preemphasis and deemphasis filtering are used**.

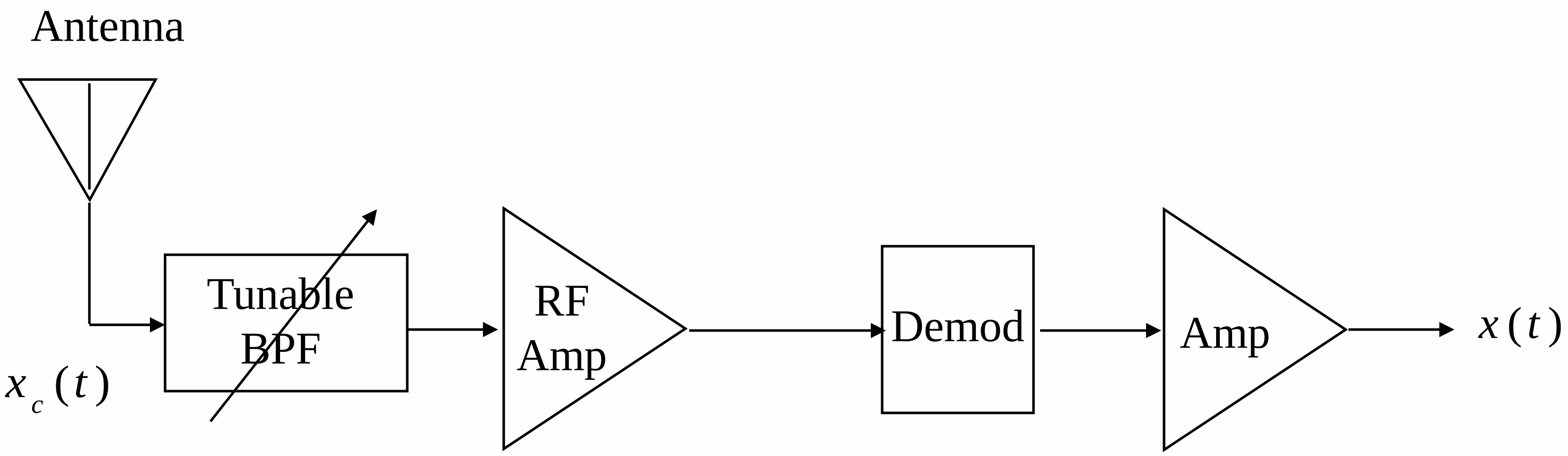
UNIT V
RECEIVERS AND SAMPLING THEORM

Types of Receivers

- Early days, the radio communication suffered from co-channel and adjacent-channel interference (CCI, ACI) from other stations. Then, resonance phenomenon was exploited to select desired signal at the Rx antenna and following circuit.
- Tuned RF receiver
- Superhetrodyne
- Direct conversion

Tuned RF Receiver (TRF)

- RF amplifier + demodulator
- All gain is accomplished with RF amplifier
- For the product detector TRF, selectivity and station selection is accomplished via a tuned RF amplifier
- “Crystal radio” is classic TRF



7.1.1 Superhetrodyne Receiver

- Input bandpass signal at f_c is mixed with a LO output to be hetrodyned (or “beat”) down to an intermediate frequency f_{IF} before detection.
- Tuning done via changing the local oscillator

- Adjacent channels are rejected via a selective BPF, called a CS (channel select) filter in the IF stage.
- The IF stage $\Rightarrow$ additional stage of gain $\Rightarrow$ the RF amplifier is not required to supply all the gain $\Rightarrow$ better stability

Superhetrodyne Equations

Superhet based on property of

$$\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta)$$

Multiplier output $\Rightarrow f_{LO} \pm f_c$ (sum and difference)

We usually choose "high side conversion" $\Rightarrow f_{IF} = f_{LO} - f_c$

$\Rightarrow$ For fixed $f_{IF} \Rightarrow$ tuning done via varying f_{LO}

Note: $f_{IF} = \left| f_{LO} - f_c \right|$

Superhet continued

Often the case we use

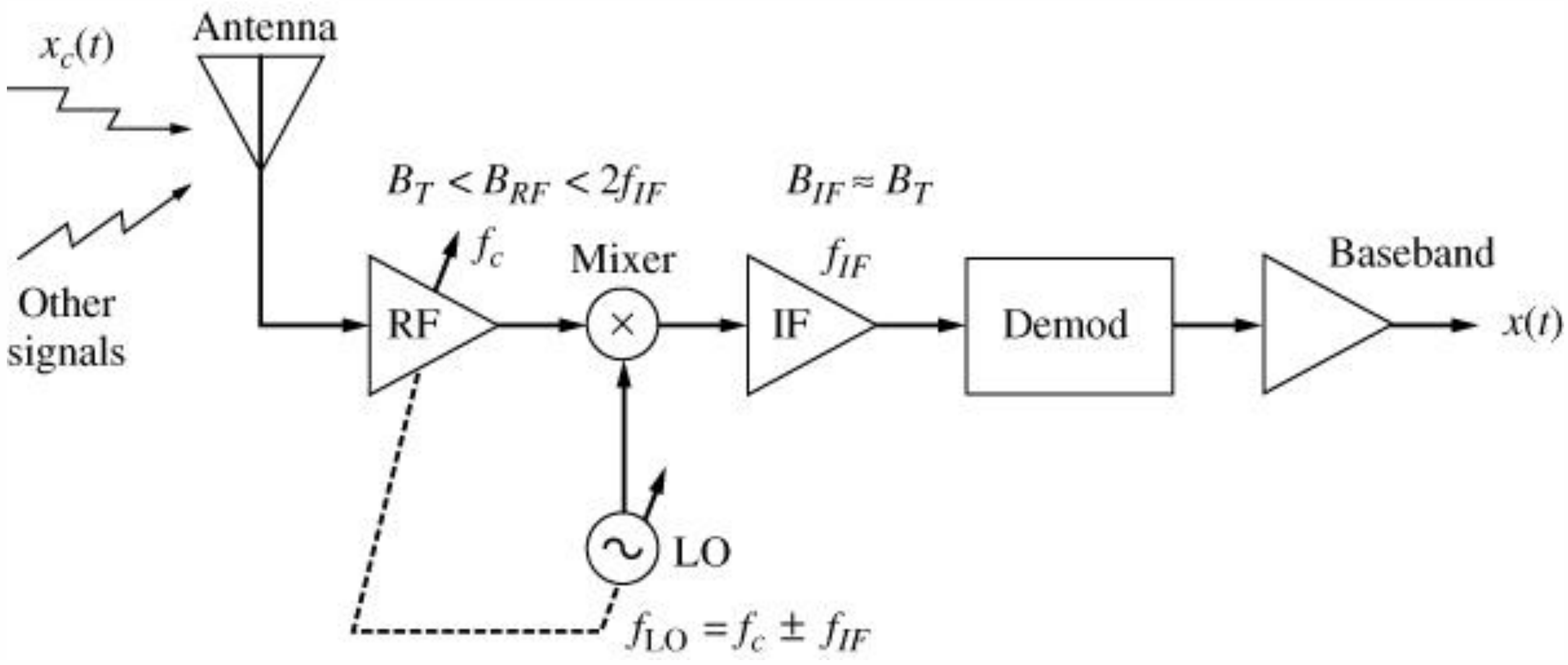
$$f_{LO} > f_{IF}$$

$$f_{IF} < f_c \text{ (the difference selected)}$$

$$f_c = f_{LO} - f_{IF} \Rightarrow \text{"high side conversion"}$$
$$\Rightarrow \text{increasing } f_{LO} \Rightarrow \text{increases } f_c$$

High side conversion does cause sideband reversal for SSB

7.1.1 Superhetrodyne Receiver



Why the Superhetrodyne vs the Tuned RF

- Easier to design a CS filter, = a selective IF BPF, with fixed frequency for adjacent channel rejection versus a tunable filter

- 2 stages of gain versus 1 stage of gain $\Rightarrow$ inherently more stable (i.e. it's more difficulty to design a high gain stable RF amp)

Superhets and Images (spurious signals)

Recall mixer output $f_{IF} = \left| f_{LO} - f_c \right| \Rightarrow$

$\Rightarrow f_{IF} = f_{LO} - f_c$ and $f_{IF} = f_c - f_{LO}$

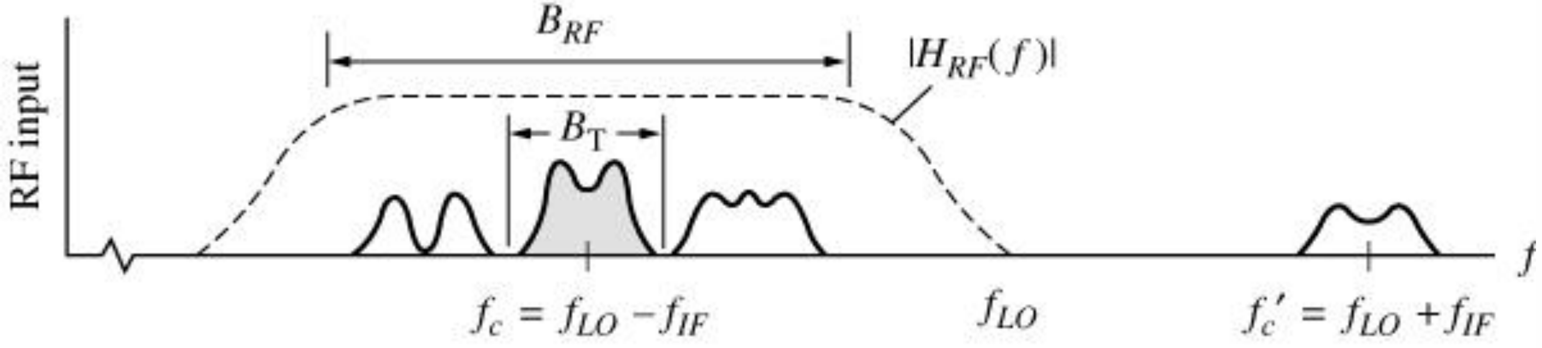
For fixed $f_{IF} \Rightarrow$

$$f_c = f_{LO} - f_{IF}$$

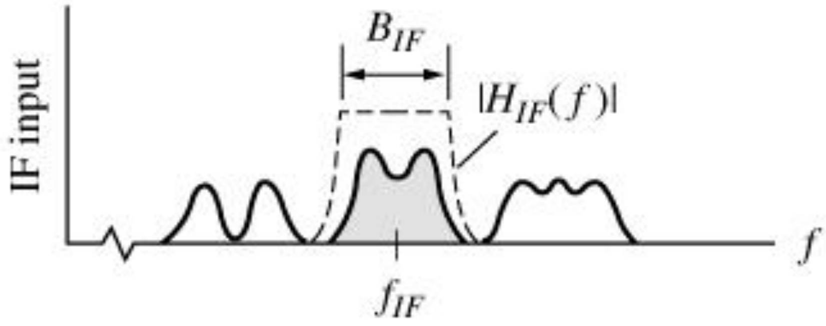
and

$$f_c' = f_c + 2 f_{IF} \Rightarrow \text{image frequency}$$

Superhet receiver waveforms



(a)



(b)

Image frequency,
image band

The signal at the image frequency $f_c^{'}$ is also passed through the IF BPF

$\Rightarrow$ the listener may not be certain of the input signal's frequency, is it f_c or $f_c^{'}$?

Image Example

Broadcast AM receiver with $f_{IF} = 455$ kHz, $f_c = 1000$ kHz

$\Rightarrow \; f_{LO} = 1455$ kHz.

$\left| f_{LO} - f_c \right| = \left| 1455\text{-}1000 \right| = 455$ kHz $\Rightarrow$ expected signal is accepted

What other input frequency is accepted?

With the above values of $f_{LO} = 1455$ and $f_{IF} = 455 \Rightarrow \; f_c^{'} = 1910$ kHz

Test: If the receiver input is 1910 kHz $\Rightarrow$ multiplier output is

$\left| f_{LO} - f_c^{'} \right| = \left| 1455\text{-}1910 \right| = 455$ kHz $\Rightarrow$ image signal is accepted

Image Minimization

- fd
- Add selective BPF or LPF at front end, called an IR (image reject) filter
- Use higher value of IF in combination with a lower order LPF or BPF at front end

Miscellaneous for Superhet

- **LO Harmonics**

The superhet is further subject to spurious inputs if the local oscillator has harmonics.
Harmonics may leak into the mixer stage.

- **Interfering signal feedthrough by nonlinearity of IF amp**

- **Gain Control**

an automatic gain control (AGC)
an automatic volume control (AVG) in an AM radio
an automatic frequency control (AFC) in an FM radio

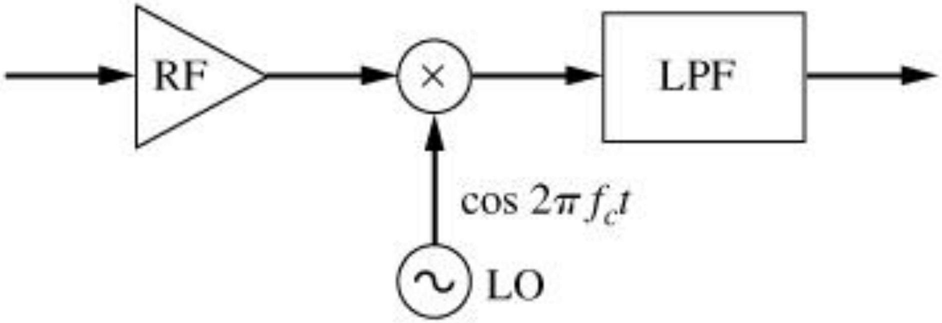
7.1.2 Direct Conversion Receivers

- TRF using a product detector
- Station selected via the local oscillator
- Selectivity for adjacent channel rejection in LPF stage
- Also called zero-IF or homodyne receiver
- Strictly speaking there are no images, but is subject to interference on the other sideband (see next 2 slides)
- Simple design and is often used

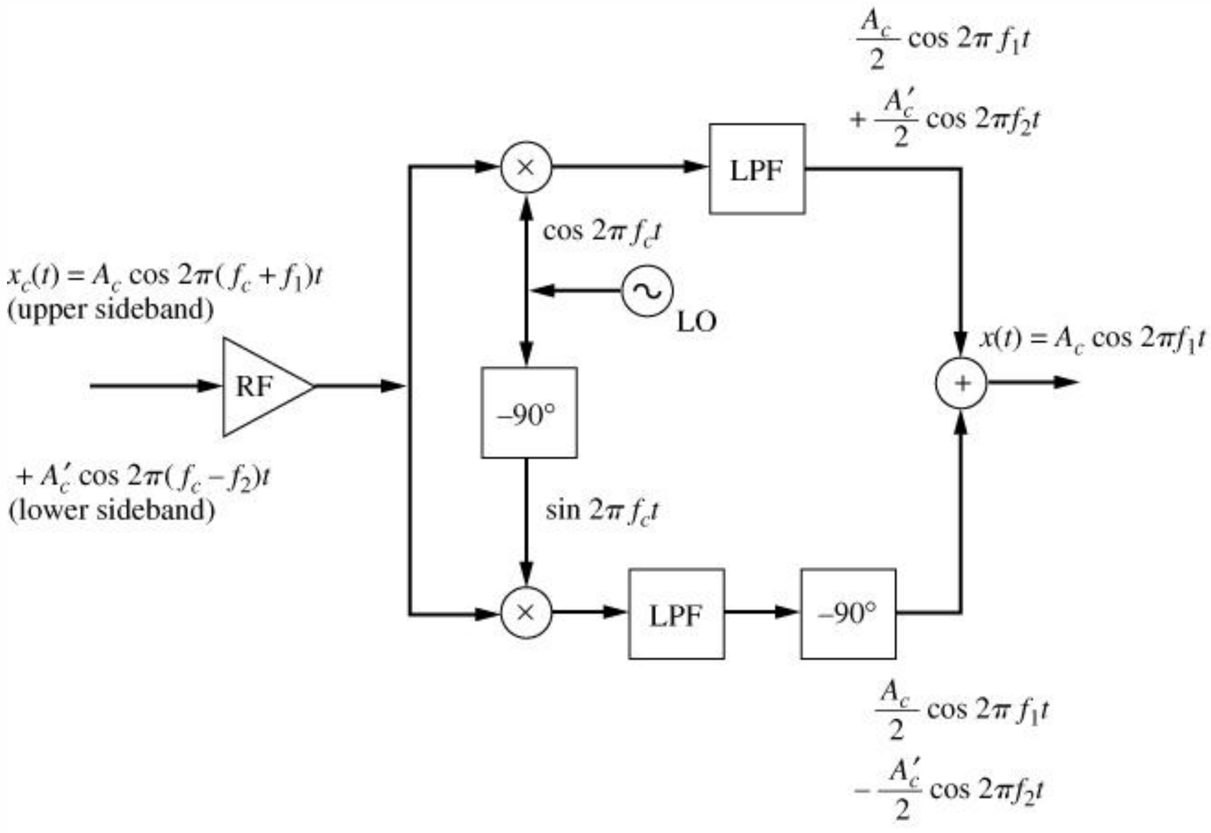
Direct conversion receiver

$$x_c(t) = A_c \cos 2\pi(f_c + f_1)t \text{ (upper sideband)}$$
$$+ A_c' \cos 2\pi(f_c - f_2)t \text{ (lower sideband)}$$

$$x(t) = \frac{A_c}{2} \cos 2\pi f_1 t + \frac{A_c'}{2} \cos 2\pi f_2 t$$



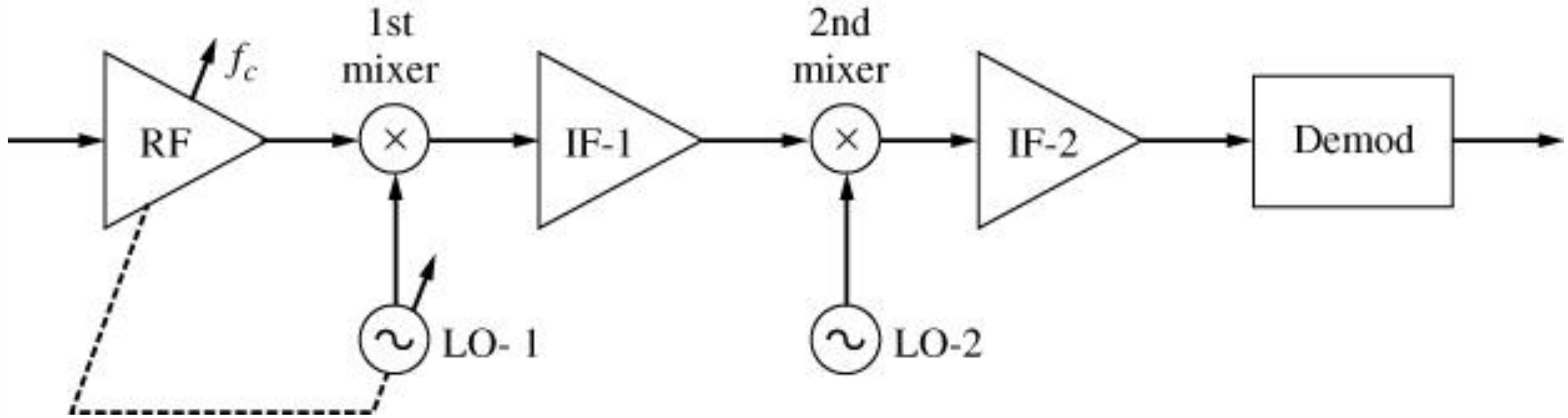
Direct conversion receiver with opposite sideband rejection



7.1.3 Double Conversion Receiver

- Additional IF stage, first stage with high IF for better image rejection.
- Put adjacent channel selectivity in the second IF stage BPF
- Additional gain
- Add a frequency converter to an existing receiver
- Can be subject to more spurious inputs

Double conversion receiver



Hetrodyne Receiver

- Superhet without the RF amplifier
- Often used at microwave frequencies with diode mixer

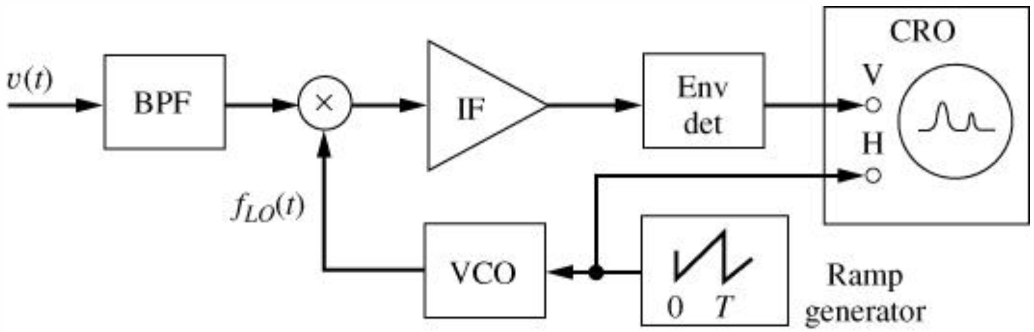
7.1.4 Receiver Performance Specifications

- **Sensitivity**: minimum input voltage required for a given signal-to-noise ratio
- **Dynamic range**: ability to retain linearity for varying signal strengths
- **Selectivity**: ability to reject adjacent channel signals
- **Noise figure**: how much noise does the receiver add to the signal
- **Image rejection**

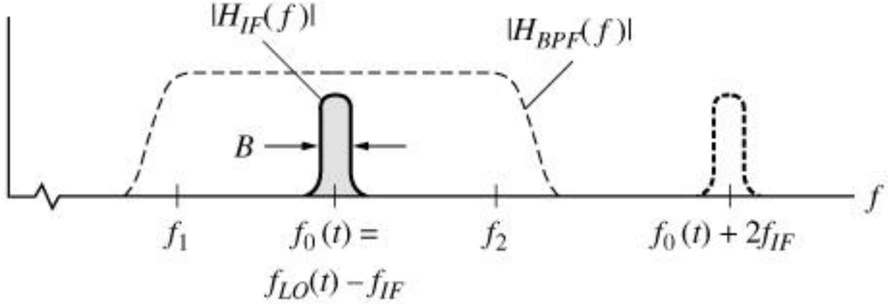
7.1.5 Scanning Spectrum Analyzer

- Power Spectral Density (PSD)
- Spectrum Analyzer
  - Scanning spectrum analyzer
  - DFT/FFT spectrum analyzer

Scanning spectrum analyzer (a) block diagram (b) amplitude response

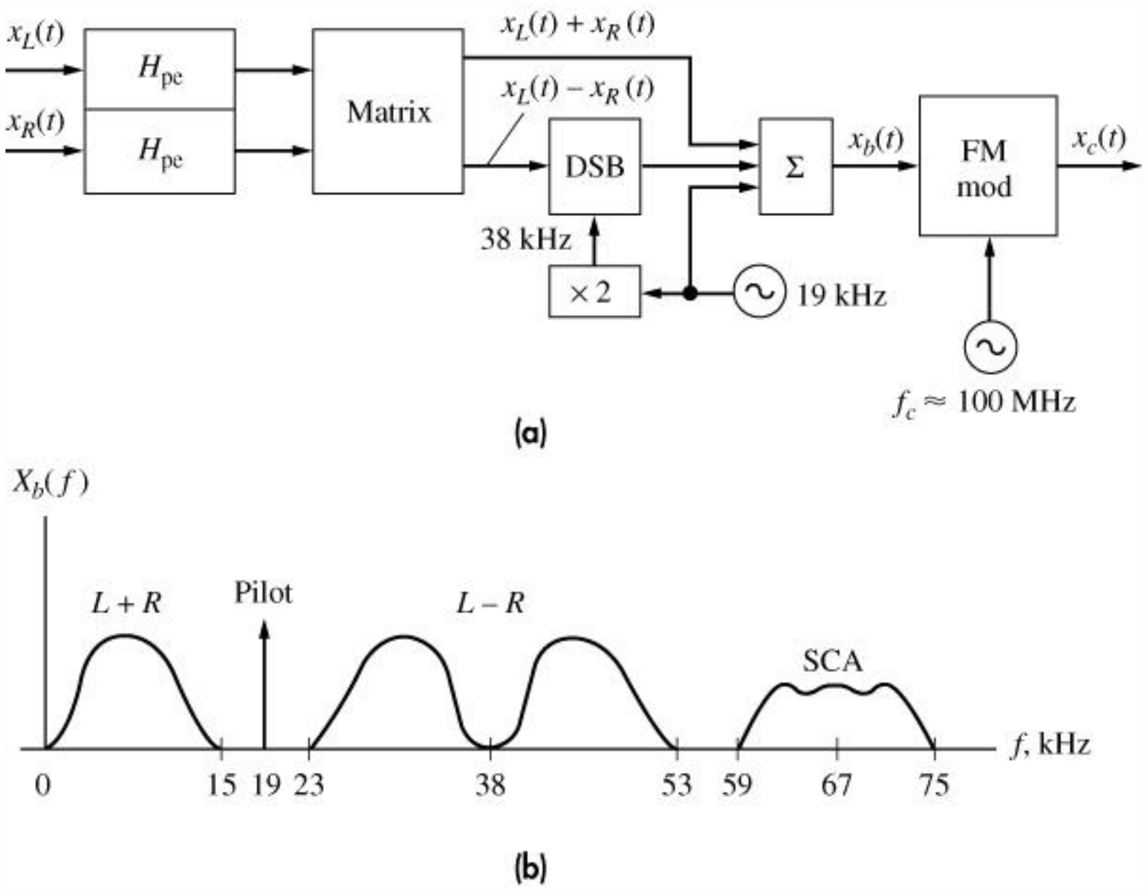


(a)

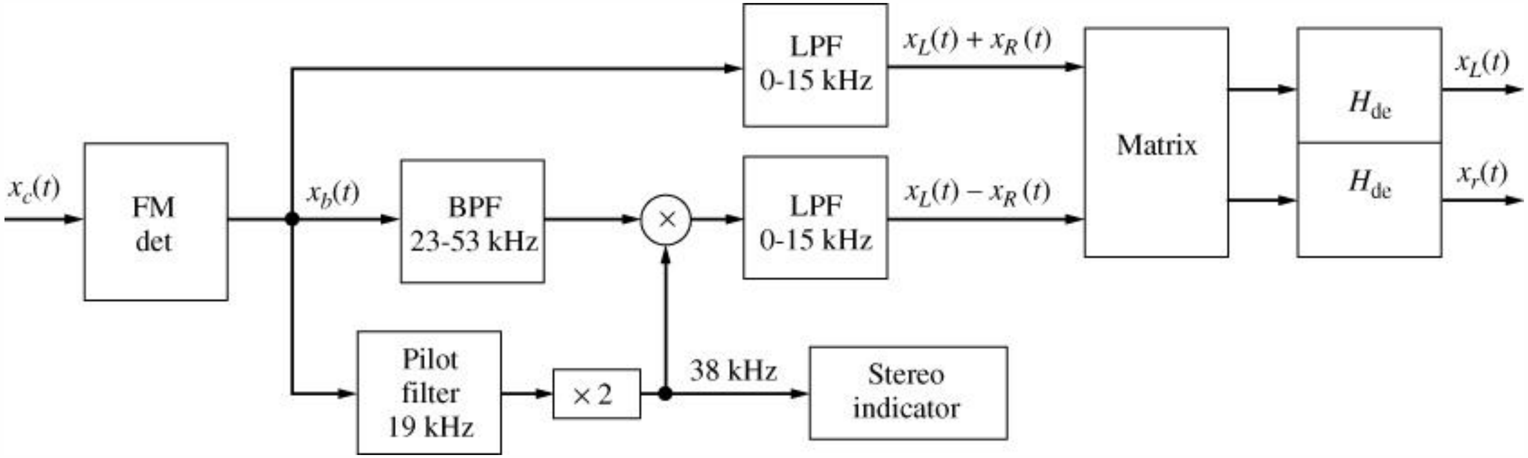


(b)

FM stereo multiplexing (a) transmitter (b) baseband spectrum



FM stereo multiplex receiver



Phase Locked Loops (PLL)

- Modulators,
- Demodulators,
- Frequency Synthesizers,
- Multiplexers, etc.

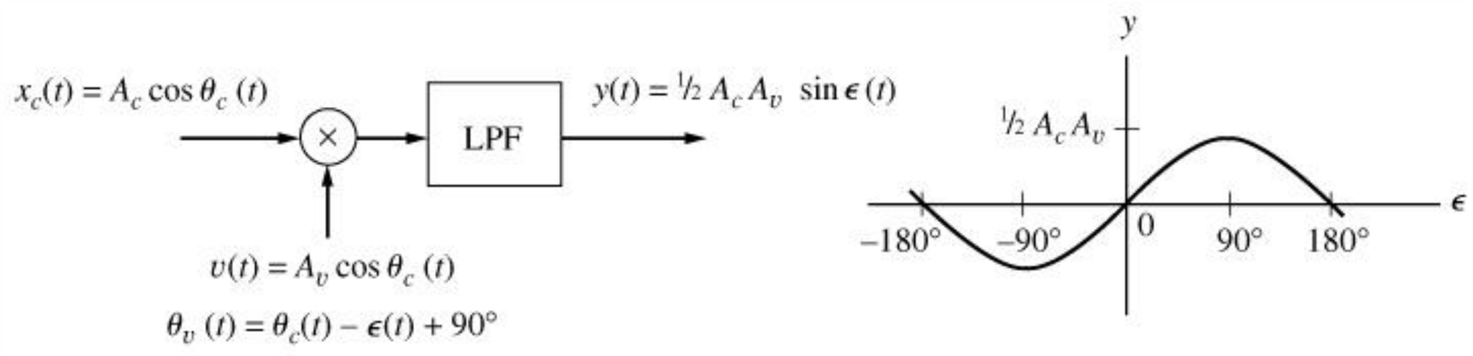
PLL Operations

- To lock or synchronize the instantaneous angle of a VCO output to that of an external bandpass signal
- Phase comparison is performed.
- DSB detection (Costas loop)
- Frequency synchronizer

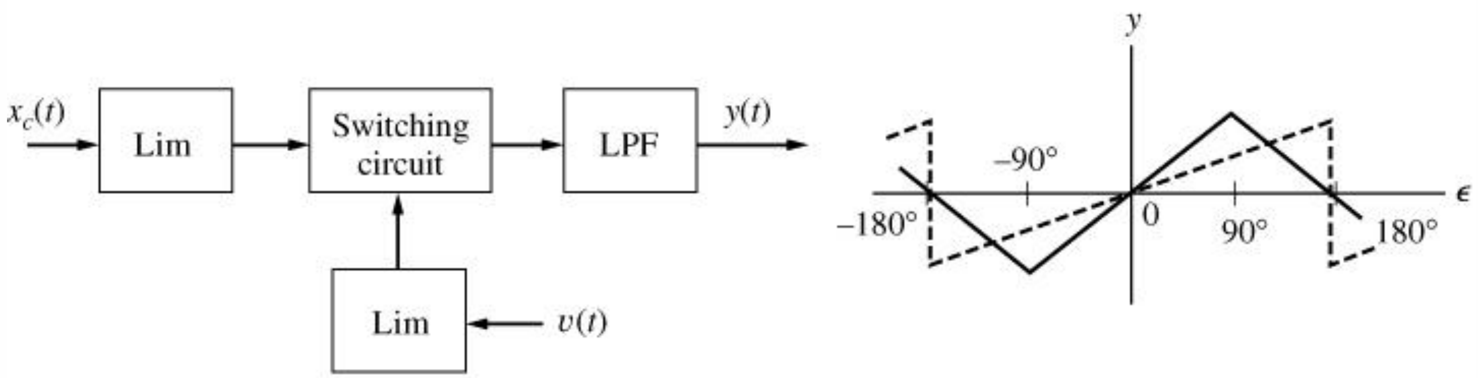
PLL Applications

- Synchronous detection
- FM and PM detection
- DSB detection (Costas loop)
- Frequency synchronizer

Phase comparators (a) analog (b) digital

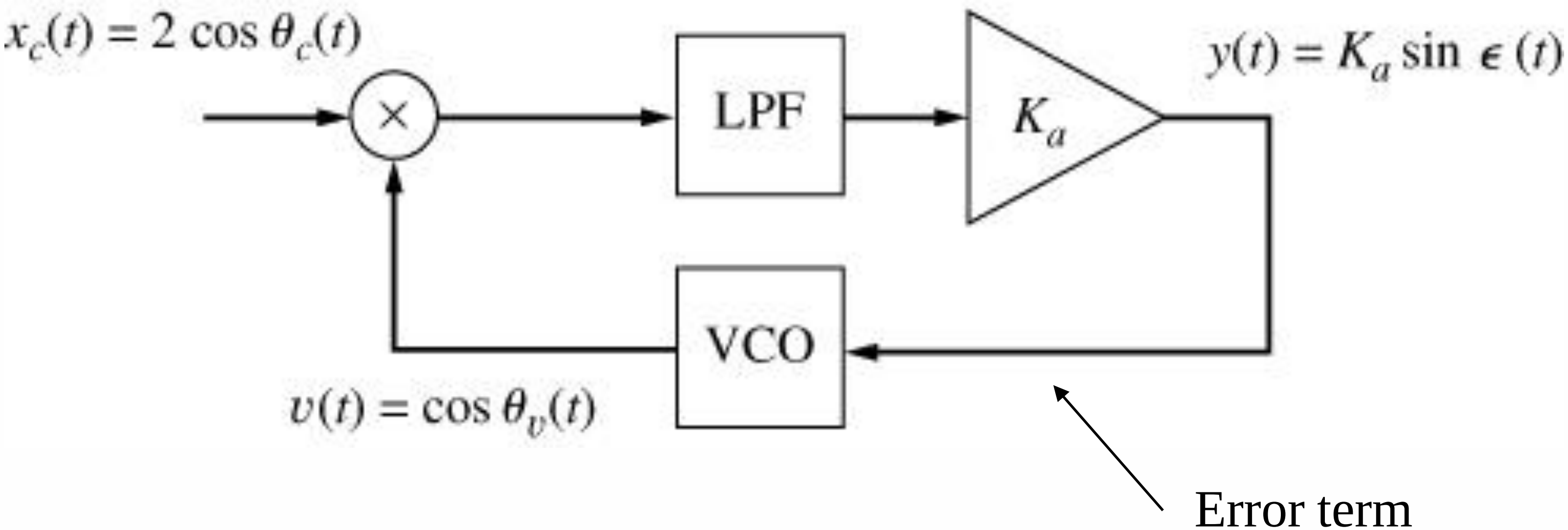


(a)



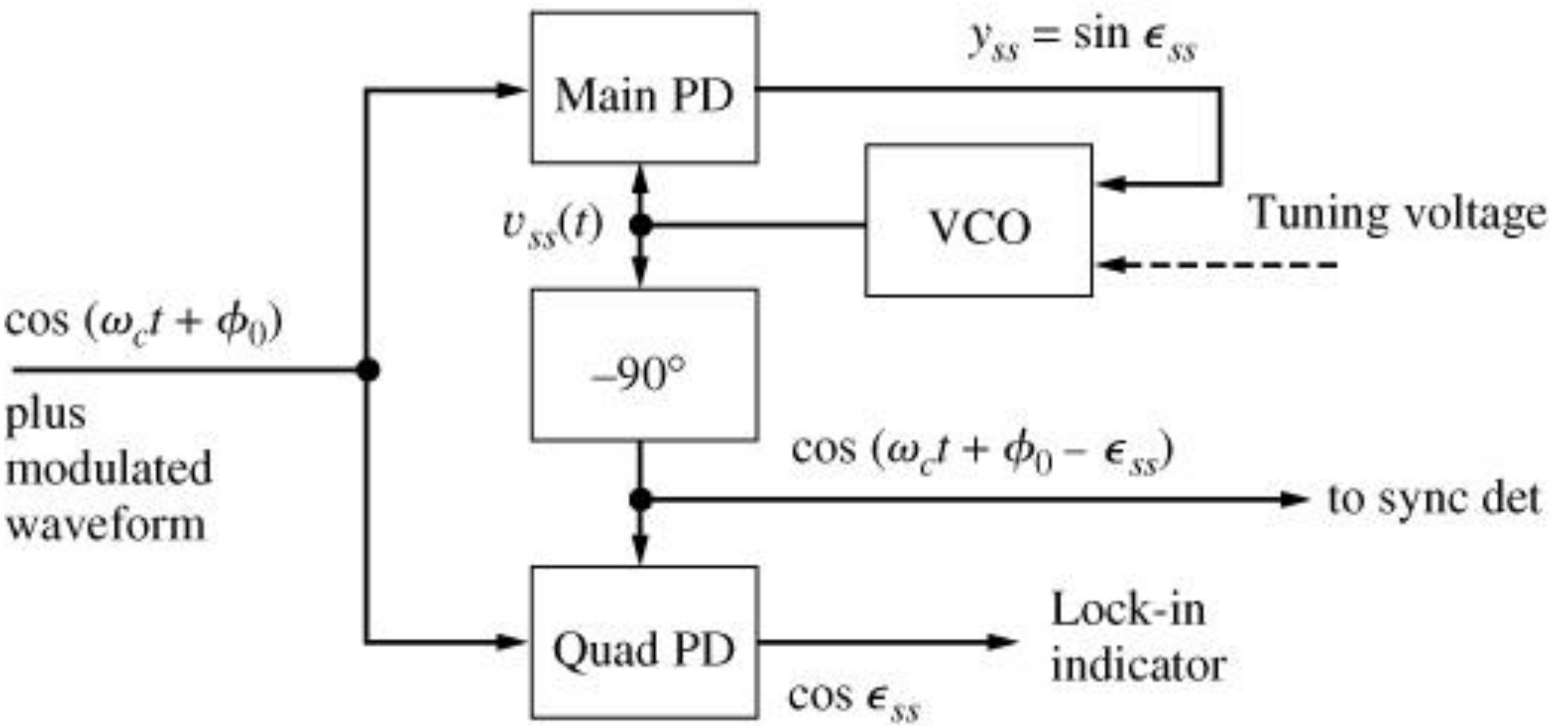
(b)

Phase-locked loop

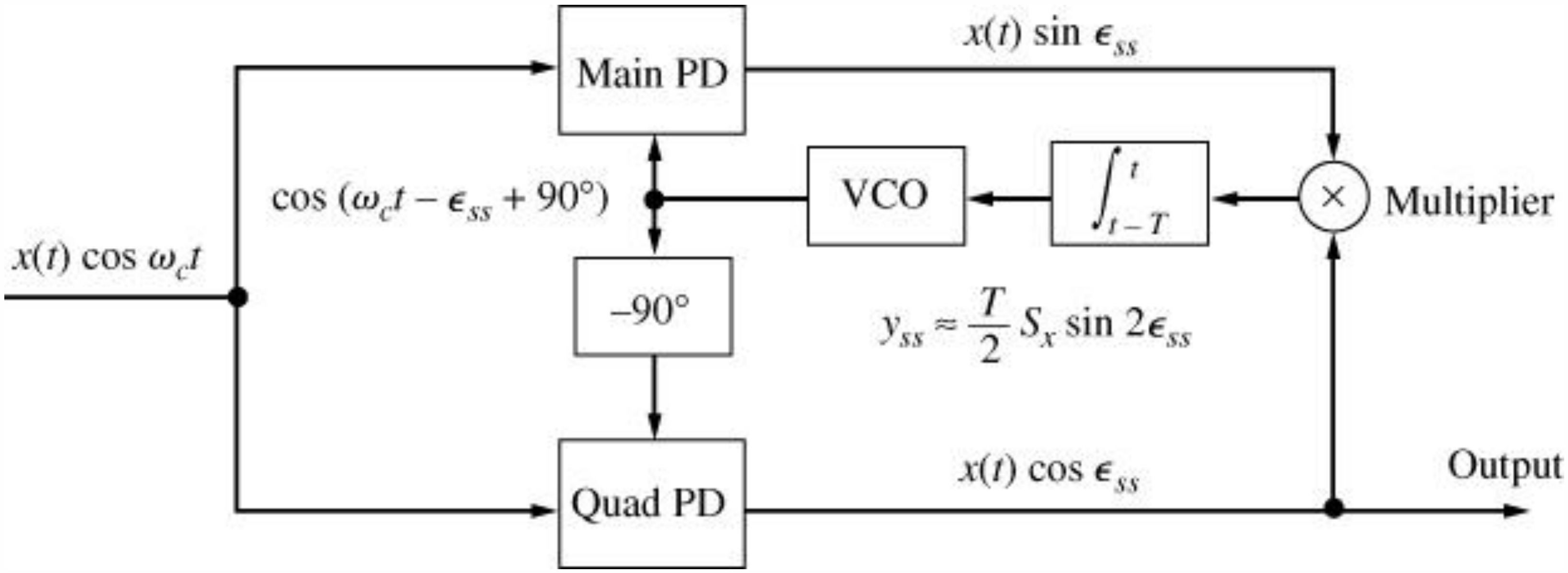


Note: $\sin[\varepsilon(t)] \approx \varepsilon(t)$ if $|\varepsilon(t)| < 1$

PLL pilot filter with two phase discriminators

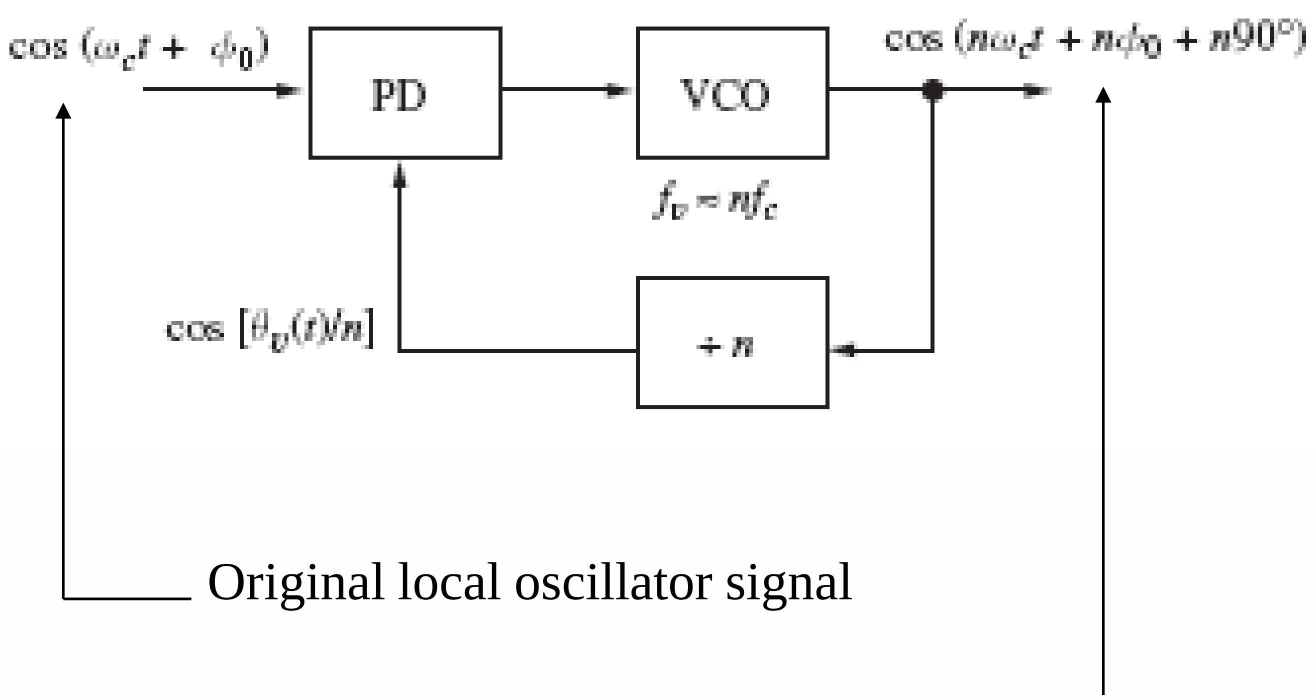


Costas PLL System For Synchronous Detection (DSB)*



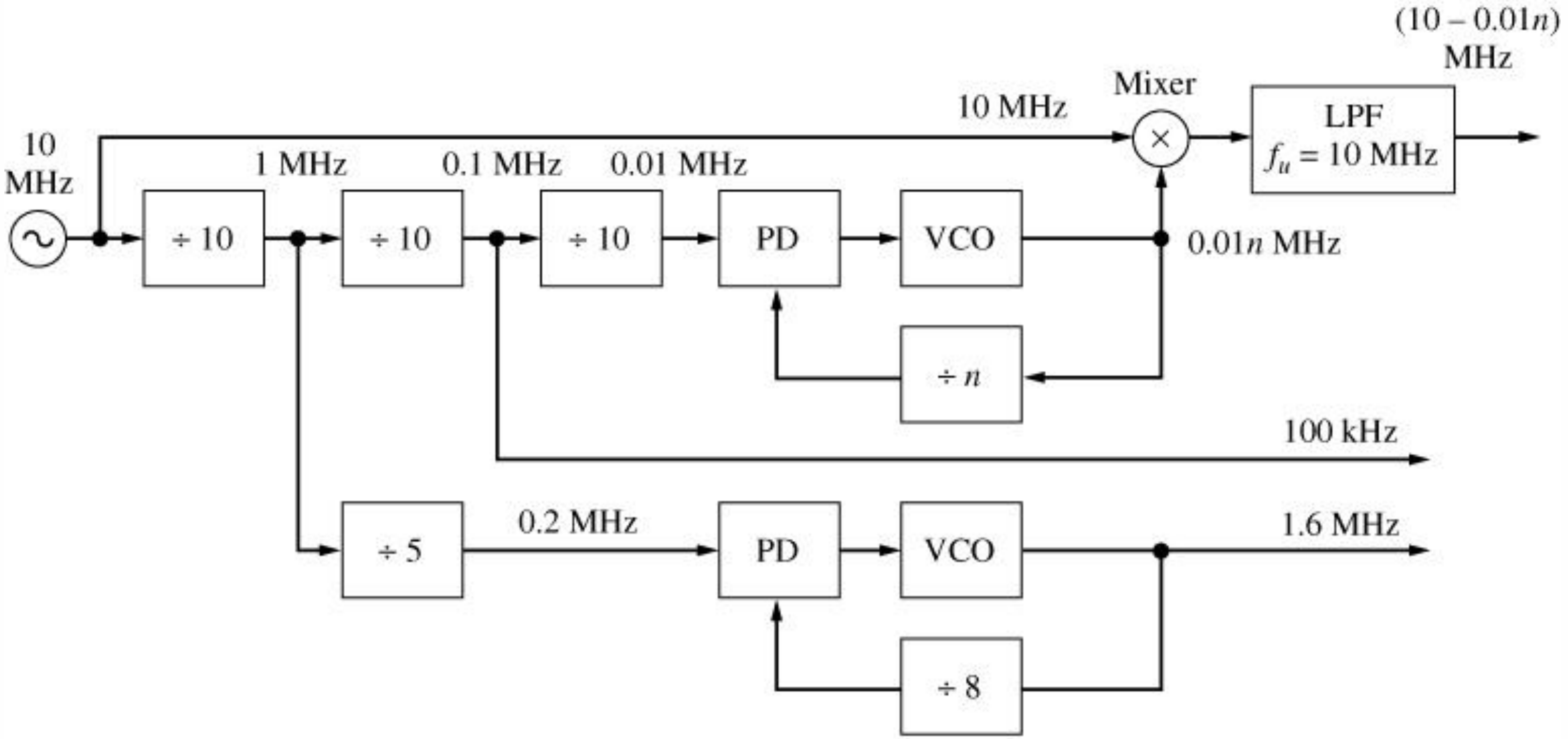
*Cannot be used to detect SSB

Adjustable Local Oscillator Using a Frequency Synthesizer (e.g. for double conversion receiver)

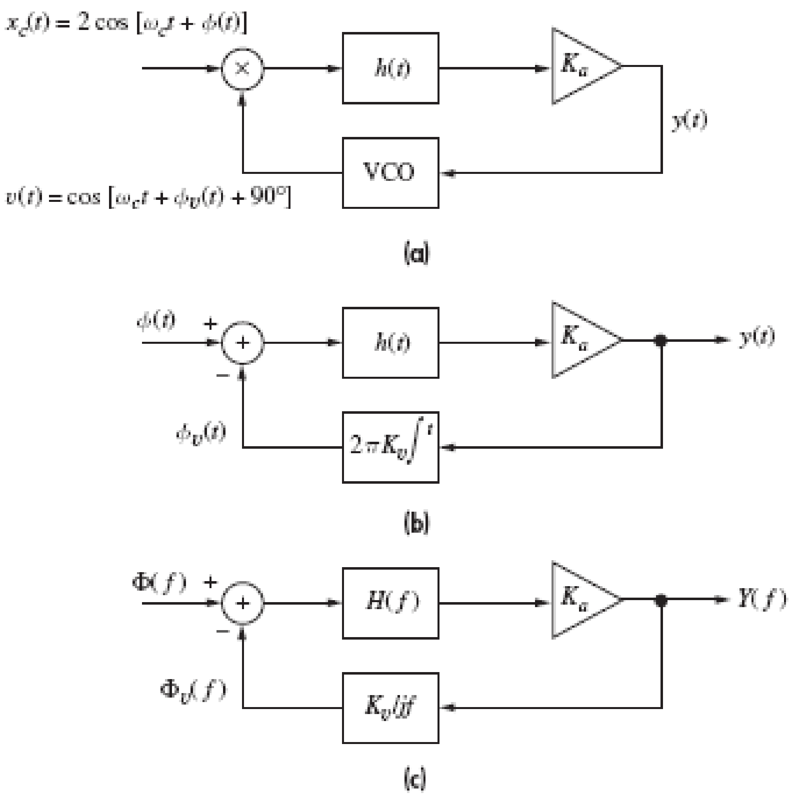


Adjustable LO in increments of 0.01MHz

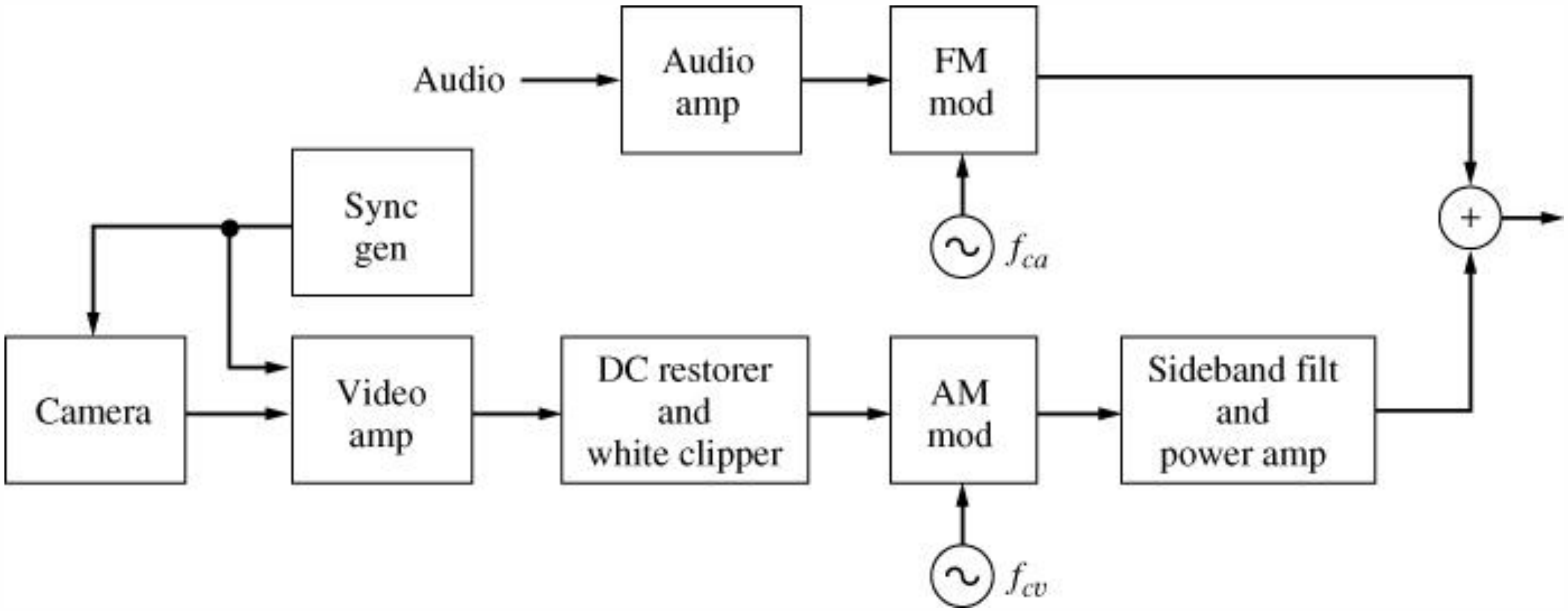
Frequency synthesizer with fixed and adjustable outputs



Linearized PLL models (a) time domain (b) phase (c) frequency domain



Monochrome TV transmitter



Mixer.

Again a new addition to the superhet radio, but is the critical addition, as it combines the received modulated radio frequency carrier (f_c) from the R.F.Amplifier, and the Local Oscillator (f_o). The output of the mixer produces at its output four different frequency signals containing the following frequencies, f_c, f_o-f_c, f_o+f_c, f_o. Three of these frequencies f_c, f_o-f_c, f_o+f_c are amplitude modulated signals each containing all the information about the original audio signal. The only one that does **not** contain the original signal is f_o, the local oscillator frequency which is a pure sine wave. The most important of these is f_o-f_c because irrespective of the carrier frequency that is tuned in, this frequency will always be the same, since the output of the local oscillator tracks the carrier frequency tuned in. This modulated frequency is called the **intermediate frequency (I.F.)** and contains the audio signal from the original radio station no matter what station is tuned in.

Antenna

As with all radio stations, the antenna will pick up the electromagnetic radio waves from the atmosphere and convert these into very small electrical currents.

Tuned R.F. Amplifier.

This block amplifies the very small currents created in the antenna, to improve the sensitivity of the radio receiver, in the same way that it was used in the TRF radio.

Local Oscillator.

A new addition to the superhet radio. This is a sine wave generator which is mechanically linked to the tuning capacitor. This ensures that it always produces a frequency at a fixed amount above the resonant frequency of the tuned amplifier. This is typically in the range 450 kHz to 480 kHz.

Mixer.

Again a new addition to the superhet radio, but is the critical addition, as it combines the received modulated radio frequency carrier (f_c) from the R.F.Amplifier, and the Local Oscillator (f_o). The output of the mixer produces at its output four different frequency signals containing the following frequencies, f_c, f_o-f_c, f_o+f_c, f_o. Three of these frequencies f_c, f_o-f_c, f_o+f_c are amplitude modulated signals each containing all the information about the original audio signal. The only one that does **not** contain the original signal is f_o, the local oscillator frequency which is a pure sine wave. The most important of these is f_o-f_c because irrespective of the carrier frequency that is tuned in, this frequency will always be the same, since the output of the local oscillator tracks the carrier frequency tuned in. This modulated frequency is called the **intermediate frequency (I.F.)** and contains the audio signal from the original radio station no matter what station is tuned in.

Characteristics of AM Radio Receiver

The performance of radio receiver is determined by its characteristics/ parameters.

- These are of three types.

Characteristics of Radio Receiver

- Sensitivity
- Selectivity
- Fidelity

Sensitivity

The ability to amplify the weak signals is called sensitivity.

It is the function of the overall receiver gain.

Sensitivity of radio receiver is decided by the gain of the RF and IF amplifiers.

- Practically, it is defined as the carrier voltage, which must be applied to the receiver input terminals to get standard output power at output terminals.
- The loudspeaker is replaced by load resistance of equal value of speaker.
- The sensitivity is expressed in **m volt** or **millivolt**.
- It may be measured at various frequencies in the radioband.
- **Improvement in Sensitivity:**
- The high gain IF amplifiers provides better sensitivity. Hence, smaller input signal is required to produce desired level of output.

Procedure to Measure Sensitivity:

fig.Set-up to plot sensutivity curve

Continued…..

- Adjust the output of AM generator to 30% modulation index, with
modulating signal frequency 400 Hz. Observe and note this AM wave on
CRO.
- Connect the external AM generator output at the antenna terminal.
- Adjust carrier frequency of AM input at 540 kHz. Then adjust the output voltage of the signal generator to get a standard output of 50 mW across
R_{eq} Measure the corresponding input voltage.
- Repeat Step – 3 for various values of carrier frequency from 540 kHz to 1640 kHz.
- Plot the graph of carrier frequency on X-axis versus receiver input on Y-axis. This is the sensitivity curve shown in Fig. 3.7.

Continued.....

Carrier Frequency of AM Signal (kHz)	(Input voltage to get the standard output) Sensitivity (µV)
540 kHz	
640 kHz	
:	
:	
1640 kHz	

Continued....

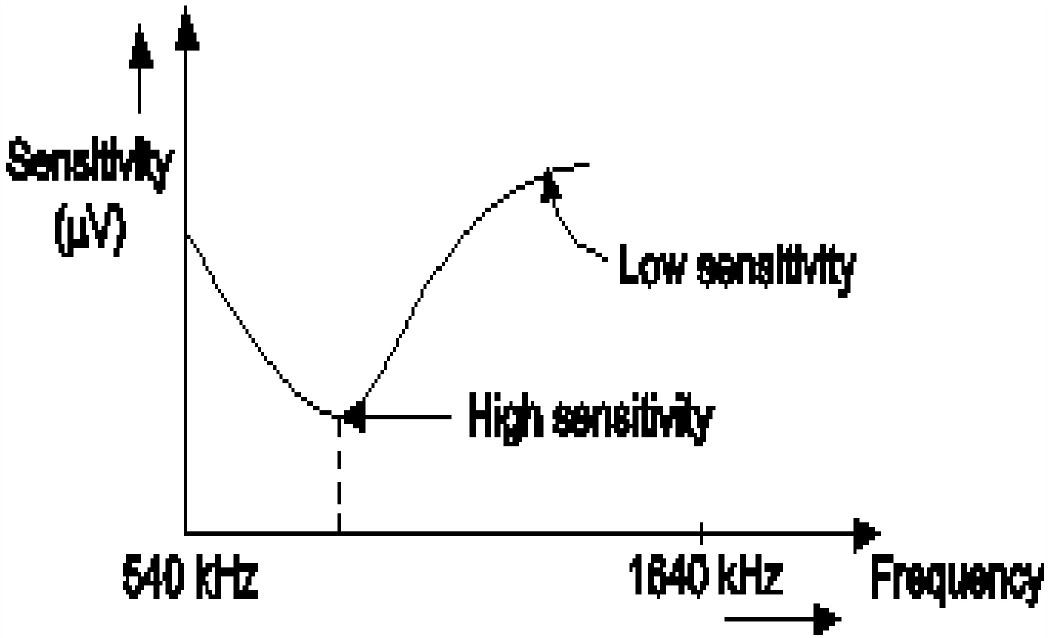


Fig. sensitivity Curve

Selectivity

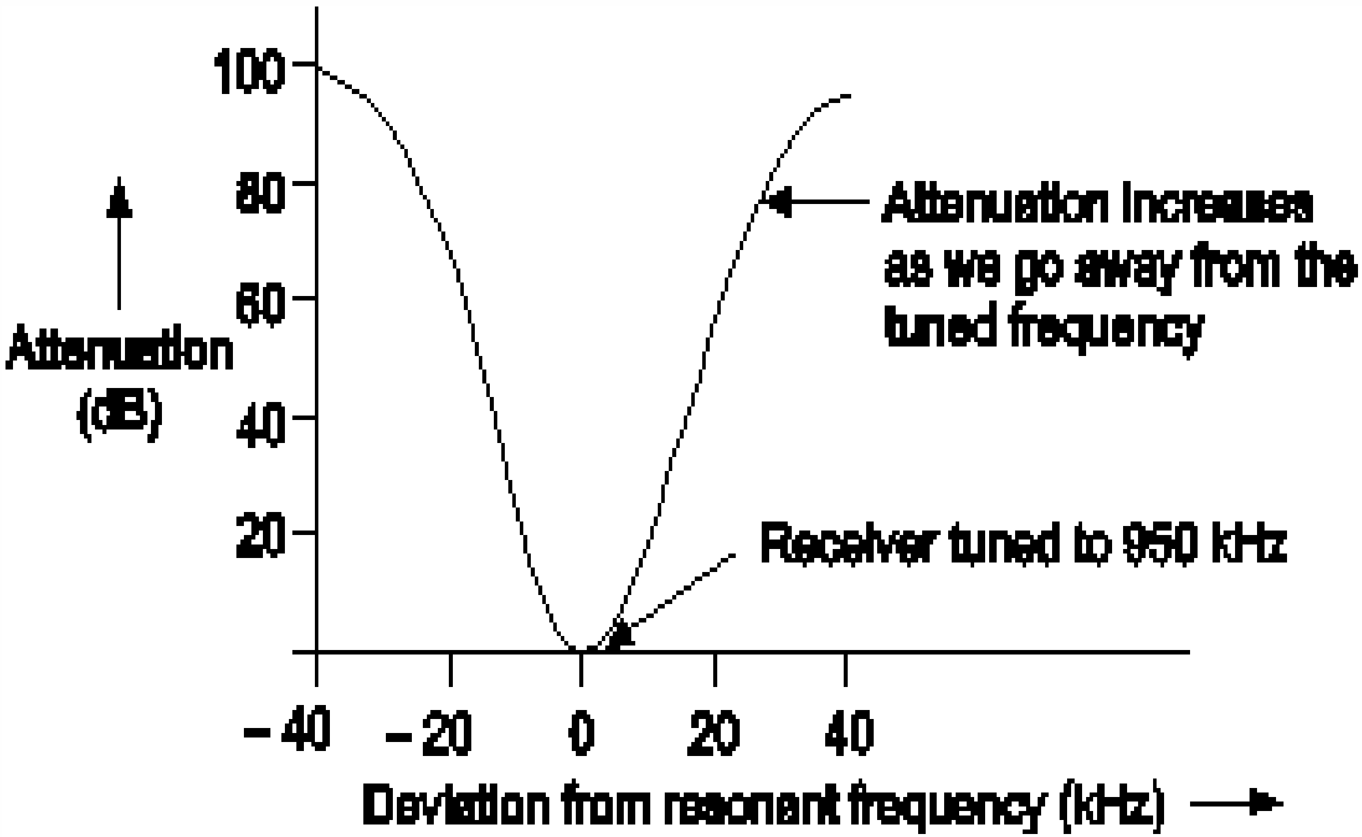
Selectivity is the ability of radio receiver to reject the unwanted signals.

•Selectivity depends on IF amplifier. Higher the 'Q' of the tuned circuit better is the selectivity.

•It is used to distinguish between two adjacent carrier frequencies.

•It shows how perfectly the receiver is able to select the desired carrier frequency and reject other frequencies

Continued....



100
80
60
40
20
Attenuation (dB)
Attenuation increases as we go away from the tuned frequency
Receiver tuned to 950 kHz
– 40
– 20
0
20
40
Deviation from resonant frequency (kHz)

Double Spotting

- The phenomenon related to image problem is **double spotting**.
- This is usually biggest problem for receivers with a low value of IF.
- This means that the image frequency is near to the signal frequency and image rejection is not as good as it could be.
- When the receiver is tuned across the band, a strong signal appears to be at two different frequencies, once at the desired frequency and again when the receiver is tuned to two times IF (i.e. 2IF) below the desired frequency.
- In this second case, the signal becomes the image, reduced in strength by the image rejection, thus, **it appears the same signal nearby (i.e. same station) that is located at two frequencies in the band.**
- So that, two station programs will appear at a time through loudspeaker which can not be understandable and irritating to the hears.
- Better to avoid double spotting.

Image Frequency Rejection

In the broadcast AM receives the local oscillator frequency is higher than the incoming by intermediate frequency i.e.

$$f_o = f_s + IF$$

$$\text{or} \quad IF = (f_o - f_s)$$

- Assume that the local oscillator frequency is set to 'f_o' and an unwanted signal at frequency $f_{si} = (f_o + IF)$ manages to reach at the input of the mixer. Then the mixer output consists of the four frequency components of

$$f_o, (f_o + IF), (2f_o + IF) \text{ and } IF$$

Where the last component at IF is the difference between f_{si} and f_o
[i.e. IF = $f_{si} - f_o$]. This component will also be amplified by the IF amplifier alongwith the desired signal at frequency f_s. This will create interference because both the stations corresponding to carrier frequencies f_s and f_{si} will be tuned at the same position.

- This **unwanted signal at frequency f_{si} is known as Image frequency and it is said to be the image of the signal f_s**. The relation between f_s and f_{si} is

Image frequency= fsi= fs +2IF

Double Spotting

- The phenomenon related to image problem is **double spotting**.
- This is usually biggest problem for receivers with a low value of IF.
- This means that the image frequency is near to the signal frequency and image rejection is not as good as it could be.
- When the receiver is tuned across the band, a strong signal appears to be at two different frequencies, once at the desired frequency and again when the receiver is tuned to two times IF (i.e. 2IF) below the desired frequency.
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- So that, two station programs will appear at a time through loudspeaker which can not be understandable and irritating to the hears.
- Better to avoid double spotting.

Intermediate Frequency

Choice of IF:

- The intermediate frequency (IF) of a receiving system is usually a compromise, since there are reasons why it should be neither low, nor high, nor in a certain range between these two.

The choice of IF depends on the factors:

1.If the intermediate frequency is too high, results in poor selectivity and poor adjacent channel rejection results.

2.High value of IF increases tracking difficulties.

3.As the IF is lowered, image-frequency rejection becomes poorer.

4.A very low IF can make the selectivity too sharp, cutting-off the sidebands.

5.If the IF is very low, the frequency stability of the local oscillator must be made corresponding higher.

6.The IF must not fall within the tuning range of the receiver, else instability will occur and heterodyne whistles will be heard, making it impossible to tune the frequency band immediately adjacent to the IF.

IF frequencies used:

Sr. No.	Receiver Type	Tuning Range	IF
1.	MW broadcast AM	540 kHz to 1640 kHz	455 kHz
2.	FM Radio Receiver	88 MHz to 108 MHz	10.7 MHz
3.	Television Receivers	54 to 233 MHz (VHF)	36 MHz
		470 to 940 MHz	40 MHz
4.	Microwave and Radar Receivers	1 to 10 GHz	30/60/70 MHz

Continued….

Why IF has constant value?

(a) If the IF is too high, poor selectivity and poor adjacent channel rejection results.

(b) High value of IF increases tracking difficulties.

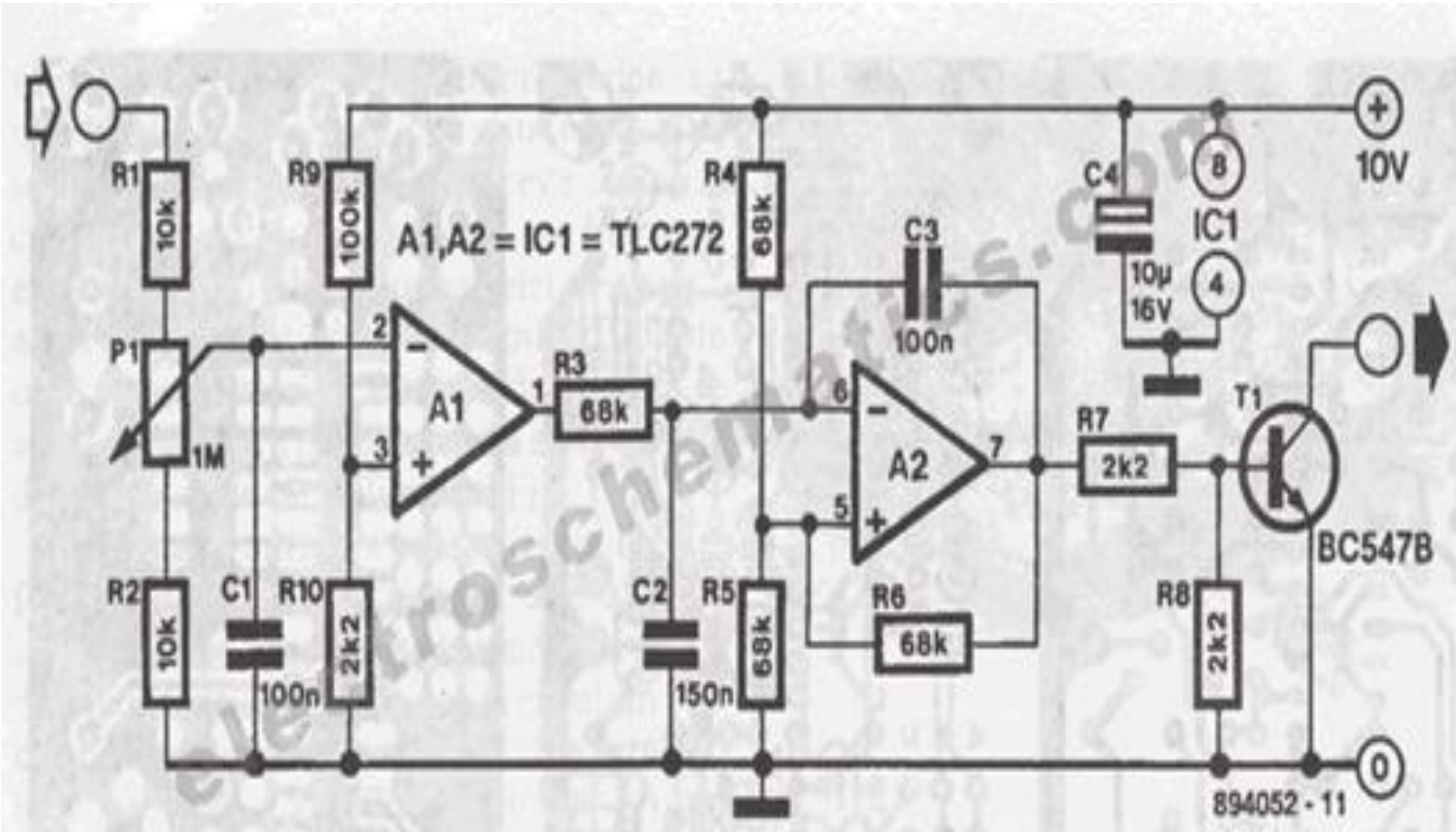
(c) As the IF is lowered, image-frequency rejection becomes poor.

(d) A very low IF can make the selectivity too sharp, cutting of the sidebands.

(e) It must not fall within the tuning range of the receiver, else instability occur. This IF has constant value.

Squelch Circuit

- Squelch is a circuit that acts to suppress the audio (or video) output of a receiver in the absence of a sufficiently strong desired input signal. This **squelch circuit** is simple and has an amplification for sufficiently large to be incorporated into an assembly of automatically control from a wide range of radio receivers. The input signal derived from the RAA circuit from a receiver is tempered by the network R1-R2-P1.

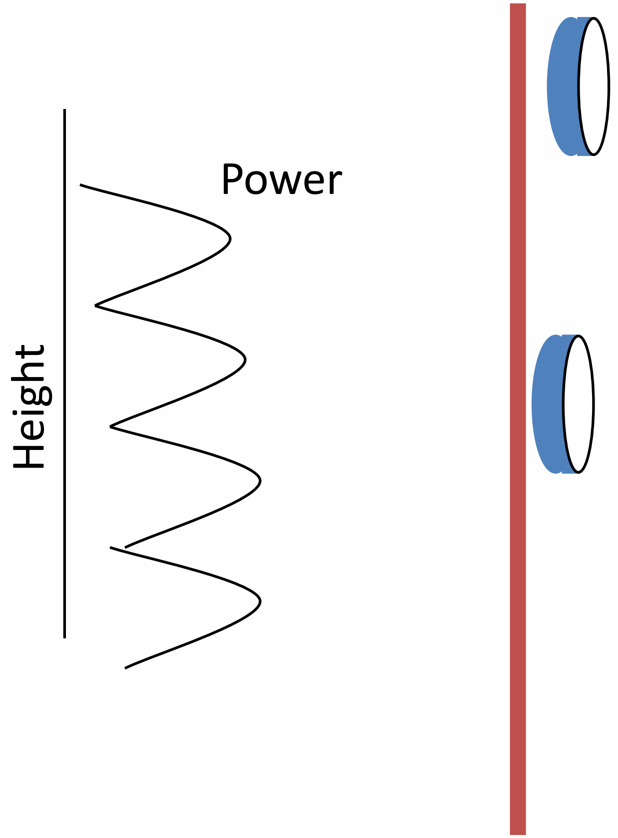


R1
10k
R9
100k
A1,A2 = IC1 = TLC272
R4
68k
C3
100n
C4
10µ
16V
8
IC1
4
10V
P1
1M
2
3
A1
1
R3
68k
6
5
A2
7
R7
2k2
T1
BC547B
R2
10k
C1
100n
R10
2k2
C2
150n
R5
68k
R6
68k
R8
2k2
0
894052 - 11

Diversity Reception

- To reduce fading effects, diversity reception techniques are used. Diversity means the provision of two or more uncorrelated (independent) fading paths from transmitter to receiver
- These uncorrelated signals are combined in a special way, exploiting the fact that it is unlikely that all the paths are poor at the same time. The probability of outage is thus reduced.
- Uncorrelated paths are created using polarization, space, frequency, and time diversity

Space Diversity



- A method of transmission or reception, or both, in which the effects of fading are minimized by the simultaneous use of two or more physically separated antennas
- Antennas must be spatially distant so that no correlation exists between fading phases (one or more wavelengths).

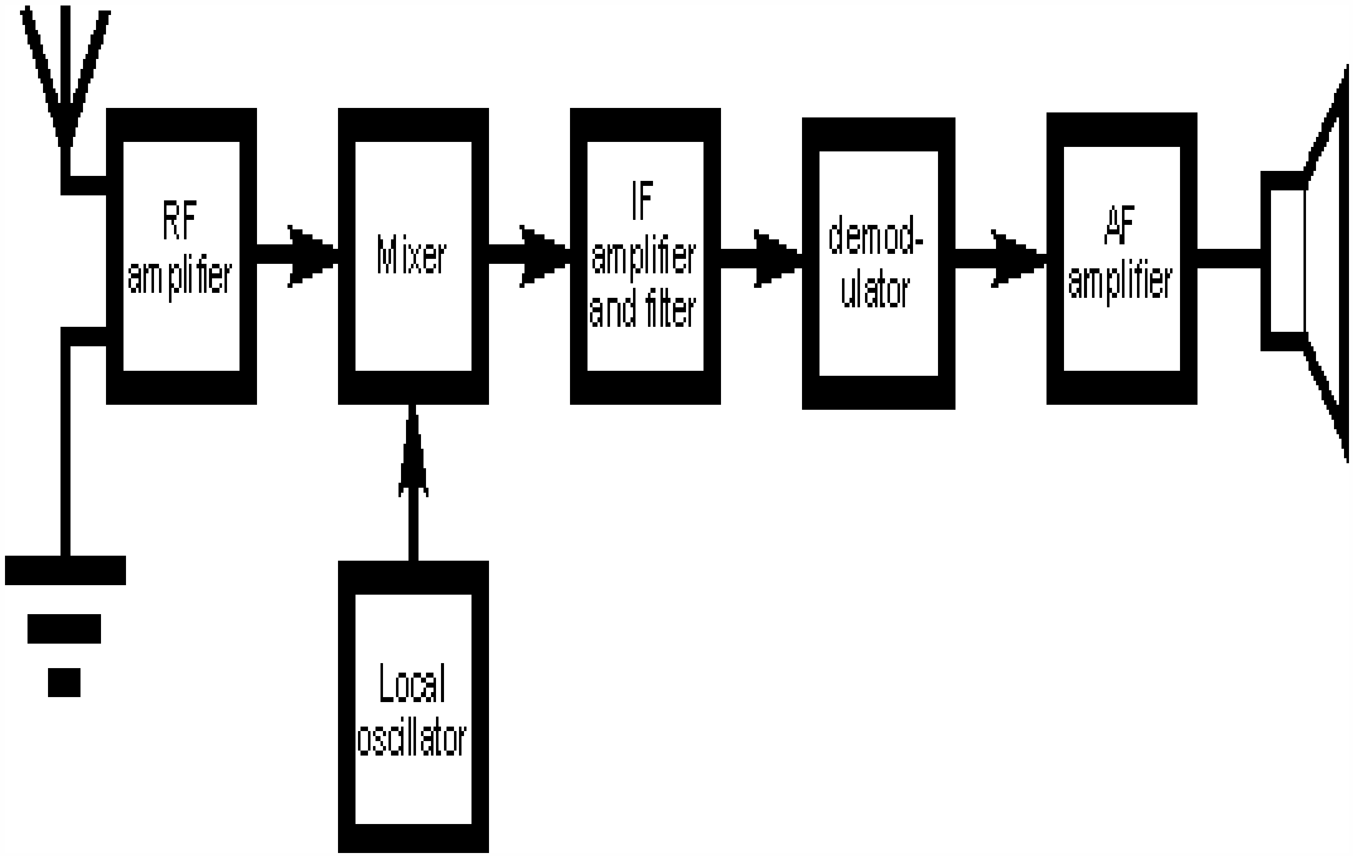
Frequency Diversity

- Transmission and reception in which the same information signal is transmitted and received simultaneously on two or more carrier frequencies.
- The frequency difference must be high enough so there no correlation exists between their fading phases

- ***Amplification:*** In terms of amplification, the level is carefully chosen so that it does not overload the mixer when strong signals are present, but enables the signals to be amplified sufficiently to ensure a good signal to noise ratio is achieved.

FM receiver

- ***Tuning:*** Broadband tuning is applied to the RF stage. The purpose of this is to reject the signals on the image frequency and accept those on the wanted frequency. It must also be able to track the local oscillator so that as the receiver is tuned, so the RF tuning remains on the required frequency.


RF amplifier
Mixer
IF amplifier and filter
demod-ulator
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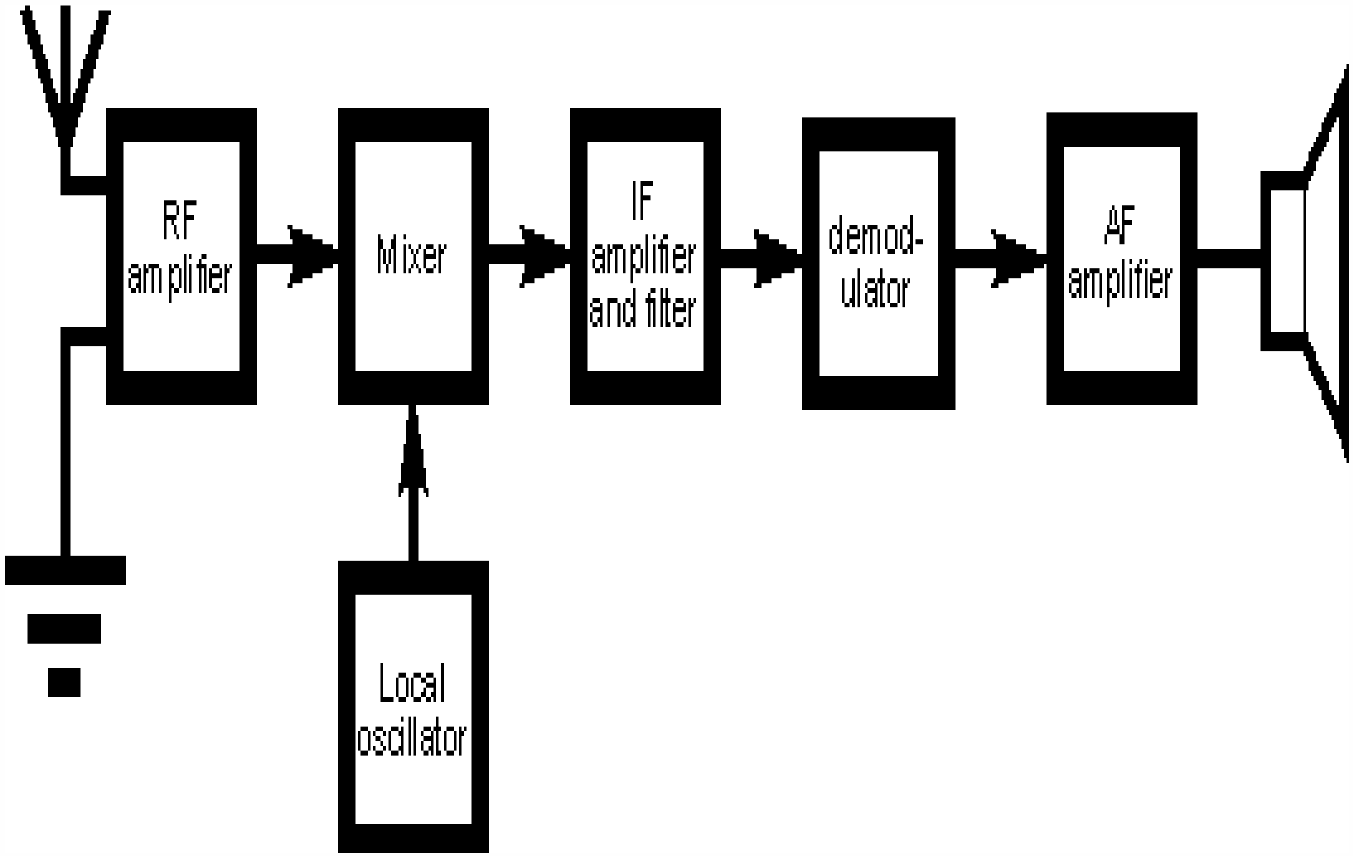
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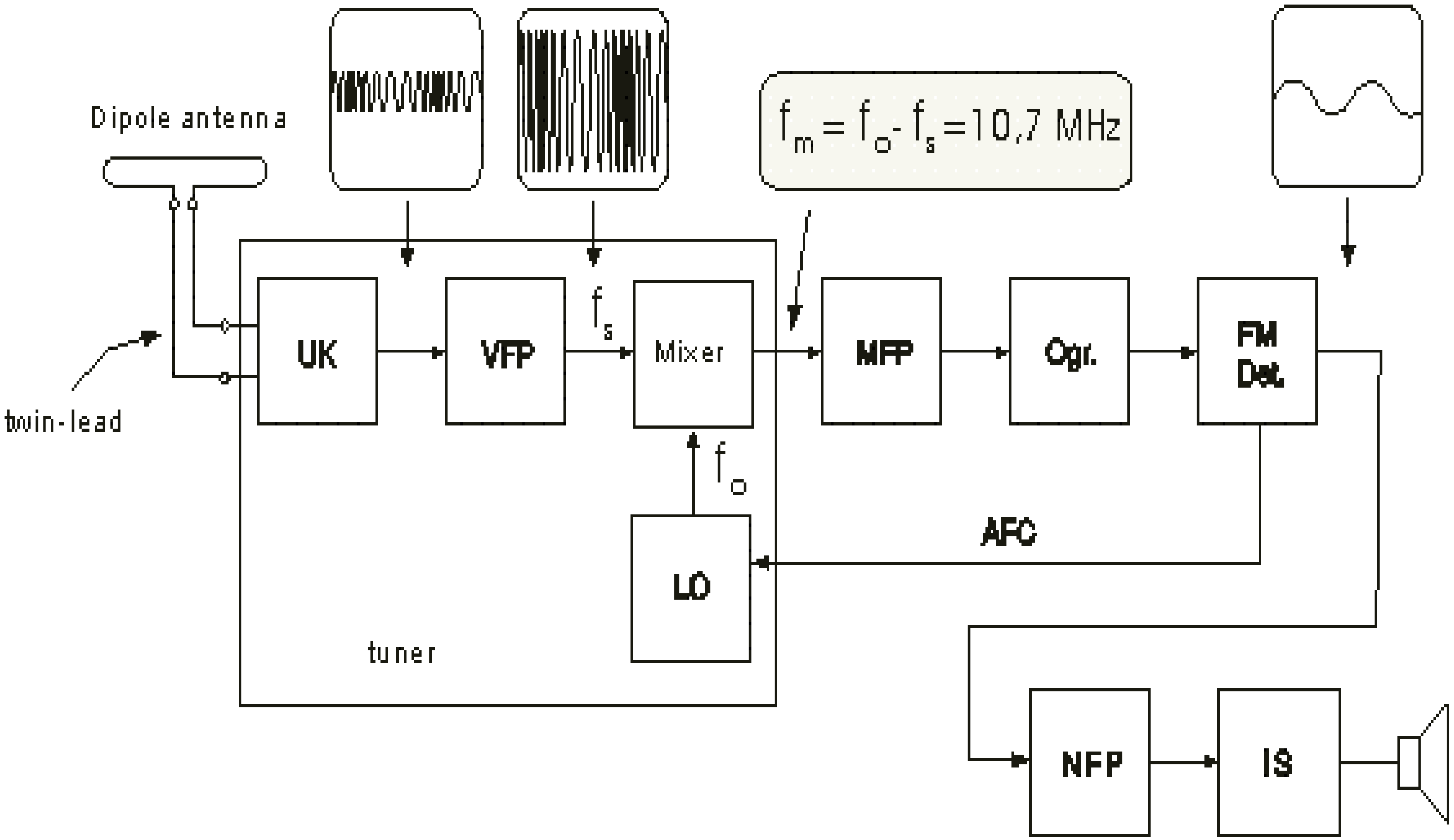


Pic. 4.6. Block diagram of the monophonic superheterodyne FM receiver

Sampling Process

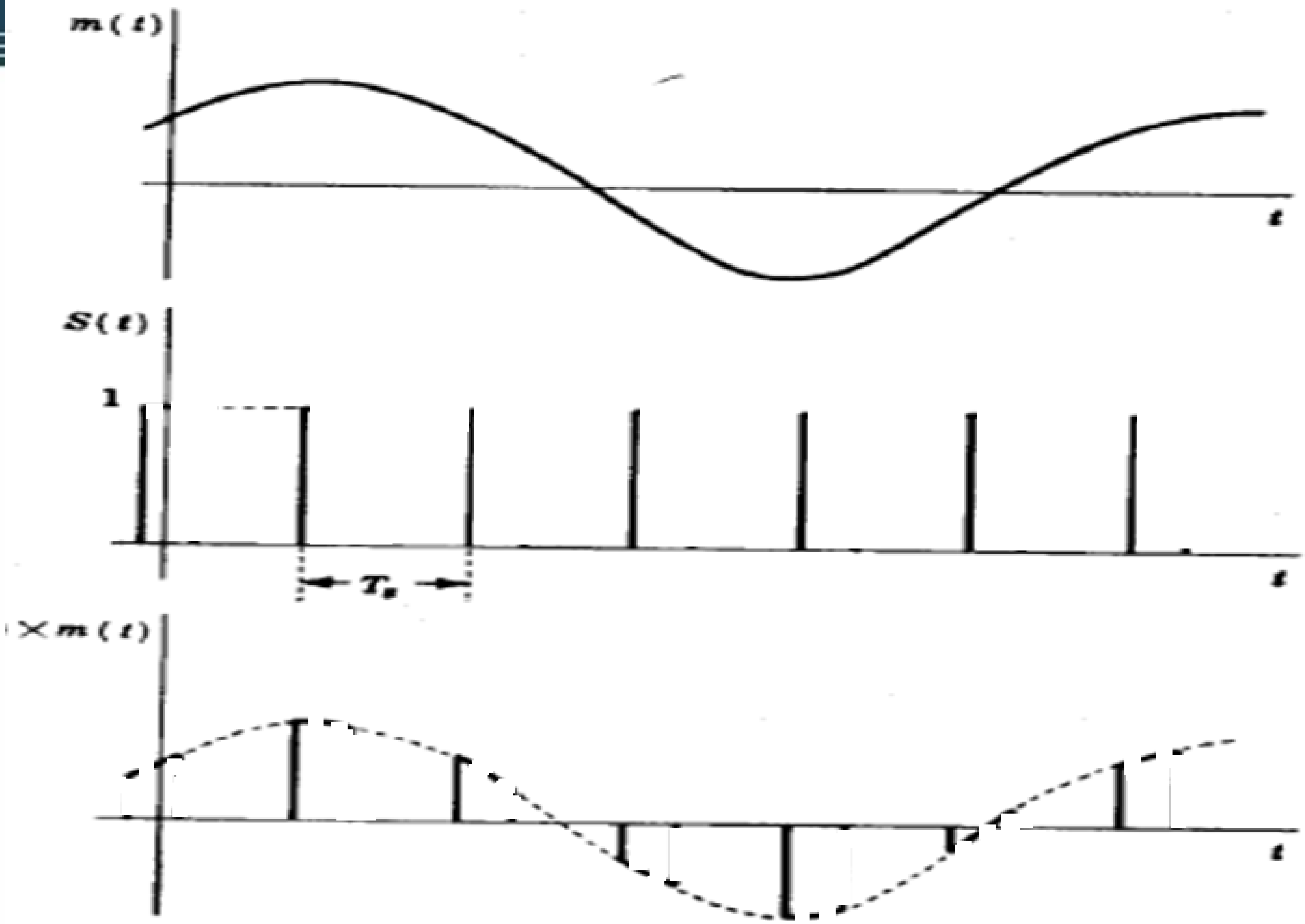
In sampling process, an analog signal is converted into a corresponding sequence of samples that are usually spaced uniformly in time.

Consider an arbitrary signal $g(t)$ of finite energy, which is specified for all time. Suppose that we sample the signal $g(t)$ instantaneously and at a uniform rate, once every T_s seconds We refer to T_s as the *sampling period*, and to its reciprocal $f_s = 1/T_s$ as the *sampling rate*. This ideal form of sampling is called *instantaneous sampling*.

The sampled signal s given by

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t - nT_s) \qquad \text{-----------------}(1)$$

Ideal (or) Instantaneous Sampling

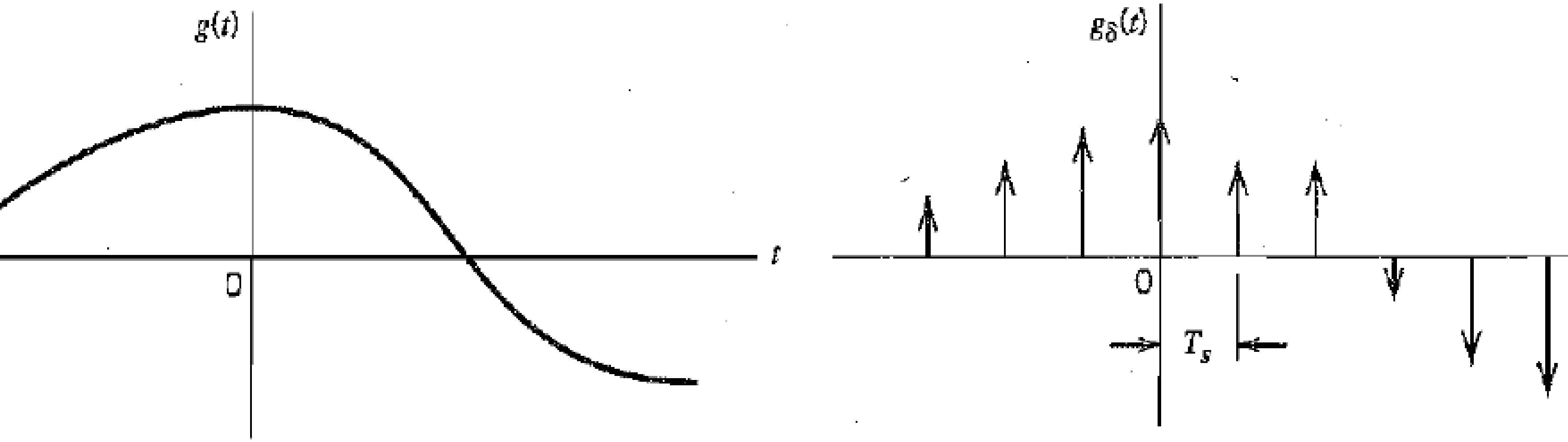


$$S(t) = \frac{dt}{T_s} + \frac{2\,dt}{T_s}\left(\cos 2\pi \frac{t}{T_s} + \cos 2 \times 2\pi \frac{t}{T_s} + \cdots\right)$$

dt is pulse width

For $T_s = 1/2f_M$,

$$S(t)m(t) = \frac{dt}{T_s} m(t) + \frac{dt}{T_s}\left[2m(t)\cos 2\pi(2f_M)t + 2m(t)\cos 2\pi(4f_M)t + \cdots\right]$$



(a) Analog signal. (b) sampled signal

Using the table of Fourier-transform pairs, we may write

$$g_\delta(t) \rightleftharpoons f_s \sum_{m=-\infty}^{\infty} G(f - mf_s) \quad \text{-------------------}(2)$$

where $G(f)$ is the Fourier transform of the original signal $g(t)$, and f_s is the sampling rate. Equation (3.2) states that *the process of uniformly sampling a continuous-time signal of finite energy results in a periodic spectrum with a period equal to the sampling rate.*

Let $G_\delta(f)$ denote the Fourier transform of $g_\delta(t)$. We may therefore write

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s) \exp(-j2\pi nfT_s) \quad \text{----------------(3)}$$

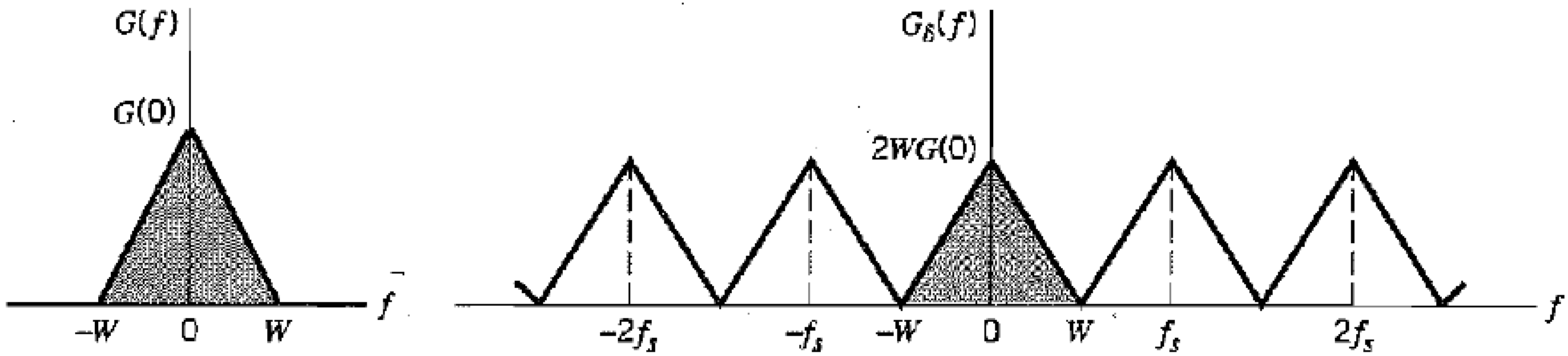
This relation is called the *discrete-time Fourier transform*. It may be viewed as a complex Fourier series representation of the periodic frequency function $G_\delta(f)$, with the sequence of samples $\{g(nT_s)\}$ defining the coefficients of the expansion.

Suppose, that the signal $g(t)$ is *strictly band-limited*, with no frequency components higher than W Hertz. Suppose also that $T_s = 1/2W$. Then the spectrum $G_\delta(f)$ becomes

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi nf}{W}\right) \quad \text{---------------------(4)}$$

Fourier transform of $g_\delta(t)$ may also be expressed as

$$G_\delta(f) = f_s G(f) + f_s \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} G(f - mf_s) \quad \text{--------------(5)}$$



(*a*) Spectrum of a band-limited signal $g(t)$ (*b*) Spectrum of the sampled

Hence, under the following two conditions:

1. $G(f) = 0$ for $|f| \geq W$
2. $f_s = 2W$

we find from Equation (5) that

$$G(f) = \frac{1}{2W} G_\delta(f), \qquad -W < f < W \quad \text{----------------(6)}$$

Substituting Equation (4) into (6), we may also write

$$G(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi n f}{W}\right), \quad -W < f < W \quad \text{---------(7)}$$

Therefore, if the sample values $g(n/2W)$ of a signal $g(t)$ are specified for all n, then the Fourier transform $G(f)$ of the signal is uniquely determined by using the discrete-time Fourier transform of Equation (7). Because $g(t)$ is related to $G(f)$ by the inverse Fourier transform, it follows that the signal $g(t)$ is itself uniquely determined by the sample values $g(n/2W)$ for $-\infty < n < \infty$. In other words, the sequence $\{g(n/2W)\}$ has all the information contained in $g(t)$.

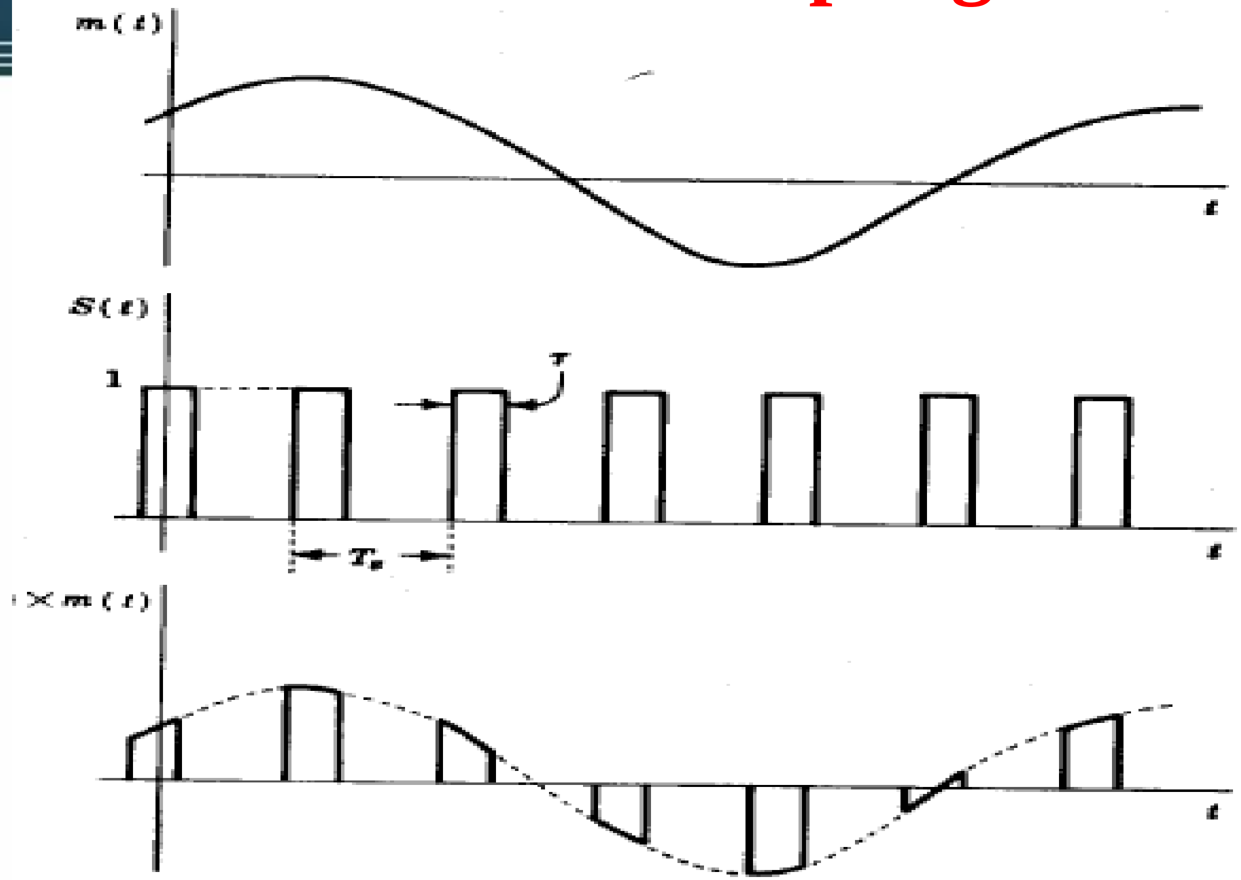
Sampling Theorem

1. *A band-limited signal of finite energy, which has no frequency components higher than W Hertz, is completely described by specifying the values of the signal at instants of time separated by 1/2W seconds.*
2. *A band-limited signal of finite energy, which has no frequency components higher than W Hertz, may be completely recovered from a knowledge of its samples taken at the rate of 2W samples per second.*

To combat the effects of aliasing in practice, we may use two corrective measures,

1. Prior to sampling, a low-pass *anti-aliasing filter* is used to attenuate those high-frequency components of the signal that are not essential to the information being conveyed by the signal.
2. The filtered signal is sampled at a rate slightly higher than the Nyquist rate.
   In this case, the design of the *reconstruction filter* becomes easy.

Natural Sampling



$$S(t) = \frac{\tau}{T_s} + \frac{2\tau}{T_s}\left(C_1 \cos 2\pi \frac{t}{T_s} + C_2 \cos 2 \times 2\pi \frac{t}{T_s} + \cdots\right)$$

with the constant C_n given by

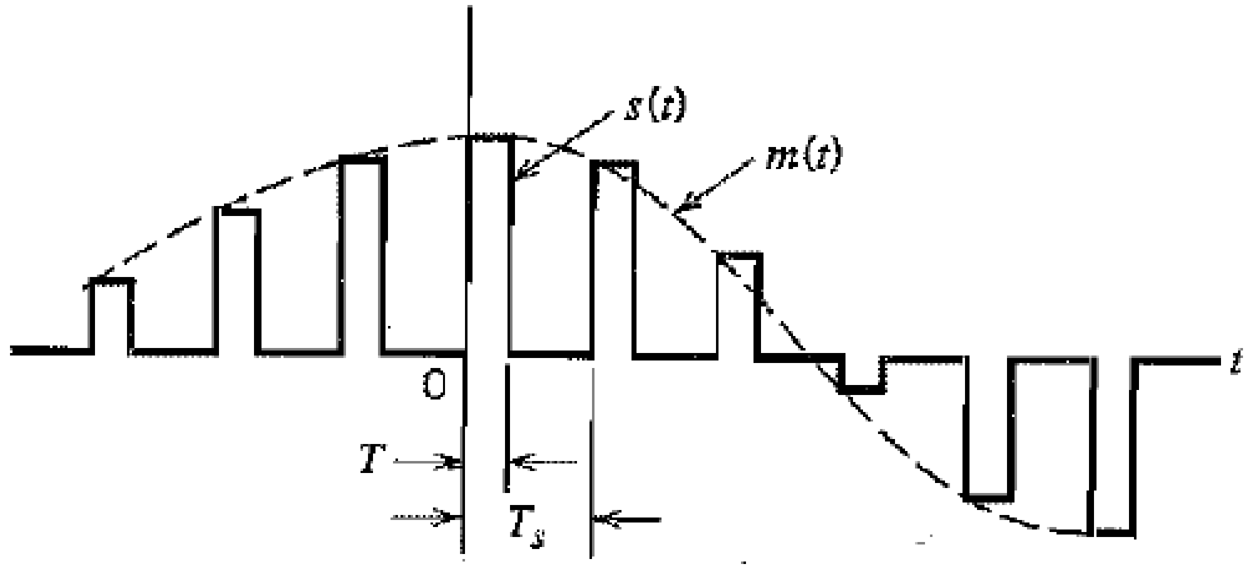
$$C_n = \frac{\sin (n\pi\tau/T_s)}{n\pi\tau/T_s}$$

$$S(t)m(t) = \frac{\tau}{T_s} m(t) + \frac{2\tau}{T_s} [m(t)C_1 \cos 2\pi(2f_M)t + m(t)C_2 \cos 2\pi(4f_M)t + \cdots]$$

Therefore, a low-pass filter with cutoff at f_M will deliver an output signal $s_o(t)$ given by

$$s_o(t) = \frac{\tau}{T_s} m(t)$$

Flat Top Sampling

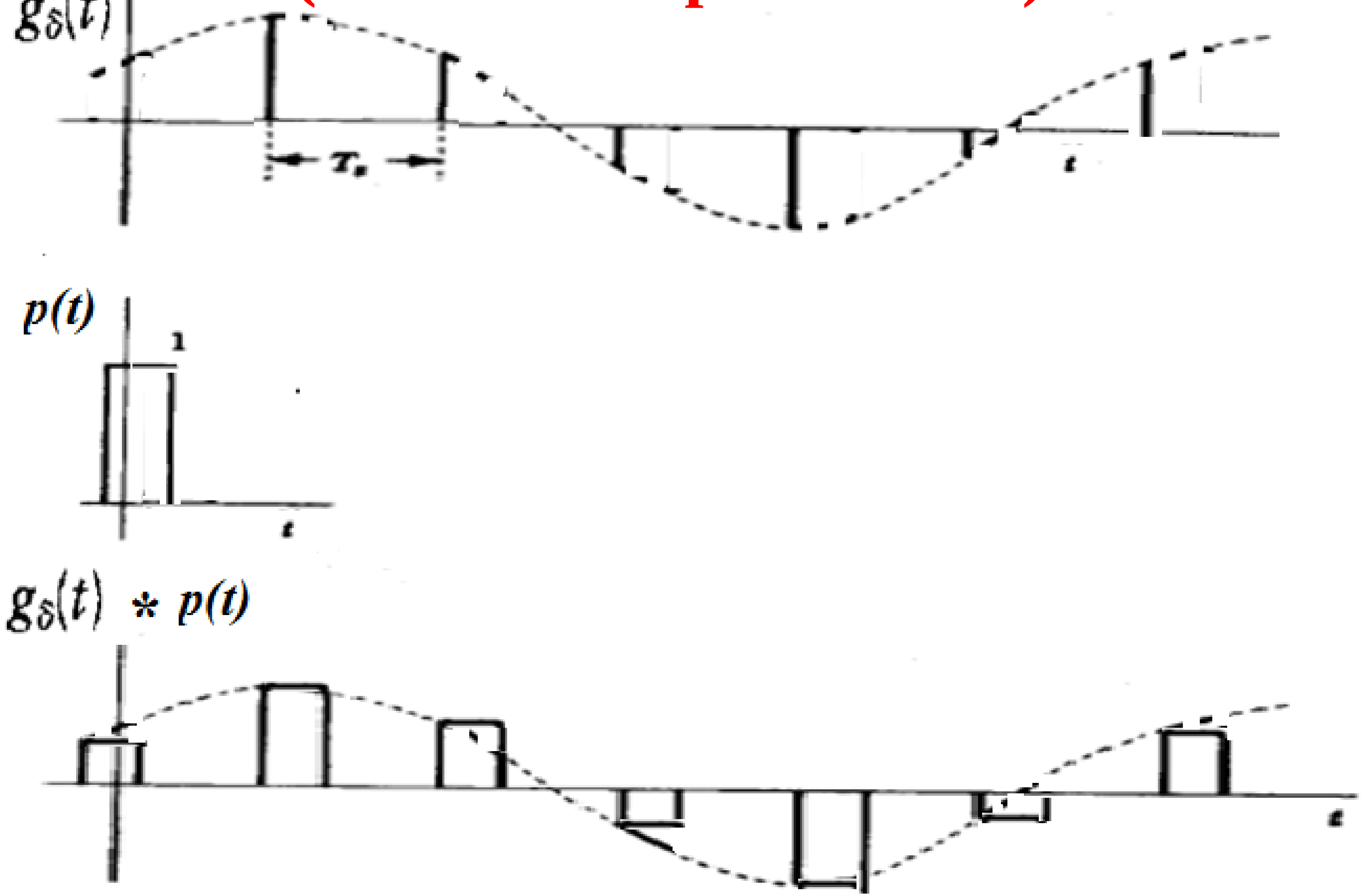


There are two operations involved

1. *Instantaneous sampling* of the message signal $m(t)$ every T_s seconds, where the sampling rate $f_s = 1/T_s$ is chosen in accordance with the sampling theorem.
2. *Lengthening* the duration of each sample so obtained to some constant value T.

In digital circuit technology, these two operations are jointly referred to as "sample and hold." One important reason for intentionally lengthening the duration of each sample is to avoid the use of an excessive channel bandwidth, since bandwidth is inversely proportional to pulse duration.

Aperture Effect
(due to finite pulse width)



Aperture Effect
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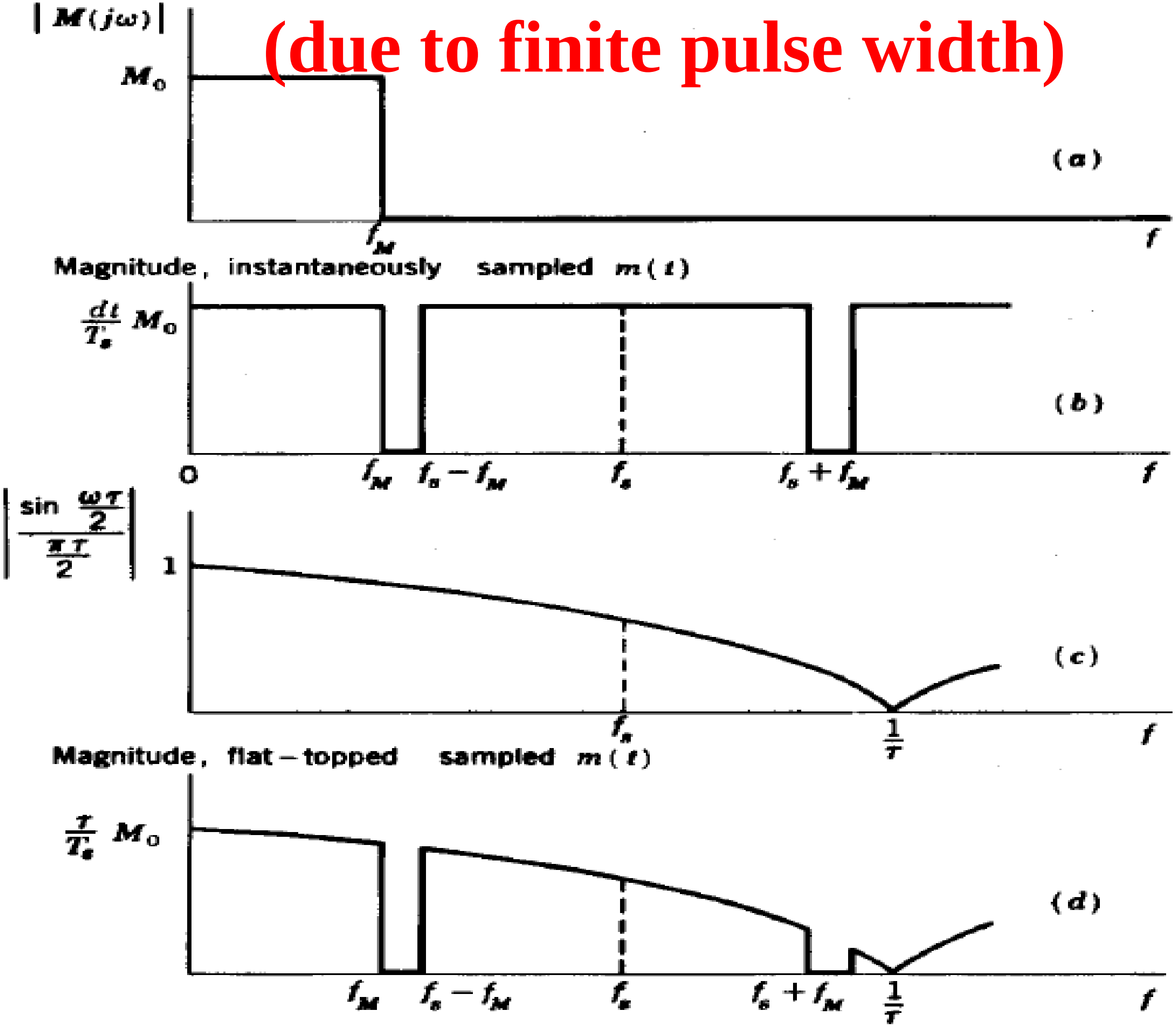


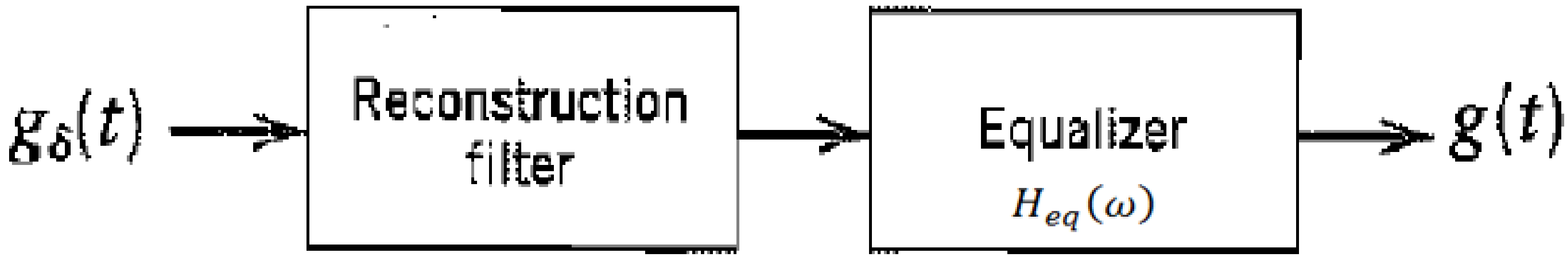
This distortion results because the original signal was "observed" through a finite rather than an infinitesimal time "aperture" and is hence referred to as *aperture effect* distortion.

The distortion results from the fact that the spectrum is multiplied by the sampling function $Sa(x) \equiv (\sin x)/x$ (with $x = \omega\tau/2$). The magnitude of the sampling function falls off slowly with increasing x in the neighborhood of $x = 0$

To minimize the distortion due to the aperture effect, it is advantageous to arrange that $x = \pi$ correspond to a frequency very large in comparison with f_M. Since $x = \pi f \tau$, the frequency f_0 corresponding to $x = \pi$ is $f_0 = 1/\tau$. If $f_0 \gg f_M$, or, correspondingly, if $\tau \ll 1/f_M$, the aperture distortion will be small. The distortion becomes progressively smaller with decreasing τ. And, of course, as $\tau \rightarrow 0$ (instantaneous sampling), the distortion similarly approaches zero.

Thus the effect of *flat-top sampling* is the unequal transmission of spectral components in the range 0 to f_M. If the distortion is not acceptable, then, it may be corrected by an $x/\sin x$ equalizer.

Digital-to-Analog Converter (DAC)



$$H(\omega) = T\ sinc\left(\frac{\omega T}{2}\right) = T\ sinc\left(\frac{2\pi f T}{2}\right)$$

$$= T\ sinc\left(\frac{2\pi f_s T}{4}\right) = T\ sinc\left(0.5\,\pi\frac{T}{T_s}\right)$$

when normalized to that value at zero frequency i.e. T.

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The equalizer must have the response as

$$H_{eq}(\omega) = \frac{1}{H(\omega)} = \frac{1}{sinc\left(0.5\,\pi\frac{T}{T_s}\right)}$$

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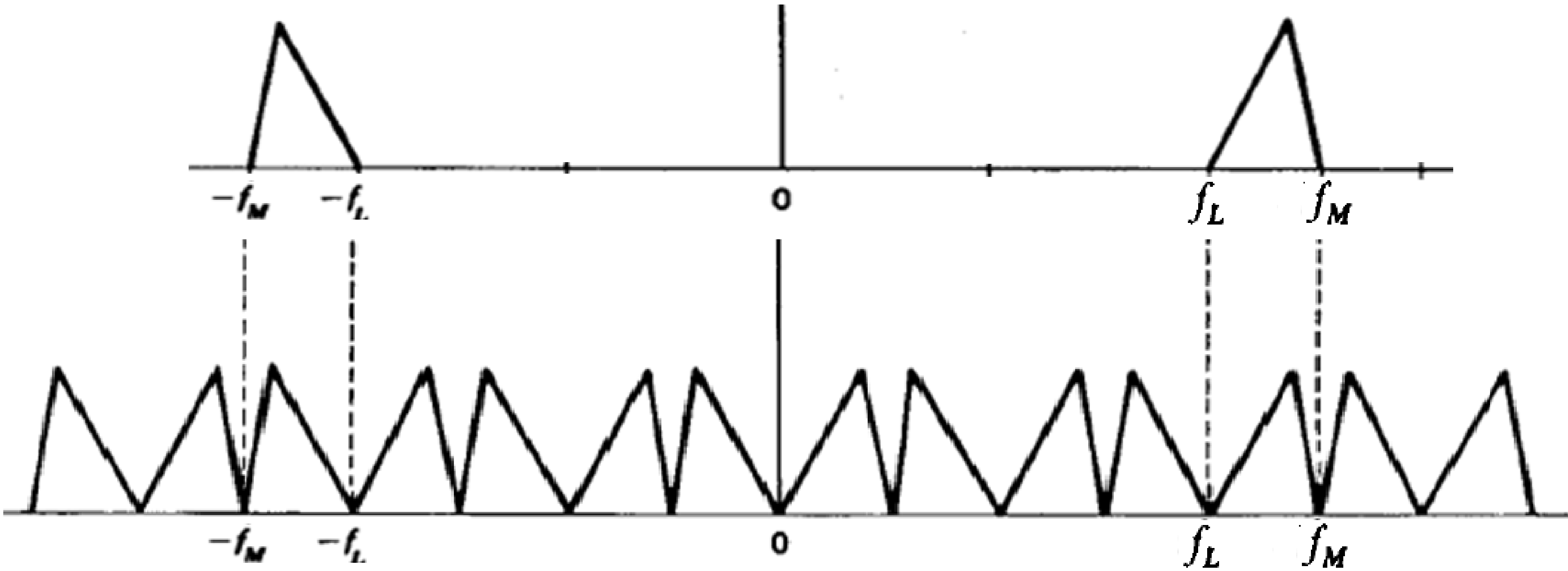
Sampling of Band Pass Signals



$f_s = 2(f_M - f_L)$ provided that either f_M or f_L is a harmonic of f_s

$= 2(10.1 - 10.0) = 0.2$ MHz

Sampling Process

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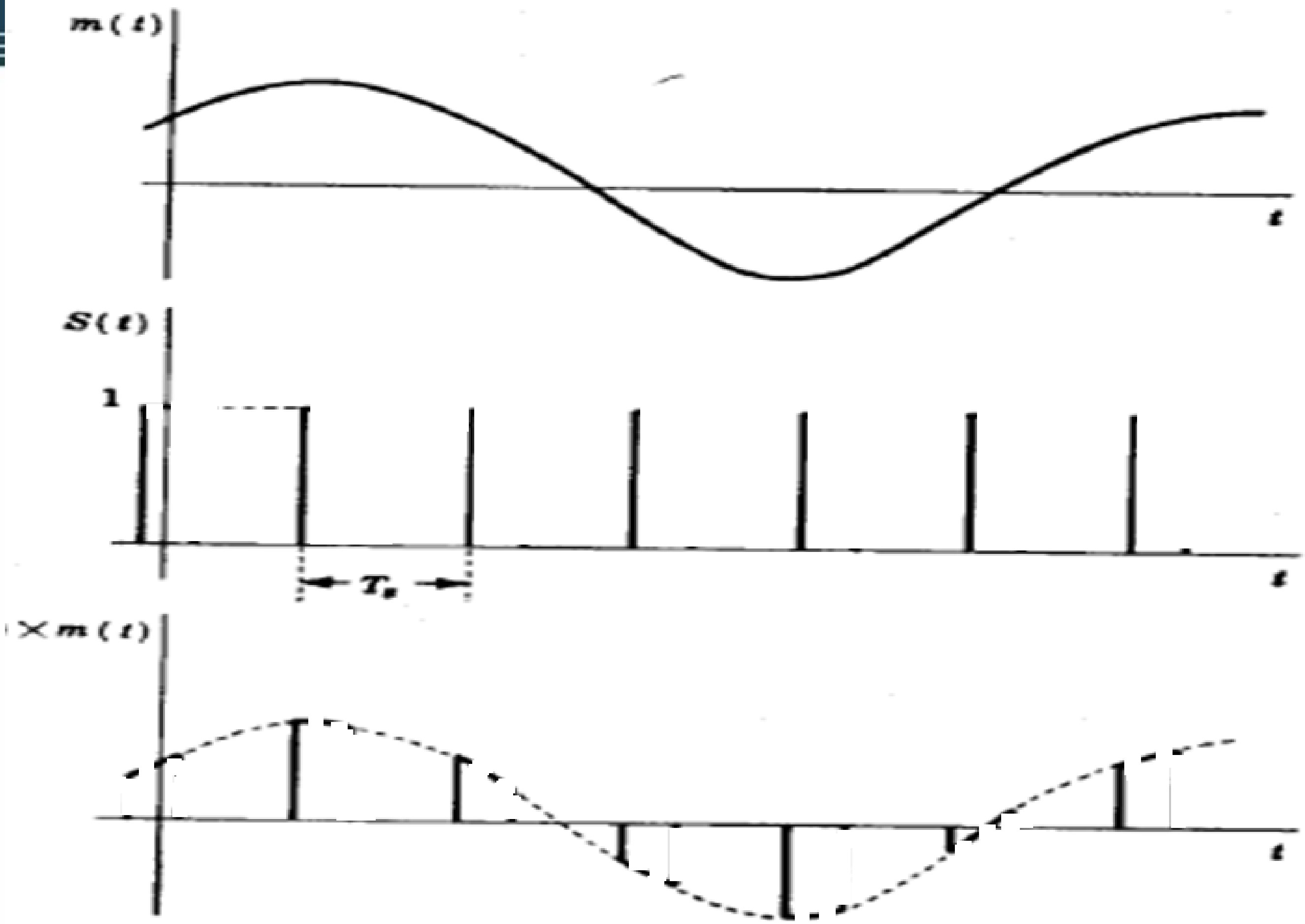
The sampled signal s given by

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\, \delta(t - nT_s) \qquad \text{-----------------(1)}$$

Ideal (or) Instantaneous Sampling



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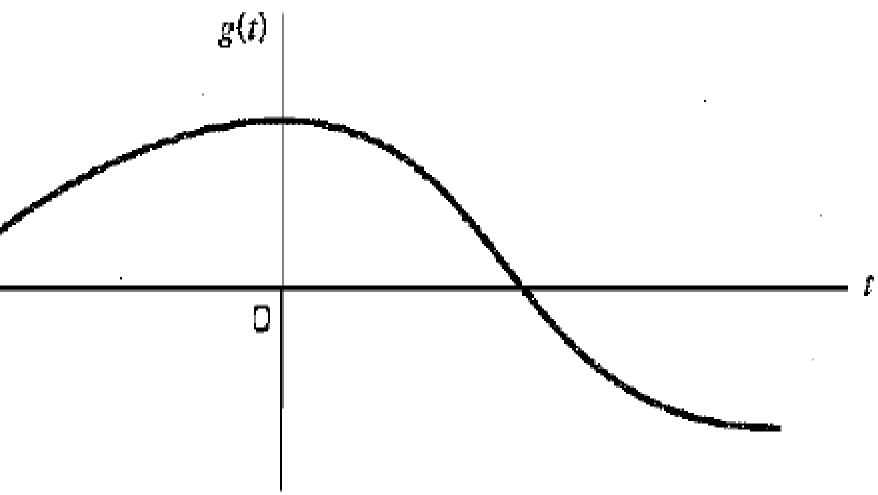
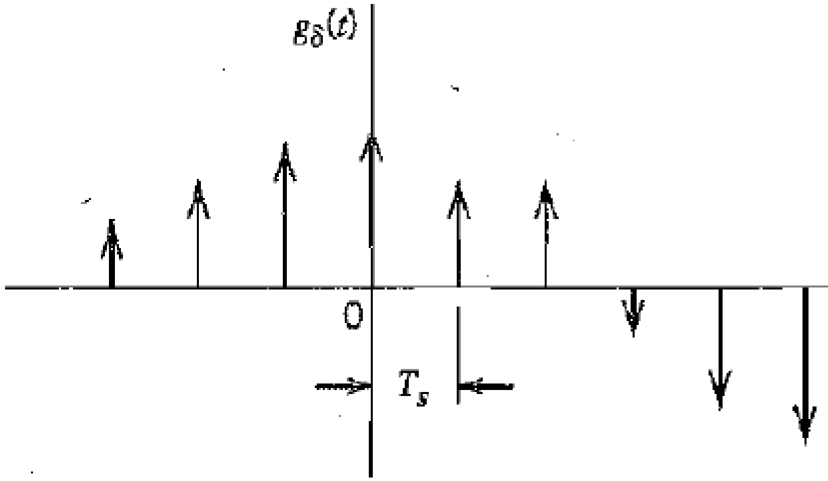


(a) Analog signal.



(b) sampled signal

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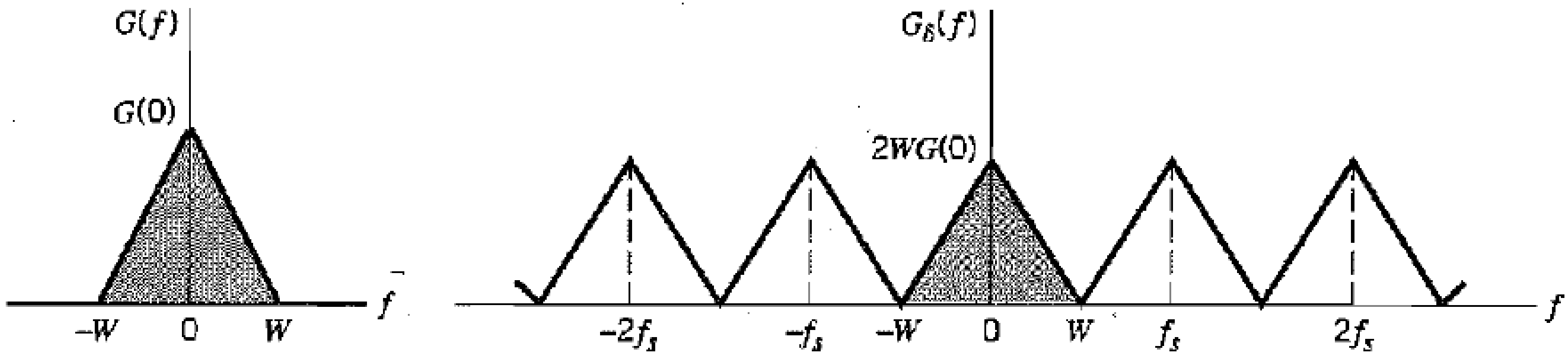

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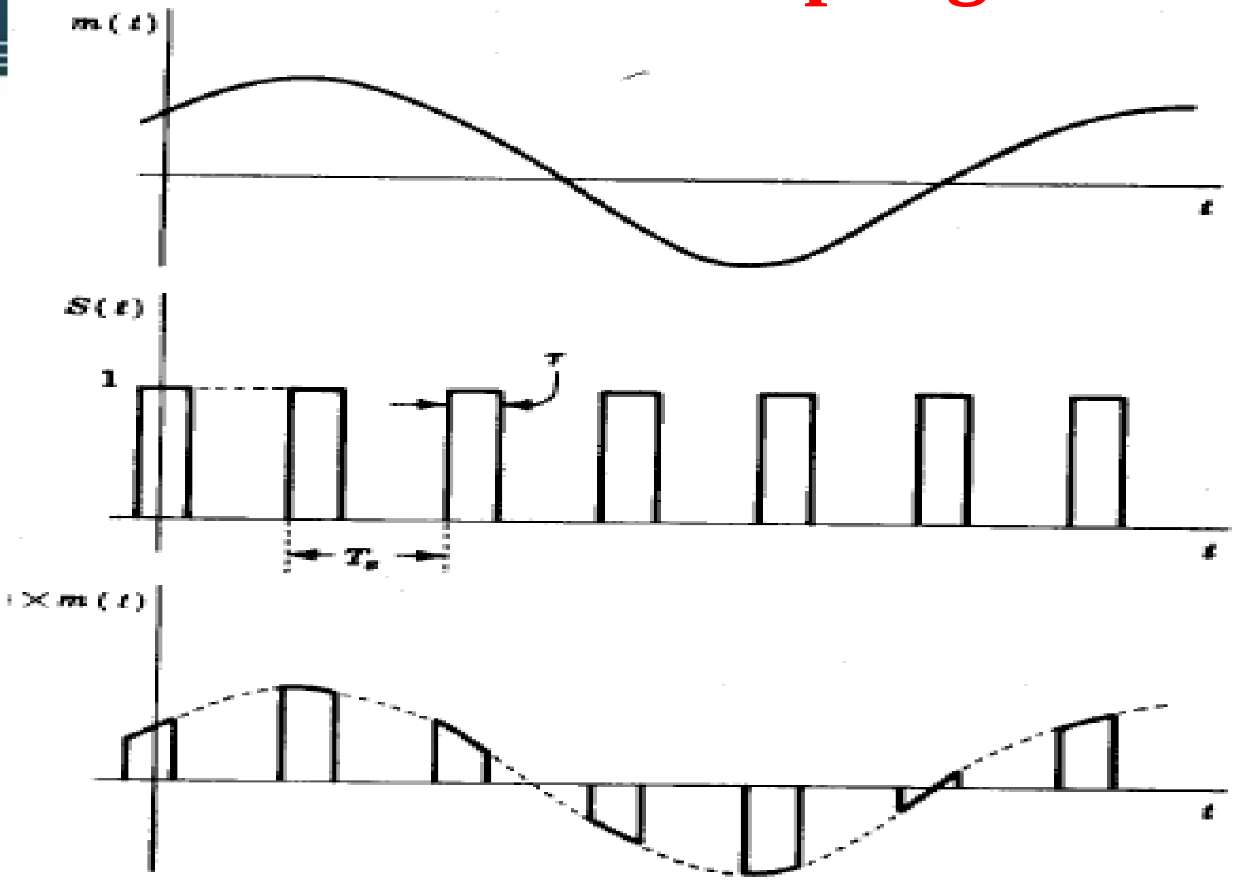
Natural Sampling



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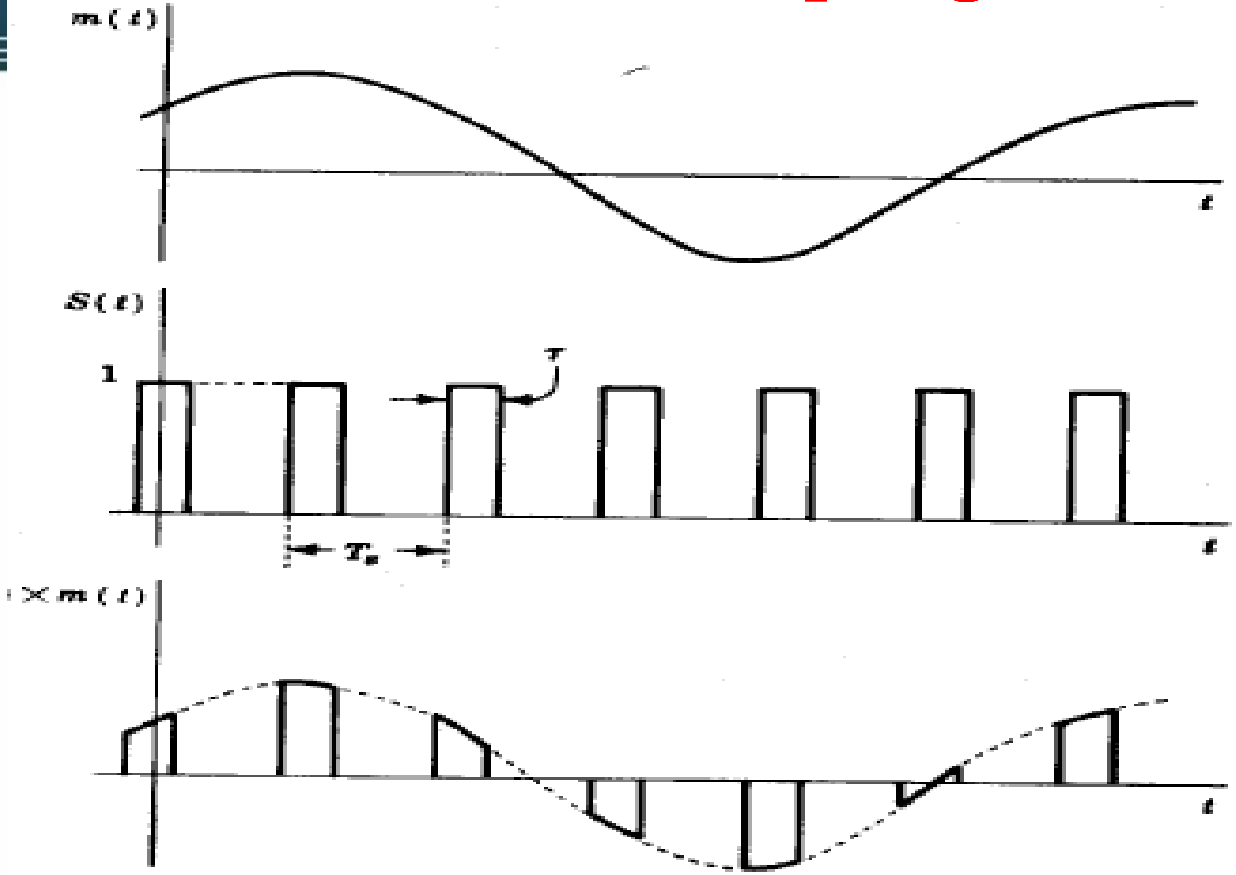

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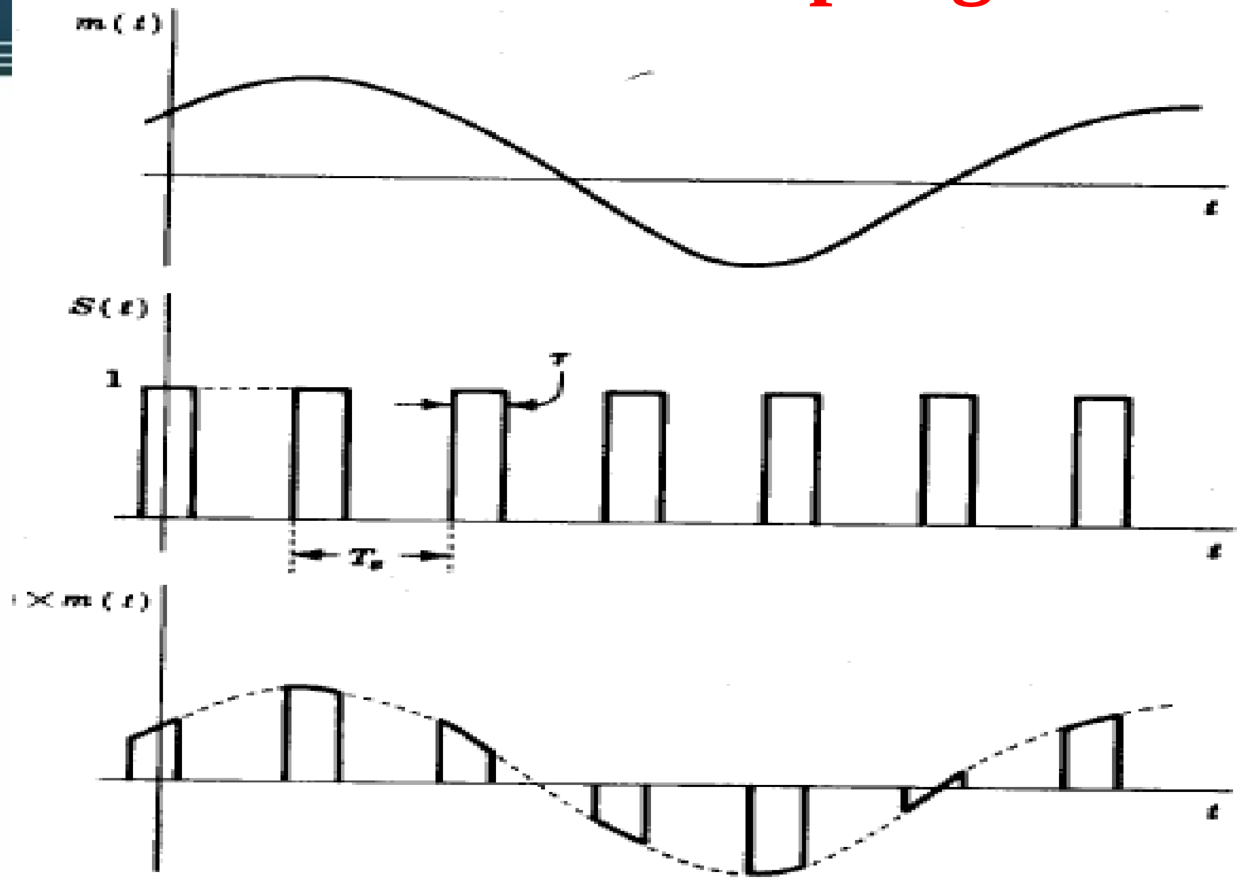

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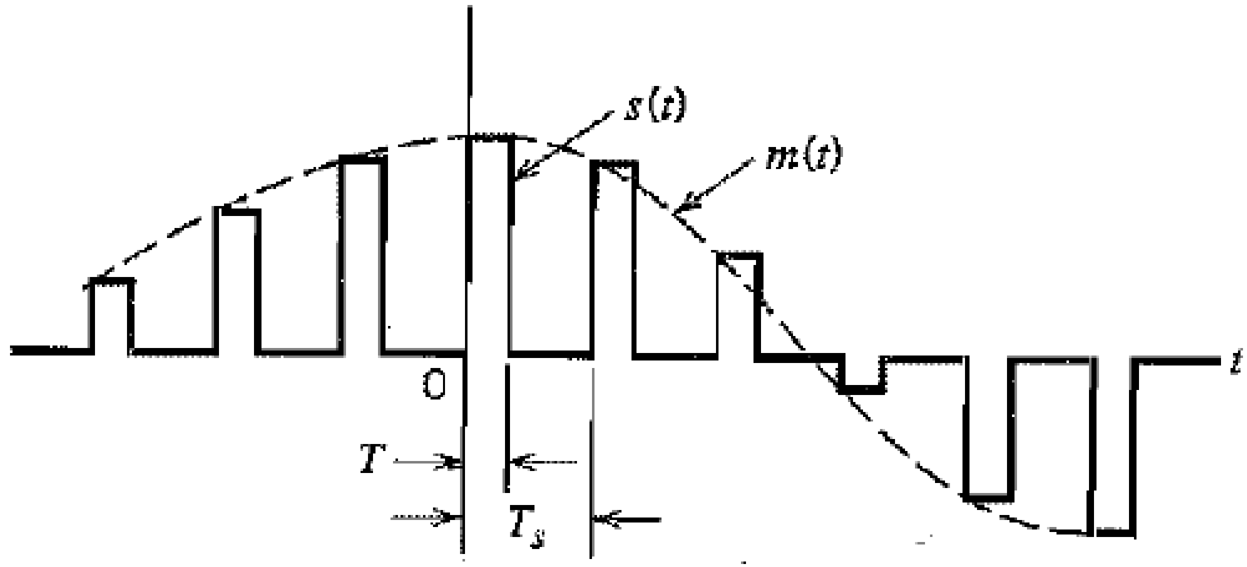

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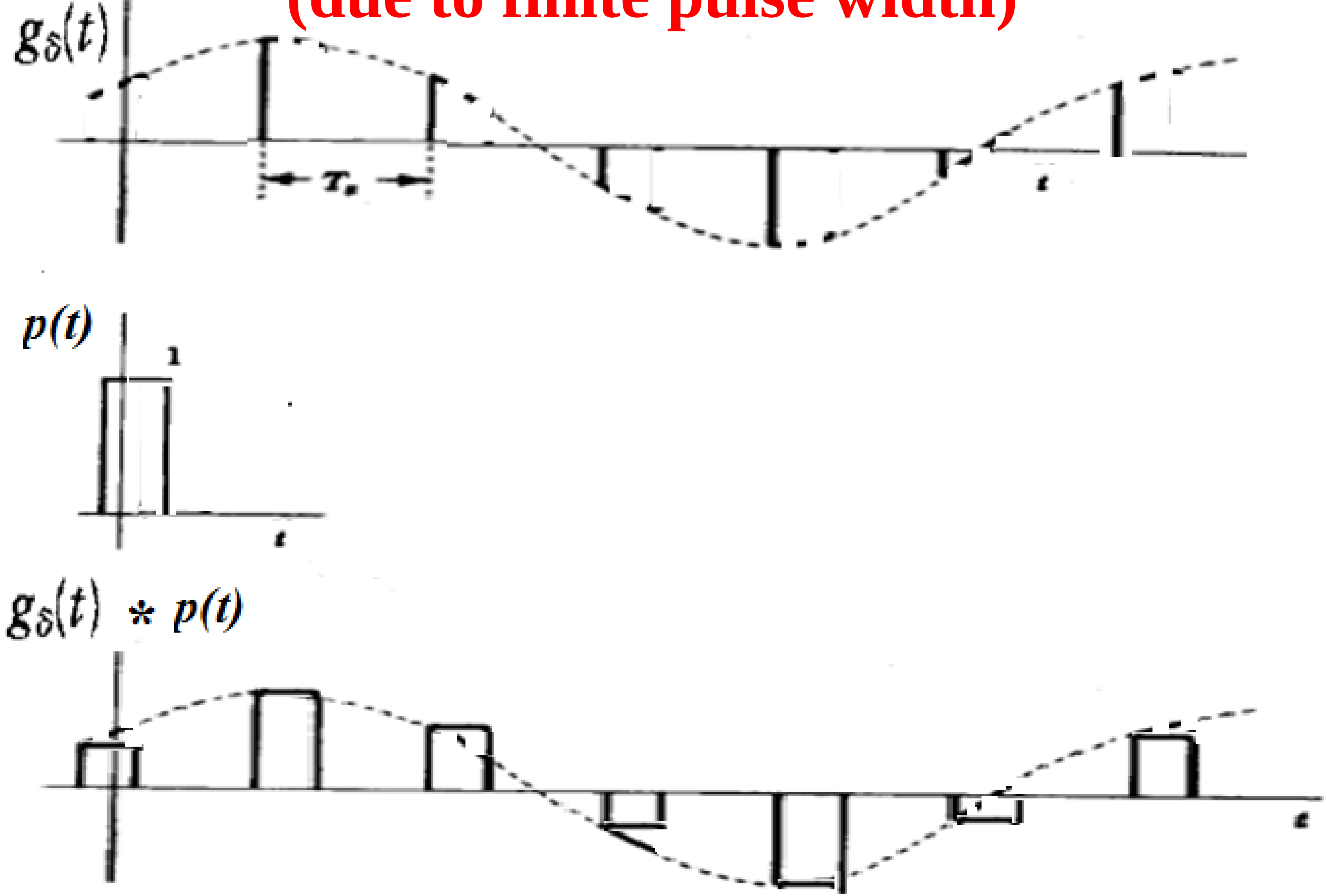

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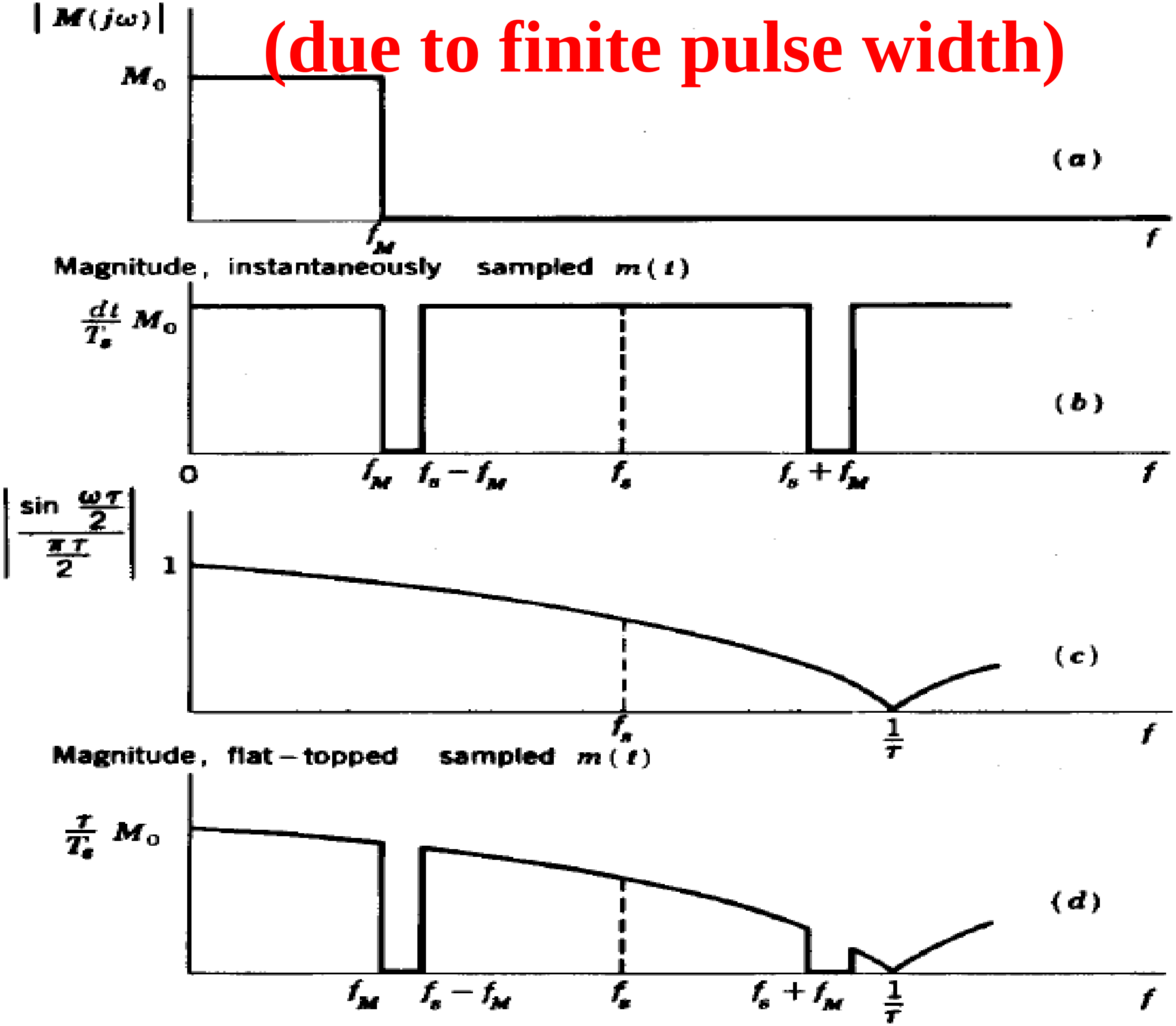

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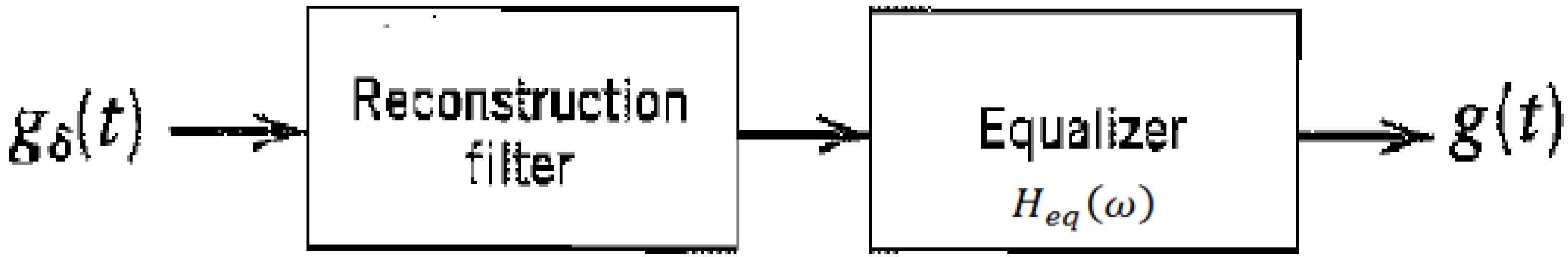
Digital-to-Analog Converter (DAC)



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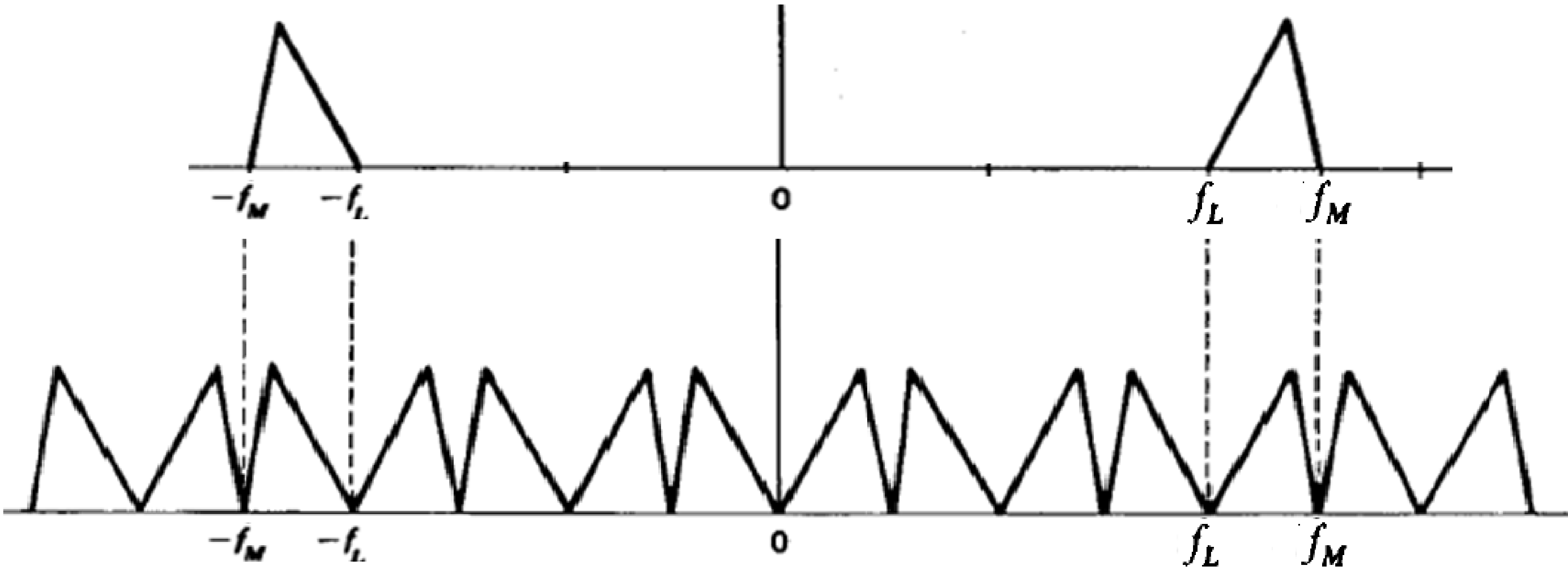
Sampling of Band Pass Signals



$f_s = 2(f_M - f_L)$ provided that either f_M or f_L is a harmonic of f_s

$= 2(10.1 - 10.0) = 0.2$ MHz

Sampling Process

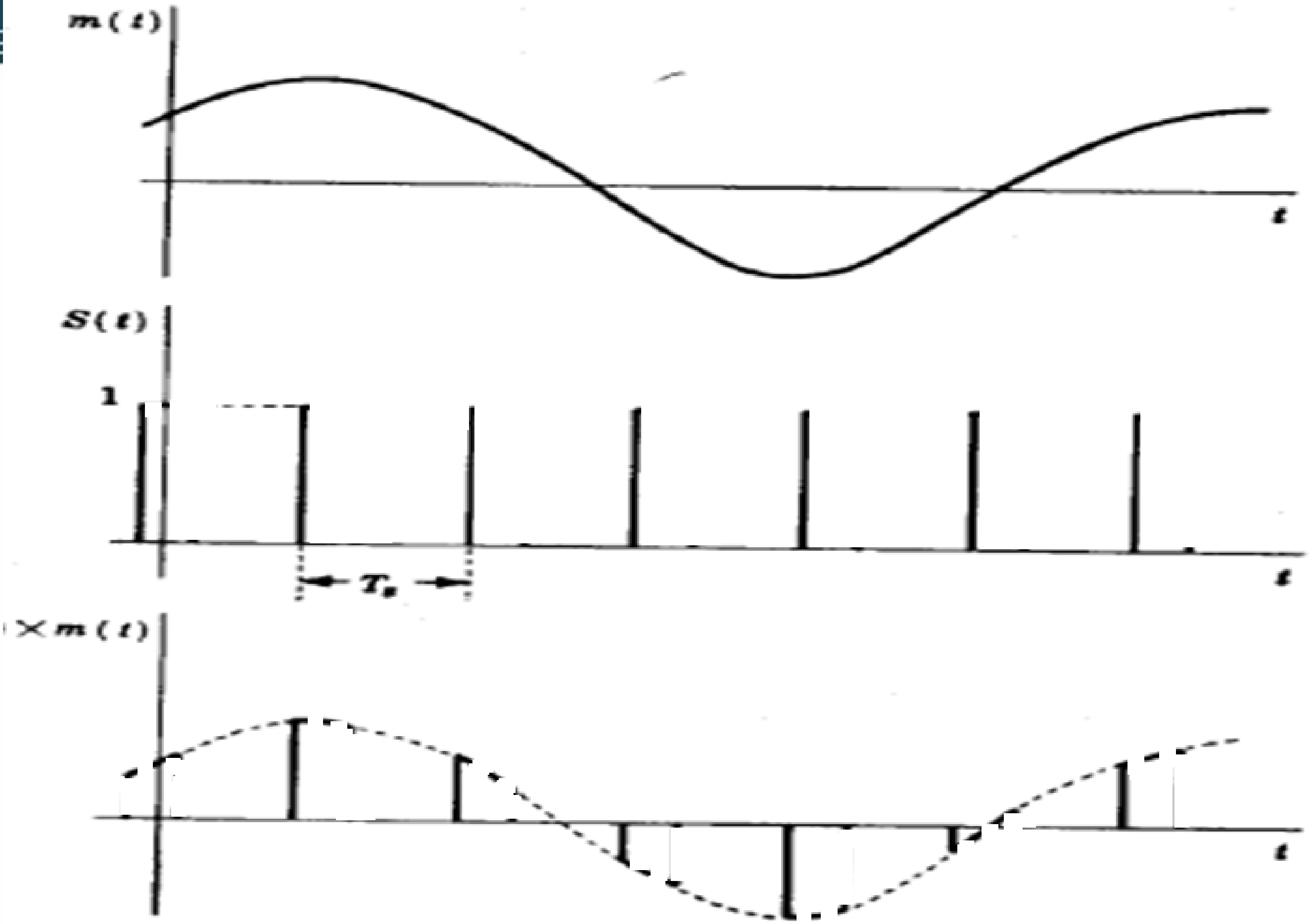
In sampling process, an analog signal is converted into a corresponding sequence of samples that are usually spaced uniformly in time.

Consider an arbitrary signal $g(t)$ of finite energy, which is specified for all time. Suppose that we sample the signal $g(t)$ instantaneously and at a uniform rate, once every T_s seconds We refer to T_s as the *sampling period*, and to its reciprocal $f_s = 1/T_s$ as the *sampling rate*. This ideal form of sampling is called *instantaneous sampling*.

The sampled signal s given by

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\, \delta(t - nT_s) \qquad \text{-----------------(1)}$$

Ideal (or) Instantaneous Sampling

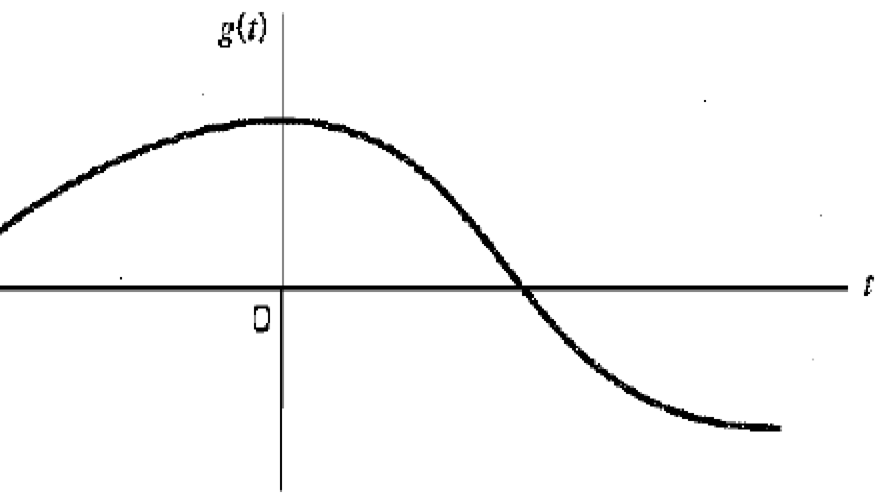


$$S(t) = \frac{dt}{T_s} + \frac{2\,dt}{T_s}\left(\cos 2\pi \frac{t}{T_s} + \cos 2 \times 2\pi \frac{t}{T_s} + \cdots\right)$$

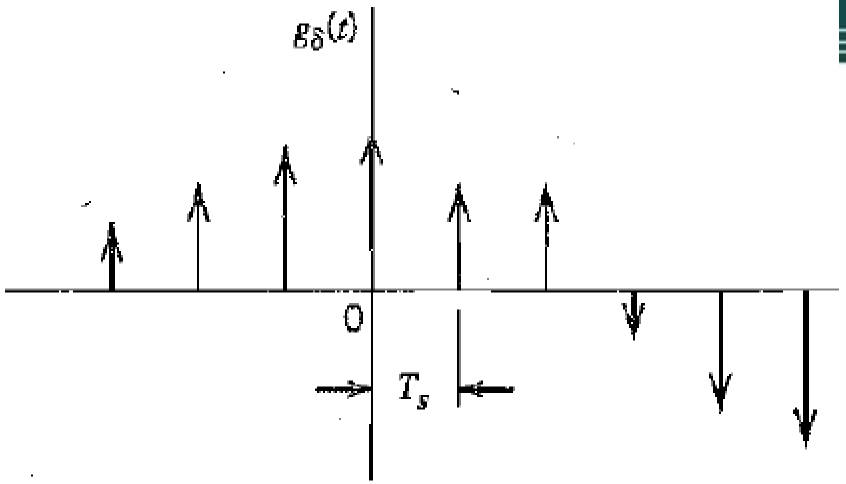
dt is pulse width

For $T_s = 1/2f_M$,

$$S(t)m(t) = \frac{dt}{T_s}\, m(t) + \frac{dt}{T_s}\,[2m(t) \cos 2\pi(2f_M)t + 2m(t) \cos 2\pi(4f_M)t + \cdots]$$



(a) Analog signal.



(b) sampled signal

Using the table of Fourier-transform pairs, we may write

$$g_\delta(t) \rightleftharpoons f_s \sum_{m=-\infty}^{\infty} G(f - mf_s) \qquad \text{------------------}(2)$$

where $G(f)$ is the Fourier transform of the original signal $g(t)$, and f_s is the sampling rate. Equation (3.2) states that *the process of uniformly sampling a continuous-time signal of finite energy results in a periodic spectrum with a period equal to the sampling rate.*

Let $G_\delta(f)$ denote the Fourier transform of $g_\delta(t)$. We may therefore write

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s) \exp(-j2\pi n f T_s) \quad \text{----------------(3)}$$

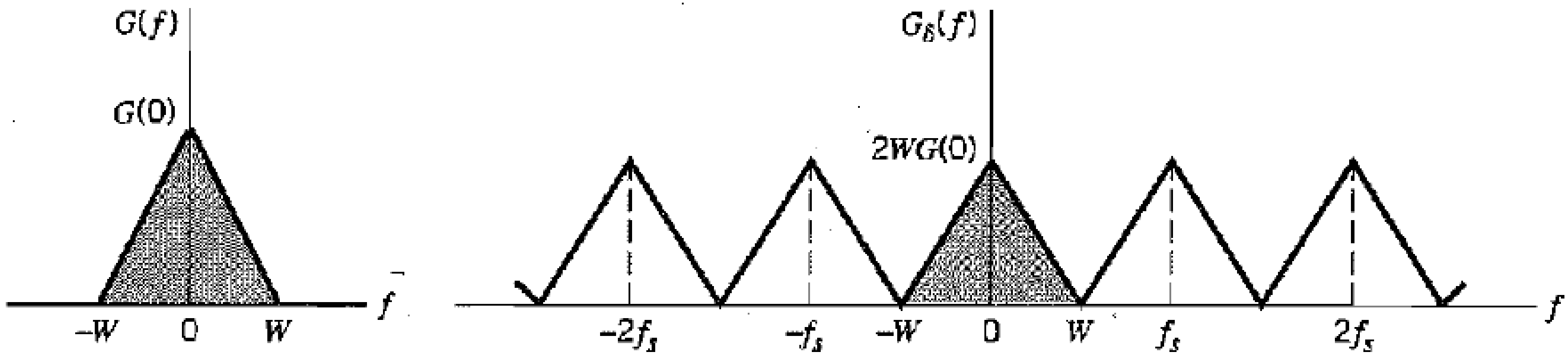
This relation is called the *discrete-time Fourier transform*. It may be viewed as a complex Fourier series representation of the periodic frequency function $G_\delta(f)$, with the sequence of samples $\{g(nT_s)\}$ defining the coefficients of the expansion.

Suppose, that the signal $g(t)$ is *strictly band-limited*, with no frequency components higher than W Hertz. Suppose also that $T_s = 1/2W$. Then the spectrum $G_\delta(f)$ becomes

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi n f}{W}\right) \quad \text{---------------------(4)}$$

Fourier transform of $g_\delta(t)$ may also be expressed as

$$G_\delta(f) = f_s G(f) + f_s \sum_{\substack{m=-\infty \\ m \neq 0}}^{\infty} G(f - m f_s) \quad \text{--------------(5)}$$



(*a*) Spectrum of a band-limited signal $g(t)$ (*b*) Spectrum of the sampled

Hence, under the following two conditions:

1. $G(f) = 0$ for $|f| \geq W$
2. $f_s = 2W$

we find from Equation (5) that

$$G(f) = \frac{1}{2W} G_\delta(f), \qquad -W < f < W \quad \text{----------------(6)}$$

Substituting Equation (4) into (6), we may also write

$$G(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi n f}{W}\right), \quad -W < f < W \quad \text{---------(7)}$$

Therefore, if the sample values $g(n/2W)$ of a signal $g(t)$ are specified for all n, then the Fourier transform $G(f)$ of the signal is uniquely determined by using the discrete-time Fourier transform of Equation (7). Because $g(t)$ is related to $G(f)$ by the inverse Fourier transform, it follows that the signal $g(t)$ is itself uniquely determined by the sample values $g(n/2W)$ for $-\infty < n < \infty$. In other words, the sequence $\{g(n/2W)\}$ has all the information contained in $g(t)$.

Sampling Theorem

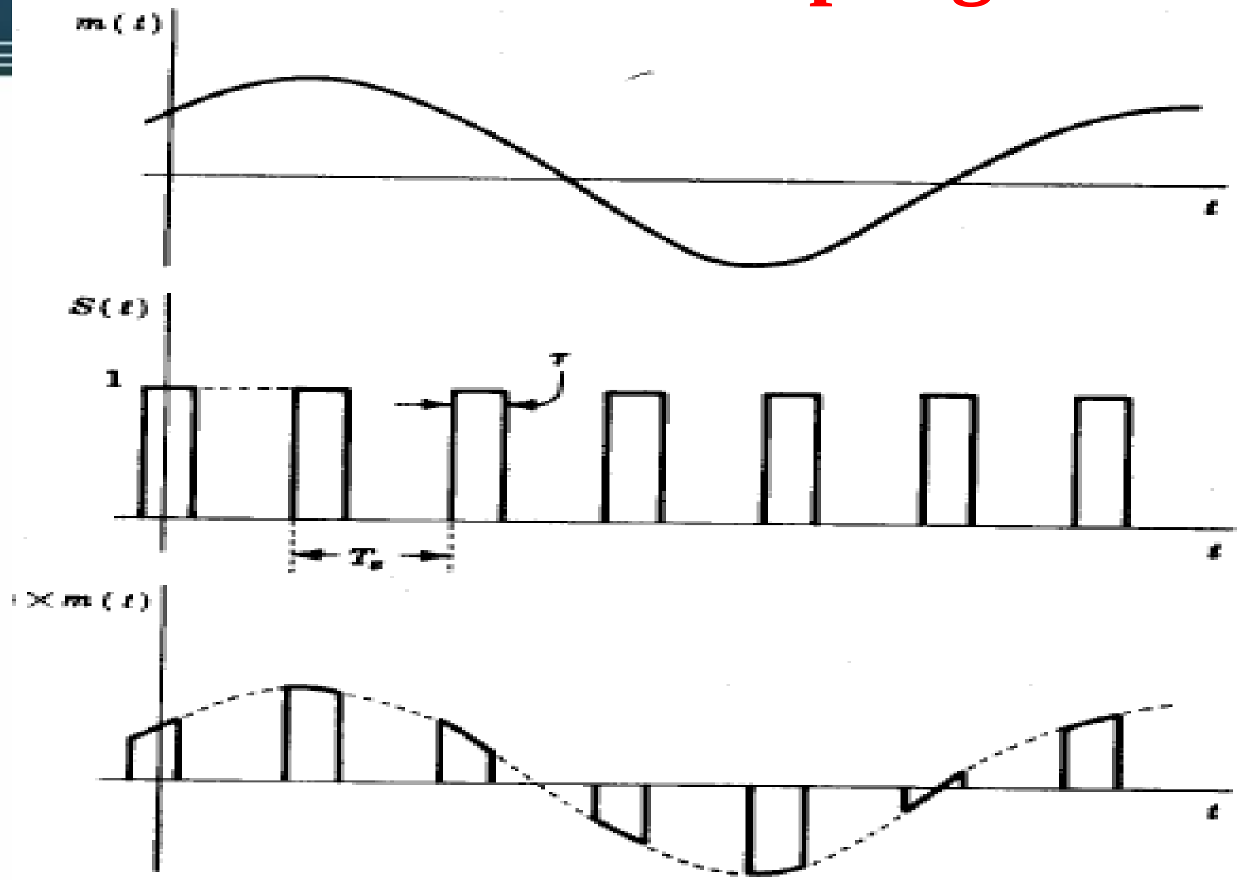
1. *A band-limited signal of finite energy, which has no frequency components higher than W Hertz, is completely described by specifying the values of the signal at instants of time separated by 1/2W seconds.*
2. *A band-limited signal of finite energy, which has no frequency components higher than W Hertz, may be completely recovered from a knowledge of its samples taken at the rate of 2W samples per second.*

To combat the effects of aliasing in practice, we may use two corrective measures,

1. Prior to sampling, a low-pass *anti-aliasing filter* is used to attenuate those high-frequency components of the signal that are not essential to the information being conveyed by the signal.
2. The filtered signal is sampled at a rate slightly higher than the Nyquist rate.

In this case, the design of the *reconstruction filter* becomes easy.

Natural Sampling



$$S(t) = \frac{\tau}{T_s} + \frac{2\tau}{T_s}\left(C_1 \cos 2\pi \frac{t}{T_s} + C_2 \cos 2 \times 2\pi \frac{t}{T_s} + \cdots\right)$$

with the constant C_n given by

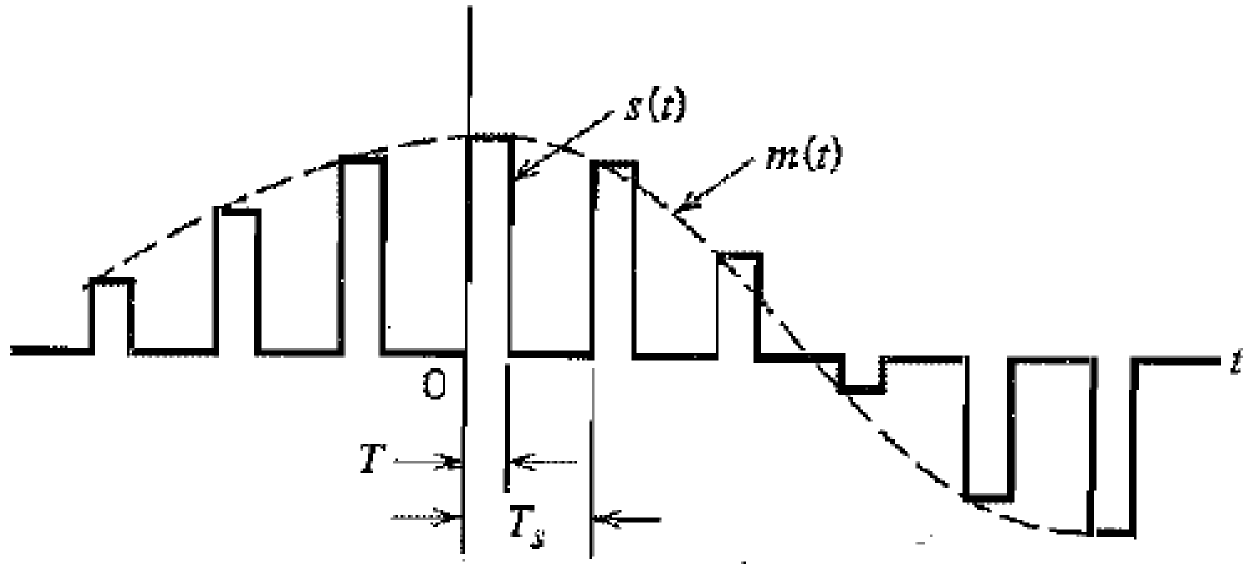
$$C_n = \frac{\sin\,(n\pi\tau/T_s)}{n\pi\tau/T_s}$$

$$S(t)m(t) = \frac{\tau}{T_s} m(t) + \frac{2\tau}{T_s} [m(t)C_1 \cos 2\pi(2f_M)t + m(t)C_2 \cos 2\pi(4f_M)t + \cdots]$$

Therefore, a low-pass filter with cutoff at f_M will deliver an output signal $s_o(t)$ given by

$$s_o(t) = \frac{\tau}{T_s} m(t)$$

Flat Top Sampling

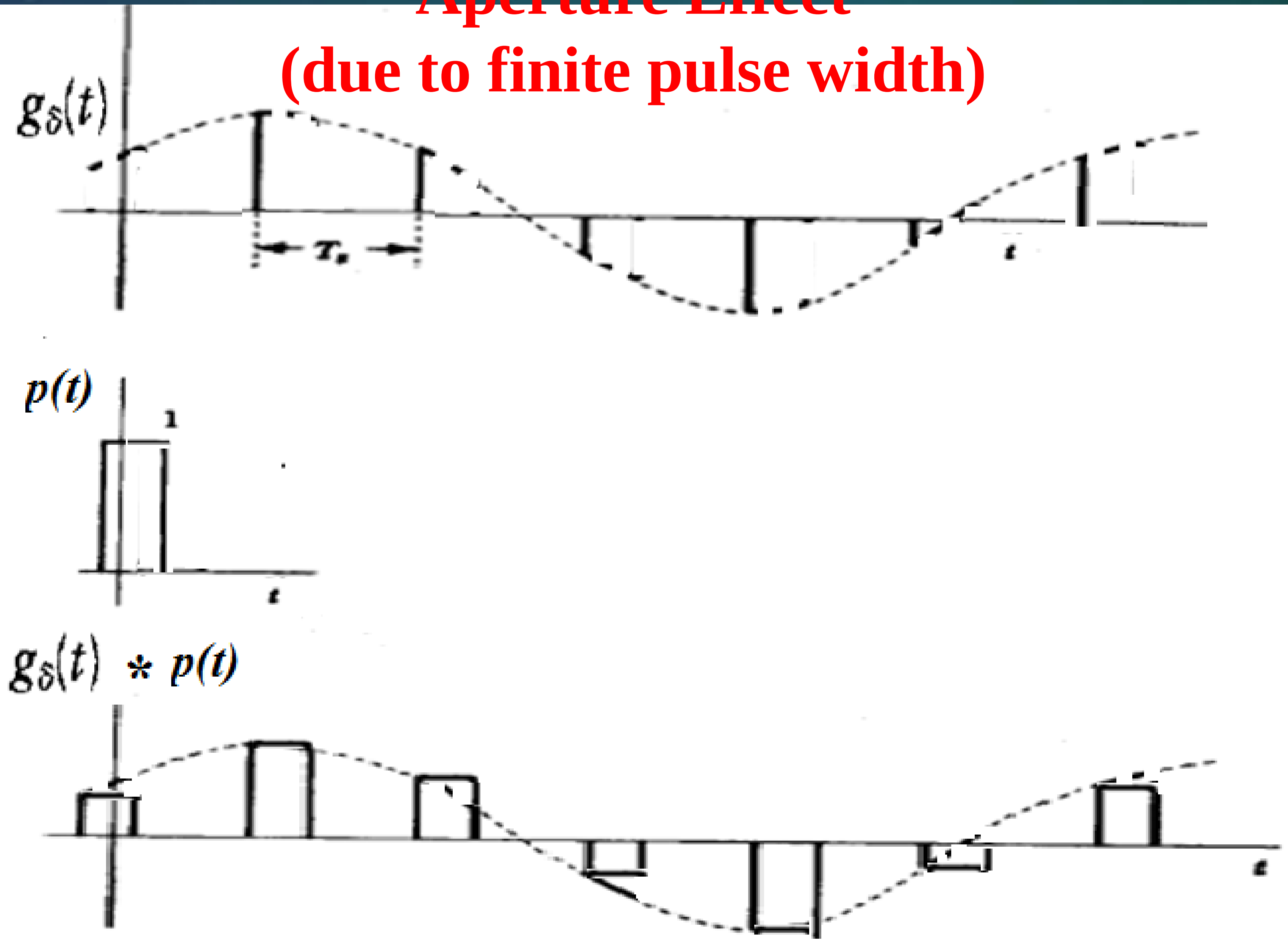


There are two operations involved

1. *Instantaneous sampling* of the message signal $m(t)$ every T_s seconds, where the sampling rate $f_s = 1/T_s$ is chosen in accordance with the sampling theorem.
2. *Lengthening* the duration of each sample so obtained to some constant value T.

In digital circuit technology, these two operations are jointly referred to as "sample and hold." One important reason for intentionally lengthening the duration of each sample is to avoid the use of an excessive channel bandwidth, since bandwidth is inversely proportional to pulse duration.

Aperture Effect
(due to finite pulse width)



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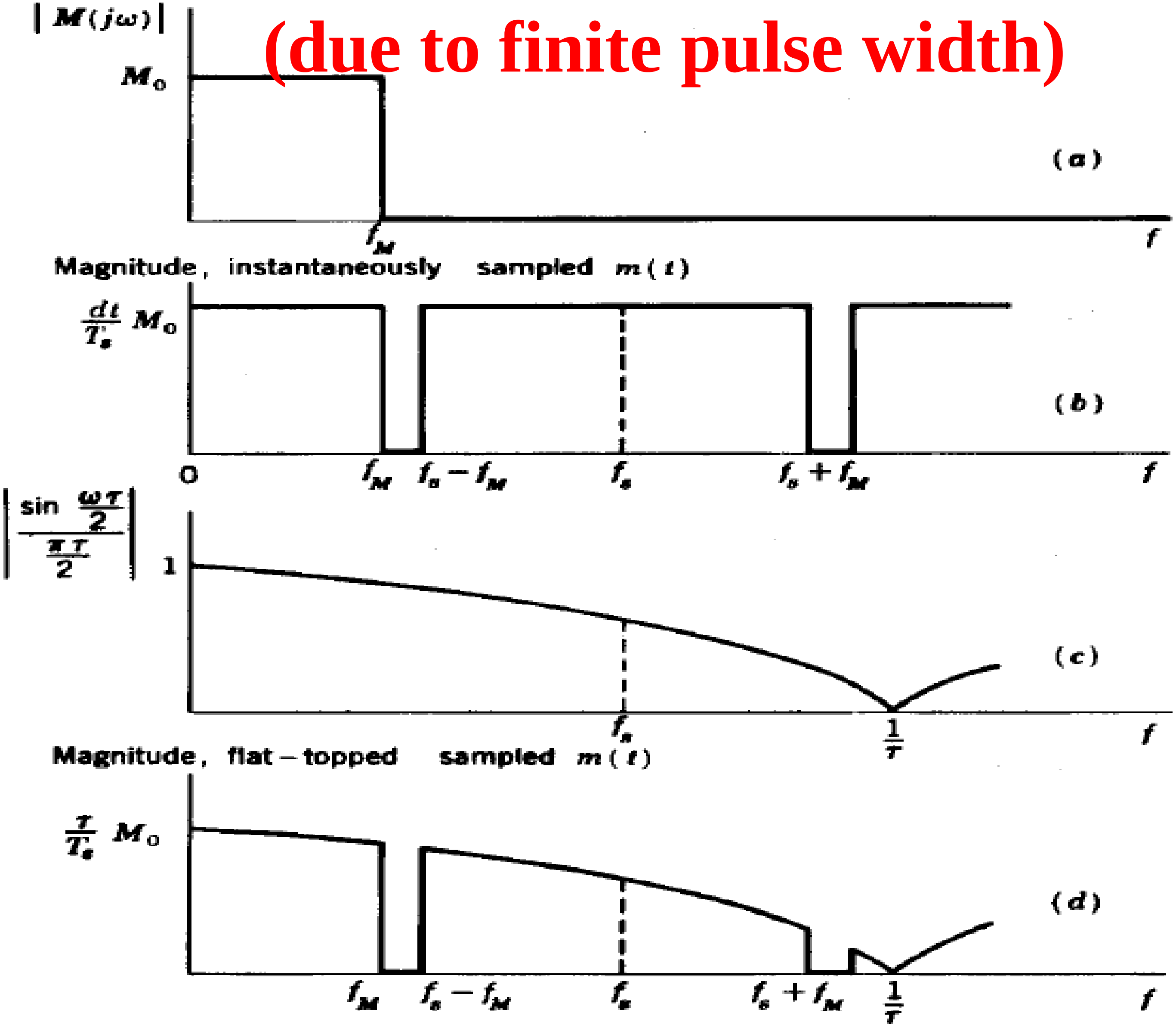


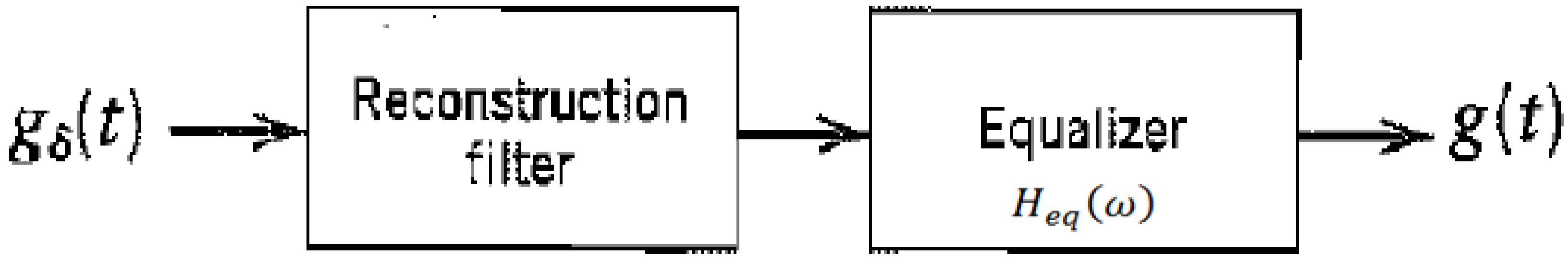
This distortion results because the original signal was "observed" through a finite rather than an infinitesimal time "aperture" and is hence referred to as *aperture effect* distortion.

The distortion results from the fact that the spectrum is multiplied by the sampling function $Sa(x) \equiv (\sin x)/x$ (with $x = \omega\tau/2$). The magnitude of the sampling function falls off slowly with increasing x in the neighborhood of $x = 0$

To minimize the distortion due to the aperture effect, it is advantageous to arrange that $x = \pi$ correspond to a frequency very large in comparison with f_M. Since $x = \pi f\tau$, the frequency f_0 corresponding to $x = \pi$ is $f_0 = 1/\tau$. If $f_0 \gg f_M$, or, correspondingly, if $\tau \ll 1/f_M$, the aperture distortion will be small. The distortion becomes progressively smaller with decreasing τ. And, of course, as $\tau \rightarrow 0$ (instantaneous sampling), the distortion similarly approaches zero.

Thus the effect of *flat-top sampling* is the unequal transmission of spectral components in the range 0 to f_M. If the distortion is not acceptable, then, it may be corrected by an $x/\sin x$ equalizer.

Digital-to-Analog Converter (DAC)



$$H(\omega) = T\ sinc\left(\frac{\omega T}{2}\right) = T\ sinc\left(\frac{2\pi f T}{2}\right)$$

$$= T\ sinc\left(\frac{2\pi f_s T}{4}\right) = T\ sinc\left(0.5\,\pi\frac{T}{T_s}\right)$$

when normalized to that value at zero frequency i.e. T.

$$H(\omega) = \ sinc\left(0.5\,\pi\frac{T}{T_s}\right)$$

The equalizer must have the response as

$$H_{eq}(\omega) = \frac{1}{H(\omega)} = \frac{1}{sinc\left(0.5\,\pi\frac{T}{T_s}\right)}$$

$$= \frac{0.5\,\pi\frac{T}{T_s}}{sin\left(0.5\,\pi\frac{T}{T_s}\right)}$$

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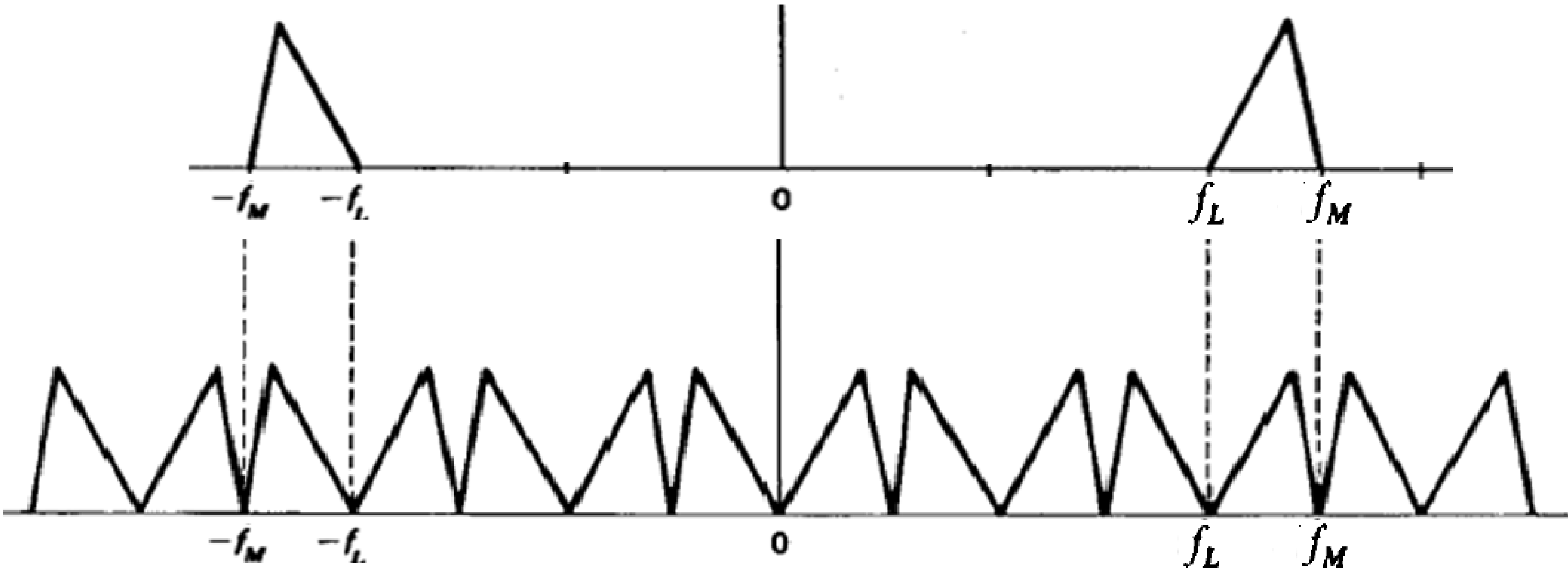

$f_s = 2(f_M - f_L)$ provided that either f_M or f_L is a harmonic of f_s

$= 2(10.1 - 10.0) = 0.2$ MHz