INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)
Dundigal, Hyderabad - 500 043

Course : ENGINEERING MECHANICS (AME002)

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Subject

- Graduates:
  - Midterm exam 30%
  - Final exam 70%
- Course Materials
  - Lecture notes
    - Power points slides
    - Class notes
  - Textbooks
    - Engineering Mechanics: Statics 10th Edition by R.C. Hibbeler

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COURSE OBJECTIVES

The course should enable the students to:

I. Develop the ability to work comfortably with basic engineering mechanics concepts required for analysing static structures.
II. Identify an appropriate structural system to studying a given problem and isolate it from its environment, model the problem using good free body diagrams and accurate equilibrium equations.
III. Identify and model various types of loading and support conditions that act on structural systems, apply pertinent mathematical, physical and engineering mechanical principles to the system to solve and analyze the problem.
IV. Solve the problem of equilibrium by using the principle of work and energy in mechanical design and structural analysis.
V. Apply the concepts of vibrations to the problems associated with dynamic behavior.

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COURSE OUTCOMES

After completing this course the student must demonstrate the knowledge and ability to:

1. **Classifying** different types of motions in kinematics.
2. **Categorizing** the bodies in kinetics as a particle, rigid body in translation and rotation.
3. **Choosing principle** of impulse momentum and virtual work for equilibrium of ideal systems, stable and unstable equilibriums
4. **Appraising** work and energy method for particle motion and plane motion.
5. **Apply** the concepts of vibrations.

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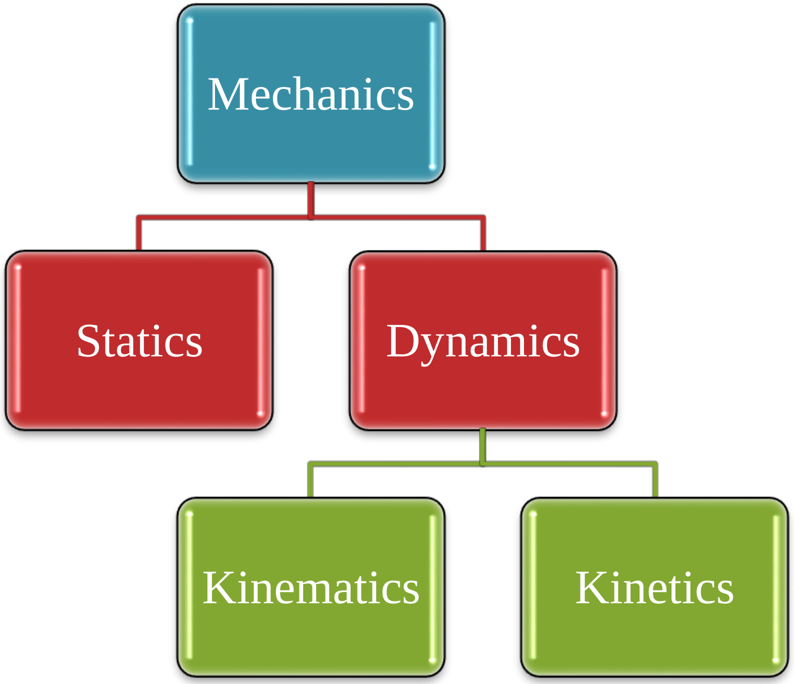
Course Outline

	PARTICLE	SYSTEM OF PARTICLES	RIGID BODIES
KINEMATICS	Chapter 1	↑	Chapter 3
KINETICS NEWTON'S LAW	Chapter 2	Chapter 4	Chapter 4
KINETICS ENERGY & MOMENTUM	Chapter 3	↓	Chapter 5

Introduction to Mechanics

- **What is mechanics?**
  - Physical science deals with the state of rest or motion of bodies under the action of force



- **Why we study mechanics?**
  - This science form the groundwork for further study in the design and analysis of structures

Basic Terms

- Essential basic terms to be understood
  - **Statics:** dealing with the equilibrium of a rigid-body at rest
  - **Rigid body:** the relative movement between its parts are negligible
  - **Dynamics:** dealing with a rigid-body in motion
  - **Length:** applied to the linear dimension of a straight line or curved line
  - **Area:** the two dimensional size of shape or surface
  - **Volume:** the three dimensional size of the space occupied by substance
  - **Force:** the action of one body on another whether it's a push or a pull force
  - **Mass:** the amount of matter in a body
  - **Weight:** the force with which a body is attracted toward the centre of the Earth
  - **Particle:** a body of negligible dimension

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Units of Measurement

- Four fundamental quantities in mechanics
  - Mass
  - Length
  - Time
  - Force
- Two different systems of units we dealing with during the course
  - ______Units (CGS)
    - Length in centimeter(cm)
    - Time in Seconds (s)
    - Force in kilograms (kg)
  - International System of Units or Metric Units (SI)
    - Length in metre (m)
    - Time in Seconds (s)
    - Force in Newton (N)

Units of Measurement

- Summery of the four fundamental quantities in the two system

Quantity	SI Units		US Units	
	Unit	Symbol	Unit	Symbol
Mass	kilogram	kg	slug	-
Length	meter	m	foot	ft
Time	second	s	second	sec
Force	newton	N	pound	lb

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Units of Measurement

- Metric System (SI)
  - SI System offers major advantages relative to the FPS system
    - Widely used throughout the world
    - Use one basic unit for length ☞ meter; while FPS uses many basic units ☞ inch, foot, yard, mile
    - SI based on multiples of 10, which makes it easier to use & learn whereas FPS is complicated, for example
      - SI system ➔ 1 meter = 100 centimeters, 1 kilometer = 1000 meters, etc
      - FPS system ➔ 1 foot = 12 inches, 1 yard = 3 feet, 1 mile = 5280 feet, etc
- Metric System (SI)
  - Newton's second law F = m.a
    - Thus the force (N) = mass (kg) × acceleration (m/s^2)
  - Therefore 1 Newton is the force required to give a mass of 1 kg an acceleration of 1 m/s^2

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Units of Measurement

- U.S. Customary System (FPS)
  - Force (lb) = mass (slugs) × acceleration (ft/sec^2)
    - Thus (slugs) = lb.sec^2/ft
  - Therefore 1 slug is the mass which is given an acceleration of 1 ft/sec^2 when acted upon by a force of 1 lb
- Conversion of Units
  - Converting from one system of unit to another;

Quantity	FPS	Equals	SI
Force	**1 lb**	⇨	**4.448 N**
Mass	**1 slug**	⇨	**14.593 kg**
Length	**1 ft**	⇨	**0.304 m**

- The standard value of g (gravitational acceleration)
  - SI units g = 9.806 m/s2
  - FPS units g = 32.174 ft/sec2

Objectives

To provide an introduction of:

※ Fundamental concepts,

※ General principles,

※ Analysis methods,

※ Future Studies

in Engineering Mechanics.

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Outline

- 1. Engineering Mechanics
- 2. Fundamental Concepts
- 3. General Principles
- 4. StaticAnalysis
- 5. DynamicAnalysis
- 6. Future Studies

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1. Engineering Mechanics

- Mechanics :

  - Rigid-body Mechanics
  - Deformable-body Mechanics
  - Fluid Mechanics

- Rigid-body Mechanics :

  - Statics
  - Dynamics

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1. Engineering Mechanics

- Statics – Equilibrium Analysis of particles and bodies
- Dynamics – Accelerated motion of particles and bodies

  Kinematics and Kinetics
- Mechanics of Materials…
- Theory of Vibration…

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2. Fundamentals Concepts

Basic Quantities

- Length,   Mass, Time, Force

Units of Measurement

- m, kg,s, N…   (SI, Int.System of Units)

\- Dimensional Homogeneity

\- Significant Figures

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2. Fundamentals Concepts

Idealizations

- Particles
  – Consider mass but neglect size
- Rigid Body
  – Neglect material properties
- Concentrated Force
- Supports and Reactions

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3. General Principles

- Newton's Laws of Motion
  - First Law, Second Law, Third Law
  - Law of Gravitational Attraction

- D'Alembert Principle : $F+(-ma)=0$

- Impulse and Momentum

- Work and Energy

- Principle of Virtual Work (Equilibrium)

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4. Static Analysis

- Force and Equilibrium
- Force System Resultants
- Structural Analysis
- Internal forces
- Friction
- Centroid and Moments of Inertia
- Virtual Work and Stability

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5. Dynamic Analysis

- Kinematics of a Particle
- Kinetics: Force and Acceleration
- Work and Energy
- Impulse and Momentum (Impact)
- Planar Kinematics and Kinetics
- 3-D Kinematics and Kinetics
- Vibrations

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UNIT-I

KINEMATICS OF PARTICLES IN RECTILINEAR MOTION

Motion of a particle, rectilinear motion, motion curves, rectangular components of curvilinear motion, kinematics of rigid body, types of rigid body motion, angular motion, fixed axis rotation.

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INTRODUCTION TO DYNAMICS

- Galileo and Newton (Galileo's experiments led to Newton's laws)
- Kinematics – study of motion
- Kinetics – the study of what causes changes in motion
- Dynamics is composed of kinematics and kinetics

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Introduction

- Dynamics includes:

  - ***Kinematics***: study of the motion (displacement, velocity, acceleration, & time) without reference to the cause of motion (i.e. *regardless of forces*).

  - ***Kinetics***: study of the forces acting on a body, and the resulting motion caused by the given forces.

- ***Rectilinear*** motion: position, velocity, and acceleration of a particle as it moves along a **straight line**.

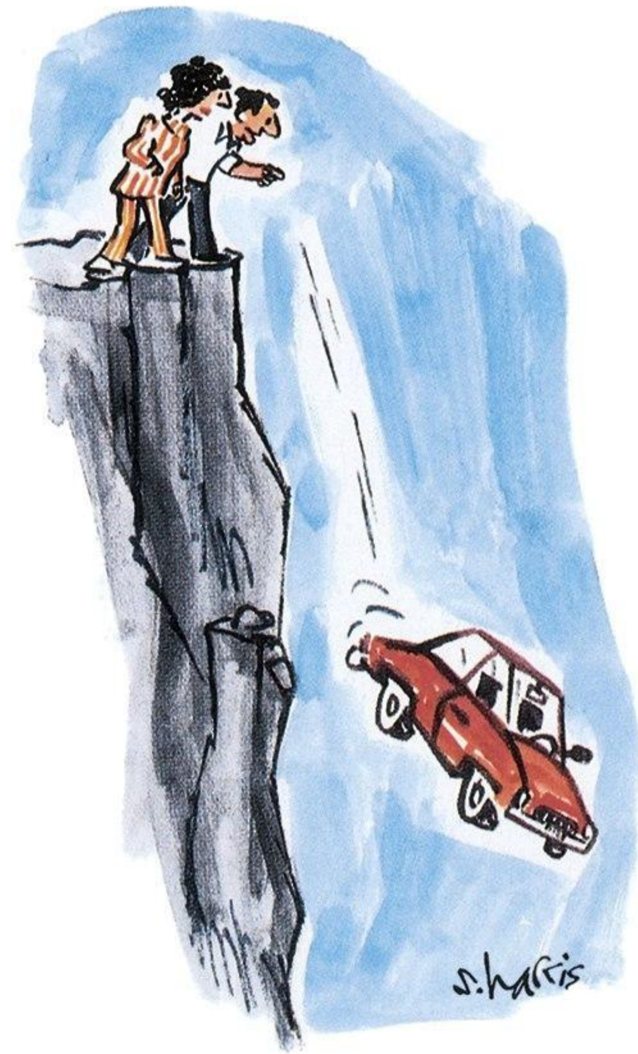
- ***Curvilinear*** motion: position, velocity, and acceleration of a particle as it moves along a **curved line**.

RECTILINEAR MOTION OF PARTICLES

Rectilinear Motion: Position, Velocity & Acceleration

MECHANICS
Kinematics of Particles
Motion in One Dimension

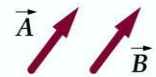
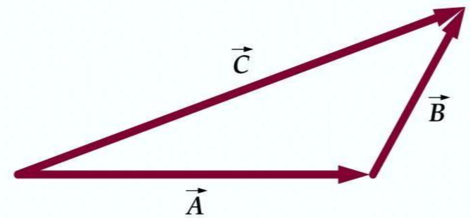
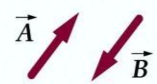
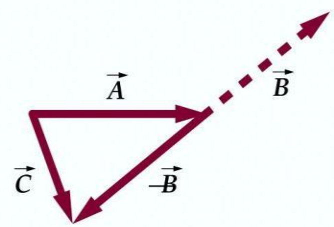
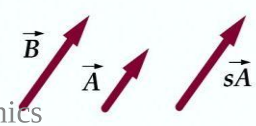
Acceleration



"It goes from zero to 60 in about 3 seconds."

Summary of properties of vectors

Properties of Vectors

Property	Explanation	Figure	Component representation
Equality	$\vec{A} = \vec{B}$ if $\lvert\vec{A}\rvert = \lvert\vec{B}\rvert$ and their directions are the same		$A_x = B_x$ $A_y = B_y$ $A_z = B_z$
Addition	$\vec{C} = \vec{A} + \vec{B}$		$C_x = A_x + B_x$ $C_y = A_y + B_y$ $C_z = A_z + B_z$
Negative of a vector	$\vec{A} = -\vec{B}$ if $\lvert\vec{B}\rvert = \lvert\vec{A}\rvert$ and their directions are opposite		$A_x = -B_x$ $A_y = -B_y$ $A_z = -B_z$
Subtraction	$\vec{C} = \vec{A} - \vec{B}$		$C_x = A_x - B_x$ $C_y = A_y - B_y$ $C_z = A_z - B_z$
Multiplication by a scalar	$\vec{B} = s\vec{A}$ has magnitude $\lvert\vec{B}\rvert = \lvert s\rvert\lvert\vec{A}\rvert$ and has the same direction as $\vec{A}$ if s is positive or $-\vec{A}$ if s is negative		$B_x = sA_x$ $B_y = sA_y$ $B_z = sA_z$

POSITION, VELOCITY, AND ACCELERATION

For linear motion x marks the position of an object. Position units would be m, ft, etc.

Average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

Velocity units would be in m/s, ft/s, etc.
The instantaneous velocity is

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

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The average acceleration is

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

The units of acceleration would be m/s², ft/s², etc.

The instantaneous acceleration is

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\frac{dx}{dt} = \frac{d^2 x}{dt^2}$$

Notice If v is a function of x, then

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

One more derivative

$$\frac{da}{dt} = Jerk$$

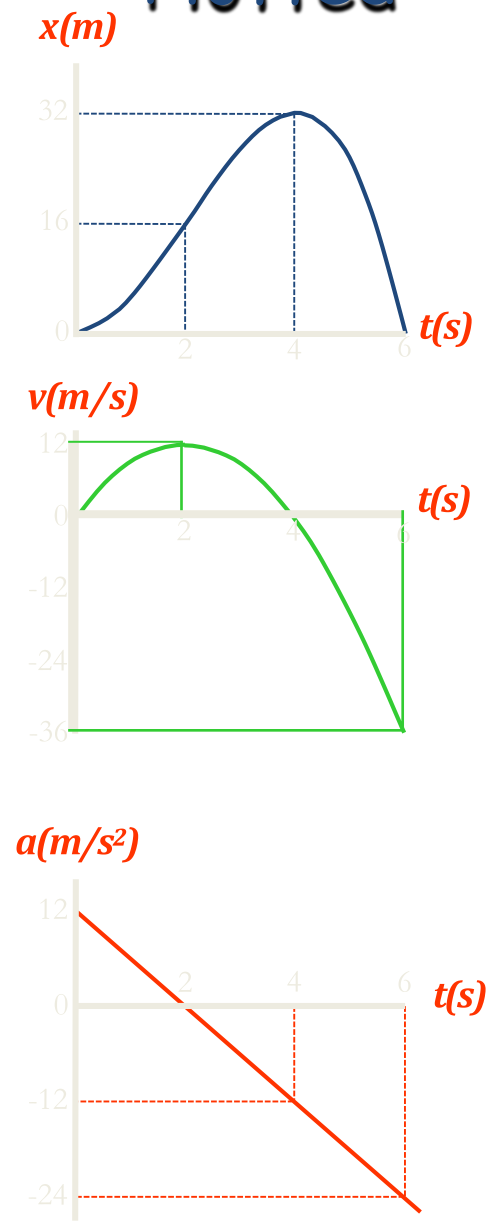
Consider the function

$$x = -t^3 + 6t^2$$

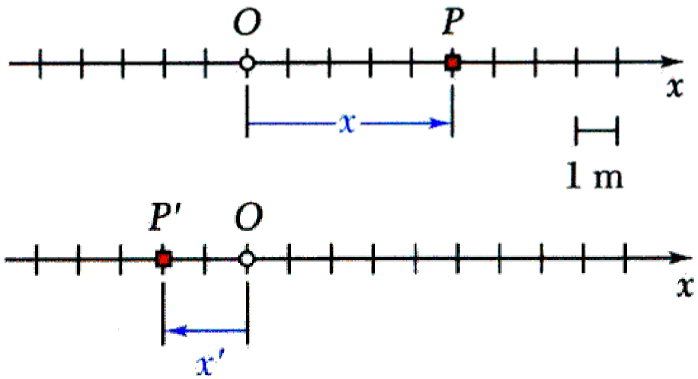
$$v = -3t^2 + 12t$$

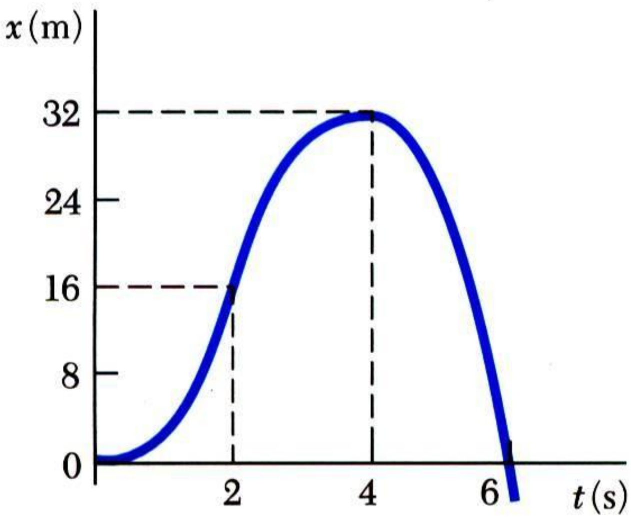
$$a = -6t + 12$$

Plotted



Rectilinear Motion: Position, Velocity & Acceleration



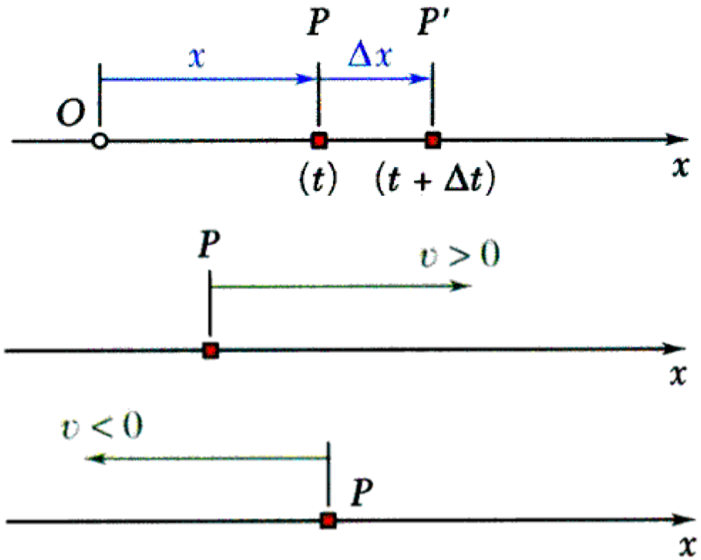


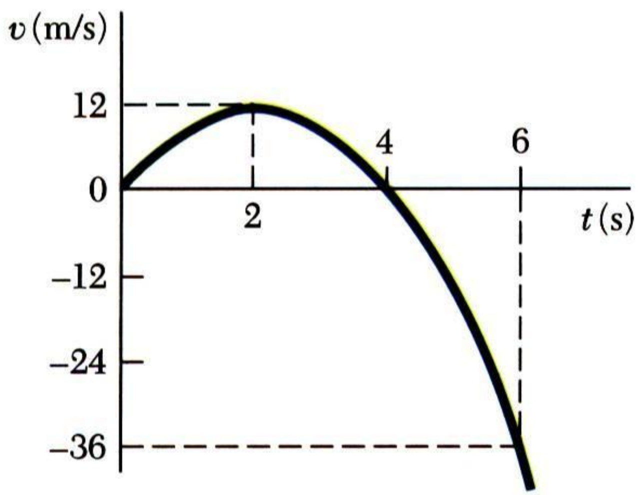
- Particle moving along a straight line is said to be in *rectilinear motion*.
- *Position coordinate* of a particle is defined by (+ or -) distance of particle from a fixed origin on the line.
- The *motion* of a particle is known if the position coordinate for particle is known for every value of time *t*. Motion of the particle may be expressed in the form of a function, e.g.,

$$x = 6t^2 - t^3$$

or in the form of a graph *x* vs. *t*.

Rectilinear Motion: Position, Velocity & Acceleration





- Consider particle which occupies position *P* at time *t* and *P'* at *t*+*Δt*,

$$Average\ velocity = \frac{\Delta x}{\Delta t}$$

$$Instantaneous\ velocity = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

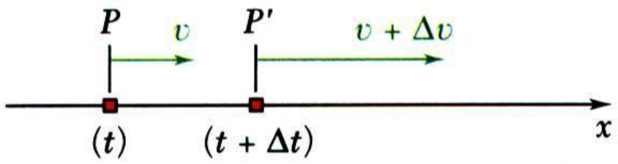
- Instantaneous velocity may be positive or negative. Magnitude of velocity is referred to as *particle speed*.
- From the definition of a derivative,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

e.g., $x = 6t^2 - t^3$

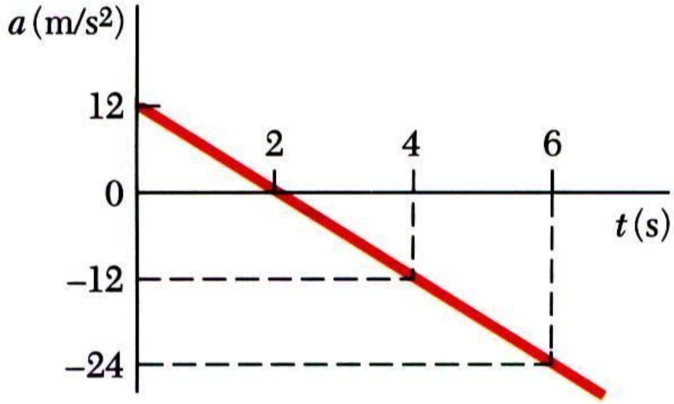
$$v = \frac{dx}{dt} = 12t - 3t^2$$

Rectilinear Motion: Position, Velocity & Acceleration



- Consider particle with velocity v at time t and v' at $t+\Delta t$,

  Instantaneous acceleration $= a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$



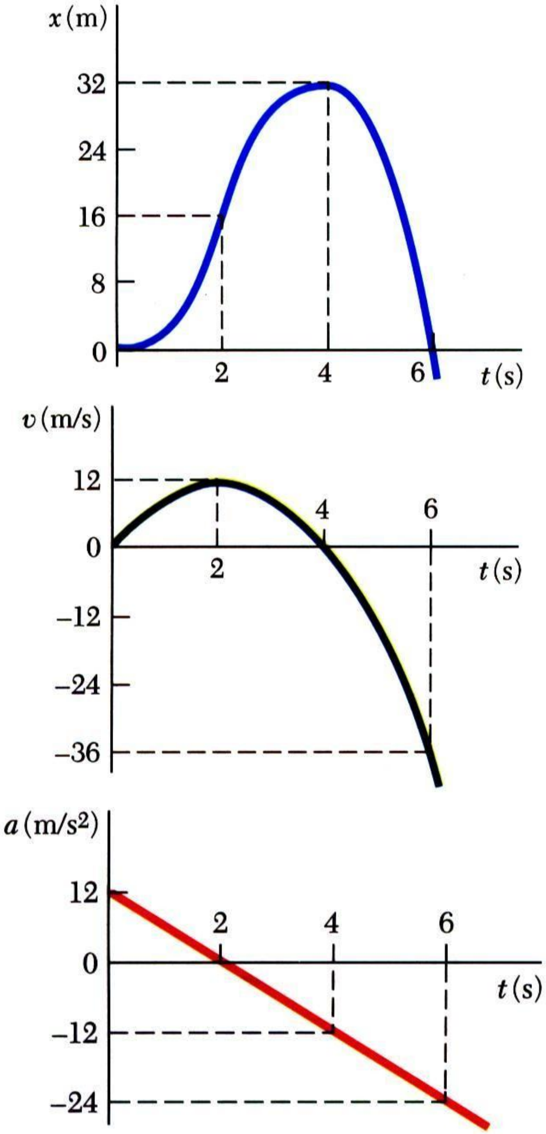
- From the definition of a derivative,

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

e.g. $v = 12t - 3t^2$

$$a = \frac{dv}{dt} = 12 - 6t$$

Rectilinear Motion: Position, Velocity & Acceleration



- Consider particle with motion given by

$$x = 6t^2 - t^3$$

$$v = \frac{dx}{dt} = 12t - 3t^2$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 12 - 6t$$

- at $t = 0$, $x = 0$, $v = 0$, $a = 12$ m/s^2
- at $t = 2$ s, $x = 16$ m, $v = v_{max} = 12$ m/s, $a = 0$
- at $t = 4$ s, $x = x_{max} = 32$ m, $v = 0$, $a = -12$ m/s^2
- at $t = 6$ s, $x = 0$, $v = -36$ m/s, $a = -24$ m/s^2

DETERMINATION OF THE MOTION OF A PARTICLE

Three common classes of motion

1. $$a = f(t) = \frac{dv}{dt}$$

$$dv = a\,dt = f(t)\,dt$$

$$v - v_0 = \int_0^t f(t)\,dt = \frac{dx}{dt} - v_0$$

$$\frac{dx}{dt} = v_0 + \int_0^t f(t)\,dt$$

$$\frac{dx}{dt} = v_o + \int_0^t f(t)dt$$

$$dx = v_o dt + \left[\int_0^t f(t)dt \right] dt$$

$$x - x_o = v_o t + \int_0^t \left[\int_0^t f(t)dt \right] dt$$

$$x = x_0 + v_0 t + \int_0^t \left[\int_0^t f(t)dt \right] dt$$

$$2. \quad a = f(x) = v\frac{dv}{dx}$$

$$vdv = adx = f(x)dx$$

$$\tfrac{1}{2}(v^2 - v_0^2) = \int_{x_o}^{x} f(x)dx$$

with $v = \dfrac{dx}{dt}$ then get $x = x(t)$

3. $$a = f(v) = \frac{dv}{dt} = v\frac{dv}{dx}$$

$$\int_{v_0}^{v} \frac{dv}{f(v)} = \int_{0}^{t} dt = t$$

or $$\int_{x_0}^{x} dx = \int_{v_0}^{v} \frac{v\,dv}{f(v)}$$ Both can lead to

$$x = x(t)$$

UNIFORM RECTILINEAR MOTION

$$v = constant$$

$$a = 0$$

$$v = \frac{dx}{dt}$$

$$x - x_0 = \int v dt = vt$$

$$x = x_0 + vt$$

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UNIFORMLY ACCELERATED RECTILINEAR MOTION

$$a = constant$$

$$v = v_0 + at$$

$$x = x_o + v_0 t + \tfrac{1}{2} a t^2$$

Also

$$v \frac{dv}{dx} = a$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

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Determining the Motion of a Particle

- Recall, *motion* is defined if position *x* is known for all time *t*.

$$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt} \qquad a = \frac{d^2x}{dt^2} \qquad a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

- If the acceleration is given, we can determine velocity and position by two successive integrations.

- Three classes of motion may be defined for:
  - acceleration given as a function of *time*, $a = f(t)$
  - acceleration given as a function of *position*, $a = f(x)$
  - acceleration given as a function of *velocity*, $a = f(v)$

Determining the Motion of a Particle

- **Acceleration given as a function of *time*, *a* = *f*(*t*):**

$$a = f(t) = \frac{dv}{dt} \quad \Rightarrow dv = f(t)dt \quad \Rightarrow \int_{v_0}^{v} dv = \int_{0}^{t} f(t)dt \quad \Rightarrow v - v_0 = \int_{0}^{t} f(t)dt$$

$$v = \frac{dx}{dt} \quad \Rightarrow dx = vdt \quad \Rightarrow \int_{x_0}^{x} dx = \int_{0}^{t} vdt \quad \Rightarrow x - x_0 = \int_{0}^{t} vdt$$

- **Acceleration given as a function of *position*, *a* = *f*(*x*):**

$$a = f(x) = v\frac{dv}{dx} \quad \Rightarrow vdv = f(x)dx \quad \Rightarrow \int_{v_0}^{v} vdv = \int_{x_0}^{x} f(x)dx \quad \Rightarrow \frac{1}{2}v^2 - \frac{1}{2}v_0^2 = \int_{x_0}^{x} f(x)dx$$

$$v = \frac{dx}{dt} \quad \Rightarrow \frac{dx}{v} = dt \quad \Rightarrow \int_{x_0}^{x} \frac{dx}{v} = \int_{0}^{t} dt$$

Determining the Motion of a Particle

- **Acceleration given as a function of velocity, *a* = *f*(*v*):**

$$a = f(v) = \frac{dv}{dt} \quad \Rightarrow \frac{dv}{f(v)} = dt \quad \Rightarrow \int_{v_0}^{v} \frac{dv}{f(v)} = \int_{0}^{t} dt \quad \Rightarrow \int_{v_0}^{v} \frac{dv}{f(v)} = t$$

$$a = f(v) = v\frac{dv}{dx} \quad \Rightarrow dx = \frac{v\,dv}{f(v)} \quad \Rightarrow \int_{x_0}^{x} dx = \int_{v_0}^{v} \frac{v\,dv}{f(v)} \quad \Rightarrow x - x_0 = \int_{v_0}^{v} \frac{v\,dv}{f(v)}$$

Summary

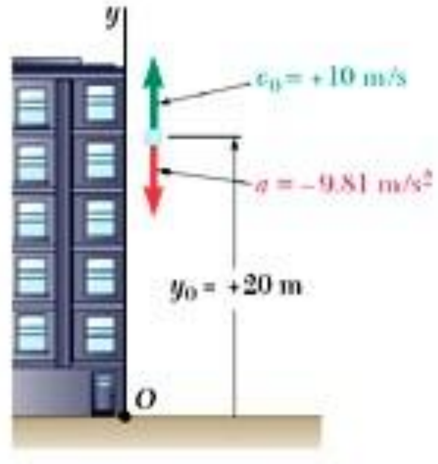
Procedure:

1. Establish a coordinate system & specify an origin
2. Remember: x, v, a, t are related by:

$$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt} \qquad a = \frac{d^2x}{dt^2} \qquad a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

3. When integrating, either use limits (if known) or add a constant of integration

Sample Problem 1

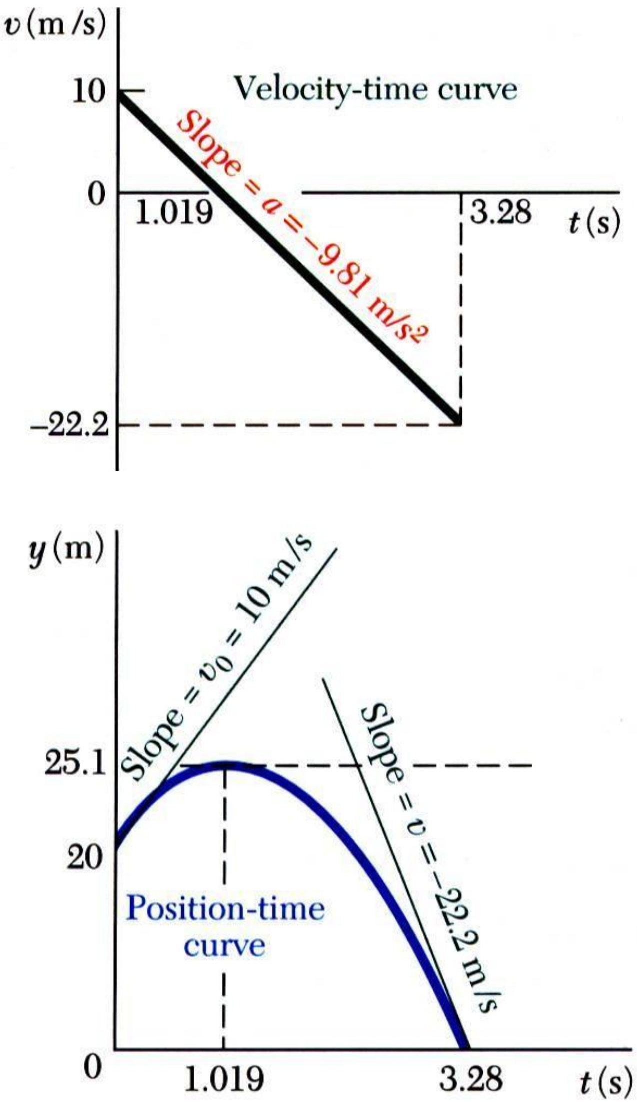


Ball tossed with 10 m/s vertical velocity from window 20 m above ground.

Determine:

- velocity and elevation above ground at time *t*,
- highest elevation reached by ball and corresponding time, and
- time when ball will hit the ground and corresponding velocity.

Sample Problem 1



SOLUTION:

- Integrate twice to find *v*(*t*) and *y*(*t*).

$$\frac{dv}{dt} = a = -9.81\,\mathrm{m/s^2}$$

$$\int_{v_0}^{v(t)} dv = -\int_0^t 9.81\,dt \qquad v(t) - v_0 = -9.81t$$

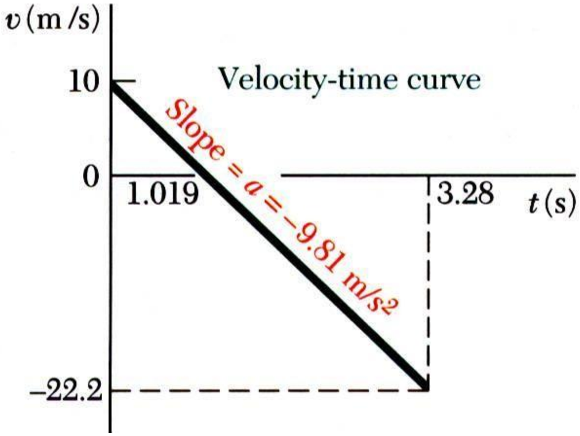
$$v(t) = 10\,\frac{\mathrm{m}}{\mathrm{s}} - \left(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)t$$

$$\frac{dy}{dt} = v = 10 - 9.81t$$

$$\int_{y_0}^{y(t)} dy = \int_0^t (10 - 9.81t)\,dt \qquad y(t) - y_0 = 10t - \tfrac{1}{2}9.81t^2$$

$$y(t) = 20\,\mathrm{m} + \left(10\,\frac{\mathrm{m}}{\mathrm{s}}\right)t - \left(4.905\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)t^2$$

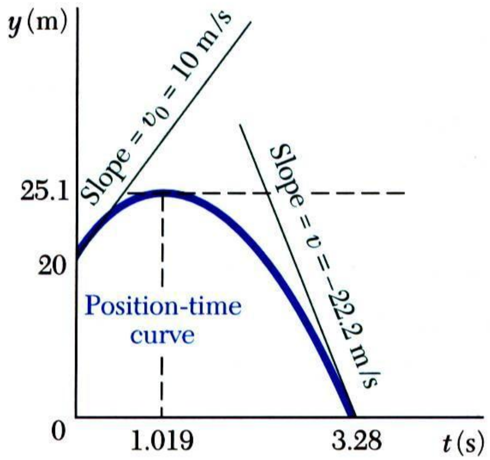
Sample Problem 1



- Solve for t at which velocity equals zero and evaluate corresponding altitude.

$$v(t) = 10\,\frac{\mathrm{m}}{\mathrm{s}} - \left(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)t = 0$$

$$\boxed{t = 1.019\mathrm{s}}$$

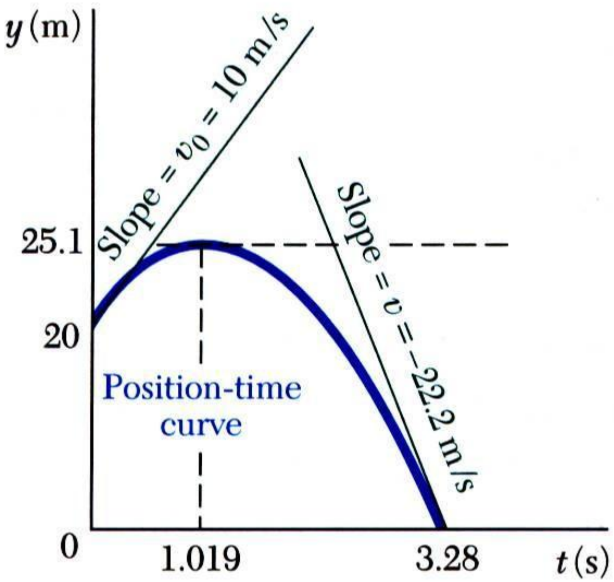


$$y(t) = 20\,\mathrm{m} + \left(10\,\frac{\mathrm{m}}{\mathrm{s}}\right)t - \left(4.905\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)t^2$$

$$y = 20\,\mathrm{m} + \left(10\,\frac{\mathrm{m}}{\mathrm{s}}\right)(1.019\,\mathrm{s}) - \left(4.905\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)(1.019\,\mathrm{s})^2$$

$$\boxed{y = 25.1\mathrm{m}}$$

Sample Problem 1



- Solve for t at which altitude equals zero and evaluate corresponding velocity.

$$y(t) = 20\,\mathrm{m} + \left(10\,\frac{\mathrm{m}}{\mathrm{s}}\right)t - \left(4.905\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)t^2 = 0$$

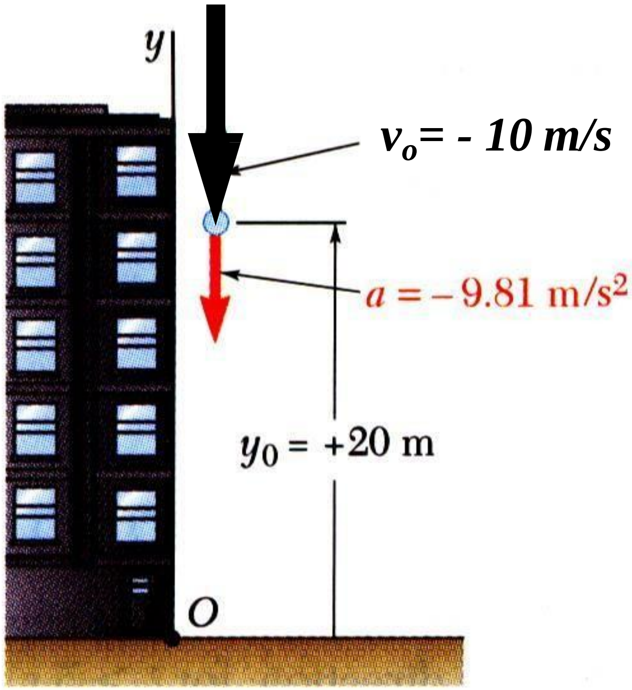
$$t = -1.243\mathrm{s}\ (\text{meaningless})$$
$$t = 3.28\mathrm{s}$$

$$v(t) = 10\,\frac{\mathrm{m}}{\mathrm{s}} - \left(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)t$$

$$v(3.28\mathrm{s}) = 10\,\frac{\mathrm{m}}{\mathrm{s}} - \left(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)(3.28\mathrm{s})$$

$$v = -22.2\,\frac{\mathrm{m}}{\mathrm{s}}$$

What if the ball is tossed downwards with the same speed? (The audience is thinking …)



Uniform Rectilinear Motion

Uniform rectilinear motion ➡ acceleration = 0 ➡ velocity = constant

$$\frac{dx}{dt} = v = \text{constant}$$

$$\int_{x_0}^{x} dx = v\int_{0}^{t} dt$$

$$x - x_0 = vt$$

$$x = x_0 + vt$$

Uniformly Accelerated Rectilinear Motion

Uniformly accelerated motion ⇒ acceleration = constant

$$\frac{dv}{dt} = a = \text{constant} \;\Rightarrow\; \int_{v_0}^{v} dv = a\int_{0}^{t} dt \;\Rightarrow\; v - v_0 = at$$

$$\Rightarrow\; v = v_0 + at$$

$$\frac{dx}{dt} = v_0 + at \;\Rightarrow\; \int_{x_0}^{x} dx = \int_{0}^{t} (v_0 + at)\,dt \;\Rightarrow\; x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

$$\Rightarrow\; x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

Also:

$$v\frac{dv}{dx} = a = \text{constant} \;\Rightarrow\; \int_{v_0}^{v} v\,dv = a\int_{x_0}^{x} dx \;\Rightarrow\; \tfrac{1}{2}\left(v^2 - v_0^2\right) = a(x - x_0)$$

$$\Rightarrow\; v^2 = v_0^2 + 2a(x - x_0)$$

Application: free fall

MOTION OF SEVERAL PARTICLES

When independent particles move along the same line, independent equations exist for each. Then one should use the same origin and time.

Relative motion of two particles.

The relative position of B with respect to A

$$x_{B/A} = x_B - x_A$$

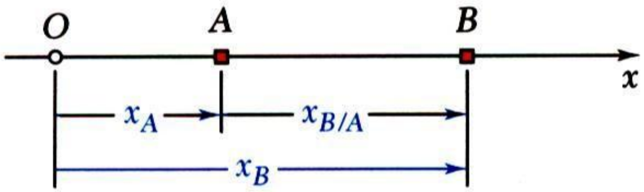
The relative velocity of B with respect to A

$$v_{B/A} = v_B - v_A$$

The relative acceleration of B with respect to A

$$a_{B/A} = a_B - a_A$$

Motion of Several Particles: Relative Motion



- For particles moving along the same line, displacements should be measured from the same origin in the same direction.

$$x_{B/A} = x_B - x_A = \text{relative position of } B \text{ with respect to } A$$

$$x_B = x_A + x_{B/A}$$

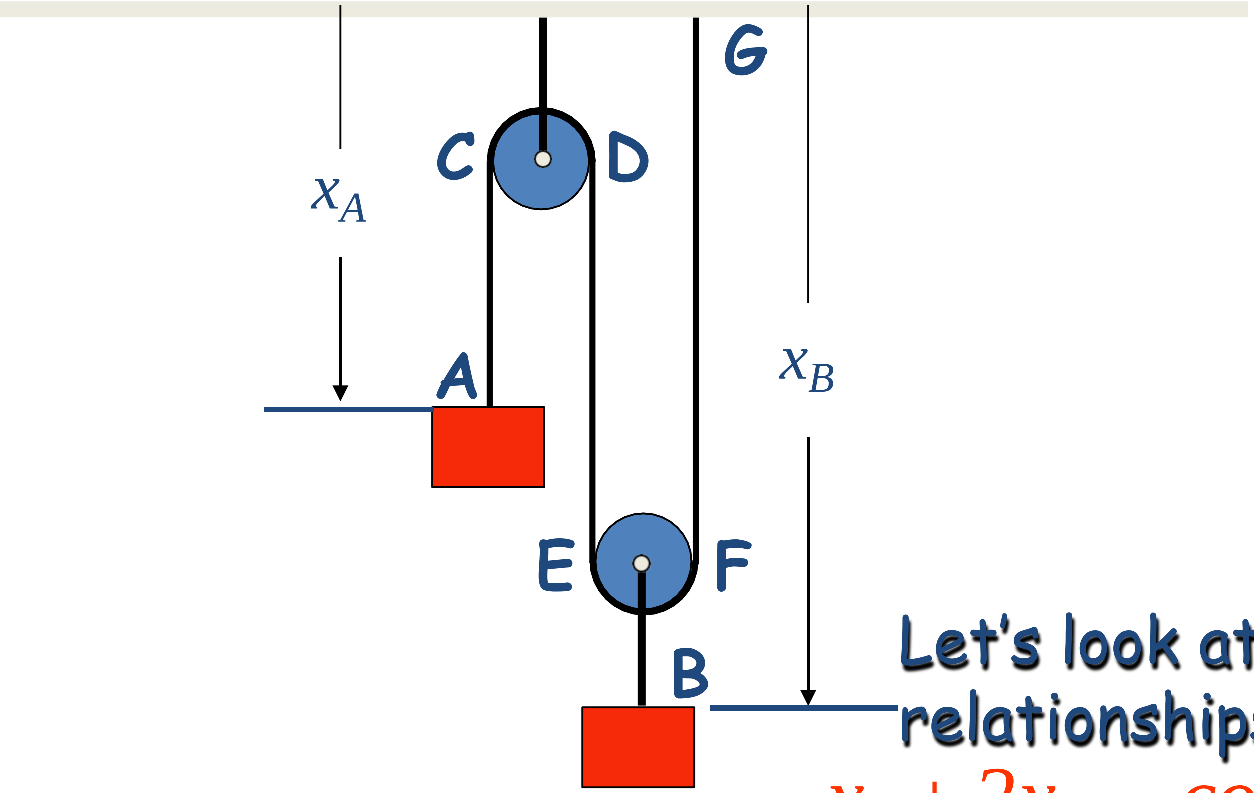
$$v_{B/A} = v_B - v_A = \text{relative velocity of } B \text{ with respect to } A$$

$$v_B = v_A + v_{B/A}$$

$$a_{B/A} = a_B - a_A = \text{relative acceleration of } B \text{ with respect to } A$$

$$a_B = a_A + a_{B/A}$$

Let’s look at some dependent motions.



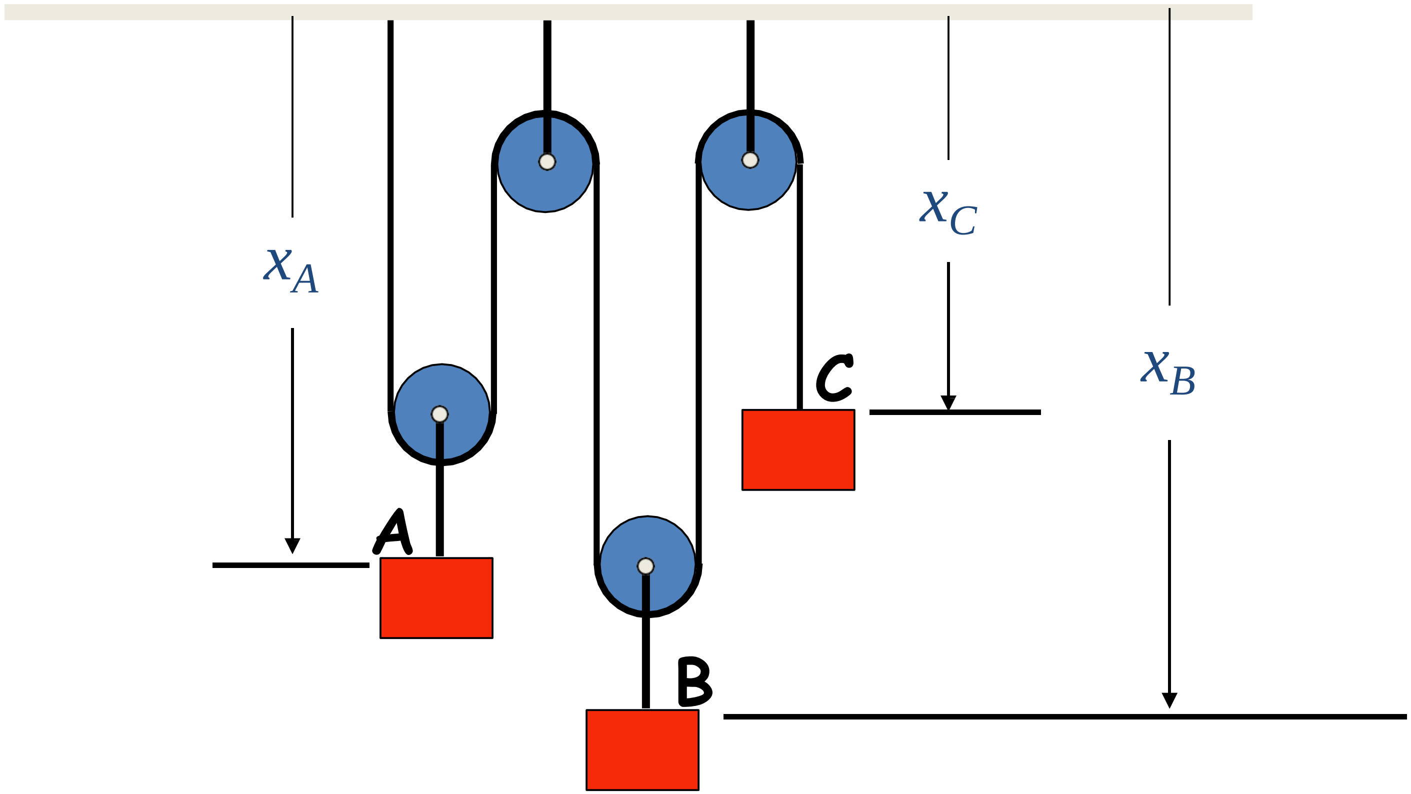
Let's look at the relationships.

$$x_A + 2x_B = constant$$

$$v_A + 2v_B = 0$$

$$a_A + 2a_B = 0$$

System has one degree of freedom since only one coordinate can be chosen independently.



System has 2 degrees of freedom.

Let's look at the relationships.

$$2x_A + 2x_B + x_C = constant$$

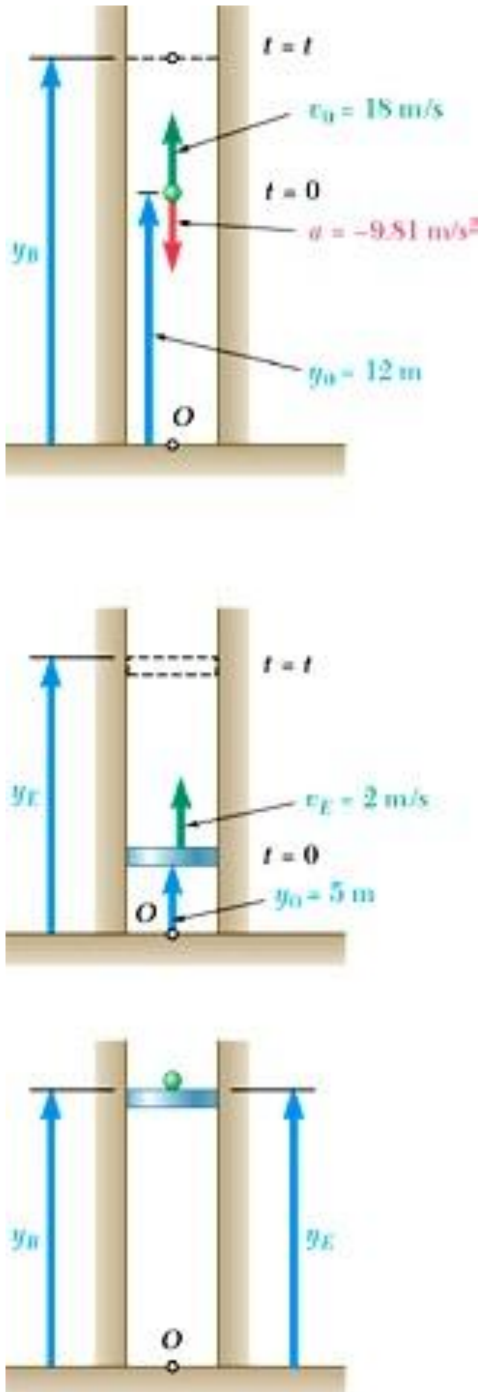
$$2v_A + 2v_B + v_C = 0$$

$$2a_A + 2a_B + a_C = 0$$

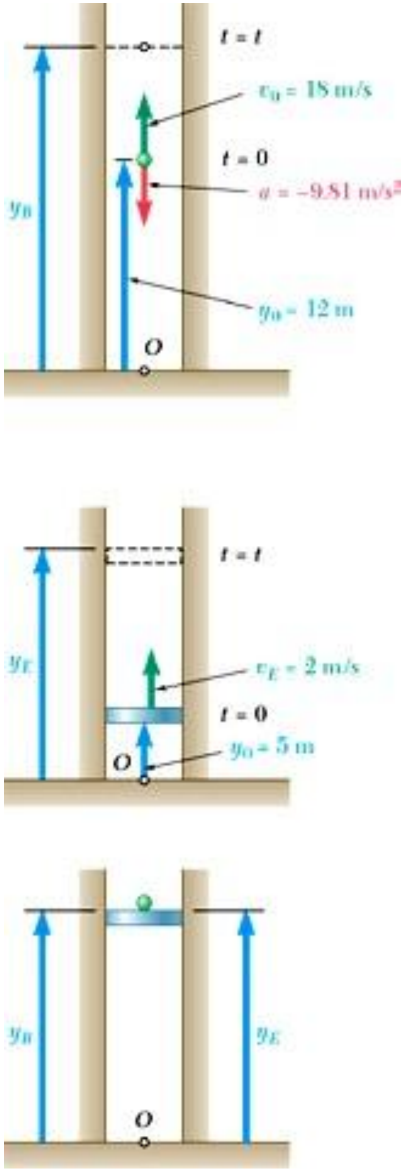
Sample Problem 2

Ball thrown vertically from 12 m level in elevator shaft with initial velocity of 18 m/s. At same instant, open-platform elevator passes 5 m level moving upward at 2 m/s.

Determine *(a)* when and where ball hits elevator and *(b)* relative velocity of ball and elevator at contact.

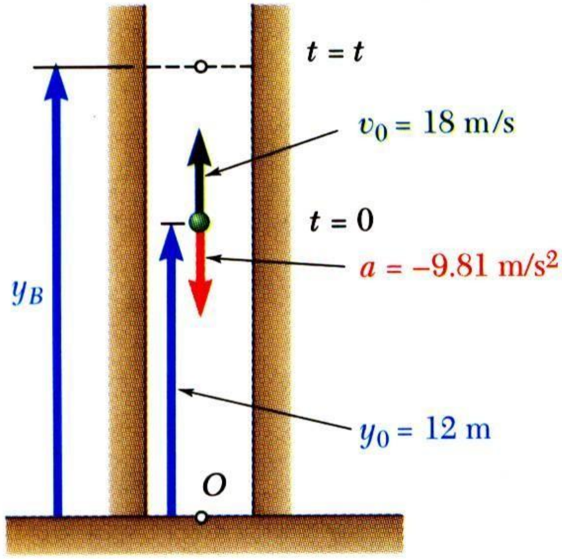


SOLUTION: Sample Problem 2



- Ball: uniformly accelerated motion (given initial position and velocity).
- Elevator: constant velocity (given initial position and velocity)
- Write equation for relative position of ball with respect to elevator and solve for zero relative position, i.e., impact.
- Substitute impact time into equation for position of elevator and relative velocity of ball with respect to elevator.

Sample Problem 3

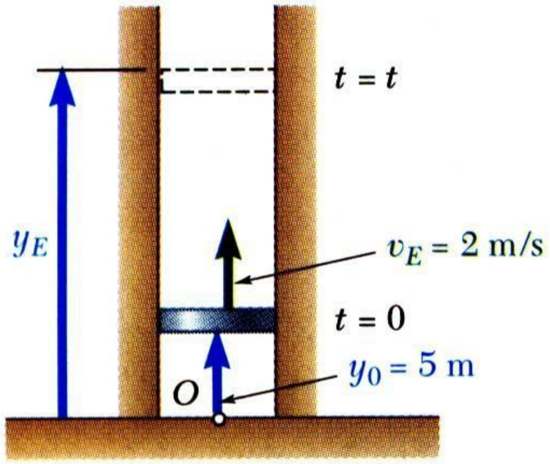


SOLUTION:

- Ball: uniformly accelerated rectilinear motion.

$$v_B = v_0 + at = 18\,\frac{\text{m}}{\text{s}} - \left(9.81\,\frac{\text{m}}{\text{s}^2}\right)t$$

$$y_B = y_0 + v_0 t + \tfrac{1}{2}at^2 = 12\,\text{m} + \left(18\,\frac{\text{m}}{\text{s}}\right)t - \left(4.905\,\frac{\text{m}}{\text{s}^2}\right)t^2$$

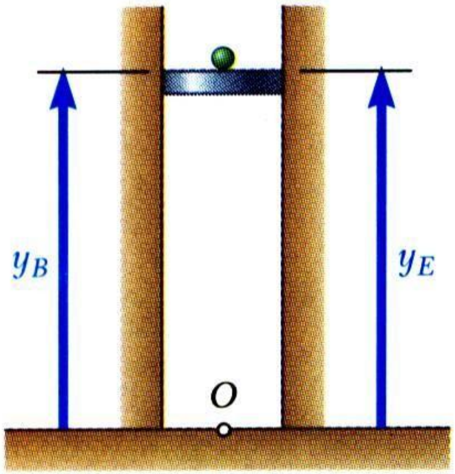


- Elevator: uniform rectilinear motion.

$$v_E = 2\,\frac{\text{m}}{\text{s}}$$

$$y_E = y_0 + v_E t = 5\,\text{m} + \left(2\,\frac{\text{m}}{\text{s}}\right)t$$

Sample Problem 3



- Relative position of ball with respect to elevator:

$$y_{B/E} = \left(12 + 18t - 4.905t^2\right) - (5 + 2t) = 0$$

$$t = -0.39\text{s} \ (\text{meaningless})$$
$$t = 3.65\text{s}$$

- Substitute impact time into equations for position of elevator and relative velocity of ball with respect to elevator.

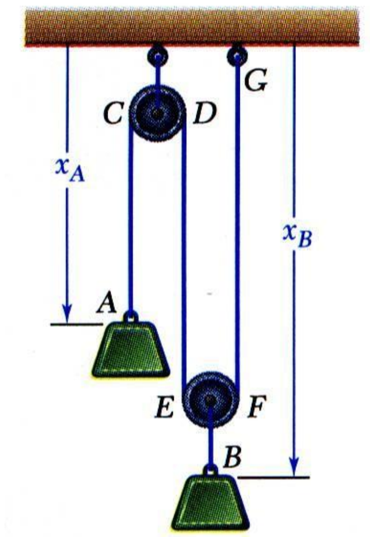
$$y_E = 5 + 2(3.65)$$

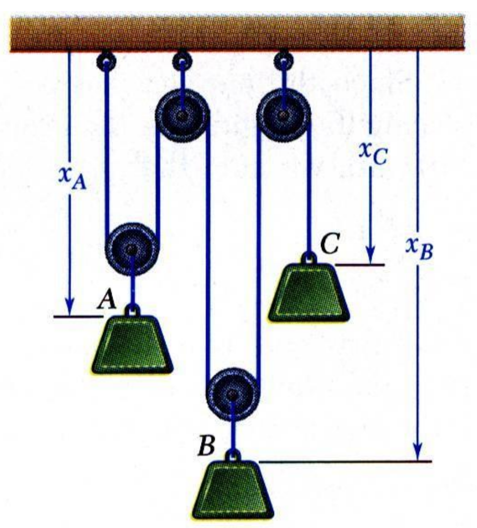
$$y_E = 12.3\text{m}$$

$$v_{B/E} = (18 - 9.81t) - 2$$
$$= 16 - 9.81(3.65)$$

$$v_{B/E} = -19.81 \frac{\text{m}}{\text{s}}$$

Motion of Several Particles: Dependent Motion





- Position of a particle may *depend* on position of one or more other particles.
- Position of block *B* depends on position of block *A*. Since rope is of constant length, it follows that sum of lengths of segments must be constant.

$$x_A + 2x_B = \text{constant (one degree of freedom)}$$

- Positions of three blocks are dependent.

$$2x_A + 2x_B + x_C = \text{constant (two degrees of freedom)}$$

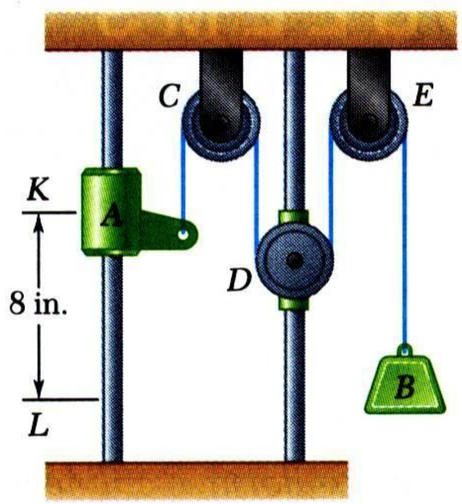
- For linearly related positions, similar relations hold between velocities and accelerations.

$$2\frac{dx_A}{dt} + 2\frac{dx_B}{dt} + \frac{dx_C}{dt} = 0 \quad \text{or} \quad 2v_A + 2v_B + v_C = 0$$

$$2\frac{dv_A}{dt} + 2\frac{dv_B}{dt} + \frac{dv_C}{dt} = 0 \quad \text{or} \quad 2a_A + 2a_B + a_C = 0$$

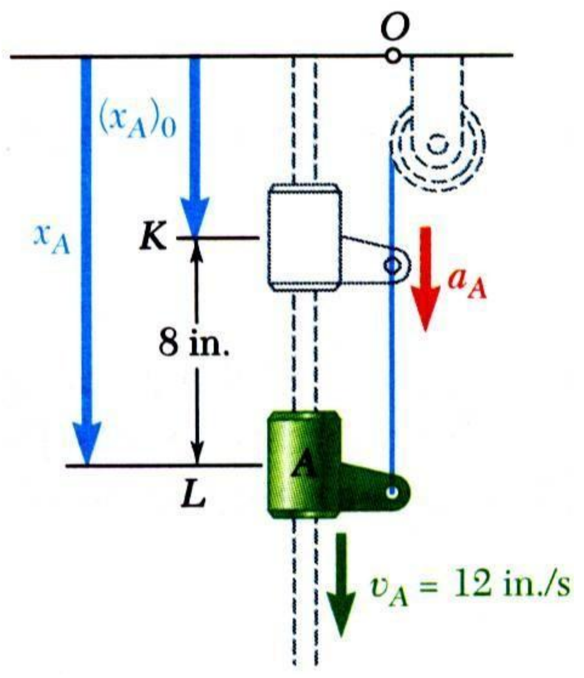
Applications

Sample Problem 4



Pulley *D* is attached to a collar which is pulled down at 3 in./s. At $t = 0$, collar *A* starts moving down from *K* with constant acceleration and zero initial velocity. Knowing that velocity of collar *A* is 12 in./s as it passes *L*, determine the change in elevation, velocity, and acceleration of block *B* when block *A* is at *L*.

Sample Problem 4



SOLUTION:

- Define origin at upper horizontal surface with positive displacement downward.

- Collar *A* has uniformly accelerated rectilinear motion. Solve for acceleration and time *t* to reach *L*.

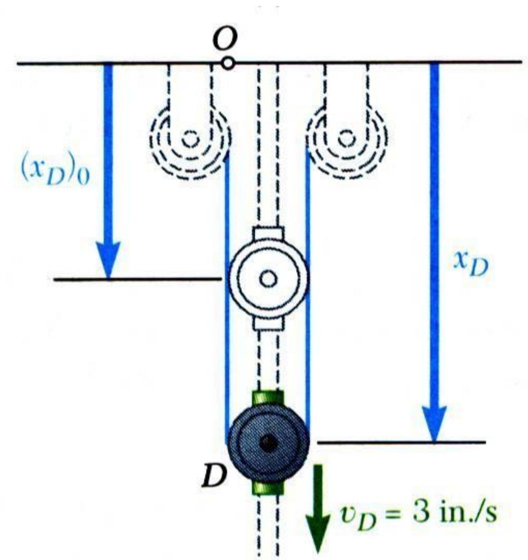
$$v_A^2 = (v_A)_0^2 + 2a_A\left[x_A - (x_A)_0\right]$$

$$\left(12\,\frac{\text{in.}}{\text{s}}\right)^2 = 2a_A(8\,\text{in.}) \qquad a_A = 9\,\frac{\text{in.}}{\text{s}^2}$$

$$v_A = (v_A)_0 + a_A t$$

$$12\,\frac{\text{in.}}{\text{s}} = 9\,\frac{\text{in.}}{\text{s}^2}\,t \qquad t = 1.333\,\text{s}$$

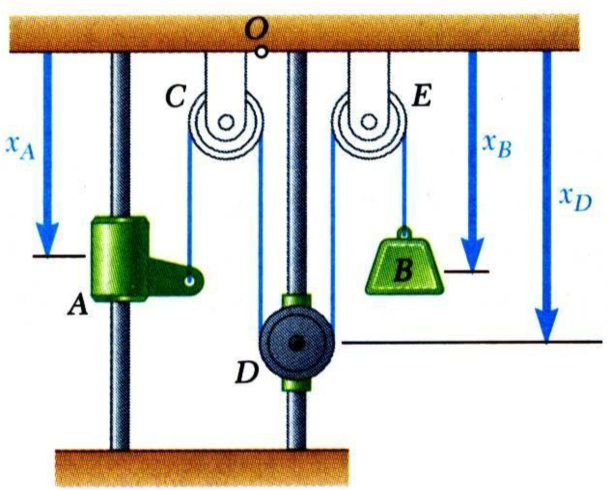
Sample Problem 4



- Pulley *D* has uniform rectilinear motion. Calculate change of position at time *t*.

$$x_D = (x_D)_0 + v_D t$$

$$x_D - (x_D)_0 = \left(3\,\frac{\text{in.}}{\text{s}}\right)(1.333\,\text{s}) = 4\,\text{in.}$$



- Block *B* motion is dependent on motions of collar *A* and pulley *D*. Write motion relationship and solve for change of block *B* position at time *t*.

Total length of cable remains constant,

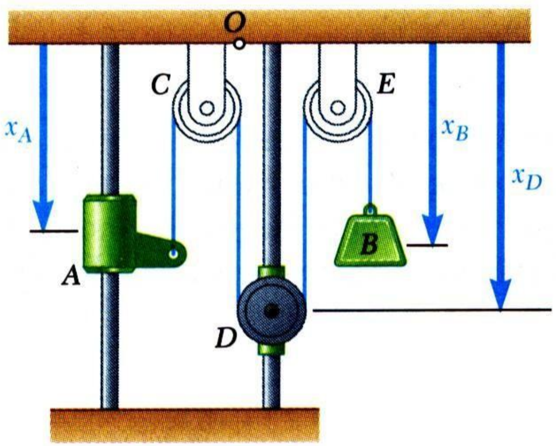
$$x_A + 2x_D + x_B = (x_A)_0 + 2(x_D)_0 + (x_B)_0$$

$$\left[x_A - (x_A)_0\right] + 2\left[x_D - (x_D)_0\right] + \left[x_B - (x_B)_0\right] = 0$$

$$(8\,\text{in.}) + 2(4\,\text{in.}) + \left[x_B - (x_B)_0\right] = 0$$

$$\boxed{x_B - (x_B)_0 = -16\,\text{in.}}$$

Sample Problem 4



- Differentiate motion relation twice to develop equations for velocity and acceleration of block *B*.

$$x_A + 2x_D + x_B = \text{constant}$$

$$v_A + 2v_D + v_B = 0$$

$$\left(12\,\frac{\text{in.}}{\text{s}}\right) + 2\left(3\,\frac{\text{in.}}{\text{s}}\right) + v_B = 0$$

$$v_B = -18\,\frac{\text{in.}}{\text{s}}$$

$$a_A + 2a_D + a_B = 0$$

$$\left(9\,\frac{\text{in.}}{\text{s}^2}\right) + a_B = 0$$

$$a_B = -9\,\frac{\text{in.}}{\text{s}^2}$$

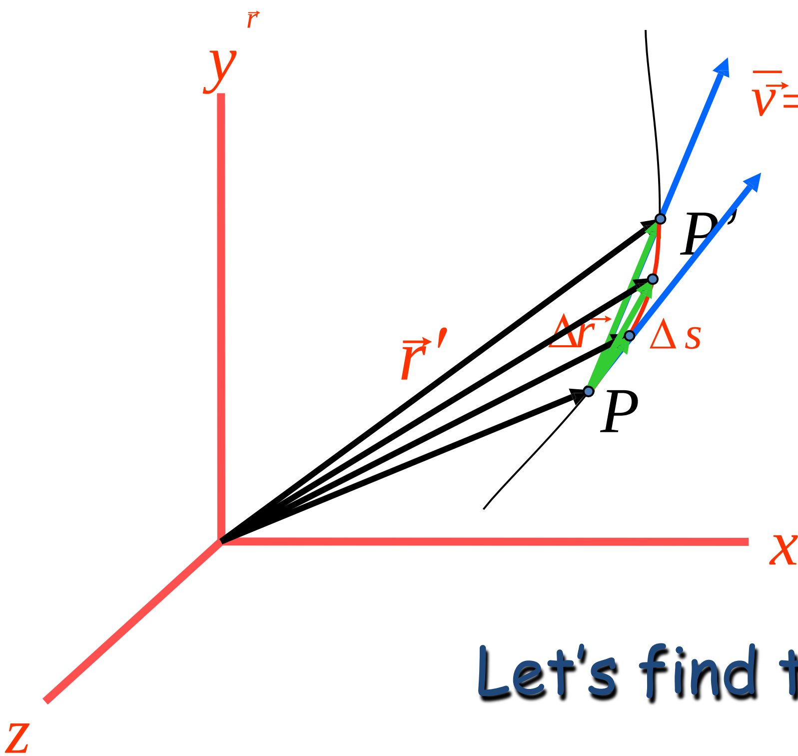
Curvilinear Motion

A particle moving along a curve other than a straight line is said to be in *curvilinear motion*.

http://news.yahoo.com/photos/ss/441/im:/070123/ids_photos_wl/r2207709100.jpg

CURVILINEAR MOTION OF PARTICLES

POSITION VECTOR, VELOCITY, AND ACCELERATION



$$\vec{v} = \frac{\Delta \vec{r}}{\Delta t}$$

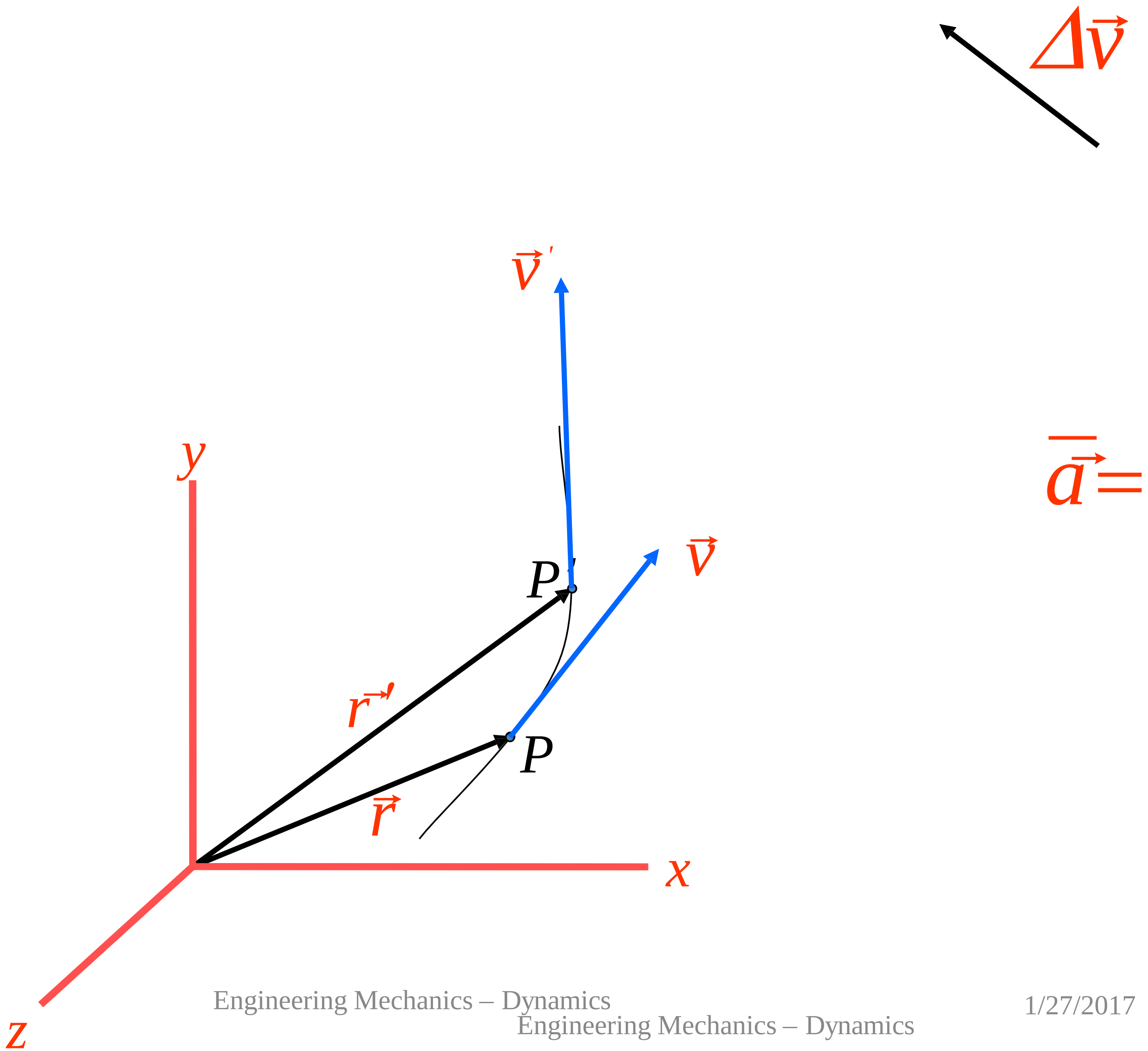
$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\dot{s} = \frac{\Delta s}{\Delta t}$$

$$v = \frac{ds}{dt}$$

Let's find the instantaneous velocity.

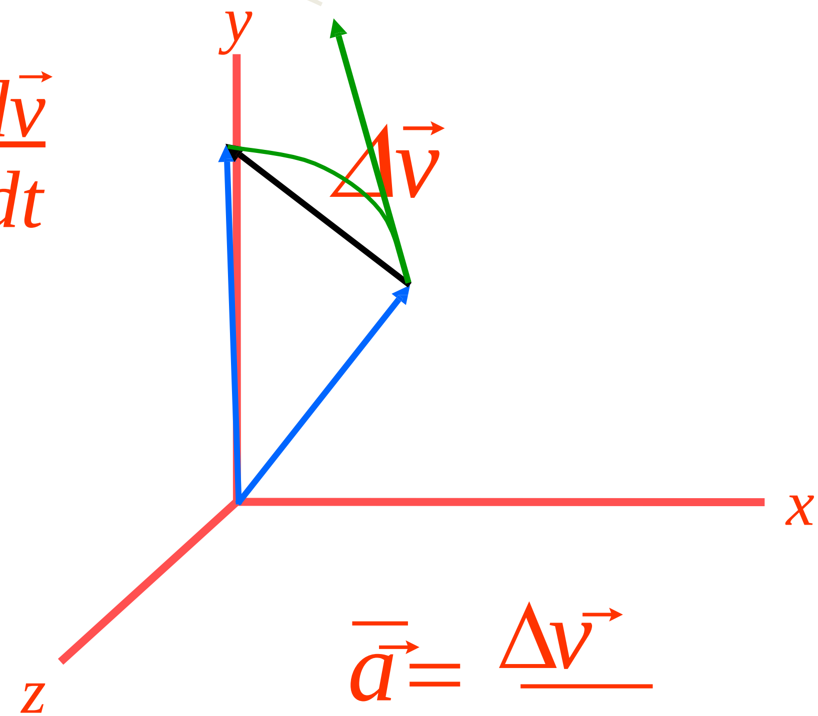
Engineering Mechanics – Dynamics



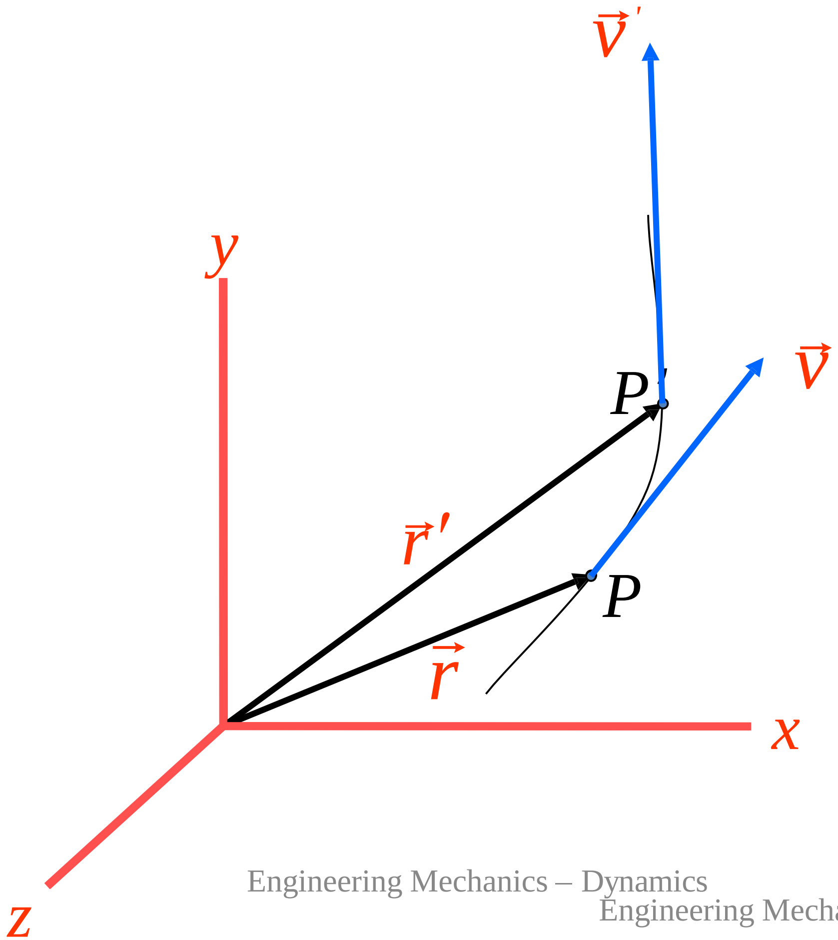
$$\bar{\vec{a}} = \frac{\Delta\vec{v}}{\Delta t}$$

Engineering Mechanics – Dynamics 74

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$



$$\overline{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t}$$



Note that the acceleration is not necessarily along the direction of the velocity.

Engineering Mechanics – Dynamics

DERIVATIVES OF VECTOR FUNCTIONS

$$\frac{d\vec{P}}{du} = \lim_{\Delta u \to 0} \frac{\Delta \vec{P}}{\Delta u} = \lim_{\Delta u \to 0} \left[\frac{\vec{P}(u + \Delta u) - \vec{P}(u)}{\Delta u} \right]$$

$$\frac{d(\vec{P} + \vec{Q})}{du} = \frac{d\vec{P}}{du} + \frac{d\vec{Q}}{du} + f\frac{d\vec{P}}{du}$$

$$\frac{d(f\vec{P})}{du} = \frac{df}{du}\vec{P}$$

Engineering Mechanics – Dynamics

$$\frac{d(\vec{P}\cdot\vec{Q})}{du} = \frac{d\vec{P}}{du}\cdot\vec{Q} + \vec{P}\cdot\frac{d\vec{Q}}{du}$$

$$\frac{d(\vec{P}\times\vec{Q})}{du} = \frac{d\vec{P}}{du}\times\vec{Q} + \vec{P}\times\frac{d\vec{Q}}{du}$$

$$\frac{d\vec{P}}{du} = \frac{dP_x}{du}\hat{i} + \frac{dP_y}{du}\hat{j} + \frac{dP_z}{du}\hat{k}$$

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Rate of Change of a Vector

$$\dot{\vec{P}} = \dot{P}_x \hat{i} + \dot{P}_y \hat{j} + \dot{P}_z \hat{k}$$

The rate of change of a vector is the same with respect to a fixed frame and with respect to a frame in translation.

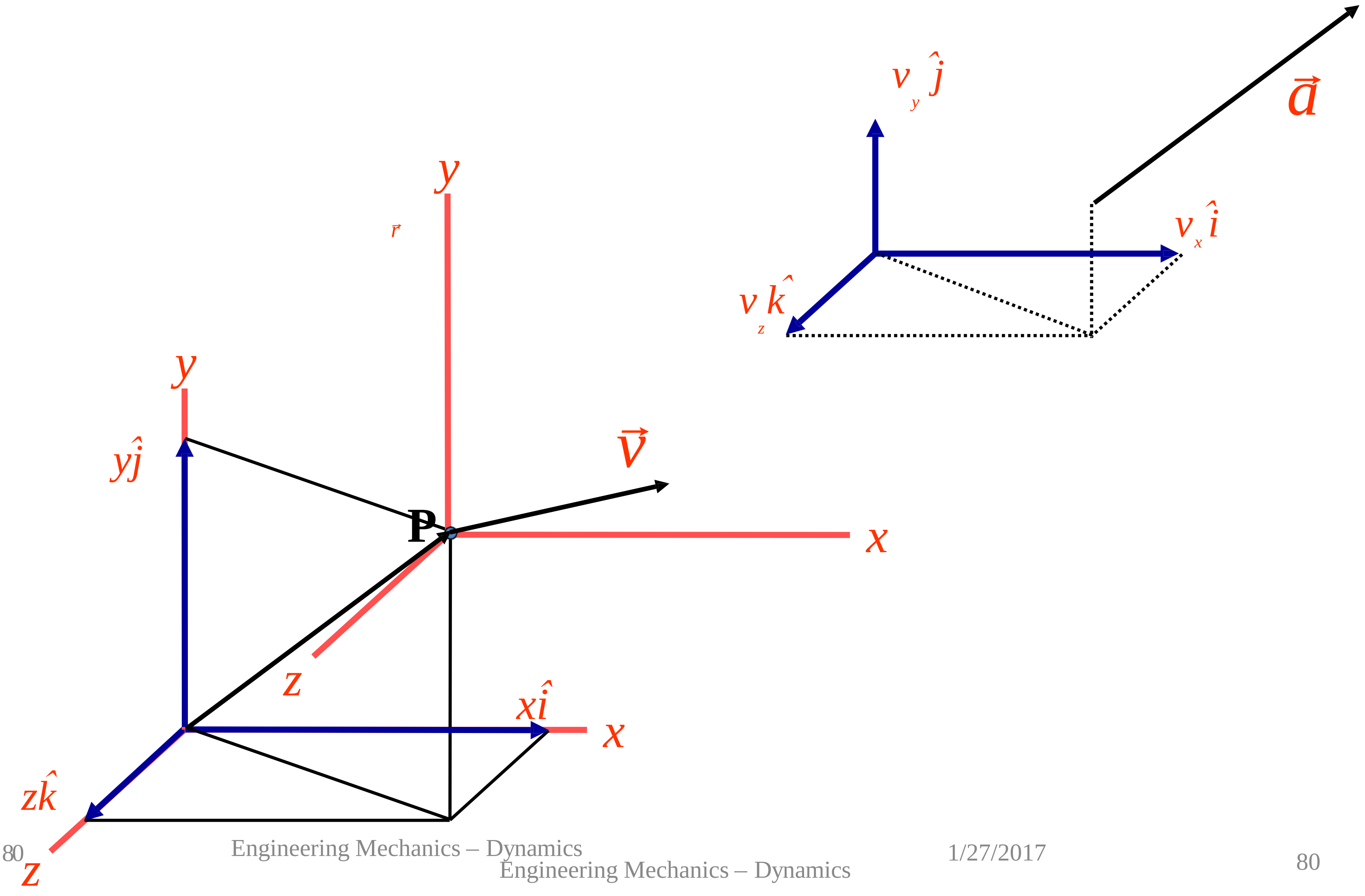
Engineering Mechanics – Dynamics 78

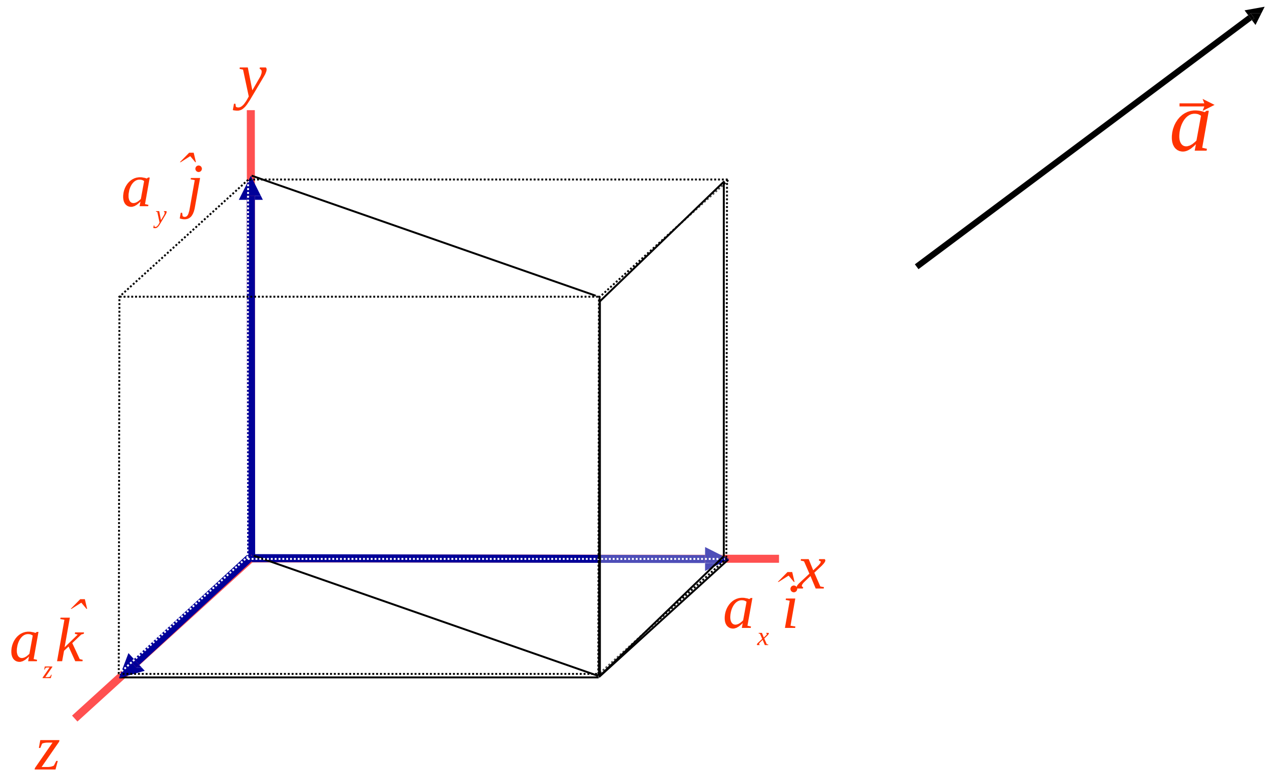
RECTANGULAR COMPONENTS OF VELOCITY AND ACCELERATION

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

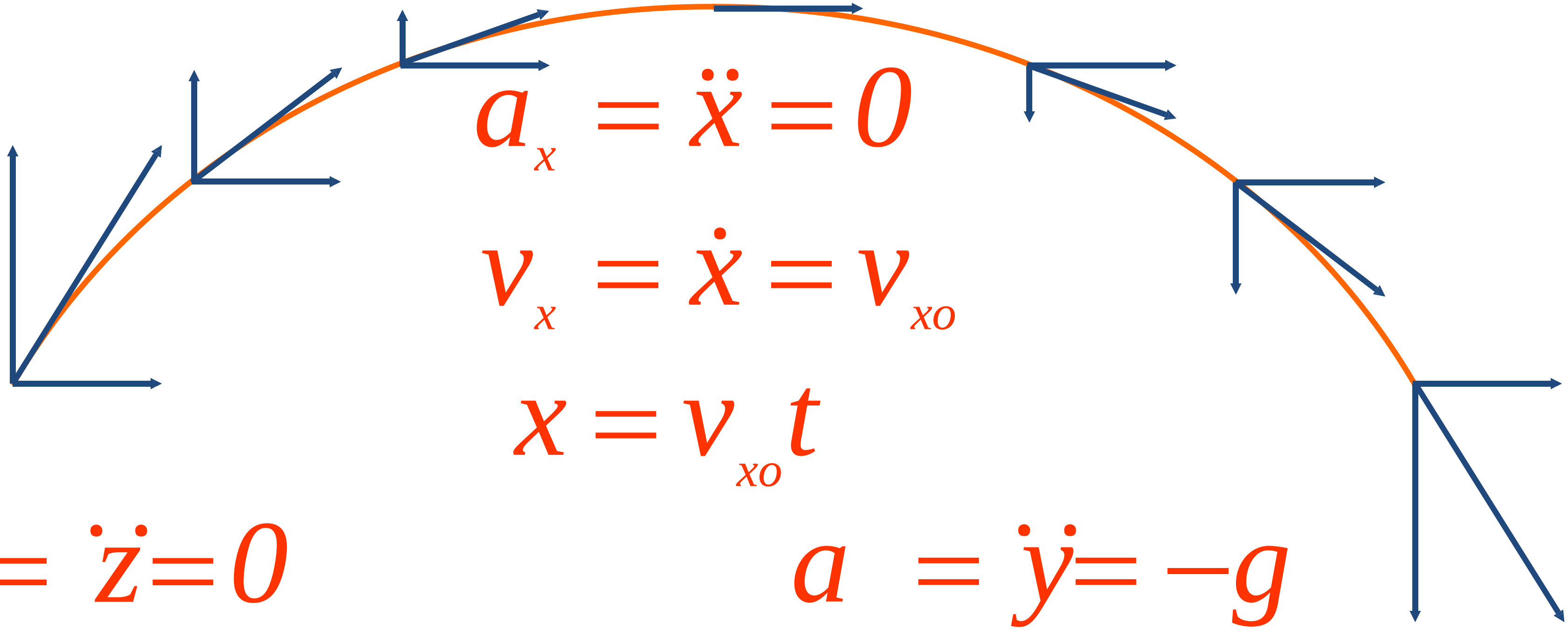
$$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$$


$v_y\hat{j}$
$\vec{a}$
y
$\vec{r}$
$v_x\hat{i}$
$v_z\hat{k}$
y
$\vec{v}$
$y\hat{j}$
P
x
z
$x\hat{i}$
x
$z\hat{k}$
z



Engineering Mechanics – Dynamics
81

Velocity Components in Projectile Motion



$$a_x = \ddot{x} = 0$$

$$v_x = \dot{x} = v_{xo}$$

$$x = v_{xo} t$$

$$a_z = \ddot{z} = 0$$

$$a_y = \ddot{y} = -g$$

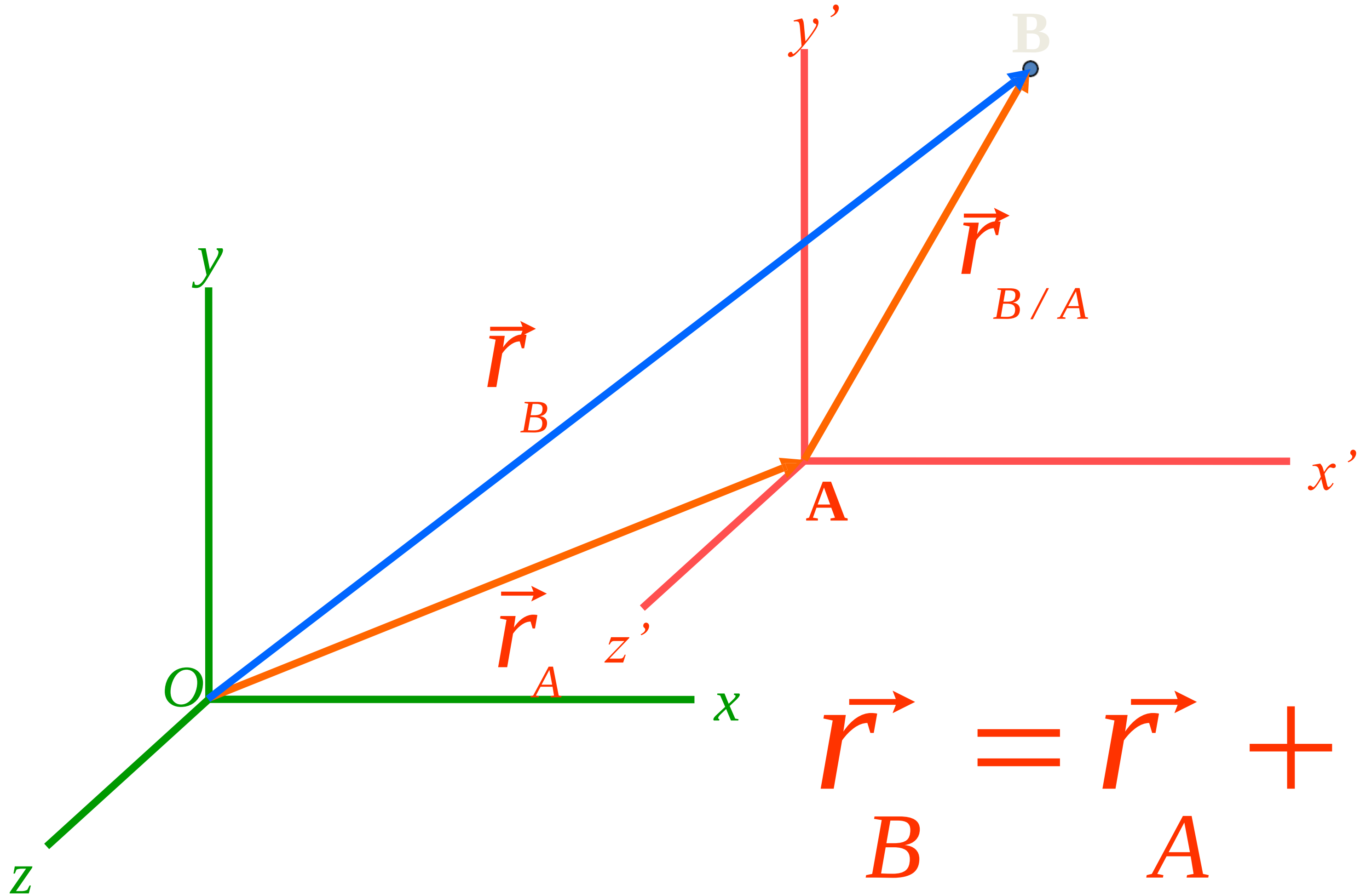
$$v_z = \dot{z} = v_{zo} = 0$$

$$v_y = \dot{y} = v_{yo} - gt$$

$$z = 0$$

$$y = v_{yo} t - \tfrac{1}{2} g t^2$$

MOTION RELATIVE TO A FRAME IN TRANSLATION



$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

$$\dot{\vec{r}}_B = \dot{\vec{r}}_A + \dot{\vec{r}}_{B/A}$$

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\dot{\vec{v}}_B = \dot{\vec{v}}_A + \dot{\vec{v}}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\ddot{\vec{r}}_B = \ddot{\vec{r}}_A + \ddot{\vec{r}}_{B/A}$$

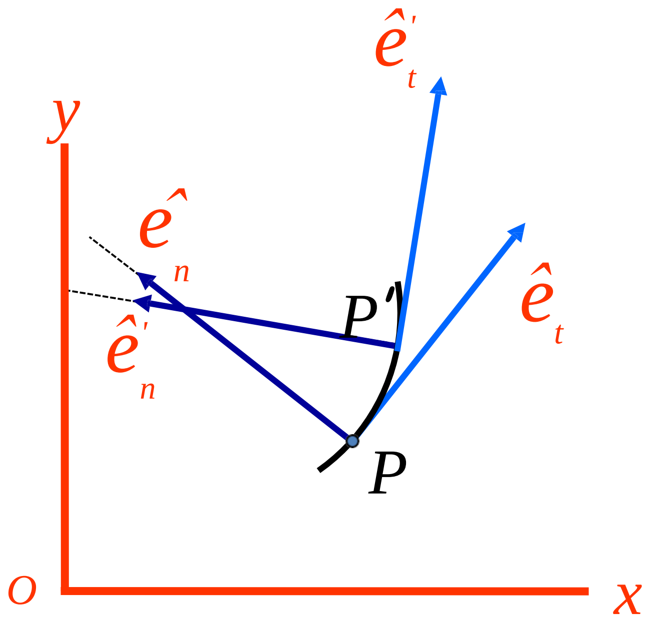
TANGENTIAL AND NORMAL COMPONENTS

Velocity is tangent to the path of a particle.

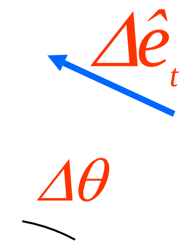
Acceleration is not necessarily in the same direction.

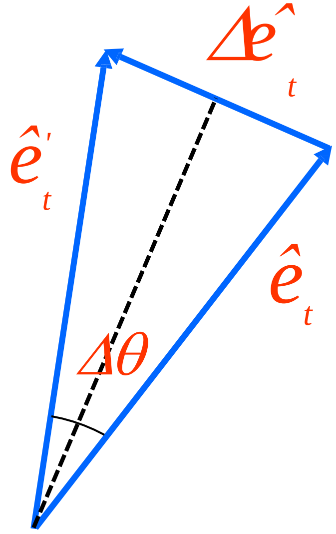
It is often convenient to express the acceleration in terms of components tangent and normal to the path of the particle.

Plane Motion of a Particle



$$\vec{v} = v\hat{e}_t$$





$$\lim_{\Delta\theta\to 0}\frac{\Delta\hat{e}_t}{\Delta\theta}=\hat{e}_n\lim_{\Delta\theta\to 0}\frac{\left|\Delta\hat{e}_t\right|}{\Delta\theta}=\hat{e}_n\lim_{\Delta\theta\to 0}\left[\frac{2\sin(\Delta\theta/2)}{\Delta\theta}\right]$$

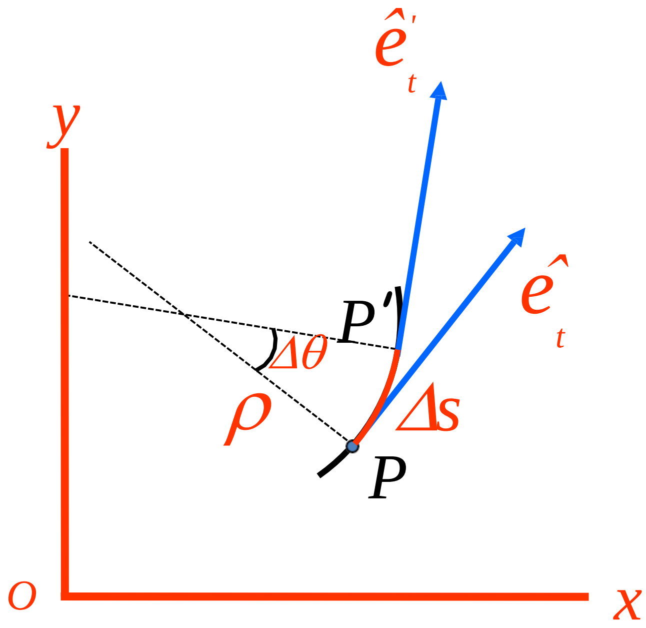
$$=\hat{e}_n\lim_{\Delta\theta\to 0}\left[\frac{\sin(\Delta\theta/2)}{\Delta\theta/2}\right]=\hat{e}_n$$

$$\hat{e}_n=\frac{d\hat{e}_t}{d\theta}$$

$$\hat{e}_n = \frac{d\hat{e}_t}{d\theta}$$

$$\vec{v} = v\hat{e}_t$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv}{dt}\hat{e}_t + v\frac{d\hat{e}_t}{dt}$$



$$\vec{a} = \frac{dv}{dt}\hat{e}_t + v\frac{d\hat{e}_t}{dt}$$

$$\Delta s = \rho\Delta\theta$$

$$\rho = \lim_{\Delta\theta\to 0}\frac{\Delta s}{\Delta\theta} = \frac{ds}{d\theta}$$

$$\frac{d\hat{e}_t}{dt} = \frac{d\hat{e}_t}{d\theta}\frac{d\theta}{ds}\frac{ds}{dt} = \frac{d\hat{e}_t}{d\theta}\frac{v}{\rho} = \frac{v}{\rho}\hat{e}_n$$

$$\vec{a} = \frac{dv}{dt}e_t + \frac{v^2}{\rho}e_n$$

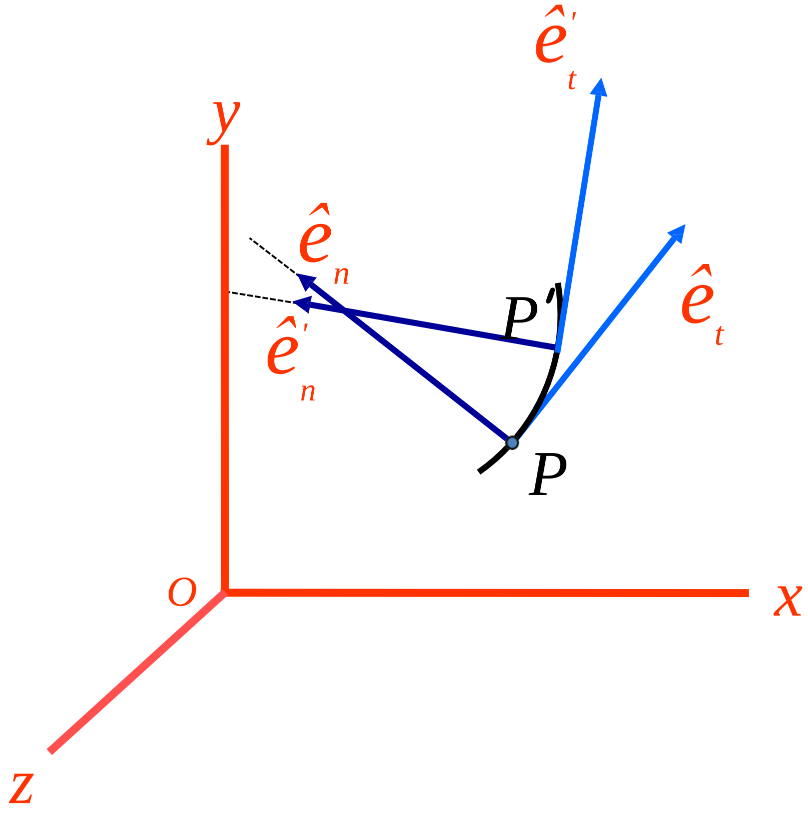
$$\vec{a} = \frac{dv}{dt}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

$$\vec{a} = a_t \hat{e}_t + a_n \hat{e}_n$$

$$a_t = \frac{dv}{dt} \qquad a_n = \frac{v^2}{\rho}$$

Discuss changing radius of curvature for highway cur

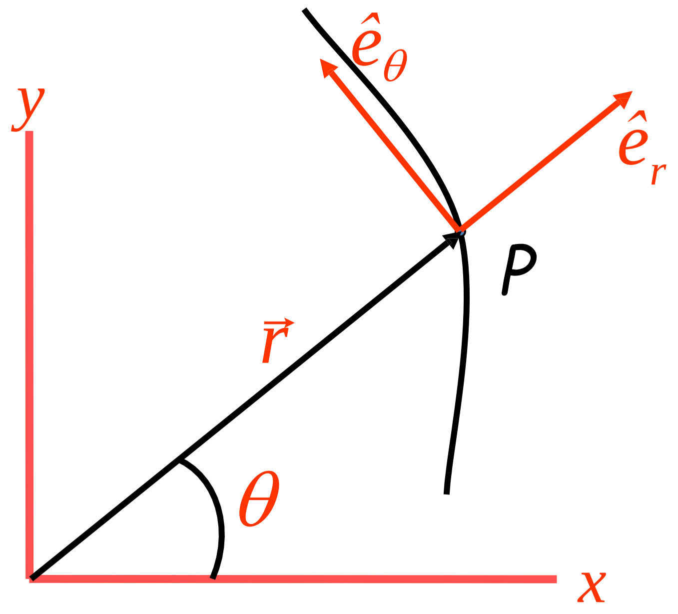
Motion of a Particle in Space

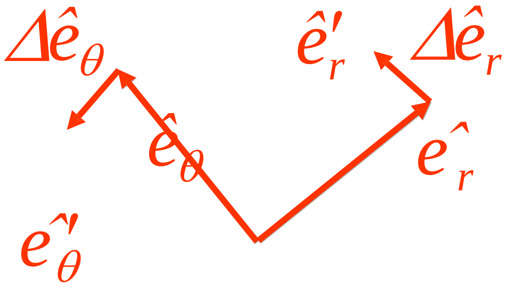


The equations are the same.

RADIAL AND TRANSVERSE COMPONENTS

Plane Motion





$$\frac{d\hat{e}_r}{d\theta} = \hat{e}_\theta \qquad \frac{d\hat{e}_\theta}{d\theta} = -\hat{e}_r$$

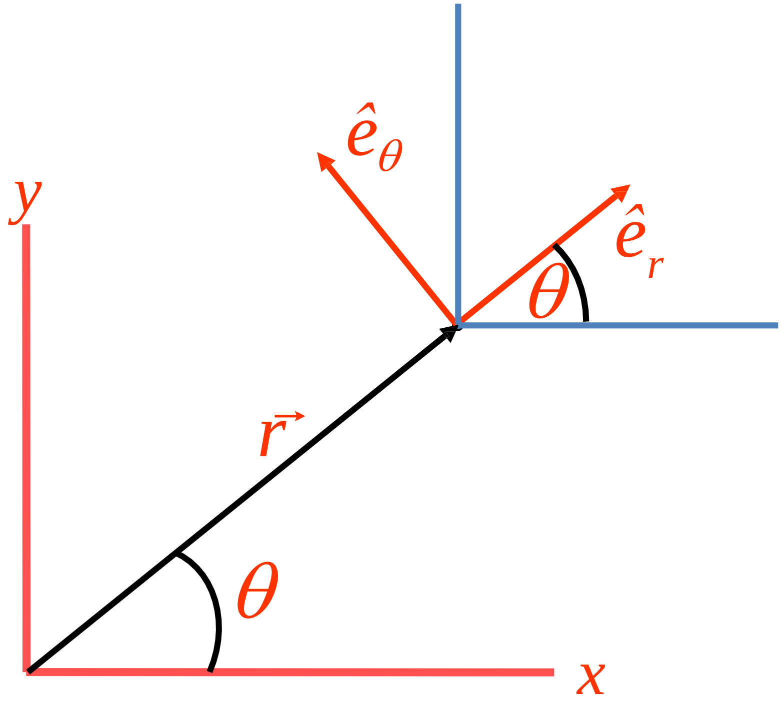
$$\frac{d\hat{e}_r}{dt} = \frac{d\hat{e}_r}{d\theta}\frac{d\theta}{dt} = \dot{\theta}\,\hat{e}_\theta$$

$$\frac{d\hat{e}_\theta}{dt} = \frac{d\hat{e}_\theta}{d\theta}\frac{d\theta}{dt} = -\dot{\theta}\,\hat{e}_r$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(r\hat{e}_r) = \dot{r}\hat{e}_r + r\dot{\hat{e}}_r$$

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta = v_r\hat{e}_r + v_\theta\hat{e}_\theta$$

$$v_r = \dot{r} \qquad\qquad v_\theta = r\dot{\theta}$$



$$\hat{e}_r = \hat{i}\cos\theta + \hat{j}\sin\theta$$

$$\frac{d\hat{e}_r}{d\theta} = -\hat{i}\sin\theta + \hat{j}\cos\theta = \hat{e}_\theta$$

$$\frac{d\hat{e}_\theta}{d\theta} = -\hat{i}\cos\theta - \hat{j}\sin\theta = -\hat{e}_r$$

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$

$$\vec{a} = \ddot{r}\hat{e}_r + \dot{r}\,\dot{\hat{e}}_r + \dot{r}\,\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\,\hat{e}_\theta + r\dot{\theta}\,\dot{\hat{e}}_\theta$$

$$\vec{a} = \ddot{r}\hat{e}_r + \dot{r}\,\dot{\theta}\,\hat{e}_\theta + \dot{r}\,\dot{\theta}\hat{e}_\theta + r\,\ddot{\theta}\hat{e}_\theta - r\,\dot{\theta}^2\,\hat{e}_r$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\,\dot{\theta})\hat{e}_\theta$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \qquad a_\theta = r\ddot{\theta} + 2\dot{r}\,\dot{\theta}$$

Note $\quad a_r \neq \dfrac{dv_r}{dt} \qquad a_\theta \neq \dfrac{dv_\theta}{dt}$

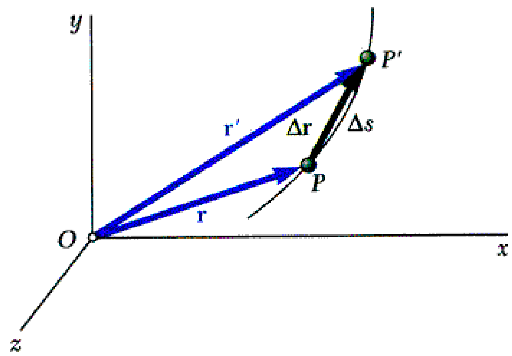
Extension to the Motion of a Particle in Space: Cylindrical Coordinates

$$\vec{r} = R\hat{e}_r + z\hat{k}$$

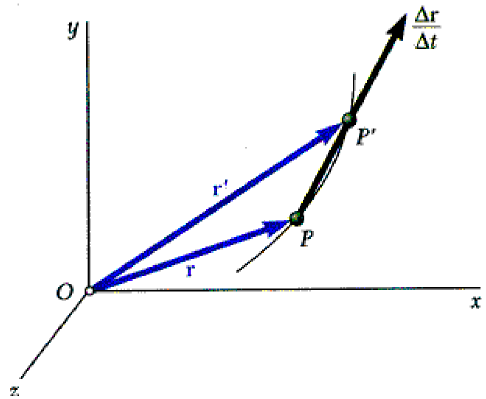
$$\vec{v} = \dot{R}\hat{e}_R + R\dot{\theta}\hat{e}_\theta + \dot{z}\hat{k}$$

$$\vec{a} = (\ddot{R} - R\dot{\theta}^2)\hat{e}_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\hat{e}_\theta + \ddot{z}\hat{k}$$

Curvilinear Motion: Position, Velocity & Acceleration



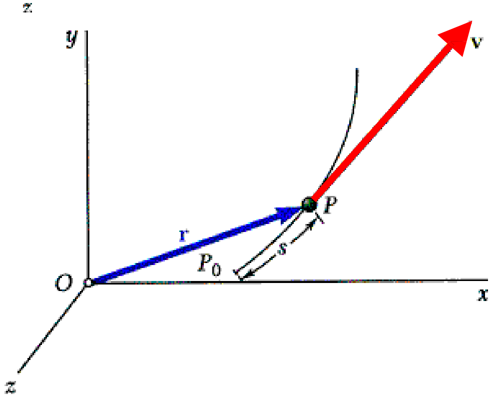
- *Position vector* of a particle at time *t* is defined by a vector between origin *O* of a fixed reference frame and the position occupied by particle.



- Consider particle which occupies position *P* defined by $\vec{r}$ at time *t* and *P'* defined by $\vec{r}\,'$ at $t + \Delta t$,

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$=$ instantaneous velocity (vector)

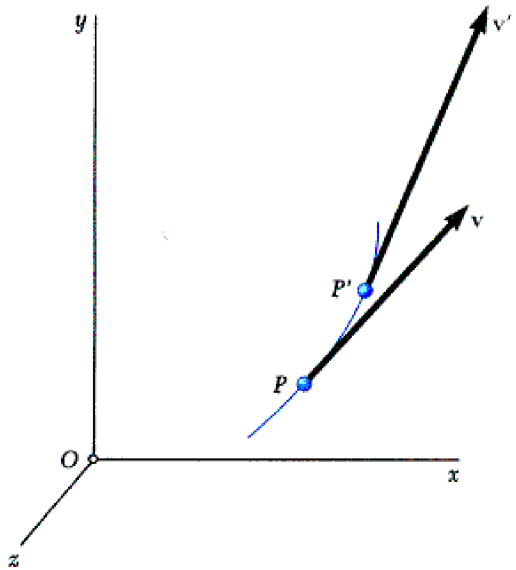


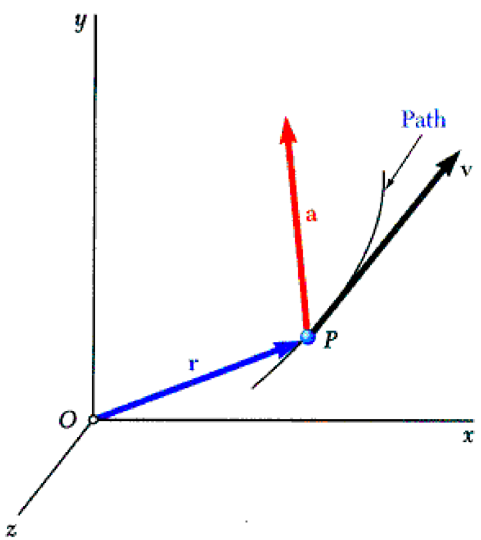
$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

$=$ instantaneous speed (scalar)

Velocity is <u>tangent</u> to path

Curvilinear Motion: Position, Velocity & Acceleration





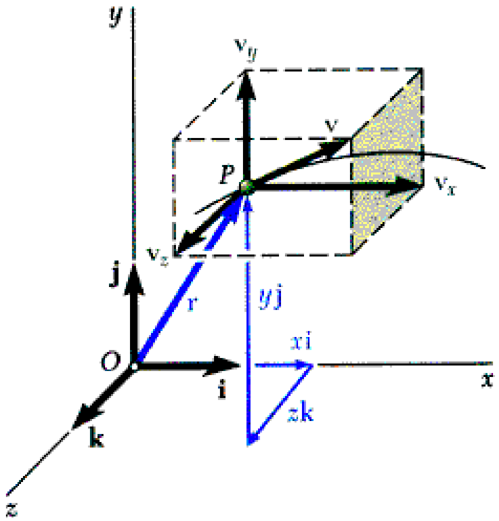
- Consider velocity $\vec{v}$ of particle at time t and velocity $\vec{v}'$ at $t + \Delta t$,

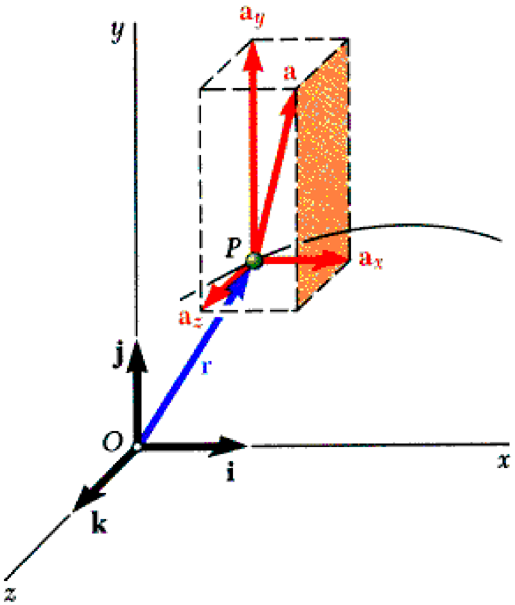
$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$= \text{instantaneous acceleration (vector)}$$

- In general, acceleration vector is not tangent to particle path and velocity vector.

Rectangular Components of Velocity & Acceleration





- Position vector of particle *P* given by its rectangular components:

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

- Velocity vector,

$$\vec{v} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}$$

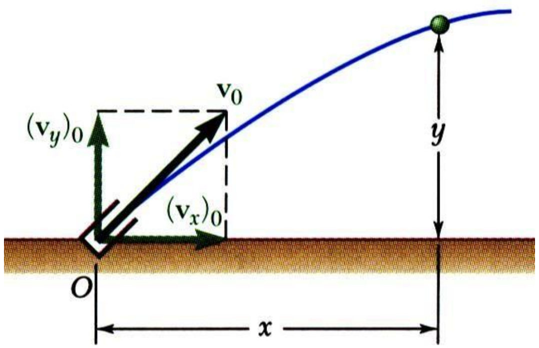
$$= v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$$

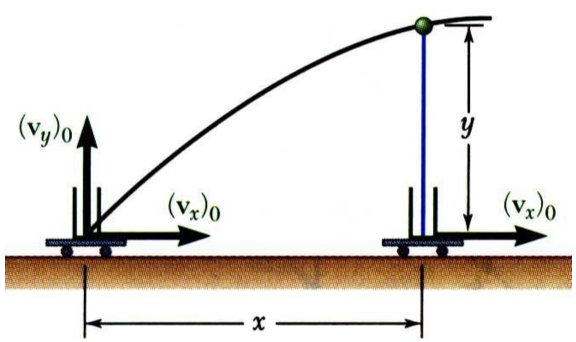
- Acceleration vector,

$$\vec{a} = \frac{d^2x}{dt^2}\vec{i} + \frac{d^2y}{dt^2}\vec{j} + \frac{d^2z}{dt^2}\vec{k} = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k}$$

$$= a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

Rectangular Components of Velocity & Acceleration





- Rectangular components are useful when acceleration components can be integrated independently, ex: motion of a **projectile**.

$$a_x = \ddot{x} = 0 \qquad a_y = \ddot{y} = -g \qquad a_z = \ddot{z} = 0$$

with initial conditions,

$$x_0 = y_0 = z_0 = 0 \qquad (v_x)_0 = (v_y)_0 = given$$

Therefore:

$$v_x = (v_x)_0 \qquad v_y = (v_y)_0 - gt$$

$$x = (v_x)_0 t \qquad y = (v_y)_0 t - \frac{1}{2} g t^2$$

- Motion in horizontal direction is uniform.
- Motion in vertical direction is uniformly accelerated.
- Motion of projectile could be replaced by two independent rectilinear motions.

Example

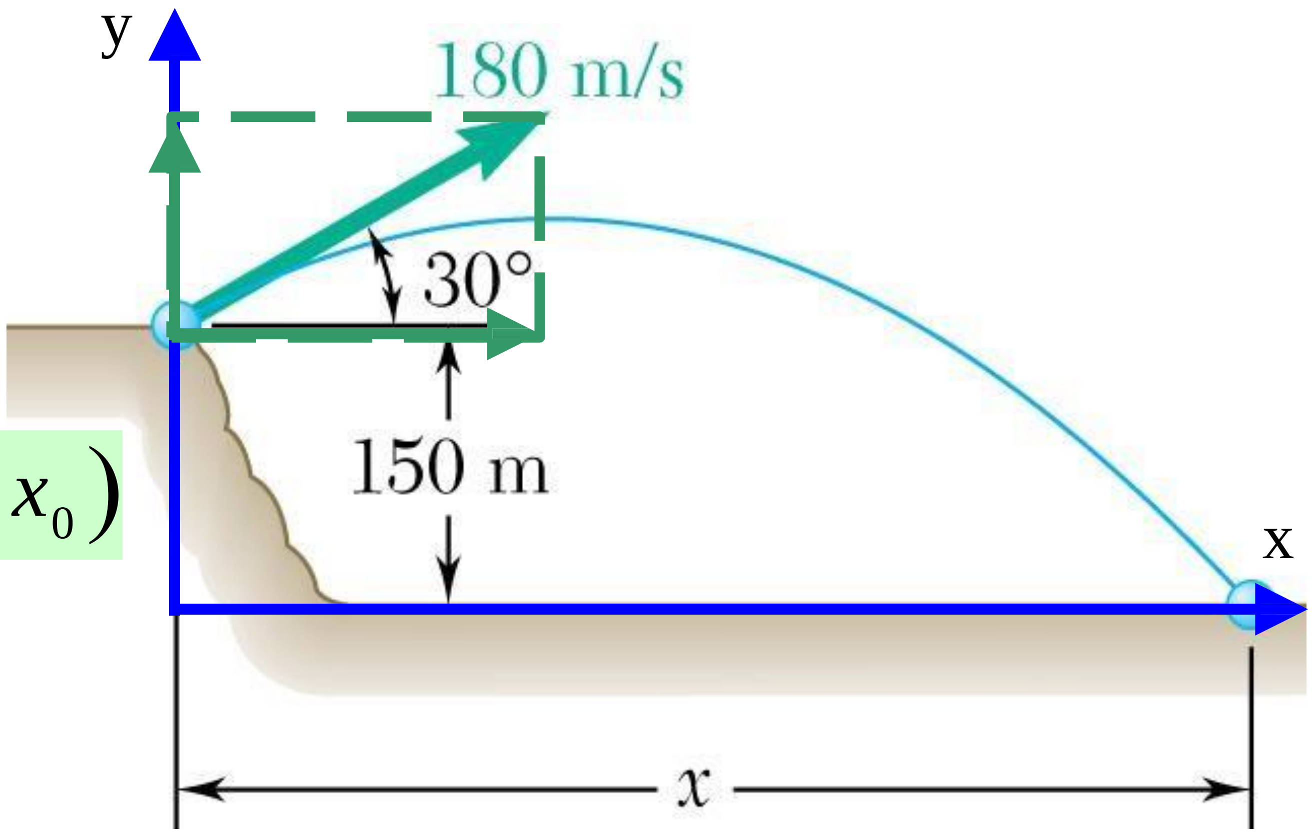
A projectile is fired from the edge of a 150-m cliff with an initial velocity of 180 m/s at an angle of 30° with the horizontal. Find (a) the range, and (b) maximum height.

Remember:

$$v = v_0 + at$$

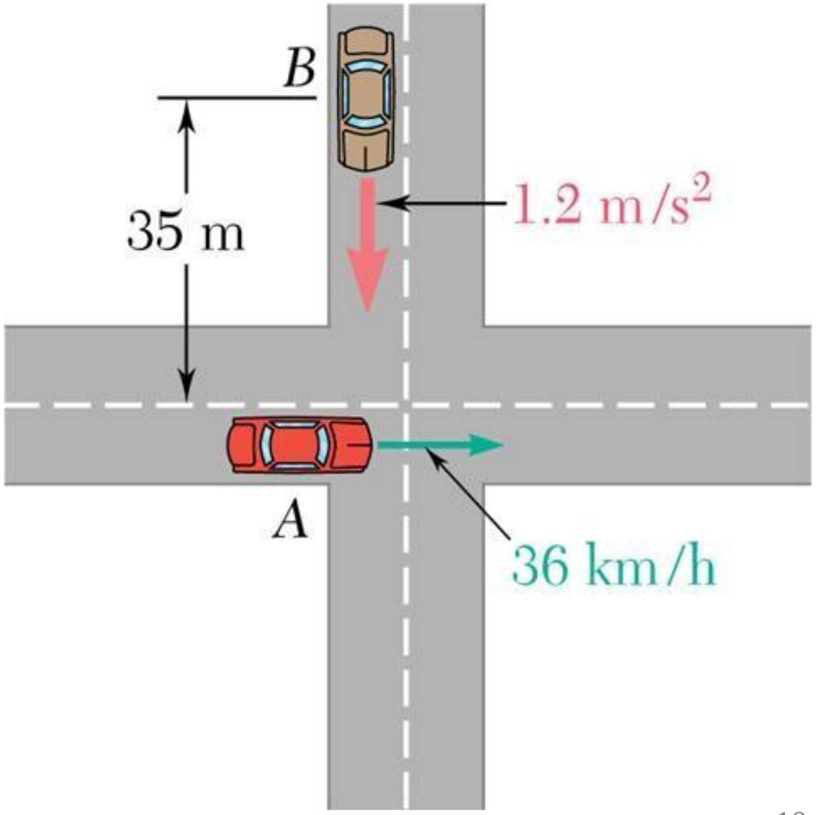
$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2a\left(x - x_0\right)$$

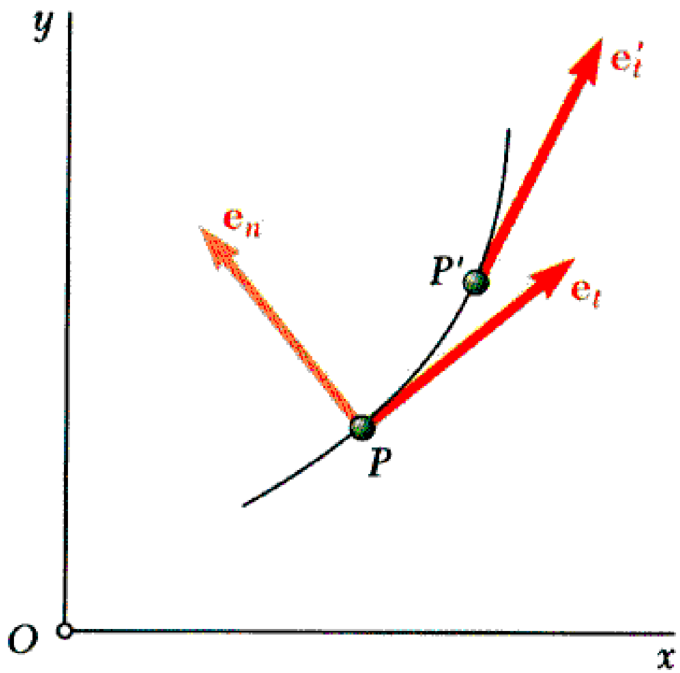


Example

Car A is traveling at a constant speed of 36 km/h. As A crosses intersection, B starts from rest 35 m north of intersection and moves with a constant acceleration of 1.2 m/s². Determine the speed, velocity and acceleration of B relative to A 5 seconds after A crosses intersection.

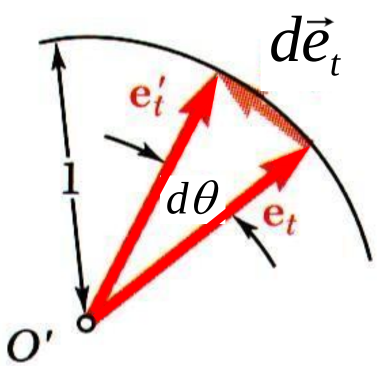


Tangential and Normal Components



- Velocity vector of particle is tangent to path of particle. In general, acceleration vector is not. Wish to express acceleration vector in terms of **tangential** and **normal** components.

- $\vec{e}_t$ and $\vec{e}_t'$ are **tangential** unit vectors for the particle path at *P* and *P'*. When drawn with respect to the same origin, $d\vec{e}_t = \vec{e}_t' - \vec{e}_t$



$$\vec{e}_t' = \vec{e}_t + d\vec{e}_t$$

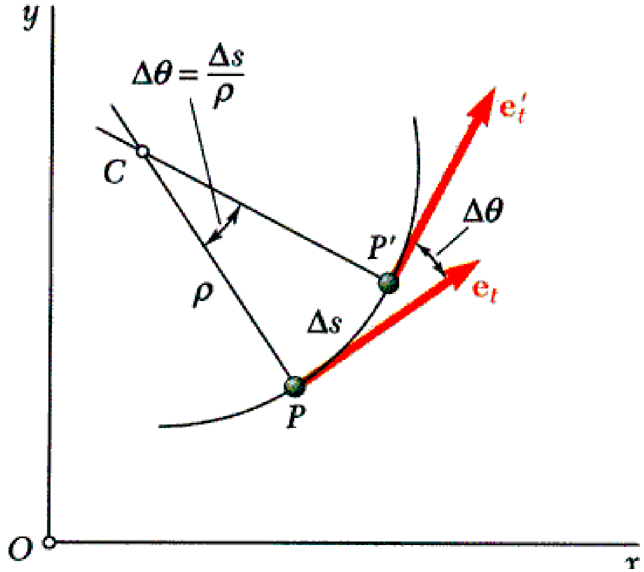
From geometry:

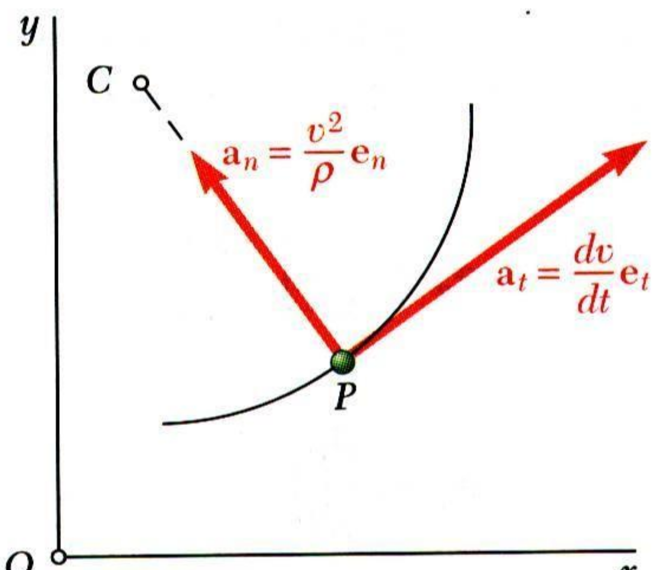
$$de_t = d\theta$$

$$d\vec{e}_t = d\theta\vec{e}_n$$

$$\frac{d\vec{e}_t}{d\theta} = \vec{e}_n$$

Tangential and Normal Components





- With the velocity vector expressed as $\vec{v} = v\vec{e}_t$ the particle acceleration may be written as

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv}{dt}\vec{e}_t + v\frac{d\vec{e}_t}{dt} = \frac{dv}{dt}\vec{e}_t + v\frac{d\vec{e}_t}{d\theta}\frac{d\theta}{ds}\frac{ds}{dt}$$

but

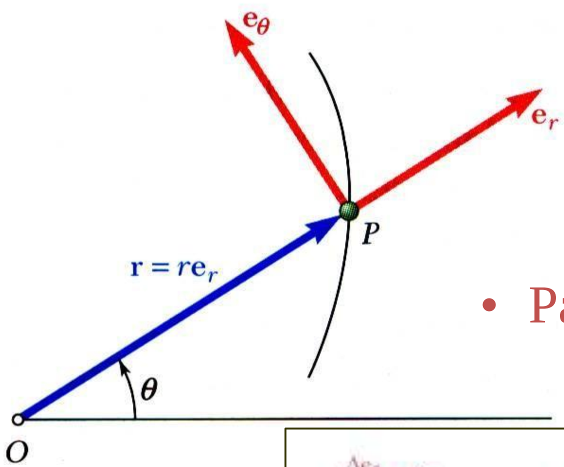
$$\frac{d\vec{e}_t}{d\theta} = \vec{e}_n \qquad \rho\, d\theta = ds \qquad \frac{ds}{dt} = v$$

After substituting,

$$\vec{a} = \frac{dv}{dt}\vec{e}_t + \frac{v^2}{\rho}\vec{e}_n \qquad a_t = \frac{dv}{dt} \qquad a_n = \frac{v^2}{\rho}$$

- Tangential component of acceleration reflects change of speed and normal component reflects change of direction.
- Tangential component may be positive or negative. Normal component always points toward center of path curvature.

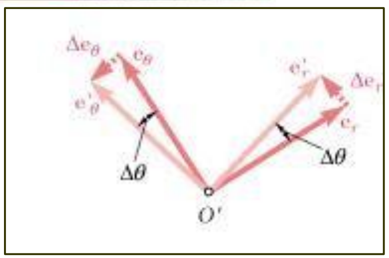
Radial and Transverse Components



- If particle position is given in **polar coordinates**, we can express velocity and acceleration with components parallel and perpendicular to *OP*.
- Particle position vector: $\vec{r} = r\vec{e}_r$
- Particle velocity vector: $\vec{v} = \dfrac{d}{dt}\left(r\vec{e}_r\right) = \dfrac{dr}{dt}\vec{e}_r + r\dfrac{d\vec{e}_r}{dt}$

$$v = \frac{dr}{dt}\vec{e}_r + r\frac{d\theta}{dt}\vec{e}_\theta = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$$

$$\vec{r} = r\vec{e}_r$$



$$\frac{d\vec{e}_r}{d\theta} = \vec{e}_\theta \qquad \frac{d\vec{e}_\theta}{d\theta} = -\vec{e}_r$$

$$\frac{d\vec{e}_r}{dt} = \frac{d\vec{e}_r}{d\theta}\frac{d\theta}{dt} = \vec{e}_\theta\frac{d\theta}{dt}$$

$$\frac{d\vec{e}_\theta}{dt} = \frac{d\vec{e}_\theta}{d\theta}\frac{d\theta}{dt} = -\vec{e}_r\frac{d\theta}{dt}$$

- Similarly, particle acceleration:

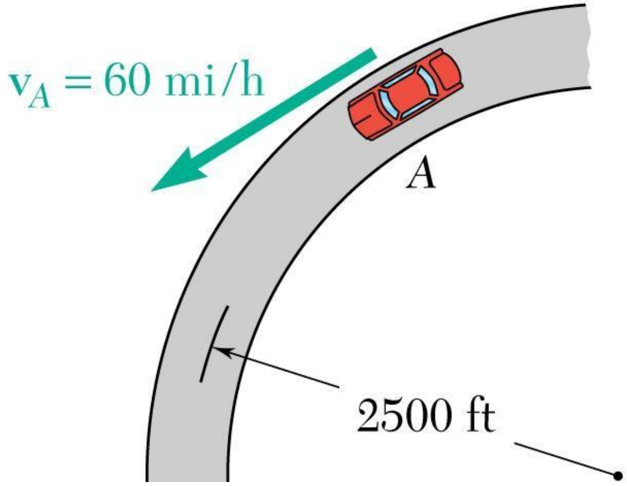
$$\vec{a} = \frac{d}{dt}\left(\dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta\right)$$

$$= \ddot{r}\vec{e}_r + \dot{r}\frac{d\vec{e}_r}{dt} + \dot{r}\dot{\theta}\vec{e}_\theta + r\ddot{\theta}\vec{e}_\theta + r\dot{\theta}\frac{d\vec{e}_\theta}{dt}$$

$$= \ddot{r}\vec{e}_r + \dot{r}\vec{e}_\theta\frac{d\theta}{dt} + \dot{r}\dot{\theta}\vec{e}_\theta + r\ddot{\theta}\vec{e}_\theta - r\dot{\theta}\vec{e}_r\frac{d\theta}{dt}$$

$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\vec{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\vec{e}_\theta$$

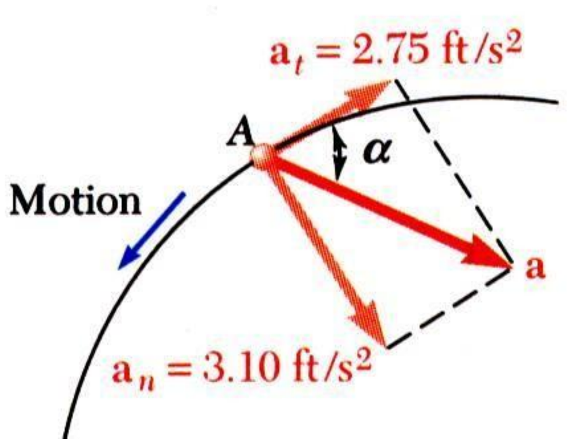
Sample Problem



A motorist is traveling on curved section of highway at 60 mph. The motorist applies brakes causing a constant deceleration.

Knowing that after 8 s the speed has been reduced to 45 mph, determine the acceleration of the automobile immediately after the brakes are applied.

Sample Problem



$$60 \text{ mph} = 88 \text{ft/s}$$
$$45 \text{ mph} = 66 \text{ ft/s}$$

SOLUTION:

- Calculate tangential and normal components of acceleration.

$$a_t = \frac{\Delta v}{\Delta t} = \frac{(66 - 88)\text{ft/s}}{8\text{ s}} = -2.75 \frac{\text{ft}}{\text{s}^2}$$

$$a_n = \frac{v^2}{\rho} = \frac{(88 \text{ ft/s})^2}{2500 \text{ ft}} = 3.10 \frac{\text{ft}}{\text{s}^2}$$

- Determine acceleration magnitude and direction with respect to tangent to curve.

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-2.75)^2 + 3.10^2}$$

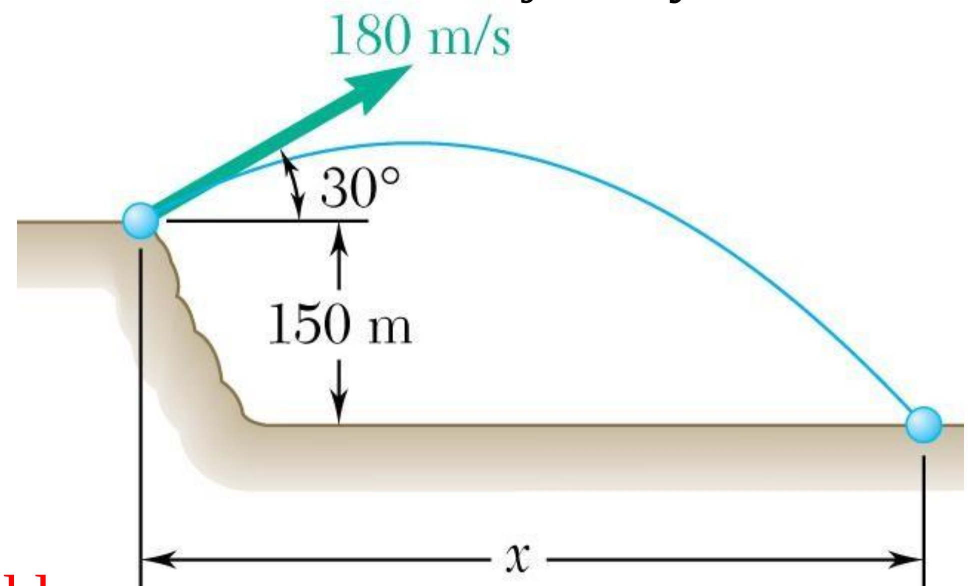
$$a = 4.14 \frac{\text{ft}}{\text{s}^2}$$

$$\alpha = \tan^{-1} \frac{a_n}{a_t} = \tan^{-1} \frac{3.10}{2.75}$$

$$\alpha = 48.4°$$

Sample Problem

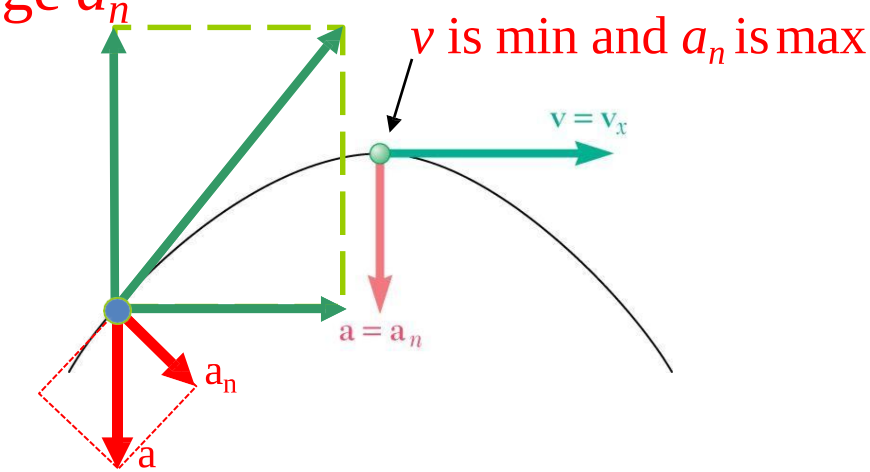
Determine the minimum radius of curvature of the trajectory described by the projectile.



Recall: $a_n = \dfrac{v^2}{\rho}$

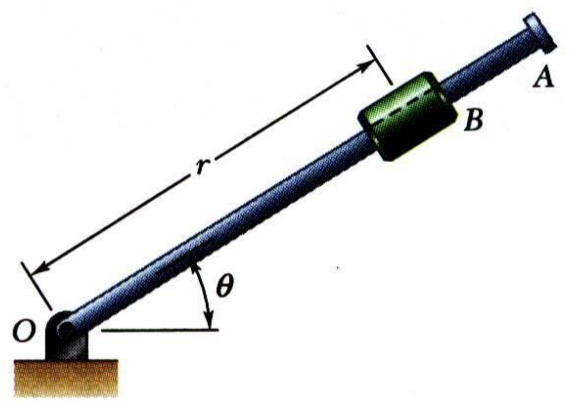
$$\rho = \frac{v^2}{a_n}$$

Minimum r, occurs for small v and large a_n



$$\rho = \frac{(155.9)^2}{9.81} = 2480\,m$$

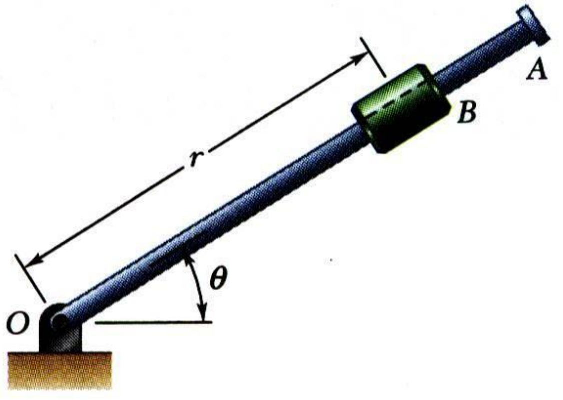
Sample Problem



Rotation of the arm about O is defined by $\theta = 0.15t^2$ where θ is in radians and t in seconds. Collar B slides along the arm such that $r = 0.9 - 0.12t^2$ where r is in meters.

After the arm has rotated through 30^o, determine *(a)* the total velocity of the collar, *(b)* the total acceleration of the collar, and *(c)* the relative acceleration of the collar with respect to the arm.

Sample Problem



SOLUTION:

- Evaluate time t for $\theta = 30^{\circ}$.

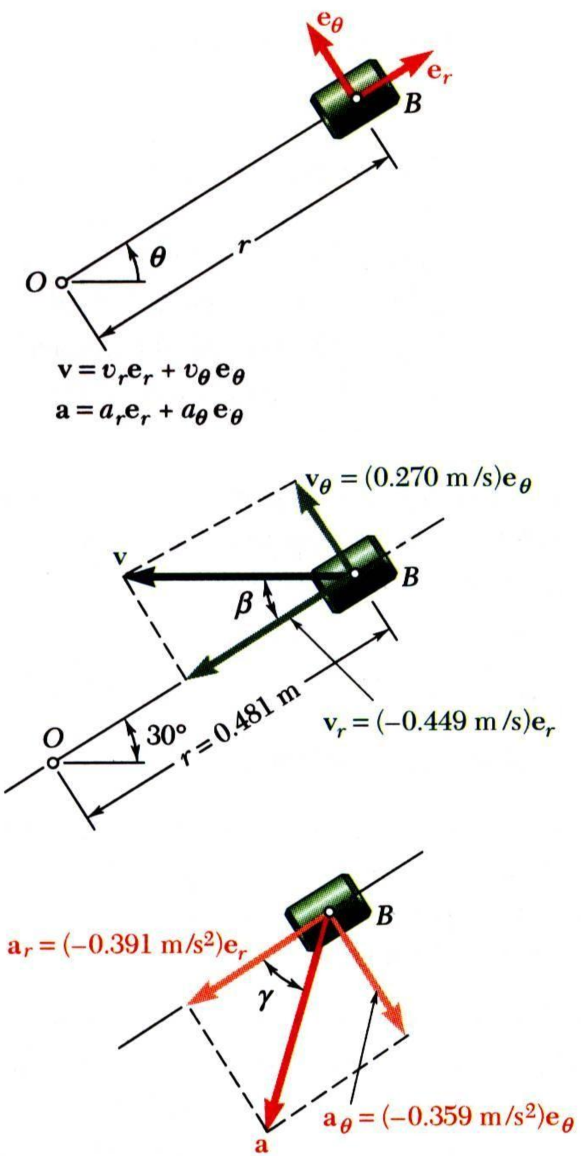
$$\theta = 0.15t^2$$
$$= 30^{\circ} = 0.524\,\text{rad} \qquad t = 1.869\,\text{s}$$

- Evaluate radial and angular positions, and first and second derivatives at time t.

$$r = 0.9 - 0.12t^2 = 0.481\,\text{m}$$
$$\dot{r} = -0.24\,t = -0.449\,\text{m/s}$$
$$\ddot{r} = -0.24\,\text{m/s}^2$$

$$\theta = 0.15t^2 = 0.524\text{rad}$$
$$\dot{\theta} = 0.30\,t = 0.561\text{rad/s}$$
$$\ddot{\theta} = 0.30\,\text{rad/s}^2$$

Sample Problem



- Calculate velocity and acceleration.

$$v_r = \dot{r} = -0.449\ \mathrm{m/s}$$

$$v_\theta = r\dot{\theta} = (0.481\mathrm{m})(0.561\mathrm{rad/s}) = 0.270\ \mathrm{m/s}$$

$$v = \sqrt{v_r^2 + v_\theta^2} \qquad \beta = \tan^{-1}\frac{v_\theta}{v_r}$$

$$v = 0.524\mathrm{m/s} \qquad \beta = 31.0^\circ$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$= -0.240\ \mathrm{m/s^2} - (0.481\mathrm{m})(0.561\mathrm{rad/s})^2$$

$$= -0.391\mathrm{m/s^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

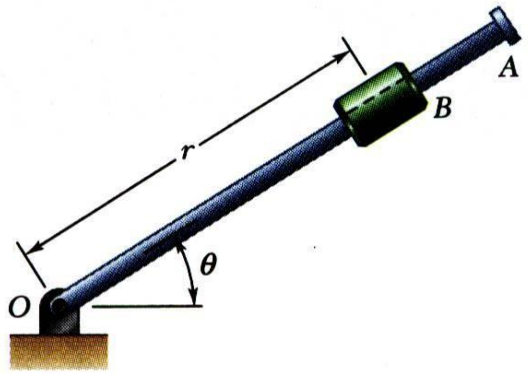
$$= (0.481\mathrm{m})\left(0.3\mathrm{rad/s^2}\right) + 2(-0.449\ \mathrm{m/s})(0.561\mathrm{rad/s})$$

$$= -0.359\ \mathrm{m/s^2}$$

$$a = \sqrt{a_r^2 + a_\theta^2} \qquad \gamma = \tan^{-1}\frac{a_\theta}{a_r}$$

$$a = 0.531\mathrm{m/s} \qquad \gamma = 42.6^\circ$$

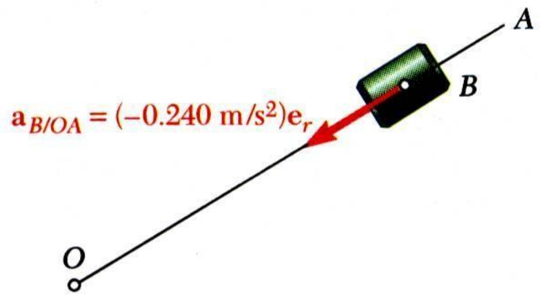
Sample Problem



- Evaluate acceleration with respect to arm.

Motion of collar with respect to arm is rectilinear and defined by coordinate *r*.

$$a_{B/OA} = \ddot{r} = -0.240 \text{ m/s}^2$$

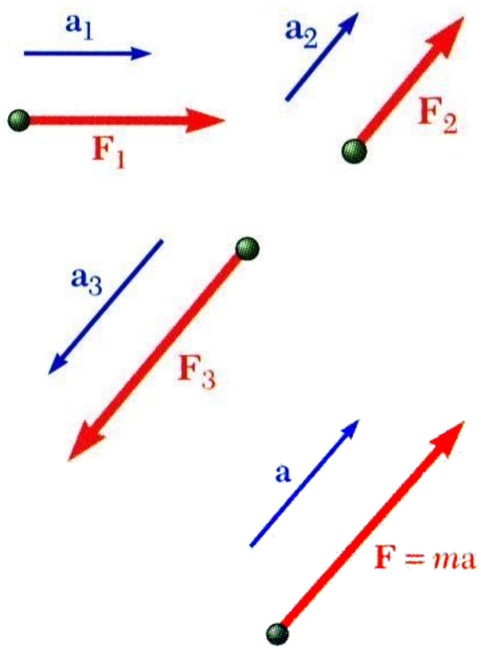


UNIT-II

KINETICS OF PARTICLE

Introduction, definitions of matter, body, particle, mass, weight, inertia, momentum, Newton’s law of motion, relation between force and mass, motion of a particle in rectangular coordinates, D’Alembert’s principle, motion of lift, motion of body on an inclined plane, motion of connected bodies.

Newton's Second Law of Motion



- If the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of resultant and in the direction of the resultant.

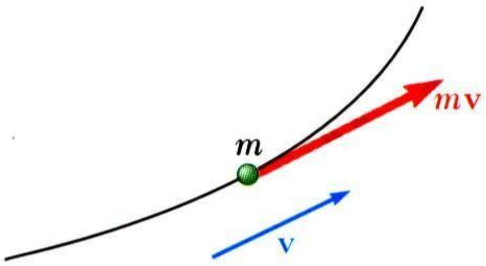
$$\vec{F} = m\vec{a}$$

- If particle is subjected to several forces:

$$\sum \vec{F} = m\vec{a}$$

- We must use a <u>Newtonian frame of reference</u>, i.e., one that is not accelerating or rotating.
- If no force acts on particle, particle will not accelerate, i.e., it will remain stationary or continue on a straight line at constant velocity.

Linear Momentum of a Particle



$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$= \frac{d}{dt}(m\vec{v}) = \frac{d}{dt}(\vec{L})$$

$$\vec{L} = m\vec{v}$$ Linear momentum

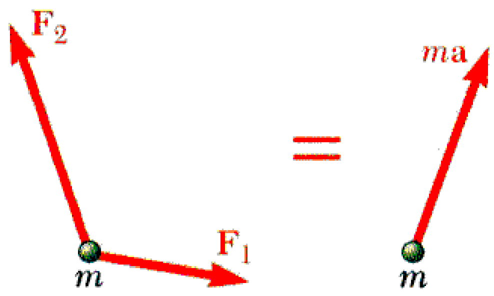
Sum of forces = rate of change of linear momentum

$$\sum \vec{F} = \dot{\vec{L}}$$

If $\sum \vec{F} = 0$ ⇒ linear momentum is <u>constant</u>

<u>Principle of conservation of linear momentum</u>

Equations of Motion



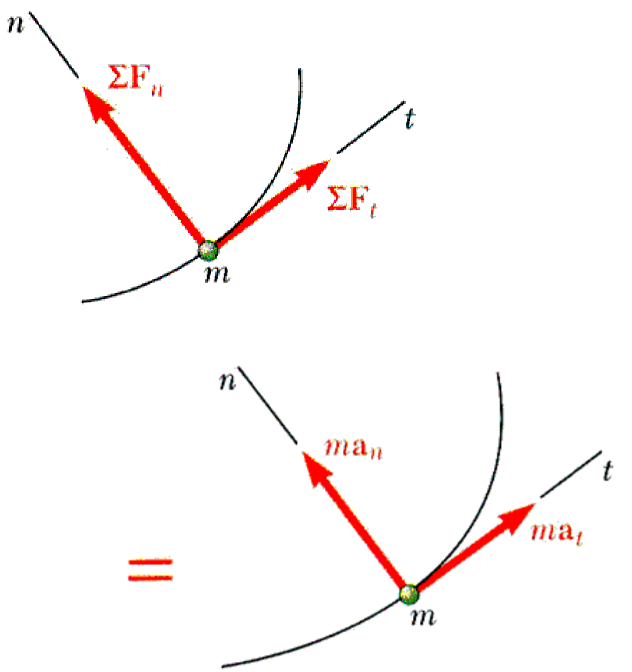
- Newton's second law $\sum \vec{F} = m\vec{a}$

- Convenient to resolve into components:

$$\sum \left(F_x \vec{i} + F_y \vec{j} + F_z \vec{k}\right) = m\left(a_x \vec{i} + a_y \vec{j} + a_z \vec{k}\right)$$

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z$$

$$\sum F_x = m\ddot{x} \qquad \sum F_y = m\ddot{y} \qquad \sum F_z = m\ddot{z}$$

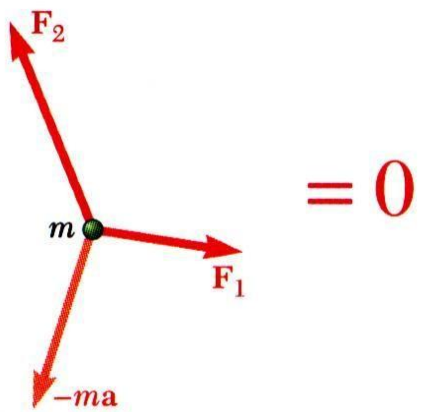


- For tangential and normal components:

$$\sum F_t = ma_t \qquad \sum F_n = ma_n$$

$$\sum F_t = m\frac{dv}{dt} \qquad \sum F_n = m\frac{v^2}{\rho}$$

Dynamic Equilibrium

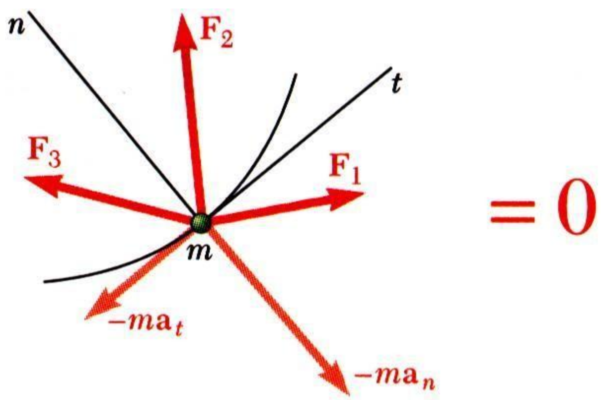


- Alternate expression of Newton's law:

$$\sum \vec{F} - m\vec{a} = 0$$

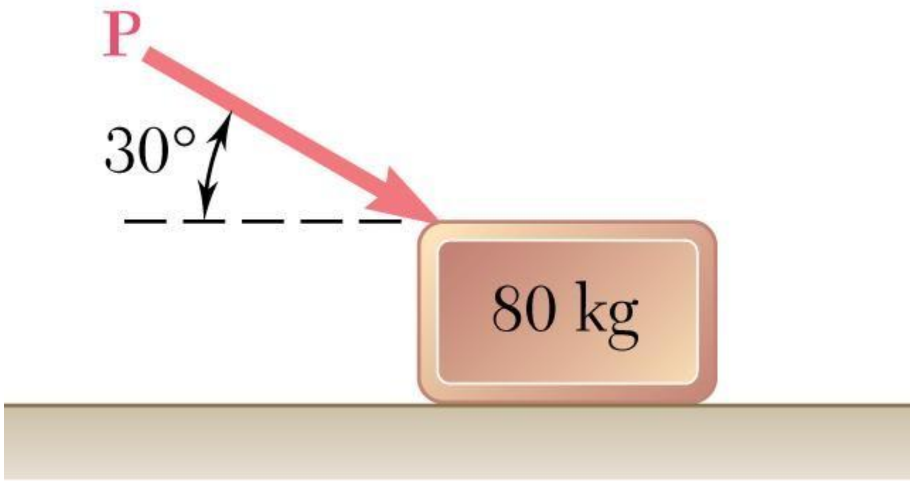
$-m\vec{a}$ → inertia vector

- If we include inertia vector, the system of forces acting on particle is equivalent to zero. The particle is said to be in *dynamic equilibrium*.



- Inertia vectors are often called *inertia forces* as they measure the resistance that particles offer to changes in motion.

Sample Problem 1

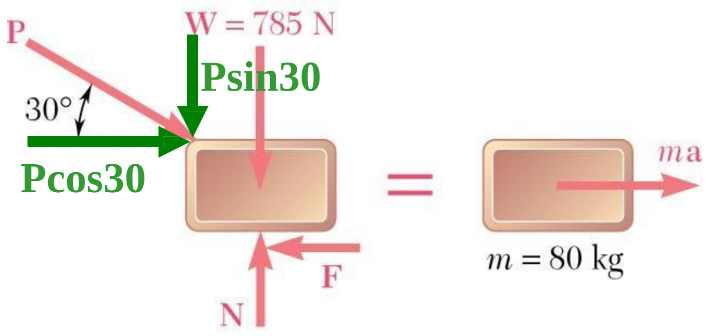


SOLUTION:

- Draw a free body diagram
- Apply Newton’s law. Resolve into rectangular components

An 80-kg block rests on a horizontal plane. Find the magnitude of the force **P** required to give the block an acceleration of 2.5 m/s^2 to the right. The coefficient of kinetic friction between the block and plane is m_k = 0.25.

Sample Problem 12.2



$$W = mg = 80 \times 9.81 = 785N$$

$$F = \mu_k N = 0.25N$$

$$\sum F_x = ma:$$

$$P\cos 30° - 0.25N = (80)(2.5)$$
$$= 200$$

$$\sum F_y = 0:$$

$$N - P\sin 30° - 785 = 0$$

Solve for P and N

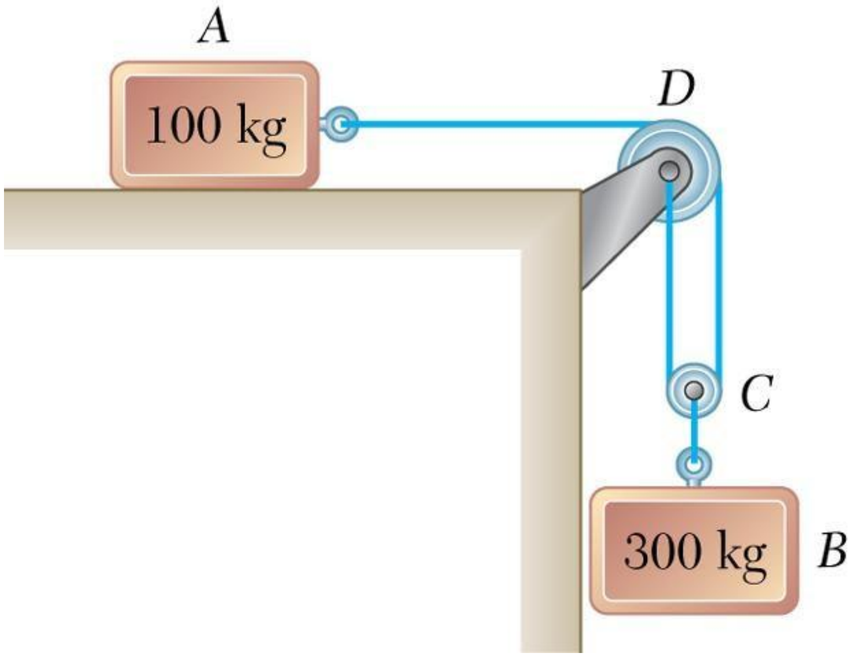
$$N = P\sin 30° + 785$$

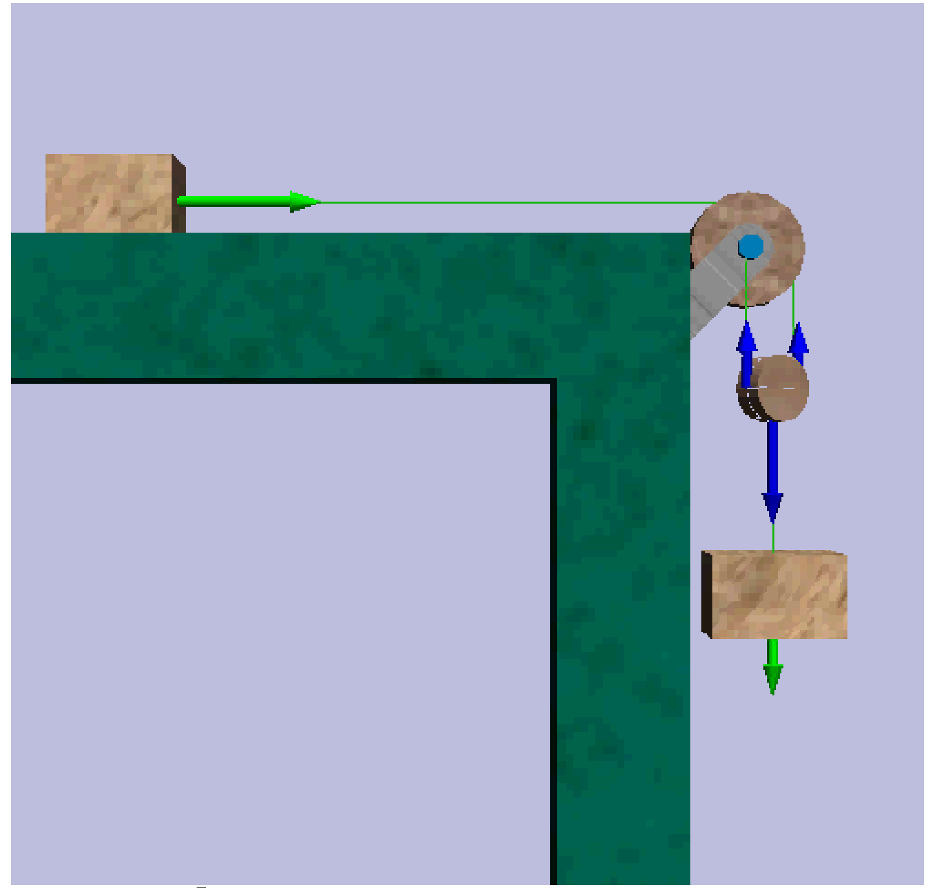
$$P\cos 30° - 0.25(P\sin 30° + 785) = 200$$

$$P = 534.7 \quad N$$

$$N = 1052.4 \quad N$$

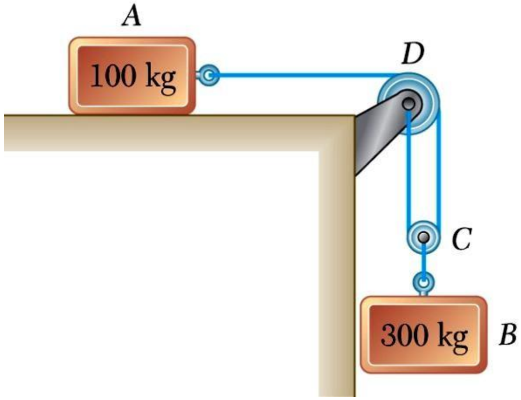
Sample Problem 12.3





The two blocks shown start from rest. The horizontal plane and the pulley are frictionless, and the pulley is assumed to be of negligible mass. Determine the acceleration of each block and the tension in the cord.

Sample Problem 2



- Kinematic relationship: If A moves x_A to the right, B moves down 0.5 x_A

$$x_B = \tfrac{1}{2} x_A \qquad a_B = \tfrac{1}{2} a_A$$

Draw free body diagrams & apply Newton's law:

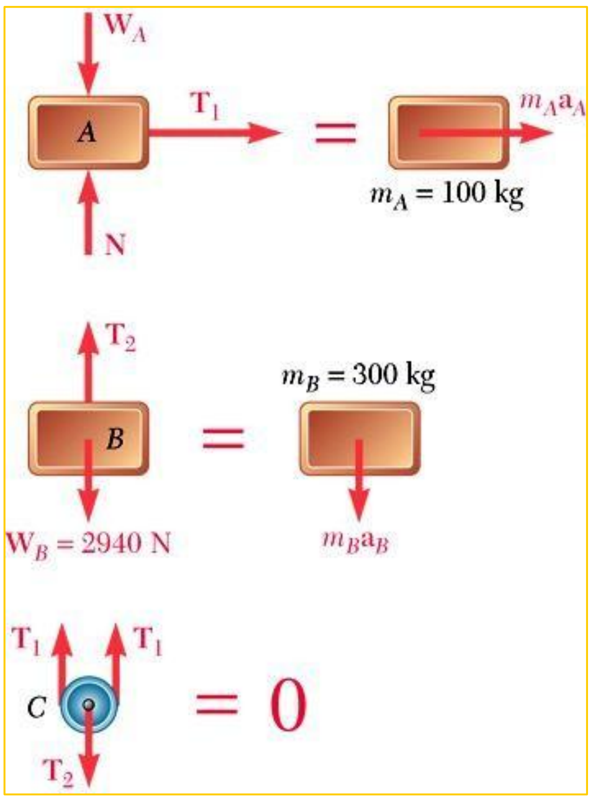
$$\sum F_x = m_A a_A \Rightarrow T_1 = (100) a_A$$

$$\sum F_y = m_B a_B \Rightarrow m_B g - T_2 = m_B a_B$$

$$300 \times 9.81 - T_2 = (300) a_B$$

$$T_2 = 2940\text{-}(300) a_B$$

$$\sum F_y = m_C a_C \Rightarrow T_2 - 2T_1 = 0$$

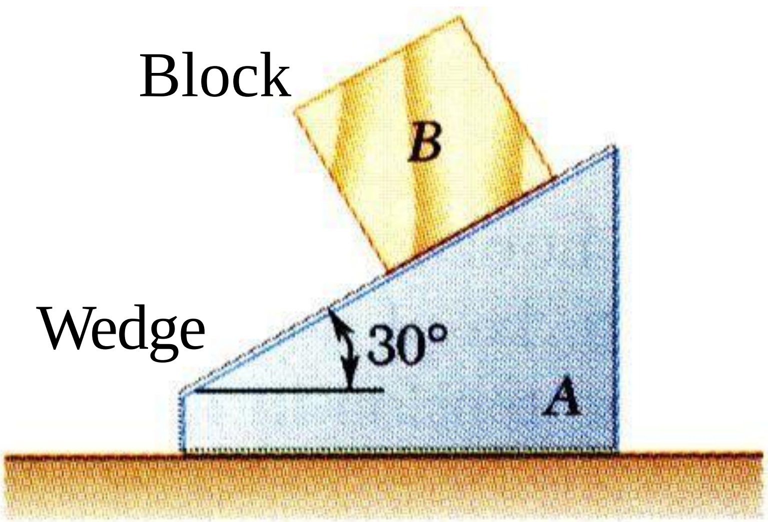


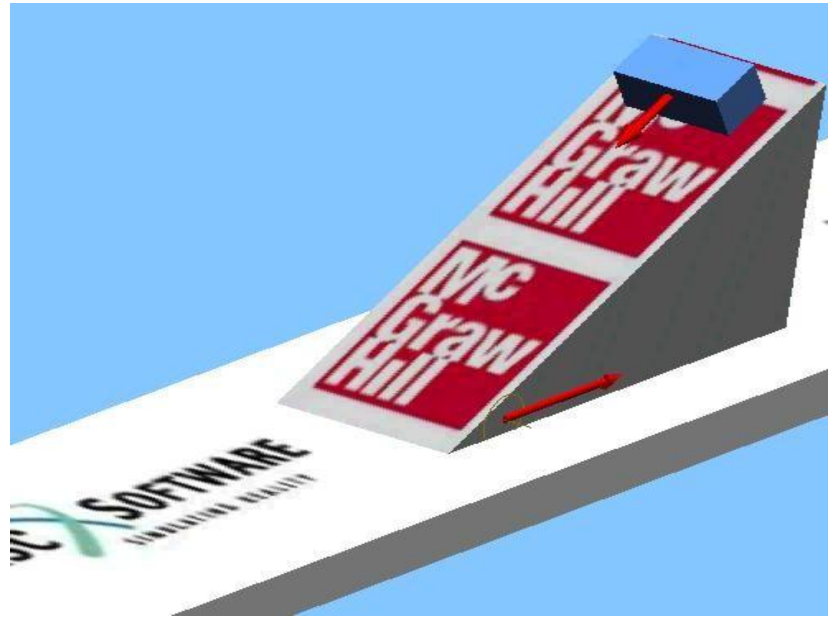
$$2940\text{-}(300) a_B - 2T_1 = 0 \qquad 2940\text{-}(300) a_B - 200 a_A = 0$$

$$2940\text{-}(300) a_B - 2 \times 200 a_B = 0$$

$a_B = 4.2\ m/s^2$ $\quad a_A = 8.4\ m/s^2$ $\quad T_1 = 840\ N$ $\quad T_2 = 1680\ N$

Sample Problem 3

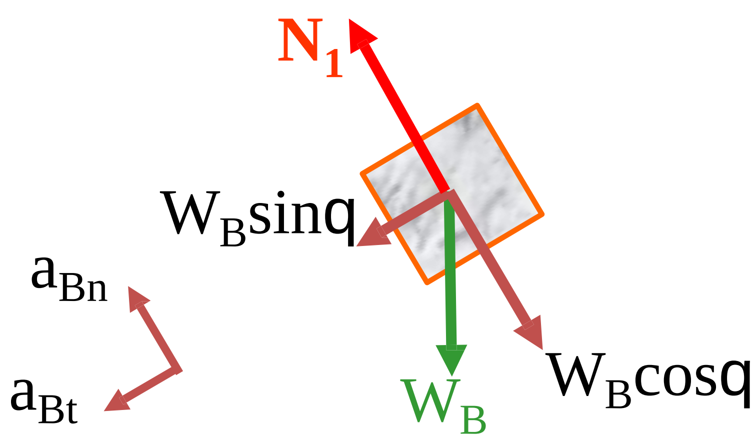


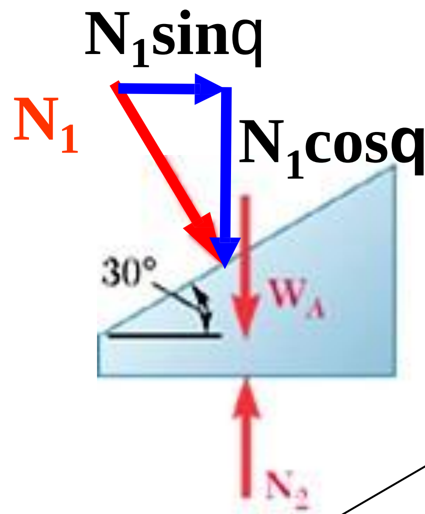


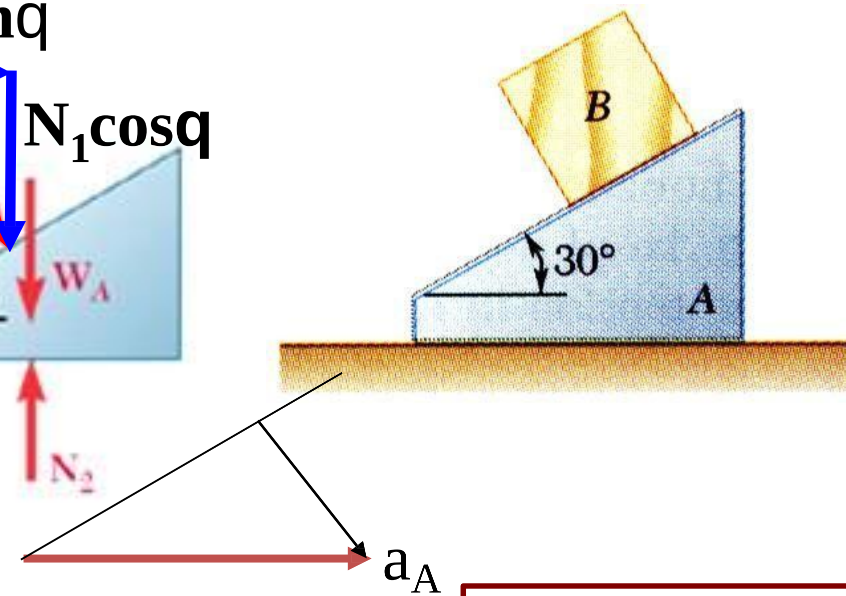
The 12-lb block *B* starts from rest and slides on the 30-lb wedge *A*, which is supported by a horizontal surface.

Neglecting friction, determine *(a)* the acceleration of the wedge, and *(b)* the acceleration of the block relative to the wedge.

Draw free body diagrams for block & wedge







$$W_B \sin\theta = m_B a_{Bt}$$

$$12 \times 0.5 = \frac{12}{32.2} a_{Bt} \quad \Rightarrow a_{Bt} = 16.1 \ ft/s^2$$

$$N_1 \sin\theta = m_A a_A \quad \Rightarrow \quad 0.5 N_1 = \frac{30}{32.2} a_A$$

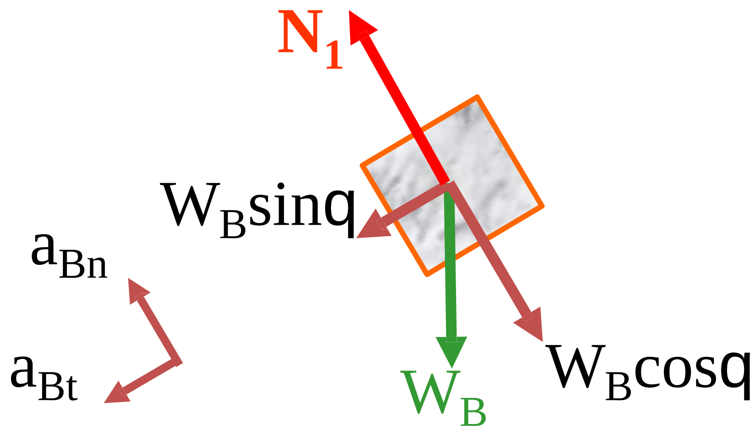
$$N_1 - W_B \cos\theta = m_B a_{Bn}$$

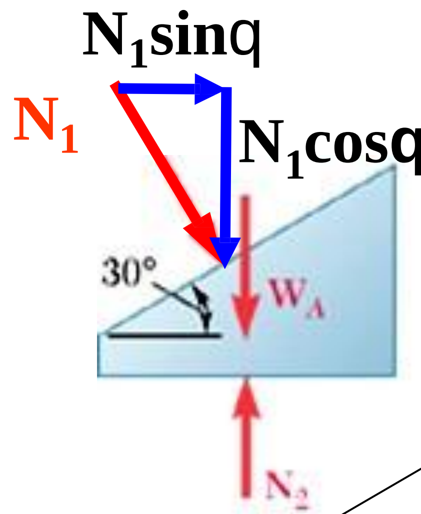
$$N_1 \cos\theta + W_A = N_2$$

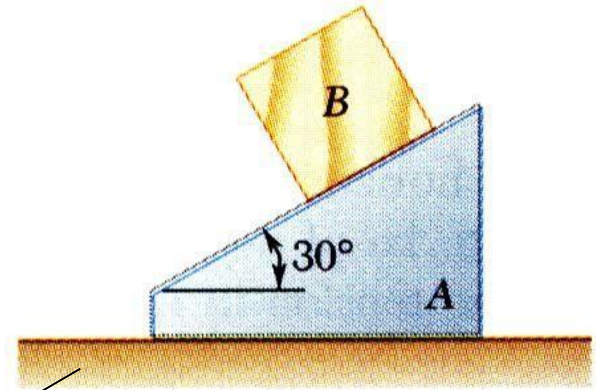
But $a_{Bn} = -a_A \sin\theta$ Same normal acceleration (to maintain contact)

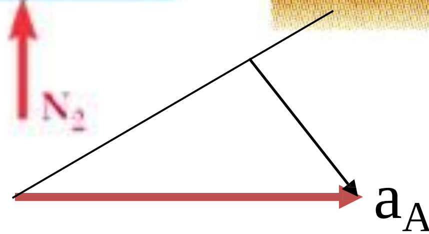
$$N_1 - W_B \cos\theta = -m_B a_A \sin\theta \quad \Rightarrow \quad N_1 - 10.39 = -\frac{12 \times 0.5}{32.2} a_A$$

$$\Rightarrow a_A = 5.08 \ ft/s^2 \longrightarrow \quad a_{Bn} = -2.54 \ ft/s^2$$









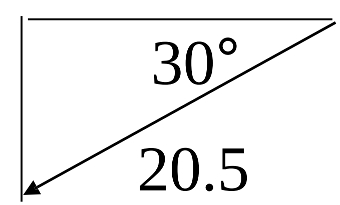
$$a_{Bx} = -a_{Bt}\cos\theta - a_{Bn}\sin\theta = -12.67\ ft/s^2$$

$$a_{By} = -a_{Bt}\sin\theta + a_{Bn}\cos\theta = -10.25\ ft/s^2$$

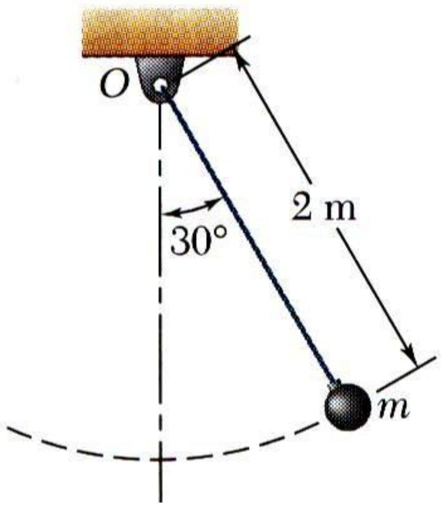
$$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$

$$\vec{a}_{B/A} = \left(-12.67\vec{i} - 10.25\,\vec{j}\right) - \left(5.08\vec{i}\right)$$

$$= -17.75\vec{i} - 10.25\,\vec{j}$$

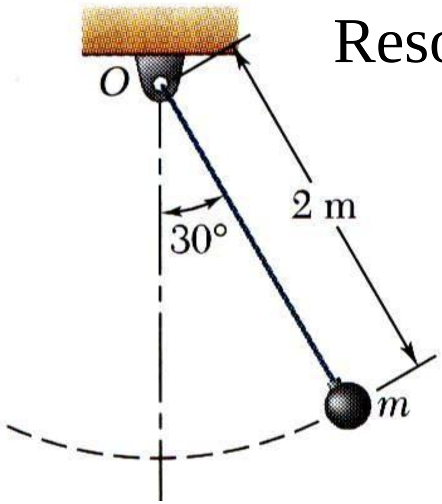


Sample Problem 4



The bob of a 2-m pendulum describes an arc of a circle in a vertical plane. If the tension in the cord is 2.5 times the weight of the bob for the position shown, find the velocity and acceleration of the bob in that position.

Sample Problem 5



Resolve into tangential and normal components:

$$\sum F_t = ma_t : \qquad mg\sin 30° = ma_t$$

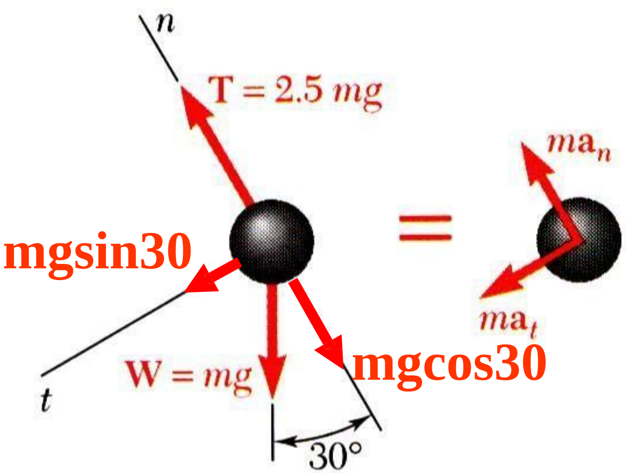
$$a_t = g \sin 30°$$

$$\boxed{a_t = 4.9 \text{ m/s}^2}$$

$$\sum F_n = ma_n : \qquad 2.5mg - mg\cos 30° = ma_n$$

$$a_n = g(2.5 - \cos 30°)$$

$$\boxed{a_n = 16.03 \text{ m/s}^2}$$

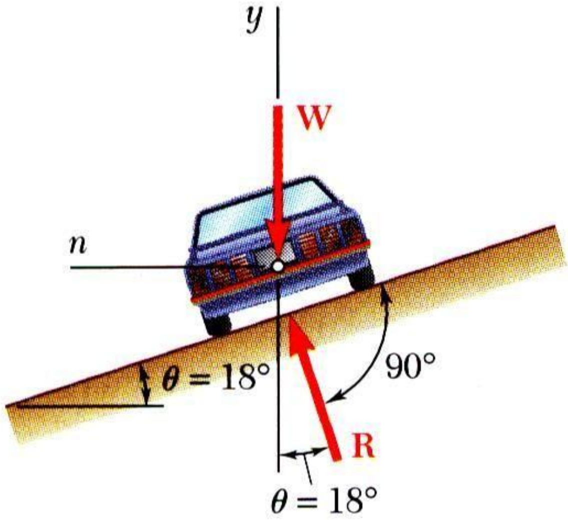


- Solve for velocity in terms of normal acceleration.

$$a_n = \frac{v^2}{\rho} \qquad v = \sqrt{\rho a_n} = \sqrt{(2 \text{ m})\left(16.03 \text{ m/s}^2\right)}$$

$$\boxed{v = \pm 5.66 \text{ m/s}}$$

Sample Problem 6



Determine the rated speed of a highway curve of radius $\rho = 400$ ft banked through an angle $\theta = 18^{o}$. The rated speed of a banked highway curve is the speed at which a car should travel if no lateral friction force is to be exerted at its wheels.

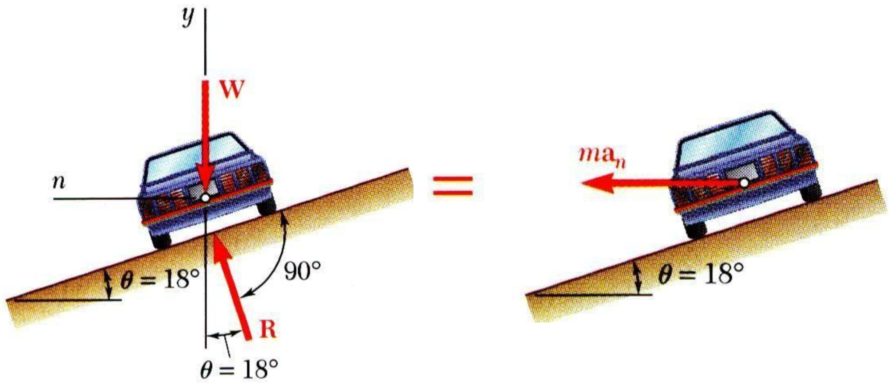
SOLUTION:

- The car travels in a horizontal circular path with a normal component of acceleration directed toward the center of the path.The forces acting on the car are its weight and a normal reaction from the road surface.

- Resolve the equation of motion for the car into vertical and normal components.

- Solve for the vehicle speed.

Sample Problem 7



SOLUTION:

- The car travels in a horizontal circular path with a normal component of acceleration directed toward the center of the path.The forces acting on the car are its weight and a normal reaction from the road surface.

- Resolve the equation of motion for the car into vertical and normal components.

$$\sum F_y = 0: \qquad R\cos\theta - W = 0$$

$$R = \frac{W}{\cos\theta}$$

$$\sum F_n = ma_n: \qquad R\sin\theta = \frac{W}{g}a_n$$

$$\frac{W}{\cos\theta}\sin\theta = \frac{W}{g}\frac{v^2}{\rho}$$

- Solve for the vehicle speed.

$$v^2 = g\rho\tan\theta$$

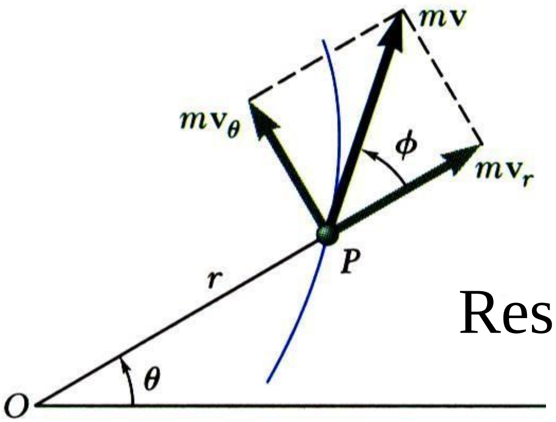
$$= \left(32.2\,\text{ft}/\text{s}^2\right)(400\,\text{ft})\tan 18^\circ$$

$$\boxed{v = 64.7\ \text{ft/s} = 44.1\,\text{mi/h}}$$

Angular Momentum

From before, linear momentum: $\vec{L} = m\vec{v}$

Now <u>angular momentum</u> is defined as the *moment of momentum*



$$\vec{H}_O = \vec{r} \times m\vec{v}$$

$\vec{H}_O$ is a vector perpendicular to the plane containing $\vec{r}$ and $m\vec{v}$

Resolving into radial & transverse components:

$$H_O = mv_\theta r = mr^2\dot{\theta}$$

Derivative of angular momentum with respect to time:

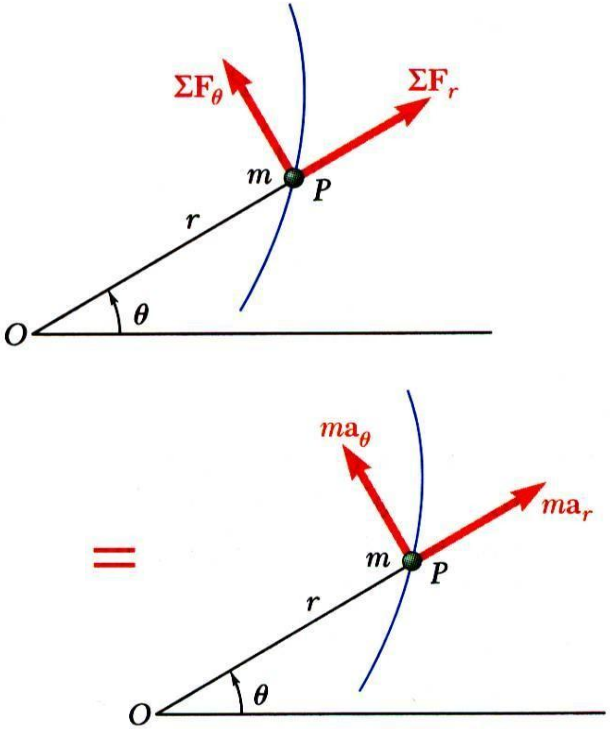
$$\dot{\vec{H}}_O = \dot{\vec{r}} \times m\vec{v} + \vec{r} \times m\dot{\vec{v}} = \vec{v} \times m\vec{v} + \vec{r} \times m\vec{a}$$

$$= r \times \sum \vec{F}$$ ⟵ Moment of $\vec{F}$ about O

$$= \sum \vec{M}_O$$

Sum of moments about *O* = rate of change of angular momentum

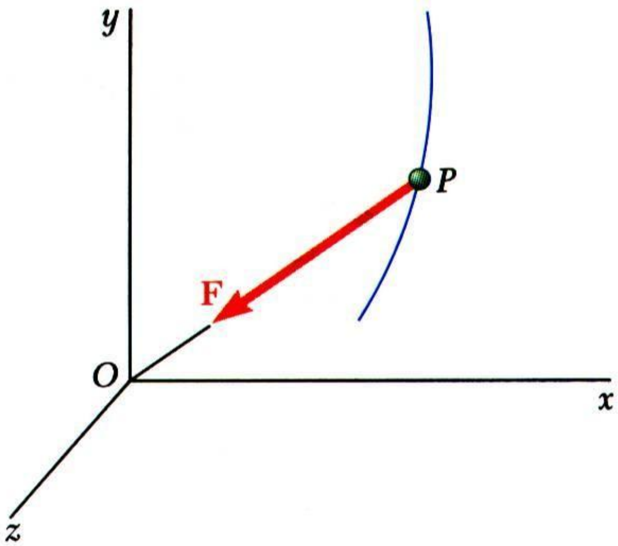
Equations of Motion in Radial & Transverse Components



$$\sum F_r = ma_r = m\left(\ddot{r} - r\dot{\theta}^2\right)$$

$$\sum F_\theta = ma_\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$

Central Force



When force acting on particle is directed toward or away from a fixed point *O*, the particle is said to be *moving under a central force.*

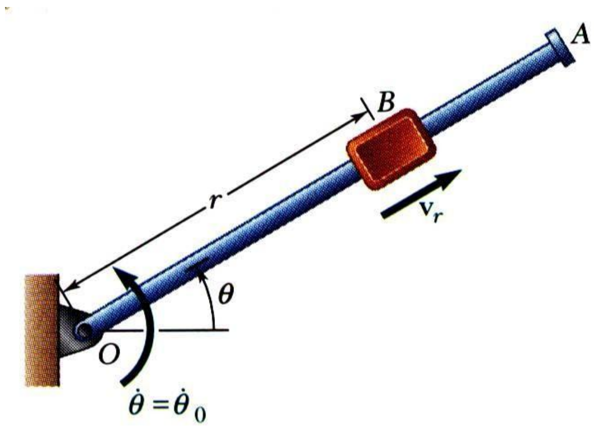
O = center of force

Since line of action of the central force passes through *O*:

$$\sum \vec{M}_O = \dot{\vec{H}}_O = 0$$

$$\vec{r} \times m\vec{v} = \vec{H}_O = \text{constant}$$

Sample Problem 8



A block B of mass m can slide freely on a frictionless arm OA which rotates in a horizontal plane at a constant rate $\dot{\theta}_0$.

Knowing that B is released at a distance r_0 from O, express as a function of r

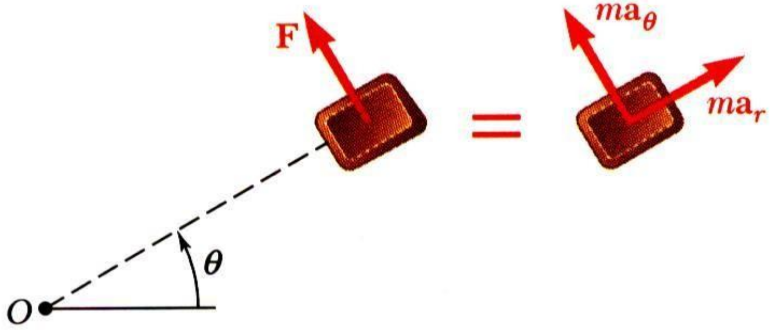
a) the component v_r of the velocity of B along OA, and

b) the magnitude of the horizontal force exerted on B by the arm OA.

SOLUTION:

- Write the radial and transverse equations of motion for the block.
- Integrate the radial equation to find an expression for the radial velocity.
- Substitute known information into the transverse equation to find an expression for the force on the block.

Sample Problem 8



$$\Rightarrow \quad \ddot{r} = r\dot{\theta}^2$$

$$\ddot{r} = \dot{v}_r = \frac{dv_r}{dt} = \frac{dv_r}{dr}\frac{dr}{dt} = v_r\frac{dv_r}{dr}$$

But $v_r = \dot{r}$

$$r\dot{\theta}^2 = v_r\frac{dv_r}{dr} \qquad\qquad r\dot{\theta}^2 dr = v_r dv_r$$

$$\int_0^{v_r} v_r dv_r = \int_{r_o}^{r} r\dot{\theta}_o^2 dr$$

$$v_r^2 = \dot{\theta}_0^2\left(r^2 - r_0^2\right)$$

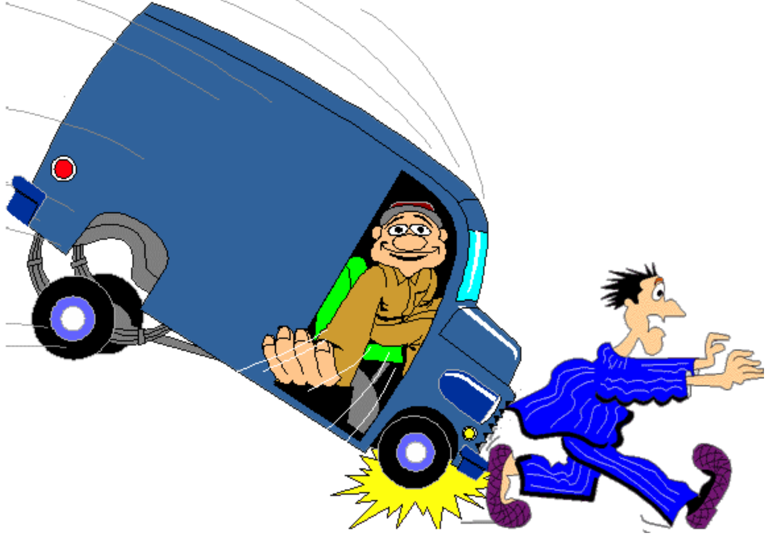
Write radial and transverse equations of motion:

$$\sum F_r = m \quad a_r \Rightarrow 0 = m\left(\ddot{r} - r\dot{\theta}^2\right)$$

$$\sum F_\theta = m \quad a_\theta \Rightarrow F = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$

$$\Rightarrow \quad v_r = \dot{\theta}_0\left(r^2 - r_0^2\right)^{1/2}$$

$$\Rightarrow \quad F = 2m\dot{\theta}_0^2\left(r^2 - r_0^2\right)^{1/2}$$



UNIT-III

IMPULSE AND MOMENTUM, VIRTUAL WORK

Impulse and momentum: Introduction; Impact, momentum, impulse, impulsive forces, units, law of conservation of momentum, Newton's law of collision of elastic bodies.

Coefficient of restitution, recoil of gun, impulse momentum equation; Virtual work: Introduction, principle of virtual work, applications, beams, lifting machines, simple framed structures.

Impulse = Momentum

Consider Newton's 2nd Law and the

Impulse-Momentum Theorem

$J = \Delta p$

$Ft = \Delta mv$

$$\frac{F_{Net}}{m} = a, \quad a = \frac{\Delta v}{t}$$

$$\frac{F_{Net}}{m} = \frac{\Delta v}{t} \rightarrow Ft = \Delta mv$$

$$Ft = \text{Impulse(J)}$$

$$\Delta mv = \text{Momentum(p)}$$

Ns

Kg x m/s

Momentum is defined as *"Inertia in Motion"*

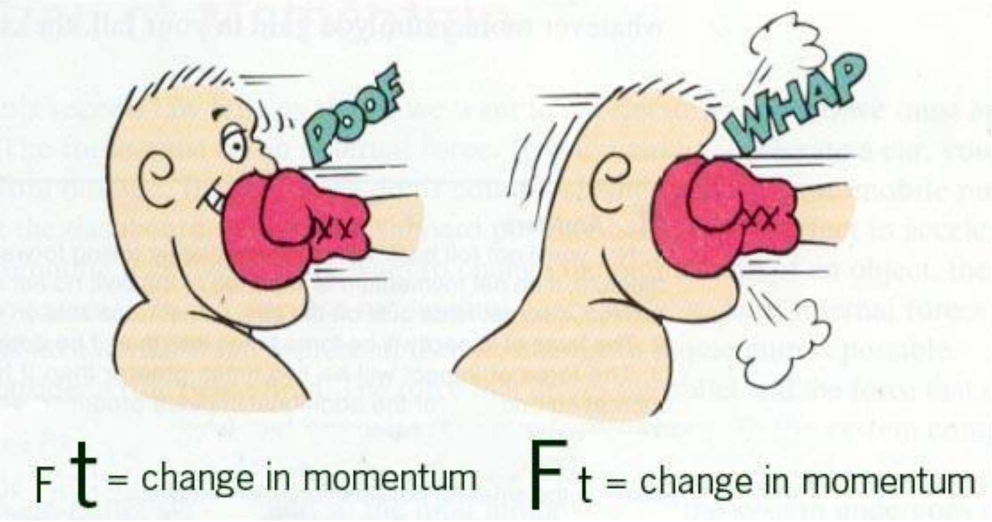
Units of Impulse:

Units of Momentum:

Impulse – Momentum Theorem

$$Ft = \boxed{m\Delta v}$$

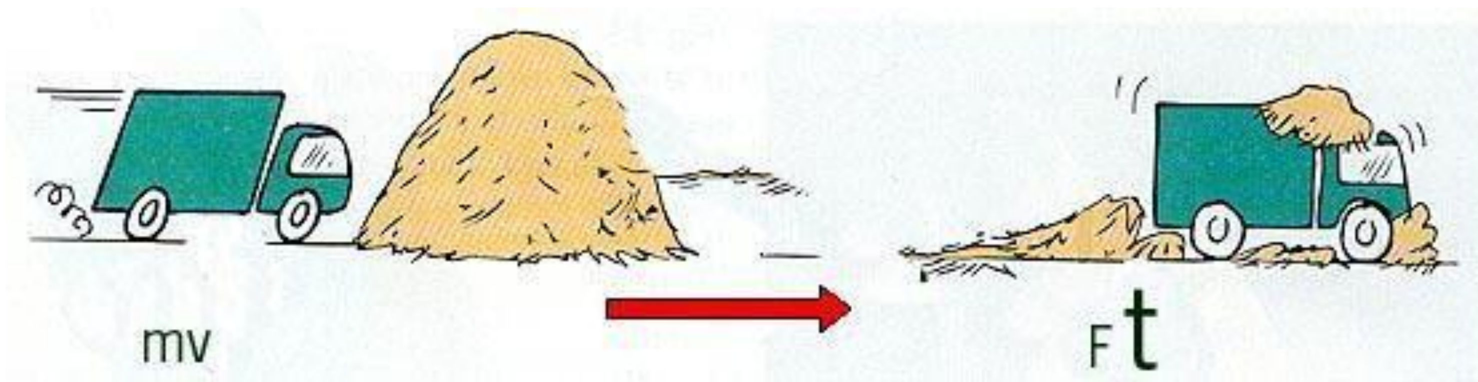
IMPULSE — **CHANGE IN MOMENTUM**

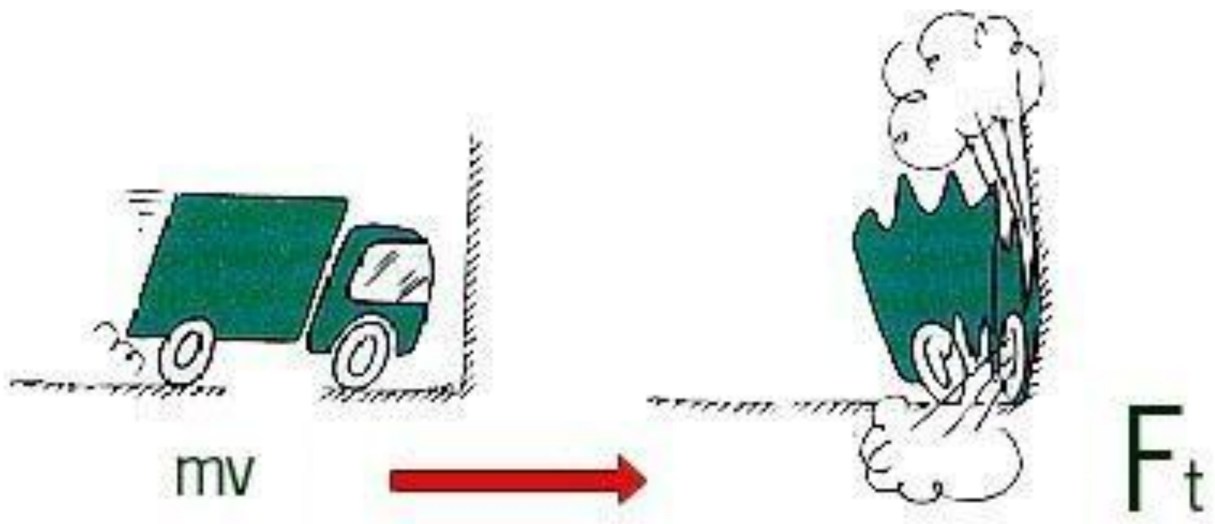


This theorem reveals some interesting relationships such as the INVERSE relationship between FORCE and TIME

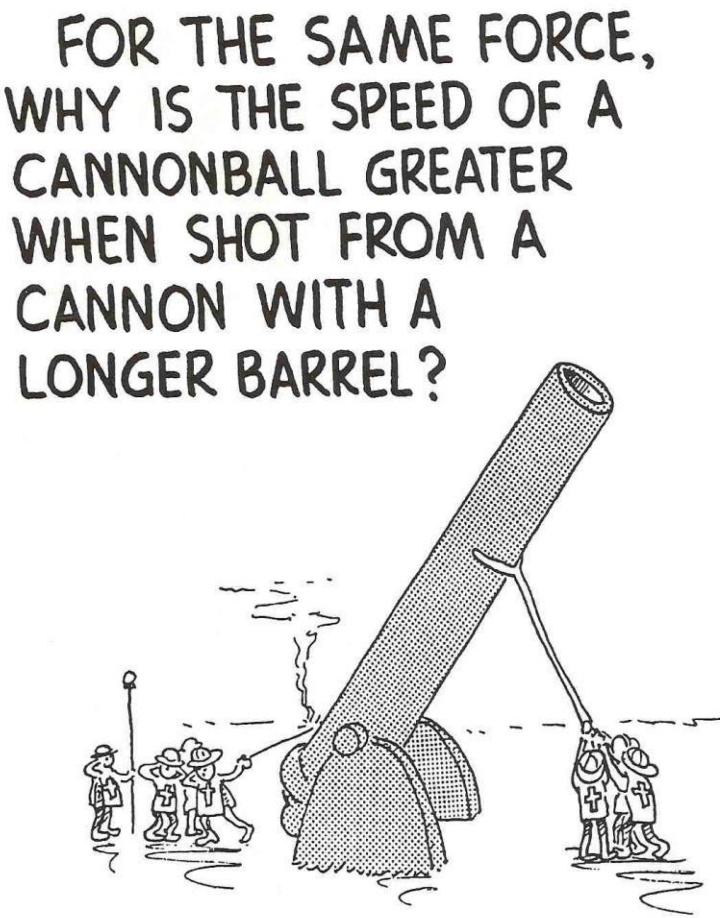
$$F = \frac{m\Delta v}{t}$$

Impulse – Momentum Relationships





Impulse – Momentum Relationships



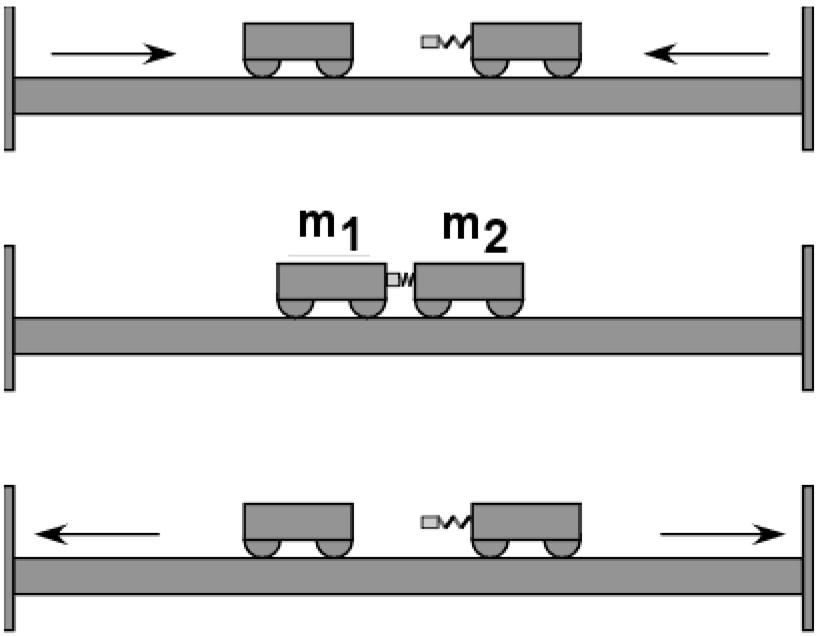
$$fT = m\Delta V$$

Constant

Since TIME is directly related to the VELOCITY when the force and mass are constant, the LONGER the cannonball is in the barrel the greater the velocity.

Also, you could say that the force acts over a larger displacement, thus there is

How about a collision?



$$F_1 = -F_2 \qquad t_1 = t_2$$

$$(Ft)_1 = -(Ft)_2$$

$$J_1 = -J_2$$

Consider 2 objects speeding toward each other. When they collide......

Due to Newton's 3rd Law the FORCE they exert on each other are EQUAL and OPPOSITE.

The TIMES of impact are also equal.

Therefore, the IMPULSES of the 2 objects colliding are also EQUAL

How about a collision?

If the Impulses are equal then the MOMENTUMS are also equal!

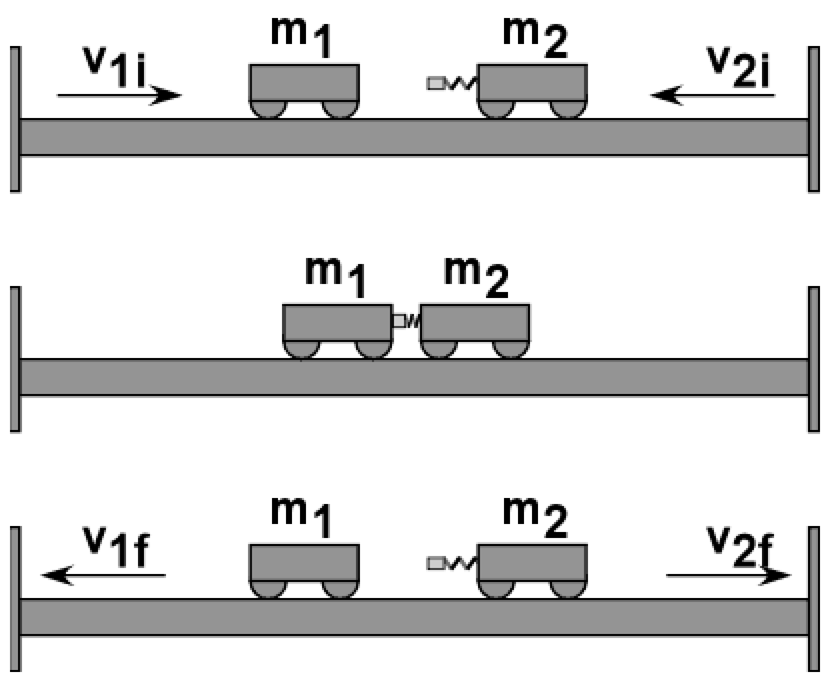
$$J_1 = -J_2$$

$$p_1 = -p_2$$

$$m_1 \Delta v_1 = -m_2 \Delta v_2$$

$$m_1 (v_1 - v_{o1}) = -m_2 (v_2 - v_{o2})$$

$$m_1 v_1 - m_1 v_{o1} = -m_2 v_2 + m_2 v_{o2}$$

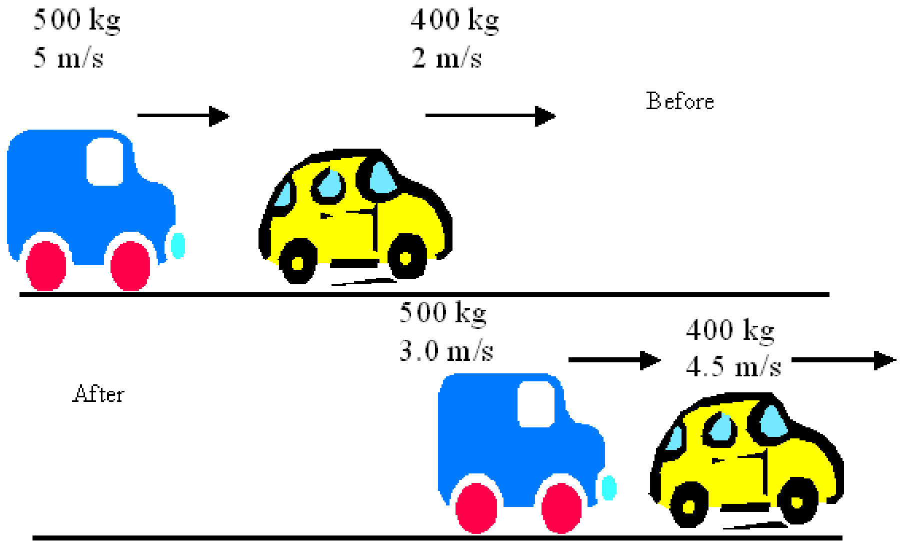


$$\sum p_{before} = \sum p_{after}$$

$$m_1 v_{o1} + m_2 v_{o2} = m_1 v_1 + m_2 v_2$$

Momentum is conserved!

The Law of Conservation of Momentum: ***"In the absence of an external force (gravity, friction), the total momentum before the collision is equal to the total momentum after the collision."***



$$p_{o(truck)} = mv_o = (500)(5) = 2500 kg*m/s$$

$$p_{o(car)} = (400)(2) = 800 kg*m/s$$

$$p_{o(total)} = 3300 kg*m/s$$

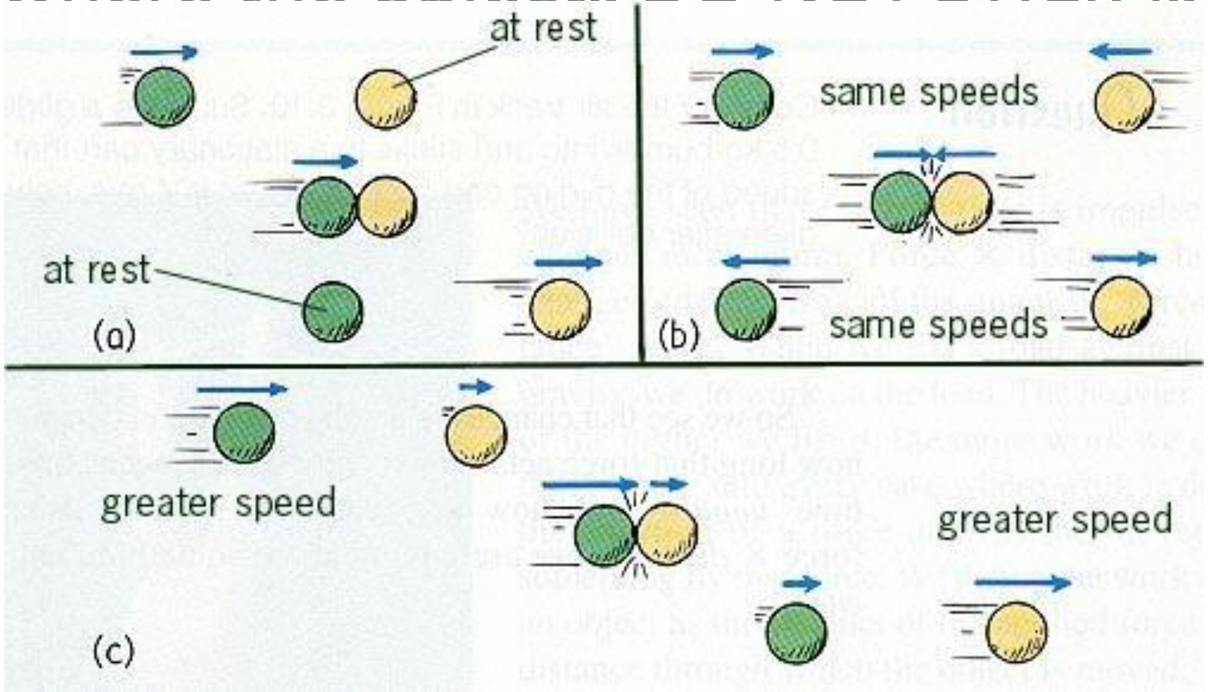
$$p_{truck} = 500*3 = 1500 kg*m/s$$

$$p_{car} = 400*4.5 = 1800 kg*m/s$$

$$p_{total} = 3300 kg*m/s$$

Types of Collisions

A situation where the objects DO NOT STICK is one type



Notice that in EACH case, you have TWO objects BEFORE and AFTER the collision.

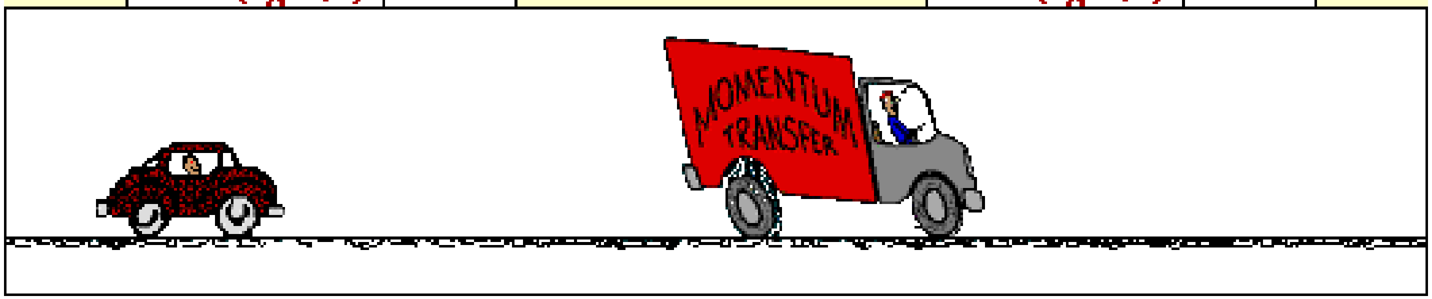
A “no stick” type collision

Car

mass (kg)	1000
vel. (m/s)	20.0
mom. (kg m/s)	20 000

Truck

mass (kg)	3000
vel. (m/s)	0.0
mom. (kg m/s)	0



$$\Sigma p_{before} = \Sigma p_{after}$$

$$m_1 v_{o1} + m_2 v_{o2} = m_1 v_1 + m_2 v_2$$

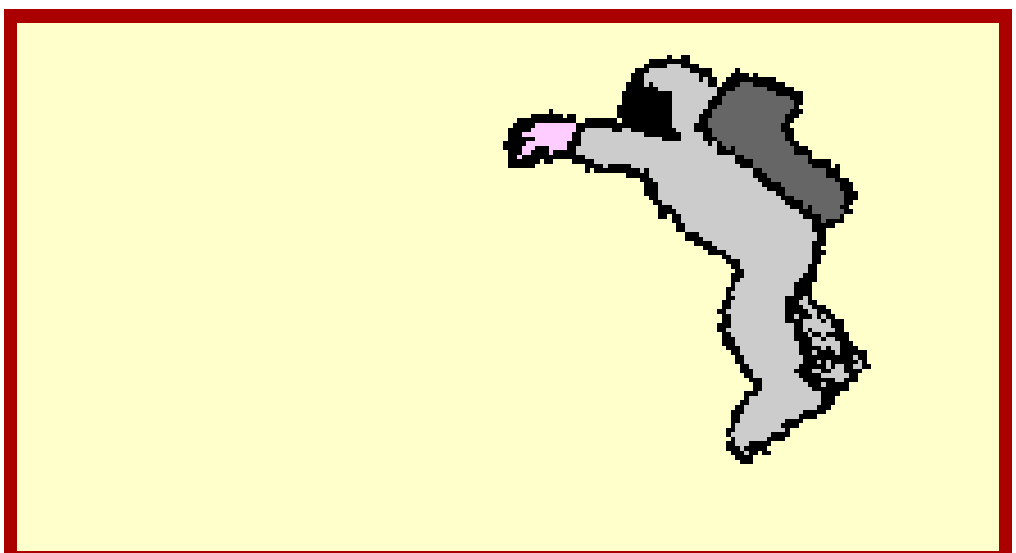
$$(1000)(20) + 0 = (1000)(v_1) + (3000)(10)$$

$$-10000 = 1000 v_1$$

$$v_1 = \textbf{-10 m/s}$$

Types of Collisions

Another type of collision is one where the objects “STICK” together. Notice you have TWO objects before the collision and ONE object after the collision.



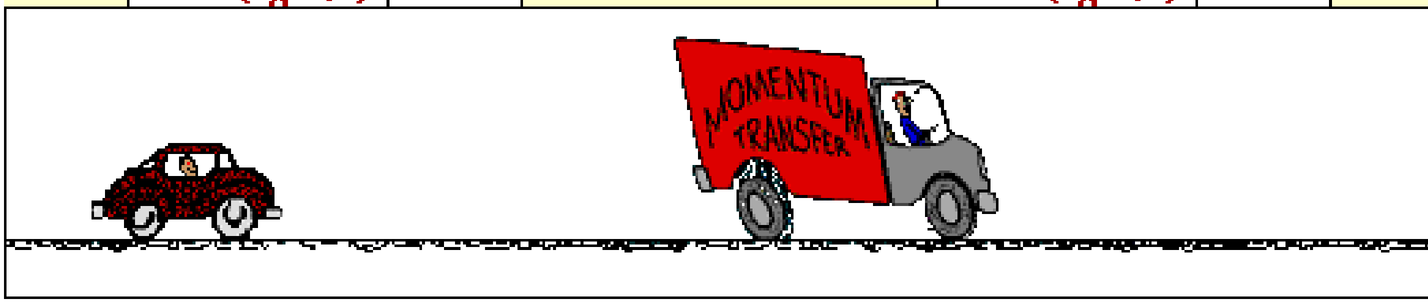
A "stick" type of collision

Car

mass (kg)	1000
vel. (m/s)	20.0
mom. (kg m/s)	20 000

Truck

mass (kg)	3000
vel. (m/s)	0.0
mom. (kg m/s)	0



$$\Sigma p_{before} = \Sigma p_{after}$$

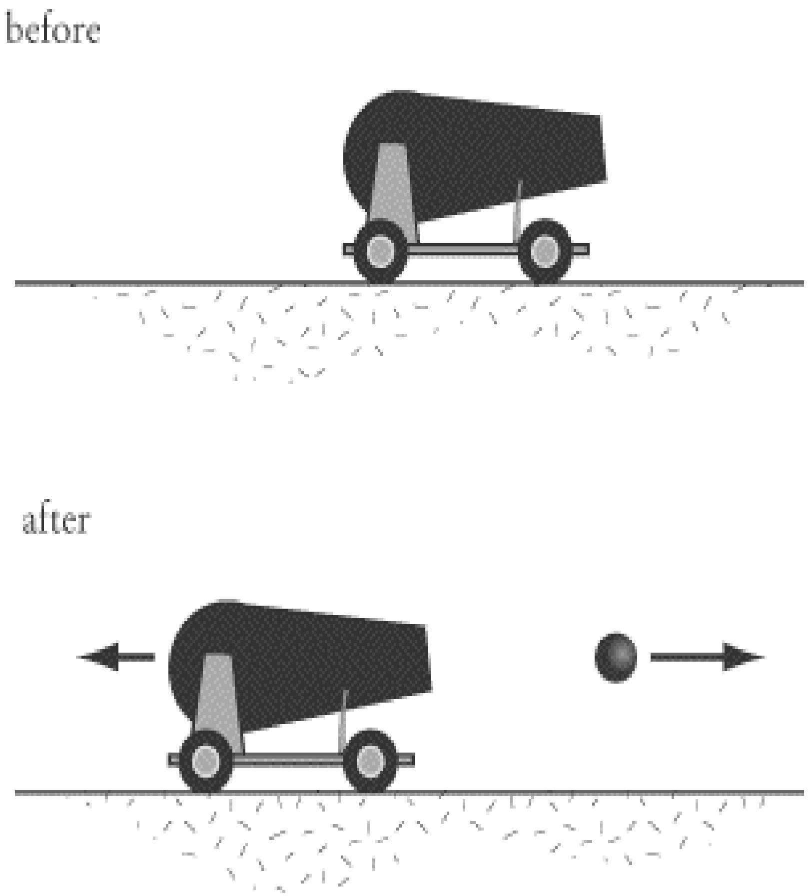
$$m_1 v_{o1} + m_2 v_{o2} = m_T v_T$$

$$(1000)(20) + 0 = (4000) v_T$$

$$20000 = 4000 v_T$$

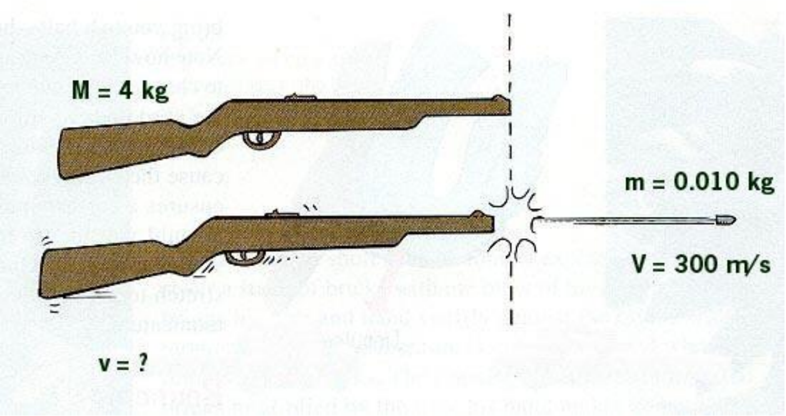
$$v_T = \textbf{5 m/s}$$

The “explosion” type



This type is often referred to as “backwards inelastic”. Notice you have ONE object (we treat this as a SYSTEM) before the explosion and TWO objects after the explosion.

Backwards Inelastic - Explosions



Suppose we have a 4-kg rifle loaded with a 0.010 kg bullet. When the rifle is fired the bullet exits the barrel with a velocity of 300 m/s. How fast does the gun RECOIL backwards?

Σp_{before} =	Σp_{after}
$m_T v_T =$	$m_1 v_1 + m_2 v_2$
$(4.010)(0) =$	$(0.010)(300) + (4)(v_2)$
$0 =$	$3 + 4v_2$
$v_2 =$	**-0.75 m/s**

Collision Summary

Sometimes objects stick together or blow apart. In this case, momentum is ALWAYS conserved.

$$\sum p_{before} = \sum p_{after}$$

$$m_1 v_{01} + m_2 v_{02} = m_1 v_1 + m_2 v_2$$ → When 2 objects collide and DON'

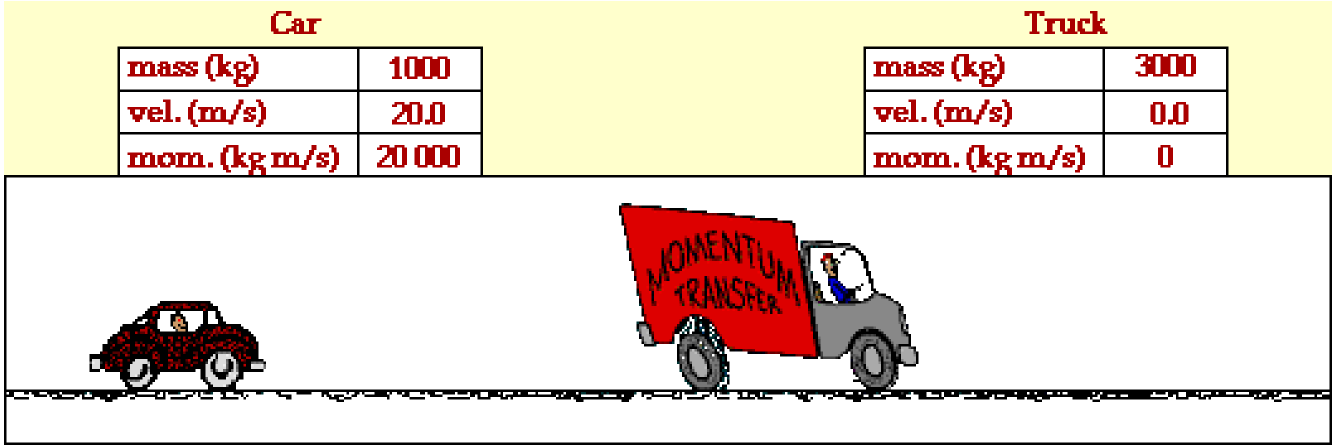
$$m_1 v_{01} + m_2 v_{02} = m_{total} v_{total}$$ → When 2 objects collide and stick to

$$m_{total} v_{o(total)} = m_1 v_1 + m_2 v_2$$ → When 1 object breaks into 2 objec

Elastic Collision = Kinetic Energy is Conserved

Inelastic Collision = Kinetic Energy is NOT Conserved

Elastic Collision



$$KE_{car}(Before) = \frac{1}{2}mv^2 = 0.5(1000)(20)^2 = 200,000J$$

$$KE_{truck}(After) = 0.5(3000)(10)^2 = 150,000J$$

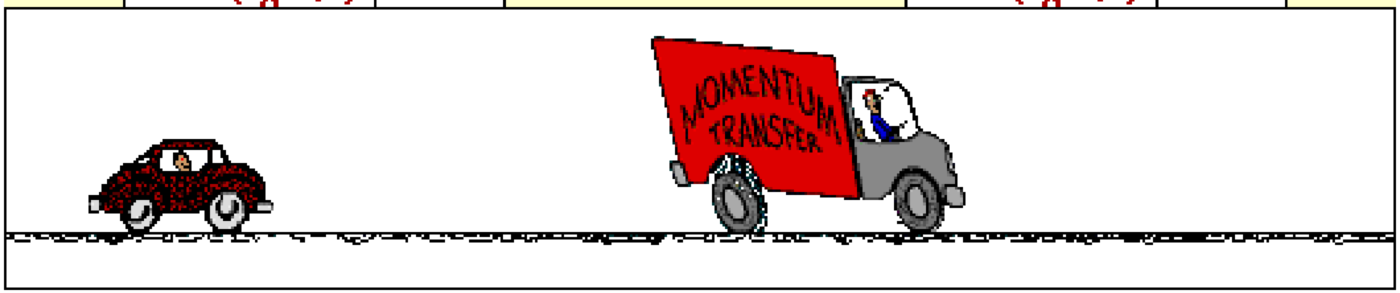
$$KE_{car}(After) = 0.5(1000)(-10)^2 = 50,000J$$

Since KINETIC ENERGY is conserved during the collision we call this an **ELASTIC COLLISION**.

Inelastic Collision

Car	
mass (kg)	1000
vel. (m/s)	20.0
mom. (kg m/s)	20 000

Truck	
mass (kg)	3000
vel. (m/s)	0.0
mom. (kg m/s)	0



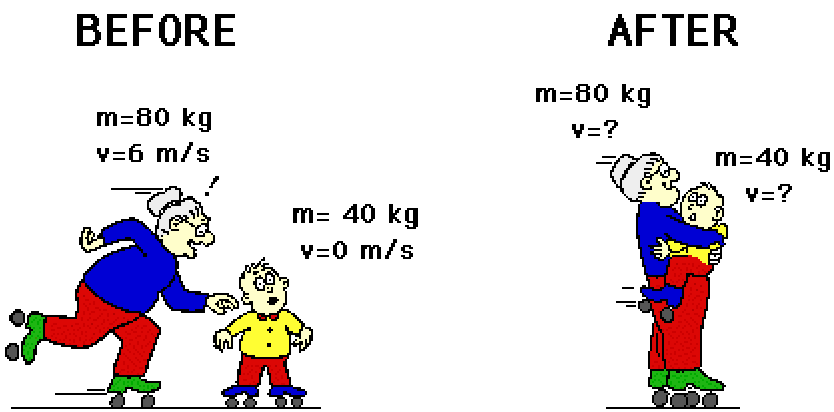
$$KE_{car}(Before) = \tfrac{1}{2}mv^2 = 0.5(1000)(20)^2 = 200{,}000J$$

$$KE_{truck/car}(After) = 0.5(4000)(5)^2 = 50{,}000J$$

Since KINETIC ENERGY was NOT conserved during the collision we call this an **INELASTIC COLLISION**.

Example

Granny (m=80 kg) whizzes around the rink with a velocity of 6 m/s. She suddenly collides with Ambrose (m=40 kg) who is at rest directly in her path. Rather than knock him over, she picks him up and continues in motion without "braking." Determine the velocity of Granny and Ambrose.



How many objects do I have before the collision?

2

$$\sum p_b = \sum p_a$$

$$m_1 v_{o1} + m_2 v_{o2} = m_T v_T$$

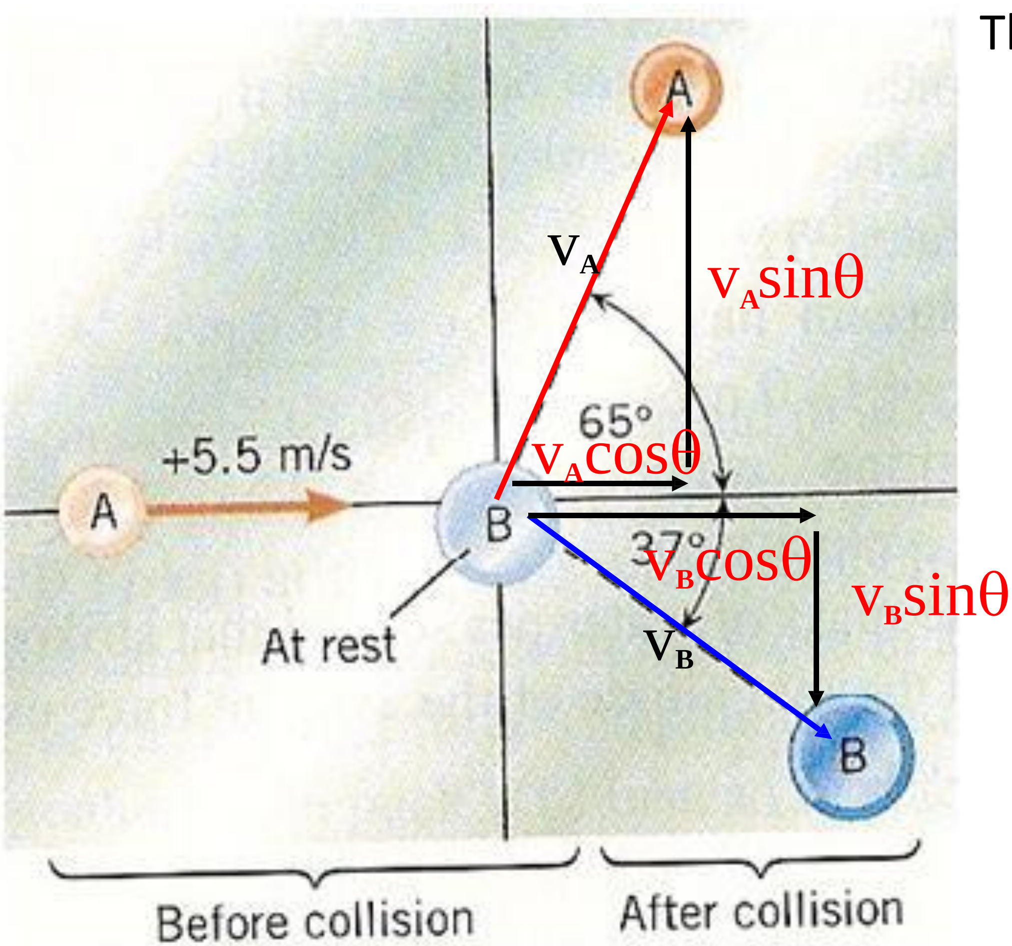
How many objects do I have after the collision?

1

$$(80)(6) + (40)(0) = 120 v_T$$

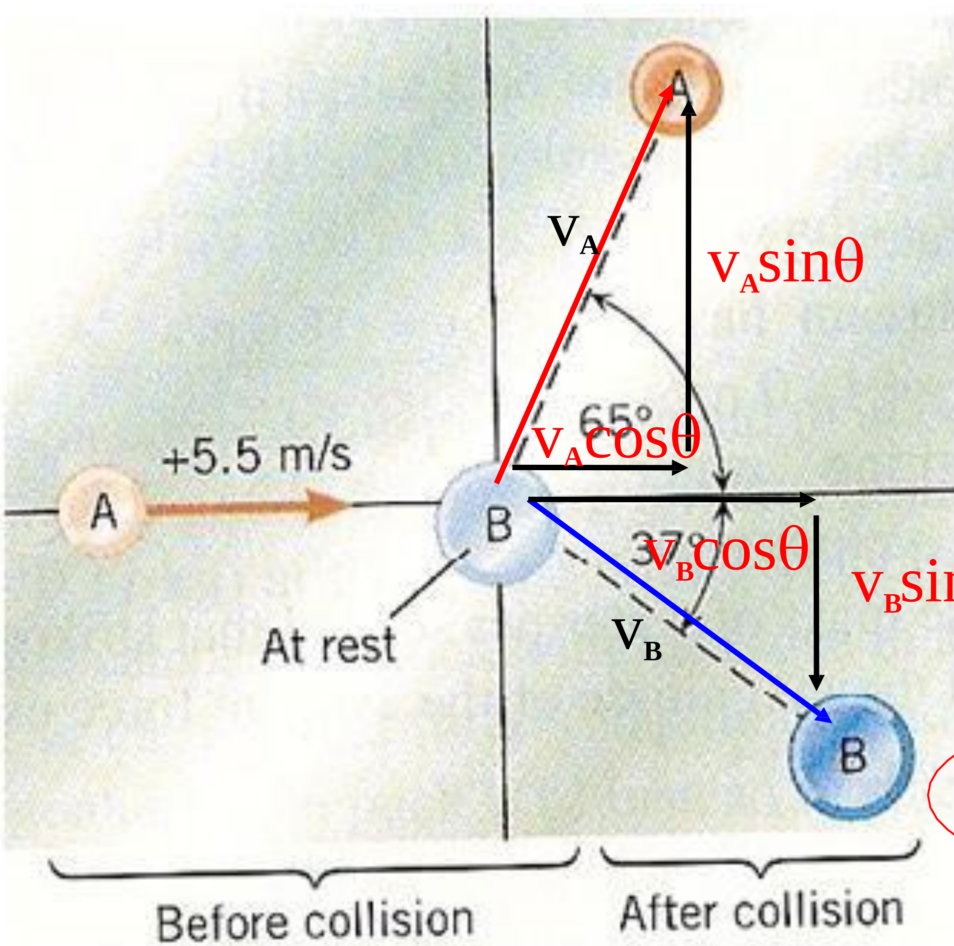
$$v_T = 4\ \mathrm{m/s}$$

Collisions in 2 Dimensions



The figure to the left shows a collision between two pucks on an air hockey table. Puck A has a mass of 0.025-kg and is moving along the x-axis with a velocity of +5.5 m/s. It makes a collision with puck B, which has a mass of 0.050-kg and is initially at rest. The collision is NOT head on. After the collision, the two pucks fly apart with angles shown in the drawing. Calculate the speeds of the pucks after the collision.

Collisions in 2 dimensions



$$\sum p_{ox} = \sum p_x$$

$$m_A v_{oxA} + m_B v_{oxB} = m_A v_{xA} + m_B v_{xB}$$

$$(0.025)(5.5) + 0 = (.025)(v_A \cos 65) + (.050)(v_B \cos 37)$$

$$0.1375 = 0.0106 v_A + 0.040 v_B$$

$$\sum p_{oy} = \sum p_y$$

$$0 = m_A v_{yA} + m_B v_{yB}$$

$$0 = (0.025)(v_A \sin 65) + (0.050)(-v_B \sin 37)$$

$$0.0300 v_B = 0.0227 v_A$$

$$v_B = 0.757 v_A$$

Collisions in 2 dimensions

$$0.1375 = 0.0106v_A + 0.040v_B$$

$$v_B = 0.757v_A$$

$$v_B = 0.757(2.84) = 2.15m/s$$

$$0.1375 = 0.0106v_A + (0.050)(0.757v_A)$$

$$0.1375 = 0.0106v_A + 0.03785v_A$$

$$0.1375 = 0.04845v_A$$

$$v_A = 2.84m/s$$

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UNIT-IV

WORK ENERGY METHOD

Work energy method: Law of conservation of energy, application of work energy method to particle motion and connected system, work energy applied to connected systems, work energy applied to fixed axis rotation.

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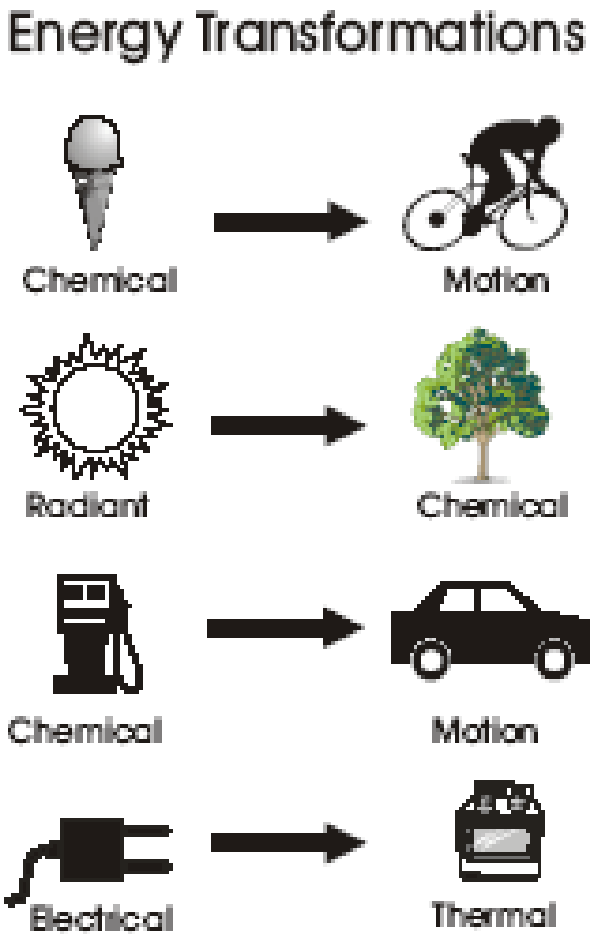
Law of Conservation of Energy



- What you put in is what you get out

- Total energy is conserved

Practical Applications

- Gasoline converts to energy which moves the car
- A battery converts stored chemical energy to electrical energy
- Dams convert the kinetic energy of falling water into electrical energy

Can You Think of Other Examples?

Conservation of Mechanical Energy

$$\frac{1}{2}mv^2 + mgh = E$$

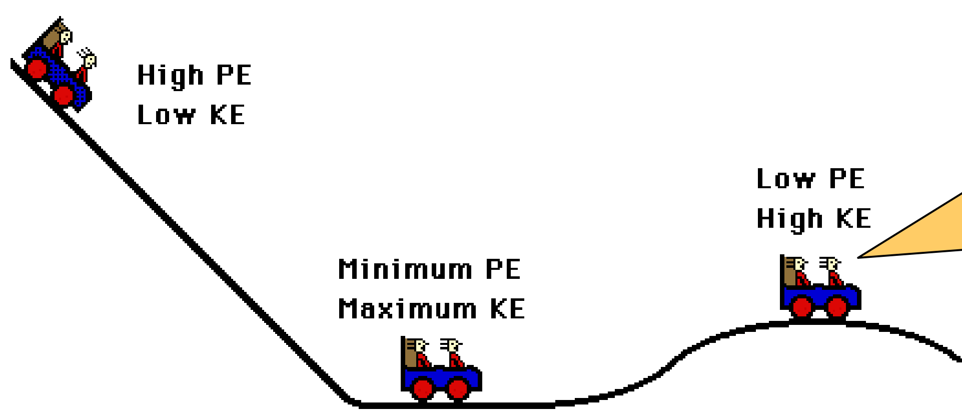
Kinetic Energy — Potential Energy — Total Energy

m = mass

v = velocity

g = gravitational acceleration

h = height

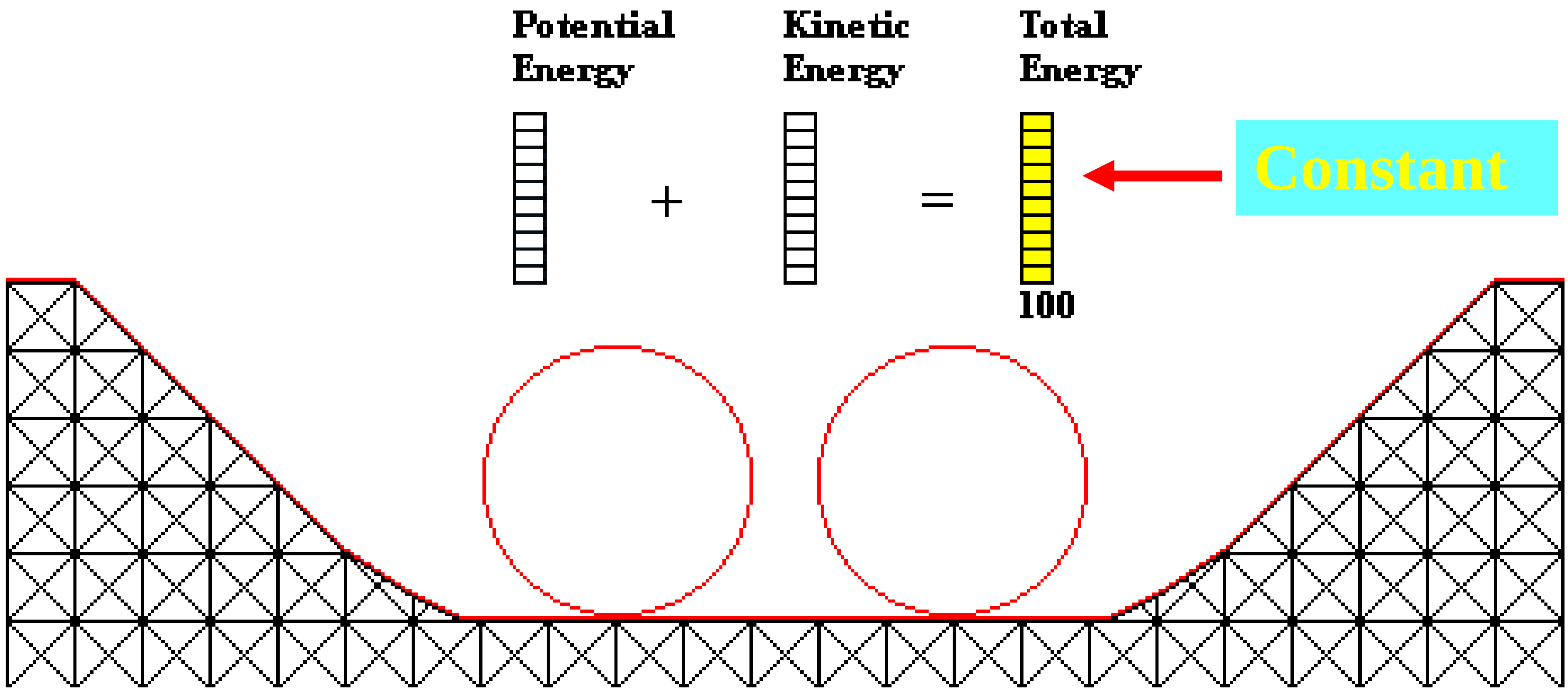


ILYA, did you know that even though it was a bumpy ride, our energy remained constant!

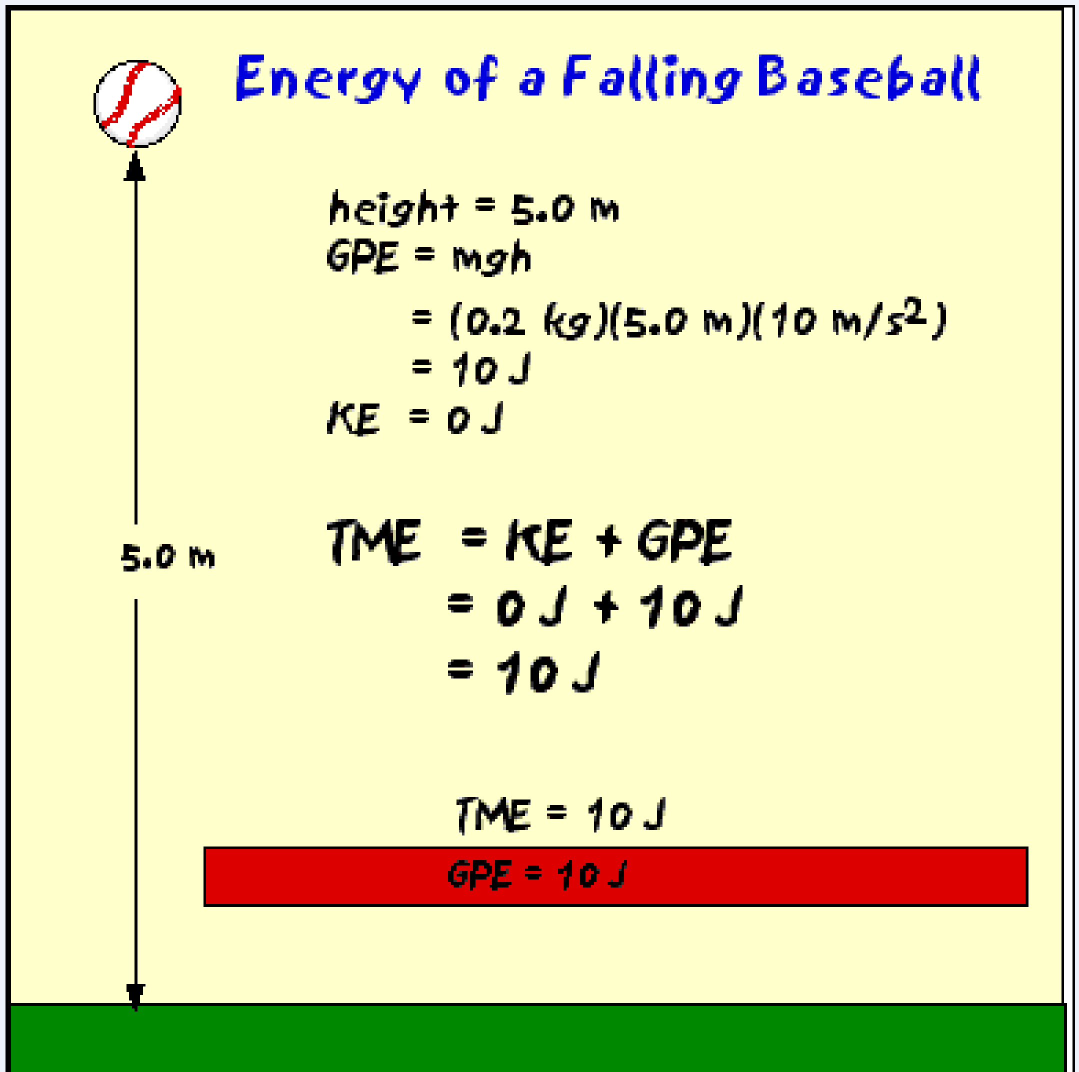
As a coaster car loses height, it gains speed; PE is transformed into KE. As a coaster car gains height it loses speed; KE is transformed into PE. The sum of the KE and PE is a constant.

Example of Conservation of Mechanical Energy

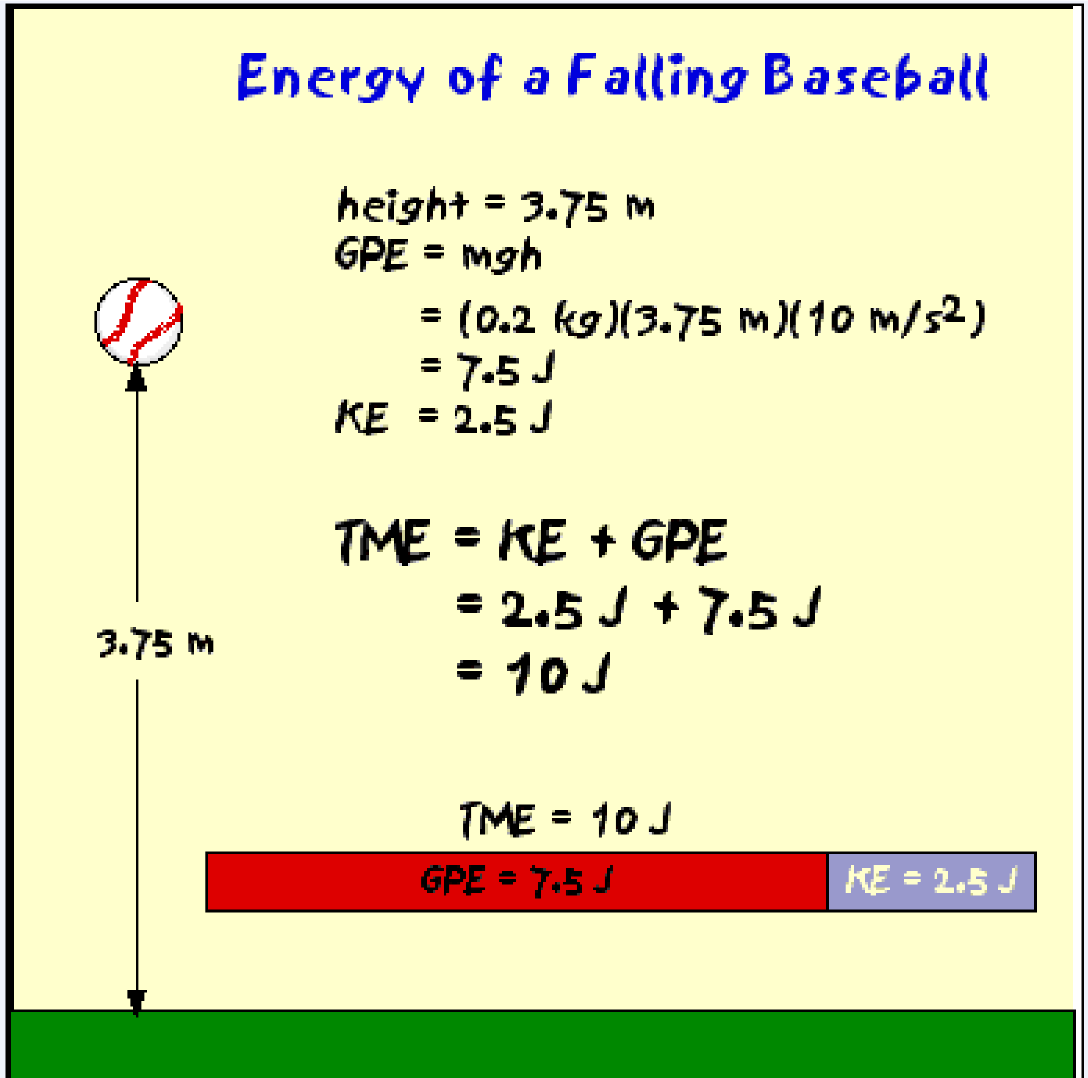
$$\frac{1}{2} m v^2 + m g h = E$$



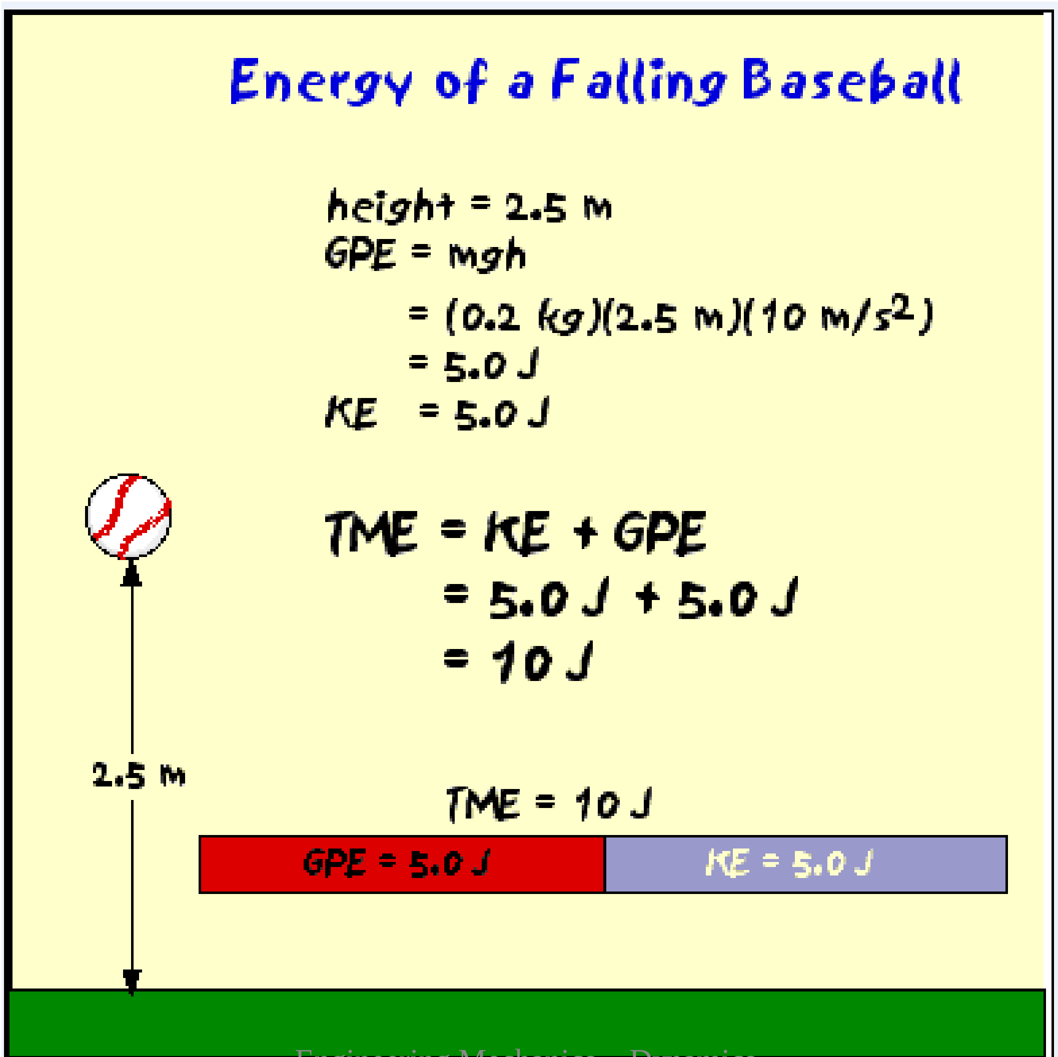
An Example


Energy of a Falling Baseball
height = 5.0 m
GPE = mgh
= (0.2 kg)(5.0 m)(10 m/s2)
= 10 J
KE = 0 J
5.0 m
TME = KE + GPE
= 0 J + 10 J
= 10 J
TME = 10 J
GPE = 10 J

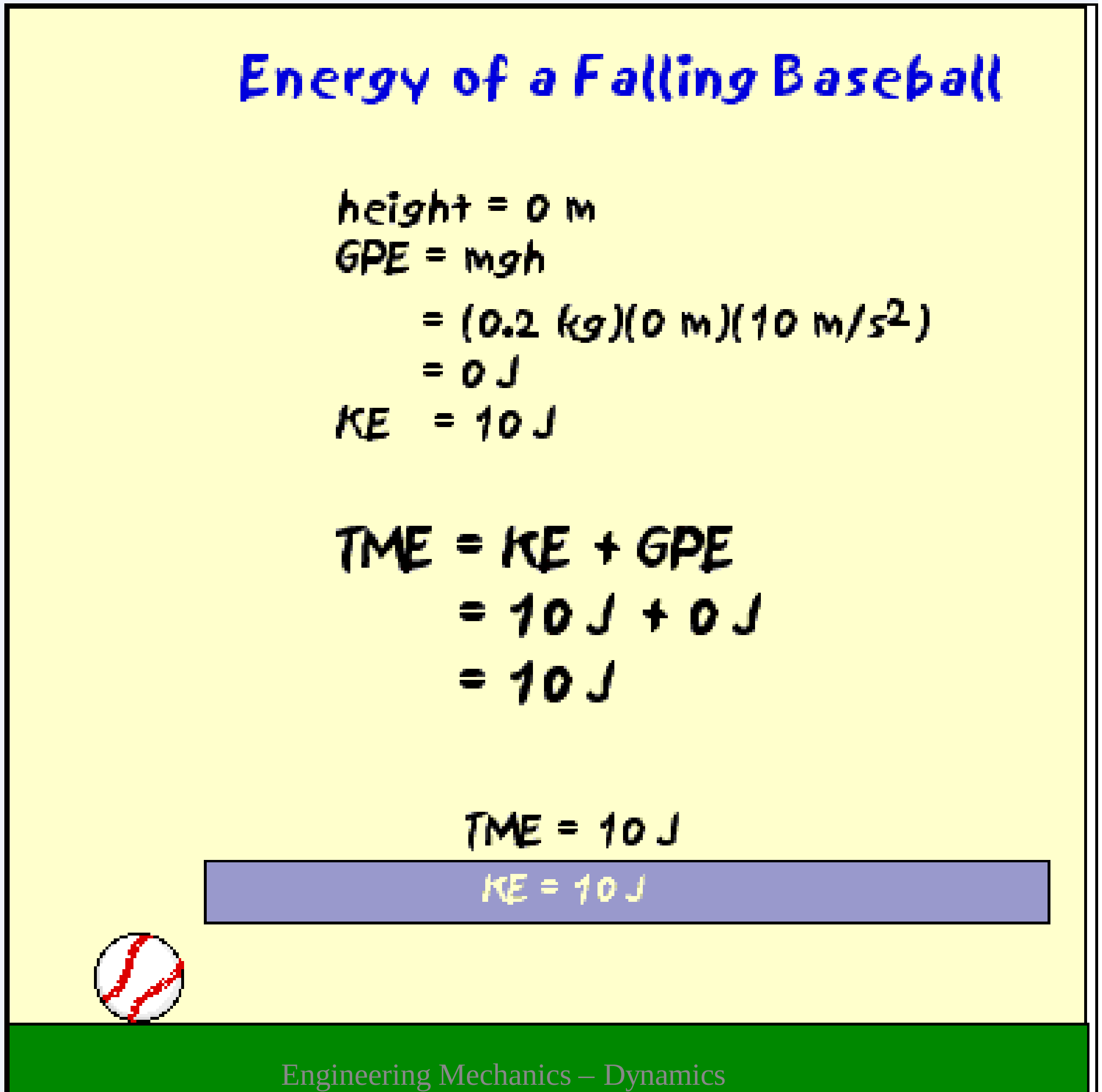
Another Example



Yet Another Example



Last Example



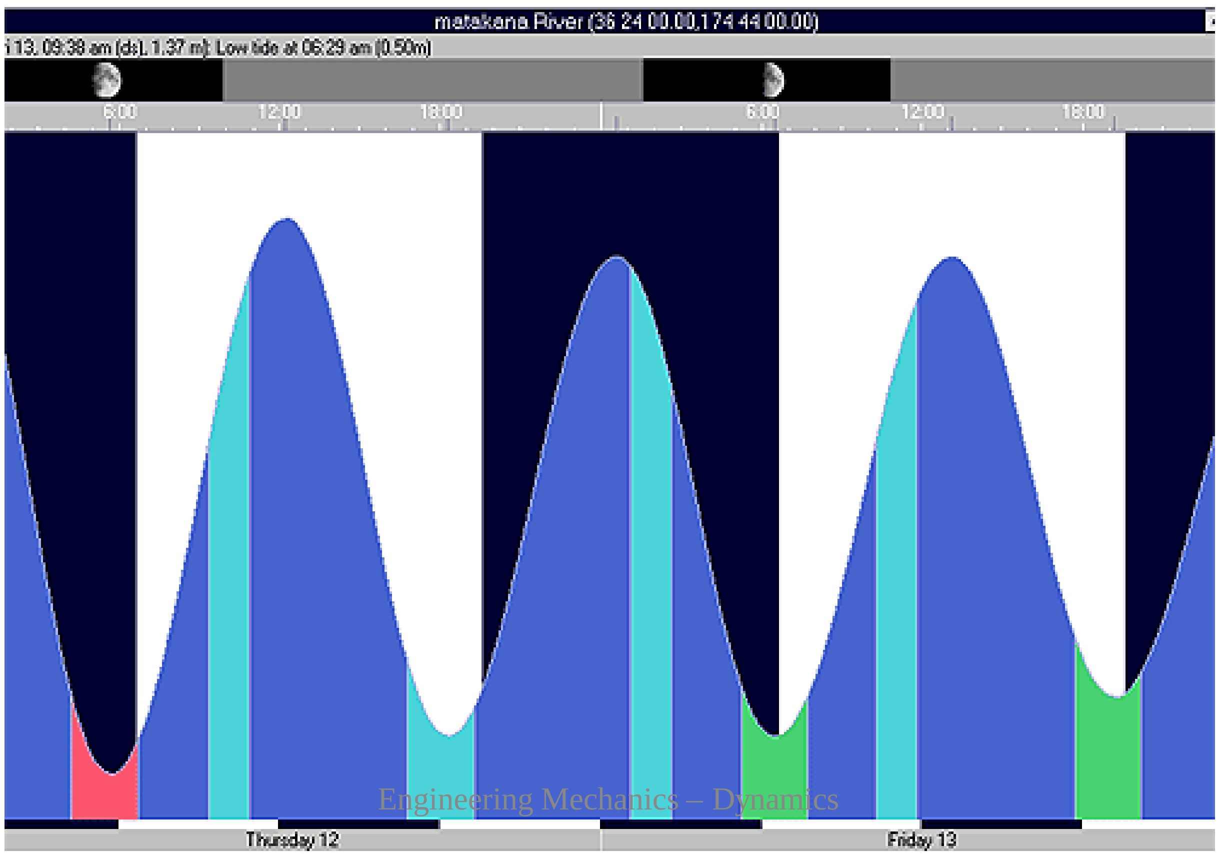
UNIT-V

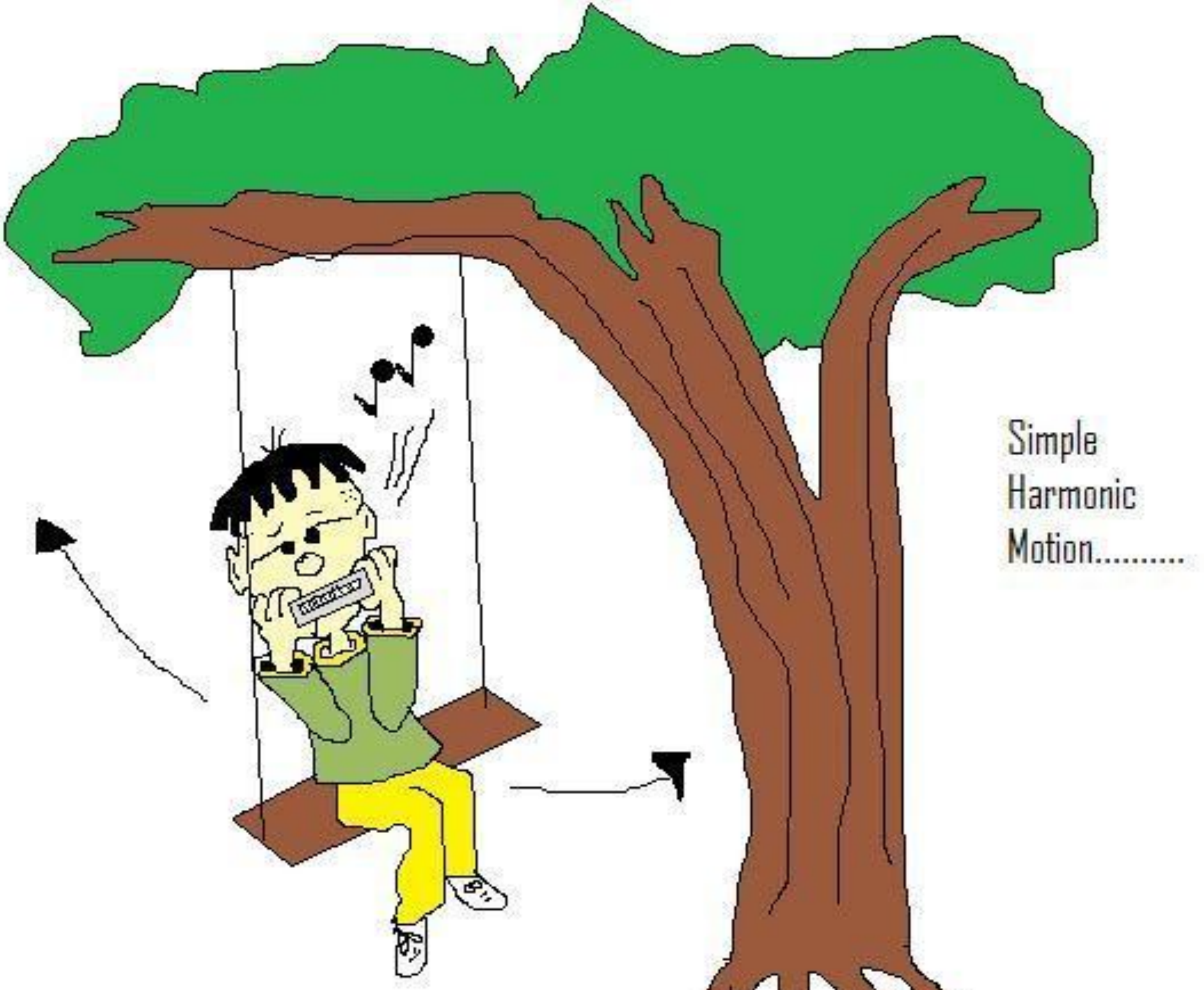
MECHANICAL VIBRATIONS

Definitions and concepts, simple harmonic motion, free vibrations, simple and compound pendulum, torsion pendulum, free vibrations without damping, general cases.

Simple Harmonic Motion

- Harmonic Motion is any motion that repeats itself.
- Examples of Harmonic Motion.

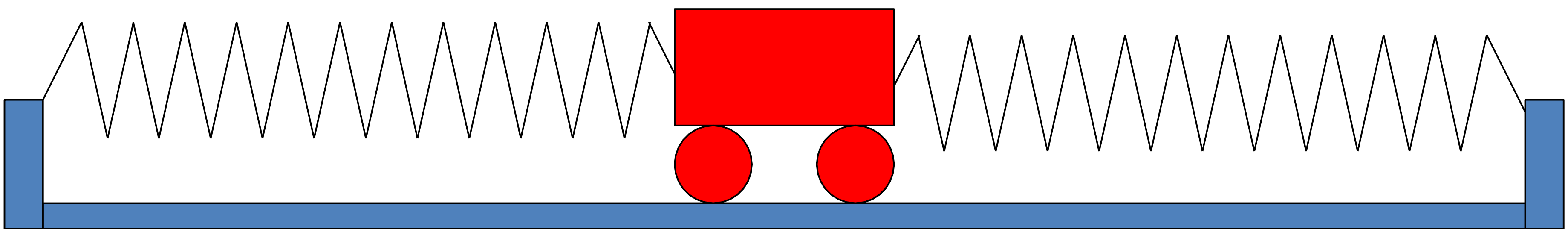



Simple
Harmonic
Motion..........

Period	Time for one oscillation
Frequency	Number of oscillations in one second
Displacement	Distance from equilibrium
Amplitude	Maximum displacement

- Simple harmonic motion is a special type of harmonic motion.

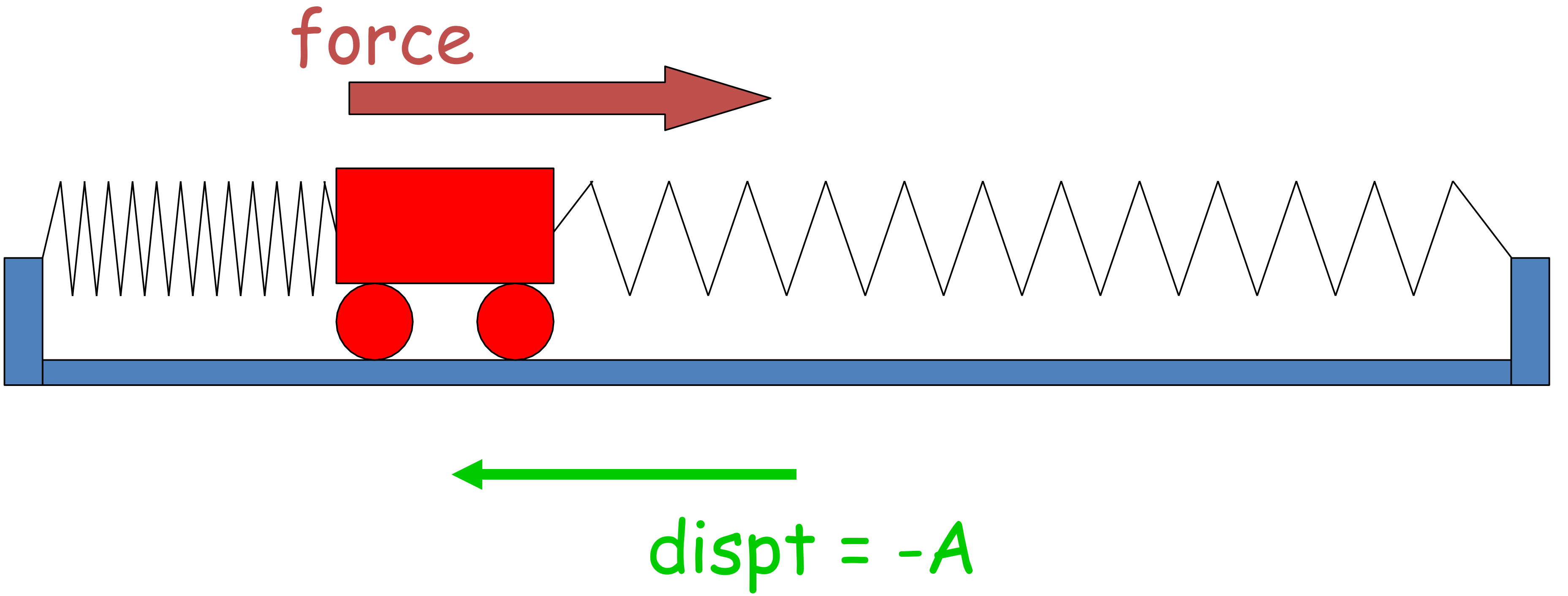
- Consider a mass on a spring.

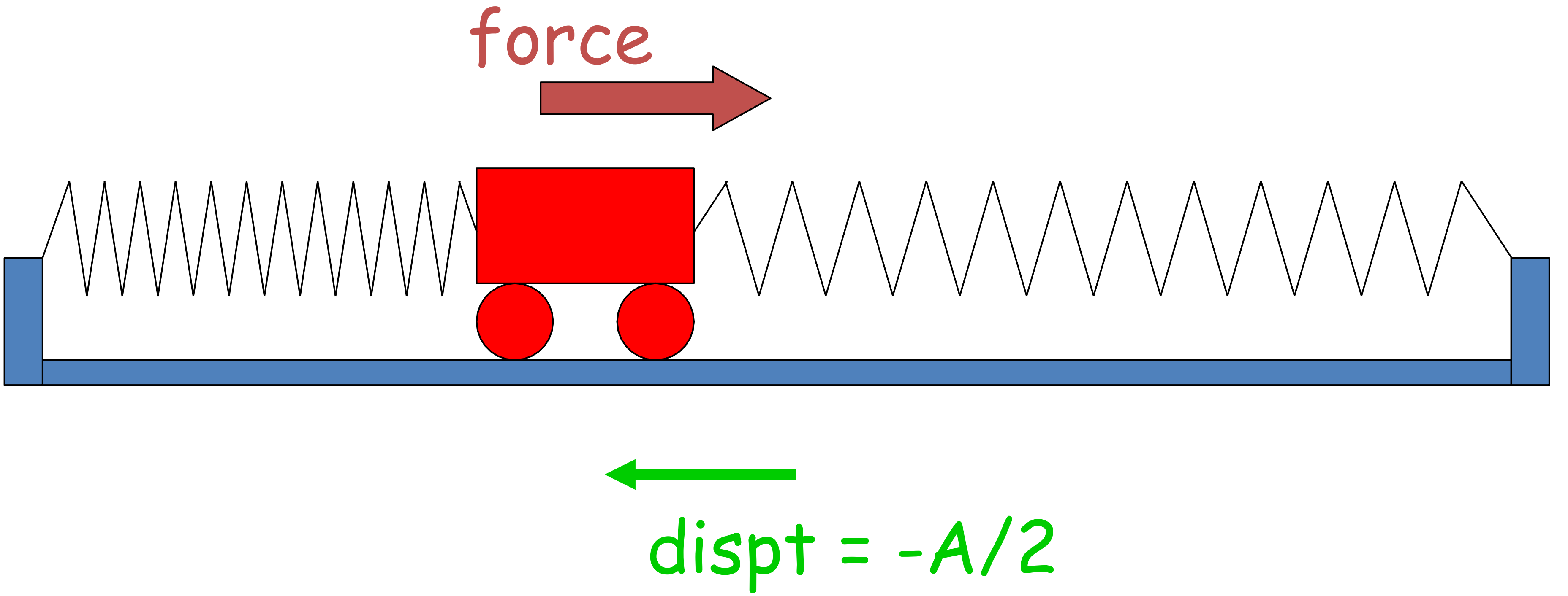


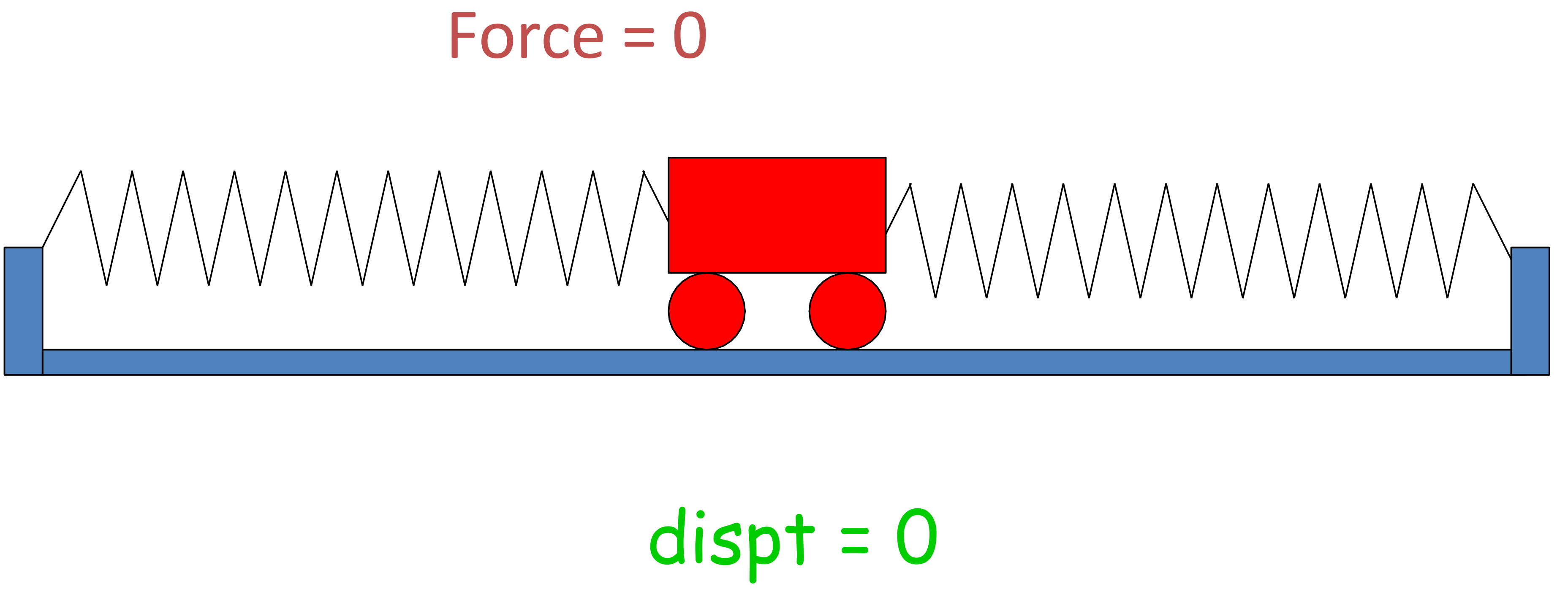
- The cart is in equilibrium, because the total force is zero.
- The acceleration is also zero.

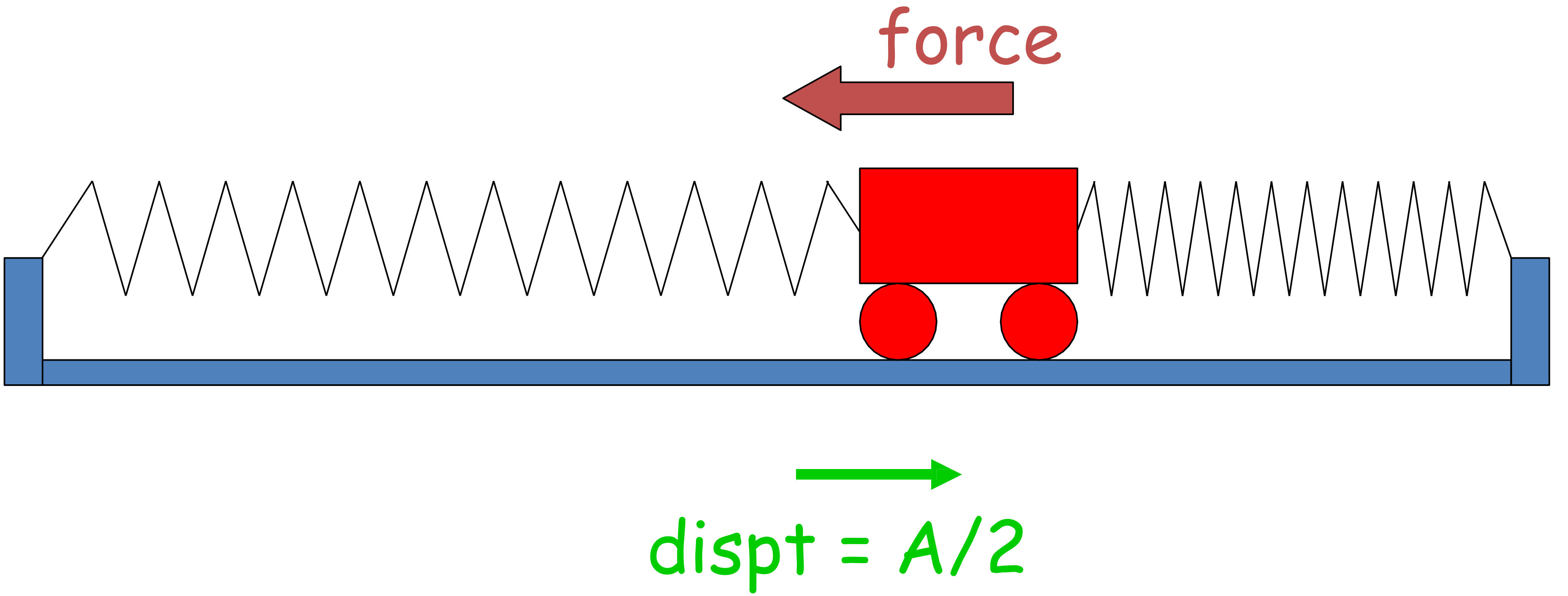
(this doesn't mean it's stationary)

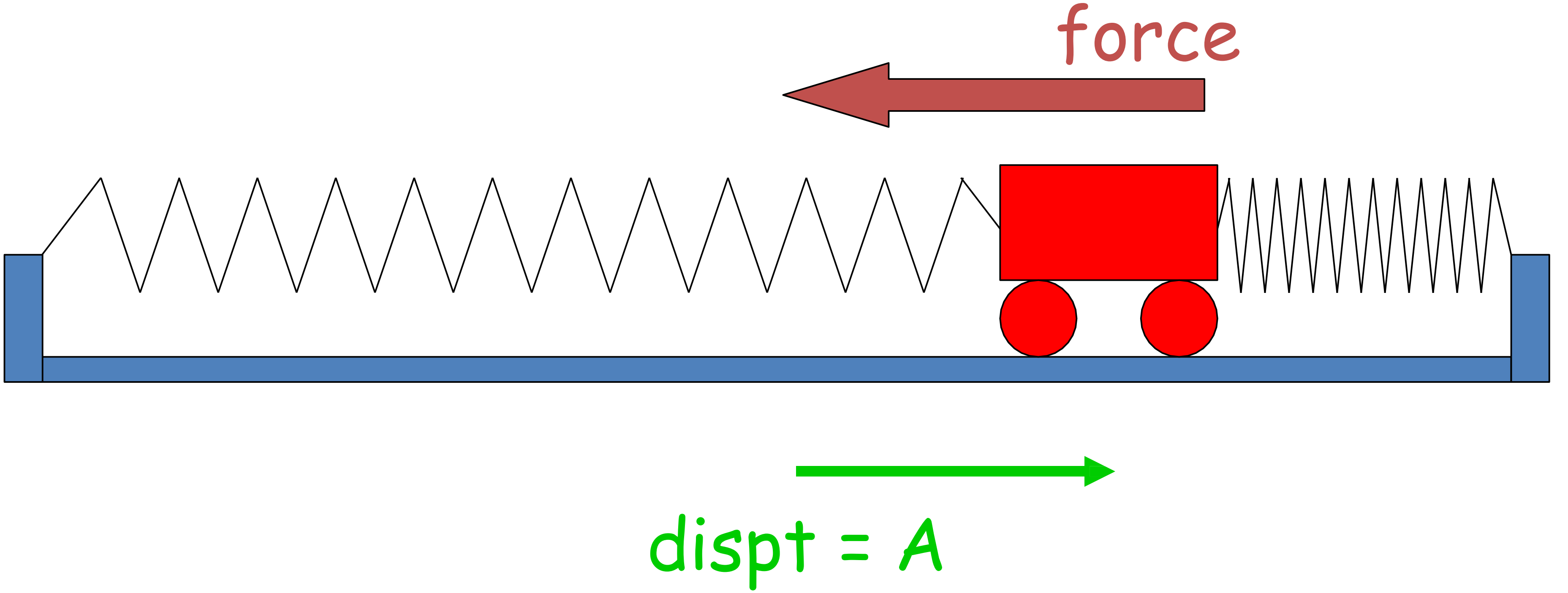
Lets look at the forces

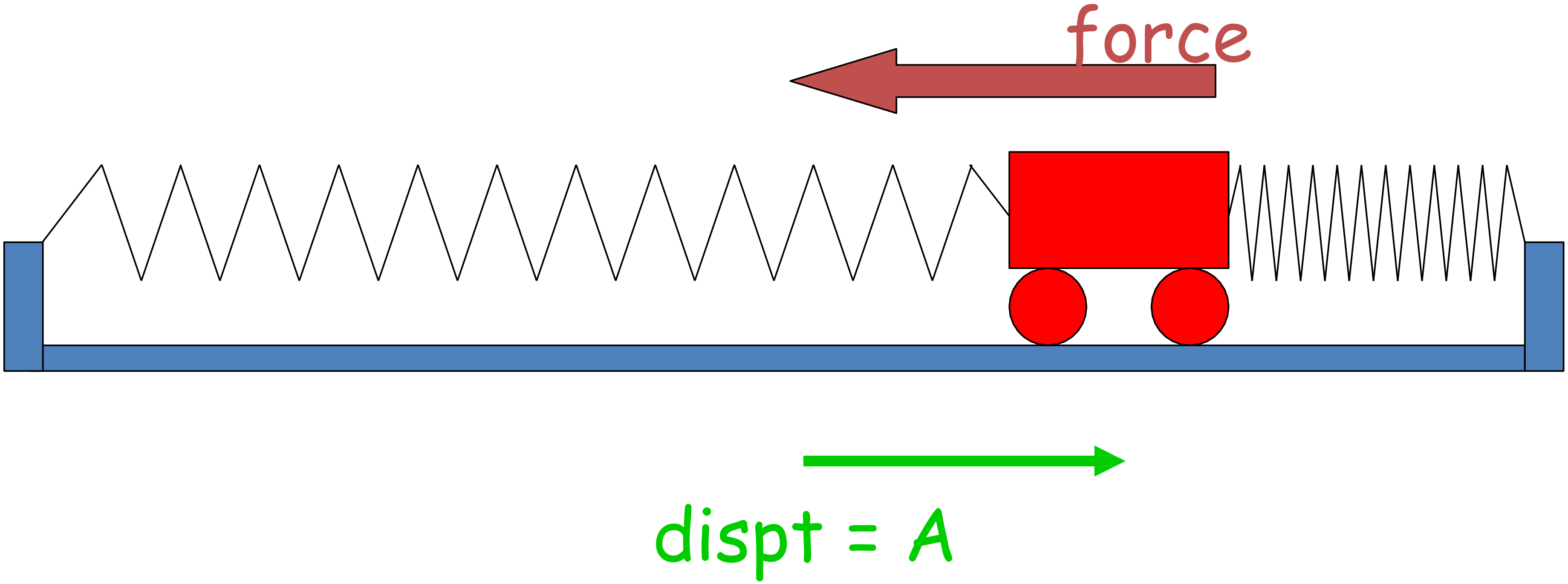



force
dispt = -A/2


Force = 0
dispt = 0


force
dispt = A/2

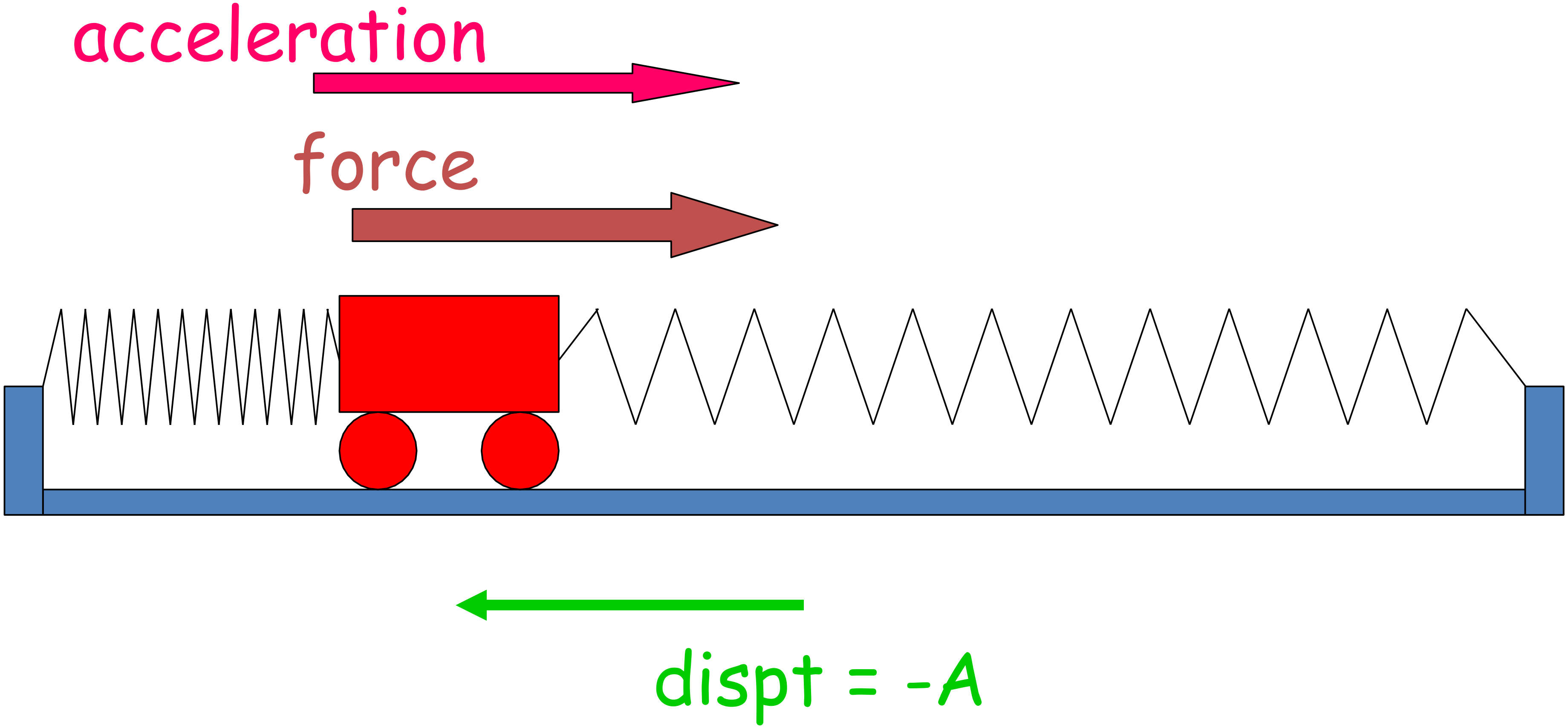

force
dispt = A

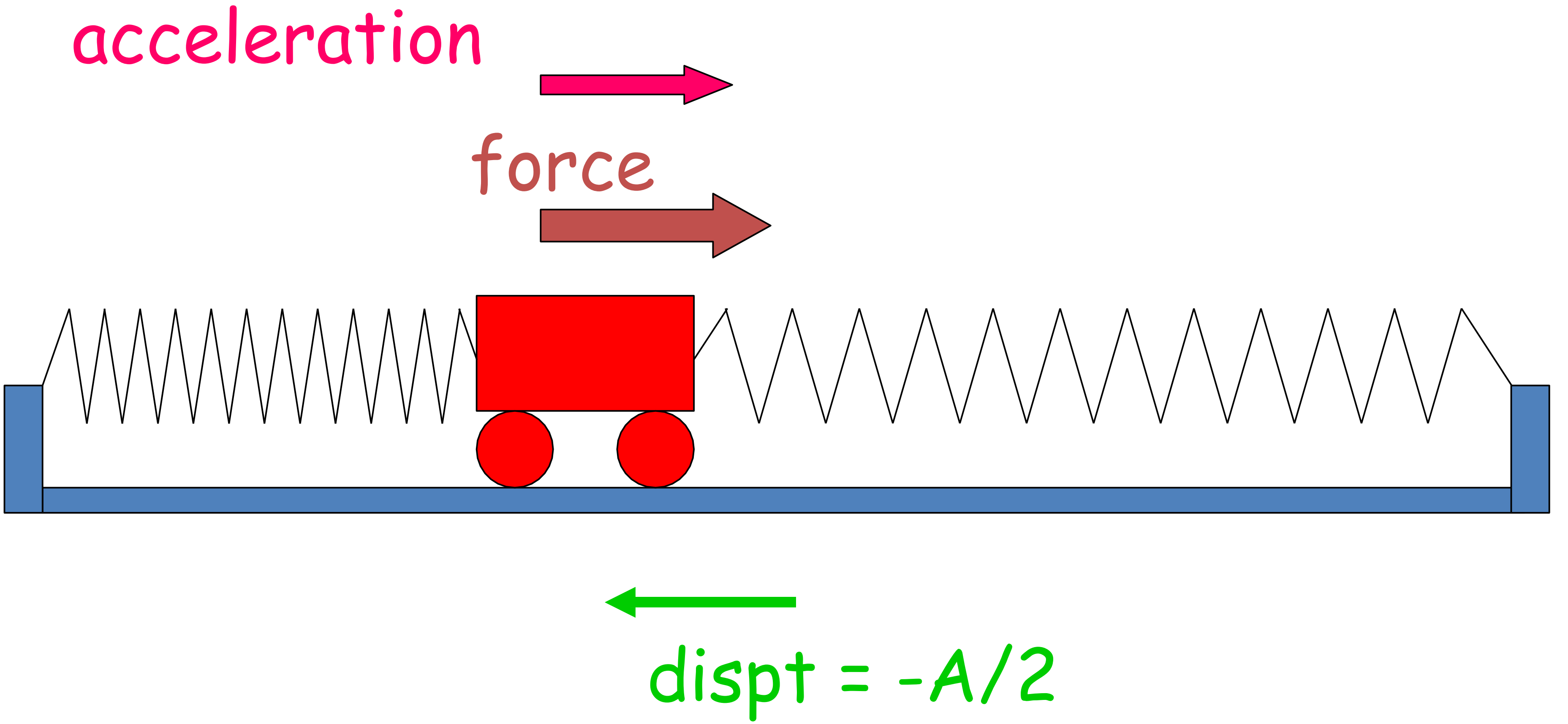


- Notice that as the displacement ***increases***, the restoring force ***increases***.

- Notice that the restoring force is always in the ***opposite direction*** to the displacement

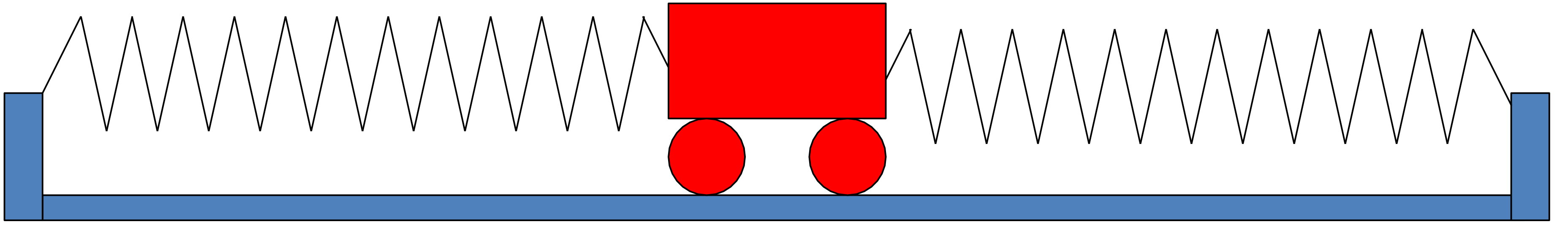
Now we'll look at the acceleration



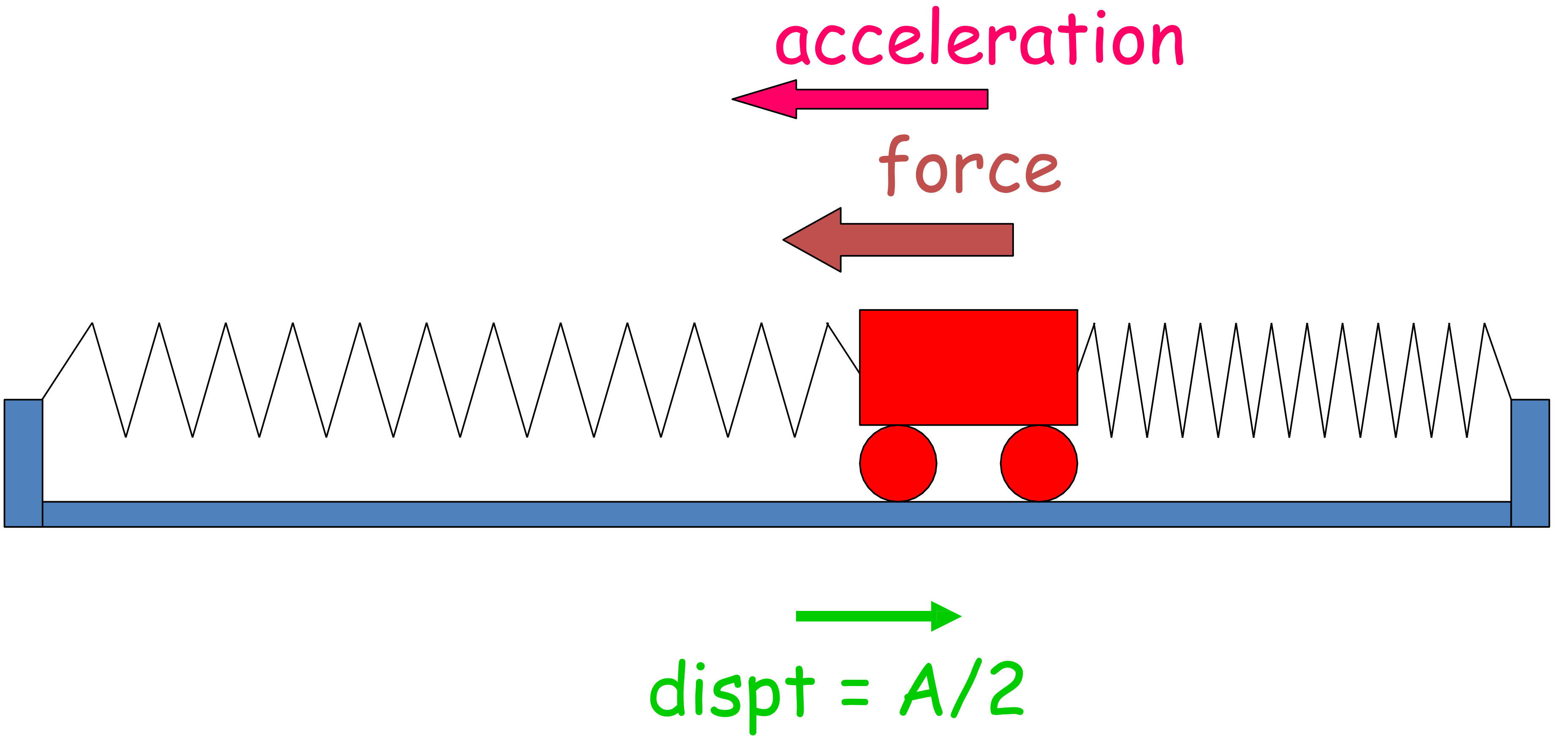

acceleration
force
dispt = -A/2

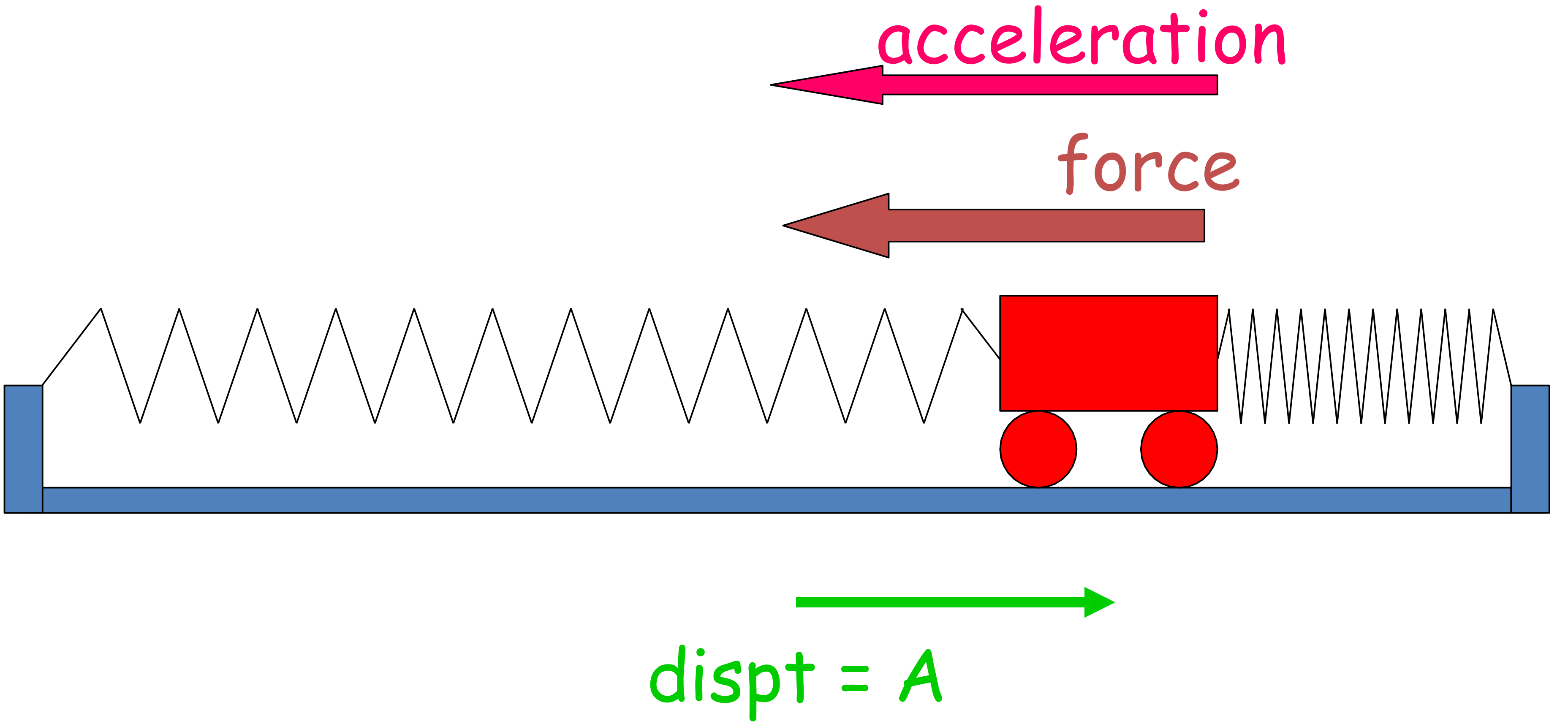
Acceleration =0

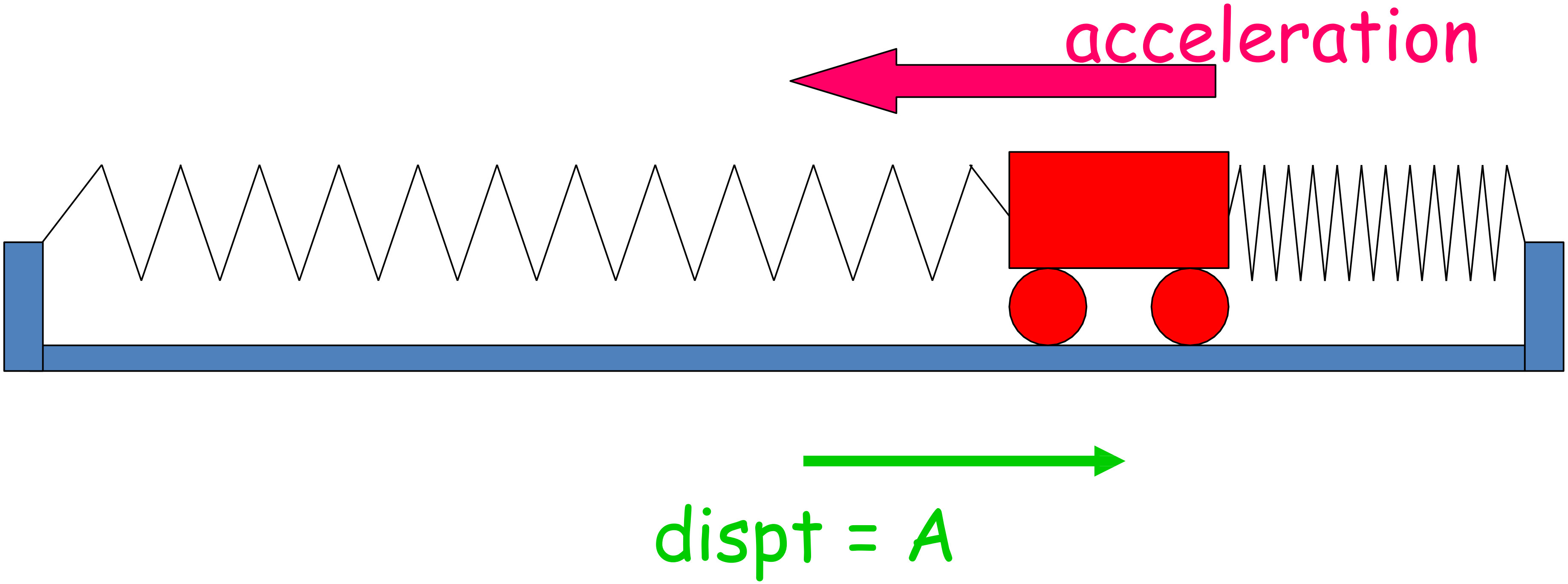
Force = 0



disp = 0


acceleration
force
dispt = A/2


acceleration
force
dispt = A



- Notice that as the displacement ***increases***, the acceleration ***increases***.

- Notice that the acceleration is always in the ***opposite direction*** to the displacement

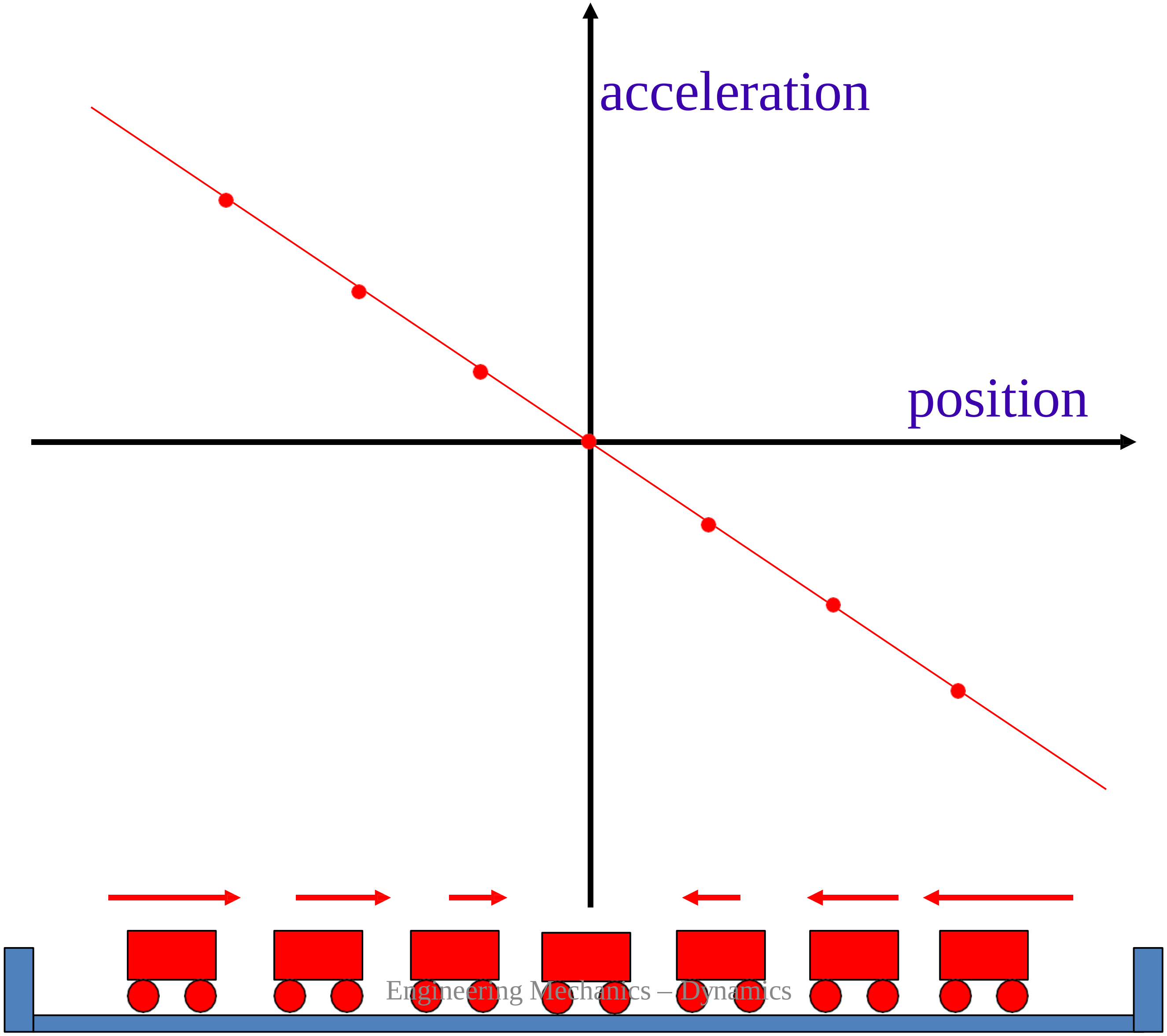
- The relation between acceleration and displacement is …..

- Acceleration is proportional to displacement
- Acceleration is in opposite direction to displacement.

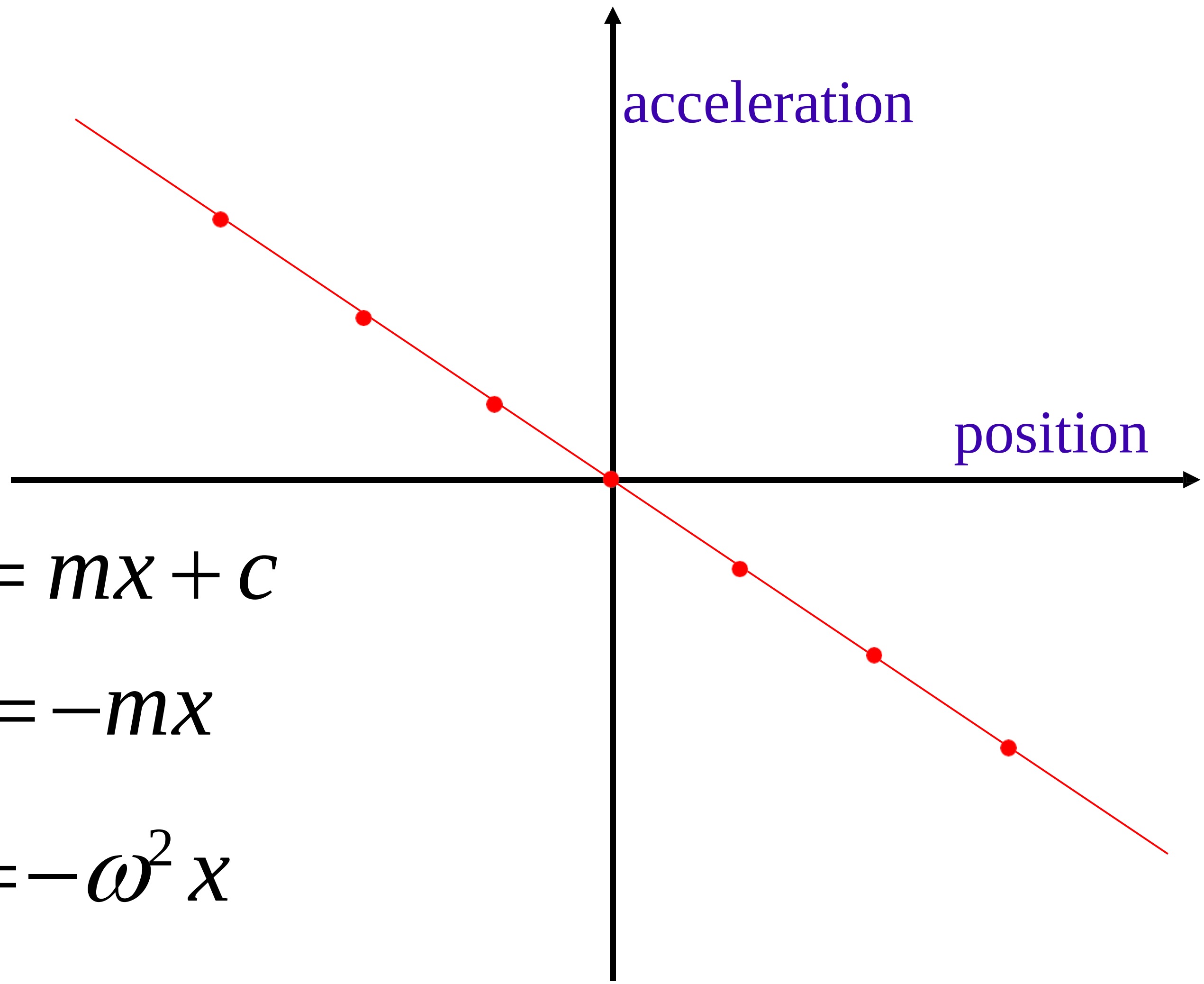
$$a = -\ \text{constant} \times \text{y}$$

$$a = -\ \omega^2 \times \text{y} \qquad \omega = \frac{2\pi}{T}$$

Acceleration/position graph



Acceleration/position graph

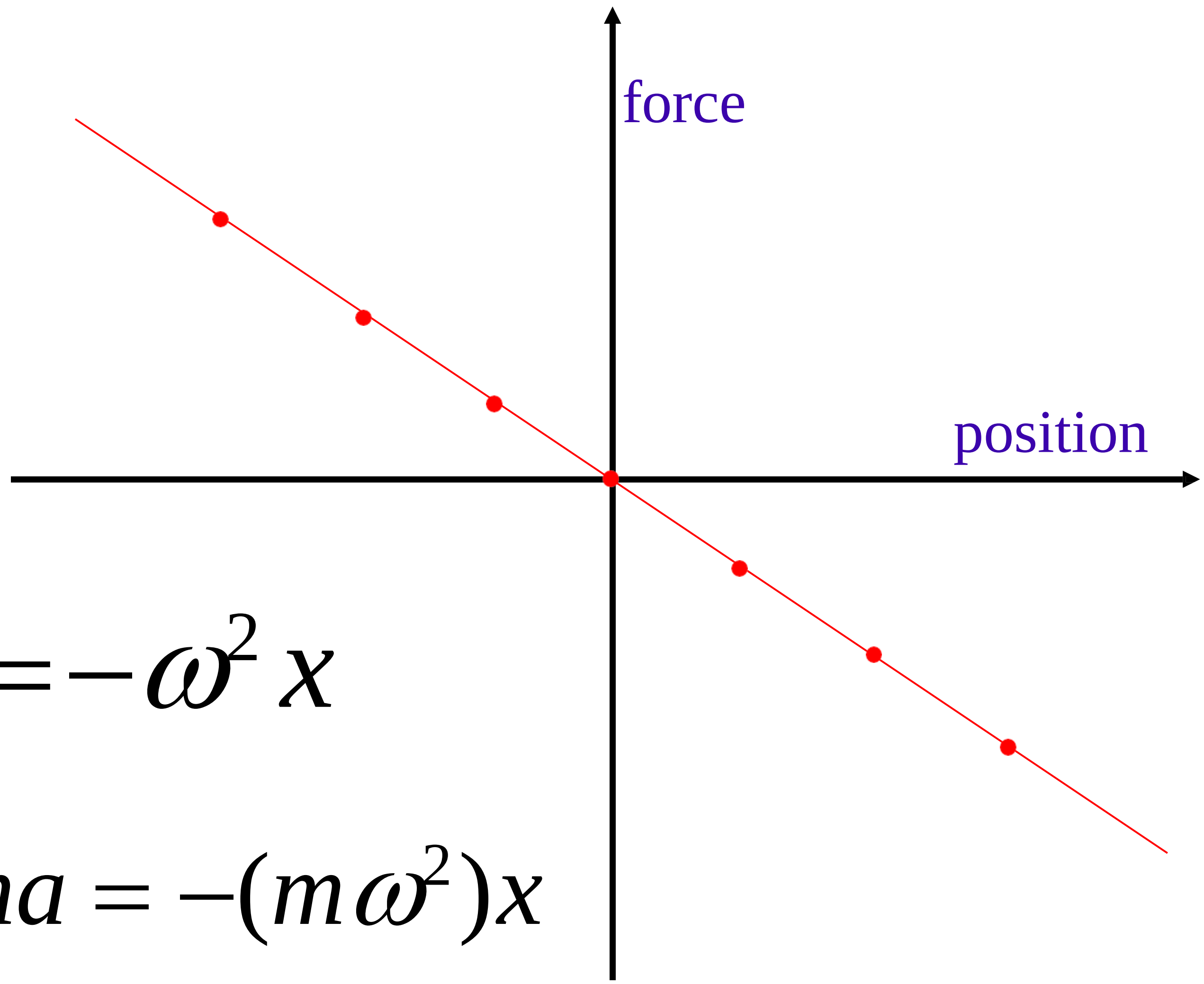


$$y = mx + c$$

$$a = -mx$$

$$a = -\omega^2 x$$

Force/position graph

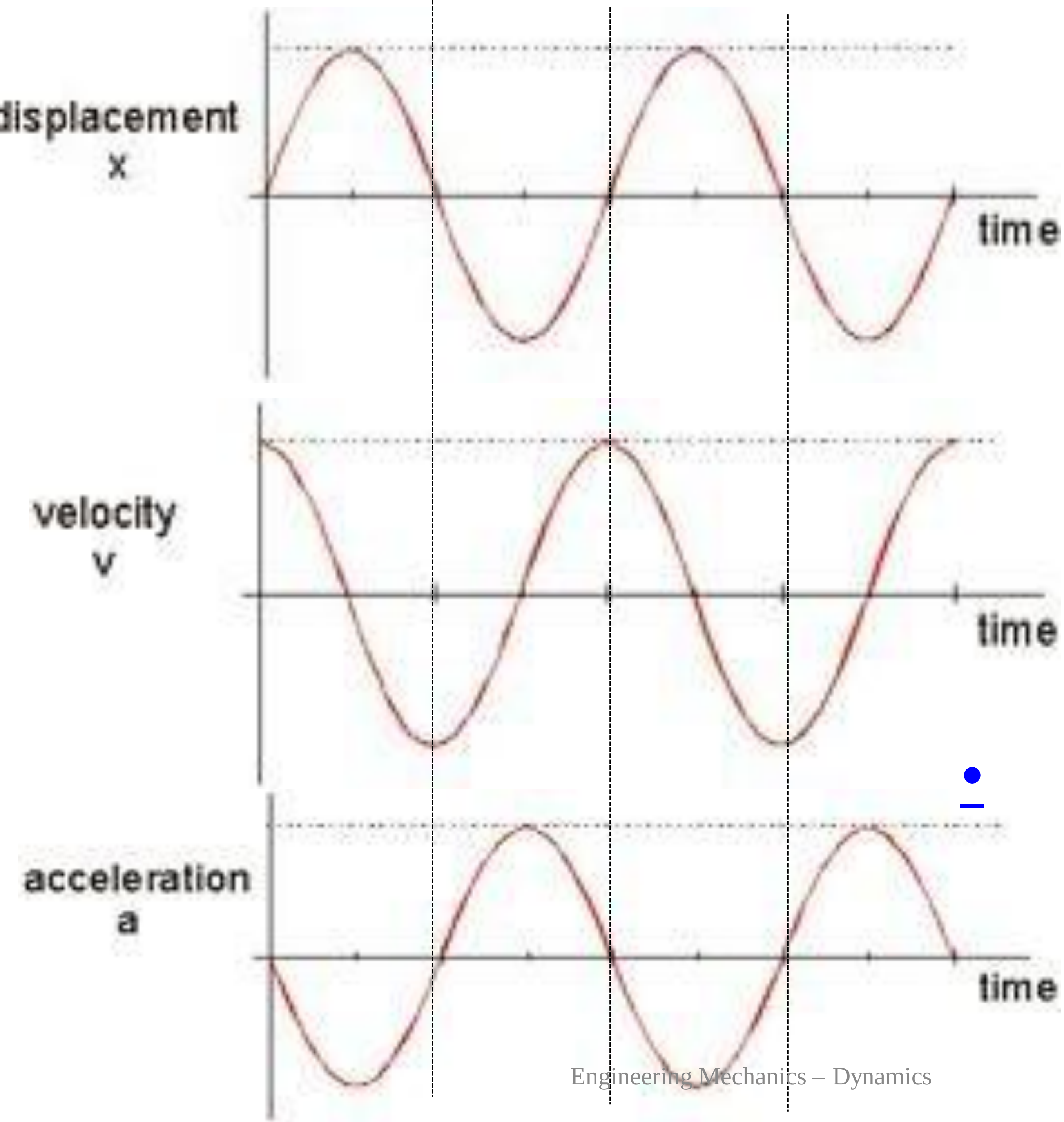


$$a = -\omega^2 x$$

$$F = ma = -(m\omega^2)x$$

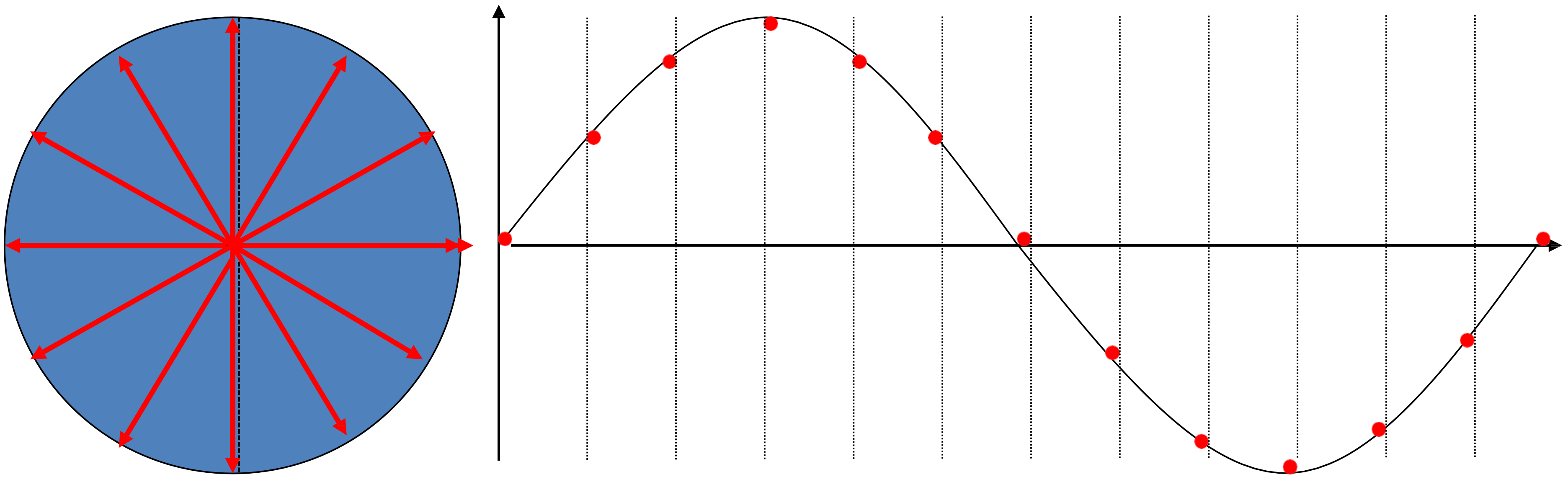
Graphs of SHM

- We have looked at simple harmonic motion as a function of ***position***.
- Now we'll look at it as a function of ***time***

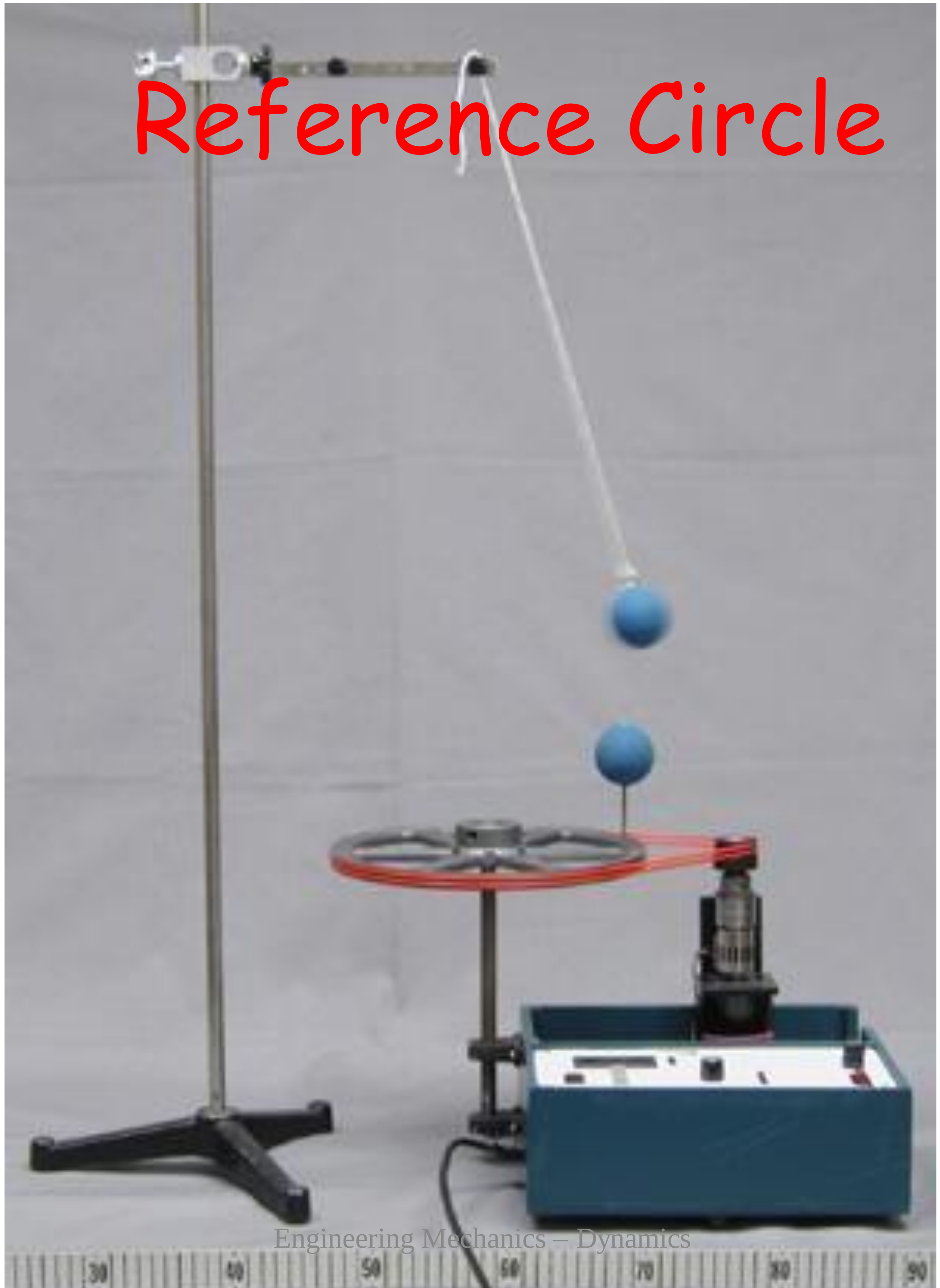


raphs

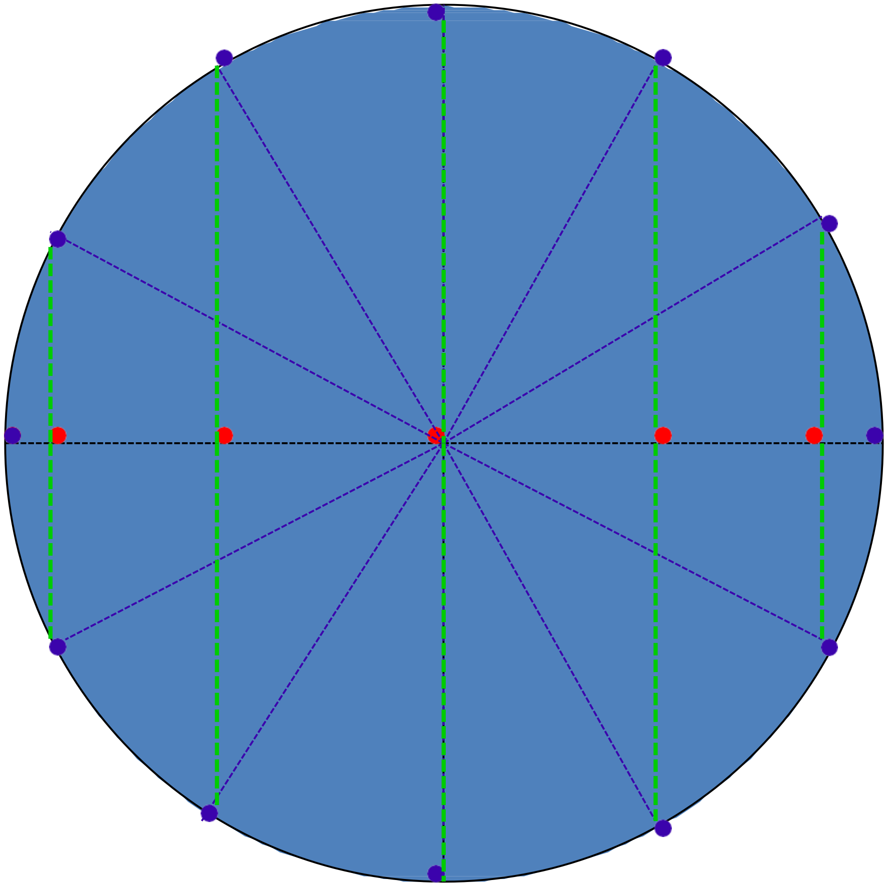

reference
equilibrium
position
reference
none
displacement
velocity
acceleration
mass
spring constant
damping
pause
slow
reset



Reference Circle



Reference Circle



Red ball moves in SHM horizontally

Blue ball moves in a circle

Both have same period

Amplitude of SHM equals radius of circle

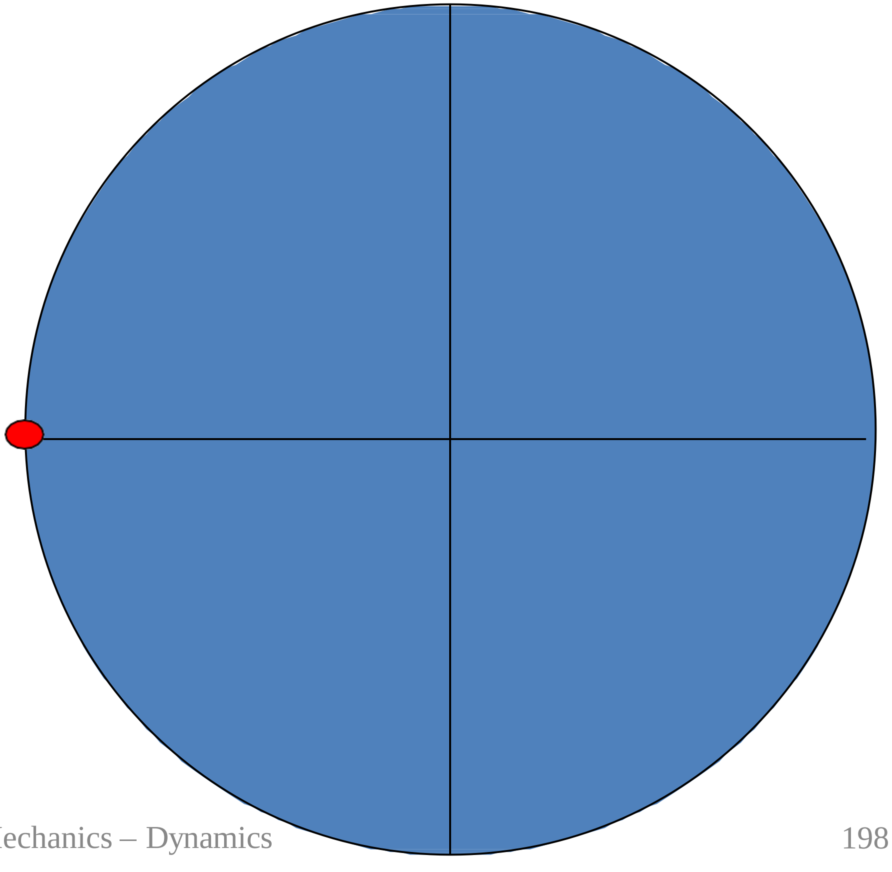
Both have same ***horizontal displacement***

To find the **position** of a swing at a certain time.

The period is 4.0s

The amplitude is 2.0m

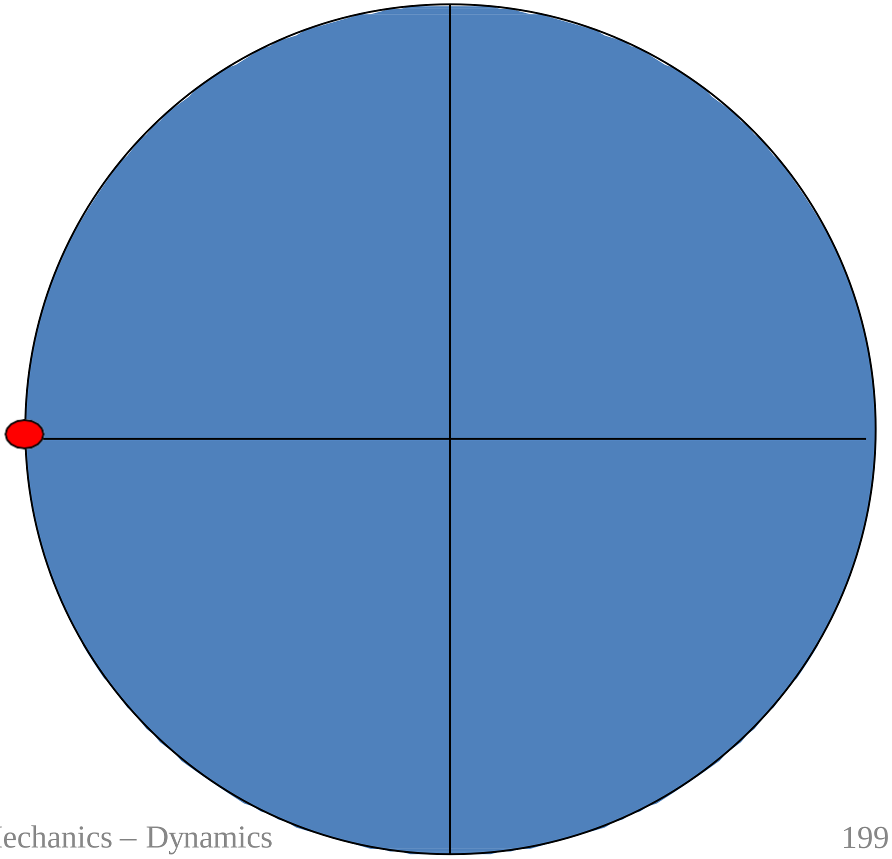
Where is the swing 2.0s after release?



The period is 4.0s

The amplitude is 2.0m

Where is the swing 1.0s after release?



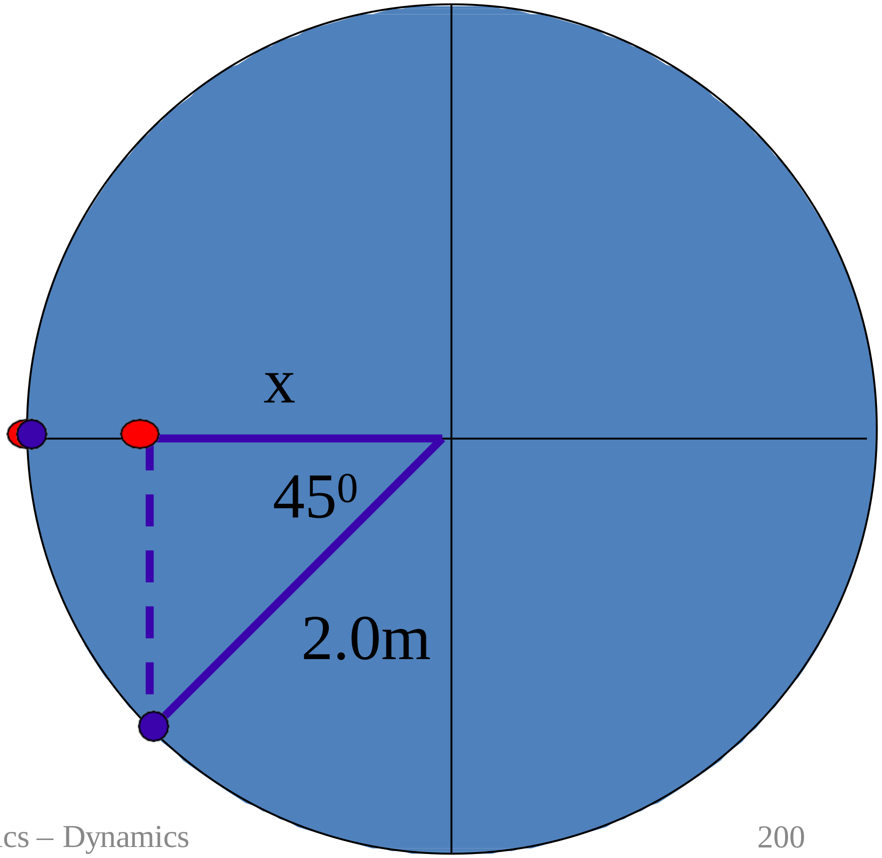
Where is the swing 0.5s after release?

Convert time to angle (1period = 360°)

$$\frac{0.5}{4.0} \times 360^0 = 45^0$$

$$0.50s = 45^0$$

$$\cos 45^0 = \frac{x}{2}$$



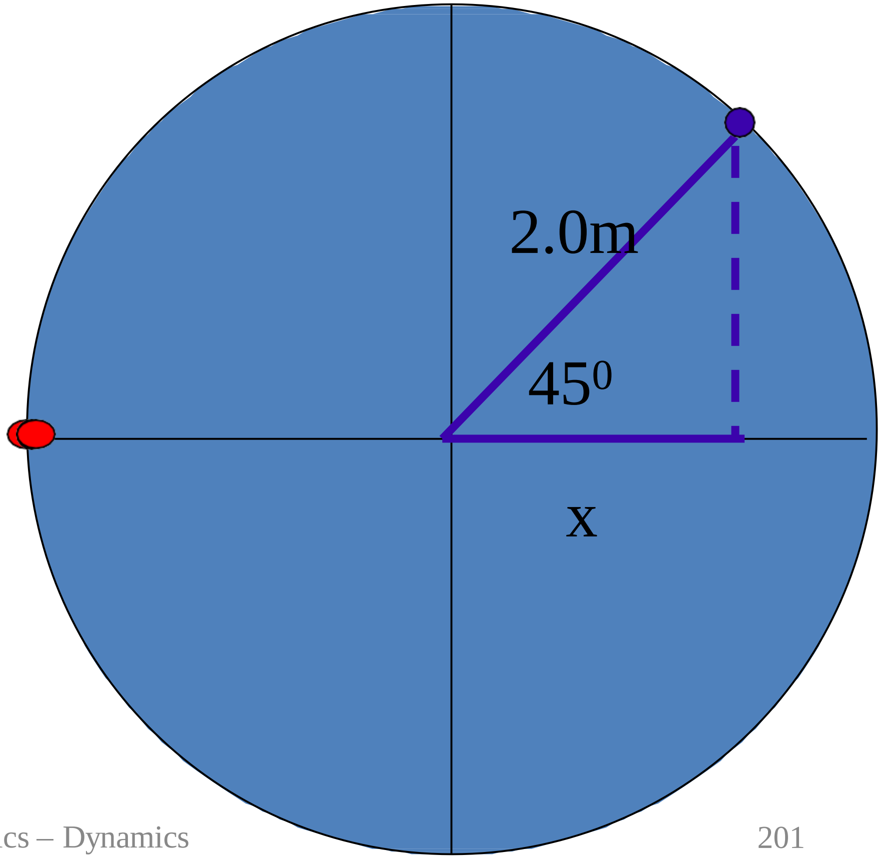
Where is the swing 2.5s after release?

Convert time to angle (1period = 360°)

$$\frac{2.5}{4.0} \times 360^0 = 225^0$$

$$2.50s = 225^0$$

$$\cos 45^0 = \frac{x}{2}$$



How long does it take to go 1.4m from the start?

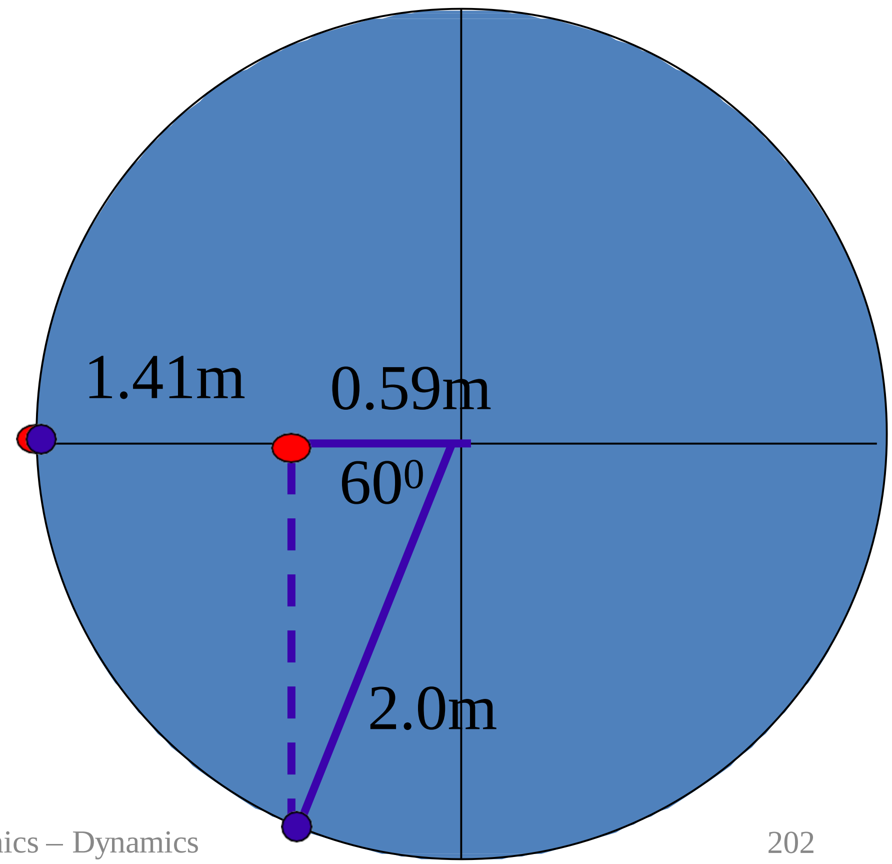
(1) Calculate angle

$$\cos\theta = \frac{0.59}{2}$$

$$\theta =$$

(2) Convert angle to time

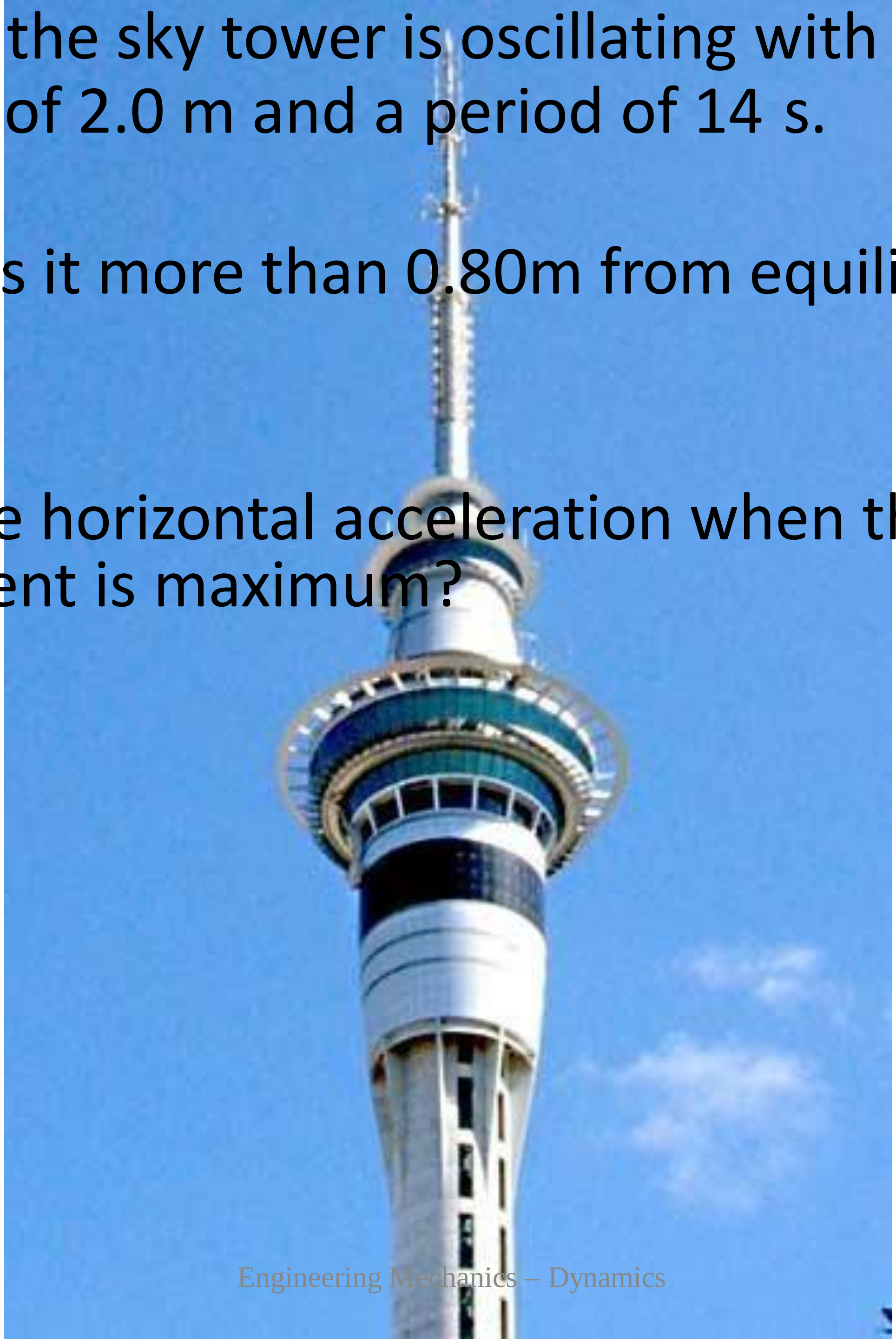
(1period = 360°)



$$60^0 = \frac{60}{360} \text{of a period}$$

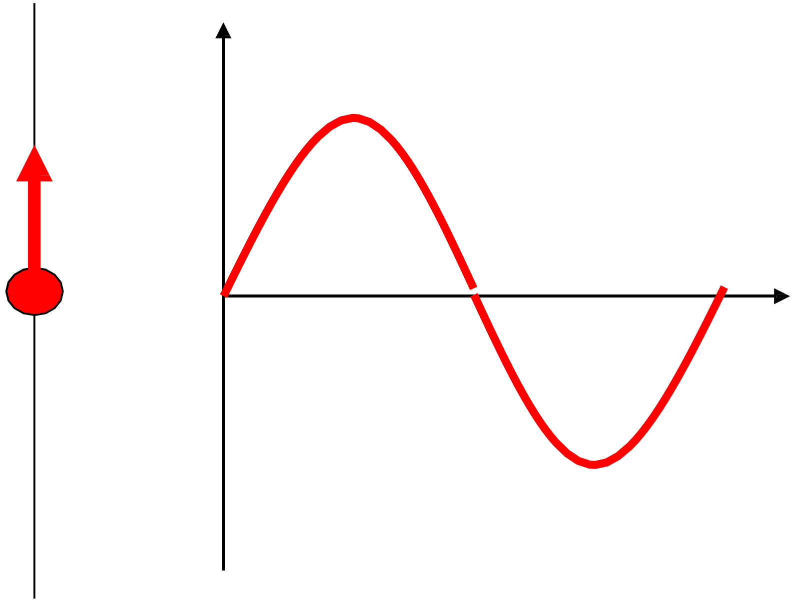
$$60^0 = {}^1\!/_6 \times 4.0s$$

- The top of the sky tower is oscillating with an amplitude of 2.0 m and a period of 14 s.
- How long is it more than 0.80m from equilibrium each cycle?
- What is the horizontal acceleration when the displacement is maximum?



Equations 1

$$y = A\sin\theta \qquad \theta = \omega t$$



$$y = A\sin\omega t$$

$$v = A\omega\cos\omega t$$

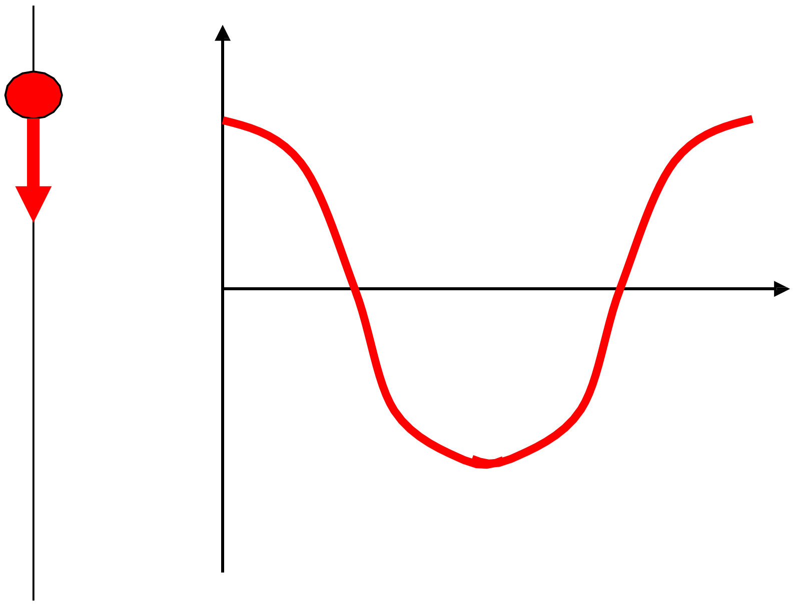
$$a = -A\omega^2\sin\omega t$$

Equations 2

$$y = A\cos\omega t$$

$$v = -A\omega\sin\omega t$$

$$a = -A\omega^2\cos\omega t$$

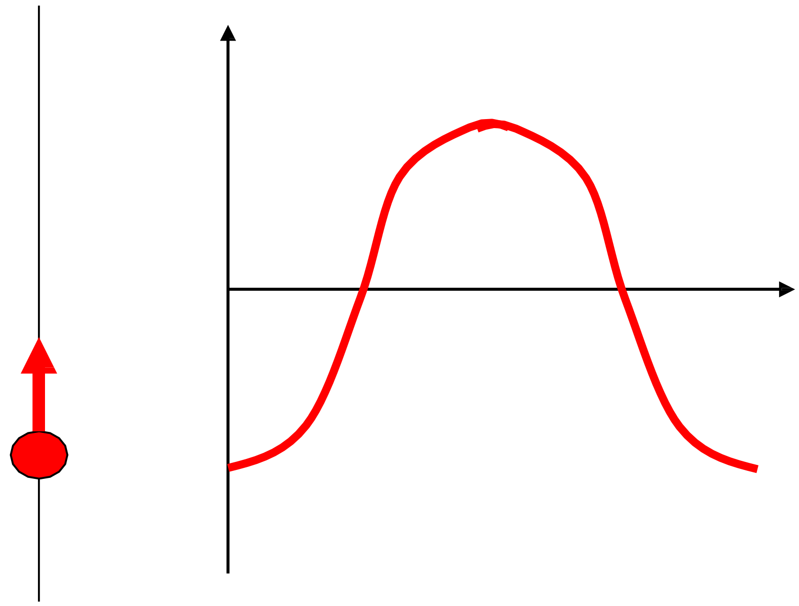


Equations 3

$$y = -A\cos\omega t$$

$$v = A\omega\sin\omega t$$

$$a = A\omega^2\cos\omega t$$



$$y = A\sin\omega t \qquad y_{\max} = A$$

$$v = A\omega\cos\omega t \qquad v_{\max} = A\omega$$

$$a = -A\omega^2\sin\omega t \qquad a_{\max} = -A\omega^2$$

$$a = -\omega^2 A\sin\omega t = -\omega^2 y$$

$$a = -\omega^2 y$$

Anisha is on a swing. Kate pulls her back 2.0m and lets her go. Her period is 4.0s.

(a) Calculate her maximum speed. (where is it?)

(b) Calculate her maximum acceleration. (where is it?)

Anisha is on a swing. Kate pulls her back 2.0m and lets her go. Her period is 4.0s.

(a) Calculate her speed 0.50s after being released
(b) Calculate her acceleration 0.50s after being released

- Nik is bungee jumping. In one oscillation he travels 12 m and it takes 8.0s.

- Tahi starts videoing him as he passes through the mid position moving UP.

(a) Calculate his velocity 1.0s after the video starts

(b) Calculate his acceleration 2.0s after the video starts.

Mass on a Spring

- As the mass increases, the period… increases

- As the spring stiffness increases the period … increases

Effect of mass:

$$a = \frac{F}{m}$$

- As the mass increases, the acceleration…

  decreases (assuming constant force)

- As the acceleration decreases the period …

  increases

A larger mass means a longer period.

Effect of spring stiffness:

$$a = \frac{F}{m} \qquad F = kx$$

- As the stiffness increases, the restoring force…

  increases (assuming same displacement)

- As the restoring force increases the acceleration …

  increases

- As the acceleration increases the period … decreases

A stiffer spring means a shorter period.

Summary

- mass ↑ acceln↓ period ↑
- stiffness ↑ force ↑ acceln ↑ period ↓

equation

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Extension .....derivation of the equation:

consider a mass on a spring.

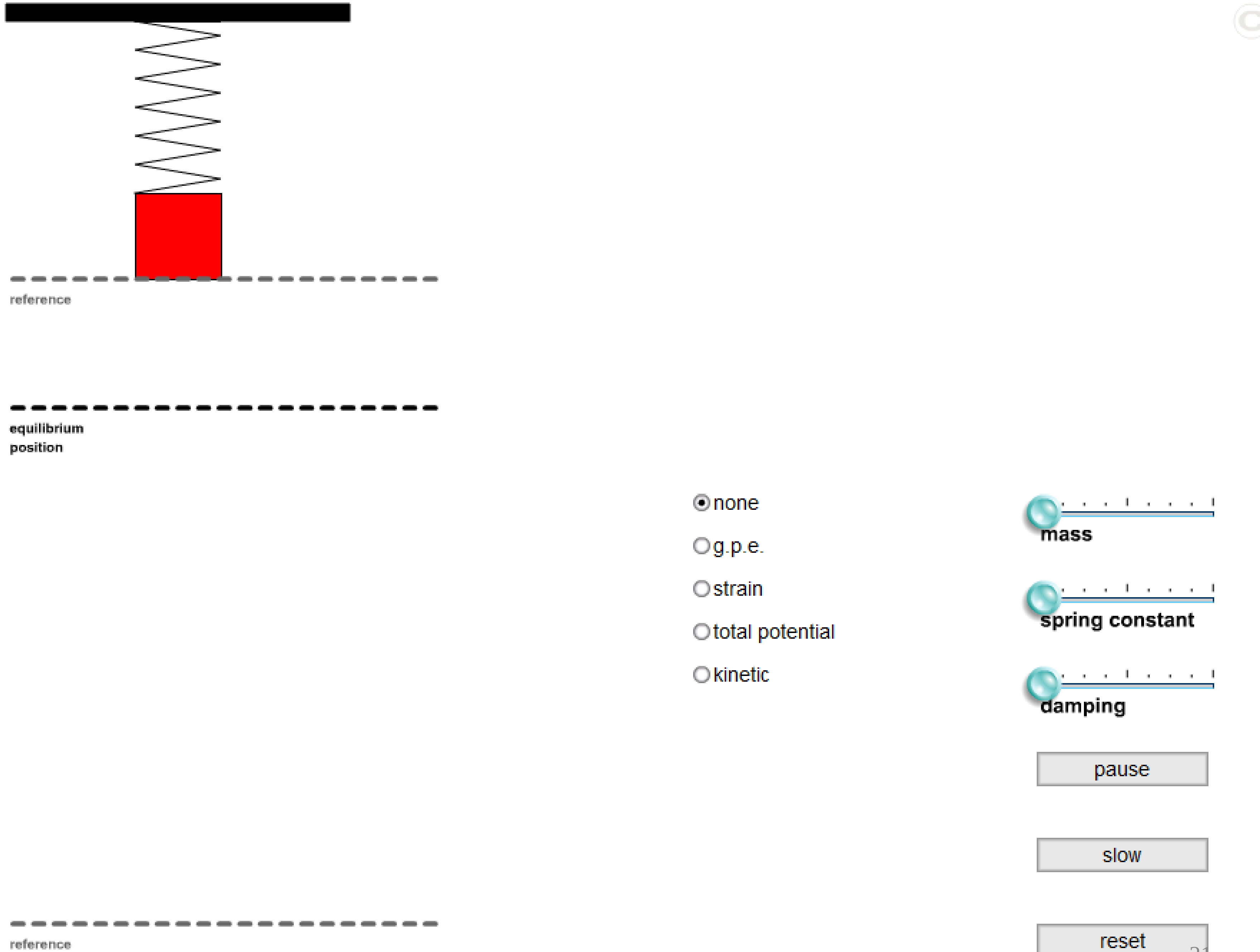
$$a = \frac{F}{m} \qquad F = -kx$$

$$a = \frac{-kx}{m} \qquad a = -\frac{k}{m}x \qquad (i.e. \quad a \propto -x)$$

$$a = \frac{-k}{m}x \qquad a = -\omega^2 x$$

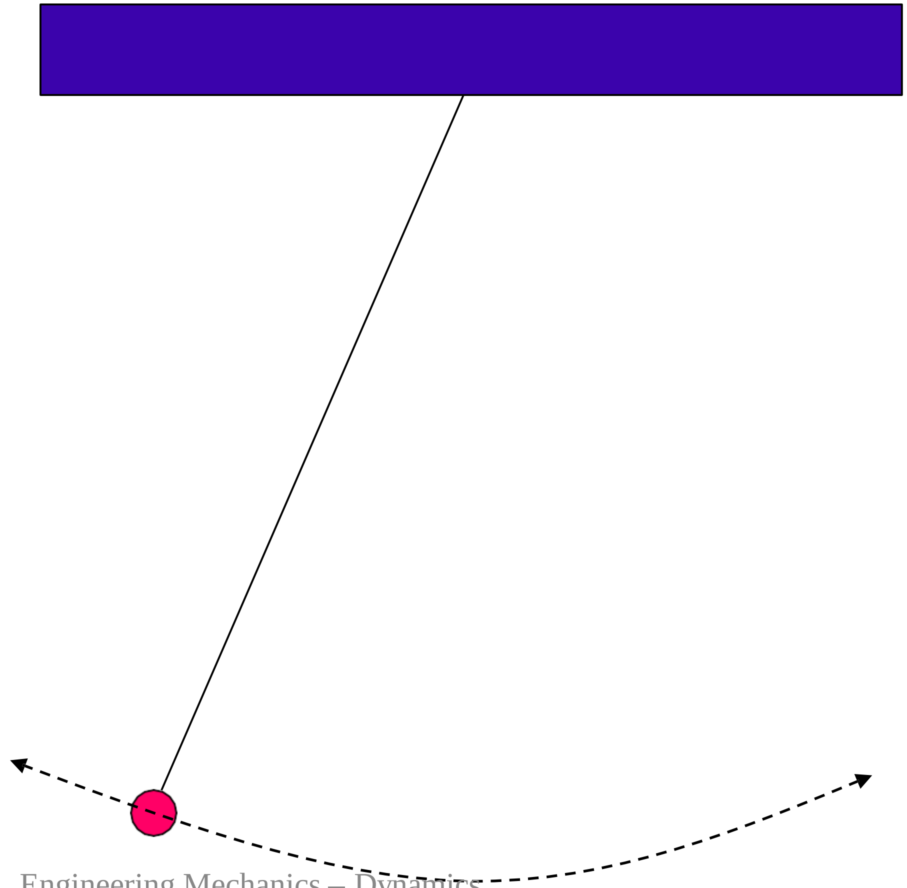
$$\frac{k}{m} = \omega^2 = (\frac{2\pi}{T})^2$$

$$\sqrt{\frac{k}{m}} = \frac{2\pi}{T} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

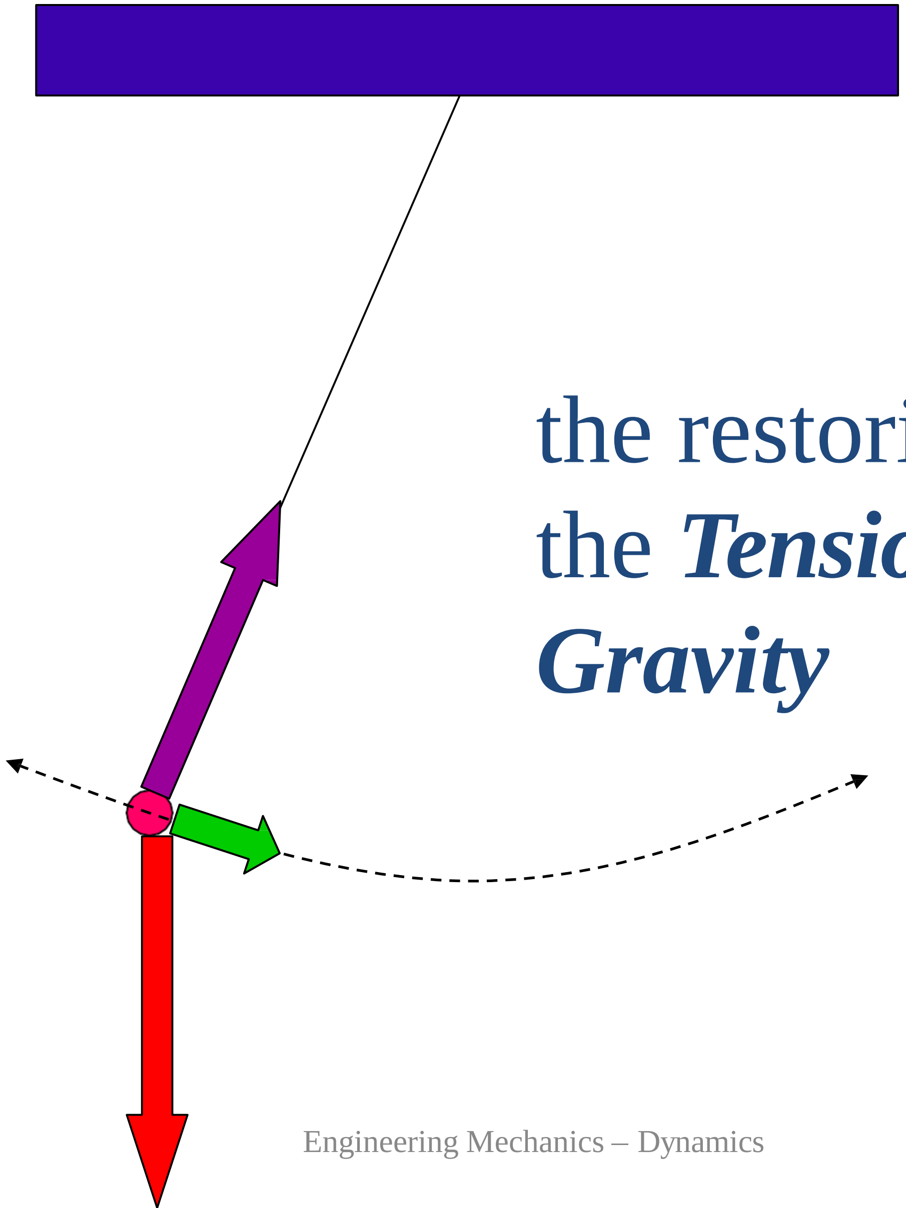

reference
equilibrium
position
none
g.p.e.
strain
total potential
kinetic
mass
spring constant
damping
pause
slow
reset
reference

Simple Pendulum

- This is where all the mass is concentrated in one point.

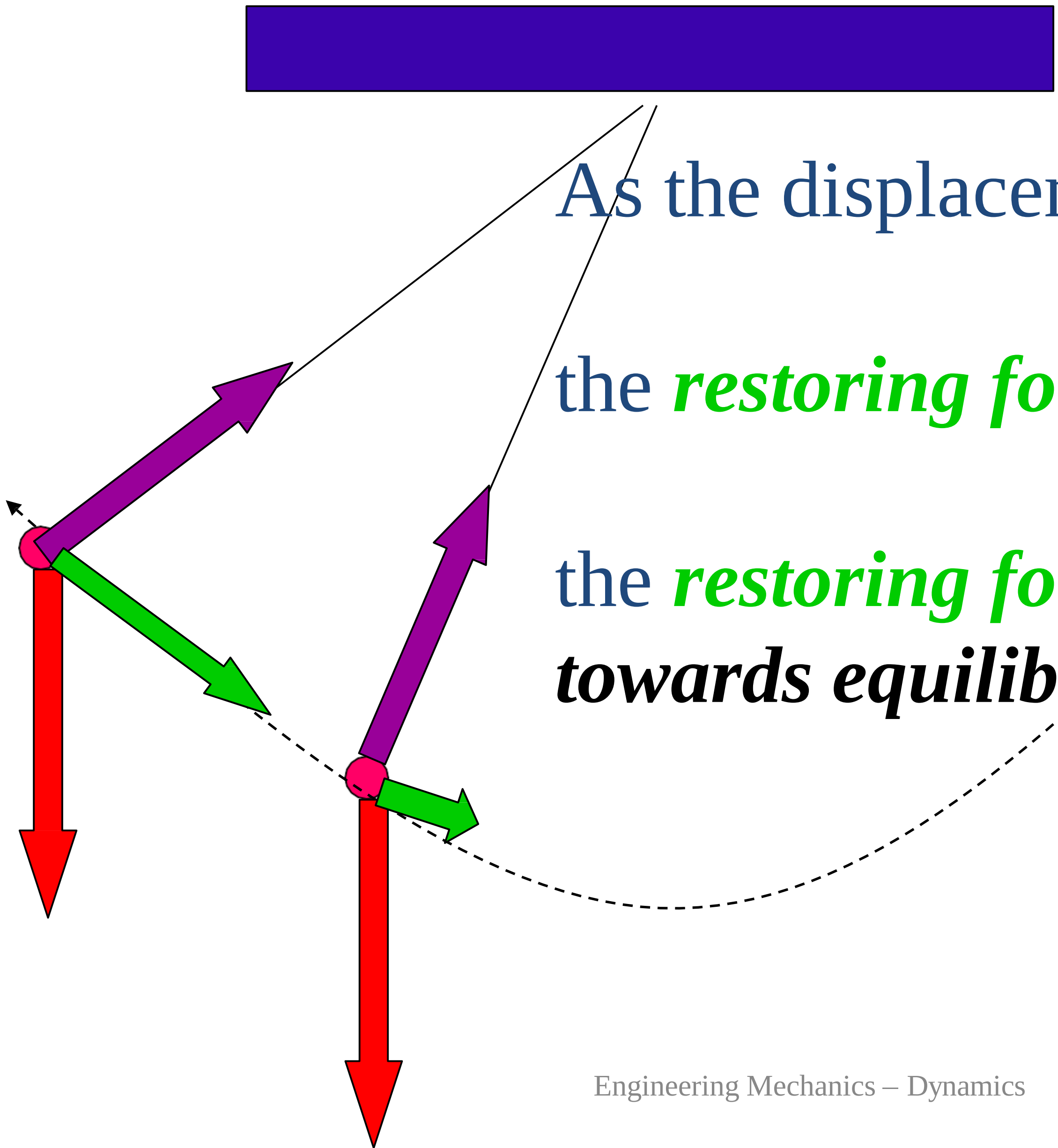


What provides the restoring force?



the restoring force is the ***Tension*** plus ***Gravity***

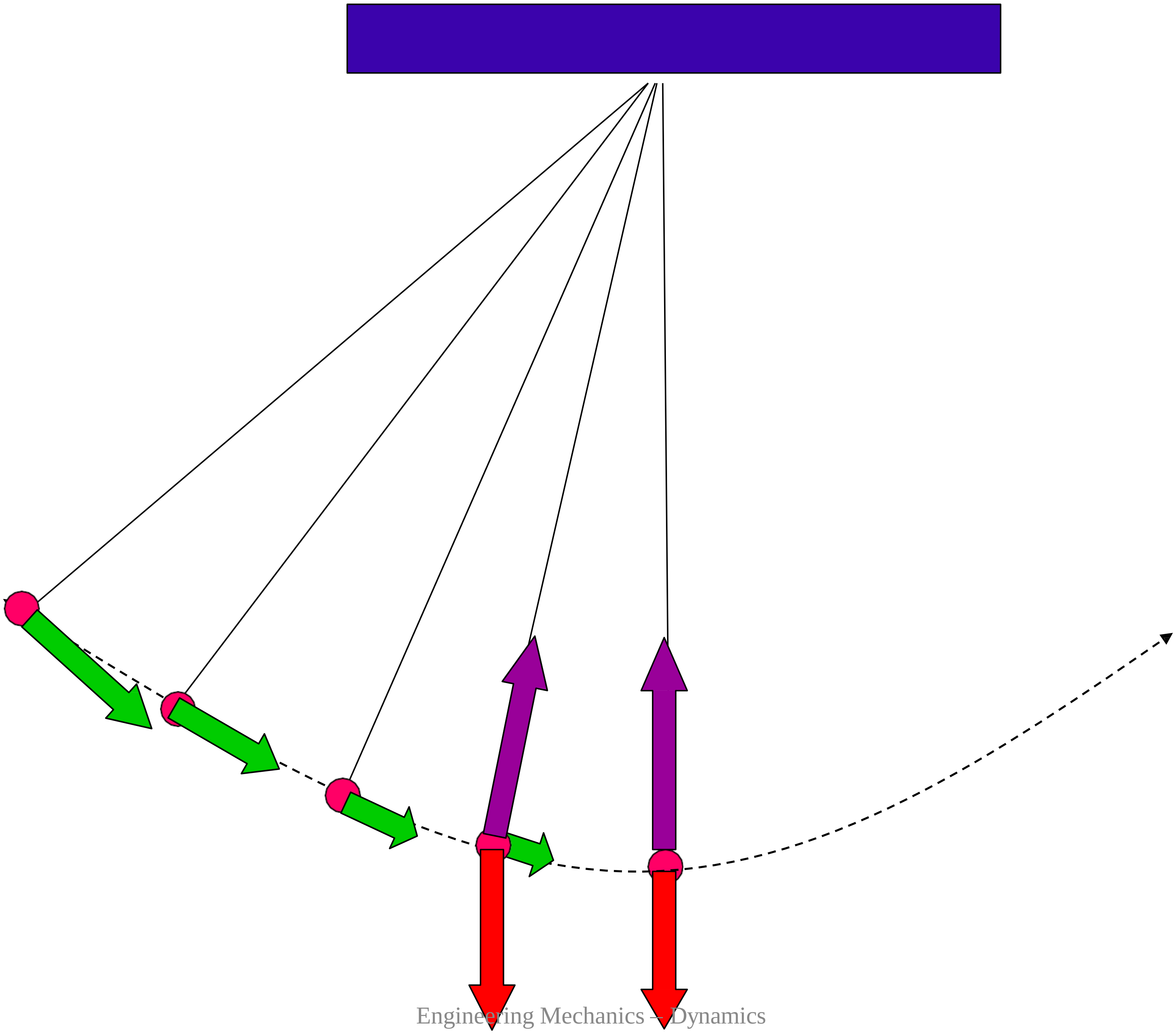
Why is the motion SHM?



As the displacement increases,

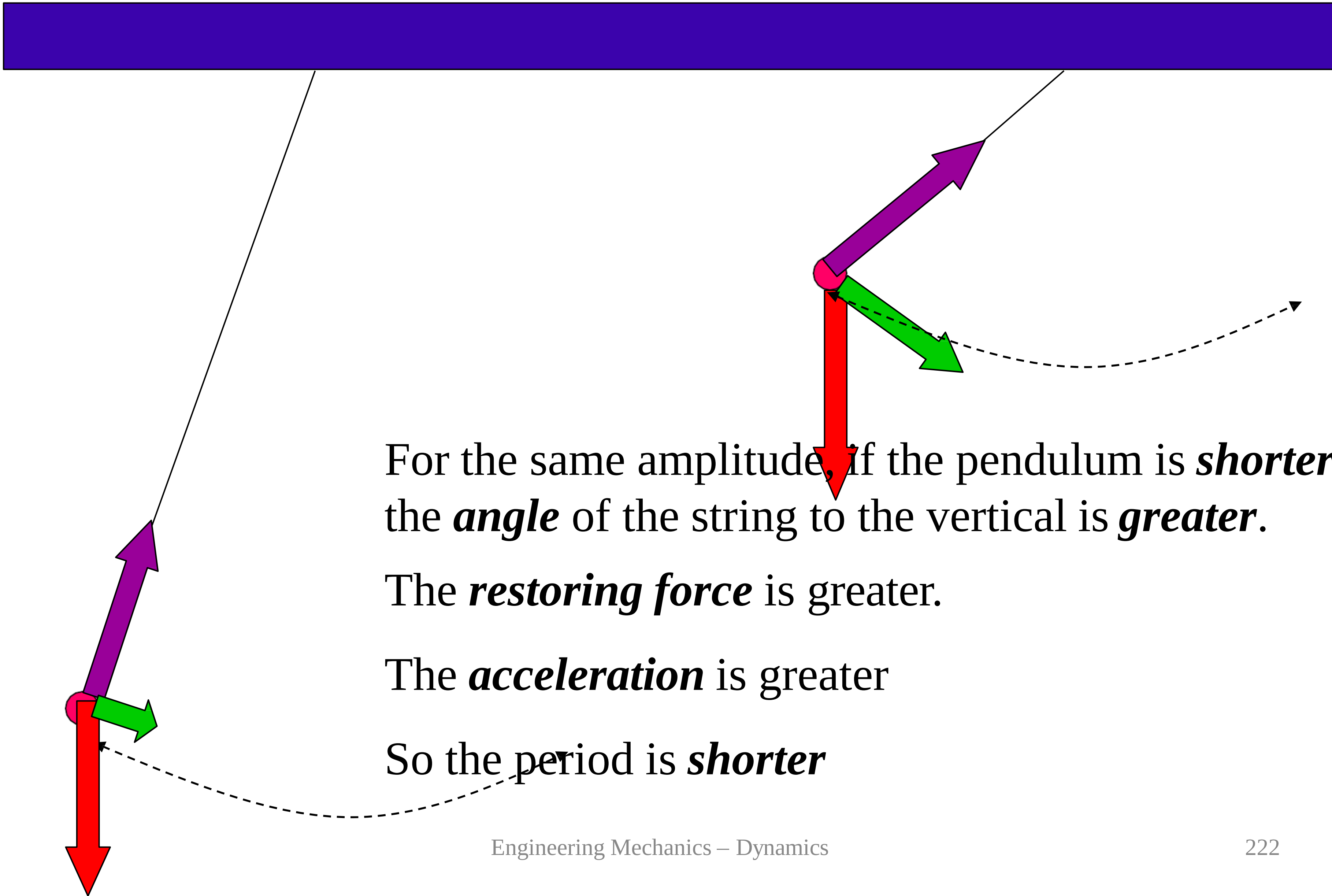
the ***restoring force.*** ***increases.***

the ***restoring force*** ***is always towards equilibrium***



- This next bit is very important

Why does length affect period?



For the same amplitude, if the pendulum is ***shorter***, the ***angle*** of the string to the vertical is ***greater***.

The ***restoring force*** is greater.

The ***acceleration*** is greater

So the period is ***shorter***

period of a pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

How is ***length*** measured?

As the pendulum expands down,

The mercury expands up

This keeps the center of mass in the same place

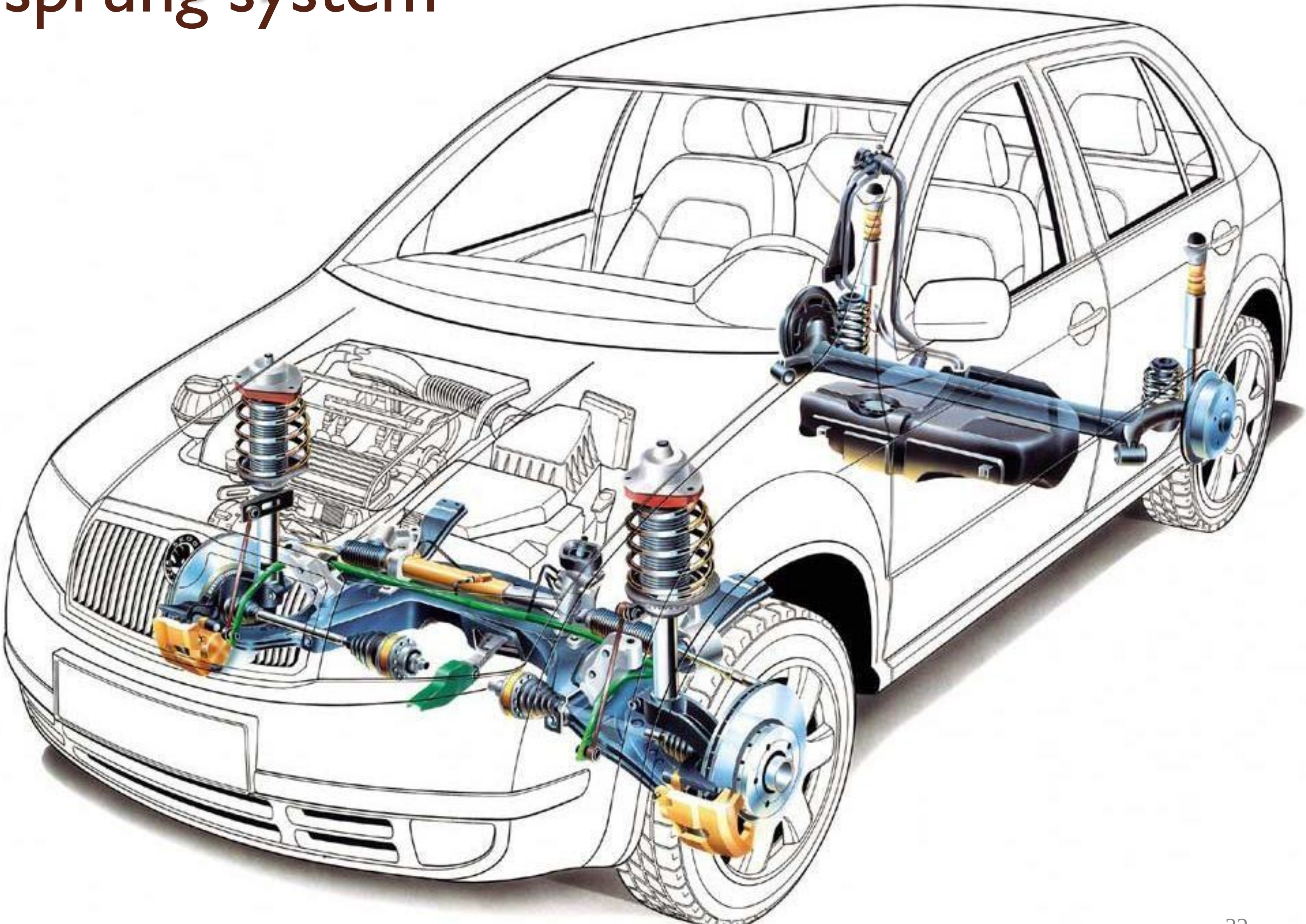
Same length same period.

Energy of SHM

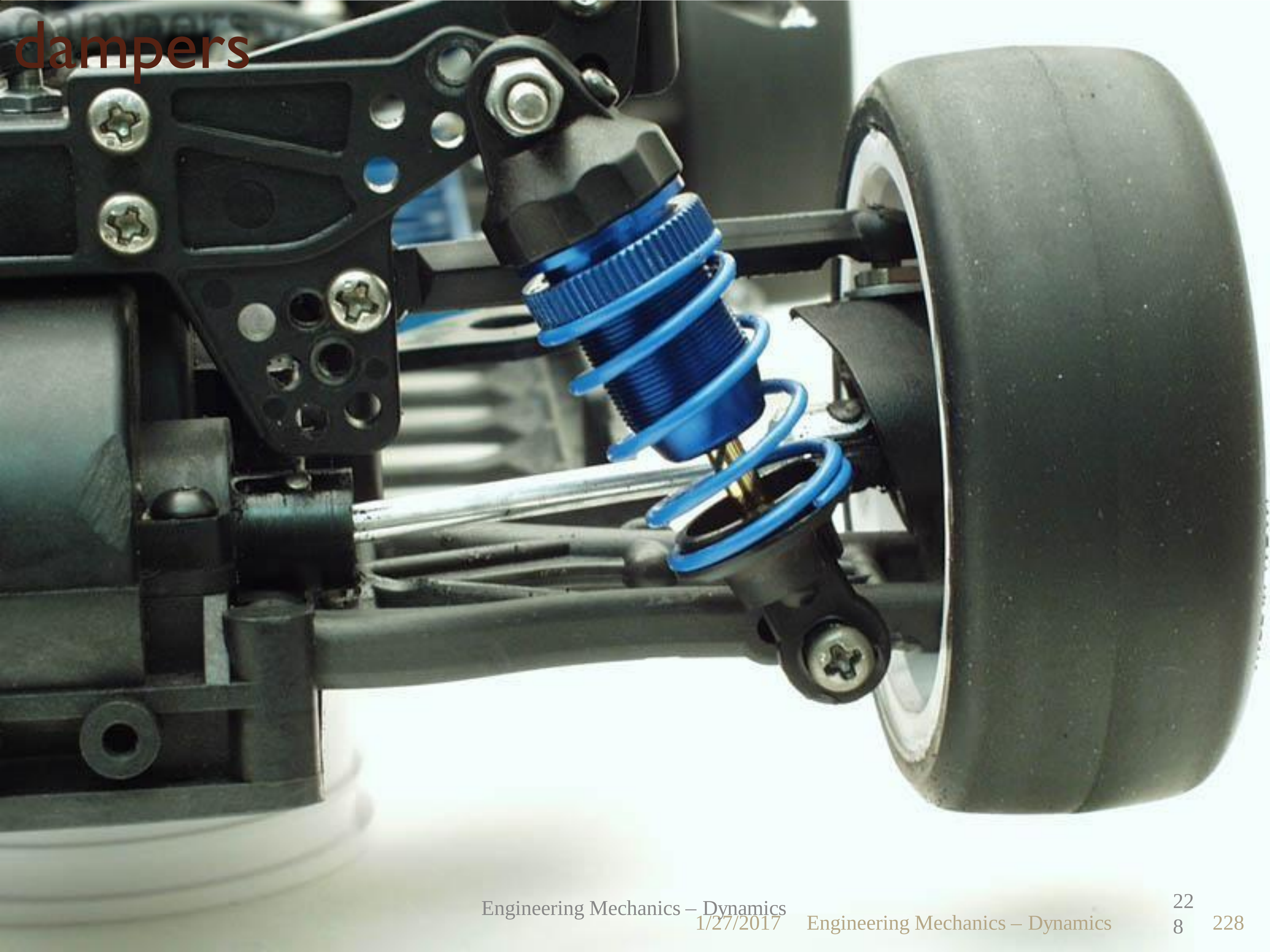

reference
equilibrium
position
reference
none
g.p.e.
strain
total potential
kinetic
mass
spring constant
damping
pause
slow
reset

a sprung system

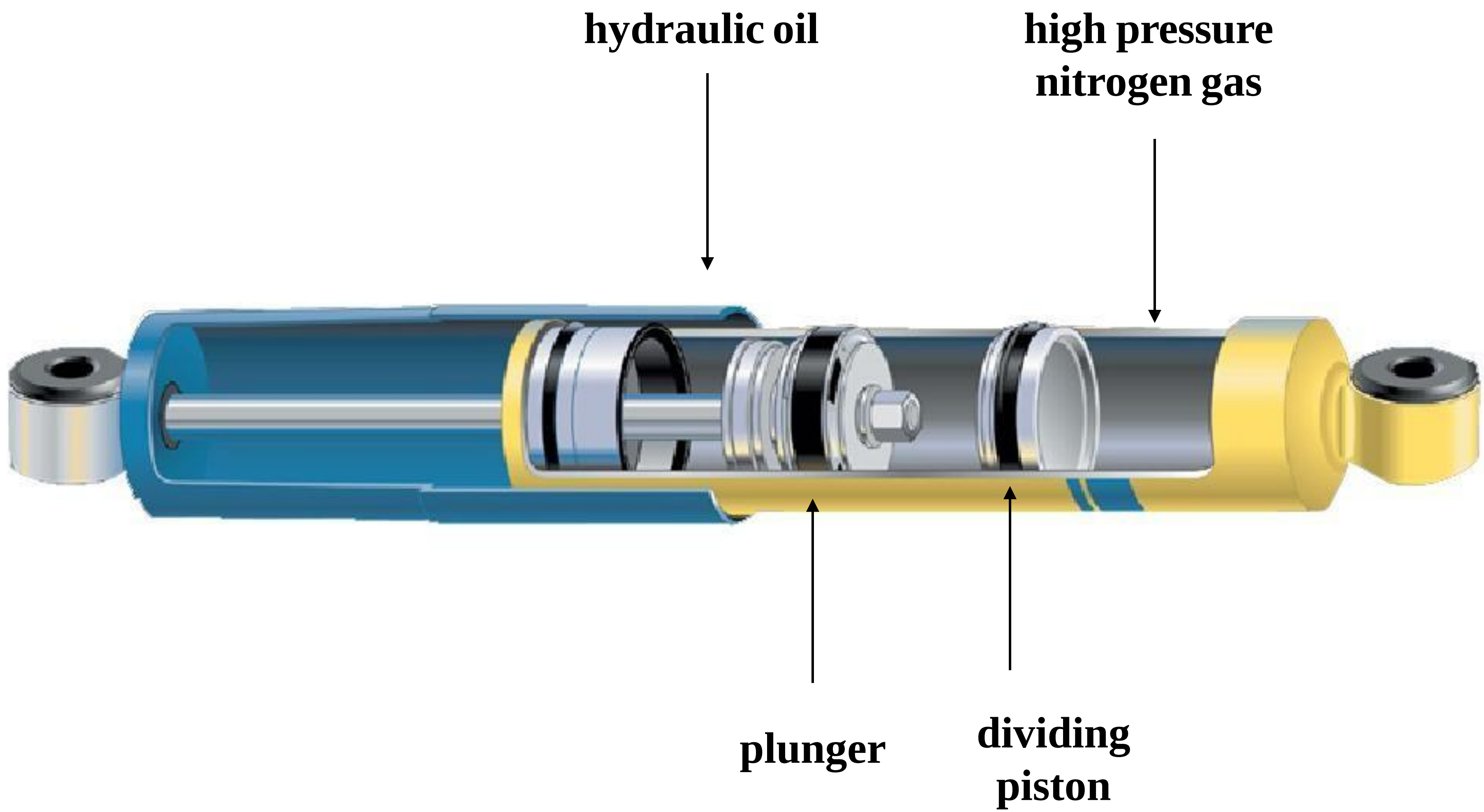
a sprung system



dampers



energy dissipation



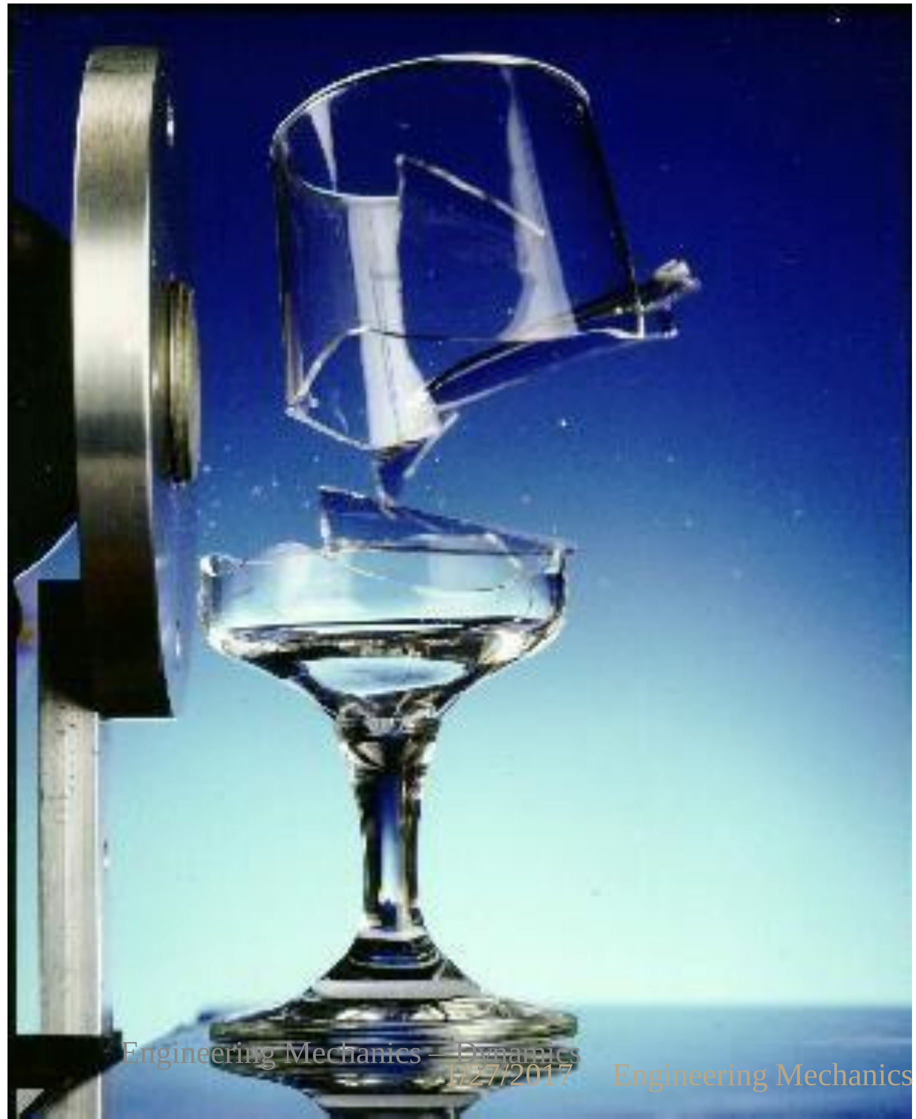
bridge dampers

Engineering Mechanics – Dynamics 230

Resonance

- Any elastic system has a natural period of oscillation.
- If bursts of energy (pushes) are supplied at the natural period, the amplitude will increase.
- This is called ***resonance***

Examples of resonance



- The glass has a natural frequency of vibration.
- If you tap the glass, it vibrates at the natural frequency causing sound.
- If you put energy in at the natural frequency, the amplitude increases. This is resonance.
- If the amplitude gets high enough, the glass can break.

Bay of Fundy

Engineering Mechanics – Dynamics

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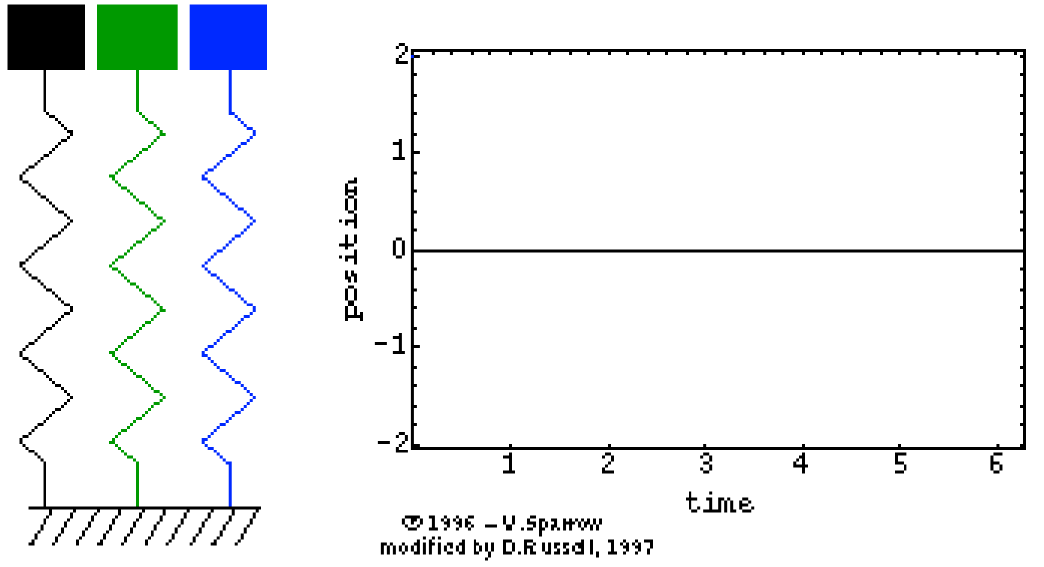
Bay of Fundy

The period of the tide is 12 hours.

The time for a wave to move up the bay and back is 12 hours

What is vibration?

- Vibrations are oscillations of a system about an equilbrium position.



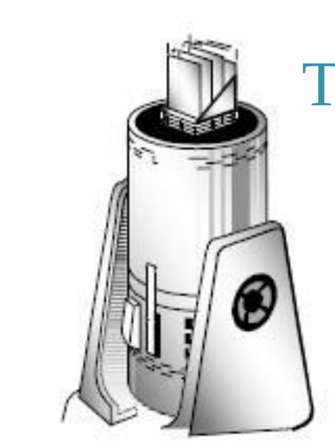
Vibration…

It is also an everyday phenomenon we meet on everyday life

Vibration …

Useful Vibration

Compressor



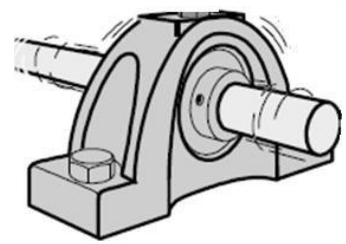
Testing

Ultrasonic
cleaning

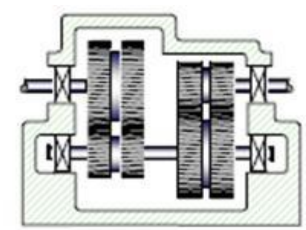
Harmful vibration

Noise

Destruction

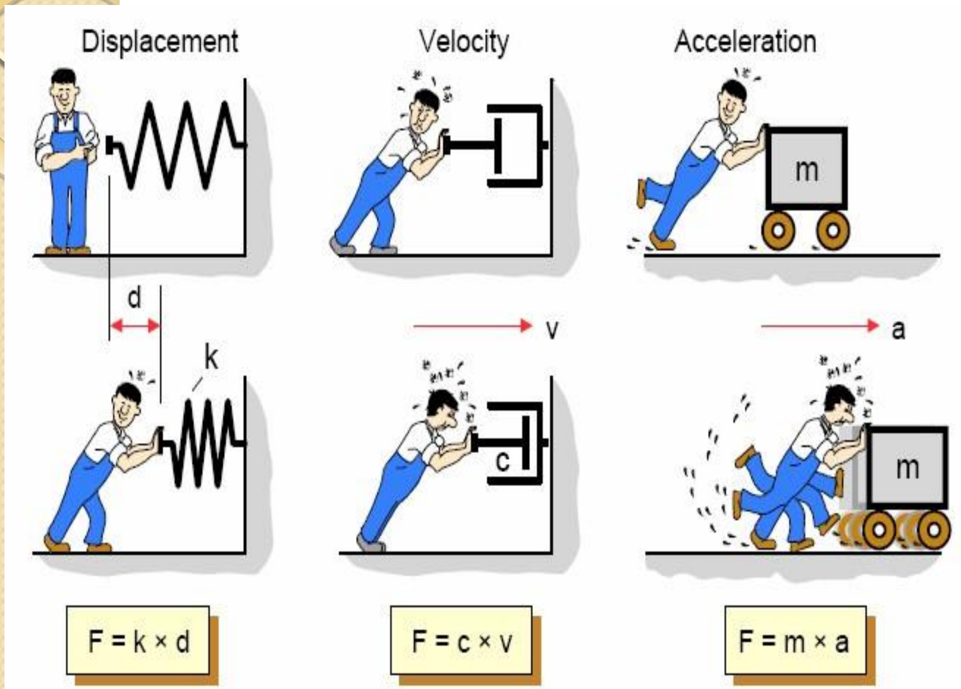


Wear



Fatigue

Vibration parameters



All mechanical systems can be modeled by containing three basic components:

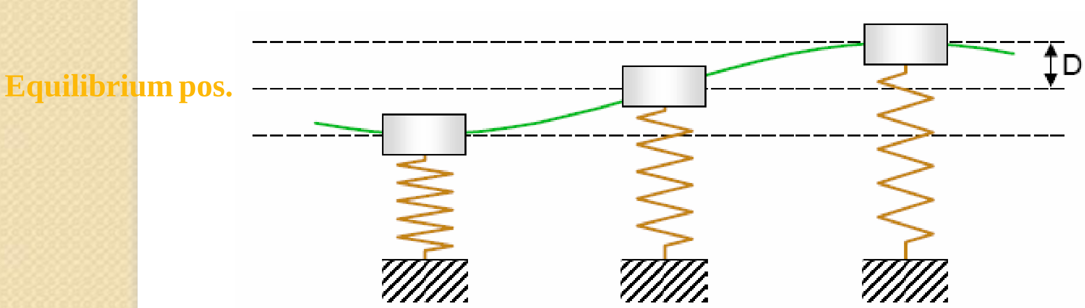
spring, damper, mass

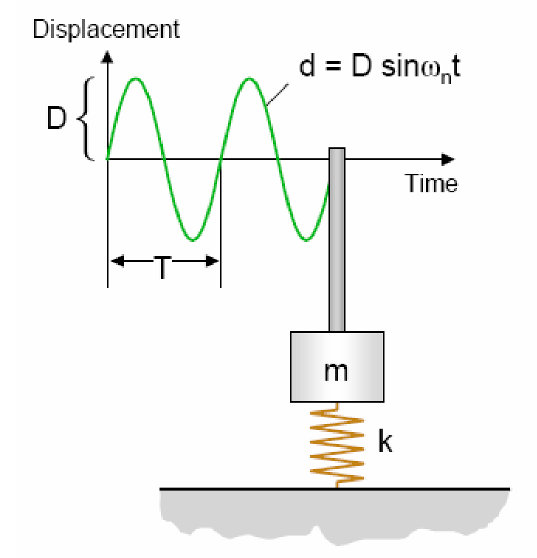
When these components are subjected to *constant* force, they react with a *constant*

displacement, velocity and acceleration

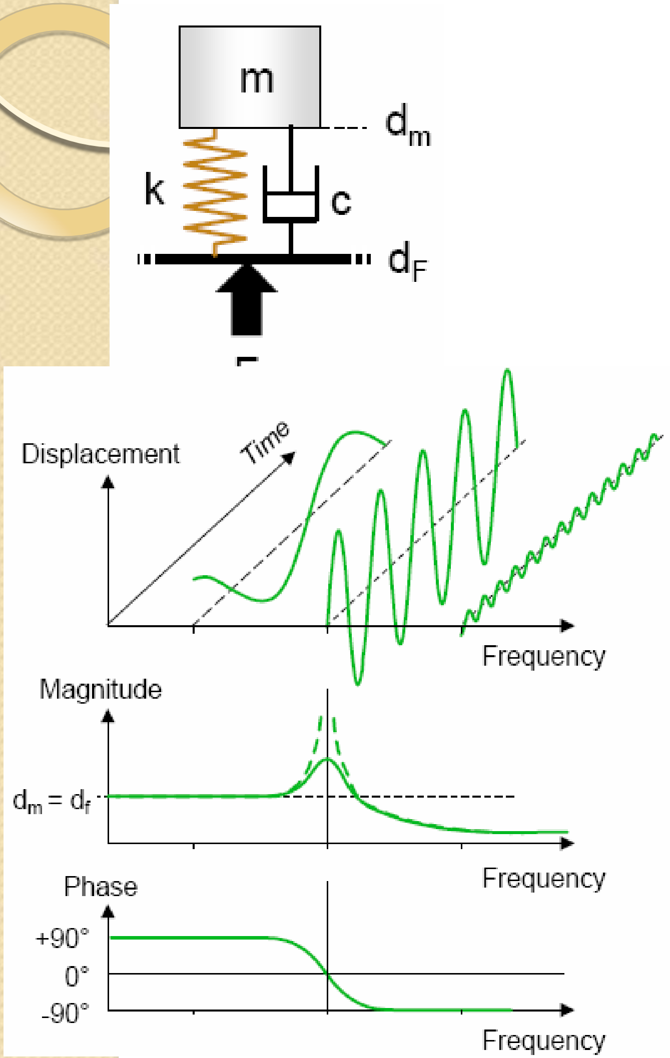
Free vibration

- When a system is initially disturbed by a displacement, velocity or acceleration, the system begins to vibrate with a constant amplitude and frequency depend on its stiffness and mass.
- This frequency is called as **natural frequency,** and the form of the vibration is called as **mode shapes**





Forced Vibration



If an external force applied to a system, the system will follow the force with the same frequency.

However, when the force frequency is increased to the system's natural frequency, amplitudes will dangerously increase in this region. This phenomenon called as **"Resonance"**

Watch these …

Bridge collapse:

http://www.youtube.com/watch?v=j-zczJXSxnw

Hellicopter resonance:

http://www.youtube.com/watch?v=0FeXjhUEXlc

Resonance vibration test:

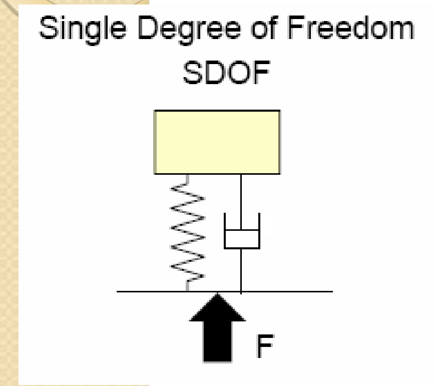
http://www.youtube.com/watch?v=LV_UuzEznHs

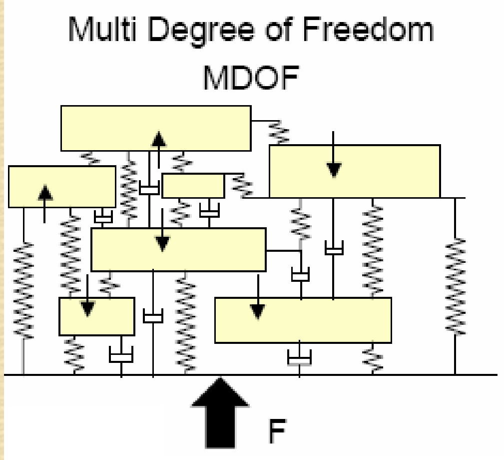
Flutter (Aeordynamically induced vibration) :

http://www.youtube.com/watch?v=OhwLojNerMU

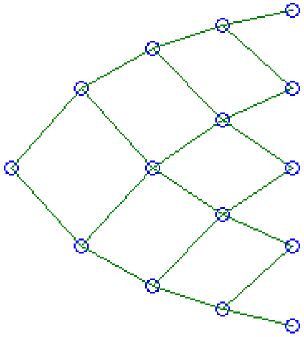
Modelling of vibrating systems

Lumped (Rigid) Modelling

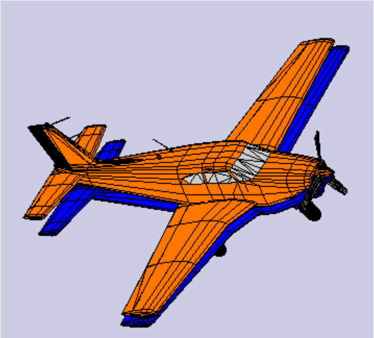




Numerical Modelling



Element-based methods (FEM, BEM)



Statistical and Energy-based methods

(SEA, EFA, etc.)

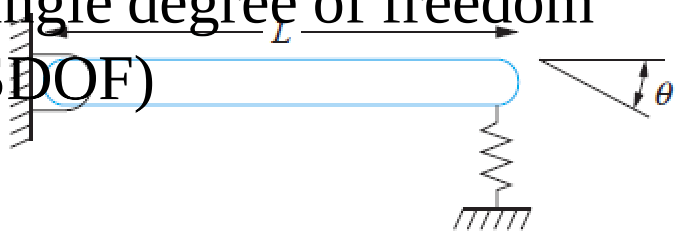
Degree of Freedom (DOF)

• Mathematical modeling of a physical system requires the selection of a set of variables that describes the behavior of the system.

• The number of *degrees of freedom* for a system is the number of kinematically independent variables necessary to completely describe the motion of every particle in the system

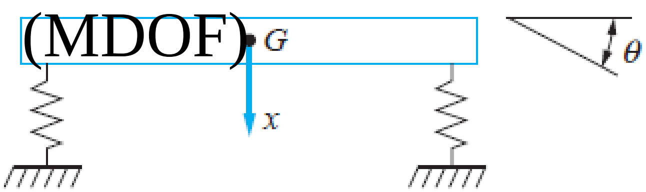
DOF=1

Single degree of freedom (SDOF)



DOF=2

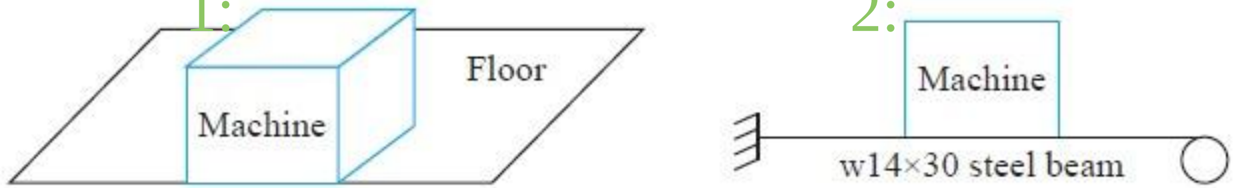
Multi degree of freedom (MDOF)



Equivalent model of systems

Example 1:

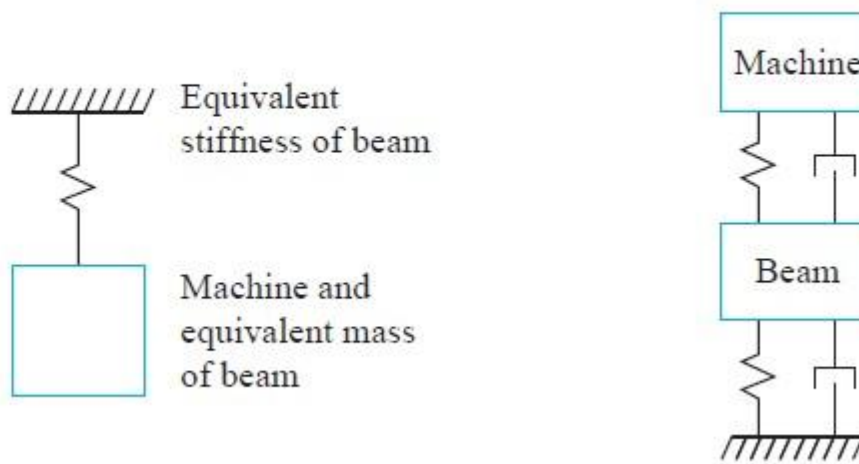
Example 2:



(a) (b)

SDOF

DOF=1



(c) (d)

MDOF

DOF=2

Equivalent model of systems

Example 3:

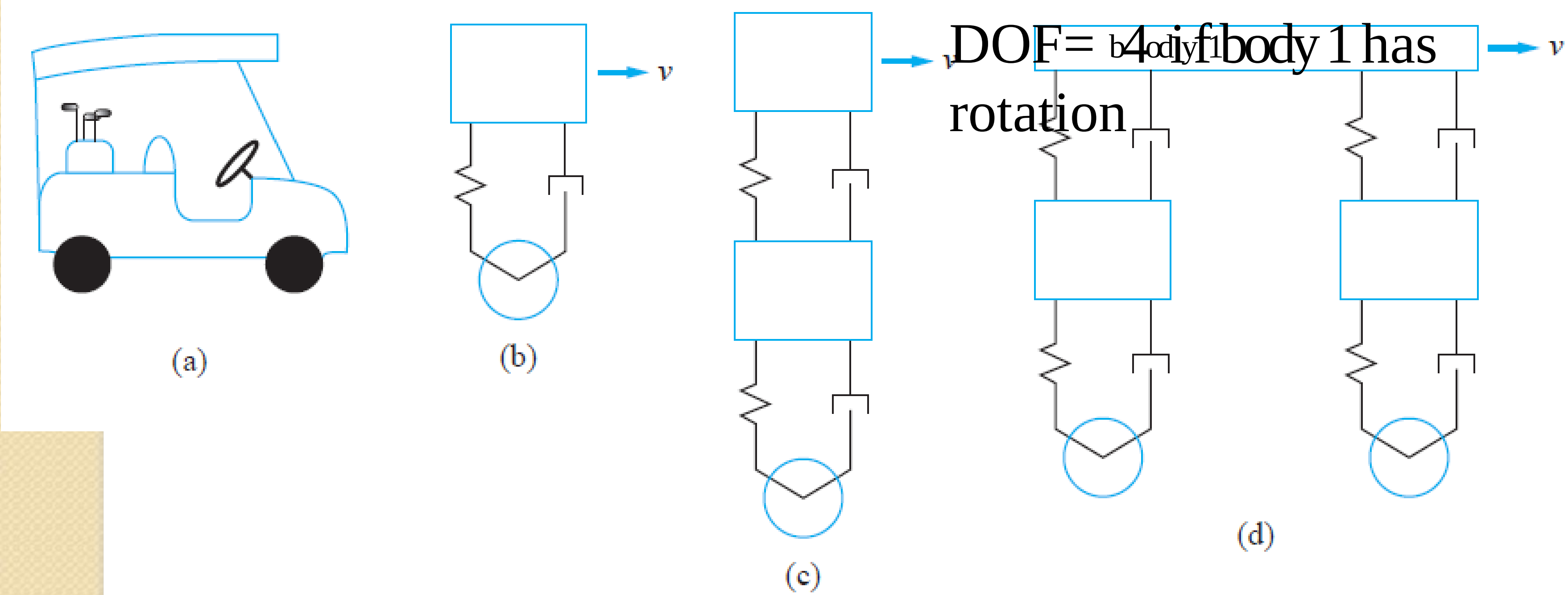
SDOF

DOF=2

MDOF

DOF= 3 if body 1 has no rotation

DOF= 4 if body 1 has rotation



SDOF systems

- Helical springs

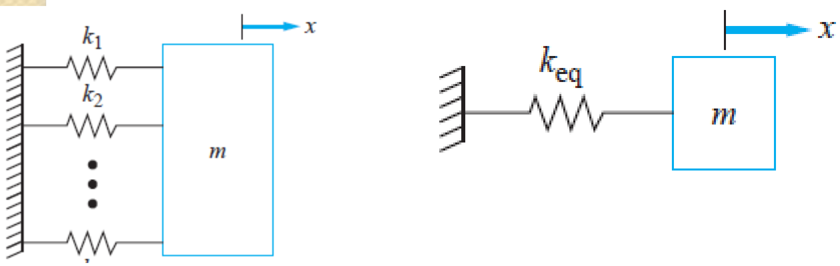
Shear stress: $\tau_{max} = \frac{FrD}{2J} = \frac{16Fr}{\pi D^3}$

Stiffness coefficient: $k = \frac{GD^4}{64Nr^3}$

F: *Force*, *D*: *D*iameter, *G*: Shear modulus of the rod,
N: Number of turns, *r* : Radius

- Springs in combinations:

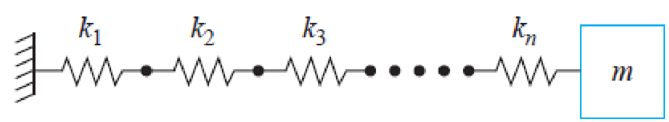
<u>Parallel combination</u>



$$F = k_1x + k_2x + \cdots + k_nx = \left(\sum_{i=1}^{n} k_i\right)x$$

$$k_{eq} = \sum_{i=1}^{n} k_i$$

<u>Series combination</u>

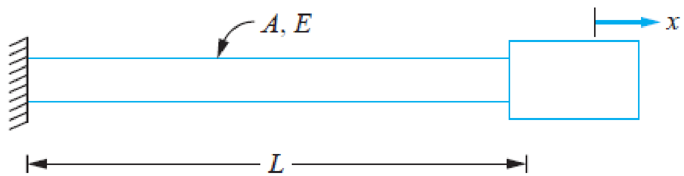


$$x = x_1 + x_2 + \cdots + x_n = \sum_{i=1}^{n} x_i$$

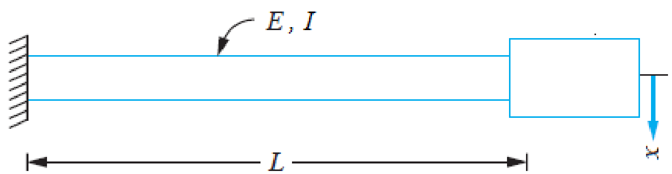
$$x = \sum_{i=1}^{n} \frac{F}{k_i} \qquad k_{eq} = \frac{1}{\sum_{i=1}^{n} \frac{1}{k_i}}$$

Elastic elements as springs

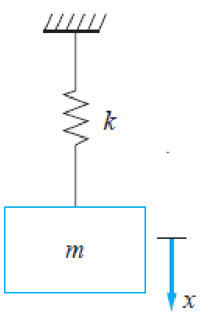
System	Stiffness Coeff.	SDOF Model

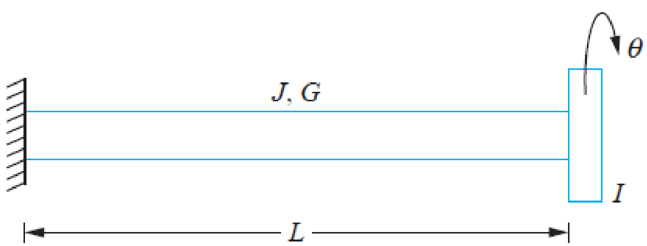


$$k = \frac{AE}{L}$$

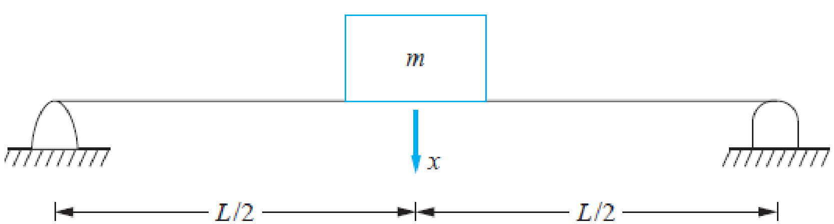


$$k = \frac{48EI}{L^3}$$





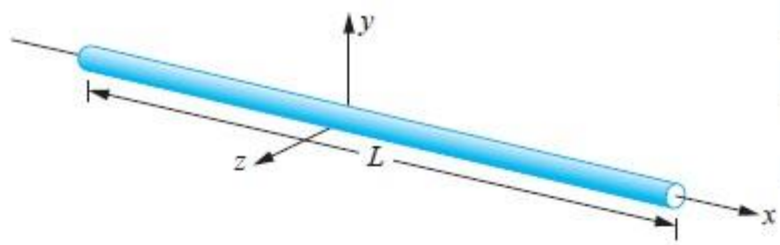
$$k = \frac{JG}{L}$$



$$k = \frac{3EI}{L^3}$$

Moment of Inertia

Slender rod

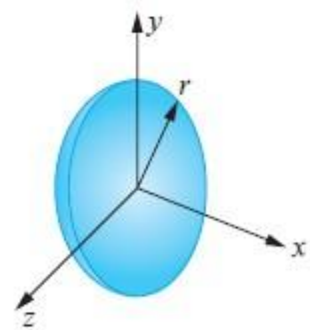


$$\bar{I}_x \approx 0$$

$$\bar{I}_y = \frac{1}{12} m L^2$$

$$\boxed{\bar{I}_z = \frac{1}{12} m L^2}$$

Thin disk

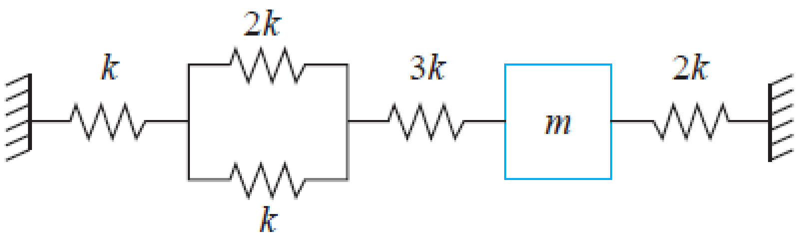


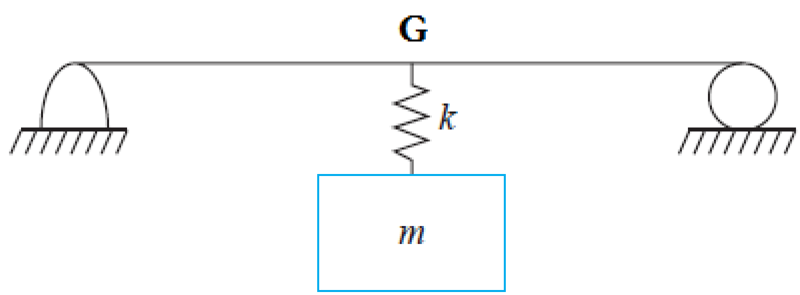
$$\boxed{\bar{I}_x = \frac{1}{2} m r^2}$$

$$\bar{I}_y = \frac{1}{4} m r^2$$

$$\bar{I}_z = \frac{1}{4} m r^2$$

What are the equivalent stiffnesses?





Do it yourself !

Do it yourself !