UNIT – I

INTRODUCTION & HYDROSTATIC FORCES

(FLUID MECHANICS)

IARE

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Definition of Stress

Consider a small area δA on the surface of a body (Fig. 1.1). The force acting on this area is δF This force can be resolved into **two perpendicular components**

- The component of force acting normal to the area called **normal** force and is denoted by δF_n
- The component of force acting along the plane of area is called **tangential** force and is denoted by δF_t

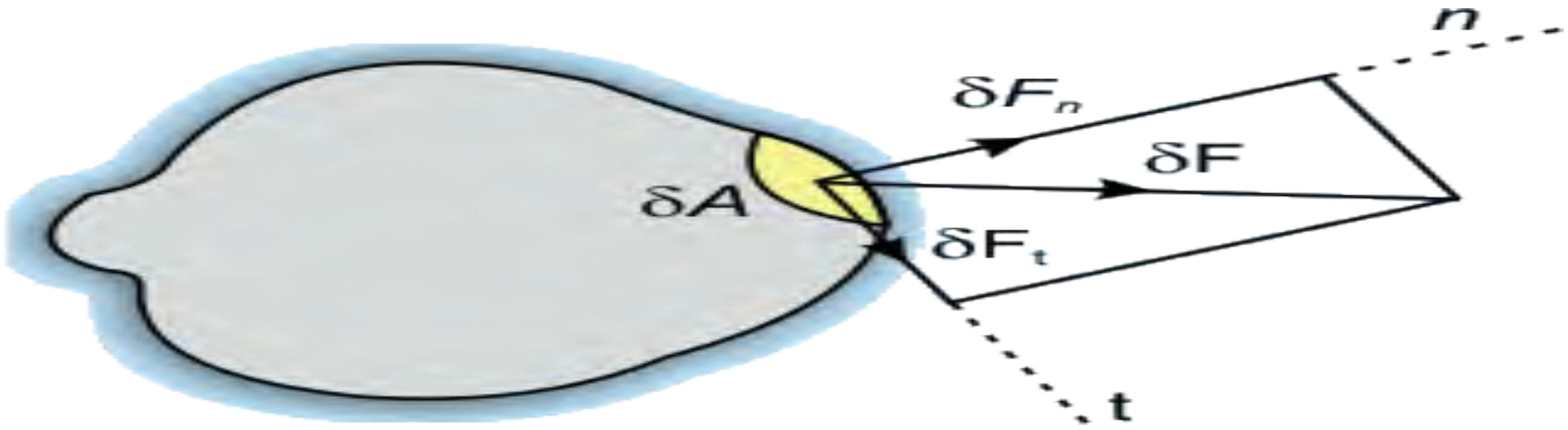


Fig 1.1 Normal and Tangential Forces on a surface

When they are expressed as force per unit area they are called as **normal stress** and **tangential stress** respectively**.** The tangential stress is also called shear stress.

- The normal stress

$$\sigma = \lim_{\delta A \to 0} \left(\frac{\delta F_n}{\delta A} \right)$$

And shear stress

$$\tau = \lim_{\delta A \to 0} \left(\frac{\delta F_t}{\delta A} \right)$$

Definition of Fluid

- A fluid is a substance that **deforms continuously** in the face of tangential or shear stress, **irrespective of the magnitude of shear stress** .This continuous deformation under the application of shear stress constitutes a flow.
- In this connection fluid can also be defined as the **state of matter that cannot sustain any shear stress.**

Example : Consider Fig 1.2

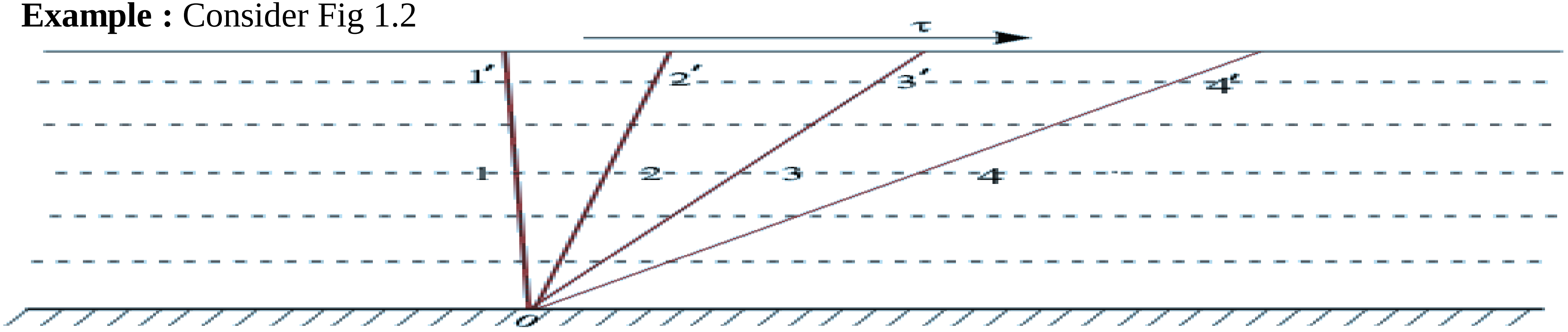


Fig 1.2 Shear stress on a fluid body

If a shear stress τ is applied at any location in a fluid, the element 011' which is initially at rest, will move to 022', then to 033'. Further, it moves to 044' and continues to move in a similar fashion.

In other words, the **tangential stress in a fluid body depends on velocity of deformation and vanishes as this velocity approaches zero. A good example is** Newton's parallel plate experiment **where dependence of shear force on the velocity of deformation was established.**

Distinction Between Solid and Fluid

Solid

- More Compact Structure
- Attractive Forces between the molecules
are larger therefore more closely packed
- Solids can resist tangential stresses in static condition
- Whenever a solid is subjected to shear stress
    a. It undergoes a definite deformationα or breaks
    b. α is proportional to shear stress upto some limiting condition
- Solid may regain partly or fully its original shape when the tangential stress is removed

Fluid

- Less Compact Structure
- Attractive Forces between the molecules
are smaller therefore more loosely packed
- Fluids cannot resist tangential stresses in static condition.
- Whenever a fluid is subjected to shear stress
    a. No fixed deformation
    b. Continious deformation takes place
until the shear stress is applied
- A fluid can never regain its original shape, once it has been distorded by the shear stress

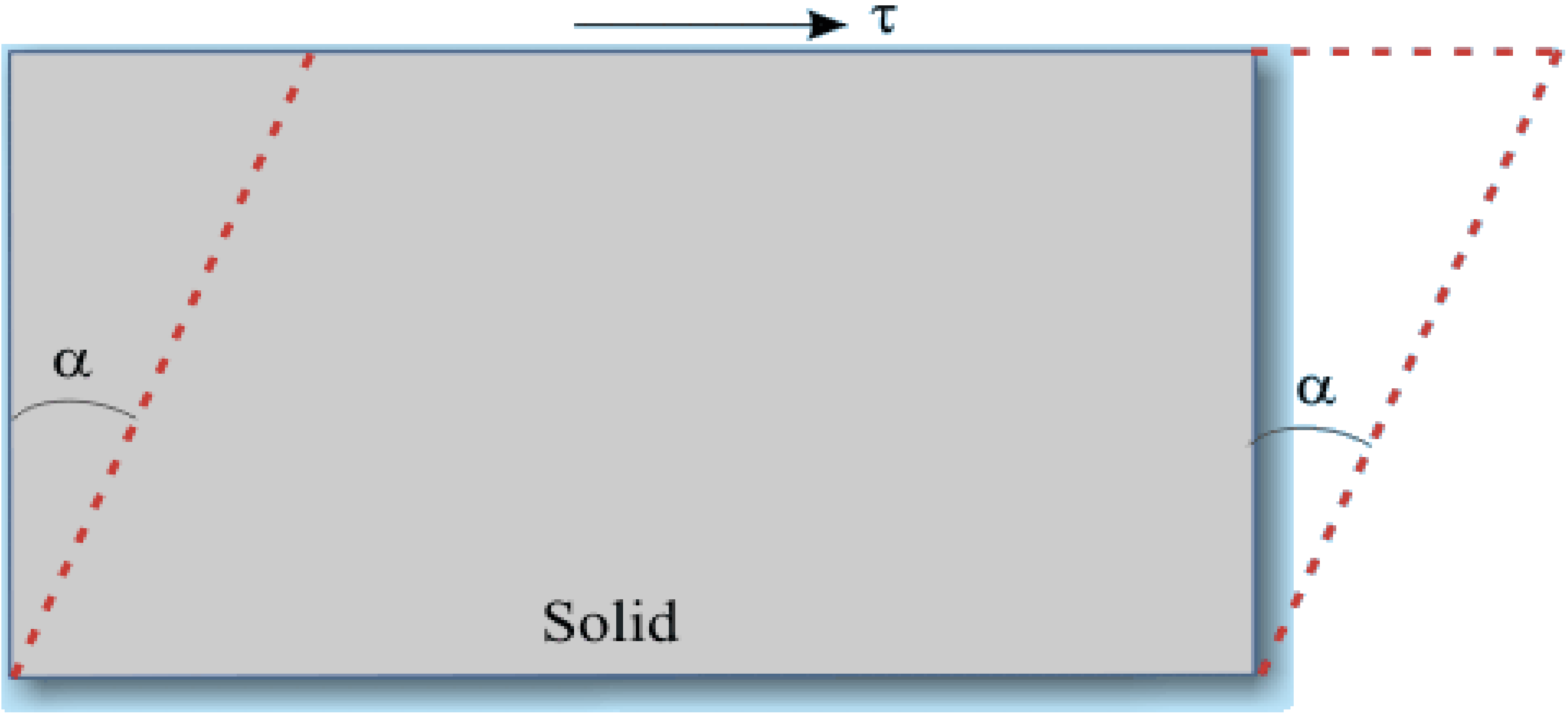
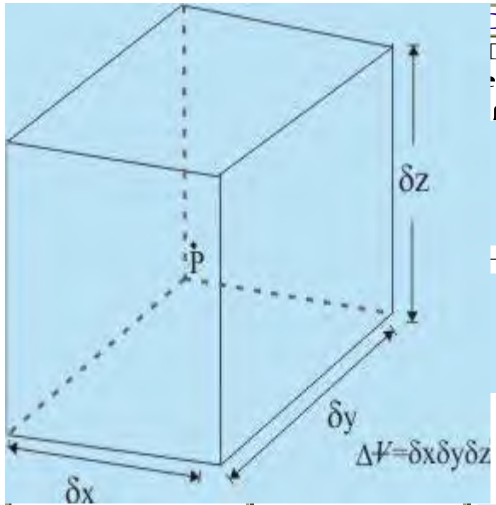
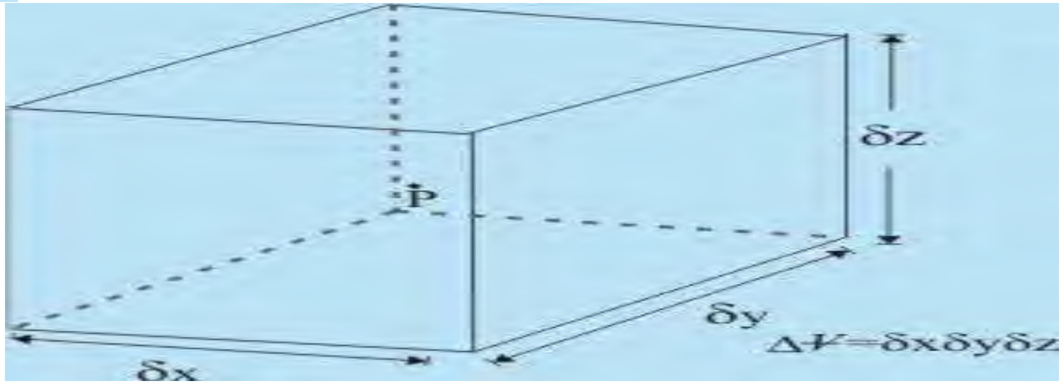


Fig 1.3 Deformation of a Solid Body

		Definition	Uni
 		The density *p* of a fluid is its mass per unit volume . If a fluid element enclosing a point *P* has a volume Δ∀ and mass Δm (Fig. 1.4), then density (ρ)at point *P* is written as $$\rho = \lim_{\Delta\forall \to \Delta\forall_C} \left(\frac{m}{\Delta\forall} \right)$$ However, in a medium where continuum model is valid one can write – $$\rho = \lim_{\Delta\forall \to 0} \left(\frac{m}{\Delta\forall} \right) = \left[\frac{dm}{d\forall} \right]_\forall \quad (1.3)$$	
Density	ρ	 **Fig 1.4 A fluid element enclosing point P**	kg/m
Specific Weight	γ	The **specific weight** is the weight of fluid per unit volume. The specific weight is given by $\gamma = \rho g$ (1.4) Where *g* is the gravitational acceleration. Just as weight must be clearly distinguished from mass, so must the specific weight be distinguished from density.	N/m

Specific Volume	**v**	The **specific volume** of a fluid is the volume occupied by unit mass of fluid. Thus $v = \frac{1}{\rho}$ (1.5)	m^3
Specific Gravity	s	For liquids, it is the ratio of density of a liquid at actual conditions to the density of pure water at 101 kN/m^2 , and at 4°C. The specific gravity of a gas is the ratio of its density to that of either hydrogen or air at some specified temperature or pressure. **However, there is no general standard; so the conditions must be stated while referring to the specific gravity of a gas.**	-

Viscosity (μ) :

- Viscosity is a fluid property whose effect is understood when the fluid is in motion.
- In a flow of fluid, when the fluid elements move with different velocities, each element will feel some resistance due to fluid friction within the elements.
- Therefore, shear stresses can be identified between the fluid elements with different velocities.
- The relationship between the shear stress and the velocity field was given by Sir Isaac Newton.

Consider a flow (Fig. 1.5) in which all fluid particles are moving in the same direction in such a way that the fluid layers move parallel with different velocities.

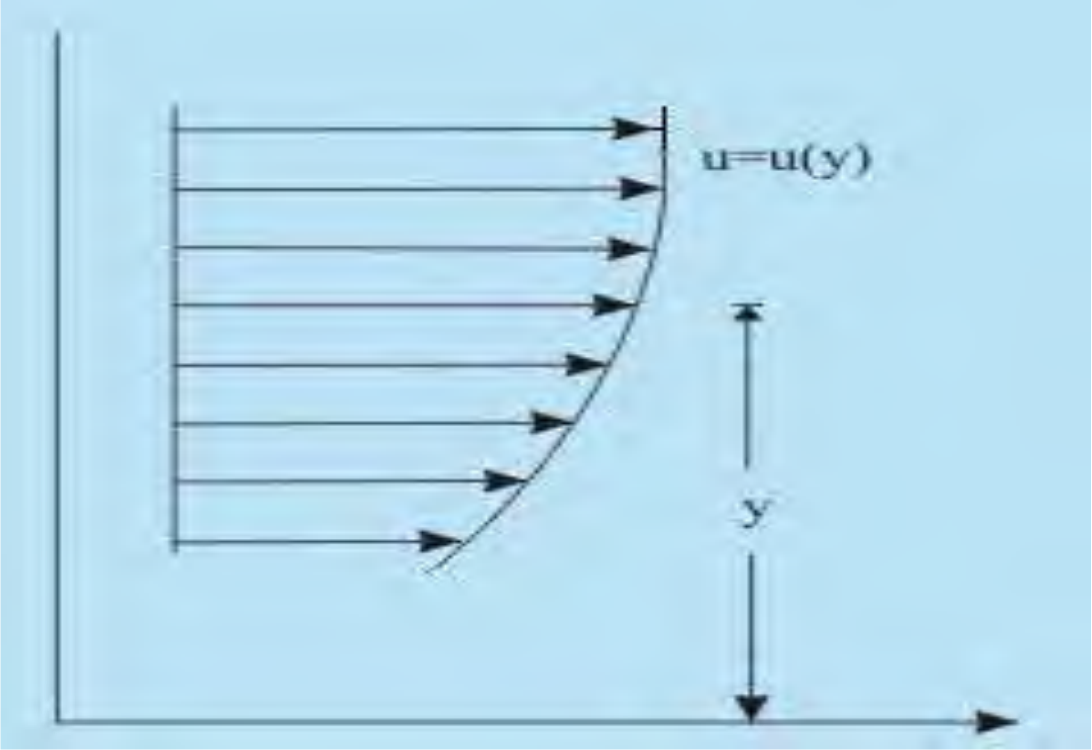


Fig 1.5 Parallel flow of a fluid

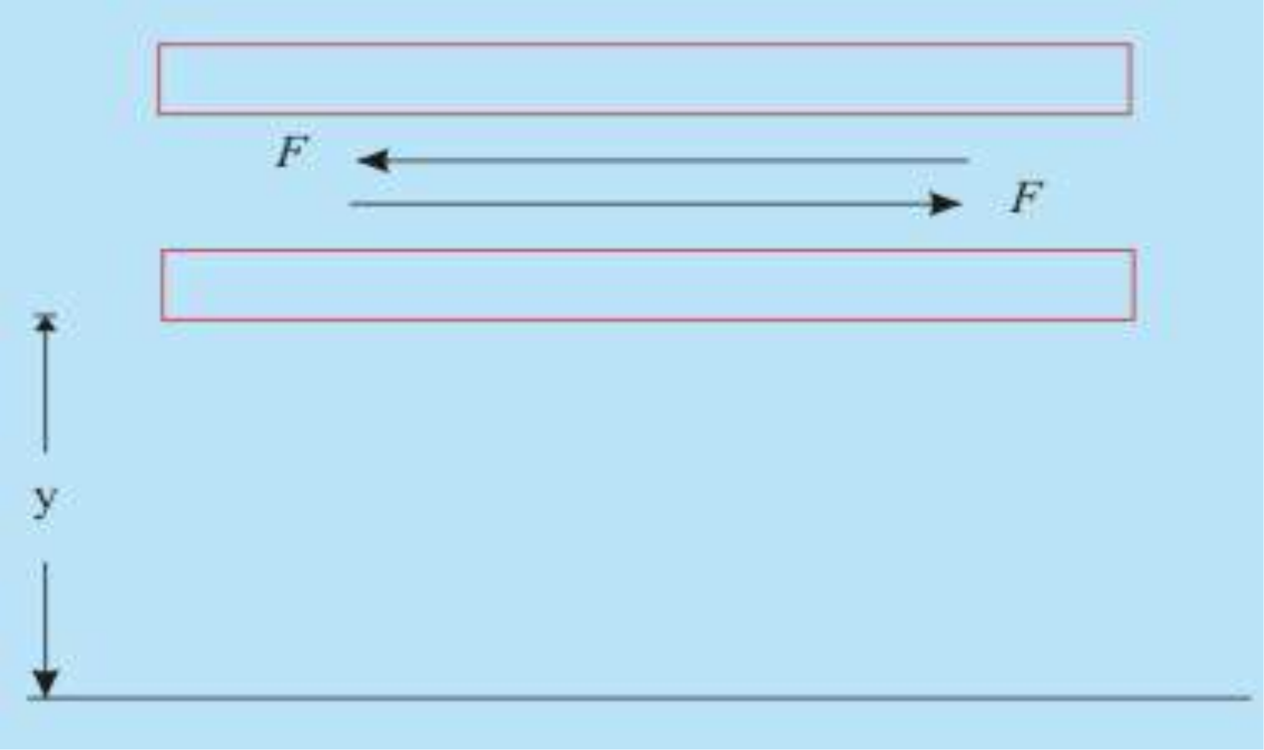


Fig 1.6 Two adjacent layers of a moving fluid.

- The upper layer, which is moving faster, tries to draw the lower slowly moving layer along with it by means of a force *F* along the direction of flow on this layer. Similarly, the lower layer tries to retard the upper one, according to Newton's third law, with an equal and opposite force *F* on it (Figure 1.6).

- Such a fluid flow where x-direction velocities, for example, change with y-coordinate is called **shear flow** of the fluid.

- Thus, the dragging effect of one layer on the other is experienced by a tangential force *F* on the respective layers. If *F* acts over an area of contact *A*, then the shear stress τ is defined as $\tau = F/A$

Viscosity (μ)

- Newton postulated that τ is proportional to the quantity Δu/ Δy where Δy is the distance of separation of the two layers and Δu is the difference in their velocities.

- In the limiting case of , Δu / Δy equals du/dy, the velocity gradient at a point in a direction perpendicular to the direction of the motion of the layer.

- According to Newton τ and du/dy bears the relation $\tau = \mu \frac{du}{dy}$

- where, the constant of proportionality μ is known as the **coefficient of viscosity** or simply viscosity which is a property of the fluid and depends on its state.
- Sign of τ depends upon the sign of du/dy.
- For the profile shown in Fig. 1.5, du/dy is positive everywhere and hence, τ is positive.
- Both the velocity and stress are considered positive in the positive direction of the coordinate parallel to them.

Equation $$\tau = \mu \frac{du}{dy}$$

Causes of Viscosity

- The causes of viscosity in a fluid are possibly attributed to two factors:

(i) intermolecular force of cohesion

(ii) molecular momentum exchange

Due to strong cohesive forces between the molecules, any layer in a moving fluid tries to drag the adjacent layer to move with an equal speed and thus produces the effect of viscosity as discussed earlier. Since cohesion decreases with temperature, the liquid viscosity does likewise

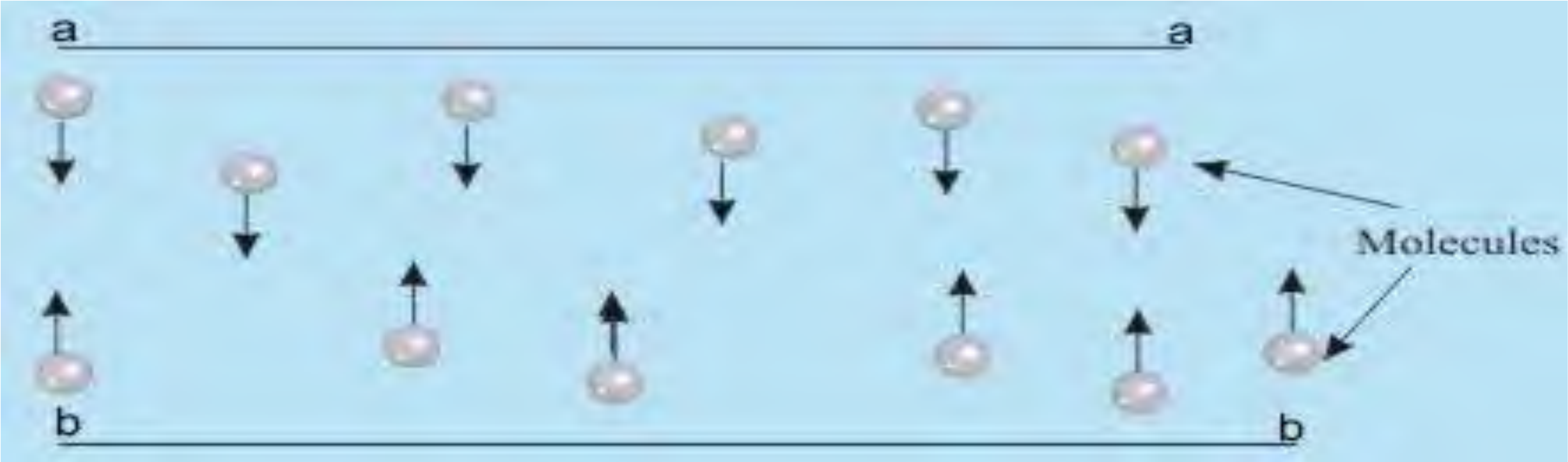


Fig 1.7 Movement of fluid molecules between two adjacent moving layers

- As the random molecular motion increases with a rise in temperature, the viscosity also increases accordingly. Except for very special cases (e.g., at very high pressure) the viscosity of both liquids and gases ceases to be a function of pressure.
- For Newtonian fluids, the coefficient of viscosity depends strongly on temperature but varies very little with pressure.
- For liquids, molecular motion is less significant than the forces of cohesion, thus **viscosity of liquids decrease with increase in temperature.**
- For gases, molecular motion is more significant than the cohesive forces, thus **viscosity of gases increase with increase in temperature.**

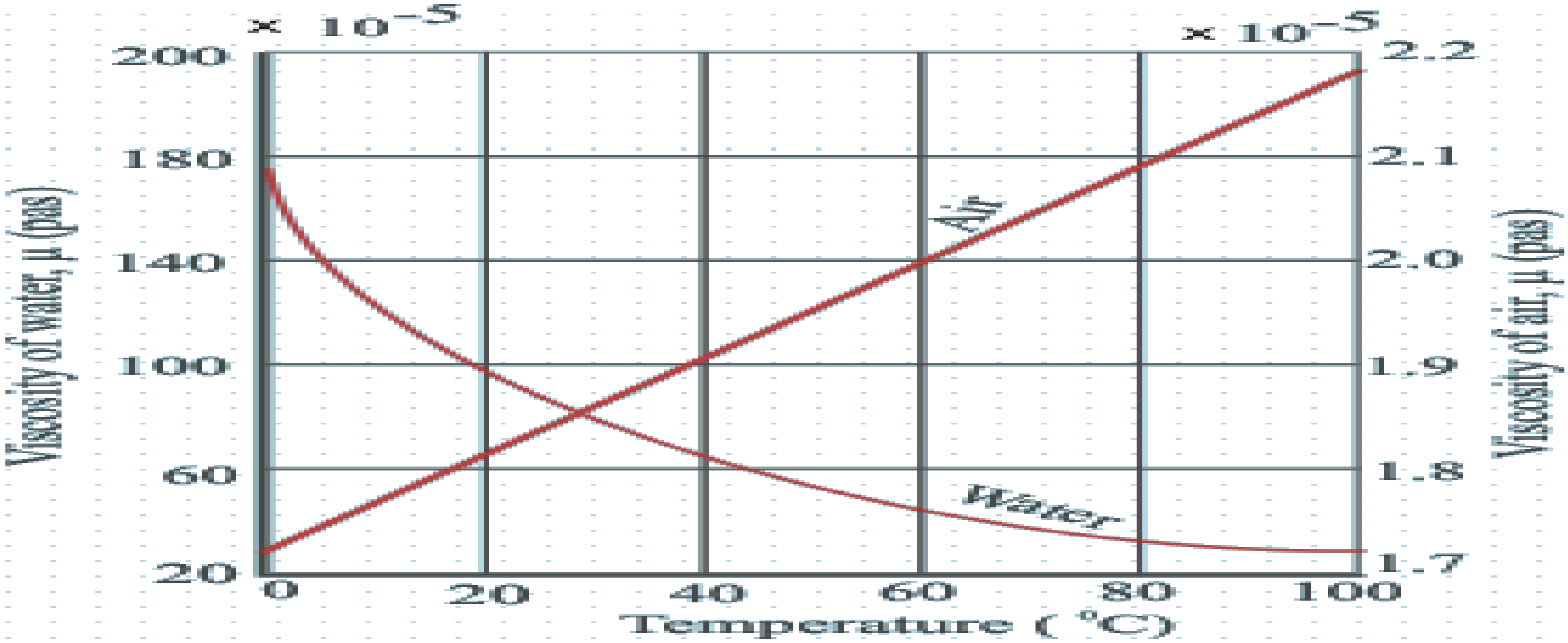


Fig 1.8: Change of Viscosity of Water and Air under 1 atm

No-slip Condition of Viscous Fluids

- It has been established through experimental observations that the relative velocity between the solid surface and the adjacent fluid particles is zero whenever a viscous fluid flows over a solid surface. This is known as no-slip condition.

- This behavior of no-slip at the solid surface is not same as the wetting of surfaces by the fluids. For example, mercury flowing in a stationary glass tube will not wet the surface, but will have zero velocity at the wall of the tube.

- The wetting property results from surface tension, whereas the no-slip condition is a consequence of fluid viscosity.

Ideal Fluid

- Consider a hypothetical fluid having a zero viscosity ($\mu = 0$). Such a fluid is called an ideal fluid and the resulting motion is called as **ideal** or **inviscid flow**. **In an ideal flow, there is no existence of shear force because of vanishing viscosity.**

$$\tau = \mu \frac{du}{dy} = 0 \qquad \text{since } \mu = 0$$

- All the **fluids in reality have viscosity** ($\mu > 0$) and hence they are termed as real fluid and their motion is known as viscous flow.

- Under certain situations of very high velocity flow of viscous fluids, an accurate analysis of flow field away from a solid surface can be made from the ideal flow theory.

Deformation of Fluids

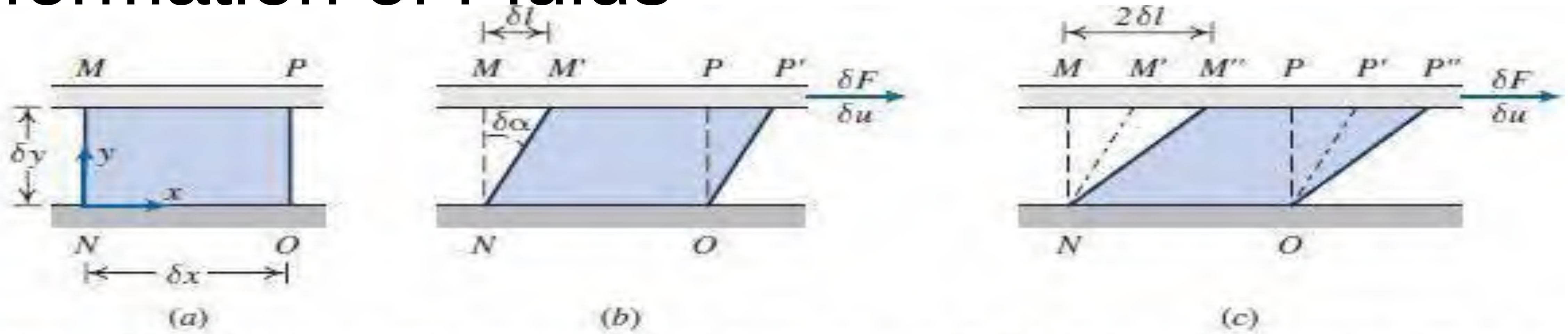


Fig. 2.9 (*a*) Fluid element at time t, (*b*) deformation of fluid element at time $t + \delta t$, and (*c*) deformation of fluid element at time $t + 2\delta t$.

$$\tau_{yx} = \lim_{\delta A_y \to 0} \frac{\delta F_x}{\delta A_y} = \frac{dF_x}{dA_y}$$

$$\text{deformation rate} = \lim_{\delta t \to 0} \frac{\delta \alpha}{\delta t} = \frac{d\alpha}{dt}$$

$$\frac{d\alpha}{dt} = \frac{du}{dy}$$

Newtonian Fluids

- Fluids in which shear stress is **directly proportional to rate of deformation are called Newtonian fluids**
- Most common fluids such as water, air, and gasoline are Newtonian under normal conditions
- If a fluid is Newtonian then:

$$\tau_{yx} \propto \frac{du}{dy}$$

$$\tau_{yx} = \mu \frac{du}{dy}$$

- The constant of proportionality is called **Absolute or Dynamic viscosity** denoted by μ.
- The ratio of absolute viscosity to density is called **Kinematic Viscosity** and is denoted by ν.

Non Newtonian Fluids

- Fluids in which shear stress is not directly proportional to deformation rate are non-Newtonian
- Examples are toothpaste and Lucite5 paint.
- The paint is very “thick” when in the can, but becomes “thin” when sheared by brushing.
- Toothpaste behaves as a “fluid” when squeezed from the tube. However, it does not run out by itself when the cap is removed.
- There is a threshold or yield stress below which

Apparent Viscosity

- The viscosity is normally constant but apparent viscosity depends upon shear rate and may be much higher at certain shear rates for non Newtonian fluids
- Mathematically : $\eta = k|du/dy|^{n-1}$

Types of Non Newtonian fluids

- Fluids in which the apparent viscosity decreases with increasing deformation rate ($n<1$) are called **pseudoplastic** (or shear thinning) fluids.
- Examples are polymer solutions, colloidal suspensions, and paper pulp in water
- If the apparent viscosity increases with increasing deformation rate ($n>1$) the fluid is termed **dilatant (or shear thickening)**. Suspensions of starch and of sand are examples of dilatant fluids
- On the beach—if you walk slowly (and hence generate a low shear rate) on very wet sand, you sink into it, but if you jog on it (generating a high shear rate), it's very firm.

Types of Non Newtonian fluids

- A "fluid" that behaves as a solid until a minimum yield stress, τ_y, is exceeded and subsequently exhibits a linear relation between stress and rate of deformation is referred to as **an ideal or Bingham plastic**. The corresponding shear stress model is:

$$\tau_{yx} = \tau_y + \mu_p \frac{du}{dy}$$

- Clay suspensions, drilling muds, and toothpaste are examples of substances exhibiting this behavior

Types of Non Newtonian fluids

- **Thixotropic fluids:** **N**on-Newtonian fluids in which apparent viscosity may be time-dependent i.e. show a decrease in η with time under a constant applied shear stress; many paints are thixotropic.
- **Rheopectic:** Non Newtonian fluids that show an increase in η with time hence called Rheopectic.
- **Viscoelastic:** After deformation some fluids partially return to their original shape when the applied stress is released; such fluids are called viscoelastic (many biological fluids work this way).

Surface tension

You can tell when your car needs waxing: Water droplets tend to appear somewhat flattened out. After waxing, you get a nice "beading" effect. These two cases are shown in Fig. 2.11. We define a liquid as "wetting" a surface when the *contact angle* $\theta < 90°$. By this definition, the car's surface was wetted before waxing, and not wetted after. This is an example of effects due to *surface tension*. Whenever a liquid is in contact with other liquids or gases, or in this case a gas/solid surface, an interface develops that acts like a stretched elastic membrane, creating surface tension.

There are two features to this membrane: the contact angle, θ, and the magnitude of the surface tension, σ (N/m). Both of these depend on the type of liquid and the type of solid surface (or other liquid or gas) with which it shares an interface. In the car-waxing example, the contact angle changed from being smaller than 90° to larger than 90° because, in effect, the waxing changed the nature of the solid surface. Factors that affect the contact angle include the cleanliness of the surface and the purity of the liquid.

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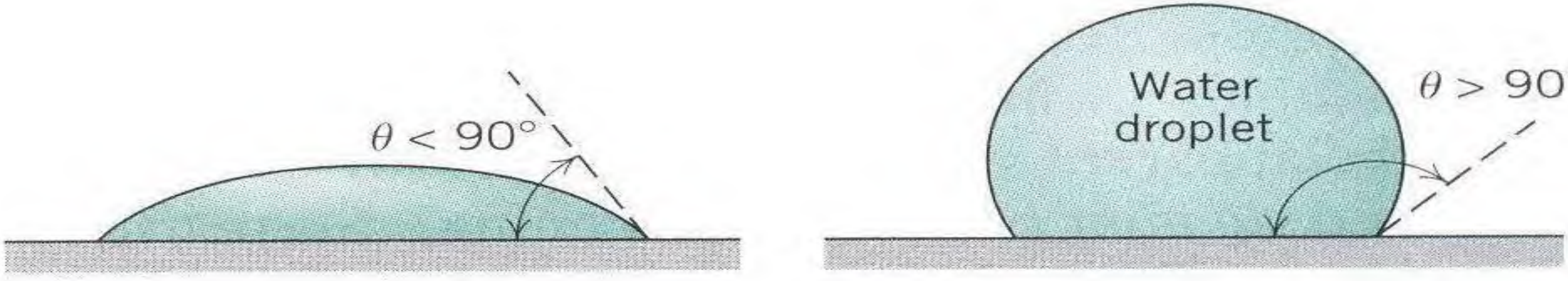

(*a*) A "wetted" surface (*b*) A nonwetted surface

Surface tension effects on water droplets.

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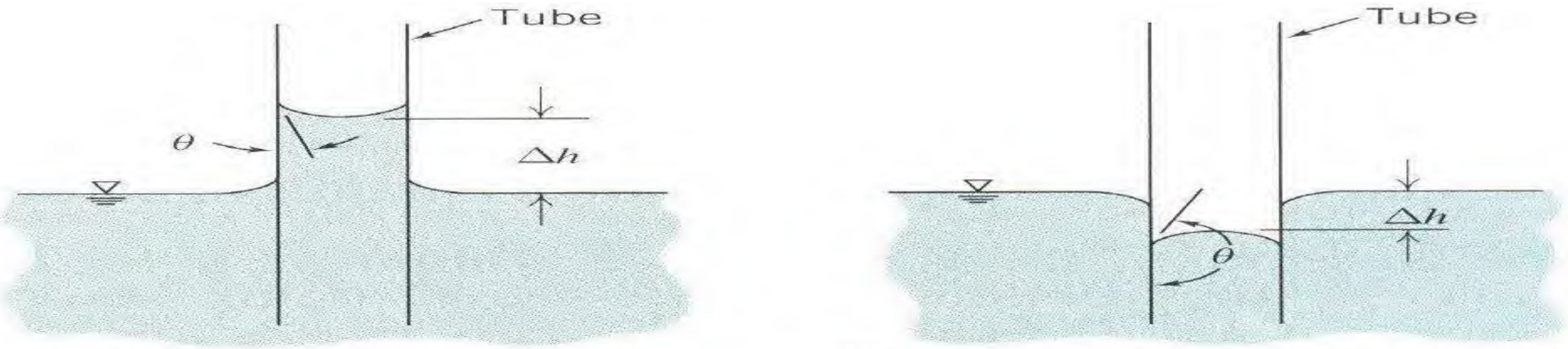

(a) Capillary rise ($\theta < 90°$) (b) Capillary depression ($\theta > 90°$)

Capillary rise and capillary depression inside and outside a circular tube.

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Example 2.3 ANALYSIS OF CAPILLARY EFFECT IN A TUBE

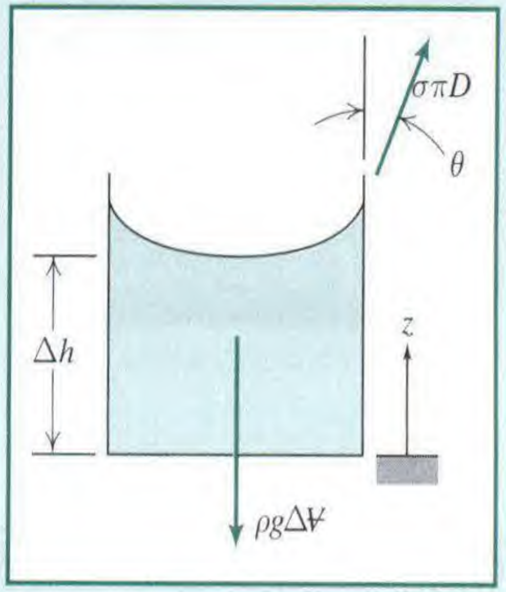
Create a graph showing the capillary rise or fall of a column of water or mercury, respectively, as a function of tube diameter D. Find the minimum diameter of each column required so that the height magnitude will be less than 1 mm.

Given: Tube dipped in liquid as in Fig. 2.12.

Find: A general expression for Δh as a function of D.

Solution:

Apply free-body diagram analysis, and sum vertical forces.



Governing equation:

$$\sum F_z = 0$$

Assumptions: (1) Measure to middle of meniscus

(2) Neglect volume in meniscus region

Summing forces in the z direction:

$$\sum F_z = \sigma \pi D \cos\theta - \rho g \Delta \forall = 0 \qquad (1)$$

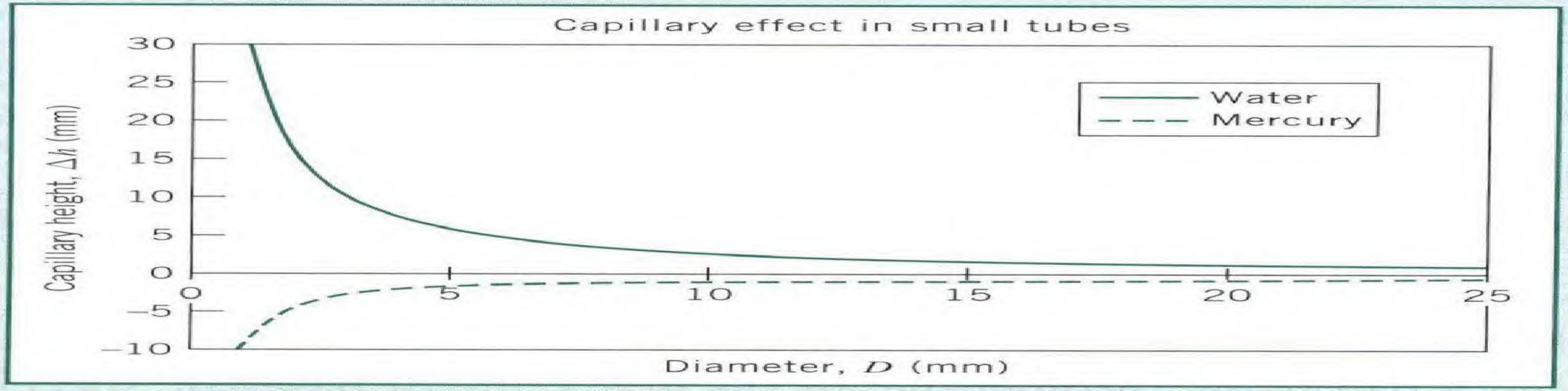
If we neglect the volume in the meniscus region:

$$\Delta V \approx \frac{\pi D^2}{4}\Delta h$$

Substituting in Eq. (1) and solving for Δh gives

$$\Delta h = \frac{4\sigma \cos\theta}{\rho g D}$$

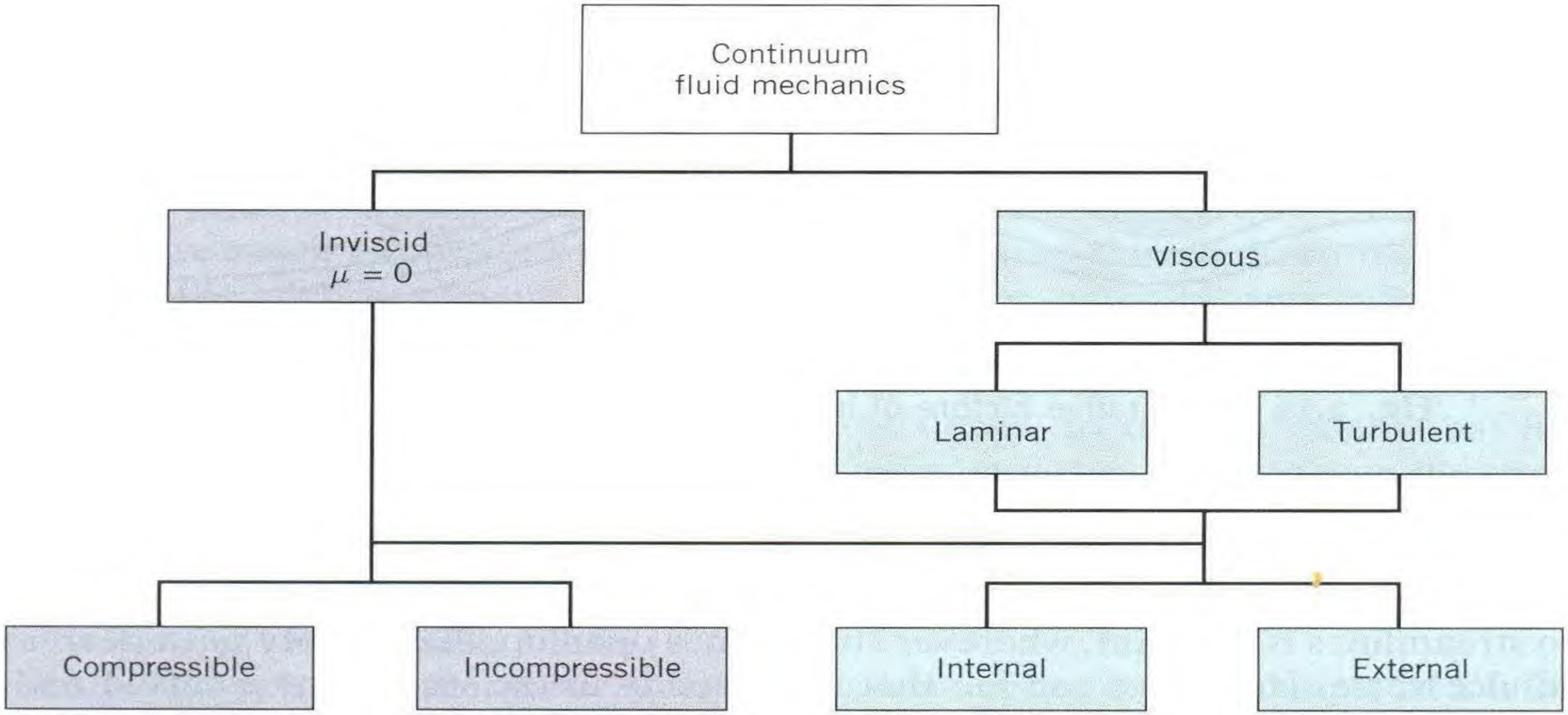
For water, $\sigma = 72.8$ mN/m and $\theta \approx 0°$, and for mercury, $\sigma = 484$ mN/m and $\theta = 140°$ (Table A.4).



Using the above equation to compute D_{min} for $\Delta h = 1$ mm, we find for mercury and water

$$D_{M_{min}} = 11.2 \text{ mm} \quad \text{and} \quad D_{W_{min}} = 30 \text{ mm}$$

Viscous and Invicid flows



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Reynolds No

- A number given by $Re = \rho \frac{VL}{\mu}$
- It is used to predict whether viscous forces acting on a body are negligible as compared to pressure forces or not
- If Re is high, viscous forces are negligible
- If it is low then the viscous forces are not negligible
- If it is neither small nor large, no general conclusion can be drawn

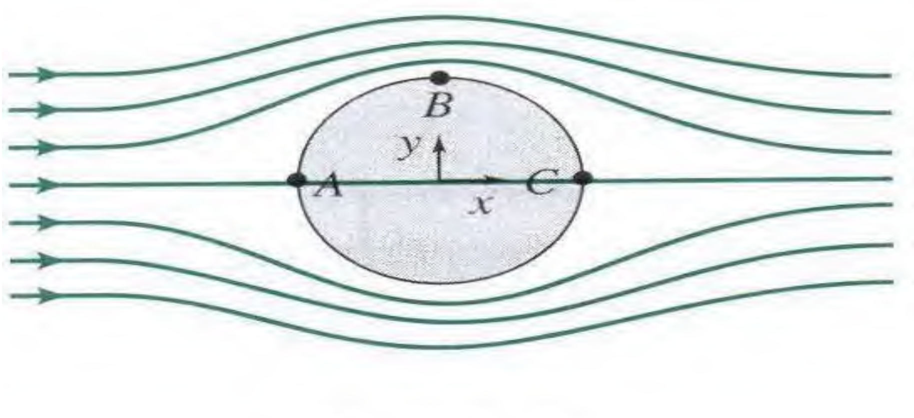
Reynolds No

$$Re = \rho \frac{VL}{\mu}$$

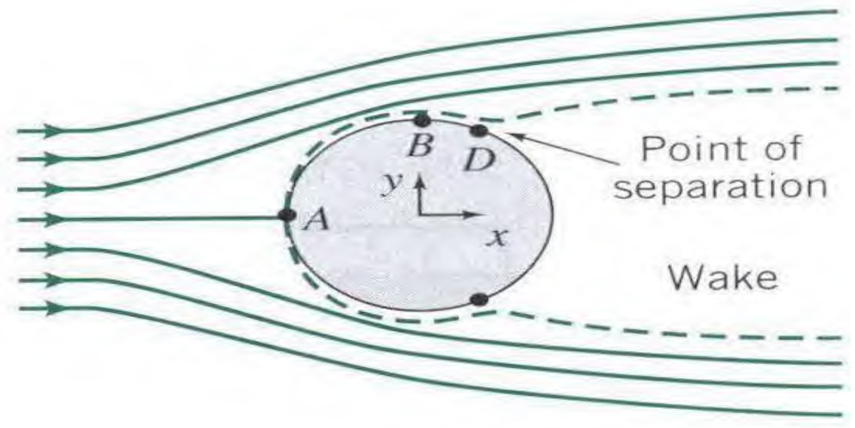
To illustrate this very powerful idea, consider two simple examples. First, the drag on your ball: Suppose you kick a soccer ball (diameter = 22.23 cm) so it moves at 97 km/h. The Reynolds number (using air properties from Table A.10) for this case is about 400,000—by any measure a large number; hence the drag on the soccer ball is almost entirely due to the pressure build-up in front of it. For our second example, consider a dust particle (modeled as a sphere of diameter 1 mm) falling under gravity at a terminal velocity of 1 cm/s: In this case $Re \approx 0.7$—a quite small number; hence the drag is mostly due to the friction of the air. Of course, in both of these examples, if we wish to *determine* the drag force, we would have to do substantially more analysis.

Various concepts

- **Inviscid Flow**: A friction less flow is called inviscid flow. It has no Viscosity effects
- **Viscous Flow**: A flow which involves force of friction is called viscous flow
- **Stagnation points**: where velocity is zero



(a) Inviscid flow



(b) Viscous flow

Boundary layer

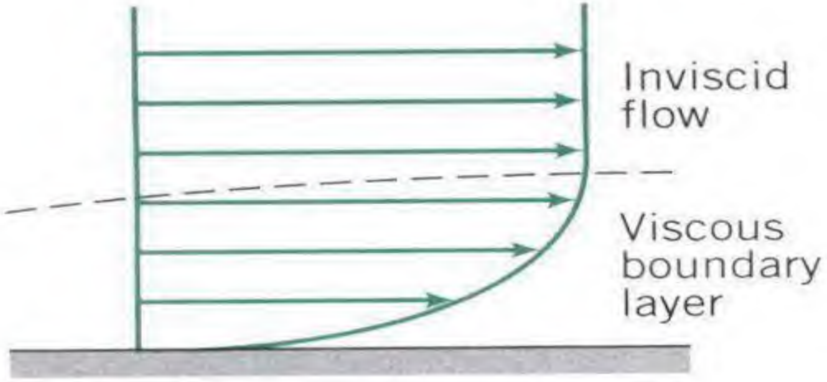


Fig. 2.15 Schematic of a boundary layer.

Prandtl suggested that even though friction is negligible in general for high-Reynolds number flows, there will always be a thin *boundary layer*, in which friction is significant and across the width of which the velocity increases rapidly from zero (at the surface) to the value inviscid flow theory predicts (on the outer edge of the boundary layer). This is shown in Fig. 2.14*b* from point *A* to point *B*, and in more detail in Fig. 2.15.

Boundary layer

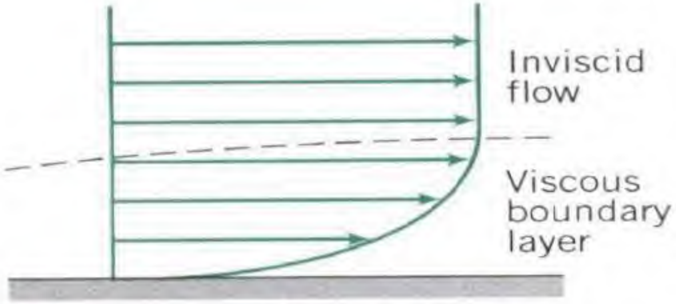
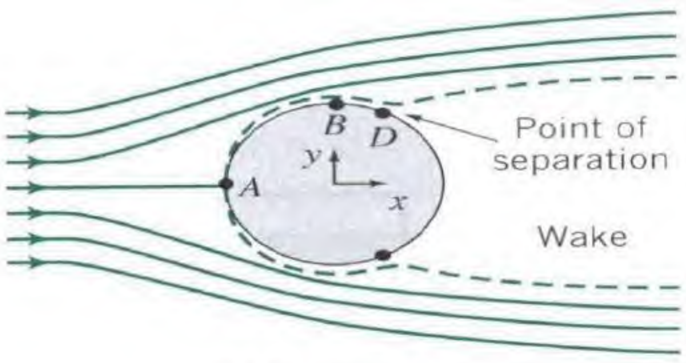


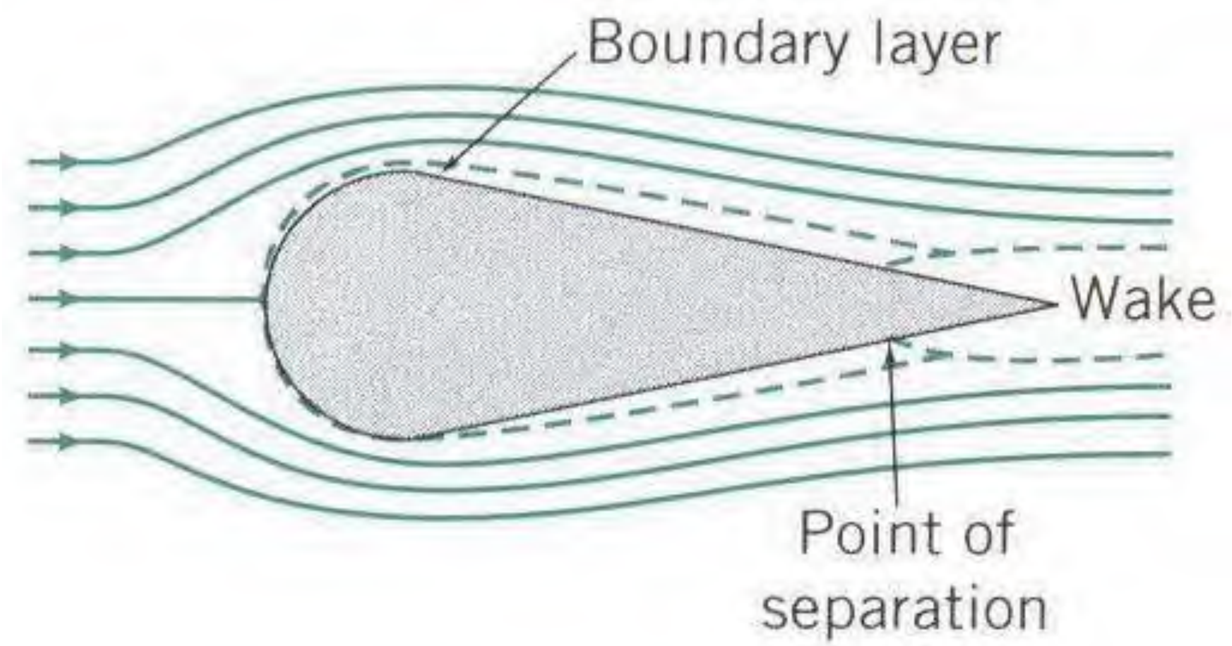
Fig. 2.15 Schematic of a boundary layer.



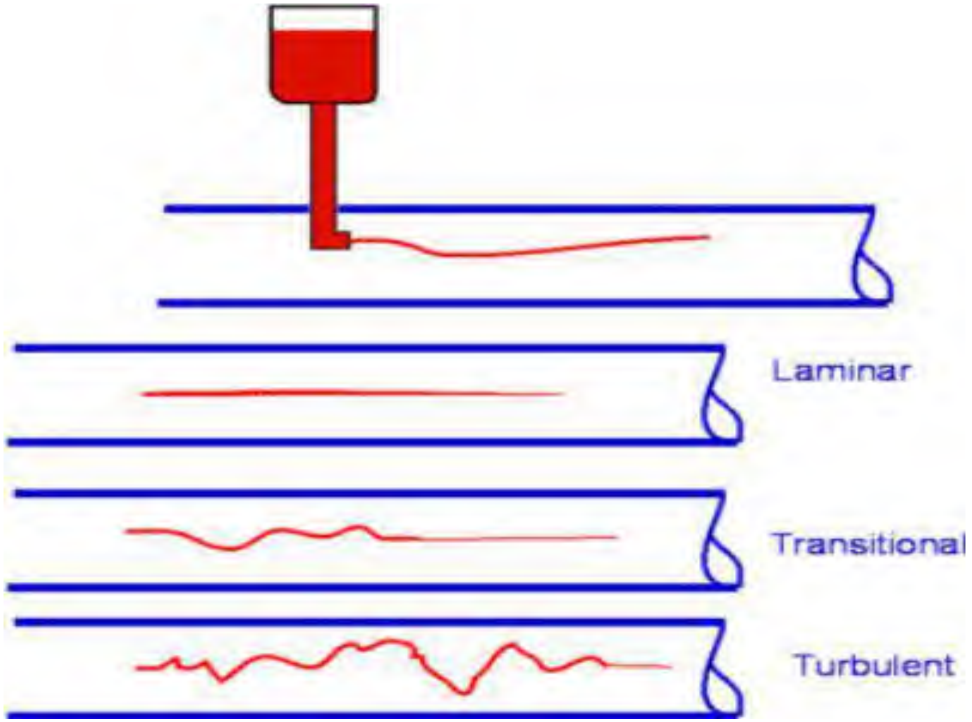
(*b*) Viscous flow

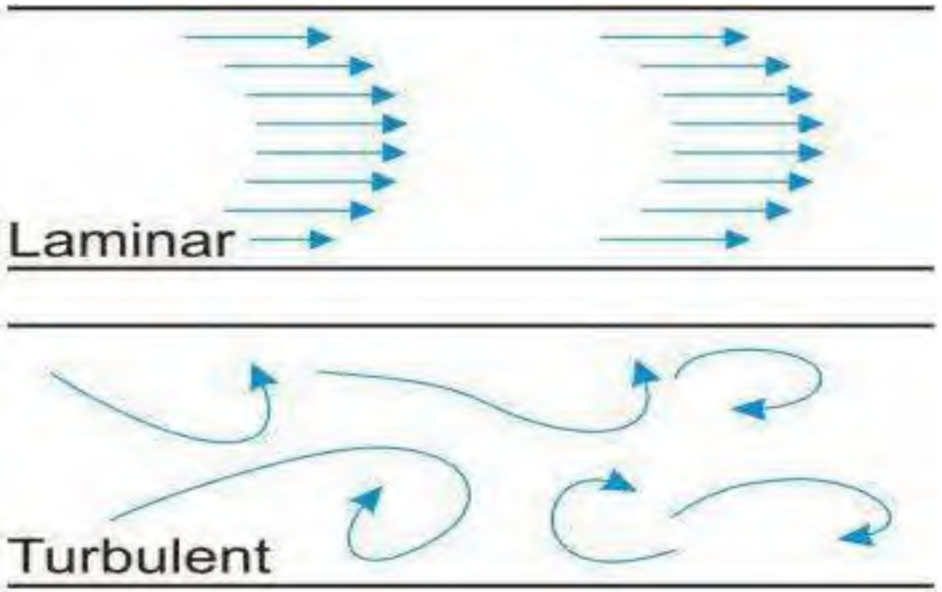
Once we have friction in a boundary layer we will have drag. However, this boundary layer has another important consequence: It often leads to bodies having a *wake*, as shown in Fig. 2.14*b* from point *D* onwards. Point *D* is a *separation point*, where fluid particles are pushed off the object and cause a wake to develop. Consider once again the original inviscid flow (Fig. 2.14*a*): As a particle moves along the surface from point *B* to *C*, it moves from low to high pressure. This *adverse pressure gradient* (a pressure change opposing fluid motion) causes the particles to slow down as they move along the rear of the sphere. If we now add to this the fact that the particles are moving in a boundary layer with friction that also slows down the fluid, the particles will eventually be brought to rest and then pushed off the sphere by the following particles, forming the wake. This is generally very bad news: It turns out that the wake will always be relatively low pressure, but the front of the sphere will still have relatively high pressure. Hence, the sphere will now have a quite large *pressure drag* (or *form drag*—so called because it's due to the shape of the object).

Boundary layer over a streamlined object



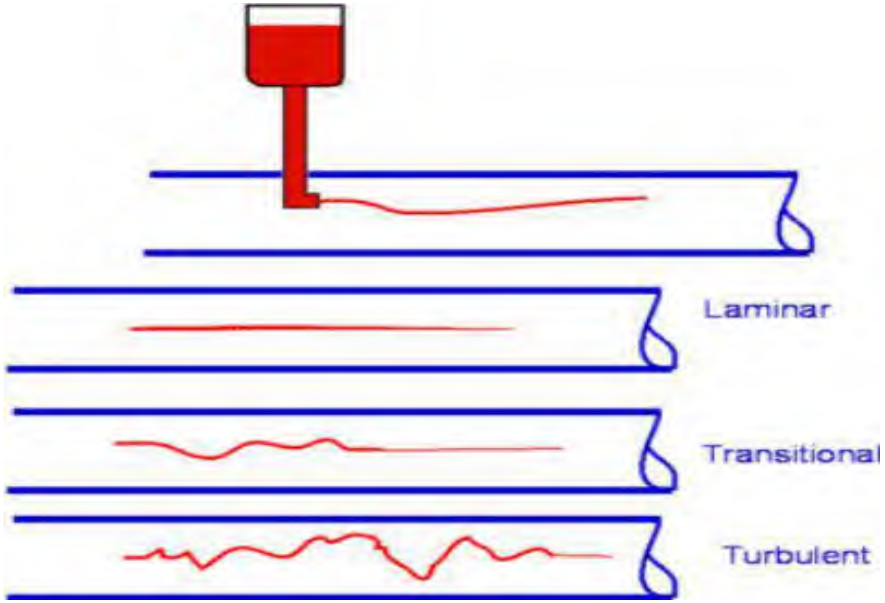
Laminar and Turbulent Flows

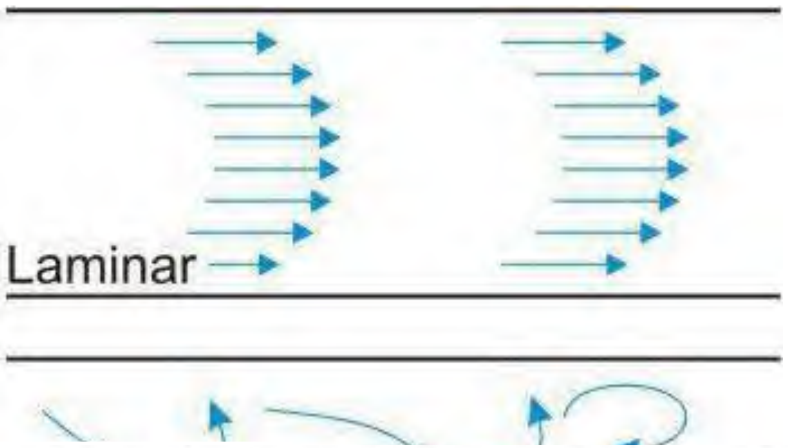




A *laminar* flow is one in which the fluid particles move in smooth layers, or laminas; a *turbulent* flow is one in which the fluid particles rapidly mix as they move along due to random three-dimensional velocity fluctuations.

Laminar and Turbulent Flows





The velocity of the laminar flow is simply u; the velocity of the turbulent flow is given by the mean velocity $\bar{u}$ plus the three components of randomly fluctuating velocity u', v', and w'.

In a one-dimensional laminar flow, the shear stress is related to the velocity gradient by the simple relation

$$\tau_{yx} = \mu \frac{du}{dy} \tag{2.15}$$

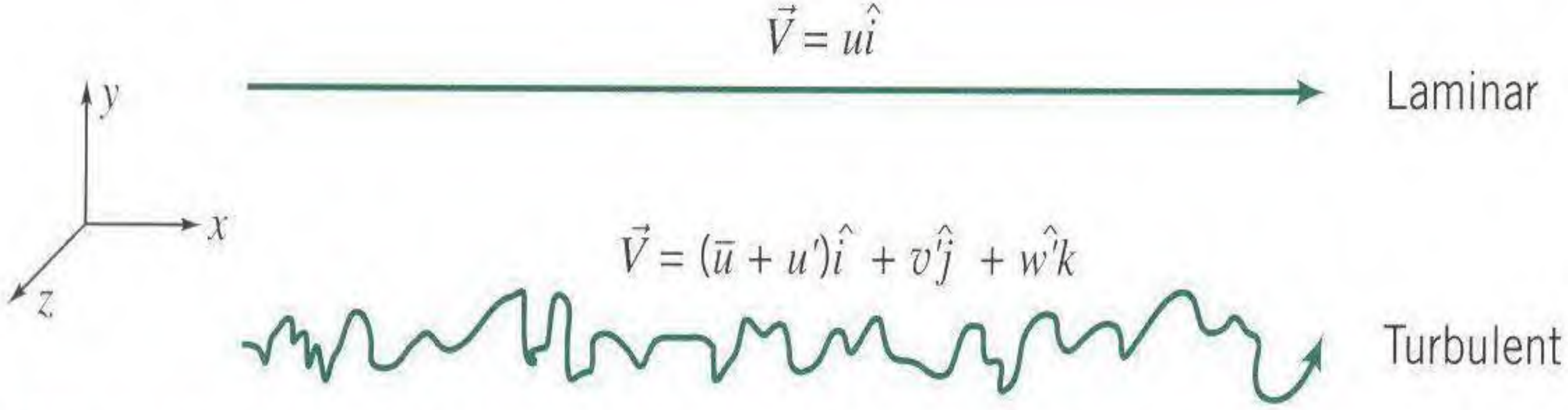


Fig. 2.17 Particle pathlines in one-dimensional laminar and turbulent flows.

Compressible and incompressible flows

Flows in which variations in density are negligible are termed *incompressible*; when density variations within a flow are not negligible, the flow is called *compressible*. The most common example of compressible flow concerns the flow of gases, while the flow of liquids may frequently be treated as incompressible.

compressibility effects in liquids can be important. Pressure and density changes in liquids are related by the *bulk compressibility modulus*, or modulus of elasticity,

$$E_v \equiv \frac{dp}{(d\rho/\rho)} \tag{2.19}$$

If the bulk modulus is independent of temperature, then density is only a function of pressure (the fluid is *barotropic*). Bulk modulus data for some common liquids are given in Appendix A.

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Compressible and incompressible flows

the ratio of the flow speed, V, to the local speed of sound, c, in the gas is defined as the Mach number,

$$M \equiv \frac{V}{c}$$

For $M < 0.3$, the maximum density variation is less than 5 percent. Thus gas flows with $M < 0.3$ can be treated as incompressible; a value of $M = 0.3$ in air at standard conditions corresponds to a speed of approximately 100 m/s. For example, although it might

Internal and External Flows

Flows completely bounded by solid surfaces are called *internal* or *duct flows*. Flows over bodies immersed in an unbounded fluid are termed *external flows*. Both internal and external flows may be laminar or turbulent, compressible or incompressible.

we have a Reynolds number for pipe flows defined as $Re = \rho \bar{V} D/\mu$, where $\bar{V}$ is the average flow velocity and D is the pipe diameter (note that we do *not* use the pipe length!). This Reynolds number indicates whether a pipe flow will be laminar or turbulent. Flow will generally be laminar for $Re \leq 2300$ and turbulent for larger values: Flow in a pipe of constant diameter will be entirely laminar or entirely turbulent, depending on the value of the velocity $\bar{V}$. We will explore internal flows in

Summary and Useful equations

- ✓ How to describe flows (timelines, pathlines, streamlines, streaklines).
- ✓ Forces (surface, body) and stresses (shear, normal).
- ✓ Types of fluids (Newtonian, non-Newtonian—dilatant, pseudoplastic, thixotropic, rheopectic, Bingham plastic) and viscosity (kinematic, dynamic, apparent).
- ✓ Types of flow (viscous/inviscid, laminar/turbulent, compressible/incompressible, internal/external).

We also briefly discussed some interesting phenomena, such as surface tension, boundary layers, wakes, and streamlining. Finally, we introduced two very useful dimensionless groups—the Reynolds number and the Mach number.

Summary and Useful equations

Definition of specific gravity:	$SG = \frac{\rho}{\rho_{H_2O}}$
Definition of specific weight:	$\gamma = \frac{mg}{\forall} \rightarrow \gamma = \rho g$
Definition of streamlines (2D):	$\left.\frac{dy}{dx}\right)_{\text{streamline}} = \frac{v(x,y)}{u(x,y)}$
Definition of pathlines (2D):	$\left.\frac{dx}{dt}\right)_{\text{particle}} = u(x,y,t) \qquad \left.\frac{dy}{dt}\right)_{\text{particle}} = v(x,y,t)$
Definition of streaklines (2D):	$x_{\text{streakline}}(t_0) = x(t, x_0, y_0, t_0) \qquad y_{\text{streakline}}(t_0) = y(t, x_0, y_0, t_0)$
Newton's law of viscosity (1D flow):	$\tau_{yx} = \mu \frac{du}{dy}$
Shear stress for a non-Newtonian fluid (1D flow):	$\tau_{yx} = k \left\lvert \frac{du}{dy} \right\rvert^{n-1} \frac{du}{dy} = \eta \frac{du}{dy}$

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UNIT-II
Fluid Kinematics

Overview

- Fluid Kinematics deals with the motion of fluids without considering the forces and moments which create the motion.
- Items discussed in this Chapter.
  - Material derivative and its relationship to Lagrangian and Eulerian descriptions of fluid flow.
  - Flow visualization.
  - Plotting flow data.
  - Fundamental kinematic properties of fluid motion and deformation.
  - Reynolds Transport Theorem

Lagrangian Description

- Lagrangian description of fluid flow tracks the position and velocity of individual particles.
- Based upon Newton's laws of motion.
- Difficult to use for practical flow analysis.
  - Fluids are composed of *billions* of molecules.
  - Interaction between molecules hard to describe/model.
- However, useful for specialized applications
  - Sprays, particles, bubble dynamics, rarefied gases.
  - Coupled Eulerian-Lagrangian methods.
- Named after Italian mathematician Joseph Louis Lagrange (1736-1813).

Eulerian Description

- Eulerian description of fluid flow: a **flow domain** or **control volume** is defined by which fluid flows in and out.
- We define **field variables** which are functions of space and time.
  - Pressure field, P=P(x,y,z,t)
  - Velocity field,

$$\vec{V} = \vec{V}(x, y, z, t)$$

$$\vec{V} = u(x, y, z, t)\vec{i} + v(x, y, z, t)\vec{j} + w(x, y, z, t)\vec{k}$$

  - Acceleration field, $\vec{a} = \vec{a}(x, y, z, t)$

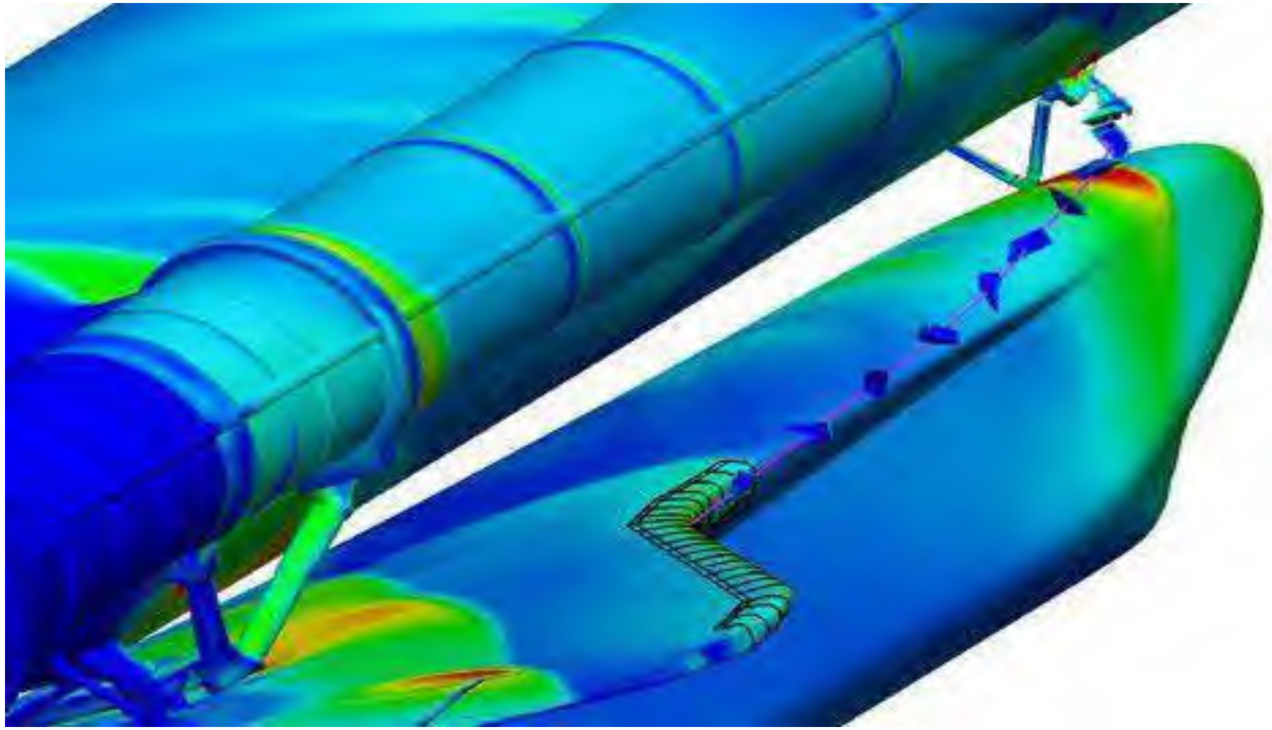
$$\vec{a} = a_x(x, y, z, t)\vec{i} + a_y(x, y, z, t)\vec{j} + a_z(x, y, z, t)\vec{k}$$

  - These (and other) field variables define the **flow field**.
- Well suited for formulation of initial boundary-value problems (PDE's).
- Named after Swiss mathematician Leonhard Euler (1707-1783).

Example: Coupled Eulerian-Lagrangian Method

- Global Environmental MEMS Sensors (GEMS)
- Simulation of micron-scale airborne probes. The probe positions are tracked using a Lagrangian particle model embedded within a flow field computed using an Eulerian CFD code.

Example: Coupled Eulerian-Lagrangian Method



Forensic analysis of Columbia accident: simulation of shuttle debris trajectory using Eulerian CFD for flow field and Lagrangian method for the debris.

Acceleration Field

- Consider a fluid particle and Newton's second law,

$$\vec{F}_{particle} = m_{particle} \vec{a}_{particle}$$

- The acceleration of the particle is the time derivative of the particle's velocity.

$$\vec{a}_{particle} = \frac{d\vec{V}_{particle}}{dt}$$

- To take the time derivative of, chain rule must be used.

$$\vec{a}_{particle} = \frac{\partial \vec{V}}{\partial t}\frac{dt}{dt} + \frac{\partial \vec{V}}{\partial x}\frac{dx_{particle}}{dt} + \frac{\partial \vec{V}}{\partial y}\frac{dy_{particle}}{dt} + \frac{\partial \vec{V}}{\partial z}\frac{dz_{particle}}{dt}$$

Acceleration Field

- Since $\frac{dx_{particle}}{dt} = u, \frac{dy_{particle}}{dt} = v, \frac{dz_{particle}}{dt} = w$

$$\vec{a}_{particle} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

- In vector form, the acceleration can be written as

$$\vec{a}(x, y, z, t) = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \cdot \vec{\nabla}\right)\vec{V}$$

- First term is called the **local acceleration** and is nonzero only for unsteady flows.
- Second term is called the **advective acceleration** and accounts for the effect of the fluid particle moving to a new location in the flow, where the

Material Derivative

- The total derivative operator d/dt is call the **material derivative** and is often given special notation, D/Dt.

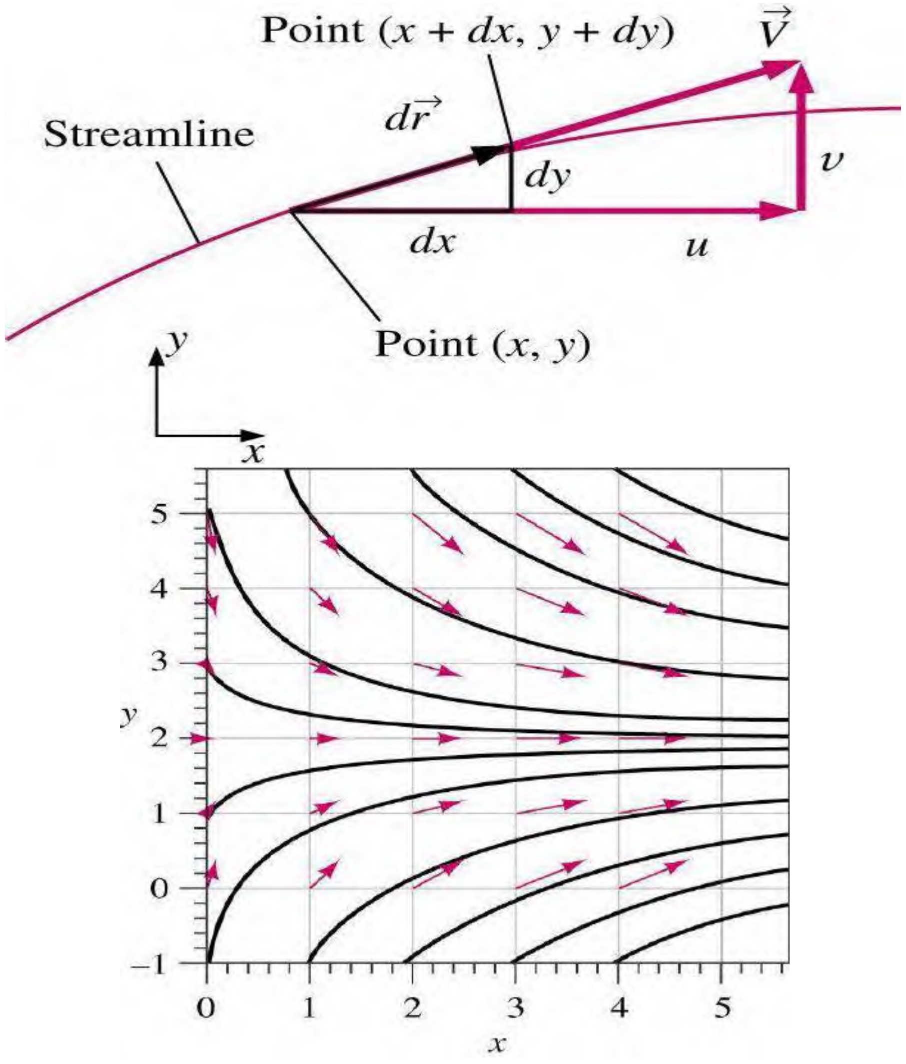
$$\frac{D\vec{V}}{Dt} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \cdot \vec{\nabla}\right)\vec{V}$$

- Advective acceleration is nonlinear: source of many phenomenon and primary challenge in solving fluid flow problems.
- Provides ``transformation'' between Lagrangian and Eulerian frames.
- Other names for the material derivative include: **total, particle, Lagrangian, Eulerian,** and **substantial** derivative.

Flow Visualization

- Flow visualization is the visual examination of flow-field features.
- Important for both physical experiments and numerical (CFD) solutions.
- Numerous methods
  - Streamlines and streamtubes
  - Pathlines
  - Streaklines
  - Timelines
  - Refractive techniques
  - Surface flow techniques

Streamlines



- A **Streamline** is a curve that is everywhere tangent to the *instantaneous* local velocity vector.
- Consider an arc length

$$d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k}$$

- $d\vec{r}$ must be parallel to the local velocity vector

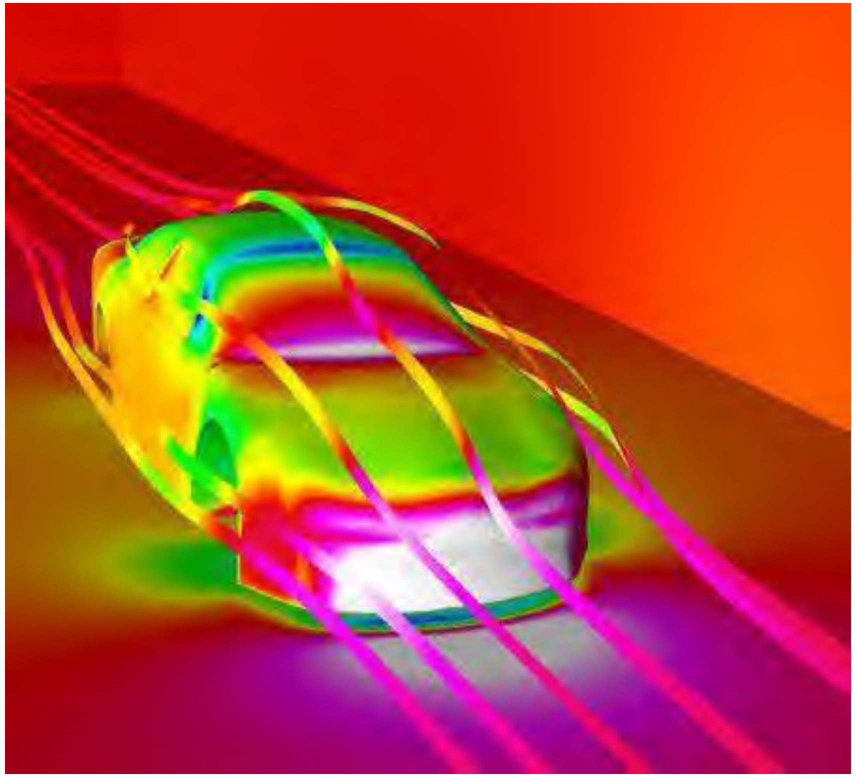
$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

- Geometric arguments results in the equation for a streamline

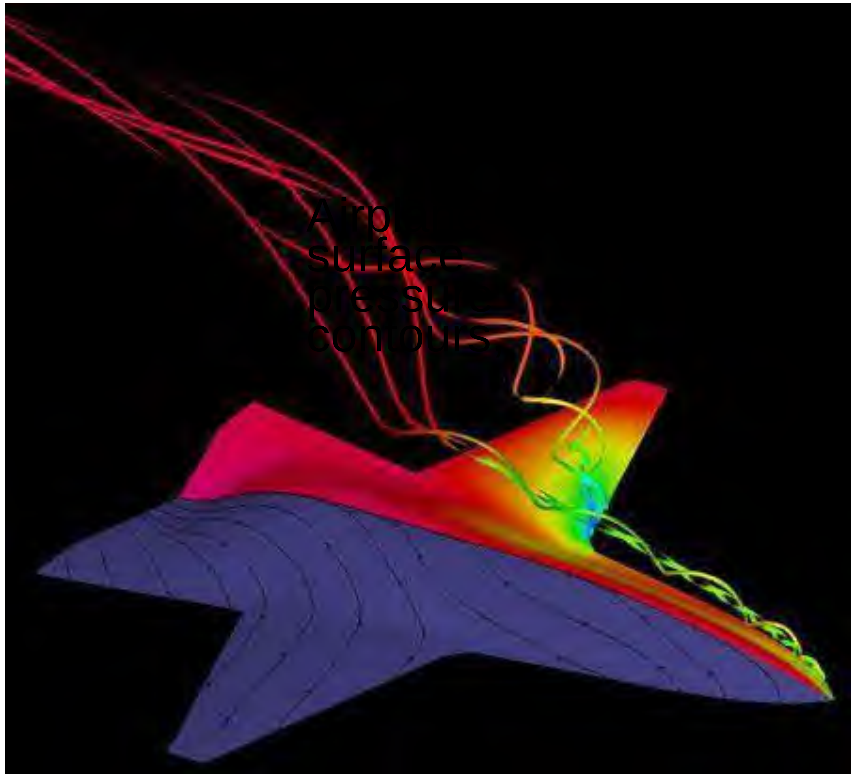
$$\frac{dr}{V} = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Streamlines

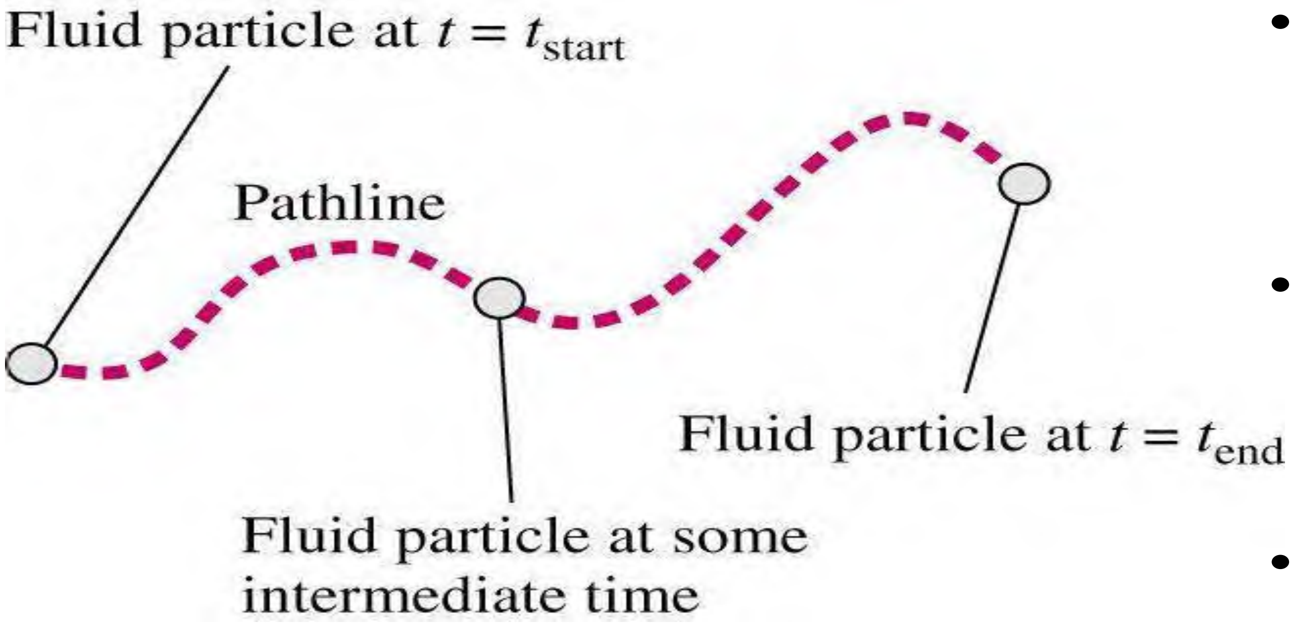
NASCAR surface pressure contours and streamlines



Airplane surface pressure contours, Volume stream lines



Pathlines



- A **Pathline** is the actual path traveled by an individual fluid particle over some time period.
- Same as the fluid particle's material position vector

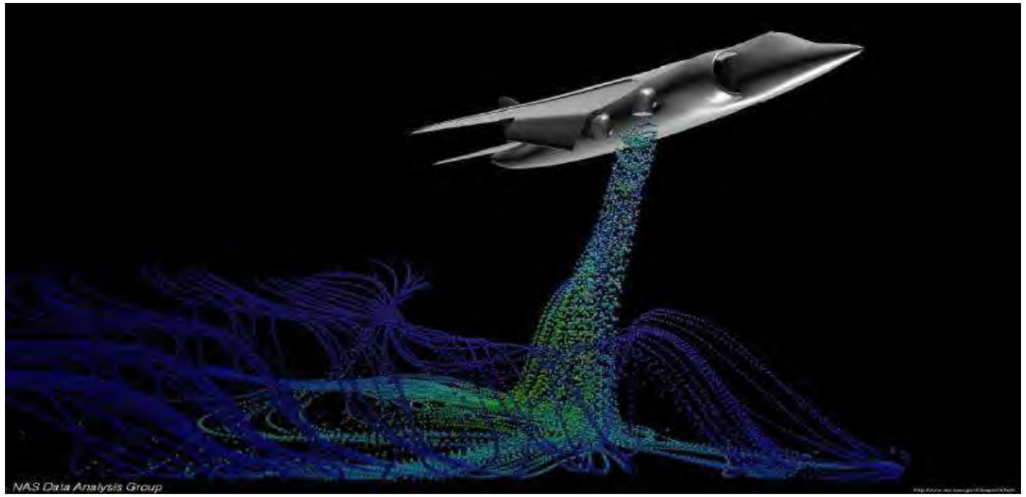
$$\left(x_{particle}(t), y_{particle}(t), z_{particle}(t) \right)$$

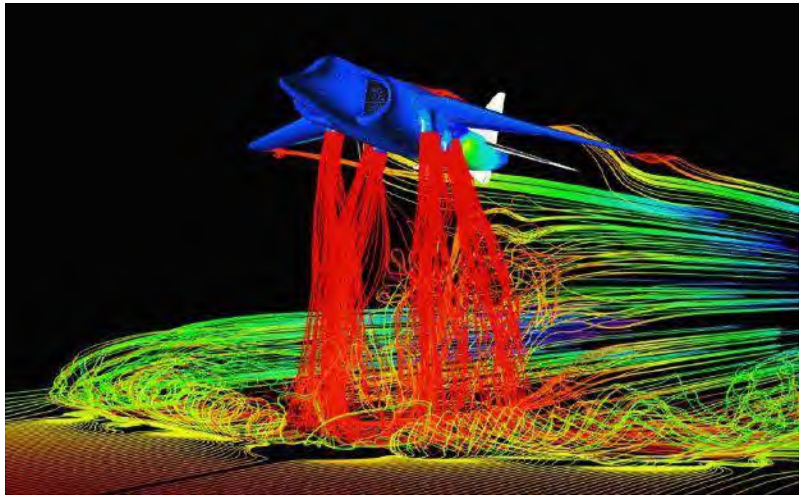
- Particle location at time t:

$$\vec{x} = \vec{x}_{start} + \int_{t_{start}}^{t} \vec{V}\, dt$$

- Particle Image Velocimetry (PIV) is a modern experimental technique to measure velocity field over a plane in the flow field.

Streaklines



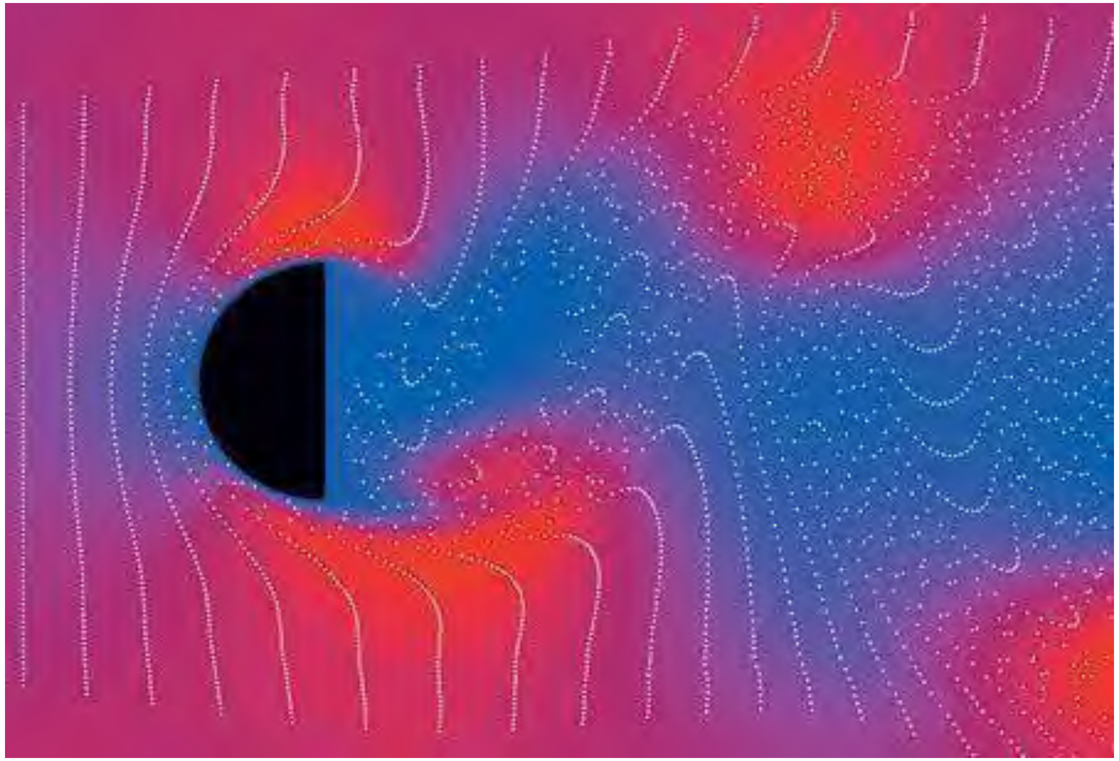


- A **Streakline** is the locus of fluid particles that have passed sequentially through a prescribed point in the flow.
- Easy to generate in experiments: dye in a water flow, or smoke in an airflow.

Comparisons

- For steady flow, streamlines, pathlines, and streaklines are identical.
- For unsteady flow, they can be very different.
  - Streamlines are an instantaneous picture of the flow field
  - Pathlines and Streaklines are flow patterns that have a time history associated with them.
  - Streakline: instantaneous snapshot of a time-integrated flow pattern.
  - Pathline: time-exposed flow path of an individual particle.

Timelines

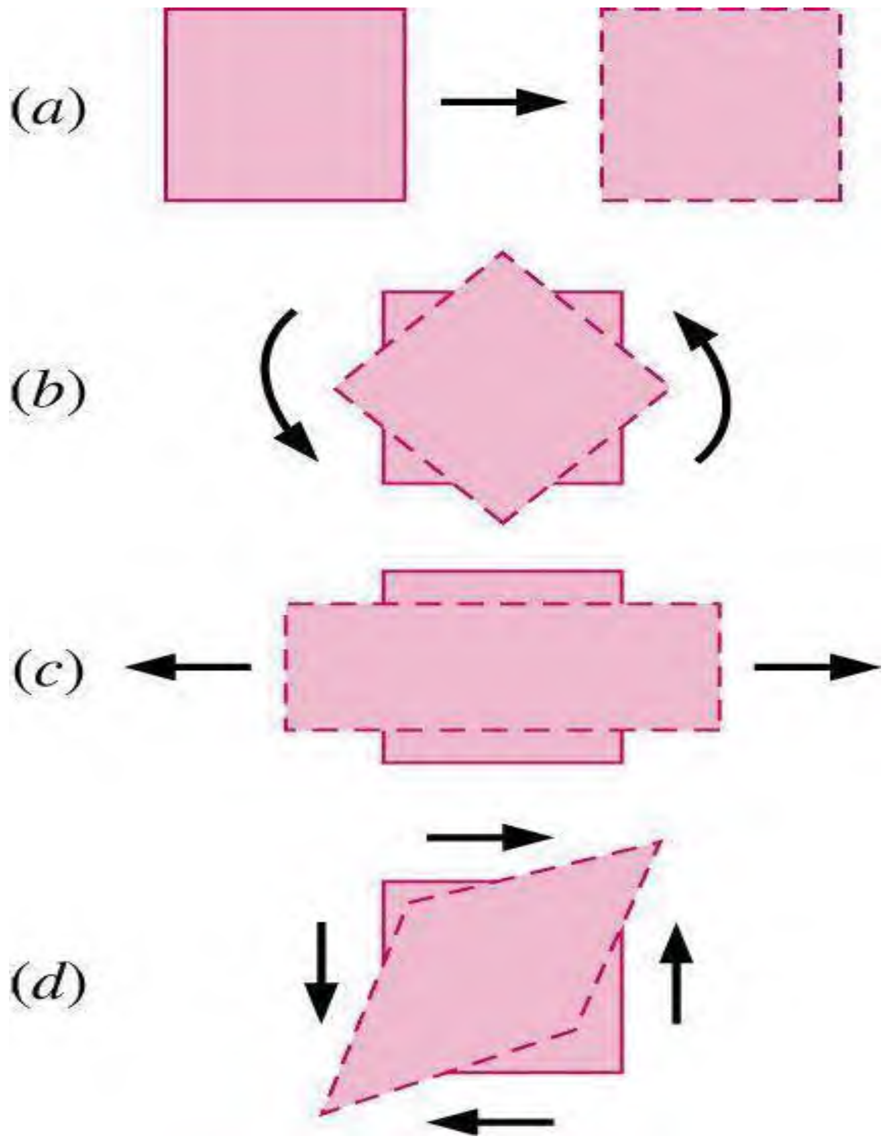


- A **Timeline** is the locus of fluid particles that have passed sequentially through a prescribed point in the flow.
- Timelines can be generated using a hydrogen bubble wire.

Plots of Data

- A **Profile plot** indicates how the value of a scalar property varies along some desired direction in the flow field.
- A **Vector plot** is an array of arrows indicating the magnitude and direction of a vector property at an instant in time.
- A **Contour plot** shows curves of constant values of a scalar property for magnitude of a vector property at an instant in time.

Kinematic Description



- In fluid mechanics, an element may undergo four fundamental types of motion.
  a) Translation
  b) Rotation
  c) Linear strain
  d) Shear strain
- Because fluids are in constant motion, motion and deformation is best described in terms of rates
  a) velocity: rate of translation
  b) angular velocity: rate of rotation
  c) linear strain rate: rate of linear strain
  d) shear strain rate: rate of shear strain

Rate of Translation and Rotation

- To be useful, these rates must be expressed in terms of velocity and derivatives of velocity
- The **rate of translation vector** is described as the velocity vector. In Cartesian coordinates:

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

- **Rate of rotation** at a point is defined as the average rotation rate of two initially perpendicular lines that intersect at that point. The rate of rotation vector in Cartesian coordinates:

$$\vec{\omega} = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$

Linear Strain Rate

- **Linear Strain Rate** is defined as the rate of increase in length per unit length.
- In Cartesian coordinates

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \varepsilon_{yy} = \frac{\partial v}{\partial y}, \varepsilon_{zz} = \frac{\partial w}{\partial z}$$

- Volumetric strain rate in Cartesian coordinates

$$\frac{1}{V}\frac{DV}{Dt} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

- Since the volume of a fluid element is constant for an incompressible flow, the volumetric strain rate must be zero.

Shear Strain Rate

- **Shear Strain Rate** at a point is defin ed as *half of the rate of decrease of the angle between two initially perpendicular lines that intersect at a point* .
- Shear strain rate can be expressed in Cartesian coordinates as:

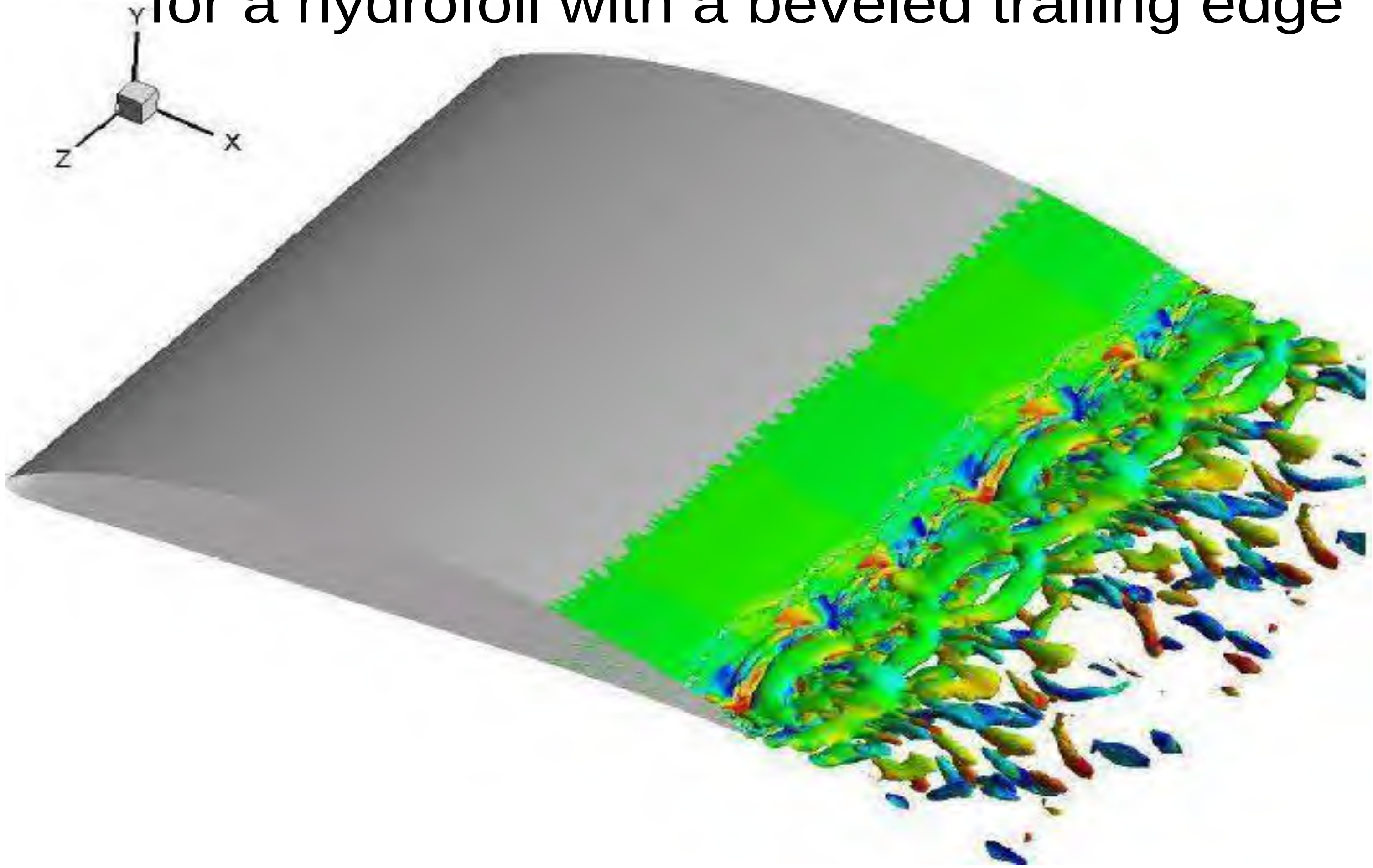
$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \varepsilon_{zx} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right), \varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

Shear Strain Rate

- Purpose of our discussion of fluid element kinematics:
  - Better appreciation of the inherent complexity of fluid dynamics
  - Mathematical sophistication required to fully describe fluid motion
- Strain-rate tensor is important for numerous reasons. For example,
  - Develop relationships between fluid stress and strain rate.
  - Feature extraction and flow visualization in CFD simulations.

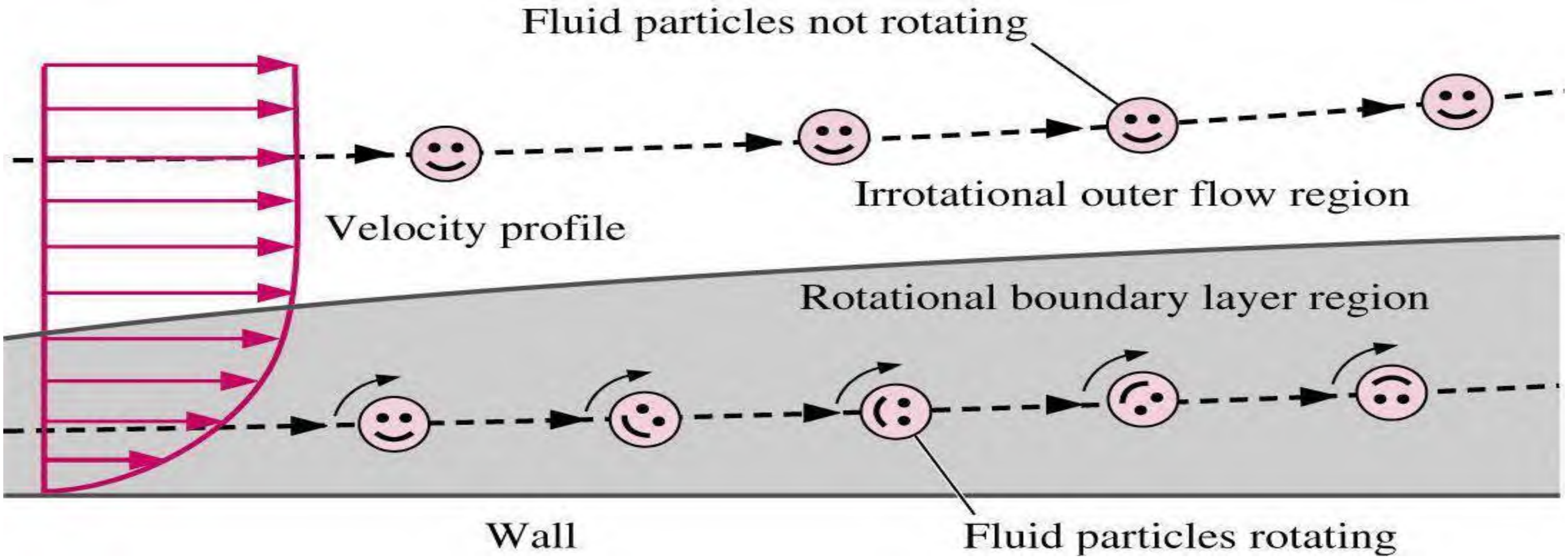
Shear Strain Rate

Example: Visualization of trailing-edge turbulent eddies for a hydrofoil with a beveled trailing edge



Feature extraction method is based upon eigen-analysis of the strain-rate tensor.

Vorticity and Rotationality



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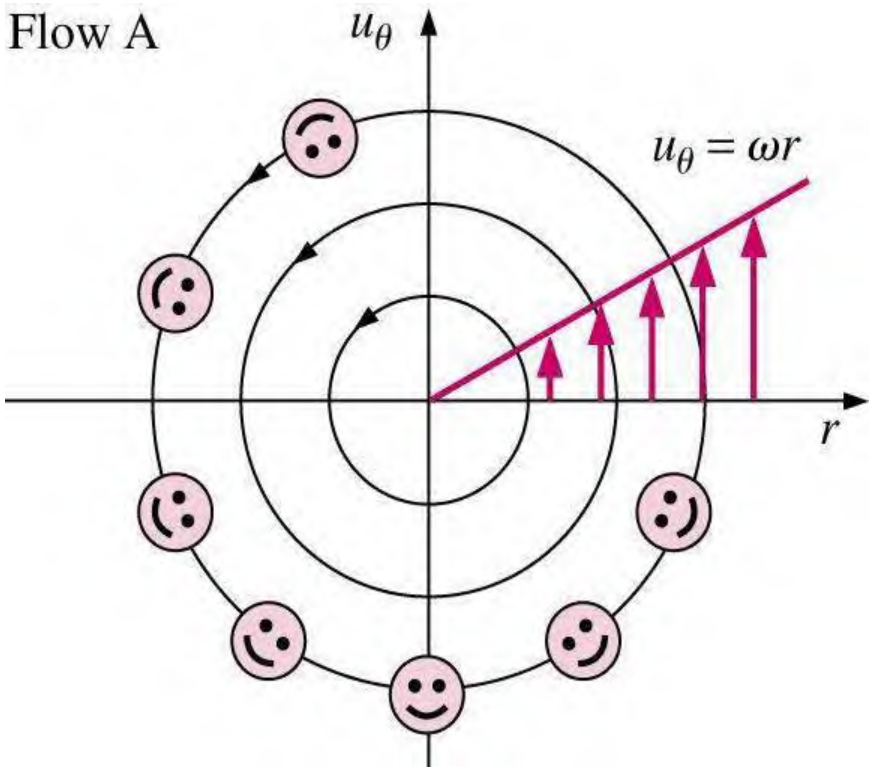
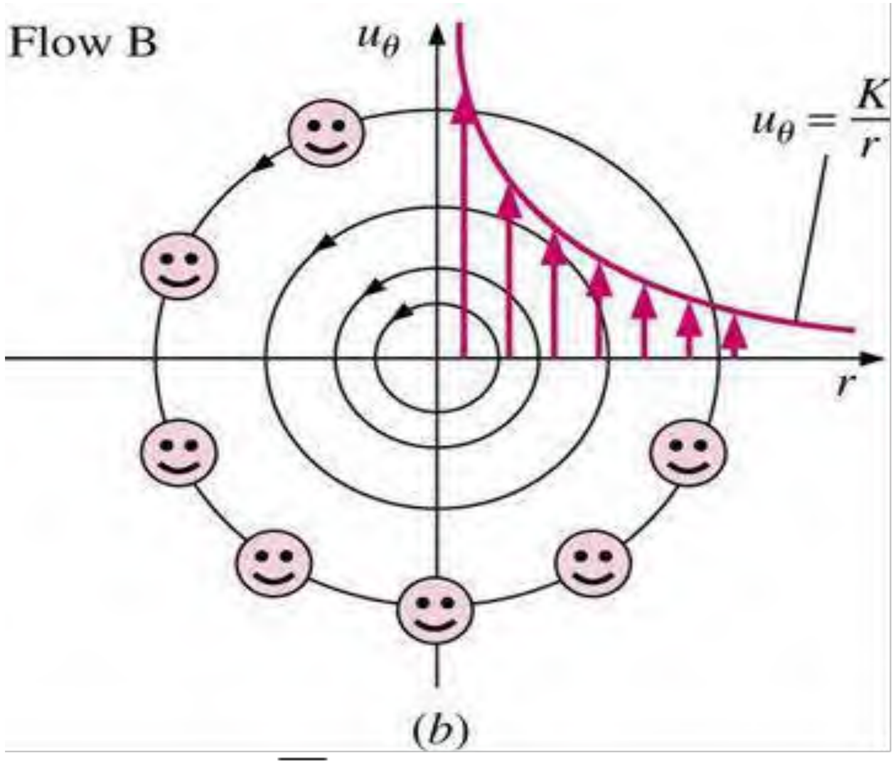
Comparison of Two Circular Flows

Special case: consider two flows with circular streamlines

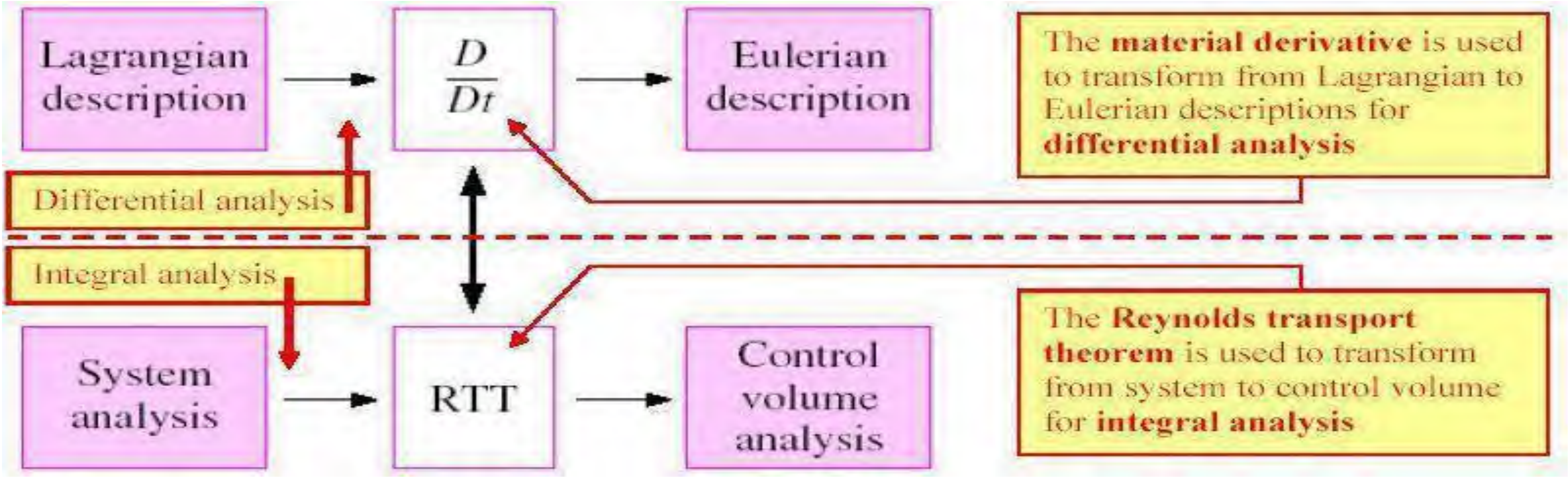




Reynolds—Transport Theorem (RTT)

- A **system** is a quantity of matter of fixed identity. *No mass can cross a system boundary.*
- A **control volume** is a region in space chosen for study. Mass can cross a control surface.
- The fundamental conservation laws (conservation of mass, energy, and momentum) apply directly to systems.
- However, in most fluid mechanics problems, control volume analysis is preferred over system analysis (for the same reason that the Eulerian description is usually preferred over the Lagrangian description).
- Therefore, we need to transform the conservation laws from a system to a control volume. This is accomplished with the Reynolds transport theorem (RTT).

Reynolds—Transport Theorem (RTT)



There is a direct analogy between the transformation from Lagrangian to Eulerian descriptions (for differential analysisusing infinitesimally small fluid elements) and the transformation from systems to control volumes (for integral analysis using large, finite flow fields).

Reynolds—Transport Theorem (RTT)

- Material derivative (differential analysis): $\frac{Db}{Dt} = \frac{\partial b}{\partial t} + (\vec{V} \cdot \vec{\nabla}) b$

- General RTT, nonfixed CV (integral analysis):

$$\frac{dB_{sys}}{dt} = \int_{CV} \frac{\partial}{\partial t}(\rho b)\, d\mathcal{V} + \int_{CS} \rho b\, \vec{V} \cdot \vec{n}\, dA$$

	Mass	Momentum	Energy	Angular momentum
B, Extensive properties	m	$m\vec{V}$	E	$\vec{H}$
b, Intensive properties	1	$\vec{V}$	e	$(\vec{r} \times \vec{V})$

Reynolds—Transport Theorem (RTT)

- Interpretation of the RTT:
  - Time rate of change of the property B of the system is equal to (Term 1) + (Term 2)
  - Term 1: the time rate of change of B of the control volume
  - Term 2: the net flux of B out of the control volume by mass crossing the control surface

RTT Special Cases

For **moving** and/or **deforming** control volumes,

$$\frac{dB_{sys}}{dt} = \int_{CV} \frac{\partial}{\partial t}(\rho b)\, dV + \int_{CS} \rho b \vec{V}_r \cdot \vec{n}\, dA$$

- Where the absolute velocity *V* in the second term is replaced by the **relative velocity**

  $V_r = V - V_{CS}$
- V_r is the fluid velocity expressed relative to a coordinate system moving **with** the control volume.

RTT Special Cases

For steady flow, the time derivative drops out,

$$\frac{dB_{sys}}{dt} = \int_{CV} \frac{\partial}{\partial t}(\rho b)\, dV^{\,0} + \int_{CS} \rho b \vec{V}_r \cdot \vec{n}\, dA = \int_{CS} \rho b \vec{V}_r \cdot \vec{n}\, dA$$

For control volumes with well-defined inlets and outlets

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b\, dV + \sum_{out} \rho_{avg}\, b_{avg}\, V_{r,avg}\, A - \sum_{in} \rho_{avg}\, b_{avg}\, V_{r,avg}\, A$$

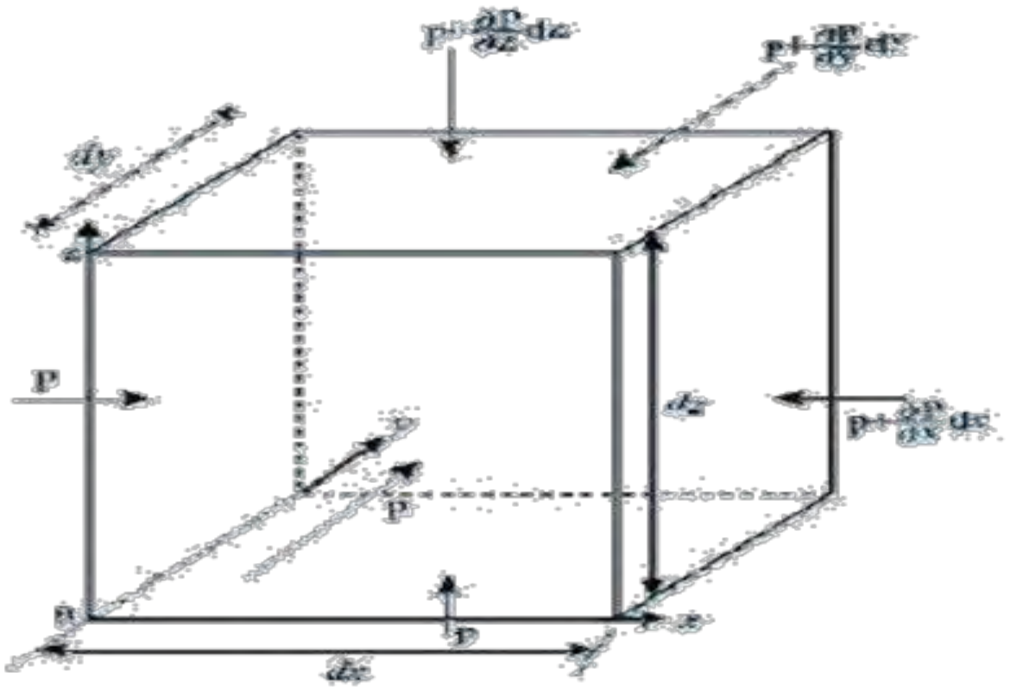
UNIT-III
Fluid Dynamics

Euler and Navier Stokes Equation:

Euler’s Equation: The Equation of Motion of an Ideal Fluid

Using the Newton's second law of motion the relationship between the velocity and pressure field for a flow of an inviscid fluid can be derived. The resulting equation, in its differential form, is known as Euler‟s Equation. The equation is first derived by the scientist Euler.

Derivation:



The net forces acting on the fluid element along x, y and z directions can be written as

$$F_x = \rho X_x dx\,dy\,dz + p\,dy\,dz - \left(p + \frac{\partial p}{\partial x}dx\right)dy\,dz = \left(\rho X_x - \frac{\partial p}{\partial x}\right)dx\,dy\,dz$$

$$F_y = \rho X_y dx\,dy\,dz + p\,dx\,dz - \left(p + \frac{\partial p}{\partial y}dy\right)dx\,dz = \left(\rho X_y - \frac{\partial p}{\partial y}\right)dx\,dy\,dz$$

$$F_z = \rho X_z dx\,dy\,dz + p\,dx\,dy - \left(p + \frac{\partial p}{\partial z}dz\right)dx\,dy = \left(\rho X_z - \frac{\partial p}{\partial z}\right)dx\,dy\,dz$$

Since each component of the force can be expressed as the rate of change of momentum in the respective directions, we have

$$\frac{D}{Dt}(\rho\,dx\,dy\,dz\,u) = \left(\rho X_x - \frac{\partial p}{\partial x}\right)dxdydz$$

$$\frac{D}{Dt}(\rho\,dx\,dy\,dz\,v) = \left(\rho X_y - \frac{\partial p}{\partial y}\right)dxdydz$$

$$\frac{D}{Dt}(\rho\,dx\,dy\,dz\,w) = \left(\rho X_z - \frac{\partial p}{\partial z}\right)dxdydz$$

Expanding the material accelerations in Eqs in terms of their respective temporal and convective components we get

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=X_x-\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=X_y-\frac{1}{\rho}\frac{\partial p}{\partial y}$$

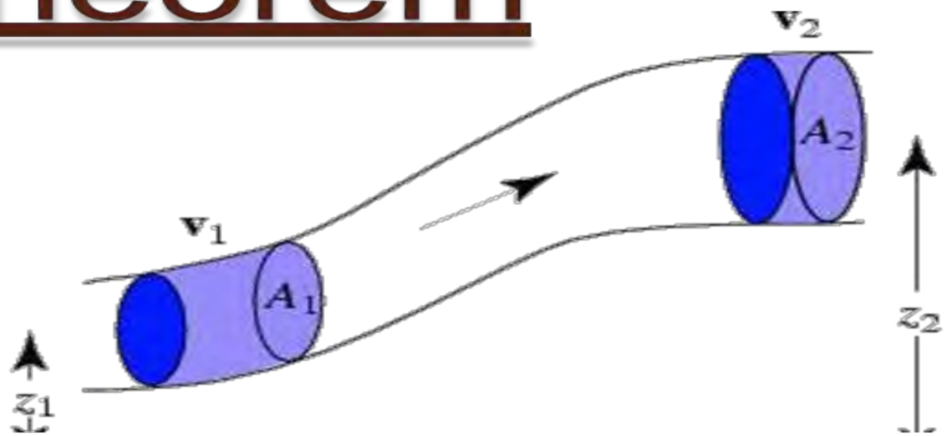
$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=X_z-\frac{1}{\rho}\frac{\partial p}{\partial z}$$

$$\frac{D\vec{V}}{Dt}=-\frac{\nabla p}{\rho}+\vec{X}$$

$$\frac{\partial \vec{V}}{\partial t}+(\vec{V}\cdot\nabla)\vec{V}=\vec{X}-\frac{1}{\rho}\nabla p$$

✓ Bernoulli's Theorem

Assumptions:



- no work or heat interaction between a fluid element and the surrounding takes place.
- The flow must be incompressible
- Friction by viscous forces has to be negligible.

✓Bernoulli's Theorem

- This equation was developed first by Daniel Bernoulli in 1738.

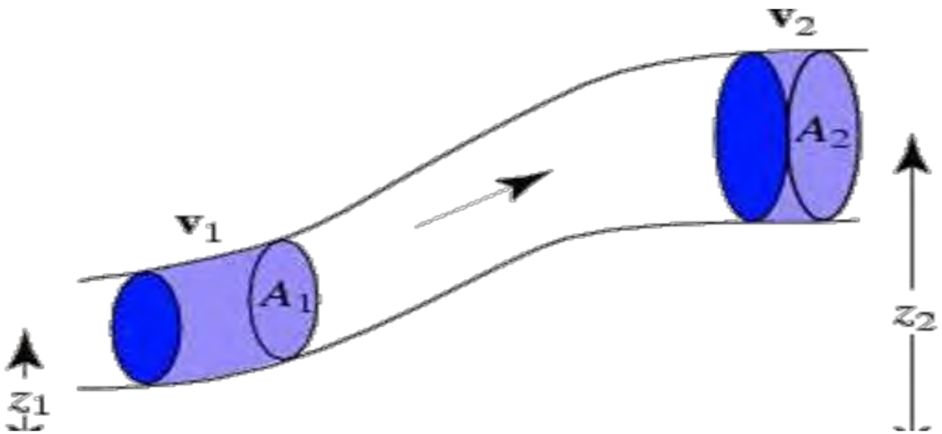
$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = C_1 \text{(constant)}$$

flow work per unit mass

kinetic energy per unit mass

potential energy per unit mass

✓ Bernoulli's Theorem with Head Loss

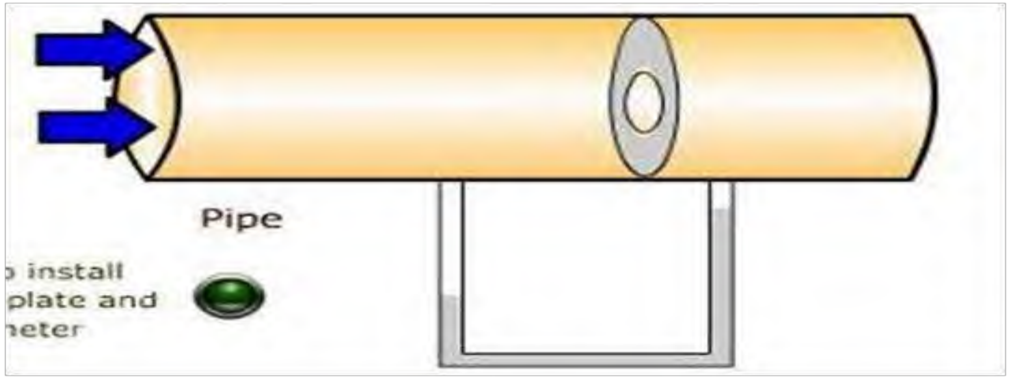


$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_f$$

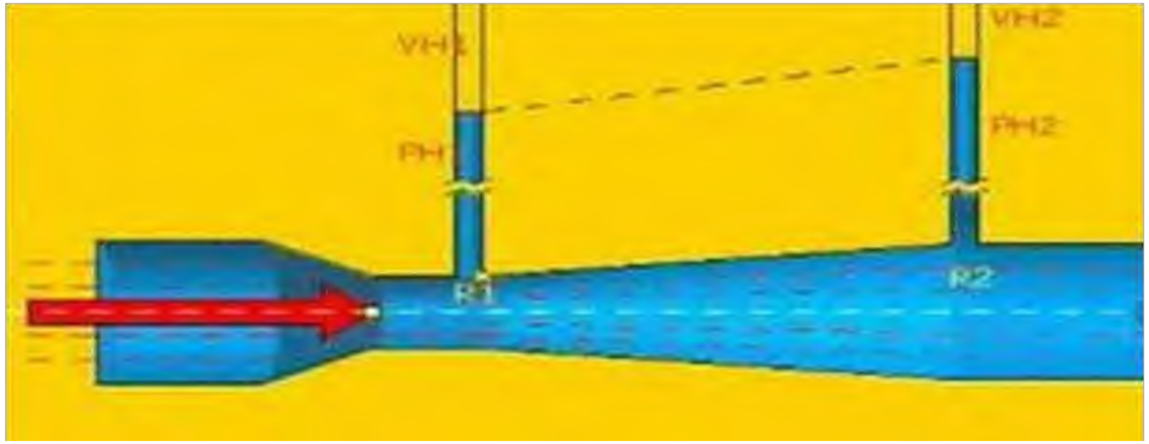
Where, h_f represents the frictional work done (the work done against the fluid friction) per unit weight of a fluid element while moving from a station 1 to 2 along a streamline in the direction of flow.

Application of Bernoulli's Law

- Orifice meter



- Venturi meter



Momentum Equation in Integral Form:

Conservation of Momentum: Momentum Theorem

In Newtonian mechanics, the conservation of momentum is defined by Newton‟s second law of motion.

Newton's Second Law of Motion

The rate of change of momentum of a body is proportional to the impressed action and takes place in the direction of the impressed action.

If a force acts on the body ,linear momentum is implied.

If a torque (moment) acts on the body,angular momentum is implied.

Reynolds Transport Theorem

A study of fluid flow by the Eulerian approach requires a mathematical modeling for a control volume either in differential or in integral form. Therefore the physical statements of the principle of conservation of mass, momentum and energy with reference to a control volume become necessary. This is done by invoking a theorem known as the Reynolds transport theorem which relates the control volume concept with that of a control mass system in terms of a general property of the system.

Statement of Reynolds Transport Theorem

The theorem states that "the time rate of increase of property N within a control mass system is equal to the time rate of increase of property N within the control volume plus the net rate of efflux of the property N across the control surface”.

Reynolds Transport Theorem

After deriving Reynolds Transport Theorem according to the above statement we get

$$\left(\frac{dN}{dt}\right)_{CMS} = \frac{\partial}{\partial t}\iiint_{CV} \eta\rho\, dV + \iint_{CS} \eta\rho\vec{V}\cdot d\vec{A}$$

In this equation
N - flow property which is transported
η - intensive value of the flow property

Application of the Reynolds Transport Theorem to Conservation of Mass and Momentum

Angular Momentum Equation in Integral Form:

Angular Momentum

The angular momentum or moment of momentum theorem is also derived from below Eq in consideration of the property N as the angular momentum and accordingly η as the angular momentum per unit mass. Thus,

$$\frac{d}{dt}(A_{cms}) = \frac{\partial}{\partial t}\iiint_{cv} \rho(\vec{r} \times \vec{V}_r)\,dV + \iint_{cs} (\vec{r} \times \vec{V}_r)\rho\vec{V}_r \cdot d\vec{A}$$

where

Control mass system is the **angular momentum of the control mass system**. . It has to be noted that the origin for the angular momentum is the origin of the position vector

Flow Measurement

Pipes (pressure conduits)	Open channel (flumes, canals and rivers etc)
1. Venturimeter 2. Orifices 3. Orifice meter 4. Mouth pieces/tubes 5. Nozzle 6. Pitot static tube	1. Notches (Rectangular notch, V notch) 2. Weirs

2

Flow Measurement in Pipes

- Venturimeter

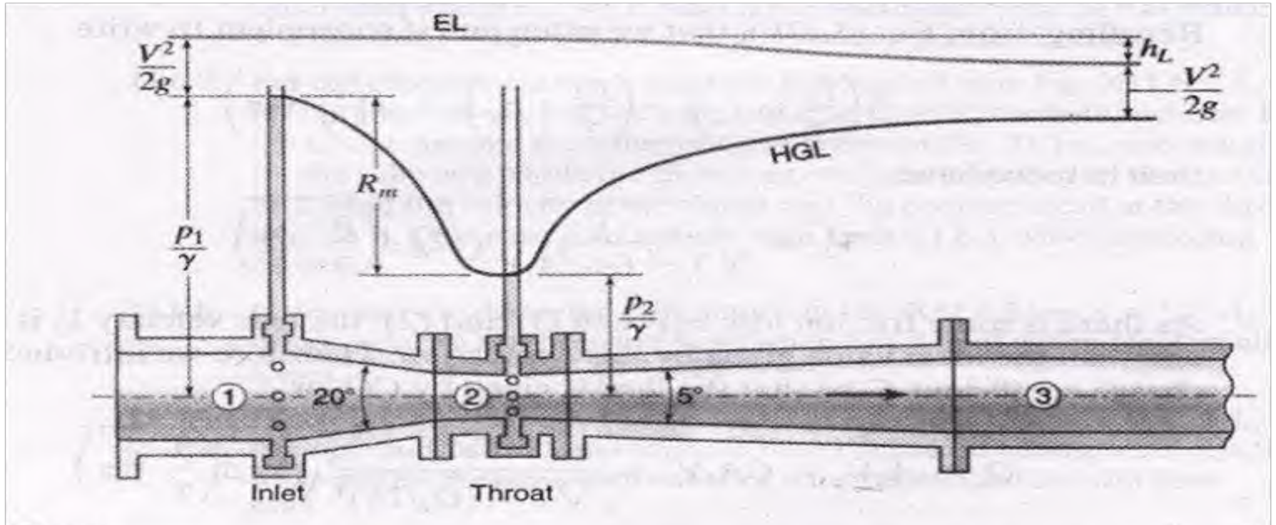


Figure
Venturi meter with conical entrance and flow coefficients for $D_2/D_1 = 0.5$.

The venturi tube provides an accurate means for measuring flow in pipelines.

Aside from the installation cost, the only disadvantage of the venturi meter is that it introduces a permanent frictional resistance in the pipeline. Practically all this loss occurs in the diverging part between sections (2) and (3), and is ordinarily from $0.1h$ to $0.2h$, where h is the static-head differential between the upstream section and the throat

Values of D_2/D_1 may vary from $\frac{1}{4}$ to $\frac{3}{4}$, but a common ratio is $\frac{1}{2}$. A small ratio gives increased accuracy of the gage reading, but is accompanied by a higher friction loss and may produce an undesirably low pressure at the throat

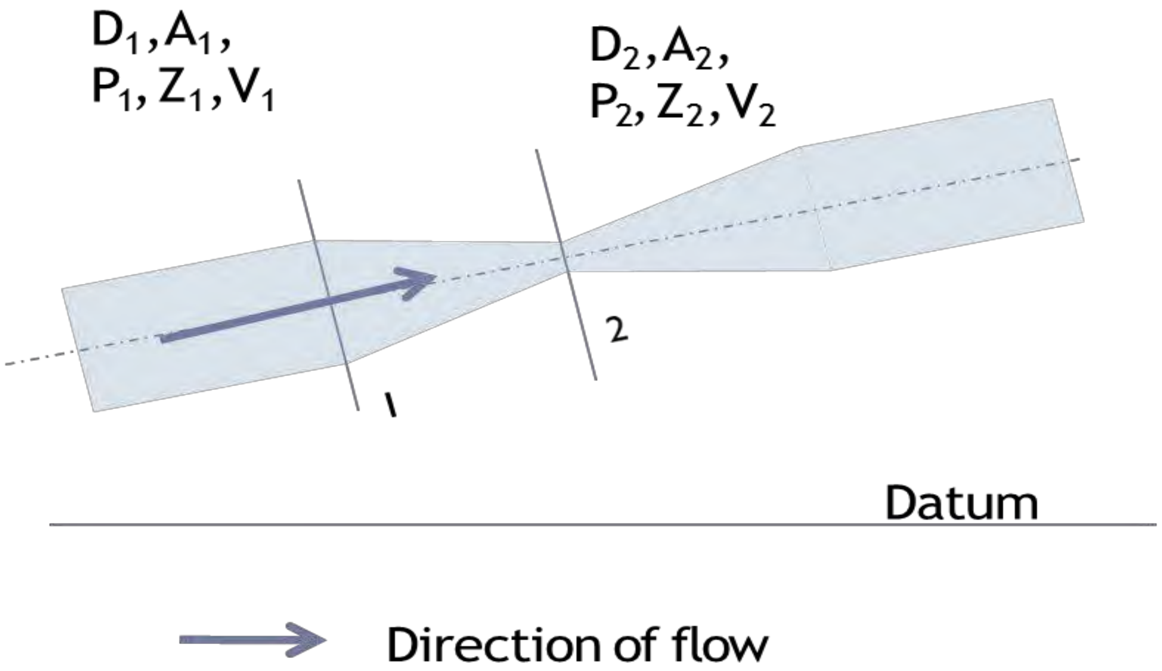
3

Flow Measurements in Pipes

c Venturimeter

$$Q_{act} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right) + 2g(z_1 - z_2)}$$

Figure shows a venturimeter in which discharge Q is flowing,
Let, D_1 is diameter, A_1 is cross-section area, P_1 is pressure, z_1 is elevation head V_1 is velocity at section 1. Similarly D_2 ,A_2, P_2, z_2 & V_2 are corresponding values at section 2



According to Bernoulli's Equation between section 1 and 2 we can write;

$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

4 $$2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right) + 2g(z_1 - z_2) = v_2^2 - v_1^2$$

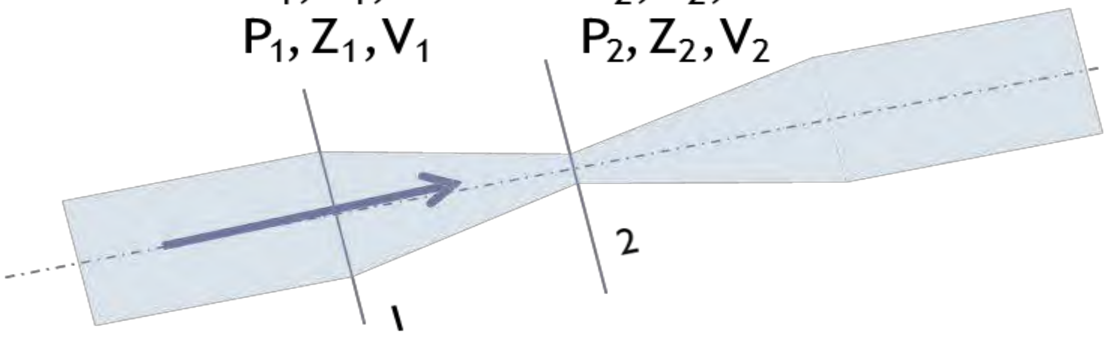
Flow Measurements in Pipes

- **Venturimeter**

$$Q_{act} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right) + 2g(z_1 - z_2)}$$

$$2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right) + 2g(z_1 - z_2) = \frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2}$$

$$\because Q = A_1 V_1 = A_2 V_2$$



$$2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right) + 2g(z_1 - z_2) = \left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) Q^2$$

$$2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right) + 2g(z_1 - z_2) = \left(\frac{A_1^2 - A_2^2}{A_1^2 A_2^2}\right) Q^2 \Rightarrow Q^2 = \frac{A_1^2 A_2^2}{A_1^2 - A_2^2} 2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right) + 2g(z_1 - z_2)$$

$$Q_{th} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right) + 2g(z_1 - z_2)}$$

5

Flow Measurements in Pipes

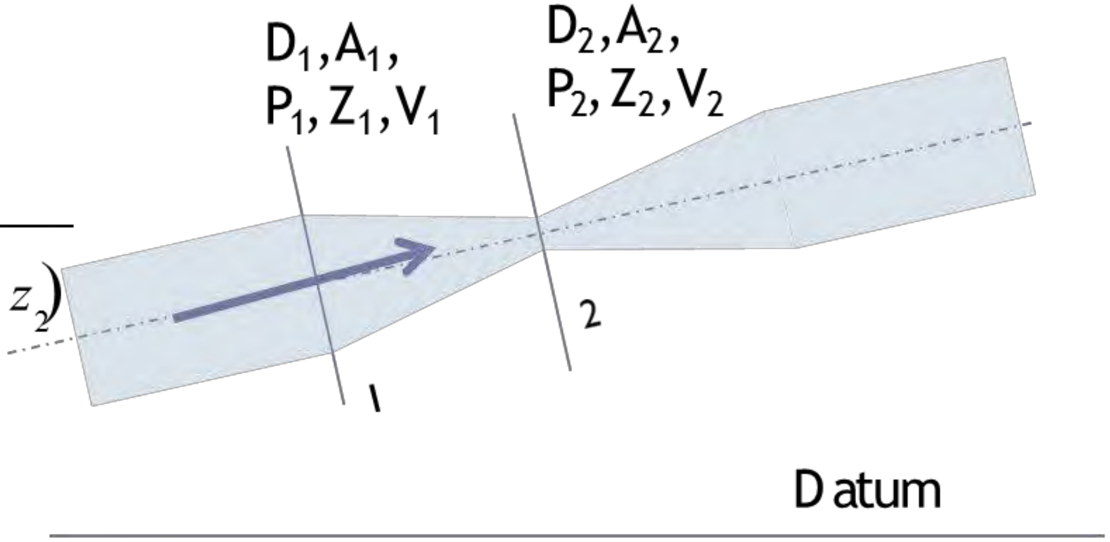
- Venturimeter

$$Q_{act} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right) + 2g(z_1 - z_2)}$$

Since

$$Q_{act} = c_d Q_{th}$$

$$Q_{act} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right) + 2g(z_1 - z_2)}$$



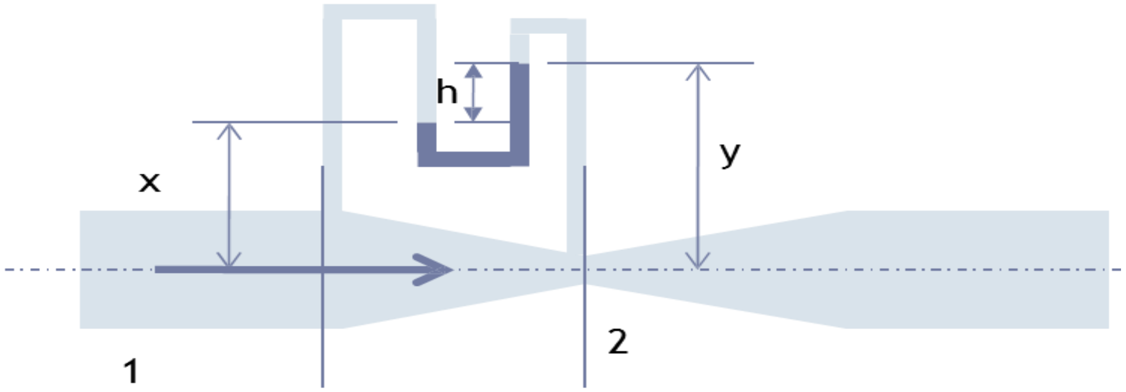
Where C_d is coefficient of discharge and is defined as ratio of actual discharge to theoretical discharge.

Flow Measurements in Pipes

- Types of Venturimeter
- *a. Horizontal Venturimeter*
- *b. Vertical Venturimeter*

$$Q_{act} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right)}$$

- **a. Horizontal Venturimeter**
- Figure shows a venturimeter connected with a differential manometer.
- At section 1, diameter of pipe is D_1, and pressure is P_1 and similar D_2 and P_2 are respective values at section 2.



According to gauge pressure equation

$$\frac{P_1}{\gamma} - x - S_m h + y = \frac{P_2}{\gamma}$$

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} = S_m h - (y - x) = S_m h - (h)$$

7

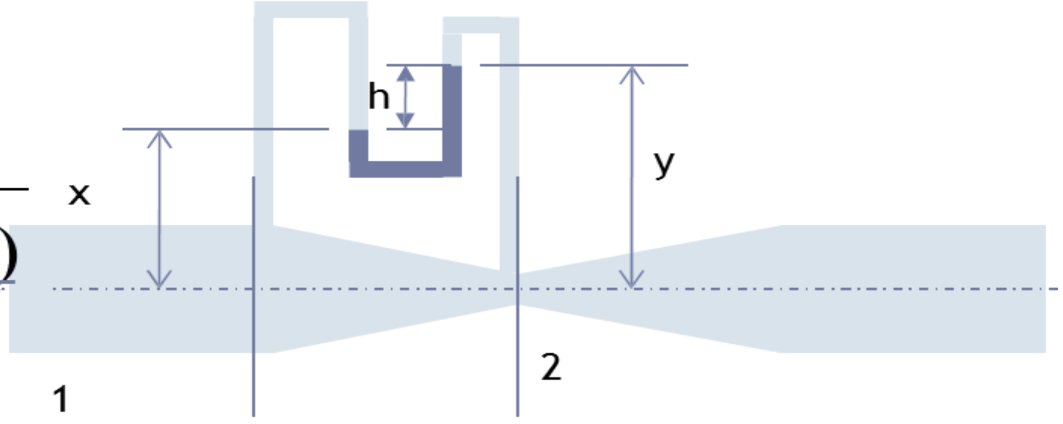
Flow Measurements in Pipes

- Types of Venturimeter
- *a. Horizontal Venturimeter*
- *b. Vertical Venturimeter*

$$Q_{act} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right)}$$

- **a. Horizontal Venturimeter**

$$Q_{act} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right) + 2g(z_1 - z_2)}$$



For horizontal venturimeter, $(z_1 - z_2) = 0$

$$Q_{act} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right)}$$

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} = S_m h - (y - x) = S_m h - (h)$$

8

According to gauge pressure equation

$$\frac{P_1}{\gamma} - x - S_m h + y = \frac{P_2}{\gamma}$$

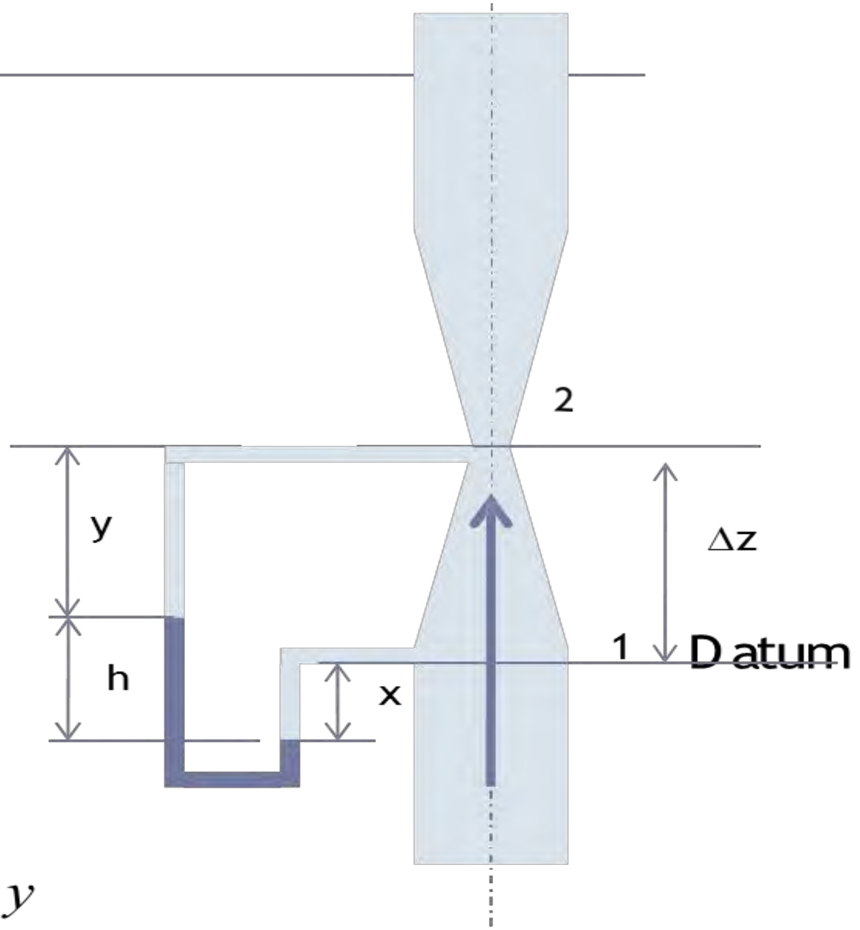
$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} = S_m h - (y - x) = S_m h - (h)$$

Flow Measurements in Pipes

- Types of Venturimeter
- *a. Horizontal Venturimeter*
- *b. Vertical Venturimeter*

$$Q_{act} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right) + 2g(z_1 - z_2)}$$

- **b. Vertical Venturimeter**
- Figure shows a venturimeter connected with a differential manometer.



According to gauge pressure equation

$$\frac{P_1}{\gamma} + x - S_m h - y = \frac{P_2}{\gamma}$$

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} = S_m h + y - x$$

9 $$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} = S_m h + \Delta z - h$$

Q $x + \Delta z = h + y$

Flow Measurements in Pipes

- Types of Venturimeter
- *a. Horizontal Venturimeter*
- *b. Vertical Venturimeter*

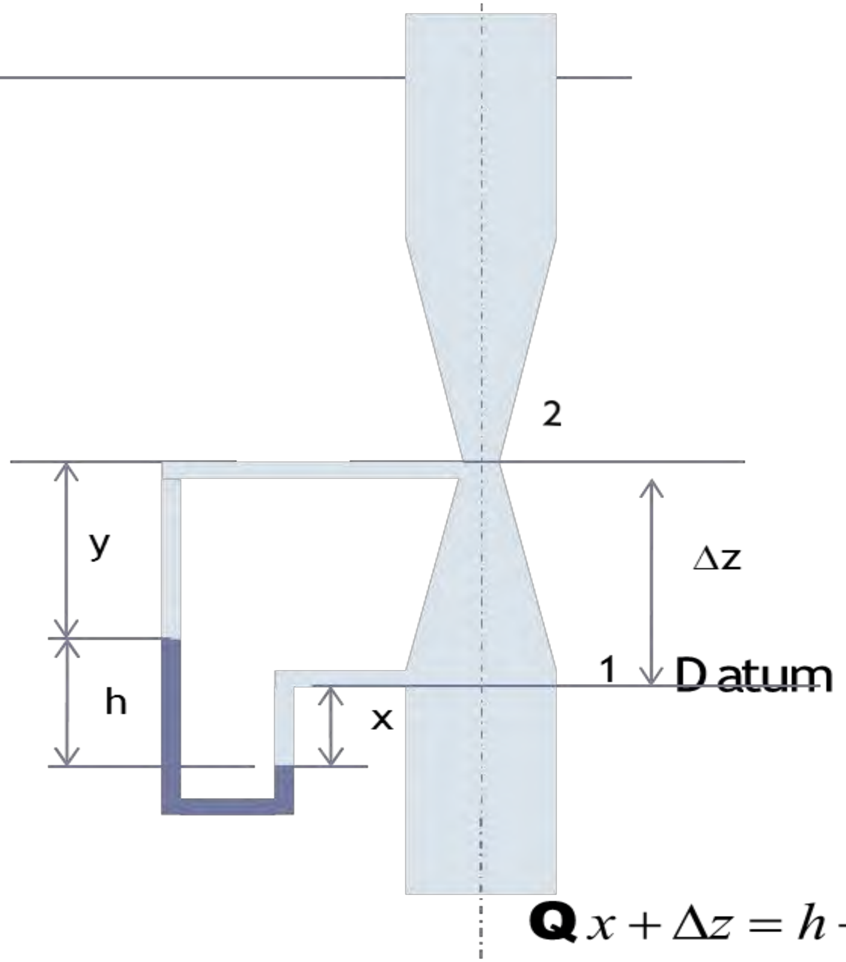
$$Q_{act} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right) + 2g(z_1 - z_2)}$$

- **b. Vertical Venturimeter**

$$Q_{act} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right) + 2g(z_1 - z_2)}$$

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} = S_m h + \Delta z - h$$

$$(z_1 - z_2) = \Delta z$$



$$\because x + \Delta z = h + y$$

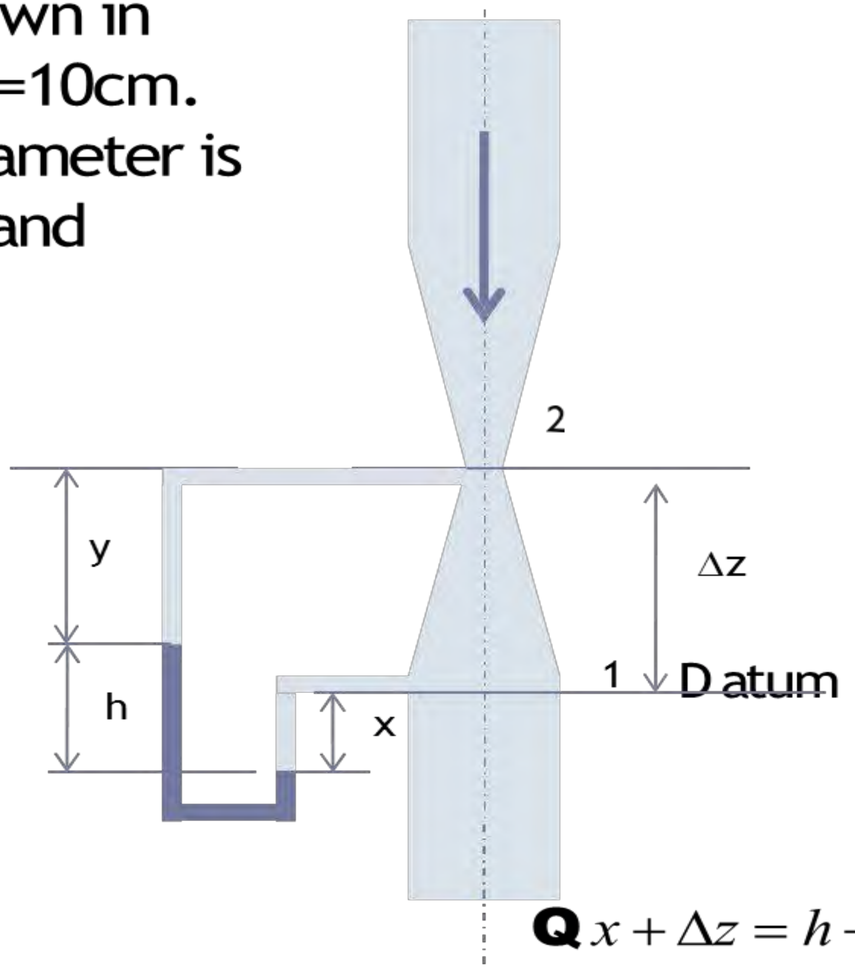
10

Numerical Problem

- Find the flow rate in venturimeter as shown in figure if the mercury manometer reads h=10cm. The pipe diameter is 20cm and throat diameter is 10 cm and Δz =0.45m. Assume C_d=0.98 and direction of flow is downward.

$$Q_{act} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right) + 2g(z_1 - z_2)}$$

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} = S_m h + \Delta z - h$$



$\because x + \Delta z = h + y$

Orifice

- An orifice is an opening (usually circular) in wall of a tank or in plate normal to the axis of pipe, the plate being either at the end of the pipe or in some intermediate location.
- An orifice is characterized by the fact that the thickness of the wall or plate is very small relative to the size of opening.

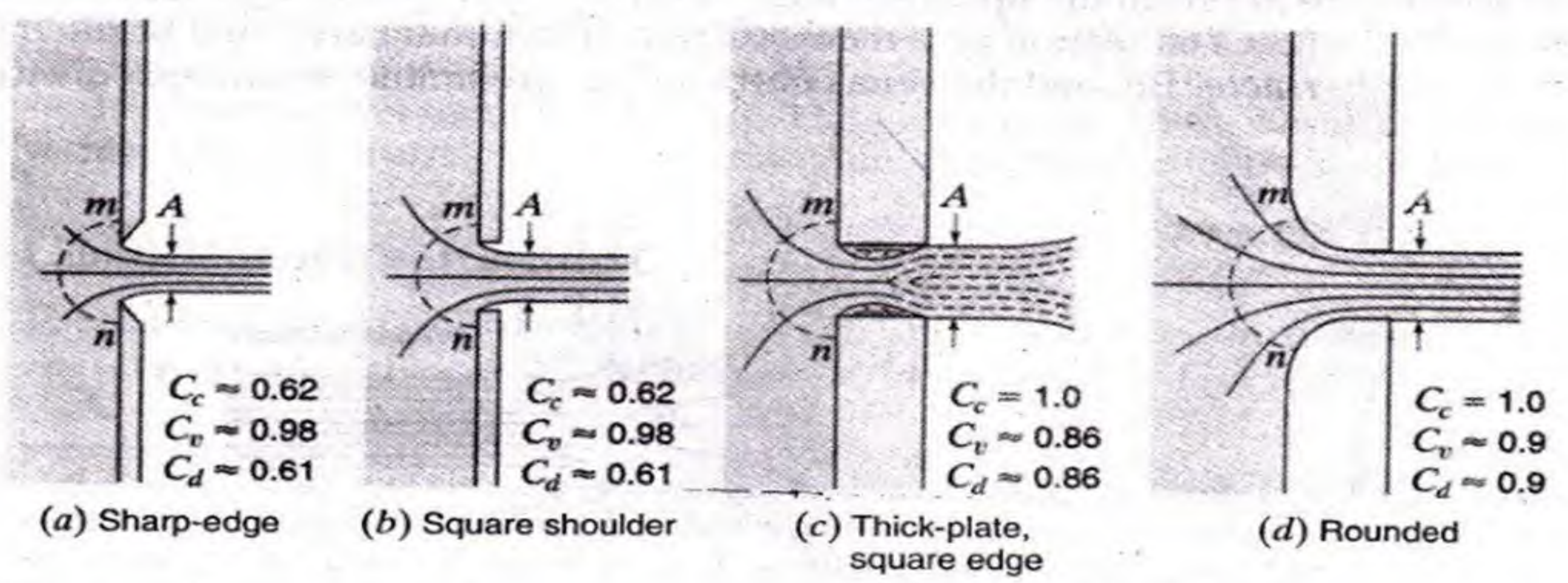


Figure
Orifices.

Orifice

- A standard orifice is one with a sharp edge as in Fig (a) or an absolutely square shoulder (Fig. b) so that there in only a line contact with the fluid

- Those shown in Fig. c and d are not standard because the flow through them is affected by the thickness of plate, the roughness of surface and radius of curvature (Fig. d).
- Hence such orifices should be calibrated if high accuracy is desired.

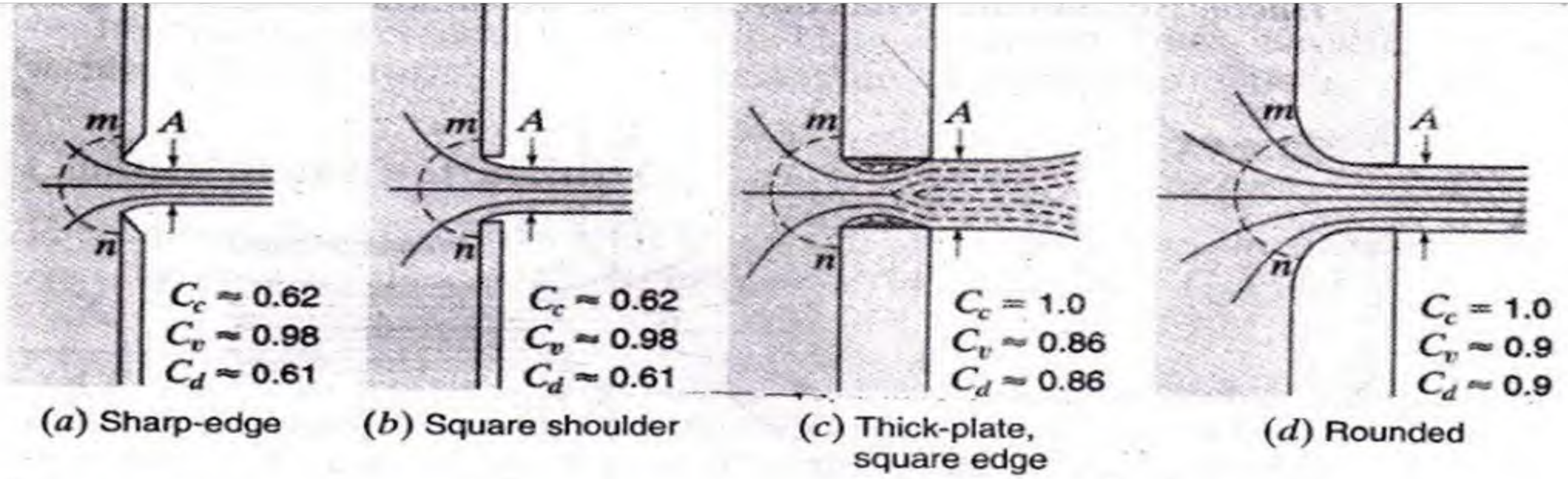


Figure
Orifices.

Classification of Orifice

- **According to size**
- **1. Small orifice**
- **2. Large orifice**

- An orifice is termed as small when its size is small compared to head causing flow. The velocity does not vary appreciably from top to bottom edge of the orifice and is assumed to be uniform.

- The orifice is large if the dimensions are comparable with the head causing flow. The variation in the velocity from top to bottom edge is considerable.

- **According to shape**
- 1. Circular orifice
- 2. Rectangular orifice
- 3. Square orifice
- 4. Triangular orifice

- **According to shape of upstream edge**
- 1. Sharp-edged orifice
- 2. bell-mouthed orifice
- **According to discharge condition**
- 1. Free discharge orifice
- 2. Submerged orifice

Coefficients

- **Coefficient of contraction:** It is the ratio of area A_c *of jet*, to the area A_o of the orifice or other opening.

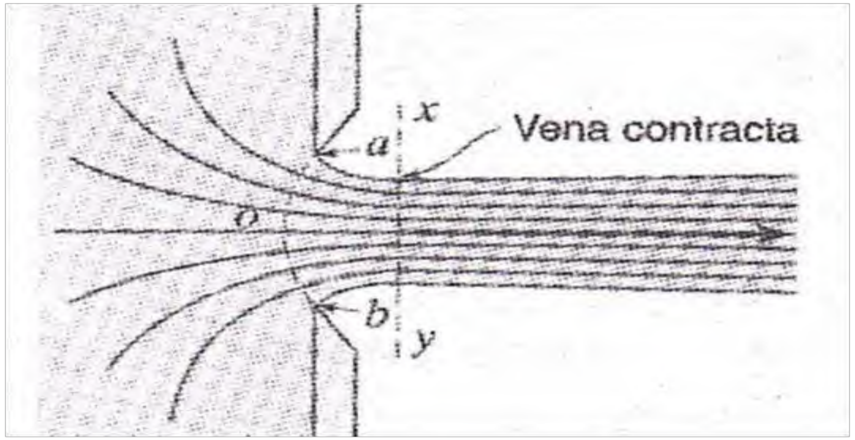
$$C_c = A_c / A_o$$

- **Coefficient of velocity:** It is ratio of actual velocity to ideal velocity

$$C_v = \frac{V_{act}}{V_{th}}$$

- **Coefficient of discharge:** It is the ratio of actual discharge to ideal discharge.

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{V_{act} A_{act}}{V_{th} A_{th}} = C_v C_c$$



Vena-Contracta is section of jet of minimum area. This section is about 0.5Do from upstream edge of the opening, where Do is diameter of orifice

Orifice

- Small orifice
- Figure shows a tank having small orifice at it bottom. Let the flow in tanks is steady.
- Let's take section 1 (at the surface) and 2 just outside of tank near orifice.
- According to Bernoulli's equation

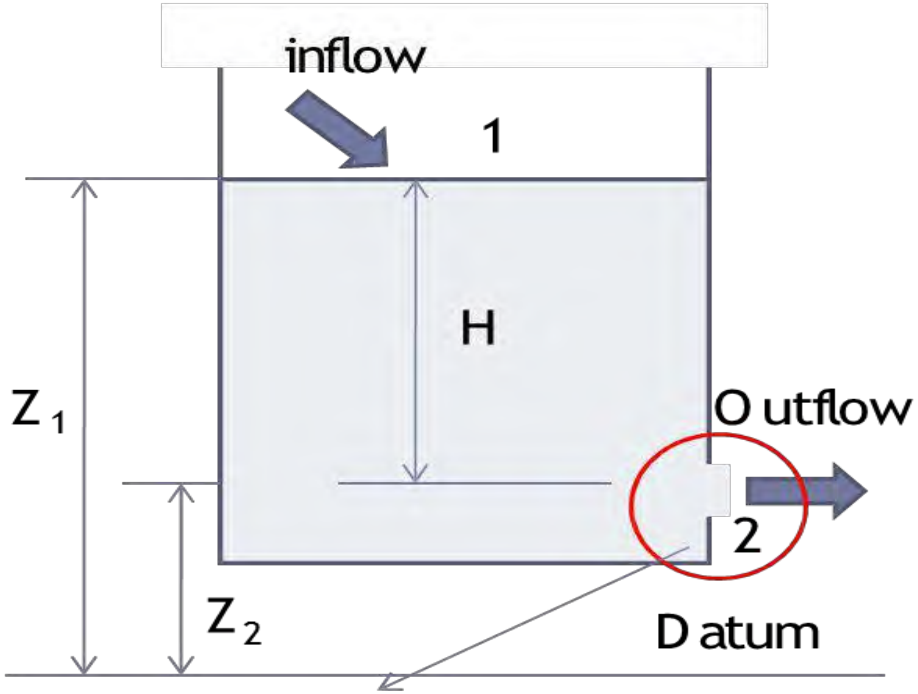
$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

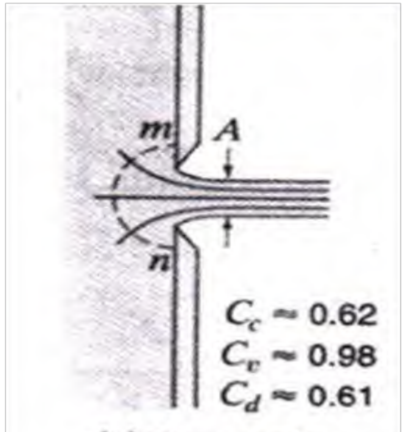
$$0 + z_1 + 0 = 0 + z_2 + \frac{v_2^2}{2g}$$

$$\frac{v_2^2}{2g} = z_1 - z_2 = H$$

$$v_{th} = \sqrt{2gH}$$

Where, H is depth of water above orifice





(a) Sharp-edge

Cross-sectional area

Orifice

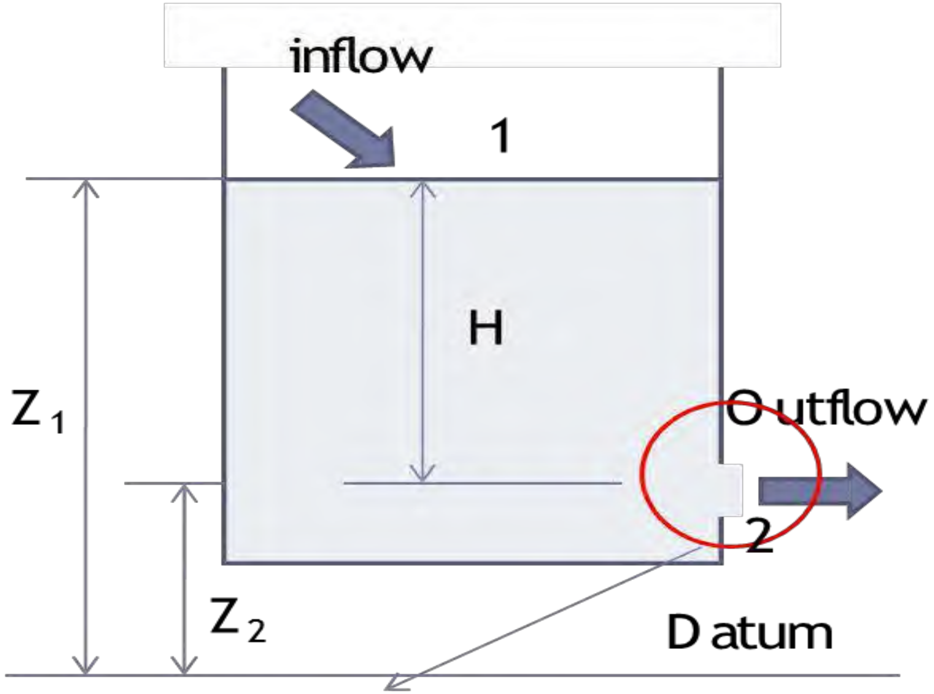
- Small orifice

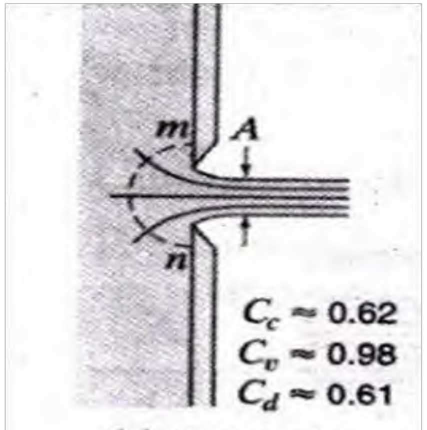
$$Q_{th} = Av_{th} = A\sqrt{2gH}$$

$$Q_{act} = C_d Av_{th} = C_d A\sqrt{2gH}$$

Q $v_{th} = \sqrt{2gH}$

Where, A is cross-sectional are of orifice and C_d is coefficient of discharge.





(a) Sharp-edge


C ross-sectional area, A

Mouthpieces/tubes

- A tube/mouth piece is a short pipe whose length is not more than **two or three diameters.**
- There is no sharp distinction between a tube and a thick walled orifices.
- A tube may be uniform diameter or it may diverge.

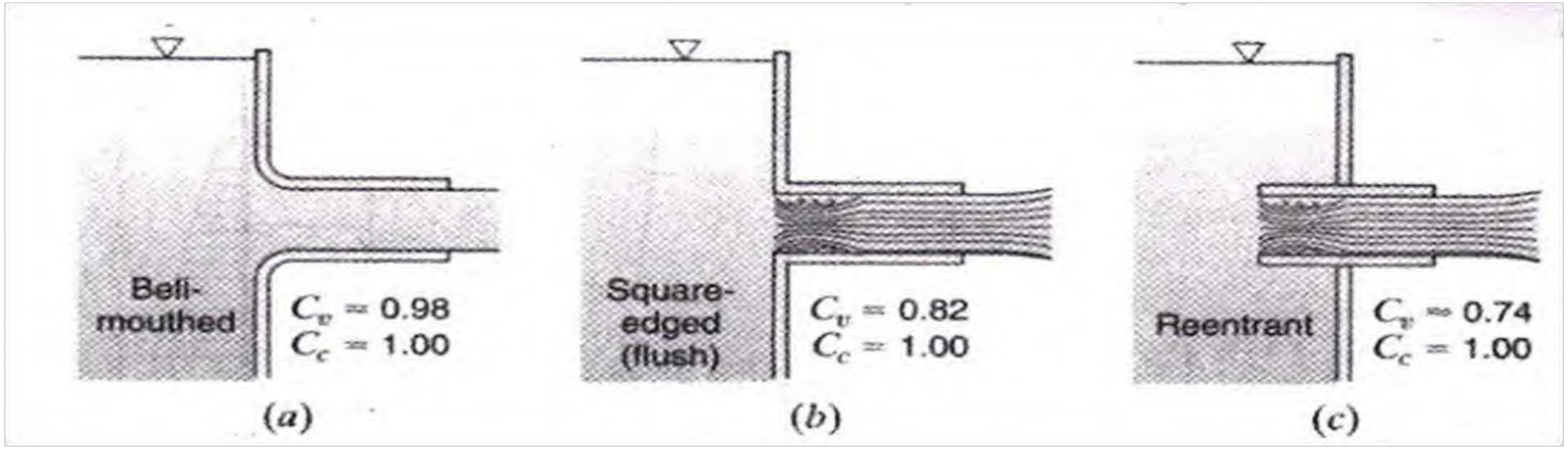
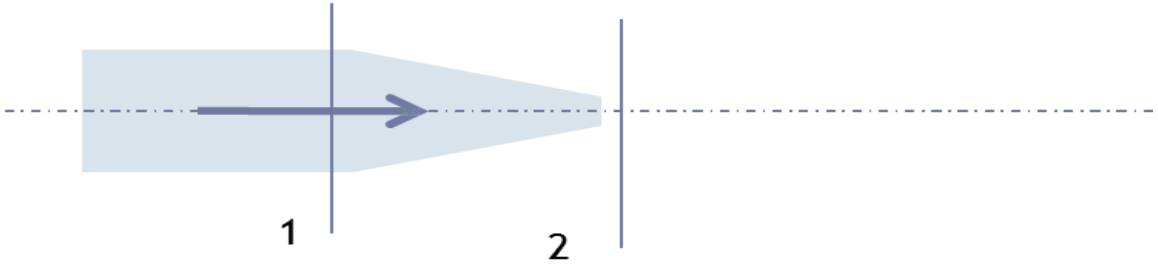


Figure: types and coefficients of tubes/mouthpieces

Nozzle

- **A nozzle is a tube of changing diameter, usually converging as shown in figure if used for liquids.**

Figure shows a nozzle. At section 1, diameter of pipe is D_1, and pressure is P_1 and similar D_2 and P_2 are respective values at section 2.



$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

$$\frac{P_1}{\gamma} + 0 + \frac{v_1^2}{2g} = 0 + 0 + \frac{v_2^2}{2g}$$

$$\frac{v_2^2}{2g} - \frac{v_1^2}{2g} = \frac{P_1}{\gamma}$$

$$\frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2} = 2g\frac{P_1}{\gamma}$$

According to continuity eq.

$$Q = Q_1 = Q_2$$

$$Q = A_1V_1 = A_2V_2$$

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Nozzle

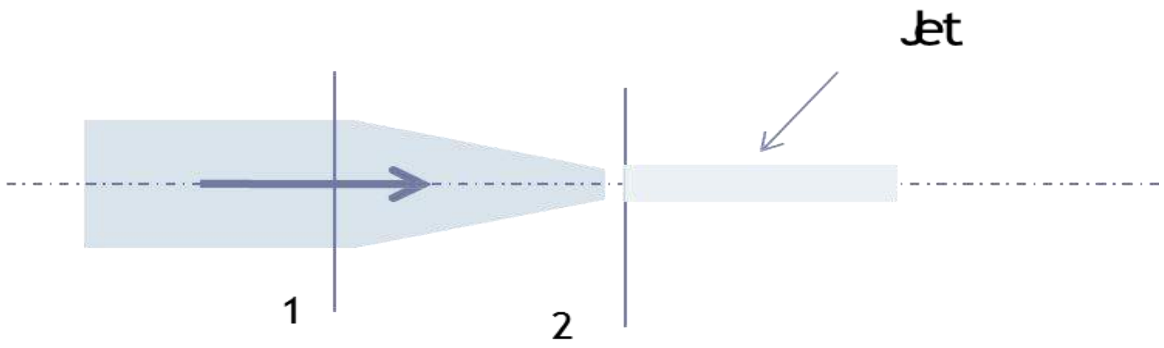
Jet: It is a stream issuing from a orifice, nozzle, or tube.

$$\frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2} = 2g\frac{P_1}{\gamma}$$

$$Q^2\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) = 2g\frac{P_1}{\gamma}$$

$$Q_{th} = \left(\frac{\square_1 A_2 A}{\sqrt{A_1^2 - A_2^2}}\right)\sqrt{2g\frac{P_1}{\gamma}}$$

$$Q_{act} = C_d\left(\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\right)\sqrt{2g\frac{P_1}{\gamma}}$$



According to continuity eq.

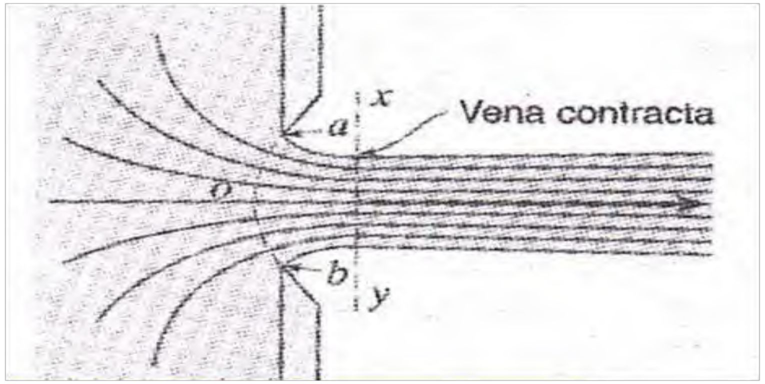
$$Q = Q_1 = Q_2$$

$$Q = A_1V_1 = A_2V_2$$

20

Nozzle

Vena-contracta is section of jet of minimum area. This section is about *0.5Do* from upstream edge of the opening, where *Do* is diameter of orifice



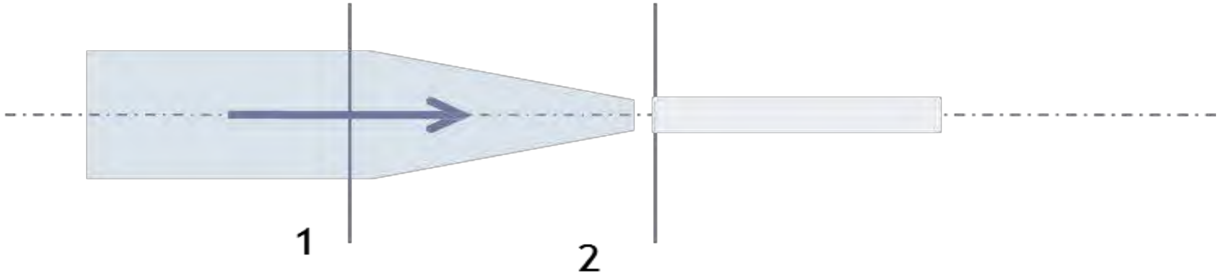
$$Q_{act} = C_d\left(\frac{A_1\,A_2}{\sqrt{A_1^2 - A_2^2}}\right)\sqrt{2g\frac{P_1}{\gamma}}$$

$$Q_{act} = C_d\left(\frac{A_1\left(C_c A_o\right)}{\sqrt{A_1^2 - \left(C_c^2 A_o^2\right)}}\right)\sqrt{2g\frac{P_1}{\gamma}}$$

$$Q_{act} = K\sqrt{2g\frac{P_1}{\gamma}}$$

$$K = C_d\left(\frac{A_1\left(C_c A_o\right)}{\sqrt{A_1^2 - \left(C_c^2 A_o^2\right)}}\right)$$

Where, K is coefficient of nozzle



$A_2 = C_c A_o$

Ao= cross-section area at nozzle

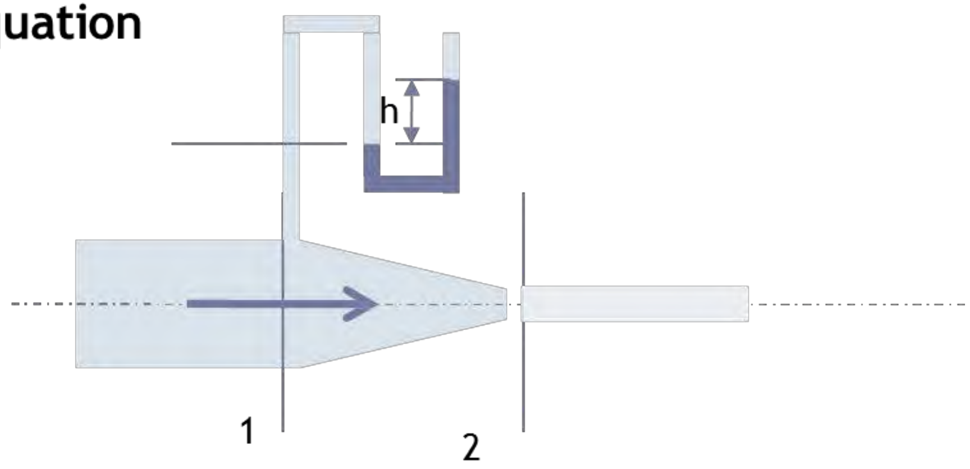
Pressure, P_1 is then measured with the help of piezometer or manometer

Nozzle

According to gauge pressure equation

$$\frac{P_1}{\gamma} - x - S_m h = 0$$

$$\frac{P_1}{\gamma} = x + S_m h$$



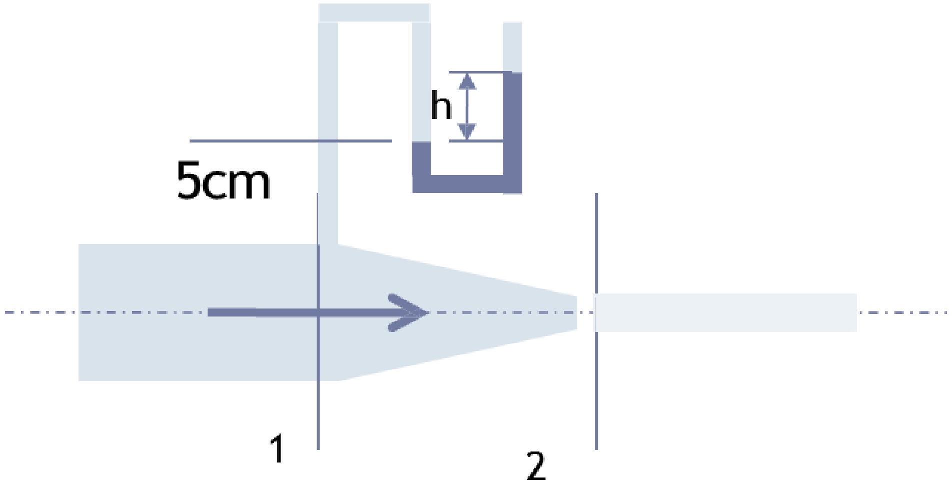
Calibration and Calibration Curves

- **Calibration :** Determine coefficients of flow measuring devices, e.g.,
  - Cd, Cc, Cv, etc

- **Calibration curve:** Plotting calibration curve
  - e.g., $h^{1/2}$ Vs Q_{act}
  - $h^{3/2}$ Vs Q_{act}

Numerical Problems

- Discharge and headloss in nozzle are 20L/s and 0.5m respectively. If dia of pipe is 10cm and dia of nozzle is 4cm, determine the manometric reading. Manometric fluid is mercury.



Solution:

$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g} + H_L$$

$$\frac{P_1}{\gamma} = x + S_m h$$

$$Q_{act} = C_d \left(\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \right) \sqrt{2g \frac{P_1}{\gamma}}$$

Numerical Problem

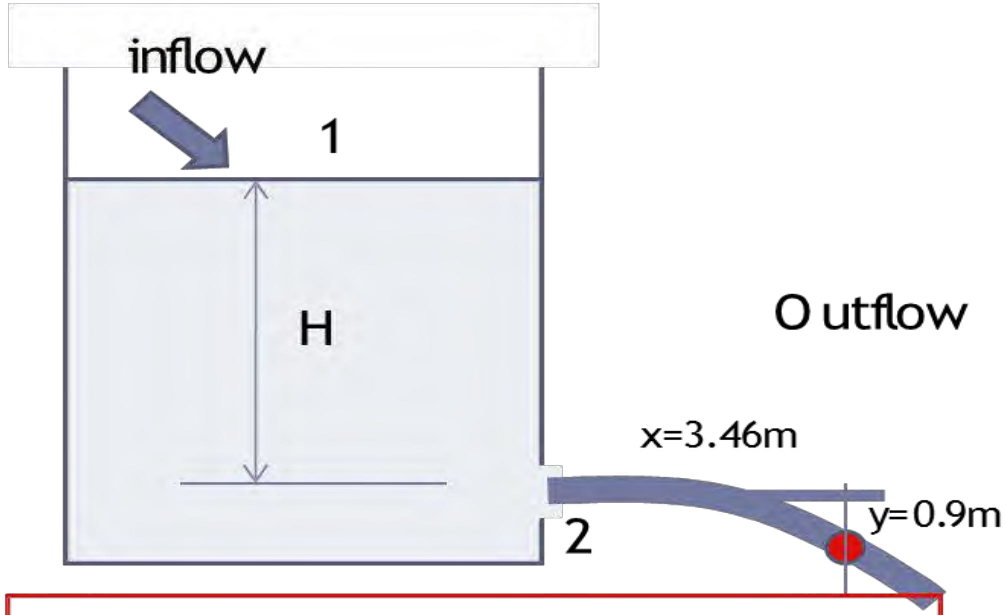
- A jet discharges from an orifice in a vertical plane under a head of 3.65m. The diameter of orifice is 3.75 cm and measured discharge is 6m³/s. The co-ordinates of centerline of jet are 3.46m horizontally from the vena-contracta and 0.9m below the center of orifice.
- Find the coefficient of discharge, velocity and contraction.

$$Q_{act} = C_d A v_{th} = C_d A\sqrt{2gH}$$

$$C_d = Q_{act} / \left(A\sqrt{2gH}\right)$$

$$C_v = \frac{v_{act}}{v_{th}} = \frac{\sqrt{gx^2 / 2y}}{\sqrt{2gH}}$$

$$C_c = C_d / C_v$$



$$x = v_{act} t$$

$$y = \frac{1}{2} g t^2 \Rightarrow V_{act} = \sqrt{gx^2 / 2y}$$

Bernoulli's Equation

$$\frac{P}{\gamma} + z + \frac{V^2}{2g} = H$$

Pressure head + Elevation head + Velocity head = Total Head

Multiplying with unit weight, γ,

$$P + \rho g z + \rho \frac{V^2}{2} = contt$$

- Static Pressure : P
- Dynamic pressure : $\rho V^2 / 2$
- Hydrostatic Pressure: $\rho g Z$
- Stagnation Pressure: Static pressure + dynamic Pressure

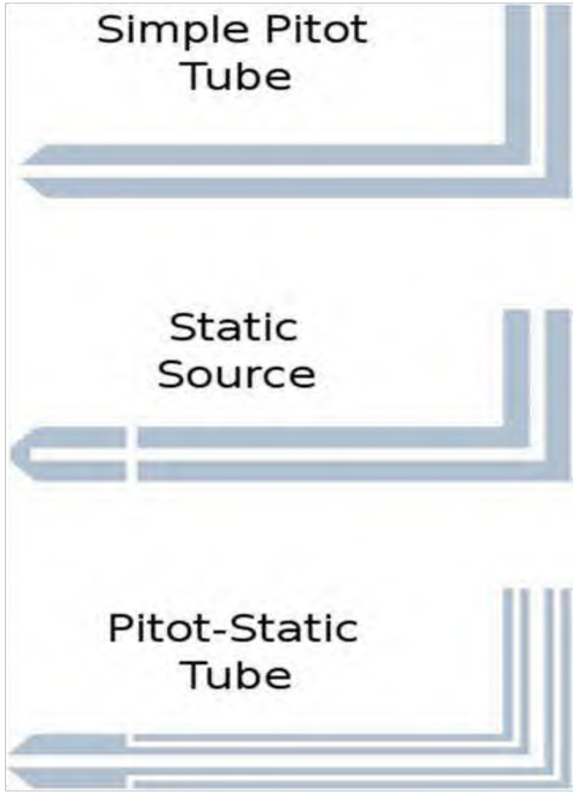
$$P + \rho \frac{V^2}{2} = P_{stag}$$

Pitot Tube and Pitot Static Tube

- Pitot Tube: It measures sum of velocity head and pressure head

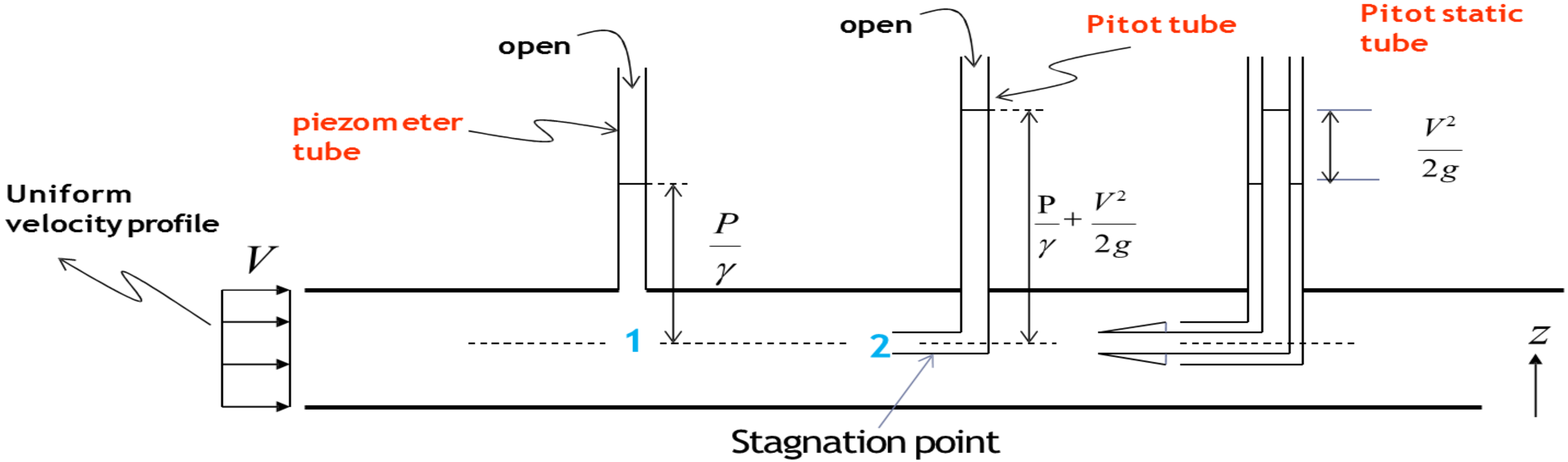
- Piezoemeter: It measures pressure head

- Pitot-Static tube: It is combination of piezometer and pitot tube. It can measure velocity head.



Pitot Tube and Pitot Static Tube

Consider the following closed channel flow (neglect friction):



$$\frac{V^2}{2g}=\left(\frac{V^2}{2g}+\frac{P}{\gamma}\right)-\frac{P}{\gamma}$$

$$V_{th}=\sqrt{2g\left(\frac{P_{stag}}{\gamma}-\frac{P}{\gamma}\right)}$$

Theoretical/ideal flow velocity at elevation z in pipe.

Remember !!

$$P+\rho\frac{V^2}{2}=P_{stag}$$

$$\frac{P}{\gamma}+\frac{V^2}{2g}=\frac{P_{stag}}{\gamma}$$

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Pitot Static Tube

- In reality, directional velocity fluctuations increase pitot-tube readings so that we must multiply Vth with factor C varying from 0.98 to 0.995 to give true (actual) velocity

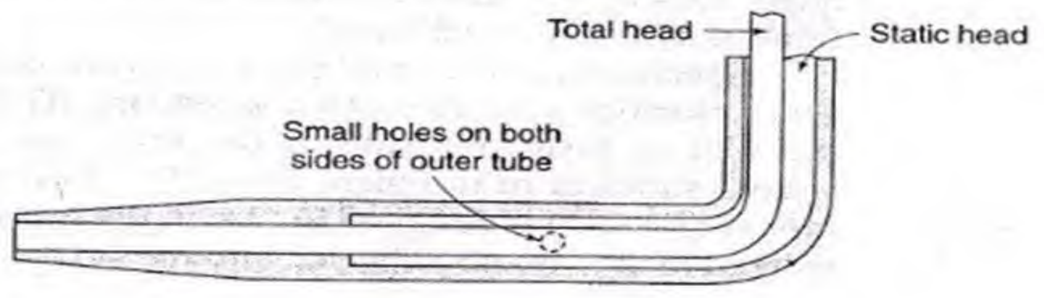

Figure
Pitot-static tube.

$$V_{act} = C\sqrt{2g\left(\frac{P_{stag}}{\gamma} - \frac{P}{\gamma}\right)}$$

- However, piezometer holes are rarely located in precisely correct position to indicate true value of P/γ, we modify above equation as;

$$V_{act} = C_1\sqrt{2g\left(\frac{P_{stag}}{\gamma} - \frac{P}{\gamma}\right)}$$

- Where C_1 is coefficient of instrument to account for discrepancy.

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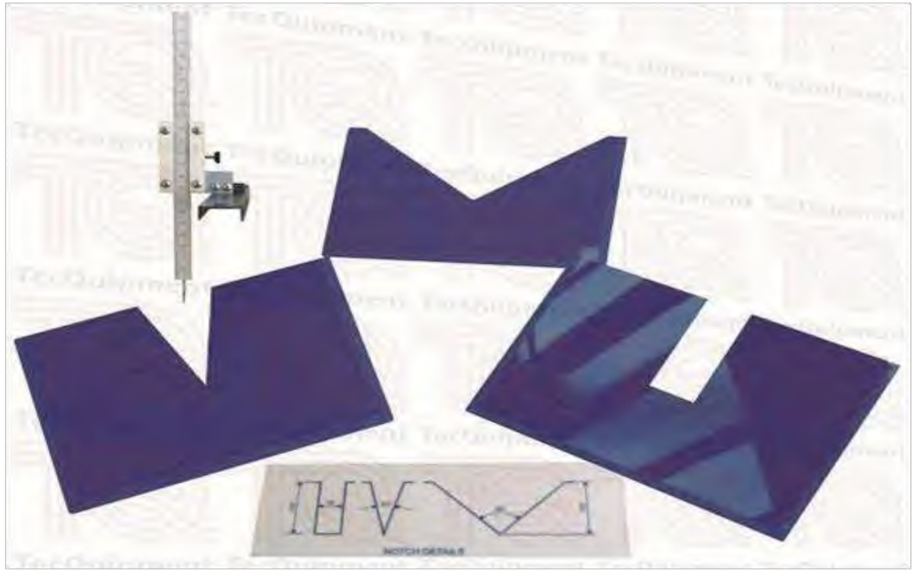
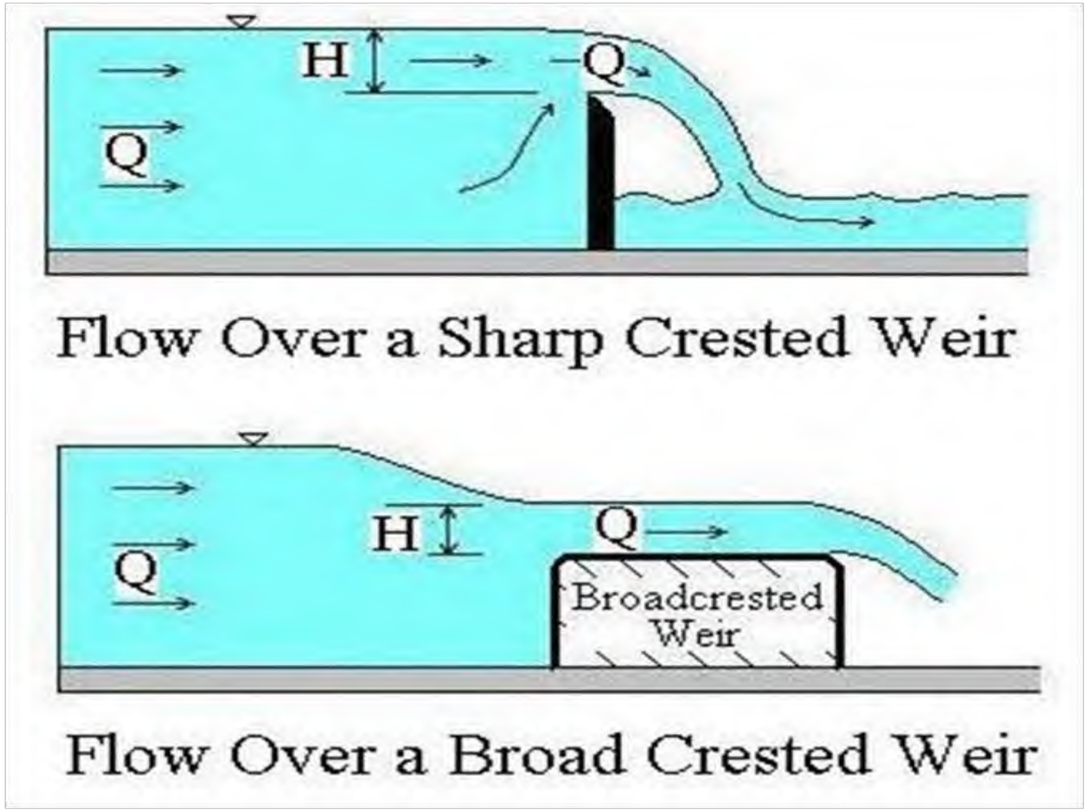
Notches and Weirs

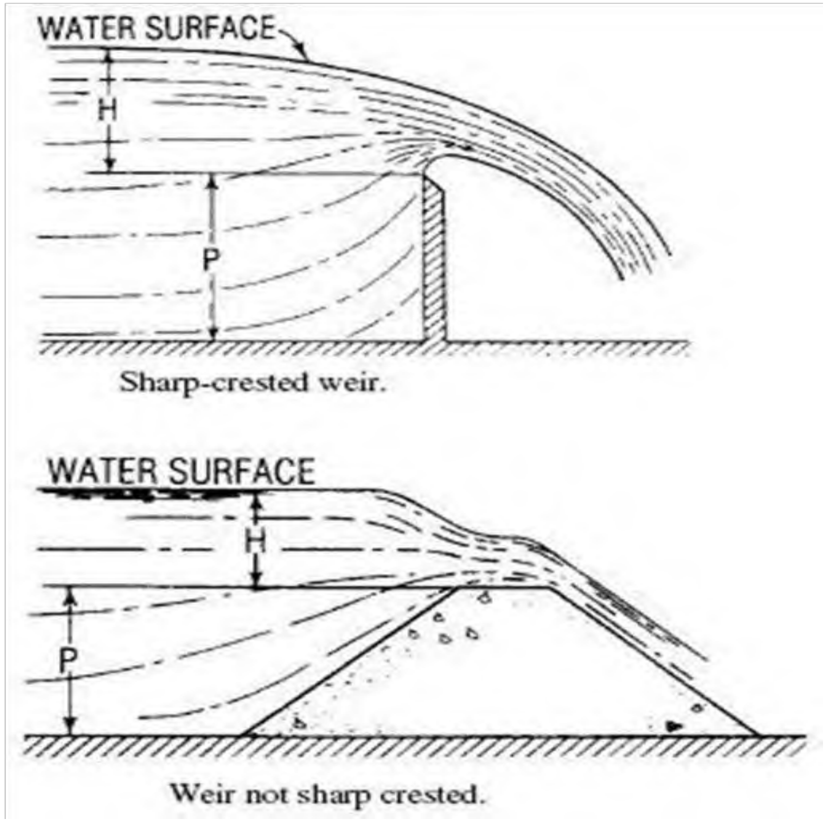


V Notch

30

Notches and Weirs





Notches and Weirs

- **Notch.** A notch may be defined as an opening in the side of a tank or vessel such that the liquid surface in the tank is below the top edge of the opening.
- A notch may be regarded as an orifice with the water surface below its upper edge. It is generally made of metallic plate. It is used for measuring the rate of flow of a liquid through a small channel of tank.
- **Weir:** It may be defined as any regular obstruction in an open stream over which the flow takes place. It is made of masonry or concrete.The condition of flow, in the case of a weir are practically same as those of a rectangular notch.

- **Nappe:** The sheet of water flowing through a notch or over a weir
- **Sill or crest.** The top of the weir over which the water flows is known as sill or crest.

- Note:The main difference between notch and weir is that the notch is smaller in size compared to weir.

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Classification of Notches/Weirs

- **Classification of Notches**
- 1. Rectangular notch
- 2. Triangular notch
- 3. Trapezoidal Notch
- 4. Stepped notch

- **Classification of Weirs**
- **According to shape**
- 1. Rectangular weir
- 2. Cippoletti weir
- **According to nature of discharge**
- 1. Ordinary weir
- 2. Submerged weir
- **According to width of weir**
- 1. Narrow crested weir
- 2. Broad crested weir
- **According to nature of crest**
- 1. Sharp crested weir
- 2. Ogee weir

Discharge over Rectangular Notch/Weir

- Consider a rectangular notch or weir provided in channel carrying water as shown in figure. In order to obtain discharge over whole area we must integrate above equation from $h=0$ to $h=H$, therefore;

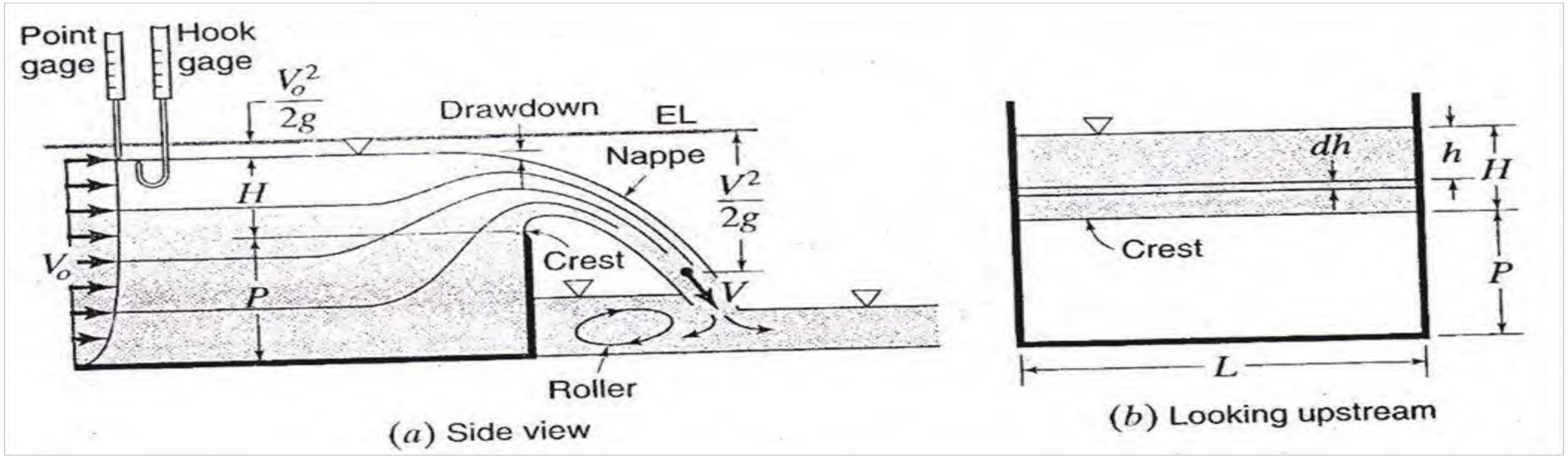

Figure: Flow over rectangular notch/weir

$$Q_{act} = C_d \frac{2}{3} \sqrt{2g} L H^{3/2}$$

Note: The expression of discharge (Q) for rectangular weir and sharp crested weirs are same.

Numerical Problems

- A rectangular notch 2m wide has a constant head of 500mm. Find the discharge over the notch if coefficient of discharge for the notch is 0.62.

Solution. Length of the notch, $L = 2{\cdot}0$ m

Head over notch, $H = 500$ mm $= 0{\cdot}5$ m

Co-efficient of discharge, $C_d = 0{\cdot}62$

Discharge, Q:

Using the relation,

$$Q = \frac{2}{3} C_d \, . \, L\sqrt{2g}\, (H)^{3/2}$$

$$= \frac{2}{3} \times 0{\cdot}62 \times 2{\cdot}0 \times \sqrt{2 \times 9{\cdot}81} \times (0{\cdot}5)^{3/2}$$

$$= \mathbf{1{\cdot}294\ m^3/s\ (Ans.)}$$

Numerical Problems

- A rectangular notch has a discharge of $0.24 m^3/s$, when head of water is 800mm. Find the length of notch. Assume C_d=0.6

Solution. Discharge, $Q = 0 \cdot 24\ m^3/s$

Head over notch, $H = 800\ mm = 0 \cdot 8\ m$

Co-efficient of discharge, $C_d = 0 \cdot 6$

Length of the notch, L:

Using the relation :

$$Q = \frac{2}{3} C_d \cdot L \times \sqrt{2g}\ (H)^{3/2}$$

$$0 \cdot 24 = \frac{2}{3} \times 0 \cdot 6 \times L \times \sqrt{2 \times 9 \cdot 81}\ (0 \cdot 8)^{3/2} = 1 \cdot 267\ L$$

$$\therefore \quad L = \frac{0 \cdot 24}{1 \cdot 267} = 0 \cdot 189 \text{ m or } 189 \text{ mm}$$

i.e. **L = 189 mm (Ans.)**

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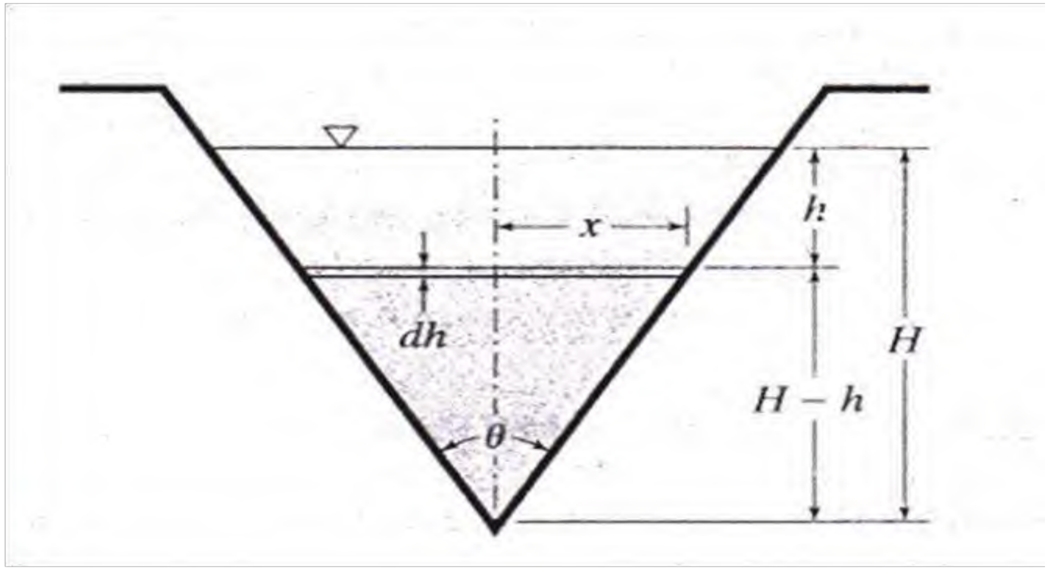
Discharge over Triangular Notch (V-Notch)

- In order to obtain discharge over whole area we must integrate above equation from $h=0$ to $h=H$, therefore;



$$Q_{act} = \frac{8}{15} C_d \sqrt{2g} \tan(\theta/2) \left[H^{5/2} \right]$$

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Numerical Problems

- Find the discharge over a triangular notch of angle 60°, when head over triangular notch is 0.2m. Assume C_d=0.6

Solution. Angle of notch, $\theta = 60°$

Depth of water, $H = 0{\cdot}2$ m

Co-efficient of discharge, $C_d = 0{\cdot}6$

Discharge, Q:

Using the relation :

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} \times H^{5/2}$$

$$= \frac{8}{15} \times 0{\cdot}6 \times \sqrt{2 \times 9{\cdot}81} \times \tan\frac{60°}{2} \times (0{\cdot}2)^{5/2}$$

$$= \frac{8}{15} \times 0{\cdot}6 \times 4{\cdot}429 \times 0{\cdot}577 \times 0{\cdot}01788$$

$= \mathbf{0{\cdot}01462\ m^3/s}$ **(Ans.)**

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Numerical Problems

- During an experiment in a laboratory, $0.05 m^3$ of water flowing over a right angled notch was collected in one minute. If the head over sill is 50mm calculate the coefficient of discharge of notch.
- **Solution:**
- Discharge=$0.05 m^3$/min=$0.000833 m^3/s$
- Angle of notch, $\theta=90^o$
- Head of water=H=50mm=0.05m
- Cd=?

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan \frac{\theta}{2} \times H^{5/2}$$

$$0 \cdot 000833 = \frac{8}{15} \times C_d \times \sqrt{2 \times 9 \cdot 81} \times \tan \left(\frac{90°}{2} \right) \times (0 \cdot 05)^{5/2}$$

$$= \frac{8}{15} \times C_d \times 4 \cdot 429 \times 1 \times 0 \cdot 000559 = 0 \cdot 00132 \, C_d$$

$$C_d = \frac{0 \cdot 000833}{0 \cdot 00132} = 0 \cdot 63 \text{ (Ans.)}$$

Numerical Problems

- A rectangular channel 1.5m wide has a discharge of 0.2m³/s, which is measured in right-angled V notch, Find position of the apex of the notch from the bed of the channel. Maximum depth of water is not to exceed 1m. Assume C_d=0.62

Width of rectangular channel, L=1.5m
Discharge=Q=0.2m³/s
Depth of water in channel=1m
Coefficient of discharge=0.62
Angle of notch= 90°

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan \frac{\theta}{2} \times H^{5/2}$$

$$0 \cdot 2 = \frac{8}{15} \times 0 \cdot 62 \times \sqrt{2 \times 9 \cdot 81} \times \tan \left(\frac{90^\circ}{2} \right) \times H^{5/2}$$

$$= 1 \cdot 465\, H^{5/2}$$

$$H = \left(\frac{0 \cdot 2}{1 \cdot 465} \right)^{2/5} = 0 \cdot 45 \text{ m}$$

Height of apex of notch from bed=Depth of water in channel-height of water over V-notch
=1-0.45= 0.55m

Discharge over Rectangular Notch/Weir

- Consider a rectangular notch or weir provided in channel carrying water as shown in figure.

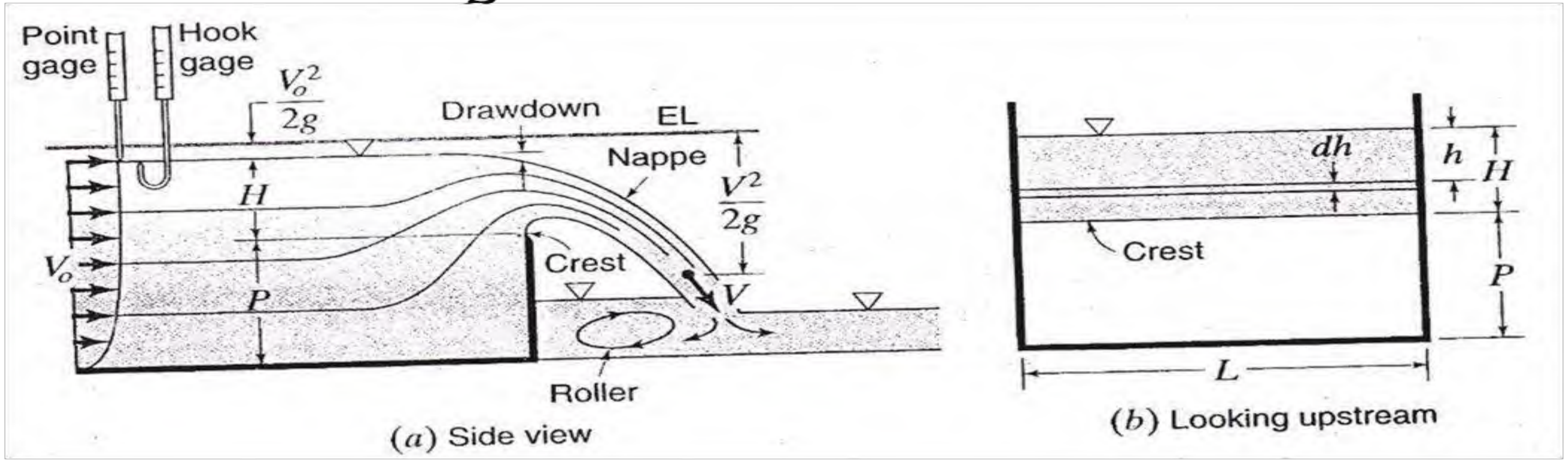

Figure: flow over rectangular notch/weir

H=height of water above crest of notch/weir
P =height of notch/weir
L =length of notch/weir
dh=height of strip
h= height of liquid above strip
L(dh)=area of strip

Vo = Approach velocity
Theoretical velocity of strip neglecting approach velocity = $\sqrt{2gh}$

Thus,
discharge passing through strips
= $Area \times velocity$

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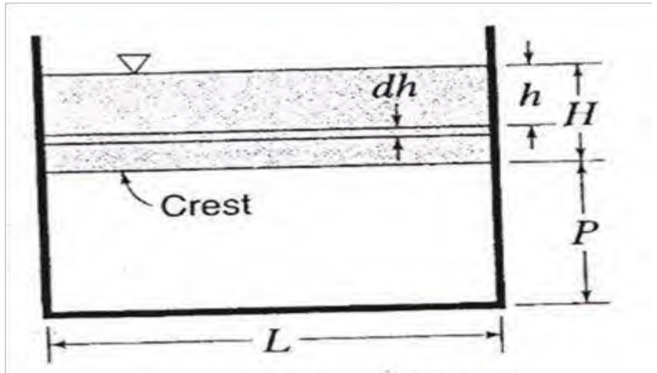
Discharge over Rectangular Notch/Weir

- Therefore, discharge of strip

$$dQ = Ldh\left(\sqrt{2gh}\right)$$

$$v_{strip} = \sqrt{2gh}$$

$$A_{strip} = Ldh$$



(b) Looking upstream

- In order to obtain discharge over whole area we must integrate above eq. from h=0 to h=H, therefore;

$$Q = \sqrt{2g}L\int_0^H \sqrt{h}\,dh$$

$$Q = \frac{2}{3}\sqrt{2g}LH^{3/2}$$

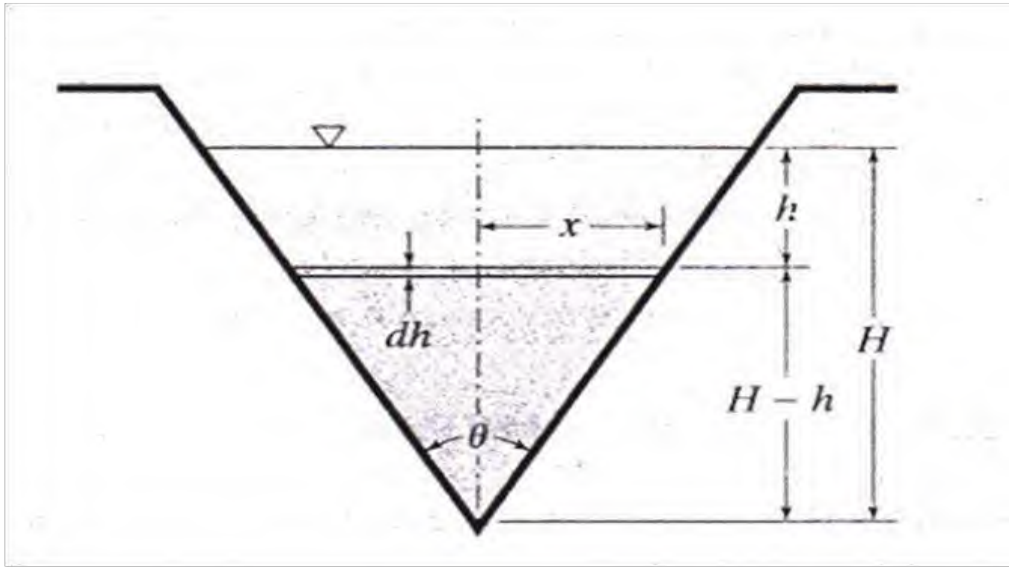
$$Q_{act} = C_d\,\frac{2}{3}\sqrt{2g}LH^{3/2}$$

Where, *Cd* = Coefficient of discharge

Note: The expression of discharge (Q) for rectangular weir and sharp crested weirs are same.

Discharge over Triangular Notch (V-Notch)

- In order to obtain discharge over whole area we must integrate above equation from $h=0$ to $h=H$, therefore;



$$Q=\int_0^H dh\big(2(H-h)\tan(\theta/2)\big)\left(\sqrt{2gh}\right)$$

$$Q=2\sqrt{2g}\,\tan(\theta/2)\int_0^H (H-h)\sqrt{h}\,dh$$

$$Q=2\sqrt{2g}\,\tan(\theta/2)\int_0^H \left(Hh^{1/2}-h^{3/2}\right)dh$$

$$Q=2\sqrt{2g}\,\tan(\theta/2)\left[\frac{4}{15}H^{5/2}\right]$$

$$Q=\frac{8}{15}\sqrt{2g}\,\tan(\theta/2)\left[H^{5/2}\right]$$

$$Q_{act}=\frac{8}{15}C_d\sqrt{2g}\,\tan(\theta/2)\left[H^{5/2}\right]$$

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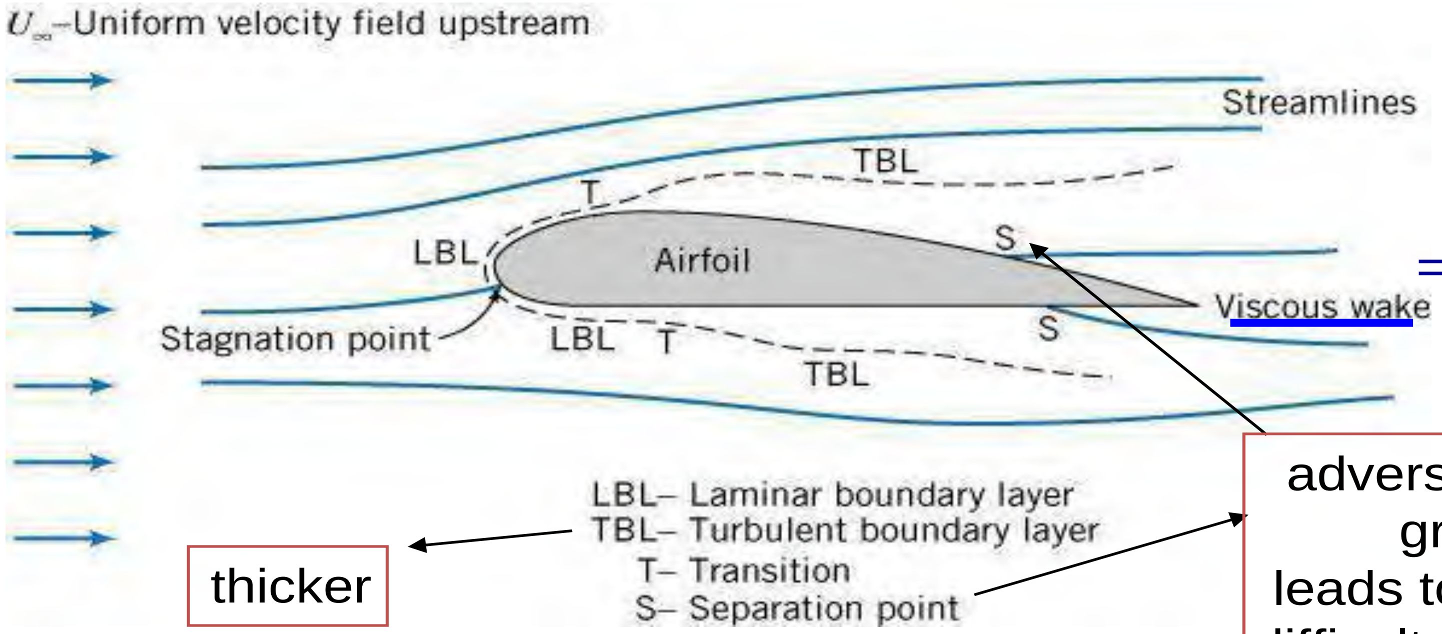
UNIT-IV
Boundary Layer Theory

EXTERNAL INCOMPRESSIBLE VISCOUS FLOWS



= separated bdy layer

thicker

adverse pressure gradient
leads to separation
difficult to use theory

- $Re = U_\infty x/\nu$; $Re = U_\infty c/\nu$; …
- laminar and turbulent boundary layers
- displaced inviscid outer flow
- adverse pressure gradient and separation

Boundary Layer Provides Missing Link Between Theory and Practice

Boundary layer, d, where viscous stresses
(i.e. velocity gradient) are important we’ll define
as where u(x,y) = 0 to 0.99 $U_{¥}$ above boundary.

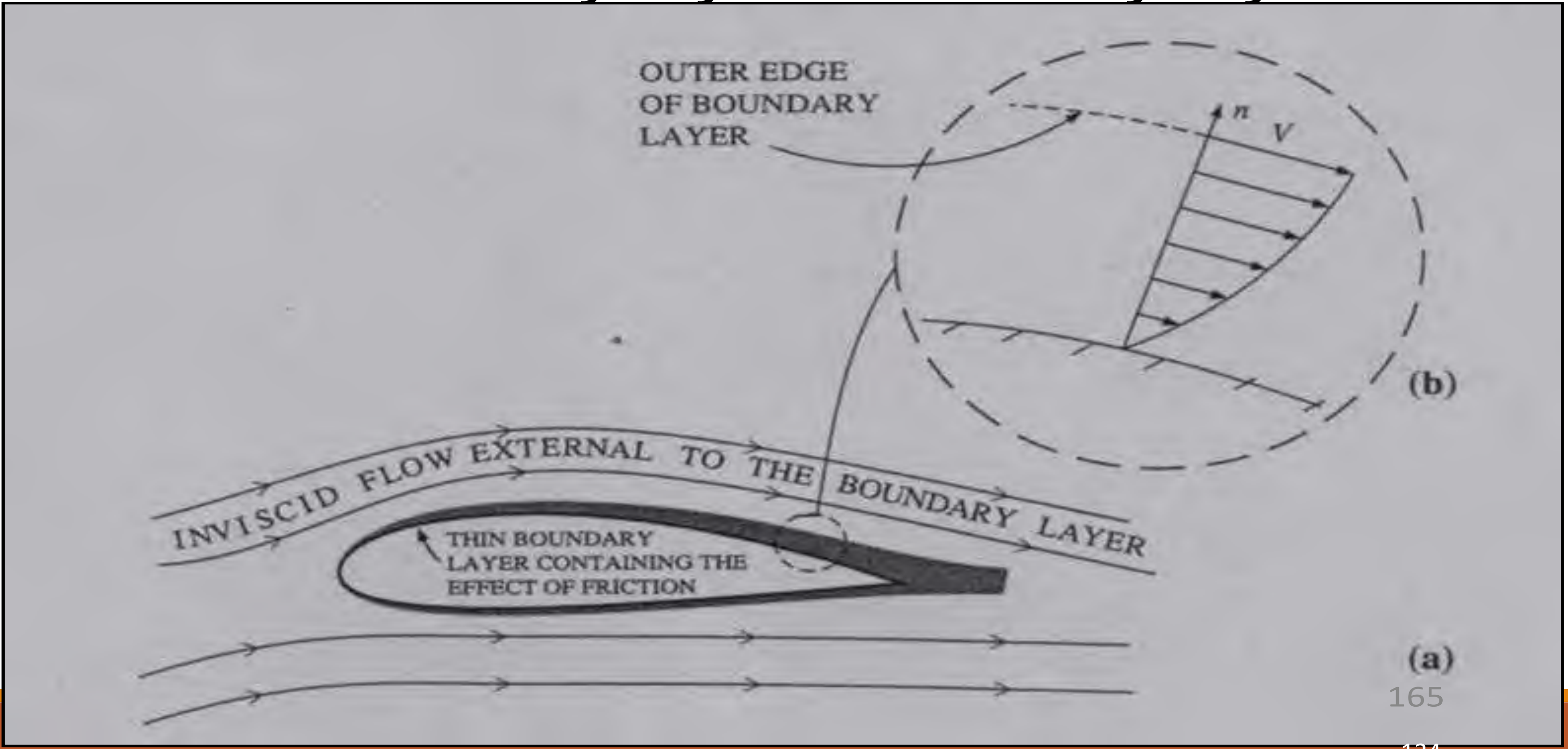
Ludwig Prandtl

originator of boundary layer theory and advisor to von Kármán, Blasius, Nikuradse and others

In August of 1904 Ludwig Prandtl, a 29-year old professor presen a remarkable paper at the 3rd International Mathematical Congres Heidelberg. Although initially largely ignored, by the 1920s and 19 the powerful ideas of that paper helped create modern fluid dyna out of ancient hydraulics and 19th-century hydrodynamics.

- Prandtl assumed no slip condition
- Prandtl assumed thin boundary layer region where shear force are important because of large velocity gradient
- Prandtl assumed inviscid external flow
- Prandtl assumed boundary so thin that within it $\partial p/\partial y \approx 0$; $v << u$ and $\partial/\partial x << \partial/\partial y$
- Prandtl outer flow drives boundary layer boundary layer can greatly effe

Extraordinary insights: Ludwig Prandtl in 1936.



BOUNDARY LAYER HISTORY

- 1904 Prandtl
  Fluid Motion with Very Small Friction
  2-D boundary layer equations

- 1908 Blasius
  The Boundary Layers in Fluids with Little Friction
  Solution for laminar, 0-pressure gradient flow

- 1921 von Karman
  Integral form of boundary layer equations

- 1924 Sir Horace Lamb
  Hydrodynamics ~ one paragraph on bdy layers

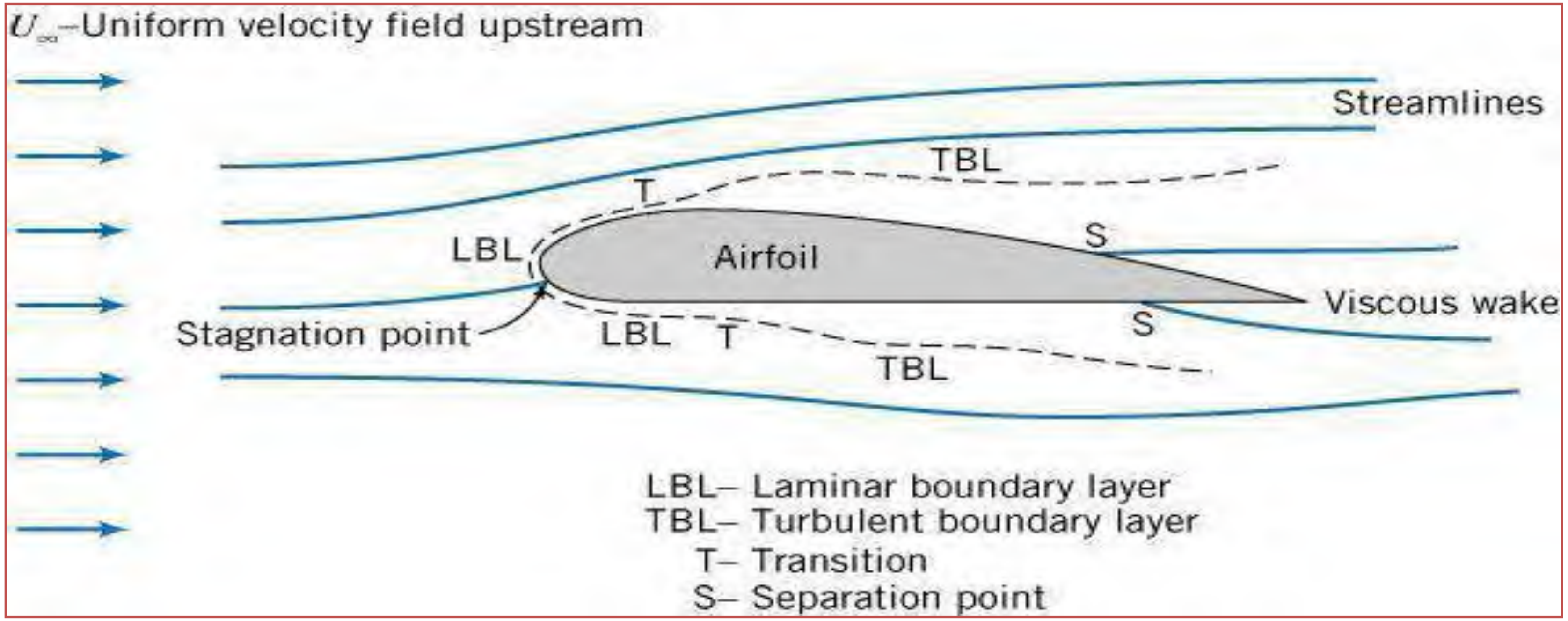
- 1932 Sir Horace Lamb
  Hydrodynamics ~ entire section on bdy layers

Theodore von Karman

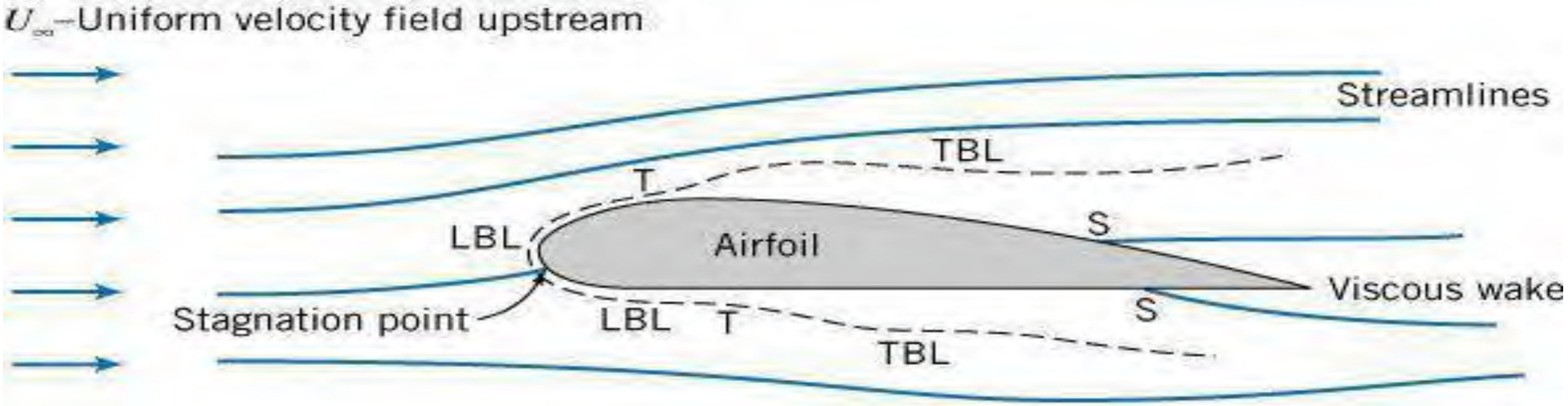
	INTERNAL	EXTERNAL
FULLY DEVELOPED?	CAN BE	NEVER
WAKE?	NEVER	USUALLY - PLATE IS EXCEPTION
THEORY LAMINAR	PIPES, DUCTS,..	FLAT PLATE & ZERO PRESSURE GRADIENT
GROWING BOUNDARY LAYER?	NOT WHEN FULLY DEVELOPED	ALWAYS
ADVERSE PRESSURE GRADIENT	PIPE/DUCT=N0 DIFFUSER=YES	PLATE=MAYBE BODIES=USUALLY

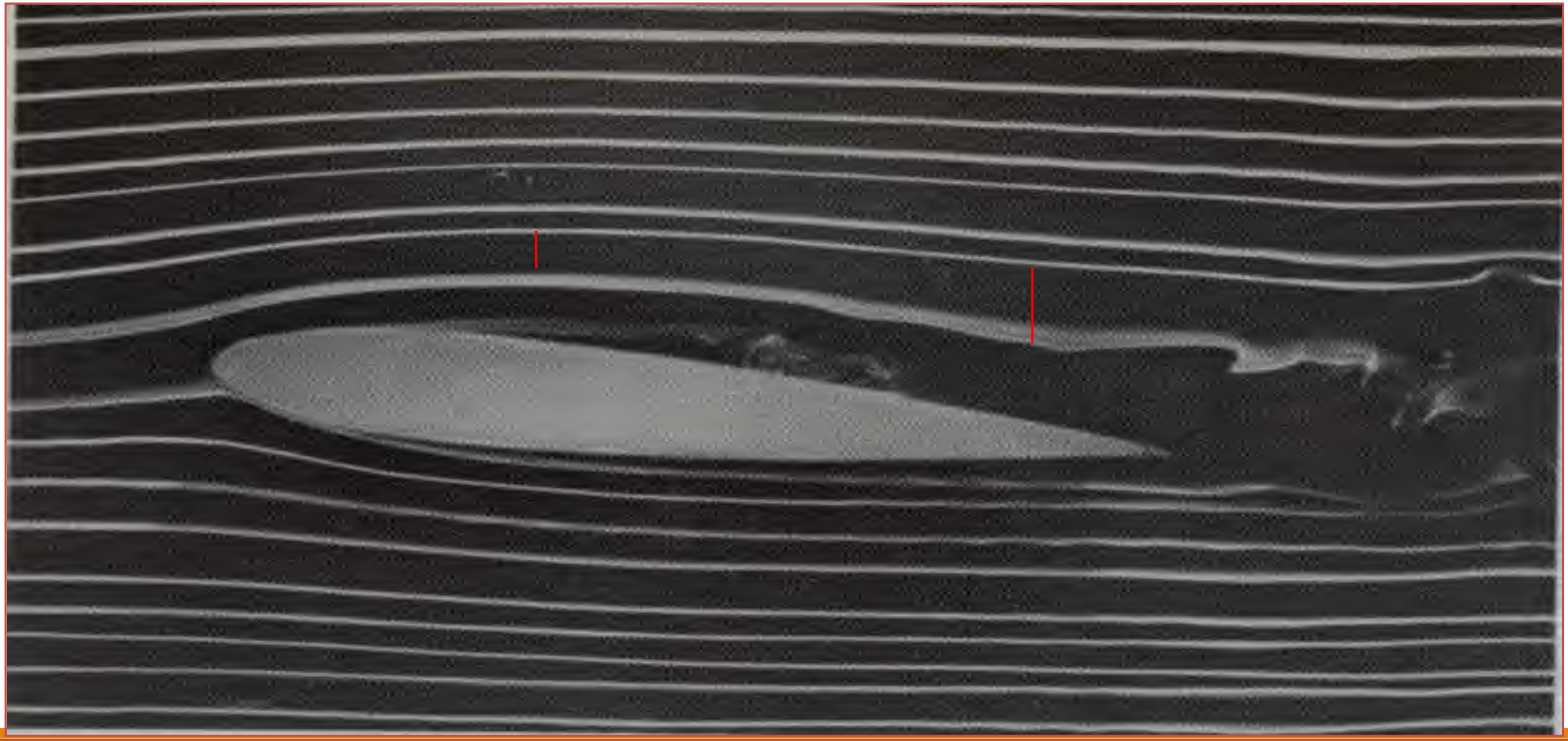
ote *– throughout figures the oundary layer thickness*, δ, is greatly exaggerated!*

(*disturbance layer**)



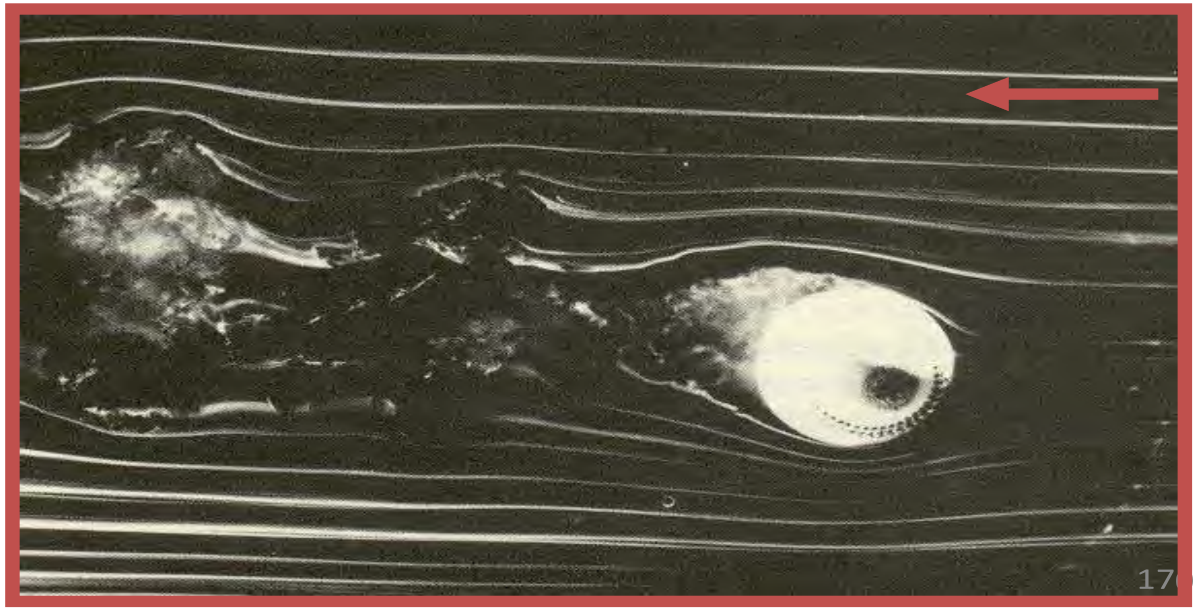
Airline industry had to develop flat face rivets.





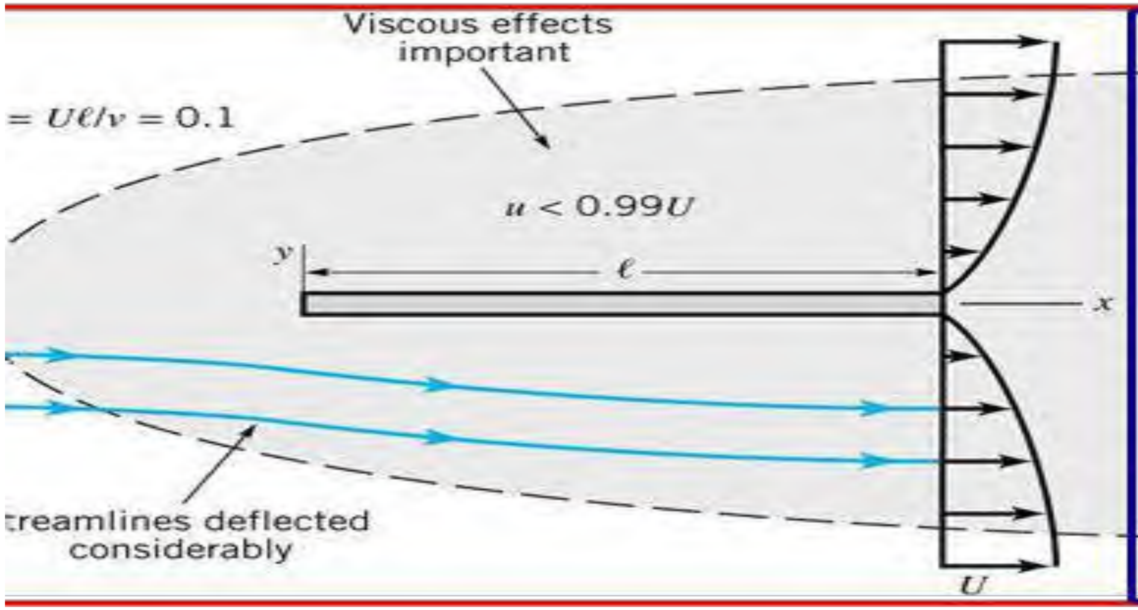
Re = 20,000
Angle of attack = 6o
Symmetric Airfoil
16% thick

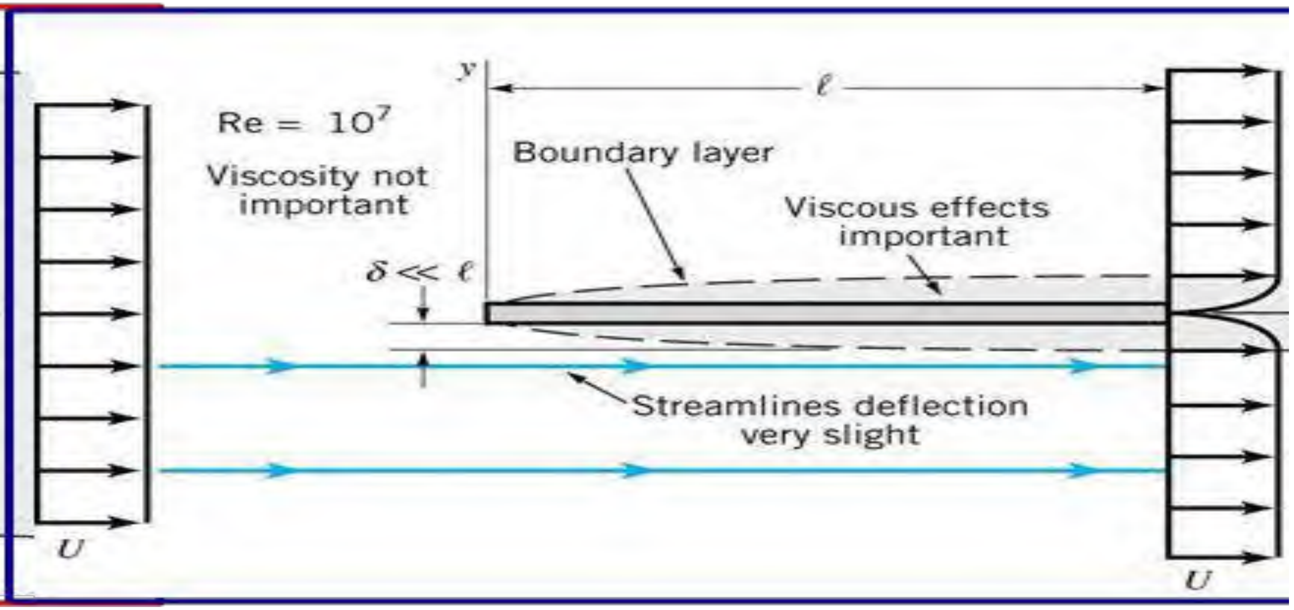
Flat Plate (no pressure gradient)
~ what is velocity profile?
~ wall shear stress/drag?
~ displacement of free stream?
~ laminar vs turbulent flow?



Immersed Bodies
~ wall shear stress/drag?
~ lift?
~ minimize wake

FLAT PLATE – ZERO PRESSURE GRADIENT





Laminar Flow

$\delta/x \sim 5.0/Re_x^{1/2}$

THEORY

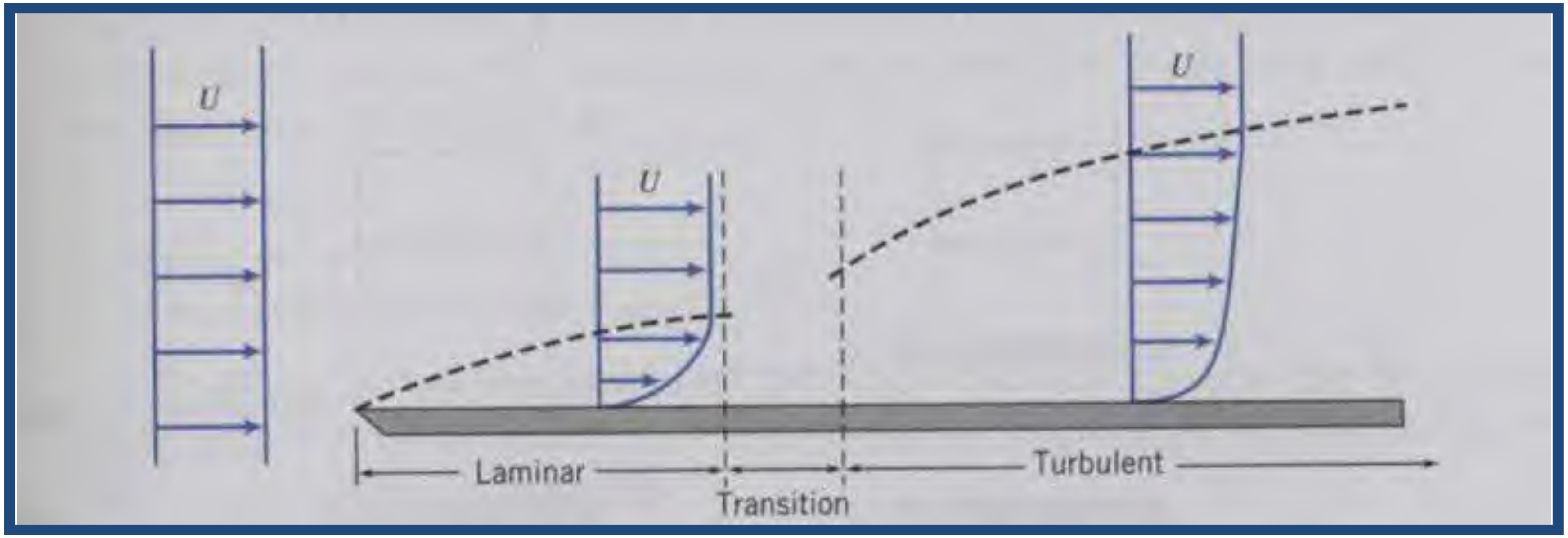
Turbulent Flow

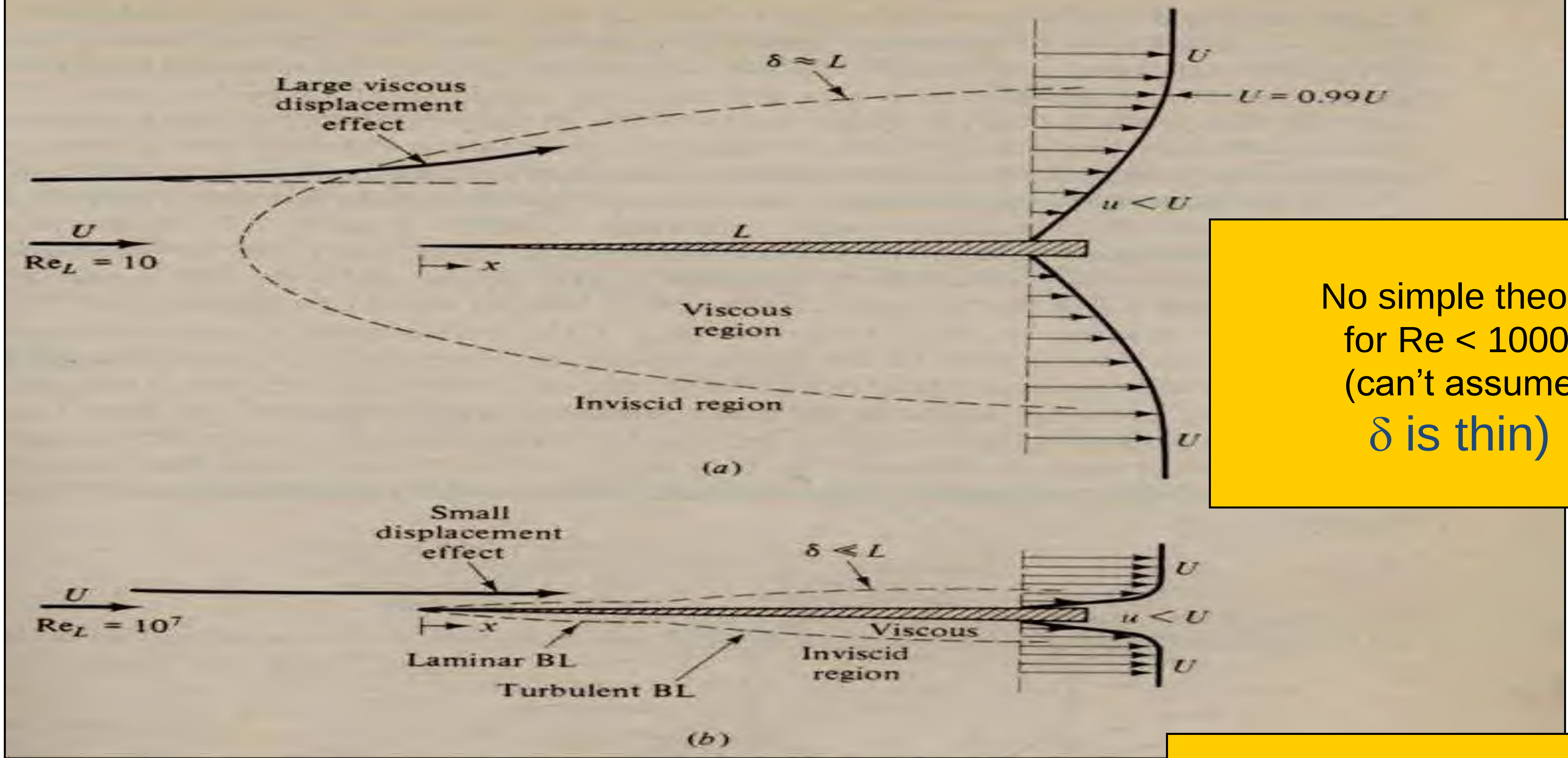
$Re_{xtransition} > 500{,}000$

$u(y)/U_\infty = (y/\delta)^{1/7}$

$\delta/x \sim 0.382/Re_x^{1/5}$

EXPERIMENTAL



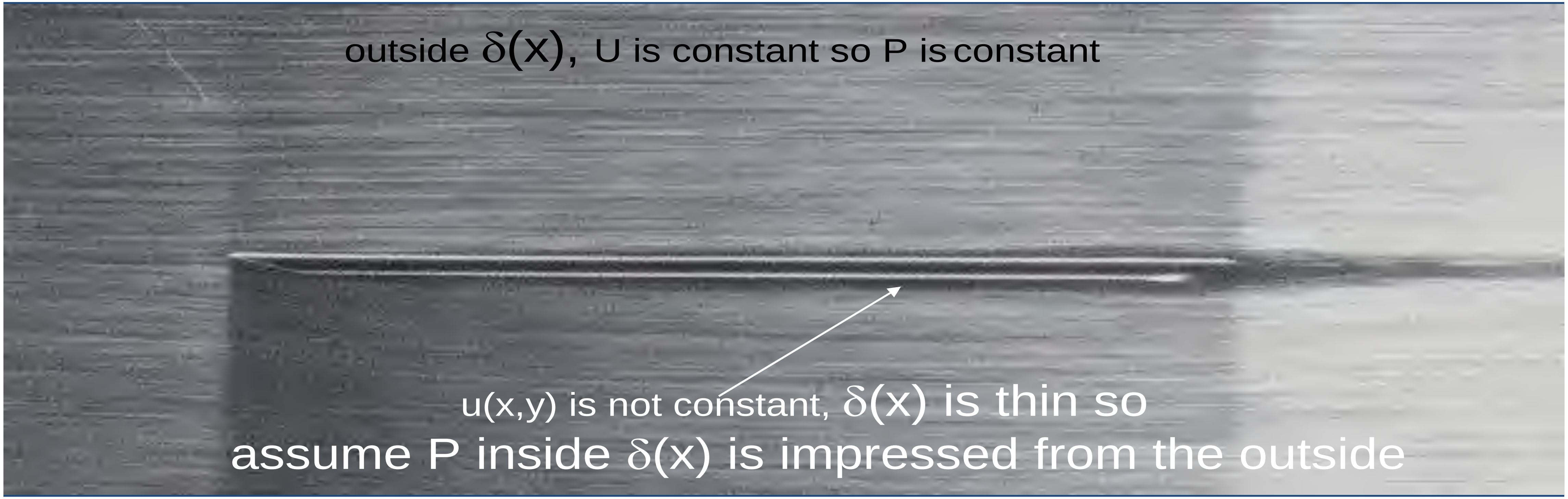


No simple theory for Re < 1000; (can't assume δ is thin)

Re_x	10^4	10^5	10^6	10^7	10^8
$(\delta/x)_{lam}$	0.050	0.016	0.005		
$(\delta/x)_{turb}$			0.022	0.016	0.011

"At these Re_x number bdy layers so thin that displacement effect o outer inviscid layer is

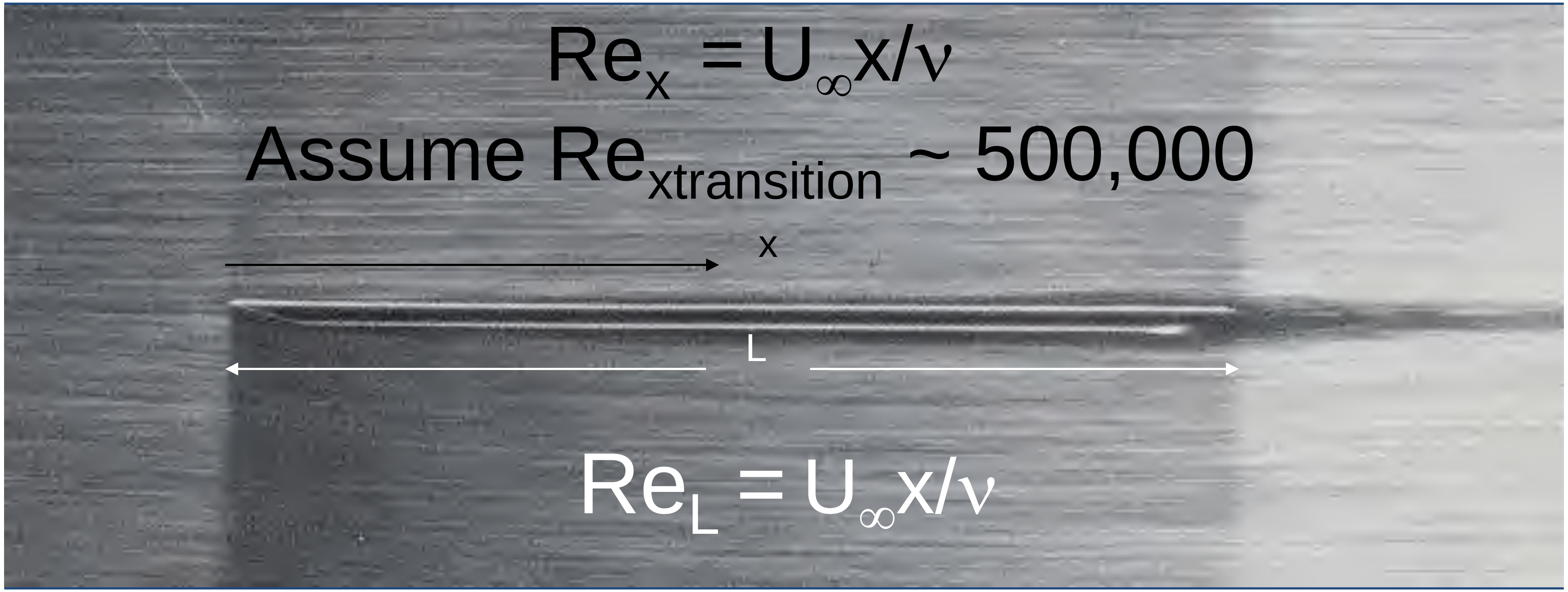
FLAT PLATE – ZERO PRESSURE GRADIENT

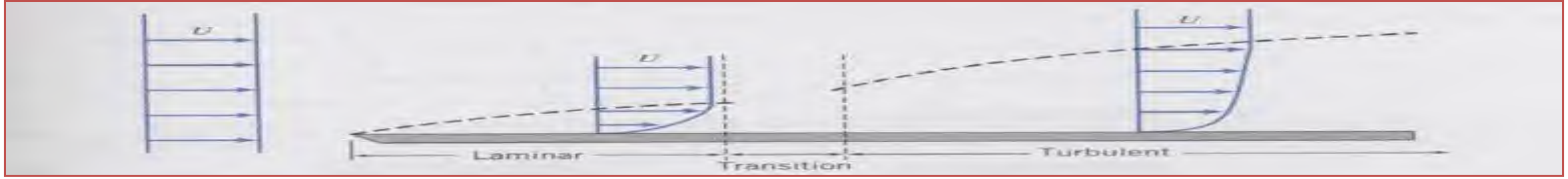


e_L = 10,000 Visualization is by air bubbles see that boundary+ layer, is thin and that outer free stream is displaced, δ^*, very little.

+ Disturbance Thickness, δ(x) (pg 412); boundary layer thickness, δ(x) (pg 415

FLAT PLATE – ZERO PRESSURE GRADIENT





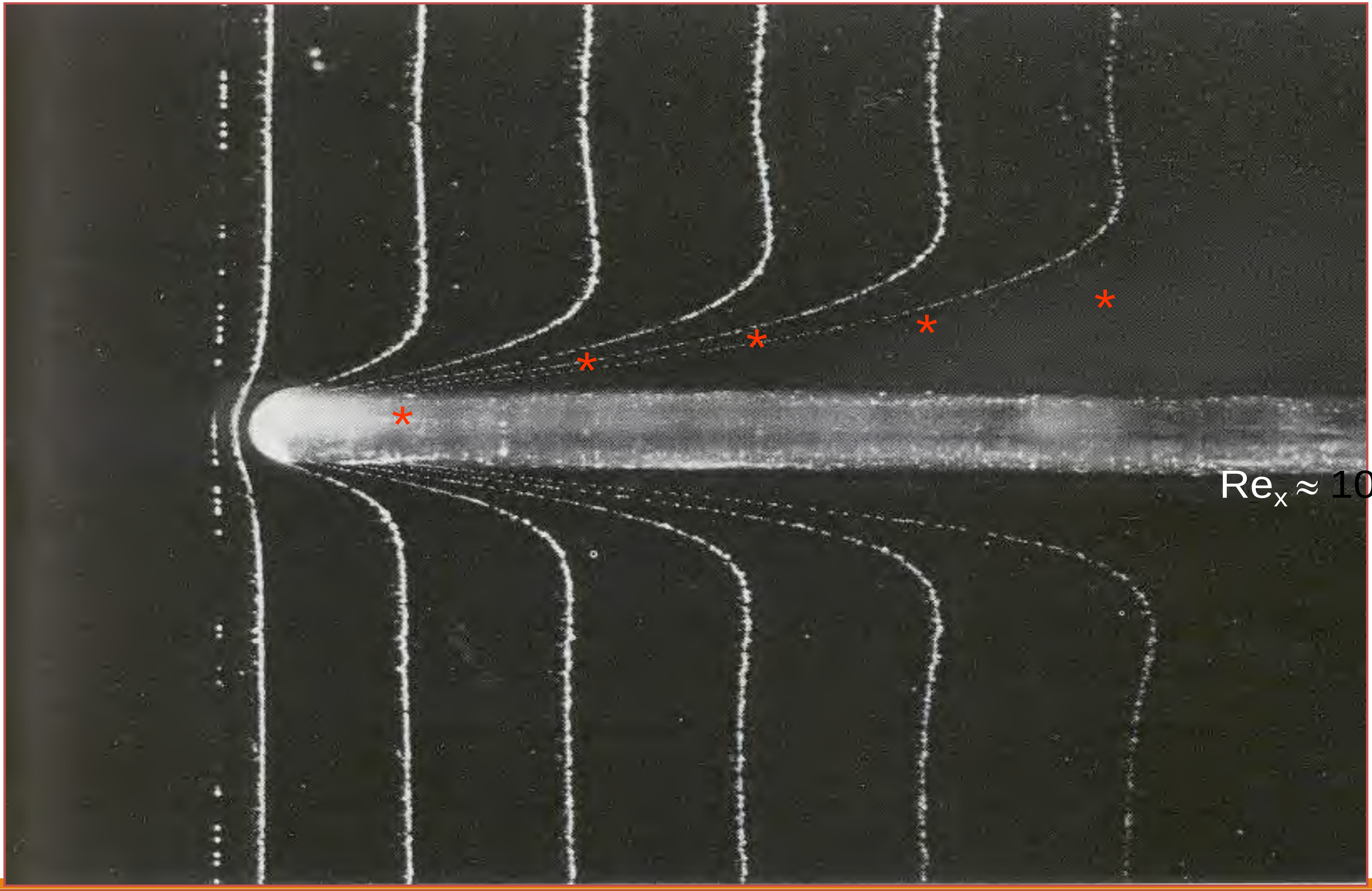
SIMPLIFYING ASSUMPTIONS OFTEN MADE FOR ENGINERING ANALYSIS OF BOUNDARY LAYER FLOWS

1. $u \rightarrow U$ at $y = \delta$
2. $\partial u/\partial y \rightarrow 0$ at $y = \delta$
3. $v \ll U$ within the boundary layer

Results of the analyses developed in the next two sections show that the boundary layer is very thin compared with its development length along the surface. Therefore it is also reasonable to assume:

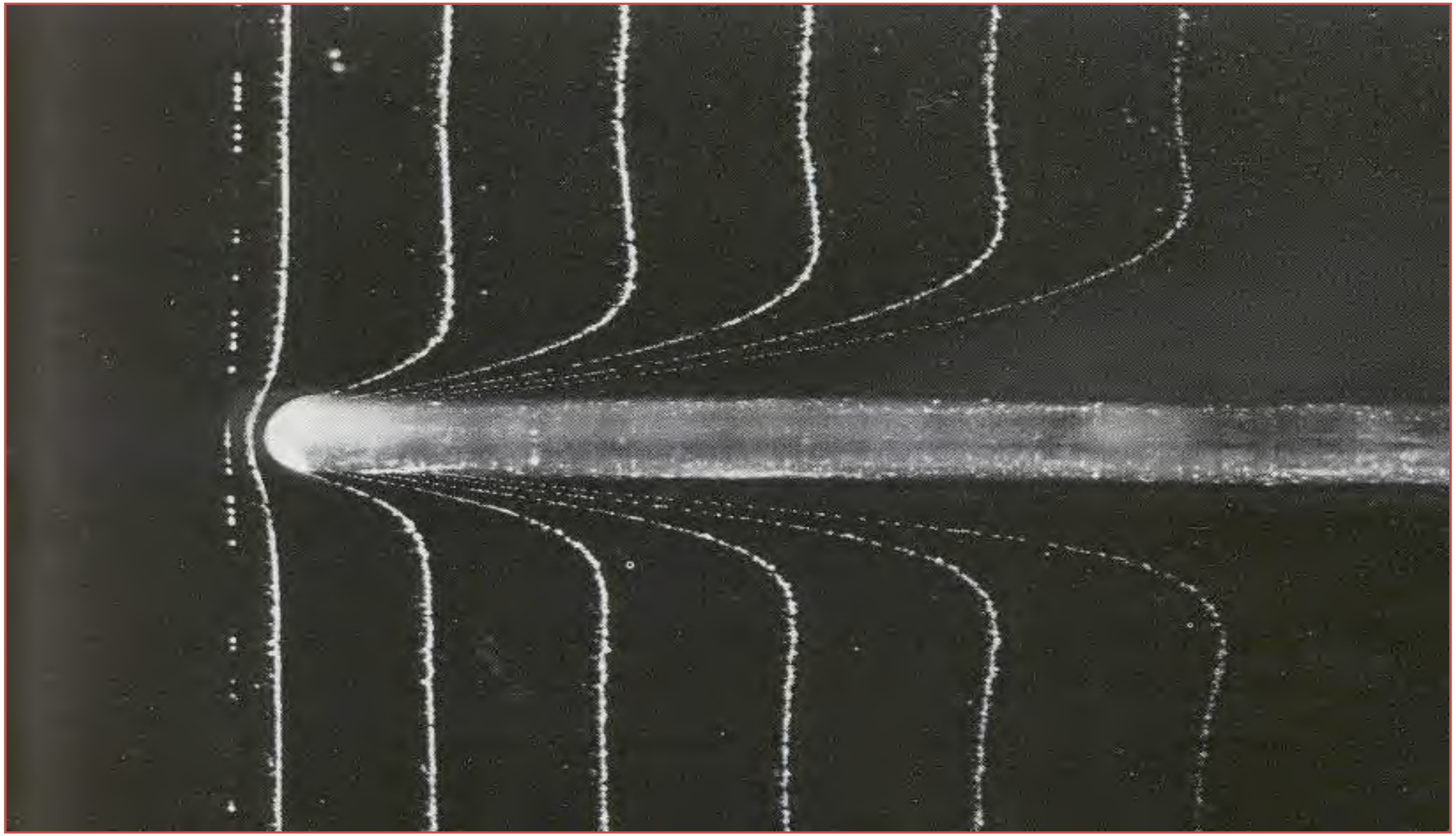
4. Pressure variation across the thin boundary layer is negligible. The freestream pressure distribution is *impressed* on the boundary layer.

Development of laminar boundary layer
(0.01% salt water, free stream velocity 0.6 cm/s, thickness of the plate 0.5 mm, hydrogen bubble method).



FLAT PLATE – ZERO PRESSURE GRADIENT: $\delta(x)$

$\delta(x)$ δ^* θ



BOUNDARY OR DISTURBANCE LAYER

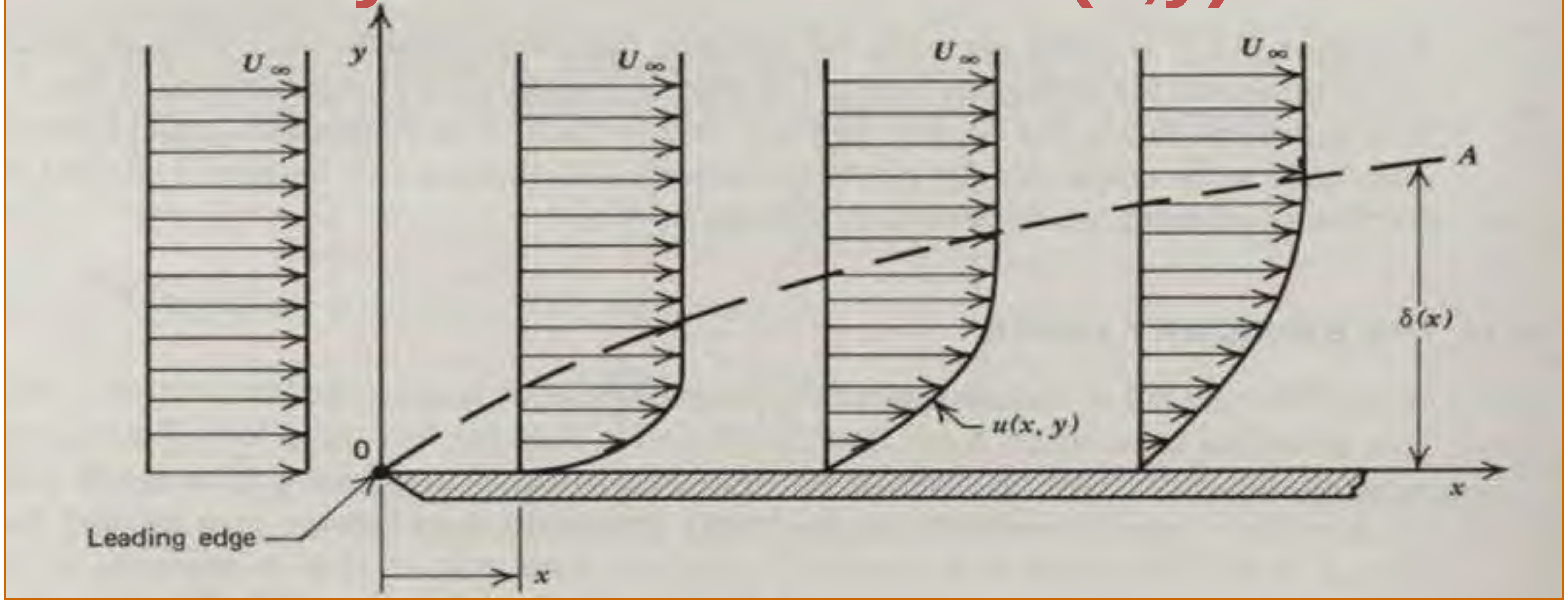
Boundary Layer Thickness $\delta(x)$

Definition:

$u(x,\delta) = 0.99$ of $U=U_\infty=U_e$

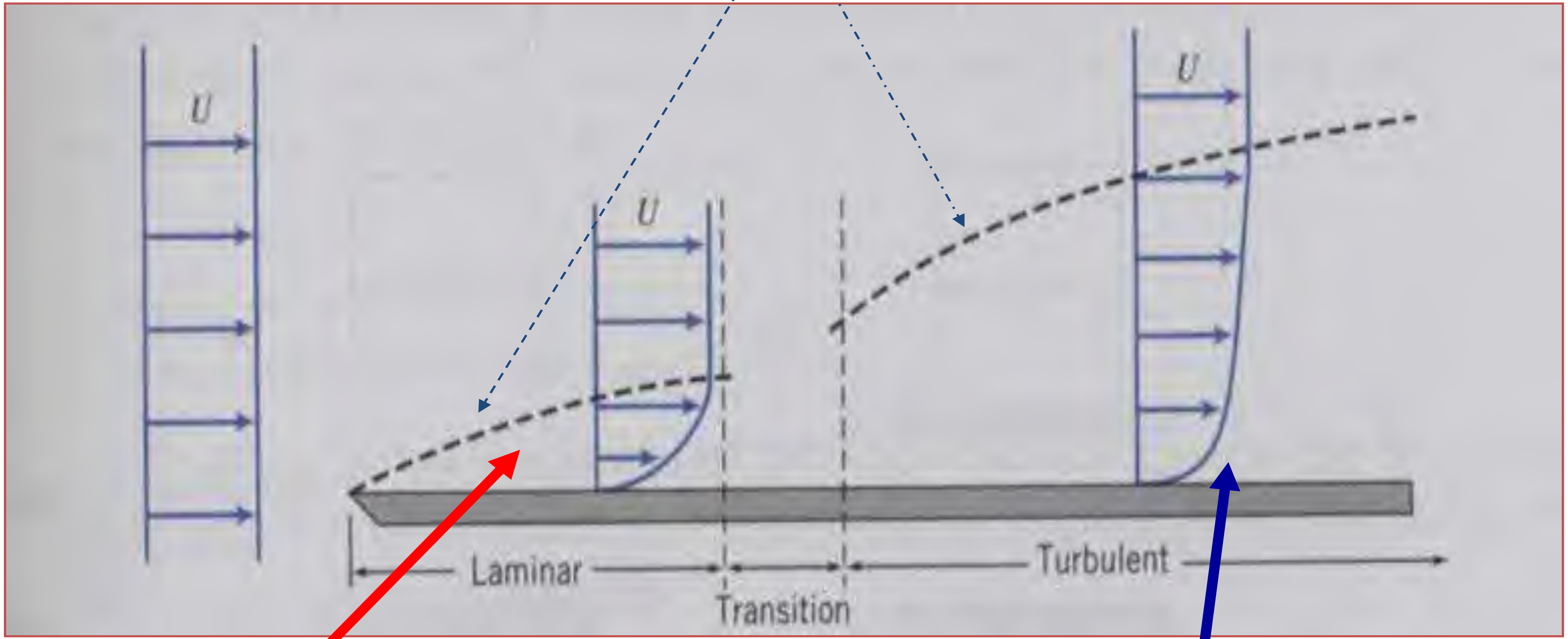
(within 1 % of U_∞)

δ is at y location where u(x,y) = 0.99 U



Because the change in u in the boundary layer takes place asymptotically, there is some indefiniteness in determining δ exactly.

NOTE: boundary layer is much thicker in turbulent flow.



Blasius showed theoretically for laminar flow that

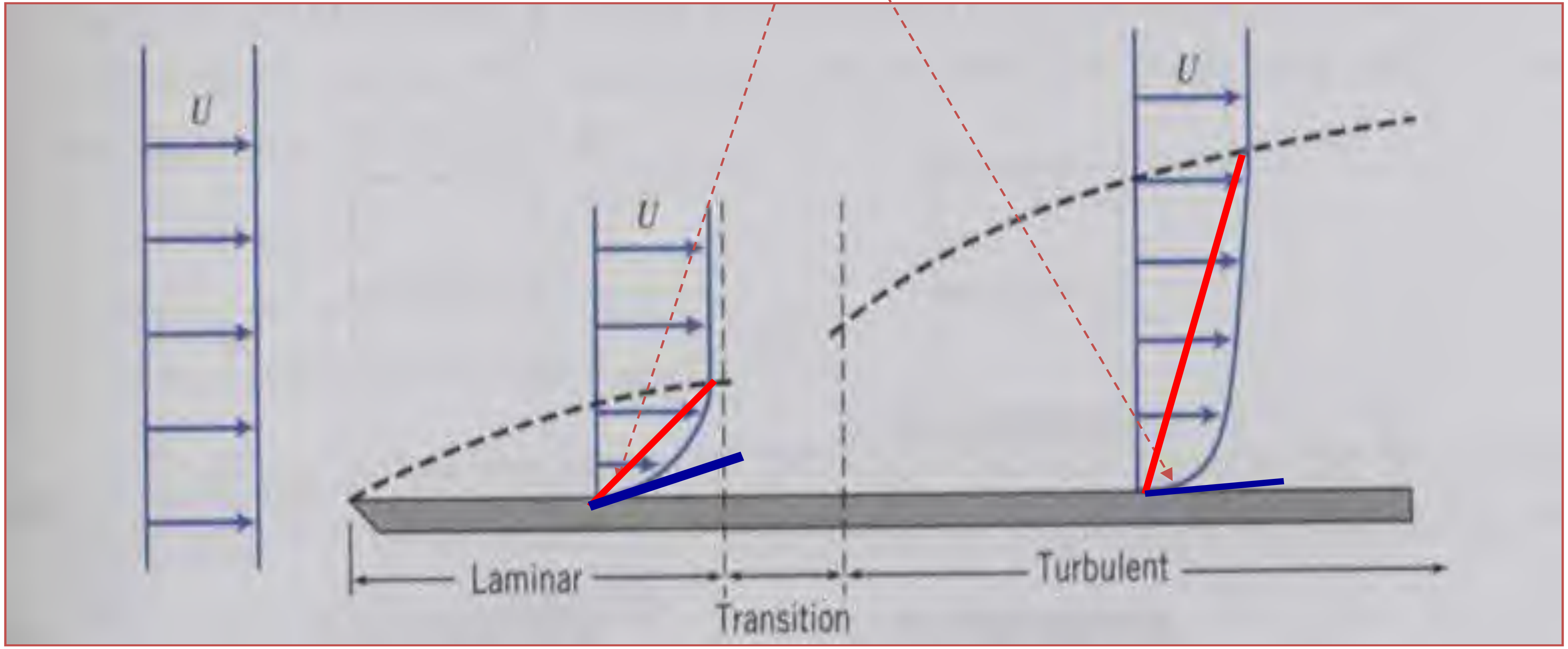
$\delta/x = 5/(Re_x)^{1/2}$ ($Re_x = \rho U_\infty x/\mu$)

$\delta \ \alpha \ \mathbf{x^{1/2}}$

Experimentally found* for turbulent flow that

$\delta \ \alpha \ \mathbf{x^{4/5}}$

NOTE: velocity gradient at wall ($\tau_w = \mu \, du/dy$) is significantly greater.



At same x: $U/\delta_L > U/\delta_T$

At same x: $\tau_{wL} < \tau_{wT}$

Note, boundary layer is not a streamlin

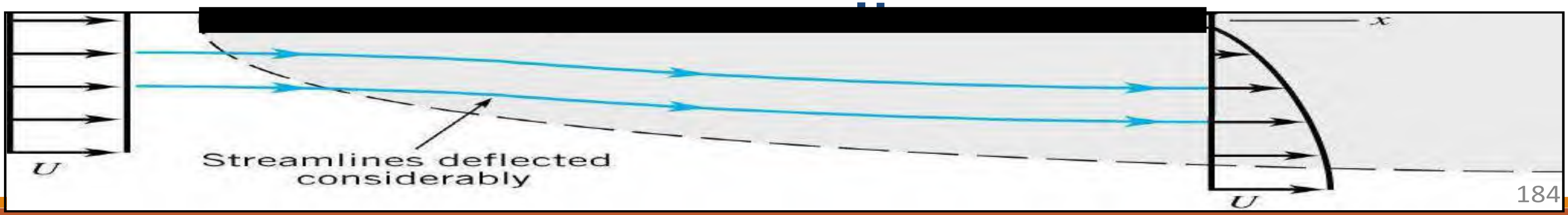
From **theory** (Blasius 1908, student of Prandtl):

$$\delta = 5x/(Re_x^{1/2}) = 5x/(U/[\nu x])^{1/2} = 5\nu^{1/2}x^{1/2}/U^{1/2}$$

$$d\delta/dx = 5\,(\nu/U)^{1/2}\,(½)\,x^{-1/2} = 2.5/(Re_x)^{1/2}$$

$$V/U = dy/dx\,|_{streamline} = 0.84/(Re_x^{1/2})$$

$dy/dx\,|_{streamline} \neq d\delta/dx$ so δ **not**



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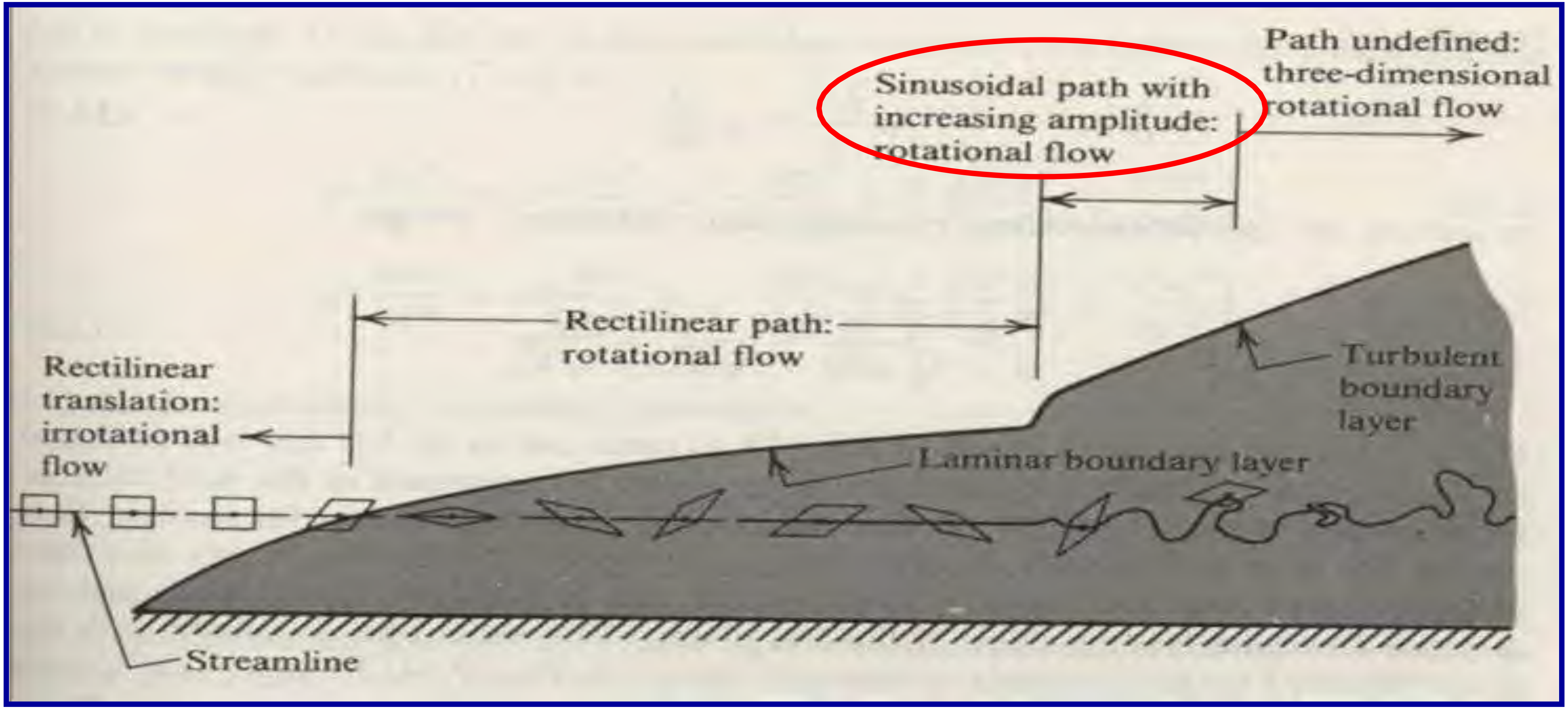
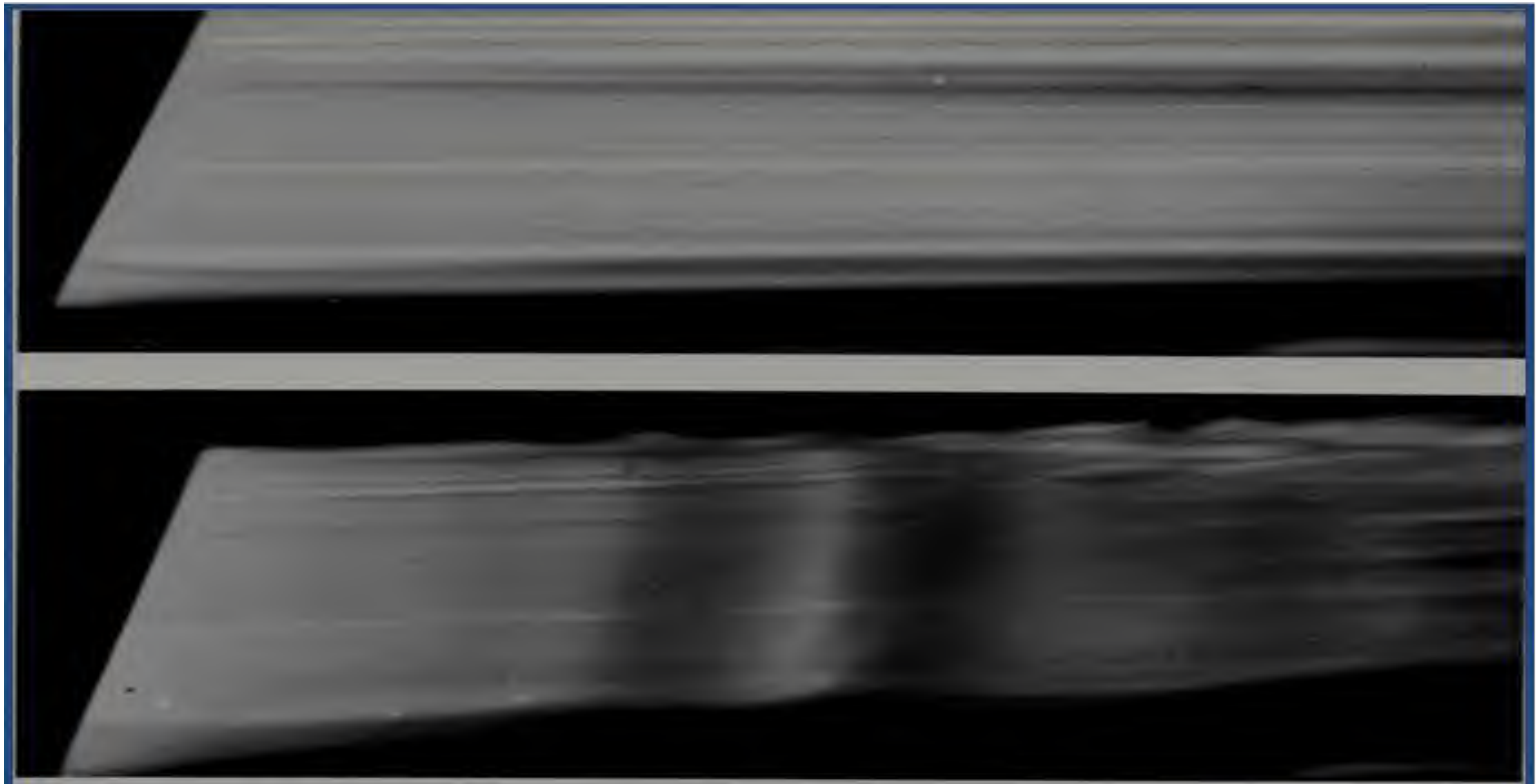


Behavior of a fluid particle traveling along a streamline **through** a boundary layer along a flat plate.

LAMINAR TO TURBULENT TRANSITION

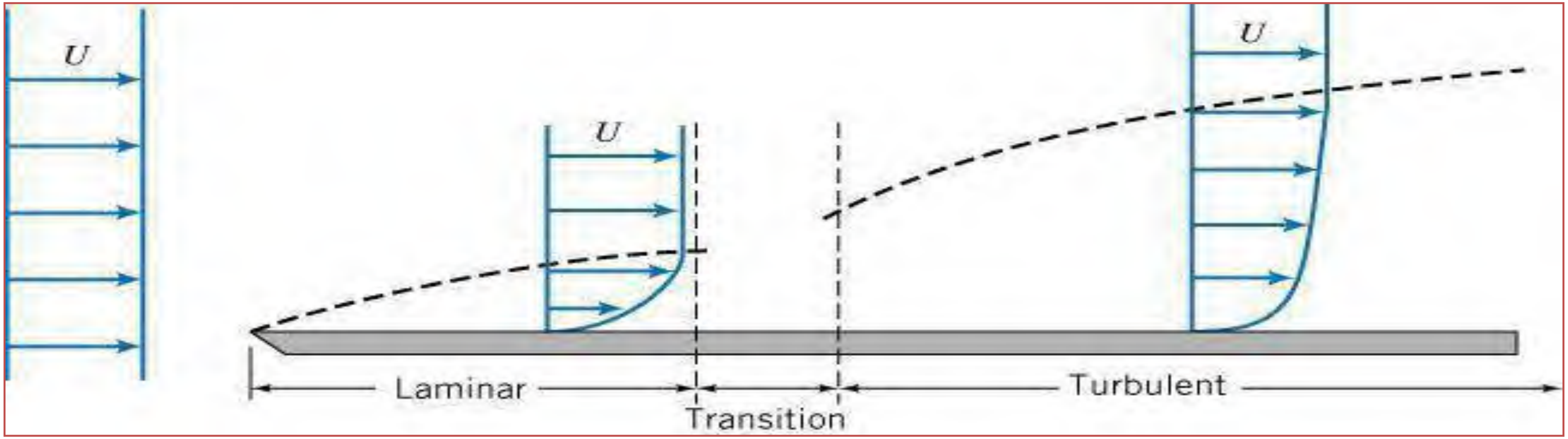


104. Instability of the boundary layer on a plate. At $R=20{,}000$ based on length (upper photograph) the boundary layer is laminar over a flat plate aligned with the stream. At $R=100{,}000$ (lower photograph) two-dimensional Tollmien-Schlichting waves appear. They are made visible by colored fluid in water. *ONERA photographs, Werlé 1980*

NOTE: Turbulence is **not** initiated at Re_{tr} all along the width of the plate

Emmons spot ~ $Re_x = 200{,}000$

Spots grow approximately linearly downstream at downstream speed that is a fraction of the free stream velocity.



x=0

Turbulent boundary layer is thicker and grows fast

Transition **not fixed** but usually around Re_x ~500,000

($2x10^5$-$3x10^6$, MYO)

For air at standard conditions and U = 30 m/s, x_{tr} ~ 0.24 m

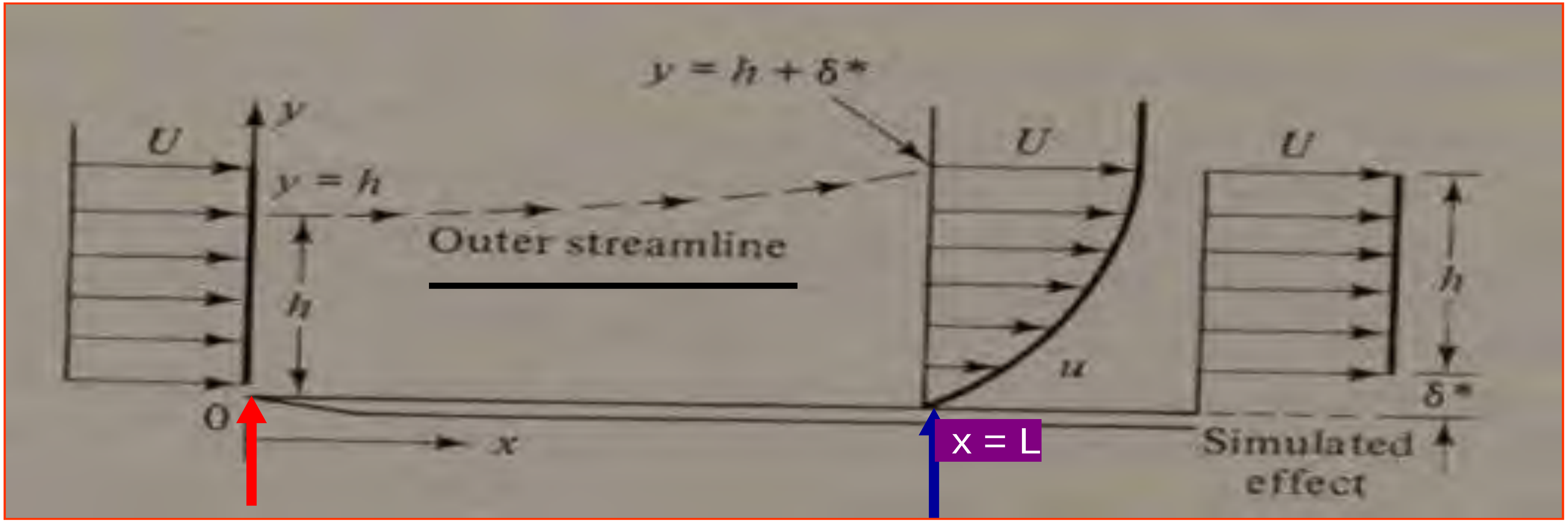
Displacement Thickness $\delta^*(x)$

Definition:

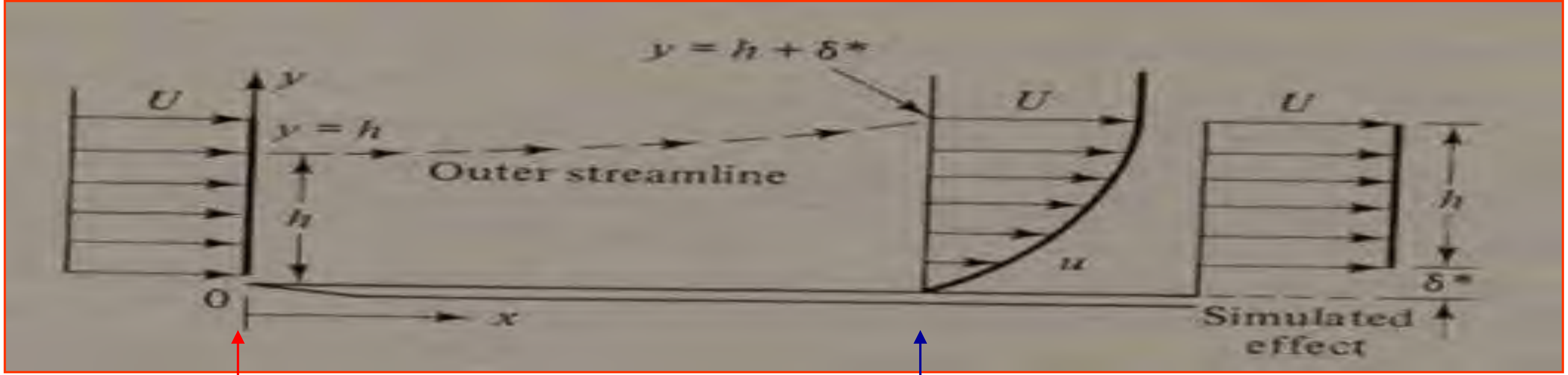
$$\delta^* = \int_0^{\infty} (1 - u/U)\,dy$$

δ^* is displacement of outer streamlines due to boundary layer

Displacement thickness δ^*



By definition, no flow passes through streamline, so mass through 0 to h at x = 0

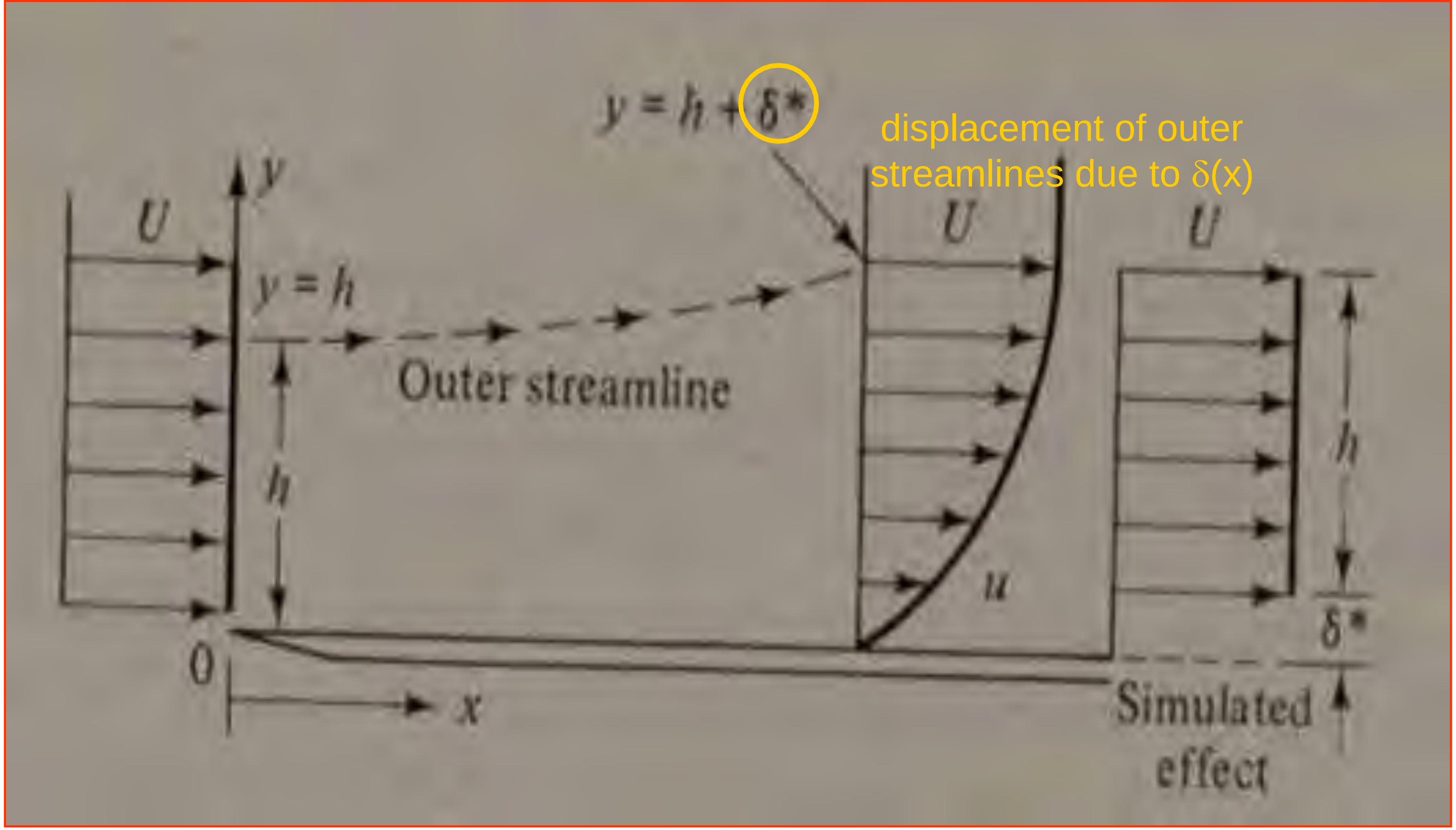


$$\rho U h = \int_0^{h+\delta^*} \rho u\,dy = \int_0^{h+\delta^*} \rho\,(U + u - U)\,dy$$

$$Uh = \int_0^{h+\delta^*} U\,dy + \int_0^{h+\delta^*} (u - U)\,dy$$

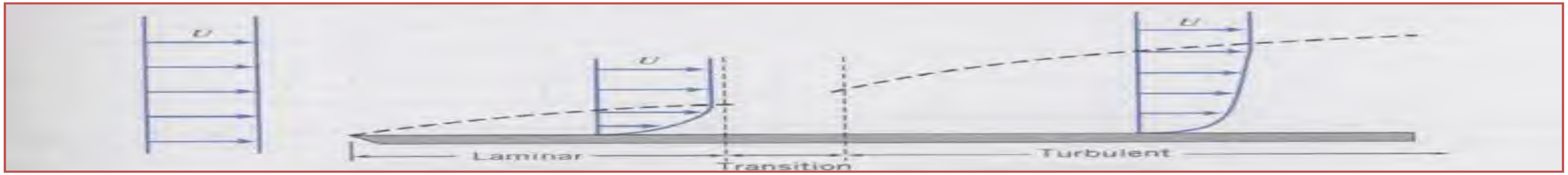
$$\not{U}\not{h} = U(\not{h} + \delta^*) + \int_0^{h+\delta^*} (u - U)\,dy$$

$$-U\delta^* = \int_0^{h+\delta^*} (u - U)\,dy$$



$$\delta^* \approx \int_0^{\infty} (1 - u/U)dy \approx \int_0^{\delta} (1 - u/U)dy$$

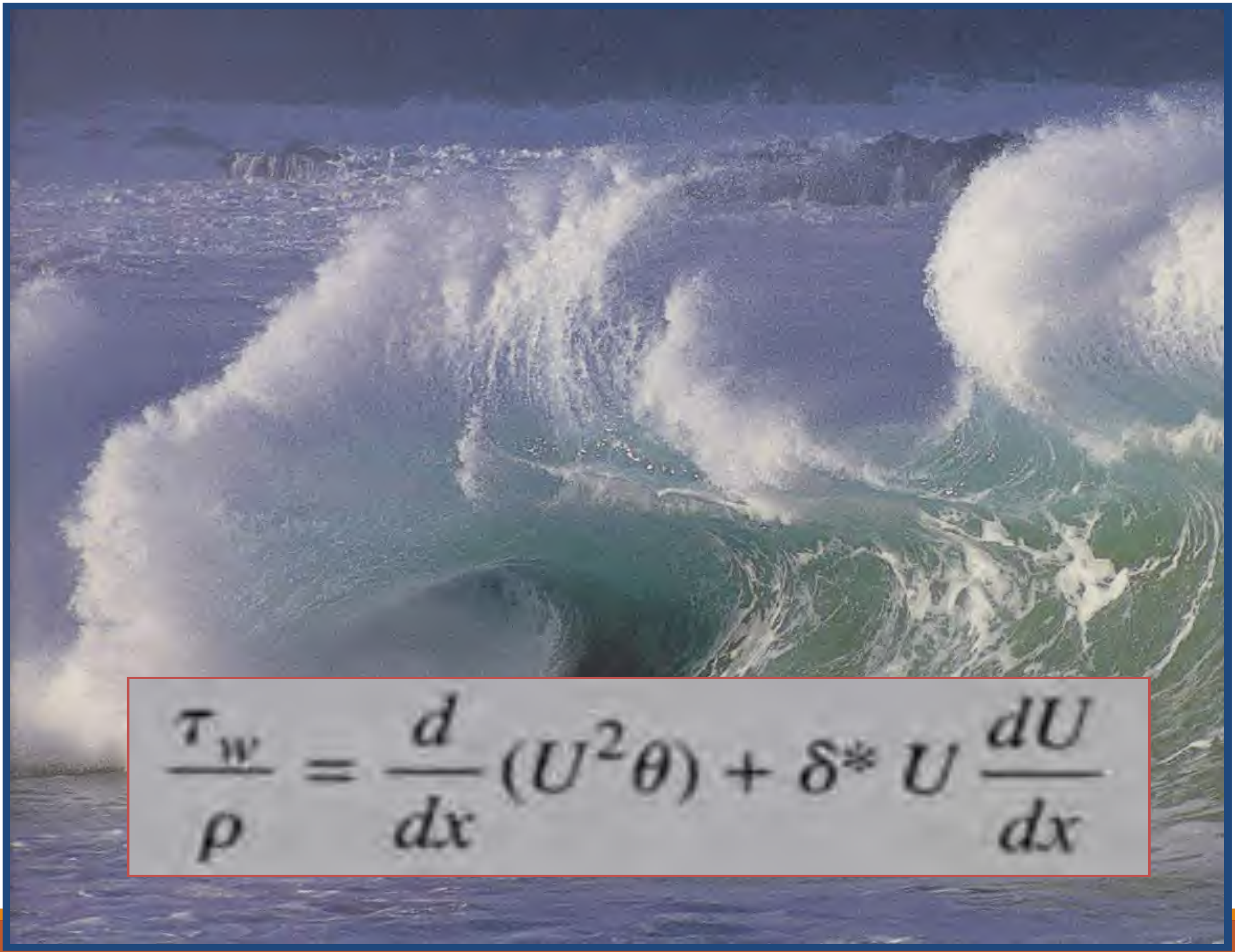
function of x!



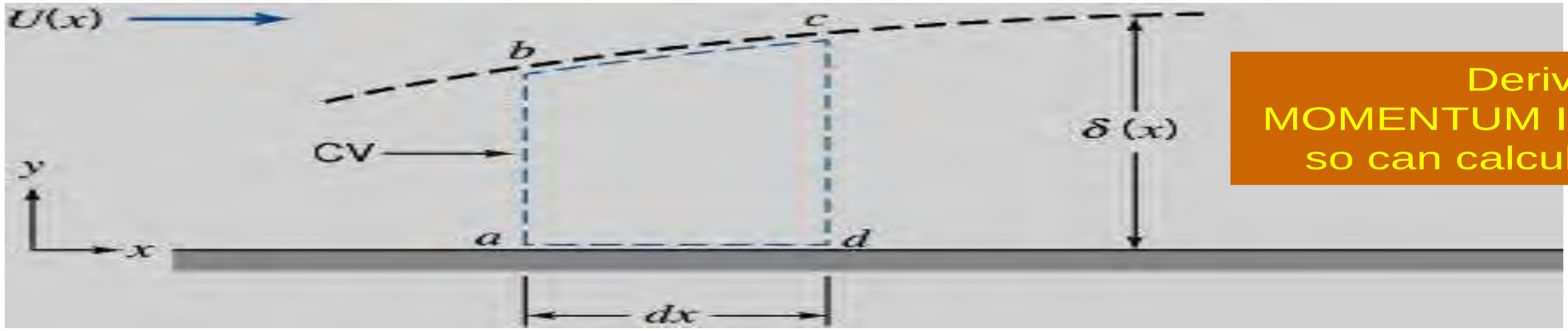
Blasius developed an exact solution (but numerical integration was necessary) for laminar flow with no pressure variation. Blasius could theoretically predict boundary layer thickness $\delta(x)$, velocity profile $u(x,y)/U_\infty$ vs y/δ, and wall shear stress $\tau_w(x)$.

Von Karman and Poulhausen derived momentum integral equation (approximation) which can be used for both laminar (with and without pressure gradient) and

MOMENTUM INTEGRAL EQUATION
dP/dx is not a constant!



$$\frac{\tau_w}{\rho} = \frac{d}{dx}(U^2\theta) + \delta^* U \frac{dU}{dx}$$



Deriving:
MOMENTUM INTEGRAL EQ
so can calculate $\delta(x)$, τ_w.

a. Continuity Equation

Basic equation:

$$\overbrace{\frac{\partial}{\partial t}\int_{CV} \rho \, d\forall}^{=0(1)} + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0$$

Assumptions: (1) Steady flow.
(2) Two-dimensional flow.

Then

$$\int_{CS} \rho \vec{V} \cdot d\vec{A} = 0$$

$$\dot{m}_{ab} + \dot{m}_{bc} + \dot{m}_{cd} = 0$$

or

$$\dot{m}_{bc} = -\dot{m}_{ab} - \dot{m}_{cd}$$

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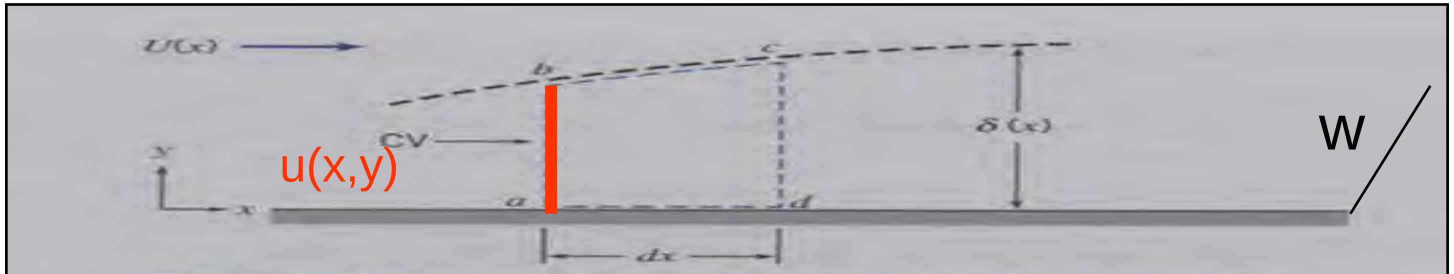


Fig. 9.4 Differential control volume in a boundary layer.

Surface Mass Flux Through Side ab

ab Surface *ab* is located at *x*. Since the flow is two-dimensional (no variation with *z*), the mass flux through *ab* is

$$\dot{m}_{ab} = -\left\{\int_0^{\delta} \rho u \, dy\right\} w$$

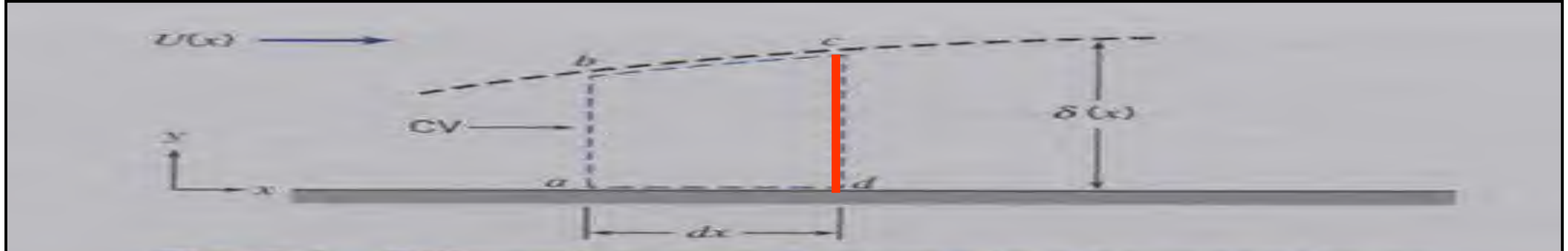


Fig. 9.4 Differential control volume in a boundary layer.

Surface Mass Flux Through Side cd

cd Surface *cd* is located at $x + dx$. Expanding $\dot{m}$ in a Taylor series about location x, we obtain

$$\dot{m}_{x+dx} = \dot{m}_x + \left.\frac{\partial \dot{m}}{\partial x}\right]_x dx$$

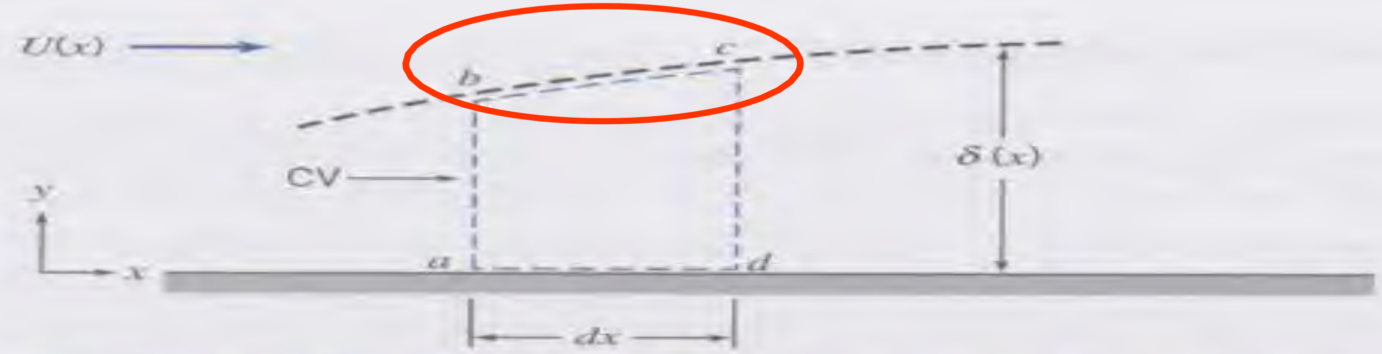


Fig. 9.4 Differential control volume in a boundary layer.

Surface Mass Flux Through Side bc

bc Thus for surface *bc* we obtain

$$\dot{m}_{bc} = -\dot{m}_{ab} - \dot{m}_{cd}$$

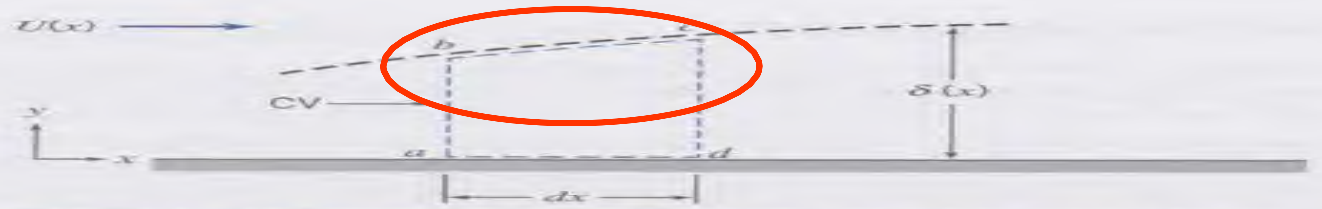


Fig. 9.4 Differential control volume in a boundary layer.

Surface Mass Flux Through Side bc

$$\dot{m}_{bc} = -\dot{m}_{ab} - \dot{m}_{cd}$$

$$\dot{m}_{ab} = -\left\{\int_0^{\delta} \rho u \, dy\right\} w$$

$$\dot{m}_{cd} = \left\{\int_0^{\delta} \rho u \, dy + \frac{\partial}{\partial x}\left[\int_0^{\delta} \rho u \, dy\right] dx\right\} w$$

bc Thus for surface *bc* we obtain

$$\dot{m}_{bc} = -\left\{\frac{\partial}{\partial x}\left[\int_0^{\delta} \rho u \, dy\right] dx\right\} w$$

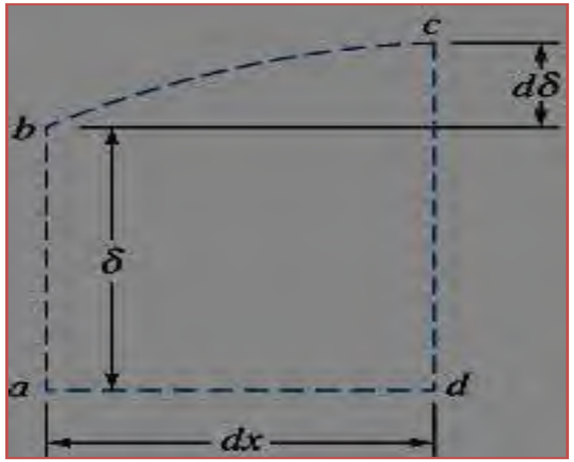
b. Momentum Equation

Apply x-component of momentum eq. to differential control volume *abcd*

$$F_{S_x} + \underbrace{F_{B_x}}_{=0(3)} = \underbrace{\frac{\partial}{\partial t}}_{=0(1)} \int_{CV} u \rho \, dV + \int_{CS} u \rho \vec{V} \cdot d\vec{A}$$

Assumption : (1) *steady* (3) *no body forces*

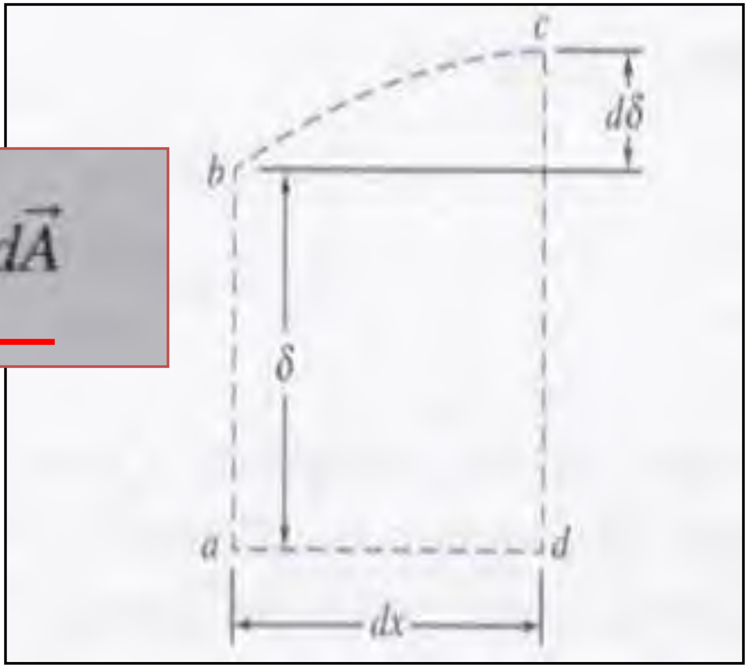
u →



b. Momentum Equation

$$F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\forall + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$$

$$F_{S_x} = \mathrm{mf}_{ab} + \mathrm{mf}_{bc} + \mathrm{mf}_{cd}$$



mf represents x-component of momentum fl

F_{sx} will be composed of shear force on boun

Surface Momentum Flux Through Side ab

ab Surface *ab* is located at *x*. Since the flow is two-dimensional, the *x* momentum flux through *ab* is

$$\mathrm{mf}_{ab} = -\left\{\int_0^{\delta} u\,\rho u\,dy\right\} w$$

$$\int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$$

X-momentum Flux = $\int_{cv} u\rho\vec{V}\bullet d\vec{A}$

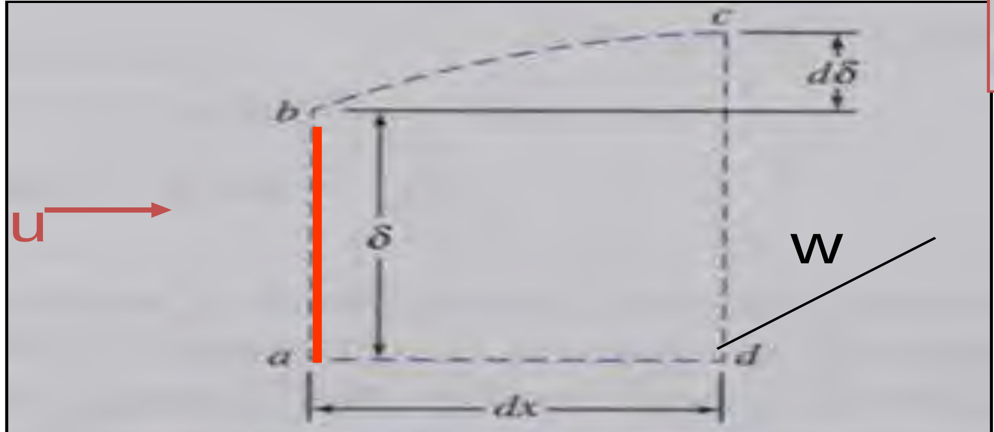


Fig. 9.5 Differential control volume.

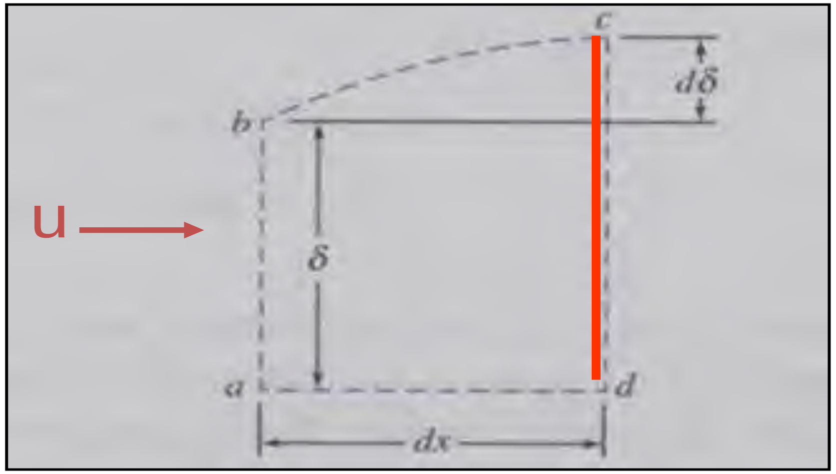
Surface Momentum Flux Through Side cd

cd Surface *cd* is located at $x + dx$. Expanding the x momentum flux (mf) in a Taylor series about location x, we obtain

$$\mathrm{mf}_{x+dx} = \mathrm{mf}_x + \left.\frac{\partial \mathrm{mf}}{\partial x}\right]_x dx$$

$$\mathrm{mf}_{cd} = \left\{\int_0^\delta u\,\rho u\,dy + \frac{\partial}{\partial x}\left[\int_0^\delta u\,\rho u\,dy\right]dx\right\}w$$

X-momentum Flux = $\int_{CV} u\rho \vec{V} \bullet d\vec{A}$



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Surface Momentum Flux Through Side bc

bc Since the mass crossing surface *bc* has velocity component U in the x direction, the x momentum flux across *bc* is given by

$$\text{mf}_{bc} = U\dot{m}_{bc}$$

$$\text{mf}_{bc} = -U\left\{\frac{\partial}{\partial x}\left[\int_0^{\delta} \rho u\, dy\right]dx\right\}w$$

$U=U_e=U_\infty$

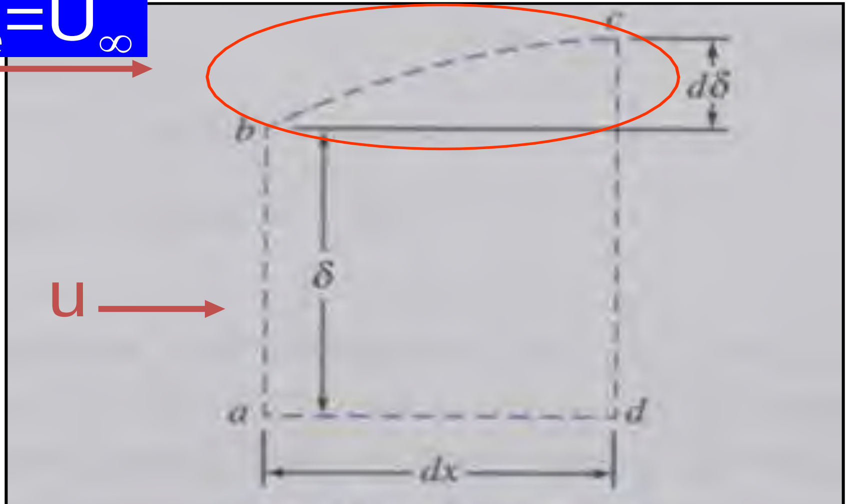


Fig. 9.5 Differential control volume.

X-momentum Flux = $\int_{cv} u\rho\vec{V}\bullet d\vec{A}$

X-Momentum Flux Through Control Surface

From the above we can evaluate the net x momentum flux through the control surface as

a-b c-d

$$\int_{CS} u\,\rho\vec{V}\cdot d\vec{A} = -\left\{\int_0^\delta u\,\rho u\,dy\right\}w + \left\{\int_0^\delta u\,\rho u\,dy\right\}w$$

c-d b-c

$$+\left\{\frac{\partial}{\partial x}\left[\int_0^\delta u\,\rho u\,dy\right]dx\right\}w - U\left\{\frac{\partial}{\partial x}\left[\int_0^\delta \rho u\,dy\right]dx\right\}w$$

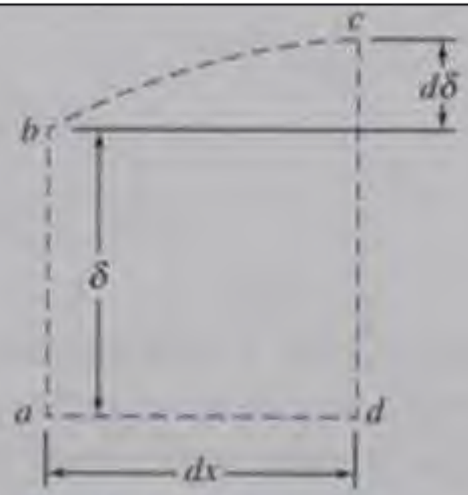
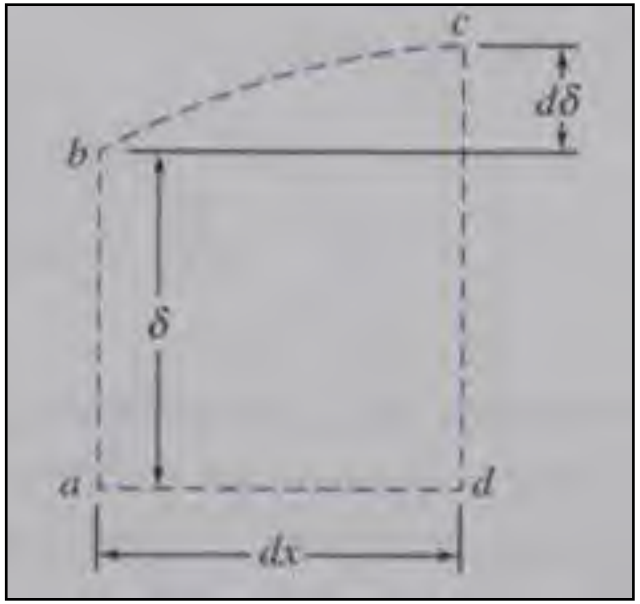


Fig. 9.5 Differential control volume.

IN SUMMARY
X-Momentum Equation

$$\int_{CS} u\,\rho\vec{V}\cdot d\vec{A} = +\left\{\frac{\partial}{\partial x}\left[\int_0^{\delta} u\,\rho u\,dy\right]dx - U\,\frac{\partial}{\partial x}\left[\int_0^{\delta}\rho u\,dy\right]dx\right\}w$$

$$F_{S_x} = \mathrm{mf}_{ab} + \mathrm{mf}_{bc} + \mathrm{mf}_{cd}$$



UNIT-V
Closed Conduit Flow

Closed Conduit Flow

- Energy equation
- EGL and HGL
- Head loss
  - major losses
  - minor losses
- Non circular conduits

Conservation of Energy

- Kinetic, potential, and thermal energy

$h_p =$ head supplied by a pump

$h_t =$ head given to a turbine

$h_L =$ head loss between sections 1 and 2

Cross section 2 is downstream from cross section 1!

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Energy Equation Assumptions

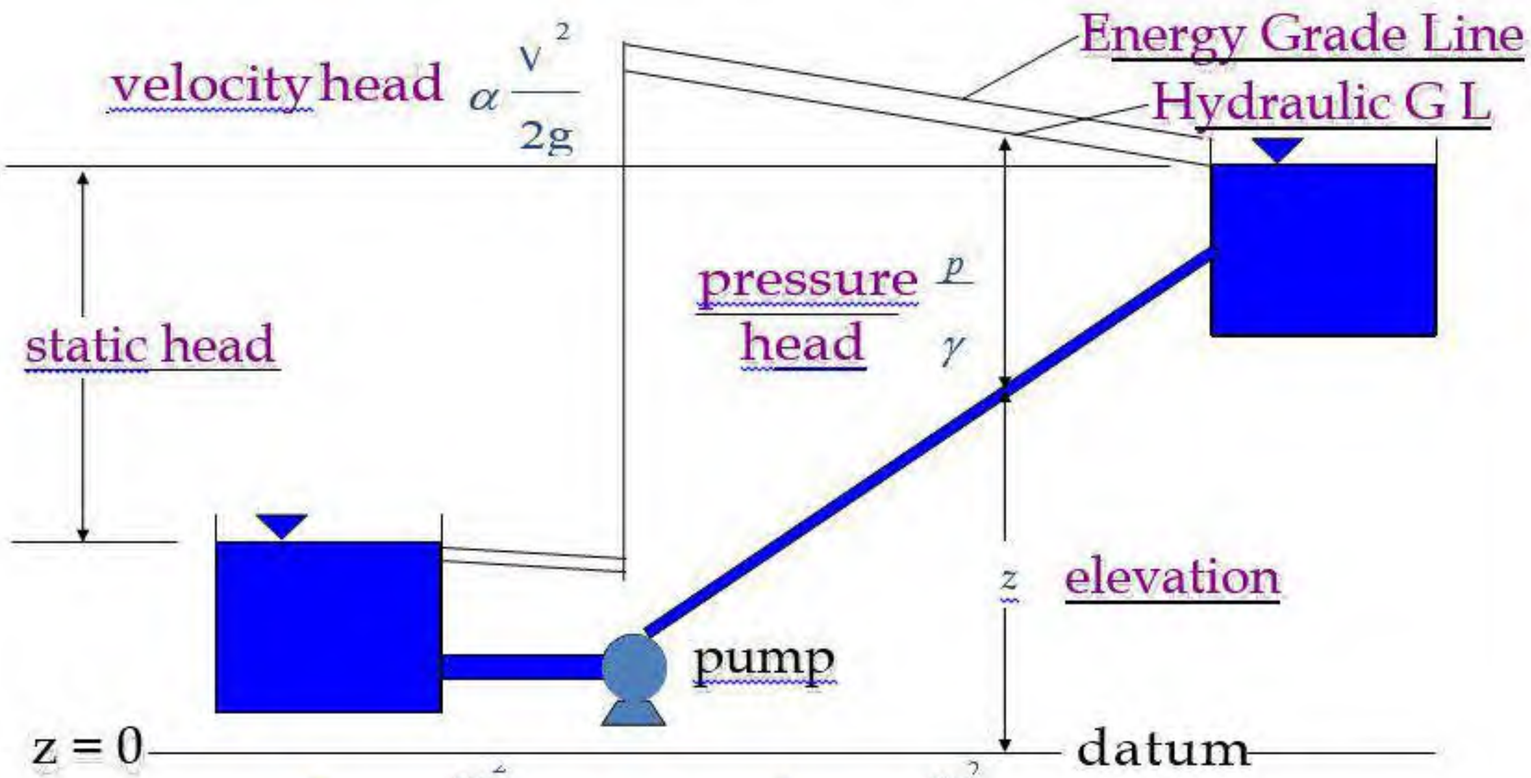
- Pressure is __hydrostatic__ in both cross sections
hydrostatic
  - pressure changes are due to elevation only $p = \gamma h$
- section is drawn perpendicular to the streamlines (otherwise the __kinetic__ energy term is incorrect)

density

- Constant __density__ at the cross section
- __Steady__ flow

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Energy equation



$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Bernoulli Equation Assumption

- ___Frictionless___ (viscosity can't be a significant parameter!)
- Along a ___streamline___
- ___Steady___ flow
- Constant ___density___

$$z + \frac{V^2}{2g} + \frac{p}{\gamma} = const$$

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Pipe Flow: Review

- We have the control volume energy equation for pipe flow.
- We need to be able to predict the head loss term.
- How do we predict head loss?
  Dimensional analysis ________ ___ __________.

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Pipe Flow Energy Losses

$$\frac{p_1}{\gamma} + \cancel{\alpha_1 \frac{V_1^2}{2g}} + \cancel{z_1} + \cancel{h_p} = \frac{p_2}{\gamma} + \cancel{\alpha_2 \frac{V_2^2}{2g}} + \cancel{z_2} + \cancel{h_t} + h_l$$

$$h_f = -\frac{\Delta p}{\gamma}$$

Horizontal pipe

$$f = \left(C_p \frac{D}{L} \right) = \text{function of} \left(\frac{\varepsilon}{D}, \mathrm{Re} \right)$$

Dimensional Analysis

$$C_p = \frac{-2\Delta p}{\rho V^2} \qquad C_p = \frac{2gh_f}{V^2}$$

$$f = \frac{2gh_f}{V^2} \frac{D}{L} \qquad h_f = f \frac{L}{D} \frac{V^2}{2g}$$

Darcy-Weisbach equation

Friction Factor: Major losses

- Laminar flow
  - Hagen-Poiseuille
- Turbulent (Smooth, Transition, Rough)
  - Colebrook Formula
  - Moody diagram
  - Swamee-Jain

Laminar Flow Friction Factor

$$V = \frac{\gamma D^2}{32\mu}\frac{h_l}{L}$$

Hagen-Poiseuille

$$h_f = \frac{32 m L V}{r g D^2} \qquad h_f = \frac{128 m L Q}{p r g D^4}$$

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

Darcy-Weisbach

$$\frac{32 m L V}{r g D^2} = f\frac{L}{D}\frac{V^2}{2g}$$

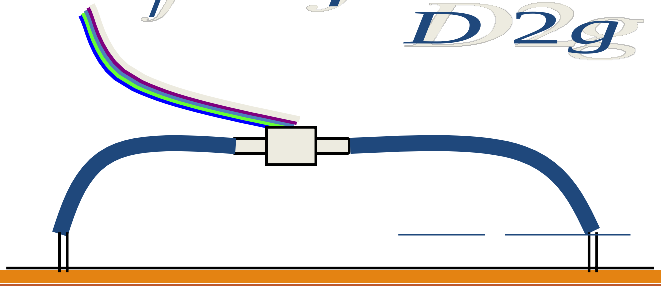
$$f = \frac{64 m}{r V D} = \frac{64}{\mathrm{Re}}$$

Slope of _-1_ on log-log plot

Turbulent Pipe Flow Head Loss

- Proportional to the length of the pipe
- Proportional to the square of the velocity (almost)
- Increases with surface roughness
- Is a function of density and viscosity
- Is independent of pressure

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$



Smooth, Transition, Rough Turbulent Flow

$$h_f = f \frac{L}{D}\frac{V^2}{2g}$$

- Hydraulically smooth pipe law (von Karman, 1930)

$$\frac{1}{\sqrt{f}} = 2\log\left(\frac{\mathrm{Re}\sqrt{f}}{2.51}\right)$$

- Rough pipe law (von Karman, 1930)

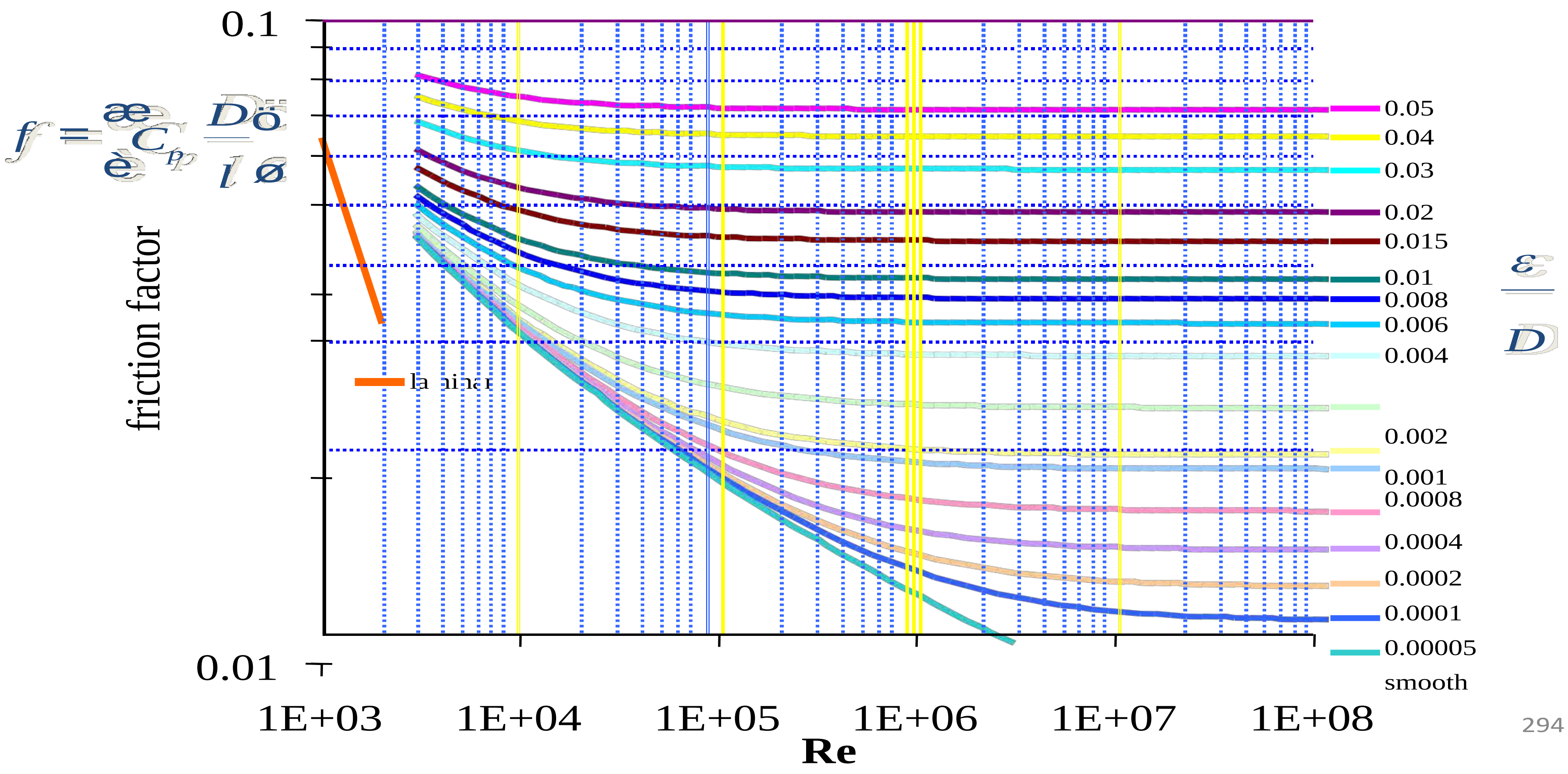
$$\frac{1}{\sqrt{f}} = 2\log\left(\frac{3.7D}{e}\right)$$

- Transition function for both smooth and rough pipe laws (Colebrook)

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{e/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

(used to draw the Moody diagram)

Moody Diagram



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Pipe roughness

pipe material	pipe roughness ε (mm)
glass, drawn brass, copper	0.0015
commercial steel or wrought iron	0.045
asphalted cast iron	0.12
galvanized iron	0.15
cast iron	0.26
concrete	0.18-0.6
rivet steel	0.9-9.0
corrugated metal	45
PVC	0.12

Exponential Friction Formulas

- Commonly used in commercial and industrial settings

$$h_f = \frac{RLQ^n}{D^m}$$

- Only applicable over __range__ of data collected
- Hazen-Williams exponential friction formula

$$R = \begin{cases} \frac{4.727}{C^n} & \text{USC units} \\ \frac{10.675}{C^n} & \text{SI units} \end{cases}$$

$$h_f = \frac{10.675L}{D^{4.8704}} \left(\frac{Q}{C} \right)^{1.852} \quad \text{SI units}$$

C = Hazen-Williams coefficient 296

Head loss:
Hazen-Williams Coefficient

C	Condition
150	PVC
140	Extremely smooth, straight pipes; asbestos cement
130	Very smooth pipes; concrete; new cast iron
120	Wood stave; new welded steel
110	Vitrified clay; new riveted steel
100	Cast iron after years of use
95	Riveted steel after years of use
60-80	Old pipes in bad condition

Hazen-Williams

vs

Darcy-Weisbach

$$h_f = \frac{10.675L}{D^{4.8704}}\left(\frac{Q}{C}\right)^{1.852} \quad \text{SI units}$$

$$h_f = f\,\frac{8}{\pi^2 g}\,\frac{LQ^2}{D^5}$$

- Both equations are empirical
- Darcy-Weisbach is rationally based, dimensionally correct, and preferred .

- Hazen-Williams can be considered valid only over the range of gathered data.
- Hazen-Williams can’t be extended to other fluids without further experimentation.

Head Loss: Minor Losses

- Head loss due to outlet, inlet, bends, elbows, valves, pipe size changes
- Losses due to expansions are greater than losses due to contractions
- Losses can be minimized by gradual transitions

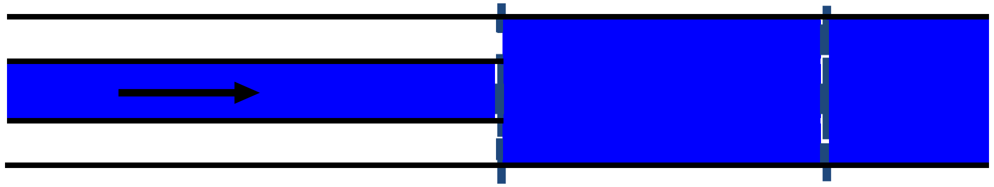
Minor Losses

- Most minor losses can not be obtained analytically, so they must be measured
- Minor losses are often expressed as a loss coefficient, K, times the velocity head.

$$C_p = f(\text{geometry}, \text{Re})$$

$$C_p = \frac{-2\Delta p}{\rho V^2} \qquad C_p = \frac{2gh_l}{V^2} \qquad h_l = C_p \frac{V^2}{2g} \qquad h = K\frac{V^2}{2g}$$

Head Loss due to Sudden Expansion: Conservation of Energy



$$\frac{p_1}{\gamma_1} + z_1 + \alpha_1 \frac{V_1^2}{2g} + H_p = \frac{p_2}{\gamma_2} + z_2 + \alpha_2 \frac{V_2^2}{2g} + H_t + h_l$$

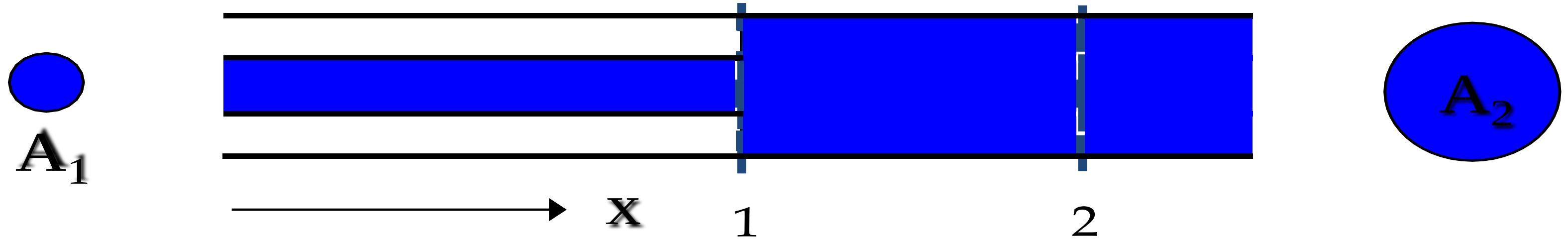
$$\frac{p_1 - p_2}{\gamma} = \frac{V_2^2 - V_1^2}{2g} + h_l$$

$z_1 = z_2$

$$h_l = \frac{p_1 - p_2}{\gamma} + \frac{V_1^2 - V_2^2}{2g}$$

What is $p_1 - p_2$?

Head Loss due to Sudden Expansion: Conservation of Momentum



$$M_1 + M_2 = W + F_{p_1} + F_{p_2} + F_{ss}$$

Apply in direction of flow

$$M_{1x} + M_{2x} = F_{p_{1x}} + F_{p_{2x}}$$

Neglect surface shear

$$M_{1x} = -\rho V_1^2 A_1 \qquad M_{2x} = \rho V_2^2 A_2$$

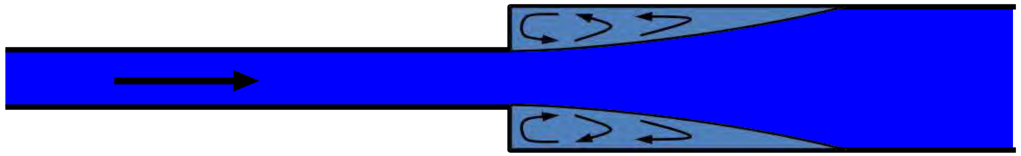
Pressure is applied over all of section 1.
Momentum is transferred over area corresponding to upstream pipe diameter.
area corresponding to upstream pipe diameter.
V_1 is velocity upstream.

$$-\rho V_1^2 A_1 + \rho V_2^2 A_2 = p_1 A_2 - p_2 A_2$$

Divide by $(A_2 \gamma)$

$$\frac{p_1 - p_2}{\gamma} = \frac{V_2^2 - V_1^2 \frac{A_1}{A_2}}{g}$$

Head Loss due to Sudden Expansion



Energy

$$h_l = \frac{p_1 - p_2}{\gamma} + \frac{V_1^2 - V_2^2}{2g}$$

Mass

$$\frac{A_1}{A_2} = \frac{V_2}{V_1}$$

Momentum

$$\frac{p_1 - p_2}{\gamma} = \frac{V_2^2 - V_1^2 \frac{A_1}{A_2}}{g}$$

$$h_l = \frac{V_2^2 - V_1^2 \frac{V_2}{V_1}}{g} + \frac{V_1^2 - V_2^2}{2g}$$

$$h_l = \frac{V_2^2 - 2V_1V_2 + V_1^2}{2g}$$

$$h_l = \frac{(V_1 - V_2)^2}{2g}$$

$$h_l = \frac{V_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2$$

$$K = \left(1 - \frac{A_1}{A_2}\right)^2$$

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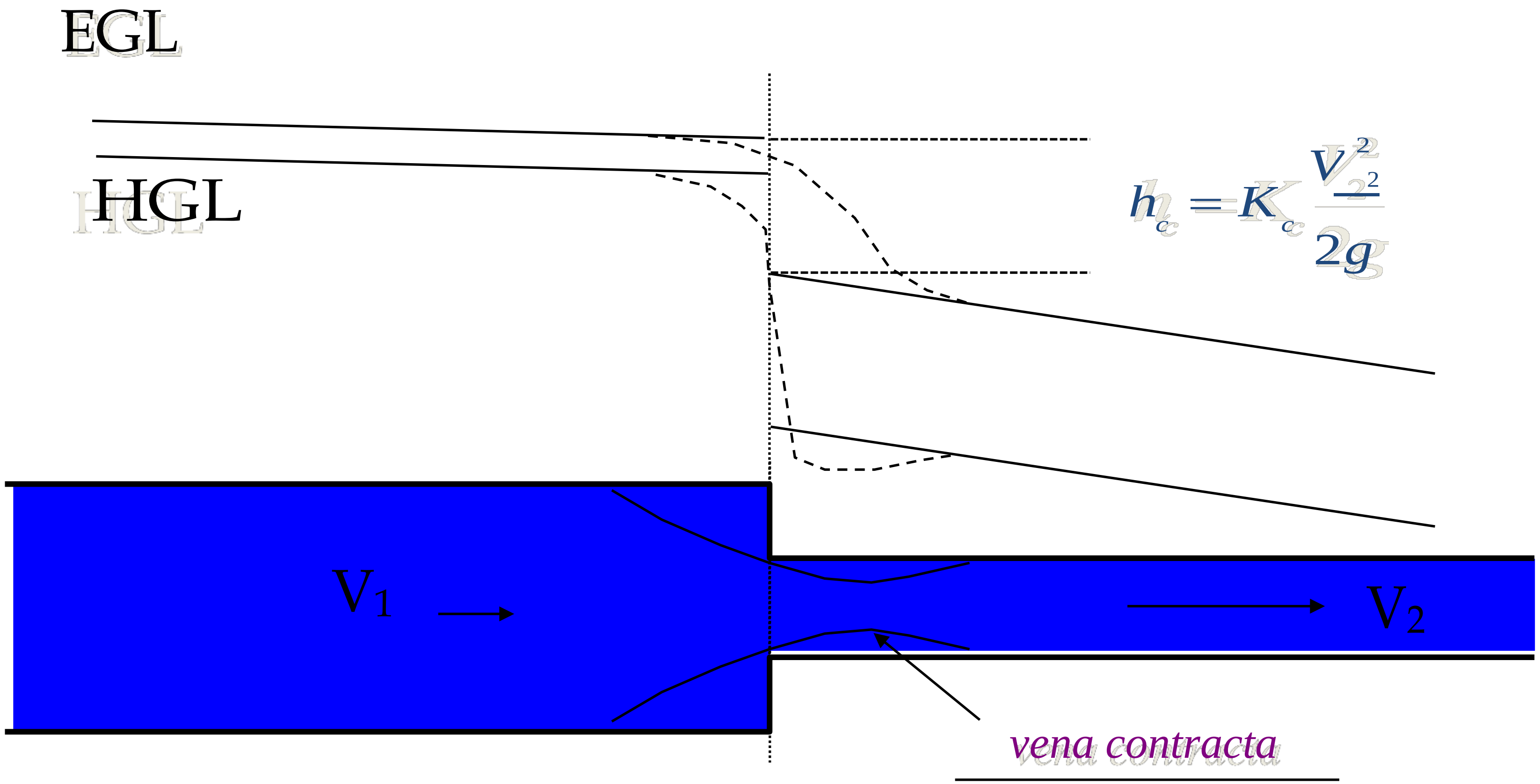
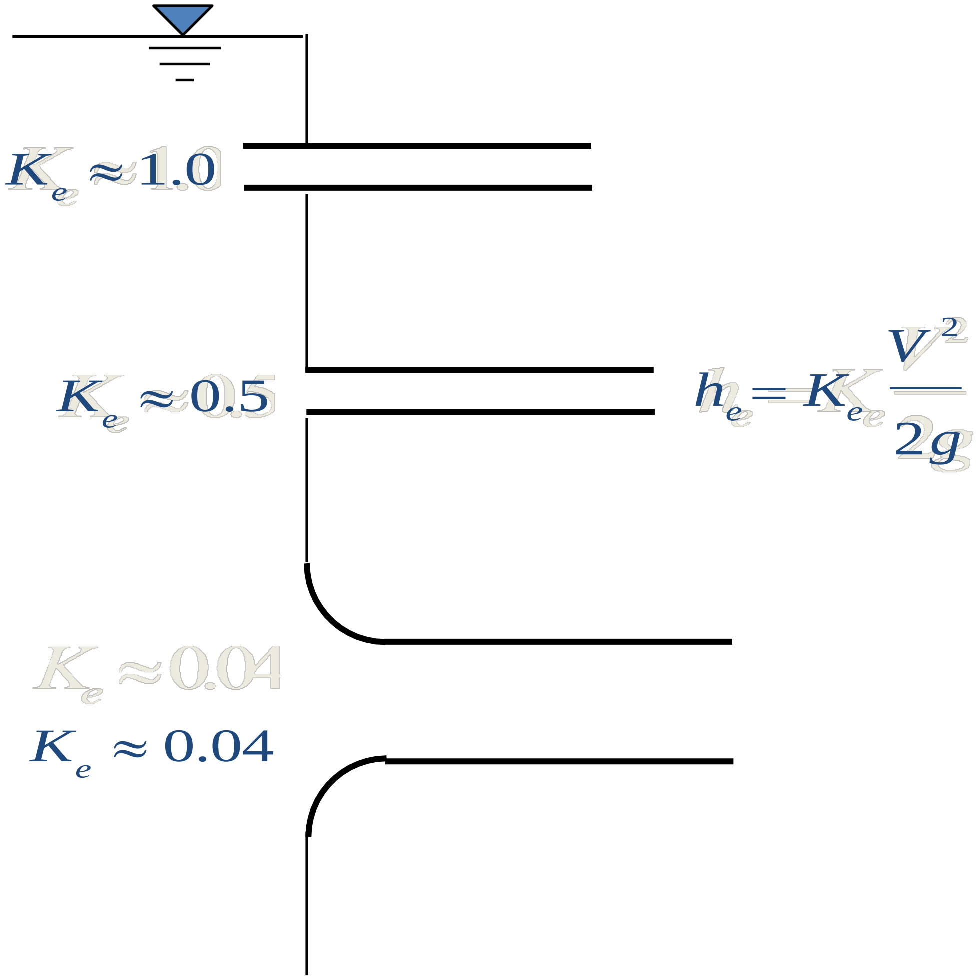
Contraction



losses are reduced with a gradual contraction

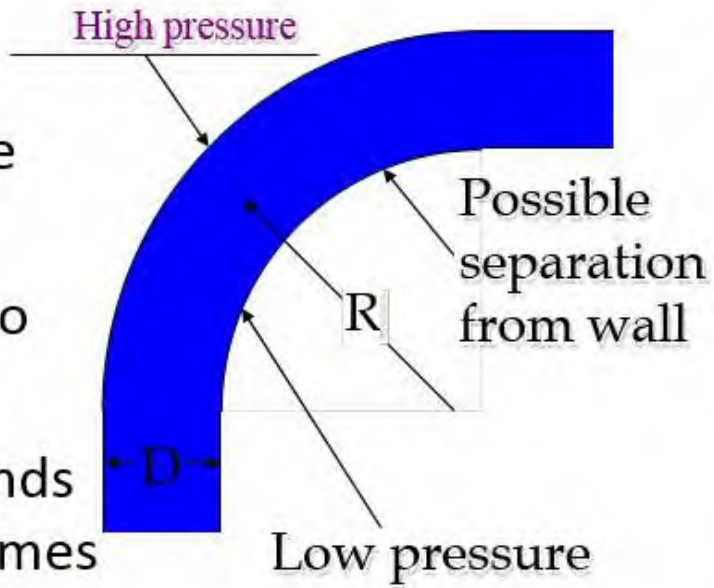
Entrance Losses

- Losses can be reduced by accelerating the flow gradually and eliminating the *vena contracta*



Head Loss in Bends

- Head loss is a function of the ratio of the bend radius to the pipe diameter (R/D)
- Velocity distribution returns to normal far downstream
- Head loss from a series of bends is not the number of bends times the loss through a single bend



$$h_b = K_b \frac{V^2}{2g}$$

Head Loss in Valves

- Function of valve type and valve position
- The complex flow path through valves often results in high head loss
- What is the maximum value that K_v can have?

$$h_v = K_v \frac{V^2}{2g}$$

Non-Circular Conduits: Hydraulic Radius Concept

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

- A is cross sectional area
- P is wetted perimeter
- R_h is the “Hydraulic Radius” (Area/Perimeter)
- Don’t confuse with radius!

$$R_h = \frac{A}{P} = \frac{\frac{\pi}{4}D^2}{\pi D} = \frac{D}{4}$$

For a pipe

$$D = 4R_h$$

$$h_f = f\frac{L}{4R_h}\frac{V^2}{2g}$$

We can use Moody diagram or Swamee Jain with D = 4R!