INSTITUTE OF AERONAUTICALENGINEERING
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OPERATIONS RESEARCH

(Accelerated Course)

B.Tech VI Semester

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SYLLABUS

UNIT-I

Development – Definition– Characteristics and Phases– Types of models - Operations Research models –applications.

Allocation: Linear Programming Problem Formulation – Graphical solution – Simplex method – Artificial variables techniques: Two–phase method, Big-M method.

UNIT-II

Transportation Problem - Formulation – Optimal solution, unbalanced transportation problem –Degeneracy.

Assignment problem – Formulation – Optimal solution – Variants of Assignment Problem- Traveling Salesman problem.

UNIT-III

Sequencing - Introduction – Flow –Shop sequencing – n jobs through two machines – n jobs through three machines – Job shop sequencing – two jobs through „m" machines.

Replacement: Introduction – Replacement of items that deteriorate with time – when money value is not counted and counted – Replacement of items that fail completely, Group Replacement.

UNIT-IV

Theory Of Games: Introduction – Terminology - Solution of games with saddle points and without saddle points – 2×2 games – dominance principle – m X 2 & 2 X n games - Graphical method.

Inventory: Introduction – Single item – Deterministic models – Purchase inventory models with one price break and multiple price breaks – Stochastic models – demand may be discrete variable or continuous variable – Single period model and no setup cost.

UNIT-V

Waiting Lines: Introduction – Terminology - Single Channel – Poisson arrivals and exponential service times – with infinite population and finite population models– Multichannel – Poisson arrivals and exponential service times with infinite population.

Dynamic Programming: Introduction –Terminology - Bellman‟sPrinciple of optimality –Applications of dynamic programming – shortest path problem – linear programming problem. **Simulation:** Introduction, Definition, types of simulationmodels,

steps involved in the simulation process - Advantages and Disadvantages – Application of Simulation to queuing and inventory.

Unit – I: Linear Programming Problem

- As its name implies, operations research involves "research on operations." Thus, operations research is applied to problems that concern how to conduct and coordinate the operations (i.e., the activities) within anorganization.
- The nature of the organization is essentially immaterial, in fact, OR has been applied extensively in such diverse areas as manufacturing, transportation, construction, telecommunications, financial planning, health care, the military, and public services, to name just a few.

OPERATIONS RESEARCH

- The process begins by carefully observing and formulating the problem, including gathering all relevant data. Then construct a scientific model to represent the real problem, while explaining its objectives with the systemconstraints.

- It attempts to resolve the conflicts of interest among the componentsofthe organization in a way that is best for the organization as awhole.

HISTORY OF OPERATIONS RESEARCH

Operations Research cameinto existence during World War II, when the British and American military management called upon a group of scientists with diverse educational background namely,Physics, Biology, Statistics, Mathematics, Psychology, etc., to develop and apply a scientific approach to deal with strategic and tactical problems of various militaryoperations.

HISTORY OF OPERATIONS RESEARCH

The objective was to allocate scarce resources in an effective manner to various military operations and to the activities within each operation. The name Operations Research (OR) came directly from the context in which it was used and developed, viz., and research on military operations

During the 1950s, Operations Research achieved recognitionas a subject for study in the universities. Since then the subject has gained increasing importance for the students of Management, Public Administration, Behavioral Sciences, Engineering, Mathematics, Economics and Commerce, etc. Today, Operations Research is also widely used in regional planning, transportation, public health, communication etc., besides military and industrialoperations.

In India, Operations Research came into existence in1949 with the opening of an Operations Research Unit at the Regional Research Laboratory at Hyderabad and also in the Defence Science Laboratory at Delhi which devoted itself to the problems of stores, purchase and planning. For national planning and survey, an Operations Research Unit was established in 1953 at the India Statistical Institute, Calcutta. In 1957, Operations Research Society of India was formed. Almost all the universities and institutions in India are offering the input of Operations Research in their curriculum.

Definition

- **OPERATIONS RESEARCH (OR)**

  It is a scientific approach to determine the optimum (best) solution to a decision problem under the restriction of limited resources, using the mathematical techniques to model, analyze, and solve the problem

PHASES OF OR

- Definition of theproblem
- ModelConstruction
- Solution of themodel
- Modelvalidity
- Implementation of thesolution

BASIC COMPONENTS OF THE MODEL

1. ObjectiveFunction
2. Constraints
3. DecisionVariables

O.R IS USEFUL IN THE FOLLOWING VARIOUS IMPORTANT FIELDS.

1. In Agriculture:
   (i) Optimum allocation of land to various crops in accordance with the climatic conditions, and
   (ii) Optimum allocation of water from various resources like canal for irrigation purposes.
2. In Finance:
   (i) To maximize the per capita income with minimum resources
      (i) To find the profit plan for the country
      (ii) To determine the best replacement policies, etc.

SCOPE OF OPERATIONS RESEARCH

3. InIndustry:

   - O.R is useful for optimum allocations oflimited resources such as men materials, machines, money, time, etc. to arrive at theoptimumdecision.

4. InMarketing:

   With the help of O.R Techniques a marketing Administrator (manager) can decide where to distribute the products for sale so that the total Costof transportation etc. is minimum.

SCOPE OF OPERATIONS RESEARCH

Continuation…

- The minimum per unit sale price
- The size of the stock to meet the futuredemand
- How to select the best advertising media withrespect to time, costetc.
- How when and what to purchase at the min. possiblecost?

5. In PersonnelManagement:

- To appoint the most suitable persons on min.salary
- To determine the best age of retirement for the employees
- To find out the number of persons to be appointed onfull time basis when the work load is seasonal.

Scope of Operations Research

Continuation…

6. In ProductionManagement:
   - To find out the number and size of the items to beproduced
   - In scheduling and sequencing the production run byproper allocation ofmachines
   - In calculating the optimum product mix,and
   - To select, locate and design the sites for the productionplants

7. <u>InL.I.C.:</u>
   - **What** should be the premium rates for various modesof policies

- How best the profits could be distributed in the casesofwith profit policy

Example 1:

- A company manufactures two products A&B. with 4 & 3 units of price. A&B take 3&2 minutes respectively to be machined. The total time available at machining department is 800 hours (100 days or 20 weeks). A market research showed that at least 10000 units of A and not more than 6000 units of B are needed. It is required to determine the number of units of A&B tobe produced to maximizeprofit.

- Decisionvariables

  X1 = number of units produced of A.

  X2 = number of units produced of B.

- ObjectiveFunction

  Maximize $Z = 4X_1 + 3X_2$

- Constraints

$$
\begin{aligned}
3X_1 + 2X_2 &\leq 800 \times 60 \\
X_1 &\geq 10000 \\
X_2 &\leq 6000 \\
X_1,\ X_2 &\geq 0
\end{aligned}
$$

Decisionvariables

X1 = weight of feed A kg/day/animal

X2 = weight of feed B kg/day/animal

- ObjectiveFunction

Minimize $Z = 20X_1 + 50X_2$

Cost function

- Constraints

Protein	$0.1 X_1 + 0.4X_2$	$\geq$	0.4
Carbohydrates	$0.8 X_1 + 0.6X_2$	$\geq$	0.8
Fats	$0.1X_1$	$\leq$	0.1
	$X_1, \quad X_2$	$\geq$	0

Example 3: Blending Problem

- An iron ore from 4 mines will be blended. The analysis has shown that, in order to obtain suitable tensile properties, minimum requirements must be met for 3 basic elements A, B, and C. Each of the 4 mines contains different amounts of the 3 elements Formulate to find theleast cost(Minimize) blend for one ton of iron ore.

Problem Formulation

- **Decisionvariables**

    X1= Fraction of ton to be selected from mine number 1

    X2= Fraction of ton to be selected from mine number 2

    X3= Fraction of ton to be selected from mine number 3

    X4= Fraction of ton to be selected from mine number 4

- **ObjectiveFunction**

    Minimize Z= 800 X1 + 400 X2 + 600 X3 + 500 X4

- **Constraints**

$$10X_1 + 3X_2 + 8X_3 + 2X_4 \quad 5$$

$$90\,X_1 + 150X_2 + 75X_3 + 175X_4 \quad 10 \geq$$

$$45X_1 + 25X_2 + 20X_3 + 37X_4 \quad 30 \geq$$

$$X_1 + X_2 + X_3 + X_4 \quad 1 \quad \geq$$

$$X_1, X_2, X_3, X_4 \quad \underline{0}$$

$$\geq$$

Example 4: Inspection Problem

- A company has 2 grades of inspectors 1&2. It is required that at least 1800 pieces be inspected per 8 hour day. Grade 1 inspectors can check pieces at the rate of 25 per hour with an accuracy of 98%. Grade 2 inspectors can check at the rate of 15 pieces per hour with an accuracy of 95%. Grade 1 costs 4 L.E/hour L.E/hour, grade 2 costs 3 L.E/hour. Each time an error is madeby an inspector costs the company 2 L.E. There are 8 grade 1 and 10 grade 2 inspectors available. The company wants to determine the optimal assignment of inspectors which will minimize the total cost of inspection/day.

Problem Formulation

- **Decisionvariables**

  X1= Number of grade 1 inspectors/day.

  X2= Number of grade 2 inspectors/day.

- **ObjectiveFunction**

  Cost of inspection = Cost of error + Inspector salary/day

  Cost of grade 1/hour = 4 + (2 X 25 X 0.02) = 5 L.E

  Cost of grade 2/hour = 3 + (2 X 15 X 0.05) = 4.5 L.E

  Minimize Z= 8 (5 X1 + 4.5 X2) = 40 X1 + 36 X2

- **Constraints**

$$
\begin{aligned}
X_1 &\leq 8 \\
X_2 &\leq 10 \\
8(25)\,X_1 + 8(15)X_2 &\geq 1800 \\
200\,X_1 + 120\,X_2 &\geq 1800 \\
X_1,\ X_2 &\geq 0
\end{aligned}
$$

Example 5: Trim-loss Problem.

- A company produces paper rolls with a standard width of 20 feet. Each special customer orders with different widths are produced by slitting the standard rolls. Typical orders are summarized in the following tables.

Order	Desired Width	Desired Number of Rolls
1	5	150
2	7	200
3	9	300

Possible knife settings

Required Width	Knife settings						Minimum Number of rolls
	X1	X2	X3	X4	X5	X6	
5	0	2	2	4	1	0	150
7	1	1	0	0	2	0	200
9	1	0	1	0	0	2	300
Trim loss/roll	4	3	1	0	1	2	

- Formulate to minimize the trim loss and thenumber of rolls needed to satisfy theorder.

Problem Formulation

Decision variables

X_j = Number of standard rolls to be cut according to

settingj j = 1, 2, 3, 4, 5,6

- Number of 5 feet rolls produced= $2X_2 + 2X_3 + 4X_4 + X_5$
- Number of 7 feet rolls produced = $X_1 + X_2 + 2X_5$
- Number of 9 feet rolls produced = $X_1 + X_3 + 2X_6$
- Let Y_1, Y_2, Y_3 be the number of surplus rolls of the 5, 7, 9 feet rollsthus
  - $Y_1 = 2X_2 + 2X_3 + 4X_4 + X_5 - 150$
  - $Y_2 = X_1 + X_2 + 2X_5 - 200$
  - $Y_3 = X_1 + X_3 + 2X_6 - 300$
- The total trim losses = L $(4X_1 + 3X_2 + X_3 + X_5 + 2X_6 + 5Y_1 + 7Y_2 + 9Y_3)$

*Where L is the length of the standardroll.

- **ObjectiveFunction**

$$\text{Minimize } Z = L(4X_1 + 3X_2 + X_3 + X_5 + 2X_6 + 5Y_1 + 7Y_2 + 9Y_3)$$

- **Constraints**

$$2X_2 + 2X_3 + 4X_4 + X_5 - Y_1 = 150$$

$$X_1 + X_2 + 2X_5 - Y_2 = 200$$

$$X_1 + X_3 + 2X_6 - Y_3 = 300$$

$$X1, X2, X3, X4, X5, X6 \geq 0$$

$$Y1, Y2, Y3 \geq 0$$

General form of a LP problem with m constraints and n decision variables is:

$$\text{Maximize } Z = C_1X_1 + C_2X_2 + \dots + C_nX_n$$

- **Constraints**

$$A_{11}X_1 + A_{12}X_2 + \dots\dots\dots\dots\dots\dots + A_{1n}X_n \leq B_1$$

$$A_{21}X_1 + A_{22}X_2 + \dots\dots\dots\dots\dots\dots + A_{2n}X_n \leq B_2$$

.

.

.

.

$$A_{m1}X_1 + A_{m2}X_2 + \dots\dots\dots\dots\dots\dots + A_{mn}X_n \leq B_m$$

$$X_1, X_2, \dots\dots\dots\dots\dots\dots\dots\dots, X_n \geq 0$$

OR

Maximize $$z = \sum_{j=1}^{n} c_j x_j$$

- **Constraints**

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad , i = 1,2,\ldots, m$$

$$x_j \geq 0 \quad , j = 1, 2,\ldots, n$$

Terminology of solutions for a LP model:

- **ASolution**

  Any specifications ofvaluesof $X_1, X_2, \ldots, X_n$ is called asolution.
- **A FeasibleSolution**

  Is a solution for which all the constraints are satisfied

- **An OptimalSolution**

  Is a feasible solution that has the most favorable value of the Objective function (largest of maximize or smallest for minimize)

Graphical Solution

- **Construction of the LPmodel**
  - *<u>Example 1: The Reddy MikksCompany</u>*

Reddy Labs produces both interior and exterior paints from two raw materials, M1&M2. The following table provides the basic data of the problem.

	Tons of raw material per ton of		Maximum daily availability (tons)
	Exterior paint	Interior paint	
Raw Material, M1	6	4	24
Raw Material, M2	1	2	6
Profit per ton ($1000)	5	4	

- A market survey indicates that the daily demand for interior paint cannot exceed that of exterior paint by more than 1 ton. Also, the maximum daily demandof interior paint is 2ton.
- Reddy Labs wants to determine the optimum(best) product mix of interior and exterior paints that maximizes the total dailyprofit

Problem Formulation

- **<u>Decisionvariables</u>**

  X_1= Tons produced daily of exterior paint.

  X_2= Tons produced daily of interior paint.

- **<u>ObjectiveFunction</u>**

  Maximize $Z= 5\,X_1 + 4X_2$

- **<u>Constraints</u>**

  $6X_1+4X_2$ 24

  ≤

  X_1+2X_2 6

  ≤

  $-X_1+\ X_2$ 1

  X_2 ≤□2

  $X_1,\ \ X_2$ 0 ≤

  ≥

- Any solution that satisfies all the constraints of the model is a *feasible solution*. For example, X1=3 tons and X2=1 ton is a feasible solution. We have an infinite number of feasible solutions, but we are interested in the optimum feasible solution thatyields the maximum totalprofit.

Graphical Solution

- The graphical solution is valid only for two-variable problem.
- The graphical solution includes two basicsteps:
  1. The determination of the solution space that defines the feasible solutions that satisfy all theconstraints.
  2. The determination of the optimum solution from among all the points in the feasible solutionspace.

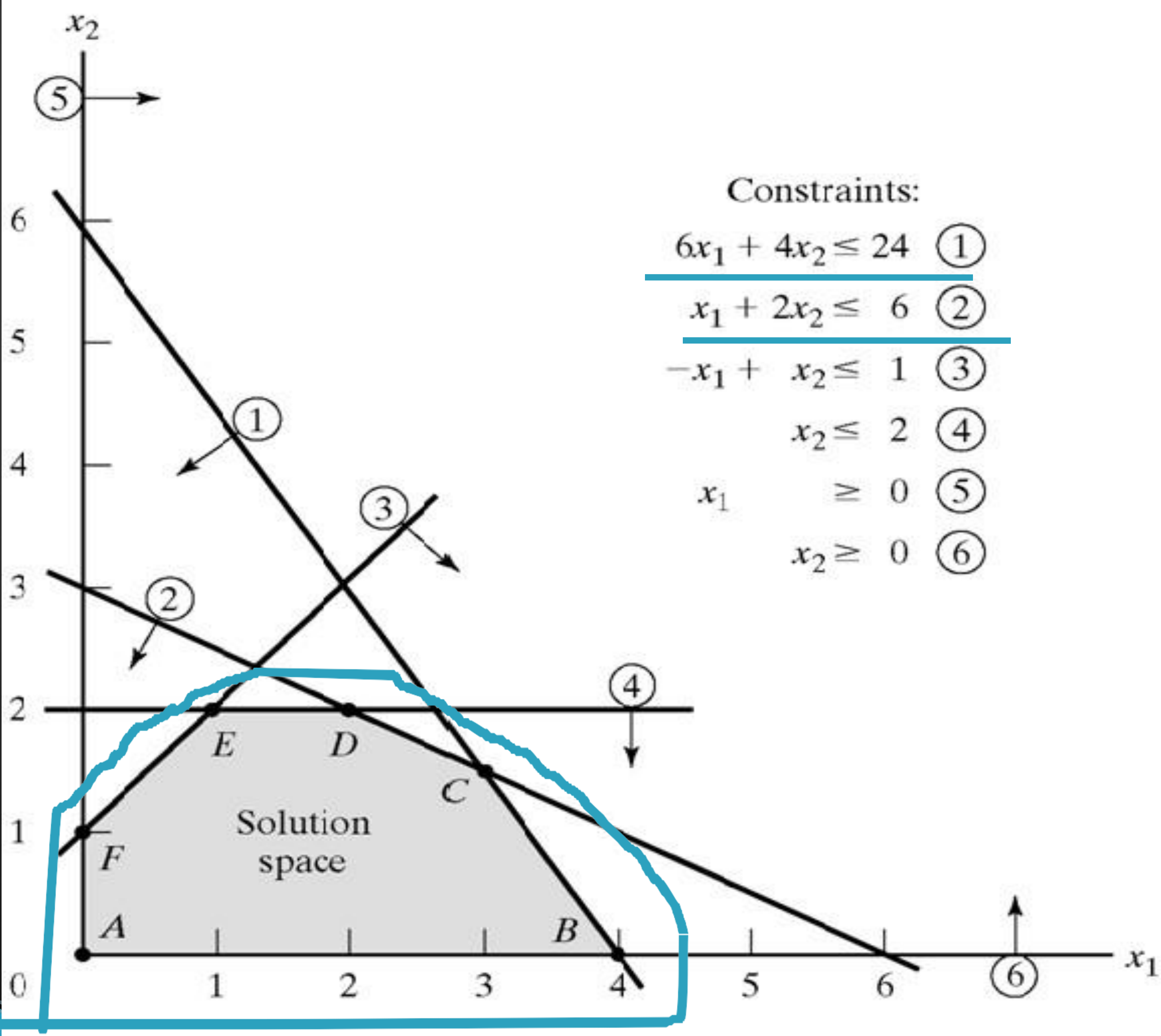


Figure 2.1
Feasible space of the Reddy Mikks model.

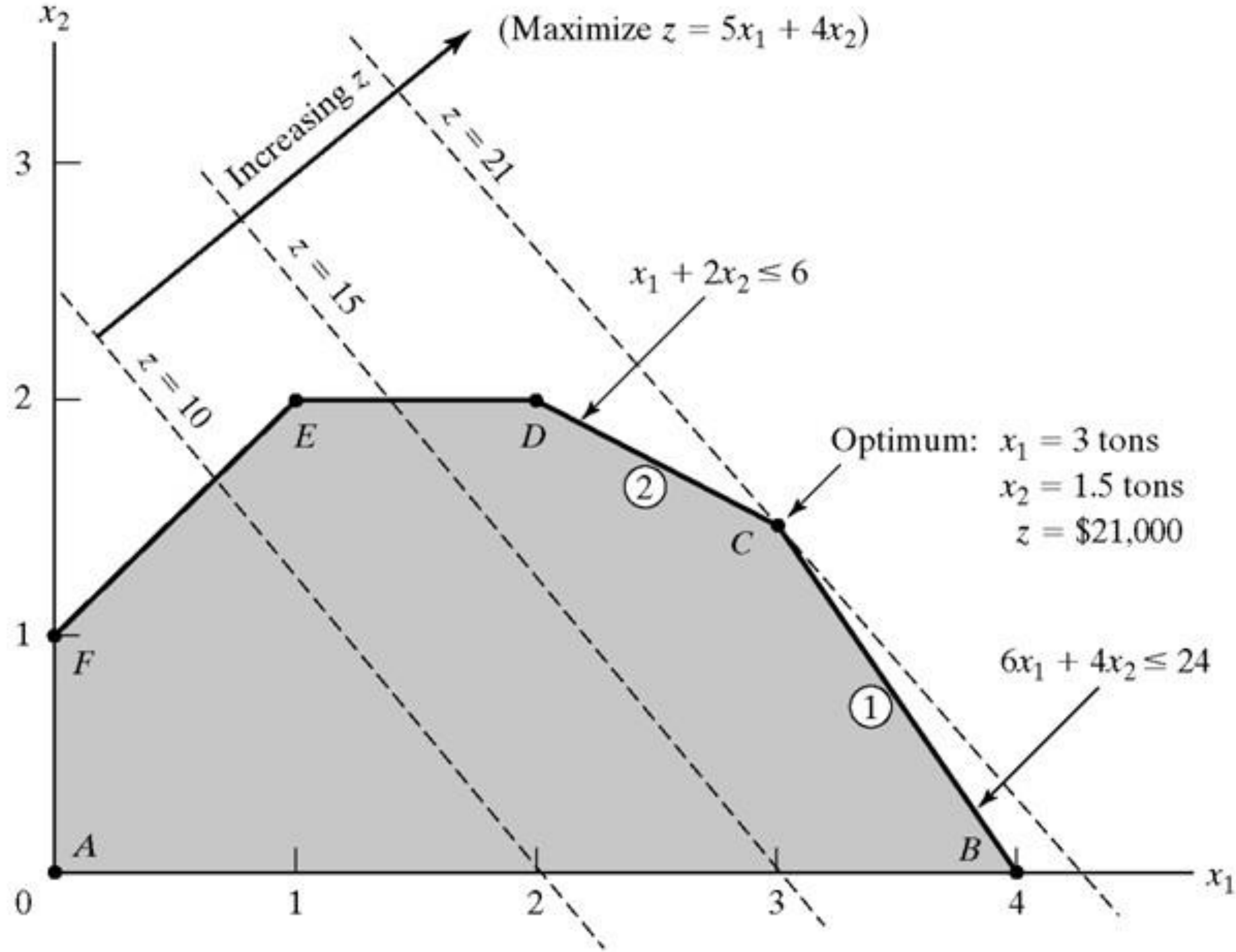


Figure 2.2
Optimum solution of the Reddy Mikks model.

- ABCDEF consists of an infinite number of points;we need a systematic procedure that identifies the optimum solutions. The optimum solution is associated with a corner point of the solutionspace.
- Todetermine the direction in which the profit function increases we assign arbitraryincreasing values of 10 and15

$$5X_1 + 4X_2 = 10$$

And $$5X_1 + 4X_2 = 15$$

- The optimum solution is mixture of 3 tons of exterior and 1.5 tons of interior paints will yield a daily profit of 21000$.

Consider the following linear programming problem.

Minimize $c_1x_1 + c_2x_2 + \cdots + c_nx_n$

Subject to

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &\geq b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &\geq b_2 \\
\vdots \qquad \vdots \qquad \qquad \vdots & \quad \vdots \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &\geq b_m \\
x_1, \quad x_2, \quad \ldots, \quad x_n &\geq 0
\end{aligned}$$

THE LINEAR PROGRAMMING PROBLEM

INTRODUCTION

A linear programming problem is a problem of minimizing or maximizing a linear function in the presence of linear constraints of the inequality and/or the equality type.

Here $c_1x_1 + c_2x_2 +, \ldots, + c_nx_n$ is the *objective function* (or criterion function) to be minimized and will be denoted by z. The coefficients $c_1, c_2, \ldots, c_n$ are the (known) *cost coefficients* and $x_1, x_2, \ldots, x_n$ are the *decision variables* (variables, or activity levels) to be determined. The inequality $\sum_{j=1}^{n} a_{ij}x_j \geqslant b_i$ denotes the ith *constraint* (or restriction). The coefficients a_{ij} for $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$ are called the *technological coefficients*. These technological coefficients form the *constraint matrix* **A** given below.

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

The column vector whose ith component is b_i, which is referred to as the *right-hand-side vector*, represents the minimal requirements to be satisfied. The constraints $x_1, x_2, \ldots, x_n \geqslant 0$ are the *nonnegativity constraints*. A set of variables $x_1, \ldots, x_n$ satisfying all the constraints is called a *feasible point* or a *feasible vector*. The set of all such points constitutes the *feasible region* or the *feasible space*.

Using the foregoing terminology, the linear programming problem can be

Linear Programming in Matrix Notation

A linear programming problem can be stated in a more convenient form using matrix notation. To illustrate, consider the following problem.

$$\text{Minimize} \quad \sum_{j=1}^{n} c_j x_j$$

$$\text{Subject to} \quad \sum_{j=1}^{n} a_{ij} x_j = b_i \quad i = 1, 2, \ldots, m$$

$$x_j \geqslant 0 \quad j = 1, 2, \ldots, n$$

Denote the row vector $(c_1, c_2, \ldots, c_n)$ by $\mathbf{c}$, and consider the following column vectors $\mathbf{x}$ and $\mathbf{b}$, and the $m \times n$ matrix $\mathbf{A}$.

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix} \qquad \mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Then the above problem can be written as follows.

$$\begin{aligned} \text{Minimize} \quad & \mathbf{cx} \\ \text{Subject to} \quad & \mathbf{Ax} = \mathbf{b} \\ & \mathbf{x} \geqslant \mathbf{0} \end{aligned}$$

The problem can also be conveniently represented via the columns of $\mathbf{A}$. Denoting $\mathbf{A}$ by $[\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n]$ where $\mathbf{a}_j$ is the jth column of $\mathbf{A}$, the problem can be formulated as follows.

Table 1.1 Standard and Canonical Forms

	MINIMIZATION PROBLEM	MAXIMIZATION PROBLEM
Standard Form	Minimize $\sum_{j=1}^{n} c_j x_j$ Subject to $\sum_{j=1}^{n} a_{ij} x_j = b_i \quad i = 1, \ldots, m$ $x_j \geqslant 0 \quad j = 1, \ldots, n$	Maximize $\sum_{j=1}^{n} c_j x_j$ Subject to $\sum_{j=1}^{n} a_{ij} x_j = b_i \quad i = 1, \ldots, m$ $x_j \geqslant 0 \quad j = 1, \ldots, n$
Canonical Form	Minimize $\sum_{j=1}^{n} c_j x_j$ Subject to $\sum_{j=1}^{n} a_{ij} x_j \geqslant b_i \quad i = 1, \ldots, m$ $x_j \geqslant 0 \quad j = 1, \ldots, n$	Maximize $\sum_{j=1}^{n} c_j x_j$ Subject to $\sum_{j=1}^{n} a_{ij} x_j \leqslant b_i \quad i = 1, \ldots, m$ $x_j \geqslant 0 \quad j = 1, \ldots, n$

Maximize $Z=6X1 + 8X2$

Subject to $30X1 + 20X2 \leq 300$

$5X1 + 10X2 \leq 110$

And $X1, X2 \geq 0$

Method :

Step1: Convert the above inequality constraint into equality constraint by adding slack variables S1 andS2

The constraint equations are now

$30X1 + 20X2 + S1 = 300$, $S1 \geq 0$

$5X1+10X2 \quad + S2 = 110$, $S2 \geq 0$

The LP problem in standard isnow

$Z = 6X1 + 8X2 + 0 \times S1 + 0 \times S2$

$30X1 + 20X2 + S1 = 300$

$5X1+10X2 \quad + S2 = 110$

And $X1, X2, S1, S2 \geq 0$

Variables withnon-zero values are calledbasic variables.

Variables withzero values are called non-basic variables.

If there is no redundant constraint equation in the problem, there will be as many basic variables as many constraints, provided a basic feasible solution exists.

Step 2 : Form a table

Table I

Basic	Z	X1	X2	S1	S2	Solution	Ratio
Z	1	-6	-8	0	0	0	
S1	0	30	20	1	0	300	(300/20=15)
S2	0	5	10	0	1	110	(110/10=11)

Start with the current solution at the origin X1=0,X2=0 And therefore Z = 0. S1,S2 are the basic variables and X1,X2 are the non-basic variables.

- Z is 0 and it is not maximum . It has scopefor improvement
- Z is found to be most sensitive to X2 since its coefficient is -8, so this is chosen as the pivotcolumn .X2 **enter**s into the basic variable column. This becomes the pivot column.
- Search for leaving variable in the firstcolumn

by choosing the row which has the least value in the ratio column. It is S2 which leaves the basic variable

The modified table is now obtained by

1.New pivot row =current pivot row/pivotelement

2. All other new row (including Z row)=

current row- its pivot column coefficient*new pivot-row

Table II

Basic	Z	X1	X2	S1	S2	Solution	Ratio
Z	1	-2	0	0	8/10	88	
S1	0	20	0	1	-2	80	(80/20=4)
X2	8	5/10	1	0	1/10	11	(11/.5 =22)

Unit – II: Transportation Problem

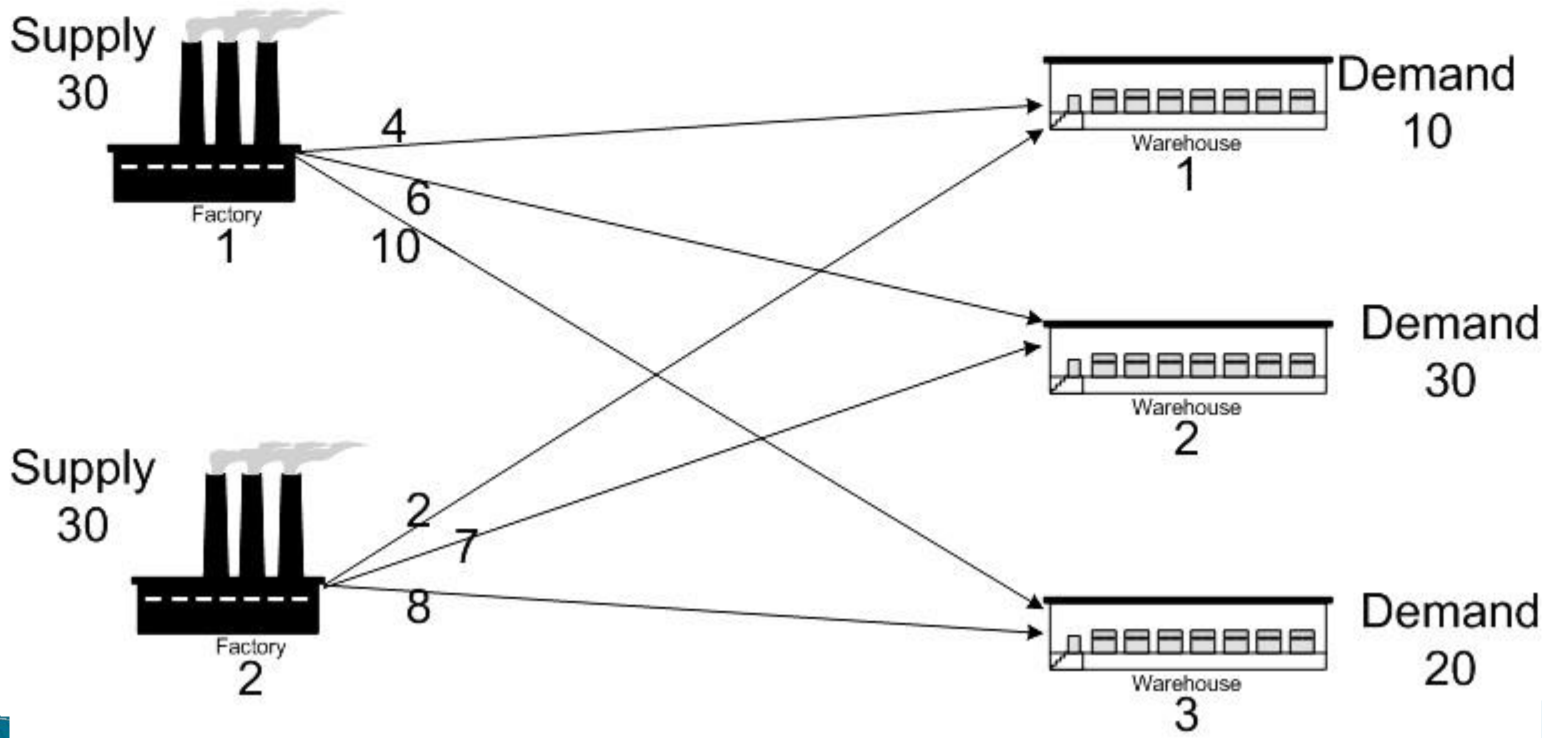
The Transportation Problem is a classic Operations Research problem where the objective is to determine the schedule for transporting goods from source to destination in a way that minimizes the shipping cost while satisfying supply and demand constraints.

Transportation Problem

A typical Transportation Problem has the following elements:

1. Source(s)
2. Destination(s)
3. Weighted edge(s) showing cost of transportation

Transportation Problem



The Assignment Problem

- In many business situations, management needsto assign - personnel to jobs, - jobs to machines, - machines to job locations, or - salespersons to territories.
- Consider the situation of assigning *n* jobs to*n* machines.
- When a job i (=1,2,....,n) is assigned to machine j (=1,2, ...n) that incurs a cost Cij.
- The objective is to assign the jobs to machines at the least possible total cost.

ASSIGNMENT PROBLEM

- This situation is a special case of the Transportation Model And it is known as the *assignment problem*.
- Here, jobs represent “sources” and machines represent “destinations.”
- The supply available at each source is 1unit and demand at each destination is 1unit.

The Assignment Problem

Assignment Problem is a special type of linear programming problem where the objective is to minimize the cost or time of completing a number of jobs by a number of persons.

The assignment problem in the general form can be stated as follows:

"Given *n* facilities, *n* jobs and the effectiveness of each facility for each job, the problem is to assign each facility to one and only one job in such a way that the measure of effectiveness is optimized (Maximized or Minimized)."Several problems of management have a structure identical with the assignment problem.

EXAMPLE OF ASSIGNMENT PROBLEM:

Example-1: A manager has four persons (i.e. facilities) available for four separate jobs (i.e. jobs) and the cost of assigning (i.e. effectiveness) each job to each person is given. His objective is to assign each person to one and only one job in such a way that the total cost of assignment is minimized.

Example-2: A manager has four operators for four separate jobs and the time of completion of each job by each operator is given. His objective is to assign each operator to one and only one job in such a way that the total time of completion is minimized.

Example-3:A tourist car operator has four cars in each of the four cities and four customers in four different cities. The distance between different cities is given. His objective is to assign each car to one and only one customer in such a way that the total distance covered is minimized

HUNGARIAN METHOD

Although an assignment problem can be formulated as a linear programming problem, it is solved by a special method known as Hungarian Method because of its special structure.. The Hungarian Method is based on the principle that if a constant is added to every element of a row and/or a column of cost matrix, the optimum solution of the resulting assignment problem is the same as the original problem and vice versa. The original cost matrix can be reduced to another cost matrix by adding constants to the elements of rows and columns where the total cost or the total completion time of an assignment is zero. Since the optimum solution remains unchanged after this reduction, this assignment is also the optimum solution of the original problem.

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STEPS FOR SOLVING MINIMIZATION ASSIGNMENT PROBLEM

Step 1: See whether Number of Rows are equal to Number of Column. If yes, problem is balanced one; if not, then add a Dummy Row or Column to make the problem a balanced one by allotting zero value or specific value (if any given) to each cell of the Dummy Row or Column, as the case may be.

Step 2: Row Subtraction: Subtract the minimum element of each row from all elements of that row.

Note: If there is zero in each row, there is no need for row subtraction.

Step 3: Column Subtraction: Subtract the minimum element of each column from all elements of that column.

Note: If there is zero in each column, there is no need for column subtraction.

Step 4: Draw minimum number of Horizontal and/or Vertical Lines to cover all zeros.

LINE DRAWING PROCEDURE

1. Select a row containing exactly one uncovered zero and draw a vertical line through the column containing this zero and repeat the process till no such row is left.
2. Select a column containing exactly one uncovered zero and draw a horizontal line through the row containing the zero and repeat the process till no such column is left.

Step 5: If the total lines covering all zeros are equal to the size of the matrix of the Table, we have got the optimal solution; if not, subtract the minimum uncovered element from all uncovered elements and add this element to all elements at the intersection point of the lines covering zeros.

Step 6: Repeat Steps 4 and 5 till minimum number of lines covering all zeros is equal to the size of the matrix of the Table.

Step 7: Assignment: Select a row containing exactly one unmarked zero and surround it by ,‘and draw a vertical line through the column containing this zero. Repeat this process till no such row is left; then select a column containing exactly one unmarked zero and surround it by, and draw a horizontal line through the row containing this zero and repeat this process till no such column is left.

Note: If there is more than one unmarked zero in any row or column, it indicates that an alternative solution exists. In this case, select anyone arbitrarily and pass two lines horizontally and vertically.

Step 8: Add up the value attributable to the allocation, which shall be the minimum value.

Step 9 : Alternate Solution: If there are more than one unmarked zero in any row or column, select the other one (i.e., other than the one selected in Step 7) and pass two lines horizontally and vertically.

Step 10:Add up the value attributable to the allocation, which shall be the minimum value.

Problem

ABC Corporation has four plants each of which can manufacture anyone of the four products. Product costs differ from one plant to another as follow:

Plant	Product 1	Product 2	Product 3	Product 4
A	33	40	43	32
B	45	28	31	23
C	42	29	36	29
D	27	42	44	38

You are required:

a. to obtain which product each plant should produce to minimisecost, Step 1 : Row Deduction: Subtracting the minimum element of each row from all the elements of that row

1	8	11	0
22	5	8	0
13	0	7	0
0	15	17	11

Step 2 : Column Deduction: Subtracting the minimum element of each column of above matrix from all the elements of that column and then drawing the minimum number of lines (whether horizontal/vertical) to cover all the zeros:

1	8	4	0
22	5	1	0
13	0	0	0
0	15	10	11

Since number of lines drawn =3 and order of matrix =4, we will have to take the step to increase the number of zeros.

Step 3 ; Subtracting the minimum uncovered element (1 in this case) from all the uncovered elements and adding to the elements at intersection points, and then drawing the minimum numbers of lines to cover all zeros.

0	7	3	0
21	4	0	0
13	0	0	1
0	15	10	12

Since the number of lines drawn (4) = order of matrix (4), the above matrix will provide the optimal solution.

Step 4 :*Assignment:* Selecting a row containing exactly one unmarked zero and surrounding it by ‘0’ and draw a vertical line thorough the column containing this zero.

Repeatingthisprocesstillnosuchrowisleft;then selectingacolumncontainingexactlyone unmarked zero and surrounding it by ‘0’ and draw a horizontal line through the row containingthiszeroandrepeatingthisprocesstillnosuc hcolumnisleft.

0	7	3	$\boxed{0}$
21	4	$\boxed{0}$	0
13	$\boxed{0}$	0	11
$\boxed{0}$	15	10	12

Step-5: the optimal assignment is as follows:

Assign Plant –a to product -4

Plant B to product -3

plant	product	Cost (Rs)
Assign plant -A	4	32
Assign plant -B	3	31
Assign plant - C	2	29
Assign plant -D	1	27
Total cost		Rs 119

STEPS INVOLVED SOLVING IN MAXIMISATION TYPE ASSIGNMENT PROBLEM

Step 1: See whether Number of Rows is equal to Number of Columns. If yes, problem is a balanced one; if not, then adds a Dummy Row or Column to make the problem a balanced one by allotting zero value or specific value (if any given) to each cell of the Dummy Row or Column, as the case may be.
Step 2: Derive Profit Matrix by deducting cost from revenue.

Step 3: Derive Loss Matrix by deducting all elements from the largest element.
Step 4: Follow the same Steps 2 to 9 as involved in solving Minimization Problems.
Minimization Problem

Continued…

Example 1.*(Maximization Problem)*.

		Jobs				
		A	B	C	D	E
	1	5	11	10	12	4
	2	2	4	6	3	5
Machines	3	3	12	5	14	6
	4	6	14	4	11	7
	5	7	9	8	12	5

A company has 5jobs to be done. The following matrix shows the return in rupees on assigning I th (i = 1,2,3,4,5) machine to the jth job (j =A, B, C, D, E). Assign the five jobs to the five machines so as to maximize the total expected profit

Continued…

Solution.*Step* 1.*Converting from Maximization to Minimization:*

9	3	4	2	10
12	10	8	11	9
11	2	9	0	8
8	0	10	3	7
7	5	6	2	5

Since the highest element is]4, so subtracting all the elements from]4, the following reduced cost (opportunity loss ofmaximum profit) matrix is obtained.

Continued…

Step 2. Now following the usual procedure of solving an assignment problem, an optimal assign- ment is obtained in the following table:

1	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	5
$\cancel{0}$	13	$\cancel{0}$	5	$\boxed{0}$
5	1	7	$\boxed{0}$	5
3	$\boxed{0}$	9	4	5
$\boxed{0}$	3	3	1	5

This table gives the optimum assignment as : $1 \to C$, $2 \to E$, $3 \to D$, $4 \to B$, $5 \to A$; with maximum profit of Rs. 50.

Continued…

UNIT- III SEQUENCING AND REPLACEMENT

Introduction

Suppose there are n jobs to perform, each of which requires processing on some or all of m different machines. The effectiveness (i.e. cost, time or mileage, etc.) can be measured for any given sequence of jobs at each machine, and the most suitable sequence is to be selected (which optimizes the effectiveness measure) among all (n!)m theoretically possible sequences. Although, theoretically, it is a1ways possible to select the best sequence by testing each one, but it is practically impossible because of large number of computations.

DEFINITION OF SEQUENCING :Suppose there are n jobs (1, 2, 3,..., n), each of which has to be processed one at a time at each of m machines A, B, C, ... The order of processing each job through machines is given (for example, job I is processed through machines A, C, B-in this order). The time that each job must require on each machine is known. The problem is to find a sequence among (n!)m number of all possible sequences (or combinations) (or order) for processing the jobs so that the total elapsed time for all the jobs will be minimum.

Mathematically, let
Ai = time for job i on machine A,
Bi = time for job i on machine B, etc.
T = time from start of first job to completion of the last job.
Then, the problem is to determine for each machine a sequence of jobs i1, i2, i3, ..., in- where
(i1,i2, i3, ..., in) is the permutation of the integers which will minimize T.

TERMINOLOGY AND NOTATIONS

- Number of Machines. It means the service facilities through which. a job must pass before it is completed. .
- For example, a book to be published has to be processed through composing, printing, binding, etc. In this example, the book constitutes the job and the different processes constitute the number of machines. .
- Processing Order. It refers to the order in which various machines are required for completing the job.
- Processing Time. It means the time required by each job on each machine. The notation Tij will denote the processing time required for Ith job b yjth machine (i = 1,2,..., n;j = 1, 2, ...,m).
- Idle Time on a Machine. This is the time for which a machine remains idle during the total elapsed time. The notation Xij shall be (used to denote the idle time of machine j between the end of the (i - 1)th job and the start of the ith job.
- Total Elapsed Time. This is the time between starting the first job and completing the last job. This also includes idle time, if exists. It will be denoted by the symbol T.
- No Passing Rule. This rule means that P1lssing is not allowed, i.e. the same order of jobs is maintained over each machine. If each of the n-jobs is to be processed through two machines A and B in the order AB, then this rule means

that each job will go to machine A first and then to B.

PRINCIPAL ASSUMPTIONS

- No machine can process more than one operation at a time.
- Each operation, once started, must be performed till completion.
- A job is an entity, i.e. even though the job represents a lot of individual parts, no lot may be processed by more than one machine at a time.
- Each operation must be completed before any other operation, which it must precede, can begin.
- Time intervals for processing are independent of the order in which operations are performed.
- There is only one of each type of machine.
- A job is processed as soon as possible subject to ordering requirements.
- All jobs are known and are ready to start processing before the period under consideration begins.
- The time required to transfer jobs between machines is negligible.

SEQUENCING PROBLEM MODELS

1. n jobs and two machines A and B, all jobs processed in the order AB.
2. n jobs and three machines A, Band C, all jobs processed in the order ABC.
3. Two jobs and m machines. Each job is to be processed through the machines in a prescribed order (which is not necessarily the same for both the jobs)
4. Problems with n jobs and m-machines.

Our syllabus focusses on following models

- Processing n jobs and 2 Machines
- Processing n jobs and 3 Machines
- Processing n jobs and m Machines

Processing n Jobs Through Two Machines

The problem can be described as: (i) only two machines A and B are involved, (ii) each job is pro- cessed in the order AB, and (iii) the exact or expected processing times A), Az, A3, ..., An; B), B2, B3,..., Bn are known

Processing Times	Job (i)				
	1	2	3	...	n
A_i	A_1	A_2	A_3	...	A_n
B_i	B_1	B_2	B_3	...	B_n

The problem is to sequence (order) the jobs so as to minimize the total elapsed time T.

Solution Procedure

Step 1. Select, the least processing time occurring in the list A I, Az, A3,..., Ar and Bt, B2, B3' ..., BII' If there is a tie, either of the smallest processing time should be selected.

Step 2. If the least processing time is Ar select r th job first. If it is Bs, do the s th job last (as the given order is AB).

Step 3. There are now n - I jobs left to be ordered. Again repeat steps I and n for the reduced set of processing times obtained by deleting processing times for both the machines corresponding to the job already assigned. .

Continue till all jobs have been ordered. The resulting ordering will minimize the elapsed time T.

Proof. Since passing is not allowed, all n jobs must be processed on machine A without any idle time for it. On the other hand, machine B is subject to its remaining idle time at various stages. Let Yj be the time for which m machine B remains idle after completing (i - l)th job and before starting processing the ith job (i=1,2, ..., n). Hence, the total

$$T = \sum_{i=1}^{n} B_i + \sum_{i=1}^{n} Y_i$$

elapsed time T is given by

JOHNSON’S ALGORITHM FOR N JOBS 2 MACHINES

The Johnson’s iterative procedure for determining the optimal sequence for an n-job 2-machine sequencing problem can be outlined as follows:

Step 1. Examine the A;’s and B;’s for i = 1,2, ..., n and find out min [Ai,Bi]

Step2.

i.If this minimum be Ak for some i =k, do (process) the kth job first of all.

ii.If this minimum be Br for some i =r, do (process) the rth job last of all.

Step 3.

i.If there is a tie for minima Ak =Bn process the kth job first of all and rth job in the last.

ii.If the tie for the minimum occurs among the A;’s, select the job corresponding to the minimum of B;’s and process it first of all.

iii.If the tie for minimum occurs among the B;’s, select the job corresponding to the minimum of Ai’s and process it in the last. Go to next step.

Step4. Cross-out the jobs already assigned and repeat steps 1 to 3 arranging the jobs next to first or next to last, until all the jobs have been assigned.

Example 1. There are five jobs each of which must go through the two machines A and B in the order AB. Processing times are given below:

Processing time (hours)					
Job	1	2	3	4	5
Time for *A*	5	1	9	3	10
Time for *B*	2	6	7	8	4

Determine a sequence for five jobs that will minimize the elapsed time T. Calculate the total idle time for the machines in this period.

Solution. Apply steps I and II of solution procedure. It is seen that the smallest processing time is one hour for job 2 on the machine A. So list the job 2 at first place as shown below.

2				

Now, the reduced list of processing times becomes

Job	*A*	*B*
1	5	$\boxed{2}$
3	9	7
4	3	8
5	10	4

Again, the smallest processing time in the reduced list is 2 for job 1 on the machine *B*. So place job 1 last.

2				1

Continuing in the like manner, the next reduced list is obtained

Job	*A*	*B*
3	9	7
4	$\boxed{3}$	8
5	10	4

leading to sequence	2	4			1

and the list

Job	A	B
3	9	$\boxed{7}$
5	10	4

gives rise to sequence	2	4		5	1

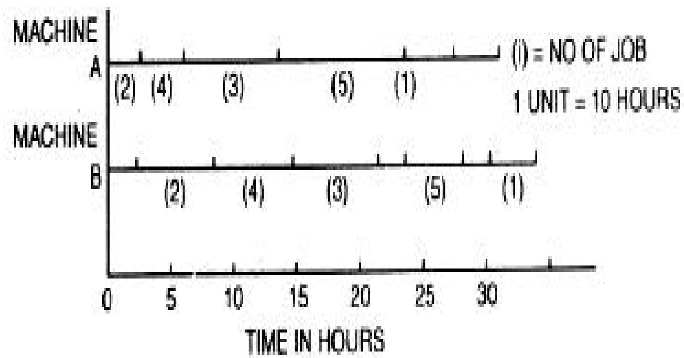
Finally, the optimal sequence is obtained,

2	4	3	5	1

Further, it is also possible to calculate the minimum elapsed time corresponding to the optimal sequencing, using the individual processing time given in the statement of the problem. The details are given in Table

Job sequence	Machine A		Machine B	
	Time in	Time out	Time in	Time out
2	0	1	1	7
4	1	4	7	15
3	4	13	15	22
5	13	23	23	27
1	23	28	28	$\boxed{30}$

Thus, the minimum time, i.e. the time for starting of job 2 to completion of the last job 1, is 30 hrs only. During this time, the machine A remains idle for 2 hrs (from 28 to 30 hrs) and the machine B remains idle for 3 hrs only (from 0-1,22-23, and 27-28 hrs). The total elapsed time can also be calculated by using Gantt chart as follows:



From the Fig it can be seen that the total elapsed time is 30 hrs, and the idle time of the machine B is 3 hrs. In this problem, it is observed that job may be held in inventory before going to the machine. For example, 4th job will be free on machine A after 4th hour and will start on machine B after 7th hr. Therefore, it will be kept in inventory for 3 hrs.

PROCESSING N JOBS THROUGH THREE MACHINES

The problem can be described as: (i) Only three machines A, B and C are involved, (ii) each job is processed in the prescribed order ABC, (iii) transfer of jobs is not permitted, i.e. adhere strictly the order over each machine, and (iv) exact or expected processing times are given in Table

Job	Machine A	Machine B	Machine C
1	A_1	B_1	C_1
2	A_2	B_2	C_2
3	A_3	B_3	C_3
:	:	:	:
n	A_n	B_n	C_n

However, the earlier method adopted by Johnson(1954) can be extended to cover the special cases where either one or both of the following condition should:

i. The minimum time on machine A the maximum time on machine B.
ii. The minimum time on machine C the maximum time on machine B.

The procedure explained here (without proof) is to

$$G_i = A_i + B_i, \quad H_i = B_i + C_i.$$

replace the problem with an equivalent problem, involving n jobs and two fictitious machines denoted by G and H, and corresponding time Gj and Hj are defined by

If this problem with prescribed ordering *GH* is solved, the resulting optimal sequence will also be optimal for the original problem.

Rules for deleting the programs which cannot be optimal.

Rule no.	Technological orderings for		Delete programs containing
	Job 1	Job 2	
I	X...Y	Y...	xy
II	X...Y...	...XY	$\bar{x}y$
III	...X...Y	...XY...	$\bar{x}y$
IV	...XY...	X...Y...	xy
V	...XY...Z...	...X...YZ...	$x\bar{y}z$
VI	...X...YZ...	...XY...Z	$\bar{x}y\bar{z}$

Example. There are five jobs, each of which must go through machines A, Band C in the order ABC.

Job i	Processing Times		
	A_i	B_i	C_i
1	8	5	4
2	10	6	9
3	6	2	8
4	7	3	6
5	11	4	5

Processing times are given in Table

determine a sequence for five jobs that will minimize the elapsed time T.

Solution. Here min Ai = 6, max Bi = 6, min Ci = 4.
Since one of two conditions is satisfied by min Ai =max Bi, so the procedure adopted in
Example 1 can be followed.

The equivalent problem, involving five jobs and two fictitious machine G and H, becomes:

Job i	Processing Times	
	$G_i (= A_i + B_i)$	$H_i (= B_i + C_i)$
1	13	9
2	16	15
3	8	10
4	10	9
5	15	9

This new problem can be solved by the procedure described earlier. Because of ties, possible optimal sequences are :

(*i*) | 3 | 2 | 1 | 4 | 5 |

(*ii*) | 3 | 2 | 4 | 1 | 5 |

(*iii*) | 3 | 2 | 4 | 5 | 1 |

(*iv*) | 3 | 2 | 5 | 4 | 1 |

(*v*) | 3 | 2 | 1 | 5 | 4 |

(*vi*) | 3 | 2 | 5 | 1 | 4 |

It is possible to calculate the minimum elapsed time for first sequence as shown in Table

Job	Machine A		Machine B		Machine C	
	Time in	Time out	Time in	Time out	Time in	Time out
3	0	6	6	8	8	16
2	6	16	16	22	22	31
1	16	24	24	29	31	35
4	24	31	31	34	35	41
5	31	42	42	46	46	51

Thus, any of the sequences from (i) to (vi) may be used to order the jobs through machines A, B and C. and they all will give a minimum elapsed time of 51 hrs. Idle time for machine A is 9 hrs, for B 31 hrs, for C 19hrs.

PROCESSING 2 JOBS THROUGH M MACHINES

Graphical Method
In the two job m-machine problem, there is a graphical procedure, which is rather simple to apply and usually provides good (though not necessarily optimal) results. The following example will make the graphical procedure clear.

Example 3. Use graphical method to minimize the time needed to process the following jobs on the machines shown below, i.e. for each machine find the job, which should be done first. Also calculate the total time needed to complete both thejobs.

Job 1	Sequence of Machines	:	A	B	C	D	E
	Time	:	2	3	4	6	2
Job 2	Sequence of Machines	:	C	A	D	E	B
	Time	:	4	5	3	2	6

Solution.

Step 1. First, draw a set of axes, where the horizontal axis represents processing time on job 1 and the vertical axis represents processing time on job 2

Step 2. Layout the machine time for two jobs on the corresponding axes in the given techno-logicalorderMachineAtakes2hrsforjob1and5hrsforjob2.ConstructtherectanglePQRS for the machine A. Similarly, other rectangles for machines B, C, D and E are constructed as shown.

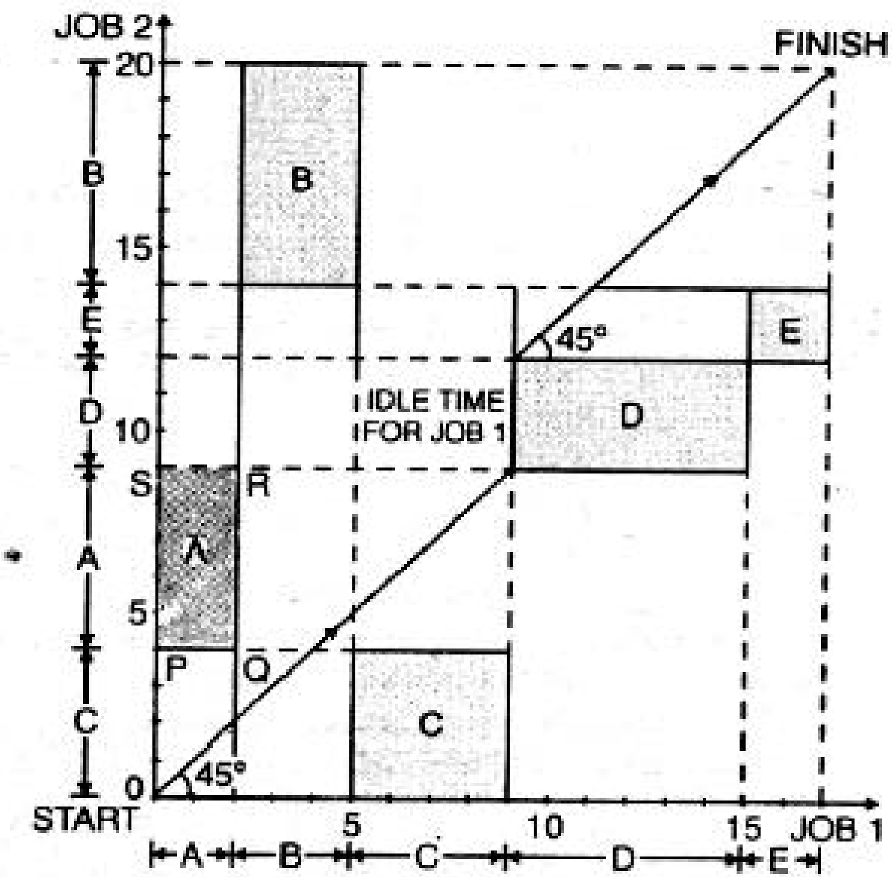
Step 3. Make programme by starting from the origin 0 and moving through various states of completion (points) until the point marked

Graphical solution for the 2-job 5-machine sequencing problem. 'finish' is obtained. Physical interpretation of the path thus chosen involves the series of segments, which are horizontal or vertical or diagonal making an angle of 45° with the horizontal. Moving to the right means that job 1is proceeding while job 2 is idle, and moving upward means that job 2 is proceeding while job 1 is idle, and moving diagonally means that both the jobs are proceeding simultaneously.

Further,boththejobscannotbeprocessedsimultaneouslyonthesamemachine.Graphically, diagonalmovementthroughtheblocked-out(shaded)areaisnotallowed,andsimilarlyfor other machinestoo.

Step4.To find an optimal path. An optimal path (programme) is one that minimizes idle time for job (horizontal movement). Similarly, an optimal path is one that minimizes idle time for job 2 (vertical movement). Choose such a path on which diagonal movement is as much as possible. According to this, choose a good path by inspection as shown by arrows.

Step5. To find the elapsed time. The elapsed time is obtained by adding the idle time for either of the job to the processing time for that job. In this problem, the idle time for the chosen path is seen to be 3 hrs. for the job I, and zero for the job 2. Thus, the total elapsed time, 17 + 3 = 20 hrs is obtained.



UNIT- IV THEORY OF GAMES AND INVENTORY

Game is defined as an activity between two or more persons involving activities by each person according to a set of rule at the end of which each person receives some benefit or satisfaction or suffers loss (negative benefit). The set of rules defines the game. Going through the set of rules once by the participants defines a play.

CHARACTERISTICSOF GAME THEORY

Games. They can be classified on the basis of the following characteristics.

i. Chance of strategy:
ii. Number of persons:
iii. Number of activities:
iv. These may be finite or infinite.

- Number of alternatives (choices) .. .
- Information to the players about the past activities of other .
- Payoff: A quantitative measure of satisfaction a person gets at the end of each play is called a payoff.

Competitive Game. A competitive situation is called acompetitive game if it has the following four properties

2. Zero-sum and Non-zero-sum Games. Competitive games are classified according to the number of players involved,
i.e. as a two person game.. three person game, etc. Another important distinction is between zero-sum games and nonzero-sum games. If the players make payments only to each other, i.e. the loss of one is the gain of others, and nothing comes from outside, the competitive game is said to be zero-sum.

Strategy. A strategy of a player has been loosely defined as a rule for decision-making in advance of all the plays by which he decides the activities he should adopt. In other words, a strategy for a given player is a set of rules (programmes) that specifies which of the available course of action he should make at each play.

i. Pure Strategy. : If a player knows exactly what the other player is going to do, a deterministic situation is obtained and objective function is to maximize the gain. Therefore, the pure strategy is a decision rule always to select a particular course of action. A pure strategy is usually represented by a number with which the course of action is associated.

ii. Mixed Strategy: If a player is guessing as to which activity is to be selected by the other on any particular occasion, a probabilistic situation is obtained and objective function is to maximize the expected gain. Thus, the mixed strategy is a selection among pure strategies with fixed probabilities.

TWO-PERSON, ZERO-SUM (RECTANGULAR) GAMES:

A game with only two players (say, Player A and Player B) is called a ‘two- person, zero-sum game’ if the losses of one player are equivalent to the gains of the other, so that the sum of their net gains is zero.

Two-person, zero-sum games are also called rectangular games as these are usually represented by a payoff matrix in rectangular form.

Pay-off Matrix. Suppose the player A has m activities and the player B has n activities. Then a payoff matrix can be formed by adopting the following rules:

- Row designations for each matrix are activities available to player A.
- Column designations for each matrix are activities available to player B.
- Cell entry ‘vij , is the payment to player A in A’s payoff matrix when A chooses the activity i and B chooses the activity.

- With a ‘zero-sum, two person game’, the cell entry in the player B’ s payoff matrix will be negative of the corresponding cell entry ‘Vi}, in the player A’s payoff matrix so that sum of payoff matrices for player A and player B is ultimately zero.

MINIMAX (MAXIMIN) CRITERION AND OPTIMAL STRATEGY

The ‘Minimax criterion of optimality’ states that if a player lists the worst possible outcomes of all his potential strategies, he will choose that strategy to be most suitable for him which corresponds to the best of these worst outcomes. Such a strategy is called an optimal strategy.

- Example 1. Consider (two-person, zero-sum) game matrix, which represents payoff to the player A. Find, the optimal strategy for the payoff matrix given below.
  -

B

	i \ j	1	2	3
	1	3	−4	8
A	2	−8	5	−6
	3	6	−7	6

SADDLE POINT

- A saddle point of a payoff matrix is the position of such an element in the payoff matrix, which is minimum in its row and maximum in its column.

RULES FOR DETERMINING A SADDLE POINT:

- Select the minimum element of each row of the payoff matrix and mark them by ‘0’.
- Select the greatest element of each column of the payoff matrix and mark them by ‘0 ‘.
- there appears an element in the payoff matrix marked by ‘0’ and ‘D’ both, the position of that element is a saddle point of the payoff matrix.

SOLUTION OF GAMES WITH SADDLE POINTS

To obtain a solution of a rectangular game, it is feasible to find out:

- the best strategy for player A
- the best strategy for player B
- the value of the game (V).

Player A can choose his strategies from {AI, A2, A3} only, while B can choose from the set (B1, B2) only. The rules of the game state that the payments should be made in accordance with the selection of strategies:

RECTANGULAR GAMES WITHOUT SADDLE POINT

RECTANGULAR GAMES WITHOUT SADDLE POINT

PRINCIPLE OF DOMINANCE:

- A given strategy can also be said to be dominated if it is inferior to some convex linear combination of two or more strategies

RULES OF DOMINANCE:

- Delete the minimum row: if all the elements of one row is less than or equal to the corresponding element in the other row
- Delete the maximum Column : if all the if all the elements of one column is greater than or equal to the corresponding element in the other column)
- A given strategy can be dominated if it is inferior to an average of two or more other pure strategies.

- If the ith row dominates the convex linear combination of some other rows, then one of the rows involving in the combination may be deleted. Similar arguments follow for columns also.
    - The optimal strategy for player A is reduced
    - The optimal strategy for player B also is reduced
    - Finally find the strategy of player-A and player-B

GRAPHICAL METHOD FOR (2 X N) AND (MX2) GAMES

The optimal strategies for a (2 x n) or (m x 2) matrix game can be located easily by a simple graphical method. This method enables us to reduce the 2 x n or m x 2 matrix game to 2 x 2 game that could be easily solved by the earlier methods.

STEPS INVOLVED:

- Step 1. Construct two vertical axes, axis I at the point x) =0 and axis 2 at the point x) :;: 1.
- Step 2. Represent the payoffs V2j'j = 1. 2, ..., n on axis I and payoff line vlj ,j =I, 2, ..., n on axis 2. Step 3. Join the point representing vij on Axis 2 to the point representing V2jon axis I. The resulting straight-line is the expected payoff line
- Step 4. Mark the lowest boundary of the lines E.i(x) so plotted, by thick line segments. The highest point
- on this lowest boundary gives the maximin point P and identifies the two critical. moves of player B.
- If there are more than two lines passing through the maximum point P. there are ties for the optimum mixed strategies for player B. Thus any two such lines with opposite sign slopes will define an alternative optimum .

UNIT –IV B INVENTORY

INVENTORY:

- In broad sense, inventory may be defined as the stock of goods, commodities or other economic resources that are stored or reserved in order to ensure smooth and efficient running of business affairs.

Why Inventory is maintained?

- Inventory helps in smooth and efficient running of business.
- Inventory provides service to the customers immediately or at a short notice.
- Due to absence of stock, the company may have to pay high prices because of piece-wise purchasing.
- Maintainingof inventory may earn price discount because of bulk- purchasing.
- Inventory also acts as a buffer stock when raw materials are received late and so many sale-orders are likely to be rejected.

- Inventory also reduces product costs because there is an additional advantage of batching and long smooth running production runs.
- Inventory helps in maintaining the economy by absorbing some of the

FORMS OF INVENTORY:

- The inventory or stock of goods may be kept in any of the following forms:
- Raw material inventory, i.e. raw materials which are kept in stock for using in the production of\ goods.
- Work-in-process inventory, i.e. semi finished goods or goods in process, which are stored during the production process.
- Finished goods inventory, i.e. finished goods awaiting shipment from the factory.
- Inventory also include: furniture, machinery, fixtures, etc. The term inventory may be classified in two main categories

I. DIRECT INVENTORY:

The items which playa direct role in the manufacture and become an integral part of finished goods are included in the category of direct inventories

CLASSIFICATION OF INVENTORIES:

A).Raw material inventories are provided:

- for economical bulk purchasing,
- to enable production rate changes
- to provide production buffer against delays in transportation,
- for seasonal fluctuations.

B) Work-in-process inventories are provided:

- to enable economical lot production,
- to cater to the variety of products
- for replacement of wastages,
- to maintain uniform production even if amount of sales may vary.

C) Finished-goods inventories are provided:

- for maintaining off-self delivery,
- to allow stabilization of the production level
- for sales promotion.

D)Spare parts.

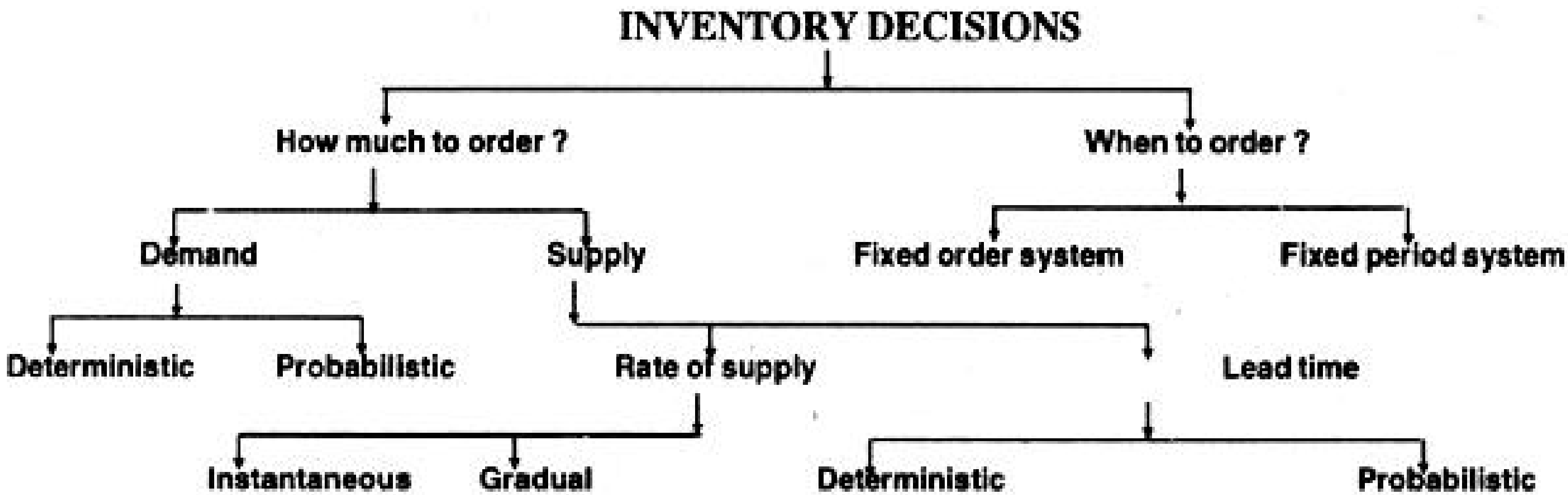
II. INDIRECT INVENTORIES

Indirect inventories include those items, which are necessarily required for manufacturing but do not become the component of finished production, like: oil, grease, lubricants, petrol, and office-material maintenance material, etc.

INVENTORY DECISIONS:

The managers must take two basic decisions in order to accomplish the functions of inventory. The decisions made for every item in the inventory are:

- How much amount of an item should be ordered when the inventory of that item is to be replenished?
- When to replenish the inventory of that item



Before taking inventory decisions, it is necessary to develop an inventory model.

INVENTORY COSTS:

I. Holding Cost (C or Ch):

The cost associated with carrying or holding the goods in stock is known as holding or carrying cost which is usually denoted by C. or Ch per unit of goods for a unit of time components of holding cost:,

- Invested Capital Cost. This is the interest charge over the capital investment.
- Record Keeping and Administrative Cost.
- Handling Costs. These include all costs associated with movement of stock such as: cost of labour over-head cranes, gantries and other machinery required for this purpose.
- Storage Costs. These involve the rent of storage space or depreciation and interest even if the own space is used.
- Depreciation, Deterioration and Obsolescence Costs.

- Taxes and Insurance Costs. All these costs require careful study and generally amounts to I% to 2% of the invested capital.
- Purchase Price or Production Costs. Purchase price per- unit item is affected by the quantity

II. Shortage Costs or Stock-out Costs (C2 or C,).

The penalty costs that are incurred as a result of running out of stock (i.e., shortage) are known as shortage or stock-out costs. These are denoted by C2 or Cs per unit of goods (or a specified period.

These costs arise due to shortage of goods, sales may be lost, and good will may be lost either by a delay in meeting the demand or being quite unable to meet the demand at all.

III. SET-UP COSTS (C3 or Co):

these include the fixed cost associated with obtaining goods through placing of an order or purchasing or manufacturing or setting up a machinery before starting production. So they include costs of purchase, requisition, follow-up, receiving the goods, quality control, etc. These are also called order costs or replenishment costs, usually denoted by C3 or Co per production run (cycle). They are assumed to be independent of the quantity ordered or produced.

- Purchased due to quantity discounts or price-breaks. Production cost per unit item depends upon the length of production runs. For long smooth production runs this cost in lower due to more efficiency of men and machines. So the order quantity must be suitably modified to take the advantage of these price discounts. If P is the purchase price of an item and I is the stock holding cost per unit item expressed as a fraction of stock value (in rupees), then the holding cost C. = IP.

- Salvage Costs or Selling Price. When the demand for an item is affected by its quantity in stock, the decision model of the problem depends upon the profit maximization criterion and includes the revenue (sales tax etc.) from the sale of the item.

ECONOMIC ORDERING QUANTITY (EOQ)

- Economic Ordering Quantity (EOQ) is that size of order which minimizes total annual (or other time period as determined by individual firms) cost of carrying inventory and cost of ordering under the assumed conditions of certainty and that annual demands are known.

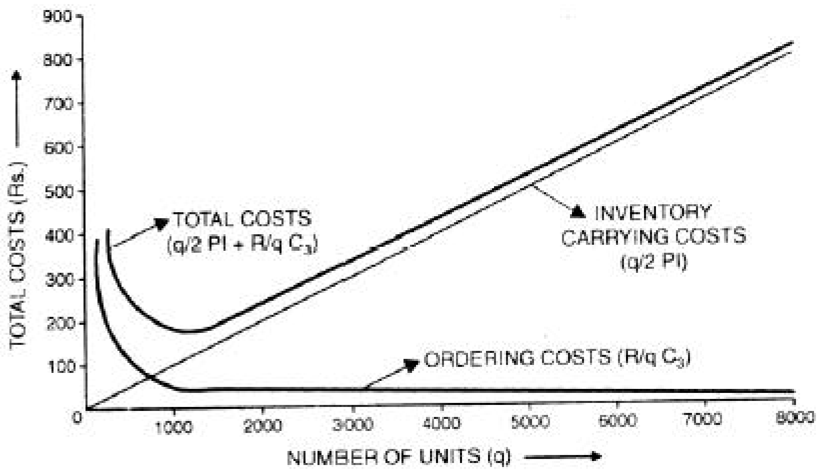
CONDITION TO APPLY EOQ MODEL:

a. The item is replenished in lots or batches, either by purchasing or by manufacturing.

b. Consumption of items (or sales or usage rate) is uniform and continuous.

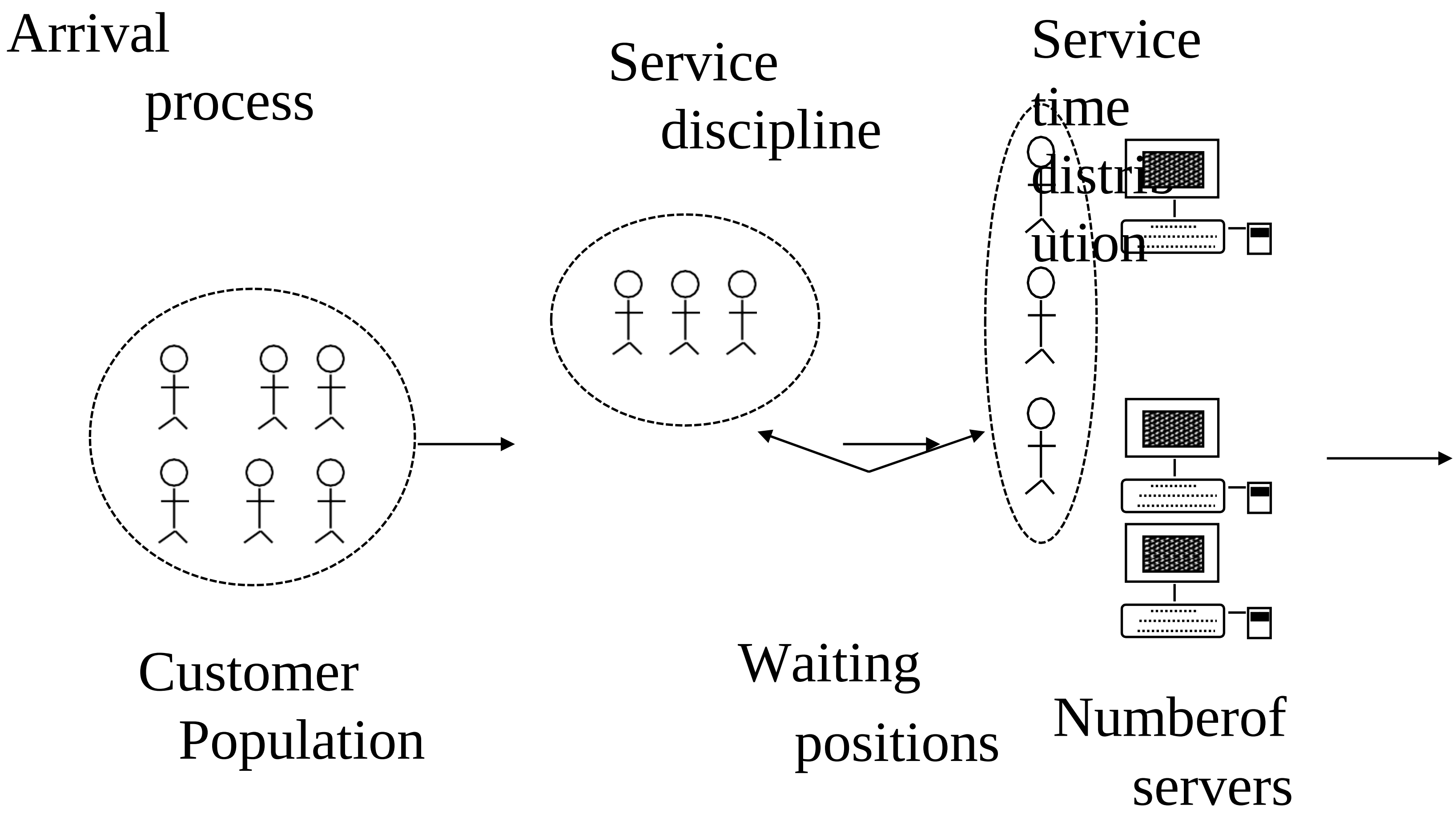
EOQ MODEL IS DESCRIBED UNDER THE FOLLOWING SITUATIONS:

- Planning period is one year.
- Demand is deterministic and indicated by parameter D units per year.
- Cost of purchases, or of one unit is C.
- Cost of ordering (or procurement cost of replenishment cost) is C3or Co ‘For manufacturing goods, it is known as set-up cost.
- Cost of holding stock (also known as inventory carrying cost) is C1or Ch per unit per year expressed either in items of cost per unit per period or in terms of percentage charge of the purchase price.
- Shortage cost (mostly it is back order cost) is C2or Cs per unit per year.
- Lead time is L, expressed in unit of time.
- Cycle period in replenishment is t.
- Order size is Q.



TOTAL COSTS (Rs.)
900
800
700
600
500
400
300
200
100
0
1000
2000
3000
4000
5000
6000
7000
8000
NUMBER OF UNITS (q)
TOTAL COSTS
(q/2 PI + R/q C3)
INVENTORY
CARRYING COSTS
(q/2 PI)
ORDERING COSTS (R/q C3)

UNIT-V WAITING LINES AND SIMULATION


Arrival process
Service discipline
Service time distribution
Customer Population
Waiting positions
Numberof servers

, Example-1: waiting of customer at cinema ticket counter .

- The arriving people are called the customers
- The person issuing the tickets is called a server.

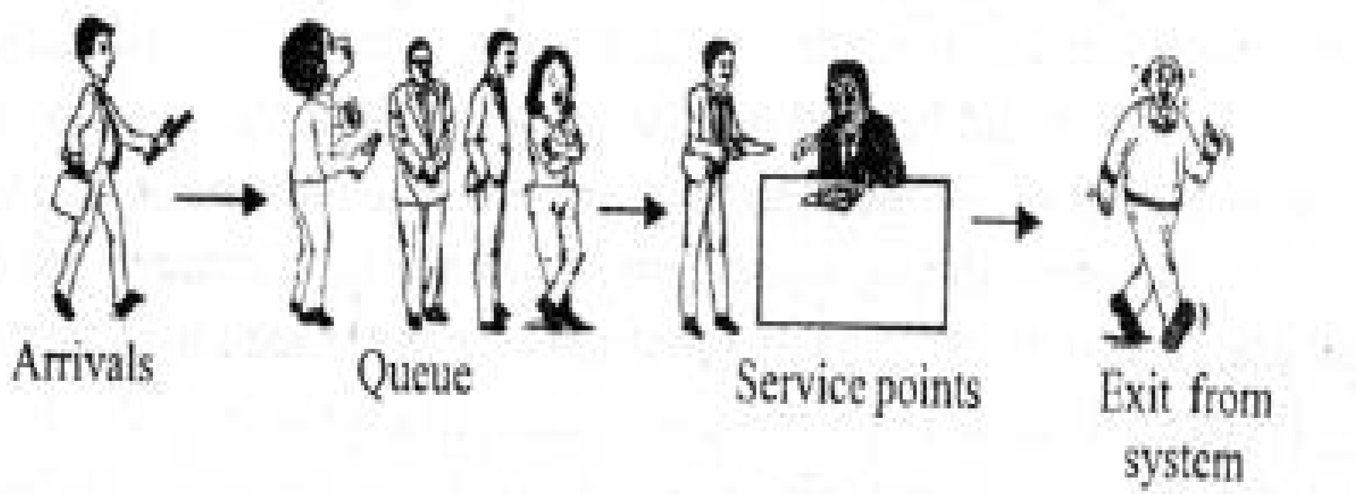
Example-2 : In a office letters arriving at a typist’s desk.

- the letters represent the customers
- the typist represents the server.

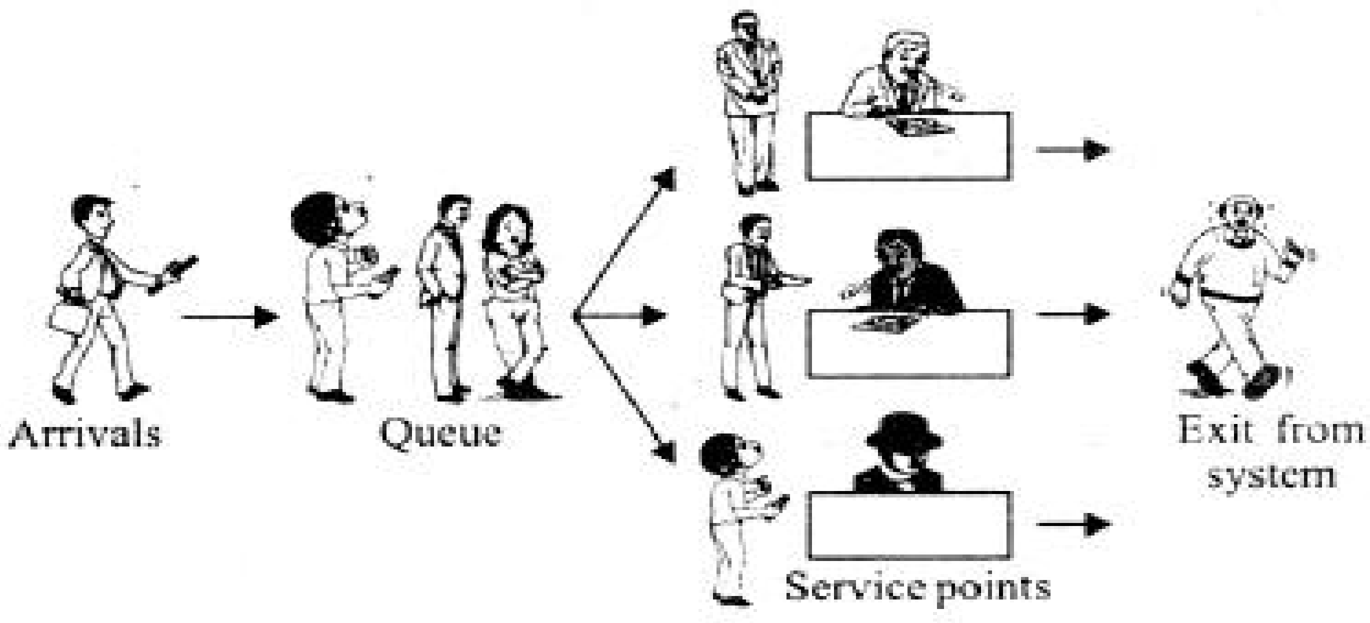
Example-3: machine breakdown situation.

- A broken machine represents a customer calling for the service of a repairman
- Service mechanic/Engineer is the server

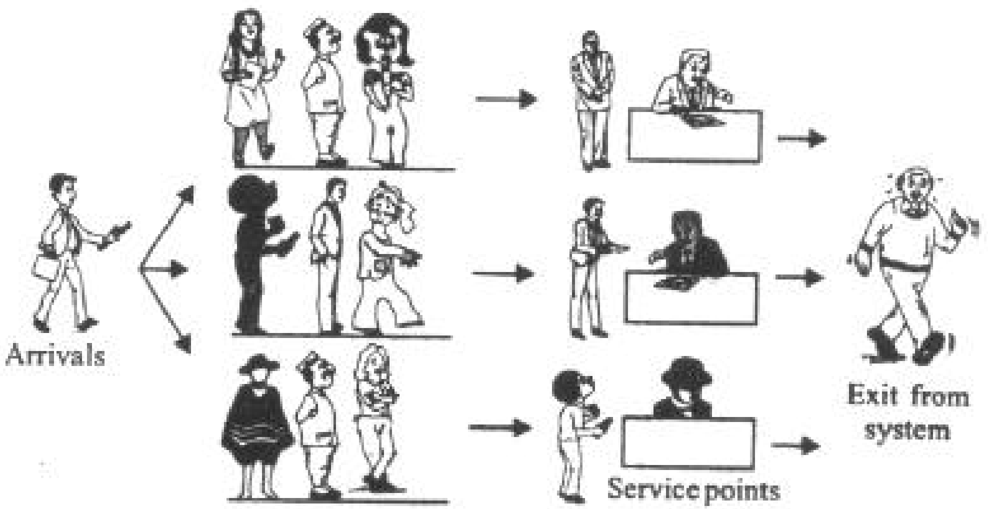
1. QUEUING SYSTEM WITH SINGLE QUEUE AND SINGLE SERVICE STATION.



2. QUEUING SYSTEM WITH SINGLE QUEUE AND SEVERAL SERVICE STATIONS.



3. QUEUING SYSTEM WITH SEVERAL QUEUES AND SEVERAL QUEUES



QUEUING THEORY:

Queuing theory is concerned with the statistical description of the behavior of queues with finding, *e.g.*, the probability distribution of the number in the queue from which the mean and variance of queue length and the probability distribution of waiting time for a customer, or the distribution of a server’s busy periods can be found.

Meaning of a Queuing Model

A Queuing Model is a suitable model to represent a service- oriented problem where customers arrive randomly to receive some service, the service time being also a random var

Objective of a Queuing Model

The objective of a queuing model is to find out the optimum service rate and the number of servers so that the average cost of being in queuing system and the cost of service are minimized

PARAMETERS INVOLVED IN QUEUING MODEL

- Customer
- Server
- Queue Discipline
- Time Spent in the Queuing System

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KENDALL'S NOTATION

Kendall's Notation is a system of notation according to which the various characteristics of a queuing model are identified. Kendall (Kendall, 1951) has introduced a set of notations, which have become standard in the literature of queuing models.

A general queuing system is denoted by (a/b/c) :(d/e) where

a = probability distribution of the inter arrival time. b = probability distribution of the service time.

c = number of servers in the system.

d = maximum number of customers allowed in the system.

e = queue discipline

. Traditionally, the exponential distribu- tion in queuing problems is denoted by M.

Model-1: Thus (MIMI!): (∞/ FIFO) indicates a queuing system when the inter arrival times and service times are exponentially distributed having one server in the system with first in first out discipline and the number of customers allowed in the system can be infinite

State of Queuing System

The transient state of a queuing system is the state where the probability of the number of customers in the system depends upon time. The steady state of a queuing system is the state where the probability of the number of customers in the system is independent of*t*.

Let Pn (t) indicate the probability of having n customers in the system at time *t*. Then if Pn(t) depends upon *t*, the queuing system is said to be in the transient state. After the queuing system has become operative *for* a considerable period of time, the probability Pn(t) may become independent of t. The

Poisson Process

When the number of arrivals in a time interval of length *t* follows a Poisson distribution with parameter *(At),* which is the product of the arrival rate (A-)and the length of the interval *t,* the arrivals are said to follow a poison process

M/M/1 Queueing Model

The M/M/1 queuing model is a queuing model where the arrivals follow a Poisson process, service times are exponentially distributed and there is one server.

ASSUMPTION OF M/M/1 QUEUING MODEL:

1. The number of customers arriving in a time interval *t* follows a Poisson Process with parameter A.
2. The interval between any two successive arrivals is exponentially distributed with parameterA.
3. Thetimetakentocompleteasingleserviceisexponentially distributed with parameter
4. The number of server isone.
5. Althoughnotexplicitlystatedboththepopulationandthe queue size can beinfinity.
6. The order of service is assumed to beFIFO.
7.

If $\frac{\lambda}{\mu} < 1$, the steady state probabilities exist and P_n the number of customers in the system follows a geometric distribution with parameter $\frac{\lambda}{\mu}$ **(also known as traffic intensity).**

The probabilities are :

$$P_n = P\,(\text{No. of customers in the sytem} = n)$$

$$= \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right); n = 1, 2, \ldots\ldots$$

$$P_0 = 1 - \frac{\lambda}{\mu}$$

$$L_s = E(n) = \sum_{n=1}^{\infty} nP_n = \sum_{n=1}^{\infty} n\left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n$$

$$= \frac{\lambda}{\mu - \lambda} = \frac{\rho}{1 - \rho}$$

The expected number of customers in the queue is given by –

$$L_q = \sum_{n=1}^{\infty} (n-1)P_n = \sum_{n=1}^{\infty} nP_n = \sum_{n=1}^{\infty} P_n$$

$$= \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{\rho^2}{1 - \rho}$$

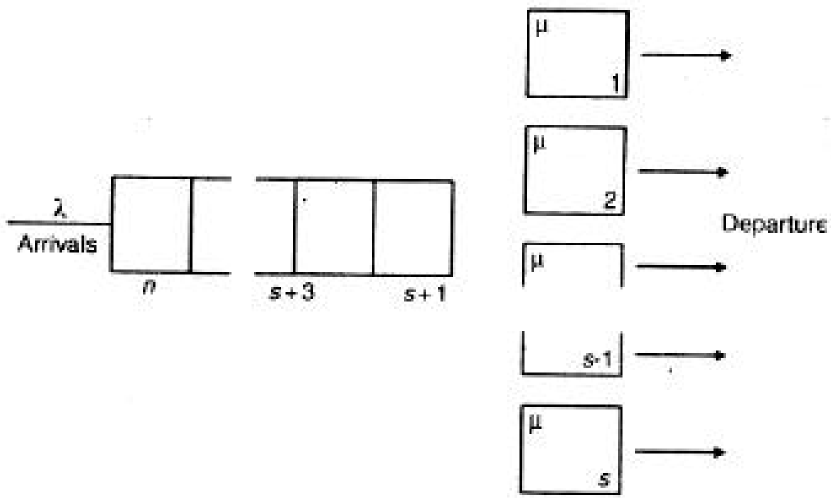
Average waiting time of a customer in the system $W_s = \dfrac{1}{\mu - \lambda}$.

Average waiting time of a customer in the queue $W_q = \dfrac{\lambda}{\mu(\mu - \lambda)}$.

PRACTICAL FORMULAE INVOLVED IN QUEUEING THEORY

1.	Arrival Rate per hour	$= \lambda$
2.	Service Rate per hour	$= \mu$
3.	Average Utilisation Rate (*or* Utilisation Factor), ρ	$= \dfrac{\lambda}{\mu}$
4.	Average Waiting Time in the System, (waiting and servicing Time) W_s	$= \dfrac{1}{(\mu - \lambda)}$
5.	Average Waiting Time in the Queue, W_q	$= \dfrac{\lambda}{\mu(\mu - \lambda)}$
6.	Average Number of Customers (including the one who is being served) in the System, L_s	$= \dfrac{\lambda}{(\mu - \lambda)}$

MODEL IV (A) : $(M \mid M \mid S) : (\infty \mid FCFS)$



SIMULATION

SIMULATION: simulation is the representative model for real situations.

Example: the testing of an air craft model in a wind tunnel from which the performance of the real aircraft is determined for being under fit under real operating conditions.

In the laboratories, we often perform a number of experiments on simulated models to predict the behavior of the real system under true environments.

Definition: Simulation is a representation of reality through the use of a model or other device which will react in the same manner as reality under a given set of conditions

Simulation is the use of system model that has designed the characteristics of reality in order to produce the essence of actual operation

According to Donald G. Malcolm, a simulated model may be defined as one which depicts the working of a large scale system of men, machines, materials and information operating over period of time in a simulated environment of the actual real world conditions.

TYPES OF SIMULATION

- Analog simulation
- Computer simulation (System Simulation)
  - *i.* Deterministic models
  - *ii.* Stochastic model
  - *iii.* Static models
  - *iv.* Dynamic models

NEED FOR SIMULATION

- Simulation techniques allow experimentation with a model of real-life system rather than the actual operating system.
- Sometimes there is no sufficient time to allow the actual system to operate extensively.
- The non-technical manager can comprehend simulation more easily than a complex mathematical model.

v. The use of simulation enables a manager to provide insights into certain managerial problems.

SIMULATION PROCESS

Step-1: First define and identify the problem clearly.

Step-2.Secondly, list the decision variable and decision rules of the problem.

Step-3. Formulate the suitable model for the given problem.

Step-4. Test the model and compare its behavior with the behavior of real problem situation.

Step-5. Collect and identify the data required to test the model.

Step-6. Execute (run) the simulation model.

Step-7’ the results of simulation run are then analyzed. If the simulation run is complete, then choose the best course of action, other wise , required changes are done in model decision variables, design or parameters and go to step-4.

Step-8. Run the simulation again to find the new solution.
Step-9. Validate the simulation.

LIMITATION OF SIMULATION

- Optimum results cannot
- The another difficulty lies in the quantification of the variables.
- In very large and complex problems, it becomes difficult to make the computer program on account of large number of variables and the involved inter-relationships among them.
- Simulation is comparatively costlier, time consuming method in many situations.

TYPES OF SIMULATION MODELS

- Deterministic models

  In this type of models, the input and output variables cannot be random variables and can be described by exact functional relationships.

- Probabilistic models

  In these models, method of random sampling is used. This technique is called ‘Monte-Carlo Technique.

- Static Models:

  In these types of models, the variable time cannot be taken into account consideration.

- Dynamic Models:

  These models deal with time varying interaction.

PHASES OF SIMULATION MODEL

Phase-1: Data collection

Data generation involves the sample obserervation of variables and can be carried with the help of following methods.

- Using random number tables.
- Resorting to mechanical devices.)example: roulettes wheel)
- Using electronic computers

Phase -2. Book-keeping

Book-keeping phase of a simulation model deals with updating the system when new events occur, monitoring and recording the system states as and when they change, and keeping track of quantities of our interest (such as idle time and waiting time) to compute the measure of effectiveness.

GENERATION OF RANDOM NUMBERS

For clear understanding, the following perameters are be defined

- Random variable:
  it refers to a particular outcome of an experiment.
- Random Number:
  it refers to a uniform random variable or numerical value assigned to a random variable following uniform probability density function. (ie., normal, poisson, exponential, etc).
- Pseudo-random Numbers:
- Random numbers are called pseudo-random numbers when they are generated by aome

deterministic process but they qualify the pre-determined statistical test for randomness.

MONTE-CARLO SIMULATION- STEPS INVOLVED

1) First define the problem by
   - Identifying the objectives of the problem
   - Identifying the main factors having the greatest effect on the objective of the problem
2) Construct an appropriate model by
   - Specifying the variables and parameters of the model
   - Formulating the suitable decision rules.
   - Identifying the distribution that will be used.
   - Specifying the number in which time will change,
   - Defining the relationship between the

variables and parameters.

3) Prepare the model for experimentation
    - Defining the starting conditions for the simulation
    - Specifying the number of runs of simulation.
4) Using step1 to step 3, test the model by
    - Defining a coding system that will correlate the factors defined in step1 with random numbers to be generated for simulation.
    - Selecting a random generator and creating the random numbers to be used in the simulation.
    - Associating the generated random numbers with the factors as identified in step1 and coded in step4.

5) Summarize and examine the results as obtained in.
6) Evaluate the results of the simulation
7) Formulate proposals for advice to management on the course of action to be adopted and modify the model, if required.

APPLICATION OF SIMULATION

- Simulation model can be applied for Solving Inventory problems
- Simulation model can be applied for solving Queuing problems

THANK YOU