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Question Paper Code: AECB08



# INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad - 500 043

## MODEL QUESTION PAPER – I

Four Year B.Tech III Semester End Examinations(Supplementary), November - 2019

**Regulations: IARE-R18**

### PROBABILITY THEORY AND STOCHASTIC PROCESS

(ECE)

**Time: 3 hours**

**Max. Marks: 70**

Answer ONE Question from each Unit

All Questions Carry Equal Marks

All parts of the question must be answered in one place only

#### UNIT – I

- 1 a) Define following types of events. i) Simple events ii) Conditional events iii) Independent events iv) Joint events with examples. [7M]
- b) Five men in a company of 20 are graduates. Three men are picked out of 20 at random. What is the probability that all are graduates? [7M]  
What is the probability of at least 1 is a graduate?
- 2 a) A random variable X has the following probability function shown in Table 1: [7M]

Table 1

X	0	1	2	3	4
P(x)	1/25	3/25	1/5	7/25	9/25

Find (i) The distribution function of X. (ii)  $P(X < 3)$  and  $P(0 < X < 4)$

- b) If X and Y are independent Poisson random variable show that the conditional distribution of X given X+Y is a binomial distribution. [7M]

#### UNIT – II

- 3 a) Explain monotonic and non monotonic transformations of a single random variable and derive density expression for transformed random variable. [7M]
- b) The life time of a certain brand of an electric bulb may be considered as a random variable with mean 1200 and standard deviation 250. Find the probability using Central Limit Theorem that the average lifetime of 60 bulbs exceeds 1250 hours. [7M]
- 4 a) Derive the expression for distribution and density function of a sum of two random variables. [7M]
- b) If a pdf of a random variable X is given by  $f_X(x) = be^{-ax}$ , where a and b are real constants, find the moment generating function, mean and variance. [7M]

### UNIT – III

- 5 a) Explain the joint moments of random variables and write the expression for  $n^{\text{th}}$  moment about the origin. [7M]
- b) Statistically independent random variables X and Y have moments  $m_{10}=2$ ,  $m_{20}=14$  and  $m_{11} = -6$ . Find second central moment  $\mu_{22}$ . [7M]
- 6 a) Explain characteristic function and moment generating function and state its properties? [7M]
- b) The characteristic function for a Gaussian random variable X, having a mean value of 0, is  $\Phi_X(\omega) = \exp(-\omega^2 / \sigma^2)$  [7M]  
Find all the moments of X using  $\Phi_X(\omega)$ .

### UNIT – IV

- 7 a) When does the time average converge to the ensemble average? Justify the answer. Briefly explain about Gaussian random process. [7M]
- b) A random process is defined as  $X(t) = A \cos(\omega t + \theta)$  where  $\theta$  is a uniform random variable over  $(0, 2\pi)$ . Verify the process is ergodic in the mean sense and auto correlation sense [7M]
- 8 a) Define strict sense stationary random process, auto correlation and cross correlation function of a random process. [7M]
- b) Consider two random processes  $X(t) = A \cos \omega t + B \sin \omega t$  and  $Y(t) = B \cos \omega t - A \sin \omega t$  where A and B are uncorrelated, zero mean random variables with same variance and ' $\omega$ ' is a constant. Show that X(t) and Y(t) are jointly stationary [7M]

### UNIT – V

- 9 a) State and prove Winner-Khinchine theorem. [7M]
- b) The auto correlation of a stationary random process is given by  $R_{XX}(\tau) = ae^{-a|\tau|}$ ,  $a > 0$ . Find the spectral density function. [7M]
- 10 a) Explain power spectrums for discrete-time random processes and sequences and state any two properties of cross-power density spectrum. [7M]
- b) A WSS random process X(t) with autocorrelation function  $R_{XX}(\tau) = e^{-a|\tau|}$  where a is a real positive constant, is applied to the input of an LTI system with impulse response  $h(t) = e^{-bt}u(t)$ , where b is a real positive constant. Find the autocorrelation function of the output Y(t) of the system. [7M]



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## COURSE OBJECTIVES:

The course should enable the students to:

S.No	Description
I	Understand the random experiments, sample space and event probabilities.
II	Study the random variables, density and distribution functions, moments and transformation of random variables.
III	Understand the concept of random process and sample functions (signals)
IV	Explore the temporal and spectral characteristics of random processes

## COURSE OUTCOMES (COs):

CO Code	Description
CO 1	Appreciate the concept of the random experiments, event probabilities, random variables and their description, functions of random variables.
CO 2	Learn and understand the single random variable transformation- multiple random variables.
CO 3	Understand the operations multiple random variables and their expectations.
CO 4	Understand the concept of random processes and their time domain description.
CO 5	Explore the spectral characteristics of random processes, and filtered random processes.

## COURSE LEARNING OUTCOMES:

Students who complete the course will have demonstrated the ability to do the following

CLO Code	Description
AECB08.01	Describe the basic concepts of the random experiments, event probabilities, joint and conditional probabilities- Bayes theorem.
AECB08.02	Learn and understand the concept of random variables, continuous and discrete variables, the probability density functions (pdfs), Probability Distribution Functions (PDFs), different random variables and their properties.
AECB08.03	Learn and understand the functions of a random variable, standard and central moments, and their physical significance.
AECB08.04	Understand the Characteristic and Moment Generating Functions; Understand and apply the transformations on continuous and discrete random variables – Expectations.
AECB08.05	Learn and understanding of Vector random variables, joint, Marginal and Conditional distribution functions, joint, Marginal and Conditional density functions.
AECB08.06	Learn and understand the Conditional distribution and density functions: point and interval conditioning.

AECB08.07	State and Explain the Central limit theorem ,Sum of several random variables.
AECB08.08	Learn and understanding of functions of vector random variables, Joint standard and central moments, joint characteristic functions.
AECB08.09	Learn and understanding of Jointly Gaussian random variables; and Transformations of multiple random variables.
AECB08.10	Learn and understanding of Random Process, sample functions and time domain characteristics: Stationary, Independence and Ergodicity.
AECB08.11	Contrasting of Correlation and Covariance functions, Gaussian and Poisson Random Processes.
AECB08.12	Distinguish between Auto- and Cross- power density spectra, properties, relationship between Correlation functions and Power density spectra.
AECB08.13	Understand and Discuss the linear time invariant (LTI) systems driven by random process, Input-output Spectral relations, White and Colored noises.

#### MAPPING OF SEMESTER END EXAMINATION TO COURSE LEARNING OUTCOMES:

SEE Question No.	CLO Code	Course learning Outcomes	CO code	Blooms Taxonomy Level
1	a	AECB08.01 Describe the basic concepts of the random experiments, event probabilities, joint and conditional probabilities- Bayes theorem	CO 1	Remember
	b	AECB08.01 Describe the basic concepts of the random experiments, event probabilities, joint and conditional probabilities- Bayes theorem	CO 1	Understand
2	a	AECB08.02 Learn and understand the concept of random variables, continuous and discrete variables, the probability density functions (pdfs), Probability Distribution Functions (PDFs), different random variables and their properties	CO 1	Understand
	b	AECB08.02 Learn and understand the concept of random variables, continuous and discrete variables, the probability density functions (pdfs), Probability Distribution Functions (PDFs), different random variables and their properties	CO 1	Understand
3	a	AECB08.04 Understand the Characteristic and Moment Generating Functions; Understand and apply the transformations on continuous and discrete random variables – Expectations	CO 2	Understand
	b	AECB08.07 State and Explain the Central limit theorem ,Sum of several random variables	CO 2	Understand
4	a	AECB08.07 State and Explain the Central limit theorem ,Sum of several random variables	CO 2	Understand
	b	AECB08.04 Understand the Characteristic and Moment Generating Functions; Understand and apply the transformations on continuous and discrete random variables – Expectations	CO 2	Understand
5	a	AECB08.08 Learn and understanding of functions of vector random variables, Joint standard and central moments, joint characteristic functions	CO 3	Understand
	b	AECB08.08 Learn and understanding of functions of vector random variables, Joint standard and central moments, joint characteristic functions	CO 3	Understand

SEE Question No.		CLO Code	Course learning Outcomes	CO code	Blooms Taxonomy Level
6	a	AECB08.08	Learn and understanding of functions of vector random variables, Joint standard and central moments, joint characteristic functions	CO 3	Remember
	b	AECB08.08	Learn and understanding of functions of vector random variables, Joint standard and central moments, joint characteristic functions	CO 3	Understand
7	a	AECB08.10	Learn and understanding of Random Process, sample functions and time domain characteristics: Stationary, Independence and Ergodicity	CO 4	Understand
	b	AECB08.10	Learn and understanding of Random Process, sample functions and time domain characteristics: Stationary, Independence and Ergodicity	CO 4	Understand
8	a	AECB08.11	Contrasting of Correlation and Covariance functions, Gaussian and Poisson Random Processes	CO 4	Understand
	b	AECB08.11	Contrasting of Correlation and Covariance functions, Gaussian and Poisson Random Processes	CO 4	Understand
9	a	AECB08.12	Distinguish between Auto- and Cross- power density spectra, properties, relationship between Correlation functions and Power density spectra	CO 5	Understand
	b	AECB08.12	Distinguish between Auto- and Cross- power density spectra, properties, relationship between Correlation functions and Power density spectra	CO 5	Understand
10	a	AECB08.12	Distinguish between Auto- and Cross- power density spectra, properties, relationship between Correlation functions and Power density spectra	CO 5	Understand
	b	AECB08.13	Understand and Discuss the linear time invariant (LTI) systems driven by random process, Input- output Spectral relations, White and Colored noises	CO 5	Understand

**Signature of Course Coordinator**

**HOD, ECE**