INSTITUTE OF AERONAUTICAL ENGINEERING

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Regulation: R15

STRUCTURAL ANALYSIS

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MOMENT DISTRIBUTION METHOD

UNIT 1
MOMENT DISTRIBUTION METHOD - AN OVERVIEW

- MOMENT DISTRIBUTION METHOD - AN OVERVIEW
- INTRODUCTION
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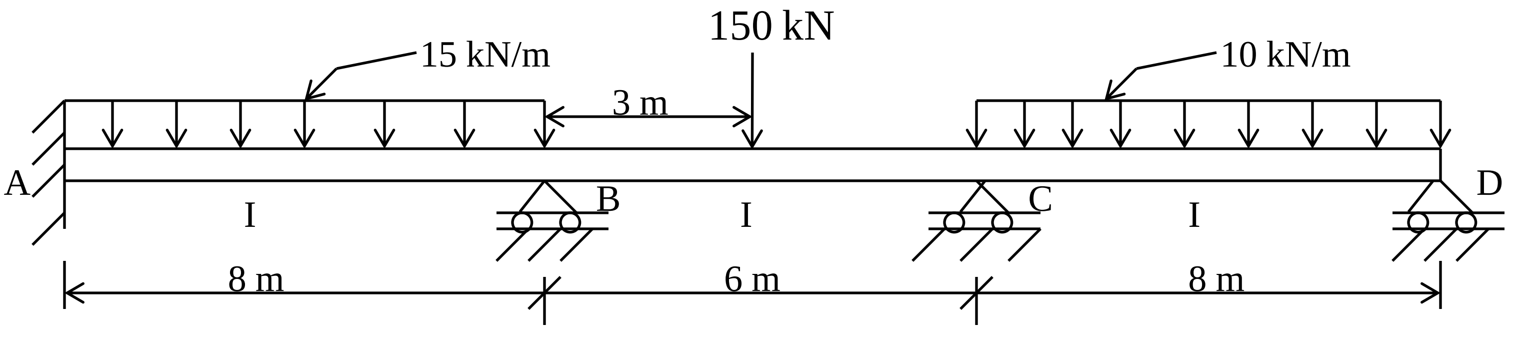
MOMENT DISTRIBUTION METHOD - INTRODUCTION AND BASIC PRINCIPLES

Introduction

(Method developed by Prof. Hardy Cross in 1932)
The method solves for the joint moments in continuous beams and rigid frames by successive approximation.

Statement of Basic Principles

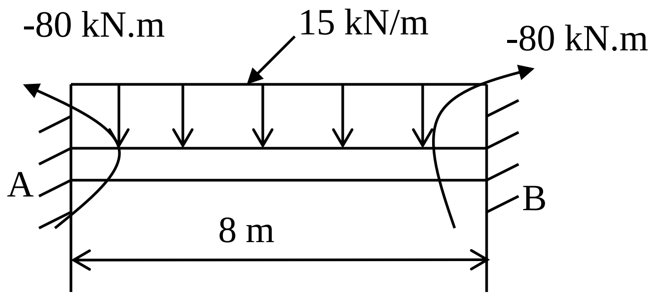
Consider the continuous beam ABCD, subjected to the given loads, as shown in Figure below. Assume that only rotation of joints occur at B, C and D, and that no support displacements occur at B, C and D. Due to the applied loads in spans AB, BC and CD, rotations occur at B, C and D.

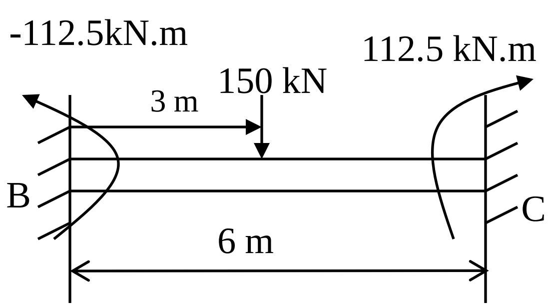


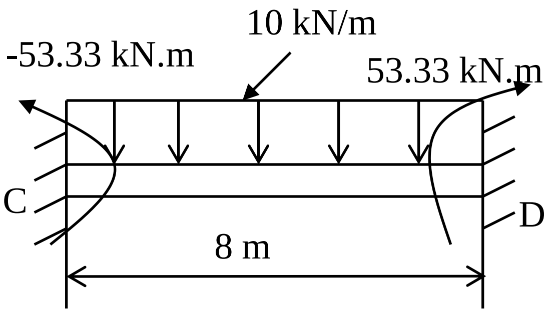
In order to solve the problem in a successively approximating manner, it can be visualized to be made up of a continued two-stage problems viz., that of locking and releasing the joints in a continuous sequence.

Step I

The joints B, C and D are locked in position before any load is applied on the beam ABCD; then given loads are applied on the beam. Since the joints of beam ABCD are locked in position, beams AB, BC and CD acts as individual and separate fixed beams, subjected to the applied loads; these loads develop fixed end moments.







<u>In beam AB</u>

Fixed end moment at A = $-wl^2/12$ = $-(15)(8)(8)/12$ = - 80 kN.m

Fixed end moment at B = $+wl^2/12$ = $+(15)(8)(8)/12$ = + 80 kN.m

<u>In beam BC</u>

Fixed end moment at B = $-(Pab^2)/l^2 = -(150)(3)(3)^2/6^2$

= -112.5 kN.m

Fixed end moment at C = $+(Pab^2)/l^2 = +(150)(3)(3)^2/6^2$

= + 112.5 kN.m

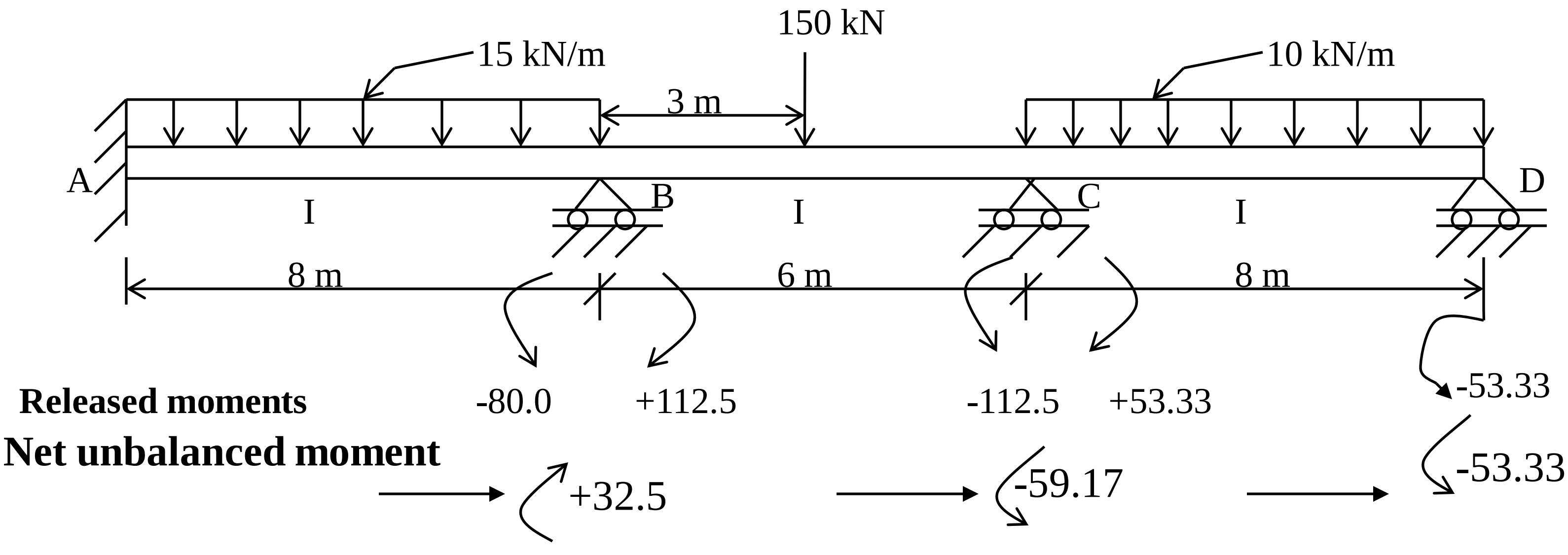
<u>In beam AB</u>

Fixed end moment at C = $-wl^2/12$ = $-(10)(8)(8)/12$ = - 53.33 kN.m

Fixed end moment at D = $+wl^2/12$ = $+(10)(8)(8)/12$ = + 53.33kN.m

Step II

Since the joints B, C and D were fixed artificially (to compute the the fixed-end moments), now the joints B, C and D are released and allowed to rotate. Due to the joint release, the joints rotate maintaining the continuous nature of the beam. Due to the joint release, the fixed end moments on either side of joints B, C and D act in the opposite direction now, and cause a net unbalanced moment to occur at the joint.



Step III

These unbalanced moments act at the joints and modify the joint moments at B, C and D, according to their relative stiffnesses at the respective joints. The joint moments are distributed to either side of the joint B, C or D, according to their relative stiffnesses. These distributed moments also modify the moments at the opposite side of the beam span, viz., at joint A in span AB, at joints B and C in span BC and at joints C and D in span CD. This modification is dependent on the carry-over factor (which is equal to 0.5 in this case); **when this carry over is made, the joints on opposite side are assumed to be fixed.**

Step IV

The carry-over moment becomes the unbalanced moment at the joints to which they are carried over. Steps 3 and 4 are repeated till the carry-over or distributed moment becomes small.

Step V

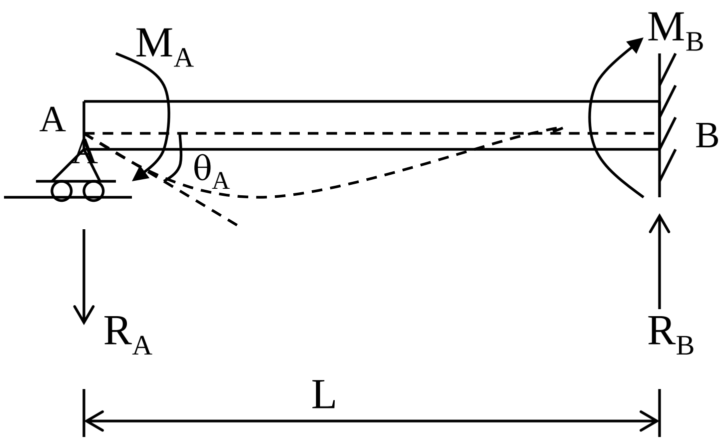
Sum up all the moments at each of the joint to obtain the joint moments.

7.3 SOME BASIC DEFINITIONS

In order to understand the five steps mentioned in section 7.3, some words need to be defined and relevant derivations made.

7.3.1 Stiffness and Carry-over Factors

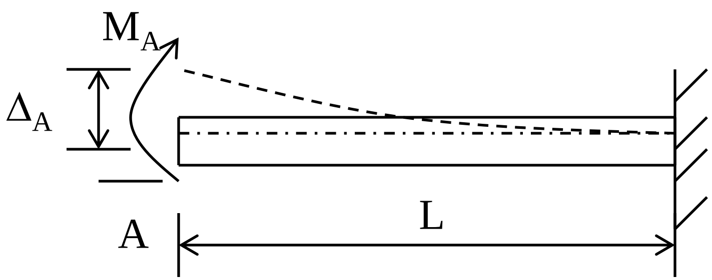
Stiffness = Resistance offered by member to a unit displacement or rotation at a point, for given support constraint conditions

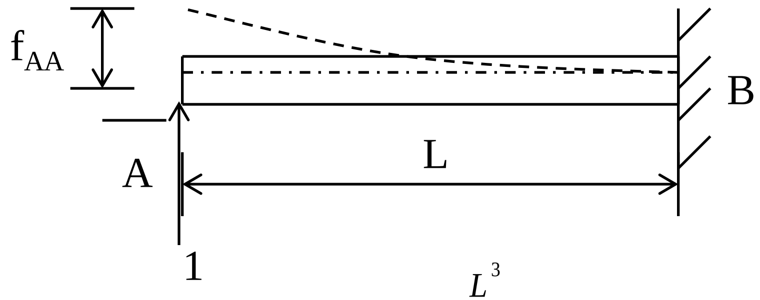


E, I – Member properties

A clockwise moment M_A is applied at A to produce a +ve bending in beam AB. Find θ_A and M_B.

Using method of consistent deformations





$$\Delta_A = +\frac{M_A L^2}{2EI} \qquad f_{AA} = \frac{L^3}{3EI}$$

Applying the principle of consistent deformation,

$$\Delta_A + \mathbf{R}_A \mathbf{f}_{AA} = 0 \quad \rightarrow \mathbf{R}_A = -\frac{3\mathbf{M}_A}{2\mathbf{L}} \downarrow$$

$$\theta_A = \frac{M_A L}{EI} + \frac{R_A L^2}{2EI} = \frac{M_A L}{4EI} \qquad \therefore M_A = \frac{4EI}{L}\theta_A; \quad hence \quad k_\theta = \frac{M_A}{\theta_A} = \frac{4EI}{L}$$

Stiffness factor = k_θ = 4EI/L

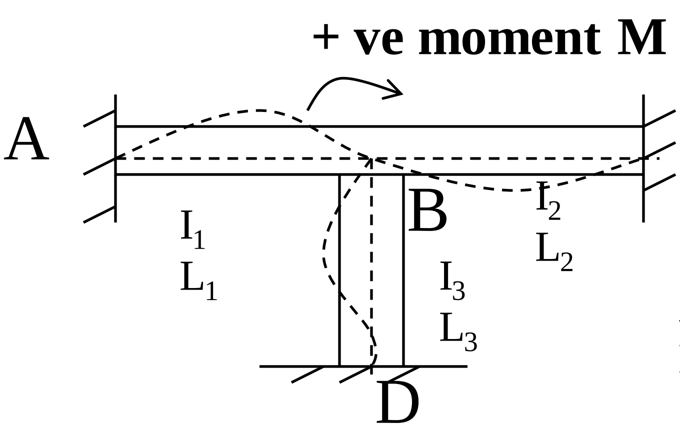
Considering moment $M_{B,}$

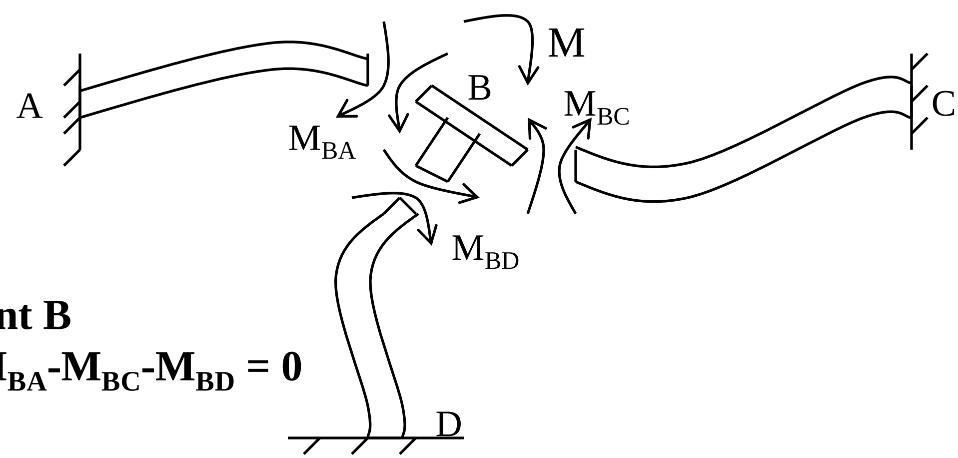
$$M_B + M_A + R_A L = 0$$
$$\therefore M_B = M_A/2 = (1/2)M_A$$

Carry - over Factor = 1/2

Distribution Factor

Distribution factor is the ratio according to which an externally applied unbalanced moment M at a joint is apportioned to the various members mating at the joint





At joint B

$$M - M_{BA} - M_{BC} - M_{BD} = 0$$

i.e., $\mathbf{M = M_{BA} + M_{BC} + M_{BD}}$

$$= \left[\left(\frac{4E_1 I_1}{L_1}\right) + \left(\frac{4E_2 I_2}{L_2}\right) + \left(\frac{4E_3 I_3}{L_3}\right)\right]\theta_B$$

$$= \left(K_{BA} + K_{BC} + K_{BD}\right)\theta_B$$

$$\therefore \quad \theta_B = \frac{M}{\left(K_{BA} + K_{BC} + K_{BD}\right)} = \frac{M}{\sum K}$$

$$M_{BA} = K_{BA}\theta_B = \left(\frac{K_{BA}}{\sum K}\right)M = (D.F)_{BA}\, M$$

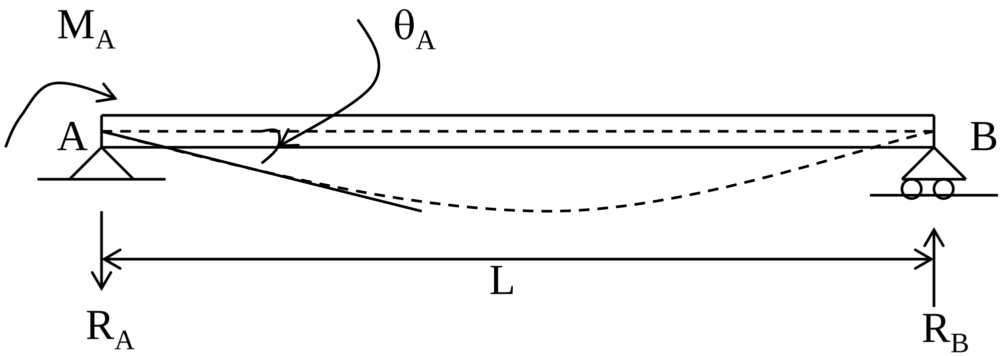
Similarly

$$M_{BC} = \left(\frac{K_{BC}}{\sum K}\right)M = (D.F)_{BC}\, M$$

$$M_{BD} = \left(\frac{K_{BD}}{\sum K}\right)M = (D.F)_{BD}\, M$$

Modified Stiffness Factor

The stiffness factor changes when the far end of the beam is simply-supported.



As per earlier equations for deformation, given in Mechanics of Solids text-books.

$$\theta_A = \frac{M_A L}{3EI}$$

$$K_{AB} = \frac{M_A}{\theta_A} = \frac{3EI}{L} = \left(\frac{3}{4}\right)\left(\frac{4EI}{L}\right)$$

$$= \frac{3}{4}(K_{AB})_{fixed}$$

7.4 SOLUTION OF PROBLEMS -

7.4.1 Solve the previously given problem by the moment distribution method

7.4.1.1: Fixed end moments

$$M_{AB} = -M_{BA} = -\frac{wl^2}{12} = -\frac{(15)(8)^2}{12} = -80 \ kN.m$$

$$M_{BC} = -M_{CB} = -\frac{wl}{8} = -\frac{(150)(6)}{8} = -112.5 \ kN.m$$

$$M_{CD} = -M_{DC} = -\frac{wl^2}{12} = -\frac{(10)(8)^2}{12} = -53.333 \ kN.m$$

7.4.1.2 Stiffness Factors (Unmodified Stiffness)

$$\mathbf{K}_{AB} = \mathbf{K}_{BA} = \frac{4\mathbf{EI}}{\mathbf{L}} = \frac{(4)(\mathbf{EI})}{8} = 0.5\mathbf{EI}$$

$$\mathbf{K}_{BC} = \mathbf{K}_{CB} = \frac{4\mathbf{EI}}{\mathbf{L}} = \frac{(4)(\mathbf{EI})}{6} = 0.667\ \mathbf{EI}$$

$$\mathbf{K}_{CD} = \left[\frac{4\mathbf{EI}}{8}\right] = \frac{4}{8}\mathbf{EI} = 0.5\mathbf{EI}$$

$$\mathbf{K}_{DC} = \frac{4\mathbf{EI}}{8} = 0.5\mathbf{EI}$$

Distribution Factors

$$\mathbf{DF}_{\mathbf{AB}} = \frac{\mathbf{K}_{\mathbf{BA}}}{\mathbf{K}_{\mathbf{BA}} + \mathbf{K}_{\mathbf{wall}}} = \frac{0.5\mathbf{EI}}{0.5 + \infty\ (\mathbf{wall\ \ stiffness})} = 0.0$$

$$\mathbf{DF}_{\mathbf{BA}} = \frac{\mathbf{K}_{\mathbf{BA}}}{\mathbf{K}_{\mathbf{BA}} + \mathbf{K}_{\mathbf{BC}}} = \frac{0.5\mathbf{EI}}{0.5\mathbf{EI} + 0.667\ \mathbf{EI}} = 0.4284$$

$$\mathbf{DF}_{\mathbf{BC}} = \frac{\mathbf{K}_{\mathbf{BC}}}{\mathbf{K}_{\mathbf{BA}} + \mathbf{K}_{\mathbf{BC}}} = \frac{0.667\ \mathbf{EI}}{0.5\mathbf{EI} + 0.667\ \mathbf{EI}} = 0.5716$$

$$\mathbf{DF}_{\mathbf{CB}} = \frac{\mathbf{K}_{\mathbf{CB}}}{\mathbf{K}_{\mathbf{CB}} + \mathbf{K}_{\mathbf{CD}}} = \frac{0.667\ \mathbf{EI}}{0.667\ \mathbf{EI} + 0.500\ \mathbf{EI}} = 0.5716$$

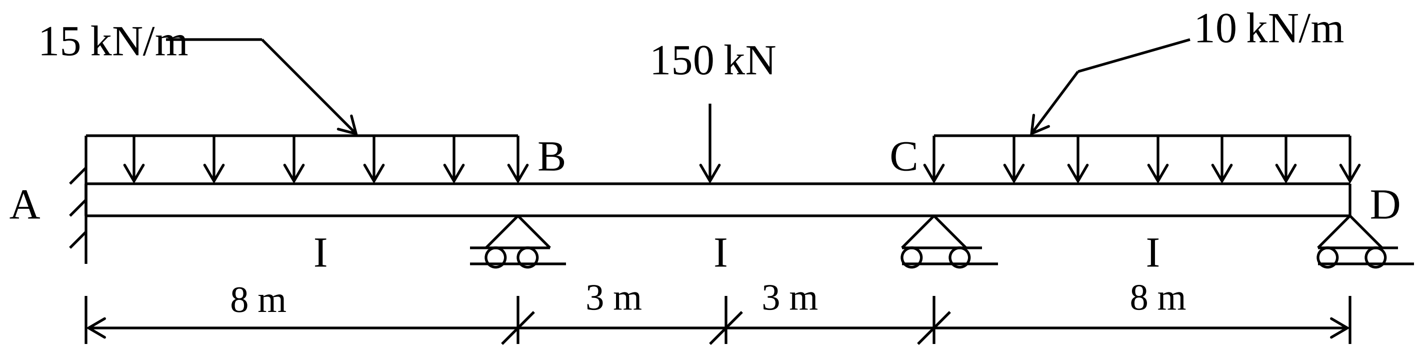
$$\mathbf{DF}_{\mathbf{CD}} = \frac{\mathbf{K}_{\mathbf{CD}}}{\mathbf{K}_{\mathbf{CB}} + \mathbf{K}_{\mathbf{CD}}} = \frac{0.500\ \mathbf{EI}}{0.667\ \mathbf{EI} + 0.500\ \mathbf{EI}} = 0.4284$$

$$\mathbf{DF}_{\mathbf{DC}} = \frac{\mathbf{K}_{\mathbf{DC}}}{\mathbf{K}_{\mathbf{DC}}} = 1.00$$

Moment Distribution Table

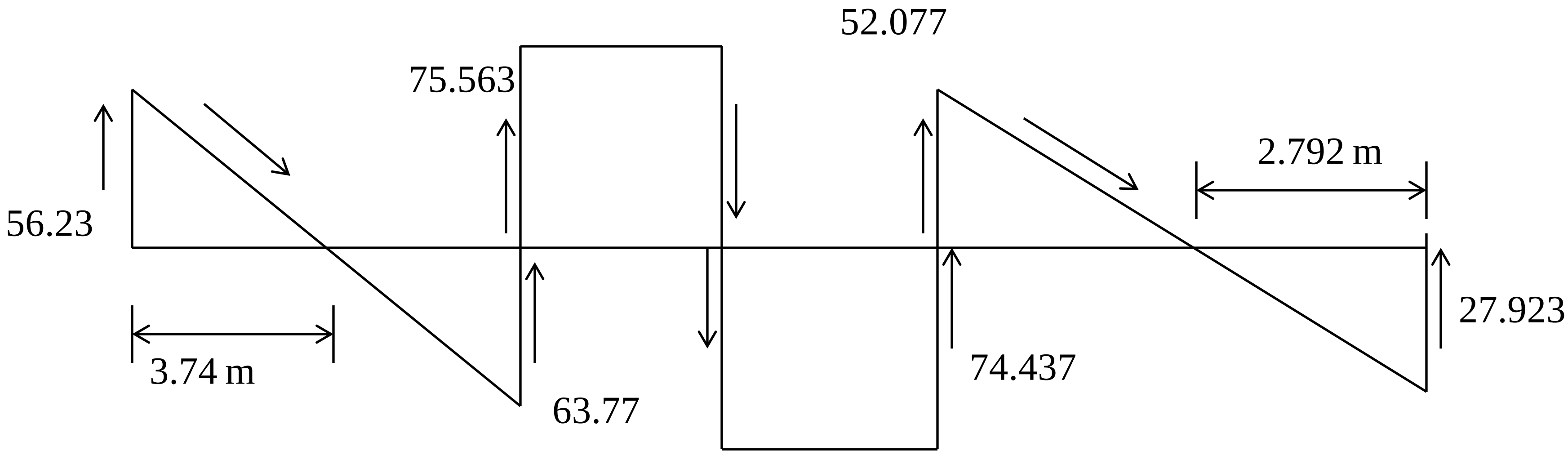
Joint		A	B		C		D
Member		AB	BA	BC	CB	CD	DC
Distribution Factors		0	0.4284	0.5716	0.5716	0.4284	1
Cycle 1	Computed end moments	-80	80	-112.5	112.5	-53.33	53.33
	Distribution		13.923	18.577	-33.82	-25.35	-53.33
Cycle 2	Carry-over moments	6.962		-16.91	9.289	-26.67	-12.35
	Distribution		7.244	9.662	9.935	7.446	12.35
Cycle 3	Carry-over moments	3.622		4.968	4.831	6.175	3.723
	Distribution		-2.128	-2.84	-6.129	-4.715	-3.723
Cycle 4	Carry-over moments	-1.064		-3.146	-1.42	-1.862	-2.358
	Distribution		1.348	1.798	1.876	1.406	2.358
Cycle 5	Carry-over moments	0.674		0.938	0.9	1.179	0.703
	Distribution		-0.402	-0.536	-1.187	-0.891	-0.703
	Summed up moments	-69.81	99.985	-99.99	96.613	-96.61	0

Computation of Shear Forces

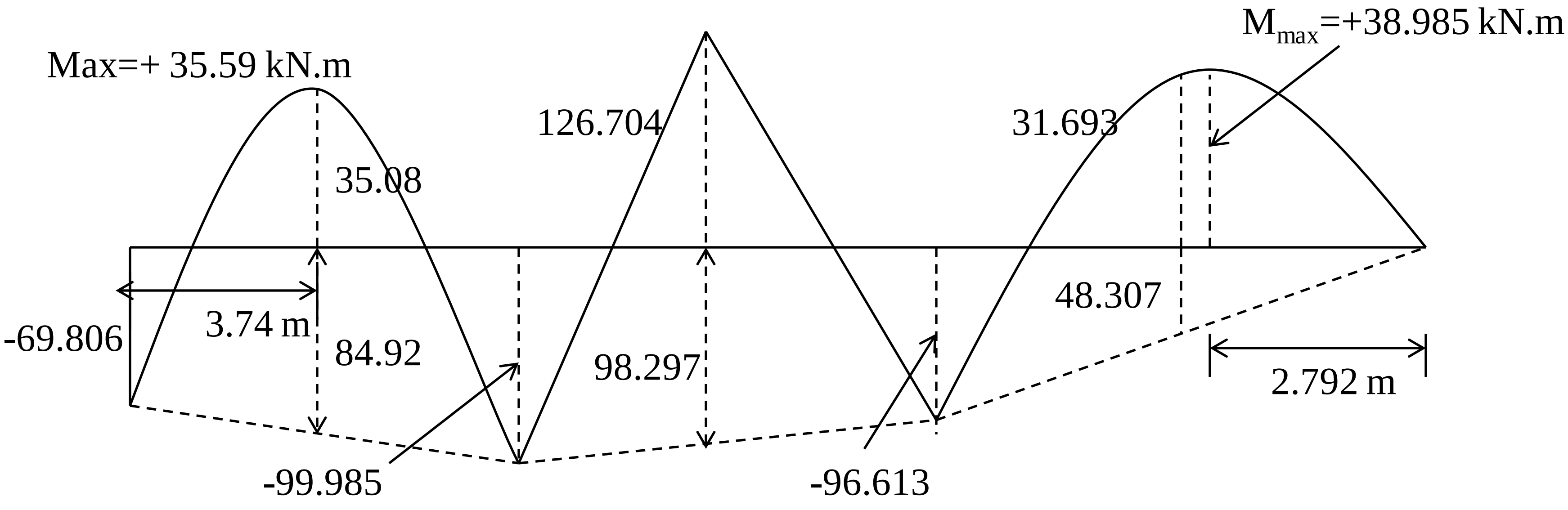


Simply-supported reaction	60	60	75	75	40	40
End reaction due to left hand FEM	8.726	-8.726	16.665	-16.67	12.079	-12.08
End reaction due to right hand FEM	-12.5	12.498	-16.1	16.102	0	0
Summed-up moments	56.228	63.772	75.563	74.437	53.077	27.923

Shear Force and Bending Moment Diagrams



S. F. D.



B. M. D

Simply-supported bending moments at center of span

M_{center} in AB = $(15)(8)^2/8 = +120$ kN.m

M_{center} in BC = $(150)(6)/4 = +225$ kN.m

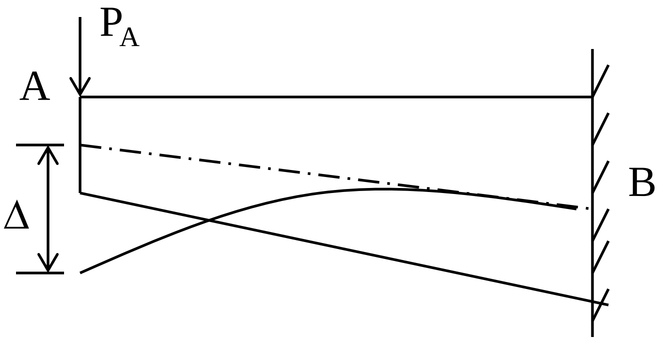
M_{center} in AB = $(10)(8)^2/8 = +80$ kN.m

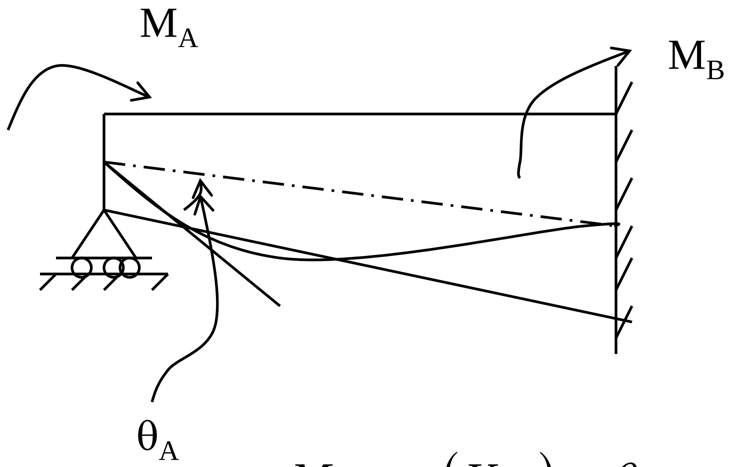
MOMENT DISTRIBUTION METHOD FOR NONPRISMATIC MEMBER (CHAPTER 12)

The section will discuss moment distribution method to analyze beams and frames composed of nonprismatic members. First the procedure to obtain the necessary carry-over factors, stiffness factors and fixed-end moments will be outlined. Then the use of values given in design tables will be illustrated. Finally the analysis of statically indeterminate structures using the moment distribution method will be outlined

Stiffness and Carry-over Factors

Use moment-area method to find the stiffness and carry-over factors of the non-prismatic beam.



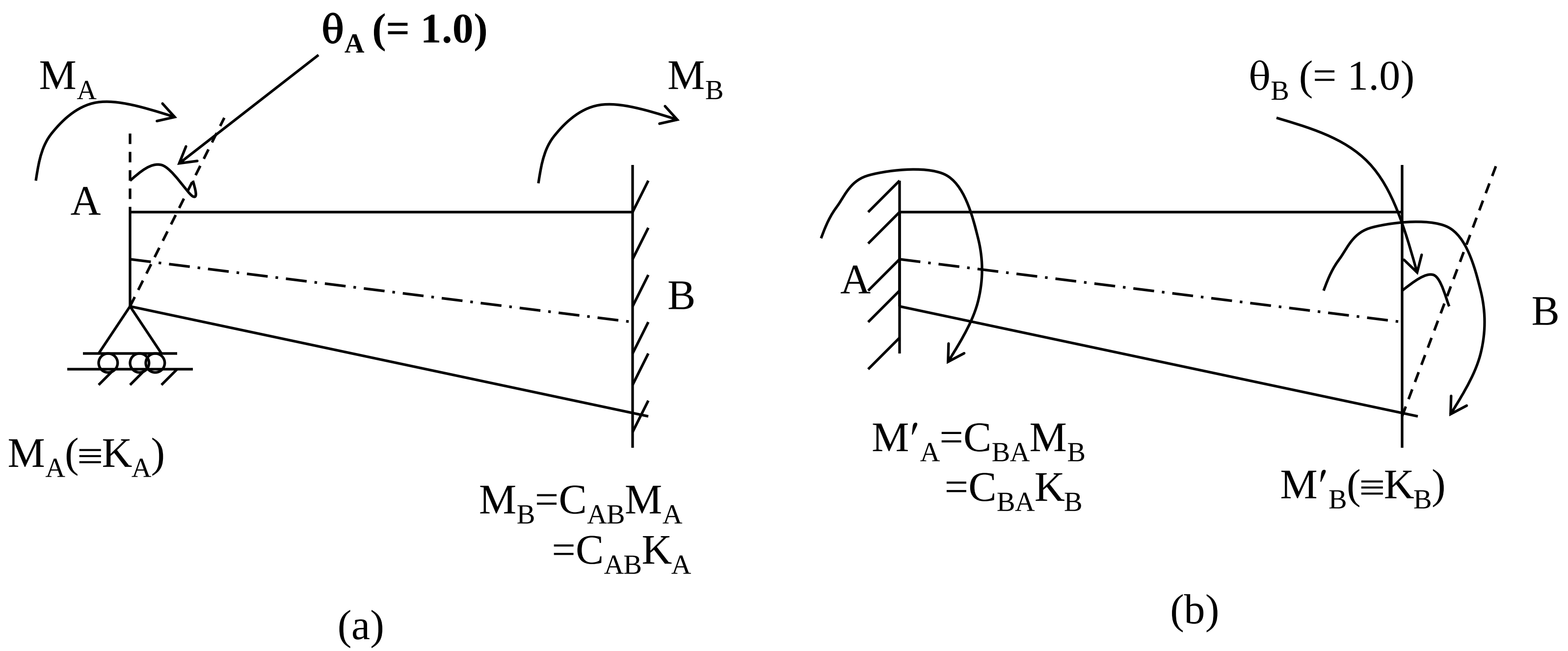


$$P_A = (K_A)_{AB}\,\Delta_A$$

$$M_A = (K_\theta)_{AB}\,\theta_A$$

$$M_B = C_{AB}\,M_A$$

C_{AB}= Carry-over factor of moment M_A from A to B



Use of Betti-Maxwell's reciprocal theorem requires that the work done by loads in case (a) acting through displacements in case (b) is equal to work done by loads in case (b) acting through displacements in case (a)

$$\left(M_A\right)(0)+\left(M_B\right)(1)=\left(M'_A\right)(1.0)+\left(M'_B\right)(0.0)$$

$$\mathrm{C}_{\mathrm{AB}}K_A=\mathrm{C}_{\mathrm{BA}}K_B$$

Tabulated Design Tables

Graphs and tables have been made available to determine fixed-end moments, stiffness factors and carry-over factors for common structural shapes used in design. One such source is the Handbook of Frame constants published by the Portland Cement Association, Chicago, Illinois, U. S. A. A portion of these tables, is listed here as Table 1 and 2

Nomenclature of the Tables

a_A a_b = ratio of length of haunch (at end A and B to the length of span

b = ratio of the distance (from the concentrated load to end A) to the length of span

h_A, h_B= depth of member at ends A and B, respectively

h_C = depth of member at minimum section

I_c = moment of inertia of section at minimum section = (1/12)B$(h_c)^3$, with B as width of beam

k_{AB}, k_{BC} = stiffness factor for rotation at end A and B, respectively

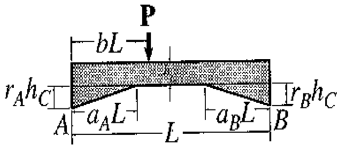
L = Length of member

M_{AB}, M_{BA} = Fixed-end moments at end A and B, respectively; specified in tables for uniform load w or concentrated force P

$$r_A = \frac{h_A - h_C}{h_C} \qquad r_B = \frac{h_B - h_C}{h_C}$$

Also $$K_A = \frac{k_{AB} EI_C}{L}, \quad K_B = \frac{k_{BA} EI_C}{L}$$

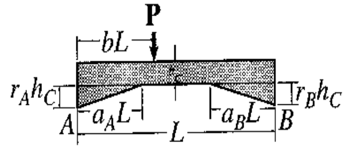
Table 12-1 Straight Haunches—Constant Width



Note: All carry-over factors are negative and all stiffness factors are positive.

Right Haunch		Carry-over Factors		Stiffness Factors		Unif. Load FEM Coef. × wL^2		Concentrated Load FEM—Coef. × PL: b = 0.1		0.3		0.5		0.7		0.9		Haunch Load at Left: FEM Coef. × $w_A L^2$		Right: FEM Coef. × $w_B L^2$	
a_B	r_B	C_{AB}	C_{BA}	k_{AB}	k_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}
$a_A = 0.3$ a_B = variable $r_A = 1.0$ r_B = variable																					
0.2	0.4	0.543	0.766	9.19	6.52	0.1194	0.0791	0.0935	0.0034	0.2185	0.0384	0.1955	0.1147	0.0889	0.1601	0.0096	0.0870	0.0133	0.0008	0.0006	0.0058
	0.6	0.576	0.758	9.53	7.24	0.1152	0.0851	0.0934	0.0038	0.2158	0.0422	0.1883	0.1250	0.0798	0.1729	0.0075	0.0898	0.0133	0.0009	0.0005	0.0060
	1.0	0.622	0.748	10.06	8.37	0.1089	0.0942	0.0931	0.0042	0.2118	0.0480	0.1771	0.1411	0.0668	0.1919	0.0047	0.0935	0.0132	0.0011	0.0004	0.0062
	1.5	0.660	0.740	10.52	9.38	0.1037	0.1018	0.0927	0.0047	0.2085	0.0530	0.1678	0.1550	0.0559	0.2078	0.0028	0.0961	0.0130	0.0012	0.0002	0.0064
	2.0	0.684	0.734	10.83	10.09	0.1002	0.1069	0.0924	0.0050	0.2062	0.0565	0.1614	0.1645	0.0487	0.2185	0.0019	0.0974	0.0129	0.0013	0.0001	0.0065
0.3	0.4	0.579	0.741	9.47	7.40	0.1175	0.0822	0.0934	0.0037	0.2164	0.0419	0.1909	0.1225	0.0856	0.1649	0.0100	0.0861	0.0133	0.0009	0.0022	0.0118
	0.6	0.629	0.726	9.98	8.64	0.1120	0.0902	0.0931	0.0042	0.2126	0.0477	0.1808	0.1379	0.0747	0.1807	0.0080	0.0888	0.0132	0.0010	0.0018	0.0124
	1.0	0.705	0.705	10.85	10.85	0.1034	0.1034	0.0924	0.0052	0.2063	0.0577	0.1640	0.1640	0.0577	0.2063	0.0052	0.0924	0.0131	0.0013	0.0013	0.0131
	1.5	0.771	0.689	11.70	13.10	0.0956	0.1157	0.0917	0.0062	0.2002	0.0675	0.1483	0.1892	0.0428	0.2294	0.0033	0.0953	0.0129	0.0015	0.0008	0.0137
	2.0	0.817	0.678	12.33	14.85	0.0901	0.1246	0.0913	0.0069	0.1957	0.0750	0.1368	0.2080	0.0326	0.2455	0.0022	0.0968	0.0128	0.0017	0.0006	0.0141
$a_A = 0.2$ a_B = variable $r_A = 1.5$ r_B = variable																					
0.2	0.4	0.569	0.714	7.97	6.35	0.1166	0.0799	0.0966	0.0019	0.2186	0.0377	0.1847	0.1183	0.0821	0.1626	0.0088	0.0873	0.0064	0.0001	0.0006	0.0058
	0.6	0.603	0.707	8.26	7.04	0.1127	0.0858	0.0965	0.0021	0.2163	0.0413	0.1778	0.1288	0.0736	0.1752	0.0068	0.0901	0.0064	0.0001	0.0005	0.0060
	1.0	0.652	0.698	8.70	8.12	0.1069	0.0947	0.0963	0.0023	0.2127	0.0468	0.1675	0.1449	0.0616	0.1940	0.0043	0.0937	0.0064	0.0002	0.0004	0.0062
	1.5	0.691	0.691	9.08	9.08	0.1021	0.1021	0.0962	0.0025	0.2097	0.0515	0.1587	0.1587	0.0515	0.2097	0.0025	0.0962	0.0064	0.0002	0.0002	0.0064
	2.0	0.716	0.686	9.34	9.75	0.0990	0.1071	0.0960	0.0028	0.2077	0.0547	0.1528	0.1681	0.0449	0.2202	0.0017	0.0975	0.0064	0.0002	0.0001	0.0065
0.3	0.4	0.607	0.692	8.21	7.21	0.1148	0.0829	0.0965	0.0021	0.2168	0.0409	0.1801	0.1263	0.0789	0.1674	0.0091	0.0866	0.0064	0.0002	0.0020	0.0118
	0.6	0.659	0.678	8.65	8.40	0.1098	0.0907	0.0964	0.0024	0.2135	0.0464	0.1706	0.1418	0.0688	0.1831	0.0072	0.0892	0.0064	0.0002	0.0017	0.0123
	1.0	0.740	0.660	9.38	10.52	0.1018	0.1037	0.0961	0.0028	0.2078	0.0559	0.1550	0.1678	0.0530	0.2085	0.0047	0.0927	0.0064	0.0002	0.0012	0.0130
	1.5	0.809	0.645	10.09	12.66	0.0947	0.1156	0.0958	0.0033	0.2024	0.0651	0.1403	0.1928	0.0393	0.2311	0.0029	0.0950	0.0063	0.0003	0.0008	0.0137
	2.0	0.857	0.636	10.62	14.32	0.0897	0.1242	0.0955	0.0038	0.1985	0.0720	0.1296	0.2119	0.0299	0.2469	0.0020	0.0968	0.0063	0.0003	0.0005	0.0141

Table 12-2 Parabolic Haunches—Constant Width



Note: All carry-over factors are negative and all stiffness factors are positive.

Right Haunch		Carry-over Factors		Stiffness Factors		Unif. Load FEM Coef. × wL^2		Concentrated Load FEM—Coef. × PL, b = 0.1		0.3		0.5		0.7		0.9		Haunch Load at Left: FEM Coef. × $w_A L^2$		Right: FEM Coef. × $w_B L^2$	
a_B	r_B	C_{AB}	C_{BA}	k_{AB}	k_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}
$a_A = 0.2$ a_B = *variable* $r_A = 1.0$ r_B = *variable*																					
0.2	0.4	0.558	0.627	6.08	5.40	0.1022	0.0841	0.0938	0.0033	0.1891	0.0502	0.1572	0.1261	0.0715	0.1618	0.0073	0.0877	0.0032	0.0001	0.0002	0.0030
	0.6	0.582	0.624	6.21	5.80	0.0995	0.0887	0.0936	0.0036	0.1872	0.0535	0.1527	0.1339	0.0663	0.1708	0.0058	0.0902	0.0032	0.0001	0.0002	0.0031
	1.0	0.619	0.619	6.41	6.41	0.0956	0.0956	0.0935	0.0038	0.1844	0.0584	0.1459	0.1459	0.0584	0.1844	0.0038	0.0935	0.0032	0.0001	0.0001	0.0032
	1.5	0.649	0.614	6.59	6.97	0.0921	0.1015	0.0933	0.0041	0.1819	0.0628	0.1399	0.1563	0.0518	0.1962	0.0025	0.0958	0.0032	0.0001	0.0001	0.0032
	2.0	0.671	0.611	6.71	7.38	0.0899	0.1056	0.0932	0.0044	0.1801	0.0660	0.1358	0.1638	0.0472	0.2042	0.0017	0.0971	0.0032	0.0001	0.0000	0.0033
0.3	0.4	0.588	0.616	6.22	5.93	0.1002	0.0877	0.0937	0.0035	0.1873	0.0537	0.1532	0.1339	0.0678	0.1686	0.0073	0.0877	0.0032	0.0001	0.0007	0.0063
	0.6	0.625	0.609	6.41	6.58	0.0966	0.0942	0.0935	0.0039	0.1845	0.0587	0.1467	0.1455	0.0609	0.1808	0.0057	0.0902	0.0032	0.0001	0.0005	0.0065
	1.0	0.683	0.598	6.73	7.68	0.0911	0.1042	0.0932	0.0044	0.1801	0.0669	0.1365	0.1643	0.0502	0.2000	0.0037	0.0936	0.0031	0.0001	0.0004	0.0068
	1.5	0.735	0.589	7.02	8.76	0.0862	0.1133	0.0929	0.0050	0.1760	0.0746	0.1272	0.1819	0.0410	0.2170	0.0023	0.0959	0.0031	0.0001	0.0003	0.0070
	2.0	0.772	0.582	7.25	9.61	0.0827	0.1198	0.0927	0.0054	0.1730	0.0805	0.1203	0.1951	0.0345	0.2293	0.0016	0.0972	0.0031	0.0001	0.0002	0.0072
$a_A = 0.5$ a_B = *variable* $r_A = 1.0$ r_B = *variable*																					
0.2	0.4	0.488	0.807	9.85	5.97	0.1214	0.0753	0.0929	0.0034	0.2131	0.0371	0.2021	0.1061	0.0979	0.1506	0.0105	0.0863	0.0171	0.0017	0.0003	0.0030
	0.6	0.515	0.803	10.10	6.45	0.1183	0.0795	0.0928	0.0036	0.2110	0.0404	0.1969	0.1136	0.0917	0.1600	0.0083	0.0892	0.0170	0.0018	0.0002	0.0030
	1.0	0.547	0.796	10.51	7.22	0.1138	0.0865	0.0926	0.0040	0.2079	0.0448	0.1890	0.1245	0.0809	0.1740	0.0056	0.0928	0.0168	0.0020	0.0001	0.0031
	1.5	0.571	0.786	10.90	7.90	0.1093	0.0922	0.0923	0.0043	0.2055	0.0485	0.1818	0.1344	0.0719	0.1862	0.0035	0.0951	0.0167	0.0021	0.0001	0.0032
	2.0	0.590	0.784	11.17	8.40	0.1063	0.0961	0.0922	0.0046	0.2041	0.0506	0.1764	0.1417	0.0661	0.1948	0.0025	0.0968	0.0166	0.0022	0.0001	0.0032
0.5	0.4	0.554	0.753	10.42	7.66	0.1170	0.0811	0.0926	0.0040	0.2087	0.0442	0.1924	0.1205	0.0898	0.1595	0.0107	0.0853	0.0169	0.0020	0.0042	0.0145
	0.6	0.606	0.730	10.96	9.12	0.1115	0.0889	0.0922	0.0046	0.2045	0.0506	0.1820	0.1360	0.0791	0.1738	0.0086	0.0878	0.0167	0.0022	0.0036	0.0152
	1.0	0.694	0.694	12.03	12.03	0.1025	0.1025	0.0915	0.0057	0.1970	0.0626	0.1639	0.1639	0.0626	0.1970	0.0057	0.0915	0.0164	0.0028	0.0028	0.0164
	1.5	0.781	0.664	13.12	15.47	0.0937	0.1163	0.0908	0.0070	0.1891	0.0759	0.1456	0.1939	0.0479	0.2187	0.0039	0.0940	0.0160	0.0034	0.0021	0.0174
	2.0	0.850	0.642	14.09	18.64	0.0870	0.1275	0.0901	0.0082	0.1825	0.0877	0.1307	0.2193	0.0376	0.2348	0.0027	0.0957	0.0157	0.0039	0.0016	0.0181

UNIT 1
PART 2
Kani’s Method

Analysis by Kani's Method:

- Framed structures are rarely symmetric and subjected to side sway, hence Kani's method is best and much simpler than other methods.
- PROCEDURE:
- 1. Rotation stiffness at each end of all members of a structure is determined depending upon the end conditions.
- a. Both ends fixed Kij= Kji= EI/L
- b. Near end fixed, far end simply supported Kij= ¾ EI/L; Kji= 0

- 2. Rotational factors are computed for all the members at each joint it is given by Uij= -0.5 (Kij/ ?Kji) {THE SUM OF ROTATIONAL FACTORS AT A JOINT IS -0.5} (Fixed end moments including transitional moments, moment releases and carry over moments are computed for members and entered. The sum of the FEM at a joint is entered in the central square drawn at the joint).

- 3. Iterations can be commenced at any joint however the iterations commence from the left end of the structure generally given by the equation M?ij = Uij [(Mfi + M??i) + ? M?ji)]

- 4. Initially the rotational components? Mji (sum of the rotational moments at the far ends of the joint) can be assumed to be zero. Further iterations take into account the rotational moments of the previous joints. 5. Rotational moments are computed at each joint successively till all the joints are processed. This process completes one cycle of iteration

- 6. Steps 4 and 5 are repeated till the difference in the values of rotation moments from successive cycles is neglected.
- 7. Final moments in the members at each joint are computed from the rotational members of the final iterations step. Mij = (Mfij + M??ij) + 2 M?ij + M?jii

- The lateral translation of joints (side sway) is taken into consideration by including column shear in the iterative procedure.
- 8. Displacement factors are calculated for each storey given by Uij = -1.5 (Kij/?Kij)

- Application Of Analysis Methods For The Portal Frame
- Application of Rotation contribution Method (Kani's Method) for the analysis of portal frame:
- Fixed end moments
- FEMAB = 0
- FEMBA = 0
- FEMBC = -120 kNm
- FEMCB = 120 kNm
- FEMCD = 0
- FEMDC = 0

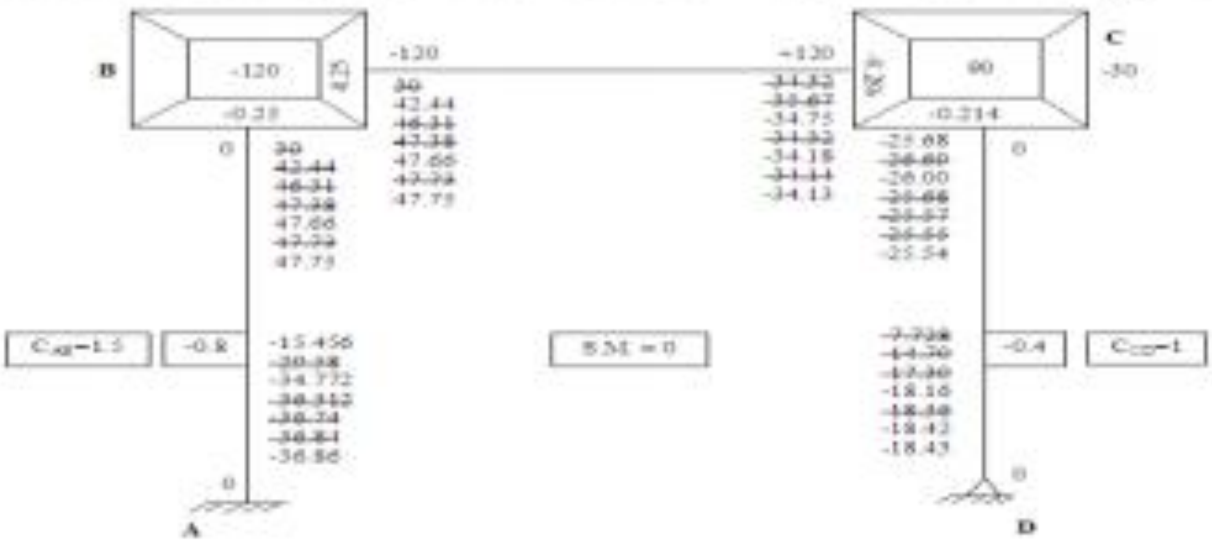
- Stiffness and rotation factor (R.F.)
- Table 1.
- Stiffness and Rotation Factors – Kani’s Method

Stiffness and rotation factor (R.F.)

Table 1. Stiffness and Rotation Factors – Kani's Method

Joint	Member	K	ΣK	RF
B	BA	0.333 I	0.666 I	-0.25
	BC	0.333 I		-0.25
C	CB	0.333 I	0.583 I	-0.286
	CD	0.25 I		-0.214

- 3. Displacement factors (δ)
- Table 2. Calculation of Displacement factors (δ)
- ΣUCD = (-1.2) + (-0.3) = -1.5
- Checked.
- Hence OK
- Storey Moment (SM) Storey moment = 0 (since lack of nodal loads and lack of loadings on columns, SM=0) Iterations by Kani's Method Figure 2. Calculations of rotation contributions in tabular form using Kani's Method



B
C
A
D
-120
-120
-120
90
-30
47.75
-34.13
-25.54
-36.86
-18.43
-0.8
-0.4

- Final End Moments For columns:
- => F.E.M + 2 (near end contribution) + far end contribution of that particular column + L.D.C. of that column
- For beams: => F.E.M + 2 (near end contribution) + far end contribution of that particular beam or slab.
- MAB = 10.89 kNm
- MBA = 58.64 kNm
- MBC = -58.63 kNm
- MCB = 99.49 kNm
- MCD = -69.51 kNm
- MDC = 0 kNm
- MCE = -30 kNm

UNIT 2
SLOPE DEFLECTION METHOD

- Introduction
- Assumptions
- Sign conventions
- Derivation of slope deflection method
- Example

Introduction

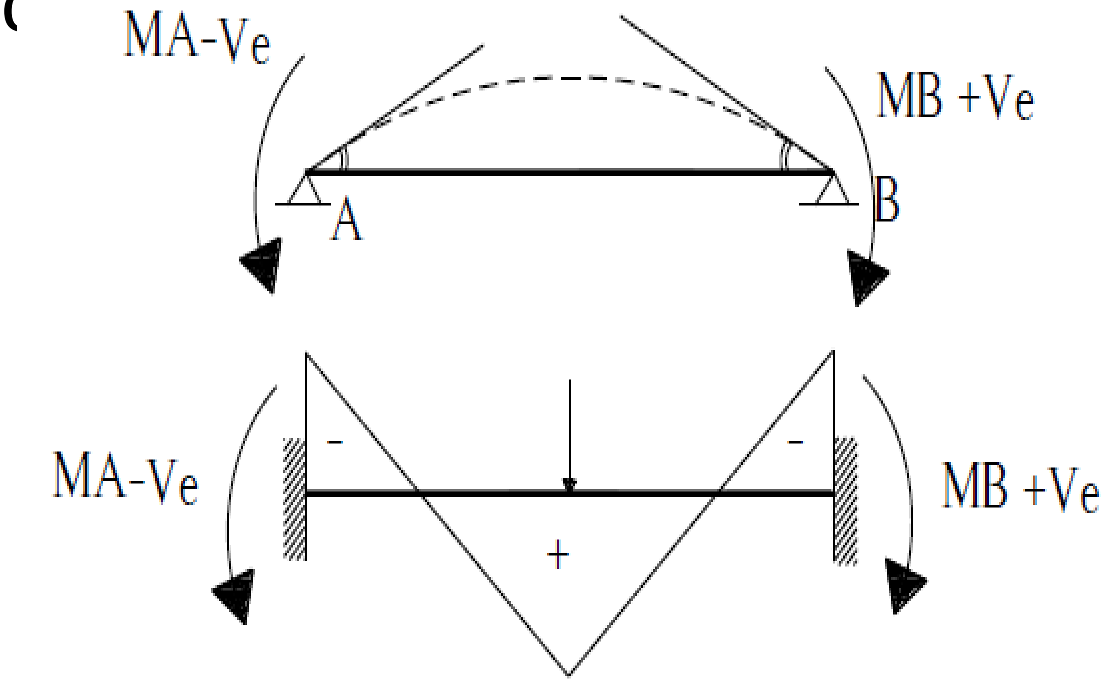
- The methods of three moment equation, and consistent deformation method represent the FORCE METHOD of structural analysis, The slope deflection method use displacements as unknowns, hence this method is the displacement method.
- In this method, if the slopes at the ends and the relative displacement of the ends are known, the end moment can be found in terms of slopes, deflection, stiffness and length of the members.

ASSUMPTIONS IN THE SLOPE DEFLECTION METHOD

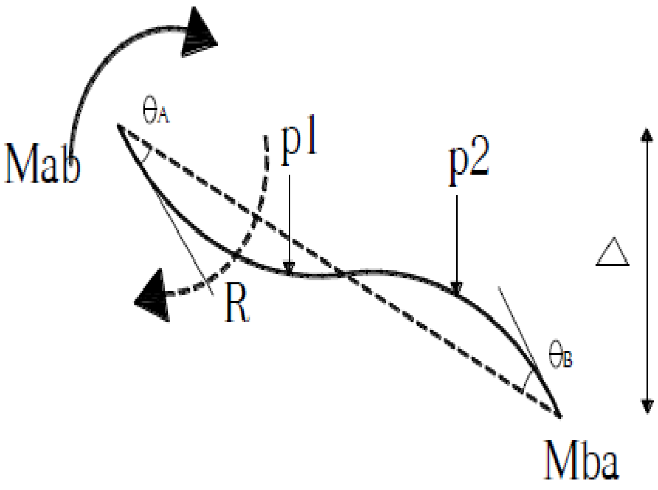
- This method is based on the following simplified assumptions.
- 1- All the joints of the frame are rigid, i.e , the angle between the members at the joints do not change, when the members of frame are loaded.
- 2- Distortion, due to axial and shear stresses, being very small, are neglected.

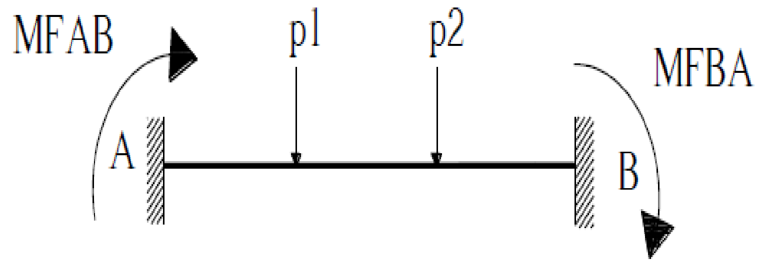
Sign Conventions:-

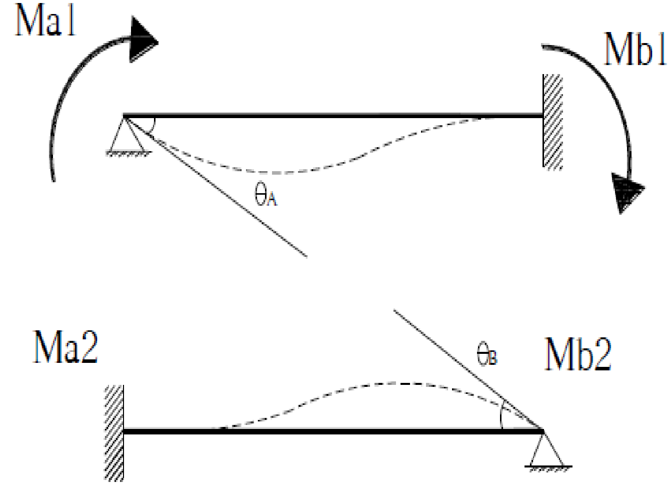
- Joint rotation & Fixed end moments are considered positive when occurring in a clo

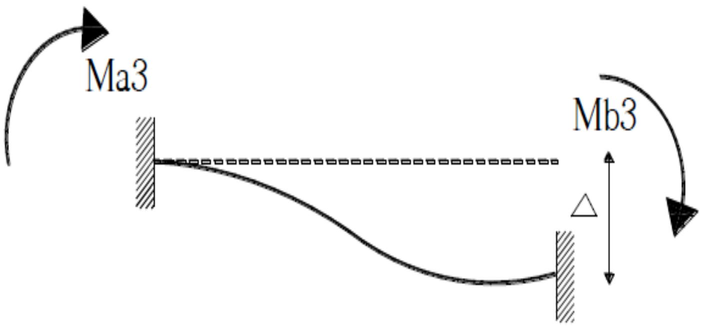


Derivation of slope deflection equation:-









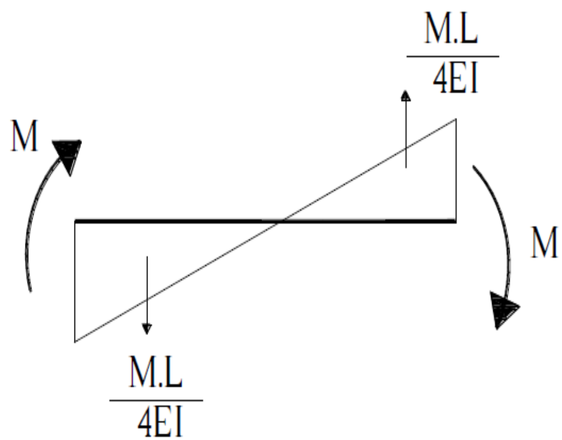
$$M_{a1} = \frac{4EI}{L}\theta_A$$

$$M_{b1} = \frac{2EI}{L}\theta_A$$

$$M_{a2} = \frac{2EI}{L}\theta_B$$

$$M_{b2} = \frac{4EI}{L}\theta_B$$

- Required M_{ab} & M_{ba} in term of
- θ_A, θ_B at joint
- rotation of member (R)
- loads acting on member
- First assume ,
- Get Mf_{ab} & Mf_{ba} due to acting loads. These fixed end moment must be corrected to allow for the end rotations θ_A,θ_B and the member rotation R.
- The effect of these rotations will be found separately



$$M_{a1} = \frac{4EI}{L}.\theta_A$$

$$M_{b1} = \frac{2EI}{L}.\theta_A$$

$$M_{b2} = \frac{4EI}{L}.\theta_B$$

$$M_{a2} = \frac{2EI}{L}.\theta_B$$

$$M_{b3} = M_{a3} = \frac{-6EI}{L^2}.\Delta$$

$$= \frac{-6EI}{L}.R$$

- by Superposition;

$$M_{ab} = Mf_{ab} + M_{a1} + M_{a2} + M_{a3}$$

$$Mf_{ab} + \frac{4EI}{L}.\theta_A + \frac{2EI}{L}\theta_B + \frac{-6EI}{L}.R$$

$$\boxed{M_{ab} = Mf_{ab} + \frac{2EI}{L}(2\theta_A + \theta_B - 3R)} \qquad \frac{\Delta}{L} = R$$

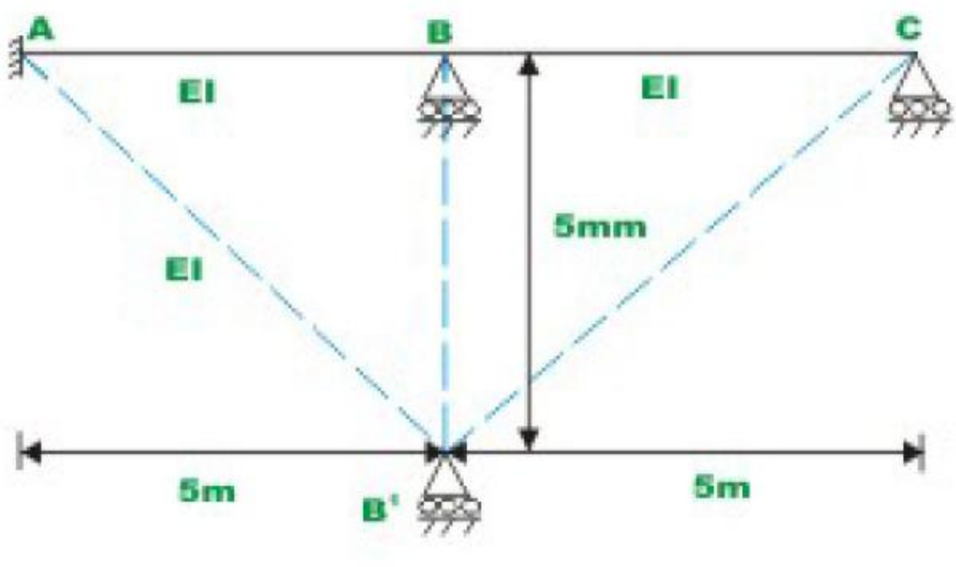
Example

- Calculate the support moments in the continuous beam having constant flexural rigidity EI throughout ,due to

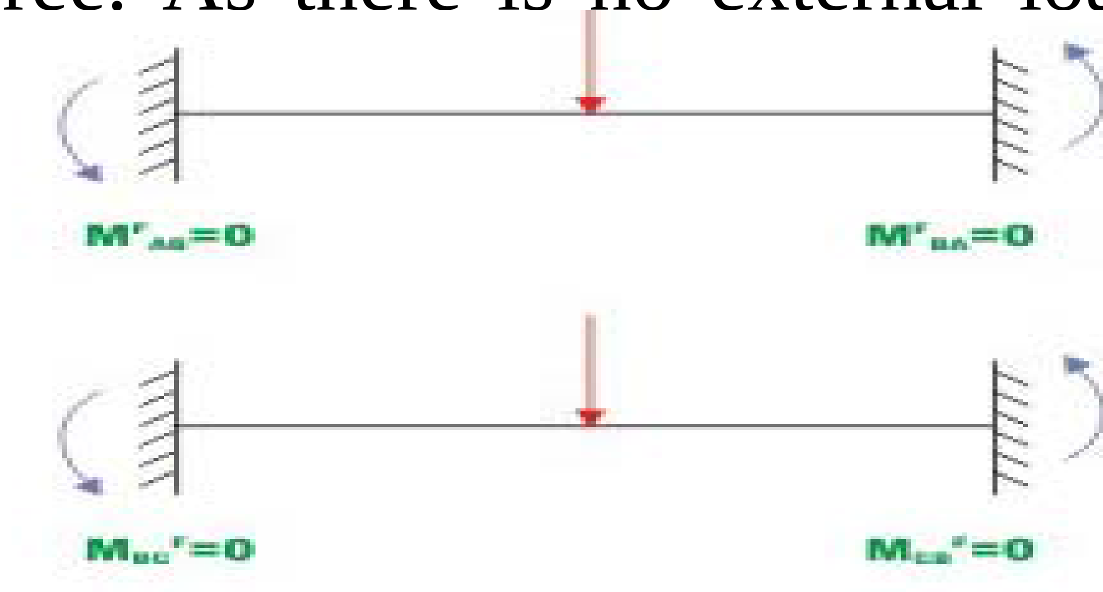
ort B by

=4* 10^-

elastic



- In the continuous beam, two rotations $B\theta$ and $C\theta$ need to be evaluated. Hence, beam is kinematically indeterminate to second degree. As there is no external load on the be   in the res



- For each span, two slope-deflection equations need to be written. In span *AB*, *B* is below *A* ... ates in clockwise ... ken as neg

$$\psi_{AB} = \frac{-5\times10^{-3}}{5} = -1\times10^{-3} \qquad (1)$$

Writing slope-deflection equation for span AB ,

$$M_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B - 3\psi_{AB}\right)$$

For span AB , $\theta_A = 0$, Hence,

$$M_{AB} = \frac{2EI}{5}\left(\theta_B + 3\times10^{-3}\right)$$

$$M_{AB} = 0.4EI\theta_B + .0012EI \qquad (2)$$

Similarly, for beam-end moment at end B , in span AB

$$M_{BA} = 0.4EI\left(2\theta_B + 3\times10^{-3}\right)$$
$$M_{BA} = 0.8EI\theta_B + 0.0012EI \qquad (3)$$

In span BC, the support C is above support B, Hence the chord joining $B'C$ rotates in anticlockwise direction.

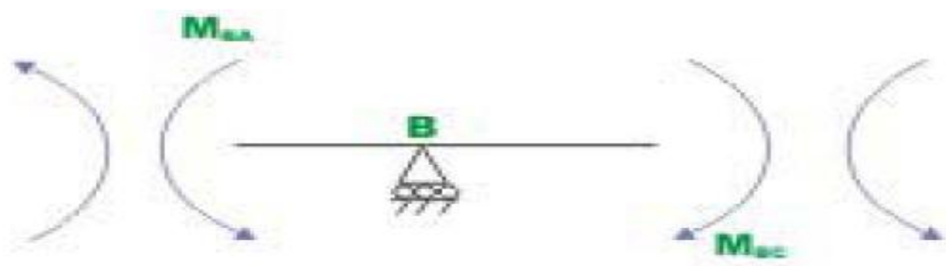
$$\psi_{BC} = \psi_{CB} = 1 \times 10^{-3} \qquad (4)$$

Writing slope-deflection equations for span BC,

$$M_{BC} = 0.8EI\theta_B + 0.4EI\theta_C - 1.2 \times 10^{-3} EI$$

$$M_{CB} = 0.8EI\theta_C + 0.4EI\theta_B - 1.2 \times 10^{-3} EI \qquad (5)$$

Now, consider the joint equilibrium of support B



$$M_{BA} + M_{BC} = 0 \qquad (6)$$

Substituting the values of M_{BA} and M_{BC} in equation (6),

$$0.8EI\theta_B + 1.2 \times 10^{-3} EI + 0.8EI\theta_B + 0.4EI\theta_C - 1.2 \times 10^{-3} EI = 0$$

Simplifying,

$$1.6\theta_B + 0.4\theta_C = 1.2 \times 10^{-3} \qquad (7)$$

Also, the support C is simply supported and hence, $M_{CB} = 0$

$$M_{CB} = 0 = 0.8\theta_C + 0.4\theta_B - 1.2 \times 10^{-3} EI$$

$$0.8\theta_C + 0.4\theta_B = 1.2 \times 10^{-3} \qquad (8)$$

We have two unknowns θ_B and θ_C and there are two equations in θ_B and θ_C. Solving equations (7) and (8)

$$\theta_B = -0.4286 \times 10^{-3} \text{ radians}$$

$$\theta_C = 1.7143 \times 10^{-3} \text{ radians} \qquad (9)$$

Substituting the values of θ_B, θ_C and EI in slope-deflection equations, Substituting the values of θ_B, θ_C and EI in slope-deflection equations,

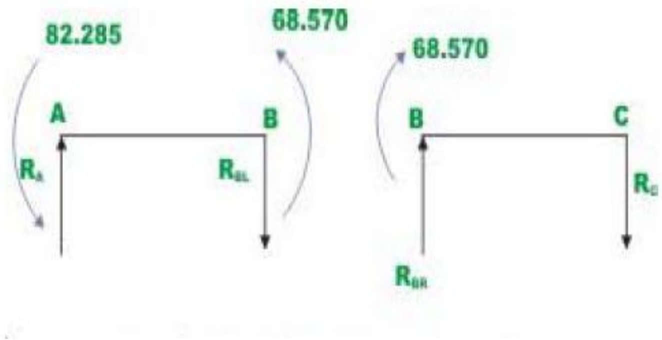
$$M_{AB} = 82.285 \text{ kN.m}$$

$$M_{BA} = 68.570 \text{ kN.m}$$

$$M_{BC} = -68.573 \text{ kN.m}$$

$$M_{CB} = 0 \text{ kN.m} \tag{10}$$

Reactions are obtained from equations of static equilibriu



In beam AB ,

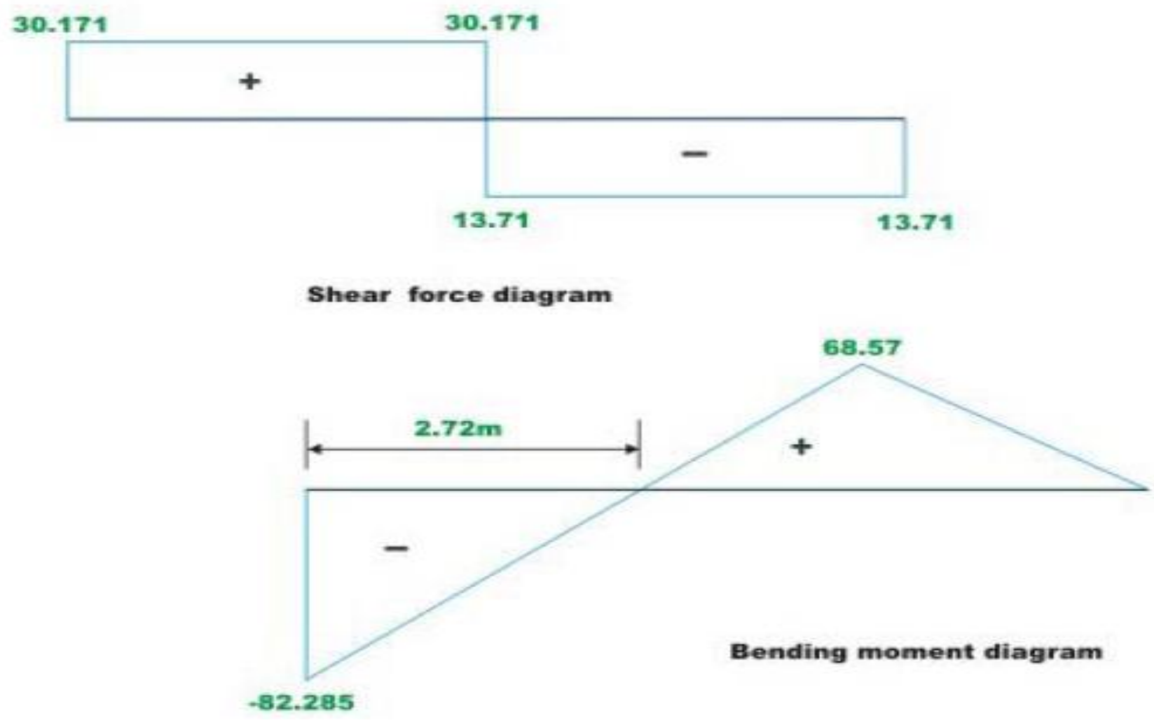
$$\sum M_B = 0,\ R_A = 30.171\,\text{kN}(\uparrow)$$

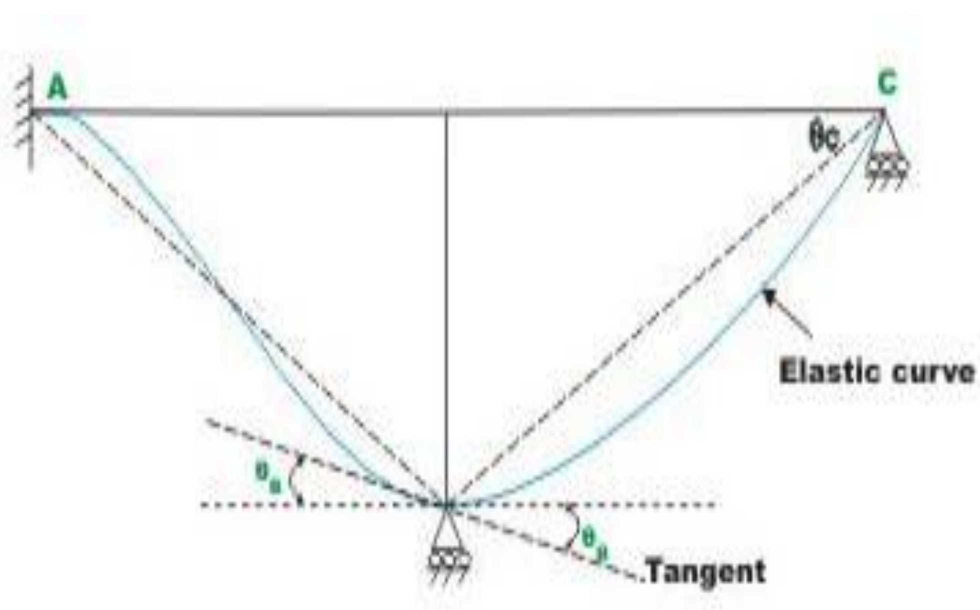
$$R_{BL} = -30.171\,\text{kN}(\downarrow)$$

$$R_{BR} = -13.714\,\text{kN}(\downarrow)$$

$$R_C = 13.714\,\text{kN}(\uparrow)$$

- The shear force and bending moment diagram and elastic curve Is shown in fig.





Elasctic curve

UNIT 2

Two-Hinged Arch

Analysis of two-hinged arch

- A typical two-hinged arch is shown in Fig. 33.
- 1a. In the case of two-hinged arch, we have four unknown reactions, but there are only three equations of equilibrium available. Hence, the degree of statical indeterminacy is one for twohinged arch.

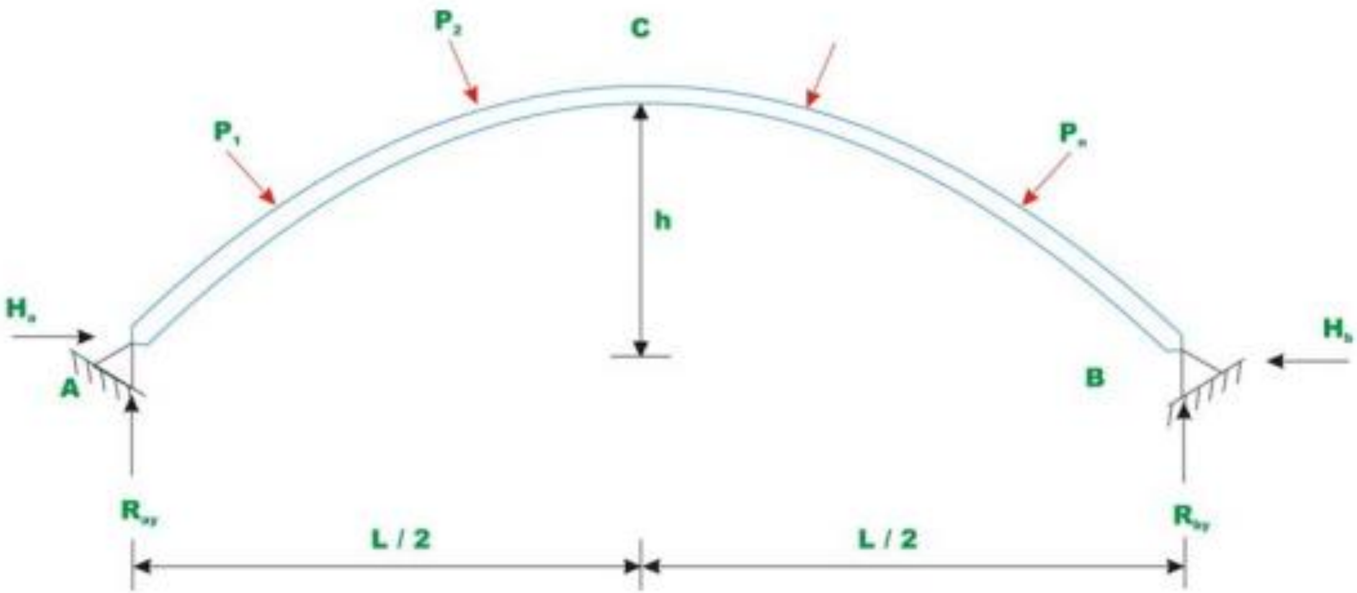


Fig. 33.1a Two - hinged arch.

- ARCH
- It transfers the load to end support by axial compression & partly by bending &shear action
- Due to equal distribution of stress, the section is fully utilized.
- In arches, its bending moment is low compare to beam.
- Bending Moment=W*xH*y

- CLASSIFICATION OF ARCHES Based on shape
- 1. Steel arches
- 2. R.C.C arches
- 3. Masonry arches
- Based on structural behavior
- [1]. Two hinged arches
- [2]. Fixed arches
- [3]. Three hinged arches

- ANALYSIS OF TWO HINGED ARCHES
- A two hinged arch is statically indeterminate to single degree, since there are four reaction components to be determined while the number of equations available from static equilibrium is only three. Considering H to be the redundant reaction, it can be found out by only by the use of Castigliano’s theorem of least work.

- Thus, assuming the horizontal span remaining unchanged, we have,
- $\partial U \ \partial H = 0$,
- Where
- U is the total strain energy stored in the arch. Here also, the strain energy stored due to thrust and shear will be

considered negligible in comparison to that due to bending.

$$U = \int \frac{M^2 ds}{2EI}$$

$$\frac{\partial U}{\partial H} = \int \frac{2M}{2EI} \cdot \frac{\partial M}{\partial H} \cdot ds = \int \frac{M}{EI} \frac{\partial M}{\partial H} \cdot ds$$

Now, $M = \mu - Hy$; $\frac{\partial M}{\partial H} = -y$

$$\frac{\partial U}{\partial H} = 0 = \int \frac{(\mu - Hy)(-y)}{EI} ds$$

$$H \int \frac{y^2 ds}{EI} = \int \frac{\mu\, yds}{EI}$$

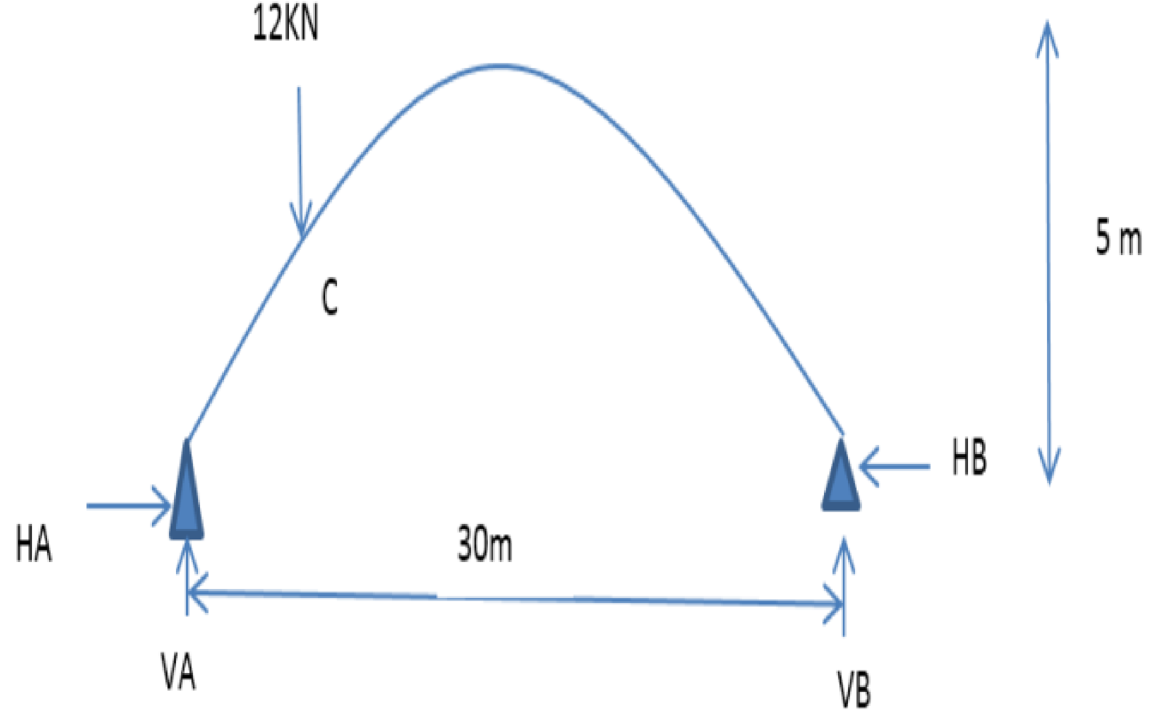
$$H = \frac{\int \frac{\mu y ds}{EI}}{\int \frac{y^2 ds}{EI}}$$

Taking $dx = ds\cos\theta$ so, $ds = dx\sec\theta$

From that we get,

$$H = \frac{\int \mu . y dx}{\int y^2 dx}$$

- A parabolic Arch hinged at the ends has a span 30 m and rise 5m. A concentrated load of 12 kN acts at 10m from the left hinge. The second moment of area varies as the secant of the slope of the rib axis. Calculate the horizontal thrust and the reactions at the hinges. Also calculate the maximum bending moment anywhere on the arch.


12KN
5 m
C
HB
HA
30m
VA
VB

$\Sigma M@A = 0,$

$V_b*30 = 12*10$

$V_b = 4$ KN

Now, $V_a + V_b = 12$ kN

So $V_a = 8$ kN

Equation of parabola,

$$Y = \frac{4hx(L-x)}{L^2}$$

$$Y = \frac{4*5*x(30-x)}{30^2}$$

$$Y = \frac{x(30-x)}{45}$$

Now, horizontal thrust can be found out from the equation

$$H = \frac{\int \mu . y dx}{\int y^2 dx}$$

$\mu = 8x$, for AC portion

$\mu = 8x - 12(x-10)$, for CB portion

$\mu = (120 - 4x)$

$$\int_0^{30} \mu\, y dx = \int_0^{10} 8xy dx + \int_{10}^{30} (120 - 4x) y dx$$

$$= \int_0^{10} \frac{8x^2(30-x)}{45} + \int_{10}^{30} \frac{4x(30-x)^2}{45}$$

$$\int_0^{30} \mu\, y dx = \frac{44000}{9}$$

$$\int_0^{30} y^2 dx = \int_0^{30} \frac{x^2(30-x)^2 dx}{45^2}$$

$$= \frac{1}{45^2} \int_0^{30} (900x^2 + x^4 - 60x^2 x) dx$$

$$\int_0^{30} y^2 dx = 400$$

So, $H = \frac{44000}{9*400}$

$H = 12.22$ kN

Now, Resultant reaction $R_a = \sqrt{8^2 + 12.22^2} = 14.61$ kN

$\tan \theta a = \frac{8}{12.22} = 0.655$

$\theta_a = 33.21°$

$R_b = \sqrt{4^2 + 12.22^2} = 12.85$ kN

$\theta_b = \tan^{-1} \frac{4}{12.22}$

Maximum BM will occur in AC, just below the load

Rise of arch at that point

$Y = \frac{x(30-x)}{45} = \frac{10(30-10)}{45} = 40/9$m

$M_{max} = 8*10 - 12.22*40/9 = 25.49$ kNm

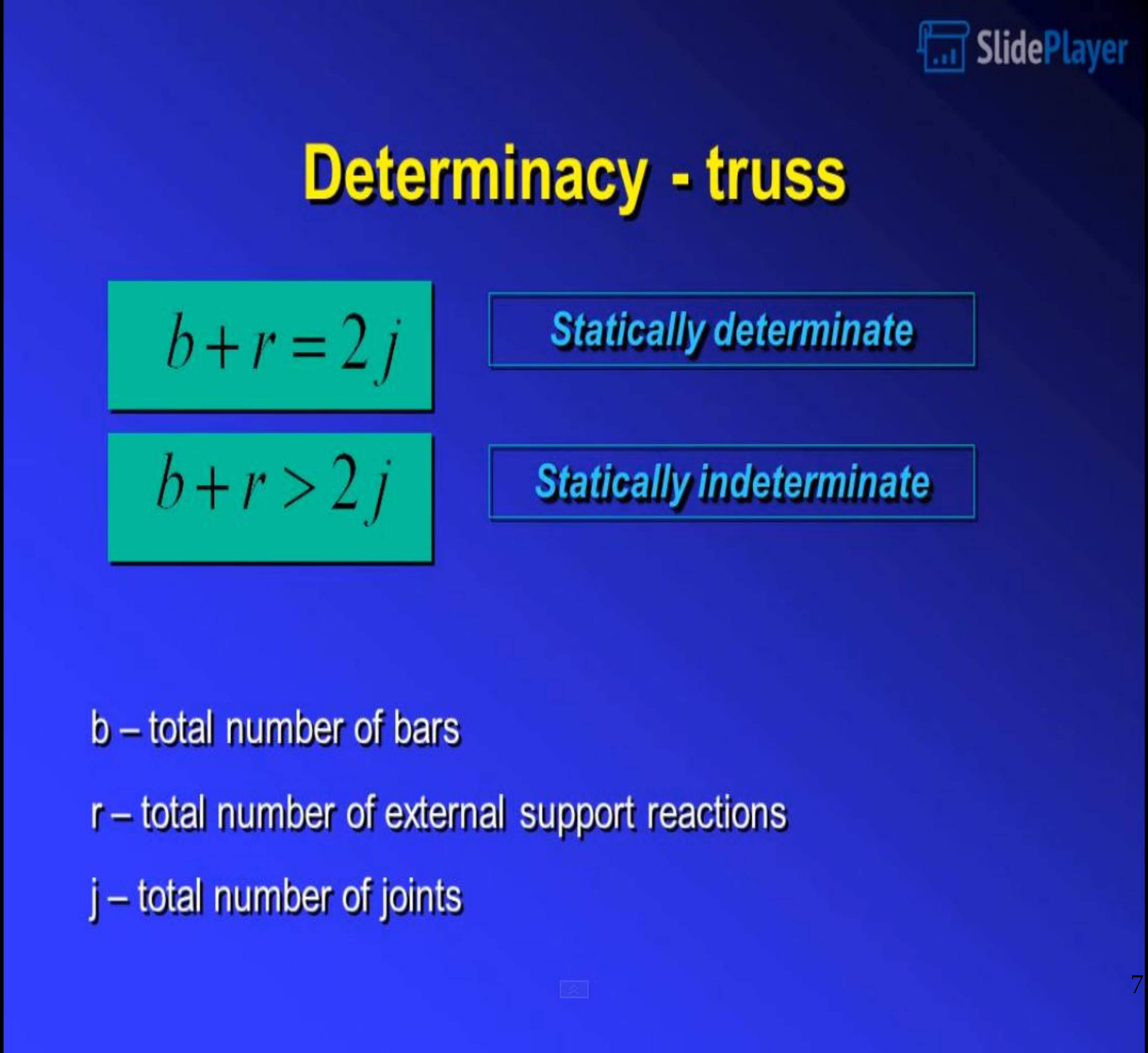
UNIT 3
APPROXIMATE METHODS OF ANALYSIS

Introduction

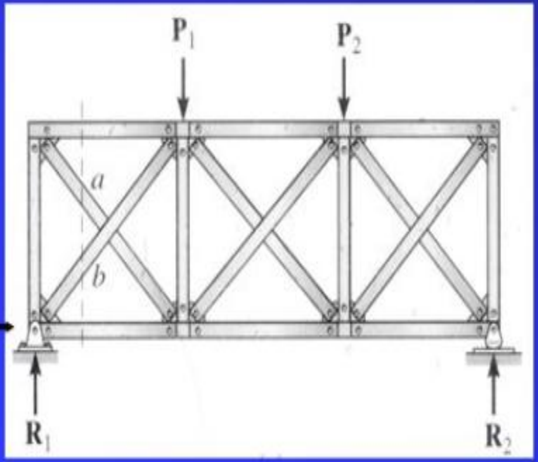
- Using approximate methods to analyse statical
  indeterminate trusses and frames
- The methods are based on the way the structu
  deforms under the load
- Trusses
- Portal frames with trusses
- Vertical loads on building frames
- Lateral loads on building frames
  - Portal method
  - Cantilever method

Approximate Analysis

- Statically determinate structure – the force equilibrium equation is sufficient to find the support reactions
- Approximate analysis – to develop a simple model of the structure which is statically determinate to solve a statically indeterminate problem
- The method is based on the way the structure deforms under loads
- Their accuracy in most cases compares favourably with more exact methods of analysis (the statically indeterminate analysis)

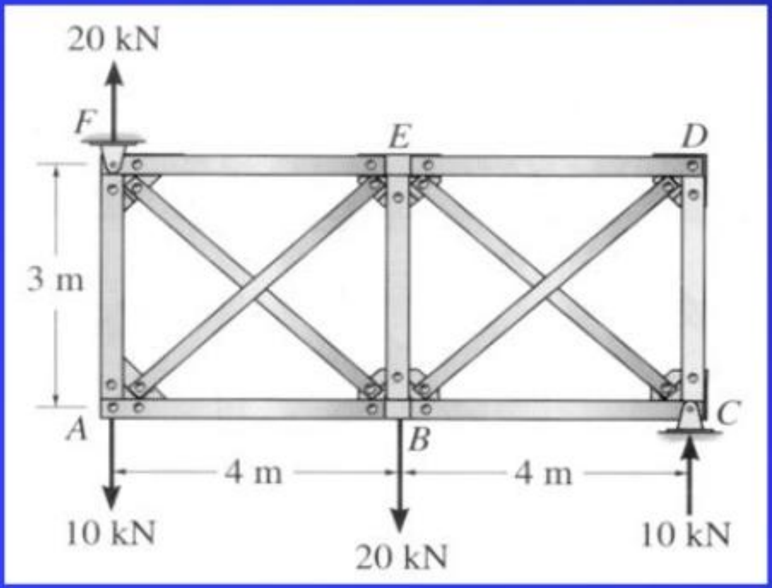

Determinacy - truss
$b + r = 2j$
Statically determinate
$b + r > 2j$
Statically indeterminate
b – total number of bars
r – total number of external support reactions
j – total number of joints

Trusses

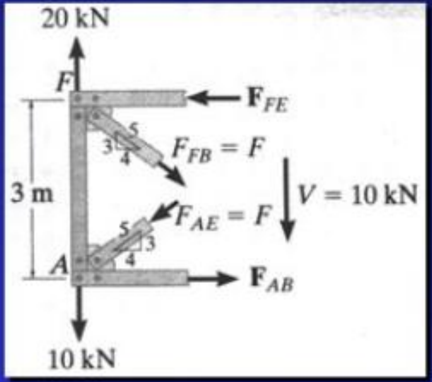
real structure	approximation
 *b=16, r=3, j=8* *b+r = 19 > 2j=16* *The truss is statically indeterminate to the third degree*	**Method 1: Design long, slender diagonals - compressive diagonals are assumed to be a zero force member and all panel shear is resisted by tensile diagonal only** **Method 2: Design diagonals to support both tensile and compressive forces - each diagonal is assumed to carry half the panel shear.** *Three assumptions regarding the bar forces will be required*

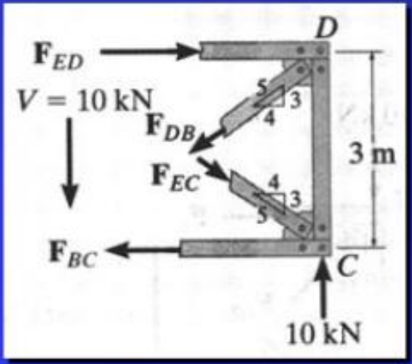
Example 1 - trusses

Determine (approximately) the forces in the members. The diagonals are to be designed to support both tensile and compressive forces.



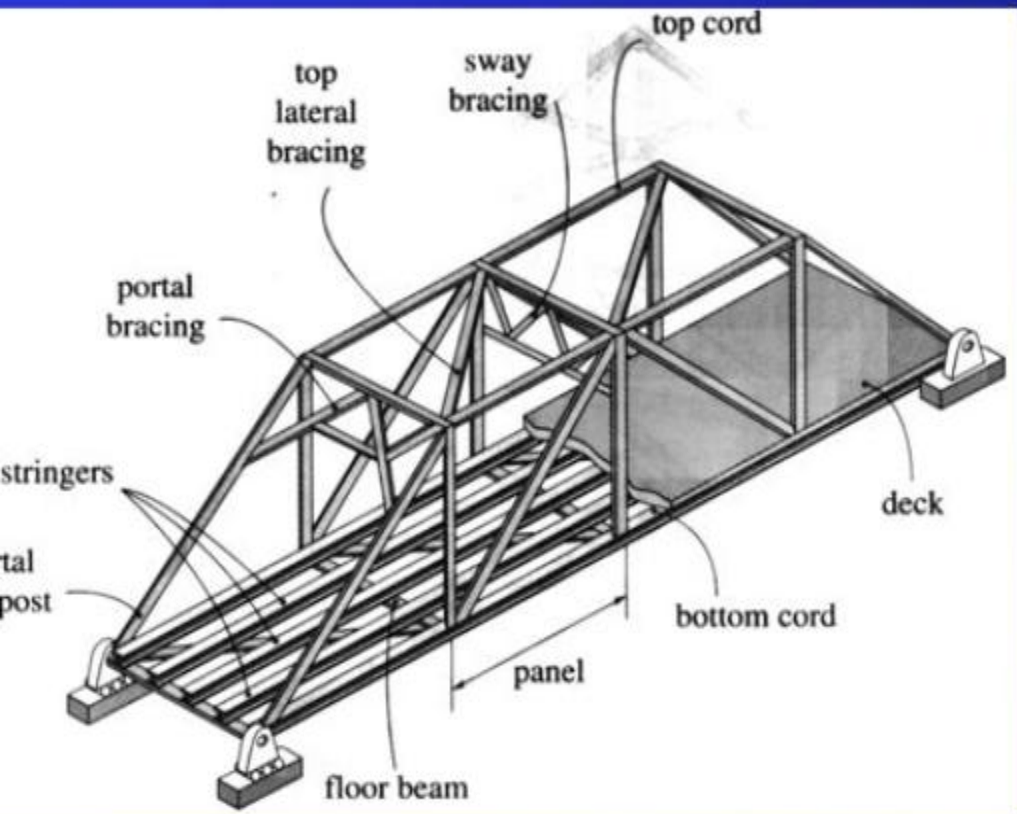
$$F_{FB} = F_{AE} = F$$





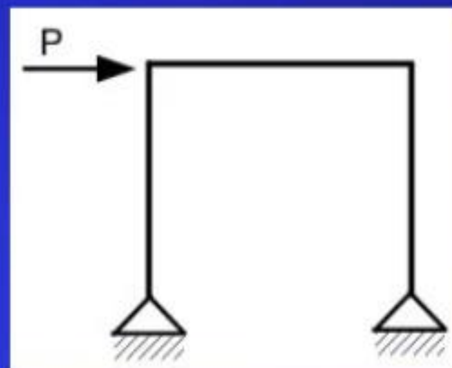
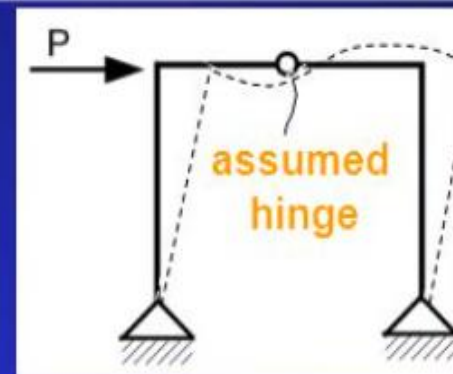
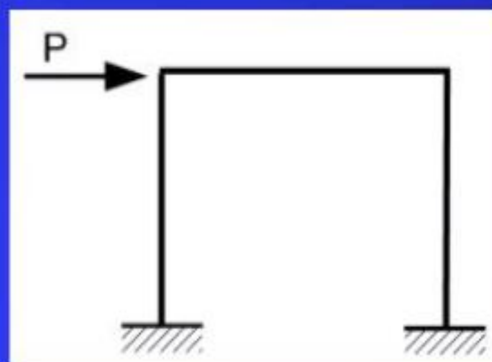
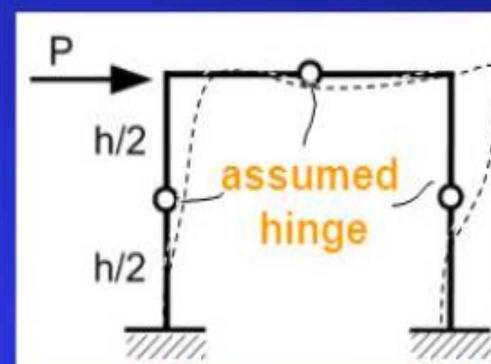
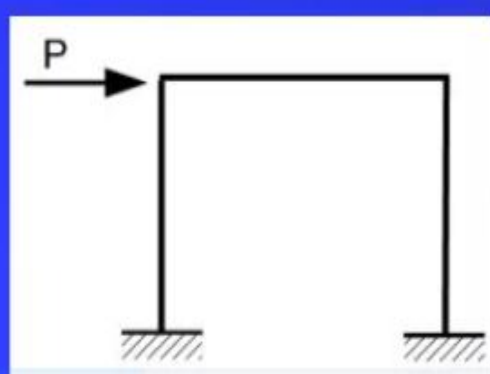
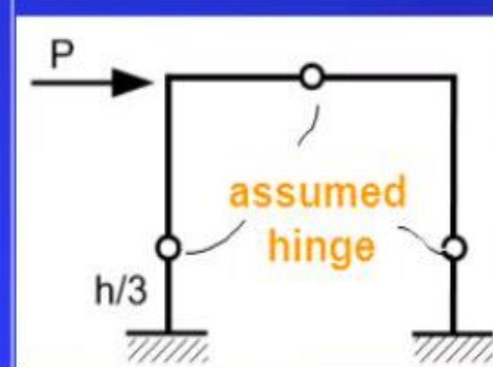
$$F_{DB} = F_{EC} = F$$

Portal frames – lateral loads



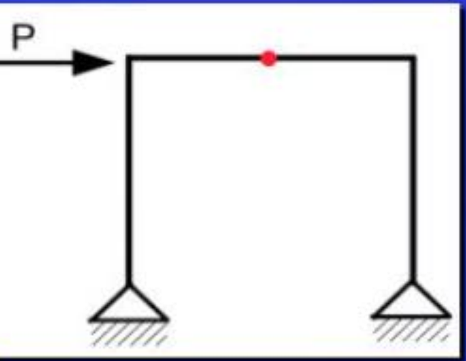
- Portal frames are frequently used over the entrance of a bridge
- Portals can be pin supported, fixed supported or supported by partial fixity

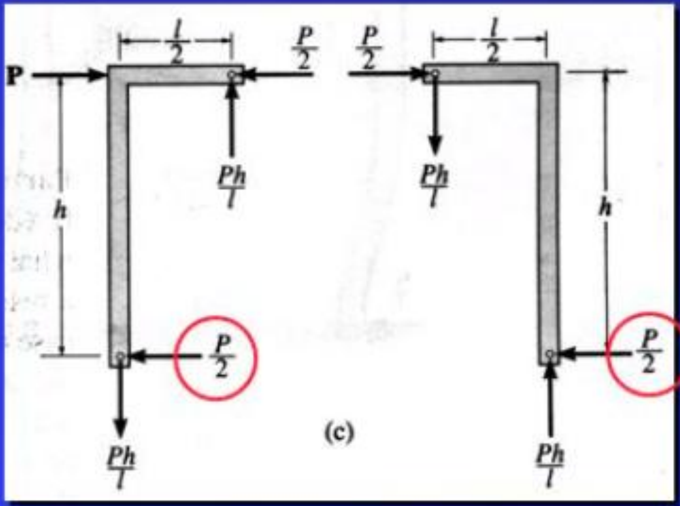
Portal frames – lateral loads

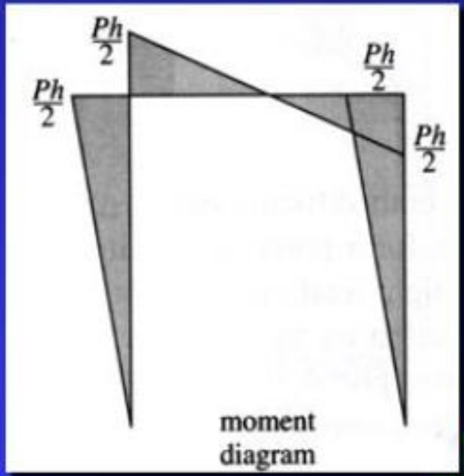
real structure		approximation	
-supported ne assumption ust be made			A point of inflection is located *approximately* at the girder's midpoint
d -supported hree assumption ust be made			Points of inflection are located *approximately* at the midpoints of all three members
tial fixity			Points of inflection for columns are located *approximately* at h/3 and the centre of the girder

Pin-Supported Portal Frames

- A point of inflection – where the moment changes from positive bending to negative bending.
- Bending moment is zero at this point.



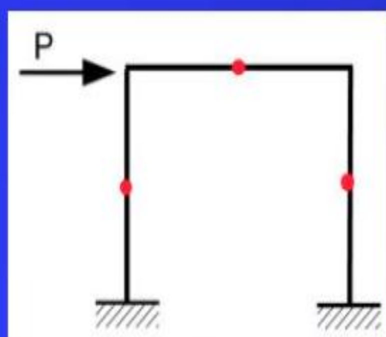


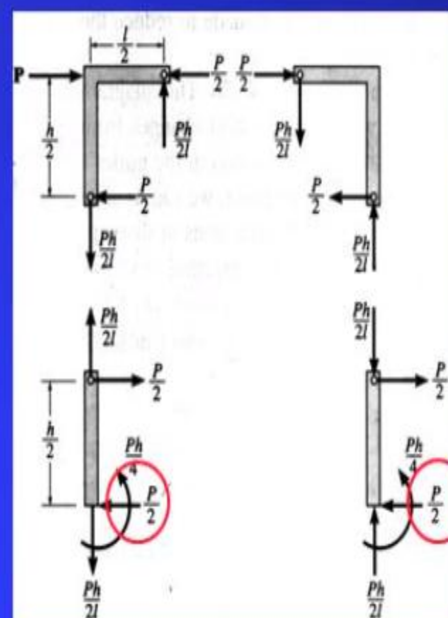


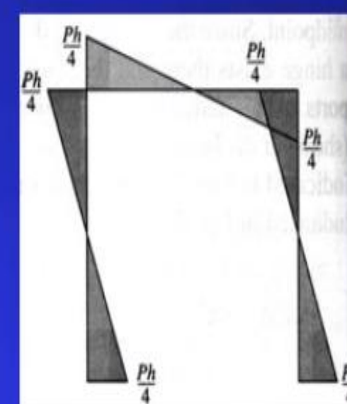
The horizontal reactions (shear) at the base of each column are ***equal***

Fixed-Supported Portal Frames

- A point of inflection – where the moment changes from positive bending to negative bending.
- Bending moment is zero at this point.



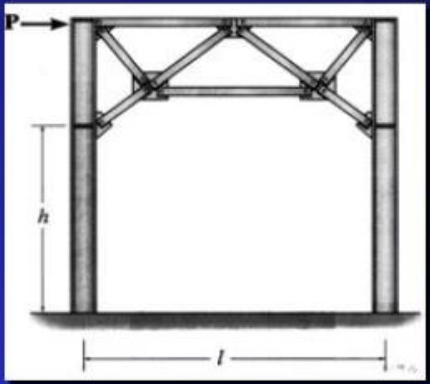


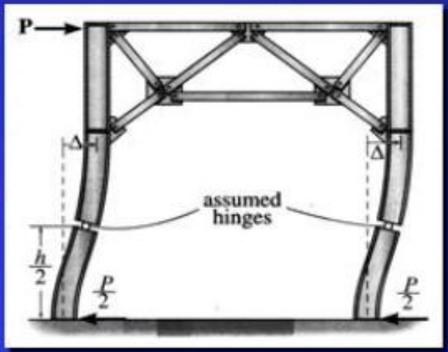


The horizontal reactions (shear) at the base of each column are *equal*

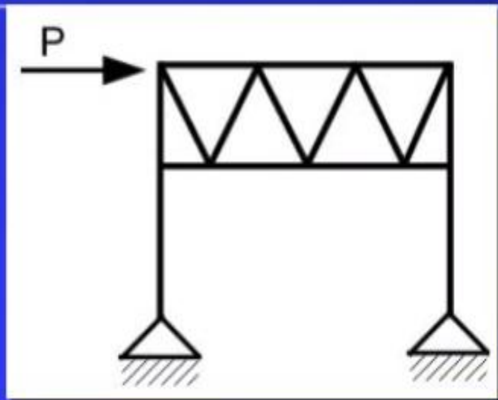
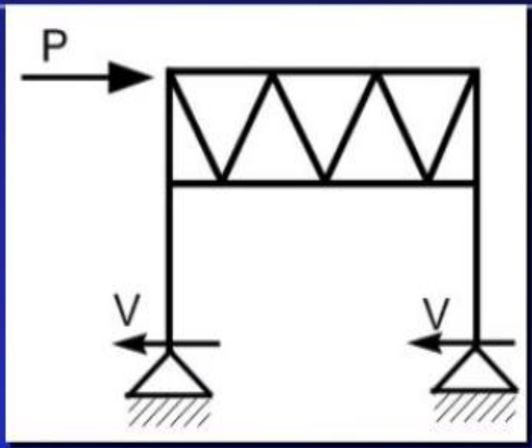
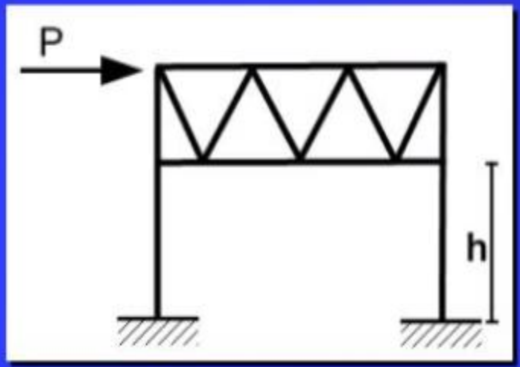
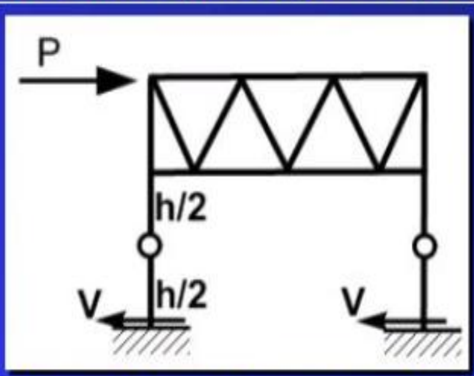
Frames with trusses

- When a portal is used to span large distance, a truss may be used in place of the horizontal girder
- The suspended truss is assumed to be pin connected at its points of attachment to the columns
- Use the same assumptions as those used for simple portal frames



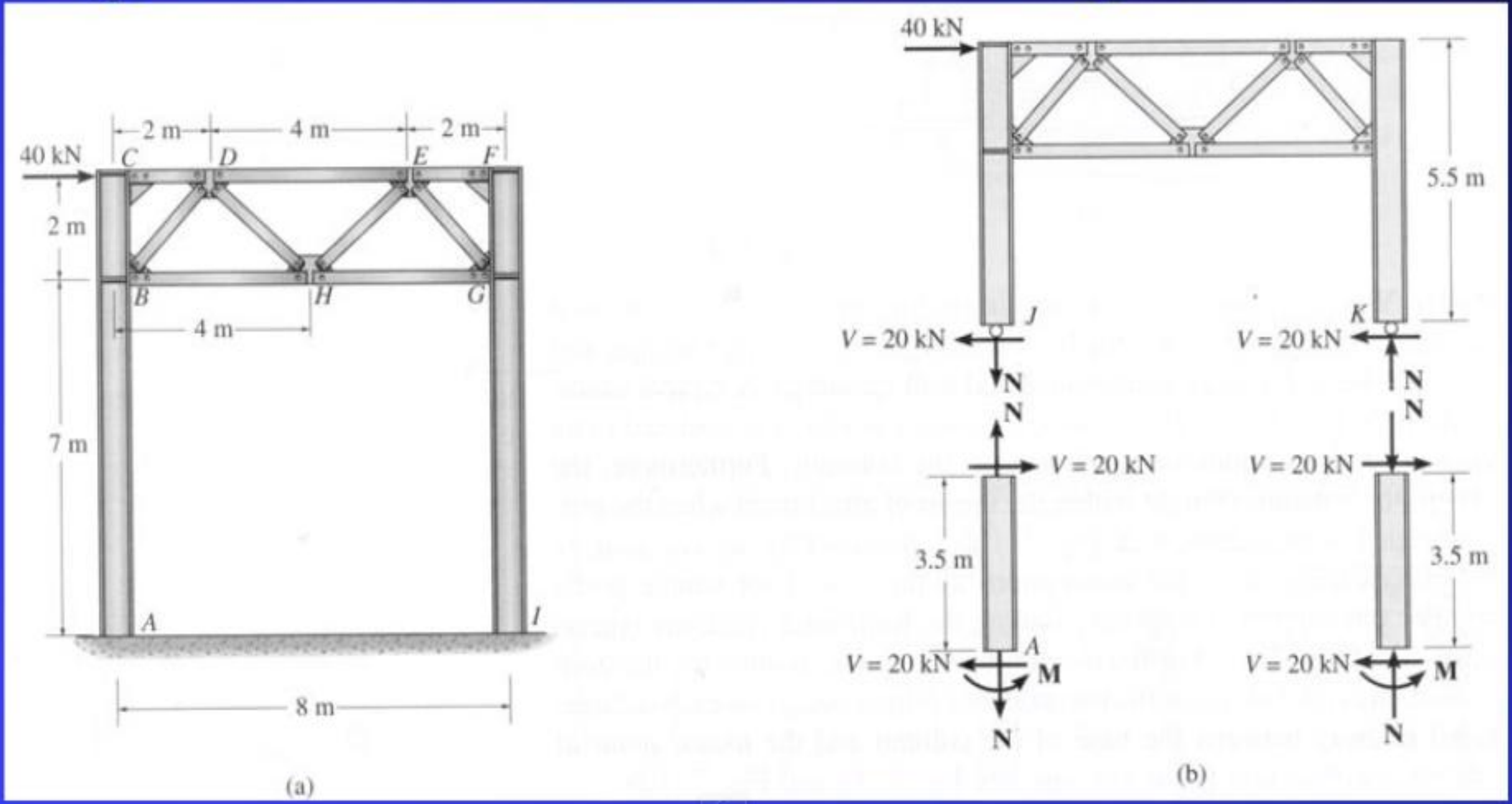


Frames with trusses

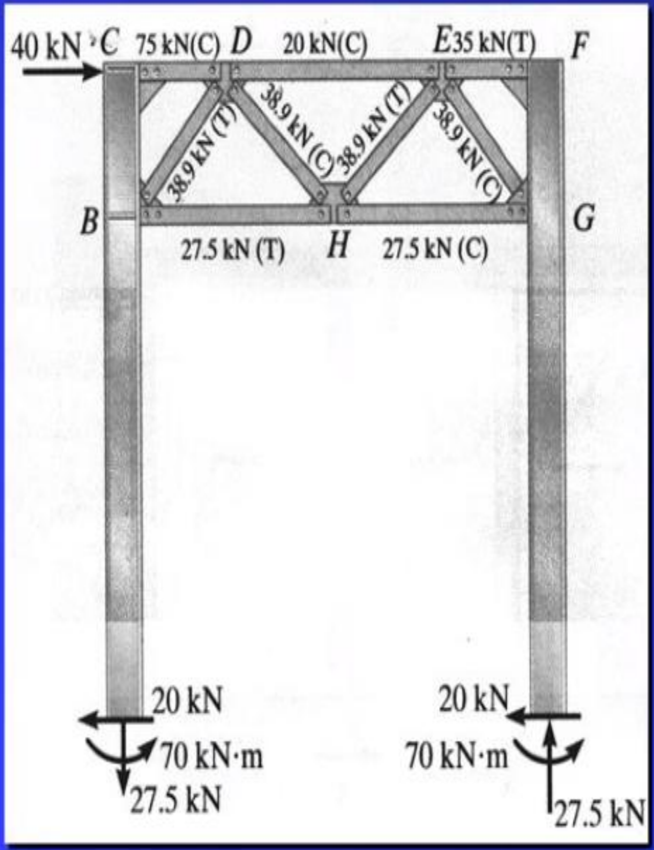
real structure	approximation
 pin supported columns pin connection truss-column	 the horizontal reactions (shear) are equal
 fixed supported columns pin connection truss-column	 horizontal reactions (shear) are equal there is a zero moment (hinge) on each column

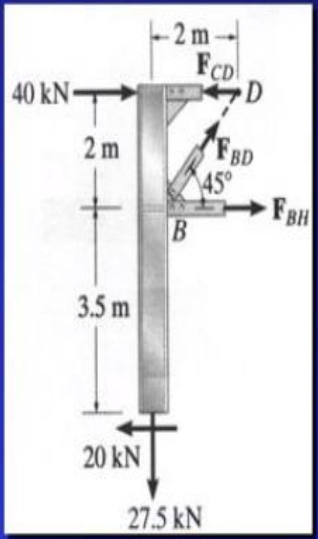
Example 2 – Frame with trusses

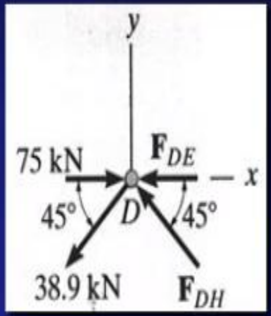
Determine by approximate methods the forces acting in the members of the Warren portal.

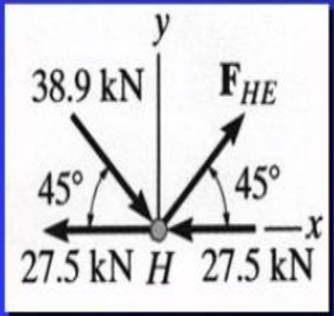


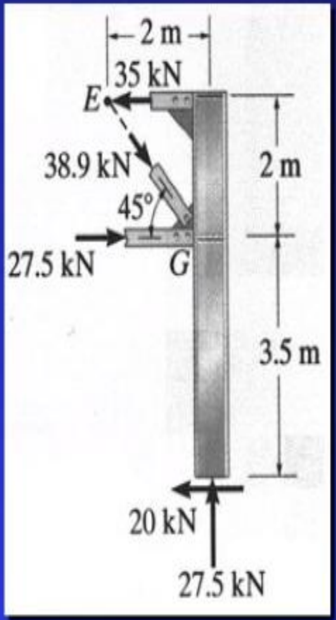
Example 2 (contd)





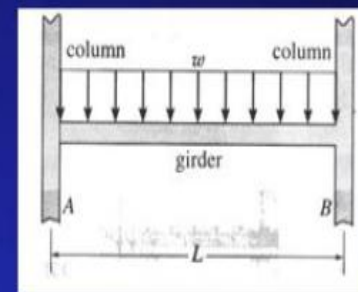


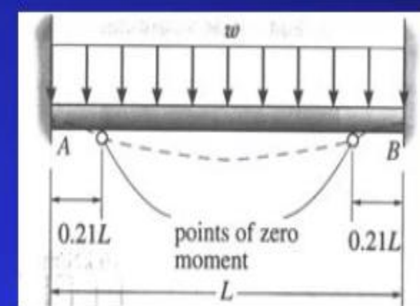




Building frames – vertical loads

- Building frames often consist of girders that are rigidly connected to columns
- The girder is statically indeterminate to the third degree – require 3 assumptions

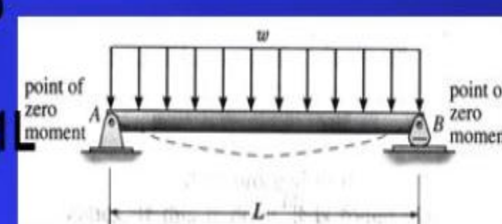




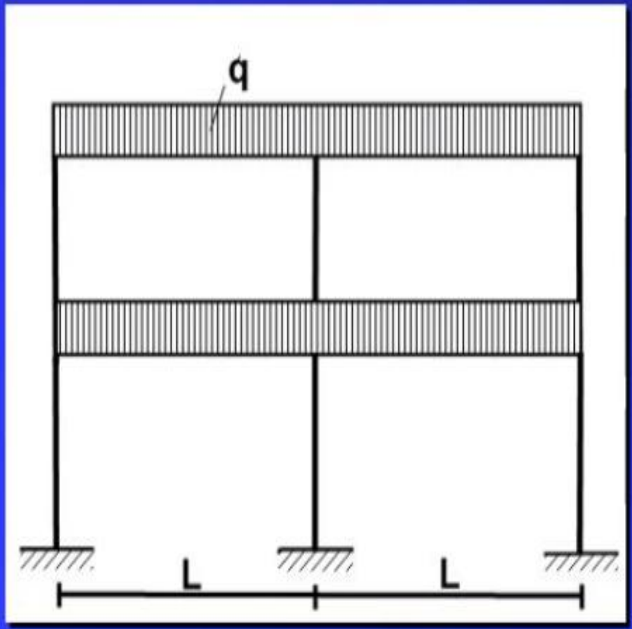
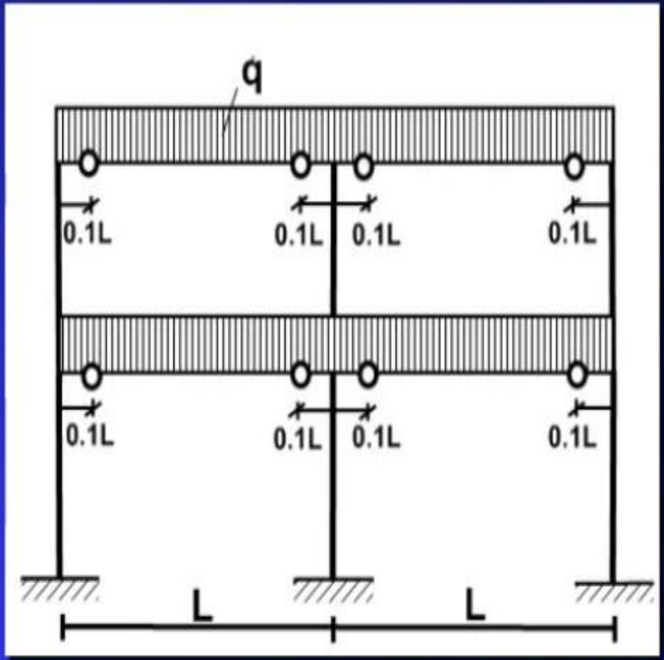
If the columns are extremely stiff

Average point between the two extremes = (0.21L+0)/2 ≈ 0.1L

If the columns are extremely flexible



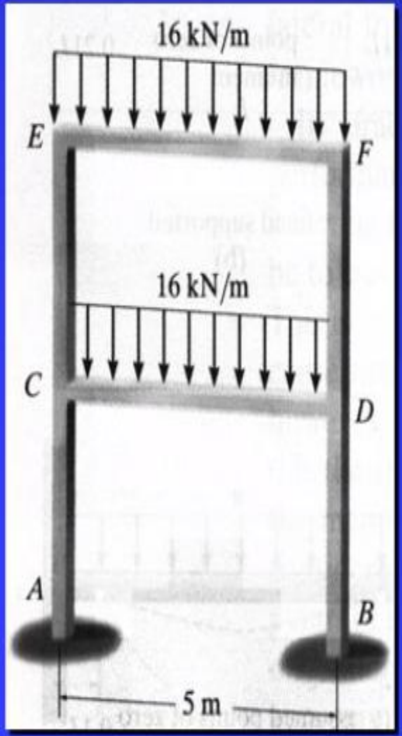
Building frames – vertical loads

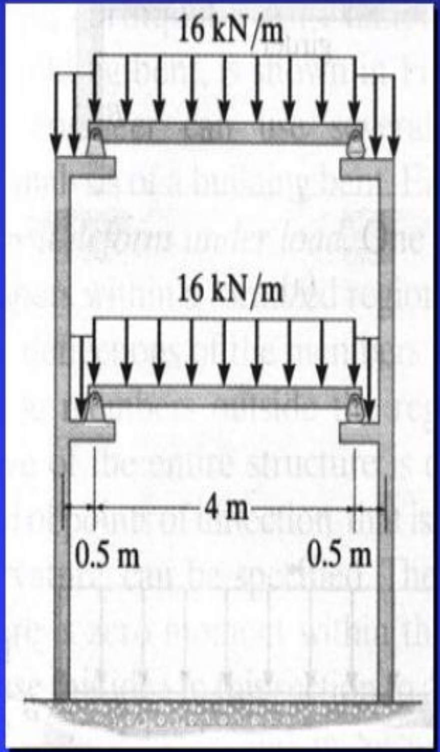
real structure	approximation
	

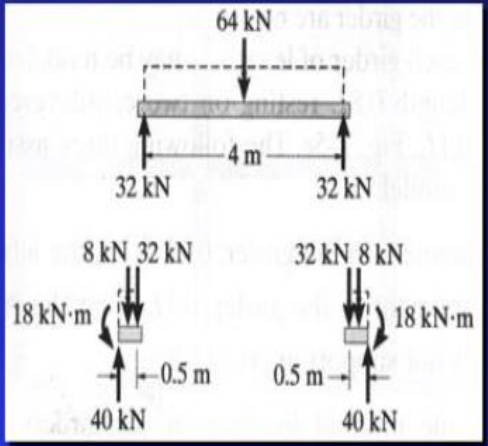
1. There is zero moment (hinge) in the girder 0.1L from the left support
2. There is zero moment (hinge) in the girder 0.1L from the right support
3. The girder does not support an axial force.

Example 3 – Vertical loads

Determine (approximately) the moment at the joints E and C caused by members EF and CD.

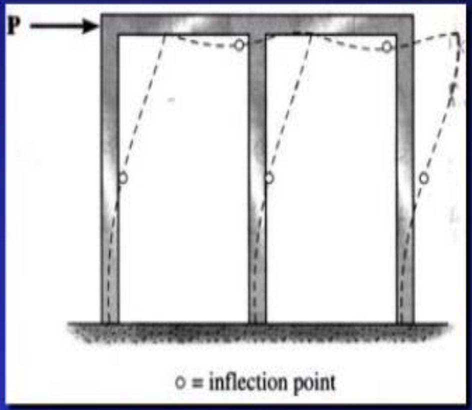


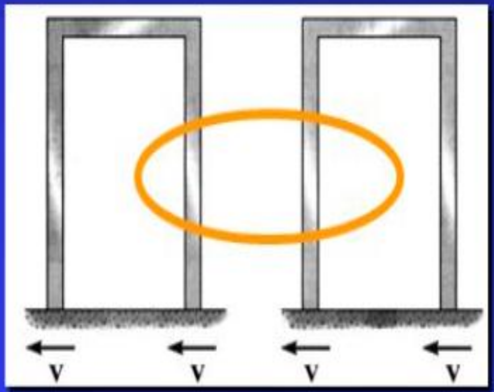




Building frames – lateral loads: Portal method

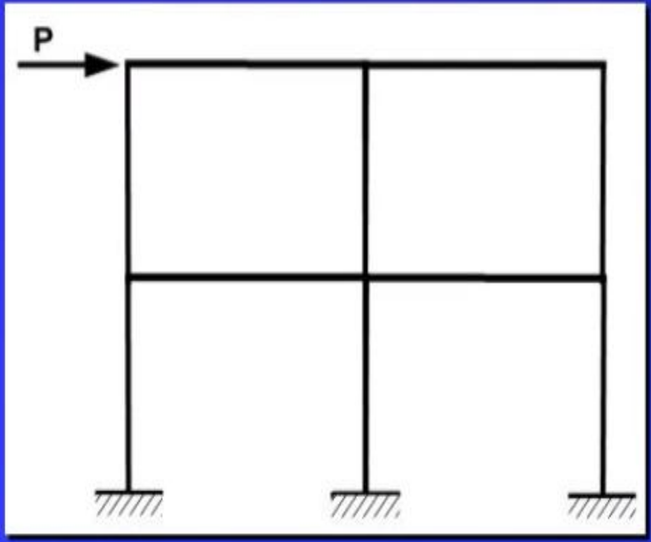
- A building bent deflects in the same way as a portal frame
- The assumptions would be the same as those used for portal frames
- The *interior columns* would represent the effect of *two portal columns*

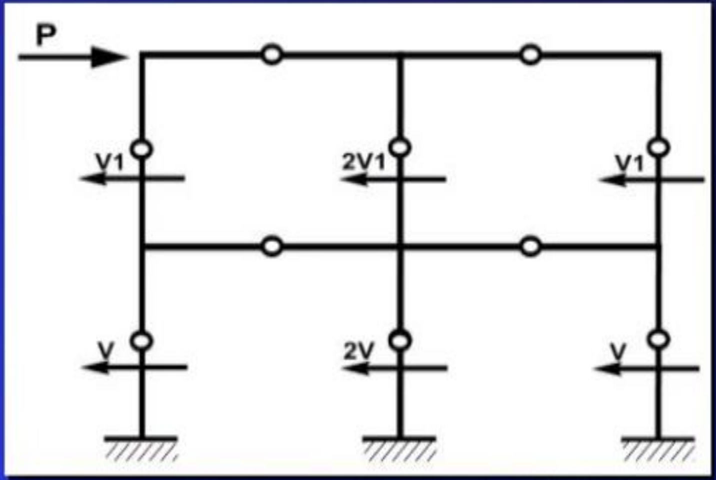




Building frames – lateral loads: Portal method

real structure	approximation





The method is most suitable for buildings having low elevation and uniform framing

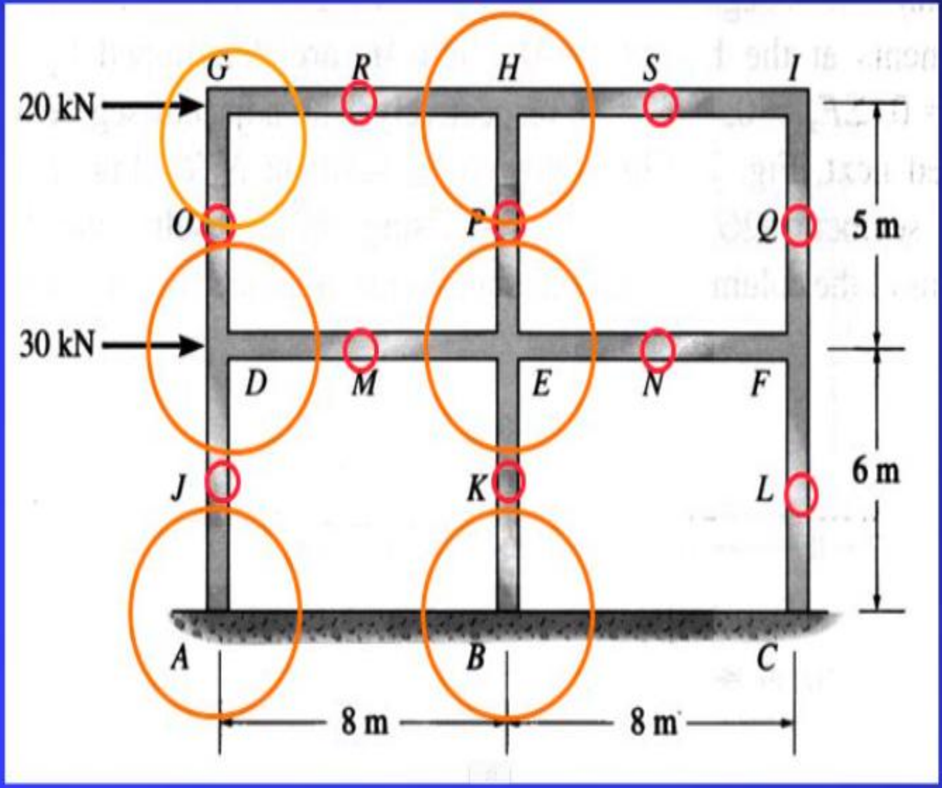
1. A hinge is placed at the centre of each girder, since this is assumed to be a point of zero moment.

2. A hinge is placed at the centre of each column, this to be a point of zero moment.

3. At the given floor level the shear at the interior column hinges is twice that at the exterior column hinges

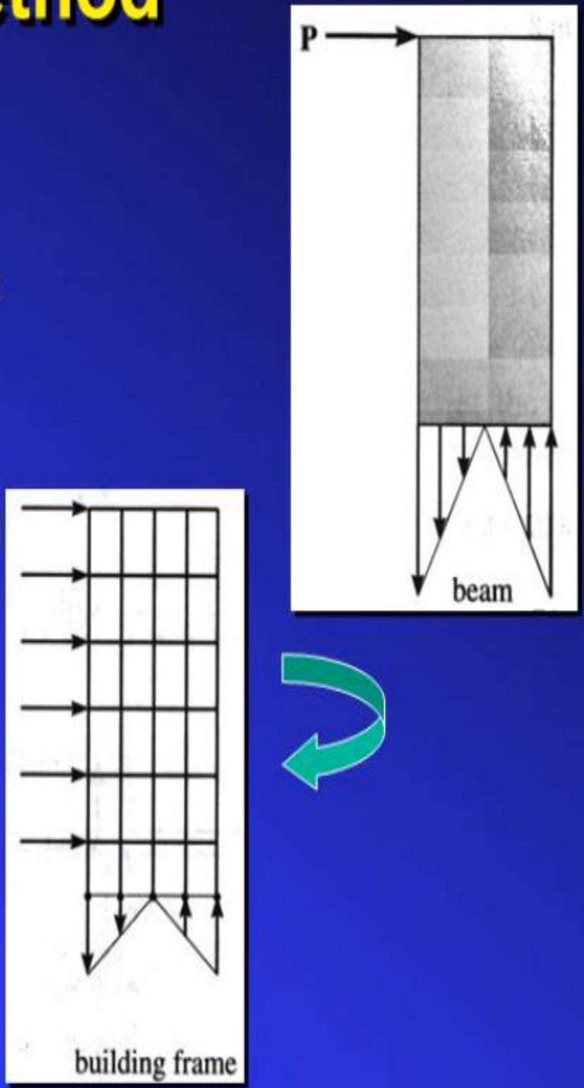
Example 4 – Portal method

Determine (approximately) the reactions at the base of the columns of the frame.



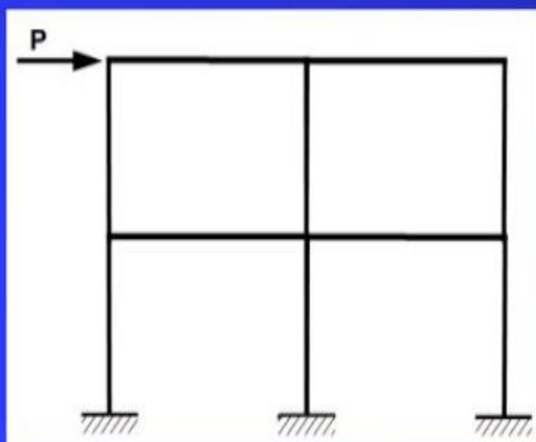
Building frames – lateral loads: Cantilever method

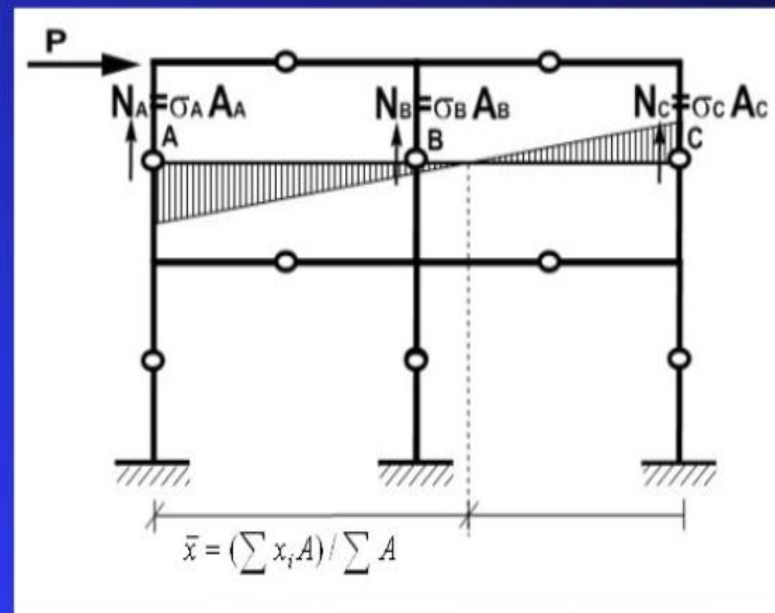
- The method is based on the same action as a long cantilevered beam subjected to a transverse load
- It is reasonable to assume the axial stress has a linear variation from the centroid of the column areas



Building frames – lateral loads: Cantilever method

real structure	approximation





The method is most suitable if the frame is tall and slender, or has columns with different cross sectional areas.

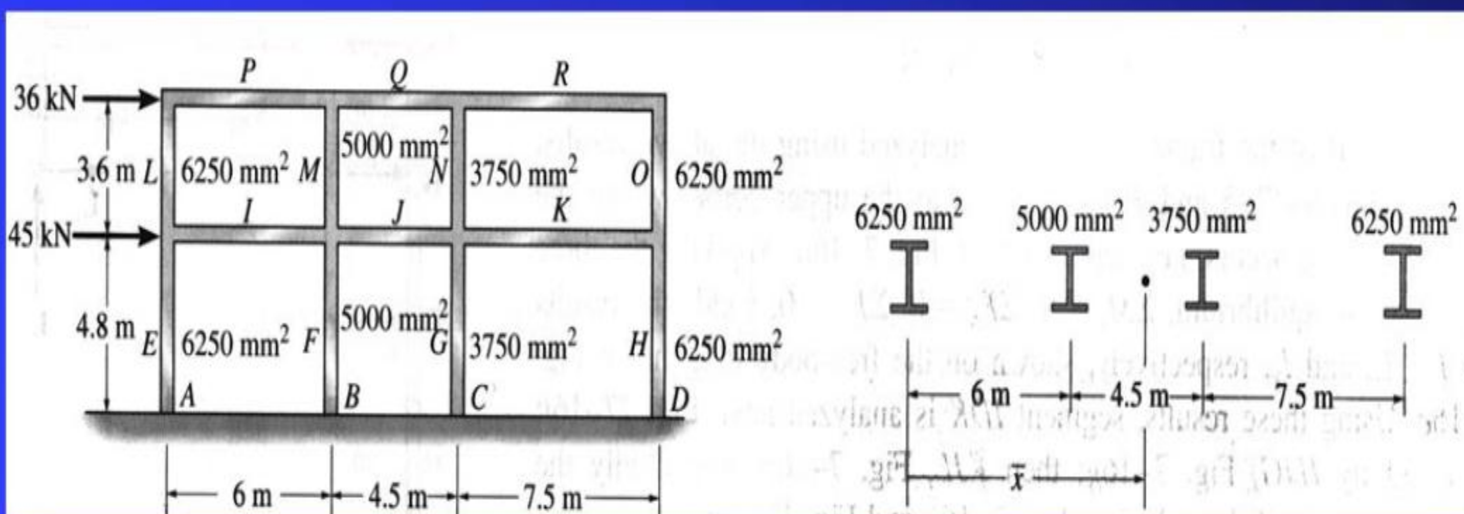
1 zero moment (hinge) at the centre of each girder

2. zero moment (hinge) at the centre of each column

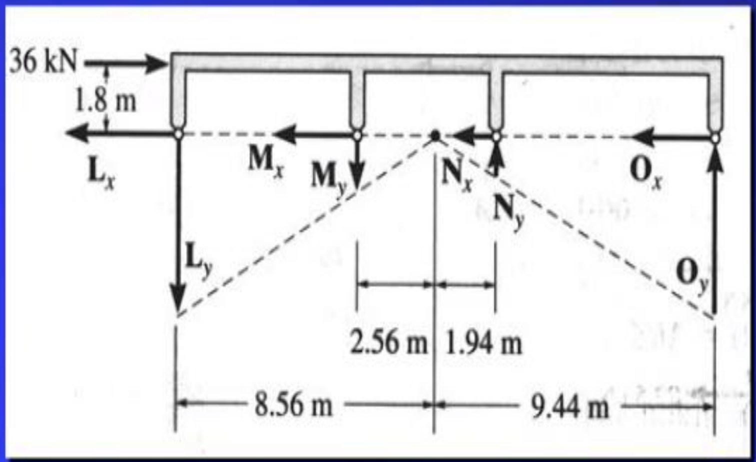
3. The axial stress in a column is proportional to its distance from the centroid of the cross-sectional areas of the columns at a given floor level

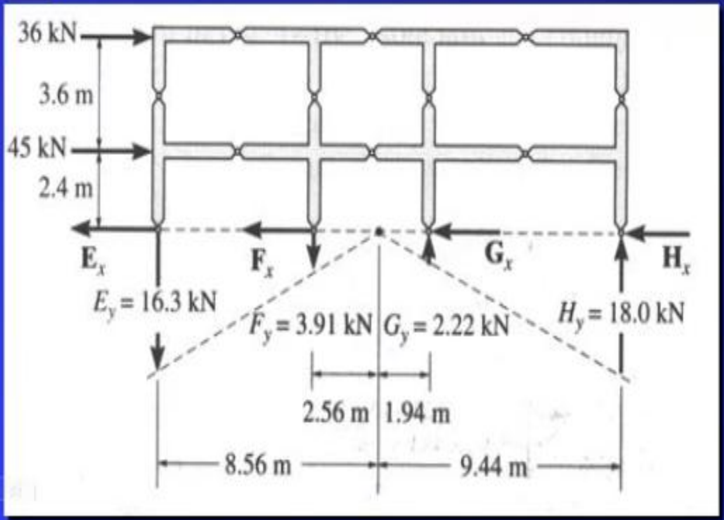
Example 5 – Cantilever method

Show how to determine (approximately) the reactions at the base of the columns of the frame.



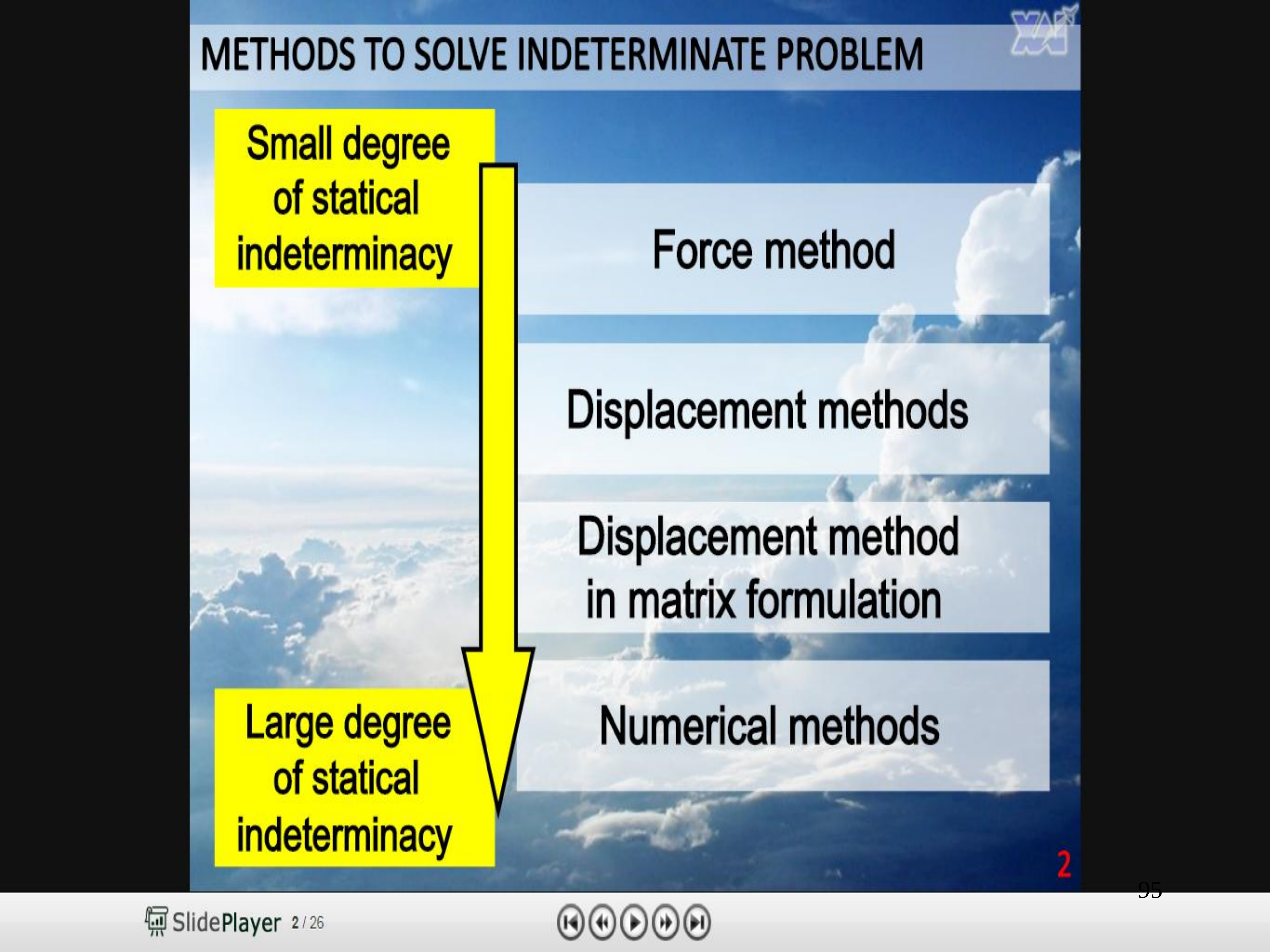
Example 5 – Cantilever method





UNIT 4

- MATRIX METHODS OF ANALYSIS


METHODS TO SOLVE INDETERMINATE PROBLEM
Small degree of statical indeterminacy
Force method
Displacement methods
Displacement method in matrix formulation
Numerical methods
Large degree of statical indeterminacy
2
SlidePlayer 2 / 26

ADVANTAGES AND DISADVANTAGES OF MATRI
METHODS

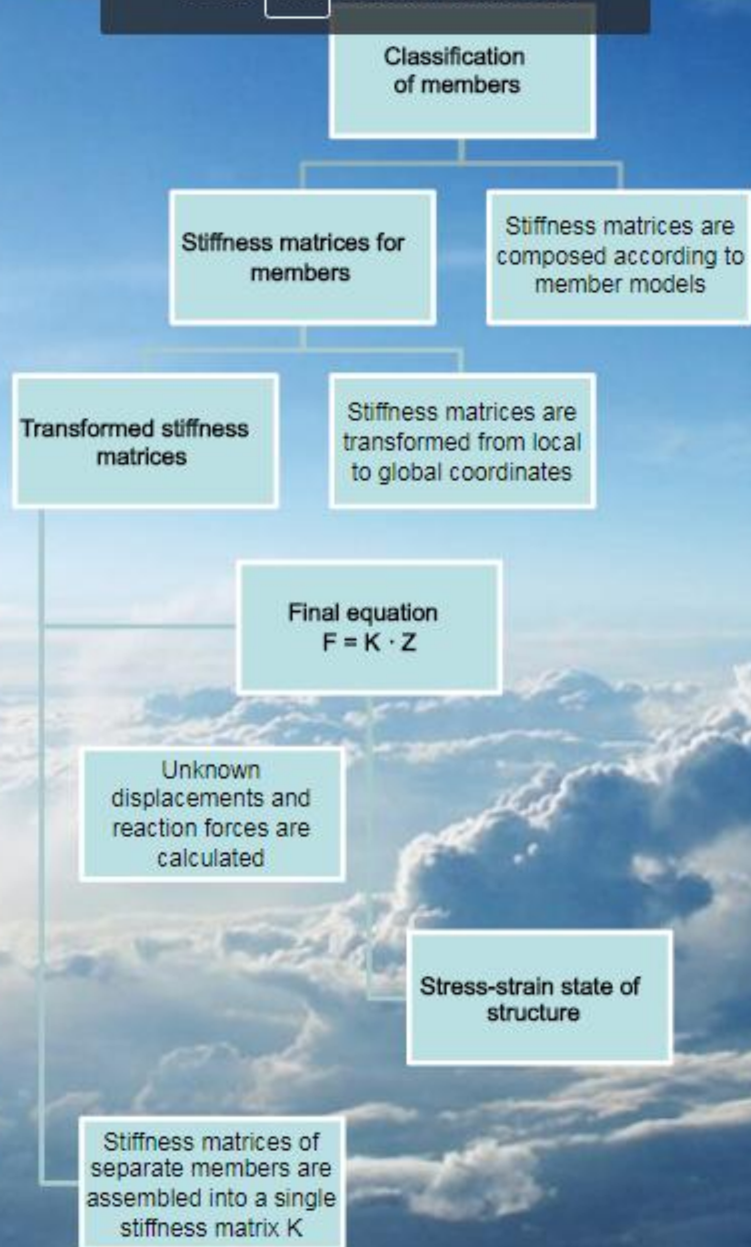
Advantages:

- very formalized and computer-friendly;
- versatile, suitable for large problems;
- applicable for both statically determinate ar indeterminate problems.

Disadvantages:

- bulky calculations (not for hand calculation
- structural members should have some cert number of unknown nodal forces and nodal displacements; for complex members such a beams and arbitrary solids this requires som discretization, so no analytical solution is pos

FLOWCHART OF MATRIX METHOD



STIFFNESS MATRIX OF STRUCTURAL MEMBER

Stiffness matrix ($\boldsymbol{K}$) gives the relation betwe
of nodal forces ($\boldsymbol{F}$) and nodal displacements

$$F = \begin{pmatrix} F_1 \\ F_2 \\ \cdots \\ F_n \end{pmatrix}$$

$$F = K \cdot Z$$

$$Z =$$

$$K = \begin{pmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ \cdots & \cdots & & \cdots \\ k_{n1} & k_{n2} & \cdots & k_{nn} \end{pmatrix}$$

EXAMPLE OF MEMBER STIFFNESS MATRIX

Stiffness relation for a rod:

$$F_i = -\frac{EA}{L}\cdot\left(x_j - x_i\right)$$

$$F_j = \frac{EA}{L}\cdot\left(x_j - x_i\right)$$

$$F = K \cdot Z \qquad\qquad F = \begin{pmatrix} F_i \\ F_j \end{pmatrix}$$

Stiffness matrix:

$$K = \begin{pmatrix} EA/L & -EA/L \\ -EA/L & EA/L \end{pmatrix}$$

ASSEMBLY OF STIFFNESS MATRICES

To assemble stiffness matrices of separate r
into a single matrix for the whole structure, w
simply add terms for corresponding displace
Physically, this procedure represent the usag
compatibility and equilibrium equations.

ASSEMBLY OF STIFFNESS MATRICES - EXAMPLE

Let's consider a system of two rods:

$$\begin{pmatrix} F_i \\ F_j \end{pmatrix} = \frac{EA}{L} \cdot \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_i \\ x_j \end{pmatrix}$$

$$\begin{pmatrix} F_j \\ F_k \end{pmatrix} = \frac{EA}{L} \cdot \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_j \\ x_k \end{pmatrix}$$

$$\begin{pmatrix} F_i \\ F_j \\ F_k \end{pmatrix} = \frac{EA}{L} \cdot \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_i \\ x_j \\ x_k \end{pmatrix}$$

SOLUTION USING MATRIX METHOD - EXAMPL

Given $\quad F_k = 10$

Boundary conditions: $\quad Z_i = 0$

Equilibrium conditions: $\quad F_j = 0$

Matrix equation: $\quad F = \frac{E \cdot A}{L} \cdot K \cdot Z$

Solution of system: $\quad \mathrm{Res} := \mathrm{Find}(F, Z)$

Forces:

$$\mathrm{Res}_0 = \begin{pmatrix} -10 \\ 0 \\ 10 \end{pmatrix}$$

Displacements

$$\mathrm{Res}_1 = \begin{pmatrix} 0 \\ 5 \\ 10 \end{pmatrix} \cdot 10^{-6}$$

SOLUTION USING MATRIX METHOD - EXAMPL

Given

Boundary conditions: $F_j = 10 \quad Z_i = 0$

Matrix equation: $F = \frac{E \cdot A}{L} \cdot K \cdot Z$

Solution of system: $Res := Find(F, Z$

Forces:

$$Res_0 = \begin{pmatrix} -5 \\ 10 \\ -5 \end{pmatrix}$$

Displacements

$$Res_1 = \begin{pmatrix} 0 \\ 2.5 \\ 0 \end{pmatrix} \cdot 10^{-}$$

TRANSFORMATION MATRIX

Transformation matrix is used to transform n
displacements and forces from local to globa
coordinate system (CS) and vice versa:

$$\overline{F} = T \cdot F \qquad \overline{Z} = T \cdot Z$$

Transformation matrix is always orthogonal,
inverse matrix is equal to transposed matrix:

$$T^{-1} = T^{M}$$

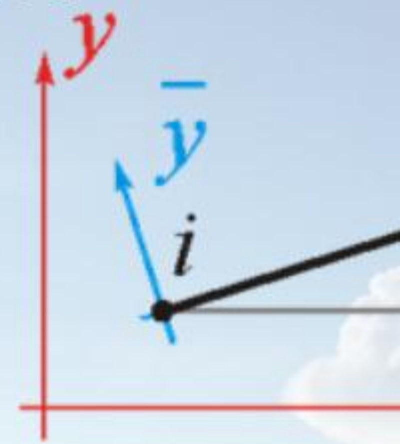
The transformation from local CS to global C

$$F = T^{T} \cdot \overline{F} \qquad Z = T^{T} \cdot \overline{Z}$$

TRANSFORMATION MATRIX EXAMPLE

For simplest member (rod) we get:

$$Z = \begin{pmatrix} x_i \\ y_i \\ x_j \\ y_j \end{pmatrix} \quad \overline{Z} = \begin{pmatrix} \overline{x}_i \\ \overline{y}_i \\ \overline{x}_j \\ \overline{y}_j \end{pmatrix}$$



$$\overline{Z} = T \cdot Z$$

$$T = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & \cos(\theta) \\ 0 & 0 & -\sin(\theta) \end{pmatrix}$$

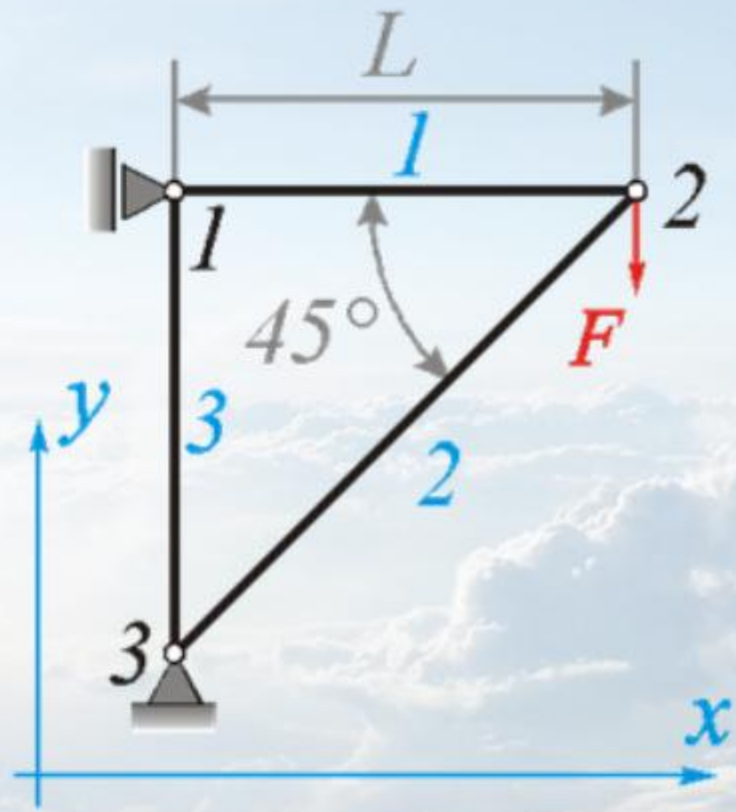
TRANSFORMATION MATRIX

To transform the stiffness matrix from local C
global CS, the following formula is used:

$$K = T^T \cdot \overline{K} \cdot T$$

EXAMPLE FOR A TRUSS

The truss has three members, thus 6 degree
freedom. The stiffness matrix will be 6x6.



EXAMPLE FOR A TRUSS

$\mathbf{ORIGIN} := 1$

Number of DOFs $\quad d := 6 \qquad u := 1 .. d$

Number of members $\quad n := 3 \qquad i := 1 .. n$

Geometrical and physical properties: $\mathbf{EA}$

Stiffness matrix in local CS:

$$K_{LCS_i} := \frac{EA}{L_i} \cdot \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

EXAMPLE FOR A TRUSS

Press Esc to exit full screen

Transormation matrix:

$$T(\theta) := \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 & 0 \\ -\sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & \cos(\theta) & \sin(\theta) \\ 0 & 0 & -\sin(\theta) & \cos(\theta) \end{pmatrix}$$

Angles and stiffness matrices for membe

$$\theta := \begin{pmatrix} 0 \\ \frac{5\cdot\pi}{4} \\ \frac{3\cdot\pi}{2} \end{pmatrix} \qquad K_{GCS_i} := T(\theta_i)^T \cdot K_{LC}$$

EXAMPLE FOR A TRUSS

Results for stiffness matrices:

$$K_{GCS_1} = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad K_{GCS_3} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & -1 \end{pmatrix}$$

$$K_{GCS_2} = \begin{pmatrix} 0.354 & 0.354 & -0.354 & -0.354 \\ 0.354 & 0.354 & -0.354 & -0.354 \\ -0.354 & -0.354 & 0.354 & 0.354 \\ -0.354 & -0.354 & 0.354 & 0.354 \end{pmatrix}$$

EXAMPLE FOR A TRUSS

Indexes of nodes for members:

$$\mathbf{Ind} := \begin{pmatrix} 1 & 2 \\ 2 & 3 \\ 1 & 3 \end{pmatrix}$$

EXAMPLE FOR A TRUSS

Press Esc to exit full screen

Assembly of stiffness matrix:

$$
\mathbf{K} := \left|
\begin{array}{l}
\text{for } j \in 1..n \\
\quad \text{for } k \in 1..n \\
\quad\quad K_{j,k} \leftarrow \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\
\text{for } i \in 1..3 \\
\quad \left| \begin{array}{l}
K_{Ind_{i,1},Ind_{i,1}} \leftarrow K_{Ind_{i,1},Ind_{i,1}} + \text{submatrix}(K \\
K_{Ind_{i,1},Ind_{i,2}} \leftarrow K_{Ind_{i,1},Ind_{i,2}} + \text{submatrix}(K \\
K_{Ind_{i,2},Ind_{i,1}} \leftarrow K_{Ind_{i,2},Ind_{i,1}} + \text{submatrix}(K \\
K_{Ind_{i,2},Ind_{i,2}} \leftarrow K_{Ind_{i,2},Ind_{i,2}} + \text{submatrix}(K
\end{array} \right. \\
K
\end{array} \right.
$$

EXAMPLE FOR A TRUSS

Result after previous step:

$$\mathbf{K} = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & \cdots \\ 0 & \cdots \end{pmatrix} \\ \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 1.354 & 0.354 \\ 0.354 & 0.354 \end{pmatrix} & \begin{pmatrix} -0.354 & \cdots \\ -0.354 & \cdots \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} & \begin{pmatrix} -0.354 & -0.354 \\ -0.354 & -0.354 \end{pmatrix} & \begin{pmatrix} 0.354 & \cdots \\ 0.354 & \cdots \end{pmatrix} \end{bmatrix}$$

EXAMPLE FOR A TRUSS

$$
\mathbf{K} := \left|\begin{array}{l}
\text{for } j \in 1..d \\
\quad \text{for } k \in 1..d \\
\quad\quad \left|\begin{array}{l}
M \leftarrow K_{\text{ceil}\left(\frac{j}{2}\right),\,\text{ceil}\left(\frac{k}{2}\right)} \\
KN_{j,k} \leftarrow M_{2-\text{mod}(j,2),\,2-\text{mod}(k,2)}
\end{array}\right. \\
KN
\end{array}\right.
$$

$$
\mathbf{K} = \begin{pmatrix}
1 & 0 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 \\
-1 & 0 & 1.354 & 0.354 & -0.354 & -0.354 \\
0 & 0 & 0.354 & 0.354 & -0.354 & -0.354 \\
0 & 0 & -0.354 & -0.354 & 0.354 & 0.354 \\
0 & -1 & -0.354 & -0.354 & 0.354 & 1.354
\end{pmatrix}
$$

THREE BASIC EQUATIONS

How are they implemented in matrix met

Equilibrium equations

- **Constitutive equations**
  - **Compatibility equations**
    - Taken into account when global stiffness matrix is assembled from member matrices
  - Through member stiffness matrices
- Taken into accou when global stiffne matrix is assembl from member matr

UNIT 5

INFLUENCE LINES FOR STATICALLY INDETERMINATE STRUCTURES

3. INFLUENCE LINES FOR STATICALLY DETERMINATE STRUCTURES - AN OVERVIEW

- Introduction - What is an influence line?
- Influence lines for beams
- Qualitative influence lines - Muller-Breslau Principle
- Influence lines for floor girders
- Influence lines for trusses
- Live loads for bridges
- Maximum influence at a point due to a series of concentrated loads
- Absolute maximum shear and moment

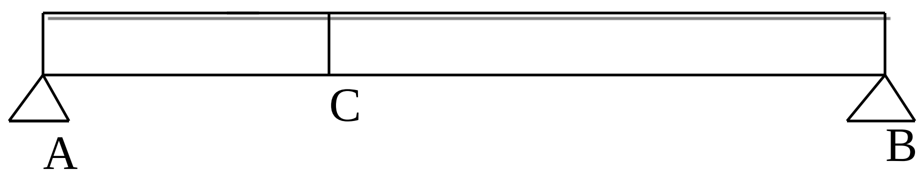
INTRODUCTION TO INFLUENCE LINES

- **<u>Influence lines describe the variation of an analysis variable</u>** (reaction, shear force, bending moment, twisting moment, deflection, etc.) **<u>at a point</u>** (say at C in Figure 6.1)

  ..

  …

  …



- Why do we need the influence lines? For instance, when loads pass over a structure, say a bridge, one needs to know when the maximum values of shear/reaction/bending-moment will occur at a point so that the section may be designed

- **<u>Notations:</u>**

  – **<u>Normal Forces</u> -** +ve forces cause +ve displacements in +ve directions

  – **<u>Shear Forces</u> -** +ve shear forces cause clockwise rotation & - ve shear force causes anti-clockwise rotation

  – **<u>Bending Moments:</u>** +ve bending moments cause "cup holding water" deformed shape

INFLUENCE LINES FOR BEAMS

- **Procedure:**

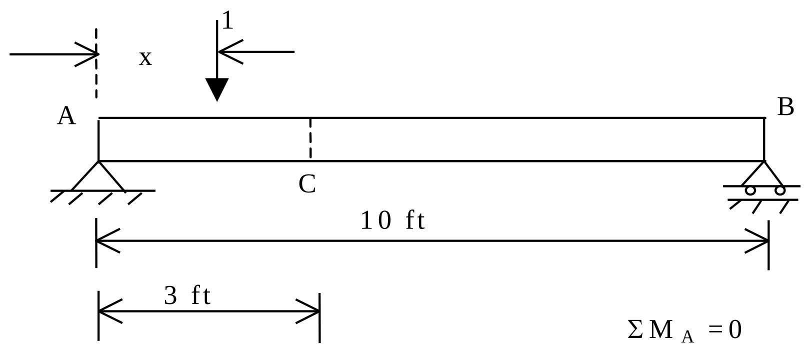
  (1) **Allow a unit load** (either 1b, 1N, 1kip, or 1 tonne) **to move over beam from left to right**

  (2) **Find the values** of shear force or bending moment, **at the point under consideration**, as the unit load moves over the beam from left to right

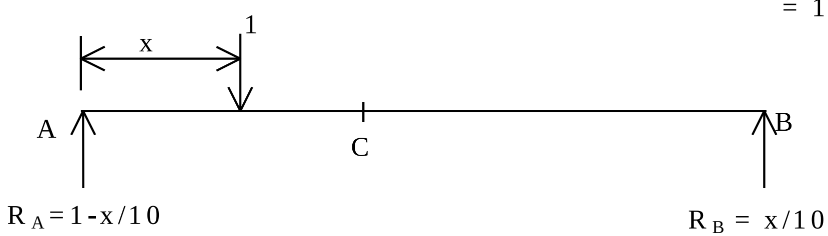
  (3) **Plot the values** of the shear force or bending moment, **over the length of the beam, computed for the point under consideration**

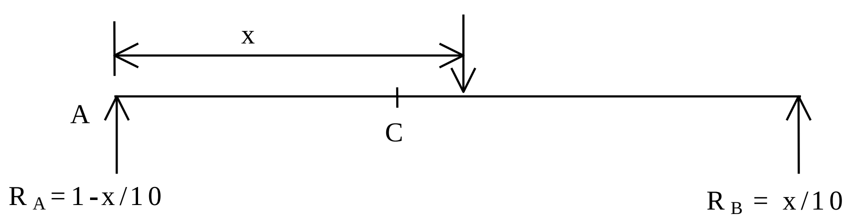
3.3 MOVING CONCENTRATED LOAD

3.3.1 Variation of Reactions R_A and R_B as functions of load position



$$\Sigma M_A = 0$$
$$(R_B)(10) - (1)(x) = 0$$
$$R_B = x/10$$
$$R_A = 1 - R_B$$
$$= 1 - x/10$$

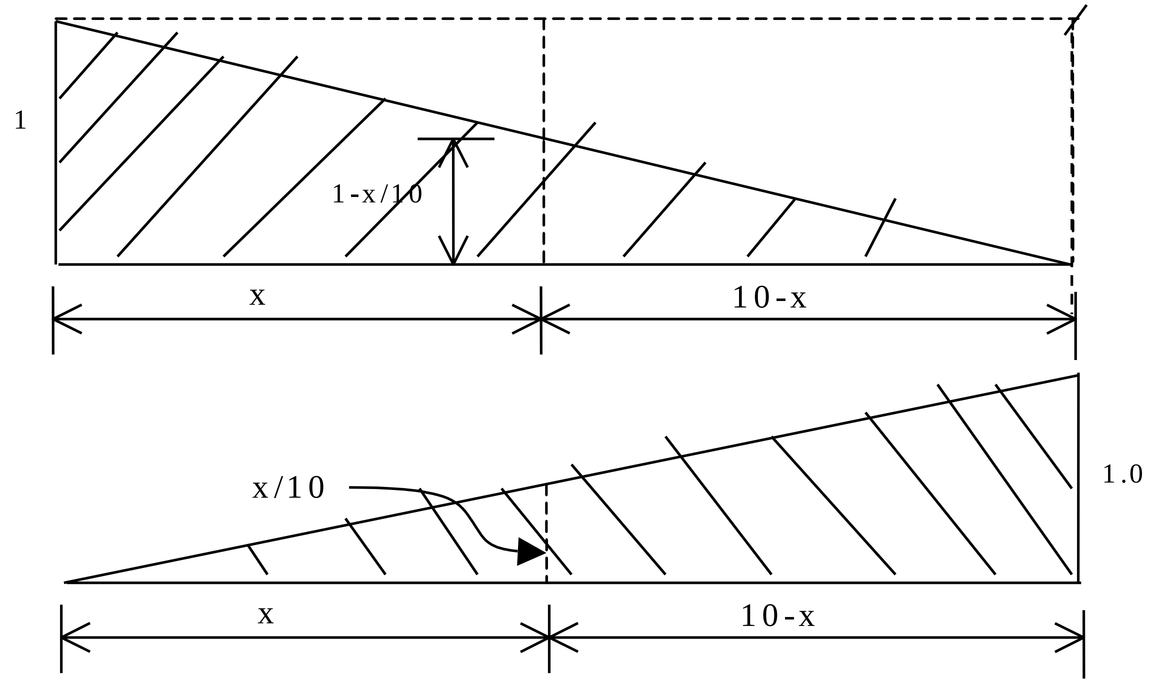




R_A occurs only at A ; R_B occurs only at B

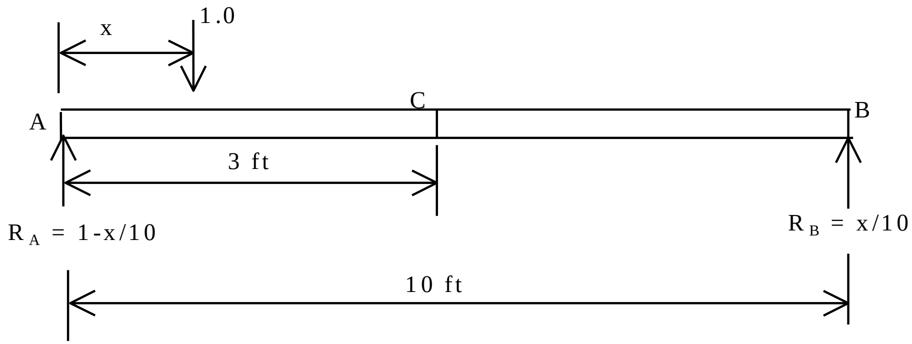
Influence line for R_A

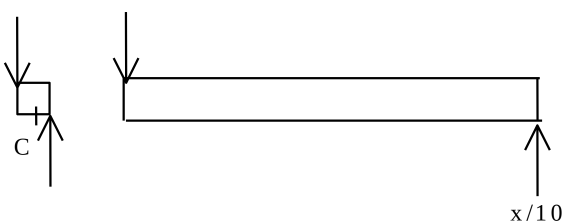
Influence line for R_B



3.3.2 Variation of Shear Force at C as a function of load position

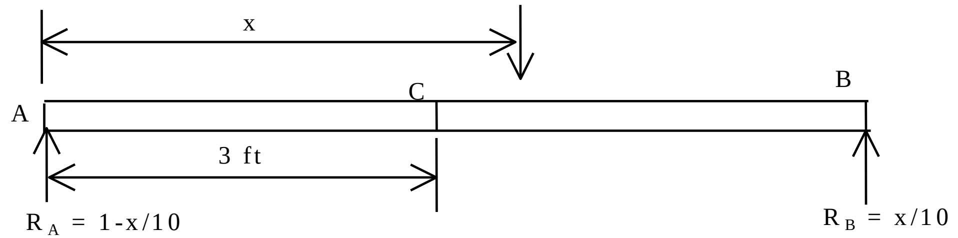
0 < x < 3 ft (unit load to the left of C)

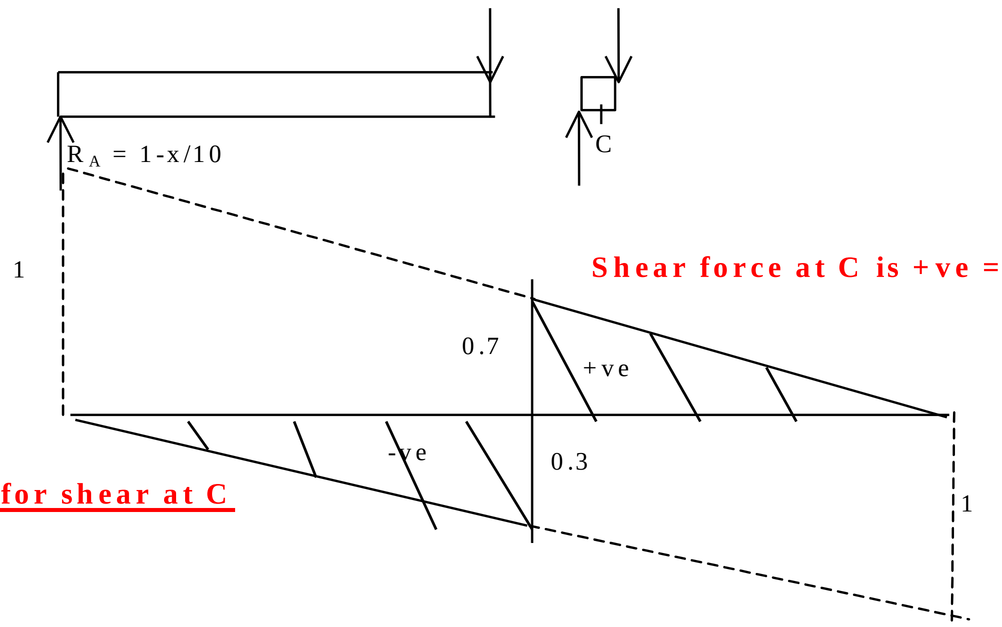




Shear force at C is –ve, $V_C = -x/10$

3 < x < 10 ft (unit load to the right of C)



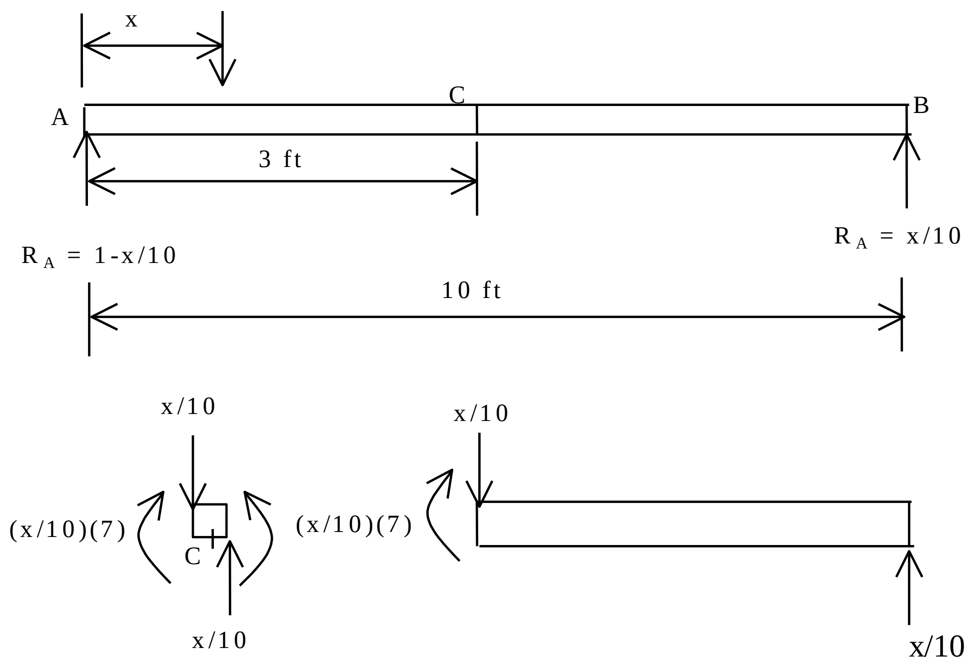


Shear force at C is +ve = 1-x/10

Influence line for shear at C

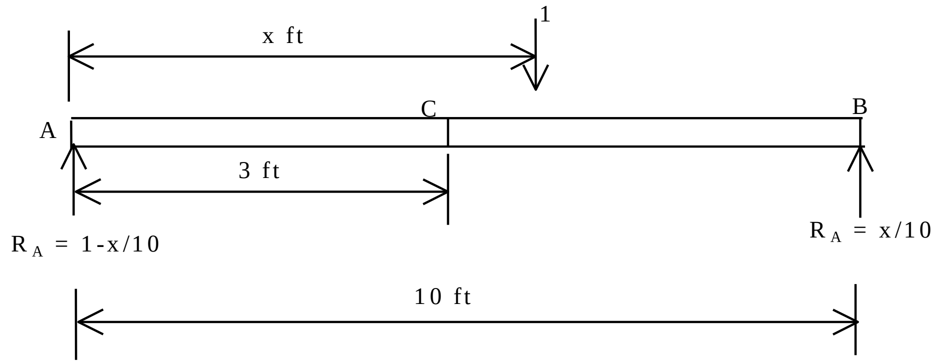
3.3.3 Variation of Bending Moment at C as a function of load position

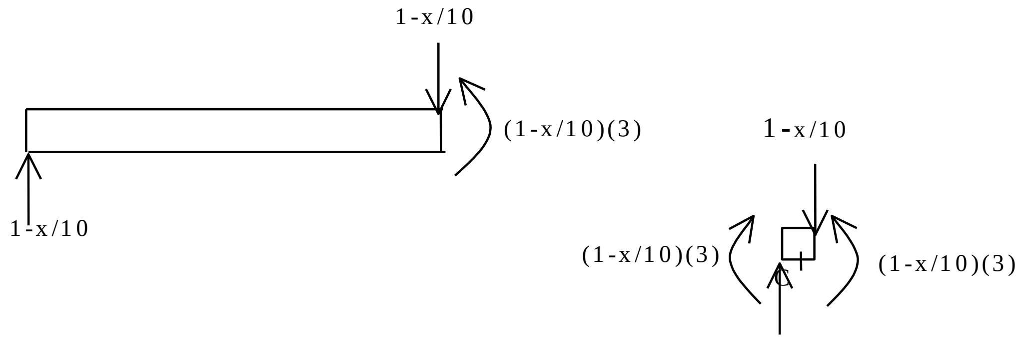
0 < x < 3.0 ft (Unit load to the left of C)



Bending moment is +ve at C

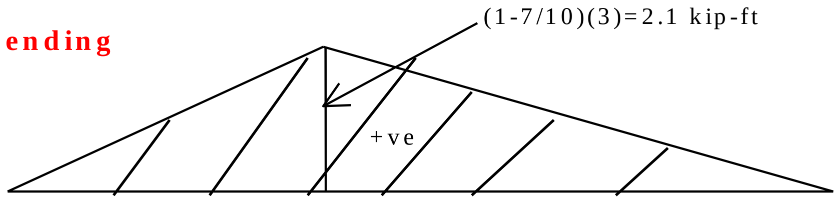
3 < x < 10 ft (Unit load to the right of C)





Moment at C is +ve

Influence line for bending Moment at C



3.4 QUALITATIVE INFLUENCED LINES - MULLER-BRESLAU'S PRINCIPLE

- The principle **gives only a procedure** to determine of the influence line of a parameter for a determinate or an indeterminate structure
- But **using the basic understanding of the influence lines**, the **magnitudes** of the influence lines also **can be computed**
- In order **to draw the shape of the influence lines properly**, the **capacity of the beam to resist the parameter investigated** (reaction, bending moment, shear force, etc.), **at that point, must be removed**
- The principle states that:**The influence line for a parameter** (say, reaction, shear or bending moment), at a point, **is to the same scale as the deflected shape of the beam**, when the beam is acted upon by that parameter.
  - The **capacity of the beam to resist that parameter**, at that point, **must be removed.**
  - **Then allow the beam to deflect under that parameter**
  - Positive **directions of the forces are the same** as before

PROBLEMS - Influence Line for a Determinate Beam by Muller-Breslau's Method

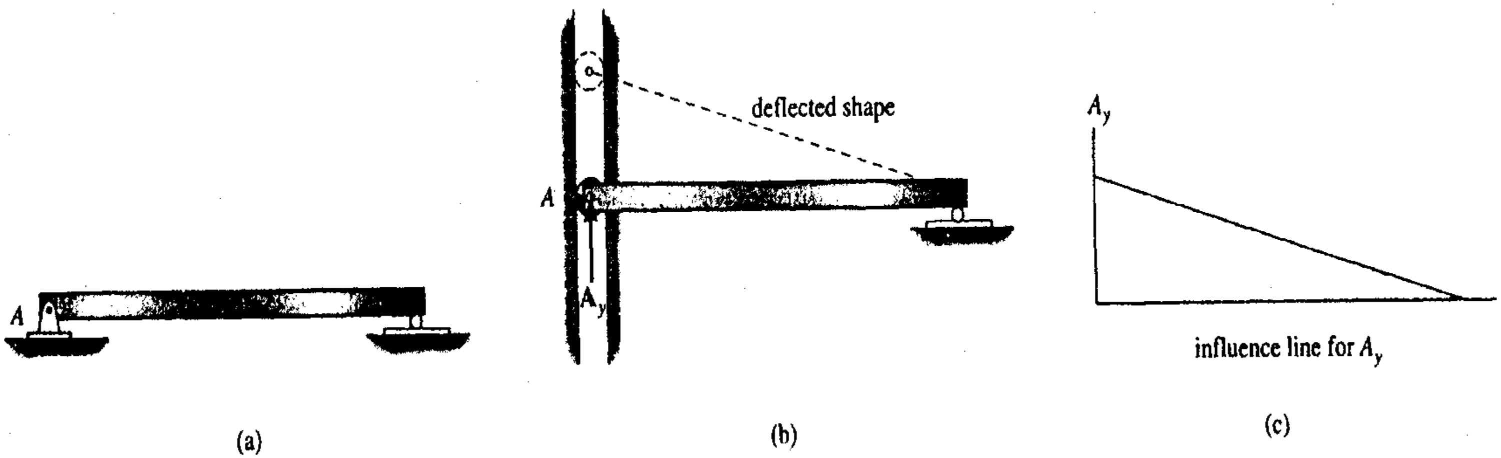


Fig. 6–12

Influence line for Reaction at A

Influence Lines for a Determinate Beam by Muller-Breslau's Method

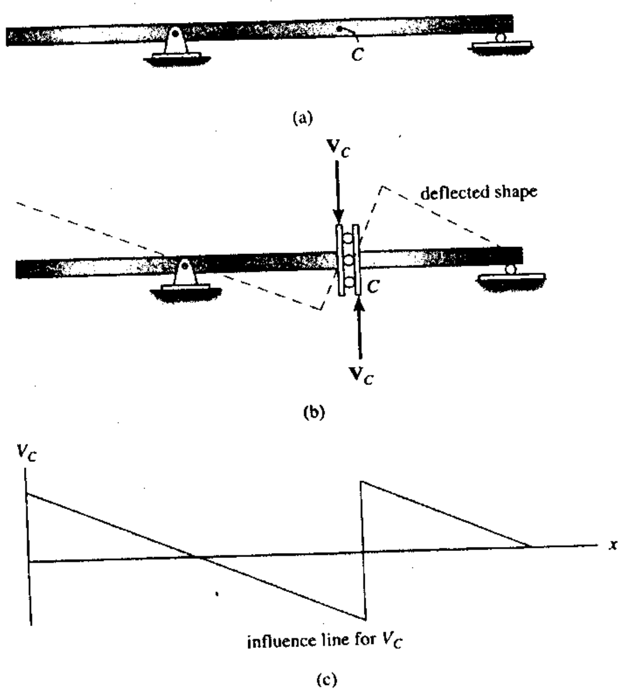


Fig. 6–13

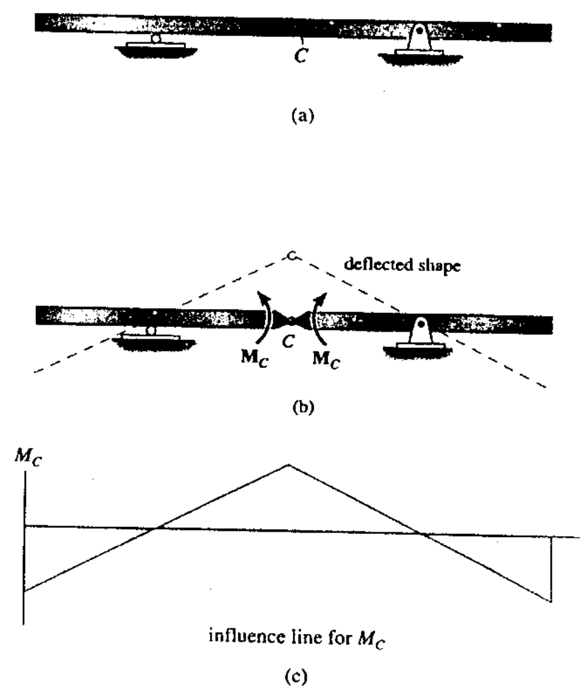


Fig. 6–14

Influence Line for Shear at C

Influence Line for Bending Moment at C

Influence Lines for an Indeterminate Beam by Muller-Breslau’s Method

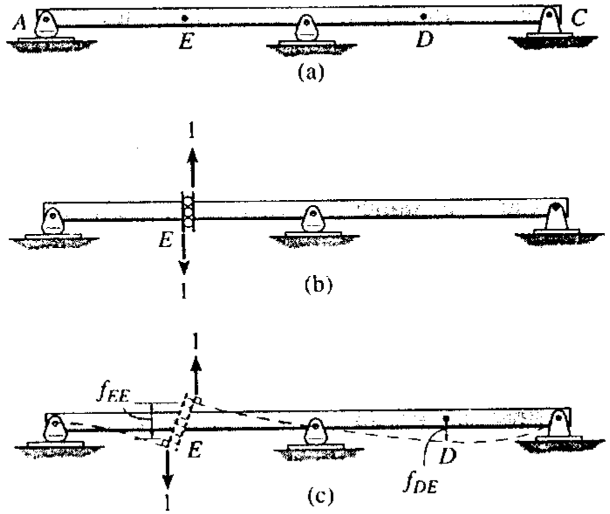


Fig. 9–24

Influence Line for Shear at E

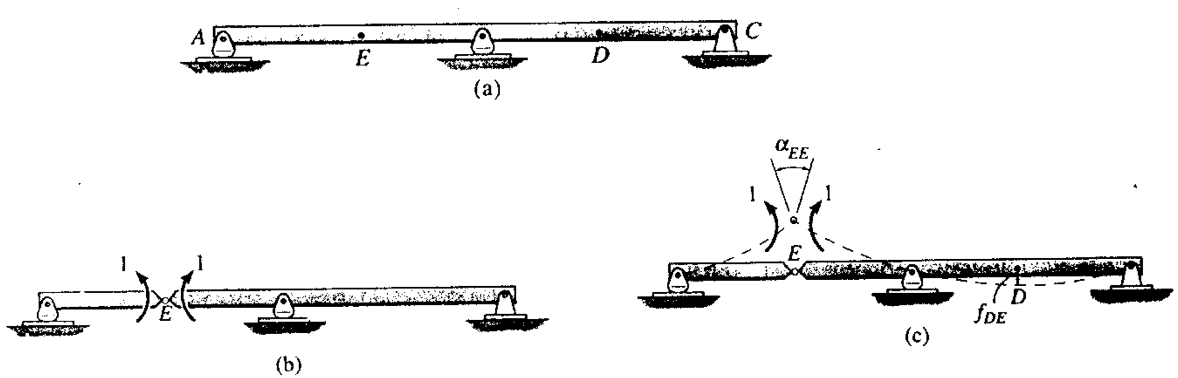
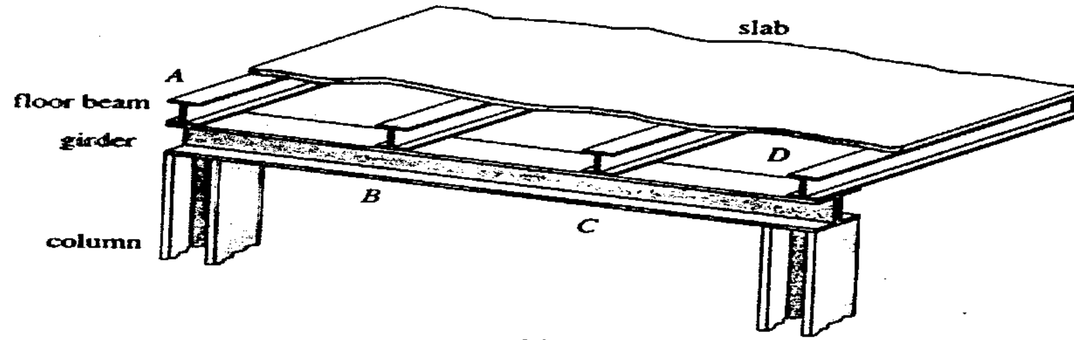


Fig. 9–25

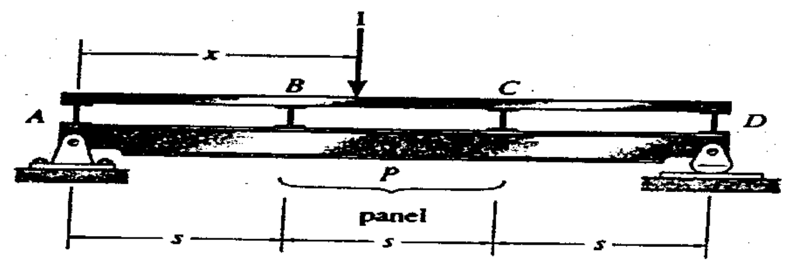
Influence Line for Bending Moment at E

INFLUENCE LINE FOR FLOOR GIRDERS

Floor systems are constructed as shown in figure below,



(a)



(b)

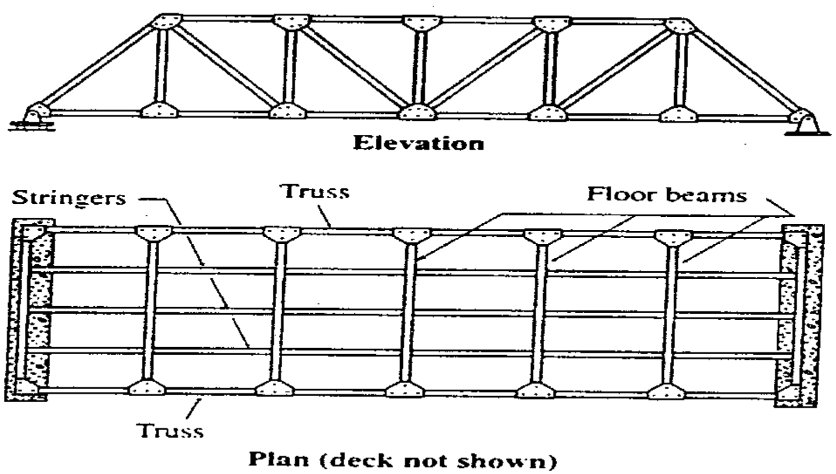
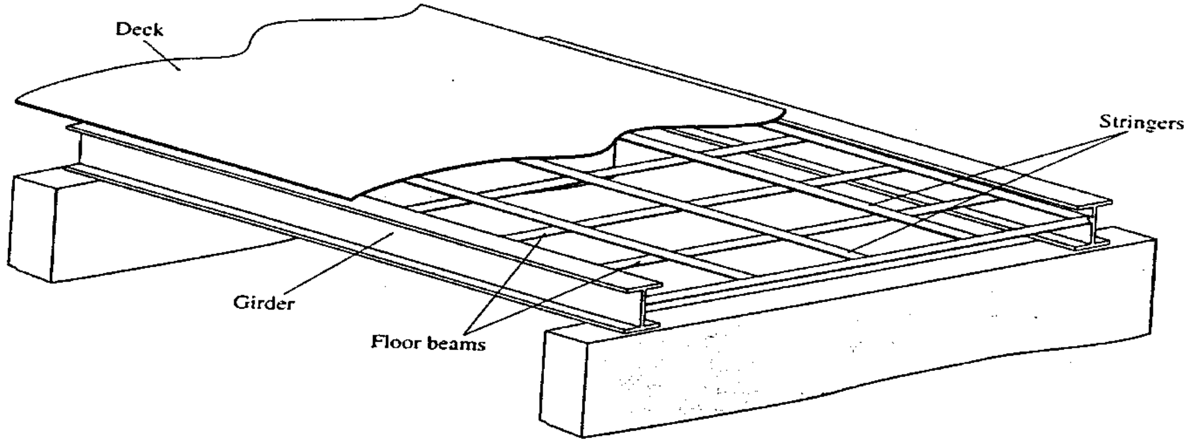
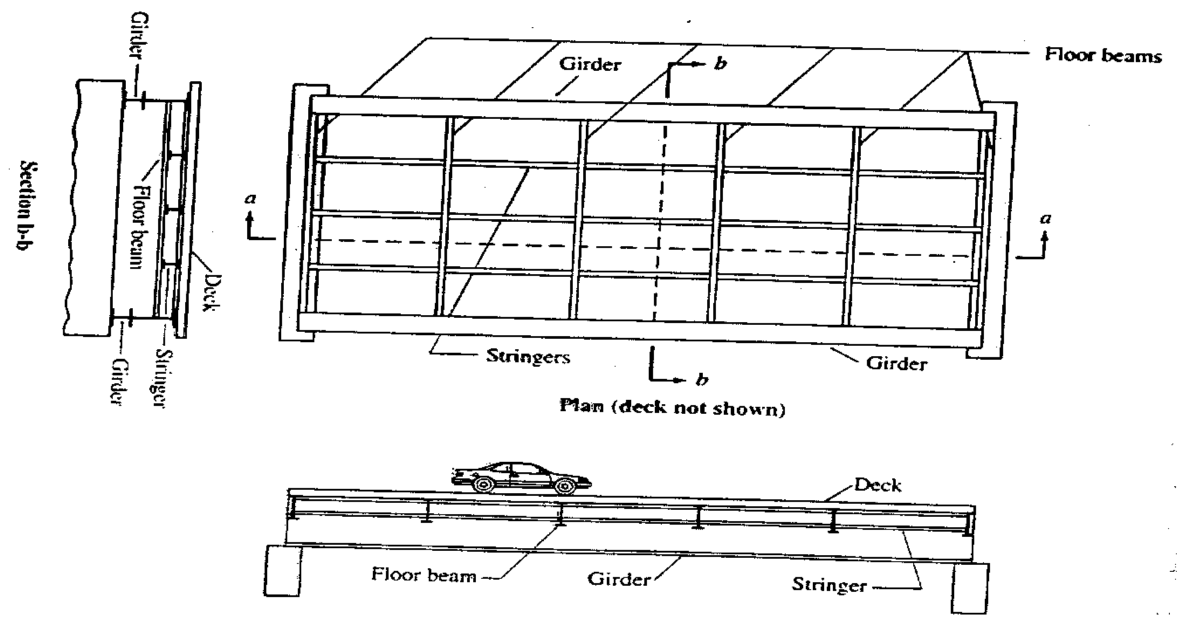


Fig. 8.14

INFLUENCE LINES FOR FLOOR GIRDERS (Cont'd)



(a)



Section a-a

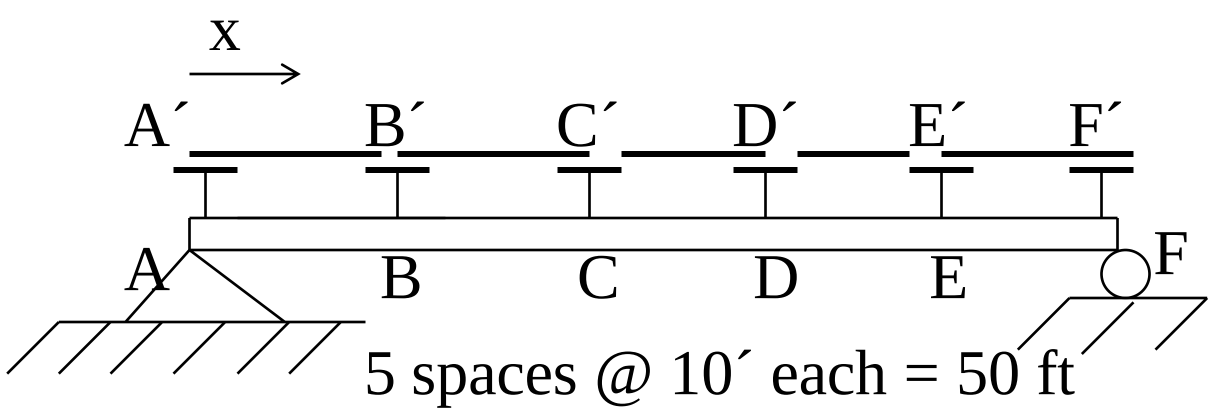
(b)

Fig. 8.10

3.6 INFLUENCE LINES FOR FLOOR GIRDERS (Cont'd)

3.6.1 Force Equilibrium Method:

Draw the Influence Lines for: (a) Shear in panel CD of the girder; and (b) the moment at E.



Place load over region A´B´ (0 < x < 10 ft)

Find the shear over panel CD

V_{CD}= - x/50

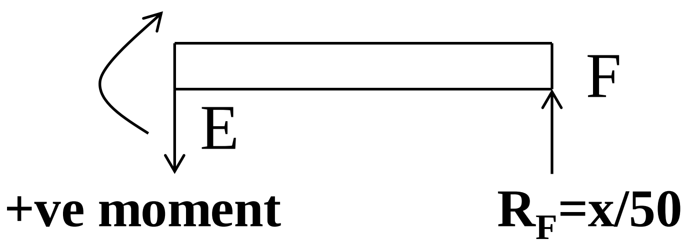
At x=0, V_{CD} = 0

At x=10, V_{CD} = -0.2

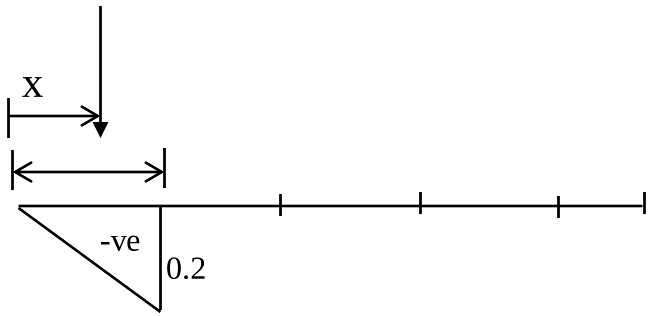
Find moment at E = +(x/50)(10)=+x/5

At x=0, M_E=0

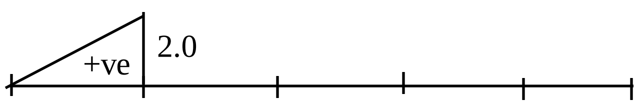
At x=10, M_E=+2.0



Continuation of the Problem



I. L. for V_{CD}



I. L. for M_E

Problem Continued -
Place load over region B´C´ (10 ft < x < 20ft)

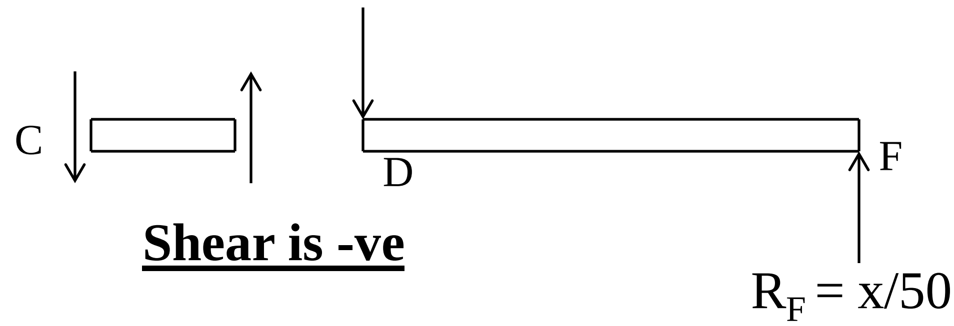
V_{CD} = -x/50 kip

At x = 10 ft

V_{CD} = -0.2

At x = 20 ft

V_{CD} = -0.4

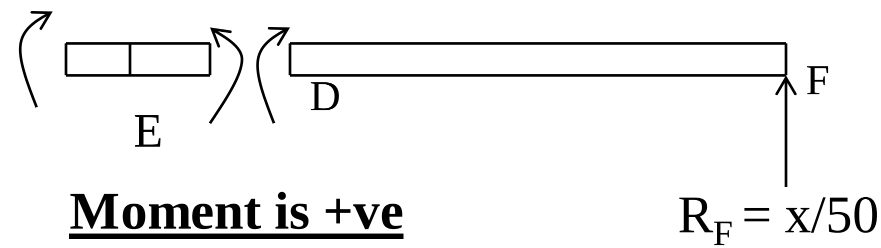


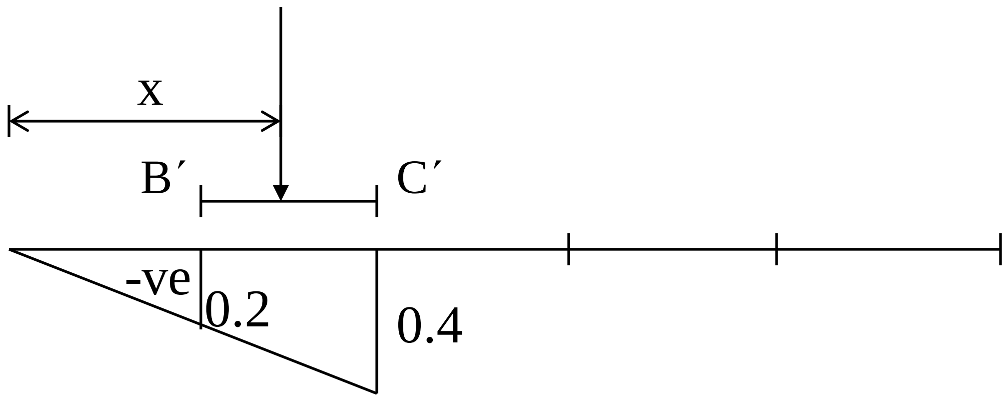
M_E = +(x/50)(10)

= +x/5 kip.ft

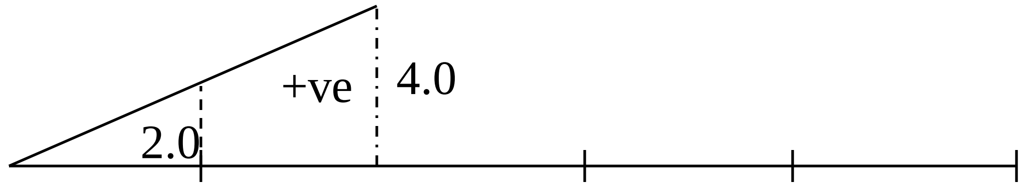
At x = 10 ft, M_E = +2.0 kip.ft

At x = 20 ft, M_E = +4.0 kip.ft





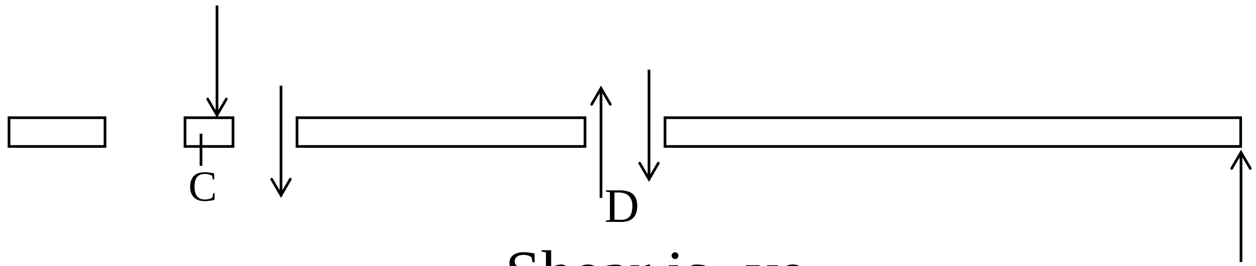
I. L. for V_{CD}



I. L. for M_E

Place load over region C´D´ (20 ft < x < 30 ft)

<u>When the load is at C’</u> (x = 20 ft)

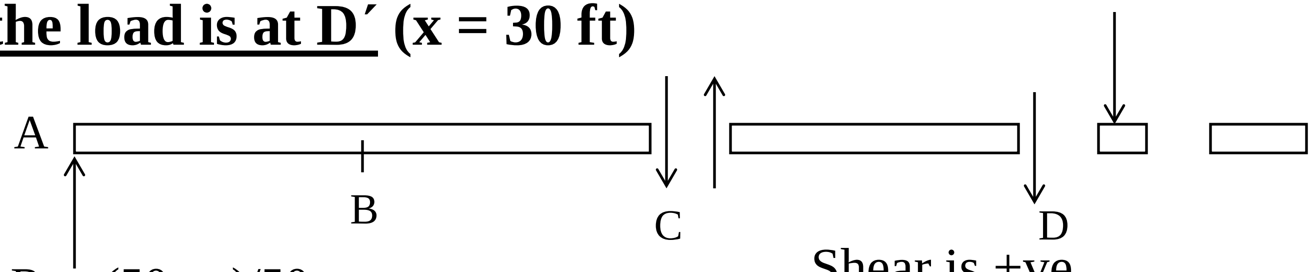


Shear is -ve

$R_F = 20/50$
$= 0.4$

$V_{CD} = -0.4$ kip

<u>When the load is at D´</u> (x = 30 ft)

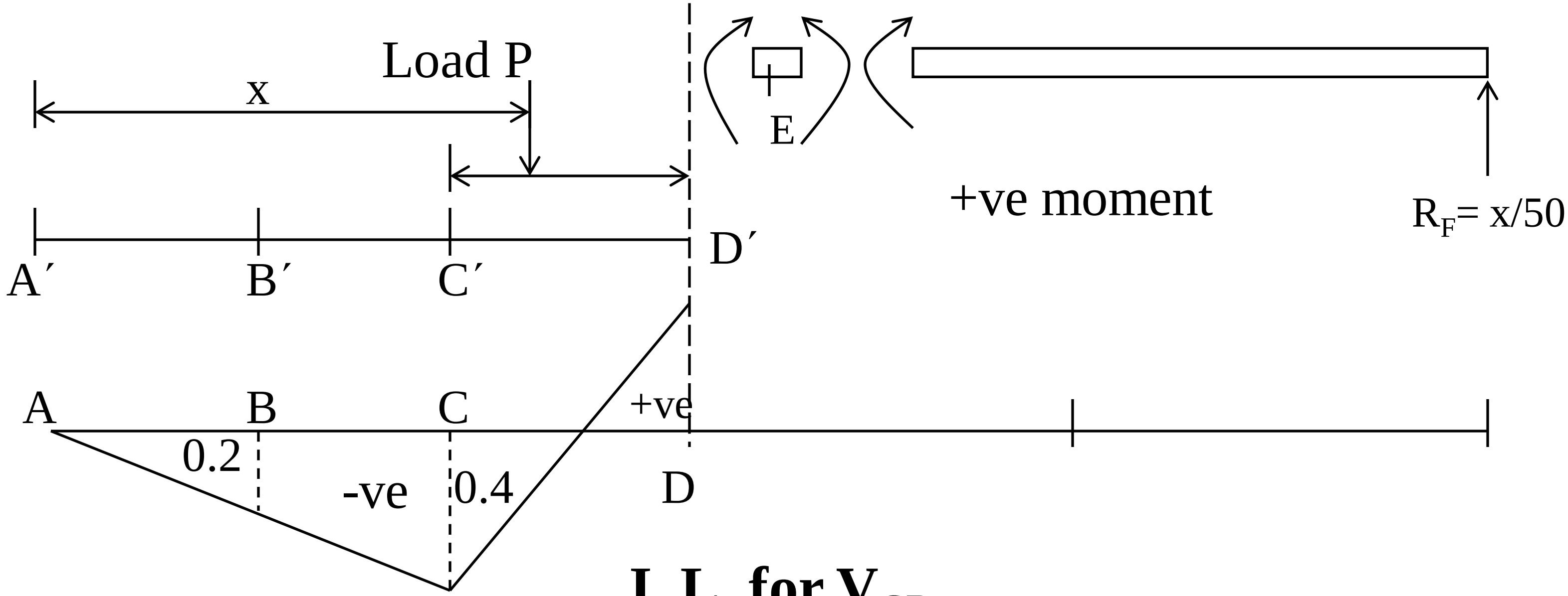


$R_A = (50 - x)/50$

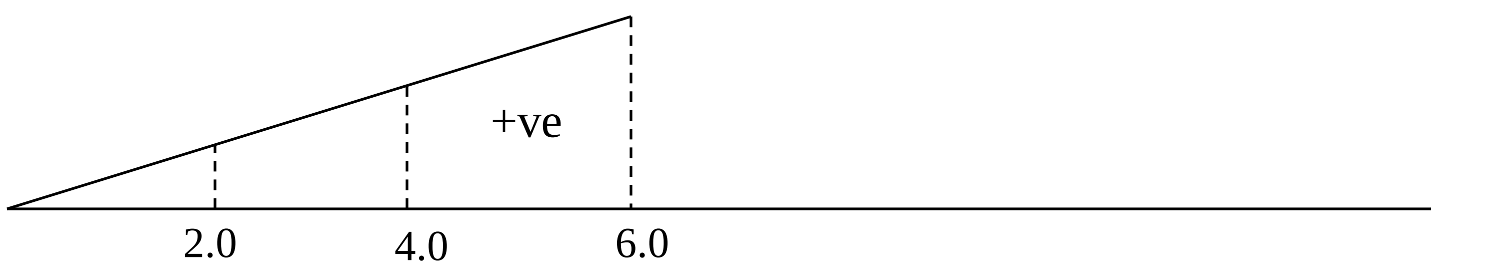
Shear is +ve

$V_{CD} = +\ 20/50$
$= +\ 0.4$ kip

$$M_E = +\,(x/50)(10) = +\,x/5$$



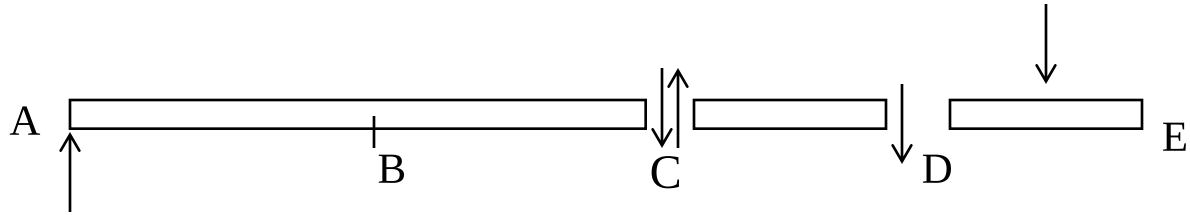
I. L. for V_{CD}



I. L. for M_E

Place load over region D´E´ (30 ft < x < 40 ft)

$V_{CD} = +\ (1 - x/50)$ kip



Shear is +ve

$R_A = (1 - x/50)$

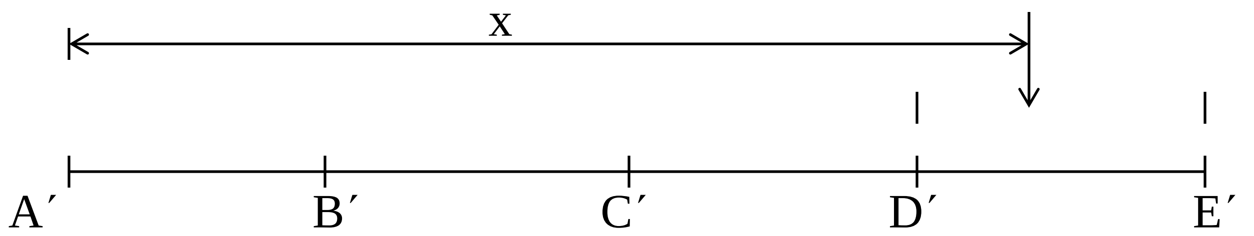
$M_E = +(x/50)(10)$
$\quad = +\ x/5$ kip.ft

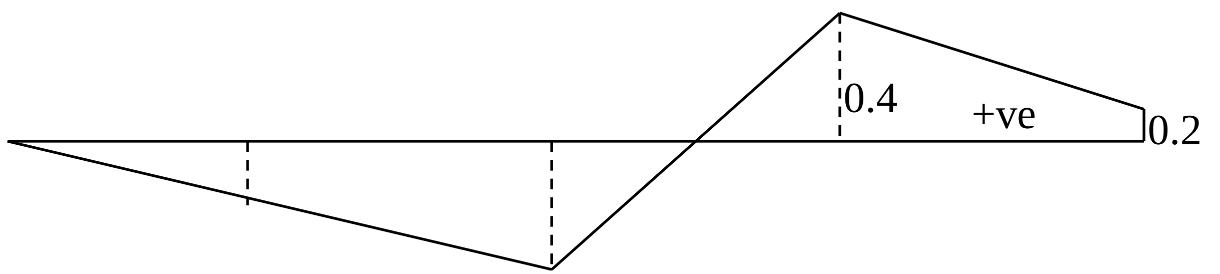
Moment is +ve

$R_F = x/50$

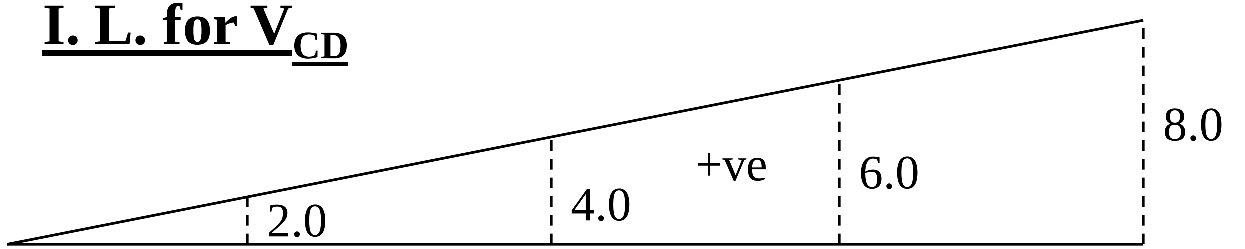
At x = 30 ft, $M_E = +6.0$
At x = 40 ft, $M_E = +8.0$

Problem continued





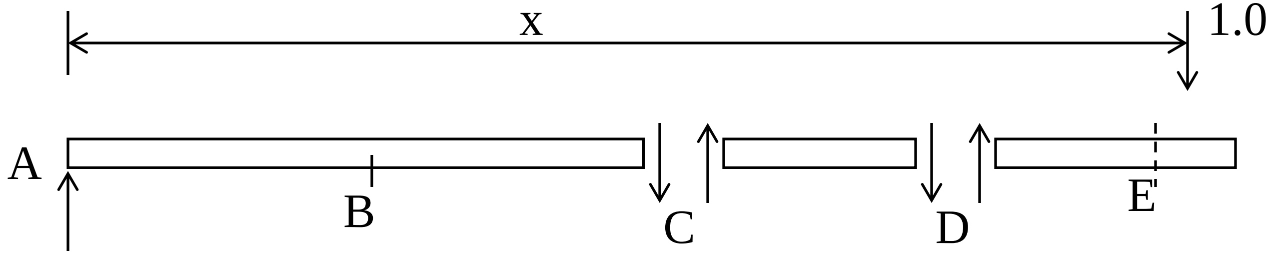
I. L. for V_{CD}



I. L. for M_E

Place load over region E´F´ (40 ft < x < 50 ft)

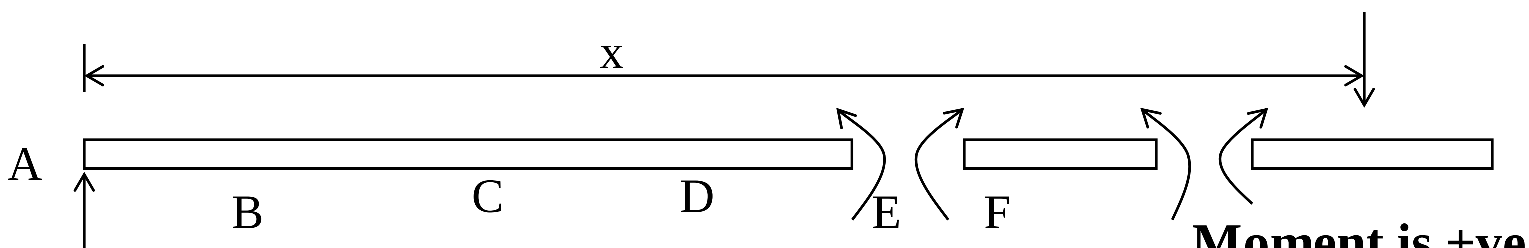
V_{CD} = + 1-x/50 At x = 40 ft, V_{CD}= + 0.2
At x = 50 ft, V_{CD} = 0.0



R_A= 1-x/50

Shear is +ve

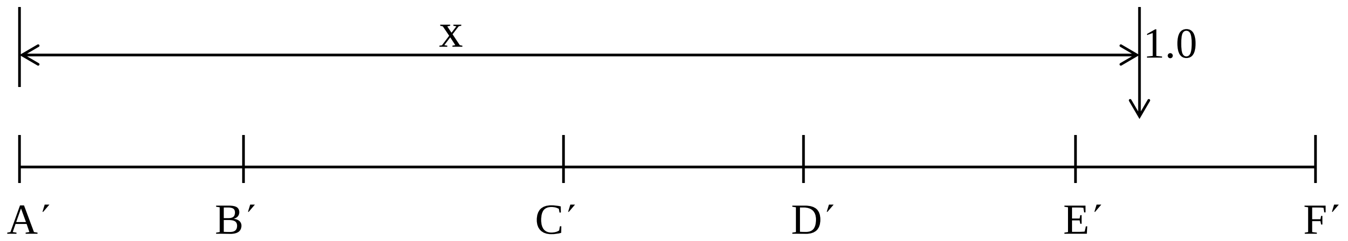
M_E= + (1-x/50)(40) = (50-x)*40/50 = +(4/5)(50-x)

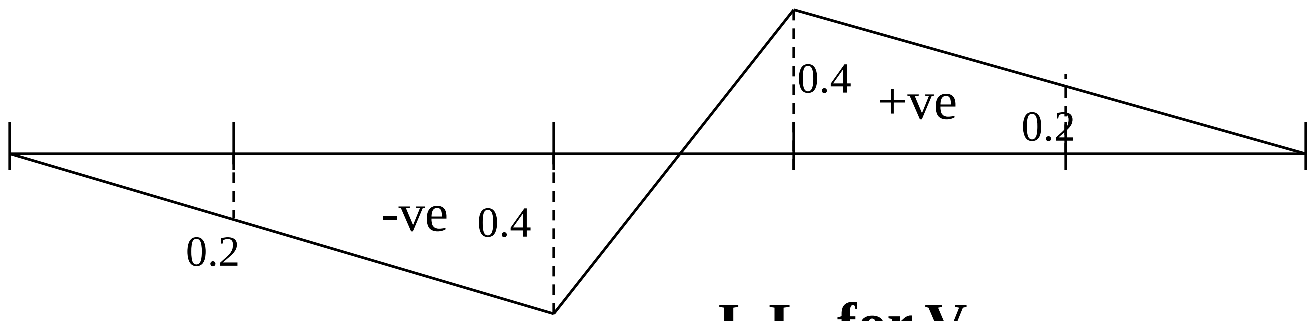


Moment is +ve

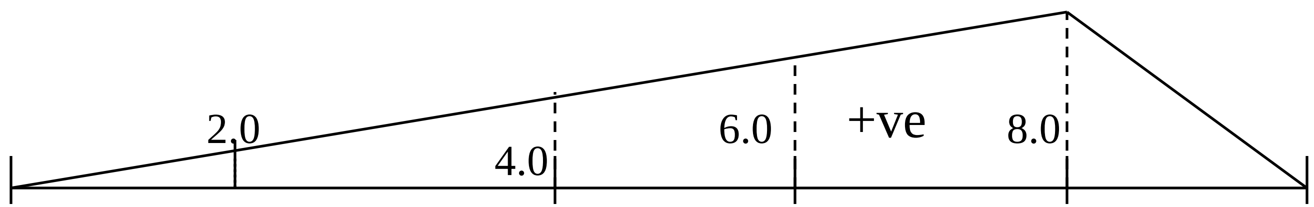
R_A=1-x/50

At x = 40 ft, M_E= + 8.0 kip.ft
At x = 50 ft, M_E = 0.0





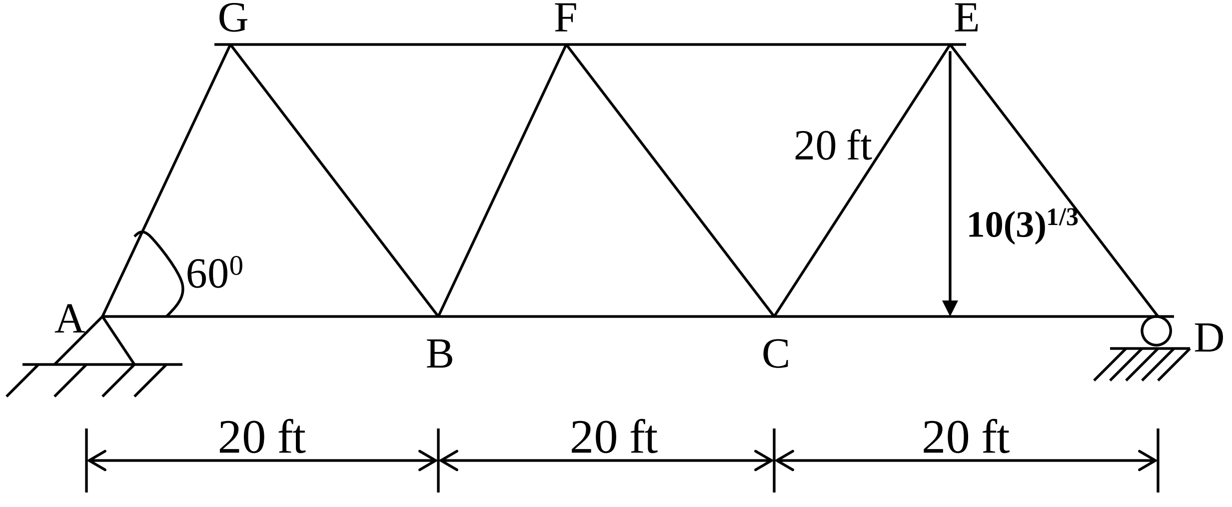
I. L. for V_{CD}



I. L. for M_E

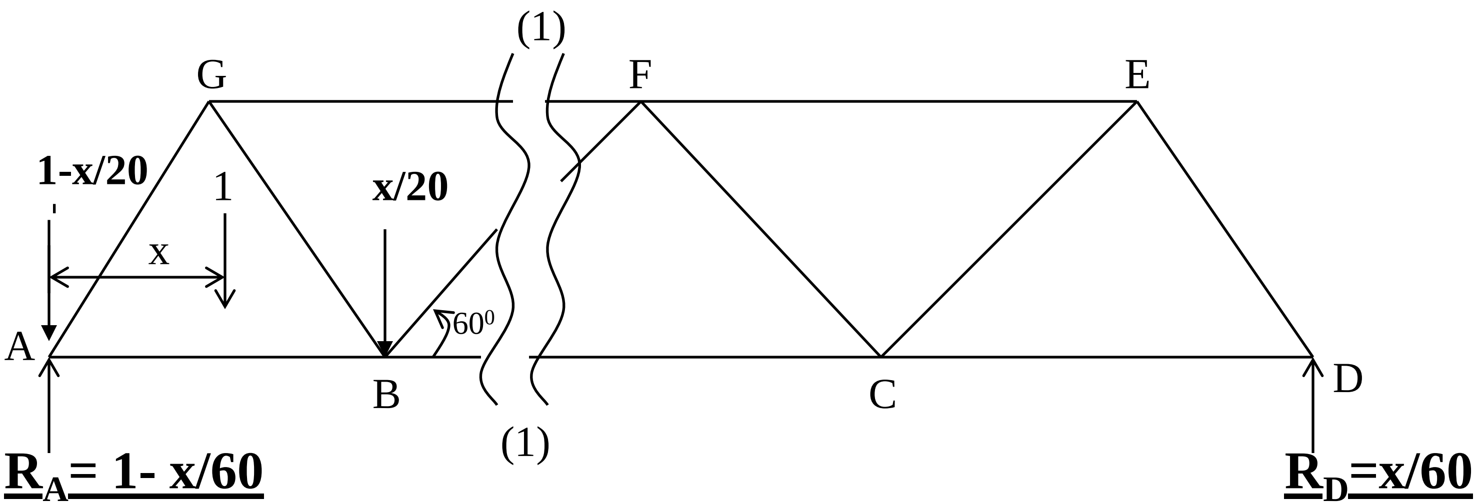
INFLUENCE LINES FOR TRUSSES

Draw the influence lines for: (a) Force in Member GF; and (b) Force in member FC of the truss shown below in Figure below



Problem 3.7 continued -
3.7.1 Place unit load over AB

(i) To compute GF, cut section (1) - (1)



At x = 0,
$F_{GF} = 0$
At x = 20 ft
$F_{GF} = - 0.77$

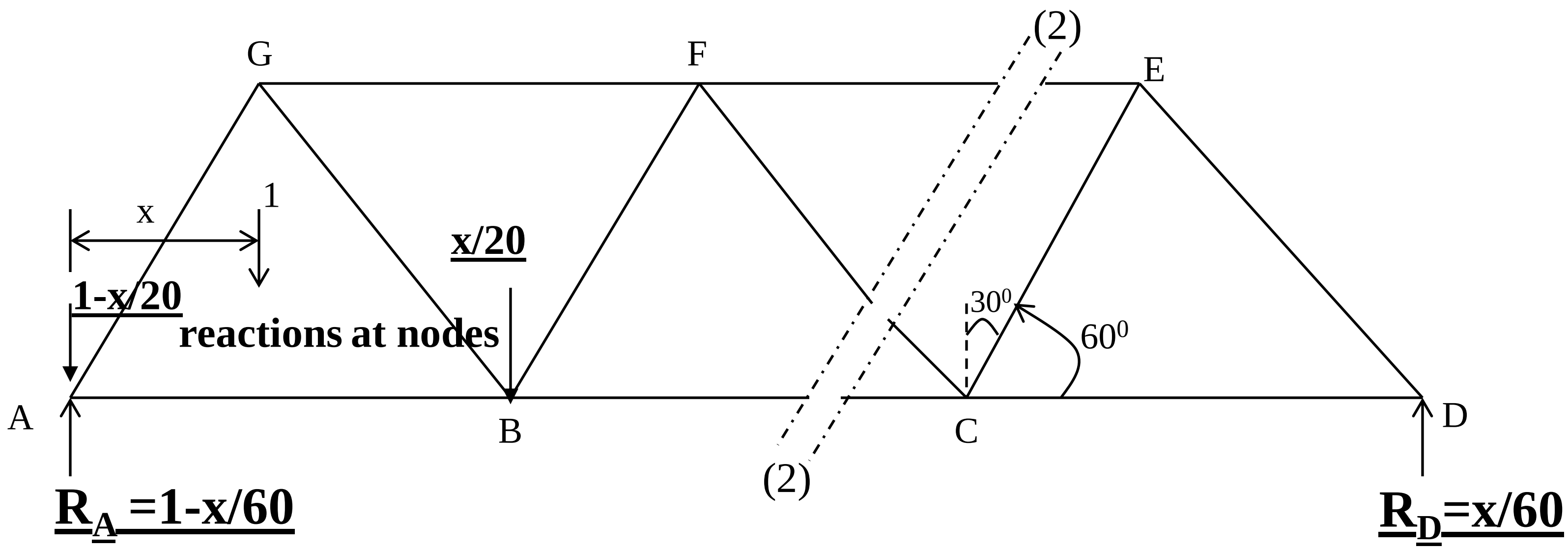
Taking moment about B to its right,

$(R_D)(40) - (F_{GF})(10\sqrt{3}) = 0$

$F_{GF} = (x/60)(40)(1/ 10\sqrt{3}) = x/(15 \sqrt{3})$ (-ve)

PROBLEM CONTINUED -

(ii) To compute F_{FC}, cut section (2) - (2)



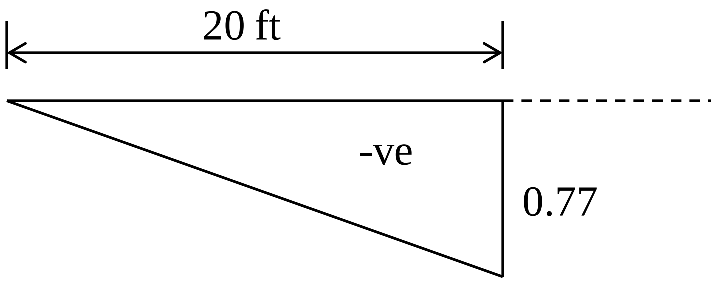
Resolving vertically over the right hand section

$$F_{FC}\cos 30^0 - R_D = 0$$

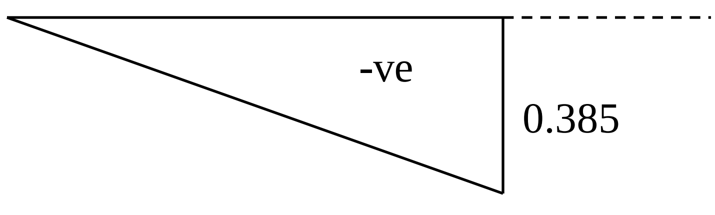
$$F_{FC} = R_D/\cos 30 = (x/60)(2/\sqrt{3}) = x/(30\sqrt{3})\ (\text{-ve})$$

At x = 0, F_{FC} = 0.0

At x = 20 ft, F_{FC} = -0.385



I. L. for F_{GF}

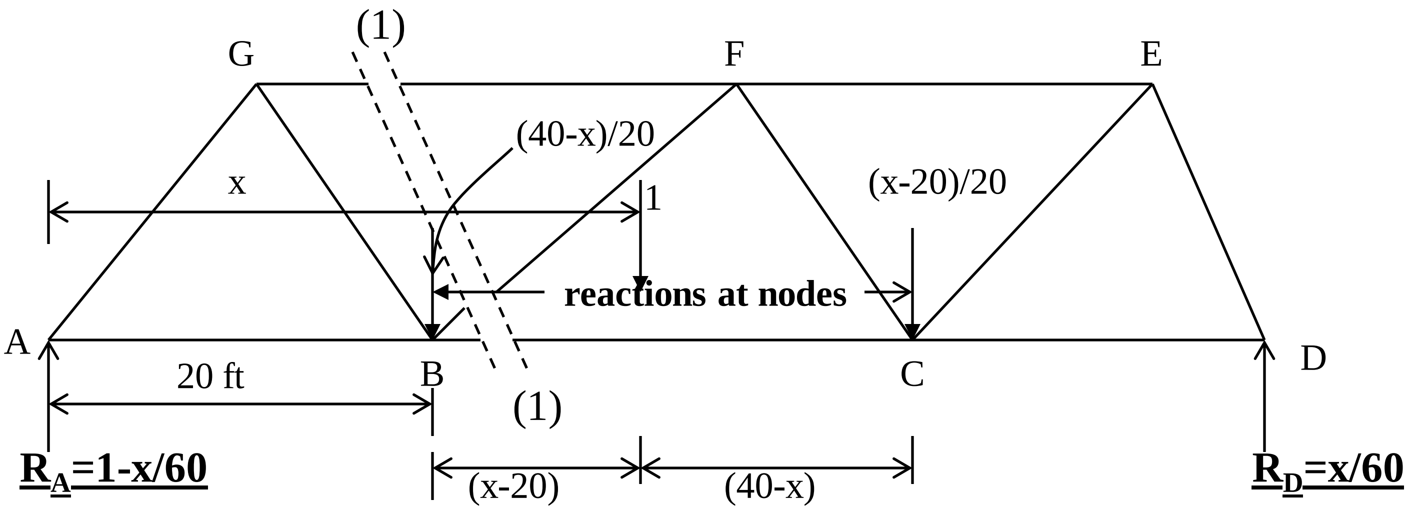


I. L. for F_{FC}

PROBLEM Continued -
Place unit load over BC (20 ft < x <40 ft)

[Section (1) - (1) is valid for 20 < x < 40 ft]

(i) To compute F_{GF} use section (1) -(1)



Taking moment about B, to its left,

$$(R_A)(20) - (F_{GF})(10\sqrt{3}) = 0$$

$$F_{GF} = (20R_A)/(10\sqrt{3}) = (1-x/60)(2/\sqrt{3})$$

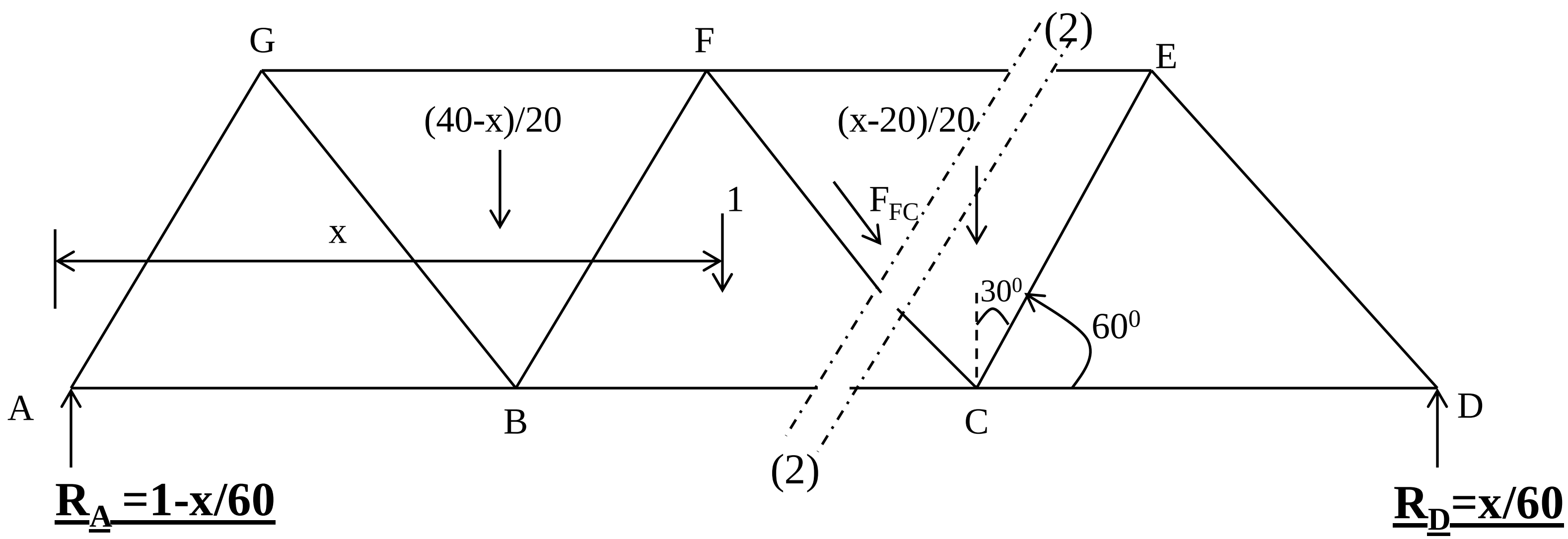
At x = 20 ft, F_{FG} = 0.77 (-ve)

At x = 40 ft, F_{FG} = 0.385 (-ve)

PROBLEM Continued -

(ii) To compute F_{FC}, use section (2) - (2)

Section (2) - (2) is valid for 20 < x < 40 ft



Resolving force vertically, over the right hand section,

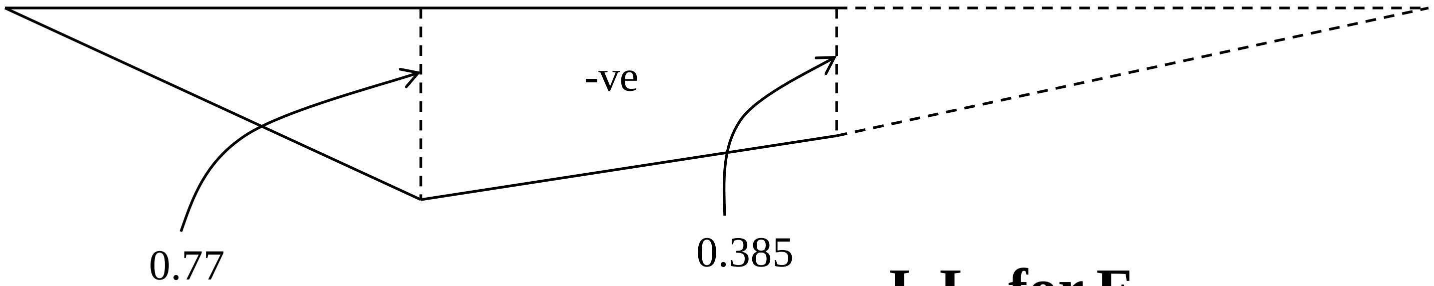
$$F_{FC}\cos 30 - (x/60) + (x-20)/20 = 0$$

$$F_{FC}\cos 30 = x/60 - x/20 + 1 = (1-2x)/60 \text{ (-ve)}$$

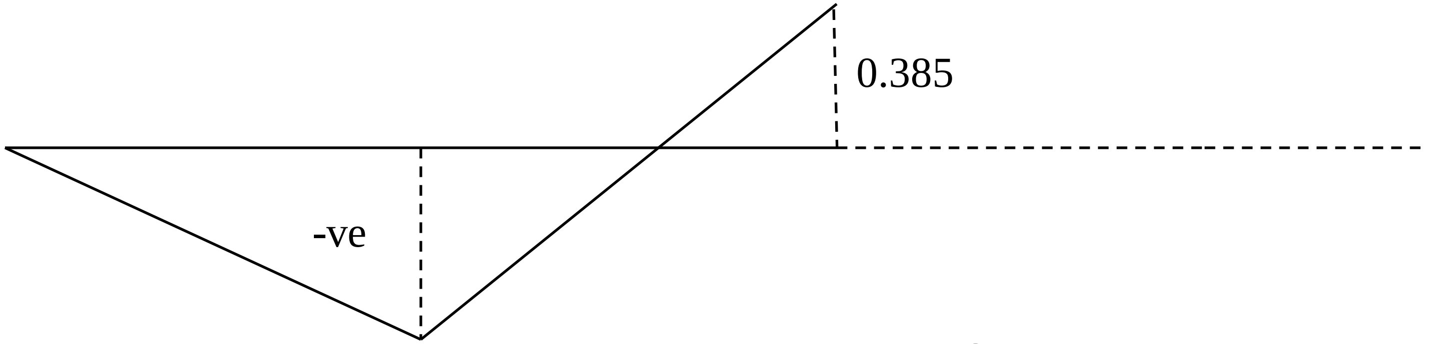
$$F_{FC} = ((60 - 2x)/60)(2/\sqrt{3}) \text{ -ve}$$

At x = 20 ft, $F_{FC} = (20/60)(2/\sqrt{3}) = 0.385$ (-ve)

At x = 40 ft, $F_{FC} = ((60-80)/60)(2/\sqrt{3}) = 0.385$ (+ve)



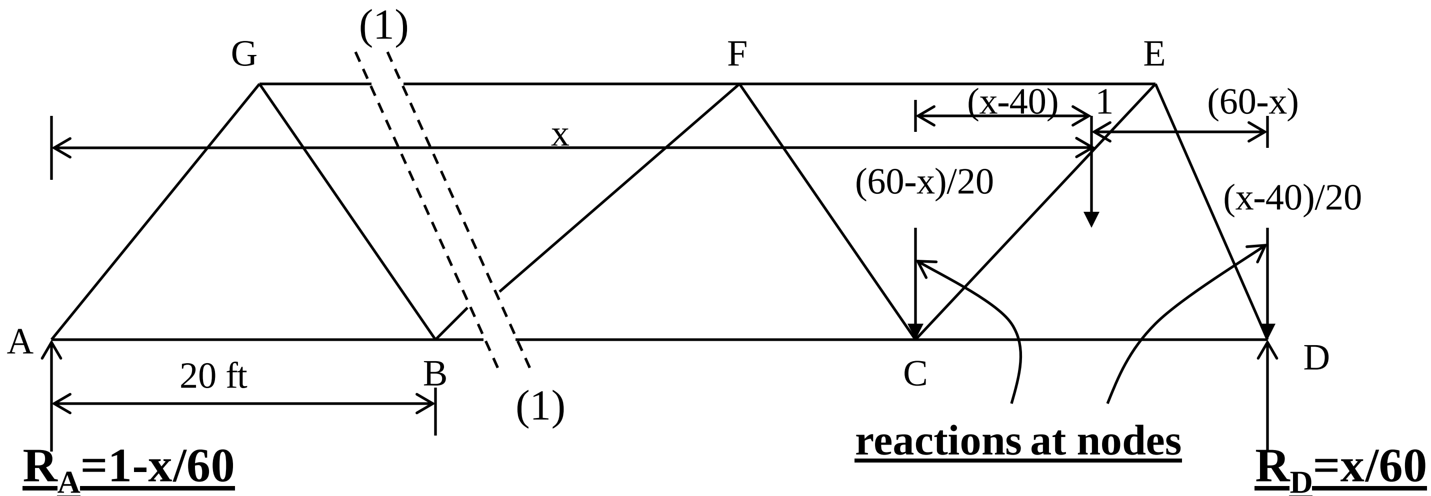
I. L. for F_{GF}



I. L. for F_{FC}

PROBLEM Continued -
3.7.3 Place unit load over CD (40 ft < x <60 ft)

(i) To compute F_{GF}, use section (1) - (1)



Take moment about B, to its left,

$$(F_{FG})(10\sqrt{3}) - (R_A)(20) = 0$$

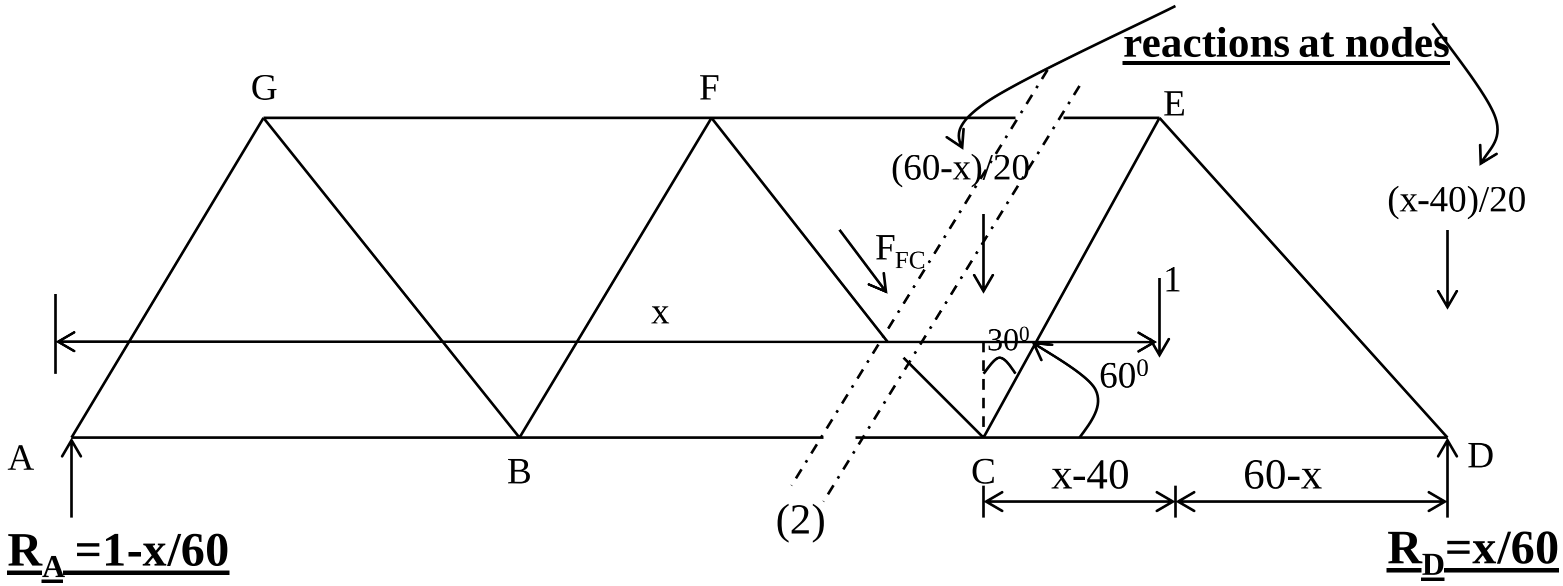
$$F_{FG} = (1\text{-}x/60)(20/10\sqrt{3}) = (1\text{-}x/60)(2/\sqrt{3}) \text{ -ve}$$

At x = 40 ft, F_{FG} = 0.385 kip (-ve)

At x = 60 ft, F_{FG} = 0.0

PROBLEM Continued -

(ii) To compute F_{FG}, use section (2) - (2)

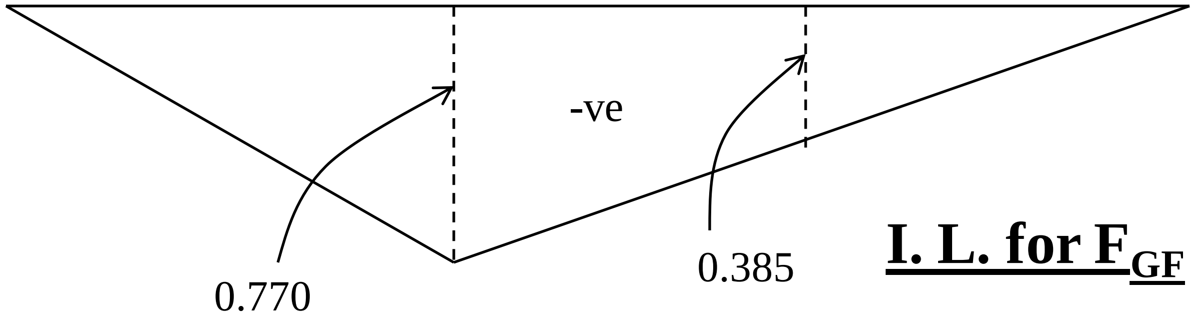


Resolving forces vertically, to the left of C,

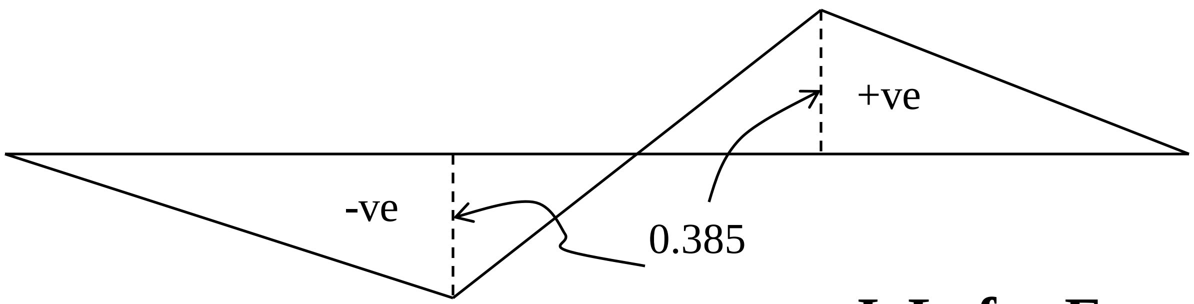
$$(R_A) - F_{FC} \cos 30 = 0$$

$$F_{FC} = R_A/\cos 30 = (1\text{-}x/10)\ (2/\sqrt{3})\ +ve$$

At x = 40 ft, F_{FC} = 0.385 (+ve)
At x = 60 ft, F_{FC} = 0.0

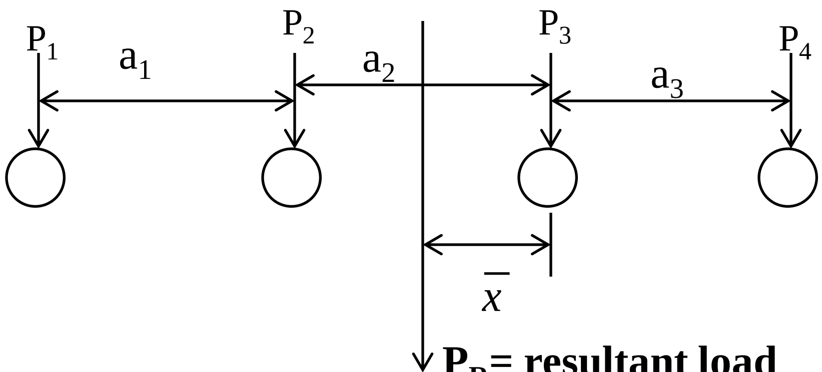


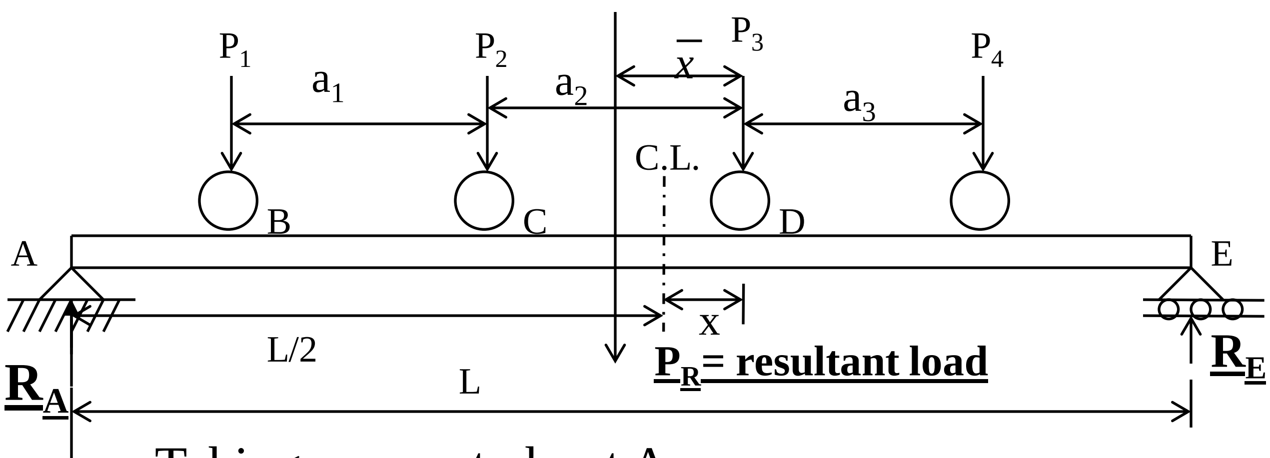
I. L. for F_{GF}



I. L. for F_{FC}

MAXIMUM SHEAR FORCE AND BENDING MOMENT UNDER A SERIES OF CONCENTRATED LOADS





Taking moment about A,

$R_E \times L = P_R \times [L/2 - (\bar{x} - x)]$

$$R_E = \frac{P_R}{L}(L/2 - \bar{x} + x)$$

Taking moment about E,

$$R_A \times L = P_R \times [L/2 + (\bar{x} - x)]$$

$$R_A = \frac{P_R}{L}(L/2 + \bar{x} - x)$$

$$M_D = R_A \times (L/2 + x) - P_1(a_1 + a_2) - P_2 \times a_2$$

$$= \frac{P_R}{L}(L/2 + \bar{x} - x)(L/2 + x) - P_1(a_1 + a_2) - P_2 \times (a_2)$$

$$\frac{dM_D}{dx} = 0$$

$$0 = \frac{P_R}{L}(L/2 + \bar{x} - x) + \frac{P_R}{L}(L/2 + x)(-1)$$

$$= \frac{P_R}{L}[(L/2) + \bar{x} - x - (L/2) - x]$$

$$i.e., \quad \bar{x} - 2x = 0$$

$$\bar{x} = 2x$$

$$x = \frac{\bar{x}}{2}$$

The centerline must divide the distance between the resultant of all the loads in the moving series of loads and the load considered under which maximum bending moment occurs.