

Hall Ticket No

Question Paper Code: AHSB02



# INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad - 500 043

## MODEL QUESTION PAPER - I

B. Tech I Semester End Examinations, December – 2019

Regulations: R18

### LINEAR ALGEBRA AND CALCULUS

(Common to All Branches)

Time: 3 hours

Max. Marks: 70

Answer ONE Question from each Module  
All Questions Carry Equal Marks  
All parts of the question must be answered in one place only

#### MODULE – I

1 (a) Find rank by reducing to Echelon form of 
$$\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$$
 [7M]

(b) Find the inverse of a matrix by using Gauss-Jordan method 
$$\begin{bmatrix} -1 & -3 & 3 & 1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$$
 [7M]

2 (a) Solve the differential equation  $D^2(D^2 + 4)y = 96x^2 + \sin 2x - k$  [7M]

(b) Solve by using method of variation of parameters  $(D^2 + 1)y = \cos ecx$  [7M]

#### MODULE – II

3 (a) Verify Cayley-Hamilton theorem for  $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$  and find  $A^{-1}$  &  $A^4$  [7M]

(b) Diagonalize the matrix  $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$  by linear transformation and find  $A^4$ . [7M]

4 (a) Evaluate the double integral  $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dx dy$ . [7M]

(b) Change the order of integration in  $\int_0^1 \int_{x^2}^{2-x} xy dx dy$  and hence evaluate the double integral [7M]

**MODULE – III**

5 (a) If  $a < b$ , P.T  $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$  using Lagrange's Mean value theorem. Deduce the following. [7M]

(i)  $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1} \frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$

ii)  $\frac{5\pi + 4}{20} < \tan^{-1} 2 < \frac{\pi + 2}{4}$

(b) Verify Rolle's theorem for the function  $\log \left( \frac{x^2 + ab}{x(a+b)} \right)$  in  $[a, b]$ ,  $a > 0, b > 0$  [7M]

6 (a) Evaluate the triple integral  $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z} dx dy dz$ . [7M]

(b) Using triple integration find the volume of the sphere  $x^2 + y^2 + z^2 = a^2$ . [7M]

**MODULE – IV**

7 (a) Prove that  $u = x + y + z, v = xy + yz + zx, w = x^2 + y^2 + z^2$  are functionally dependent. [7M]

(b) Find three positive numbers whose sum is 100 and whose product is maximum. [7M]

8 (a) A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction by using Lagrange's multipliers method [7M]

(b) Determine whether the following functions are functionally dependent or not. If functionally dependent, find the relation between them. [7M]

$u = \frac{x+y}{1-xy}, v = \tan^{-1} x + \tan^{-1} y$

**MODULE – V**

9 (a) Find the directional derivative of the function  $\phi = xy^2 + yz^3$  at the point  $P(1, -2, -1)$  in the direction to the surface  $x \log z - y^2 = -4$  at  $(-1, 2, 1)$ . [7M]

(b) If  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$  show that  $\nabla r^n = nr^{n-2}\vec{r}$ . [7M]

- 10 (a) Evaluate  $\iint_S \vec{F} \cdot d\vec{s}$  if  $f = yzi + 2y^2j + xz^2k$  and S is the Surface of the Cylinder  $x^2+y^2=9$  [7M]  
contained in the first Octant between the planes  $z=0$  and  $z=2$ .
- (b) Verify Green's Theorem in the plane for  $\int_C (x^2 - xy^3)dx + (y^2 - 2xy)dy$  where C is a [7M]  
square with vertices  $(0,0),(2,0),(2,2),(0,2)$ .



## INSTITUTE OF AERONAUTICAL ENGINEERING (Autonomous)

### COURSE OBJECTIVES:

The course should enable the students to:

I	Determine rank of a matrix and solve linear differential equations of second order.
II	Determine the characteristic roots and apply double integrals to evaluate area.
III	Apply mean value theorems and apply triple integrals to evaluate volume.
IV	Determine the functional dependence and extremum value of a function
V	Analyze gradient, divergence, curl and evaluate line, surface, volume integrals over a vector field.

### COURSE OUTCOMES (COs):

CO 1	Determine rank by reducing the matrix to Echelon and Normal forms. Determine inverse of the matrix by Gauss Jordan Method and Solving Second and higher order differential equations with constant coefficients.
CO 2	Determine a modal matrix, and reducing a matrix to diagonal form. Evaluate inverse and powers of matrices by using Cayley-Hamilton theorem. Evaluate double integral. Utilize the concept of change order of integration and change of variables to evaluate double integrals. Determine the area.
CO 3	Apply the Mean value theorems for the single variable functions. Apply triple integrals to evaluate volume.
CO 4	Determine the maxima and minima for a function of several variable with and without constraints.
CO 5	Analyze scalar and vector fields and compute the gradient, divergence and curl. Evaluate line, surface and volume integral of vectors. Use Vector integral theorems to facilitate vector integration.

### COURSE LEARNING OUTCOMES (CLOs):

AHSB02.01	Demonstrate knowledge of matrix calculation as an elegant and powerful mathematical language in connection with rank of a matrix.
AHSB02.02	Determine rank by reducing the matrix to Echelon and Normal forms.
AHSB02.03	Determine inverse of the matrix by Gauss Jordan Method.
AHSB02.04	Find the complete solution of a non-homogeneous differential equation as a linear combination of the complementary function and a particular solution.
AHSB02.05	Solving Second and higher order differential equations with constant coefficients.
AHSB02.06	Interpret the Eigen values and Eigen vectors of matrix for a linear transformation and use properties of Eigen values
AHSB02.07	Understand the concept of Eigen values in real-world problems of control field where they are pole of closed loop system.
AHSB02.08	Apply the concept of Eigen values in real-world problems of mechanical systems where Eigen values are natural frequency and mode shape.
AHSB02.09	Use the system of linear equations and matrix to determine the dependency and independency.
AHSB02.10	Determine a modal matrix, and reducing a matrix to diagonal form.
AHSB02.11	Evaluate inverse and powers of matrices by using Cayley-Hamilton theorem.
AHSB02.12	Apply double integrals to evaluate area of a given function.

AHSB02.13	Utilize the concept of change order of integration and change of variables to evaluate double integrals.
AHSB02.14	Apply the Mean value theorems for the single variable functions.
AHSB02.15	Apply triple integrals to evaluate volume of a given function.
AHSB02.16	Find partial derivatives numerically and symbolically and use them to analyze and interpret the way a function varies.
AHSB02.17	Understand the techniques of multidimensional change of variables to transform the coordinates by utilizing the Jacobian. Determine Jacobian for the coordinate transformation.
AHSB02.18	Apply maxima and minima for functions of several variable's and Lagrange's method of multipliers.
AHSB02.19	Analyze scalar and vector fields and compute the gradient, divergence and curl.
AHSB02.20	Understand integration of vector function with given initial conditions.
AHSB02.21	Evaluate line, surface and volume integral of vectors.
AHSB02.22	Use Vector integral theorems to facilitate vector integration.

### MAPPING OF SEMESTER END EXAMINATION TO COURSE LEARNING OUTCOMES:

SEE Question No		Course Learning Outcomes		Course Outcomes	Blooms Taxonomy Level
1	a	AHSB02.02	Determine rank by reducing the matrix to Echelon and Normal forms.	CO 1	Understand
	b	AHSB02.03	Determine inverse of the matrix by Gauss Jordan Method.	CO 1	Understand
2	a	AHSB02.04	Find the complete solution of a non-homogeneous differential equation as a linear combination of the complementary function and a particular solution.	CO 1	Understand
	b	AHSB02.05	Solving Second and higher order differential equations with constant coefficients.	CO 1	Understand
3	a	AHSB02.11	Evaluate inverse and powers of matrices by using Cayley-Hamilton theorem.	CO 2	Understand
	b	AHSB02.10	Determine a modal matrix, and reducing a matrix to diagonal form.	CO 2	Remember
4	a	AHSB02.12	Apply double integrals to evaluate area of a given function.	CO 2	Understand
	b	AHSB02.13	Utilize the concept of change order of integration and change of variables to evaluate double integrals.	CO 2	Understand
5	a	AHSB02.14	Apply the Mean value theorems for the single variable functions.	CO 3	Understand
	b	AHSB02.14	Apply the Mean value theorems for the single variable functions.	CO 3	Understand
6	a	AHSB02.15	Apply triple integrals to evaluate volume of a given function.	CO 3	Understand
	b	AHSB02.15	Apply triple integrals to evaluate volume of a given function.	CO 3	Understand
7	a	AHSB02.17	Understand the techniques of multidimensional change of variables to transform the coordinates by utilizing the Jacobian. Determine Jacobian for the coordinate transformation.	CO 4	Understand
	b	AHSB02.18	Apply maxima and minima for functions of several variable's and Lagrange's method of multipliers.	CO 4	Understand
8	a	AHSB02.18	Apply maxima and minima for functions of several variable's and Lagrange's method of multipliers.	CO 4	Understand

	b	AHSB02.17	Understand the techniques of multidimensional change of variables to transform the coordinates by utilizing the Jacobian. Determine Jacobian for the coordinate transformation.	CO 4	Understand
9	a	AHSB02.20	Understand integration of vector function with given initial conditions.	CO 5	Understand
	b	AHSB02.20.	Understand integration of vector function with given initial conditions.	CO 5	Understand
10	a	AHSB02.21	Evaluate line, surface and volume integral of vectors.	CO 5	Understand
	b	AHSB02.22	Use Vector integral theorems to facilitate vector integration.	CO 5	Understand

**Signature of Course Coordinator**

**HOD, CE**