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INSTITUTE OF AERONAUTICAL ENGINEERING

## MODEL QUESTION PAPER-I

M. Tech I Semester End Examinations, January - 2020

Regulations: $\mathbf{R 1 8}$
MATHEMATICAL METHODS IN ENGINEERING
(MECH)
Max. Marks: 70
Time: 3 hours

## Answer ONE Question from each UNIT

All Questions Carry Equal Marks
All parts of the question must be answered in one place only

## UNIT - I

1. a) A continuous random variable has the probability density function

$$
f(x)=\left\{\begin{array}{l}
k x e^{-\lambda x}, \text { for } x \geq 0, \lambda>0 \\
0, \text { otherwise }
\end{array}\right.
$$

Determine (i) k (ii) Mean (iii) Variance.
b) 1000 students have written an examination with the mean of test is 35 and standard deviation is 5. Assuming the distribution to be normal find i) How many students marks like between 25 and 40 ? ii) How many students get more than 40 ? iii) How many students get below 20? iv) How many students get more than 50 .
2. a) The no. of automobile accidents per week in a certain area as follows:
$12,8,20,2,14,10,15,6,9,4$. Are these frequencies in agreement with the belief that accidents were same in the during last 10 weeks.
b) Time taken by workers in performing a job by method 1 and method 2 is given below.

| Method 1 | 20 | 16 | 27 | 23 | 22 | 26 | - |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Method 2 | 27 | 33 | 42 | 35 | 32 | 34 | 38 |

Does the data show that variances of time distribution from population which these samples are drawn do not differ significantly?
UNIT - II
3. a) The means of two large samples of sizes 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same population of S.D 2.5 inches
b) According to norms established for a mechanical aptitude test persons who are 18 years
4. a) A cigarette manufacturing firm claims that brand A line of cigarettes outsells its brand B by $8 \%$.if it is found that 42 out of a sample of 200 smokers prefer brand A and 18 out of another sample of 100 smokers prefer brand B. Test whether $8 \%$ difference is a valid claim.
b) Three different methods of teaching statistics are used on three groups of students.

Random samples of size 5 are taken from each group and the results are shown below the grades are on a 10-point scale

| Group <br> A | Group B | Group C |
| :--- | :--- | :--- |
| 7 | 3 | 4 |
| 6 | 6 | 7 |
| 7 | 5 | 7 |
| 7 | 4 | 4 |
| 8 | 7 | 8 |

Determine on the basis of the above data whether there is difference in the teaching methods
UNIT - III
5. a) Using Taylor's series method find an approximate value of y at $\mathrm{x}=0.2$ for the differential equation $y^{\prime}-2 y=3 e^{x}, y(0)=0$.
b) Apply the $4^{\text {th }}$ order Runge-Kutta method to find an approximate value of y when $\mathrm{x}=1.2$ in steps of 0.1 ,given that $y^{\prime}=x^{2}+y^{2}, \mathrm{y}(1)=1.5$
6. a) Obtain the solution of $y^{\prime}=x^{2}(1+y), y(1)=1$ at $x=1(0.1) 1.2$ by any numerical method and estimate $x=1.3$ by Adam's method.
b) Find $y(1.2)$ and $y(1.4)$ by Modified Euler's method given $y^{\prime}=\log (x+y), y(0)=2$ taking $h=0.2$.
UNIT - IV
7. a) Find the temperature in a thin metal rod of length $L$, with both ends insulated and with initial temperature in the rod in $\sin \left(\frac{\pi x}{L}\right)$.
b) Solve the partial differential equation $(m z-n y) p+(n x-l z) q=(l y-m x)$.
8. a)

Solve $u_{x x}=u_{y}+2 u$ with $u(0, y)=0$ and $\frac{\partial u(0, y)}{\partial x}=1+e^{=3 y}$.
b) Solve the partial differential equation $\left(x^{2}-y z\right) p+\left(y^{2}-z x\right) q=z^{2}-x y$

## UNIT - V

9. a) Solve by the method of separation of variables $2 u_{x}+u_{y}=3 u$ and $u(0, y)=e^{-5 y}$
b) Prove that $\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}\right)|\operatorname{Realf}(z)|^{2}=2\left|f^{\prime}(z)\right|^{2}$ where $\mathrm{w}=\mathrm{f}(\mathrm{z})$ is an analytic function.
10. a) Show that real part $\mathrm{u}=x^{3}-3 x y^{2}$ of an analytic function $\mathrm{f}(\mathrm{z})$ is harmonic. Hence find the conjugate harmonic function and the analytic function.
b) A string is stretched and fastened to two points at $\mathrm{x}=0$ and $\mathrm{x}=\mathrm{L}$. Motion is started by displacing the string into the form $\mathrm{y}=\mathrm{k}\left(1 \mathrm{x}-\mathrm{x}^{2}\right)$ from which it is released at time $\mathrm{t}=0$. Find the displacement of any point on the string at a distance of $x$ from one end at time $t$.

INSTITUTE OF AERONAUTICAL ENGINEERING

## (Autonomous)

Dundigal, Hyderabad - 500043

## COURSE OBJECTIVES:

The course should enable the students to:

| I | Develop a basic understanding of a range of mathematics tools with emphasis on engineering <br> applications. |
| :---: | :--- |
| II | Solve problems with techniques from advanced linear algebra, ordinary differential equations and <br> multivariable differentiation. |
| III | Develop skills to think quantitatively and analyze problems critically. |

## COURSE OUTCOMES (COs):

| CO 1 | Describe the basic concepts of probability, discrete, continuous random variables and determine <br> probability distribution, sampling distribution of statistics like t, F and chi-square. |
| :---: | :--- |
| CO 2 | Understand the foundation for hypothesis testing to predict the significance difference in the sample <br> means and the use of ANOVA technique. |
| CO 3 | Determine Ordinary linear differential equations solvable by nonlinear ODE's. |
| CO 4 | Explore First and second order partial differential equations. |
| CO 5 | Analyze the solution methods for wave equation, D'Alembert solution, and potential equation, <br> properties of harmonic functions, maximum principle, and solution by variable separation method. |

## COURSE LEARNING OUTCOMES (CLOs):

| BCCB02.01 | Describe the basic concepts of probability, discrete and continuous random variables |
| :--- | :--- |
| BCCB02.02 | Determine the probability distribution to find mean and variance. |
| BCCB02.03 | Discuss the concept of sampling distribution of statistics like t, F and chi-square. |
| BCCB02.04 | Understand the foundation for hypothesis testing. |
| BCCB02.05 | Apply testing of hypothesis to predict the significance difference in the sample means. |
| BCCB02.06 | Understand the assumptions involved in the use of ANOVA technique. |
| BCCB02.07 | Solve differential equation using single step method. |
| BCCB02.08 | Solve differential equation using multi step methods. |
| BCCB02.09 | Understand the concept of non- linear ordinary differential equations. |
| BCCB02.10 | Understand partial differential equation for solving linear equations. |
| BCCB02.11 | Solving the heat equation in subject to boundary conditions. |
| BCCB02.12 | Solving the wave equation in subject to boundary conditions. |
| BCCB02.13 | Understand the conditions for a complex variable to be analytic and entire function. |
| BCCB02.14 | Understand the concept of harmonic functions. |
| BCCB02.15 | Analyze the concept of partial differential equations by variable separation method. |

## MAPPING OF SEMESTER END EXAMINATION - COURSE OUTCOMES

| $\begin{array}{c\|} \hline \text { SEE } \\ \text { Question } \\ \text { No } \\ \hline \end{array}$ |  | Course Learning Outcomes |  | Course Outcomes | Blooms Taxonomy Level |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | a | BCCB02.01 | Describe the basic concepts of probability, discrete and continuous random variables | CO 1 | Understand |
|  | b | BCCB02.02 | Determine the probability distribution to find mean and variance. | CO 1 | Understand |
| 2 | a | BCCB02.03 | Discuss the concept of sampling distribution of statistics like $\mathrm{t}, \mathrm{F}$ and chi-square. | CO 1 | Understand |
|  | b | BCCB02.03 | Discuss the concept of sampling distribution of statistics like $\mathrm{t}, \mathrm{F}$ and chi-square. | CO 1 | Understand |
| 3 | a | BCCB02.05 | Apply testing of hypothesis to predict the significance difference in the sample means. | CO2 | Understand |
|  | b | BCCB02.05 | Apply testing of hypothesis to predict the significance difference in the sample means. | CO2 | Remember |
| 4 | a | BCCB02.05 | Apply testing of hypothesis to predict the significance difference in the sample means. | CO2 | Understand |
|  | b | BCCB02.06 | Understand the assumptions involved in the use of ANOVA technique. | CO2 | Understand |
| 5 | a | BCCB02.07 | Solve differential equation using single step method. | CO 3 | Understand |
|  | b | BCCB02.08 | Solve differential equation using multi step methods. | CO 3 | Understand |
| 6 | a | BCCB02.08 | Solve differential equation using multi step methods. | CO 3 | Understand |
|  | b | BCCB02.08 | Solve differential equation using multi step methods. | CO 3 | Understand |
| 7 | a | BCCB02.11 | Solving the heat equation in subject to boundary conditions. | CO 4 | Understand |
|  | b | BCCB02.10 | Understand partial differential equation for solving linear equations. | CO 4 | Understand |
| 8 | a | BCCB02.11 | Solving the heat equation in subject to boundary conditions. | CO 4 | Understand |
|  | b | BCCB02.10 | Understand partial differential equation for solving linear equations. | CO 4 | Understand |
| 9 | a | BCCB02.15 | Analyze the concept of partial differential equations by variable separation method. | CO 5 | Understand |
|  | b | BCCB02.12 | Solving the wave equation in subject to boundary conditions. | CO 5 | Understand |
| 10 | a | BCCB02.14 | Understand the concept of harmonic functions. | CO 5 | Understand |
|  | b | BCCB02.13 | Understand the conditions for a complex variable to be analytic and entire function. | CO 5 | Understand |

