MECHANICS OF FLUIDS

BY

Mr.C. Satya Sandeep
Assistant Professor

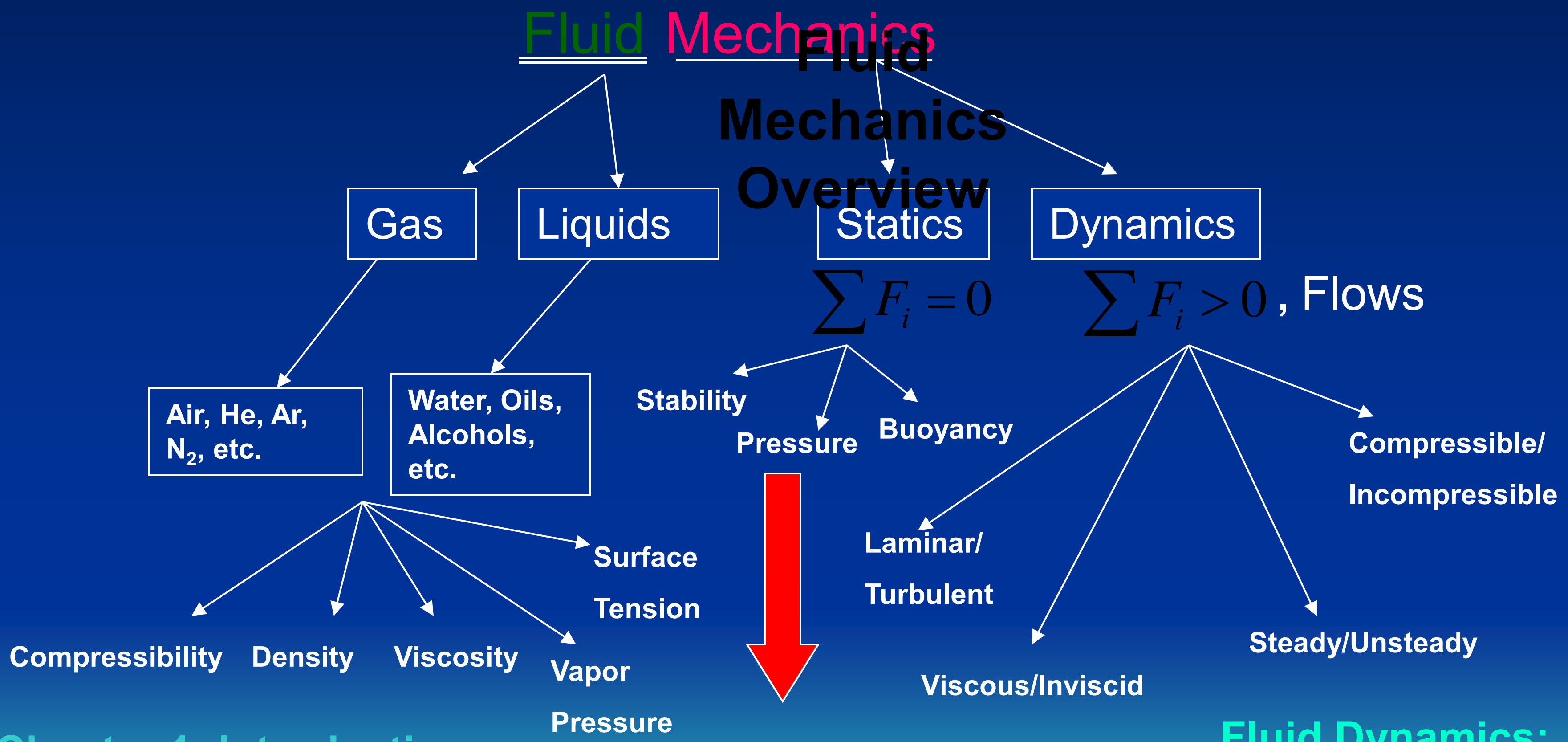
INSTITUTE OF AERONAUTICAL ENGINEERING
(Autonomous)
Dundigal, Hyderabad - 500 043
AERONAUTICAL ENGINEERING

Fluid Mechanics

The atmosphere is a fluid!

Fluid Mechanics Overview

Fluid Mechanics Overview



Fluid Mechanics

Gas

Liquids

Statics $\sum F_i = 0$

Dynamics $\sum F_i > 0$, Flows

Air, He, Ar, N_2, etc.

Water, Oils, Alcohols, etc.

Compressibility

Density

Viscosity

Vapor Pressure

Surface Tension

Stability

Pressure

Buoyancy

Laminar/ Turbulent

Viscous/Inviscid

Steady/Unsteady

Compressible/ Incompressible

Chapter 1: Introduction

Chapter 2: Fluid Statics

Fluid Dynamics: Rest of Course

Characteristics of Fluids

- Gas or liquid state
- “Large” molecular spacing relative to a solid
- “Weak” intermolecular cohesive forces
- Can not resist a shear stress in a stationary state
- Will take the shape of its container
- Generally considered a continuum
- Viscosity distinguishes different types of fluids

Measures of Fluid Mass and Weight: Density

The density of a fluid is defined as mass per unit volume.

$$\rho = \frac{m}{v}$$

m = mass, and v = volume.

- Different fluids can vary greatly in density
- Liquids densities do not vary much with pressure and temperature
- Gas densities can vary quite a bit with pressure and temperature
- Density of water at 4° C : 1000 kg/m^3
- Density of Air at 4° C : 1.20 kg/m^3

Alternatively, **Specific Volume**: $\upsilon = \frac{1}{\rho}$

Measures of Fluid Mass and Weight: Specific Weight

The specific weight of fluid is its weight per unit volume.

$$\gamma = \rho g$$

g = local acceleration of gravity, 9.807 m/s^2

- Specific weight characterizes the weight of the fluid system
- Specific weight of water at 4° C : 9.80 kN/m^3
- Specific weight of air at 4° C : 11.9 N/m^3

Measures of Fluid Mass and Weight: Specific Gravity

The specific gravity of fluid is the ratio of the density of the fluid to the density of water @ 4° C.

$$SG = \frac{\rho}{\rho_{H_2O}}$$

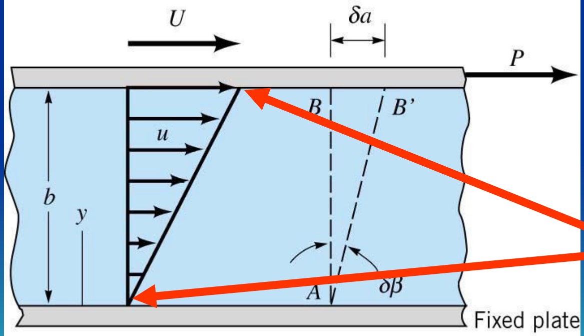
- Gases have low specific gravities
- A liquid such as Mercury has a high specific gravity, 13.2
- The ratio is unitless.
- Density of water at 4° C : 1000 kg/m^3

Viscosity: Introduction

The viscosity is measure of the "fluidity" of the fluid which is not captured simply by density or specific weight. A fluid can not resist a shear and under shear begins to flow. The shearing stress and shearing strain can be related with a relationship of the following form for common fluids such as water, air, oil, and gasoline:

$$\tau = \mu \frac{du}{dy}$$

μ is the absolute viscosity or dynamics viscosity of the fluid, u is the velocity of the fluid and y is the vertical coordinate as shown in the schematic below:



"No Slip Condition"

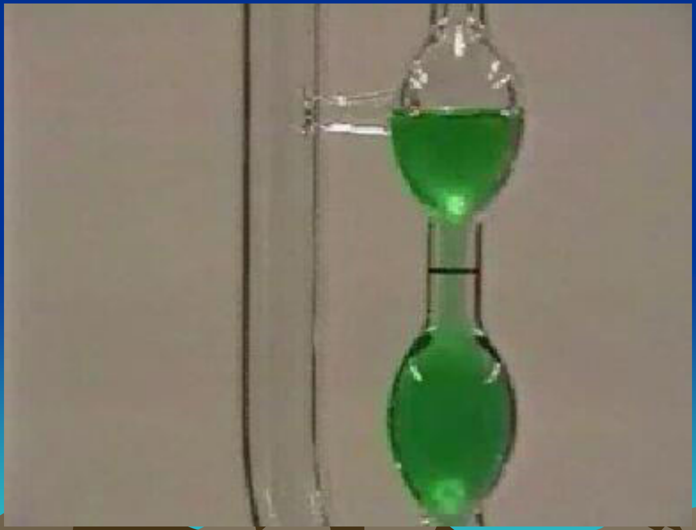
Viscosity: Measurements

A Capillary Tube Viscosimeter is one method of measuring the viscosity of the fluid.

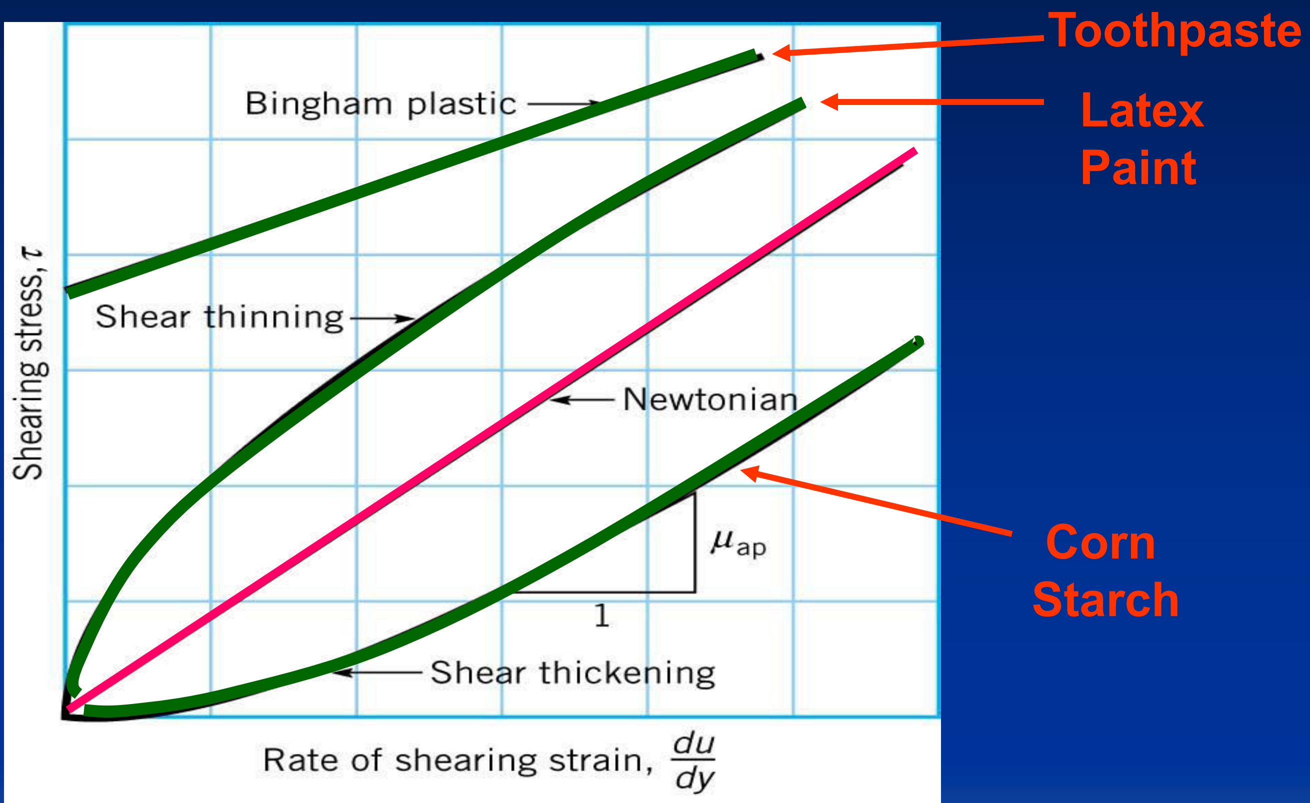
Viscosity Varies from Fluid to Fluid and is dependent on temperature, thus temperature is measured as well.

Units of Viscosity are $N \cdot s/m^2$ or $lb \cdot s/ft^2$

Movie Example using a Viscosimeter:



Viscosity: Newtonian vs. Non-Newtonian



Newtonian Fluids Linear Relationships ress and strain: Mo

Non-Newtonian Fluids Non-Linear and strain

Viscosity: Kinematic Viscosity

$$\nu = \frac{\mu}{\rho}$$

- Kinematic viscosity is another way of representing viscosity
- Used in the flow equations
- The units are of L^2/T or m^2/s and ft^2/s

Compressibility of Fluids: Bulk Modulus

$$E_\upsilon = \frac{dp}{d\rho / \rho}$$

P is pressure, and ρ is the density.

- Measure of how pressure compresses the volume/density
- Units of the bulk modulus are N/m^2 (Pa) and lb/in.^{2} (psi).
- Large values of the bulk modulus indicate incompressibility
- Incompressibility indicates large pressures are needed to compress the volume slightly
- It takes 3120 psi to compress water 1% at atmospheric pressure and 60° F.
- Most liquids are incompressible for most practical engineering problems.

Compressibility of Fluids: Bulk Modulus

$$E_{\upsilon} = \frac{dp}{d\rho / \rho}$$

P is pressure, and ρ is the density.

- Measure of how pressure compresses the volume/density
- Units of the bulk modulus are N/m^2 (Pa) and lb/in.^{2} (psi).
- Large values of the bulk modulus indicate incompressibility
- Incompressibility indicates large pressures are needed to compress the volume slightly
- It takes 3120 psi to compress water 1% at atmospheric pressure and 60° F.
- Most liquids are incompressible for most practical engineering problems.

Surface Tension

At the interface between a liquid and a gas or two immiscible liquids, forces develop forming an analogous “skin” or “membrane” stretched over the fluid mass which can support weight.

This “skin” is due to an imbalance of cohesive forces. The interior of the fluid is in balance as molecules of the like fluid are attracting each other while on the interface there is a net inward pulling force.

Surface tension is the intensity of the molecular attraction per unit length along any line in the surface.

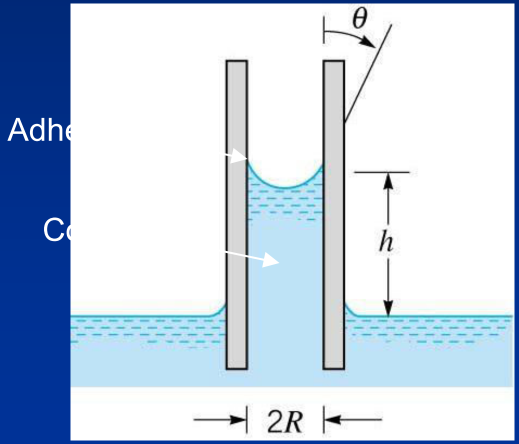
Surface tension is a property of the liquid type, the temperature, and the other fluid at the interface.

This membrane can be “broken” with a surfactant which reduces the surface tension.

Surface Tension: Capillary Action

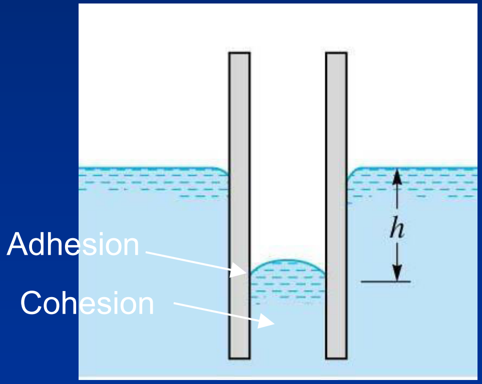
Capillary action in small tubes which involve a liquid-gas-solid interface is caused by surface tension. The fluid is either drawn up the tube or pushed down.

“Wetted”



Adhesion > Cohesion

“Non-Wetted”



Cohesion > Adhesion

h is the height, R is the radius of the tube, θ is the angle of contact.

The weight of the fluid is balanced with the vertical force caused by surface tension.

Pressure in a fluid

- Pressure is the ratio of the perpendicular force applied to an object and the surface area to which the force was applied

$$P = F/A$$

- Don’t confuse pressure (a scalar) with force (a vector).

The Pascal

- The SI unit of pressure is the pascal (Pa)

  $$1 \text{ Pa} = 1 \text{ N/m}^2$$

  named after Blaise Pascal (1623-1662). One pascal also equals 0.01 millibar or 0.00001 bar.
- Meteorologists have used the millibar as a unit of air pressure since 1929.

Millibar or hPa?

- When the change to scientific units occurred in the 1960's many meteorologists preferred to keep using the magnitude they were used to and use a prefix "hecto" (h), meaning 100.
- Therefore, 1 hectopascal (hPa) = 100 Pa = 1 millibar (mb). 100,000 Pa equals 1000 hPa which equals 1000 mb. The units we refer to in meteorology may be different, however, their numerical value remains the same.

Atmospheric Pressure

- We live at the bottom of a sea of air. The pressure varies with temperature, altitude, and other weather conditions
- The average at sea level is 1 atm (atmosphere)
- Some common units used:

1 atm = 101,325 Pa

1 atm = 1013.25 mb = 1013.25 hPa

1 atm = 760 mmHg = 29.96 inHg

1 atm = 14.7 lb/in^2

Atmospheric Layer Boundaries

- The layers of our atmosphere can vary in thickness from the equator to the poles
- The layer boundaries occur at changes in temperature profile

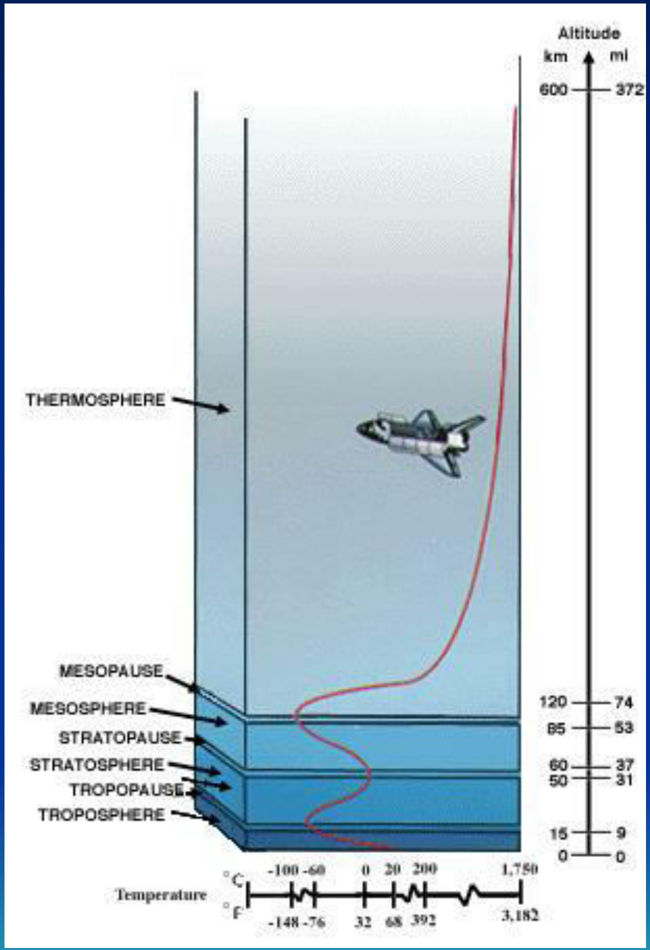
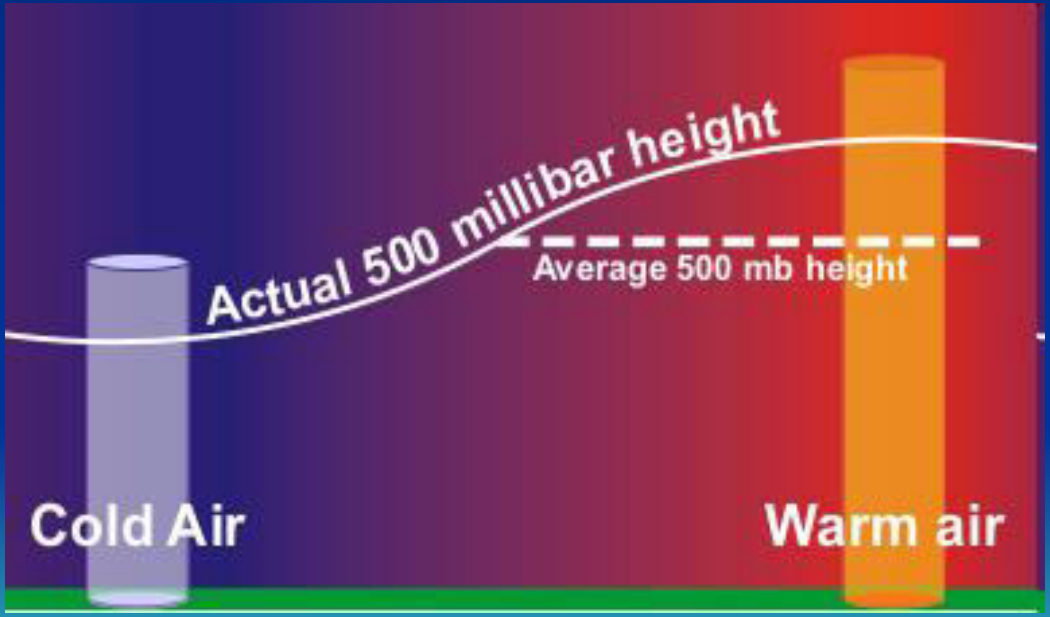


Image courtesy NASA

Temperature affects pressure

- When air warms, it expands, becoming less dense. Lower density means a volume of air weighs less, therefore applying less pressure.



← Approx 5.6 km

Image Courtesy NOAA

Fluid Pressure

- A column of fluid h = 4 m high will exert a greater pressure than a column h = 2 m
- What will the pressure be due to this fluid?

Force = mg   Area = A

But $m = \rho V$ and $A = V/h$

$P_{fluid} = \rho Vg/V/h = \rho gh$

Assuming uniform density for the fluid

$P_{fluid} = \rho g h$

- The pressure due to a fluid depends only on the average density and the height
- It does not depend on the shape of the container!
- The total pressure at the bottom of an open container will be the sum of this fluid pressure and the atmospheric pressure above

$$P = P_0 + P_{fluid} = P_0 + \rho g h$$

Compressibility

- Liquids are nearly incompressible, so they exhibit nearly uniform density over a wide range of heights (ρ only varies by a few percent)
- Gases, on the other hand, are highly compressible, and exhibit significant change in density over height

ρ_{air} at sea level ~ $3\rho_{air}$ at Mt Everest's peak

Pascal’s Law

- Pressure applied to a contained fluid is transmitted undiminished to the entire fluid and to the walls of the container
- You use this principle to get toothpaste out of the tube (squeezing anywhere will transmit the pressure throughout the tube)
- Your mechanic uses this principle to raise your car with a hydraulic lift

Absolute and Gauge Pressure

- Your tire maker recommends filling your tires to 30 psi. This is in addition to the atmospheric pressure of 14.7 psi (typical)
- Since $P = P_0 + P_{fluid}$, the absolute pressure is P, and the gauge pressure is P_{fluid}
- In this case, the gauge pressure would be 30 psi and the absolute pressure would be 44.7 psi

(psi = pounds per square inch)

Measuring Pressure

- There are two main types of instruments used to measure fluid pressure
- The Manometer
  - Blood pressure is measured with a variant called the sphygmomanometer (say that three times fast!)
- The Barometer
  - Many forms exist

The Manometer

- Open-tube manometer has a known pressure P_0 enclosed on one end and open at the other end

$$P_0 - P_a = \rho g h$$

- If $P_a > P_0$, the fluid will be forced toward the closed end (h is neg, as shown)

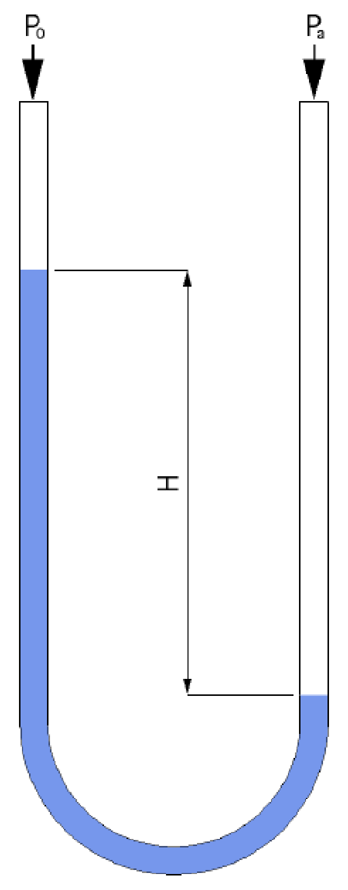


Image from Ruben Castelnuovo - Wikipedia

The Barometer

- Filling a tube (closed at one end) with liquid, then inverting it in a dish of that liquid
- A near vacuum will form at the top
- Since $P_{fluid} = \rho gh$, the column of liquid will be in equilibrium when $P_{fluid} = P_{air}$
- Meteorologist speak of 29.96 inches. This is the height of a column of Mercury which could be supported by that air pressure

MAE 3130: Fluid Mechanics
Lecture 5: Fluid Kinematics
Spring 2003

Dr. Jason Roney

Mechanical and Aerospace Engineering

Outline

- Introduction
- Velocity Field
- Acceleration Field
- Control Volume and System Representation
- Reynolds Transport Theorem
- Examples

Fluid Kinematics: Introduction

- Fluids subject to shear, flow
- Fluids subject to pressure imbalance, flow
- In kinematics we are not concerned with the force, but the motion.
- Thus, we are interested in visualization.
- We can learn a lot about flows from watching.

Velocity Field

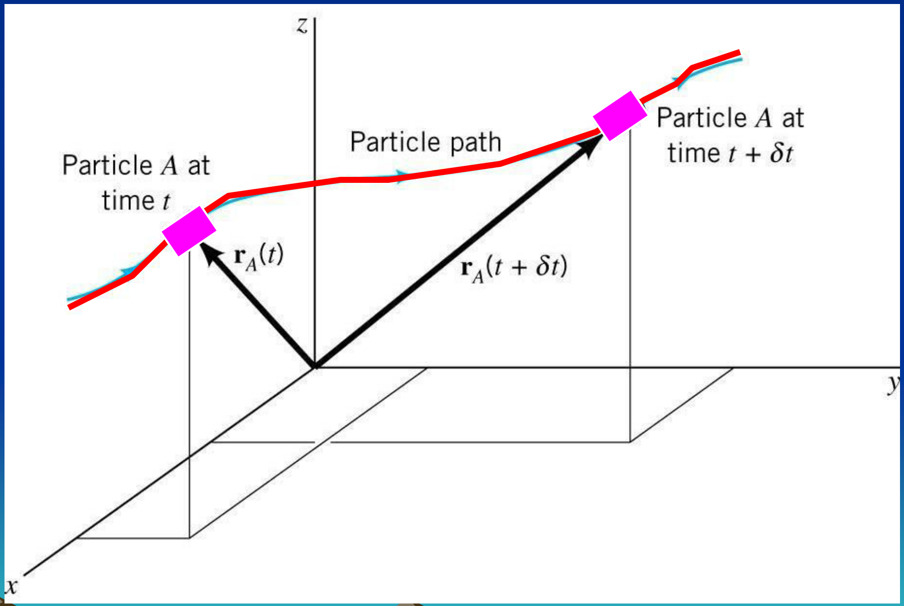
Continuum Hypothesis: the flow is made of tightly packed fluid particles that interact with each other. Each particle consists of numerous molecules, and we can describe velocity, acceleration, pressure, and density of these particles at a given time.

$$\mathbf{V} = u(x, y, z, t)\hat{\mathbf{i}} + v(x, y, z, t)\hat{\mathbf{j}} + w(x, y, z, t)\hat{\mathbf{k}}$$

$$\mathbf{V} = \mathbf{V}(x, y, z, t)$$

$$V = |\mathbf{V}| = (u^2 + v^2 + w^2)^{1/2}$$

$$d\mathbf{r}_A/dt = \mathbf{V}_A$$



Velocity Field:

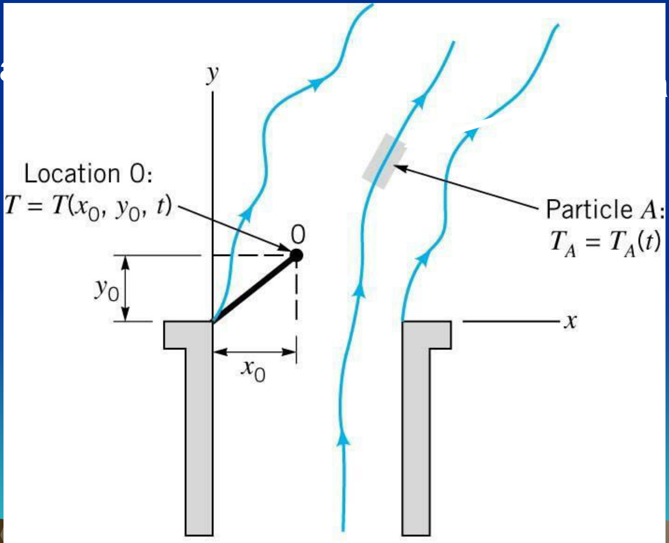
Velocity Field: Eulerian and Lagrangian

Eulerian: the fluid motion is given by completely describing the necessary properties as a function of space and time. We obtain information about the flow by noting what happens at fixed points.

Lagrangian: following individual fluid particles as they move about and determining how the fluid properties of these particles change as a function of time.

Measurement of Temperature

Euleri



ngian

If we have enough information, we can obtain Eulerian from Lagrangian or vice versa.

Eulerian methods are commonly used in fluid experiments or analysis—a probe placed in a flow.

Lagrangian methods can also be used if we “tag” fluid particles in a flow.

Velocity Field: Steady and Unsteady Flows

Steady Flow: The velocity at a given point in space does not vary with time.

$$\partial \mathbf{V}/\partial t = 0$$

Very often, we assume steady flow conditions for cases where there is only a slight time dependence, since the analysis is "easier"

Unsteady Flow: The velocity at a given point in space does vary with time.

Almost all flows have some unsteadiness. In addition, there are periodic flows, non-periodic flows, and completely random flows.

Examples:

Nonperiodic flow: "water hammer" in water pipes.

Unsteady Flow:

Periodic flow: "fuel injectors" creating a periodic swirling in the combustion chamber. Effect occurs time after time.

Random flow: "Turbulent", gusts of wind, splashing of water in the sink

Flow Visualize:

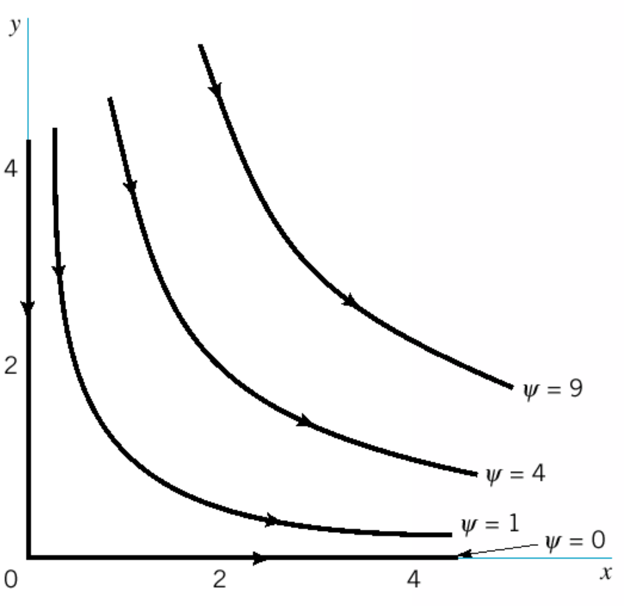
Steady or Unsteady only pertains to fixed measurements, i.e. exhaust temperature from a tail pipe is relatively constant "steady"; however, if we followed individual particles of exhaust they cool!

Velocity Field: Streamlines

Streamline: the line that is everywhere tangent to the velocity field. If the flow is steady, nothing at a fixed point changes in time. In an unsteady flow the streamlines due change in time.

Analytically, for 2D flows, integrate the equations defining lines tangent to the velocity field:

$$\frac{dy}{dx} = \frac{v}{u}$$



Experimentally, flow visualization with dyes can easily produce the streamlines for a steady flow, but for unsteady flows these types of experiments don't necessarily provide information about the streamlines.

Velocity Field: Streaklines

Streaklines: a laboratory tool used to obtain instantaneous photographs of marked particles that all passed through a given flow field at some earlier time. Neutrally buoyant smoke, or dye is continuously injected into the flow at a given location to create the picture.

If the flow is steady, the picture will look like streamlines (previous video).

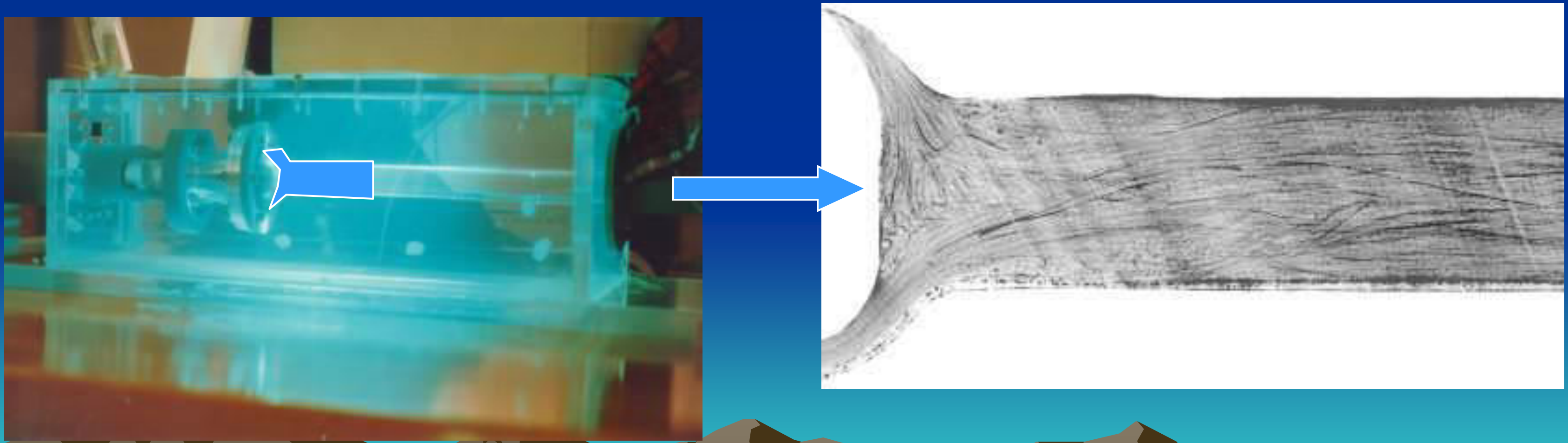
If the flow is unsteady, the picture will be of the instantaneous flow field, but will change from frame to frame, “streaklines”.

Velocity Field: Pathlines

Pathlines: line traced by a given particle as it flows from one point to another. This method is a Lagrangian technique in which a fluid particle is marked and then the flow field is produced by taking a time exposure photograph of its movement.

If the flow is steady, the picture will look like streamlines (previous video).

If the flow is unsteady, the picture will be of the instantaneous flow field, but will change from frame to frame, “pathlines”.



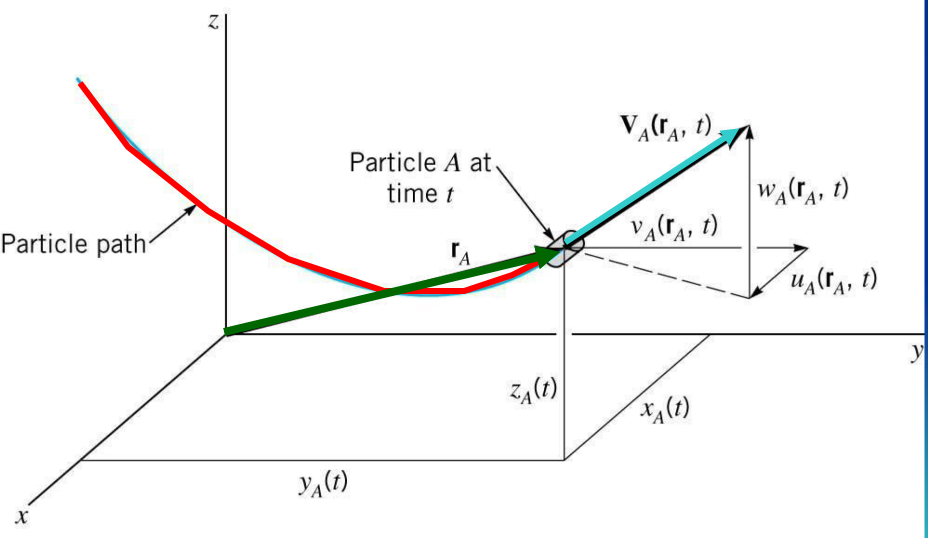
Acceleration Field

Lagrangian Frame: $\mathbf{a} = \mathbf{a}(t)$

Eulerian Frame: we describe the acceleration in terms of position and time without following an individual particle. This is analogous to describing the velocity field in terms of space and time.

$$\mathbf{V}_A = \mathbf{V}_A(\mathbf{r}_A, t) = \mathbf{V}_A[x_A(t), y_A(t), z_A(t), t]$$

A fluid particle can accelerate due to a change in velocity in time (“unsteady”) or in space (moving to a place with a greater velocity).



Acceleration Field: Material (Substantial) Derivative

$$\mathbf{a}_A(t) = \frac{d\mathbf{V}_A}{dt} = \underbrace{\frac{\partial \mathbf{V}_A}{\partial t}}_{\text{time dependence}} + \underbrace{\frac{\partial \mathbf{V}_A}{\partial x}\frac{dx_A}{dt} + \frac{\partial \mathbf{V}_A}{\partial y}\frac{dy_A}{dt} + \frac{\partial \mathbf{V}_A}{\partial z}\frac{dz_A}{dt}}_{\text{spatial dependence}}$$

We note:

$u_A = dx_A/dt$ $\quad v_A = dy_A/dt$ $\quad w_A = dz_A/dt$

Then, substituting:

$$\mathbf{a}_A = \frac{\partial \mathbf{V}_A}{\partial t} + u_A\frac{\partial \mathbf{V}_A}{\partial x} + v_A\frac{\partial \mathbf{V}_A}{\partial y} + w_A\frac{\partial \mathbf{V}_A}{\partial z}$$

The above is good for any fluid particle, so we drop “A”:

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z}$$

Acceleration Field: Material (Substantial) Derivative

Writing out these terms in vector components:

x-direction: $$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

y-direction: $$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

z-direction: $$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Fluid flows experience fairly large accelerations or decelerations, especially approaching stagnation points.

Writing these results in "short-hand": $$\mathbf{a} = \frac{D\mathbf{V}}{Dt}$$

where, $$\frac{D(\ )}{Dt} \equiv \frac{\partial(\ )}{\partial t} + u\frac{\partial(\ )}{\partial x} + v\frac{\partial(\ )}{\partial y} + w\frac{\partial(\ )}{\partial z}$$

$$\frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + (\mathbf{V}\cdot\nabla)(\ )$$

$$\nabla() = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \quad , \quad \mathbf{V}\cdot\nabla(\ ) = u\partial(\ )/\partial x + v\partial(\ )/\partial y + w\partial(\ )/\partial z$$

Acceleration Field: Material (Substantial) Derivative

Applied to the Temperature Field in a Flow:

$$T = T(x, y, z, t)$$

$$\mathbf{V} = \mathbf{V}\,(x, y, z, t)$$

The material derivative of any variable is the rate at which that variable changes with time for a given particle (as seen by one moving along with the fluid—Lagrangian description).

$$\frac{dT_A}{dt} = \frac{\partial T_A}{\partial t} + \frac{\partial T_A}{\partial x}\frac{dx_A}{dt} + \frac{\partial T_A}{\partial y}\frac{dy_A}{dt} + \frac{\partial T_A}{\partial z}\frac{dz_A}{dt}$$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{\partial T}{\partial t} + \mathbf{V}\cdot\nabla T$$

Acceleration Field: Unsteady Effects

If the flow is unsteady, its paramater values at any location may change with time (velocity, temperature, density, etc.)

The local derivative represents the unsteady portion of the flow: $[\partial(\ )/\partial t]$

If we are talking about velocity, then the above term is local acceleration.

In steady flow, the above term goes to zero.

If we are talking about temperature, and **V** = 0, we still have heat transfer because of the following term:

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \underbrace{u\frac{\partial T}{\partial x}}_{0} + \underbrace{v\frac{\partial T}{\partial y}}_{0} + \underbrace{w\frac{\partial T}{\partial z}}_{0}$$

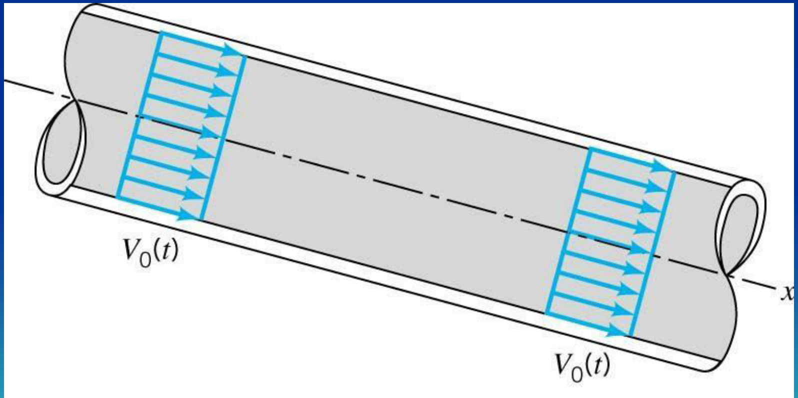
$$= \partial T/\partial t$$

Acceleration Field: Unsteady Effects

Consider flow in a constant diameter pipe, where the flow is assumed to be spatially uniform: $\mathbf{V} = V_0(t)\,\hat{\mathbf{i}}$

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z} \longrightarrow \frac{\partial V_0}{\partial t}\hat{\mathbf{i}}$$

$$\frac{\partial V_0}{\partial t}\hat{\mathbf{i}} \qquad 0 \qquad 0 \quad 0 \qquad 0 \quad 0$$



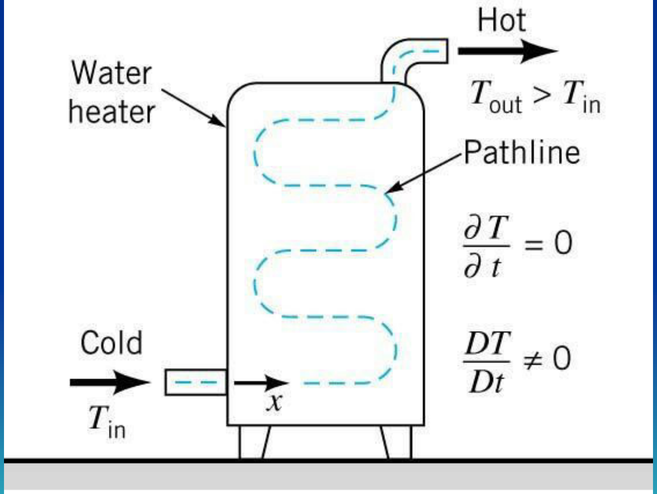
Acceleration Field: Convective Effects

The portion of the material derivative represented by the spatial derivatives is termed the convective term or convective accleration: $(\mathbf{V} \cdot \nabla)\mathbf{V}$

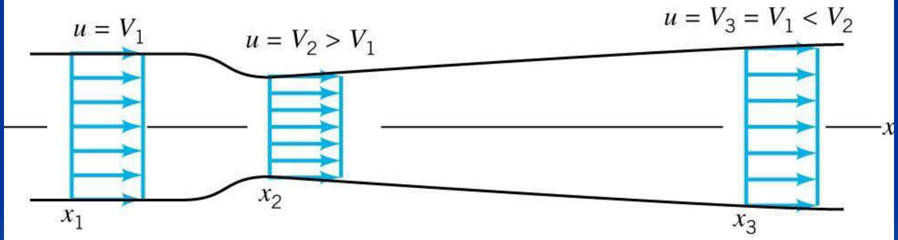
It represents the fact the flow property associated with a fluid particle may vary due to the motion of the particle from one point in space to another.

Convective effects may exist whether the flow is steady or unsteady.

Example 1:



Example 2:



$$a_x = u \, \partial u / \partial x$$

$$\partial u / \partial x > 0$$

$$a_x > 0$$

$$\partial u / \partial x < 0$$

$$a_x < 0$$

Acceleration = Deceleration

Control Volume and System Representations

Systems of Fluid: a specific identifiable quantity of matter that may consist of a relatively large amount of mass (the earth's atmosphere) or a single fluid particle. They are always the same fluid particles which may interact with their surroundings.

Example: following a system the fluid passing through a compressor

We can apply the equations of motion to the fluid mass to describe their behavior, but in practice it is very difficult to follow a specific quantity of matter.

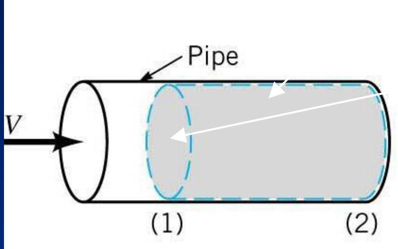
Control Volume: is a volume or space through which the fluid may flow, usually associated with the geometry.

When we are most interested in determining the the forces put on a fan, airplane, or automobile by the air flow past the object rather than following the fluid as it flows along past the object.

Identify the specific volume in space and analyze the fluid flow within, through, or around that volume.

Control Volume and System Representations

Fixed Control Volume:

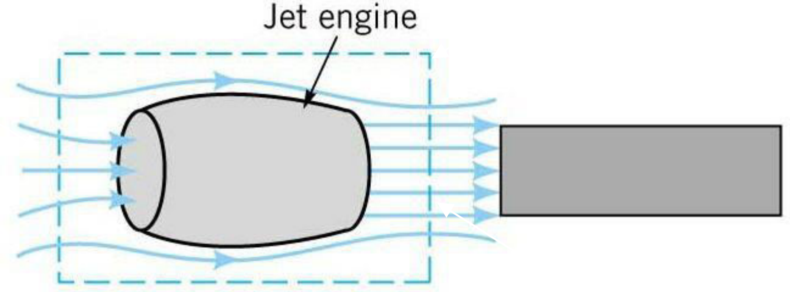


Surface of the Pipe

Surface of the Fluid

Fixed or Moving Control Volume:

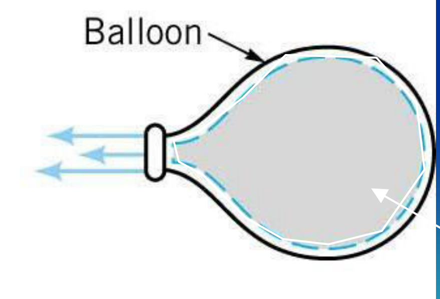
Inflow



Volume Around The

Outflow

Deforming Control Volume:



Outflow

Deforming Volume

Reynolds Transport Theorem: Preliminary Concepts

All the laws of governing the motion of a fluid are stated in their basic form in terms of a system approach, and not in terms of a control volume.

The Reynolds Transport Theorem allows us to shift from the system approach to the control volume approach, and back.

General Concepts: $B = mb$

B represents any of the fluid properties, m represent the mass, and b represents the amount of the parameter per unit volume.

Examples:

Mass	$b = 1$	$B = m$
Kinetic Energy	$b = V^2/2$	$B = mV^2/2$
Momentum	$b = \mathbf{V}$ **(vector)**	$\mathbf{B} = m\mathbf{V}$

B is termed an extensive property, and b is an intensive property. B is directly proportional to mass, and b is independent of mass.

Reynolds Transport Theorem: Preliminary Concepts

For a System: The amount of an extensive property can be calculated by adding up the amount associated with each fluid particle.

$$B_{\mathrm{sys}} = \lim_{\delta\forall \to 0} \sum_i b_i (\rho_i \, \delta\forall_i) = \int_{\mathrm{sys}} \rho b \, d\forall$$

Now, the time rate of change of that system:

$$\frac{dB_{\mathrm{sys}}}{dt} = \frac{d\left(\int_{\mathrm{sys}} \rho b \, d\forall\right)}{dt}$$

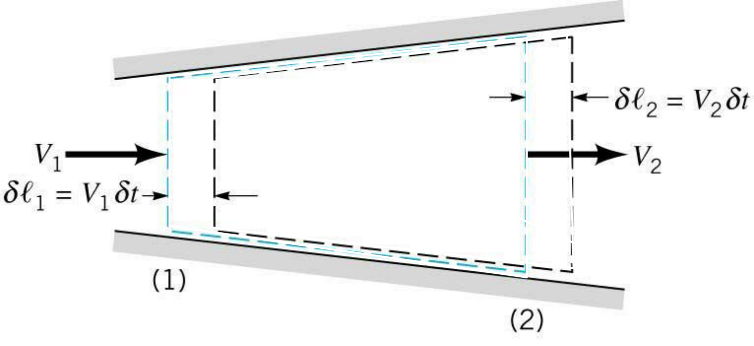
Now, for control volume:

$$\frac{dB_{\mathrm{cv}}}{dt} = \frac{d\left(\int_{\mathrm{cv}} \rho b \, d\forall\right)}{dt}$$

For the control volume, we only integrate over the control volume, this is different integrating over the system, though there are instance when they could be the same.

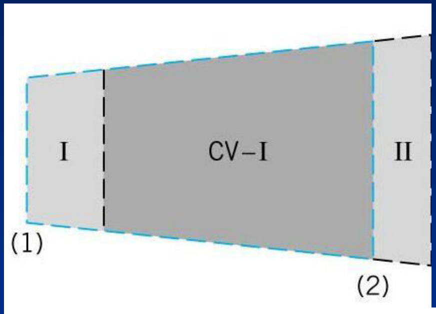
Reynolds Transport Theorem: Derivation

Consider a 1D flow through a fixed control volume between (1) and (2):



stem at t$_2$

system at t$_1$



$$\delta\ell_1 = V_1\,\delta t$$
$$\delta\ell_2 = V_2\,\delta t$$

$\Rightarrow$ “SYS = CV − I + II”

Writing equation in terms of the extensive parameter:

Originally, $B_{\mathrm{sys}}(t) = B_{\mathrm{cv}}(t)$

At time 2: $B_{\mathrm{sys}}(t + \delta t) = B_{\mathrm{cv}}(t + \delta t) - B_{\mathrm{I}}(t + \delta t) + B_{\mathrm{II}}(t + \delta t)$

Divide by δt:

$$\frac{\delta B_{\mathrm{sys}}}{\delta t} = \frac{B_{\mathrm{sys}}(t + \delta t) - B_{\mathrm{sys}}(t)}{\delta t} = \frac{B_{\mathrm{cv}}(t + \delta t) - B_{\mathrm{I}}(t + \delta t) + B_{\mathrm{II}}(t + \delta t) - B_{\mathrm{sys}}(t)}{\delta t}$$

Reynolds Transport Theorem: Derivation

Noting, $B_{\text{sys}}(t) = B_{\text{cv}}(t)$

$$\frac{\delta B_{\text{sys}}}{\delta t} = \frac{B_{\text{cv}}(t + \delta t) - B_{\text{cv}}(t)}{\delta t} - \frac{B_{\text{I}}(t + \delta t)}{\delta t} + \frac{B_{\text{II}}(t + \delta t)}{\delta t}$$

(1) (2) (3) (4)

Let, $\delta t \longrightarrow 0$

(1) $\frac{\delta B_{\text{sys}}}{\delta t} \Rightarrow DB_{\text{sys}}/Dt$

Time rate of change of mass wi

(2) $$\frac{B_{\text{cv}}(t + \delta t) - B_{\text{cv}}(t)}{\delta t} \Rightarrow \lim_{\delta t \to 0} \frac{B_{\text{cv}}(t + \delta t) - B_{\text{cv}}(t)}{\delta t} = \frac{\partial B_{\text{cv}}}{\partial t} = \frac{\partial \left(\int_{\text{cv}} \rho b \, d\forall \right)}{\partial t}$$

The rate at which the extensive property flows out of the control surface:

(4) $B_{\text{II}}(t + \delta t) = (\rho_2 b_2)(\delta \forall_{\text{II}}) = \rho_2 b_2 A_2 V_2 \, \delta t$

$\delta \forall_{\text{II}} = A_2 \, \delta \ell_2 = A_2 (V_2 \delta t)$

$$\Rightarrow \dot{B}_{\text{out}} = \lim_{\delta t \to 0} \frac{B_{\text{II}}(t + \delta t)}{\delta t} = \rho_2 A_2 V_2 \, b_2$$

Reynolds Transport Theorem: Derivation

The rate at which the extensive property flows into the control surface:

(3) $$B_{\mathrm{I}}(t + \delta t) = (\rho_1 b_1)(\delta \forall_1) = \rho_1 b_1 A_1 V_1\, \delta t$$

$$\delta \forall_{\mathrm{I}} = A_1\, \delta \ell_1 = A_1(V_1\, \delta t)$$

$$\dot{B}_{\mathrm{in}} = \lim_{\delta t \to 0} \frac{B_{\mathrm{I}}(t + \delta t)}{\delta t} = \rho_1 A_1 V_1 b_1$$

Now, collecting the terms:

$$\frac{DB_{\mathrm{sys}}}{Dt} = \frac{\partial B_{\mathrm{cv}}}{\delta t} + \dot{B}_{\mathrm{out}} - \dot{B}_{\mathrm{in}}$$

or

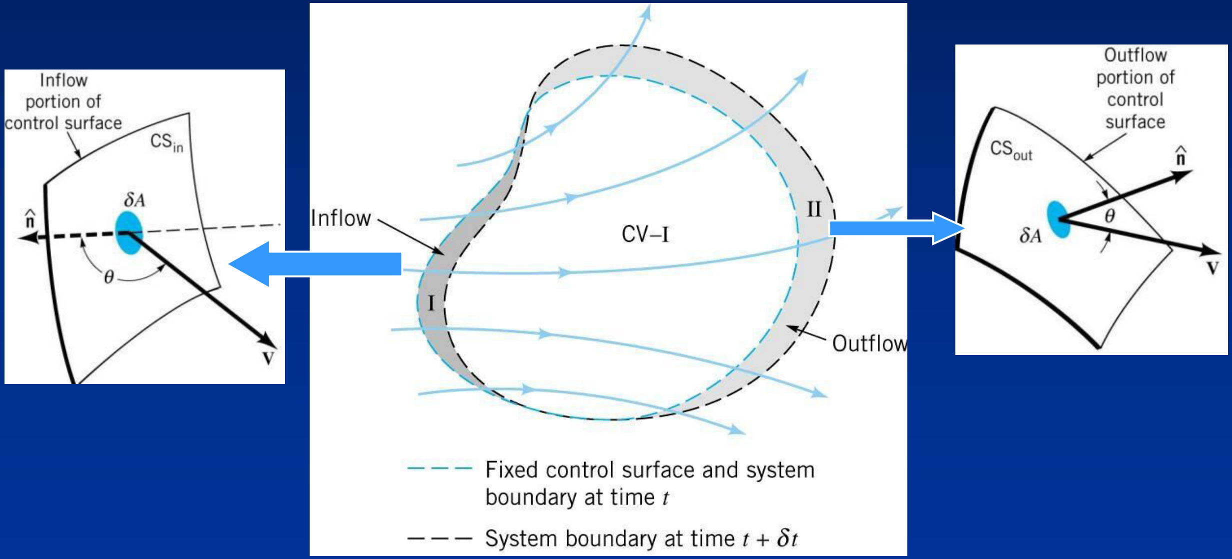
$$\boxed{\frac{DB_{\mathrm{sys}}}{Dt} = \frac{\partial B_{\mathrm{cv}}}{\partial t} + \rho_2 A_2 V_2 b_2 - \rho_1 A_1 V_1 b_1}$$

Restrictions for the above Equation:

1) Fixed control volume
2) One inlet and one outlet
3) Uniform properties
4) Normal velocity to section (1) and (2)

Reynolds Transport Theorem: Derivation

The Reynolds Transport Theorem can be derived for more general conditions.



Result:

$$\boxed{\frac{DB_{\text{sys}}}{Dt} = \frac{\partial}{\partial t}\int_{\text{cv}} \rho b \, d\forall + \int_{\text{cs}} \rho b \, \mathbf{V} \cdot \hat{\mathbf{n}} \, dA}$$

This form is for a fixed non-deforming control volume.

Reynolds Transport Theorem: Physical Interpretation

$$\frac{DB_{\text{sys}}}{Dt} = \frac{\partial}{\partial t}\int_{\text{cv}} \rho b \, d\forall + \int_{\text{cs}} \rho b \, \mathbf{V} \cdot \hat{\mathbf{n}} \, dA$$

(1) (2) (3)

(1) The time rate of change of the extensive parameter of a system, mass, momentum, energy.

(2) The time rate of change of the extensive parameter within the control volume.

(3) The net flow rate of the extensive parameter across the entire control surface.

$(\mathbf{V} \cdot \hat{\mathbf{n}} > 0)$ “outflow across the surface”

$(\mathbf{V} \cdot \hat{\mathbf{n}} < 0)$ “inflow across the surface”

$b\mathbf{V} \cdot \hat{\mathbf{n}} = 0$ “no flow across the surface”

$b = 0$ $\mathbf{V} = 0$

$\mathbf{V}$ is parallel

Mass flow rate: $\rho \mathbf{V} \cdot \hat{\mathbf{n}} \, \delta A$

Reynolds Transport Theorem: Analogous to Material Derivative

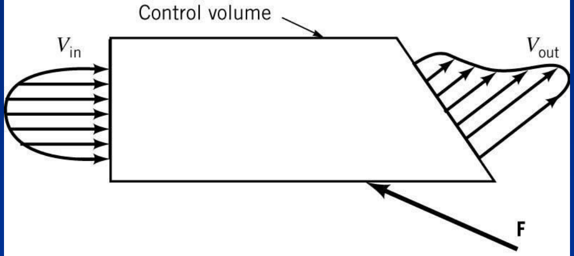
$$\frac{DB_{\text{sys}}}{Dt} = \frac{\partial}{\partial t}\int_{\text{cv}} \rho b \, d\forall + \int_{\text{cs}} \rho b \, \mathbf{V}\cdot\hat{\mathbf{n}} \, dA$$

Unsteady Portion

Convective Portion

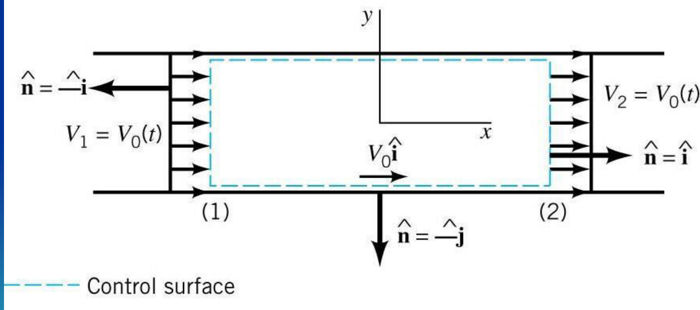
Steady Effects:

$$\frac{DB_{\text{sys}}}{Dt} = \int_{\text{cs}} \rho b \mathbf{V}\cdot\hat{\mathbf{n}} \, dA$$



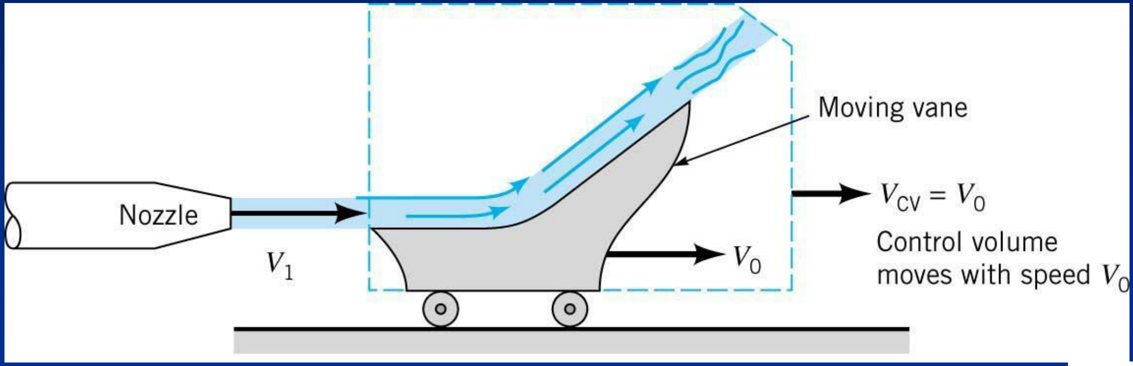
Unsteady Effects (inflow = outflow):

$$\frac{DB_{\text{sys}}}{Dt} = \frac{\partial}{\partial t}\int_{\text{cv}} \rho b \, d\forall$$



Reynolds Transport Theorem: Moving Control Volume

There are cases where it is convenient to have the control volume move. The most convenient is when the control volume moves with a constant velocity.

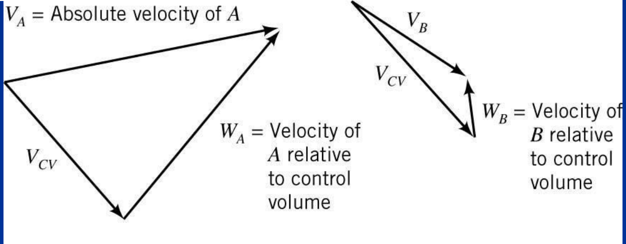


$$\mathbf{V}_{cv} = \mathbf{V} - \mathbf{W}$$

relative velocity, $\mathbf{W}$

absolute velocity, $\mathbf{V}$

$$\mathbf{V} = \mathbf{W} + \mathbf{V}_{cv}$$



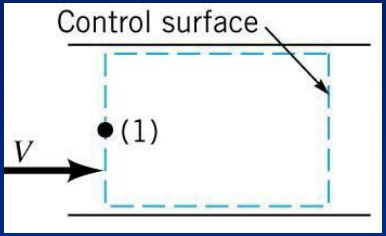
V_o = 20i ft/s, V_1 = 100i ft/s , Then W = 80i ft/s

Now, in general for a constant velocity control volume:

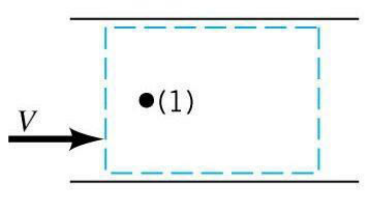
$$\frac{DB_{\mathrm{sys}}}{Dt} = \frac{\partial}{\partial t}\int_{\mathrm{cv}} \rho b \, d\forall + \int_{\mathrm{cs}} \rho b \, \mathbf{W} \cdot \hat{\mathbf{n}} \, dA$$

Reynolds Transport Theorem: Choosing a Control Volume

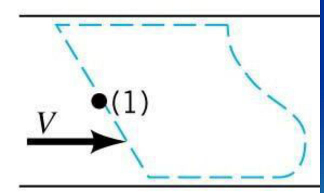
If we want to know a property at point 1, pressure or velocity for instance:



Good choice, since the point we want to know is on control surface. Likewise, the values at the inlet and exit are normal to the surface.



Valid control volume, but the point we want to know is interior. So, it unlikely we will have enough information to obtain its value.



Valid control volume, but the surfaces are not normal to the inlet and outlet.

Boundary Layer Flow

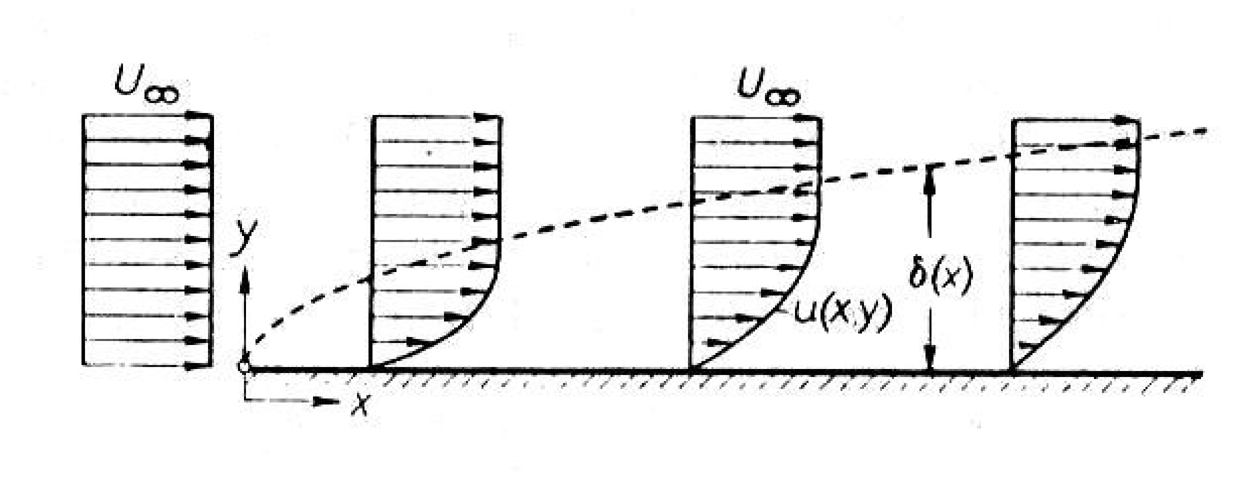
- The concept of boundary layer is due to Prandtl. It occurs on the solid boundary for high Reynolds number flows. Most high Reynolds number external flow can be divided into two regions:
  - Thin layer attached to the solid boundaries where viscous force is dominant, i.e. boundary layer flow region.
  - Other encompassing the rest region where viscous force can be neglected, i.e., the potential flow region, that has been discussed in chapter 7.

Boundary Layer Flow

- The thin layer adjacent to a solid boundary is called the boundary layer and the flow inside the layer is called the boundary layer flow
- Inside the thin layer the velocity of the fluid increases from zero at the wall (no slip) to the full value of corresponding potential flow.
- There exists a leading edge for all external flows. The boundary layer flow developing from leading edge is laminar

Boundary Layer Equations

- For simplicity of illustration, we shall consider an incompressible steady flow over a semi-infinite flat plate with an uniform incoming flow of velocity *U* in parallel to the plate.
- The flow is two dimensional.
- The coordinates are chosen such that *x* is in the incoming flow direction with *x=0* being located the leading edge and *y* is normal to the plate with *y=0* being located at the plate wall.



Boundary Layer Equations

- The continuity and Navier-Stokes equations read:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

Boundary Layer Equations

- The above equations apply generally to two dimensional steady incompressible flows for all Reynolds number over the entire flow domain.
- We now seek the equations that provide the first order approximation for high Reynolds number flows in the boundary layer.

Boundary Layer Equations

- When normalize based on the following scales, we recall the normalized governing equations with Re underneath the viscous term

$$x^* = \frac{x}{L}, y^* = \frac{y}{L}, u^* = \frac{u}{U}, v^* = \frac{v}{U}, p^* = \frac{p}{\rho U^2}$$

$$\Rightarrow \left\{ \begin{aligned} & \frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \\ & u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{\mathrm{Re}_L}\left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} \right) \\ & u^* \frac{\partial v *}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^{\circ}} = -\frac{\partial p^*}{\partial y^*} + \frac{1}{\mathrm{Re}_L}\left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right) \end{aligned} \right.$$

Boundary Layer Equations

- When the viscous terms are dropped for high Re number flows, the equations become those for potential flows outside the boundary layer. The boundary layer effect is not realized.

- Using L to normalize y cannot resolve the boundary layer near the solid boundary. We need to choose a proper length scale to normalize the y coordinate.

Boundary Layer Equations

- To this end, let *L* be the characteristic length in the *x* direction and that *L* be sufficiently long,

  such that $\mathrm{Re}_L = \frac{\rho U L}{\mu} >> 1$

- Therefore, the viscous diffusion layer thickness δ_L at $x=L$ is small compared to L, i.e., . $\delta_L << L$

- This viscous diffusion layer near the wall is the boundary layer.

Boundary Layer Equations

- To resolve the flow in the boundary layer, the proper length scale in y-direction is δ_L while that in x-direction remains as L.

- The condition of $v=0$ for potential flows near the wall outside the boundary layer and the continuity Equation also imply that the velocity v in the boundary layer is small compared to U. Let V be the scale of v in the boundary layer, then $V<<U$.

- It is clear that the non-dimensional normalized variables can now be expressed as:

$$x^* = \frac{x}{L}, y^* = \frac{y}{\delta_L}, u^* = \frac{u}{U}, v^* = \frac{v}{V}$$

Boundary Layer Equations

- For high Reynolds number flow, the proper pressure scale is ρU^2; hence, $p^* = \frac{p}{\rho U^2}$.

- In terms of the dimensionless variables, the governing equations becomes:

$$\frac{U}{L}\frac{\partial u^*}{\partial x^*} + \frac{V}{\delta_L}\frac{\partial v^*}{\partial y^*} = 0$$

$$\frac{\rho U^2}{L}\left(u^*\frac{\partial u^*}{\partial x^*} + \frac{VL}{U\delta_L}v^*\frac{\partial u^*}{\partial y^*}\right) = -\frac{\rho U^2}{L}\frac{\partial p^*}{\partial x^*} + \frac{\mu U}{\delta_L^2}\left(\frac{\delta_L^2}{L^2}\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

$$\frac{\rho UV}{L}\left(u^*\frac{\partial v*}{\partial x^*} + \frac{VL}{U\delta_L}v^*\frac{\partial v^*}{\partial y^\circ}\right) = -\frac{\rho U^2}{\delta_L}\frac{\partial p^*}{\partial y^*} + \frac{\mu V}{\delta_L^2}\left(\frac{\delta_L^2}{L^2}\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}}\right)$$

Boundary Layer Equations

- From the continuity equation, we need $\frac{U}{L} = \frac{V}{\delta_L}$ such that

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

- Therefore, $V = \frac{U\delta_L}{L}$, and the substitution of *V* into the momentum equation leads to:

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{L^2}{Re_L\, \delta_L^2}\left(\frac{\delta_L^2}{L^2} \frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} \right)$$

$$\frac{\delta_L^2}{L^2}\left(u^* \frac{\partial v *}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^\circ} \right) = -\frac{\partial p^*}{\partial y^*} + \frac{1}{Re_L}\left(\frac{\delta_L^2}{L^2} \frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right)$$

Boundary Layer Equations

- In order to balance the shear force with the inertia force, it is clear that we need, $\frac{L^2}{Re_L \, \delta_L} = 1$,i.e., $\frac{\delta_L}{L} = \frac{1}{\sqrt{Re_L}} = \frac{V}{U}$

- The momentum equations reduced further to

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \left(\frac{1}{Re_L} \frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} \right)$$

$$\frac{1}{Re_L} \left(u^* \frac{\partial v*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^\circ} \right) = -\frac{\partial p^*}{\partial y^*} + \frac{1}{Re_L} \left(\frac{1}{Re_L} \frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right)$$

- For high Reynolds number flows, the terms with Re_L to the first approximation can be neglected.

Boundary Layer Equations

- These results in the boundary layer equations that in dimensional form are given by:

Continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

X-momentum:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}$$

Y-momentum:

$$0=-\frac{\partial p}{\partial y}$$

Boundary Layer Equations

- The last equation for y-momentum equation indicates that the pressure is constant across the boundary layer, i.e., equal to that outside the boundary layer (in the free stream), i.e.,

$$p_{(outside\ \text{the boundary layer})} = p_{\infty}(x)$$

- In the free stream (outside the boundary layer), the viscous force is negligible and we also have $u = U_{\infty}(x)$, which in fact is the slip velocity of corresponding potential theory near the boundary

- The *x*-momentum boundary layer equation near the free stream becomes:

$$\rho U_{\infty}(x)\frac{dU_{\infty}(x)}{dx} = -\frac{\partial p_{\infty}(x)}{\partial x}$$

Boundary Layer Equations

- Therefore, the boundary layer equations can be re-written into:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \rho U_{\infty}\frac{dU_{\infty}}{dx} + \mu\frac{\partial^2 u}{\partial y^2}$$

and the proper boundary conditions are:

$u = v = 0$ on $y = 0$ and $u \to U_{\infty}$ as $y \to \infty$

Boundary Layer Equations

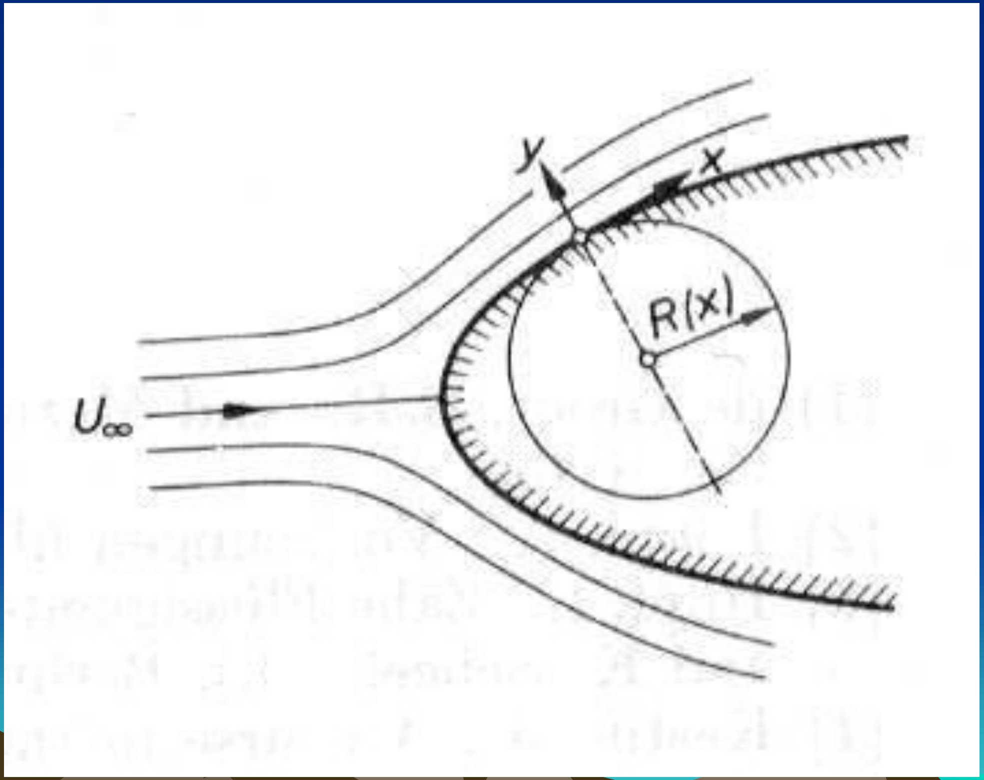
- For semi-infinite flat plate with uniform incoming velocity, $U_\infty = \text{constant}$. The boundary layer equations reduced further to:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right) = \mu\frac{\partial^2 u}{\partial y^2}$$

Boundary Layer Flows over Curve Surfaces

- In fact the boundary layer equation is also meant for curved solid boundary, given a large radius of curvature $R >> \delta_L$.

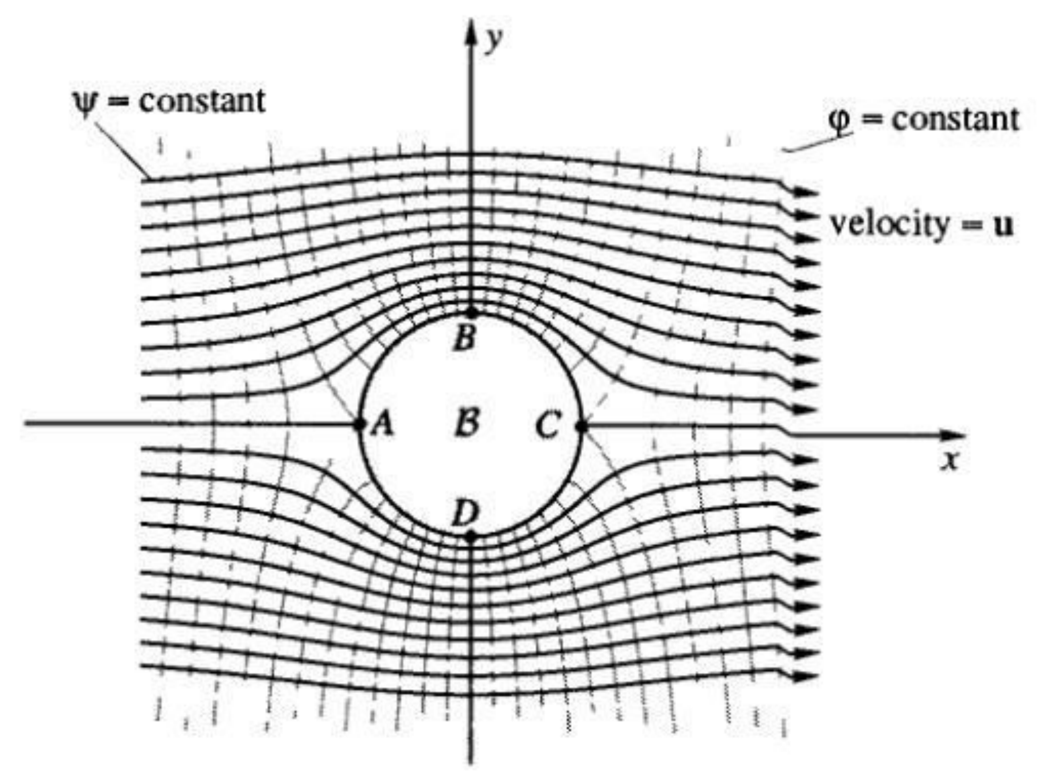


Boundary Layer Flows over Curve Surfaces

- By defining an orthogonal coordinate system with x coordinate along boundary and y coordinate normal to boundary, previous analysis is also valid for curved surface. This can be done through a coordinate transformation.
- Since radius of curvature is large, the curvature effects become higher order terms after transformation. These higher order terms can be neglected for 1st-order approximation. The same boundary layer equation can be obtained.

Boundary Layer Flows over Curve Surfaces

- For example in 2D flows, one way is to use the potential lines and streamlines to form a coordination system. x is along streamline direction, and y is the along potential lines. Such coordination system are called body-fitted coordination system.



Similarity Solution

- If *L* is considered as a varying length scale equal to *x*, then the boundary thickness varies with *x* as $\frac{\delta}{x} = \frac{1}{\sqrt{\mathrm{Re}_x}}$ where $\mathrm{Re}_x = \frac{U_\infty x}{\nu}$ is the local Reynolds number.

- A boundary layer flow is similar if its velocity profile as normalized by U_∞ depends only on the normalized distance from the wall, $\eta = \frac{y}{\delta_x}$, i.e., $= \left(\frac{U_\infty}{x\nu}\right)^{1/2} y$

$$\frac{u}{U_\infty} = g(\eta) \quad \text{and} \quad \frac{v}{V_\infty} = h(\eta)$$

where V_∞ is the velocity components outside the boundary layer normal to U_∞. Here g(η) and h(η) are called the similarity variables.

Blasius Solution

- For uniform flows past a semi-infinite flat plate, the Boundary layer flows are 2-D. It can be shown that the stream function defined by $\psi = \left(U_\infty x \nu\right)^{\frac{1}{2}} f(\eta)$ will satisfy the above conditions for similarity solution such that

$$u = \frac{\partial \psi}{\partial y} = U_\infty f^{'}(\eta) \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} = \frac{1}{2}\left(\frac{\nu U_\infty}{x}\right)^{\frac{1}{2}}\left(\eta f^{'} - f\right)$$

where the f' denotes the derivative with respect to η. Consequently,

$$V_\infty = \frac{U_\infty}{\sqrt{\mathrm{Re}_x}} = \frac{U_\infty \delta_x}{x}$$

Blasius Solution

- The boundary layer equation in term of the similarity variables becomes:

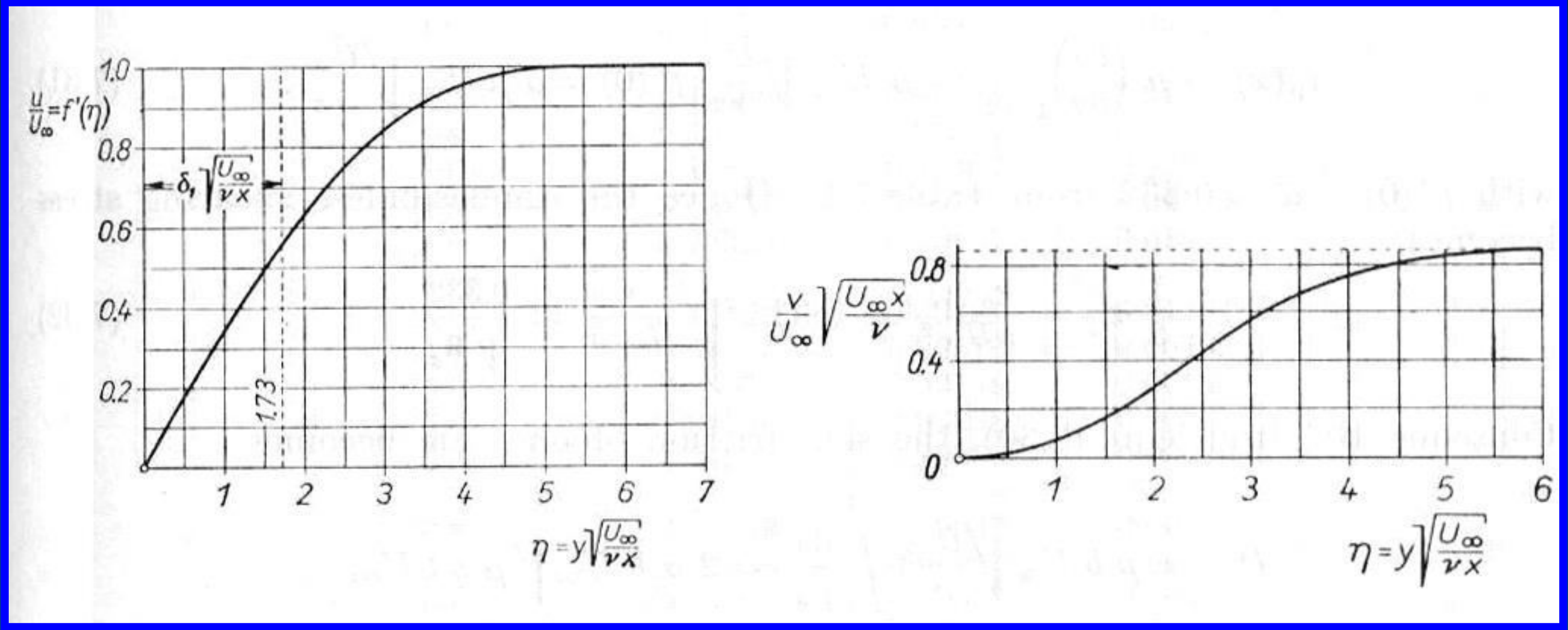
$$2f''' + ff'' = 0$$

subject to the boundary conditions:

$$f' = f = 0 \text{ at } \eta = 0 \text{ and } f' \to 1 \text{ as } \eta \to \infty$$

- The velocity profile obtained by solving the above ordinary differential equation is called the Blasius profile.

Blasius Solution Plot



streamwise and transverse velocities

Boundary Thickness and Skin Friction

- Since the velocity profile merges smoothly and asymptotically into the free stream, it is difficult to measure the boundary layer thickness δ. Conventionally, δ is defined as the distance from the surface to the point where velocity is 99% of free stream velocity.

$$\delta = 5\sqrt{\frac{\nu x}{U_\infty}} \quad \text{or} \quad \delta = \frac{5x}{\sqrt{\mathrm{Re}_x}}$$

- This occurs when $\eta = 5$, i.e., $f'(5) = 0.99$
- Therefore, for laminar boundary layer,

Boundary Thickness and Skin Friction

- The wall shear stress can be expressed as,

$$\tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \mu \frac{U_\infty}{\delta_x} f''(0) = \frac{0.332 \rho U_\infty^2}{\sqrt{\mathrm{Re}_x}}$$

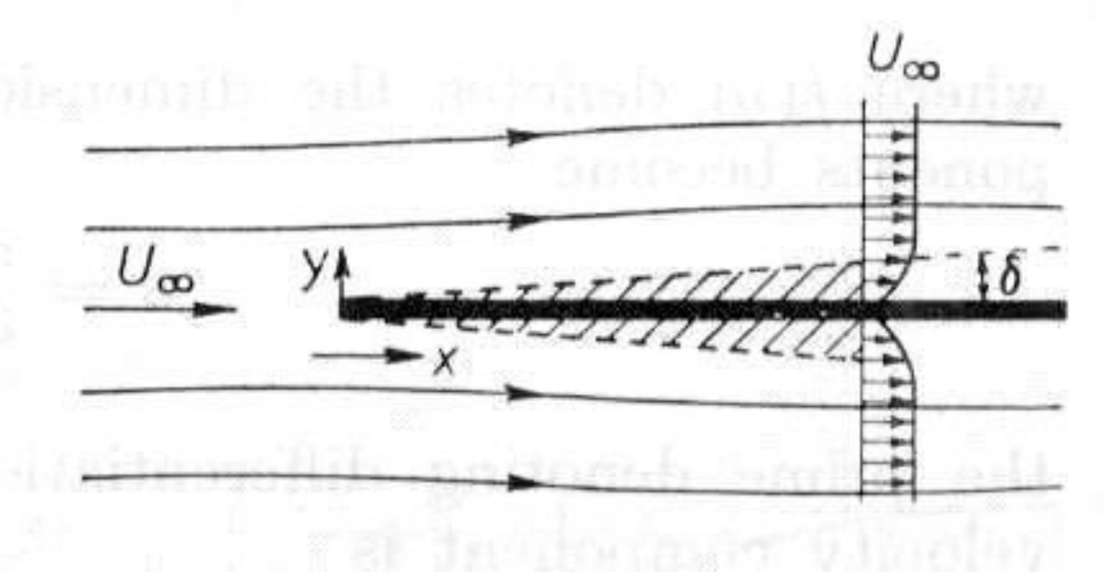
- And the friction coefficient C_f is given by,

$$C_f = \frac{\tau_w}{\rho U_\infty^{\,2}/2} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$$

Boundary Thickness and Skin Friction

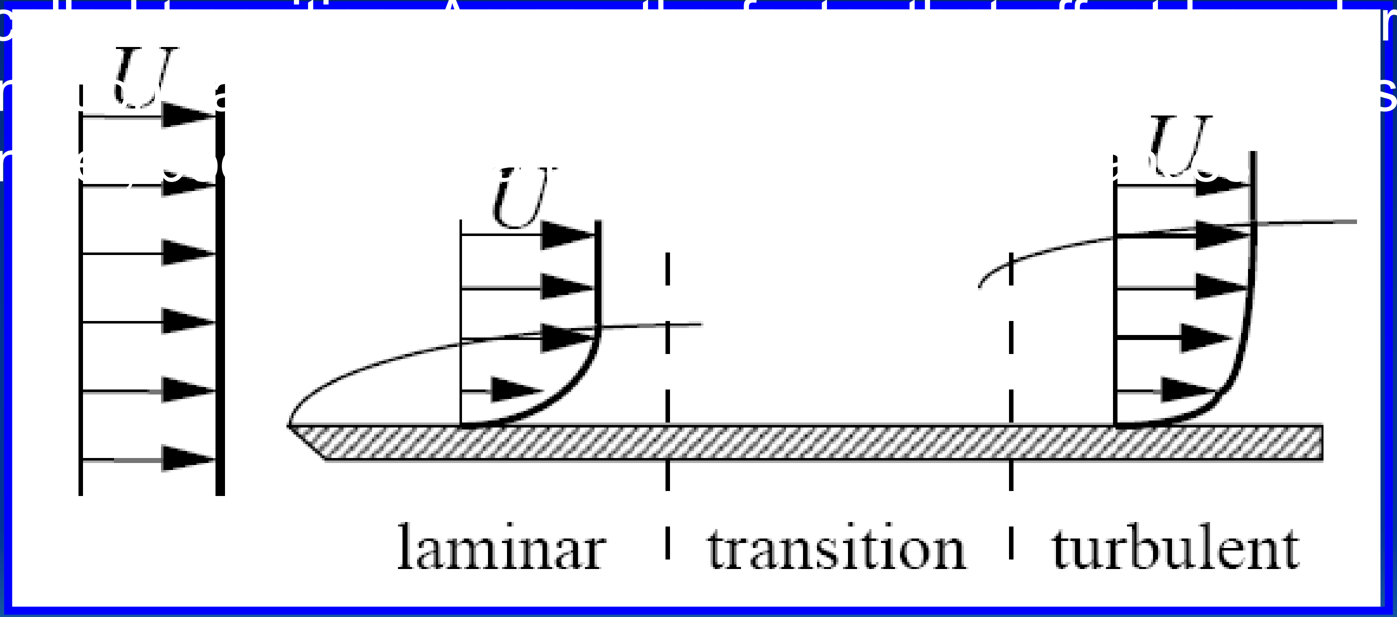
- The boundary layer thickness δ increases with $x^{1/2}$, while the wall shear stress and the skin friction coefficient vary as $x^{-1/2}$.

- These are the characteristics of a laminar boundary layer over a flat plate.



Turbulent Boundary Layer

- Laminar boundary layer flow can become unstable and evolve to turbulent boundary layer flow at down stream. This process is c ry-layer tran s, heat tran



Turbulent Boundary Layer

- Under typical flow conditions, transition usually occurs at a Reynolds number of 5 x 10^5, which can be delayed to Re between 3~ 4 x 10^6 if external disturbances are minimized.
- Velocity profile of turbulent boundary layer flows is unsteady.
- Because of turbulent mixing, the mean velocity profile of turbulent boundary layer is more flat near the outer region of the boundary layer than the profile of a laminar boundary layer.

Turbulent Boundary Layer

- A good approximation to the mean velocity profile for turbulent boundary layer is the empirical 1/7 power-law profile given by

$$\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{1/7}$$

- This profile doesn't hold in the close proximity of the wall, since at the wall it predicts $\left.\frac{du}{dy}\right|_0 = \infty$.
- Hence, we cannot use this profile in the definition of τ_w to obtain an expression in terms of δ.

Turbulent Boundary Layer

- For the drag of turbulent boundary-layer flow, we use the following empirical expression developed for circular pipe flow, $\tau_w = 0.003325\rho U_m^{\ 2}\left(\frac{v}{RU_m}\right)^{1/4}$

  where U_m is the pipe cross-sectional mean velocity and *R* the pipe radius.

- For a 1/7-power profile in a pipe, $U_m = 0.8U_\infty$ .The substitution of $\frac{U_m}{U_\infty} = 0.8$ and $R = \delta$ gives,

$$C_f = 0.045\left(\frac{v}{U_\infty \delta}\right)^{1/4} \quad \text{and} \quad \tau_w = 0.00225\rho U_\infty^{\ 2}\left(\frac{v}{U_\infty \delta}\right)^{1/4}$$

Turbulent Boundary Layer

- For turbulent boundary layer, empirically we have

$$\frac{\delta}{x} = \frac{0.37}{\left(\mathrm{Re}_x\right)^{1/5}}$$

- Therefore,

$$C_f = \frac{\tau_w}{\rho U_\infty{}^2 / 2} = \frac{0.0577}{\mathrm{Re}_x{}^{1/5}}$$

- Experiment shows that this equation predicts the turbulent skin friction on a flat plate within about 3% for 5 x $10^6 < Re_x < 10^7$

Turbulent Boundary Layer

- Note the friction coefficient for the laminar boundary layer is proportional to $Re_x^{-1/2}$, while that for the turbulent boundary layer is proportional to $Re_x^{-1/5}$, with the proportional constants different also by a factor of 10.

- The turbulent boundary layer develops more rapidly than the laminar boundary layer.

Fluid Force on Immersed Bodies

- Relative motion between a solid body and the fluid in which the body is immersed leads to a net force, ***F***, acting on the body. This force is due to the action of the fluid.

- In general, *d**F*** acting on the surface element area, will be the added results of pressure and shear forces normal and tangential to the element, respectively.

Fluid Force on Immersed Bodies

- Hence,

$$\mathbf{F} = \int_{b.s.} d\mathbf{F} = \int_{b.s.} d\mathbf{F}_{pressure} + \int_{b.s.} d\mathbf{F}_{shear}$$

- The resultant force, ***F***, can be decomposed into parallel and perpendicular components. The component parallel to the direction of motion is called the drag, ***D***, and the component perpendicular to the direction of motion is called the drag, ***D***, and the component perpendicular to the direction of motion is called the lift, ***L***.

Fluid Force on Immersed Bodies

- Now

$$d\mathbf{F}_{pressure} = pdA\,\mathbf{n}_s \text{ and } d\mathbf{F}_{shear} = \tau_w dA\,\mathbf{t}_s$$

where $\mathbf{n}_s$ is the unit vector inward normal to the body surface, and is the $\mathbf{t}_s$ unit vector tangential to the surface along the surface slip velocity direction. The total fluid force on the body becomes

$$\boldsymbol{F} = \int_{b.s.} (pdA\,\mathbf{n}_s + \tau_w dA\,\mathbf{t}_s)$$

Fluid Force on Immersed Bodies

- If $\mathbf{i}$ is the unit vector in the body motion direction, then magnitude of drag F_D becomes:

$$F_D = \boldsymbol{F} \bullet \mathbf{i} = \int_{b.s.} (p dA\, \mathbf{n}_s + \tau_w dA\, \mathbf{t}_s) \bullet \mathbf{i}$$ and

$$\mathbf{L} = \mathbf{F} - \mathbf{D} = \mathbf{F} - F_D \mathbf{i}$$

- Note that $\mathbf{L}$ is in the plane normal to $\mathbf{i}$, generally for three-dimensional flows.

- For two-dimensional flows, we can denotes $\mathbf{j}$ as the unit vector normal to the flow direction.

Fluid Force on Immersed Bodies

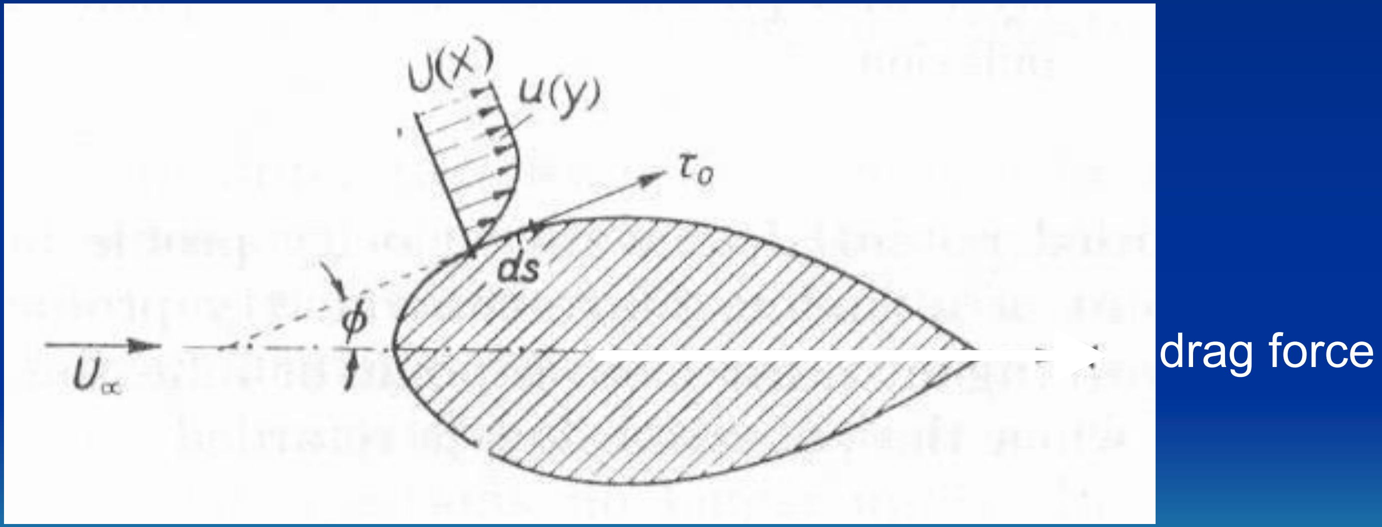
- Therefore, $L = F_L \, \mathbf{j}$ where F_L is the magnitude of lift and is determined by:

$$F_L = \boldsymbol{F} \bullet \mathbf{j} = \int_{b.s.} (p dA \, \mathbf{n}_s + \tau_w dA \, \mathbf{t}_s) \bullet \mathbf{j}$$

- For most body shapes of interest, the drag and lift cannot be evaluated analytically
- Therefore, there are very few cases in which the lift and drag can be determined without resolving by computational or experimental methods.

Drag

- The drag force is the component of force on a body acting parallel to the direction of motion.



Drag

- The drag coefficient defined as

$$C_D = \frac{F_D}{\rho U^2 A/2}$$

is a function of Reynolds number $\mathrm{Re} = \frac{UD}{\nu}$, i.e.

$$C_D = f\left(Re\right)$$

- This form of the equation is valid for incompressible flow over any body, and the length scale, *D*, depends on the body shape.

Friction Drag

- If the pressure gradient is zero and no flow separation, then the total drag is equal to the friction drag, , and,

$$F_D = \int_{b.s.} \tau_w dA$$

$$C_D = \frac{F_D}{\rho U^2 A/2} = \frac{\int_{b.s.} \tau_w dA}{\rho U^2 A/2}$$

- The drag coefficient depends on the shear stress distribution.

Friction Drag

- For a laminar flow over a flat surface, $U_\infty = U$ and the skin-friction coefficient is given by,

$$C_f = \frac{\tau_w}{\rho U^2/2} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$$

- The drag coefficient for flow with free stream velocity, *U*, over a flat plate of length, *L*, and width, *b*, is obtained by substituting into ,

τ_w C_D

$$C_D = \frac{1}{A}\int_A \frac{0.664}{\sqrt{\mathrm{Re}_x}}\,dA = \frac{1}{bL}\int_0^L 0.664b\frac{dx}{\sqrt{xU/v}}$$

$$C_D = \frac{1.328}{\sqrt{\mathrm{Re}_L}} \quad \text{where } \mathrm{Re}_L = \frac{UL}{v}$$

Friction Drag

- If the boundary layer is turbulent, the shear stress on the flat plate then is given by,

τ_w

$$C_f = \frac{\tau_w}{\rho U^2 / 2} = \frac{0.0577}{\mathrm{Re}_x^{1\!/\!5}}$$

- The substitution for results in,

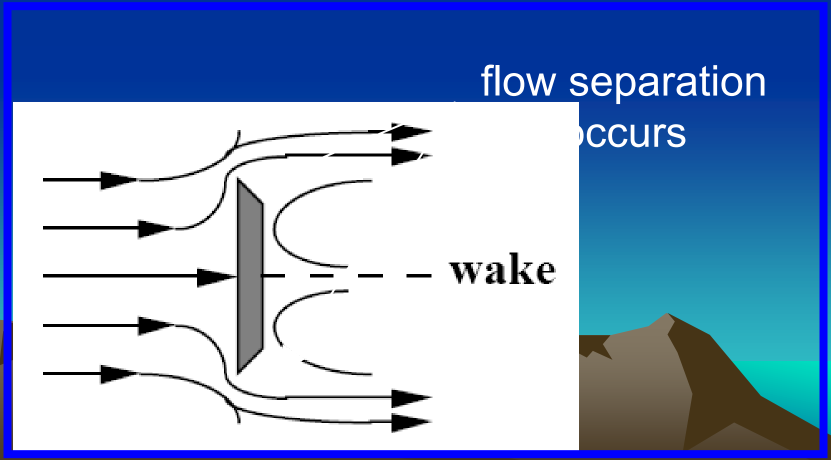
$$C_D = \frac{0.072}{\mathrm{Re}_L^{1\!/\!5}}$$

- This result agrees very well with experimental coefficient of, 0.074 for $Re_L < 10^7$

Pressure Drag (Form Drag)

- The pressure drag is usually associated with flow separation which provide the pressure difference between the front and rear faces of the body. Therefore, this type of pressure drag depends strongly on the shape of the body and is called form drag.

- In a flow over a flat plate normal to the flow as shown in the following picture, the wall shear stress contributes very little to the drag force.



Pressure Drag (Form Drag)

- The form drag is given by,

$$F_D = \int_{b.s.} p(\mathbf{n}_s \bullet \mathbf{i})\, dA$$

- As the pressure difference between front and rear faces of the plate is caused by the inertia force, the form drag depends only on the shape of the body and is independent of the fluid viscosity.

Pressure Drag (Form Drag)

- The drag coefficient for all object with shape edges is essentially independent of Reynolds number.

- Hence, C_D=constant where the constant changes with the body shape and can only be determined experimentally.

Friction and Pressure Drag for Low Reynolds Number Flows

- At very low Reynolds number, *Re*<<1, the viscous force encompass a very large region surrounding the body.
- The pressure drag is mainly caused by fluid viscosity rather than inertia.
- Hence, both friction and pressure drags contribute to the total drag force, i.e., the total drag is entirely viscous drag

Friction and Pressure Drag for Low Reynolds Number Flows

- For low velocity flows passing a sphere of diameter D, Stokes had shown that the total viscous drag is given by

$$F_D = 3\pi\mu UD$$

with 1/3 of it being contributed from normal pressure and 2/3 from frictional shear. The drag coefficient then is expressed as

$$C_D = \frac{F_D}{\rho U^2 A/2} = \frac{24}{\mathrm{Re}_D}$$

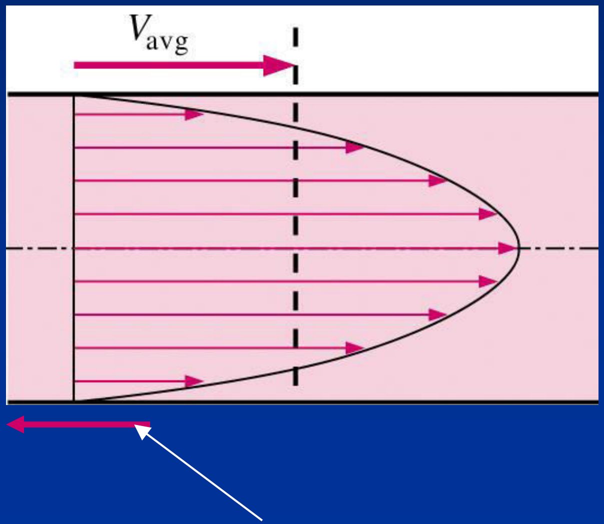
where $A = \pi D^2/4$ is the projected area of the sphere in the flow direction

- As the Re_D increases, the flow separates and the relative contribution of viscous pressure drag decreases.

Flow in Pipes

Introduction

- Average velocity in a pipe
  - Recall - because of the <u>no-slip condition</u>, the velocity at the walls of a pipe or duct flow is zero
  - We are often interested only in V_{avg}, which we usually call just V (drop the subscript for convenience)
  - Keep in mind that the no-slip condition causes shear stress and <u>friction</u> along the pipe walls



Friction force of wall on fluid

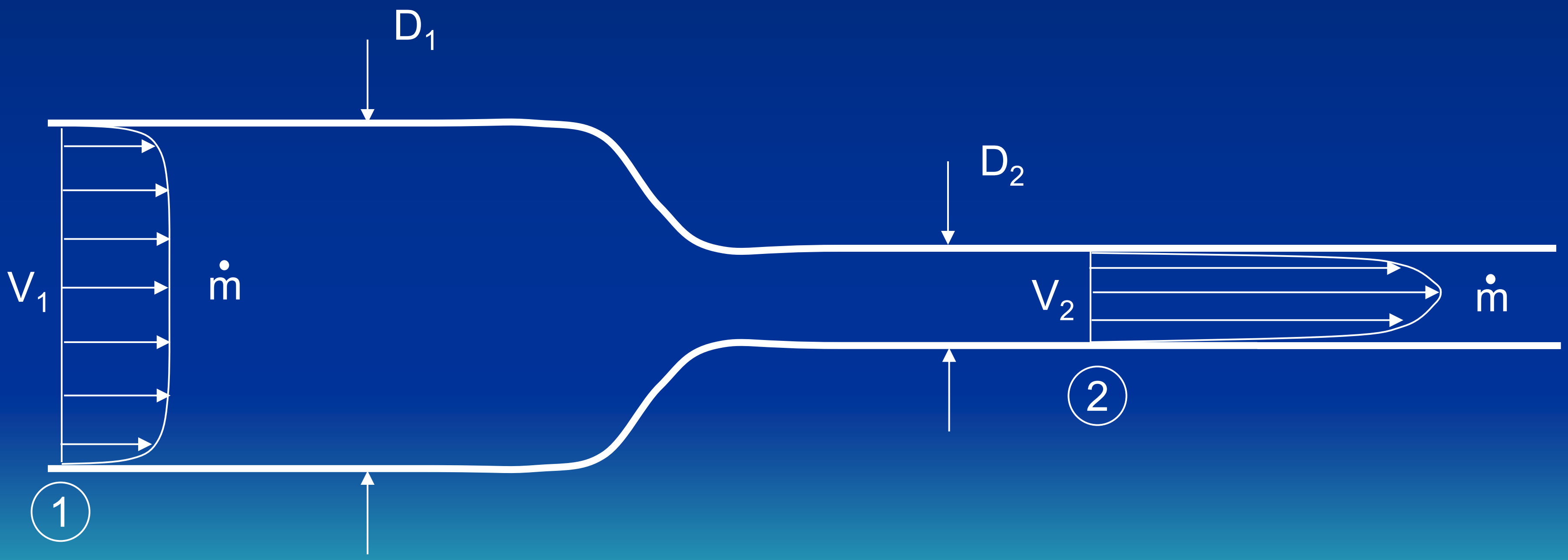
Introduction

- For pipes of constant diameter and incompressible flow
  - V_{avg} stays the same down the pipe, even if the velocity profile changes
    - Why? Conservation of Mass

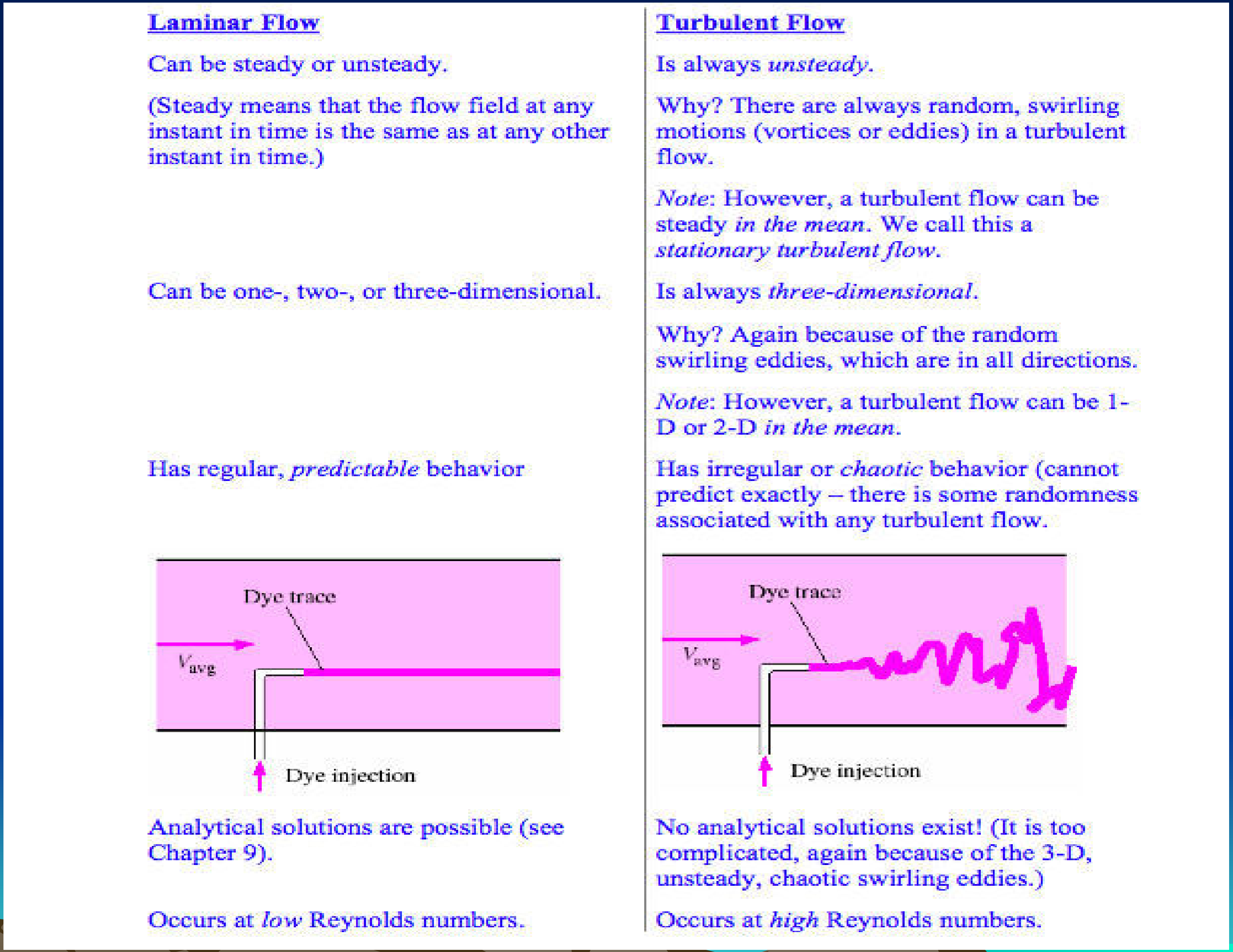
$$\dot{m} = \rho V_{avg} A = constant$$

Introduction

- For pipes with variable diameter, m is still the same due to conservation of mass, but $V_1 \neq V_2$



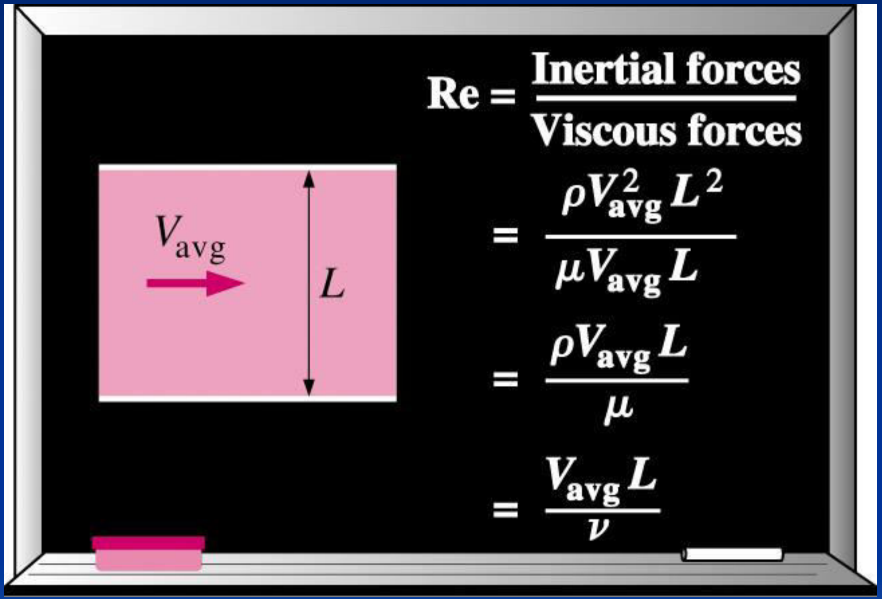
Laminar and Turbulent Flows



Laminar Flow	**Turbulent Flow**
Can be steady or unsteady. (Steady means that the flow field at any instant in time is the same as at any other instant in time.)	Is always *unsteady*. Why? There are always random, swirling motions (vortices or eddies) in a turbulent flow. *Note*: However, a turbulent flow can be steady *in the mean*. We call this a *stationary turbulent flow*.
Can be one-, two-, or three-dimensional.	Is always *three-dimensional*. Why? Again because of the random swirling eddies, which are in all directions. *Note*: However, a turbulent flow can be 1-D or 2-D *in the mean*.
Has regular, *predictable* behavior	Has irregular or *chaotic* behavior (cannot predict exactly – there is some randomness associated with any turbulent flow.
Analytical solutions are possible (see Chapter 9).	No analytical solutions exist! (It is too complicated, again because of the 3-D, unsteady, chaotic swirling eddies.)
Occurs at *low* Reynolds numbers.	Occurs at *high* Reynolds numbers.

Laminar and Turbulent Flows

Definition of Reynolds number



$$\text{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{\rho V_{\text{avg}}^2 L^2}{\mu V_{\text{avg}} L} = \frac{\rho V_{\text{avg}} L}{\mu} = \frac{V_{\text{avg}} L}{\nu}$$

- Critical Reynolds number (Re_{cr}) for flow in a round pipe

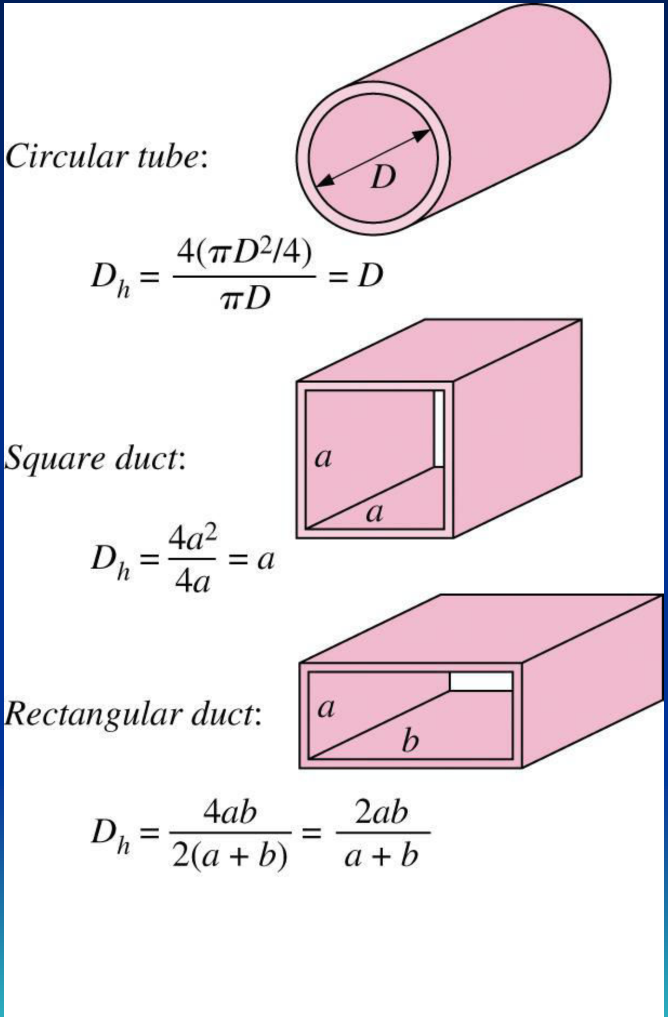
  $\text{Re} < 2300 \Rightarrow$ laminar

  $2300 \leq \text{Re} \leq 4000 \Rightarrow$ transitional

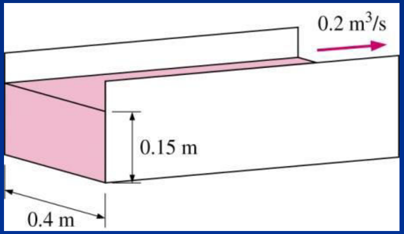
  $\text{Re} > 4000 \Rightarrow$ turbulent

- Note that these values are approximate.
- For a given application, Recr depends upon
  - Pipe roughness
  - Vibrations
  - Upstream fluctuations, disturbances (valves, elbows, etc. that may disturb the flow)

Laminar and Turbulent Flows



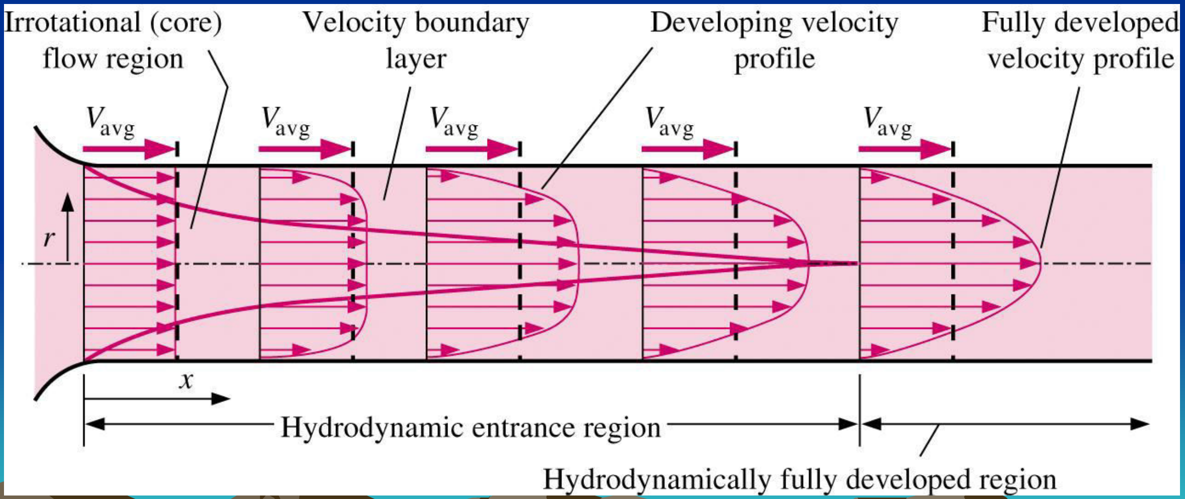
- For <u>non-round</u> pipes, define the hydraulic diameter
  $D_h = 4A_c/P$
  A_c = cross-section area
  P = wetted perimeter



- Example: open channel
  $A_c = 0.15 * 0.4 = 0.06\text{m}^2$
  $P = 0.15 + 0.15 + 0.5 = 0.8\text{m}$
  Don't count free surface, since it does not contribute to friction along pipe walls!
  $D_h = 4A_c/P = 4*0.06/0.8 = 0.3\text{m}$
  <u>What does it mean?</u> This channel flow is equivalent to a round pipe of diameter 0.3m (approximately).

The Entrance Region

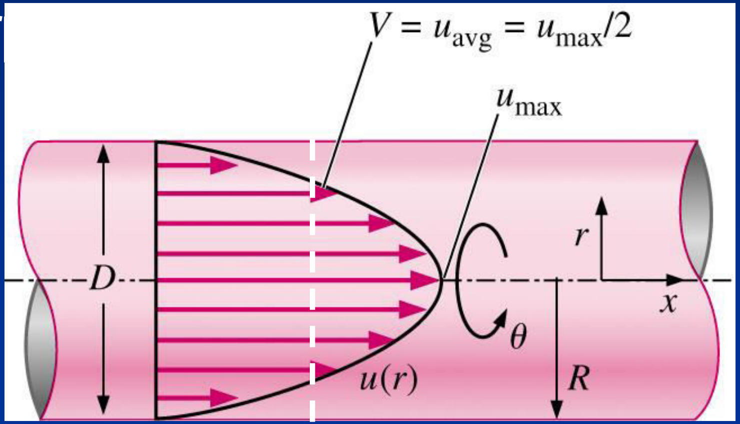
- Consider a round pipe of diameter D. The flow can be laminar or turbulent. In either case, the profile develops downstream over several diameters called the *entry length* L_h. L_h/D is a function of Re.



Fully Developed Pipe Flow

- Comparison of laminar and turbulent flow

There are some major differences between laminar and turbulent fully developed pipe f



<u>Laminar</u>

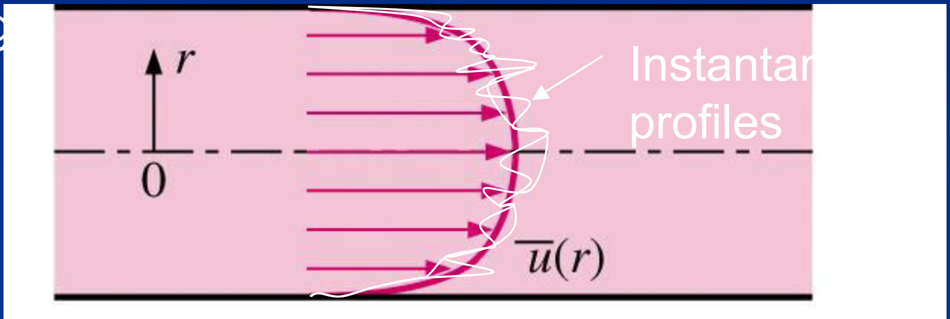
- Can solve exactly (Chapter 9)
- Flow is steady
- Velocity profile is parabolic
- Pipe roughness not important

It turns out that $V_{avg} = 1/2U_{max}$ and $u(r)= 2V_{avg}(1 - r^2/R^2)$

Fully Developed Pipe Flow

Turbulent

- *Cann*ot solve exactly (too complex)
- Flow is unsteady (3D swirling eddies), but it is steady in the mean
- Mean velocity profile is fuller (shape more like a top-hat profile, with very sharp slope at the wall)
- Pipe roug



- V_{avg} 85% of U_{max} (depends on Re a bit)
- No analytical solution, but there are some good semi-empirical expressions that approximate the velocity profile shape. See text

  Logarithmic law (Eq. 8-46)

  Power law (Eq. 8-49)

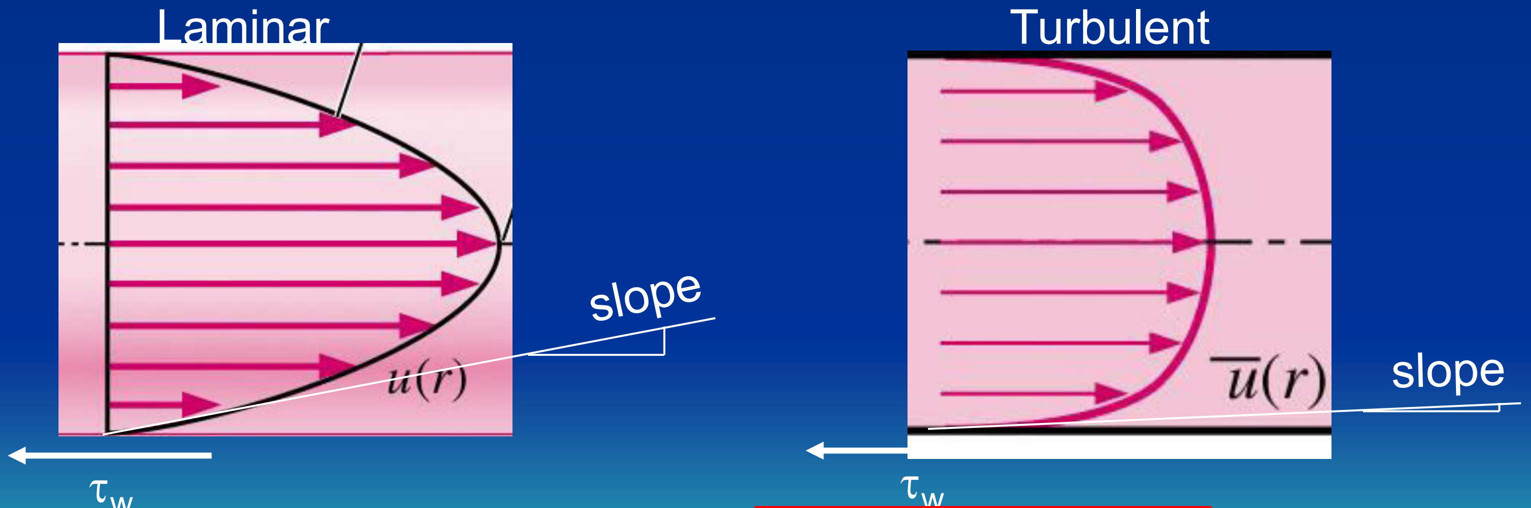
Fully Developed Pipe Flow
Wall-shear stress

- Recall, for simple shear flows $u=u(y)$, we had

$$\tau = \mu du/dy$$

- In fully developed pipe flow, it turns out that

$$\tau = \mu du/dr$$

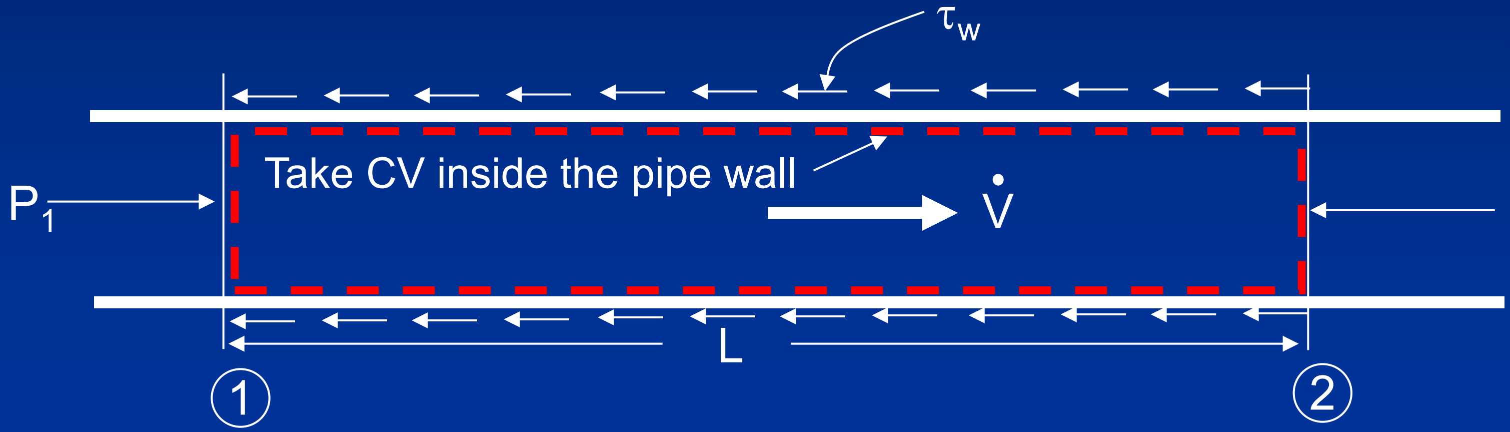


τ_w = shear stress at the wall, acting on the fluid

$$\tau_{w,turb} > \tau_{w,lam}$$

Fully Developed Pipe Flow Pressure drop

- There is a direct connection between the pressure drop in a pipe and the shear stress at the wall
- Consider a horizontal pipe, fully developed, and incompressible flow



- Let’s apply conservation of mass, momentum, and energy to this CV (good review problem!)

Fully Developed Pipe Flow Pressure drop

- Conservation of Mass

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\rho \dot{V}_1 = \rho \dot{V}_2 \rightarrow \dot{V} = const$$

$$V_1 \frac{\pi D^2}{4} = V_2 \frac{\pi D^2}{4} \rightarrow \boxed{V_1 = V_2}$$

- Conservation of x-momentum

$$\sum F_x = \sum F_{x,grav} + \sum F_{x,press} + \sum F_{x,visc} + \sum F_{x,other} = \sum_{out} \beta \dot{m} V - \sum_{in} \beta \dot{m} V$$

$$P_1 \frac{\pi D^2}{4} - P_2 \frac{\pi D^2}{4} - \tau_w \pi D L = \beta_2 \dot{m} V_2 - \beta_1 \dot{m} V_1$$

Terms cancel since $\beta_1 = \beta_2$ and $V_1 = V_2$

Fully Developed Pipe Flow
Pressure drop

- Thus, x-momentum reduces to

$$(P_1 - P_2)\frac{\pi D^2}{4} = \tau_w \pi D L$$

or

$$\boxed{P_1 - P_2 = 4\tau_w \frac{L}{D}}$$

- Energy equation (in head form)

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + \underbrace{z_1 + h_{pump,u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2}_{\text{cancel (horizontal pipe)}} + h_{turbine,e} + h_L$$

Velocity terms cancel again because $V_1 = V_2$, and $\alpha_1 = \alpha_2$ (shape not changing)

$$\boxed{P_1 - P_2 = \rho g h_L}$$

h_L = irreversible head loss & it is felt as a pressure drop in the pipe

Fully Developed Pipe Flow Friction Factor

- From momentum CV analysis

$$P_1 - P_2 = 4\tau_w \frac{L}{D}$$

- From energy CV analysis

$$P_1 - P_2 = \rho g h_L$$

- Equat

$$4\tau_w \frac{L}{D} = \rho g h_L$$

$$h_L = \frac{4\tau_w}{\rho g} \frac{L}{D}$$

- To predict head loss, we need to be able to calculate τ_w. How?
  - Laminar flow: solve exactly
  - Turbulent flow: rely on empirical data (experiments)
  - In either case, we can benefit from dimensional analysis!

Fully Developed Pipe Flow Friction Factor

- $\tau_w = \text{func}(\rho,\ V, \mu, D, \varepsilon)$

ε = average roughness of the inside wall of the pipe

- Π-analysis gives

$$\Pi_1 = f$$

$$\Pi_2 = Re$$

$$\Pi_3 = \frac{\epsilon}{D}$$

$$f = \frac{8\tau_w}{\rho V^2}$$

$$Re = \frac{\rho V D}{\mu}$$

$$\epsilon/D = \text{roughness factor}$$

$$\Pi_1 = func(\Pi_2, \Pi_3)$$

$$f = func(Re, \epsilon/D)$$

Fully Developed Pipe Flow Friction Factor

- Now go back to equation for h_L and substitute f for τ_w

$$h_L = \frac{4\tau_w}{\rho g}\frac{L}{D}$$

$$f = \frac{8\tau_w}{\rho V^2} \rightarrow \tau_w = f\rho V^2/8$$

$$\boxed{h_L = f\frac{L}{D}\frac{V^2}{2g}}$$

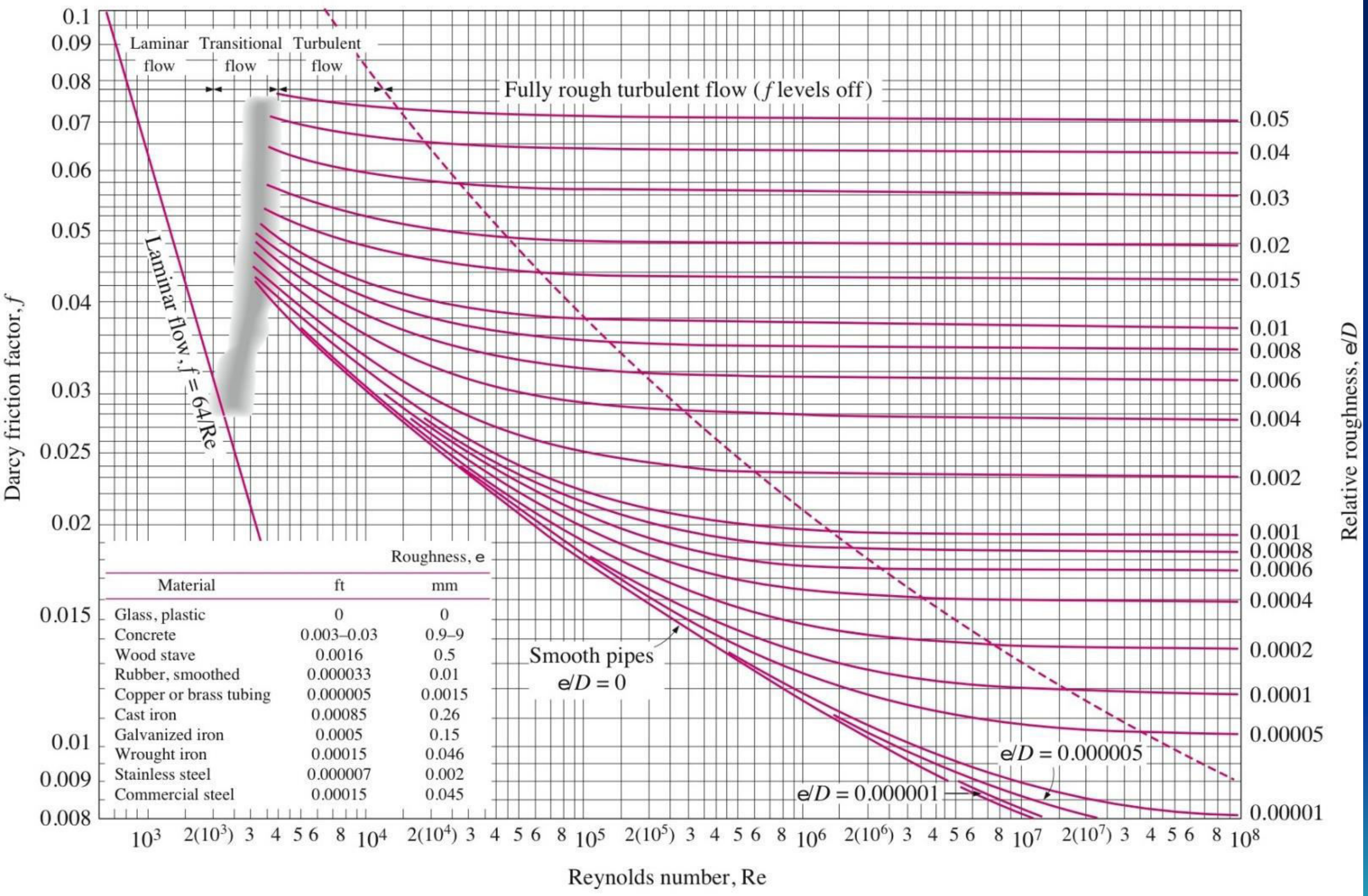
- Our problem is now solving for Darcy friction factor *f*
  - Recall

$$f = func(Re, \epsilon/D)$$

  - Therefore
    - Laminar flow: f = 64/Re (exact)
    - Turbulent flow: Use charts or empirical equations (Moody Chart, a famous plot of *f* vs. Re and ε/D, See Fig. A-12, p. 898 in text)

But for laminar flow, roughness does not affect the flow unless it is huge

The Moody Chart



	Roughness, e	
Material	ft	mm
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

Fully Developed Pipe Flow Friction Factor

- Moody chart was developed for circular pipes, but can be used for non-circular pipes using hydraulic diameter
- Colebrook equation is a curve-fit of the data which is conveni

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\epsilon / D}{3.7} + \frac{2.51}{Re\sqrt{f}} \right)$$

Implicit equation for f which can be solved using the root-finding algorithm in EES

- Both Moody chart and Colebrook equation are accurate to ±15% due to roughness size, experimental error, curve fitting of data, etc.

Types of Fluid Flow Problems

- In design and analysis of piping systems, 3 problem types are encountered
  1. Determine Δp (or h_L) given L, D, V (or flow rate)
     Can be solved directly using Moody chart and Colebrook equation
  2. Determine V, given L, D, Δp
  3. Determine D, given L, Δp, V (or flow rate)
- Types 2 and 3 are common engineering design problems, i.e., selection of pipe diameters to minimize construction and pumping costs
- However, iterative approach required since both *V* and *D* are in the Reynolds number.

Types of Fluid Flow Problems

- Explicit relations have been developed which eliminate iteration. They are useful for quick, direct calculation, but introduce an additional 2% error.

$$h_L = 1.07\frac{\dot{\mathcal{V}}^2 L}{gD^5}\left\{\ln\left[\frac{\epsilon}{3.7D} + 4.62\left(\frac{\nu D}{\dot{\mathcal{V}}}\right)^{0.9}\right]\right\}^{-2}$$

$10^{-6} < \epsilon/D < 10^{-2}$

$3000 < Re < 3 \times 10^8$

$$\dot{\mathcal{V}} = -0.965\left(\frac{gD^5 h_L}{L}\right)^{0.5}\ln\left[\frac{\epsilon}{3.7D} + \left(\frac{3.17\nu^2 L}{gD^3 h_L}\right)^{0.5}\right]$$

$Re > 2000$

$$D = 0.66\left[\epsilon^{1.25}\left(\frac{L\dot{\mathcal{V}}^2}{gh_L}\right)^{4.75} + \nu\dot{\mathcal{V}}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04}$$

$10^{-6} < \epsilon/D < 10^{-2}$

$5000 < Re < 3 \times 10^8$

Minor Losses

- Piping systems include fittings, valves, bends, elbows, tees, inlets, exits, enlargements, and contractions.
- These components interrupt the smooth flow of fluid and cause additional losses because of flow separation and mixing
- We introduce a relation for the minor losses associated with these components

$$\boxed{h_L = K_L \frac{V^2}{2g}}$$

- K_L is the loss coefficient.
- Is different for each component.
- Is assumed to be independent of Re.
- Typically provided by manufacturer or generic table (e.g., Table 8-4 in text).

Minor Losses

- Total head loss in a system is comprised of major losses (in the pipe sections) and the minor losses (in th

$$h_L = h_{L,major} + h_{L,minor}$$

$$h_L = \sum_i f_i \frac{L_i}{D_i} \frac{V_i^2}{2g} + \sum_j K_{L,j} \frac{V_j^2}{2g}$$

i pipe sections

j components

- If the pipi diameter

$$h_L = \left(f \frac{L}{D} + \sum K_L \right) \frac{V^2}{2g}$$

Minor Losses

Here are some sample loss coefficients for various minor loss components. More values are listed in Table 8-4, page 350 of the Çengel-Cimbala textbook:

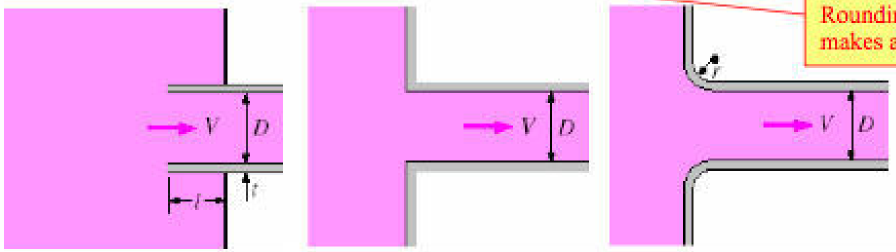
Pipe Inlet

Reentrant: $K_L = 0.80$
($t \ll D$ and $l \approx 0.1D$)

Sharp-edged: $K_L = 0.50$

Well-rounded ($r/D > 0.2$): $K_L = 0.03$
Slightly rounded ($r/D = 0.1$): $K_L = 0.12$
(see Fig. 8–36)

Rounding of an inlet makes a big difference.



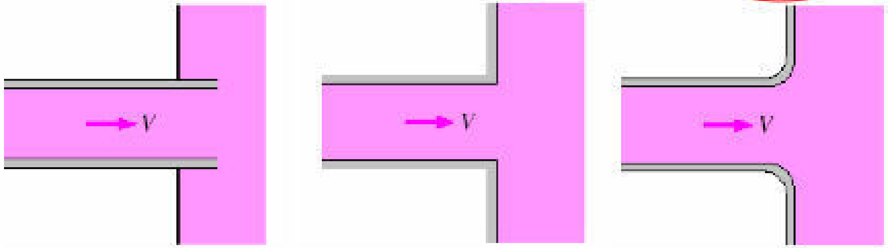
Pipe Exit

Reentrant: $K_L = \alpha$

Sharp-edged: $K_L = \alpha$

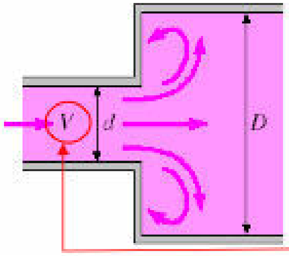
Rounded: $K_L = \alpha$

Rounding of an outlet makes no difference.



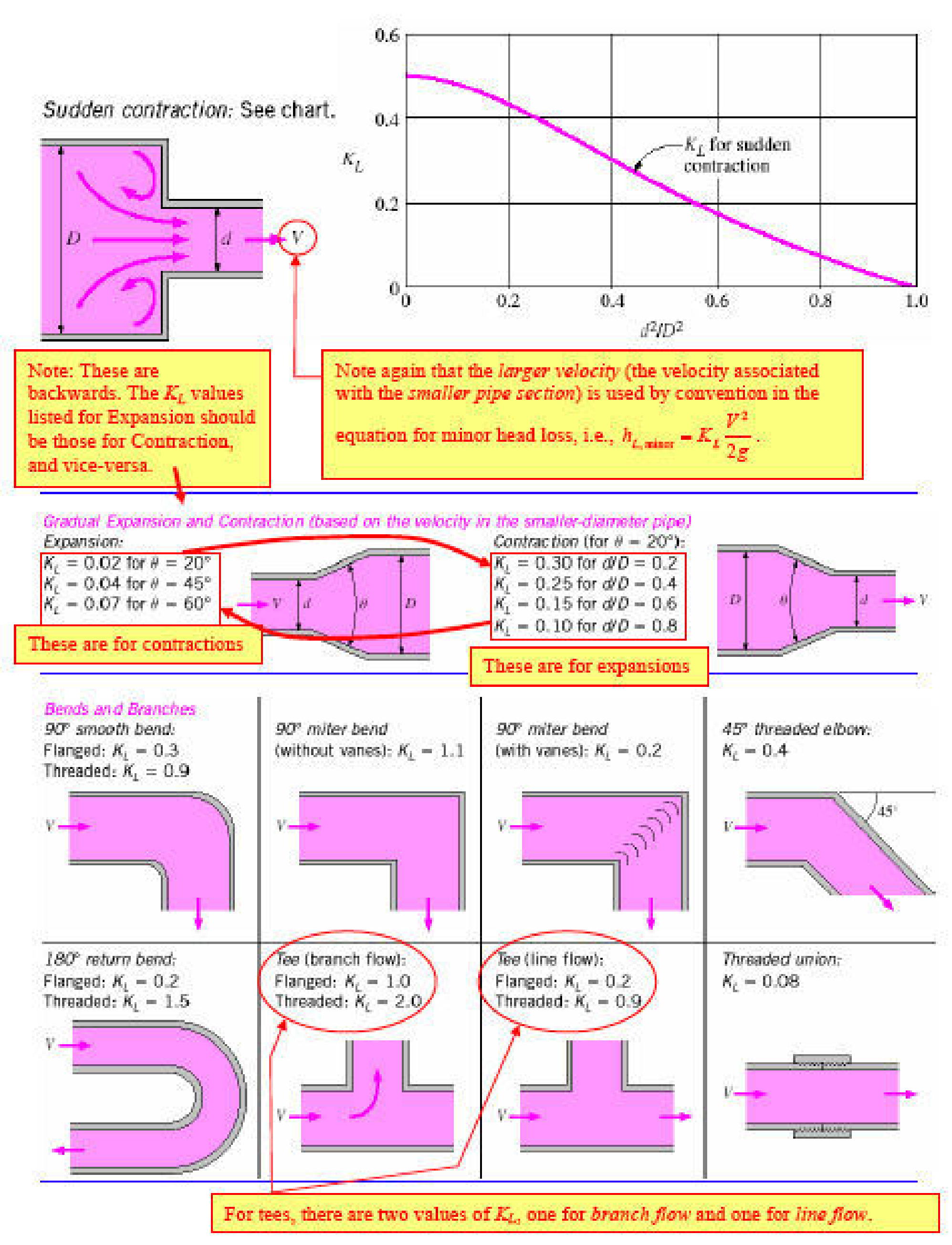
Sudden Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

Sudden expansion: $K_L = \left(1 - \frac{d^2}{D^2}\right)^2$



Note that the *larger velocity* (the velocity associated with the *smaller pipe section*) is used by convention in the equation for minor head loss, i.e.,

$$h_{L,\text{minor}} = K_L \frac{V^2}{2g}.$$



Sudden contraction: See chart.

Note: These are backwards. The K_L values listed for Expansion should be those for Contraction, and vice-versa.

Note again that the *larger velocity* (the velocity associated with the *smaller pipe section*) is used by convention in the equation for minor head loss, i.e., $h_{L,\text{minor}} = K_L \dfrac{V^2}{2g}$.

Gradual Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

Expansion:

$K_L = 0.02$ for $\theta = 20°$
$K_L = 0.04$ for $\theta = 45°$
$K_L = 0.07$ for $\theta = 60°$

These are for contractions

Contraction (for $\theta = 20°$):

$K_L = 0.30$ for $d/D = 0.2$
$K_L = 0.25$ for $d/D = 0.4$
$K_L = 0.15$ for $d/D = 0.6$
$K_L = 0.10$ for $d/D = 0.8$

These are for expansions

Bends and Branches

90° smooth bend:
Flanged: $K_L = 0.3$
Threaded: $K_L = 0.9$

90° miter bend
(without vanes): $K_L = 1.1$

90° miter bend
(with vanes): $K_L = 0.2$

45° threaded elbow:
$K_L = 0.4$

180° return bend:
Flanged: $K_L = 0.2$
Threaded: $K_L = 1.5$

Tee (branch flow):
Flanged: $K_L = 1.0$
Threaded: $K_L = 2.0$

Tee (line flow):
Flanged: $K_L = 0.2$
Threaded: $K_L = 0.9$

Threaded union:
$K_L = 0.08$

For tees, there are two values of K_L, one for *branch flow* and one for *line flow*.