MECHANICS OF SOLIDS

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UNIT-I

MECHANICS OF SOLIDS

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PART - I

Syllabus:- Part - A

1. Simple Stresses & Strains:-

Introduction, Stress, Strain,

Tensile, Compressive & Shear Stresses,

Elastic Limit, Hooke's Law, Poisson's Ratio,

Modulus of Elasticity, Modulus of Rigidity,

Bulk Modulus, Bars of Varying Sections,

Extension of Tapering Rods, Hoop Stress,

Stresses on Oblique Sections.

2. Principle Stresses & Strains:-

State of Simple Shear,

Relation between Elastic Constants, Compound Stresses, Principle Planes

Principle Stresses,

Mohr’s Circle of Stress, Principle Strains,

Angle of Obliquity of Resultant Stresses, Principle Stresses in beams.

3. Torsion:-

Torsion of Circular, Solid, Hollow Section Shafts

Shear Stress, Angle of Twist,

Torsional Moment of Resistance,

Power Transmitted by a Shaft,

Keys & Couplings,

Combined Bending & Torsion,

Close Coiled Helical Springs,

Principle Stresses in Shafts Subjected to Bending, Torsion & Axial Force.

Syllabus:- Part - B

1. **Bending Moment & Shear Force:-**

Bending Moment,

Shear Force in Statically Determinate Beams Subjected to Uniformly Distributed, Concentrated & Varying Loads,

Relation Between Bending Moment,

Shear force & Rate of Loading.

2. Moment of Inertia:-

Concept Of Moment of Inertia,

Moment of Inertia of Plane Areas,

Polar Moment of Inertia,

Radius of Gyration of an Area,

Parallel Axis Theorem,

Moment of Inertia of Composite Areas,

Product of Inertia,

Principle Axes & Principle Moment of Inertia.

3. Stresses in Beams:-

Theory of Simple Bending, Bending Stresses,

Moment of Resistance,

Modulus of Section,

Built up & Composite Beam Section,

Beams of Uniform Strength.

4. Shear stresses in Beams:-

Distribution of Shear Stresses in Different Sections.

5. Mechanical Properties of Materials:-

Ductility, Brittleness, Toughness, Malleability, Behaviour of Ferrous & Non-Ferrous metals in Tension & Compression, Shear & Bending tests, Standard Test Pieces, Influence of Various Parameters on Test Results, True & Nominal Stress, Modes of Failure, Characteristic Stress-Strain Curves, Izod, Charpy & Tension Impact Tests,

Fatigue, Creep, Corelation between Different Mechanical Properties, Effect of Temperature, Testing Machines & Special Features, Different Types of Extensometers & Compressemeters, Measurement of Strain by Electrical Resistance Strain Gauges.

Text Books:-

1. Mechanics of Structures Vol.-1:- S.B.Junarkar & H.J. Shah
2. Strength of Materials:- S.Ramamurtham.

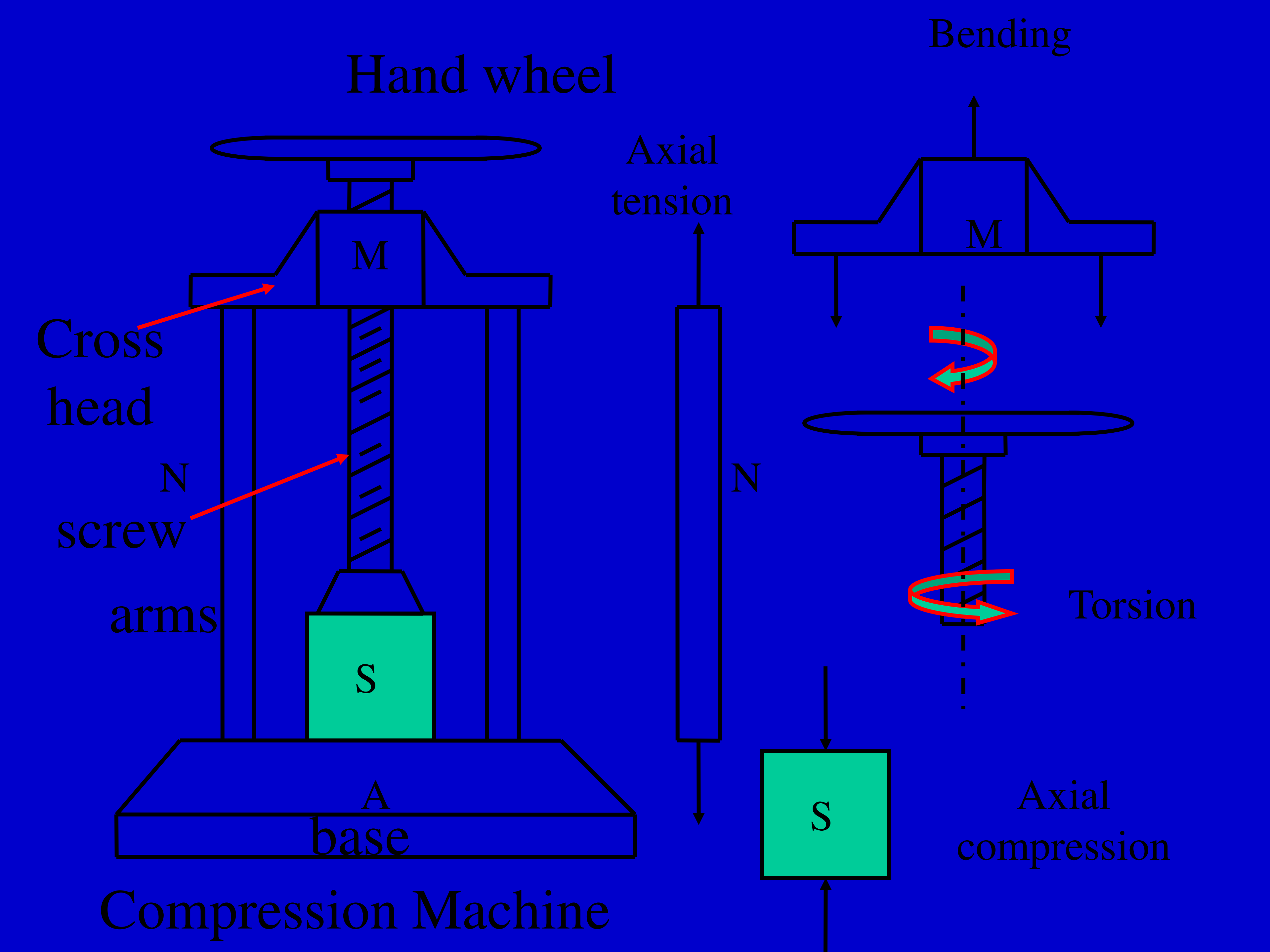
MECHANICS OF SOLIDS

Introduction:-

- Structures /Machines
- Numerous Parts / Members
- Connected together
- perform useful functions/withstand applied loads

AIM OF MECHANICS OF SOLIDS:

Predicting how geometric and physical properties of structure will influence its behaviour under service conditions.


Hand wheel
Bending
Axial
tension
M
M
Cross
head
N
N
screw
arms
S
Torsion
A
base
S
Axial
compression
Compression Machine

- Stresses can occur isolated or in combination.

- Is structure strong enough to withstand loads applied to it ?

- Is it stiff enough to avoid excessive deformations and deflections?

- Engineering Mechanics----> Statics----->

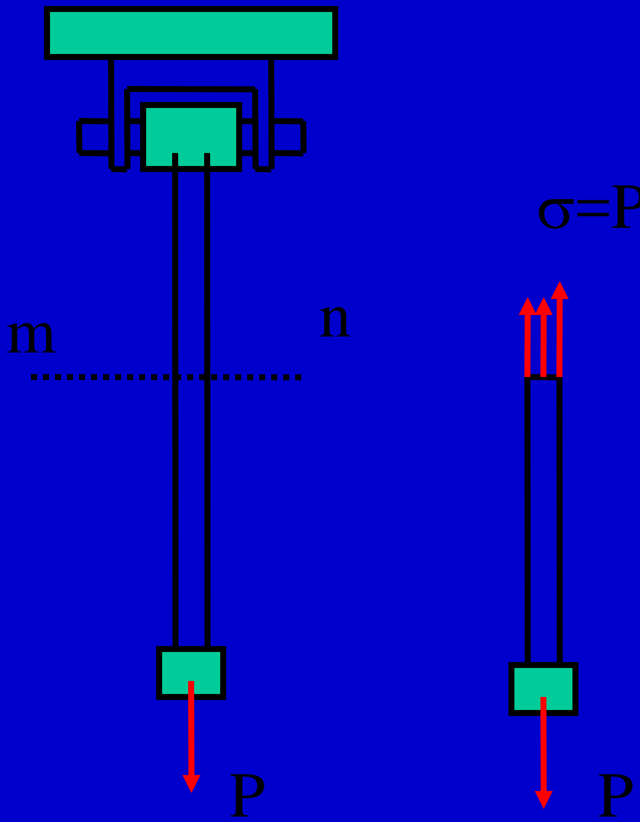
deals with rigid bodies

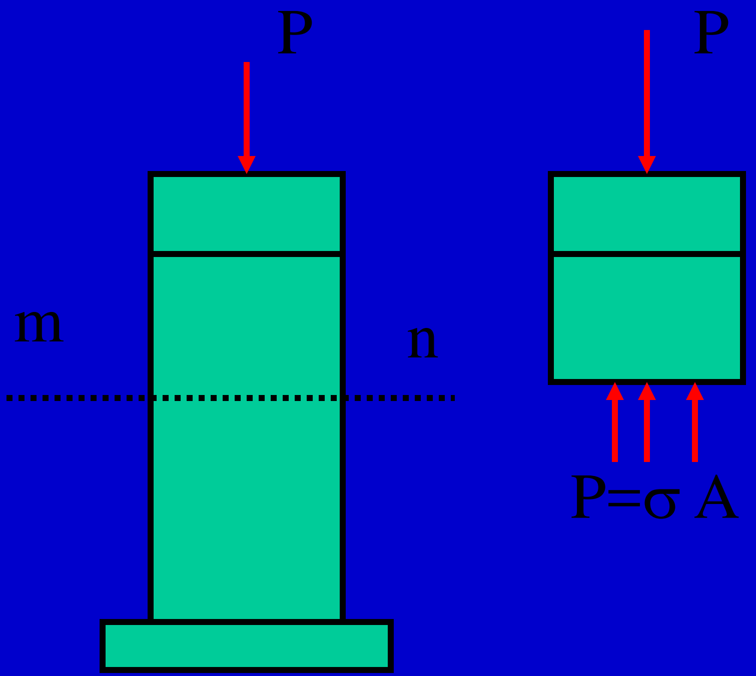
- All materials are deformable and mechanics of solids takes this into account.

- Strength and stiffness of structures is function of size and shape, certain physical properties of material.

- **Properties of Material:-**
  - Elasticity
  - Plasticity
  - Ductility
  - Malleability
  - Brittleness
  - Toughness
  - Hardness

INTERNAL FORCE:- STRESS





- Axial tension
- Stretches the bars & tends to pull it apart
- Rupture

- Axial Compression
- Shortens the bar
- Crushing
- Buckling

- Resistance offered by the material per unit cross-sectional area is called STRESS.

$$\sigma = P/A$$

Unit of Stress:

Pascal = 1 N/m^2

kN/m^2 , MN/m^2 , GN/m^2

Permissible stress or allowable stress or working stress = yield stress or ultimate stress /factor of safety.

1 MPa = 1 N/mm^2

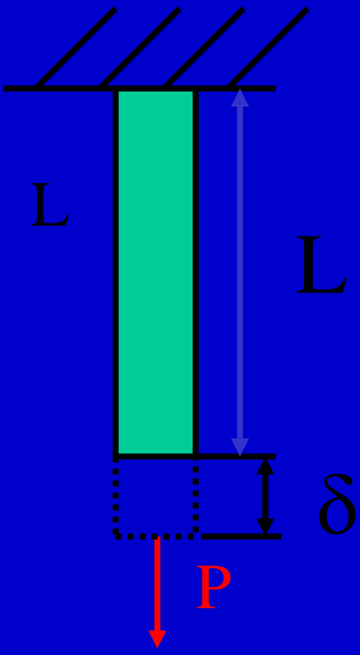
• Strain

•It is defined as deformation per unit length

• it is the ratio of change in length to original length

•Tensile strain (+ Ve) (ε) $= \dfrac{\text{increase in length}}{\text{Original length}} = \dfrac{\delta}{L}$

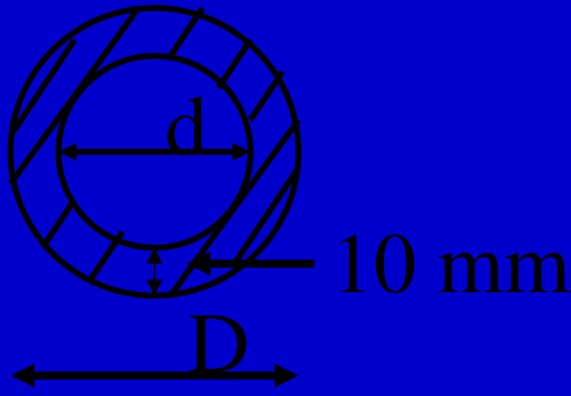
Compressive strain (- Ve) (ε) $= \dfrac{\text{decrease in length}}{\text{Original length}} = \dfrac{\delta}{L}$



•Strain is dimensionless quantity.

Example : 1

A short hollow, cast iron cylinder with wall thickness of 10 mm is to carry compressive load of 100 kN. Compute the required outside diameter `D' , if the working stress in compression is 80 N/mm^2. (D = 49.8 mm).



Solution: $\sigma = 80 N/mm^2$;

$P = 100\ kN = 100*10^3\ N$

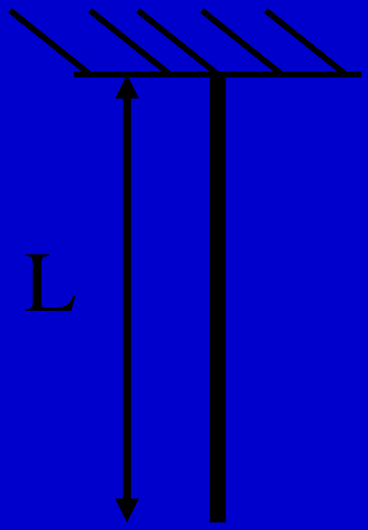
$A = (\pi/4) * \{D^2 - (D-20)^2\}$

as $\sigma = P/A$

substituting in above eq. and solving. D = 49.8 mm

Example: 2

A Steel wire hangs vertically under its weight. What is the greatest length it can have if the allowable tensile stress σ_t =200 MPa? Density of steel γ=80 kN/m^3.(ans:-2500 m)



Solution:

σ_t =200 MPa= 200*10^3 kN/m^2 ;

γ=80 kN/m^3.

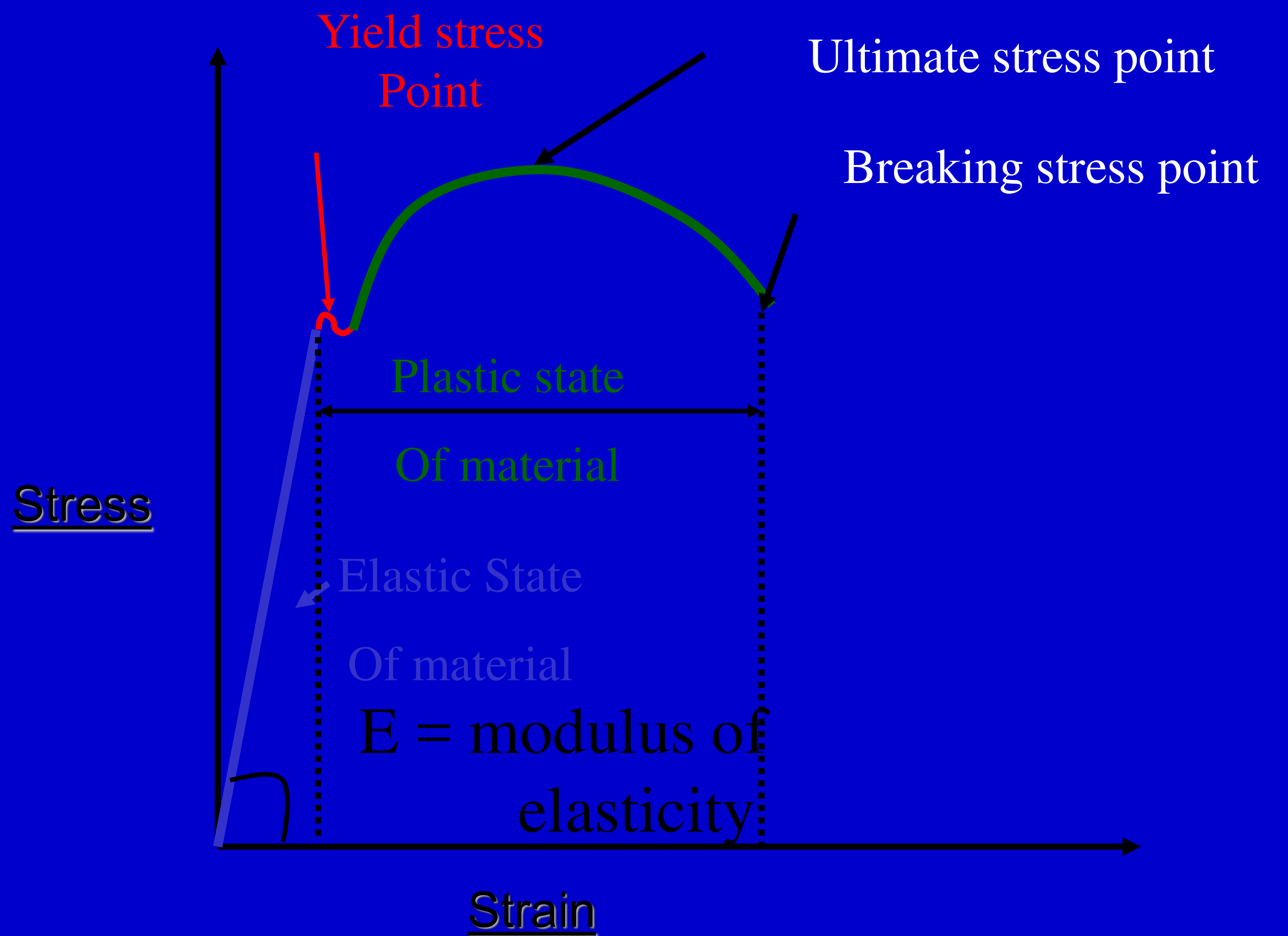
Wt. of wire P=(π/4)*D^2*L* γ

c/s area of wire A=(π/4)*D^2

σ_t = P/A

solving above eq. L =2500m

Stress- Strain Curve for Mild Steel (Ductile Material)

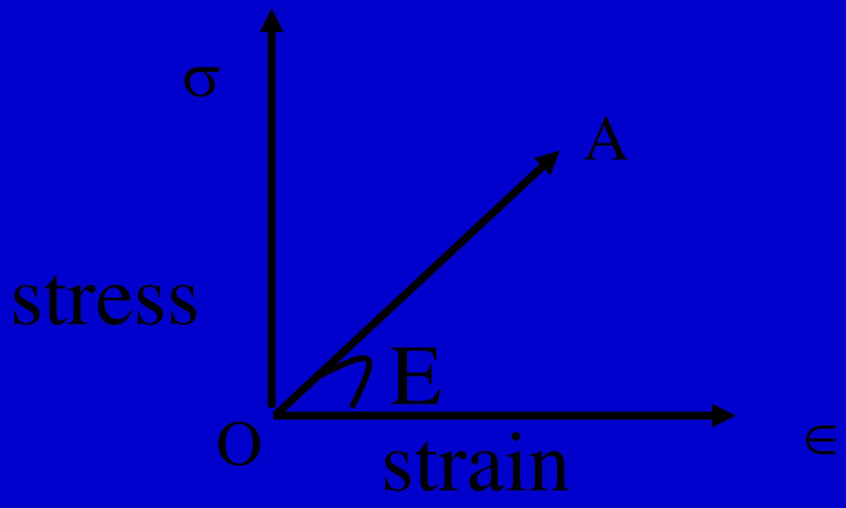


Modulus of Elasticity: $\sigma = E \in$

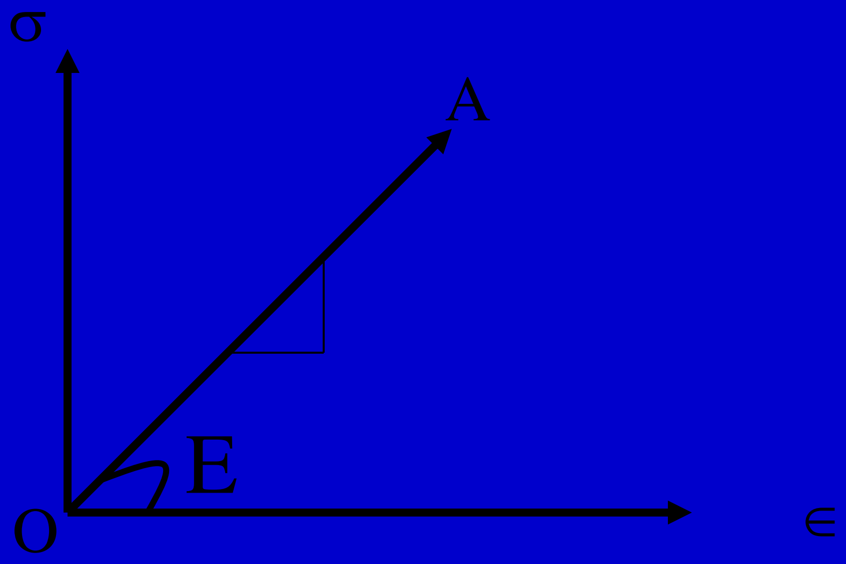
- Stress required to produce a strain of unity.

- i.e. the stress under which the bar would be stretched to twice its original length . If the material remains elastic throughout , such excessive strain.

- Represents slope of stress-strain line OA.



Value of E is same in Tension & Compression.



- Hooke's Law:-

Up to elastic limit, Stress is proportional to strain

$\sigma \ \alpha \ \in$

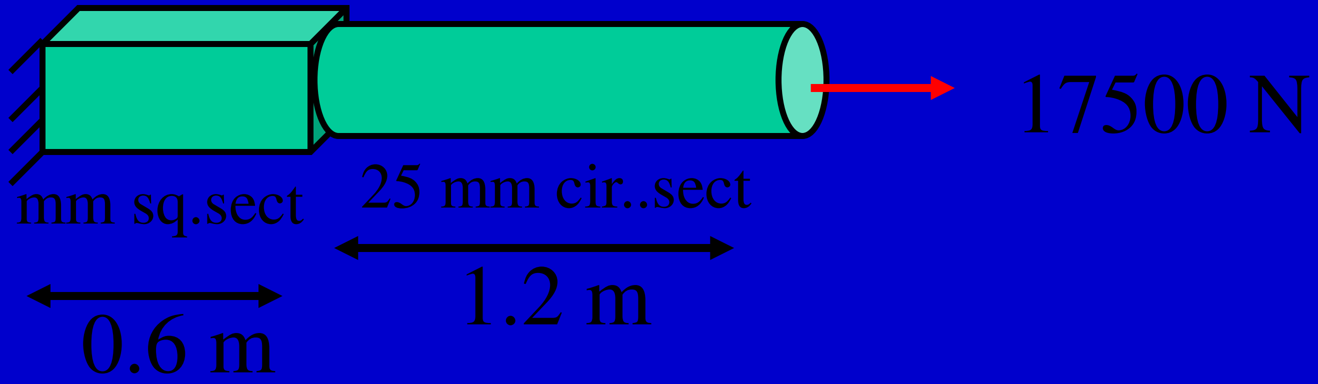
$\sigma = E \in$; where E=Young's modulus

$\sigma = P/A$ and $\in = \delta / L$

$P/A = E (\delta / L)$

$\delta = PL / AE$

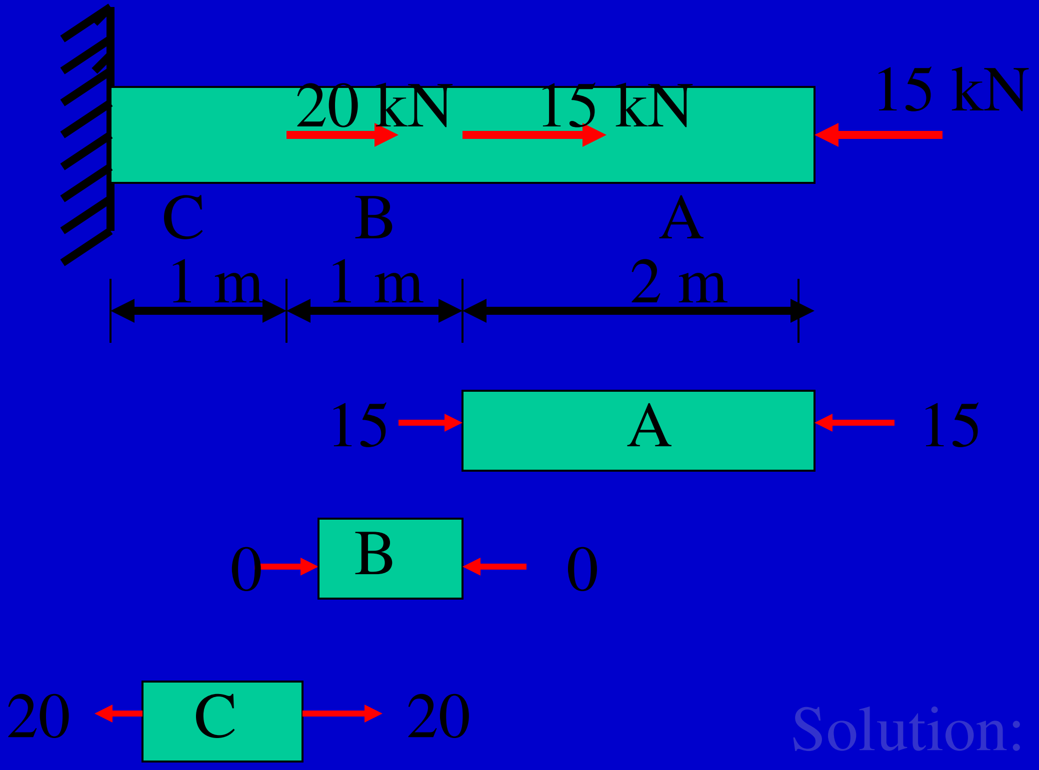
Example:4 An aluminium bar 1.8 meters long has a 25 mm square c/s over 0.6 meters of its length and 25 mm circular c/s over other 1.2 meters . How much will the bar elongate under a tensile load P=17500 N, if E = 75000 Mpa.



Solution :- $\delta = \sum PL/AE$

$=17500*600 / (25^2*75000) + 17500*1200/(0.785*25^2*75000) = 0.794$ mm

Example: 5 A prismatic steel bar having cross sectional area of A=300 mm^2 is subjected to axial load as shown in figure . Find the net increase δ in the length of the bar. Assume $E = 2 \times 10^5$ MPa.(Ans δ = -0.17mm)

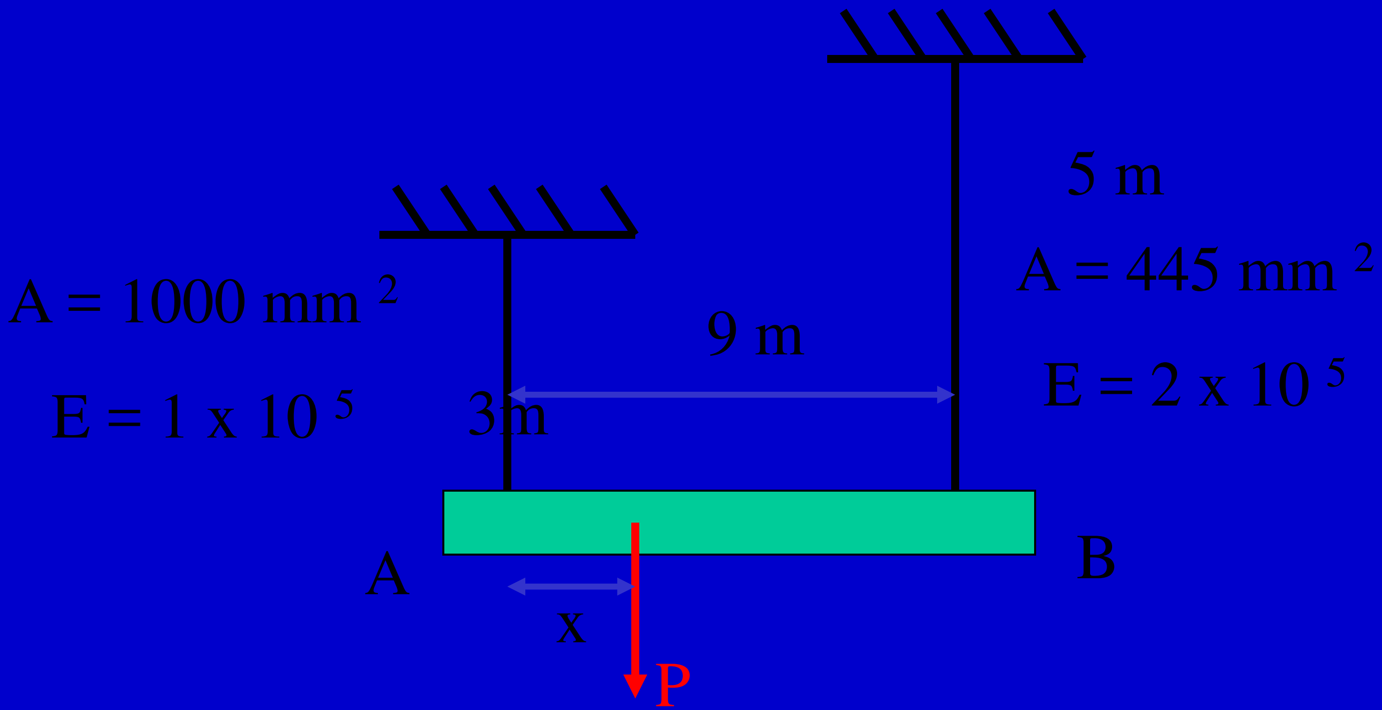


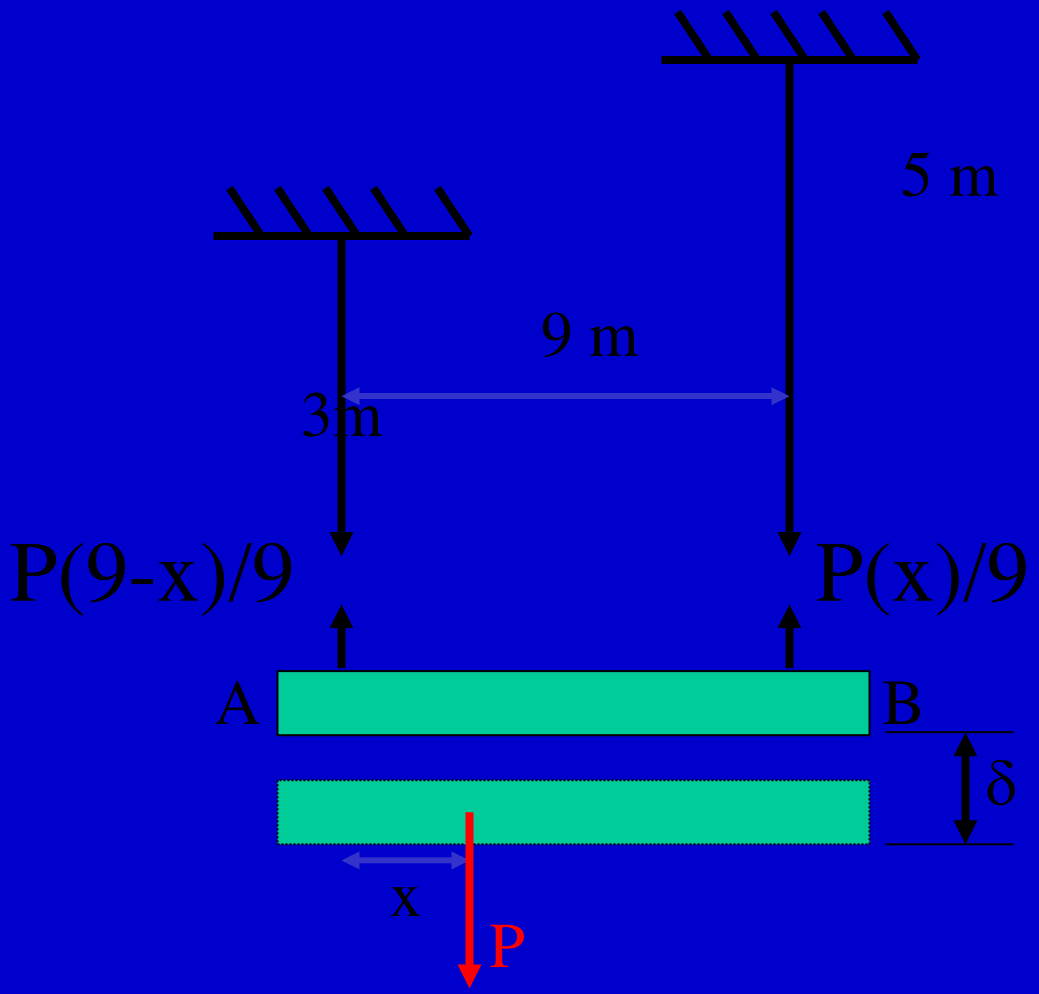
Solution:

$$\delta = 20000*1000/(300*2\times10^5) - 15000*2000/(300*2\times10^5)$$

$$= 0.33 - 0.5 = -0.17 \text{ mm} \quad \text{(i.e.contraction)}$$

Example: 6 A rigid bar AB, 9 m long, is supported by two vertical rods at its end and in a horizontal position under a load P as shown in figure. Find the position of the load P so that the bar AB remains horizontal.




5 m
9 m
3m
P(9-x)/9
P(x)/9
A
B
δ
x
P

For the bar to be in horizontal position, Displacements at A & B should be same,

$$\delta_A = \delta_B$$

$$(PL/AE)_A = (PL/AE)_B$$

$$\frac{\{P(9-x)/9\}*3}{(0.001*1*10^5)} = \frac{\{P(x)/9\}*5}{0.000445*2*10^5}$$

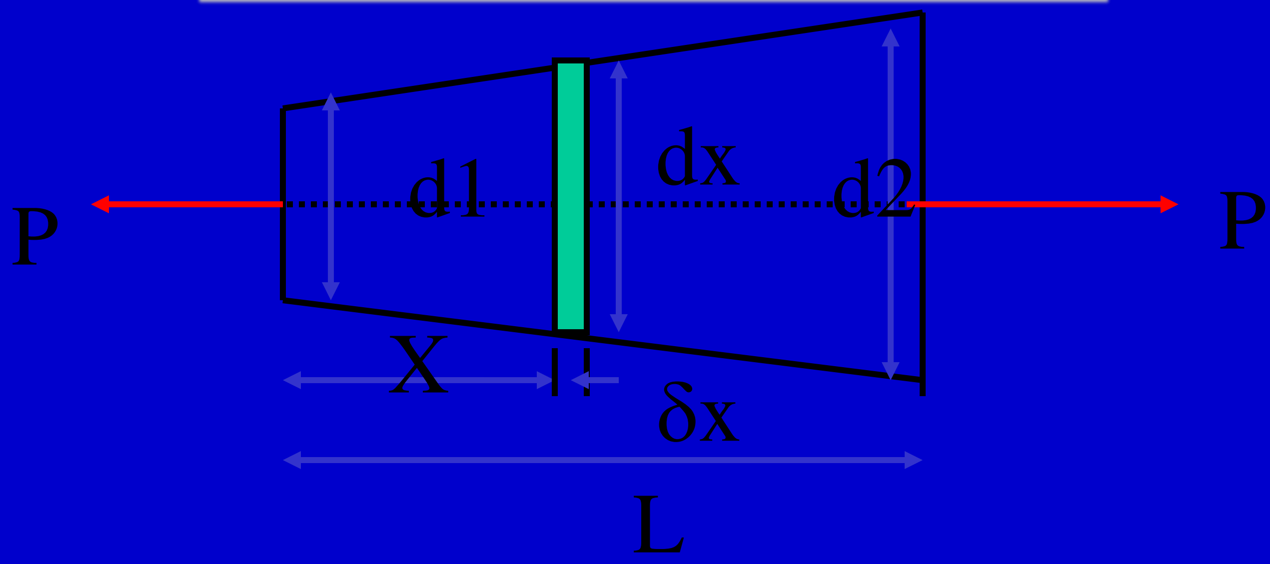
$$(9 - x)*3 = x*5*1.1236$$

$$27-3x=5.618\ x$$

$$8.618\ x=27$$

$$x = 3.13\ m$$

Extension of Bar of Tapering cross Section from diameter d1 to d2:-



Bar of Tapering Section:

$$dx = d1 + [(d2 - d1) / L] * X$$

$$\delta\Delta = P\delta x / E[\pi /4\{d1 + [(d2 - d1) / L] * X\}^2]$$

$$\Delta = \int_0^L 4\,P\,dx\,/[E\,\pi\{d1+kx\}^2]$$

$$= -\,[4P/\,\pi\,E]\;x\;\;1/k\;\;[\,\{1\,/(d1+kx)\}]_0^L\;\;dx$$

$$=-\,[4PL/\,\pi\,E(d2-d1)]\,\{1/(d1+d2\,-d1)\,-\,1/d1\}$$

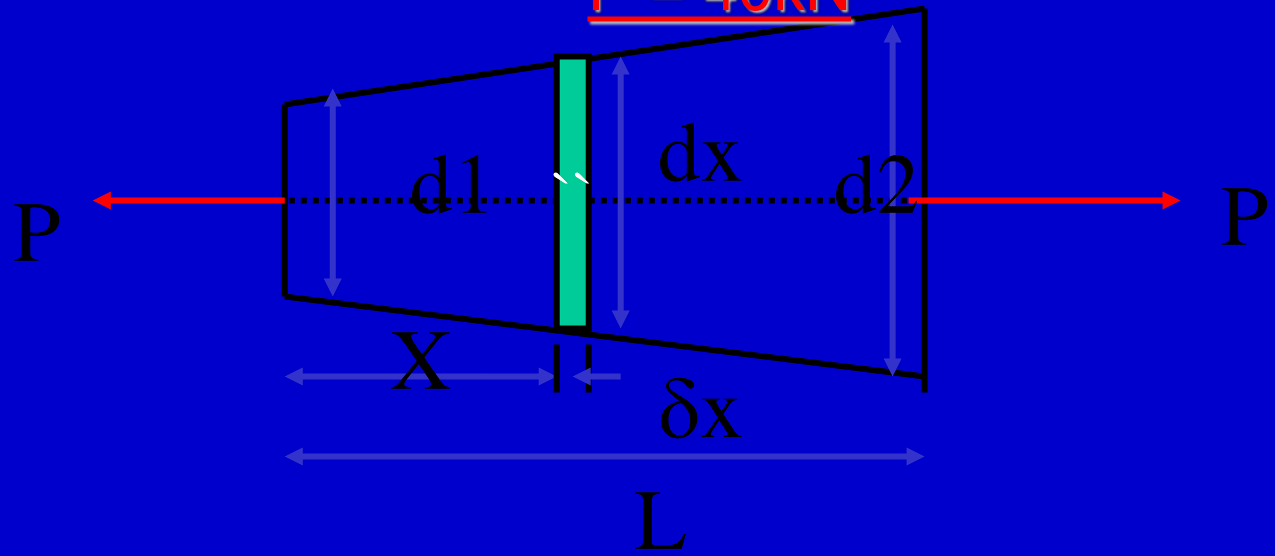
$$\Delta = 4PL/(\pi\,E\,d1\,d2)$$

Check :-

When $d = d1 = d2$

$$\Delta = PL/\,[(\pi\,/4)^*\,d^2E\,] = PL\,/AE\;\;(\text{refer -24})$$

Q. Find extension of tapering circular bar under axial pull for the following data: d1 = 20mm, d2 = 40mm, L = 600mm, E = 200GPa. P = 40kN

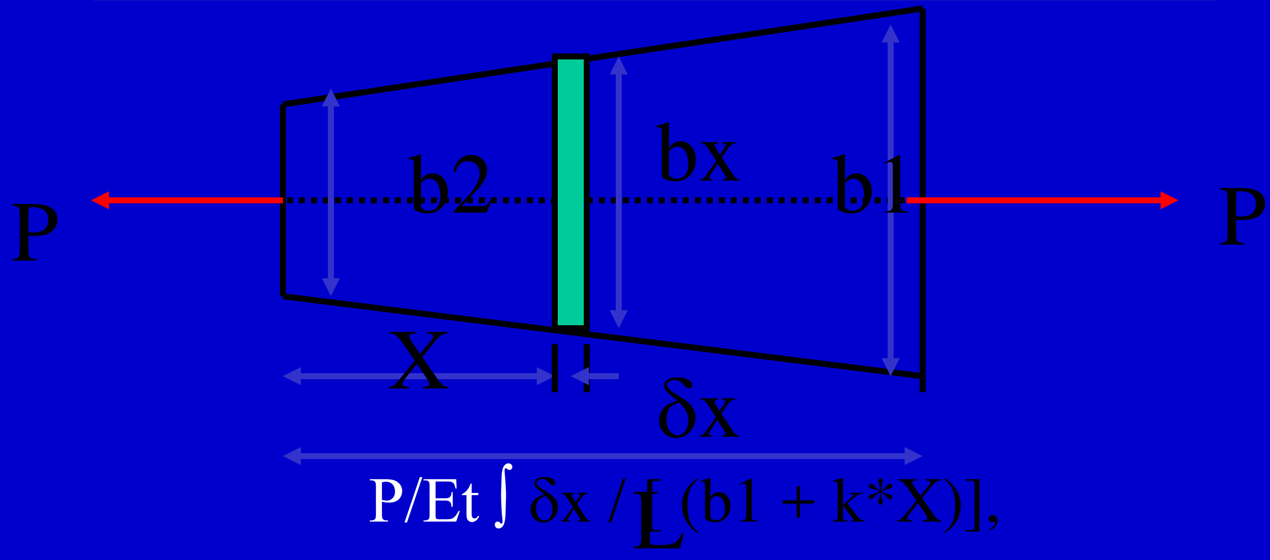


$\Delta L = 4PL/(\pi E\, d1\, d2)$

$= 4*40{,}000*600/(\pi* 200{,}000*20*40)$

$= 0.38$mm. Ans.

Extension of Tapering bar of uniform thickness t, width varies from b1 to b2:-



P/Et ∫ δx / [(b1 + k*X)],

Bar of Tapering Section:

$$bx = b1 + [(b2 - b1) / L] * X = b1 + k*x,$$

$$\delta\Delta = P\delta x / [Et(b1 + k*X)], \quad k = (b2 - b1) / L$$

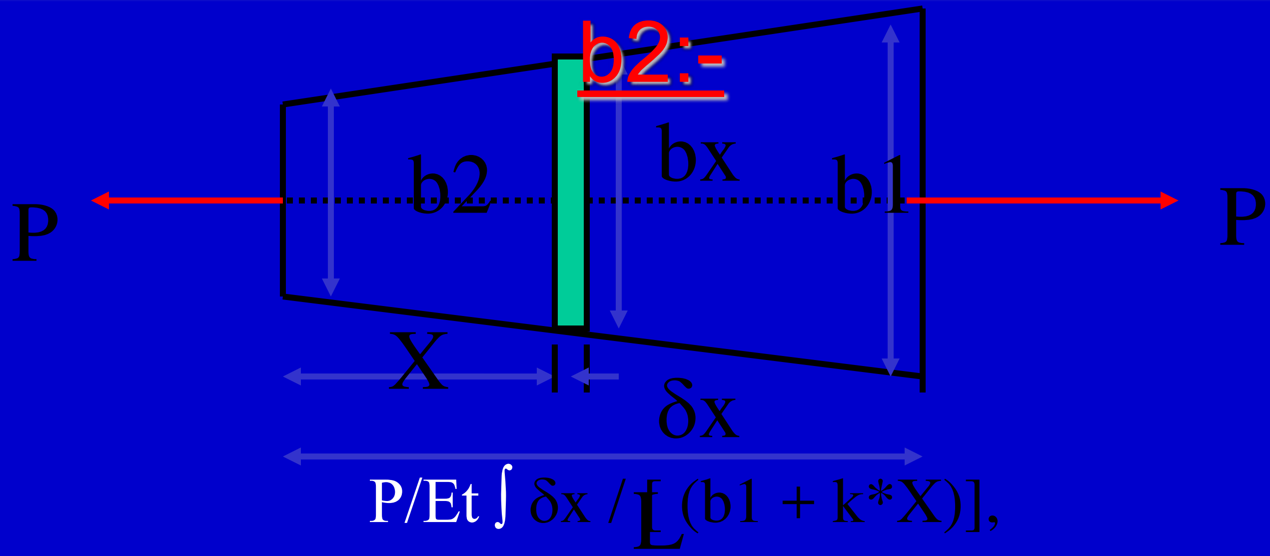
$$\int_0^L \int_0^L \Delta L = \int_0^L \Delta L = P\delta x / [Et(b1 - k^*\mathbf{X})],$$

$$= P/Et \int \delta x / [\, (b1 - k^*\mathbf{X})],$$

$$= - P/Etk * \log_e \; [\, (b1 - k^*\mathbf{X})]_0^L ,$$

$$= PL\log_e(b1/b2) / [Et(b1 - b2)]$$

Q. Calculate extension of Tapering bar of uniform thickness t, width varies from b1 to b2:-



P/Et ∫ δx / [(b1 + k*X)],

Take b1 = 200mm, b2 = 100mm, L = 500mm

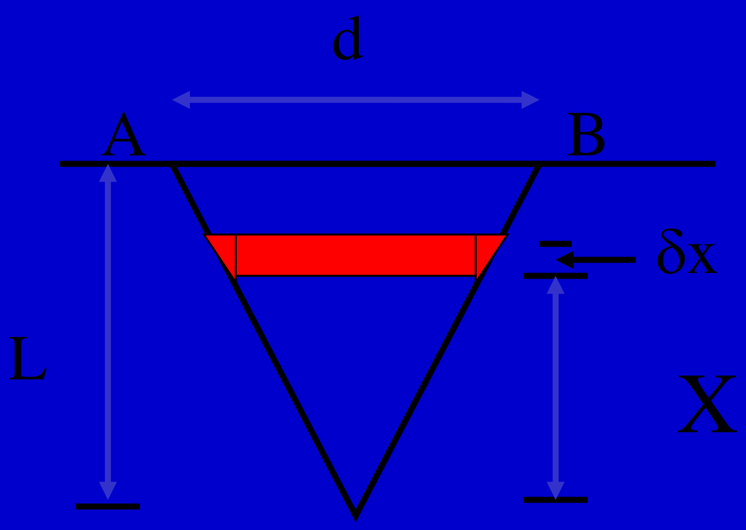
P = 40kN, and E = 200GPa, t = 20mm

δL= PLloge(b1/b2) / [Et(b1 – b2)]

= 40000*500loge(200/100)/[200000*20 *100]

= 0.03465mm

Elongation of a Bar of circular tapering section due to self weight:



$$\delta\Delta = W_x * \delta_x / (A_x E)$$

(from $\Delta = PL/AE$)

now $W_x = 1/3 * A_x X \gamma$

where W_x = Wt. of the bar

so $\delta\Delta = X \gamma * \delta_x / (3E)$

so now

$$\Delta L = \int_0^L X \gamma * \delta_x / (3E)$$

$$= \int_0^L \gamma/(3E) \; X dx = [\gamma/3E] \, [X^2/2]$$

$$= \gamma L^2/(6E)$$

Let W=total weight of bar = $(1/3)*(\pi/4*d^2)L\,\gamma$

$$\gamma = 12W/(\pi*d^2L)$$

so,

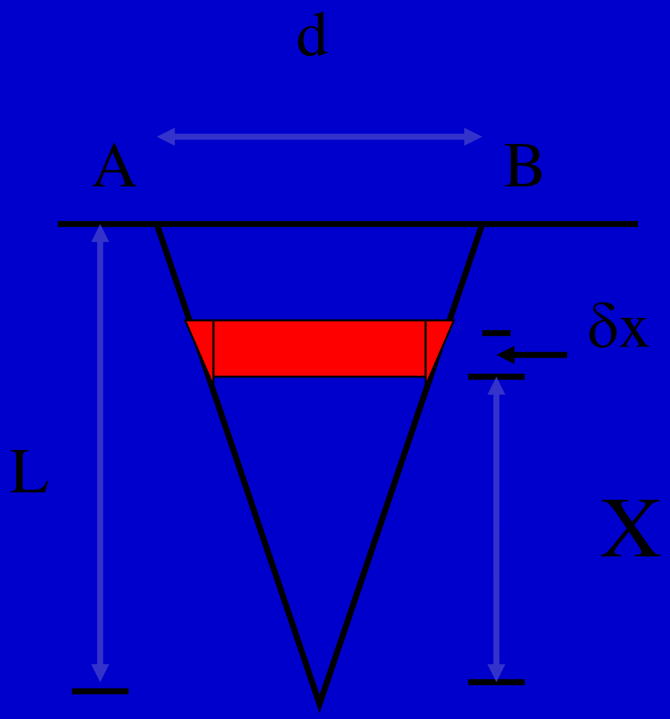
$$\Delta L = [12W/(\pi*d^2L)]*(L^2/6E)$$

$$=2WL/(\pi*d^2E)$$

$$=WL/[2*(\pi*d^2/4)*E]$$

$$=WL/2*A*E$$

Calculate elongation of a Bar of circular tapering section due to self weight:Take L =10m, d = 100mm, γ = 7850kg/m^3

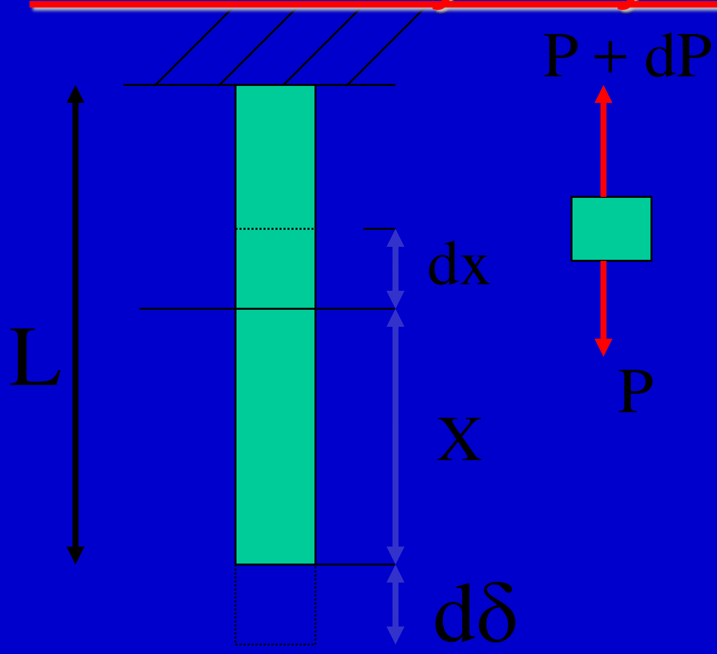


$$\Delta L = \gamma L^2/(6E)$$

$$7850*9.81*10000*10000*/ [6*200000*1000^3]$$

$$= 0.006417\text{mm}$$

Extension of Uniform cross section bar subjected to uniformly varying tension due to self weight



$$P_X = \gamma A x$$

$$d\delta = P_X \, dx / A E;$$

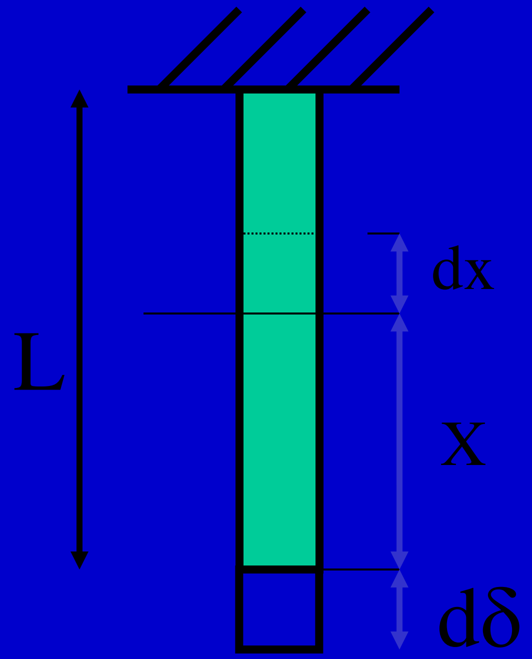
$$\delta = \int_0^L P_X \, dx/AE = \int_0^L \gamma A x \, dx/AE$$

$$\delta = (\gamma / E) \int_0^L x \, dx = (\gamma L^2/2E)$$

If total weight of bar $W \Rightarrow \gamma A L$ $\quad \gamma = W/AL$

$\delta = WL/2AE$ (compare this results with slide-26)

Q. Calculate extension of Uniform cross section bar subjected to uniformly varying tension due to self weight



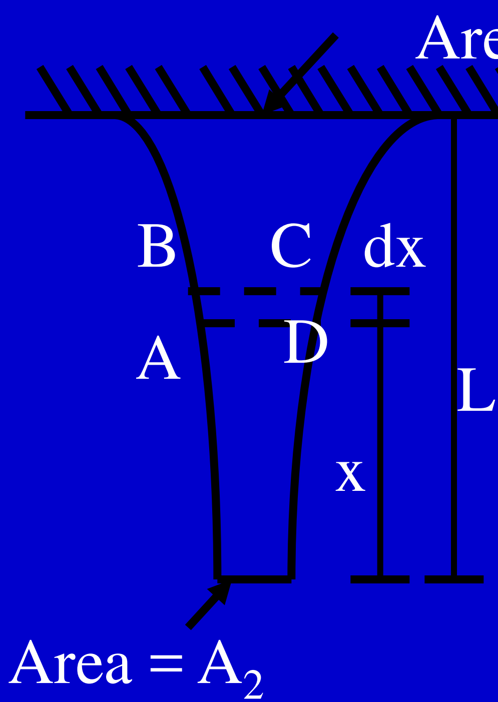
Take L = 100m, A = 100mm^2 , density = 7850kg/m^3

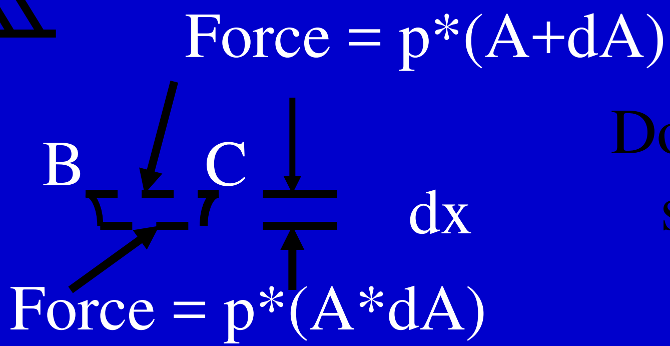
$$\delta = (\gamma L^2/2E)$$

δ = 850*9.81*100000*100000/ [2*200000*1000^3]

= 1.925mm

Bar of uniform strenght:(i.e.stress is constant at all points of the bar.)





Down ward force of strip = w*A*dx,

comparing force at BC level of strip

of thickness dx

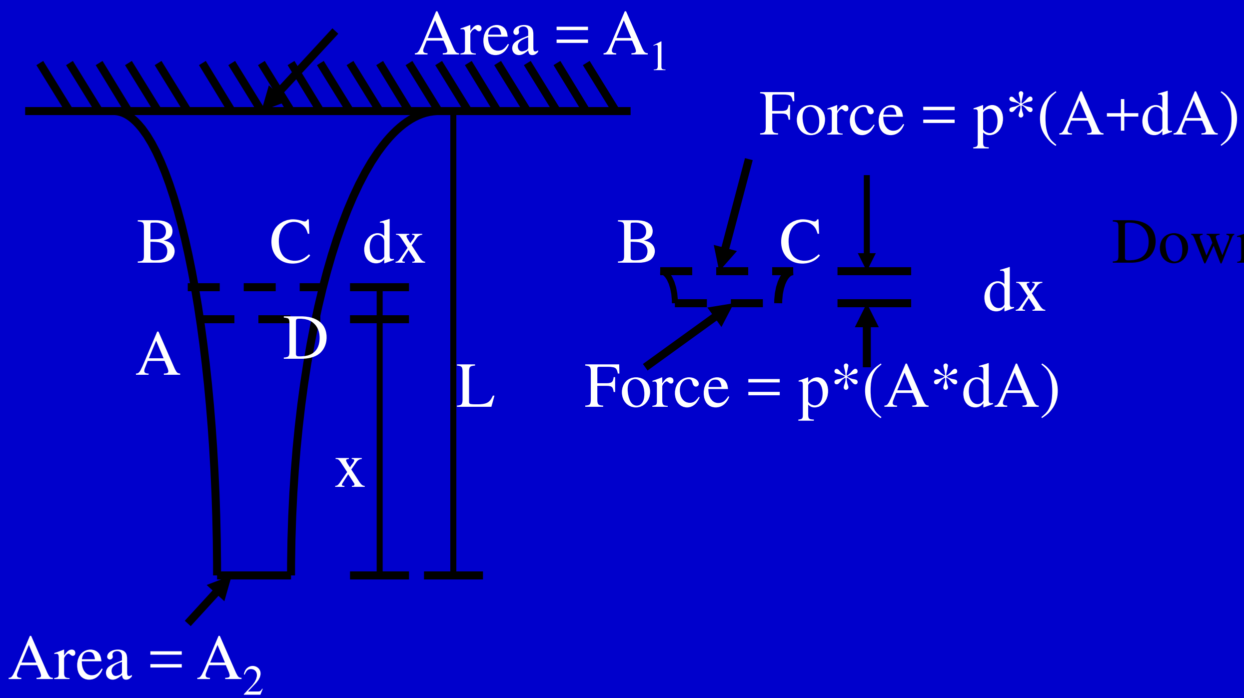
P(A + dA) = Pa + w*A*dx,

where w is density of the material hence

dA/A = wdx/p, Integrating $\log_e A = wx/p + C$,

at x = 0, A = A_2 and x = L, A = $A_{1,}$ C = A_2

$\log_e(A/A_2) = wx/p$ OR $A = e^{wx/p}$

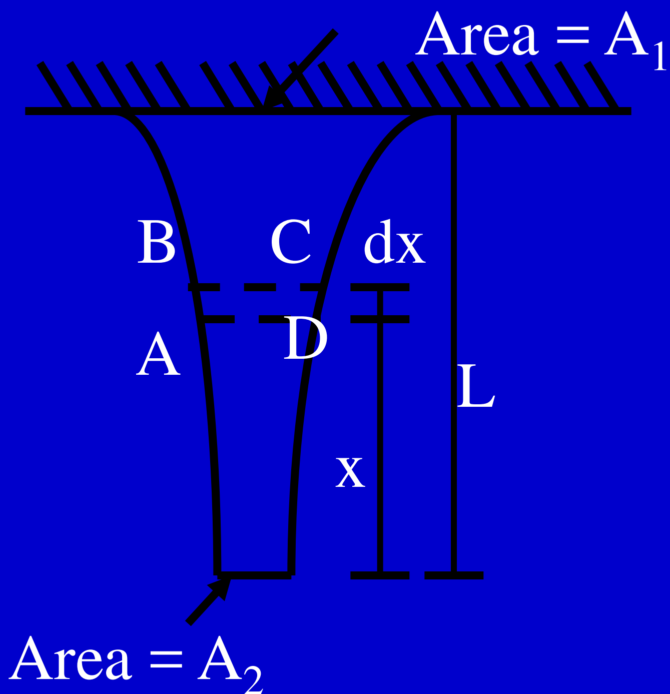


Down ward force of strip = w*A*dx,

$$A = e^{wx/p}$$

(where A is cross section area at any level x of bar of uniform strenght)

Q. A bar of uniform strength has following data. Calculate cross sectional area at top of the bar.



$A_2 = 5000mm^2$, L = 20m, load at lower end = 700kN, density of the material = $8000kg/m^3$

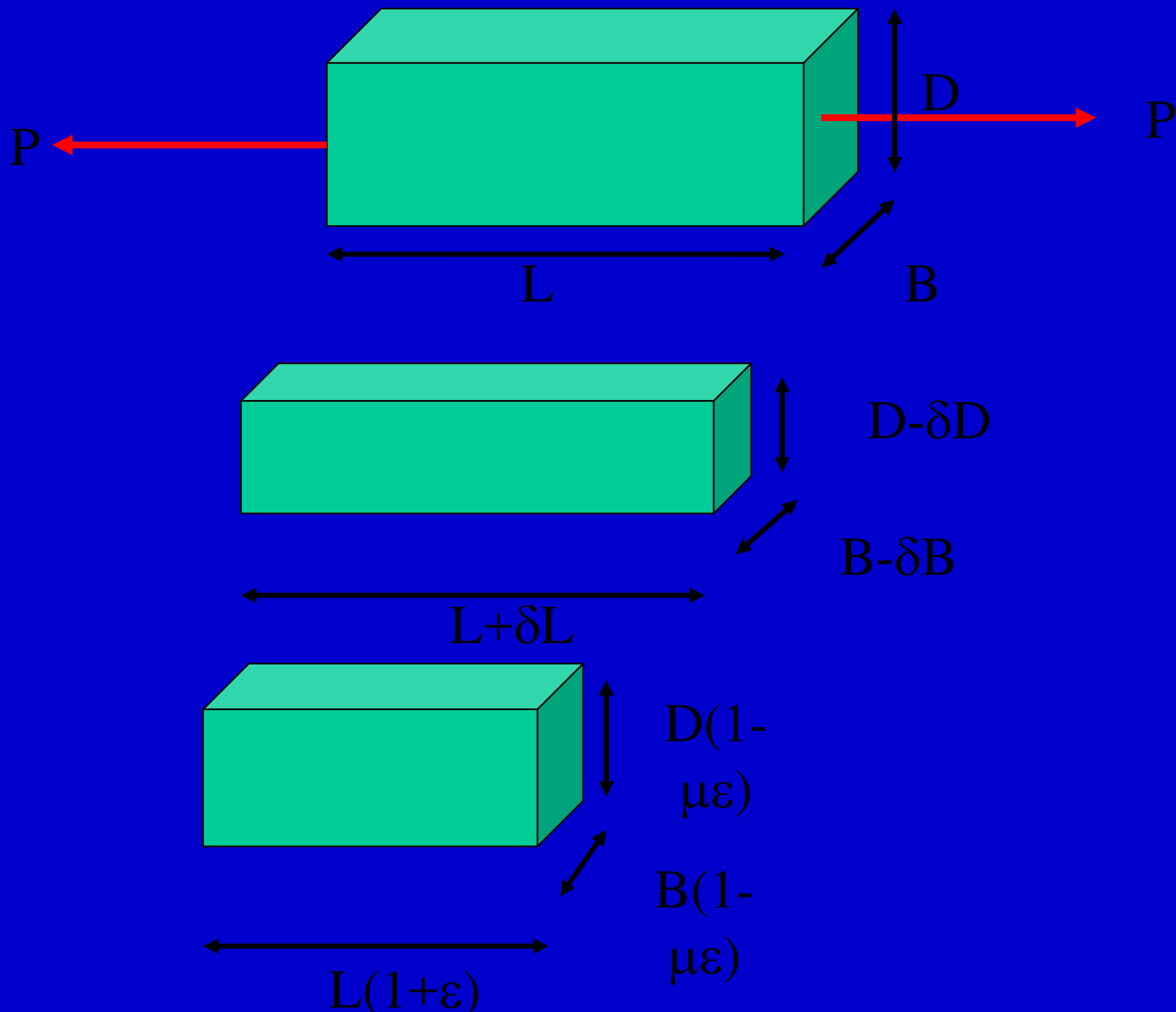
p = 700000/5000 = 140MPa

$$A_1 = A_2\, e^{wx/p}$$

$$A_1 = 5000 * e^{8000*9.81*20000/[140*1000^3]}$$

$$= 5056.31mm^2$$

POISSONS RATIO:- μ = lateral contraction per Unit axial elongation, (with in elastic limit)



$\mu = (\delta B/B)/(\delta L/L);$

$= (\delta B/B)/(\varepsilon)$

So $\delta B = \varepsilon\, \mu B;$

New breadth =

$B - \delta B = B - \varepsilon\, \mu B$

$= B(1 - \mu\, \varepsilon)$

Sim., New depth=

$D(1 - \mu\, \varepsilon)$

for isotropic materials $\mu = ¼$ for steel $\mu = 0.3$

Volume of bar before deformation $V = L * B*D$

new length after deformation $L_1 = L + \delta L = L + \varepsilon L = L(1+\varepsilon)$

new breadth $B_1 = B - \delta B = B - \varepsilon \mu B = B(1 - \mu \varepsilon)$

new depth $D_1 = D - \delta D = D - \varepsilon \mu D = D(1 - \mu \varepsilon)$

new cross-sectional area $= A_1 = B(1-\mu \varepsilon)*D(1-\mu \varepsilon) = A(1-\mu \varepsilon)^2$

new volume $V1 = V - \delta V = L(1+\varepsilon)* A(1-\mu \varepsilon)^2$

$$\approx AL(1+\varepsilon - 2\mu \varepsilon)$$

Since ε is small

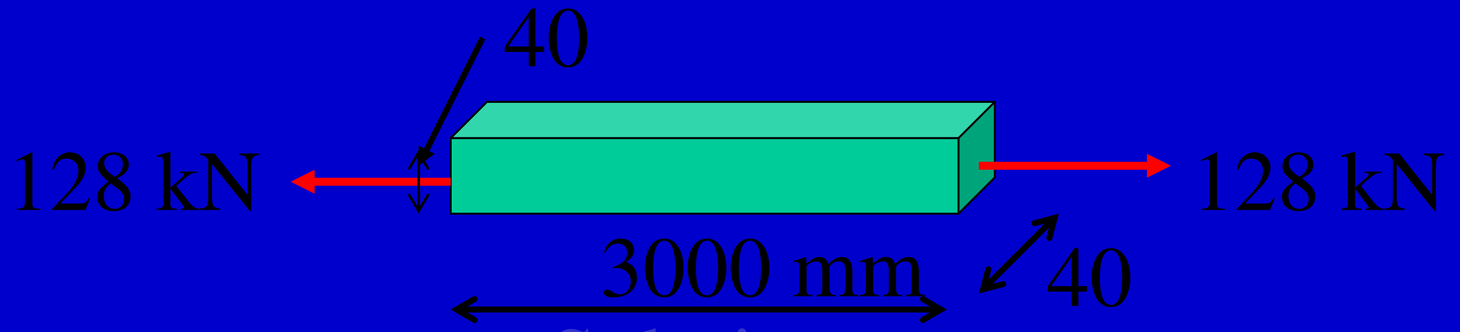
change in volume $= \delta V = V1 - V \quad = AL\varepsilon(1-2\mu)$

and unit volume change $= \delta V/V \quad = \{AL\varepsilon(1-2\mu)\}/AL$

$$\delta V/V \quad = \varepsilon(1-2\mu)$$

In case of uniformly varying tension, the elongation ‘δ’ is just half what it would be if the tension were equal throughout the length of the bar.

Example: 7 A steel bar having 40mm*40mm*3000mm dimension is subjected to an axial force of 128 kN. Taking $E=2*10^5 N/mm^2$ and $\mu = 0.3$, find out change in dimensions.



Solution:

given b=40 mm, t=40mm, L=3000mm

$P=128$ kN$=128*10^3$ N, $E=2*10^5$ mm^2, $\mu=0.3$

$\delta L=?$, $\delta b=?$, $\delta t=?$

$\sigma_t = P/A = 128*10^3 / 40*40 = 80$ N/mm^2

now $\varepsilon = \sigma_t/E=80/(2*10^5)=4*10^{-4}$

$\varepsilon = \delta L/L \Rightarrow \delta L= \varepsilon *L=4*10^{-4} *3000 = 1.2$ mm

(increase)

$\delta b= - \mu*(\varepsilon *b)= -0.3*4*10^{-4}*40 = 4.8*10^{-3}$ mm

(decrease)

$\delta t = - \mu*(\varepsilon *t)= -0.3*4*10^{-4}*40 = 4.8*10^{-3}$ mm

(decrease)

Change in volume = [3000 + 1.2) * (40 – 0.0048) *

(40 – 0.0048)] – 3000*40*40

= 767.608 mm^3

OR by using equation (derivation is in chapter of volumetric stresses and strains)

$$dv = p*(1-2\mu)v/E$$

$$= (128000/40*40)*0.4*3000*40*40/200000$$

$$= 768 mm^3$$

Example: 8 A strip of 20 mm*30 mm c/s and 1000mm length is subjected to an axial push of 6 kN. It is shorten by 0.05 mm. Calculate (1) the stress induced in the bar. (2) strain and young's modulus & new cross-section. Take μ =0.3

Solution:given,

c/s =20 mm*30 mm, A =600mm^2,L=1000 mm,

P=6 kN=6*10^3 N, δL =0.05 mm, ε = ?, σ=?,E =?.

1. σ = P/A =6000/600 =10 N/mm^2 -----(1)

2 ε = δL /L=0.05/1000 =0.00005 -----(2)

σ =E ε ==>E = σ/ ε =10/0.00005 = 2*10^5 N/mm^2

3 Now,

New breadth B1 $=B(1-\mu\ \varepsilon)$

$=20(1-0.3*0.00005)$

$=19.9997$ mm

New Depth D1 $= D(1-\mu\ \varepsilon)$

$=30(1-0.3*0.00005)$

$= 29.9995$mm

Example: 9 A iron bar having 200mm*10 mm c/s,and 5000 mm long is subjected to an axial pull of 240 kN.Find out change in dimensions of the bar. Take E $=2*10^5$ N/mm^2 and $\mu = 0.25$.

Solution: b =200 mm,t = 10mm,so A = $2000mm^2$

$$\sigma = P/A=240*10^3 / 2000 =120N/mm^2$$

now $\sigma = E\,\varepsilon$ $\qquad$ $\varepsilon = \sigma/E =120/2*10^5=0.0006$

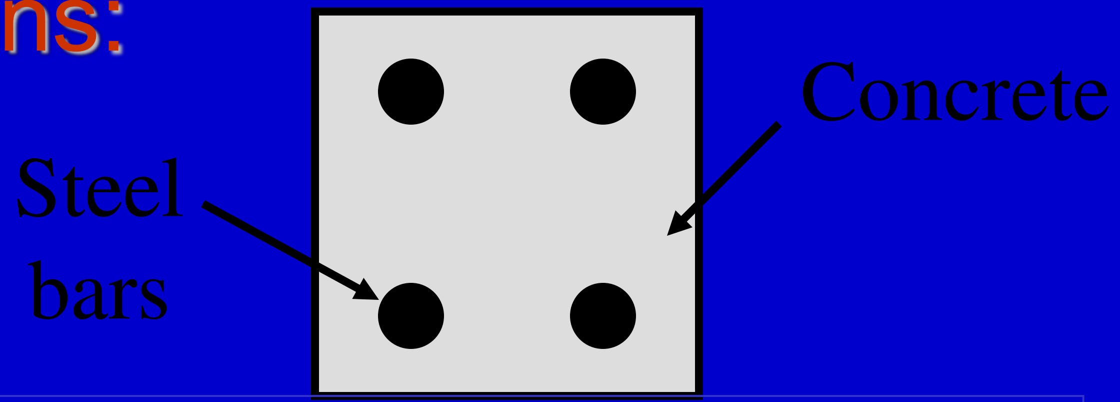
$\varepsilon= \delta L /L$ $\qquad$ $\delta L = \varepsilon *L=0.0006*5000=3$ mm

$$\delta b = -\mu*(\varepsilon *b)= -0.25*6*10^{-4}*200$$

$$= 0.03 \text{ mm(decrease)}$$

$$\delta t = -\mu*(\varepsilon *t) = -0.25*6*10^{-4}*10$$

Composite Sections:



- as both the materials deforms axially by same value strain in both materials are same.

$$\varepsilon s = \varepsilon c = \varepsilon$$

$$\sigma s / Es = \sigma c / E \ (= \varepsilon = \delta L / L) \text{ ______ } (1) \ \& \ (2)$$

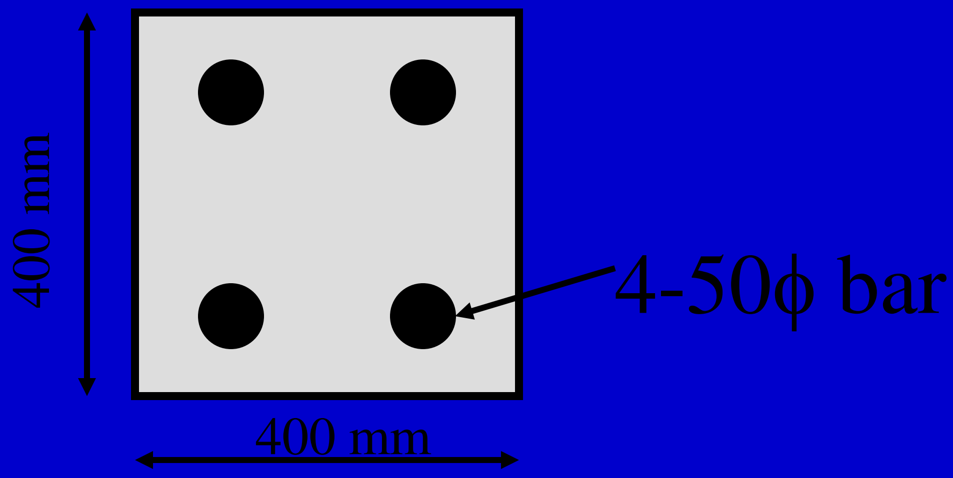
- Load is shared between the two materials.

$$P_s + P_c = P \text{ i.e. } \sigma_s * A_s + \sigma_c * A_c = P \text{ ---}(3)$$

(unknowns are σs, σc and δL)

Example: 10 A Concrete column of C.S. area 400 x 400 mm reinforced by 4 longitudinal 50 mm diameter round steel bars placed at each corner of the column carries a compressive load of 300 kN. Calculate (i) loads carried by each material & compressive stresses produced in each material. Take Es = 15 Ec Also calculate change in length of the column. Assume the column in 2m long.

Take Es = 200GPa



Solution:-

Gross C.S. area of column =0.16 m^2

C.S. area of steel = $4*\pi*0.025^2$ = 0.00785 m^2

Area of concrete =0.16 - 0.00785=0.1521m^2

Steel bar and concrete shorten by same amount. So,

$\varepsilon_s = \varepsilon_c \Rightarrow \sigma_s /Es = \sigma_c /Ec \Rightarrow \sigma_s = \sigma_c x (Es /Ec)$

$= 15\sigma_c$

load carried by steel +concrete=300000 N

Ws +Wc= 300000

σ_s As + σ_c Ac = 300000

15 σ_c x 0.00785 + σ_c x0.1521 = 300000

σ_c = 1.11 x 10 ^{6} N/ m^2

σ_s =15x σ_c=15 x1.11x 10 ^{6}=16.65 x10 ^{6} N/ m^2

Ws =16.65x10 ^{6} x0.00785 / 10 ^{3} =130.7 kN

Wc = 1.11x 10 ^{6} x 0.1521/10^3= 168.83 kN

(error in result is due to less no. of digits considered in stress calculation.)

we know that,

$\sigma s / Es = \sigma c / E$ (= ε = δL /L) ______(1) & (2)

σ_c = 1.11 MPa

σ_s =15x σ_c=15 x1.11x 10 ^{6}=16.65 MPa

The length of the column is 2m

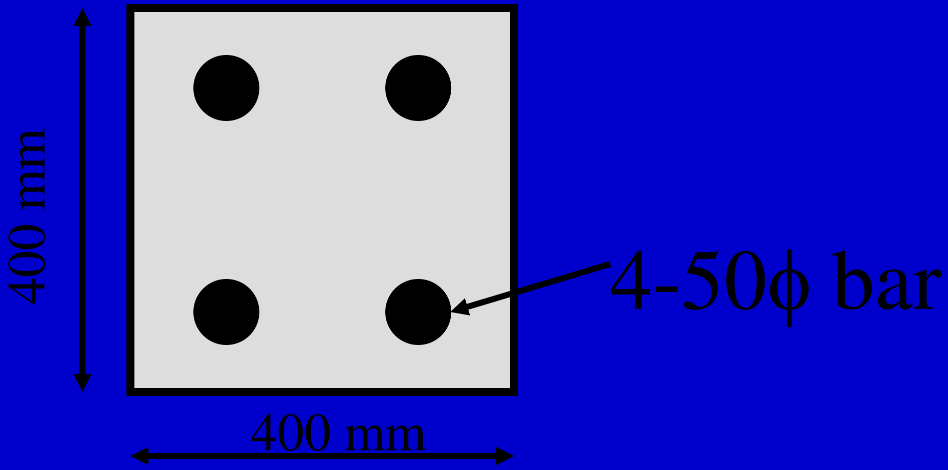
Change in length

dL = 1.11*2000/[13.333*1000] = 0.1665mm

OR

dL = 16.65*2000/[200000] = 0.1665mm

Example: 10 A Concrete column of C.S. area 400 x 400 mm reinforced by 4 longitudinal 50 mm diameter round steel bars placed at each corner of the column. Calculate (1) maximum axial compressive load the column can support &(ii) loads carried by each material & compressive stresses produced in each material. Take Also calculate change in length of the column. Assume the column in 2m long. Permissible stresses in steel and concrete are 160 and 5MPa respectively. Take Es = 200GPa and Ec = 14GPa.



Solution:-

Gross C.S. area of column =0.16 m^2

C.S. area of steel = $4*\pi*0.025^2$ = 0.00785 m^2

Area of concrete =0.16 - 0.00785=0.1521m^2

Steel bar and concrete shorten by same amount. So,

$\varepsilon_s = \varepsilon_c \Rightarrow \sigma_s / Es = \sigma_c / Ec \Rightarrow \sigma_s = \sigma_c x\ (Es / Ec)$

$= 14.286\ \sigma_c$

Solution:-

Gross C.S. area of column =0.16 m^2

C.S. area of steel = $4*\pi*0.025^2$ = 0.00785 m^2

Area of concrete =0.16 - 0.00785=0.1521m^2

Steel bar and concrete shorten by same amount. So,

$\varepsilon_s = \varepsilon_c \Rightarrow \sigma_s /Es = \sigma_c /Ec \Rightarrow \sigma_s = \sigma_c x\ (Es /Ec) = \sigma c x\ (200/14)$

$= 14.286\sigma_c$

So $\sigma_s = 14.286\sigma_c$

σs = 160 then σc = 160/14.286 = 11.2MPa > 5MPa, Not valid

σc = 5MPa then σs = 14.286*5 = 71.43 MPa <120MPa, Valid

Permissible stresses in each material are

σc = 5MPa & σs = 71.43 MPa

We know that

$$\sigma s\ As + \sigma c\ Ac = W$$

$[71.43 \times 0.00785 + 5 \times 0.1521]*1000^2 / 1000 = 1321.22kN$

Load in each materials are

$Ws = 71.43 \times 0.00785 \times 1000 = 560.7255$ kN

$Wc = 5 \times 0.1521 \times 1000 = 760.5kN$

we know that,

$\sigma s / Es = \sigma c / E$ (= ε = $\delta L / L$) ______(1) & (2)

$\sigma_c = 5$ MPa

$\sigma_s = 71.43$ MPa

The length of the column is 2m

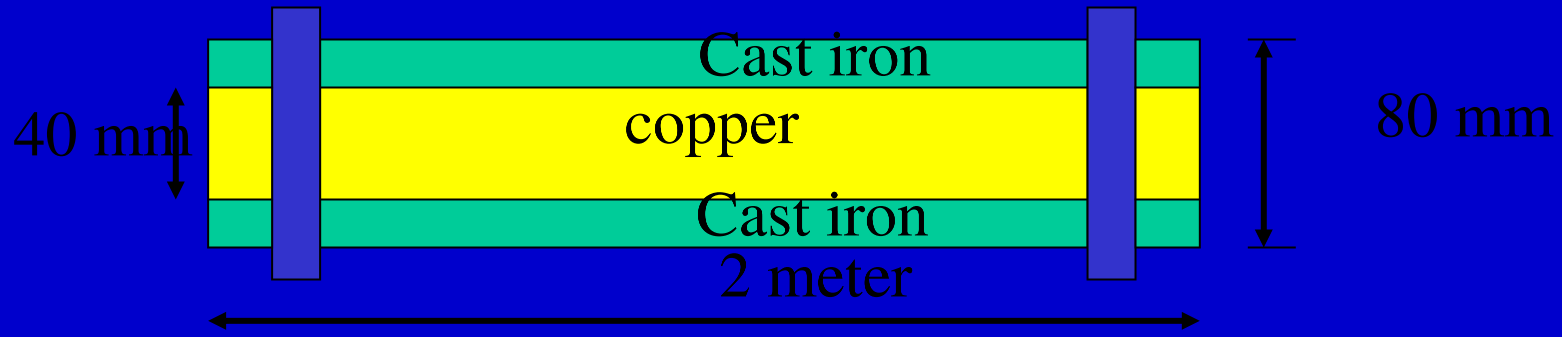
Change in length

dL = 5*2000/[14000] = 0.7143mm

OR

dL = 71.43*2000/[200000] = 0.7143mm

Example: 11 A copper rod of 40 mm diameter is surrounded tightly by a cast iron tube of 80 mm diameter, the ends being firmly fastened together. When it is subjected to a compressive load of 30 kN, what will be the load shared by each? Also determine the amount by which a compound bar shortens if it is 2 meter long. Eci=175 GN/m^2,Ec= 75 GN/m^2 .



Area of Copper Rod $= A_c = (\pi/4) * 0.04^2 = 0.0004\pi \text{ m}^2$

Area of Cast Iron $= A_{ci} = (\pi/4) * (0.08^2 - 0.04^2) = 0.0012\pi \text{ m}^2$

$\sigma_{ci} / E_{ci} = \sigma_c / E_c$ or

$$\sigma_{ci} / \sigma_c = E_{ci}/E_c = \frac{175 \times 10^9}{75 \times 10^9} = 2.33$$

$\sigma_{ci} = 2.33\, \sigma_c$

Now,

$$W = Wci + Wc$$

$$30 = (2.33\ \sigma_c)\ \text{x}\ 0.012\ \pi + \sigma_c\ \text{x}\ 0.0004\ \pi$$

$$\sigma_c = 2987.5\ kN/m^2$$

$$\sigma_{ci} = 2.33\ \text{x}\ \sigma_c = 6960.8 kN/m^2$$

load shared by copper rod $= Wc = \sigma_c\ Ac$

$$= 2987.5\ \text{x}\ 0.0004\ \pi$$

$$= 3.75\ kN$$

$$Wci = 30 - 3.75 = 26.25\ kN$$

Strain $\varepsilon_c = \sigma_c / Ec = \delta L / L$

$\delta L = (\sigma_c / Ec) \times L \quad = [2987.5/(75 \times 10^{9})] \times 2$

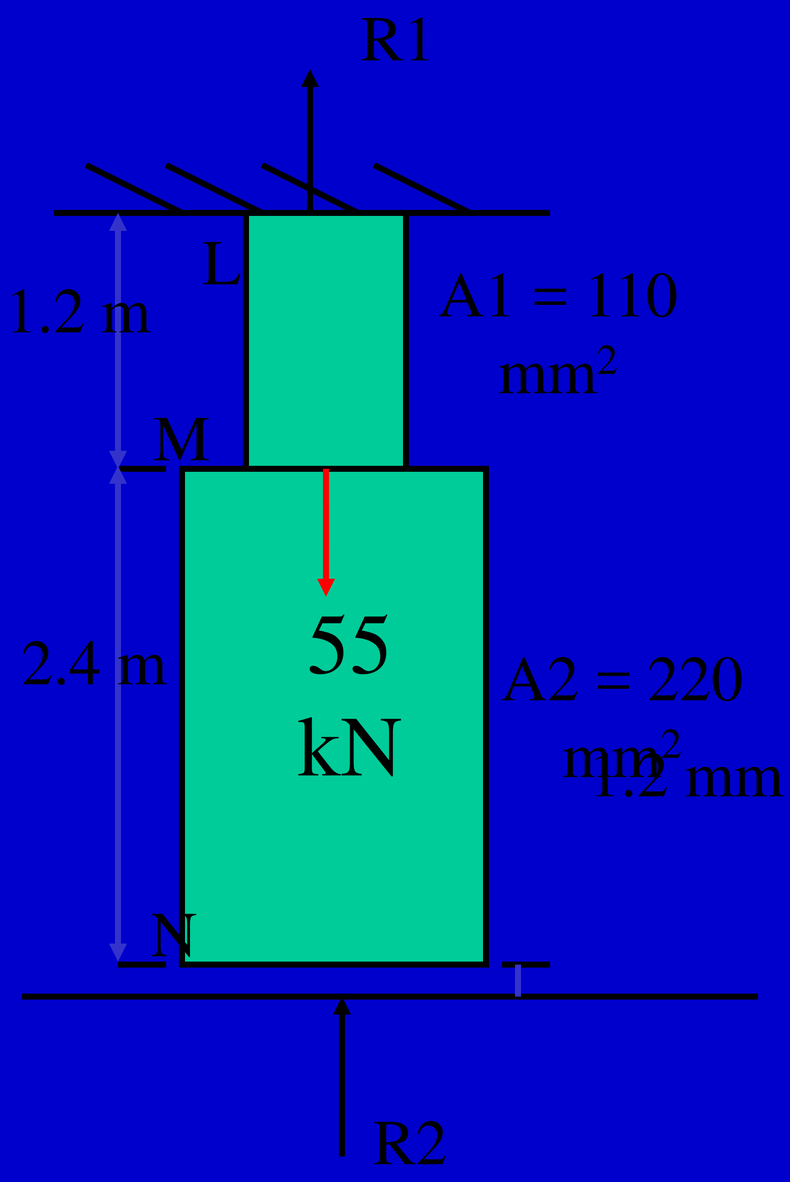
$= 0.0000796$ m

$= 0.0796$ mm

Decrease in length = 0.0796 mm

Example: 12

For the bar shown in figure, calculate the reaction produced by the lower support on the bar. Take E= $2*10^8$ kN/m^2.Find also stresses in the bars.



Solution:-

R1+R2 = 55

δ L1 =(55-R2)*1.2 / $(110*10^{-6})*2*10^{8}$ (LM extension)

δ L2 =R2*2.4 / $(220*10^{-6})*2*10^{8}$ (MN contraction)

(Given: δ L1- δ L2 =1.2 /1000=0.0012)

(55-R2)*1.2 / $[(110*10^{-6})*2*10^{8}]$ -R2*2.4 /$[(220*10^{-6})*2*10^{8}]$

=0.0012

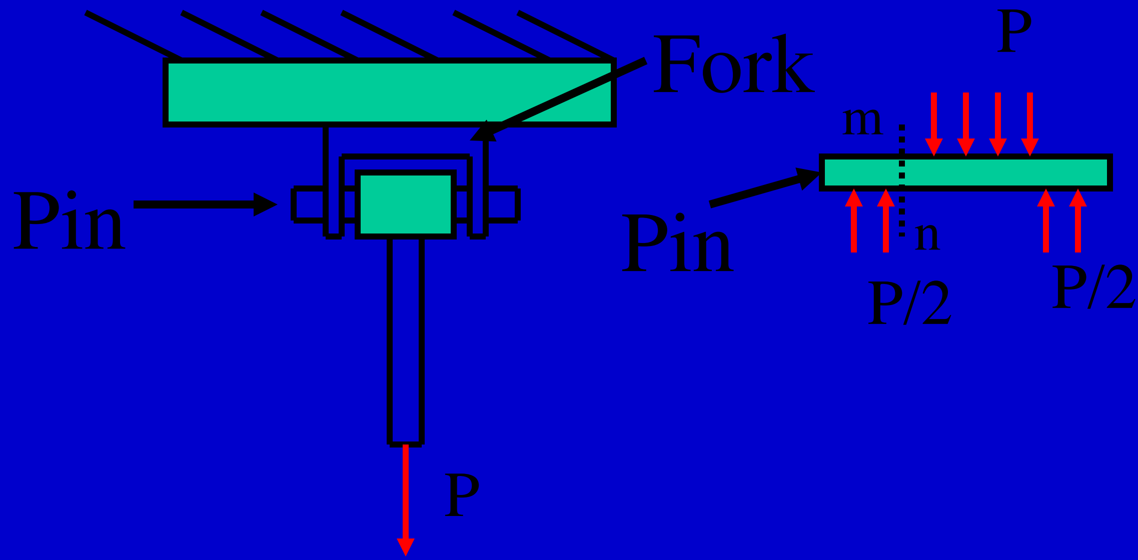
so R2 = 16.5 kN Since R1+R2 = 55 kN,

R1=38.5 kN

Stress in LM = Force/area = 350000 kN/m^2

Stress in MN =75000 kN/m^2

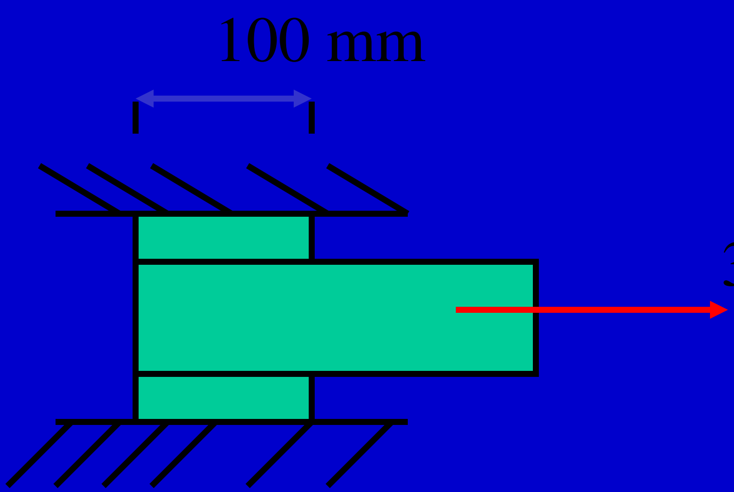
Direct Shear:--



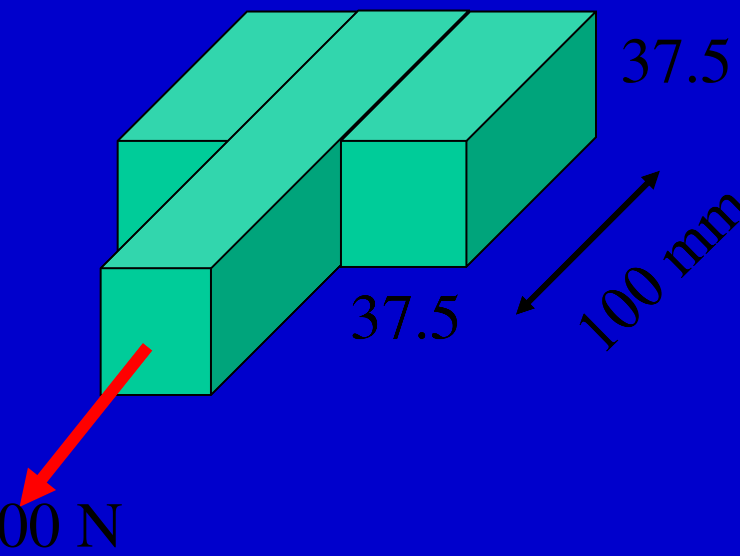
- Connection should withstand full load P transferred through the pin to the fork .
- Pin is primarily in shear which tends to cut it across at section m-n .
  - Average shear Stress => $\tau = P/(2A)$ (where A is cross sectional area of pin)
- Note: Shearing conditions are not as simple as that for direct stresses.

- Dealing with machines and structures an engineer encounters members subjected to tension, compression and shear.
- The members should be proportioned in such a manner that they can safely & economically withstand loads they have to carry.

Example: 3 Three pieces of wood having 37.5 x 37.5 mm square C.S. are glued together and to the foundation as shown in figure. If the horizontal force P=30000 N is applied to it, what is the average shear stress in each of the glued joints.(ans=4 N/mm^2)



Plan



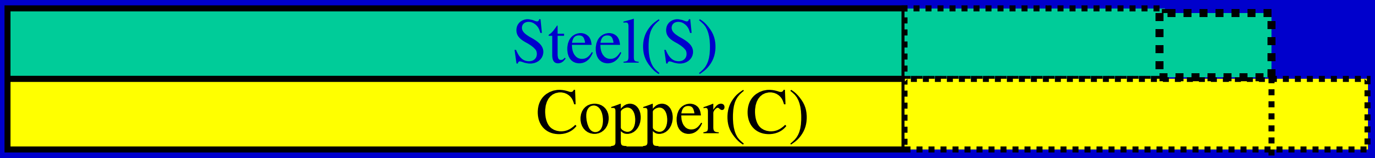
Solution:-

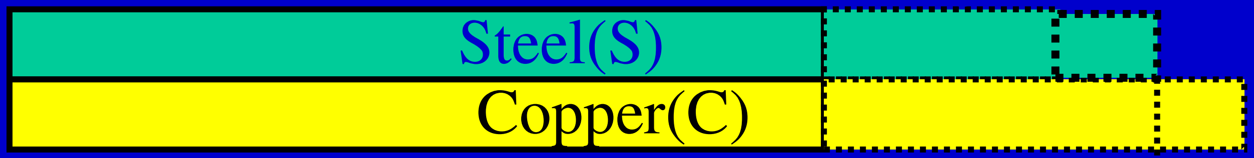
P=30000N;glued c.s area=37.5x100mm x2 surfaces

Shear stress τ = P/c.s area = 4N/mm^2

Temperature stresses:-

Composite Section:-




Steel(S)
Copper(C)

Example: 13

A railway is laid so that there is no stress in rail at 10° C. If rails are 30 m long Calculate,

1. The stress in rails at 60 ° C if there is no allowance for expansion.

2. The stress in the rails at 60 ° C if there is an expansion allowance of 10 mm per rail.

3. The expansion allowance if the stress in the rail is to be zero when temperature is 60 ° C.

4. The maximum temp. to have no stress in the rails if the expansion allowance is 13 mm/rail.

Take $\alpha = 12 \times 10^{-6}$ per 1°C $E = 2 \times 10^{5}\ N/mm^{2}$

Solution:

1. Rise in temp. $= 60° - 10° = 50 °C$

so stress $= \alpha\, t\, E = 12 \times 10^{-6} \times 50 \times 2 \times 10^{5}$

$= 120$ MPa

2. $\sigma_{tp} \times L/E = \Delta = (L\alpha\, t - 10)$

$= (30000 \times 12 \times 10^{-6} \times 50 - 10)$

$= 18 - 10 = 8$ mm

$\sigma_{tp} = \Delta E / L = 8 \times 2 \times 10^{5} / 30000$

$= 53.3$ MPa

3. If stresses are zero ,

Expansion allowed $=(L\alpha t)$

$= (30000 \times 12 \times 10^{-6} \times 50)$

$=18$ mm

4. If stresses are zero

$\sigma_{tp} = E/L^*(L\alpha t - 13) = 0$

$L\alpha t = 13$

so $t = 13/(30000 \times 12 \times 10^{-6}) = 36^0$ C

allowable temp. $= 10 + 36 = 46^0$c.

Example: 14

A steel bolt of length L passes through a copper tube of the same length, and the nut at the end is turned up just snug at room temp. Subsequently the nut is turned by 1/4 turn and the entire assembly is raised by temp 55^0C. Calculate the stress in bolt if L=500mm,pitch of nut is 2mm, area of copper tube =500sq.mm,area of steel bolt=400sq.mm

$Es=2 * 10^5\ N/mm^2;\alpha_s = 12*10^{-6}\ /^0C$

$Ec=1 * 10^5\ N/mm^2;\alpha_c = 17.5*10^{-6}\ /^0C$

Solution:-

Two effects

(i) tightening of nut

(ii)raising temp.

tensile stress in steel = compressive force in copper

[Total extension of bolt
+Total compression of tube] =Movement of Nut

$[\Delta s + \Delta c] = np$ (where p = pitch of nut)

$$(PL/A_sE_s + \alpha_s L t) + (PL/A_cE_c - \alpha_c L t) = np$$

$$P (1/A_sE_s + 1/A_cE_c) = t(\alpha_c - \alpha_s) + np/L$$

$$\text{so } P[1/(400*2*10^5) + 1/(500*1*10^5)]$$

$$=(17.5-12)*10^{-6} + (1/4)*2/500$$

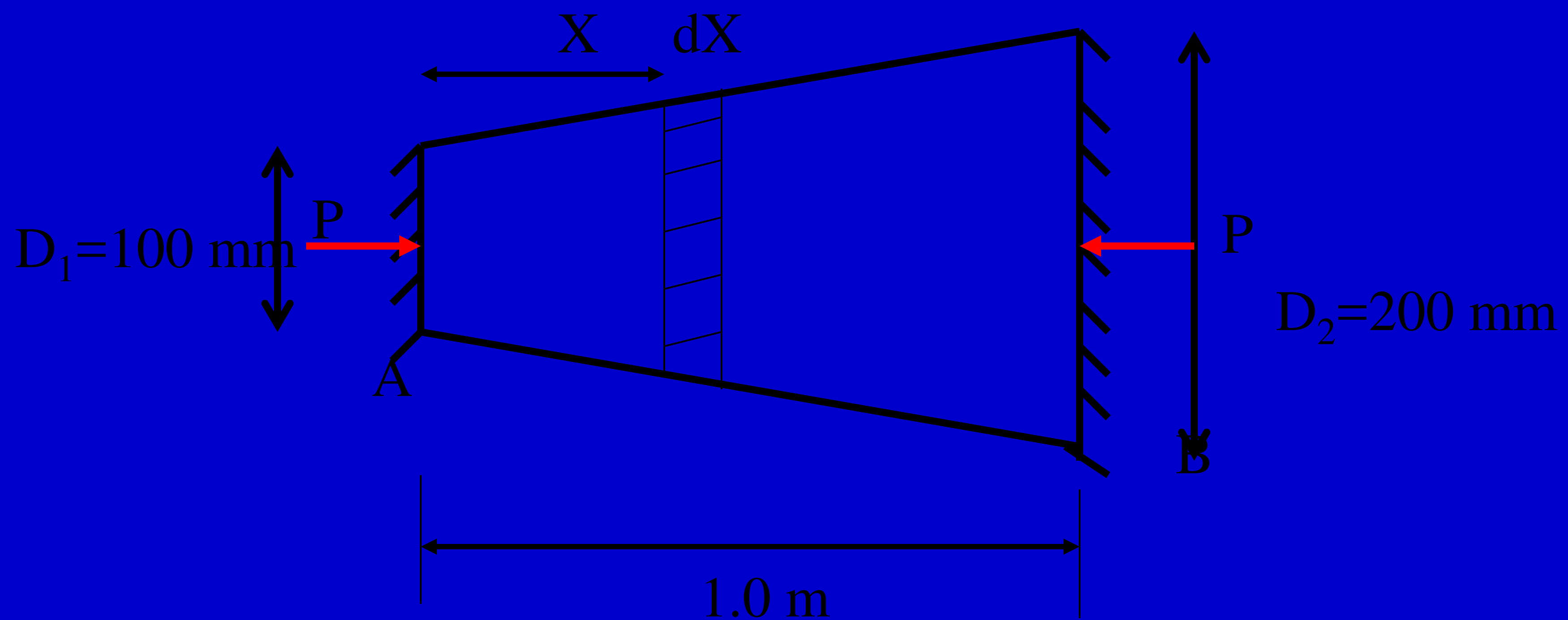
so P=40000N

so p_s=40000/400 = 100 MPa(tensile)

and p_c=40000/500=80 MPa(compressive)

Example: 15 A circular section tapered bar is rigidly fixed as shown in figure. If the temperature is raised by 30^0 C, calculate the maximum stress in the bar. Take

$$E=2*10^5\ N/mm^2\ ;\alpha=12*10^{-6}\ /^0C$$



With rise in temperature compressive force P is induced which is same at all c/s.

Free expansion = $L \alpha t = 1000*12*10^{-6}*30$

$=0.36$ mm

Force P induced will prevent a expansion of 0.36 mm

$\Delta = 4PL/(\pi E*d1*d2) = L \alpha t$

Or $P = (\pi/4)*d1*d2\ \alpha\ t\ E = 1130400$ N

Now Maximum stress = P/(least c/s area)

$=1130400/(.785*100^2) = 144$MPa

Example: 16 A composite bar made up of aluminum and steel is held between two supports.The bars are stress free at 40^0c. What will be the stresses in the bars when the temp. drops to 20^0C, if

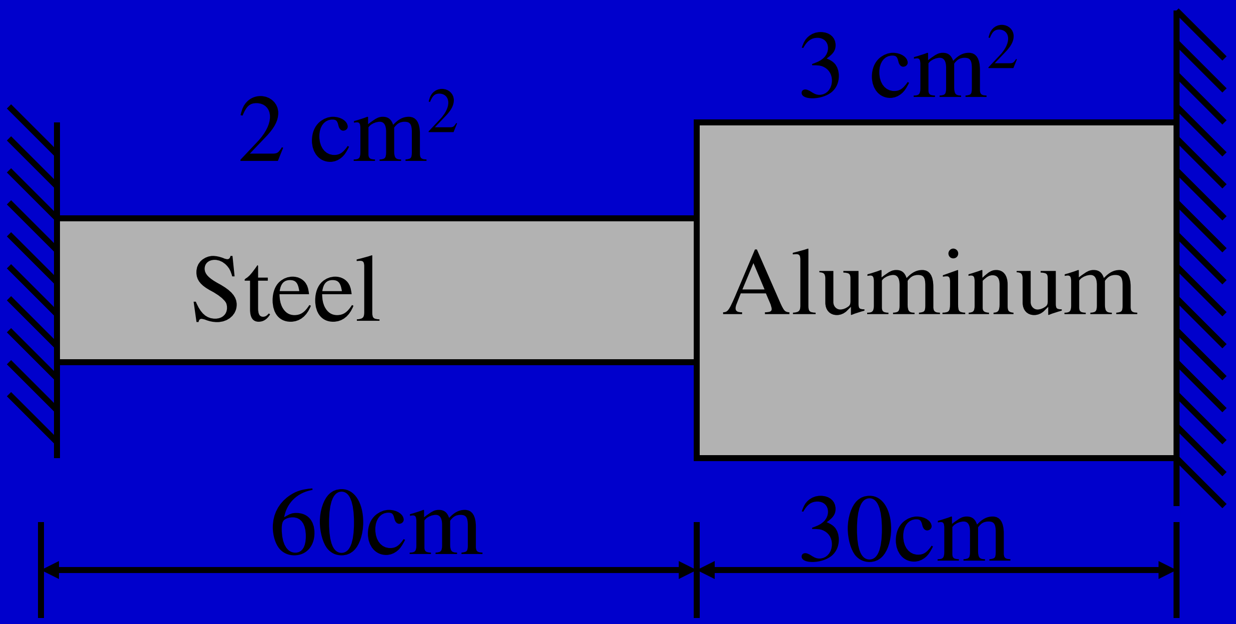
(a) the supports are unyielding

(b)the supports come nearer to each other by 0.1 mm.

Take $E_{al} = 0.7*10^5$ N/mm^2 ; $\alpha_{al} = 23.4*10^{-6}$ /^{0}C

$E_S = 2.1*10^5$ N/mm^2 $\alpha_s = 11.7*10^{-6}$ /^{0}C

$A_{al} = 3$ cm^2 $A_s = 2$ cm^2


3 cm^2
2 cm^2
Steel
Aluminum
60cm
30cm

Free contraction $\Delta = L_s \alpha_s t + L_{AL} \alpha_{Al} t$

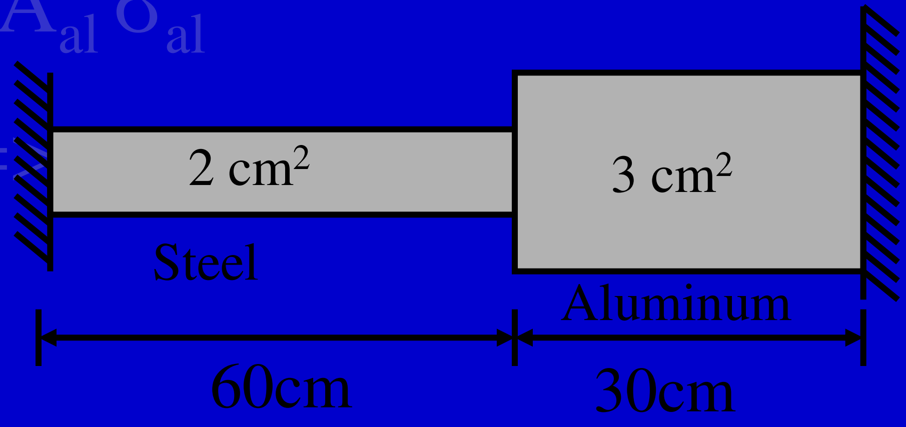
$\Delta = 600*11.7*10^{-6}*(40-20)+300*23.4*$

$10^{-6}*(40-20)=0.2808$ mm.

Since contraction is checked tensile stresses will be set up. Force being same in both

$A_s \sigma_s = A_{al} \sigma_{al}$

$2 \sigma_s = 3 \sigma_{al}$ ==



contraction of steel bar $\Delta_s^{\sigma} = (\sigma_s/E_s)*L_s$

$=[600/(2.1*10^5)]*\sigma_s$

contra.of aluminum bar $\Delta_{al}^{\sigma} = (\sigma_{al}/E_{al})*L_{al}$

$=[300/(0.7*10^5)]*\sigma_{al}$

(a) When supports are unyielding

$\Delta_s^{\sigma} + \Delta_{al}^{\sigma} = \Delta$ (free contraction)

$=[600/(2.1*10^5)]*\sigma_s + [300/(0.7*10^5)]*\sigma_{al}$

$=0.2808$ mm

$$=[600/(2.1*10^5)]*\ \sigma_s + [300/(0.7*10^5)]*\ \sigma_{al}$$

$$=0.2808;\ \text{but}$$

$$\sigma_s = 1.5\ \sigma_{al}$$

σ_{al} =32.76 N/mm^2(tensile)

σ_s =49.14 N/mm^2(tensile)

(b) Supports are yielding

$$\Delta_s^{\ \sigma} + \Delta_{al}^{\ \sigma} = (\Delta - 0.1\text{mm})$$

σ_{al} =21.09 N/mm^2(tensile)

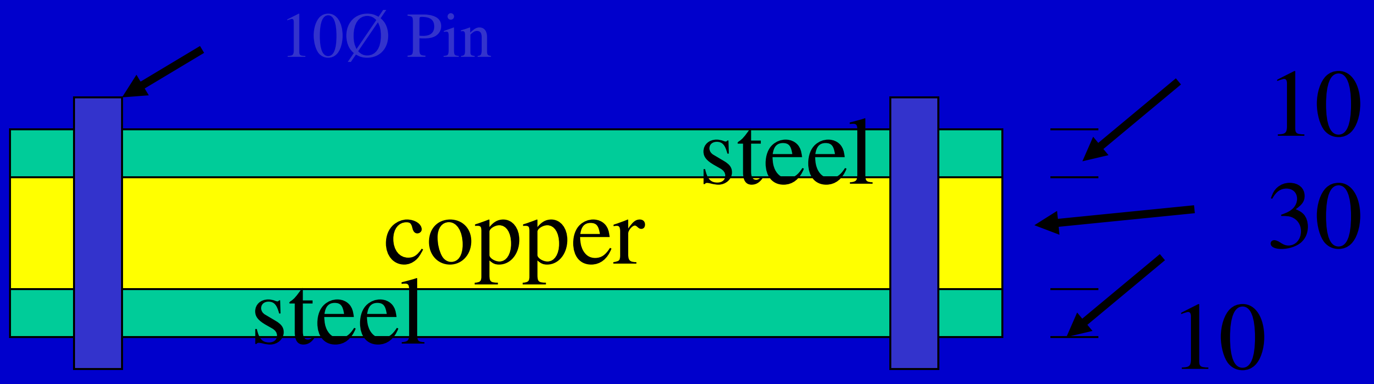
σ =31.64 N/mm^2(tensile)

Example: 17 A copper bar 30 mm dia. Is completely enclosed in a steel tube 30mm internal dia. and 50 mm external dia. A pin 10 mm in dia.,is fitted transversely to the axis of each bar near each end. To secure the bar to the tube.Calculate the intensity of shear stress induced in the pins when the temp of the whole assembly is raised by 50^0K

$Es=2 * 10^5 N/mm^2 ;\alpha_s =11*10^{-6} /^0K$

$Ec=1 * 10^5 N/mm^2 ;\alpha_c= 17*10^{-6} /^0K$

Solution



Copper bar $Ac = 0.785 * 30^2 = 706.9\ mm^2$

steel bar $As = 0.785 * (50^2 - 30^2) = 1257.1\ mm^2$

$[\sigma_s / Es] + [\sigma_c / Ec] = (\alpha_c - \alpha_s) * t$

$[\sigma_s / 2 * 10^5] + [\sigma_c / 1 * 10^5] = (17-11) * 10^{-6} * 50$

$\sigma_s + 2\sigma_c = 60$-----(1)

Since no external force is present

$$\sigma_s A_s = \sigma_c A_c$$

$$\sigma_s = \sigma_c A_c / A_s = [706.9/1257.1] * \sigma_c$$

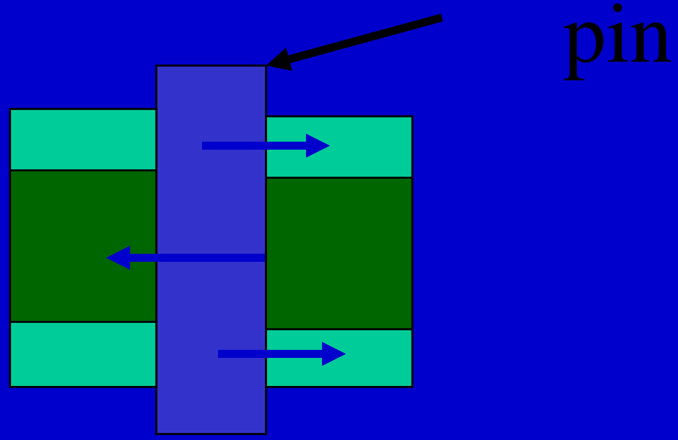
$$= 0.562\ \sigma_c \text{---}(2)$$

substituting in eq.(1)

$$\sigma_c = 23.42\ N/mm^2$$

Hence force in between copper bar &steel tube

$$= \sigma_c A_c = 23.42 * 706.9 = 16550N$$



C.S. area of pin = $0.785*10^2 = 78.54\ mm^2$

pin is in double shear

so shear stress in pin

$=16550/(2*78.54)=105.4 N/mm^2$

SHRINKING ON:

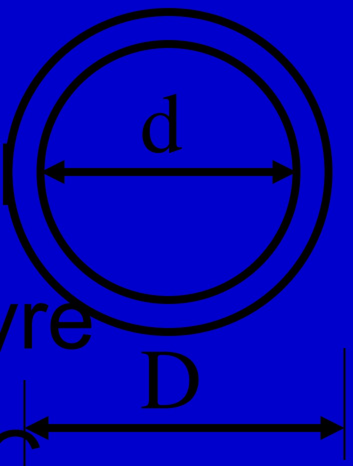
d<D

D=diameter of wheel

d = diameter of steel tyre

increase in temp = t^oC

dia increases from d--->D



- tyre slipped on to wheel, temp. allowed to fall
- Steel tyre tries to come back to its original position
- hoop stresses will be set up.

Tensile strain

$$\varepsilon = (\pi D - \pi d) / \pi d = (D-d)/d$$

so hoop stress = $\sigma = E\varepsilon$

$$\sigma = E*(D - d)/d$$

Example: 18

A thin steel tyre is to be shrunk onto a rigid wheel of 1m dia. If the hoop stress is to be limited to 100N/mm^2, calculate the internal dia. of tyre. Find also the least temp. to which the tyre must be heated above that of the wheel before it could be slipped on.

Take α for the tyre = $12*10^{-6}/^{o}C$

$E = 2.04 * 10^5 N/mm^2$

Solution:

$$\sigma = E*(D - d)/d$$

$$100 = 2.04*10^6(D - d)/d$$

or

$$(D - d)/d = 4.9*10^{-4}$$

or $D/d = (1+4.9*10^{-4})$

so $d = 0.99951D = 0.99951*1000 = 999.51$ mm

Now

$$\pi D = \pi d(1 + \alpha t)$$

or

$$\alpha t = (D/d) - 1 = (D-d)/d = 4.9*10^{-4}$$

$$t = (D-d)/d * 1/\alpha$$

$$= 4.9*10^{-4}/12*^{-6}$$

$$= 40.85\ ^{0}C$$

ELASTIC CONSTANTS:

Any direct stress produces a strain in its own direction and opposite strain in every direction at right angles to it.

Lateral strain /Longitudinal strain

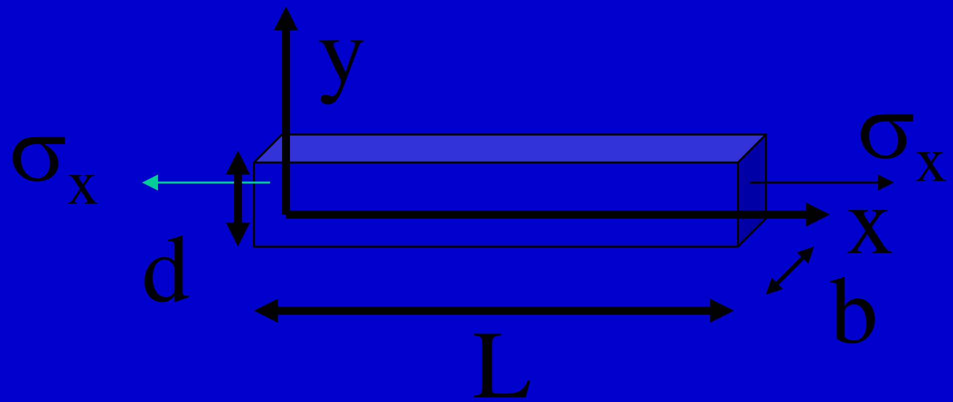
= Constant

= 1/m = μ = Poisson's ratio

Lateral strain = Poisson's ratio x Longitudinal strain

$\varepsilon_y = \mu\, \varepsilon_x$ ------------(1)

Single direct stress along longitudinal axis



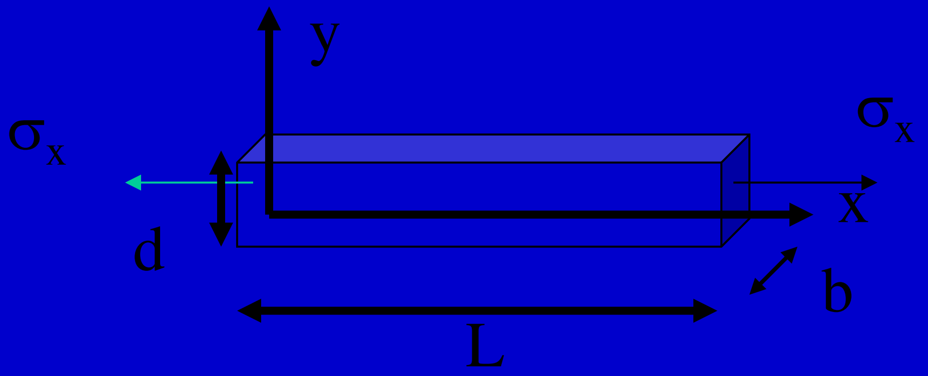
$\varepsilon_x = \sigma_x/E$ (tensile)

$\varepsilon_y = \mu\, \varepsilon_x = \mu\, [\sigma_x/E]$ (compressive)

Volume = L b d

$\delta V = bd\, \delta L - d\, L\delta b - L\, b\delta d$

$\delta V/ V = \delta L/L - \delta b/b - \delta d/d$



$$= \varepsilon_x - \varepsilon_y - \varepsilon_z = \varepsilon_x - \mu\,\varepsilon_x - \mu\varepsilon_x = \varepsilon_x - 2\mu\,\varepsilon_x = \varepsilon_x(1-2\,\mu)$$

$$= [\sigma_x/E] \times (1-2\,\mu)$$

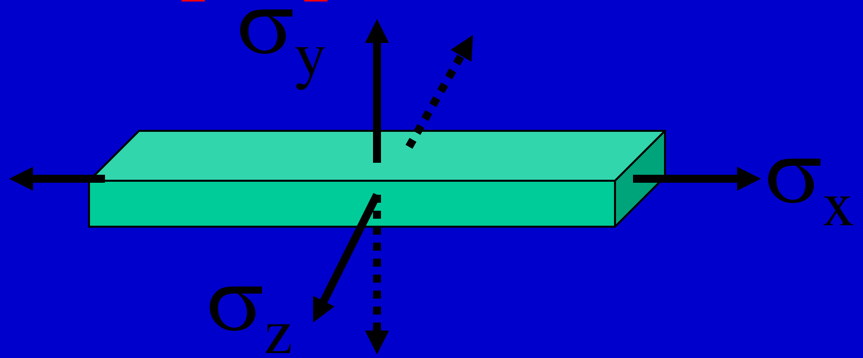
Volumetric strain= $\varepsilon_v = [\sigma_x/E] \times (1-2\,\mu)$ _

-----(2)

or $\varepsilon_v = [\sigma_x/E] \times (1-2/m)$

$$\varepsilon_v = [\sigma_x/E] \times (1-2/m)$$

Stress σ_x along the axis and σ_y and σ_z perpendicular to it.



$$\varepsilon_x = \sigma_x/E - \sigma_y/mE - \sigma_z/mE\text{------(i)}$$

-------(3)

$$\varepsilon_y = \sigma_y/E - \sigma_z/mE - \sigma_x/mE\text{----}$$

Note:- If some of the stresses have opposite sign necessary changes in algebraic signs of the

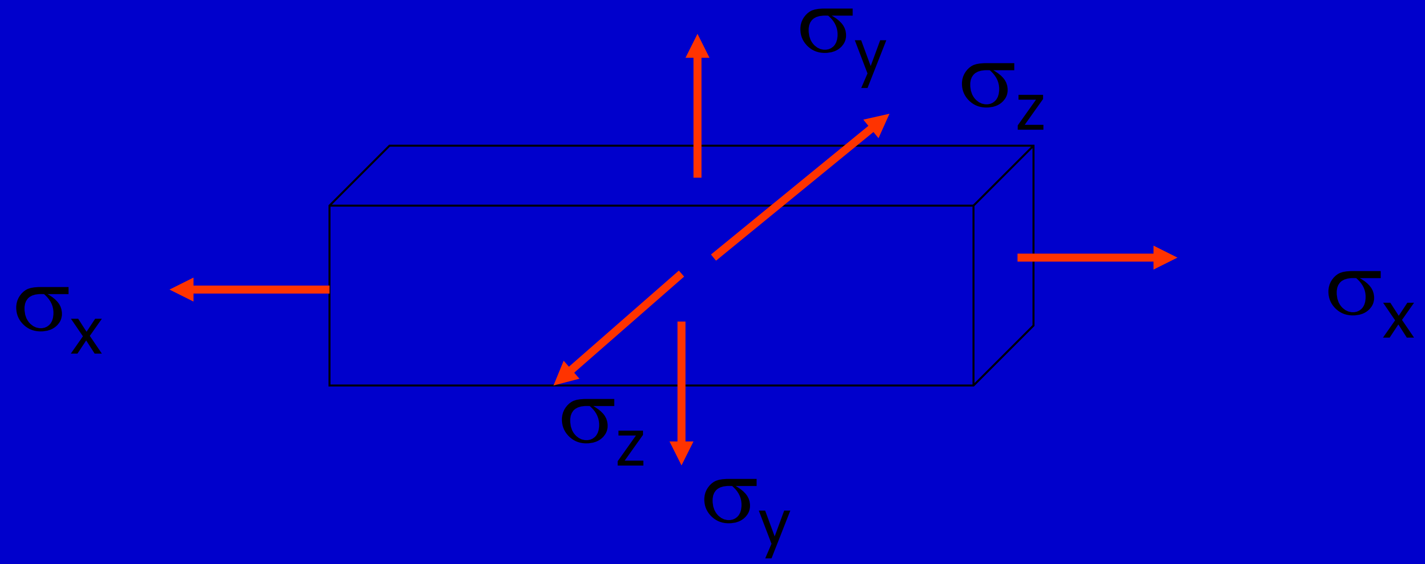
Upper limit of Poisson's Ratio:

adding (i),(ii) and (iii)

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = (1 - 2/m)(\sigma_x + \sigma_y + \sigma_z)/ E_- \quad \text{--------(4)}$$

known as DILATATION

For small strains represents the change in volume /unit volume.



	ε_x	ε_y	ε_z
σ_x	σ_x/E	$-\mu\ \sigma_x/E$	$-\mu\ \sigma_x/E$
σ_y	$-\mu\ \sigma_y/E$	σ_y/E	$-\mu\ \sigma_y/E$
σ_z	$-\mu\ \sigma_z/E$	$-\mu\ \sigma_z/E$	σ_z/E

Sum all

Example: 19

A steel bar of size 20 mm x 10mm is subjected to a pull of 20 kN in direction of its length. Find the length of sides of the C.S. and decrease in C.S. area. Take E=2 x 10^{5} N/mm^2 and m=10/3.

$\varepsilon_x = \sigma_x/E = (P/A_x) \times (1/E)$

$= (20000/(20 \times 10)) \times 1/(2 \times 10^5) = 5 \times 10^{-4}$ (T)

Lateral Strain $= \varepsilon_y = -\mu\, \varepsilon_x = -\varepsilon_x/m = -1.5 \times 10^{-4}$ (C)

side decreased by $20 \times 1.5 \times 10^{-4} = 0.0030$mm

side decreased by $10 \times 1.5 \times 10^{-4} = 0.0015$mm

new C.S$=(20-0.003)(10-.0015)=199.94$mm^2

Example: 20

A steel bar 200x20x20 mm C.S. is subjected to a tensile force of 40000N in the direction of its length. Calculate the change in volume.

Take 1/m =0.3 and E = 2.05 $*10^5$ MPa.

Solution:

$$\varepsilon_x = \sigma_x/E = (P/A) \times (1/E)$$

$$=40000/20*20*2.05*10^5 = 4.88*10^{-4}$$

$$\varepsilon_y = \varepsilon_z = -(1/m)*\varepsilon_x = -0.3*4.88*10^{-4}$$

Change in volume:

$$\delta V / V = \varepsilon_x + \varepsilon_y + \varepsilon_z = (4.88 - 2*1.464)*10^{-4}$$

$$= 1.952 * 10^{-4}$$

$$V = 200*20*20 = 80000 \text{ mm}^3$$

$$\delta V = 1.952*10^{-4}*80000 = 15.62 \text{ mm}^3$$

YOUNG'S MODULUS (E):--

Young's Modulus (E) is defined as the Ratio of Stress (σ) to strain (ε).

$$E = \sigma / \varepsilon \qquad \text{-------------(5)}$$

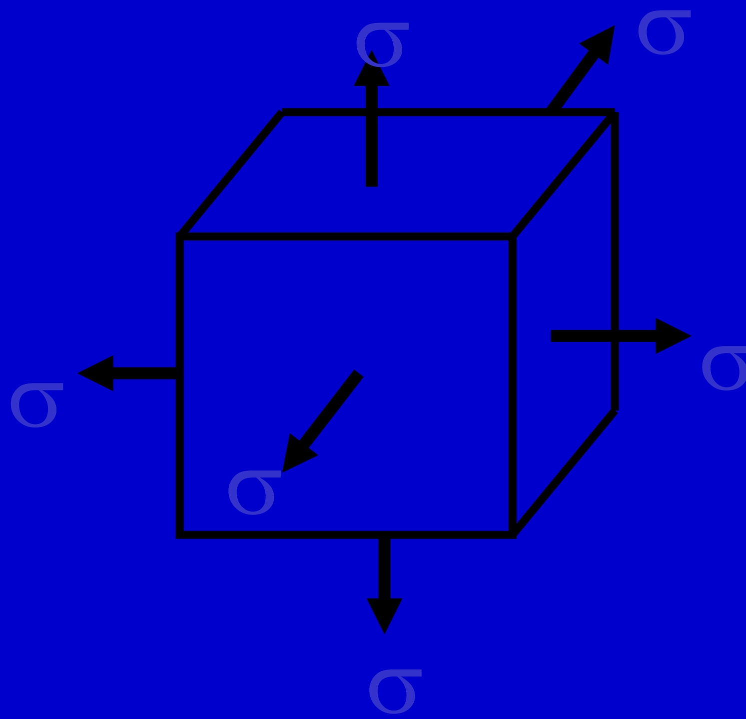
BULK MODULUS (K):--

- When a body is subjected to the identical stress σ in three mutually perpendicular directions, the body undergoes uniform changes in three directions without the distortion of the shape.

- The ratio of change in volume to original volume has been defined as volumetric strain(ε_v)

- Then the bulk modulus, K is defined as $K = \sigma / \varepsilon_v$

BULK MODULUS (K):--



$$K = \sigma / \varepsilon_v \quad \text{-------------(6)}$$

Where, $\varepsilon_v = \Delta V/V$

$$= \frac{\text{Change in volume}}{\text{Original volume}}$$

= Volumetric Strain

MODULUS OF RIGIDITY (N): OR MODULUS OF TRANSVERSE ELASTICITY OR SHEARING MODULUS

Up to the elastic limit,

shear stress (τ) $\propto$ shearing strain(ϕ)

$$\tau = N\,\phi$$

Expresses relation between shear stress and shear strain.

where

Modulus of Rigidity = $N = \tau / \phi$ ----------------(7)

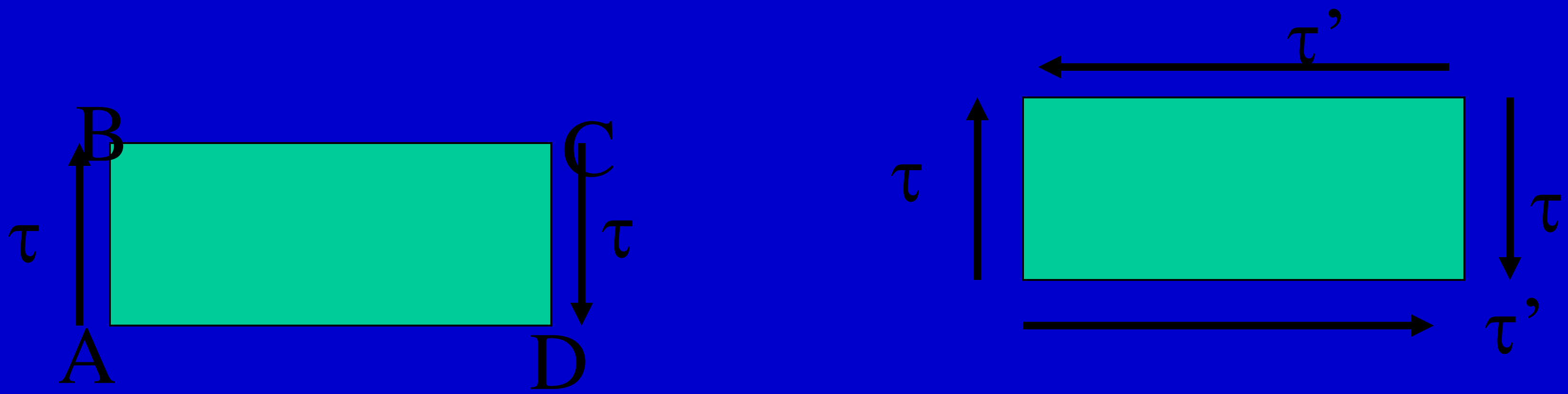
ELASTIC CONSTANTS

YOUNG'S MODULUS $E = \sigma / \varepsilon$ -------------(5)

BULK MODULUS $K = \sigma / \varepsilon_v$ -------------(6)

MODULUS OF RIGIDITY $N = \tau / \phi$ -------------(7)

COMPLEMENTRY STRESSES:"A stress in a given direction cannot exist without a balancing shear stress of equal intensity in a direction at right angles to it."



Moment of given couple=Force *Lever arm

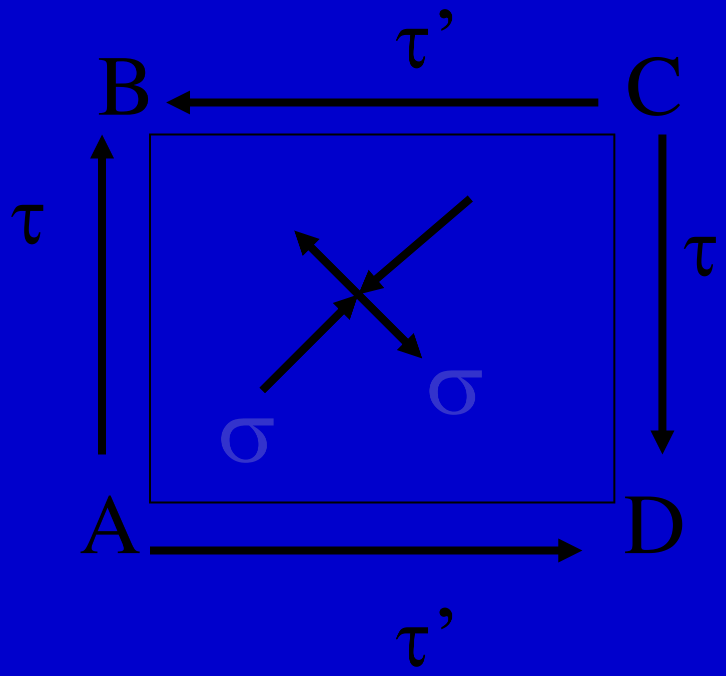
$= (\tau.AB)*AD$

Moment of balancing couple$= (\tau'.AD)*AB$

so $(\tau.AB)*AD=(\tau'.AD)*AB \Rightarrow \tau= \tau'$

Where τ=shear stress & τ'=Complementary shear stress

State of simple shear: Here no other stress is acting - only simple shear.



Let side of square = b

length of diagonal AC = $\sqrt{2}\,.b$

consider unit thickness perpendicular to block.

Equilibrium of piece ABC

the resolved sum of τ perpendicular to the diagonal = $2*(\tau*b*1)\cos 45^0 = \sqrt{2}\ \tau.b$

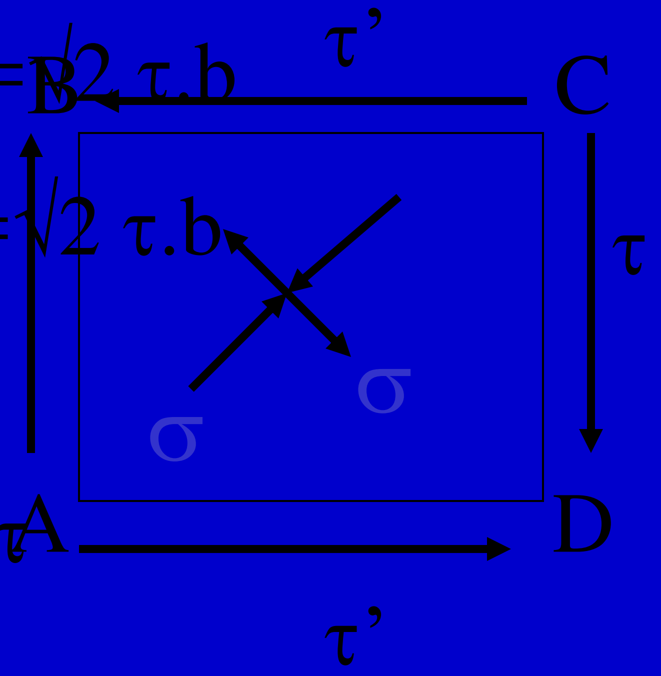
if σ is the tensile stress so produced on the diagonal

$$\sigma(AC*1)=\sqrt{2}\ \tau.b$$

$$\sigma(\sqrt{2}\ .b)=\sqrt{2}\ \tau.b$$

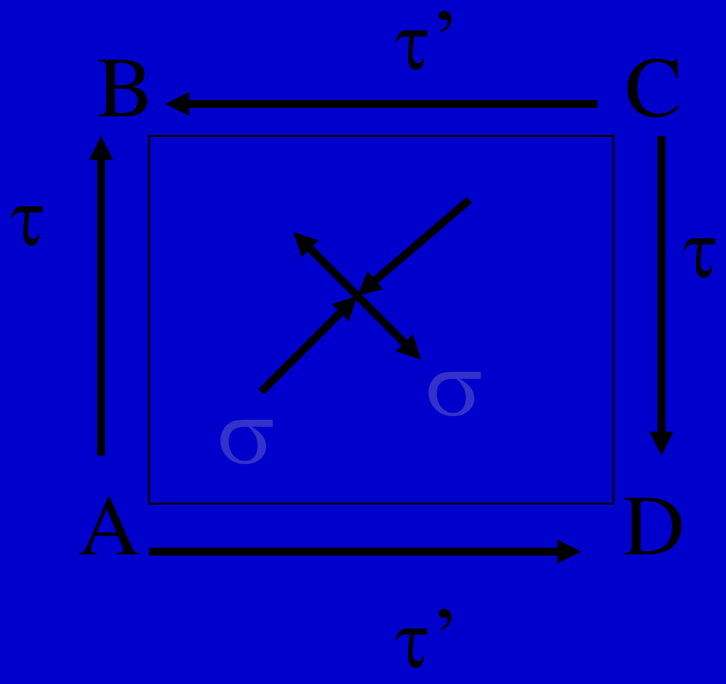
so

$$\sigma=\tau$$

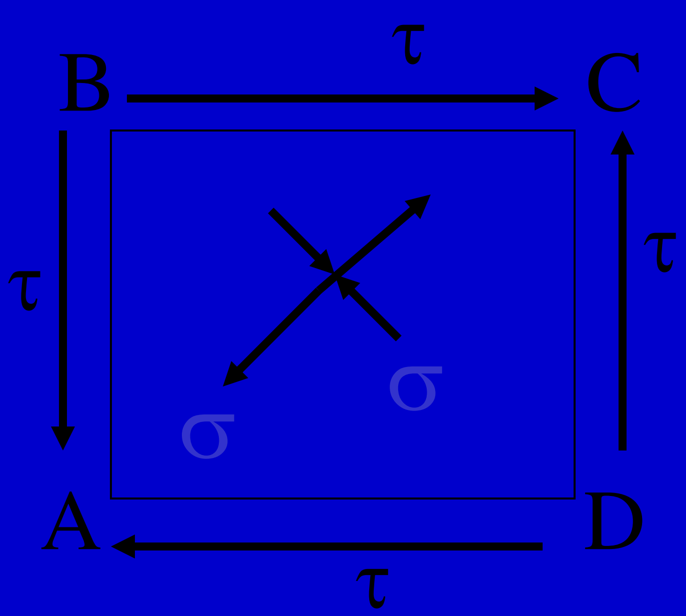


Similarly the intensity of compressive stress on plane BD is numerically equal to τ.

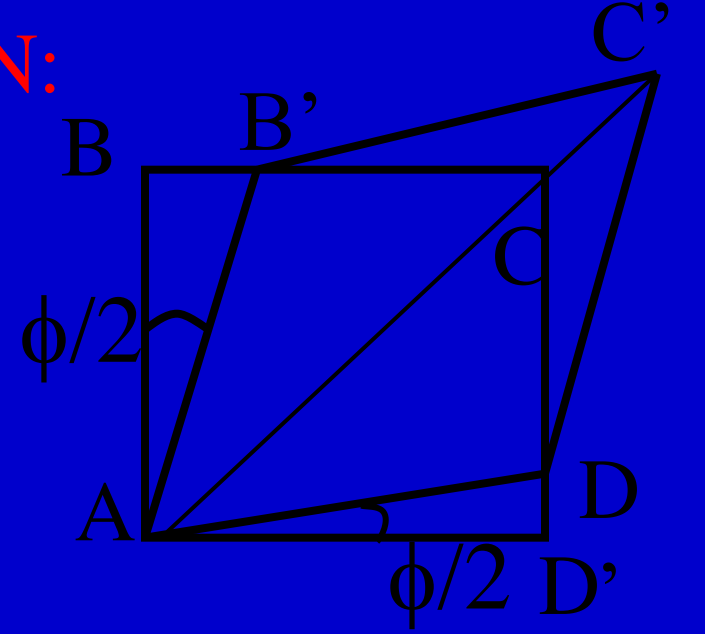
"Hence a state of simple shear produces pure tensile and compressive stresses across planes inclined at 45 ^{0} to those of pure shear, and intensities of these direct stresses are each equal to pure shear stress."



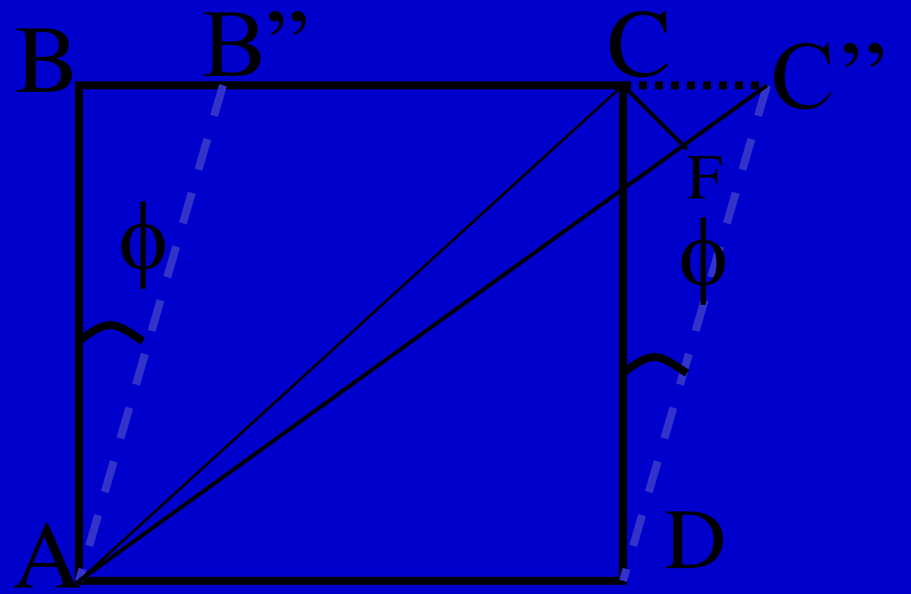
SHEAR STRAIN:



Total change in corner angles +/- ϕ



State of simple Shear on Block



Distortion with side AD fixed

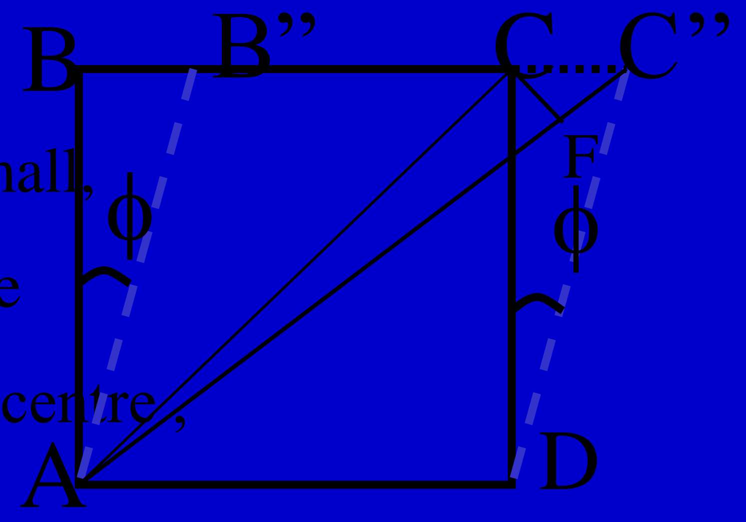
Since

ϕ is extremely small,

we can assume

BB" = arc with A as centre ,

AB as radius.



So, ϕ=BB"/AB=CC"/CD

Elongation of diagonal AC can be nearly taken as FC".

Linear strain of diagonal = FC"/AC

= CC"cos 45/CDsec45

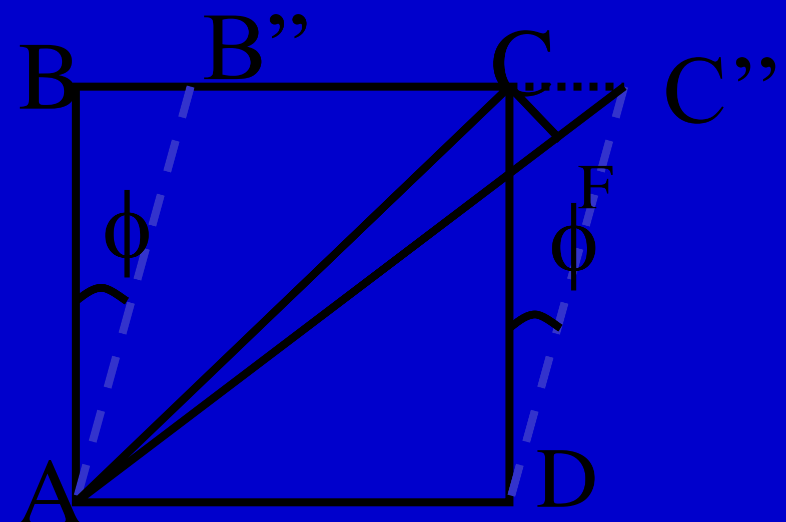
$$\varepsilon = CC''/2CD = (1/2)\,\phi$$

but $\phi = \tau / N$ (we know $N = \tau / \phi$)

so

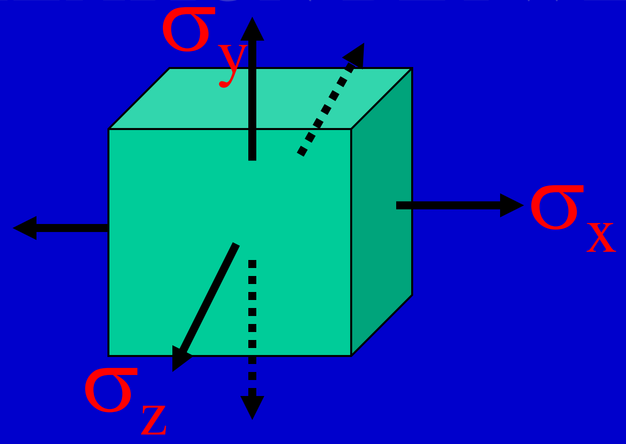
$$\varepsilon = \tau / 2N \quad \text{------(8)}$$

Linear strain ‘ε’ is half the shear strain ‘ϕ’.



RELATION BETWEEN ELASTIC CONSTANTS

(A) RELATION BETWEEN E and K



Let a cube having a side L be subjected to three mutually perpendicular stresses of intensity σ

By definition of bulk modulus

$$K = \sigma / \varepsilon_v$$

Now $\varepsilon_v = \delta_v / V = \sigma / K$ ------------------------------(i)

The total linear strain for each side

$$\varepsilon = \sigma/E - \sigma/(mE) - \sigma/(mE)$$

so $\delta L / L = \varepsilon = (\sigma/E) * (1-2/m)$ ---------------(ii)

now $V = L^3$

$$\delta V = 3 L^2 \delta L$$

$$\delta V/V = 3 L^2 \delta L / L^3 = 3 \delta L/L$$

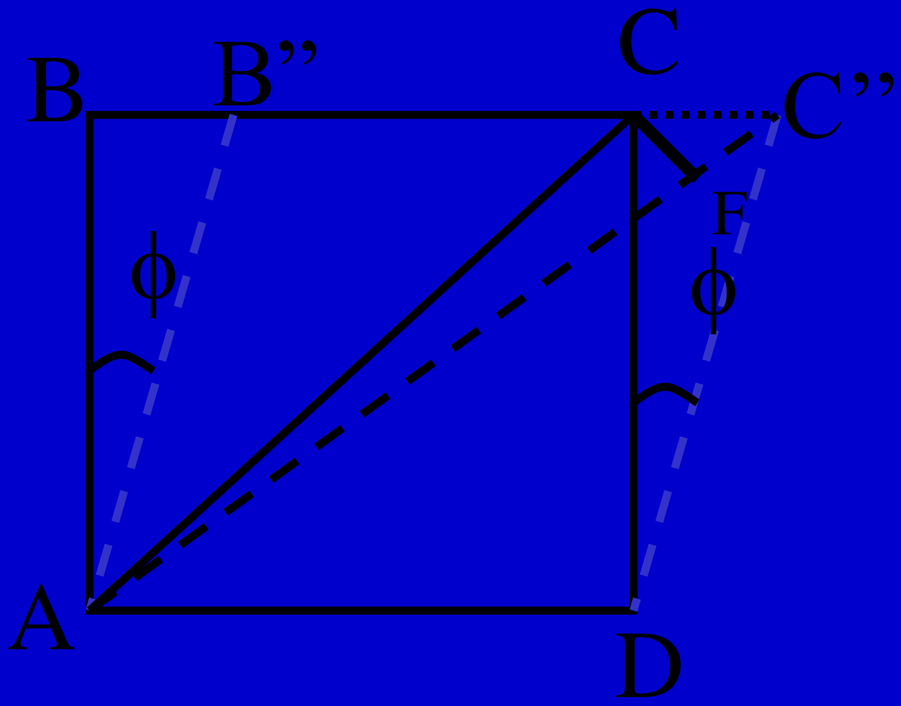
$= 3 (\sigma/E) * (1-2/m)$ -------------------(iii)

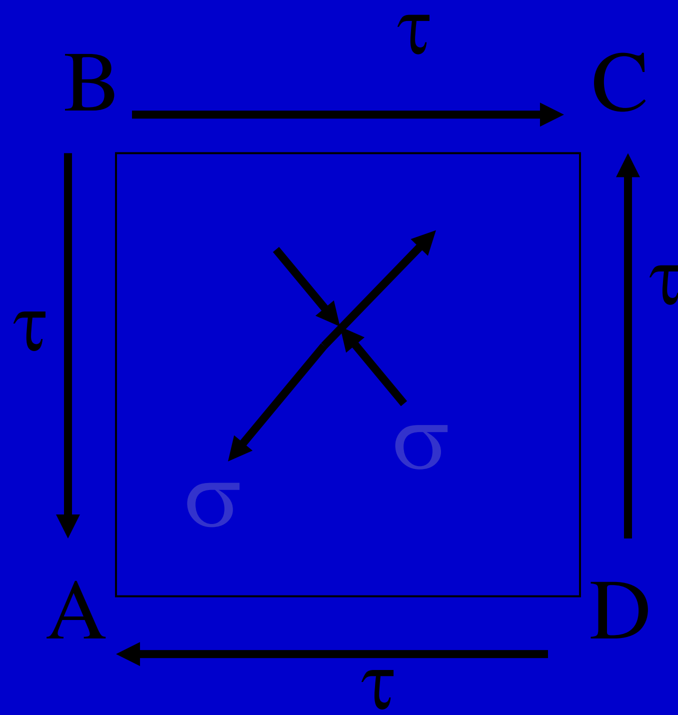
Equating (i) and (iii)

$\sigma/K = 3(\sigma/E)(1-2/m)$

$$E = 3K(1-2/m) \quad \text{-----(9)}$$

(B) Relation between E and N





Linear strain of diagonal AC,

$\varepsilon = \phi/2 = \tau/2N$ ---------------------------------(i)

State of simple shear produces tensile and compressive stresses along diagonal planes and

$$\sigma = \tau$$

Strain ε of diagonal AC, due to these two mutually perpendicular direct stresses

$$\varepsilon = \sigma/E - (-\sigma/mE) = (\sigma/E)*(1+1/m) \quad ---(ii)$$

But $\sigma = \tau$

so $\varepsilon = (\tau /E)*(1+1/m)$ -------------------(iii)

From equation (i) and (iii)

$$\tau /2N = (\tau /E)(1+1/m)$$

OR

$$E = 2N(1+1/m) \text{--------}(10)$$

$$\text{But } E = 3K(1-2/m) \text{-------}(9)$$

Eliminating E from --(9) & --(10)

$$\mu = 1/m = (3K - 2N) / (6K + 2N) \text{------}(11)$$

Eliminating m from –(9) & --(10)

$$E = 9KN / (N+3K) \qquad \text{----------}(12)$$

(C) Relation between E ,K and N:--

$$E = 2N(1+1/m) \quad \text{--------}(10)$$

$$E = 3K\,(1-2/m) \quad \text{---------}(9)$$

$$E = 9KN / (N+3K) \quad \text{--------}(12)$$

(D) Relation between μ,K and N:--

$$\mu = 1/m = (3K-2N)/(6K+2N) \text{------}(11)$$

Example: 21

(a) Determine the % change in volume of a steel bar of size 50 x 50 mm and 1 m long, when subjected to an axial compressive load of 20 kN.

(b) What change in volume would a 100 mm cube of steel suffer at a depth of 5 km in sea water?

Take $E = 2.05 \times 10^{5}\ N/mm^2$ and

$N = 0.82 \times 10^{5}\ N/mm^2$

Solution: (a)

$\delta V/V = \varepsilon_v = (\sigma/E)(1\text{-}2\ /m)$

$[\sigma = P/A = 20000/50 \text{ x } 50 = 8 \text{ kN/cm2}]$

so now

$\delta V/V = -\ (8\ /\ 2.05 \text{ x } 10^{5})(1 - 2/m)$

$= -3.902 * 10^{-5}(1 - 2/m)$------------------------(i)

Also $E = 2N(1+1/m)$ -------------------------(10)

$(1 + 1/m) = E/2N = 2.05 \text{ x } 10^{5} / (2 * 0.82 \text{ x } 10^{5})$

so $1/m = 0.25$

Substituting in ----(i)

$\delta V/V = -3.902*10^{-5}(1-2(0.25))=-1.951*10^{-5}$

Change in volume$=-1.951*10^{-5}*1000*50*50$

$\delta V = 48.775\ mm^2$

% Change in volume=(48.775/ 50*50*1000)*100

=0.001951 %

Solution:(b)

Pressure in water at any depth 'h' is given by

$p=wh$ taking $w= 10080 N/m^3$ for sea water

and $h = 5km=5000m$

$p=10080*5000=50.4 *10^6 N/m^2 = 50.4 N/mm^2$

$E = 3K(1-2/m)$

We have 1/m =0.25

so E = 3K(1-0.5) or K=E/1.5 = 2/3(E)

$K=2/3 * 2.05* 10^5 = 1.365 * 10^5 = N/mm^2$

now by definition of bulk modulus

$K = \sigma/\varepsilon_v$ or $\varepsilon_v = \sigma/K$

but $\varepsilon_v = \delta V/V$

$\delta V/V = \sigma/K$

$\delta V = 50.4 / 1.365 * 10^5 * 100^3 = 369.23\ mm^3$

Example: 22 A bar 30 mm in diameter was subjected to tensile load of 54 kN and measured extension of 300 mm gauge length was 0.112 mm and change in diameter was 0.00366 mm. Calculate Poisson's Ratio and the value of three moduli.

Solution:

$$\text{Stress} = 54 * 10^3/(\pi/4 * d^2) = 76.43\ N/mm^2$$

$$\varepsilon = \text{Linear strain} = \delta L/L = 0.112/300$$

$$= 3.733 * 10^{-4}$$

$E = stress/strain = 76.43/3.733 * 10^{-4}$

$= 204741\ N/mm^2 = 204.7\ kN/mm^2$

Lateral strain= $\delta d/d = 0.00366/30 = 1.22*10^{-4}$

But lateral strain $= 1/m * \varepsilon$

so $1.22*10^{-4} = 1/m * 3.733*10^{-4}$

so $1/m = 0.326$

$E = 2N(1+1/m)$ or $N = E/[2*(1+1/m)]$

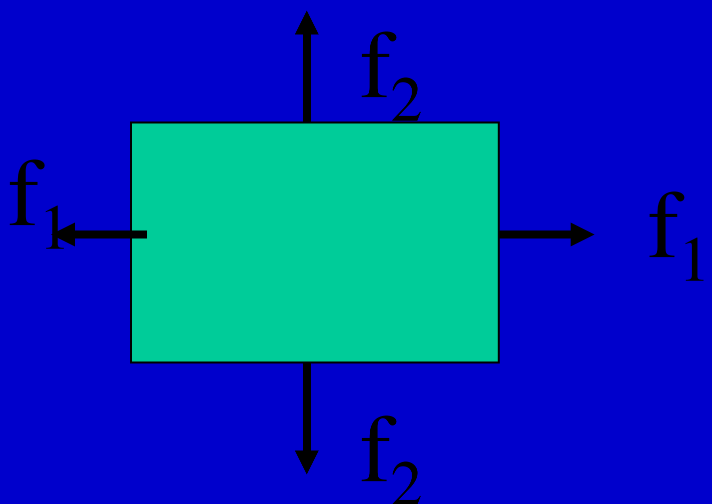
so $N = 204.7/[2*(1+0.326)] = 77.2\ kN/mm^2$

$$E = 3K*(1-2/m)$$

$$\text{so } K=E/[3*(1-2/m)]=204.7/[3*(1-2*0.326)]$$

$$K=196\,kN/mm^2$$

Example: 23 Tensile stresses f1 and f2 act at right angles to one another on a element of isotropic elastic material. The strain in the direction of f1 is twice the direction of f2. If E for the material is 120 kN/mm3, find the ratio of f1:f2. Take 1/m=0.3



$$\varepsilon_1 = 2\,\varepsilon_2$$

$$\text{So}, f_1/E - f_2/mE = 2(f_2/E - f_1/mE)$$

$$f_1/E + 2f_1/mE = 2f_2/E + f_2/mE$$

So

$$(f_1/E)(1+2/m) =(f_2/E)(2+1/m)$$

$$f_1(1+2*0.3) =f_2(2+0.3)$$

$$1.6f_1=2.3f_2$$

So $f_1:f_2 = 1:1.4375$

Example: 24 A rectangular block 250 mmx100 mmx80mm is subjected to axial loads as follows.

480 kN (tensile in direction of its length)

900 kN (tensile on 250mm x 80 mm faces)

1000kN (comp. on 250mm x100mm faces)

taking E=200 GN/m2 and 1/m=0.25 find

(1) Change in volume of the block

(2) Values of N and K for material of the block.

$$\sigma_x = 480x10^3/(0.1*0.08) = 60 *10^6 N/m^2 \text{ (tens.)}$$

$$\sigma_y = 1000x10^3/(0.25*0.1) = 40*10^6 N/m^2 \text{(comp)}$$

$$\sigma_z = 900x10^3/(0.25*0.08) = 45*10^6 N/m^2 \text{(tens.)}$$

$$\varepsilon_x = (60 *10^6/E)+(0.25* 40*10^6/E)$$

$$- (0.25* 45*10^6/E) = (58.75* 10^6/E)$$

$$\varepsilon_y = -(40 *10^6/E)-(0.25* 45*10^6/E)$$

$$- (0.25* 60*10^6/E) = (- 66.25* 10^6/E)$$

$$\varepsilon_z = (45 *10^6/E)-(0.25* 60*10^6/E)$$

$$+ (0.25* 40*10^6/E) = (40* 10^6/E)$$

$$\text{Volumetric strain} = \varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$=(58.75 * 10^6/E) - (66.25 * 10^6/E) + (40 * 10^6/E)$$

$$=32.5*10^6/E$$

$$\varepsilon_v = \delta V/V$$

$$\text{so } \delta V = \varepsilon_v V$$

$$=32.5*10^6*[(0.25*0.10*0.08)/(200*10^9)]*10^9$$

$$=325 \text{ mm}^3 \text{(increase)}$$

Modulus of Rigidity

$$E = 2N(1+1/m)$$

so

$$N=E/[2*(1+1/m)]=200/[2(1+0.25)]=80GN/m^2$$

Bulk Modulus:

$$E = 3K(1-2/m)$$

so $K=E/[3(1-2/m)]=200/[3(1-2*0.25)=133.33\ GN/m^2$

Example: 25 For a given material $E=110GN/m^2$ and $N=42\ GN/M^2$. Find the bulk modulus and lateral contraction of a round bar of 37.5 mm diameter and 2.4 m long when stretched by 2.5 mm.

Solution:

$$E=2N(1+1/m)$$

$$110*10^9=2*42*10^9(1+1/m)$$

gives $1/m=0.32$

Now $E = 3K(1-2/m)$

$110 \times 10^9=3K(1-2*0.31)$

gives $K=96.77\ GN/m^2$

Longitudinal strain = $\delta L/L=0.0025/2.4=0.00104$

Lateral strain=.00104*1/m=0.00104*0.31

=0.000323

Lateral Contraction=0.000323*37.5=0.0121mm

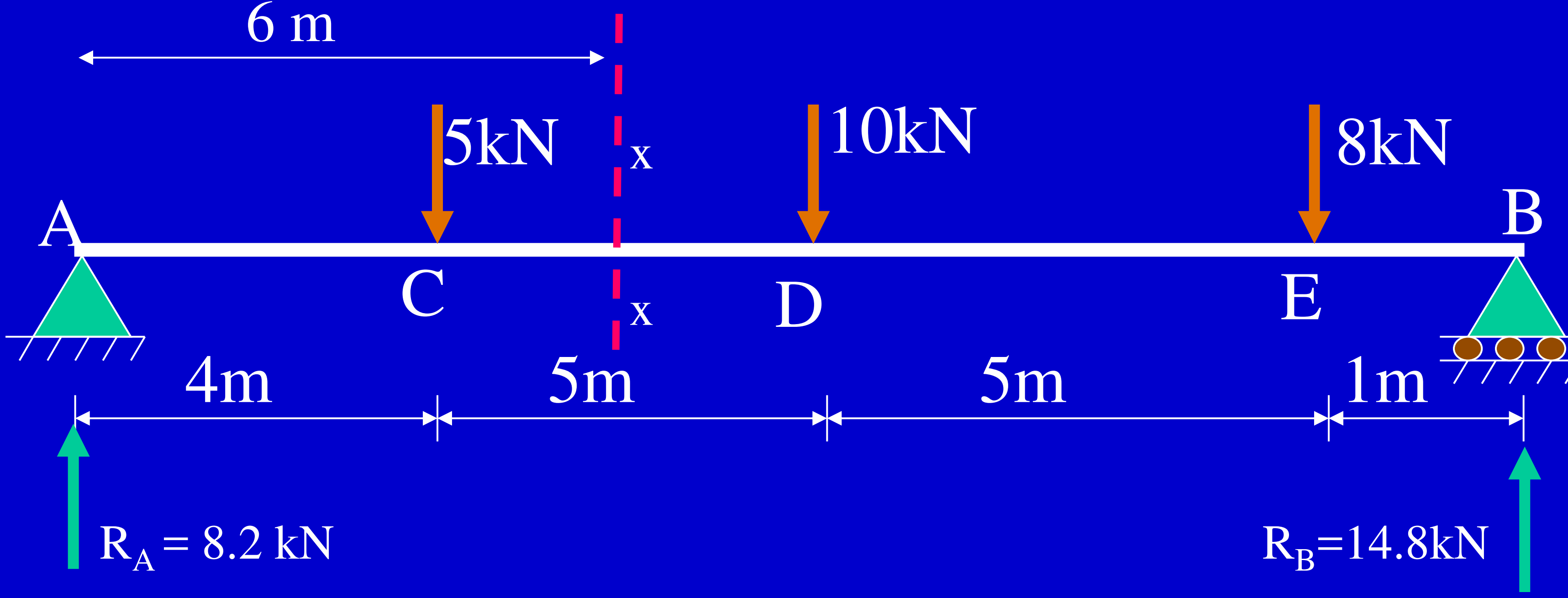
UNIT-II

Shear Force and Bending Moment Diagrams

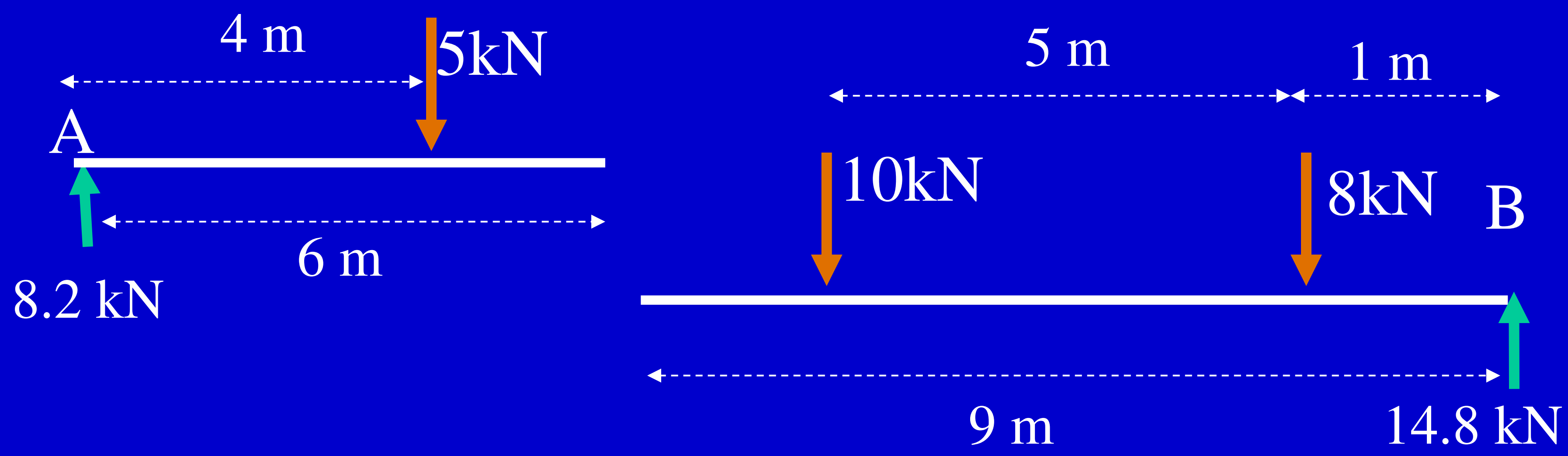
[SFD & BMD]

Shear Force and Bending Moments

Consider a section x-x at a distance 6m from left hand support A



Imagine the beam is cut into two pieces at section x-x and is separated, as shown in figure



To find the forces experienced by the section, consider any one portion of the beam. Taking left hand portion

Transverse force experienced = 8.2 – 5 = 3.2 kN (upward)

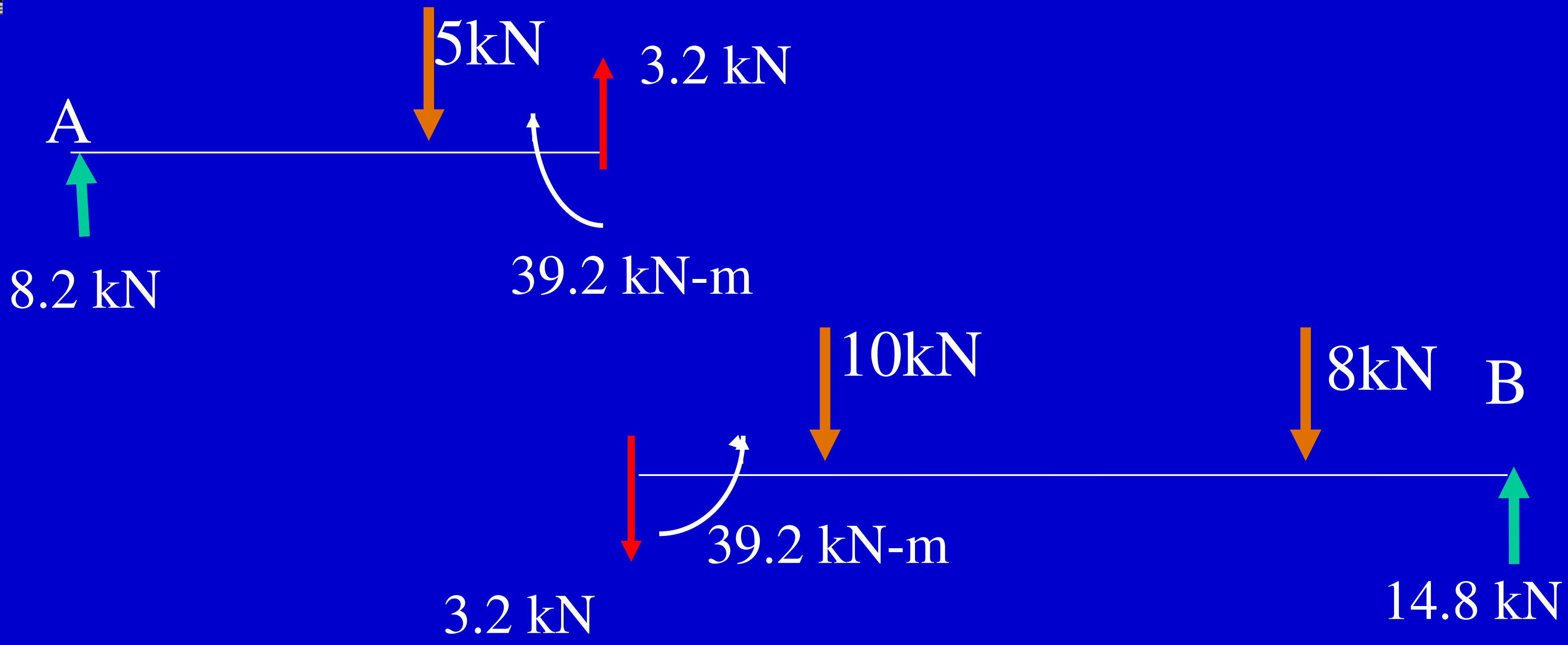
Moment experienced = 8.2 × 6 – 5 × 2 = 39.2 kN-m (clockwise)

If we consider the right hand portion, we get

Transverse force experienced = 14.8 – 10 – 8 =-3.2 kN = 3.2 kN (downward)

Moment experienced = - 14.8 × 9 +8 × 8 + 10 × 3 = -39.2 kN-m = 39.2 kN-m

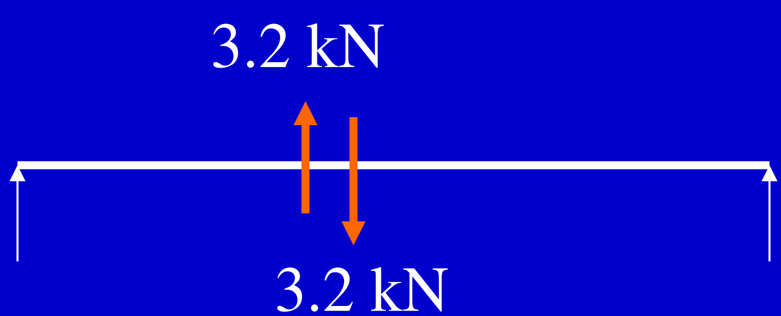
(anticlockwise)

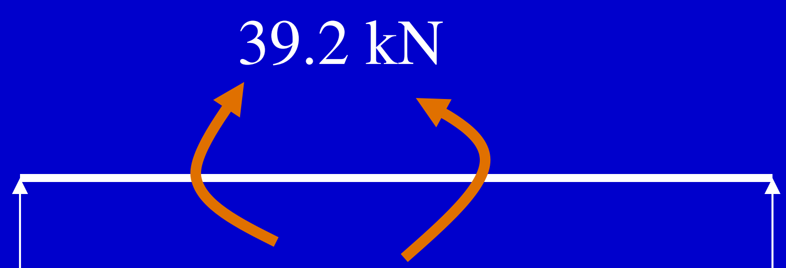


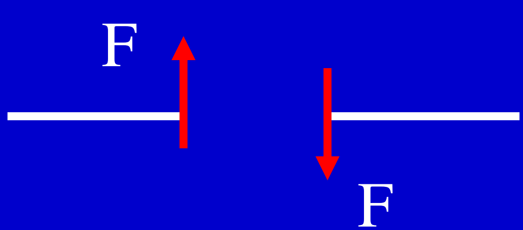
Thus the section x-x considered is subjected to forces 3.2 kN and moment 39.2 kN-m as shown in figure. The force is trying to shear off the section and hence is called shear force. The moment bends the section and hence, called bending moment.

<u>Shear force at a section</u>: The algebraic sum of the vertical forces acting on the beam either to the left or right of the section is known as the *shear force at a section.*

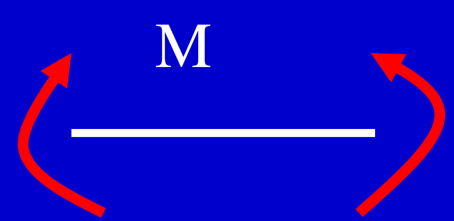
<u>Bending moment (BM) at section</u>: The algebraic sum of the moments of all forces acting on the beam either to the left or right of the section is known as the *bending moment at a section*







Shear force at x-x



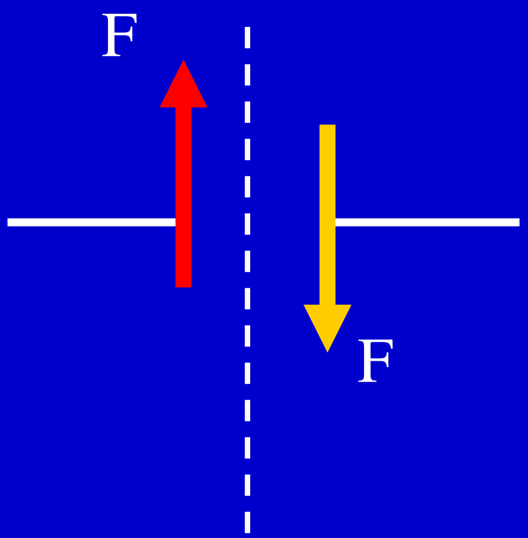
Bending moment at x-x

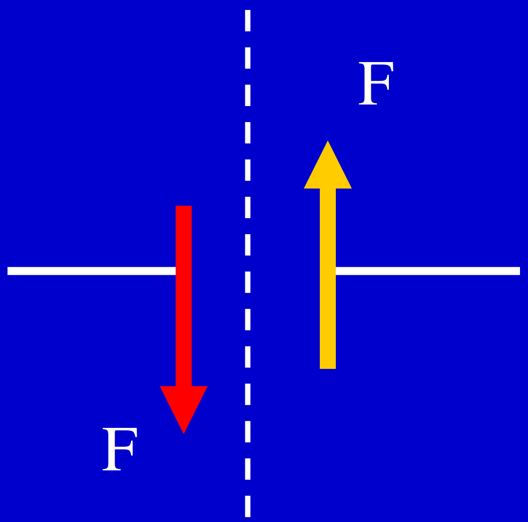
Moment and Bending moment

Moment: It is the product of force and perpendicular distance between line of action of the force and the point about which moment is required to be calculated.

Bending Moment (BM): The moment which causes the bending effect on the beam is called *Bending Moment*. It is generally denoted by 'M' or 'BM'.

Sign Convention for shear force





+ ve shear force

- ve shear force

Sign convention for bending moments:

The bending moment is considered as Sagging Bending Moment if it tends to bend the beam to a curvature having convexity at the bottom as shown in the Fig. given below. Sagging Bending Moment is considered as positive bending moment.

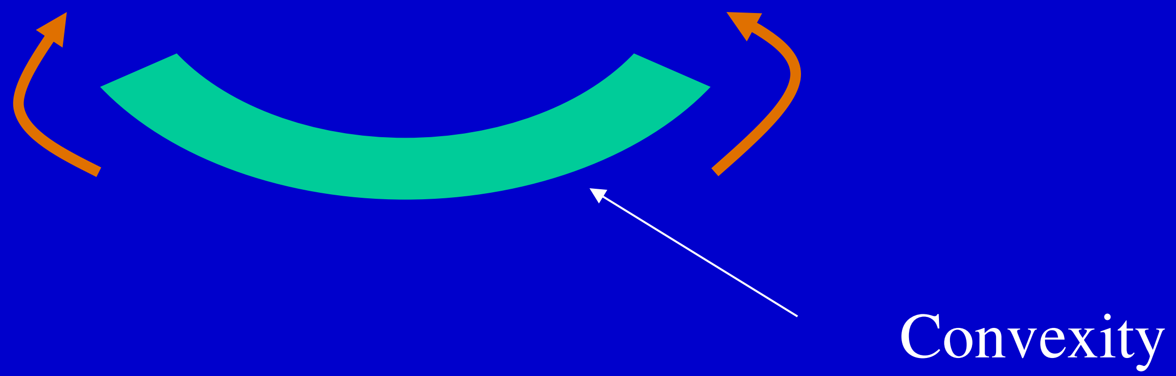


Fig. Sagging bending moment [Positive bending moment]

Sign convention for bending moments:

Similarly the bending moment is considered as hogging bending moment if it tends to bend the beam to a curvature having convexity at the top as shown in the Fig. given below. Hogging Bending Moment is considered as Negative Bending Moment.

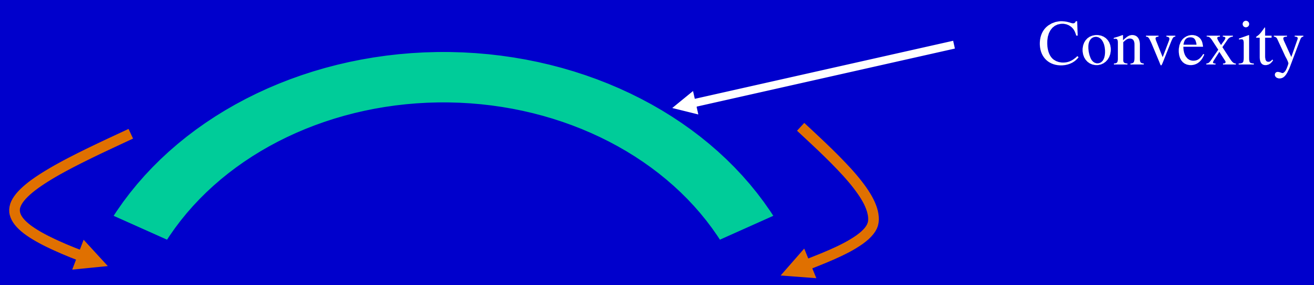


Fig. Hogging bending moment [Negative bending moment]

Shear Force and Bending Moment Diagrams (SFD & BMD)

Shear Force Diagram (SFD):

The diagram which shows the variation of shear force along the length of the beam is called ***Shear Force Diagram (SFD).***

Bending Moment Diagram (BMD):

The diagram which shows the variation of bending moment along the length of the beam is called ***Bending Moment Diagram (BMD).***

Point of Contra flexure [Inflection point]:

It is the point on the bending moment diagram where bending moment changes the sign from positive to negative or vice versa.

It is also called ‘Inflection point’. At the point of inflection point or contra flexure the bending moment is zero.

Relationship between load, shear force and bending moment

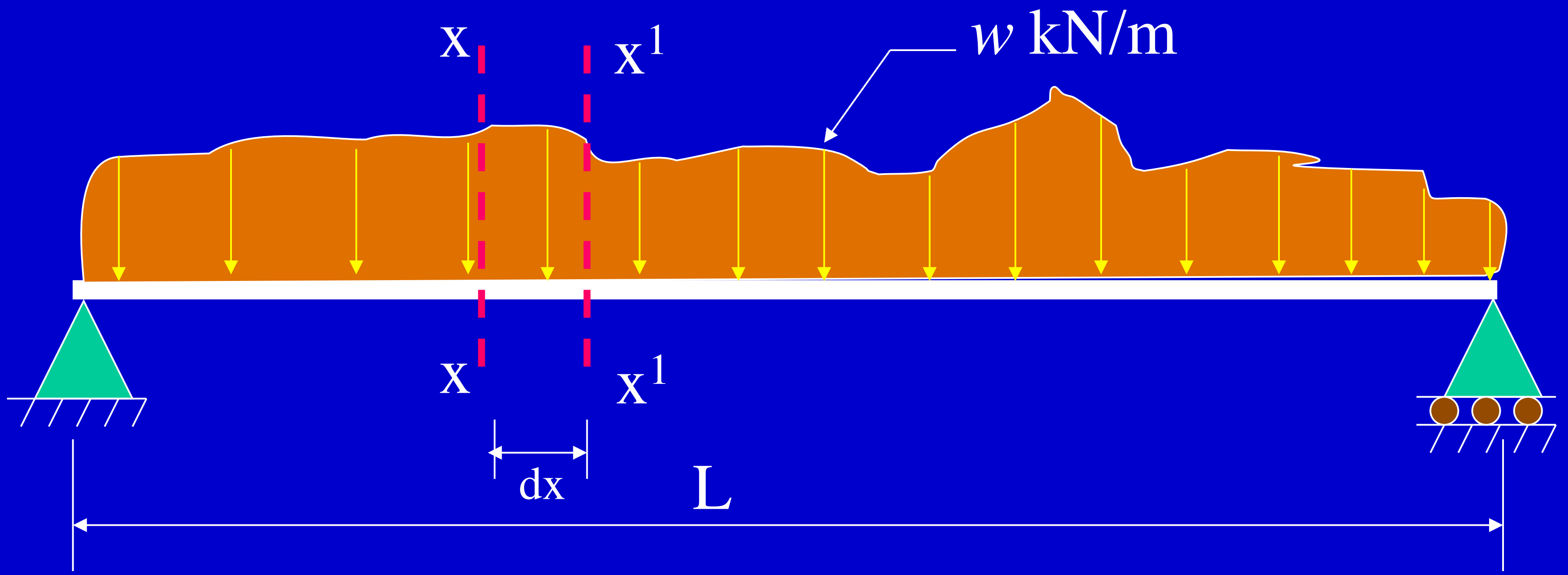


Fig. A simply supported beam subjected to general type loading

The above Fig. shows a simply supported beam subjected to a general type of loading. Consider a differential element of length ‘dx’ between any two sections x-x and x^1-x^1 as shown.

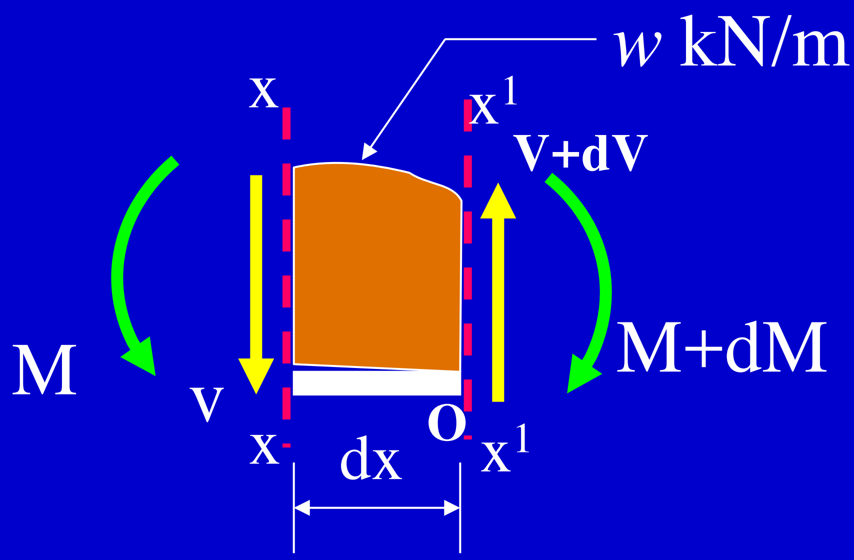


Fig. FBD of Differential element of the beam

Taking moments about the point 'O' [Bottom-Right corner of the differential element]

$- M + (M+dM) – V.dx – w.dx.dx/2 = 0$

Neglecting the small quantity of higher order

$V.dx = dM \rightarrow v = \frac{dM}{dx}$

It is the relation between shear force and BM

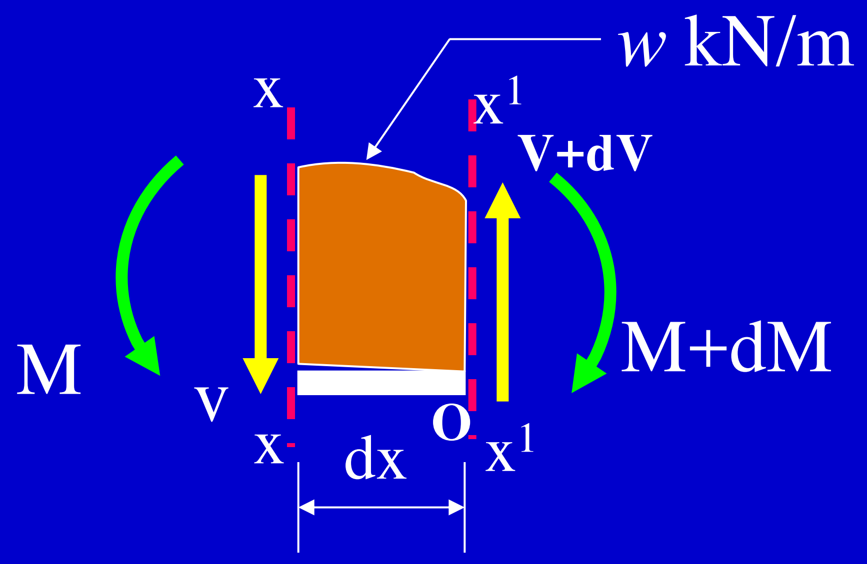


Fig. FBD of Differential element of the beam

Considering the Equilibrium Equation $\Sigma Fy = 0$

$- V + (V+dV) - w\, dx = 0$ ➔ $dv = w.dx$ ➔

$$w = \frac{dv}{dx}$$

It is the relation Between intensity of Load and shear force

Variation of Shear force and bending moments

Variation of Shear force and bending moments for various standard loads are as shown in the following Table

Table: Variation of Shear force and bending moments

Type of load / SFD/BMD	Between point loads OR for no load region	Uniformly distributed load	Uniformly varying load
Shear Force Diagram	Horizontal line	Inclined line	Two-degree curve (Parabola)
Bending Moment Diagram	Inclined line	Two-degree curve (Parabola)	Three-degree curve (Cubic-parabola)

Sections for Shear Force and Bending Moment Calculations:

Shear force and bending moments are to be calculated at various sections of the beam to draw shear force and bending moment diagrams.

These sections are generally considered on the beam where the magnitude of shear force and bending moments are changing abruptly.

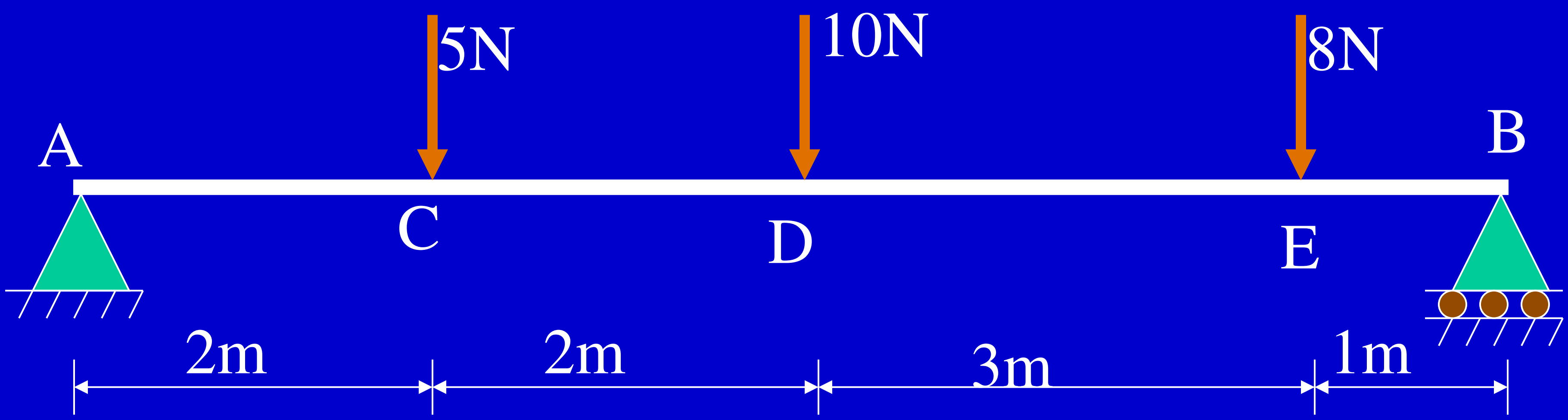
Therefore these sections for the calculation of shear forces include sections on either side of point load, uniformly distributed load or uniformly varying load where the magnitude of shear force changes abruptly.

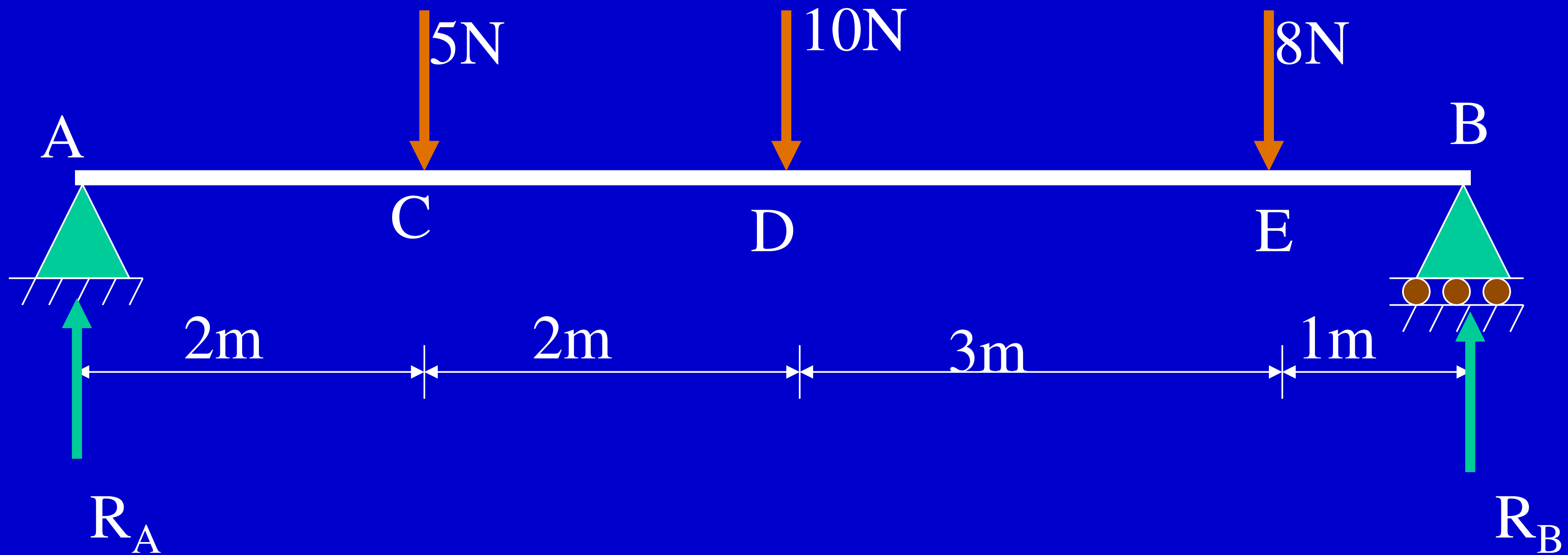
The sections for the calculation of bending moment include position of point loads, either side of uniformly distributed load, uniformly varying load and couple

Note: While calculating the shear force and bending moment, only the portion of the udl which is on the left hand side of the section should be converted into point load. But while calculating the reaction we convert entire udl to point load

Example Problem 1

1. Draw shear force and bending moment diagrams [SFD and BMD] for a simply supported beam subjected to three point loads as shown in the Fig. given below.





Solution:

[Clockwise moment is Positive]

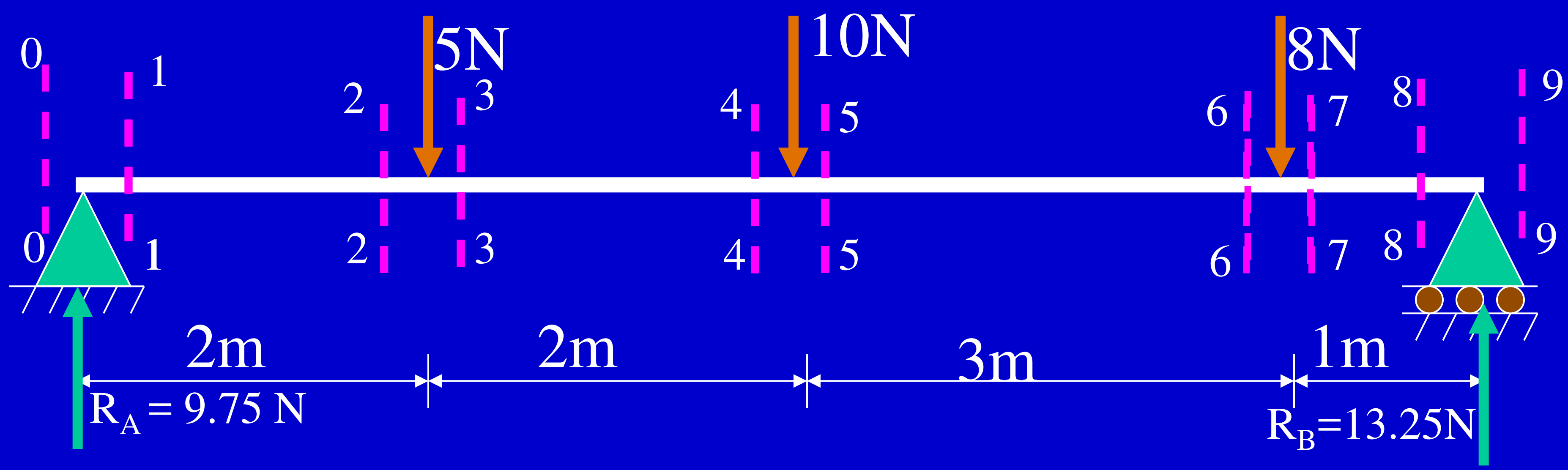
Using the condition: $\Sigma M_A = 0$

$$- R_B \times 8 + 8 \times 7 + 10 \times 4 + 5 \times 2 = 0 \quad \rightarrow \quad R_B = 13.25 \text{ N } [\uparrow]$$

Using the condition: $\Sigma F_y = 0$

$$R_A + 13.25 = 5 + 10 + 8 \quad \rightarrow \quad R_A = 9.75 \text{ N } [\uparrow]$$

Shear Force Calculation:



Shear Force at the section 1-1 is denoted as V_{1-1}

Shear Force at the section 2-2 is denoted as V_{2-2} and so on...

$V_{0-0} = 0$; $V_{1-1} = +9.75$ N

$V_{2-2} = +9.75$ N

$V_{3-3} = +9.75 - 5 = 4.75$ N

$V_{4-4} = +4.75$ N

$V_{5-5} = +4.75 - 10 = -5.25$ N

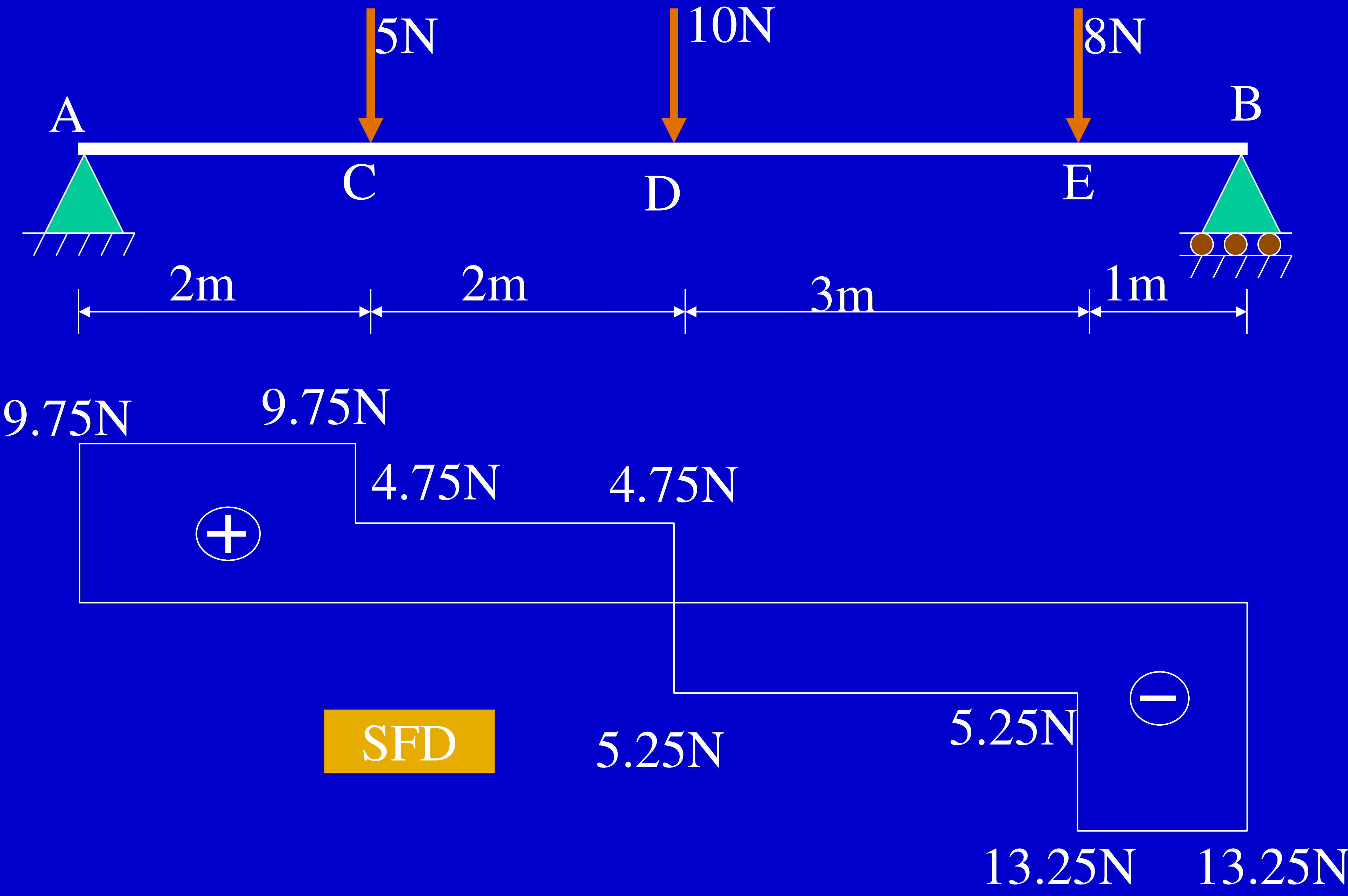
$V_{6-6} = -5.25$ N

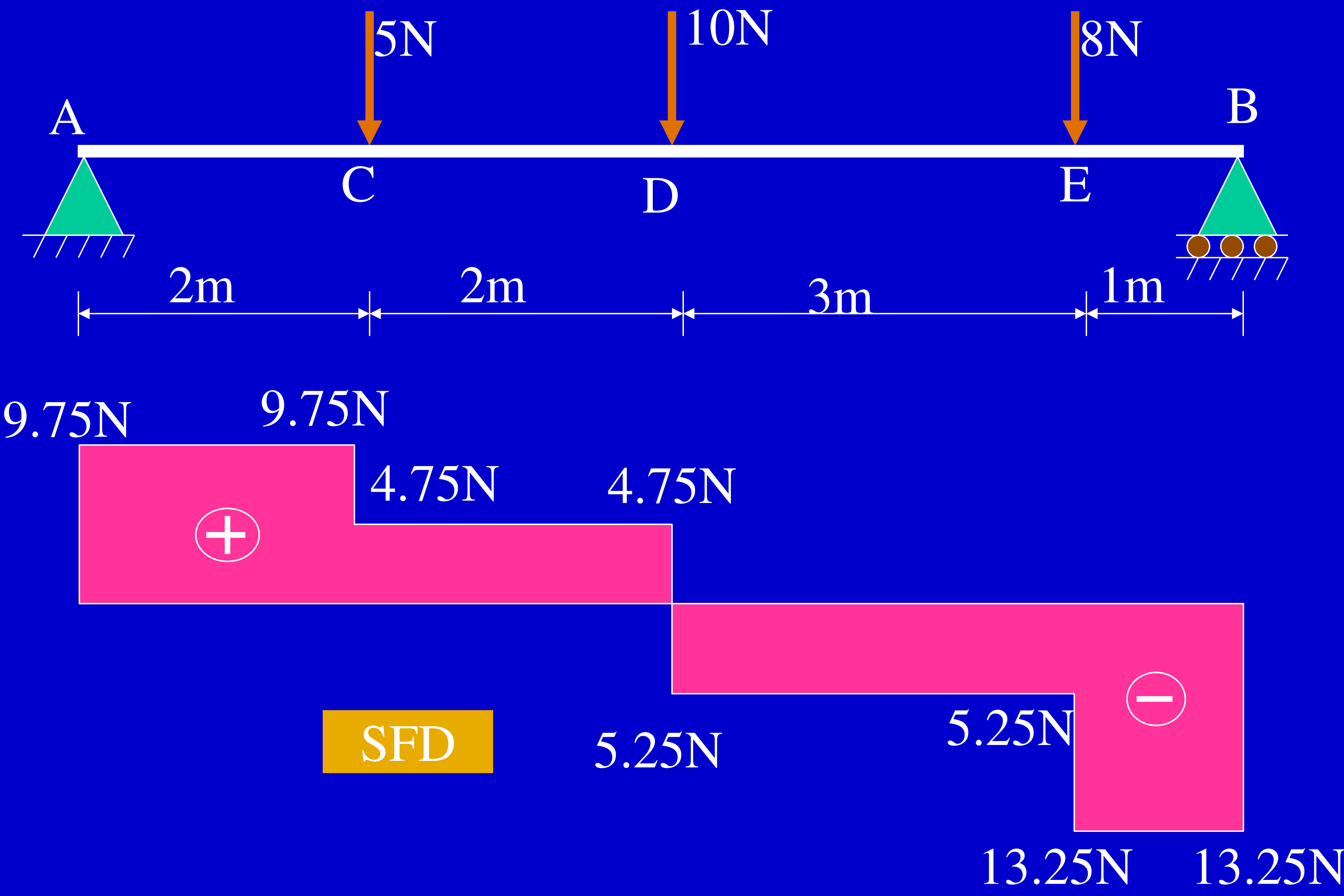
$V_{7-7} = 5.25 - 8 = -13.25$ N

$V_{8-8} = -13.25$

$V_{9-9} = -13.25 + 13.25 = 0$

(Check)


5N
10N
8N
A
B
C
D
E
2m
2m
3m
1m
9.75N
9.75N
4.75N
4.75N
+
5.25N
5.25N
−
SFD
13.25N
13.25N


5N
10N
8N
A
B
C
D
E
2m
2m
3m
1m
9.75N
9.75N
4.75N
4.75N
+
−
SFD
5.25N
5.25N
13.25N
13.25N

Bending Moment Calculation

Bending moment at A is denoted as M_A

Bending moment at B is denoted as M_B

and so on…

$M_A = 0$ [since it is simply supported]

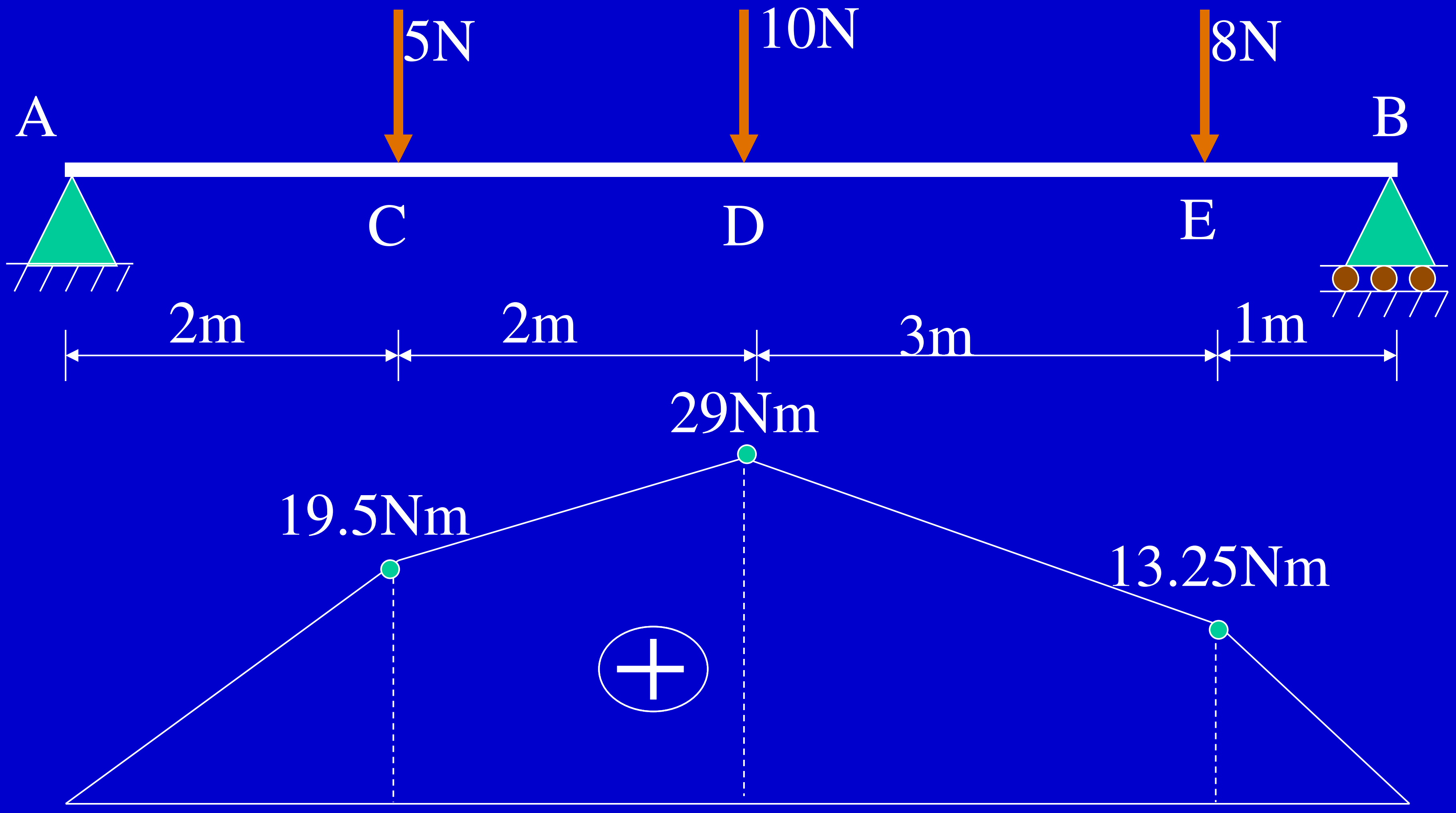
$M_C = 9.75 \times 2 = 19.5$ Nm

$M_D = 9.75 \times 4 - 5 \times 2 = 29$ Nm

$M_E = 9.75 \times 7 - 5 \times 5 - 10 \times 3 = 13.25$ Nm

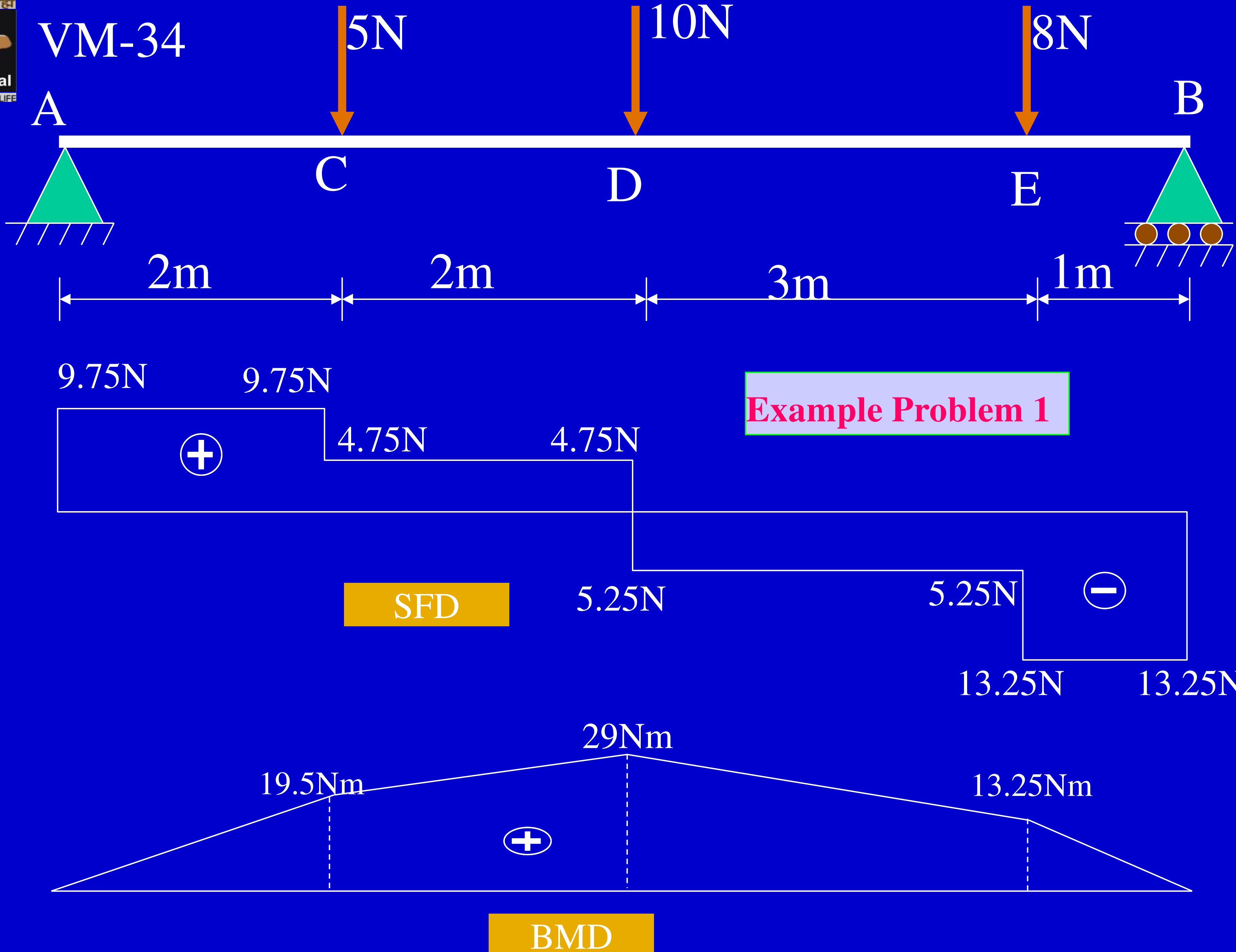
$M_B = 9.75 \times 8 - 5 \times 6 - 10 \times 4 - 8 \times 1 = 0$

or $M_B = 0$ [since it is simply supported]

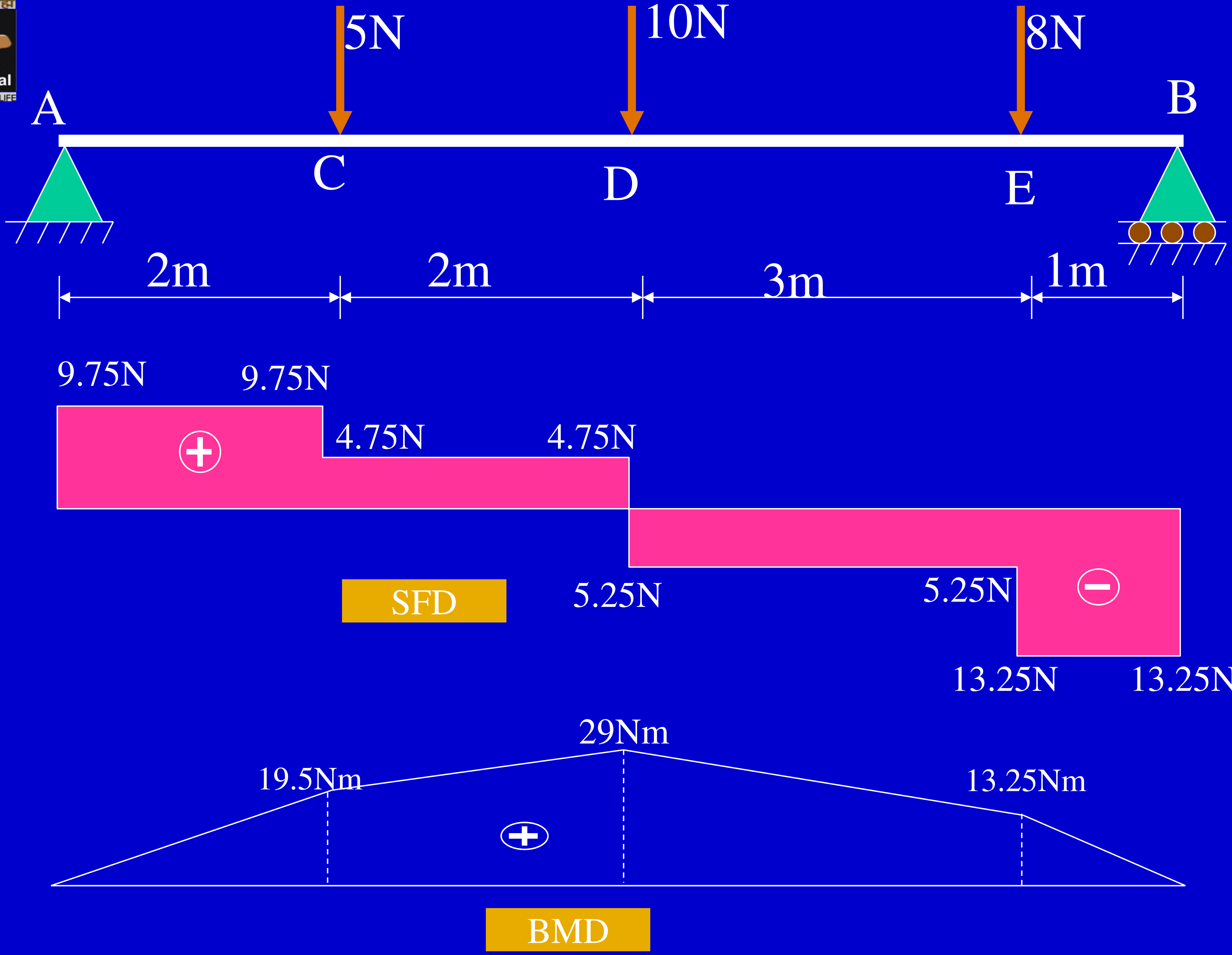


BMD

VM-34

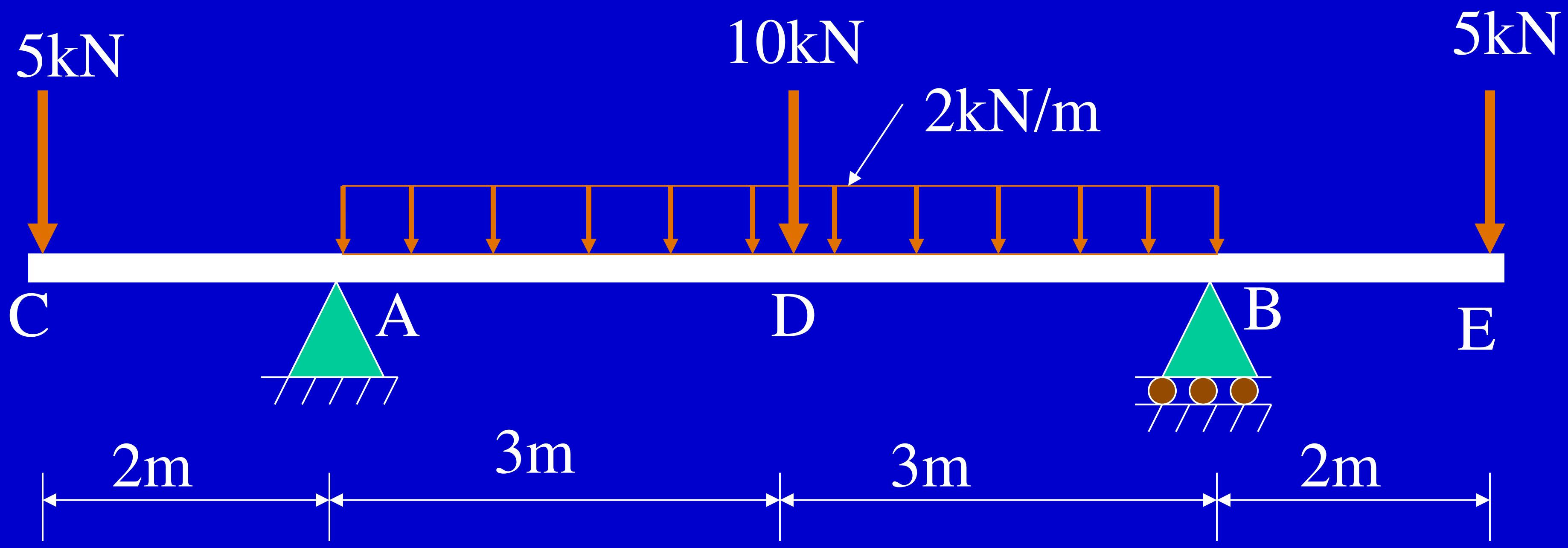


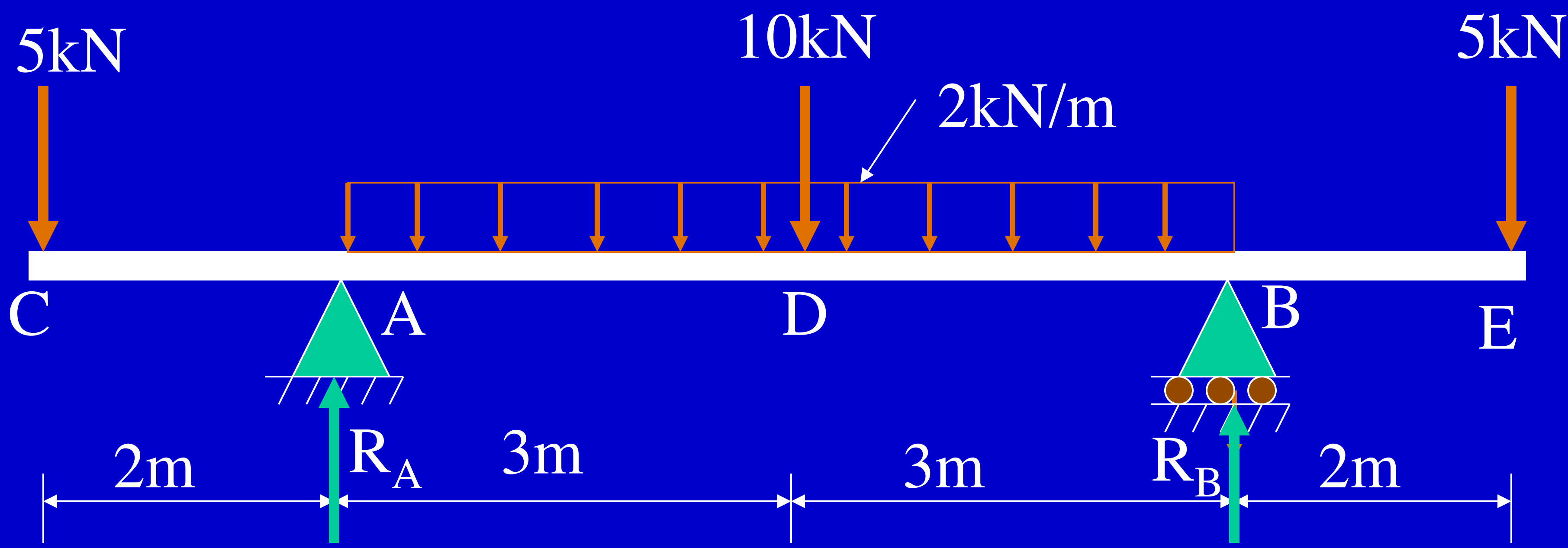
Example Problem 1


5N
10N
8N
A
B
C
D
E
2m
2m
3m
1m
9.75N
9.75N
4.75N
4.75N
5.25N
5.25N
13.25N
13.25N
SFD
29Nm
19.5Nm
13.25Nm
BMD

Example Problem 2

2. **Draw SFD and BMD for the double side overhanging beam subjected to loading as shown below. Locate points of contraflexure if any.**



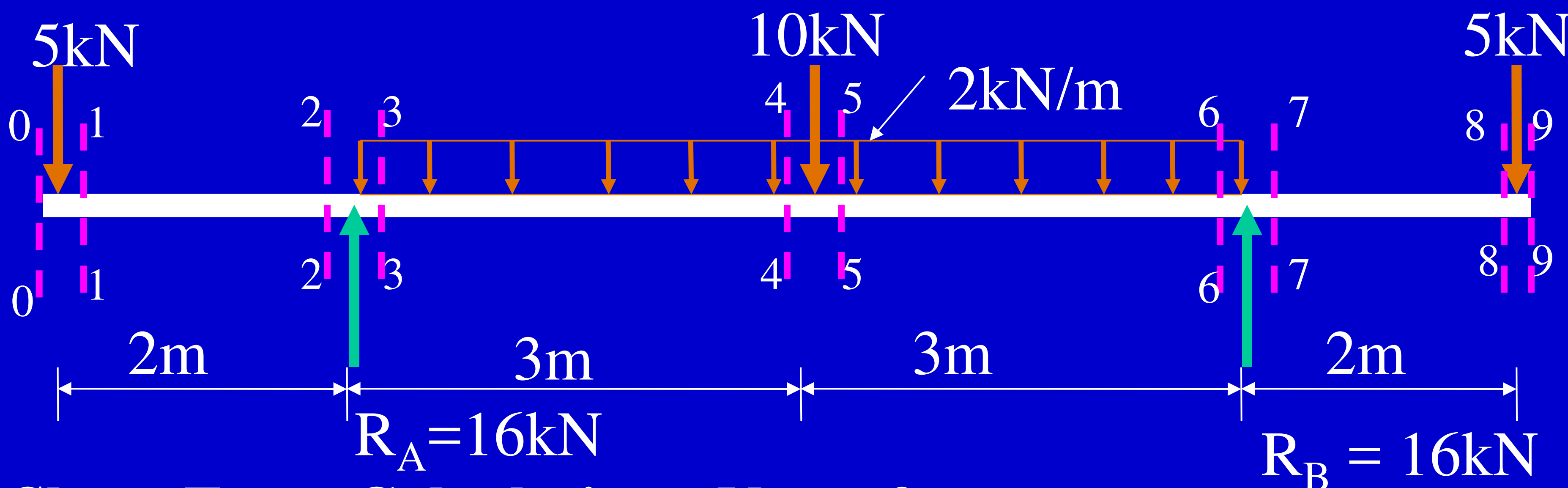


Solution:

Calculation of Reactions:

Due to symmetry of the beam, loading and boundary conditions, reactions at both supports are equal.

$$\therefore \; R_A = R_B = ½(5+10+5+2 \times 6) = 16 \text{ kN} \; [\uparrow]$$



Shear Force Calculation: $V_{0-0} = 0$

$V_{1-1} = -\ 5\text{kN}$

$V_{2-2} = -\ 5\text{kN}$

$V_{3-3} = -\ 5 + 16 = 11\ \text{kN}$

$V_{4-4} = 11 - 2 \times 3 = +5\ \text{kN}$

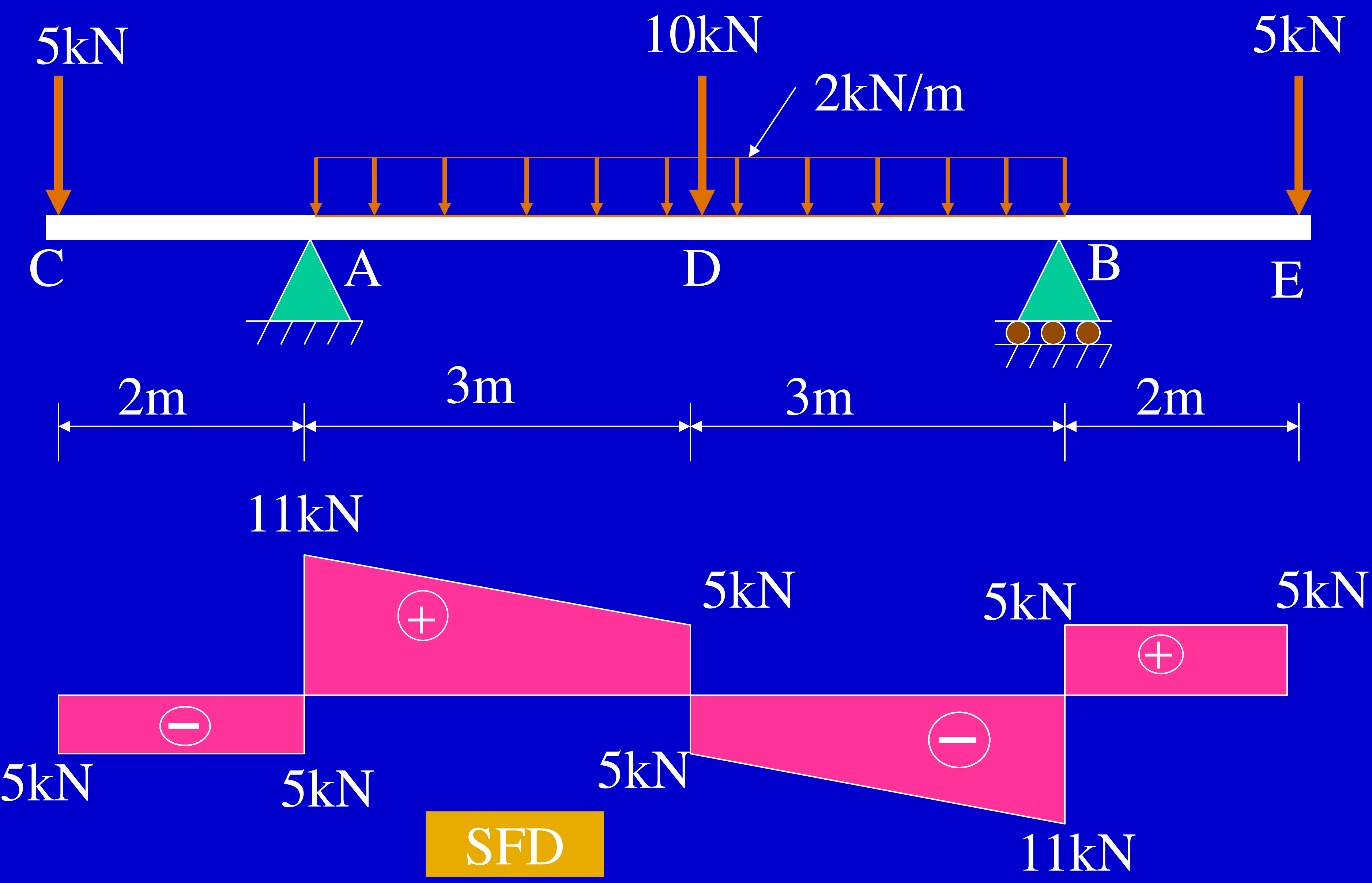
$V_{5-5} = 5 - 10 = -\ 5\text{kN}$

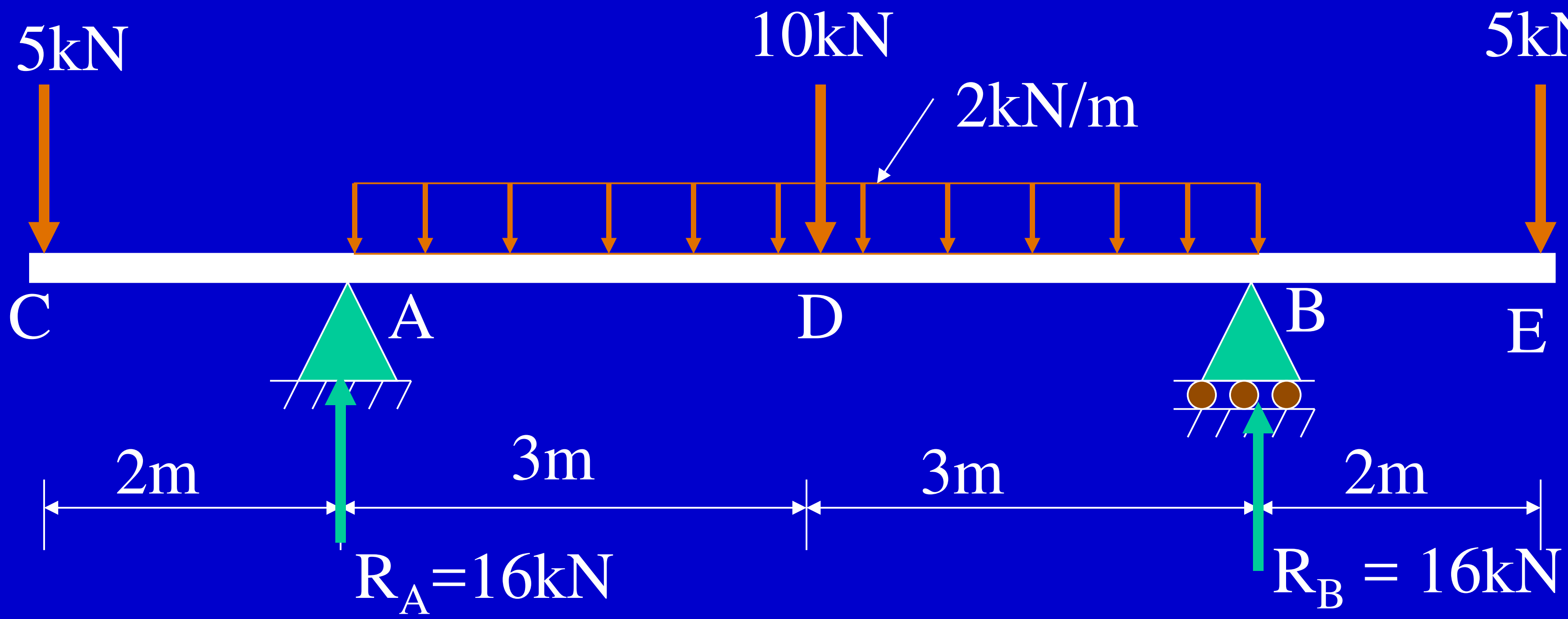
$V_{6-6} = -\ 5 - 6 = -\ 11\text{kN}$

$V_{7-7} = -\ 11 + 16 = 5\text{kN}$

$V_{8-8} = 5\ \text{kN}$

$V_{9-9} = 5 - 5 = 0$ (Check)


5kN
10kN
5kN
2kN/m
C
A
D
B
E
2m
3m
3m
2m
11kN
5kN
5kN
5kN
5kN
5kN
5kN
11kN
SFD

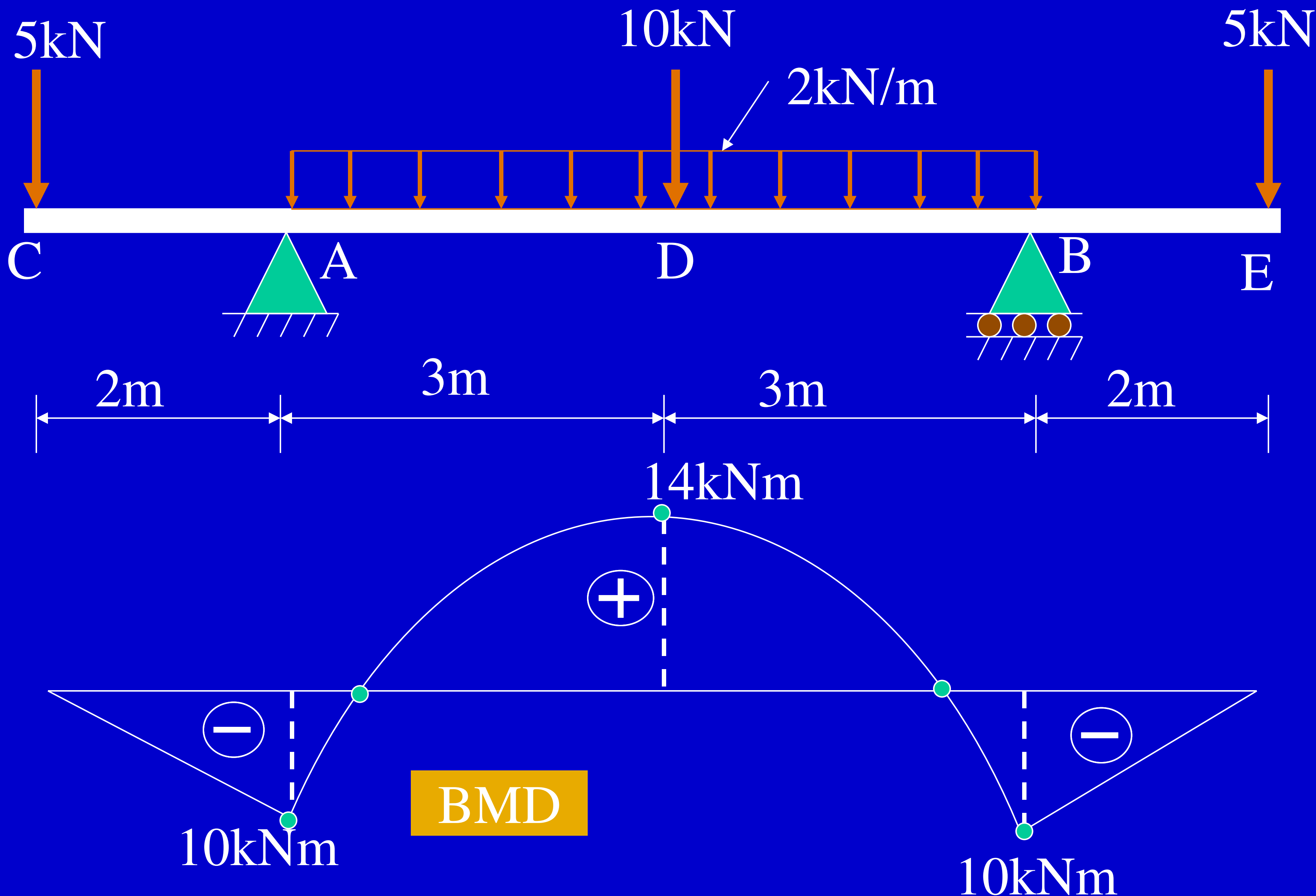


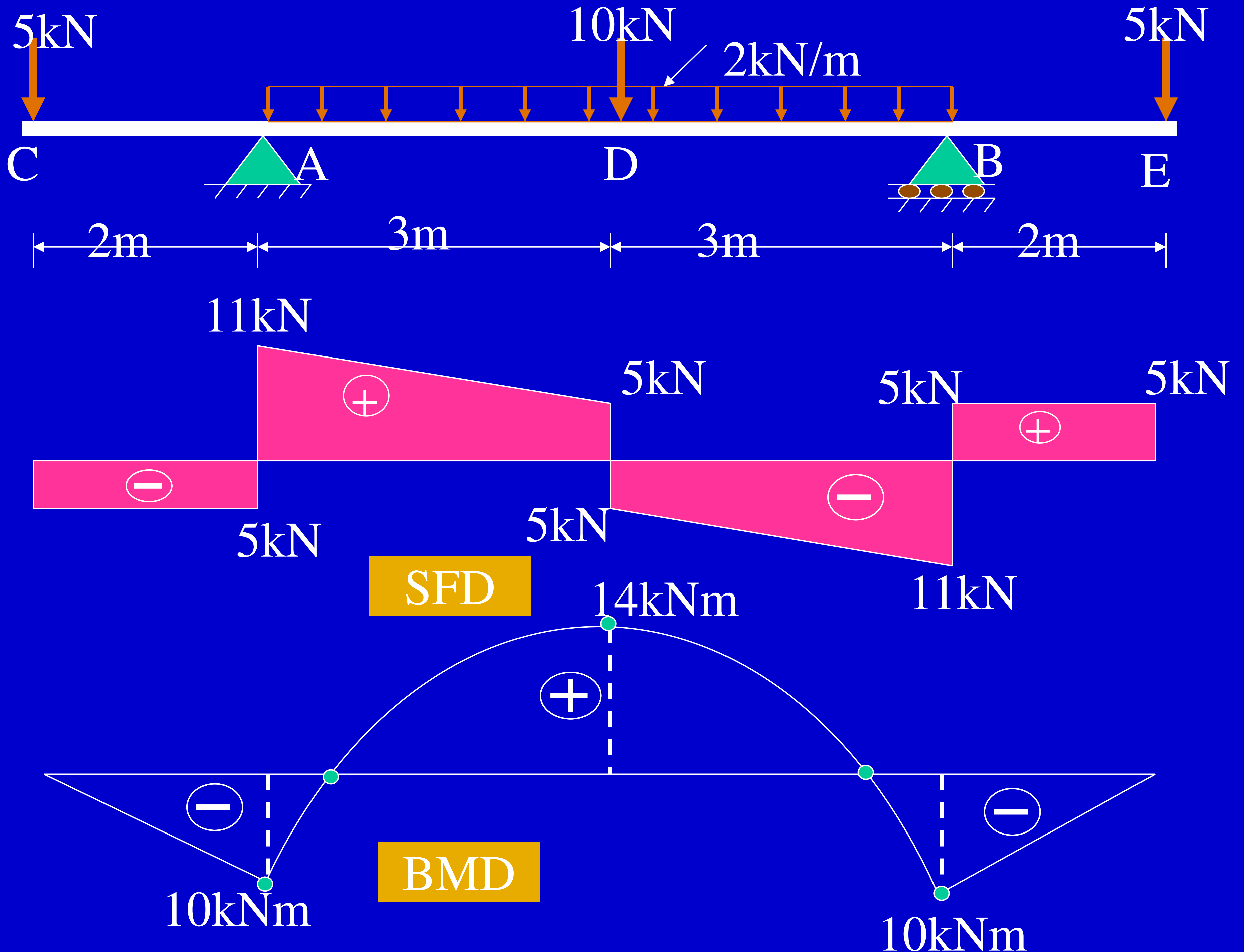
Bending Moment Calculation:

$M_C = M_E = 0$ [Because Bending moment at free end is zero]

$M_A = M_B = -5 \times 2 = -10$ kNm

$M_D = -5 \times 5 + 16 \times 3 - 2 \times 3 \times 1.5 = +14$ kNm


5kN
10kN
5kN
2kN/m
C
A
D
B
E
2m
3m
3m
2m
14kNm
10kNm
BMD
10kNm


5kN
10kN
5kN
2kN/m
C
A
D
B
E
2m
3m
3m
2m
11kN
5kN
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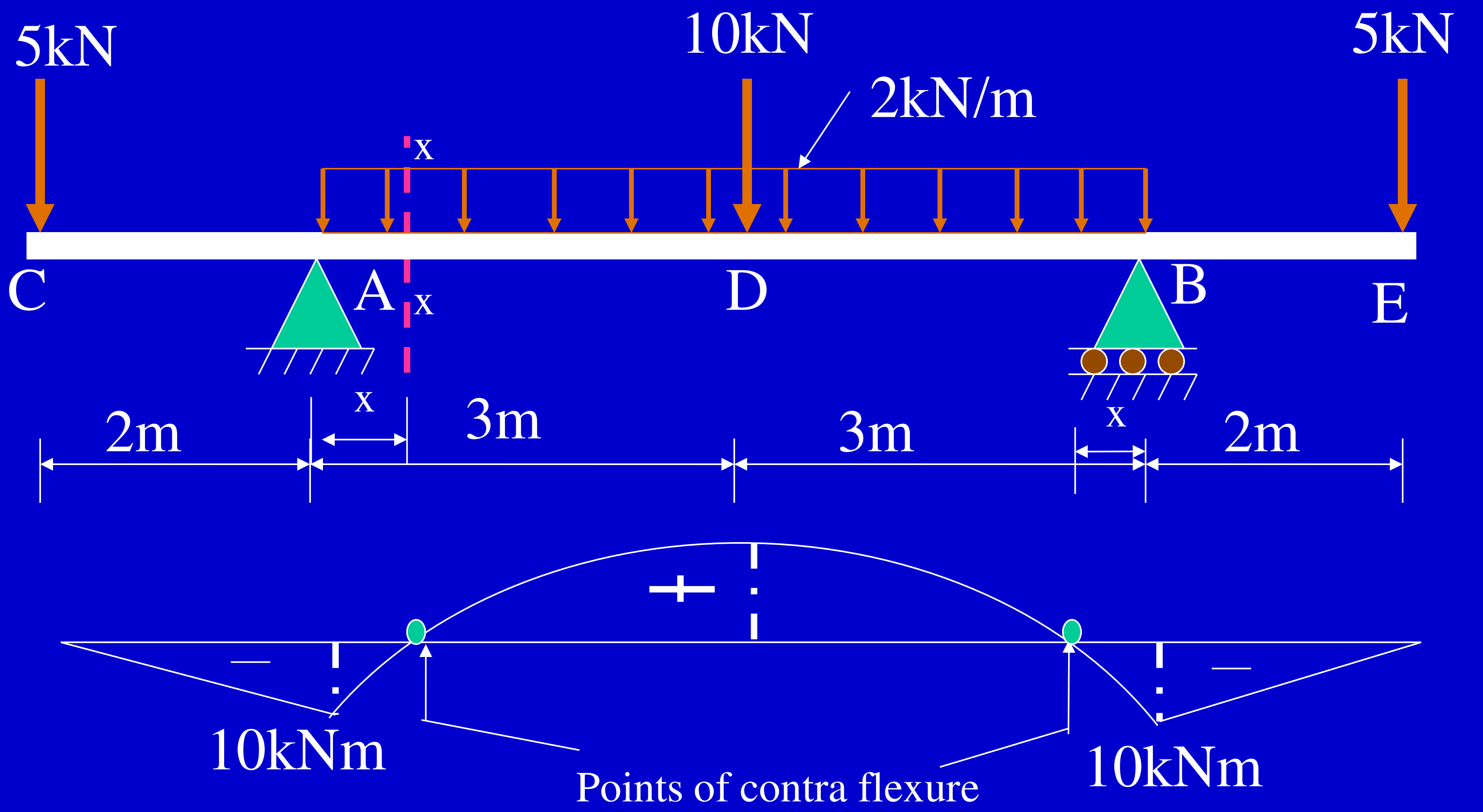
5kN
5kN
5kN
11kN
SFD
14kNm
BMD
10kNm
10kNm



Let x be the distance of point of contra flexure from support A

Taking moments at the section x-x (Considering left portion)

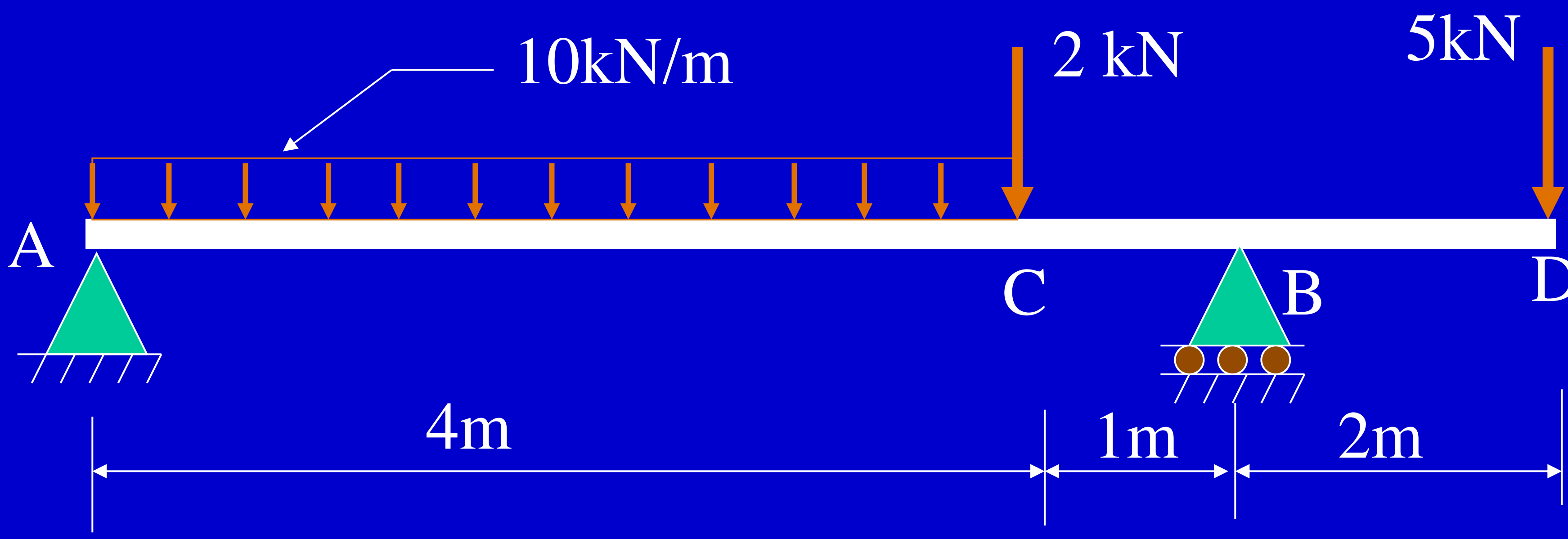
$$M_{x-x} = -5(2+x) + 16x - 2\frac{x^2}{2} = 0$$

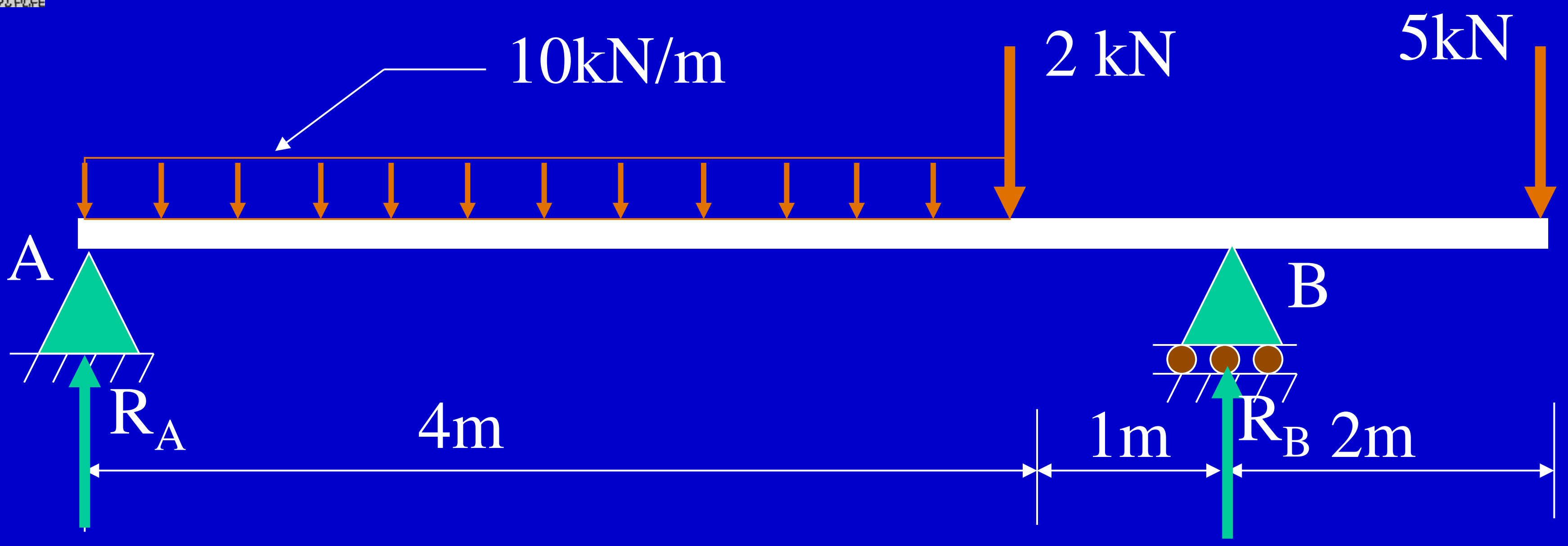
x = 1 or 10

$\therefore$ x = 1 m

Example Problem 3

3. Draw SFD and BMD for the single side overhanging beam subjected to loading as shown below. Determine the absolute maximum bending moment and shear forces and mark them on SFD and BMD. Also locate points of contra flexure if any.





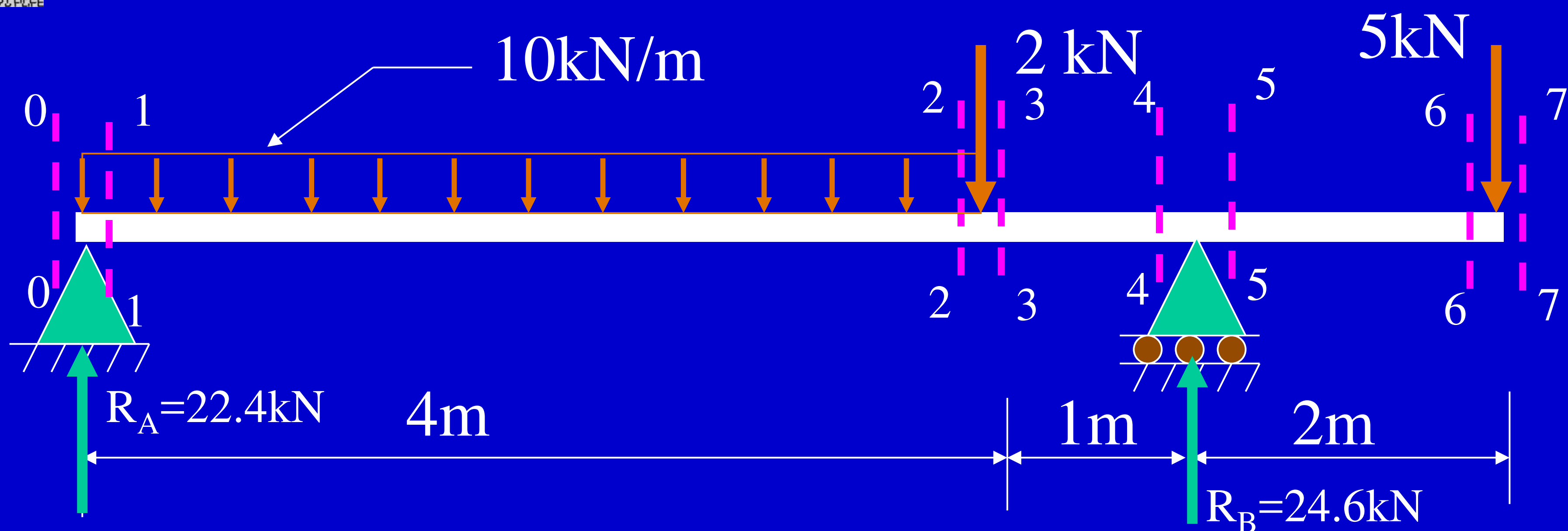
Solution : Calculation of Reactions:

$\Sigma M_A = 0$

$- R_B \times 5 + 10 \times 4 \times 2 + 2 \times 4 + 5 \times 7 = 0$ ➔ $R_B = 24.6$ kN [↑]

$\Sigma F_y = 0$

$R_A + 24.6 - 10 \text{ x } 4 - 2 + 5 = 0$ ➔ $R_A = 22.4$ kN [↑]



Shear Force Calculations:

$V_{0-0} = 0$; $V_{1-1} = 22.4$ kN

$V_{2-2} = 22.4 - 10 \times 4 = -17.6$ kN

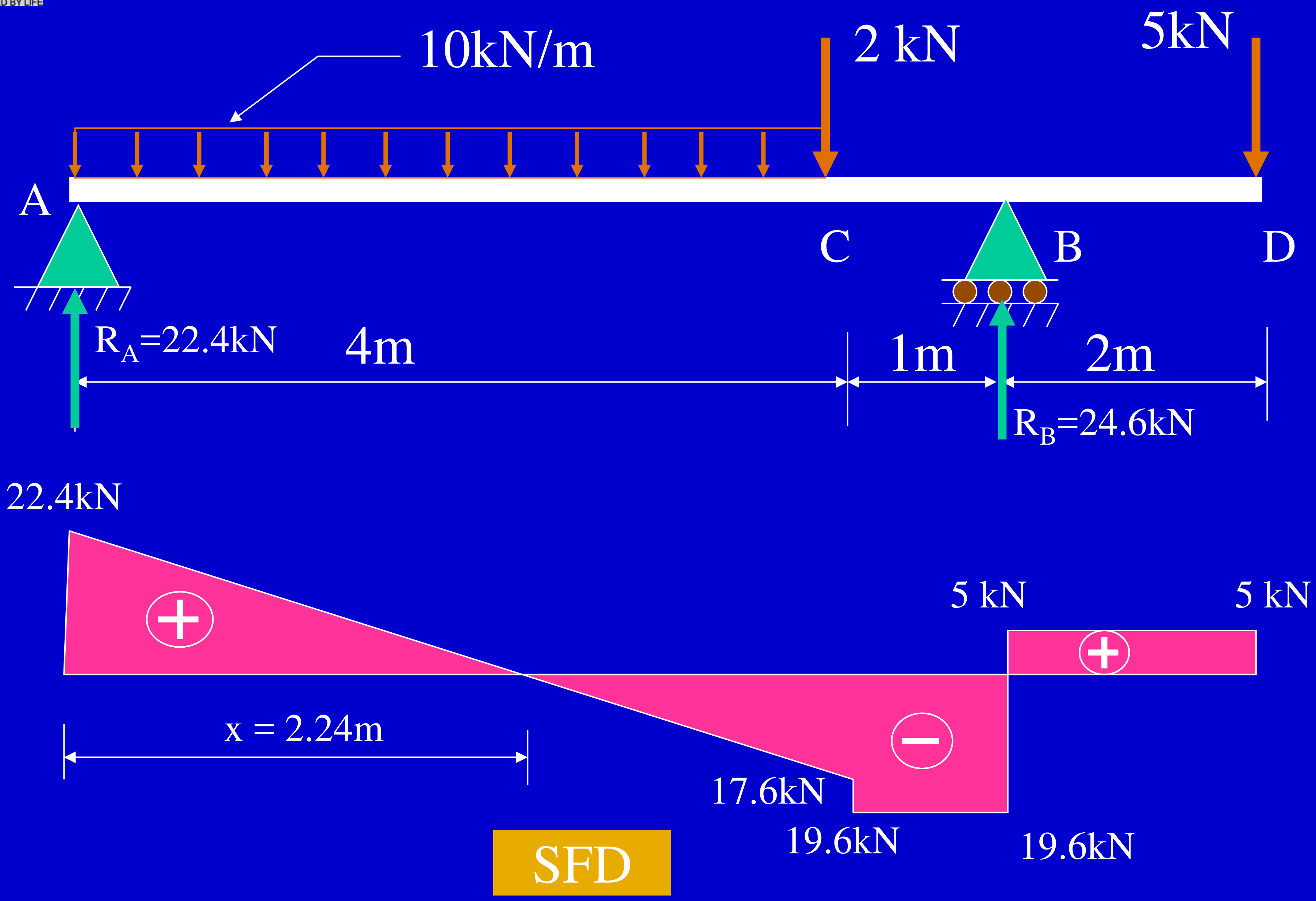
$V_{3-3} = -17.6 - 2 = -19.6$ kN

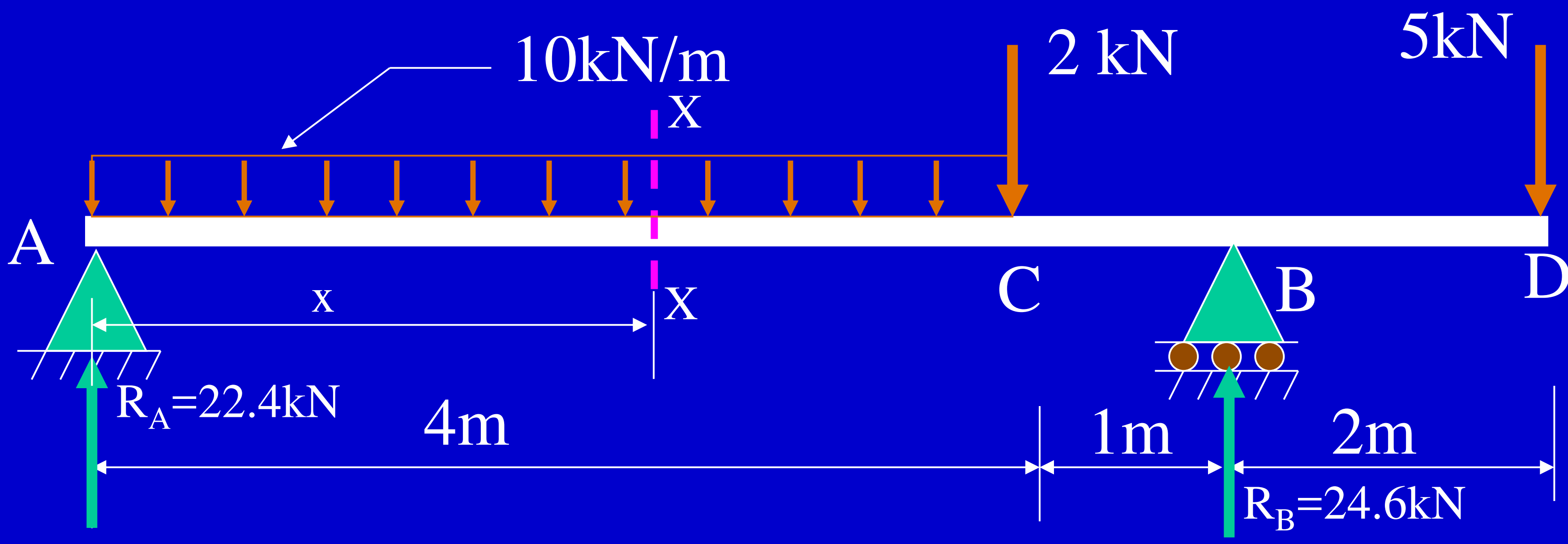
$V_{4-4} = -19.6$ kN

$V_{5-5} = -19.6 + 24.6 = 5$ kN

$V_{6-6} = 5$ kN

$V_{7-7} = 5 - 5 = 0$ (Check)

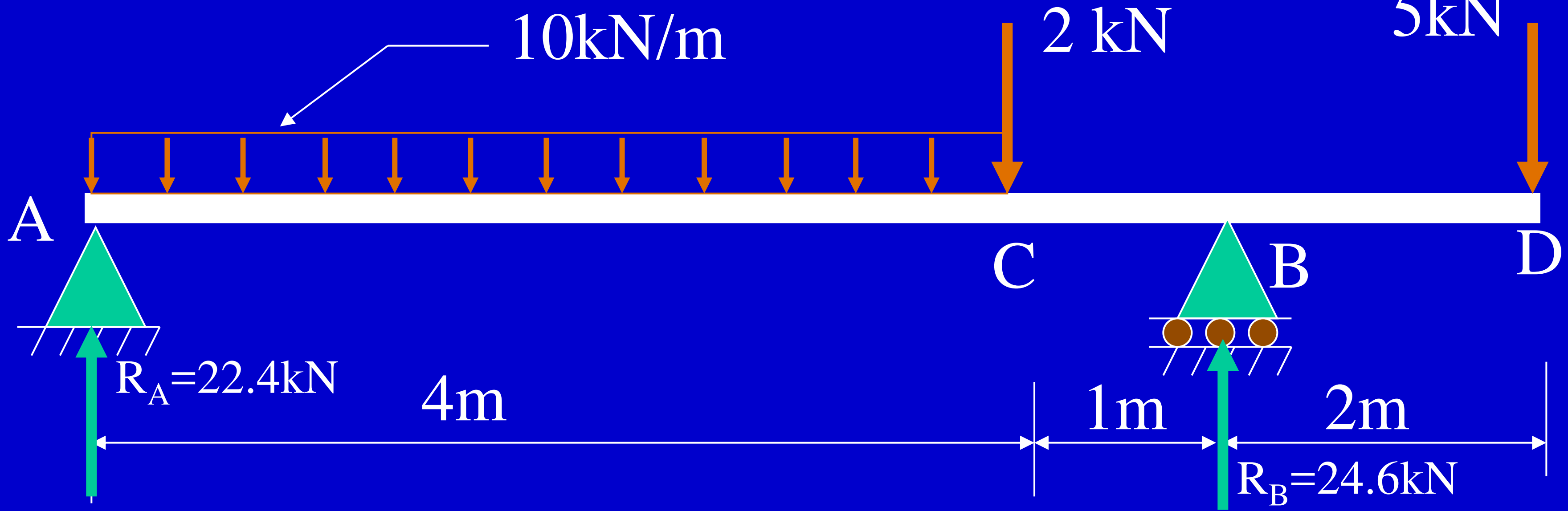

10kN/m
2 kN
5kN
A
C
B
D
R_A=22.4kN
4m
1m
2m
R_B=24.6kN
22.4kN
5 kN
5 kN
x = 2.24m
17.6kN
19.6kN
19.6kN
SFD



Max. bending moment will occur at the section where the shear force is zero. The SFD shows that the section having zero shear force is available in the portion AC. Let that section be X-X, considered at a distance x from support A as shown above.

The shear force at that section can be calculated as

$V_{x-x} = 22.4 - 10. x = 0 \quad \rightarrow \quad x = 2.24$ m



Calculations of Bending Moments:

$M_A = M_D = 0$

$M_C = 22.4 \times 4 - 10 \times 4 \times 2 = 9.6$ kNm

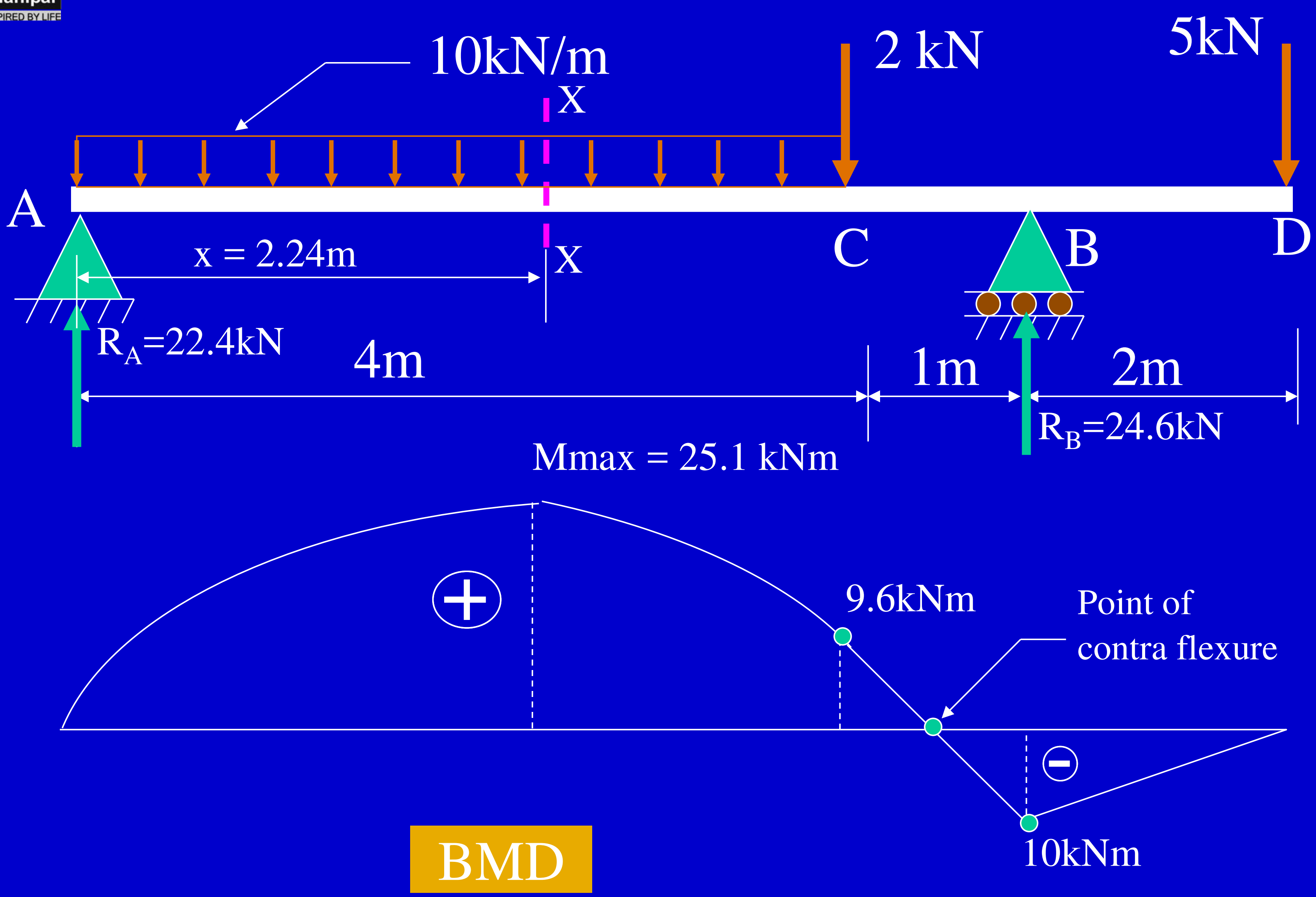
$M_B = 22.4 \times 5 - 10 \times 4 \times 3 - 2 \times 1 = -10$kNm (Considering Left portion of the section)

Alternatively

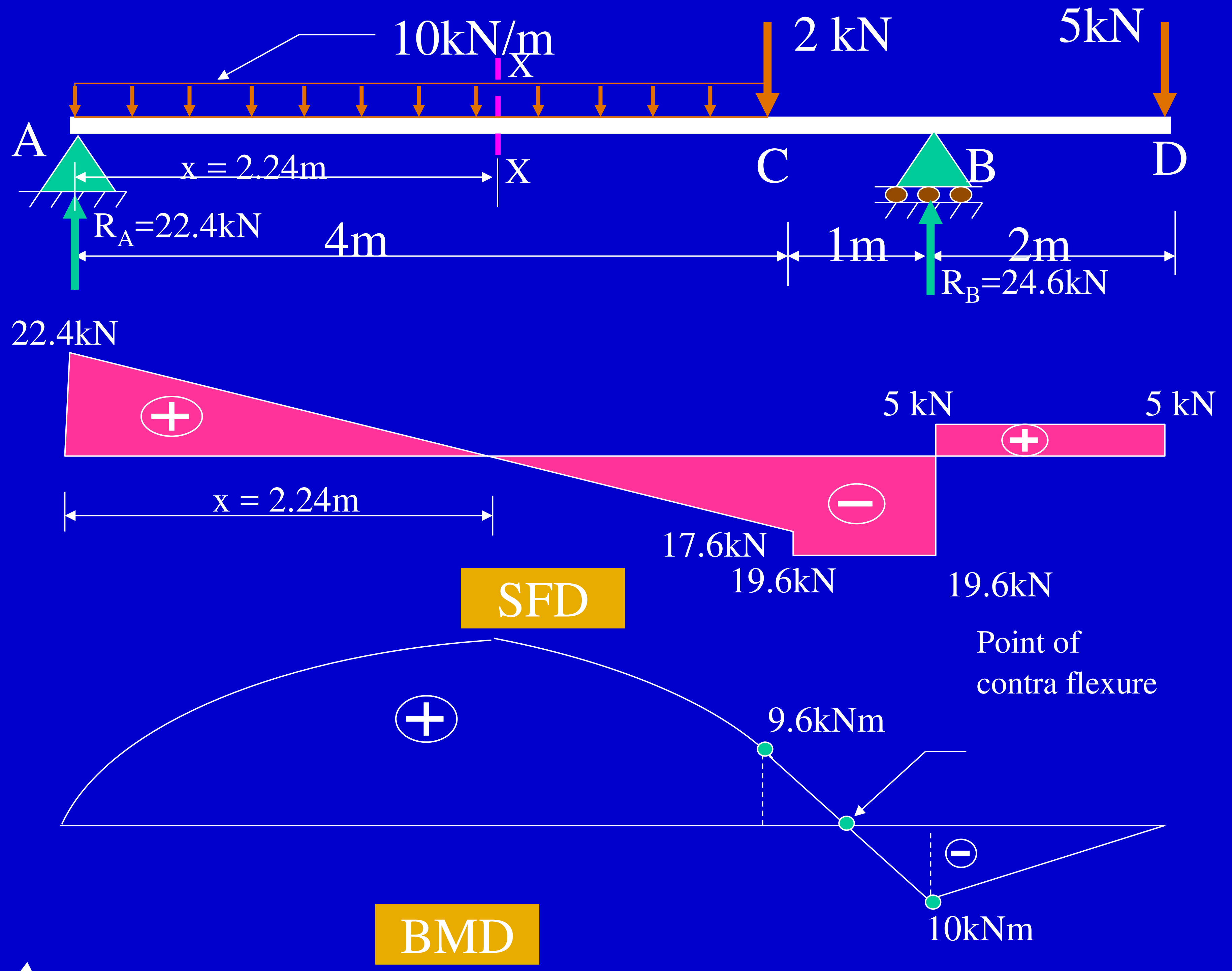
$M_B = -5 \times 2 = -10$ kNm (Considering Right portion of the section)

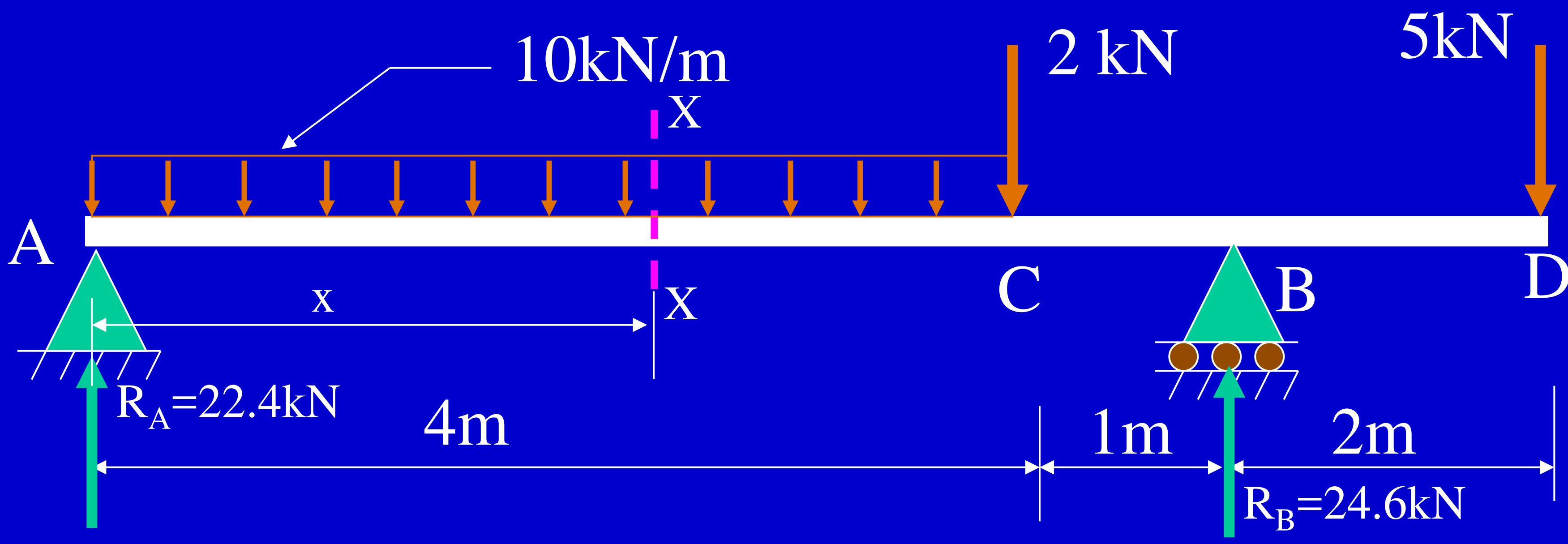
Absolute Maximum Bending Moment is at X- X ,

Mmax = 22.4 × 2.24 – 10 × (2.24)2 / 2 = 25.1 kNm



10kN/m
2 kN
5kN
X
A
C
B
D
x = 2.24m
X
R_A=22.4kN
4m
1m
2m
R_B=24.6kN
Mmax = 25.1 kNm
+
9.6kNm
Point of
contra flexure
−
10kNm
BMD


10kN/m
X
2 kN
5kN
A
x = 2.24m
X
C
B
D
RA=22.4kN
4m
1m
2m
RB=24.6kN
22.4kN
5 kN
5 kN
x = 2.24m
17.6kN
19.6kN
19.6kN
SFD
Point of
contra flexure
9.6kNm
10kNm
BMD



Calculations of Absolute Maximum Bending Moment:

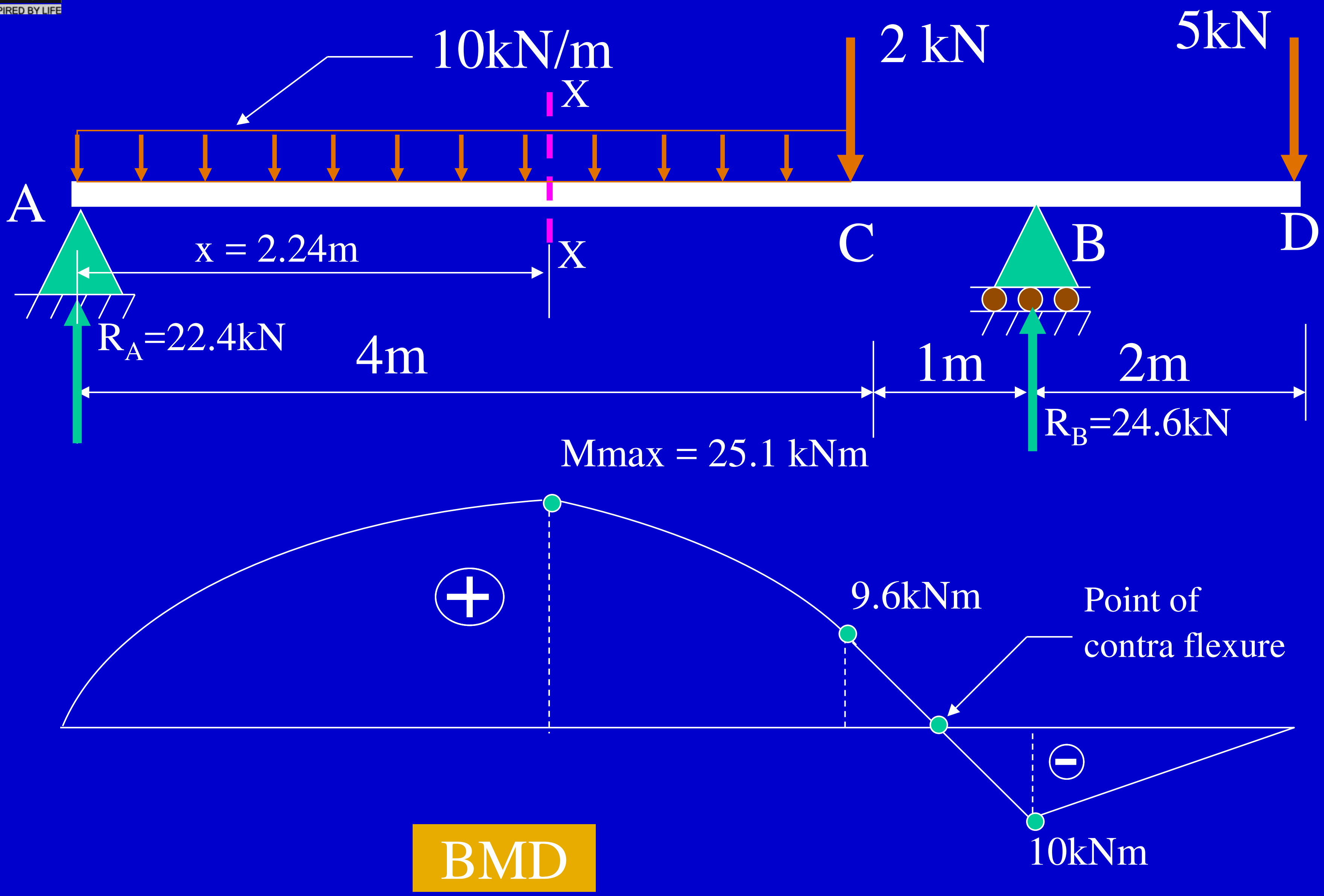
Max. bending moment will occur at the section where the shear force is zero. The SFD shows that the section having zero shear force is available in the portion AC. Let that section be X-X, considered at a distance x from support A as shown above.

The shear force at that section can be calculated as

$V_{x-x} = 22.4 - 10.\,x = 0 \;\rightarrow\; x = 2.24$ m

Max. BM at X- X ,

$M_{max} = 22.4 \times 2.24 - 10 \times (2.24)^2 / 2 = 25.1$ kNm



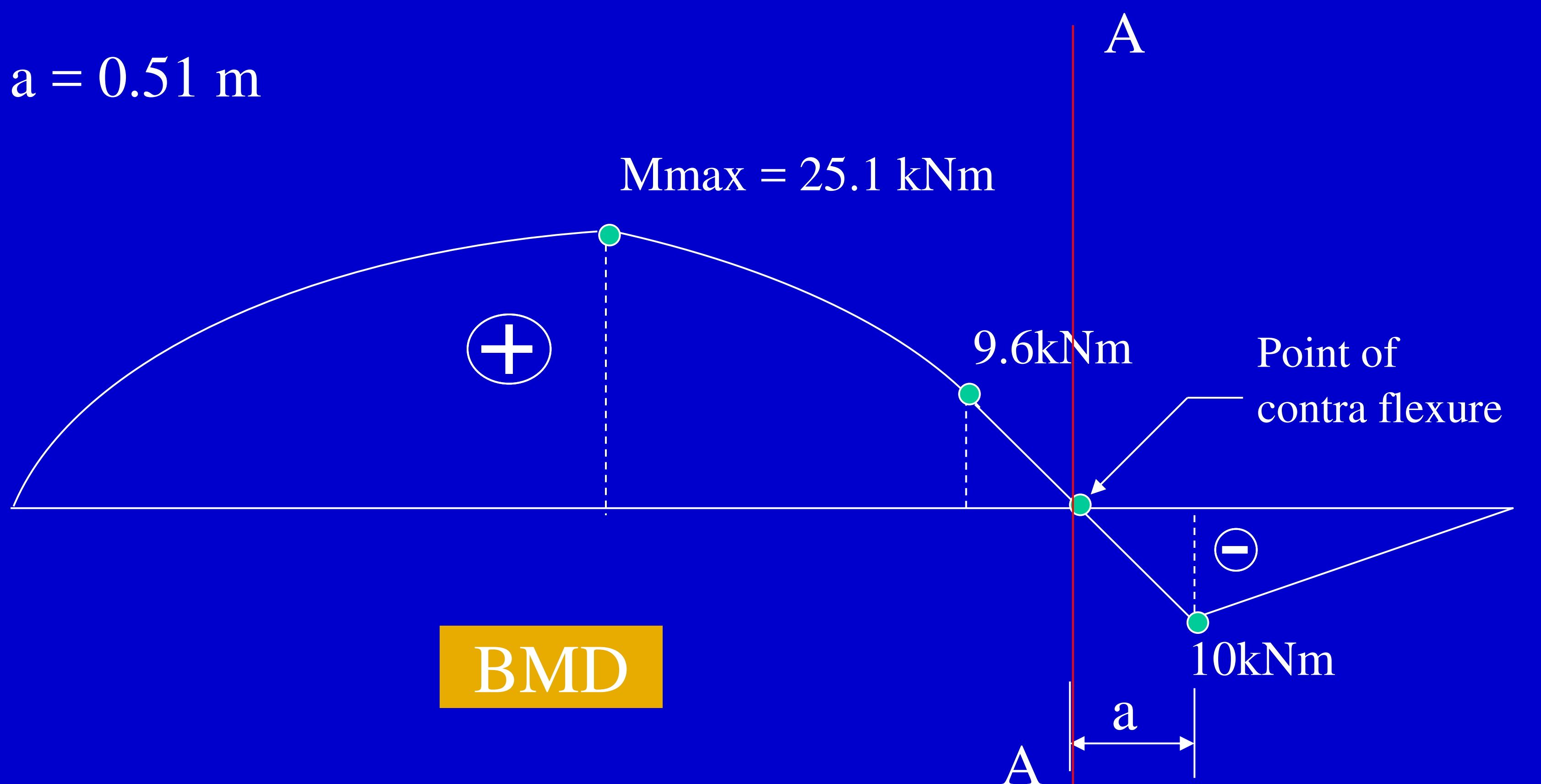
10kN/m
2 kN
5kN
X
A
C
B
D
X
x = 2.24m
R_A=22.4kN
4m
1m
2m
R_B=24.6kN
Mmax = 25.1 kNm
+
9.6kNm
Point of
contra flexure
−
10kNm
BMD

Let a be the distance of point of contra flexure from support B

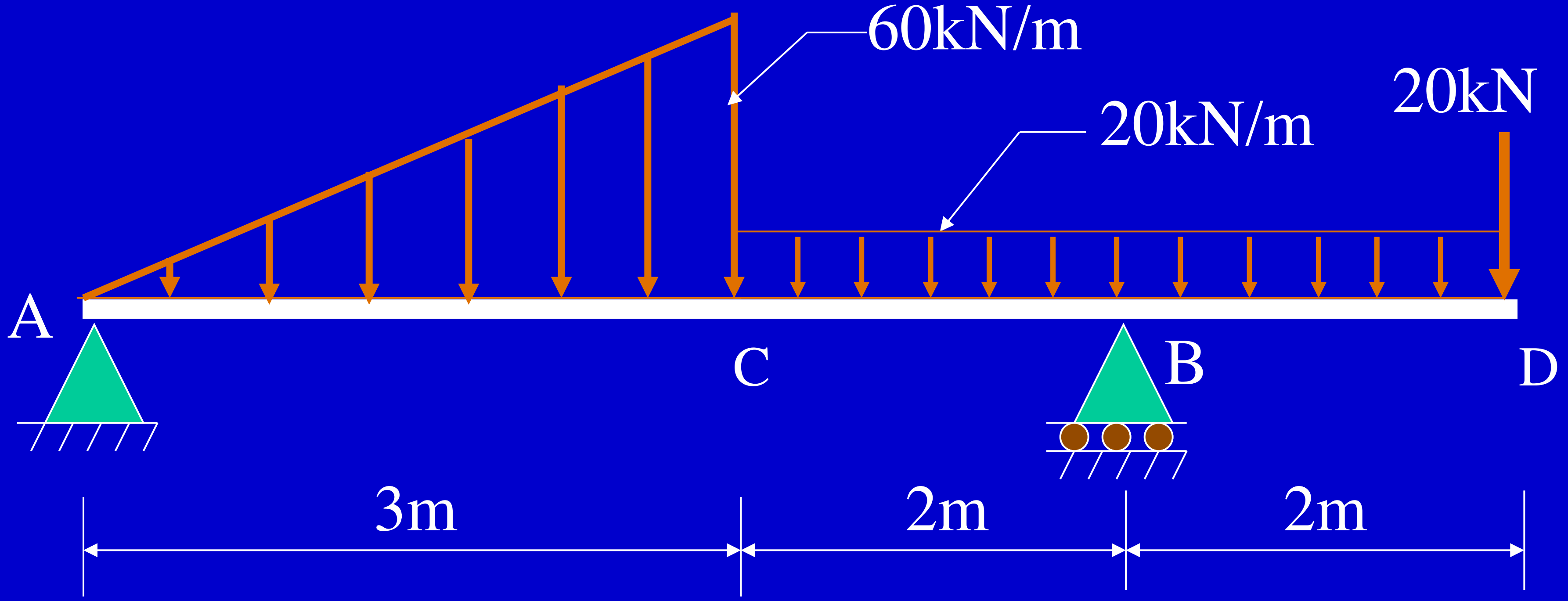
Taking moments at the section A-A (Considering left portion)

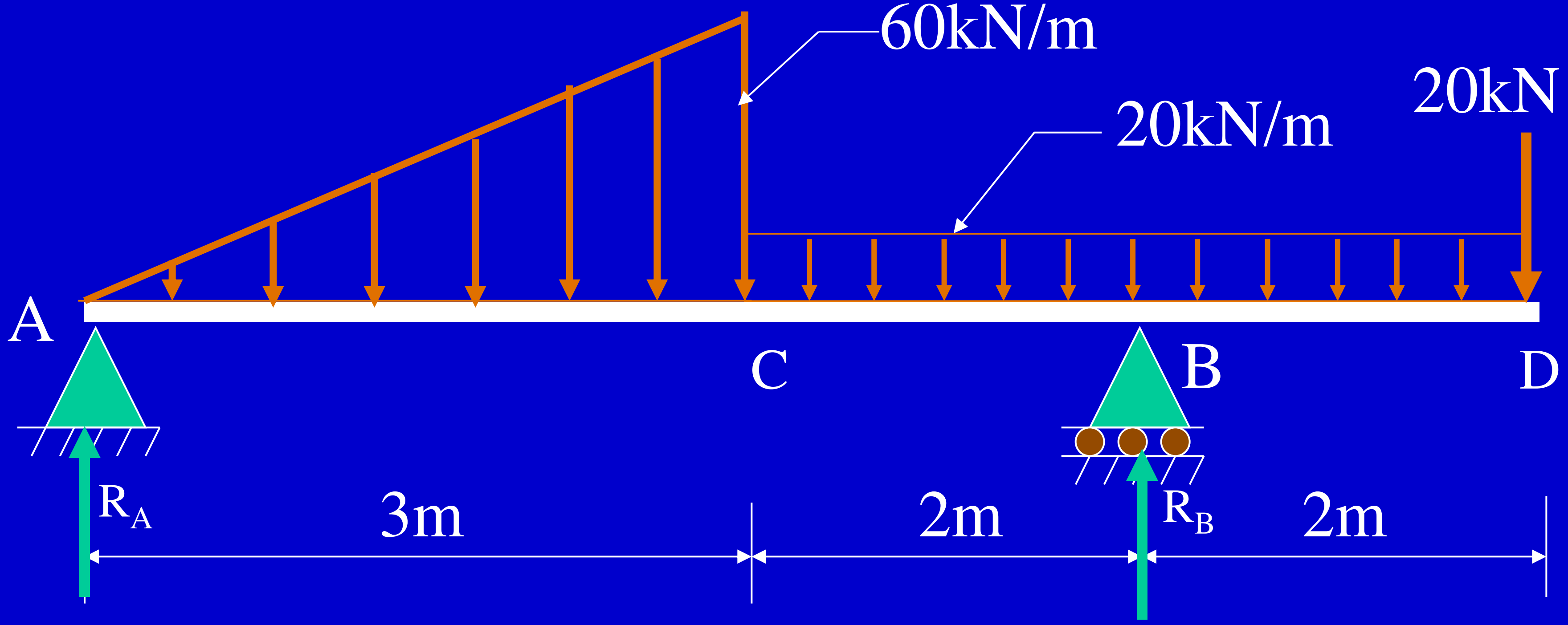
$$M_{A-A} = -5(2+a) + 24.6a = 0$$

a = 0.51 m



4. Draw SFD and BMD for the single side overhanging beam subjected to loading as shown below. Mark salient points on SFD and BMD.





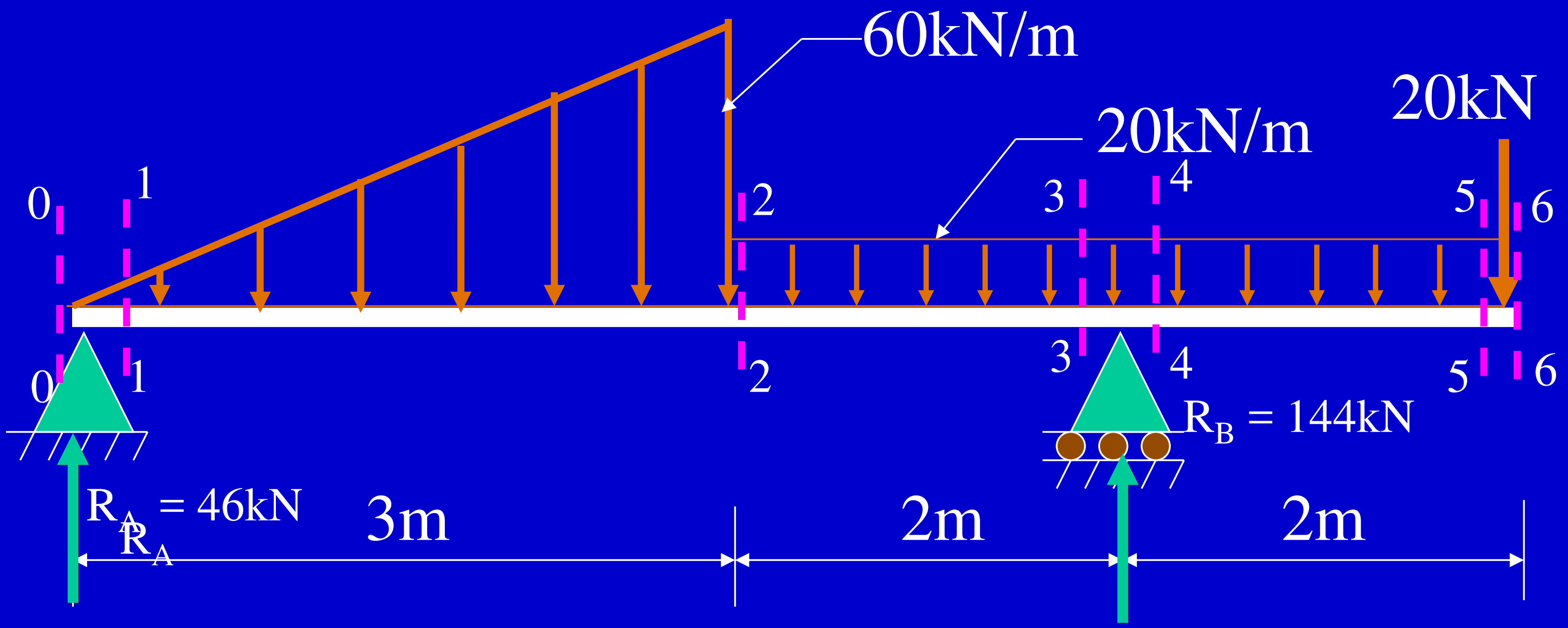
Solution: Calculation of reactions:

$\Sigma MA = 0$

$-R_B \times 5 + ½ \times 3 \times 60 \times (2/3) \times 3 + 20 \times 4 \times 5 + 20 \times 7 = 0 \rightarrow R_B = 144kN \uparrow$

$\Sigma Fy = 0$

$R_A + 144 – ½ \times 3 \times 60 – 20 \times 4 - 20 = 0 \rightarrow R_A = 46kN \uparrow$



Shear Force Calculations:

$V_{0-0} = 0$; $V_{1-1} = + 46$ kN

$V_{2-2} = +46 - \frac{1}{2} \times 3 \times 60 = - 44$ kN

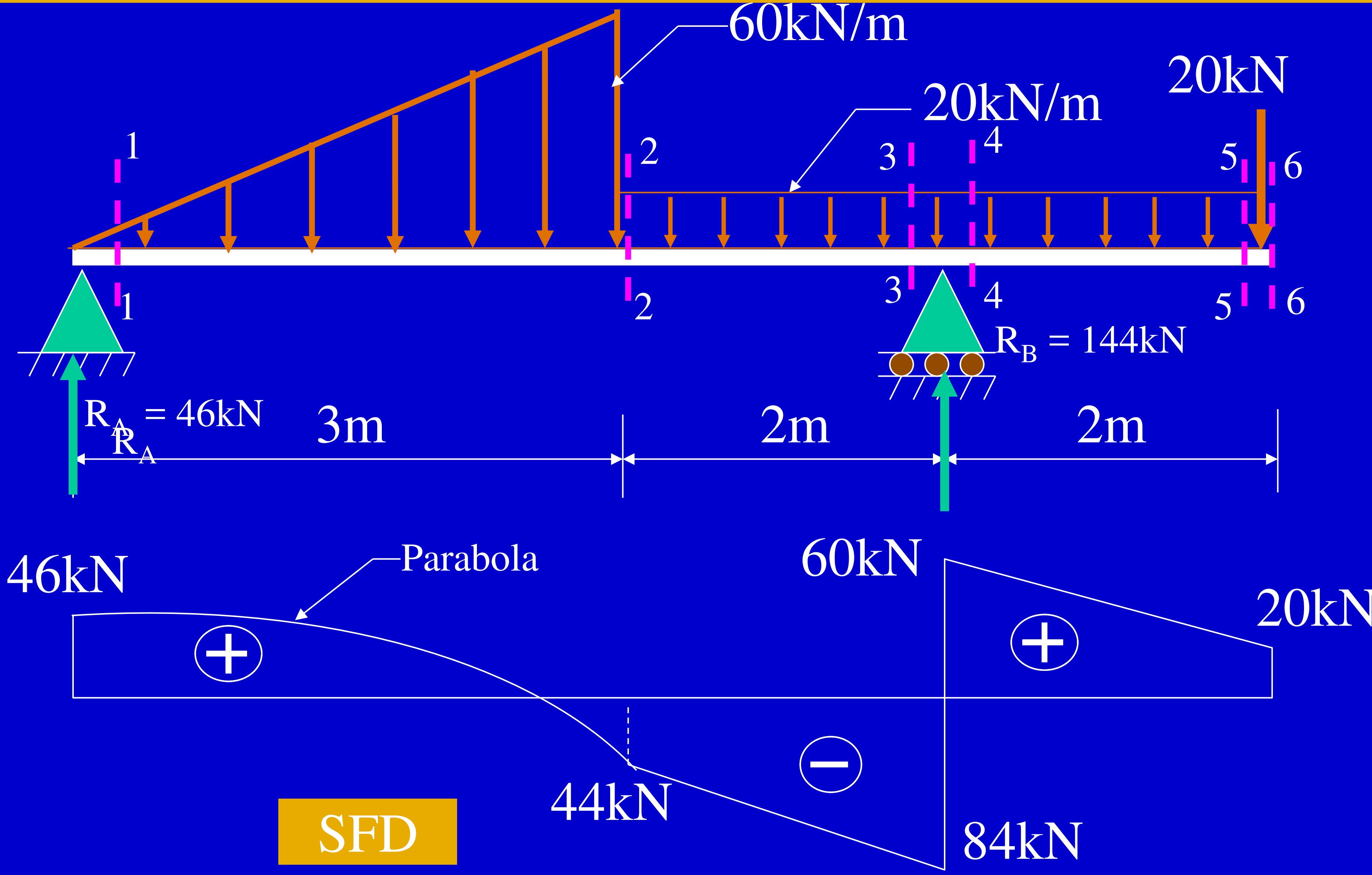
$V_{3-3} = - 44 - 20 \times 2 = - 84$ kN

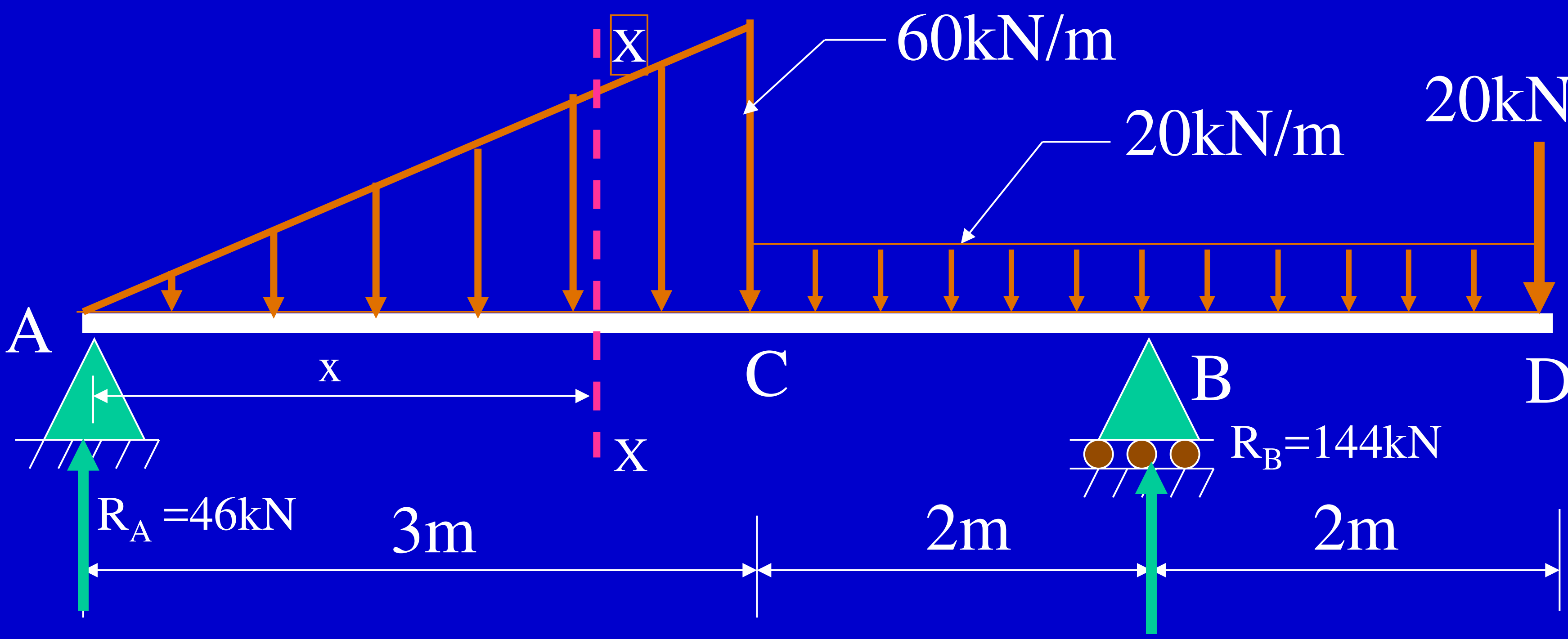
$V_{4-4} = - 84 + 144 = + 60$kN

$V_{5-5} = +60 - 20 \times 2 = + 20$ kN

$V_{6-6} = 20 - 20 = 0$ (Check)

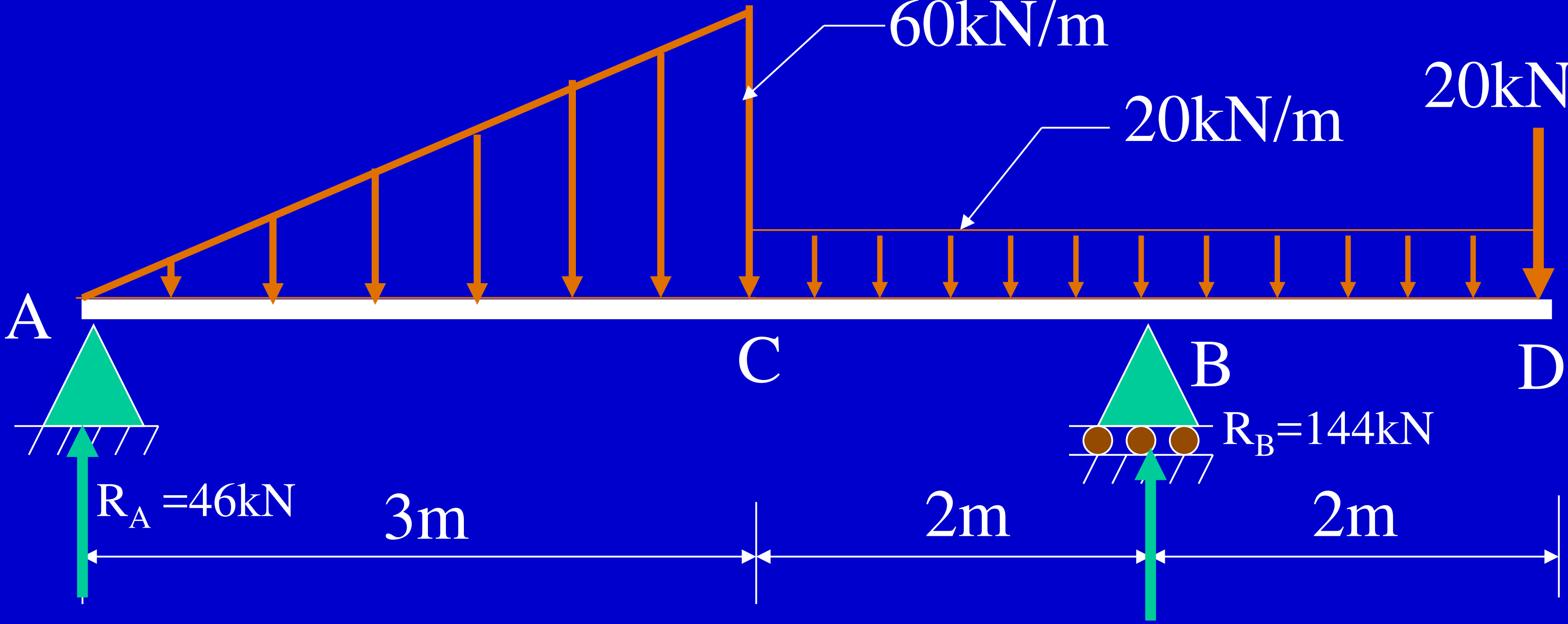
Example Problem 4





Max. bending moment will occur at the section where the shear force is zero. The SFD shows that the section having zero shear force is available in the portion AC. Let that section be X-X, considered at a distance 'x' from support A as shown above. The shear force expression at that section should be equated to zero. i.e.,

$V_{x-x} = 46 - \frac{1}{2} . x . (60/3) x = 0 \rightarrow x = 2.145$ m



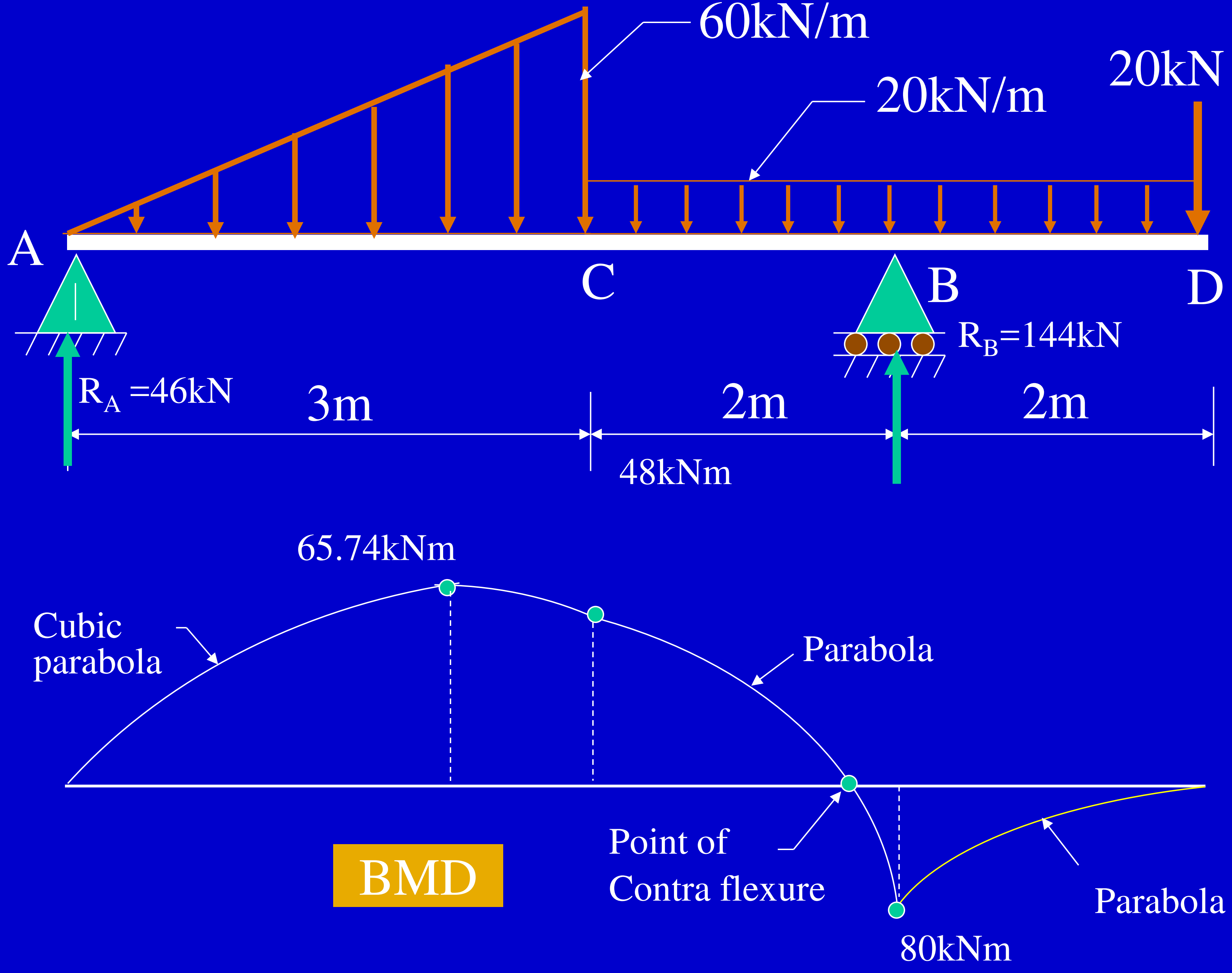
Calculation of bending moments:

$M_A = M_D = 0$

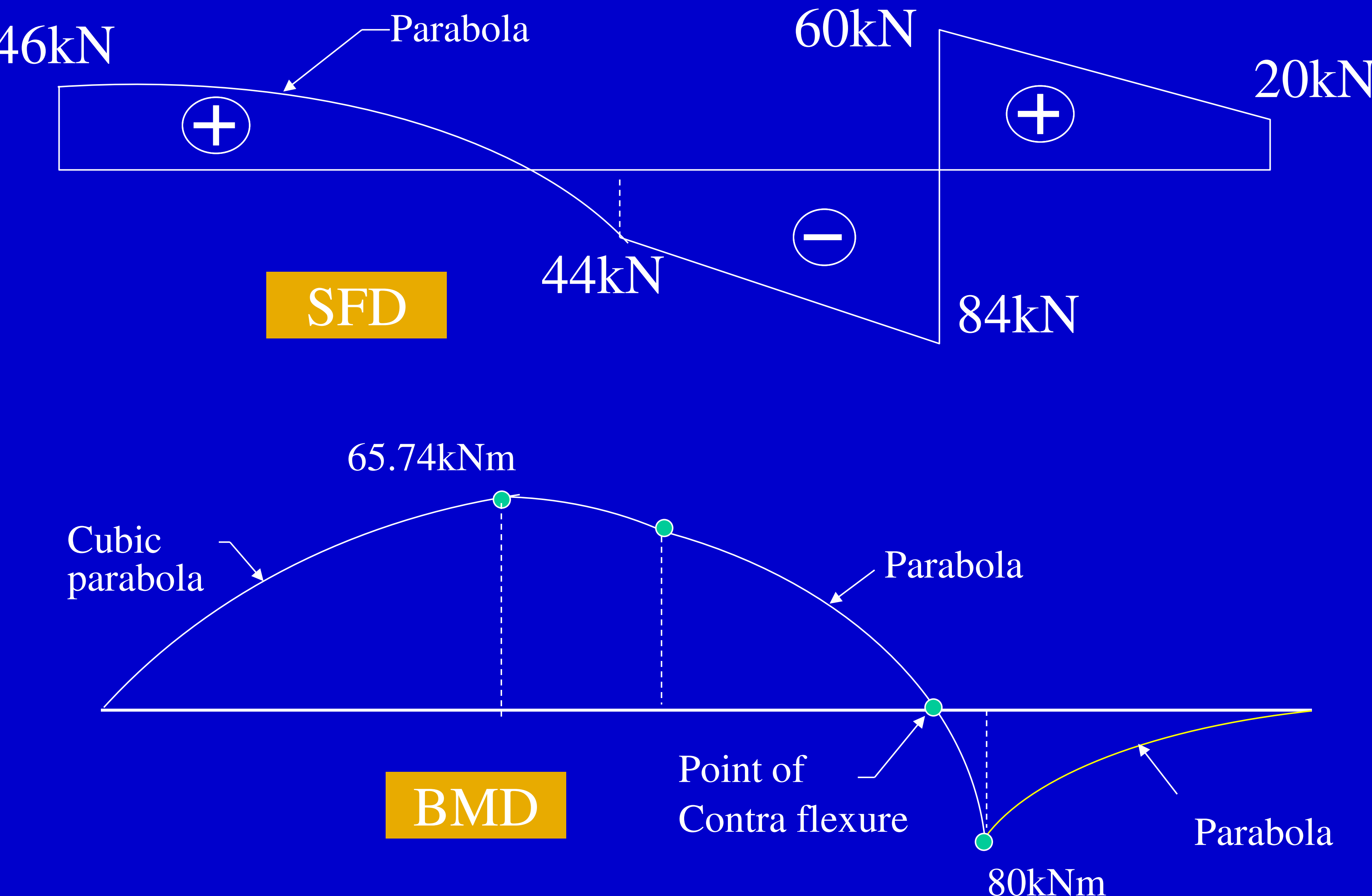
$M_C = 46 \times 3 - ½ \times 3 \times 60 \times (1/3 \times 3) = 48$ kNm[Considering LHS of section]

$M_B = -20 \times 2 - 20 \times 2 \times 1 = -80$ kNm [Considering RHS of section]

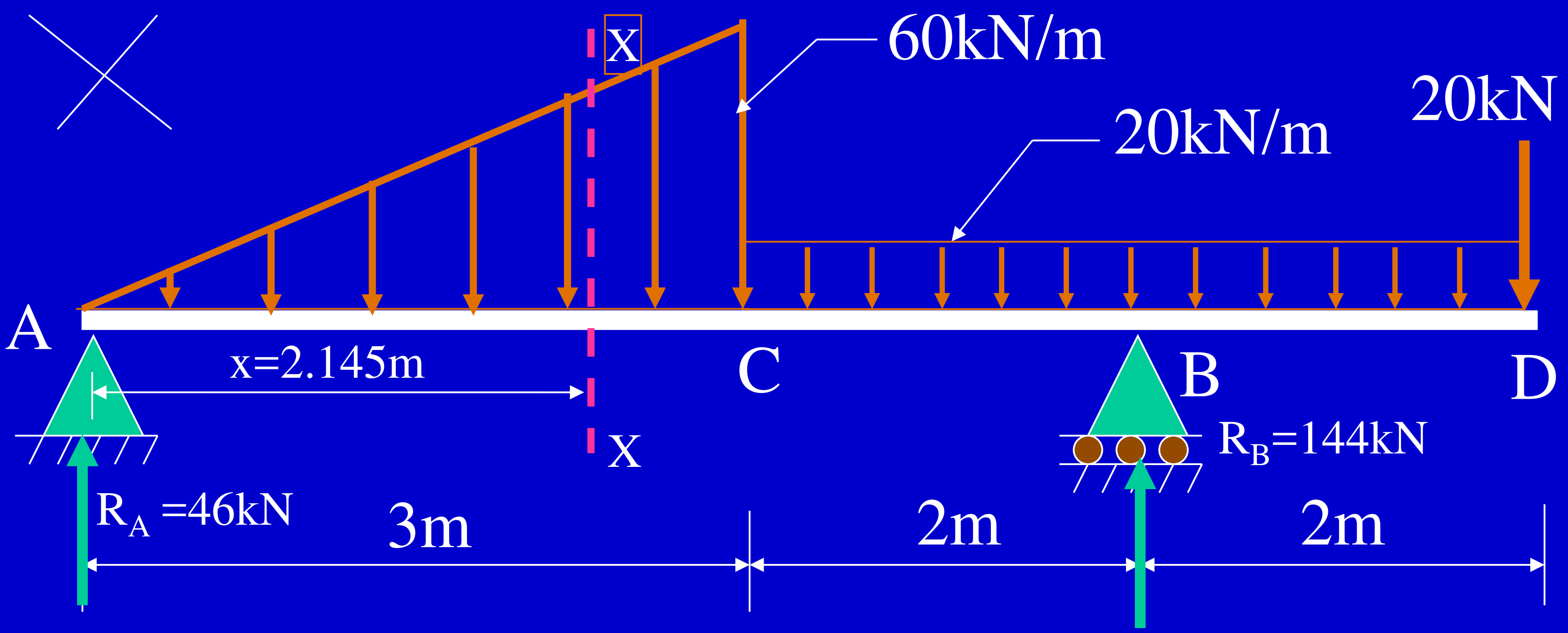
Absolute Maximum Bending Moment, Mmax $= 46 \times 2.145 - ½ \times 2.145 \times (2.145 \times 60/3) \times (1/3 \times 2.145) = 65.74$ kNm



60kN/m
20kN/m
20kN
A
C
B
D
R_B=144kN
R_A =46kN
3m
2m
2m
48kNm
65.74kNm
Cubic parabola
Parabola
Point of Contra flexure
BMD
Parabola
80kNm



46kN
Parabola
60kN
20kN
44kN
84kN
SFD
65.74kNm
Cubic parabola
Parabola
Point of Contra flexure
BMD
Parabola
80kNm

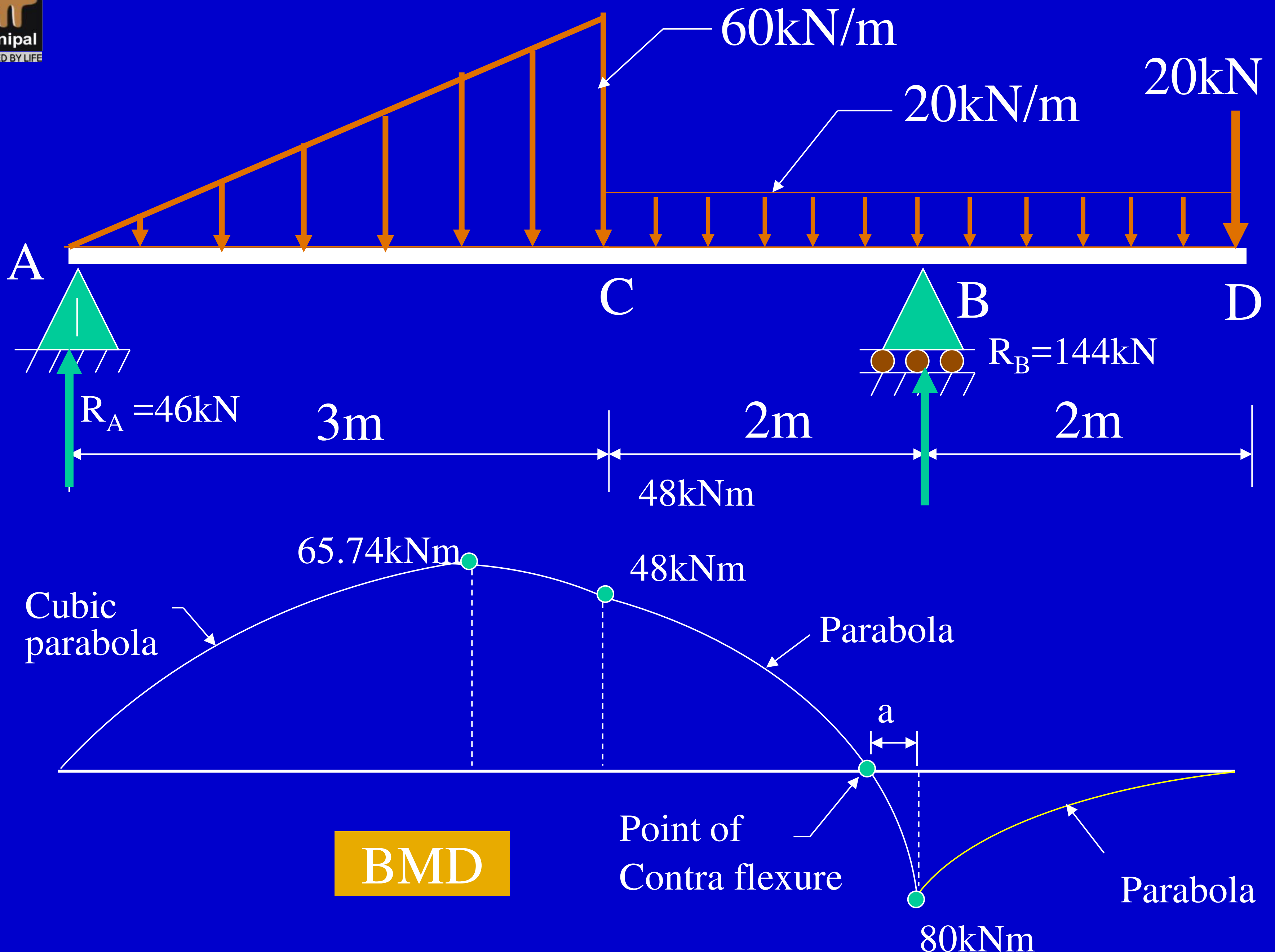


<u>Calculations of Absolute Maximum Bending Moment</u>:

Max. bending moment will occur at the section where the shear force is zero. The SFD shows that the section having zero shear force is available in the portion AC. Let that section be X-X, considered at a distance ‘x’ from support A as shown above. The shear force expression at that section should be equated to zero. i.e.,

$V_{x-x} = 46 – ½ .x. (60/3)x = 0$ → $x = 2.145$ m

BM at X- X , $M_{max} = 46 \times 2.145 – ½ \times 2.145 \times(2.145 \times 60/3) \times (1/3 \times 2.145)=65.74$ kNm


60kN/m
20kN/m
20kN
A
C
B
D
R$_B$=144kN
R$_A$ =46kN
3m
2m
2m
48kNm
65.74kNm
48kNm
Cubic parabola
Parabola
a
Point of Contra flexure
BMD
Parabola
80kNm

Point of contra flexure:

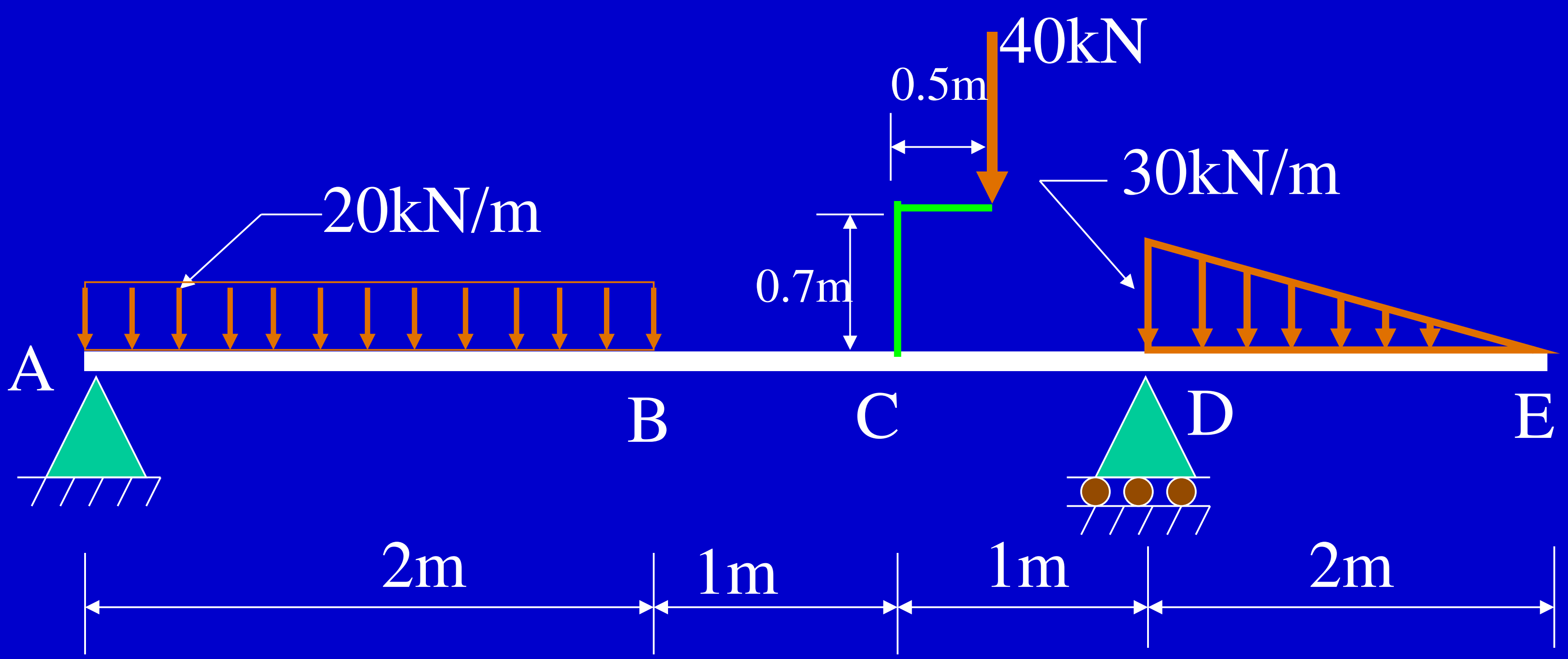
BMD shows that point of contra flexure is existing in the portion CB. Let '*a*' be the distance in the portion CB from the support B at which the bending moment is zero. And that '*a*' can be calculated as given below.

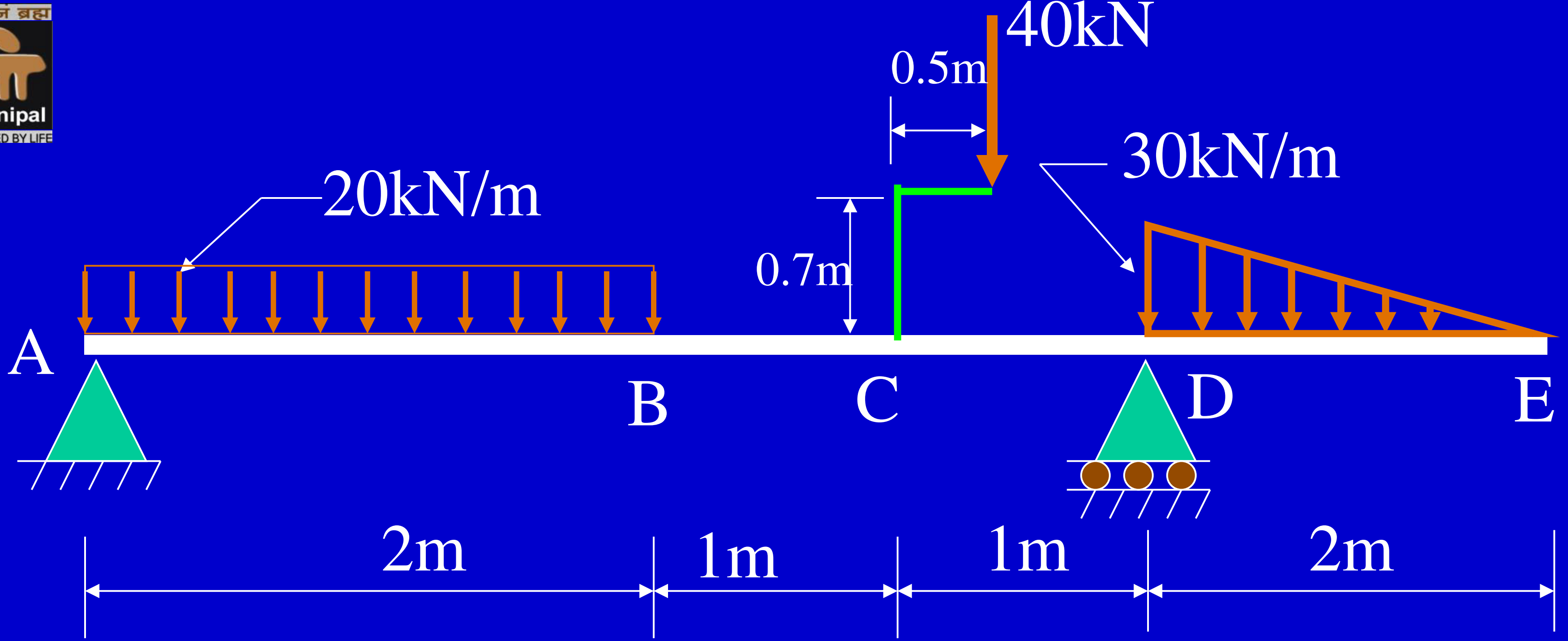
$\Sigma M_{x\text{-}x} = 0$

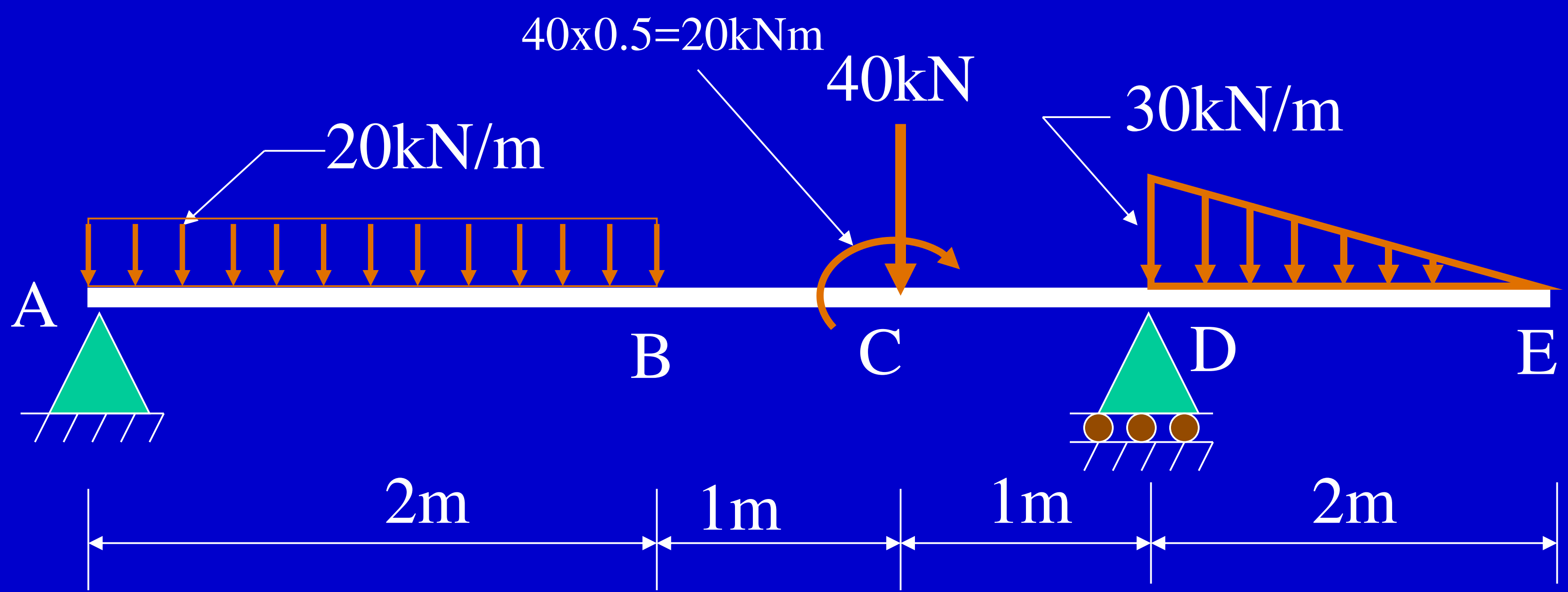
$$144a - 20(a+2) - 20\frac{(2+a)^2}{2} = 0$$

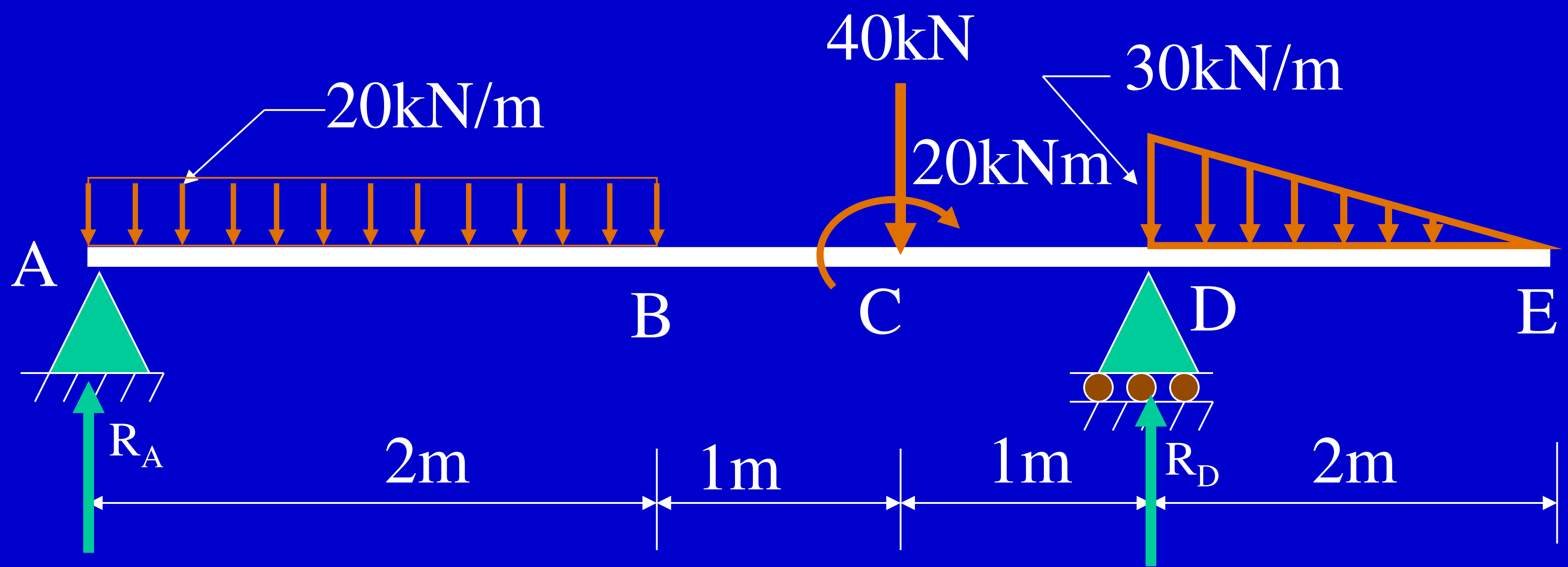
$a = 1.095$ m

5. Draw SFD and BMD for the single side overhanging beam subjected to loading as shown below. Mark salient points on SFD and BMD.




40kN
0.5m
30kN/m
20kN/m
0.7m
A
B
C
D
E
2m
1m
1m
2m


40x0.5=20kNm
40kN
30kN/m
20kN/m
A
B
C
D
E
2m
1m
1m
2m



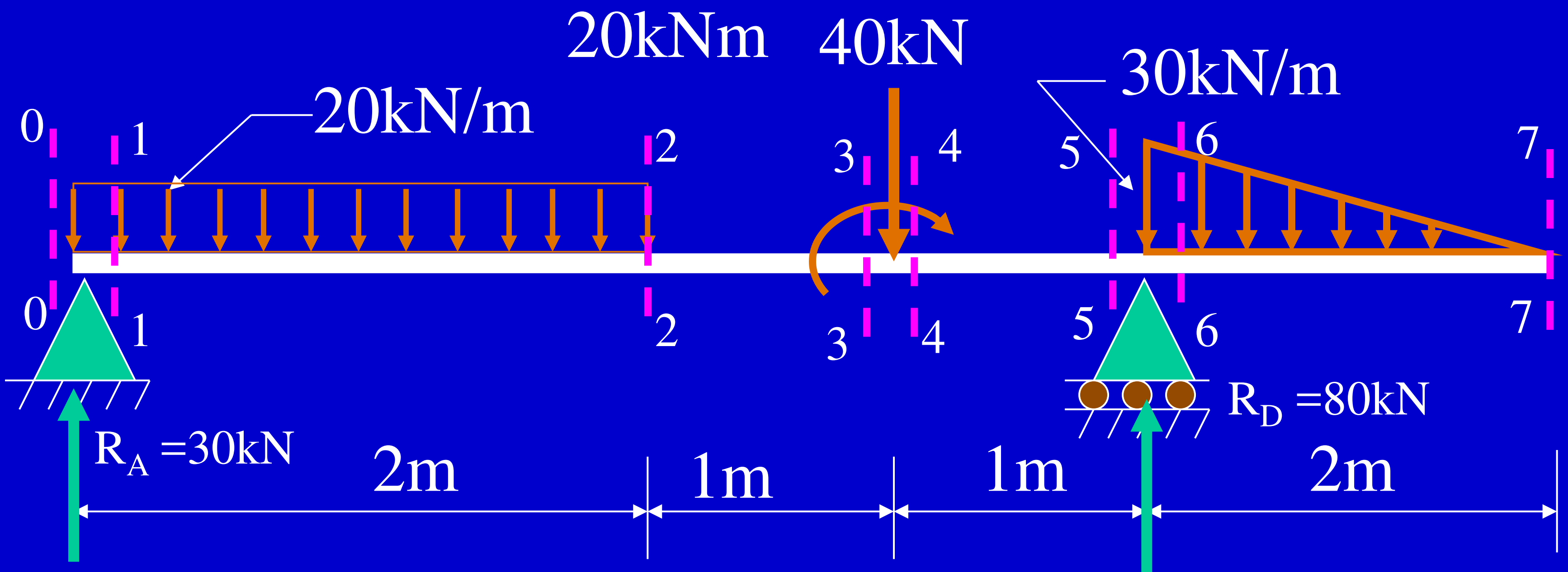
Solution: Calculation of reactions:

$\Sigma M_A = 0$

$-R_D \times 4 + 20 \times 2 \times 1 + 40 \times 3 + 20 + ½ \times 2 \times 30 \times (4+2/3) = 0 \rightarrow R_D = 80k$ [↑]

$\Sigma Fy = 0$

$R_A + 80 – 20 \times 2 - 40 - ½ \times 2 \times 30 = 0 \rightarrow R_A = 30$ kN [↑]



Calculation of Shear Forces: $V_{0\text{-}0} = 0$

$V_{1\text{-}1} = 30\text{ kN}$

$V_{2\text{-}2} = 30 - 20 \times 2 = -10\text{kN}$

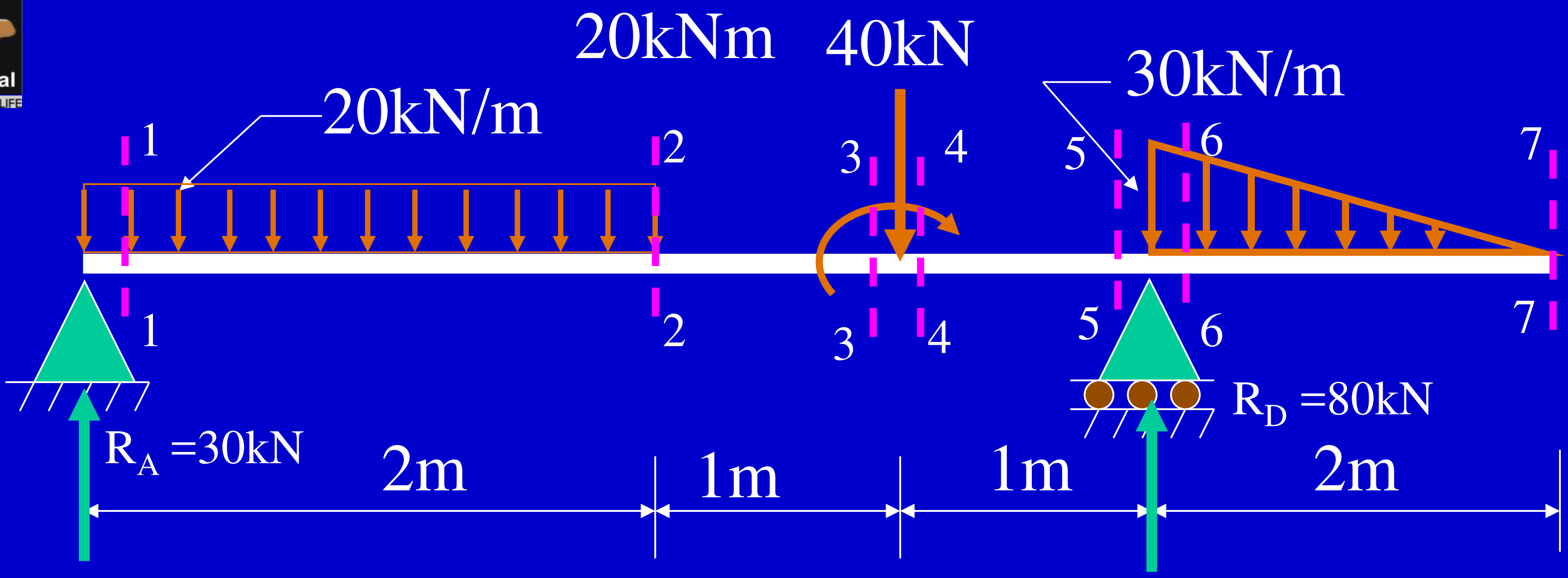
$V_{3\text{-}3} = -10\text{kN}$

$V_{4\text{-}4} = -10 - 40 = -50\text{ kN}$

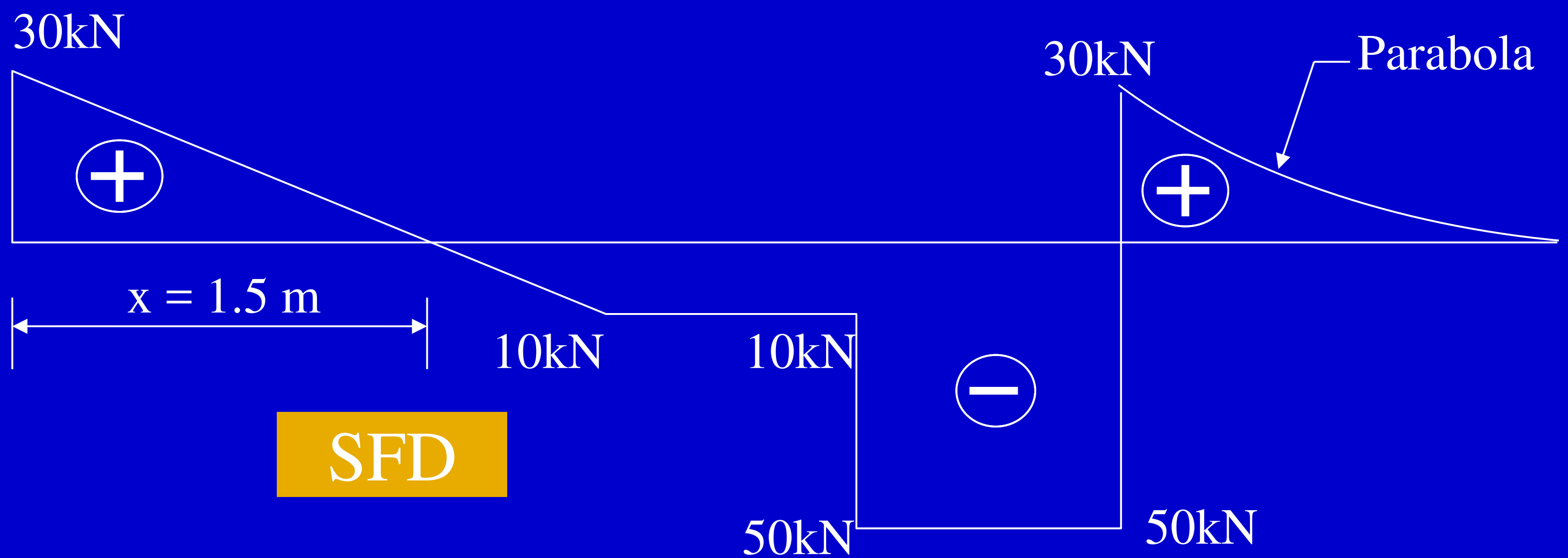
$V_{5\text{-}5} = -50\text{ kN}$

$V_{6\text{-}6} = -50 + 80 = +30\text{kN}$

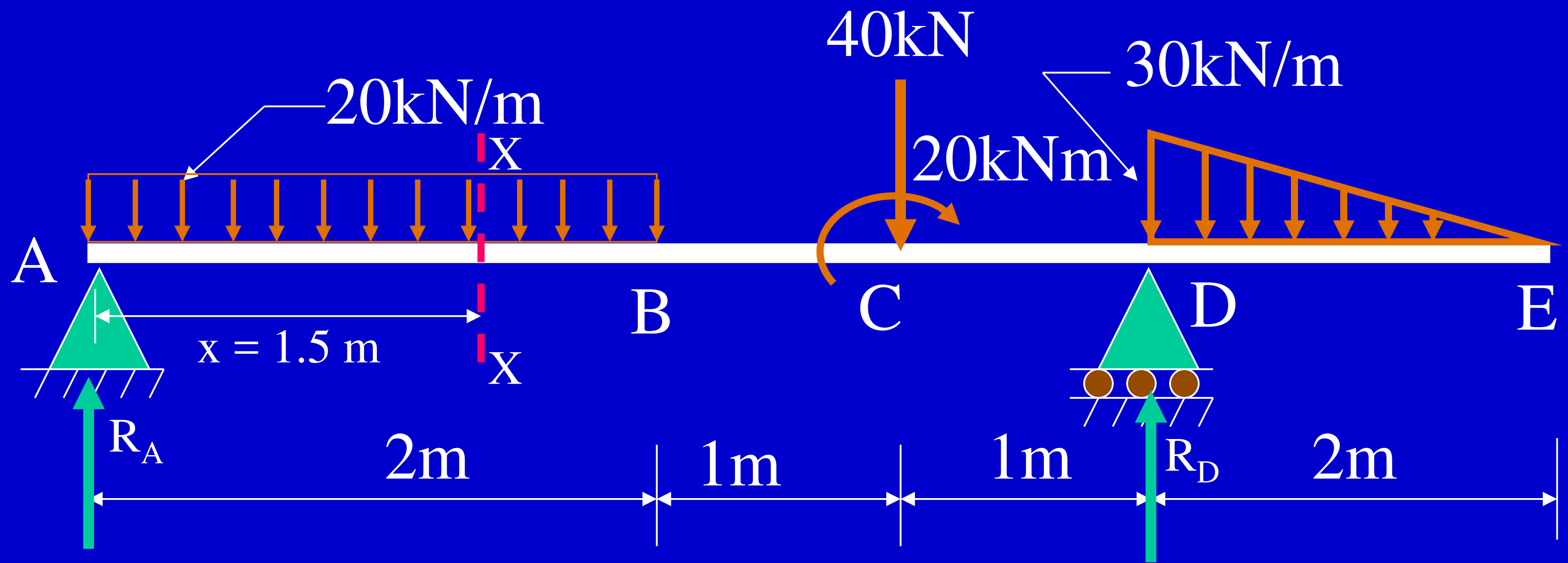
$V_{7\text{-}7} = +30 - ½ \times 2 \times 30 = 0\text{(check)}$



20kNm
40kN
30kN/m
20kN/m
1
2
3
4
5
6
7
RD =80kN
RA =30kN
2m
1m
1m
2m



30kN
30kN
Parabola
x = 1.5 m
10kN
10kN
50kN
50kN
SFD



<u>Calculation of bending moments</u>:

$M_A = M_E = 0$

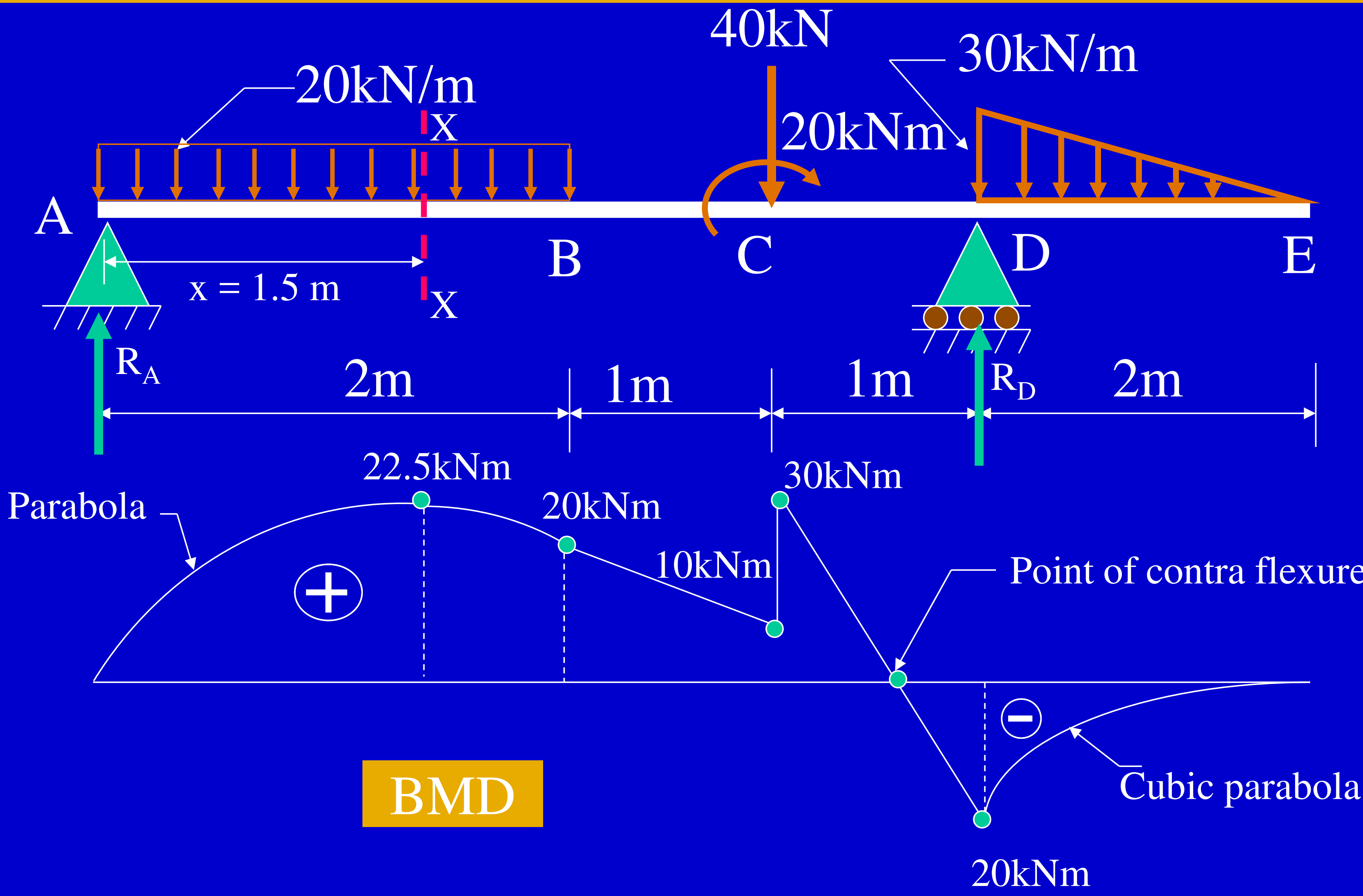
$M_X = 30 \times 1.5 - 20 \times 1.5 \times 1.5/2 = 22.5$ kNm

$M_B = 30 \times 2 - 20 \times 2 \times 1 = 20$ kNm

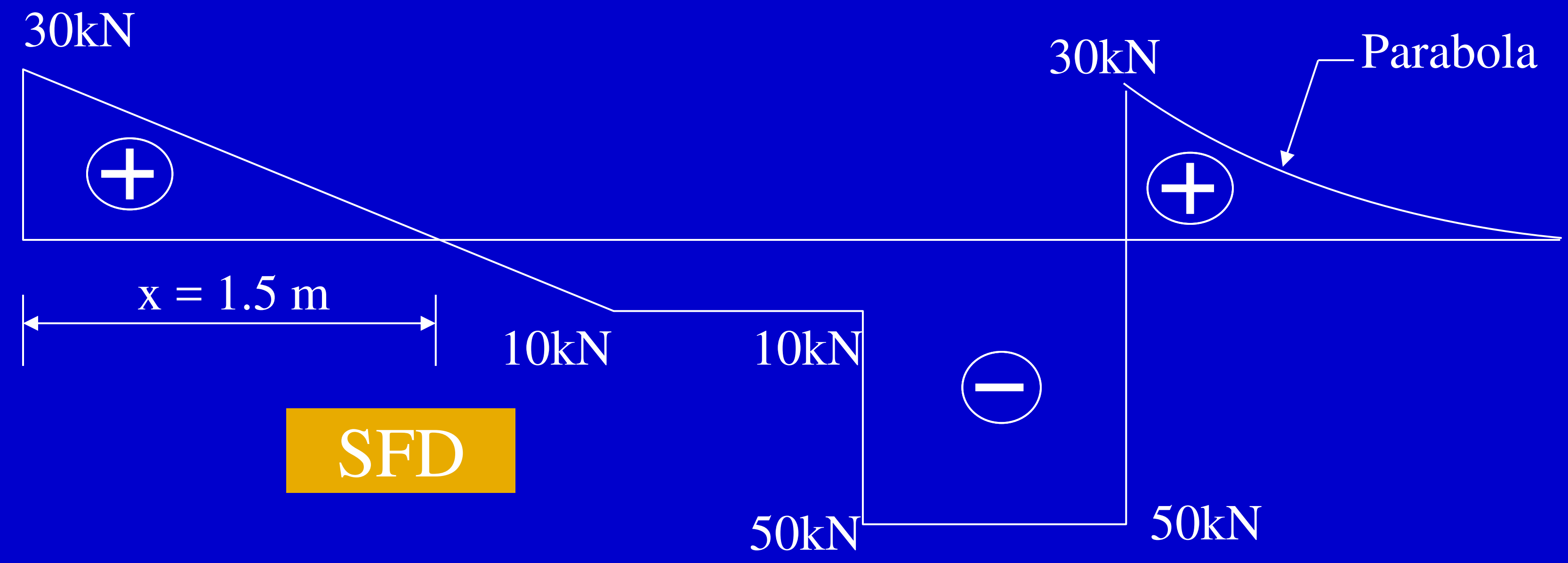
$M_C = 30 \times 3 - 20 \times 2 \times 2 = 10$ kNm (section before the couple)

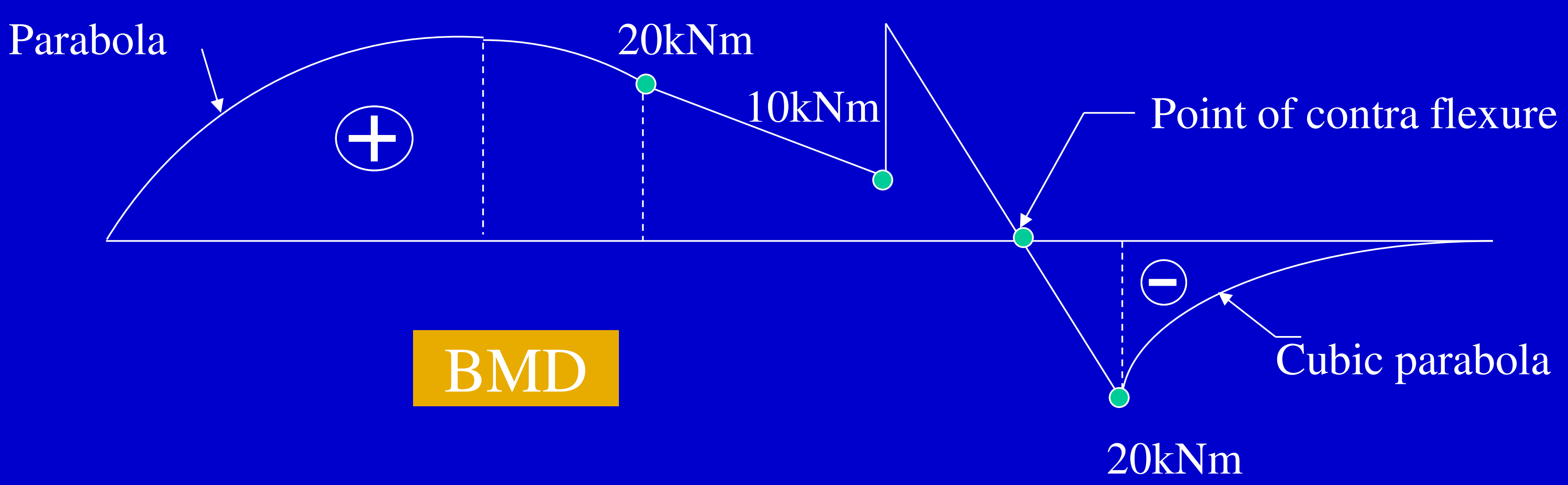
$M_C = 10 + 20 = 30$ kNm (section after the couple)

$M_D = -\frac{1}{2} \times 30 \times 2 \times (1/3 \times 2) = -20$ kNm(Considering RHS of the sectio

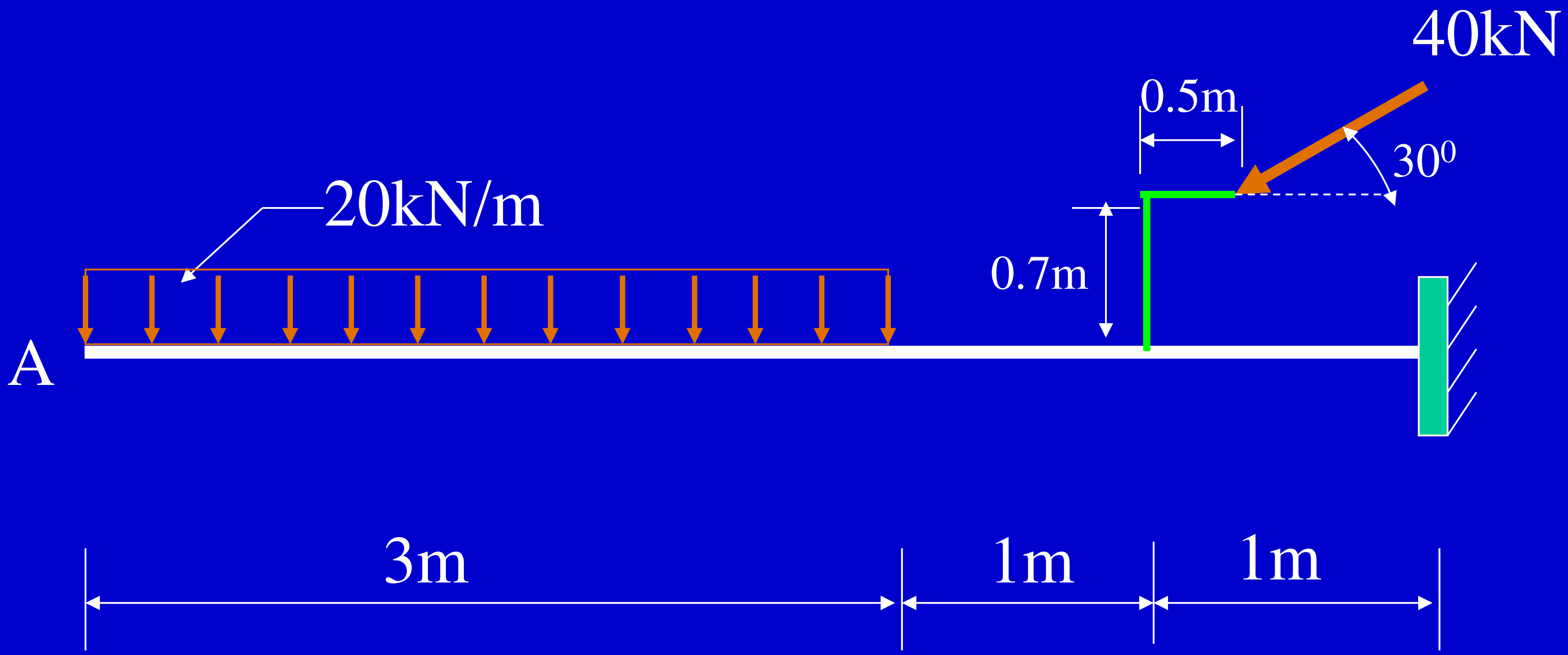


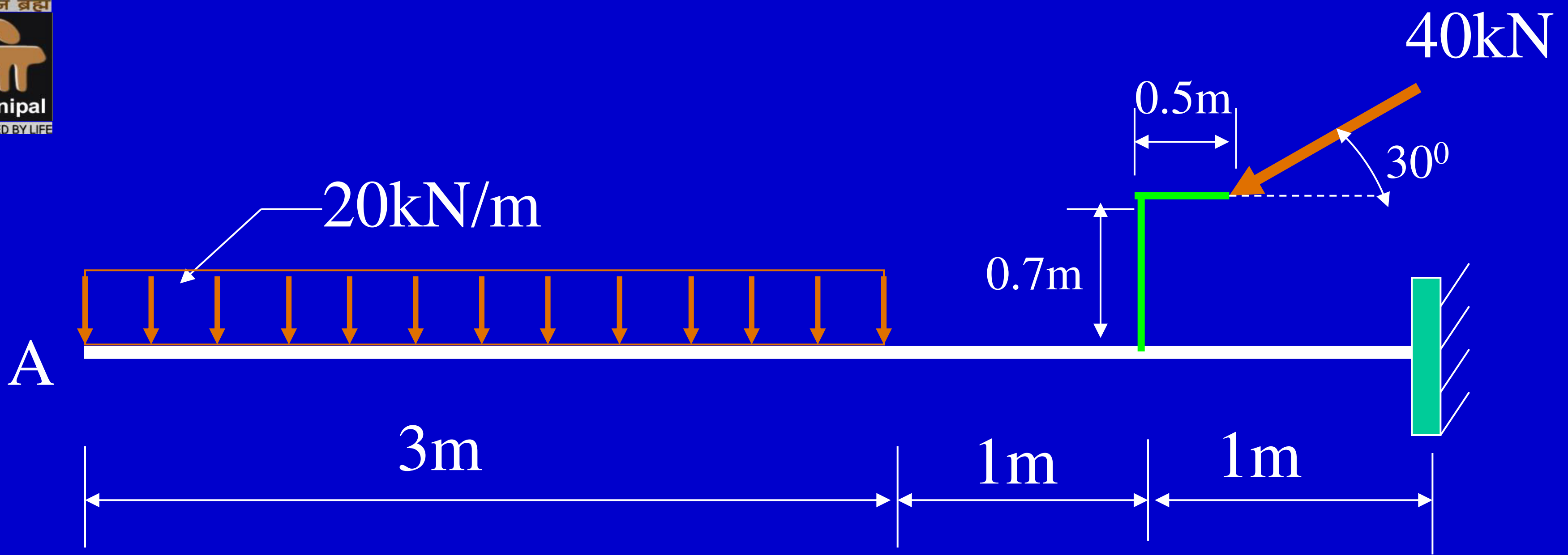
40kN
30kN/m
20kN/m
20kNm
X
A
B
C
D
E
x = 1.5 m
X
R_A
R_D
2m
1m
1m
2m
22.5kNm
30kNm
Parabola
20kNm
10kNm
Point of contra flexure
+
−
Cubic parabola
BMD
20kNm

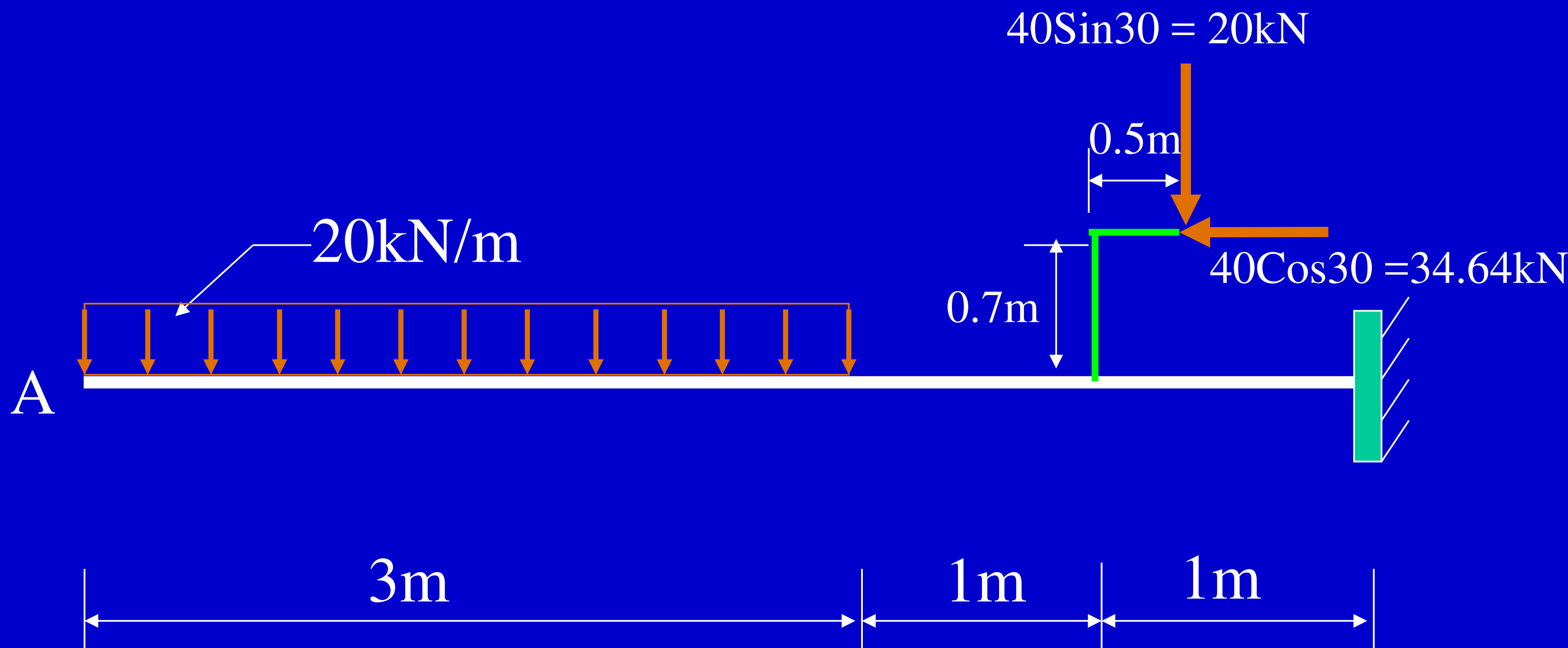

30kN
30kN
Parabola
+
+
x = 1.5 m
10kN
10kN
−
SFD
50kN
50kN

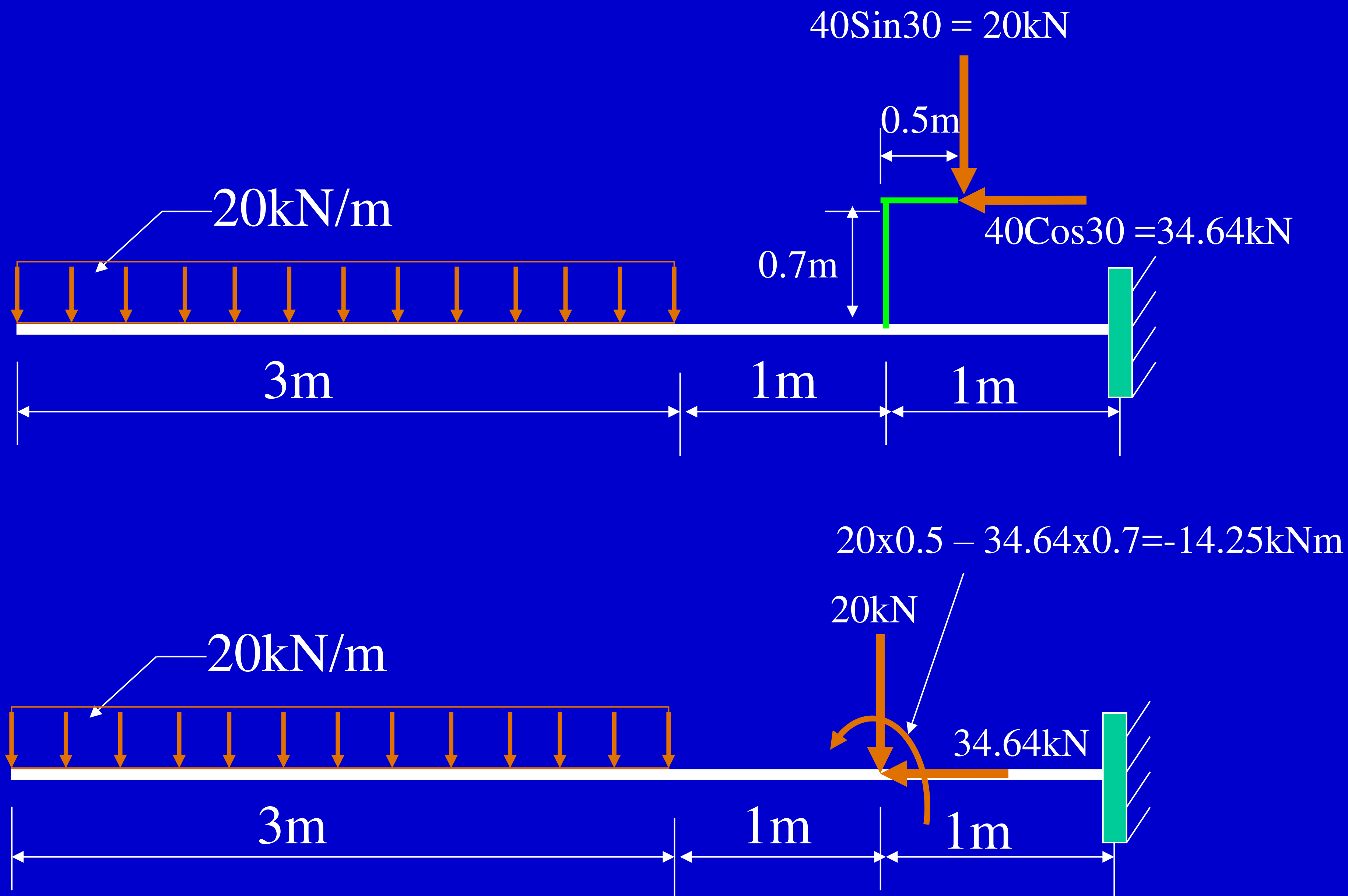

Parabola
20kNm
10kNm
+
Point of contra flexure
−
Cubic parabola
BMD
20kNm

6. Draw SFD and BMD for the cantilever beam subjected to loading as shown below.

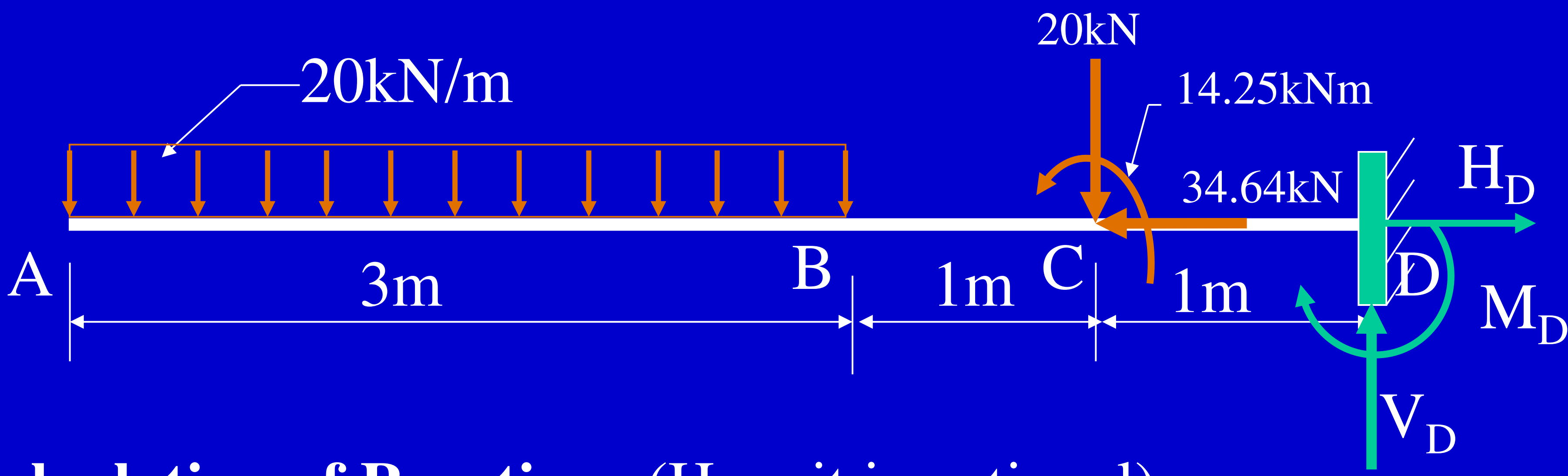



40kN
0.5m
30^0
20kN/m
0.7m
A
3m
1m
1m


40Sin30 = 20kN
0.5m
20kN/m
40Cos30 =34.64kN
0.7m
A
3m
1m
1m



40Sin30 = 20kN
0.5m
20kN/m
0.7m
40Cos30 =34.64kN
3m
1m
1m
20x0.5 – 34.64x0.7=-14.25kNm
20kN
20kN/m
34.64kN
3m
1m
1m

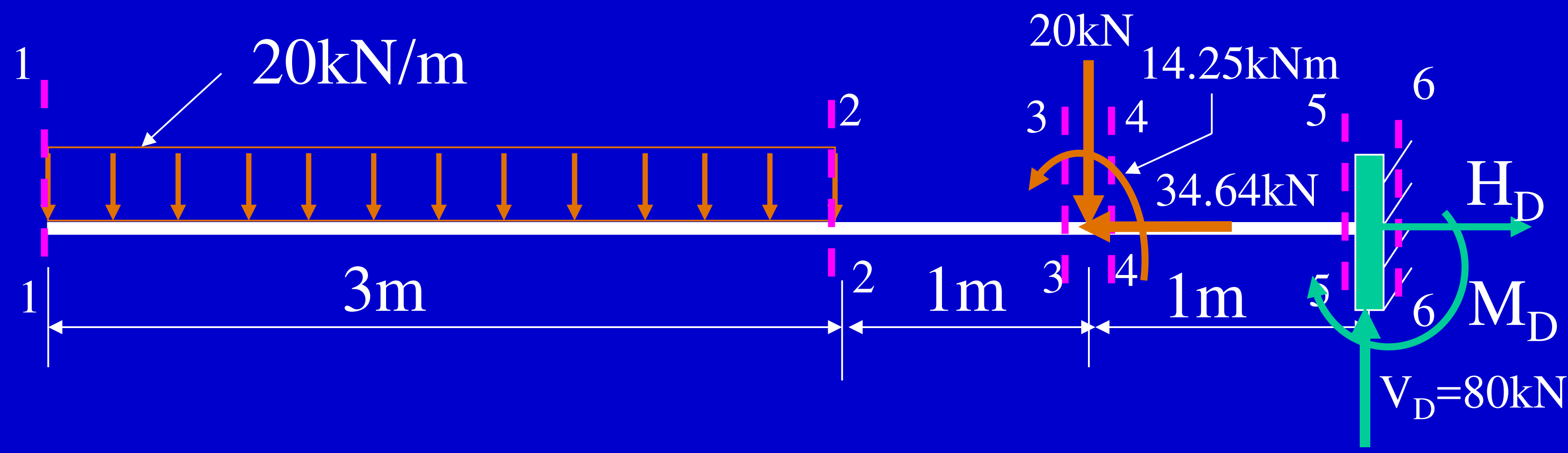


Calculation of Reactions (Here it is optional):

$\Sigma F_x = 0 \rightarrow \quad H_D = 34.64$ kN

$\Sigma F_y = 0 \rightarrow \quad V_D = 20 \times 3 + 20 = 80$ kN

$\Sigma M_D = 0 \rightarrow \quad M_D - 20 \times 3 \times 3.5 - 20 \times 1 - 14.25 = 244.25$kNm



Shear Force Calculation:

$V_{1-1} = 0$

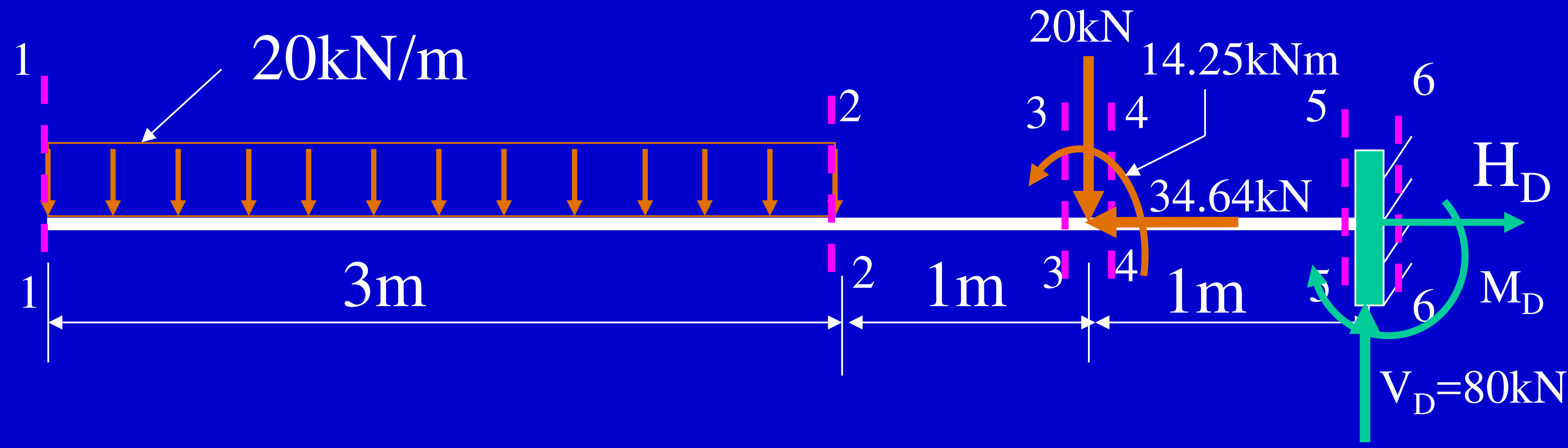
$V_{2-2} = -20 \times 3 = -60kN$

$V_{3-3} = -60\ kN$

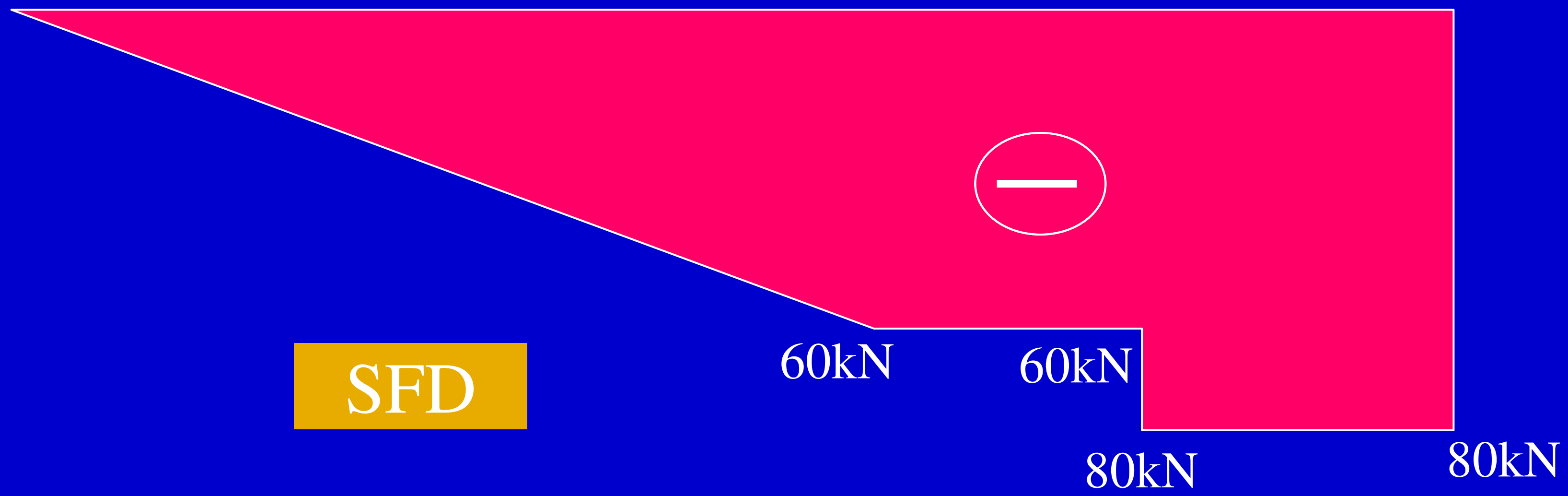
$V_{4-4} = -60 - 20 = -80\ kN$

$V_{5-5} = -80\ kN$

$V_{6-6} = -80 + 80 = 0$ (Check)

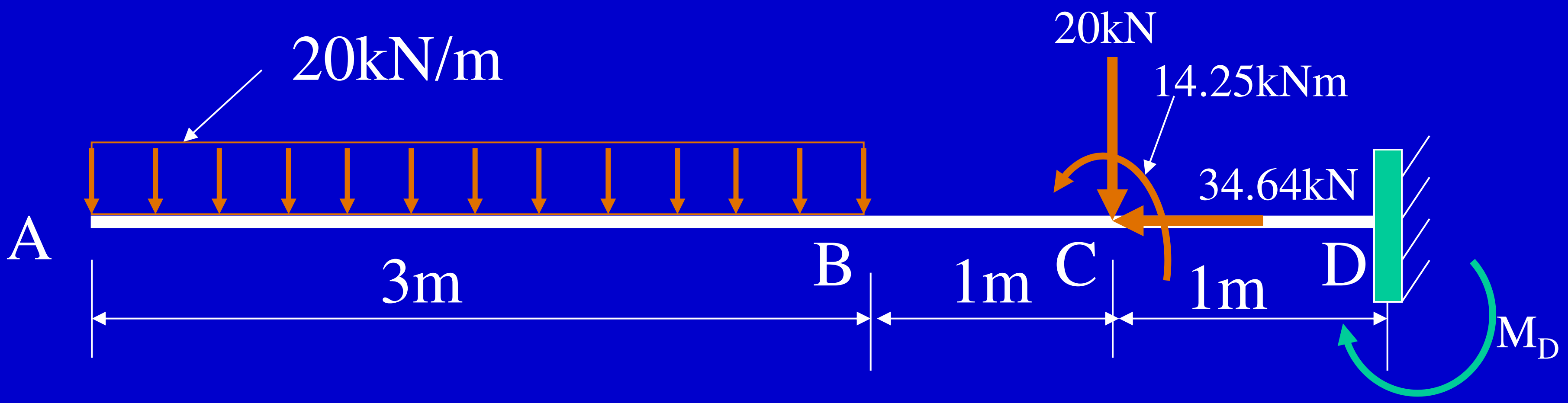


20kN/m
20kN
14.25kNm
34.64kN
H_D
M_D
V_D=80kN
1
2
3
4
5
6
3m
1m
1m



60kN
60kN
80kN
80kN

SFD



Bending Moment Calculations:

$M_A = 0$

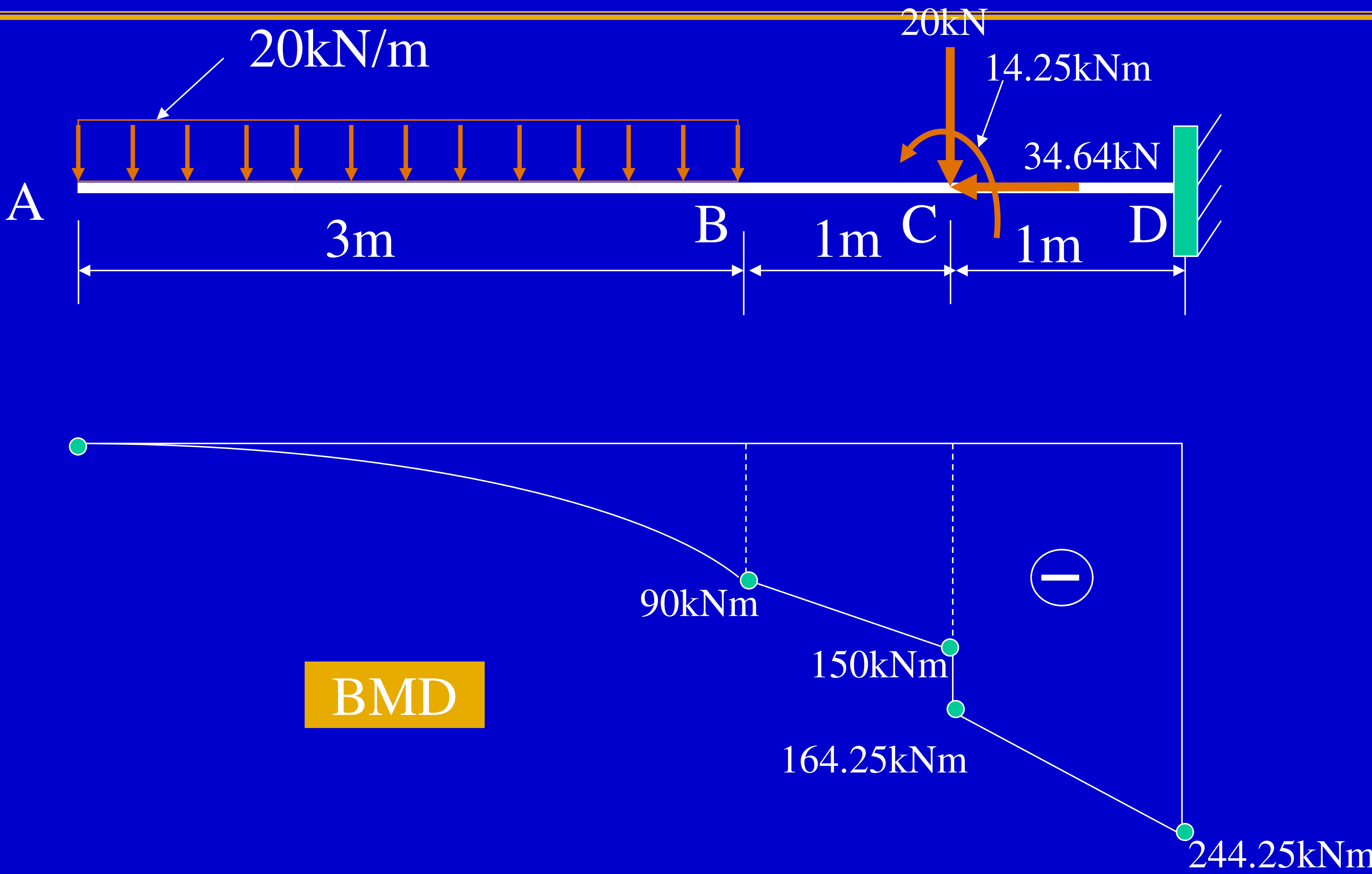
$M_B = - 20 \times 3 \times 1.5 = - 90$ kNm

$M_C = - 20 \times 3 \times 2.5 = - 150$ kNm (section before the couple)

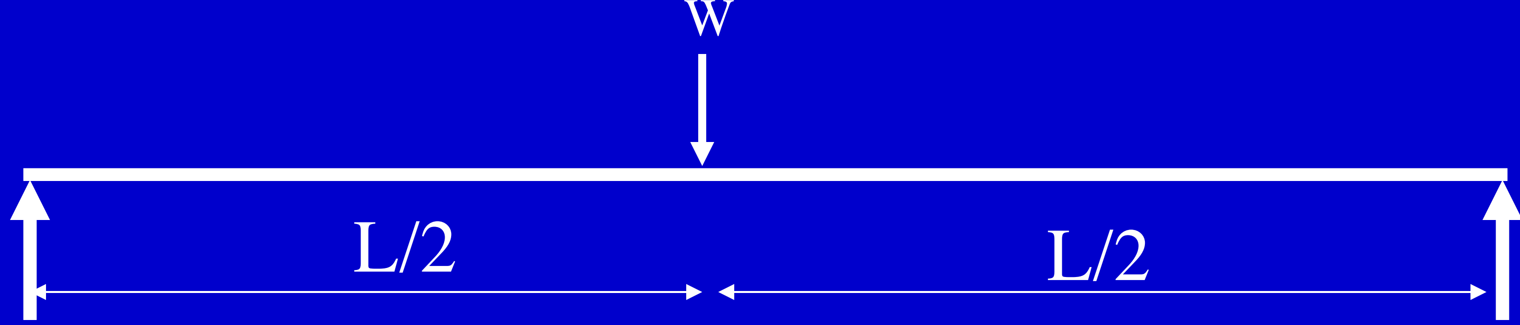
$M_C = - 20 \times 3 \times 2.5 – 14.25 = -164.25$ kNm (section after the couple)

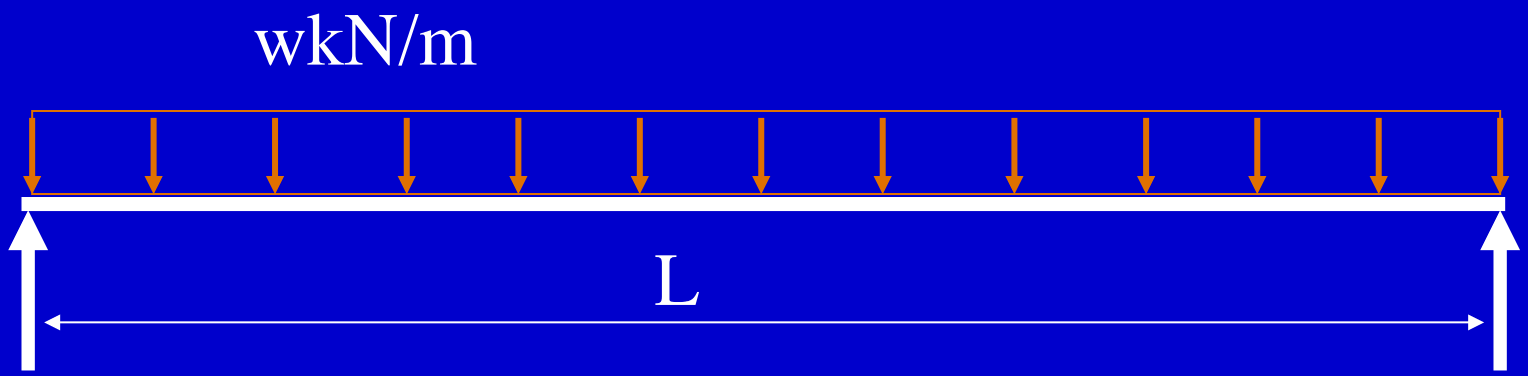
$M_D = - 20 \times 3 \times 3.5 -14.25 – 20 \times 1 = -244.25$ kNm (section before M_D) moment)

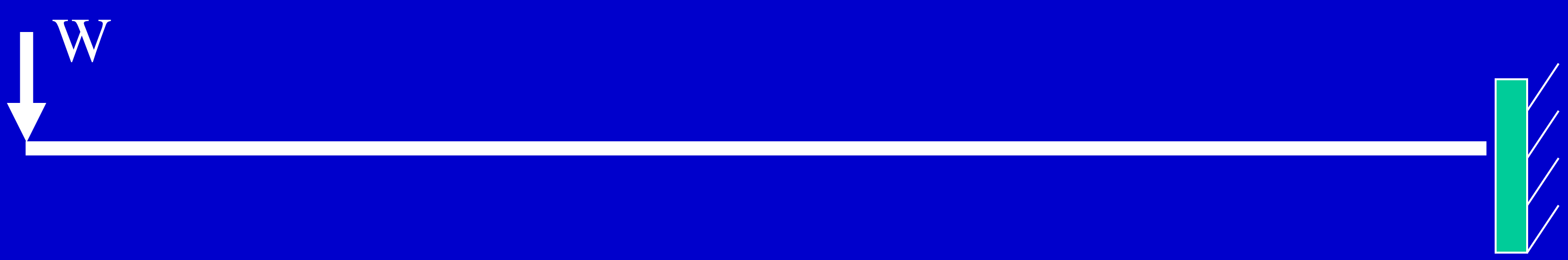
MD = -244.25 +244.25 = 0 (section after M_D)

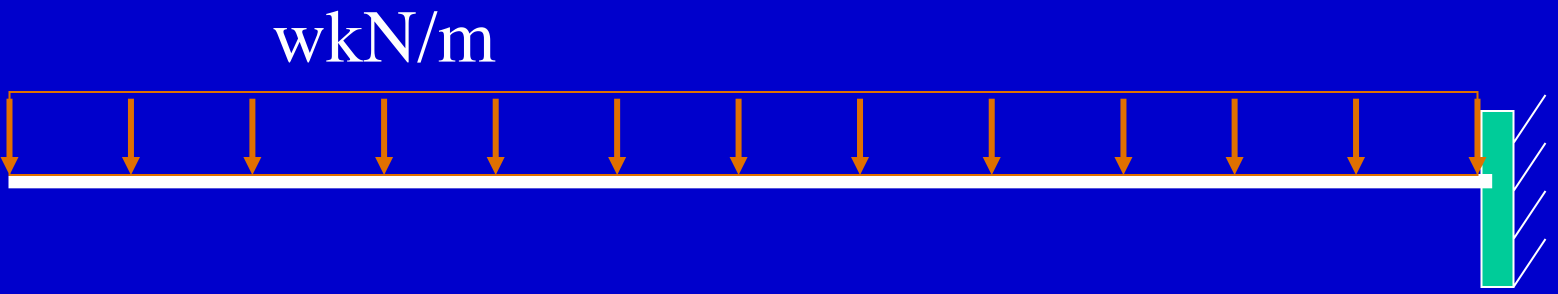


20kN/m
20kN
14.25kNm
34.64kN
A
B
C
D
3m
1m
1m
90kNm
150kNm
164.25kNm
244.25kNm
BMD


W
L/2
L/2

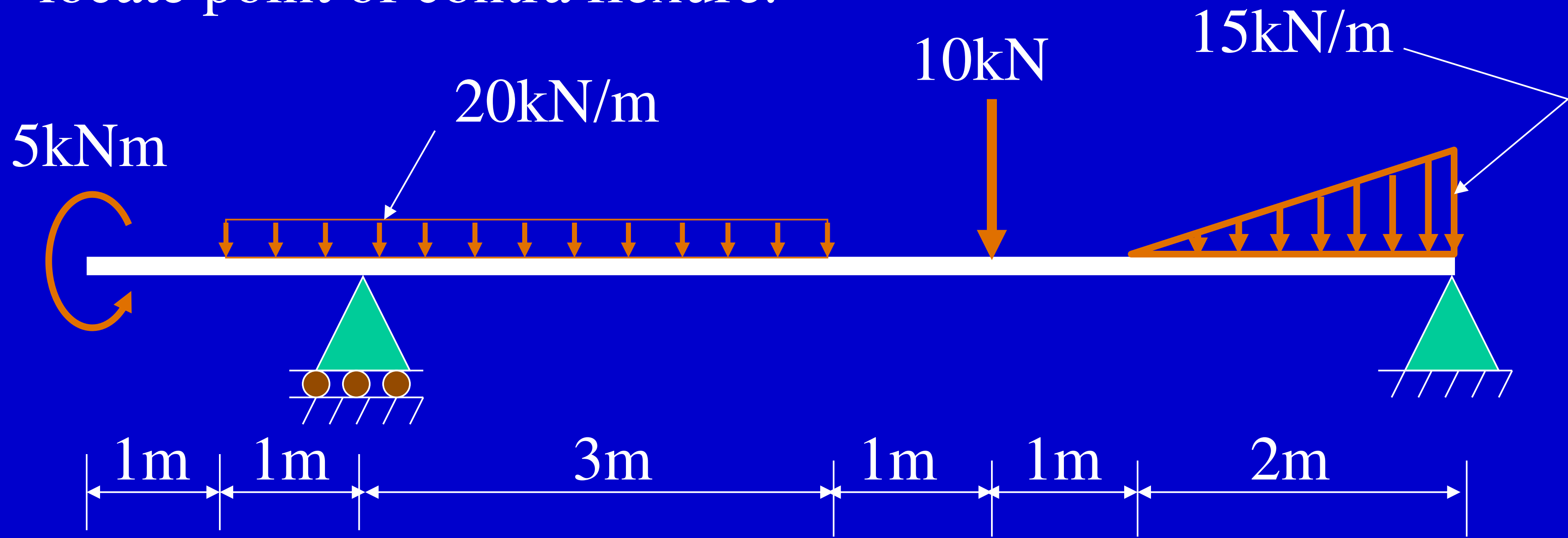

wkN/m
L


W


wkN/m

Exercise Problems

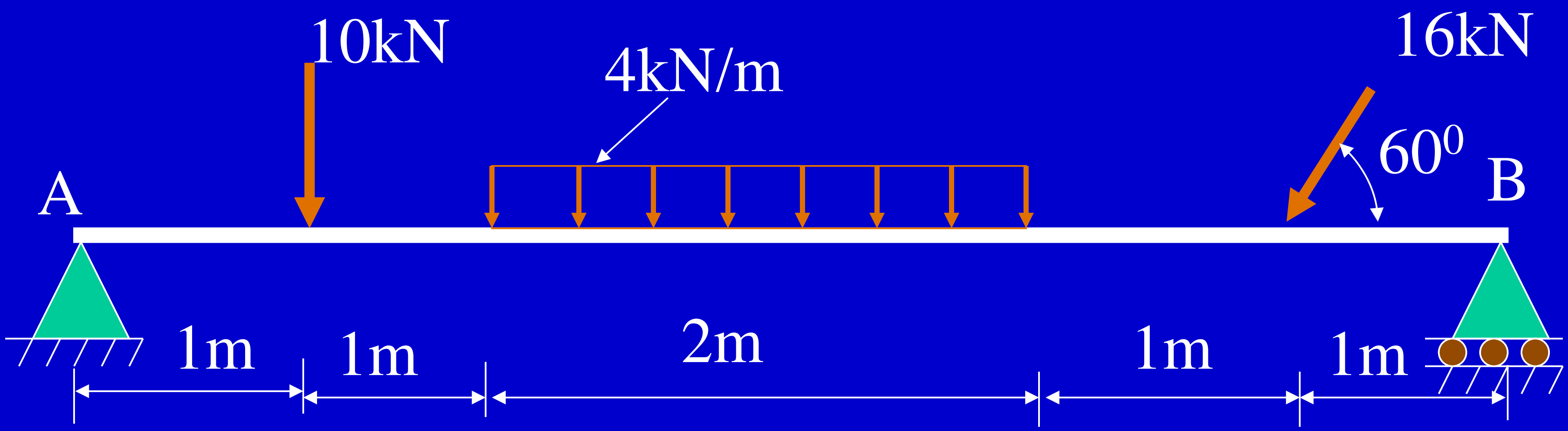
1. Draw SFD and BMD for a single side overhanging beam subjected to loading as shown below. Mark absolute maximum bending moment on bending moment diagram and locate point of contra flexure.



[Ans: Absolute maximum BM = 60.625 kNm]

Exercise Problems

2. Draw shear force and bending moment diagrams [SFD and BMD] for a simply supported beam subjected to loading as shown in the Fig. given below. Also locate and determine absolute maximum bending moment.

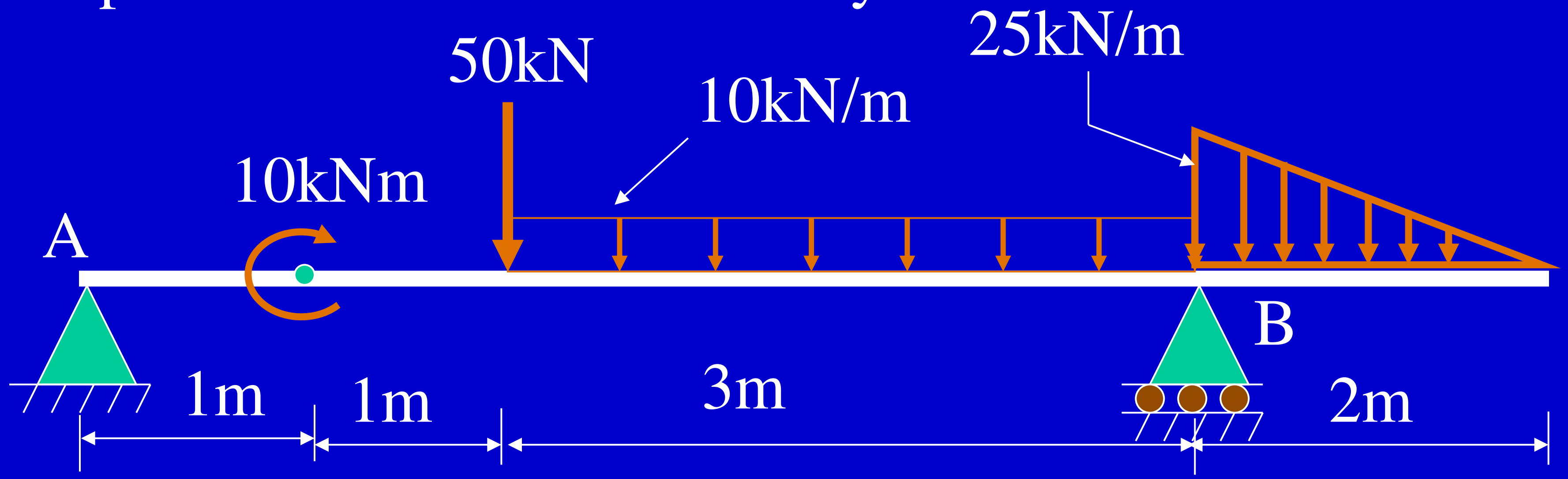


[Ans: Absolute maximum bending moment = 22.034kNm

Its position is 3.15m from Left hand support]

Exercise Problems

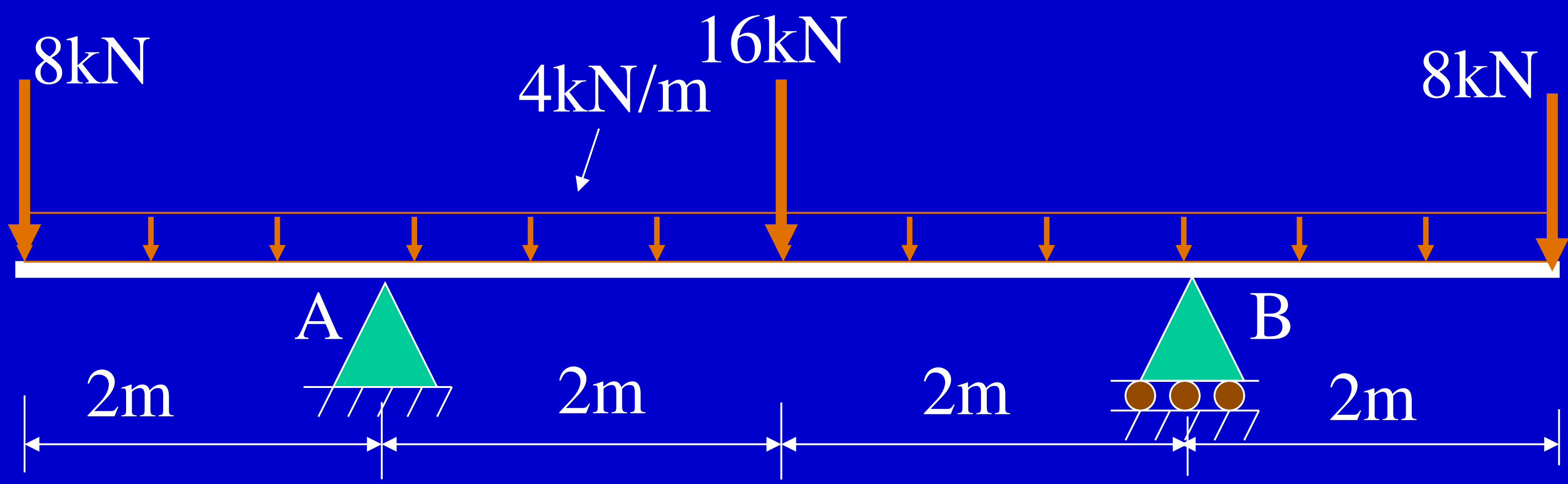
3. Draw shear force and bending moment diagrams [SFD and BMD] for a single side overhanging beam subjected to loading as shown in the Fig. given below. Locate points of contra flexure if any.



[Ans : Position of point of contra flexure from RHS = 0.375m]

Exercise Problems

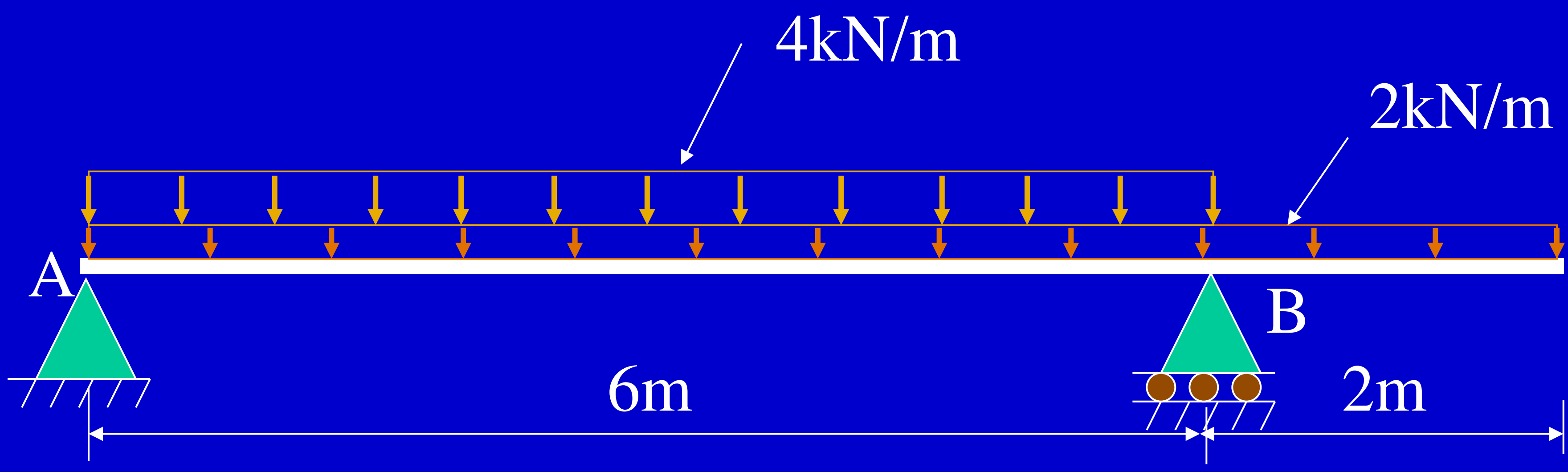
4. Draw SFD and BMD for a double side overhanging beam subjected to loading as shown in the Fig. given below. Locate the point in the AB portion where the bending moment is zero.



[Ans : Bending moment is zero at mid span]

Exercise Problems

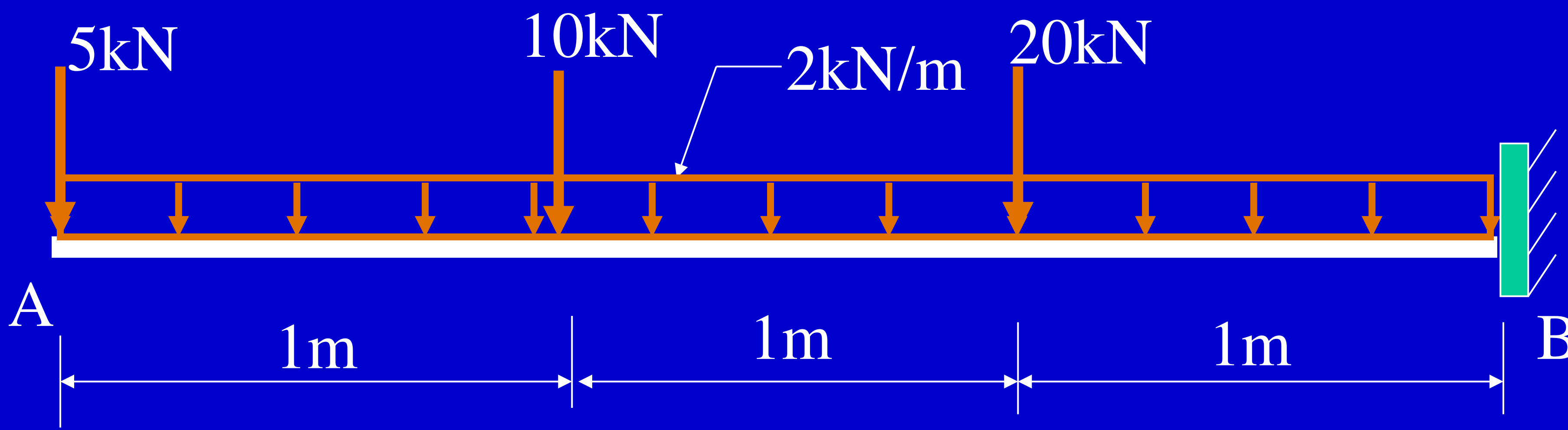
5. A single side overhanging beam is subjected to uniformly distributed load of 4 kN/m over AB portion of the beam in addition to its self weight 2 kN/m acting as shown in the Fig. given below. Draw SFD and BMD for the beam. Locate the inflection points if any. Also locate and determine maximum negative and positive bending moments.



[Ans :Max. positive bending moment is located at 2.89 m from LHS. and whose value is 37.57 kNm]

Exercise Problems

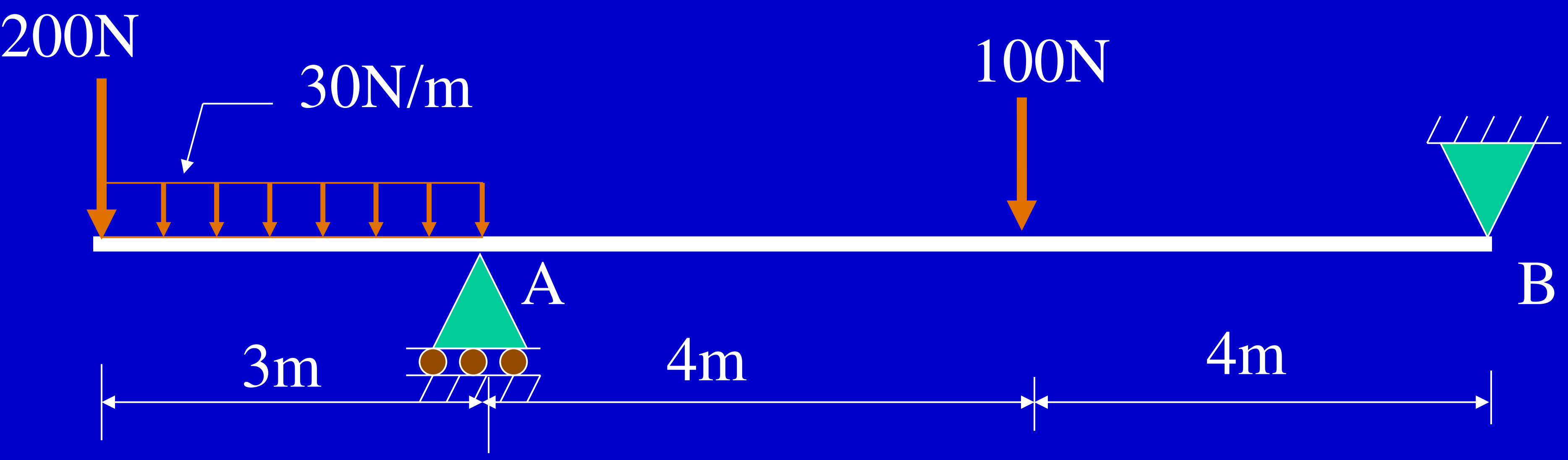
6. Three point loads and one uniformly distributed load are acting on a cantilever beam as shown in the Fig. given below. Draw SFD and BMD for the beam. Locate and determine maximum shear force and bending moments.



[Ans : Both Shear force and Bending moments are maximum at supports.]

Exercise Problems

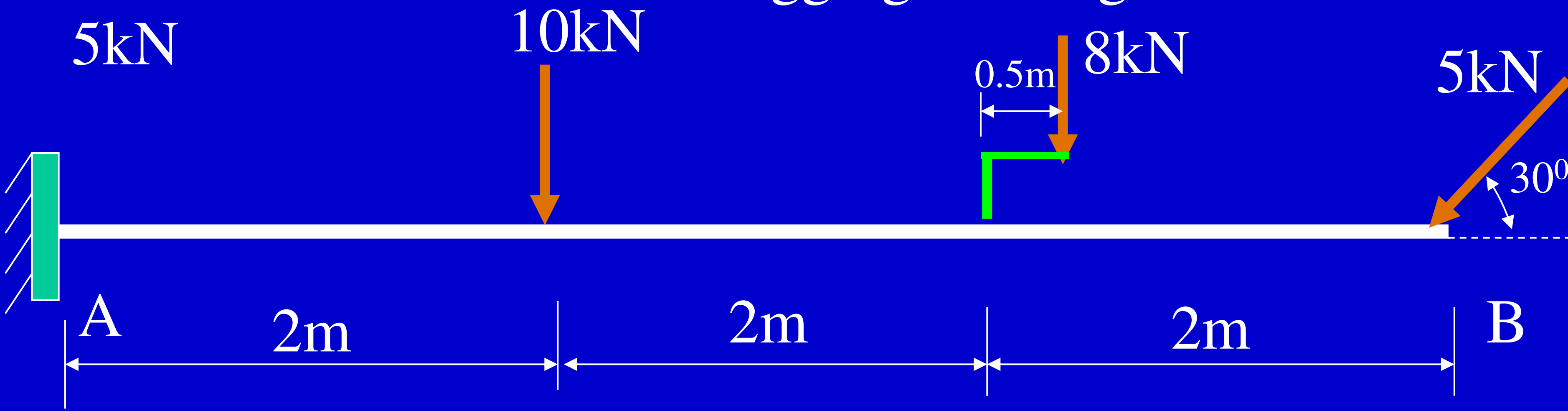
7. One side overhanging beam is subjected loading as shown below. Draw shear force and bending moment diagrams [SFD and BMD] for beam. Also determine maximum hogging bending moment.



[Ans: Max. Hogging bending moment = 735 kNm]

Exercise Problems

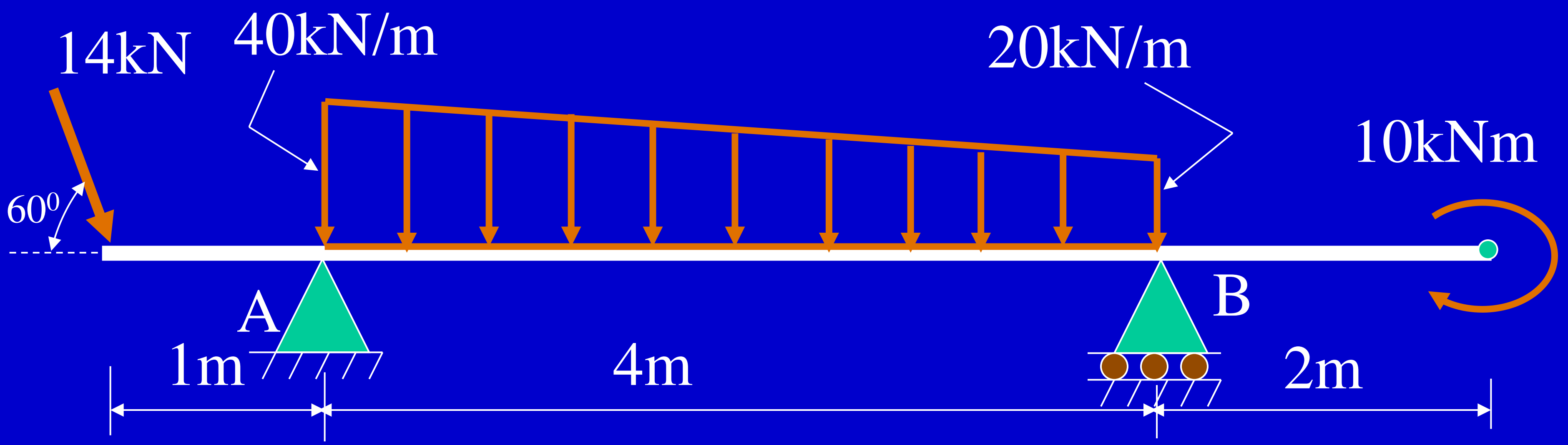
8. A cantilever beam of span 6m is subjected to three point loads at 1/3rd points as shown in the Fig. given below. Draw SFD and BMD for the beam. Locate and determine maximum shear force and hogging bending moment.



[Ans : Max. Shear force = 20.5kN, Max BM= 71kNm
Both max. shear force and bending moments will occur at supports.]

Exercise Problems

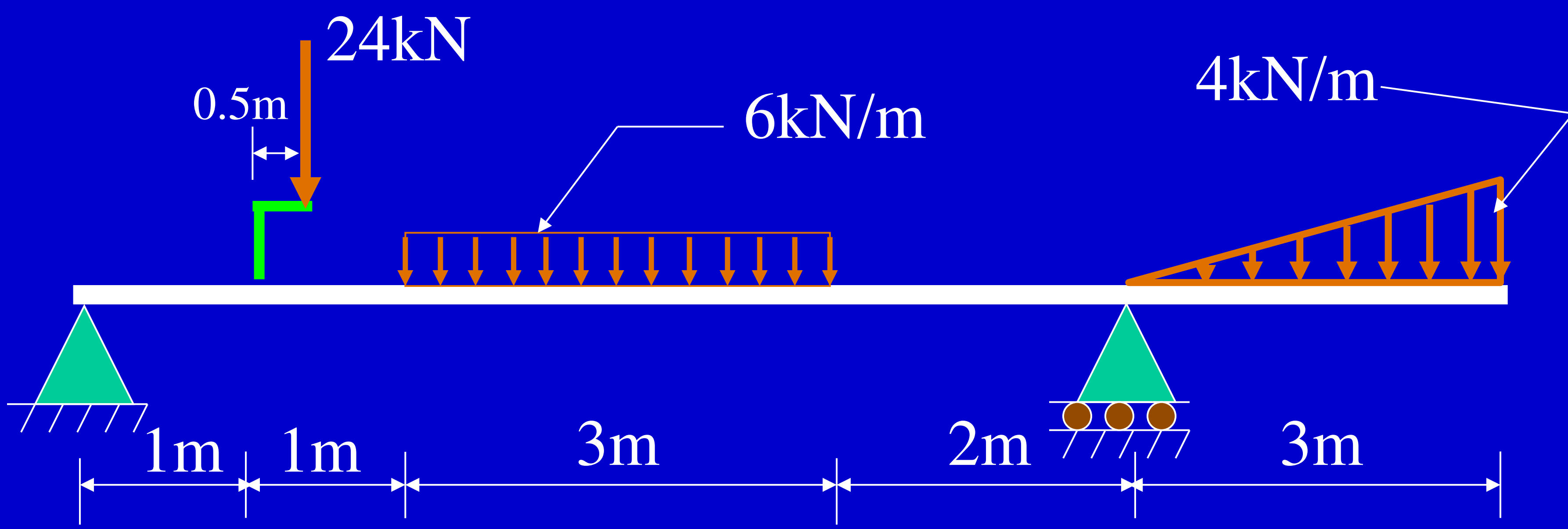
9. A trapezoidal load is acting in the middle portion AB of the double side overhanging beam as shown in the Fig. given below. A couple of magnitude 10 kNm and a concentrated load of 14 kN acting on the tips of overhanging sides of the beam as shown. Draw SFD and BMD. Mark salient features like maximum positive, negative bending moments and shear forces, inflection points if any.



[Ans : Maximum positive bending moment = 49.06 kNm

Exercise Problems

10. Draw SFD and BMD for the single side overhanging beam subjected loading as shown below.. Mark salient features like maximum positive, negative bending moments and shear forces, inflection points if any.

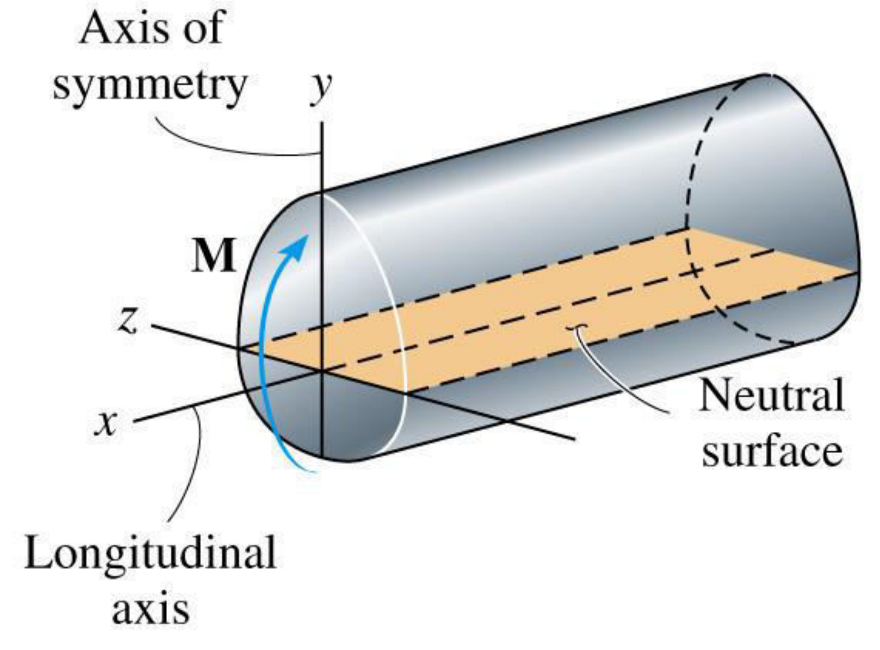


Ans: Maximum positive bending moment = 41.0 kNm

UNIT-III

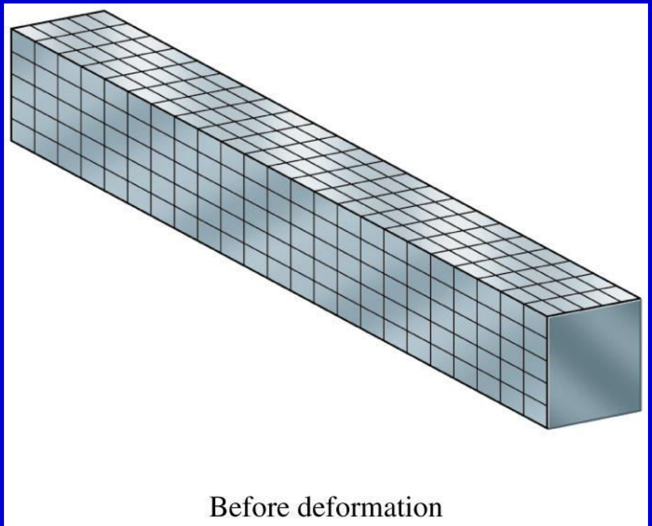
Chapter 6
Section 3,4
Bending Deformation, Strain and Stress in Beams

6.2 Bending Deformation and Strain

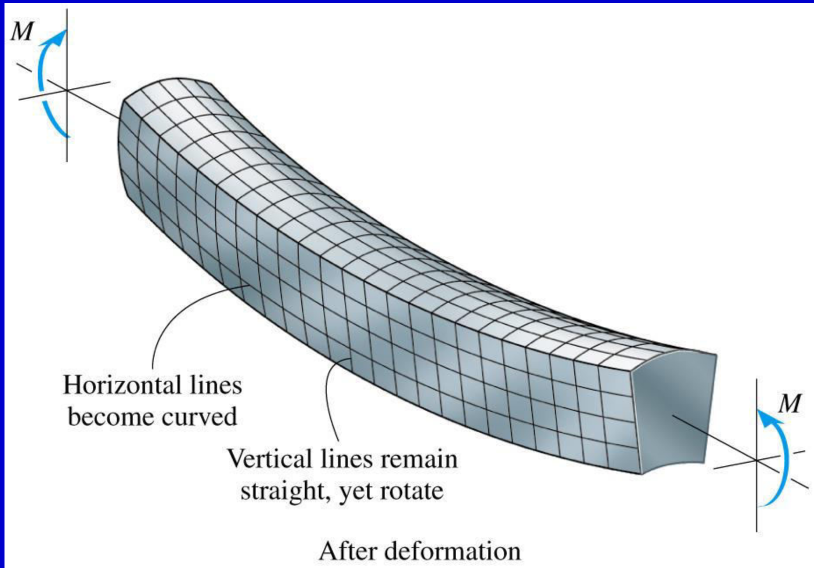


Key Points:

1. Bending moment causes beam to deform.
2. X = longitudinal axis
3. Y = axis of symmetry
4. Neutral surface – does not undergo a change in length



(a)

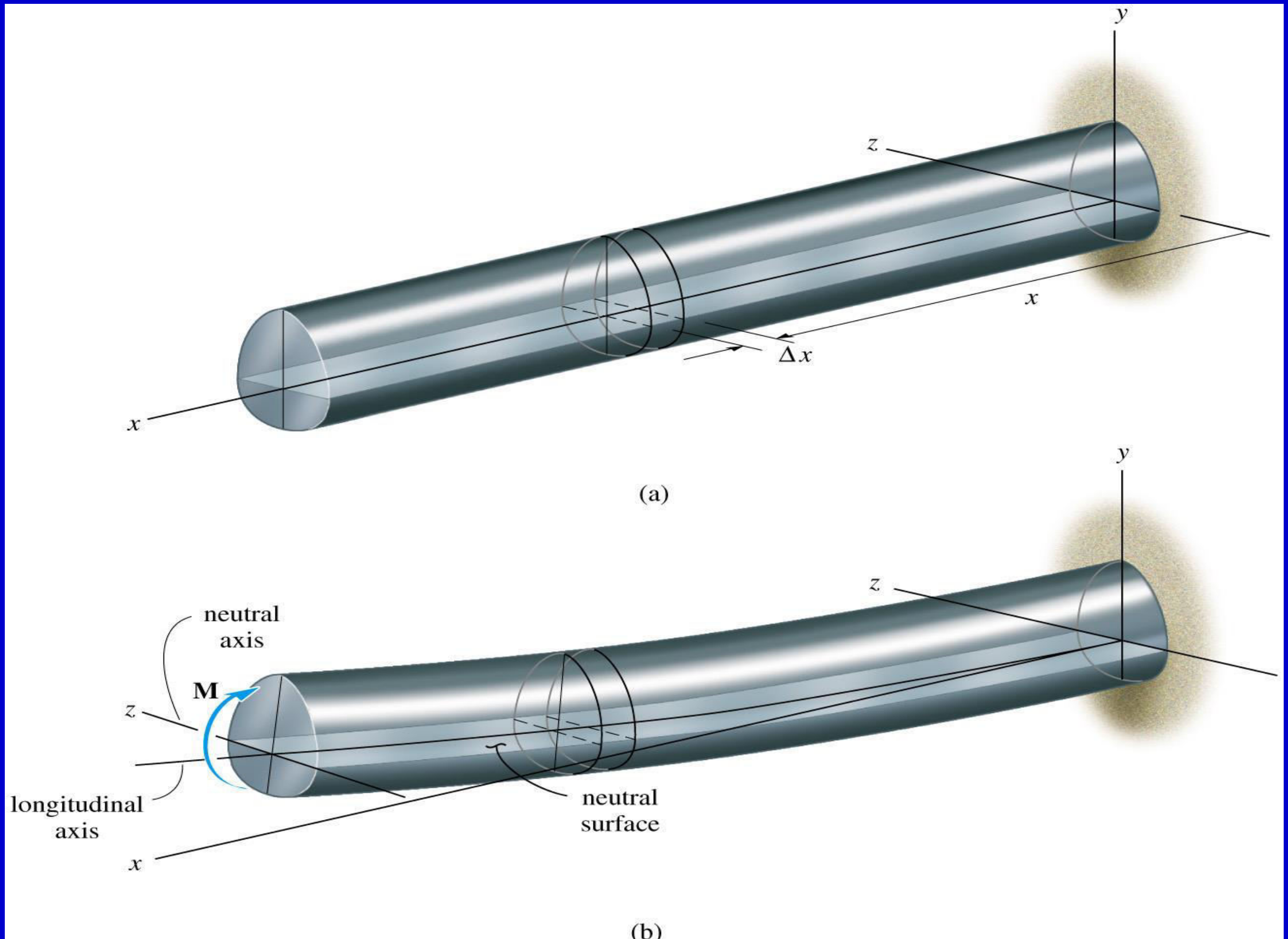


(b)

Key Points:

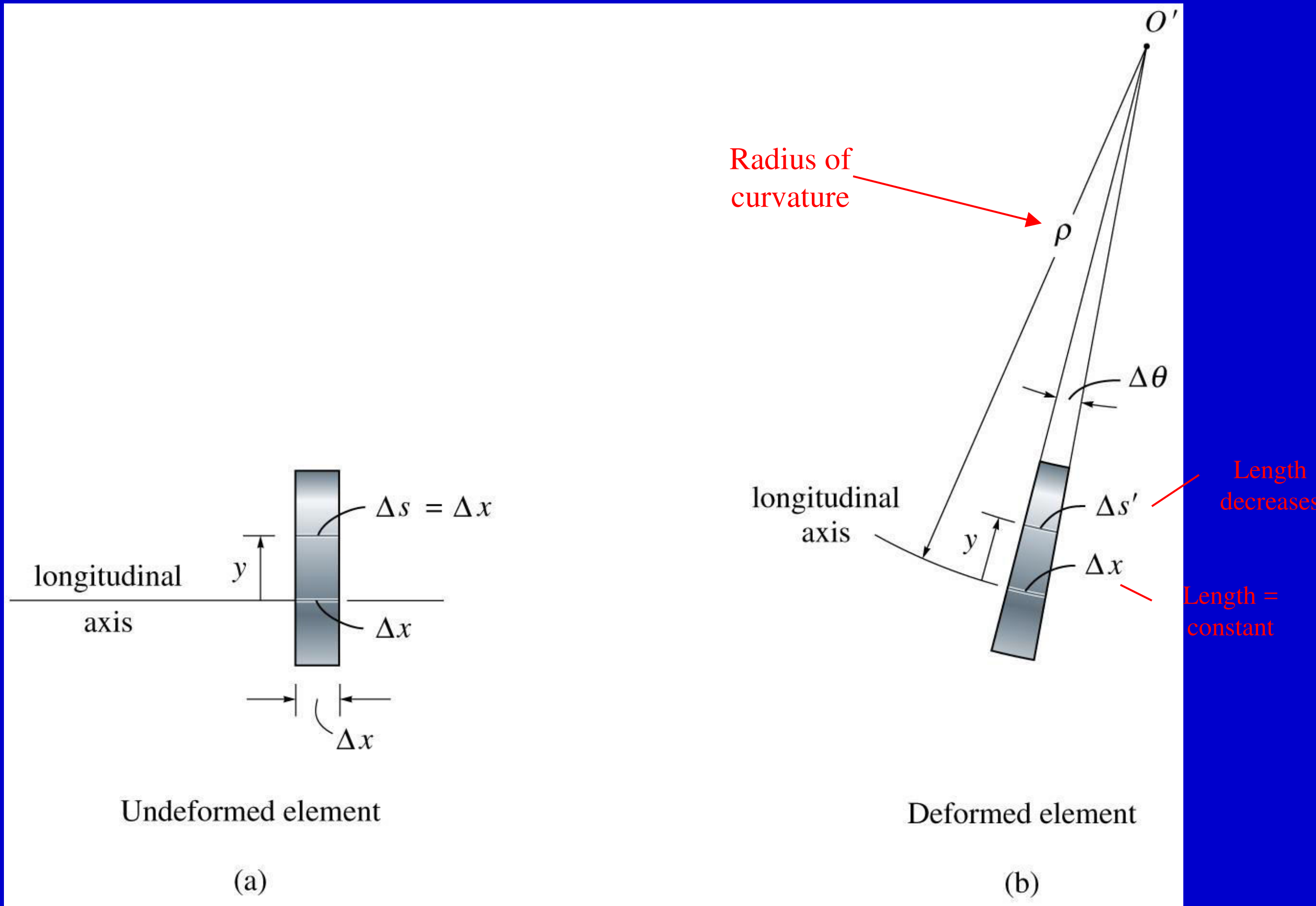
1. Internal bending moment causes beam to deform.

2. For this case, top fibers in compression, bottom in tension.



Key Points:

1. Neutral surface – no change in length.
2. All cross-sections remain plane and perpendicular to longitudinal axis.

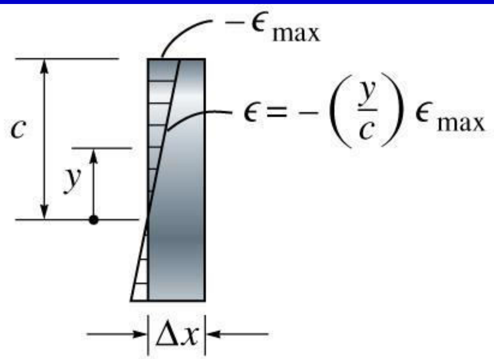


$$\varepsilon = \lim_{\Delta s \to 0} \frac{\Delta s' - \Delta s}{\Delta s} = \lim_{\Delta s \to 0} \frac{(\rho - y)\Delta\theta - \rho\Delta\theta}{\rho\Delta\theta} = -\frac{y}{\rho}$$

$$\varepsilon = -\frac{y}{\rho}$$

Says normal strain is linear

Maximum at outer surface (where y = c)



Normal strain distribution

$$\varepsilon = -\left(\frac{y}{c}\right)\varepsilon_{max}$$

6.2 Bending Stress – The Flexure Formula

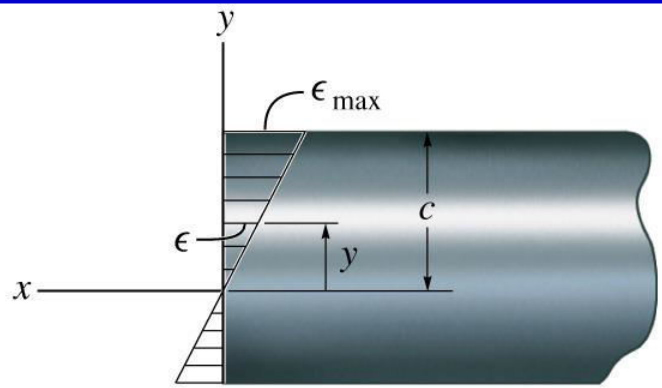
What about Stress????

Recall from section 6.1:

$$\varepsilon = -\left(\frac{y}{c}\right)\varepsilon_{\max}$$

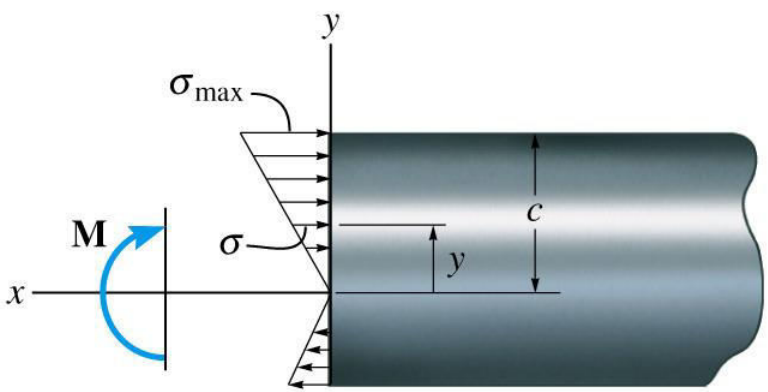
Therefore, it follows that

$$\sigma = -\left(\frac{y}{c}\right)\sigma_{\max}$$



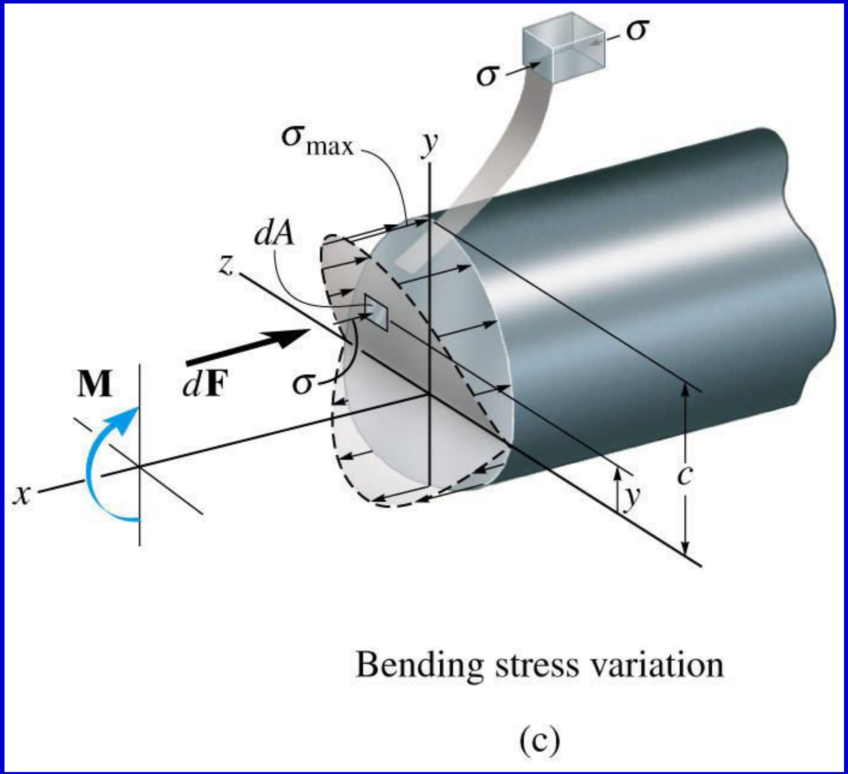
Normal strain variation
(profile view)

(a)



Bending stress variation
(profile view)

(b)



Sum moments about cut:

$$M = \int_A y\,dF = \int_A y(\sigma dA) = \int_A y\left(\frac{y}{c}\right)\sigma_{max}\,dA$$

$$M = \frac{\sigma_{max}}{c}\int_A y^2 dA$$

This is the moment of inertia, I

The Flexure Formula:

Internal bending moment, lb-in

Max bending stress, psi

Distance from NA to outer fiber, in

$$\sigma_{max} = \frac{Mc}{I}$$

Moment of inertia, in^4

Or in general:

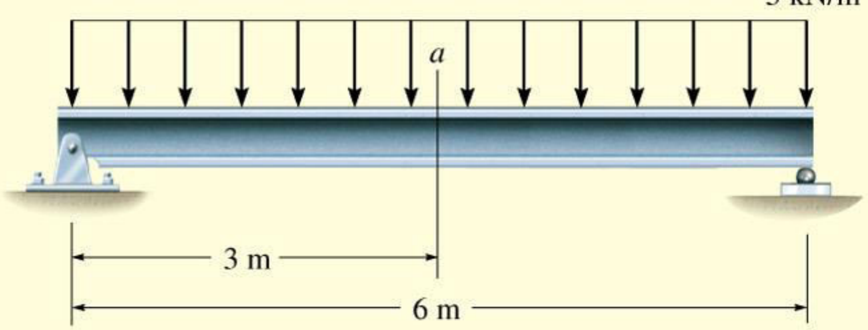
$$\sigma = \frac{My}{I}$$

Examples:

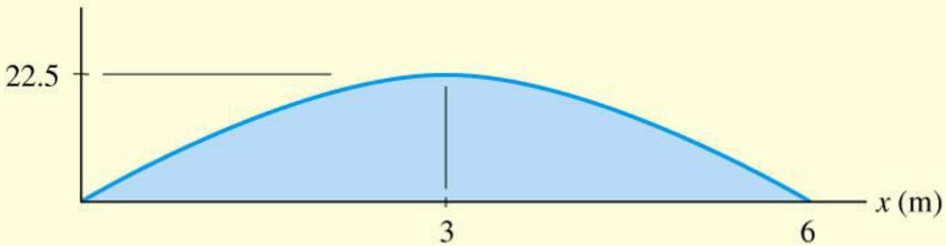
- Find maximum moment
- Find area properties, I and c
- Calculate stress

EXAMPLE 6.15

The simply supported beam in Fig. 6–28*a* has the cross-sectional area shown in Fig. 6–28*b*. Determine the absolute maximum bending stress in the beam and draw the stress distribution over the cross section at this location.



(a)



(c)

Fig. 6–28

WHERE IS BENDING STRESS MAXIMUM???

Answer:

- Outer surface (furthest away from Neutral Axis)
- Value of x along length where moment is maximum!!

Solution

Maximum Internal Moment. The maximum internal moment in the beam, $M = 22.5 \text{ kN}\cdot\text{m}$, occurs at the center as shown on the bending moment diagram, Fig. 6–28*c*. See Example 6.3.

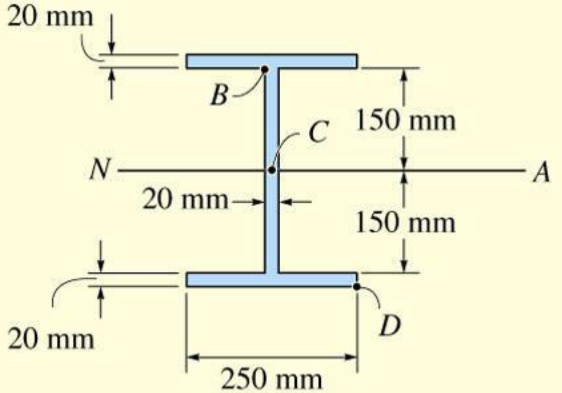
Section Property. By reasons of symmetry, the centroid C and thus the neutral axis pass through the midheight of the beam, Fig. 6–28*b*. The area is subdivided into the three parts shown, and the moment of inertia of each part is computed about the neutral axis using the parallel-axis theorem. (See Eq. A–5 of Appendix A.) Choosing to work in meters, we have

$$I = \Sigma(\bar{I} + Ad^2)$$

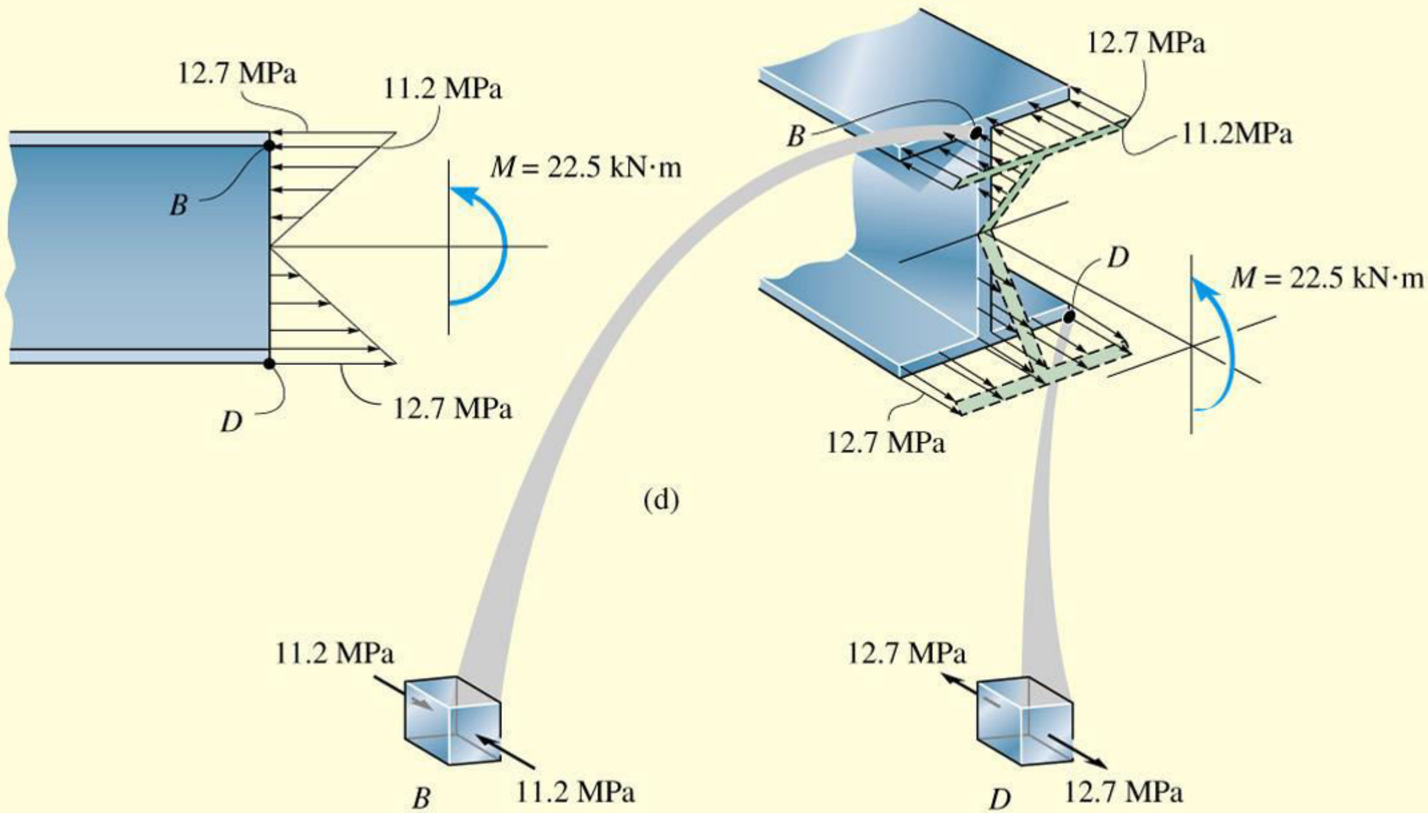
$$= 2\left[\frac{1}{12}(0.25 \text{ m})(0.020 \text{ m})^3 + (0.25 \text{ m})(0.020 \text{ m})(0.160 \text{ m})^2\right]$$

$$+ \left[\frac{1}{12}(0.020 \text{ m})(0.300 \text{ m})^3\right]$$

$$= 301.3(10^{-6}) \text{ m}^4$$



(b)


12.7 MPa
11.2 MPa
B
M = 22.5 kN·m
D
12.7 MPa
12.7 MPa
B
11.2MPa
D
M = 22.5 kN·m
12.7 MPa
11.2 MPa
B
11.2 MPa
12.7 MPa
D
12.7 MPa

(d)

(e)

Bending Stress. Applying the flexure formula, with $c = 170$ mm, the absolute maximum bending stress is

$$\sigma_{max} = \frac{Mc}{I}; \qquad \sigma_{max} = \frac{22.5 \text{ kN} \cdot \text{m}(0.170 \text{ m})}{301.3(10^{-6}) \text{ m}^4} = 12.7 \text{ MPa} \qquad \textit{Ans.}$$

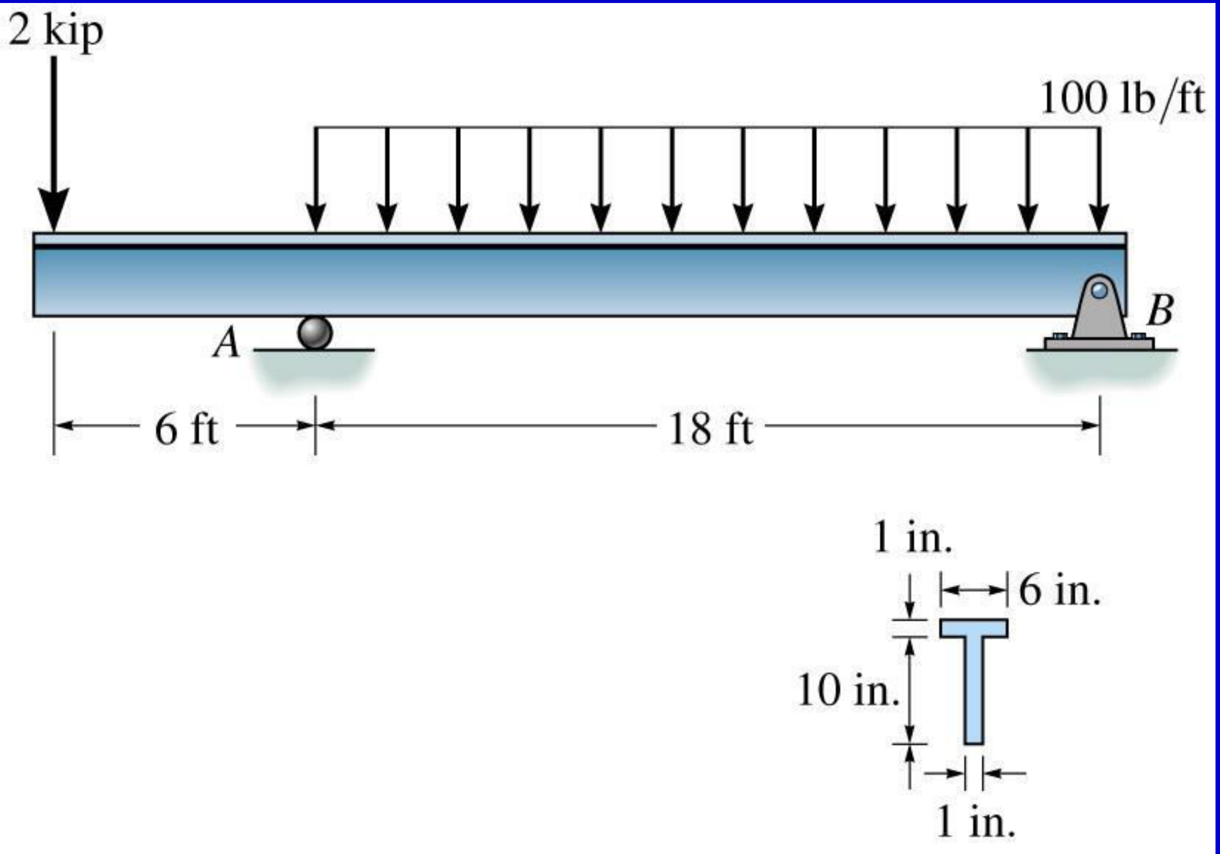
Two-and-three-dimensional views of the stress distribution are shown in Fig. 6–28*d*. Notice how the stress at each point on the cross section develops a force that contributes a moment $d\mathbf{M}$ about the neutral axis such that it has the same direction as **M**. Specifically, at point B, $y_B = 150$ mm, and so

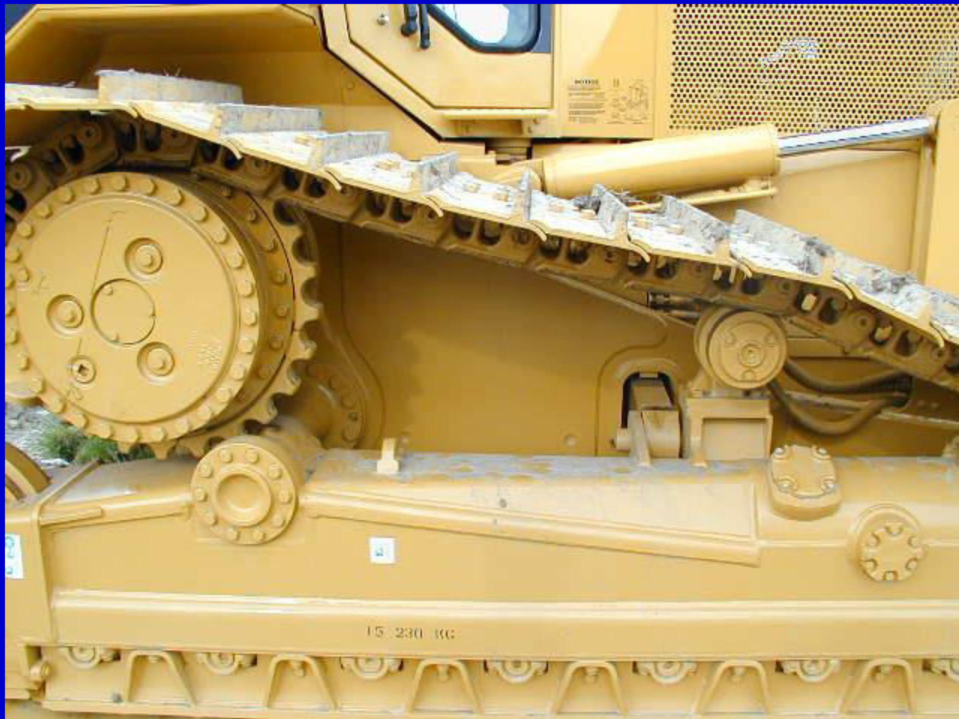
$$\sigma_B = \frac{My_B}{I}; \quad \sigma_B = \frac{22.5 \text{ kN} \cdot \text{m}(0.150 \text{ m})}{301.3(10^{-6}) \text{ m}^4} = 11.2 \text{ MPa}$$

The normal stress acting on elements of material located at points B and D is shown in Fig. 6–28*e*.

Example: The T-shape beam is subjected to the loading below.

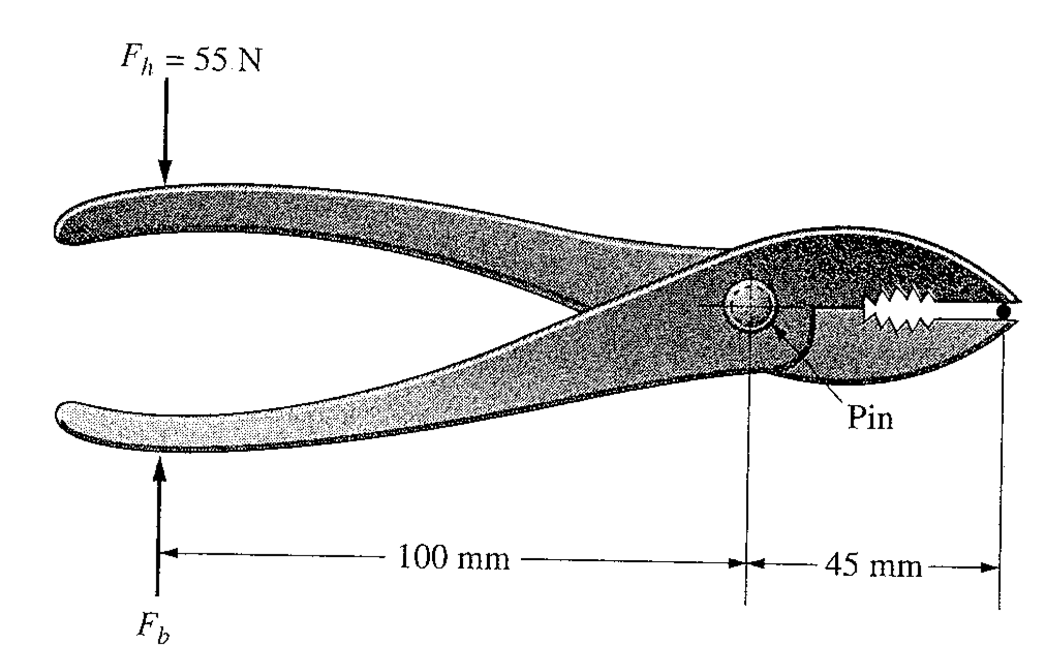
1. Draw shear and moment diagram. Identify location and magnitude of Mmax.
2. Determine location and magnitude of maximum bending stress and draw stress profile. Is the beam safe if the material is aluminum w/ σ_y = 15 ksi?
3. What is the largest internal moment the beam can resist if σ_{allow} = 2 ksi?





45K

Statics: Example 1 - Pliers



Given: Typical household pliers as shown.

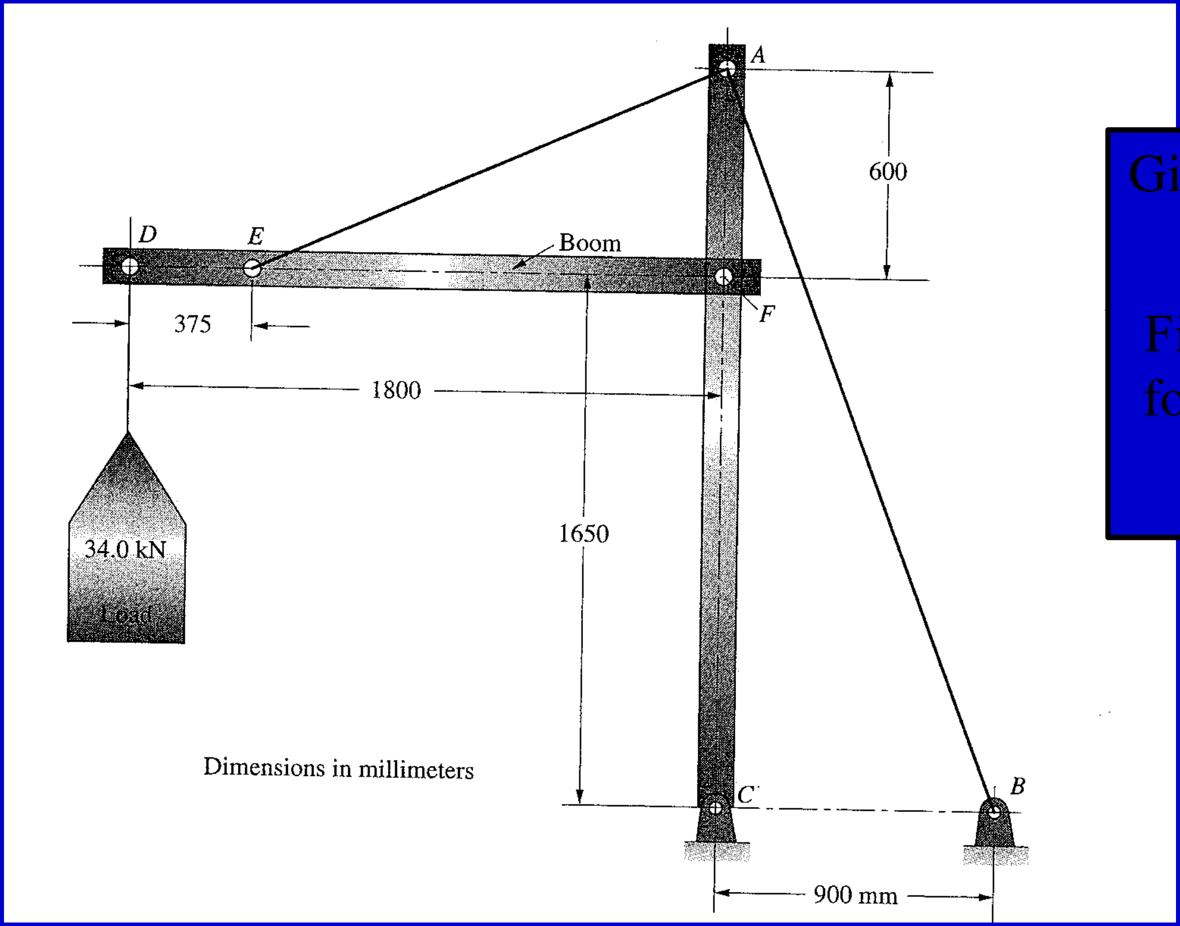
Find: Force applied to wire and force in pin that connects the two parts of the pliers.

Do this for homework.

See solution Link

Side: what is the shear stress in pin and bending stress in handle? SofM

Statics: Example 2 – Crane Structure



Given: Crane structure as shown.

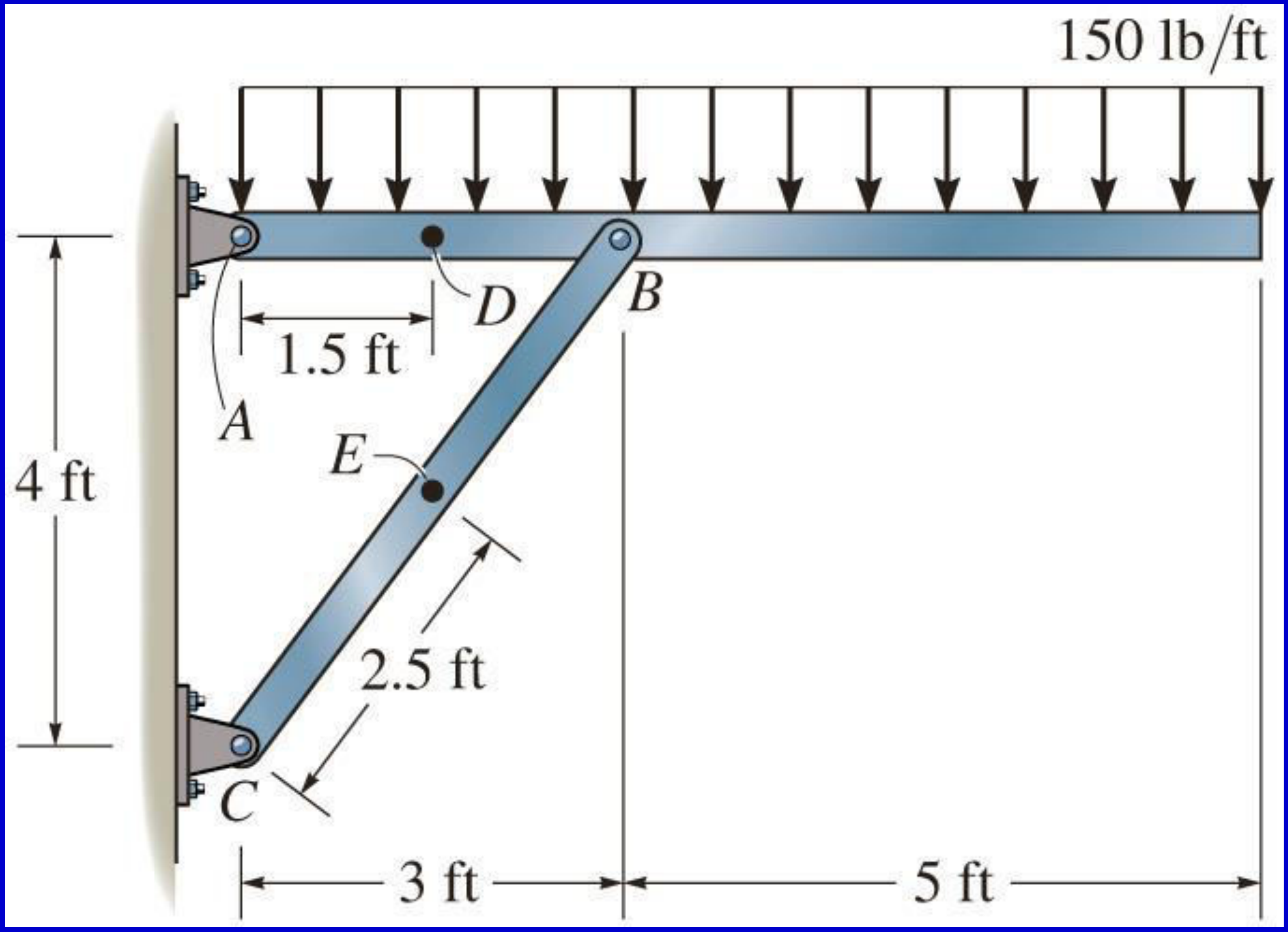
Find: Forces and FBD's for cables A-B and A-E, boom DEF and post AFC.

Do this for homework.

Side: what is the normal stress in cables (average normal only) and normal stress in boom and post (combined loading)? SofM

See solution Link

Example 4: Determine the resultant internal loadings acting on the cross sections located through points D and E of the frame. (1-114)



UNIT_IV

Chapter 3 Torsion

Introduction

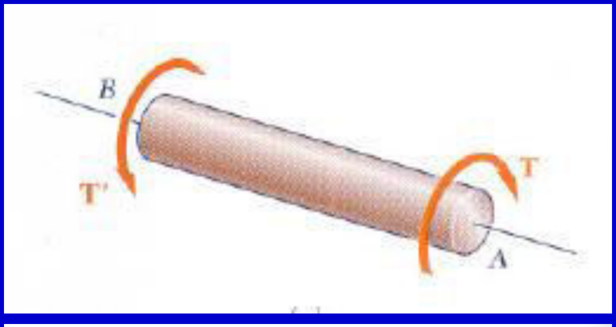
-- Analyzing the stresses and strains in machine parts which are subjected to torque T

-- Cross-section
- Circular
- Non-circular
- Irregular shapes

-- Material
- (1) Elastic
- (2) Elasto-plastic

-- Shaft
- (1) Solid
- (2) Hollow

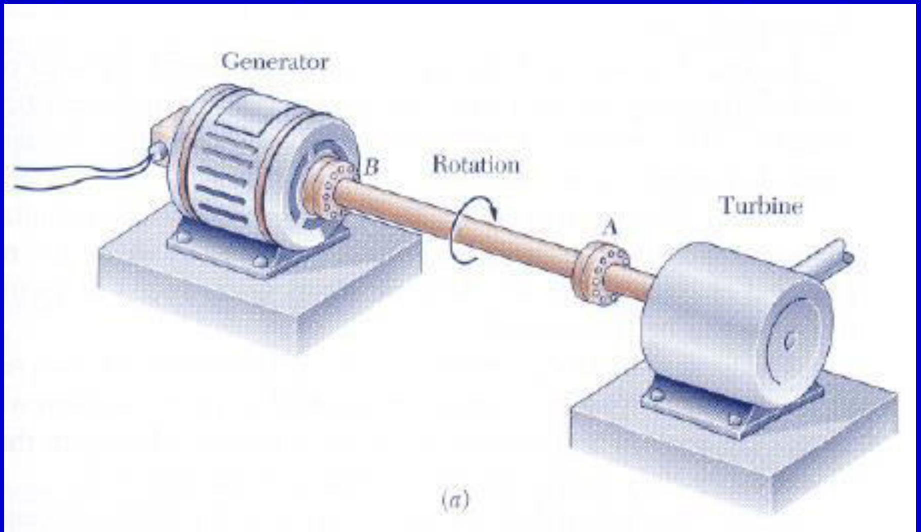
3.1 Introduction



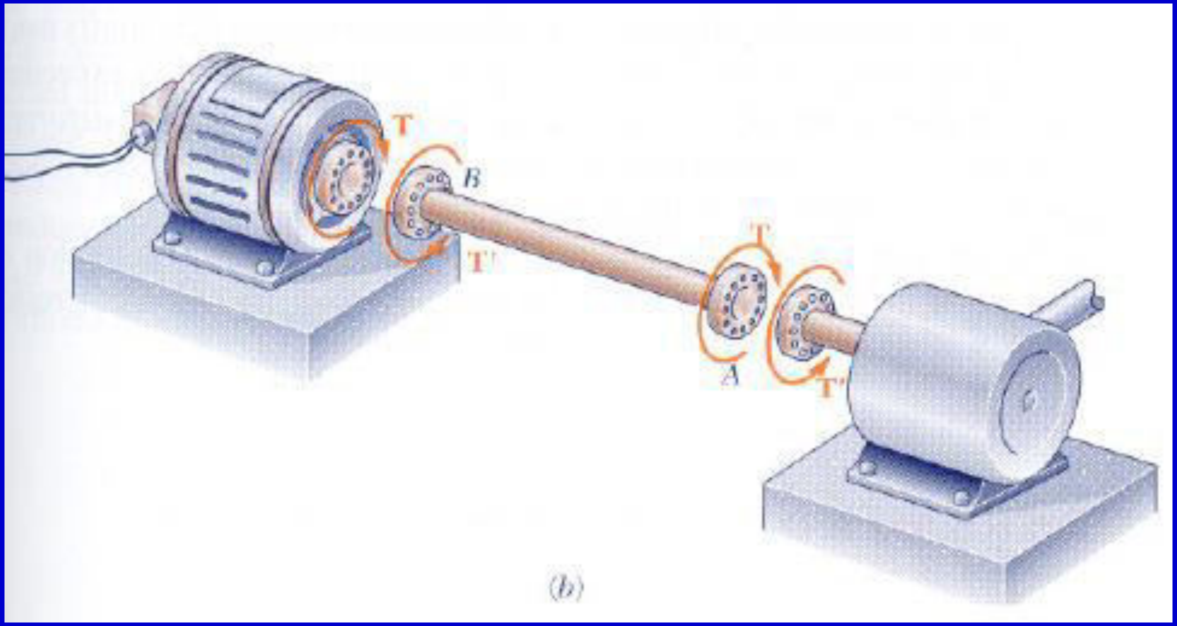
- T is a vector
- Two ways of expression

-- Applications:

a. Transmission of torque in shafts,
e.g. in automobiles



(a)



Assumptions in Torque Analysis:

a. Every cross section remains plane and undistorted.

b. Shearing strain varies linearly along the axis of the shaft.

3.2 Preliminary Discussion of the Stresses in a Shaft

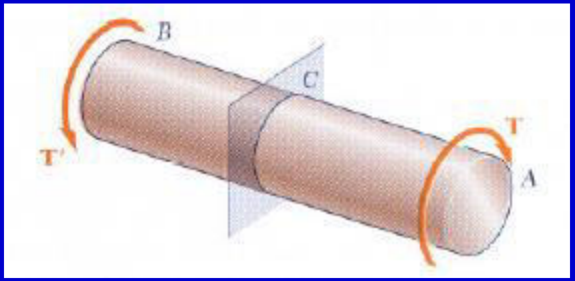
$$\int \rho dF = T$$

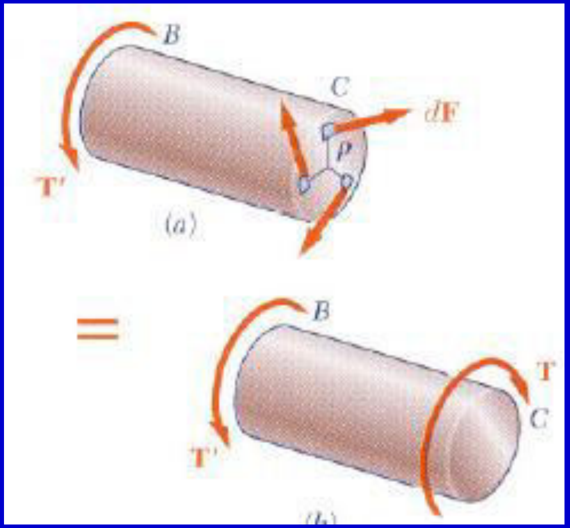
Where ρ = distance (torque arm)

Since dF = τ dA

$$\int \rho(\tau dA) = T$$

The stress distribution is Statically Indeterminate.

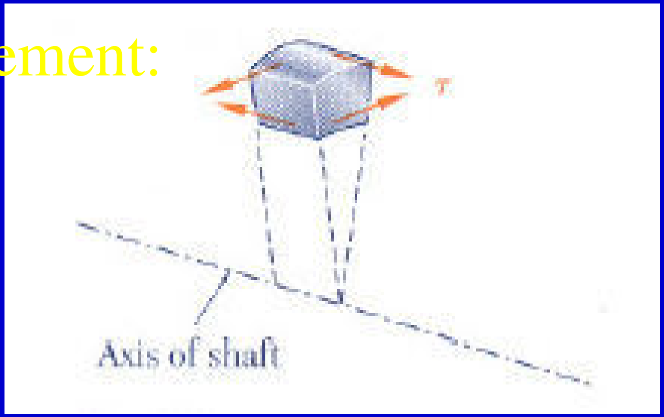


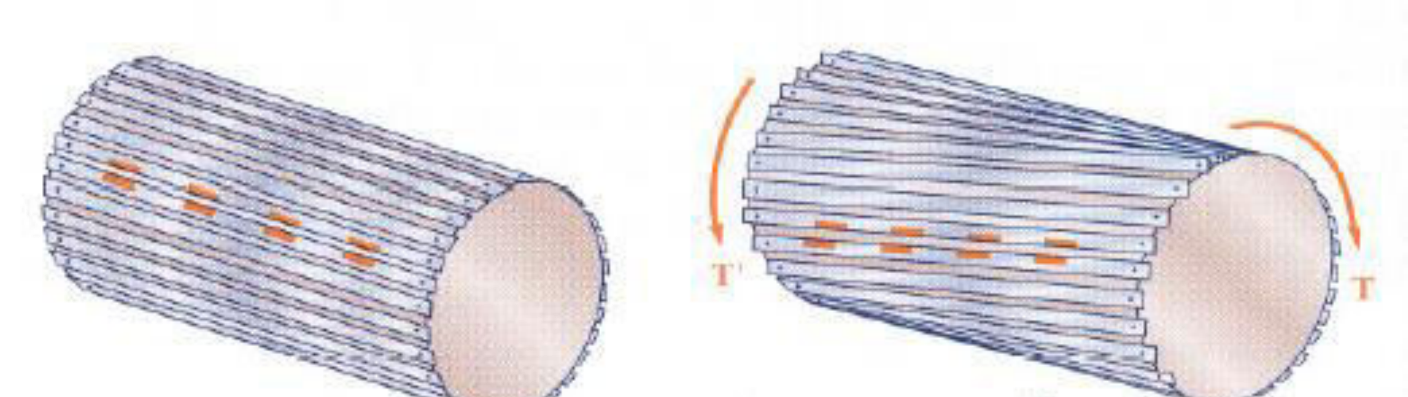


Free-body Diagram

-- Must rely on “deformation” to solve the problem.

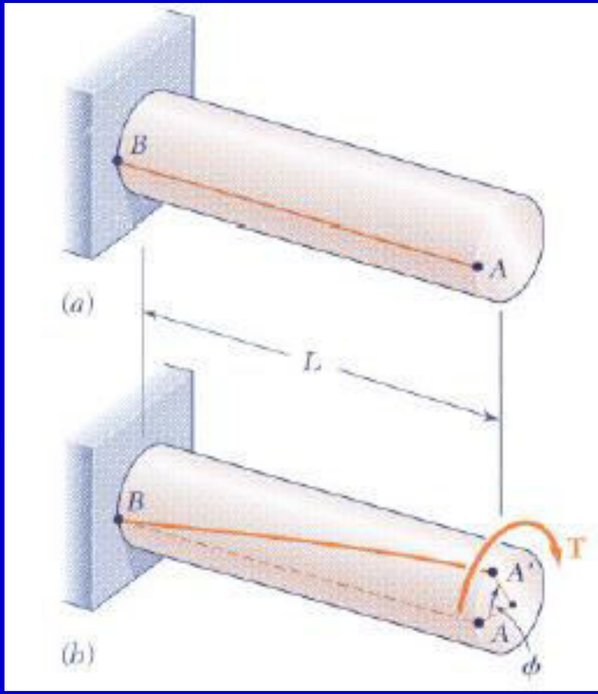
Analyzing a small element:

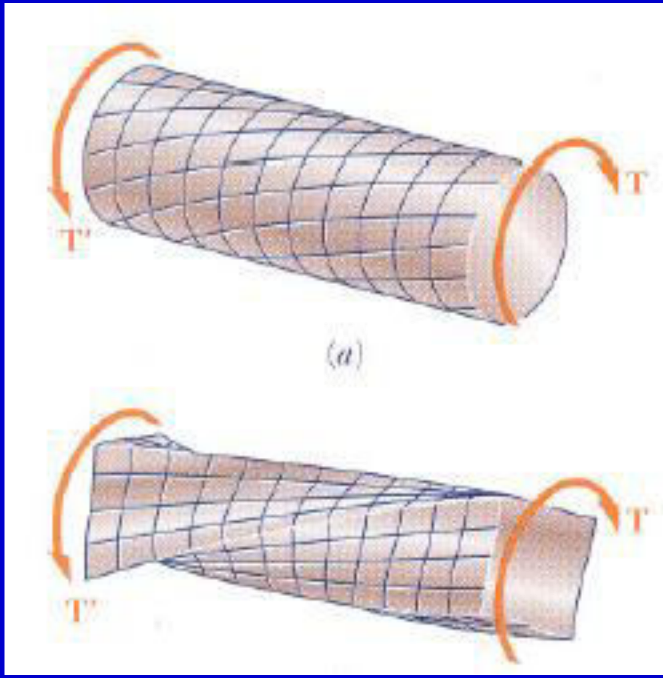




(a) (b)

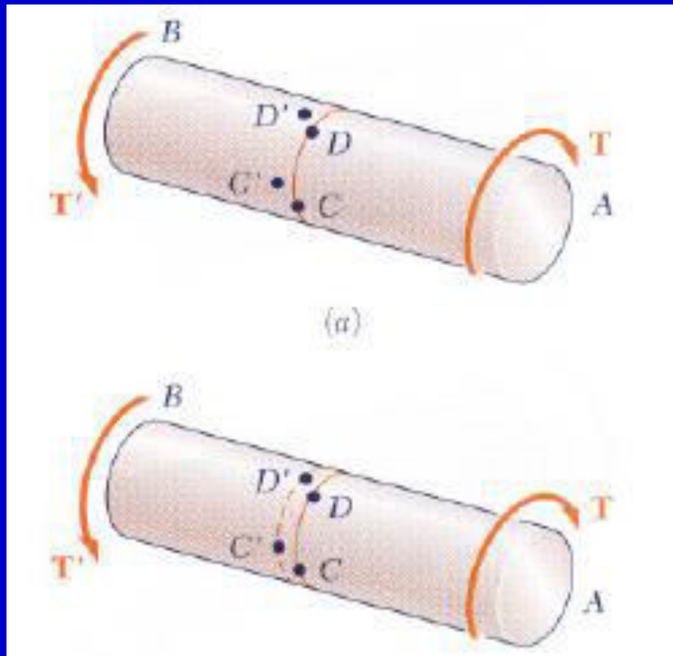
3.3 Deformations in a Circular Shaft





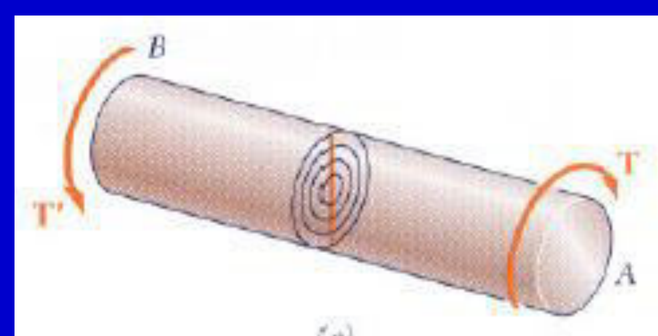
$\phi = \phi\,(T, L)$ -- the angle of twist (deformation)

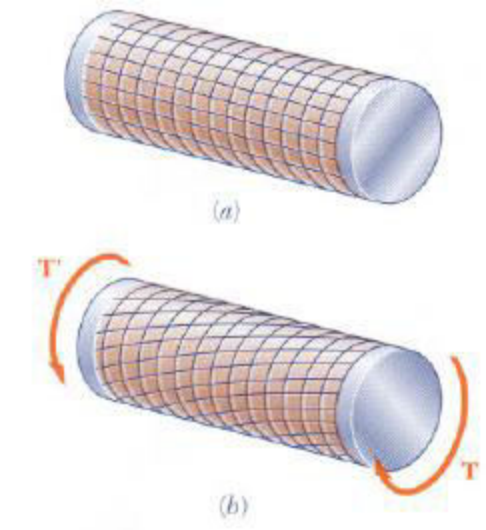
Rectangular cross section warps under torsion

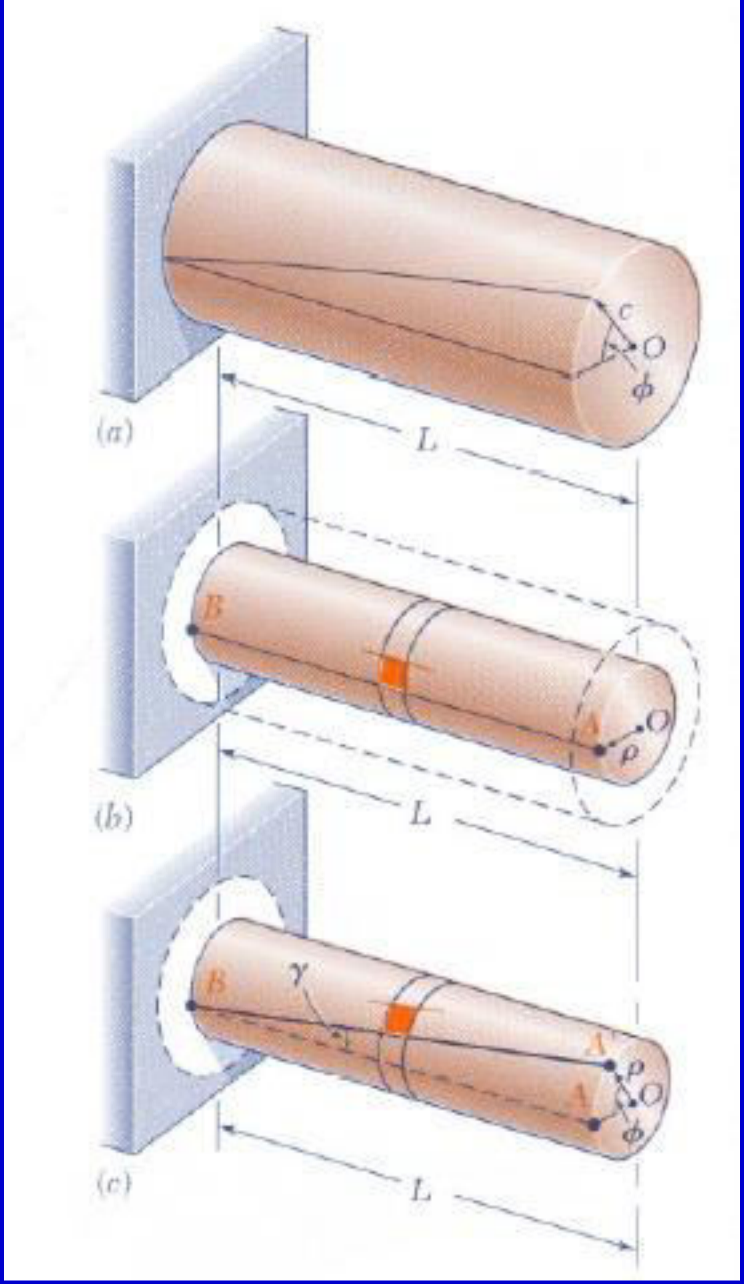


$$CD = C'D'$$

$\therefore$ A circular plane remains circular plane







Determination of Shear Strain γ

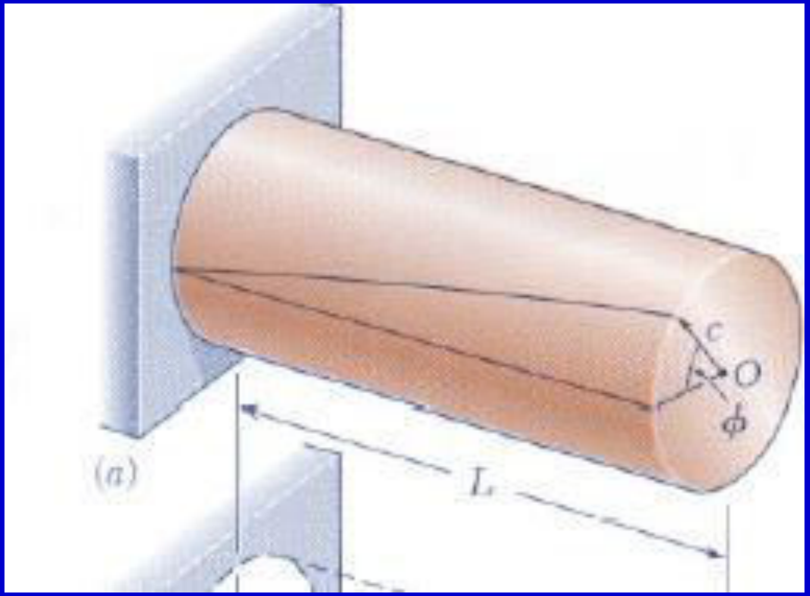
$$\gamma = \frac{\rho\phi}{L} \quad \text{(in radians)}$$

The shear strain γ ∝ ρ

$$\gamma_{max} = \frac{c\phi}{L}$$

ρ = c = radius of the shaft

$$\therefore \ \phi = \frac{\gamma_{max} L}{c}$$



Since

$$\gamma = \frac{\rho\phi}{L}$$

$$\gamma = \frac{\rho}{c}\gamma_{max}$$

3.4 Stresses in the Elastic Range

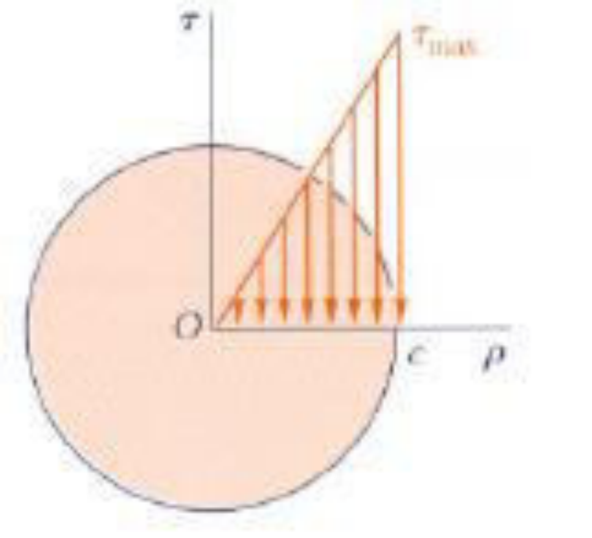
Hooke's Law $\tau = G\gamma$

$$\gamma = \frac{\rho}{c}\gamma_{\max}$$

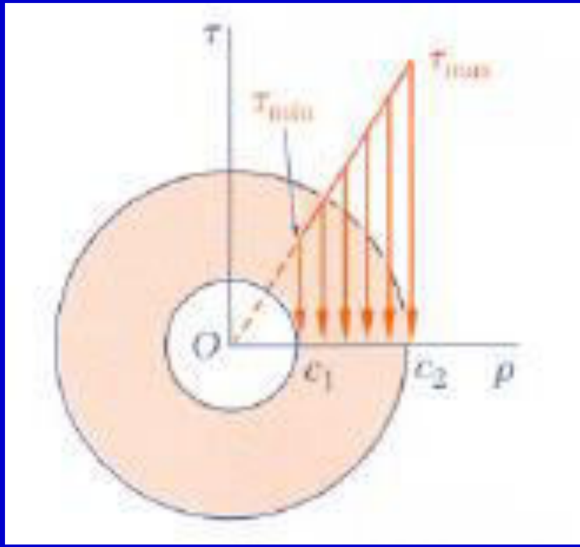
$$\tau = G\gamma = G\frac{\rho}{c}\gamma_{\max}$$

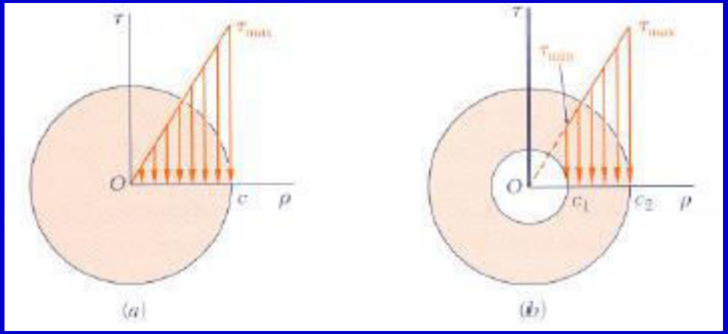
$$\tau = G\gamma \quad \rightarrow \tau_{\max} = G\gamma_{\max}$$

Therefore, $$\tau = \frac{\rho}{c}\tau_{\max} \qquad (3.6)$$



$$\tau_{\min} = \frac{c_1}{c_2}\tau_{\max}$$





$$\int \rho(\tau dA) = T \qquad (3.1) \qquad \tau = \frac{\rho}{c}\tau_{\max} \qquad (3.6)$$

$$T = \int \rho \tau dA = \int \rho \frac{\rho}{c}\tau_{\max} dA = \frac{\tau_{\max}}{c}\int \rho^2 dA$$

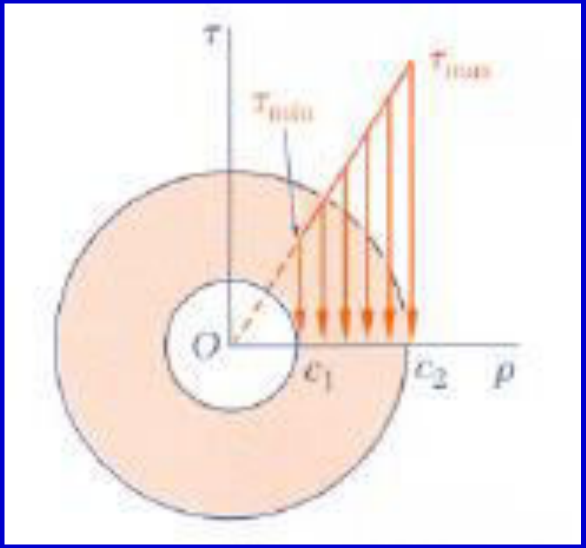
But $\int \rho^2 dA = J$

Therefore, $T = \dfrac{\tau_{\max} J}{c}$ Or, $\tau_{\max} = \dfrac{Tc}{J}$ (3.9)

Substituting Eq. (3.9) into Eq. (3.6)

$$\tau_{\max} = \frac{Tc}{J} \qquad (3.10)$$

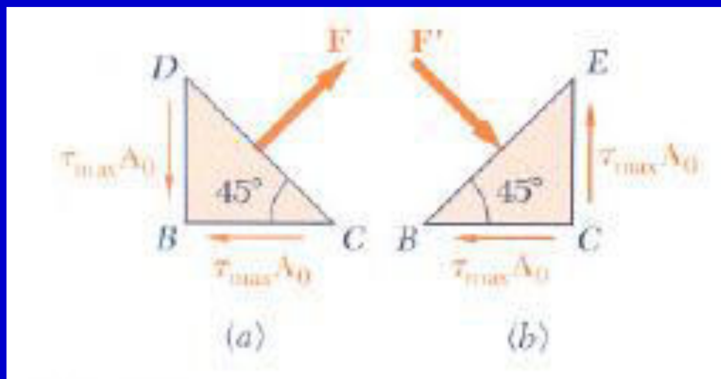
$$\tau = \frac{T\rho}{J} \qquad (3.9)$$

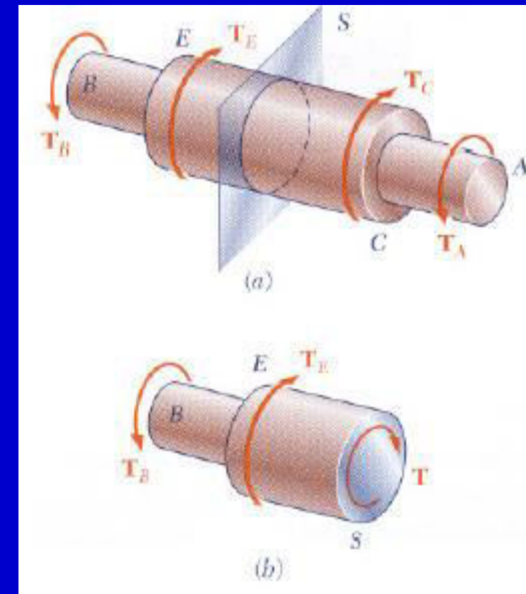


These are *elastic torsion formulas*.

For a solid cylinder: $J = \frac{1}{2}\pi c^4$

For a hollow cylinder: $J = \frac{1}{2}\pi (c_2^4 - c_1^4)$





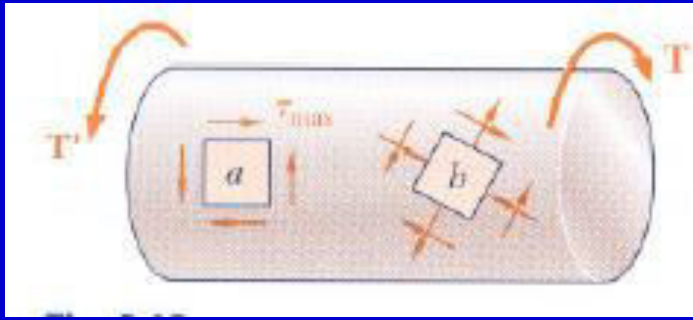
$$F = 2(\tau_{max} A_0)\cos 45° = \tau_{max} A_0 \sqrt{2} \qquad (3\text{-}13)$$

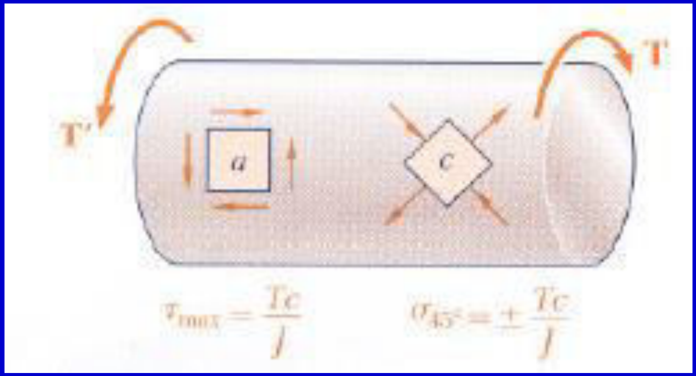
Since $A = A_o\sqrt{2} \rightarrow \dfrac{Eq.(3-13)}{A}$

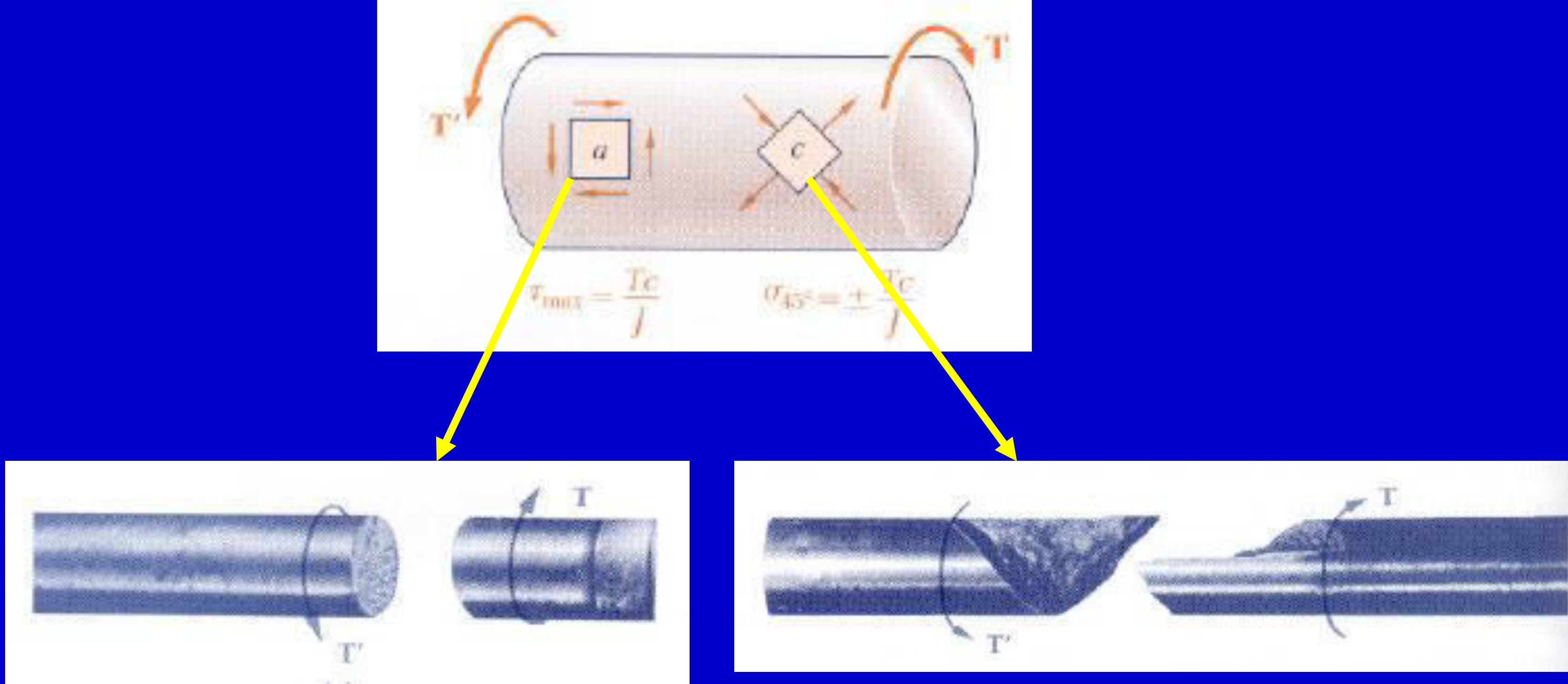
$$\rightarrow \quad \sigma = \frac{F}{A} = \frac{\tau_{max} A_0 \sqrt{2}}{A_0\sqrt{2}} = \tau_{max}$$

Mohr's Circle (Sec. 7.4)

-- Pure Shear Condition



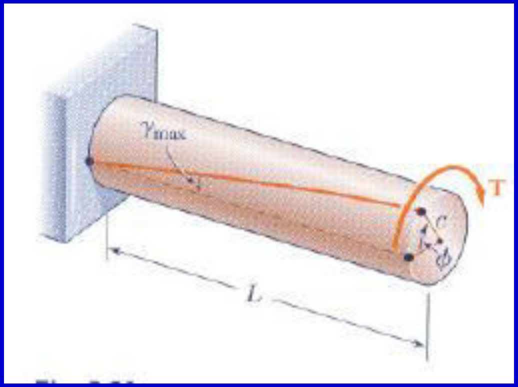




Ductile materials fail in shear (90° fracture)

Brittle materials are weaker in tension (45° fracture)

3.5 Angle of Twist in the Elastic Range



$$\gamma_{\max} = \frac{c\phi}{L} \qquad (3.3)$$

$$\gamma_{\max} = \frac{\tau_{\max}}{G} \qquad \text{since} \quad \tau_{\max} = \frac{Tc}{J}$$

Therefore,

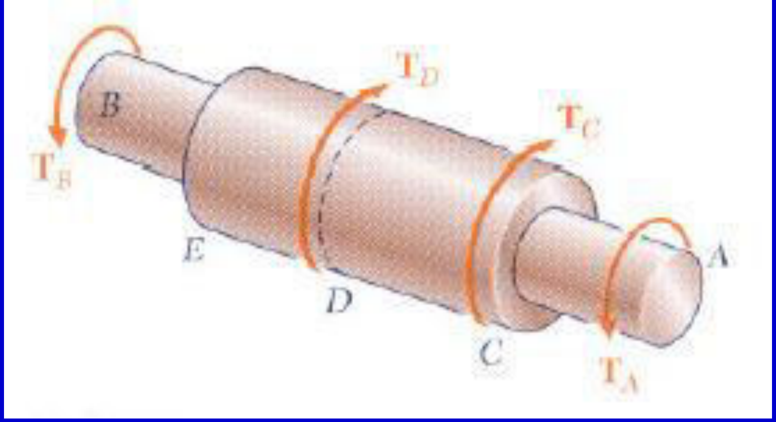
$$\gamma_{\max} = \frac{Tc}{JG} \qquad (3.15)$$

Eq. (3.3) = Eq. (3.15) $\rightarrow$ $\gamma_{\max} = \dfrac{c\phi}{L} = \dfrac{Tc}{JG}$

Hence,

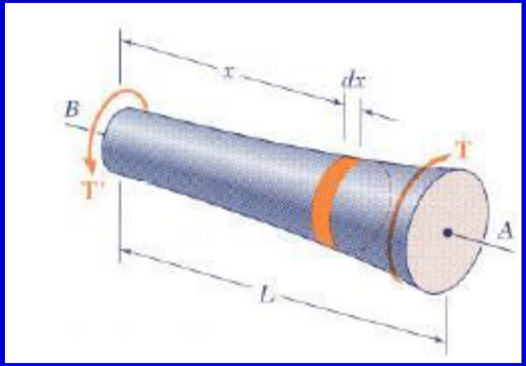
$$\phi = \frac{TL}{JG}$$

For Multiple-Section Shafts:



$$\phi = \sum_i \frac{T_i J_i}{J_i G_i}$$

Shafts with a Variable Circular Cross Section



$$d\phi = \frac{T\,dx}{JG}$$

$$\phi = \int_0^L \frac{T\,dx}{JG}$$

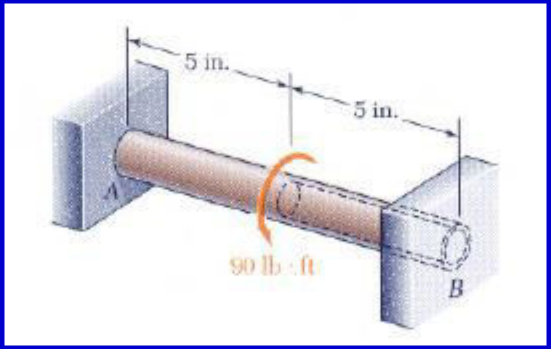
3.6 Statically Indeterminate Shafts

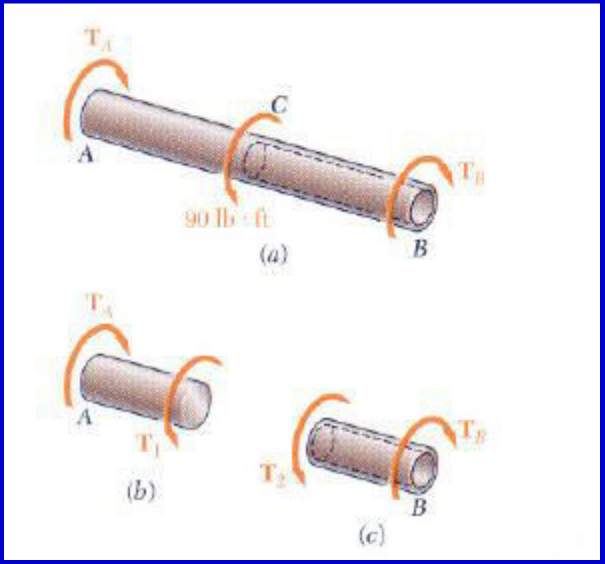
-- Must rely on both

(1) Torque equations and $\Sigma T = 0$

(2) Deformation equation, i.e. $\phi = \dfrac{TL}{JG}$

Example 3.05





3.7 Design of Transmission Shafts

-- Two Parameters in Transmission Shafts:

a. Power P

b. Speed of rotation

$$P = power = T\omega$$

where ω = angular velocity (radians/s) = 2π*f*

***f* = frequency (Hz)**

$$P = 2\pi\ f\ T$$

$$T = \frac{P}{2\pi f} \qquad \text{[N.m/s = watts (W)]} \qquad (3.21)$$

$$T = \frac{P}{2\pi f} \qquad (3.21)$$

$$\tau_{max} = \frac{Tc}{J} \qquad (3.9)$$

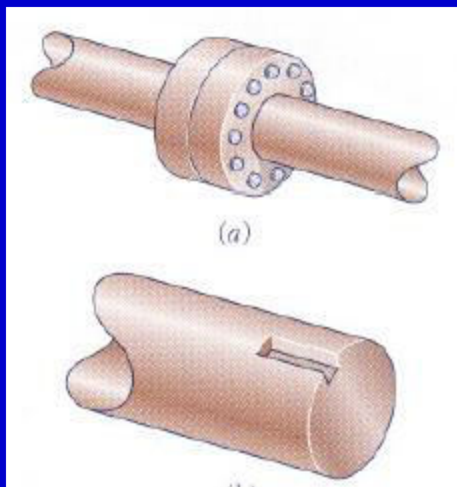
Therefore, $\dfrac{J}{c} = \dfrac{T}{\tau_{max}}$

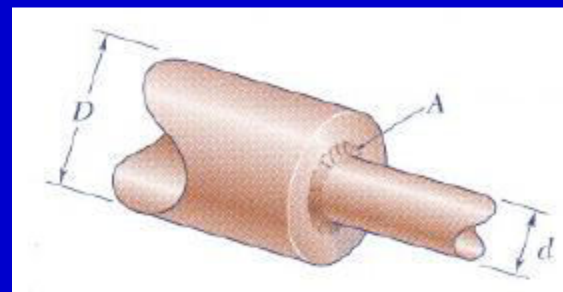
For a Solid Circular Shaft:

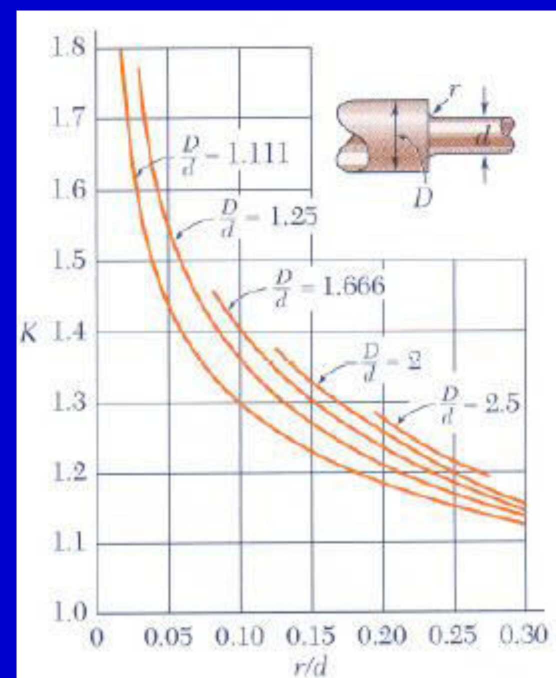
$$J = \frac{1}{2}\pi c^4 \quad and \quad J/c = \frac{1}{2}\pi c^3$$

$$\frac{1}{2}\pi c^3 = \frac{T}{\tau_{max}} \quad \rightarrow \quad c = \left(\frac{2T}{\pi \tau_{max}}\right)^{1/3}$$

3.8 Stress Concentrations in Circular Shafts





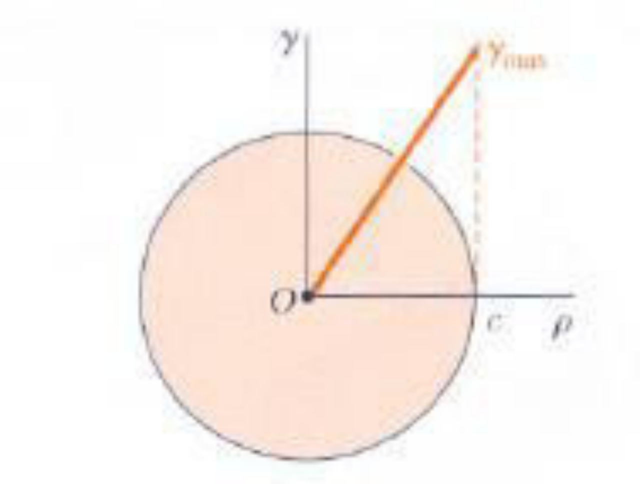


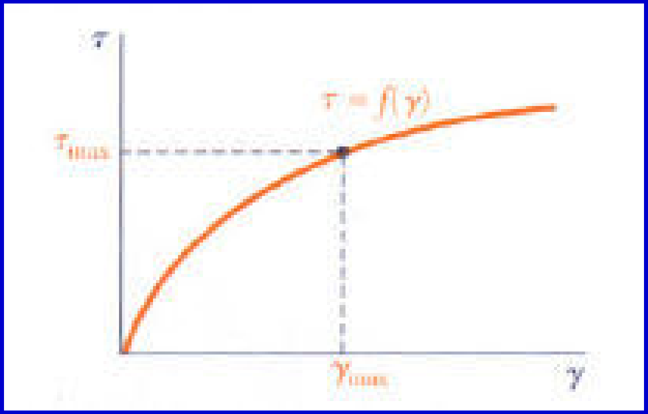
$$\tau_{max} = K\frac{Tc}{J}$$

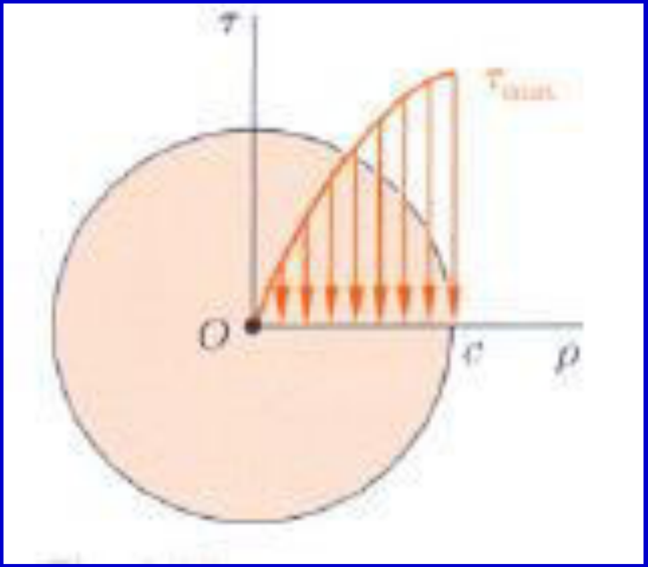
3.9 Plastic Deformation sin Circular Shafts

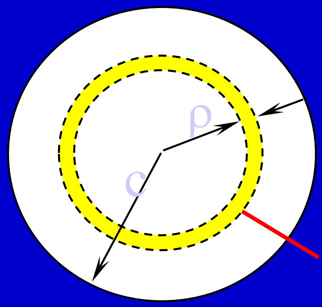
$$\gamma = \frac{\rho}{c}\gamma_{max} \qquad (3.4)$$

c = radius of the shaft









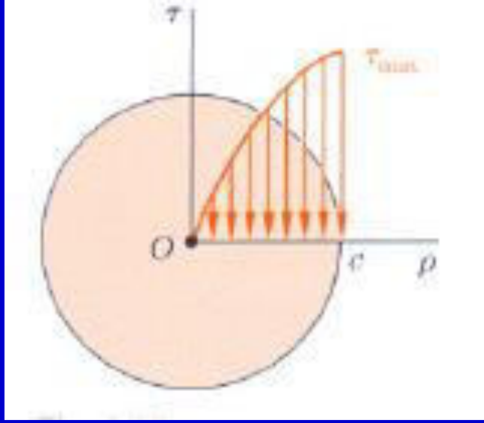
$$\int \rho dF = T \qquad (3.1)$$

Knowing dF = τ dA

$$T = \int \rho dF = \int \rho \tau dA = \int \rho \tau (2\pi \rho d\rho)$$

$$T = 2\pi \int_0^c \rho^2 \tau d\rho \qquad (3.26)$$

Where τ = τ(ρ)



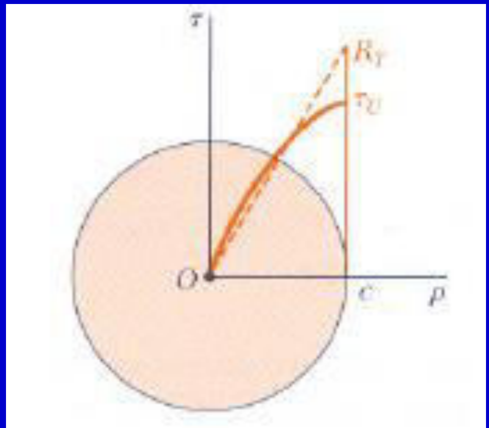
$$\tau_{max} = \frac{Tc}{J} \qquad (3.9)$$

If we can determine experimentally an Ultimate Torque, T_U,

then by means of Eq. (3.9), we have

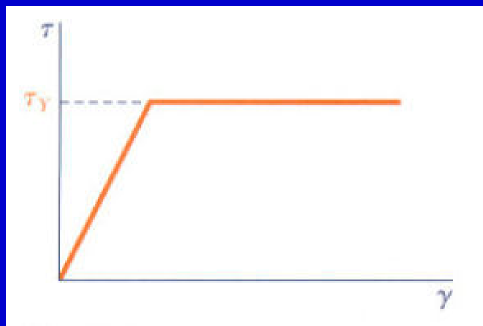
$$R_T = \frac{T_U c}{J}$$

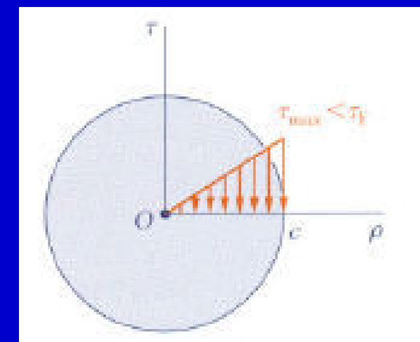
R_T = Modulus of Rupture in Torsion



$$\gamma = \frac{\rho\phi}{L}$$

3.10 Circular Shafts Made of an Elasto-Plastic Material

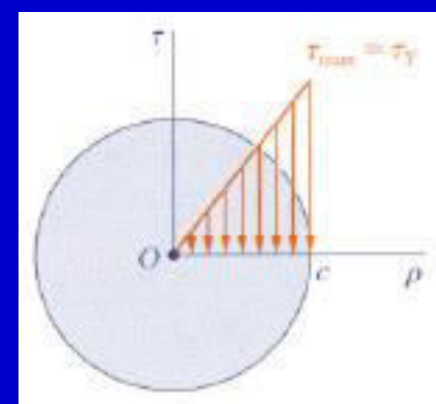




Case I

Case I: $\tau < \tau_Y$ Hooke's Law applies, $\tau < \tau_{max}$

$$\tau_{max} = \frac{Tc}{J}$$



Case II: $\tau < \tau_Y$ Hooke's Law applies, $\tau = \tau_{max}$

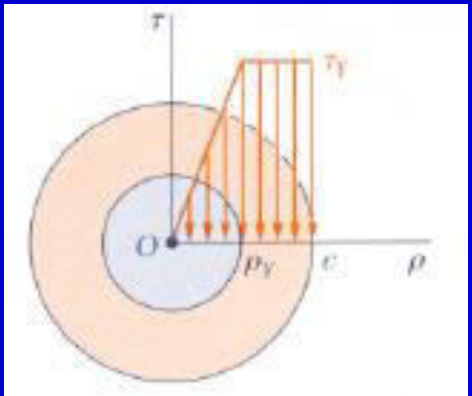
$$T_Y = \frac{J}{c}\tau_Y$$

T_Y = max elastic torque

Case II

$$J/C = \frac{1}{2}\pi c^3$$ **Since**

$$T_Y = \frac{1}{2}\pi c^3 \tau_Y \qquad (3\text{-}29)$$



Case III

Case III: Entering Plastic Region

$0 \le \rho \le \rho_Y$: $$\tau = \frac{\tau_Y}{\rho_Y}\rho$$

$\rho_Y \le \rho \le c$: $$\tau = \tau_Y$$

ρ_Y – region within the plastic range

By evoking Eq. (3.26)

$$T = 2\pi \int_0^c \rho^2 \tau \, d\rho \qquad (3.26)$$

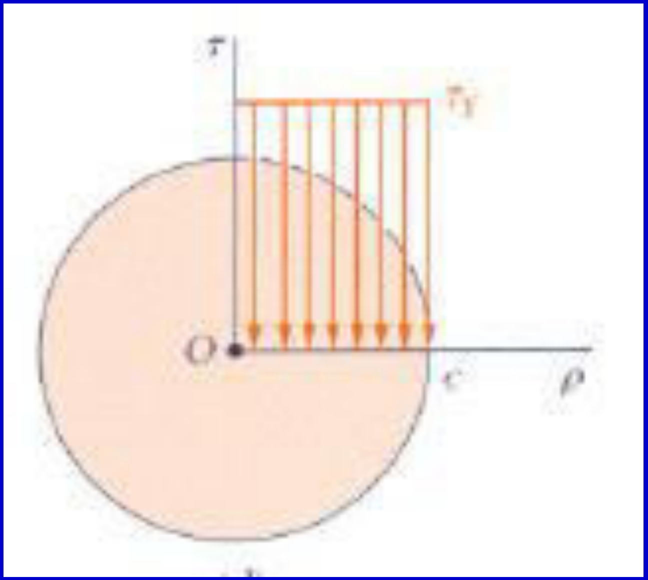
$$T = T_{elastic} + T_{plastic} = 2\pi \int_0^{\rho_Y} \rho^2 \left(\frac{\tau_Y}{\rho_Y} \rho \right) d\rho + 2\pi \int_{\rho_Y}^{c} \rho^2 \tau_Y \, d\rho$$

$$= \frac{1}{2}\pi \rho_Y^3 \tau_Y + \frac{2}{3}\pi c^3 \tau_Y - \frac{2}{3}\pi \rho_Y^3 \tau_Y$$

$$T = \frac{2}{3}\pi c^3 \tau_Y (1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}) \qquad (3.31)$$

$$T = \frac{4}{3} T_Y (1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}) \quad \leftarrow \quad T_Y = \frac{1}{2}\pi c^3 \tau_Y$$

Case IV -- Fully Plastic

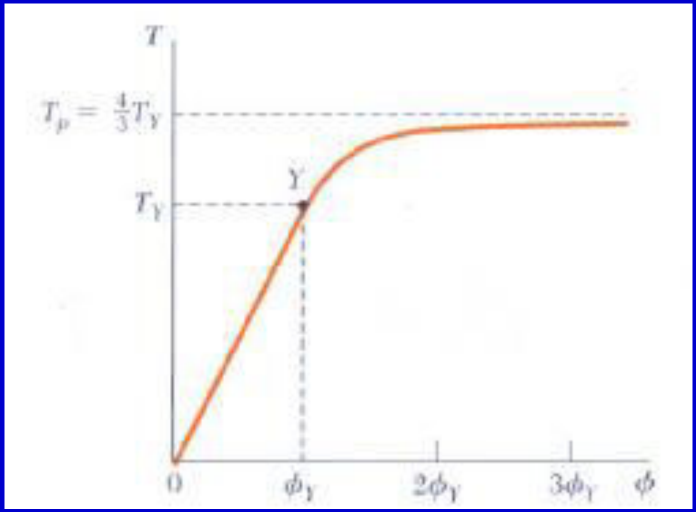


$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right)$$

$\rho_Y \rightarrow 0$:

$$T_P = \frac{4}{3} T_Y$$ = Plastic Torque (3-33)

Case IV



$$\rho_Y = \frac{L\gamma_Y}{\phi}$$

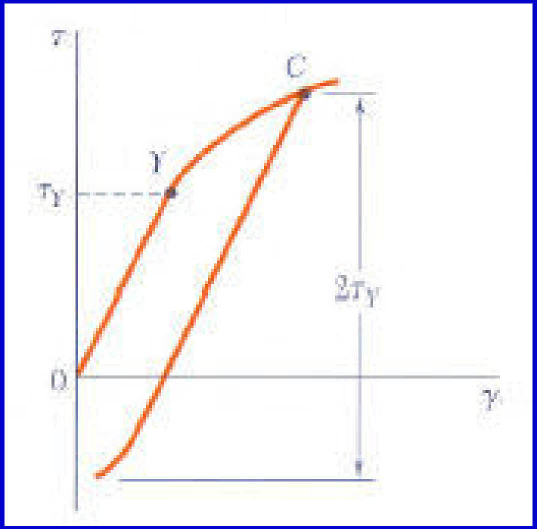
$$c = \frac{L\gamma_Y}{\phi_Y}$$

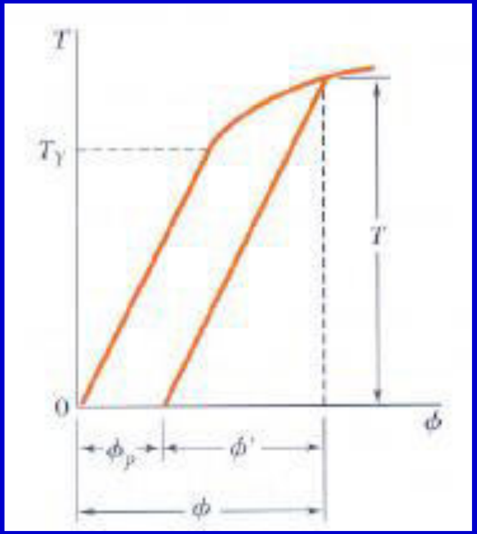
$$\frac{\rho_Y}{c} = \frac{\phi_Y}{\phi}$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\phi_Y^3}{\phi^3}\right)$$

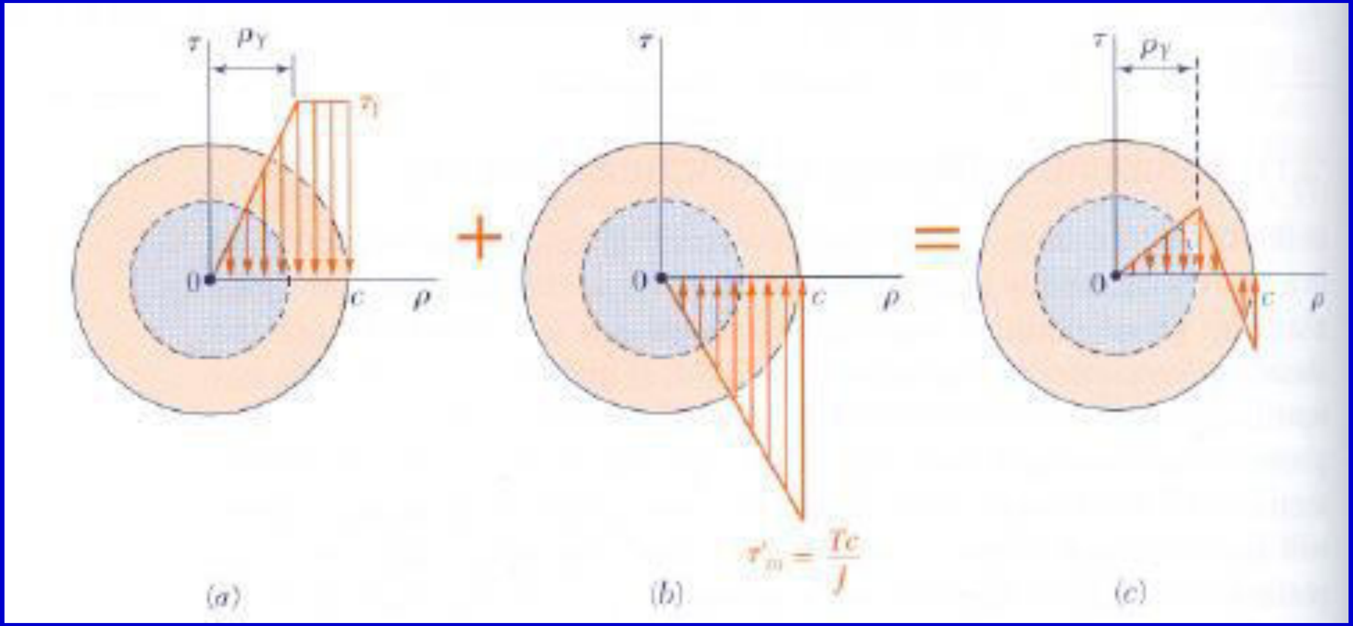
$$T = \rho A \tau$$

3.11 Residual Stresses in Circular Shafts

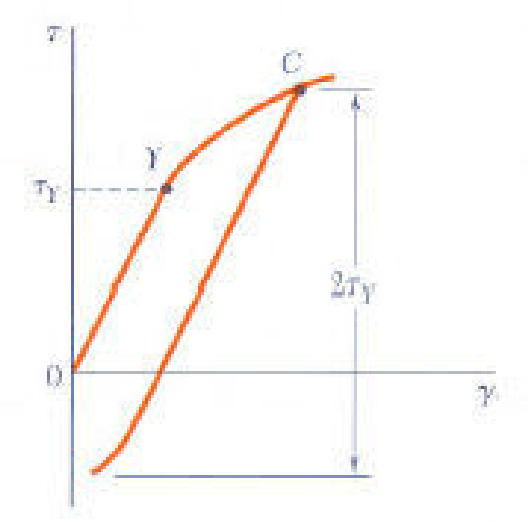




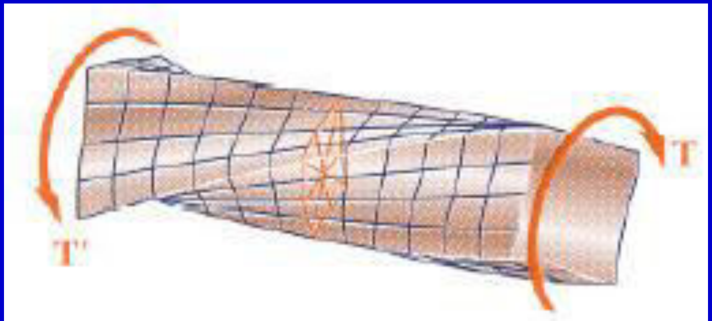
$$\phi_P = \phi - \phi^{'}$$

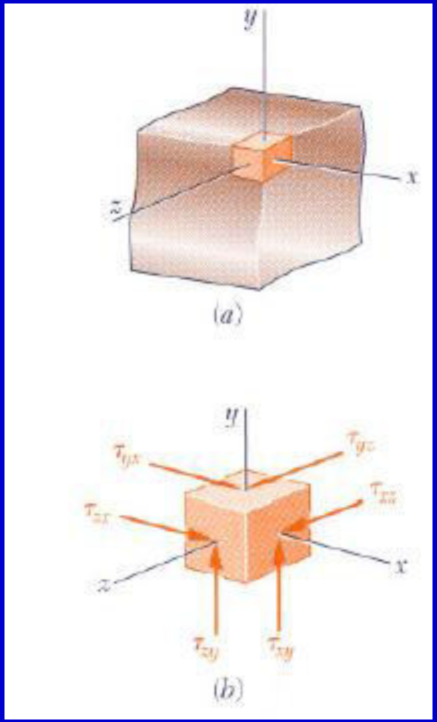


$$\int \rho(\tau dA) = 0$$



3.12 Torsion of Noncircular Members



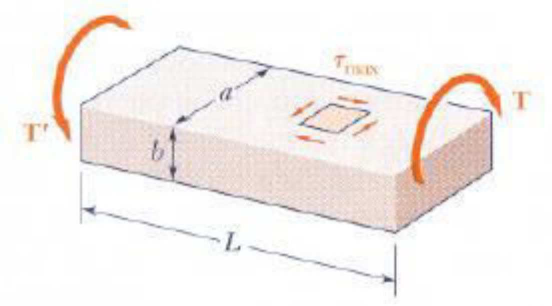


A rectangular shaft does not axisymmetry.

$$\tau_{zx} = 0 \qquad \tau_{zy} = 0$$

$$\tau_{yx} = 0 \qquad \tau_{yz} = 0$$

$$\tau_{xy} = 0 \qquad \tau_{xz} = 0$$



From Theory of Elasticity:

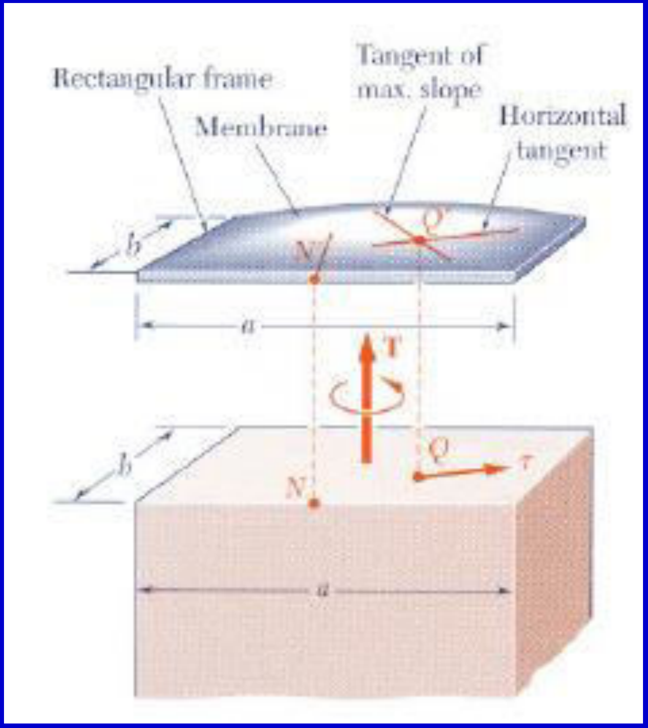
$$\tau_{\max} = \frac{T}{c_1 a b^2}$$

$$\phi = \frac{TL}{c_2 a b^3 G}$$

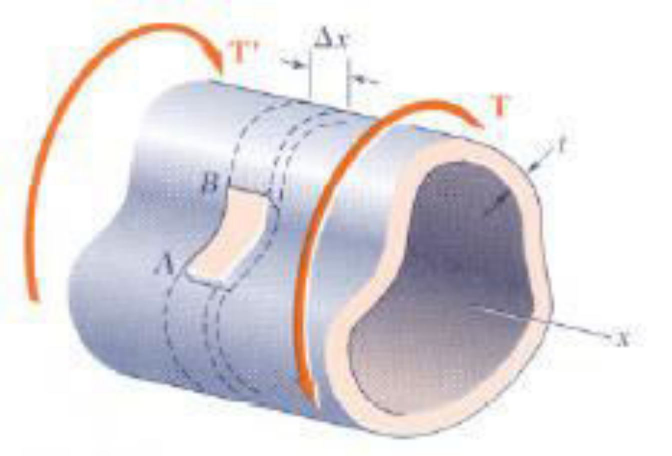
TABLE 3.1. Coefficients for Rectangular Bars in Torsion

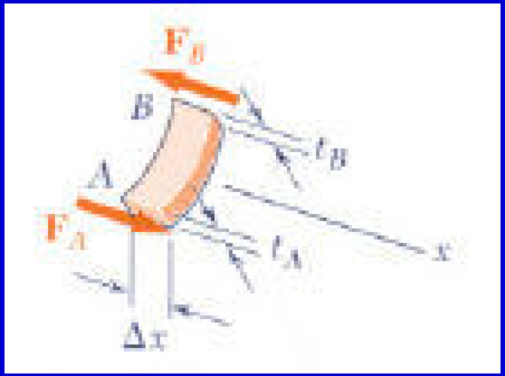
a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

$$c_1 = c_2 = \frac{1}{3}(1 - 0.630 b/a) \quad \textbf{(for b/a = 5 only)} \quad \textbf{3.45}$$



3.13 Thin-Walled Hollow Shafts



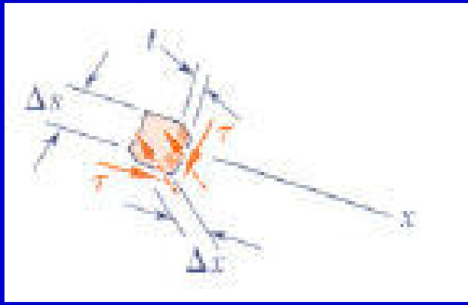


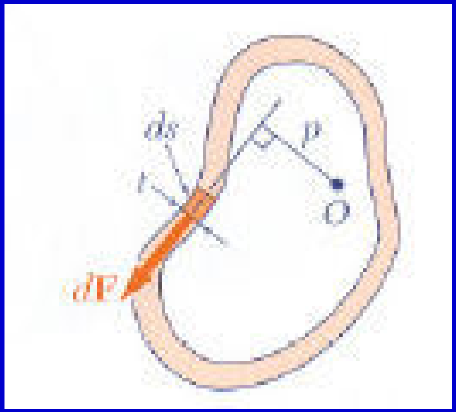
$\Sigma F_x = 0$ $\quad$ $F_A - F_B = 0$ $\quad$ $F_A = \tau_A (t_A \Delta x)$

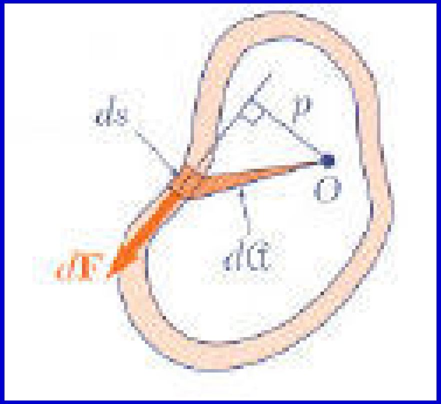
$$\tau_A (t_A \Delta x) - \tau_B (t_B \Delta x) = 0$$

$$\tau_A t_A = \tau_B t_B$$

$$q = \tau t = constant$$





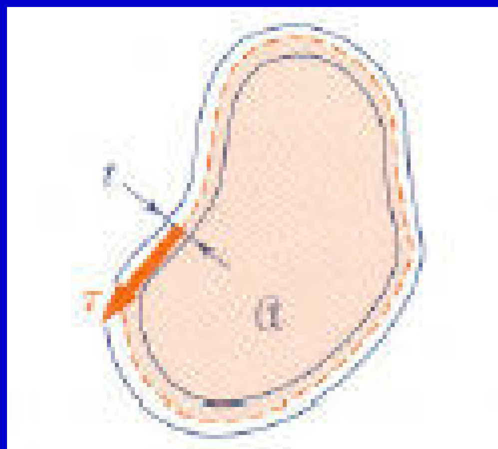


$$dF = \tau dA = \tau(tds) = (\tau t)ds = qds$$

$$dM_O = pdF = p(qds) = q(pds)$$

$$dM_o = q(2d\mathcal{A})$$

$$T = 2q\mathcal{A}$$



$$\phi = \frac{TL}{4\mathcal{A}^2 G}\oint \frac{ds}{t}$$

$$\tau = \frac{T}{2t\mathcal{A}}$$

UNIT-V

THIN AND THICK CYLINDERS

INTRODUCTION:

In many engineering applications, cylinders are frequently used for transporting or storing of liquids, gases or fluids.

Eg: Pipes, Boilers, storage tanks etc.

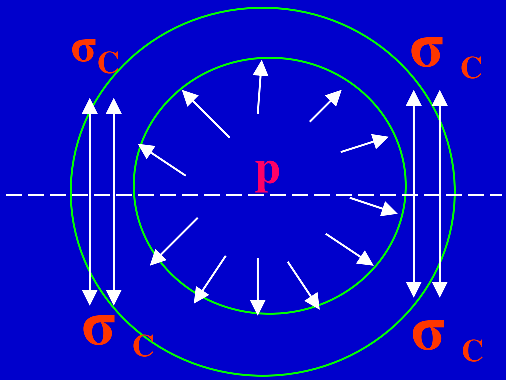
These cylinders are subjected to fluid pressures. When a cylinder is subjected to a internal pressure, at any point on the cylinder wall, three types of stresses are induced on three mutually perpendicular planes.

They are,

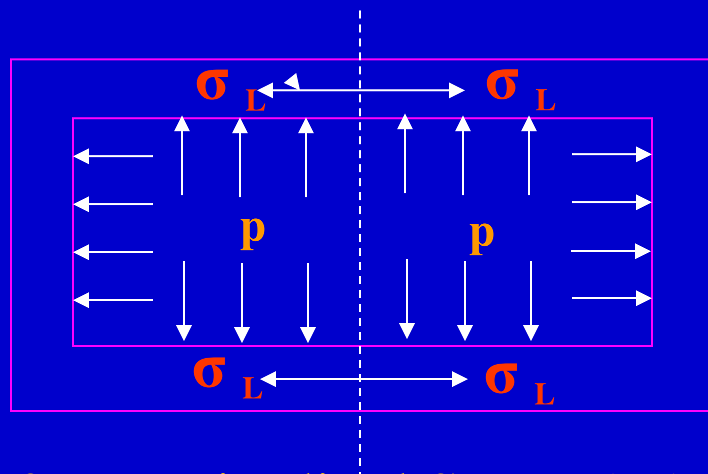
1. Hoop or Circumferential Stress (σ_C) – This is directed along the tangent to the circumference and tensile in nature. Thus, there will be increase in diameter.

2. Longitudinal Stress (σ_L) – This stress is directed along the length of the cylinder. This is also tensile in nature and tends to increase the length.

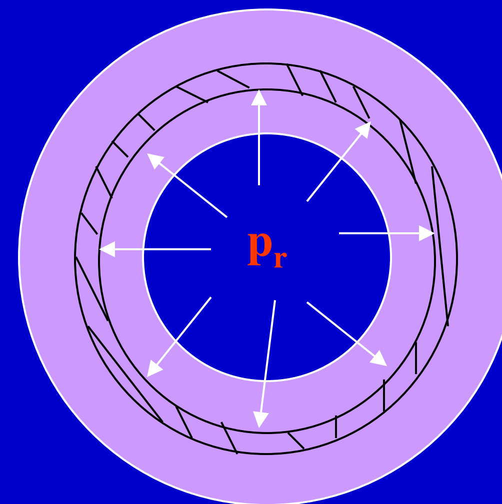
3. Radial pressure (p_r) – It is compressive in nature. Its magnitude is equal to fluid pressure on the inside wall and zero on the outer wall if it is open to atmosphere.



1. Hoop Stress (♦$_C$)

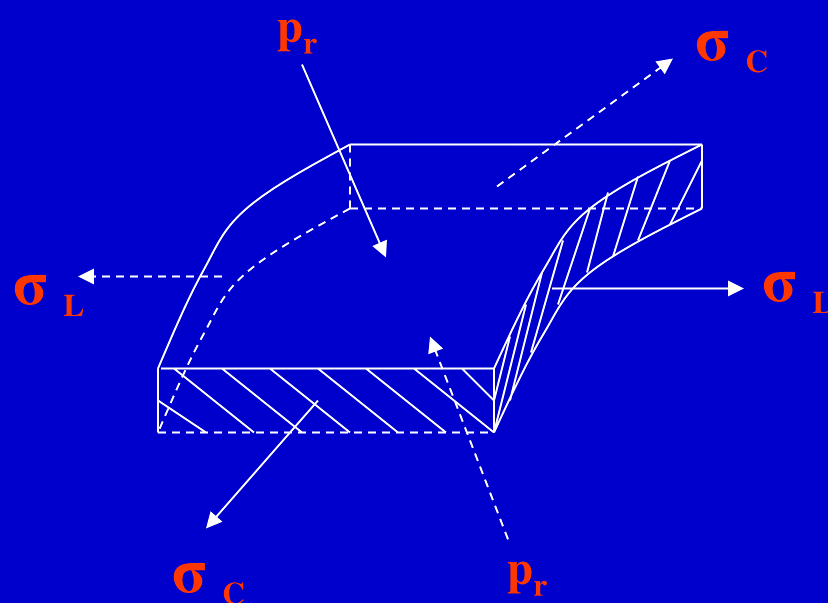


2. Longitudinal Stress (♦$_L$)



3. Radial Stress (p_r)

Element on the cylinder wall subjected to these three stresses



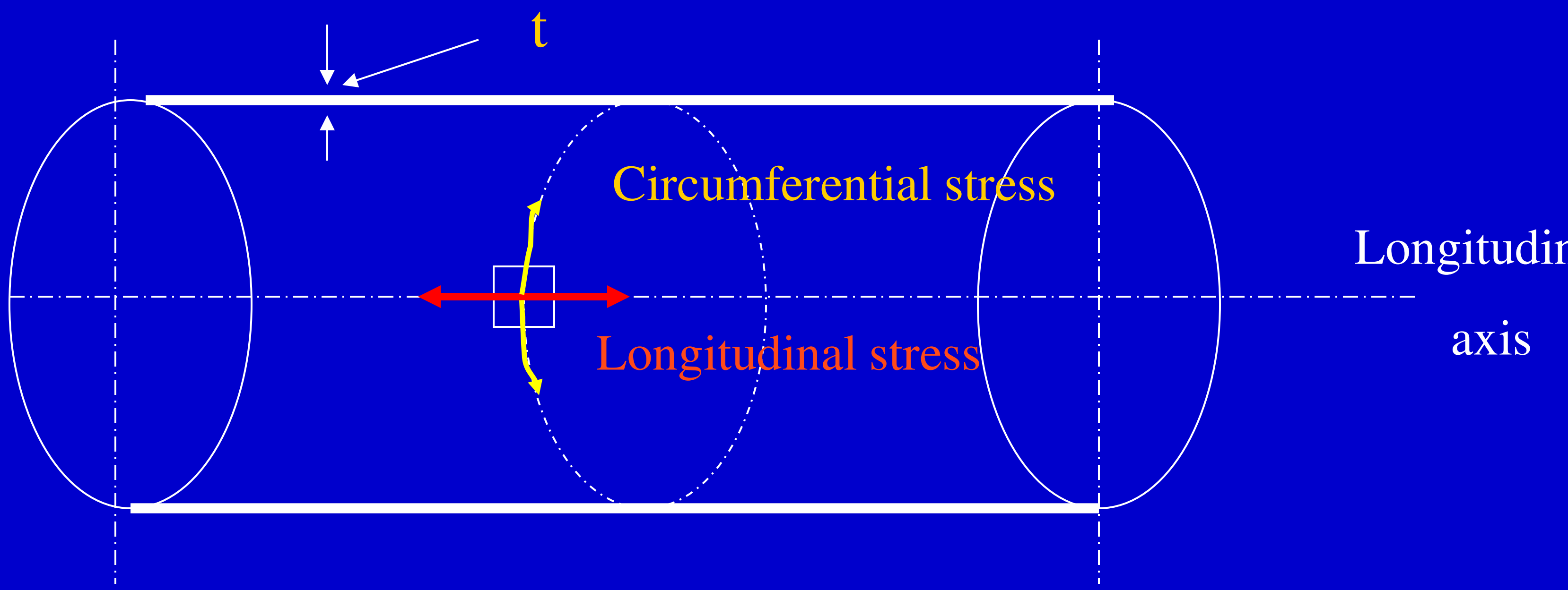
THIN CYLINDERS

INTRODUCTION:

A cylinder or spherical shell is considered to be thin when the metal thickness is small compared to internal diameter.

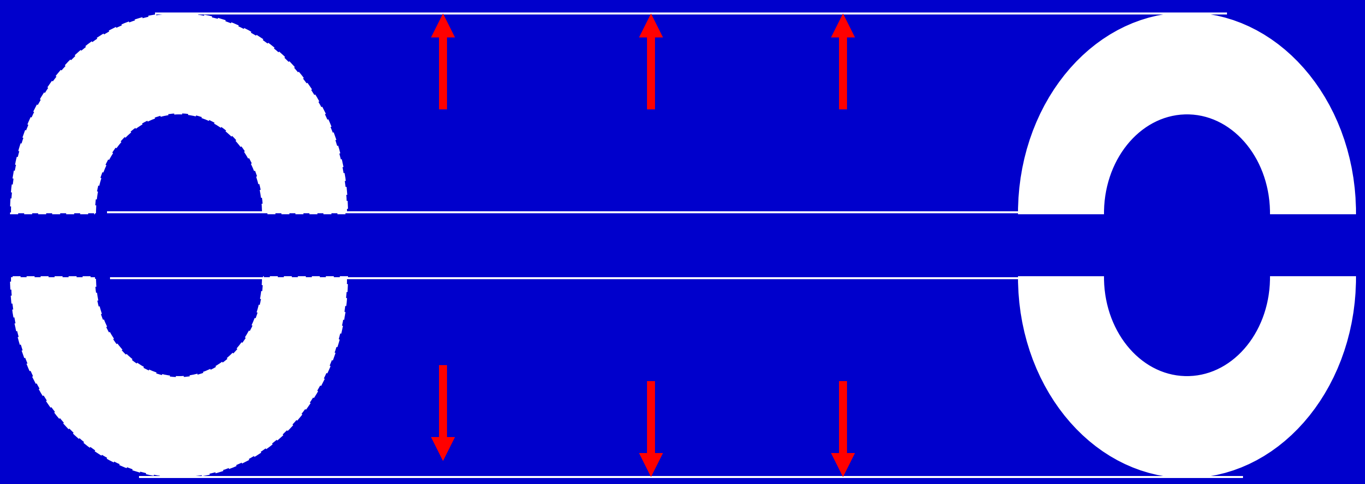
i. e., when the wall thickness, ‘t’ is equal to or less than ‘d/20’, where ‘d’ is the internal diameter of the cylinder or shell, we consider the cylinder or shell to be thin, otherwise thick.

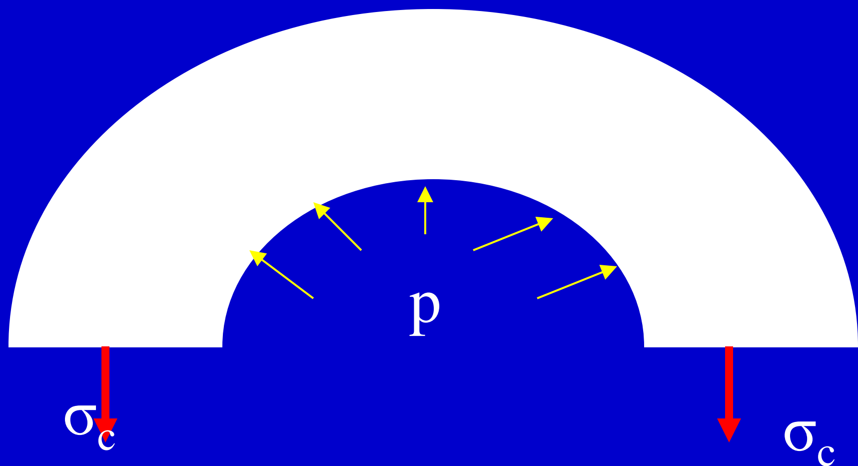
Magnitude of radial pressure is very small compared to other two stresses in case of thin cylinders and hence neglected.



The stress acting along the circumference of the cylinder is called circumferential stresses whereas the stress acting along the length of the cylinder (i.e., in the longitudinal direction) is known as longitudinal stress

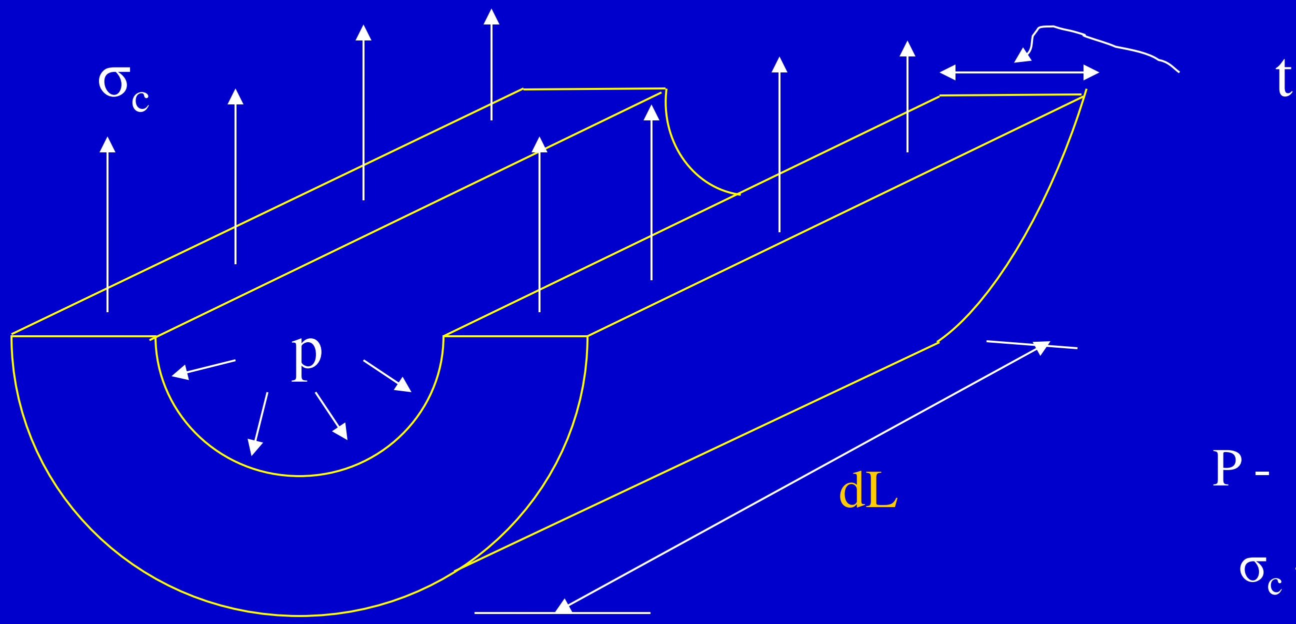
The bursting will take place if the force due to internal (fluid) pressure (acting vertically upwards and downwards) is more than the resisting force due to circumferential stress set up in the material.





P - internal pressure (str

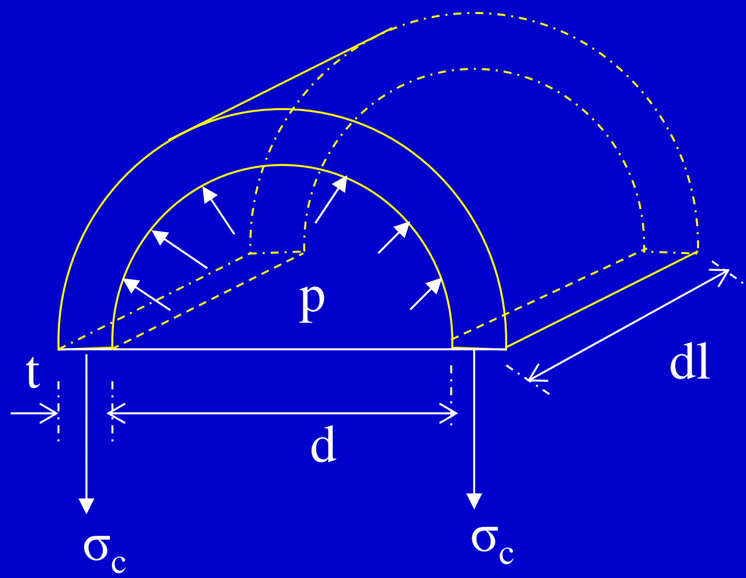
σ_c –circumferential stre

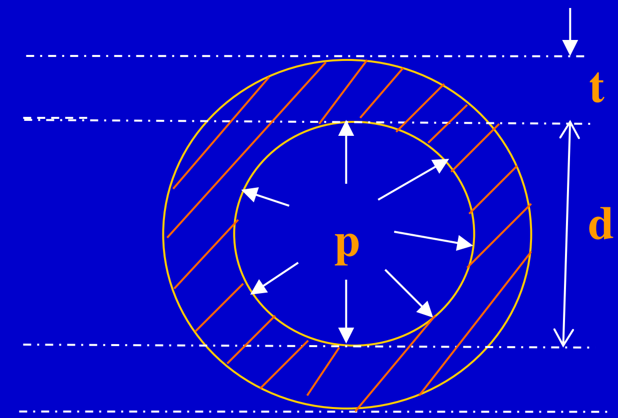


P - internal pressure (str

σ_c – circumferential stre

EVALUATION OF CIRCUMFERENTIAL or HOOP STRESS (σ_C):





Consider a thin cylinder closed at both ends and subjected to internal pressure ‘p’ as shown in the figure.

Let d=Internal diameter, t = Thickness of the wall

L = Length of the cylinder.

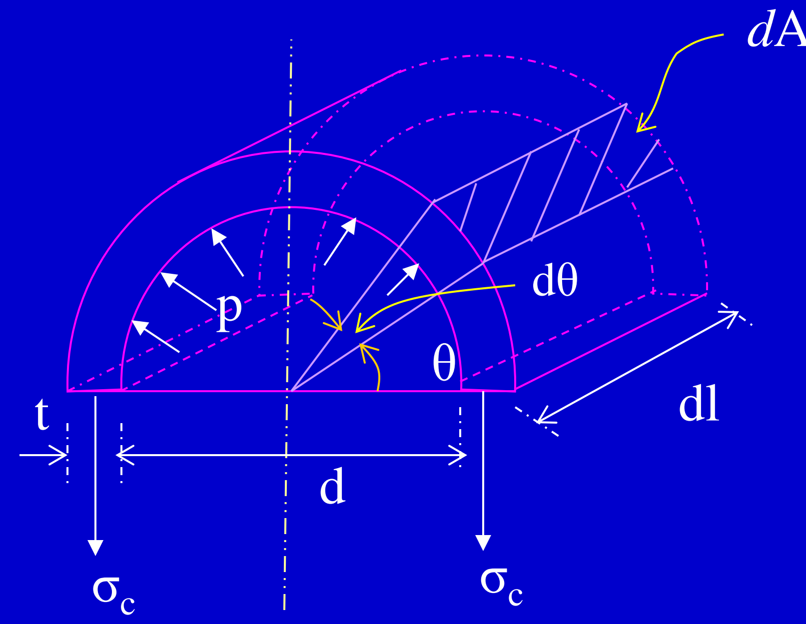
To determine the Bursting force across the diameter:

Consider a small length '*dl*' of the cylinder and an elementary area '*dA*' as shown in the figure.

Force on the elementary area,

$$dF = p \times dA = p \times r \times dl \times d\theta$$

$$= p \times \frac{d}{2} \times dl \times d\theta$$



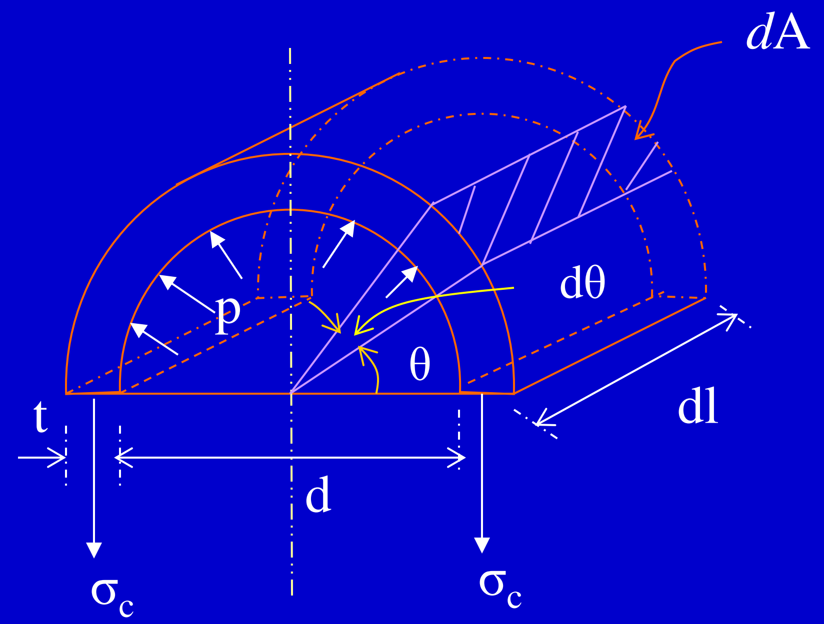
Horizontal component of this force

$$dF_x = p \times \frac{d}{2} \times dl \times \cos\theta \times d\theta$$

Vertical component of this force

$$dF_y = p \times \frac{d}{2} \times dl \times \sin\theta \times d\theta$$

The horizontal components cancel out when integrated over semi-circular portion as there will be another equal and opposite horizontal component on the other side of the vertical axis.



$$\therefore \text{Total diametrical bursting force} = \int_0^{\pi} p \times \frac{d}{2} \times dl \times \sin\theta \times d\theta$$

$$= p \times \frac{d}{2} \times dl \times \left[-\cos\theta\right]_0^{\pi} = \underline{p \times d \times dl}$$

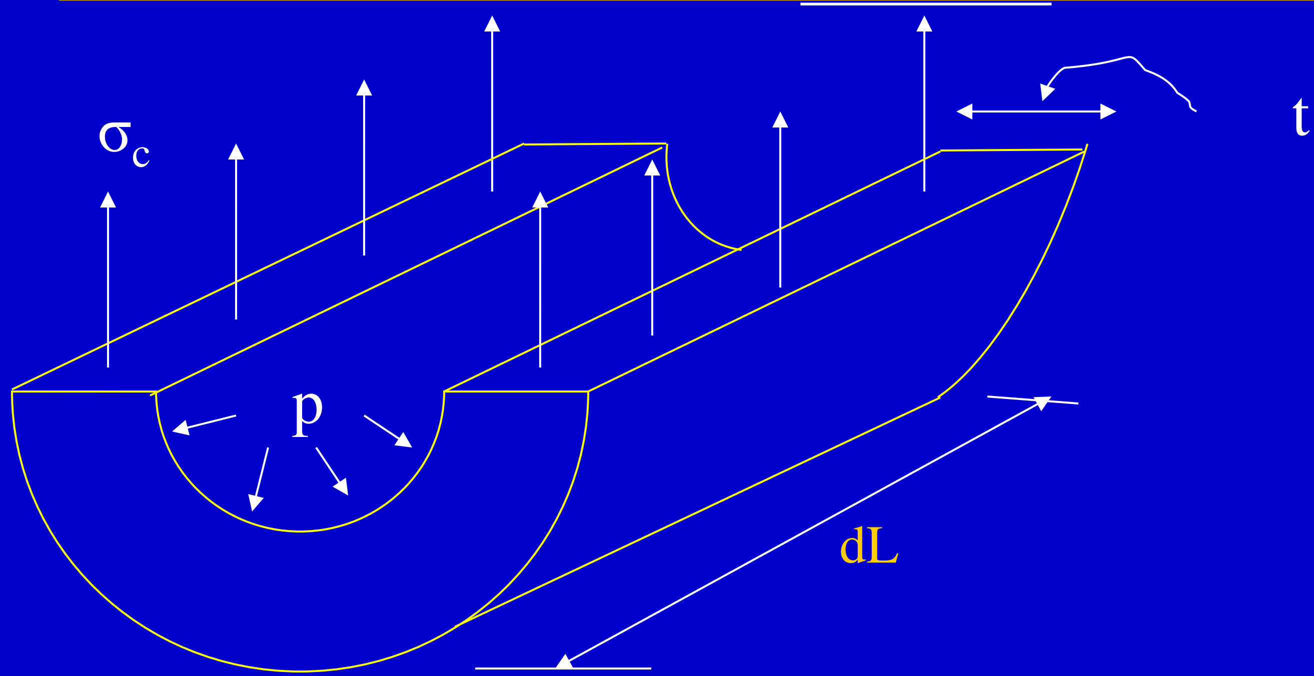
$$= p \times \text{projected area of the curved surface.}$$

$$\therefore \text{Resisting force (due to circumferential stress } \sigma_c) = 2 \times \sigma_c \times t \times dl$$

$$\text{Under equillibrium, Resisting force} = \text{Bursting force}$$

$$\text{i.e., } 2 \times \sigma_c \times t \times dl = p \times d \times dl$$

$$\therefore \text{Circumferential stress, } \sigma_c = \frac{p \times d}{2 \times t} \text{.......................} (1)$$



Assumed as rectangular

Force due to fluid pressure = p × area on which p is acting = p ×(d ×L)

(bursting force)

Force due to circumferential stress = σ_c × area on which σ_c is acting

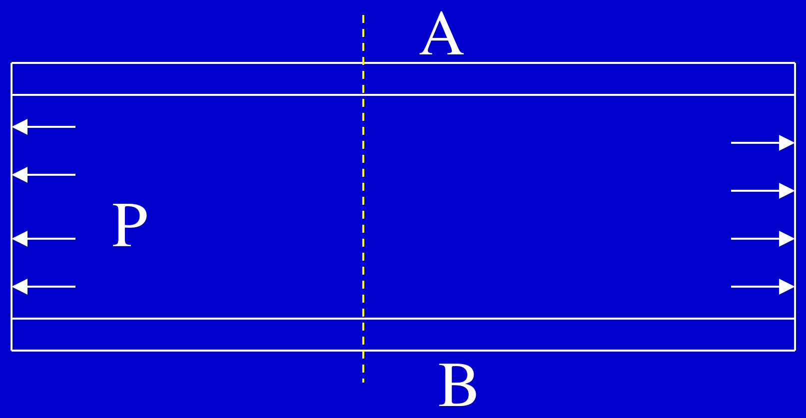
(resisting force) $= \sigma_c \times (L \times t + L \times t) = \sigma_c \times 2\,L \times t$

Under equilibrium bursting force = resisting force

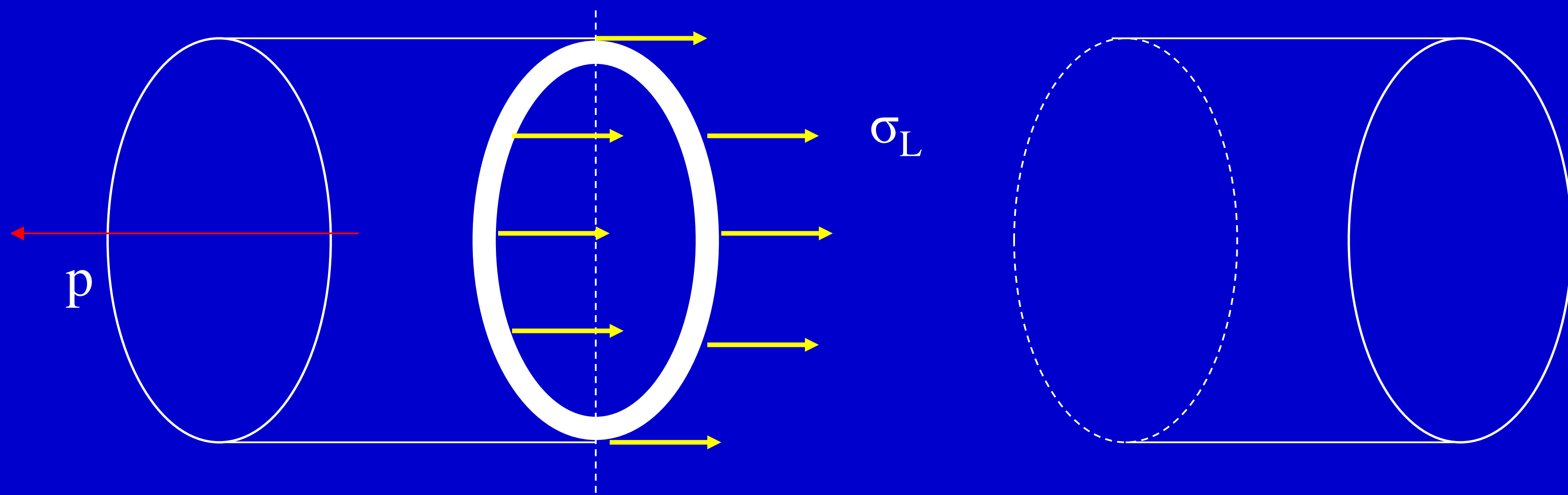
$$p \times (d \times L) = \sigma_c \times 2\,L \times t$$

$$\therefore \text{Circumferential stress, } \sigma_c = \frac{p \times d}{2 \times t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

LONGITUDINAL STRESS (σ_L):

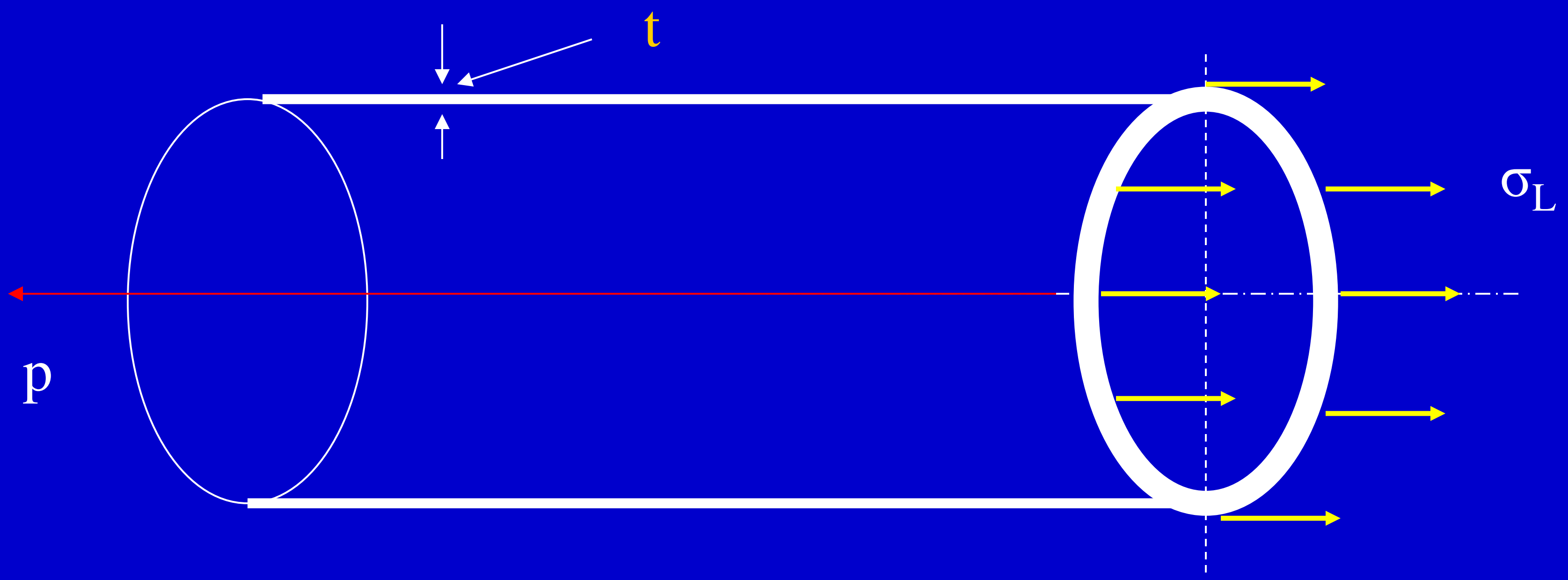


The bursting of the cylinder takes place along the section AB



The force, due to pressure of the fluid, acting at the ends of the thin cylinder, tends to burst the cylinder as shown in figure

EVALUATION OF LONGITUDINAL STRESS (σ_L):



$$\text{Longitudinal bursting force (on the end of cylinder)} = p \times \frac{\pi}{4} \times d^2$$

$$\text{Area of cross section resisting this force} = \pi \times d \times t$$

$$\text{Let } \sigma_L = \text{Longitudinal stress of the material of the cylinder.}$$

$$\therefore \text{Resisting force} = \sigma_L \times \pi \times d \times t$$

Under equillibrium, bursting force = resisting force

$$\text{i.e., } p \times \frac{\pi}{4} \times d^2 = \sigma_L \times \pi \times d \times t$$

$$\therefore \quad \text{Longitudinal stress, } \sigma_L = \frac{p \times d}{4 \times t} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

From eqs (1) & (2), $\sigma_C = 2 \times \sigma_L$

$$\text{Force due to fluid pressure} = p \times \text{area on which p is acting}$$

$$= p \times \frac{\pi}{4} \times d^2$$

$$\text{Re}\,\textit{sisting force} = \sigma_L \times \textit{area on which } \sigma_L \textit{ is acting}$$

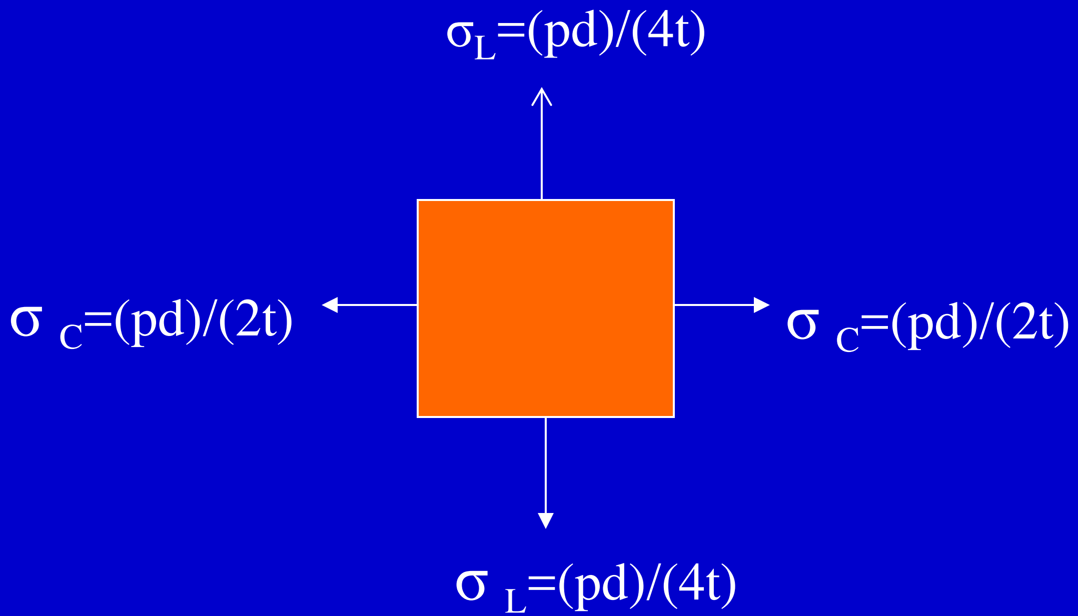
$$= \sigma_L \times \boxed{\pi \times d} \times t$$

<u>circumference</u>

$$\text{Under equillibrium, bursting force} = \text{resisting force}$$

$$\therefore \quad \text{Longitudinal stress, } \sigma_L = \frac{p \times d}{4 \times t} \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

EVALUATION OF STRAINS



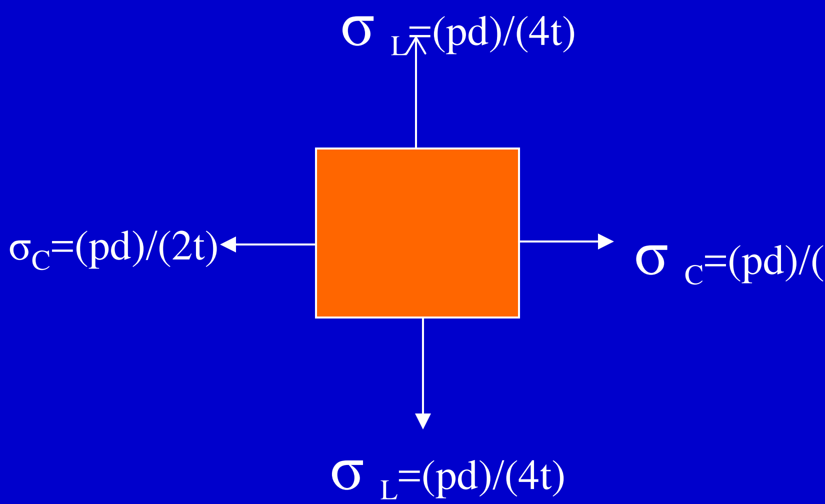
A point on the surface of thin cylinder is subjected to biaxial stress system, (Hoop stress and Longitudinal stress) mutually perpendicular to each other, as shown in the figure. The strains due to these stresses i.e., circumferential and longitudinal are obtained by applying Hooke's law and Poisson's theory for elastic materials.

Circumferential strain, ε_C :

$$\varepsilon_C = \frac{\sigma_C}{E} - \mu \times \frac{\sigma_L}{E}$$

$$= 2 \times \frac{\sigma_L}{E} - \mu \times \frac{\sigma_L}{E}$$

$$= \frac{\sigma_L}{E} \times (2 - \mu)$$



i.e., $$\varepsilon_C = \frac{\delta d}{d} = \frac{p \times d}{4 \times t \times E} \times (2 - \mu) \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Note: Let δd be the change in diameter. Then

$$\varepsilon_c = \frac{final\ \ circumference - original\ \ circumference}{original\ \ circumference}$$

$$= \left[\frac{\pi\left(d + \delta d\right) - \pi d}{\pi d} \right] = \frac{\delta d}{d}$$

Longitudinal strain, ε_L :

$$\varepsilon_L = \frac{\sigma_L}{E} - \mu \times \frac{\sigma_C}{E}$$

$$= \frac{\sigma_L}{E} - \mu \times \frac{(2 \times \sigma_L)}{E} = \frac{\sigma_L}{E} \times (1 - 2 \times \mu)$$

i.e., $\varepsilon_L = \frac{\delta l}{L} = \frac{p \times d}{4 \times t \times E} \times (1 - 2 \times \mu)$..............................(4)

VOLUMETRIC STRAIN, $\frac{\delta V}{V}$

Change in volume = δV = final volume – original volume

original volume = V = area of cylindrical shell × length

$$= \frac{\pi d^2}{4} L$$

final volume = final area of cross section × final length

$$= \frac{\pi}{4}\left[d + \delta d\right]^2 \times \left[L + \delta L\right]$$

$$= \frac{\pi}{4}\left[d^2 + (\delta d)^2 + 2 d \delta d\right] \times \left[L + \delta L\right]$$

$$= \frac{\pi}{4}\left[d^2 L + (\delta d)^2 L + 2 L d \delta d + d^2 \delta L + (\delta d)^2 \delta L + 2 d \delta d \, \delta L\right]$$

neglecting the smaller quantities such as $(\delta d)^2 L, (\delta d)^2 \delta L$ *and* $2 d \delta d \, \delta L$

$$Final\ volume = \frac{\pi}{4}\left[d^2 L + 2 L d \delta d + d^2 \delta L\right]$$

$$change\ in\ volume\ \delta V = \frac{\pi}{4}\left[d^2 L + 2 L d \delta d + d^2 \delta L\right] - \frac{\pi}{4}\left[d\right]^2 L$$

$$\delta V = \frac{\pi}{4}\left[2 L d \delta d + d^2 \delta L\right]$$

$$\frac{dv}{V} = \frac{\frac{\pi}{4}\left[2\,d\,L\,\delta\,d + \delta\,L\,d^{2}\right]}{\frac{\pi}{4} \times d^{2} \times L}$$

$$= \frac{\delta L}{L} + 2 \times \frac{\delta d}{d}$$

$$\frac{\mathbf{dV}}{\mathbf{V}} = \varepsilon_L + 2 \times \varepsilon_C$$

$$= \frac{p \times d}{4 \times t \times E}(1 - 2 \times \mu) + 2 \times \frac{p \times d}{4 \times t \times E}(2 - \mu)$$

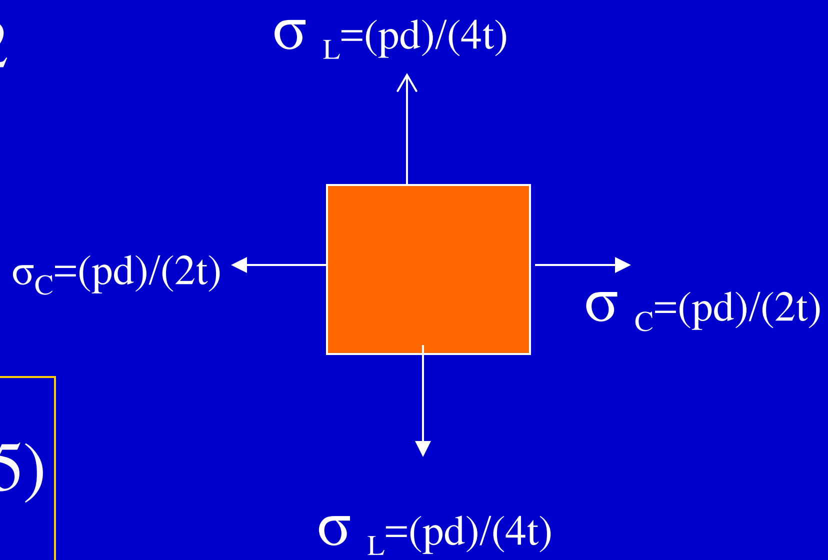
i.e., $$\frac{dv}{V} = \frac{p \times d}{4 \times t \times E}(5 - 4 \times \mu) \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Maximum Shear stress :

There are two principal stresses at any point, viz., Circumferential and longitudinal. Both these stresses are normal and act perpendicular to each other.

$$\therefore \text{ Maximum Shear stress, } \tau_{max} = \frac{\sigma_C - \sigma_L}{2}$$

$$= \frac{\frac{pd}{2t} - \frac{pd}{4t}}{2}$$



i.e., $$\tau_{max} = \frac{pd}{8t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Maximum Shear stress :

$$\therefore \text{ Maximum Shear stress, } \tau_{max} = \frac{\sigma_C - \sigma_L}{2}$$

$$= \frac{\frac{pd}{2t} - \frac{pd}{4t}}{2}$$

$$\text{i.e.,} \qquad \tau_{max} = \frac{pd}{8t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

ILLUSTRATIVE PROBLEMS

PROBLEM 1:

A thin cylindrical shell is 3m long and 1m in internal diameter. It is subjected to internal pressure of 1.2 MPa. If the thickness of the sheet is 12mm, find the circumferential stress, longitudinal stress, changes in diameter, length and volume . Take E=200 GPa and μ= 0.3.

SOLUTION:

1. Circumferential stress, σ_C:

$$\sigma_C = (p \times d) / (2 \times t)$$

$$= (1.2 \times 1000) / (2 \times 12)$$

$$= \underline{50\ N/mm^2 = 50\ MPa}\ \text{(Tensile)}.$$

2. Longitudinal stress, σ_L:

$$\sigma_L = (p \times d) / (4 \times t)$$

$$= \sigma_C / 2 = 50/2$$

$$= 25\ N/mm^2 = 25\ MPa\ \text{(Tensile)}$$

3. Circumferential strain, ε_c:

$$\varepsilon_c = \frac{(p \times d)}{(4 \times t)} \times \frac{(2-\mu)}{E}$$

$$= \frac{(1.2 \times 1000)}{(4 \times 12)} \times \frac{(2-0.3)}{200 \times 10^3}$$

$$= \underline{2.125 \times 10^{-04}} \text{ (Increase)}$$

Change in diameter, $\delta d = \varepsilon_c \times d$

$= 2.125 \times 10^{-04} \times 1000 = \underline{0.2125}$ mm (Increase).

4. Longitudinal strain, ε_L:

$$\varepsilon_L = \frac{(p \times d)}{(4 \times t)} \times \frac{(1-2 \times \mu)}{E}$$

$$= \frac{(1.2 \times 1000)}{(4 \times 12)} \times \frac{(1-2 \times 0.3)}{200 \times 10^3}$$

$$= \underline{5 \times 10^{-05}} \text{ (Increase)}$$

Change in length $= \varepsilon_L \times L = 5 \times 10^{-05} \times 3000 = \underline{0.15}$ mm (Increase).

Volumetric strain, $\frac{dv}{V}$:

$$\frac{dv}{V} = \frac{(p \times d)}{(4 \times t) \times E} \times (5 - 4 \times \mu)$$

$$= \frac{(1.2 \times 1000)}{(4 \times 12) \times 200 \times 10^{3}} \times (5 - 4 \times 0.3)$$

$$= 4.75 \times 10^{-4} \text{ (Increase)}$$

$$\therefore \text{Change in volume, } dv = 4.75 \times 10^{-4} \times V$$

$$= 4.75 \times 10^{-4} \times \frac{\pi}{4} \times 1000^{2} \times 3000$$

$$= 1.11919 \times 10^{6} \text{ mm}^{3} = 1.11919 \times 10^{-3} \text{ m}^{3}$$

$$= \underline{1.11919 \text{ Litres}}.$$

A copper tube having 45mm internal diameter and 1.5mm wall thickness is closed at its ends by plugs which are at 450mm apart. The tube is subjected to internal pressure of 3 MPa and at the same time pulled in axial direction with a force of 3 kN. Compute: i) the change in length between the plugs ii) the change in internal diameter of the tube. Take E_{CU} = 100 GPa, and μ_{cu} = 0.3.

SOLUTION:

A] Due to Fluid pressure of 3 MPa:

Longitudinal stress, $\sigma_L = (p\times d) / (4\times t)$

$= (3\times 45) / (4\times 1.5) = 22.50\ N/mm^2 = 22.50\ MPa.$

Long. strain, $\varepsilon_L = \frac{(p\times d)}{4\times t} \times \frac{(1-2\times\mu)}{E}$

$$= \frac{22.5\times(1-2\times 0.3)}{100\times 10^3} = \underline{9\times 10^{-5}}$$

Change in length, $\delta_L = \varepsilon_L \times L = 9\times 10^{-5}\times 450 = \underline{+0.0405\ mm}$ (increase)

$Pd/4t = 22.5$

$$\text{Circumferential strain } \varepsilon_C = \frac{(p \times d)}{(4 \times t)} \times \frac{(2-\mu)}{E}$$

$$= \frac{22.5 \times (2-0.3)}{100 \times 10^3} = \underline{3.825 \times 10^{-4}}$$

Change in diameter, $\delta_d = \varepsilon_c \times d = 3.825 \times 10^{-4} \times 45$

$= +\ \underline{0.0172\ \text{mm}}$ (increase)

B] Due to Pull of 3 kN (P=3kN):

Area of cross section of copper tube, $A_c = \pi \times d \times t$

$= \pi \times 45 \times 1.5 = 212.06\ \text{mm}^2$

Longitudinal strain, ε_L = direct stress/E = $\sigma/E = P/(A_c \times E)$

$= 3 \times 10^3/(212.06 \times 100 \times 10^3)$

$= \underline{1.415 \times 10^{-4}}$

Change in length, $\delta_L = \varepsilon_L \times L = 1.415 \times 10^{-4} \times 450 = \underline{+0.0637\text{mm}}$ (increase)

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Lateral strain, $\varepsilon_{lat} = -\mu \times$ Longitudinal strain $= -\mu \times \varepsilon_L$

$= -0.3 \times 1.415 \times 10^{-4} = -4.245 \times 10^{-5}$

Change in diameter, $\delta_d = \varepsilon_{lat} \times d = -4.245 \times 10^{-5} \times 45$

$= \underline{-1.91 \times 10^{-3}\ \text{mm}}$ (decrease)

C) Changes due to combined effects:

Change in length $= 0.0405 + 0.0637 = \underline{+0.1042\ \text{mm}}$ (increase)

Change in diameter $= 0.01721 - 1.91 \times 10^{-3} = \underline{+0.0153\ \text{mm}}$ (increase)

PROBLEM 3:

A cylindrical boiler is 800mm in diameter and 1m length. It is required to withstand a pressure of 100m of water. If the permissible tensile stress is $20N/mm^2$, permissible shear stress is $8N/mm^2$ and permissible change in diameter is 0.2mm, find the minimum thickness of the metal required. Take E = 200GPa, and $\mu = 0.3$.

SOLUTION:

Fluid pressure, $p = 100m$ of water $= 100\times9.81\times10^3\ N/m^2$

$= 0.981N/mm^2$.

1. Thickness from Hoop Stress consideration: (Hoop stress is critical than long. Stress)

$$\sigma_C = (p\times d)/(2\times t)$$

$$20 = (0.981\times800)/(2\times t)$$

2. Thickness from Shear Stress consideration:

$$\tau_{max} = \frac{(p \times d)}{(8 \times t)}$$

$$8 = \frac{(0.981 \times 800)}{(8 \times t)}$$

$$\therefore t = \underline{12.26mm.}$$

3. Thickness from permissible change in diameter consideration

(δd=0.2mm):

$$\frac{\delta d}{d} = \frac{(p \times d)}{(4 \times t)} \times \frac{(2-\mu)}{E}$$

$$\frac{0.2}{800} = \frac{(0.981 \times 800)}{(4 \times t)} \times \frac{(2-0.3)}{200 \times 10^3}$$

$$t = \underline{6.67mm}$$

Therefore, required thickness, t = 19.62 mm.

PROBLEM 4:

A cylindrical boiler has 450mm in internal diameter, 12mm thick and 0.9m long. It is initially filled with water at atmospheric pressure. Determine the pressure at which an additional water of 0.187 liters may be pumped into the cylinder by considering water to be incompressible. Take E = 200 GPa, and $\mu = 0.3$.

SOLUTION:

Additional volume of water, $\delta V = 0.187$ liters $= 0.187\times10^{-3}\ m^3$

$= 187\times10^{3}\ mm^3$

$$V = \frac{\pi}{4}\times 450^2\times(0.9\times10^3) = 143.14\times10^6\ mm^3$$

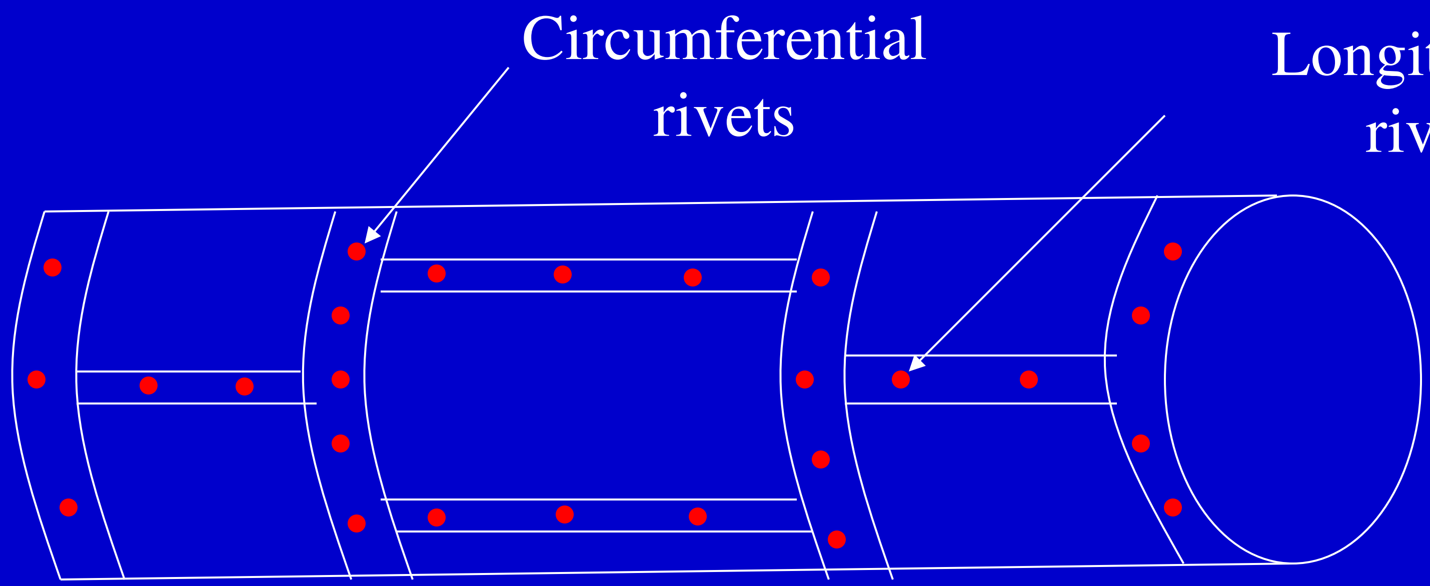
$$\frac{dV}{V} = \frac{p\times d}{4\times t\times E}(5-4\times\mu)$$

$$\frac{187\times10^3}{143.14\times10^6} = \frac{p\times450}{4\times12\times200\times10^3}(5-4\times0.33)$$

Solving, $p=7.33\ N/mm^2$

JOINT EFFICIENCY

Steel plates of only particular lengths and width are available. Hence whenever larger size cylinders (like boilers) are required, a number of plates are to be connected. This is achieved by using riveting in circumferential and longitudinal directions as shown in figure. Due to the holes for rivets, the net area of cross section decreases and hence the stresses increase.



JOINT EFFICIENCY

The cylindrical shells like boilers are having two types of joints namely Longitudinal and Circumferential joints. Due to the holes for rivets, the net area of cross section decreases and hence the stresses increase. If the efficiencies of these joints are known, the stresses can be calculated as follows.

Let η_L = Efficiency of Longitudinal joint

and η_C = Efficiency of Circumferential joint.

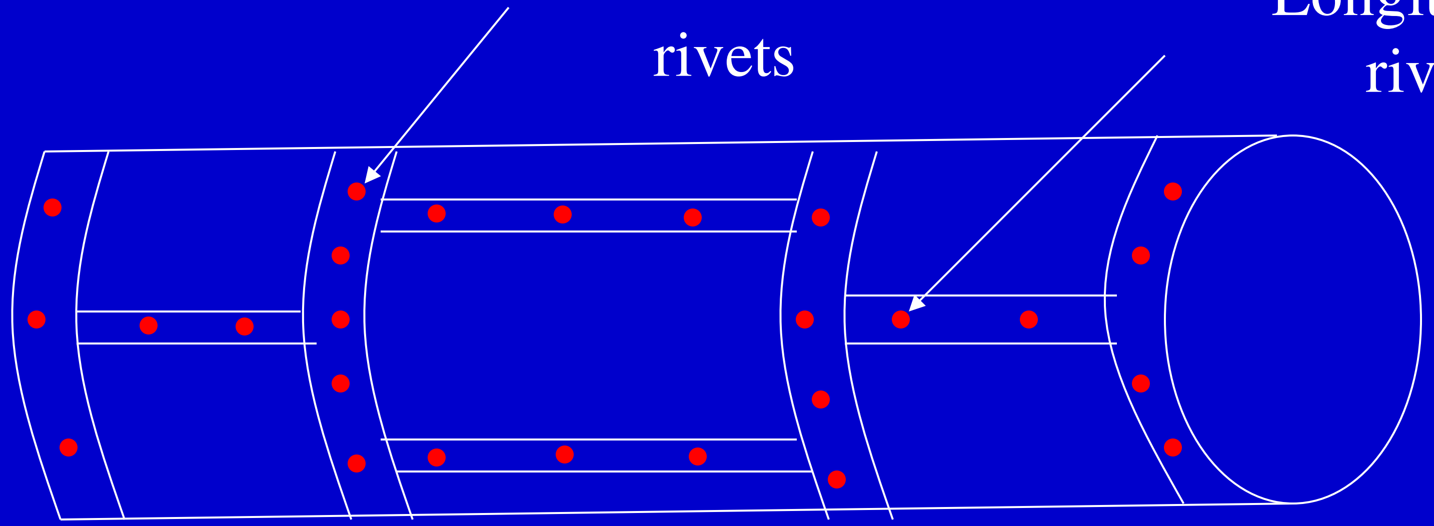
Circumferential stress is given by,

$$\sigma_C = \frac{p \times d}{2 \times t \times \eta_L} \quad \text{.............(1)}$$

Longitudinal stress is given by,

$$\sigma_L = \frac{p \times d}{4 \times t \times \eta_C} \quad ....(2)$$

Note: In longitudinal joint, the circumferential stress is developed and in circumferential joint, longitudinal stress is developed.



If A is the gross area and A_{eff} is the effective resisting area then,

$$Efficiency = A_{eff}/A$$

$$Bursting\ force = p\ L\ d$$

$$Resisting\ force = \sigma_c \times A_{eff} = \sigma_c \times \eta_L \times A = \sigma c \times \eta_L \times 2\ t\ L$$

Where η_L =Efficiency of Longitudinal joint

Bursting force = Resisting force

$$p\ L\ d = \sigma c \times \eta_L \times 2\ t\ L$$

$$\sigma_C = \frac{p \times d}{2 \times t \times \eta_L} \quad ....(1)$$

If η_c =Efficiency of circumferential joint

Efficiency = A_{eff}/A

Bursting force = $(\pi d^2/4)p$

Resisting force = $\sigma_L \times A'_{eff} = \sigma_L \times \eta_c \times A' = \sigma_L \times \eta_c \times \pi\, d\, t$

Where η_L =Efficiency of circumferential joint

Bursting force = Resisting force

$$\sigma_L = \frac{p \times d}{4 \times t \times \eta_C} \quad \text{.............(2)}$$

A cylindrical tank of 750mm internal diameter, 12mm thickness and 1.5m length is completely filled with an oil of specific weight 7.85 kN/m^3 at atmospheric pressure. If the efficiency of longitudinal joints is 75% and that of circumferential joints is 45%, find the pressure head of oil in the tank. Also calculate the change in volume. Take permissible tensile stress of tank plate as 120 MPa and E = 200 GPa, and μ = 0.3.

SOLUTION:

Let p = max permissible pressure in the tank.

Then we have, $\sigma_L = (p \times d)/(4 \times t)\ \eta_C$

$$120 = (p \times 750)/(4 \times 12)\ 0.45$$

$$p = 2.456 \text{ MPa.}$$

Also, $\sigma_C = (p \times d)/(2 \times t)\ \eta_L$

$$120 = (p \times 750)/(2 \times 12)\ 0.75$$

$$p = 2.88 \text{ MPa.}$$

Max permissible pressure in the tank, p = 2.88 MPa.

$$\text{Vol. Strain, } \frac{dv}{V} = \frac{(p \times d)}{(4 \times t \times E)} \times (5 - 4 \times \mu)$$

$$= \frac{(2.88 \times 750)}{(4 \times 12 \times 200 \times 10^{3})} \times (5 - 4 \times 0.3) = 8.55 \times 10^{-4}$$

$$dv = 8.55 \times 10^{-4} \times V = 8.55 \times 10^{-4} \times \frac{\pi}{4} \times 750^{2} \times 1500 = 0.567 \times 10^{6} \text{ mm}^{3}.$$

$$= 0.567 \times 10^{-3} \text{ m}^{3} = \underline{0.567} \text{ litres.}$$

A boiler shell is to be made of 15mm thick plate having a limiting tensile stress of 120 N/mm^2. If the efficiencies of the longitudinal and circumferential joints are 70% and 30% respectively determine;

i) The maximum permissible diameter of the shell for an internal pressure of 2 N/mm^2.

(ii) Permissible intensity of internal pressure when the shell diameter is 1.5m.

SOLUTION:

(i) To find the maximum permissible diameter of the shell for an internal pressure of 2 N/mm^2:

a) Let limiting tensile stress = Circumferential stress = σ_c = 120N/mm^2.

$$\text{i. e.,} \quad \sigma_c = \frac{p \times d}{2 \times t \times \eta_L}$$

$$120 = \frac{2 \times d}{2 \times 15 \times 0.7}$$

d = 1260 mm

b) Let limiting tensile stress = Longitudinal stress = σ_L = 120N/mm^2.

$$\text{i.e.,} \quad \sigma_L = \frac{p \times d}{4 \times t \times \eta_C}$$

$$120 = \frac{2 \times d}{4 \times 15 \times 0.3} \qquad . \qquad d = 1080 \text{ mm}$$

The maximum diameter of the cylinder in order to satisfy both the conditions = 1080 mm.

(ii) To find the permissible pressure for an internal diameter of 1.5m: (d=1.5m=1500mm)

a) Let limiting tensile stress = Circumferential stress = σ_c = 120N/mm^2.

$$\text{i. e.,} \quad \sigma_c = \frac{p \times d}{2 \times t \times \eta_L}$$

$$120 = \frac{p \times 1500}{2 \times 15 \times 0.7}$$

$$p = 1.68 \text{ N/mm}^2.$$

b) Let limiting tensile stress = Longitudinal stress = σ_L = 120N/mm^2.

$$\text{i. e.,} \quad \sigma_L = \frac{p \times d}{4 \times t \times \eta_C}$$

$$120 = \frac{p \times 1500}{4 \times 15 \times 0.3}$$

$$p = 1.44 \text{ N/mm}^2.$$

The maximum permissible pressure = 1.44 N/mm^2.

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PROBLEMS FOR PRACTICE

PROBLEM 1:

Calculate the circumferential and longitudinal strains for a boiler of 1000mm diameter when it is subjected to an internal pressure of 1MPa. The wall thickness is such that the safe maximum tensile stress in the boiler material is 35 MPa. Take E=200GPa and μ= 0.25.

(Ans: ε_C=0.0001531, ε_L=0.00004375)

PROBLEM 2:

A water main 1m in diameter contains water at a pressure head of 120m. Find the thickness of the metal if the working stress in the pipe metal is 30 MPa. Take unit weight of water = 10 kN/m^3.

(Ans: t=20mm)

PROBLEM 3:

A gravity main 2m in diameter and 15mm in thickness. It is subjected to an internal fluid pressure of 1.5 MPa. Calculate the hoop and longitudinal stresses induced in the pipe material. If a factor of safety 4 was used in the design, what is the ultimate tensile stress in the pipe material?

(Ans: $\blacklozenge_C$=100 MPa, $\blacklozenge_L$=50 MPa, σ_U=400 MPa)

PROBLEM 4:

At a point in a thin cylinder subjected to internal fluid pressure, the value of hoop strain is 600×10^{-4} (tensile). Compute hoop and longitudinal stresses. How much is the percentage change in the volume of the cylinder? Take E=200GPa and μ= 0.2857.

(Ans: $\blacklozenge_C$=140 MPa, $\blacklozenge_L$=70 MPa, %age change=0.135%.)

PROBLEM 5:

A cylindrical tank of 750mm internal diameter and 1.5m long is to be filled with an oil of specific weight 7.85 kN/m3 under a pressure head of 365 m. If the longitudinal joint efficiency is 75% and circumferential joint efficiency is 40%, find the thickness of the tank required. Also calculate the error of calculation in the quantity of oil in the tank if the volumetric strain of the tank is neglected. Take permissible tensile stress as 120 MPa, E=200GPa and μ= 0.3 for the tank material. (Ans: t=12 mm, error=0.085%.)

THICK CYLINDERS

INTRODUCTION:

The thickness of the cylinder is large compared to that of thin cylinder.

i. e., in case of thick cylinders, the metal thickness ‘t’ is more than ‘d/20’, where ‘d’ is the internal diameter of the cylinder.

Magnitude of radial stress (p_r) is large and hence it cannot be neglected. The circumferential stress is also not uniform across the cylinder wall. The radial stress is compressive in nature and circumferential and longitudinal stresses are tensile in nature. Radial stress and circumferential stresses are computed by using ‘Lame’s equations’.

LAME'S EQUATIONS (Theory) :

ASSUMPTIONS:

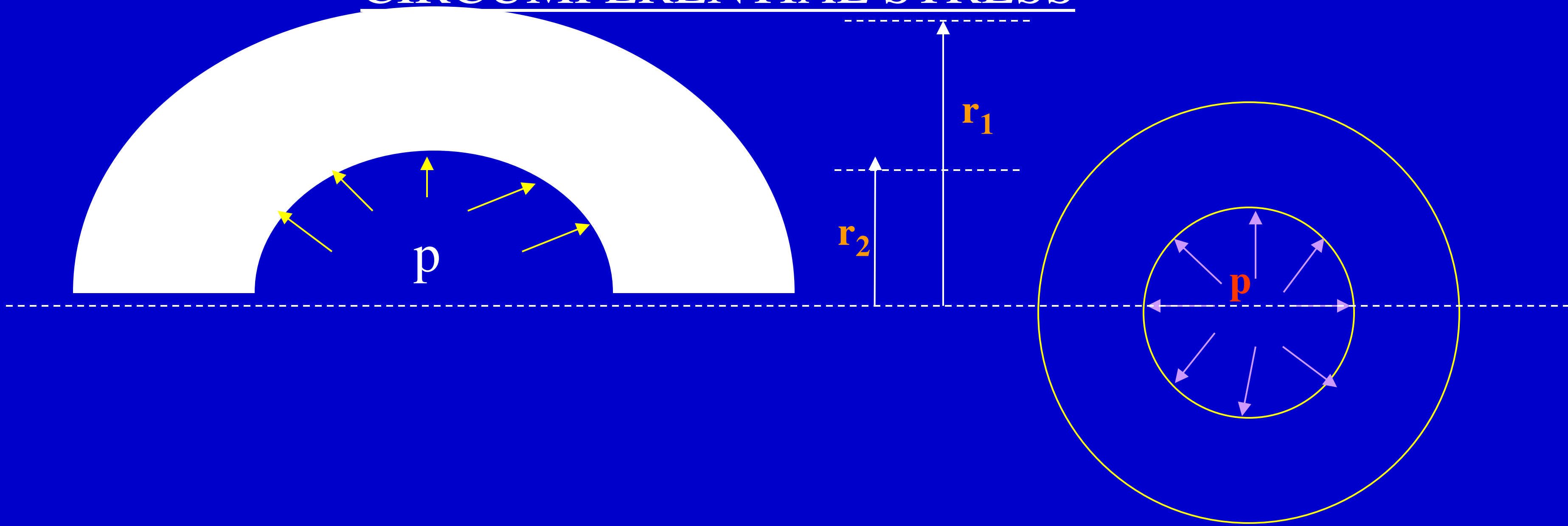
1. Plane sections of the cylinder normal to its axis remain plane and normal even under pressure.

2. Longitudinal stress (σ_L) and longitudinal strain (ε_L) remain constant throughout the thickness of the wall.

3. Since longitudinal stress (σ_L) and longitudinal strain (ε_L) are constant, it follows that the difference in the magnitude of hoop stress and radial stress (p_r) at any point on the cylinder wall is a constant.

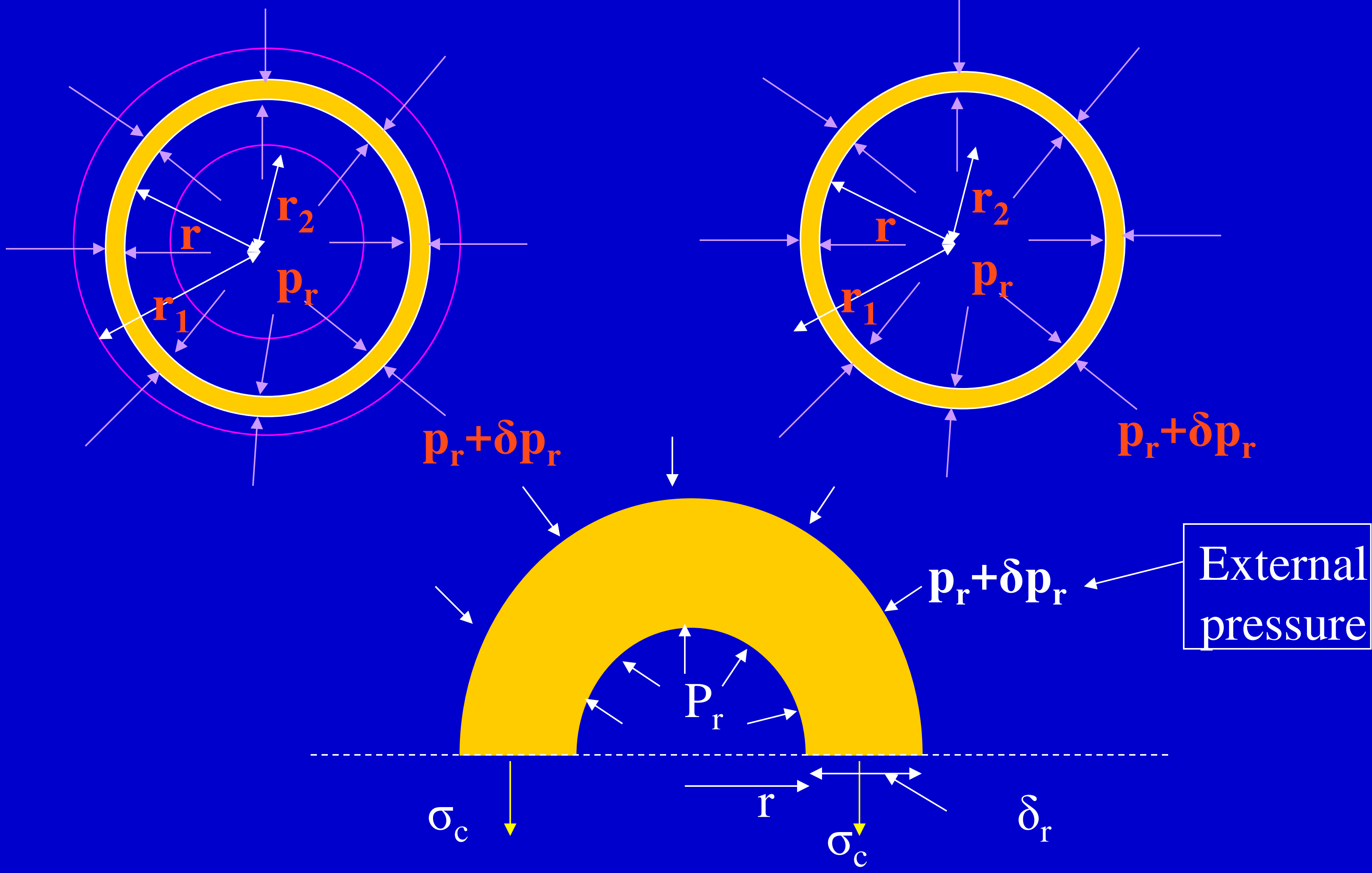
4. The material is homogeneous, isotropic and obeys Hooke's law. (The stresses are within proportionality limit).

LAME'S EQUATIONS FOR RADIAL PRESSURE AND CIRCUMFERENTIAL STRESS



Consider a thick cylinder of external radius r_1 and internal radius r_2, containing a fluid under pressure 'p' as shown in the fig.

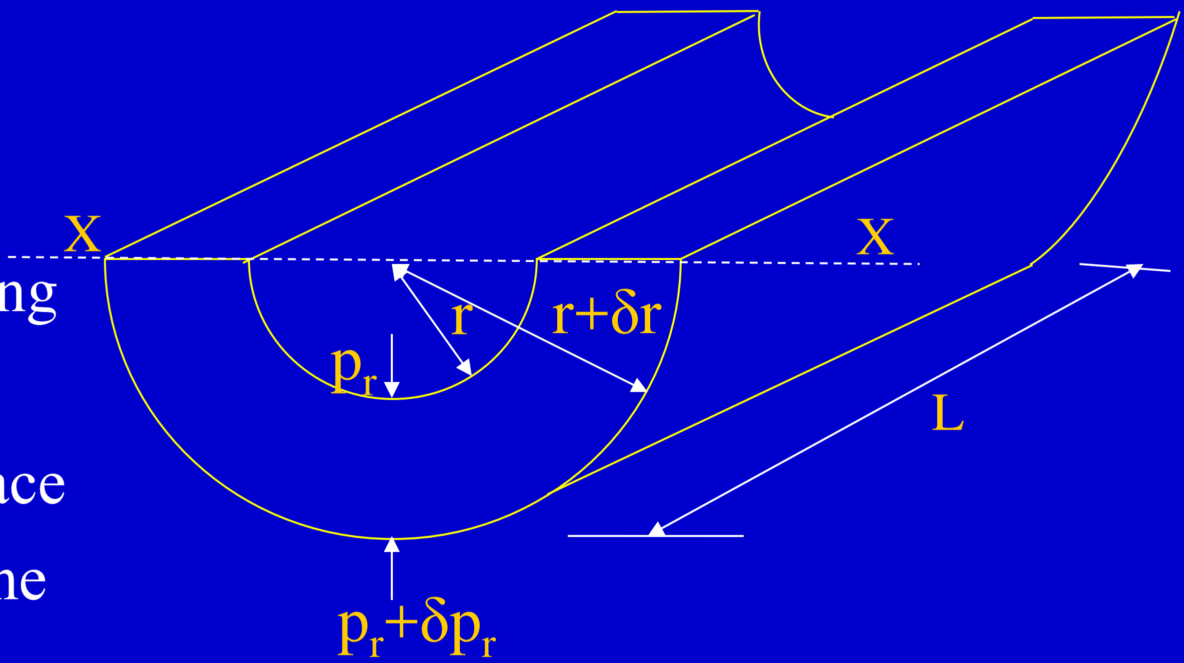
Let 'L' be the length of the cylinder.



Consider an elemental ring of radius 'r' and thickness 'δ_r' as shown in the above figures. Let p_r and (p_r+ δp_r) be the intensities of radial

Consider the longitudinal section XX of the ring as shown in the fig.

The bursting force is evaluated by considering the projected area, ‘$2\times r\times L$’ for the inner face and ‘$2\times(r+\delta_r)\times L$’ for the outer face .



The net bursting force, $P = p_r\times 2\times r\times L - (p_r+\delta p_r)\times 2\times(r+\delta_r)\times L$

$$=(-p_r\times \delta_r - r\times\delta p_r - \delta p_r \times \delta_r)\, 2L$$

Bursting force is resisted by the hoop tensile force developing at the level of the strip i.e., Thus, for equilibrium, $P = F_r$

$$(-p_r \times \delta_r - r \times \delta p_r - \delta p_r \times \delta_r)\, 2L = \sigma_c \times 2 \times \delta_r \times L$$

$$-pr \times \delta r - r \times \delta p_r - \delta p_r \times \delta_r = \sigma_c \times \delta r$$

Neglecting products of small quantities, (i.e., $\delta p_r \times \delta r$)

$$\sigma_c = - p_r - (r \times \delta p_r)/\delta_r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Longitudinal strain is constant. Hence we have,

$$\varepsilon_L = \frac{\sigma_L}{E} - \mu \times \frac{\sigma_C}{E} + \mu \times \frac{p_r}{E} = \text{constant}$$

Since P_r is compressive

$$\varepsilon_L = \frac{\sigma_L}{E} - \frac{\mu}{E}(\sigma_C - p_r) = \text{constant}$$

$$\sigma_c - p_r = 2a,$$

$$\text{i.e., } \sigma_c = p_r + 2a, \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

From (1), $p_r + 2a = - p_r - (r \times \delta p_r) / \delta_r$

i. e., $$2(p_r + a) = -r \times \frac{\delta p_r}{\delta_r}$$

$$-2 \times \frac{\delta_r}{r} = \frac{\delta p_r}{(p_r + a)} \ldots\ldots\ldots\ldots(3)$$

Integrating, $(-2 \times \log_e r) + c = \log_e (p_r + a)$

Where c is constant of integration. Let it be taken as $\log_e b$, where 'b' is another constant.

Thus, $\log_e (p_r + a) = -2 \times \log_e r + \log_e b = - \log_e r^2 + \log_e b = \log_e \frac{b}{r^2}$

i.e., $p_r + a = \frac{b}{r^2}$ or, radial stress, $p_r = \frac{b}{r^2} - a$ ....(4)

Substituting it in equation 2, we get

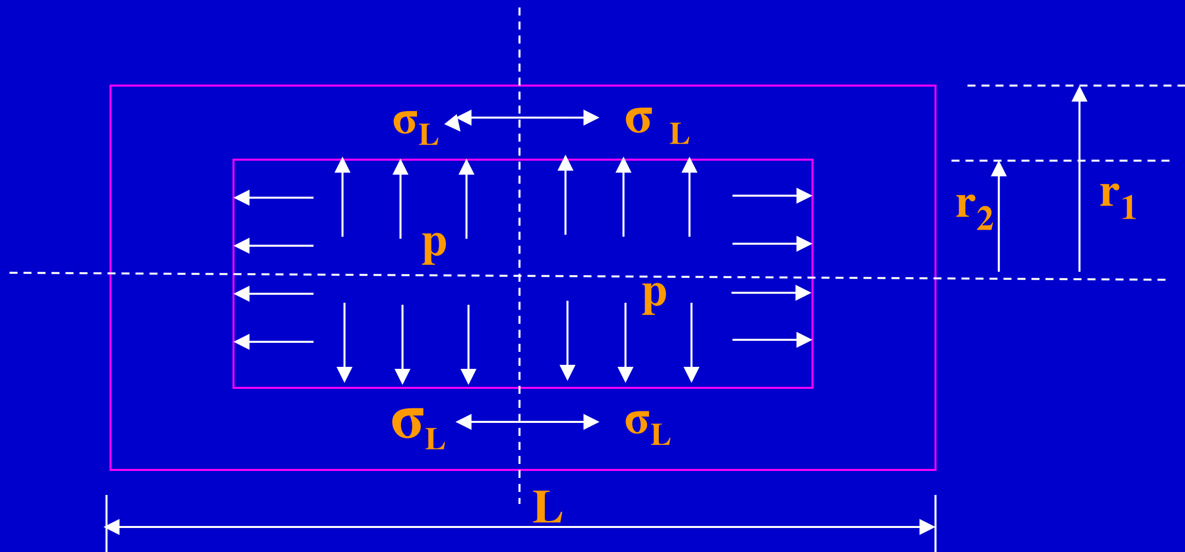
Hoop stress, $\sigma_c = p_r + 2a = \frac{b}{r^2} - a + 2a$

i.e., $\sigma_c = \frac{b}{r^2} + a$ ....(5)

The equations (4) & (5) are known as "Lame's Equations" for radial pressure and hoop stress at any specified point on the cylinder wall.

Thus, $r_1 \leq r \leq r_2$.

ANALYSIS FOR LONGITUDINAL STRESS



Consider a transverse section near the end wall as shown in the fig.

Bursting force, $P = \pi \times r_2^2 \times p$

Resisting force is due to longitudinal stress 'σ_L'.

i.e., $F_L = \sigma_L \times \pi \times (r_1^2 - r_2^2)$

For equilibrium, $F_L = P$

$\sigma_L \times \pi \times (r_1^2 - r_2^2) = \pi \times r_2^2 \times p$

$$\sigma_L = \frac{p \times r_2^2}{(r_1^2 - r_2^2)}$$ (Tensile)

NOTE:

1. Variations of Hoop stress and Radial stress are parabolic across the cylinder wall.
2. At the inner edge, the stresses are maximum.
3. The value of 'Permissible or Maximum Hoop Stress' is to be considered on the inner edge.
4. The maximum shear stress (σ_{max}) and Hoop, Longitudinal and radial strains (ε_c, ε_L, ε_r) are calculated as in thin cylinder but separately for inner and outer edges.

ILLUSTRATIVE PROBLEMS

PROBLEM 1:

A thick cylindrical pipe of external diameter 300mm and internal diameter 200mm is subjected to an internal fluid pressure of 20N/mm^2 and external pressure of 5 N/mm^2. Determine the maximum hoop stress developed and draw the variation of hoop stress and radial stress across the thickness. Show at least four points for each case.

SOLUTION:

External diameter = 300mm. External radius, r_1=150mm.

Internal diameter = 200mm. Internal radius, r_2=100mm.

Lame's equations:

$$\sigma_c = \frac{b}{r^2} + a$$

For Hoop stress,

$$p_r = \frac{b}{r^2} - a \;.........(1)$$

Boundary conditions:

At r =100mm (on the inner face), radial pressure = 20N/mm^2

i.e.,

$$20 = \frac{b}{100^2} - a \;\;\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Similarly, at r =150mm (on the outer face), radial pressure = 5N/mm^2

i.e.,

$$5 = \frac{b}{150^2} - a \;\;\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Solving equations (3) & (4), we get a = 7, b = 2,70,000.

Lame's equations are, for Hoop stress,

$$\sigma_c = \frac{2{,}70{,}000}{r^2} + 7 \;\;\ldots\ldots\ldots\ldots(5)$$

$$p_r = \frac{2{,}70{,}000}{r^2} - 7 \;\;\ldots\ldots\ldots\ldots(6)$$

To draw variations of Hoop stress & Radial stress :

At r =100mm (on the inner face),

$$\text{Hoop stress, } \sigma_c = \frac{2,70,000}{100^2} + 7 = 34 \text{ MPa (Tensile)}$$

$$\text{Radial stress, } p_r = \frac{2,70,000}{100^2} - 7 = 20 \text{ MPa (Comp)}$$

At r =120mm,

$$\text{Hoop stress, } \sigma_c = \frac{2,70,000}{120^2} + 7 = 25.75 \text{ MPa (Tensile)}$$

$$\text{Radial stress, } p_r = \frac{2,70,000}{120^2} - 7 = 11.75 \text{ MPa (Comp)}$$

At r =135mm,

$$\text{Hoop stress, } \sigma_c = \frac{2,70,000}{135^2} + 7 = 21.81 \text{ MPa (Tensile)}$$

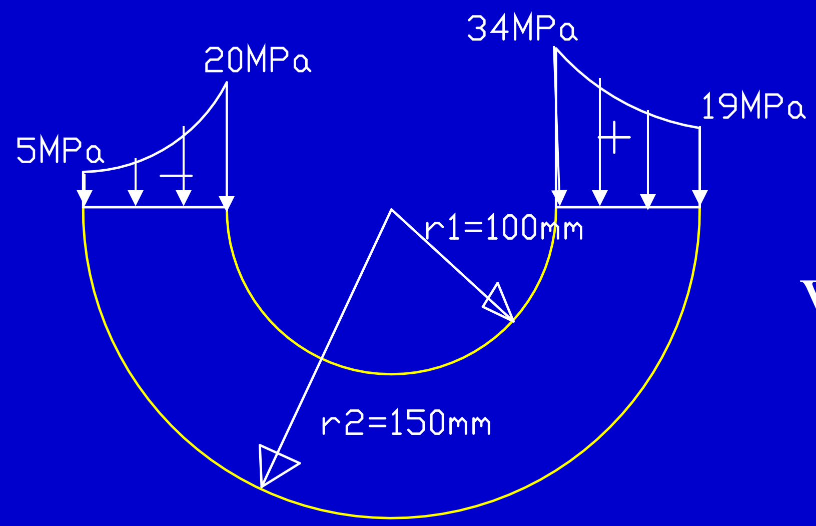
$$\text{Radial stress, } p_r = \frac{2,70,000}{135^2} - 7 = 7.81 \text{ MPa (Comp)}$$

At $r = 150\text{mm}$,

$$\text{Hoop stress,}\quad \sigma_c = \frac{2,70,000}{150^2} + 7 = 19\ \text{MPa (Tensile)}$$

$$\text{Radial stress,}\quad p_r = \frac{2,70,000}{150^2} - 7 = 5\ \text{MPa (Comp)}$$

Variation of Radial Stress –Comp (Parabolic)



Variation of Hoop Stress-Tensile (Parabolic)

Variation of Hoop stress & Radial stress

PROBLEM 2:

Find the thickness of the metal required for a thick cylindrical shell of internal diameter 160mm to withstand an internal pressure of 8 N/mm^2. The maximum hoop stress in the section is not to exceed 35 N/mm^2.

SOLUTION:

Internal radius, r_2=80mm.

Lame's equations are,

$$\text{for Hoop Stress, } \sigma_c = \frac{b}{r^2} + a \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$\text{for Radial stress, } p_r = \frac{b}{r^2} - a \quad \ldots\ldots\ldots\ldots\ldots(2)$$

Boundary conditions are,

at $r = 80\text{mm}$, radial stress $p_r = 8\ \text{N/mm}^2$,

and Hoop stress, $\sigma_C = 35\ \text{N/mm}^2$. ($\because$ Hoop stress is max on inner face)

i.e.,
$$8 = \frac{b}{80^2} - a \quad \text{.................(3)}$$

$$35 = \frac{b}{80^2} + a \quad \text{.................(4)}$$

Solving equations (3) & (4), we get $a = 13.5$, $b = 1{,}37{,}600$.

$\therefore$ Lame's equations are, $$\sigma_c = \frac{1{,}37{,}600}{r^2} + 13.5 \quad \text{...........(5)}$$

and $$p_r = \frac{1{,}37{,}600}{r^2} - 13.5 \quad \text{...........(6)}$$

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On the outer face, pressure = 0.

i.e., $p_r = 0$ at $r = r_1$.

$$\therefore \quad 0 = \frac{1,37,600}{r_1^2} - 13.5$$

$$\therefore \quad r_1 = \underline{100.96 mm.}$$

$$\therefore \quad \text{Thickness of the metal} = r_1 - r_2$$

$$= \underline{20.96 mm.}$$

PROBLEM 3:

A thick cylindrical pipe of outside diameter 300mm and internal diameter 200mm is subjected to an internal fluid pressure of 14 N/mm^2. Determine the maximum hoop stress developed in the cross section. What is the percentage error if the maximum hoop stress is calculated by the equations for thin cylinder?

SOLUTION:

Internal radius, r_2=100mm. External radius, r_1=150mm

.

Lame's equations: $\sigma_c = \frac{b}{r^2} + a$

For Hoop stress, ...(1)

$$p_r = \frac{b}{r^2} - a$$

For radial pressure, ...(2)

Boundary conditions:

At x =100mm $\qquad P_r = 14 N/mm^2$

$$14 = \frac{b}{100^2} - a \; ....(1)$$

i.e.,

Similarly, at x =150mm $\qquad P_r = 0$

$$0 = \frac{b}{150^2} - a \; ....(2)$$

i.e.,

Solving, equations (1) & (2), we get a =11.2, b = 2,52,000.

$$\therefore \text{Lame's equation for Hoop stress, } \sigma_r = \frac{22,500}{r^2} + 11.2 \; ....(3)$$

Max hoop stress on the inner face (where x=100mm):

$$\sigma_{max} = \frac{252000}{100^2} + 11.2 = \underline{36.4\ MPa.}$$

By thin cylinder formula, $\sigma_{max} = \frac{p \times d}{2 \times t}$

where $D = 200mm,\ t = 50mm$ and $p = 14MPa.$

$$\therefore \sigma_{max} = \frac{14 \times 200}{2 \times 50} = \underline{28MPa.}$$

$$\text{Percentage error} = \left(\frac{36.4 - 28}{36.4}\right) \times 100 = \underline{23.08\%}.$$

PROBLEM 4:

The principal stresses at the inner edge of a cylindrical shell are 81.88 MPa (T) and 40MPa (C). The internal diameter of the cylinder is 180mm and the length is 1.5m. The longitudinal stress is 21.93 MPa (T). Find,

(i) Max shear stress at the inner edge.

(ii) Change in internal diameter.

(iii) Change in length.

(iv) Change in volume.

Take E=200 GPa and μ=0.3.

SOLUTION:

i) Max shear stress on the inner face :

$$\tau_{max} = \frac{\sigma_C - p_r}{2} = \frac{81.88 - (-40)}{2}$$

$$= 60.94 \text{ MPa}$$

ii) Change in inner diameter :

$$\frac{\delta d}{d} = \frac{\sigma_C}{E} - \frac{\mu}{E} \times p_r - \frac{\mu}{E} \times \sigma_L$$

$$= \frac{81.88}{200 \times 10^3} - \frac{0.3}{200 \times 10^3} \times 21.93 - \frac{0.3}{200 \times 10^3} \times (-40)$$

$$= 4.365 \times 10^{-4}$$

$$\therefore \quad \delta d = \underline{+0.078\text{mm.}}$$

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iii) Change in Length :

$$\frac{\delta l}{L} = \frac{\sigma_L}{E} - \frac{\mu}{E} \times p_r - \frac{\mu}{E} \times \sigma_C$$

$$= \frac{21.93}{200 \times 10^3} - \frac{0.3}{200 \times 10^3} \times (-40) - \frac{0.3}{200 \times 10^3} \times 81.88$$

$$= 46.83 \times 10^{-6}$$

$$\therefore \quad \delta l = \underline{+0.070\text{mm.}}$$

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iv) Change in volume :

$$\frac{\delta V}{V} = \frac{\delta l}{L} + 2 \times \frac{\delta d}{D}$$

$$= 9.198 \times 10^{-4}$$

$$\therefore \quad \delta V = 9.198 \times 10^{-4} \times \left(\frac{\pi \times 180^2 \times 1500}{4}\right)$$

$$= \underline{35.11 \times 10^3} \text{ mm}^3.$$

PROBLEM 5:

Find the max internal pressure that can be allowed into a thick pipe of outer diameter of 300mm and inner diameter of 200mm so that tensile stress in the metal does not exceed 16 MPa if, (i) there is no external fluid pressure, (ii) there is a fluid pressure of 4.2 MPa.

SOLUTION:

External radius, r_1=150mm.

Internal radius, r_2=100mm.

Case (i) – When there is no external fluid pressure:

Boundary conditions:

At r=100mm , $\sigma_c = 16N/mm^2$

At r=150mm , $P_r = 0$

i.e., $$16 = \frac{b}{100^2} + a \quad \text{..................}(1)$$

$$0 = \frac{b}{150^2} - a \quad \text{..................}(2)$$

Solving we get, a = 4.92 & $b = 110.77 \times 10^3$

so that $$\sigma_c = \frac{110.77 \times 10^3}{r^2} + 4.92 \quad \text{..................}(3)$$

$$p_r = \frac{110.77 \times 10^3}{r^2} - 4.92 \quad \text{..................}(4)$$

Fluid pressure on the inner face where r = 100mm,

$$p_r = \frac{110.77 \times 10^3}{100^2} - 4.92 = \underline{6.16 \text{ MPa.}}$$

Case (ii) – When there is an external fluid pressure of 4.2 MPa:

Boundary conditions:

At r=100mm , σ_c= 16 N/mm^2

At r=150mm , p_r= 4.2 MPa.

i.e., $$16 = \frac{b}{100^2} + a \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$4.2 = \frac{b}{150^2} - a \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Solving we get, a = 2.01 & $b=139.85\times10^3$

so that $$\sigma_r = \frac{139.85\times10^3}{r^2} + 2.01 \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$p_r = \frac{139.85\times10^3}{r^2} - 2.01 \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Fluid pressure on the inner face where $r = 100$mm,

$$p_r = \frac{139.85 \times 10^3}{100^2} - 2.01 = \underline{11.975} \text{ MPa.}$$

PROBLEMS FOR PRACTICE

PROBLEM 1:

A pipe of 150mm internal diameter with the metal thickness of 50mm transmits water under a pressure of 6 MPa. Calculate the maximum and minimum intensities of circumferential stresses induced.

(Ans: 12.75 MPa, 6.75 MPa)

PROBLEM 2:

Determine maximum and minimum hoop stresses across the section of a pipe of 400mm internal diameter and 100mm thick when a fluid under a pressure of 8N/mm^2 is admitted. Sketch also the radial pressure and hoop stress distributions across the thickness.

(Ans: $\blacklozenge_{max}$=20.8 N/mm^2, $\blacklozenge_{min}$=12.8 N/mm^2)

PROBLEM 3:

A thick cylinder with external diameter 240mm and internal diameter

PROBLEM 4:

A thick cylinder of 1m inside diameter and 7m long is subjected to an internal fluid pressure of 40 MPa. Determine the thickness of the cylinder if the maximum shear stress in the cylinder is not to exceed 65 MPa. What will be the increase in the volume of the cylinder? E=200 GPa, μ=0.3. (Ans: t=306.2mm, $\delta v = 5.47 \times 10^{-3} m^3$)

PROBLEM 5:

A thick cylinder is subjected to both internal and external pressure. The internal diameter of the cylinder is 150mm and the external diameter is 200mm. If the maximum permissible stress in the cylinder is 20 N/mm^2 and external radial pressure is 4 N/mm^2, determine the intensity of internal radial pressure. (Ans: 10.72 N/mm^2)

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THANK YOU