NETWORK THEORY
POWER POINT PRESENTATION

PREPARED BY:

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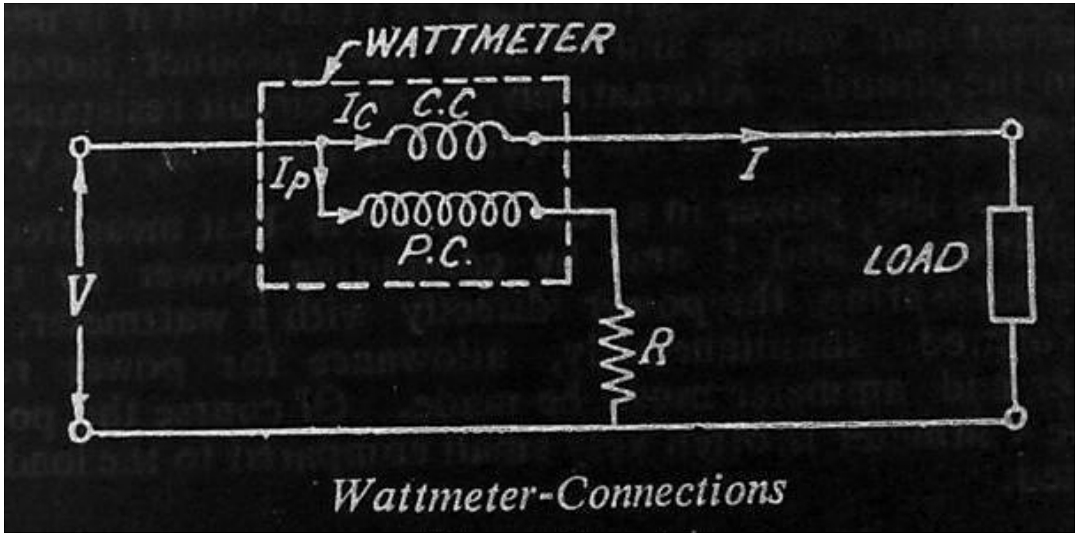
Wattmeter

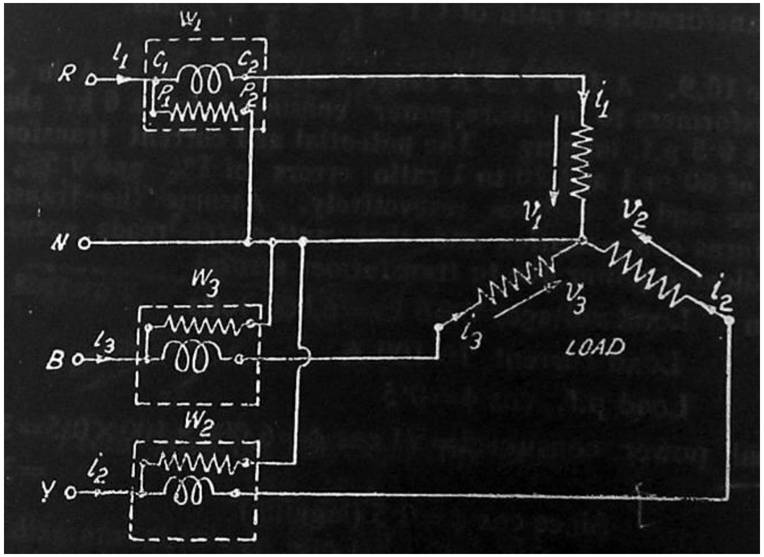
- A wattmeter is essentially an inherent combination of an ammeter and a voltmeter and, therefore , consists of two coils known as *current coil* and *pressure coil*.
- Wattmeter connection:



Wattmeter-Connections

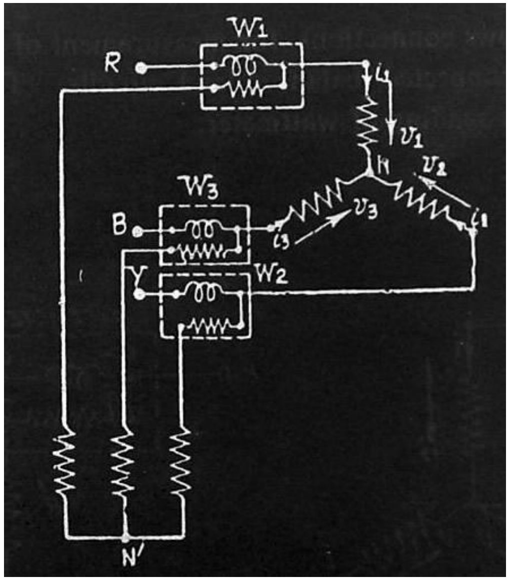
Measurement of Power in 3-Phase Circuit

- Measurement of power in 3-phase, 4-wire circuits----------→



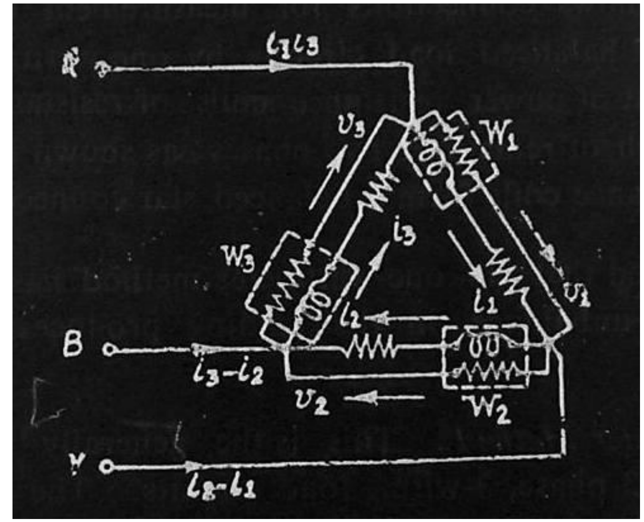
- $P=W_1+W_2+W_3$

- Measurement of power in 3-phase, 3-wire circuits------------→

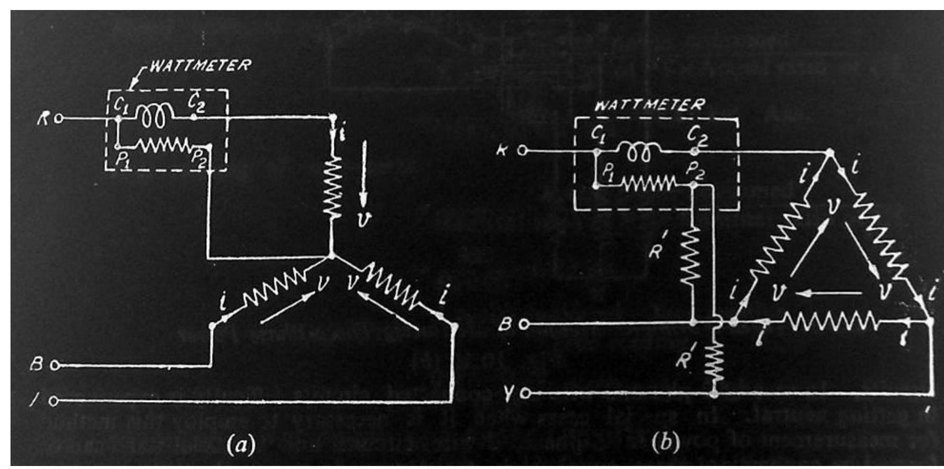


- $P=W_1+W_2+W_3$

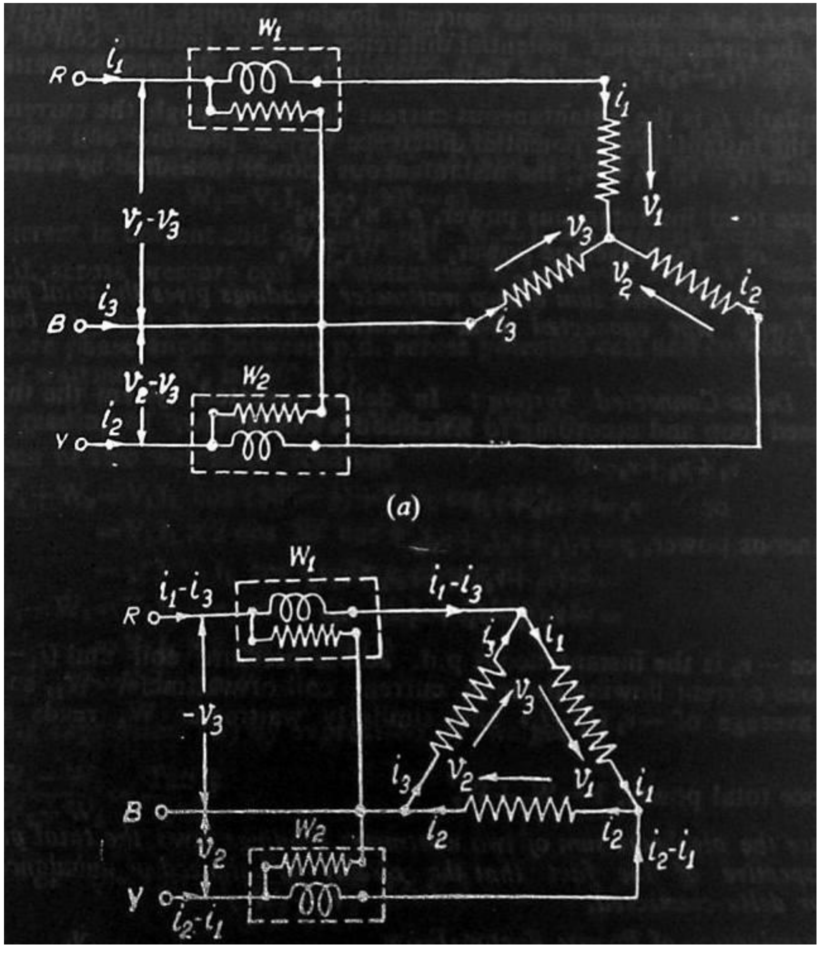
- 3-wattmeter method of measuring 3-phase power of delta connected
- $P=W_1+W_2+W_3$



- 1-wattmeter method of measuring balanced 3-phase power (a) star connected, (b) delta connected
- $P=3W$

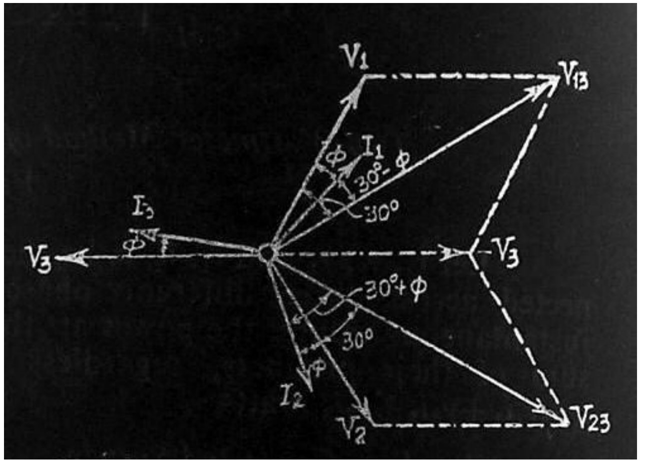


- 2-wattmeter method of measuring 3-phase 3-wire power :
  - (a) star connected,
  - $P=W_1+W_2$
  - (b) delta connected
  - $P=W_1+W_2$



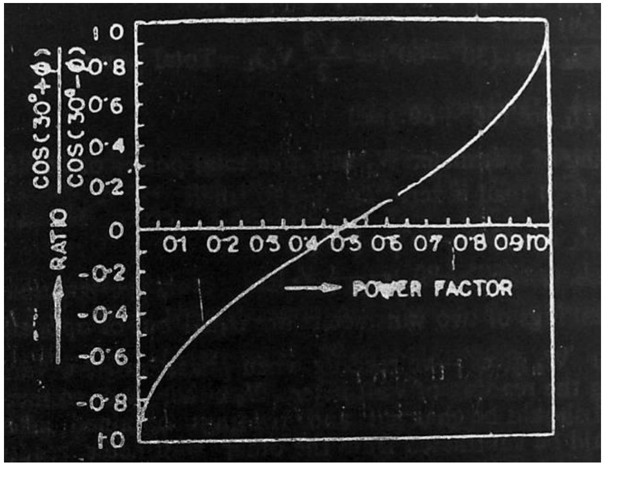
Determination of P.F. from Wattmeter Reading

- If load is balanced, then p.f. of the load can be determined from the wattmeter readings
- Vector diagram for balanced star connected inductive load -----→



$$\cos\varphi = \cos\tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$$

- The watt-ratio Curve ----------→



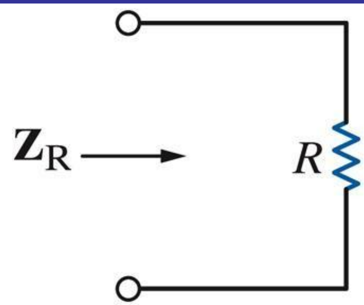
- p.f. can be determined from reading of two wattmeters

AC STEADY STATE ANALYSIS

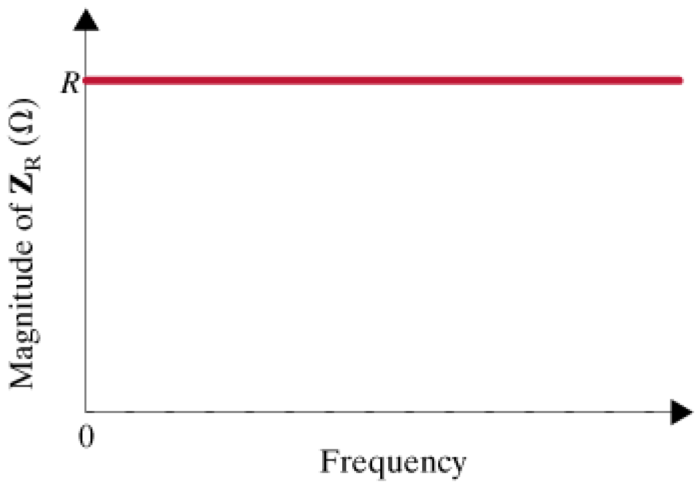
In AC steady state analysis the frequency is assumed constant (e.g., 60Hz). Here we consider the frequency as a variable and examine how the performanc varies with the frequency.

Variation in impedance of basic components

Resistor



$$Z_R = R = R\angle 0°$$



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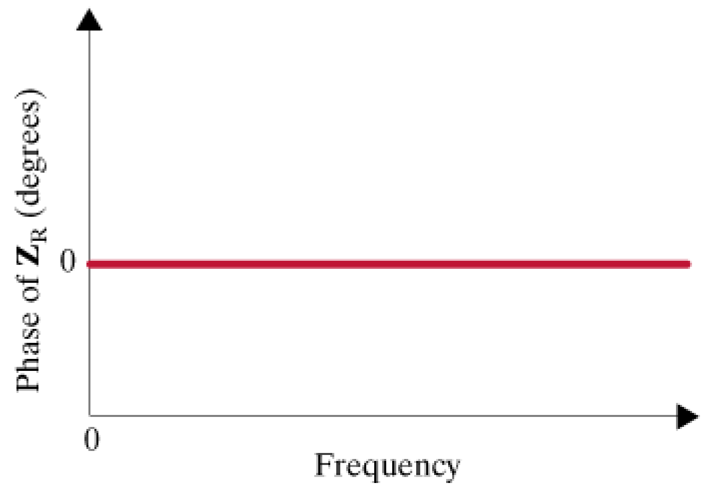

(c)

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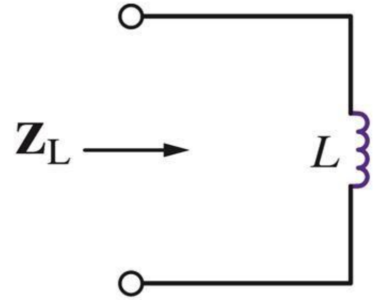
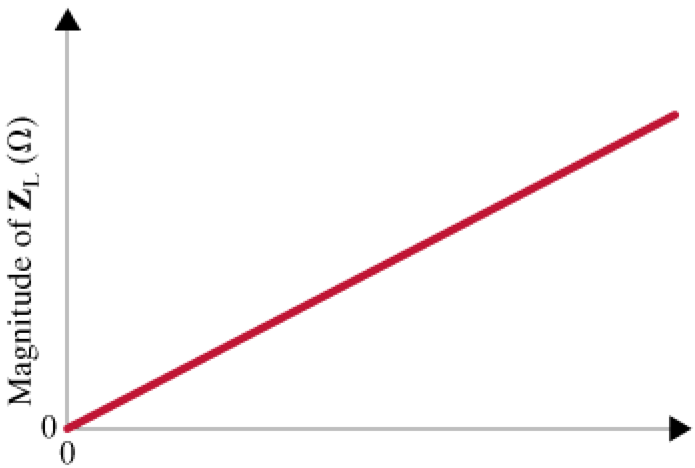
Inductor



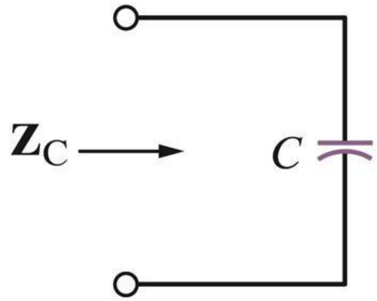
$$Z_L = j\omega L = \omega L \angle 90°$$



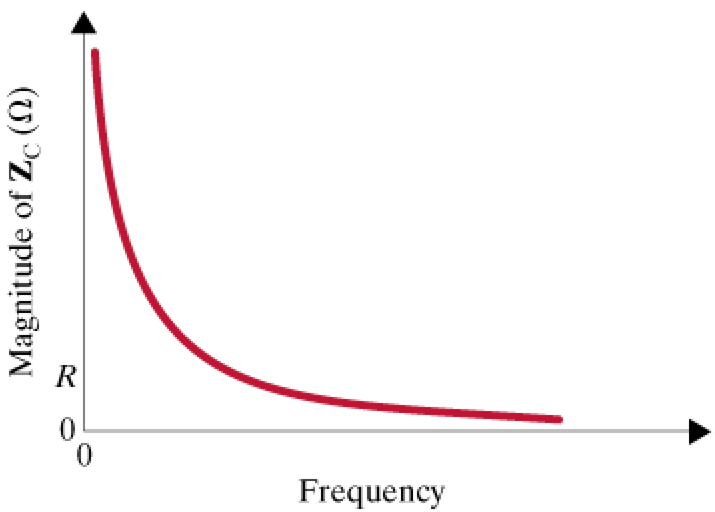
(b)

(c)

Capacitor



$$\mathbf{Z}_c = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90^\circ$$



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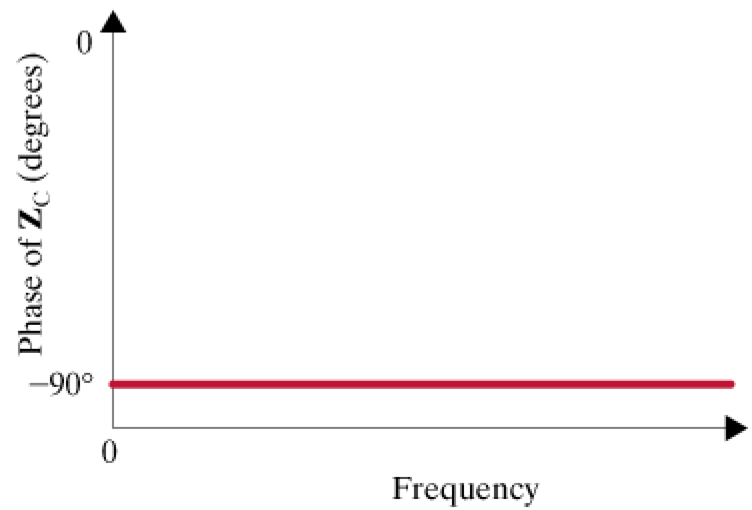
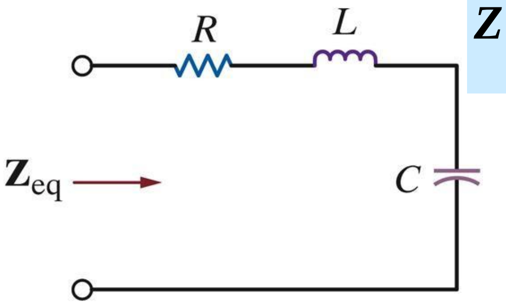


(c)

Frequency dependent behavior of series RLC network



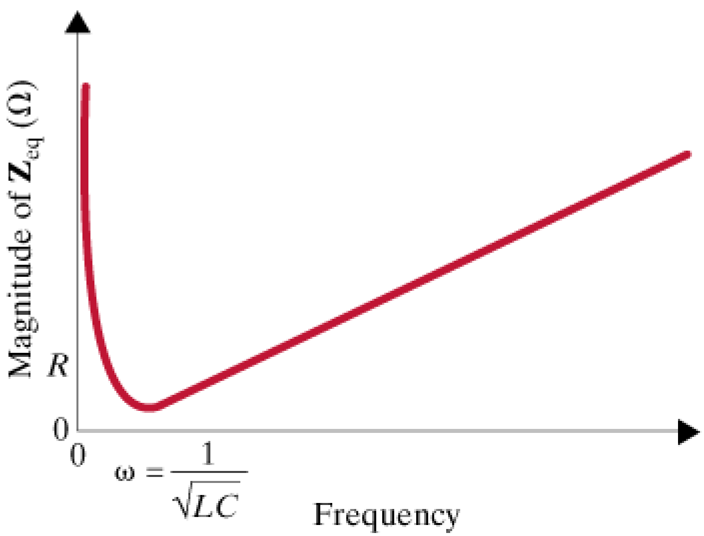
$$Z_{eq} = R + j\omega L + \frac{1}{j\omega C} = \frac{(j\omega)^2 LC + j\omega RC + 1}{j\omega C} \times \frac{-j}{-j} = \frac{\omega RC + j(\omega^2 LC - 1)}{\omega C}$$

"Simplification in notation" $j\omega \approx s$

$$Z_{eq}(s) = \frac{s^2 LC + sRC + 1}{sC}$$

$$|Z_{eq}| = \frac{\sqrt{(\omega RC)^2 + (1 - \omega^2 LC)^2}}{\omega C}$$

$$\angle Z_{eq} = \tan^{-1}\left(\frac{\omega^2 LC - 1}{\omega RC}\right)$$



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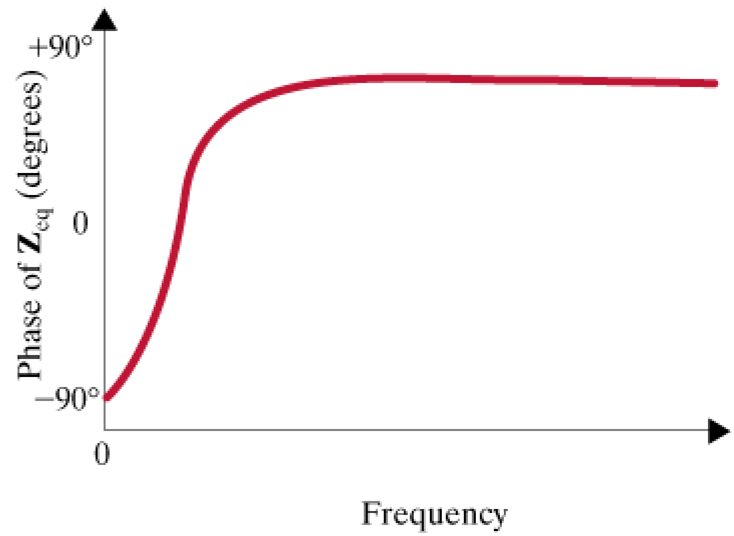


(c)

Simplified notation for basic components

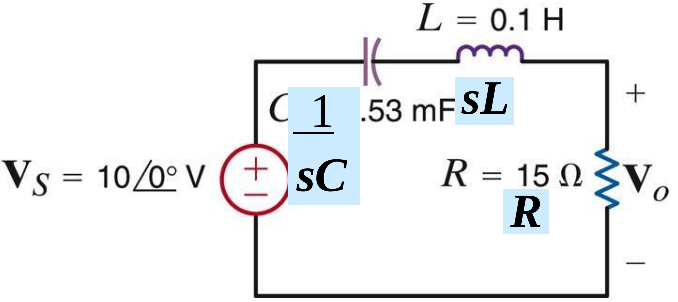
$$Z_R(s) = R,\ Z_L(s) = sL,\ Z_C = \frac{1}{sC}$$

For all cases seen, and all cases to be studied, the impedance is of the form

$$Z(s) = \frac{a_m s^m + a_{m-1}s^{m-1} + \ldots + a_1 s + a_0}{b_n s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + b_0}$$

Moreover, if the circuit elements (L,R,C, dependent sources) are real then the expression for any voltage or current will also be a rational function in *s*

LEARNING EXAMPLE



$$V_o(s) = \frac{R}{R + sL + 1/sC} V_S = \frac{sRC}{s^2 LC + sRC + 1} V_S$$

$$s \approx j\omega$$

$$V_o = \frac{j\omega RC}{(j\omega)^2 LC + j\omega RC + 1} V_S$$

$$V_o = \frac{j\omega(15 \times 2.53 \times 10^{-3})}{(j\omega)^2(0.1 \times 2.53 \times 10^{-3}) + j\omega(15 \times 2.53 \times 10^{-3}) + 1} 10\angle 0^\circ$$

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NETWORK FUNCTIONS

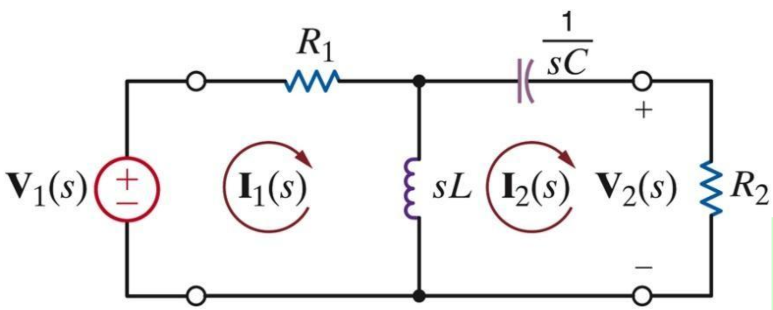
Some nomenclature

When voltages and currents are defined at different terminal pairs we define the ratios as Transfer Functions

INPUT	OUTPUT	TRANSFER FUNCTION	SYMBOL
Voltage	Voltage	Voltage Gain	**Gv(s)**
Current	Voltage	Transimpedance	**Z(s)**
Current	Current	Current Gain	**Gi(s)**
Voltage	Current	Transadmittance	**Y(s)**

If voltage and current are defined at the same terminals we define Driving Point Impedance/Admittance

EXAMPLE



To compute the transfer functions one must sol the circuit. Any valid technique is acceptable

$$Y_T(s) = \frac{I_2(s)}{V_1(s)} \begin{cases} \text{Transadmittance} \\ \text{Transfer admittance} \end{cases}$$

$$G_v(s) = \frac{V_2(s)}{V_1(s)} \quad \text{Voltage gain}$$

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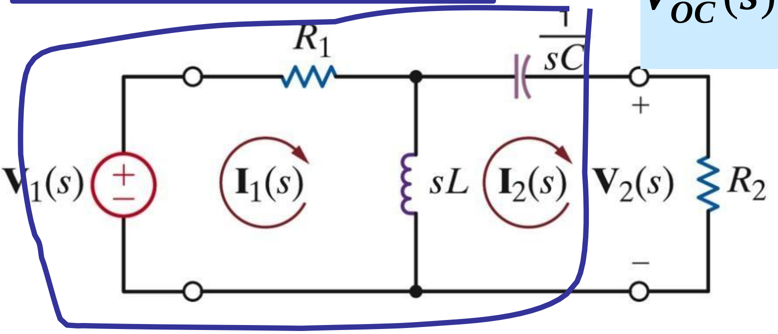
LEARNING EXAMPLE



$$V_{OC}(s) = \frac{sL}{sL + R_1} V_1(s)$$

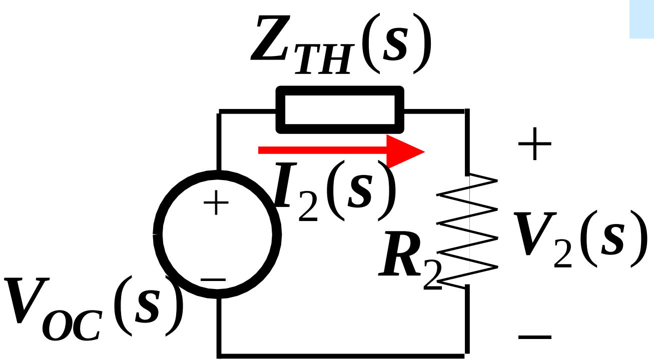
The textbook uses mesh analysis. We will use Thevenin's theorem

$$Z_{TH}(s) = \frac{1}{sC} + R_1 \parallel sL = \frac{1}{sC} + \frac{sLR_1}{sL + R_1}$$

$$Z_{TH}(s) = \frac{s^2 LCR_1 + sL + R_1}{sC(sL + R_1)}$$

$$Y_T(s) = \frac{I_2(s)}{V_1(s)} \begin{cases} \text{Transadmittance} \\ \text{Transfer admittance} \end{cases}$$

$$G_v(s) = \frac{V_2(s)}{V_1(s)} \quad \text{Voltage gain}$$



$$I_2(s) = \frac{V_{OC}(s)}{R_2 + Z_{TH}(s)} = \frac{\dfrac{sL}{sL + R_1} V_1(s)}{R_2 + \dfrac{s^2 LCR_1 + sL + R_1}{sC(sL + R_1)}} \times \frac{sC(sL + R_1)}{sC(sL + R_1)}$$

$$Y_T(s) = \frac{s^2 LC}{s^2 (R_1 + R_2) LC + s(L + R_1 R_2 C) + R_1}$$

$$G_v(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_2 I_2(s)}{V_1(s)} = R_2 Y_T(s)$$

POLES AND ZEROS

(More nomenclature)

$$H(s) = \frac{a_m s^m + a_{m-1}s^{m-1} + \ldots + a_1 s + a_0}{b_n s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + b_0}$$

Arbitrary network function

Using the roots, every (monic) polynomial can be expressed as a product of first order terms

$$H(s) = K_0 \frac{(s - z_1)(s - z_2)\ldots(s - z_m)}{(s - p_1)(s - p_2)\ldots(s - p_n)}$$

$z_1, z_2, \ldots, z_m =$ zeros of the network function

$p_1, p_2, \ldots, p_n =$ poles of the network function

The network function is uniquely determined by its poles and zeros and its value at some other value of s (to compute the gain)

EXAMPLE

zeros : $z_1 = -1,$

poles : $p_1 = -2 + j2,\ p_2 = -2 - j2$

$H(0) = 1$

$$H(s) = K_0 \frac{(s+1)}{(s+2-j2)(s+2+j2)} = K_0 \frac{s+1}{s^2+4s+8}$$

$$H(0) = K_0 \frac{1}{8} = 1 \Rightarrow$$

$$H(s) = 8\frac{s+1}{s^2+4s+8}$$

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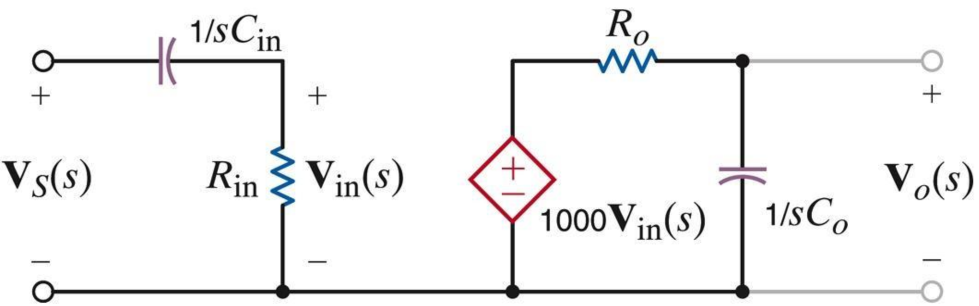
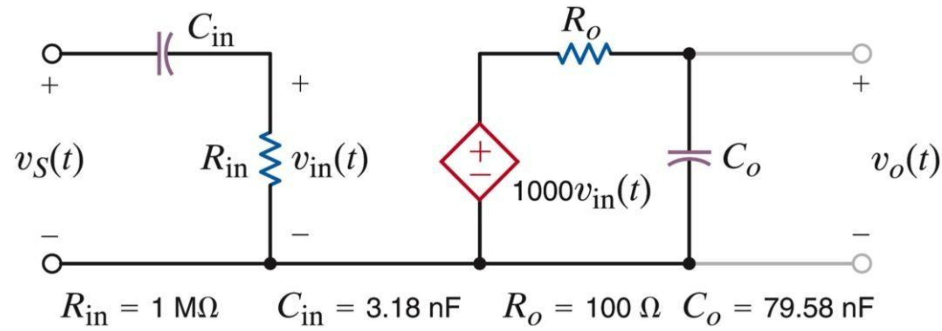
LEARNING EXTENSION

Find the pole and zero locations and the value of K_o for the voltage gain $G(s) = \dfrac{V_o(s)}{V_S(s)}$





$$H(s) = K_0 \frac{(s-z_1)(s-z_2)\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)}$$

Zeros = roots of numerator
Poles = roots of denominator

For this case the gain was shown to be

$$G(s) = \left[\frac{sC_{in}R_{in}}{1+sC_{in}R_{in}}\right][1000]\left[\frac{1}{1+sC_oR_o}\right] = \left[\frac{s}{s+100\pi}\right][1000]\left[\frac{40{,}000\pi}{s+40{,}000\pi}\right]$$

zero : $z_1 = 0$

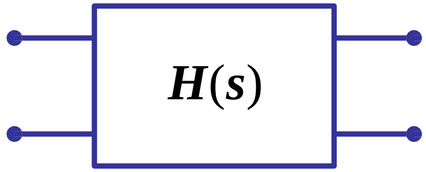
poles : $p_1 = -50Hz,\ p_2 = -20{,}000Hz$

$K_0 = (4\times 10^7)\pi$

Variable Frequency Response

SINUSOIDAL FREQUENCY ANALYSIS

$$\left.\begin{array}{l} A_0 e^{j(\omega t+\theta)} \\ B_0 \cos(\omega t+\theta) \end{array}\right\}$$



$$\left\{\begin{array}{l} A_0 H(j\omega) e^{j(\omega t+\theta)} \\ B_0 \,|H(j\omega)|\cos\left(\omega t+\theta+\angle H(j\omega)\right) \end{array}\right.$$

Circuit represented by network function

To study the behavior of a network as a function of the frequency we analyze the network function $H(j\omega)$ as a function of ω.

Notation

$$M(\omega) = |H(j\omega)|$$

$$\phi(\omega) = \angle H(j\omega)$$

$$H(j\omega) = M(\omega) e^{j\phi(\omega)}$$

Plots of $M(\omega), \phi(\omega)$, as function of ω are generally called magnitude and phase characteristics.

BODE PLOTS $\left\{\begin{array}{c} 20\log_{10}(M(\omega)) \\ \phi(\omega) \end{array}\right.$ $vs \ \log_{10}(\omega)$

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HISTORY OF THE DECIBEL

Originated as a measure of relative (radio) power

$$P_2|_{dB}\ (\text{over } P_1) = 10\log\frac{P_2}{P_1}$$

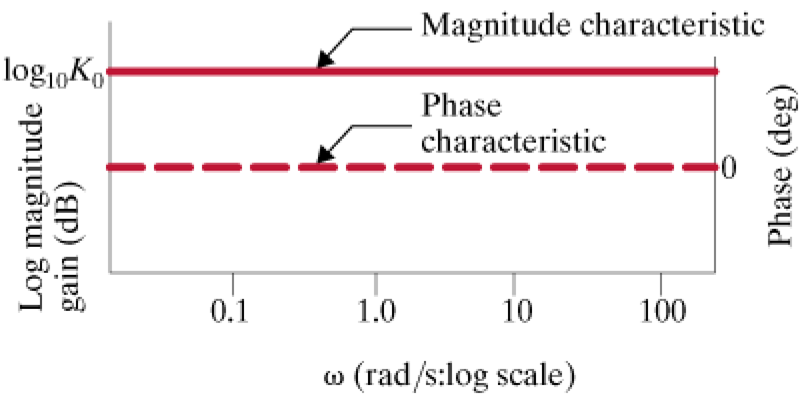
$$P = I^2R = \frac{V^2}{R} \Rightarrow P_2|_{dB}\ (\text{over } P_1) = 10\log\frac{V_2^2}{V_1^2} = 10\log\frac{I_2^2}{I_1^2}$$

By extension

$$V|_{dB} = 20\log_{10}|V|$$
$$I|_{dB} = 20\log_{10}|I|$$
$$G|_{dB} = 20\log_{10}|G|$$

Using log scales the frequency characteristics of network functions have simple asymptotic behavior.
The asymptotes can be used as reasonable and efficient approximations

Constant Term



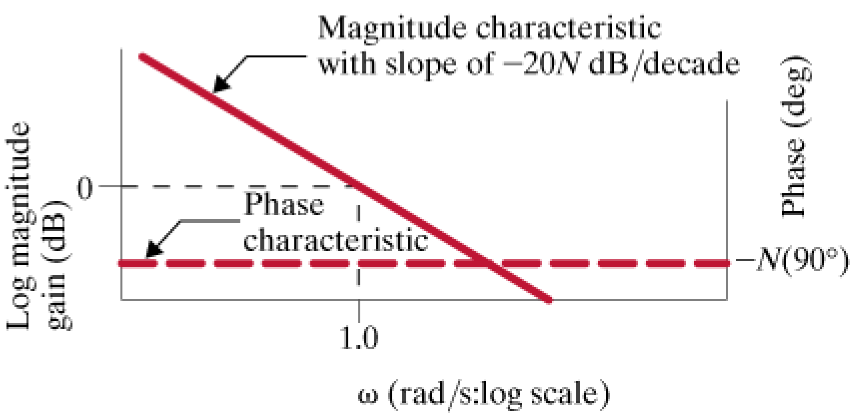
(a)

the x - axis is $\log_{10}\omega$

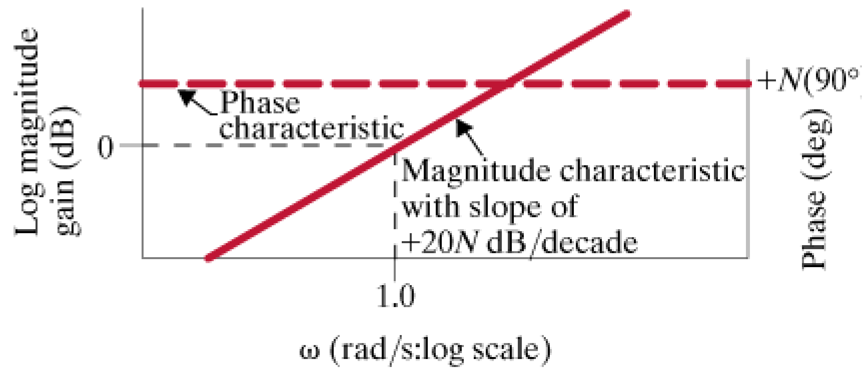
this is a straight line

Poles/Zeros at the origin

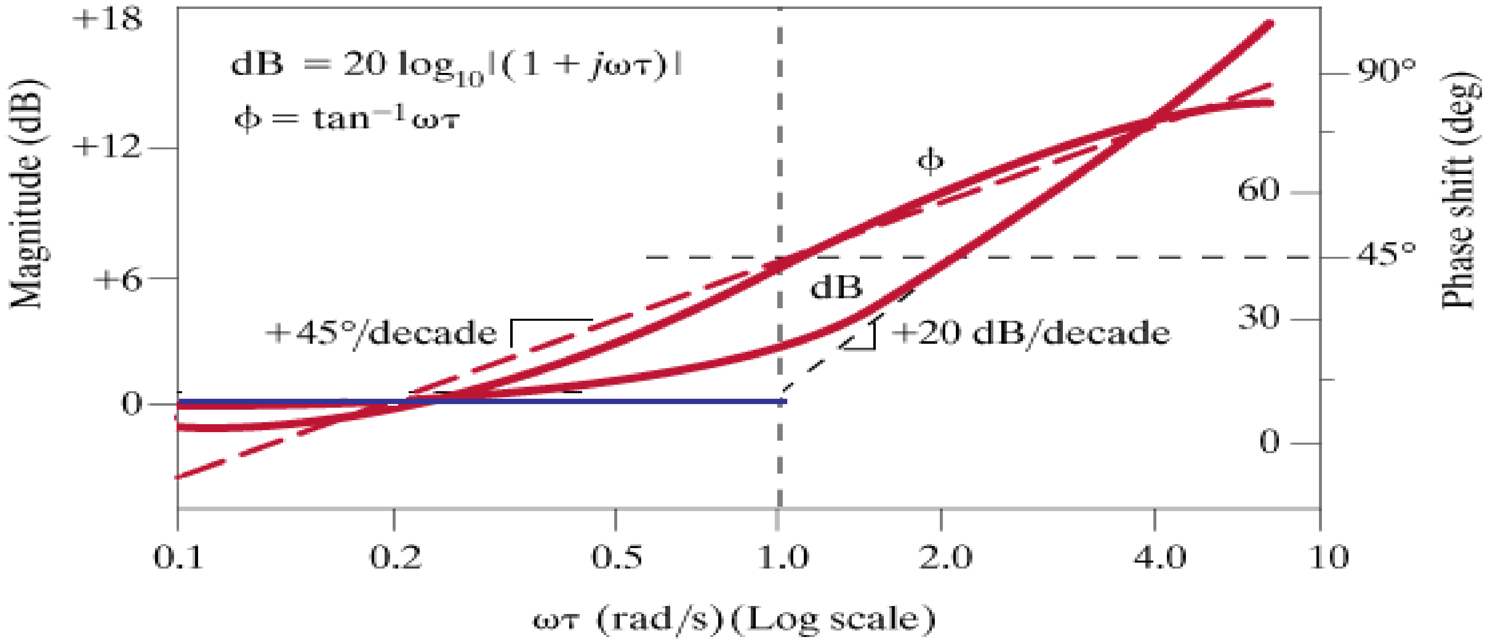
$$(j\omega)^{\pm N} \rightarrow \begin{cases} |(j\omega)^{\pm N}|_{dB} = \pm N \times 20\log_{10}(\omega) \\ \angle(j\omega)^{\pm N} = \pm N 90^\circ \end{cases}$$



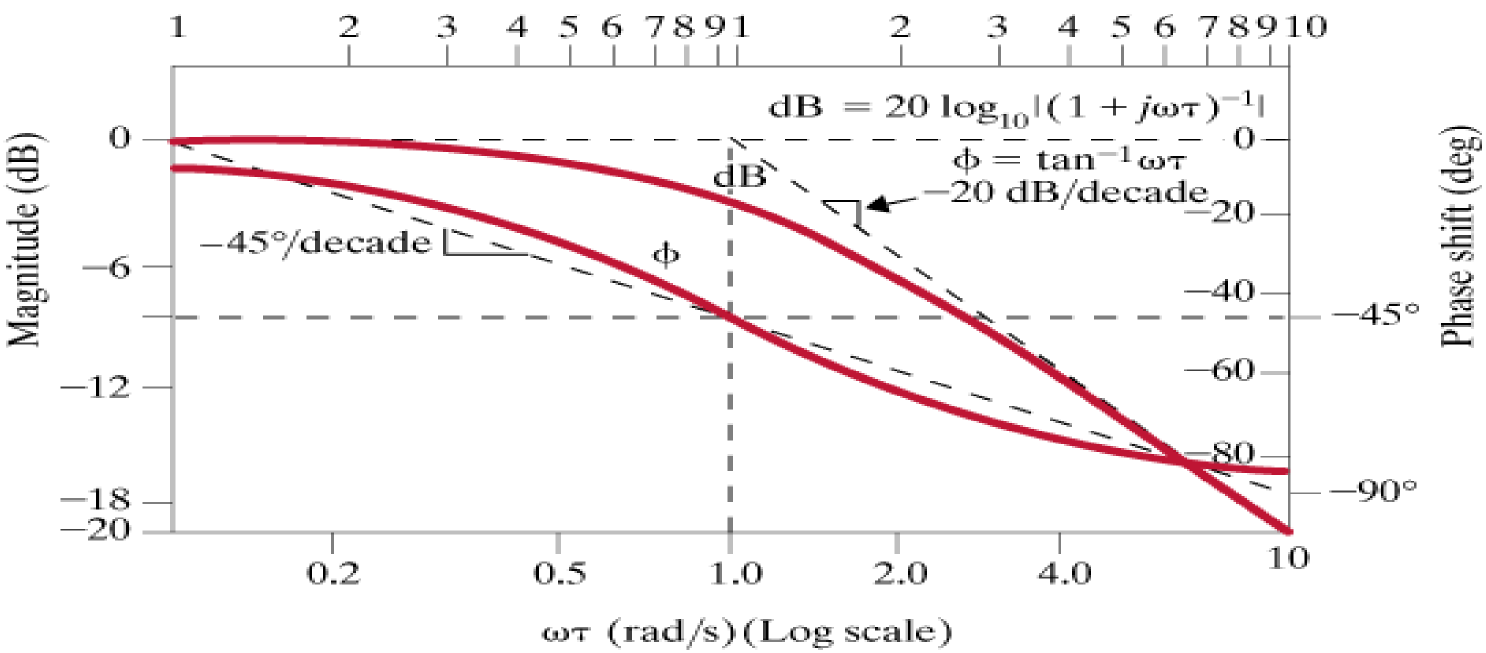
(b)



(c)



Simple zero



Simple pole

(a)

LEARNING EXAMPLE

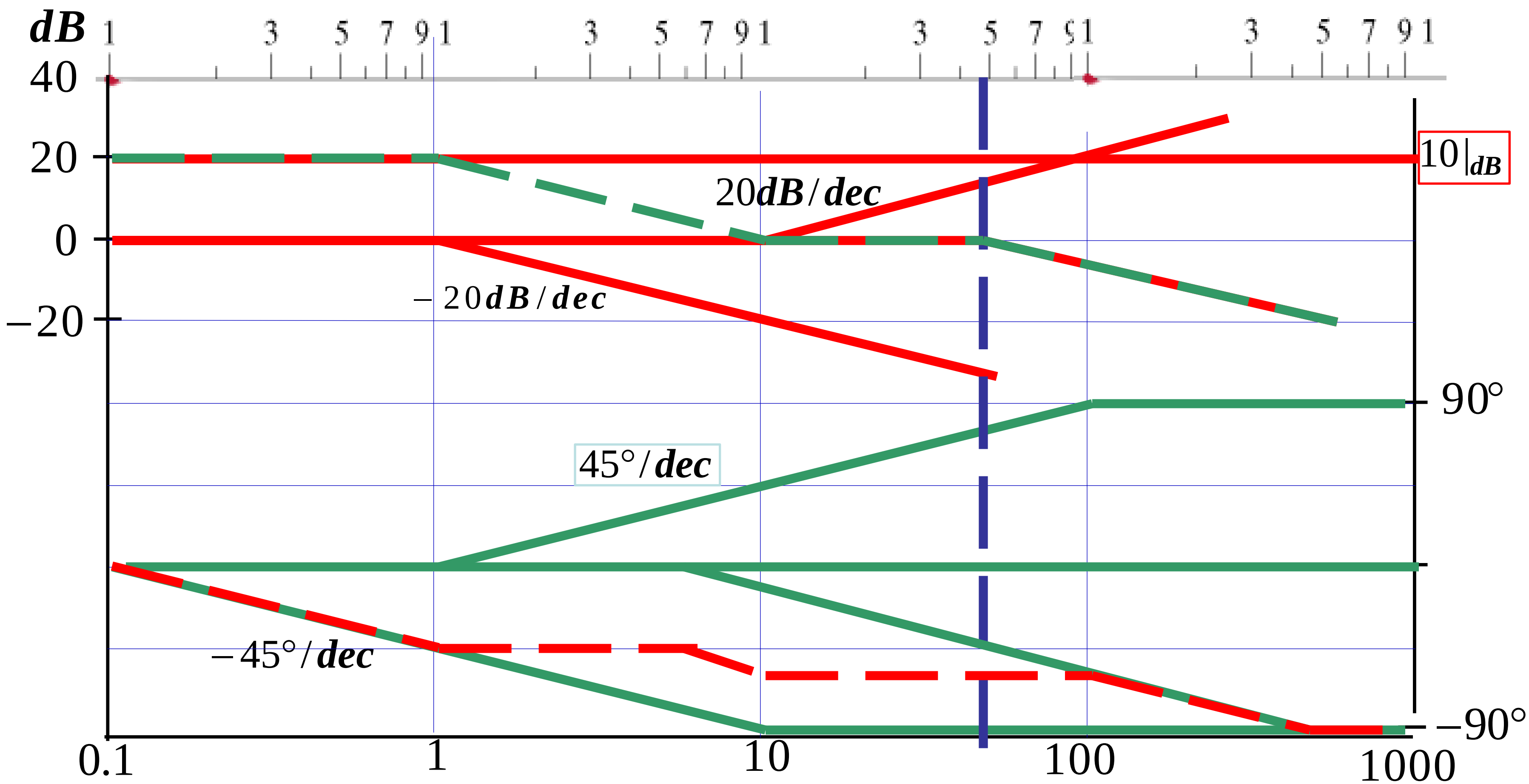
Generate magnitude and phase plots

$$G_v(j\omega) = \frac{\boxed{10}\,\boxed{(0.1 j\omega + 1)}}{\boxed{(j\omega+1)}\,\boxed{(0.02 j\omega + 1)}}$$

Draw asymptotes for each term

Breaks/corners : 1,10,50

Draw composites



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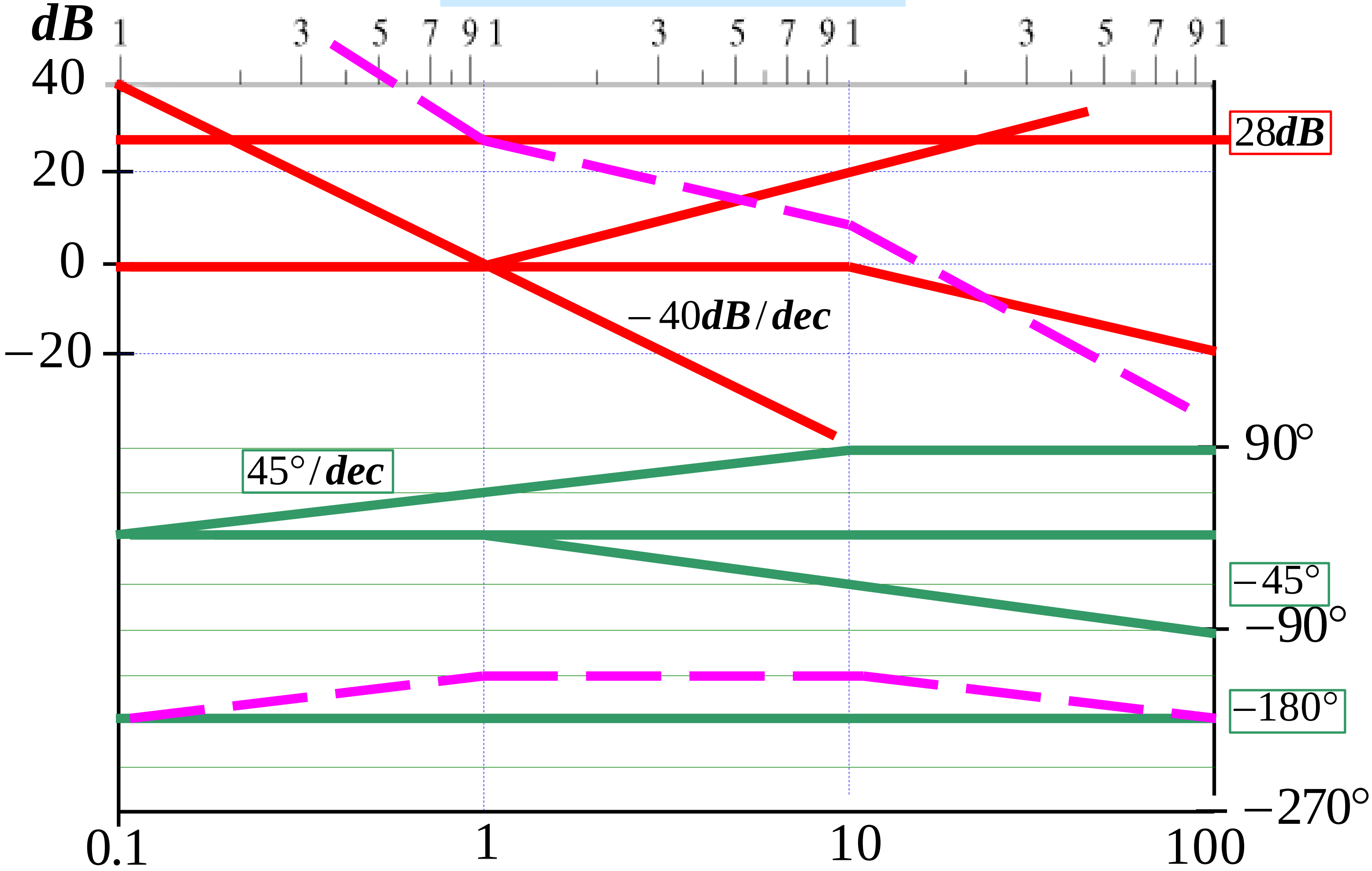
LEARNING EXAMPLE

Generate magnitude and phase plots

$$G(j\omega) = \frac{25(j\omega+1)}{(j\omega)^2(0.1j\omega+1)}$$

Breaks (corners) : 1, 10

Form composites



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LEARNING EXTENSION

Sketch the magnitude characteristic

$$G(j\omega) = \frac{10^4(j\omega + 2)}{(j\omega+10)(j\omega+100)}$$

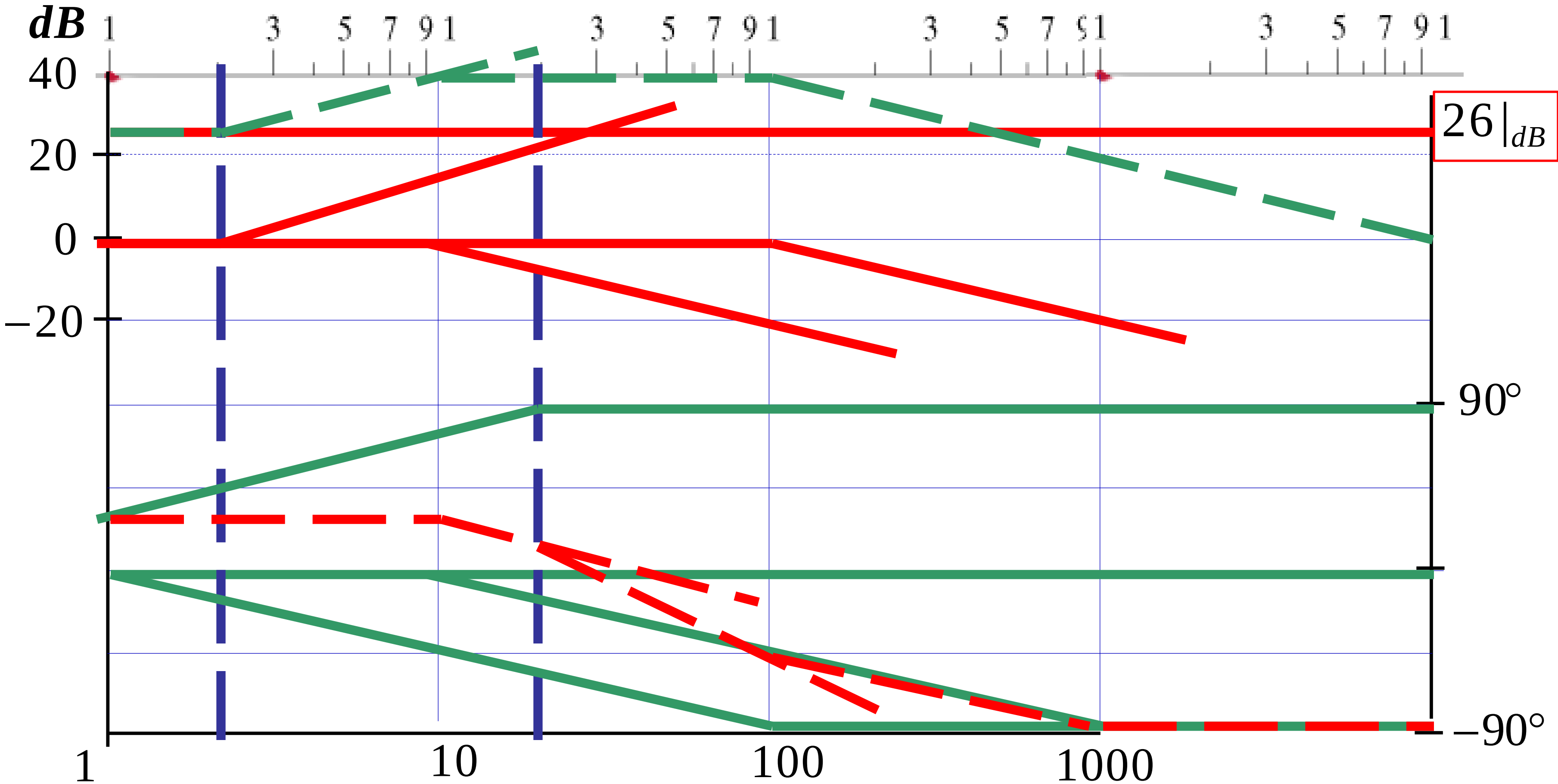
breaks : 2, 10, 100

But the function is NOT in standard form

Put in standard form

$$G(j\omega) = \frac{20\ (j\omega/2+1)}{(j\omega/10+1)(j\omega/100+1)}$$

We need to show about 4 decades



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LEARNING EXTENSION

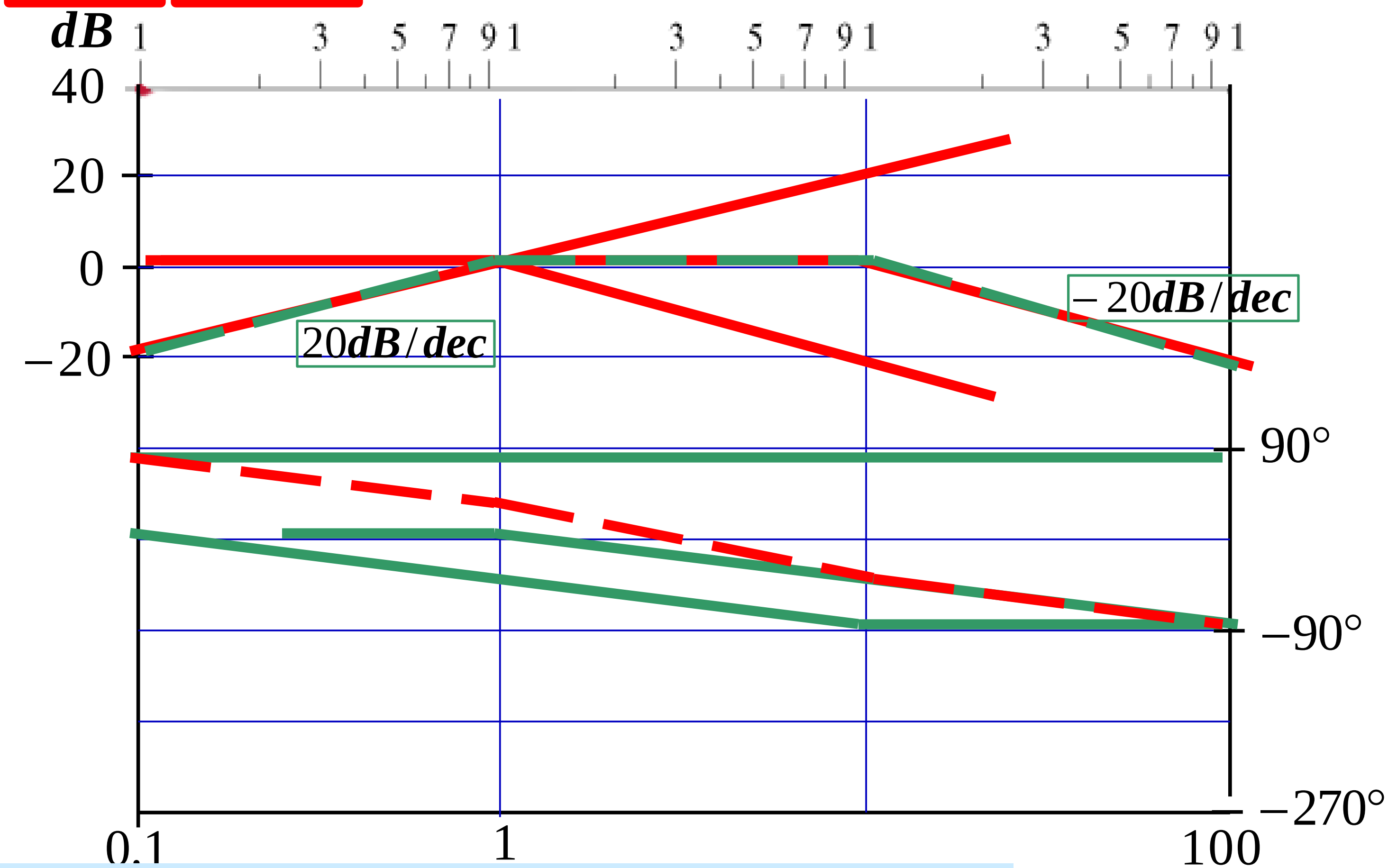
Sketch the magnitude characteristic

$$G(j\omega)=\frac{10j\omega}{(j\omega+1)(j\omega+10)}$$

not in standard form
zero at the origin
breaks: 1, 10

Put in standard form

$$G(j\omega)=\frac{\boxed{j\omega}}{\boxed{(j\omega+1)}\boxed{(j\omega/10+1)}}$$



Once each term is drawn we form the composites

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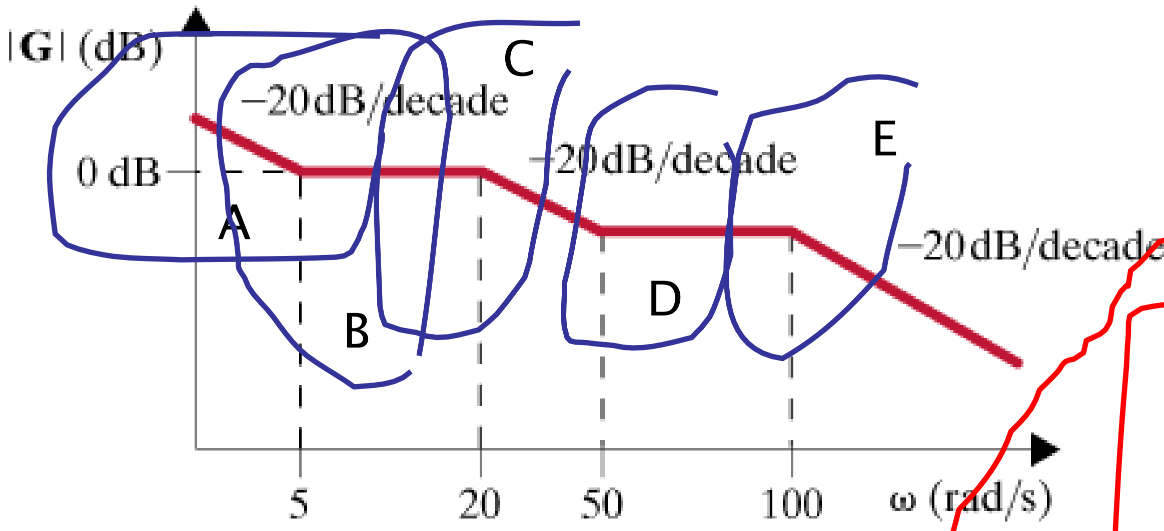
LEARNING EXTENSION

Determine a transfer function from the composite magnitude asymptotes plot



A. Pole at the origin. Crosses 0dB line at 5

$$\frac{5}{j\omega}$$

B. Zero at 5

C. Pole at 20

D. Zero at 50

E. Pole at 100

$$G(j\omega)=\frac{5(j\omega/5+1)(j\omega/50+1)}{j\omega(j\omega/20+1)(j\omega/100+1)}$$

Sinusoidal

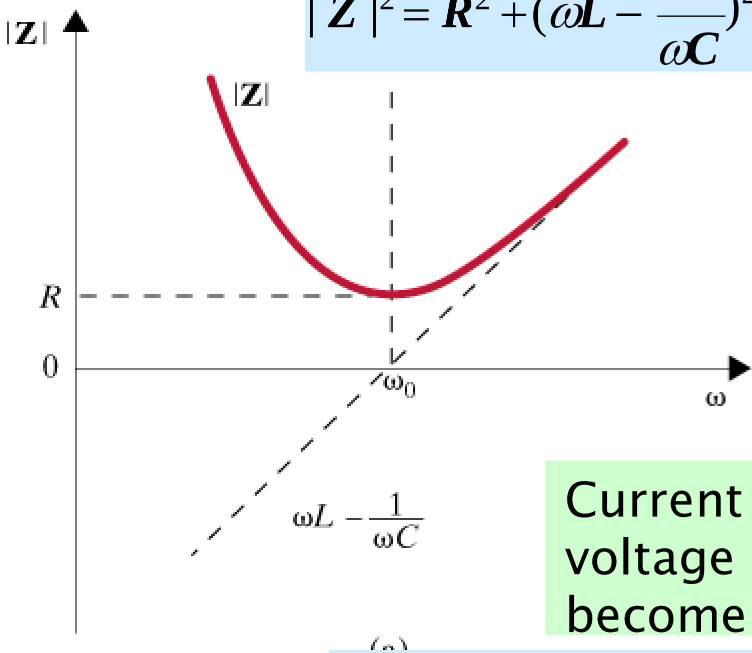
STOP

Properties of resonant circuits

At resonance the impedance/admittance is minimal

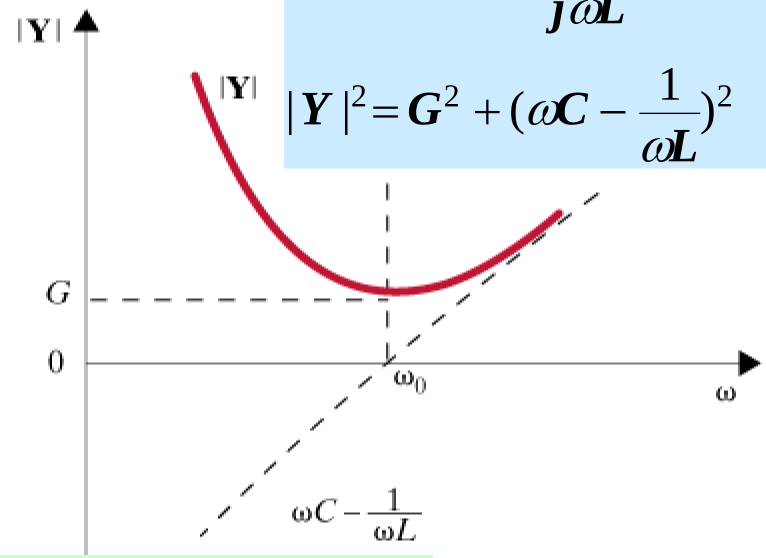
$$Z(j\omega) = R + j\omega L + \frac{1}{j\omega C}$$

$$|Z|^2 = R^2 + (\omega L - \frac{1}{\omega C})^2$$



$$Y(j\omega) = G + \frac{1}{j\omega L} + j\omega C$$

$$|Y|^2 = G^2 + (\omega C - \frac{1}{\omega L})^2$$



Current through the serial circuit/ voltage across the parallel circuit can become very large

Quality Factor: $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$

Given the similarities between series and parallel resonant circuits, we will focus on serial circuits

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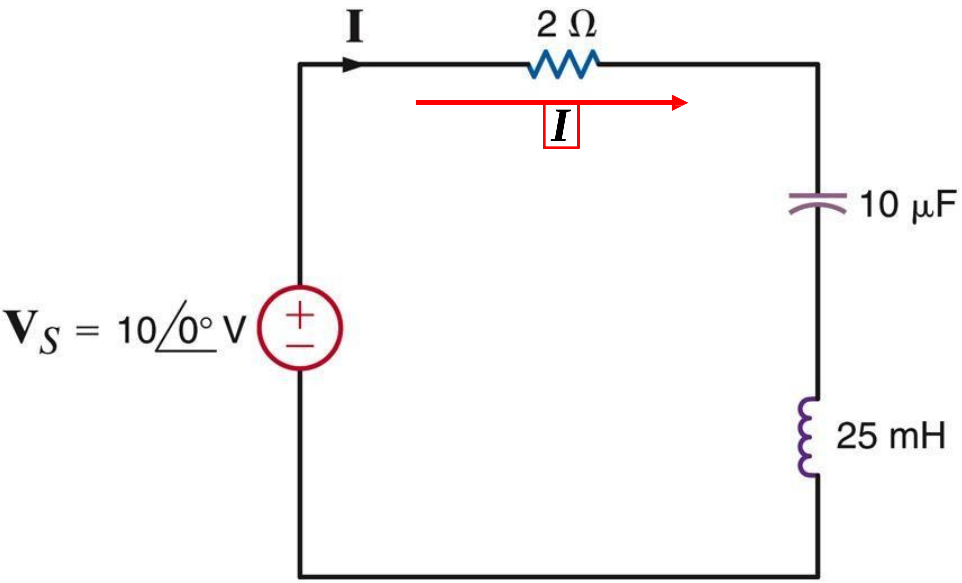
LEARNING EXAMPLE

Determine the resonant frequency, the voltage across each element at resonance and the value of the quality factor



$$\frac{1}{\omega_0 C} = \omega_0 L = 50\Omega$$

$$V_C = \frac{1}{j\omega_0 C} I = -j50 \times 5 = 250\angle -90°$$

$$Q = \frac{\omega_0 L}{R} = \frac{50}{2} = 25$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(25\times 10^{-3} H)(10\times 10^{-6} F)}} = 2000 rad/\sec$$

At resonance $Z = 2\Omega$

$$I = \frac{V_S}{Z} = \frac{10\angle 0°}{2} = 5A$$

$$\omega_0 L = (2\times 10^3)(25\times 10^{-3}) = 50\Omega$$

$$V_L = j\omega_0 LI = j50\times 5 = 250\angle 90°(V)$$

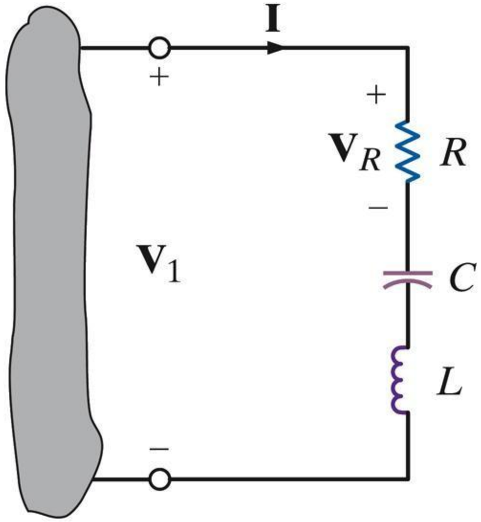
At resonance

$$|V_L| = \omega_0 L\left|\frac{V_S}{R}\right| = Q|V_S|$$

$$|V_C| = Q|V_S|$$

GEAUX

Resonance for the series circuit



$$Z(j\omega) = R + j\omega L + \frac{1}{j\omega C}$$

$$|Z|^2 = R^2 + (\omega L - \frac{1}{\omega C})^2$$

Claim: The voltage gain

$$G_v = \frac{V_R}{V_1} = \frac{1}{1 + jQ(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega})}$$

$$G_v = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} = \frac{R}{Z(j\omega)}$$

At resonance:

$$\omega_0 L = QR,\ \omega_0 C = \frac{1}{QR}$$

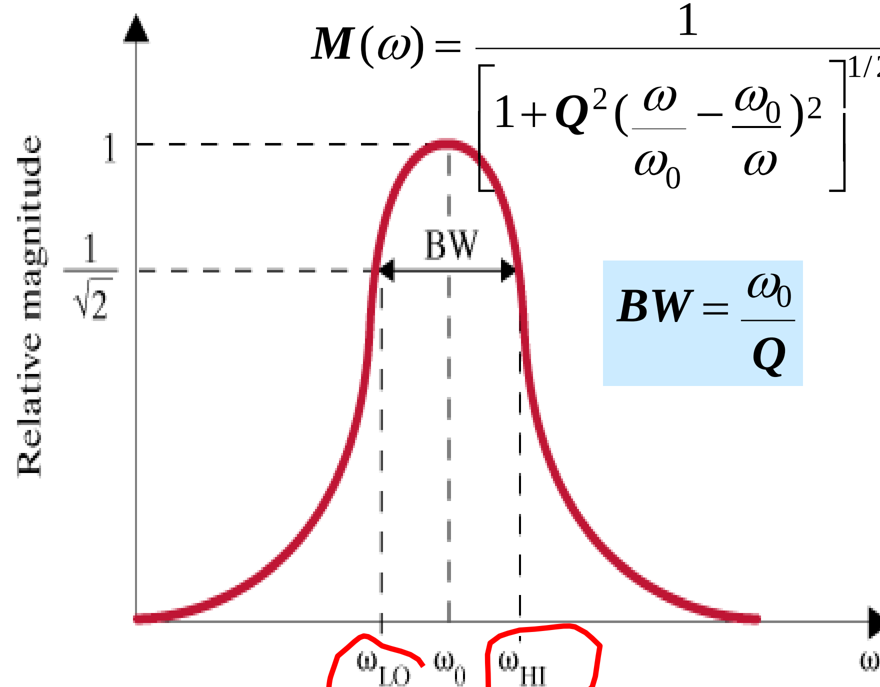
$$Z(j\omega) = R + j\frac{\omega}{\omega_0}QR - j\frac{\omega_0}{\omega}QR$$

$$= R\left[1 + jQ(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega})\right]$$

$$G_v = \frac{R}{Z}$$

$$M(\omega) = |G_v|,\ \phi(\omega) = \angle G_v$$

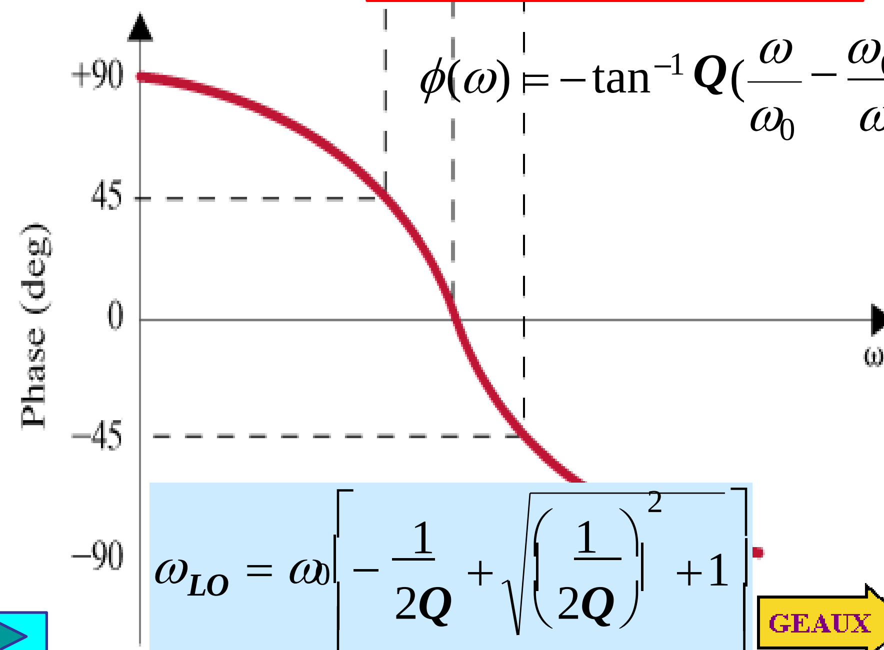
$$M(\omega) = \frac{1}{\left[1 + Q^2(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega})^2\right]^{1/2}}$$



$$BW = \frac{\omega_0}{Q}$$

Half power frequencies

$$\phi(\omega) = -\tan^{-1} Q(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega})$$



$$\omega_{LO} = \omega_0\left[-\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]$$

GEAUX

LEARNING EXTENSION

A series RLC circuit as the following properties:

$$R = 4\Omega, \omega_0 = 4000 \, rad/\sec, BW = 100 \, rad/\sec$$

Determine the values of L,C.

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} \qquad BW = \frac{\omega_0}{Q}$$

1. Given resonant frequency and bandwidth determine Q.
2. Given R, resonant frequency and Q determine L, C.

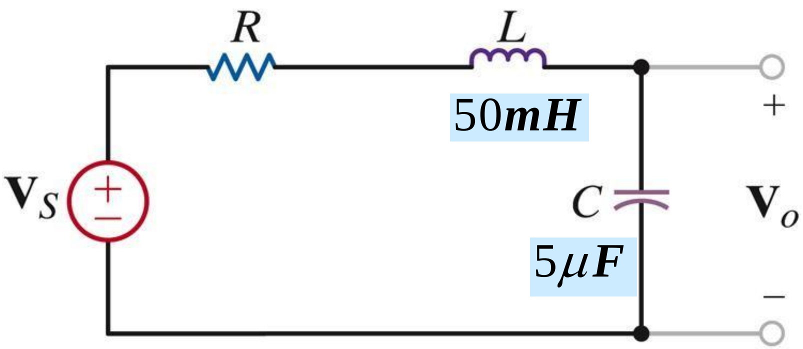
$$Q = \frac{\omega_0}{BW} = \frac{4000}{100} = 40$$

$$L = \frac{QR}{\omega_0} = \frac{40 \times 4}{4000} = 0.040H$$

$$C = \frac{1}{L\omega_0^2} = \frac{1}{\omega_0 RQ} = \frac{1}{4 \times 10^{-2} \times 16 \times 10^6} = 1.56 \times 10^{-6} F$$

GEAUX

LEARNING EXAMPLE Determine ω_0 , ω_{max} when $R = 50\Omega$ and $R = 1\Omega$



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

$$u_{max} = \frac{\omega_{max}}{\omega_0} = \sqrt{1 - \frac{1}{2Q^2}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(5\times10^{-2})(5\times10^{-6})}} = 2000 rad/s$$

$$Q = \frac{2000 \times 0.050}{R}$$

$$\omega_{max} = 2000 \times \sqrt{1 - \frac{1}{2Q^2}}$$

R	Q	Wmax
50	2	1871
1	100	2000

Evaluated with EXCEL and rounded to zero decimals

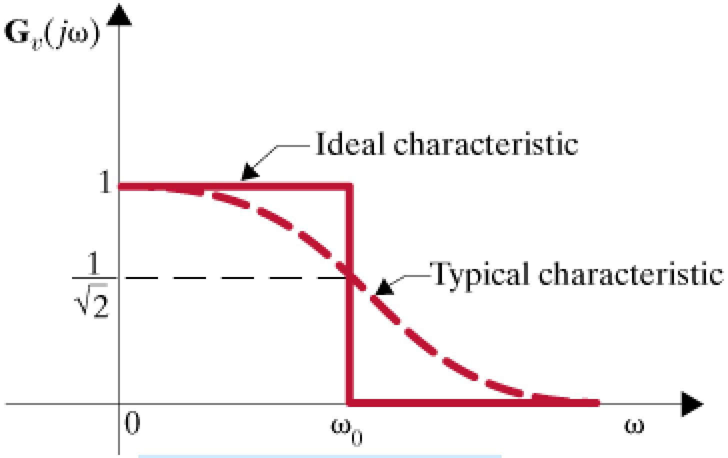
Using MATLAB one can display the frequency response

GEAUX

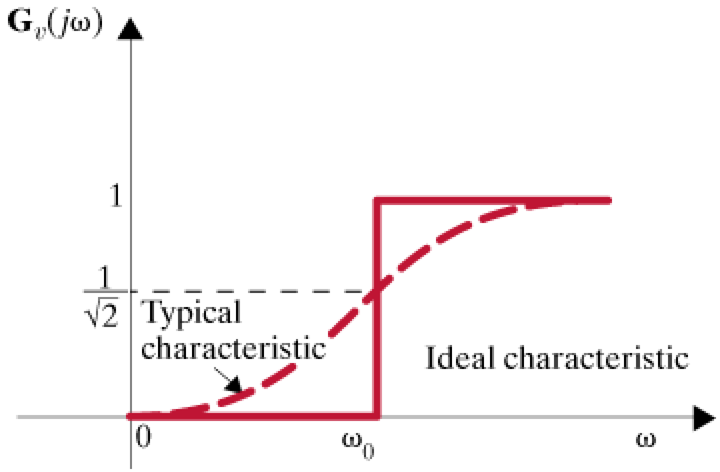
FILTER NETWORKS

Networks designed to have frequency selective behavior

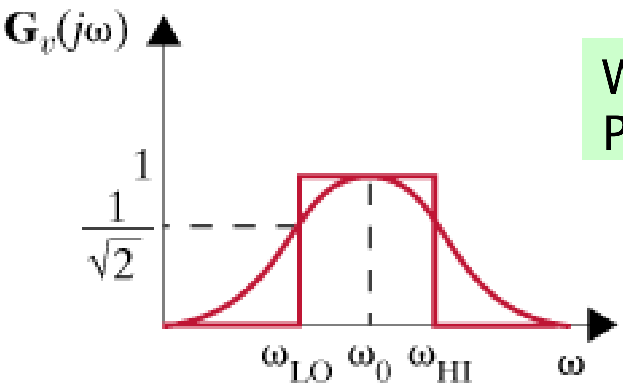
COMMON FILTERS



Low-pass filter

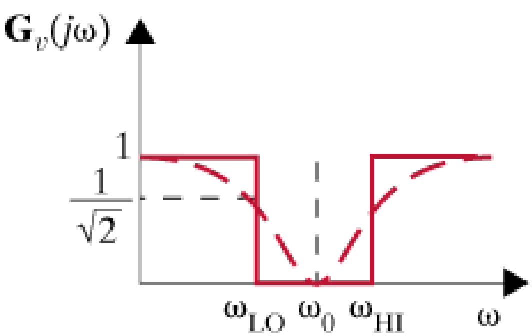


High-pass filter



Band-pass filter

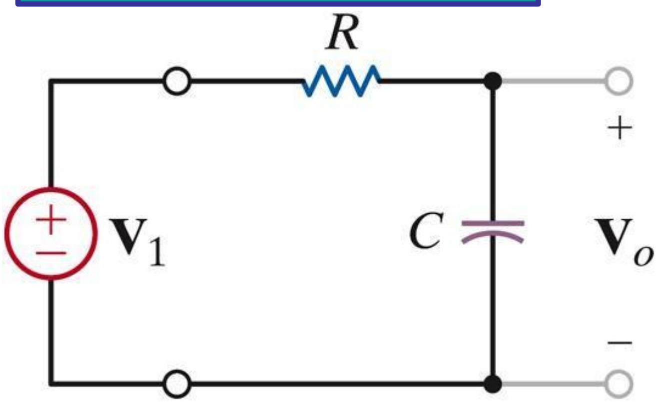
We focus first on PASSIVE filters



Band-reject filter

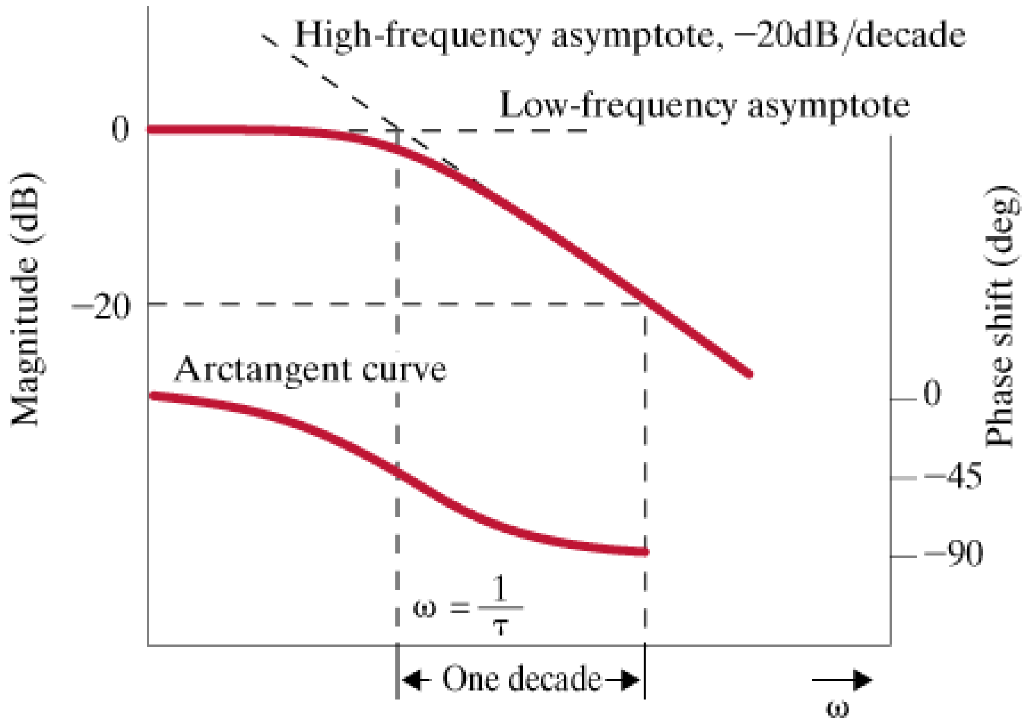
GEAUX

Simple low-pass filter



$$G_v = \frac{V_0}{V_1} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

$$G_v = \frac{1}{1 + j\omega\tau}; \quad \tau = RC$$



(c)

$$M(\omega) = |G_v| = \frac{1}{\sqrt{1 + (\omega\tau)^2}}$$

$$\angle G_v = \phi(\omega) = -\tan^{-1} \omega\tau$$

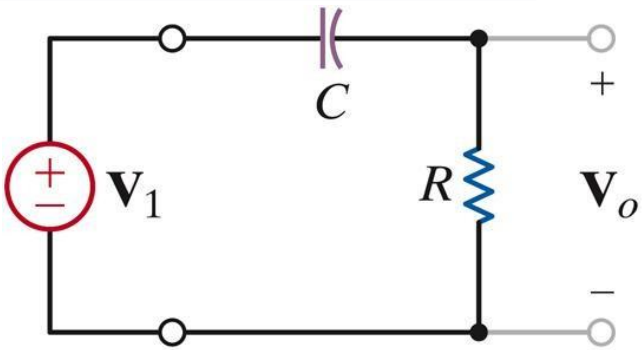
$$M_{\max} = 1, \quad M\left(\omega = \frac{1}{\tau}\right) = \frac{1}{\sqrt{2}}$$

$$BW = \frac{1}{\tau}$$

$\omega = \frac{1}{\tau}$ = half power frequency

GEAUX

Simple high-pass filter



$$G_v = \frac{V_0}{V_1} = \frac{R}{R+\frac{1}{j\omega C}} = \frac{j\omega CR}{1+j\omega CR}$$

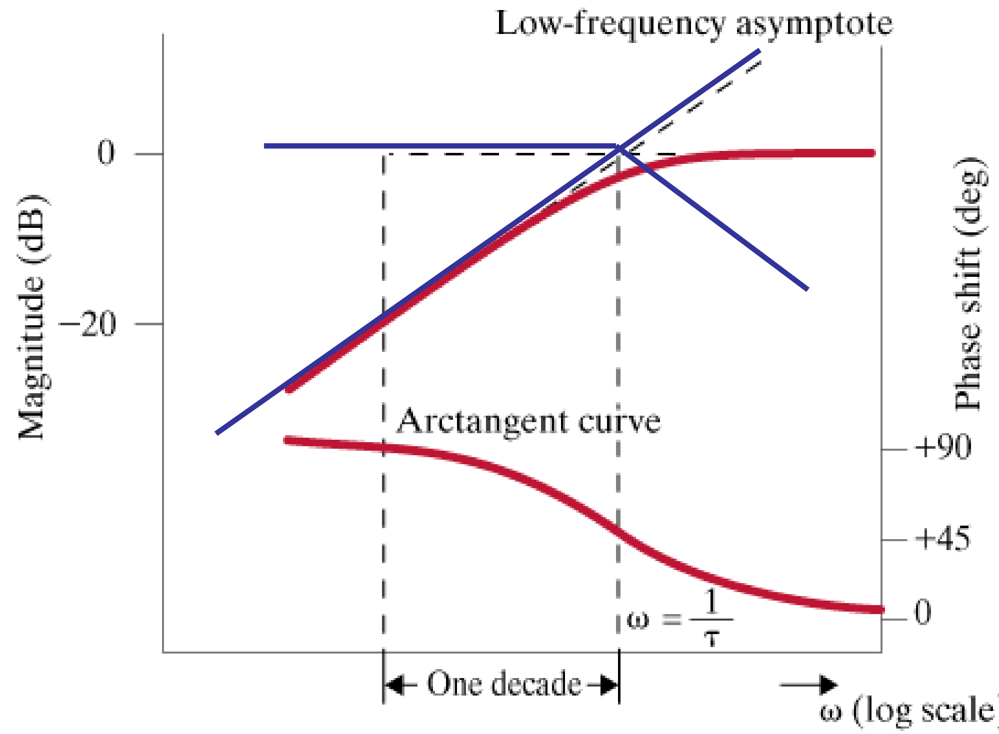
$$G_v = \frac{j\omega\tau}{1+j\omega\tau};\ \tau = RC$$

$$M(\omega) = |G_v| = \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}$$

$$\angle G_v = \phi(\omega) = \frac{\pi}{2} - \tan^{-1}\omega\tau$$

$$M_{\max} = 1,\ M\left(\omega = \frac{1}{\tau}\right) = \frac{1}{\sqrt{2}}$$

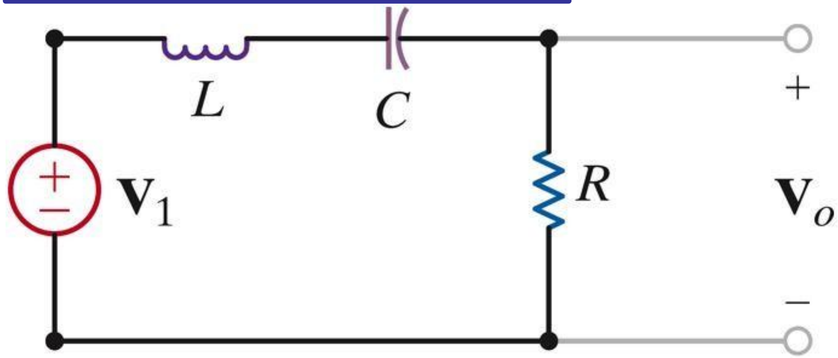
$\omega = \frac{1}{\tau}$ = half power frequency



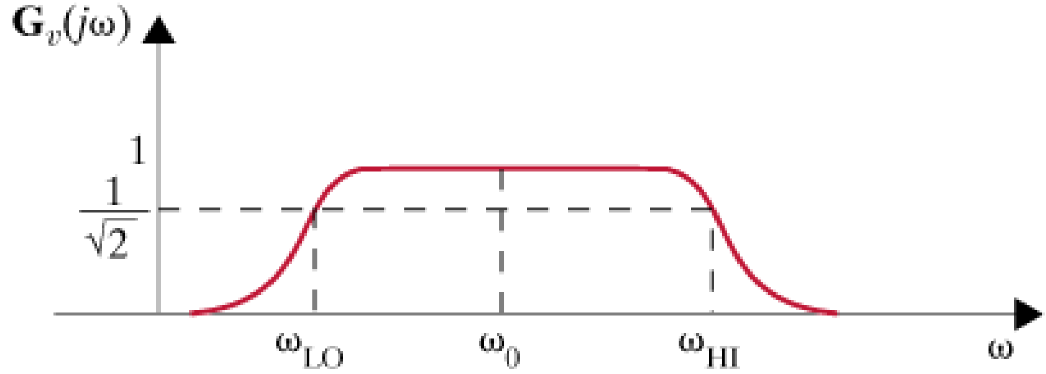
(c)

$$\omega_{LO} = \frac{1}{\tau}$$

Simple band-pass filter



Band-pass



(e)

$$G_v = \frac{V_0}{V_1} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$M(\omega) = \frac{\omega RC}{\sqrt{(\omega RC)^2 + (\omega^2 LC - 1)^2}}$$

$$M\left(\omega = \frac{1}{\sqrt{LC}}\right) = 1 \quad M(\omega = 0) = M(\omega = \infty) = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

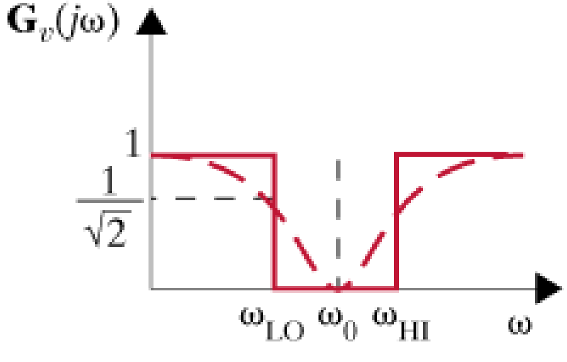
$$M(\omega_{LO}) = \frac{1}{\sqrt{2}} = M(\omega_{HI})$$

$$\omega_{LO} = \frac{-(R/L) + \sqrt{(R/L)^2 + 4\omega_0^2}}{2}$$

$$\omega_{HI} = \frac{(R/L) + \sqrt{(R/L)^2 + 4\omega_0^2}}{2}$$

$$BW = \omega_{HI} - \omega_{LO} = \frac{R}{L}$$

Simple band-reject filter



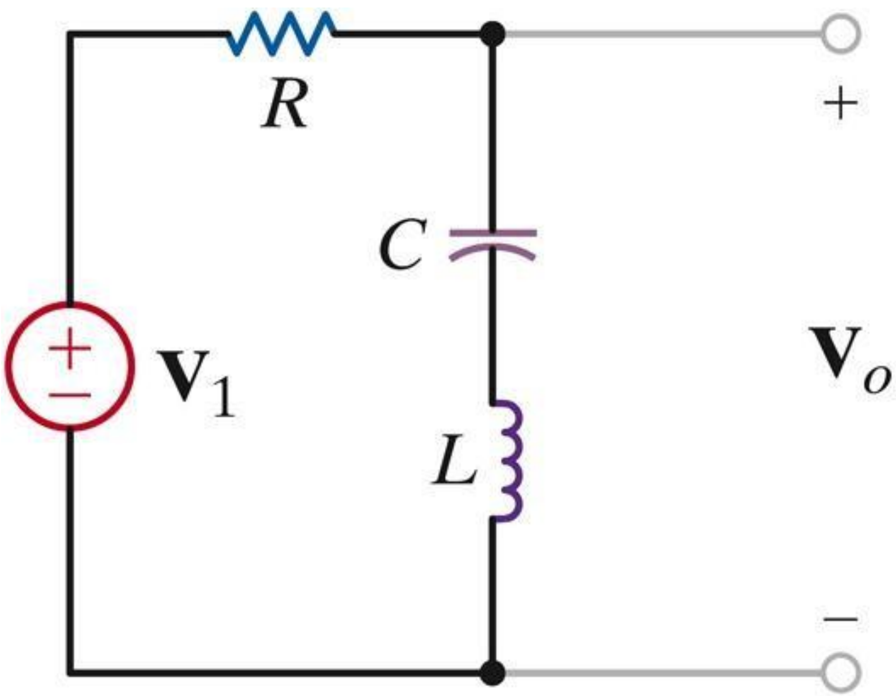
(b)

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow j\left|\left(\omega_0 L - \frac{1}{\omega_0 C}\right)\right| = 0$$

at $\omega = 0$ the capacitor acts as open circuit $\Rightarrow V_0 = V_1$

at $\omega = \infty$ the inductor acts as open circuit $\Rightarrow V_0 = V_1$

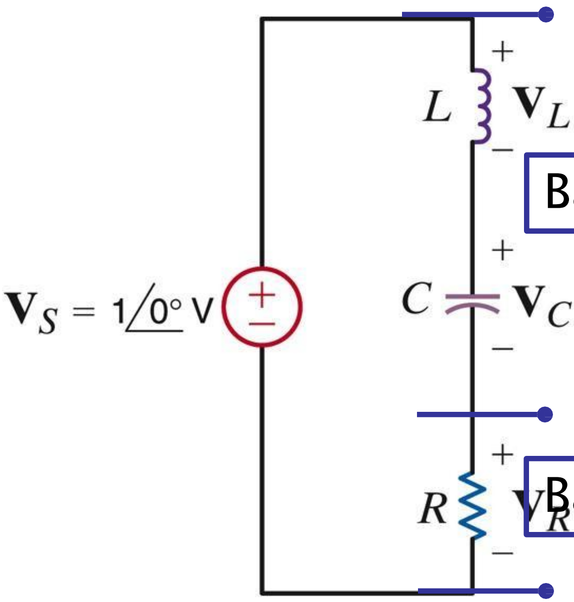
ω_{LO}, ω_{HI} are determined as in the band-pass filter



GEAUX

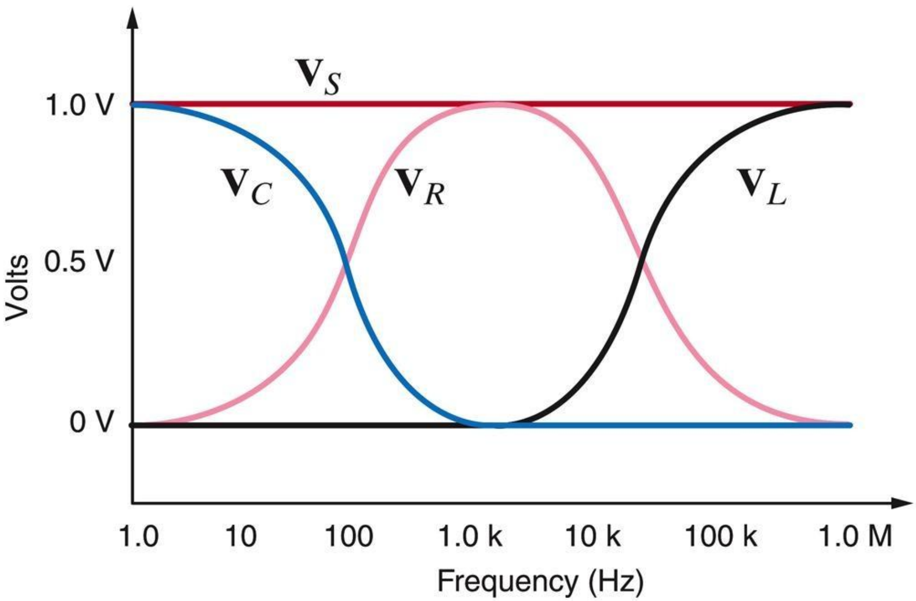
LEARNING EXAMPLE

Depending on where the output is taken, this circuit can produce low-pass, high-pass or band-pass or band-reject filters



Band-reject filter

Band-pass



Bode plot for $R = 10\Omega,\ L = 159\mu H,\ C = 159\mu F$

$$\frac{V_L}{V_S} = \frac{j\omega L}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\frac{V_L}{V_S}(\omega = 0) = 0,\quad \frac{V_L}{V_S}(\omega = \infty) = 1$$

High-pass

$$\frac{V_C}{V_S} = \frac{\frac{1}{j\omega C}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\frac{V_C}{V_S}(\omega = 0) = 1,\quad \frac{V_C}{V_S}(\omega = \infty) = 0$$

Low-pass

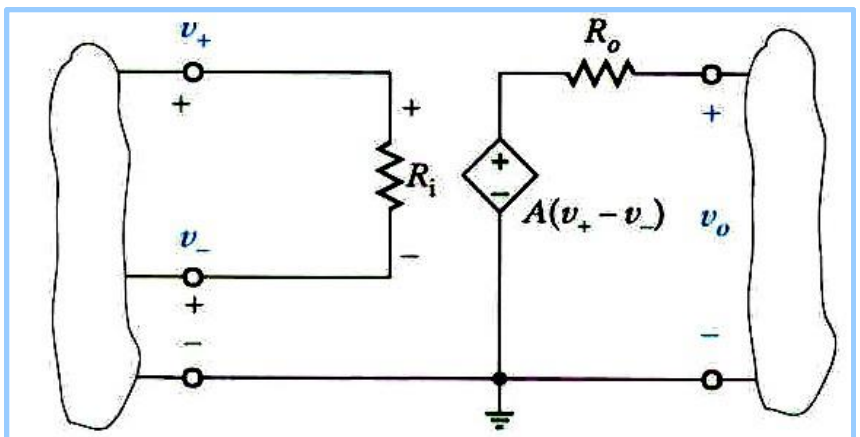
ACTIVE FILTERS

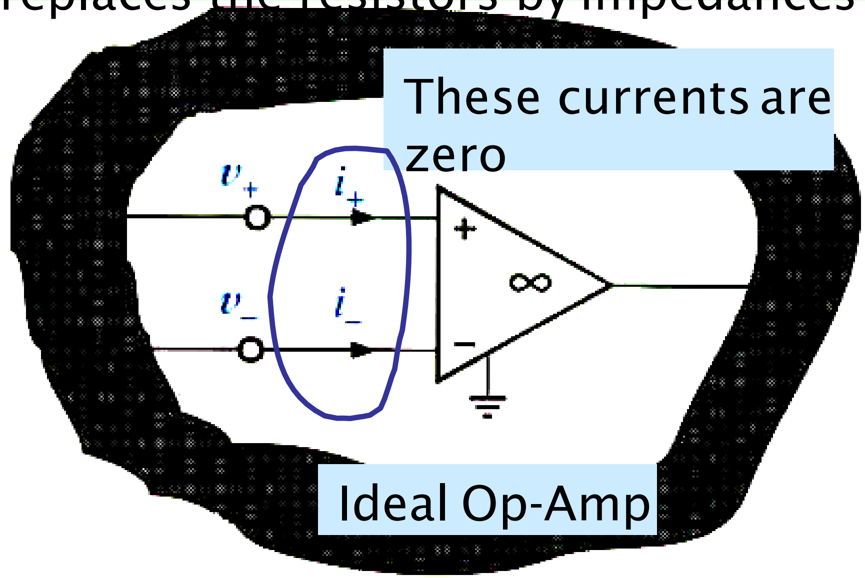
Passive filters have several limitations

1. Cannot generate gains greater than one
2. Loading effect makes them difficult to interconnect
3. Use of inductance makes them difficult to handle

Using operational amplifiers one can design all basic filters, and more, with only resistors and capacitors

The linear models developed for operational amplifiers circuits are valid, in a more general framework, if one replaces the resistors by impedances

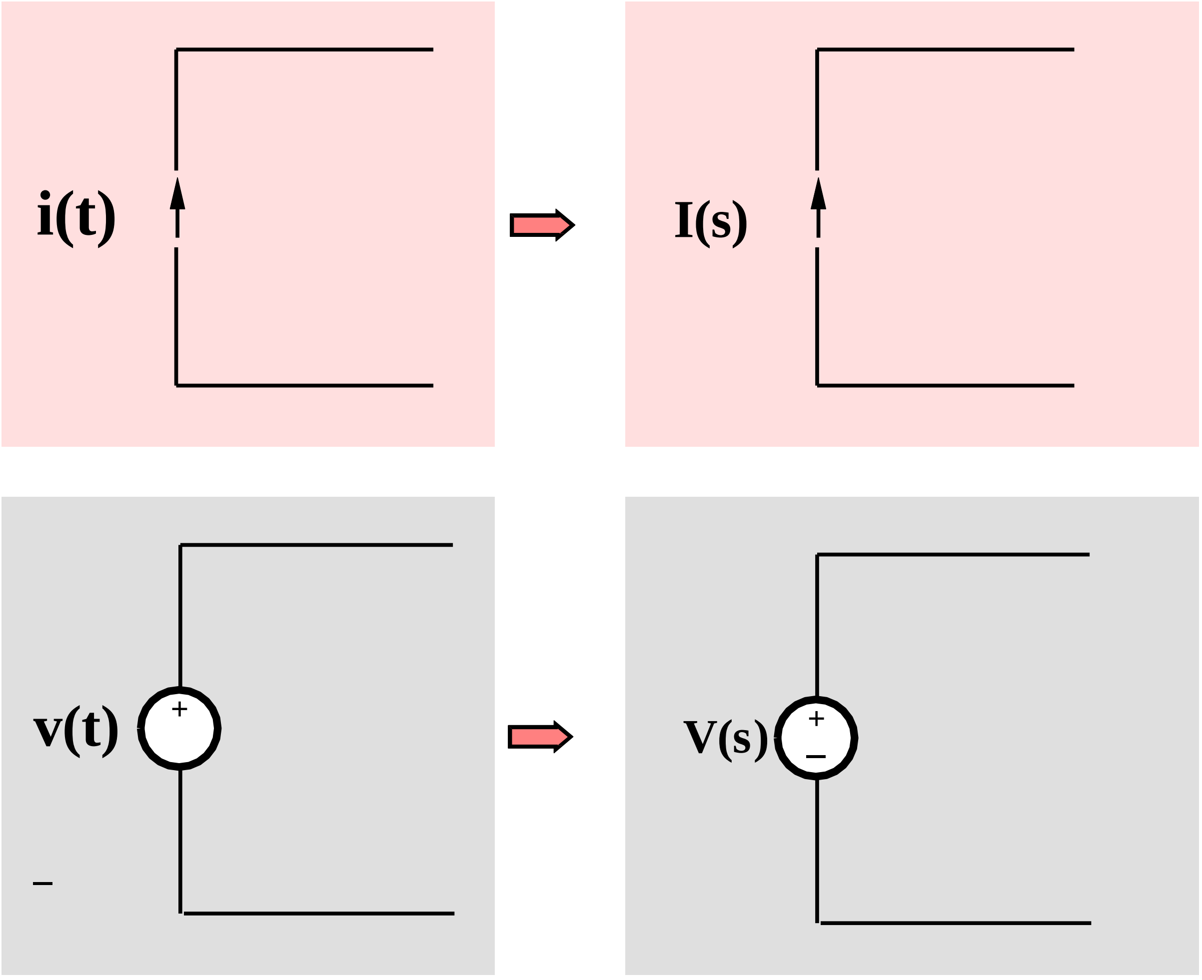




GEAUX

Laplace Circuit Analysis

Circuit Element Modeling

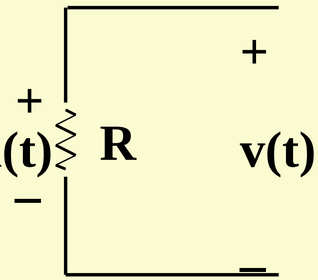


Laplace Circuit Analysis

Circuit Element Modeling

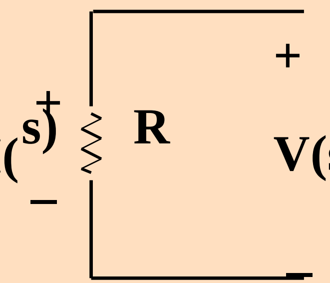
Resistance

$v(t) = Ri(t)$



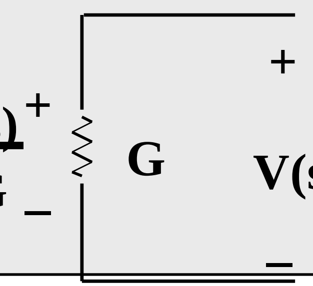
Time Domain

$V(s) = RI(s)$



Complex Frequency Domain

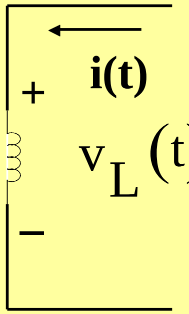
$V(s) = \dfrac{I(s)}{G}$



Laplace Circuit Analysis

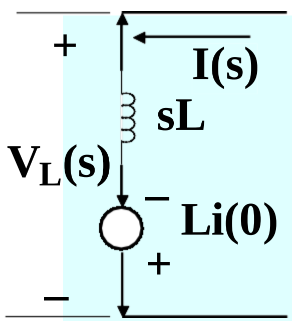
Circuit Element Modeling

Inductor



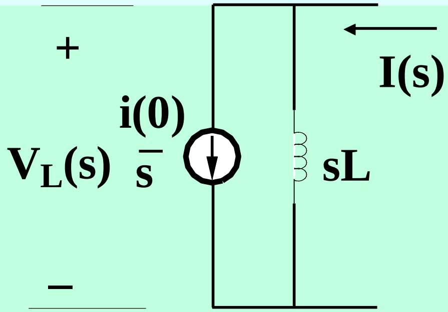
$$v_L(t) = L\frac{di(t)}{dt}$$

Best for mesh



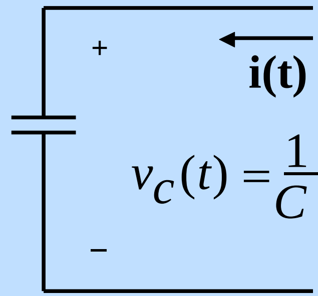
$$V_L(s) = sLI(s) - Li(0)$$

Best for nodal



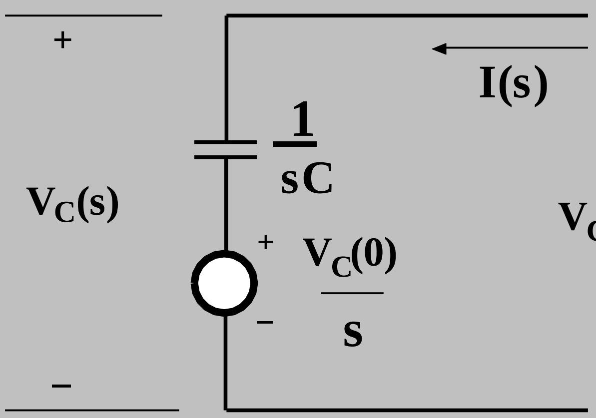
Laplace Circuit Analysis

Capacitor



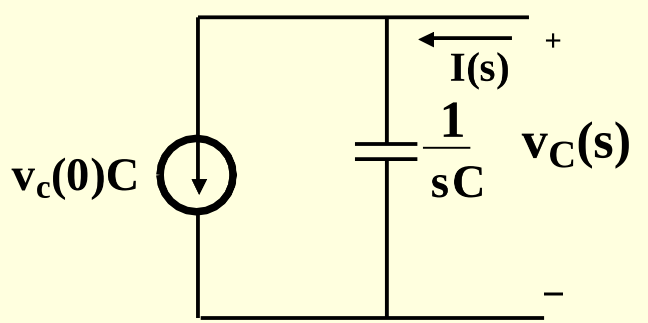
$$v_C(t) = \frac{1}{C}\int_0^t i(t)dt \ + v_C(0)$$

Mesh



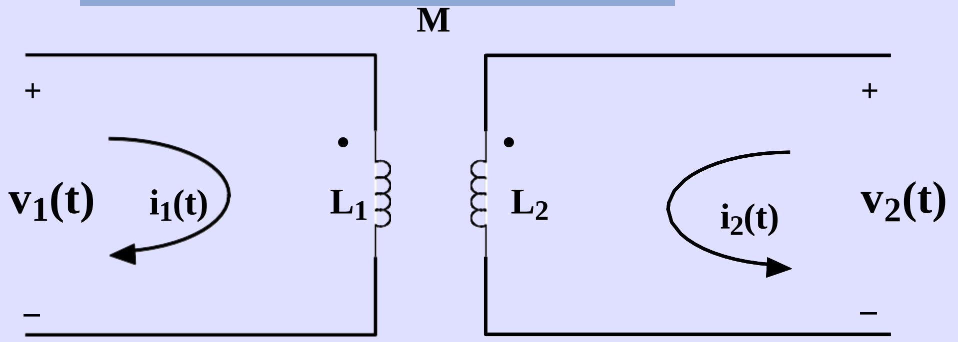
$$V_C(s) = \frac{I(s)}{sC} + \frac{v_c(0)}{s}$$

Nodal



Laplace Circuit Analysis

Linear Transformer

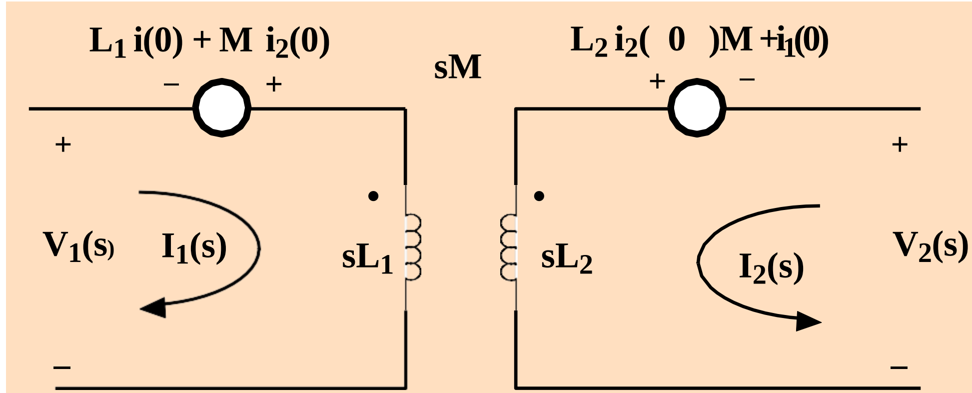


$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt}$$

$$V_1(s) = sL_1 I_1(s) - L_1 i_1(0) + sMI_2(s) - Mi_2(0)$$

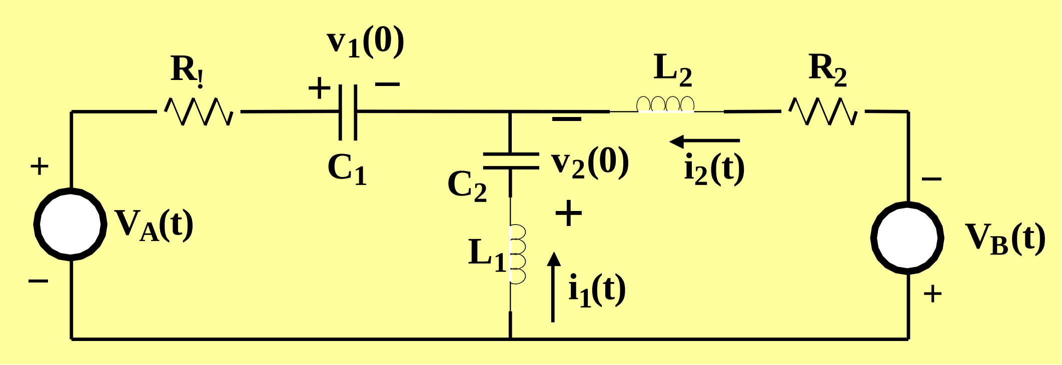
$$v_2(t) = L_2 \frac{di_2(t)}{dt} + M\frac{di_1(t)}{dt}$$

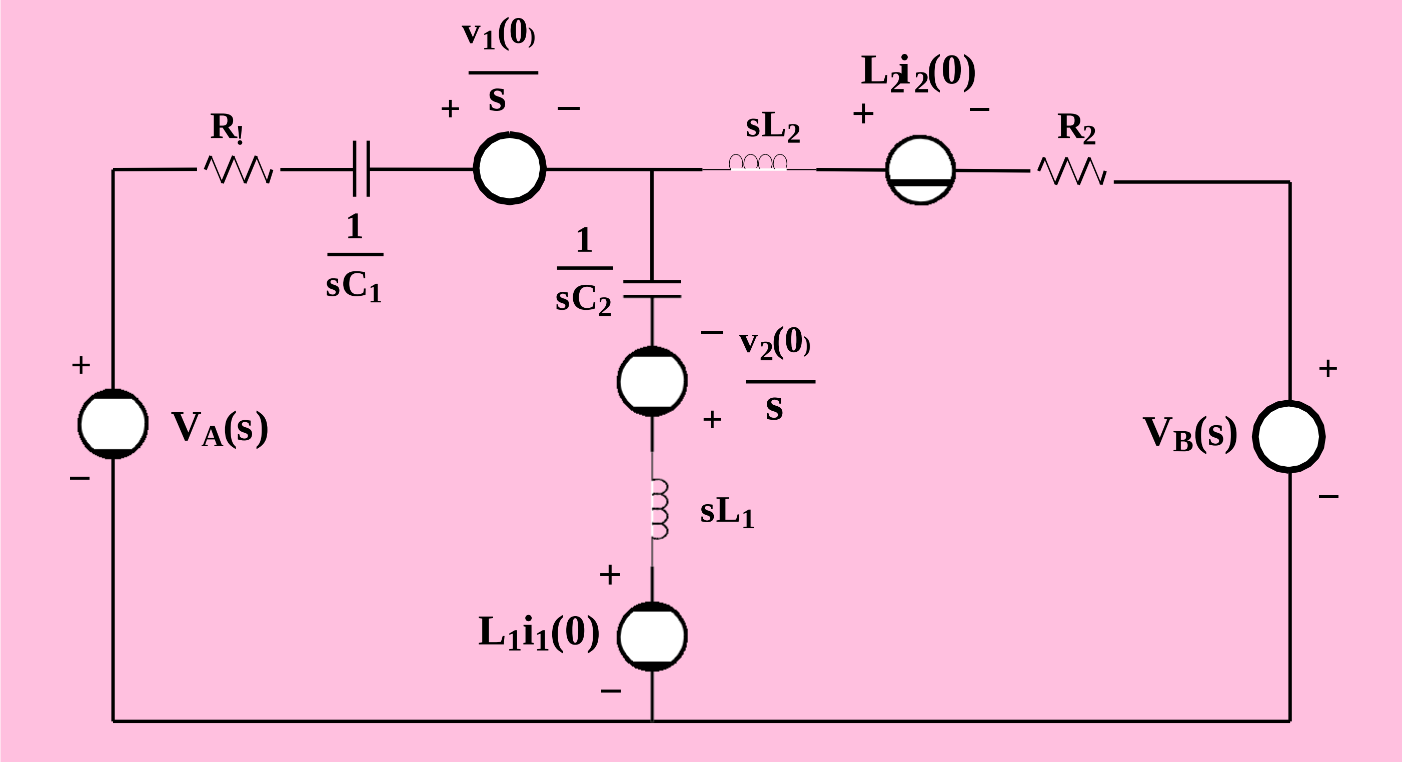
$$V_2(s) = sL_2 I_2(s) - L_2 i_2(0) + sMI_1(s) - Mi_1(0)$$



Laplace Circuit Analysis

Time domain to complex frequency domain



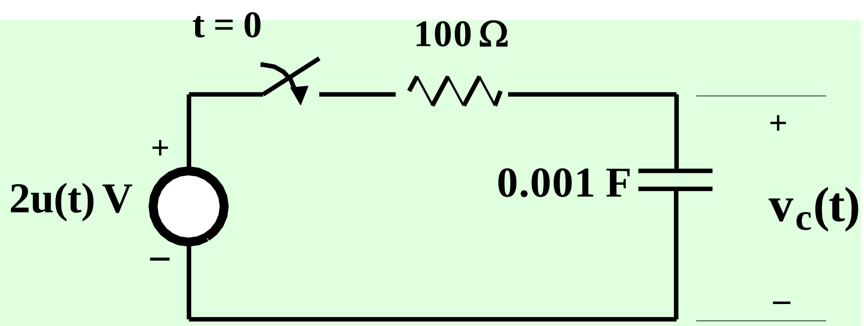


Laplace Circuit Analysis

Circuit Application:

Given the circuit below. Assume zero IC's. Use Laplace to find $v_c(t)$.

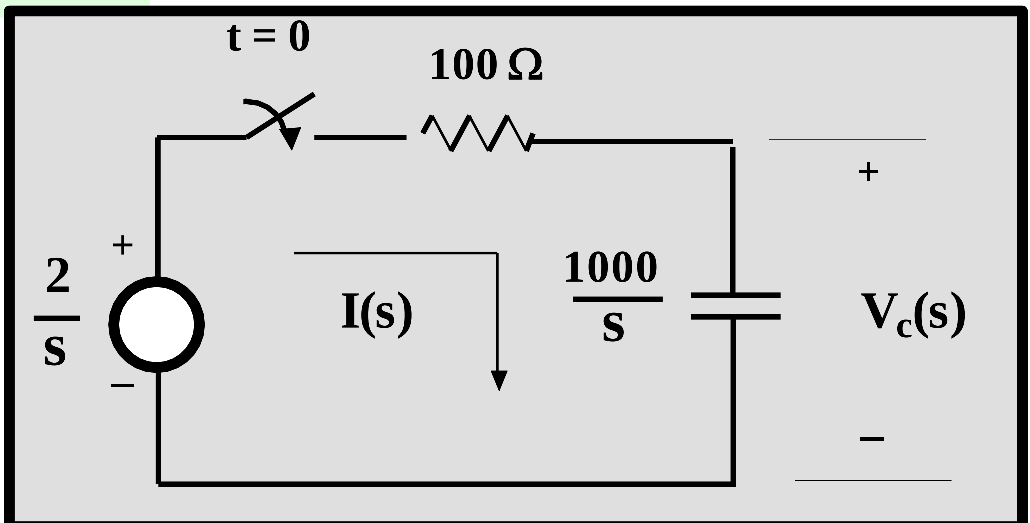
The time domain circuit:



$$V_c(s) = \frac{\left(\frac{2}{s}\right)\left(\frac{1000}{s}\right)}{100 + \frac{1000}{s}}$$

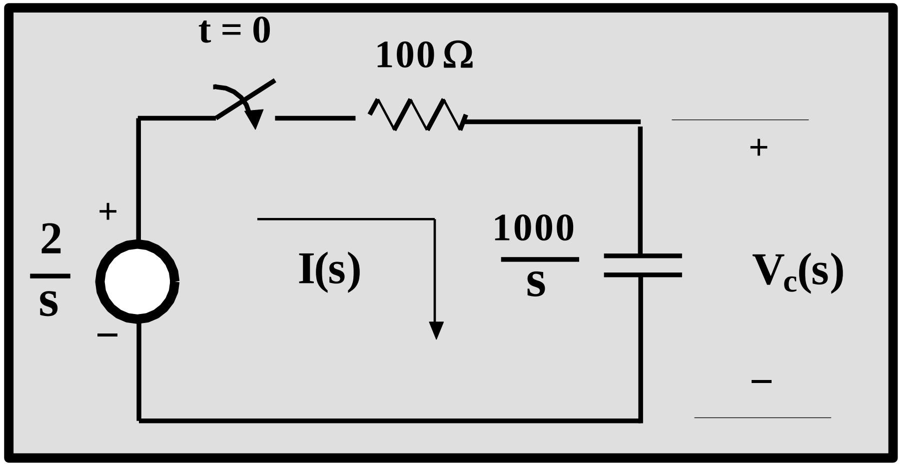
$$V_c(s) = \frac{20}{s(s+10)}$$

Laplace circuit



Laplace Circuit Analysis

Circuit Application:



$$V_c(s) = \frac{20}{s(s+10)} = \frac{2}{s} - \frac{2}{s+10}$$

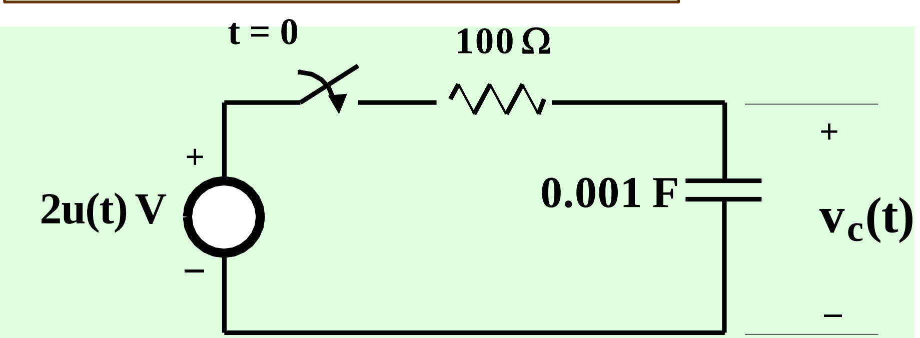
$$v_c(t) = \left[2 - 2e^{-10t}\right]u(t)$$

Laplace Circuit Analysis

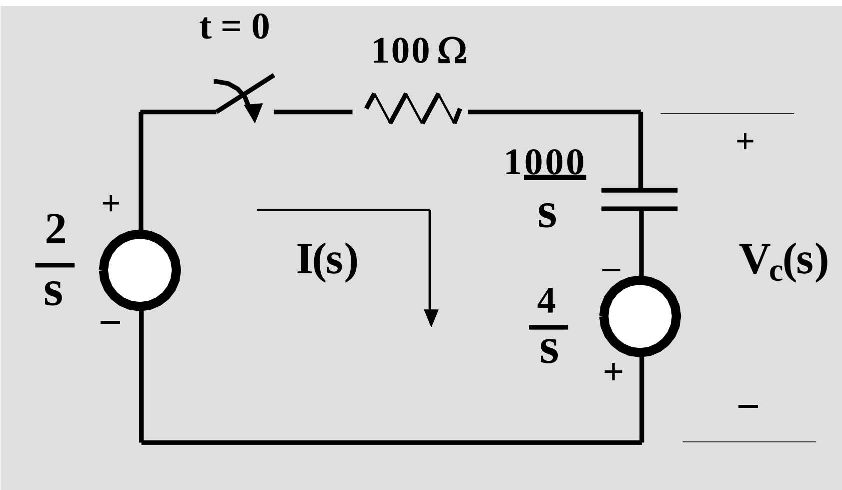
Circuit Application:

Given the circuit below. Assume $v_c(0) = -4$ V. Use Laplace to find $v_c(t)$.

The time domain circuit:



Laplace circuit:



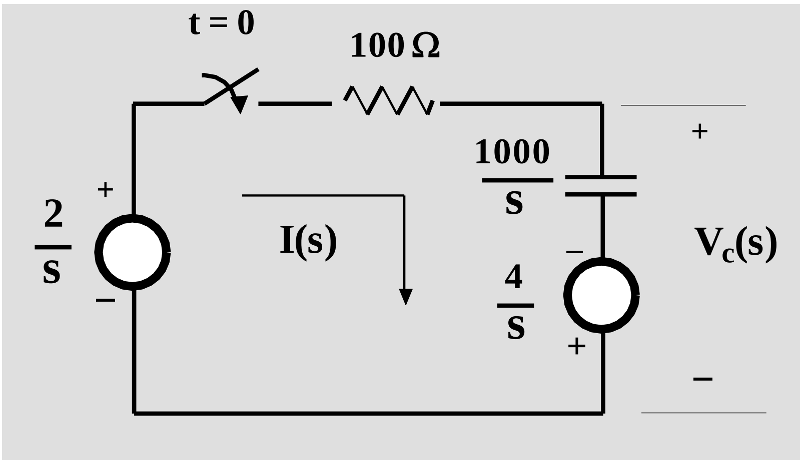
$$\frac{2}{s}+\frac{4}{} = I(s)\left[100+\frac{1000}{s}\right]$$

s

$$100I(s) = \frac{6}{s+10}$$

Laplace Circuit Analysis

Circuit Application:



1

$$\frac{2}{s} - 100I(s) - V_c(s) = 0$$

$$\frac{2}{s} - \frac{6}{s+10} = V_c(s)$$

2

$$V_c(s) = \frac{-4s + 20}{s(s+10)} = \frac{A}{s} + \frac{B}{s+10}$$

$$V_c(s) = \frac{2}{s} - \frac{6}{s+10}$$

$$v(t) = \left[2 - 6e^{-10t}\right]u(t)$$

Check the boundary conditions

3

$v_c(0) = -4\,V$

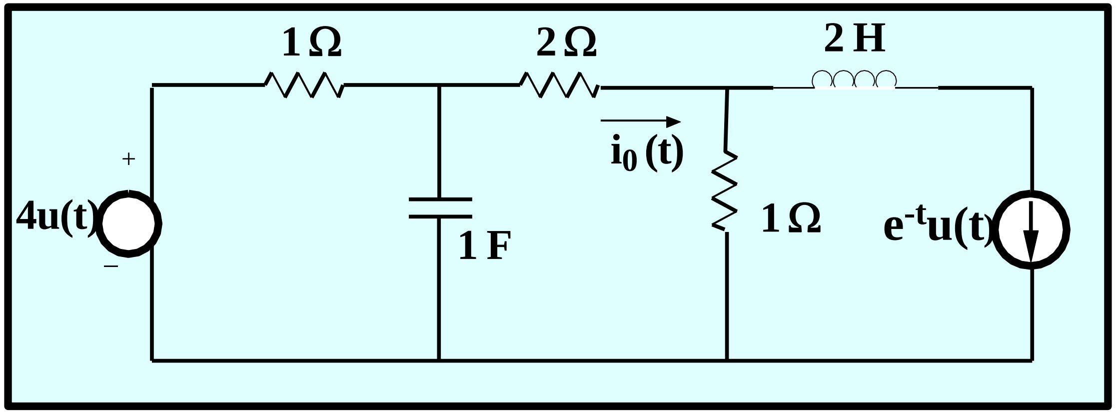
$v_c(oo) = 2\,V$

Laplace Circuit Analysis

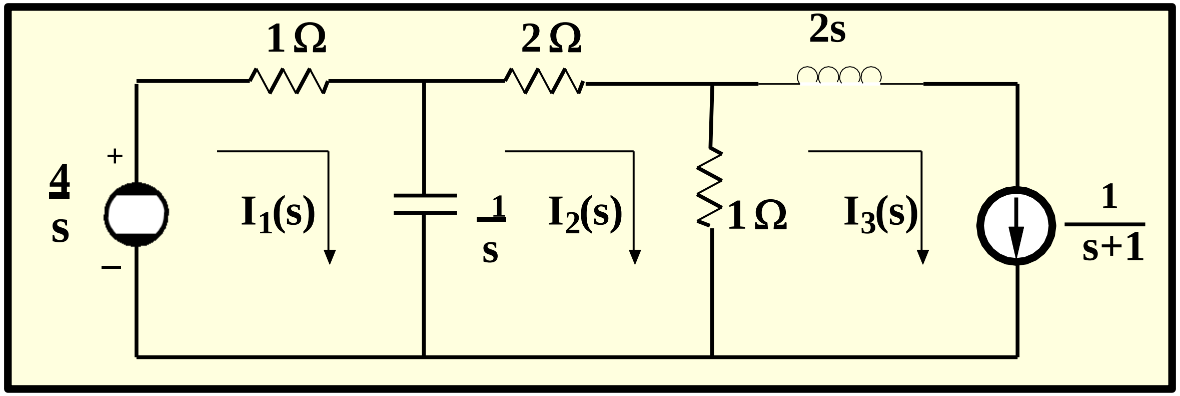
Circuit Application:

Find $i_0(t)$ using Laplace

Time Domain



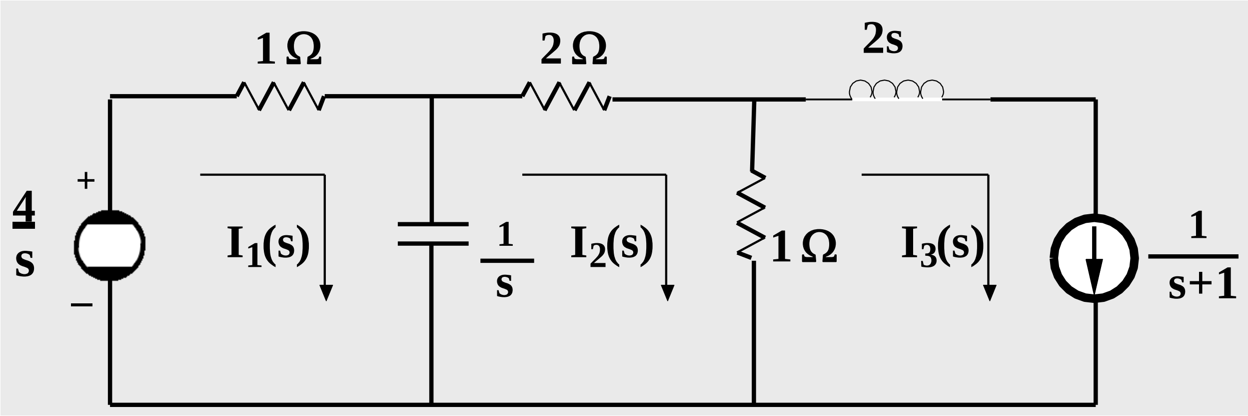
Laplace



Laplace Circuit Analysis

Circuit Application:

Find $i_0(t)$ using Laplace



Mesh 1

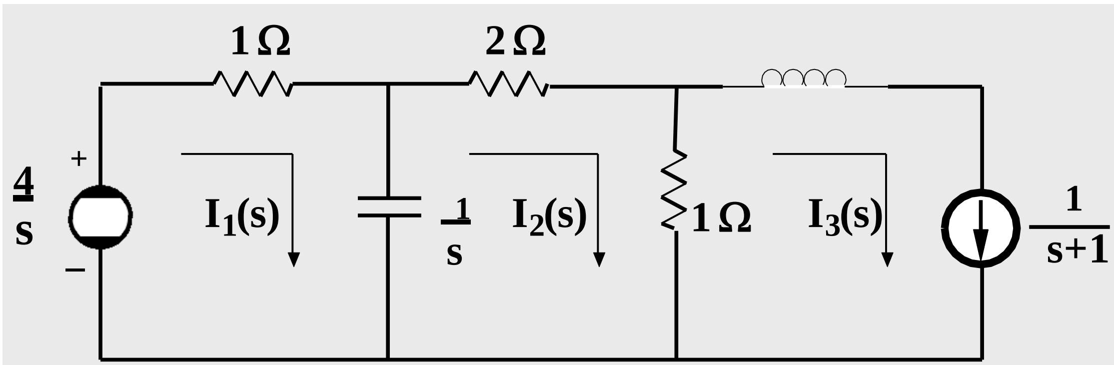
$$\frac{(s+1)}{s} I_1(s) - \frac{I_2(s)}{s} = \frac{4}{s}$$

$$(s+1) I_1(s) - I_2(s) = 4$$

Laplace Circuit Analysis

Circuit Application:

Find $i_0(t)$ using Laplace



Mesh 2

$$\frac{-1}{s} I_1(s) + \frac{3s+1}{s} I_2(s) - I_3(s) = 0$$

$$\frac{-1}{s} I_1(s) + \frac{3s+1}{s} I_2(s) - \frac{1}{s+1} = 0$$

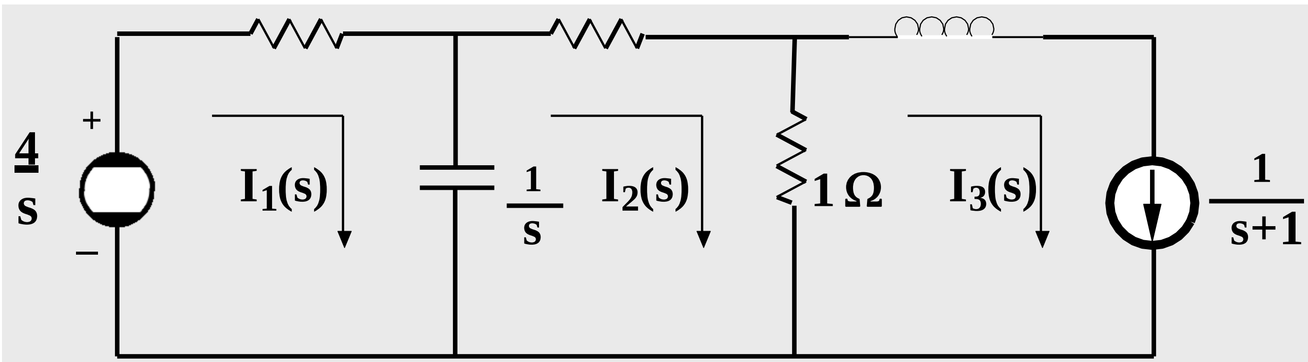
$$-I_1(s) + (3s+1) I_2(s) = \frac{s}{s+1}$$

$$-(s+1)I_1(s) + (s+1)(3s+1) I_2(s) = s$$

Laplace Circuit Analysis

Circuit Application:

Find $i_0(t)$ using Laplace



$$(s+1)I_1(s) - I_2(s) = 4$$

$$-(s+1)I_1(s) + (s+1)(3s+1)I_2(s) = s$$

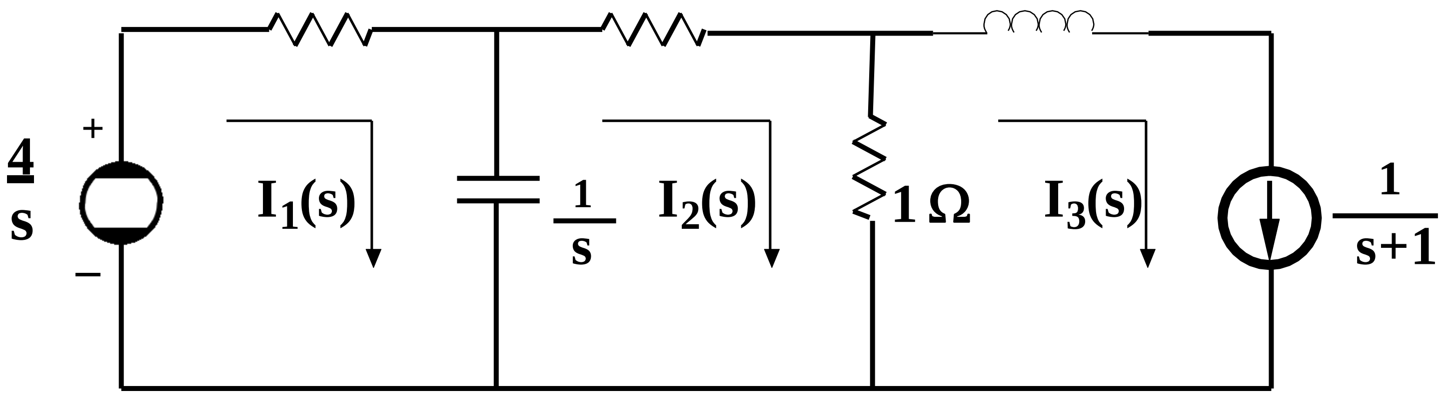
Add these 2 equations

$$\boxed{s(3s+4)I_2(s) = s+4}$$

Laplace Circuit Analysis

Circuit Application:

Find $i_0(t)$ using Laplace



$$s(3s+4)I_2(s) = s+4$$

$$I_2(s) = \frac{\left(\frac{1}{3}\right)(s+4)}{s\left(s+\frac{4}{3}\right)} = \frac{1}{s} - \frac{\frac{2}{3}}{s+\frac{4}{3}}$$

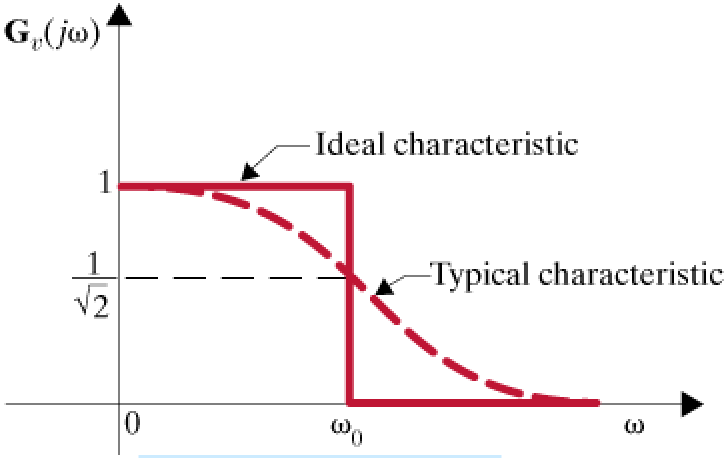
$$i_2(t) = \left[1 - \frac{2}{3}e^{-\frac{4}{3}t}\right]u(t)$$

Is final value of $i_2(t)$ reasonable?

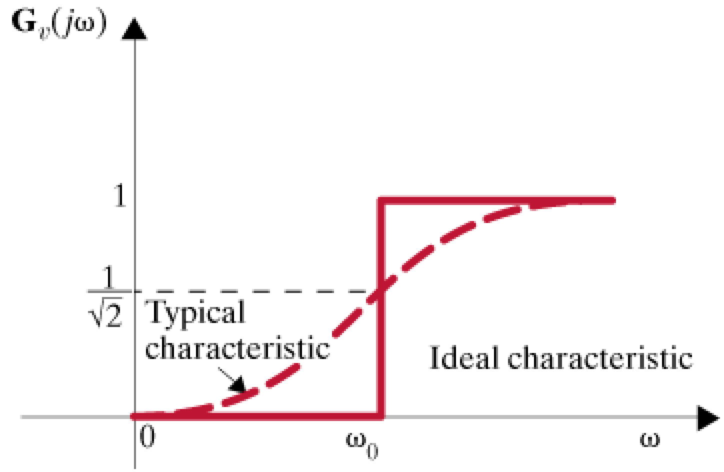
FILTER NETWORKS

Networks designed to have frequency selective behavior

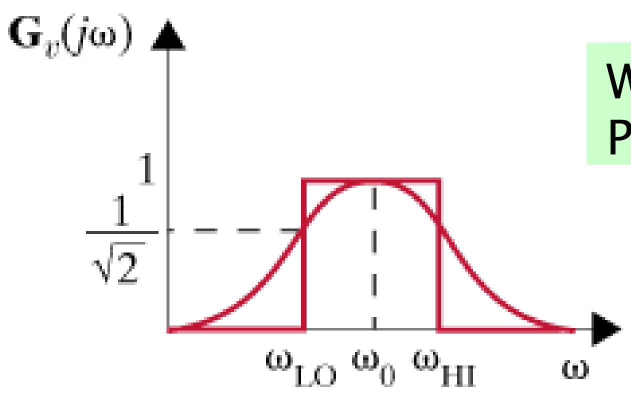
COMMON FILTERS



Low-pass filter

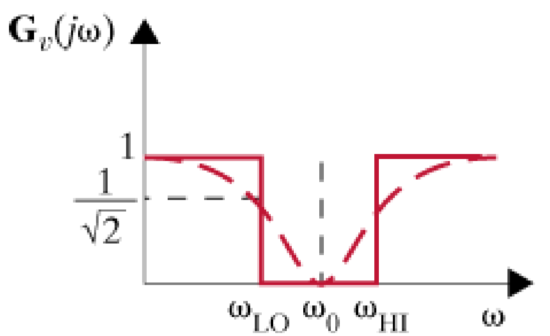


High-pass filter



Band-pass filter

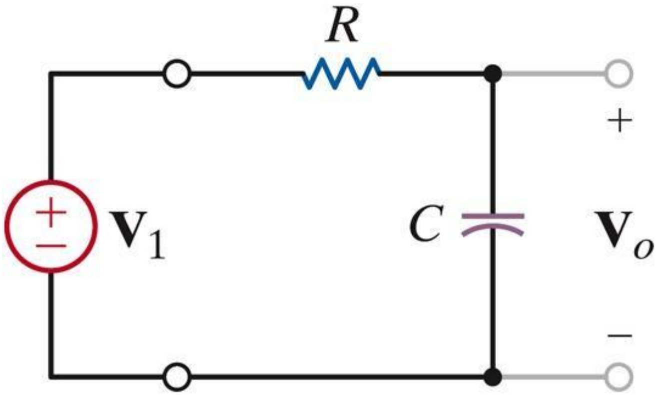
We focus first on PASSIVE filters



Band-reject filter

GEAUX

Simple low-pass filter



$$G_v = \frac{V_0}{V_1} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

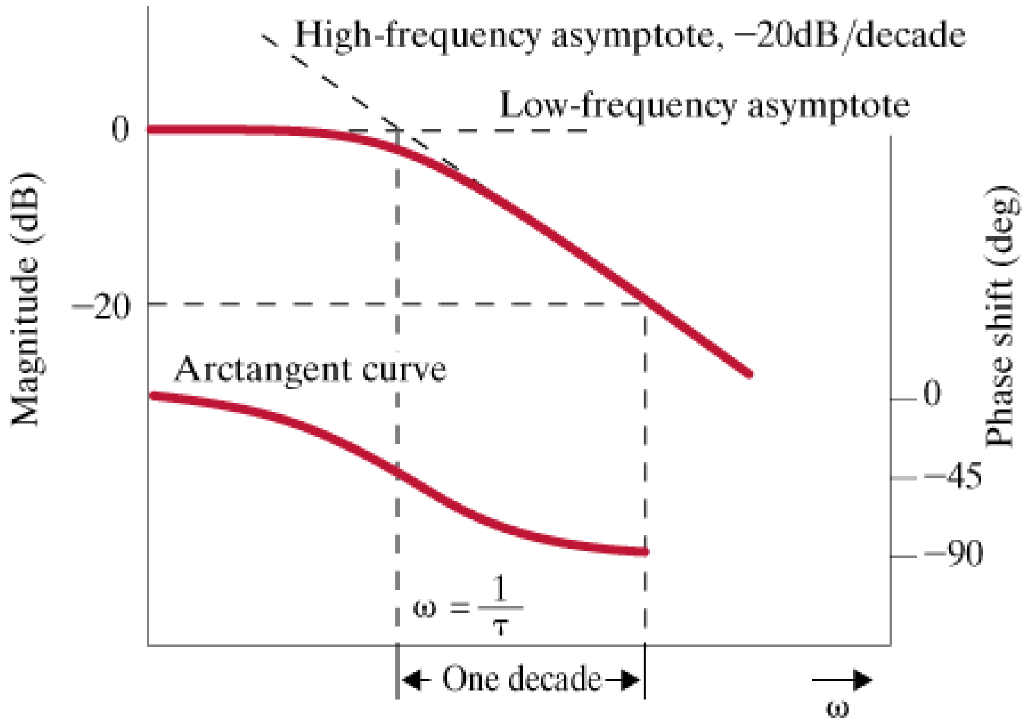
$$G_v = \frac{1}{1 + j\omega\tau}; \quad \tau = RC$$

$$M(\omega) = |G_v| = \frac{1}{\sqrt{1 + (\omega\tau)^2}}$$

$$\angle G_v = \phi(\omega) = -\tan^{-1} \omega\tau$$

$$M_{\max} = 1, \; M\left(\omega = \frac{1}{\tau}\right) = \frac{1}{\sqrt{2}}$$

$$\omega = \frac{1}{\tau} = \text{half power frequency}$$

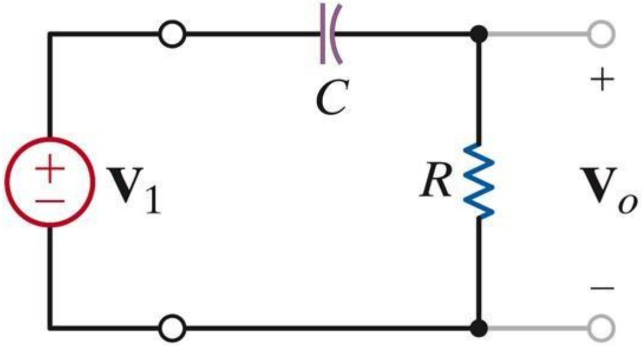


(c)

$$BW = \frac{1}{\tau}$$

GEAUX

Simple high-pass filter



$$G_v = \frac{V_0}{V_1} = \frac{R}{R+\frac{1}{j\omega C}} = \frac{j\omega CR}{1+j\omega CR}$$

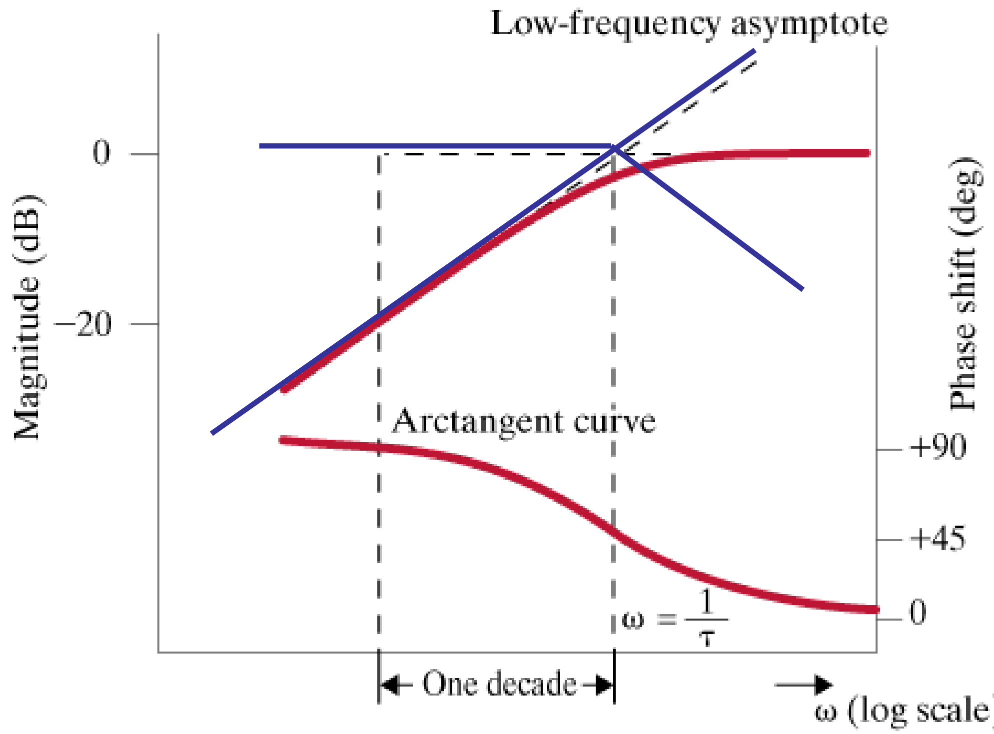
$$G_v = \frac{j\omega\tau}{1+j\omega\tau}; \ \tau = RC$$

$$M(\omega) = |G_v| = \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}$$

$$\angle G_v = \phi(\omega) = \frac{\pi}{2} - \tan^{-1}\omega\tau$$

$$M_{\max} = 1, \ M\left(\omega = \frac{1}{\tau}\right) = \frac{1}{\sqrt{2}}$$

$\omega = \frac{1}{\tau}$ =half power frequency

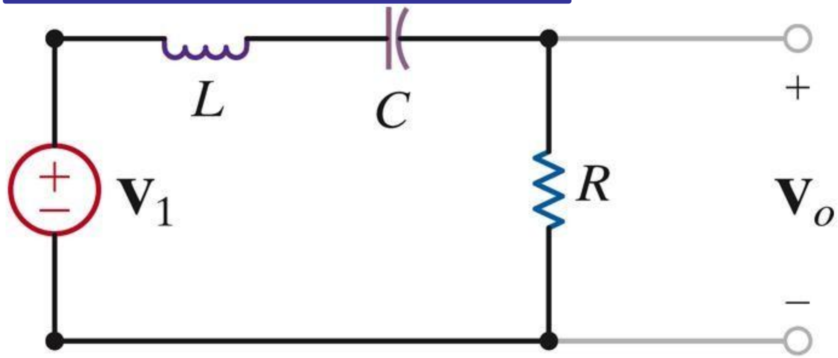


(c)

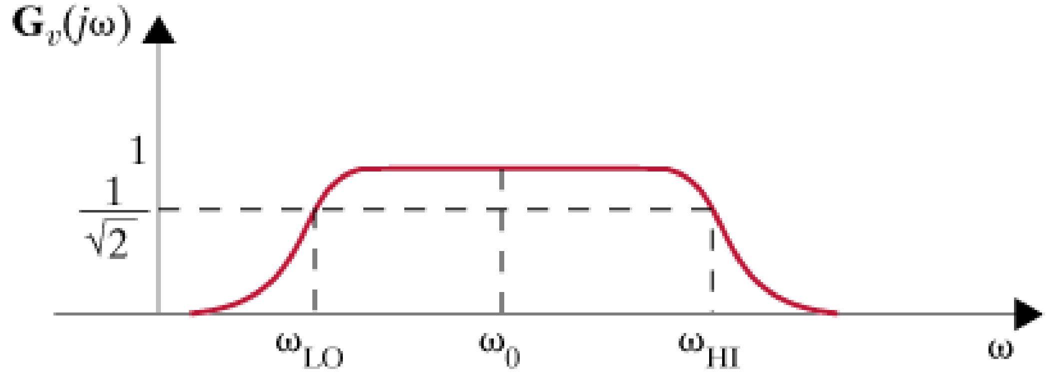
$$\omega_{LO} = \frac{1}{\tau}$$

GEAUX

Simple band-pass filter



Band-pass



(e)

$$G_v = \frac{V_0}{V_1} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$M(\omega) = \frac{\omega RC}{\sqrt{(\omega RC)^2 + (\omega^2 LC - 1)^2}}$$

$$M\left(\omega = \frac{1}{\sqrt{LC}}\right) = 1 \qquad M(\omega = 0) = M(\omega = \infty) = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$M(\omega_{LO}) = \frac{1}{\sqrt{2}} = M(\omega_{HI})$$

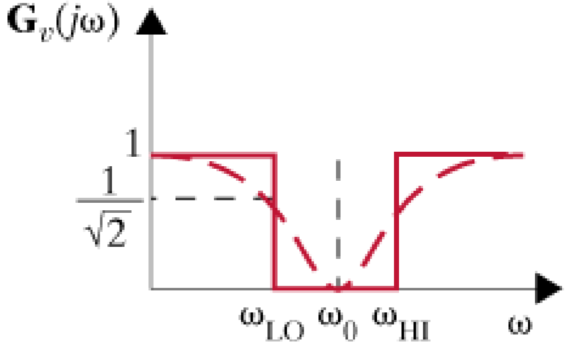
$$\omega_{LO} = \frac{-(R/L) + \sqrt{(R/L)^2 + 4\omega_0^2}}{2}$$

$$\omega_{HI} = \frac{(R/L) + \sqrt{(R/L)^2 + 4\omega_0^2}}{2}$$

$$BW = \omega_{HI} - \omega_{LO} = \frac{R}{L}$$

GEAUX

Simple band-reject filter

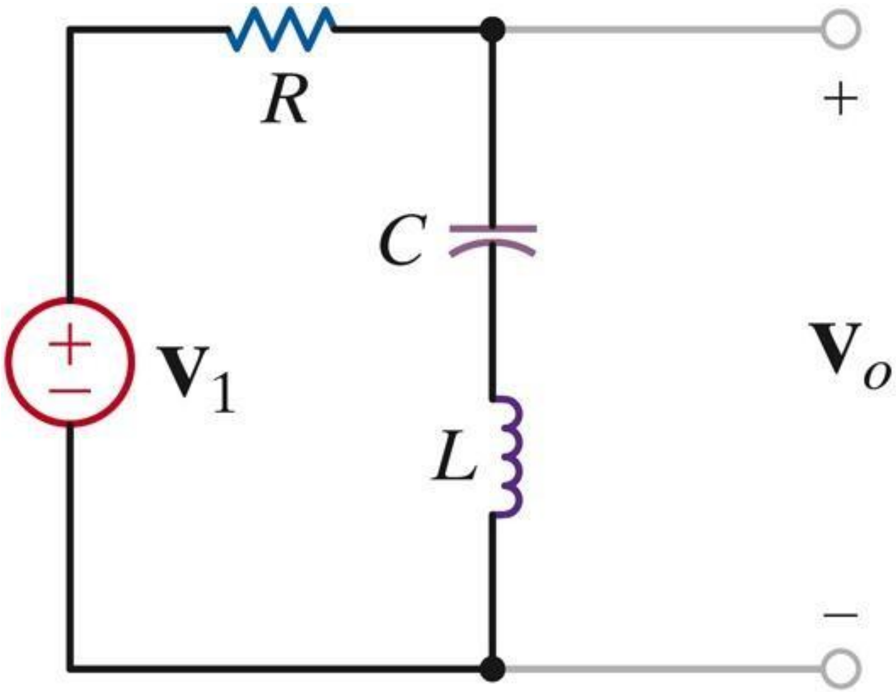


(b)

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow j\left(\omega_0 L - \frac{1}{\omega_0 C}\right) = 0$$

at $\omega = 0$ the capacitor acts as open circuit $\Rightarrow V_0 = V_1$

at $\omega = \infty$ the inductor acts as open circuit $\Rightarrow V_0 = V_1$

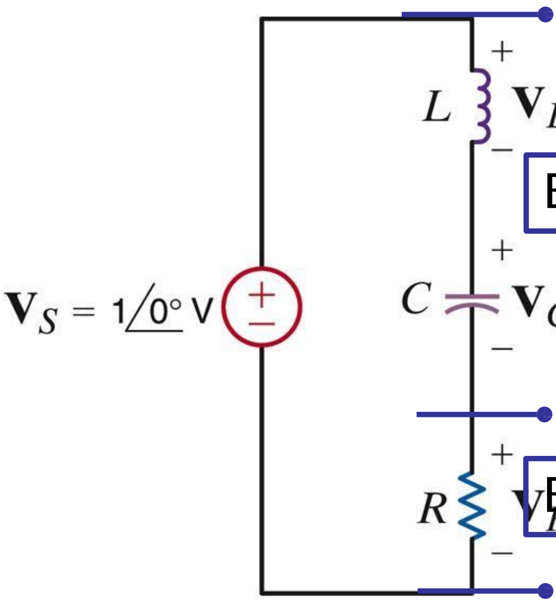


ω_{LO}, ω_{HI} are determined as in the band-pass filter

GEAUX

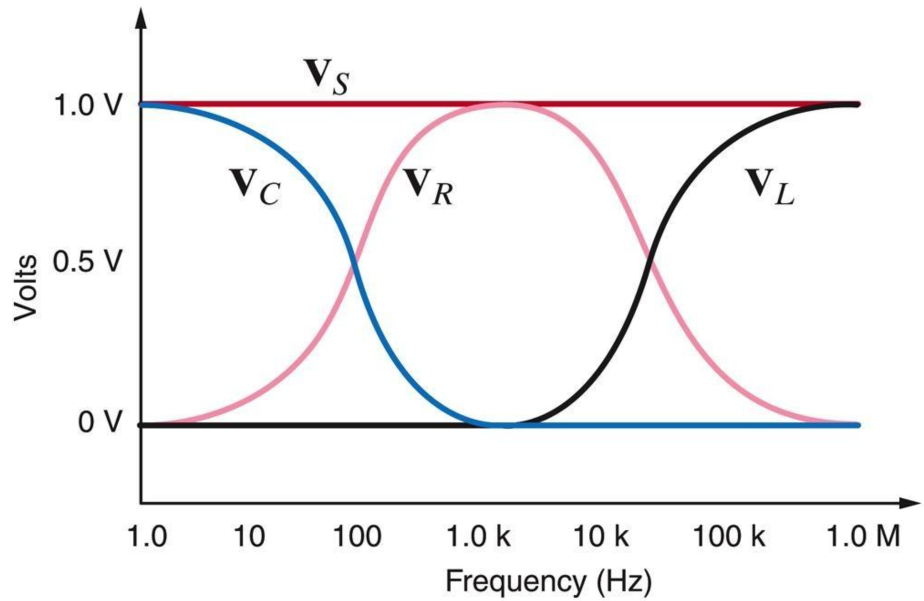
LEARNING EXAMPLE

Depending on where the output is taken, this circuit can produce low-pass, high-pass or band-pass or band-reject filters



Band-reject filter

Band-pass



Bode plot for $R = 10\Omega,\ L = 159\mu H,\ C = 159\mu F$

$$\frac{V_L}{V_S} = \frac{j\omega L}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\frac{V_L}{V_S}(\omega = 0) = 0, \quad \frac{V_L}{V_S}(\omega = \infty) = 1$$

High-pass

$$\frac{V_C}{V_S} = \frac{\frac{1}{j\omega C}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\frac{V_C}{V_S}(\omega = 0) = 1, \quad \frac{V_C}{V_S}(\omega = \infty) = 0$$

Low-pass

GEAUX

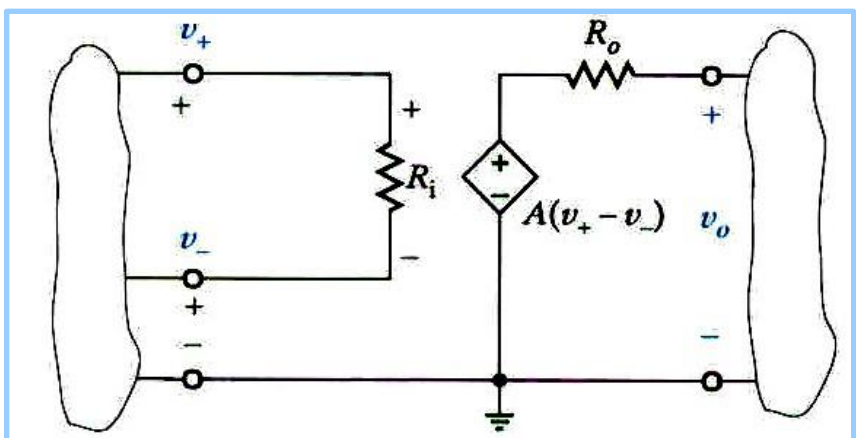
ACTIVE FILTERS

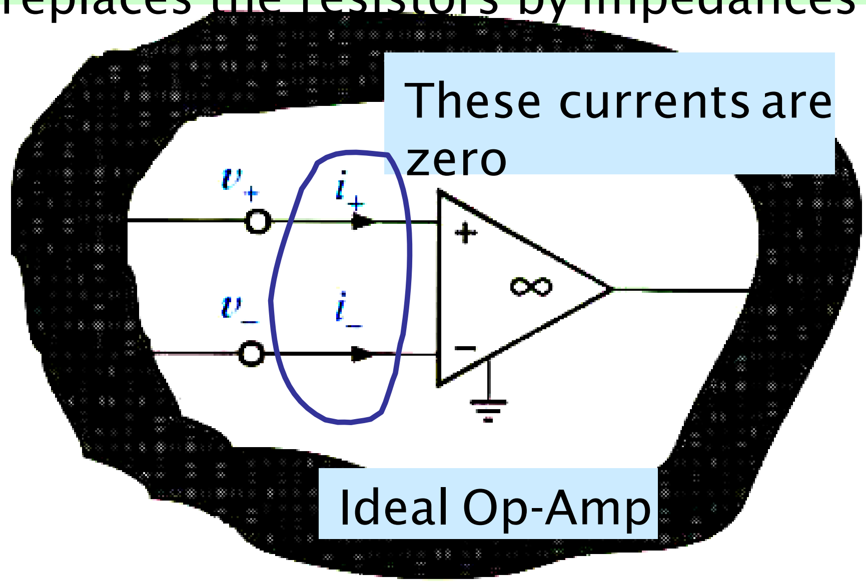
Passive filters have several limitations

1. Cannot generate gains greater than one
2. Loading effect makes them difficult to interconnect
3. Use of inductance makes them difficult to handle

Using operational amplifiers one can design all basic filters, and more, with only resistors and capacitors

The linear models developed for operational amplifiers circuits are valid, in a more general framework, if one replaces the resistors by impedances





DC TRANSIENT ANALYSIS

SUB - TOPICS

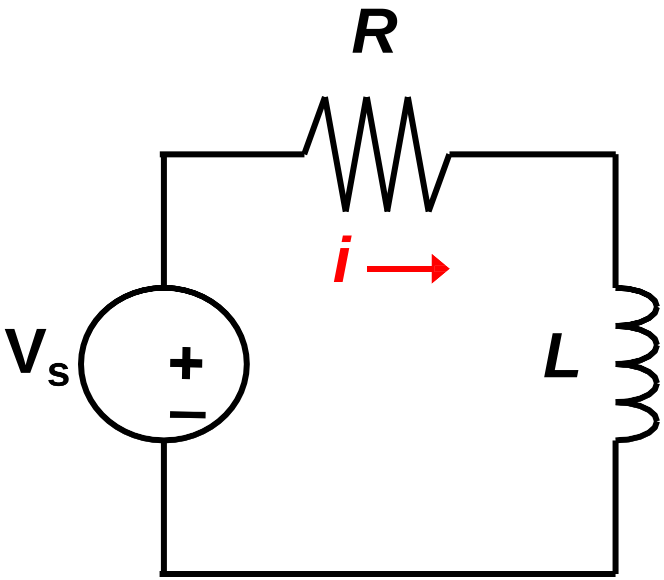
- NATURAL RESPONSE OF RL CIRCUIT
- NATURAL RESPONSE OF RC CIRCUIT
- STEP RESPONSE OF RL CIRCUIT
- STEP RESPONSE OF RC CIRCUIT

OBJECTIVES

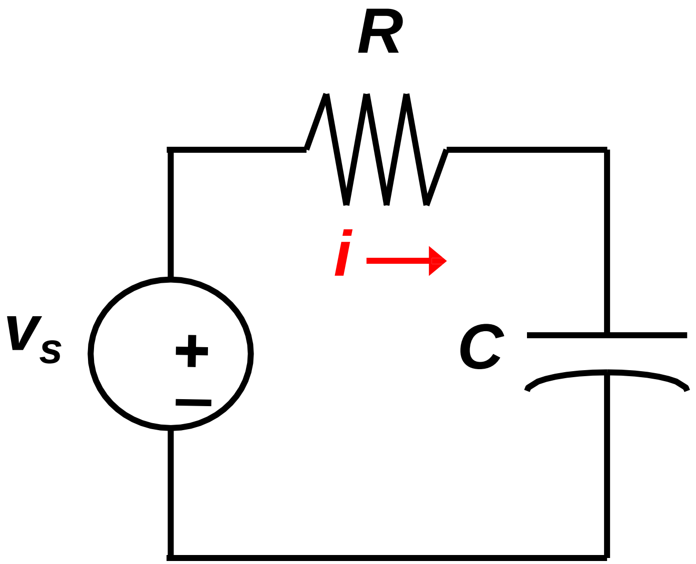
- To investigate the behavior of currents and voltages when energy is either released or acquired by inductors and capacitors when there is an abrupt change in dc current or voltage source.
- To do an analysis of natural response and step response of RL and RC circuit.

FIRST – ORDER CIRCUITS

- A circuit that contains only sources, resistor and inductor is called and RL circuit.
- A circuit that contains only sources, resistor and capacitor is called an RC circuit.
- RL and RC circuits are called first –order circuits because their voltages and currents are describe by first order differential equations.



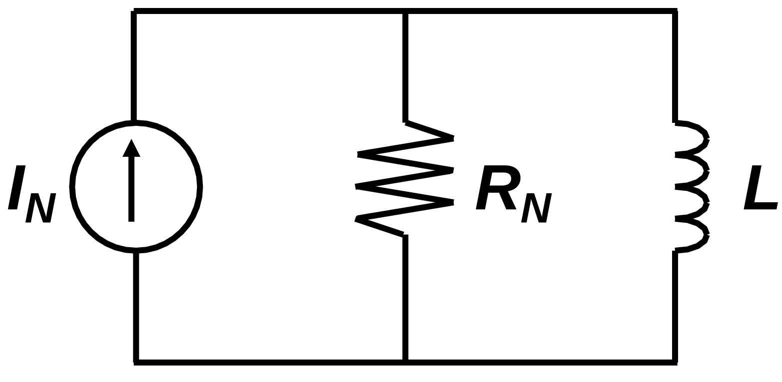
An RL circuit

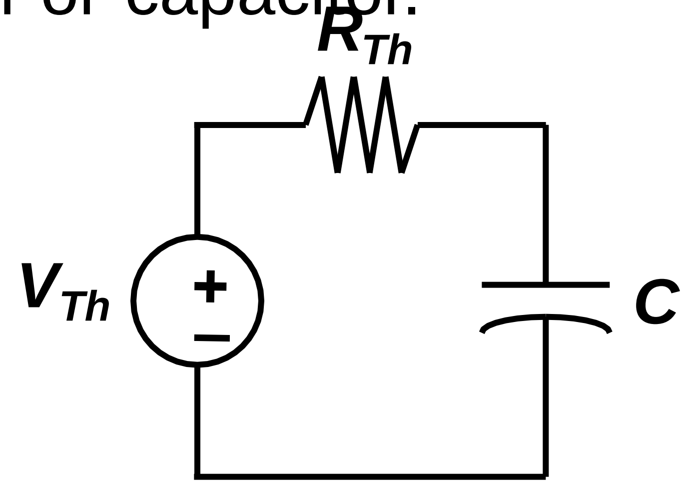


An RC circuit

Review (conceptual)

- Any first – order circuit can be reduced to a Thévenin (or Norton) equivalent connected to either a single equivalent inductor or capacitor.



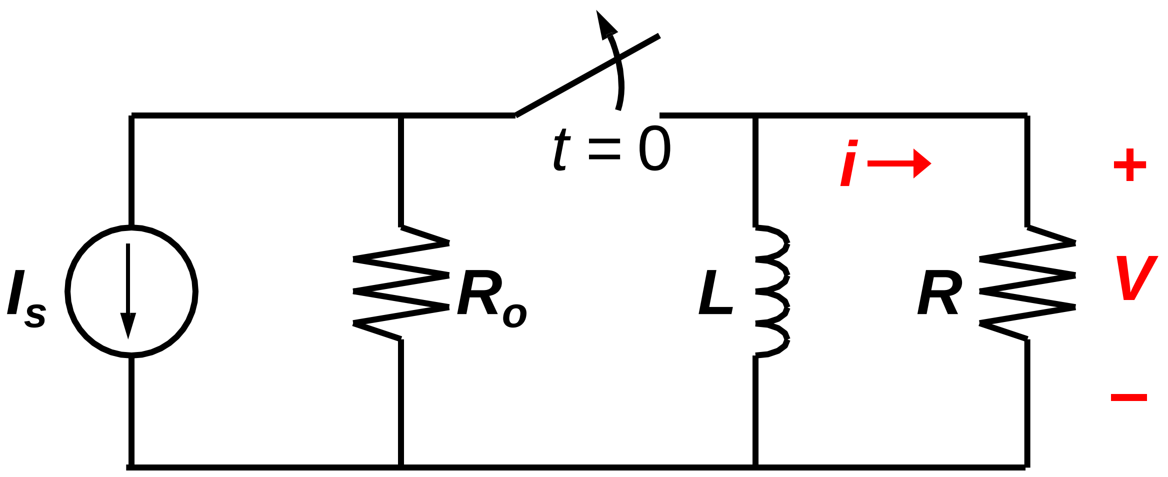


- In steady state, an inductor behave like a short circuit.
- In steady state, a capacitor behaves like an open circuit.

- The natural response of an RL and RC circuit is its behavior (i.e., current and voltage) when stored energy in the inductor or capacitor is released to the resistive part of the network (containing no independent sources)
- The steps response of an RL and RC circuits is its behavior when a voltage or current source step is applied to the circuit, or immediately after a switch state is changed.

NATURAL RESPONSE OF AN RL CIRCUIT

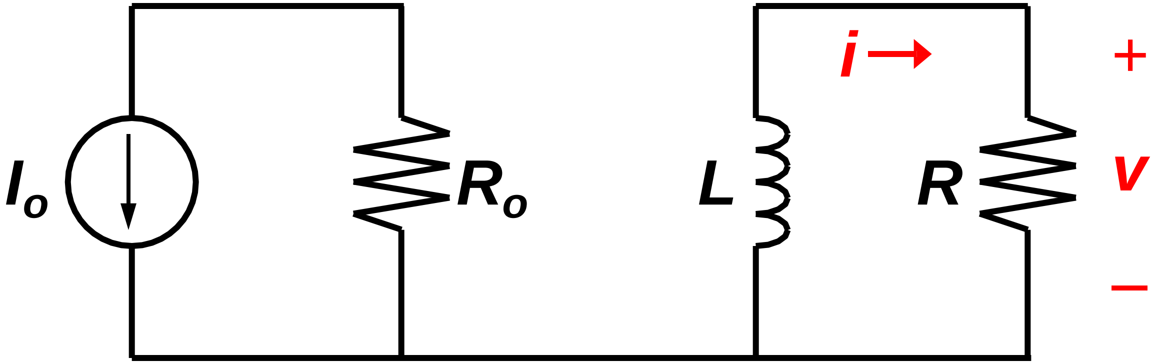
- Consider the following circuit, for which the switch is closed for *t*<*0,* and then opened at *t* = *0*:



- The dc voltage V, has been supplying the RL circuit with constant current for a long time

Solving for the circuit

- For $t \leq 0$, $i(t) = I_o$
- For $t \geq 0$, the circuit reduce to



Notation:

- 0^- is used to denote the time just prior to switching.
- 0^+ is used to denote the time immediately after switching.

Continue…

- Applying KVL to the circuit:

$$v(t) + Ri(t) = 0 \qquad \text{(1)}$$

$$L\frac{di(t)}{dt} + Ri(t) = 0 \qquad \text{(2)}$$

$$L\frac{di(t)}{dt} = -Ri(t) \qquad \text{(3)}$$

$$\frac{di(t)}{i(t)} = -\frac{R}{L}dt \qquad \text{(4)}$$

Continue

- From equation (4), let say;

$$\frac{du}{u} = -\frac{R}{L}dv \qquad \text{———————} \quad \textbf{(5)}$$

- Integrate both sides of equation (5);

$$\int_{i(t_o)}^{i(t)} \frac{du}{u} = -\frac{R}{L}\int_{t_o}^{t} dv \qquad \text{———————} \quad \textbf{(6)}$$

- Where:

❖ *i(*t_o*)* is the current corresponding to time t_o

❖ *i(t)* ia the current corresponding to time *t*

Continue

- Therefore,

$$\ln\frac{i(t)}{i(0)} = -\frac{R}{L}t \qquad \text{——————} \qquad \textbf{(7)}$$

- hence, the currentis

$$\boxed{i(t) = i(0)e^{-(R/L)t} = I_0 e^{-(R/L)t}}$$

Continue

- From the Ohm's law, the voltage across the resistor R is:

$$v(t) = i(t)R = I_0 \,\mathrm{R}e^{-(R/L)t}$$

- And the power dissipated in the resistor is:

$$p = v_R i(t) = I_0^2 \,\mathrm{R}e^{-2(R/L)t}$$

Continue

- Energy absorb by the resistor is:

$$\boxed{w = \frac{1}{2} L I_0^2 \left(1 - e^{-2(R/L)t}\right)}$$

Time Constant, τ

- Time constant, τ determines the rate at which the current or voltage approaches zero.

- Time constant, $\boxed{\tau = \frac{L}{R}}$ (sec)

- The expressions for current, voltage, power and energy using time constant concept:

$$i(t) = I_0 e^{-t/\tau}$$

$$v(t) = I_0 \,\mathrm{R} e^{-t/\tau}$$

$$p = I_0^2 \,\mathrm{R} e^{-2t/\tau}$$

$$w = \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau})$$

Switching time

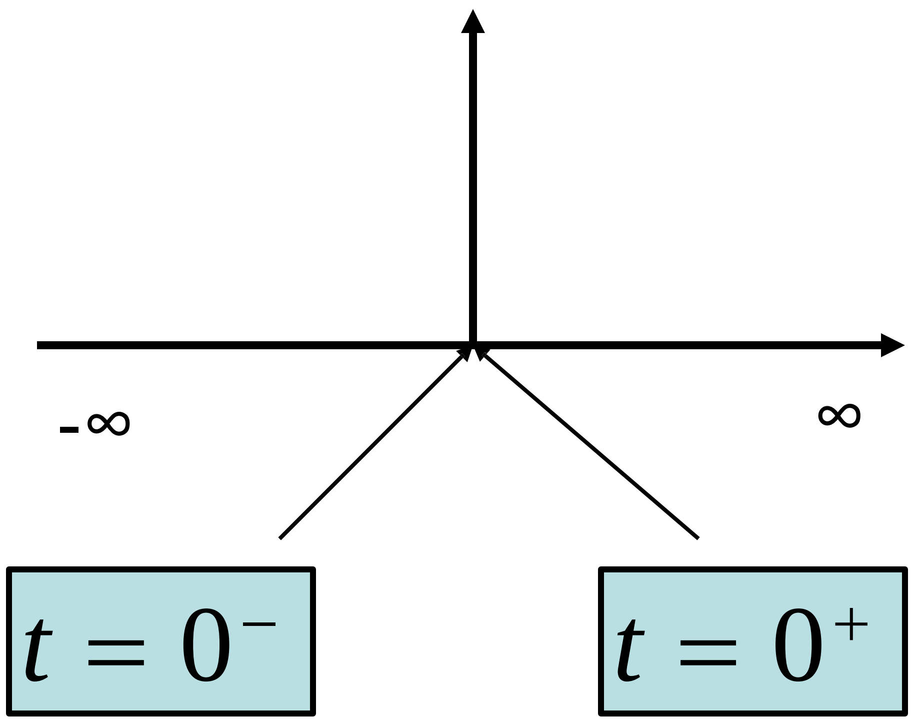
- For all transient cases, the following instants of switching times are considered.

✓ $t = 0^-$, this is the time of switching between $-\infty$ to 0 or time before.

✓ $t = 0^+$, this is the time of switching at the instant just after time t = 0s (taken as initial value)

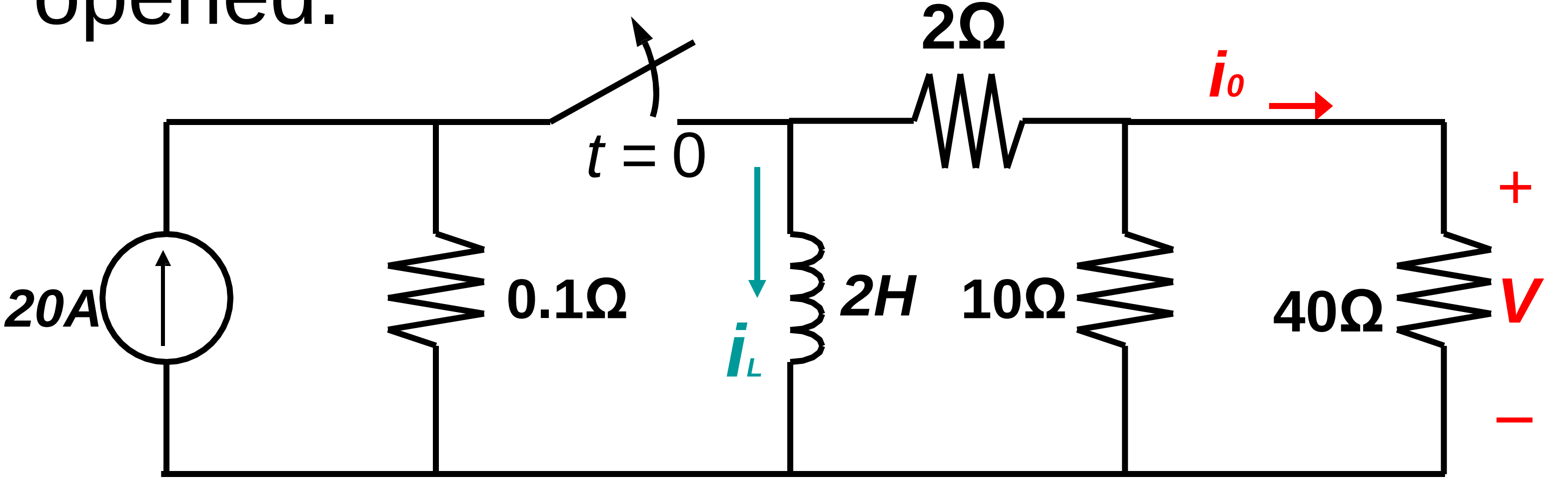
✓ $t = \infty$, this is the time of switching between $t = 0^+$ to ∞ (taken as final value for step response)

- The illustration of the different instance of switching times is:



Example

- For the circuit below, find the expression of $i_o(t)$ and $V_o(t)$. The switch was closed for a long time, and at $t = 0$, the switch was opened.



Solution :

Step 1:

Find τ for *t* > *0*. Draw the equivalent circuit. The switch is opened.

$$\boxed{R_T = (2 + 10 \,//\, 40) = 10\Omega}$$

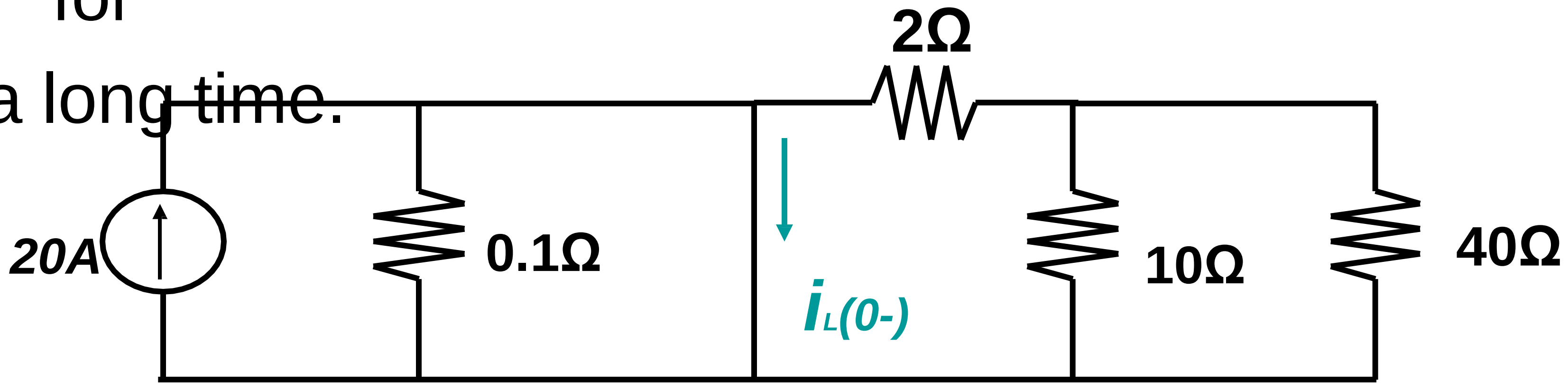
So;

$$\boxed{\tau = \frac{L}{R_T} = \frac{2}{10} = 0.2} \quad \text{sec}$$

Step 2:

At $t = 0^-$, time from $-\infty$ to 0^-, the switch was closed for

a long time.



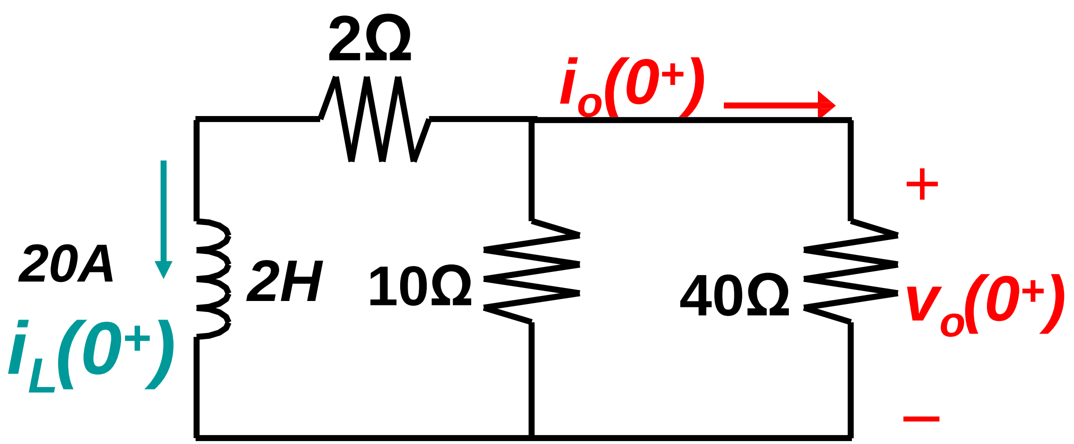
The inductor behave like a short circuit as it being

supplied for a long time by a dc current source. Current

20A thus flows through the short circuit until the switch

Step 3:

At the instant when the switch is opened, the time t = 0^+,



The current through the inductor remains the same (continuous).

Thus, $$i_L(0^+) = i_L(0^-) = 20A$$ which is the initial current.

Only at this particular instant the value of the current through the

inductor is the same.

Since, there is no other supply in the circuit after the switch is

By using current division, the current in the 40Ω resistor is:

$$i_o = -i_L \frac{10}{10+40} = -4A$$

So, $$\boxed{i_o(t) = -4e^{-5t}\,A}$$

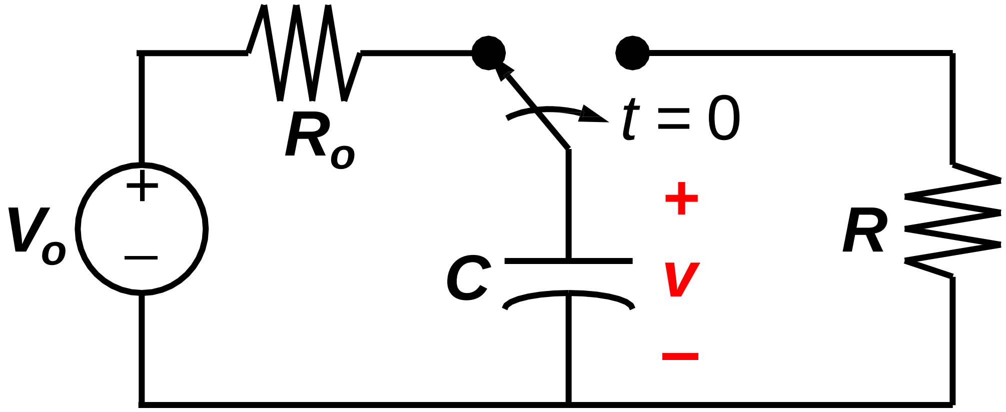
Using Ohm's Law, the V_o is:

$$V_o(t) = -4\times 40 = -160$$

So, $$\boxed{V_0(t) = -160e^{-5t}}$$

NATURAL RESPONSE OF AN RC CIRCUIT

- Consider the following circuit, for which the switch is closed for $t < 0$, and then opened at $t = 0$:

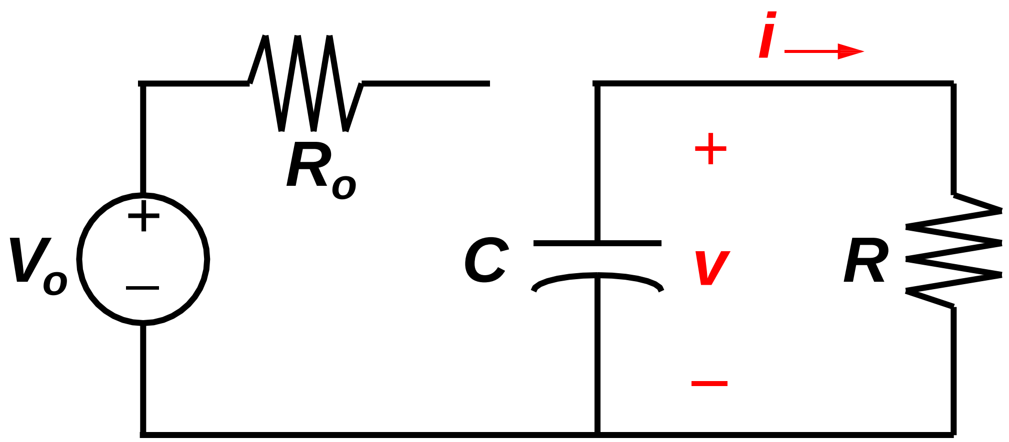


Notation:

- 0^- is used to denote the time just prior to switching
- 0^+ is used to denote the time immediately after switching.

Solving for the voltage *($t \geq 0$)*

- For $t \leq 0$, $v(t) = V_o$
- For t > 0, the circuit reduces to



Continue

- Applying KCL to the RC circuit:

$$i_C + i_R = 0 \longrightarrow \textbf{(1)}$$

$$C\frac{dv(t)}{dt} + \frac{v(t)}{R} = 0 \longrightarrow \textbf{(2)}$$

$$\frac{dv(t)}{dt} + \frac{v(t)}{RC} = 0 \longrightarrow \textbf{(3)}$$

$$\frac{dv(t)}{dt} = -\frac{v(t)}{RC} \longrightarrow \textbf{(4)}$$

$$\frac{dv(t)}{v(t)} = -\frac{1}{RC}dt \longrightarrow \textbf{(5)}$$

Continue

- From equation (5), let say:

$$\frac{dx}{x} = -\frac{1}{RC}dy \longrightarrow \textbf{(6)}$$

- Integrate both sides of equation (6):

$$\int_{V_o}^{v(t)} \frac{1}{x}du = -\frac{1}{RC}\int_0^t dy \longrightarrow \textbf{(7)}$$

- Therefore:

$$\ln \frac{v(t)}{V_o} = -\frac{t}{RC} \longrightarrow \textbf{(8)}$$

Continue

- Hence, the voltage is:

$$\boxed{v(t) = v(0)e^{-t/RC} = V_o e^{-t/RC}}$$

- Using Ohm's law, the current is:

$$\boxed{i(t) = \frac{v(t)}{R} = \frac{V_o}{R} e^{-t/RC}}$$

Continue

- The power dissipated in the resistor is:

$$\boxed{p(t) = v i_R = \frac{V_o^2}{R} e^{-2t/RC}}$$

- The energy absorb by the resistor is:

$$\boxed{w = \frac{1}{2} C V_o^2 (1 - e^{-2t/RC})}$$

Continue

- The time constant for the RC circuit equal the product of the resistance and capacitance,

- Time constant, $\boxed{\tau = RC}$ sec

- The expressions for voltage, current, power and energy using time constant concept:

$$v(t) = V_o e^{-t/\tau}$$

$$i(t) = \frac{V_o}{R} e^{-t/\tau}$$

$$p(t) = \frac{V_o^2}{R} e^{-2t/\tau}$$

$$w(t) = \frac{1}{2} C V_o^2 (1 - e^{-2t/\tau})$$

- For the case of capacitor, two important observation can be made,

  1)capacitor behaves like an open circuit when being supplied by dc source

  (From, $i_c = Cdv/dt$, when v is constant, $dv/dt = 0$. When current in circuit is zero, the circuit is open circuit.)

  2) in capacitor, the voltage is continuous / stays the same that is, $V_c(0^+) = V_c(0^-)$

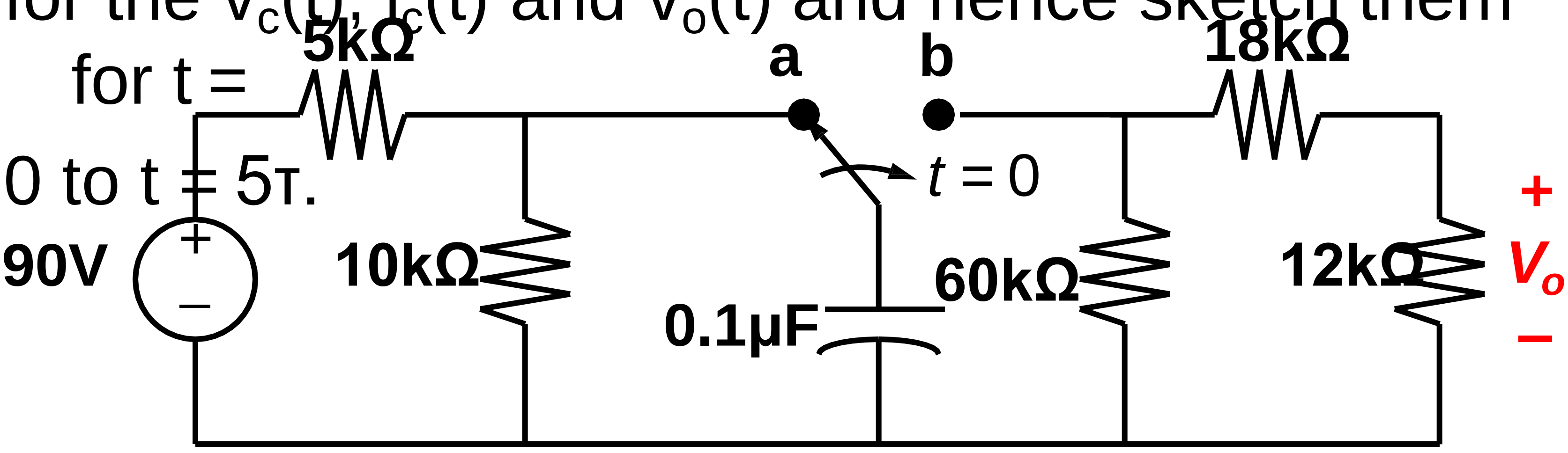
Example

The switch has been in position *a* for a long time. At

Time t = 0, the switch moves to *b*. Find the expressions

for the $v_c(t)$, $i_c(t)$ and $v_o(t)$ and hence sketch them for t =

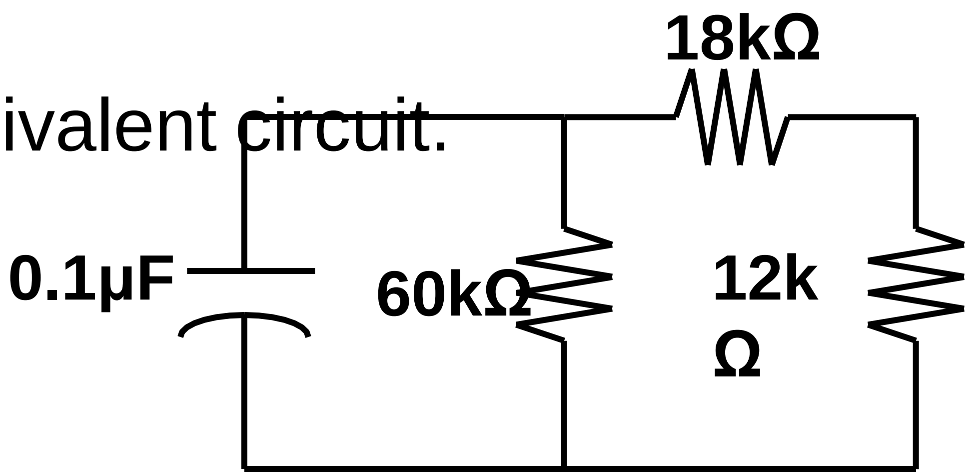
0 to t = 5τ.



Solution

Step 1:

Find t for t > 5τ that is when the switch was at *a*. Draw the equivalent circuit.



$$R_T = (18k\Omega + 12k\Omega) \,//\, 60k\Omega = 20k\Omega$$

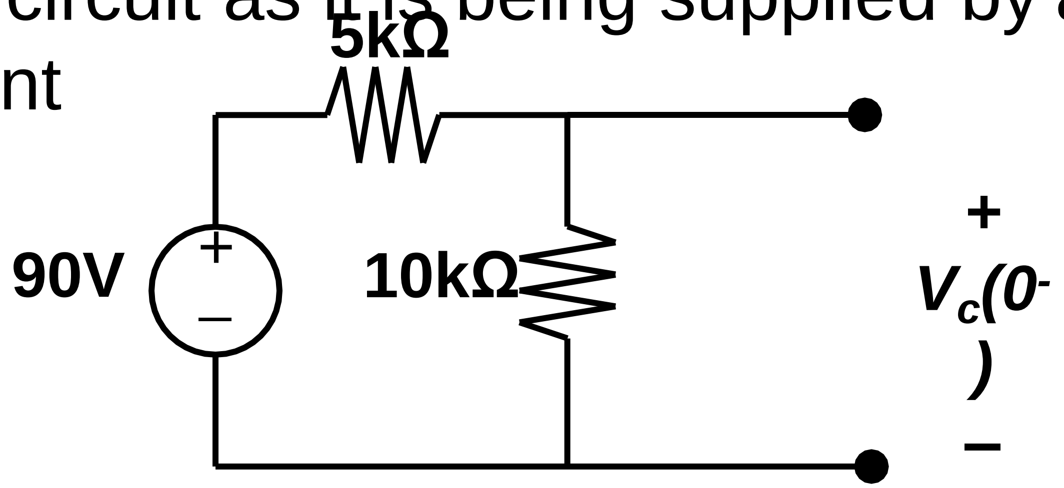
$$\tau = R_T C = 20\times10^{3}\times0.1\times10^{-6} = 2ms$$

Step 2:

At t = 0, the switch was at *a*. the capacitor behaves like

An open circuit as it is being supplied by a constant

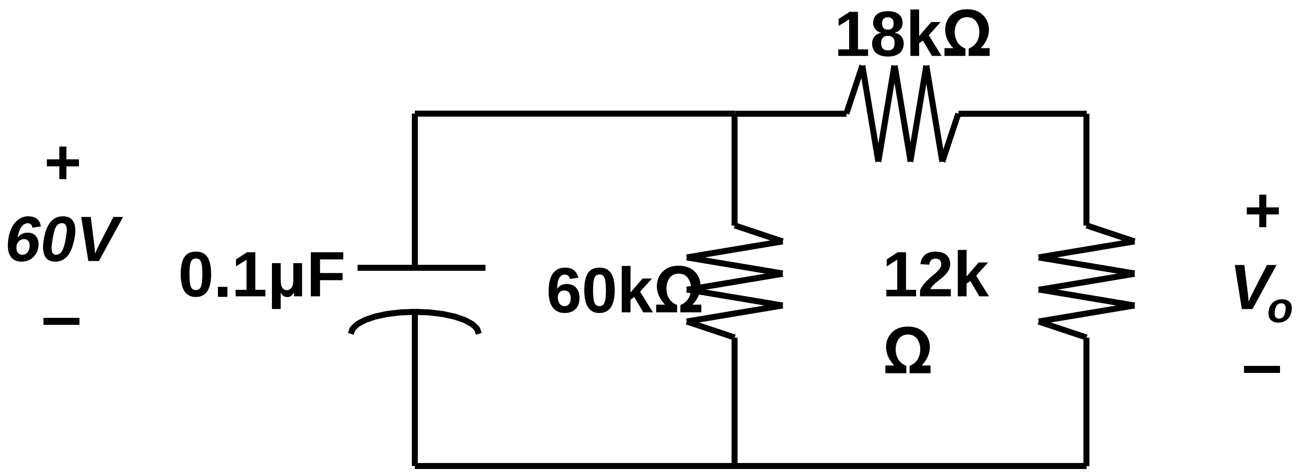
source.



$$v_c(0^-) = \frac{10}{15} \times 90 = 60V$$

Step 3:

At t = 0^+, the instant when the switch is at *b*.



The voltage across capacitor remains the same at this particular instant.

$$v_c(0^+) = v_c(0^-) = 60V$$

Using voltage divider rule,

$$V_o(0^+) = \frac{12}{30} \times 60 = 24V$$

Hence;

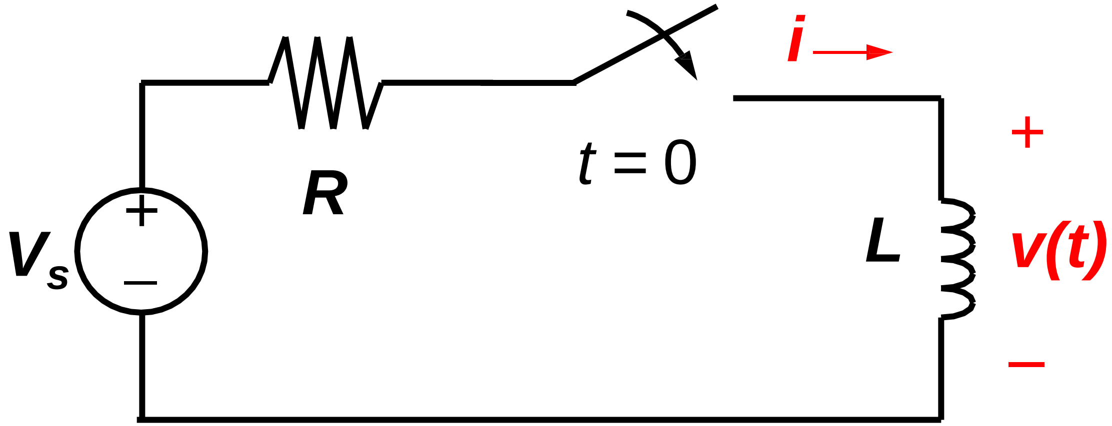
$$\boxed{\begin{aligned} v_c(t) &= 60e^{-500t}\,\mathrm{V} \\ v_o(t) &= 24e^{-500t}\,\mathrm{V} \\ i_c(t) &= -0.03e^{-500t}\,A \end{aligned}}$$

Summary

No	RL circuit	RC circuit
1	$\tau = \dfrac{L}{R}$	$\tau = RC$
2	Inductor behaves like a short circuit when being supplied by dc source for a long time	Capacitor behaves like an open circuit when being supplied by dc source for a long time
3	Inductor current is continuous $i_L(0^+) = i_L(0^-)$	Voltage across capacitor is continuous $v_C(0^+) = v_C(0^-)$

Step Response of RL Circuit

- The switch is closed at time t = 0.



- After switch is closed, using KVL

$$V_s = Ri(t) + L\frac{di}{dt} \longrightarrow \textbf{(1)}$$

Continue

- Rearrange the equation;

$$\frac{di(t)}{dt} = \frac{-Ri(t)+V_s}{L} = \frac{-R}{L}\left(i(t)-\frac{V_s}{R}\right) \longrightarrow \textbf{(2)}$$

$$di = \frac{-R}{L}\left(i-\frac{V_s}{R}\right)dt \longrightarrow \textbf{(3)}$$

$$-\frac{R}{L}dt = \frac{di}{i(t)-V_s/R} \longrightarrow \textbf{(4)}$$

$$-\frac{R}{L}\int_0^t dv = \int_0^{i(t)} \frac{du}{u-(V_s/R)} \longrightarrow \textbf{(5)}$$

Continue

- Therefore:

$$-\frac{R}{L}t = \ln\frac{i(t)-(V_s/R)}{I_0-(V_s/R)} \longrightarrow \textbf{(5)}$$

- Hence, the current is;

$$\boxed{i(t) = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-(R/L)t}}$$

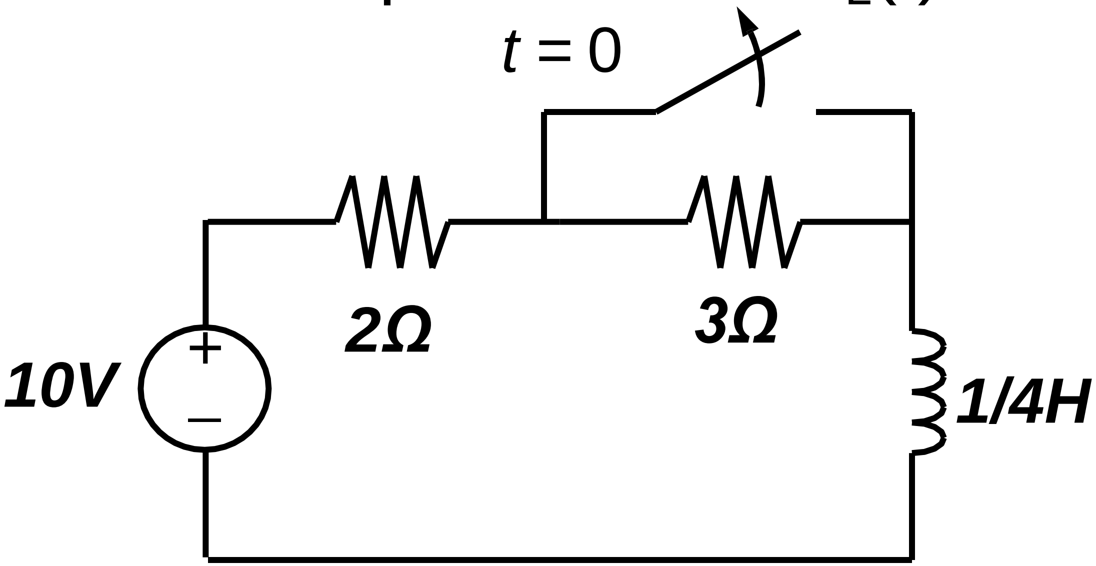
- The voltage;

$$\boxed{v(t) = (V_s - I_o R)e^{-(R/L)t}}$$

Example

The switch is closed for a long time at t = 0, the switch

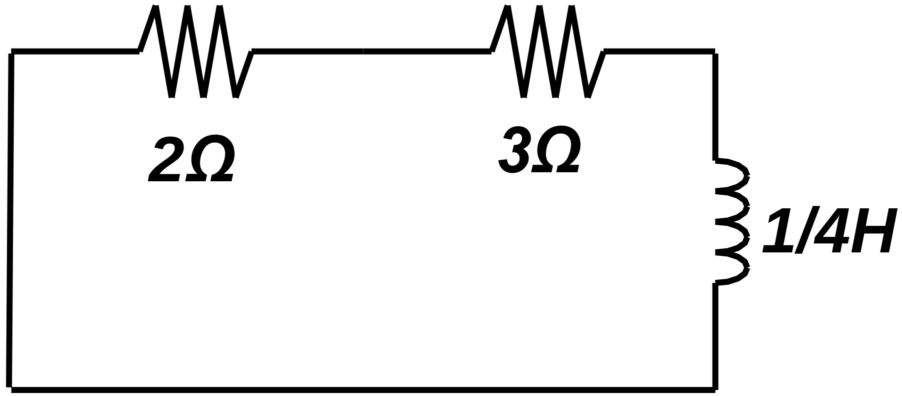
opens. Find the expressions for $i_L(t)$ and $v_L(t)$.



Solution

Step 1:

Find τ for t > 0. The switch was opened. Draw the equivalent circuit. Short circuit the voltage source.



$$R_T = (2+3)\Omega = 5\Omega$$

$$\tau = \frac{L}{R_T} = \frac{1}{20}s$$

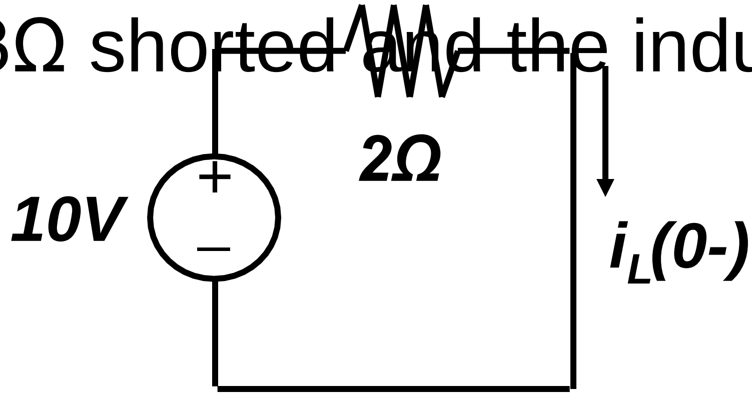
Continue

Step 2:

At t = 0^-, the switch was closed. Draw the equivalent

circuit with 3Ω shorted and the inductor behaves like a

short circuit.



$$i_L(0^-) = 10/2 = 5A$$

Continue

Step 3:

At t = 0^+, the instant switch was opened. The current in

inductor is continuous. $I_0 = i_L(0^+) = i_L(0^-) = 5A$

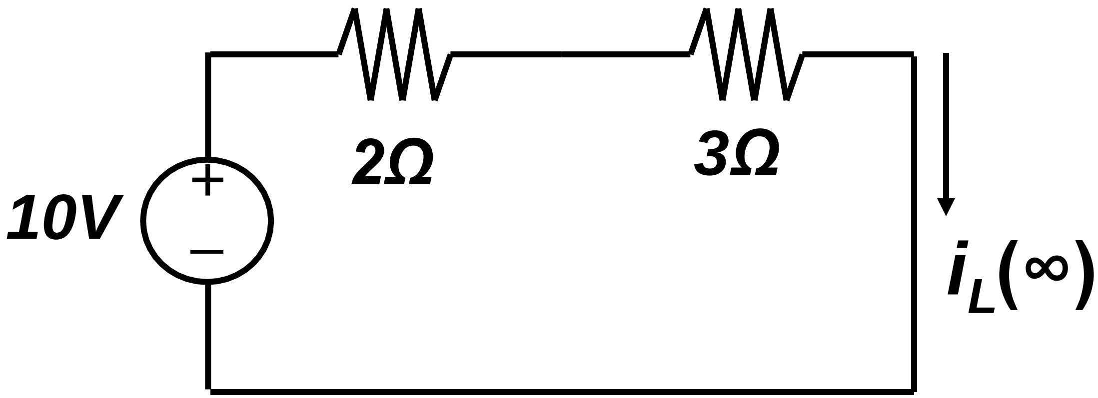
Step 4:

At t =∞, that is after a long time the switch has been left

opened. The inductor will once again be behaving like a

short circuit.

Continue



$$i_L(\infty) = V_s / R_T = 2A$$

Hence:

$$i_L(t) = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R} \right) e^{-(R/L)t}$$

$$\boxed{i_L(t) = 2 + 3e^{-20t}\,A}$$

Continue

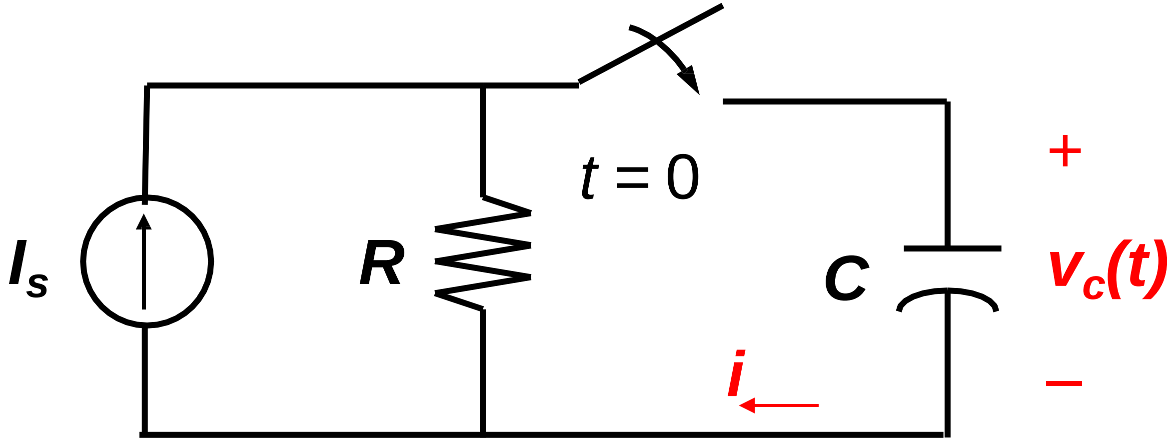
- And the voltage is:

$$v_L(t) = (V_s - I_o R)e^{-(R/L)t}$$

$$\boxed{v_L(t) = -15e^{-20t}V}$$

Step Response of RL Circuit

- The switch is closed at time t = 0



- From the circuit;

$$\boxed{I_s = C\frac{dv_c}{dt} + \frac{v_c}{R}} \longrightarrow \textbf{(1)}$$

Continue

- Division of Equation (1) by C gives;

$$\boxed{\frac{I_s}{C} = \frac{dv_c}{dt} + \frac{v_c}{RC}} \longrightarrow \textbf{(2)}$$

- Same mathematical techniques with RL, the voltage is:

$$\boxed{v_c(t) = I_s R + (V_o - I_s R)e^{-t/RC}}$$

- And the current is:

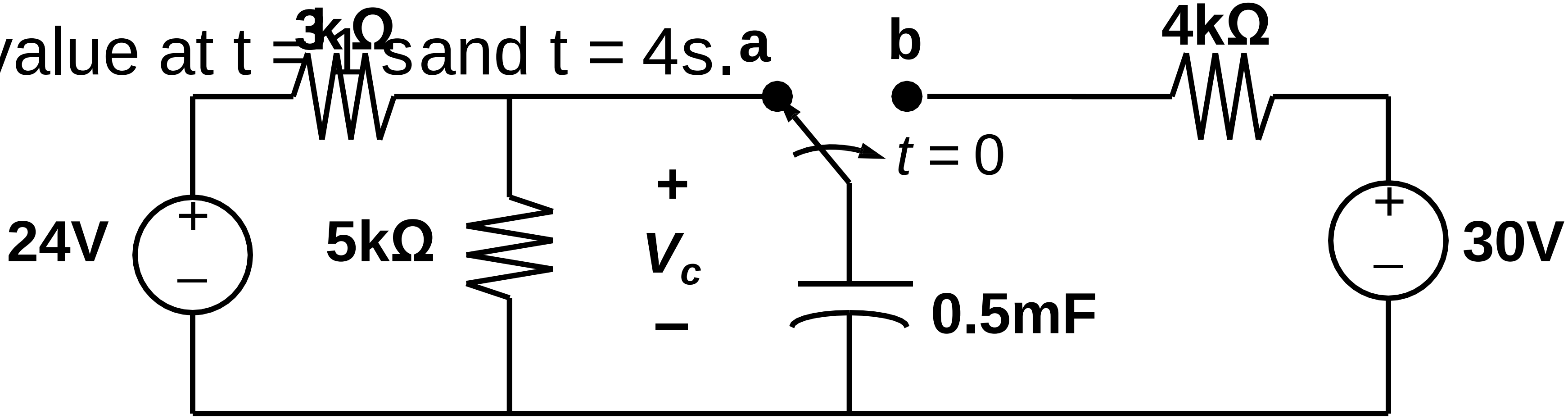
$$\boxed{i(t) = \left(I_s - \frac{V_o}{R} \right) e^{-t/RC}}$$

Example

The switch has been in position *a* for a long time. At t = 0,

the switch moves to *b*. Find $V_c(t)$ for t > 0 and calculate its

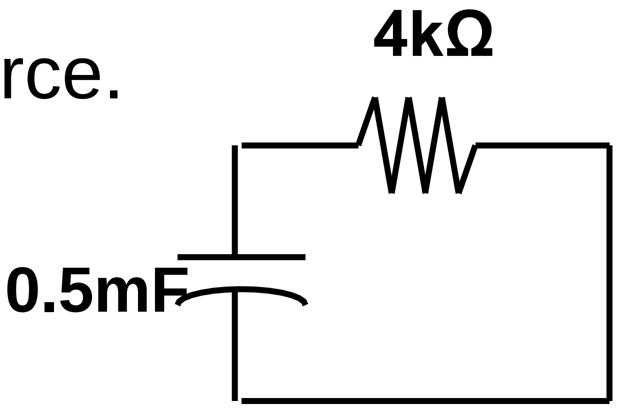
value at t = 1s and t = 4s.



Solution

Step 1:

To find τ for t > 0, the switch is at *b* and short circuit the voltage source.



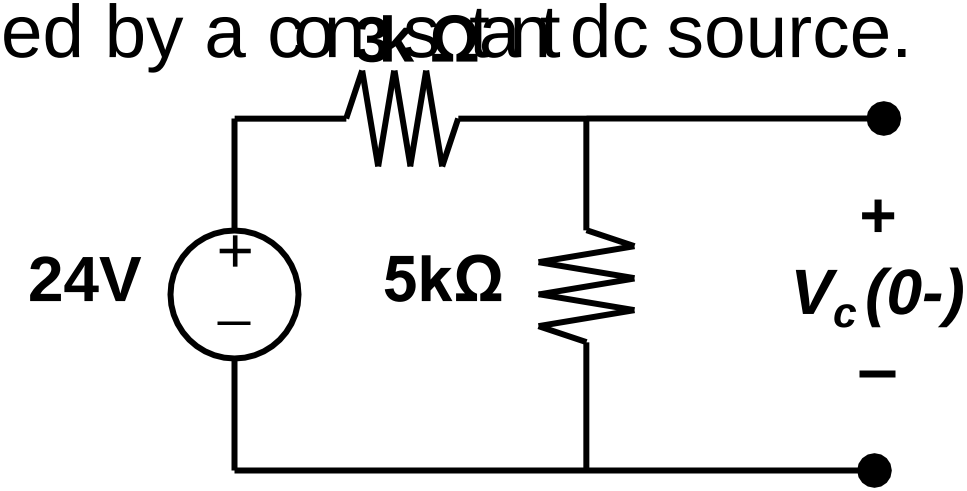
$$\tau = RC = 2s$$

Continue

Step 2:

The capacitor behaves like an open circuit as it is being

supplied by a constant dc source.



From the circuit, $V_c(0^-) = 24\times\frac{5}{8} = 15V$

Continue

Step 3:

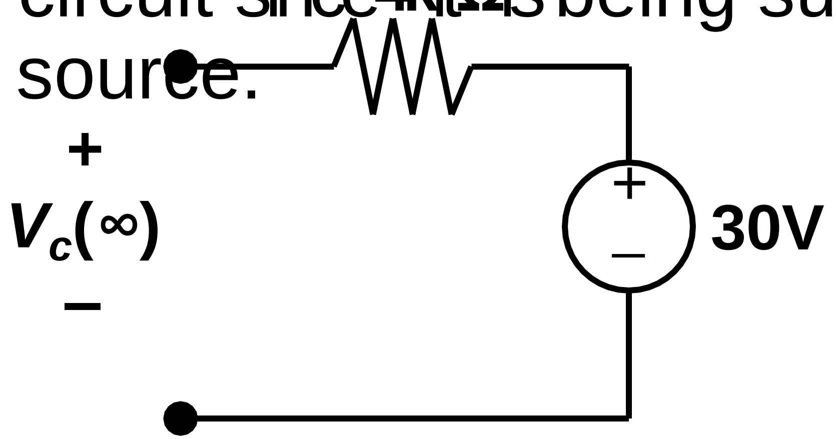
At t = 0^+, the instant when the switch is just moves to *b*.

Voltage across capacitor remains the same.

$$V_c(0^+) = V_c(0^-) = 15V$$

Step 4:

At t = ∞, the capacitor again behaves like an open circuit since it is being supplied by a constant source.



$$V_c(\infty) = 30V$$

Continue

Step 5:

Hence,

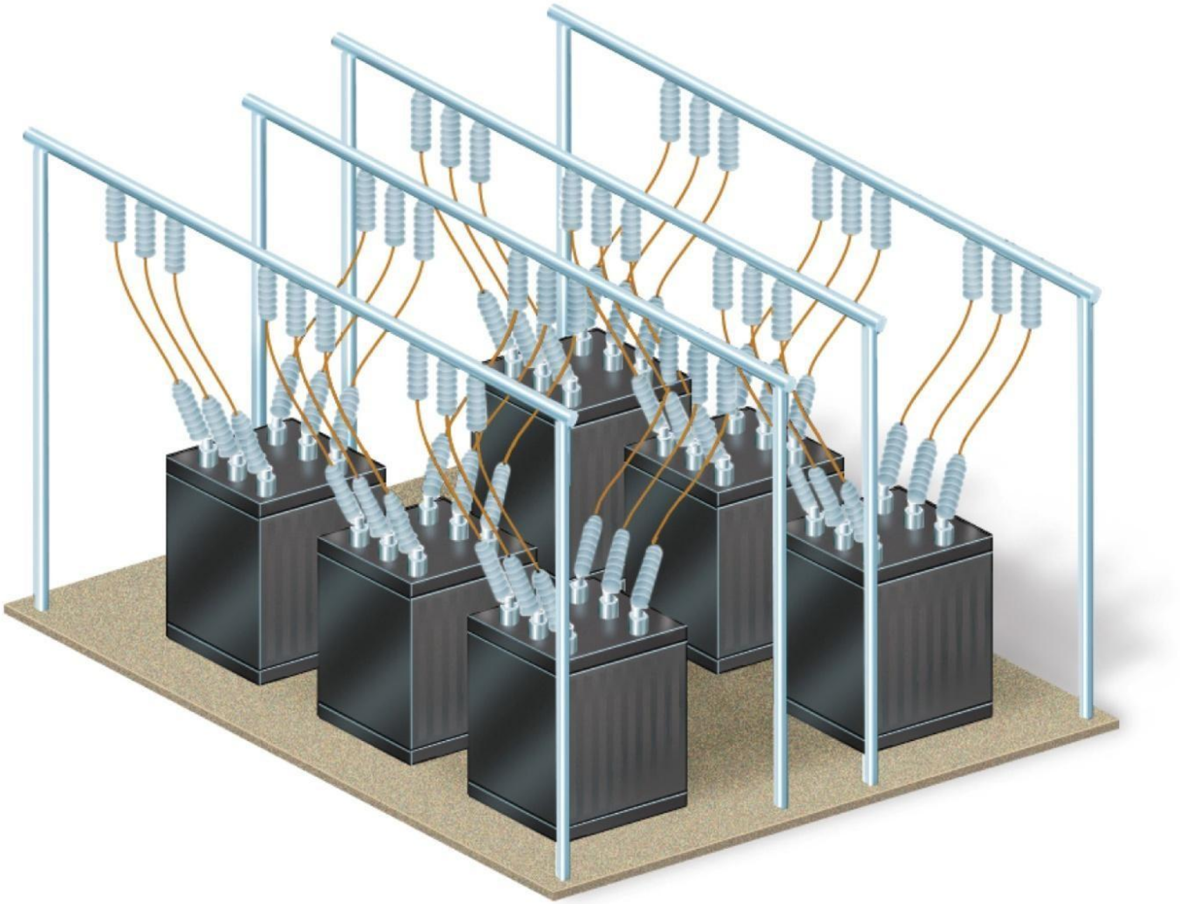
$$V_c(t) = 30 + (15 - 30)e^{-0.5t} = 30 - 15e^{-0.5t} V$$

At t = 1s, $V_c(t)$ = 20.9V

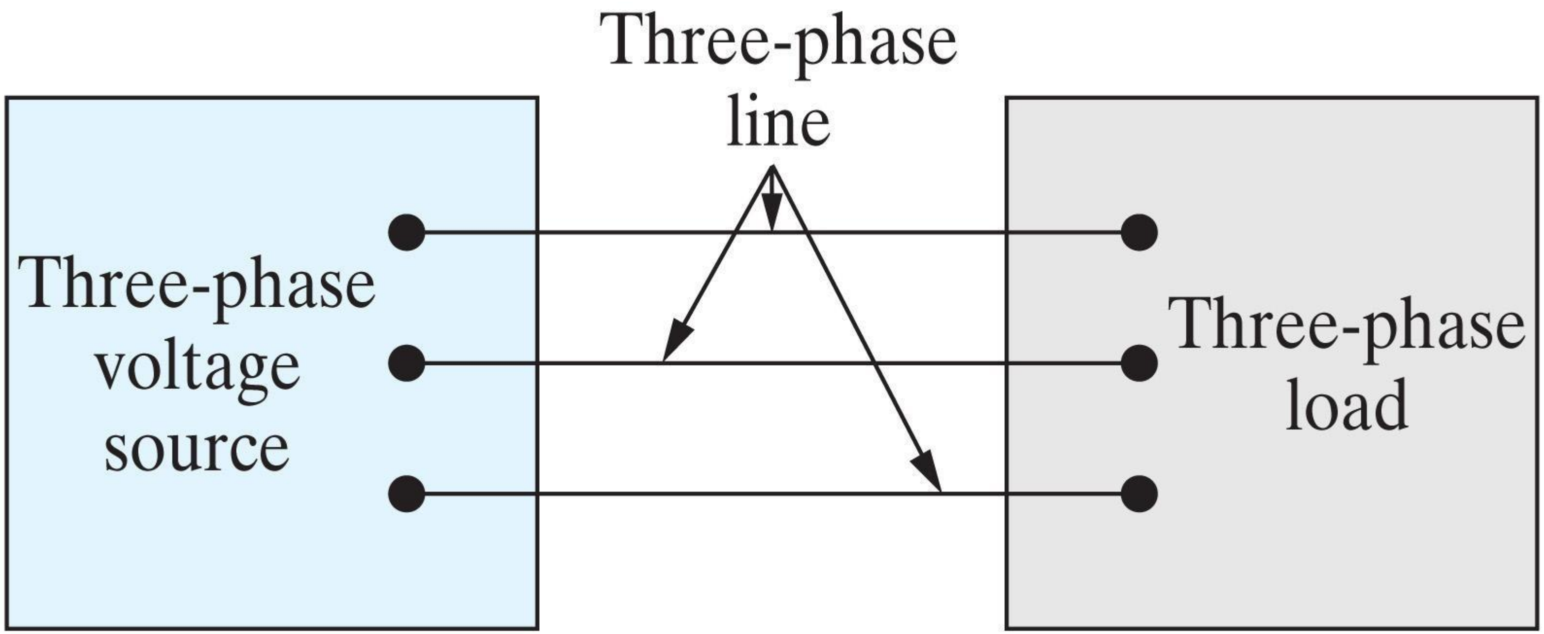
At t = 4s, $V_c(t)$ = 28 V

Introduction to Three-Phase Power

Typical Transformer Yard



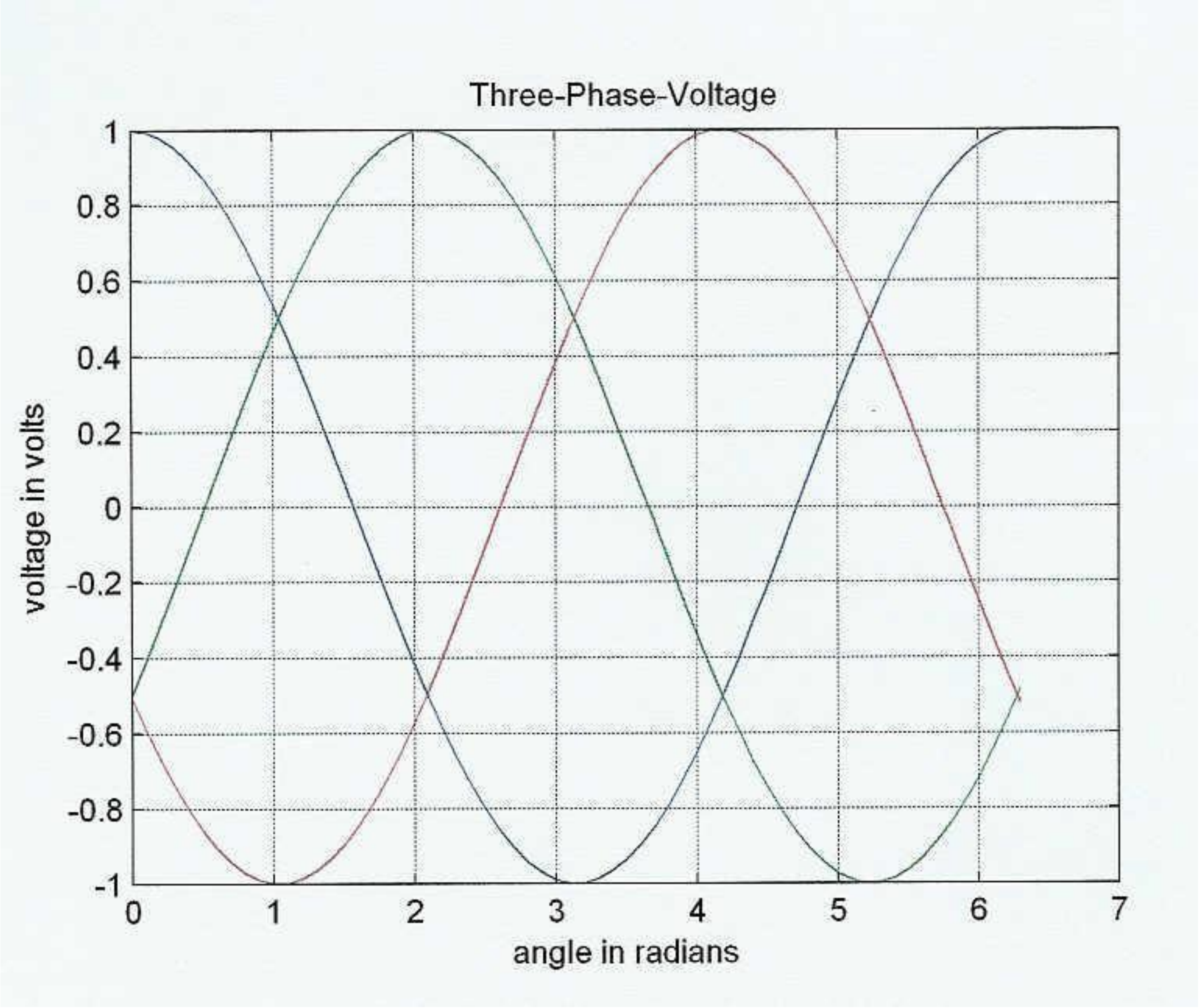
Basic Three-Phase Circuit



What is Three-Phase Power?

- Three sinusoidal voltages of equal amplitude and frequency out of phase with each other by 120°. Known as “balanced”.
- Phases are labeled A, B, and C.
- Phases are sequenced as A, B, C (positive) or A, C, B (negative).

Three-Phase Power

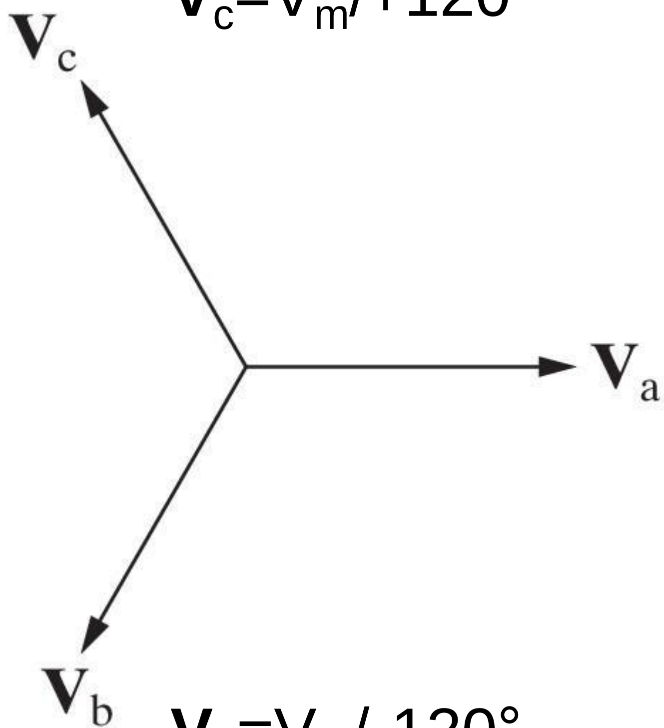


Definitions

- 4 wires
  - 3 “active” phases, A, B, C
  - 1 “ground”, or “neutral”
- Color Code
  - Phase A Red
  - Phase B Black
  - Phase C Blue
  - Neutral White or Gray

Phasor (Vector) Form for abc

$\mathbf{V}_c = V_m/+120°$

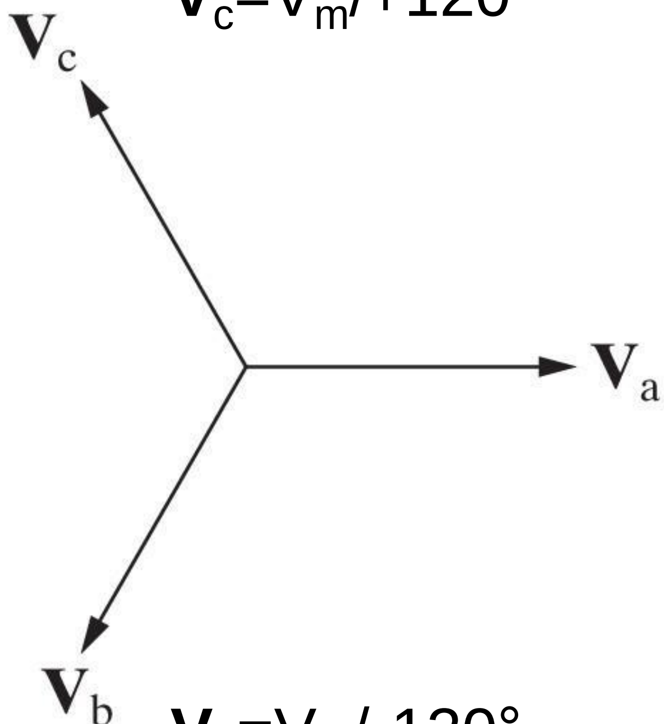


$\mathbf{V}_a = V_m/0°$

$\mathbf{V}_b = V_m/-120°$

Phasor (Vector) Form for abc

$\mathbf{V}_c = V_m / {+120°}$



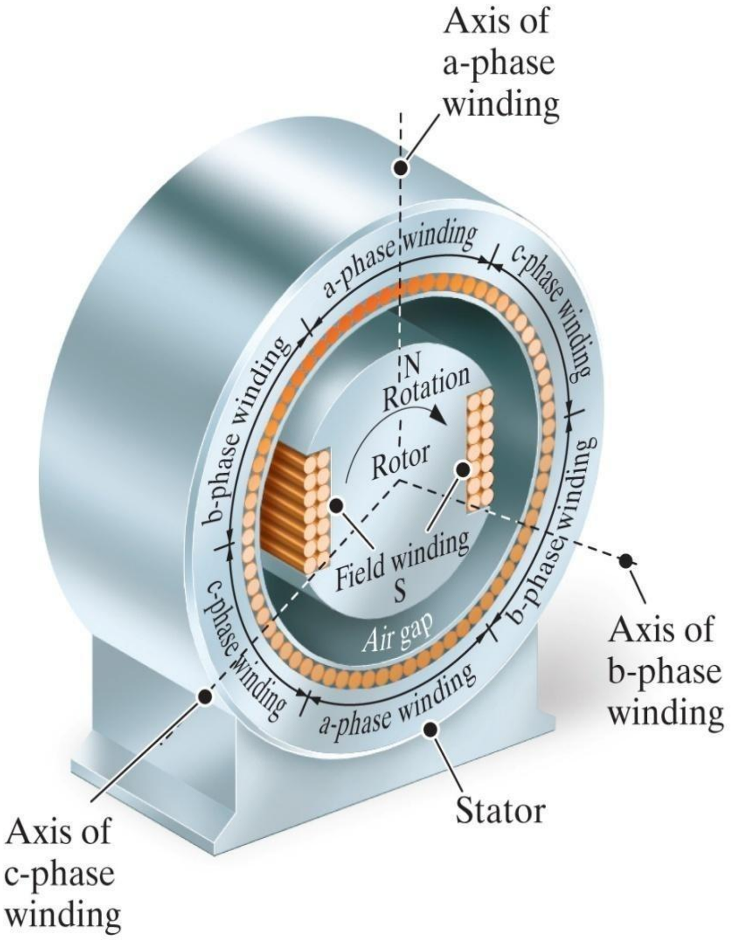
$\mathbf{V}_a = V_m / 0°$

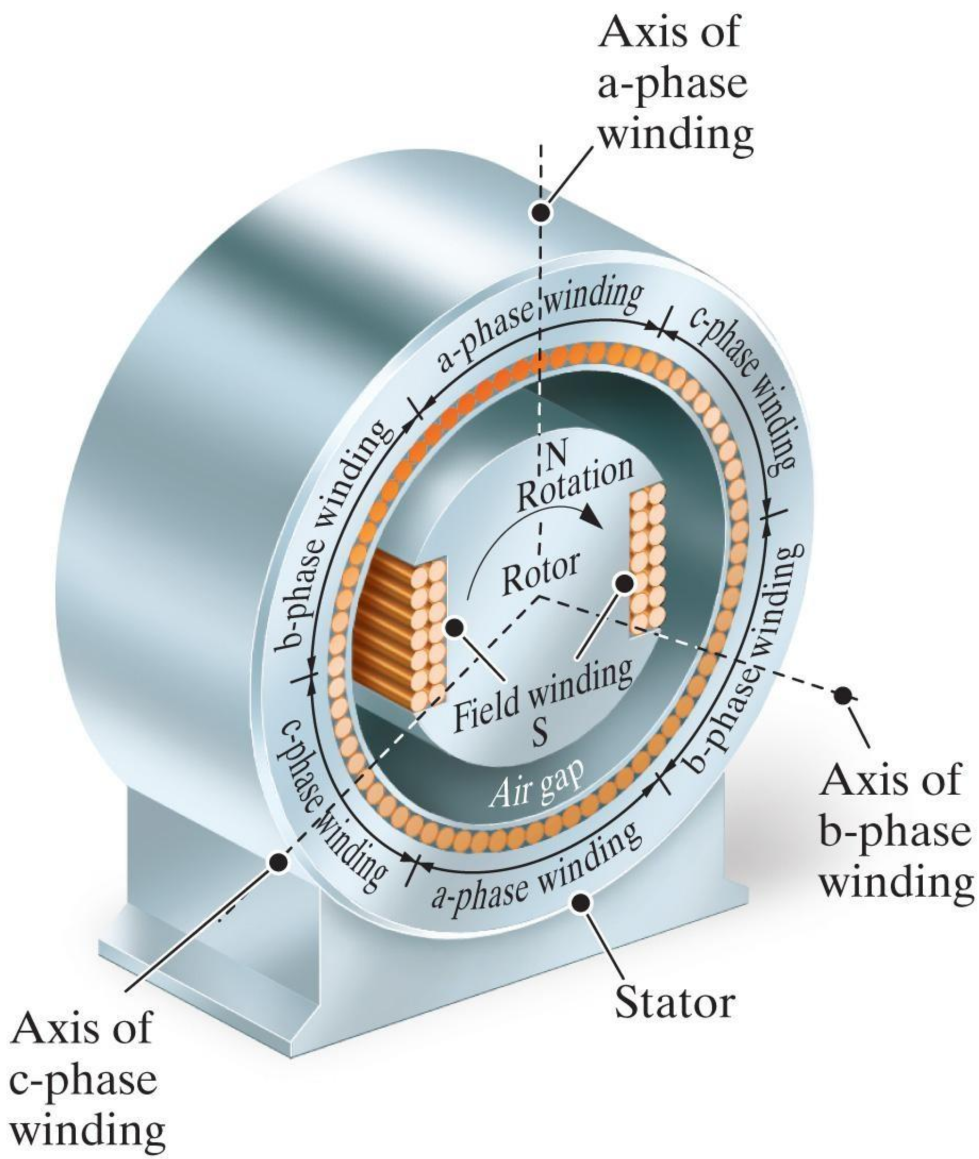
$\mathbf{V}_b = V_m / {-120°}$

Note that KVL applies .... $\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c = 0$

Three-Phase Generator

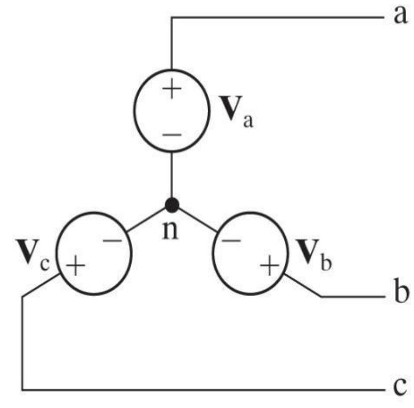
- 2-pole (North-South) rotor turned by a “prime mover”
- Sinusoidal voltages are induced in each stator winding



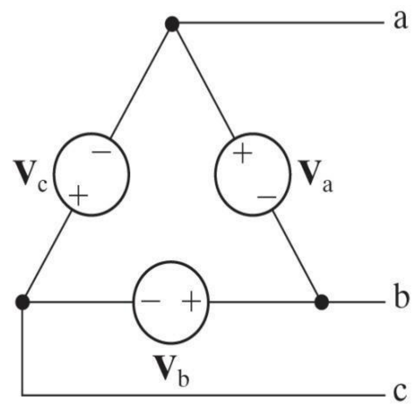

Axis of
a-phase
winding
a-phase winding
c-phase winding
b-phase winding
N
Rotation
Rotor
Field winding
S
Air gap
b-phase winding
c-phase winding
a-phase winding
Axis of
b-phase
winding
Stator
Axis of
c-phase
winding

How are the sources connected?

- (a) shows the sources (phases) connected in a wye (Y).
  - Notice the fourth terminal, known as Neutral.
- (b) shows the sources (phases) connected in a delta (Δ).
  - Three terminals

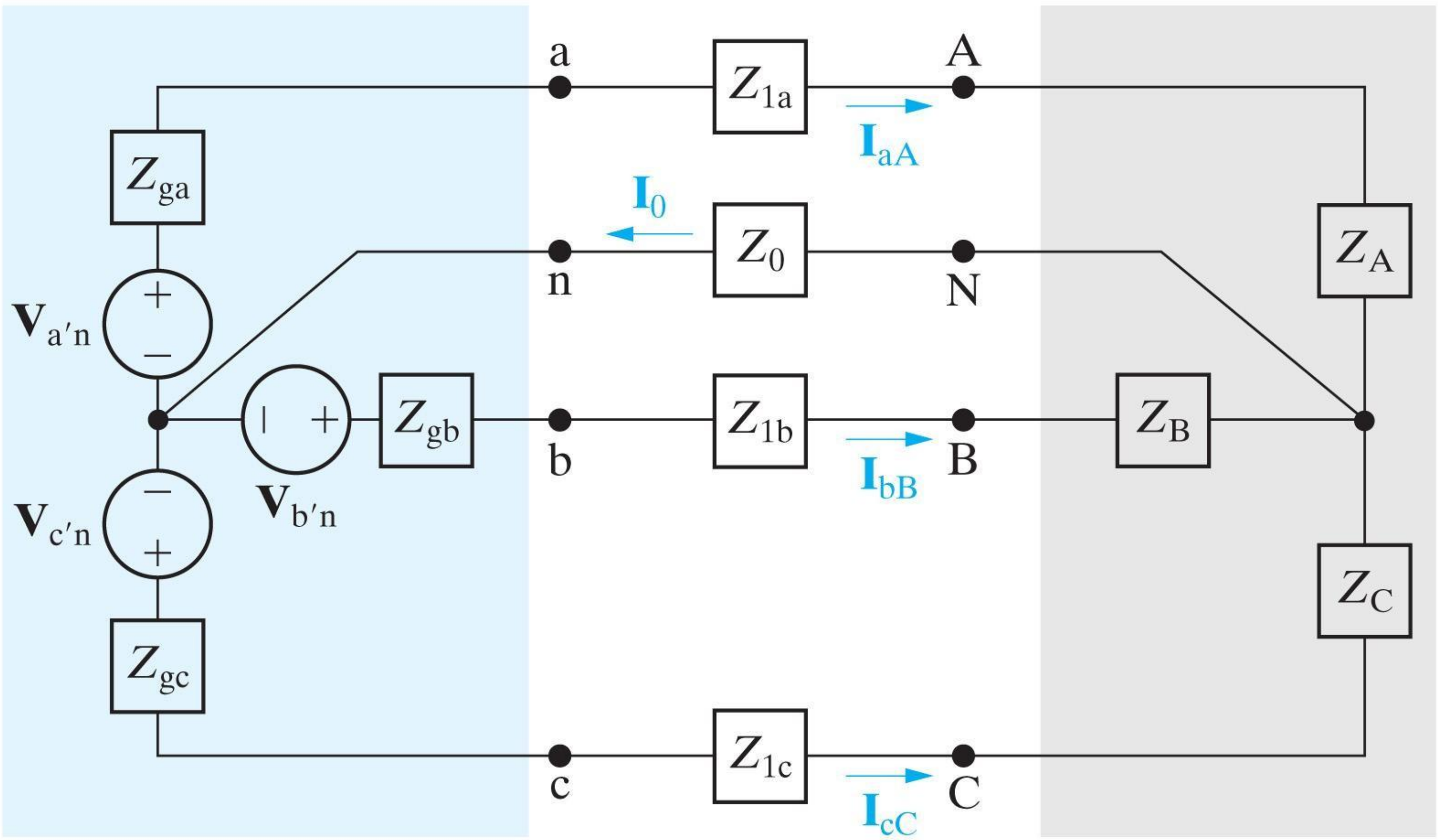


(a)

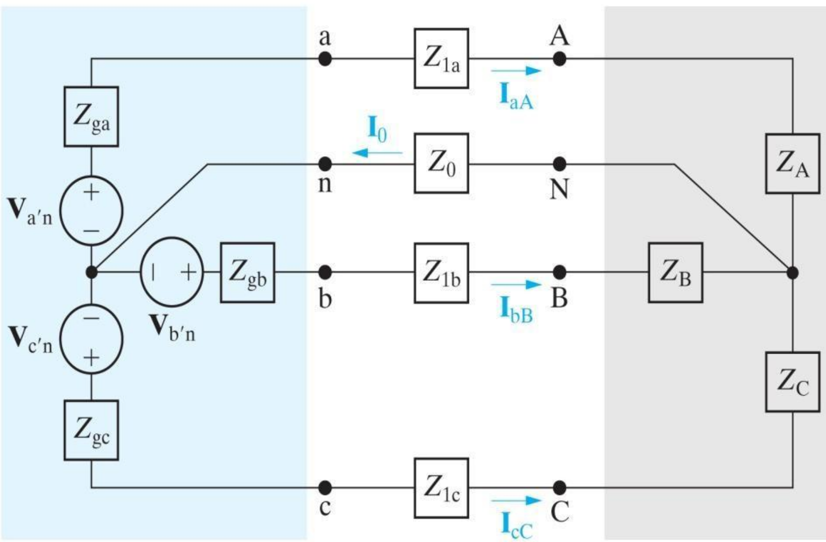


(b)

Look at a Y-Y System

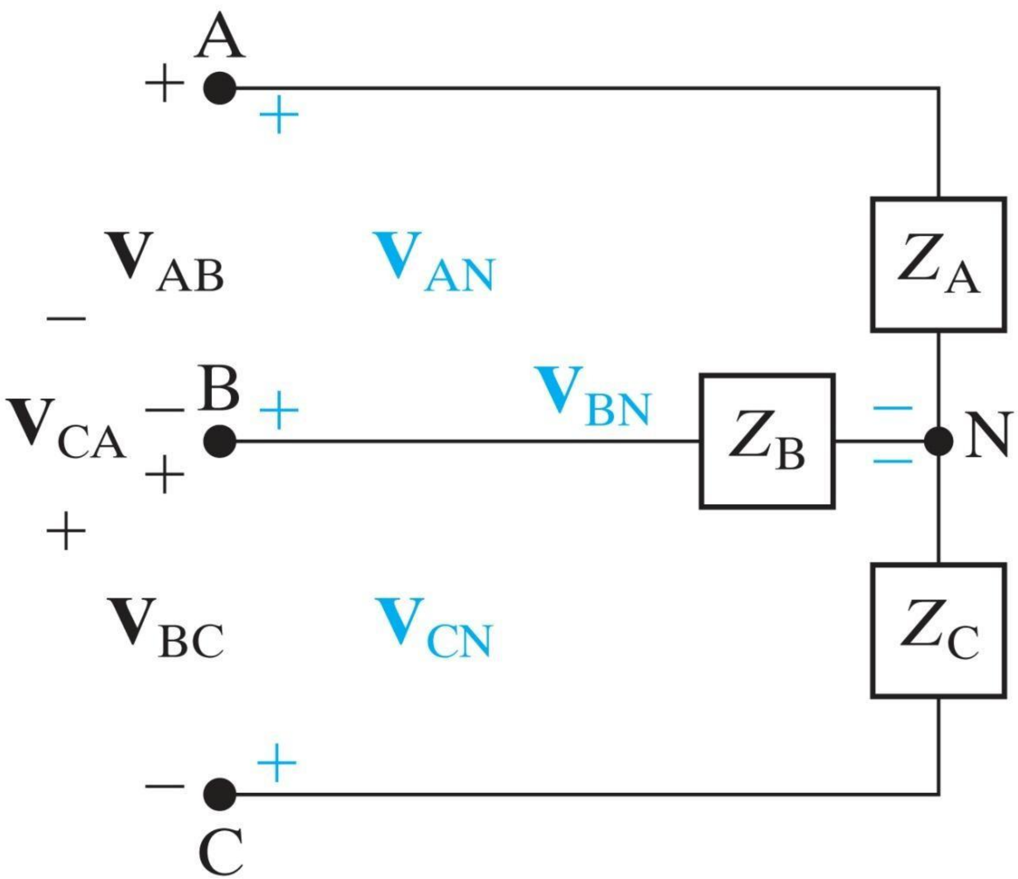


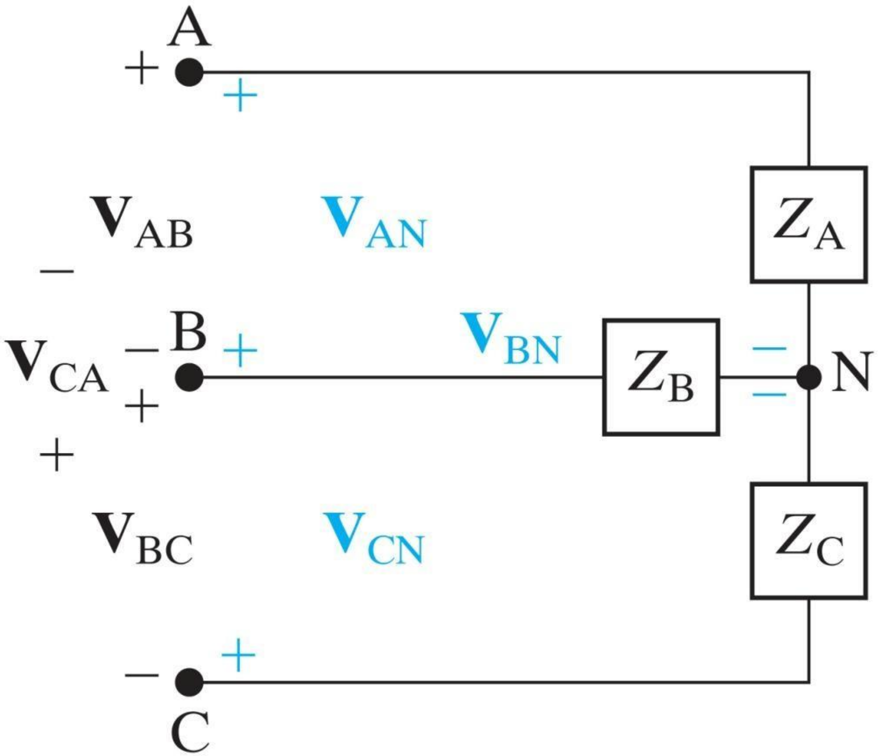
Definitions



- Z_g represents the internal generator impedance per phase
- Z_l represents the impedance of the line connecting the generator to the load
- $Z_{A,B,C}$ represents the load impedance per phase
- Z_o represents the impedance of the neutral conductor

Look at the Line and Load Voltages





Line Voltages

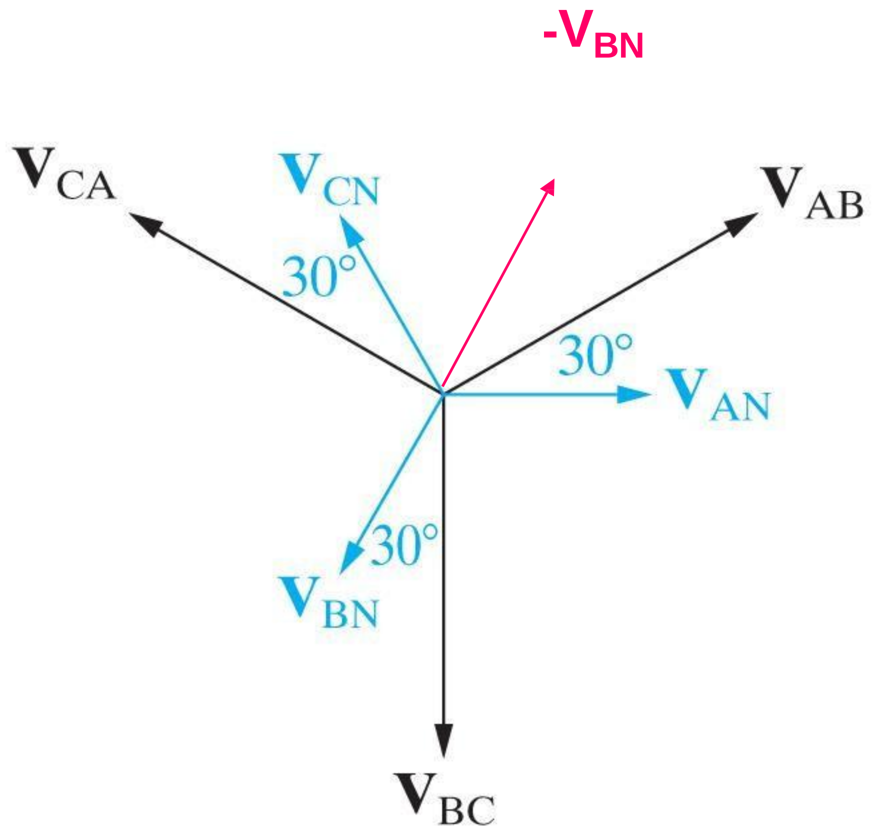
$$V_{AB} = V_{AN} - V_{BN}$$

$$V_{BC} = V_{BN} - V_{CN}$$

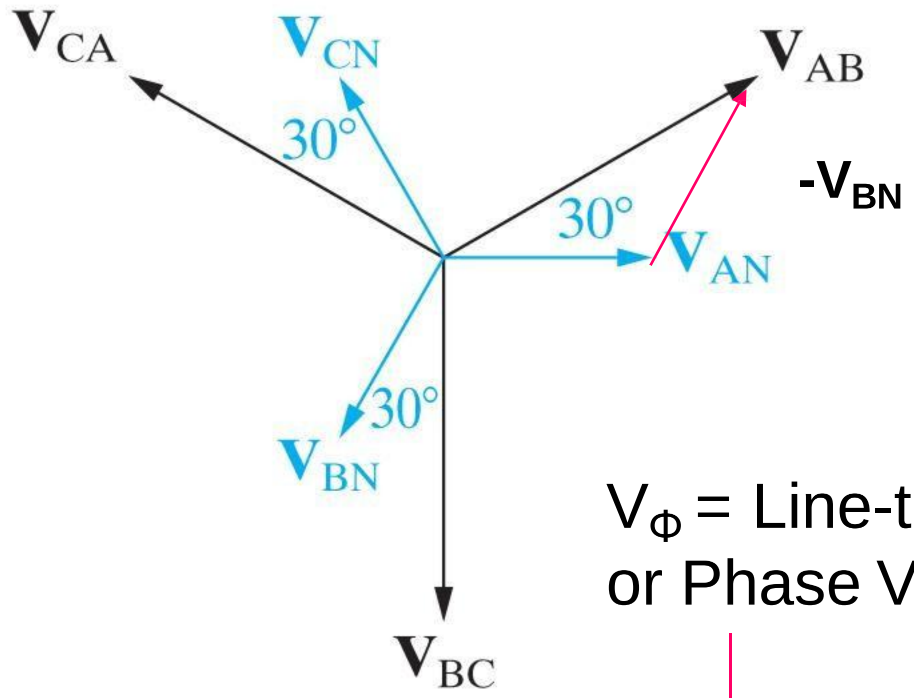
$$V_{CA} = V_{CN} - V_{AN}$$

Phase Voltages

Vector addition to find $V_{AB}=V_{AN}-V_{BN}$



Using the Tip-to-Tail Method



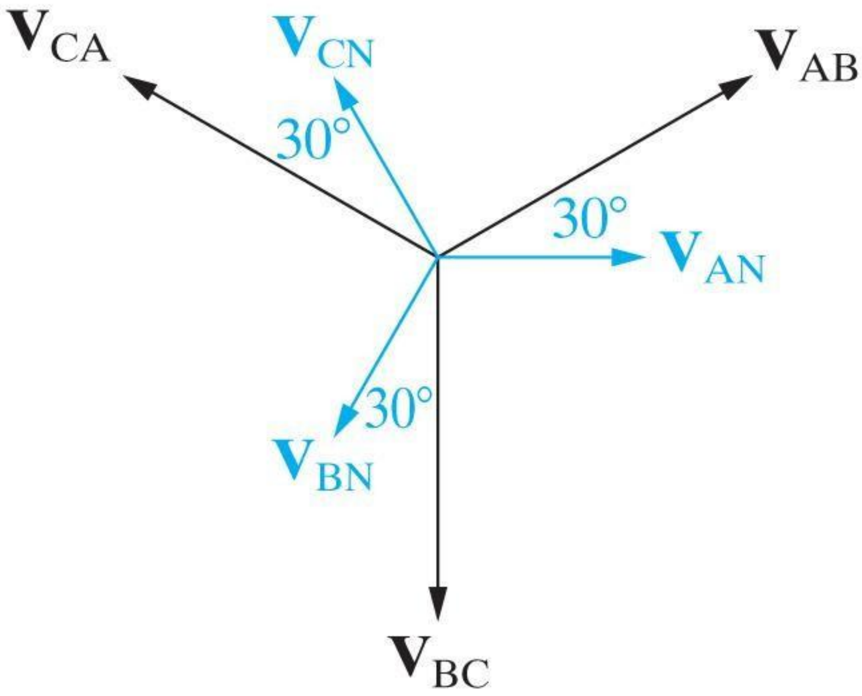
V_{Φ} = Line-to-Neutral, or Phase Voltage

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN} = \sqrt{3}V_{\Phi}$$

Conclusions for the Y connection

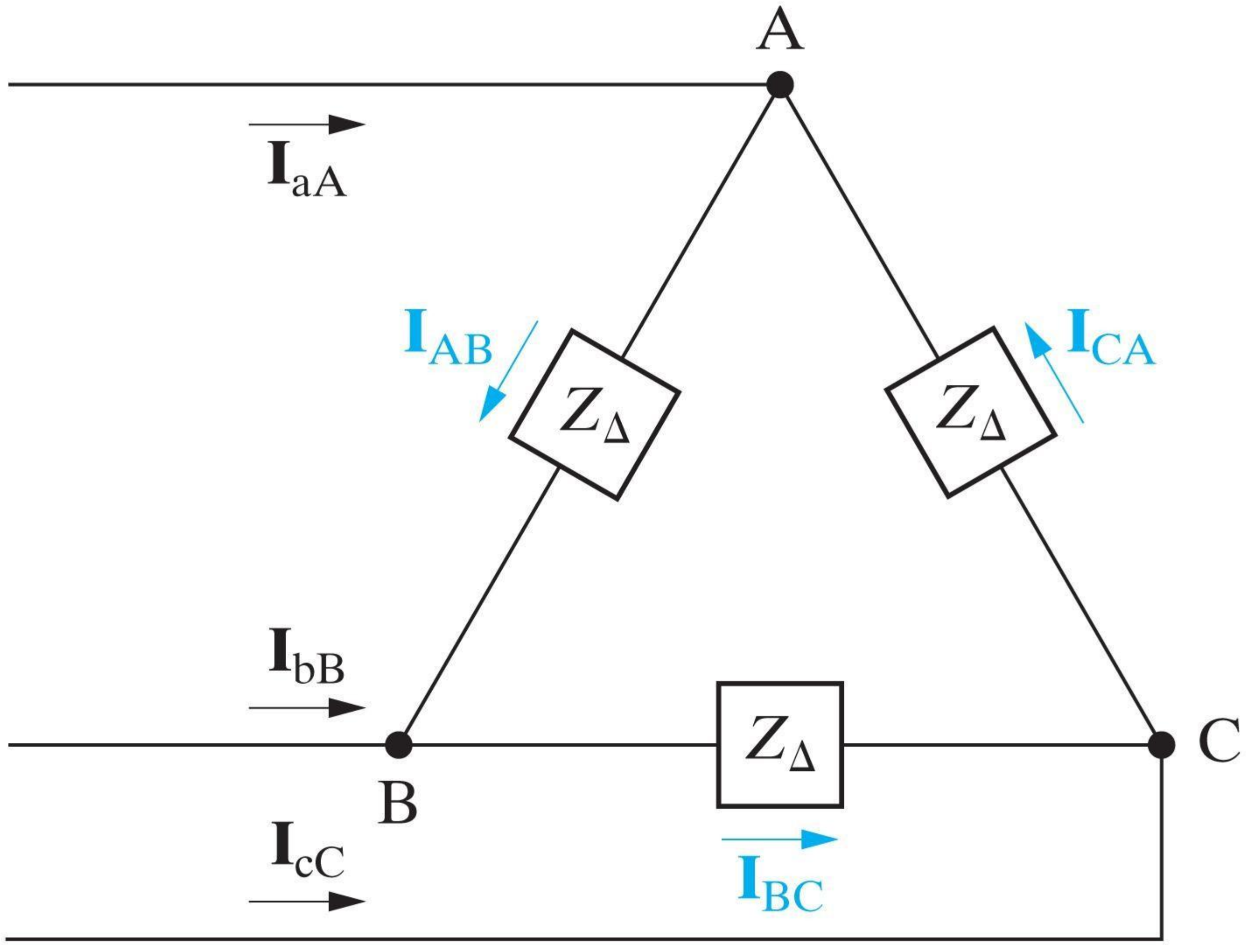
- The amplitude of the line-to-line voltage is equal to $\sqrt{3}$ times the amplitude of the phase voltage.
- The line-to-line voltages form a balanced set of 3-phase voltages.
- The set of line-to-line voltages leads the set of line-to-neutral (phase) voltages by 30°.

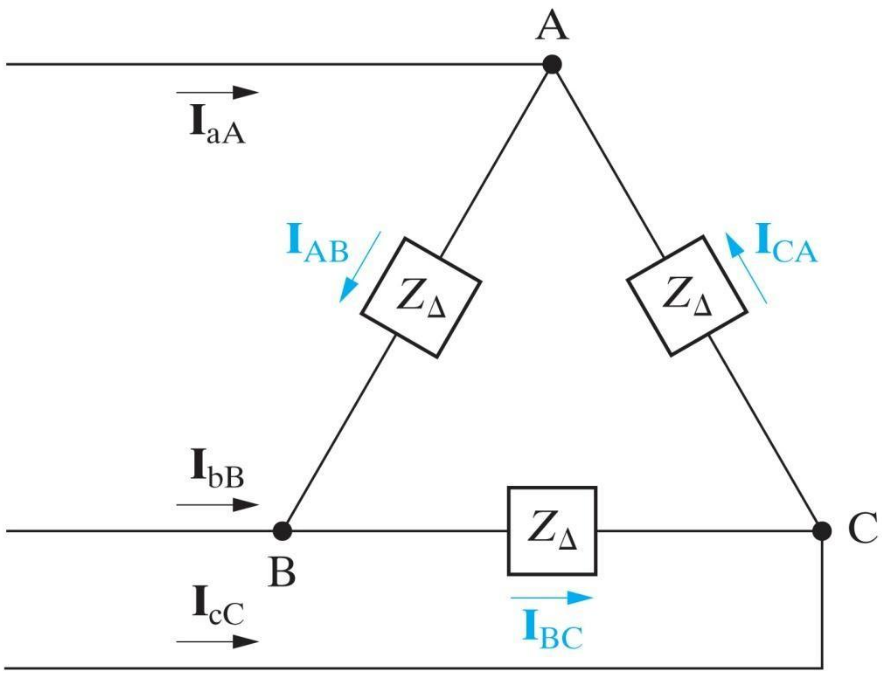
Summary



$\mathbf{V}_{CA}$
$\mathbf{V}_{CN}$
$\mathbf{V}_{AB}$
30°
30°
$\mathbf{V}_{AN}$
30°
$\mathbf{V}_{BN}$
$\mathbf{V}_{BC}$

Look at the Delta-Connected Load

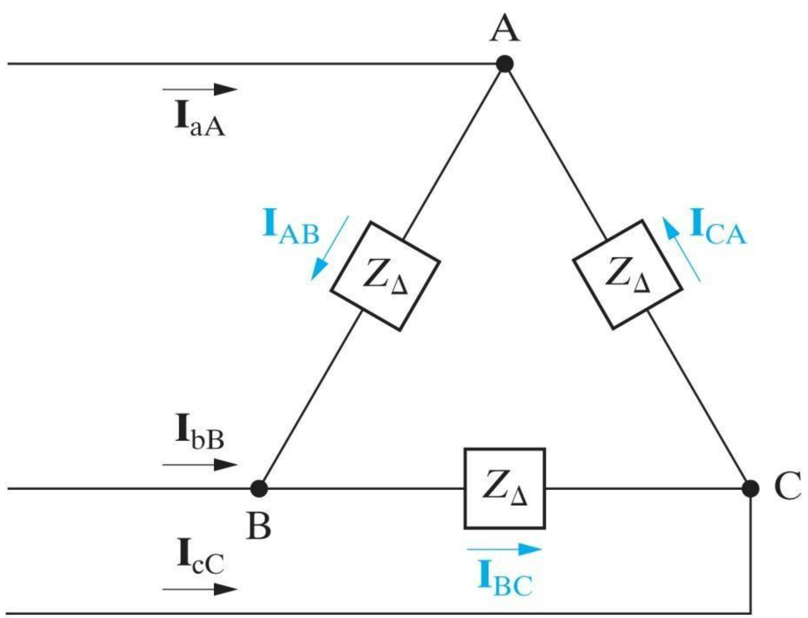




$$I_{AB} = I_\phi \angle 0°$$

$$I_{BC} = I_\phi \angle -120°$$

$$I_{CA} = I_\phi \angle 120°$$



Line Currents

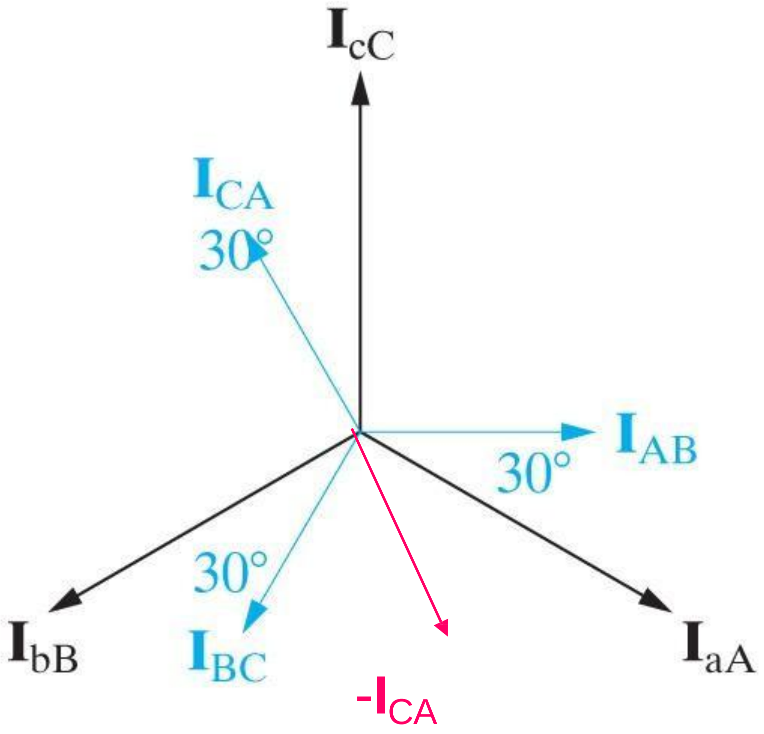
$$I_{aA} = I_{AB} - I_{CA}$$

$$I_{bB} = I_{BC} - I_{AB}$$

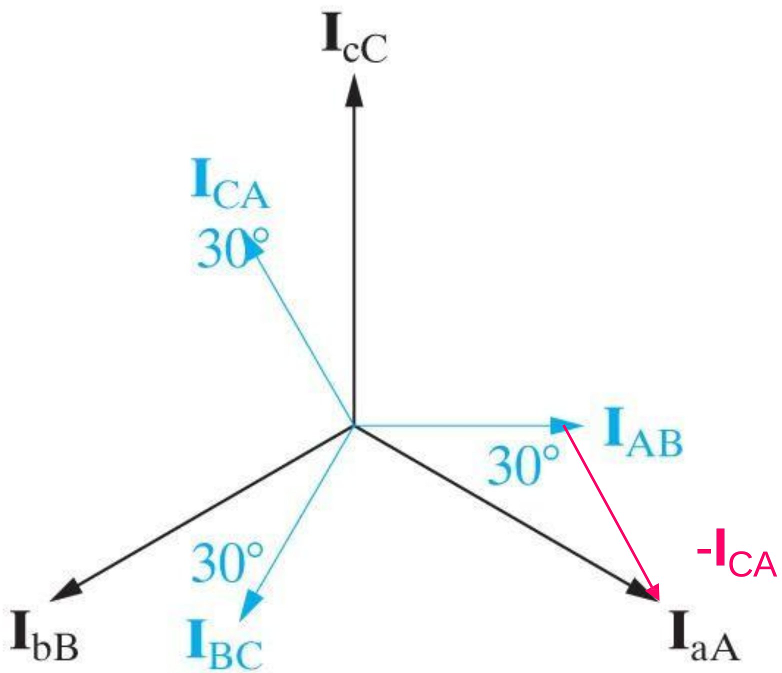
$$I_{cC} = I_{CA} - I_{BC}$$

Phase Currents

Vector Addition to find $I_{aA}=I_{AB}-I_{CA}$



Using the Tip-to-Tail Method



$$I_{aA} = \sqrt{3}I_{\Phi}/\!-30°$$

Conclusions for the Delta Connection

- The amplitude of the line current is equal to $\sqrt{3}$ times the phase current.
- The set of line currents lags the phase currents by 30°.

Fourier analysis deals with the representation of signals by ***sinewave*** components. The simplest type of Fourier analysis is that of the ***Fourier series*** where a ***periodic*** signal ***x(t)*** is represented as a ***sum*** of sinewaves.

$$x(t) = \sum \text{sine waves} \;.$$

The sine waves could be **sin ()**, **cos ()** or a *combination* thereof:

$$e^{j(\,)} = \cos(\,) + j\sin(\,).$$

The sine wave components in the summation will be a complex sine wave at various frequencies:

$$e^{jn\omega_0 t} \quad n = 0, \pm 1, \pm 2, \ldots$$

The complex sinewave frequencies are *integral multiples* of

$$\omega_0 \equiv \frac{2\pi}{T}$$

Where ***T*** is the ***period*** of the signal to be represented.

$$x(t) = x(t+T).$$

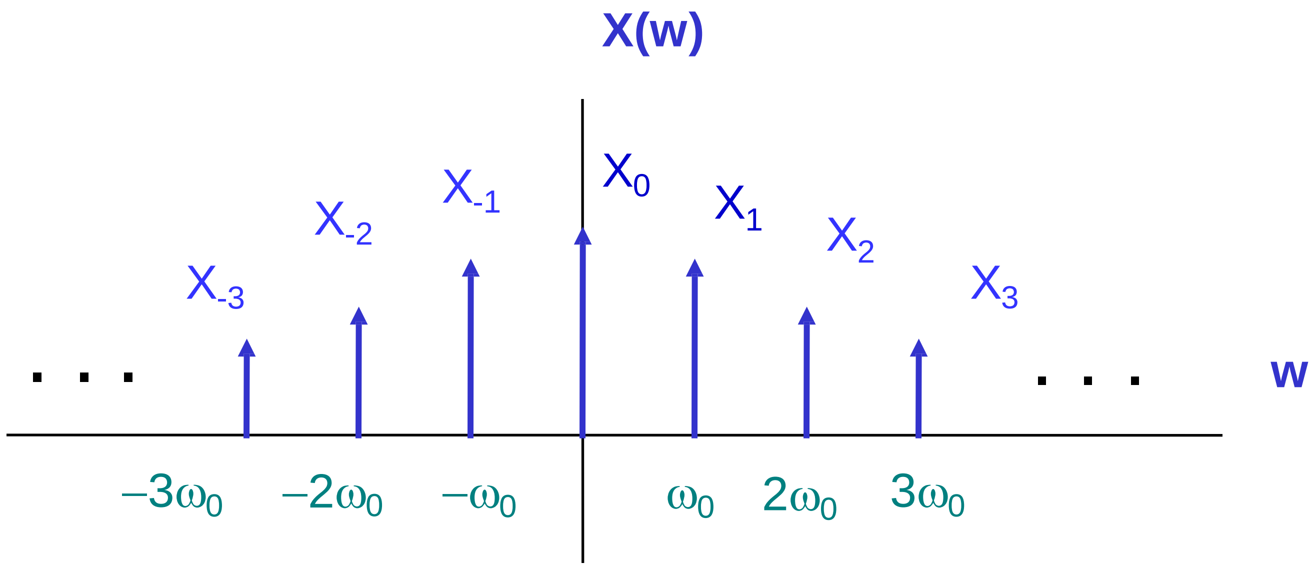
The resultant summation is a ***weighted*** *sum* of the complex sinewaves $e^{j\omega_0 t}$:

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t},$$

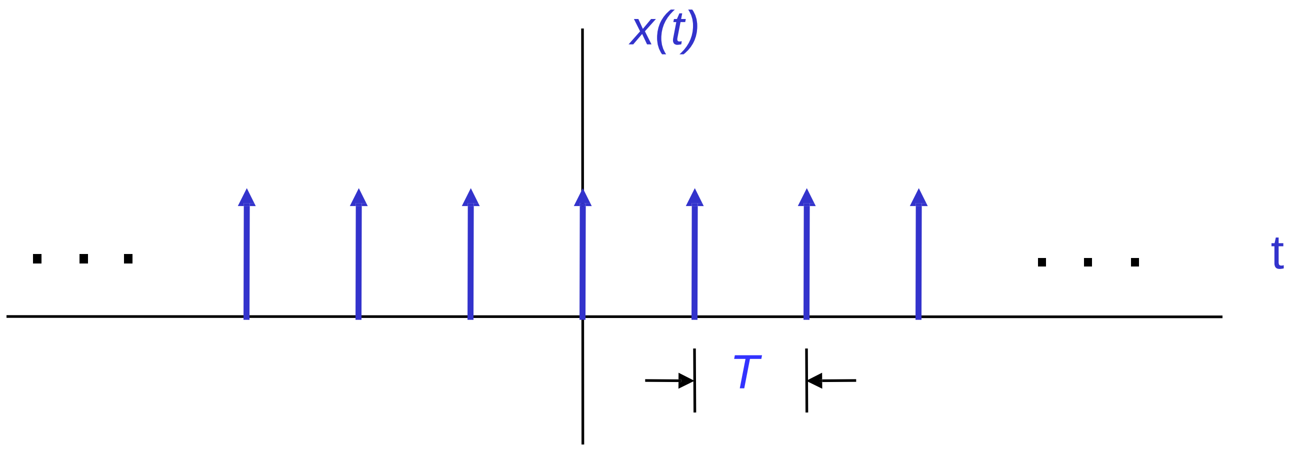
The *weights* or coefficients X_n are found from

$$X_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t) e^{-jn\omega_0 t} dt.$$

The weights ***X_n*** correspond to the ***magnitudes*** of the frequency components of the ***spectrum*** of ***x(t)***.



Example: Find the Fourier series for the following function:



$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT).$$

Solution: this function is clearly periodic. We calculate the coefficients as follows:

$$X_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t) e^{-jn\omega_0 t} dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} \delta(t) e^{-jn\omega_0 t} dt$$

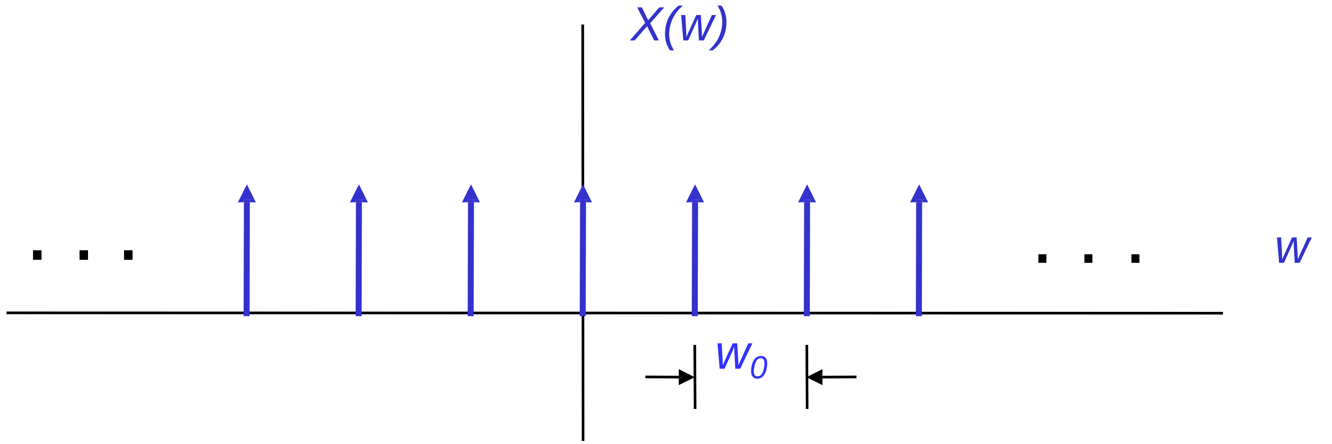
$$= \frac{1}{T}\int_{-T/2}^{T/2} \delta(t) e^{-jn\omega_0 (0)} dt$$

$$= \frac{1}{T} e^{-jn\omega_0 (0)} \int_{-T/2}^{T/2} \delta(t) dt$$

$$= \frac{1}{T}(1)(1) = \frac{1}{T}.$$

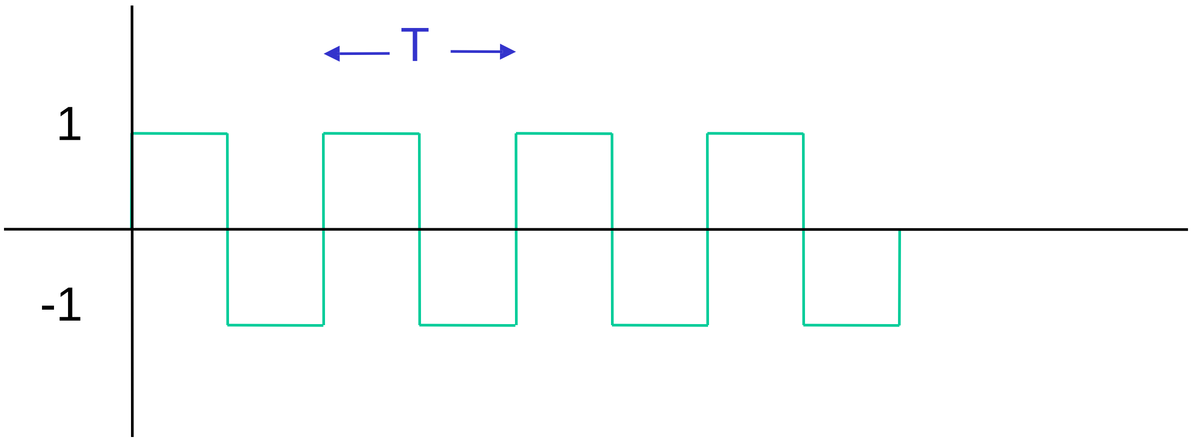
So,

$$x(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T} e^{jn\omega_0 t} = \frac{1}{T} \sum_{n=-\infty}^{\infty} e^{jn\omega_0 t}.$$



X(w)
w
w0

Example: Find the Fourier series for the following function:



Solution: The coefficients are calculated in much the same way as before.

$$
\begin{aligned}
X_n &= \frac{1}{T}\int_0^T x(t)e^{-jn\omega_0 t}\,dt \\
&= \frac{1}{T}\int_0^{T/2}(1)e^{-jn\omega_0 t}\,dt + \frac{1}{T}\int_{T/2}^{T}(-1)e^{-jn\omega_0 t}\,dt \\
&= \frac{1}{T}\frac{1}{(-jn\omega_0)}e^{-jn\omega_0 t}\Big|_0^{T/2} - \frac{1}{T}\frac{1}{(-jn\omega_0)}e^{-jn\omega_0 t}\Big|_{T/2}^{T} \\
&= \frac{1}{T(-jn\omega_0)}\left[e^{-jn\omega_0 T/2} - e^{-j0} - e^{-jn\omega_0 T} + e^{-jn\omega_0 T/2}\right]
\end{aligned}
$$

$$= \frac{1}{T}\frac{1}{(-jn\omega_0)}\left[e^{-jn\pi} - e^{-j0} - e^{-jn2\pi} + e^{-jn\pi}\right]$$

$$= \frac{1}{T}\frac{2}{(-jn\omega_0)}\left[e^{-jn\pi} - e^{-j0}\right]$$

$$= \frac{1}{T}\frac{2}{(jn\omega_0)}\left[e^{-j0} - e^{-jn\pi}\right]$$

$$= \frac{1}{(jn\pi)}\left[1 - e^{-jn\pi}\right]$$

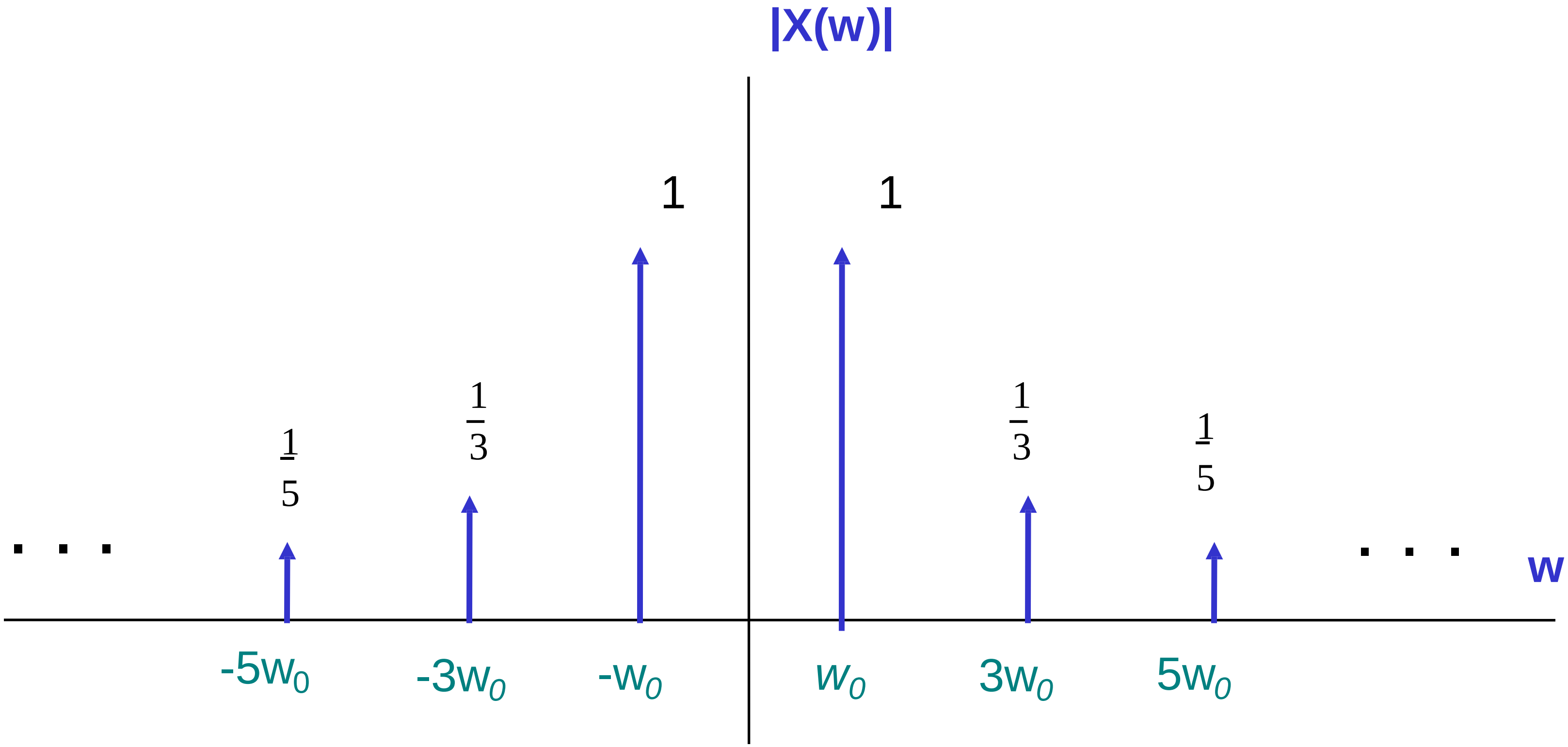
$$= \frac{1}{(jn\pi)}\left[1 - (-1)^n\right].$$

$$X_n = \begin{cases} \dfrac{2}{jn\pi} & (n \text{ odd}), \\ 0 & (n \text{ even}). \end{cases}$$

Thus,

$$x(t) = \frac{2}{j\pi} \sum_{\substack{n=-\infty \\ n\,odd}}^{\infty} \frac{1}{n} e^{jn\Omega_0 t}.$$

The spectrum of the square wave is shown below



Fourier Transforms

Fourier transforms allow the frequency analysis of *non-periodic* as well as periodic functions. The Fourier transformation is derived from Fourier series.

To derive the Fourier transform, let us take the Fourier series, and take the **limit** as **$T \rightarrow \infty$**. As $T \rightarrow \infty$, we also have $\omega_0 \rightarrow 0$ (since $\omega_0 = 2\pi/T$).

If we let $\omega_0 = \Delta\omega$, $T=2p/w$ and $c_n T$ = $X(n\Delta\omega)$, we have

$$X(n\Delta\omega) = \int_{-\infty}^{\infty} x(t) e^{-jn\Delta\omega t}\, dt.$$

and,

$$x(t) = \lim_{\Delta\omega \to 0} \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} X(n\Delta\omega) e^{jn\Delta\omega t} \Delta\omega.$$

This last expression is a Riemann sum which is a definite integral:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t}\, d\omega,$$

where ***w = nΔω***. Using this last piece of notation in ***X(nΔω)***, we have

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt.$$

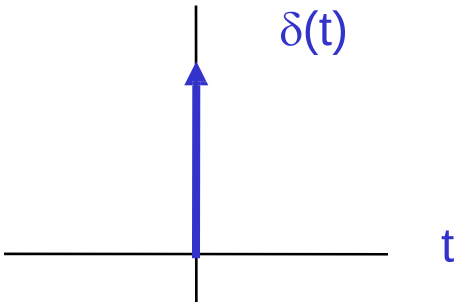
This last expression is the definition of the ***Fourier transform*** of *x(t)*:

$$F\{x(t)\} \equiv X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt.$$

In the process of deriving and expression for the Fourier transform, we have also derived an expression for the *inverse Fourier transformation*:

$$F^{-1}\{X(\omega)\} = x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega .$$

Example: find the Fourier transform of a delta function, ***x(t)=δ(t)***.



Solution: using the definition of the Fourier transform, we have

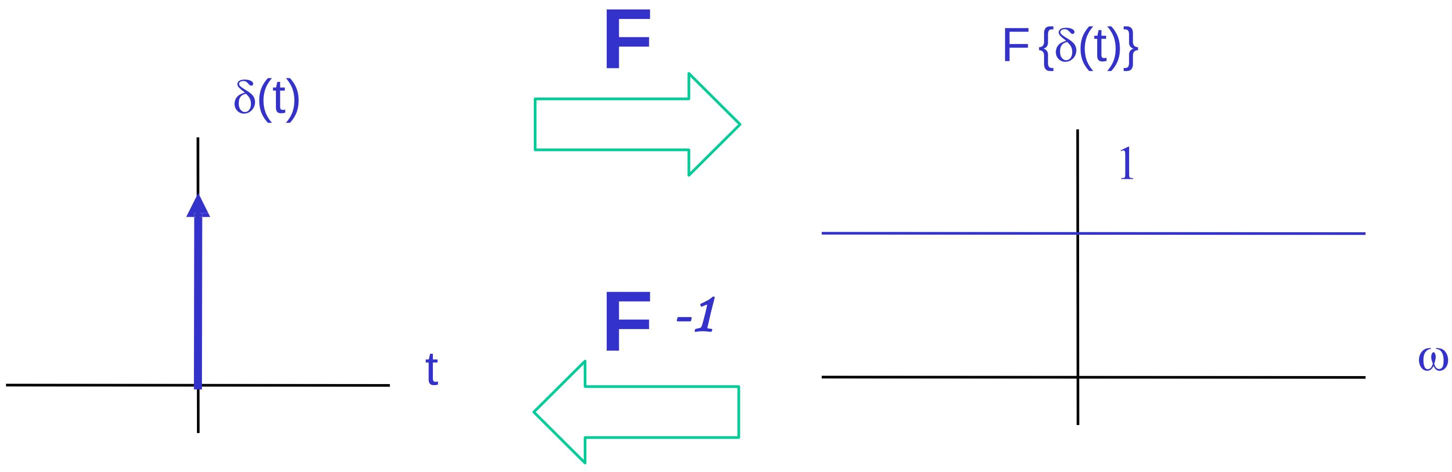
$$X(\omega) = F\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt$$

$$
\begin{aligned}
&= \int_{-\infty}^{\infty} \delta(t) e^{-j\omega(0)}\, dt \\
&= e^{-j\omega(0)} \int_{-\infty}^{\infty} \delta(t)\, dt \\
&= (1)(1) = 1.
\end{aligned}
$$

As corollary of this last problem, we have the following: the *inverse* Fourier transformation of one is a delta function:

$$F^{-1}\{1\} = \delta(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega t}\, d\omega\,.$$

This will be a ***very useful fact*** in other Fourier transform derivations.



We know that the Fourier transform of a delta function is one. What is the Fourier transform of one? The answer may be obtained by noticing the *similarities* between the forward transformation and the reverse transformation.

Let us do a few manipulations on the inverse Fourier transformation:

$$F^{-1}\{X(\omega)\} = x(t) = \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega .$$

$$F^{-1}\{X(t)\} = x(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(t) e^{j\omega t} dt .$$

$$x(-\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(t)e^{-j\omega t}\,dt\ .$$

Thus we have the Fourier transform *of* a Fourier transform:

$$F\{X(t)\} = 2\pi x(-\omega).$$

If we know the forward Fourier transform, we can find the reverse Fourier transform. This property is called ***duality.***

Applying this result to the Fourier transform of $\delta(t)$, we have

$$F\{1\} = 2\pi\,\delta(\omega)\ .$$

A *physical* interpretation can be given to the Fourier transforms of ***δ(t)*** and *1*. The function **1** is a D.C. signal whose sole frequency component is 0 Hz. This component is represented by a spike at **0** rad/sec in the frequency domain. The function ***δ(t)*** is like a lighting bolt whose spectrum covers the entire frequency band (from $f = 0$ to ∞).

Suppose we had a function *x(t)* whose Fourier transform we know, *X(ω)*. We then wished to know the Fourier transform of a *shifted version* of *x(t)*, *x(t-a)* :

$$
\begin{aligned}
F\{x(t-a)\} &= \int_{-\infty}^{\infty} x(t-a)\, e^{-j\omega t}\, dt \\
&= \int_{-\infty}^{\infty} x(t)\, e^{-j\omega(t+a)}\, dt \\
&= e^{-j\omega a} \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \\
&= e^{-j\omega a} X(\omega).
\end{aligned}
$$

Applying this principle to $\mathcal{F}\{\delta(t)\}$, we have

$$F\{\delta(t-a)\} = e^{-j\omega a}.$$

Using duality, we also have

$$\begin{aligned} F\{e^{jat}\} &= \delta(-\omega - a) \\ &= \delta(\omega + a). \end{aligned}$$

The previous two results could also have been had by *direct* application of the Fourier transform:

$$
\begin{aligned}
F\{\delta(t-a)\} &= \int_{-\infty}^{\infty} \delta(t-a)e^{-j\omega t}\,dt \\
&= \int_{-\infty}^{\infty} \delta(t-a)e^{-j\omega a}\,dt \\
&= e^{-j\omega a}\int_{-\infty}^{\infty} \delta(t-a)\,dt \\
&= e^{-j\omega a}\,.
\end{aligned}
$$

$$
\begin{aligned}
F\left\{e^{-jat}\right\} &= \int_{-\infty}^{\infty} e^{-jat} e^{-j\omega t} dt \\
&= \int_{-\infty}^{\infty} e^{-j(a+\omega)t}\, dt \\
&= \delta(\omega+a).
\end{aligned}
$$

The last step is true because of the “very useful fact”:

$$
\delta(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega t}\, d\omega .
$$

As a corollary of the previous Fourier transformations, we also have

$$F\{\delta(t+a)\} = e^{+j\omega a}.$$

$$F\{e^{+jat}\} = 2\pi\delta(\omega - a).$$

A summary of the Fourier transforms derived so far are shown in the table on the following slide.

$x(t)$	$X(\omega)$
$\delta(t)$	1
1	$2\pi\delta(\omega)$
$\delta(t-a)$	$e^{-j\omega a}$
e^{jat}	$2\pi\delta(\omega-a)$

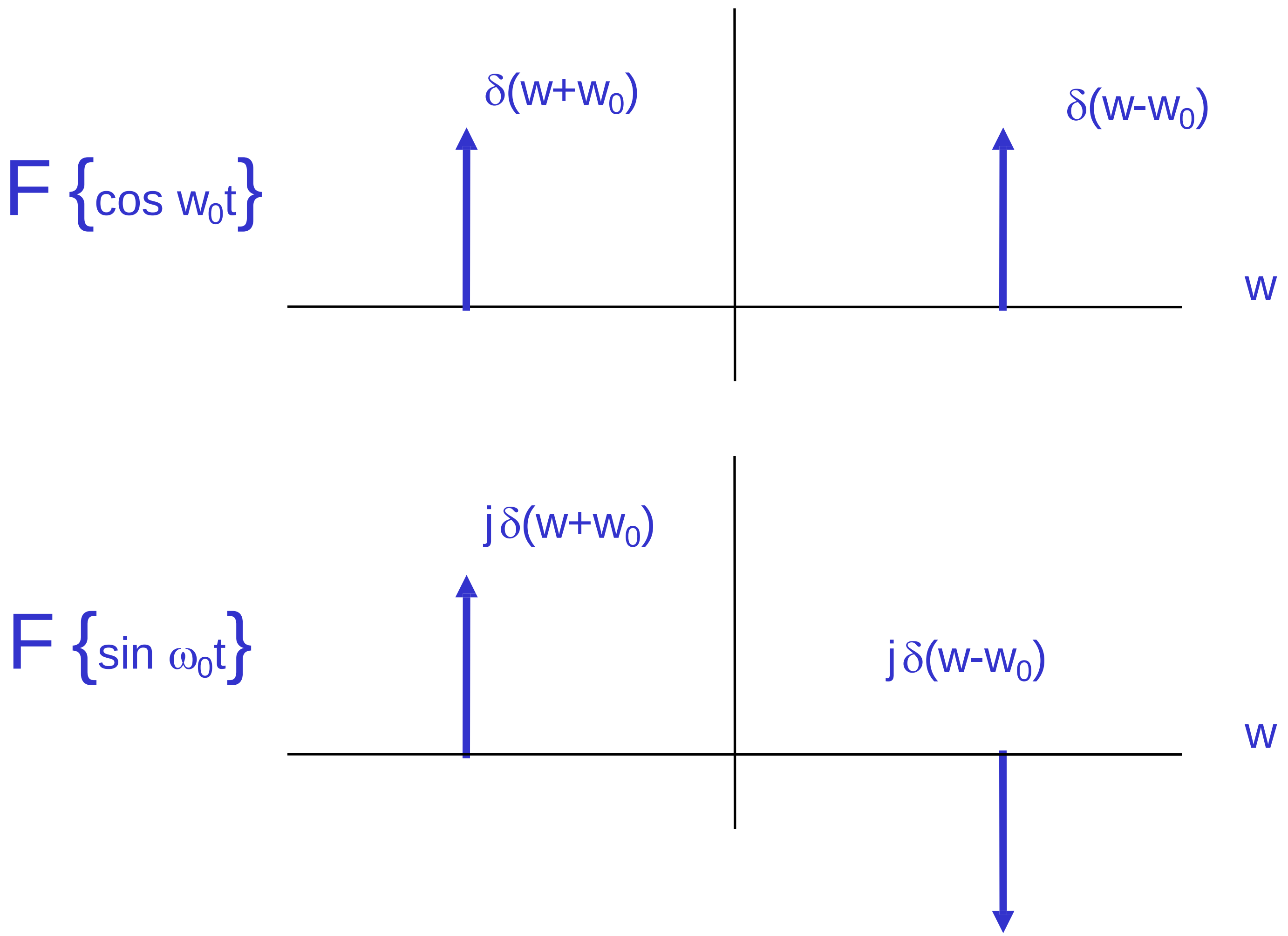
Let's find the Fourier transform of a sinewave:

$$
\begin{aligned}
F\{\cos \omega_o t\} &= F\left\{\frac{1}{2}\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right]\right\} \\
&= \frac{1}{2}\left[F\{e^{j\omega_0 t}\} + F\{e^{-j\omega_0 t}\}\right] \\
&= \frac{2\pi}{2}\left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right] \\
&= \pi\left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right].
\end{aligned}
$$

Similarly we have

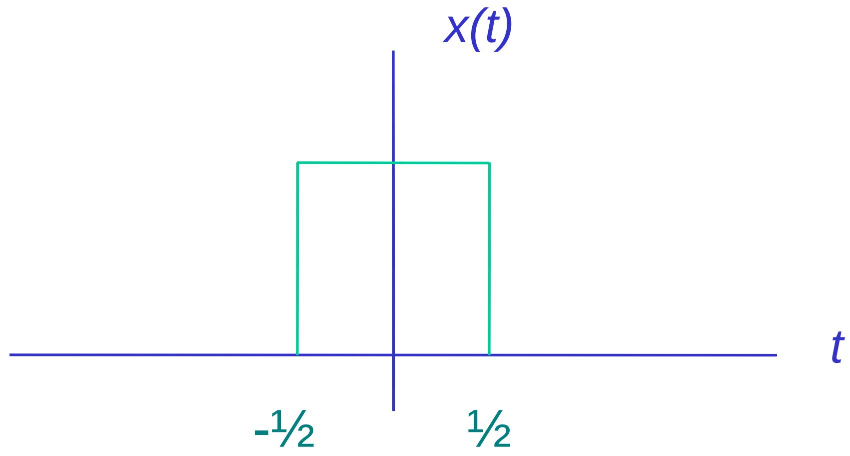
$$
\begin{aligned}
\mathcal{F}\{\sin \omega_o t\} &= \mathcal{F}\left\{\frac{1}{2j}\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right]\right\} \\
&= \frac{2\pi}{2j}\left[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)\right] \\
&= j\pi\left[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)\right].
\end{aligned}
$$

The plots of these Fourier transforms are shown below.



The physical interpretation of these transforms should be clear: the spectrum of a sinewave consists of spikes at the frequency of the sinewave along with a spike at the corresponding negative frequency. The (generally complex) coefficients of the spikes depend upon the *phase* of the sinewave.

Now, let's find the Fourier transform of a pulse:

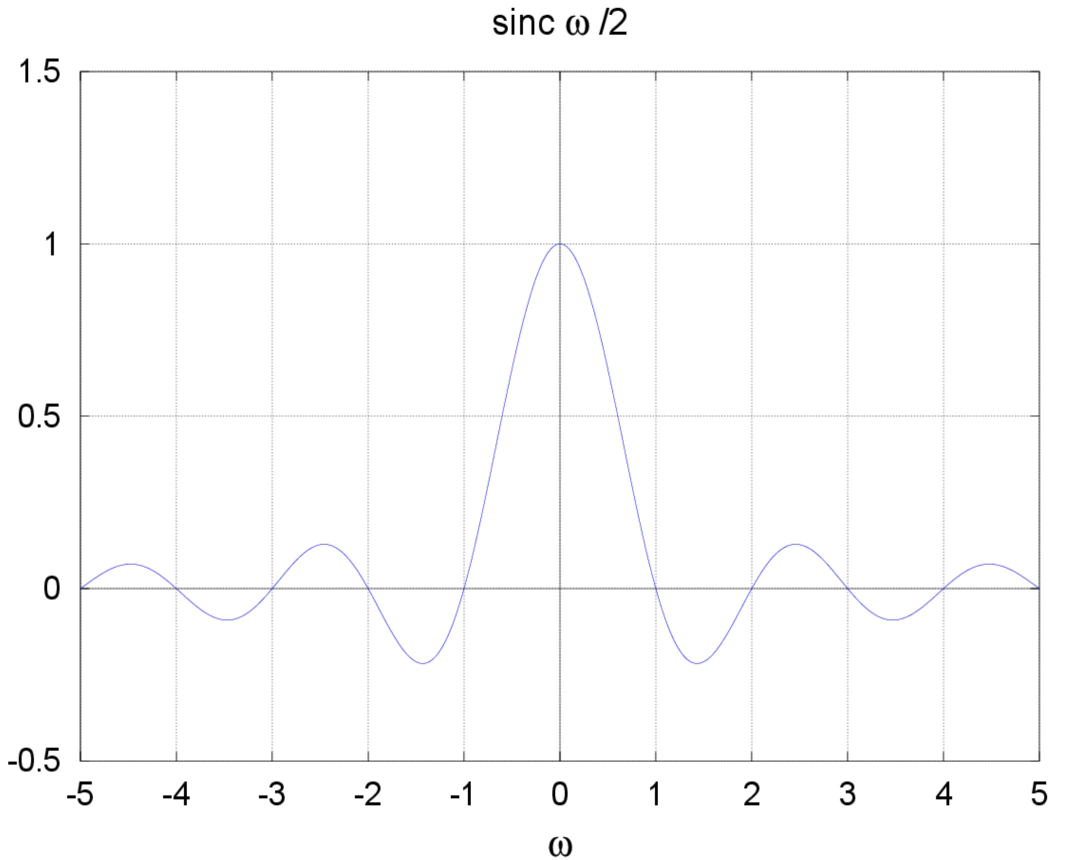


This function is sometimes referred to as $\Pi(t)$.

Applying the definition of the Fourier transform, we have

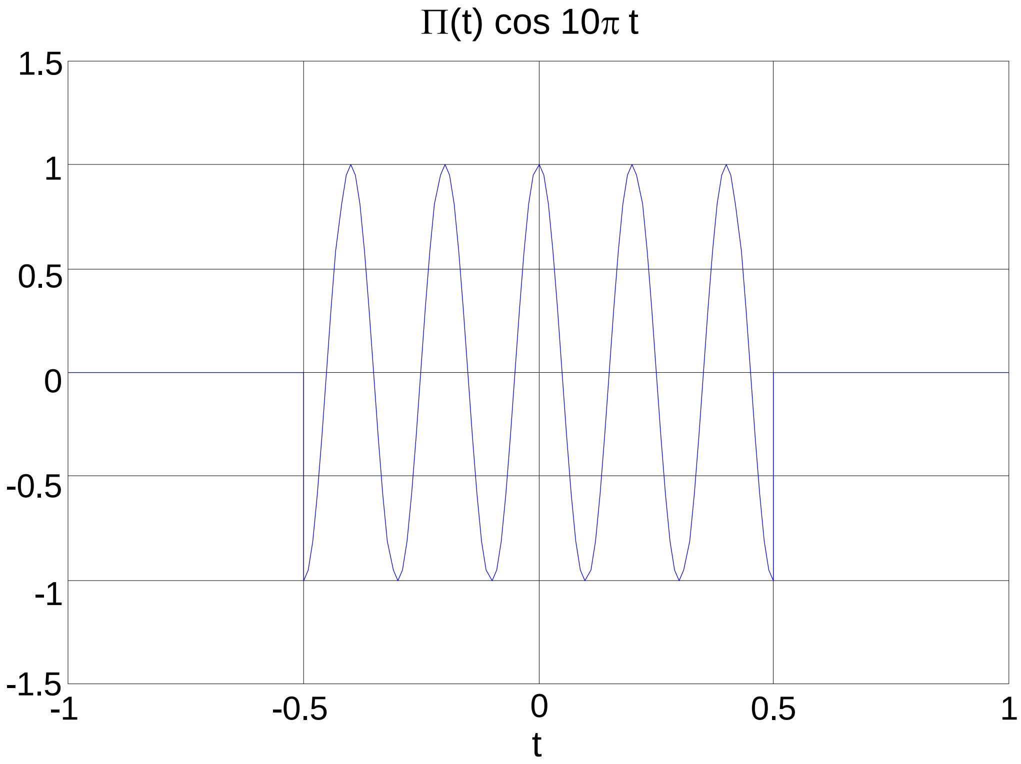
$$
\begin{aligned}
F\{\Pi(t)\} &= \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-j\omega t}\,dt \\
&= \frac{1}{-j\omega} e^{-j\omega t}\Big|_{-\frac{1}{2}}^{\frac{1}{2}} \\
&= \frac{1}{j\omega}\left[e^{j\omega/2} - e^{-j\omega/2}\right] \\
&= \frac{1}{2j\frac{\omega}{2}}\left[e^{j\omega/2} - e^{-j\omega/2}\right] \\
&= \frac{\sin\frac{\omega}{2}}{\frac{\omega}{2}} \equiv \operatorname{sinc}\frac{\omega}{2}.
\end{aligned}
$$

A plot of this function is shown below:



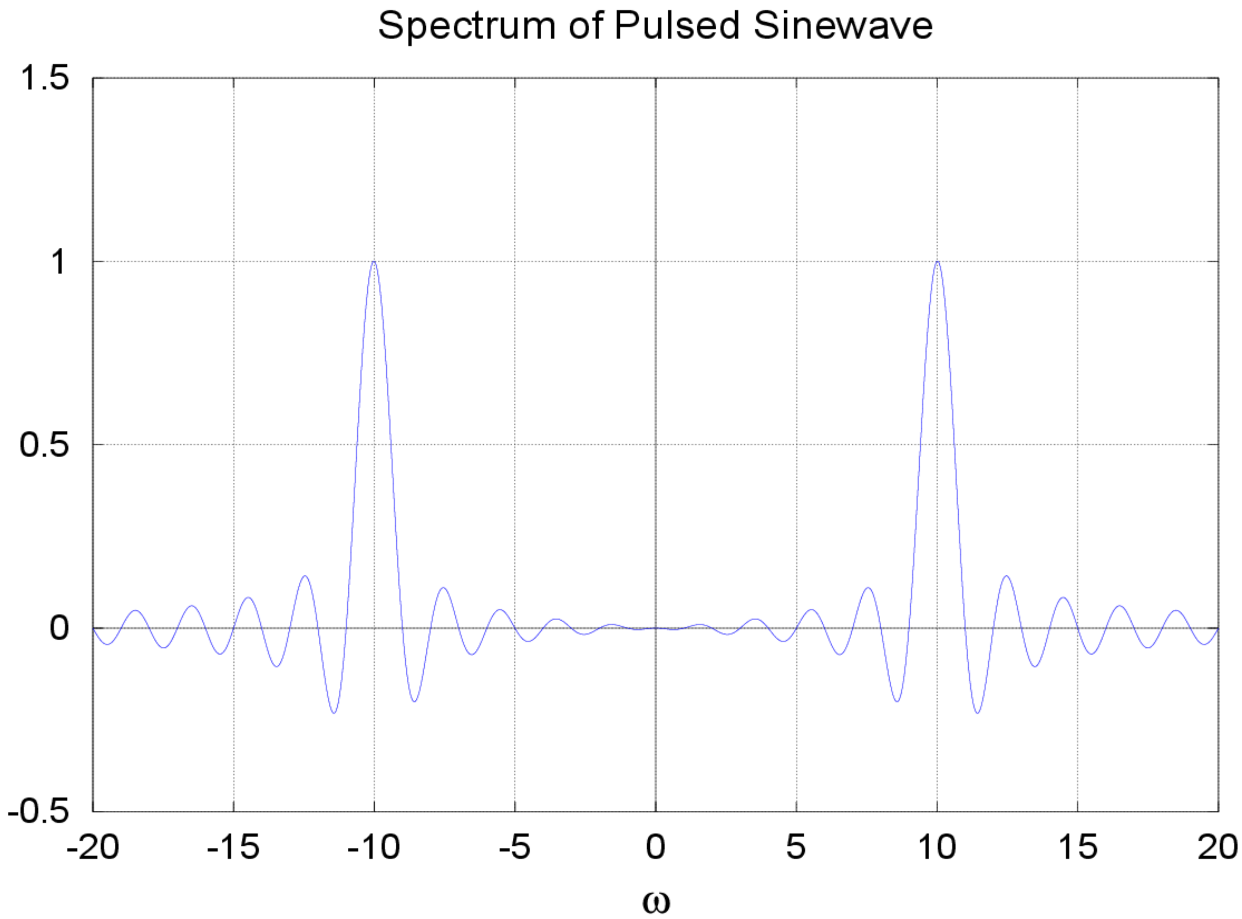
Now, let's find the Fourier transform of a *pulsed* sinewave:

$$x(t) = \Pi(t)\cos 10\pi t.$$



$$
\begin{aligned}
F\{\Pi(t)\cos 10t\} &= \frac{1}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} \left(e^{j10\pi t}+e^{-j10\pi t}\right)e^{-j\omega t}\,dt \\
&= \frac{1}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} \left(e^{-j(\omega-10)t}+e^{-j(\omega+10)t}\right)dt \\
&= \frac{1}{2}\operatorname{sinc}(\omega-10)+\frac{1}{2}\operatorname{sinc}(\omega-10).
\end{aligned}
$$

A plot of this function is shown below:



Exercise: find the Fourier transform of the following *pulsed* sine wave:

$$x(t) = \Pi\left(\frac{t}{20}\right)\sin 100\pi t\,.$$