POWER POINT PRESENTATION
ON
POWER ELECTRONICS

III B. Tech I semester (JNTUH-R15)

Prepared

By

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ELECTRICAL AND ELECTRONICS ENGINEERING

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Power Electronics

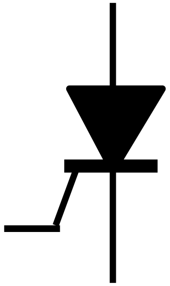
(A50220)

Unit-I

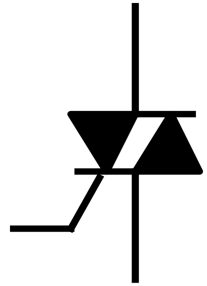
Power Semiconductor Devices & Commutation Circuits

- Diodes
- Transistors
  - Power BJTs
  - Power MOSFETs
  - Insulated-Gate BJT
    - IGBT
  - Static Induction Transistors
    - SITs
- Thyristors
  - Force-Commutated
  - Line-Commutated
  - Gate Turn Off--GTO
  - Reverse-Conducting
    - RCT
  - Gate-Assisted Turn-off
    - GATI

Thyristor/Triac



SCR1
2N3668



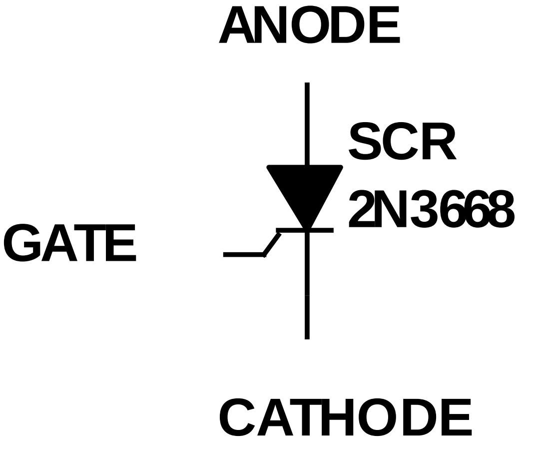
MT1
2N6346

Power Electronic Circuits

- Diode Rectifiers (AC to Fixed DC)
- AC-DC Converters (Controlled Rectifiers)
- AC-AC Converters (AC Voltage Controllers)
- DC-DC Converters (DC Choppers)
- DC-AC Converters (Inverters)
- Static Switches

SCR / Thyristor

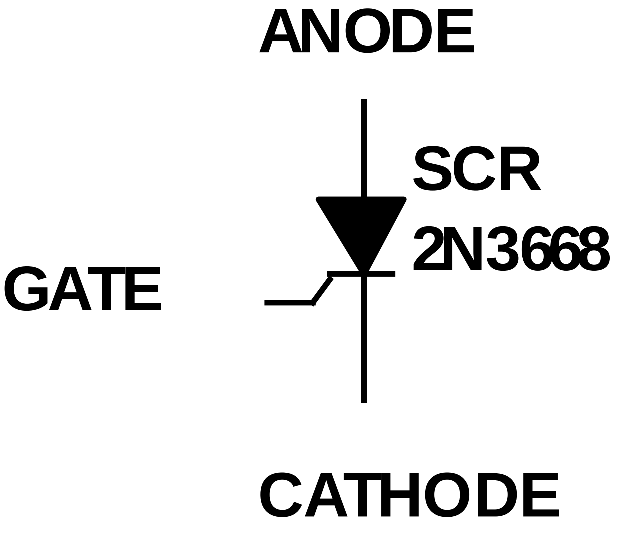
- Circuit Symbol and Terminal Identification



SCR / Thyristor

- Anode and Cathode terminals as conventional pn junction diode

- Gate terminal for a controlling input signal



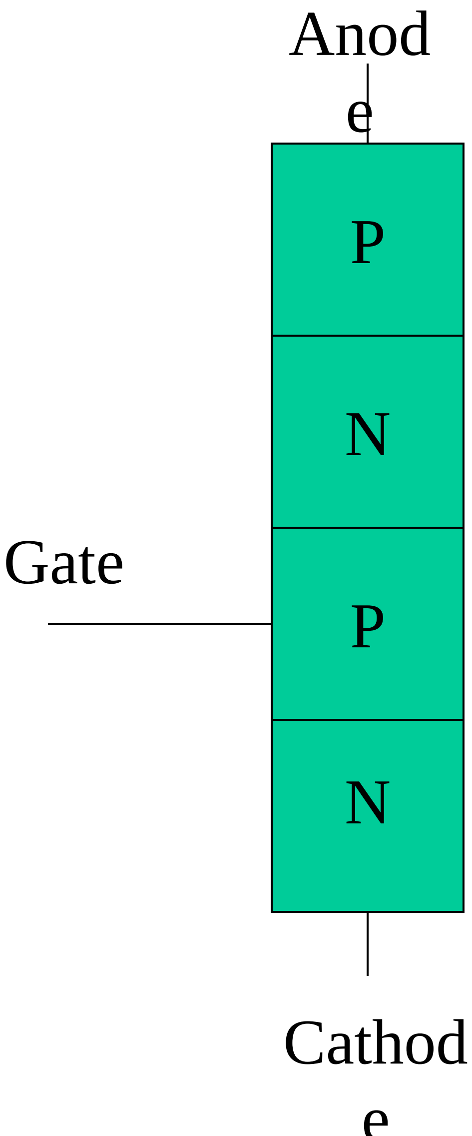
SCR/ Thyristor

- An SCR (Thyristor) is a “controlled” rectifier (diode)
- Control the conduction under forward bias by applying a current into the Gate terminal
- Under reverse bias, looks like conventional pn junction diode

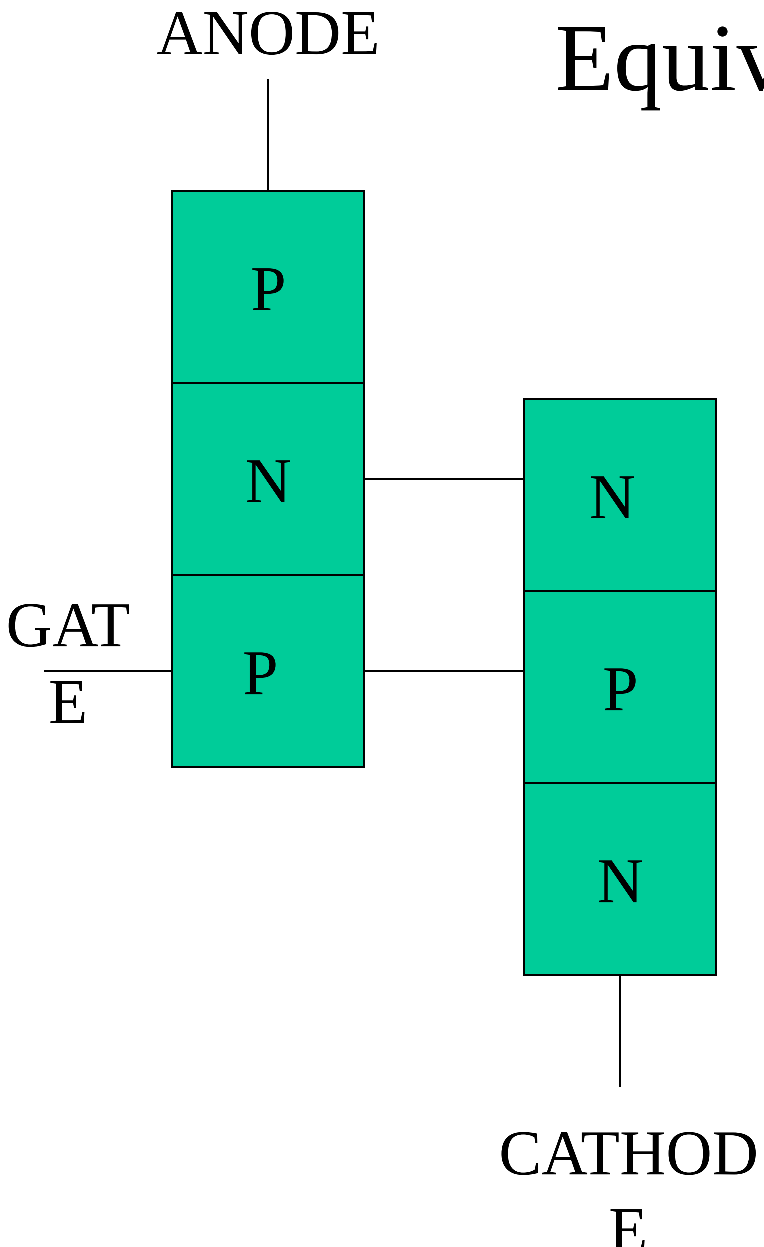
SCR / Thyristor

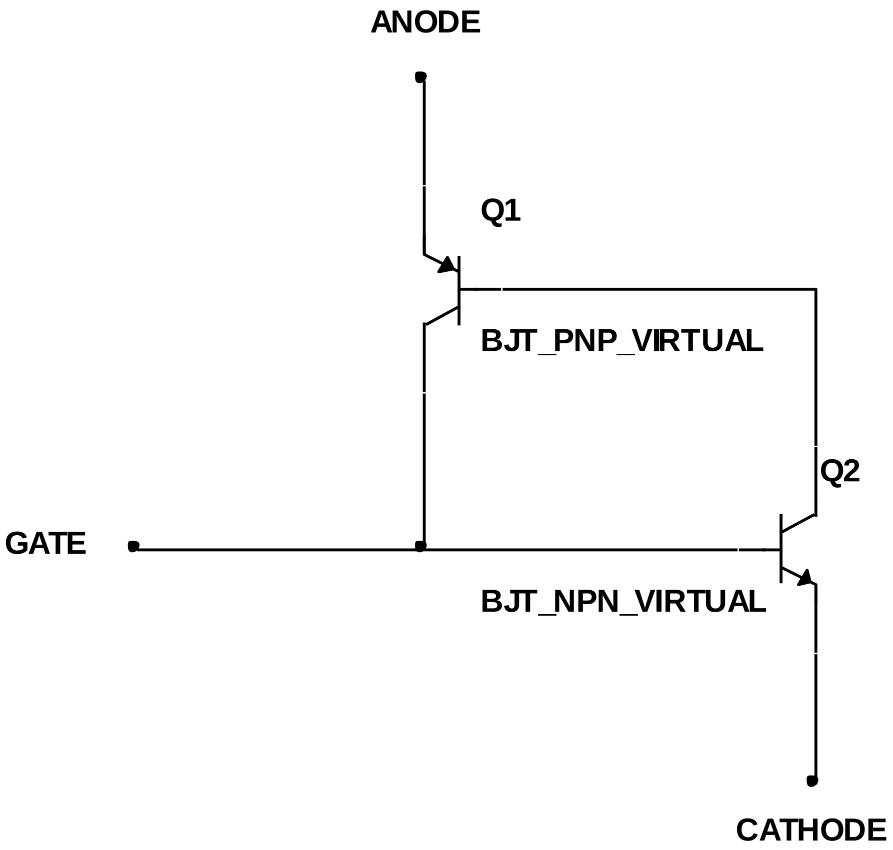
- 4-layer (pnpn) device
- Anode, Cathode as for a conventional pn junction diode

- Cathode Gate brought out for controlling input



Equivalent Circuit

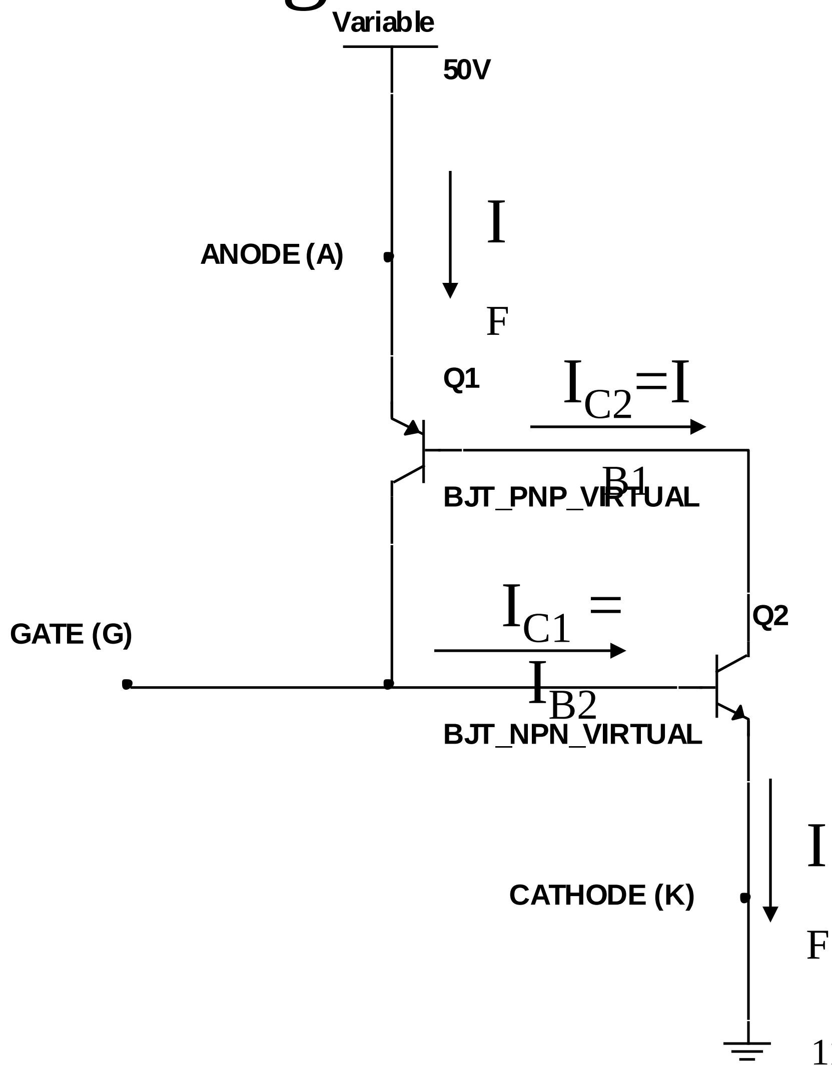




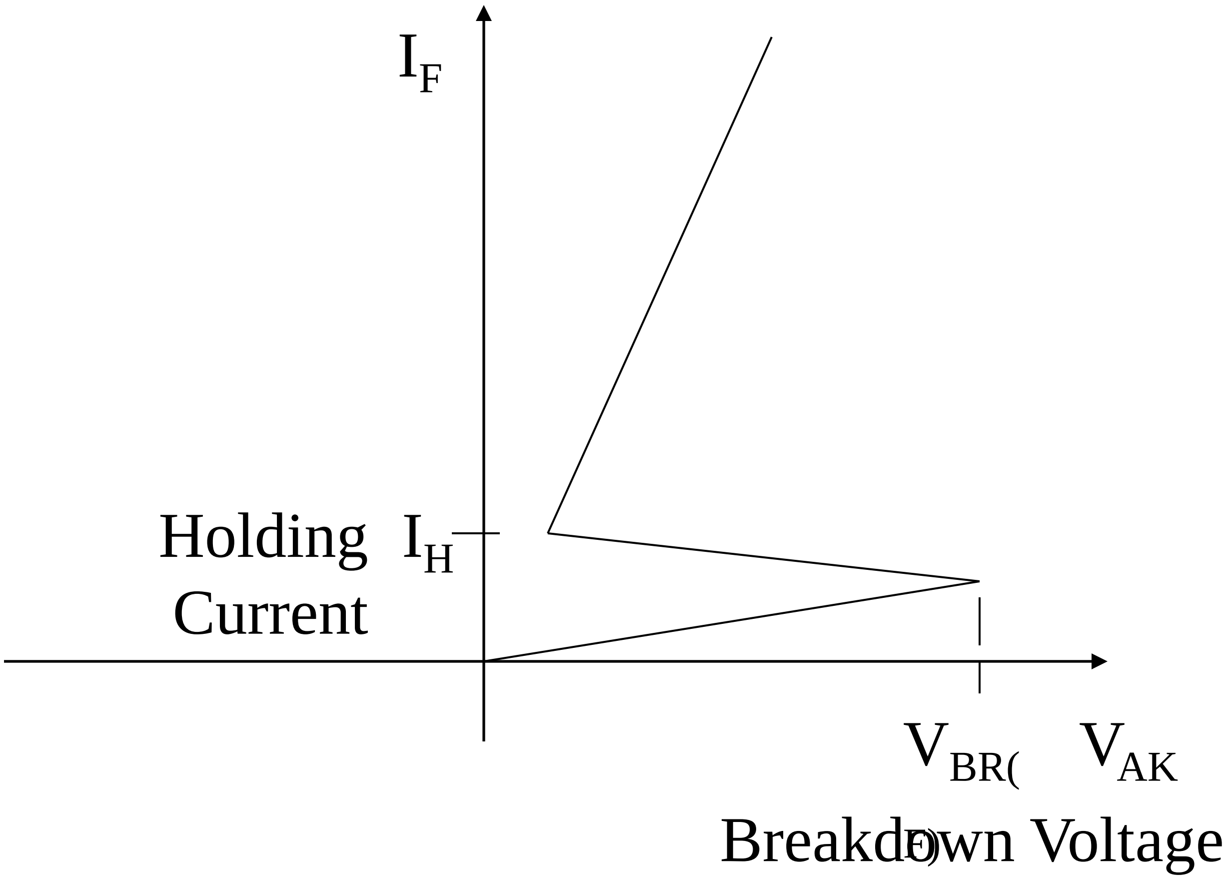
Apply Biasing

With the Gate terminal OPEN, both transistors are OFF. As the applied voltage increases, there will be a "breakdown" that causes both transistors to conduct (saturate) making $I_F > 0$ and $V_{AK} = 0$.

$V_{Breakdown} = V_{BR(F)}$



Volt-Ampere Characteristic



Apply a Gate Current

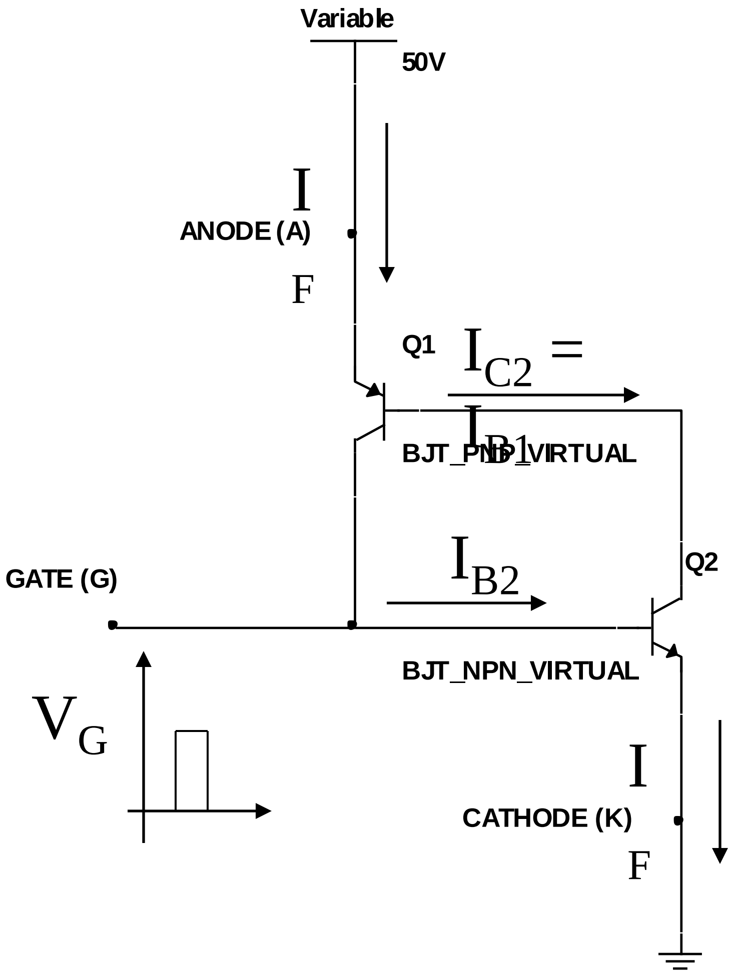
For $0 < V_{AK} < V_{BR(F)}$,

Turn Q_2 ON by applying a current into the Gate

This causes Q_1 to turn ON, and eventually both transistors SATURATE

$V_{AK} = V_{CEsat} + V_{BEsat}$

If the Gate pulse is removed, Q_1 and Q_2 still stay ON!

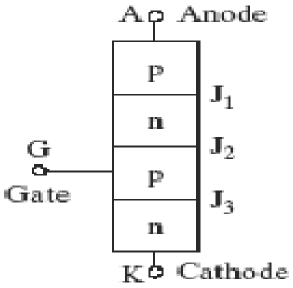


How do you turn it OFF?

- Cause the forward current to fall below the value if the “holding” current, I_H

- Reverse bias the device

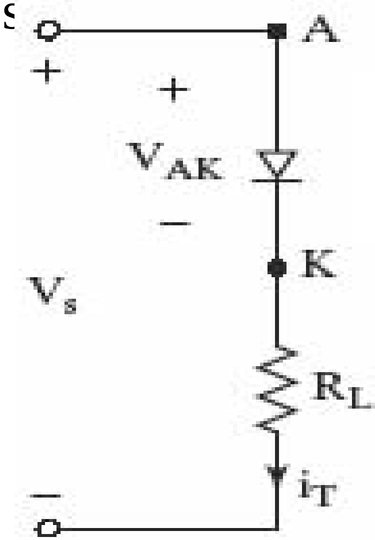
Characteristics of thyristors

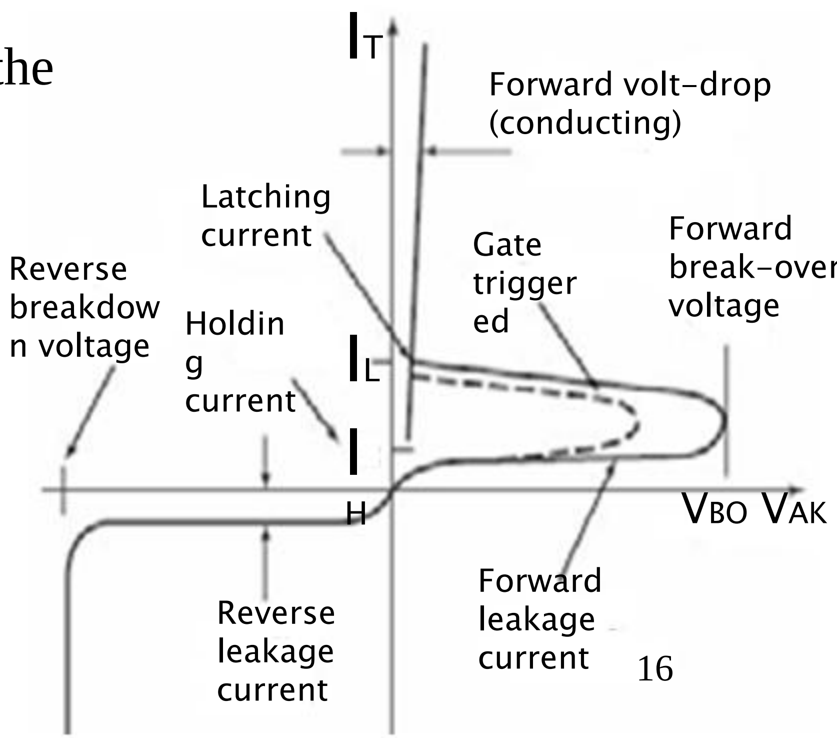


- When the anode is at a positive potential V_{AK} with respect to the cathode with no voltage applied at the gate, junctions J_1 and J_3 are forward biased, while junction J_2 is reverse biased. As J_2 is reverse biased, no conduction takes place.
- Now if V_{AK} is increased beyond the breakdown voltage V_{BO} of the thyristor, avalanche breakdown of J_2 takes place and the thyristor starts conducting.
- If a positive potential V_G is applied at the gate terminal with respect to the cathode, the breakdown of the junction J_2 occurs at a lower value of V_{AK}. By selecting an appropriate value of V_G, the thyristor can be switched into the on state suddenly.

Switching Characteristic (IV)

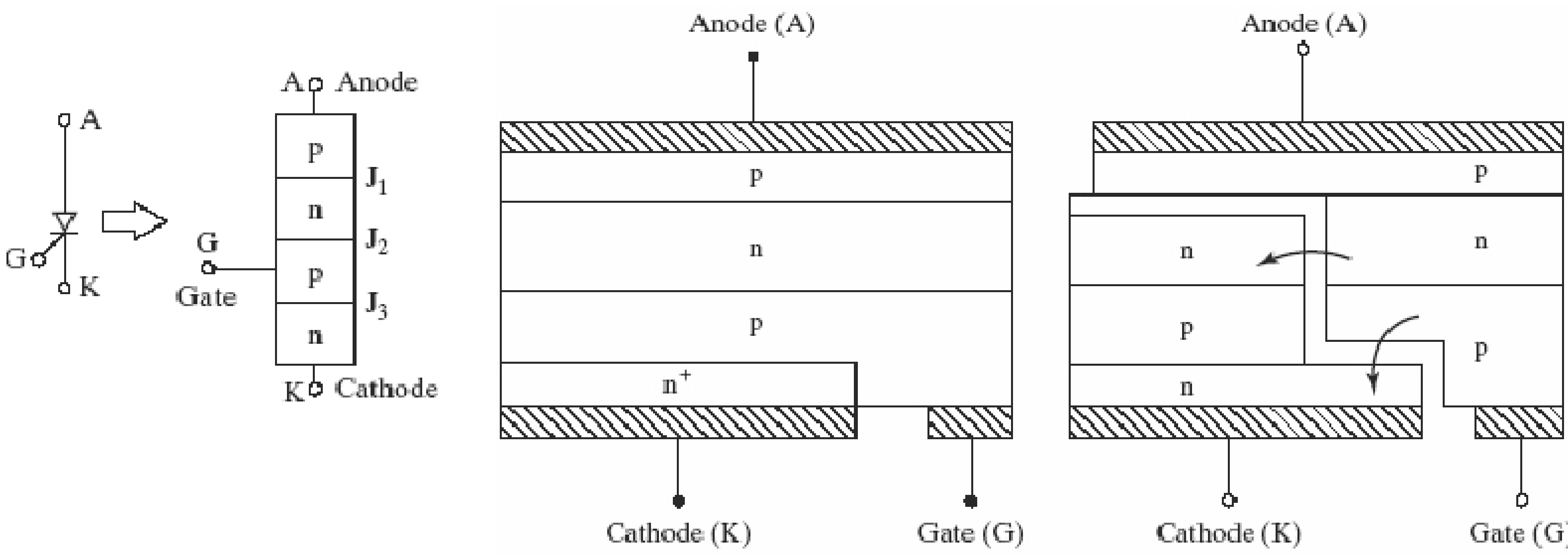
- Forward breakdown voltage V_{BO}
  - The voltage of avalanche breakdown
- Latching current I_L
  - The minimum anode current required to maintain the thyristor in the on-state immediately after it is turned on and the gate signal has been removed
- Holding current I_H
  - The minimum anode current to maintain the thyristor in the on-s
- $I_L > I_H$





Symbol and construction

The thyristor is a four-layer, three terminal semiconducting device, with each layer consisting of alternately N-type or P-type material, for example P-N-P-N. The main terminals, labeled anode and cathode, are across the full four layers, and the control terminal, called the gate, is attached to p-type material near to the cathode.

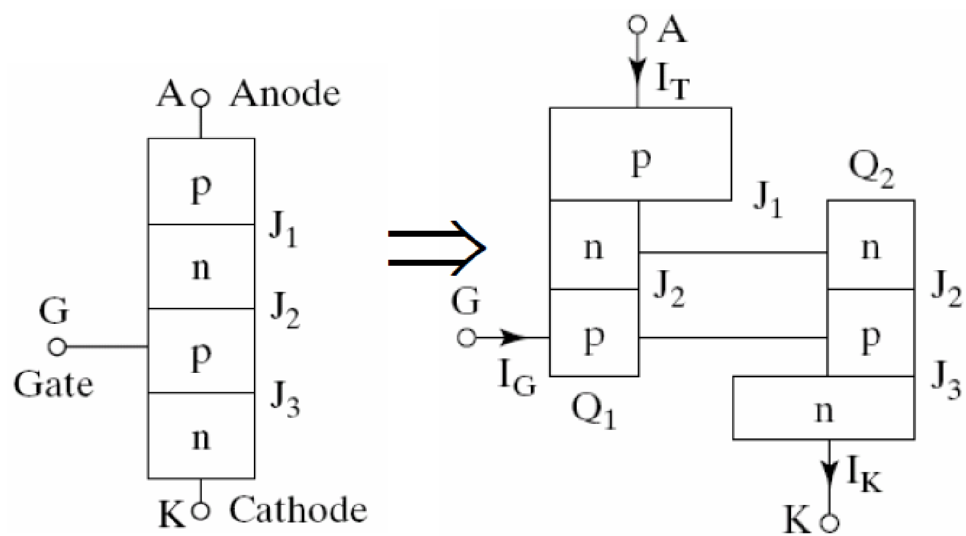


Cross section of *pnpn* structure

Split sections of *npn* and *pnp*

Different types of Thyristors

- Silicon Controlled Rectifier (SCR).
- TRIAC.
- DIAC.
- Silicon Unilateral
  in low voltage av



Construction of
SUS

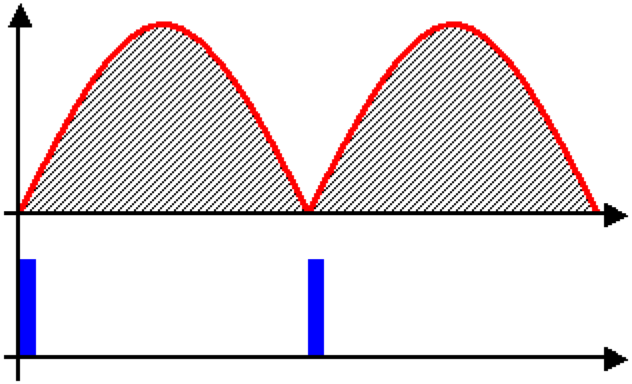
Application

- Mainly used where high currents and voltages are involved, and are often used to control alternating currents, where the change of polarity of the current causes the device to switch off automatically; referred to as Zero Cross operation.

- Thyristors can be used as the control elements for phase angle triggered controllers, also known as phase fired controllers.

Cntd…

- In power supplies for digital circuits, thyristor can be used as a sort of "circuit breaker" or "crowbar" to prevent a failure in the power supply from damaging downstream components, by shorting the power supply output to ground



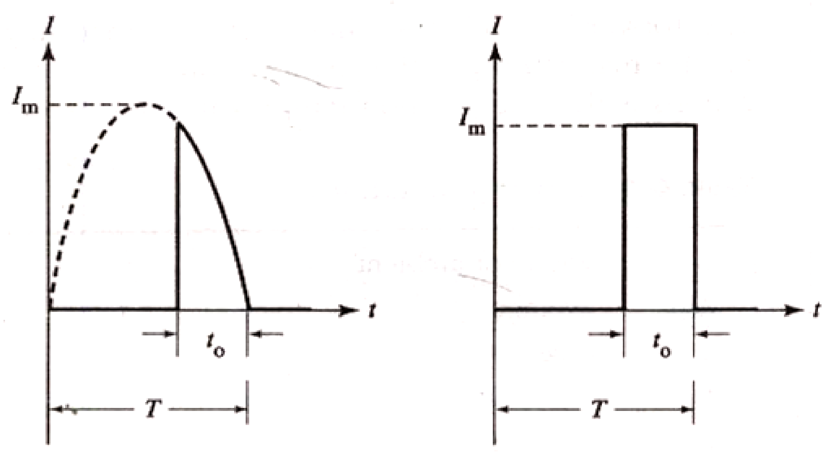
Load voltage regulated by thyristor phase control.
Red trace: load voltage
Blue trace: trigger signal.

SCR Ratings

(a) SCR Current Ratings

1- Maximum Repetitive RMS current Rating

- Average on-state current is the maximum average current value that can be carried by the SCR in its on state.
- RMS value of nonsinusoidal waveform is simplified by approximating it by rectangular waveform.
- This approximation give higher RMS value, but leaves slight safety factor.



After approximating, the RMS value of the current can be found from

$$I_{RMS} = \sqrt{\frac{I_m{}^2 t_o}{T}}$$

- Average value of pulse is

$$I_{AVE} = \frac{I_m t_o}{T}$$

- Form factor is

$$f_o = \frac{I_{RMS}}{I_{AVE}}$$

- Knowing the form factor for given waveform, RMS current can be obtained from

$$I_{RMS} = fo(I_{AVE})$$

- Maximum repetitive RMS current is given by

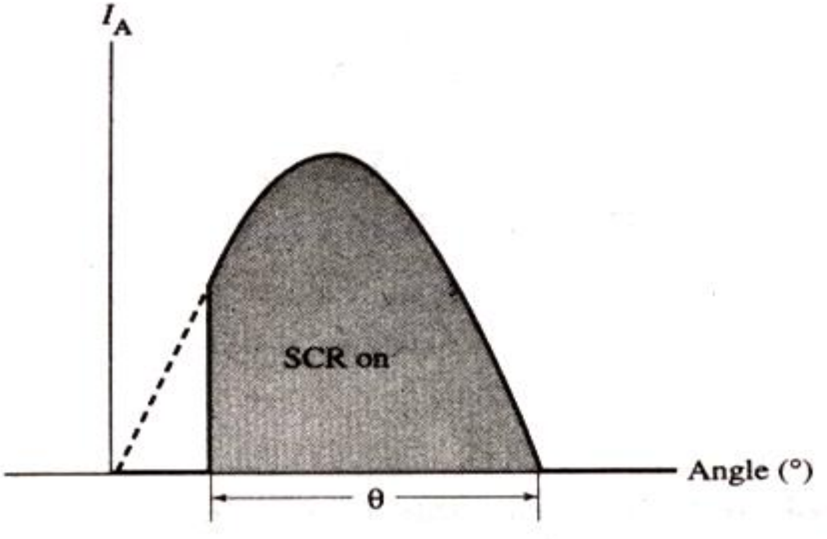
$$I_{T(RMS)} = fo(I_{T(AVE)})$$

- Conduction angle verses form factor

Conduction angle (θ)	Form factor (*f*o)
20°	5.0
40°	3.5
60°	2.7
80°	2.3
100°	2.0
120°	1.8
140°	1.6
160°	1.4
180°	1.3

Conduction Angle

- *Duration for which SCR is on*. It is measured as shown



2- *Surge Current Rating*

Peak anode current that SCR can handle for brief duration.

3- *Latching current*

Minimum anode current that must flow through the SCR in order for it to stay on initially after gate signal is removed.

4- *Holding Current*

Minimum value of anode current, required to maintain SCR in conducting state.

(b) SCR Voltage Ratings

1- Peak repetitive forward blocking voltage

Maximum instantaneous voltage that SCR can block in forward direction.

2- Peak Repetitive Reverse Voltage

Maximum instantaneous voltage that SCR can withstand, without breakdown, in reverse direction.

3- Non-repetitive peak reverse voltage

Maximum transient reverse voltage that SCR can withstand.

(c) SCR Rate-of-Change Ratings

1- (di/dt rating)

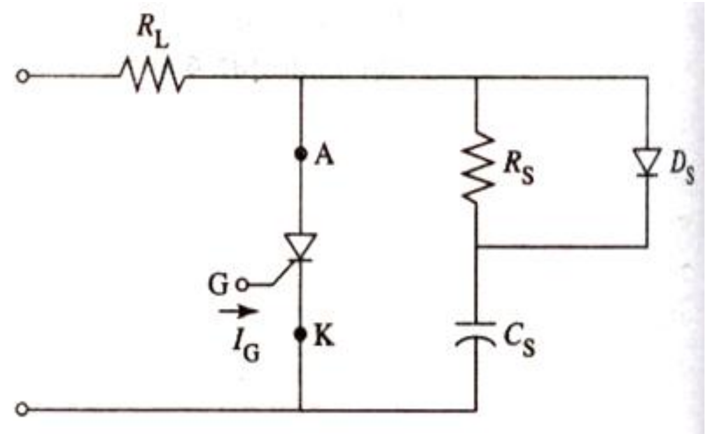
Critical rate of rise of on-state current. It is the rate at which anode current increases and must be less than rate at which conduction area increases.

To prevent damage to SCR by high di/dt value, small inductance is added in series with device. Vaue of required inductance is

$$L \geq \frac{V_p}{(di/dt)_{max}}$$

2- dv/dt rating

Maximum rise time of a voltage pulse that can be applied to the SCR in the off state without causing it to fire. Unscheduled firing due to high value of dv/dt can be prevented by using RC snubber circuit.



(d) Gate Parameters

1- Maximum Gate Peak Inverse Voltage

Maximum value of negative DC voltage that can be applied without damaging the gate-cathode junction.

2-Maximum Gate Trigger Current

Maximum DC gate current allowed to turn on the device.

3- Maximum gate trigger voltage

DC voltage necessary to produce maximum gate trigger current.

4- Maximum Gate Power Dissipation

Maximum instantaneous product of gate current and gate voltage that can exist during forward-bias.

5- Minimum gate trigger voltage

Minimum DC gate-to-cathode voltage required to trigger the SCR.

6-Minimum gate trigger current

Minimum DC gate current necessary to turn SCR on.

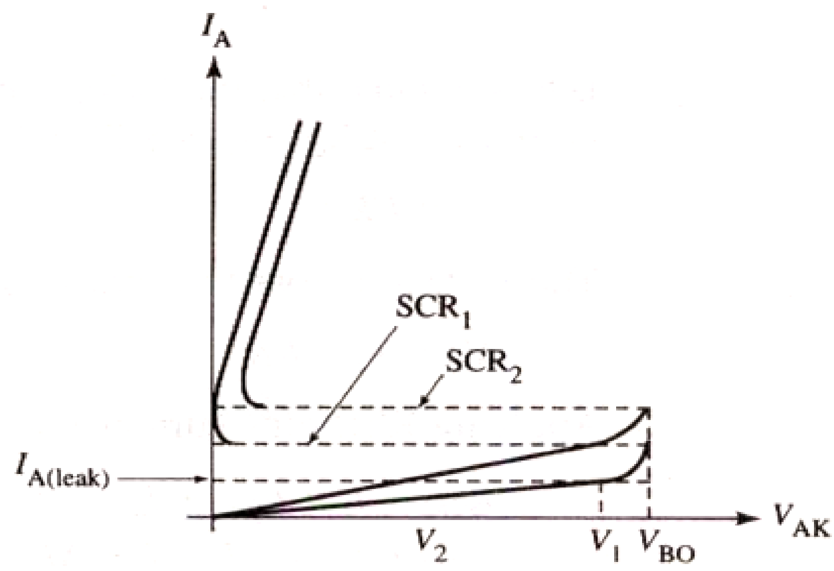
Series and Parallel SCR Connections

SCRs are connected in series and parallel to extend voltage and current ratings.

For high-voltage, high-current applications, series-parallel combinations of SCRs are used.

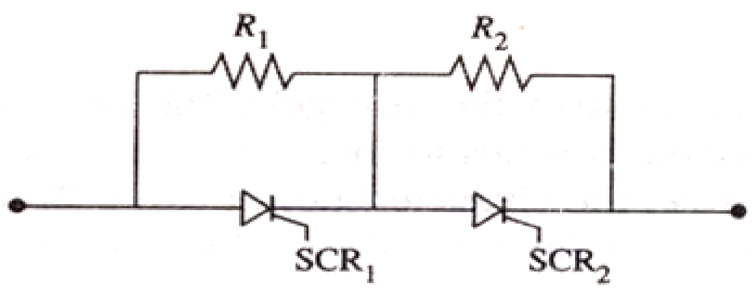
SCRs in Series

- Unequal distribution of voltage across two series SCRs.

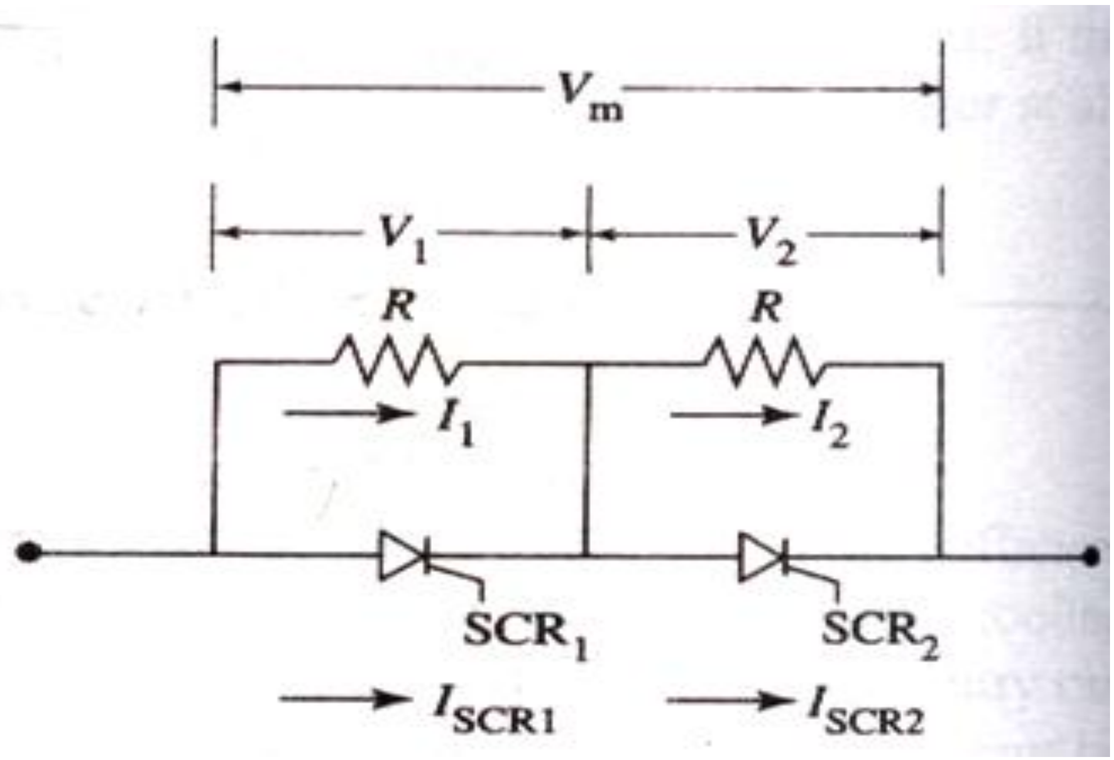


- Two SCRs do not share the same supply voltage. Maximum voltage that SCRs can block is V_1+V_2, not $2V_{BO}$.

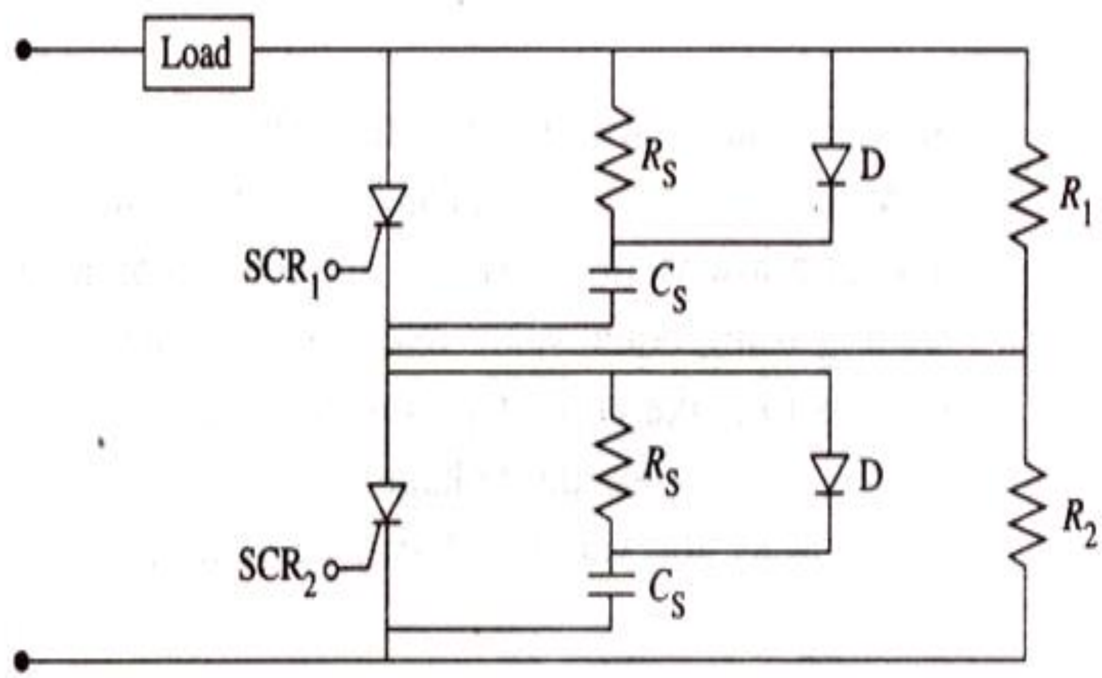
- **Resistance equalization**



- **Voltage equalization**

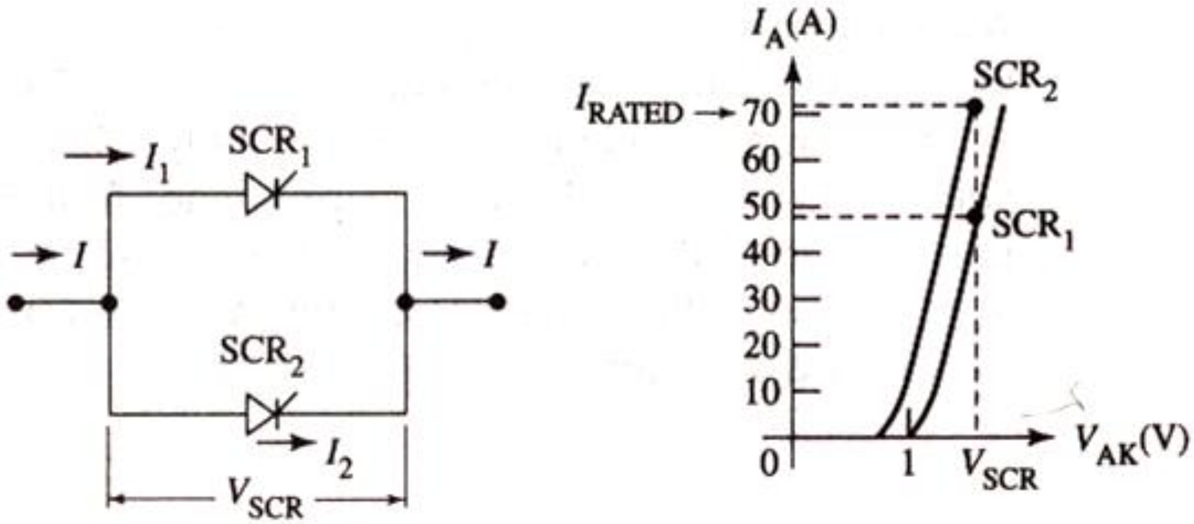


- **<u>RC equalization for SCRs connected in series.</u>**



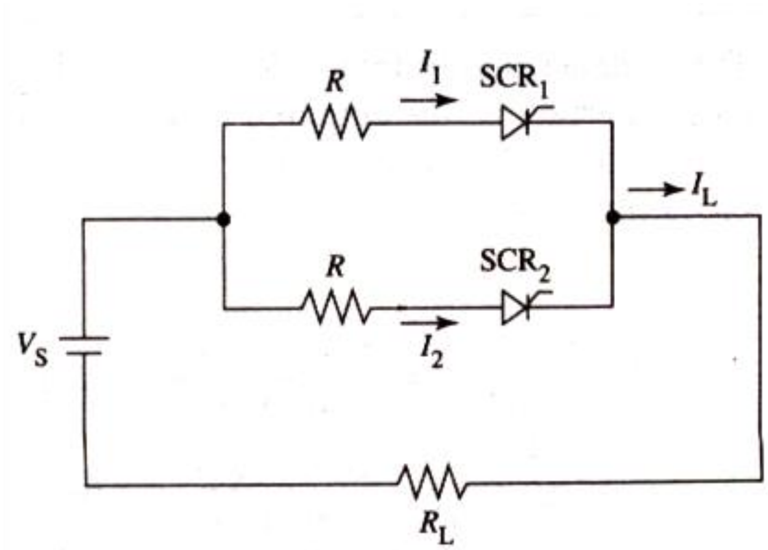
SCRs In Parallel

- Unequal current sharing between two SCRs is shown:



- Total rated current of parallel connection is I_1+I_2, not $2I_2$.

- With unmatched SCRs, equal current sharing is achieved by adding low value resistor or inductor in series with each SCR, as shown below.



- Value of resistance

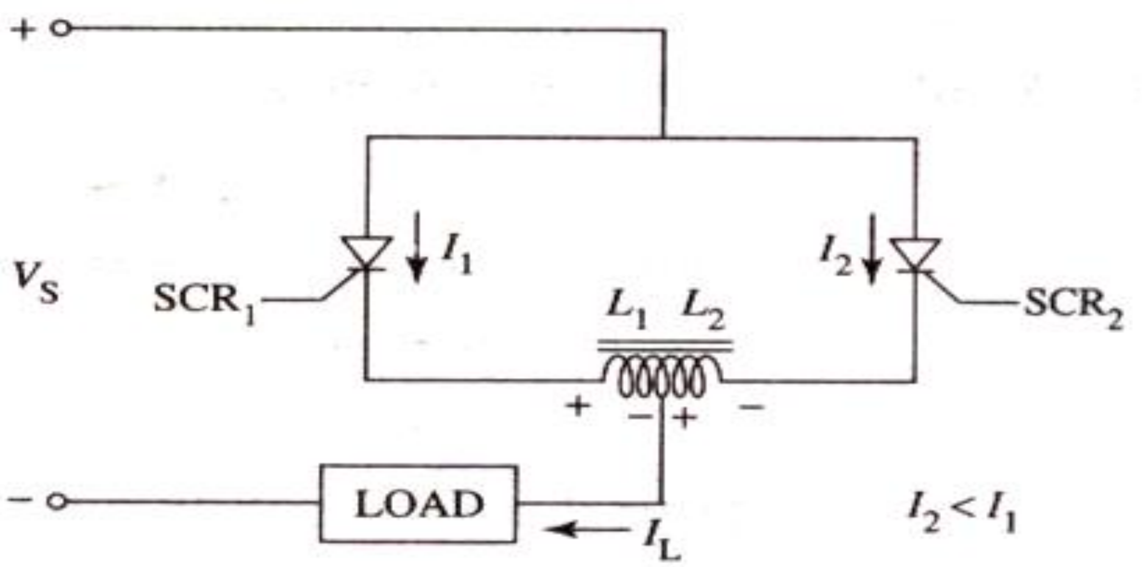
$$R=\frac{V_1-V_2}{I_2-I_1}$$

Current sharing in SCRs with parallel reactors

Equalization using resistors is inefficient due to

- *Extra power loss*
- *Noncompansation for unequal SCR turn-on and turn-off times.*
- *Damage due to overloading*

SCRs with center-tapped reactors is shown below.



SCR Gate-Triggering Circuits

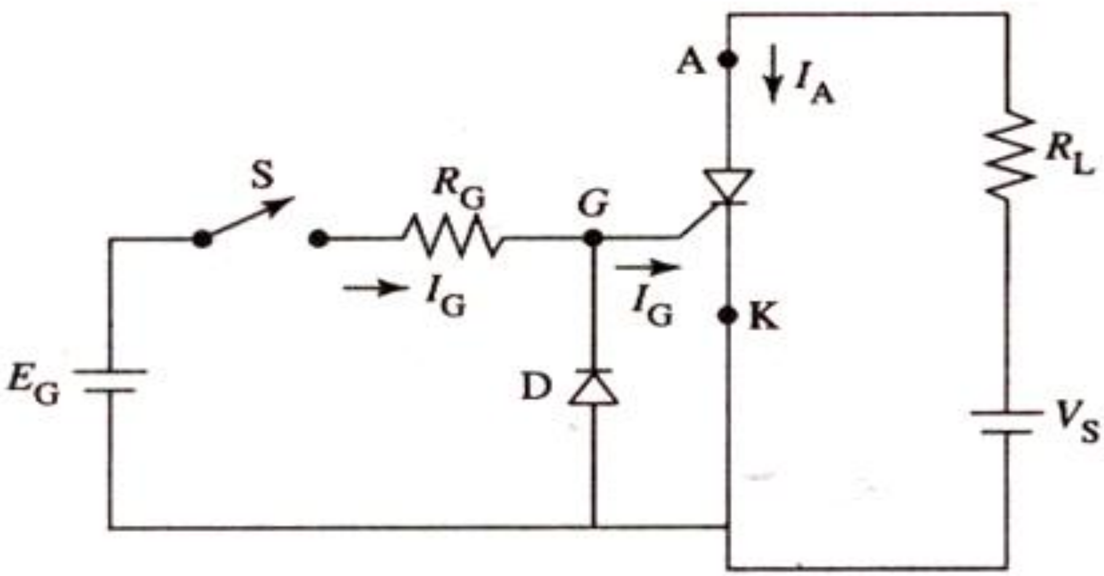
- Triggering circuits provide firing signal to turn on the SCR at precisely the correct time.
- Firing circuits must have following properties

1. Produce gate signal of suitable magnitude and sufficiently short rise time.
2. Produce gate signal of adequate duration.
3. Provide accurate firing control over the required range.
4. Ensure that triggering does not occur from false signals or noise
5. In AC applications, ensure that the gate signal is applied when the SCR is forward-biased
6. In three-phase circuits, provide gate pulses that are 120° apart with respect to the reference point
7. Ensure simultaneous triggering of SCRs connected in series or in parallel.

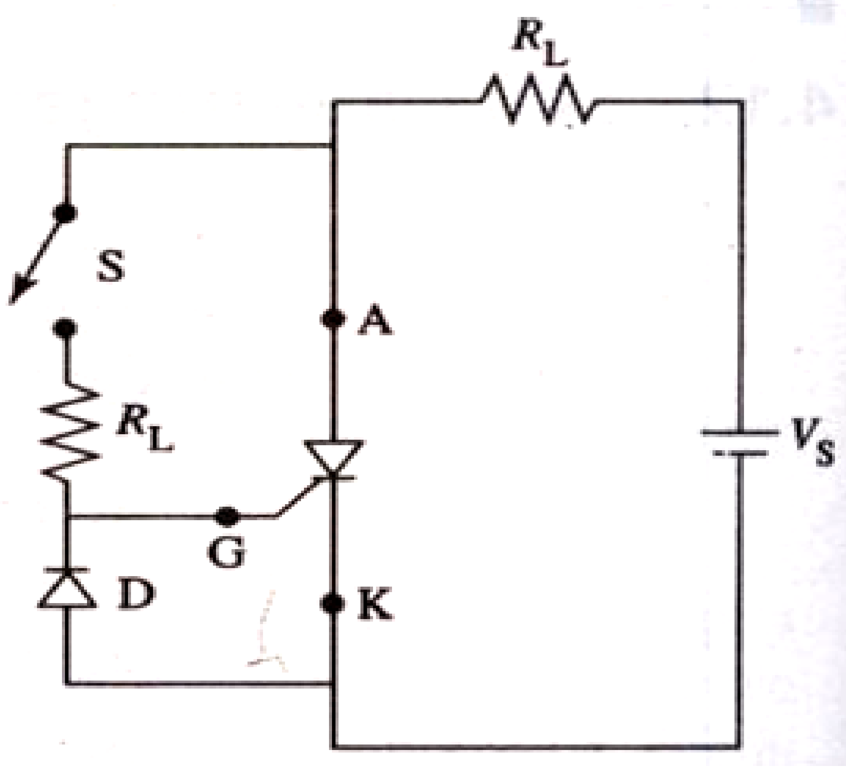
Types Of Gate Firing Signals

1. DC signals
2. Pulse signals
3. AC signals

(a) DC Gating Signal From Separate Source



DC Gating signals from Same Source



Disadvantage of DC gating Signals

1. Constant DC gate signal causes gate power dissipation

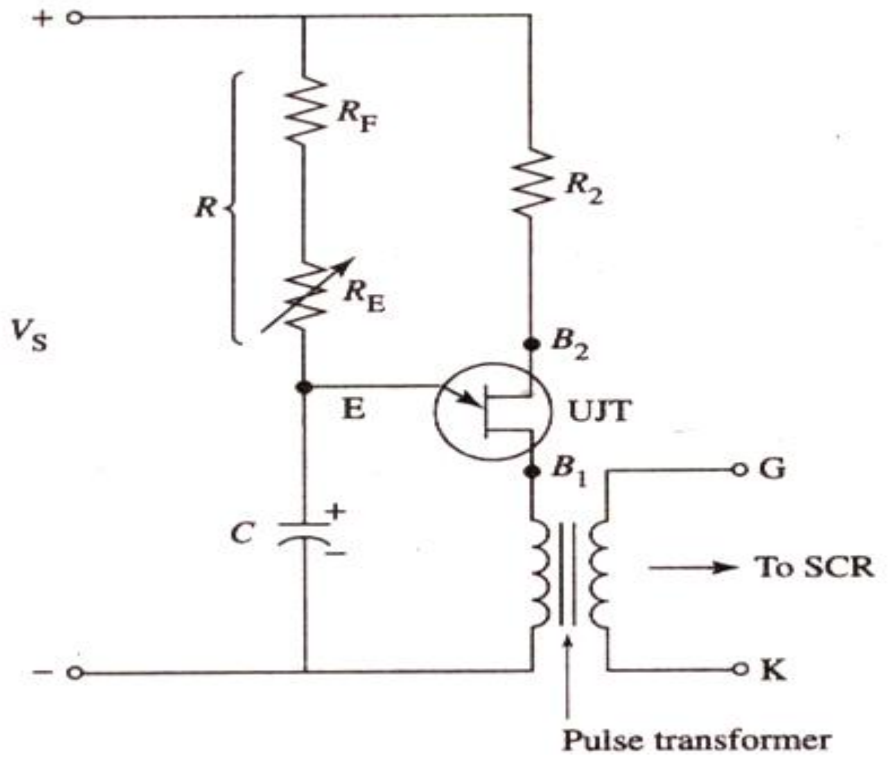
2. DC gate signals are not used for firing SCRs in AC applications, because presence of positive gate signal during negative half cycle would increase the reverse anode current and possibly destroy the device.

(2) Pulse Signals

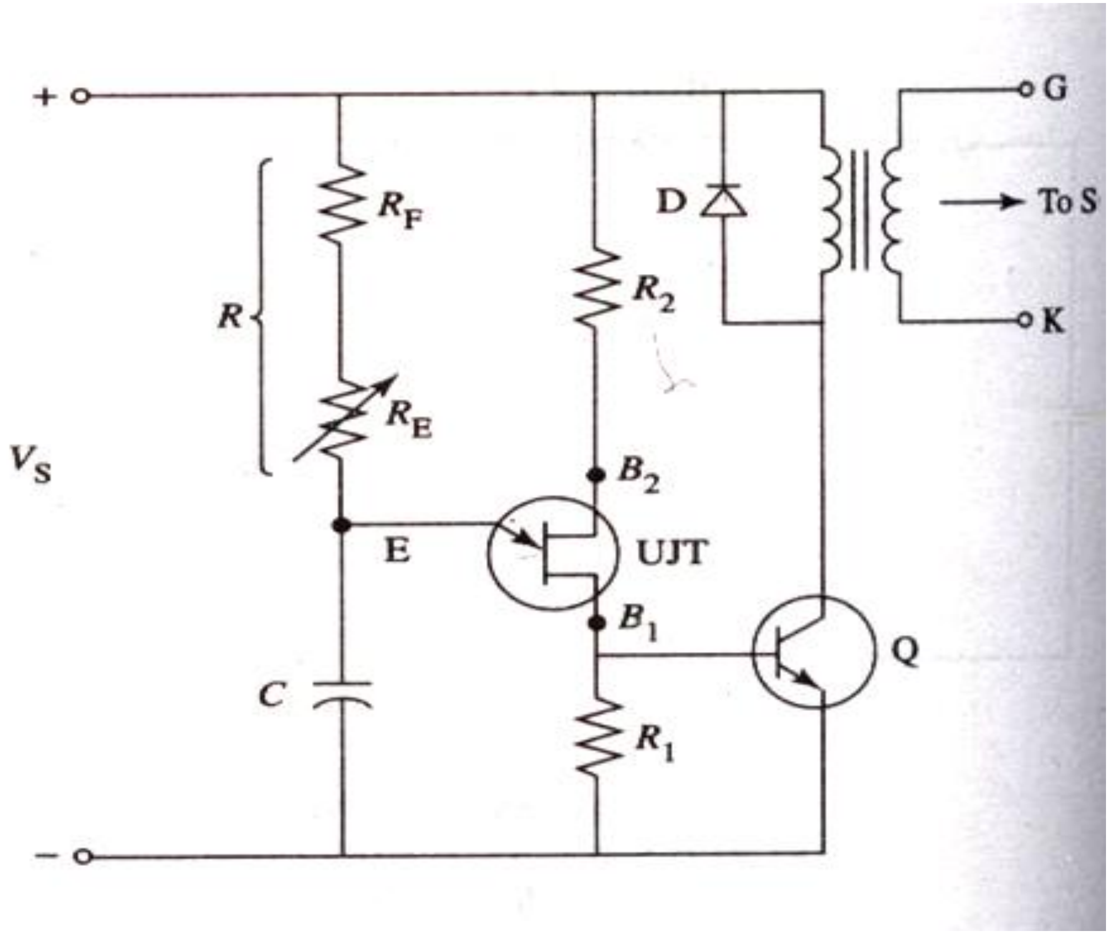
1. Instead of continuous DC signal, single pulse or train of pulses is generated.
2. It provides precise control of point at which SCR is fired.
3. It provides electrical isolation between SCR and gate-trigger circuit.

SCR trigger circuits using UJT oscillator

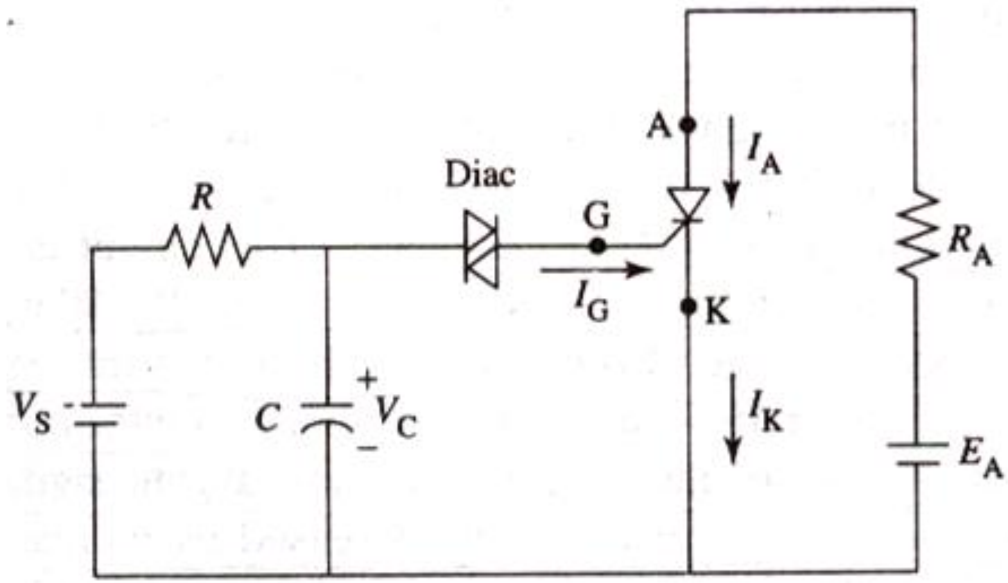
- Circuit A



Circuit B

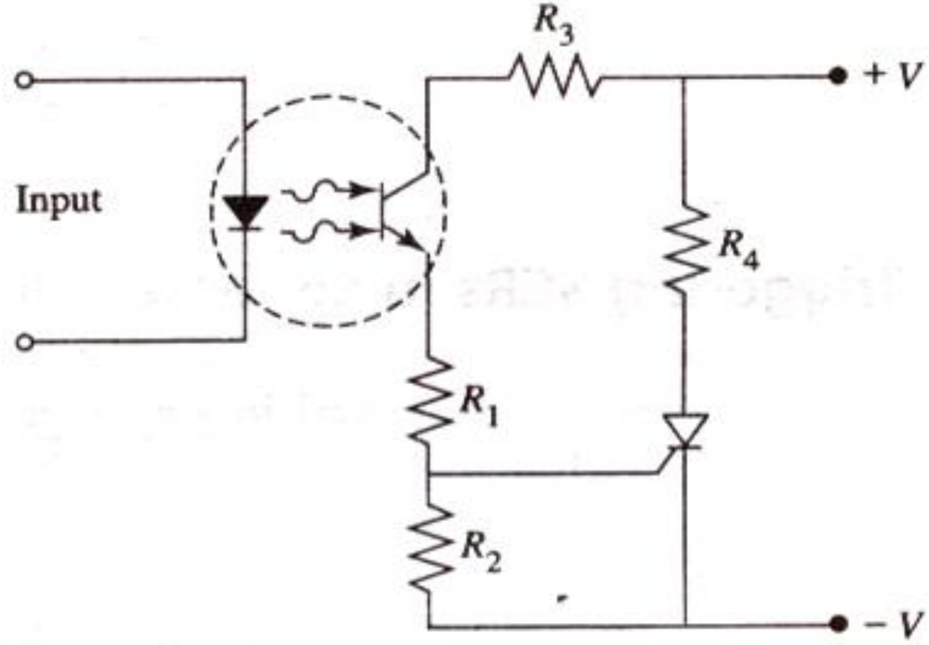


SCR trigger circuit using DIAC

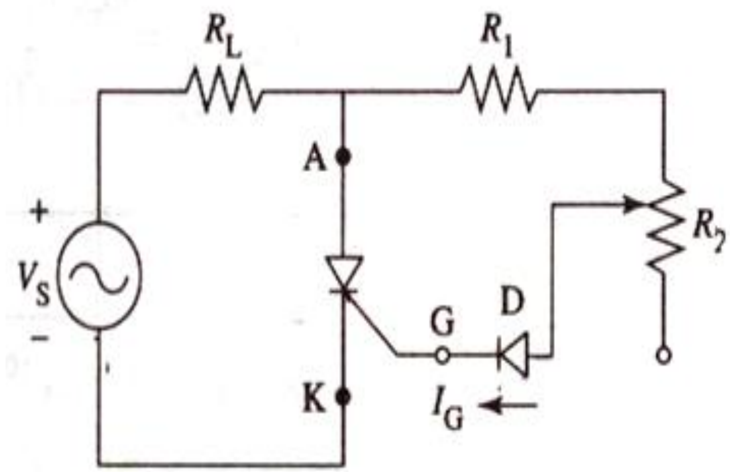


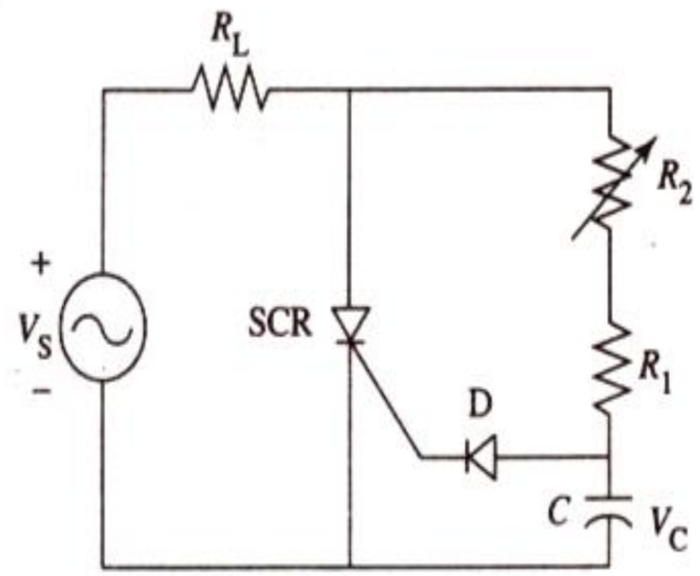
(a)

SCR trigger circuit using Optocoupler



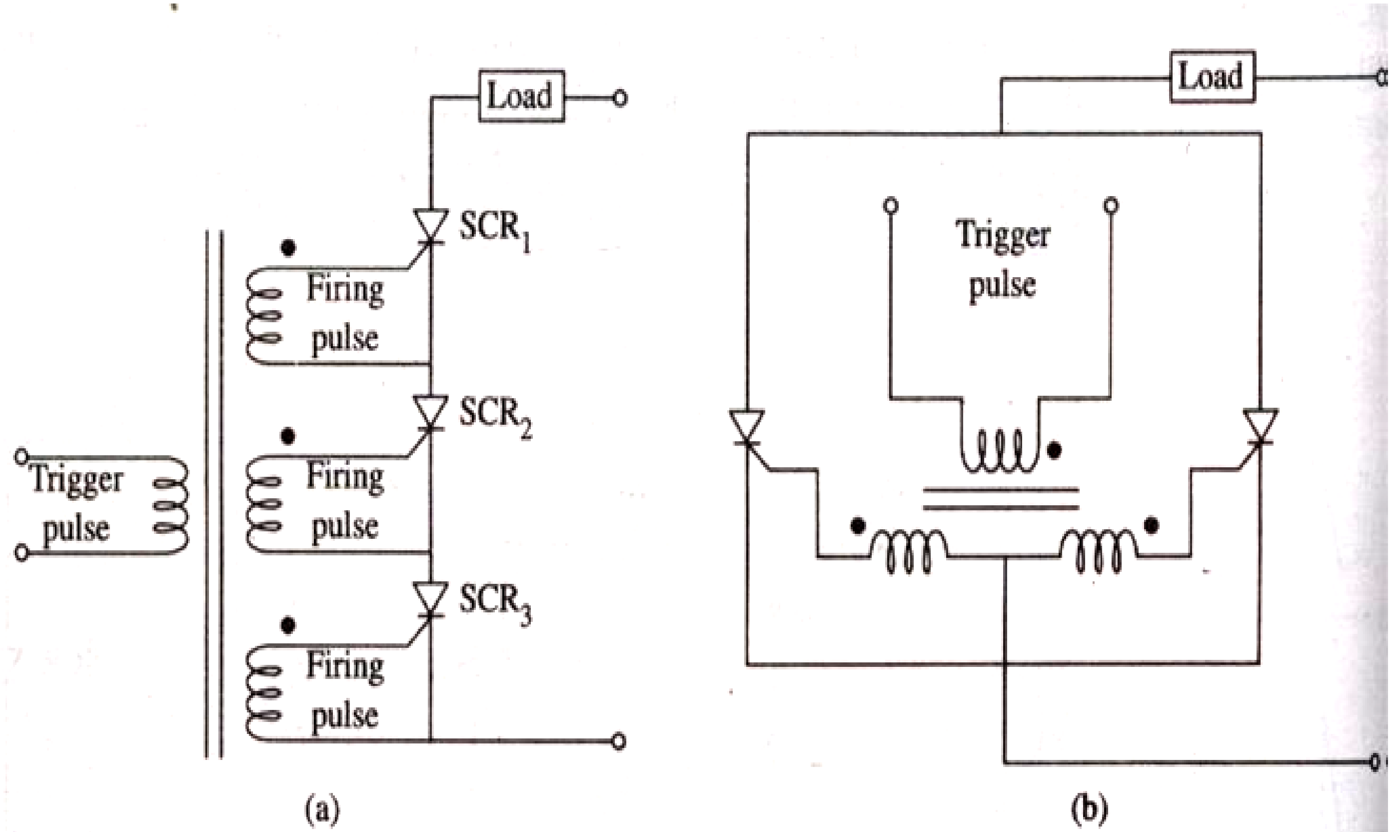
(c) AC Signals





Resistive phase control RC
phase control

Triggering SCRs in Series and in Parallel



SCR Turnoff (Commutation) Circuits

- What is Commutation?

  The process of turning off an SCR is called commutation.

  It is achieved by

1. Reducing anode current below holding current
2. Make anode negative with respect to cathode

- Types of commutation are:

1. Natural or line commutation
2. Forced commutation

SCR Turnoff Methods

1. Diverting the anode current to an alternate path

2. Shorting the SCR from anode to cathode

3. Applying a reverse voltage (by making the cathode positive with respect to the anode) across the SCR

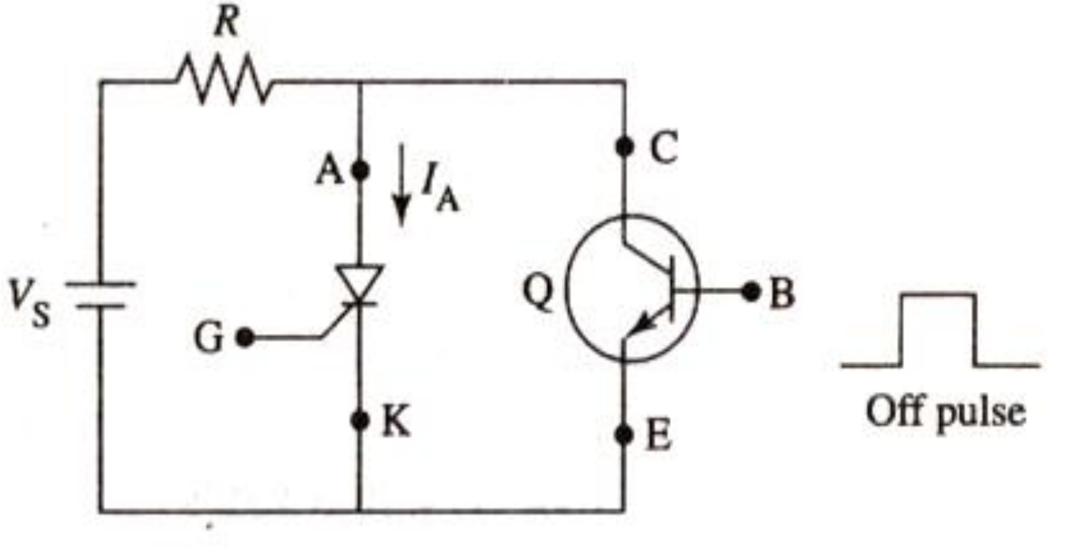
4. Forcing the anode current to zero for a brief period

5. Opening the external path from its anode supply voltage

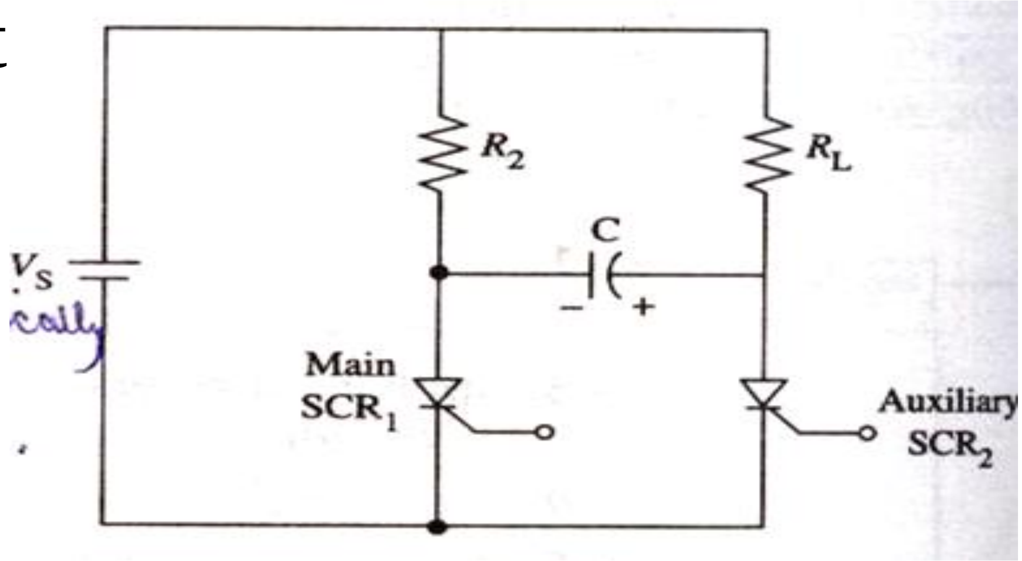
6. Momentarily reducing supply voltage to zero

(1) Capacitor Commutation

- SCR turnoff circuit using a transistor switch



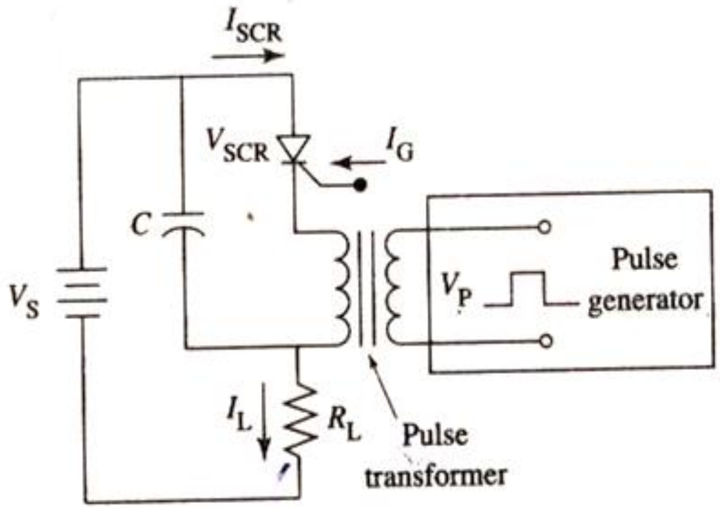
- SCR turnoff circuit using commutation capacit



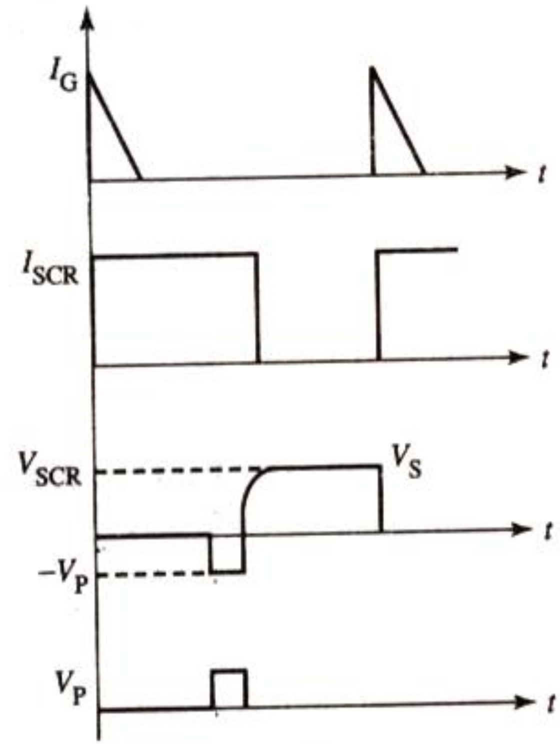
- Value of capacitance is determined by:

$$C >= \frac{t_{OFF}}{0.693R_L}$$

(2) Commutation By External Source



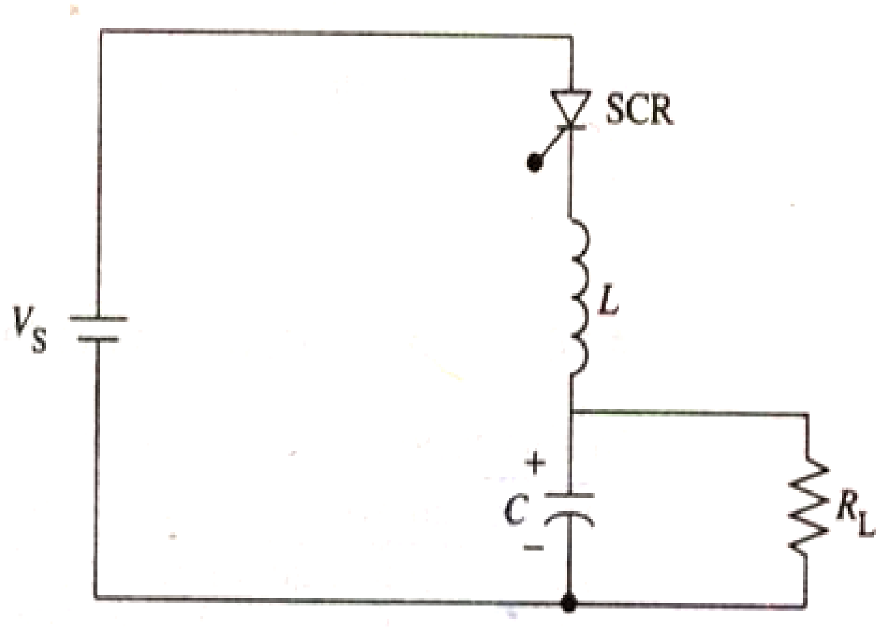
(a)



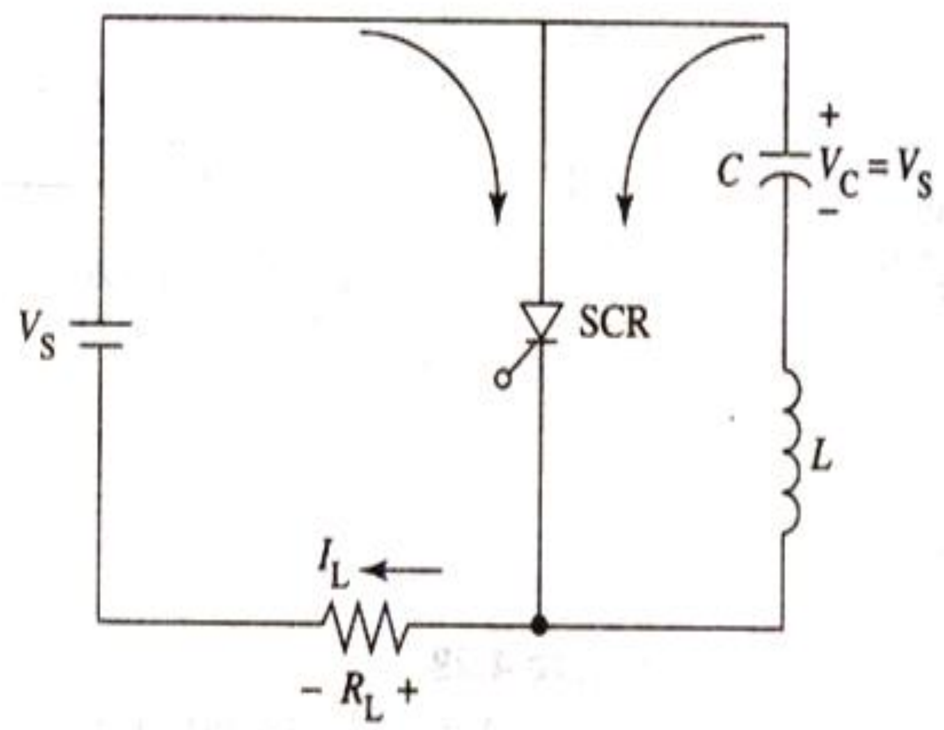
(b)

(3) Commutation by Resonance

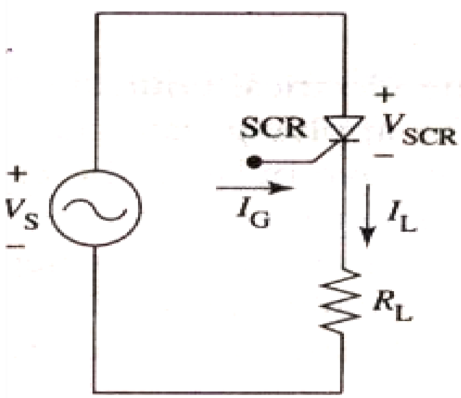
. Series resonant turnoff circuit

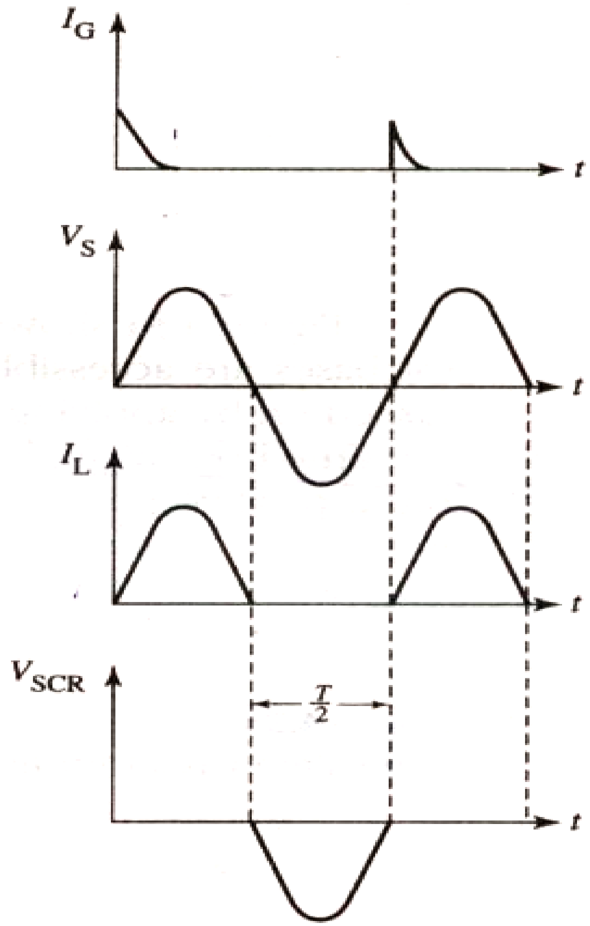


- Parallel resonant turnoff circuit



(4) AC line commutation





Other members of Thyristor Family

Power Semiconductor Switches

Power Diodes
2 layer device

Power Transistors
3 layer Device

Thyristors
4 layer Device

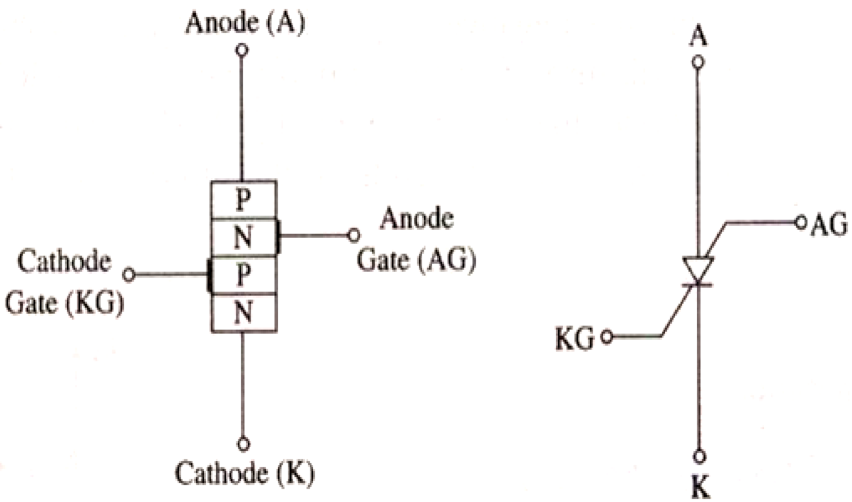
- **Thyristor devices can convert and control large amounts of power in AC or DC systems while using very low power for control.**
- **Thyristor family includes**

  1- Silicon controlled switch (SCR)
  2- Gate-turnoff thyristor (GTO)
  3- Triac
  4- Diac
  5- Silicon controlled switch (SCS)
  6- Mos-controlled switch (MCT)

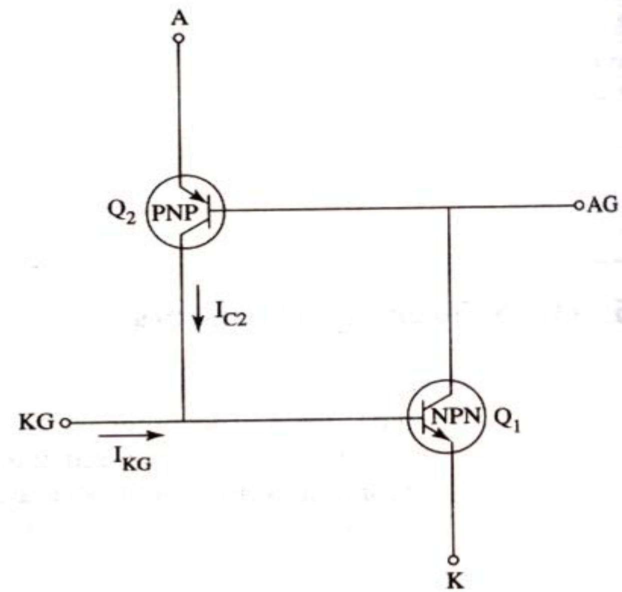
Other Types of Thyristors

1. Silicon Controlled Switch (SCS)
2. Gate Turnoff Thyristor (GTO)
3. DIAC
4. TRIAC
5. MOS-Controlled Thyristor (MCT)

1. SCS



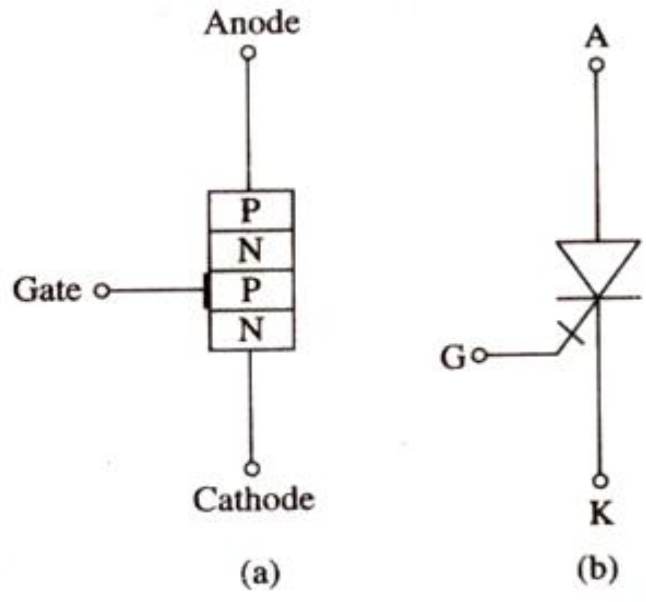
(a) (b)

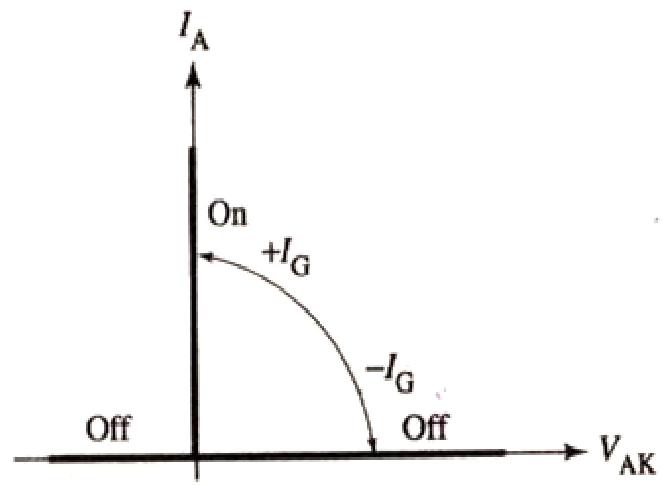


Structure
Symbol

Equivalent circuit
for SCS

(2) GTO

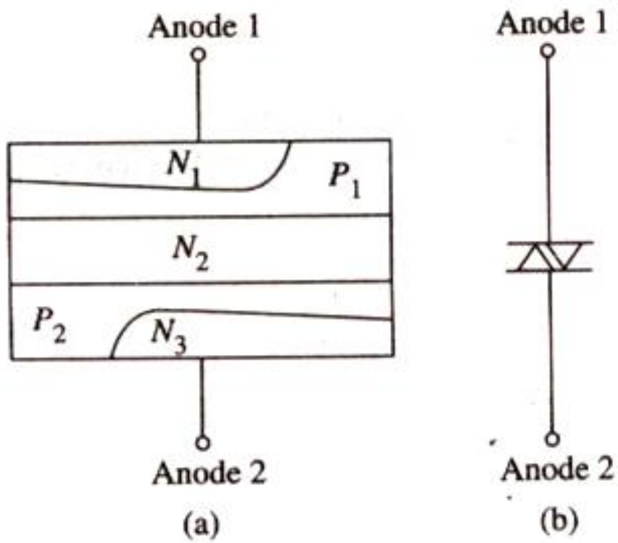


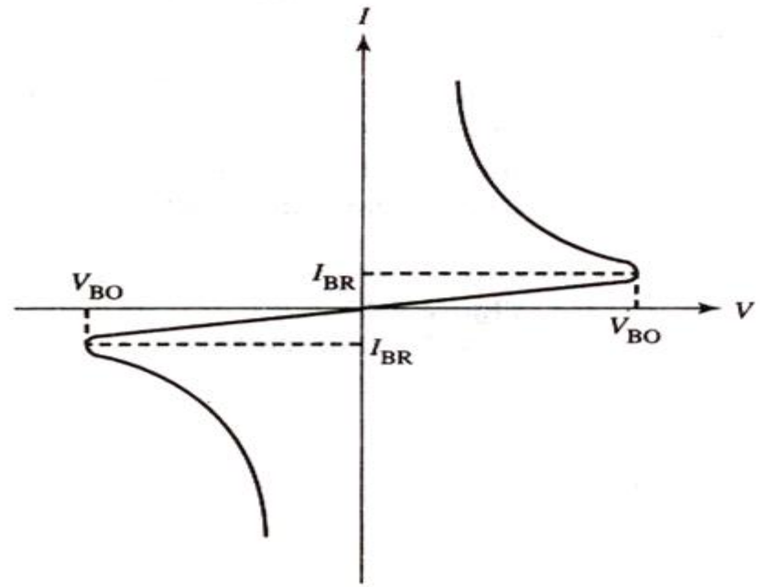


Structure
Symbol

GTO Ideal VI
characteristiccs

(3) DIAC

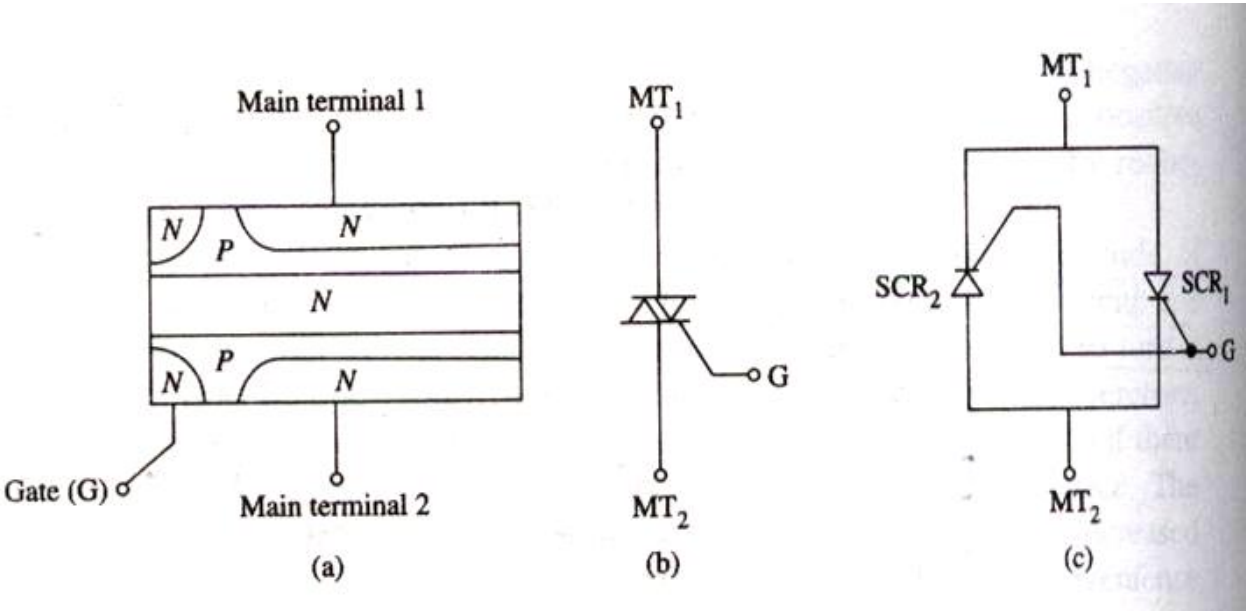




Structure
Symbol

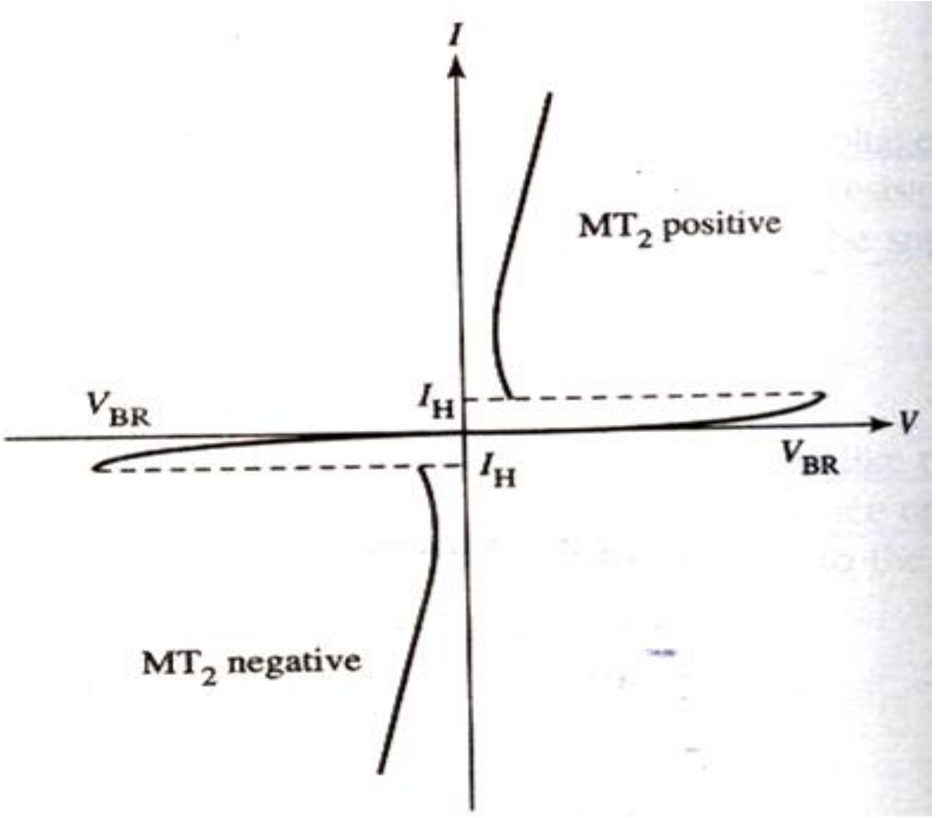
VI characteristics
of diac

(4) Triac

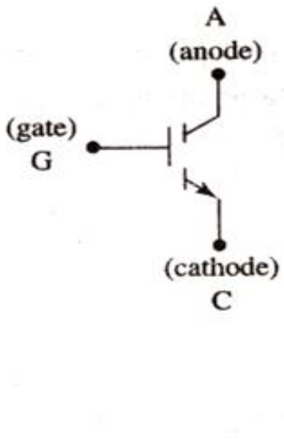


Structure Symbol SCR
equivalent circuit

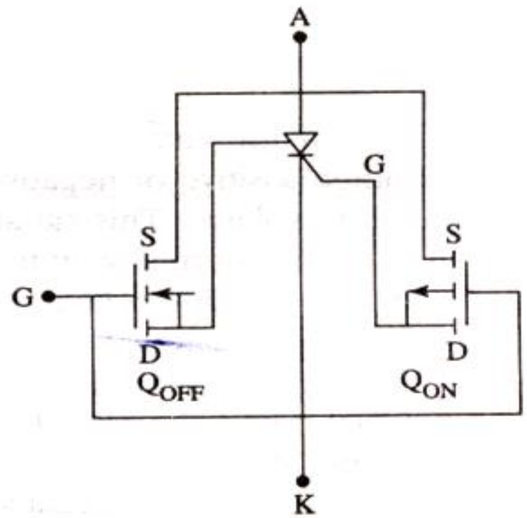
Triac VI characteristics



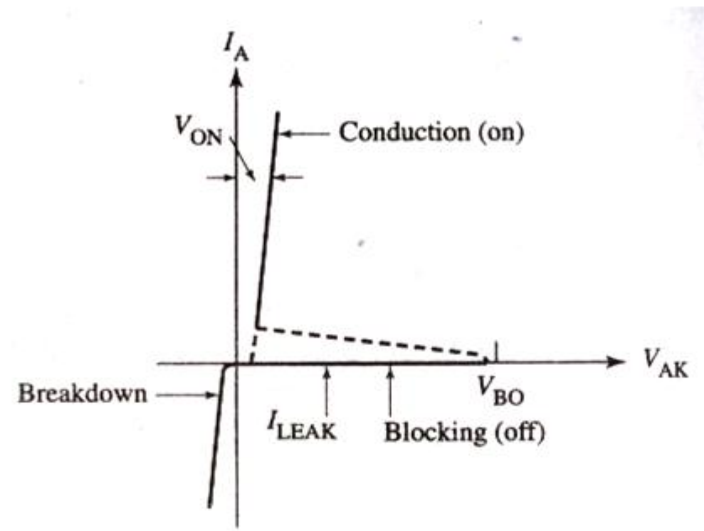
(5) MCT



(a)



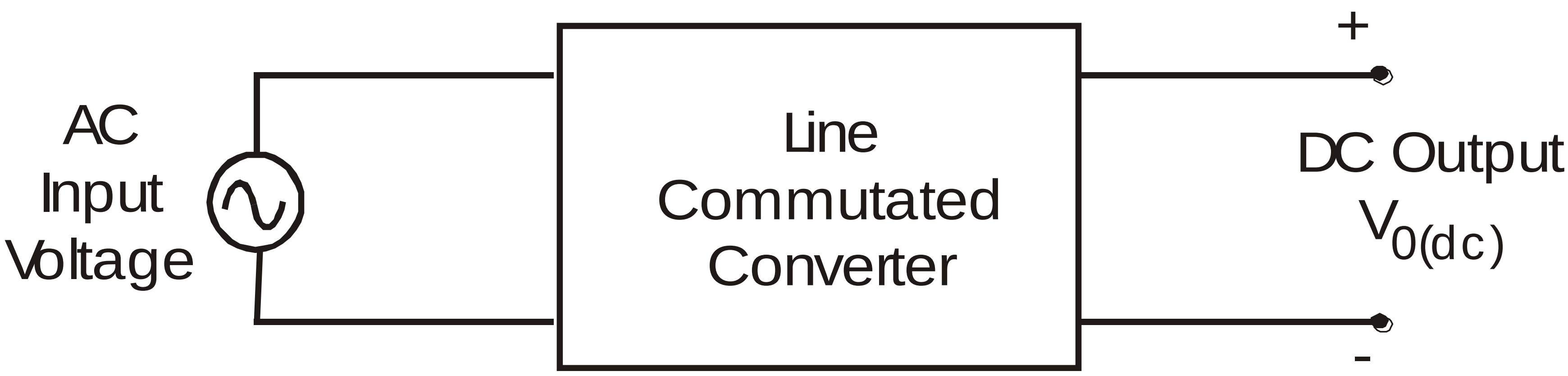
(b)



Symbol equivalent MCT VI
circuit characteristics

UNIT-II AC-DC CONVERTERS (1-PHASE & 3-PHASE CONTROLLED RECTIFIERS)

Introduction to Line commutated Inverter



- Type of input: Fixed voltage, fixed frequency ac power supply.
- Type of output: Variable dc output voltage
- Type of commutation: Natural / AC line commutation

Different types of
Line Commutated Converters

- AC to DC Converters (Phase controlled rectifiers)
- AC to AC converters (AC voltage controllers)
- AC to AC converters (Cyclo-converters) at low output frequency.

Differences Between
Diode Rectifiers
&
Phase Controlled Rectifiers

Cntd…

- The diode rectifiers are referred to as uncontrolled rectifiers .
- The diode rectifiers give a fixed dc output voltage .
- Each diode conducts for one half cycle.
- Diode conduction angle = 180^0 or π radians.
- We can not control the dc output voltage or the average dc load current in a diode rectifier circuit.

Cntd…

Single phase half wave diode rectifier gives an

Average dc output voltage $V_{O(dc)} = \dfrac{V_m}{\pi}$

Single phase full wave diode rectifier gives an

Average dc output voltage $V_{O(dc)} = \dfrac{2V_m}{\pi}$

Applications of Phase Controlled Rectifiers

- DC motor control in steel mills, paper and textile mills employing dc motor drives.
- AC fed traction system using dc traction motor.
- Electro-chemical and electro-metallurgical processes.
- Magnet power supplies.
- Portable hand tool drives.

Classification of Phase Controlled Rectifiers

- Single Phase Controlled Rectifiers.
- Three Phase Controlled Rectifiers.

Different types of Single Phase Controlled Rectifiers.

- Half wave controlled rectifiers.
- Full wave controlled rectifiers.
  - Using a center tapped transformer.
  - Full wave bridge circuit.
    - Semi converter.
    - Full converter.

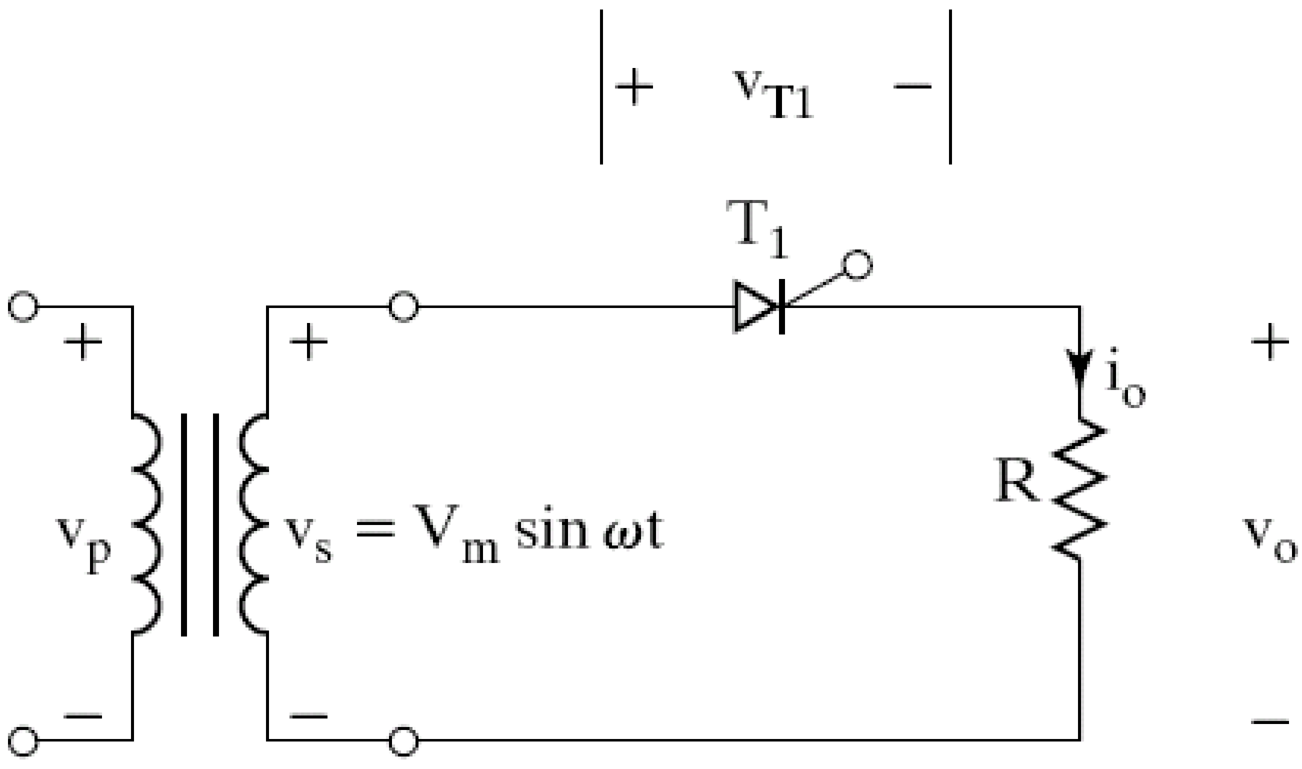
Different Types of Three Phase Controlled Rectifiers

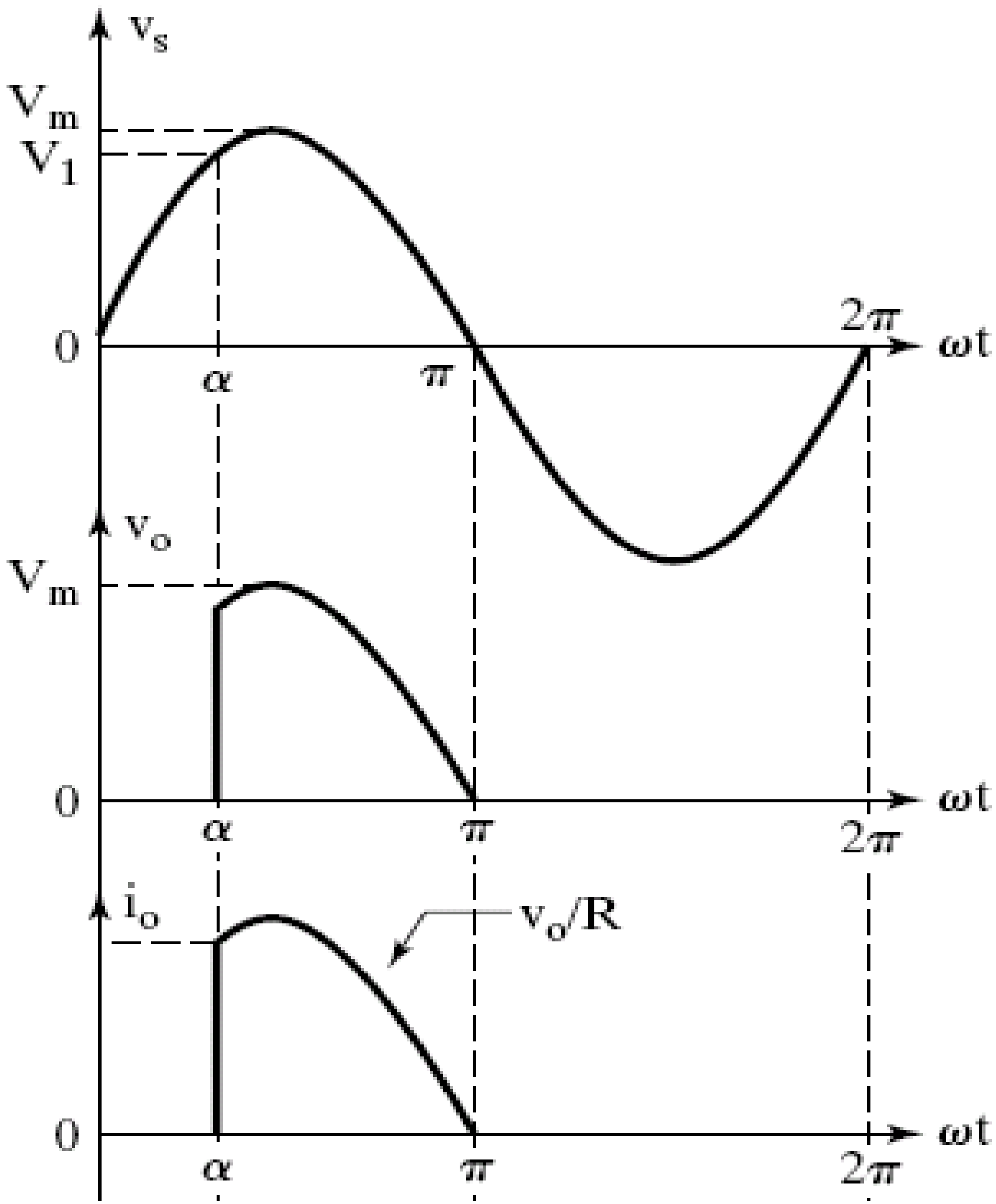
- Half wave controlled rectifiers.
- Full wave controlled rectifiers.
    - Semi converter (half controlled bridge converter).
    - Full converter (fully controlled bridge converter).

Principle of Phase Controlled Rectifier Operation

Principle of Phase Controlled Rectifier Operation

Single Phase Half-Wave Thyristor Converter with a Resistive Load

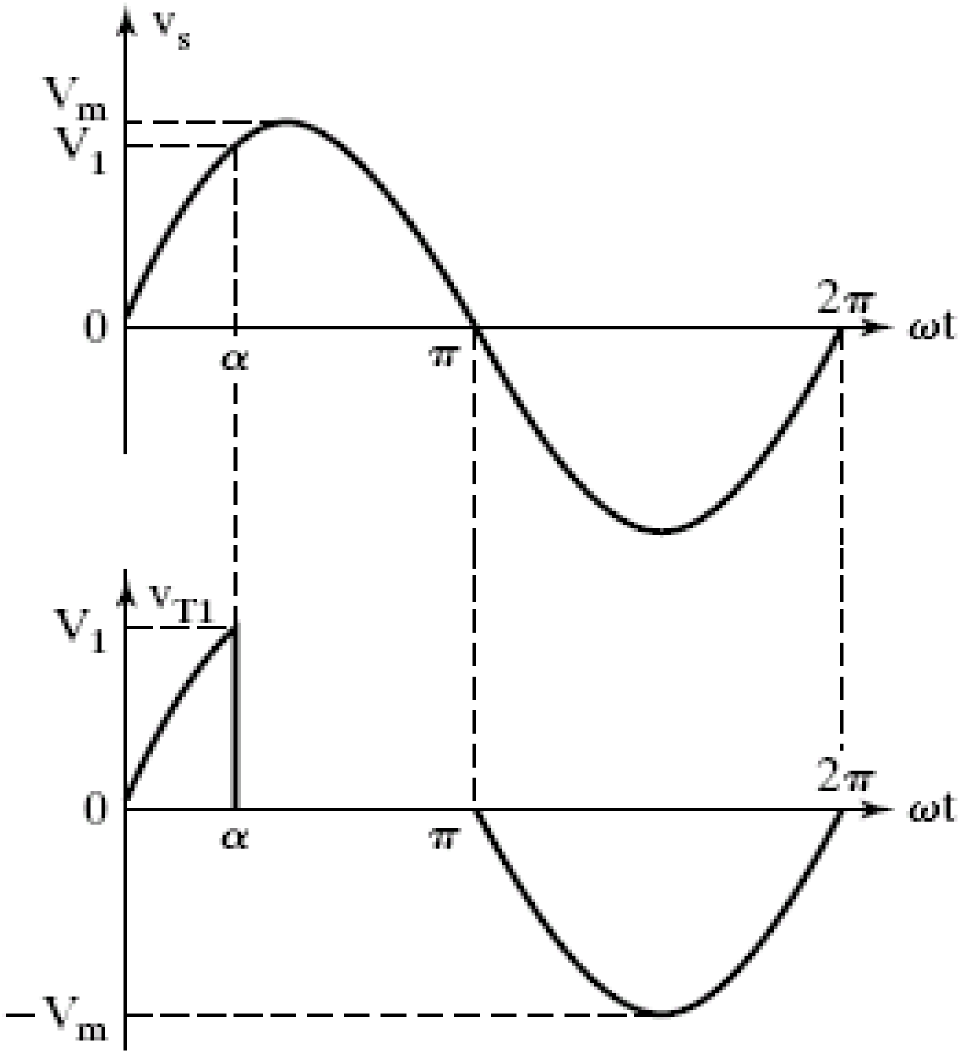




Supply Voltage

Output Voltage

Output (load) Current



Supply Voltage

Thyristor Voltage

Equations

$$v_s = V_m \sin \omega t = \text{ i/p ac supply voltage}$$

$$V_m = \text{max. value of i/p ac supply voltage}$$

$$V_S = \frac{V_m}{\sqrt{2}} = \text{RMS value of i/p ac supply voltage}$$

$$v_O = v_L = \text{ output voltage across the load}$$

When the thyristor is triggered at $\omega t = \alpha$

$$v_O = v_L = V_m \sin \omega t; \;\; \omega t = \alpha \text{ to } \pi$$

$$i_O = i_L = \frac{v_O}{R} = \text{Load current}; \;\; \omega t = \alpha \text{ to } \pi$$

$$i_O = i_L = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t; \omega t = \alpha \text{ to } \pi$$

Where $I_m = \dfrac{V_m}{R} =$ max. value of load current

To Derive an Expression for the Average (DC) Output Voltage Across The Load

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\int_{0}^{2\pi} v_O . d\left(\omega t\right);$$

$$v_O = V_m \sin\omega t \ for \ \ \omega t = \alpha \ to \ \pi$$

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin\omega t . d\left(\omega t\right)$$

$$V_{O(dc)} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin\omega t . d\left(\omega t\right)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\int_{\alpha}^{\pi} \sin\omega t.d\left(\omega t\right)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos\omega t\Big/_{\alpha}^{\pi}\right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos\pi + \cos\alpha\right]; \;\; \cos\pi = -1$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[1+\cos\alpha\right] \; ; \; V_m = \sqrt{2}V_S$$

Maximum average (dc) o/p voltage is obtained when $\alpha = 0$ and the maximum dc output voltage

$$V_{dc(\max)} = V_{dm} = \frac{V_m}{2\pi}\left(1+\cos 0\right); \quad \cos\left(0\right) = 1$$

$$\therefore V_{dc(\max)} = V_{dm} = \frac{V_m}{\pi}$$

Cntd…

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[1+\cos\alpha\right] \; ; \; V_m = \sqrt{2}V_S$$

The average dc output voltage can be varied by varying the trigger angle α from 0 to a maximum of $180^0 \left(\pi \text{ radians}\right)$

We can plot the control characteristic $\left(V_{O(dc)} \text{ vs } \alpha\right)$ by using the equation for $V_{O(dc)}$

Control Characteristic
of
Single Phase Half Wave Phase
Controlled Rectifier
with
Resistive Load

Cntd…

The average dc output voltage is given by the expression

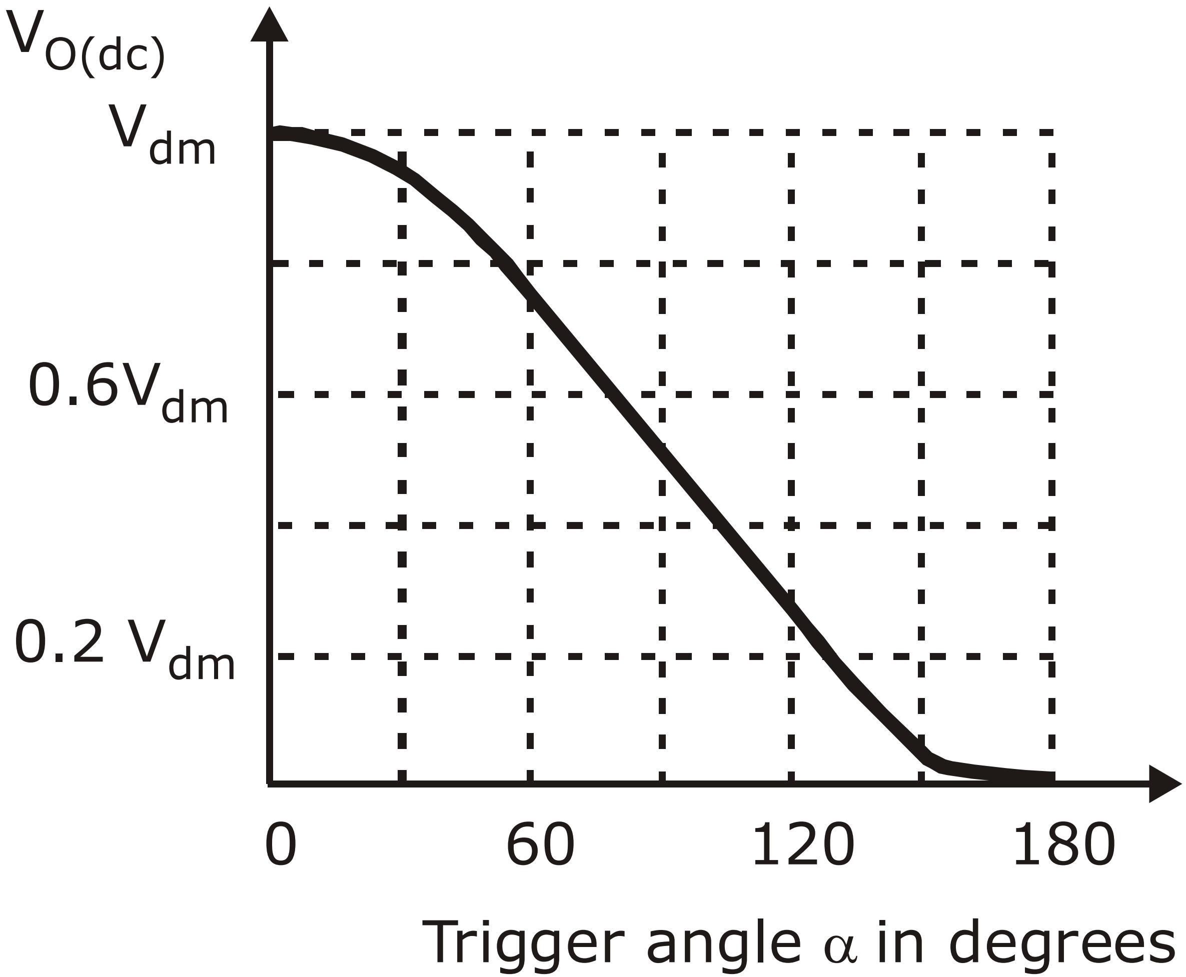
$$V_{O(dc)} = \frac{V_m}{2\pi}\left[1 + \cos\alpha\right]$$

We can obtain the control characteristic by plotting the expression for the dc output voltage as a function of trigger angle α

Trigger angle α in degrees	$V_{O(dc)}$	**%**
0	$V_{dm} = \dfrac{V_m}{\pi}$	$100\% \; V_{dm}$
30^0	$0.933 \; V_{dm}$	$93.3 \; \% \; V_{dm}$
60^0	$0.75 \; V_{dm}$	$75 \; \% \; V_{dm}$
90^0	$0.5 \; V_{dm}$	$50 \; \% \; V_{dm}$
120^0	$0.25 \; V_{dm}$	$25 \; \% \; V_{dm}$
150^0	$0.06698 \; V_{dm}$	$6.69 \; \% \; V_{dm}$
180^0	0	0

$$V_{dm} = \frac{V_m}{\pi} = V_{dc(\max)}$$

Control Characteristic



Normalizing the dc output voltage with respect to V_{dm}, the Normalized output voltage

$$V_n = \frac{V_{dc}}{V_{dm}} = \frac{\frac{V_m}{2\pi}\left(1+\cos\alpha\right)}{\frac{V_m}{\pi}}$$

$$V_n = \frac{V_{dc}}{V_{dm}} = \frac{1}{2}\left(1+\cos\alpha\right) = V_{dcn}$$

To Derive An Expression for the RMS Value of Output Voltage of a Single Phase Half Wave Controlled Rectifier With Resistive Load

Cntd…

The RMS output voltage is given by

$$V_{O(RMS)} = \left[\frac{1}{2\pi} \int_{0}^{2\pi} v_O^2 . d(\omega t) \right]$$

Output voltage $v_O = V_m \sin \omega t$; for $\omega t = \alpha$ to π

$$V_{O(RMS)} = \left[\frac{1}{2\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t . d(\omega t) \right]^{\frac{1}{2}}$$

Cntd…

By substituting $\sin^2 \omega t = \dfrac{1-\cos 2\omega t}{2}$, we get

$$V_{O(RMS)} = \left[\frac{1}{2\pi} \int_{\alpha}^{\pi} V_m^2 \frac{(1-\cos 2\omega t)}{2} .d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{4\pi} \int_{\alpha}^{\pi} (1-\cos 2\omega t) .d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{4\pi} \left\{ \int_{\alpha}^{\pi} d(\omega t) - \int_{\alpha}^{\pi} \cos 2\omega t .d(\omega t) \right\} \right]^{\frac{1}{2}}$$

Cntd…

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left\{\left(\omega t\right)\Big/_{\alpha}^{\pi} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\pi}\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left(\left(\pi-\alpha\right) - \frac{\left(\sin 2\pi - \sin 2\alpha\right)}{2}\right)\right]^{\frac{1}{2}} ; \sin 2\pi = 0$$

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left(\left(\pi-\alpha\right) + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\left(\left(\pi-\alpha\right) + \frac{\sin 2\alpha}{2}\right)^{\frac{1}{2}}$$

Performance Parameters
Of
Phase Controlled Rectifiers

Cntd…

Output dc power (avg. or dc o/p power delivered to the load)

$$P_{O(dc)} = V_{O(dc)} \times I_{O(dc)} \ ; \ i.e., \ P_{dc} = V_{dc} \times I_{dc}$$

Where

$V_{O(dc)} = V_{dc} =$ avg./ dc value of o/p voltage.

$I_{O(dc)} = I_{dc} =$ avg./dc value of o/p current

Cntd…

Output ac power

$$P_{O(ac)} = V_{O(RMS)} \times I_{O(RMS)}$$

Efficiency of Rectification (Rectification Ratio)

Efficiency $\eta = \dfrac{P_{O(dc)}}{P_{O(ac)}}$; % Efficiency $\eta = \dfrac{P_{O(dc)}}{P_{O(ac)}} \times 100$

The o/p voltage consists of two components

The dc component $V_{O(dc)}$

The ac /ripple component $V_{ac} = V_{r(rms)}$

Cntd…

The total RMS value of output voltage is given by

$$V_{O(RMS)} = \sqrt{V_{O(dc)}^2 + V_{r(rms)}^2}$$

$$\therefore \qquad V_{ac} = V_{r(rms)} = \sqrt{V_{O(RMS)}^2 - V_{O(dc)}^2}$$

Form Factor (FF) which is a measure of the shape of the output voltage is given by

$$FF = \frac{V_{O(RMS)}}{V_{O(dc)}} = \frac{\text{RMS output (load) voltage}}{\text{DC load output (load) voltage}}$$

Cntd…

The Ripple Factor (RF) w.r.t. o/p voltage w/f

$$r_v = RF = \frac{V_{r(rms)}}{V_{O(dc)}} = \frac{V_{ac}}{V_{dc}}$$

$$r_v = \frac{\sqrt{V_{O(RMS)}^2 - V_{O(dc)}^2}}{V_{O(dc)}} = \sqrt{\left[\frac{V_{O(RMS)}}{V_{O(dc)}}\right]^2 - 1}$$

$$\therefore \qquad r_v = \sqrt{FF^2 - 1}$$

Cntd…

Current Ripple Factor $r_i = \dfrac{I_{r(rms)}}{I_{O(dc)}} = \dfrac{I_{ac}}{I_{dc}}$

Where $I_{r(rms)} = I_{ac} = \sqrt{I_{O(RMS)}^2 - I_{O(dc)}^2}$

$V_{r(pp)} =$ peak to peak ac ripple output voltage

$$V_{r(pp)} = V_{O(\max)} - V_{O(\min)}$$

$I_{r(pp)} =$ peak to peak ac ripple load current

$$I_{r(pp)} = I_{O(\max)} - I_{O(\min)}$$

Cntd…

Transformer Utilization Factor (TUF)

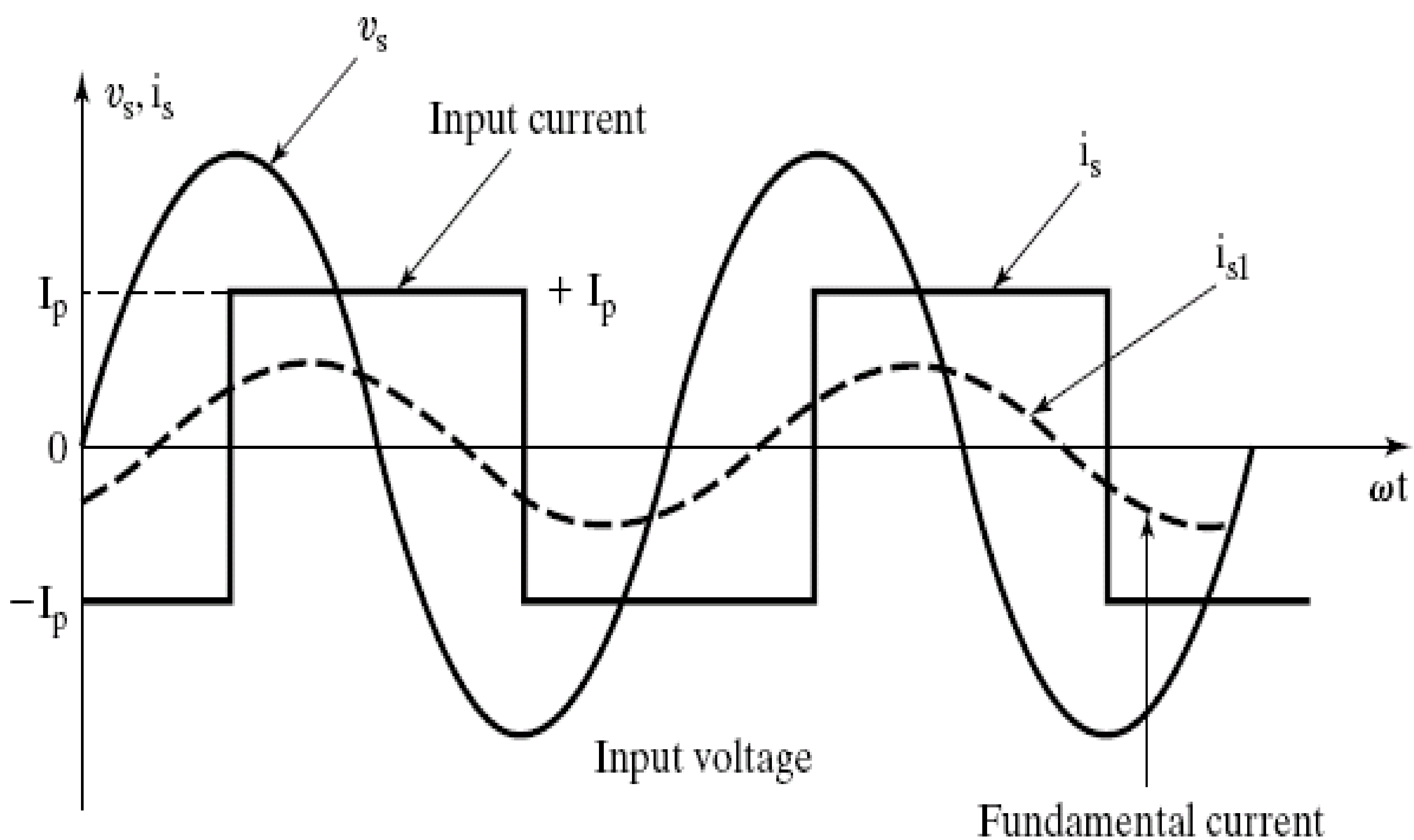
$$TUF = \frac{P_{O(dc)}}{V_S \times I_S}$$

Where

V_S = RMS supply (secondary) voltage

I_S = RMS supply (secondary) current

Cntd…



v_s
v_s, i_s
Input current
i_s
i_{s1}
I_p
$+ I_p$
0
ωt
$-I_p$
Input voltage
Fundamental current

Cntd…

Where

$v_S =$ Supply voltage at the transformer secondary side

$i_S =$ i/p supply current

(transformer secondary winding current)

$i_{S1} =$ Fundamental component of the i/p supply current

$I_P =$ Peak value of the input supply current

$\phi =$ Phase angle difference between (sine wave components) the fundamental components of i/p supply current & the input supply voltage.

Cntd…

$$\phi = \text{Displacement angle (phase angle)}$$

For an RL load

$$\phi = \text{ Displacement angle = Load impedance angle}$$

$$\therefore \quad \phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \text{ for an RL load}$$

Displacement Factor (DF) or

Fundamental Power Factor

$$DF = Cos\phi$$

Cntd…

Harmonic Factor (HF) or

Total Harmonic Distortion Factor ; THD

$$HF = \left[\frac{I_S^2 - I_{S1}^2}{I_{S1}^2} \right]^{\frac{1}{2}} = \left[\left(\frac{I_S}{I_{S1}} \right)^2 - 1 \right]^{\frac{1}{2}}$$

Where

$I_S =$ RMS value of input supply current.

$I_{S1} =$ RMS value of fundamental component of the i/p supply current.

Cntd…

Input Power Factor (PF)

$$PF = \frac{V_S I_{S1}}{V_S I_S} \cos\phi = \frac{I_{S1}}{I_S} \cos\phi$$

The Crest Factor (CF)

$$CF = \frac{I_{S(peak)}}{I_S} = \frac{\text{Peak input supply current}}{\text{RMS input supply current}}$$

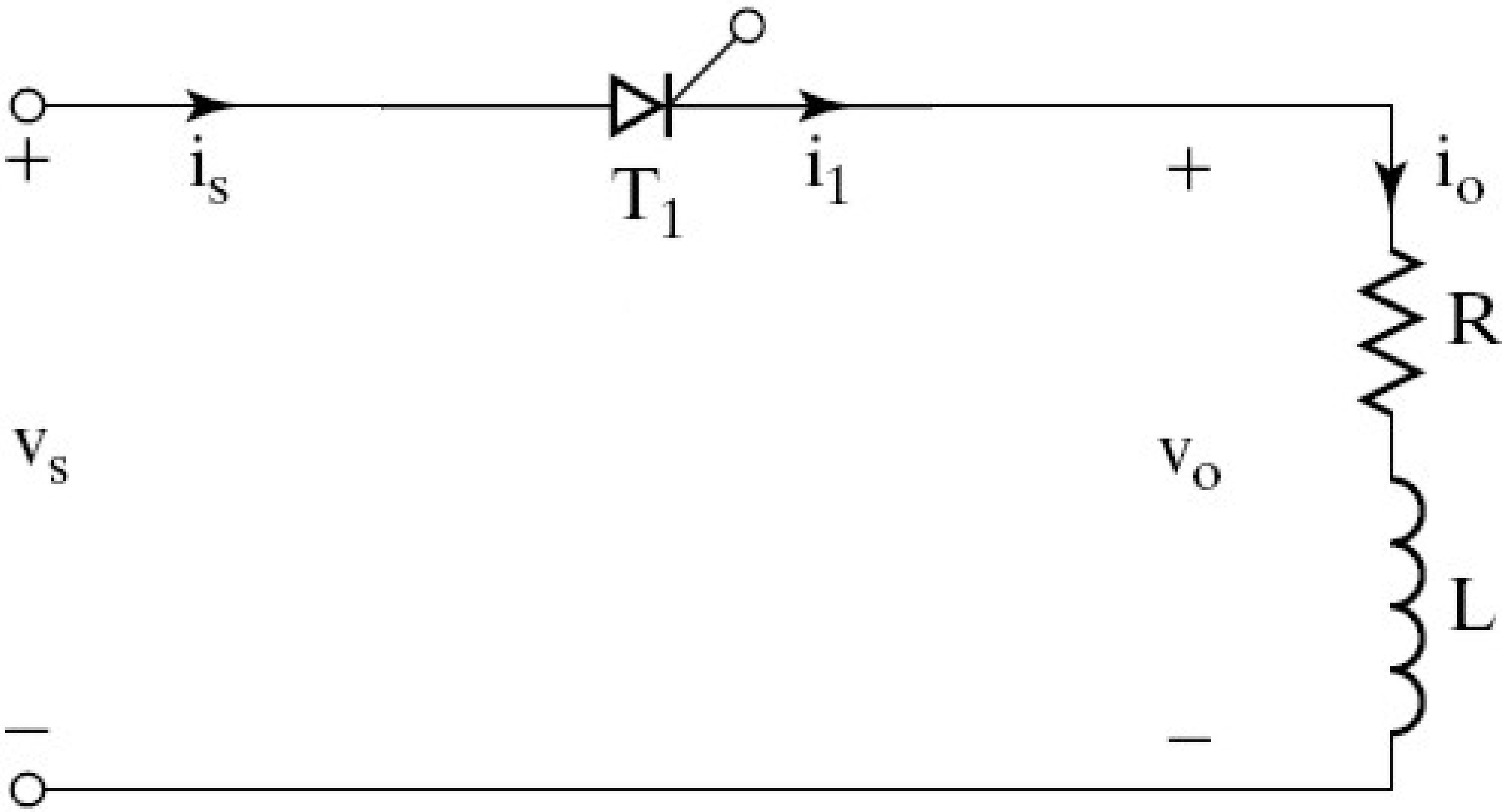
For an Ideal Controlled Rectifier

$$FF = 1;\ \eta = 100\%\ ;\ V_{ac} = V_{r(rms)} = 0\ ;\ TUF = 1;$$

$$RF = r_v = 0\ ;\ HF = THD = 0;\ \ PF = DPF = 1$$

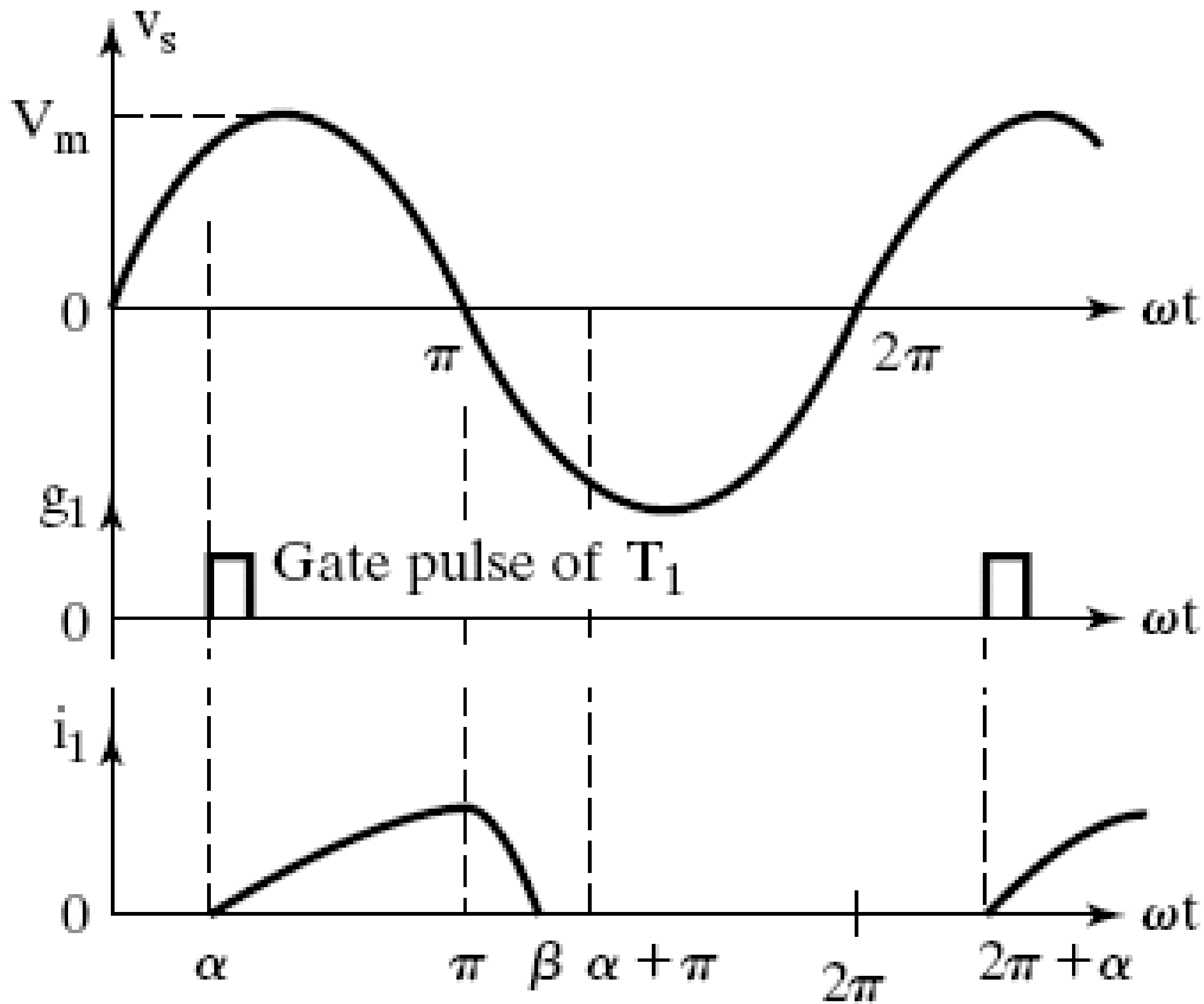
Single Phase Half Wave Controlled Rectifier With An RL Load

Cntd…



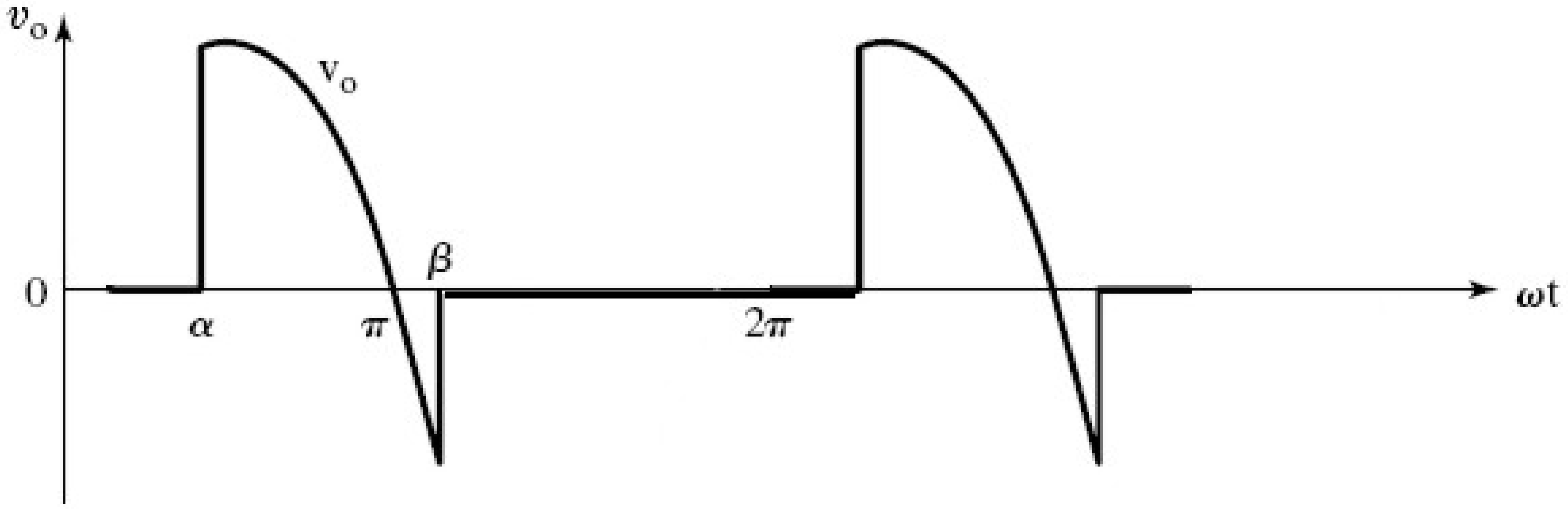
Input Supply Voltage (V_s) & Thyristor (Output) Current Waveforms

Cntd…



v_s
V_m
0
ωt
π
2π
g_1
Gate pulse of T_1
0
ωt
i_1
0
ωt
α
π
β
$\alpha + \pi$
2π
$2\pi + \alpha$

Output (Load) Voltage Waveform



Cntd…

To Derive An Expression For The Output (Load) Current, During $\omega t = \alpha$ to β When Thyristor T_1 Conducts

Cntd…

Assuming T_1 is triggered $\omega t = \alpha$,

we can write the equation,

$$L\left(\frac{di_O}{dt}\right) + Ri_O = V_m \sin \omega t \;;\; \alpha \le \omega t \le \beta$$

General expression for the output current,

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + A_1 e^{\frac{-t}{\tau}}$$

Cntd…

$$V_m = \sqrt{2}V_S = \text{maximum supply voltage.}$$

$$Z = \sqrt{R^2 + (\omega L)^2} = \text{Load impedance.}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \text{Load impedance angle.}$$

$$\tau = \frac{L}{R} = \text{Load circuit time constant.}$$

$\therefore$ general expression for the output load current

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-R}{L}t}$$

Cntd…

Constant A_1 is calculated from

initial condition $i_O = 0$ at $\omega t = \alpha$; t=$\left(\dfrac{\alpha}{\omega}\right)$

$$i_O = 0 = \frac{V_m}{Z}\sin(\alpha-\phi) + A_1 e^{\frac{-R}{L}t}$$

$$\therefore \qquad A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin(\alpha-\phi)$$

We get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin(\alpha-\phi)\right]$$

Cntd…

Substituting the value of constant A_1 in the general expression for i_O

$$i_O = \frac{V_m}{Z}\sin\left(\omega t-\phi\right)+e^{\frac{-R}{\omega L}(\omega t-\alpha)}\left[\frac{-V_m}{Z}\sin\left(\alpha-\phi\right)\right]$$

$\therefore$ we obtain the final expression for the inductive load current

$$i_O = \frac{V_m}{Z}\left[\sin\left(\omega t-\phi\right)-\sin\left(\alpha-\phi\right)e^{\frac{-R}{\omega L}(\omega t-\alpha)}\right];$$

Where $\alpha \le \omega t \le \beta$

Cntd…

Extinction angle β can be calculated by using the condition that $i_O = 0 \ at \ \omega t = \beta$

$$i_O = \frac{V_m}{Z}\left[\sin\left(\omega t-\phi\right)-\sin\left(\alpha-\phi\right)e^{\frac{-R}{\omega L}\left(\omega t-\alpha\right)}\right]=0$$

$$\therefore \sin\left(\beta-\phi\right)=e^{\frac{-R}{\omega L}\left(\beta-\alpha\right)}\times\sin\left(\alpha-\phi\right)$$

β can be calculated by solving the above eqn.

To Derive An Expression For Average (DC) Load Voltage of a Single Half Wave Controlled Rectifier with RL Load

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\int_0^{2\pi} v_O.d\left(\omega t\right)$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_0^{\alpha} v_O.d\left(\omega t\right) + \int_{\alpha}^{\beta} v_O.d\left(\omega t\right) + \int_{\beta}^{2\pi} v_O.d\left(\omega t\right)\right]$$

$$v_O = 0 \text{ for } \omega t = 0 \text{ to } \alpha \text{ \& for } \omega t = \beta \text{ to } 2\pi$$

$$\therefore V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} v_O.d\left(\omega t\right)\right];$$

$$v_O = V_m \sin\omega t \text{ for } \omega t = \alpha \text{ to } \beta$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} V_m \sin\omega t.d(\omega t)\right]$$

$$V_{O(dc)} = V_L = \frac{V_m}{2\pi}\left[-\cos\omega t\Big/_{\alpha}^{\beta}\right]$$

$$V_{O(dc)} = V_L = \frac{V_m}{2\pi}\left(\cos\alpha - \cos\beta\right)$$

$$\therefore V_{O(dc)} = V_L = \frac{V_m}{2\pi}\left(\cos\alpha - \cos\beta\right)$$

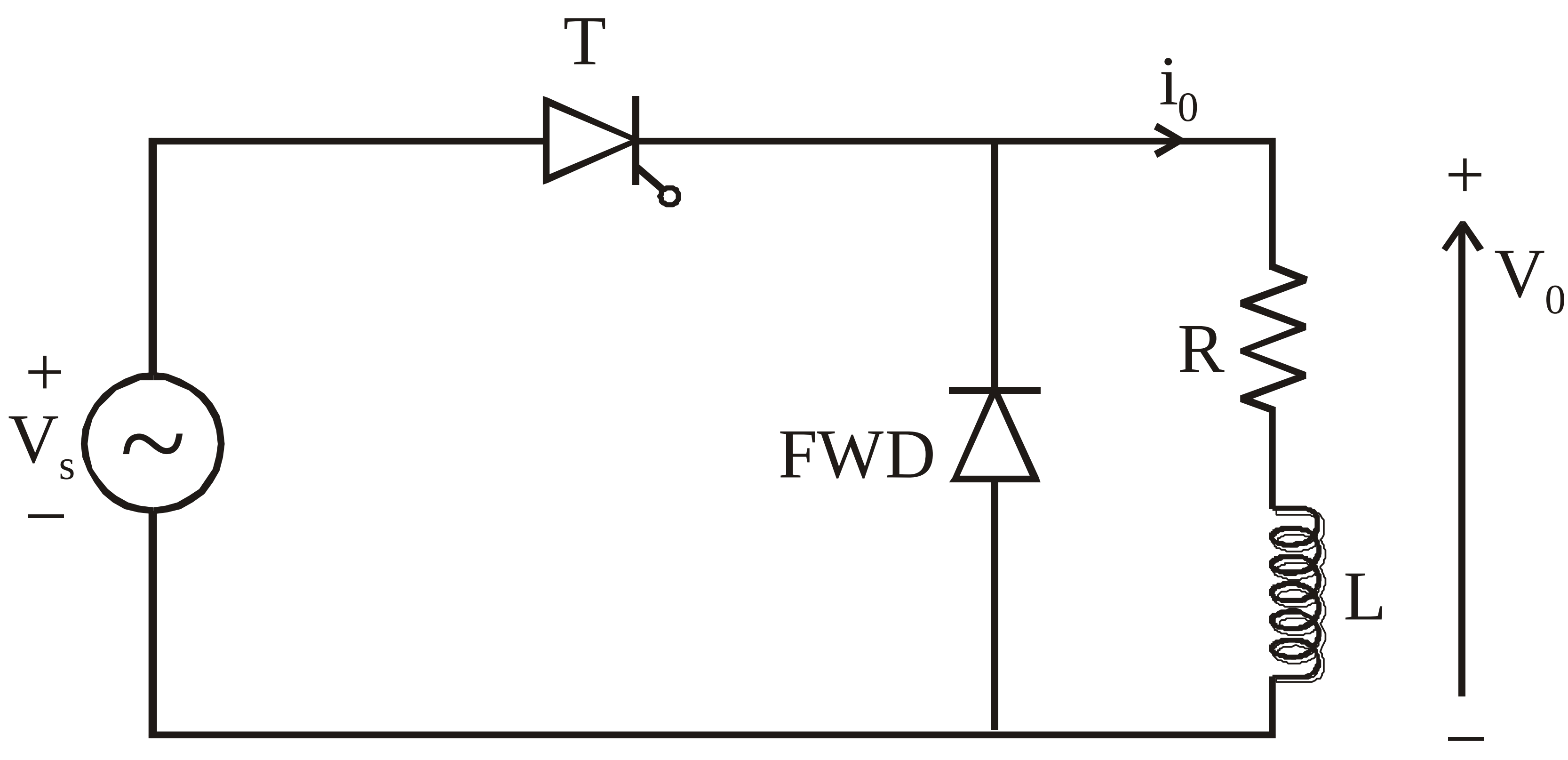
Effect of Load Inductance on the Output

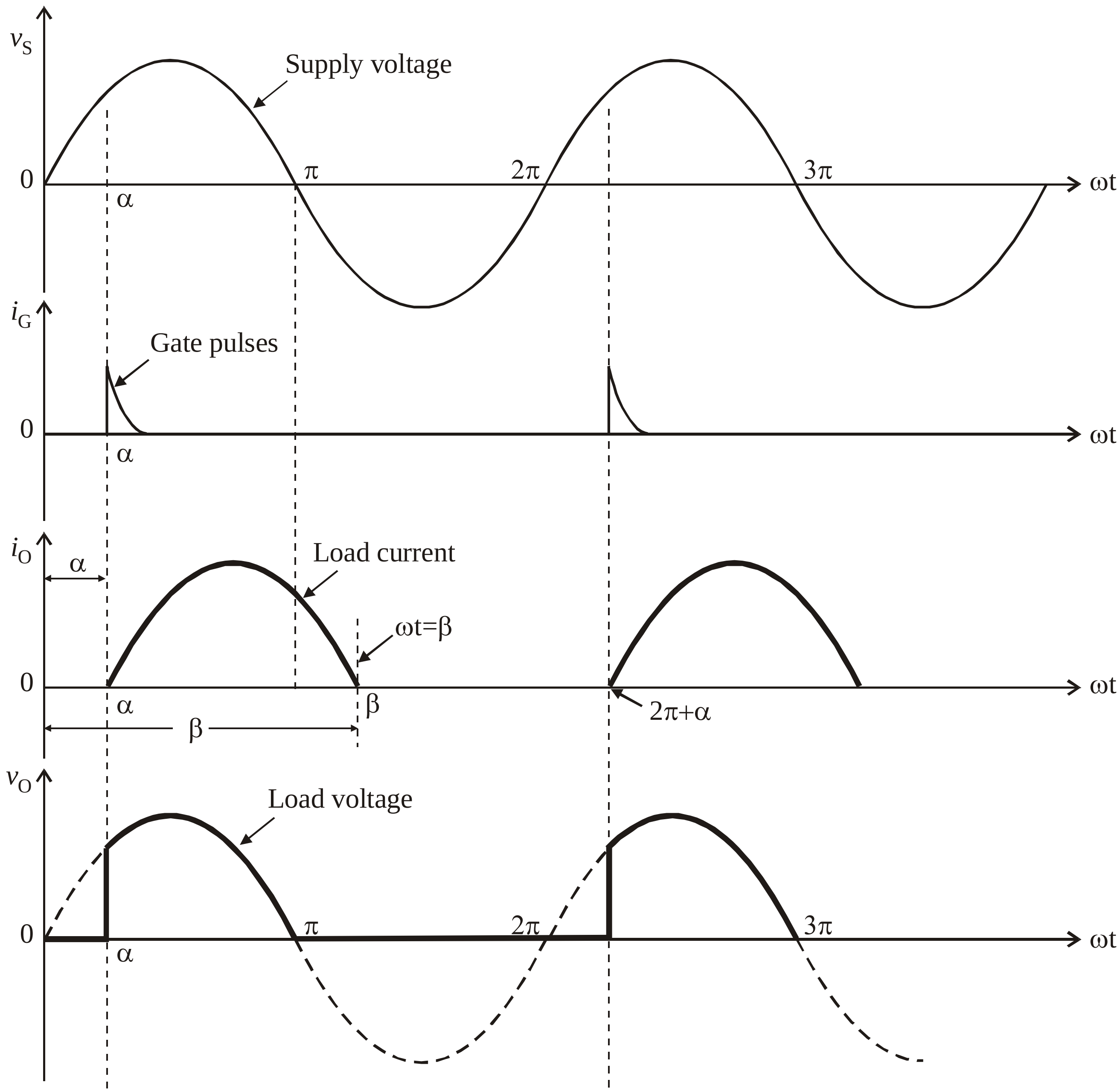
During the period $\omega t = \pi$ to β the instantaneous o/p voltage is negative and this reduces the average or the dc output voltage when compared to a purely resistive load.

Average DC Load Current

$$I_{O(dc)} = I_{L(Avg)} = \frac{V_{O(dc)}}{R_L} = \frac{V_m}{2\pi R_L}\left(\cos\alpha - \cos\beta\right)$$

Single Phase Half Wave Controlled Rectifier With RL Load & Free Wheeling Diode


T
i_0
+
V_0
R
FWD
L
+
V_s
−
−



v_S
Supply voltage
0
π
2π
3π
ωt
α
i_G
Gate pulses
0
ωt
α
i_O
α
Load current
ωt=β
0
ωt
α
β
β
2π+α
v_O
Load voltage
0
π
2π
3π
ωt
α

The average output voltage

$V_{dc} = \frac{V_m}{2\pi}\left[1+\cos\alpha\right]$ which is the same as that

of a purely resistive load.

The following points are to be noted

For low value of inductance, the load current

tends to become discontinuous.

During the period α to π

the load current is carried by the SCR.

During the period π to β load current is

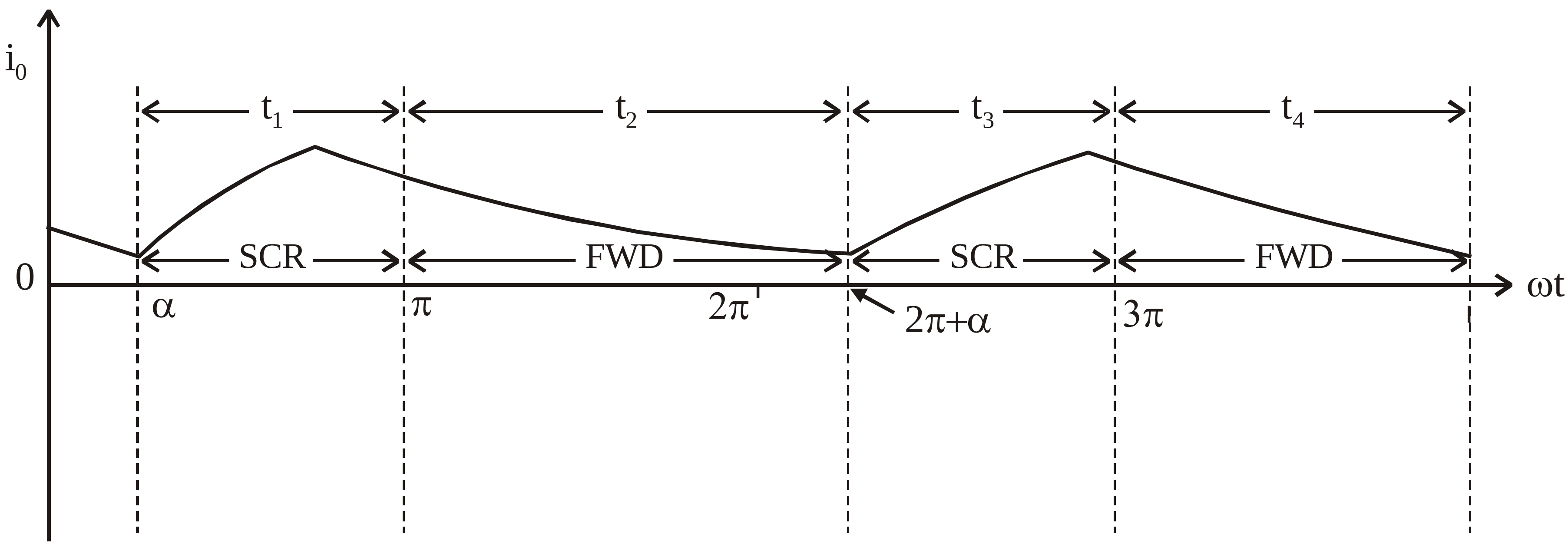
carried by the free wheeling diode.

The value of β depends on the value of

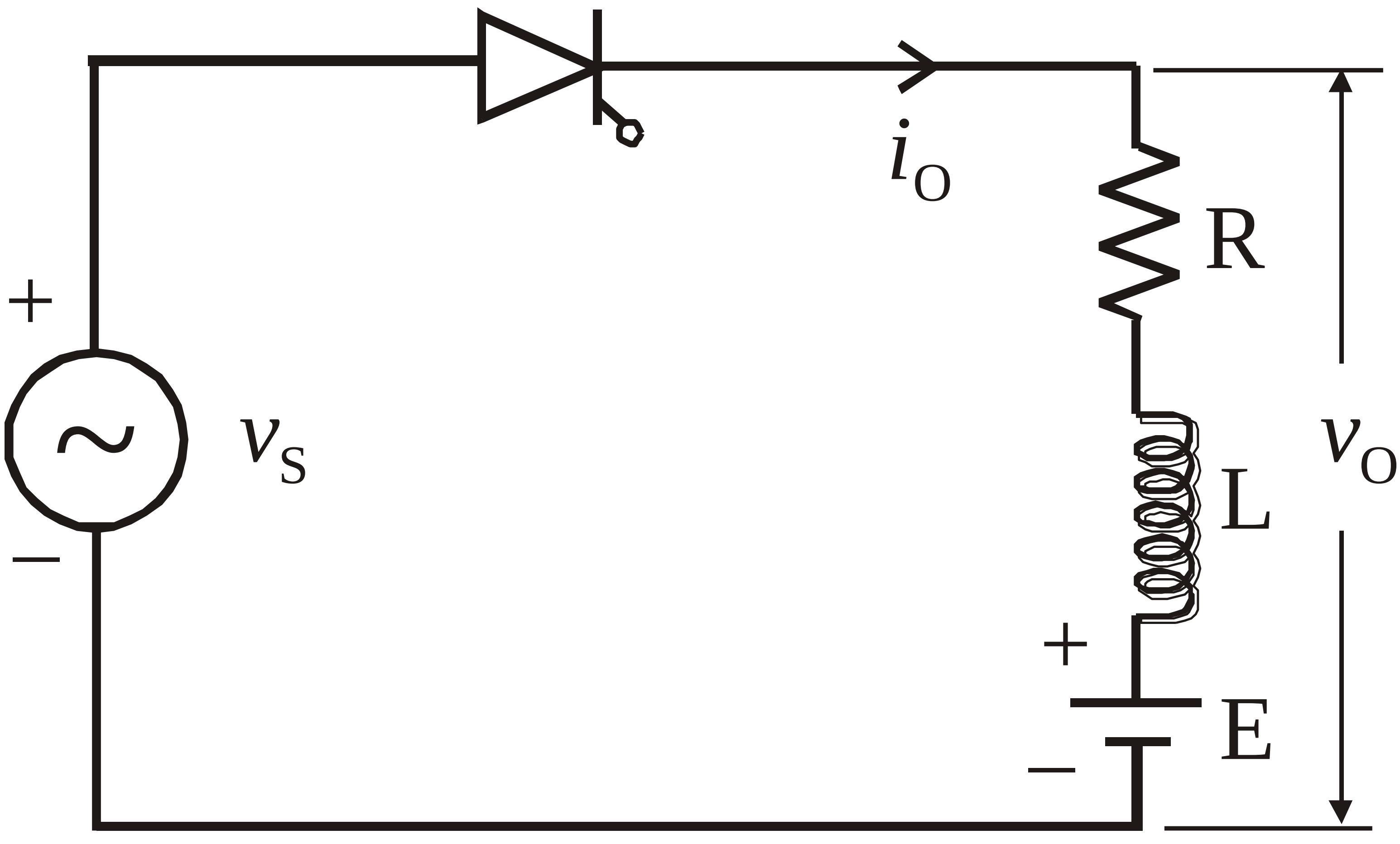
R and L and the forward resistance

of the FWD.

For Large Load Inductance
the load current does not reach zero, &
we obtain continuous load current



Single Phase Half Wave Controlled Rectifier With A General Load


+
−
v_S
i_O
R
L
+
−
E
v_O

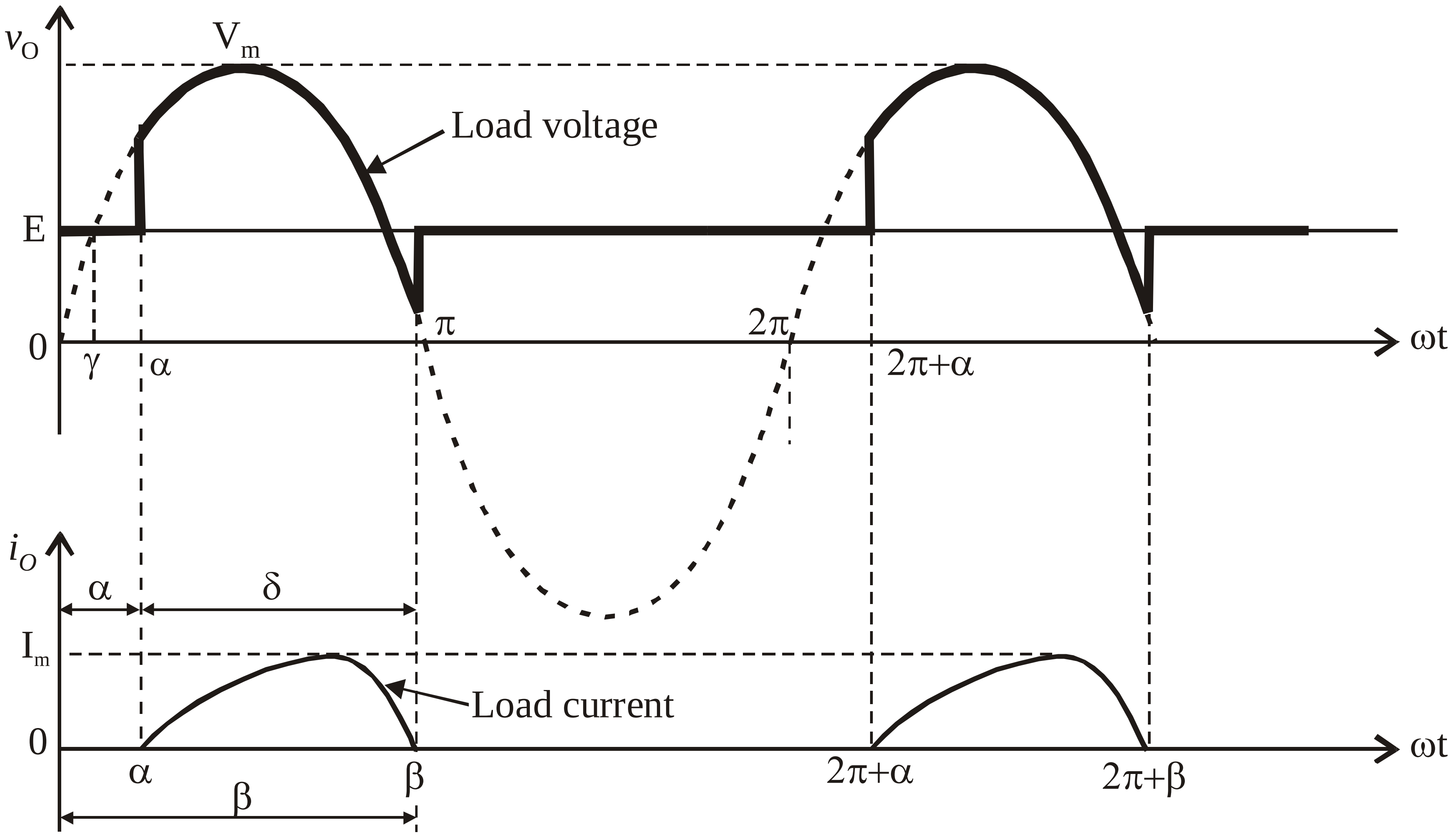
$$\gamma = \sin^{-1}\left(\frac{E}{V_m}\right)$$

For trigger angle $\alpha < \gamma$,

the Thyristor conducts from $\omega t = \gamma$ to β

For trigger angle $\alpha > \gamma$,

the Thyristor conducts from $\omega t = \alpha$ to β



v_O
V_m
Load voltage
E
0
γ
α
π
2π
ωt
2π+α
i_O
α
δ
I_m
Load current
0
α
β
β
2π+α
2π+β
ωt

Equations

$$v_S = V_m \sin \omega t = \text{ Input supply voltage.}$$

$$v_O = V_m \sin \omega t = \text{ o/p } \left(\text{load}\right) \text{ voltage}$$

$$\text{for } \omega t = \alpha \text{ to } \beta.$$

$$v_O = E \;\; \text{for } \omega t = 0 \text{ to } \alpha \;\&$$

$$\text{for } \omega t = \beta \text{ to } 2\pi.$$

Expression for the Load Current

When the thyristor is triggered at a delay angle of $\alpha > \gamma$, the eqn. for the circuit can be written as

$$V_m \sin\omega t = i_O \times R + L\left(\frac{di_O}{dt}\right) + \mathrm{E}\ ;\ \alpha \le \omega t \le \beta$$

The general expression for the output load current can be written as

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) - \frac{E}{R} + Ae^{\frac{-t}{\tau}}$$

Where

$$Z = \sqrt{R^2 + (\omega L)^2} = \text{Load Impedance.}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \text{ Load impedance angle.}$$

$$\tau = \frac{L}{R} = \text{ Load circuit time constant.}$$

The general expression for the o/p current can

be written as $i_O = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R} + Ae^{\frac{-R}{L}t}$

To find the value of the constant 'A' apply the initial conditions at $\omega t = \alpha$, load current $i_O = 0$, Equating the general expression for the load current to zero at $\omega t = \alpha$, we get

$$i_O = 0 = \frac{V_m}{Z}\sin\left(\alpha - \phi\right) - \frac{E}{R} + Ae^{\frac{-R}{L}\times\frac{\alpha}{\omega}}$$

We obtain the value of constant 'A' as

$$A = \left[\frac{E}{R} - \frac{V_m}{Z} \sin\left(\alpha - \phi\right) \right] e^{\frac{R}{\omega L}\alpha}$$

Substituting the value of the constant 'A' in the expression for the load current; we get the complete expression for the output load current as

$$i_O = \frac{V_m}{Z} \sin\left(\omega t - \phi\right) - \frac{E}{R} + \left[\frac{E}{R} - \frac{V_m}{Z} \sin\left(\alpha - \phi\right) \right] e^{\frac{-R}{\omega L}\left(\omega t - \alpha\right)}$$

To Derive An Expression For The Average Or DC Load Voltage

$$V_{O(dc)} = \frac{1}{2\pi}\int_0^{2\pi} v_O.d(\omega t)$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[\int_0^{\alpha} v_O.d(\omega t) + \int_{\alpha}^{\beta} v_O.d(\omega t) + \int_{\beta}^{2\pi} v_O.d(\omega t)\right]$$

$v_O = V_m \sin\omega t =$ Output load voltage for $\omega t = \alpha$ to β

$v_O = E$ for $\omega t = 0$ to α & for $\omega t = \beta$ to 2π

$$V_{O(dc)} = \frac{1}{2\pi}\left[\int_0^{\alpha} E.d(\omega t) + \int_{\alpha}^{\beta} V_m \sin\omega t + \int_{\beta}^{2\pi} E.d(\omega t)\right]$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[E(\omega t)\Big/_{0}^{\alpha} + V_m(-\cos\omega t)\Big/_{\alpha}^{\beta} + E(\omega t)\Big/_{\beta}^{2\pi} \right]$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[E(\alpha - 0) - V_m(\cos\beta - \cos\alpha) + E(2\pi - \beta) \right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[(\cos\alpha - \cos\beta) \right] + \frac{E}{2\pi}\left[(2\pi - \beta + \alpha) \right]$$

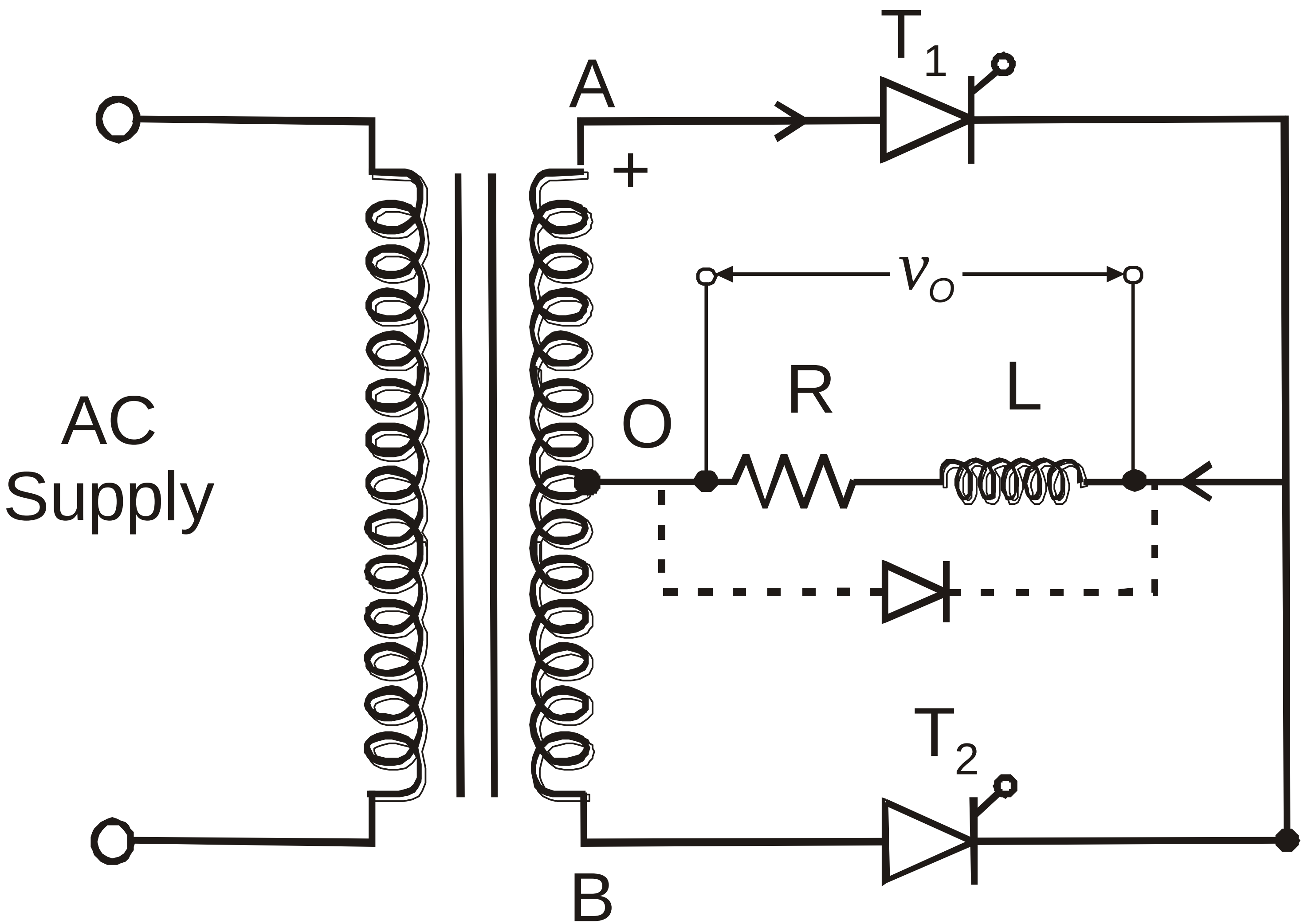
$$V_{O(dc)} = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta) + \left[\frac{2\pi - (\beta - \alpha)}{2\pi} \right] E$$

Conduction angle of thyristor $\delta = (\beta - \alpha)$

RMS Output Voltage can be calculated by using the expression

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_{0}^{2\pi} v_O^2 . d(\omega t)\right]}$$

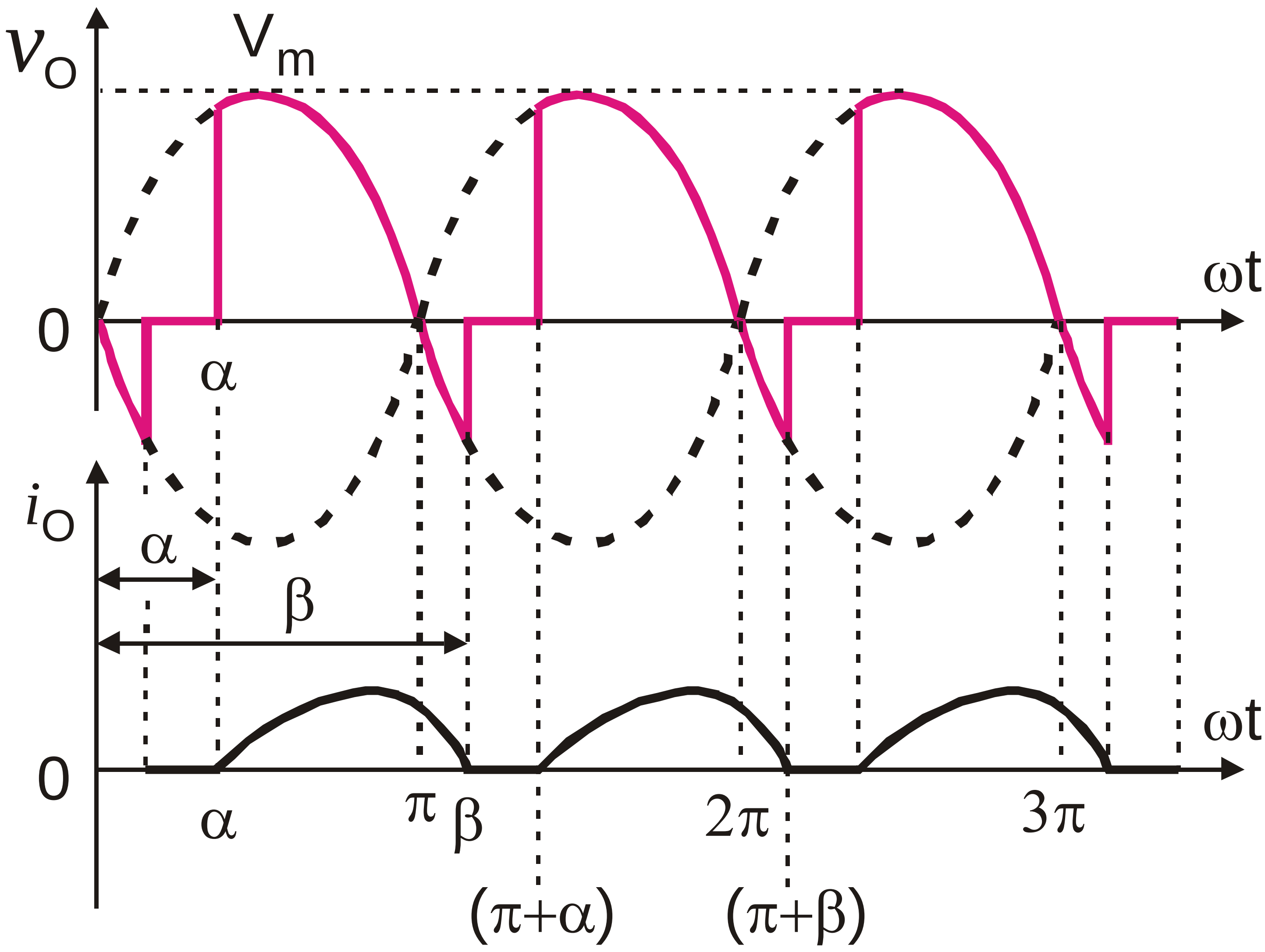
Single Phase Full Wave Controlled Rectifier Using A Center Tapped Transformer



T_1
A
+
v_O
AC Supply
O
R
L
T_2
B

Discontinuous Load Current Operation without FWD for

$$\pi < \beta < \left(\pi + \alpha\right)$$



v_O
V_m
ωt
0
α
i_O
α
β
ωt
0
α
π
β
2π
3π
(π+α)
(π+β)

To Derive An Expression For The Output (Load) Current, During $\omega t = \alpha$ to β When Thyristor T_1 Conducts

Assuming T_1 is triggered $\omega t = \alpha$,

we can write the equation,

$$L\left(\frac{di_O}{dt}\right) + Ri_O = V_m \sin \omega t \; ; \; \alpha \le \omega t \le \beta$$

General expression for the output current,

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + A_1 e^{\frac{-t}{\tau}}$$

$$V_m = \sqrt{2}V_S = \text{maximum supply voltage.}$$

$$Z = \sqrt{R^2 + (\omega L)^2} = \text{Load impedance.}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \text{Load impedance angle.}$$

$$\tau = \frac{L}{R} = \text{Load circuit time constant.}$$

$\therefore$ general expression for the output load current

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-R}{L}t}$$

Constant A_1 is calculated from

initial condition $i_O = 0$ at $\omega t = \alpha$; t= $\left(\dfrac{\alpha}{\omega}\right)$

$$i_O = 0 = \frac{V_m}{Z}\sin\left(\alpha - \phi\right) + A_1 e^{\frac{-R}{L}t}$$

$$\therefore \qquad A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin\left(\alpha - \phi\right)$$

We get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

Substituting the value of constant A_1 in the general expression for i_O

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + e^{\frac{-R}{\omega L}\left(\omega t - \alpha\right)}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

$\therefore$ we obtain the final expression for the inductive load current

$$i_O = \frac{V_m}{Z}\left[\sin\left(\omega t - \phi\right) - \sin\left(\alpha - \phi\right)e^{\frac{-R}{\omega L}\left(\omega t - \alpha\right)}\right];$$

Where $\alpha \le \omega t \le \beta$

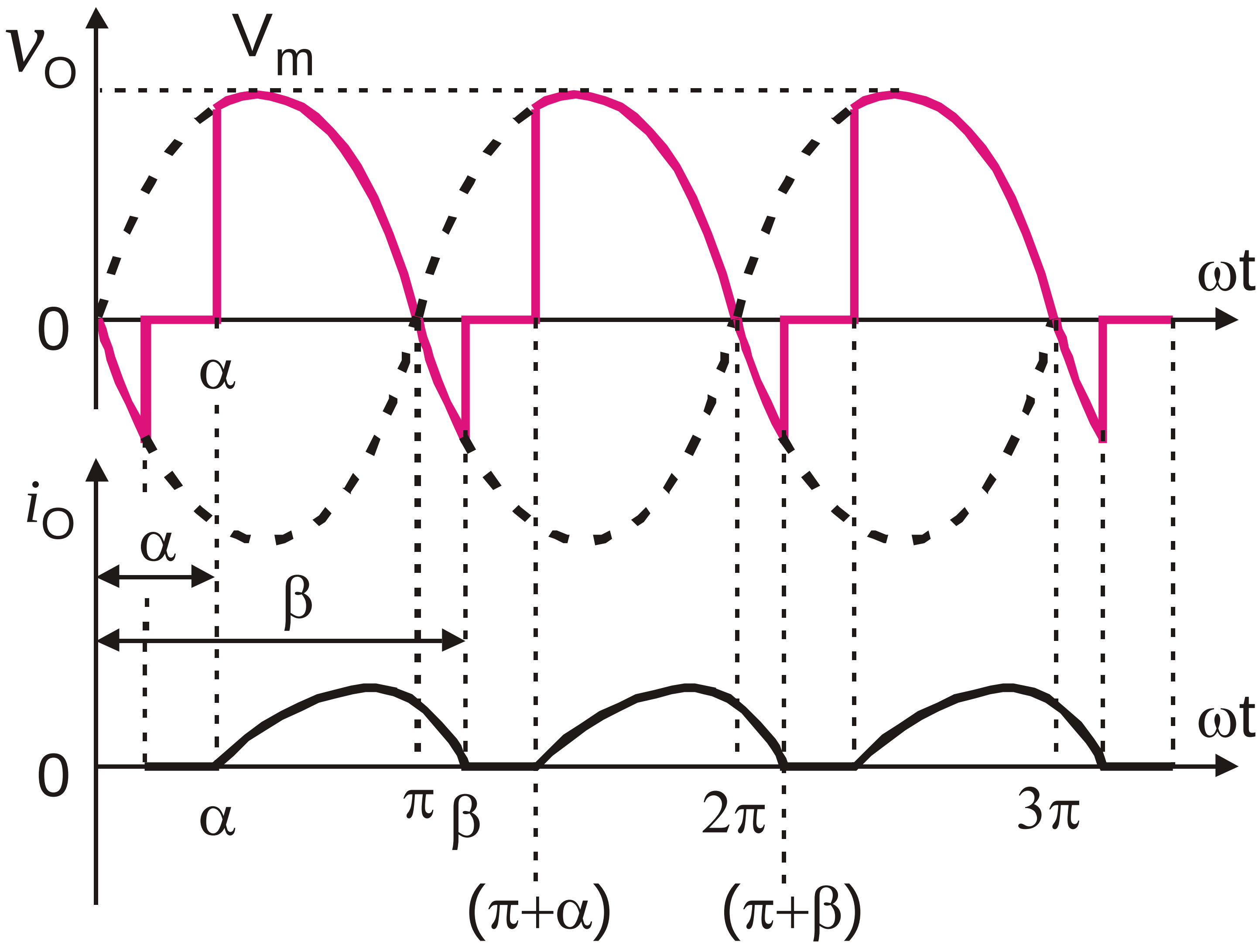
Extinction angle β can be calculated by using the condition that $i_O = 0 \ at \ \omega t = \beta$

$$i_O = \frac{V_m}{Z}\left[\sin\left(\omega t - \phi\right) - \sin\left(\alpha - \phi\right)e^{\frac{-R}{\omega L}\left(\omega t - \alpha\right)}\right] = 0$$

$$\therefore \sin\left(\beta - \phi\right) = e^{\frac{-R}{\omega L}\left(\beta - \alpha\right)} \times \sin\left(\alpha - \phi\right)$$

β can be calculated by solving the above eqn.

To Derive An Expression For The DC Output Voltage Of A Single Phase Full Wave Controlled Rectifier With RL Load *(Without FWD)*



v_O
V_m
ωt
0
α
i_O
α
β
ωt
0
α
π
β
2π
3π
(π+α)
(π+β)

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=\alpha}^{\beta} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{\beta} V_m \sin \omega t . d(\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\beta} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (\cos \alpha - \cos \beta)$$

When the load inductance is negligible $\left(\text{ i.e., } L \approx 0 \right)$

Extinction angle $\beta = \pi$ radians

Hence the average or dc output voltage for R load

$$V_{O(dc)} = \frac{V_m}{\pi}\left(\cos\alpha - \cos\pi\right)$$

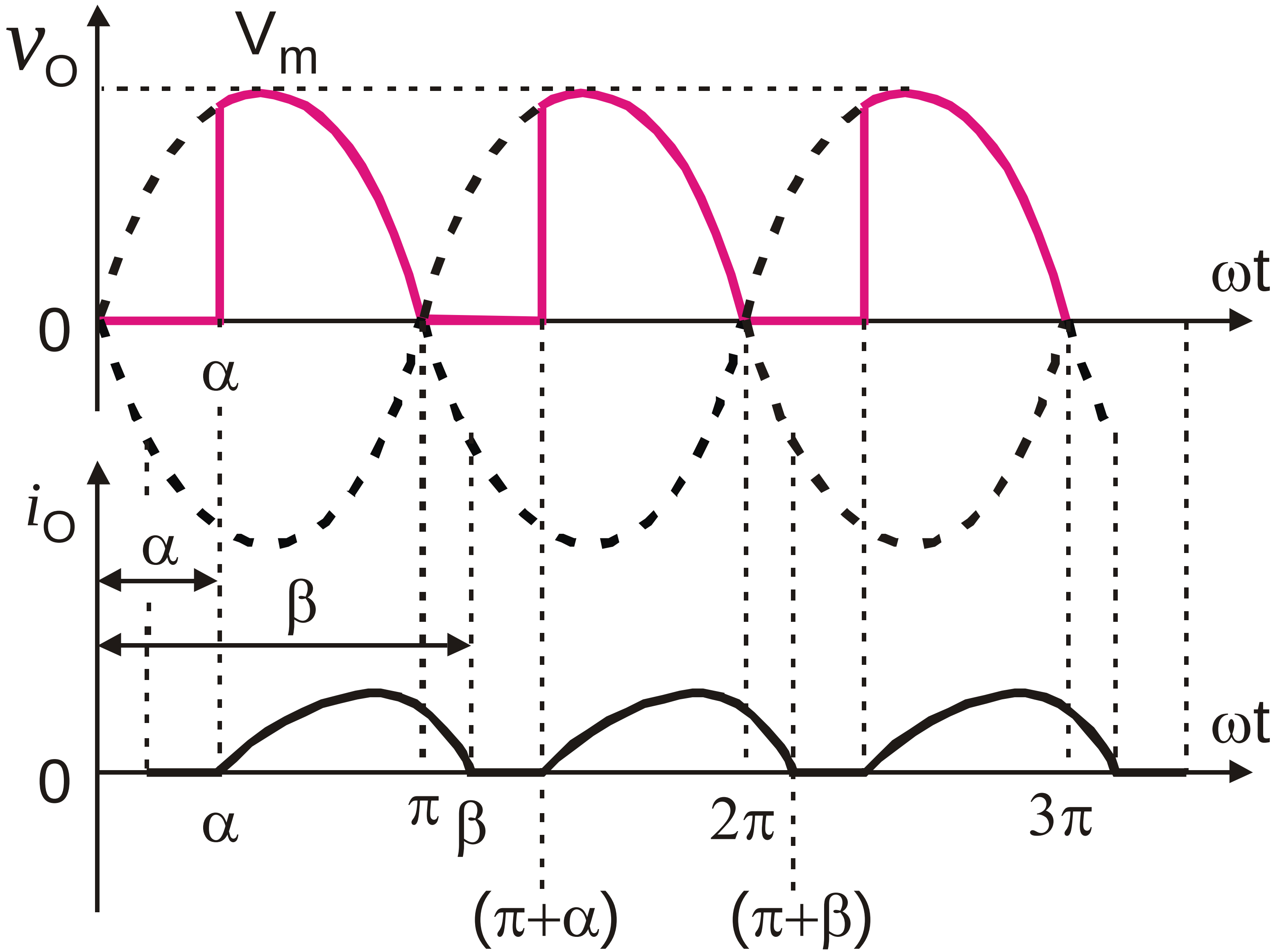
$$V_{O(dc)} = \frac{V_m}{\pi}\left(\cos\alpha - \left(-1\right)\right)$$

$$V_{O(dc)} = \frac{V_m}{\pi}\left(1 + \cos\alpha\right); \text{ for R load, when } \beta = \pi$$

To calculate the RMS output voltage we use the expression

$$V_{O(RMS)} = \sqrt{\frac{1}{\pi}\left[\int_{\alpha}^{\beta} V_m^2 \sin^2 \omega t.d\left(\omega t\right)\right]}$$

Discontinuous Load Current Operation with FWD


v_O
V_m
ωt
0
α
i_O
α
β
ωt
0
α
π
β
2π
3π
(π+α)
(π+β)

Thyristor T_1 is triggered at $\omega t = \alpha$;

T_1 conducts from $\omega t = \alpha$ to π

Thyristor T_2 is triggered at $\omega t = (\pi + \alpha)$;

T_2 conducts from $\omega t = (\pi + \alpha)$ to 2π

FWD conducts from $\omega t = \pi$ to β &

$v_O \approx 0$ during discontinuous load current.

To Derive an Expression For The DC Output Voltage For A Single Phase Full Wave Controlled Rectifier With RL Load & FWD

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=0}^{\pi} v_O . d(\omega t)$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t . d(\omega t)$$

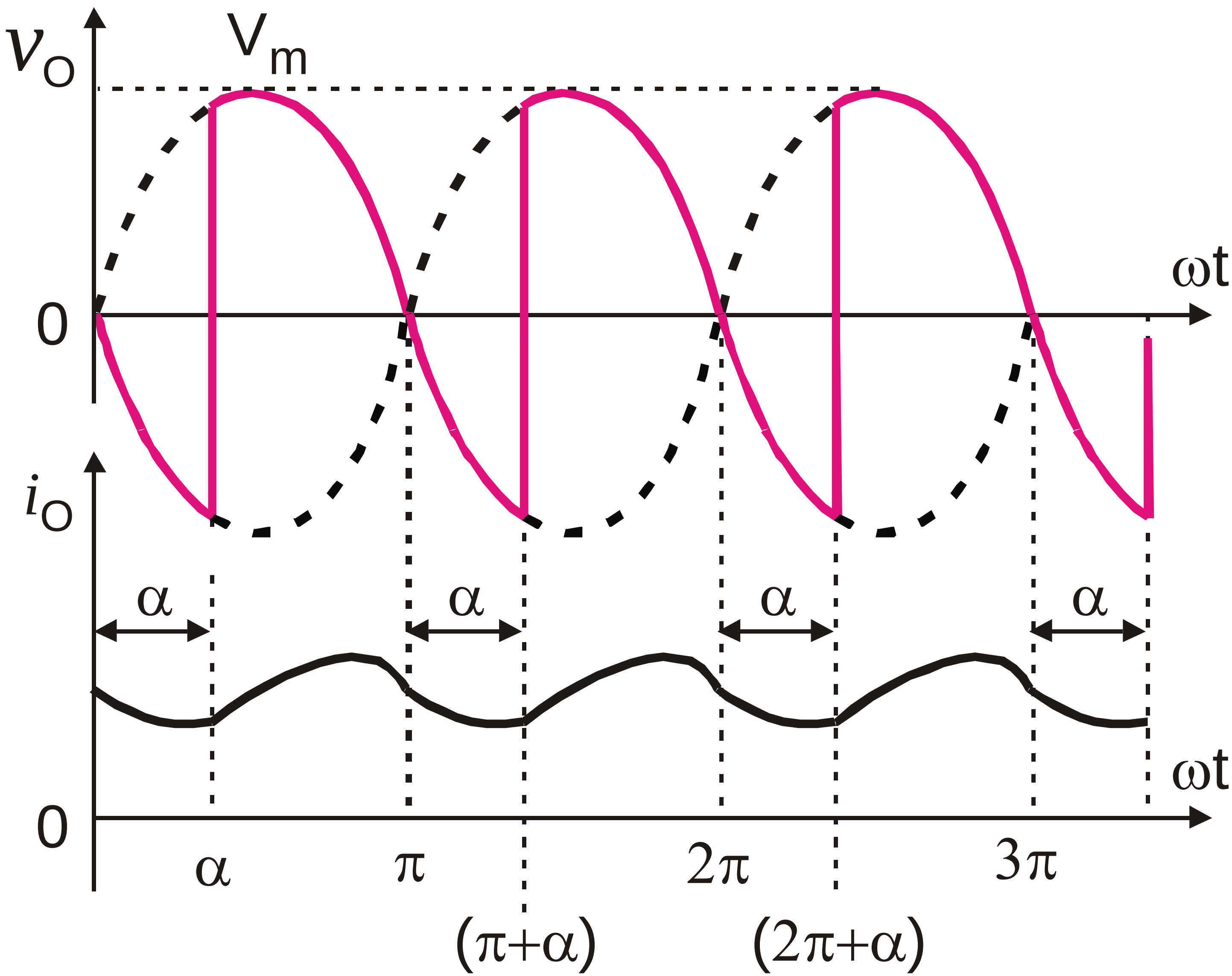
$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\pi} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \pi + \cos \alpha \right] \;\; ; \; \cos \pi = -1$$

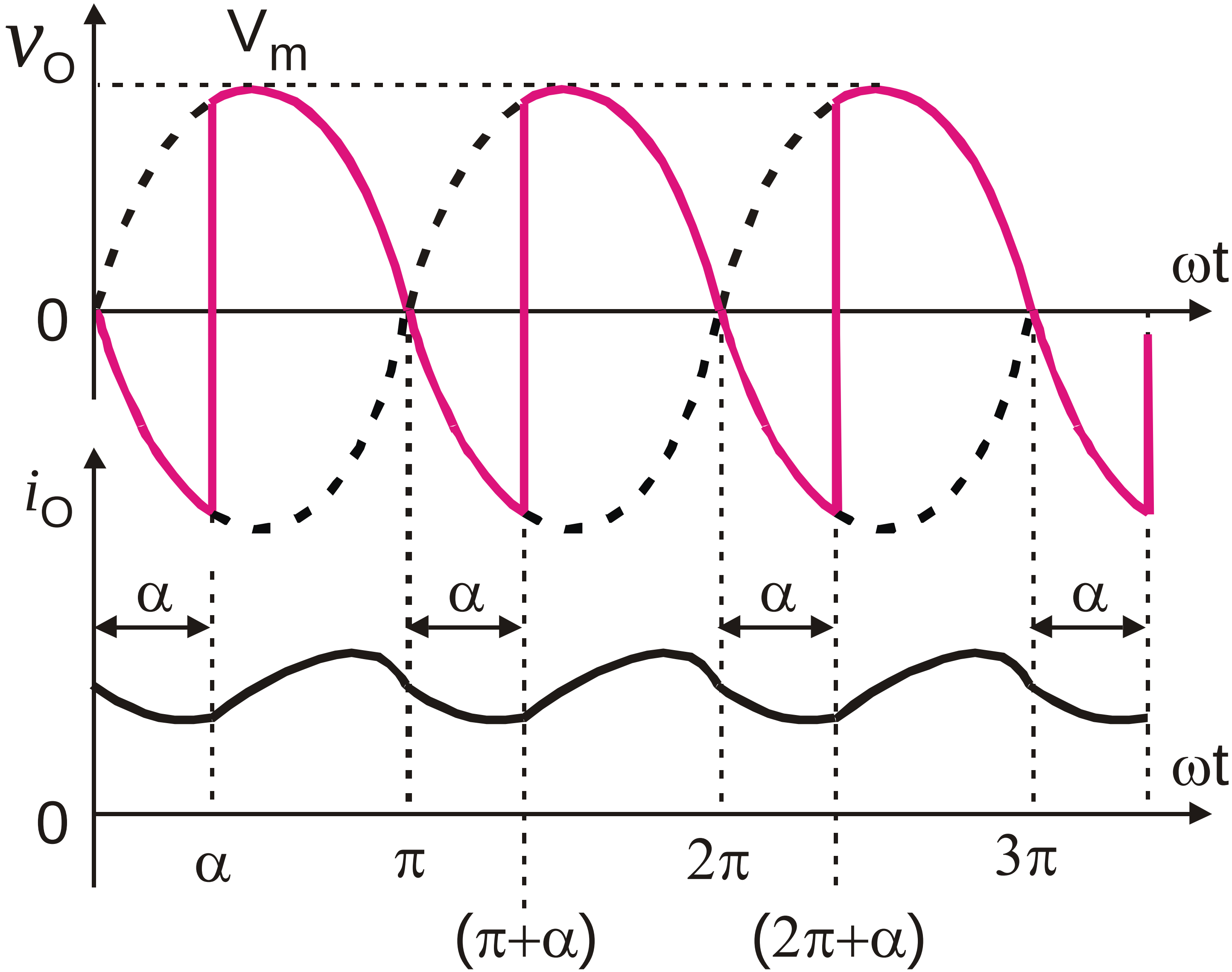
$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha)$$

- The load current is discontinuous for low values of load inductance and for large values of trigger angles.
- For large values of load inductance the load current flows continuously without falling to zero.
- Generally the load current is continuous for large load inductance and for low trigger angles.

Continuous Load Current Operation (Without FWD)


v_O
V_m
0
ωt
i_O
α
α
α
α
ωt
0
α
π
2π
3π
(π+α)
(2π+α)

To Derive
An Expression For
Average / DC Output Voltage
Of
Single Phase Full Wave Controlled Rectifier For Continuous Current Operation without FWD



v_O
V_m
ωt
0
i_O
α
α
α
α
ωt
0
α
π
2π
3π
(π+α)
(2π+α)

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi}\int_{\omega t=\alpha}^{(\pi+\alpha)} v_O.d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi}\left[\int_{\alpha}^{(\pi+\alpha)} V_m \sin\omega t.d(\omega t)\right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[-\cos\omega t\Big/_{\alpha}^{(\pi+\alpha)}\right]$$

$$V_{O(dc)} = V_{dc}$$

$$= \frac{V_m}{\pi}\left[\cos\alpha - \cos\left(\pi + \alpha\right)\right] \; ;$$

$$\cos\left(\pi + \alpha\right) = -\cos\alpha$$

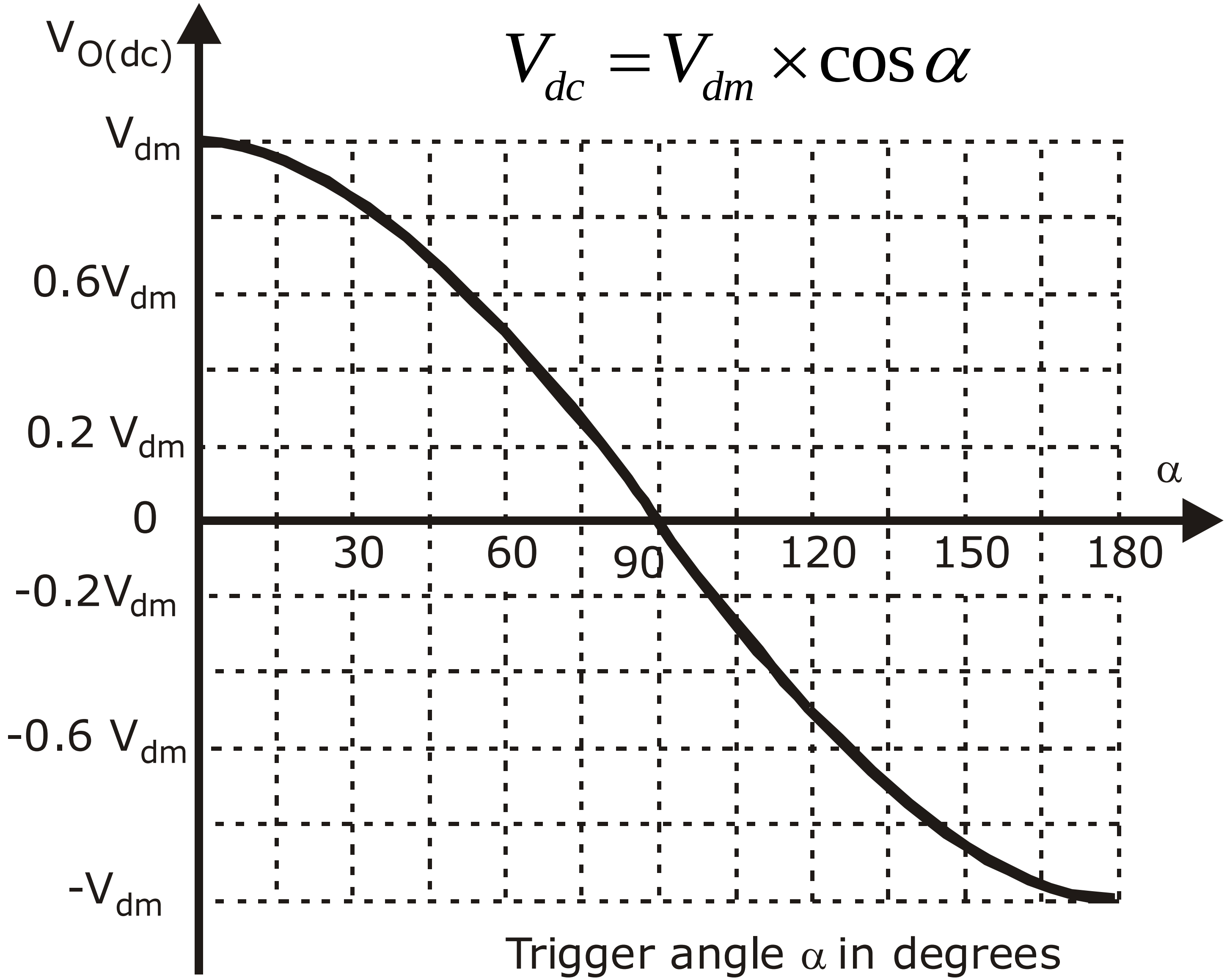
$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[\cos\alpha + \cos\alpha\right]$$

$$\therefore \qquad V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

- By plotting $V_{O(dc)}$ *versus* α,

  we obtain the control characteristic of a single phase full wave controlled rectifier with RL load for continuous load current operation without FWD

Trigger angle α in degrees	$V_{O(dc)}$	Remarks
0	$V_{dm} = \left(\frac{2V_m}{\pi}\right)$	Maximum dc output voltage $V_{dc(\max)} = V_{dm} = \left(\frac{2V_m}{\pi}\right)$
30^0	$0.866\ V_{dm}$	$V_{dc} = V_{dm} \times \cos\alpha$
60^0	$0.5\ V_{dm}$	
90^0	$0\ V_{dm}$	
120^0	$-0.5\ V_{dm}$	
150^0	$-0.866\ V_{dm}$	
180^0	$-V_{dm} = -\left(\frac{2V_m}{\pi}\right)$	



$V_{O(dc)}$
$V_{dc} = V_{dm} \times \cos\alpha$
V_{dm}
$0.6V_{dm}$
$0.2\ V_{dm}$
α
0
30
60
90
120
150
180
$-0.2V_{dm}$
$-0.6\ V_{dm}$
$-V_{dm}$
Trigger angle α in degrees

By varying the trigger angle we can vary the output dc voltage across the load. Hence we can control the dc output power flow to the load.

For trigger angle α, 0 to 90^0 $\left(i.e.,\ 0 \le \alpha \le 90^0\right)$;

$\cos\alpha$ is positive and hence V_{dc} is positive

V_{dc} & I_{dc} are positive ; $P_{dc} = \left(V_{dc} \times I_{dc}\right)$ is positive

Converter operates as a Controlled Rectifier.

Power flow is from the ac source to the load.

For trigger angle α, 90^0 to 180^0

$$\left(i.e.,\ 90^0 \le \alpha \le 180^0\right),$$

$\cos\alpha$ is negative and hence

V_{dc} is negative; I_{dc} is positive ;

$$P_{dc} = \left(V_{dc} \times I_{dc}\right) \text{ is negative.}$$

In this case the converter operates

as a Line Commutated Inverter.

Power flows from the load ckt. to the i/p ac source.

The inductive load energy is fed back to the

i/p source.

Drawbacks Of Full Wave Controlled Rectifier With Centre Tapped Transformer

- We require a centre tapped transformer which is quite heavier and bulky.
- Cost of the transformer is higher for the required dc output voltage & output power.
- Hence full wave bridge converters are preferred.

Single Phase
Full Wave Bridge Controlled Rectifier

Single Phase
Full Wave Bridge Controlled Rectifier

2 types of FW Bridge Controlled Rectifiers are

- Half Controlled Bridge Converter

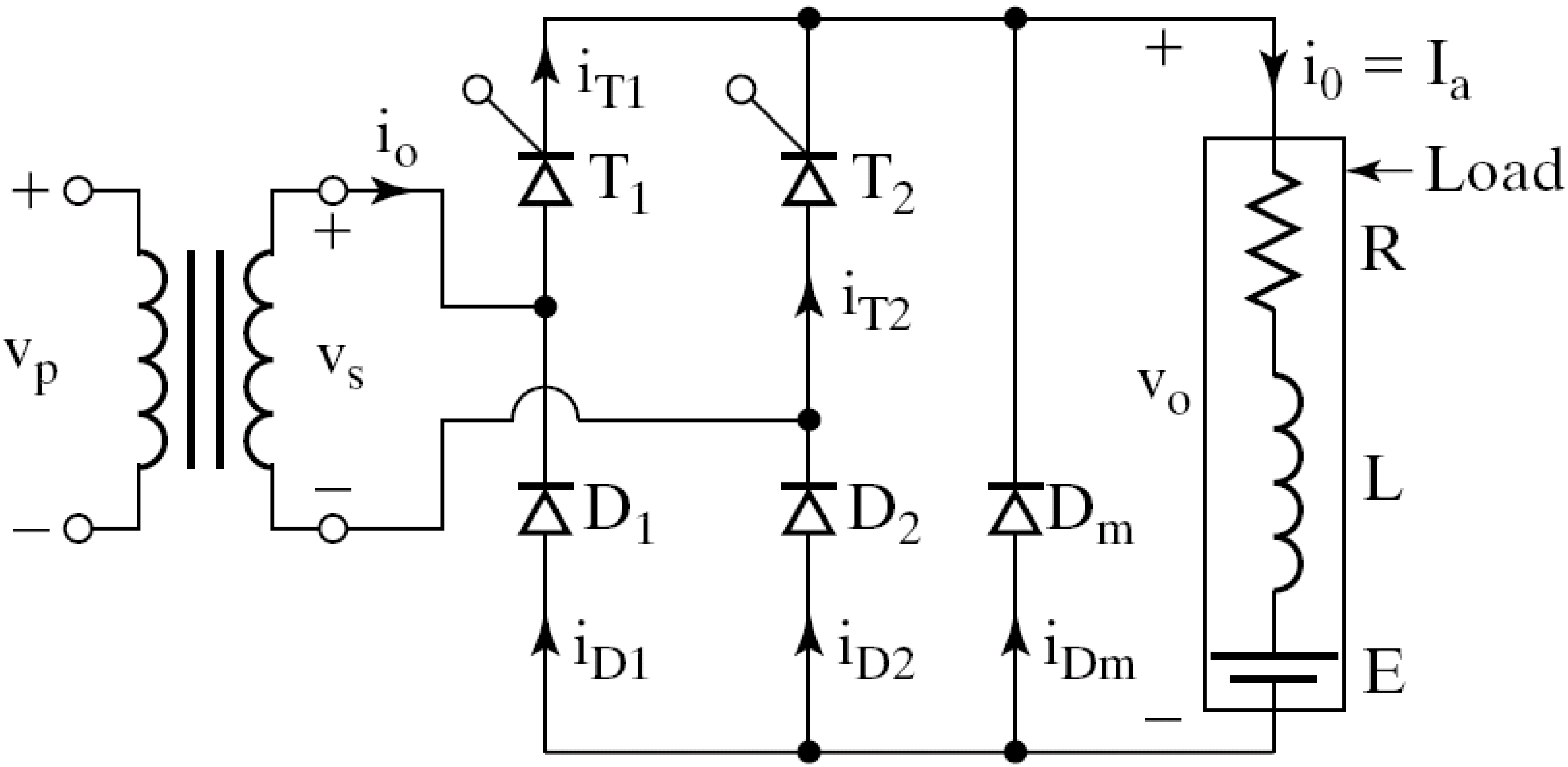
  (Semi-Converter)

- Fully Controlled Bridge Converter

  (Full Converter)

The bridge full wave controlled rectifier does not require a centre tapped transformer

Single Phase
Full Wave Half Controlled Bridge Converter
(Single Phase Semi Converter)



+
−
v_p
+
−
v_s
i_o
i_{T1}
T_1
i_{T2}
T_2
D_1
D_2
D_m
i_{D1}
i_{D2}
i_{Dm}
+
$i_0 = I_a$
Load
R
v_o
L
E
−

Trigger Pattern of Thyristors

$$Thyristor\ T_1\ is\ triggered\ at$$

$$\omega t = \alpha,\ at\ \omega t = \left(2\pi + \alpha\right),...$$

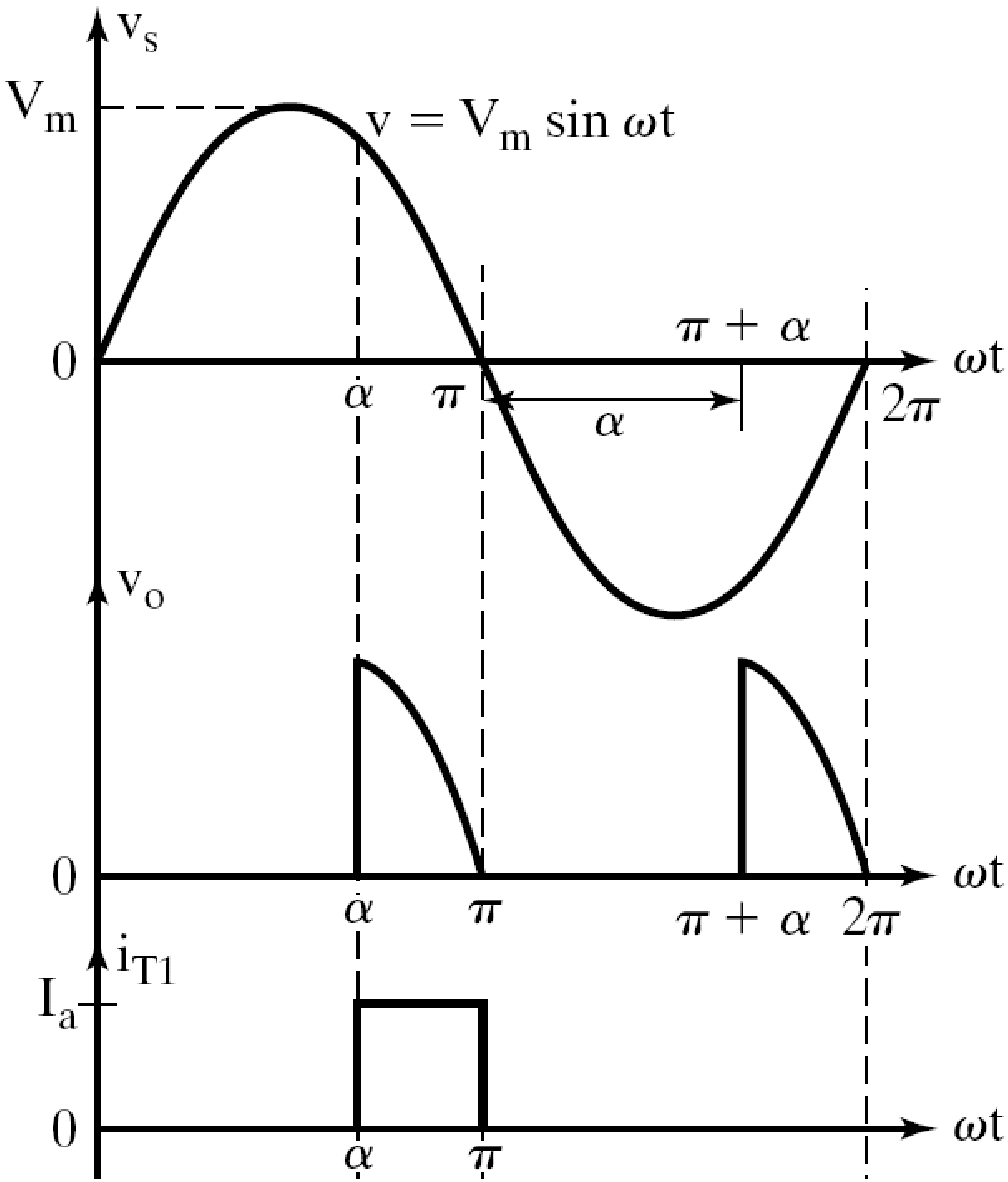
$$Thyristor\ T_2\ is\ triggered\ at$$

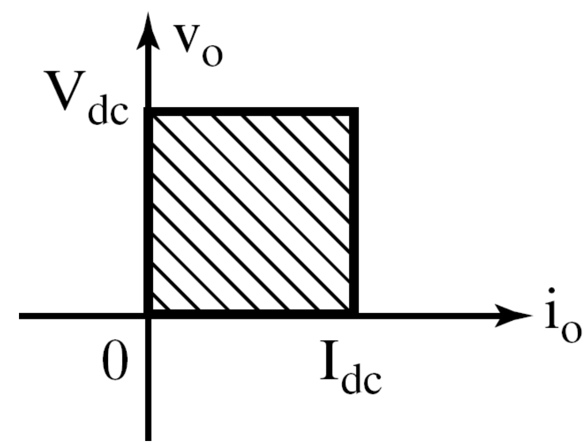
$$\omega t = \left(\pi + \alpha\right),\ at\ \omega t = \left(3\pi + \alpha\right),...$$

$$The\ time\ delay\ between\ the\ gating$$

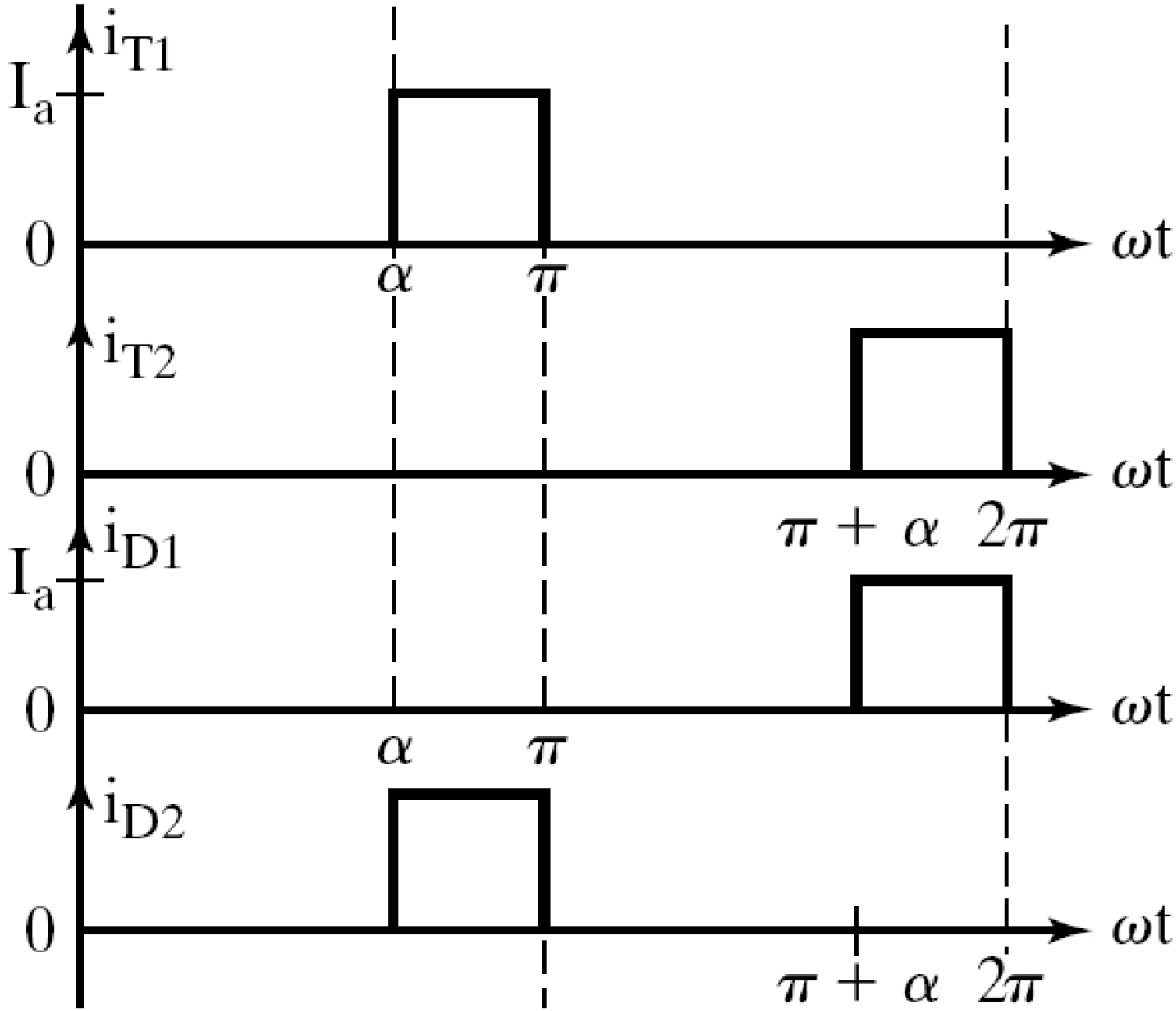
$$signals\ of\ T_1\ \&\ T_2 = \pi\ radians\ or\ 180^0$$

Waveforms of single phase semi-converter with general load & FWD for $\alpha > 90^0$

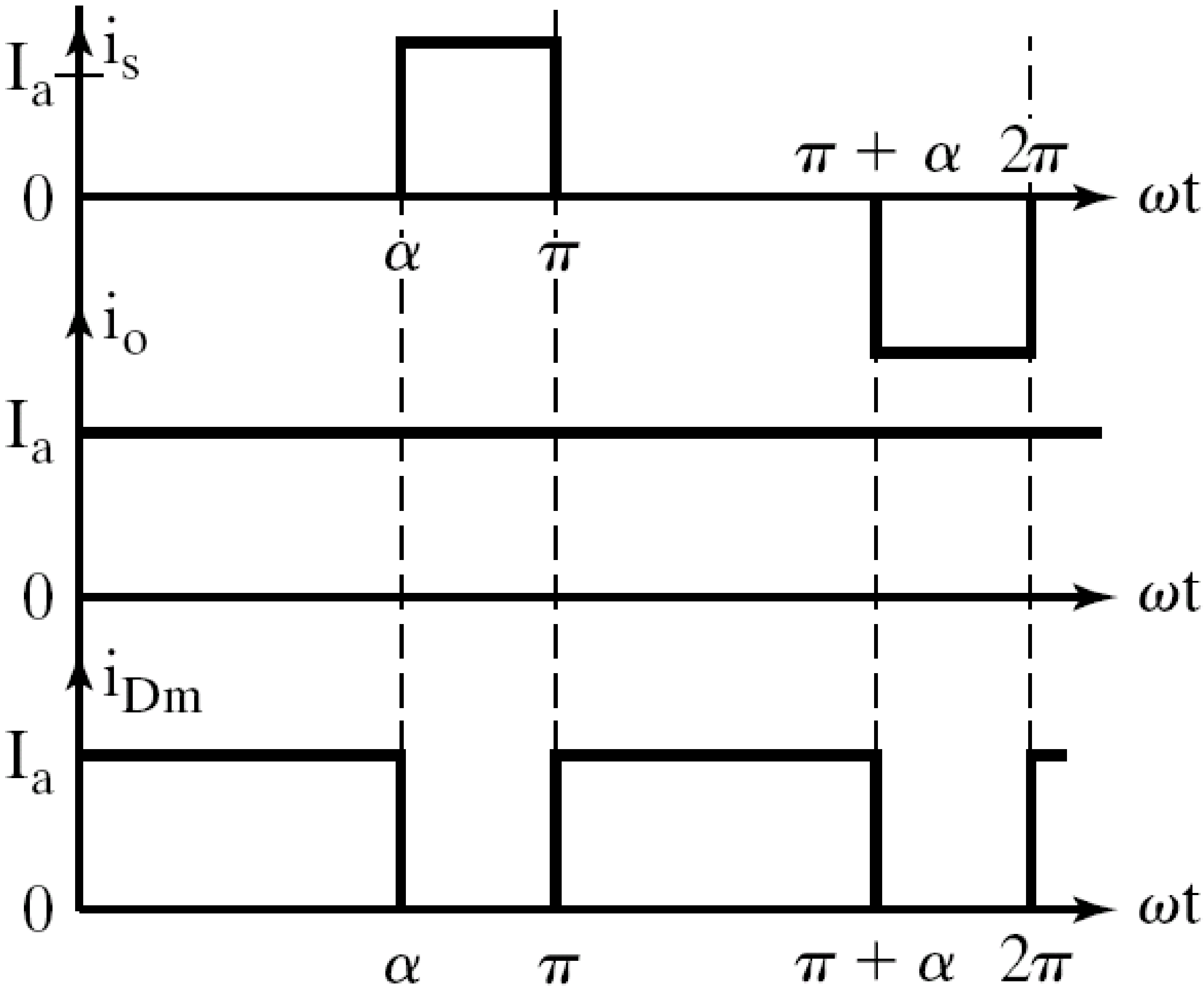




Single Quadrant Operation



i_{T1}
I_a
0
ωt
α
π
i_{T2}
0
ωt
$\pi + \alpha$
2π
i_{D1}
I_a
0
ωt
α
π
i_{D2}
0
ωt
$\pi + \alpha$
2π



I_a
i_s
0
α
π
$\pi + \alpha$
2π
ωt
i_o
I_a
0
ωt
i_{Dm}
I_a
0
ωt
α
π
$\pi + \alpha$
2π

Thyristor T_1 & D_1 conduct

$$\text{from } \omega t = \alpha \ to \ \pi$$

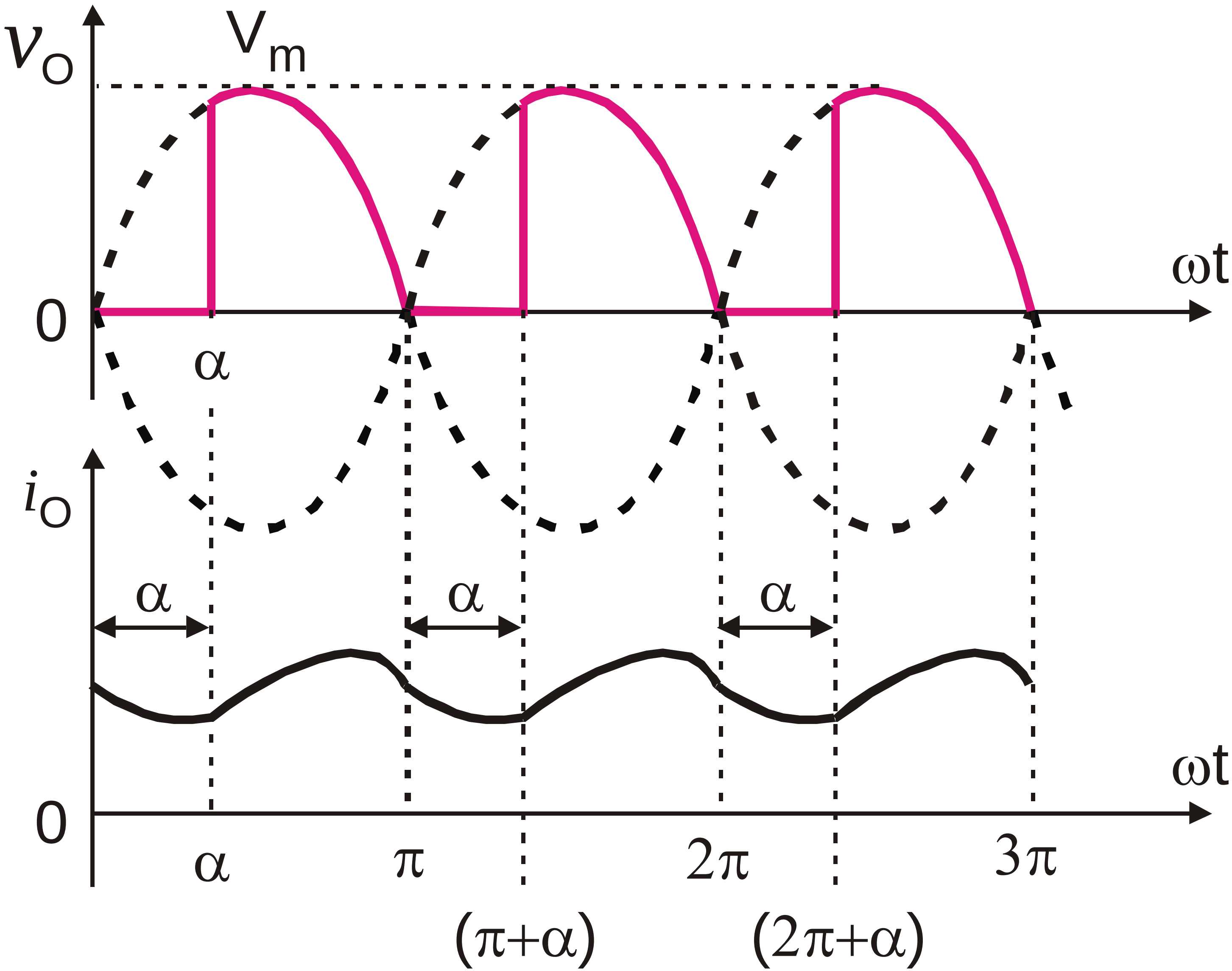
Thyristor T_2 & D_2 conduct

$$\text{from } \omega t = \left(\pi + \alpha\right) \ to \ 2\pi$$

FWD conducts during

$$\omega t = 0 \text{ to } \alpha, \ \pi \ to \ \left(\pi + \alpha\right), ...$$

Load Voltage & Load Current Waveform of Single Phase Semi Converter for $\alpha < 90^0$ & Continuous load current operation



v_O
V_m
ωt
0
α
i_O
α
α
α
ωt
0
α
π
2π
3π
(π+α)
(2π+α)

To Derive an Expression
For The
DC Output Voltage of
A
Single Phase Semi-Converter With
R,L, & E Load & FWD
For Continuous, Ripple Free Load
Current Operation

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=0}^{\pi} v_O . d\left(\omega t\right)$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t . d\left(\omega t\right)$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\pi} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \pi + \cos \alpha \right] \;\; ; \;\; \cos \pi = -1$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left(1 + \cos \alpha \right)$$

V_{dc} can be varied from a max.

value of $\dfrac{2V_m}{\pi}$ *to* 0 by varying α from 0 to π.

For $\alpha = 0,$ The max. dc o/p voltage obtained is

$$V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi}$$

Normalized dc o/p voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dn}} = \frac{\frac{V_m}{\pi}(1+\cos\alpha)}{\left(\frac{2V_m}{\pi}\right)} = \frac{1}{2}(1+\cos\alpha)$$

RMS O/P Voltage $V_{O(RMS)}$

$$V_{O(RMS)} = \left[\frac{2}{2\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t . d\left(\omega t\right) \right]^{\frac{1}{2}}$$

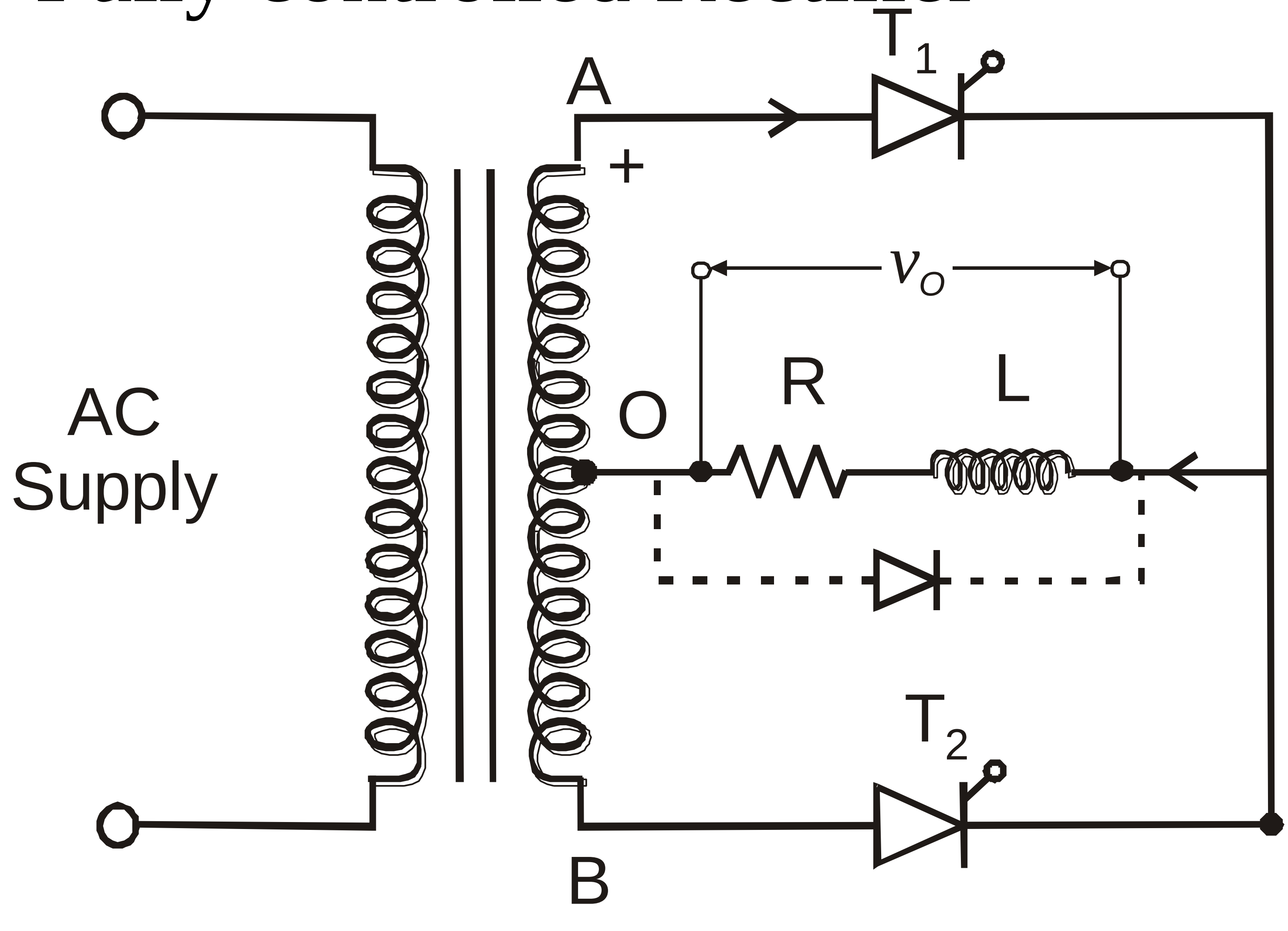
$$V_{O(RMS)} = \left[\frac{V_m^2}{2\pi} \int_{\alpha}^{\pi} \left(1 - \cos 2\omega t\right) . d\left(\omega t\right) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \left[\frac{1}{\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{\frac{1}{2}}$$

Single Phase Full Wave Controlled Rectifier

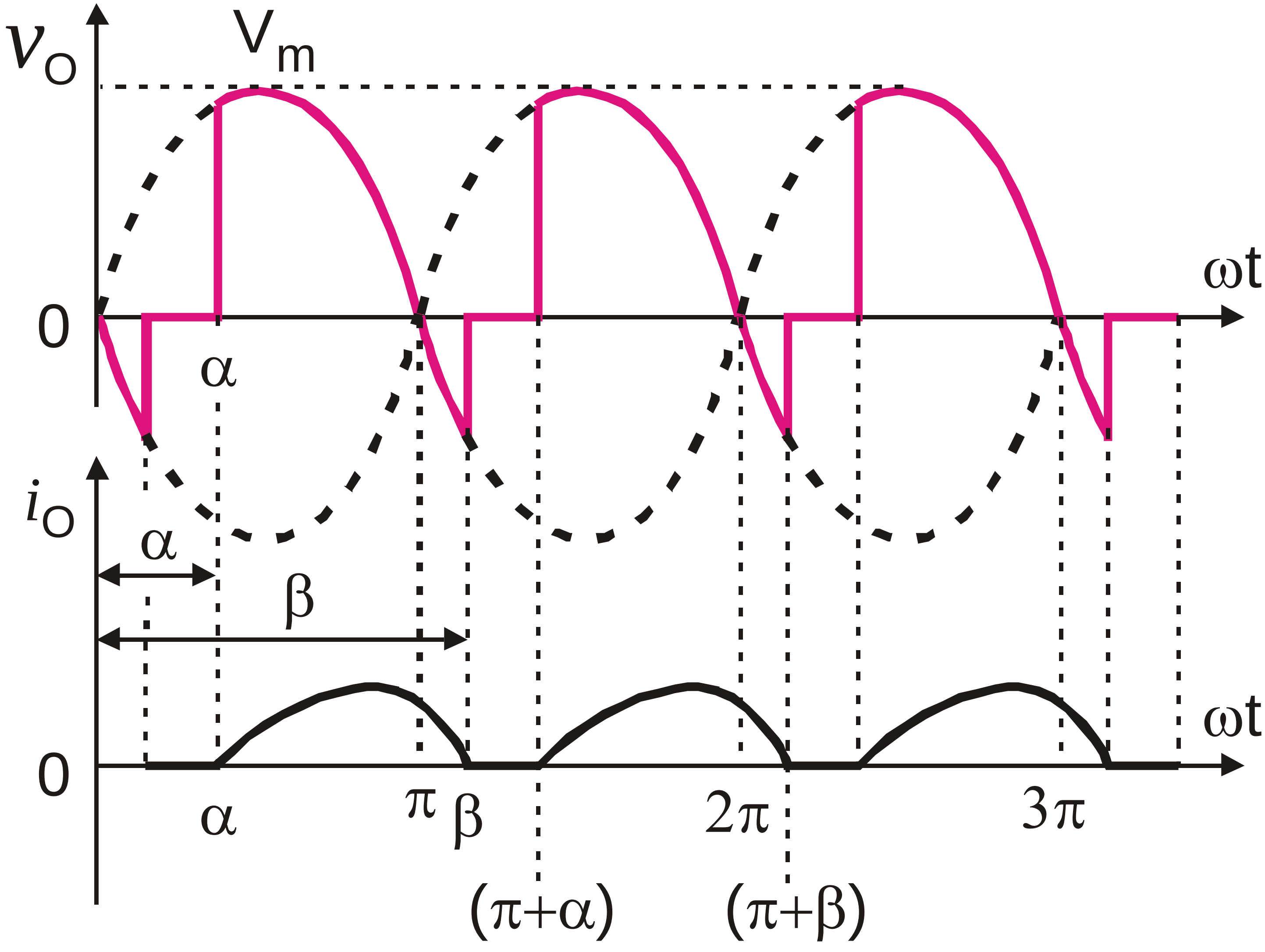
Single Phase Full Wave Controlled Rectifier Using A Center Tapped Transformer

Single Phase Midpoint type Fully controlled Rectifier



Discontinuous Load Current Operation without FWD for

$$\pi < \beta < (\pi + \alpha)$$



v_O
V_m
ωt
0
α
i_O
α
β
ωt
0
α
π
β
2π
3π
(π+α)
(π+β)

To Derive An Expression For The Output (Load) Current, During $\omega t = \alpha$ to β When Thyristor T_1 Conducts

Assuming T_1 is triggered $\omega t = \alpha$,

we can write the equation,

$$L\left(\frac{di_O}{dt}\right) + Ri_O = V_m \sin \omega t \;;\; \alpha \le \omega t \le \beta$$

General expression for the output current,

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + A_1 e^{\frac{-t}{\tau}}$$

 202

$$V_m = \sqrt{2}V_S = \text{maximum supply voltage.}$$

$$Z = \sqrt{R^2 + (\omega L)^2} = \text{Load impedance.}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \text{Load impedance angle.}$$

$$\tau = \frac{L}{R} = \text{Load circuit time constant.}$$

$\therefore$ general expression for the output load current

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-R}{L}t}$$

Constant A_1 is calculated from

initial condition $i_O = 0$ at $\omega t = \alpha$; t= $\left(\dfrac{\alpha}{\omega}\right)$

$$i_O = 0 = \frac{V_m}{Z}\sin\left(\alpha - \phi\right) + A_1 e^{\frac{-R}{L}t}$$

$$\therefore \qquad A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin\left(\alpha - \phi\right)$$

We get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

Substituting the value of constant A_1 in the general expression for i_O

$$i_O = \frac{V_m}{Z}\sin\left(\omega t-\phi\right)+e^{\frac{-R}{\omega L}\left(\omega t-\alpha\right)}\left[\frac{-V_m}{Z}\sin\left(\alpha-\phi\right)\right]$$

$\therefore$ we obtain the final expression for the inductive load current

$$i_O = \frac{V_m}{Z}\left[\sin\left(\omega t-\phi\right)-\sin\left(\alpha-\phi\right)e^{\frac{-R}{\omega L}\left(\omega t-\alpha\right)}\right];$$

Where $\alpha \le \omega t \le \beta$

Extinction angle β can be calculated by using the condition that $i_O = 0 \; at \; \omega t = \beta$

$$i_O = \frac{V_m}{Z}\left[\sin\left(\omega t - \phi\right) - \sin\left(\alpha - \phi\right) e^{\frac{-R}{\omega L}\left(\omega t - \alpha\right)}\right] = 0$$

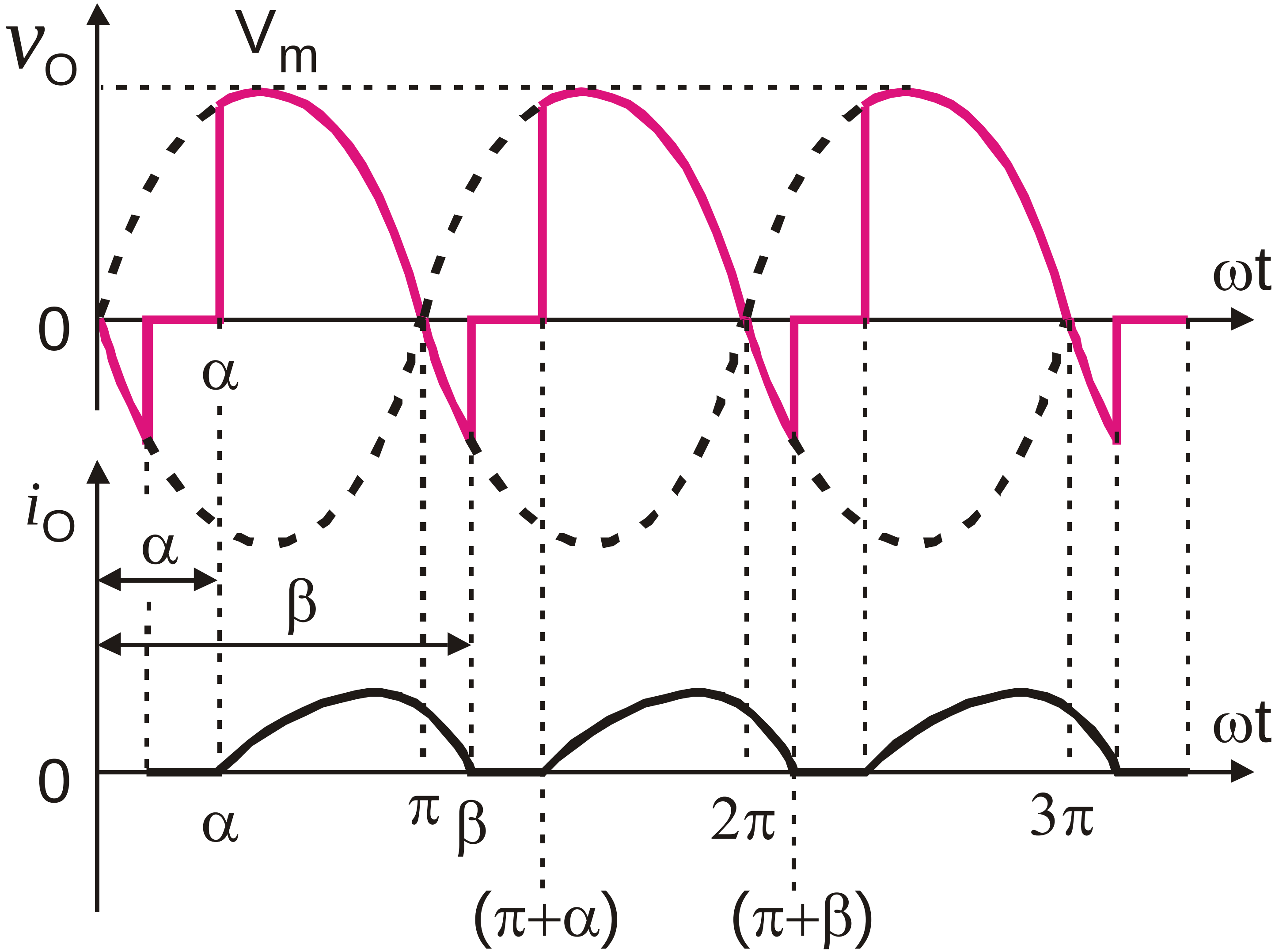
$$\therefore \sin\left(\beta - \phi\right) = e^{\frac{-R}{\omega L}\left(\beta - \alpha\right)} \times \sin\left(\alpha - \phi\right)$$

β can be calculated by solving the above eqn.

To Derive An Expression For The DC Output Voltage Of A Single Phase Full Wave Controlled Rectifier With RL Load

(Without FWD)

 207


v_O
V_m
ωt
0
α
i_O
α
β
ωt
0
α
π
β
2π
3π
(π+α)
(π+β)

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=\alpha}^{\beta} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{\beta} V_m \sin \omega t . d(\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\beta} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (\cos \alpha - \cos \beta)$$

When the load inductance is negligible $\left(\text{ i.e., } L \approx 0\right)$

Extinction angle $\beta = \pi$ radians

Hence the average or dc output voltage for R load

$$V_{O(dc)} = \frac{V_m}{\pi}\left(\cos\alpha - \cos\pi\right)$$

$$V_{O(dc)} = \frac{V_m}{\pi}\left(\cos\alpha - \left(-1\right)\right)$$

$$V_{O(dc)} = \frac{V_m}{\pi}\left(1 + \cos\alpha\right); \text{ for R load, when } \beta = \pi$$

 210

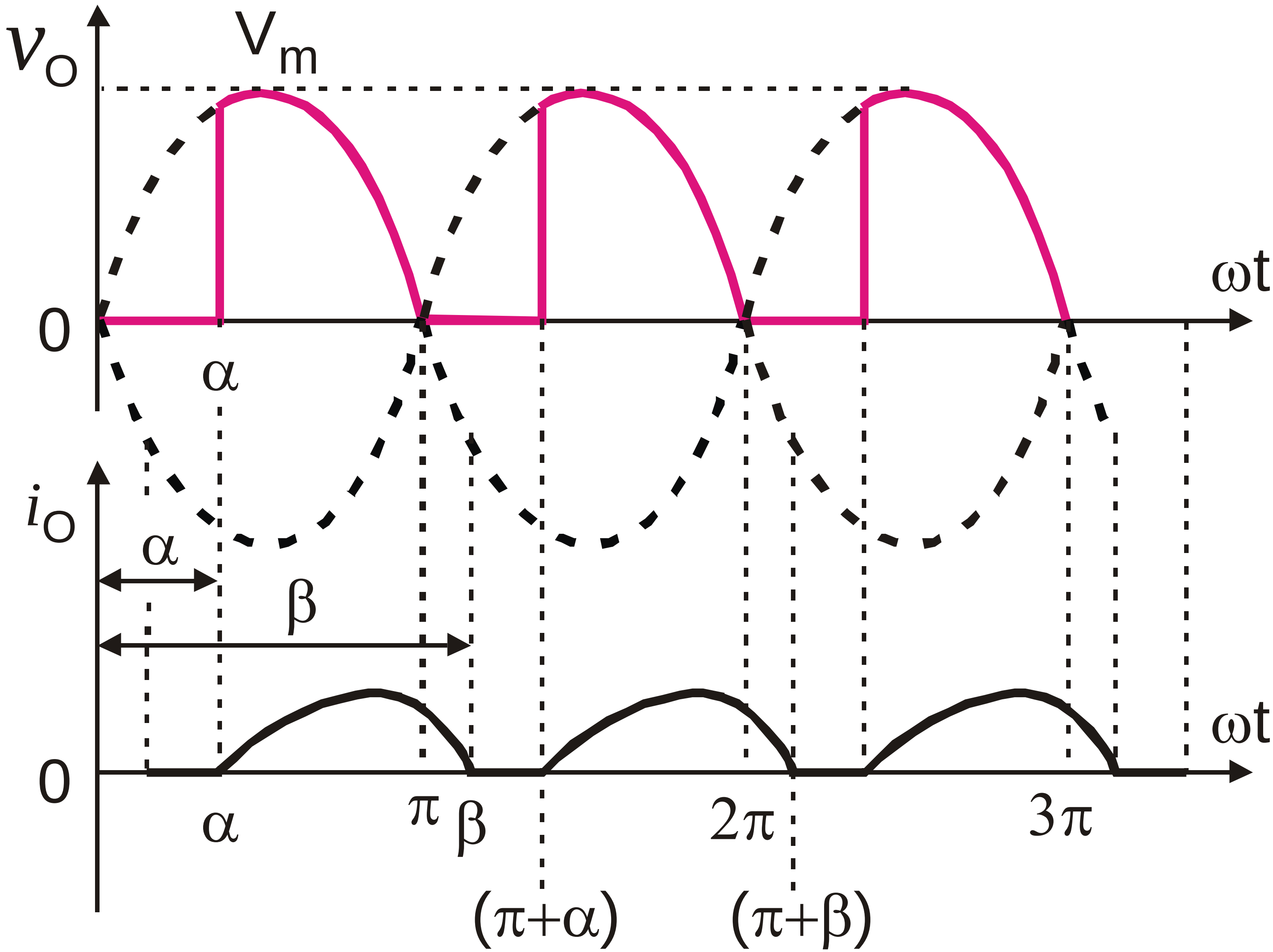
To calculate the RMS output voltage we use the expression

$$V_{O(RMS)} = \sqrt{\frac{1}{\pi}\left[\int_{\alpha}^{\beta} V_m^2 \sin^2 \omega t.d\left(\omega t\right)\right]}$$

211

Discontinuous Load Current Operation with FWD

 212



vO
Vm
ωt
0
α
iO
α
β
ωt
0
α
π
β
2π
3π
(π+α)
(π+β)

Thyristor T_1 is triggered at $\omega t = \alpha$;

T_1 conducts from $\omega t = \alpha$ to π

Thyristor T_2 is triggered at $\omega t = (\pi + \alpha)$;

T_2 conducts from $\omega t = (\pi + \alpha)$ to 2π

FWD conducts from $\omega t = \pi$ to β &

$v_O \approx 0$ during discontinuous load current.

214

To Derive an Expression For The DC Output Voltage For A Single Phase Full Wave Controlled Rectifier With RL Load & FWD

215

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=0}^{\pi} v_O . d\left(\omega t\right)$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t . d\left(\omega t\right)$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\pi} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \pi + \cos \alpha \right] \;\; ; \; \cos \pi = -1$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left(1 + \cos \alpha \right)$$

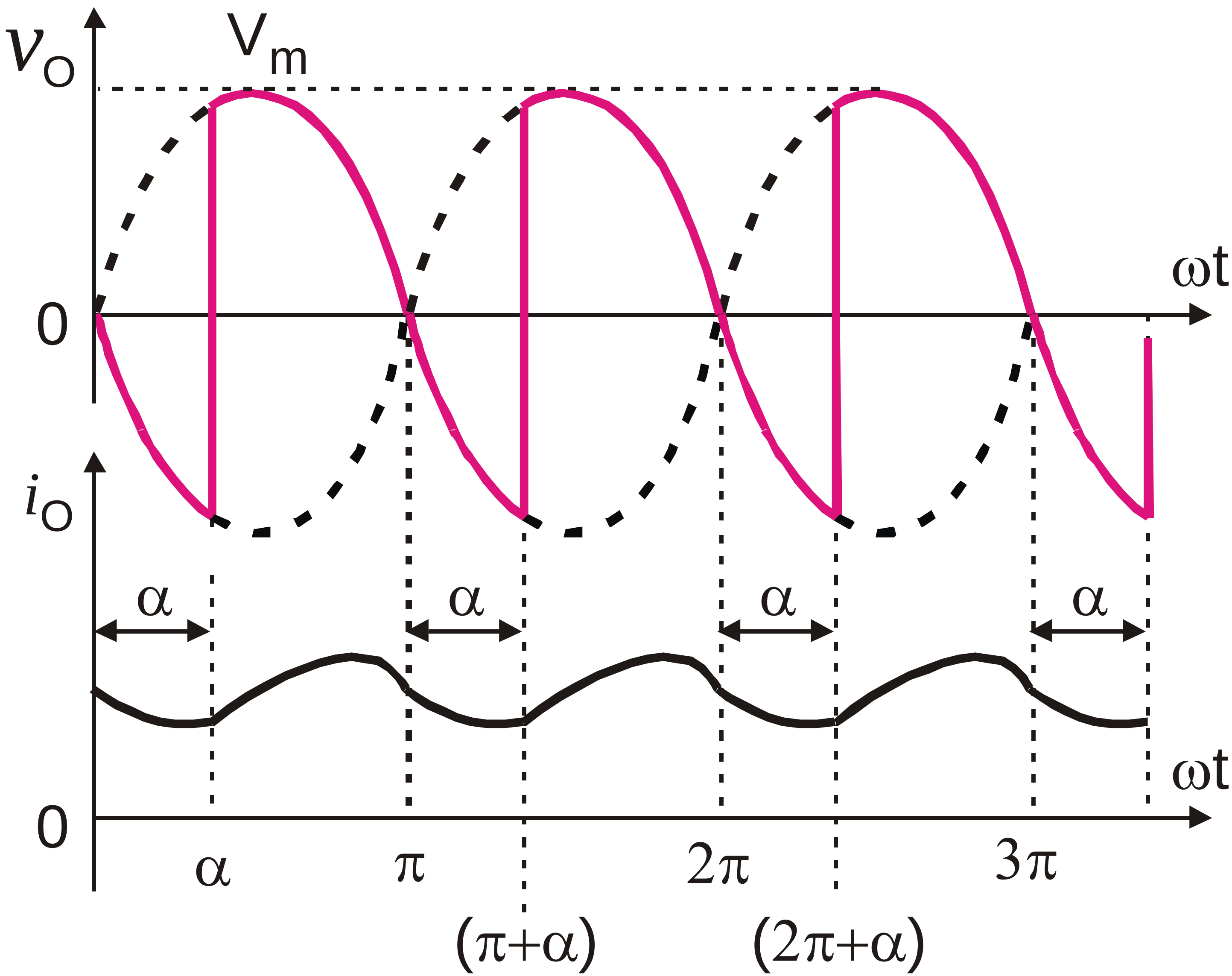
216

- The load current is discontinuous for low values of load inductance and for large values of trigger angles.
- For large values of load inductance the load current flows continuously without falling to zero.
- Generally the load current is continuous for large load inductance and for low trigger angles.

217

Continuous Load Current Operation (Without FWD)

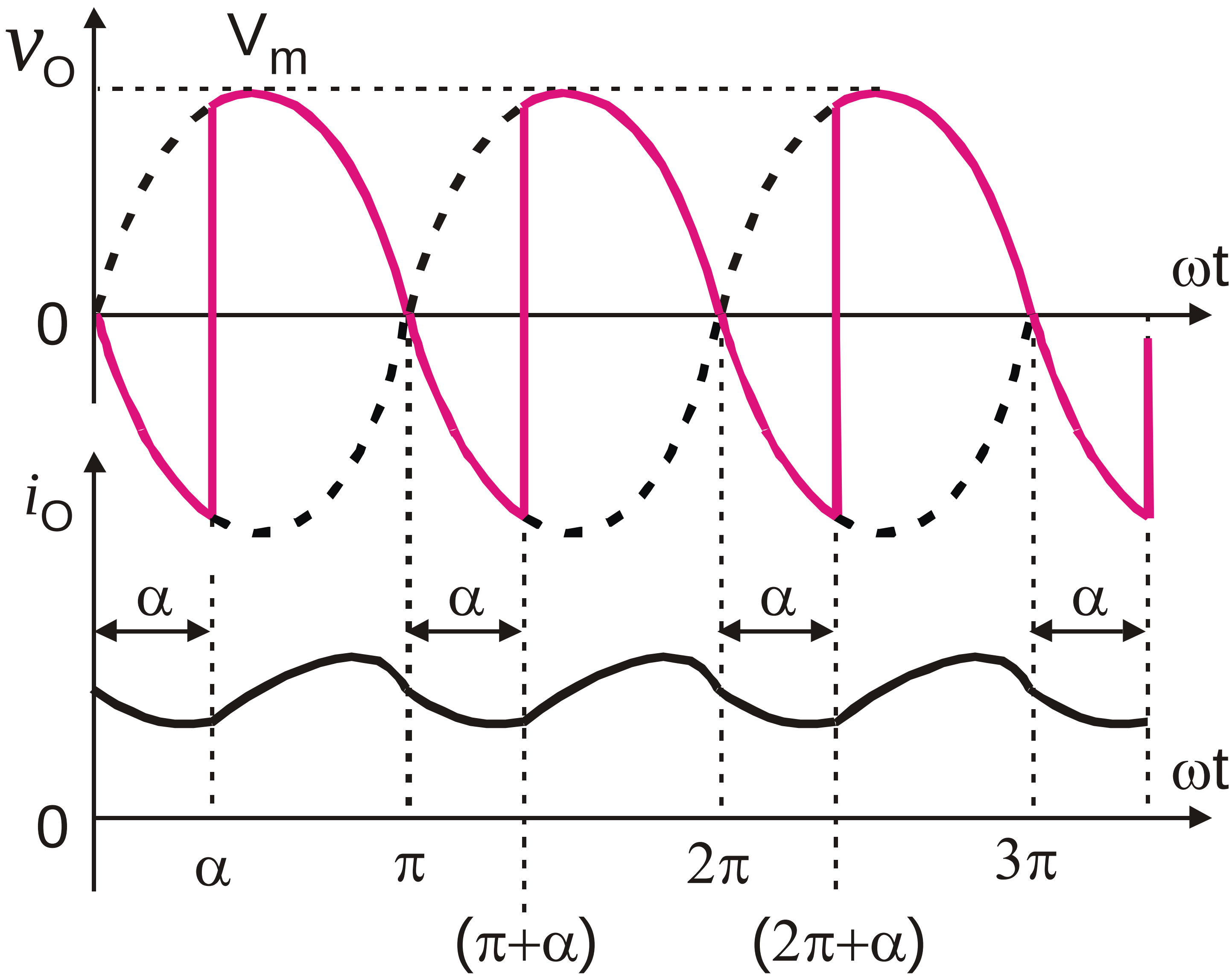
218



v_O
V_m
ωt
0
i_O
α
α
α
α
ωt
0
α
π
2π
3π
$(\pi+\alpha)$
$(2\pi+\alpha)$

To Derive An Expression For Average / DC Output Voltage Of Single Phase Full Wave Controlled Rectifier For Continuous Current Operation without FWD

220



vO
Vm
ωt
0
iO
α
α
α
α
ωt
0
α
π
2π
3π
(π+α)
(2π+α)

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=\alpha}^{(\pi+\alpha)} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{(\pi+\alpha)} V_m \sin \omega t . d(\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{(\pi+\alpha)} \right]$$

$$V_{O(dc)} = V_{dc}$$

$$= \frac{V_m}{\pi}\left[\cos\alpha - \cos(\pi + \alpha)\right] \ ;$$

$$\cos(\pi + \alpha) = -\cos\alpha$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[\cos\alpha + \cos\alpha\right]$$

$$\therefore \qquad V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

223

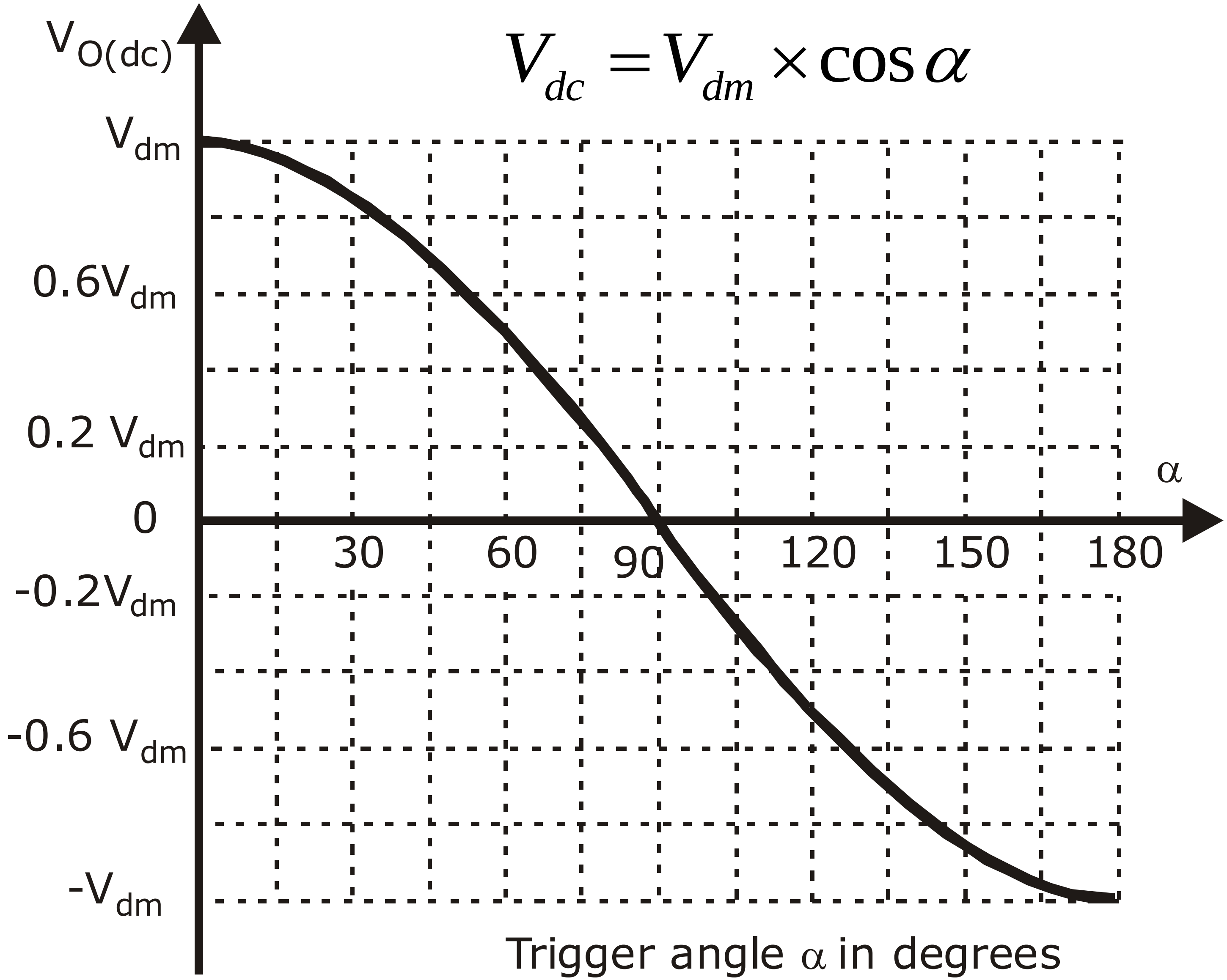
- By plotting $V_{O(dc)}$ *versus* α,
we obtain the control characteristic of a single phase full wave controlled rectifier with RL load for continuous load current operation without FWD

 224

Trigger angle α in degrees	$V_{O(dc)}$	**Remarks**
0	$V_{dm} = \left(\frac{2V_m}{\pi}\right)$	Maximum dc output voltage $V_{dc(\max)} = V_{dm} = \left(\frac{2V_m}{\pi}\right)$
30^0	$0.866\ V_{dm}$	$V_{dc} = V_{dm} \times \cos\alpha$
60^0	$0.5\ V_{dm}$	
90^0	$0\ V_{dm}$	
120^0	$-0.5\ V_{dm}$	
150^0	$-0.866\ V_{dm}$	
180^0	$-V_{dm} = -\left(\frac{2V_m}{\pi}\right)$	

225

$$V_{dc} = V_{dm} \times \cos\alpha$$



By varying the trigger angle we can vary the output dc voltage across the load. Hence we can control the dc output power flow to the load.

For trigger angle α, 0 to 90^0 $\left(i.e.,\ 0 \le \alpha \le 90^0\right)$;

$\cos\alpha$ is positive and hence V_{dc} is positive

V_{dc} & I_{dc} are positive ; $P_{dc} = \left(V_{dc} \times I_{dc}\right)$ is positive

Converter operates as a Controlled Rectifier.

Power flow is from the ac source to the load.

227

For trigger angle α, 90^0 to 180^0

$$\left(i.e., \ 90^0 \le \alpha \le 180^0 \right),$$

$\cos\alpha$ is negative and hence

V_{dc} is negative; I_{dc} is positive ;

$$P_{dc} = \left(V_{dc} \times I_{dc} \right) \text{ is negative.}$$

In this case the converter operates

as a Line Commutated Inverter.

Power flows from the load ckt. to the i/p ac source.

The inductive load energy is fed back to the

i/p source.

228

Single Phase
Full Wave Bridge Controlled Rectifier

Drawbacks Of Full Wave Controlled Rectifier With Centre Tapped Transformer

- We require a centre tapped transformer which is quite heavier and bulky.
- Cost of the transformer is higher for the required dc output voltage & output power.
- Hence full wave bridge converters are preferred.

Single Phase Full Wave Bridge Controlled Rectifier

2 types of FW Bridge Controlled Rectifiers are

- Half Controlled Bridge Converter

  (Semi-Converter)

- Fully Controlled Bridge Converter

  (Full Converter)

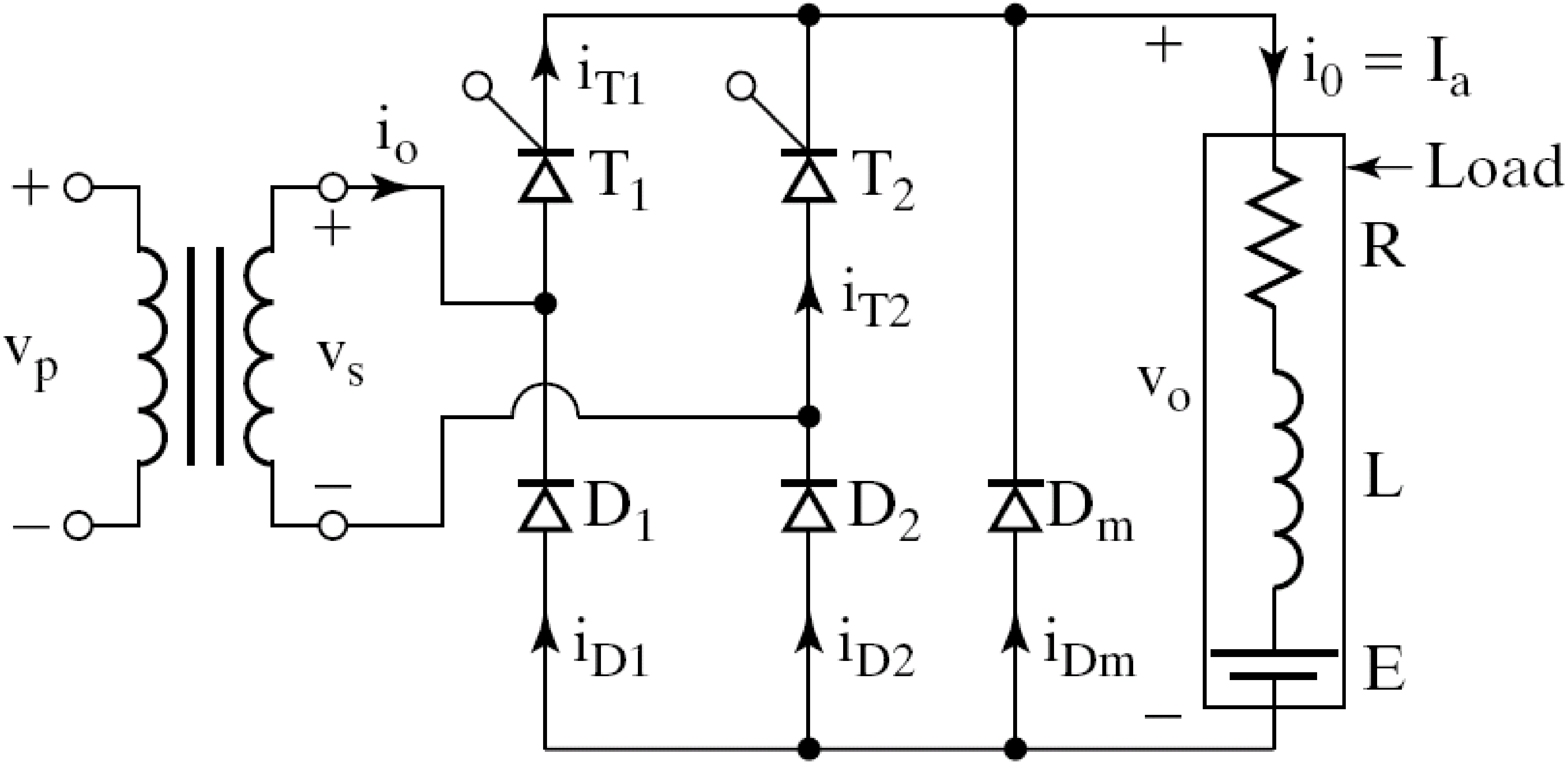
The bridge full wave controlled rectifier does not require a centre tapped transformer

 231

Single Phase
Full Wave Half Controlled Bridge Converter
(Single Phase Semi Converter)

232

Single Phase Full Wave Half Controlled Bridge Converter



233

Trigger Pattern of Thyristors

Thyristor T_1 *is triggered at*

$$\omega t = \alpha, \; at \; \omega t = \left(2\pi + \alpha\right), \ldots$$

Thyristor T_2 *is triggered at*

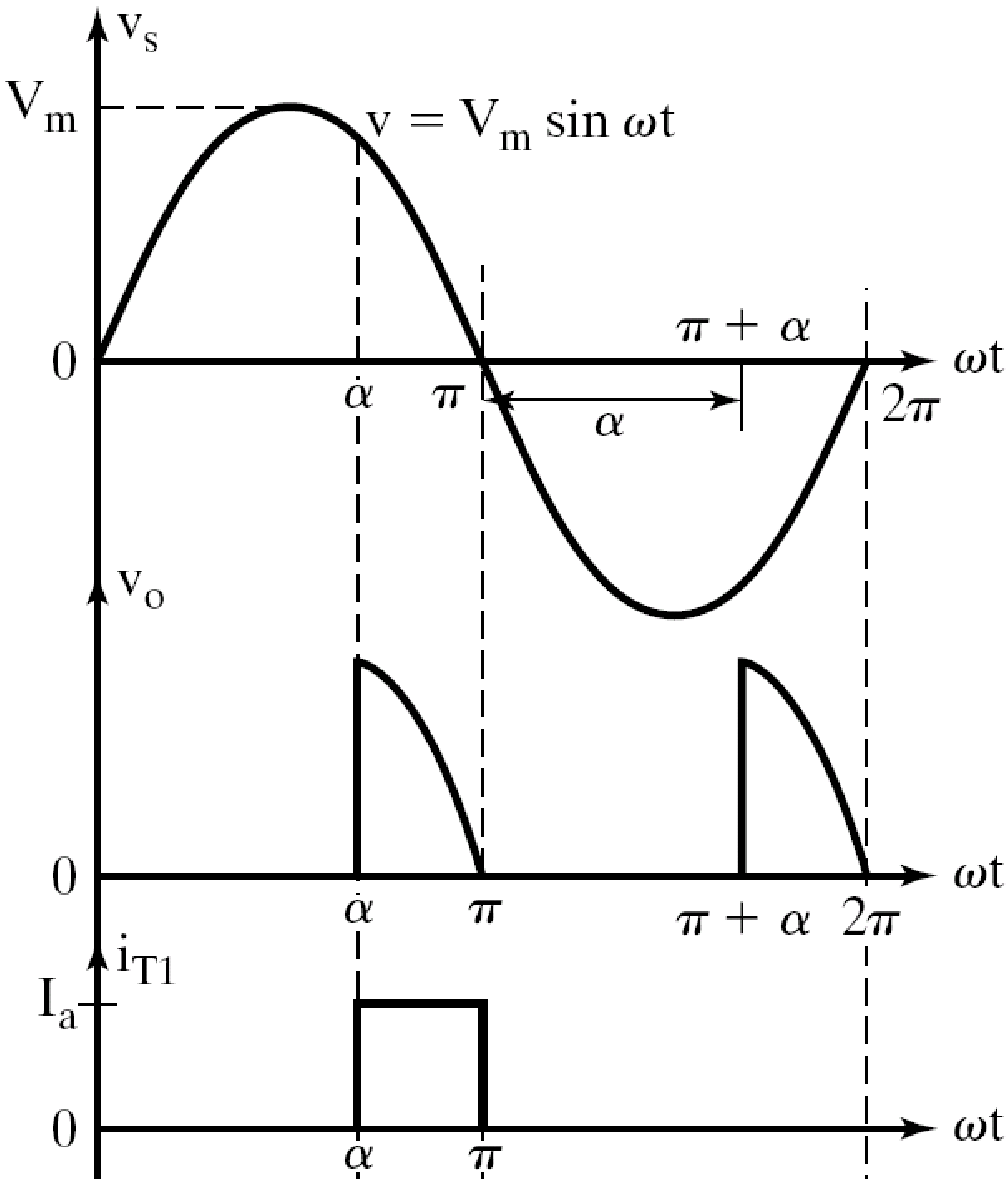
$$\omega t = \left(\pi + \alpha\right), \; at \; \omega t = \left(3\pi + \alpha\right), \ldots$$

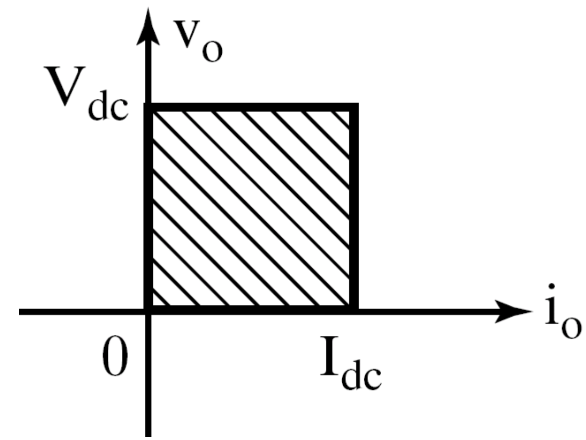
The time delay between the gating signals of T_1 & $T_2 = \pi$ *radians or* 180^0

234

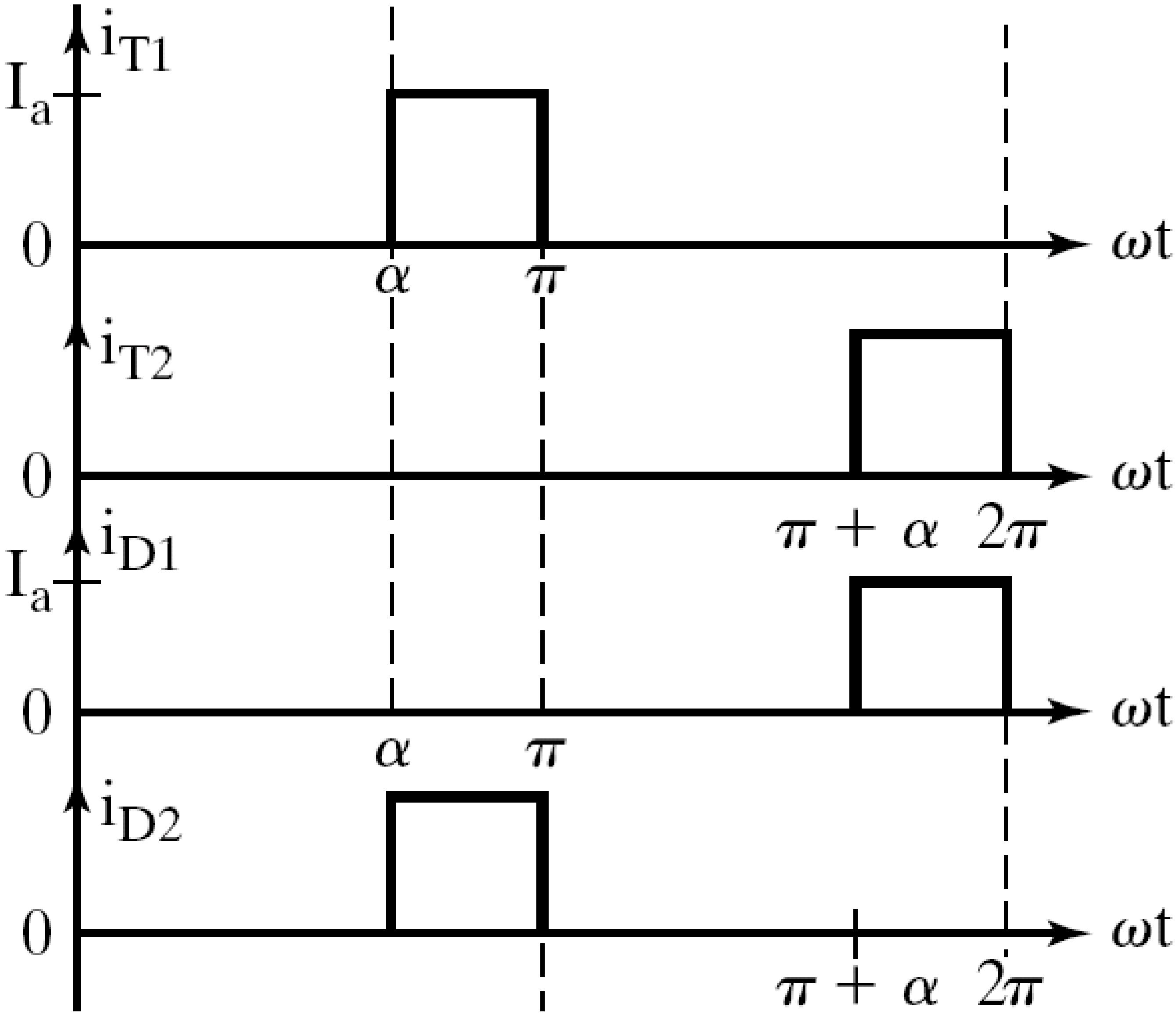
Waveforms of single phase semi-converter with general load & FWD for $\alpha > 90^0$

235

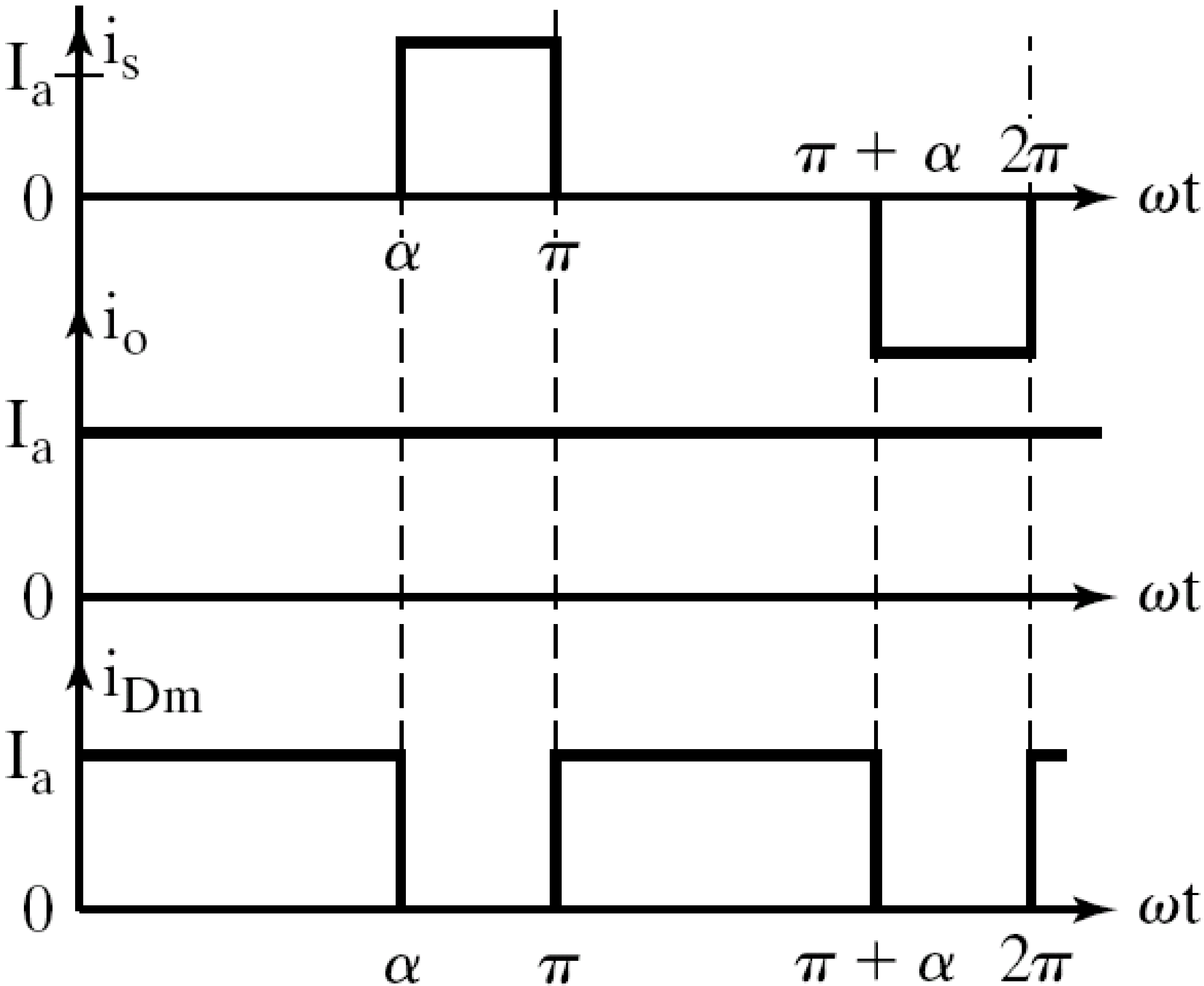




Single Quadrant Operation



i_T1
I_a
0
ωt
α
π
i_T2
0
ωt
π + α
2π
i_D1
I_a
0
ωt
α
π
i_D2
0
ωt
π + α
2π


i_s
I_a
0
$\pi + \alpha$
2π
ωt
α
π
i_o
I_a
0
ωt
i_{Dm}
I_a
0
ωt
α
π
$\pi + \alpha$
2π

Thyristor T_1 & D_1 conduct

$$\text{from } \omega t = \alpha \ to \ \pi$$

Thyristor T_2 & D_2 conduct

$$\text{from } \omega t = (\pi + \alpha) \ to \ 2\pi$$

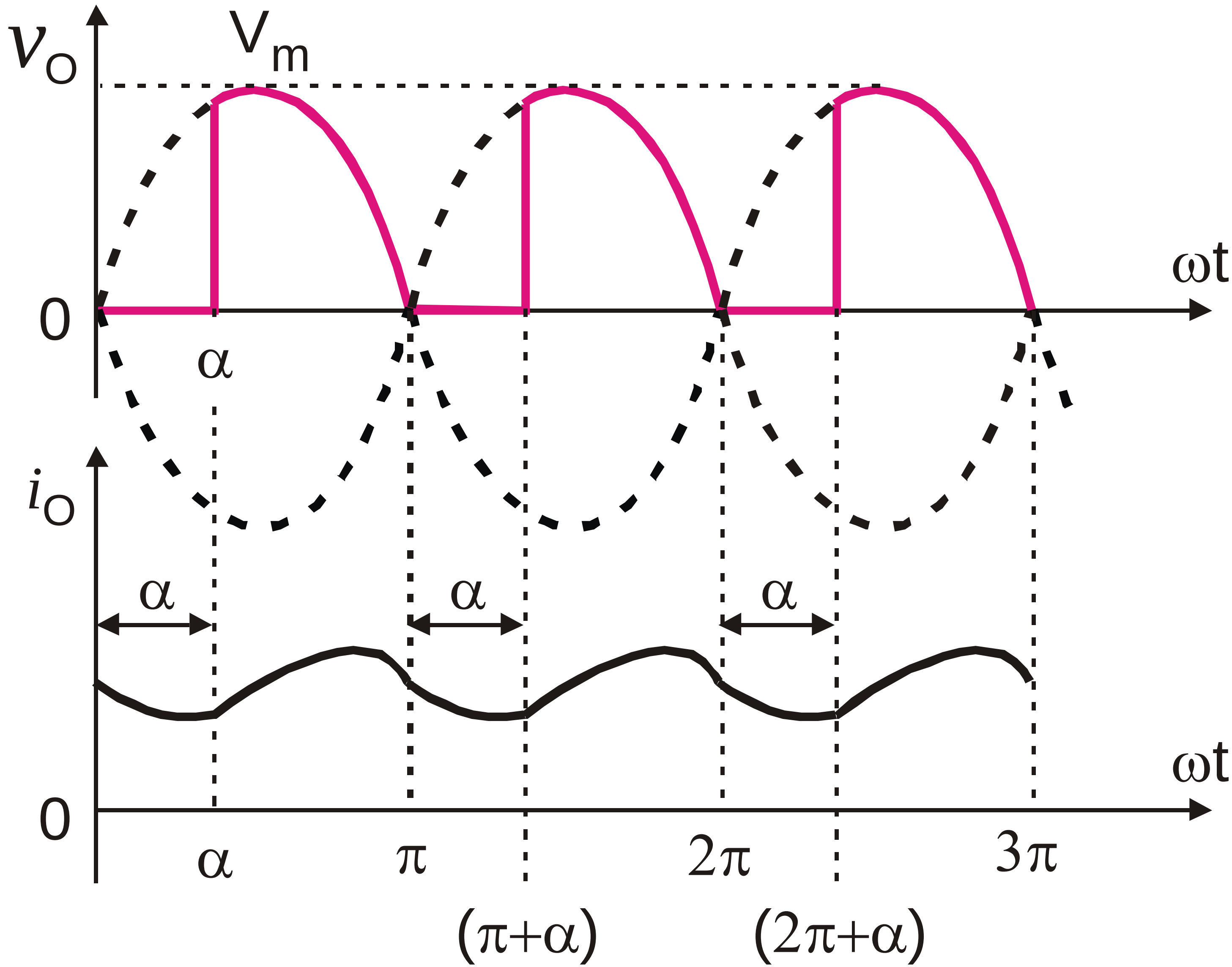
FWD conducts during

$$\omega t = 0 \text{ to } \alpha, \ \pi \ to \ (\pi + \alpha), \ldots$$

239

Load Voltage & Load Current Waveform of Single Phase Semi Converter for $\alpha < 90^0$ & Continuous load current operation

240



241

To Derive an Expression For The DC Output Voltage of A Single Phase Semi-Converter With R,L, & E Load & FWD For Continuous, Ripple Free Load Current Operation

242

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=0}^{\pi} v_O . d(\omega t)$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\pi} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \pi + \cos \alpha \right] \; ; \; \cos \pi = -1$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha)$$

243

V_{dc} can be varied from a max.

value of $\dfrac{2V_m}{\pi}$ *to* 0 by varying α from 0 to π.

For $\alpha = 0$, The max. dc o/p voltage obtained is

$$V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi}$$

Normalized dc o/p voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dn}} = \frac{\frac{V_m}{\pi}\left(1+\cos\alpha\right)}{\left(\frac{2V_m}{\pi}\right)} = \frac{1}{2}\left(1+\cos\alpha\right)$$

RMS O/P Voltage $V_{O(RMS)}$

$$V_{O(RMS)} = \left[\frac{2}{2\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t . d\left(\omega t\right) \right]^{\frac{1}{2}}$$

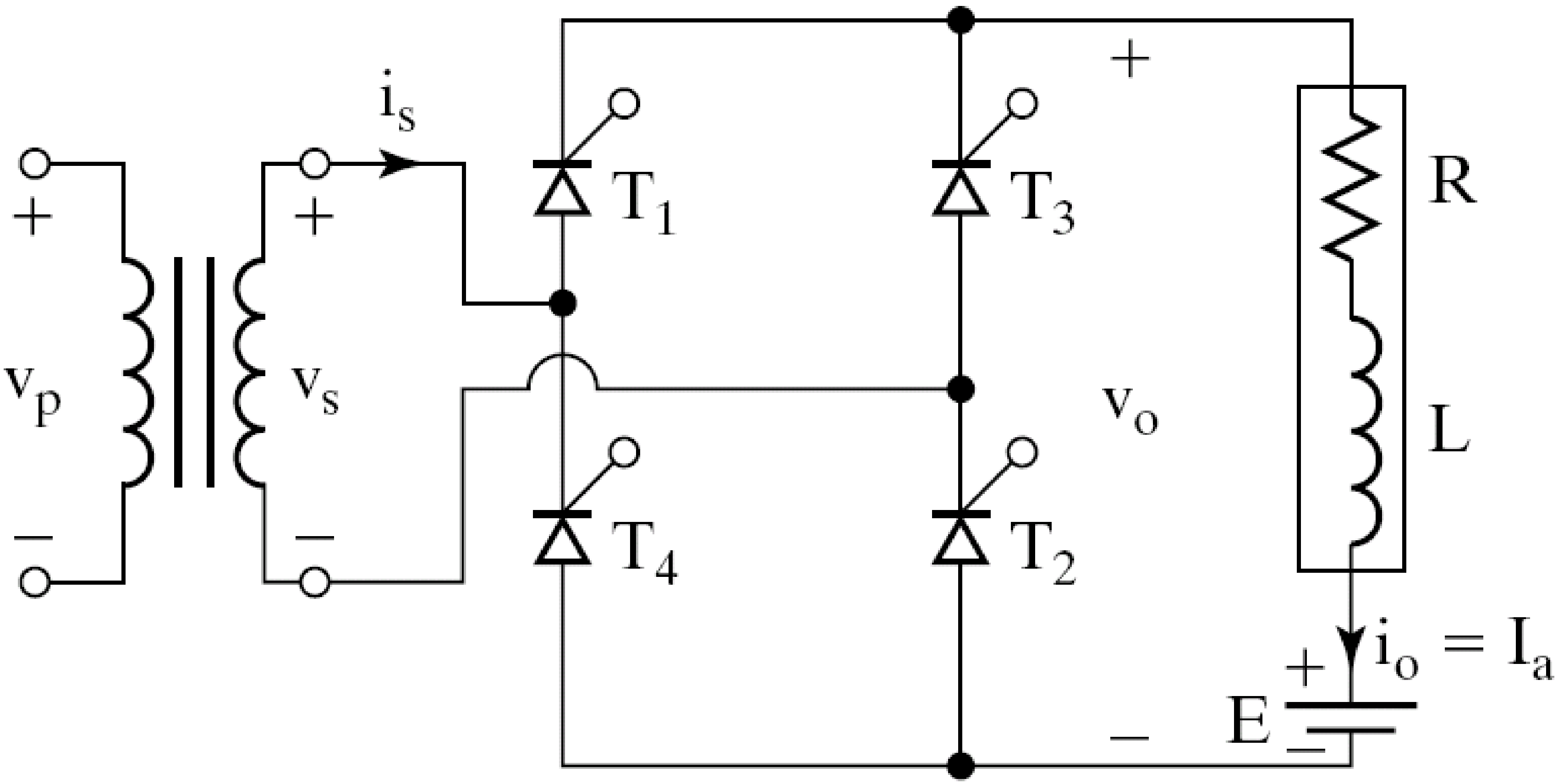
$$V_{O(RMS)} = \left[\frac{V_m^2}{2\pi} \int_{\alpha}^{\pi} \left(1 - \cos 2\omega t\right) . d\left(\omega t\right) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \left[\frac{1}{\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{\frac{1}{2}}$$

 245

Single Phase Full Converter

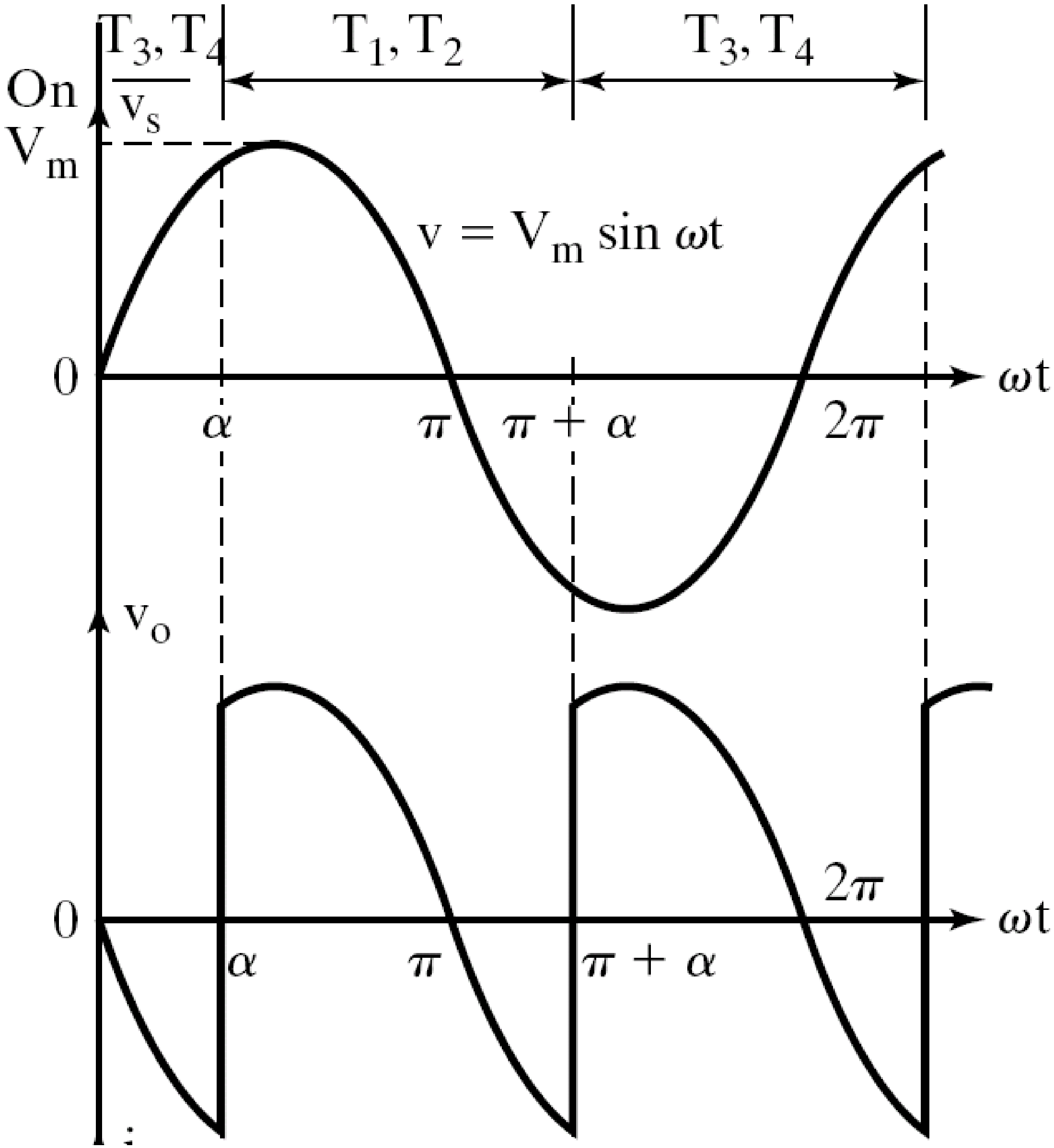
Single Phase Full Converter

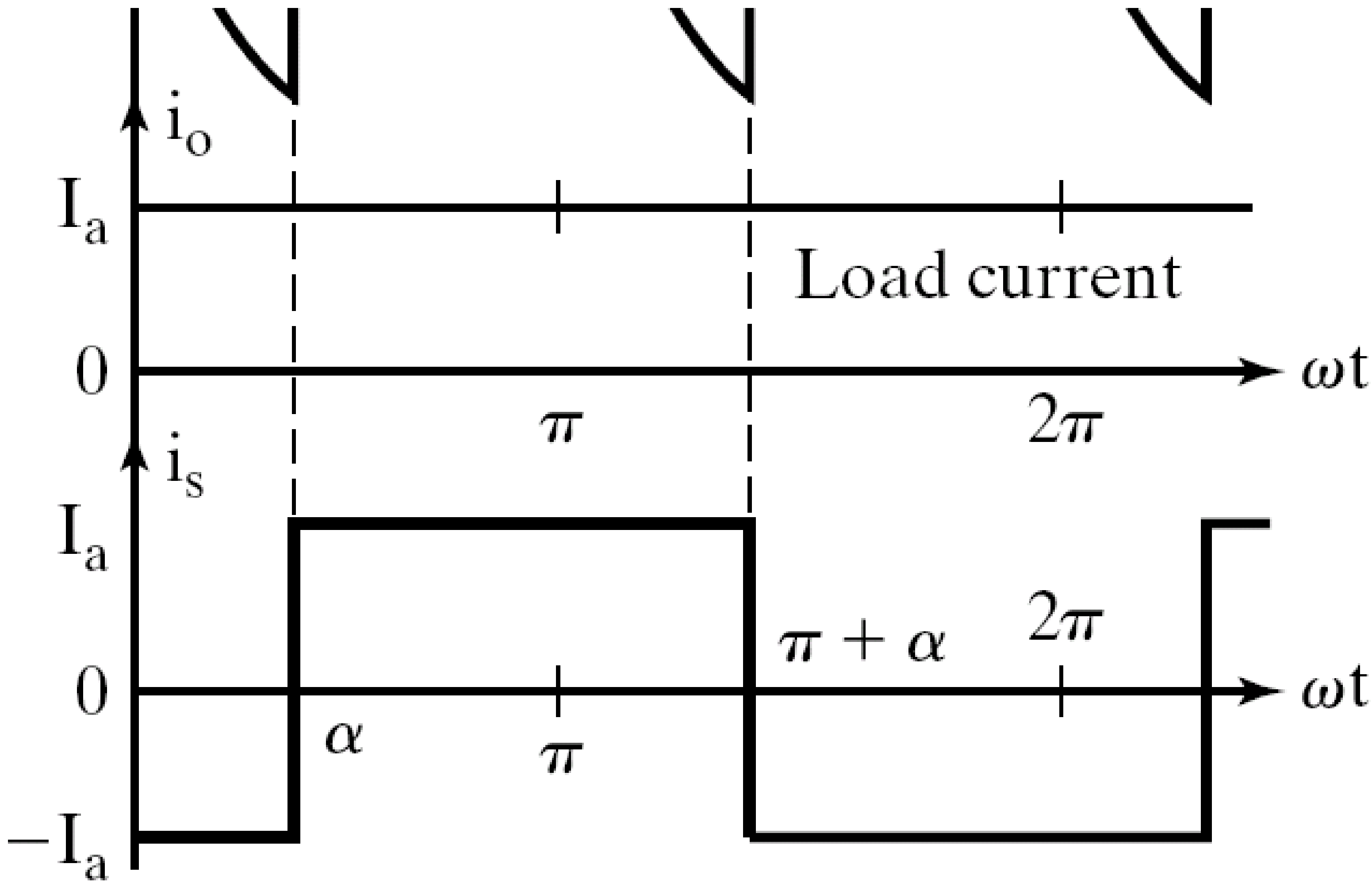


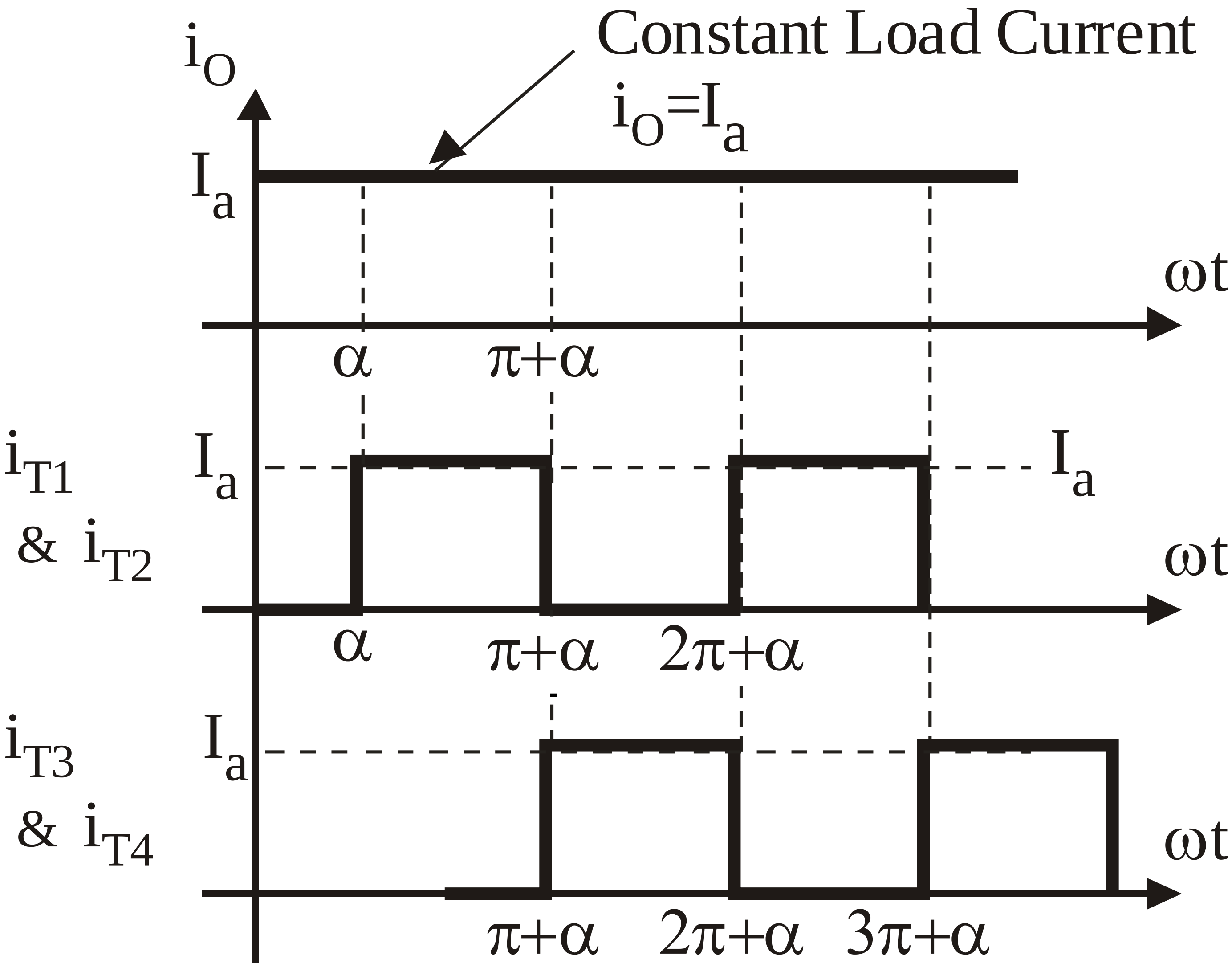
247

Waveforms of Single Phase Full Converter Assuming Continuous (Constant Load Current) & Ripple Free Load Current

248


T3, T4
T1, T2
T3, T4
On
vs
Vm
v = Vm sin ωt
0
ωt
α
π
π + α
2π
vo
0
2π
ωt
α
π
π + α


i_o
I_a
Load current
0
ωt
π
2π
i_s
I_a
$\pi + \alpha$
2π
0
ωt
α
π
$-I_a$


Constant Load Current
iO=Ia
iO
Ia
ωt
α
π+α
iT1
& iT2
Ia
Ia
ωt
α
π+α
2π+α
iT3
& iT4
Ia
ωt
π+α
2π+α
3π+α

To Derive An Expression For The Average DC Output Voltage of a Single Phase Full Converter assuming Continuous & Constant Load Current

252

The average dc output voltage

can be determined by using the expression

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\left[\int_{0}^{2\pi} v_O . d\left(\omega t\right)\right];$$

The o/p voltage waveform consists of two o/p pulses during the input supply time period of 0 to 2π radians. Hence the Average or dc o/p voltage can be calculated as

253

$$V_{O(dc)} = V_{dc} = \frac{2}{2\pi}\left[\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t . d\left(\omega t\right)\right]$$

$$V_{O(dc)} = V_{dc} = \frac{2V_m}{2\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi+\alpha}$$

$$V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

254

Maximum average dc output voltage is calculated for a trigger angle $\alpha = 0^0$ and is obtained as

$$V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi} \times \cos(0) = \frac{2V_m}{\pi}$$

$$\therefore V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi}$$

255

The normalized average output voltage is given by

$$V_{dcn} = V_n = \frac{V_{O(dc)}}{V_{dc(\max)}} = \frac{V_{dc}}{V_{dm}}$$

$$\therefore V_{dcn} = V_n = \frac{\dfrac{2V_m}{\pi}\cos\alpha}{\dfrac{2V_m}{\pi}} = \cos\alpha$$

 256

By plotting $V_{O(dc)}$ *versus* α,

we obtain the control characteristic of a single phase full wave fully controlled bridge converter

(single phase full converter)

for constant & continuous

load current operation.

257

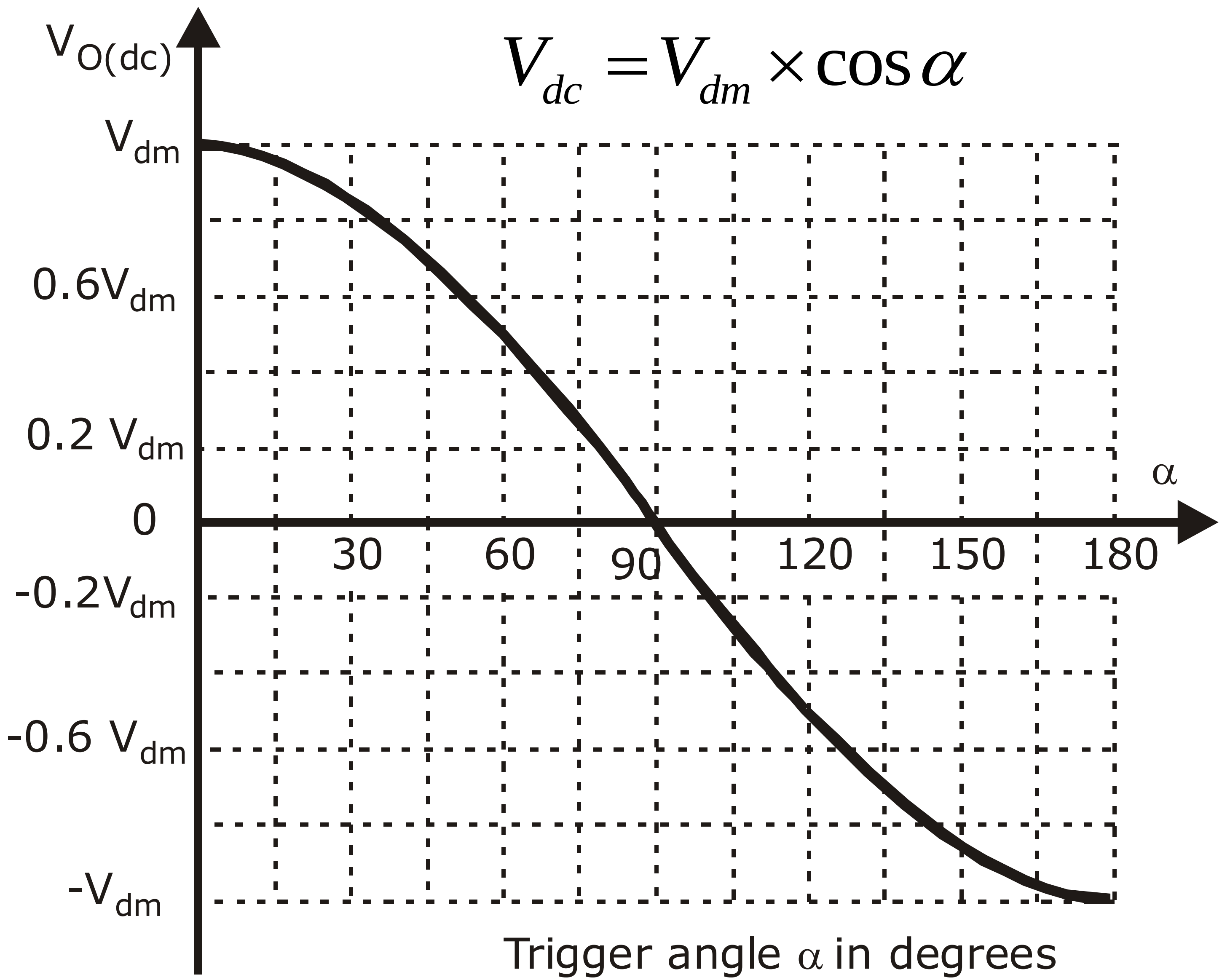
To plot the control characteristic of a Single Phase Full Converter for constant & continuous load current operation.

We use the equation for the average/ dc output voltage

$$V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

Trigger angle α in degrees	$V_{O(dc)}$	**Remarks**
0	$V_{dm} = \left(\dfrac{2V_m}{\pi}\right)$	Maximum dc output voltage $V_{dc(\max)} = V_{dm} = \left(\dfrac{2V_m}{\pi}\right)$
30^0	$0.866\ V_{dm}$	
60^0	$0.5\ V_{dm}$	
90^0	$0\ V_{dm}$	
120^0	$-0.5\ V_{dm}$	
150^0	$-0.866\ V_{dm}$	
180^0	$-V_{dm} = -\left(\dfrac{2V_m}{\pi}\right)$	

259



$V_{O(dc)}$
$V_{dc} = V_{dm} \times \cos\alpha$
V_{dm}
$0.6V_{dm}$
$0.2\ V_{dm}$
0
$-0.2V_{dm}$
$-0.6\ V_{dm}$
$-V_{dm}$
30
60
90
120
150
180
α
Trigger angle α in degrees

- During the period from $\omega t = \alpha$ to π the input voltage v_S and the input current i_S are both positive and the power flows from the supply to the load.
- The converter is said to be operated in the rectification mode

Controlled Rectifier Operation

for $0 < \alpha < 90^0$

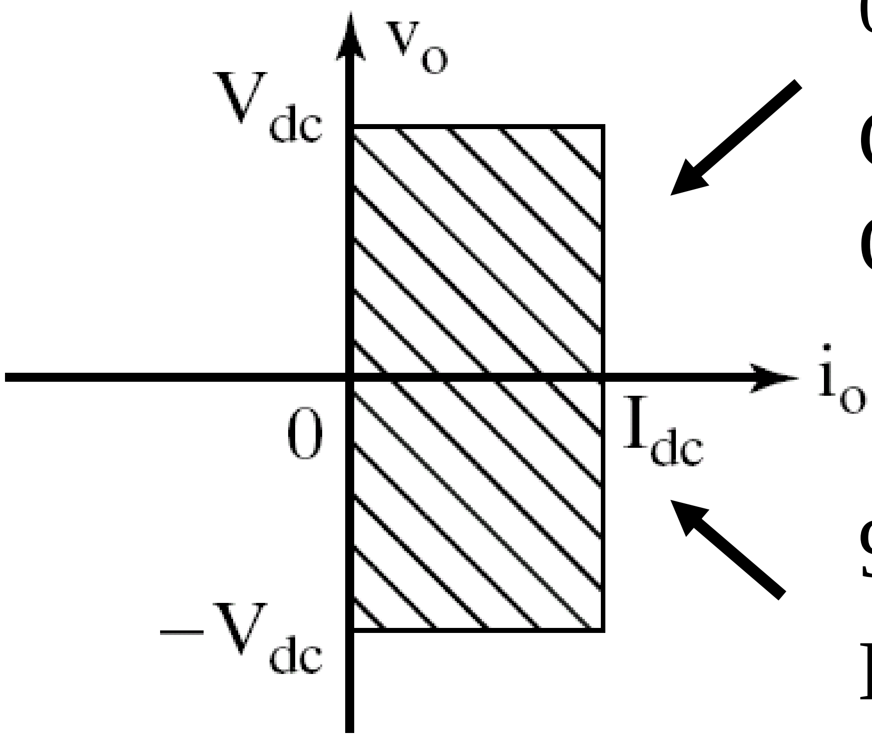
- During the period from $\omega t = \pi$ to $(\pi + \alpha)$, the input voltage v_S is negative and the input current i_S is positive and the output power becomes negative and there will be reverse power flow from the load circuit to the supply.
- The converter is said to be operated in the inversion mode.

Line Commutated Inverter Operation

for $90^0 < \alpha < 180^0$

Two Quadrant Operation of a Single Phase Full Converter

Two Quadrant Operation of a Single Phase Full Converter



$0 < \alpha < 90^0$

Controlled Rectifier Operation

$90^0 < \alpha < 180^0$

Line Commutated Inverter Operation

To Derive An Expression For The RMS Value Of The Output Voltage

The rms value of the output voltage is calculated as

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_0^{2\pi} v_O^2 . d(\omega t)\right]}$$

The single phase full converter gives two output voltage pulses during the input supply time period and hence the single phase full converter is referred to as a two pulse converter. The rms output voltage can be calculated as

$$V_{O(RMS)} = \sqrt{\frac{2}{2\pi}\left[\int_{\alpha}^{\pi+\alpha} v_O^2 . d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{1}{\pi}\left[\int_{\alpha}^{\pi+\alpha} V_m^2 \sin^2 \omega t.d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi}\left[\int_{\alpha}^{\pi+\alpha} \sin^2 \omega t.d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi}\left[\int_{\alpha}^{\pi+\alpha} \frac{\left(1-\cos 2\omega t\right)}{2}.d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[\int_{\alpha}^{\pi+\alpha} d\left(\omega t\right) - \int_{\alpha}^{\pi+\alpha} \cos 2\omega t.d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[\left(\omega t\right) \Big/_{\alpha}^{\pi+\alpha} - \left(\frac{\sin 2\omega t}{2} \right) \Big/_{\alpha}^{\pi+\alpha} \right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[\left(\pi + \alpha - \alpha\right) - \left(\frac{\sin 2\left(\pi + \alpha\right) - \sin 2\alpha}{2} \right) \right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[\left(\pi\right) - \left(\frac{\sin\left(2\pi + 2\alpha\right) - \sin 2\alpha}{2} \right) \right]};$$

$$\sin\left(2\pi + 2\alpha\right) = \sin 2\alpha$$

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$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[(\pi) - \left(\frac{\sin 2\alpha - \sin 2\alpha}{2}\right)\right]}$$

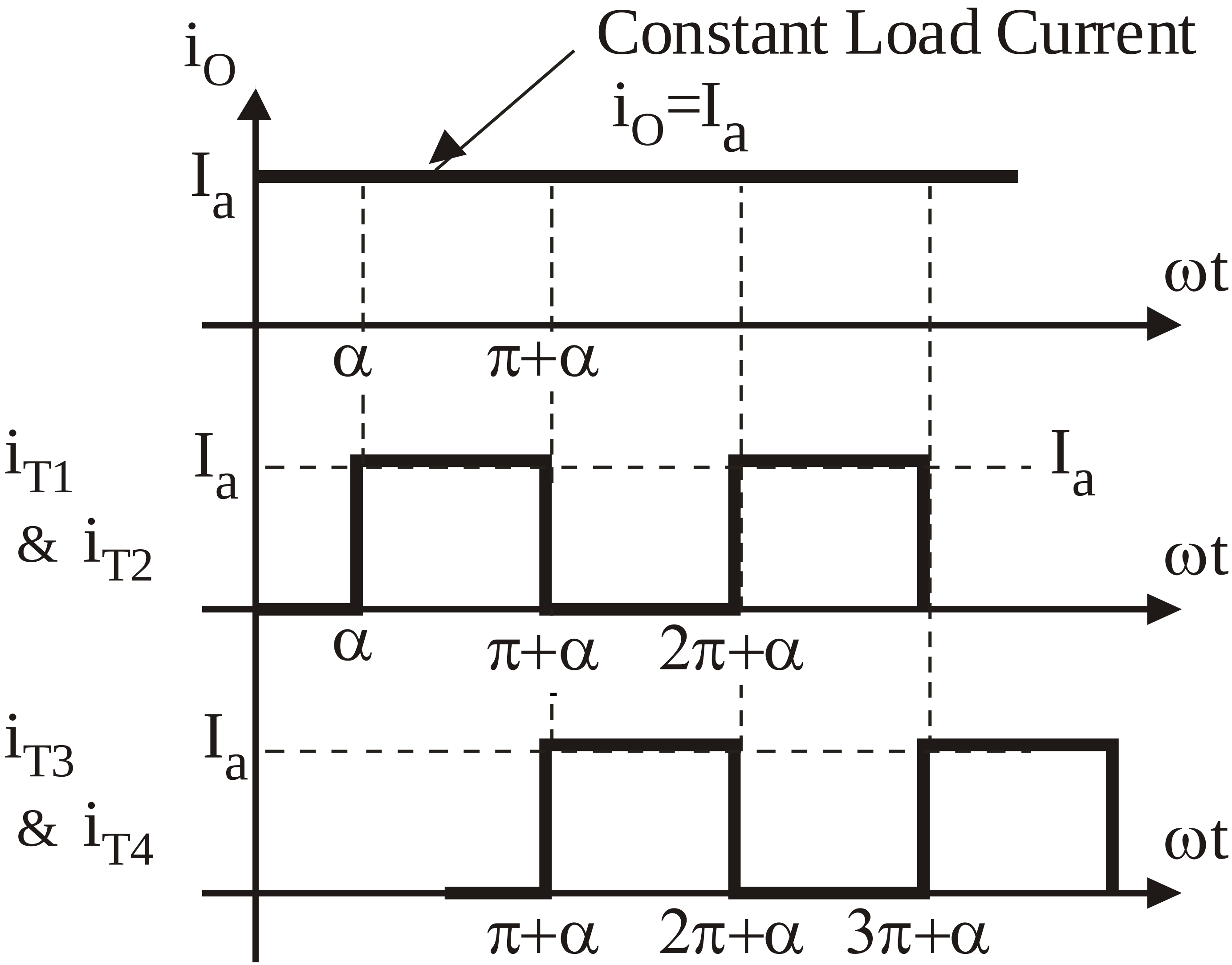
$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}(\pi) - 0} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$

$$\therefore V_{O(RMS)} = \frac{V_m}{\sqrt{2}} = V_S$$

Hence the rms output voltage is same as the rms input supply voltage

Thyristor Current Waveforms

270


Constant Load Current
$i_O = I_a$
i_O
I_a
ωt
α
$\pi+\alpha$
i_{T1} & i_{T2}
I_a
I_a
ωt
α
$\pi+\alpha$
$2\pi+\alpha$
i_{T3} & i_{T4}
I_a
ωt
$\pi+\alpha$
$2\pi+\alpha$
$3\pi+\alpha$

271

The rms thyristor current can be calculated as

$$I_{T(RMS)} = \frac{I_{O(RMS)}}{\sqrt{2}}$$

The average thyristor current can be calculated as

$$I_{T(Avg)} = \frac{I_{O(dc)}}{2}$$

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THREE PHASE LINE COMMUTATED CONVERTERS

Introduction to Three phase converters

3 Phase Controlled Rectifiers

- Three phase converters are 3-phase controlled rectifiers which are used to convert ac input power supply into dc output power across the load

Features of 3-phase controlled rectifiers

- Operate from 3 phase ac supply voltage.
- They provide higher dc output voltage.
- Higher dc output power.
- Higher output voltage ripple frequency.
- Filtering requirements are simplified for smoothing out load voltage and load current.

- Extensively used in high power variable speed industrial dc drives.
- Three single phase half-wave converters can be connected together to form a three phase half-wave converter.

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Classification of 3-phase converters

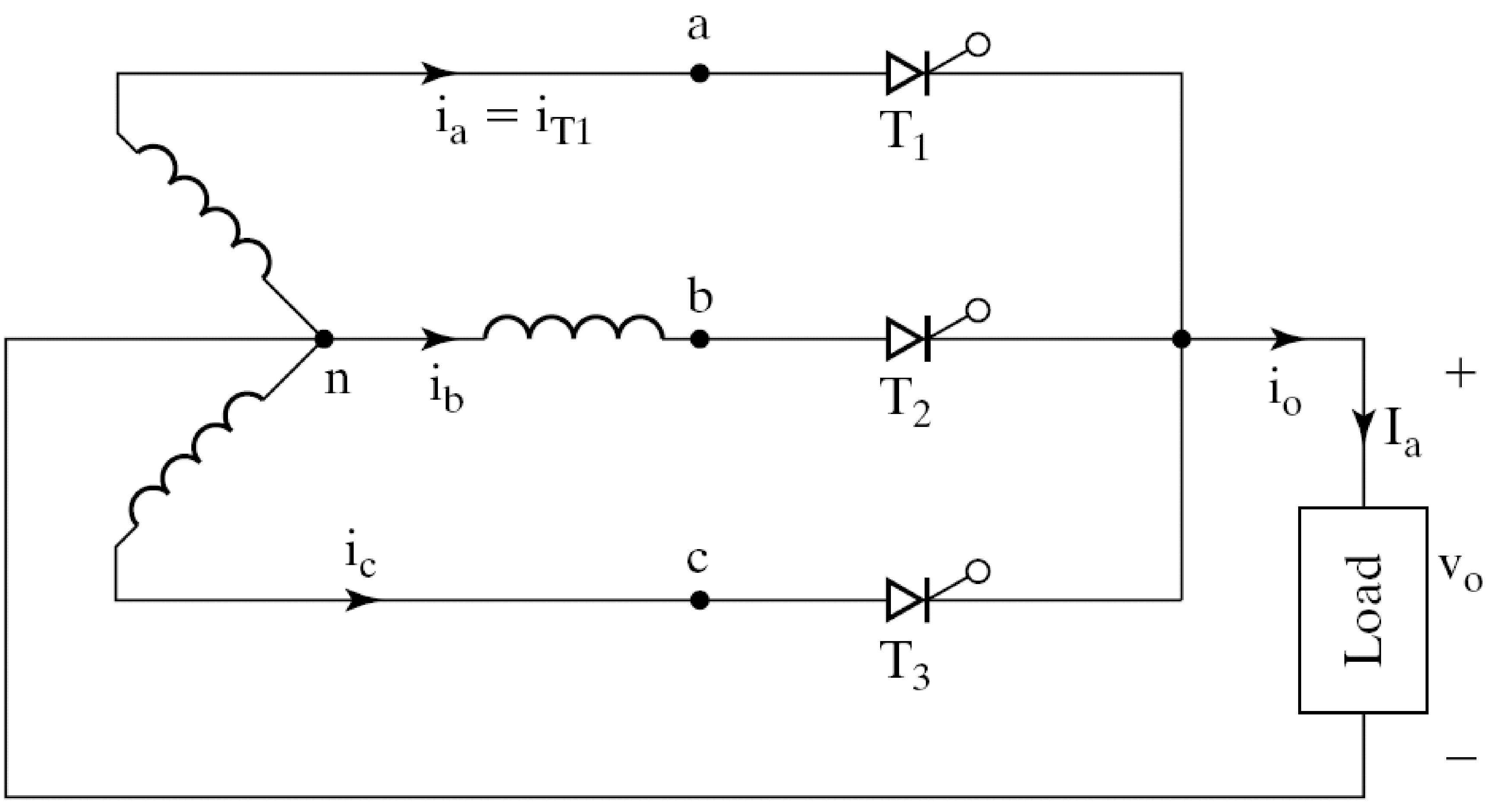
- 3-phase half wave converter
- 3-phase semi converter
- 3-phase full converter
- 3- phase dual converter

Classification according to no of pulses in the output wave

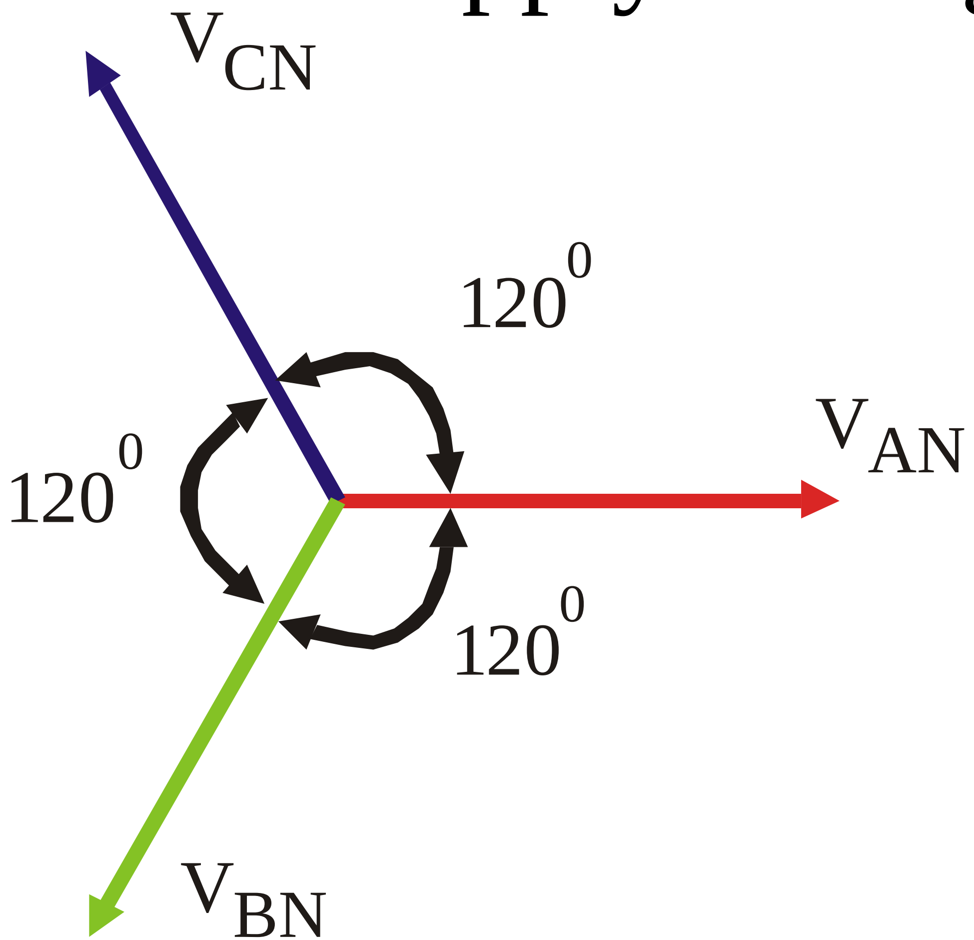
- 3- pulse converter
- 6-pulse converter
- 12- pulse converter

3-Phase Half Wave Converter (3-Pulse Converter) with R-L Load Continuous & Constant Load Current Operation

Circuit Diagram of 3- pulse converter



Vector Diagram of 3 Phase Supply Voltages



$$v_{RN} = v_{AN}$$

$$v_{YN} = v_{BN}$$

$$v_{BN} = v_{CN}$$

3 Phase Supply Voltage Equations

We deifine three line to neutral voltages (3 phase voltages) as follows

283

$$v_{RN} = v_{an} = V_m \sin \omega t;$$

$$V_m = \text{ Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right)$$

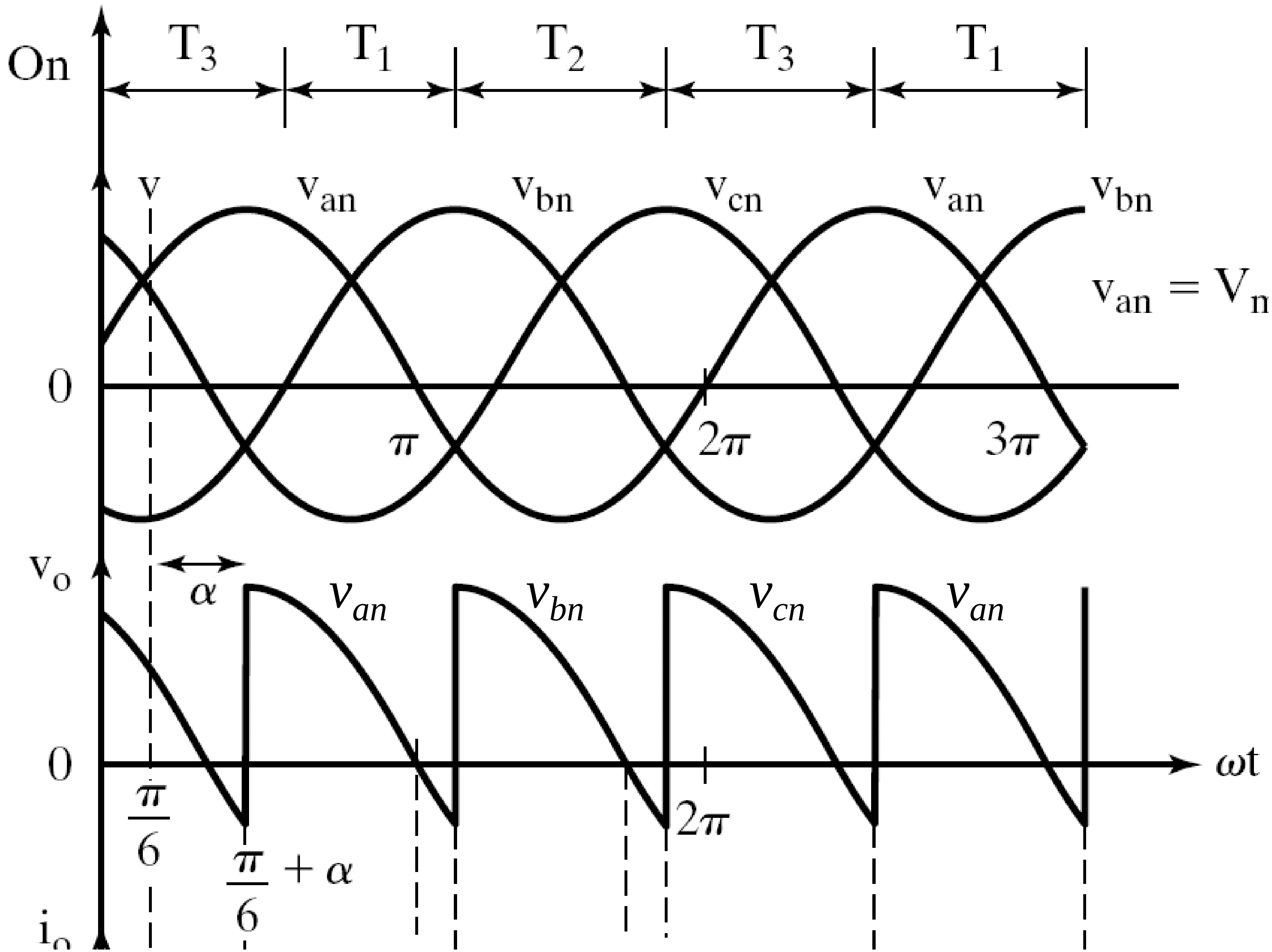
$$= V_m \sin\left(\omega t - 120^0\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right)$$

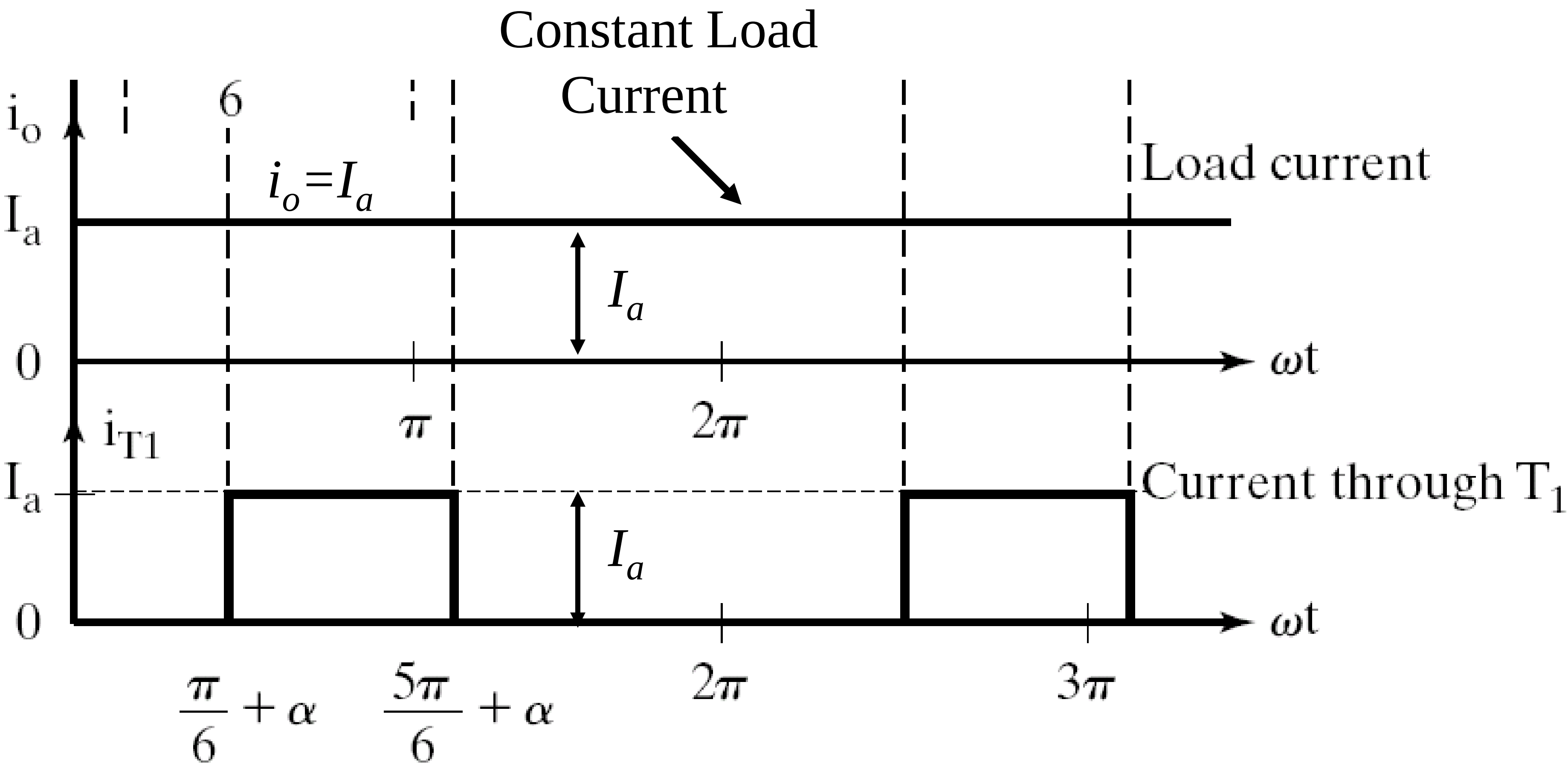
$$= V_m \sin\left(\omega t + 120^0\right)$$

$$= V_m \sin\left(\omega t - 240^0\right)$$

284


On
T3
T1
T2
T3
T1
v
van
vbn
vcn
van
vbn
van = Vm
0
π
2π
3π
vo
α
van
vbn
vcn
van
0
ωt
π/6
π/6 + α
2π
io

Each thyristor conducts for $2\pi/3$ (120^0)



286

To Derive an Expression for the Average Output Voltage of a 3-Phase Half Wave Converter with RL Load for Continuous Load Current

$$T_1 \; is \; triggered \; at \; \omega t = \left(\frac{\pi}{6} + \alpha \right) = \left(30^0 + \alpha \right)$$

$$T_2 \; is \; triggered \; at \; \omega t = \left(\frac{5\pi}{6} + \alpha \right) = \left(150^0 + \alpha \right)$$

$$T_3 \; is \; triggered \; at \; \omega t = \left(\frac{7\pi}{6} + \alpha \right) = \left(270^0 + \alpha \right)$$

$$\text{Each thytistor conducts for } 120^0 \text{ or } \frac{2\pi}{3} \text{ radians}$$

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If the reference phase voltage is $v_{RN} = v_{an} = V_m \sin \omega t$, the average or dc output voltage for continuous load current is calculated using the equation

$$V_{dc} = \frac{3}{2\pi} \left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin \omega t . d\left(\omega t\right) \right]$$

289

$$V_{dc} = \frac{3V_m}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} \sin\omega t . d\left(\omega t\right)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\left(-\cos\omega t\right)\Big/_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\left(\frac{5\pi}{6}+\alpha\right)+\cos\left(\frac{\pi}{6}+\alpha\right)\right]$$

290

Note from the trigonometric relationship

$$\cos(A+B) = (\cos A.\cos B - \sin A.\sin B)$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\left(\frac{5\pi}{6}\right)\cos(\alpha) + \sin\left(\frac{5\pi}{6}\right)\sin(\alpha) + \cos\left(\frac{\pi}{6}\right).\cos(\alpha) - \sin\left(\frac{\pi}{6}\right)\sin(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos(150^0)\cos(\alpha) + \sin(150^0)\sin(\alpha) + \cos(30^0).\cos(\alpha) - \sin(30^0)\sin(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\begin{array}{l} -\cos\left(180^0 - 30^0\right)\cos(\alpha) + \sin\left(180^0 - 30^0\right)\sin(\alpha) \\ \qquad\qquad + \cos\left(30^0\right).\cos(\alpha) - \sin\left(30^0\right)\sin(\alpha) \end{array}\right]$$

Note: $\cos\left(180^0 - 30^0\right) = -\cos\left(30^0\right)$

$\sin\left(180^0 - 30^0\right) = \sin\left(30^0\right)$

$$\therefore V_{dc} = \frac{3V_m}{2\pi}\left[\begin{array}{l} +\cos\left(30^0\right)\cos(\alpha) + \cancel{\sin\left(30^0\right)\sin(\alpha)} \\ \qquad\qquad + \cos\left(30^0\right).\cos(\alpha) - \cancel{\sin\left(30^0\right)\sin(\alpha)} \end{array}\right]$$

292

$$V_{dc} = \frac{3V_m}{2\pi}\left[2\cos\left(30^0\right)\cos\left(\alpha\right)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[2\times\frac{\sqrt{3}}{2}\cos\left(\alpha\right)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\sqrt{3}\cos\left(\alpha\right)\right] = \frac{3\sqrt{3}V_m}{2\pi}\cos\left(\alpha\right)$$

$$V_{dc} = \frac{3V_{Lm}}{2\pi}\cos\left(\alpha\right)$$

Where $V_{Lm} = \sqrt{3}V_m =$ Max. line to line supply voltage

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The maximum average or dc output voltage is obtained at a delay angle $\alpha = 0$ and is given by

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3}\ V_m}{2\pi}$$

Where V_m is the peak phase voltage.

And the normalized average output voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \cos\alpha$$

294

The rms value of output voltage is found by using the equation

$$V_{O(RMS)} = \left[\frac{3}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m^2 \sin^2 \omega t . d(\omega t) \right]^{\frac{1}{2}}$$

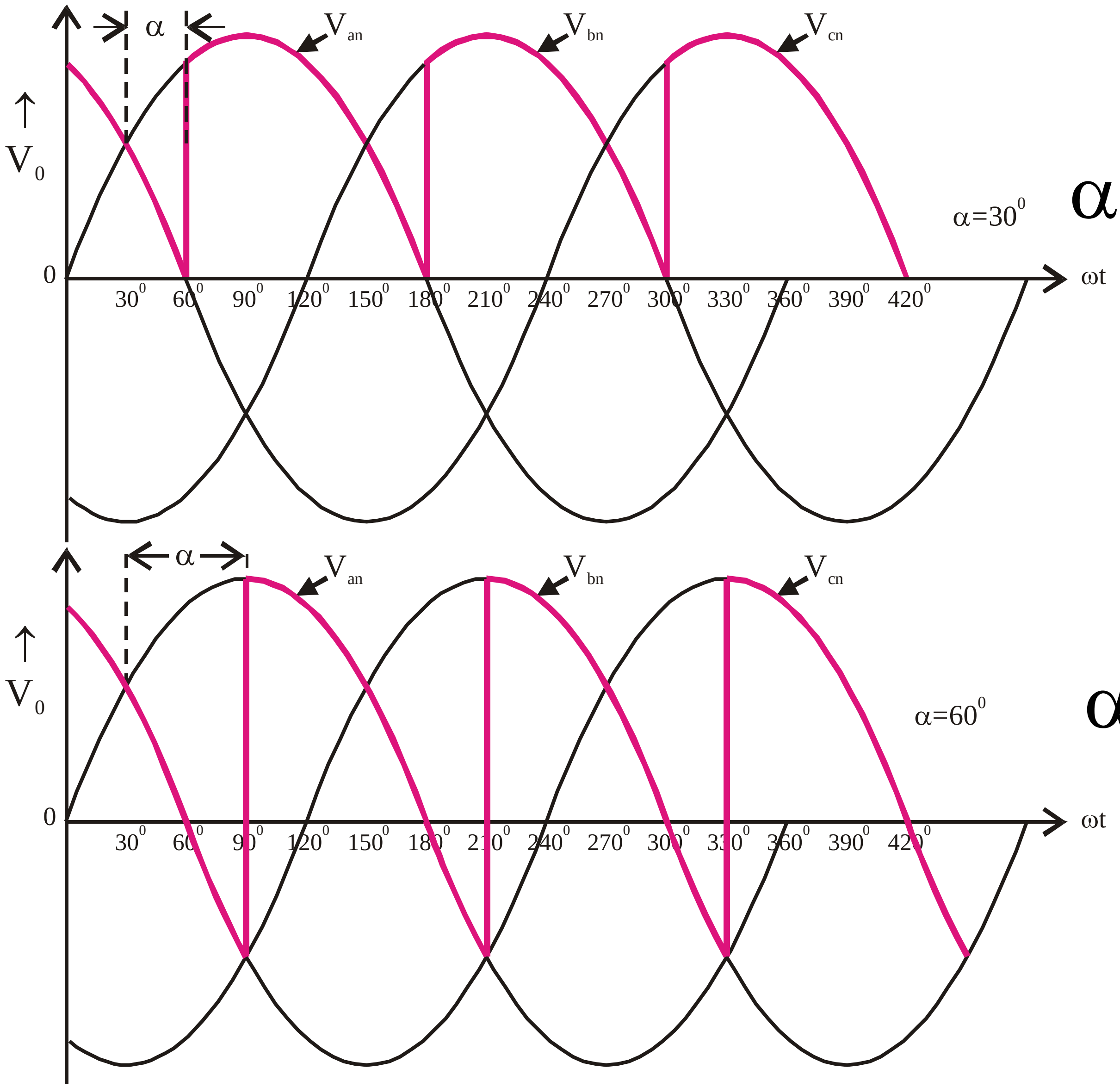
and we obtain

$$V_{O(RMS)} = \sqrt{3} V_m \left[\frac{1}{6} + \frac{\sqrt{3}}{8\pi} \cos 2\alpha \right]^{\frac{1}{2}}$$

295

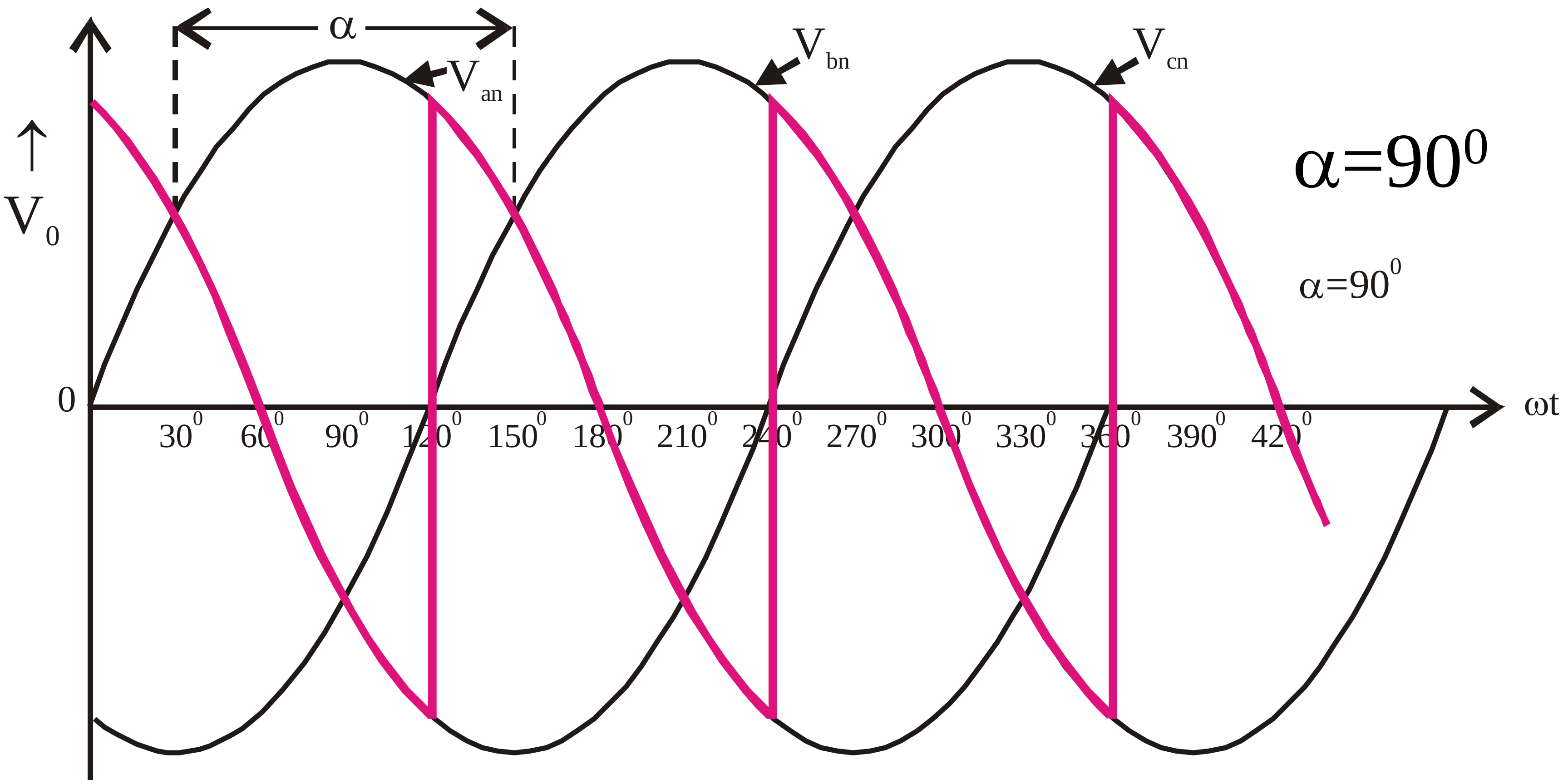
3 Phase Half Wave Controlled Rectifier Output Voltage Waveforms For RL Load at Different Trigger Angles

296



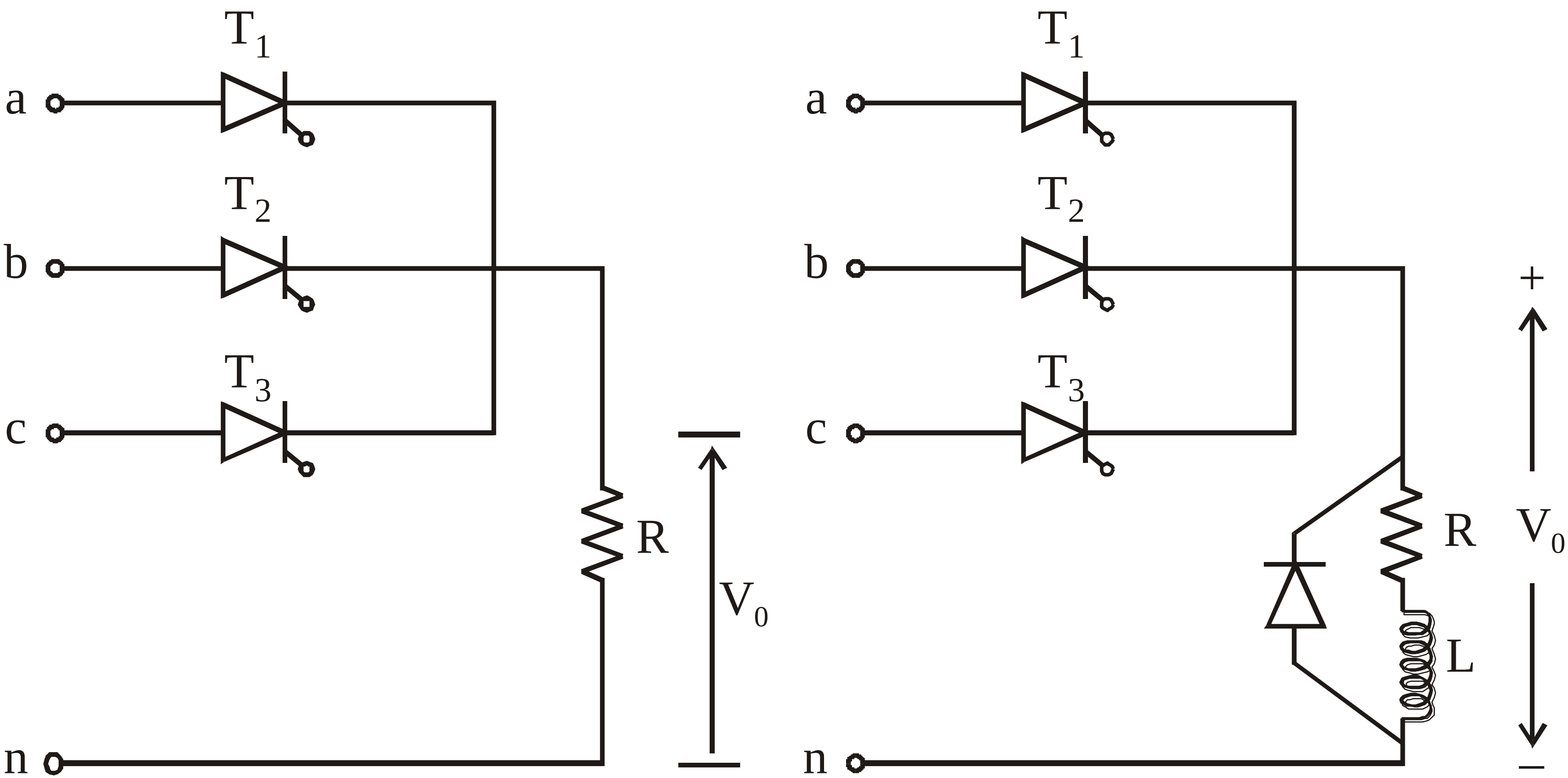
$\alpha=30^0$

$\alpha=60^0$



α
V an
V bn
V cn
V 0
0
ωt
30 0
60 0
90 0
120 0
150 0
180 0
210 0
240 0
270 0
300 0
330 0
360 0
390 0
420 0
α=90 0
α=90 0

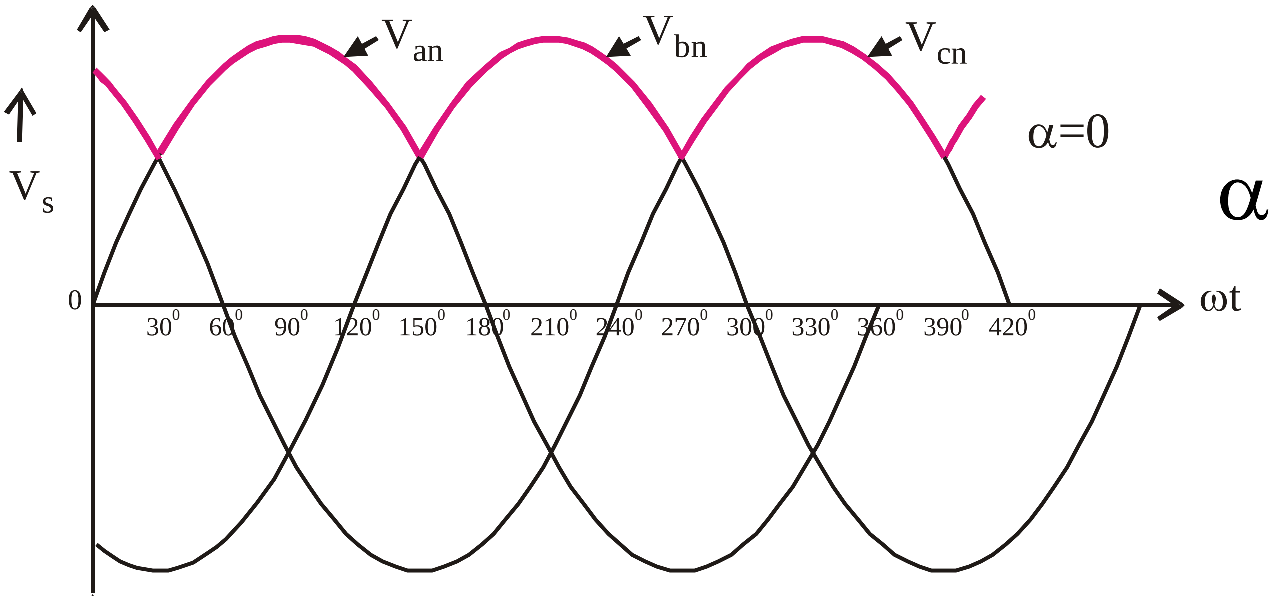
3 Phase Half Wave Controlled Rectifier With R Load and RL Load with FWD



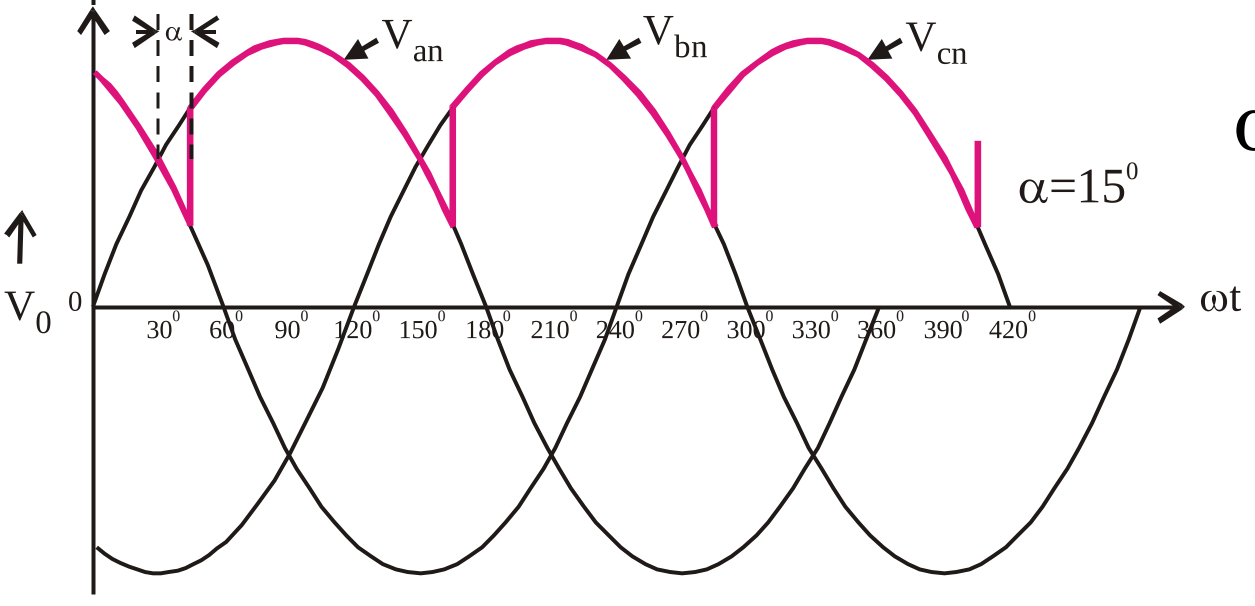
300

3 Phase Half Wave
Controlled Rectifier Output
Voltage Waveforms For R Load
or RL Load with FWD
at
Different Trigger Angles

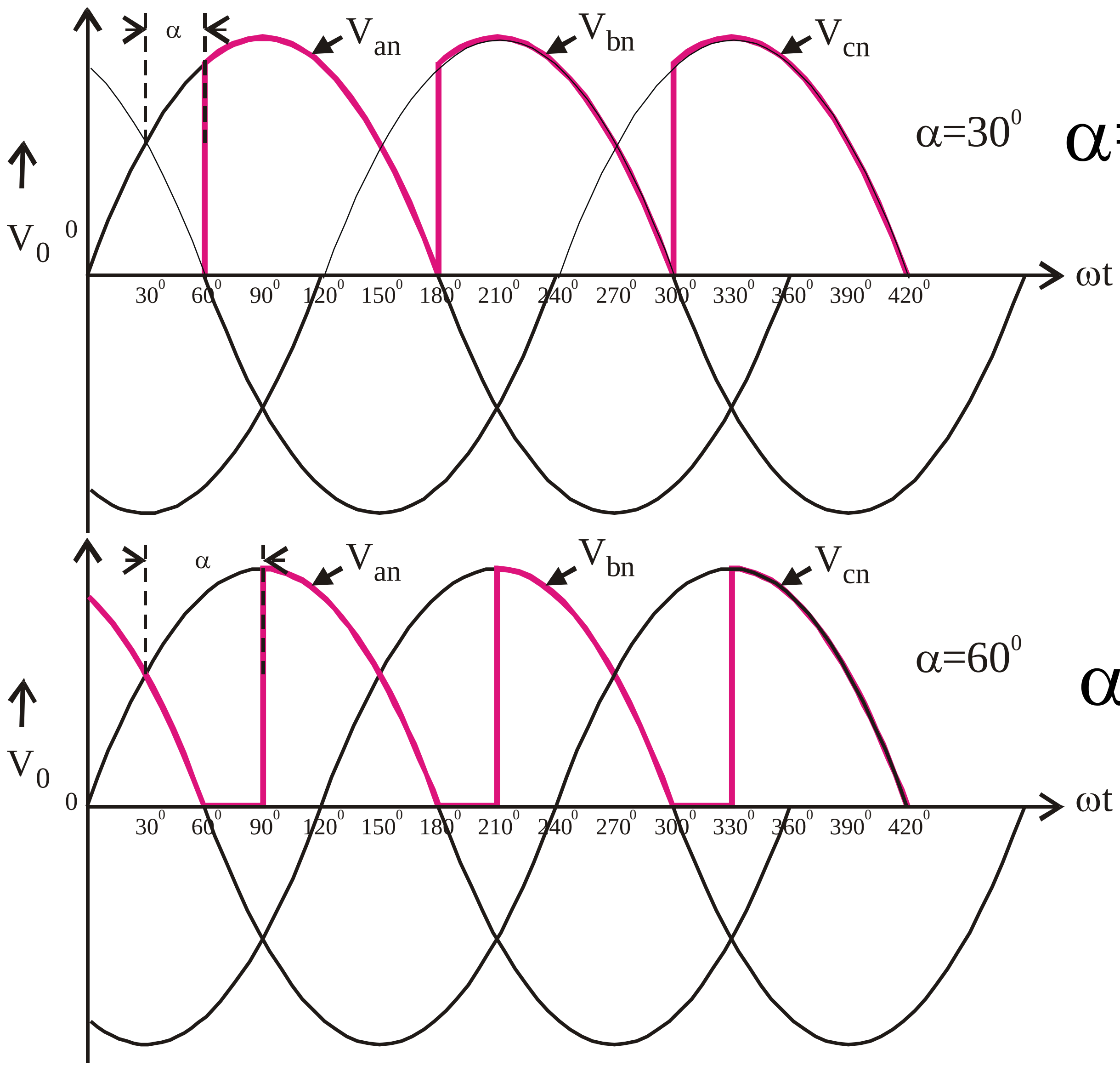
301



$\alpha = 0^0$



$\alpha = 15^0$



$\alpha=30^0$

$\alpha=60^0$

To Derive An Expression For The Average Or Dc Output Voltage Of A
3 Phase Half Wave Converter With Resistive Load Or RL Load With FWD

 304

$$T_1 \text{ is triggered at } \omega t = \left(\frac{\pi}{6}+\alpha\right) = \left(30^0+\alpha\right)$$

$$T_1 \text{ conducts from } \left(30^0+\alpha\right) \text{ to } 180^0 \quad ;$$

$$v_O = v_{an} = V_m \sin\omega t$$

$$T_2 \text{ is triggered at } \omega t = \left(\frac{5\pi}{6}+\alpha\right) = \left(150^0+\alpha\right)$$

$$T_2 \text{ conducts from } \left(150^0+\alpha\right) \text{ to } 300^0 \quad ;$$

$$v_O = v_{bn} = V_m \sin\left(\omega t - 120^0\right)$$

305

$$T_3 \ \textit{is triggered at} \ \omega t = \left(\frac{7\pi}{6} + \alpha \right) = \left(270^0 + \alpha \right)$$

$$T_3 \ \textit{conducts from} \ \left(270^0 + \alpha \right) \ \textit{to} \ 420^0 \quad ;$$

$$v_O = v_{cn} = V_m \sin\left(\omega t - 240^0 \right)$$

$$= V_m \sin\left(\omega t + 120^0 \right)$$

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\alpha+30^0}^{180^0} v_O . d\left(\omega t\right)\right]$$

$$v_O = v_{an} = V_m \sin\omega t; \text{ for } \omega t = \left(\alpha + 30^0\right) \text{ to } \left(180^0\right)$$

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\alpha+30^0}^{180^0} V_m \sin\omega t . d\left(\omega t\right)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\int_{\alpha+30^0}^{180^0} \sin\omega t . d\left(\omega t\right)\right]$$

307

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\omega t \Big/_{\alpha+30^0}^{180^0}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos 180^0 + \cos\left(\alpha + 30^0\right)\right]$$

$$\because \quad \cos 180^0 = -1, \text{ we get}$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[1 + \cos\left(\alpha + 30^0\right)\right]$$

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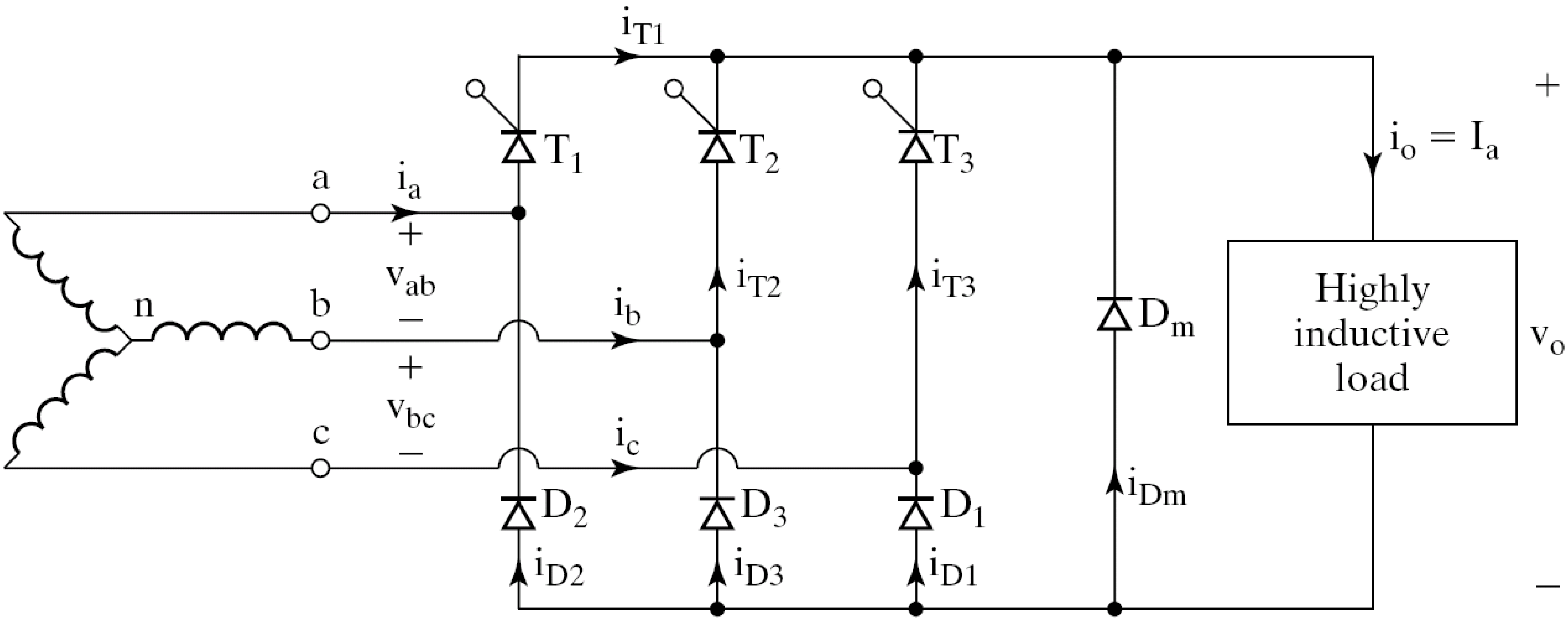
Three Phase Semi-converters

Three Phase Semi-converters

- 3 Phase semi-converters are used in Industrial dc drive applications upto 120kW power output.
- Single quadrant operation is possible.
- Power factor decreases as the delay angle increases.
- Power factor is better than that of 3 phase half wave converter.

3 Phase
Half Controlled Bridge Converter
(Semi Converter)
with Highly Inductive Load &
Continuous Ripple free Load Current

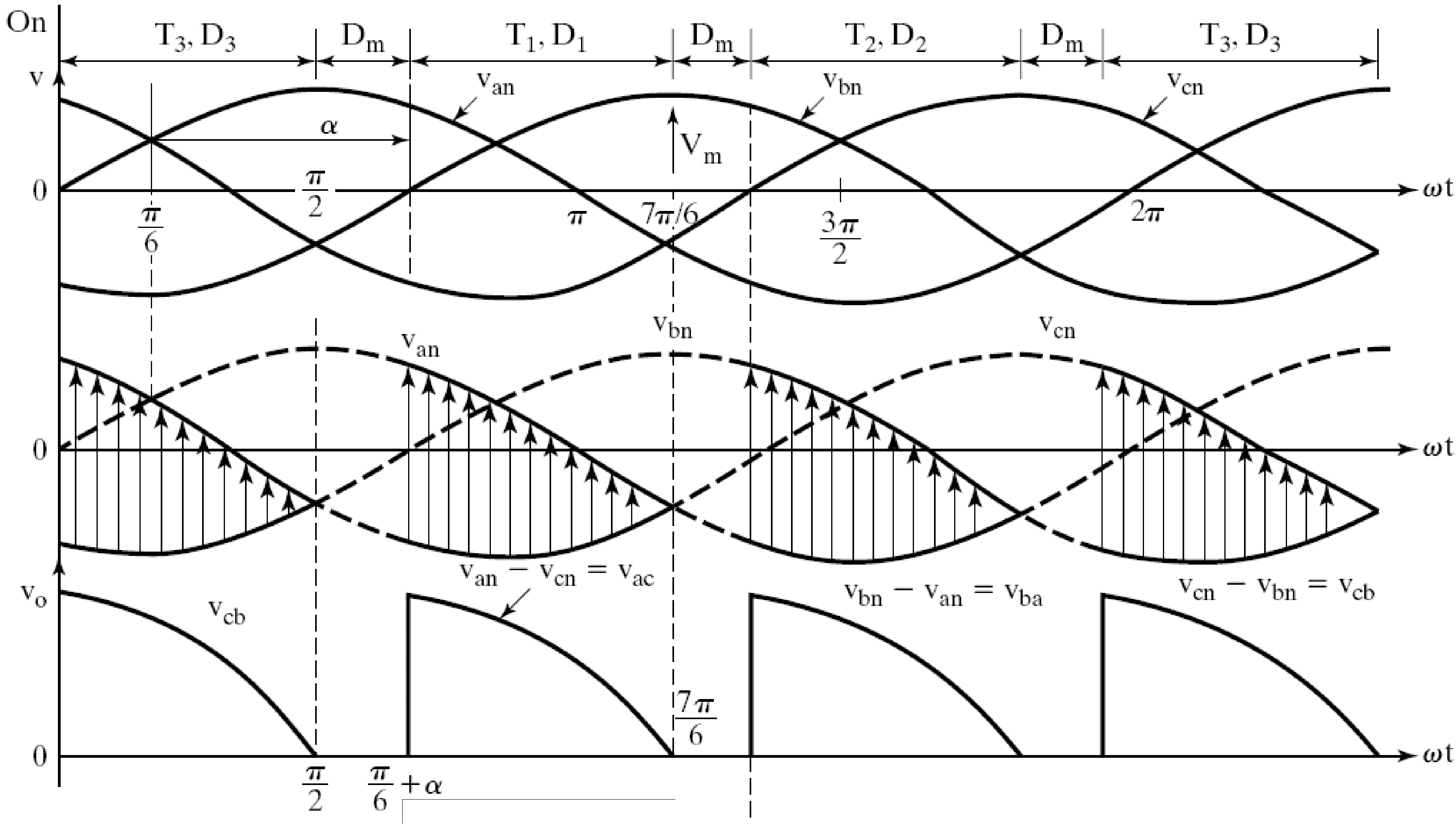
311

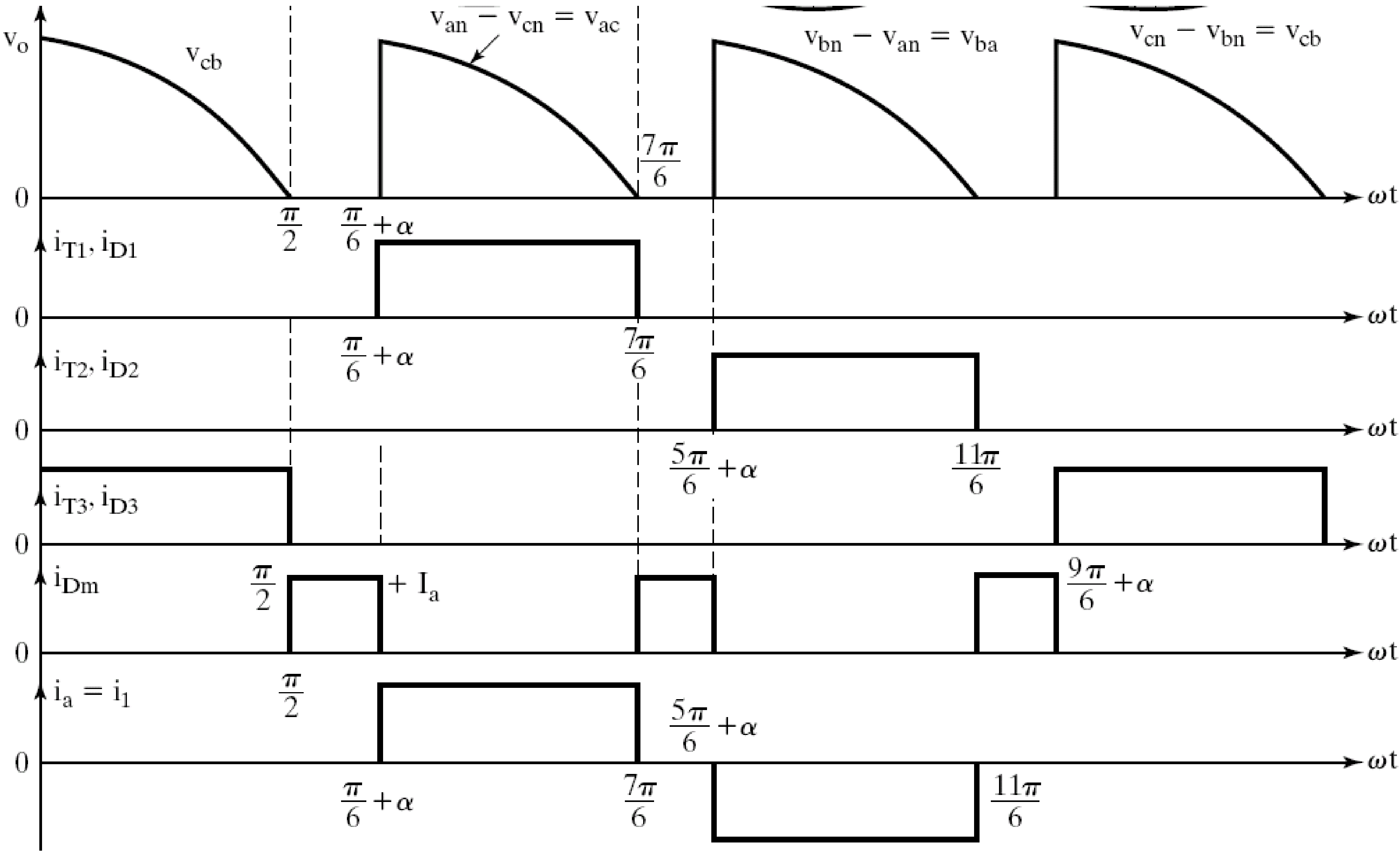

iT1
T1
T2
T3
a
ia
+
vab
−
n
b
ib
iT2
iT3
+
vbc
−
c
ic
D2
D3
D1
iD2
iD3
iD1
Dm
iDm
io = Ia
+
Highly inductive load
vo
−

312

Wave forms of 3 Phase Semiconverter for $\alpha > 60^0$

313


On
T3, D3
Dm
T1, D1
Dm
T2, D2
Dm
T3, D3
v
van
vbn
vcn
α
Vm
0
π/2
π/6
π
7π/6
3π/2
2π
ωt
vbn
vcn
van
0
ωt
van − vcn = vac
vbn − van = vba
vcn − vbn = vcb
vo
vcb
7π/6
0
π/2
π/6 + α
ωt



v_o
v_{cb}
$v_{an} - v_{cn} = v_{ac}$
$v_{bn} - v_{an} = v_{ba}$
$v_{cn} - v_{bn} = v_{cb}$
$\frac{7\pi}{6}$
0
ωt
$\frac{\pi}{2}$
$\frac{\pi}{6} + \alpha$
i_{T1}, i_{D1}
i_{T2}, i_{D2}
$\frac{7\pi}{6}$
$\frac{5\pi}{6} + \alpha$
$\frac{11\pi}{6}$
i_{T3}, i_{D3}
i_{Dm}
$+ I_a$
$\frac{9\pi}{6} + \alpha$
$i_a = i_1$

3 phase semiconverter output ripple frequency of output voltage is $3f_S$

The delay angle α can be varied from 0 to π

During the period

$$30^0 \le \omega t < 210^0$$

$$\frac{\pi}{6} \le \omega t < \frac{7\pi}{6}, \text{ thyristor } T_1 \text{ is forward biased}$$

If thyristor T_1 is triggered at $\omega t = \left(\frac{\pi}{6} + \alpha\right)$,

T_1 & D_1 conduct together and the line to line voltage v_{ac} appears across the load.

At $\omega t = \frac{7\pi}{6}$, v_{ac} becomes negative & FWD D_m conducts.

The load current continues to flow through FWD D_m;

T_1 and D_1 are turned off.

317

If FWD D_m is not used the T_1 would continue to conduct until the thyristor T_2 is triggered at $\omega t = \left(\frac{5\pi}{6}+\alpha\right)$, and Free wheeling action would be accomplished through T_1 & D_2.

If the delay angle $\alpha \le \frac{\pi}{3}$, each thyristor conducts for $\frac{2\pi}{3}$ and the FWD D_m does not conduct.

318

We deifine three line neutral voltages

(3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t \quad ; \qquad V_m = \text{ Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3} \right) = V_m \sin\left(\omega t - 120^0 \right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3} \right) = V_m \sin\left(\omega t + 120^0 \right)$$

$$= V_m \sin\left(\omega t - 240^0 \right)$$

V_m is the peak phase voltage of a wye-connected source

$$v_{RB} = v_{ac} = \left(v_{an} - v_{cn}\right) = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{6}\right)$$

$$v_{YR} = v_{ba} = \left(v_{bn} - v_{an}\right) = \sqrt{3}V_m \sin\left(\omega t - \frac{5\pi}{6}\right)$$

$$v_{BY} = v_{cb} = \left(v_{cn} - v_{bn}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$v_{RY} = v_{ab} = \left(v_{an} - v_{bn}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

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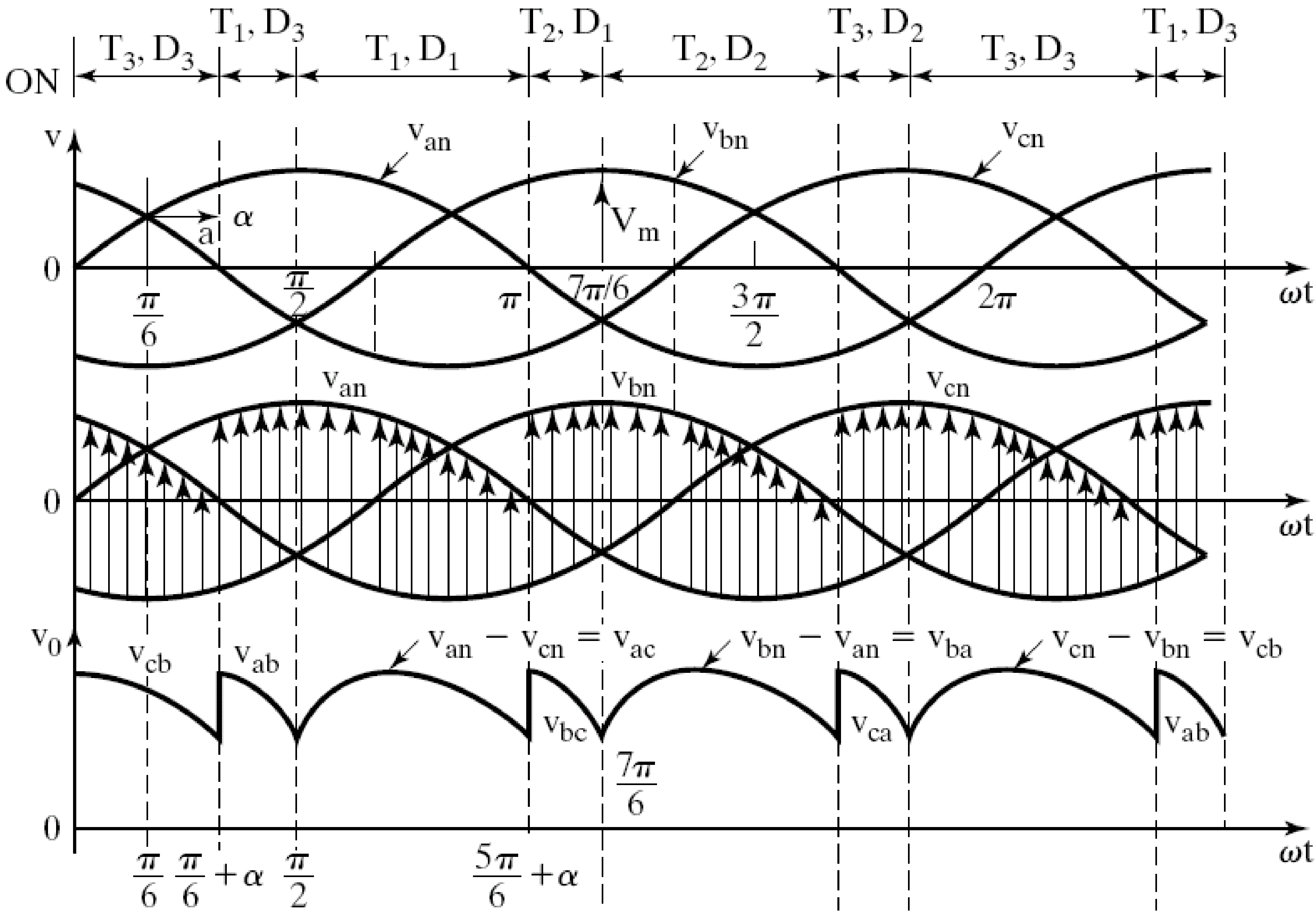
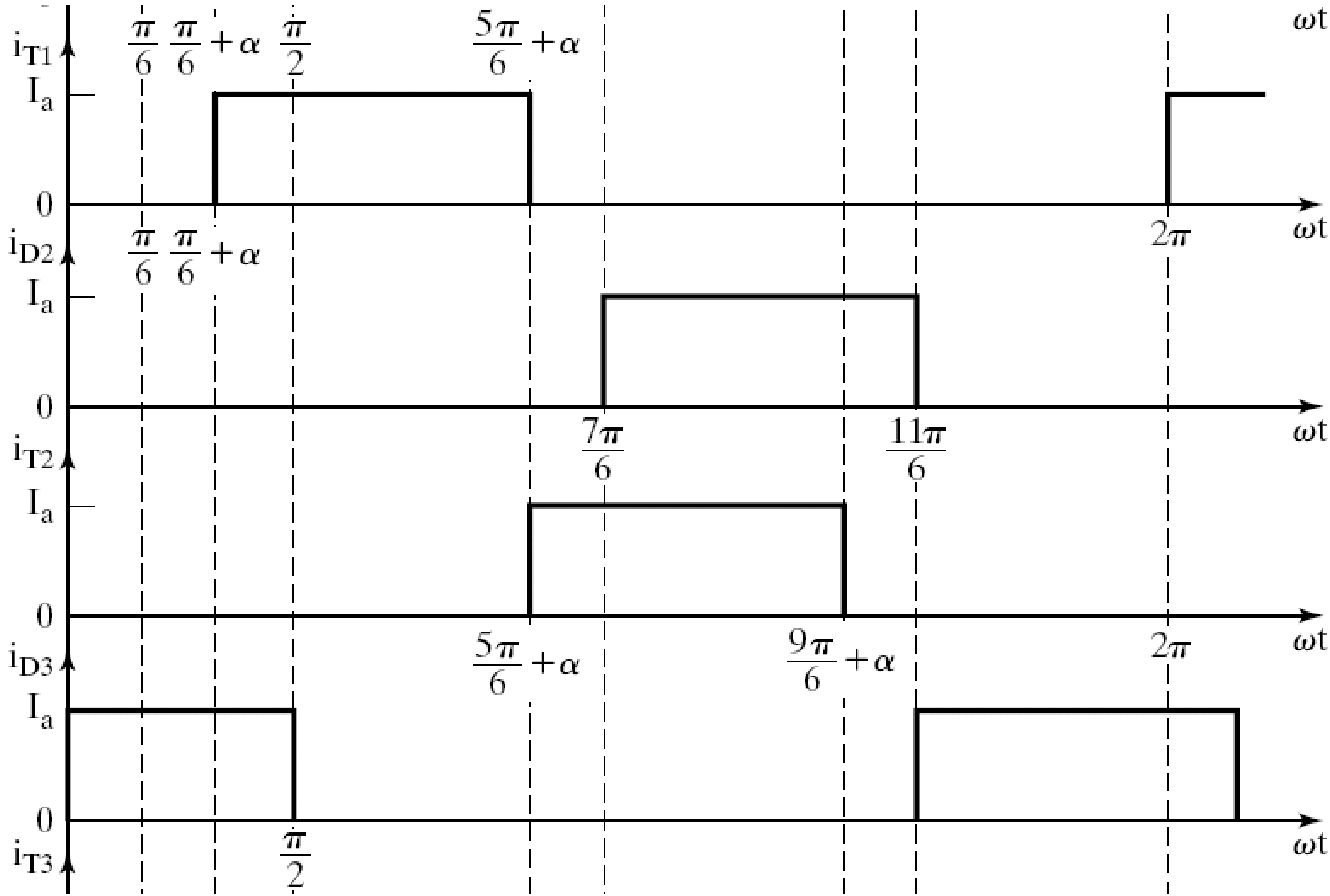
Wave forms of 3 Phase Semiconverter for $\alpha \leq 60^0$

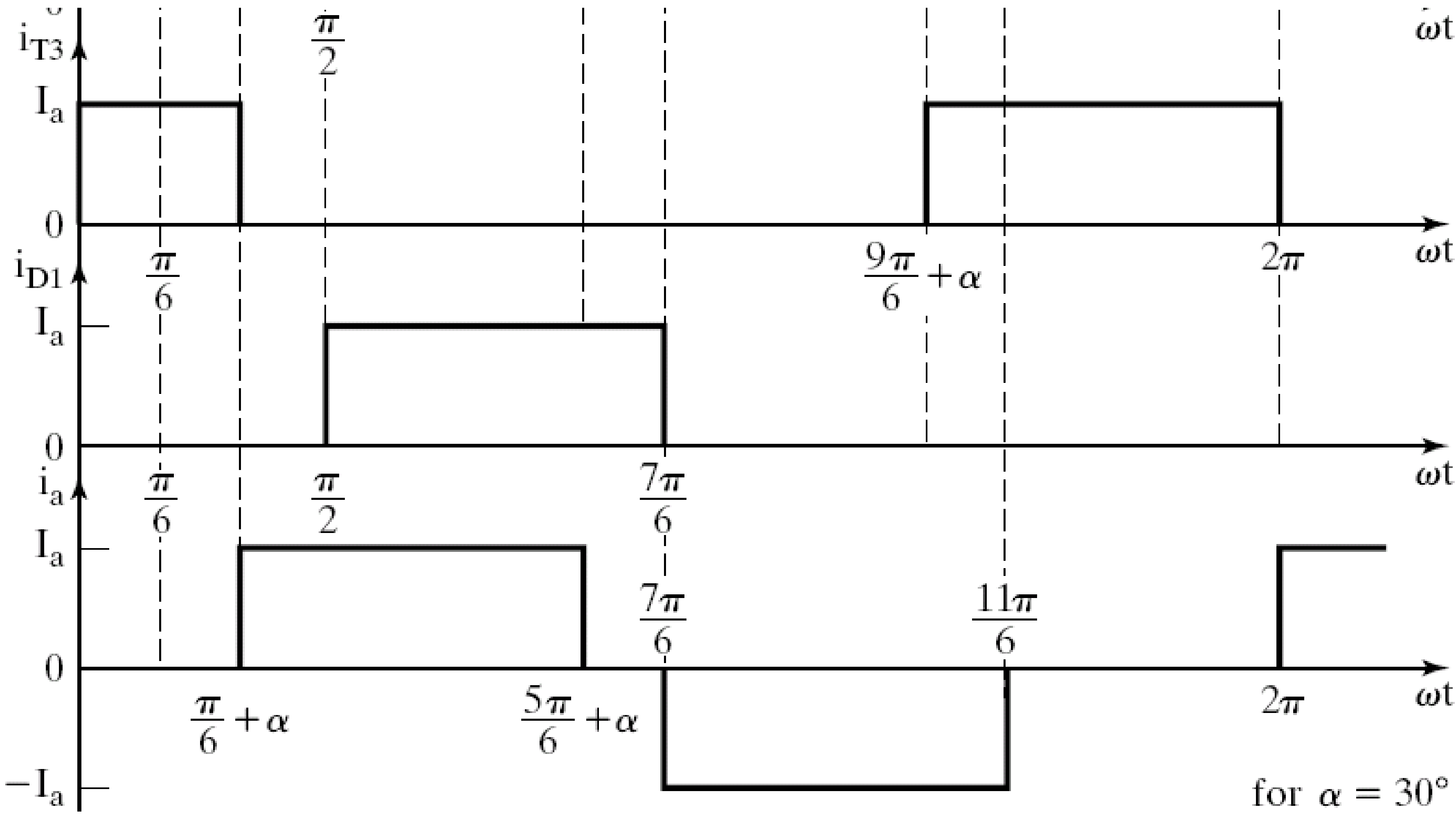
321


ON
T3, D3
T1, D3
T1, D1
T2, D1
T2, D2
T3, D2
T3, D3
T1, D3
v
van
vbn
vcn
α
a
Vm
0
π/6
π/2
π
7π/6
3π/2
2π
ωt
van
vbn
vcn
0
ωt
v0
vcb
vab
van − vcn = vac
vbn − van = vba
vcn − vbn = vcb
vbc
vca
vab
7π/6
0
π/6
π/6 + α
π/2
5π/6 + α
ωt

322



i_{T1}
$\frac{\pi}{6}$ $\frac{\pi}{6}+\alpha$ $\frac{\pi}{2}$
$\frac{5\pi}{6}+\alpha$
I_a
0
ωt
i_{D2}
$\frac{\pi}{6}$ $\frac{\pi}{6}+\alpha$
2π
I_a
0
ωt
$\frac{7\pi}{6}$
$\frac{11\pi}{6}$
i_{T2}
I_a
0
ωt
$\frac{5\pi}{6}+\alpha$
$\frac{9\pi}{6}+\alpha$
2π
i_{D3}
I_a
0
ωt
$\frac{\pi}{2}$
i_{T3}
ωt



i_{T3}
$\frac{\pi}{2}$
ωt
I_a
0
$\frac{\pi}{6}$
$\frac{9\pi}{6}+\alpha$
2π
ωt
i_{D1}
I_a
0
$\frac{\pi}{6}$
$\frac{\pi}{2}$
$\frac{7\pi}{6}$
ωt
i_a
I_a
$\frac{7\pi}{6}$
$\frac{11\pi}{6}$
0
$\frac{\pi}{6}+\alpha$
$\frac{5\pi}{6}+\alpha$
2π
ωt
$-I_a$
for $\alpha = 30°$

To derive an Expression for the Average Output Voltage of 3 Phase Semi-converter for $\alpha > \pi / 3$ and Discontinuous Output Voltage

For $\alpha \geq \dfrac{\pi}{3}$ and discontinuous output voltage:

the Average output voltage is found from

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\pi/6+\alpha}^{7\pi/6} v_{ac}.d\left(\omega t\right)\right]$$

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\pi/6+\alpha}^{7\pi/6} \sqrt{3}\; V_m \sin\left(\omega t - \frac{\pi}{6}\right) d\left(\omega t\right)\right]$$

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$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}\left(1+\cos\alpha\right)$$

$$V_{dc} = \frac{3V_{mL}}{2\pi}\left(1+\cos\alpha\right)$$

$V_{mL} = \sqrt{3}V_m =$ Max. value of line-to-line supply voltage

The maximum average output voltage that occurs at a delay angle of $\alpha = 0$ is

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3}V_m}{\pi}$$

327

The normalized average output voltage is

$$V_n = \frac{V_{dc}}{V_{dm}} = 0.5\left(1+\cos\alpha\right)$$

The rms output voltage is found from

$$V_{O(rms)} = \left[\frac{3}{2\pi}\int_{\pi/6+\alpha}^{7\pi/6} v_{ac}^2 . d\left(\omega t\right)\right]^{\frac{1}{2}}$$

328

$$V_{O(rms)} = \left[\frac{3}{2\pi} \int_{\pi/6+\alpha}^{7\pi/6} 3V_m^2 \sin^2\left(\omega t - \frac{\pi}{6}\right) d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \sqrt{3} V_m \left[\frac{3}{4\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{\frac{1}{2}}$$

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Average or DC Output Voltage of a 3-Phase Semi-converter for $\alpha \leq \pi / 3$, and Continuous Output Voltage

For $\alpha \le \dfrac{\pi}{3}$, and continuous output voltage

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\pi/6+\alpha}^{\pi/2} v_{ab}.d\left(\omega t\right) + \int_{\pi/2}^{5\pi/6+\alpha} v_{ac}.d\left(\omega t\right)\right]$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}\left(1+\cos\alpha\right)$$

$$V_n = \frac{V_{dc}}{V_{dm}} = 0.5\left(1+\cos\alpha\right)$$

RMS value of o/p voltage is calculated by using the equation

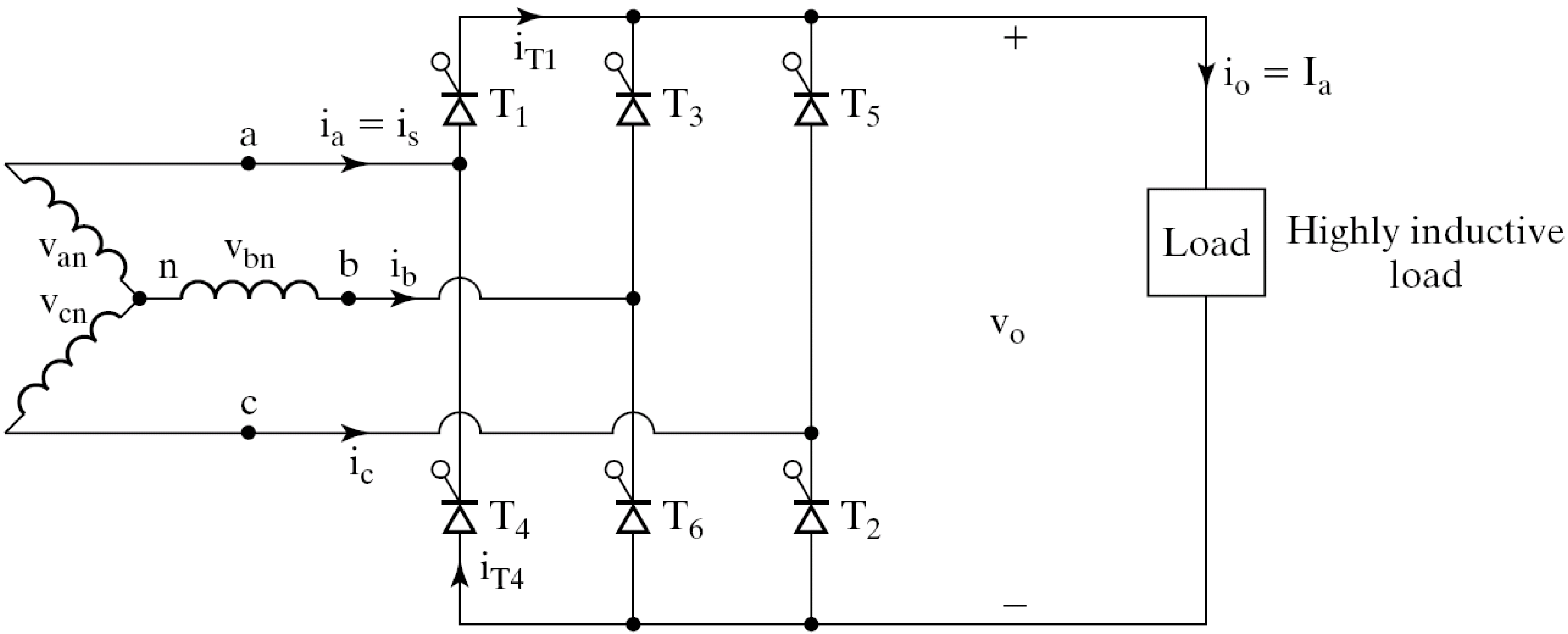
$$V_{O(rms)} = \left[\frac{3}{2\pi}\int_{\pi/6+\alpha}^{\pi/2} v_{ab}^2 .d\left(\omega t\right) + \int_{\pi/2}^{5\pi/6+\alpha} v_{ac}^2 .d\left(\omega t\right)\right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \sqrt{3}V_m \left[\frac{3}{4\pi}\left(\frac{2\pi}{3} + \sqrt{3}\cos^2\alpha\right)\right]^{\frac{1}{2}}$$

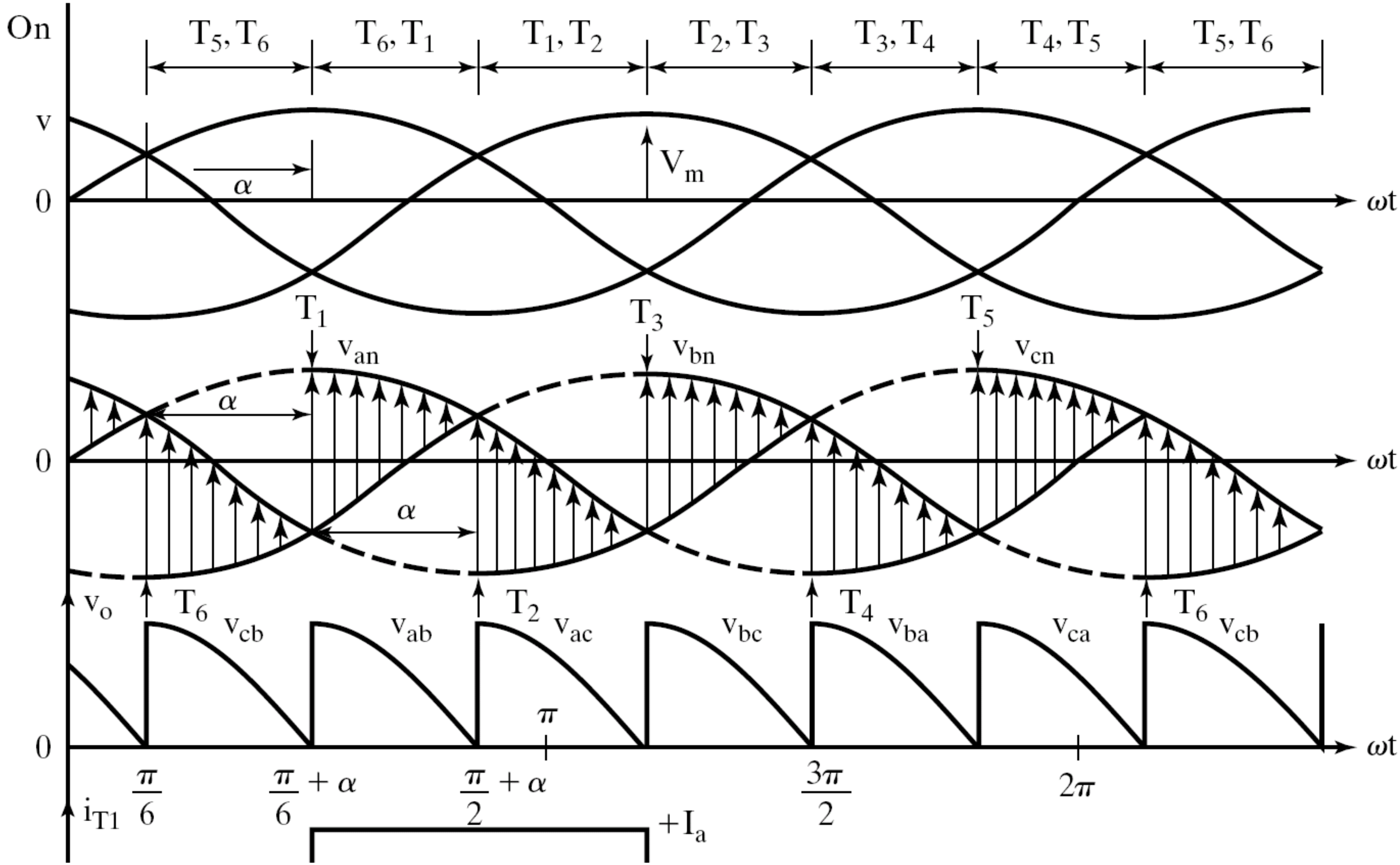
Three Phase Full Converter

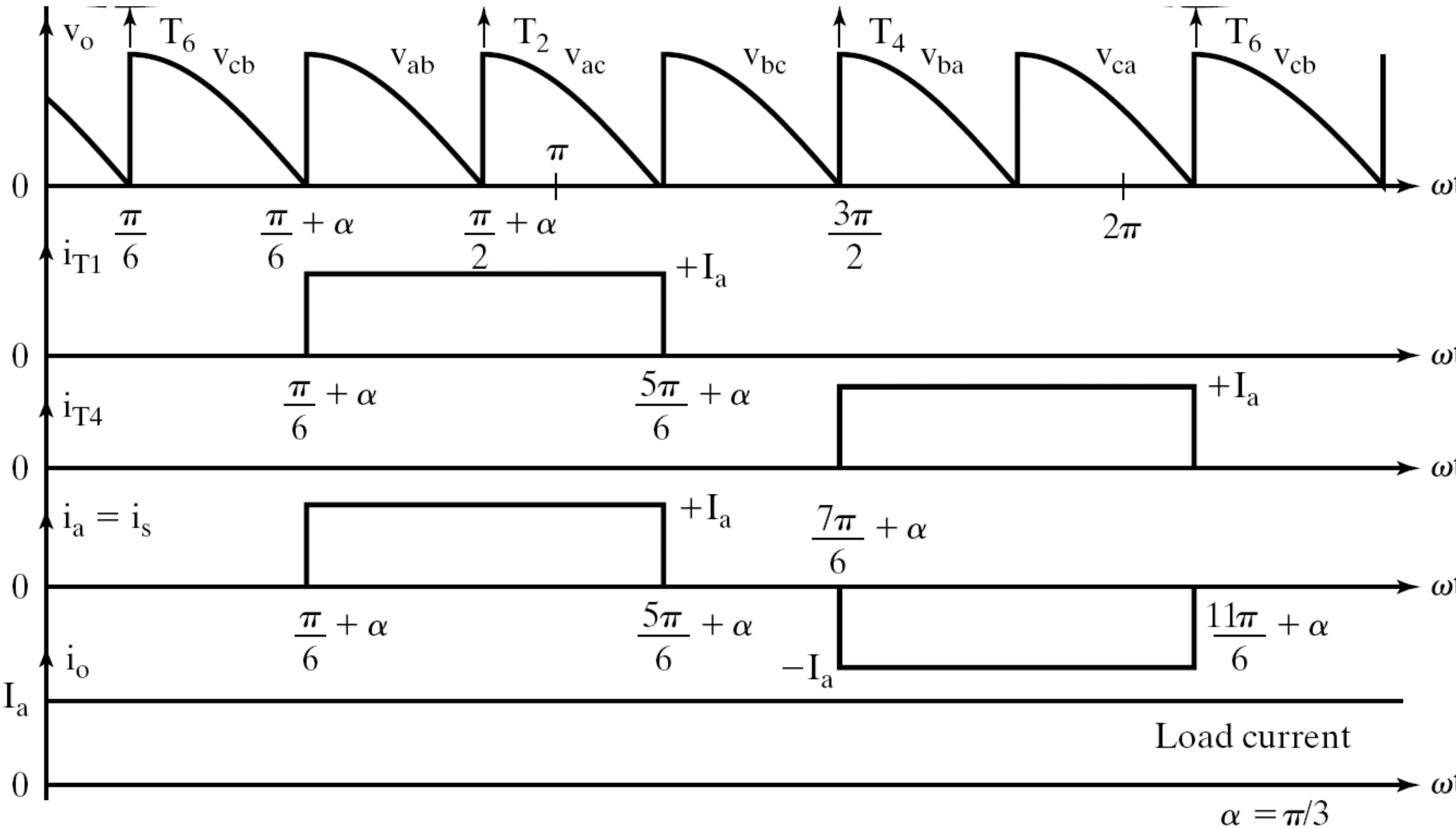
Three Phase Full Converter

- 3 Phase Fully Controlled Full Wave Bridge Converter.
- Known as a 6-pulse converter.
- Used in industrial applications up to 120kW output power.
- Two quadrant operation is possible.



i_{T1}
T_1
T_3
T_5
+
$i_o = I_a$
a
$i_a = i_s$
Load
Highly inductive load
v_{an}
n
v_{bn}
b
i_b
v_{cn}
v_o
c
i_c
T_4
T_6
T_2
i_{T4}
−


On
T5, T6
T6, T1
T1, T2
T2, T3
T3, T4
T4, T5
T5, T6
v
Vm
α
0
ωt
T1
T3
T5
van
vbn
vcn
α
α
0
ωt
vo
T6
T2
T4
T6
vcb
vab
vac
vbc
vba
vca
vcb
π
0
ωt
π/6
π/6 + α
π/2 + α
3π/2
2π
iT1
+Ia


v_o
T_6 v_{cb}
v_{ab}
T_2 v_{ac}
v_{bc}
T_4 v_{ba}
v_{ca}
T_6 v_{cb}
π
0
ωt
$\frac{\pi}{6}$
$\frac{\pi}{6} + \alpha$
$\frac{\pi}{2} + \alpha$
$\frac{3\pi}{2}$
2π
i_{T1}
$+I_a$
0
ωt
$\frac{\pi}{6} + \alpha$
$\frac{5\pi}{6} + \alpha$
i_{T4}
$+I_a$
0
ωt
$i_a = i_s$
$+I_a$
$\frac{7\pi}{6} + \alpha$
0
ωt
$\frac{\pi}{6} + \alpha$
$\frac{5\pi}{6} + \alpha$
$-I_a$
$\frac{11\pi}{6} + \alpha$
i_o
I_a
Load current
0
ωt
$\alpha = \pi/3$

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- The thyristors are triggered at an interval of $\pi / 3$.
- The frequency of output ripple voltage is $6f_S$.
- T_1 is triggered at $\omega t = (\pi/6 + \alpha)$, T_6 is already conducting when T_1 is turned ON.
- During the interval $(\pi/6 + \alpha)$ to $(\pi/2 + \alpha)$, T_1 and T_6 conduct together & the output load voltage is equal to $v_{ab} = (v_{an} - v_{bn})$

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- T_2 is triggered at $\omega t = (\pi/2 + \alpha)$, T_6 turns off naturally as it is reverse biased as soon as T_2 is triggered.
- During the interval $(\pi/2 + \alpha)$ to $(5\pi/6 + \alpha)$, T_1 and T_2 conduct together & the output load voltage $v_O = v_{ac} = (v_{an} - v_{cn})$
- Thyristors are numbered in the order in which they are triggered.
- The thyristor triggering sequence is 12, 23, 34, 45, 56, 61, 12, 23, 34, ………

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We deifine three line neutral voltages

(3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t \quad ; \qquad V_m = \text{ Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) = V_m \sin\left(\omega t - 120^0\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right) = V_m \sin\left(\omega t + 120^0\right)$$

$$= V_m \sin\left(\omega t - 240^0\right)$$

V_m is the peak phase voltage of a wye-connected source.

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The corresponding line-to-line supply voltages are

$$v_{RY} = v_{ab} = \left(v_{an} - v_{bn}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$v_{YB} = v_{bc} = \left(v_{bn} - v_{cn}\right) = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$v_{BR} = v_{ca} = \left(v_{cn} - v_{an}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

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To Derive An Expression For The Average Output Voltage Of 3-phase Full Converter With Highly Inductive Load Assuming Continuous And Constant Load Current

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The output load voltage consists of 6 voltage pulses over a period of 2π radians, Hence the average output voltage is calculated as

$$V_{O(dc)} = V_{dc} = \frac{6}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} v_O . d\omega t \quad ;$$

$$v_O = v_{ab} = \sqrt{3} V_m \sin\left(\omega t + \frac{\pi}{6} \right)$$

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$$V_{dc} = \frac{3}{\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} \sqrt{3} V_m \sin\left(\omega t + \frac{\pi}{6} \right) .d\omega t$$

$$V_{dc} = \frac{3\sqrt{3} V_m}{\pi} \cos\alpha = \frac{3 V_{mL}}{\pi} \cos\alpha$$

Where $V_{mL} = \sqrt{3} V_m =$ Max. line-to-line supply voltage

The maximum average dc output voltage is obtained for a delay angle $\alpha = 0$,

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3} V_m}{\pi} = \frac{3 V_{mL}}{\pi}$$

The normalized average dc output voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \cos\alpha$$

The rms value of the output voltage is found from

$$V_{O(rms)} = \left[\frac{6}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} v_O^2 . d\left(\omega t\right) \right]^{\frac{1}{2}}$$

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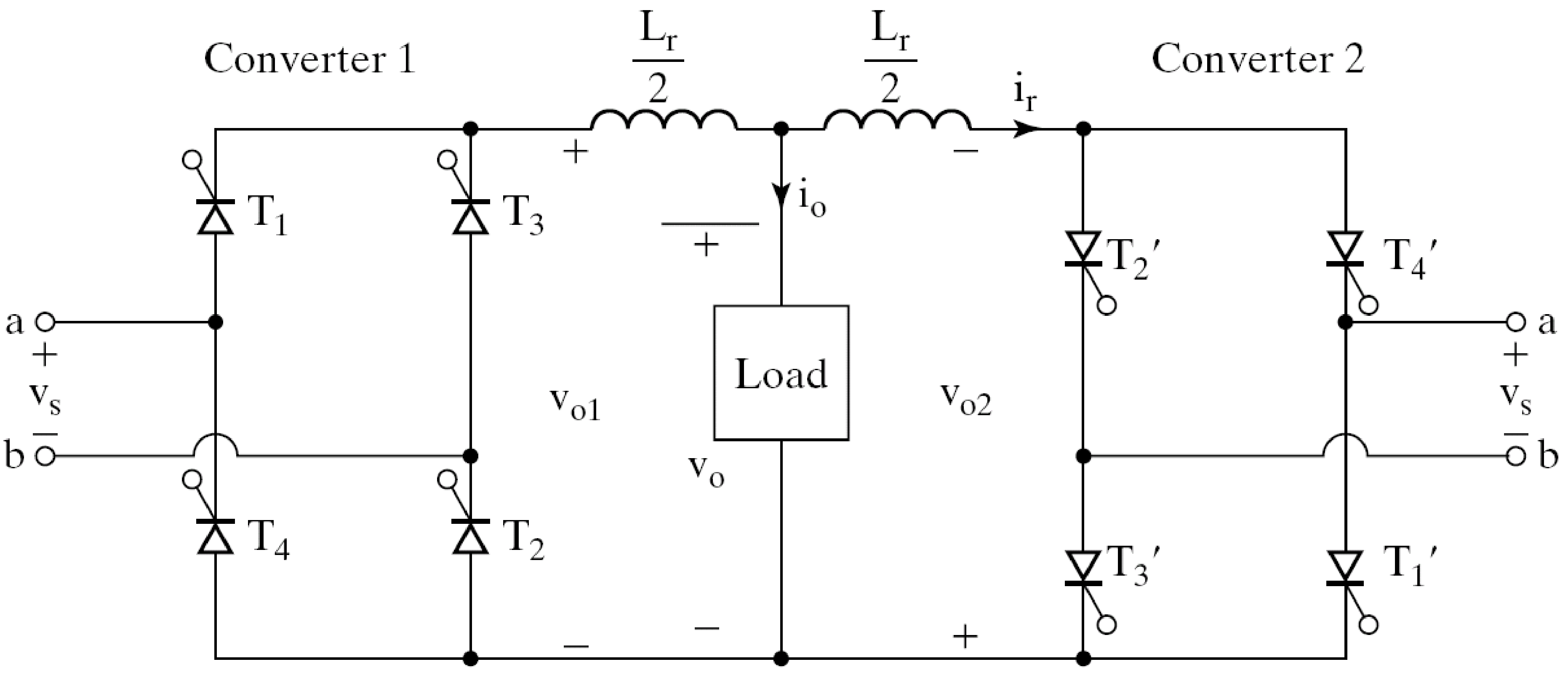
$$V_{O(rms)} = \left[\frac{6}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} v_{ab}^{2}.d\left(\omega t\right) \right]^{\frac{1}{2}}$$

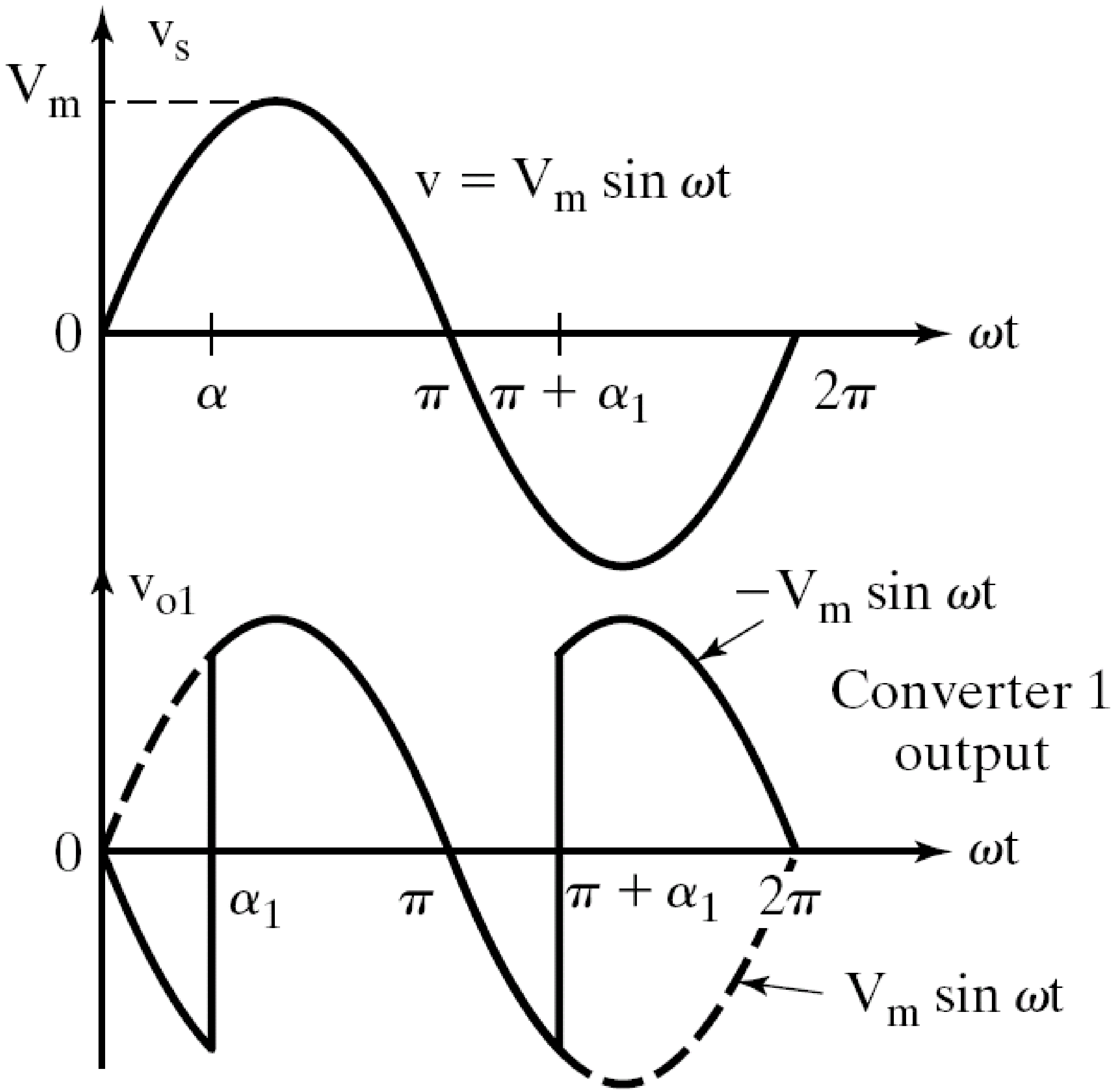
$$V_{O(rms)} = \left[\frac{3}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} 3V_m^2 \sin^2\left(\omega t + \frac{\pi}{6}\right).d\left(\omega t\right) \right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \sqrt{3}V_m \left(\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}\cos 2\alpha \right)^{\frac{1}{2}}$$

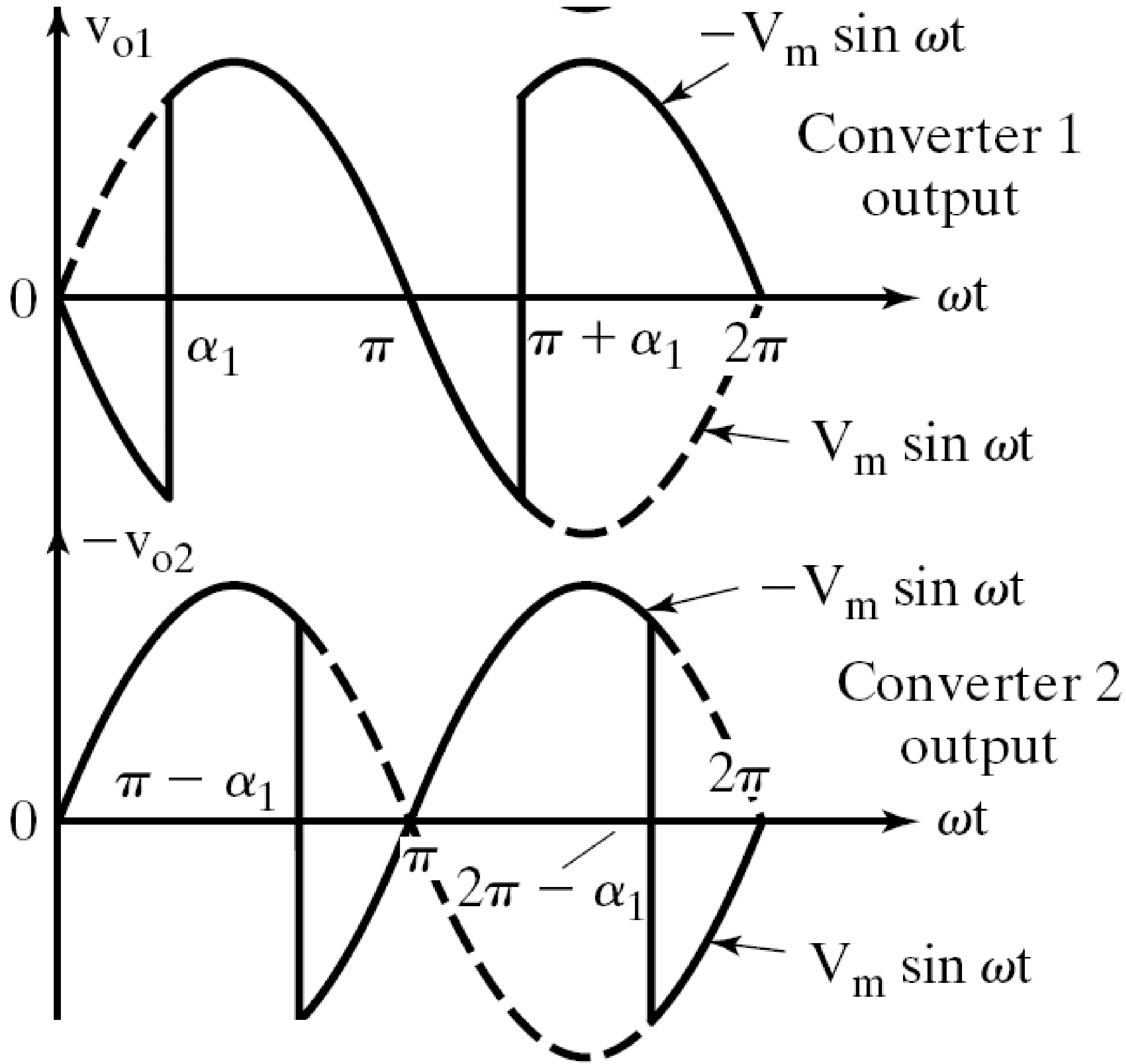
Single Phase Dual Converter

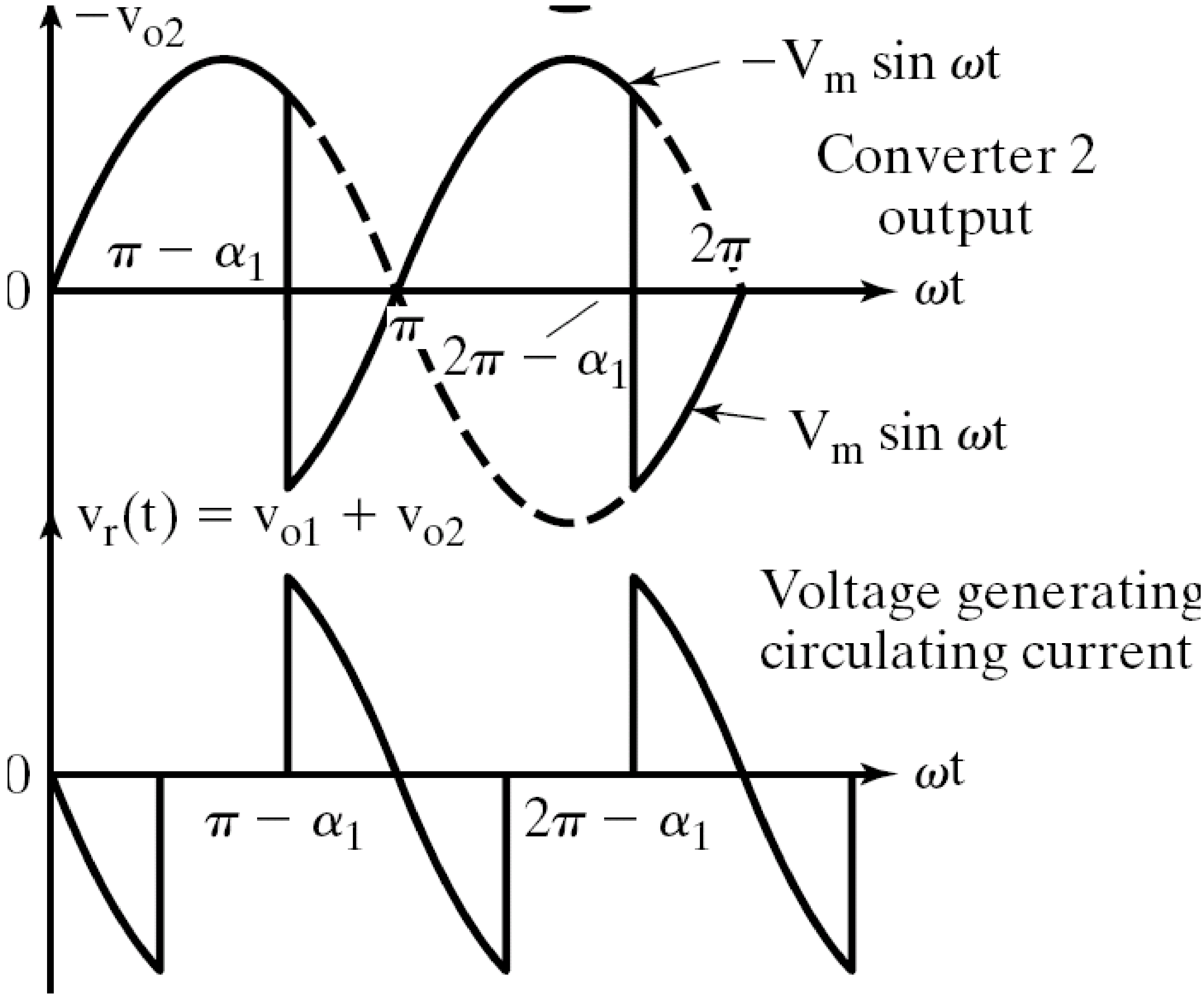
Single Phase Dual Converter





v_s
V_m
$v = V_m \sin \omega t$
0
ωt
α
π
$\pi + \alpha_1$
2π
v_{o1}
$-V_m \sin \omega t$
Converter 1 output
0
ωt
α_1
π
$\pi + \alpha_1$
2π
$V_m \sin \omega t$


v_{o1}
$-V_m \sin \omega t$
Converter 1 output
0
ωt
α_1
π
$\pi + \alpha_1$
2π
$V_m \sin \omega t$
$-v_{o2}$
$-V_m \sin \omega t$
Converter 2 output
$\pi - \alpha_1$
2π
0
ωt
π
$2\pi - \alpha_1$
$V_m \sin \omega t$


$-v_{o2}$
$-V_m \sin \omega t$
Converter 2
output
$\pi - \alpha_1$
2π
0
ωt
π
$2\pi - \alpha_1$
$V_m \sin \omega t$
$v_r(t) = v_{o1} + v_{o2}$
Voltage generating
circulating current
0
ωt
$\pi - \alpha_1$
$2\pi - \alpha_1$

The average dc output voltage of converter 1 is

$$V_{dc1} = \frac{2V_m}{\pi}\cos\alpha_1$$

The average dc output voltage of converter 2 is

$$V_{dc2} = \frac{2V_m}{\pi}\cos\alpha_2$$

In the dual converter operation one converter is operated as a controlled rectifier with $\alpha < 90^0$ & the second converter is operated as a line commutated inverter in the inversion mode with $\alpha > 90^0$

$$\therefore \qquad V_{dc1} = -V_{dc2}$$

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$$\frac{2V_m}{\pi}\cos\alpha_1 = \frac{-2V_m}{\pi}\cos\alpha_2 = \frac{2V_m}{\pi}\left(-\cos\alpha_2\right)$$

$$\therefore \qquad \cos\alpha_1 = -\cos\alpha_2$$

or

$$\cos\alpha_2 = -\cos\alpha_1 = \cos\left(\pi - \alpha_1\right)$$

$$\therefore \qquad \alpha_2 = \left(\pi - \alpha_1\right) \text{ or}$$

$$\left(\alpha_1 + \alpha_2\right) = \pi \text{ radians}$$

Which gives

$$\alpha_2 = \left(\pi - \alpha_1\right)$$

To Obtain an Expression for the Instantaneous Circulating Current

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- v_{O1} = Instantaneous o/p voltage of converter 1.
- v_{O2} = Instantaneous o/p voltage of converter 2.
- The circulating current i_r can be determined by integrating the instantaneous voltage difference (which is the voltage drop across the circulating current reactor L_r), starting from $\omega t = (2\pi - \alpha_1)$.
- As the two average output voltages during the interval $\omega t = (\pi + \alpha_1)$ to $(2\pi - \alpha_1)$ are equal and opposite their contribution to the instantaneous circulating current i_r is zero.

$$i_r = \frac{1}{\omega L_r}\left[\int_{(2\pi-\alpha_1)}^{\omega t} v_r . d(\omega t)\right]; \quad v_r = (v_{O1} - v_{O2})$$

As the o/p voltage v_{O2} is negative

$$v_r = (v_{O1} + v_{O2})$$

$$\therefore \quad i_r = \frac{1}{\omega L_r}\left[\int_{(2\pi-\alpha_1)}^{\omega t} (v_{O1} + v_{O2}).d(\omega t)\right];$$

$$v_{O1} = -V_m \sin \omega t \text{ for } (2\pi - \alpha_1) \text{ to } \omega t$$

$$i_r = \frac{V_m}{\omega L_r}\left[\int_{(2\pi-\alpha_1)}^{\omega t} -\sin \omega t . d(\omega t) - \int_{(2\pi-\alpha_1)}^{\omega t} \sin \omega t . d(\omega t)\right]$$

$$i_r = \frac{2V_m}{\omega L_r}\left(\cos \omega t - \cos \alpha_1\right)$$

The instantaneous value of the circulating current depends on the delay angle.

For trigger angle (delay angle) $\alpha_1 = 0$,

the magnitude of circulating current becomes min.

when $\omega t = n\pi,\ n = 0, 2, 4, ...$ & magnitude becomes

max. when $\omega t = n\pi,\ n = 1, 3, 5, ...$

If the peak load current is $I_p,$ one of the

converters that controls the power flow

may carry a peak current of

$$\left(I_p + \frac{4V_m}{\omega L_r} \right),$$

where

$$I_p = I_{L(\max)} = \frac{V_m}{R_L},$$

&

$$i_{r(\max)} = \frac{4V_m}{\omega L_r} = \text{ max. circulating current}$$

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Different Modes Of Operation of Dual converter

- Non-circulating current (circulating current free) mode of operation.
- Circulating current mode of operation.

Non-Circulating Current Mode of Operation

- In this mode only one converter is operated at a time.
- When converter 1 is ON, $0 < \alpha_1 < 90^0$
- V_{dc} is positive and I_{dc} is positive.
- When converter 2 is ON, $0 < \alpha_2 < 90^0$
- V_{dc} is negative and I_{dc} is negative.

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Circulating Current Mode Of Operation

- In this mode, both the converters are switched ON and operated at the same time.
- The trigger angles α_1 and α_2 are adjusted such that $(\alpha_1 + \alpha_2) = 180^0$; $\alpha_2 = (180^0 - \alpha_1)$.

- When $0 < \alpha_1 < 90^0$, converter 1 operates as a controlled rectifier and converter 2 operates as an inverter with $90^0 < \alpha_2 < 180^0$.
- In this case V_{dc} and I_{dc}, both are positive.
- When $90^0 < \alpha 1 < 180^0$, converter 1 operates as an Inverter and converter 2 operated as a controlled rectifier by adjusting its trigger angle α_2 such that $0 < \alpha_2 < 90^0$.
- In this case V_{dc} and I_{dc}, both are negative.

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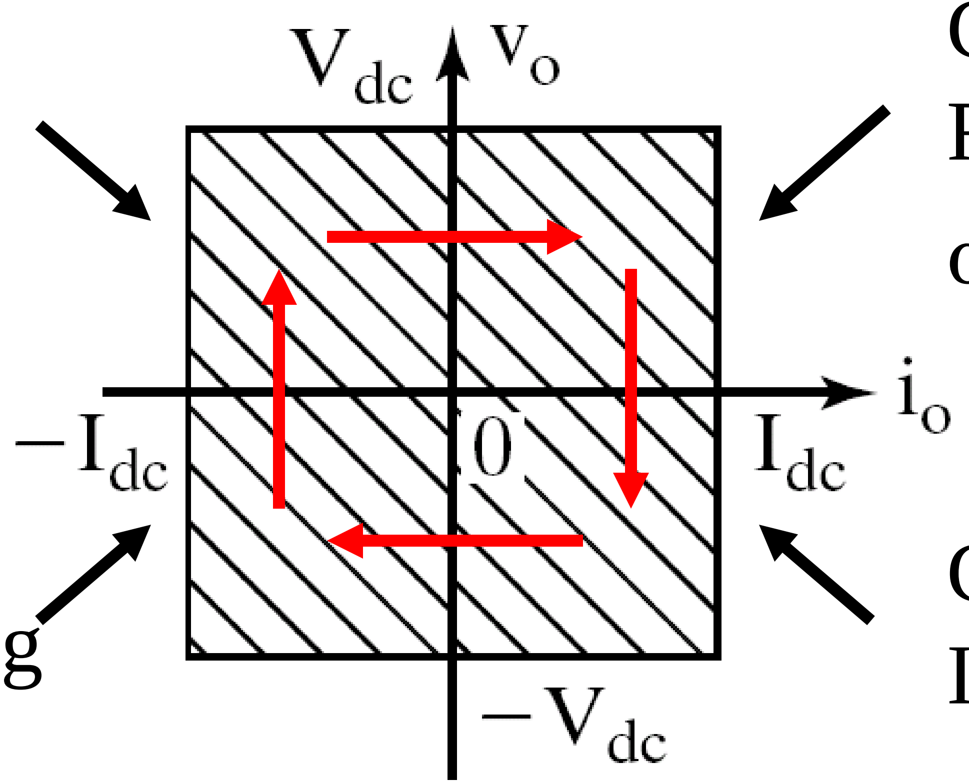
Four Quadrant Operation

Conv. 2
Inverting
$\alpha_2 > 90^0$

Conv. 1
Rectifying
$\alpha_1 < 90^0$



Conv. 2
Rectifying
$\alpha_2 < 90^0$

Conv. 1
Inverting
$\alpha_1 > 90^0$

Advantages of Circulating Current Mode Of Operation

- The circulating current maintains continuous conduction of both the converters over the complete control range, independent of the load.
- One converter always operates as a rectifier and the other converter operates as an inverter, the power flow in either direction at any time is possible.

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- As both the converters are in continuous conduction we obtain faster dynamic response. i.e., the time response for changing from one quadrant operation to another is faster.

Disadvantages of Circulating Current Mode Of Operation

- There is always a circulating current flowing between the converters.
- When the load current falls to zero, there will be a circulating current flowing between the converters so we need to connect circulating current reactors in order to limit the peak circulating current to safe level.
- The converter thyristors should be rated to carry a peak current much greater than the peak load current.

Three Phase Dual Converters

Three Phase Dual Converters

- For four quadrant operation in many industrial variable speed dc drives , 3 phase dual converters are used.
- Used for applications up to 2 mega watt output power level.
- Dual converter consists of two 3 phase full converters which are connected in parallel & in opposite directions across a common load.

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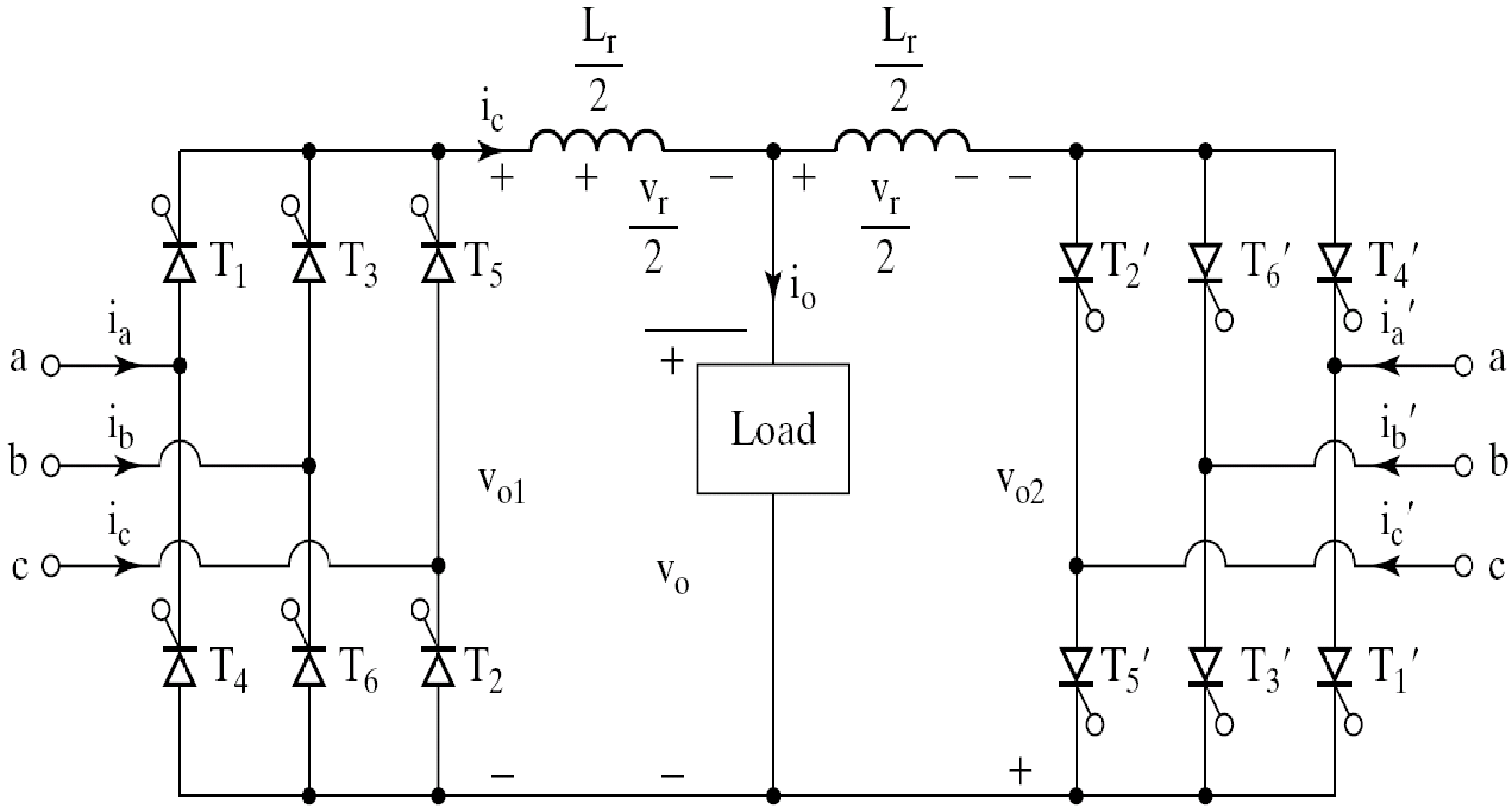
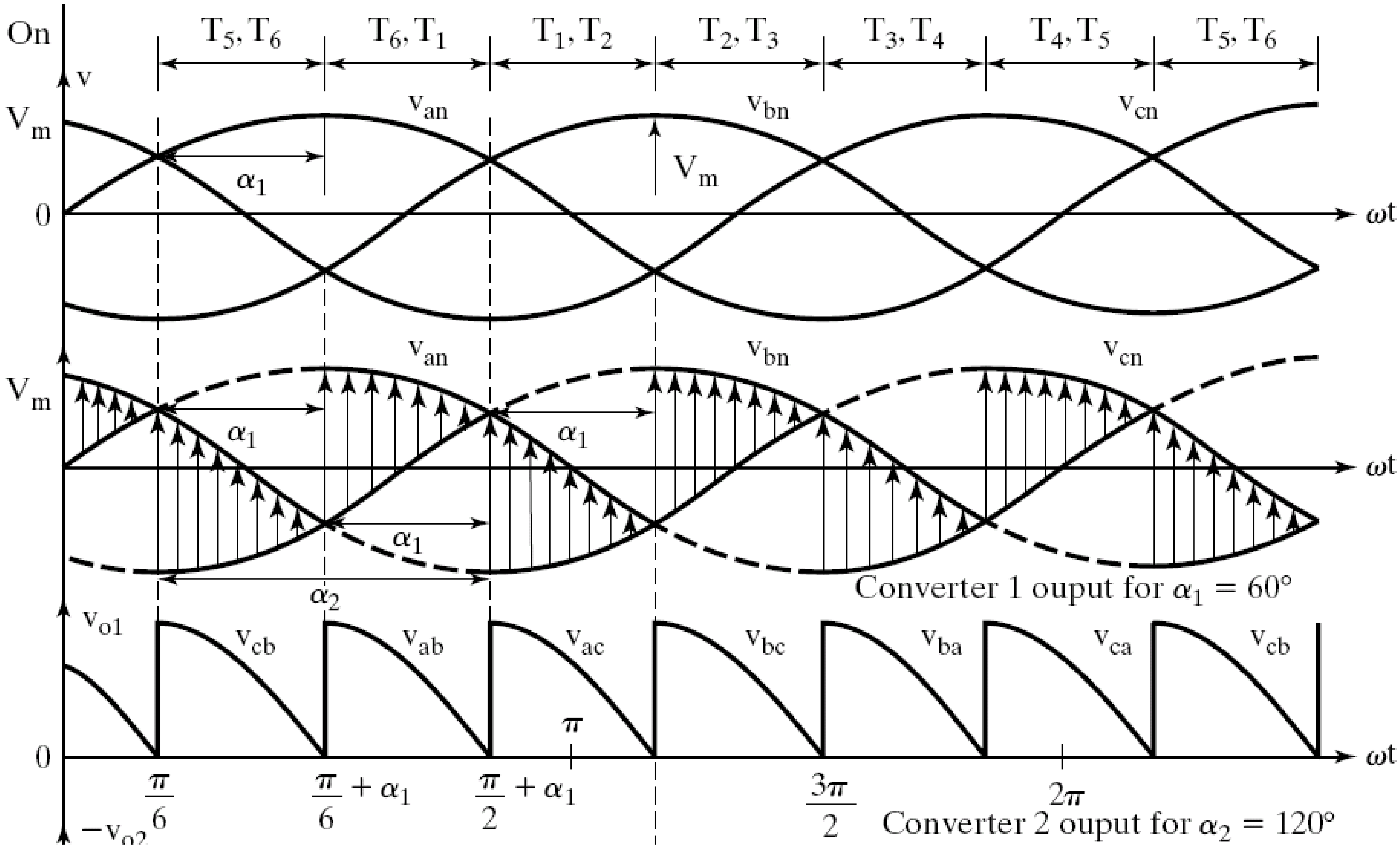
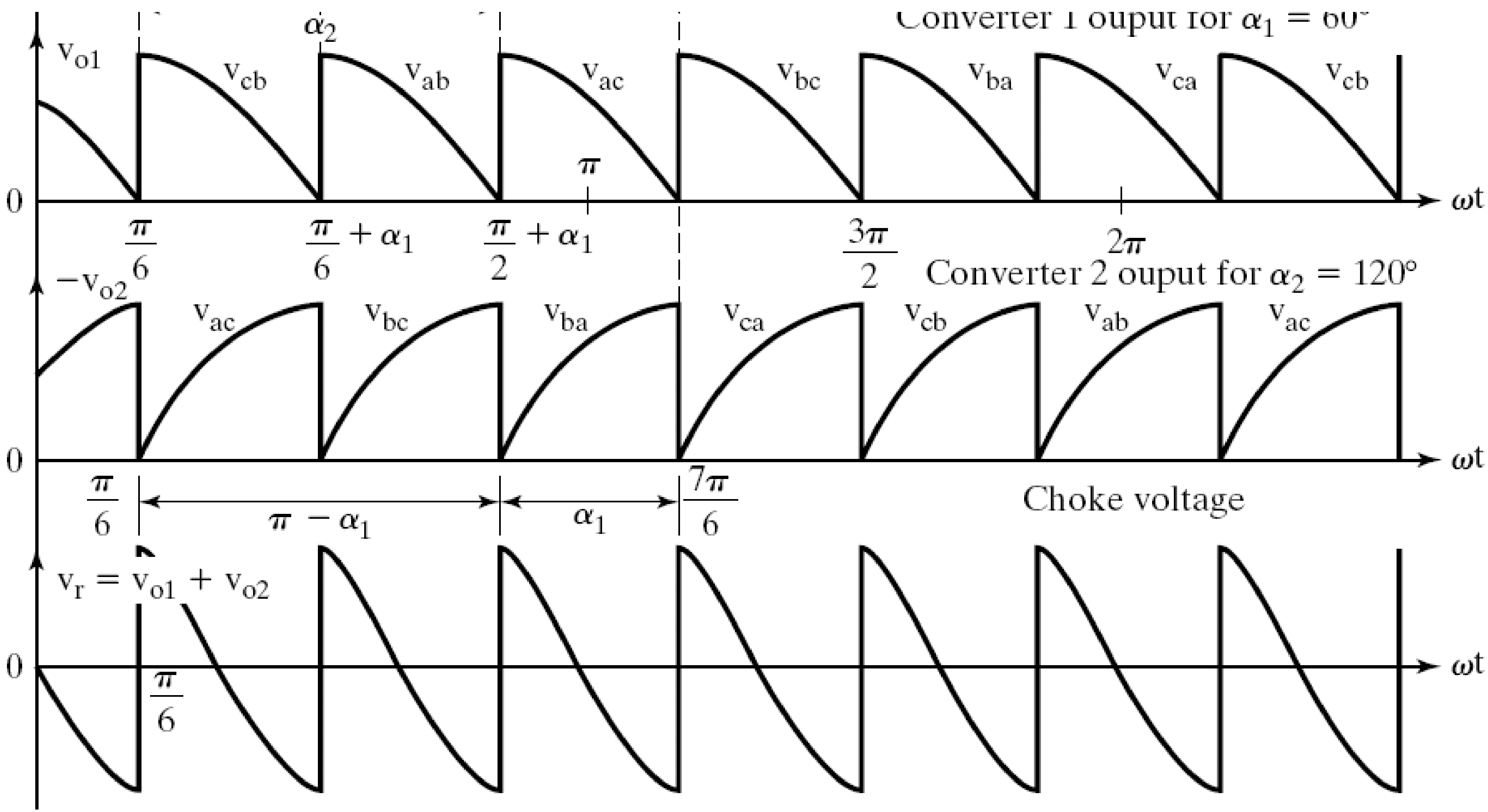


$\frac{L_r}{2}$
$\frac{L_r}{2}$
i_c
$\frac{v_r}{2}$
$\frac{v_r}{2}$
i_o
T_1
T_3
T_5
T_4
T_6
T_2
T_2'
T_6'
T_4'
T_5'
T_3'
T_1'
a
b
c
i_a
i_b
i_c
i_a'
i_b'
i_c'
Load
v_{o1}
v_{o2}
v_o


On
T_5, T_6
T_6, T_1
T_1, T_2
T_2, T_3
T_3, T_4
T_4, T_5
T_5, T_6
v
V_m
v_{an}
v_{bn}
v_{cn}
α_1
V_m
0
ωt
V_m
v_{an}
v_{bn}
v_{cn}
α_1
α_1
ωt
α_1
α_2
Converter 1 ouput for $\alpha_1 = 60°$
v_{o1}
v_{cb}
v_{ab}
v_{ac}
v_{bc}
v_{ba}
v_{ca}
v_{cb}
π
0
ωt
$\frac{\pi}{6}$
$\frac{\pi}{6} + \alpha_1$
$\frac{\pi}{2} + \alpha_1$
$\frac{3\pi}{2}$
2π
$-v_{o2}$
Converter 2 ouput for $\alpha_2 = 120°$


Converter 1 ouput for $\alpha_1 = 60°$
α_2
v_{o1}
v_{cb} v_{ab} v_{ac} v_{bc} v_{ba} v_{ca} v_{cb}
π
0
ωt
$\frac{\pi}{6}$
$\frac{\pi}{6} + \alpha_1$
$\frac{\pi}{2} + \alpha_1$
$\frac{3\pi}{2}$
2π
Converter 2 ouput for $\alpha_2 = 120°$
$-v_{o2}$
v_{ac} v_{bc} v_{ba} v_{ca} v_{cb} v_{ab} v_{ac}
0
ωt
$\frac{\pi}{6}$
$\pi - \alpha_1$
α_1
$\frac{7\pi}{6}$
Choke voltage
$v_r = v_{o1} + v_{o2}$
0
ωt
$\frac{\pi}{6}$

Outputs of Converters 1 & 2

- During the interval $(\pi/6 + \alpha_1)$ to $(\pi/2 + \alpha_1)$, the line to line voltage v_{ab} appears across the output of converter 1 and v_{bc} appears across the output of converter 2

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We deifine three line neutral voltages

(3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t \quad ;$$

$$V_m = \text{ Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) = V_m \sin\left(\omega t - 120^0\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right) = V_m \sin\left(\omega t + 120^0\right)$$

$$= V_m \sin\left(\omega t - 240^0\right)$$

The corresponding line-to-line supply voltages are

$$v_{RY} = v_{ab} = \left(v_{an} - v_{bn}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$v_{YB} = v_{bc} = \left(v_{bn} - v_{cn}\right) = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$v_{BR} = v_{ca} = \left(v_{cn} - v_{an}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

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To obtain an Expression for the Circulating Current

- If v_{O1} and v_{O2} are the output voltages of converters 1 and 2 respectively, the instantaneous voltage across the current limiting inductor during the interval $(\pi/6 + \alpha_1) \leq \omega t \leq (\pi/2 + \alpha_1)$ is given by

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$$v_r = v_{O1} + v_{O2} = v_{ab} - v_{bc}$$

$$v_r = \sqrt{3}V_m\left[\sin\left(\omega t + \frac{\pi}{6}\right) - \sin\left(\omega t - \frac{\pi}{2}\right)\right]$$

$$v_r = 3V_m \cos\left(\omega t - \frac{\pi}{6}\right)$$

The circulating current can be calculated by using the equation

$$i_r\left(t\right)=\frac{1}{\omega L_r}\int_{\frac{\pi}{6}+\alpha_1}^{\omega t} v_r.d\left(\omega t\right)$$

$$i_r\left(t\right)=\frac{1}{\omega L_r}\int_{\frac{\pi}{6}+\alpha_1}^{\omega t} 3V_m\cos\left(\omega t-\frac{\pi}{6}\right).d\left(\omega t\right)$$

$$i_r\left(t\right)=\frac{3V_m}{\omega L_r}\left[\sin\left(\omega t-\frac{\pi}{6}\right)-\sin\alpha_1\right]$$

$$i_{r(\max)}=\frac{3V_m}{\omega L_r}$$

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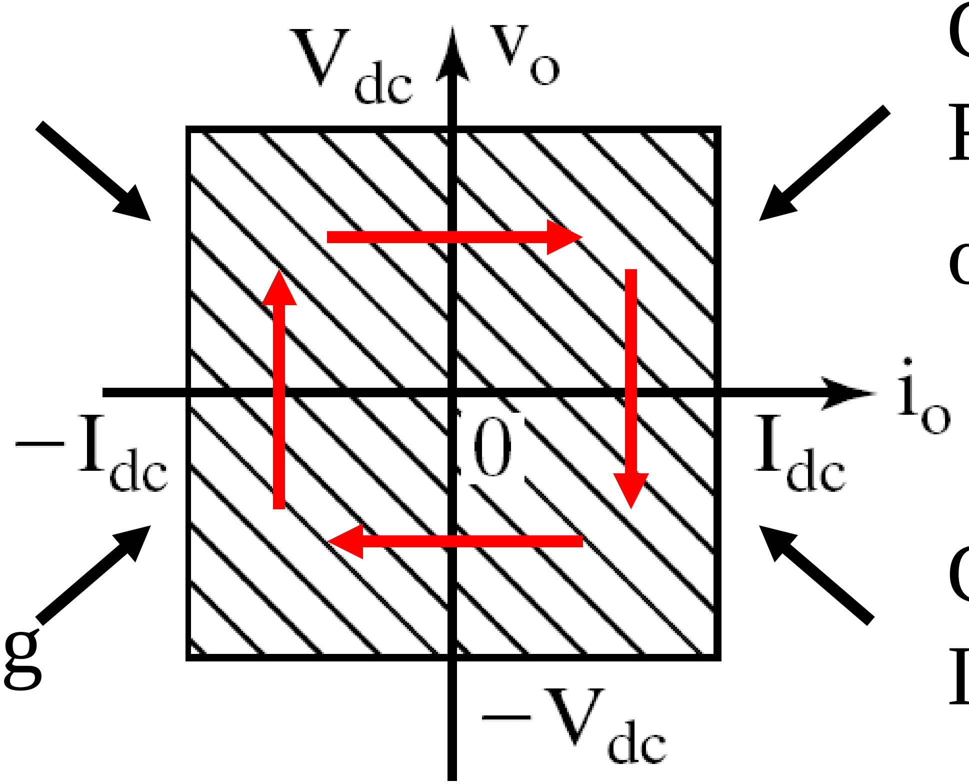
Four Quadrant Operation

Conv. 2
Inverting
$\alpha_2 > 90^0$

Conv. 1
Rectifying
$\alpha_1 < 90^0$



Conv. 2
Rectifying
$\alpha_2 < 90^0$

Conv. 1
Inverting
$\alpha_1 > 90^0$

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Contd…

- There are two different modes of operation.
  - Circulating current free (non circulating) mode of operation
  - Circulating current mode of operation

Non Circulating Current Mode Of Operation

- In this mode of operation only one converter is switched on at a time
- When the converter 1 is switched on,

  For $\alpha_1 < 90^0$ the converter 1 operates in the Rectification mode

  V_{dc} is positive, I_{dc} is positive and hence the average load power P_{dc} is positive.
- Power flows from ac source to the load

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- When the converter 1 is on,

  For $\alpha_1 > 90^0$ the converter 1 operates in the Inversion mode

  V_{dc} is negative, I_{dc} is positive and the average load power P_{dc} is negative.
- Power flows from load circuit to ac source.

- When the converter 2 is switched on,

  For $\alpha_2 < 90^0$ the converter 2 operates in the Rectification mode

  V_{dc} is negative, I_{dc} is negative and the average load power P_{dc} is positive.
- The output load voltage & load current reverse when converter 2 is on.
- Power flows from ac source to the load

- When the converter 2 is switched on,

  For $\alpha_2 > 90^0$ the converter 2 operates in the Inversion mode

  V_{dc} is positive, I_{dc} is negative and the average load power P_{dc} is negative.
- Power flows from load to the ac source.
- Energy is supplied from the load circuit to the ac supply.

Circulating Current Mode Of Operation

- Both the converters are switched on at the same time.
- One converter operates in the rectification mode while the other operates in the inversion mode.
- Trigger angles α_1 & α_2 are adjusted such that $(\alpha_1 + \alpha_2) = 180^0$

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- When $\alpha_1 < 90^0$, converter 1 operates as a controlled rectifier. α_2 is made greater than 90^0 and converter 2 operates as an Inverter.
- V_{dc} is positive & I_{dc} is positive and P_{dc} is positive.

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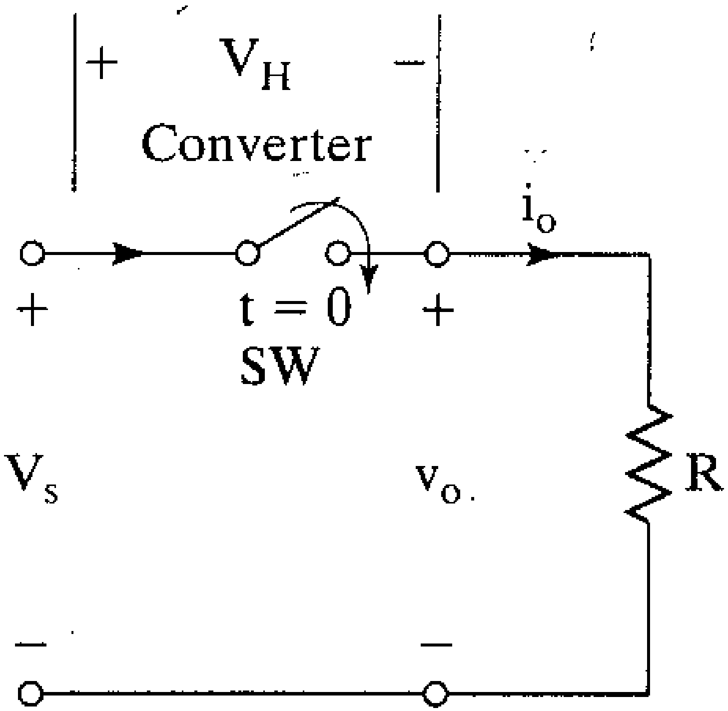
UNIT-III

DC-DC CONVERTERS (CHOPPERS)

DC-DC Converters

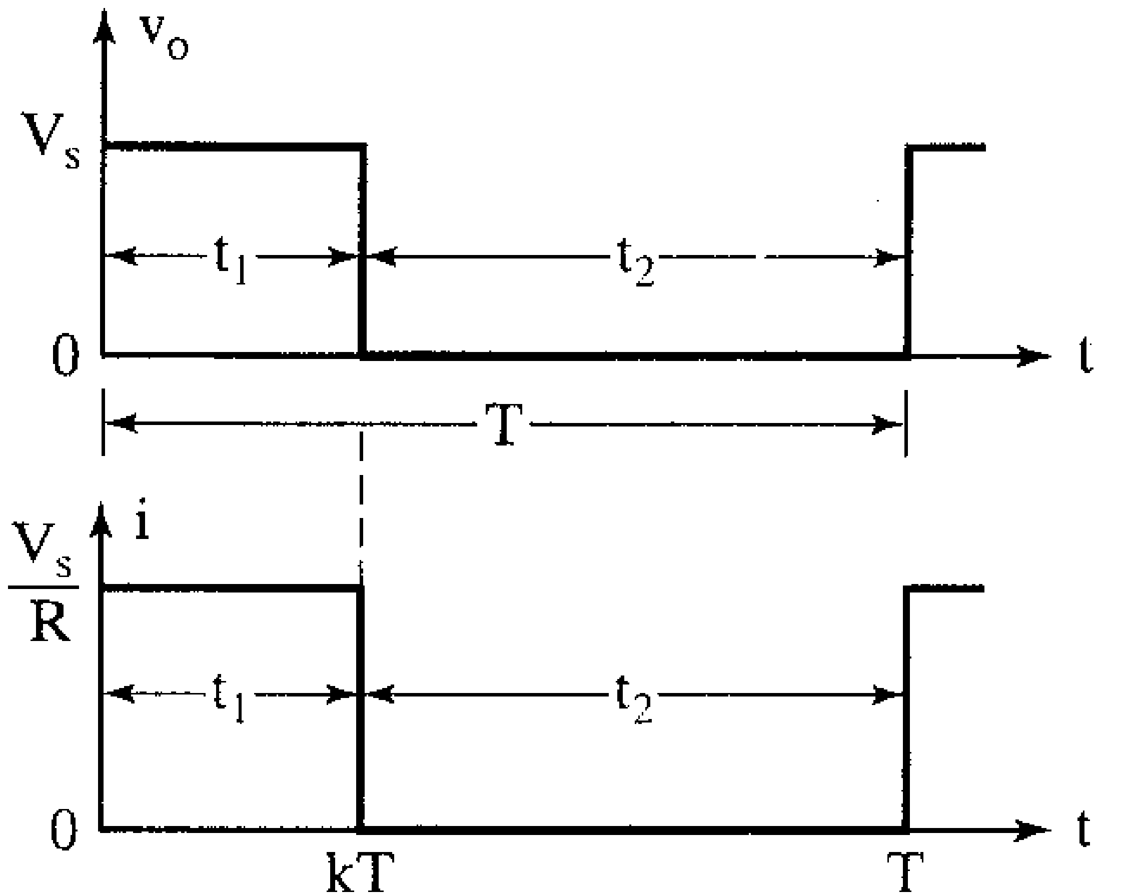
- Convert a fixed DC Source into a Variable DC Source
- DC equivalent to an AC transformer with variable turns ratio
- Step-up and Step-down versions
- Applications
  - Motor Control
  - Voltage Regulators

Step-down Operation

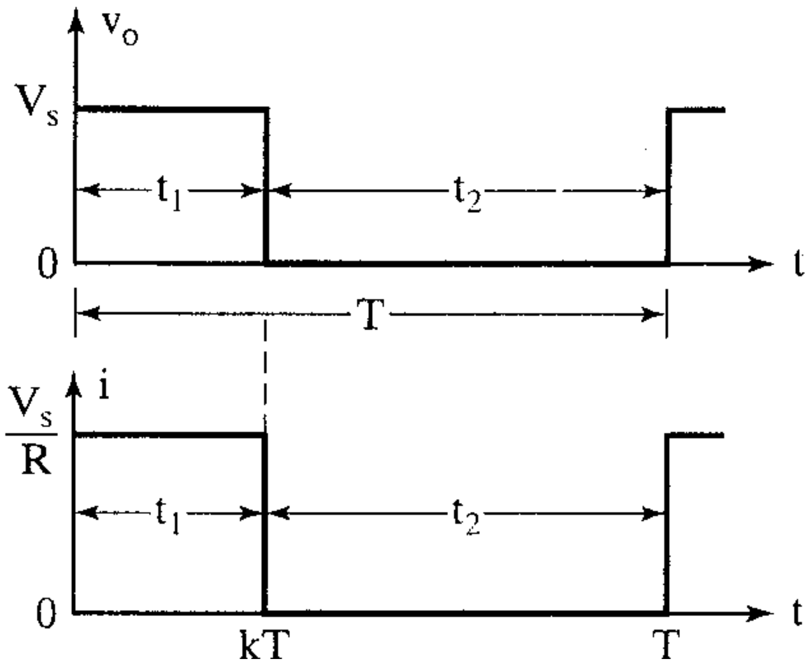


- Switch SW is known as a "Chopper"
- Use BJT, MOSFET, or IGBT
- Close for time t_1
  - V_S appears across R
- Open for time t_2
  - Voltage across R = 0
- Repeat
- Period $T = t_1 + t_2$

Waveforms for the Step-Down Converter



Average Value of the Output Voltage



$$V_a = \frac{1}{T}\int_0^{t_1} v_O dt$$

$$V_a = \frac{1}{T}\int_0^{t_1} V_S dt$$

$$V_a = \frac{t_1}{T} V_S = f t_1 V_S$$

$$\boxed{V_a = kV_S}$$

Average Value of the Load Current

$$\boxed{I_a = \frac{V_a}{R} = \frac{kV_S}{R}}$$

$$T = period$$

$$k = \frac{t_1}{T} = dutycycle$$

$$f = frequency$$

rms Value of the output voltage

$$V_O = \left(\frac{1}{T} \int_0^{kT} v_O^2 dt \right)^{\frac{1}{2}}$$

$$V_O = \left(\frac{1}{T} \int_0^{kT} V_S^2 dt \right)^{\frac{1}{2}}$$

$$V_O = \sqrt{k} V_S$$

If the converter is "lossless", $P_{in} = P_{out}$

$$P_{in} = \frac{1}{T}\int_0^{kT} v_O i\,dt$$

$$P_{in} = \frac{1}{T}\int_0^{kT} \frac{v_O^2}{R}dt$$

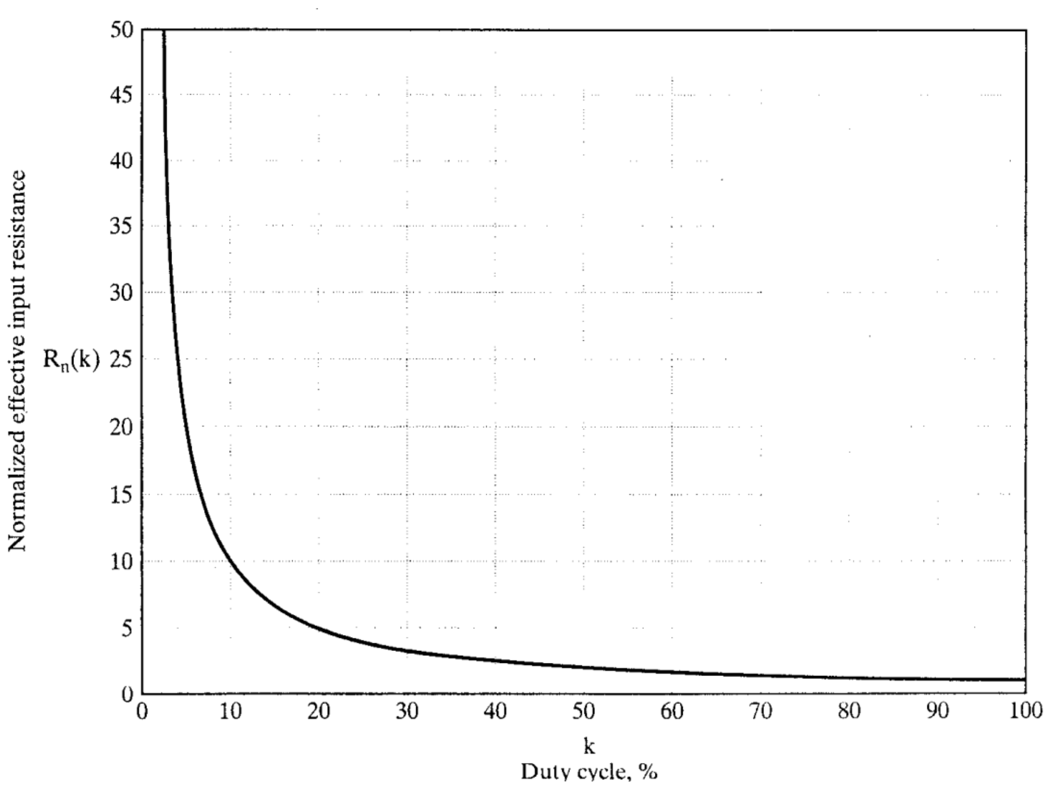
$$P_{in} = \frac{1}{T}\frac{V_S^2}{R}kT$$

$$P_{in} = k\frac{V_S^2}{R}$$

Effective Input Resistance seen by V_S

$$R_i = \frac{V_S}{I_a} = \frac{V_S}{k\,\frac{V_S}{R}}$$

$$R_i = \frac{R}{k}$$

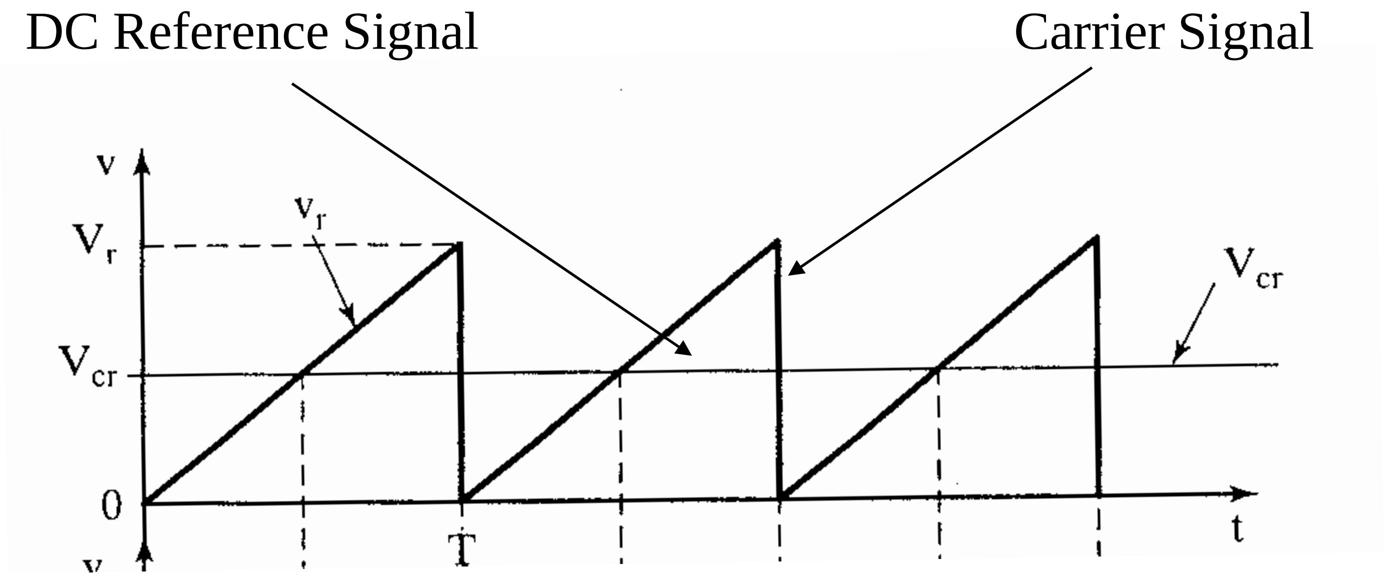


Modes of Operation

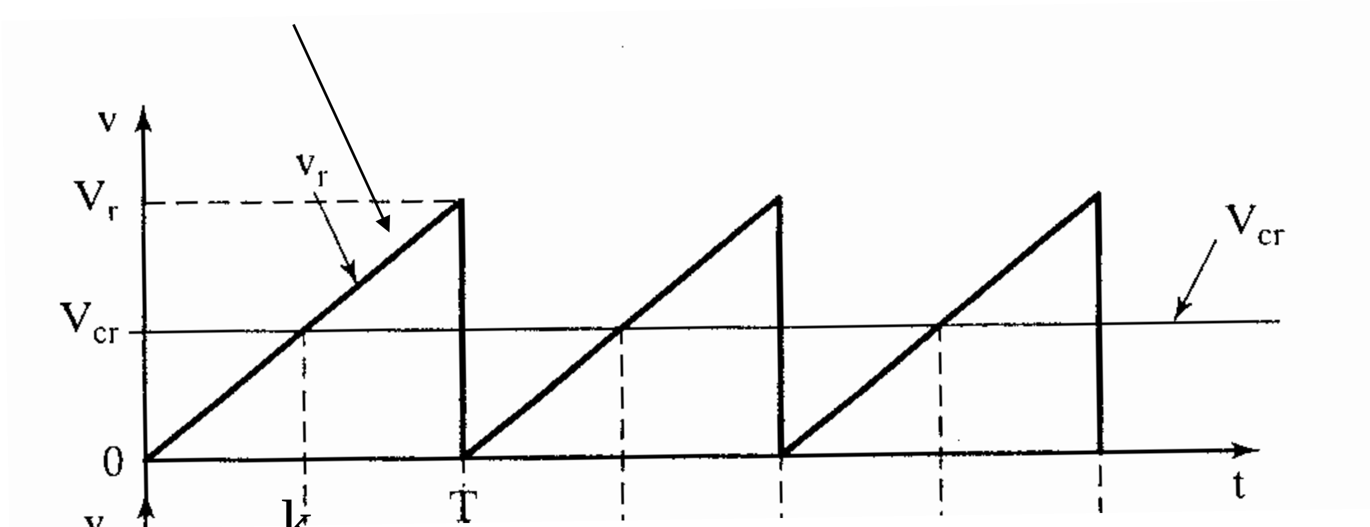
- Constant – frequency operation
  - Period T held constant, t_1 varied
  - Width of the pulse changes
  - “Pulse-width modulation”, PWM
- Variable -- frequency operation
  - Change the chopping frequency (period T)
  - Either t_1 or t_2 is kept constant
  - “Frequency modulation”

Generation of Duty Cycle

- Compare a dc reference signal with a saw-tooth carrier signal



$$v_r = \frac{V_r}{T} k$$



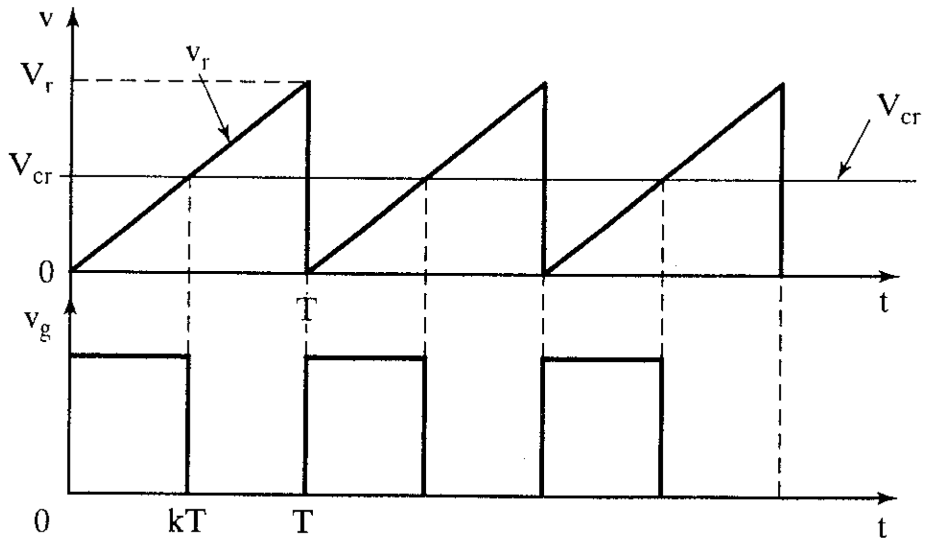
k
T

$$v_r = V_{cr} \; @ \; t = kT$$

$$V_{cr} = \frac{V_r}{T} kT$$

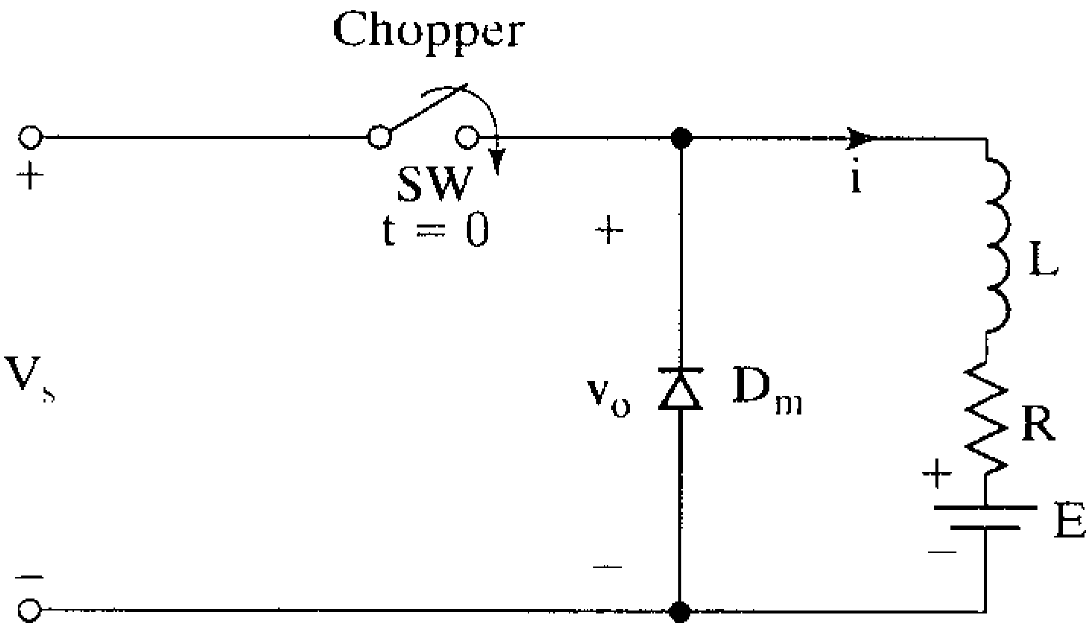
$$k = \frac{V_{cr}}{V_r} = M$$

To generate the gating signal

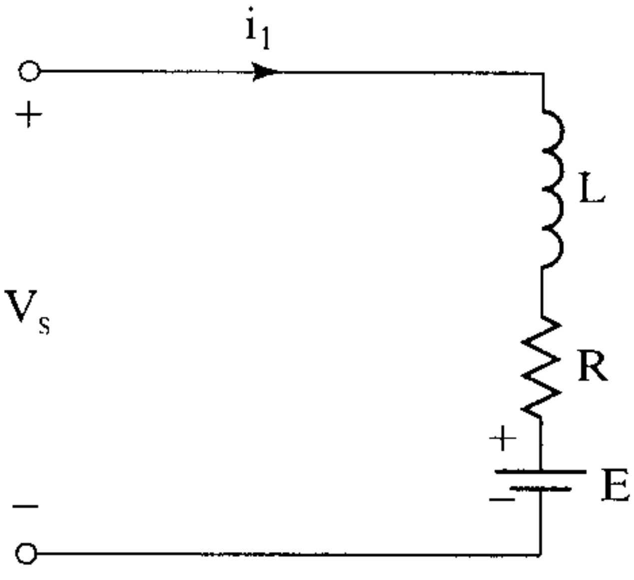


- Generate the triangular waveform of period T, v_r, and the dc carrier signal, v_{cr}
- Compare to generate the difference v_c - v_{cr}
- Apply to a "hard limiter" to "square off"

Step-Down Converter with RL Load



Mode 1: Switch Closed



Mode 1

$$V_S = R i_1 + L\frac{di_1}{dt} + E$$

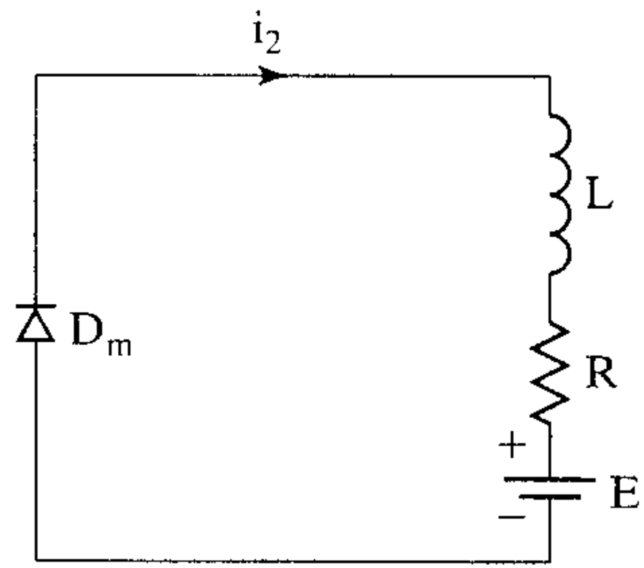
$$i_1(t)_{(t=0)} = I_1$$

$$i_1(t) = I_1 e^{-\frac{R}{L}t} + \frac{V_S - E}{R}\left(1 - e^{-\frac{R}{L}t}\right) \qquad t \le t_1(kT)$$

$$t = t_1 = kT$$

$$i_1(kT) = I_2$$

Mode 2: Switch Open



Mode 2

$$0 = R i_2 + L \frac{di_2}{dt} + E$$

$$i_2(t = 0) = I_2$$

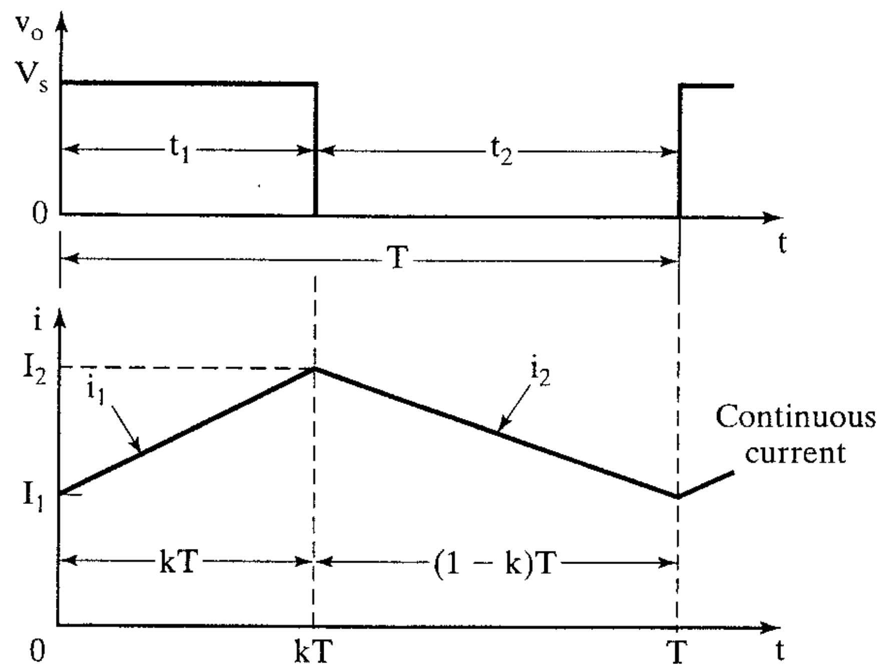
$$i_2(t) = I_2 e^{-\frac{R}{L}t} - \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$0 \le t \le t_2 = (1-k)T$$

$$@t = t_2 = (1-K)T$$

$$i_2(t_2) = I_3 = I_1$$

Current for "Continuous" Mode



$$I_1 = \frac{V_s}{R}\left(\frac{e^{kz} - 1}{e^{z} - 1}\right) - \frac{E}{R}$$

$$I_2 = \frac{V_s}{R}\left(\frac{e^{-kz} - 1}{e^{z} - 1}\right) - \frac{E}{R}$$

$$z = \frac{TR}{L}$$

$$\Delta I = \frac{V_s}{R}\left[\frac{1 - e^{-kz} + e^{-z} - e^{-(1-k)z}}{1 - e^{-z}}\right]$$

$$\Delta I_{\max} \cong \frac{V_s}{4fL}$$

For Continuous Current

$$I_1 \geq 0$$

$$\left[\frac{e^{kz} - 1}{e^{z} - 1} - \frac{E}{V_S}\right] \geq 0$$

Define the load emf ratio

$$x = \frac{E}{V_s}$$

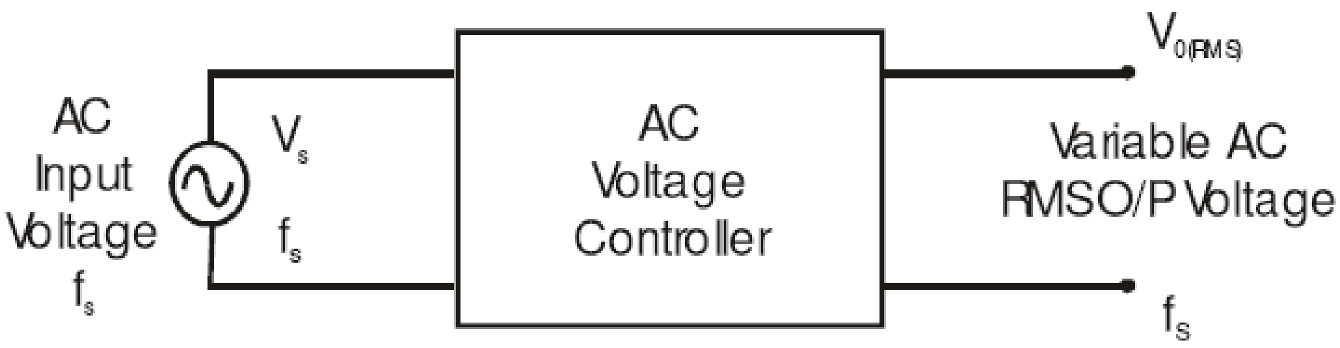
$$x = \frac{E}{V_s} \leq \frac{e^{kz} - 1}{e^{z} - 1}$$

UNIT-IV

AC-AC CONVERTERS (AC VOLTAGE CONTROLLERS) & FREQUENCY CHANGERS (CYCLO-CONVERTERS)

Ac Voltage controller circuits (RMS voltage controllers)

An ac voltage controller is a type of thyristor power converter which is used to convert a fixed voltage, fixed frequency ac input supply to obtain a variable voltage ac output



Applications Of Ac Voltage Controllers

- Lighting / Illumination control in ac power circuits.
- Induction heating.
- Industrial heating & Domestic heating.
- Transformer tap changing (on load transformer tap changing).
- Speed control of induction motors C magnet controls.

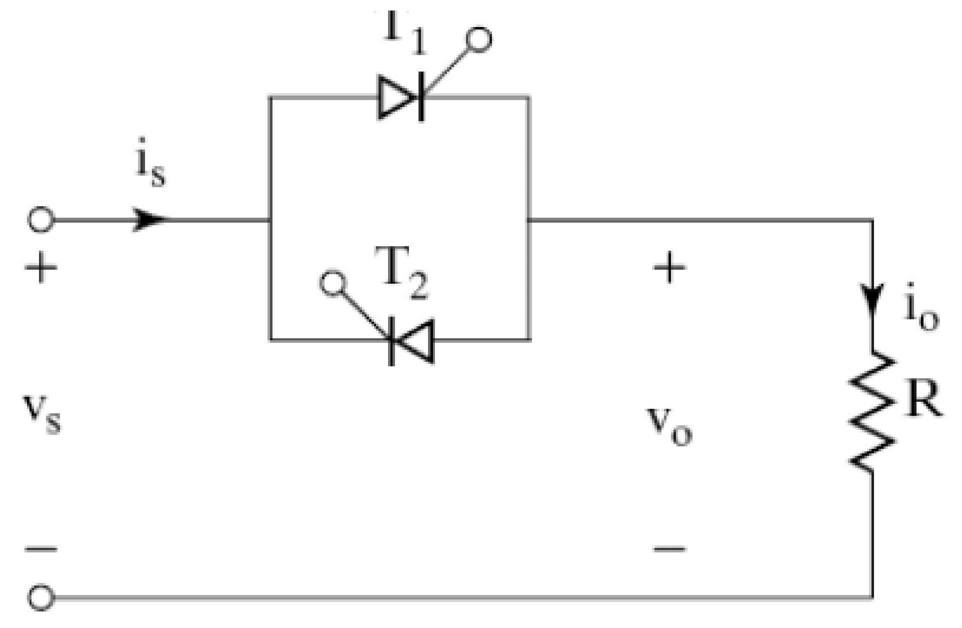
Type Of Ac Voltage Controllers

- Single phase half wave ac voltage controller (Uni-directional controller).
- Single phase full wave ac voltage controller (Bi-directional controller).
- Three phase half wave ac voltage controller (Uni-directional controller).
- Three phase full wave ac voltage controller (Bi-directional Controller)

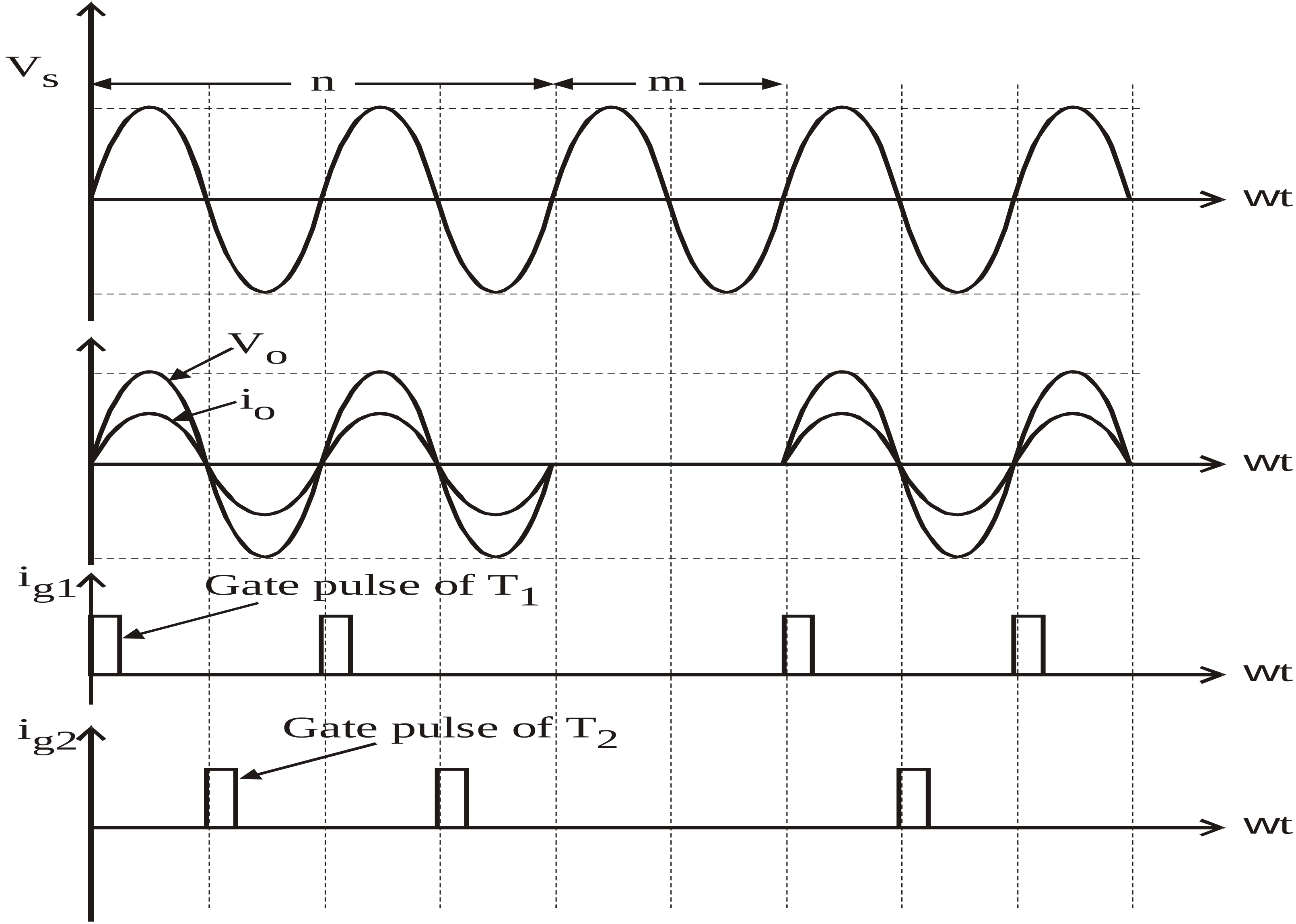
A.C voltage control technique

- On-Off control.
- Phase control.

Principle of ON-OFF Control Technique



Single phase full wave AC voltage controller circuit


Vs
n
m
wt
Vo
io
wt
ig1
Gate pulse of T1
wt
ig2
Gate pulse of T2
wt

For a sine wave i/p ac supply

$$v_s = V_m \sin \omega t = \sqrt{2} V_S \sin \omega t$$

$$V_S = \frac{V_m}{\sqrt{2}} = \text{RMS value of i/p ac supply}$$

$$t_{ON} = n \times T, \qquad t_{OFF} = m \times T$$

n = no. of i/p cycles during on time t_{ON}

m = no. of i/p cycles during off time t_{OFF}

$$T = \frac{1}{f} = \text{input cycle time (time period)}$$

$$f = \text{input supply frequency.}$$

$$t_{ON} = \text{controller on time} = n \times T$$

$$t_{OFF} = \text{controller off time} = m \times T$$

$$T_O = \text{Output time period}$$

$$= \left(t_{ON} + t_{OFF} \right) = \left(nT + mT \right)$$

For on-off control method

$$V_{O(RMS)} = V_{i(RMS)}\sqrt{\frac{t_{ON}}{T_O}} = V_S\sqrt{\frac{t_{ON}}{T_O}}$$

Expression For The RMS Value Of Output Voltage, For ON-OFF Control Method

$$V_{O(RMS)} = \sqrt{\frac{1}{\omega T_O} \int_{\omega t=0}^{\omega t_{ON}} V_m{}^2 Sin^2 \omega t . d(\omega t)}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m{}^2}{\omega T_O} \int_{0}^{\omega t_{ON}} Sin^2 \omega t . d(\omega t)}$$

Substituting for $Sin^2\theta = \dfrac{1 - Cos 2\theta}{2}$

$$V_{O(RMS)} = \sqrt{\frac{V_m{}^2}{\omega T_O} \int_{0}^{\omega t_{ON}} \left[\frac{1 - Cos 2\omega t}{2}\right] d(\omega t)}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^{\ 2}}{2\omega T_O}\left[\int_0^{\omega t_{ON}} d\left(\omega t\right) - \int_0^{\omega t_{ON}} Cos2\omega t.d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^{\ 2}}{2\omega T_O}\left[\left(\omega t\right)\Big/_{0}^{\omega t_{ON}} - \frac{Sin2\omega t}{2}\Big/_{0}^{\omega t_{ON}}\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^{\ 2}}{2\omega T_O}\left[\left(\omega t_{ON} - 0\right) - \frac{\sin 2\omega t_{ON} - \sin 0}{2}\right]}$$

Here in this case,

t_{ON} = An integral number of input cycles; Hence

$$t_{ON} = T, 2T, 3T, 4T, 5T, \ldots\ldots \quad \&$$

$$\omega t_{ON} = 2\pi, 4\pi, 6\pi, 8\pi, 10\pi, \ldots\ldots$$

$$V_{O(RMS)} = \sqrt{\frac{V_m{}^2 \cancel{\omega} t_{ON}}{2\cancel{\omega} T_O}} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{t_{ON}}{T_O}}$$

$$V_{O(RMS)} = V_{i(RMS)}\sqrt{\frac{t_{ON}}{T_O}} = V_S\sqrt{\frac{t_{ON}}{T_O}}$$

Where $V_{i(RMS)} = \frac{V_m}{\sqrt{2}} = V_S$

$$\frac{t_{ON}}{T_O} = \frac{t_{ON}}{t_{ON} + t_{OFF}}$$

$$= \frac{nT}{nT + mT} = \frac{n}{(n+m)} = k$$

$$V_{O(RMS)} = V_S \sqrt{\frac{n}{(m+n)}} = V_S \sqrt{k}$$

RMS Out put voltage

$$V_{O(RMS)} = \left[\frac{n}{2\pi(n+m)} \int_0^{2\pi} V_m{}^2 \sin^2 \omega t . d(\omega t) \right]^{1/2}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{n}{(m+n)}} = V_{i(RMS)} \sqrt{k} = V_S \sqrt{k}$$

$$V_{O(RMS)} = V_{i(RMS)} \sqrt{k} = V_S \sqrt{k}$$

Duty cycle

$$k = \frac{t_{ON}}{T_O} = \frac{t_{ON}}{\left(t_{ON} + t_{OFF}\right)} = \frac{nT}{\left(m+n\right)T}$$

$$k = \frac{n}{\left(m+n\right)}$$

RMS Load Current

$$I_{O(RMS)} = \frac{V_{O(RMS)}}{Z} = \frac{V_{O(RMS)}}{R_L}$$

Output AC (Load) Power

$$P_O = I^2_{O(RMS)} \times R_L$$

Input Power factor

$$PF = \frac{P_O}{VA} = \frac{\text{output load power}}{\text{input supply volt amperes}} = \frac{P_O}{V_S I_S}$$

$$PF = \frac{I^2_{O(RMS)} \times R_L}{V_{i(RMS)} \times I_{in(RMS)}}$$

$I_S = I_{in(RMS)}$ = RMS input supply current.

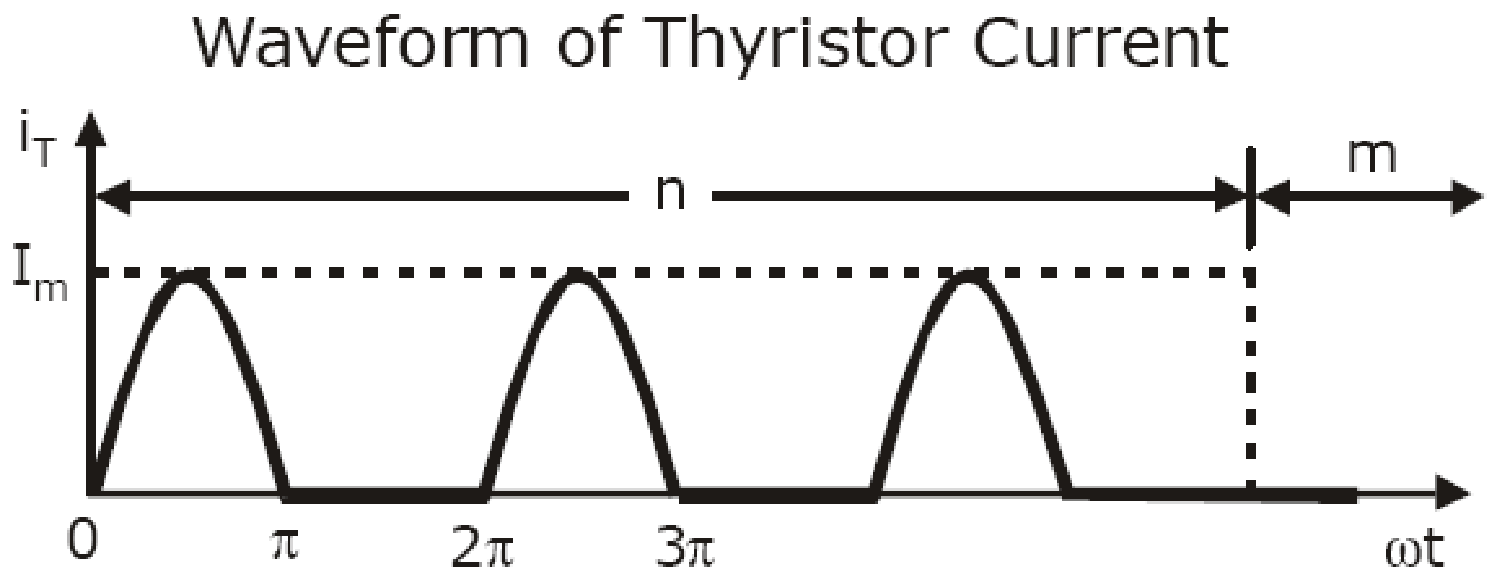
= same as RMS load current $I_{O(RMS)}$

$$PF = \frac{I^2_{O(RMS)} \times R_L}{V_{i(RMS)} \times I_{in(RMS)}} = \frac{V_{O(RMS)}}{V_{i(RMS)}}$$

$$= \frac{V_{i(RMS)}\sqrt{k}}{V_{i(RMS)}} = \sqrt{k}$$

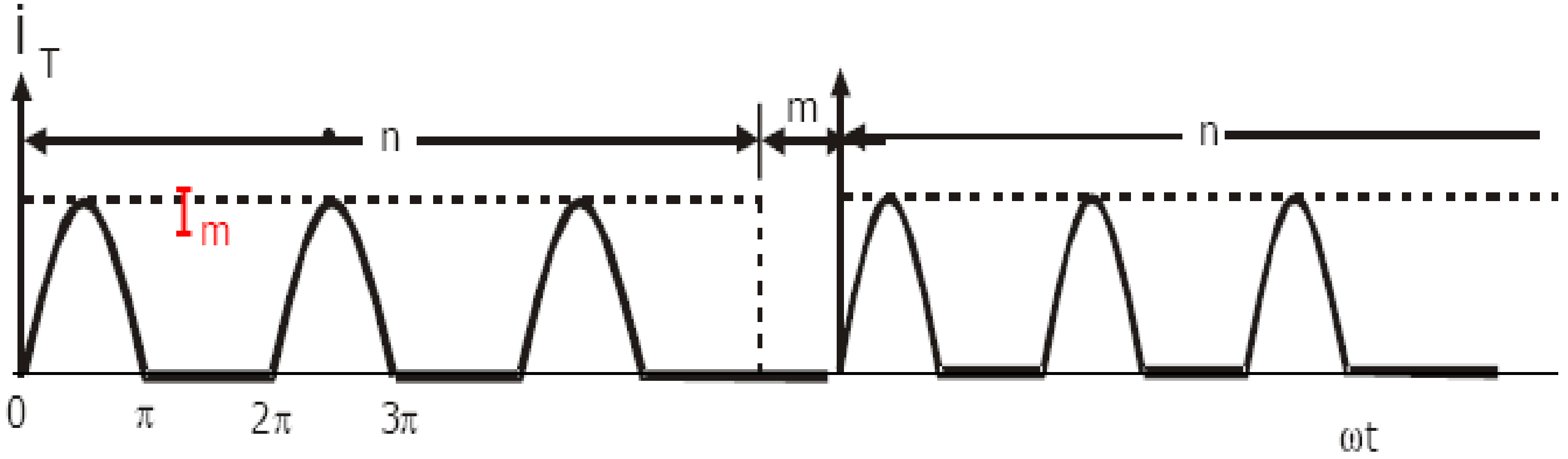
$$PF = \sqrt{k} = \sqrt{\frac{n}{m+n}}$$

The Average Current Of Thyristor



$$I_m = \frac{V_m}{R_L} = \text{max. or peak thyristor current}$$

Waveform of Thyristor current



$$I_{T(Avg)} = \frac{n}{2\pi(m+n)} \int_0^{\pi} I_m \sin \omega t . d(\omega t)$$

$$I_{T(Avg)} = \left(\frac{I_m}{\pi} \right) \frac{n}{(n+m)}$$

$$I_{T(Avg)} = \left(\frac{I_m}{\pi} \right) k$$

RMS Thyristor Current

$$I_{T(RMS)} = \left[\frac{n}{2\pi(n+m)} \int_0^{\pi} I_m^2 \sin^2 \omega t . d(\omega t) \right]^{1/2}$$

$$I_{T(RMS)} = \frac{I_m}{2} \sqrt{\frac{n}{(m+n)}} = \frac{I_m}{2} \sqrt{k}$$

$$I_{T(RMS)} = \frac{I_m}{2} \sqrt{k}$$

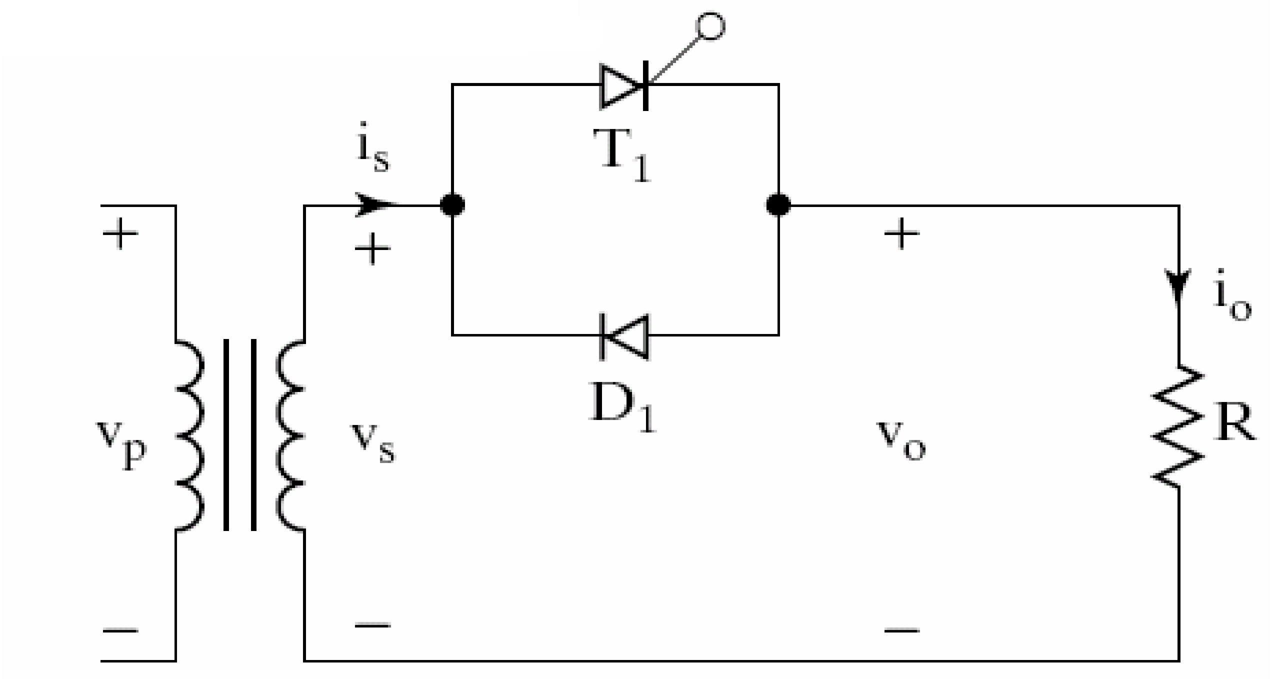
Principle Of AC Phase Control

And

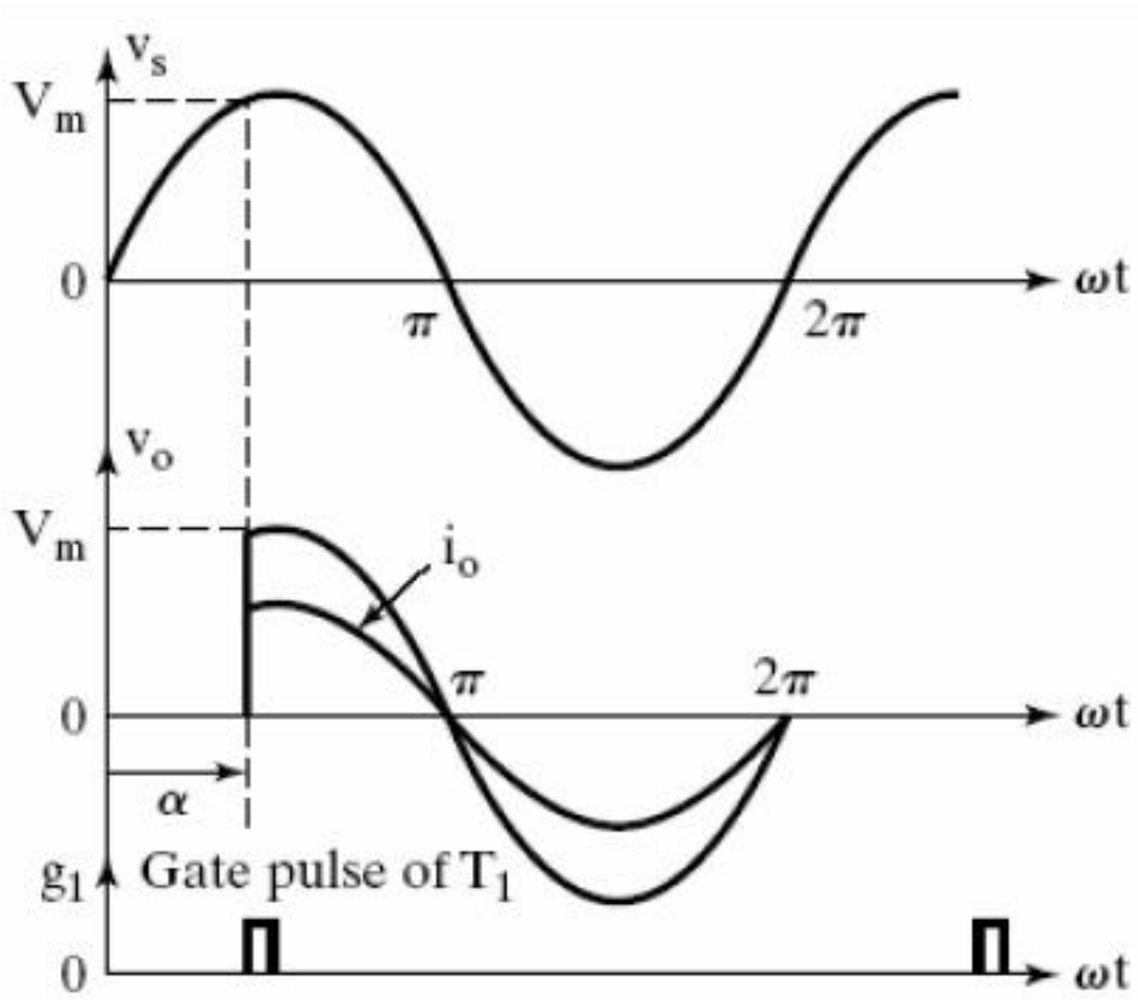
Operation of single Phase half-Wave

A.C Phase controller

Principle Of AC Phase Control



Single phase Half-wave AC phase controller
(Unidirectional Controller)



v_s
V_m
0
ωt
π
2π
v_o
V_m
i_o
π
2π
0
ωt
α
g_1
Gate pulse of T_1
0
ωt

Equations

- **Input AC Supply Voltage**

$$v_s = V_m \sin \omega t$$

$$V_S = V_{in(RMS)} = \frac{V_m}{\sqrt{2}} = \text{RMS value of}$$

input supply voltage

Output Load Voltage

$$v_o = v_L = 0;$$

$$\text{for } \omega t = 0 \text{ to } \alpha$$

$$v_o = v_L = V_m \sin \omega t;$$

$$\text{for } \omega t = \alpha \text{ to } 2\pi$$

Out Put Load Current

$$i_o = i_L = \frac{v_o}{R_L} = \frac{V_m \sin \omega t}{R_L} = I_m \sin \omega t \;\; ;$$

$$\text{for } \omega t = \alpha \text{ to } 2\pi$$

$$i_o = i_L = 0;$$

$$\text{for } \omega t = 0 \text{ to } \alpha$$

Expression For RMS Out put Load Voltage

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_{\alpha}^{2\pi} V_m{}^2 \sin^2 \omega t . d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m{}^2}{2\pi}\left[\int_{\alpha}^{2\pi} \left(\frac{1-\cos 2\omega t}{2}\right) . d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m{}^2}{4\pi}\left[\int_{\alpha}^{2\pi}(1-\cos 2\omega t).d(\omega t)\right]}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left[\int_{\alpha}^{2\pi} d(\omega t) - \int_{\alpha}^{2\pi}\cos 2\omega t.d\,\omega t\right]}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left[(\omega t)\Big/_{\alpha}^{2\pi} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{2\pi}\right]}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}} \sqrt{(2\pi - \alpha) - \left(\frac{\sin 2\omega t}{2}\right) \Big/_{\alpha}^{2\pi}}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}} \sqrt{(2\pi - \alpha) - \left\{\frac{\sin 4\pi}{2} - \frac{\sin 2\alpha}{2}\right\}}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}} \sqrt{(2\pi - \alpha) + \frac{\sin 2\alpha}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}\sqrt{2\pi}}\sqrt{(2\pi - \alpha) + \frac{\sin 2\alpha}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[(2\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]}$$

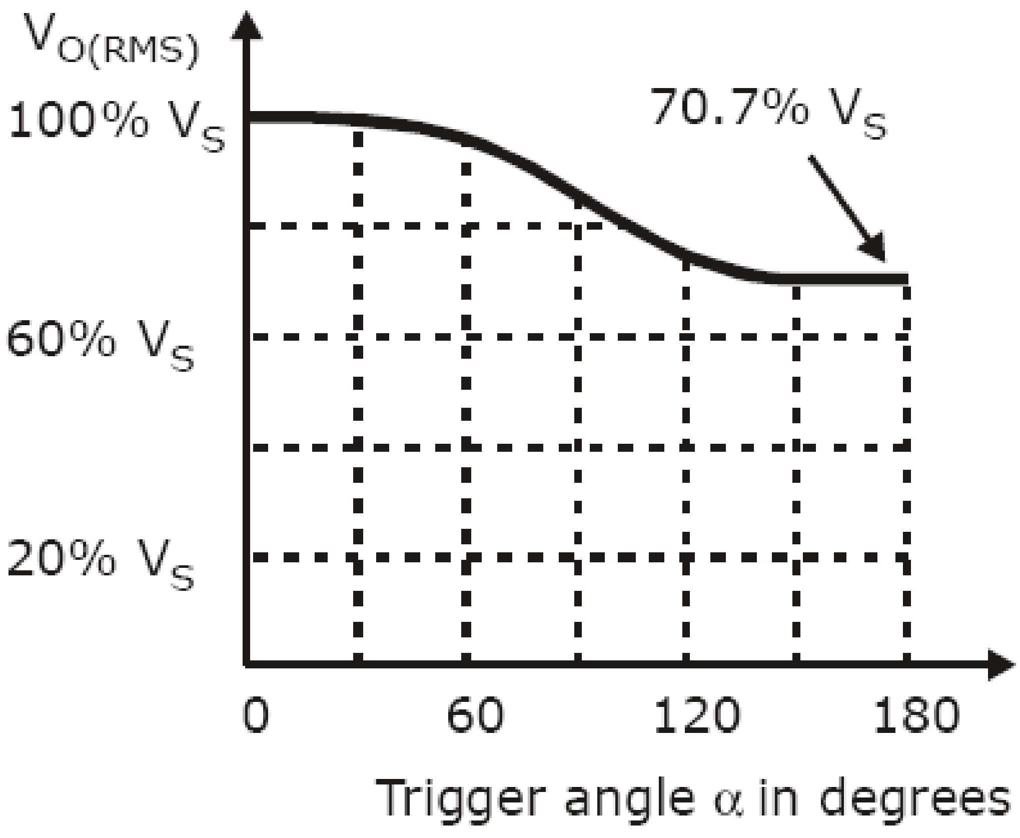
$$V_{O(RMS)} = V_{i(RMS)}\sqrt{\frac{1}{2\pi}\left[(2\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]}$$

$$V_{O(RMS)} = V_S \sqrt{\frac{1}{2\pi}\left[(2\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$$

where $V_{i(RMS)} = V_S = \dfrac{V_m}{\sqrt{2}} =$

RMS value of input supply voltage

Control Characteristics



Average Value of Out put Voltage

$$V_{O(dc)} = \frac{1}{2\pi} \int_{\alpha}^{2\pi} V_m \sin \omega t . d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi} \int_{\alpha}^{2\pi} \sin \omega t . d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos\omega t \Big/_{\alpha}^{2\pi} \right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos 2\pi + \cos\alpha \right] \ ; \ \cos 2\pi = 1$$

$$V_{dc} = \frac{V_m}{2\pi}\left[\cos\alpha - 1 \right] \ ; \ V_m = \sqrt{2} V_S$$

Hence,

$$V_{dc} = \frac{\sqrt{2}V_S}{2\pi}(\cos\alpha - 1)$$

When α is varied from 0 to π

V_{dc} varies from 0 to $\dfrac{-V_m}{\pi}$

Disadvantages

- Output load voltage has a DC component as the two halves are not symmetrical with respect to '0' level.
- Limited range of RMS output voltage control from 100% of V_S to 70.7% of V_S, when we vary the trigger angle from zero to 180 degrees.

Single Phase Full Wave Ac Voltage Controller (Bidirectional Controller) With R-Load

Single Phase Full Wave Ac Voltage Controller With R-Load

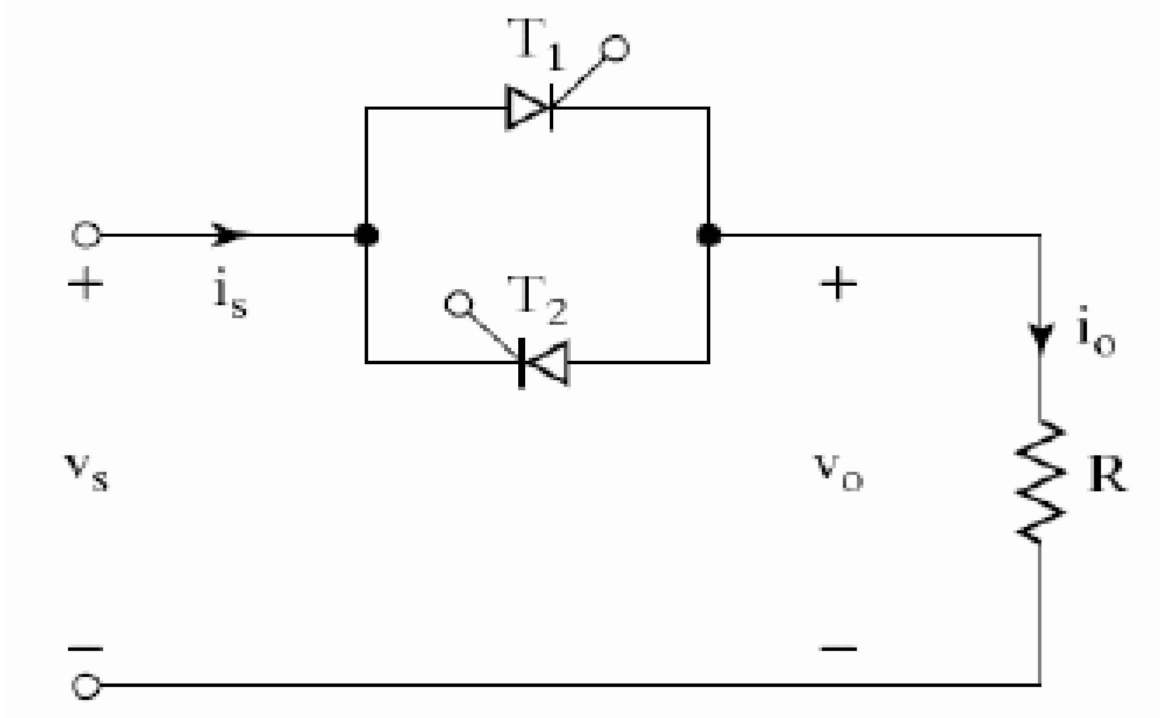
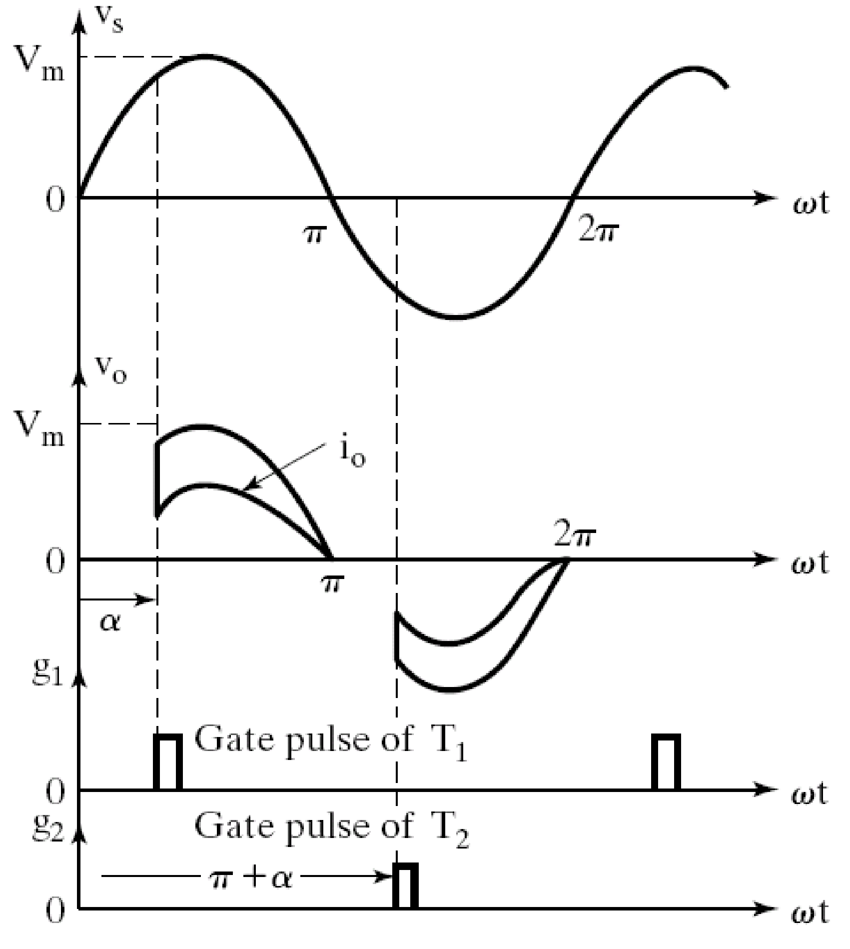


Fig.: Single phase full wave ac voltage controller (Bi-directional Controller) using SCR

Waveforms of single phase full wave ac voltage controller



Expression for RMS output voltage

$$V^2_{L(RMS)} = \frac{1}{\pi}\int_0^{\pi} V_m{}^2 \sin^2 \omega t.d\omega t$$

$$V^2_{L(RMS)} = \frac{1}{2\pi}\int_0^{2\pi} v_L{}^2 .d\left(\omega t\right)$$

$$V^2_{L(RMS)} = \frac{1}{2\pi}\left[\int_{\alpha}^{\pi}\left(V_m \sin\omega t\right)^2 d\left(\omega t\right) + \int_{\pi+\alpha}^{2\pi}\left(V_m \sin\omega t\right)^2 d\left(\omega t\right)\right]$$

Contd…

$$=\frac{1}{2\pi}\left[V_m{}^2\int_{\alpha}^{\pi}\sin^2\omega t.d\left(\omega t\right)+V_m{}^2\int_{\pi+\alpha}^{2\pi}\sin^2\omega t.d\left(\omega t\right)\right]$$

$$=\frac{1}{2\pi}\left[V_m{}^2\int_{\alpha}^{\pi}\sin^2\omega t.d\left(\omega t\right)+V_m{}^2\int_{\pi+\alpha}^{2\pi}\sin^2\omega t.d\left(\omega t\right)\right]$$

$$=\frac{V_m{}^2}{2\pi}\left[\int_{\alpha}^{\pi}\frac{1-\cos 2\omega t}{2}d\left(\omega t\right)+\int_{\pi+\alpha}^{2\pi}\frac{1-\cos 2\omega t}{2}d\left(\omega t\right)\right]$$

$$=\frac{V_m{}^2}{2\pi\times 2}\left[\int_{\alpha}^{\pi}d\left(\omega t\right)-\int_{\alpha}^{\pi}\cos 2\omega t.d\left(\omega t\right)+\int_{\pi+\alpha}^{2\pi}d\left(\omega t\right)-\int_{\pi+\alpha}^{2\pi}\cos 2\omega t.d\left(\omega t\right)\right]$$

$$=\frac{V_m{}^2}{4\pi}\left[\left(\omega t\right)\Big/_{\alpha}^{\pi}+\left(\omega t\right)\Big/_{\pi+\alpha}^{2\pi}-\left[\frac{\sin 2\omega t}{2}\right]_{\alpha}^{\pi}-\left[\frac{\sin 2\omega t}{2}\right]_{\pi+\alpha}^{2\pi}\right]$$

$$=\frac{V_m{}^2}{4\pi}\left[\left(\pi-\alpha\right)+\left(\pi-\alpha\right)-\frac{1}{2}\left(\sin 2\pi-\sin 2\alpha\right)-\frac{1}{2}\left(\sin 4\pi-\sin 2\left(\pi+\alpha\right)\right)\right]$$

$$=\frac{V_m{}^2}{4\pi}\left[2\left(\pi-\alpha\right)-\frac{1}{2}\left(0-\sin 2\alpha\right)-\frac{1}{2}\left(0-\sin 2\left(\pi+\alpha\right)\right)\right]$$

$$=\frac{V_m{}^2}{4\pi}\left[2\left(\pi-\alpha\right)+\frac{\sin 2\alpha}{2}+\frac{\sin 2\left(\pi+\alpha\right)}{2}\right]$$

$$= \frac{V_m{}^2}{4\pi}\left[2(\pi-\alpha)+\frac{\sin 2\alpha}{2}+\frac{\sin(2\pi+2\alpha)}{2}\right]$$

$$= \frac{V_m{}^2}{4\pi}\left[2(\pi-\alpha)+\frac{\sin 2\alpha}{2}+\frac{1}{2}(\sin 2\pi.\cos 2\alpha+\cos 2\pi.\sin 2\alpha)\right]$$

$$\sin 2\pi = 0 \quad \& \quad \cos 2\pi = 1$$

Therefore,

$$V^2_{L(RMS)} = \frac{V_m{}^2}{4\pi}\left[2(\pi-\alpha)+\frac{\sin 2\alpha}{2}+\frac{\sin 2\alpha}{2}\right]$$

$$V^2{}_{L(RMS)} = \frac{V_m{}^2}{4\pi}\left[(2\pi-2\alpha)+\sin 2\alpha\right]$$

- Taking the square root, we get

$$V_{L(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left[\left(2\pi - 2\alpha\right) + \sin 2\alpha\right]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}\sqrt{2\pi}}\sqrt{\left[\left(2\pi - 2\alpha\right) + \sin 2\alpha\right]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[\left(2\pi - 2\alpha\right) + \sin 2\alpha\right]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[2\left\{\left(\pi - \alpha\right) + \frac{\sin 2\alpha}{2}\right\}\right]}$$

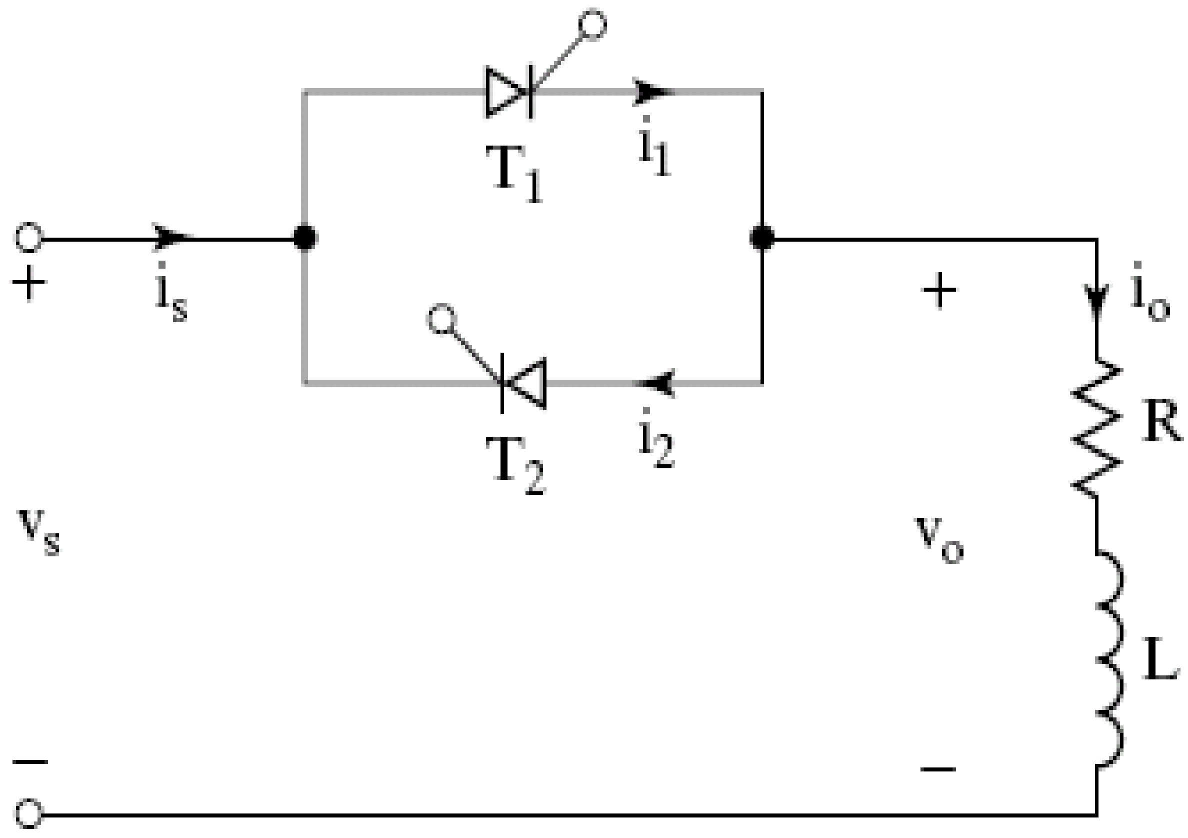
$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi}\left[\left(\pi - \alpha\right) + \frac{\sin 2\alpha}{2}\right]}$$

$$V_{L(RMS)} = V_{i(RMS)} \sqrt{\frac{1}{\pi}\left[\left(\pi - \alpha\right) + \frac{\sin 2\alpha}{2}\right]}$$

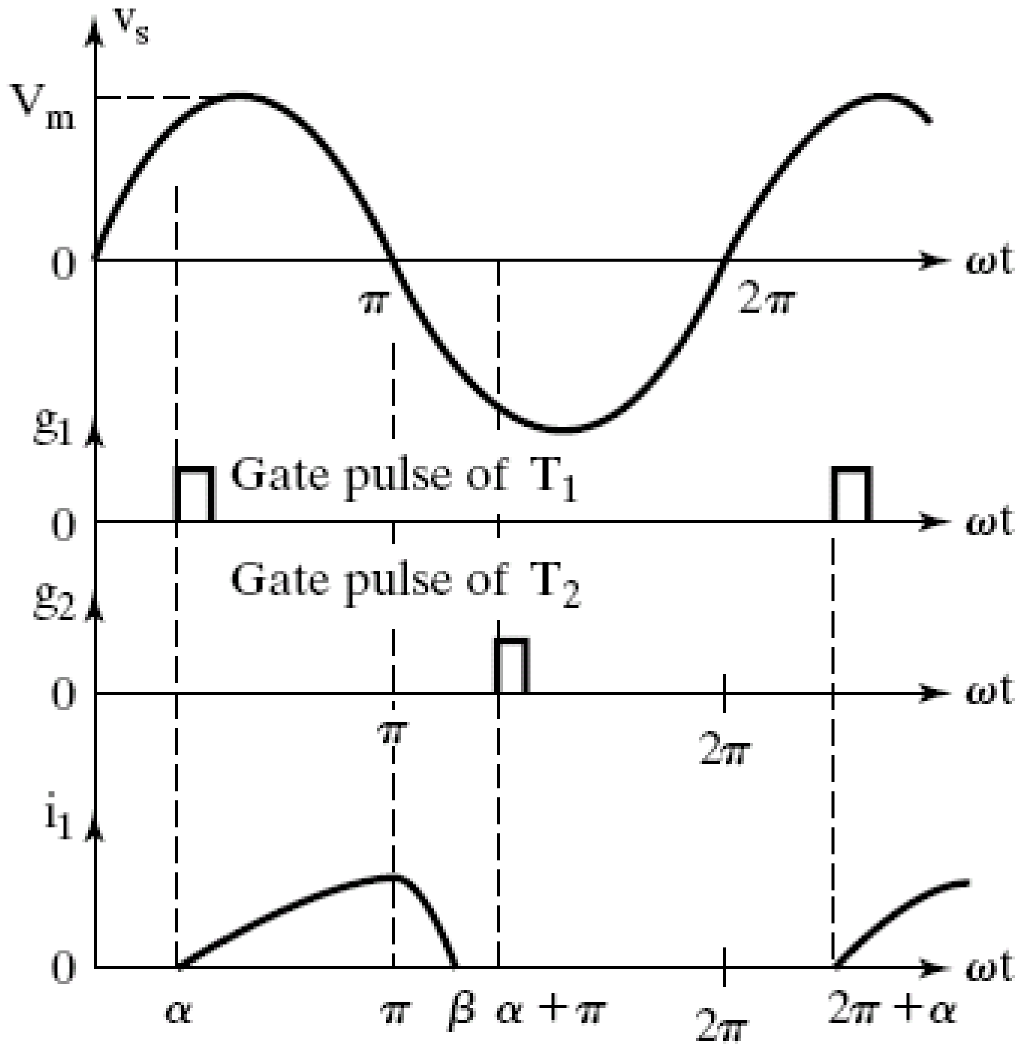
$$V_{L(RMS)} = V_S \sqrt{\frac{1}{\pi}\left[\left(\pi - \alpha\right) + \frac{\sin 2\alpha}{2}\right]}$$

Single Phase Full Wave Ac Voltage Controller (Bidirectional Controller) With R-L Load

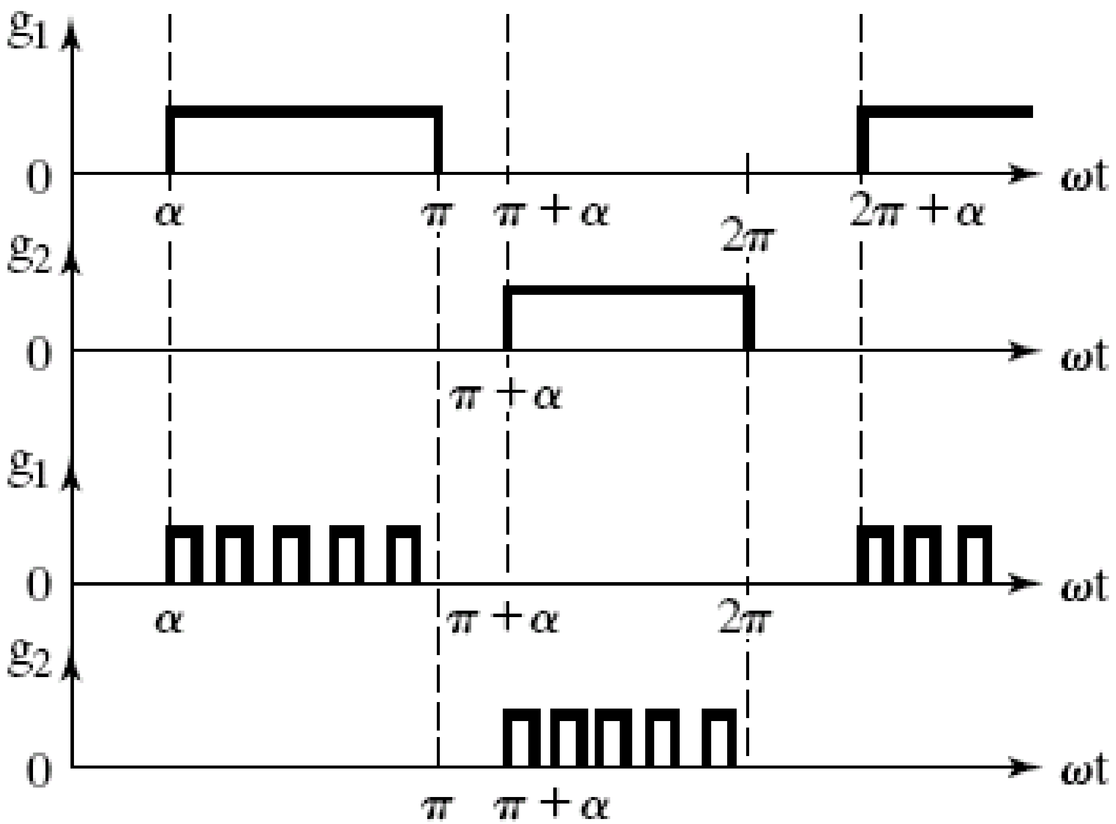
Single Phase Full Wave Ac Voltage Controller (Bidirectional Controller) With R-L Load



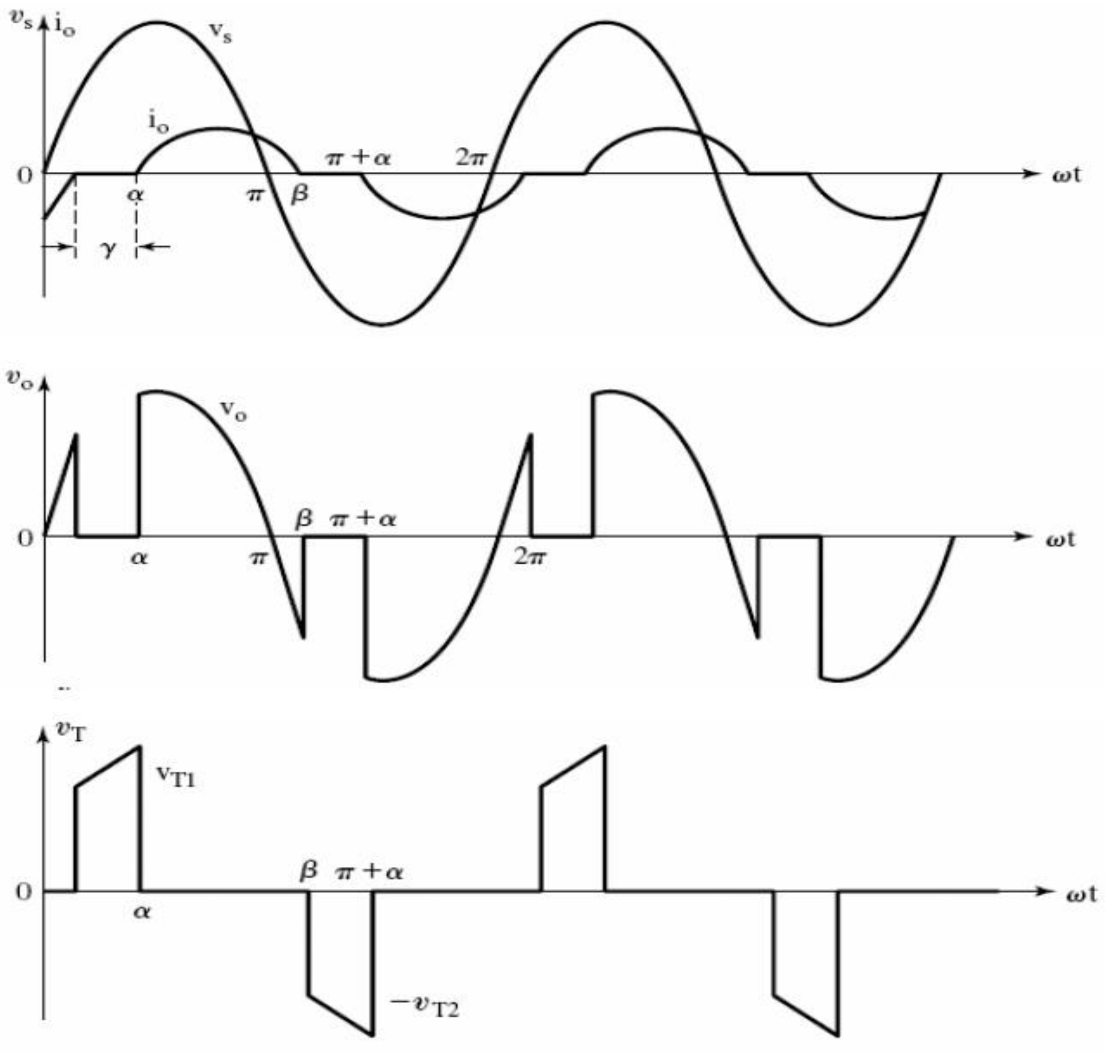
Input supply voltage & Thyristor current waveforms



Gating Signals



Waveforms For RL load for $\alpha > \phi$ and for Discontinuous Conduction



Expression for the inductive load current of a single phase full wave ac voltage controller with RL load

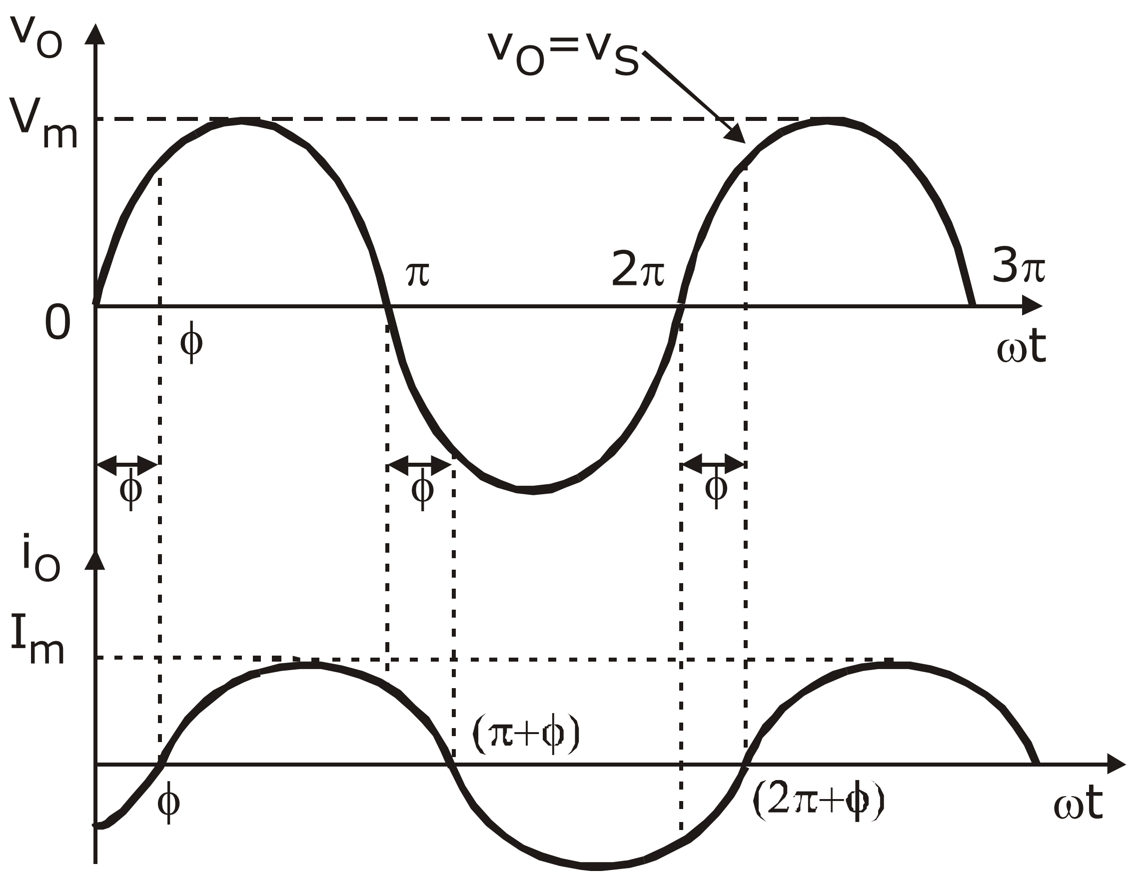
$$i_O = \frac{V_m}{Z}\left[\sin(\omega t-\phi)-\sin(\alpha-\phi)e^{\frac{-R}{\omega L}(\omega t-\alpha)}\right]$$

Where $Z=\sqrt{R^2+(\omega L)^2}$

$$\phi=\tan^{-1}\left(\frac{\omega L}{R}\right)$$

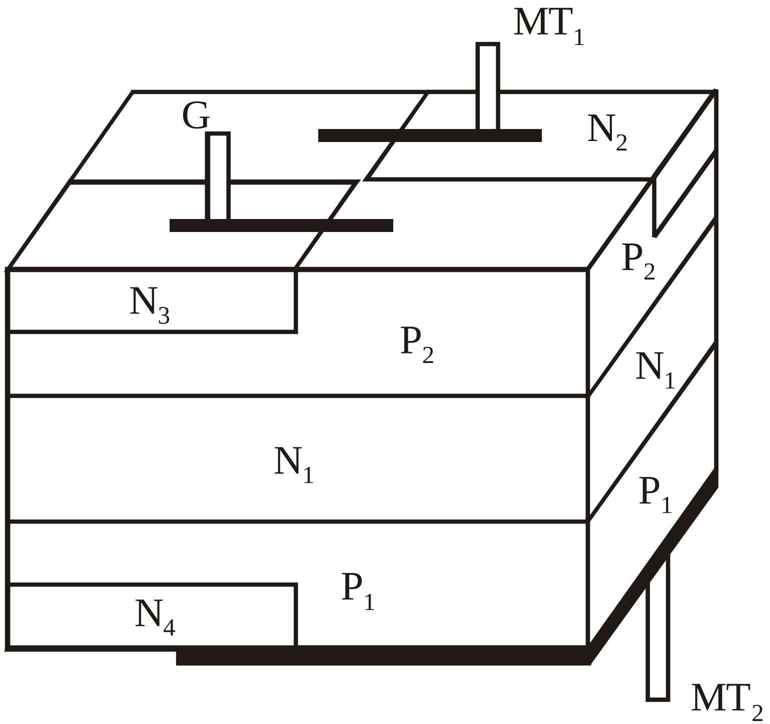
= Load impedance angle (power factor angle of load).

Output voltage and output current waveforms for a single phase full wave ac voltage controller with RL load for $\alpha \leq \phi$



TRIAC and Its Modes of Operation

TRIAC



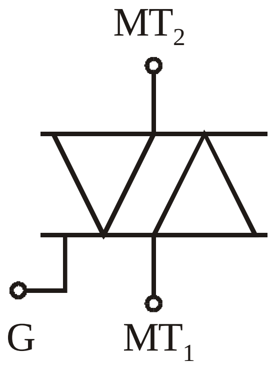
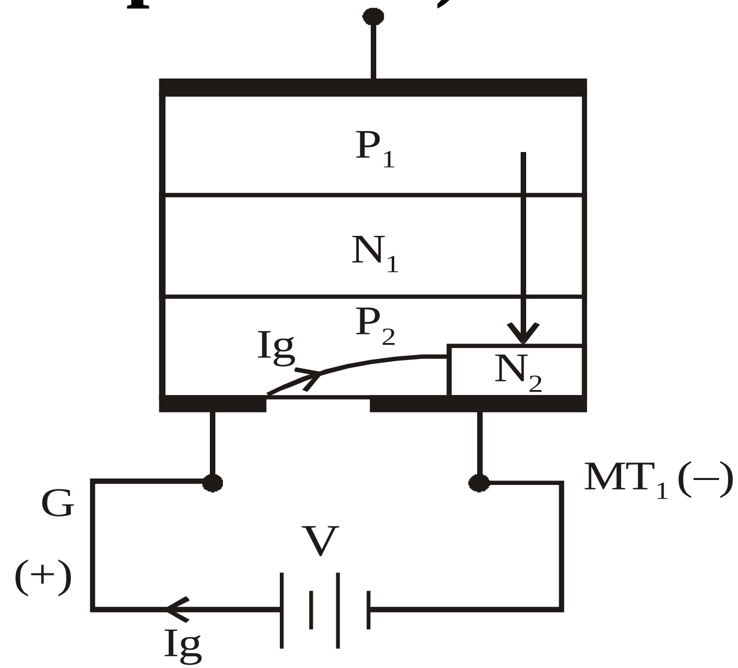


Fig.1 : Triac Structure Fig. 2 : Triac Symbol

TRIGGERING MODES OF TRIAC

- **MODE 1 : MT1 positive, Positive gate current (I^+ mode of operation)**

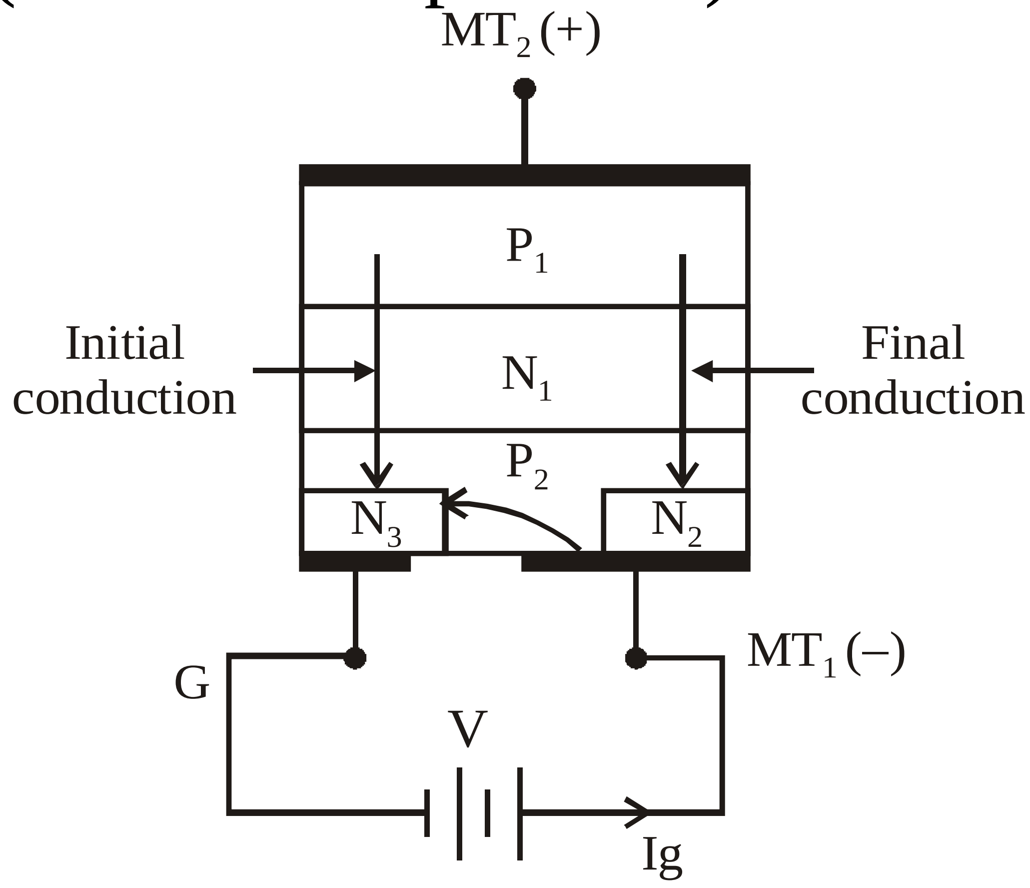


- When and gate current are positive with respect to MT1, the gate current flows through P2-N2 junction
- The junction P1-N1 and P2-N2 are forward biased but junction N1-P2 is reverse biased.
- When sufficient number of charge carriers are injected in P2 layer by the gate current the junction N1-P2 breakdown and triac starts conducting through P1N1P2N2 layers.
- Once triac starts conducting the current increases and its V-I characteristics is similar to that of thyristor. Triac in this mode operates in the first-quadrant.

MODE 2

- **MT2 positive, Negative gate current**

(I$^-$ mode of operation)

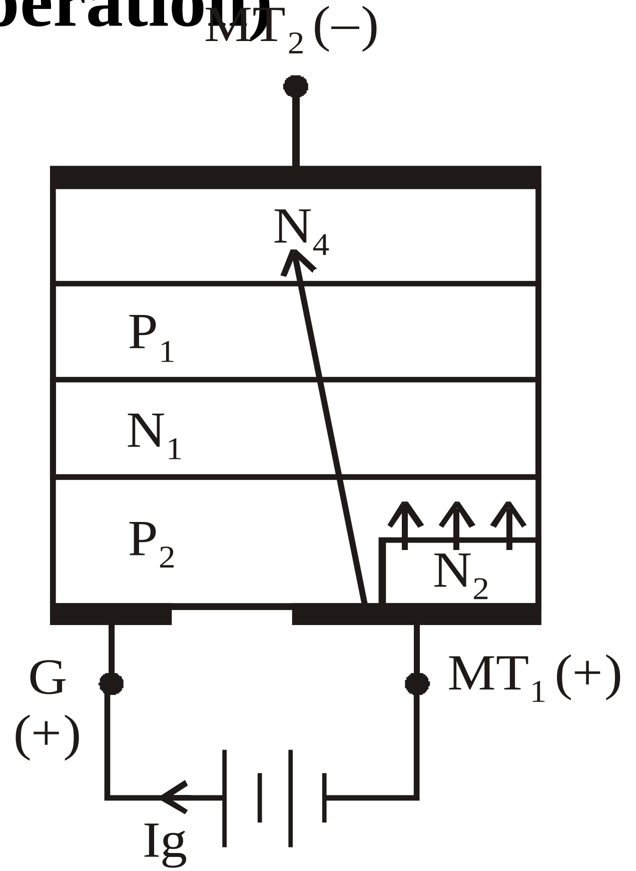


- When MT2 is positive and gate G is negative with respect to MT1 the gate current flows through P2-N3 junction
- The junction P1-N1 and P2-N3 are forward biased but junction N1-P2 is reverse biased. Hence, the triac initially starts conducting through P1N1P2N3 layers.
- As a result the potential of layer between P2-N3 rises towards the potential of MT2.
- Thus, a potential gradient exists across the layer P2 with left hand region at a higher potential than the right hand region.

- This results in a current flow in P2 layer from left to right, forward biasing the P2N2 junction. Now the right hand portion P1-N1 - P2-N2 starts conducting.
- The device operates in first quadrant. When compared to Mode 1, triac with MT2 positive and negative gate current is less sensitive and therefore requires higher gate current for triggering.

MODE 3

- **MT2 negative, Positive gate current (III$^+$ mode of operation)**

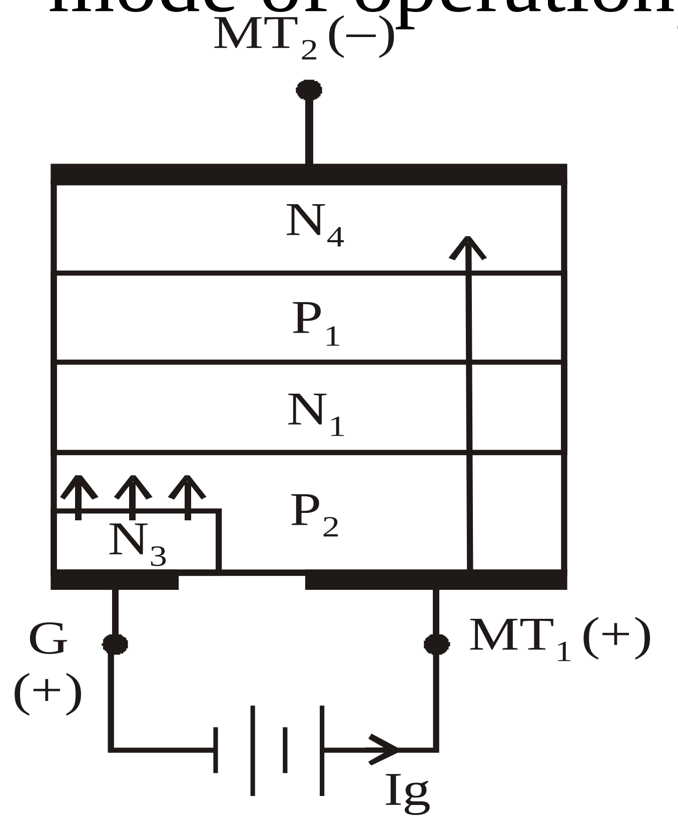


- When MT2 is negative and gate is positive with respect to MT1 junction P2N2 is forward biased and junction P1-N1 is reverse biased.
- N2 layer injects electrons into P2 layer as shown by arrows in figure below.
- This causes an increase in current flow through junction P2-N1. Resulting in breakdown of reverse biased junction N1-P1.
- Now the device conducts through layers P2N1P1N4 and the current starts increasing, which is limited by an external load.
- The device operates in third quadrant in this mode. Triac in this mode is less sensitive and requires higher gate current for triggering.

MODE 4

- **MT2 negative, Negative gate current**

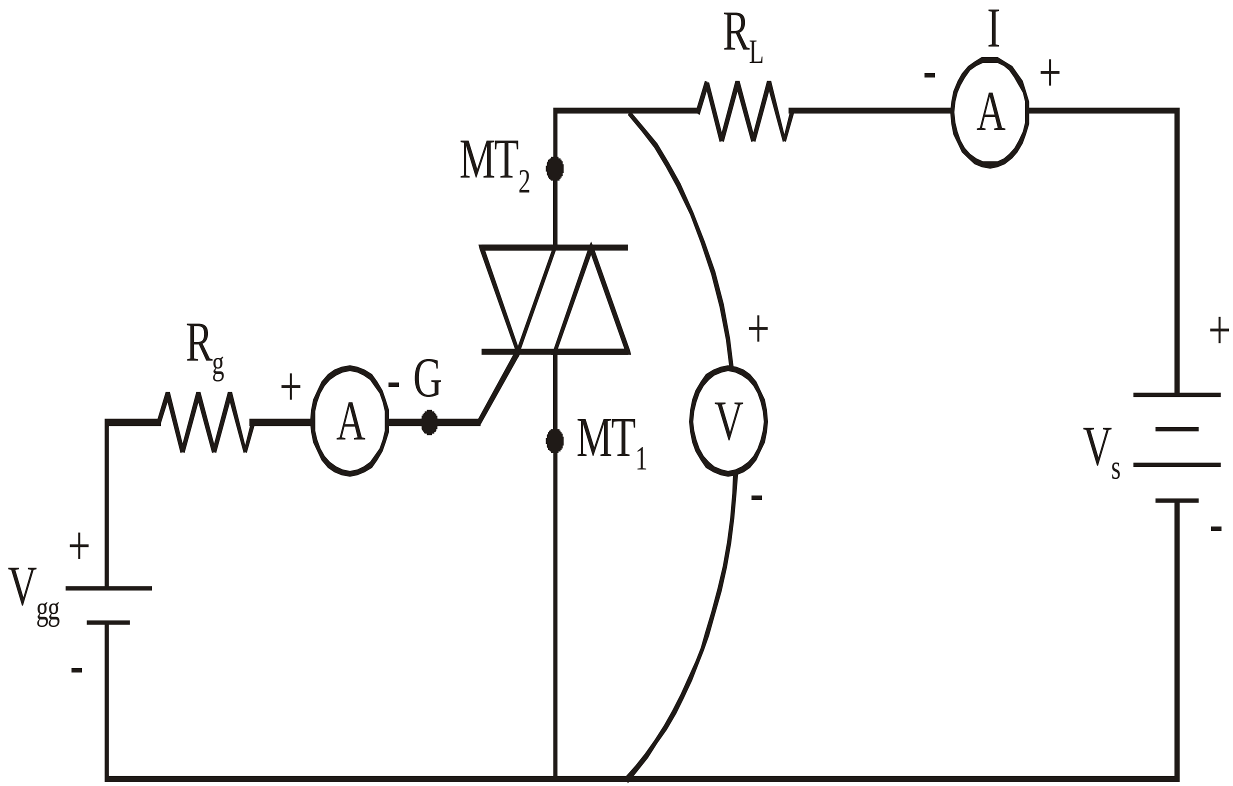
(III$^+$ mode of operation)



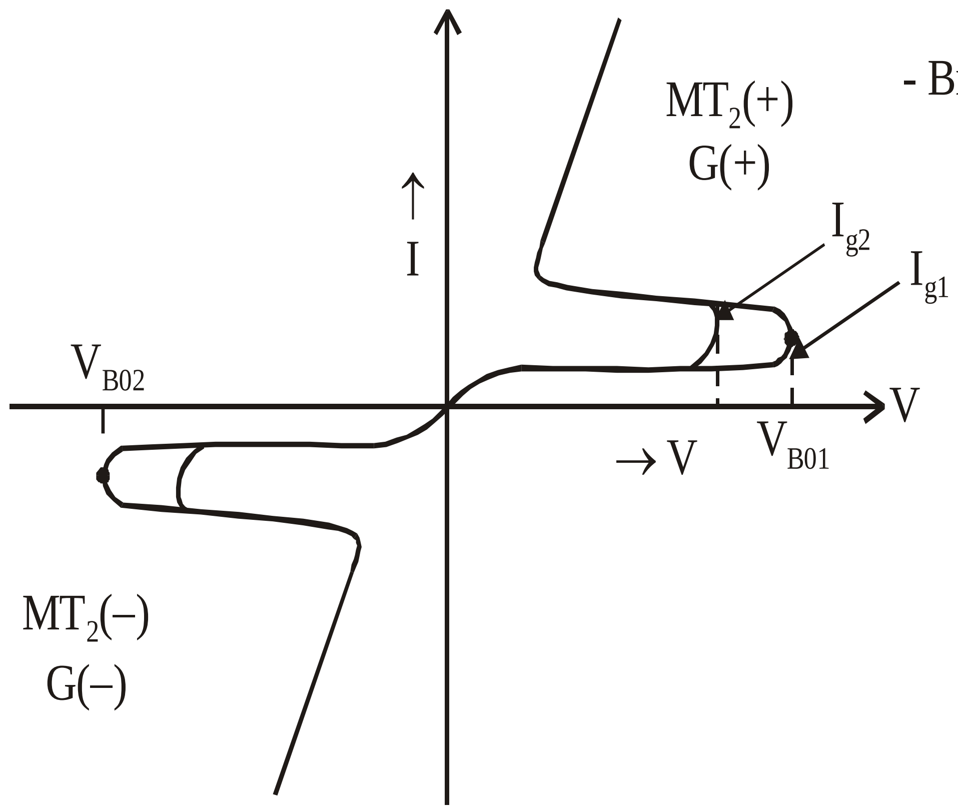
- In this mode both MT2 and gate G are negative with respect to MT1, the gate current flows through P2N3 junction as shown.
- Layer N3 injects electrons as shown by arrows into P2 layer. This results in increase in current flow across P1N1 and the device will turn ON due to increased current in layer N1.
- The current flows through layers P2N1P1N4. Triac is more sensitive in this mode compared to turn ON with positive gate current. (Mode 3).

- Triac sensitivity is greatest in the first quadrant when turned ON with positive gate current and also in third quadrant when turned ON with negative gate current. when is positive with respect to it is recommended to turn on the triac by a positive gate current.
- When is negative with respect to it is recommended to turn on the triac by negative gate current. Therefore Mode 1 and Mode 4 are the preferred modes of operation of a triac (mode and mode of operation are normally used).

Triac characteristics



V-I Characteristics of a triac

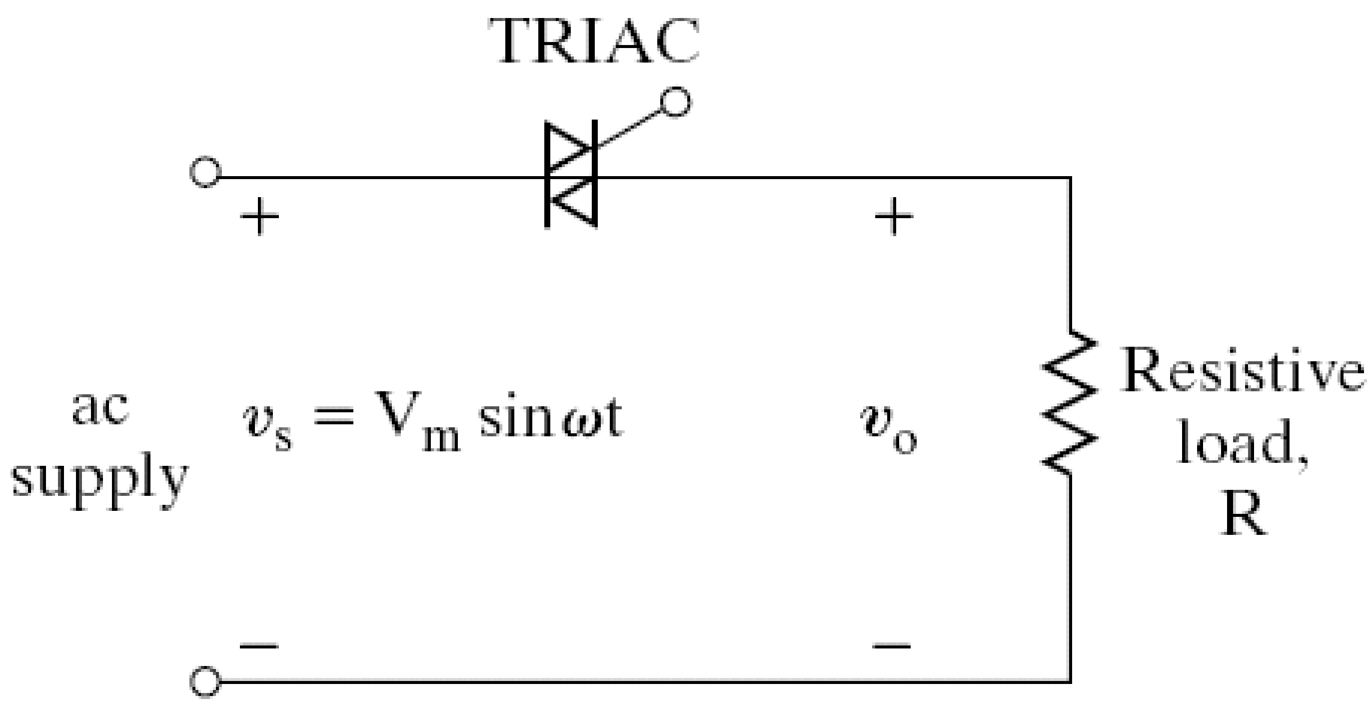


V_{B01}, V_{B01}
- Breakover voltages

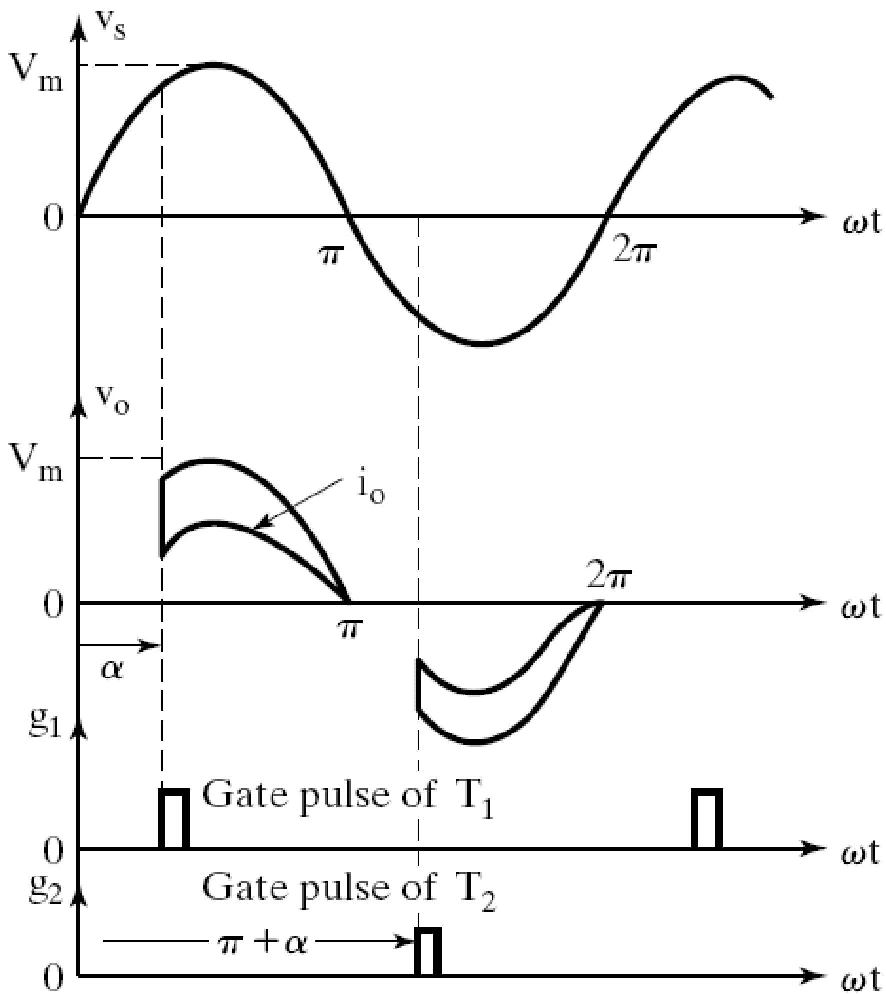
$I_{g2} > I_{g21}$

Single phase full wave ac voltage controller (Bi-directional Controller) using TRIAC

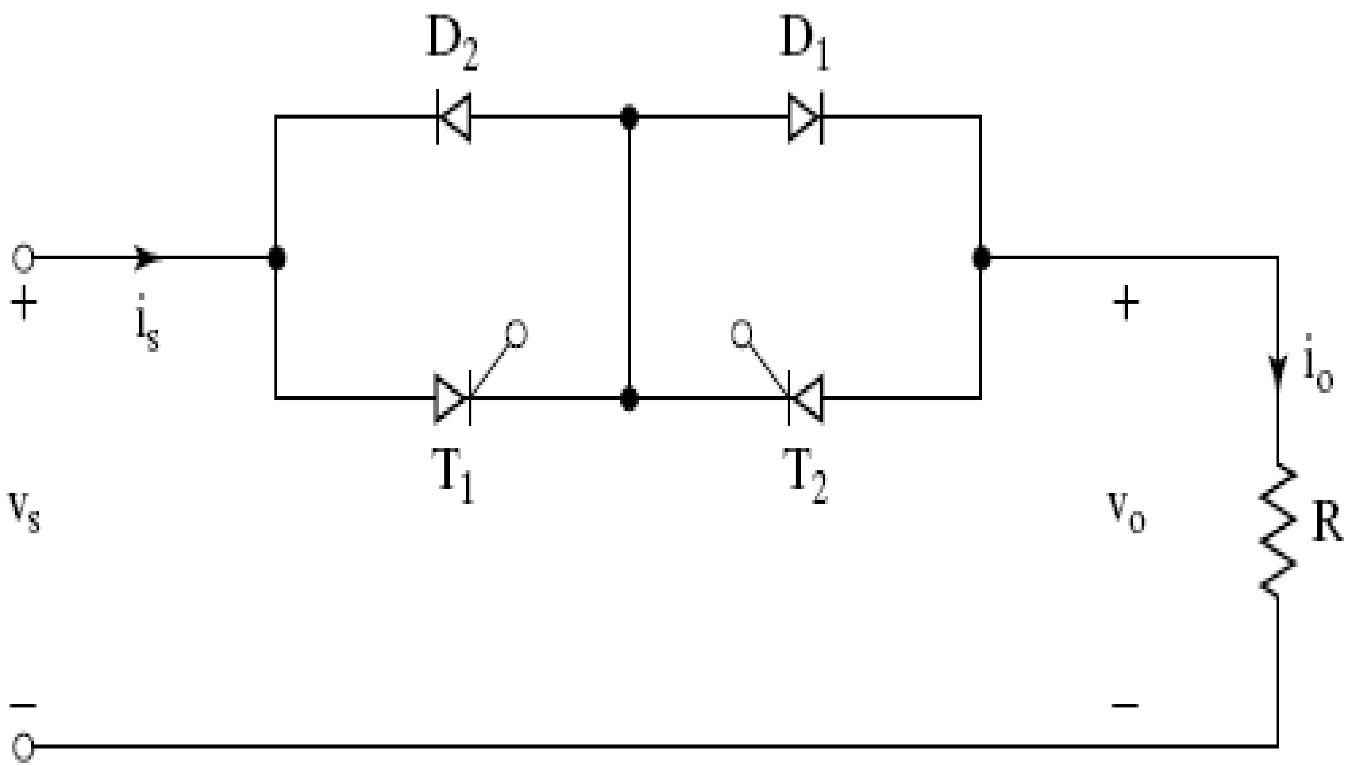
Single phase full wave ac voltage controller (Bi-directional Controller) using TRIAC



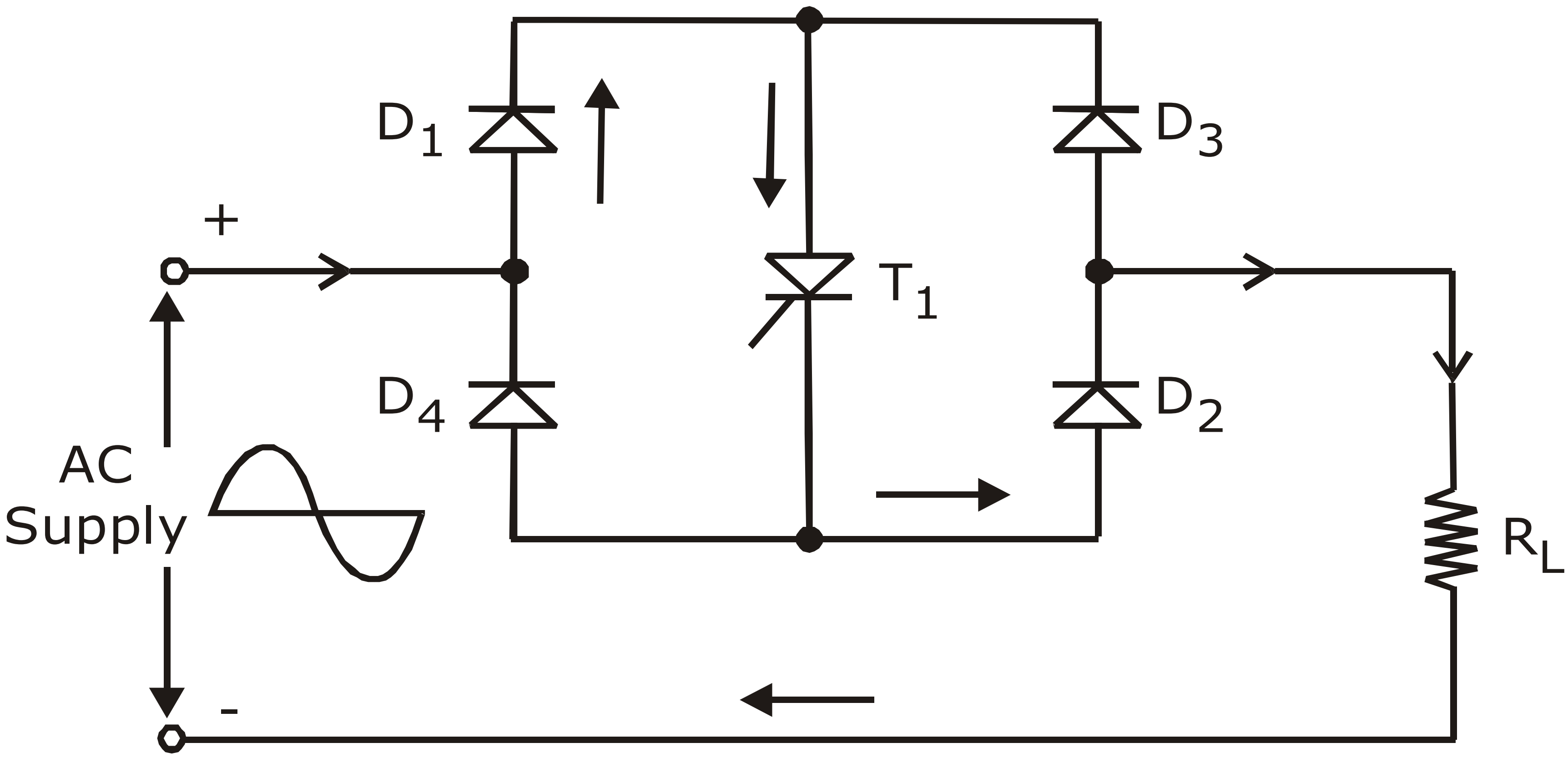
Waveforms of single phase full
wa ller



Single phase full wave ac controller with common cathode (Bidirectional controller in common cathode configuration)



Single Phase Full Wave Ac Voltage Controller Using A Single Thyristor



CYCLOCONVERTER

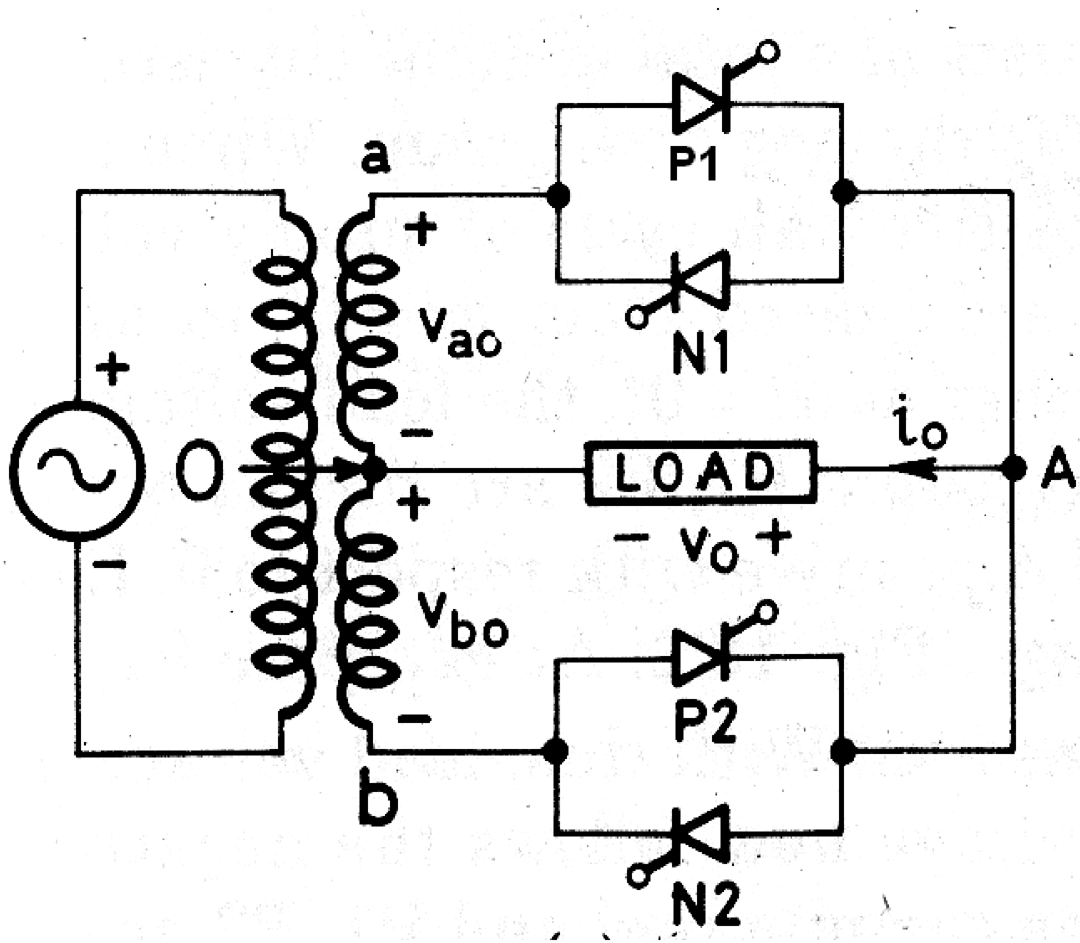
CYCLOCONVERTER

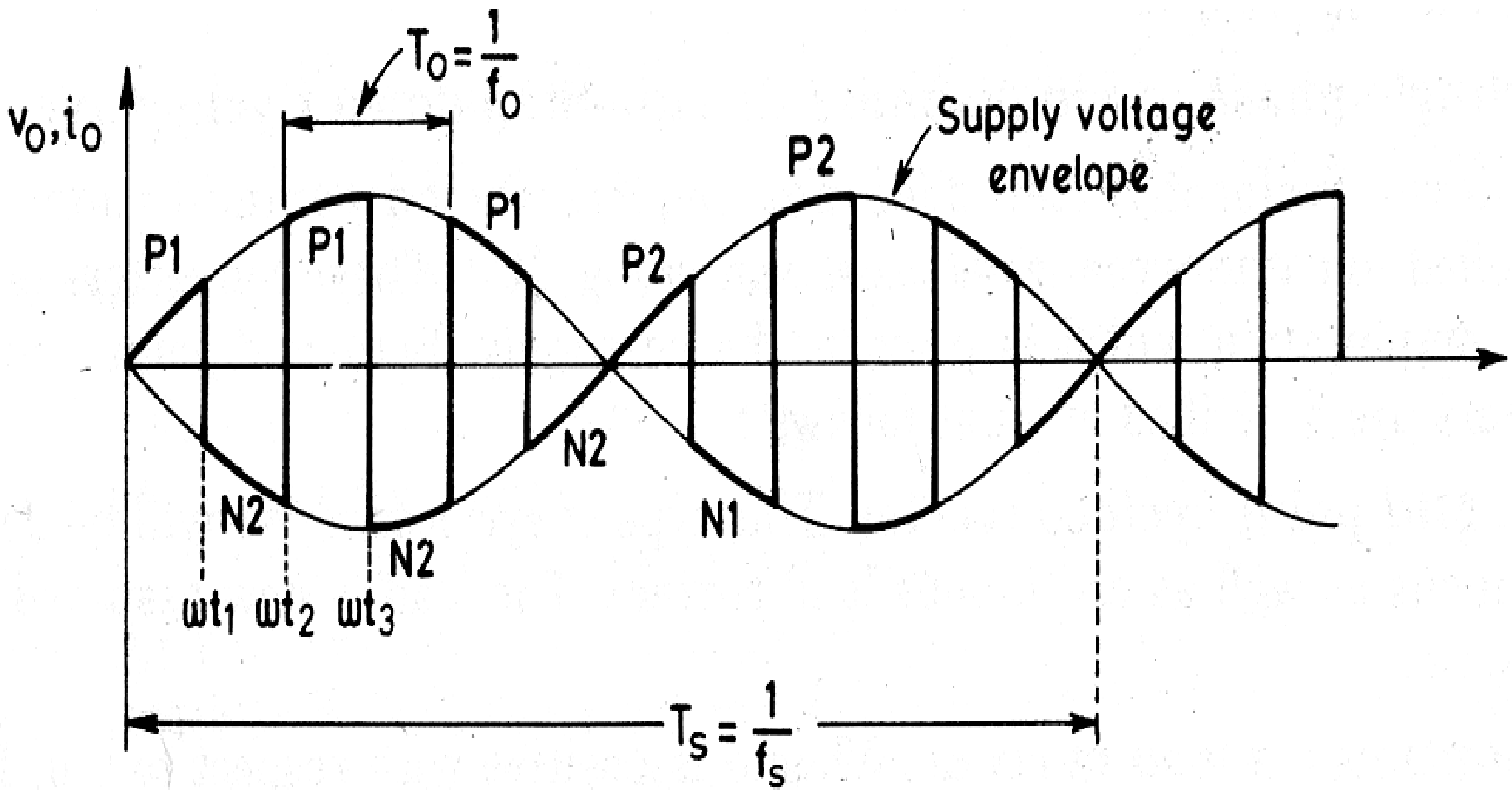
- A device which converts input power at one frequency to the out put power at different frequency with one stage conversion is called a cycloconverter.
- A cycloconverter requires one stage frequency conversion.
- Cycloconverter of two types

(i) Step-Up Cycloconverter (fo > s)

(ii) step-Down Cycloconverter (fo < fs)

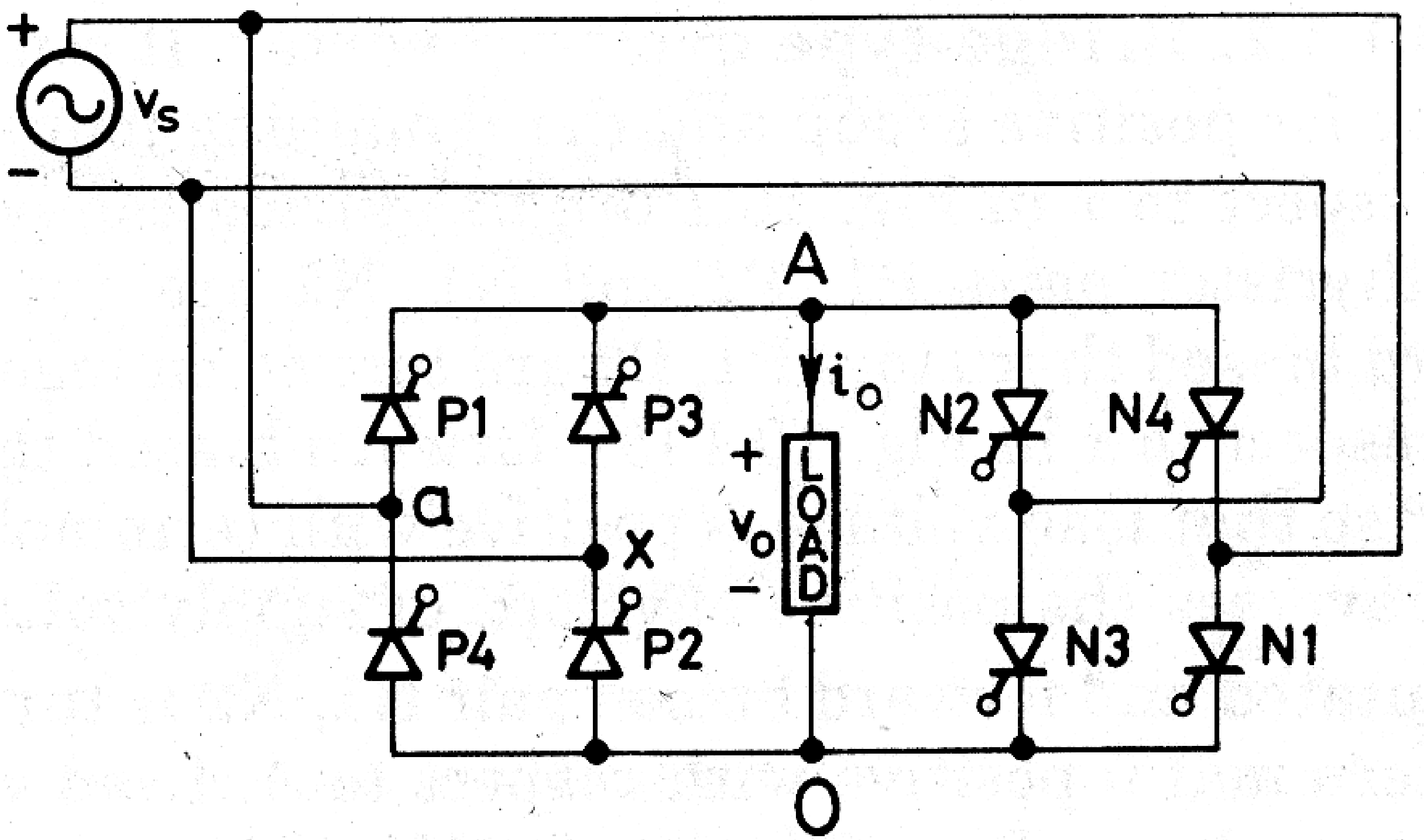
Single phase to single phase Mid point type step-up Cycloconverter with R load

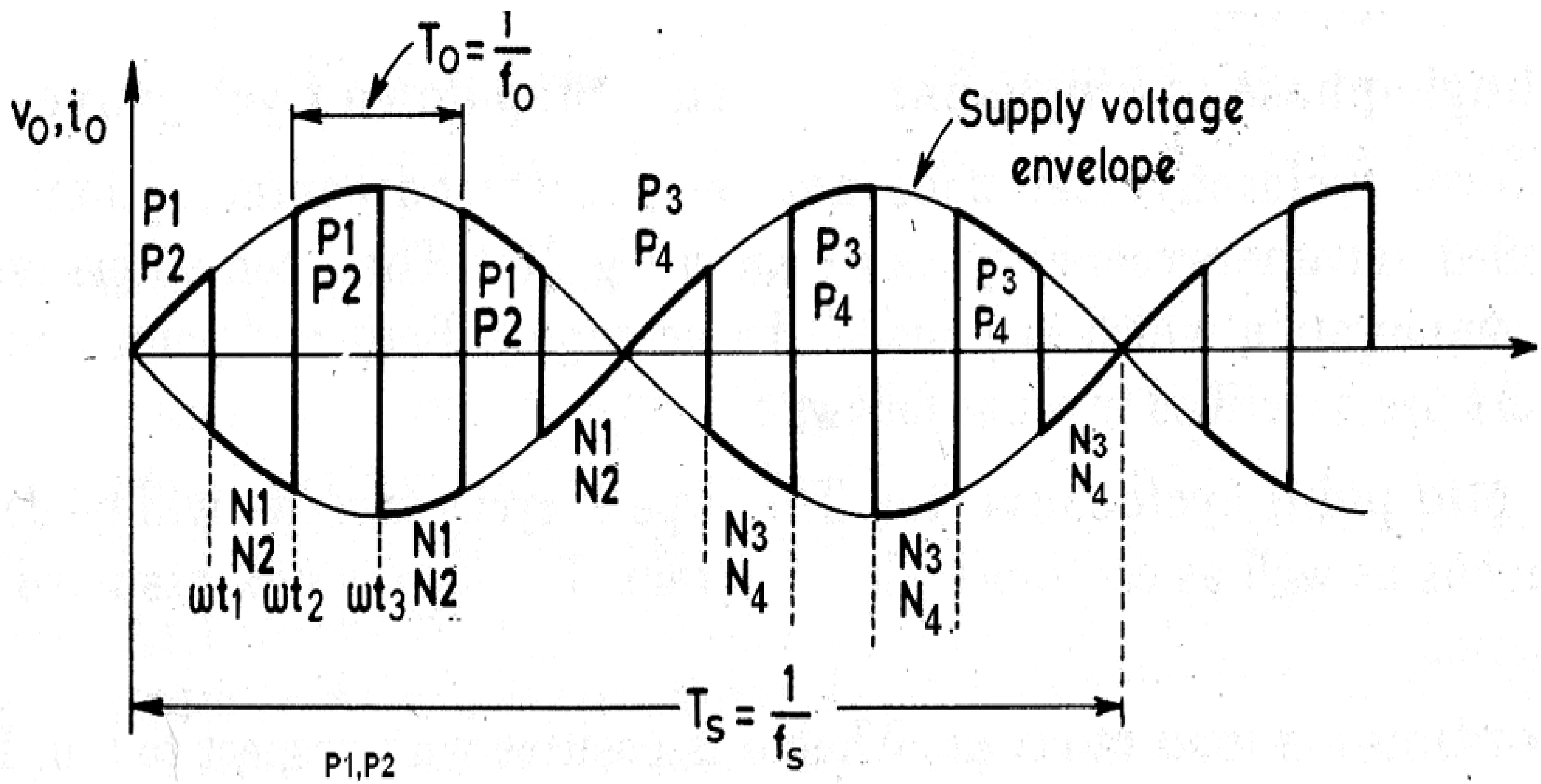




$T_O = \frac{1}{f_O}$
v_O, i_O
Supply voltage envelope
P1
P1
P1
P2
P2
N2
N2
N2
N1
ωt_1
ωt_2
ωt_3
$T_S = \frac{1}{f_S}$

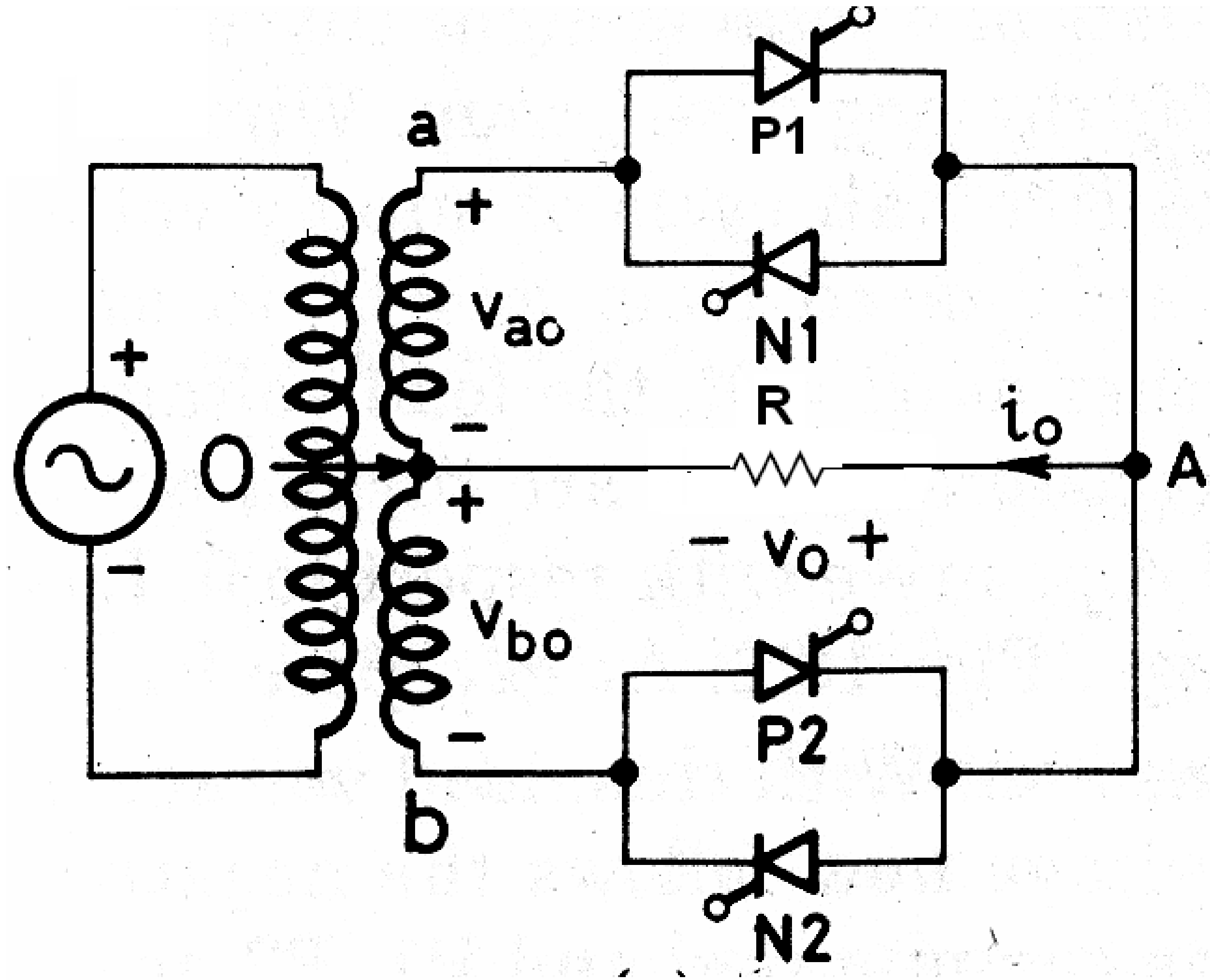
Single phase to single phase Bridge type step-up Cvcloconverter with R load





$T_O = \frac{1}{f_O}$
v_O, i_O
Supply voltage envelope
P1
P2
P1
P2
P1
P2
P3
P4
P3
P4
P3
P4
N1
N2
N1
N2
N1
N2
N3
N4
N3
N4
N3
N4
ωt_1 ωt_2 ωt_3
$T_S = \frac{1}{f_S}$
P1,P2

1-φ to 1-φ Mid point type step-Down Cycloconverter with R load



Output voltage (Vo) and current (Io) waveform

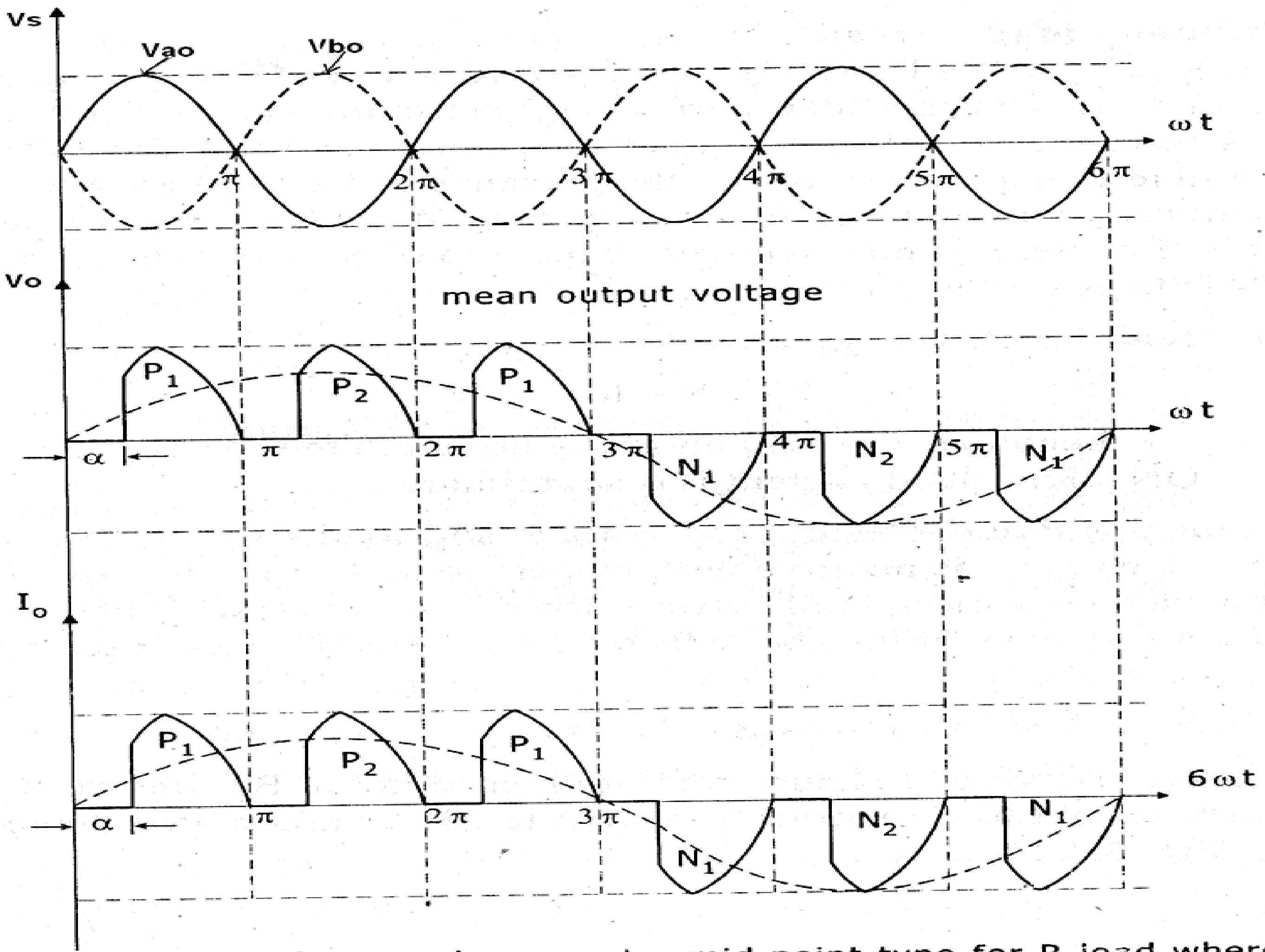
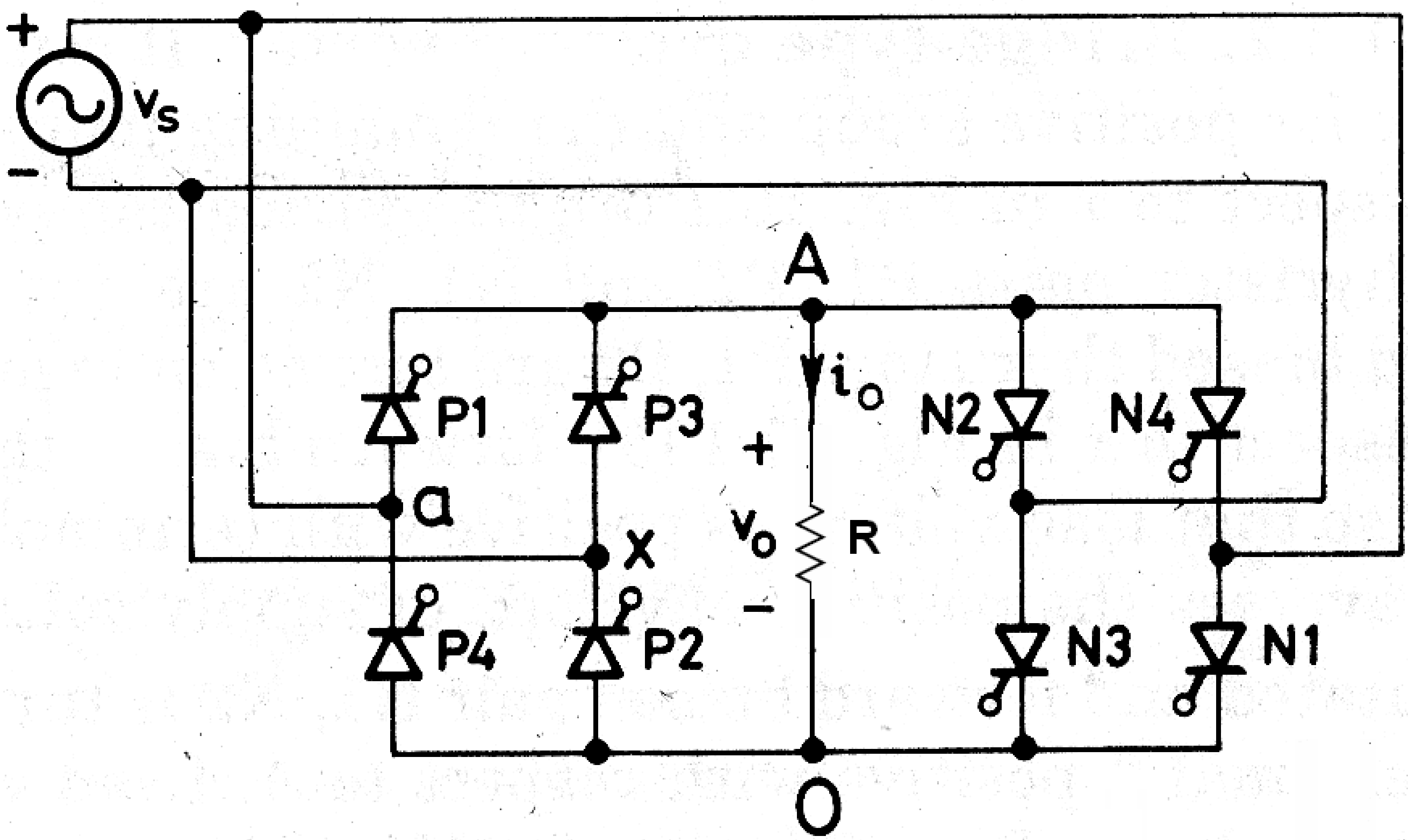


Fig. Step down cyclo converter-mid point type for R load where α is firing angle

1-φ to 1-φ Bridge type Cyclo-converter with R and R-L load

1-φ to 1-φ Bridge type step-Down Cycloconverter with R load



Output voltage (Vo) and current (Io) waveform

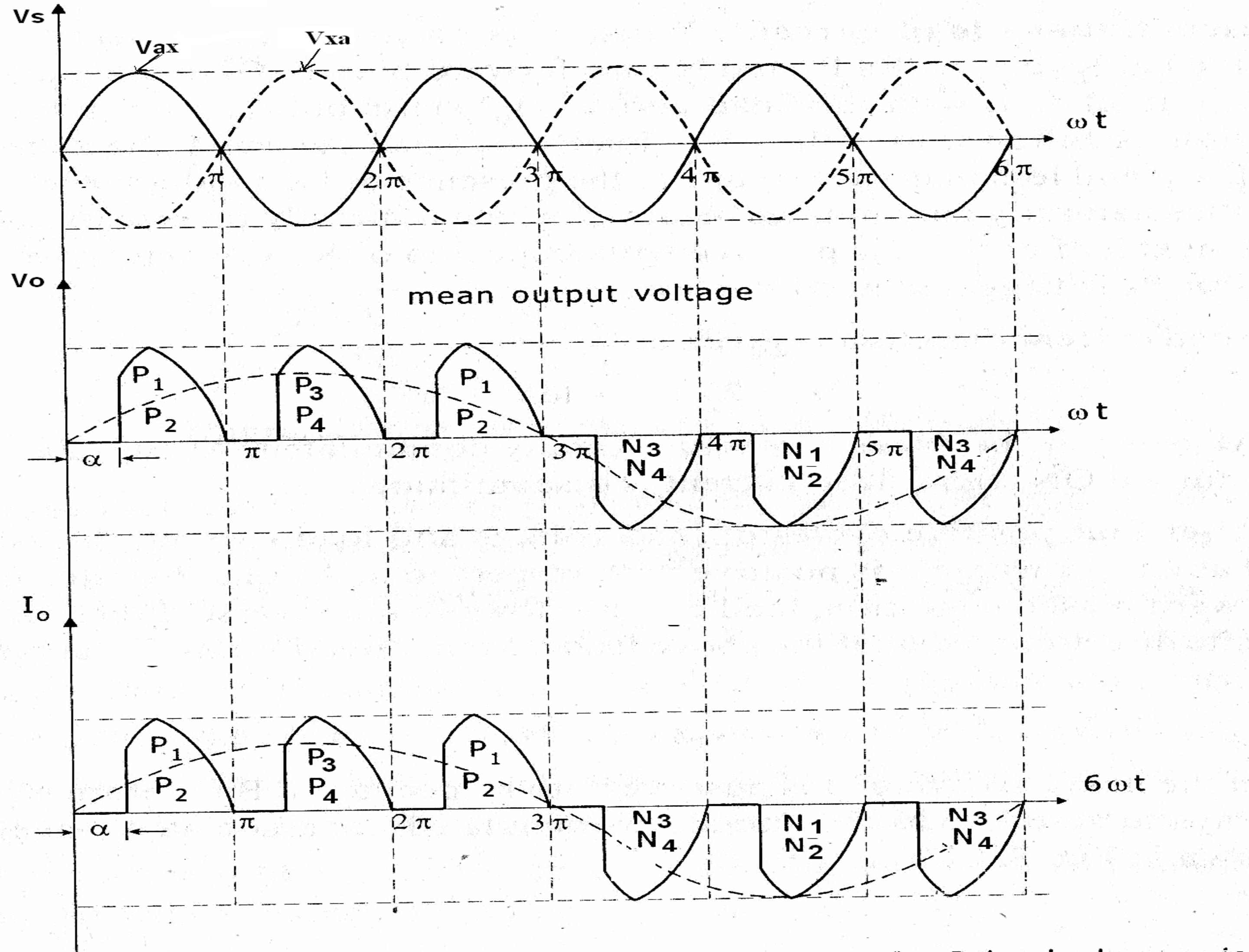
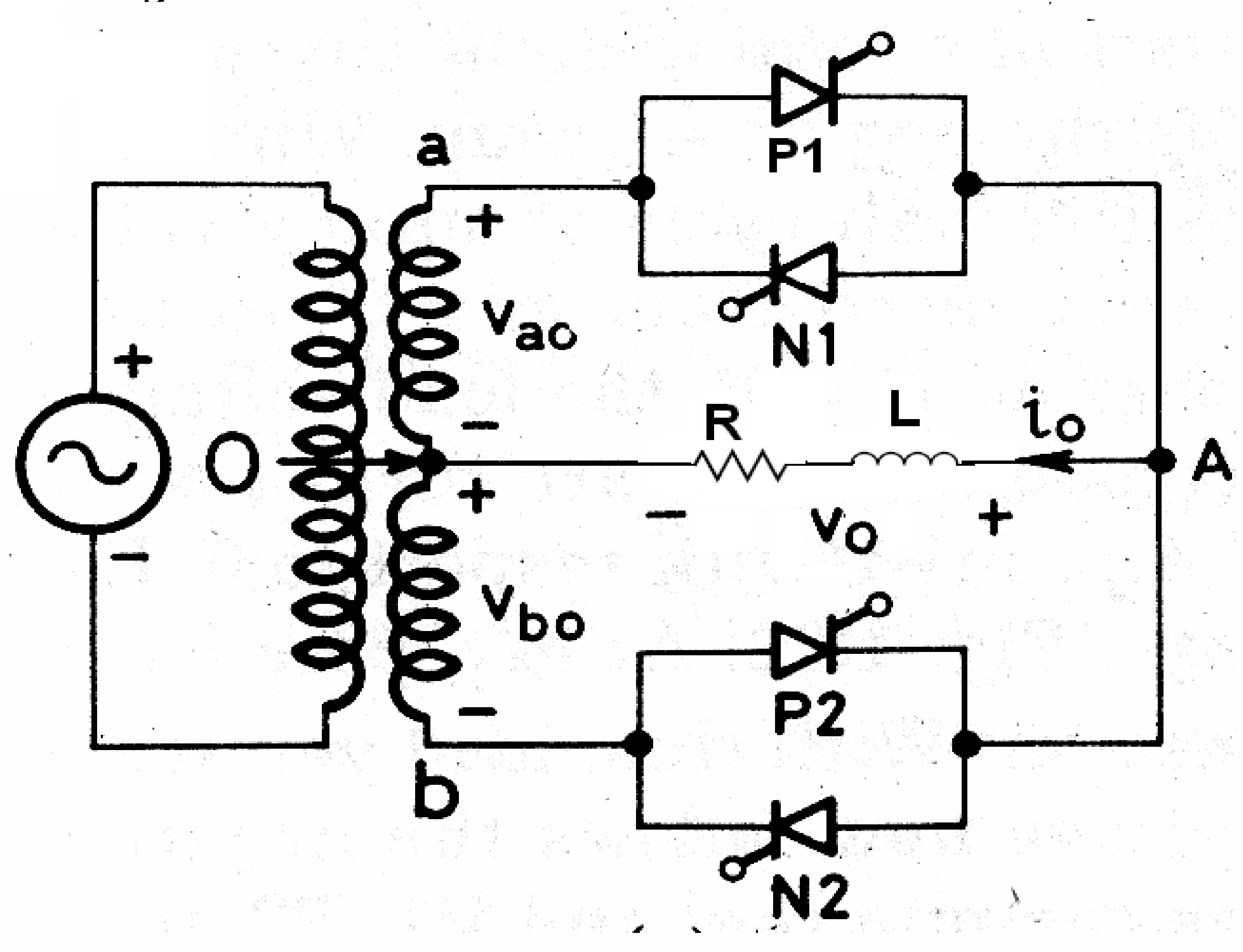
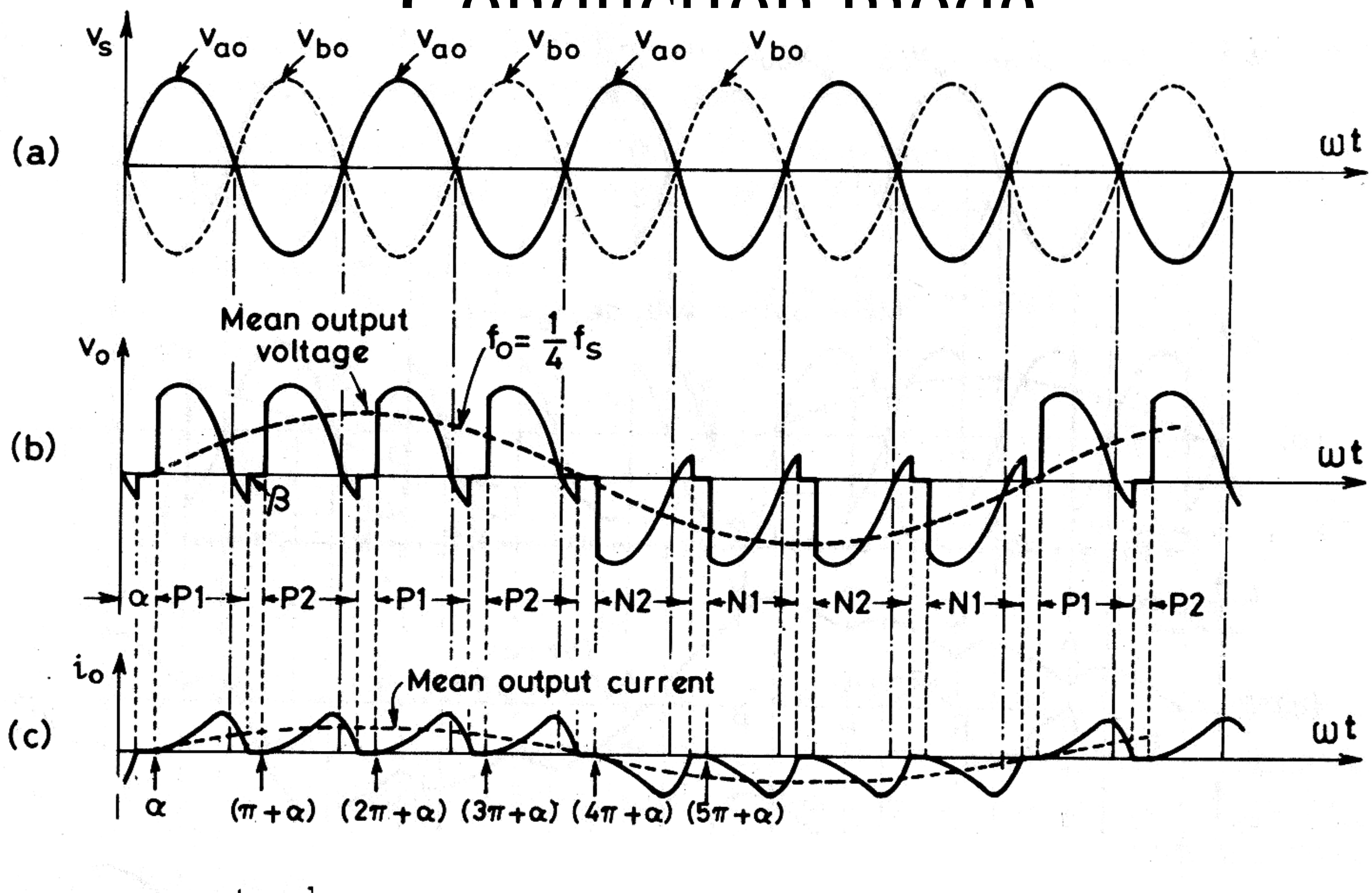


Fig. Step down cyclo converter- **Bridge** type for R load where α is firing angle

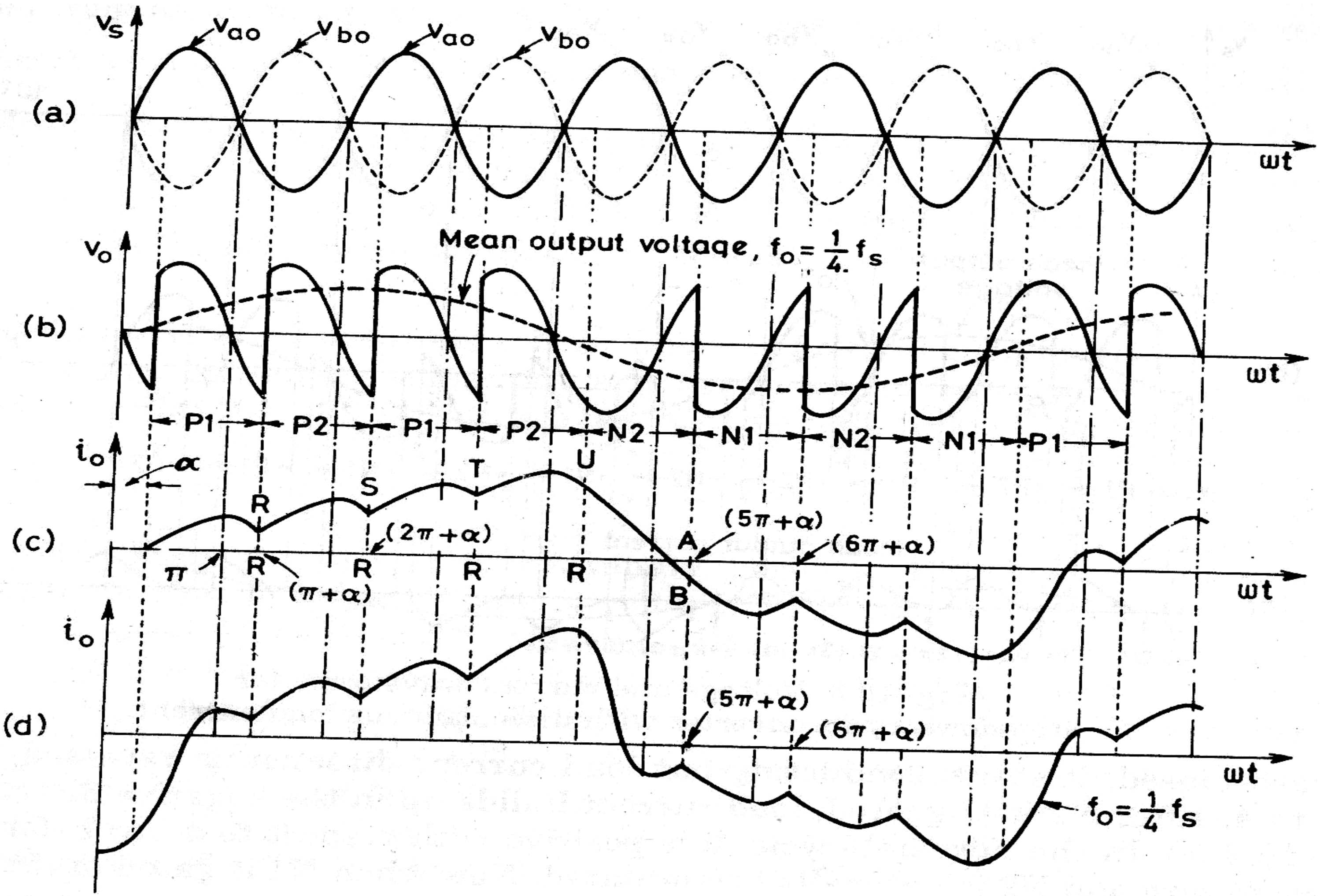
1-φ to 1-φ Midpoint type step-Down Cycloconverter with R-L load



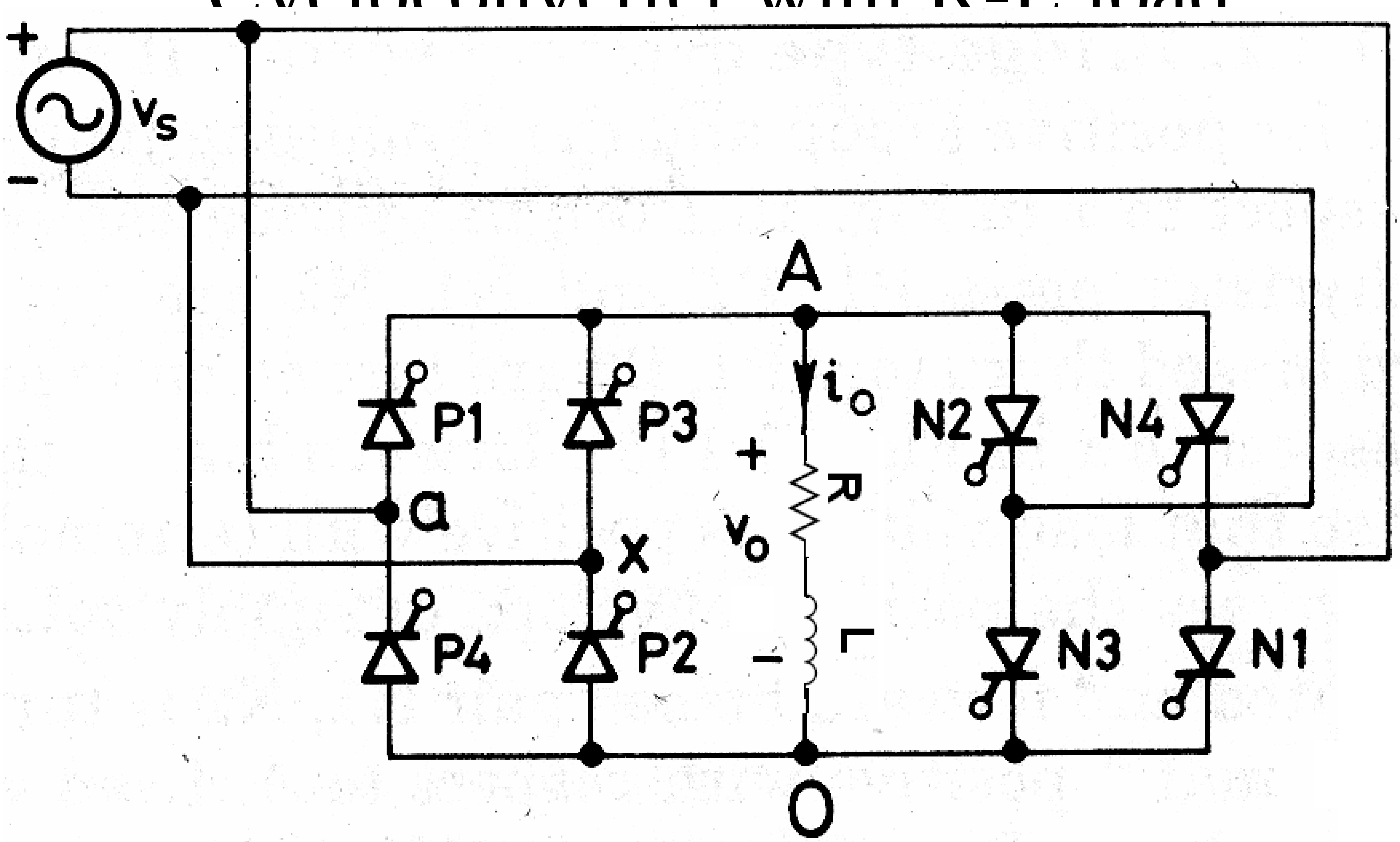
Output voltage (Vo) and current (Io) waveform for Discontinuous Conduction mode



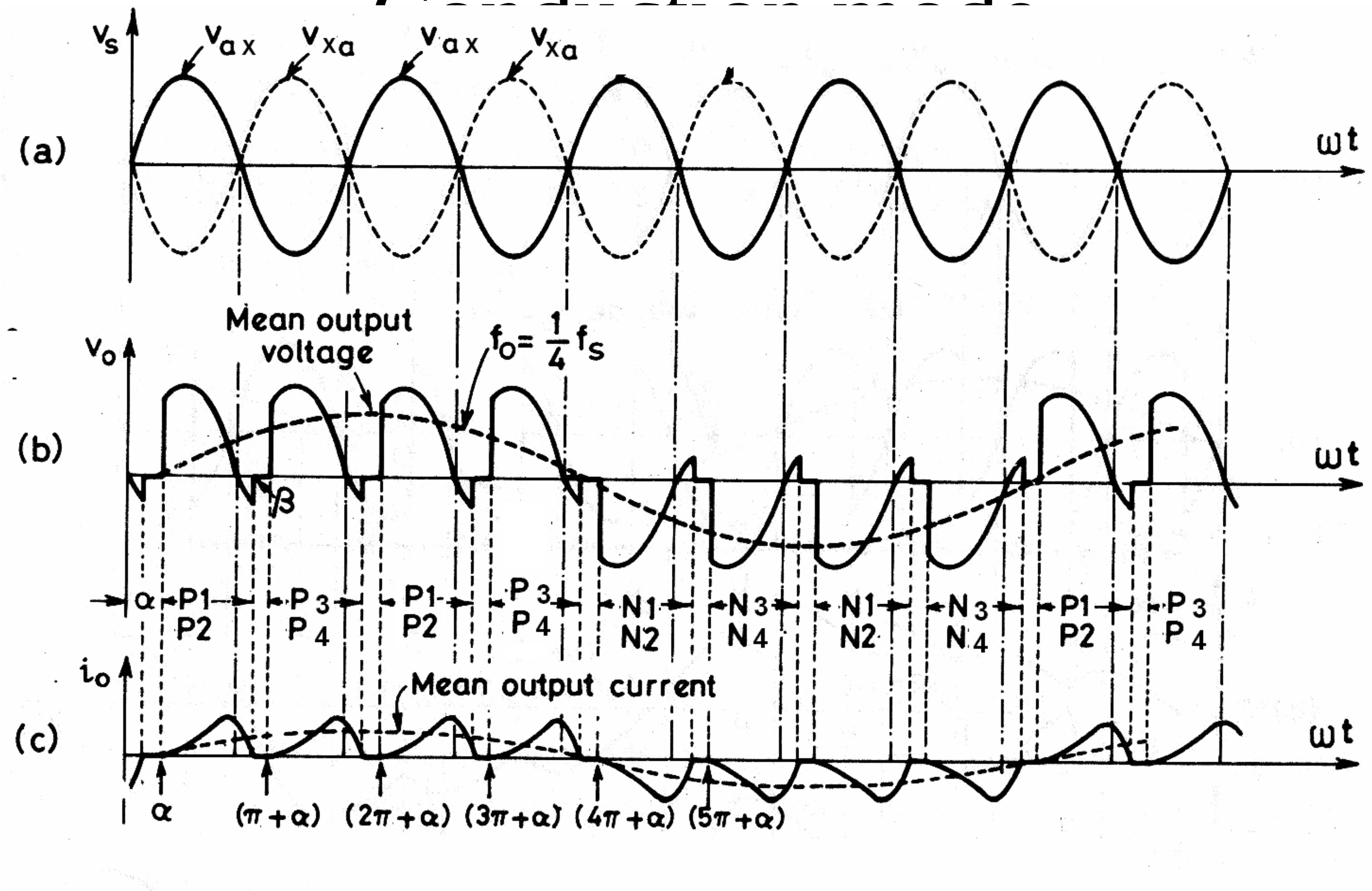
Output voltage (Vo) and current (Io) waveform for Continuous Conduction mode



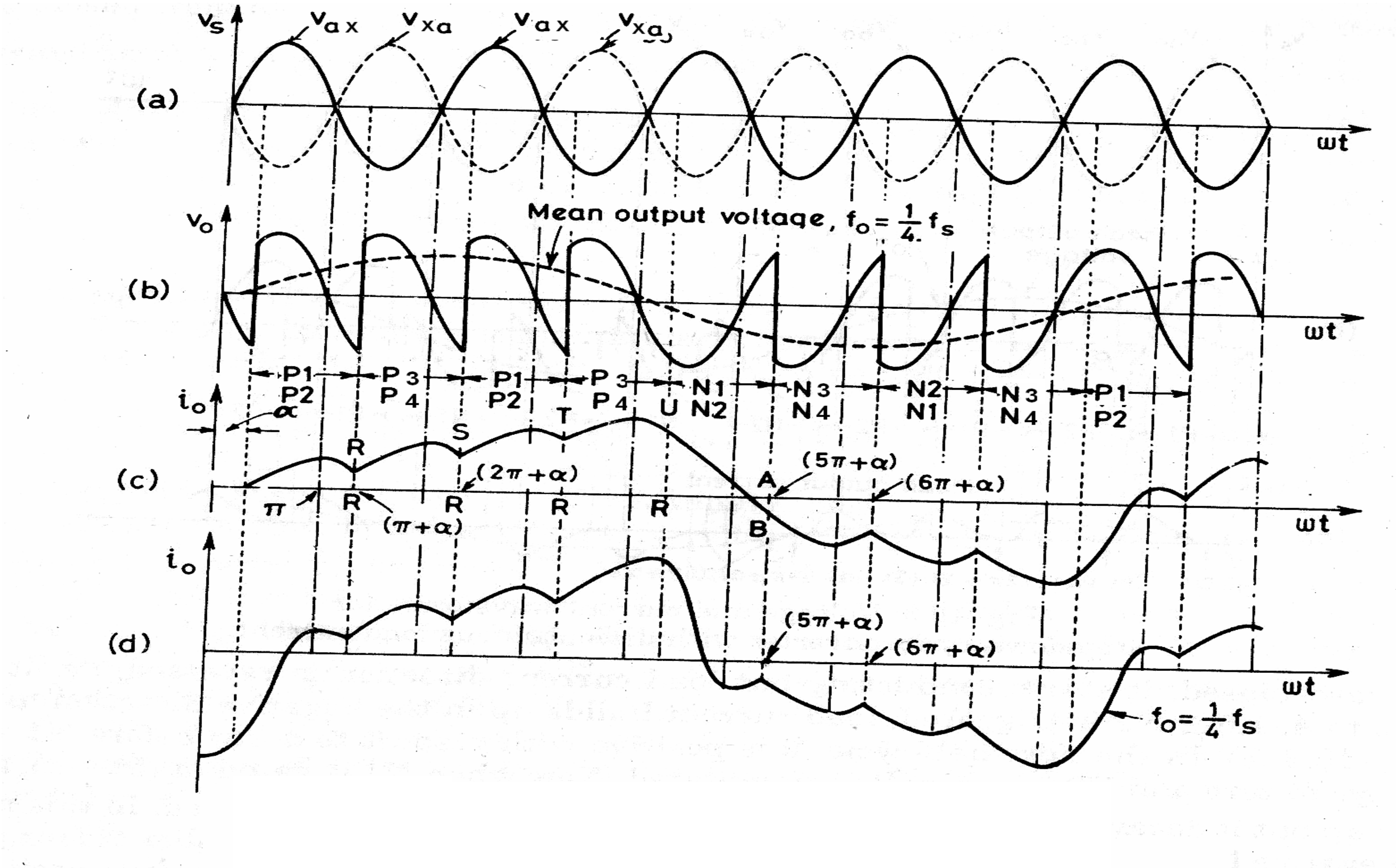
1-φ to 1-φ Bridge-type step-Down Cycloconverter with R-L load



Output voltage (Vo) and current (Io) waveform for Discontinuous Conduction mode



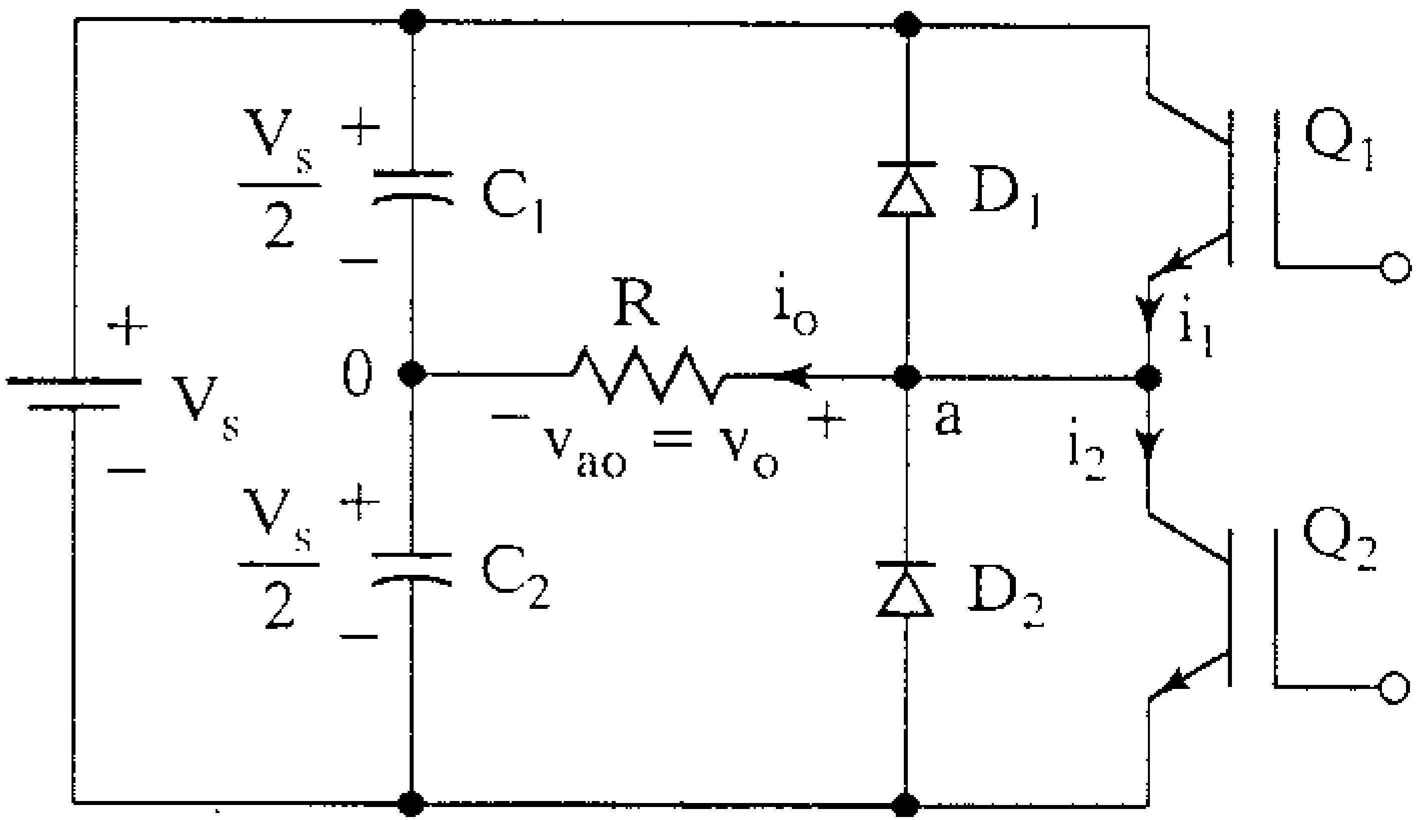
Output voltage (Vo) and current (Io) waveform for Continuous Conduction mode



UNIT-V

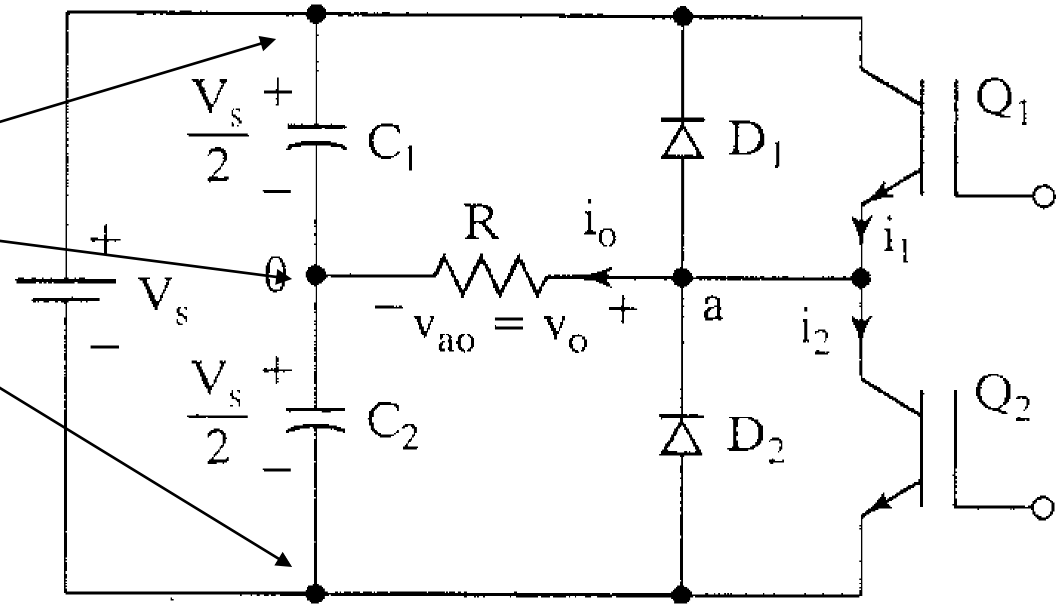
DC-AC CONVERTERS (INVERTERS)

Single-phase half-bridge inverter



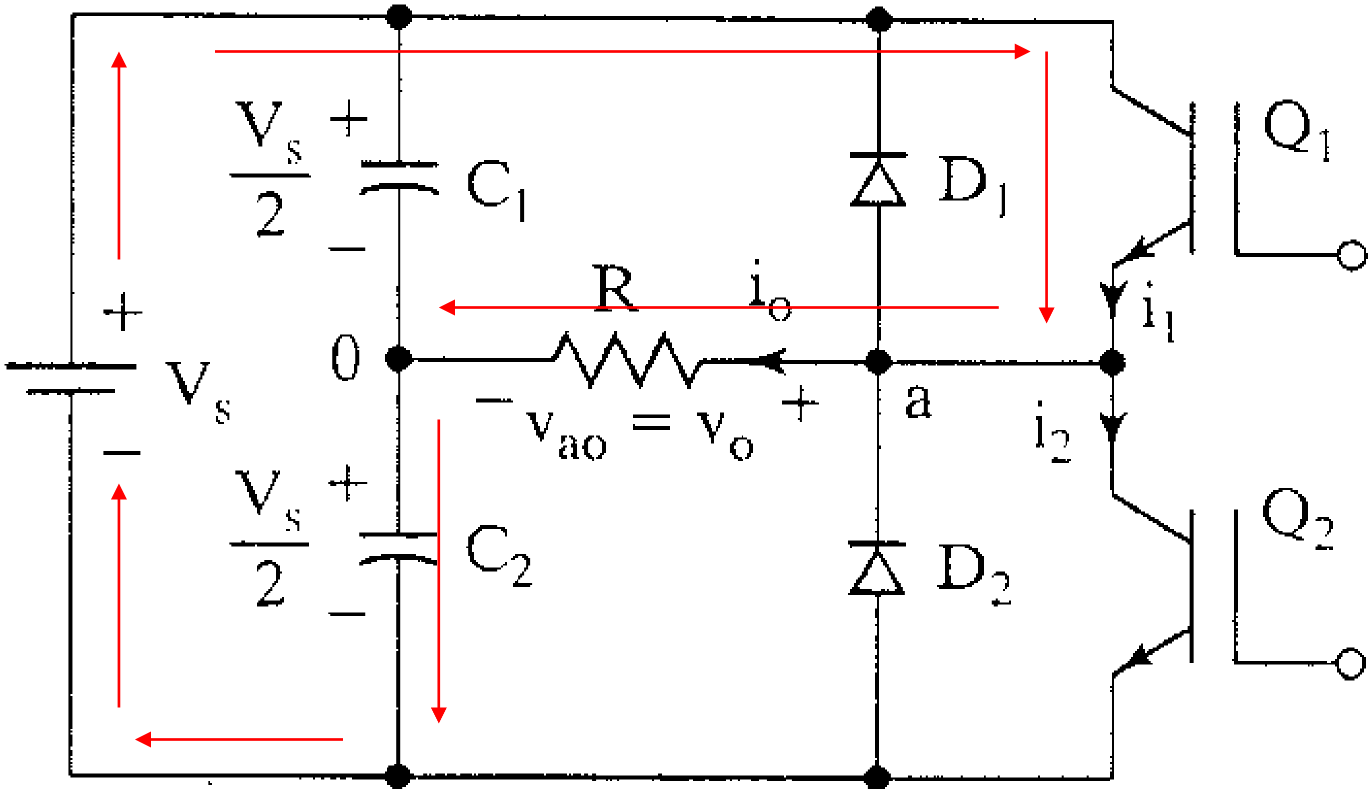
Operational Details

3-wire DC source



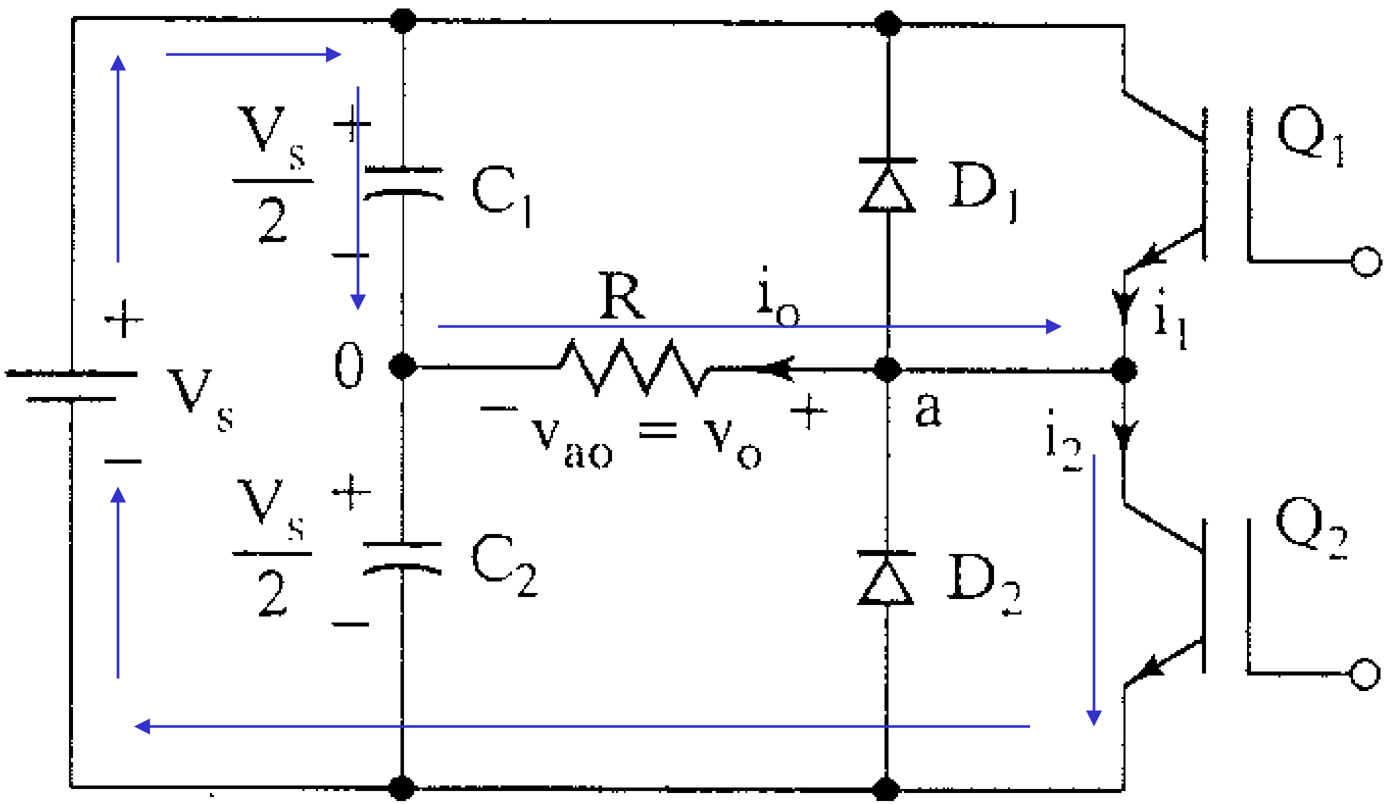
- Consists of 2 choppers, 3-wire DC source
- Transistors switched on and off alternately
- Need to isolate the gate signal for Q_1 (upper device)
- Each provides opposite polarity of $V_s/2$ across the load

Q_1 on, Q_2 off, $v_o = V_s/2$

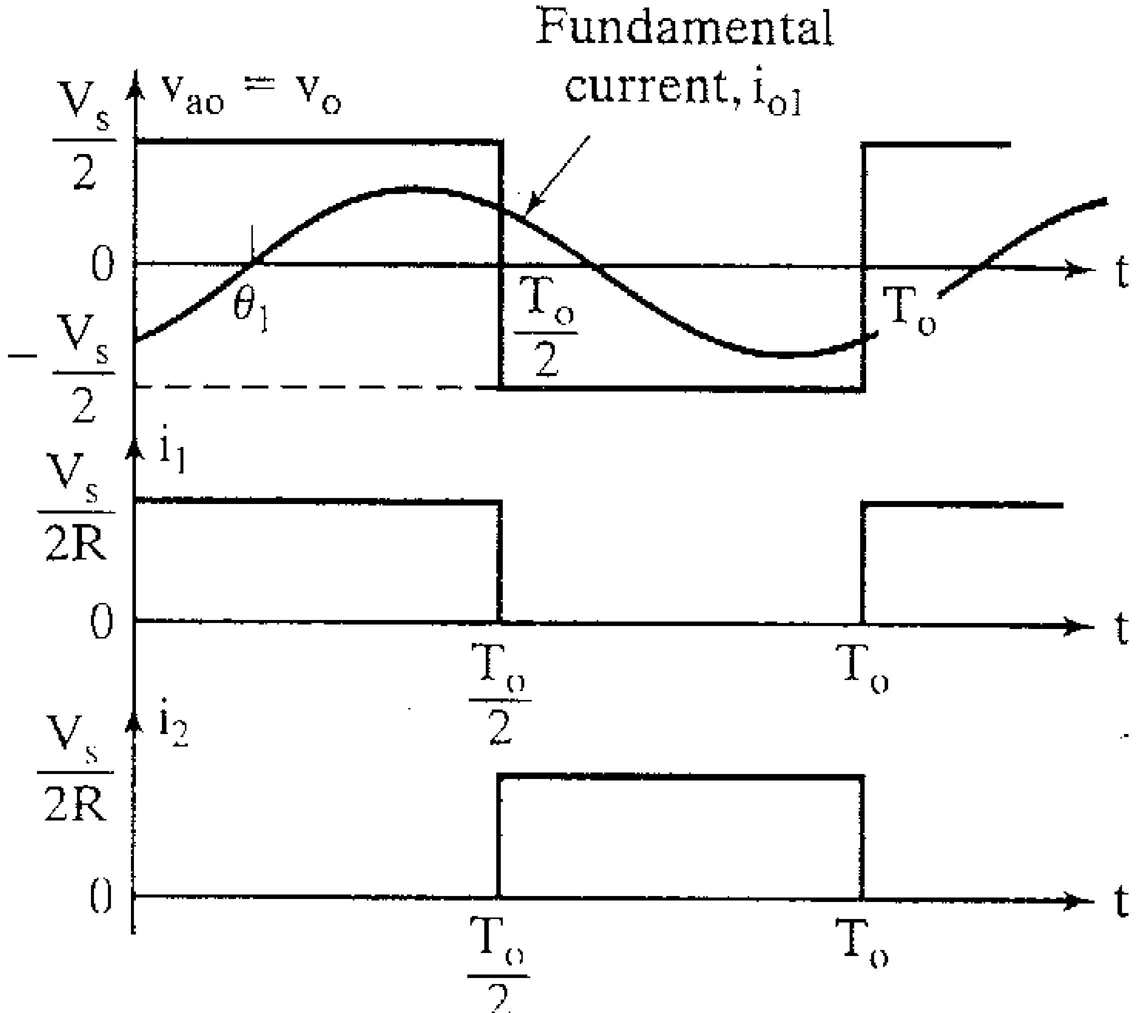


Peak Reverse Voltage of $Q_2 = V_s$

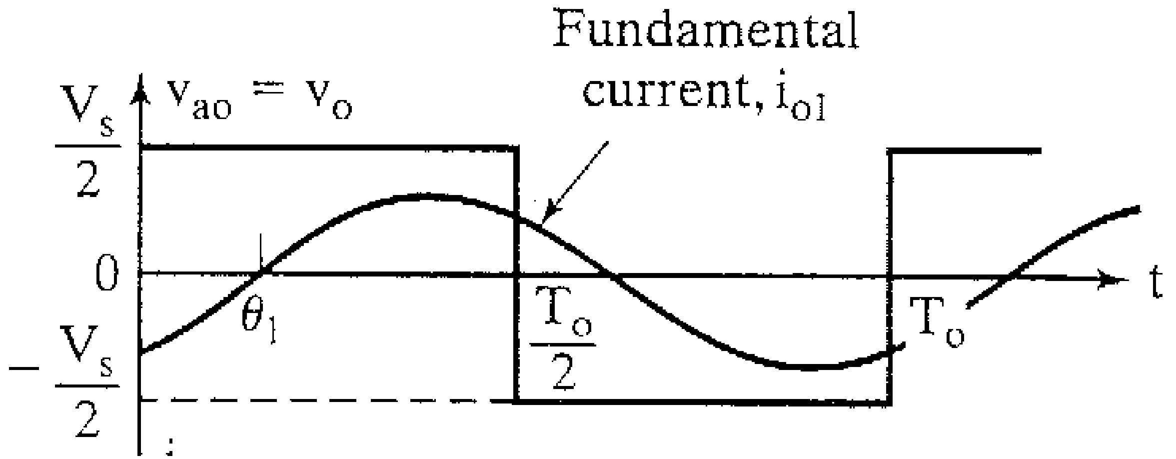
Q_1 off, Q_2 on, $v_o = -V_s/2$



Waveforms with resistive load



Look at the output voltage



rms value of the output voltage, V_o

$$V_o = \left(\frac{2}{T_o} \int_0^{\frac{T_o}{2}} \frac{V_s^2}{4} dt \right)^{\frac{1}{2}} = \frac{V_s}{2}$$

Fourier Series of the instantaneous output voltage

$$v_o = \frac{a_o}{2} + \sum_{n=1}^{\infty}\left(a_n \cos(n\omega t) + b_n \sin(n\omega t)\right)$$

$$a_o, a_n = 0$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} \frac{-V_s}{2} \sin(n\omega t) d(\omega t) + \int_{0}^{\pi} \frac{V_s}{2} \sin(n\omega t) d(\omega t)\right]$$

$$b_n = \frac{2V_s}{n\pi} \rightarrow n = 1, 3, 5, \ldots$$

$$v_o = \sum_{n=1,3,5,..}^{\infty} \frac{2V_s}{n\pi} \sin(n\omega t)$$

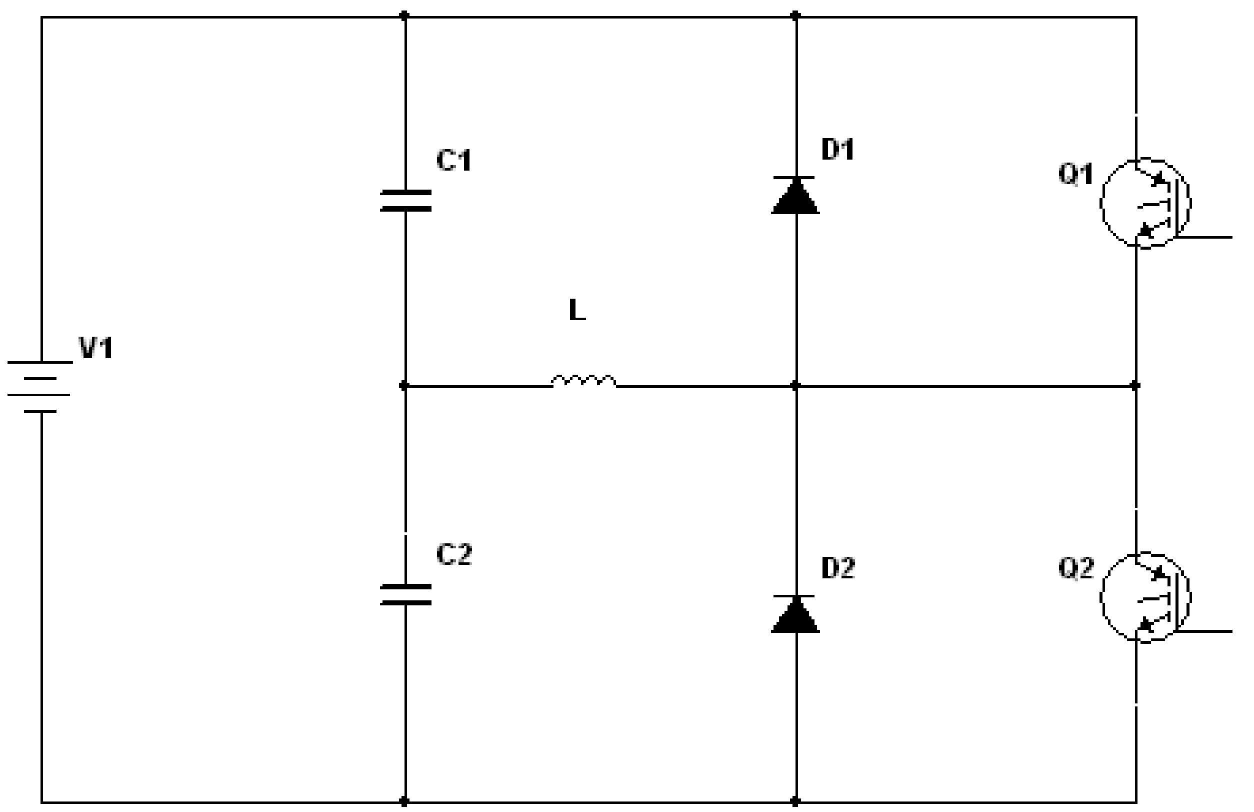
rms value of the fundamental component

$$v_o = \sum_{n=1,3,5,..}^{\infty} \frac{2V_s}{n\pi} \sin n\omega t$$

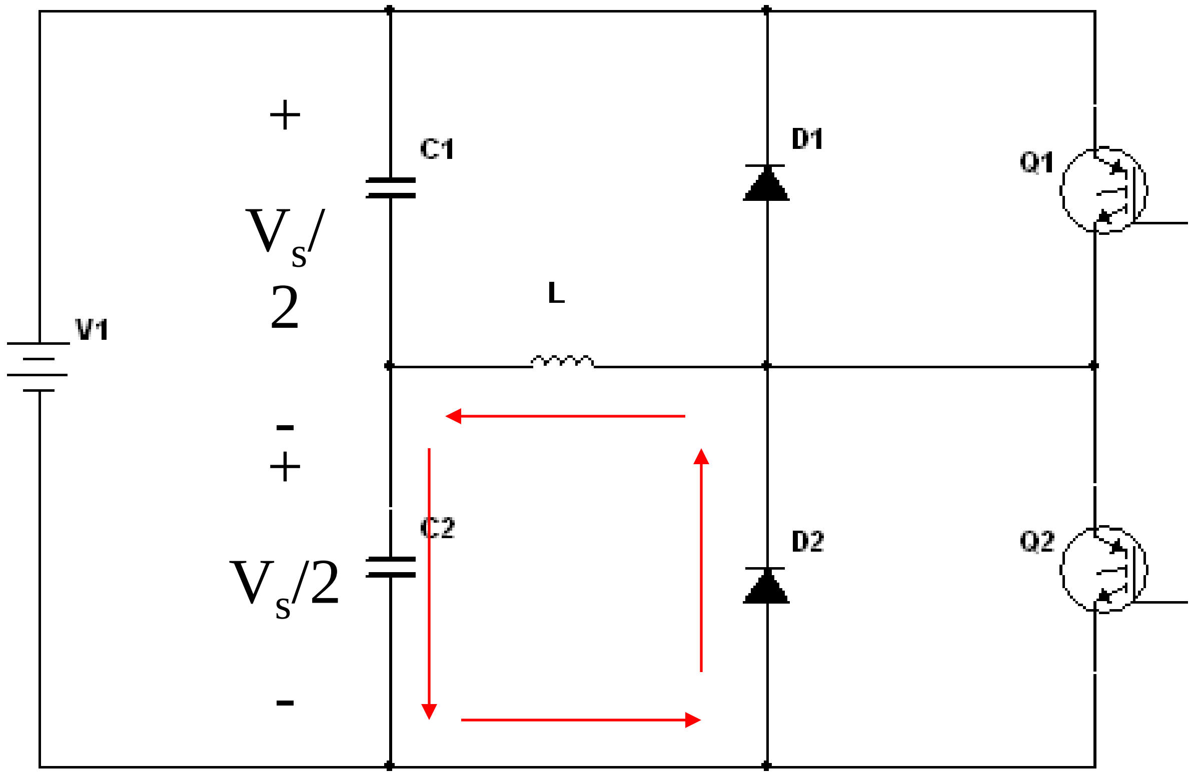
$$V_{o1} = \frac{1}{\sqrt{2}} \frac{2V_s}{\pi}$$

$$V_{o1} = 0.45V_s$$

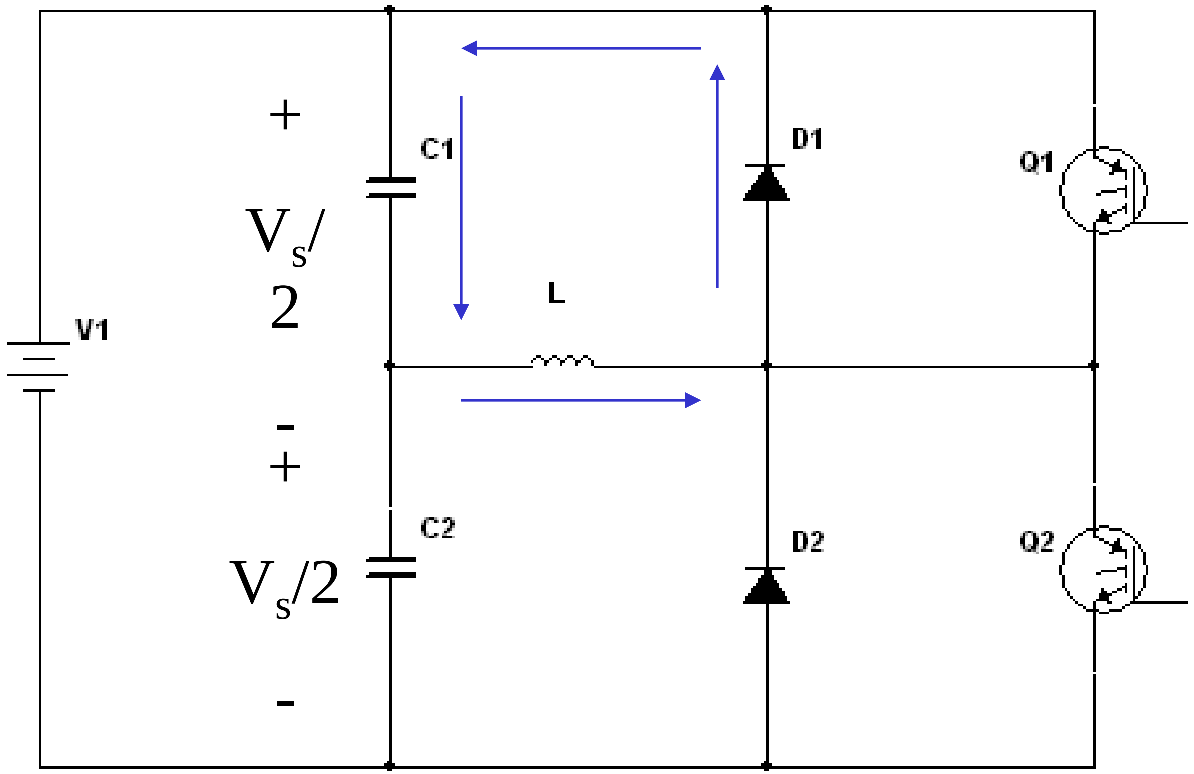
When the load is highly inductive



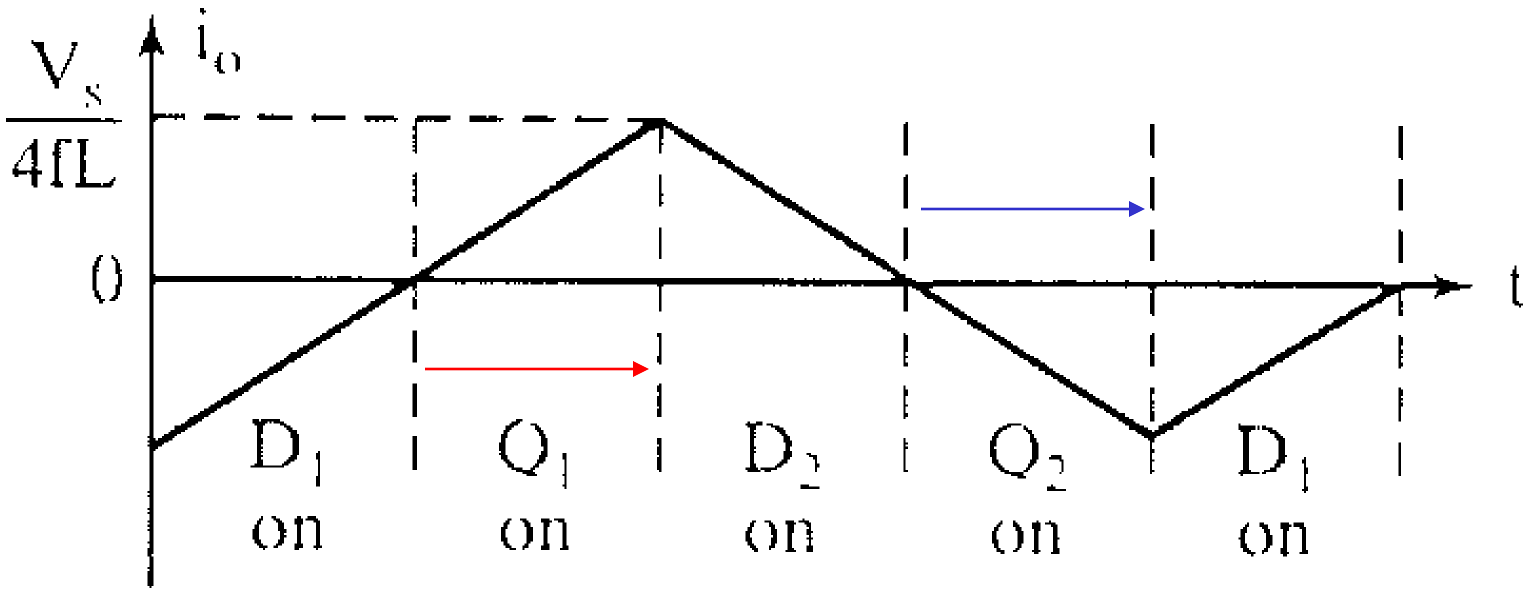
Turn off Q_1 at $t = T_o/2$
Current falls to 0 via D_2, L, $V_s/2$ lower



Turn off Q_2 at $t = T_o$
Current falls to 0 via D_1, L, $V_s/2$ upper



Load Current for a highly inductive load



Transistors are only switched on for a quarter-cycle, or 90°

Fourier Series of the output current for an RL load

$$i_o = \frac{v_o}{Z} = \frac{v_o}{R + jn\omega L} = \sum_{n=1,3,5,\ldots}^{\infty} \frac{2V_s}{n\pi\sqrt{R^2 + (n\omega L)^2}} \sin(n\omega t - \theta_n)$$

$$\theta_n = \tan^{-1}\left(\frac{n\omega L}{R}\right)$$

Fundamental Output Power

In most cases, the useful power

$$P_{o1} = V_{o1} I_{o1} \cos\theta_1 = I_{o1}^2 R$$

$$P_{o1} = \left[\frac{2V_s}{\sqrt{2}\pi\sqrt{R^2 + (\omega L)^2}} \right]^2 R$$

DC Supply Current

- If the inverter is lossless, average power absorbed by the load equals the average power supplied by the dc source.

$$\int_0^T v_s(t) i_s(t)\,dt = \int_0^T v_o(t) i_o(t)\,dt$$

- For an inductive load, the current is approximately sinusoidal and the fundamental component of the output voltage supplies the power to the load. Also, the dc supply voltage remains essentially at V_s.

DC Supply Current (continued)

$$\int_0^T i_s(t)dt = \frac{1}{V_s}\int_0^T \sqrt{2}V_{o1}\sin(\omega t)\sqrt{2}I_o\sin(\omega t-\theta_1)dt = I_s$$

$$I_s = \frac{V_{o1}}{V_s} I_o \cos(\theta_1)$$

Performance Parameters

- Harmonic factor of the nth harmonic (HF_n)

$$HF_n = \frac{V_{on}}{V_{o1}} \qquad \text{for } n>1$$

V_{on} = rms value of the nth harmonic component

V_{01} = rms value of the fundamental component

Performance Parameters (continued)

- Total Harmonic Distortion (THD)
- Measures the “closeness” in shape between a waveform and its fundamental component

$$THD = \frac{1}{V_{o1}} \left(\sum_{n=2,3,\ldots}^{\infty} V_{on}^{2} \right)^{\frac{1}{2}}$$

Performance Parameters (continued)

- Distortion Factor (DF)
- Indicates the amount of HD that remains in a particular waveform after the harmonics have been subjected to second-order attenuation.

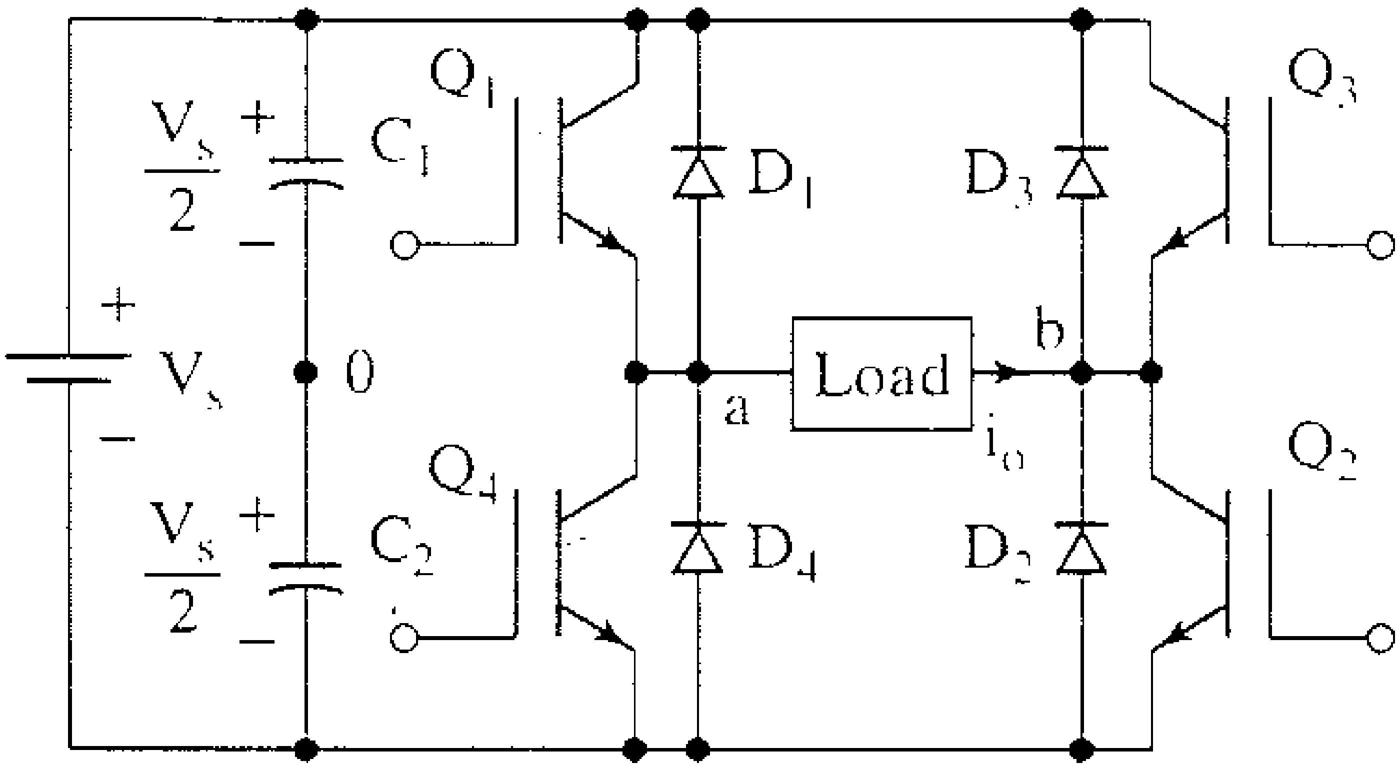
$$DF = \frac{1}{V_{o1}}\left[\sum_{n=2,3,\ldots}^{\infty}\left(\frac{V_{on}}{n^2}\right)^2\right]^{\frac{1}{2}}$$

$$DF_n = \frac{V_{on}}{V_{o1}n^2} \qquad \text{for n>1}$$

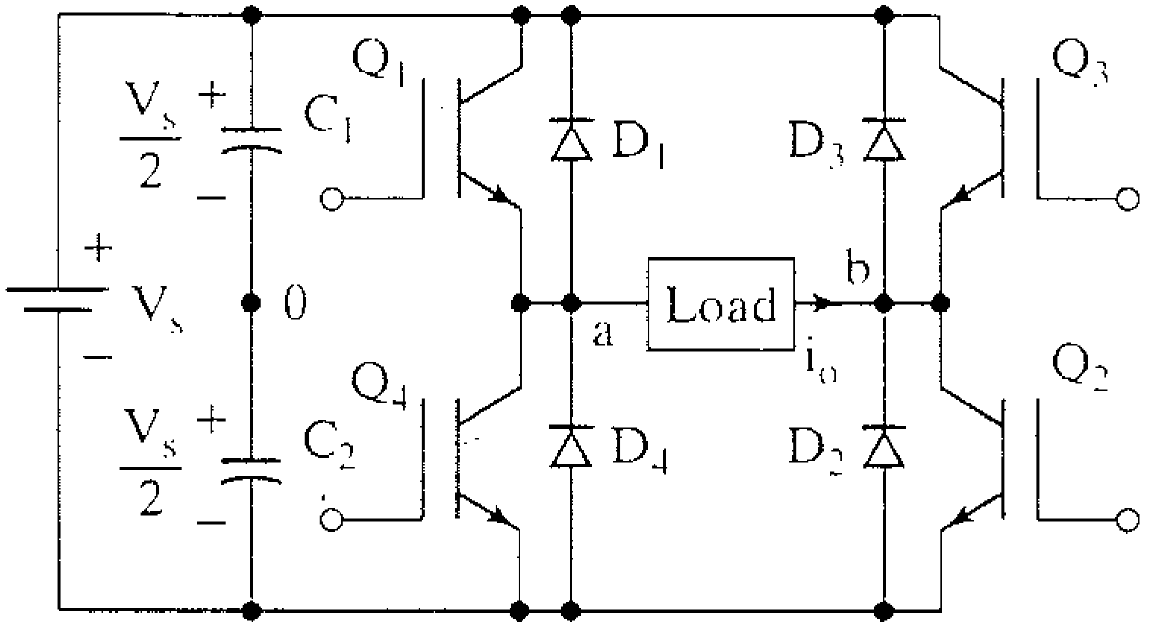
Performance Parameters (continued)

- Lowest order harmonic (LOH)
- The harmonic component whose frequency is closest to the fundamental, and its amplitude is greater than or equal to 3% of the amplitude of the fundamental component.

Single-phase full-bridge inverter

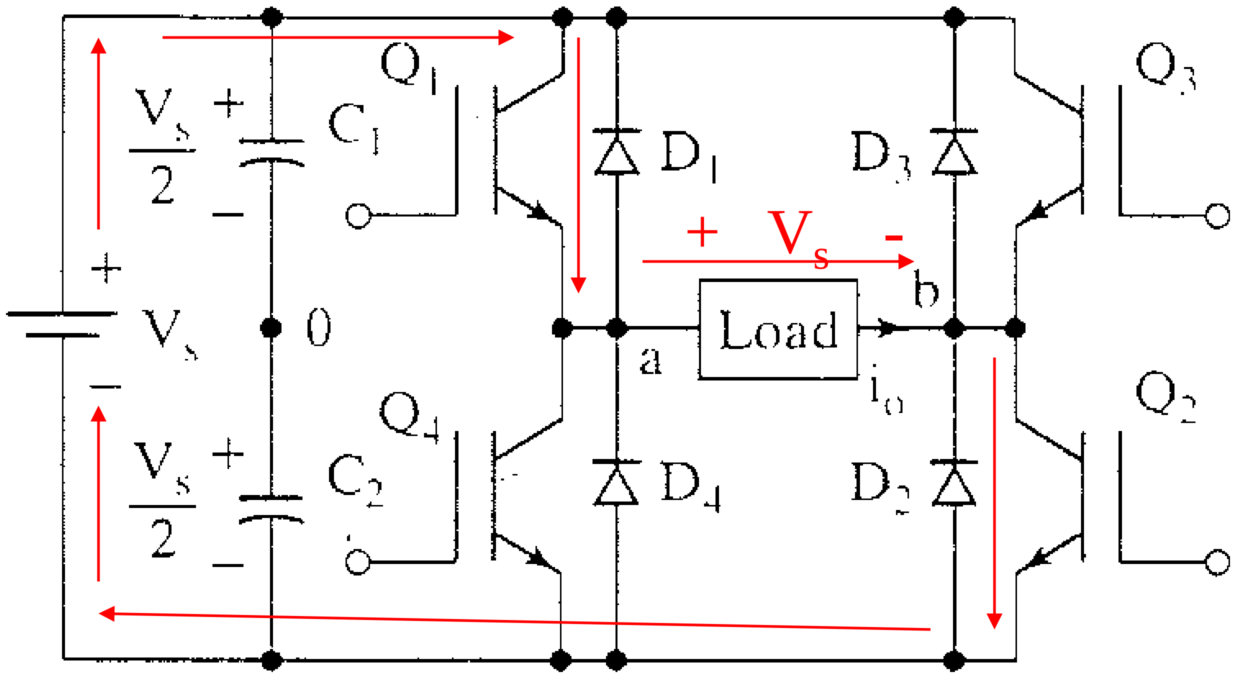


Operational Details

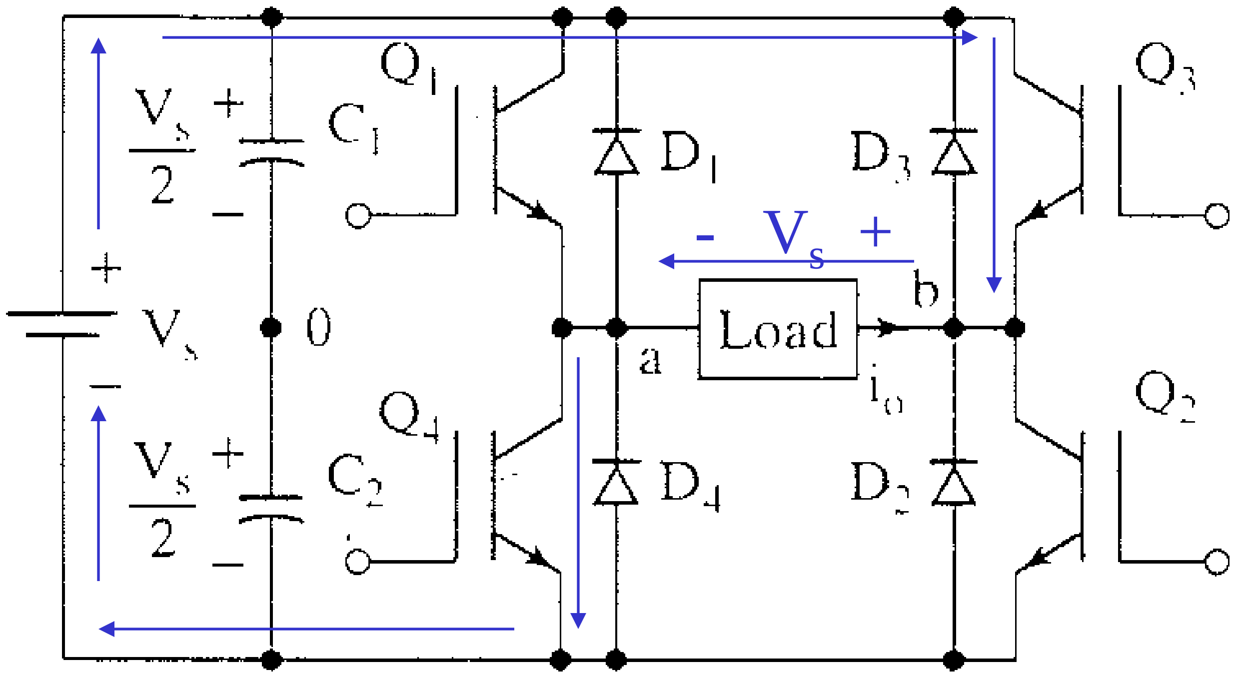


- Consists of 4 choppers and a 3-wire DC source
- Q_1-Q_2 and Q_3-Q_4 switched on and off alternately
- Need to isolate the gate signal for Q_1 and Q_3 (upper)
- Each pair provide opposite polarity of $V_s$across the load

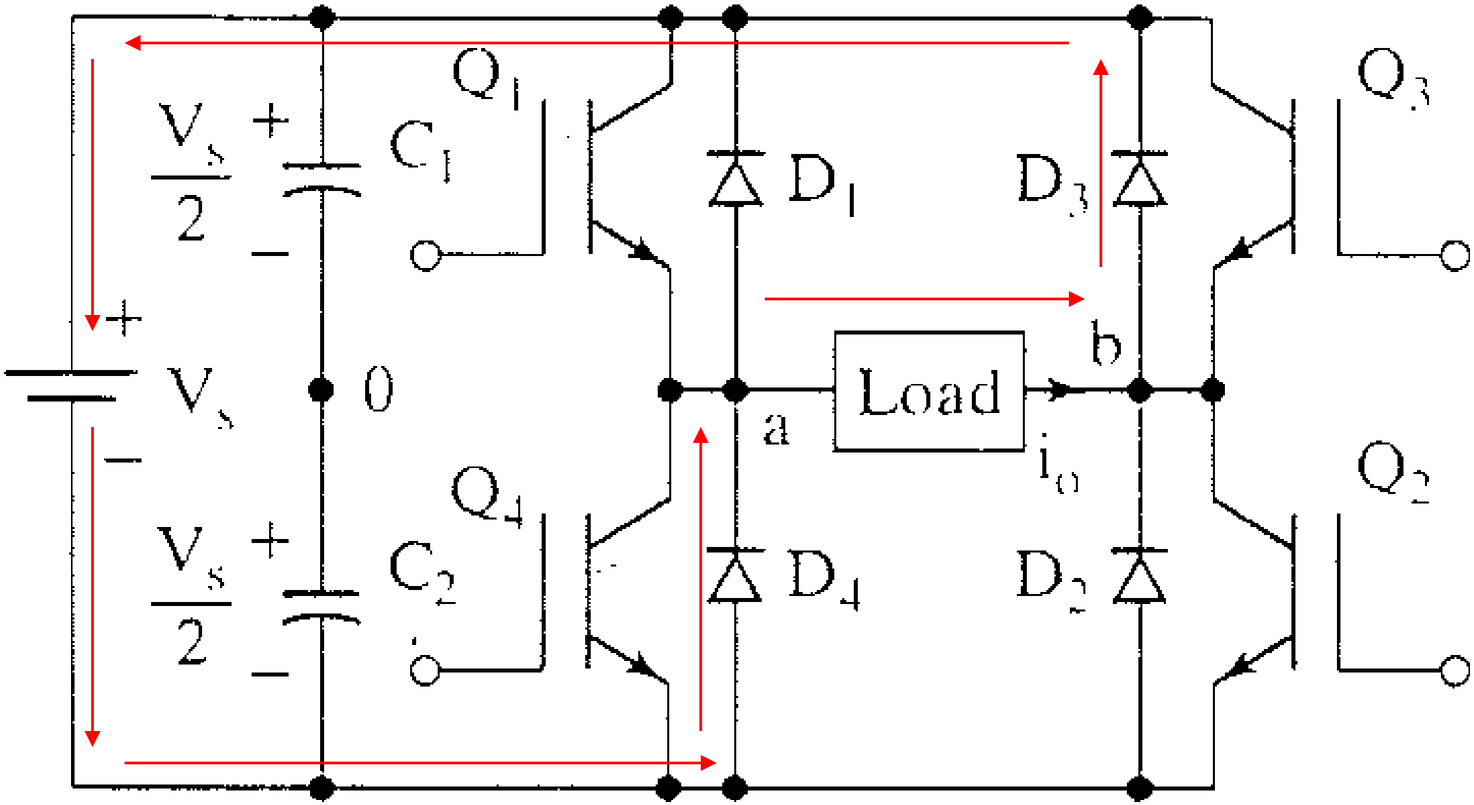
Q_1-Q_2 on, Q_3-Q_4 off, $v_o = V_s$



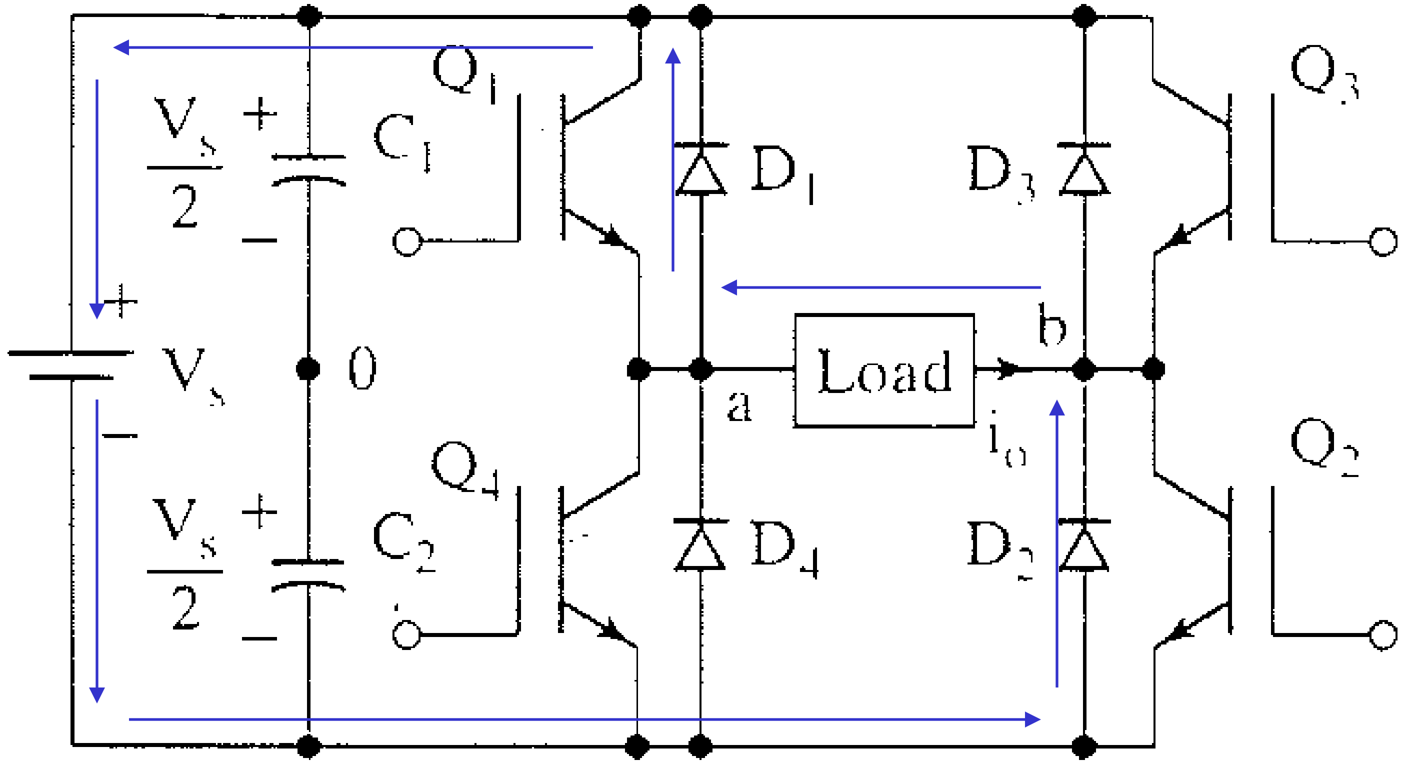
Q_3-Q_4 on, Q_1-Q_2 off, v_o = -V_s



When the load is highly inductive
Turn Q_1-Q_2 off – Q_3-Q_4 off



Turn Q_3-Q_4 off – Q_1-Q_2 off



Load current for a highly inductive load