DC TRANSIENT ANALYSIS

SUB - TOPICS

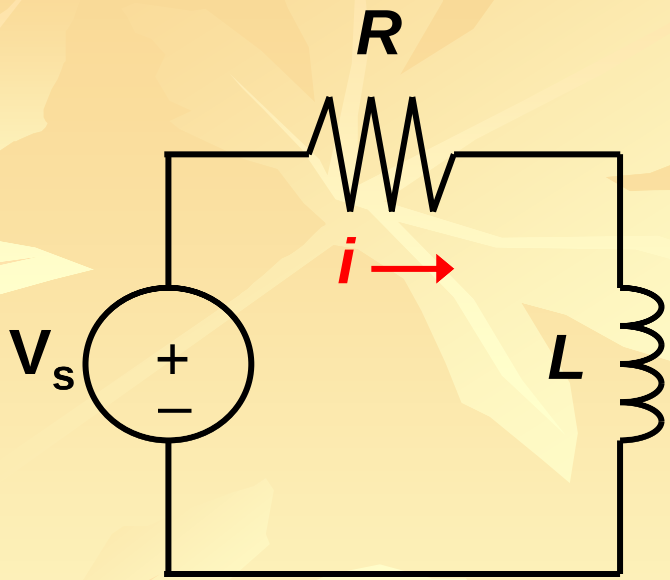
- NATURAL RESPONSE OF RL CIRCUIT
- NATURAL RESPONSE OF RC CIRCUIT
- STEP RESPONSE OF RL CIRCUIT
- STEP RESPONSE OF RC CIRCUIT

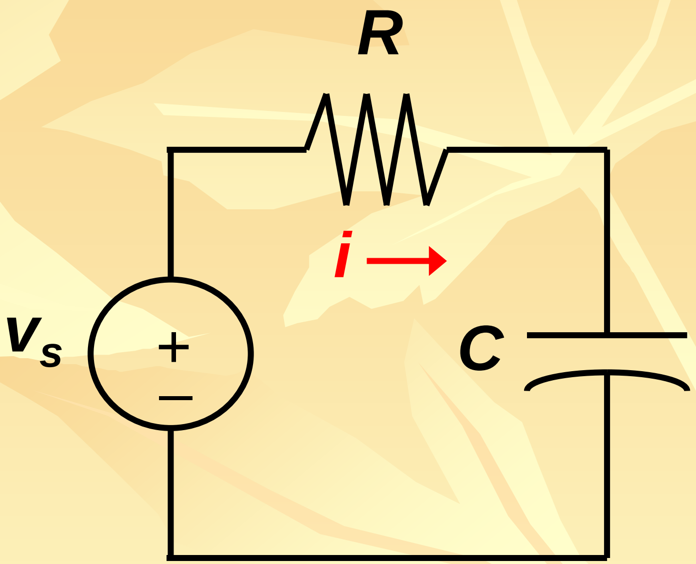
OBJECTIVES

- To investigate the behavior of currents and voltages when energy is either released or acquired by inductors and capacitors when there is an abrupt change in dc current or voltage source.
- To do an analysis of natural response and step response of RL and RC circuit.

FIRST – ORDER CIRCUITS

- A circuit that contains only sources, resistor and inductor is called and RL circuit.
- A circuit that contains only sources, resistor and capacitor is called an RC circuit.
- RL and RC circuits are called first – order circuits because their voltages and currents are describe by first order differential equations.



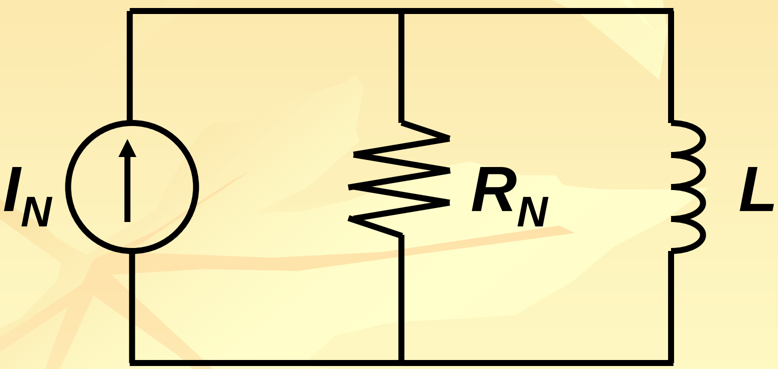


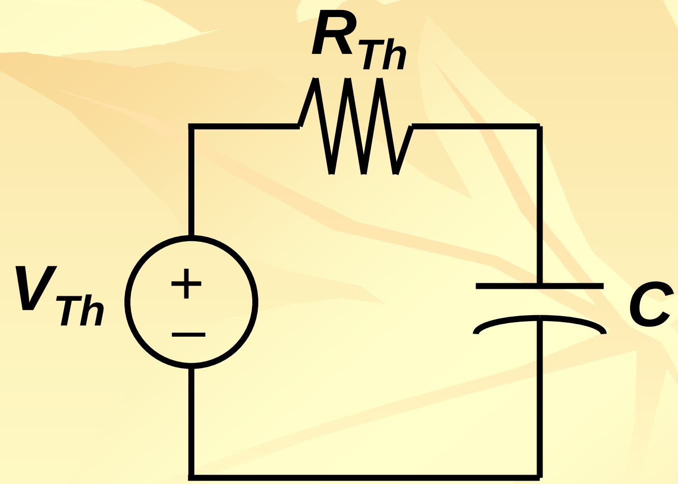
An RL circuit

An RC circuit

Review (conceptual)

- Any first – order circuit can be reduced to a Thévenin (or Norton) equivalent connected to either a single equivalent inductor or capacitor.



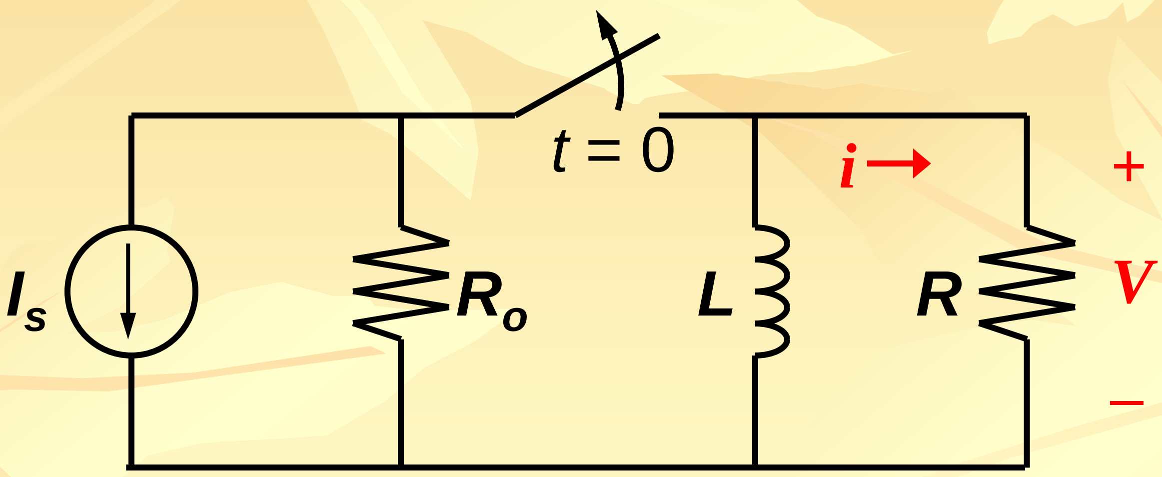


- In steady state, an inductor behave like a short circuit.
- In steady state, a capacitor behaves like an open circuit.

- The natural response of an RL and RC circuit is its behavior (i.e., current and voltage) when stored energy in the inductor or capacitor is released to the resistive part of the network (containing no independent sources)
- The steps response of an RL and RC circuits is its behavior when a voltage or current source step is applied to the circuit, or immediately after a switch state is changed.

NATURAL RESPONSE OF AN RL CIRCUIT

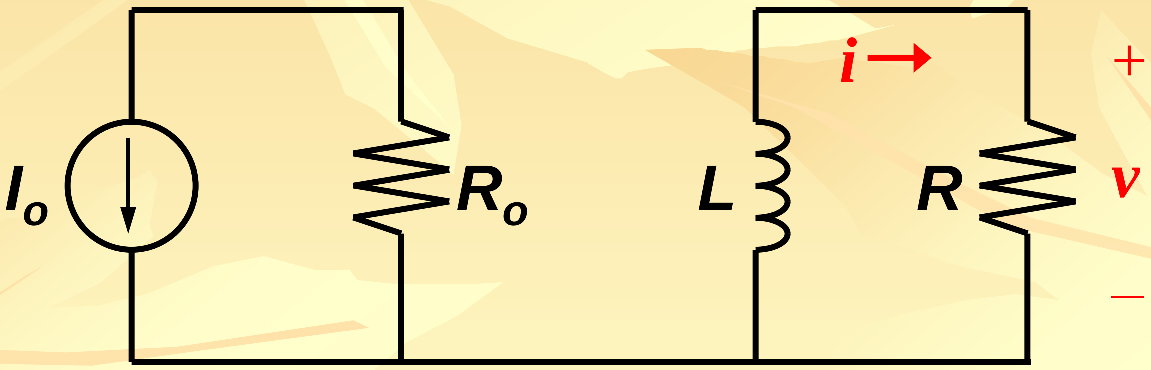
- Consider the following circuit, for which the switch is closed for $t<0$, and then opened at $t = 0$:



- The dc voltage V, has been supplying the RL circuit with constant current for a long time

Solving for the circuit

- For $t \leq 0$, $i(t) = I_o$
- For $t \geq 0$, the circuit reduce to



<u>Notation:</u>

- 0^- is used to denote the time just prior to switching.
- 0^+ is used to denote the time immediately after switching.

Continue…

- Applying KVL to the circuit:

$$v(t) + Ri(t) = 0 \quad \text{———————} \quad \textbf{(1)}$$

$$L\frac{di(t)}{dt} + Ri(t) = 0 \quad \text{———————} \quad \textbf{(2)}$$

$$L\frac{di(t)}{dt} = -Ri(t) \quad \text{———————} \quad \textbf{(3)}$$

$$\frac{di(t)}{i(t)} = -\frac{R}{L}dt \quad \text{———————} \quad \textbf{(4)}$$

Continue

- From equation (4), let say;

$$\frac{du}{u} = -\frac{R}{L}dv \qquad \text{———————} \textbf{(5)}$$

- Integrate both sides of equation (5);

$$\int_{i(t_o)}^{i(t)} \frac{du}{u} = -\frac{R}{L}\int_{t_o}^{t} dv \qquad \text{———————} \textbf{(6)}$$

- Where:

- *$i(t_o)$* is the current corresponding to time *t_o*
- *$i(t)$* ia the current corresponding to time *t*

Continue

- Therefore,

$$\ln\frac{i(t)}{i(0)} = -\frac{R}{L}t \qquad \text{——————} \qquad \textbf{(7)}$$

- hence, the current is

$$\boxed{i(t) = i(0)e^{-(R/L)t} = I_0 e^{-(R/L)t}}$$

Continue

- From the Ohm's law, the voltage across the resistor R is:

$$\boxed{v(t) = i(t)R = I_0 \, \mathrm{R} e^{-(R/L)t}}$$

- And the power dissipated in the resistor is:

$$\boxed{p = v_R i(t) = I_0^2 \, \mathrm{R} e^{-2(R/L)t}}$$

Continue

- Energy absorb by the resistor is:

$$\boxed{w = \frac{1}{2} L I_0^2 \left(1 - e^{-2(R/L)t}\right)}$$

Time Constant, τ

- Time constant, τ determines the rate at which the current or voltage approaches zero.

- Time constant, $\boxed{\tau = \dfrac{L}{R}}$ (sec)

- The expressions for current, voltage, power and energy using time constant concept:

$$i(t) = I_0 e^{-t/\tau}$$

$$v(t) = I_0 \,\mathrm{R} e^{-t/\tau}$$

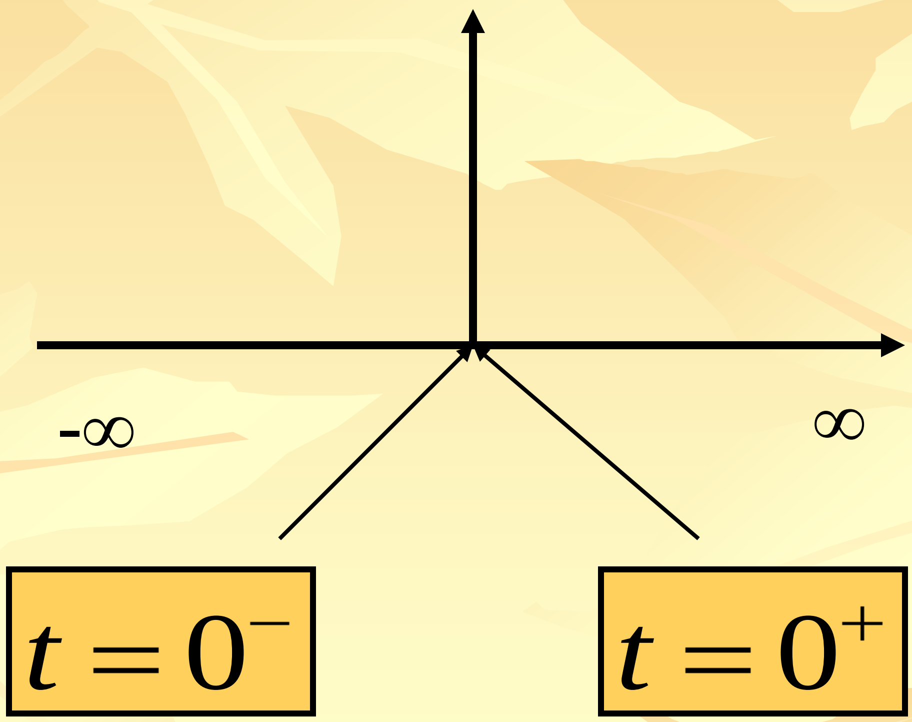
$$p = I_0^2 \,\mathrm{R} e^{-2t/\tau}$$

$$w = \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau})$$

Switching time

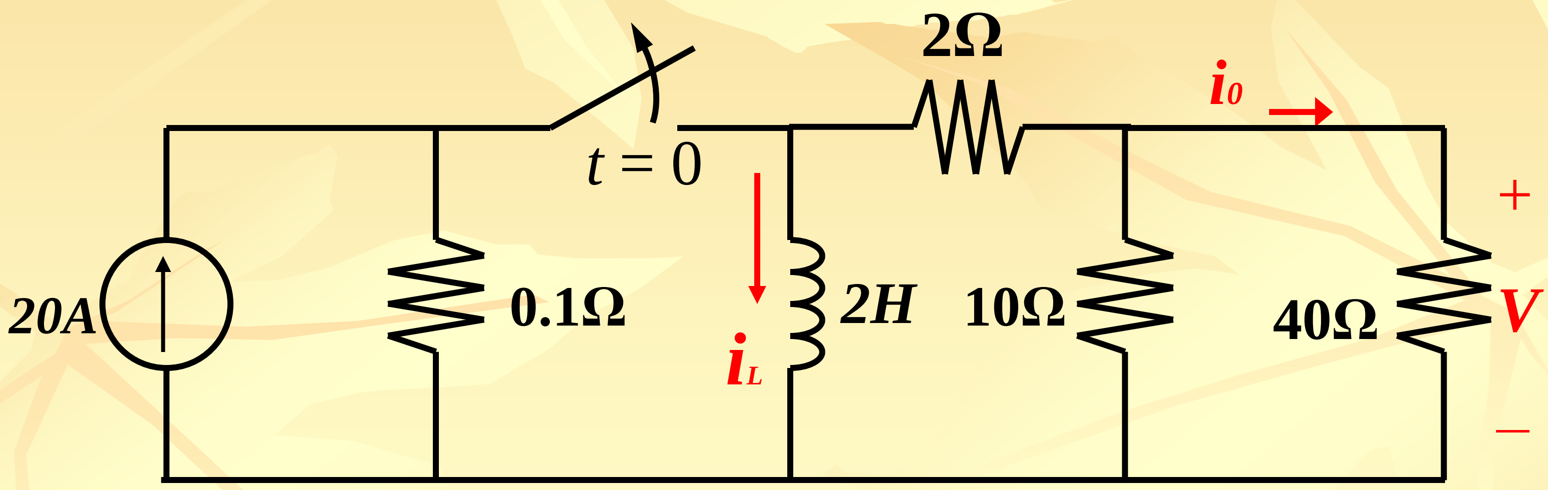
- For all transient cases, the following instants of switching times are considered.
- ✓ $t = 0^-$, this is the time of switching between $-\infty$ to 0 or time before.
- ✓ $t = 0^+$, this is the time of switching at the instant just after time t = 0s (taken as initial value)
- ✓ $t = \infty$, this is the time of switching between $t = 0^+$ to ∞ (taken as final value for step response)

- The illustration of the different instance of switching times is:



Example

- For the circuit below, find the expression of $i_o(t)$ and $V_o(t)$. The switch was closed for a long time, and at *t = 0*, the switch was opened.



Solution :

Step 1:

Find τ for $t > 0$. Draw the equivalent circuit. The switch is opened.

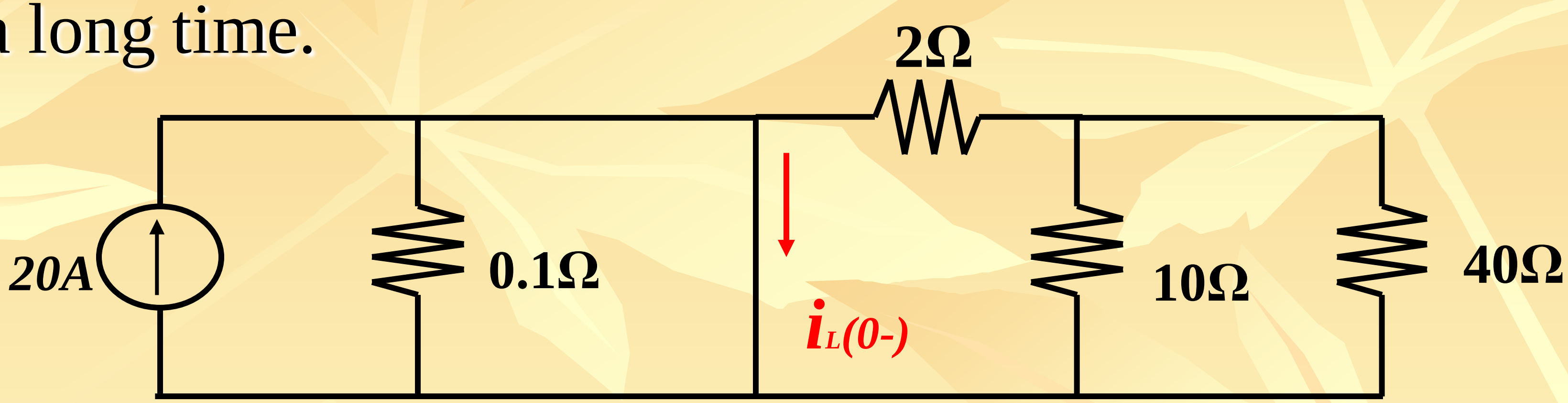
$$\boxed{R_T = (2 + 10 // 40) = 10\Omega}$$

So;

$$\boxed{\tau = \frac{L}{R_T} = \frac{2}{10} = 0.2} \text{ sec}$$

Step 2:

At $t = 0^-$, time from $-\infty$ to 0^-, the switch was closed for a long time.

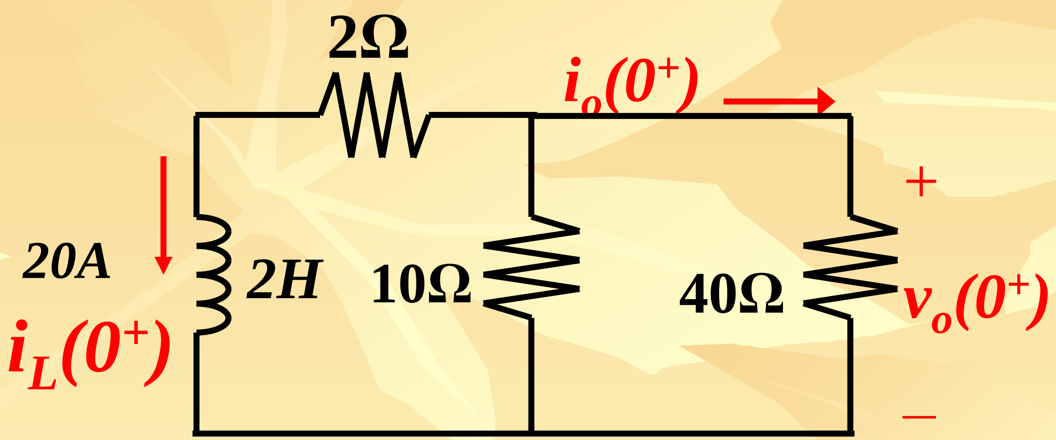


The inductor behave like a short circuit as it being supplied for a long time by a dc current source. Current 20A thus flows through the short circuit until the switch is opened.

Therefore; $i_L(0^-) = 20A$

Step 3:

At the instant when the switch is opened, the time $t = 0^+$,



The current through the inductor remains the same (continuous).

Thus, $\boxed{i_L(0^+) = i_L(0^-) = 20A}$ which is the initial current.

Only at this particular instant the value of the current through the inductor is the same.

Since, there is no other supply in the circuit after the switch is opened, this is the natural response case.

By using current division, the current in the 40Ω resistor is:

$$i_o = -i_L \frac{10}{10+40} = -4A$$

So,

$$\boxed{i_o(t) = -4e^{-5t} A}$$

Using Ohm's Law, the V_o is:

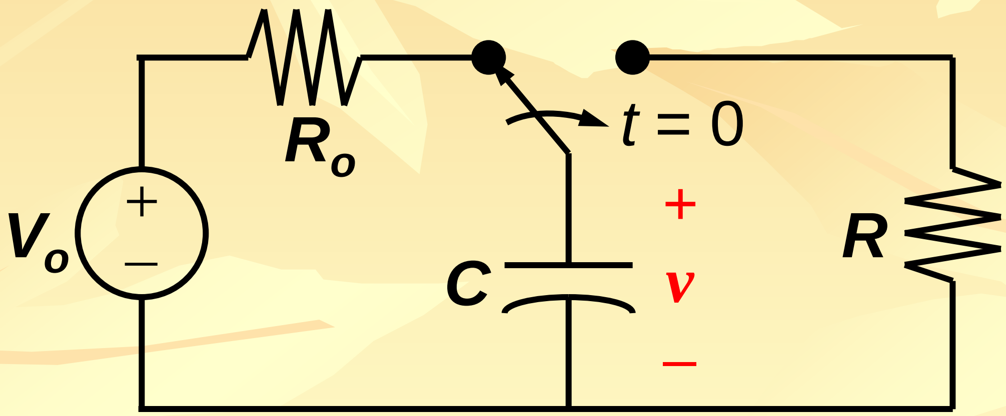
$$V_o(t) = -4 \times 40 = -160$$

So,

$$\boxed{V_0(t) = -160e^{-5t}}$$

NATURAL RESPONSE OF AN RC CIRCUIT

- Consider the following circuit, for which the switch is closed for $t < 0$, and then opened at $t = 0$:

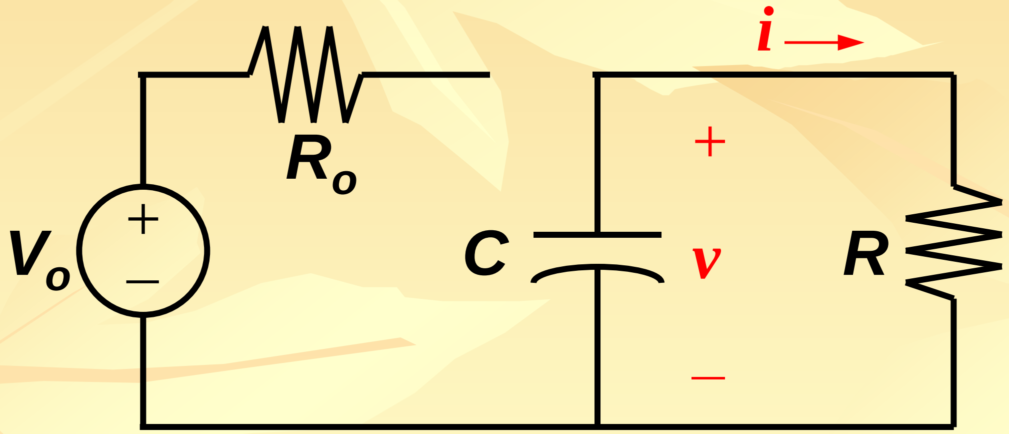


<u>Notation:</u>

- 0^- is used to denote the time just prior to switching
- 0^+ is used to denote the time immediately after switching.

Solving for the voltage $(t \geq 0)$

- For $t \leq 0$, $v(t) = V_o$
- For t > 0, the circuit reduces to



Continue

- Applying KCL to the RC circuit:

$$i_C + i_R = 0 \longrightarrow \textbf{(1)}$$

$$C\frac{dv(t)}{dt} + \frac{v(t)}{R} = 0 \longrightarrow \textbf{(2)}$$

$$\frac{dv(t)}{dt} + \frac{v(t)}{RC} = 0 \longrightarrow \textbf{(3)}$$

$$\frac{dv(t)}{dt} = -\frac{v(t)}{RC} \longrightarrow \textbf{(4)}$$

$$\frac{dv(t)}{v(t)} = -\frac{1}{RC}dt \longrightarrow \textbf{(5)}$$

Continue

- From equation (5), let say:

$$\frac{dx}{x} = -\frac{1}{RC}dy \longrightarrow \textbf{(6)}$$

- Integrate both sides of equation (6):

$$\int_{V_o}^{v(t)} \frac{1}{x} du = -\frac{1}{RC}\int_{0}^{t} dy \longrightarrow \textbf{(7)}$$

- Therefore:

$$\ln\frac{v(t)}{V_o} = -\frac{t}{RC} \longrightarrow \textbf{(8)}$$

Continue

- Hence, the voltage is:

$$\boxed{v(t) = v(0)e^{-t/RC} = V_o e^{-t/RC}}$$

- Using Ohm's law, the current is:

$$\boxed{i(t) = \frac{v(t)}{R} = \frac{V_o}{R} e^{-t/RC}}$$

Continue

- The power dissipated in the resistor is:

$$p(t) = v i_R = \frac{V_o^2}{R} e^{-2t/RC}$$

- The energy absorb by the resistor is:

$$w = \frac{1}{2} C V_o^2 (1 - e^{-2t/RC})$$

Continue

- The time constant for the RC circuit equal the product of the resistance and capacitance,

- Time constant, $\boxed{\tau = RC}$ sec

- The expressions for voltage, current, power and energy using time constant concept:

$$v(t) = V_o e^{-t/\tau}$$

$$i(t) = \frac{V_o}{R} e^{-t/\tau}$$

$$p(t) = \frac{V_o^2}{R} e^{-2t/\tau}$$

$$w(t) = \frac{1}{2} C V_o^2 (1 - e^{-2t/\tau})$$

- For the case of capacitor, two important observation can be made,

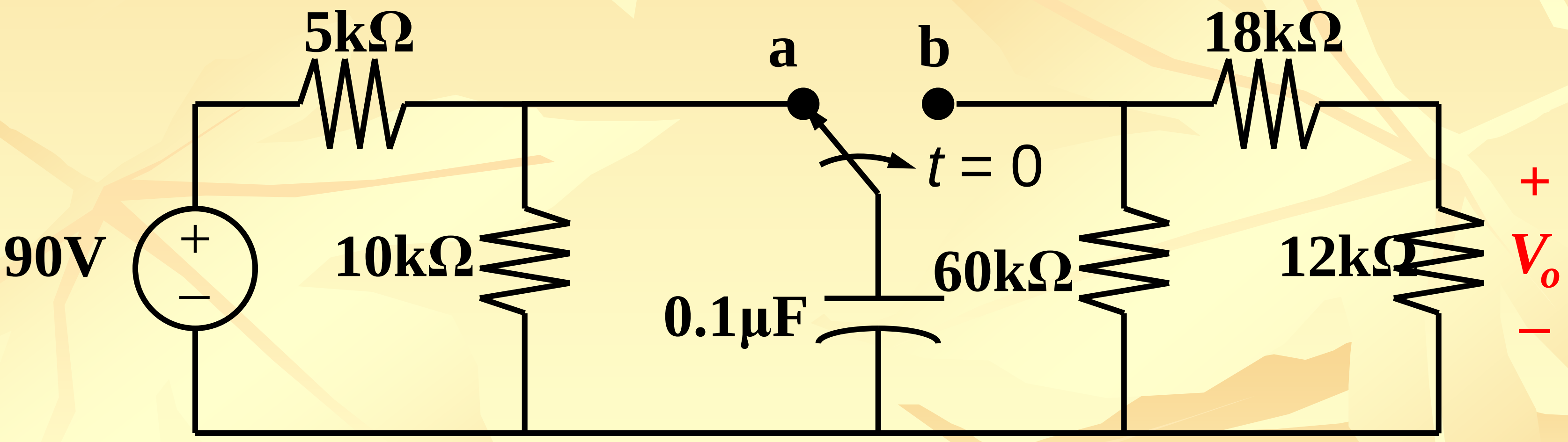
1) capacitor behaves like an open circuit when being supplied by dc source

(From, i_c = Cdv/dt, when v is constant, dv/dt = 0. When current in circuit is zero, the circuit is open circuit.)

2) in capacitor, the voltage is continuous / stays the same that is, $V_c(0^+) = V_c(0^-)$

Example

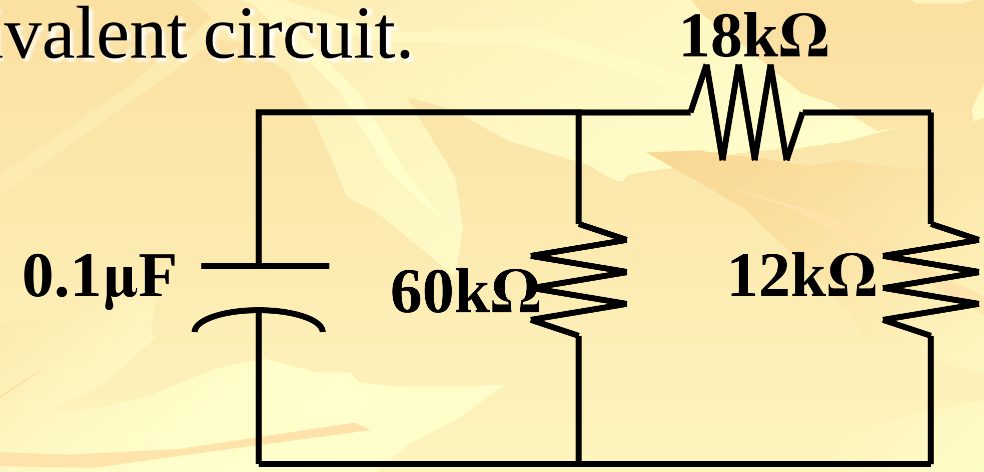
The switch has been in position *a* for a long time. At Time t = 0, the switch moves to *b*. Find the expressions for the $v_c(t)$, $i_c(t)$ and $v_o(t)$ and hence sketch them for t = 0 to t = 5τ.



Solution

Step 1:

Find t for t > 5τ that is when the switch was at *a*. Draw the equivalent circuit.

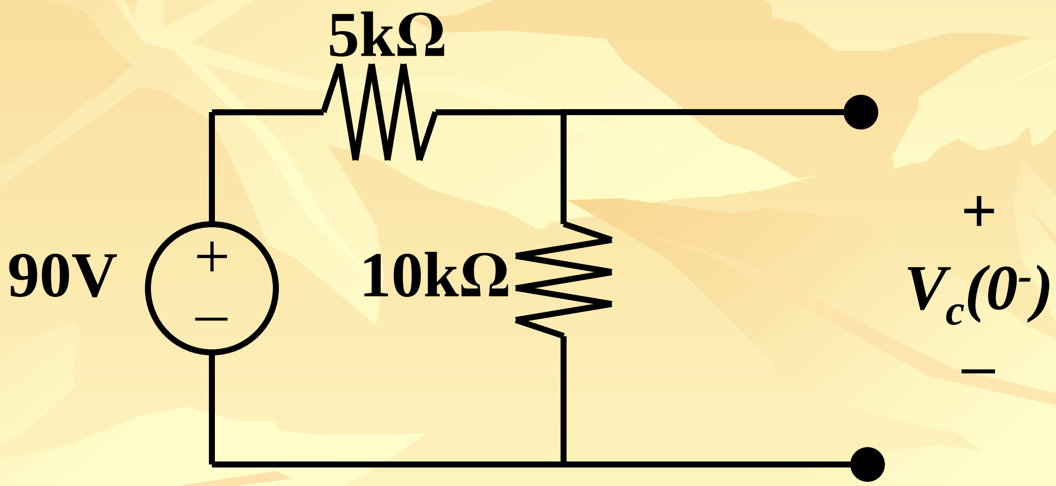


$$R_T = (18k\Omega + 12k\Omega) // 60k\Omega = 20k\Omega$$

$$\tau = R_T C = 20\times10^3 \times 0.1\times10^{-6} = 2ms$$

Step 2:

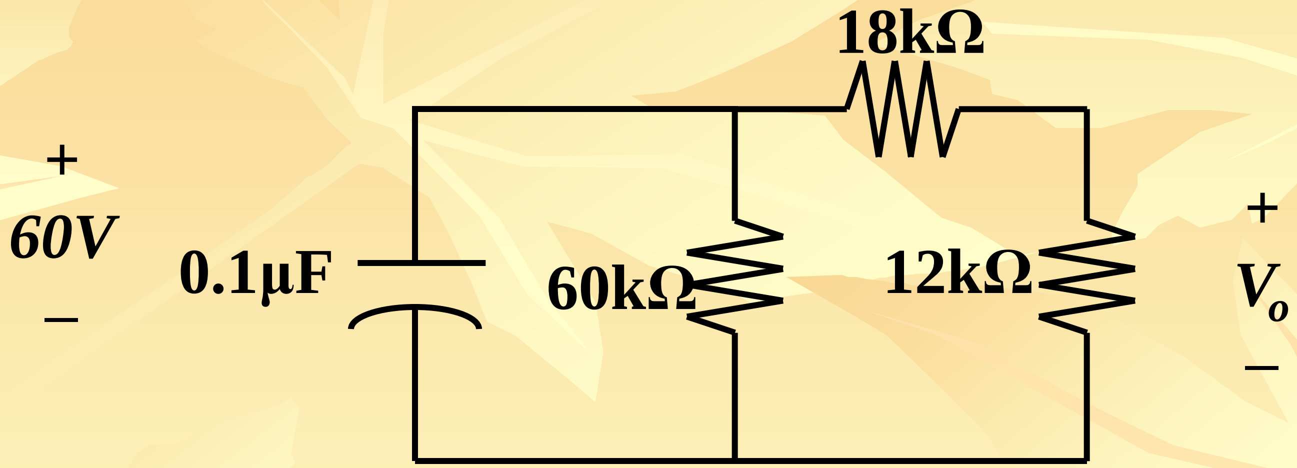
At t = 0, the switch was at *a*. the capacitor behaves like An open circuit as it is being supplied by a constant source.



$$v_c(0^-) = \frac{10}{15} \times 90 = 60V$$

Step 3:

At $t = 0^+$, the instant when the switch is at *b*.



The voltage across capacitor remains the same at this particular instant.

$$v_c(0^+) = v_c(0^-) = 60V$$

Using voltage divider rule,

$$V_o(0^+) = \frac{12}{30} \times 60 = 24V$$

Hence;

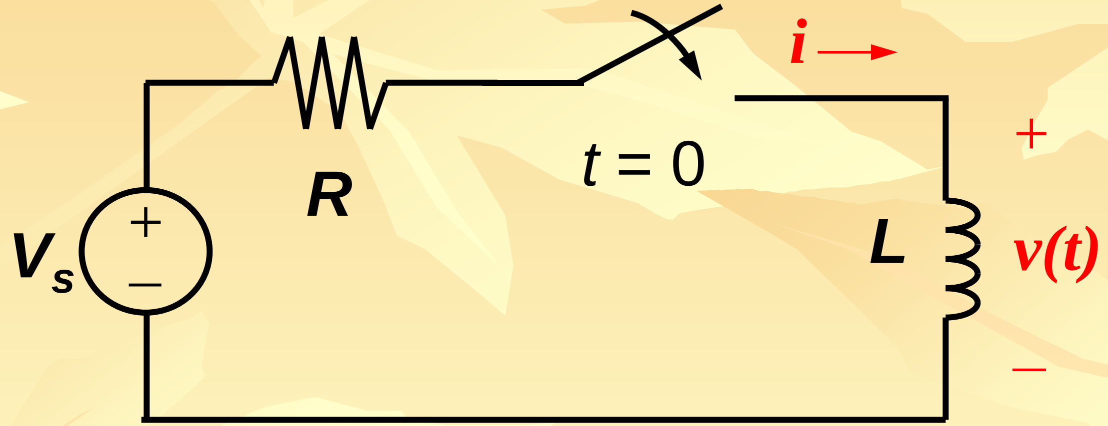
$$\boxed{\begin{aligned} v_c(t) &= 60e^{-500t}\mathrm{V} \\ v_o(t) &= 24e^{-500t}\mathrm{V} \\ i_c(t) &= -0.03e^{-500t}\,A \end{aligned}}$$

Summary

No	RL circuit	RC circuit
1	$\tau = \frac{L}{R}$	$\tau = RC$
2	Inductor behaves like a short circuit when being supplied by dc source for a long time	Capacitor behaves like an open circuit when being supplied by dc source for a long time
3	Inductor current is continuous $i_L(0^+) = i_L(0^-)$	Voltage across capacitor is continuous $v_C(0^+) = v_C(0^-)$

Step Response of RL Circuit

- The switch is closed at time t = 0.



- After switch is closed, using KVL

$$V_s = Ri(t) + L\frac{di}{dt} \longrightarrow \textbf{(1)}$$

Continue

- Rearrange the equation;

$$\frac{di(t)}{dt} = \frac{-Ri(t)+V_s}{L} = \frac{-R}{L}\left(i(t)-\frac{V_s}{R}\right) \longrightarrow \textbf{(2)}$$

$$di = \frac{-R}{L}\left(i-\frac{V_s}{R}\right)dt \longrightarrow \textbf{(3)}$$

$$\frac{-R}{L}dt = \frac{di}{i(t)-V_s/R} \longrightarrow \textbf{(4)}$$

$$-\frac{R}{L}\int_0^t dv = \int_0^{i(t)} \frac{du}{u-(V_s/R)} \longrightarrow \textbf{(5)}$$

Continue

- Therefore:

$$-\frac{R}{L}t = \ln\frac{i(t)-(V_s/R)}{I_0-(V_s/R)} \longrightarrow \textbf{(5)}$$

- Hence, the current is;

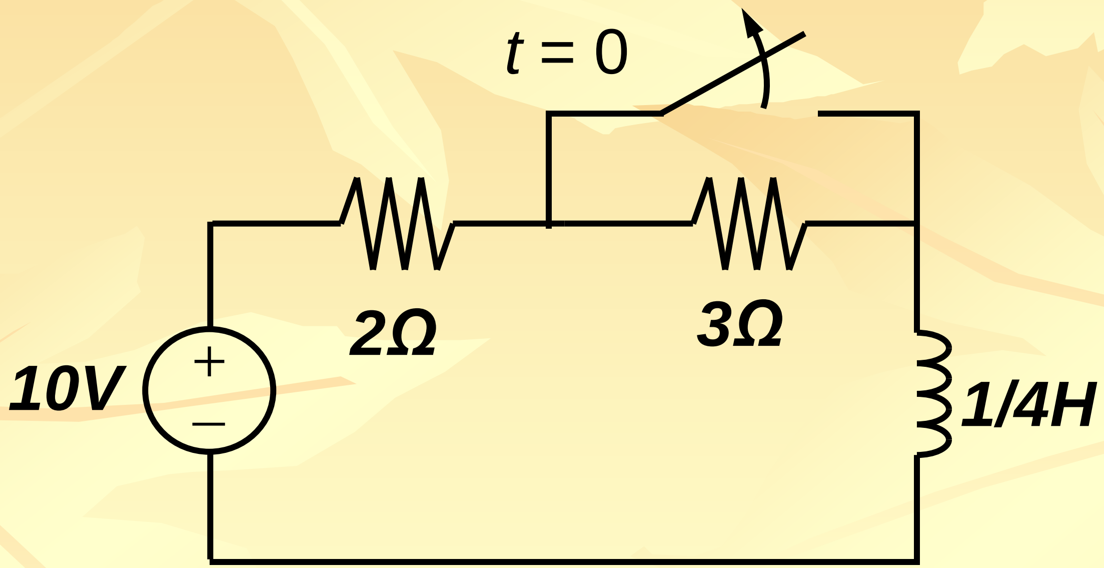
$$\boxed{i(t) = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-(R/L)t}}$$

- The voltage;

$$\boxed{v(t) = (V_s - I_o R)e^{-(R/L)t}}$$

Example

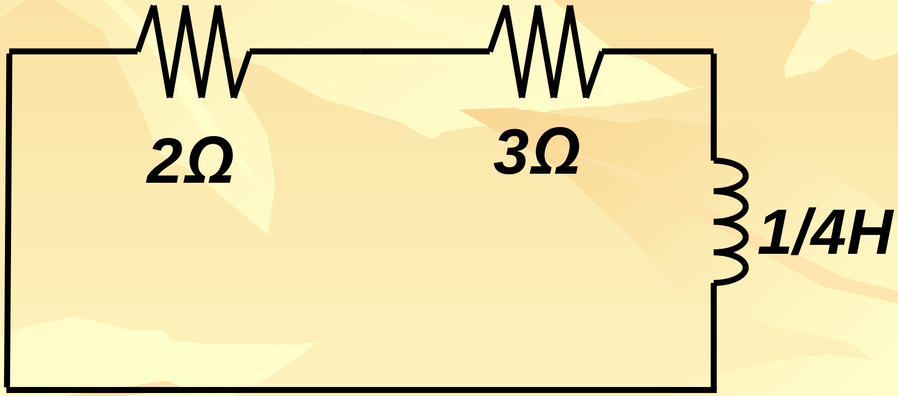
The switch is closed for a long time at t = 0, the switch opens. Find the expressions for $i_L(t)$ and $v_L(t)$.



Solution

Step 1:

Find τ for t > 0. The switch was opened. Draw the equivalent circuit. Short circuit the voltage source.



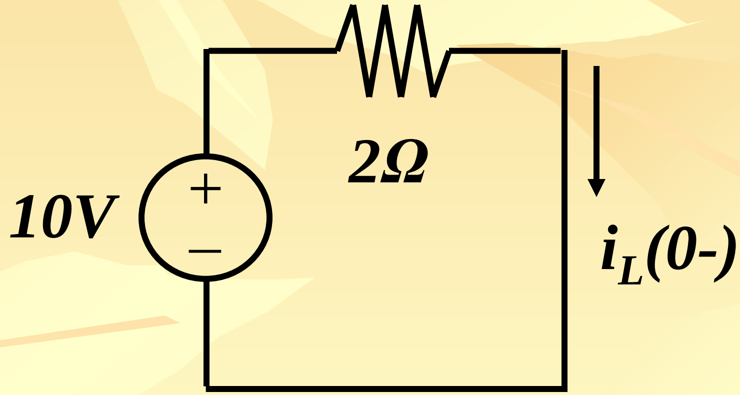
$$R_T = (2+3)\Omega = 5\Omega$$

$$\tau = \frac{L}{R_T} = \frac{1}{20}s$$

Continue

Step 2:

At $t = 0^-$, the switch was closed. Draw the equivalent circuit with 3Ω shorted and the inductor behaves like a short circuit.



$$i_L(0^-) = 10/2 = 5A$$

Continue

Step 3:

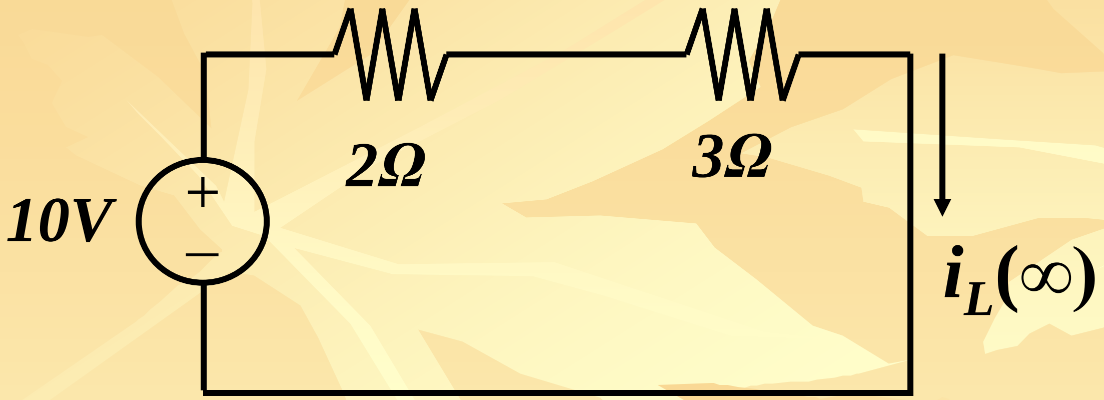
At t = 0^+, the instant switch was opened. The current in inductor is continuous.

$$I_0 = i_L(0^+) = i_L(0^-) = 5A$$

Step 4:

At t =∞, that is after a long time the switch has been left opened. The inductor will once again be behaving like a short circuit.

Continue



$$i_L(\infty) = V_s / R_T = 2A$$

Hence:

$$i_L(t) = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R} \right) e^{-(R/L)t}$$

$$\boxed{i_L(t) = 2 + 3e^{-20t}\, A}$$

Continue

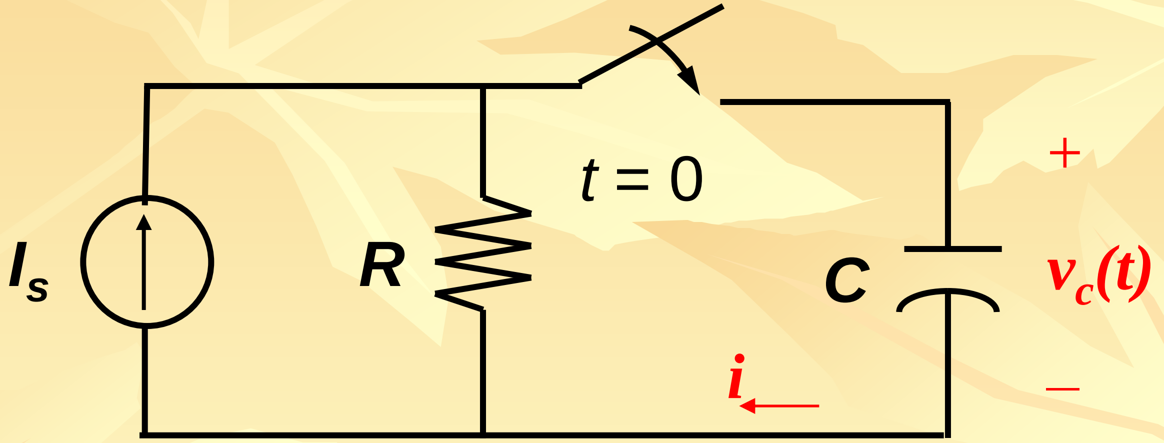
- And the voltage is:

$$v_L(t) = (V_s - I_o R)e^{-(R/L)t}$$

$$\boxed{v_L(t) = -15e^{-20t} V}$$

Step Response of RL Circuit

- The switch is closed at time t = 0



- From the circuit;

$$\boxed{I_s = C\frac{dv_c}{dt} + \frac{v_c}{R}} \longrightarrow \textbf{(1)}$$

Continue

- Division of Equation (1) by C gives;

$$\boxed{\frac{I_s}{C} = \frac{dv_c}{dt} + \frac{v_c}{RC}} \longrightarrow \textbf{(2)}$$

- Same mathematical techniques with RL, the voltage is:

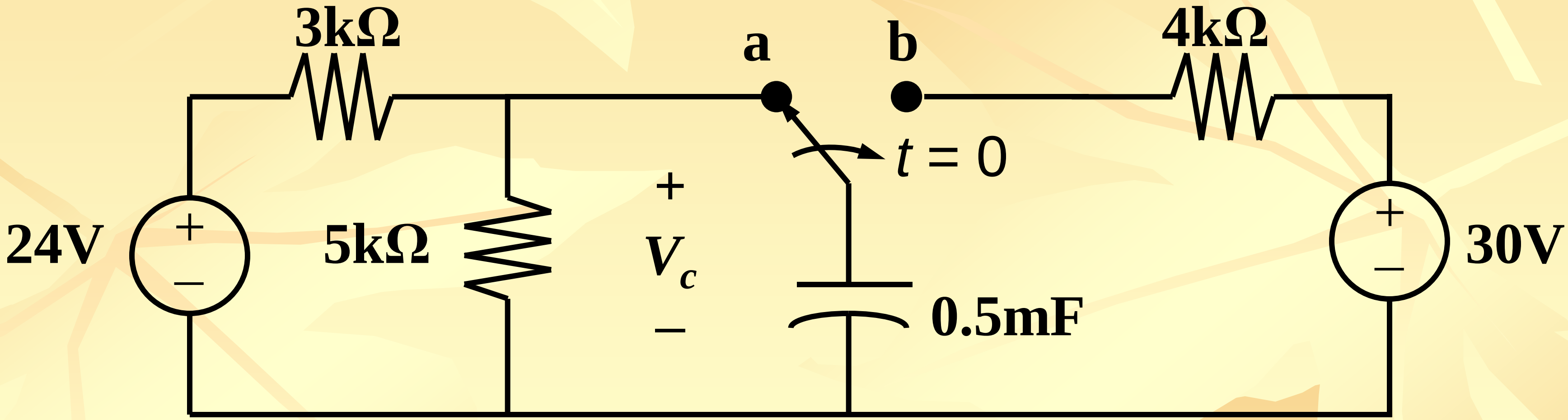
$$\boxed{v_c(t) = I_s R + (V_o - I_s R)e^{-t/RC}}$$

- And the current is:

$$\boxed{i(t) = \left(I_s - \frac{V_o}{R} \right) e^{-t/RC}}$$

Example

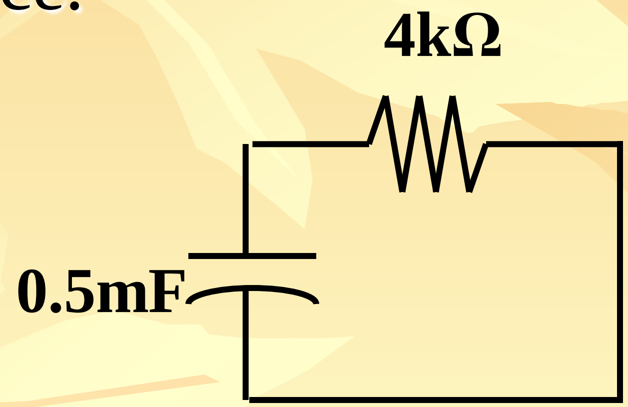
The switch has been in position *a* for a long time. At t = 0, the switch moves to *b*. Find $V_c(t)$ for t > 0 and calculate its value at t = 1s and t = 4s.



Solution

Step 1:

To find τ for t > 0, the switch is at *b* and short circuit the voltage source.

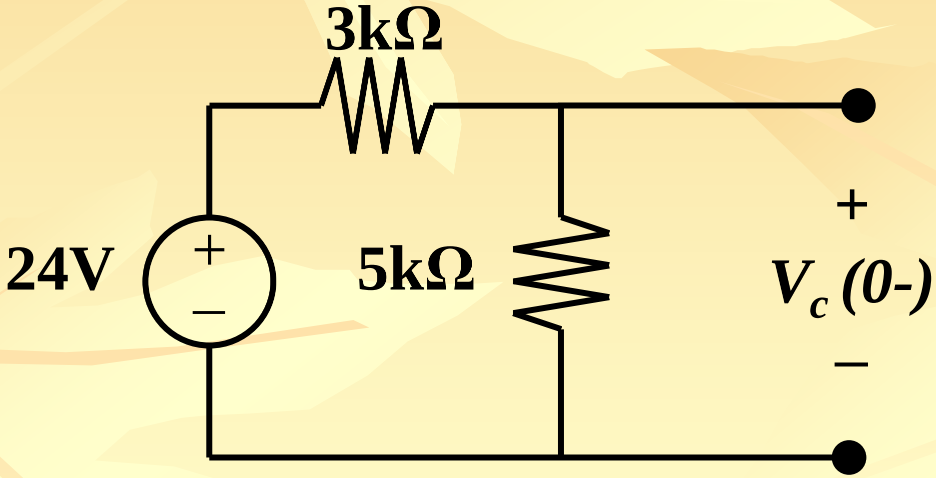


$$\tau = RC = 2s$$

Continue

Step 2:

The capacitor behaves like an open circuit as it is being supplied by a constant dc source.



From the circuit,

$$V_c(0^-) = 24 \times \frac{5}{8} = 15V$$

Continue

Step 3:

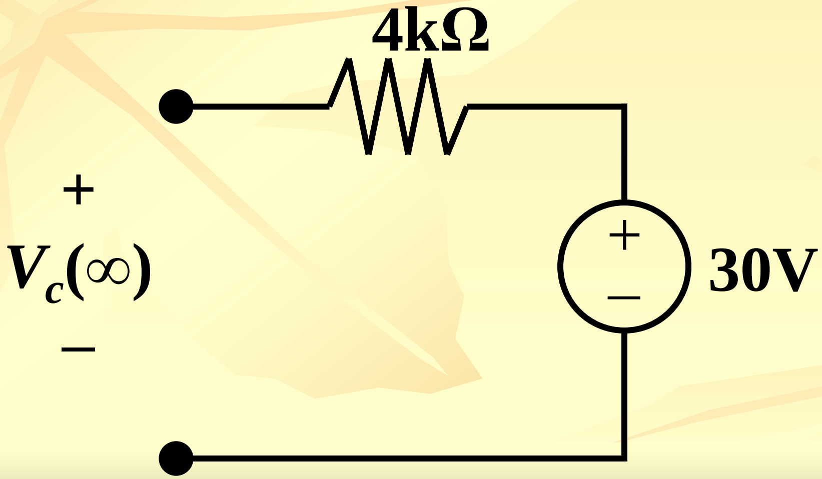
At t = 0^+, the instant when the switch is just moves to *b*.

Voltage across capacitor remains the same.

$$V_c(0^+) = V_c(0^-) = 15V$$

Step 4:

At t = ∞, the capacitor again behaves like an open circuit since it is being supplied by a constant source.



$$V_c(\infty) = 30V$$

Continue

Step 5:

Hence,

$$V_c(t) = 30 + (15 - 30)e^{-0.5t} = 30 - 15e^{-0.5t} V$$

At t = 1s, $V_c(t)$ = 20.9V

At t = 4s, $V_c(t)$ = 28 V

THE END

An Introduction To Two – Port Networks

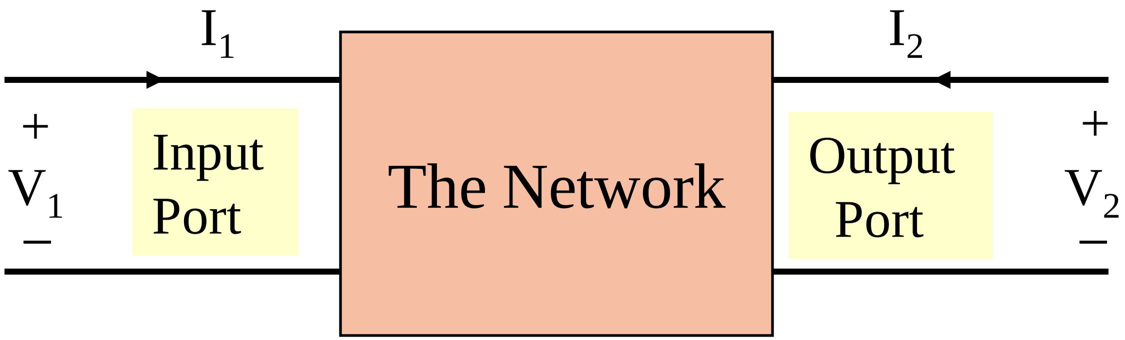
The University of Tennessee
Electrical and Computer Engineering
Knoxville, TN

wlg

Two Port Networks

Generalities:

The standard configuration of a two port:



The network ?

The voltage and current convention ?

* notes

Two Port Networks

Network Equations:

Impedance Z parameters	$V_1 = z_{11}I_1 + z_{12}I_2$ $V_2 = z_{21}I_1 + z_{22}I_2$		$V_2 = b_{11}V_1 - b_{12}I_1$ $I_2 = b_{21}V_1 - b_{22}I_1$
Admittance Y parameters	$I_1 = y_{11}V_1 + y_{12}V_2$ $I_2 = y_{21}V_1 + y_{22}V_2$	Hybrid H parameters	$V_1 = h_{11}I_1 + h_{12}V_2$ $I_2 = h_{21}I_1 + h_{22}V_2$
Transmission A, B, C, D parameters	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$		$I_1 = g_{11}V_1 + g_{12}I_2$ $V_2 = g_{21}V_1 + g_{22}I_2$

* notes

Two Port Networks

Z parameters:

$$z_{11} = \frac{V_1}{I_1} \bigg|_{I_2=0}$$

z_{11} is the impedance seen looking into port 1 when port 2 is open.

$$z_{12} = \frac{V_1}{I_2} \bigg|_{I_1=0}$$

z_{12} is a transfer impedance. It is the ratio of the voltage at port 1 to the current at port 2 when port 1 is open.

$$z_{21} = \frac{V_2}{I_1} \bigg|_{I_2=0}$$

z_{21} is a transfer impedance. It is the ratio of the voltage at port 2 to the current at port 1 when port 2 is open.

$$z_{22} = \frac{V_2}{I_2} \bigg|_{I_1=0}$$

z_{22} is the impedance seen looking into port 2 when port 1 is open.

* notes

Two Port Networks

Y parameters:

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

y_{11} is the admittance seen looking into port 1 when port 2 is shorted.

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

y_{12} is a transfer admittance. It is the ratio of the current at port 1 to the voltage at port 2 when port 1 is shorted.

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

y_{21} is a transfer impedance. It is the ratio of the current at port 2 to the voltage at port 1 when port 2 is shorted.

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

y_{22} is the admittance seen looking into port 2 when port 1 is shorted.

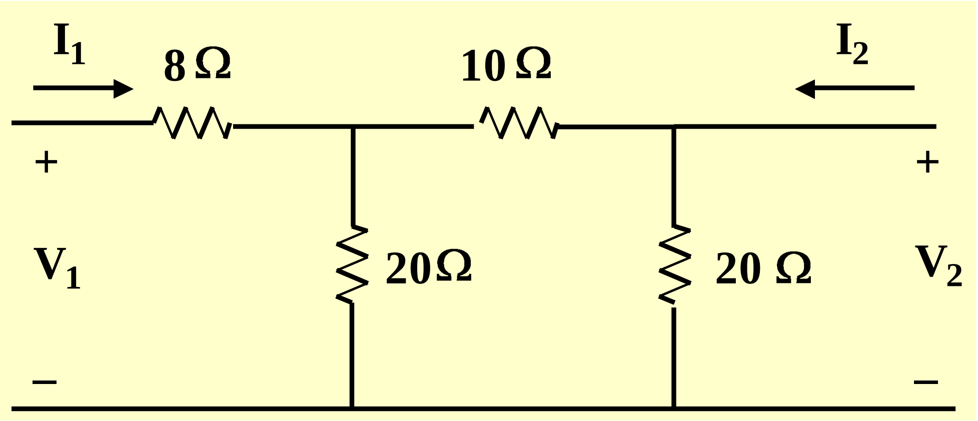
* notes

Two Port Networks

Z parameters:

Example 1

Given the following circuit. Determine the Z parameters.



Find the Z parameters for the above network.

Two Port Networks

Z parameters:

Example 1 (cont 1)

For z_{11}:

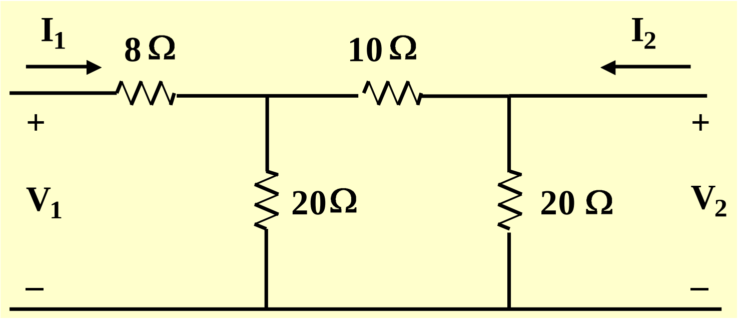
$Z_{11} = 8 + 20\|30 = 20\ \Omega$

For z_{22}:

$Z_{22} = 20\|30 = 12\ \Omega$

For z_{12}:

$$z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$



$$V_1 = \frac{20xI_2x20}{20+30} = 8xI_2$$

Therefore:

$$z_{12} = \frac{8xI_2}{I_2} = 8\ \Omega = z_{21}$$

Two Port Networks

Z parameters:

Example 1 (cont 2)

The Z parameter equations can be expressed in matrix form as follows.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

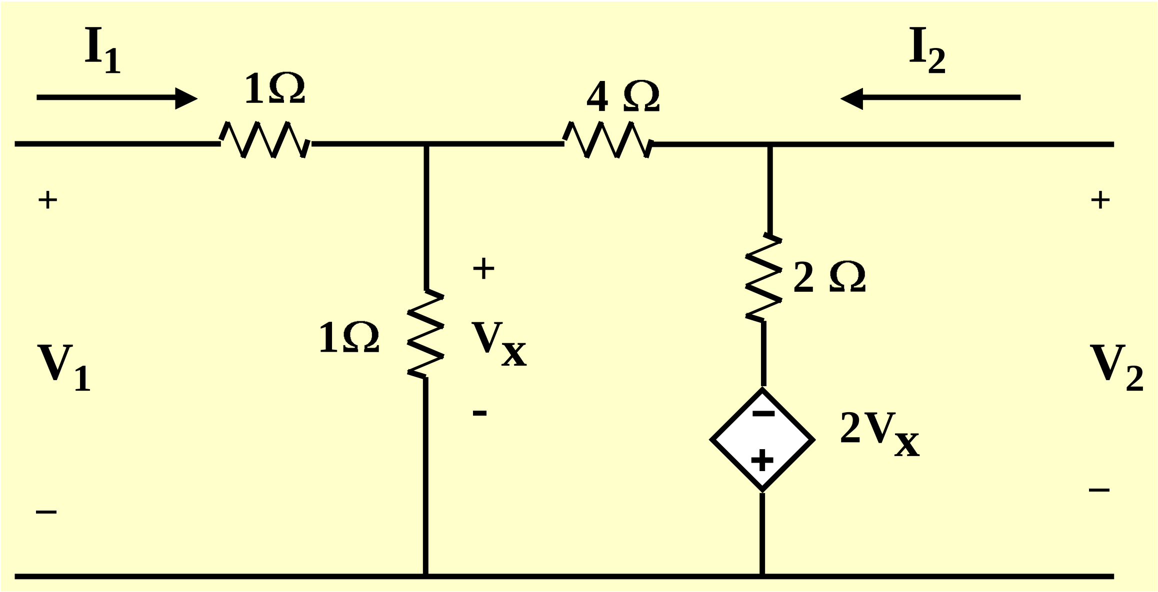
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 & 8 \\ 8 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Two Port Networks

Z parameters:

Example 2 (problem 18.7 Alexander & Sadiku)

You are given the following circuit. Find the Z parameters.

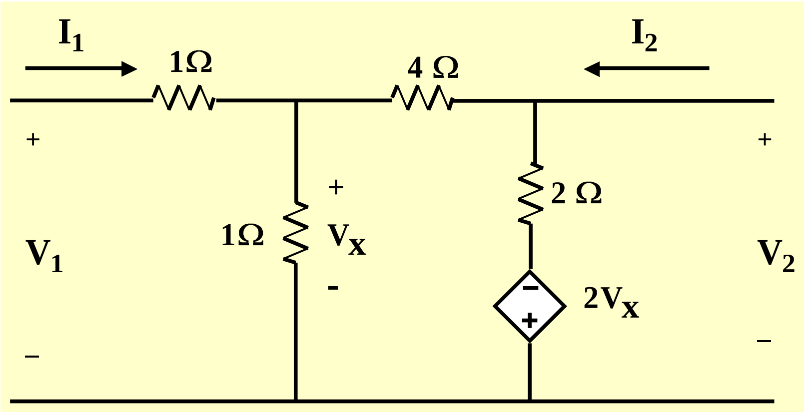


Two Port Networks

Z parameters:

Example 2 (continue p2)

$$z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0}$$

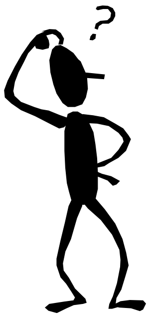


$$I_1 = \frac{V_x}{1} + \frac{V_x + 2V_x}{6} = \frac{6V_x + V_x + 2V_x}{6}$$

$$I_1 = \frac{3V_x}{2} \quad ; \quad \text{but } V_x = V_1 - I_1$$

Substituting gives;

$$I_1 = \frac{3(V_1 - I_1)}{2} \quad \text{or} \quad \frac{V_1}{I_1} = z_{11} = \frac{5}{3}\ \Omega$$



Other Answers

$Z_{21} = -0.667\ \Omega$

$Z_{12} = 0.222\ \Omega$

$Z_{22} = 1.111\ \Omega$

Two Port Networks

Transmission parameters (A,B,C,D):

The defining equations are:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$A = \frac{V_1}{V_2} \bigg|_{I_2 = 0} \qquad B = \frac{V_1}{-I_2} \bigg|_{V_2 = 0}$$

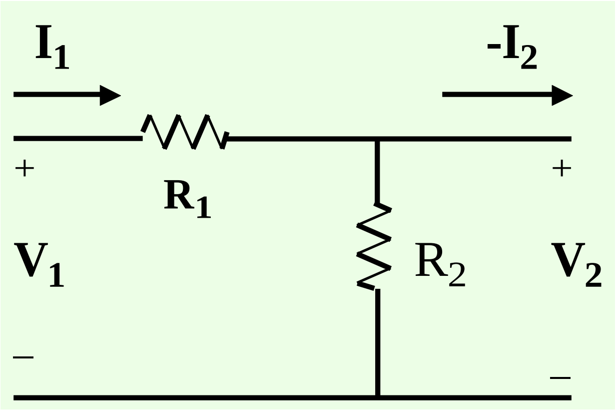
$$C = \frac{I_1}{V_2} \bigg|_{I_2 = 0} \qquad D = \frac{I_1}{-I_2} \bigg|_{V_2 = 0}$$

Two Port Networks

Transmission parameters (A,B,C,D):

Example

Given the network below with assumed voltage polarities and Current directions compatible with the A,B,C,D parameters.



We can write the following equations.

$$V_1 = (R_1 + R_2)I_1 \ + \ R_2I_2$$

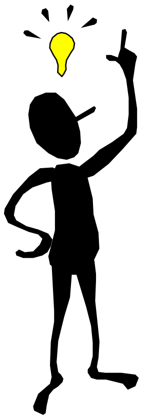
$$V_2 = R_2I_1 \ + \ R_2I_2$$

It is not always possible to write 2 equations in terms of the V’s and I’s Of the parameter set.

Two Port Networks

Transmission parameters (A,B,C,D):

Example (cont.)



$$V_1 = (R_1 + R_2)I_1 + R_2 I_2$$

$$V_2 = R_2 I_1 + R_2 I_2$$

From these equations we can directly evaluate the A,B,C,D parameters.

$$A = \frac{V_1}{V_2} \Bigg| _{I_2 = 0} = \boxed{}$$

$$B = \frac{V_1}{-I_2} \Bigg| _{V_2 = 0} = \boxed{}$$

$$C = \frac{I_1}{V_2} \Bigg| _{I_2 = 0} = \boxed{}$$

$$D = \frac{I_1}{-I_2} \Bigg| _{V_2 = 0} = \boxed{}$$

Later we will see how to interconnect two of these networks together for a final answer

* notes

Two Port Networks

Hybrid Parameters:

The equations for the hybrid parameters are:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2 = 0} \qquad h_{12} = \frac{V_1}{V_2}\bigg|_{I_1 = 0}$$

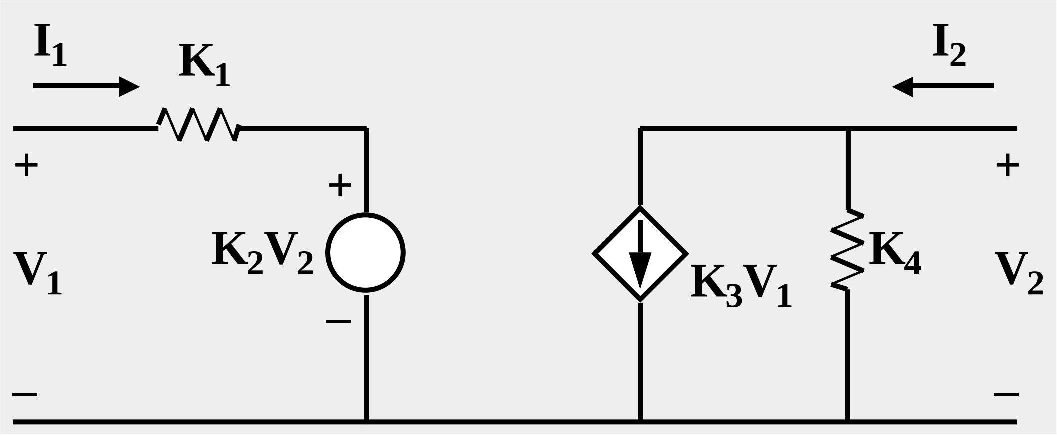
$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2 = 0} \qquad h_{22} = \frac{I_2}{V_2}\bigg|_{I_1 = 0}$$

* notes

Two Port Networks

Hybrid Parameters:

The following is a popular model used to represent a particular variety of transistors.



We can write the following equations:

$$V_1 = AI_1 + BV_2$$

$$I_2 = CI_1 + \frac{V_2}{D}$$

* notes

Two Port Networks

Hybrid Parameters:

$$V_1 = AI_1 + BV_2$$

$$I_2 = CI_1 + \frac{V_2}{D}$$

We want to evaluate the H parameters from the above set of equations.

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0} = \boxed{}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0} = \boxed{}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0} = \boxed{}$$

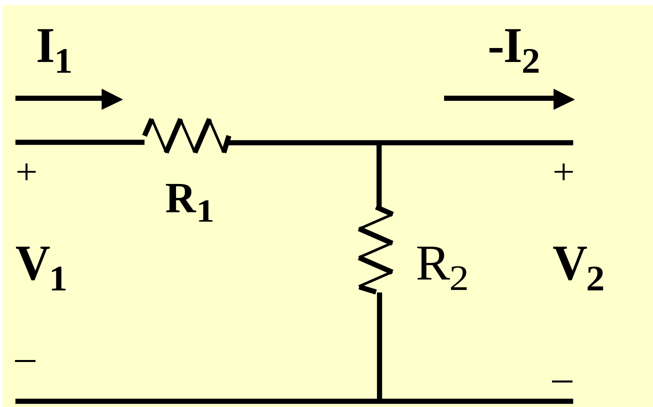
$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1 = 0} = \boxed{}$$

Two Port Networks

Hybrid Parameters:

Another example with hybrid parameters.

Given the circuit below.



The equations for the circuit are:

$$V_1 = (R_1 + R_2)I_1 + R_2 I_2$$

$$V_2 = R_2 I_1 + R_2 I_2$$

The H parameters are as follows.

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} =$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} =$$

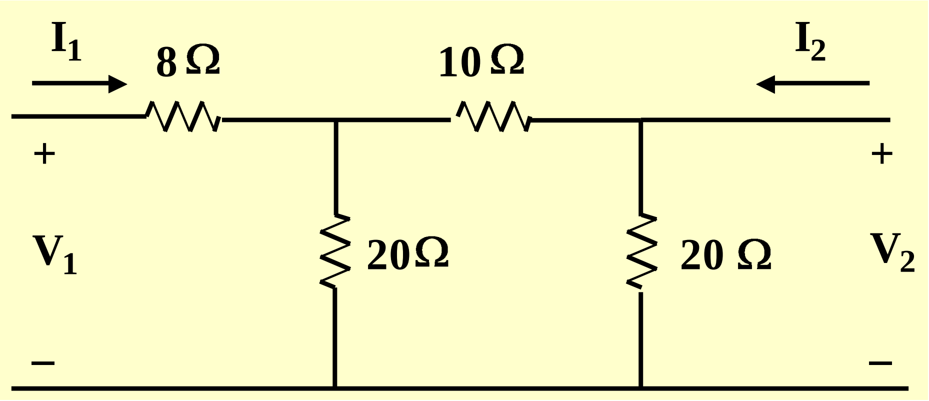
$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} =$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} =$$

Two Port Networks

Modifying the two port network:

Earlier we found the z parameters of the following network.



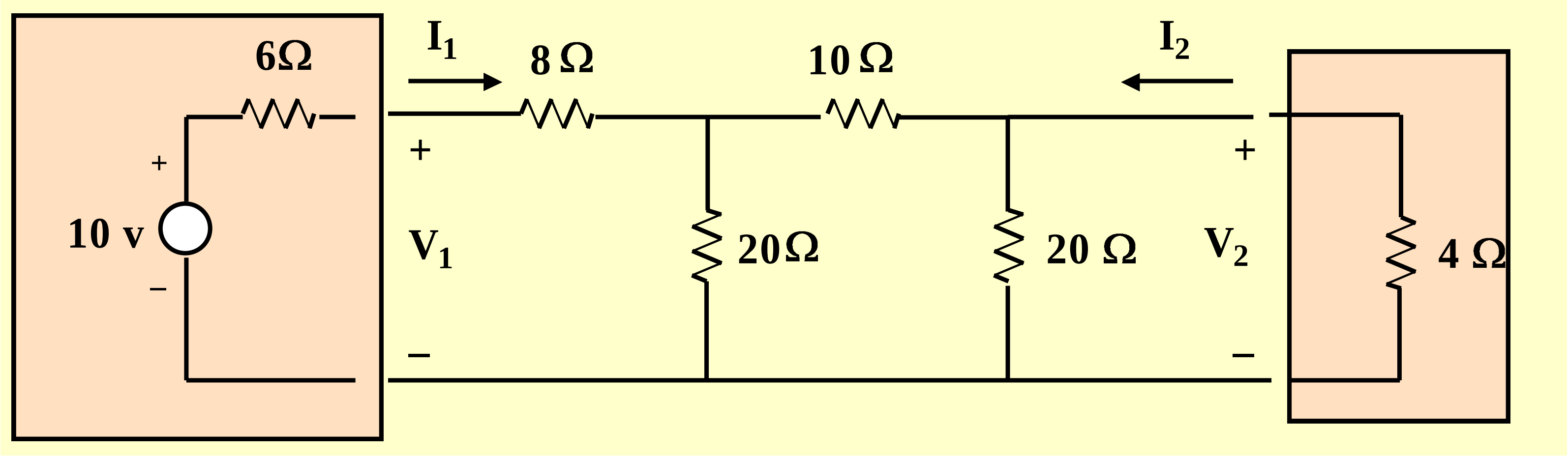
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 & 8 \\ 8 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

* notes

Two Port Networks

Modifying the two port network:

We modify the network as shown be adding elements outside the two ports



We now have:

$$V_1 = 10 - 6I_1$$

$$V_2 = - 4I_2$$

Two Port Networks

Modifying the two port network:

We take a look at the original equations and the equations describing the new port conditions.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 & 8 \\ 8 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = 10 - 6I_1$$

$$V_2 = -4I_2$$

So we have,

$$10 - 6I_1 = 20I_1 + 8I_2$$

$$-4I_2 = 8I_1 + 12I_2$$

* notes

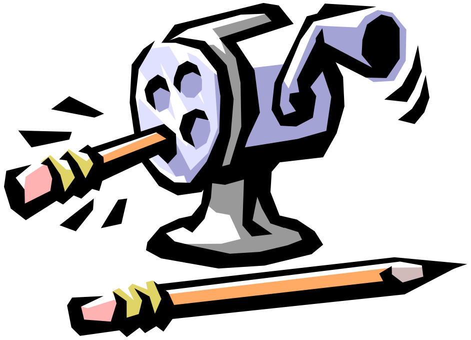
Two Port Networks

Modifying the two port network:

Rearranging the equations gives,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \left[\qquad \right]^{-1} \left[\quad \right]$$

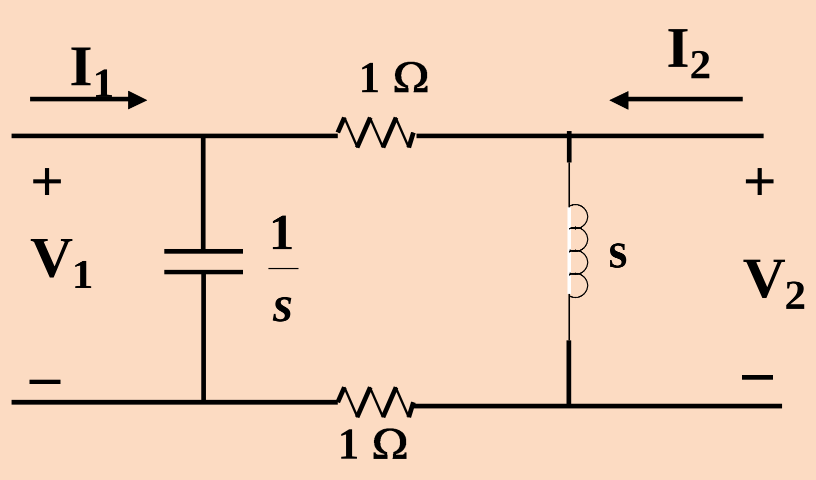
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \left[\qquad \right]$$



Two Port Networks

Y Parameters and Beyond:

Given the following network.



(a) Find the Y parameters for the network.

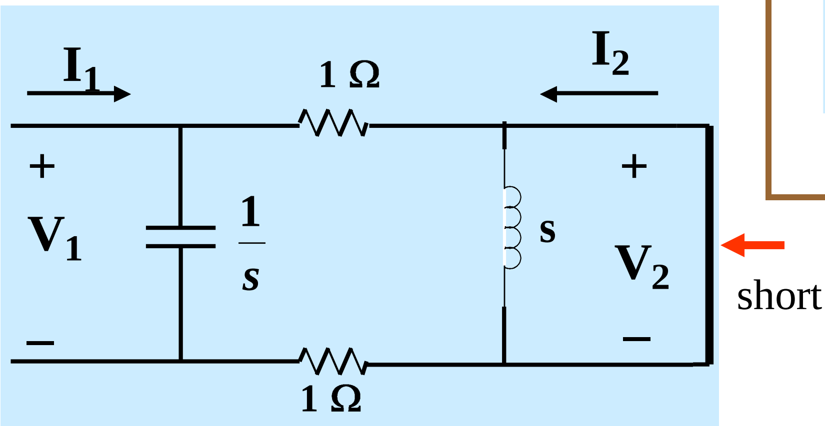
(b) From the Y parameters find the z parameters

Two Port Networks

Y Parameter Example

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$



$$y_{11} = \frac{I_1}{V_1}\Bigg|_{V_2=0} \qquad y_{12} = \frac{I_1}{V_2}\Bigg|_{V_1=0}$$

$$y_{21} = \frac{I_2}{V_1}\Bigg|_{V_2=0} \qquad y_{22} = \frac{I_2}{V_2}\Bigg|_{V_1=0}$$

We use the above equations to evaluate the parameters from the network.

To find y_{11}

$$V_1 = I_1\left(\frac{2/s}{2+1/s}\right) = I_1\left[\frac{2}{2s+1}\right]$$

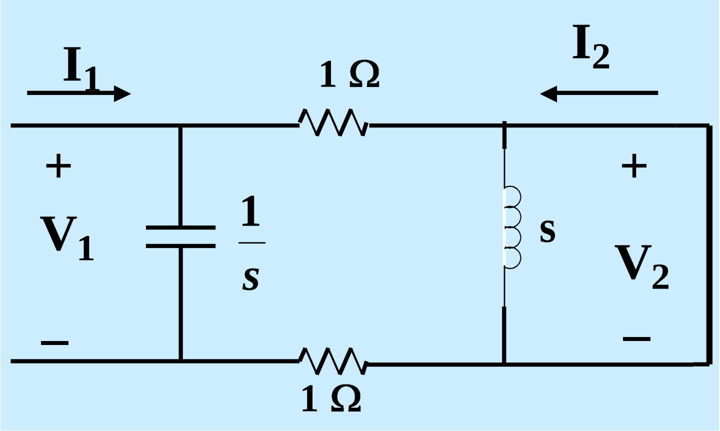
so

$$y_{11} = \frac{I_1}{V_1}\Bigg|_{V_2=0} = s + 0.5$$

Two Port Networks

Y Parameter Example

$$y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0}$$



We see

$$V_1 = -2I_2$$

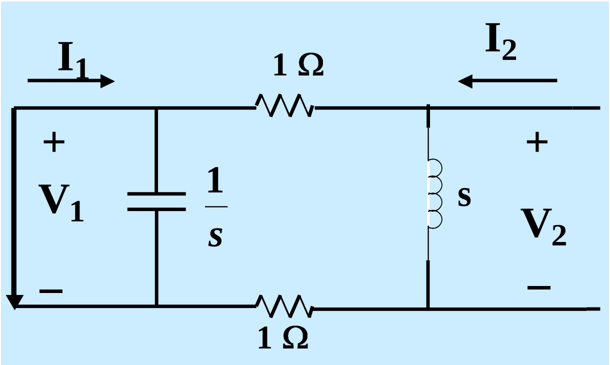
$$y_{21} = \frac{I_2}{V_1}$$ = 0.5 S

Two Port Networks

Y Parameter Example

To find y_{12} and y_{21} we reverse things and short V_1

short



$$y_{12} = \frac{I_1}{V_2}\Bigg|_{V_1=0}$$

We have

$$V_2 = -2I_1$$

$$y_{12} = \frac{I_1}{V2}$$

= 0.5 S

$$y_{22} = \frac{I_2}{V_2}\Bigg|_{V_1=0}$$

We have

$$V_2 = I_2 \frac{2s}{(s+2)}$$

$$y_{22} = 0.5 + \frac{1}{s}$$

Two Port Networks

Y Parameter Example

Summary:

$$\boxed{\mathrm{Y}} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} s+0.5 & -0.5 \\ -0.5 & 0.5+1/s \end{bmatrix}$$

Now suppose you want the Z parameters for the same network.

Two Port Networks

Going From Y to Z Parameters

For the Y parameters we have:

$$I = Y\,V$$

For the Z parameters we have:

$$V = Z\,I$$

From above;

$$V = Y^{-1} I = Z\,I$$

Therefore

$$Z = Y^{-1} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{y_{22}}{\Delta_Y} & \dfrac{-y_{12}}{\Delta_Y} \\ \dfrac{-y_{21}}{\Delta_Y} & \dfrac{y_{11}}{\Delta_Y} \end{bmatrix}$$

where

$$\Delta_Y = \det|Y|$$

Two Port Parameter Conversions:

$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta_Y} & \frac{-y_{12}}{\Delta_Y} \\ \frac{-y_{21}}{\Delta_Y} & \frac{y_{11}}{\Delta_Y} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta_T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta_H}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{22}}{\Delta_Z} & \frac{-z_{12}}{\Delta_Z} \\ \frac{-z_{21}}{\Delta_Z} & \frac{z_{11}}{\Delta_Z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta_T}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_H}{h_{11}} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_Z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta_Y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta_H}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$
$\begin{bmatrix} \frac{\Delta_Z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_Y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta_T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$

Two Port Parameter Conversions:

To go from one set of parameters to another, locate the set of parameters you are in, move along the vertical until you are in the row that contains the parameters you want to convert to – then compare element for element

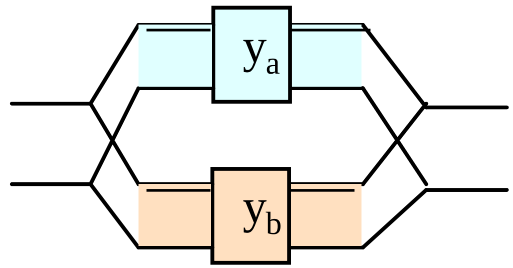
$\begin{bmatrix} \boxed{z_{11}} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta_Y} & \frac{-y_{12}}{\Delta_Y} \\ \frac{-y_{21}}{\Delta_Y} & \frac{y_{11}}{\Delta_Y} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta_T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$	$\begin{bmatrix} \boxed{\frac{\Delta_H}{h_{22}}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{22}}{\Delta_Z} & \frac{-z_{12}}{\Delta_Z} \\ \frac{-z_{21}}{\Delta_Z} & \frac{z_{11}}{\Delta_Z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta_T}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_H}{h_{11}} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_Z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta_Y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta_H}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$
$\begin{bmatrix} \frac{\Delta_Z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_Y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta_T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$

$$z_{11} = \frac{\Delta_H}{h_{22}}$$

Interconnection Of Two Port Networks

Three ways that two ports are interconnected:

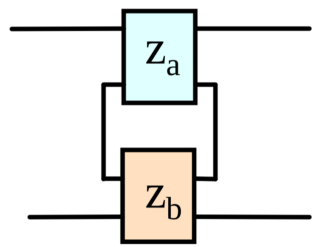
* Parallel



Y parameters

$$[Y] = [Y_a] + [Y_b]$$

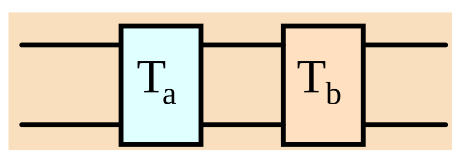
* Series



Z parameters

$$[Z] = [Z_a] + [Z_b]$$

* Cascade



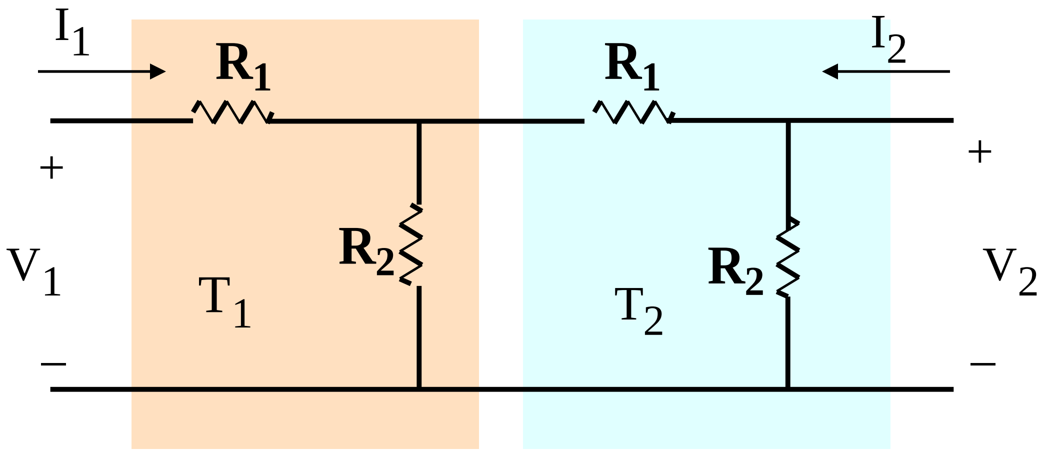
ABCD parameters

$$[T] = [T_a][T_b]$$

Interconnection Of Two Port Networks

Consider the following network:

Find $\dfrac{V_2}{V_1}$



Referring to slide 13 we have;

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \dfrac{R_1+R_2}{R_2} & R_1 \\ \dfrac{1}{R_2} & 1 \end{bmatrix} \begin{bmatrix} \dfrac{R_1+R_2}{R_2} & R_1 \\ \dfrac{1}{R_2} & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Interconnection Of Two Port Networks

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \dfrac{R_1 + R_2}{R_2} & R_1 \\ \dfrac{1}{R_2} & 1 \end{bmatrix} \begin{bmatrix} \dfrac{R_1 + R_2}{R_2} & R_1 \\ \dfrac{1}{R_2} & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Multiply out the first row:

$$V_1 = \left[\left[\left(\frac{R_1 + R_2}{R_2} \right)^2 + \frac{R_1}{R_2} \right] V_2 \; + \; \left[\left(\frac{R_1 + R_2}{R_2} \right) R_1 + R_1 \right] (-I_2) \right]$$

Set $I_2 =$ 0 (as in the diagram)

$$\frac{V_2}{V_1} = \frac{R_2^{\;2}}{R_1^{\;2} + 3R_1R_2 \;\; R_2^{\;2}}$$

Can be verified directly by solving the circuit


FINISH

Basic Laws of Circuits

End of Lesson

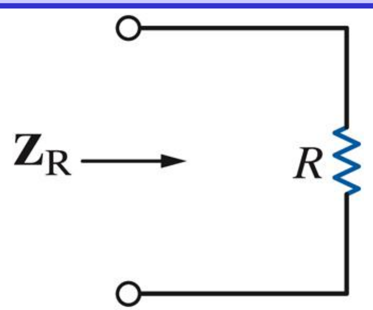
Two-Port Networks

AC STEADY STATE ANALYSIS

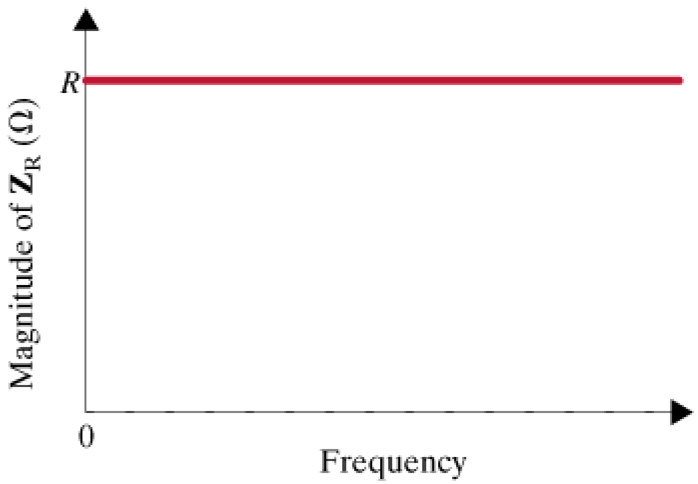
In AC steady state analysis the frequency is assumed constant (e.g., 60Hz). Here we consider the frequency as a variable and examine how the performanc varies with the frequency.

Variation in impedance of basic components

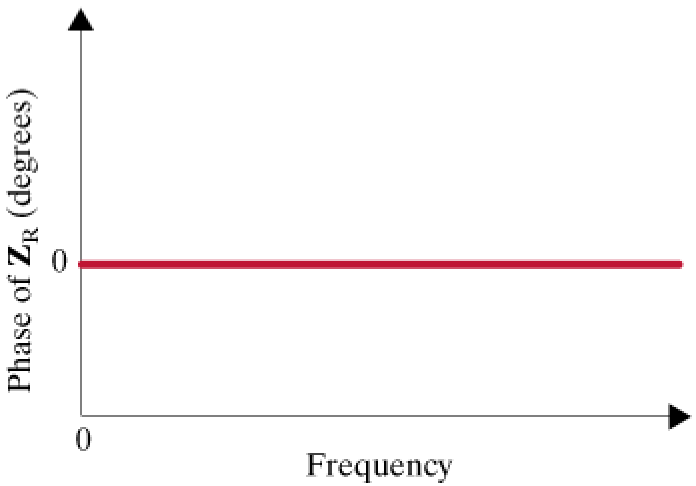
Resistor



$$Z_R = R = R\angle 0°$$



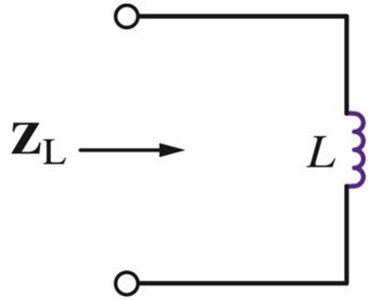
(b)



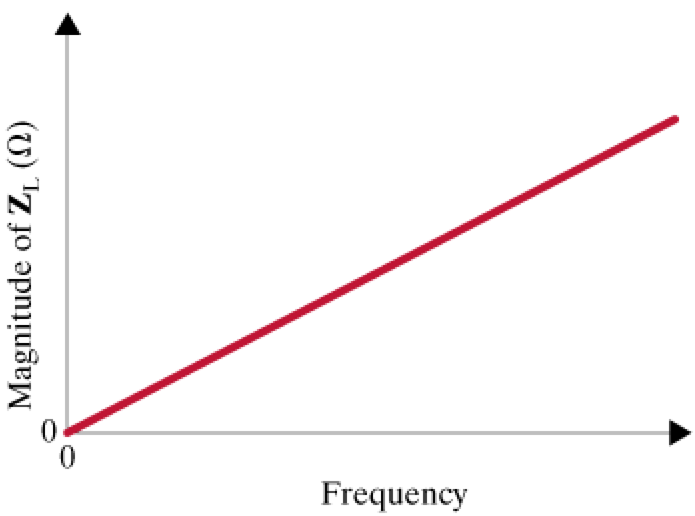
(c)

GEAUX

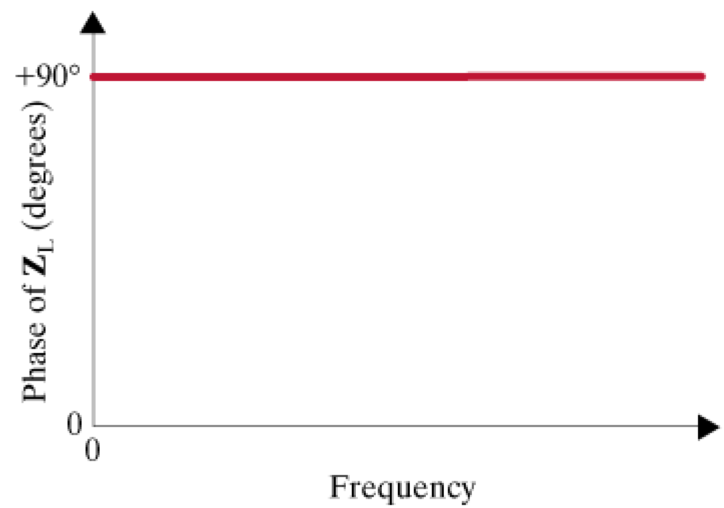
Inductor



$$\boldsymbol{Z_L} = \boldsymbol{j\omega L} = \omega \boldsymbol{L}\angle 90°$$

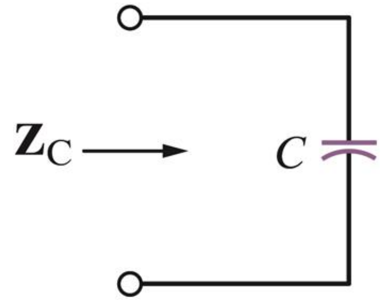


(b)

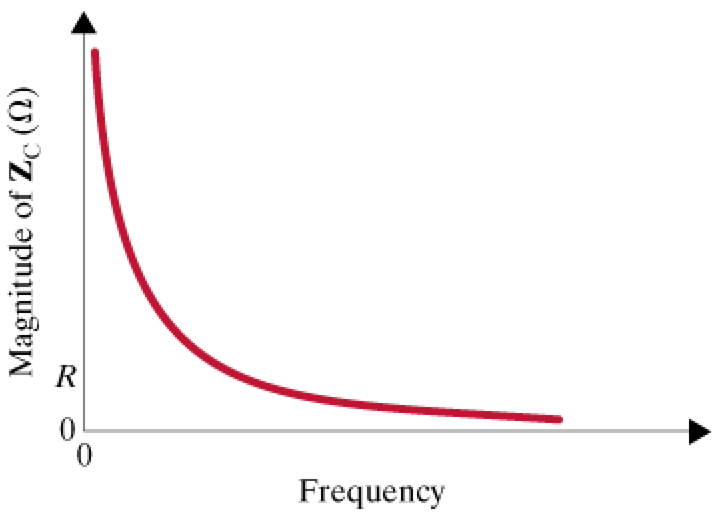


(c)

Capacitor



$$Z_c = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90°$$



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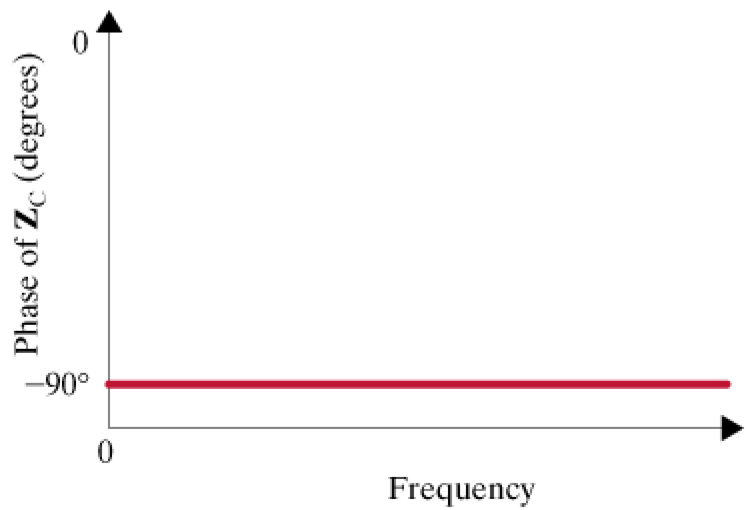
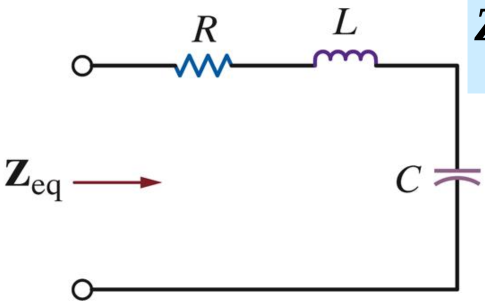


(c)

GEAUX

Frequency dependent behavior of series RLC network



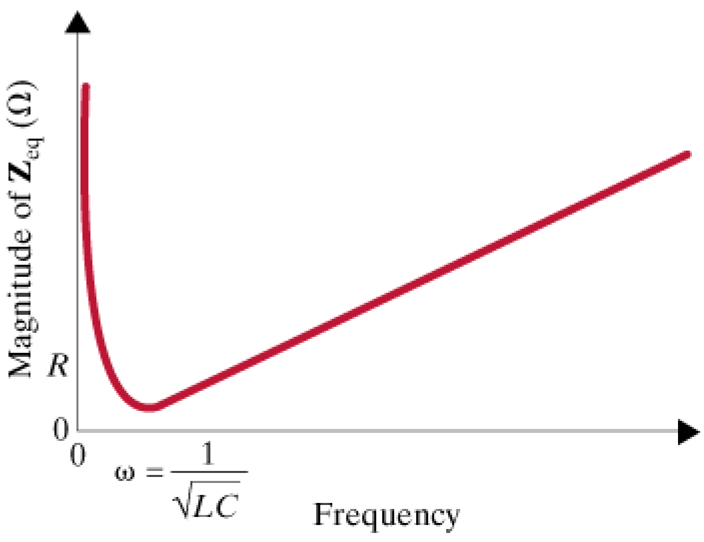
$$Z_{eq} = R + j\omega L + \frac{1}{j\omega C} = \frac{(j\omega)^2 LC + j\omega RC + 1}{j\omega C} \times \frac{-j}{-j} = \frac{\omega RC + j(\omega^2 LC - 1)}{\omega C}$$

"Simplification in notation" $j\omega \approx s$

$$Z_{eq}(s) = \frac{s^2 LC + sRC + 1}{sC}$$

$$|Z_{eq}| = \frac{\sqrt{(\omega RC)^2 + (1 - \omega^2 LC)^2}}{\omega C}$$

$$\angle Z_{eq} = \tan^{-1}\left(\frac{\omega^2 LC - 1}{\omega RC}\right)$$



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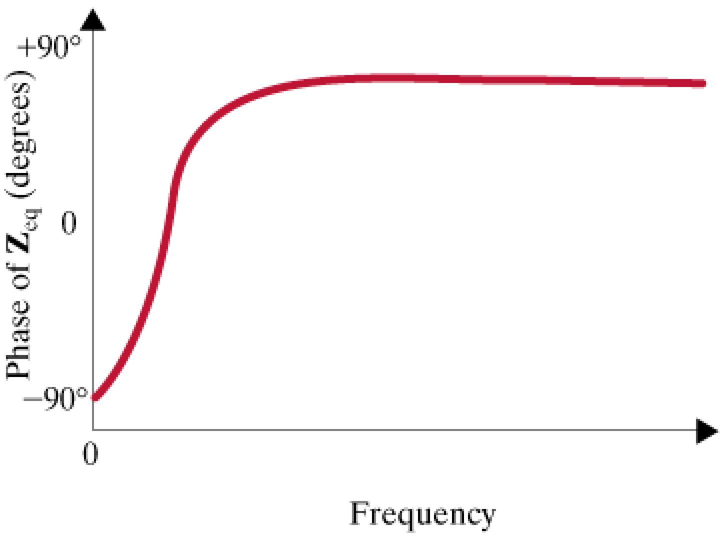


(c)

GEAUX

Simplified notation for basic components

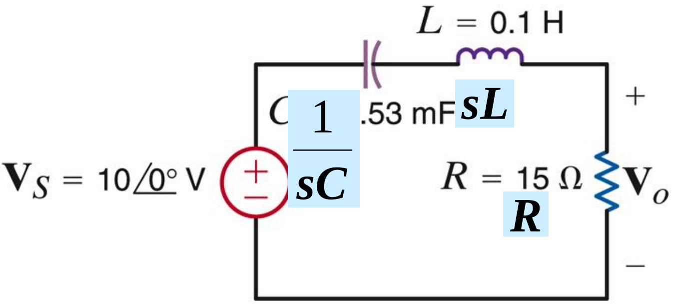
$$Z_R(s) = R,\ \ Z_L(s) = sL,\ \ Z_C = \frac{1}{sC}$$

For all cases seen, and all cases to be studied, the impedance is of the form

$$Z(s) = \frac{a_m s^m + a_{m-1}s^{m-1} + \ldots + a_1 s + a_0}{b_n s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + b_0}$$

Moreover, if the circuit elements (L,R,C, dependent sources) are real then the expression for any voltage or current will also be a rational function in *s*

LEARNING EXAMPLE



$$V_o(s) = \frac{R}{R + sL + 1/sC}V_S = \frac{sRC}{s^2 LC + sRC + 1}V_S$$

$$s \approx j\omega$$

$$V_o = \frac{j\omega RC}{(j\omega)^2 LC + j\omega RC + 1}V_S$$

$$V_o = \frac{j\omega(15 \times 2.53 \times 10^{-3})}{(j\omega)^2(0.1 \times 2.53 \times 10^{-3}) + j\omega(15 \times 2.53 \times 10^{-3}) + 1}10\angle 0°$$

NETWORK FUNCTIONS

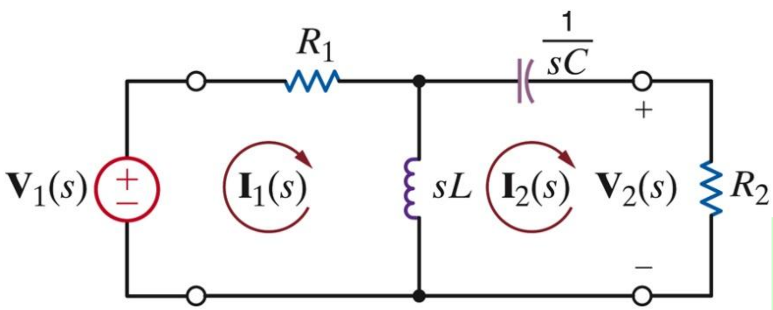
Some nomenclature

When voltages and currents are defined at different terminal pairs we define the ratios as Transfer Functions

INPUT	OUTPUT	TRANSFER FUNCTION	SYMBOL
Voltage	Voltage	Voltage Gain	**Gv(s)**
Current	Voltage	Transimpedance	**Z(s)**
Current	Current	Current Gain	**Gi(s)**
Voltage	Current	Transadmittance	**Y(s)**

If voltage and current are defined at the same terminals we define Driving Point Impedance/Admittance

EXAMPLE



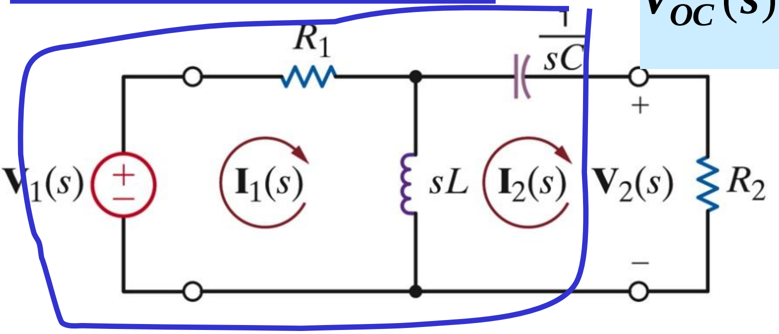
To compute the transfer functions one must solve the circuit. Any valid technique is acceptable

$$Y_T(s) = \frac{I_2(s)}{V_1(s)} \begin{cases} \text{Transadmittance} \\ \text{Transfer admittance} \end{cases}$$

$$G_v(s) = \frac{V_2(s)}{V_1(s)} \quad \text{Voltage gain}$$

GEAUX

LEARNING EXAMPLE



$$V_{OC}(s) = \frac{sL}{sL + R_1} V_1(s)$$

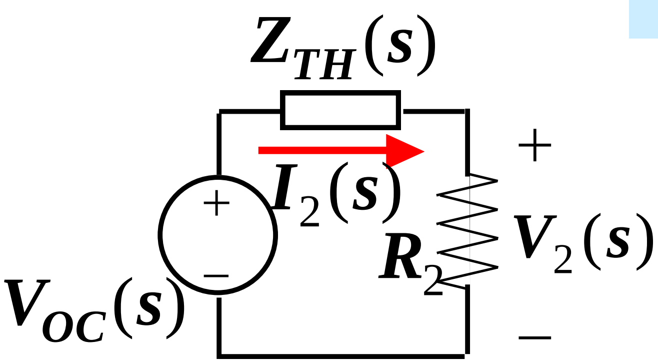
The textbook uses mesh analysis. We will use Thevenin's theorem

$$Z_{TH}(s) = \frac{1}{sC} + R_1 \| sL = \frac{1}{sC} + \frac{sLR_1}{sL + R_1}$$

$$Z_{TH}(s) = \frac{s^2 LCR_1 + sL + R_1}{sC(sL + R_1)}$$

$$Y_T(s) = \frac{I_2(s)}{V_1(s)} \begin{cases} \text{Transadmittance} \\ \text{Transfer admittance} \end{cases}$$

$$G_v(s) = \frac{V_2(s)}{V_1(s)} \quad \text{Voltage gain}$$



$$I_2(s) = \frac{V_{OC}(s)}{R_2 + Z_{TH}(s)} = \frac{\dfrac{sL}{sL + R_1} V_1(s)}{R_2 + \dfrac{s^2 LCR_1 + sL + R_1}{sC(sL + R_1)}} \times \frac{sC(sL + R_1)}{sC(sL + R_1)}$$

$$Y_T(s) = \frac{s^2 LC}{s^2 (R_1 + R_2) LC + s(L + R_1 R_2 C) + R_1}$$

$$G_v(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_2 I_2(s)}{V_1(s)} = R_2 Y_T(s)$$

GEAUX

POLES AND ZEROS (More nomenclature)

$$H(s)=\frac{a_m s^m + a_{m-1}s^{m-1}+\ldots+a_1 s+a_0}{b_n s^n + b_{n-1}s^{n-1}+\ldots+b_1 s+b_0}$$ Arbitrary network function

Using the roots, every (monic) polynomial can be expressed as a product of first order terms

$$H(s)=K_0\frac{(s-z_1)(s-z_2)\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)}$$

$z_1, z_2, \ldots, z_m$ = zeros of the network function

$p_1, p_2, \ldots, p_n$ = poles of the network function

The network function is uniquely determined by its poles and zeros and its value at some other value of s (to compute the gain)

EXAMPLE

zeros: $z_1=-1,$

poles: $p_1=-2+j2,\ p_2=-2-j2$

$H(0)=1$

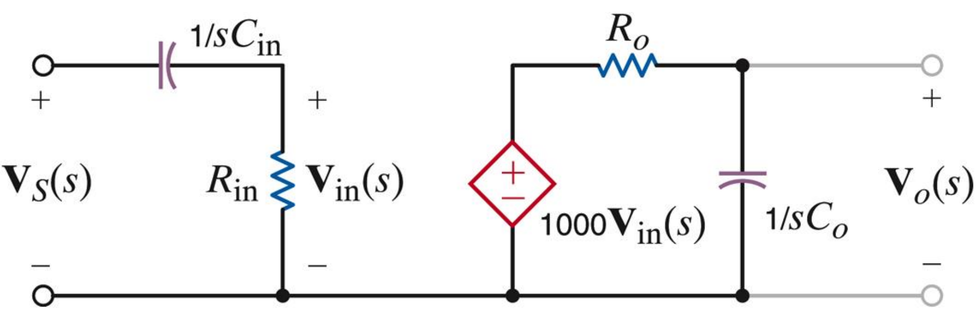
$$H(s)=K_0\frac{(s+1)}{(s+2-j2)(s+2+j2)}=K_0\frac{s+1}{s^2+4s+8}$$

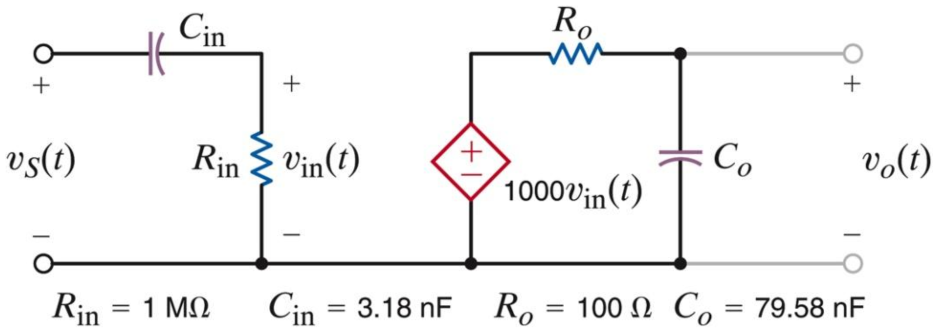
$$H(0)=K_0\frac{1}{8}=1\Rightarrow$$

$$H(s)=8\frac{s+1}{s^2+4s+8}$$

LEARNING EXTENSION

Find the pole and zero locations and the value of K_o for the voltage gain $G(s) = \dfrac{V_o(s)}{V_S(s)}$





$$H(s) = K_0 \frac{(s-z_1)(s-z_2)\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)}$$

Zeros = roots of numerator
Poles = roots of denominator

For this case the gain was shown to be

$$G(s) = \left[\frac{sC_{in}R_{in}}{1+sC_{in}R_{in}}\right][1000]\left[\frac{1}{1+sC_oR_o}\right] = \left[\frac{s}{s+100\pi}\right][1000]\left[\frac{40{,}000\pi}{s+40{,}000\pi}\right]$$

zero: $z_1 = 0$

poles: $p_1 = -50\,Hz,\ p_2 = -20{,}000\,Hz$

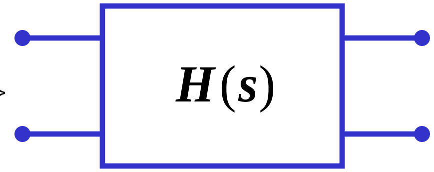
$K_0 = (4\times 10^7)\pi$

Variable Frequency Response

STOP

SINUSOIDAL FREQUENCY ANALYSIS

$$\left.\begin{array}{l} A_0 e^{j(\omega t+\theta)} \\ B_0 \cos(\omega t+\theta) \end{array}\right\}$$



$$\left\{\begin{array}{l} A_0 H(j\omega) e^{j(\omega t+\theta)} \\ B_0 \,|H(j\omega)| \cos\left(\omega t+\theta+\angle H(j\omega)\right) \end{array}\right.$$

Circuit represented by network function

To study the behavior of a network as a function of the frequency we analyze the network function $H(j\omega)$ as a function of ω.

Notation

$$M(\omega) = |H(j\omega)|$$

$$\phi(\omega) = \angle H(j\omega)$$

$$H(j\omega) = M(\omega) e^{j\phi(\omega)}$$

Plots of $M(\omega), \phi(\omega)$, as function of ω are generally called magnitude and phase characteristics.

BODE PLOTS $\left\{\begin{array}{c} 20\log_{10}(M(\omega)) \\ \phi(\omega) \end{array}\right.$ vs $\log_{10}(\omega)$

GEAUX

HISTORY OF THE DECIBEL

Originated as a measure of relative (radio) power

$$P_2|_{dB}\ (\text{over } P_1) = 10\log\frac{P_2}{P_1}$$

$$P = I^2R = \frac{V^2}{R} \Rightarrow P_2|_{dB}\ (\text{over } P_1) = 10\log\frac{V_2^2}{V_1^2} = 10\log\frac{I_2^2}{I_1^2}$$

By extension

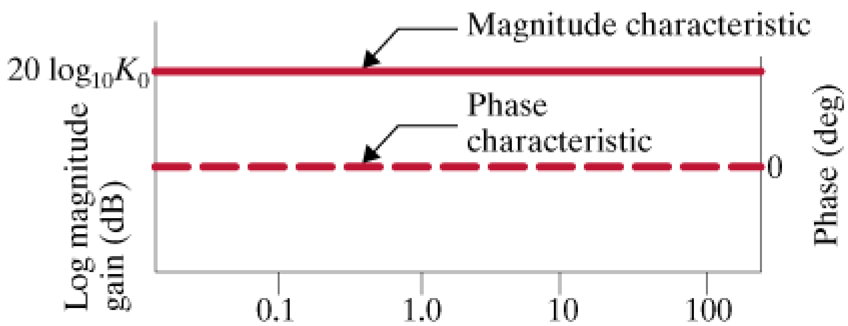
$$V|_{dB} = 20\log_{10}|V|$$

$$I|_{dB} = 20\log_{10}|I|$$

$$G|_{dB} = 20\log_{10}|G|$$

Using log scales the frequency characteristics of network functions have simple asymptotic behavior.
The asymptotes can be used as reasonable and efficient approximations

Constant Term



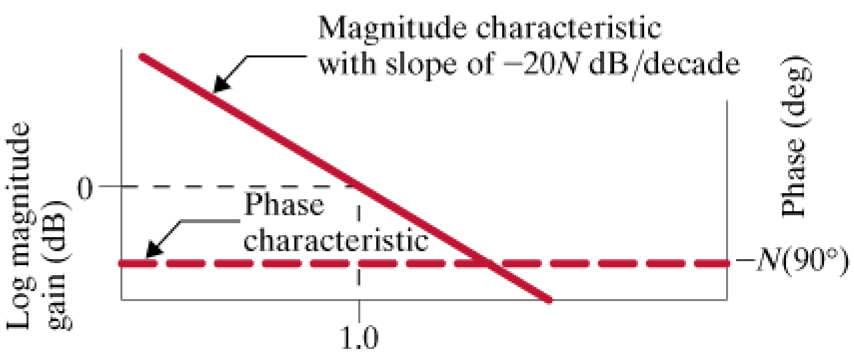
(a)

the x - axis is $\log_{10}\omega$

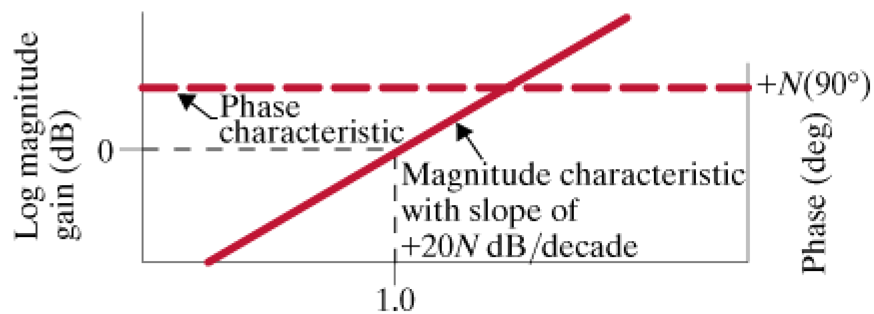
this is a straight line

Poles/Zeros at the origin

$$(j\omega)^{\pm N} \rightarrow \begin{cases} |(j\omega)^{\pm N}|_{dB} = \pm N \times 20\log_{10}(\omega) \\ \angle(j\omega)^{\pm N} = \pm N 90° \end{cases}$$

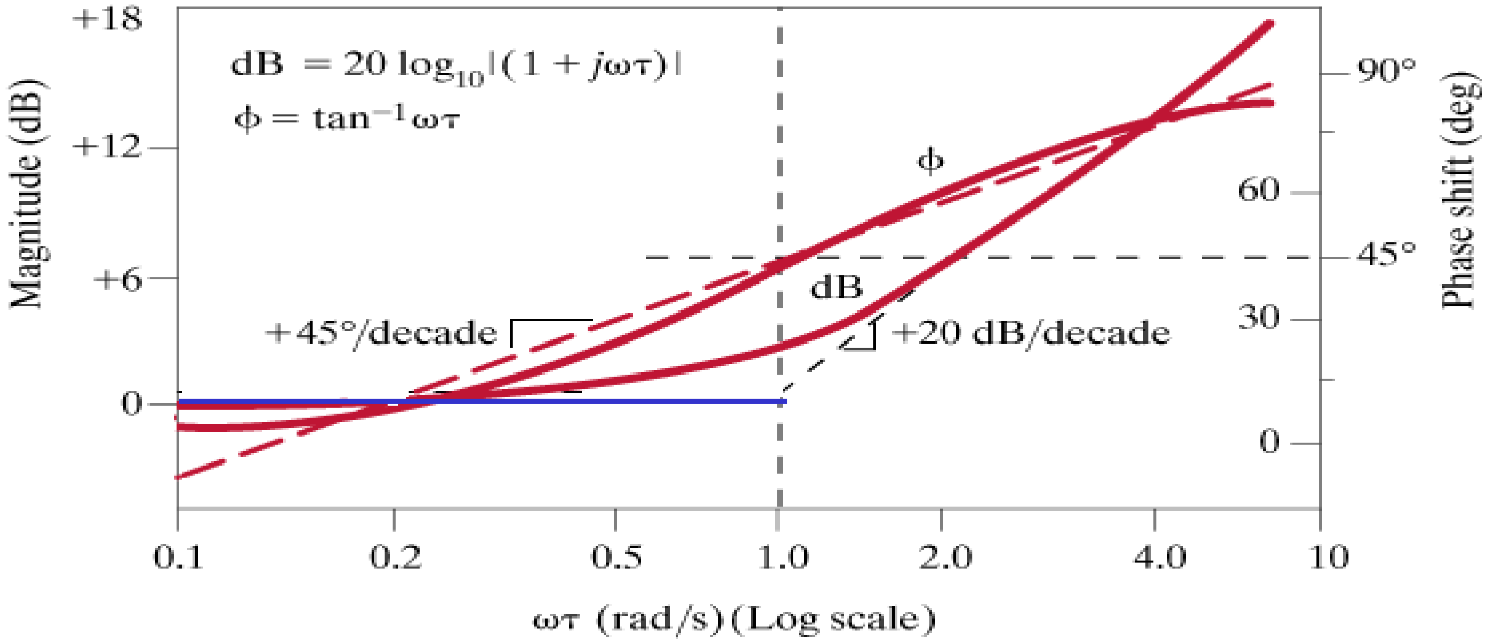


(b)

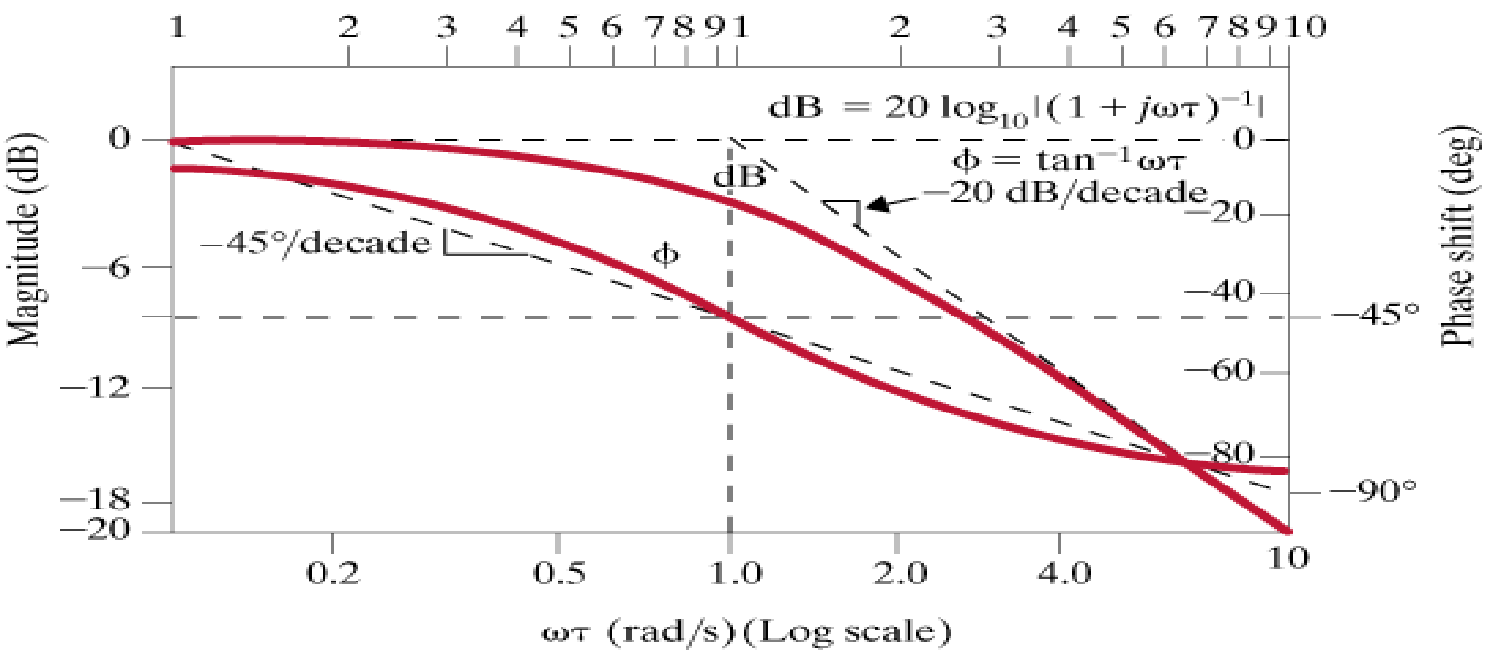


(c)

GEAUX



Simple zero



(a)

Simple pole

GEAUX

LEARNING EXAMPLE

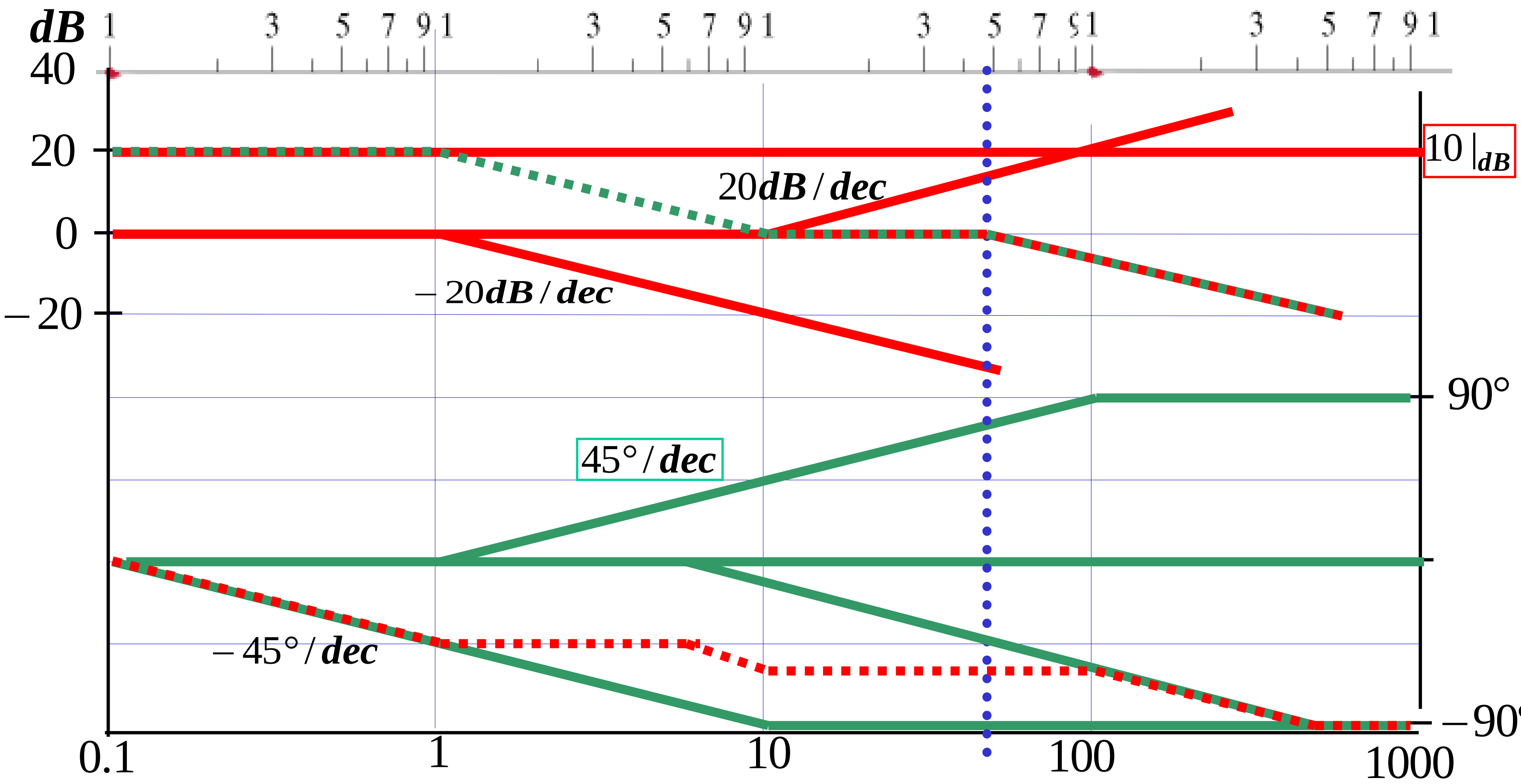
Generate magnitude and phase plots

$$G_v(j\omega) = \frac{\boxed{10}\boxed{(0.1j\omega + 1)}}{\boxed{(j\omega + 1)}\boxed{(0.02j\omega + 1)}}$$

Breaks/corners : 1,10,50

Draw asymptotes for each term

Draw composites



GEAUX

LEARNING EXAMPLE

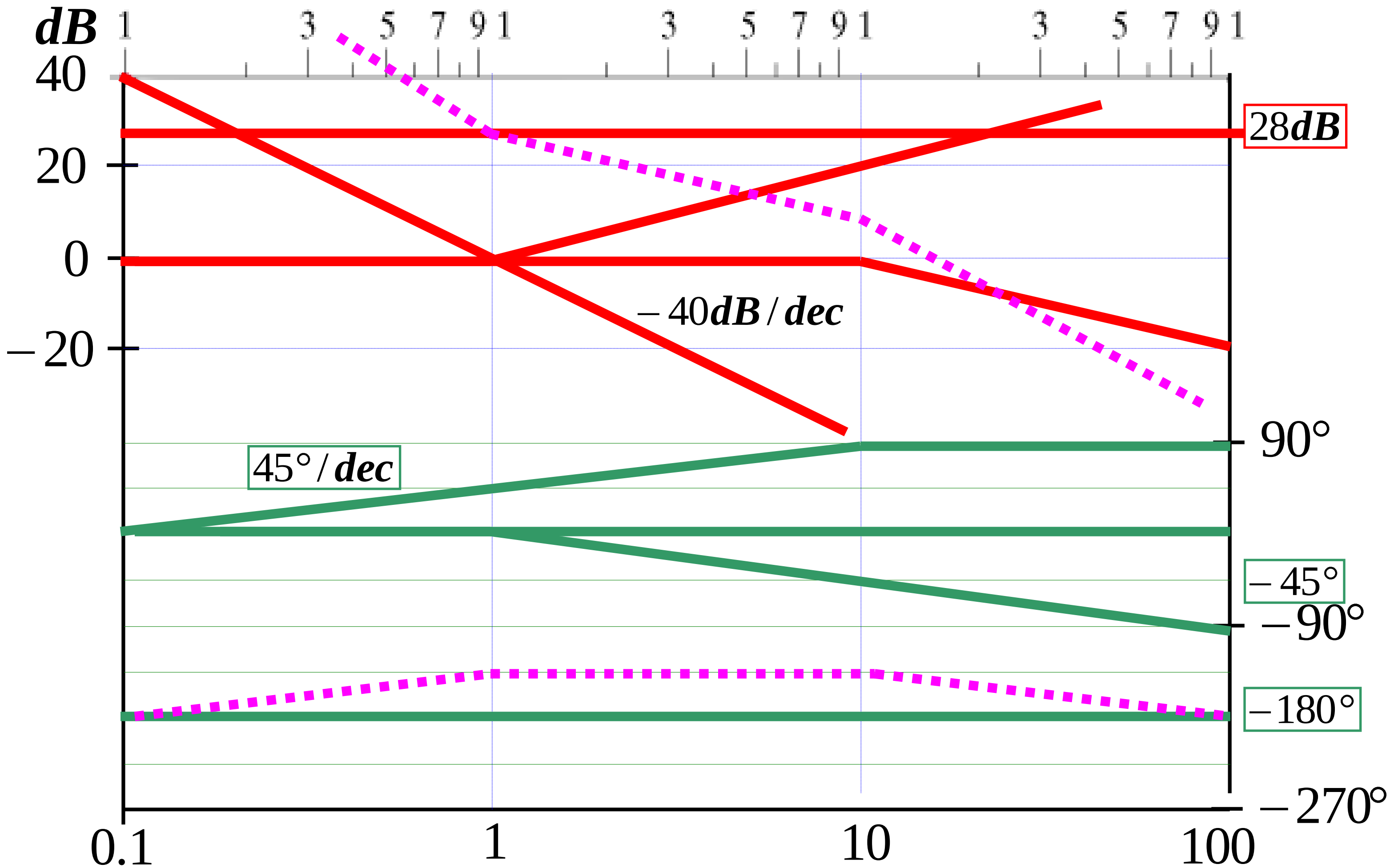
Generate magnitude and phase plots

Draw asymptotes for each

Form composites

$$G_v(j\omega) = \frac{\boxed{25}\boxed{(j\omega+1)}}{\boxed{(j\omega)^2}\boxed{(0.1j\omega+1)}}$$

Breaks (corners): 1, 10



GEAUX

LEARNING EXTENSION

Sketch the magnitude characteristic

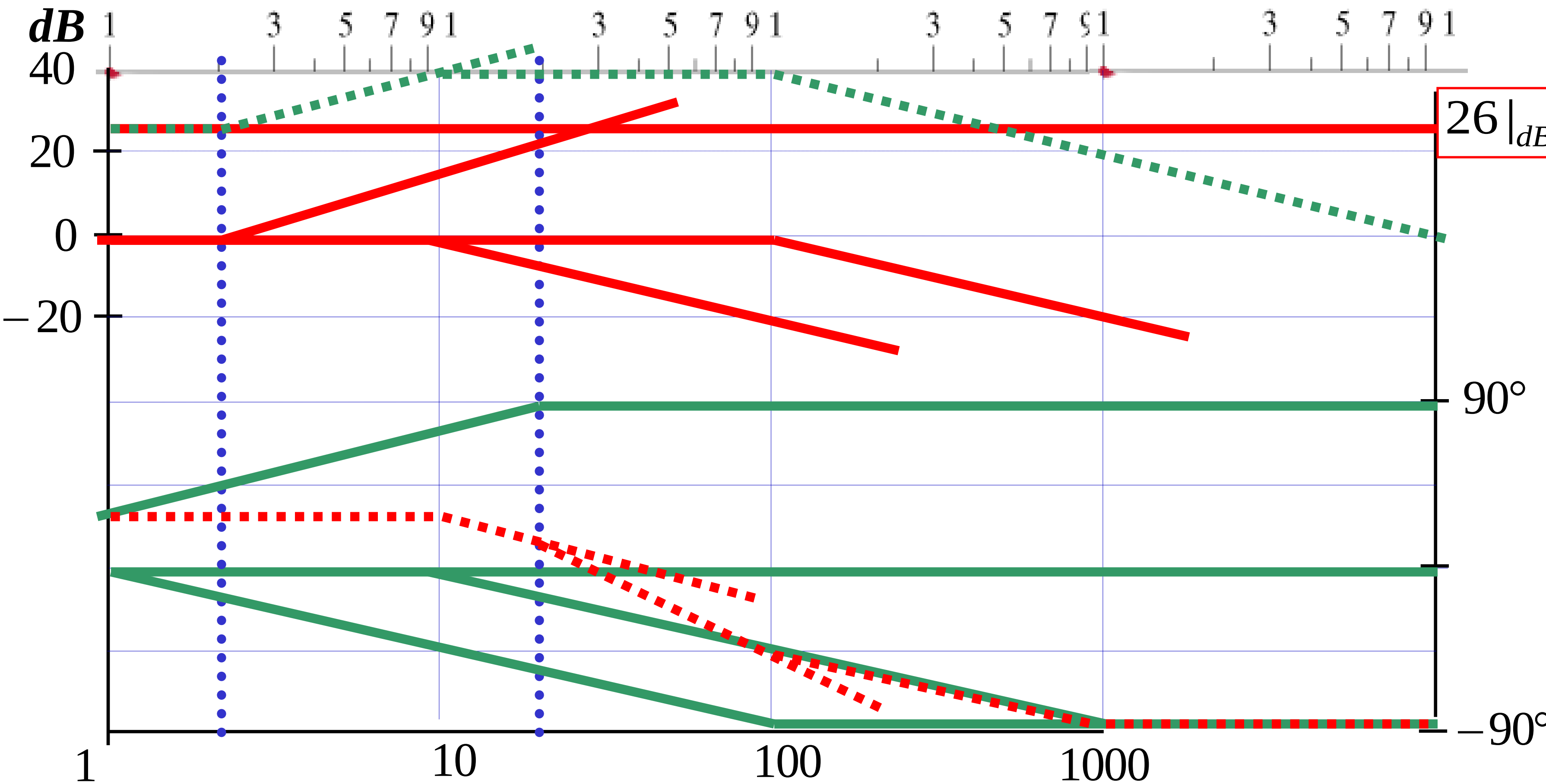
$$G(j\omega)=\frac{10^4(j\omega+2)}{(j\omega+10)(j\omega+100)}$$

breaks : 2, 10, 100

But the function is NOT in standard form

Put in standard form $G(j\omega)=\frac{20\boxed{(j\omega/2+1)}}{\boxed{(j\omega/10+1)}\boxed{(j\omega/100+1)}}$

We need to show about 4 decades



GEAUX

LEARNING EXTENSION

Sketch the magnitude characteristic

Put in standard form

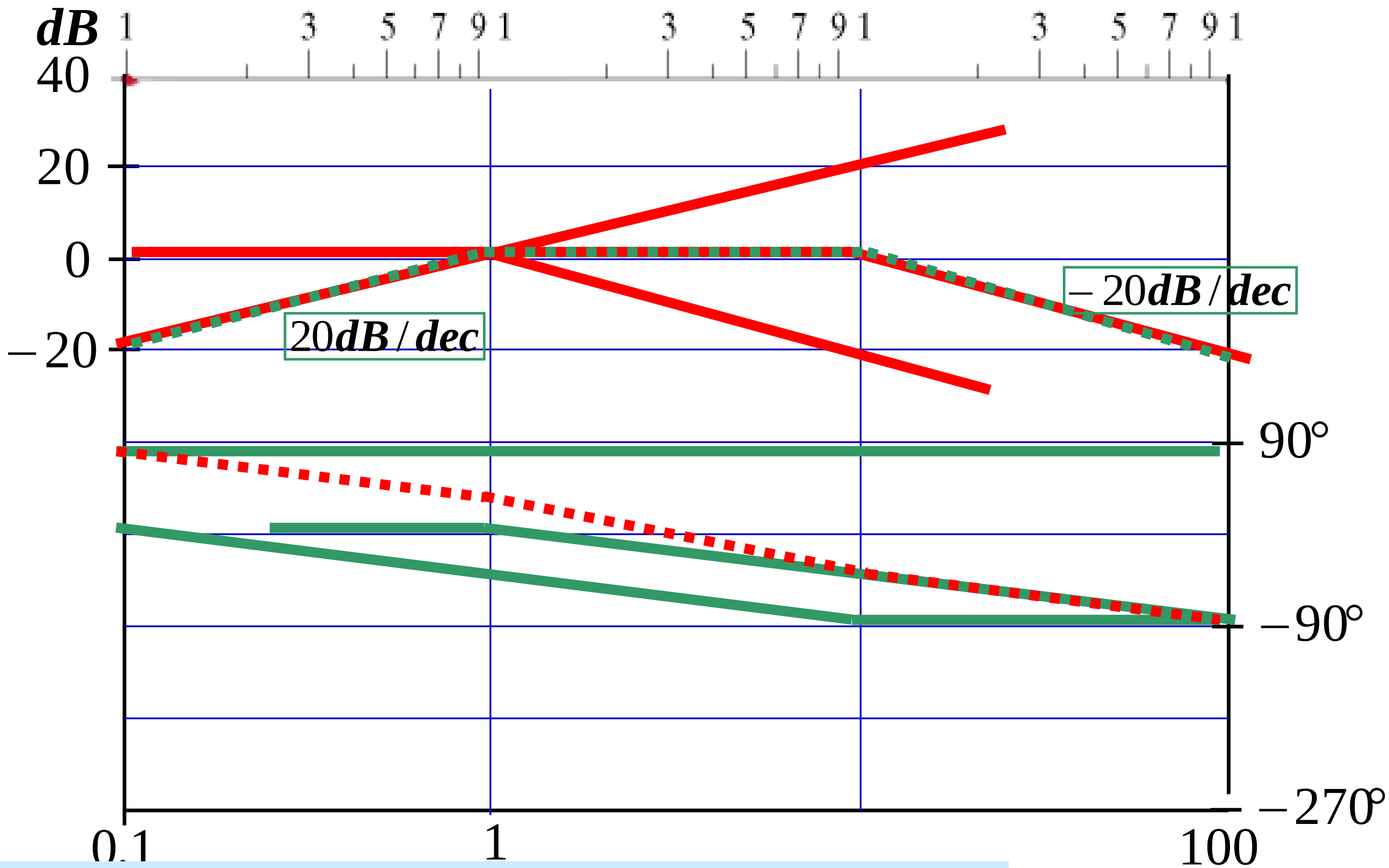
$$G(j\omega)=\frac{10\,j\omega}{(j\omega+1)(j\omega+10)}$$

$$G(j\omega)=\frac{\boxed{j\omega}}{\boxed{(j\omega+1)}\boxed{(j\omega/10+1)}}$$

not in standard form

zero at the origin

breaks : 1, 10

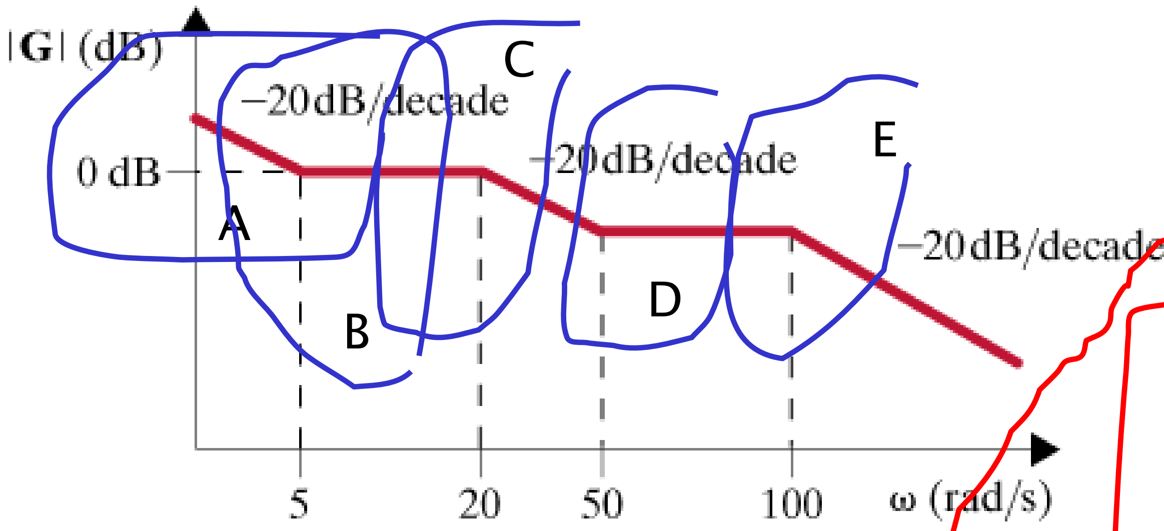


Once each term is drawn we form the composites

GEAUX

LEARNING EXTENSION

Determine a transfer function from the composite magnitude asymptotes plot



A. Pole at the origin. Crosses 0dB line at 5

$$\frac{5}{j\omega}$$

B. Zero at 5

C. Pole at 20

D. Zero at 50

E. Pole at 100

$$G(j\omega)=\frac{5(j\omega/5+1)(j\omega/50+1)}{j\omega(j\omega/20+1)(j\omega/100+1)}$$

Sinusoidal

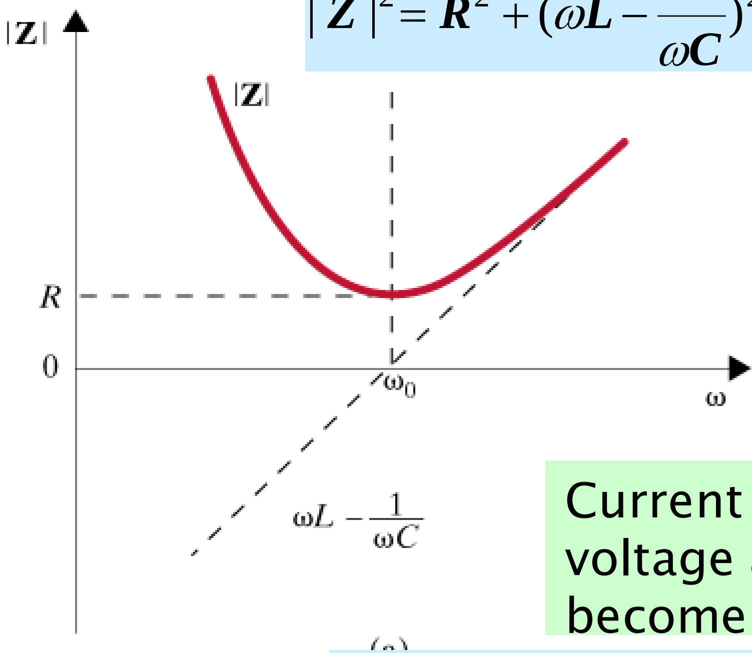
STOP

Properties of resonant circuits

At resonance the impedance/admittance is minimal

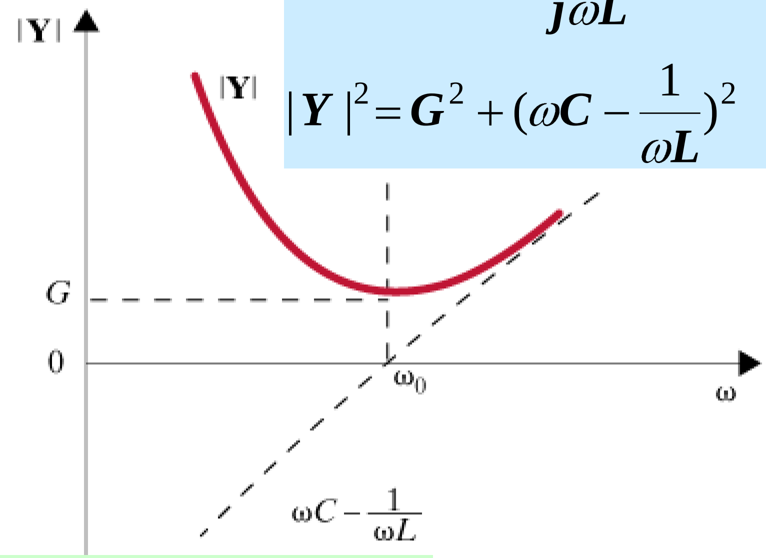
$$Z(j\omega) = R + j\omega L + \frac{1}{j\omega C}$$

$$|Z|^2 = R^2 + (\omega L - \frac{1}{\omega C})^2$$



$$Y(j\omega) = G + \frac{1}{j\omega L} + j\omega C$$

$$|Y|^2 = G^2 + (\omega C - \frac{1}{\omega L})^2$$



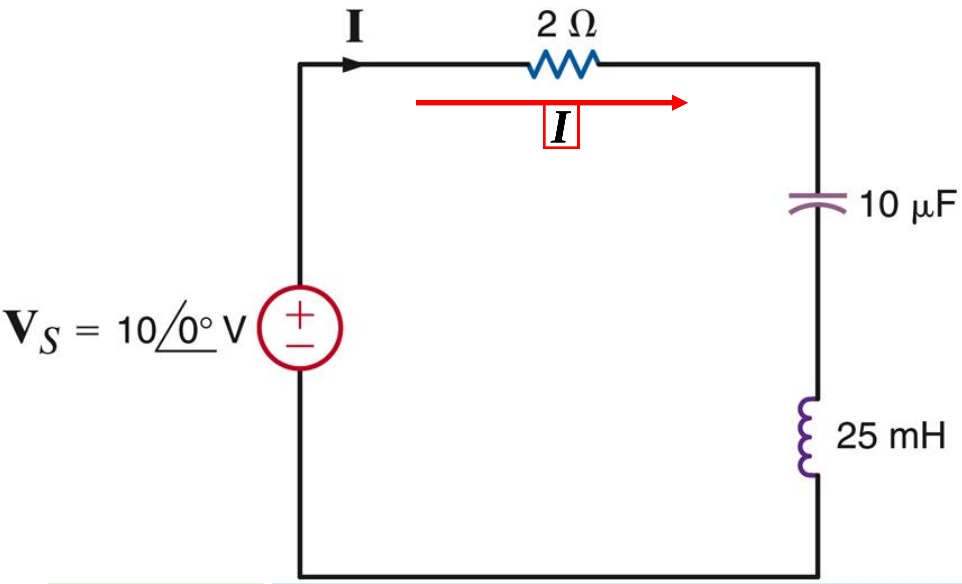
Current through the serial circuit/ voltage across the parallel circuit can become very large

Quality Factor: $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$

Given the similarities between series and parallel resonant circuits, we will focus on serial circuits

LEARNING EXAMPLE

Determine the resonant frequency, the voltage across each element at resonance and the value of the quality factor



$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(25\times 10^{-3}H)(10\times 10^{-6}F)}} = 2000\, rad/\sec$$

At resonance $Z = 2\Omega$

$$I = \frac{V_S}{Z} = \frac{10\angle 0°}{2} = 5A$$

$$\omega_0 L = (2\times 10^3)(25\times 10^{-3}) = 50\Omega$$

$$V_L = j\omega_0 LI = j50\times 5 = 250\angle 90°(V)$$

$$\frac{1}{\omega_0 C} = \omega_0 L = 50\Omega$$

$$V_C = \frac{1}{j\omega_0 C} I = -j50\times 5 = 250\angle -90°$$

$$Q = \frac{\omega_0 L}{R} = \frac{50}{2} = 25$$

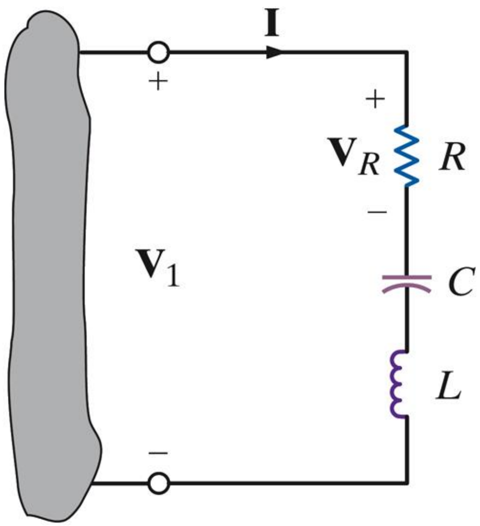
At resonance

$$|V_L| = \omega_0 L\left|\frac{V_S}{R}\right| = Q|V_S|$$

$$|V_C| = Q|V_S|$$

GEAUX

Resonance for the series circuit



$$Z(j\omega) = R + j\omega L + \frac{1}{j\omega C}$$

$$|Z|^2 = R^2 + (\omega L - \frac{1}{\omega C})^2$$

Claim: The voltage gain

$$G_v = \frac{V_R}{V_1} = \frac{1}{1 + jQ(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega})}$$

$$G_v = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} = \frac{R}{Z(j\omega)}$$

At resonance:

$$\omega_0 L = QR, \ \omega_0 C = \frac{1}{QR}$$

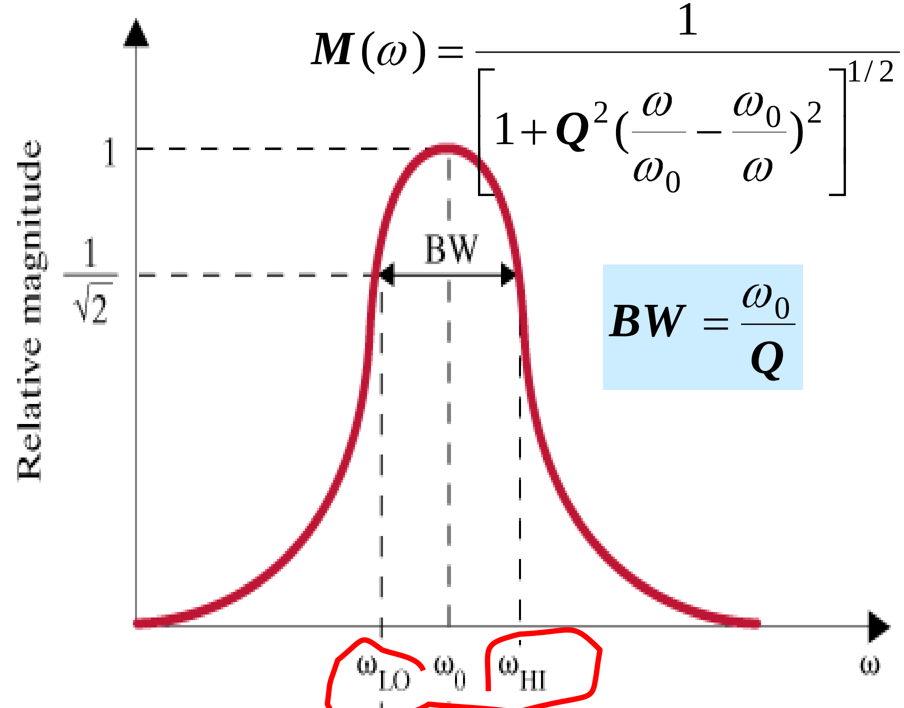
$$Z(j\omega) = R + j\frac{\omega}{\omega_0}QR - j\frac{\omega_0}{\omega}QR$$

$$= R\left[1 + jQ(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega})\right]$$

$$G_v = \frac{R}{Z}$$

$$M(\omega) = |G_v|, \ \phi(\omega) = \angle G_v$$

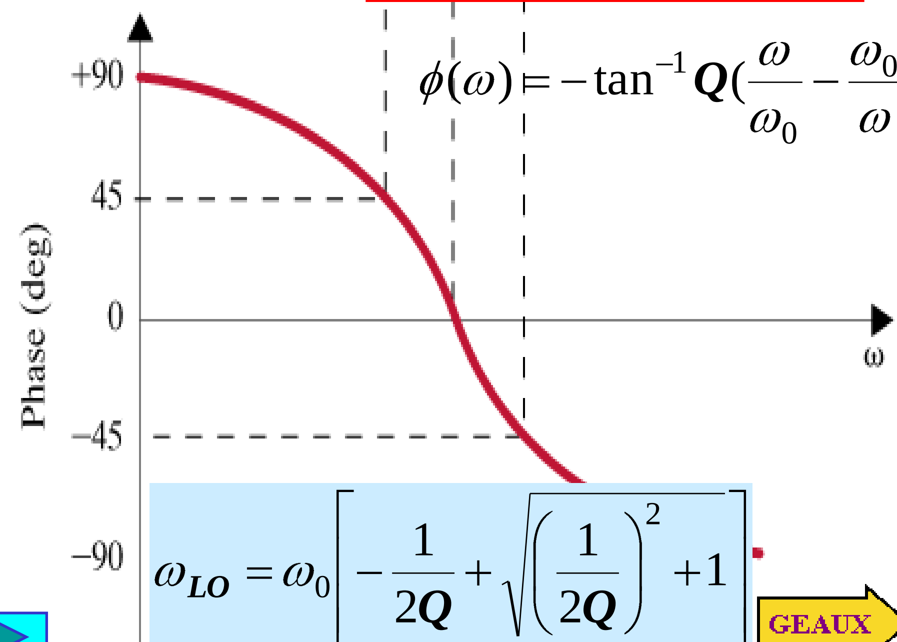
$$M(\omega) = \frac{1}{\left[1 + Q^2(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega})^2\right]^{1/2}}$$



$$BW = \frac{\omega_0}{Q}$$

Half power frequencies

$$\phi(\omega) = -\tan^{-1} Q(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega})$$



$$\omega_{LO} = \omega_0\left[-\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]$$

GEAUX

LEARNING EXTENSION

A series RLC circuit as the following properties:

$$R = 4\Omega, \omega_0 = 4000\, rad/\sec, BW = 100\, rad/\sec$$

Determine the values of L,C.

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} \qquad BW = \frac{\omega_0}{Q}$$

1. Given resonant frequency and bandwidth determine Q.
2. Given R, resonant frequency and Q determine L, C.

$$Q = \frac{\omega_0}{BW} = \frac{4000}{100} = 40$$

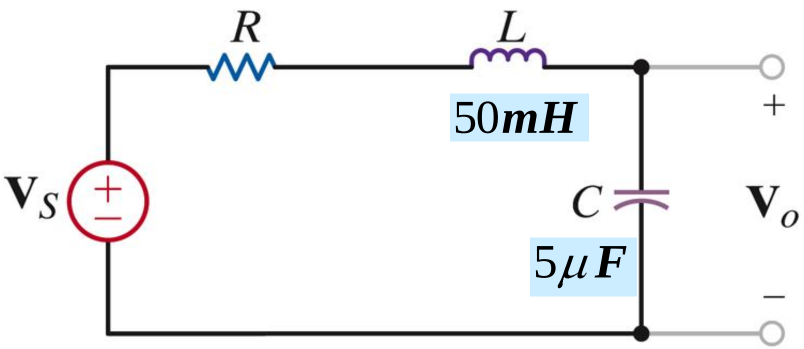
$$L = \frac{QR}{\omega_0} = \frac{40 \times 4}{4000} = 0.040H$$

$$C = \frac{1}{L\omega_0^2} = \frac{1}{\omega_0 RQ} = \frac{1}{4 \times 10^{-2} \times 16 \times 10^6} = 1.56 \times 10^{-6} F$$

GEAUX

LEARNING EXAMPLE

Determine $\omega_0, \omega_{\max}$ when $R = 50\Omega$ and $R = 1\Omega$



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

$$u_{\max} = \frac{\omega_{\max}}{\omega_0} = \sqrt{1 - \frac{1}{2Q^2}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(5\times 10^{-2})(5\times 10^{-6})}} = 2000\, rad/s$$

$$Q = \frac{2000 \times 0.050}{R} \qquad \omega_{\max} = 2000 \times \sqrt{1 - {}^{1}\!/_{2Q^2}}$$

R	Q	Wmax
50	2	1871
1	100	2000

Evaluated with EXCEL and rounded to zero decimals

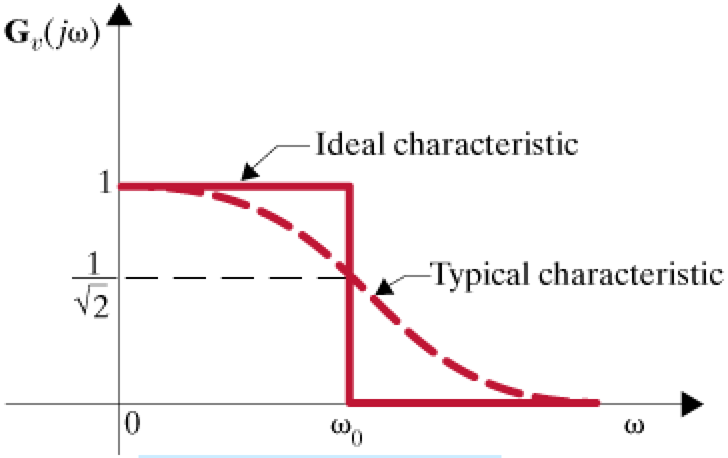
Using MATLAB one can display the frequency response

GEAUX

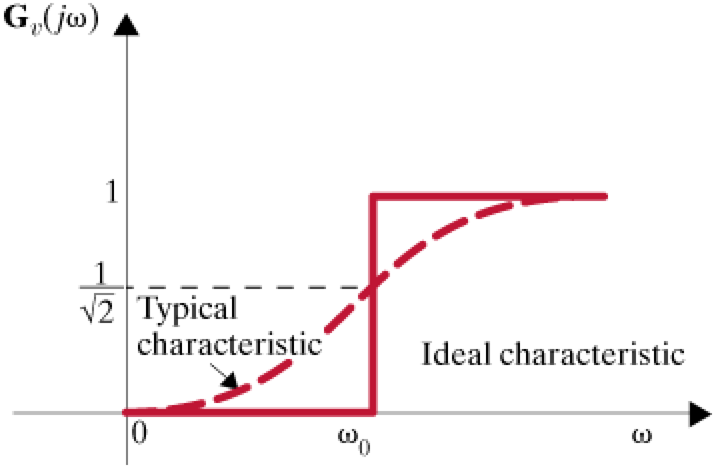
FILTER NETWORKS

Networks designed to have frequency selective behavior

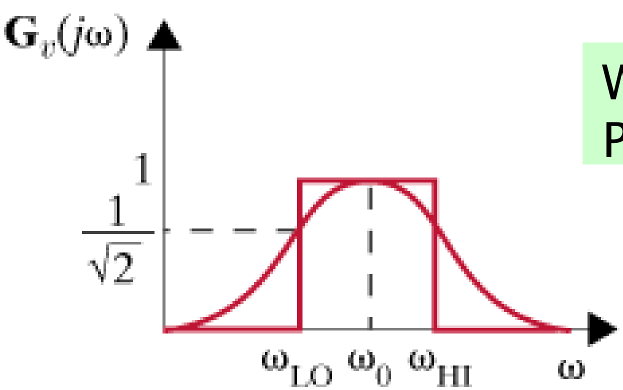
COMMON FILTERS



Low-pass filter

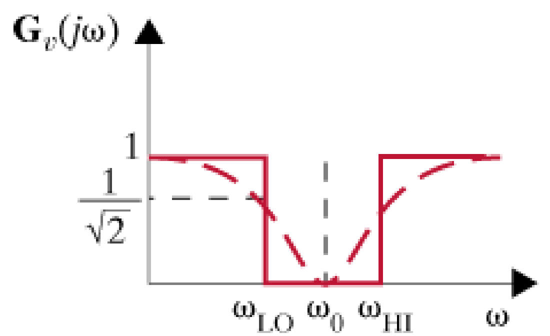


High-pass filter



Band-pass filter

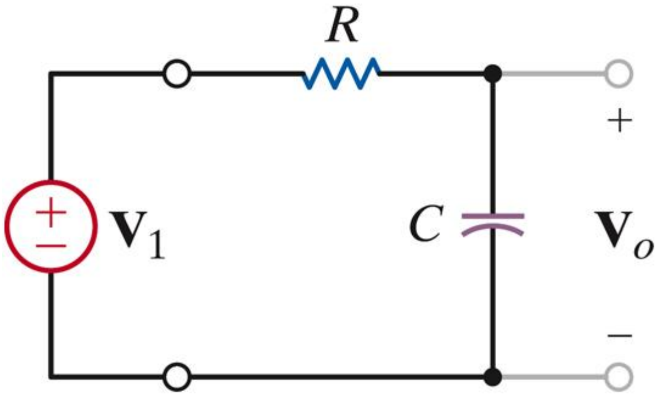
We focus first on PASSIVE filters



Band-reject filter

GEAUX

Simple low-pass filter



$$G_v = \frac{V_0}{V_1} = \frac{\dfrac{1}{j\omega C}}{R + \dfrac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

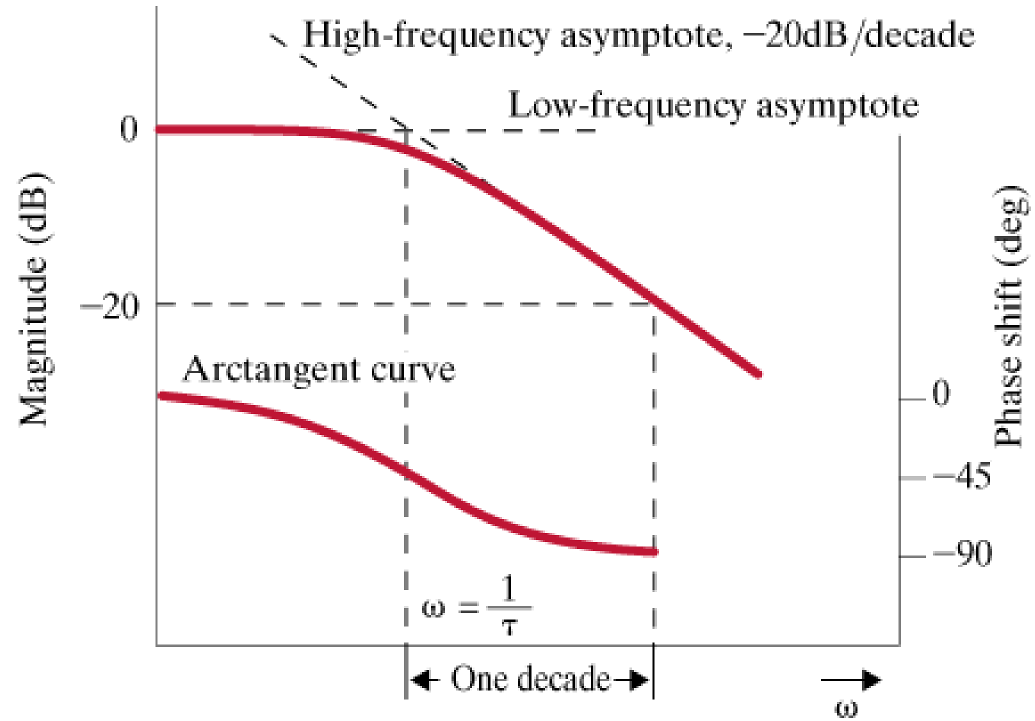
$$G_v = \frac{1}{1 + j\omega\tau}; \quad \tau = RC$$

$$M(\omega) = |G_v| = \frac{1}{\sqrt{1 + (\omega\tau)^2}}$$

$$\angle G_v = \phi(\omega) = -\tan^{-1}\omega\tau$$

$$M_{\max} = 1, \quad M\left(\omega = \frac{1}{\tau}\right) = \frac{1}{\sqrt{2}}$$

$\omega = \dfrac{1}{\tau}$ = half power frequency

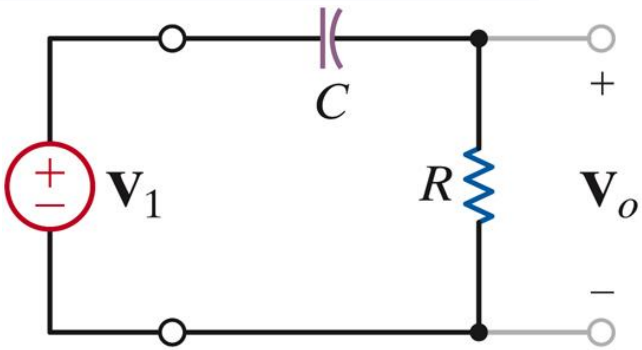


(c)

$$BW = \frac{1}{\tau}$$

GEAUX

Simple high-pass filter



$$G_v = \frac{V_0}{V_1} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega CR}{1 + j\omega CR}$$

$$G_v = \frac{j\omega\tau}{1 + j\omega\tau}; \ \tau = RC$$

$$M(\omega) = |G_v| = \frac{\omega\tau}{\sqrt{1 + (\omega\tau)^2}}$$

$$\angle G_v = \phi(\omega) = \frac{\pi}{2} - \tan^{-1}\omega\tau$$

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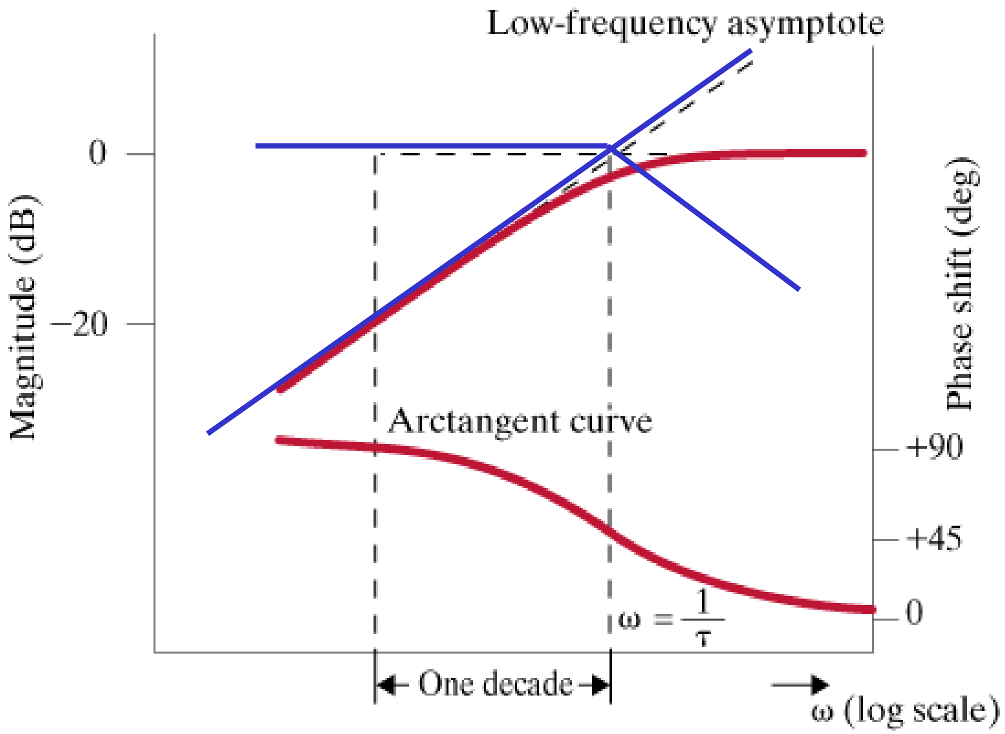
$\omega = \dfrac{1}{\tau} =$ half power frequency

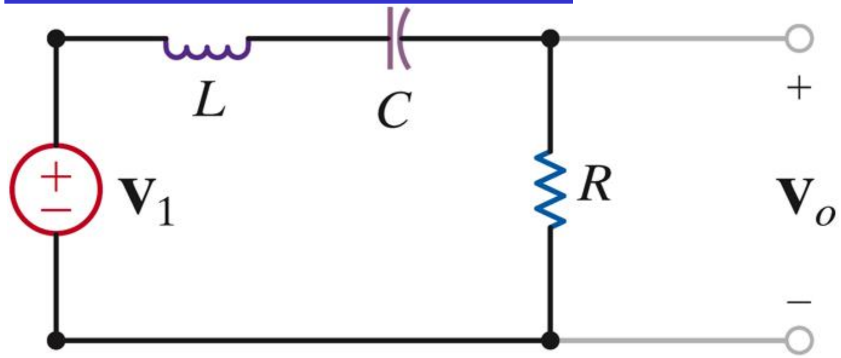


(c)

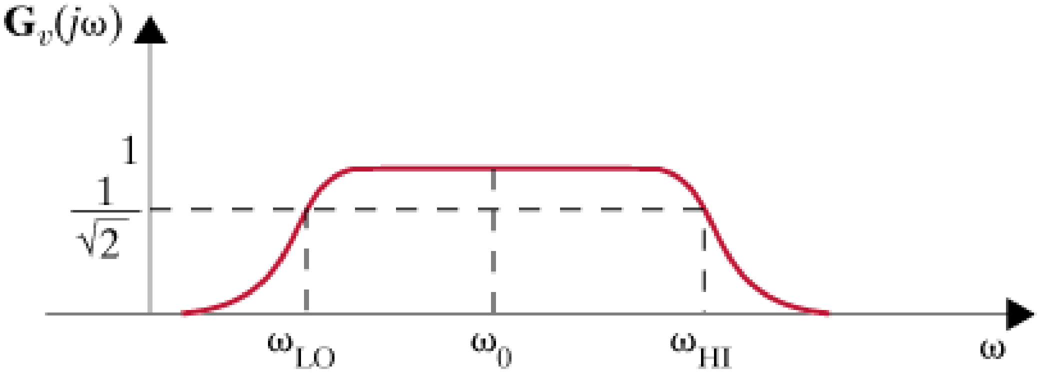
$$\omega_{LO} = \frac{1}{\tau}$$

GEAUX

Simple band-pass filter



Band-pass



(e)

$$G_v = \frac{V_0}{V_1} = \frac{R}{R + j\left(\omega L - \dfrac{1}{\omega C}\right)}$$

$$M(\omega) = \frac{\omega RC}{\sqrt{(\omega RC)^2 + (\omega^2 LC - 1)^2}}$$

$$M\left(\omega = \frac{1}{\sqrt{LC}}\right) = 1 \qquad M(\omega = 0) = M(\omega = \infty) = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$M(\omega_{LO}) = \frac{1}{\sqrt{2}} = M(\omega_{HI})$$

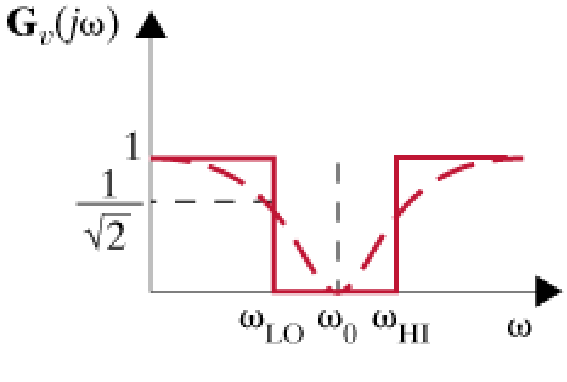
$$\omega_{LO} = \frac{-(R/L) + \sqrt{(R/L)^2 + 4\omega_0^2}}{2}$$

$$\omega_{HI} = \frac{(R/L) + \sqrt{(R/L)^2 + 4\omega_0^2}}{2}$$

$$BW = \omega_{HI} - \omega_{LO} = \frac{R}{L}$$

GEAUX

Simple band-reject filter



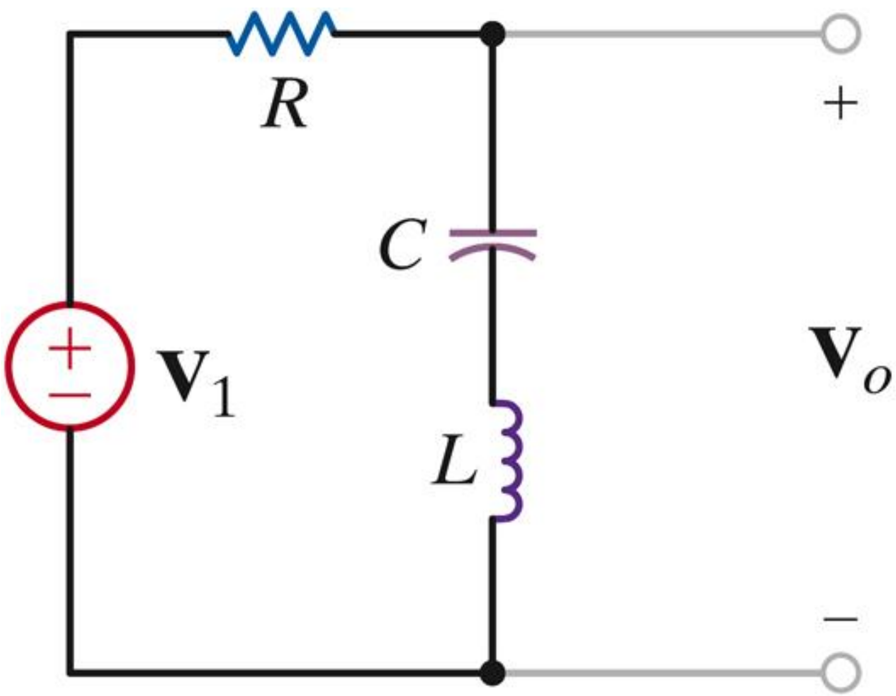
(b)

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow j\left(\omega_0 L - \frac{1}{\omega_0 C}\right) = 0$$

at $\omega = 0$ the capacitor acts as open circuit $\Rightarrow V_0 = V_1$

at $\omega = \infty$ the inductor acts as open circuit $\Rightarrow V_0 = V_1$

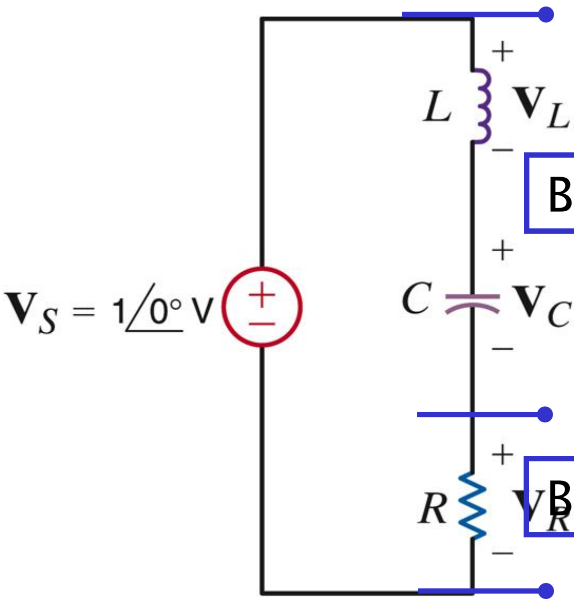
ω_{LO}, ω_{HI} are determined as in the band-pass filter



GEAUX

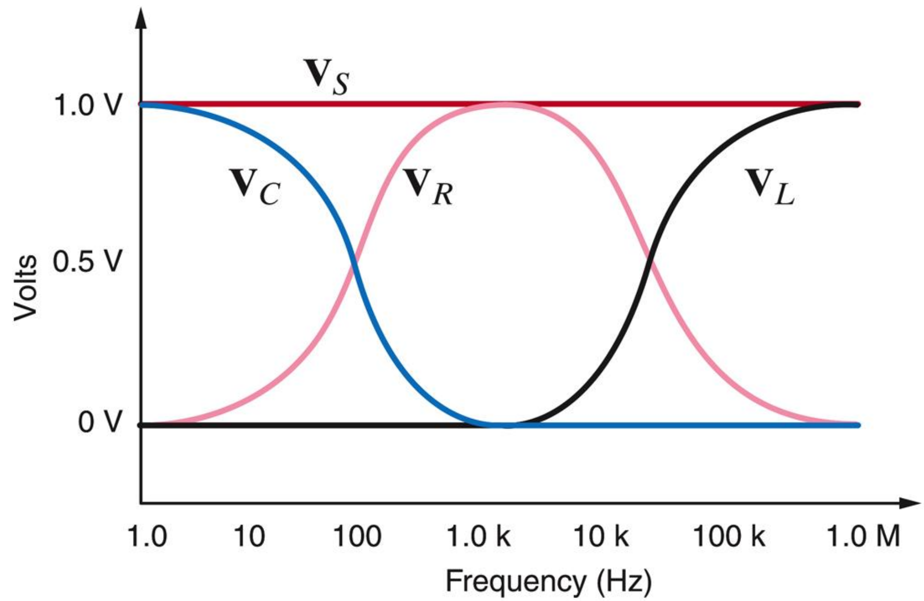
LEARNING EXAMPLE

Depending on where the output is taken, this circuit can produce low-pass, high-pass or band-pass or band-reject filters



Band-reject filter

Band-pass



Bode plot for $R = 10\Omega,\ L = 159\,\mu H,\ C = 159\,\mu F$

$$\frac{V_L}{V_S} = \frac{j\omega L}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\frac{V_L}{V_S}(\omega = 0) = 0,\ \frac{V_L}{V_S}(\omega = \infty) = 1$$

High-pass

$$\frac{V_C}{V_S} = \frac{\frac{1}{j\omega C}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\frac{V_C}{V_S}(\omega = 0) = 1,\ \frac{V_C}{V_S}(\omega = \infty) = 0$$

Low-pass

GEAUX

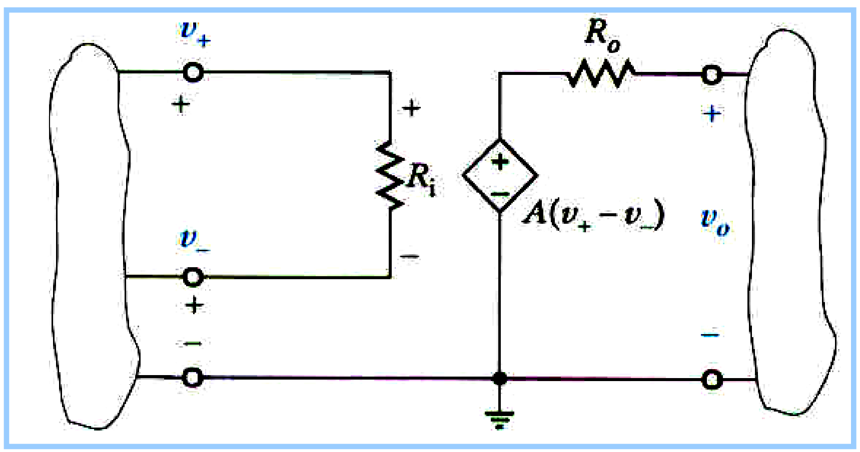
ACTIVE FILTERS

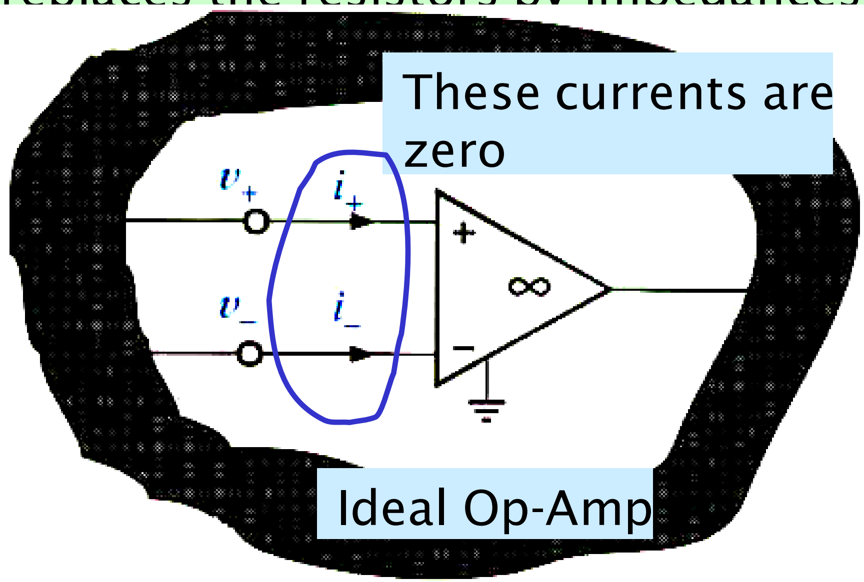
Passive filters have several limitations

1. Cannot generate gains greater than one
2. Loading effect makes them difficult to interconnect
3. Use of inductance makes them difficult to handle

Using operational amplifiers one can design all basic filters, and more, with only resistors and capacitors

The linear models developed for operational amplifiers circuits are valid, in a more general framework, if one replaces the resistors by impedances





GEAUX

DC Machines

Introduction:

The electrical machines deals with the energy transfer either from mechanical to electrical form or from electrical to mechanical form, this process is called electromechanical energy conversion.

DC GENERATOR

- An electrical machine which converts mechanical energy into electrical energy is called an electric generator and a generators works on the principle of dynamically induced emf according to Faraday’s Law of electro Magnetic Induction

Cont…

A D. C. machine consists of two main parts

a) Stationary part: It is designed mainly for producing a magnetic flux.

b) Rotating part: It is called the armature, where mechanical energy is converted into electrical Energy

Construction of a DC Generator

Parts of a Dc Generator:

- Yoke
- Magnetic Poles
  - a)Pole core
  - b)Pole Shoe
- Field Winding
- Armature Core
- Armature winding
- Commutator
- Brushes and Bearings

Yoke

1) It serves the purpose of outermost cover of the dc machine so that the Insulating materials get protected from harmful atmospheric elements like moisture, dust and various gases like SO_2, acidic fumes etc.
2) It provides mechanical support to the poles.
3) It forms a part of the magnetic circuit. It provides a path of low reluctance for magnetic flux.

Poles

Each pole is divided into two parts

a) pole core

b) pole shoe

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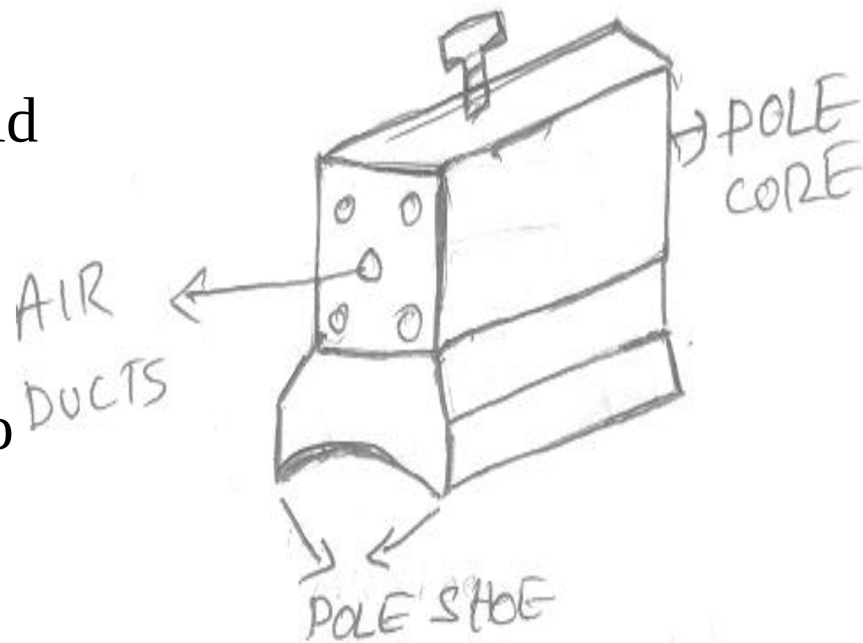
2) It directs the flux produced through air gap to armature core to the next pole.

3)Pole shoe enlarges the area of armature core to come across the flux, which is necessary to produce larger induced emf.



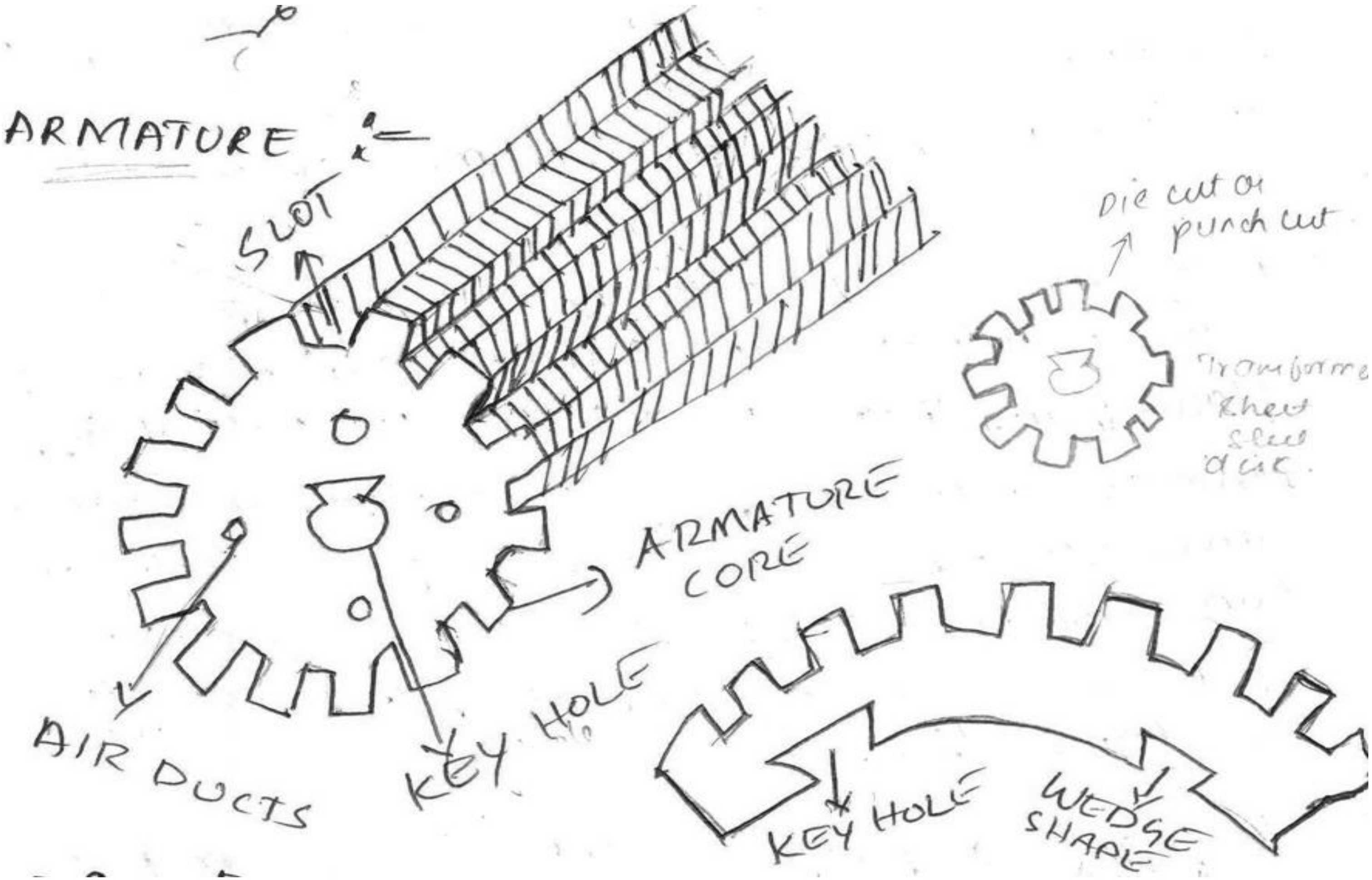
Armature

It is further divided into two parts namely,

(1) Armature core (2) Armature winding

Armature core: Armature core is cylindrical in shape mounted on the shaft. It consists of slots on its periphery and the air ducts to permit the air flow through armature which serves cooling purpose.

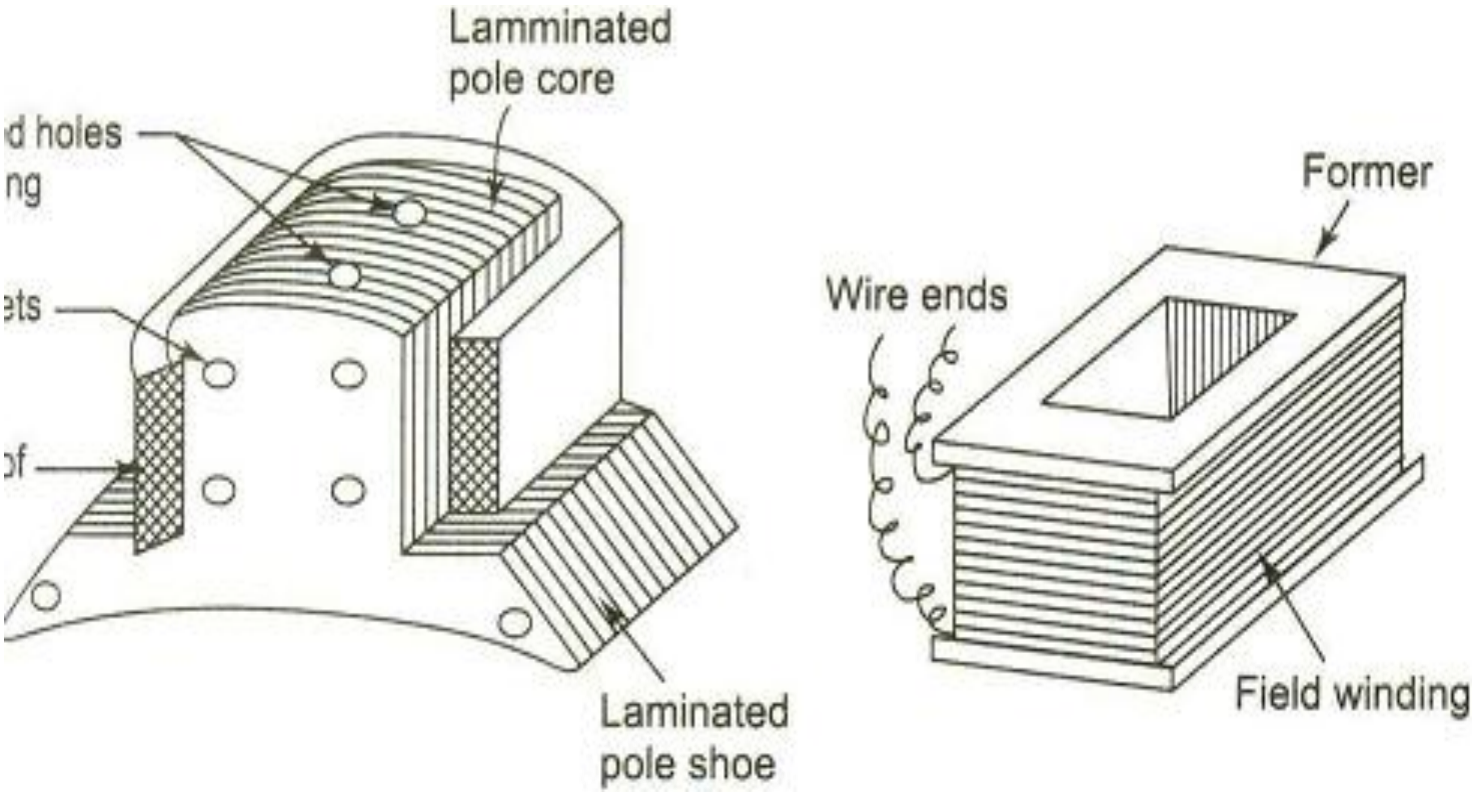
Cont…


ARMATURE :-
SLOT
Die cut or punch cut
Transformer sheet steel disk
ARMATURE CORE
AIR DUCTS
KEY HOLE
KEY HOLE
WEDGE SHAPE

Armature winding

Armature winding is nothing but the inter connection of the armature conductors, placed in the slots provided on the armature core. When the armature is rotated, in case of generator magnetic flux gets cut by armature conductors and emf gets induced in them.

Field winding



Cont..

a)The field winding is wound on the pole core with a definite direction

b)To carry current due to which pole core on which the winding is placed behaves as an electromagnet, producing necessary flux.

3)As it helps in producing the magnetic field i.e. exciting the pole as electromagnet it is called ‘**Field winding**’ or ‘**Exciting winding**’.

Commutator

The rectification in case of dc generator is done by device called as commutator.

1. To facilitate the collection of current from the armature conductors.
2. To convert internally developed alternating emf to in directional (dc) emf
3. To produce unidirectional torque in case of motor.

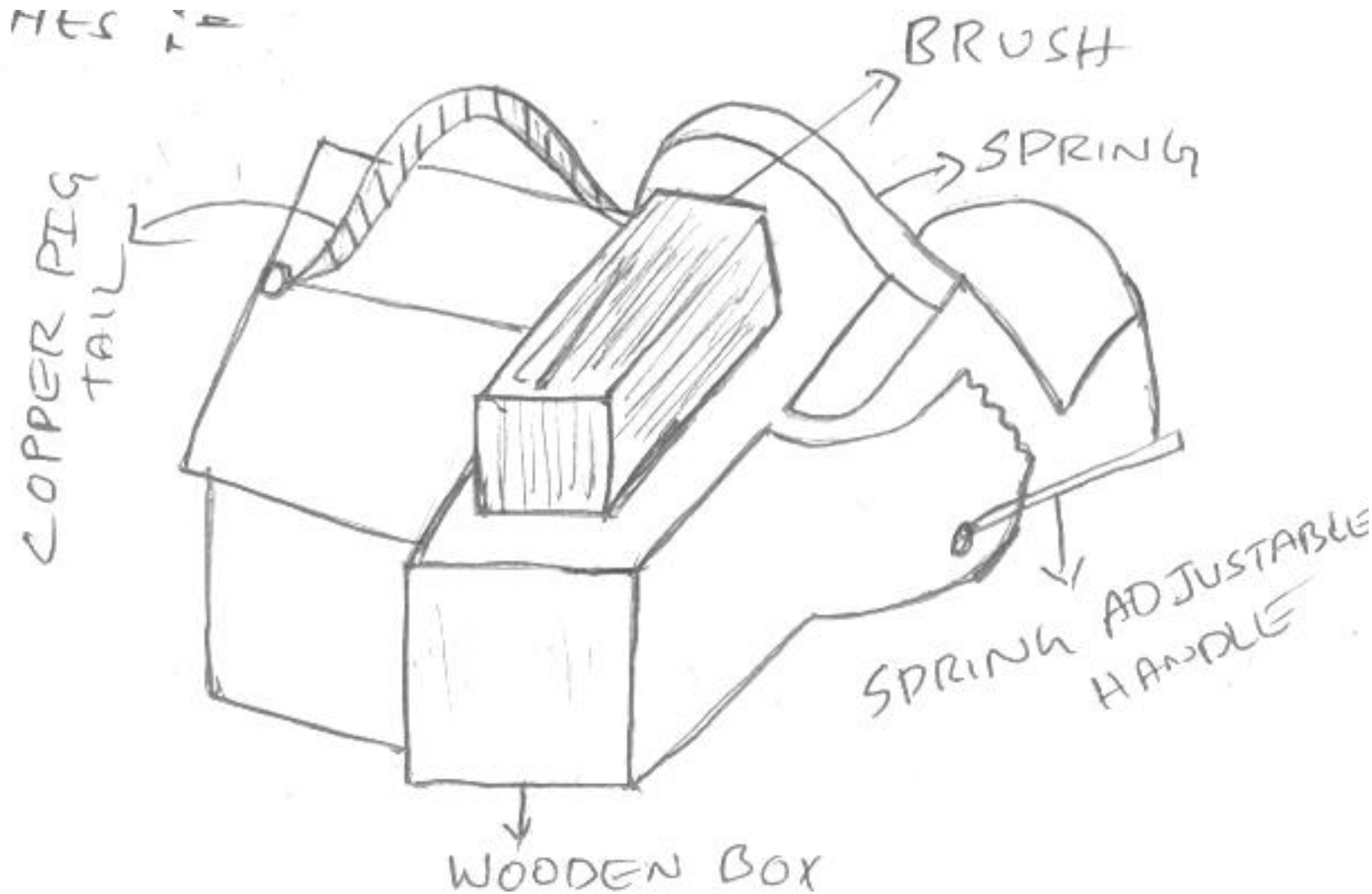
Brushes and brush gear

1) To collect current from commutator and make it available to the stationary external circuit.
2) Brushes are stationary and rest on the surface of the commutator. Brushes are rectangular in shape. They are housed in brush holders, which are usually of box type.

Cont..

- The brushes are made to press on the commutator surface by means of a spring, whose tension can be adjusted with the help of lever. A flexible copper conductor called pigtail is used to connect the brush to the external circuit

Cont…



BRUSH
SPRING
COPPER PIG TAIL
SPRING ADJUSTABLE HANDLE
WOODEN BOX

Bearings

Ball-bearings are usually used as they are more reliable. For heavy duty machines, roller bearings are preferred

Types of generators

1)Separately excited generators

2) Self-excited generators

i) Series Generator

ii)Shunt Generator

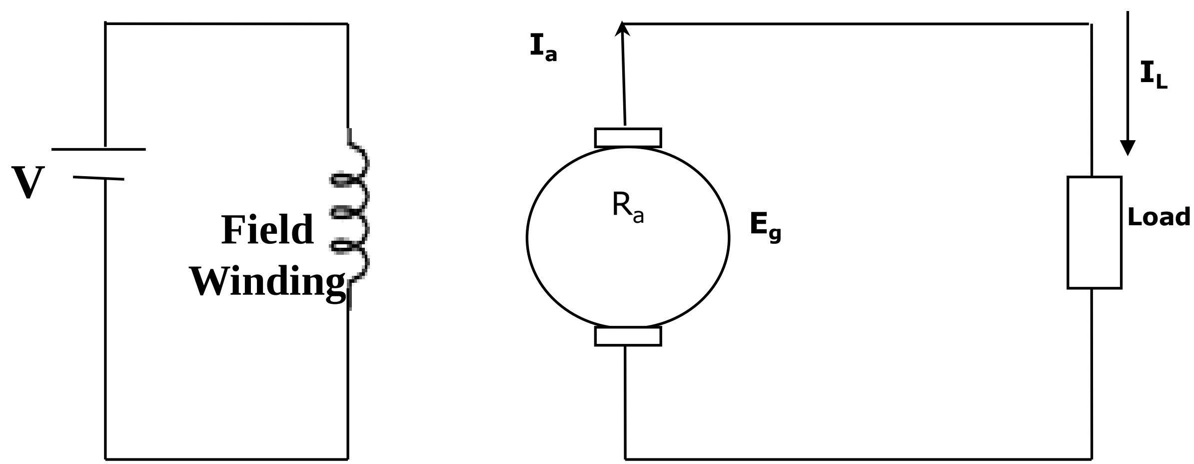
iii)Compound Generator

a) Long Shunt

b) Short Shunt

Separately excited generators:

These are generators whole field magnets are energized from an independent external source of dc current

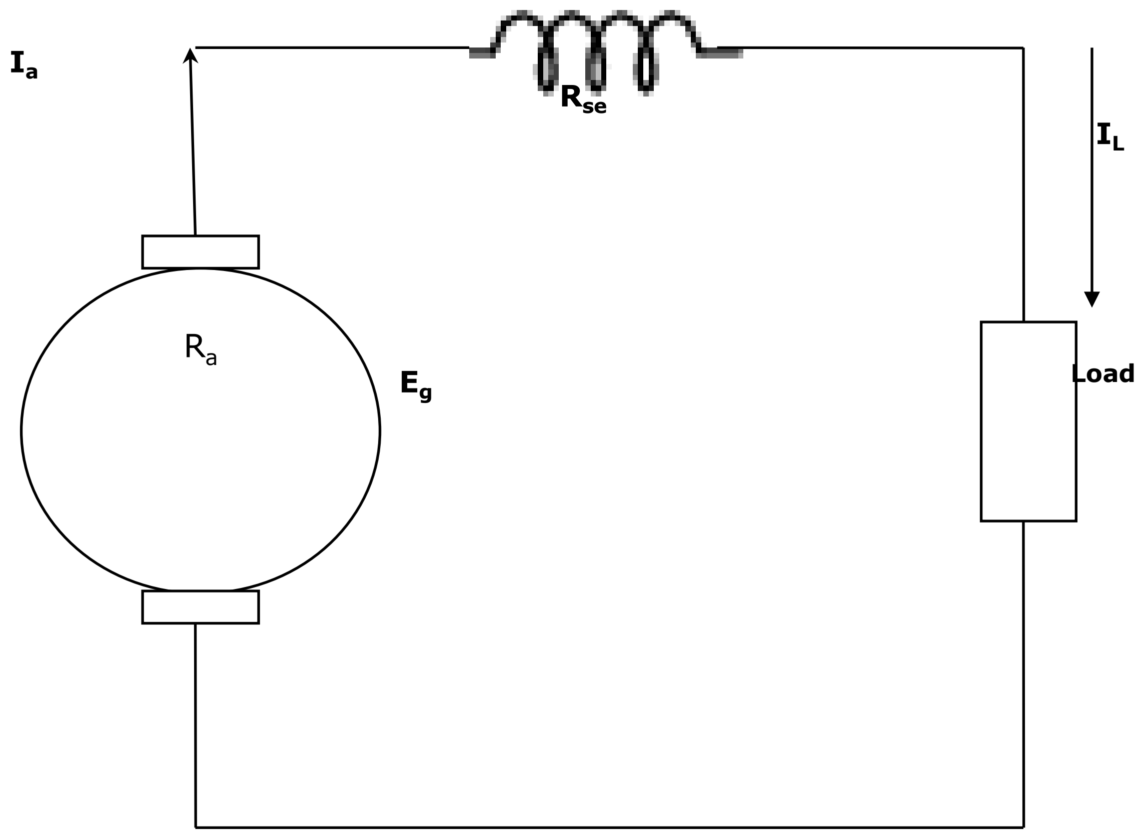


Self-excited generators

- These are the generators whose field magnets are energized by the current produced by the generators themselves.
- Due to residual magnetism, there is always present some flux in the poles when the armature is rotated, some emf and hence some induced current is produced which is partly or fully passed through the field coils thereby strengthening the residual pole flux.

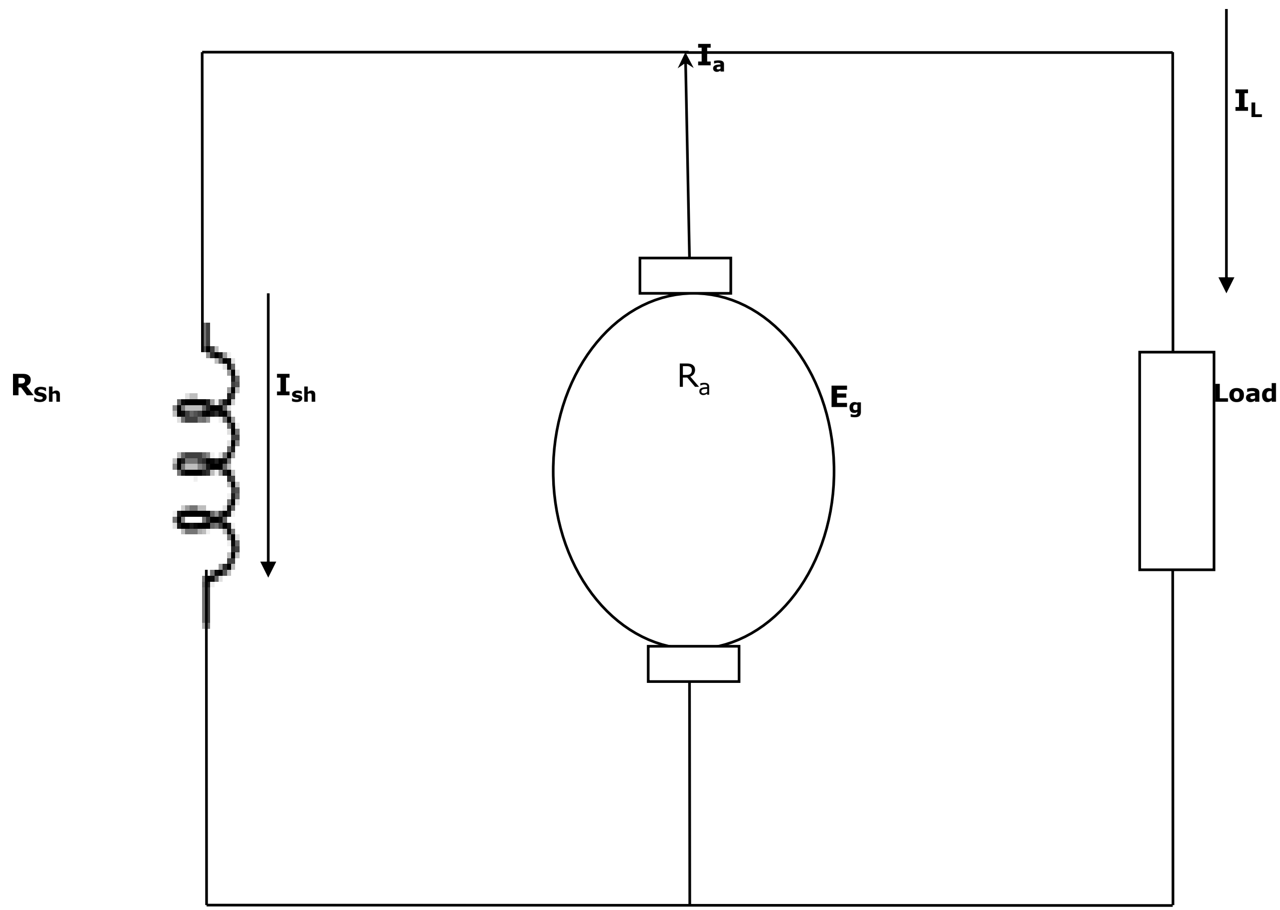
Series Generator:

It is a generator where the field winding is connected in series with the armature



Shunt Generator

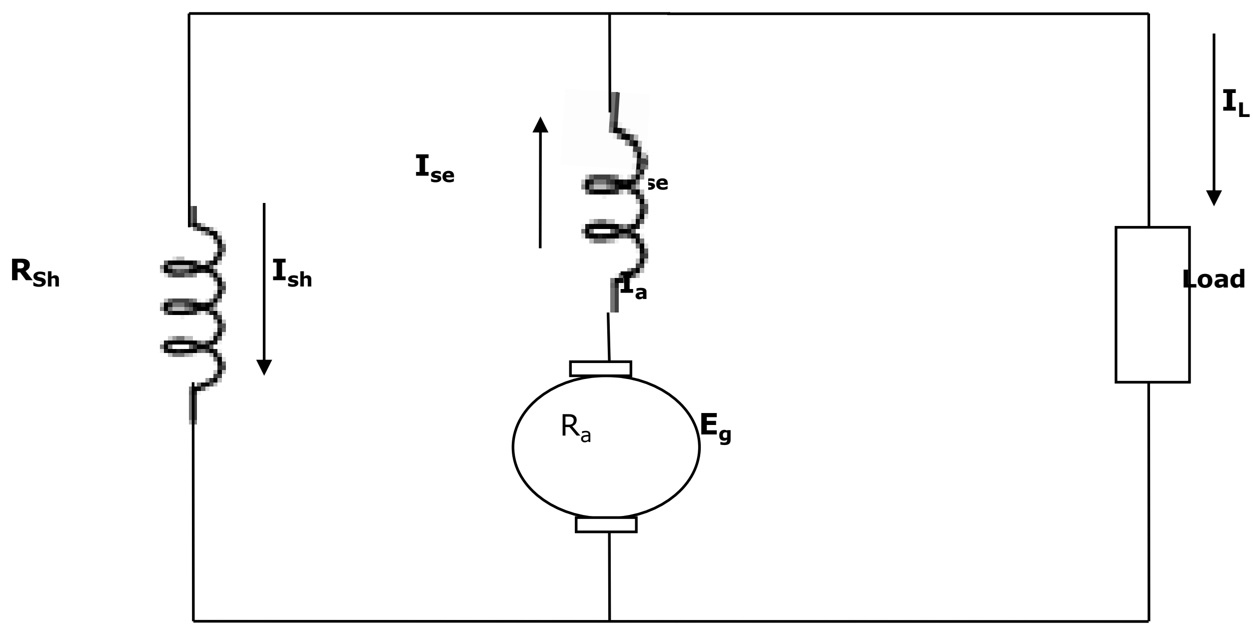
It is a generator where the field winding is connected in series with the armature



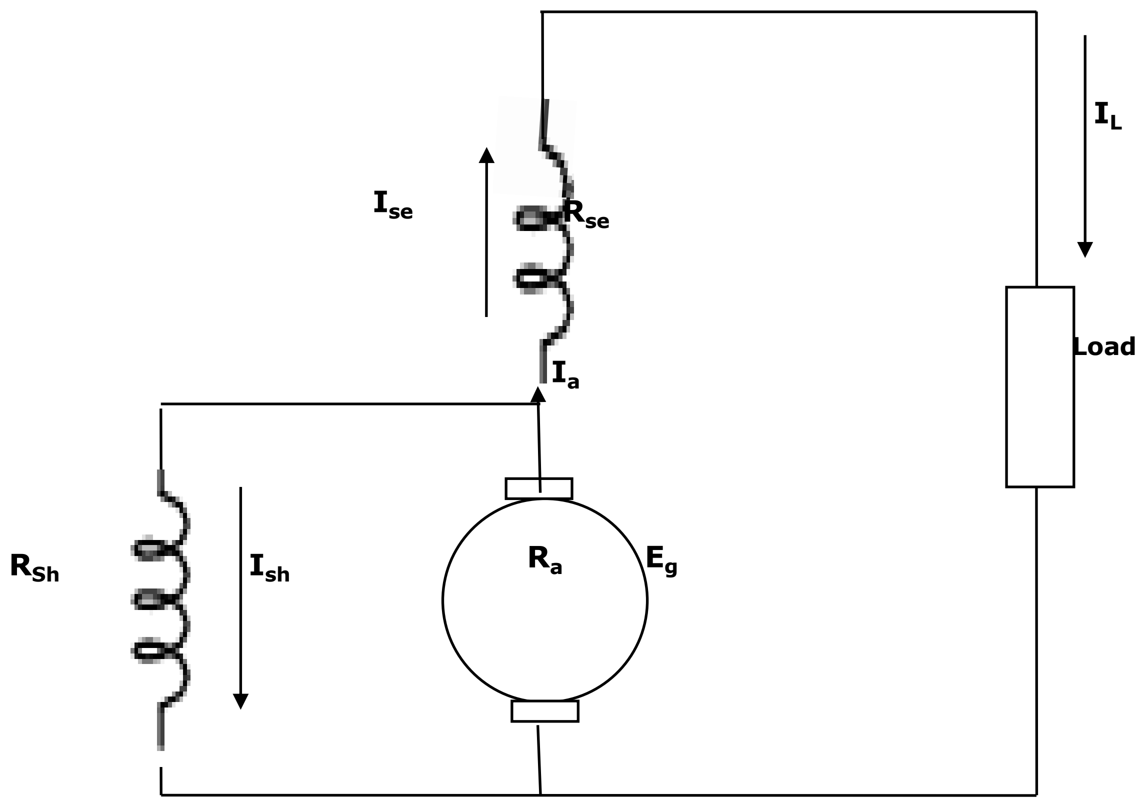
Compound Generator

It is generator which has both Series and Shunt Windings. Compound generators are of two types

Long Shunt Compound Generator



Short Shunt Compound Generator



Types of armature winding

1)Lap winding:In this case, if connection is started from conductor in slot 1 then the connections overlap each other as winding proceeds, till starting point is reached again.

2) There is overlapping of coils while proceeding. Due to such connection, the total number of conductors get divided into ‘P’ number of parallel paths, where

P = number of poles in the machine.

Wave winding

- In this type, winding always travels ahead avoiding over lapping. It travels like a progressive wave hence called wave winding
- Both coils starting from slot 1 and slot 2 are progressing in wave fashion.
- Due to this type of connection, the total number of conductors get divided into two number of parallel paths always, irrespective of number of poles of machine.
- As number of parallel paths are less, it is preferable for low current, high voltage capacity generators.

EMF equation of a generator

Let P = number of poles

ϕ = flux/pole in webers

Z = total number of armature conductors.

= number of slots x number of conductors/slot

N = armature rotation in revolutions (speed for armature) per minute (rpm)

A = No. of parallel paths into which the 'z' no. of conductors are divided.

E = emf induced in any parallel path

E_g = emf generated in any one parallel path in the armature.

Cont..

Average emf generated/conductor = $d\phi/dt$ volt

In one revolution, the conductor will cut total flux produced by all poles = $\phi \times p$

No. of revolutions/second = $N/60$

Therefore, Time for one revolution, $dt = 60/N$ second

According to Faraday's laws of Electromagnetic Induction, emf generated/conductor

$$= d\phi/dt = \phi \times p \times N / 60 \text{ volts}$$

This is emf induced in one conductor

Cont..

For a simplex wave-wound generator

No. of parallel paths = 2

No. of conductors in (series)in one path = Z/2

EMF generated/path = $\phi PN/60 \times Z/2$

$= \phi ZPN/120$ volt

Cont..

For a simple lap-wound generator

Number of parallel paths = P

Number of conductors in one path = Z/P

EMF generated/path = $\phi PN/60 \ (Z/P) = \phi ZN/60$

A = 2 for simplex – wave winding

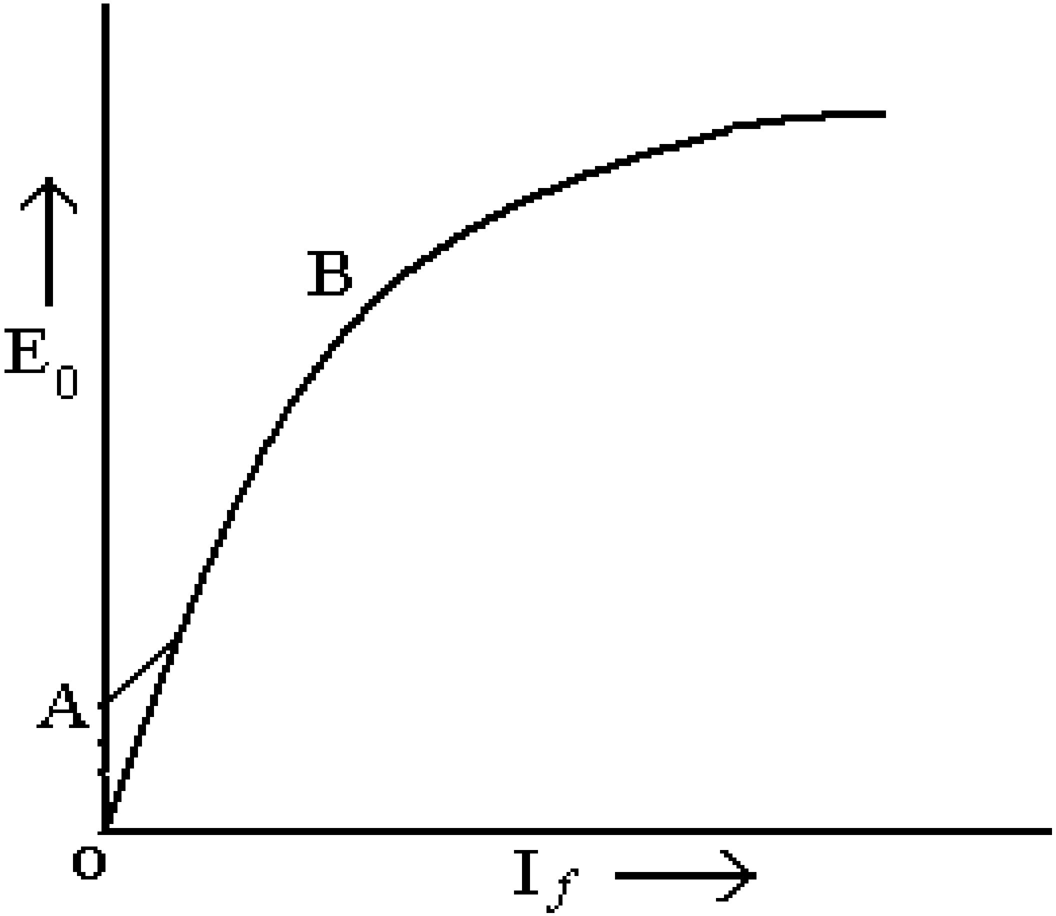
A =P for simplex-- lap winding

Characteristics of dc generators

Magnetization characteristics

- It is also known as no-load saturation characteristics or open-circuit characteristics (OCC)
- This characteristics is the graph of the no-load generated emf Eo and the field current If, at a given speed

Cont..



Losses in a Dc Generator

Variable losses :

The armature current varies with load . The copper losses that occur in the armature also vary w.r.t to load Armature Cu loss = Ia^2 Ra

Magnetic losses

- **Hysteresis :** The property of a magnetic material to retain a part of the magnetism achieved by it, one removal of the driving force is called hysteresis.
- The power required to overcome this hysteresis effect is called hysteresis loss. It is denoted by W_h.

$$W_h = \eta B_m^{1.6} f V$$

- Where , η=Steinmetz Constant
- B=Flux Density
- F=Frequency
- V=Volume Of the core

DC Motor:

- A dc motor is similar in construction to a dc generator. As a matter of fact a dc generator will run as a motor when its field & armature windings are connected to a source of direct current.

DC Motor Working principle

The principle of operation of a dc motor can be stated as when a current carrying conductor is placed in a magnetic field; it experiences a mechanical force. In a practical dc motor, the field winding produces the required magnetic held while armature conductor play the role of current carrying conductor and hence the armature conductors experience a force.

As conductors are placed in the slots which are on the periphery, the individual force experienced by the conductive acts as a twisting or turning force on the armature which is called a torque.

The torque is the product of force and the radius at which this force acts, so overall armature experiences a torque and starts rotating.

Construction of a DC Motor

Parts of a Dc Generator:

- Yoke
- Magnetic Poles
    - a)Pole core
    - b)Pole Shoe
- Field Winding
- Armature Core
- Armature winding
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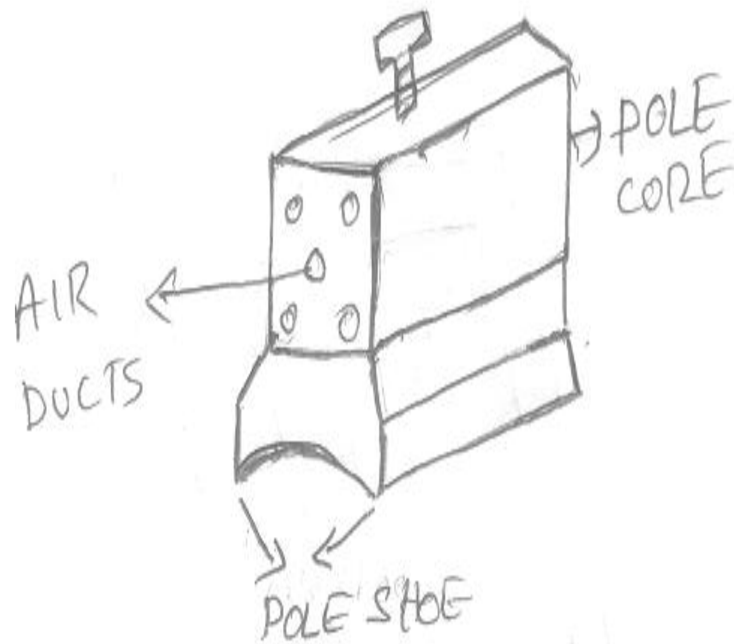

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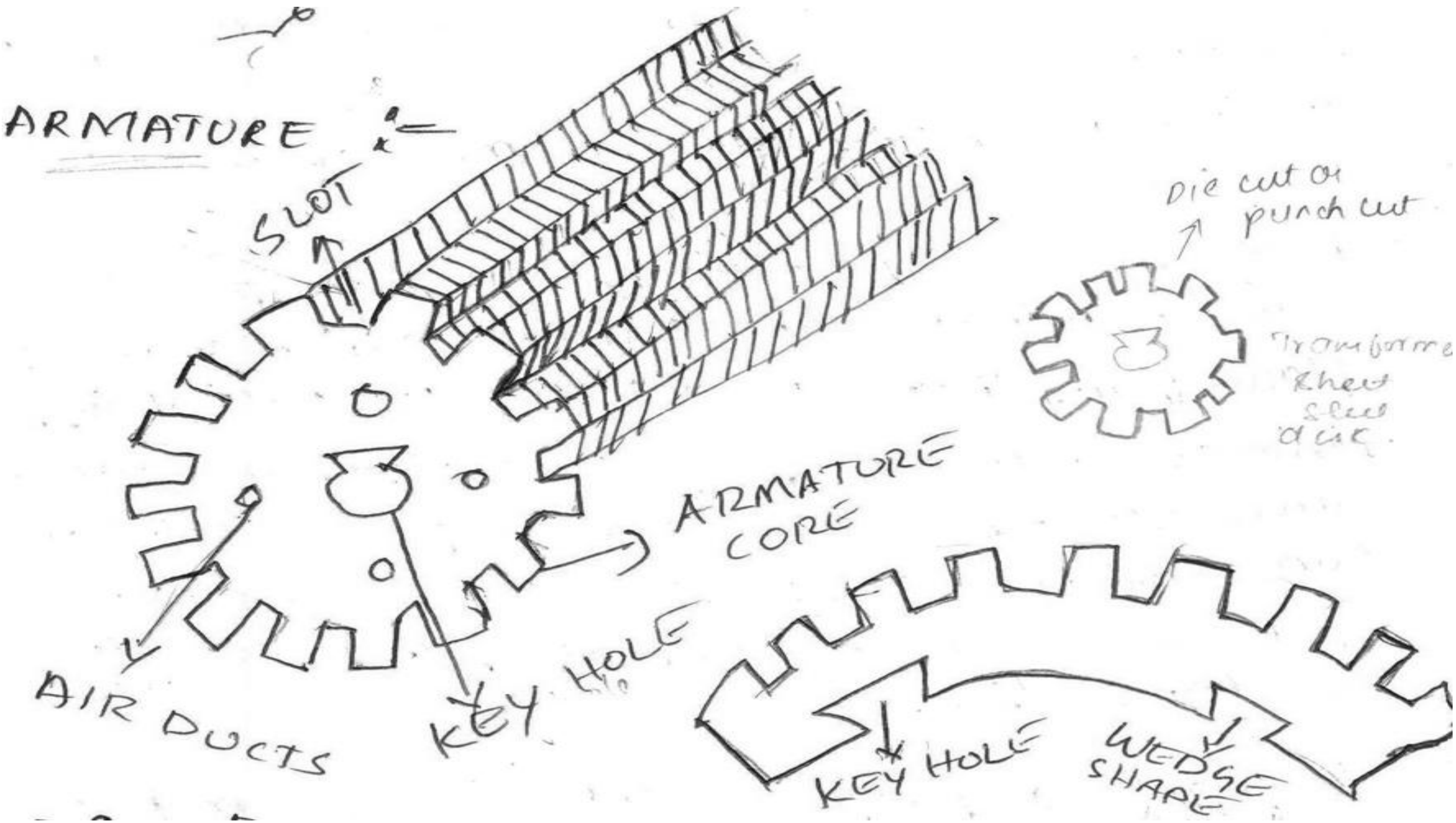
Cont…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punch cut
ARMATURE
CORE
AIR DUCTS
KEY HOLE
KEY HOLE
WEDGE
SHAPE

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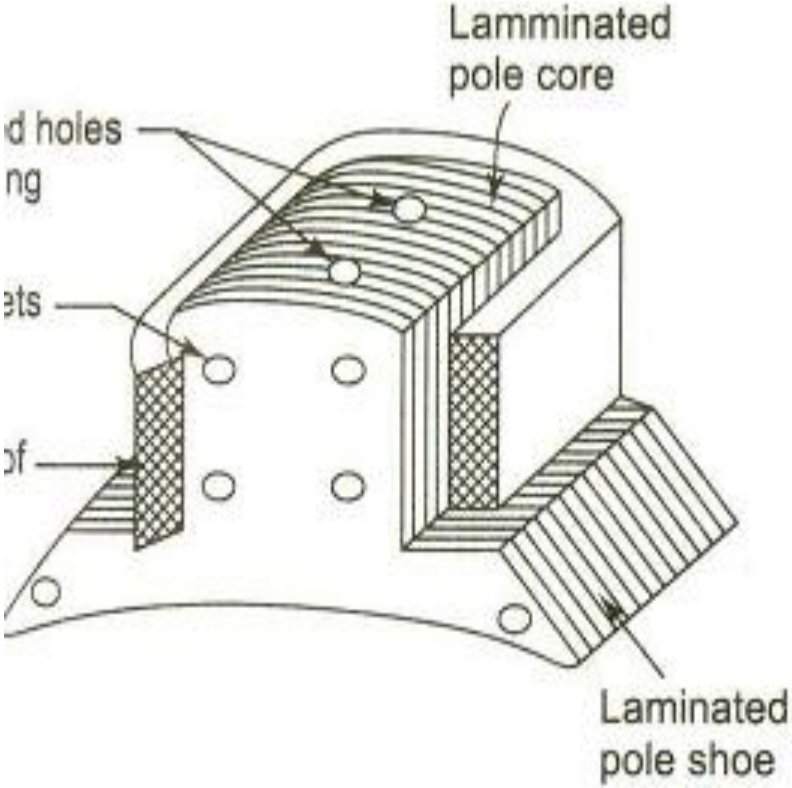
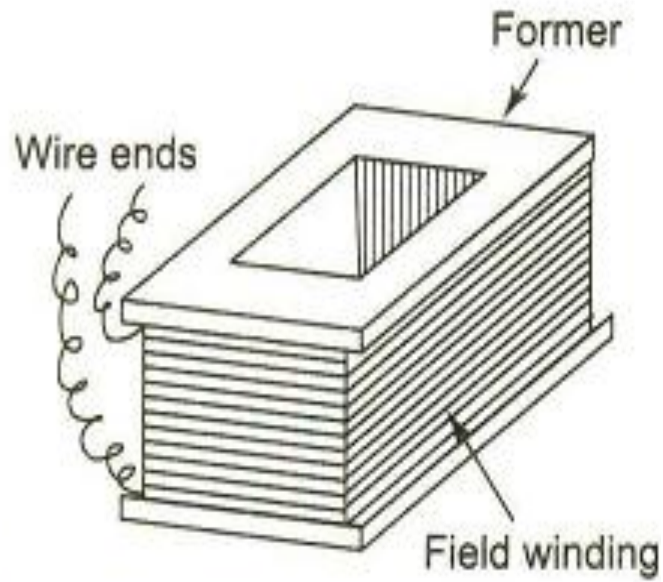
Field winding





Cont..

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Cont..

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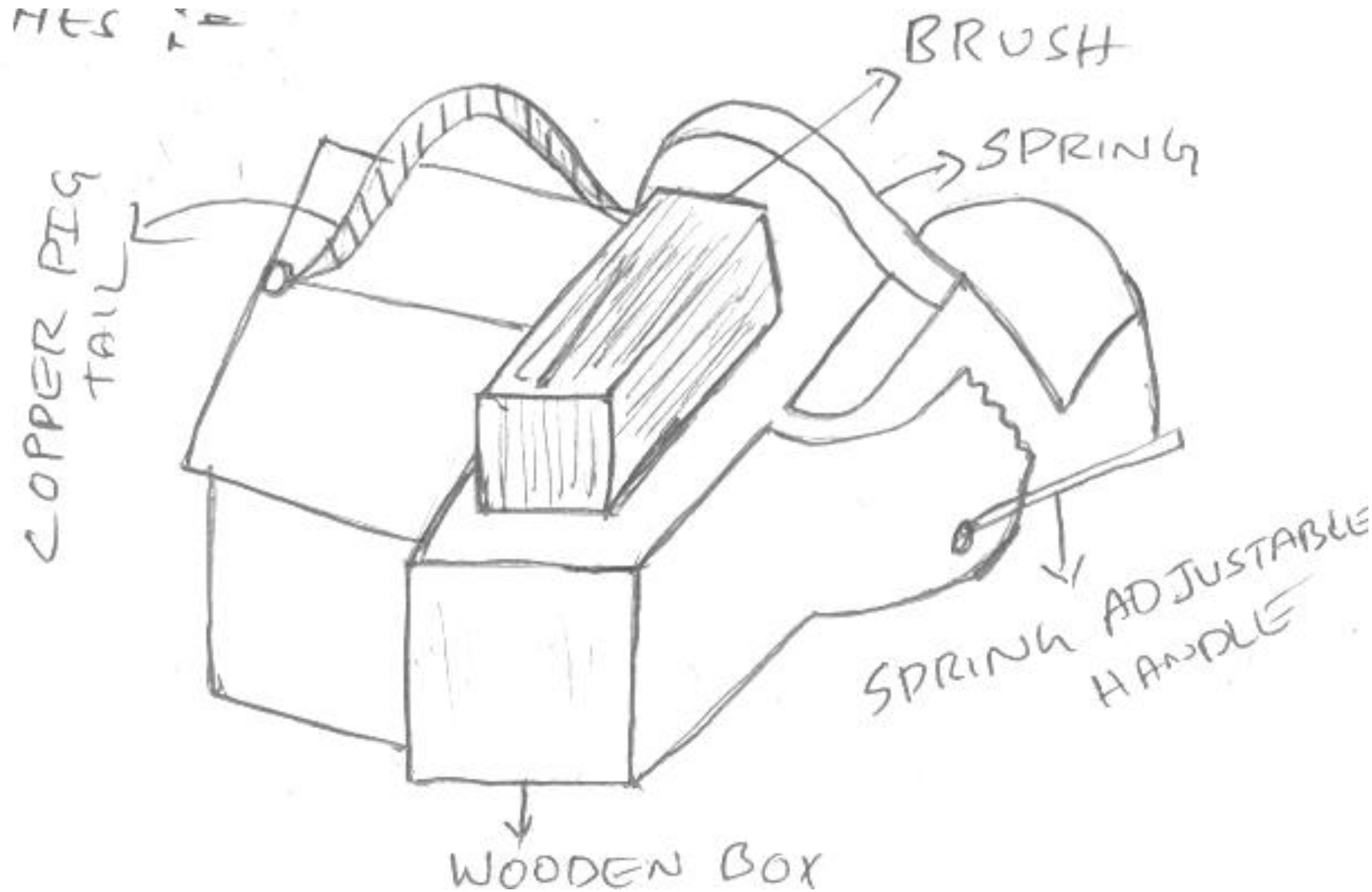
Cont…



BRUSH
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Types of motors

1) Shunt motor

2) Series motor

3) Compound motor

   i) Cumulative – compound motors

   ii) Differential compound:

Type of Motor	Characteristics	Applications
Shunt	Approximately constant speed. Adjustable speed medium starting torque	For during constant speed some centrifugal pumps machine tools blows and fans reciprocating pumps
Series	Variable speed adjustable varying speed high starting torque	For traction work i.e. electric locomotives repaid transit system trolley cars etc. Crimes and hoists conveyors.
Cumulative compound	Variable speed adjustable varying speed high starting torque	For intermittent high torque load for shears and punches elevators converge clearly planer

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P = number of poles in the machine.

Voltage equation of a Motor

- The voltage v applied across the motor armature has to (1) over core the back emf Eb and supply the armature holmic drop Ia Ro

  v = Eb + Ia Rg

  This is known as voltage equation of a motor

  Multiplying both sides by Ia, we get

  $Vi_a = E_b\ I_a + E_a^2\ R_a$

  $VI_a =$ electrical input to the armature

  $E_bI_a =$ electrical equivalent of mech. Power developed in the armature

  $I_a^2\ R_a =$ un loss in the armature

- Motor efficiency is given by the ratio of power developed by the armature to its input i.e. $E_b\ I_a\ /\ vI_a = E_b/v$.
- Higher the value of E_b as compared to v, higher the motor efficiency.

Condition for maximum power

- The gross mechanical developed by a motor

  $p_m = vI_a - I_a^2 R_a$ $\qquad\qquad$ $I_a R_a = v/2$

  As $v = E_b + I_a R_a$ and $I_a R_a = v/2$, $E_b = v/2$

- Thus gross mechanical power developed by a motor is maximum when back emf is equal to half the applied voltage. This conduction's how ever at realized in practice, because in that case current will be much beyond the normal current of the motor.

- More over, half the input would be wasted in the form of heat and taking other losses into consideration the motor efficiency will be well below 50 %.

Problems

1. **A 220v – dc machine has an armature resistance of 0.5 Ω. If the full road armature current is 20A, find the induced emf when the machine acts**

   (1)generator (2) motor.

The dc motor is assumed to be shunt connected in cash case, short current in considered negligible because its value is not given.

- (a) As generator : $E_g = v + I_a R_a = 220 + 0.5 \times 20 = 230$ v
- (b) As motor : $E_b = v - I_a R_a = 220 - 0.5 \times 20 = 210$ v

Problems

- **2. A 440 v, shunt motor has armature resistance of 0.8 a and field resistance of 200 Ω. Determine the back emf when giving an output at 7.46 kw at 85% efficiency.**

- Motor input power = $\frac{7.46x10^3}{0.85}w$

- Motor input current = $\frac{7460}{0.85x440} = 19.95\ A$

Problems

- A 4-pole dc motor has lap connected armature winding. The number of armature conductors is 250. When connected to 230 v dc supply it draws an armature current It 4 cm calculate the back emf and the speed with which motor is running. Assume armature is 0.6Ω.

- P = 4 A = P = 4 as lap connected
- ϕ = 30 m wb = 30 $\times 10^{-3}$, V = 230v, z = 250, Ia = 40 A

From voltage equation $V = E_b + I_a R_a$

$$230 = Eb + 40 \times 0.6$$

$$Eb = \phi Pnz / 60A$$

$$206 = (30 \times 10^{-3} \times 4 \times N \times 250) / (60 \times 4)$$

$$N = 1648 \text{ rpm.}$$

Power Equation

- Power developed by the machine

  = O/p + losses = VI_2 + losses

- **Variable losses :** The armature current varies with load . The copper losses that occur in the armature also vary w.r.t to load

  Ar Cu loss = Ia^2 Ra

Magnetic losses

- **Hysteresis losses :** The property of a magnetic material to retain a part of the magnetizing achieved by it, one removal of the driving force is called hysteresis.

- The power required to overcome this hysteresis effect is called hysteresis loss. It is denoted by Wn.

  $W_h \alpha f$
- $\alpha B_m^{1.6}$
- $W_h = 7\ B_m^{1.6}\ f$

- **Eddy current losses :** When the iron part (rotor) rotates in the stationary field, there is an induced emf in the iron part also apart from the emf induced in the copper windings. This emf is called eddy emf. The iron part provides a closed path for the eddy emf to circulate a current. This current is called eddy current.

Mechanical losses

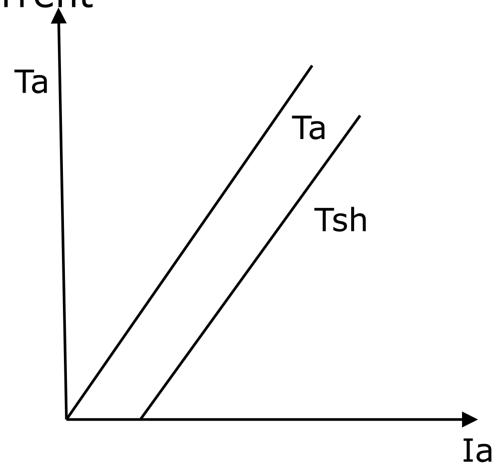
- **Friction losses :** The rotation of the armature inside the bearings causes friction to be developed. In order to over come this, the prime more has to supply more power.

- **Windage losses :** An extra mounting on the shaft of the armature such as fan to provide air for cooling purposes causes an extra burden on the prime motor. The extra power required to drive the fan is termed as windage loss. (constant)

Characteristics of motors

T_A/I_A CHARACTERISTICS.

- For series motor $\phi \; \alpha \; I_a$
- $T_a \; \alpha \; \phi \; I_a$
- $\alpha \; Ia^2$
- Thus, torque in case of series motor is proportional to the square of the armature current

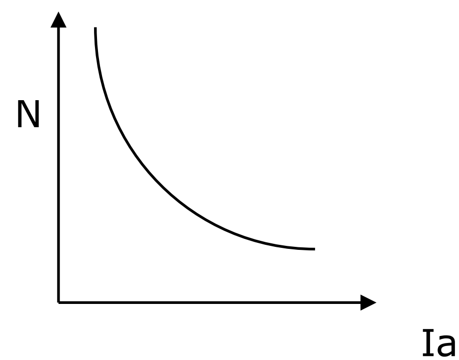


- As the load increases, armature current increases and torque produced increases proportional to the square of the armature current upto a certain limit.
- Hence Ta/Ia curve is a parabola.
- After saturation, ϕ is almost independent Ia, hence Ta α I_a only.
- So, the characteristic be comes a straight line.

Contd..

N/I_A CHARACTERISTIC

- N α (Eb/ϕ) α [v – I_a (R_a + R_{se})/I_a]
- The values R_a and Rse are so shall as ϕ α I_a in motor that the effect of change in I_a on speed avoids the effect of change in V – Ia Ra – Ia Rse on speed change in E_b for various load currents is small and hence may be neglected.



- With increased I_a, ϕ also increases Hence, speed varies inversely as armature current. When load is heavy, Ia is large. Hence speed is low (this increases Eb and allows more armature current to flow). But when load current and hence Ia falls to a small value speed becomes dangerously high.

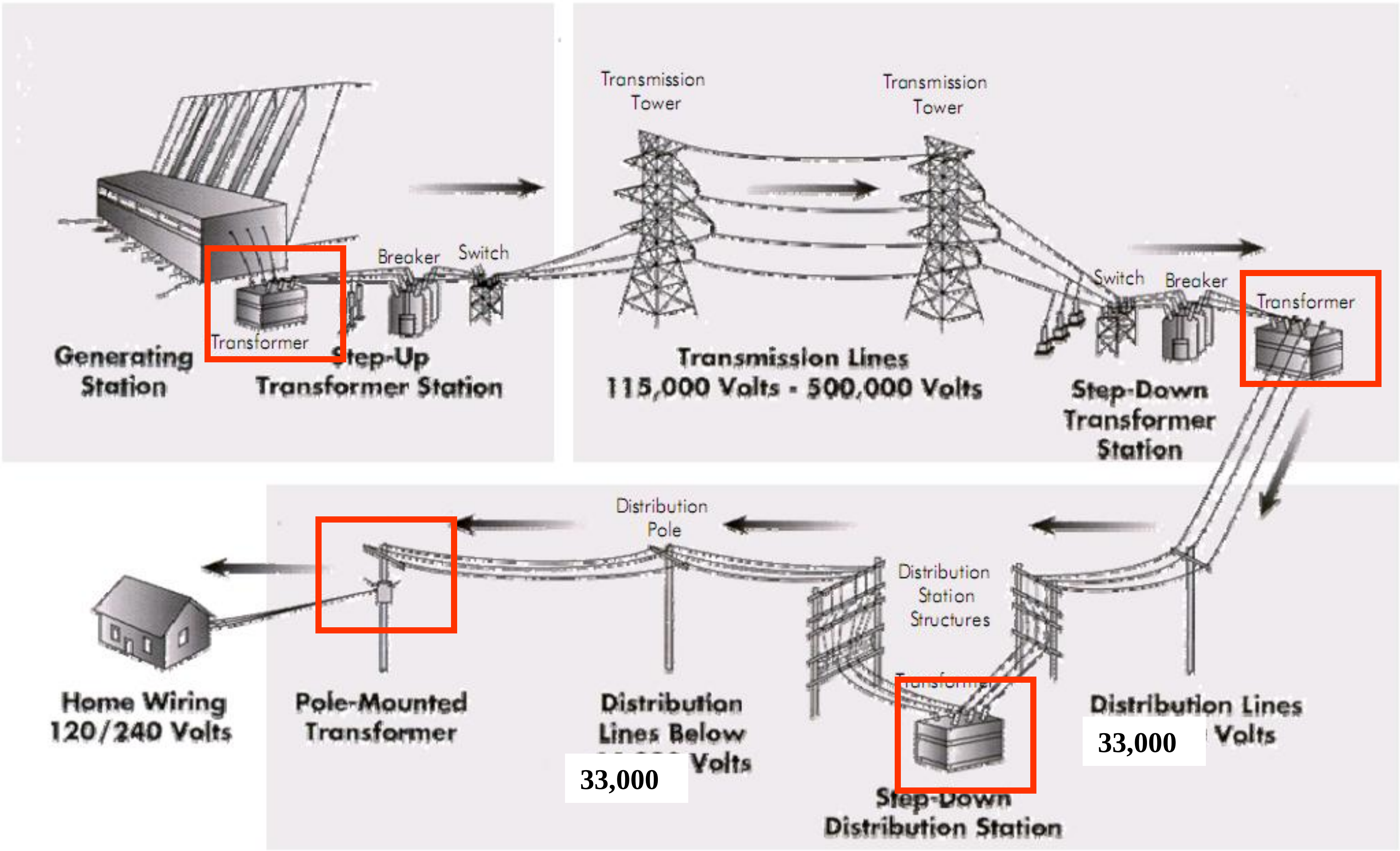
Objectives

On completion of this period, you should be able to know

- Importance of a transformer
- Features of a transformer
- Working principle of a transformer

Overview of the electric power system

Can you point where transformer is used ?



Transformer Is 120 Year Old

FIRST TRANSFORMER OF THE WORLD

60 W, 4.3 / 46.5 V, 18 Hz,

Single phase, Shell type, Dry type

Patented in 1885 by 3 Hungarian Engineers working in GANZ,

Budapest.

*** Karoly Zipernowsky**

*** Titusz Otto' Blathy**

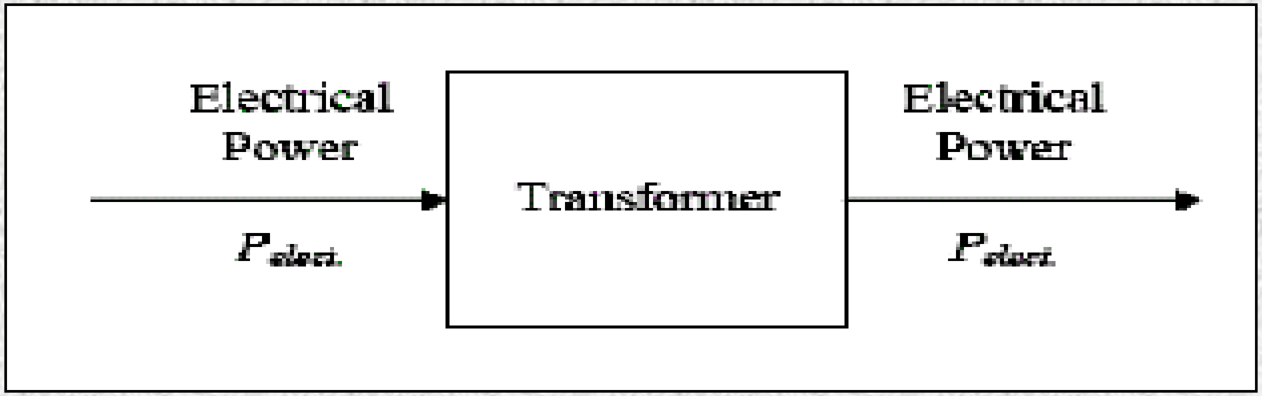
*** Miksa Deri**

Blathy coined the name "Transformer"

How It All Began

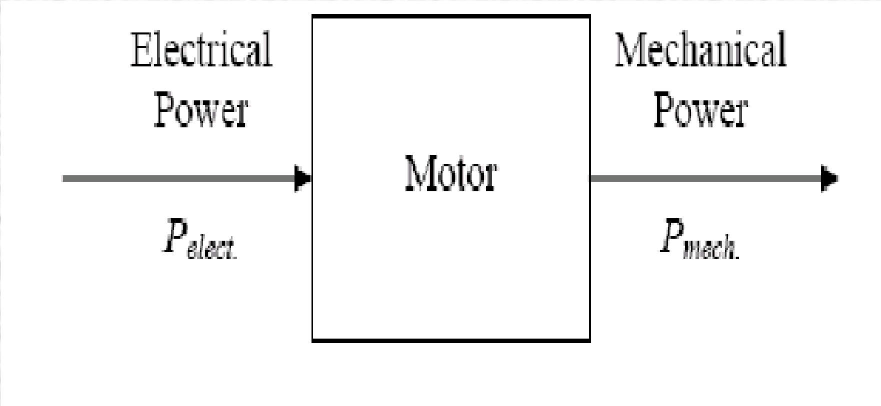
- 1831 – Faraday's law of Electromagnetic induction discovered
- 1864 – Maxwell's equations for mathematical models of electromagnetic apparatus formulated
- 1885 – First real Transformer (single phase) Patented.
- 1893 – First three phase Transformer was used in Hellsjon 9.6 kv transmission system in Sweden (dry type 3 phase transformers manufactured by ASEA)
- 1900 – Oil Immersed Transformer was born.

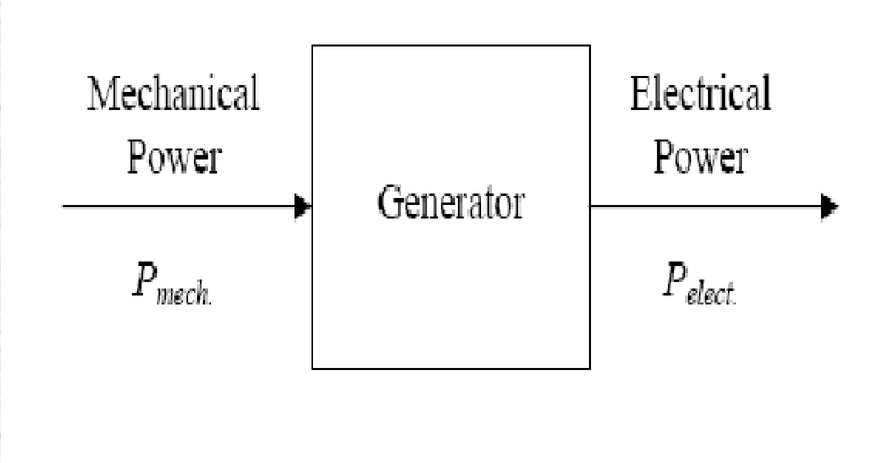
Transformer



- A transformer is a stationary electric machine which transfers electrical energy (power) from one circuit to another circuit with the same frequency but with different voltages and currents.
- A motor converts electrical power to mechanical power
- A generator converts mechanical power to electrical power

Transformer





- Unlike in rotating machines, there is no electrical to mechanical energy conversion
- A transformer is a static device and all currents and voltages are AC
- The transfer of energy takes place through the magnetic field

Introduction to Transformers

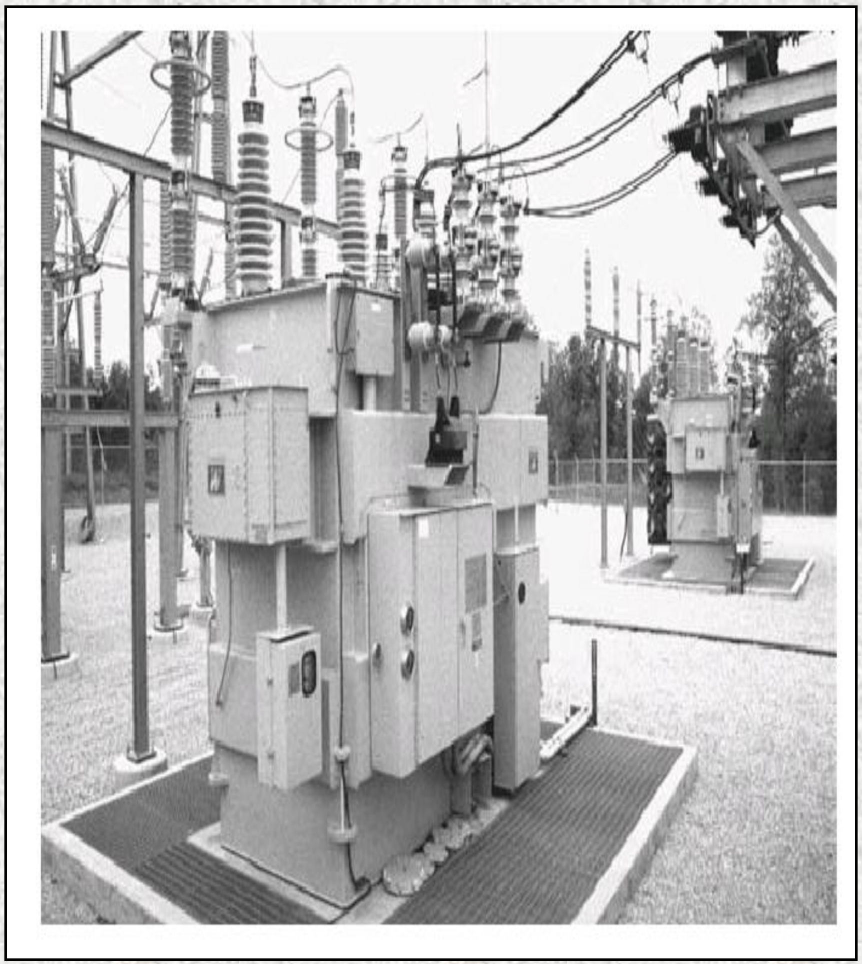
Definition

- A transformer is a static (device) AC Machine which transfers electrical energy from one electrical circuit to another electrical circuit without change of frequency through a common magnetic (field) flux

Introduction to Transformers contd…

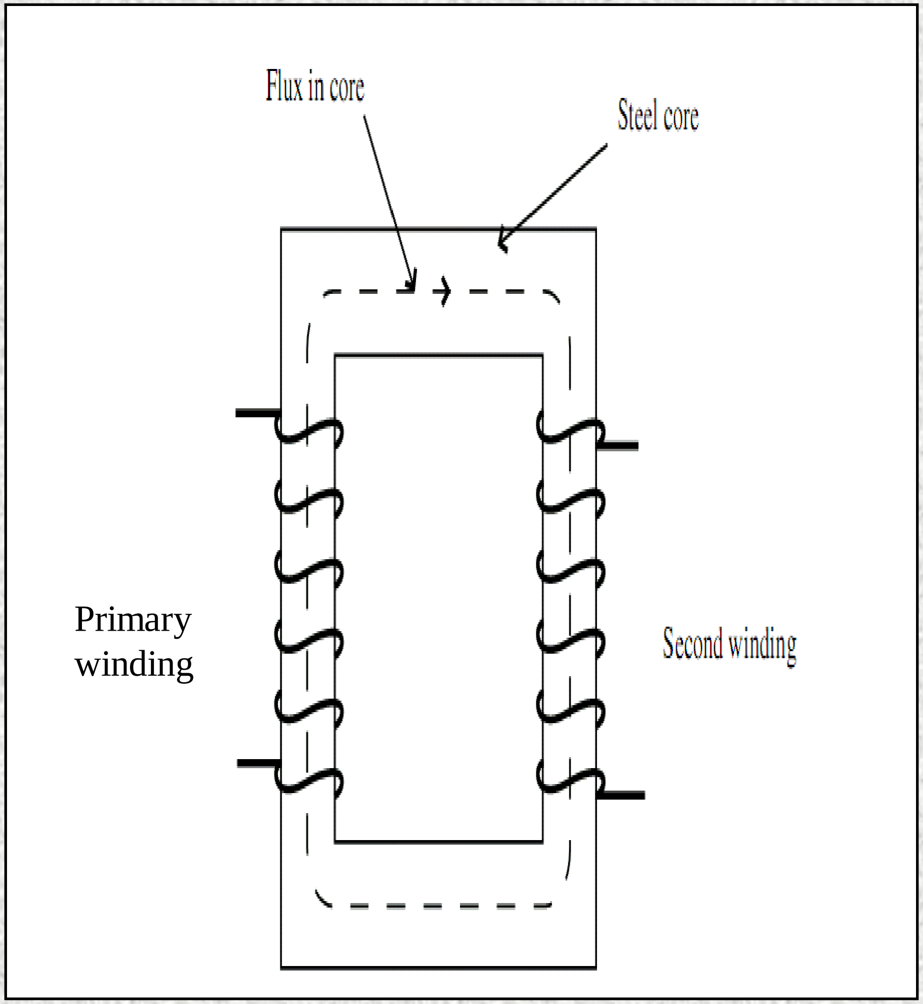
- Electrical energy is generated at places where it is easier to get water head and coal for hydro and thermal power stations respectively.
- Electrical energy generated is to be transmitted to considerable distances for use in towns, cities and villages
- Transmission of electrical energy at high voltages is economical

Introduction to Transformers contd…



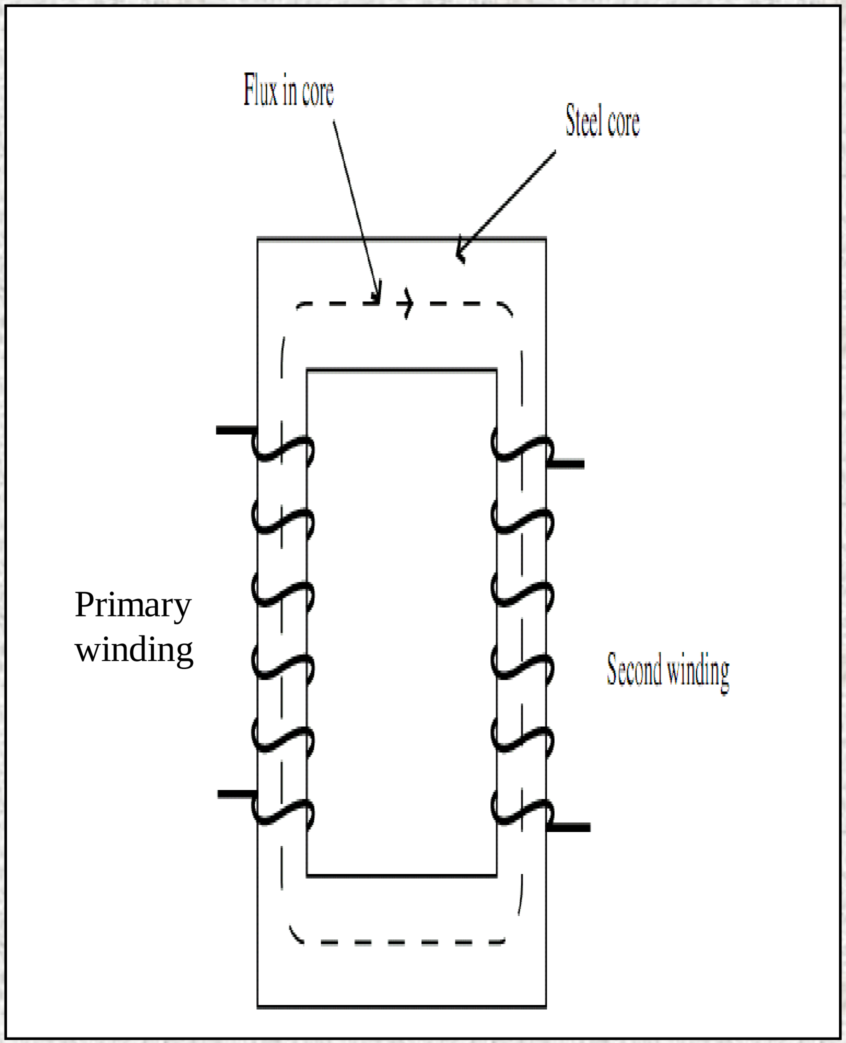
- Electrical Machines are required for stepping up the voltage at generating stations
- Electrical Machines are required for stepping down the voltage at places where it is to be used
- Electrical machine used for this purpose is known as TRANSFORMER

Introduction to Transformers contd…



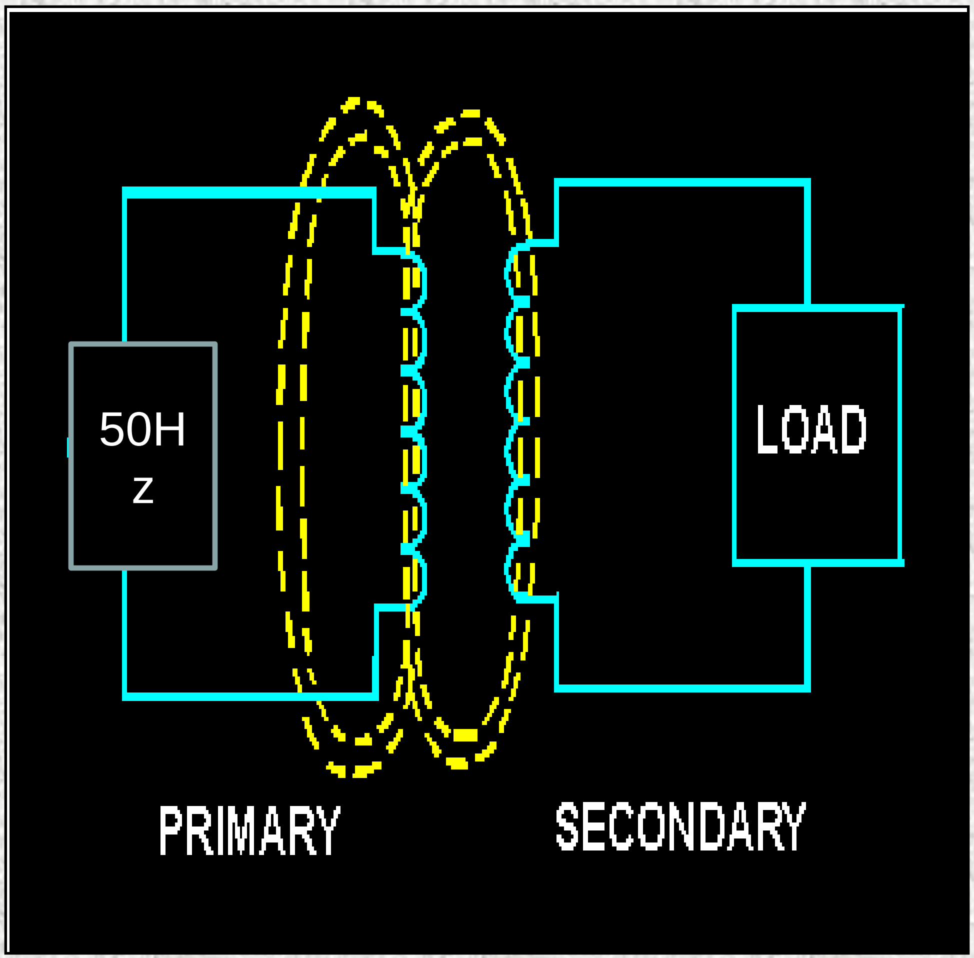
- No electrical connection is present between the two winding
- Magnetic linkage exists between the two windings
- The magnetic linkage is provided through a path of low reluctance

Introduction to Transformer contd…



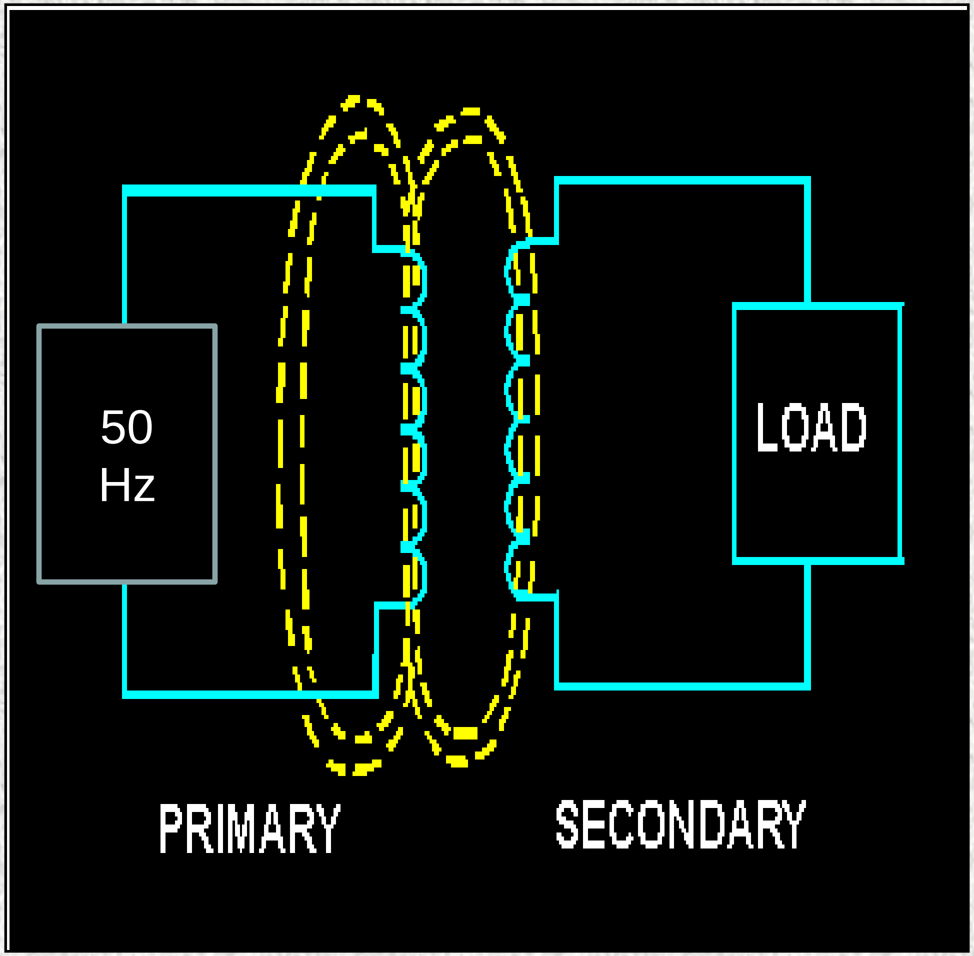
- A transformer basically consists of two windings which are wound on a soft iron or silicon steel core
- The winding which is connected to the supply mains is known as primary winding
- The winding which is connected to the load is known as secondary winding

Working Principle of Transformers contd…



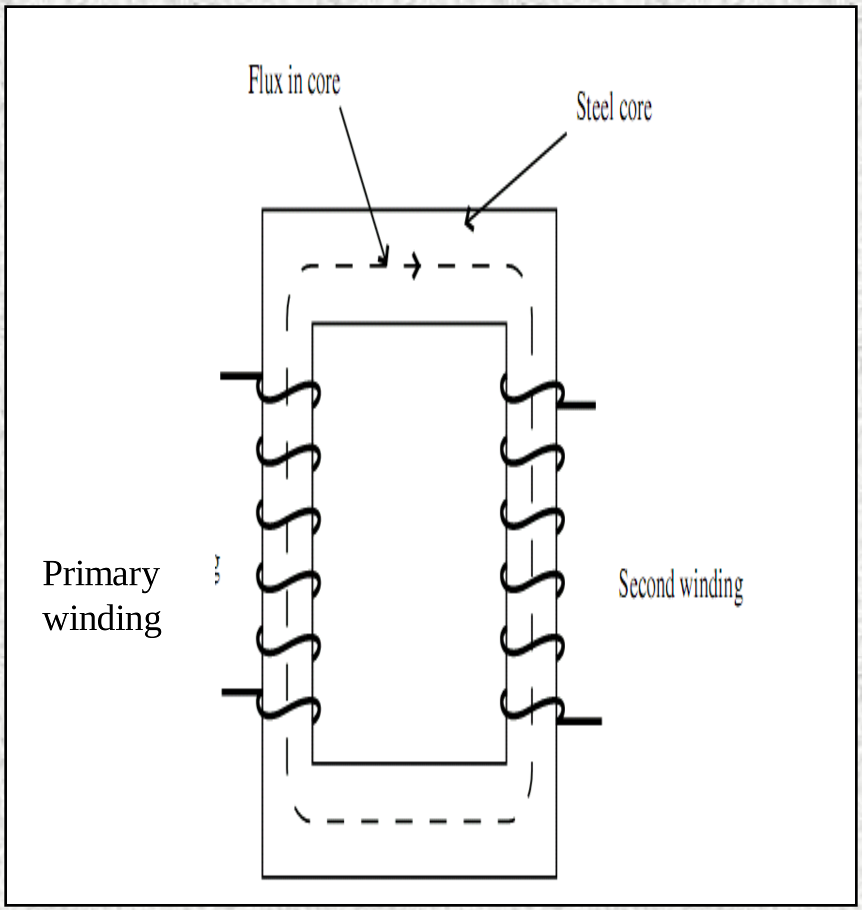
- The transformer works on the principle of Mutual induction between two magnetically coupled coils
- Is based on Faraday's laws of Electromagnetic Induction

Working Principle of Transformer contd…



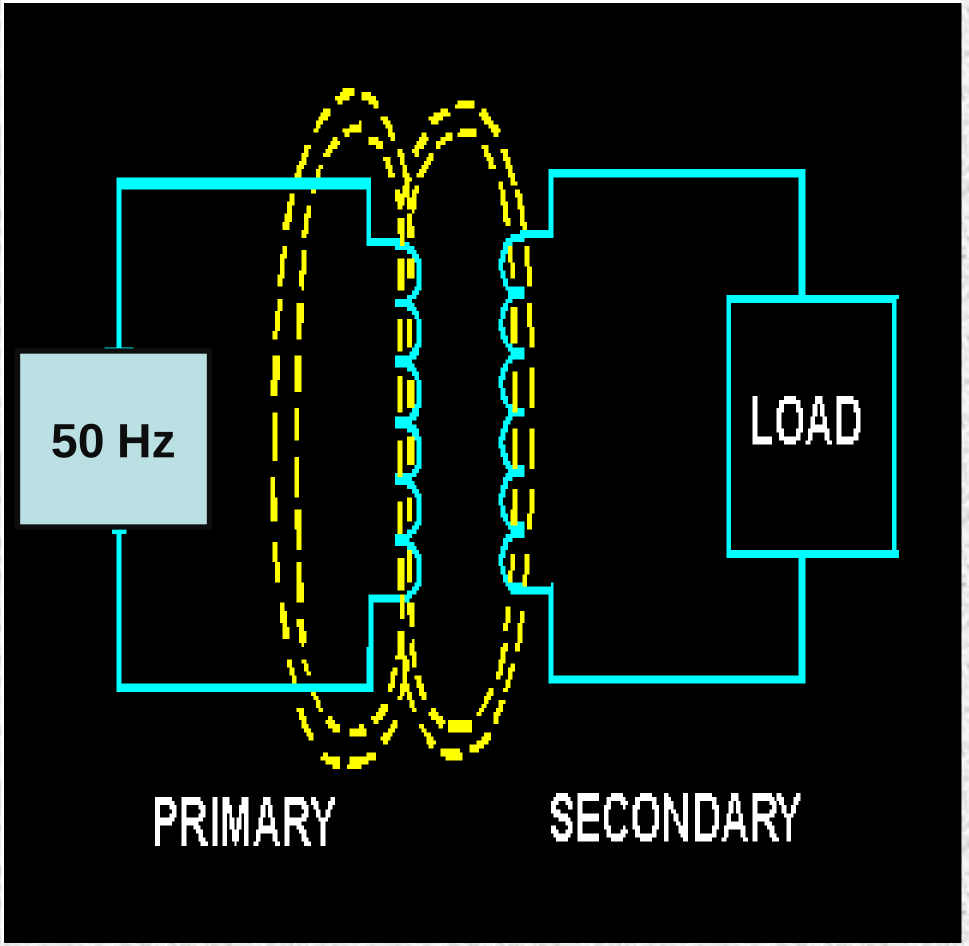
- When primary winding is connected to AC supply mains a current flows through it
- This current produces an alternating flux in the core
- This flux links with primary winding and produces self induced e.m.f in the primary winding which opposes the applied voltage

Working Principle of Transformer contd…



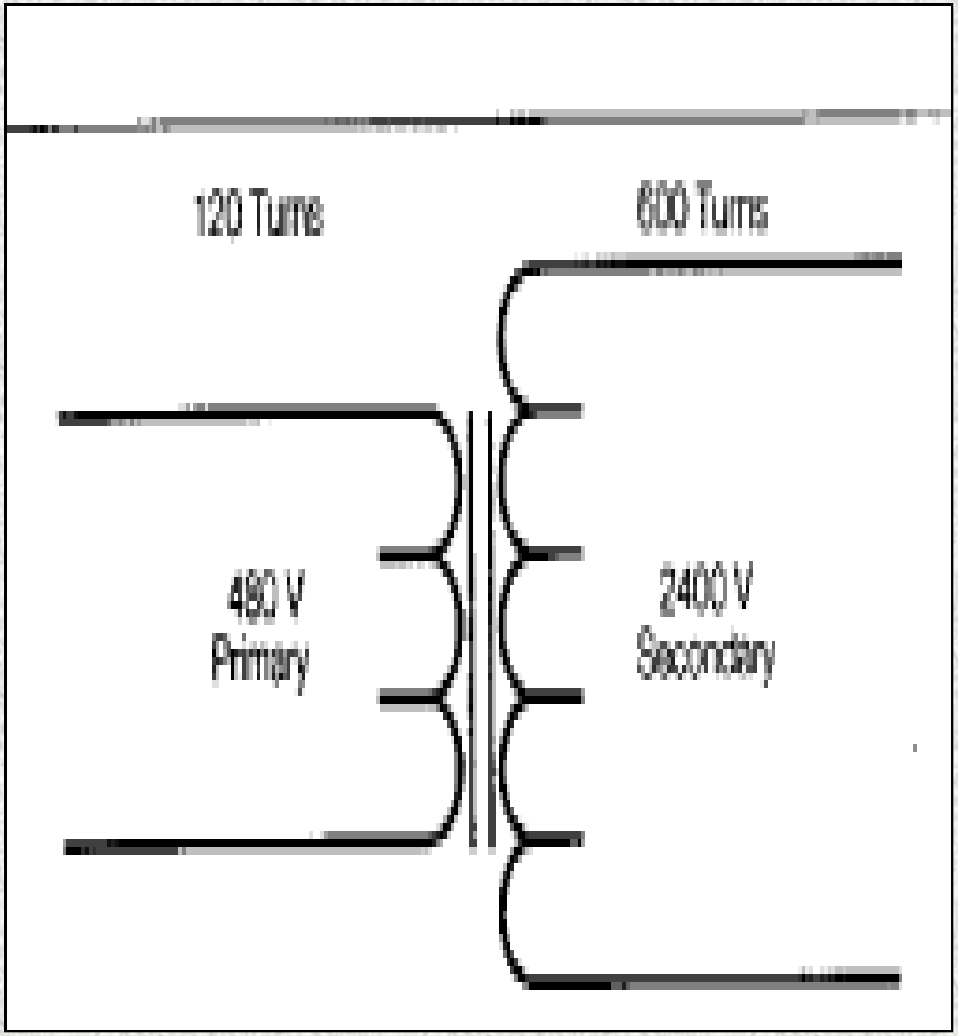
- This flux passes through the core and links with the secondary windings to induce an E.M.F called mutually induced E.M.F in the secondary winding
- The frequency of the emf induced in the secondary is same as that of the flux or that of the supply voltage

Working Principle of Transformer contd…



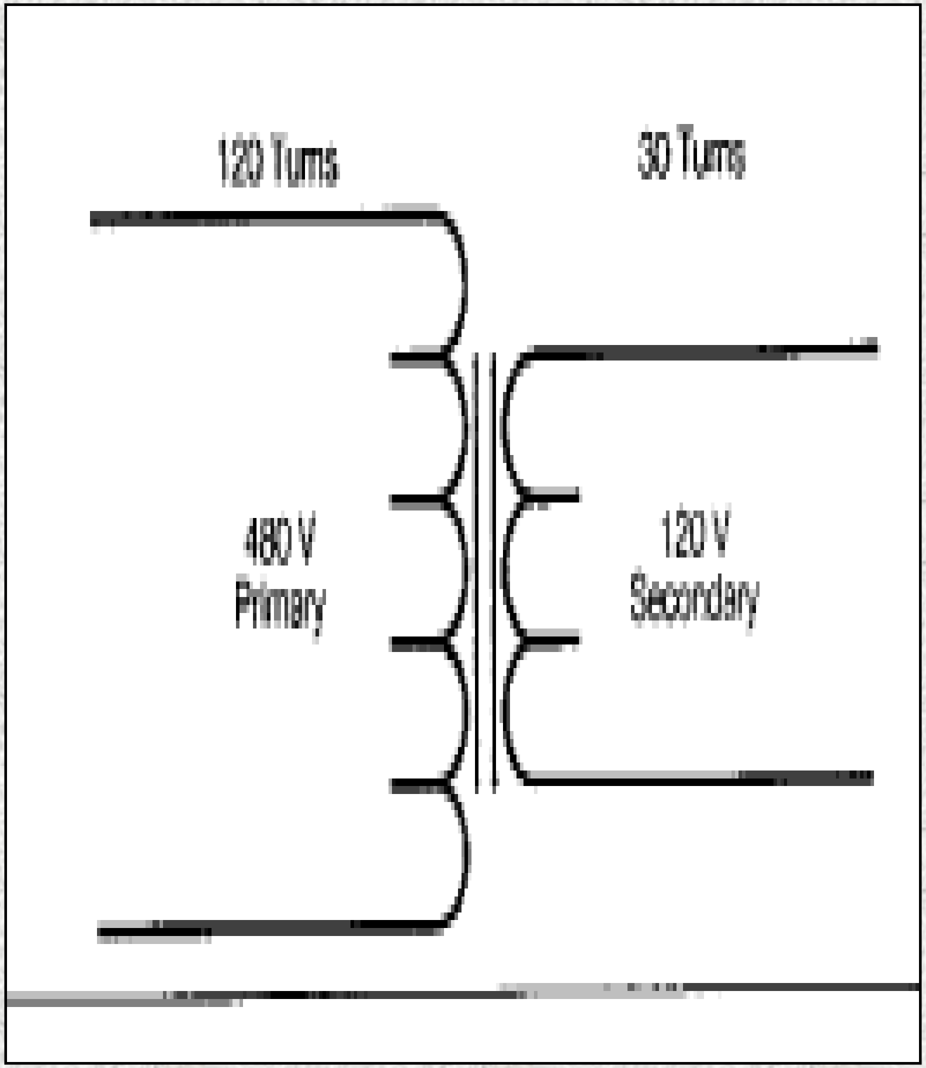
- The induced emf in the secondary winding will be able to circulate the current in the external load connected to it
- Energy is transferred from primary winding to the secondary winding by electro-magnetic induction principle without change of frequency

Step Up Transformer



- When the transformer rises the voltage it is called step-up transformer
- In step-up transformer the output voltage is higher than input voltage
- In step-up transformer secondary winding turns are more than the primary winding turns.

Step Down Transformer



- When the transformer reduces the voltage it is called step-down transformer

- In step-down transformer the output voltage is less than input voltage

- In step-down transformer secondary winding turns are less than the primary winding turns.

Summary

In this session we have learnt about

- Importance of a transformer
- Features of a Transformer
- Working principle of a transformer
- Step-up transformer
- Step-down transformer

Quiz

1) Transformer is which type of Electrical Machine

a) Static a.c. machine

b) Static d.c. machine

c) Rotating a.c. machine

d) Rotating d.c. machine

Quiz

2)Transformer works on the principle of

a) Mutual induction between two coils

b) Mutual conduction between two coils

c) Self induction between two coils

d) None of the above

Quiz

3) Transformer converts electrical energy from one electrical circuit to another electrical circuit without change in

a) Frequency

b) Voltage

c) Current

d) None of the above

Quiz

5) The winding connected to the supply mains is called

a) Secondary winding

b) Primary winding

c) Auxiliary winding

d) None of the above

Quiz

6) The winding connected to the load is known as

a) Secondary winding

b) Primary winding

c) Territory winding

d) None of the above

Quiz

7) In step-up transformer the secondary winding voltage is

a) More than the primary voltage

b) Less than the primary voltage

c) Equal than the primary voltage

d) Zero

Frequently Asked Questions

1) Define Transformer

2) Explain the working principle of transformer