Institute of Aeronautical Engineering
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Department of Civil Engineering

REINFORCED CONCRETE STRUCTURES DESIGN AND RAWING

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COURSE GOAL

- To introduce students to various concepts of Limit state design.
- To impart knowledge regarding the design of structures of buildings like slabs , beams and columns etc

Course outline

UNIT	TITLE	CONTENTS
1	INTRODUCTION	Concepts of RC Design – Limit state method – Material Stress–Strain curves – Safety factors – Characteristic values – Stress block parameters – IS-456:2000 – Working stress method. **BEAMS:** Limit state analysis and design of singly reinforced, doubly reinforced, T, and L beam sections

2	**SHEAR TORSION AND BOND**	**Limit state analysis and design of section for shear and torsion – concept of bond, anchorage and development length, I.S. code provisions. Design examples in simply supported and continuous beams, detailing Limit state design for serviceability for deflection, cracking and codal provision.**
3	DESIGN OF SLABS	Design of Two-way Slabs, one-way slabs, Continuous slabs using I.S. coefficients, Cantilever slab/ Canopy slab.

4	**SHORT AND LONG COLUMNS**	**Axial loads, uni-axial and bi-axial bending I.S. Code provisions.**
5	**DESIGN OF FOOTINGS**	Isolated(square, rectangle) and Combined Footings. Design of Stair Case

COURSE OBJECTIVES

- To develop an understanding of and appreciation for basic concepts in the behaviour and design of reinforced concrete systems and elements.
- To give an ability to differentiate between working stress design and limit state design.
- To introduce the basic concepts and steps for reinforced concrete sectional design mainly in accordance with ultimate strength design.
- To help the student develop an intuitive feeling about structural and material wise behaviour and design of reinforced concrete systems and elements.

TEACHING STRATEGIES

- The course will be taught via Lectures. Lectures will also involve the solution of tutorial questions. Tutorial questions are designed to complement and enhance both the lectures and the students appreciation of the subject.

- Course work assignments will be reviewed with the students.

- Daily assessment through questioning and class notes.

UNIT-I

INTRODUCTION-LIMIT STATE DESIGN

What is Limit State?

- ***"A limit state is a state of impending failure, beyond which a structure ceases to perform its intended function satisfactorily, in terms of either safety or serviceability".***

DESIGN CONCEPTS

►<u>***Safety***</u>***: implies that the likelihood of (partial or total) collapse of structure is acceptably low not only under (normal loads) service loads but also under overloads.***

►<u>***Serviceability***</u>***: satisfactory performance of structure under service loads without discomfort to user due to excessive deflections, cracking, vibration etc.***

- ***Other considerations such as durability, impermeability, acoustic and thermal insulation etc.***

Limit State Design

Limit States

- ***Purpose**: to achieve acceptable probability that a structure will not become unfit for its intended use i.e. that it will not reach a limit state.*

- ***Thus, a structure ceases to be fit for use will constitute a limit state and the design aims to avoid any such condition being reached during the expected life of the structure.***

- ***Two principle types of limit state are;***

    i. ***Ultimate Limit State***

    ii. ***Serviceability Limit State***

Limit State Design

Ultimate Limit State

- ***This requires that the structure must be able to withstand, with an adequate factor of safety against collapse, the loads for which it is designed.***

- ***Limit state of Collapse: flexure, shear, compression, torsion, bearing, etc.***

- ***Possibility of buckling or overturning must also be taken into account, as must the possibility of accidental damage as caused, for example, by an internal explosion.***

Serviceability Limit States

Most important serviceability limit states are

- ***Deflection:** appearance or efficiency of any part of the structure must not be adversely affected by deflections.*
- ***Cracking:** local damage due to cracking and spalling must not affect the appearance, efficiency or durability of structure.*
- ***Durability:** this must be considered in terms of the proposed life of the structure and its conditions of exposure.*

Serviceability Limit States

- ***<u>Other limit states include</u>***
- ***<u>Excessive vibration</u>: which may cause discomfort or alarm as well as damage.***
- ***<u>Fatigue</u>: must be considered if cyclic loading is likely.***
- ***<u>Fire resistance</u>: this must be considered in terms of resistance to collapse, flame penetration and heat transfer.***
- ***<u>Special circumstances</u>: any special requirements of the structure which are not covered by any of the more common limit states, such as earthquake resistance, must be taken into account.***

Assumptions for Design in Flexure

1. *At any cross-section, sections which are plane prior to bending remain plane after bending. Or strain varies linearly with distance from neutral axis i.e. plane sections remain plane in bending.*

2. *The maximum strain in concrete at the outermost fiber is 0.0035.*

3. *Stress-strain relationship in concrete could be either rectangular, parabolic or combination of rectangular and parabolic curves which should be agreeable with the experimental results.*

Assumptions for Design in Flexure

4. *The stresses in steel bars used for reinforcement are derived from the representative stress-strain curve for the type of steel used.*

5. *Perfect bond between reinforced steel and adjoining concrete.*

6. *Tensile strength of concrete is neglected.*

7. *Minimum strain in steel reinforcement should not be less than* $((0.87f_y/E_s) + 0.002)$.

What is partial safety factor?

- *In Limit State Design, the load actually used for each limit state is called the "Design Load" for that limit state*
- *"Design Load" is the product of the characteristic load and the relevant partial safety factor for loads*
- *Design load = γ_f x (characteristic load)*

Why do we use partial safety factors?

- ***Partial safety factor is intended to cover those variations in loading in design or in construction which are likely to occur after the designer and the constructor have each exercised carefully their skill and knowledge.***

- ***Also takes into account nature of limit state in question.***

Partial Factors of Safety

- **Other possible variations such as constructional tolerances are allowed for by partial factors of safety applied to the strength of materials and to loadings.**

- **Lack of adequate data, however, makes this unrealistic and in practice the values adopted are based on experience and simplified calculations.**

Partial factors of safety for loads (γ_f)

Errors and inaccuracies may be due to a number of causes:

- ***Design assumptions and inaccuracy of calculation.***
- ***Possible unusual load increases.***
- ***Unforeseen stress redistributions.***
- ***Constructional inaccuracies***

These are taken into account by applying a particular factor of safety (γ_f) on the loadings, so that

Design load = characteristic load x partial factor of safety (γ_f)

- ***This factor should also take into account the importance of the limit state under consideration and reflect to some extent the accuracy with which different type of loading can be predicted, and the probability of particular load combinations occurring.***

Partial Factor of Safety for loads

Load Combination	Limit State of Collapse			Limit State of Serviceability		
	DL	LL	WL/EL	DL	LL	WL/EL
DL + LL	1.5	1.5	1.0	1.0	1.0	--
DL + WL or EL	1.5 or 0.9*	-	1.5	1.0	---	1.0
DL + LL + WL/EL	1.2	1.2	1.2	1.0	0.8	0.8

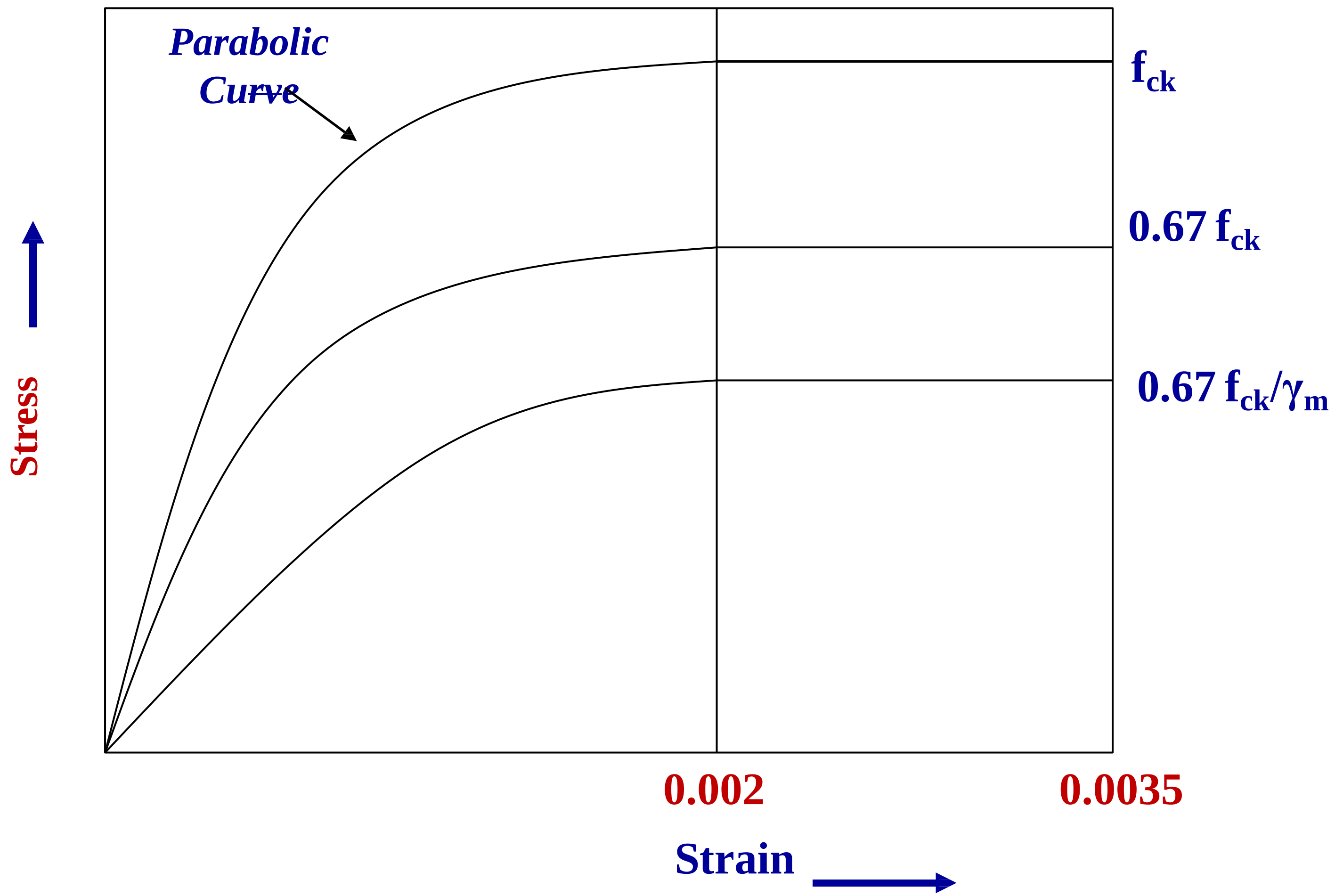
What is Design Strength?

- *In design calculations "Design Strength" for a given material and limit state is obtained by dividing the characteristic strength by the partial safety factor for strength, appropriate to that material and that limit state.*
- *When assessing the strength of a structure or structural member for the limit state of collapse, the partial safety factor should be taken as 1.5 for concrete and 1.15 for steel*

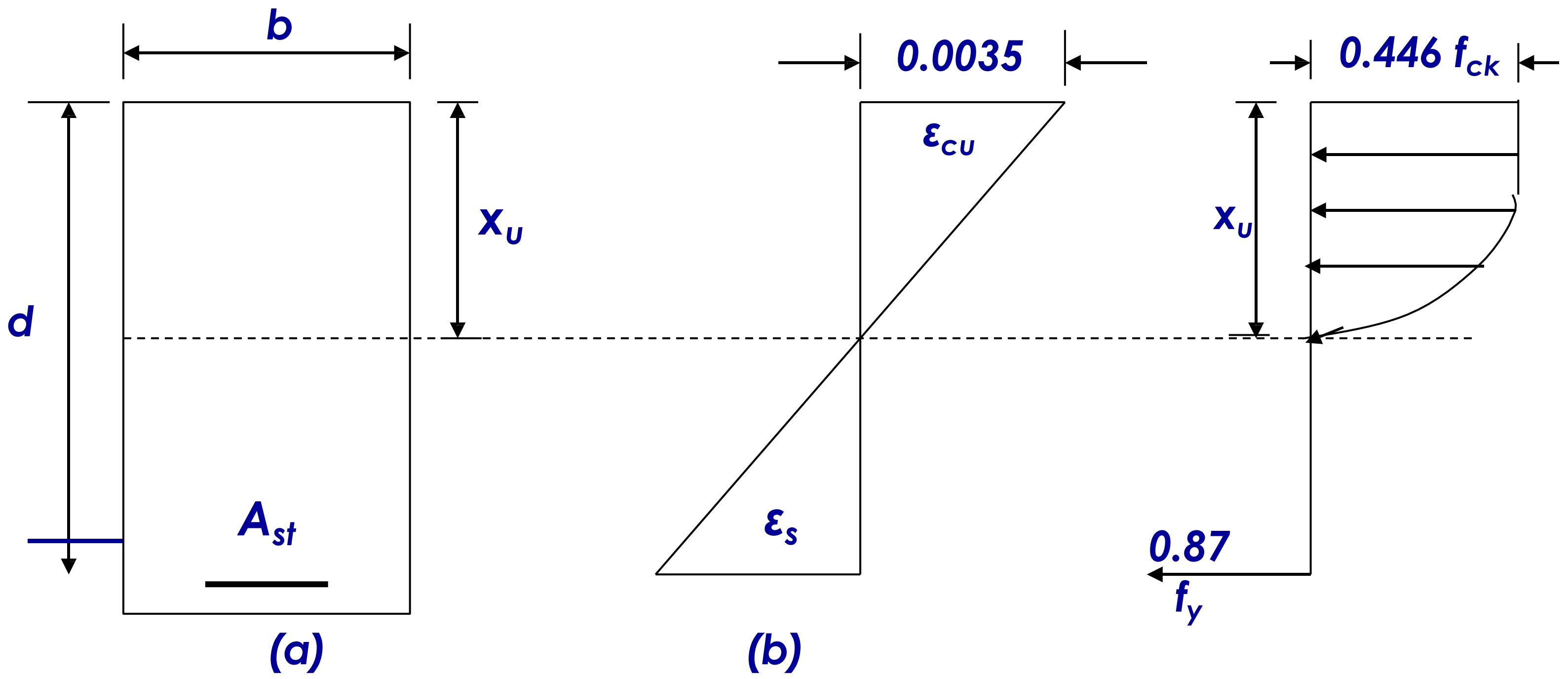
Partial Factors of Safety for Materials(γ_m)

- ***The strength of material in an actual member will differ from that measured in a carefully prepared test specimen and it is particularly true for concrete where placing, compaction and curing are so important to the strength. Steel, on the other hand, is a relatively consistent material requiring a small partial factor of safety.***

- ***The severity of the limit state being considered. Thus, higher values are taken for the ultimate limit state than for the serviceability limit state.***

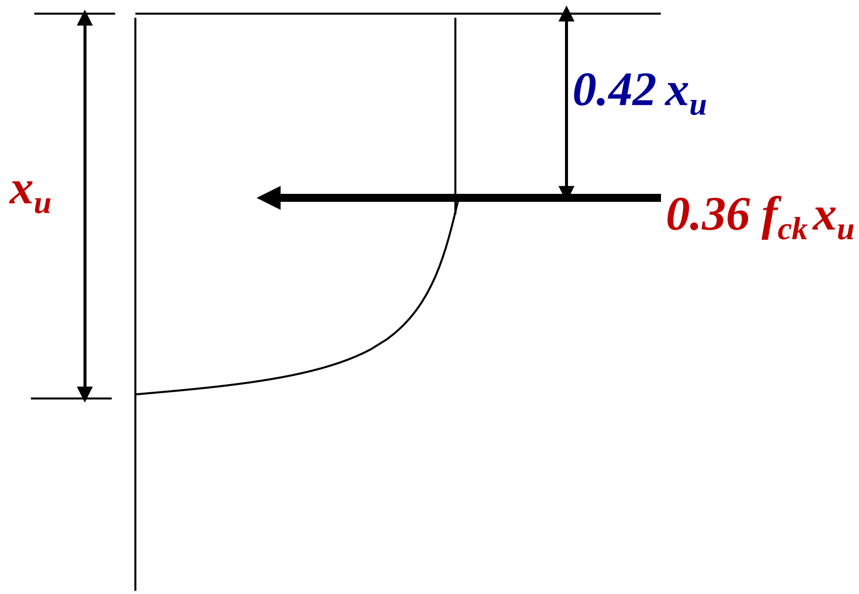


Stress-Strain Curve for Concrete in Flexural Compression



Strain diagram and Stress blocks:

(a) Section; (b) Strain diagram; (c) Stress block

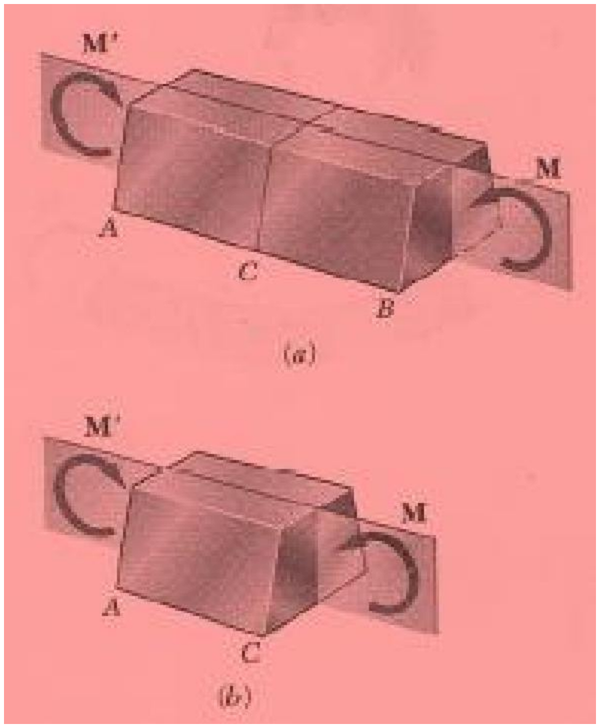


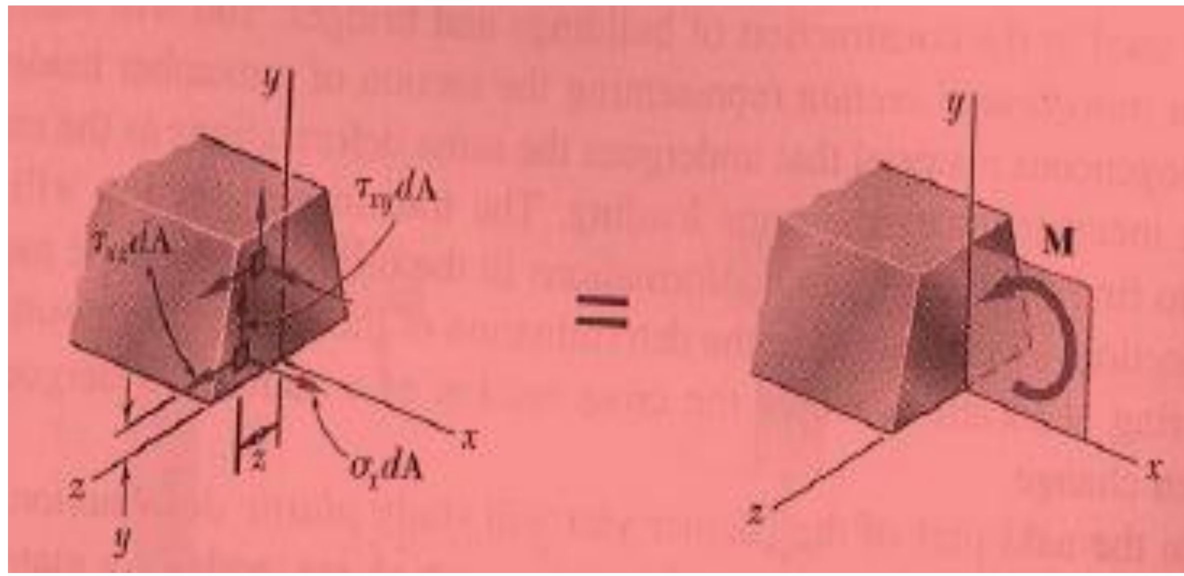
Stress Block Parameters

UNIT-II
DESIGN OF RC STRUCTURAL ELEMENTS FOR SHEAR

Bending Stresses in Beams

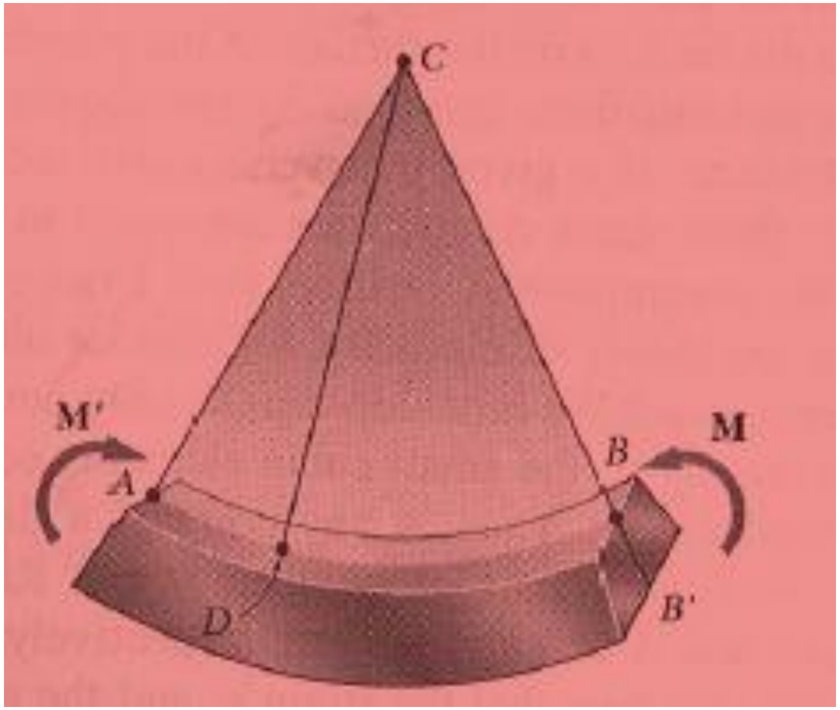
Beam subjected to pure bending moment

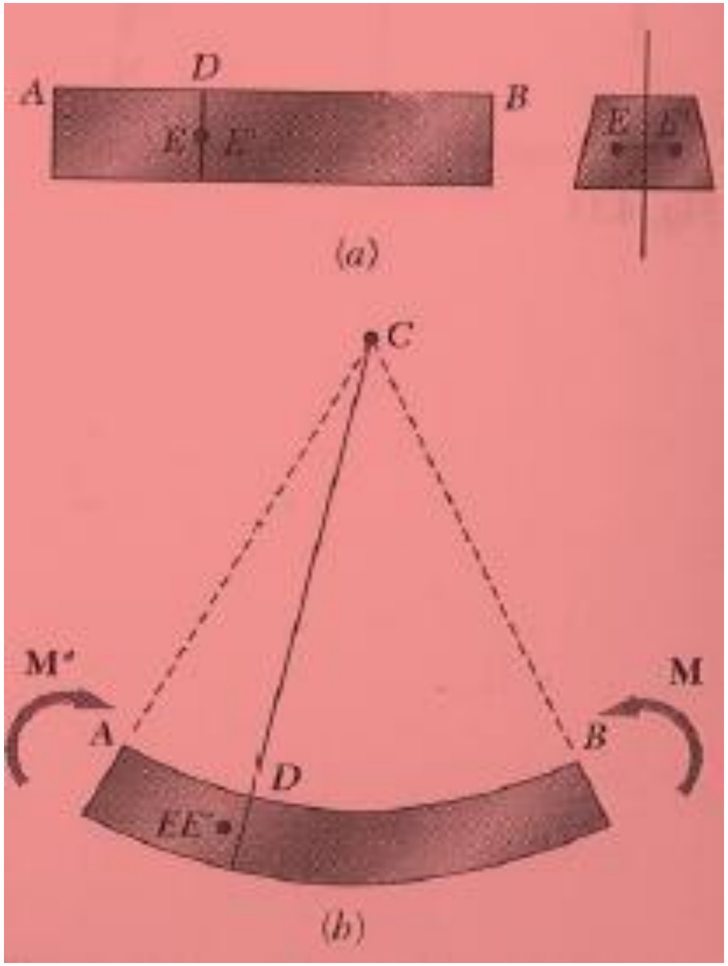




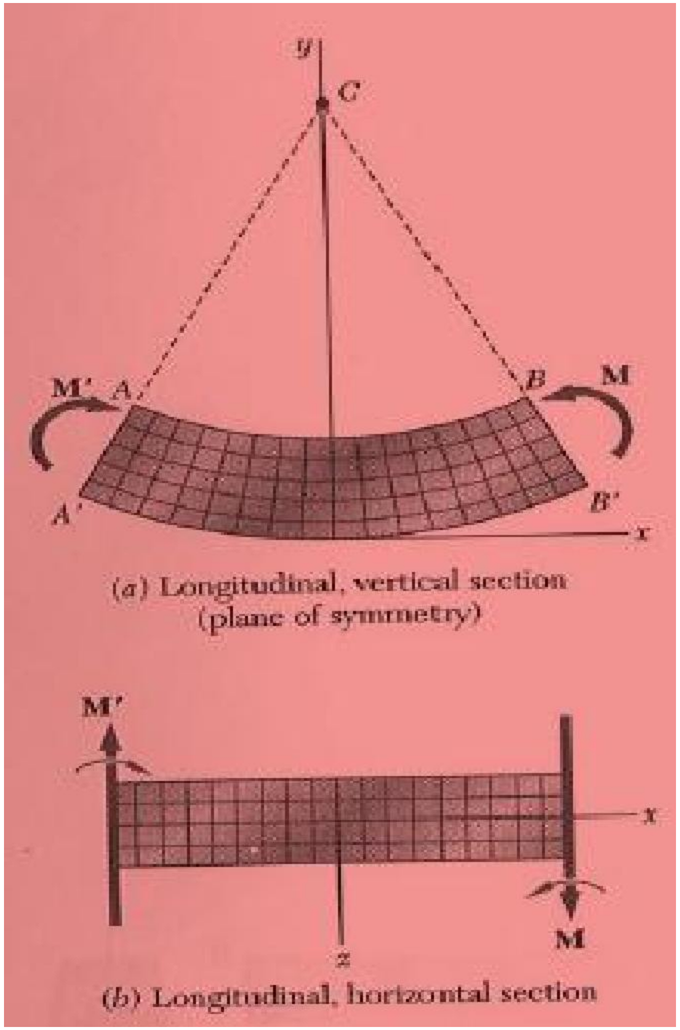
Stresses developed in beam under pure bending moment

Deflection of Beam under Pure BM

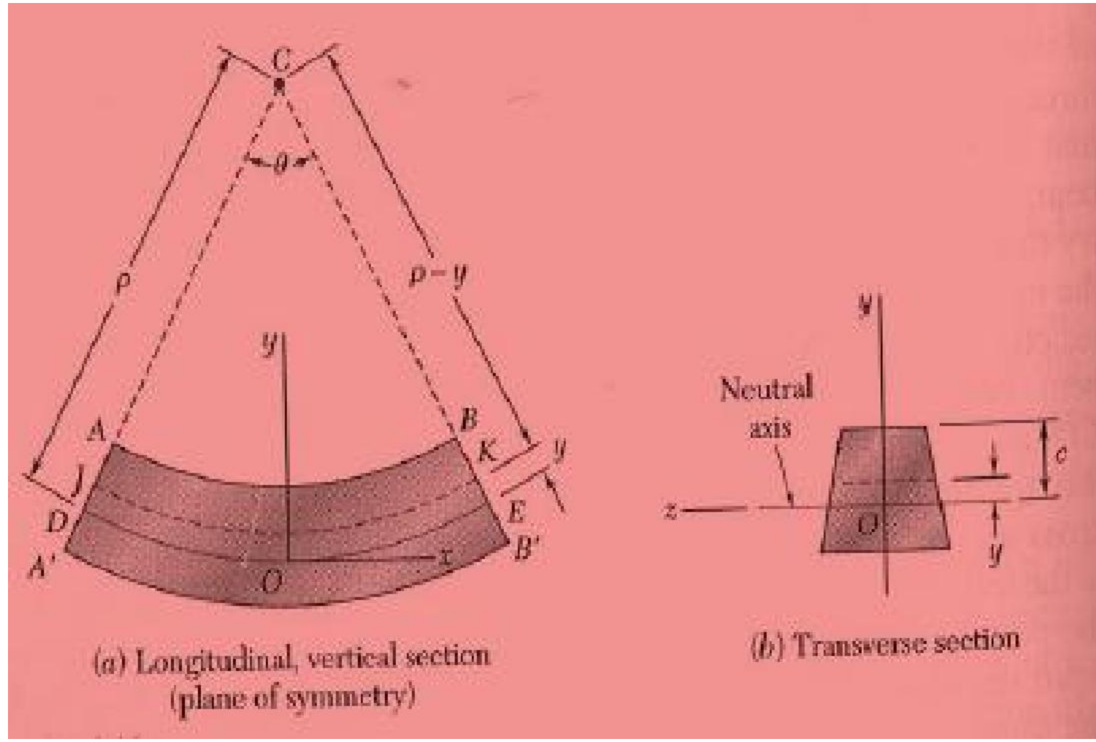




Deflected Shape of Beam

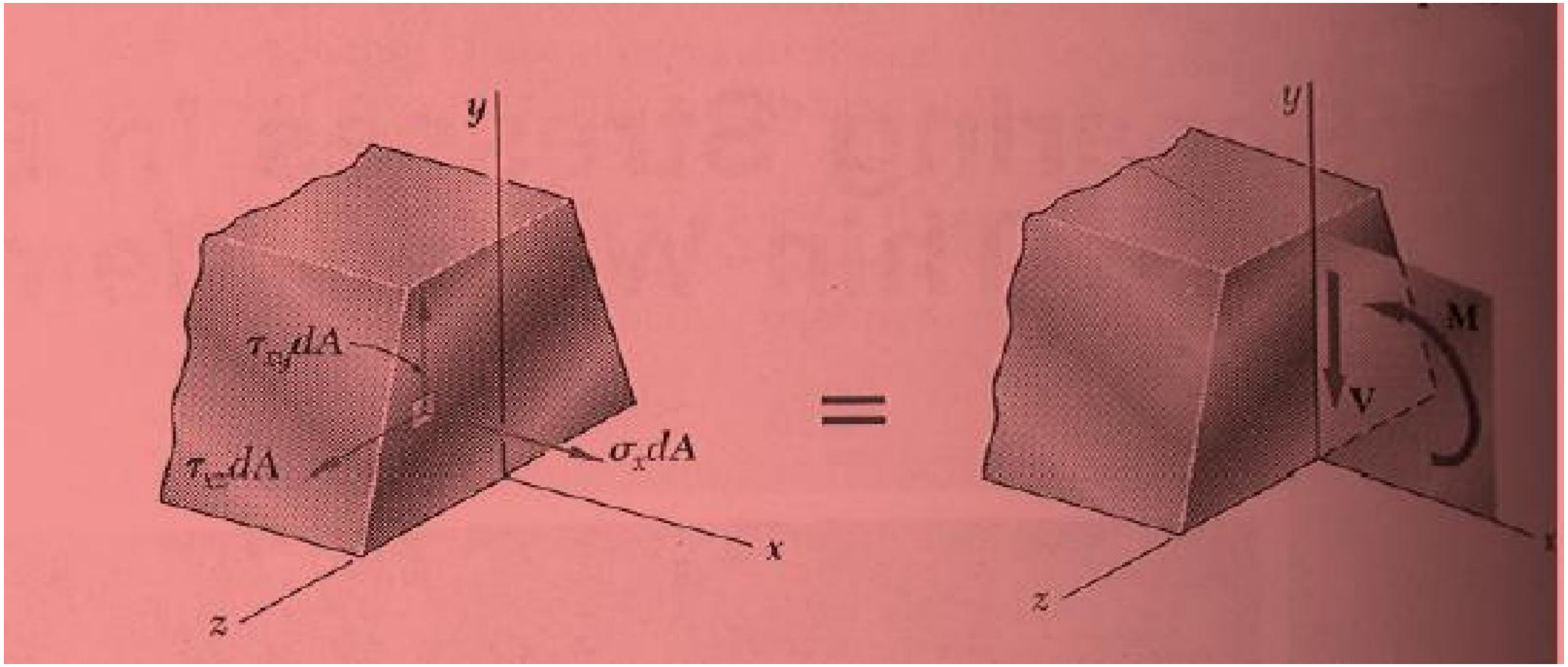


Sectional View of Beam



Neutral Surface/Axis

Shear Stress in Beams-General Loading

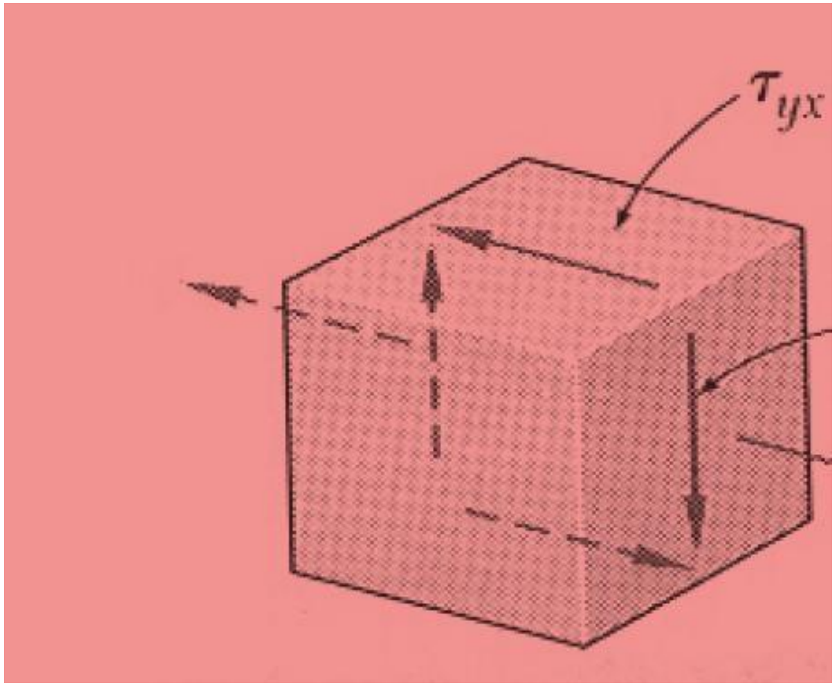


Bending and Shear Stresses

✓**Equilibrium of Forces**

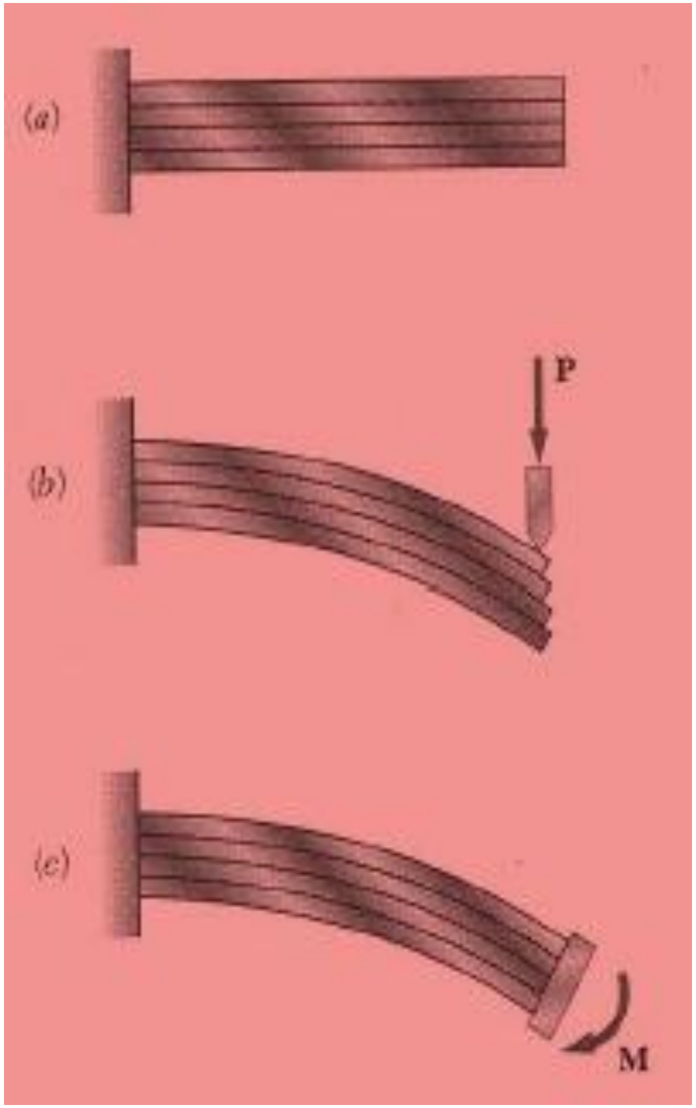
- **Vertical Shear Force**
- **Bending Moment**

Horizontal Shear Stress



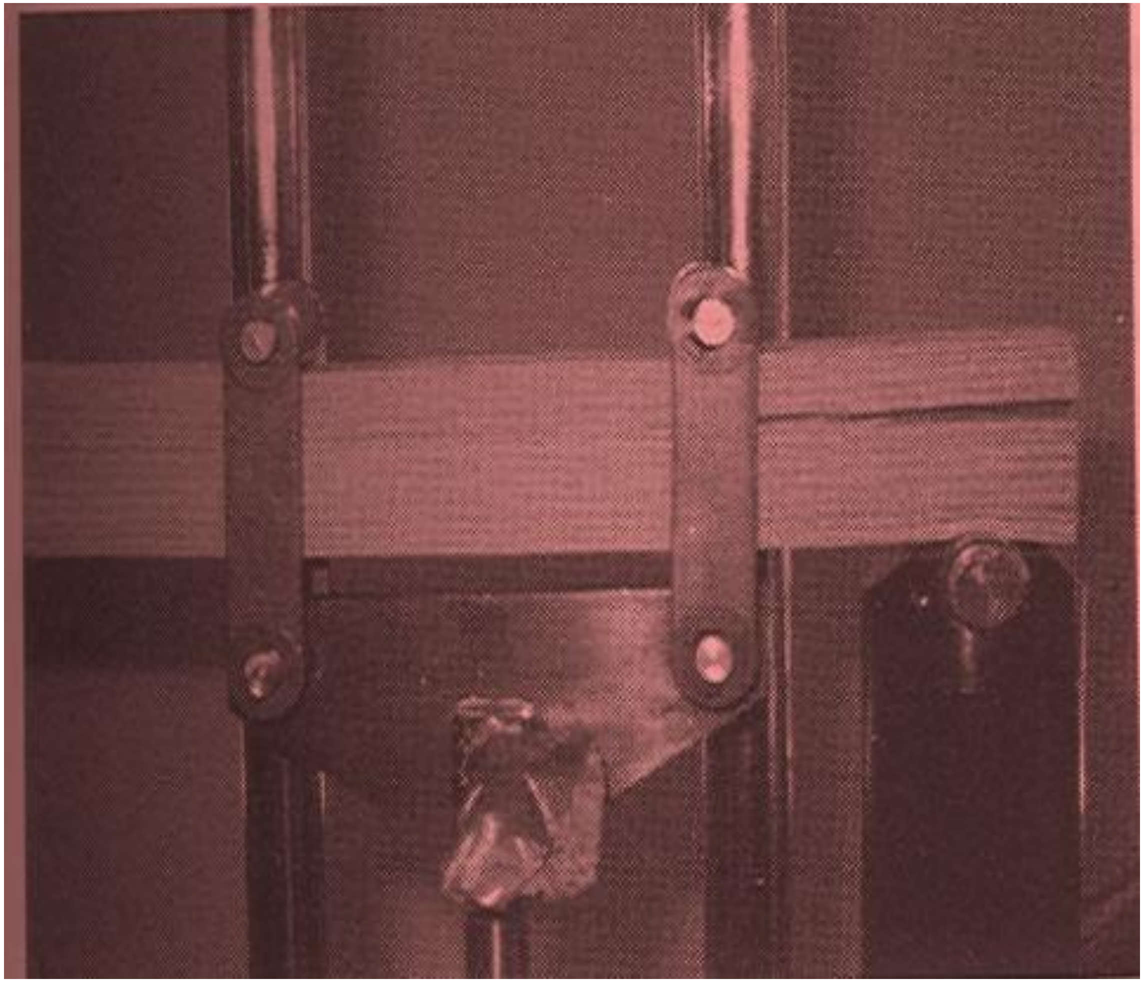
Shear stresses develop on horizontal planes

Deflection of Beam with Planks



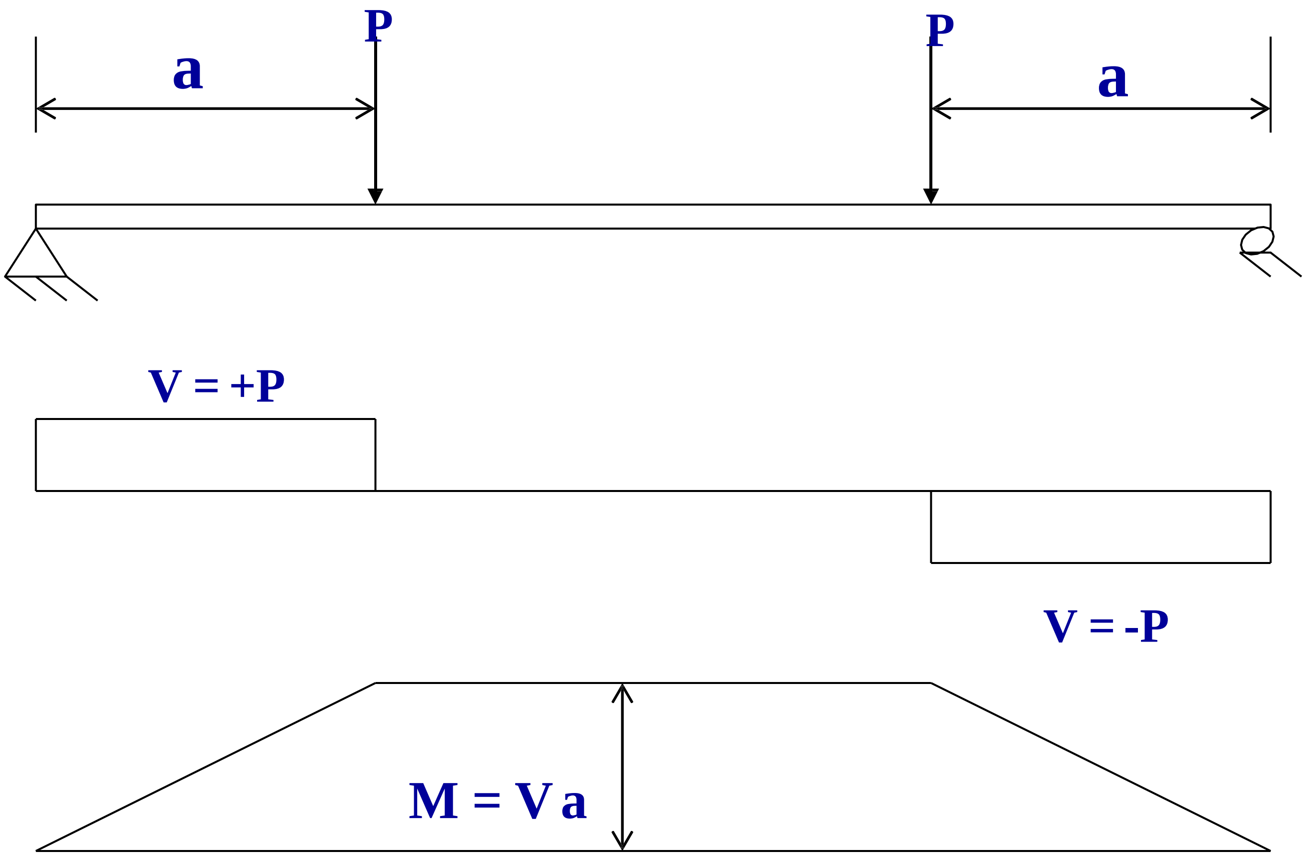
Beams with and without Planks

Horizontal Shear in Wooden Beams



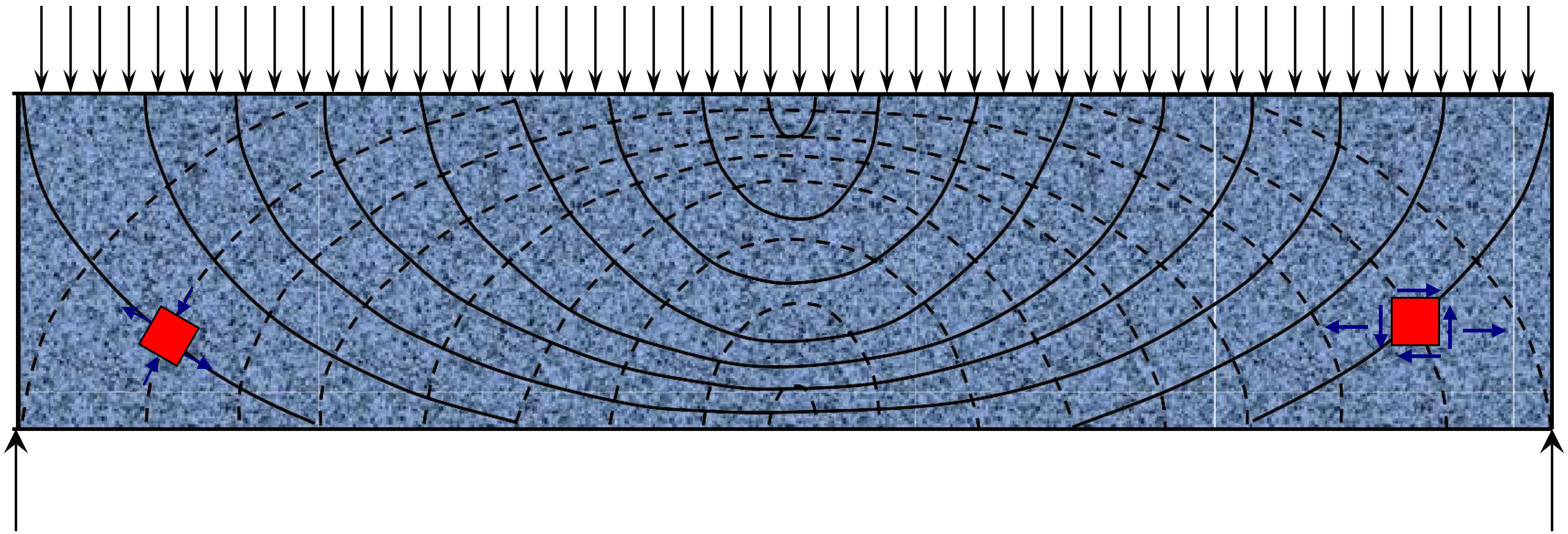
Failure of wooden beam due to delamination of fibers

Definition of Shear Span

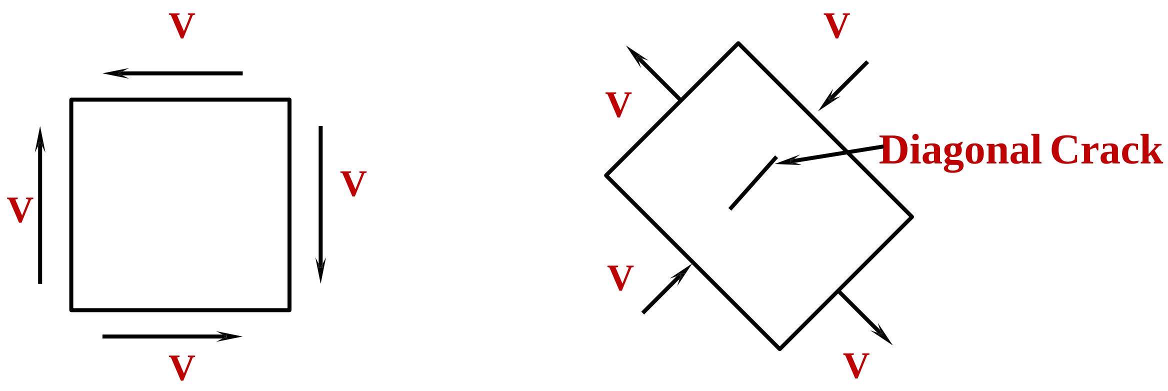


Basic Definition of Shear span, a

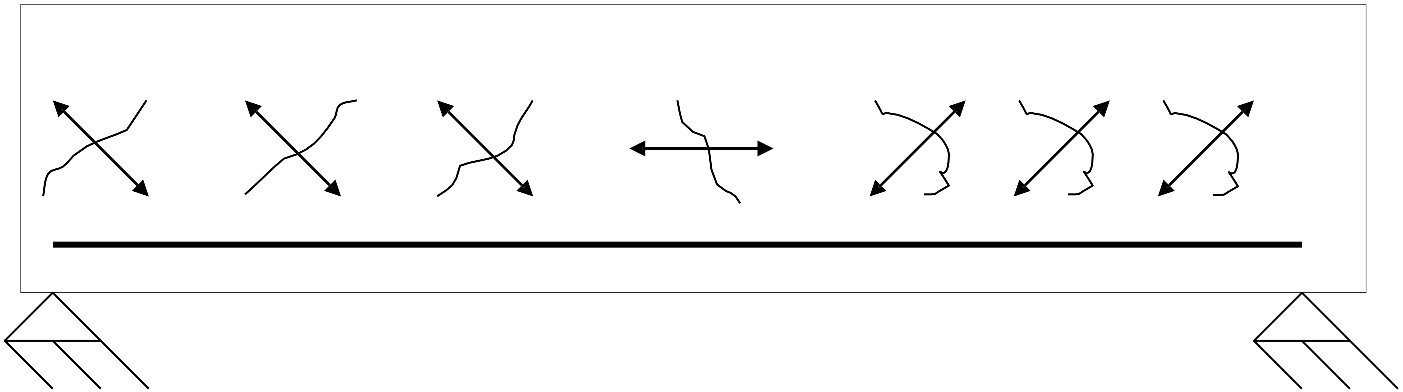
Principal Stress Trajectories



Diagonal Tension



Cracking in RC Beams



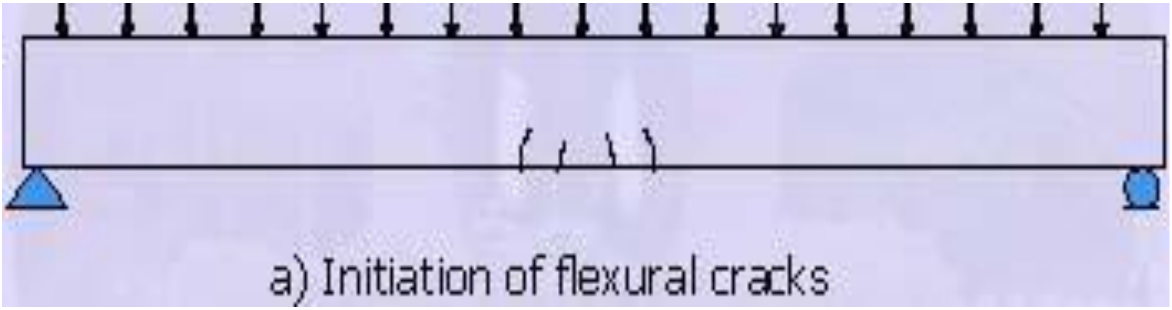
Direction of potential cracks in a simple beam.

Types of Cracks

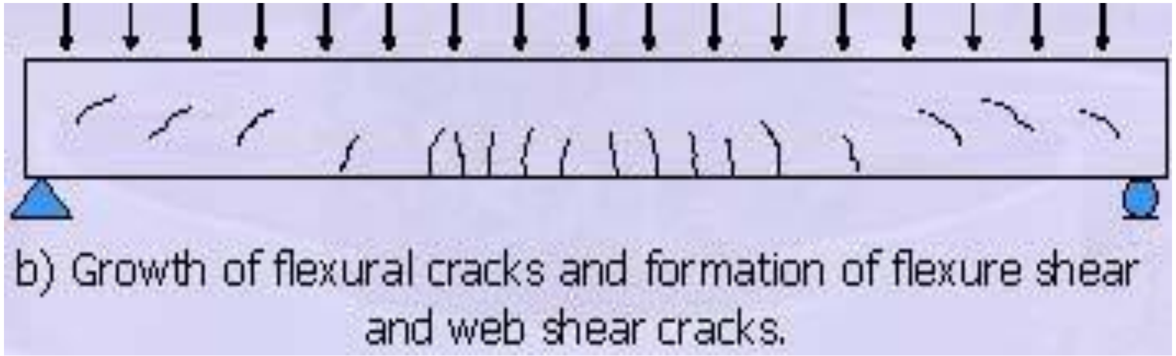
- ***Type and formation of cracks depend on span-to-depth ratio of beam and loading.***
- ***For simply supported beam under uniformly distributed load, three types of cracks are identified.***

1. ***Flexural cracks:** form at the bottom near mid span and propagate upwards.*
2. ***Web shear cracks:** form near neutral axis close to support and propagate inclined to the beam axis.*
3. ***Flexure shear cracks:** These cracks form at bottom due to flexure and propagate due to both flexure and shear.*

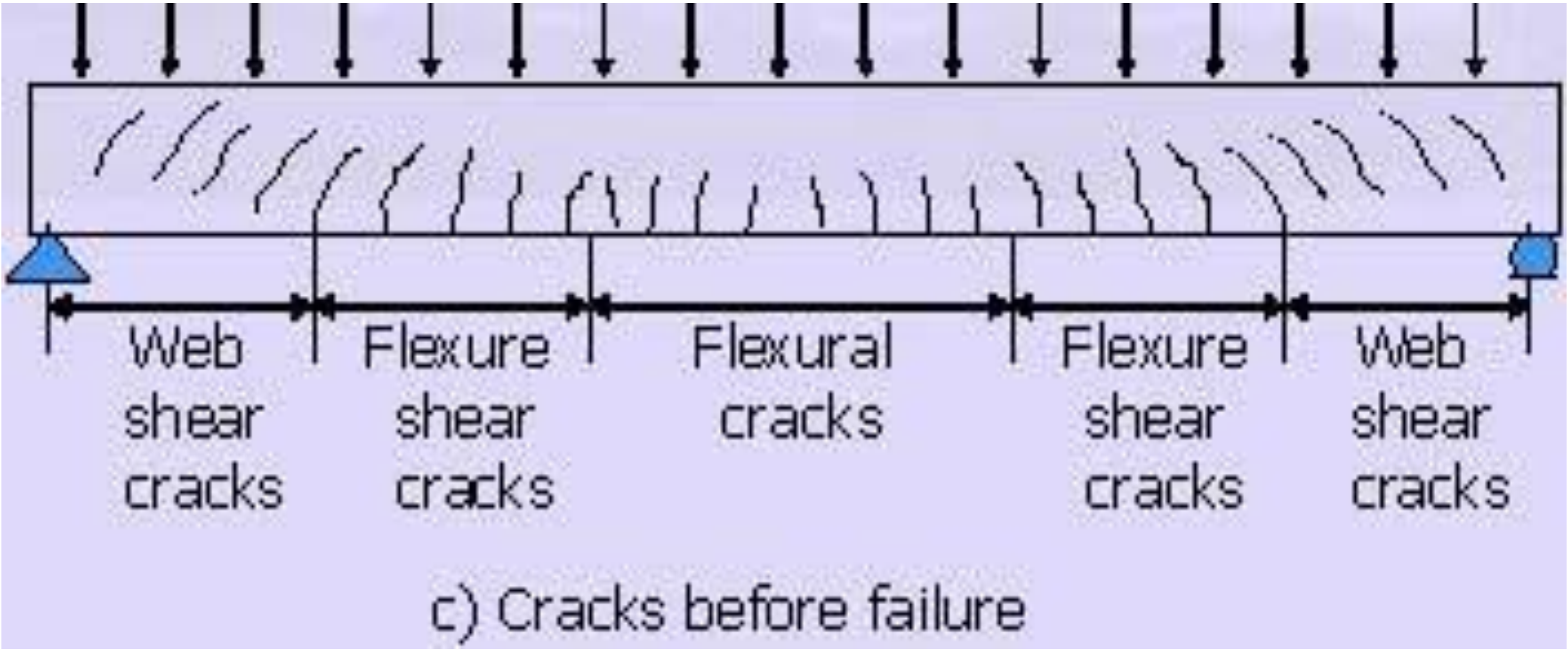
- ***Formation of cracks for a beam with large span-to-depth ratio and uniformly distributed loading is shown.***



a)Initiation of flexural cracks



b) Growth of flexural cracks and formation of flexure shear and web shear cracks.



c) Cracks before failure

Fig. 7 Formation of cracks in a reinforced concrete beam

Components of Shear Resistance

1. Components of shear resistance at a flexure shear crack are shown in the following figure.

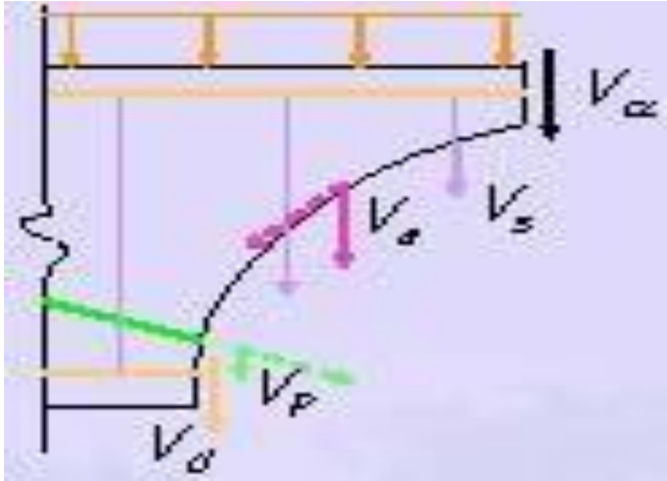


Fig. 8 Internal forces at a flexure shear crack

The notations in the previous figure are as follows.

V_{cz} = Shear carried by uncracked concrete

V_a = Shear resistance due to aggregate interlock

V_d = Shear resistance due to dowel action

V_s = Shear carried by stirrups

- Magnitude and relative value of each component change with increasing load.

Modes of Failure

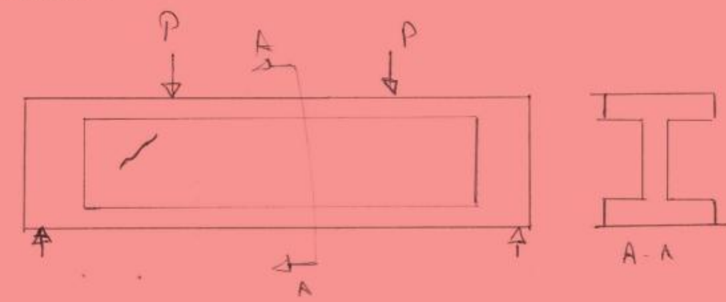
- ***For beams with low span-to-depth ratio or inadequate shear reinforcement, the failure may be due to shear.***
- ***Failure due to shear is sudden as compared to failure due to flexure.***
- **Five modes of failure due to shear are identified.**

    1. ***Diagonal tension failure***
    2. ***Shear compression failure***
    3. ***Shear tension failure***
    4. ***Web crushing failure***
    5. ***Arch rib failure***

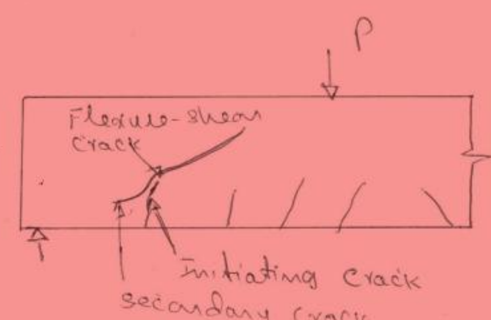
- **Mode of failure depends on span-to-depth ratio, loading, cross-section of beam, amount and anchorage of reinforcement.**

Shear in concrete

*. Direction of potential cracks in a simple beam.



(a) Web-shear Crack



(b) Flexure-shear crack.

Types of Inclined Cracks.

Diagonal tension failure: ***inclined crack propagates rapidly due to inadequate shear reinforcement.***

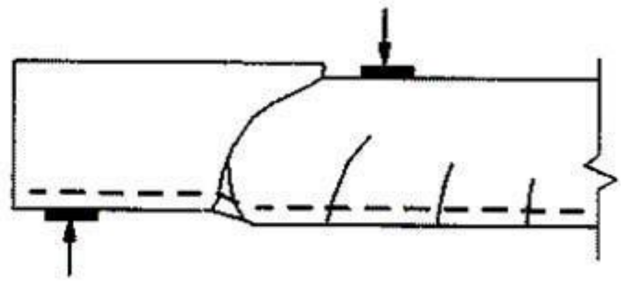


Fig. 9 Diagonal tension failure.

Shear compression failure: ***crushing of concrete near the compression flange above the tip of the inclined crack.***

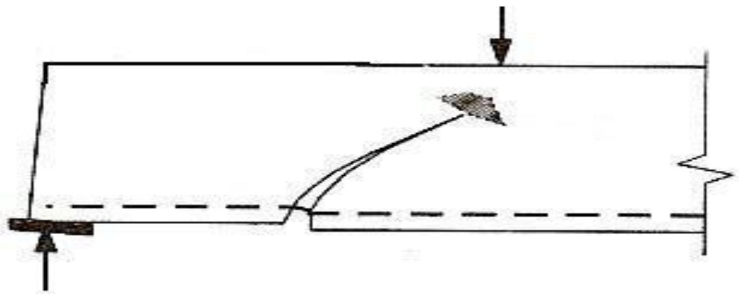


Fig. 10 Shear compression failure

Shear tension failure: inadequate anchorage of longitudinal bars, diagonal cracks propagate horizontally along the bars.

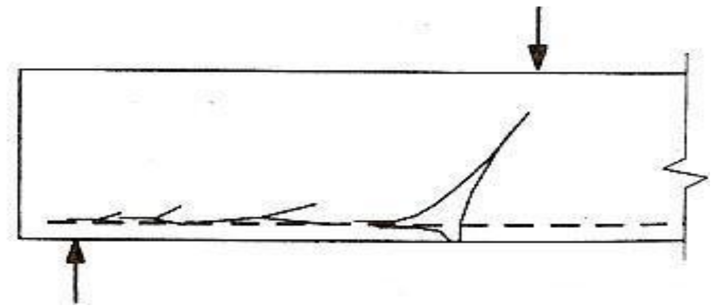


Fig. 11 Shear tension failure

Web crushing failure: concrete in the web crushes due to inadequate web thickness.

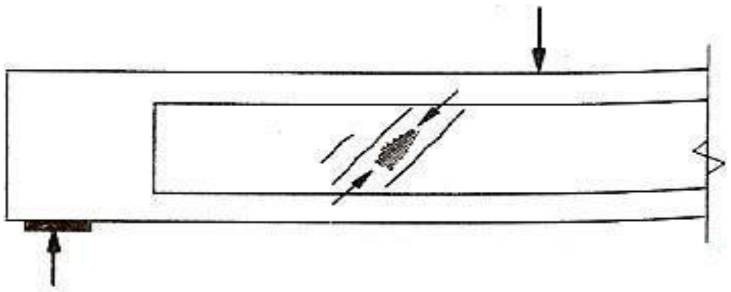


Fig. 12 Web crushing failure

Arch rib failure: in deep beams, web may buckle and subsequently crush. There can be anchorage failure or failure of the bearing.

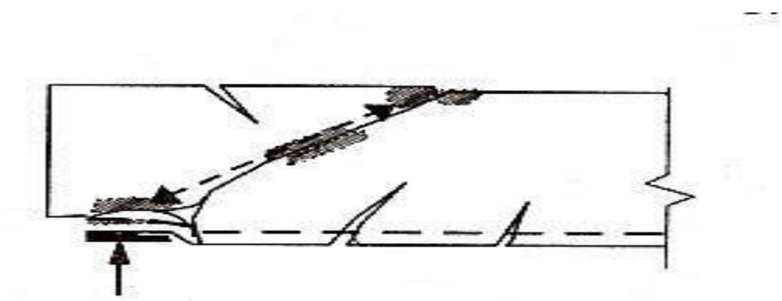


Fig. 13 Arch rib failure

- Design for shear is to avoid shear failure; beam should fail in flexure at its ultimate flexural strength.
- Design involves not only design of the stirrups, but also limiting average shear stress in concrete, providing adequate thickness of web and adequate development length of longitudinal bars.

Shear in Reinforced Concrete Members

- **Behavior of RC members under Shear (including combined loads with other loads) is very complex**

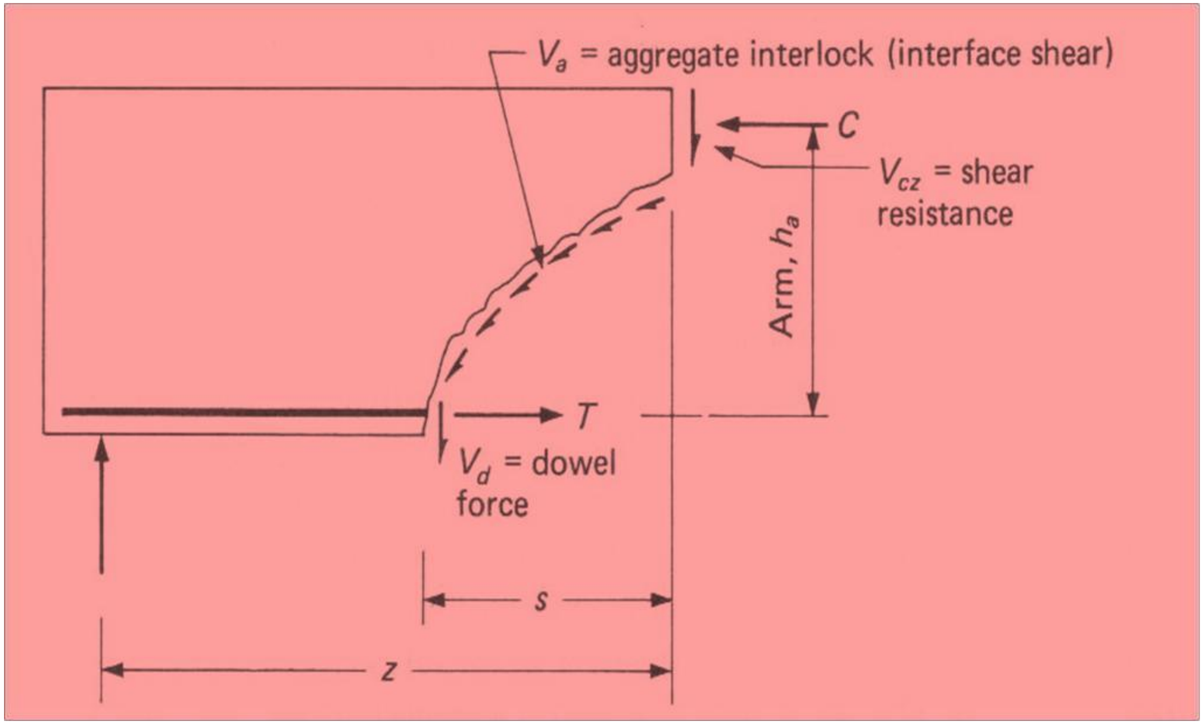
  1. *Non-homogeneity of materials*
  2. *Presence of Cracks and Reinforcement*
  3. *Nonlinearity in Material Response*

- **Current design (Code) procedures,**

  i. *Based on results of extensive tests on small size members with simplifying assumptions*

  ii. *No unified and universally accepted method for prediction of shear strength of beams without web reinforcement*

Shear Transfer Mechanisms



v_{cz} = Shear in compression zone (20-40%)

v_a = Aggregate Interlock forces (35-50%)

v_d = Dowel action from longitudinal bars (15-25%)

Total Resistance = (v_{cz} + v_{ay} +v_d) (For Beams without stirrups)

Factors Influencing Shear Strength

1. *Strength of Concrete (f'_c)*

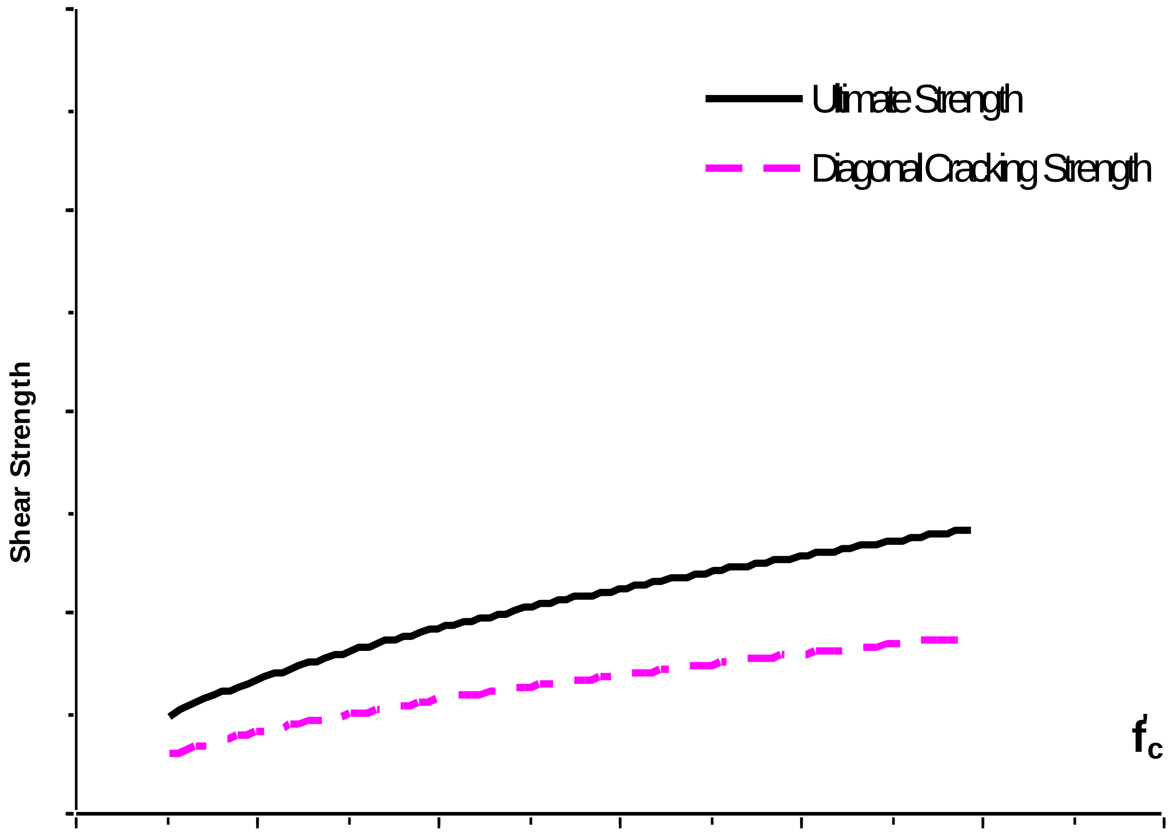
2. *Percentage of Flexural (Tensile) Reinforcement (ρ_t)*

3. *Shear Span-to-Depth Ratio (a/d)*

4. *Depth of Member (d)*

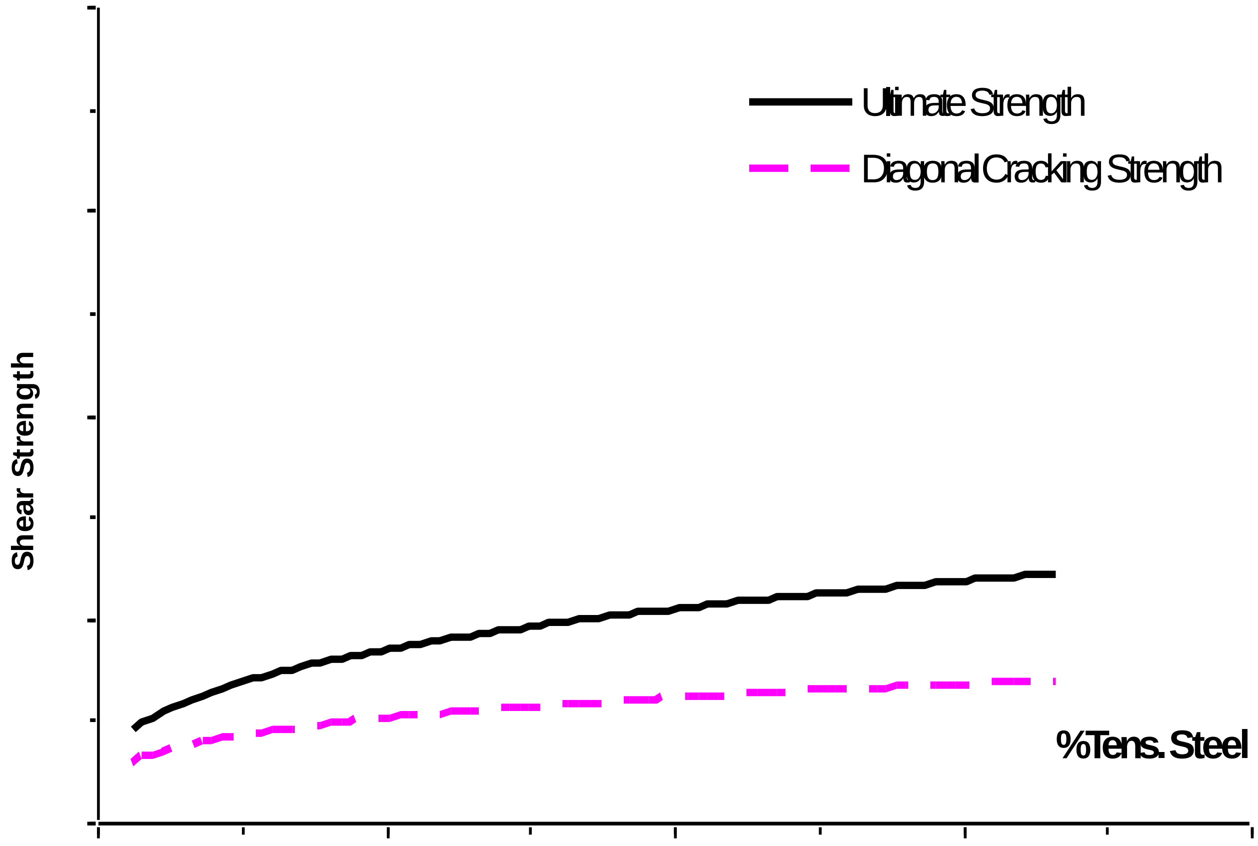
5. *Size of Aggregate (d_a) ?????*

Shear Strength with Compressive Strength of Concrete



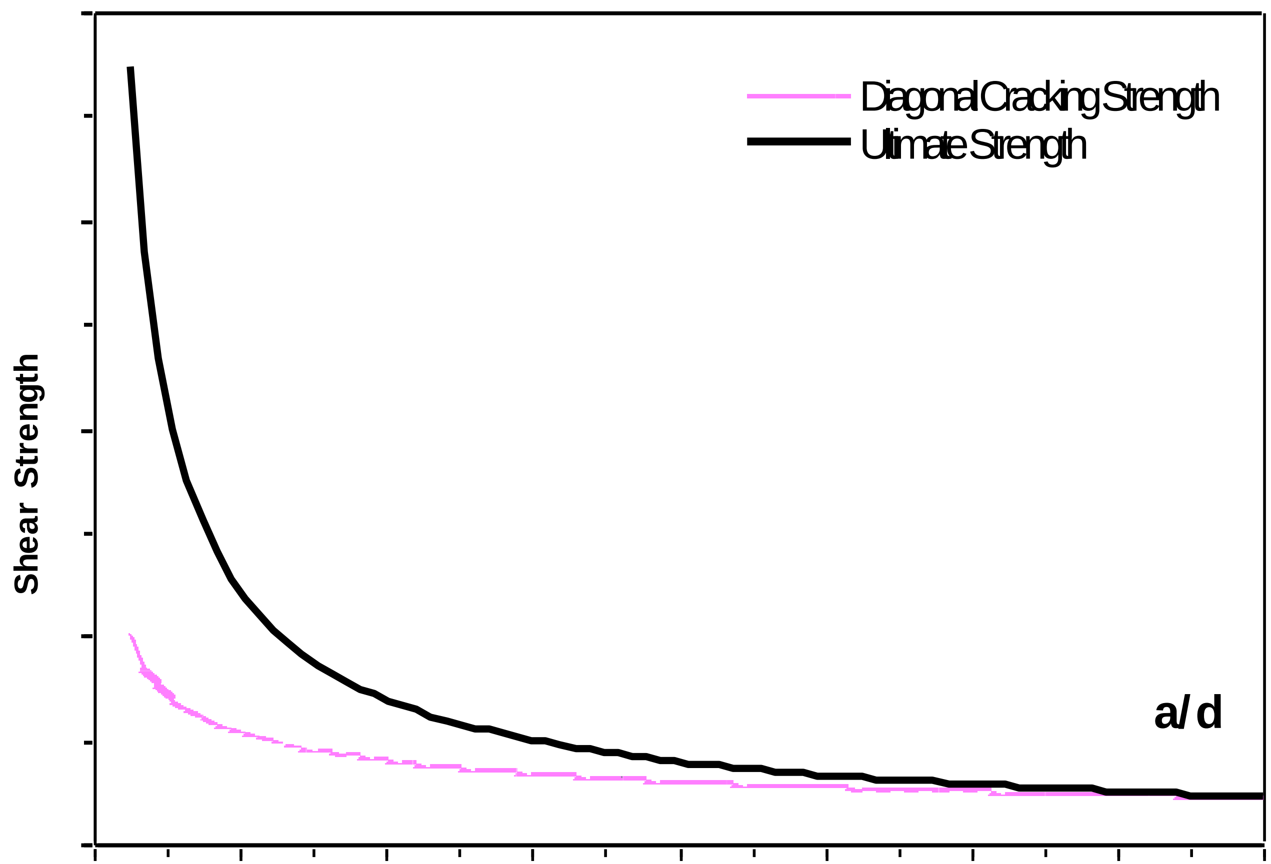
Qualitative variation of ultimate and Diagonal cracking strength with Compressive Strength of Concreet

Shear Strength with Tension Reinforcement



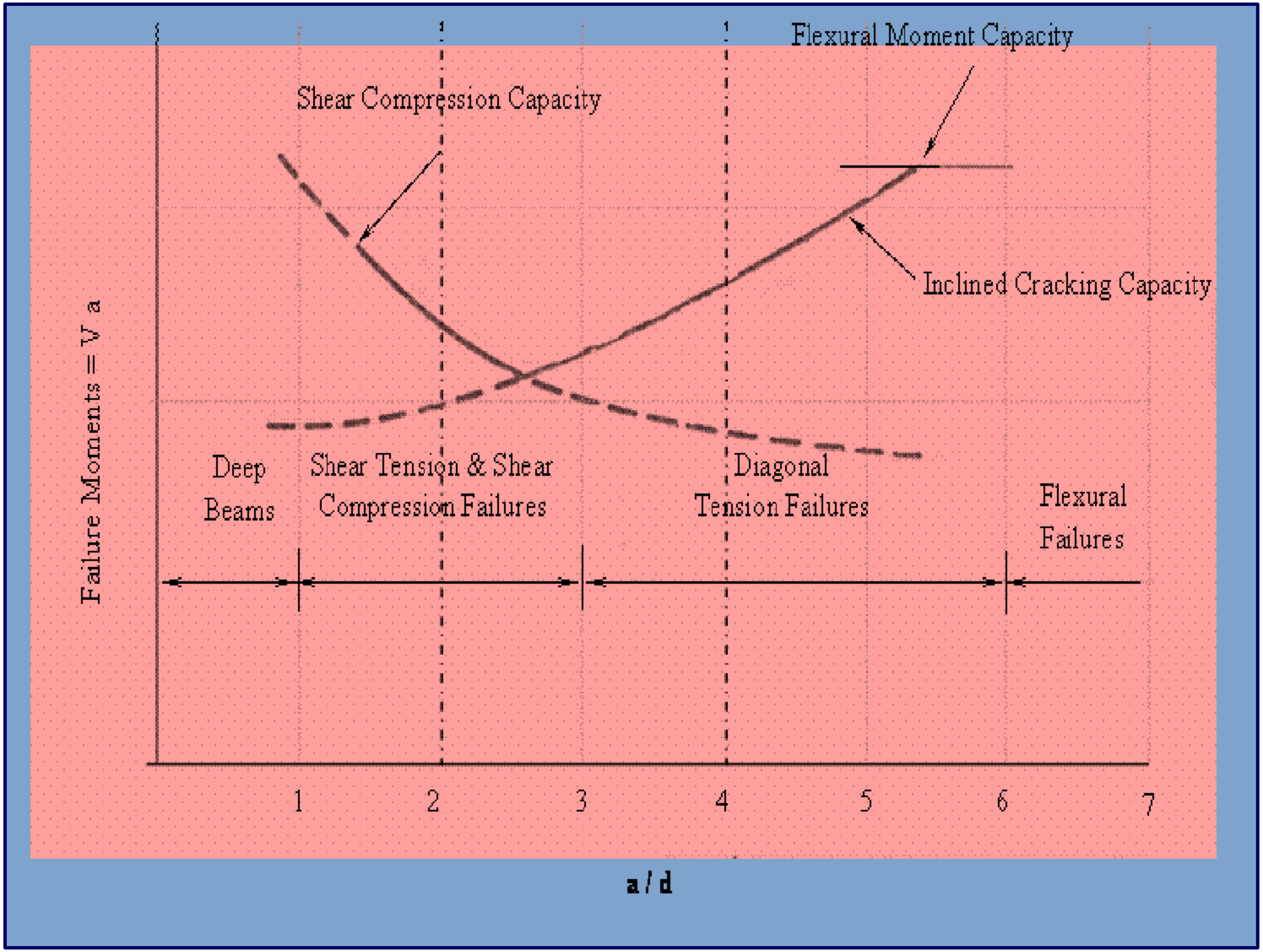
Qualitative variation of ultimate and Diagonal cracking strength with %Tension Steel

Shear Strength vs. Shear Span-to-Depth Ratio



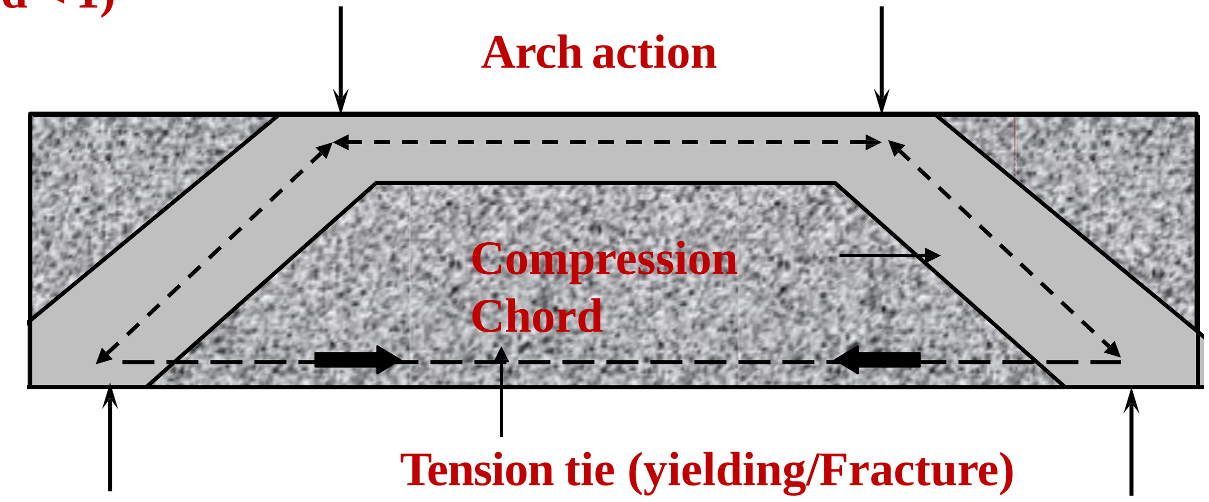
Qualitative variation of ultimate and Diagonal cracking strength with a/d ratio

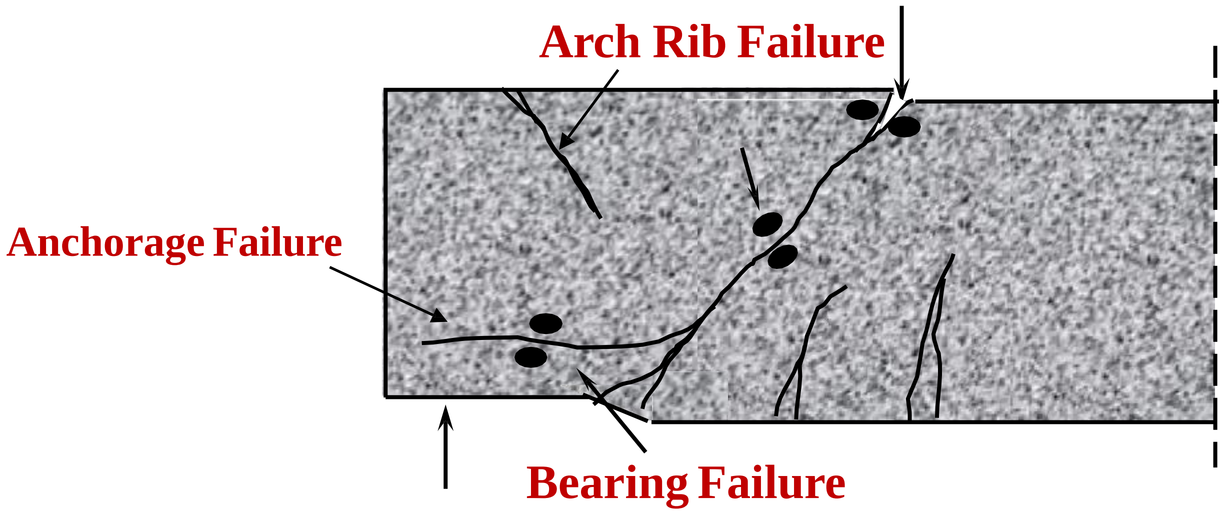
Failure Mechanism of RC Elements at different a/d Ratio



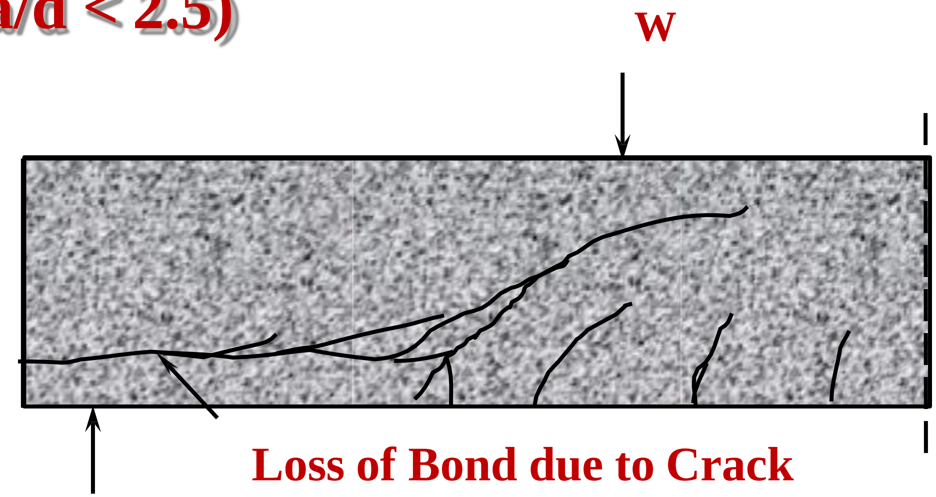
Failure Mechanisms

Deep Beams (a/d < 1)

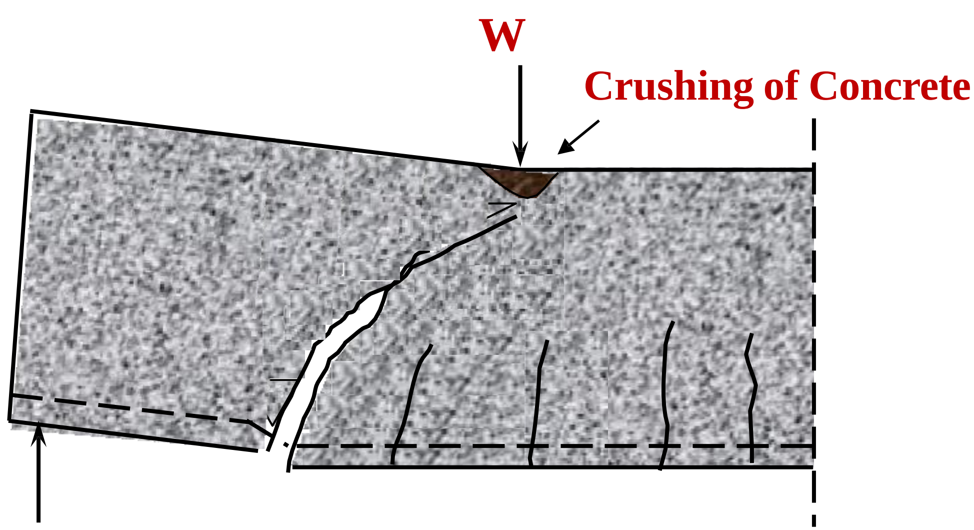




Short Beams (1 < a/d < 2.5)

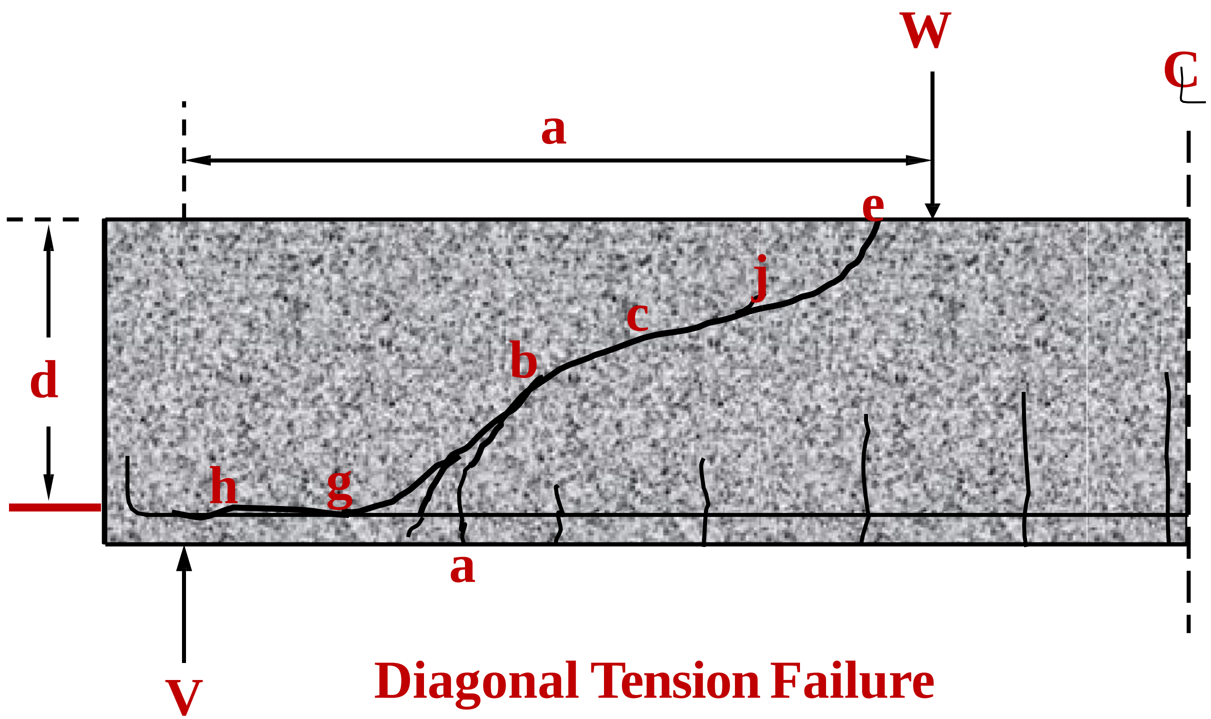


Shear Tension Failure



Shear Compression Failure

Slender Beams (2.5 < a/d < 6)



Diagonal Tension Failure

Does beam depth effect the strength?

- ***A phenomenon related to change in strength with member size,***
  - ***decrease in strength with increase in member size***
- ***Size effect in real structures:***

i) material heterogeneity and

ii) discontinuities (flaws) to flow of stress either in the form of

a. ***micro cracking before application of any load***

b. ***load induced micro cracking or***

c. ***macro cracking***

- ***Size effect in shear is more serious due to***
  - ***failure mode highly brittle -*** ***small deflections and lack of ductility***
  - ***shear strength related to tensile strength of concrete***
- **Evidence**
  - ***Sudden failure of Wilkins Air force depot warehouse in Shelby, Ohio (1955)***
  - ***Catastrophic failure of structures leading to loss of human lives and property due to Hyogo-Ken Nambu Earth Quake in 1995***

Wilkins Air Force Depot in Shelby, Ohio (1955)

Collapse of Superstructure - Hyogo-Ken Nambu EQ (1995)

Diagonal Cracking and Ultimate Strength

- ***Ultimate Strength***: *"load corresponding to the total and complete failure due to shear and diagonal tension"*

- ***Diagonal Cracking Strength***: *"load corresponding to formation of first fully developed inclined crack"*

a. An inclined crack is considered to be fully developed when it has progressed sufficiently towards both the mid span and the support while intersecting the tensile reinforcement

Diagonal Cracking and Ultimate Strength

- ***How is it measured in the laboratory?***

1. ***Normally, visual methods are deployed to measure the diagonal cracking loads in the laboratory***
2. ***load at the onset of formation of first diagonal crack***
3. ***when the diagonal crack crosses the mid height of the beam***

- **The design provisions in most of the codes**
  1) **are based on diagonal tension cracking and**
  2) **by using a suitable multiplication factor, strength of short or deep beams is obtained**
  3) **Need to be re-examined**
- **Owing to complex nature of stress distribution, Code Provisions for prediction of shear strength are Empirical in Nature for Beams without Web Reinforcement**

Prediction of Shear Strength

ACI - 2002

$$v_c = \left(0.16\sqrt{f_c'} + 17.2\,\rho \frac{V_u d}{M_u} \right) \leq 0.3\sqrt{f'_c} \quad MPa \quad \rightarrow (6)$$

$$v_c = 0.17\sqrt{f'_c} \quad MPa \quad \rightarrow 6(a)$$

Simplified

$$v_c = \left(3.5 - 2.5 \frac{M_u}{V_u d} \right) \left(0.16\sqrt{f'_c} + 17.2\rho \frac{V_u d}{M_u} \right) \quad MPa \quad \rightarrow (7)$$

$$\leq 0.5\sqrt{f'_c} \qquad (ACI - 1999)$$

$$where \left(3.5 - 2.5 \frac{M_u}{V_u d} \right) \leq 2.5 \quad and \quad \frac{V_u d}{M_u} \leq 1.0$$

BS 8110 - 1997

$$v_c = \frac{0.79}{\gamma_m}\left(\frac{100\,A_s}{b_v d}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4}\left(\frac{f_{cu}}{25}\right)^{1/3} \quad MPa \rightarrow (8)$$

Where $\frac{100\,A_s}{b_v d} \leq 3.0, \quad \frac{400}{d} \geq 1.0, \quad \gamma_m = 1.25 \quad and \quad f_{cu} \leq 40.0 \ MPa$

For Short Beams

$$v_c = (Eqn.8)\left(\frac{2}{a/d}\right) \qquad for \ \ a/d < 2.0$$

Prediction of Shear Strength

IS 456-2000

$$v_{cr} = \frac{0.85\sqrt{0.8f_{ck}}\left(\sqrt{(1+5\beta)}-1\right)}{6\beta} \quad \text{MPa} \rightarrow (9)$$

$$\leq 0.62\sqrt{f_{ck}}$$

Where 0.8 f_{ck} = Cylinder strength in terms of cube strength and

0.85 reduction factor = 1/γm ~ 1/1.2

$$and \quad \beta = \frac{0.8f_{ck}}{6.89\rho} \geq 1.0$$

For Short Beams $v_{cr} = (Eqn.9)\left[\frac{2}{a/d}\right]$

Design of Beams for Shear

- ***Nominal Shear stress*** $$\tau_v = \frac{V_u}{bd}$$

Where

- ***v_u = shear force due to design loads***
- ***b = breadth of the member which for flanged sections shall be taken as the breadth of web b_w and***
- ***d = effective depth***

Shear Strength of RC beams

- ***Calculate the nominal shear stress and compare with the shear strength of RC beams from Table 19 of IS 456-2000.***
- ***If the nominal shear stress*** τ_v ***exceeds the shear strength*** τ_c ***of RC beams without shear reinforcement, then the beam needs to be designed for shear reinforcement.***
- ***When*** τ_v ***is less than*** τ_c ***obtained from Table 19, minimum shear reinforcement is provided which is given by***

$$\frac{A_{sv}}{b s_v} > \frac{0.4}{f_y}$$

$$s_v \leq \frac{A_{sv} f_y}{0.4 b}$$

Table 19 Design Shear Strength of Concrete, τ_c, N/mm²

(*Clauses* 40.2.1, 40.2.2, 40.3, 40.4, 40.5.3, 41.3.2, 41.3.3 *and* 41.4.3)

$100\frac{A_s}{bd}$	Concrete Grade					
	M 15	M 20	M 25	M 30	M 35	M 40 and above
(1)	(2)	(3)	(4)	(5)	(6)	(7)
≤ 0.15	0.28	0.28	0.29	0.29	0.29	0.30
0.25	0.35	0.36	0.36	0.37	0.37	0.38
0.50	0.46	0.48	0.49	0.50	0.50	0.51
0.75	0.54	0.56	0.57	0.59	0.59	0.60
1.00	0.60	0.62	0.64	0.66	0.67	0.68
1.25	0.64	0.67	0.70	0.71	0.73	0.74
1.50	0.68	0.72	0.74	0.76	0.78	0.79
1.75	0.71	0.75	0.78	0.80	0.82	0.84
2.00	0.71	0.79	0.82	0.84	0.86	0.88
2.25	0.71	0.81	0.85	0.88	0.90	0.92
2.50	0.71	0.82	0.88	0.91	0.93	0.95
2.75	0.71	0.82	0.90	0.94	0.96	0.98
3.00 and above	0.71	0.82	0.92	0.96	0.99	1.01

NOTE — The term A_s is the area of longitudinal tension reinforcement which continues at least one effective depth beyond the section being considered except at support where the full area of tension reinforcement may be used provided the detailing conforms to **26.2.2** and **26.2.3**

Table 20 Maximum Shear Stress, $\tau_{c\,max}$, N/mm²

(*Clauses* 40.2.3, 40.2.3.1, 40.5.1 *and* 41.3.1)

Concrete Grade	M 15	M 20	M 25	M 30	M 35	M 40 and above
$\tau_{c\,max}$, N/mm²	2.5	2.8	3.1	3.5	3.7	4.0

Prediction of Shear Strength

IS 456-2000

$$\tau_c = \frac{0.85\sqrt{0.8f_{ck}}\,(\sqrt{(1+5\beta}-1)}{6\beta} \quad \text{MPa} \;\; \leq 0.62\sqrt{f_{ck}}$$

Where 0.8 f_{ck} = Cylinder strength in terms of cube strength and

0.85 reduction factor = 1/γm ~ 1/1.2

$$\textit{and} \qquad \beta = \frac{0.8 f_{ck}}{6.89\,\rho} \geq 1.0$$

For Short Beams $v_{cr} = (Eqn.9)\left[\frac{2}{a/d}\right]$

Design of Shear Reinforcement

I. When the shear stress is greater than shear strength given in Table 19 (IS 456), shear reinforcement shall be provided in any of the following forms

a. Vertical stirrups
b. Bent-up bars along with stirrups and
c. Inclined stirrups

- *Shear reinforcement shall be provided to carry a shear force equal to*

$$\left[V - \tau_c bd\right]$$

Forms of Shear Reinforcement

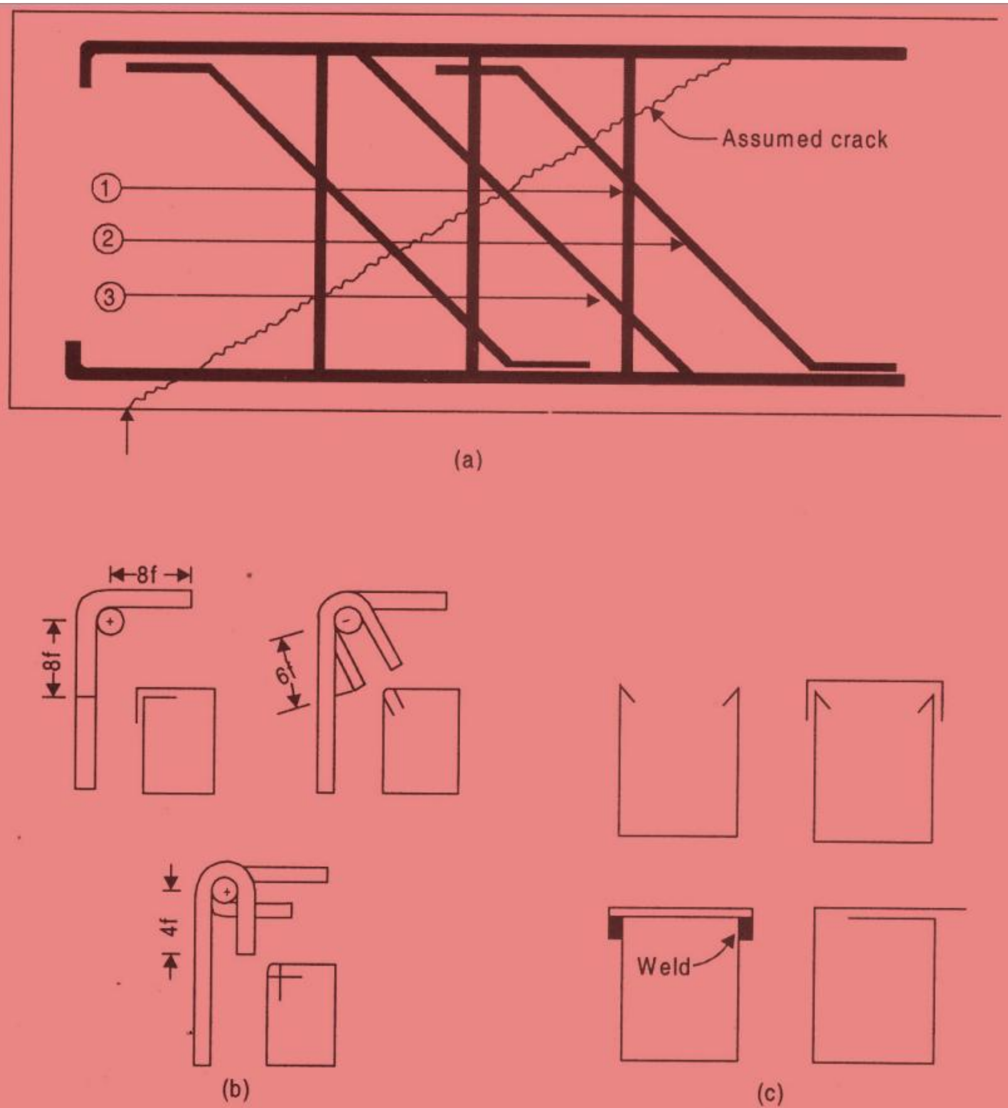


Fig. 7.4 Types of shear reinforcements in beams: (a) Shear reinforcements—(1) system of vertical stirrups; (2) system of inclined stirrups; (3) bent-up tension bar; (b) Detailing of ends of conventional types of stirrups; (c) Some special shapes of stirrups.

Forms of Shear Reinforcement

- ***For vertical stirrups***

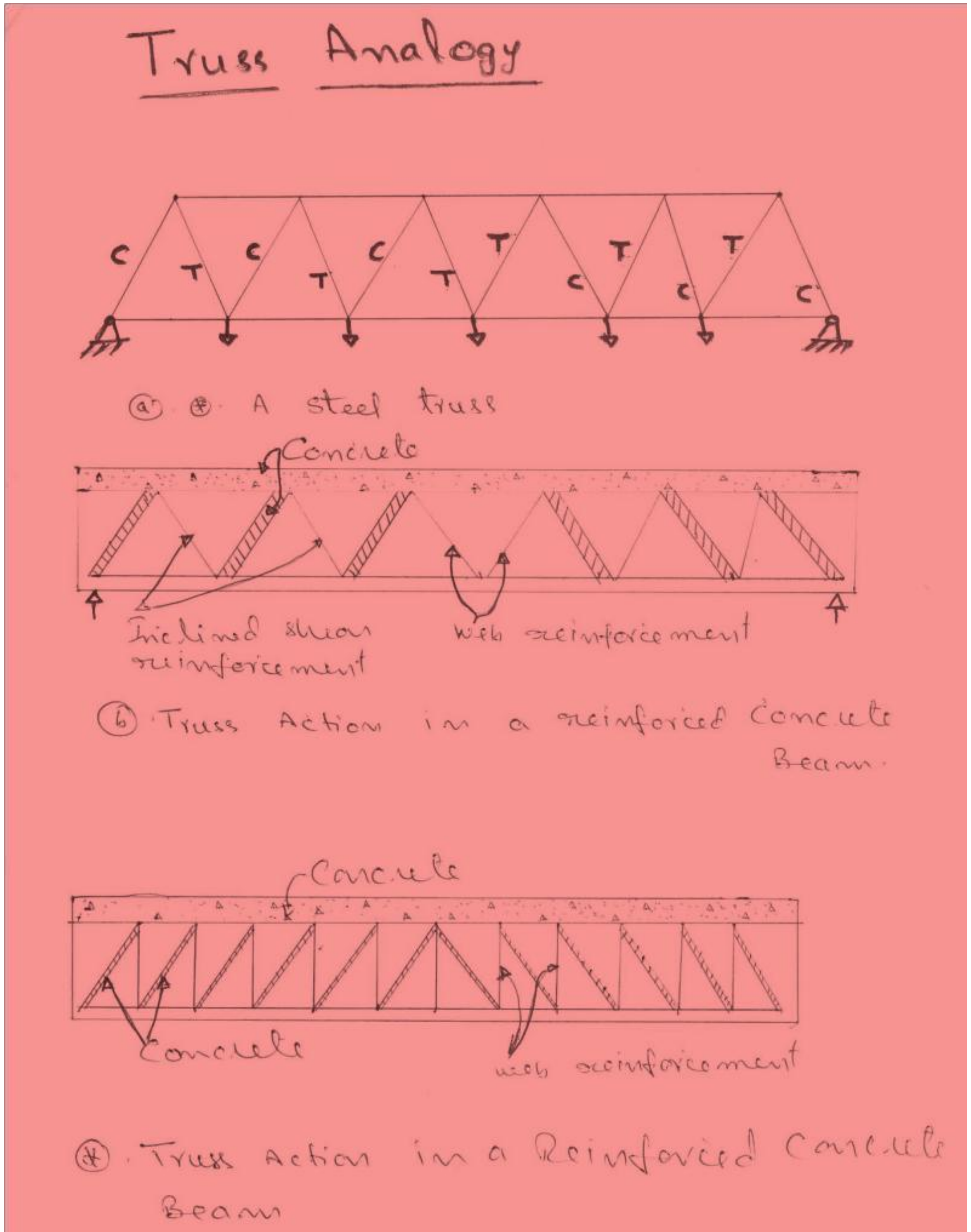
$$v_s = \frac{A_{sv} . f_{sv}}{s_v} d$$

- ***For single bar or single group of parallel bars all bent up at the same cross-section***

$$V_s = \sigma_{sv} . A_{sv} . \sin\alpha$$

- α ***= angle between the inclined stirrup or bent up bar and the axis of the member not less than*** 45^0

Truss Analogy



Truss Analogy

1. *Action of vertical and inclined (stirrups) shear reinforcement may be described by the analogous truss action.*
2. *In a simple truss, the upper and lower chords are in compression and tension respectively; the diagonal members, called web members, are alternately in compression and tension.*
3. *Shear strength of RC beam may be increased by use of shear reinforcement similar in action to tensile web members in a truss.*
4. *Shear reinforcement must be anchored in compression zone of concrete and is usually hooped around longitudinal tension reinforcement.*

Design of Stirrups

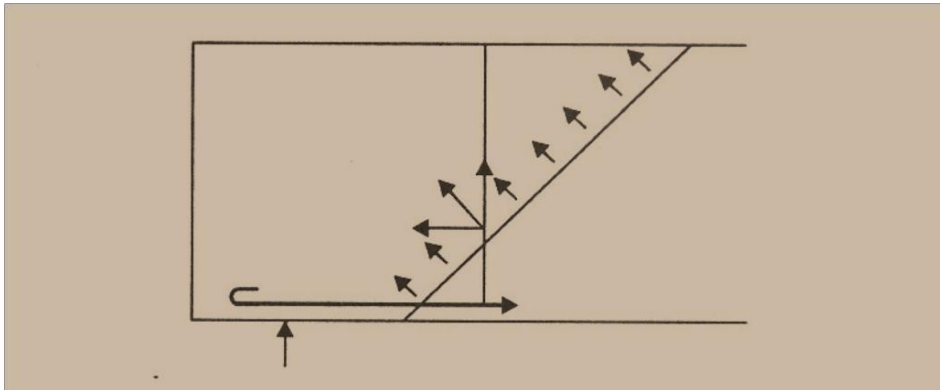


Fig. 7.6 Equilibrium of diagonal tension forces in concrete.

- A_{sv} = total area of legs of shear links
- s_v = spacing of links
- Number of links crossing 45^0 diagonal crack $N = \frac{d}{s_v}$
- Total strength of vertical stirrups $V_s = 0.87 f_y A_{SV} \frac{d}{s_v}$
- Spacing of stirrups required=

$$0.87 f_y A_{sv} \frac{d}{s_v} = (\tau_v - \tau_c) bd \Rightarrow s_v = \frac{0.87 f_y A_{sv}}{b(\tau_v - \tau_c)}$$

Design of Bent-up Bars

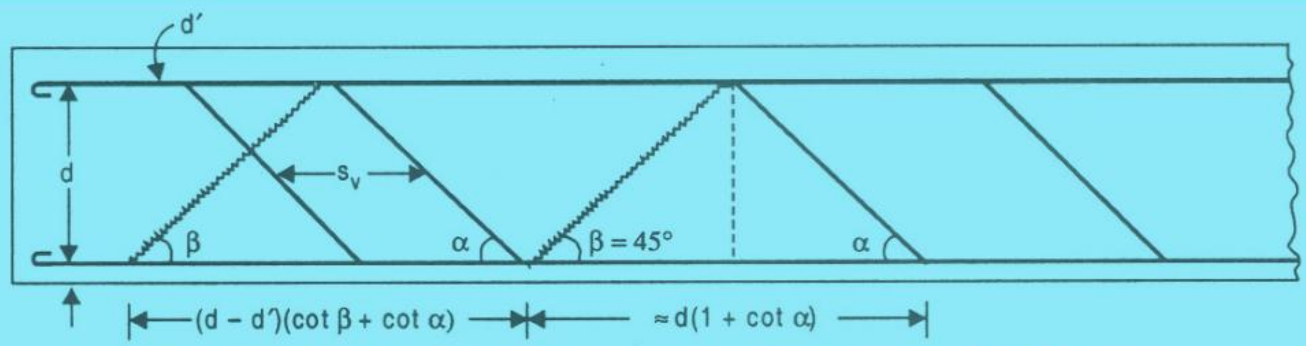


Fig. 7.7 Design of bent-up bar system for shear in RCC beams.

- ***Horizontal length over which the bar is effective can be taken as equal to d(cot β + cot α), where β = direction of shear compression, α = angle at which the bars bent***
- ***Let s_v = spacing of bent bars. Then the number of effective bars in this region are***

$$N = \frac{(\cot\beta + \cot\alpha)(d - d')}{s_v}$$

- ***The maximum shear carried by bent up bars =***

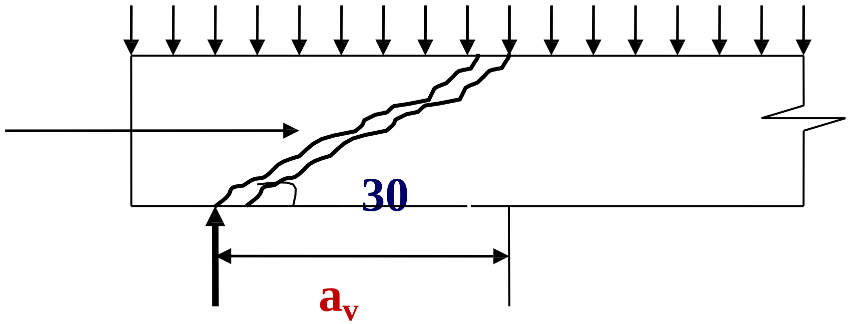
$$V_s = A_{sv}(0.87 f_y)\sin\alpha \frac{(\cot\beta + \cot\alpha)(d - d')}{s_v}$$

$$= A_{sv}(0.87 f_y)\frac{(\cos\alpha + \sin\alpha)(d)}{s_v};\therefore(\beta = 45^0; (d - d') \approx d)$$

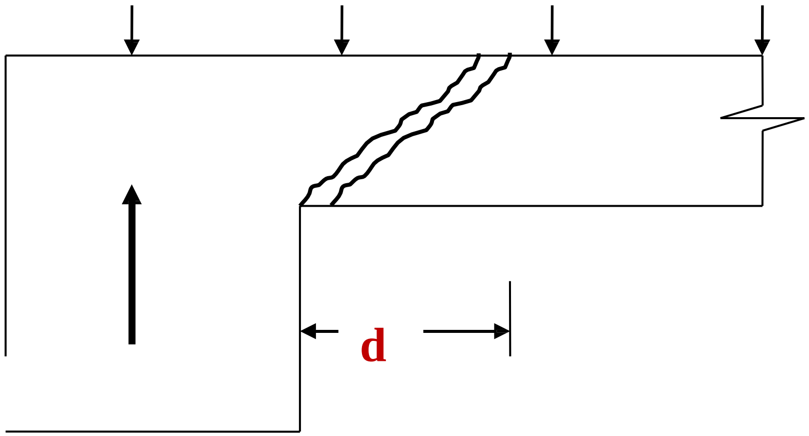
Enhanced Shear Near Supports

- ***Section near the supports can be assumed to have enhanced shear strength.***
- ***Shear failure at sections of beams and cantilevers without shear reinforcement normally takes place on a plane making an angle 30^0 with the horizontal.***
- ***Enhance shear capacity at sections near supports as would be done in design of brackets, ribs, corbels etc.***

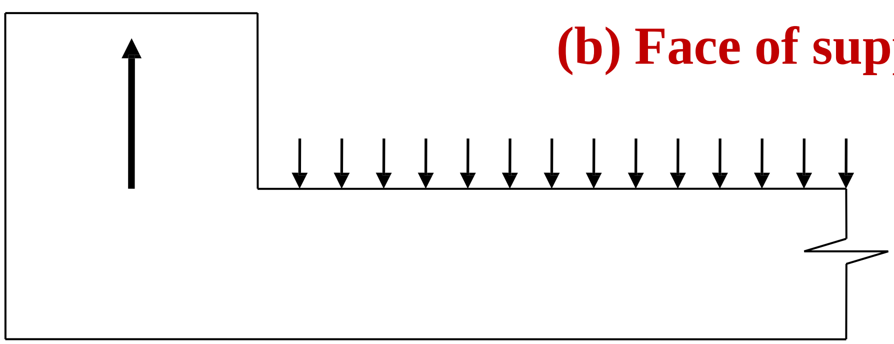
Critical Sections for shear in beams which are supported on members in compression, and tension



(a) Beams with compression at end region



(b) Face of support in compression



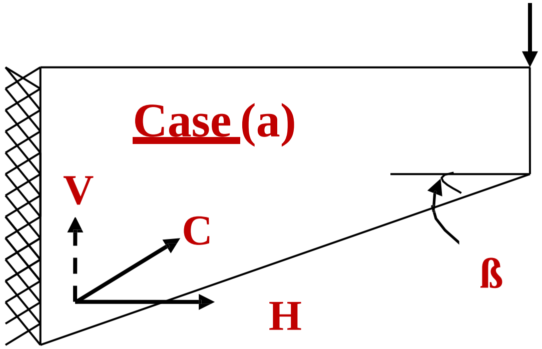
(c) Face of support in tension

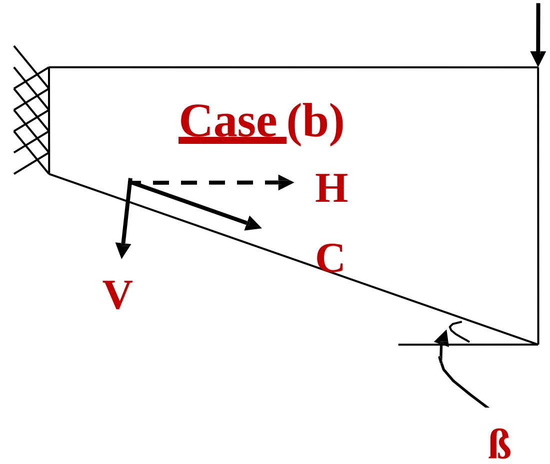
Beams of Varying Depth

- ***Beams with varying depth are encountered in RC.***
- ***Beam depth is varied according to the variation of bending moment and shear force.***

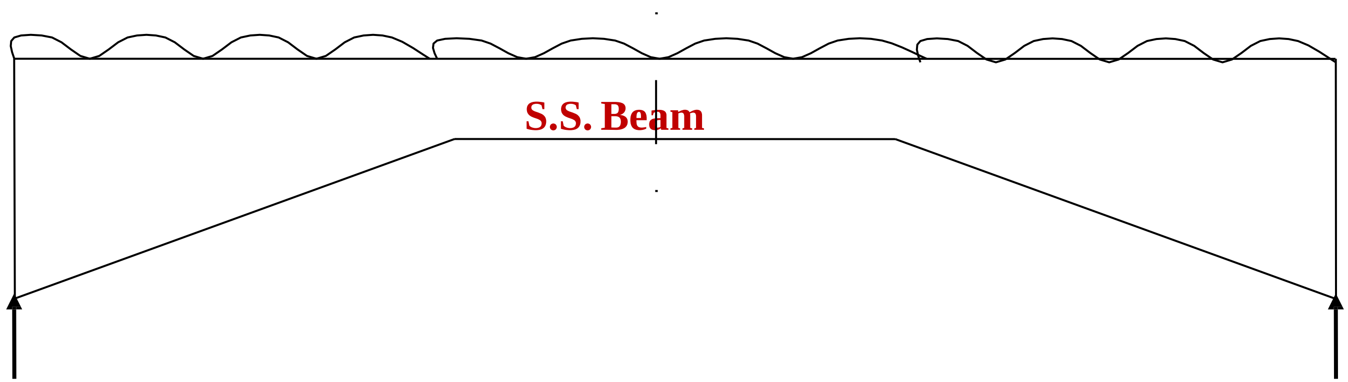
1. ***<u>Case </u>(a): Bending moment increases numerically in the direction in which effective depth increases.***

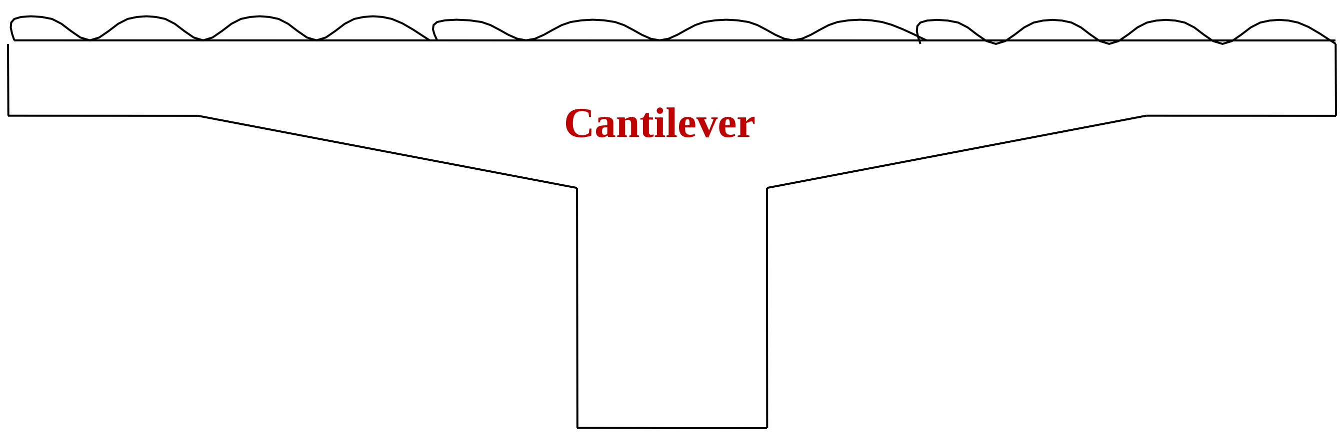
2. ***<u>Case </u>(b). Bending moment decreases numerically in the direction in which effective depth increases.***





M = H Jd





Effective shear force for determining the shear stress

$$V = V_w - \frac{M}{d} \tan\beta \text{ for case(a)}$$

$$V = V_w + \frac{M}{d} \tan\beta \text{ for case(b)}$$

Design of Stirrups at Steel Cut-off Points

- ***When flexural reinforcement in beams is terminated in tension region, at that section it should satisfy***

    a) shear at cut-off point does not exceed two-thirds of combined strength of concrete and steel. $\tau_s > \left[1.5\tau - \tau_c\right]$

    b) Additional stirrups should be provided along each terminated direction over a distance from the cut-off point equal to three-fourth effective depth, equal to $A'_{su} = \dfrac{0.4\, b\, s}{f_y}$

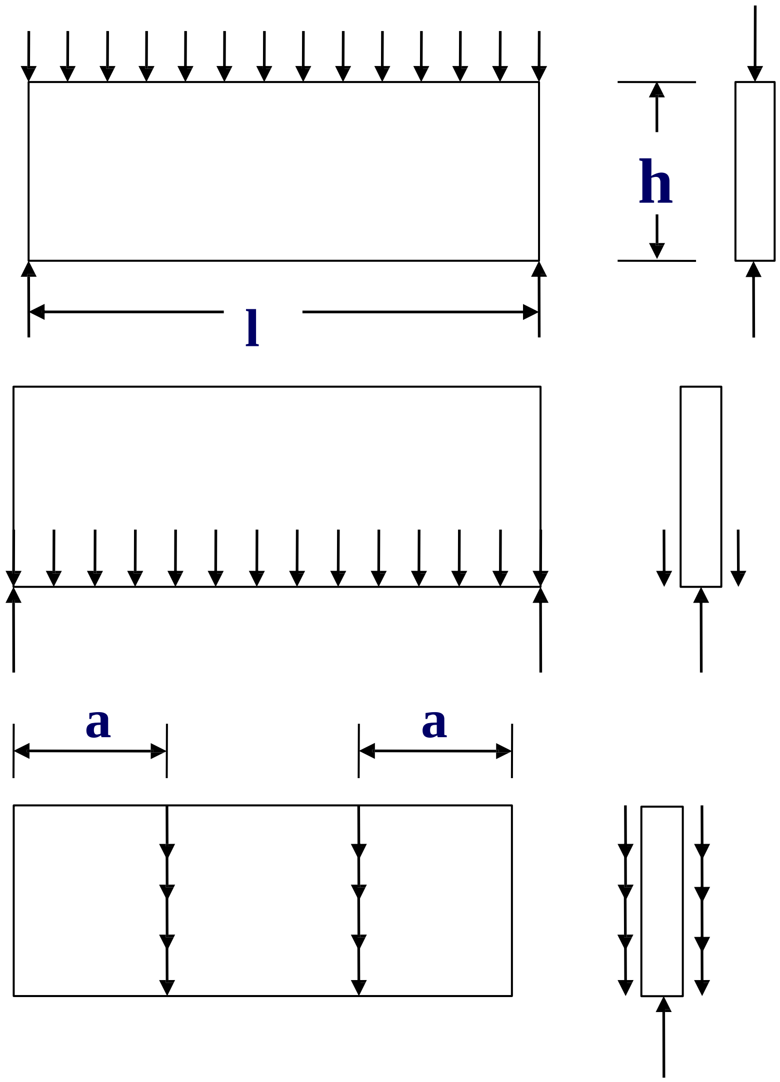
- ***Spacing of stirrups*** $< \dfrac{d}{8\,\beta}$ $\qquad \beta = \dfrac{Area\ of\ \ cut - off\ \ bars}{Total\ area\ of\ \ bars}$

Minimum Shear Reinforcement

1. ***Restrains the growth of inclined cracking.***
2. ***Ductility is increased and gives warning before failure.***
3. ***In an unreinforced web, such reinforcement is of great value if a member is subjected to an unexpected tensile force or an overload.***
4. ***A minimum area of shear reinforcement is required whenever the total factored shear force V_u is greater than one-half the shear strength provided by concrete kV_c.***
5. ***<u>Need to increase minimum shear reinforcement as concrete strength increases to prevent sudden shear failure by inclined cracking.</u>***

Deep Beams

- ***Depth much greater than normal, in relation to their span, while thickness is much smaller than either span or depth.***
- ***Main loads and reactions act in plane of member to achieve a state of plane stress in concrete***
- ***Members with span-to-depth ratio of about 5 or less, or with shear span, a, less than about twice depth are called deep beams.***



(a). loads applied along the compression edge

(b). loads suspended along the tension edge

(c). loads distributed through depth

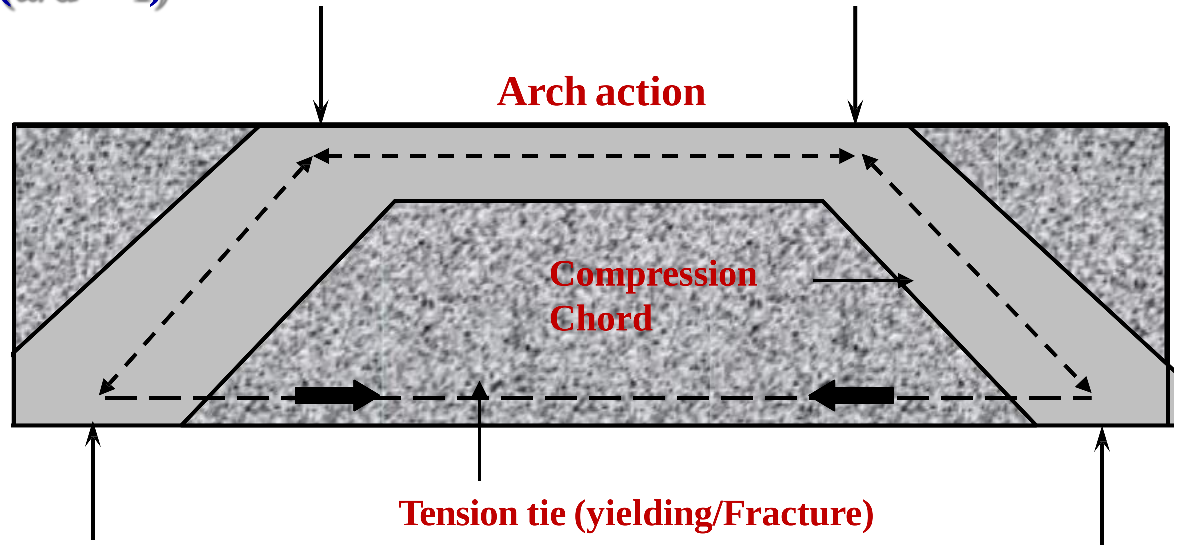
Fig. Placements of loads on deep beams.

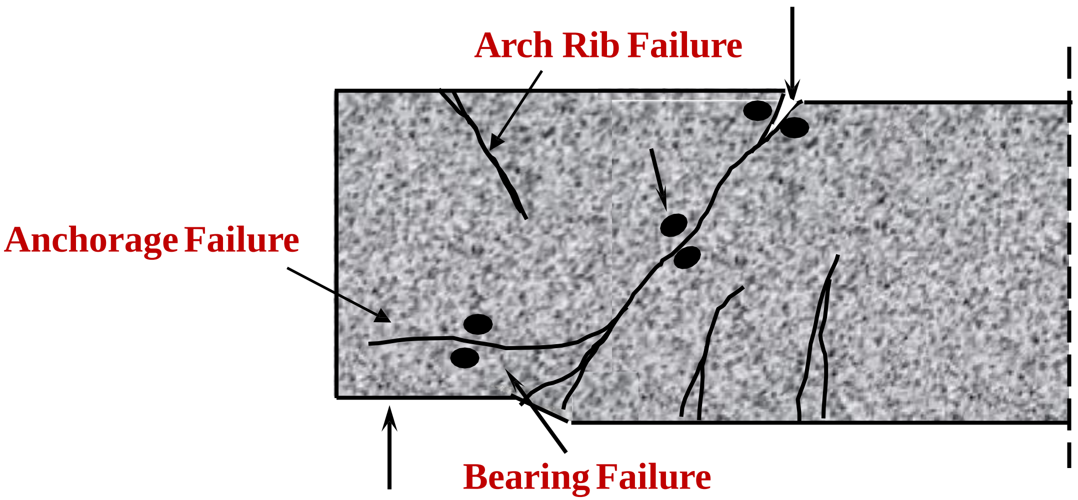
Deep Beams

- ***Examples of Deep Beams found in:***
  - ***Column offsets,***
  - ***Walls of rectangular tanks and bins,***
  - ***Floor diaphragms***
  - ***Shear walls,***
  - ***in folded plate roof structures***
- ***Behavior of deep beams is significantly different from that of the normal beams, requires special consideration in analysis, design, and detailing of reinforcement.***

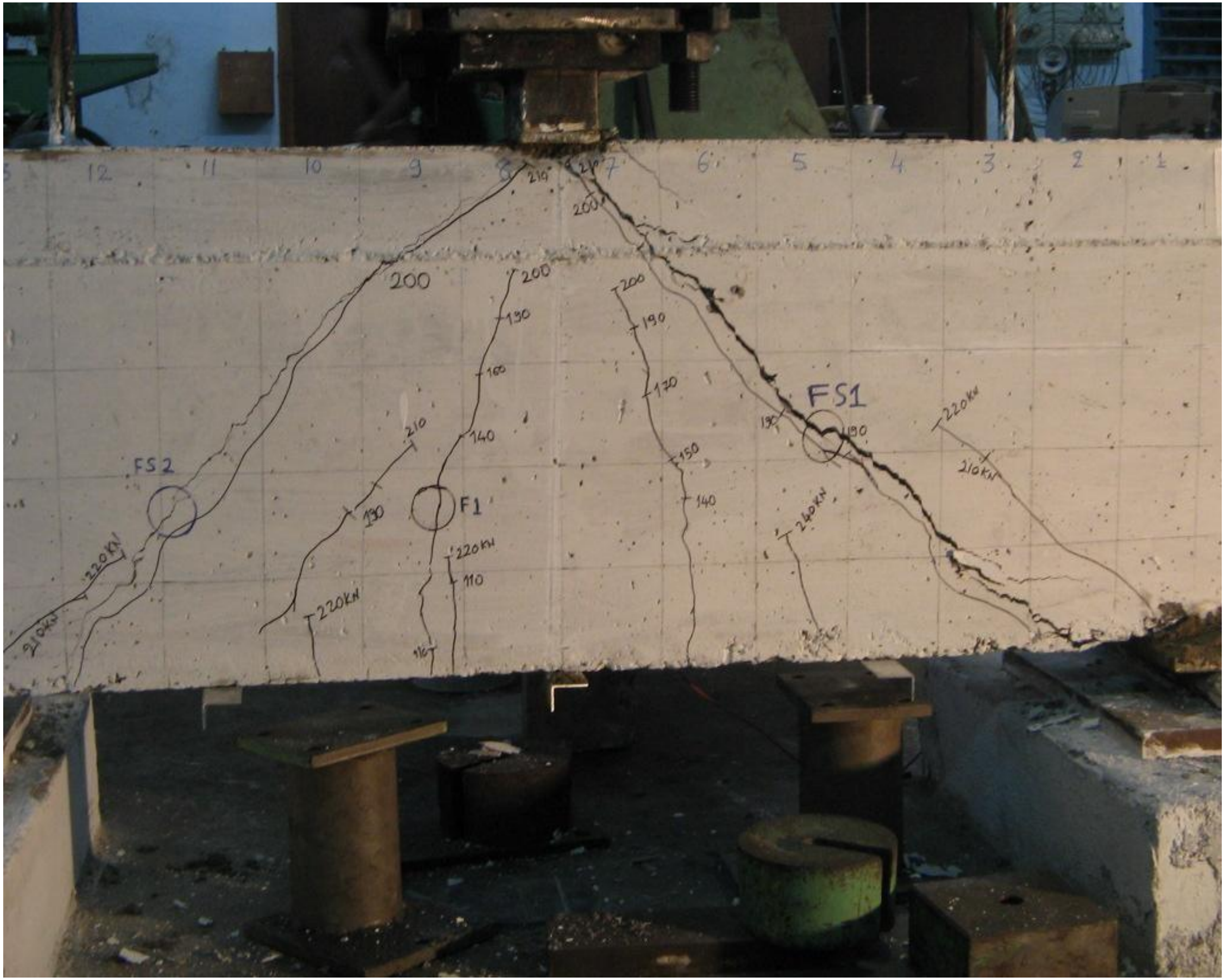
Deep Beams

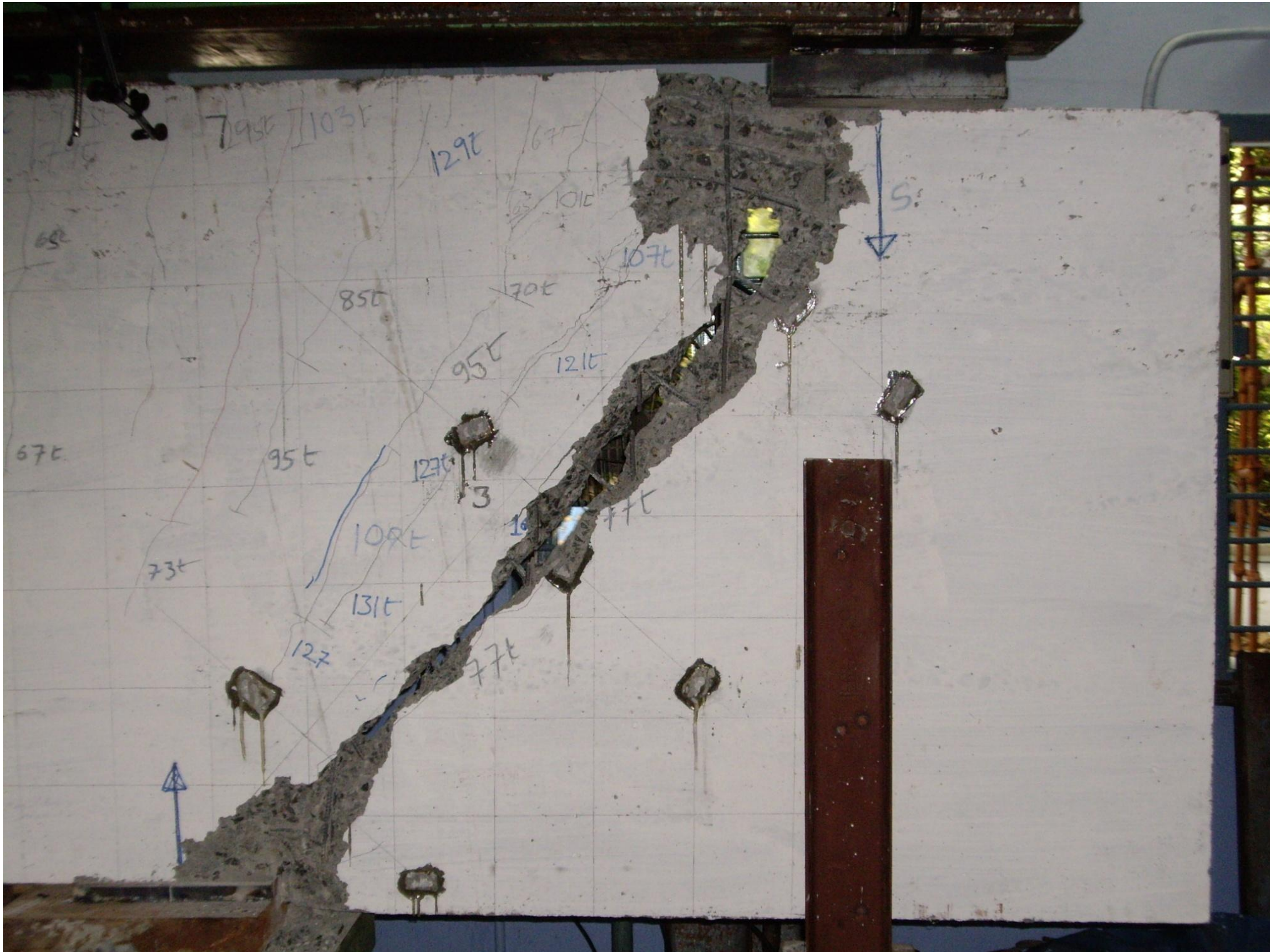
Deep Beams (a/d < 1)





Failure in Deep Beams




103t
129t
101t
107t
85t
70t
95t
121t
67t
95 t
127t
3
131t
73t
127
77t
S

Characteristic Strength

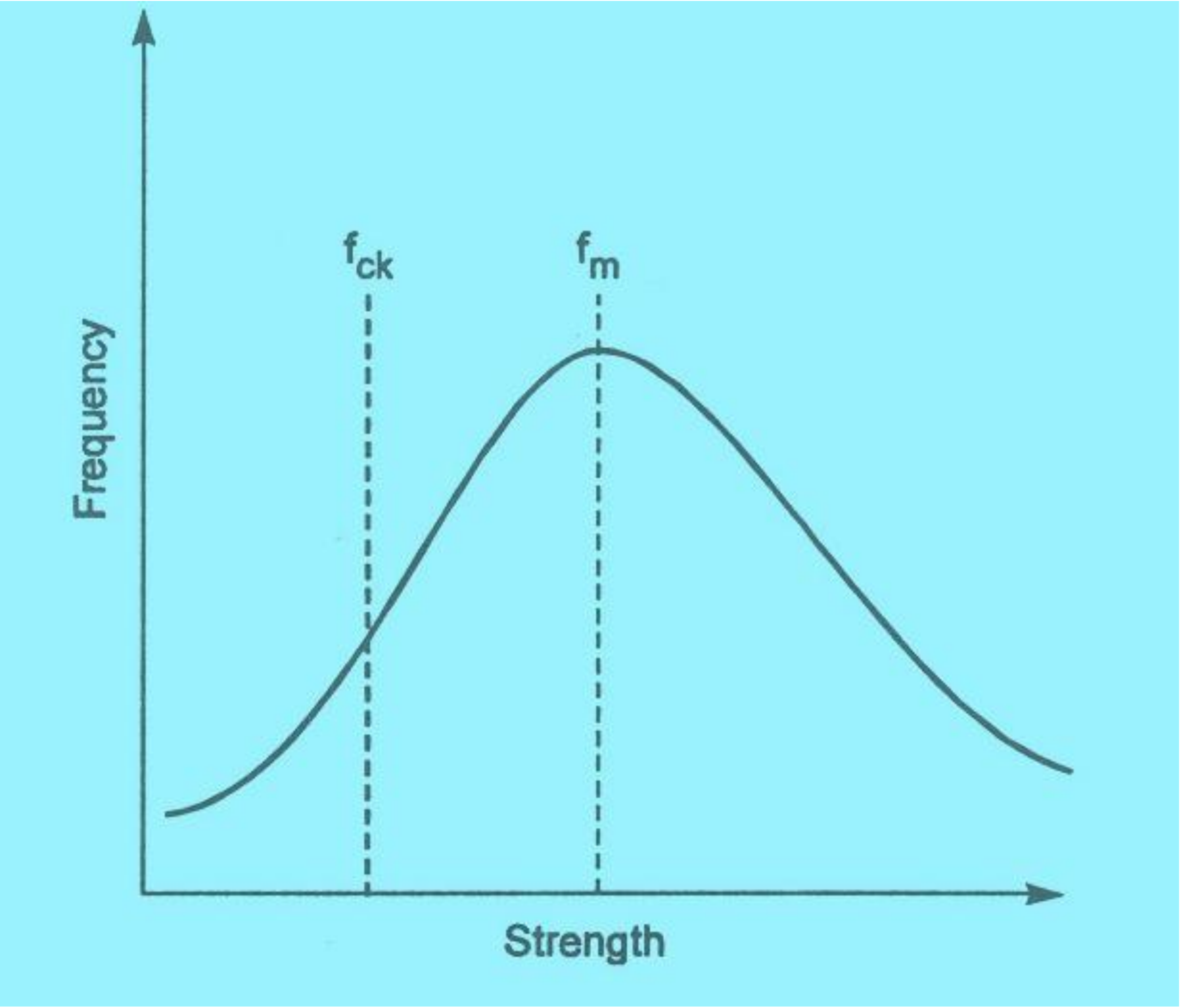
- *"Characteristic strength is defined as the strength of material below which not more than 5 percent of the test results are expected to fall".*

- *Strength of concrete varies for the same concrete mix, which give different compressive strength in laboratory tests.*

- *Variability in strength evidently depends on degree of quality control.*

- *Variability in strength is measured in terms of either the "Standard Deviation" or the Coefficient of Variation (COV), which is the ratio of standard deviation to mean strength(f_{cm}).*

Characteristic Strength



Characteristic Strength

- ***It is well established that the probability distribution of concrete strength (for a given mix) is approximately "Normal".***

- ***Coefficient of variation is generally in the range of 0.01 to 0.02.***

- ***Due to significant variability in strength, it is necessary to ensure that the designer has a reasonable assurance of a certain minimum strength of concrete.***

- ***Characteristic strength provides minimum guaranteed strength.***

Idealized Normal Distribution

- ***Accordingly, the mean strength of concrete, f_{cm} (as obtained from 28 days tests) has to be significantly greater than the 5 percentile characteristic strength, f_{ck} that is specified by the designer.***

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

$$\sigma = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{n}}$$

$$COV = \frac{\sigma}{\bar{x}} = S$$

Normal Probability Curve

$$\text{The probability function}, \; y = \frac{1}{\sigma\sqrt{(2\pi)}} \exp\left\{ -\frac{\frac{1}{2}(x-\bar{x})^2}{\sigma^2} \right\}$$

$$\text{where } e = 2.71828$$

$$\text{Let } z = \frac{x-\bar{x}}{\sigma}$$

$$\text{Then the probability function is } y = \frac{1}{\sqrt{2\pi}} \exp\left\{ -\frac{1}{2} z^2 \right\}$$

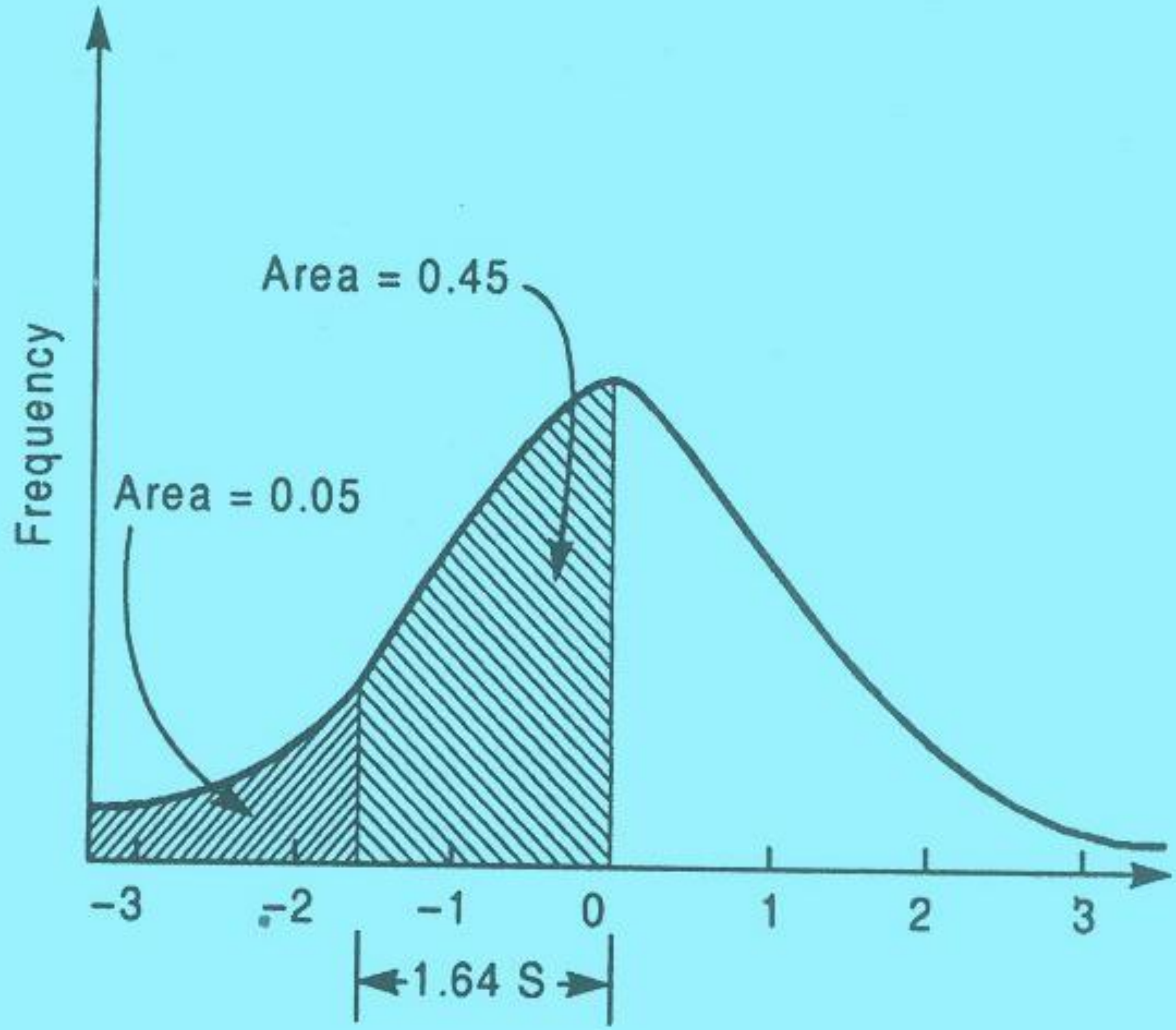


Fig. 2.1 Areas under the normal probability curve.

Normal Probability Curve

- *Strength of materials upon which design is based on that strength is assumed to be normal.*
- *Characteristic value is defined as that value below which it is unlikely that more than 5% of the results will fall.*

$$f_{ck} = f_m - 1.64\sigma$$

f_{ck} = *Characteristic Strength*

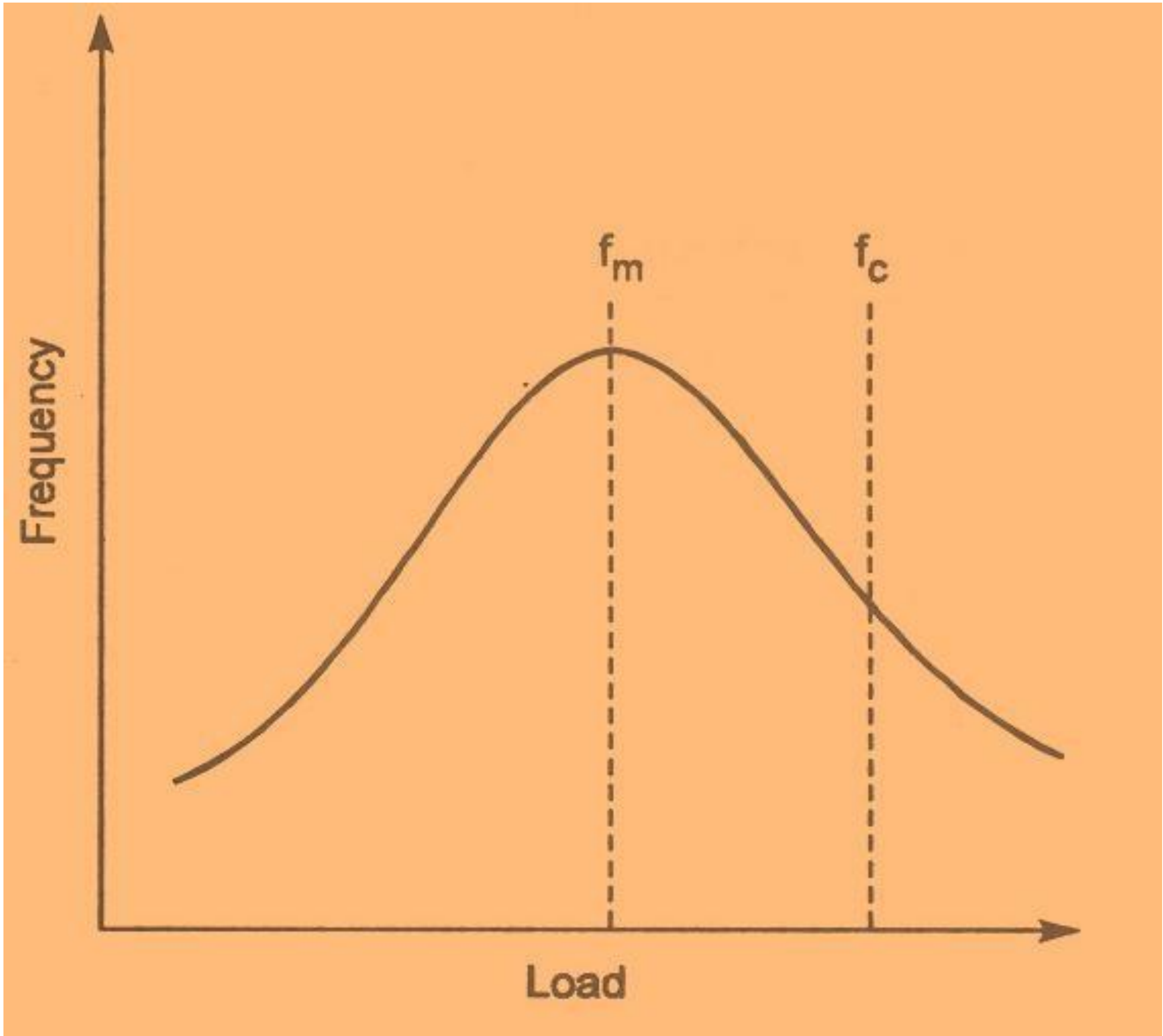
f_m = *Standard Deviation*

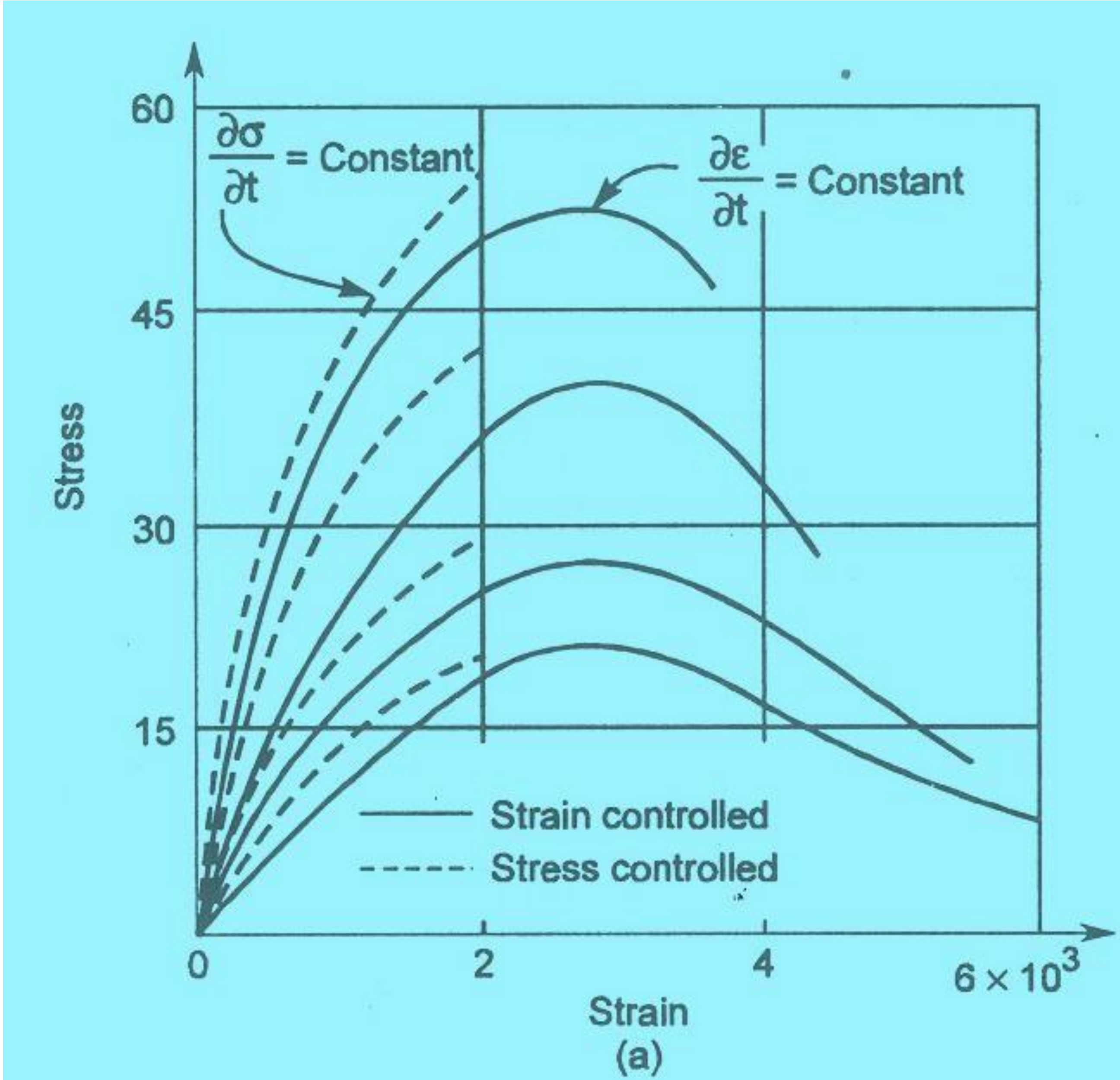
- *The relationship between* f_{ck} *and* f_m *accounts for variations in results of test specimens and with the method, and control of manufacture, quality of construction and type of materials*

Characteristic Loads

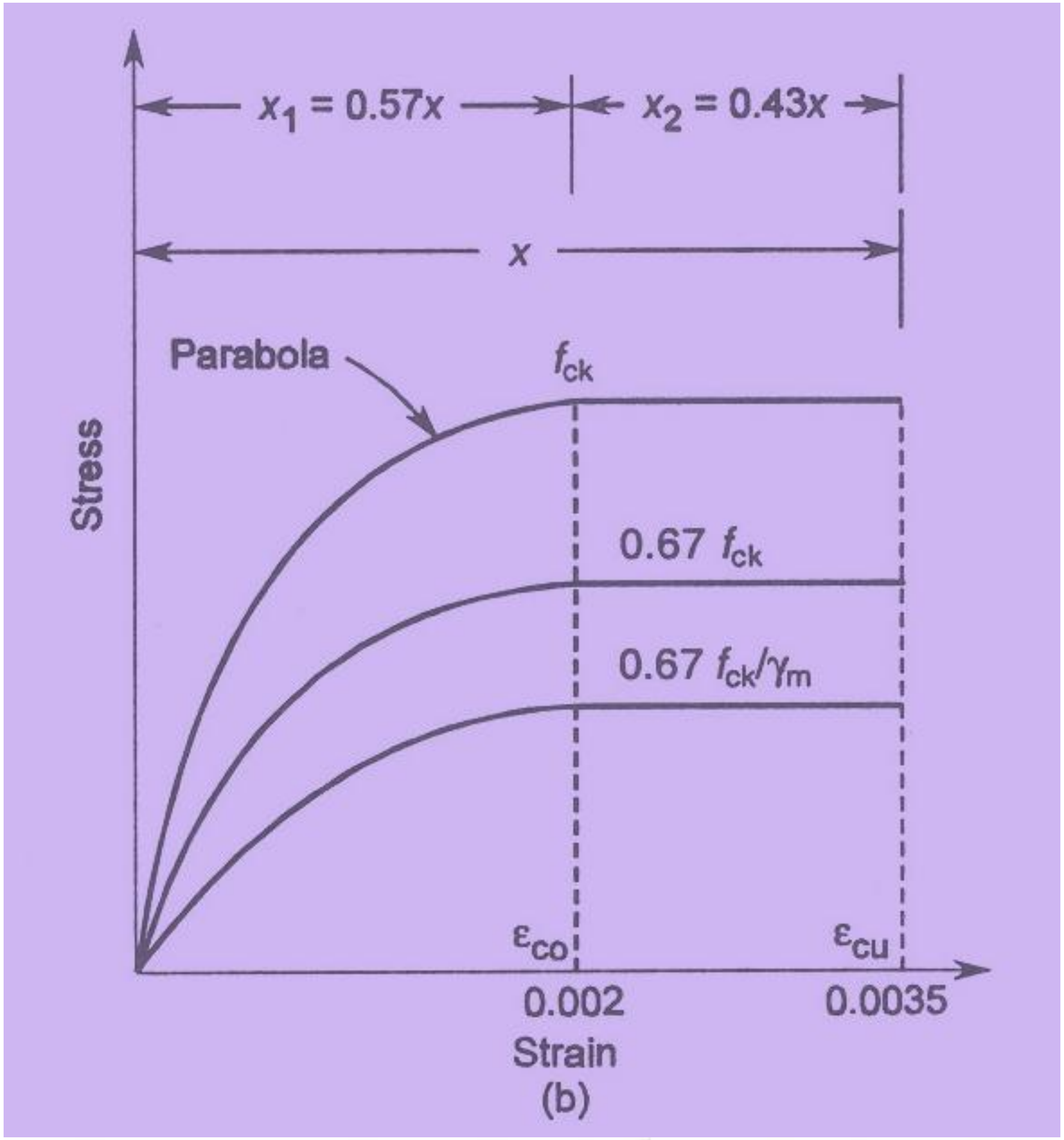
- ***Loads on structures can also be assessed stastically.***
- ***Characteristic Load = Mean Load ± 1.64 (standard deviation).***
- ***In most cases, it is the maximum loading on a structural member that is critical and the upper, positive value given by the above expression.***
- ***But the lower, minimum value may apply when considering the stability of the behaviour of continuous members.***

Characteristic Load





Design stress-strain curves for concrete in compression



Design stress-strain curves for concrete in compression

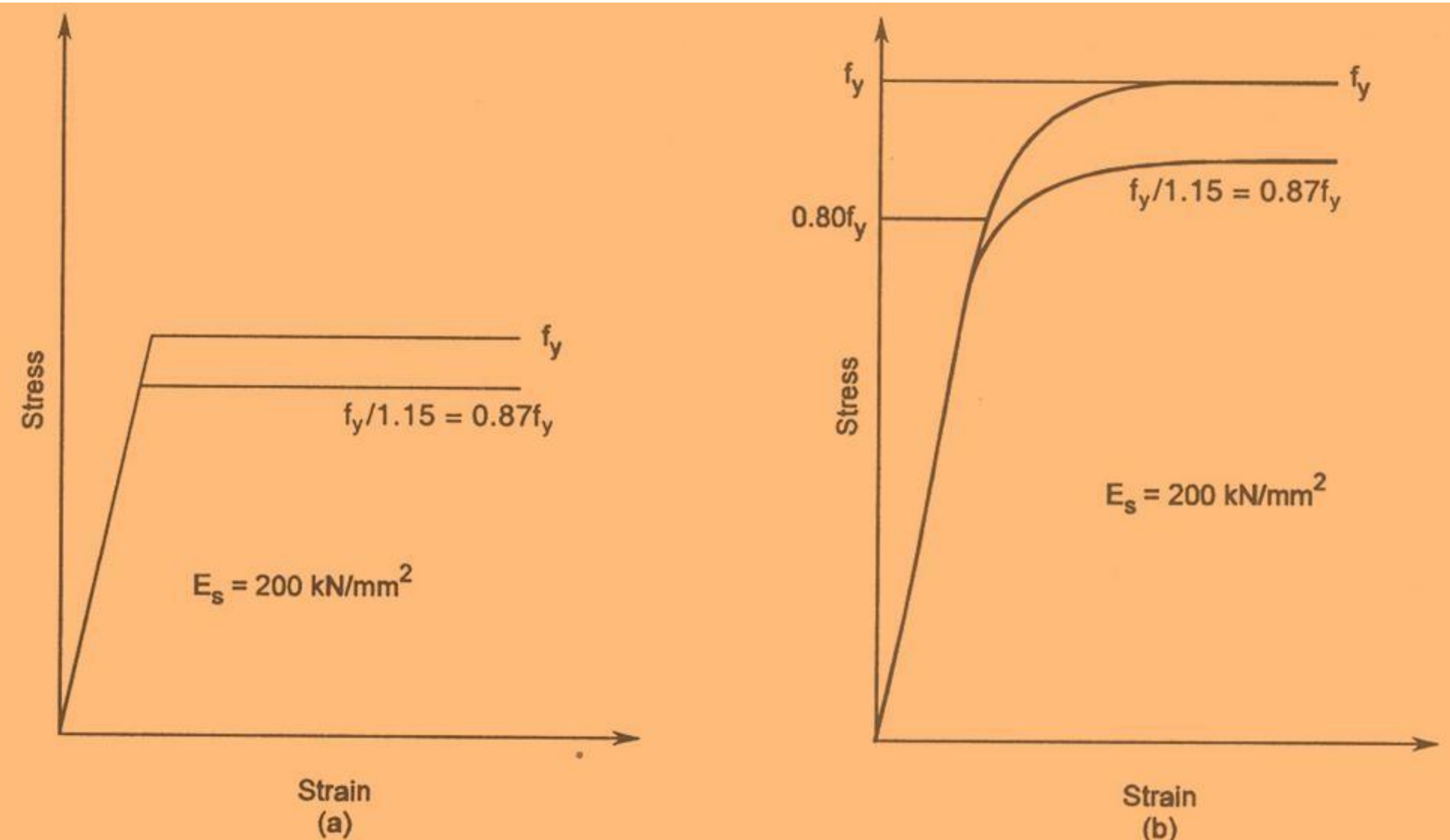


Fig. 2.4 Stress-strain curves for steel reinforcements: (a) Mild steel; and (b) Cold worked bars. (Fig. 23 of IS 456–2000)

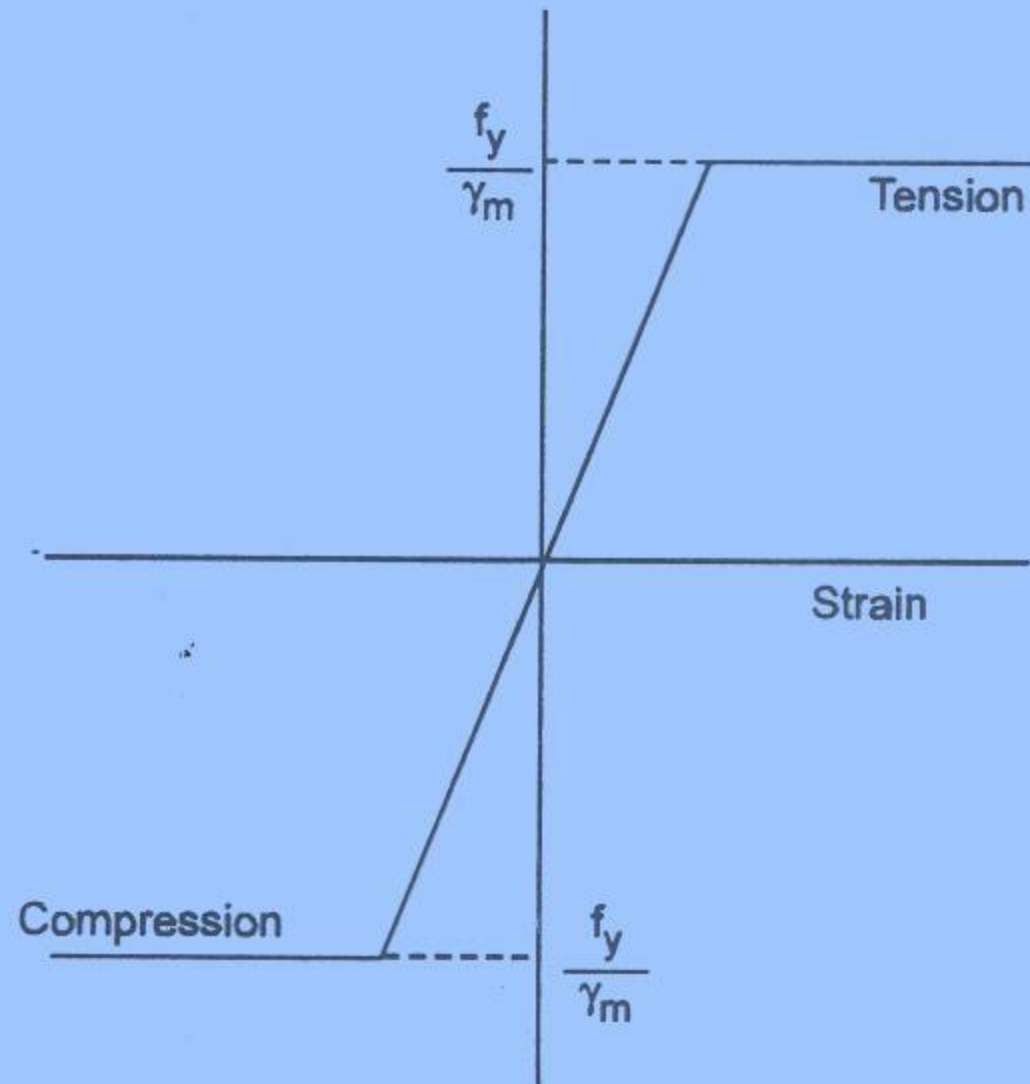


Fig. 2.5 Stress-strain curve for steel (BS 8110).

Working Stress Design

- ***The sections of the members of the structure are designed assuming straight line stress-strain relationships ensuring that at service loads the stresses in the steel and concrete do not exceed the allowable working stresses.***

- ***The allowable stresses are taken as fixed proportions of the ultimate or yield strength of the materials.***

- ***The B.Ms and forces that act on statically indeterminate structures are calculated assuming linear – elastic behaviour.***

Working Stress Design

- ***Reinforced concrete sections behave in elastically at high loads. Hence elastic theory cannot give a reliable prediction of the ultimate strength of the members because inelastic strains are not taken into account.***

- ***For structures designed by the working stress method, the exact load factor is unknown and varies from structures to structure.***

Ultimate Strength Design

- ***Sections of members of the structures are designed taking inelastic strains into account to reach ultimate (maximum) strength when an ultimate load, equal to the sum of each service load multiplied by its respective load factor, is applied to the structure.***

- ***The beginning moments and forces that act as statically indeterminate structures at the ultimate load are calculated assuming non linear elastic behaviour of the structure up to the ultimate load. i.e., redistribution of same actions are taking place due to nonlinear relationship between actions and deformations.***

Reason for Ultimate Strength Design

- *Ultimate strength design allows a more rational selection of the load factors.*

- *The stress-strain curve for concrete nonlinear and is time dependent.*

- *Ultimate strength utilizes reserves of strength resulting from a more efficient distribution of stresses allowed by inelastic strains, and at times the working stress method is very conservative.*

Reason for Ultimate Strength Design

- ***Ultimate strength design makes more efficient use of high strength reinforcement and smaller beam depths can be used without compression steel.***

- ***Ultimate strength design allows the designer to assess the ductility of the structure in the post-elastic range.***

Reasons

- *If the sections are designed based on ultimate strength design, there is a danger that although the load factor is adequate. The cracking and the deflections at the service loads may be excessive.*

- *Cracking may be excessive if the steel stresses are high or if the bars are badly distributed.*

Reasons

- ***Deflections may be critical if the shallow sections, which are possible in USD, are used and the stresses are high.***

- ***To ensure a satisfactory design, the crack widths and deflections at service loads must be checked to make sure that this lies within reasonable limiting values, as per functional requirements of the structure. This is done by use of elastic theory.***

DESIGN OF FLANGED BEAMS

Design of Flanged Beams

- *In reinforced concrete construction, slab is supported over beams.*
- *Simple concrete slabs of moderate depth and weight are limited to spans of 3m to 5m*
- *If it is desired for long spans without excessive weight and material, slab is built monolithically with RC beams and beams are considered as flanged beams.*
- *At the interior portions of floor, slab with beam acts as a T-beam and at an end the portion acts as an L-beam.*
- *Shear reinforcement of beams and bent bars extend into slab and Complete construction is cast integrally. A part of slab acts with upper part in bending compressive stresses.*

Flanged Beams

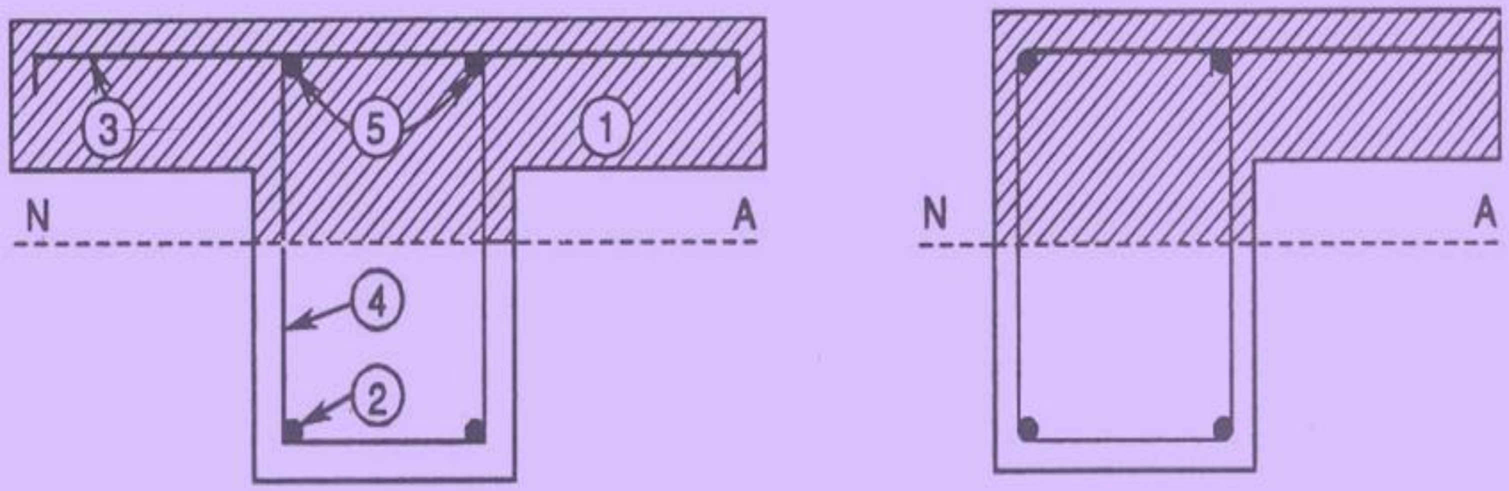


Fig. 8.1 Flanged beams: (a) T beam—1–Compression in concrete; 2–Tension steel; 3–Transverse steel; 4–Stirrups for shear; 5–Anchorage of stirrups; (b) L beam.

T-Beam

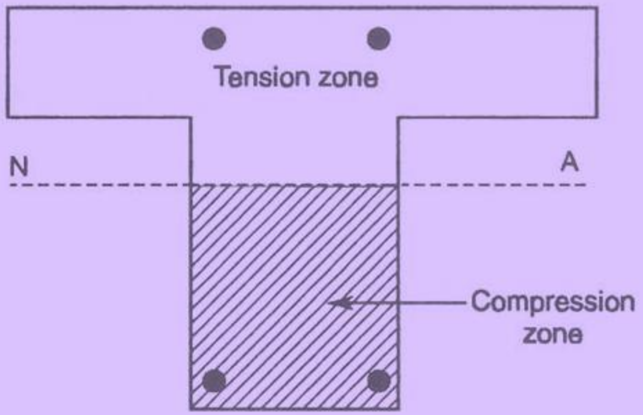


Fig. 8.2 Flanged beams over supports with negative moments.

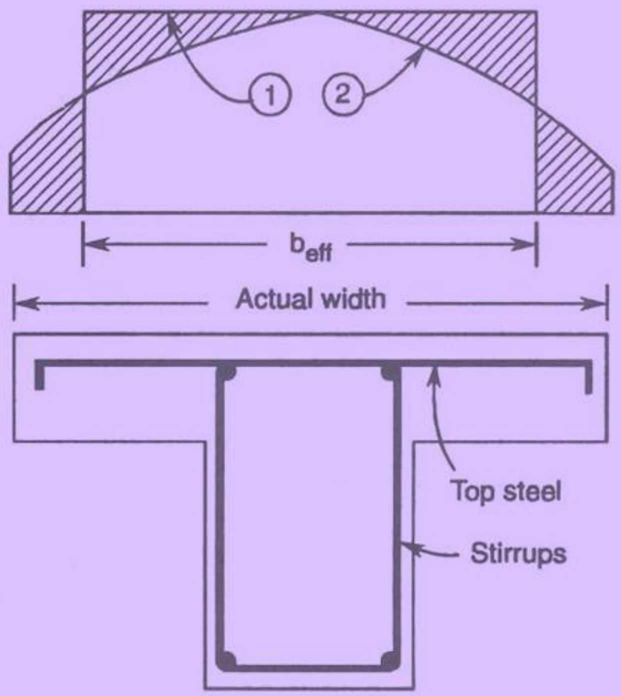


Fig. 8.3 Effective width of T beams: 1. Actual stress distribution in compression flange; 2. Assumed stress distribution in compression flange.

Effective Width of Flange

- ***Theoretically width of flange is supposed to act as top flange of beam.***

- ***Elements of flange midway between webs of two adjacent beams are less highly stressed in longitudinal compression than those elements directly over webs of beams.***

- ***An effective width of flange, b_f is used in the design of flanged beam and is treated to be uniformly stressed at the maximum value, which is smaller than actual width of flange.***

- ***Effective width of flange primarily depends on span of the beam, breadth of web, b_w and thickness of flange, D_f.***

Effective Width of Flange

- **IS: 456-2000 recommends for effective width of flanges of T- and L-beams.**
  - **For symmetrical T-beams**

    $$b_f = [(l_0\backslash 6) + b_w + 6D_f]$$

  - **For beams with slab on one side only**

    $$b_f = [(l_0\backslash 12) + b_w + 3D_f]$$

  - **For isolated T-beams**

    $$b_f = [(l_0\backslash((l_0\backslash b)+4)) + b_w]$$

  - **For Isolated L-beams**

    $$b_f = [(0.5l_0\backslash((l_0\backslash b)+4)) + b_w]$$

Effective Width of Flange

- ***Calculated effective flange width, b_f shall be not greater than the breadth of web plus half the sum of clear distances to the adjacent beams on either side***

  - $b_f < 0.5\ [l_1 + l_2] + b_w$
  - $b_f < 0.5\ [l_2 + L_3] + b_w$

Location of Neutral Axis

- *Depending upon proportions of cross-section, area of steel reinforcement in tension, strength of materials*

  1. *Neutral axis of a T-beam in one case may lie in the flange i.e. depth of NA, x_u is less than or equal to thickness of flange or depth of slab, D_f (Neutral axis lies within flange ($x_u < D_f$))*

  2. *NA may lie in web i.e. depth of neutral axis, x_u is more than thickness of slab, D_f.*

- *Stress diagram consists of a rectangular portion of depth $0.43x_u$ and a parabolic portion of depth $0.57x_u$.*

Stress Block in T-Beam

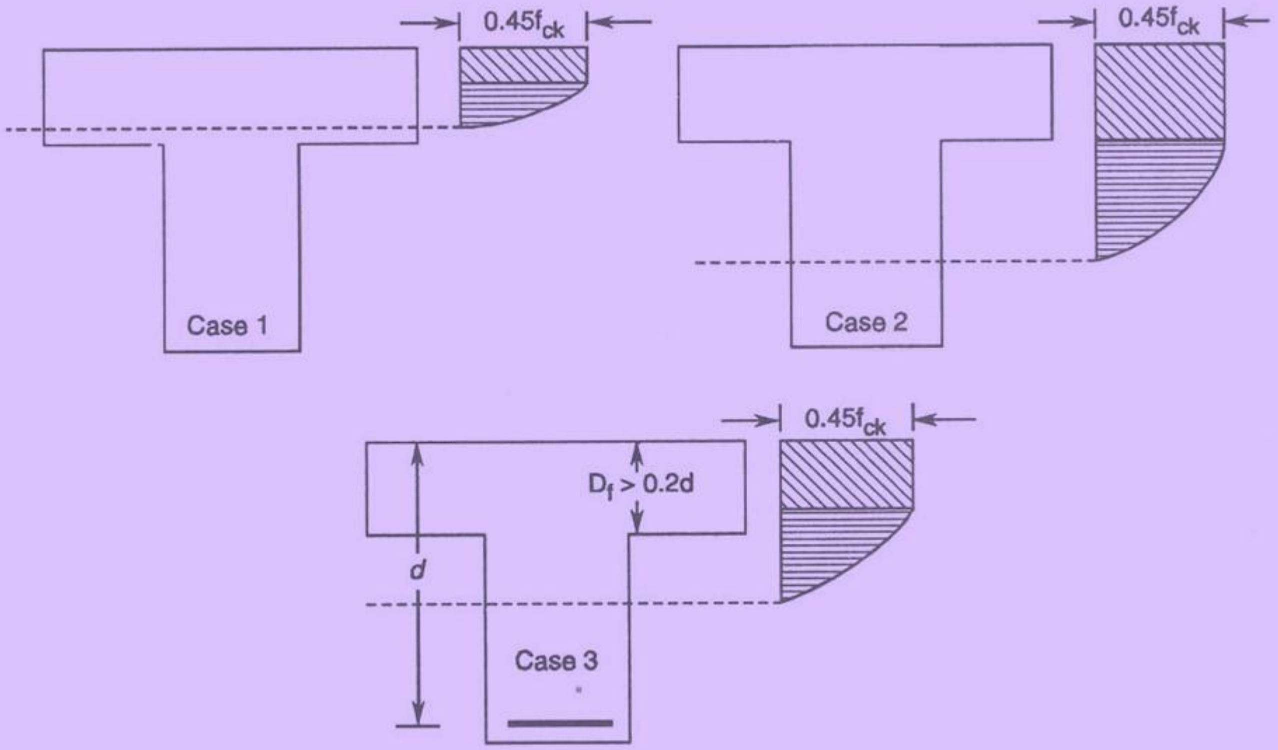


Fig. 8.4 Three possible positions of neutral axis in T beams.

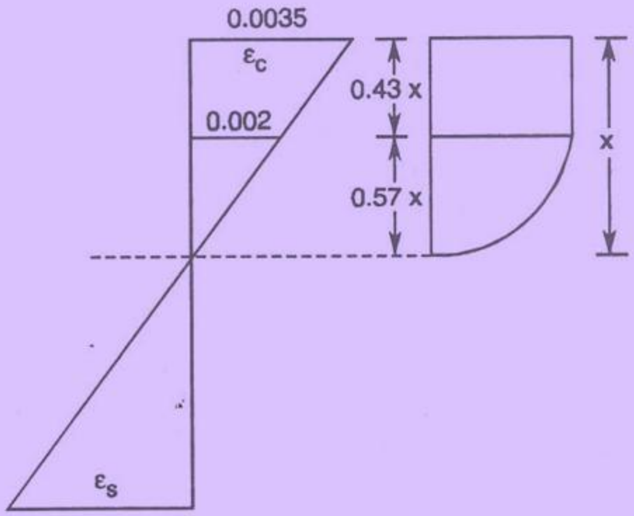


Fig. 8.5 Compression stress block in T beams.

(2) NEUTRAL AXIS LIES OUT SIDE FLANGE [i.e. $x_u > D_f$]

- *When NA of T-section lies outside flange, it lies in web of T-beam. However, there are two possibilities depending upon whether depth of flange D_f is less than or equal to $0.43x_u$ or D_f is more than $0.43\ x_u$.*
- *Comparison of D_f with $0.43x_u$ (i.e. $3/7x_u$) is more rational as $0.43x_u$ is actual depth of rectangular portion of stress block.*
- *In IS:456-2000, if (D_f/d) is less than 0.2, the flange of T-beam is considered as small.*

i. $\underline{D_f \text{ is less than } 0.43x_u}$

– *Total area in compression consists of sum of compressive force in concrete in web of width, b_w, $\underline{C_{w,\ cu}}$ and compressive force in concrete in the flange excluding web, $\underline{C_{f,\ cu}}$.*

(2) NEUTRAL AXIS LIES OUT SIDE FLANGE [i.e. $x_u > D_f$]

i. $D_f > 0.43\ x_u$ or ($D_f > 0.2d$)

i. Depth of flange D_f is more than $0.43x_u$, some portion is subjected to uniform stress equal to $0.446f_{ck}\ (0.43x_u)$ and remaining portion is subjected to parabolic stress.

i. To obtain compressive force in portion of flange, concept of modified thickness of flange equal to

$$y_f = (0.15x_u + 0.65D_f)$$

is recommended by IS456-2000.

i. Average stress is assumed to be $0.446f_{ck}$

Moment of Resistance

I. A singly reinforced slab 120mm thick is supported by T-beam spaced at 3.5m c/c has an effective depth, d= 550mm, width, b_w = 400mm. The beam is provided with steel reinforcement consisting of 5 bars of 20mm diameter in one layer, d' = 50mm. l_e = 3.7m. Use M20 grade concrete and Fe415 steel. Determine the depth of neutral axis and the moment of resistance of the beam, MR?

II. Calculate the moment of resistance of a T-beam for M20 and Fe415, D_f = 120 mm, b_f = 750mm, b_w = 250mm, d'= 50mm, D = 500mm

III. T-beam floor, D_f = 150mm, b_w = 250mm spacing = 3.5m c/c, l_e = 8.0m. LL = 6.5 kN/m. Design an intermediate beam using M20 and Fe415 steel.

IV. T-beam d = 750mm, b_f= 1400mm, D_f = 100mm, b_w = 300mm, A_{st} = ? M= 100kN-m. Use M20 and Fe 415 HYSD bars.

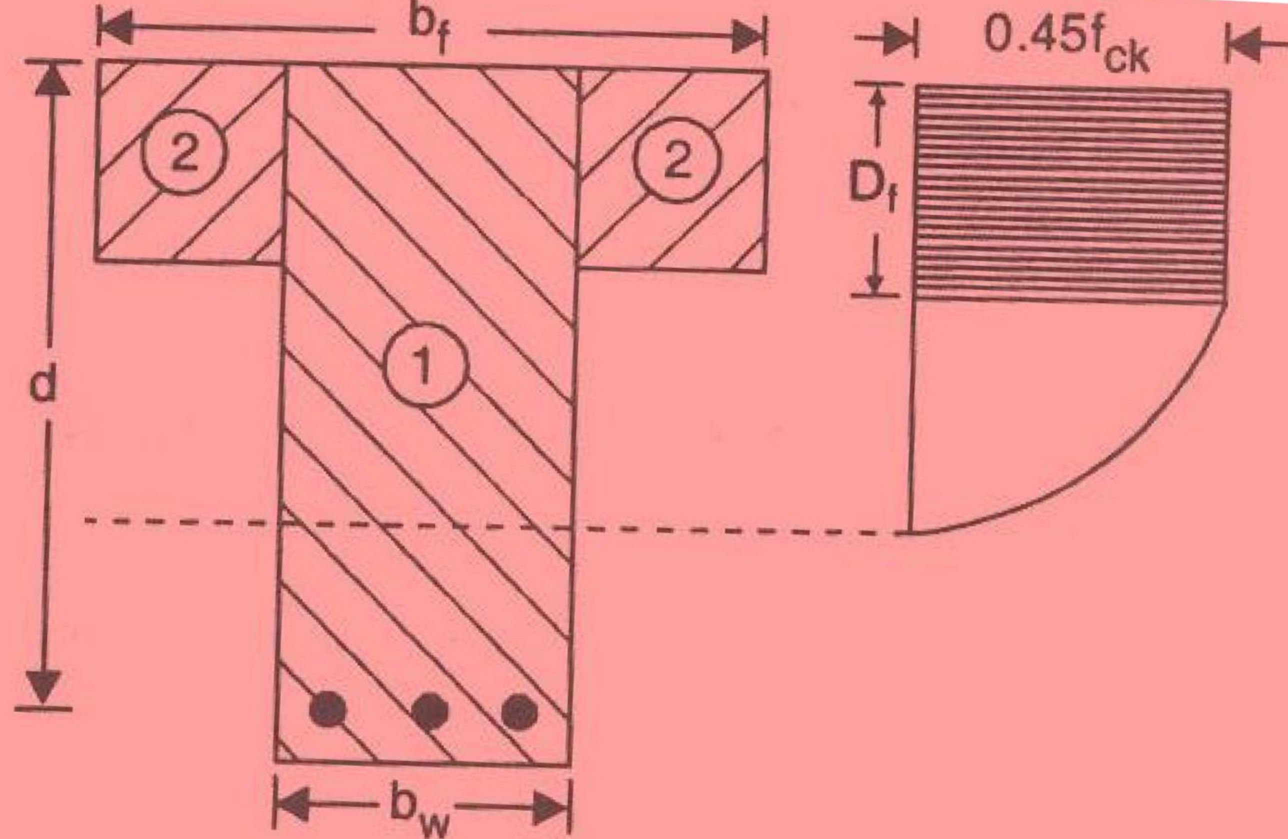


Fig. 8.6 Calculation of moment of resistance of T beams.

DESIGN OF
SINGLY REINFORCED BEAM

BEAM:-

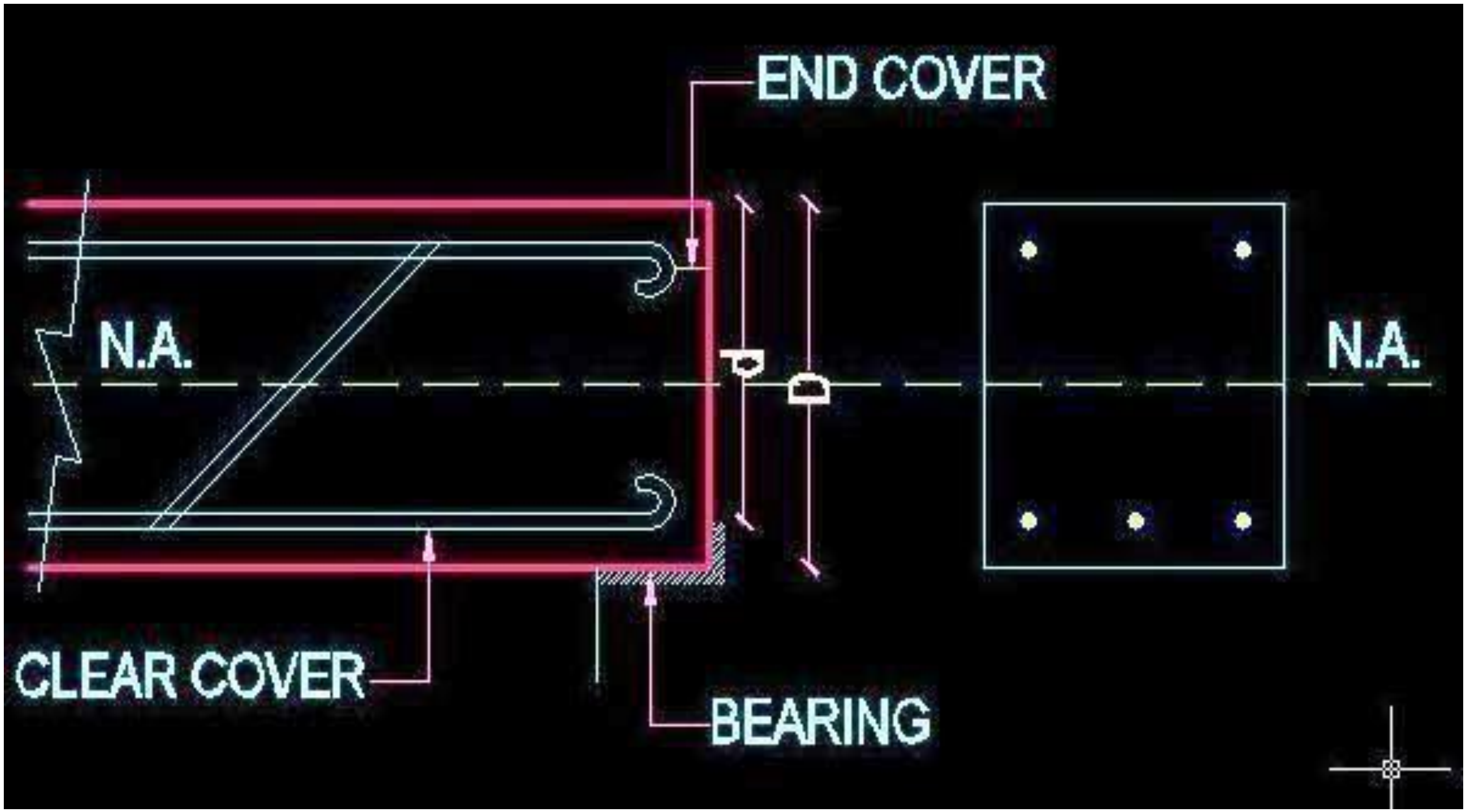
A Beam is any structural member which resists load mainly by bending. Therefore it is also called flexural member. Beam may be singly reinforced or doubly reinforced. When steel is provided only in tensile zone (i.e. below neutral axis) is called ***singly reinforced beam,*** *but when steel is provided in tension zone as well as compression zone is called doubly reinforced beam.*

The aim of design is:

- *To decide the size (dimensions) of the member and the amount of reinforcement required.*

- *To check whether the adopted section will perform safely and satisfactorily during the life time of the structure.*

FEW DEFINITIONS



OVER ALL DEPTH *:-*

THE NORMAL DISTANCE FROM THE TOP EDGE OF THE BEAM TO THE BOTTOM EDGE OF THE BEAM IS CALLED OVER ALL DEPTH. IT IS DENOTED BY ***'D'.***

EFFECTIVE DEPTH*:-*

THE NORMAL DISTANCE FROM THE TOP EDGE OF BEAM TO THE CENTRE OF TENSILE REINFORCEMENT IS CALLED EFFECTIVE DEPTH. IT IS DENOTED BY ***'d'.***

CLEAR COVER:-

THE DISTANCE BETWEEN THE BOTTOM OF THE BARS AND BOTTOM MOST THE EDGE OF THE BEAM IS CALLED CLEAR COVER.

CLEAR COVER = 25mm OR DIA OF MAIN BAR, (WHICH EVER IS GREATER).

EFFECTIVE COVER:-

THE DISTANCE BETWEEN CENTRE OF TENSILE REINFORCEMENT AND THE BOTTOM EDGE OF THE BEAM IS CALLED EFFECTIVE COVER. EFFECTIVE COVER = CLEAR COVER + ½ DIA OF BAR.

END COVER:-

END COVER = 2XDIA OF BAR OR 25mm (WHICH EVER IS GREATER)

NEUTRAL AXIS:- *THE LAYER / LAMINA WHERE NO STRESS EXIST IS KNOWN AS NEUTRAL AXIS. IT DIVIDES THE BEAM SECTION INTO TWO ZONES, COMPRESION ZONE ABOVE THE NETURAL AXIS & TENSION ZONE BELOW THE NEUTRAL AXIS.*

DEPTH OF NETURAL AXIS:- *THE NORMAL DISTANCE BETWEEN THE TOP EDGE OF THE BEAM & NEUTRAL AXIS IS CALLED DEPTH OF NETURAL AXIS. IT IS DENOTED BY* ***'n'***.

LEVER ARM:- *THE DISTANCE BETWEEN THE RESULTANT COMPRESSIVE FORCE (C) AND TENSILE FORCE (T) IS KNOWN AS LEVER ARM. IT IS DENOTED BY* ***'z'.*** *THE TOTAL COMPRESSIVE FORCE (C) IN CONCRETE ACT AT THE C.G. OF COMPRESSIVE STRESS DIAGRAM i.e. n/3 FROM THE COMPRESSION EDGE. THE TOTAL TENSILE FORCE (T) ACTS AT C.G. OF THE REINFORCEMENT.*

LEVER ARM = d-n/3

TENSILE REINFORCEMENT:-

THE REINFORCEMENT PROVIDED TENSILE ZONE IS CALLED TENSILE REINFORCEMENT. IT IS DENOTED BY A_{st}.

COMPRESSION REINFORCEMENT :-

THE REINFORCEMENT PROVIDED COMPRESSION ZONEIS CALLED COMPRESSION REINFORCEMENT. IT IS DENOTED BY A_{sc}

TYPES OF BEAM SECTION:- *THE BEAM SECTION CAN BE OF THE FOLLOWING TYPES:*

1. *BALANCED SECTION*
2. *UNBALNCED SECTION*

(a) UNDER- REINFORCED SECTION

(b) OVER-REINFORCED SECTION

1.BALANCED SECTION:- *A SECTION IS KNOWN AS BALANCED SECTION IN WHICH THE COMPRESSIVE STREE IN CONCRETE (IN COMPRESSIVE ZONES) AND TENSILE STRESS IN STEEL WILL BOTH REACH THE MAXIMUM PERMISSIBLE VALUES SIMULTANEOUSLY.*

*THE NEUTRAL AXIS OF BALANCED (OR CRITICAL) SECTION IS KNOWN AS CRITICAL NEUTRAL AXIS **(n_c)**. THE AREA OF STEEL PROVIDED AS ECONOMICAL AREA OF STEEL. REINFORCED CONCRETE SECTIONS ARE DESIGNED AS BALANCED SECTIONS.*

***2. UNBALNCED SECTION**:-THIS IS A SECTION IN WHICH THE QUANTITY OF STEEL PROVIDED IS DIFFERENT FROM WHAT IS REQUIRED FOR THE BALANCED SECTION.*

UNBALANCED SECTIONS MAY BE OF THE FOLLOWING TWO TYPES:

(a) UNDER-REINFORCED SECTION

(b) OVER-REINFORCED SECTION

(a)UNDER-REINFORCED SECTION:- *IF THE AREA OF STEEL PROVIDED IS LESS THAN THAT REQUIRED FOR BALANCED SECTION, IT IS KNOWN AS UNDER-REINFORCED SECTION. DUE TO LESS REINFORCEMENT THE POSITION OF ACTUAL NEUTRAL AXIS* ***(n)*** *WILL SHIFT ABOVE THE CRITICAL NEUTRAL AXIS* ***(n_c)****i.e.* ***$n < n_c$.*** *IN UNDER-REINFORCED SECTION STEEL IS FULLY STRESSED AND CONCRETE IS UNDER STRESSED (i.e. SOME CONCRETE REMAINS UN-UTILISED). STEEL BEING DUCTILE, TAKES SOME TIME TO BREAK. THIS GIVES SUFFICIENT WARNING BEFORE THE FINAL COLLAPSE OF THE STRUCTURE. FOR THIS REASON AND FROM ECONOMY POINT OF VIEW THE UNDER-REINFORCED SECTIONS ARE DESIGNED.*

(b) OVER-REINFORCED SECTION:- *IF THE AREA OF STEEL PROVIDED IS MORE THAN THAT REQUIRED FOR A BALANCED SECTION, IT IS KNOWN AS OVER-REINFORCED SECTION. AS THE AREA OF STEEL PROVIDED IS MORE, THE POSITION OF N.A. WILL SHIFT TOWARDS STEEL, THEREFORE ACTUAL AXIS* ***(n)*** *IS BELOW THE CRITICAL NEUTRAL AXIS* ***(n_c)****i.e.* $\mathbf{n > n_c}$. *IN THIS SECTION CONCRETE IS FULLY STRESSED AND STEEL IS UNDER STRESSED. UNDER SUCH CONDITIONS, THE BEAM WILL FAIL INITIALLY DUE TO OVER STRESS IN THE CONCRETE. CONCRETE BEING BRITTLE, THIS HAPPENS SUDDENLY AND EXPLOSIVELY WITHOUT ANY WARNING.*

Basic rules for design of beam:-

1.Effective span:- *In the case of simply supported beam the effective length,*

l = ***i****. Distance between the centre of support*

ii*. Clear span + eff. Depth*

eff. Span = least of ***i. & ii.***

2.Effective depth:- *The normal distance from the top edge of beam to the centre of tensile reinforcement is called effective depth. It is denoted by* ***'d'.***

d= D- effect. Cover

where D= over all depth

3. Bearing :- *Bearings of beams on brick walls may be taken as follow:*

Up to 3.5 m span, bearing = 200mm

Up to 5.5 m span, bearing =300mm

Up to 7.0 m span, bearing =400mm

4. Deflection control:- *The vertical deflection limits assumed to be satisfied if* ***(a)*** *For span up to 10m*

Span / eff. Depth = 20

(For simply supported beam)

Span / eff. Depth = 7

(For cantilever beam)

(b)For span above 10m, the value in (a) should be multiplied by 10/span (m), except for cantilever for which the deflection calculations should be made.

(c)Depending upon the area and type of steel the value of (a&b) modified as per modification factor.

5. Reinforcement *:-*

(a) Minimum reinforcement:- The minimum area of tensile reinforcement shall not be less than that given by the following:

$$A_{st} = 0.85\,bd / f_y$$

*(b)Maximum reinforcement:- The maximum area of tensile reinforcement shall not be more than **0.4bD***

(c)Spacing of reinforcement bars:-

i. The horizontal distance between to parallel main bars shall not be less than the greatest of the following:

- *Diameter of the bar if the bars are of same diameter.*
- *Diameter of the larger bar if the diameter are unequal.*
- *5mm more than the nominal maximum size of coarse aggregate.*

ii. When the bars are in vertical lines and the minimum vertical distance between the bars shall be greater of the following:

- *15mm.*
- *2/3rd of nominal maximum size of aggregate.*
- *Maximum diameter of the bar.*

6. Nominal cover to reinforcement :- *The Nominal cover is provided in R.C.C. design:*

- *To protect the reinforcement against corrosion.*
- *To provide cover against fire.*
- *To develop the sufficient bond strength along the surface area of the steel bar.*

As per IS 456-2000, the value of nominal cover to meet durability requirements as follow:-

Exposure conditions	***Nominal cover(mm) Not less than***
Mild	*20*
Moderate	*30*
Severe	*45*
Very severe	*50*
Extreme	*75*

Procedure for Design of Singly Reinforced Beam by Working Stress Method

Given :

(i) Span of the beam (ℓ)

(ii) Loads on the beam

(iii)Materials-Grade of Concrete and type of steel.

1. Calculate design constants for the given materials (k, j and R)

$$k = m\ \sigma_{cbc} / \ m\ \sigma_{cbc} + \sigma_{st}$$

where k is coefficient of depth of Neutral Axis

$$j = 1 - k/3$$

where j is coefficient of lever arm.

$$R = 1/2\ \sigma_{cbc}\ kj$$

where R is the resisting moment factor.

2. *Assume dimension of beam:*

d = Span/10 to Span/8

Effective cover = 40mm to 50mm

b = D/2 to 2/3D

3. *Calculate the effective span (l) of the beam.*

4. *Calculate the self weight (dead load) of the beam.*

Self weight = D x b x 25000 N/m

5. Calculate the total Load & maximum bending moment for the beam.

Total load (w) = live load + dead load

Maximum bending moment, $M = wl^2 / 8$ at the centre of beam for simply supported beam.

$M = wl^2 / 2$ at the support of beam for cantilever beam.

6. Find the minimum effective depth

$$M = M_r$$

$$= Rbd^2$$

$$d_{reqd.} = \sqrt{M / R.b}$$

7. Compare $d_{reqd.}$ With assumed depth value.

(i)If it is less than the assumed d, then assumption is correct.

(ii) If $d_{reqd.}$ is more than assumed d, then revise the depth value and repeat steps 4, 5 & 6.

8. Calculate the area of steel required (A_{st}).

$$A_{st} = M / \sigma_{st}\ jd$$

Selecting the suitable diameter of bar calculate the number of bars required

$$\text{Area of one bar} = \pi/4 \times \phi^2 = A\phi$$

$$\text{No. of bars required} = A_{st} / A\phi$$

9. *Calculate minimum area of steel (A_S) required by the relation:*

$$A_S = 0.85\, bd / f_y$$

Calculate maximum area of steel by the area relation:

Maximum area of steel = 0.04bD

Check that the actual A_{St} provided is more than minimum and less than maximum requirements.

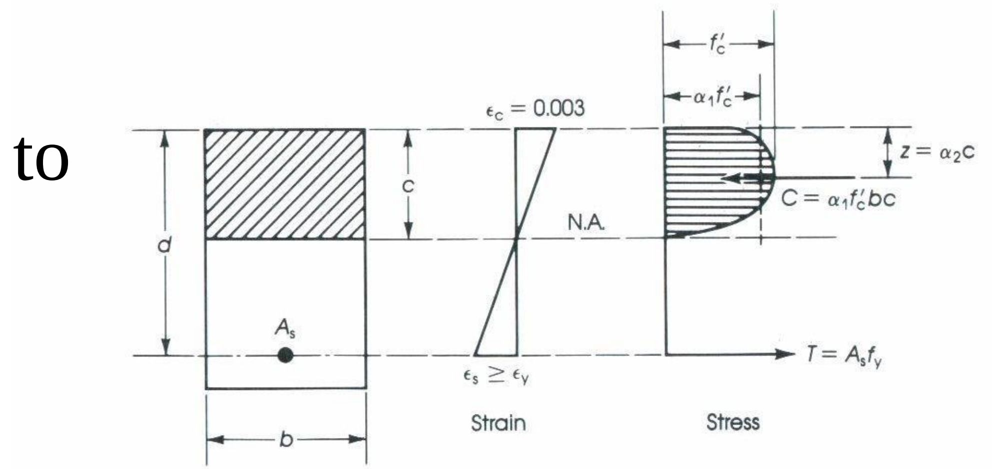
10. *Check for shear and design shear reinforcement.*
11. *Check for development length.*
12. *Check for depth of beam from deflection.*
13. *Write summary of design and draw a neat sketch.*

Lecture Goals

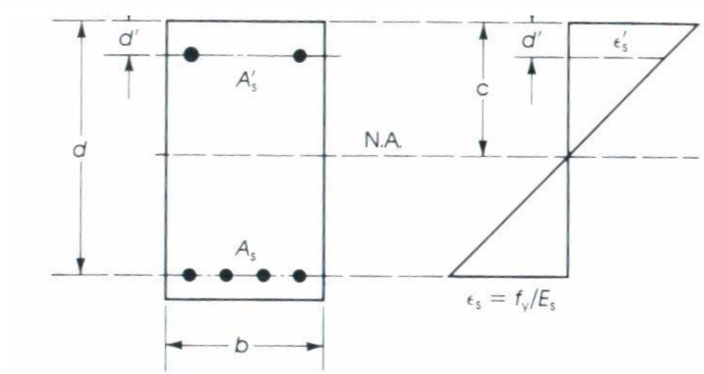
- Doubly Reinforced beams
- T Beams and L Beams

Analysis of Doubly Reinforced Sections

Effect of Compression Reinforcement on the Strength and Behavior



Less concrete is needed to resist the T and thereby moving the neutral axis (NA) up.



$$T = A_s f_y$$

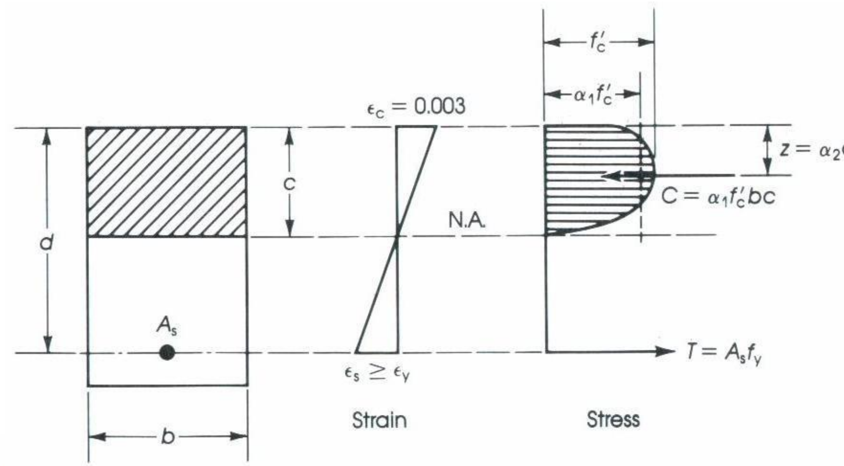
$$C = T$$

Analysis of Doubly Reinforced Sections

Effect of Compression Reinforcement on the Strength and Behavior

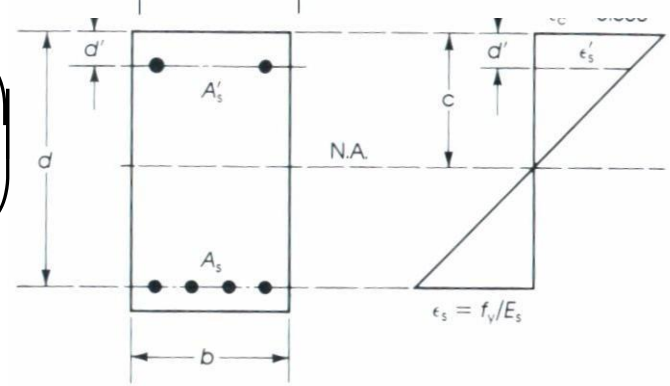
Singly Reinforced $\Rightarrow$

$$C = C_c \;\; ; M_n = A_s f_y \left(d - \frac{a_1}{2} \right)$$



Doubly Reinforced $\Rightarrow$

$$C = C_c + C'_s \;\; ; M_n = A_s f_y \left(d - \frac{a_2}{2} \right)$$



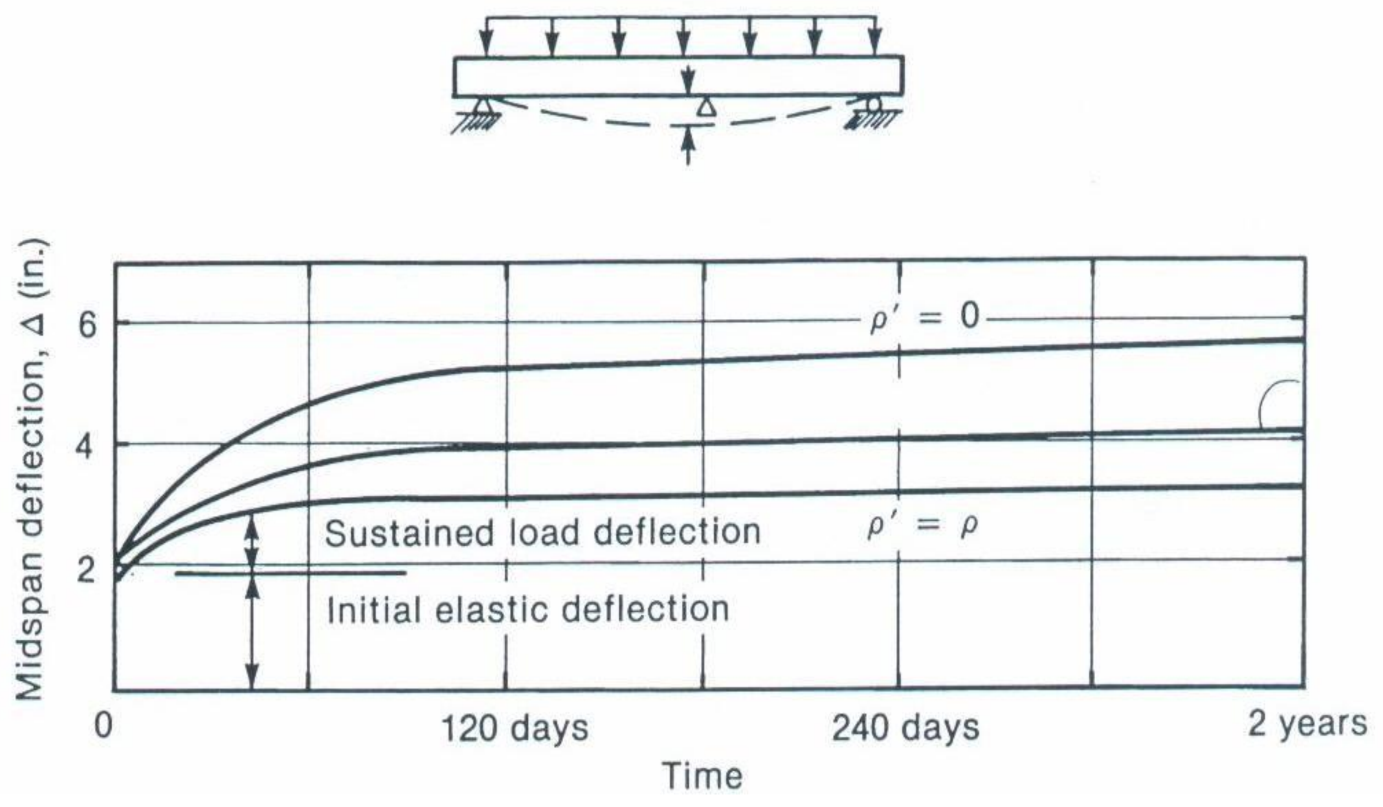
and $(a_2 < a_1)$

Reasons for Providing Compression Reinforcement

- Reduced sustained load deflections.
  - Creep of concrete in compression zone
  - transfer load to compression steel
  - reduced stress in concrete
  - less creep
  - less sustained load deflection

Reasons for Providing Compression Reinforcement

Effective of compression reinforcement on sustained load deflections.



Reasons for Providing Compression Reinforcement

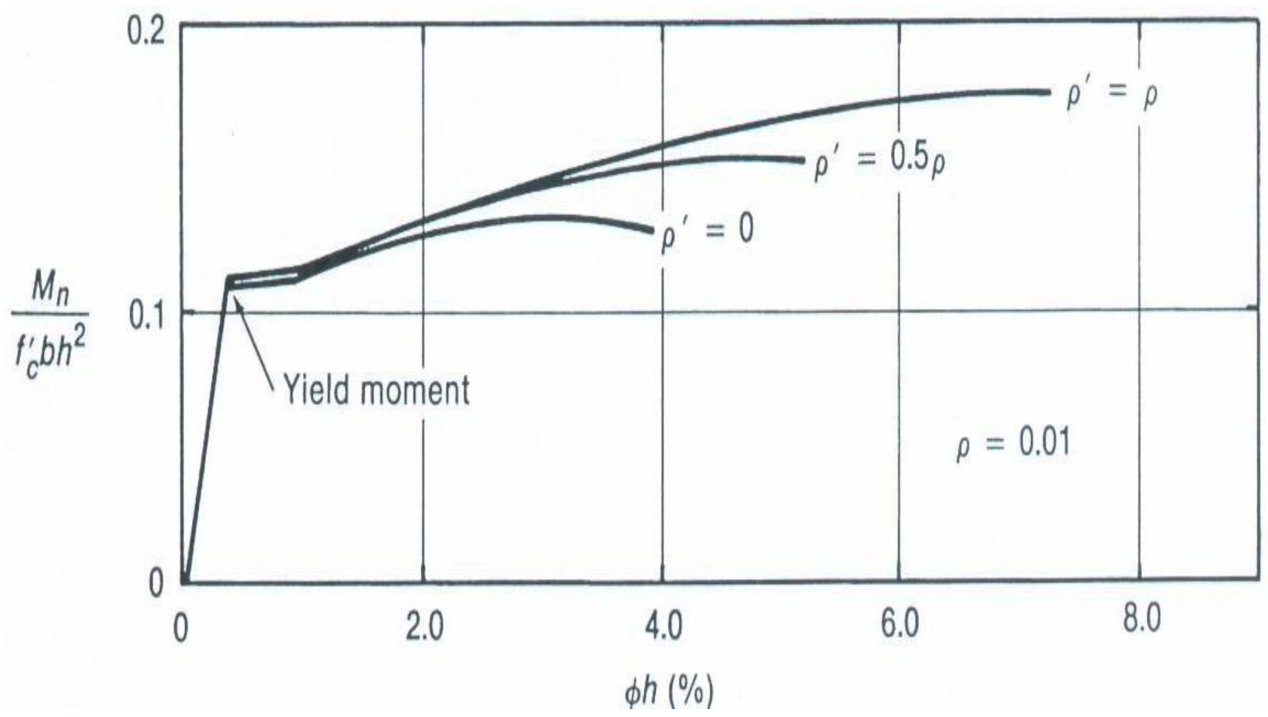
- Increased Ductility

reduced stress block depth → increase in steel strain larger curvature are obtained.

Reasons for Providing Compression Reinforcement

Effect of compression reinforcement on strength and ductility of under reinforced beams.

$\rho < \rho_b$



Reasons for Providing Compression Reinforcement

- Change failure mode from compression to tension. When $\rho > \rho_{bal}$ addition of A_s strengthens.

Compression zone → allows tension steel to yield before crushing of concrete.

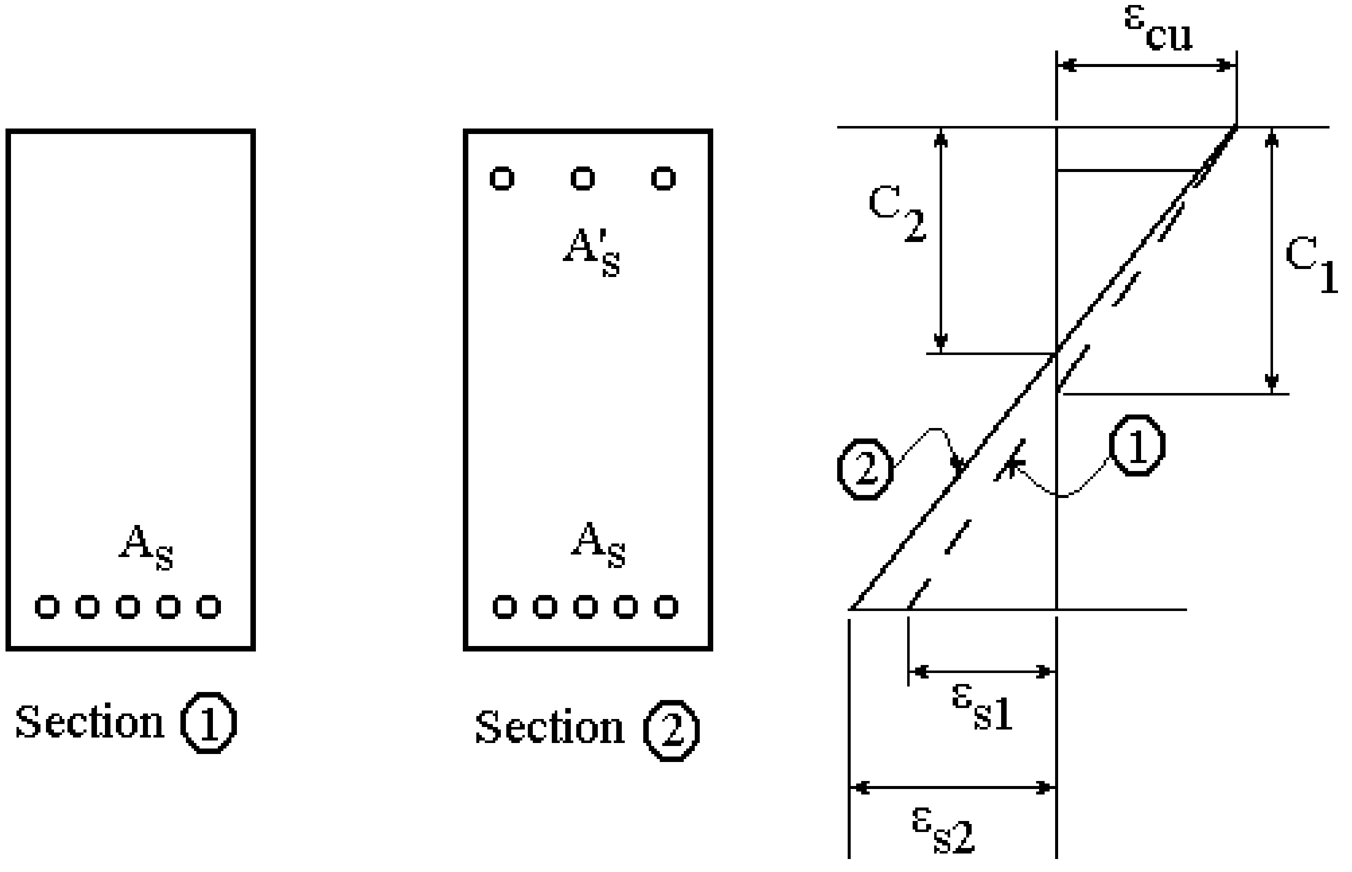
Effective reinforcement ratio $= (\rho - \rho')$

Reasons for Providing Compression Reinforcement

- Eases in Fabrication - Use corner bars to hold & anchor stirrups.

Effect of Compression Reinforcement

Compare the strain distribution in two beams with the same A_s



Effect of Compression Reinforcement

Section 1:

$$T = A_s f_s$$

$$T = C_{c1} = 0.85 f_c' ba = 0.85 f_c' b\beta_1 c_1$$

$$c_1 = \frac{A_s f_s}{0.85 f_c' b\beta_1}$$

Section 2:

$$T = A_s f_s$$

$$T = C_s' + C_{c1}$$

$$= A_s' f_s' + 0.85 f_c' ba_2$$

$$= A_s' f_s' + 0.85 f_c' b\beta_1 c_2$$

$$c_2 = \frac{A_s f_s - A_s' f_s'}{0.85 f_c' b\beta_1}$$

Addition of A'_s strengthens compression zone so that less concrete is needed to resist a given value of T. $\longrightarrow$ NA goes up ($c_2 < c_1$) and ε_s increases ($\varepsilon_{s2} > \varepsilon_{s1}$).

Doubly Reinforced Beams

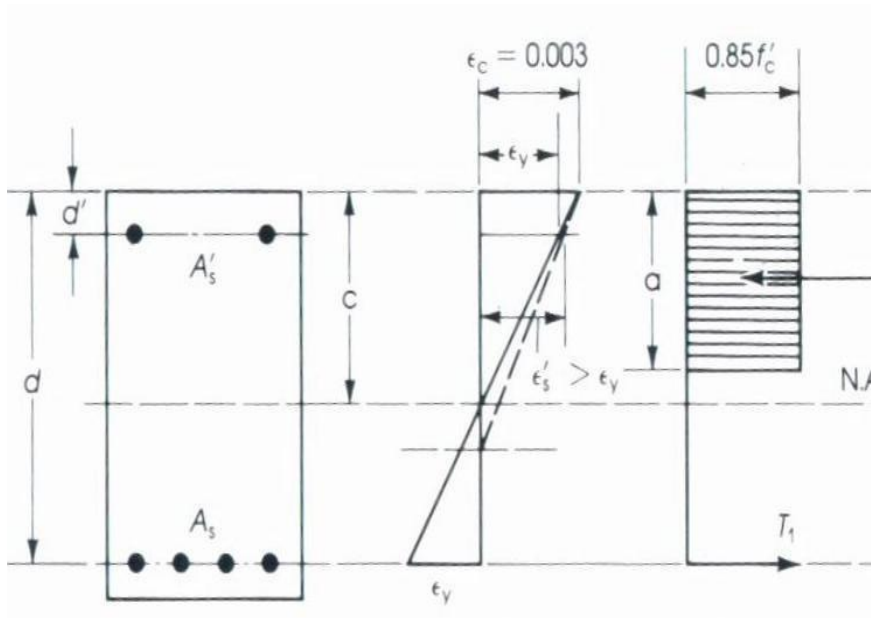
Four Possible Modes of Failure

- Under reinforced Failure
  - (Case 1) Compression and tension steel yields
  - (Case 2) Only tension steel yields
- Over reinforced Failure
  - (Case 3) Only compression steel yields
  - (Case 4) No yielding Concrete crushes

Analysis of Doubly Reinforced Rectangular Sections

<u>Strain Compatibility Check</u>

Assume ε_s' using similar triangles



$$\frac{\varepsilon'_s}{(c-d')} = \frac{0.003}{c} \Rightarrow$$

$$\varepsilon'_s = \frac{(c-d')}{c} * 0.003$$

Analysis of Doubly Reinforced Rectangular Sections

Strain Compatibility

Using equilibrium and find a

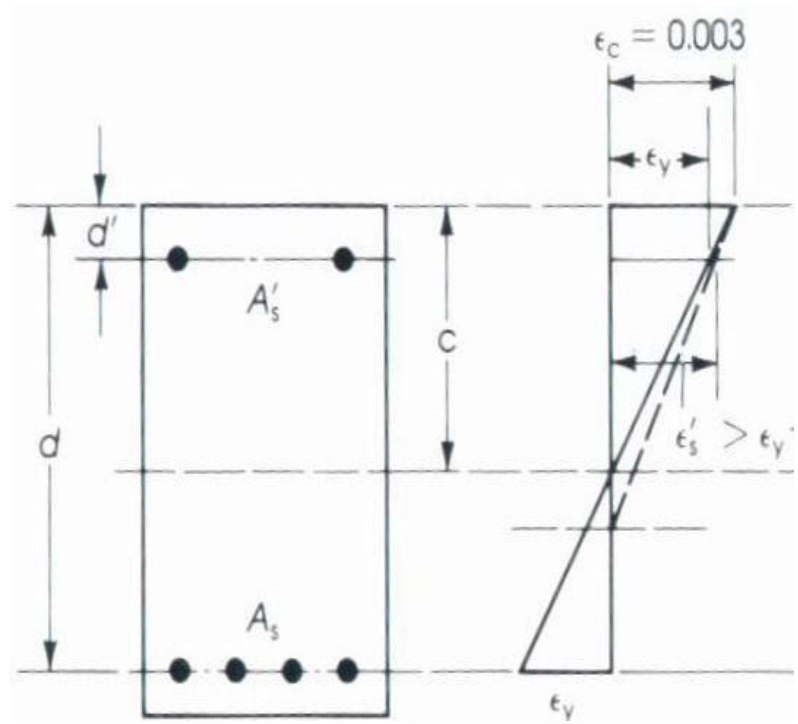
$$T = C_c' + C_s' \Rightarrow a = \frac{\left(A_s - A_s'\right) f_y}{0.85 f_c' b}$$

$$c = \frac{a}{\beta_1} = \frac{\left(A_s - A_s'\right) f_y}{\beta_1 \left(0.85 f_c' b\right)} = \frac{\left(\rho - \rho'\right) d \ f_y}{\beta_1 \left(0.85 f_c'\right)}$$

Analysis of Doubly Reinforced Rectangular Sections

<u>Strain Compatibility</u>

The strain in the compression steel is



$$\varepsilon'_{\mathrm{s}} = \left(1 - \frac{d'}{c}\right)\varepsilon_{\mathrm{cu}}$$

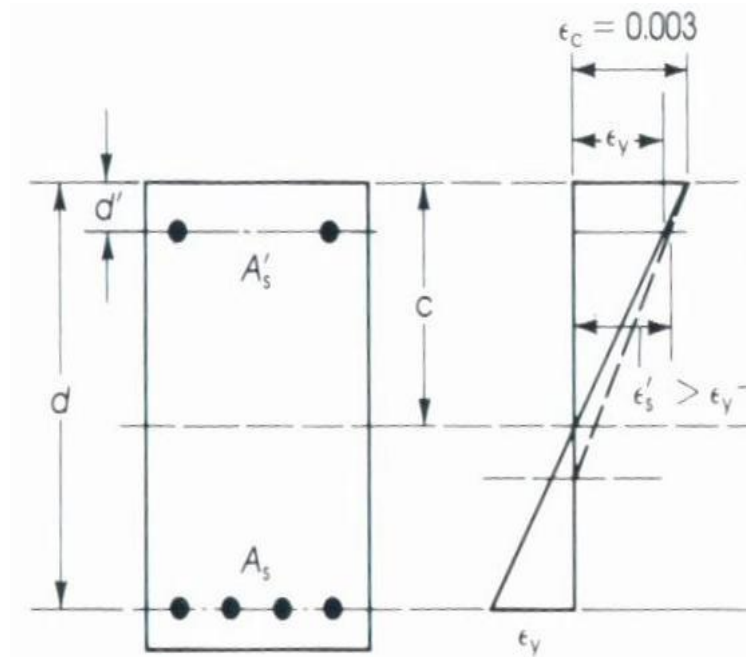
$$= \left(1 - \frac{\beta_1 \left(0.85 f'_{\mathrm{c}}\right) d'}{(\rho - \rho')d \; f_{\mathrm{y}}}\right) 0.003$$

Analysis of Doubly Reinforced Rectangular Sections

<u>Strain Compatibility</u>

Confirm

$$\varepsilon_s' \geq \varepsilon_y = \frac{f_y}{E_s}; \qquad \varepsilon_s \geq \varepsilon_y$$



$$\varepsilon_s' = \left(1 - \frac{\beta_1 (0.85 f_c') d'}{(\rho - \rho') d\ f_y}\right) 0.003 \geq \frac{f_y}{E_s} = \frac{f_y}{29 \text{ x } 10^3 \text{ ksi}}$$

Analysis of Doubly Reinforced Rectangular Sections

Strain Compatibility

Confirm

$$-\frac{\beta_1\left(0.85 f'_c\right)d'}{\left(\rho-\rho'\right)d\ f_y} \geq \frac{f_y-87}{87}$$

$$\left(\rho-\rho'\right) \geq \left(\frac{\beta_1\left(0.85 f'_c\right)d'}{d\ f_y}\right)\left(\frac{87}{87-f_y}\right)$$

Analysis of Doubly Reinforced Rectangular Sections

Find c

$$A_s' f_y + 0.85\, f_c'\, ba = A_s f_y$$

$$c = \frac{\left(A_{ss} - A_s'\right) f_y}{0.85\, f_c'\, b \beta_1} \qquad \Rightarrow a = \beta_1 c$$

confirm that the tension steel has yielded

$$\varepsilon_s = \left(\frac{d-c}{c}\right)\varepsilon_{cu} \geq \varepsilon_y = \frac{f_y}{E_s}$$

Analysis of Doubly Reinforced Rectangular Sections

If the statement is true than

$$M_{\mathrm{n}} = \left(A_{\mathrm{s}} - A_{\mathrm{s}}'\right) f_{\mathrm{y}} \left(d - \frac{a}{2} \right) + A_{\mathrm{s}}' f_{\mathrm{y}} \left(d - d' \right)$$

else the strain in the compression steel

$$f_{\mathrm{s}} = E \varepsilon_{\mathrm{s}}'$$

Analysis of Doubly Reinforced Rectangular Sections

Return to the original equilibrium equation

$$
\begin{aligned}
A_s f_y &= A_s' f_s + 0.85 f_c b a \\
&= A_s' E_s \varepsilon_s' + 0.85 f_c b \beta_1 c \\
&= A_s' E_s \left(1 - \frac{d'}{c} \right) \varepsilon_{cu} + 0.85 f_c b \beta_1 c
\end{aligned}
$$

Analysis of Doubly Reinforced Rectangular Sections

Rearrange the equation and find a quadratic equation

$$A_s f_y = A_s' E_s \left(1 - \frac{d'}{c}\right) \varepsilon_{cu} + 0.85 f_c b \beta_1 c$$

$$\Rightarrow 0.85 f_c b \beta_1 c^2 + \left(A_s' E_s \varepsilon_{cu} - A_s f_y\right) c - A_s' E_s \varepsilon_{cu} d' = 0$$

Solve the quadratic and find c.

Analysis of Doubly Reinforced Rectangular Sections

Find the f_s'

$$f_s' = \left(1 - \frac{d'}{c}\right) E_s \varepsilon_{cu} = \left(1 - \frac{d'}{c}\right) 87 \text{ ksi}$$

Check the tension steel.

$$\varepsilon_s = \left(\frac{d - c}{c}\right) \varepsilon_{cu} \geq \varepsilon_y = \frac{f_y}{E_s}$$

Analysis of Doubly Reinforced Rectangular Sections

Another option is to compute the stress in the compression steel using an iterative method.

$$f_s' = 29 \text{ x } 10^3 \left(1 - \frac{\beta_1 \left(0.85 f_c'\right) d'}{\left(\rho - \rho'\right) d \ f_y} \right) 0.003$$

Analysis of Doubly Reinforced Rectangular Sections

Go back and calculate the equilibrium with f_s'

$$T = C_c' + C_s' \Rightarrow a = \frac{\left(A_s f_y - A_s' f_s\right)}{0.85 f_c' b}$$

$$c = \frac{a}{\beta_1}$$

$$f_s' = \left(1 - \frac{d'}{c}\right) 87 \text{ ksi}$$

Iterate until the c value is adjusted for the f_s' until the stress converges.

Analysis of Doubly Reinforced Rectangular Sections

Compute the moment capacity of the beam

$$M_{\mathrm{n}} = \left(A_{\mathrm{s}} f_{\mathrm{y}} - A_{\mathrm{s}}' f_{\mathrm{s}}'\right)\left(d - \frac{a}{2}\right) + A_{\mathrm{s}}' f_{\mathrm{s}}'\left(d - d'\right)$$

Limitations on Reinforcement Ratio for Doubly Reinforced beams

Lower limit on ρ

$$\rho_{\mathrm{min}} = \frac{3\sqrt{f_{\mathrm{c}}'}}{f_{\mathrm{y}}} \geq \frac{200}{f_{\mathrm{y}}} \qquad \text{(ACI 10.5)}$$

same as for single reinforce beams.

Example: Doubly Reinforced Section

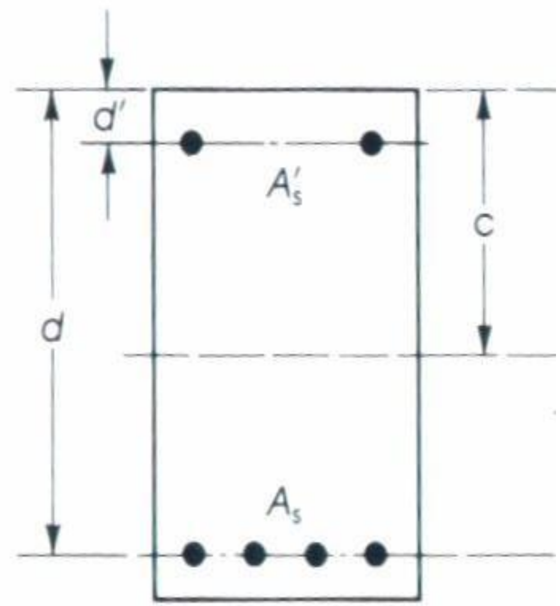
Given:

$f'_c = 4000$ psi $f_y = 60$ ksi

$A'_s = 2$ #5 $A_s = 4$ #7

$d' = 2.5$ in. $d = 15.5$ in

$h = 18$ in. $b = 12$ in.



Calculate M_n for the section for the given compression steel.

Example: Doubly Reinforced Section

Compute the reinforcement coefficients, the area of the bars #7 (0.6 in^2) and #5 (0.31 in^2)

$$A_s = 4\left(0.6\ \text{in}^2\right) = 2.4\ \text{in}^2$$

$$A_s' = 2\left(0.31\ \text{in}^2\right) = 0.62\ \text{in}^2$$

$$\rho = \frac{A_s}{bd} = \frac{2.4\ \text{in}^2}{\left(12\ \text{in.}\right)\left(15.5\ \text{in.}\right)} = 0.0129$$

$$\rho' = \frac{A_s'}{bd} = \frac{0.62\ \text{in}^2}{\left(12\ \text{in.}\right)\left(15.5\ \text{in.}\right)} = 0.0033$$

Example: Doubly Reinforced Section

Compute the effective reinforcement ratio and minimum ρ

$$\rho_{eff} = \rho - \rho' = 0.0129 - 0.0033 = 0.00957$$

$$\rho = \frac{200}{f_y} = \frac{200}{60000} = 0.00333$$

$$\text{or } \frac{3\sqrt{f_c}}{f_y} = \frac{3\sqrt{4000}}{60000} = 0.00316$$

$$\rho \geq \rho_{\min} \Rightarrow 0.0129 \geq 0.00333 \text{ OK!}$$

Example: Doubly Reinforced Section

Compute the effective reinforcement ratio and minimum ρ

$$(\rho - \rho') \geq \left(\frac{\beta_1 (0.85 f'_c) d'}{d\, f_y} \right) \left(\frac{87}{87 - f_y} \right)$$

$$\geq \left(\frac{0.85(0.85(4 \text{ ksi}))(2.5 \text{ in.})}{60 \text{ ksi}(15.5 \text{ in.})} \right) \left(\frac{87}{87 - 60} \right) = 0.0398$$

$0.00957 \not\geq 0.0398$ Compression steel has not yielded.

Example: Doubly Reinforced Section

Instead of iterating the equation use the quadratic method

$$0.85\, f_{\mathrm{c}} b \beta_1 c^2 + \left(A'_{\mathrm{s}} E_{\mathrm{s}} \varepsilon_{\mathrm{cu}} - A_{\mathrm{s}} f_{\mathrm{y}} \right) c - A'_{\mathrm{s}} E_{\mathrm{s}} \varepsilon_{\mathrm{cu}} d' = 0$$

$$0.85 \left(4 \text{ ksi} \right) \left(12 \text{ in.} \right) \left(0.85 \right) c^2 +$$

$$+ \left[\left(\left(0.62 \text{ in}^2 \right) \left(29000 \text{ ksi} \right) \left(0.003 \right) - \left(2.4 \text{ in}^2 \right) \left(60 \text{ ksi} \right) \right) \right] c$$

$$- \left(0.62 \text{ in}^2 \right) \left(29000 \text{ ksi} \right) \left(0.003 \right) \left(2.5 \text{ in.} \right) = 0$$

$$34.68 c^2 - 90.06 c - 134.85 = 0$$

$$c^2 - 2.5969 c - 3.8884 = 0$$

Example: Doubly Reinforced Section

Solve using the quadratic formula

$$c^2 - 2.5969c - 3.8884 = 0$$

$$c = \frac{2.5969 \pm \sqrt{(-2.5969)^2 - 4(-3.8884)}}{2}$$

$$c = 3.6595 \text{ in.}$$

Example: Doubly Reinforced Section

Find the f_s'

$$f_s' = \left(1 - \frac{d'}{c}\right) E_s \varepsilon_{cu} = \left(1 - \frac{2.5\text{ in.}}{3.659\text{ in.}}\right) 87\text{ ksi}$$

$$= 27.565\text{ ksi}$$

Check the tension steel.

$$\varepsilon_s = \left(\frac{15.5\text{ in.} - 3.659\text{ in.}}{3.659\text{ in.}}\right) 0.003 = 0.00971 \geq 0.00207$$

Example: Doubly Reinforced Section

Check to see if c works

$$c = \frac{A_s f_y - A_s' f_s'}{0.85 f_c \beta_1 b} = \frac{\left(2.4\ \text{in}^2\right)\left(60\ \text{ksi}\right) - \left(0.62\ \text{in}^2\right)\left(27.565\ \text{ksi}\right)}{0.85\left(4\ \text{ksi}\right)\left(0.85\right)\left(12\ \text{in.}\right)}$$

$$c = 3.659\ \text{in.}$$

The problem worked

Example: Doubly Reinforced Section

Compute the moment capacity of the beam

$$M_n = \left(A_s f_y - A_s' f_s'\right)\left(d - \frac{a}{2}\right) + A_s' f_s' \left(d - d'\right)$$

$$= \left(\begin{array}{l}\left(2.4\ \text{in}^2\right)\left(60\ \text{ksi}\right) \\ \qquad - \left(0.62\ \text{in}^2\right)\left(27.565\ \text{ksi}\right)\end{array}\right)\left(15.5\ \text{in.} - \frac{0.85\left(3.659\ \text{in.}\right)}{2}\right)$$

$$+\left(0.62\ \text{in}^2\right)\left(27.565\ \text{ksi}\right)\left(15.5\ \text{in.} - 2.5\ \text{in.}\right)$$

$$= 1991.9\ \text{k-in.} \Rightarrow 166\ \text{k-ft}$$

Example: Doubly Reinforced Section

If you want to find the M_u for the problem

$$\frac{c}{d} = \frac{3.66\text{ in.}}{15.5\text{ in.}} = 0.236$$

From ACI (figure R9.3.2)or figure (pg 100 in your text)

$$0.375 > \frac{c}{d} \quad \Rightarrow \quad \phi = 0.9$$

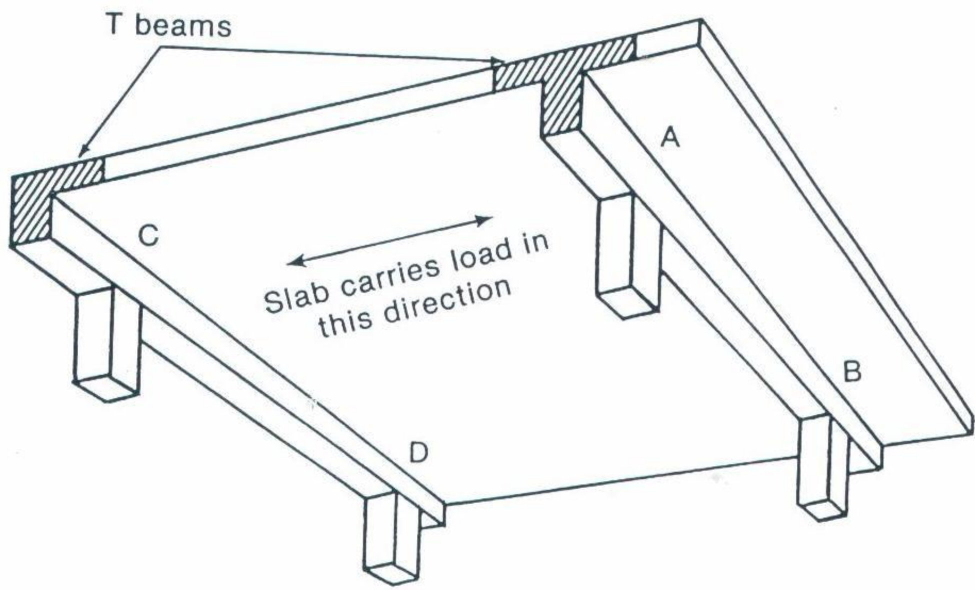
The resulting ultimate moment is

$$M_{\text{u}} = \phi M_{\text{u}} = 0.9\left(166\text{ k-ft}\right)$$

$$= 149.4\text{ k-ft}$$

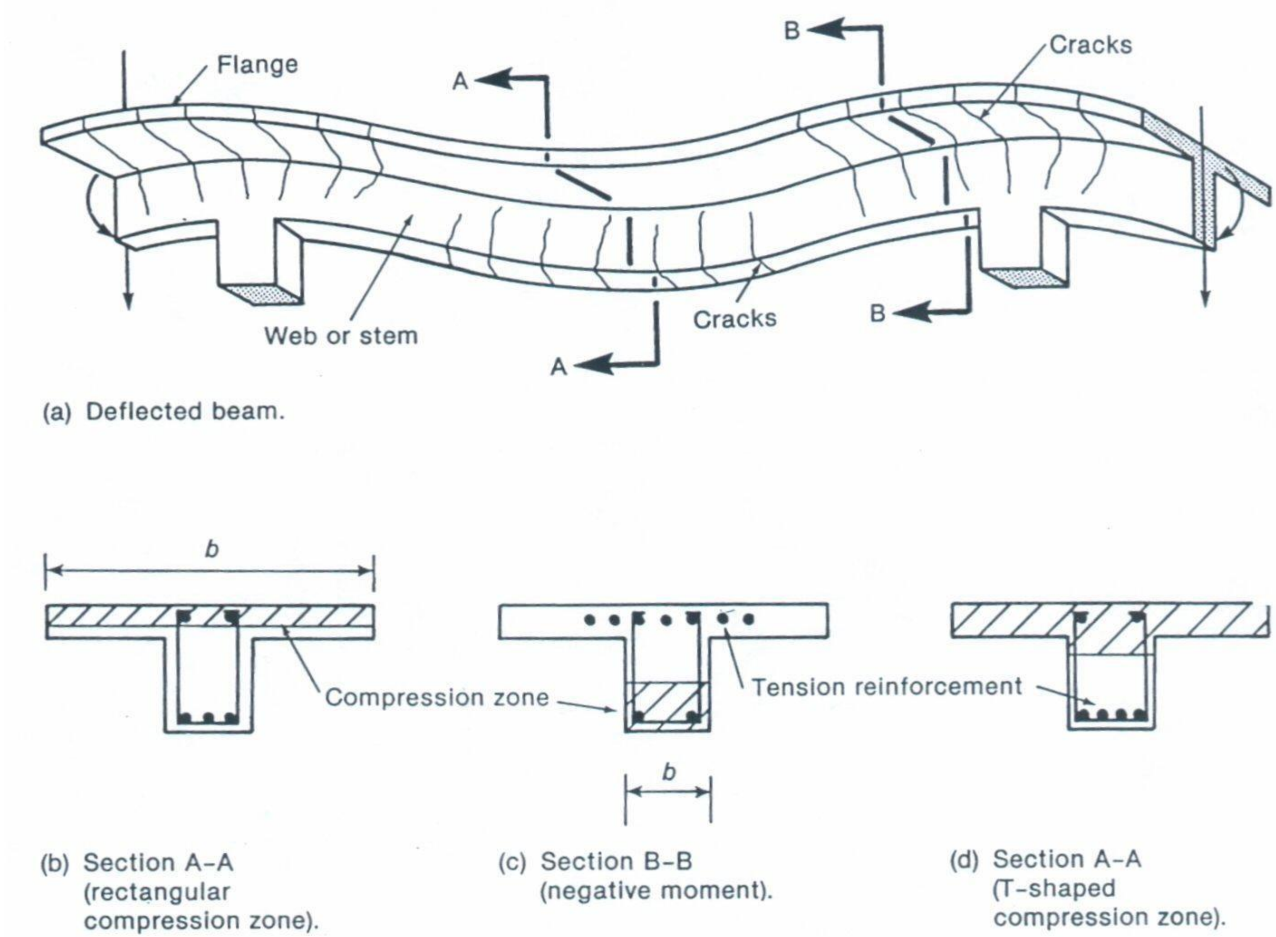
Analysis of Flanged Section

- Floor systems with slabs and beams are placed in monolithic pour.
- Slab acts as a top flange to the beam; ***T-beams***, and ***Inverted L(Spandrel) Beams***.



Analysis of Flanged Sections

Positive and Negative Moment Regions in a T-beam



(a) Deflected beam.

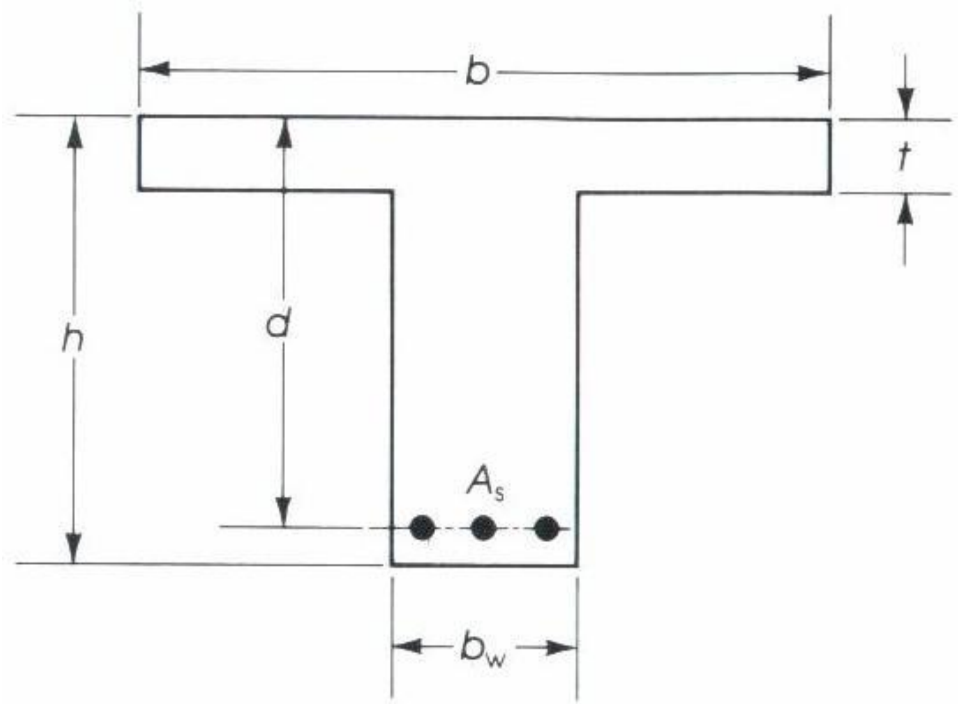
(b) Section A–A (rectangular compression zone).

(c) Section B–B (negative moment).

(d) Section A–A (T-shaped compression zone).

Analysis of Flanged Sections

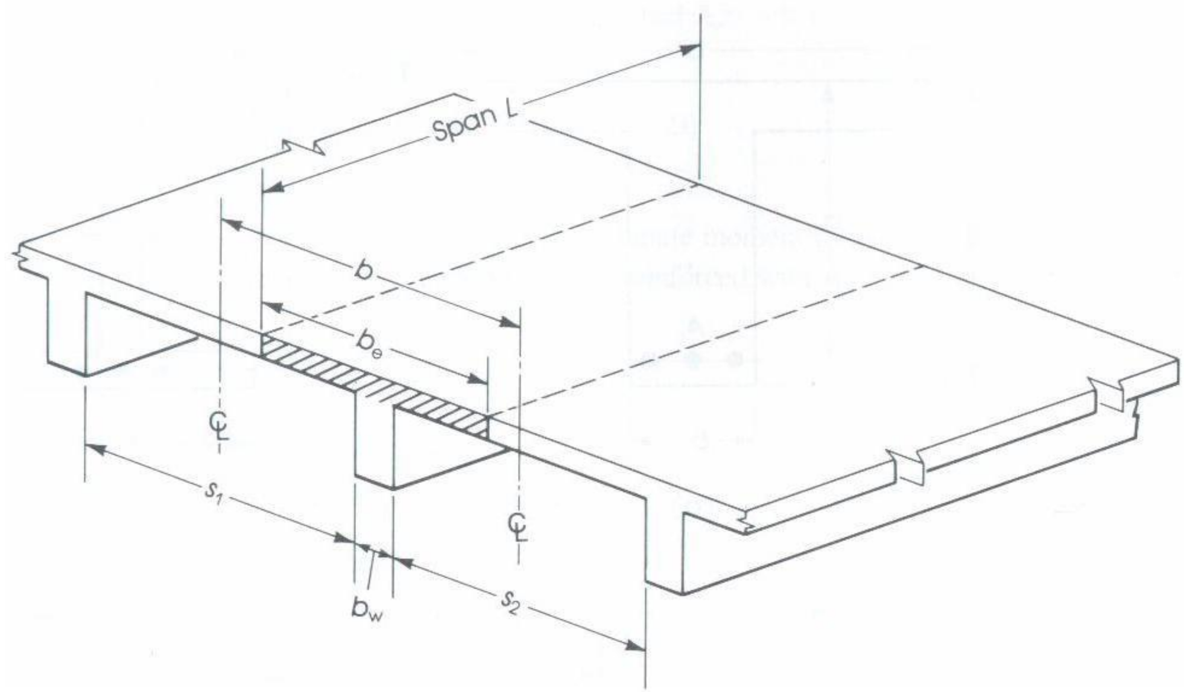
If the neutral axis falls within the slab depth analyze the beam as a rectangular beam, otherwise as a T-beam.



Analysis of Flanged Sections

<u>Effective Flange Width</u>

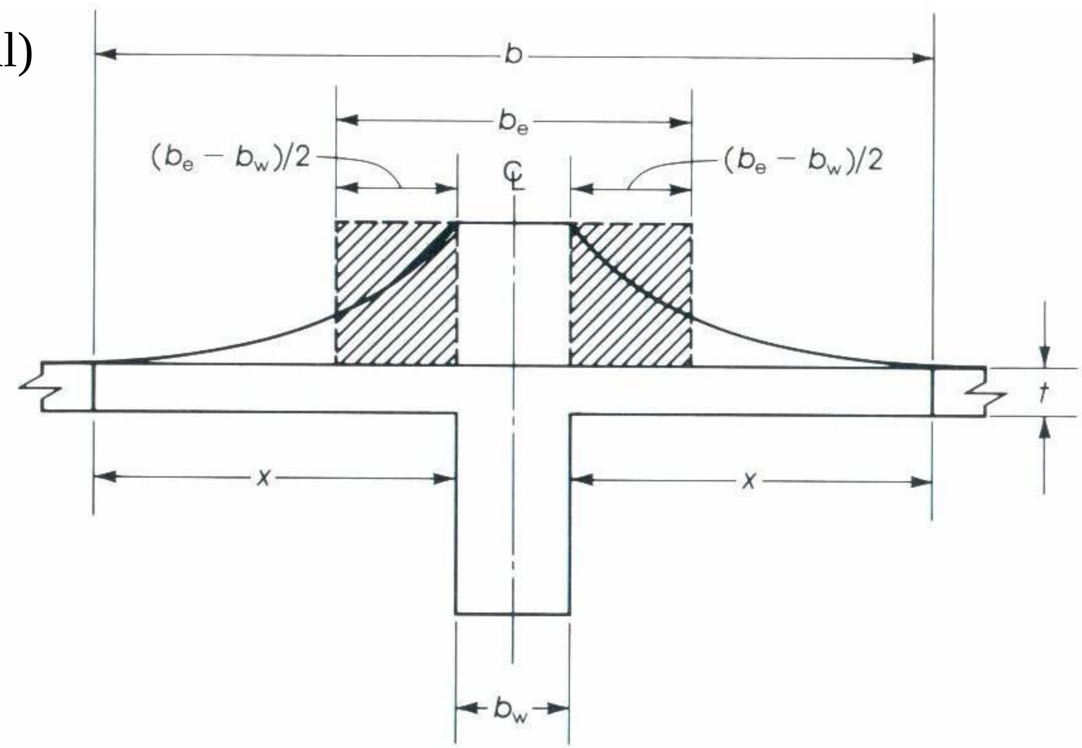
Portions near the webs are more highly stressed than areas away from the web.



Analysis of Flanged Sections

<u>Effective width</u> (b_{eff})

b_{eff} is width that is stressed uniformly to give the same compression force actually developed in compression zone of width $b_{(actual)}$



ACI Code Provisions for Estimating b_{eff}

From ACI 318, Section 8.10.2

T Beam Flange:

$$
\begin{aligned}
b_{\text{eff}} &\le \frac{L}{4} \\
&\le 16h_{\text{f}} + b_{\text{w}} \\
&\le b_{\text{actual}}
\end{aligned}
$$

ACI Code Provisions for Estimating b_{eff}

From ACI 318, Section 8.10.3

Inverted L Shape Flange

$$b_{\text{eff}} \le \frac{L}{12} + b_{\text{w}}$$

$$\le 6h_{\text{f}} + b_{\text{w}}$$

$$\le b_{\text{actual}} = b_{\text{w}} + 0.5*(\text{clear distance to next web})$$

ACI Code Provisions for Estimating b_{eff}

From ACI 318, Section 8.10

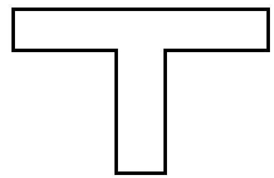
Isolated T-Beams

$$h_f \geq \frac{b_w}{2}$$

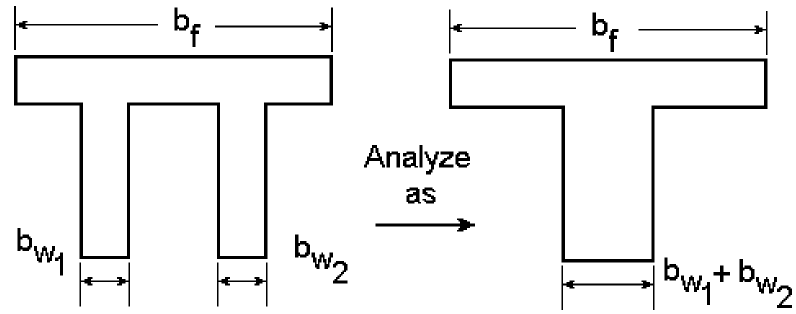
$$b_{eff} \leq 4b_w$$

Various Possible Geometries of T-Beams

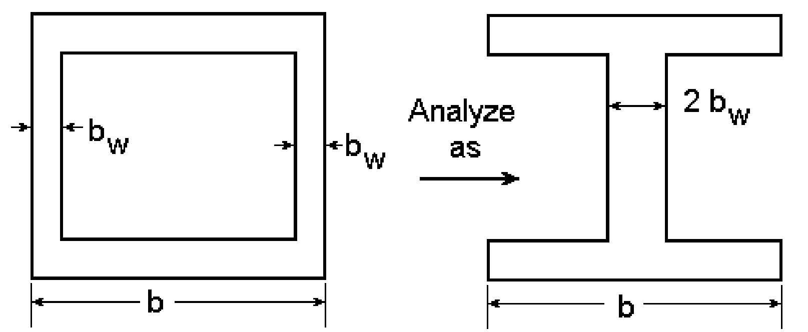
Single Tee



Twin Tee



Box



UNIT-III
ONE-WAY SLAB

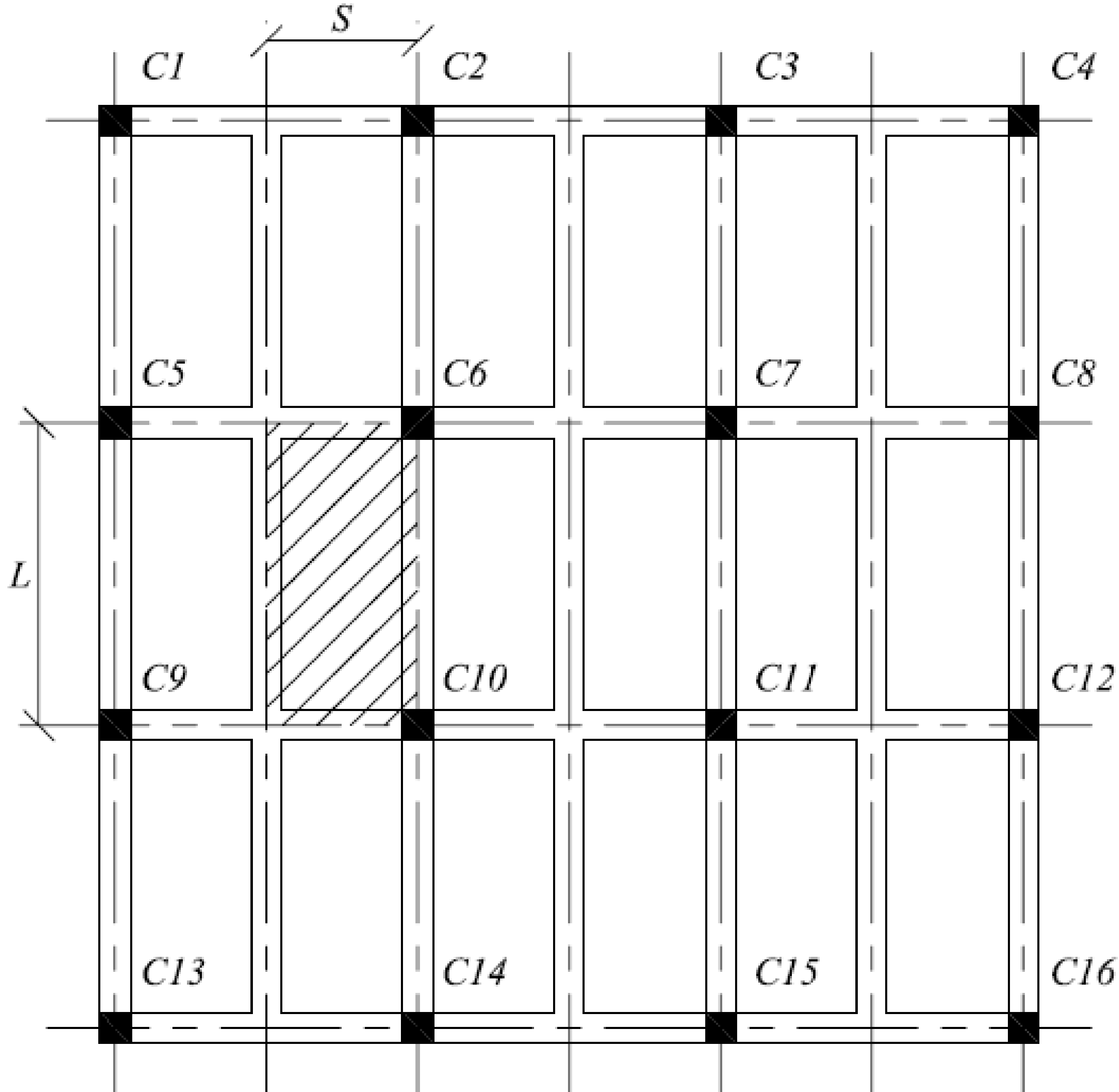
Introduction

A slab is structural element whose thickness is small compared to its own length and width. Slabs are usually used in floor and roof coverings.

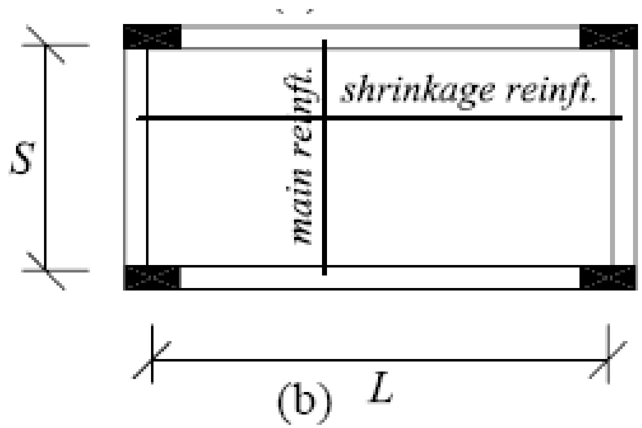
Length of the longer span(l

One-way slabs:

When the ratio of the longer to the shorter side (*L/ S*) of the slab is at least equal to 2.0, it is called one-way slab. Under the action of loads, it is deflected in the short direction only, in a cylindrical form. Therefore, main reinforcement is placed in the shorter direction, while the longer direction is provided with shrinkage reinforcement to limit cracking. When the slab is supported on two sides only, the load will be transferred to these sides regardless of its longer span to shorter span ratio, and it will be classified as one-way slab.



(a)



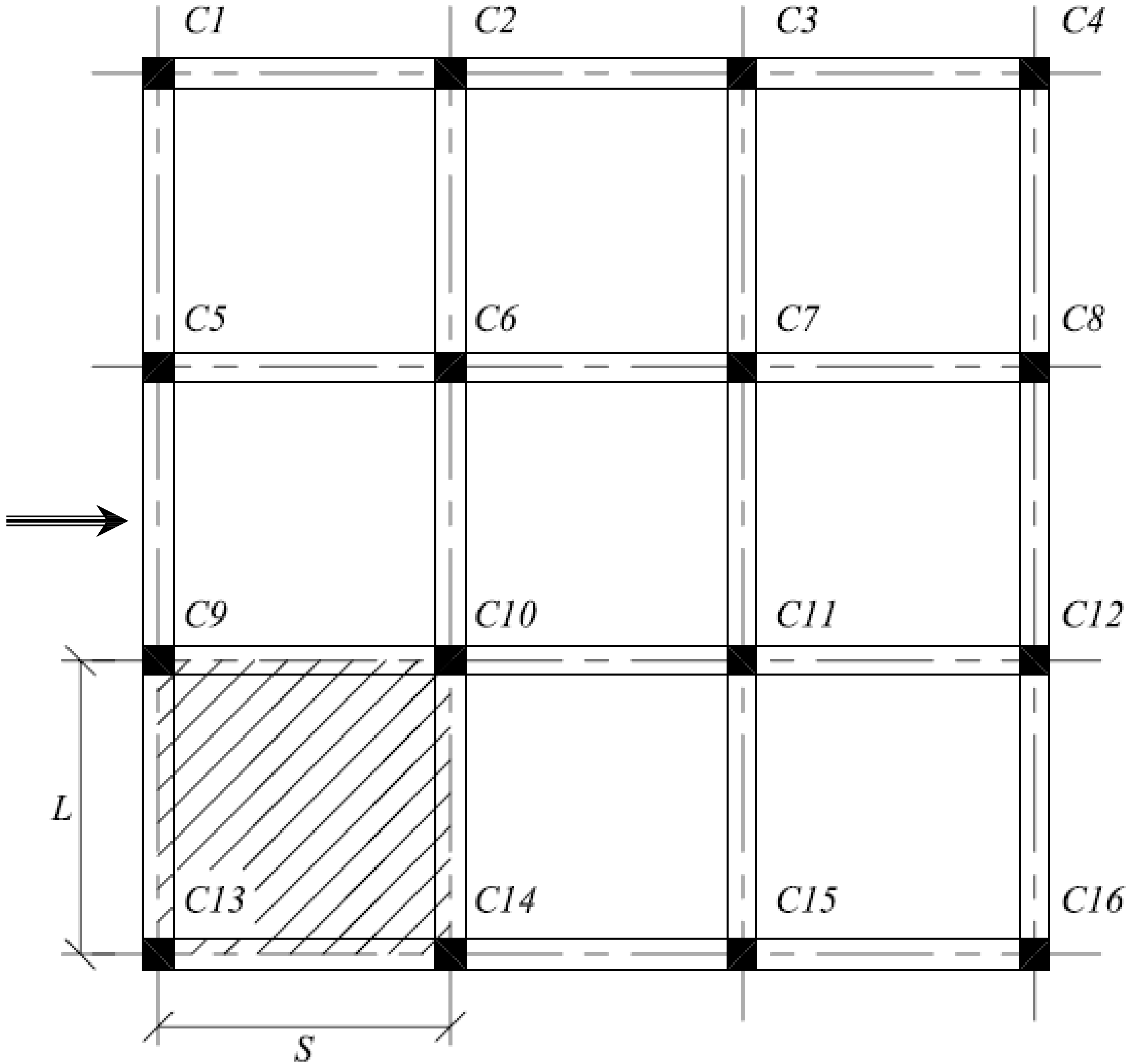
One way slab; (a) classification; (b) reinforcement

Two-way Slabs:

When the ratio (*L/ S*) is less than 2.0, it is called two-way slab. Bending will take place in the two directions in a dish-like form.

Accordingly, main reinforcement is required in the two directions.

Two way slabs



One-way Slabs

In this section, two types will be discussed, one-way solid slabs and one-way ribbed slabs.

One-way Solid Slabs

Minimum Thickness

To control deflection, *ACI Code 9.5.2.1* specifies minimum thickness values for one-way solid slabs, shown in Table.

Minimum thickness of one-way solid slabs

Element	Simply supported	One end continuous	Both ends continuous	Cantilever
One-way solid slabs	$l/20$	$l/24$	$l/28$	$l/10$

where l is the span length in the direction of bending.

Minimum Concrete Cover

According to *ACI Code 7.7.1*, the following minimum concrete cover is to be provided:

a. Concrete not exposed to weather or in contact with ground:

- ϕ36 mm and larger bar -- 4 *cm*
- ϕ36 mm and smaller bars -- 2 *cm*

b. Concrete exposed to weather or in contact with ground:

- ϕ16 mm and larger bars -- 5 *cm*
- ϕ16 mm and smaller bars -- 4 *cm*

c. Concrete cast against and permanently exposed to earth ----------- 7.5 *cm*

Design Concept

One-way solid slabs are designed as a number of independent 1 *m* wide strips which span in the short direction and supported on crossing beams.

Maximum Reinforcement Ratio

One-way solid slabs are designed as rectangular sections subjected to shear and moment. Thus, the maximum reinforcement ratio ρ_{max} is not to exceed

$$0.75\ \rho_b \text{ and } A_{s\max} = 0.75 A_{sb}$$

Shrinkage Reinforcement Ratio

According to *ACI Code 7.12.2.1* and for steels yielding at $f_y = 4200 kg/cm^2$, the shrinkage reinforcement is taken not less than 0.0018 of the gross concrete area, or $A_{s\,shrinkage} = 0.0018\, b\, h$

where, *b* = width of strip, and *h* = slab thickness.

Minimum Reinforcement Ratio

According to *ACI Code 10.5.4*, the minimum flexural reinforcement is not to be less than the shrinkage reinforcement, or

$$A_{s\,min} = 0.0018\,b\,h$$

Spacing Of Flexural Reinforcement Bars

Flexural reinforcement is to be spaced not farther than three times the slab thickness, nor farther apart than 45 *cm*, center-to-center.

Spacing Of Shrinkage Reinforcement Bars

Shrinkage reinforcement is to be spaced not farther than five times the slab thickness, nor farther apart than 45 *cm*, center-to-center.

Loads Assigned to Slabs

(1) Own weight of slab:

(2) Weight of slab covering materials:

- Sand fill with a thickness of about 5 *cm*,

$$0.05 \times 1.80\, t/m^2$$

-Cement mortar, 2.5 *cm* thick.

$$0.025 \times 2.10\ \ t/m^2$$

- Tiling

$$0.025 \times 2.30\ \ t/m^2$$

-A layer of plaster about 2 *cm* in thickness.

$$0.02 \times 2.10\, t/m^2$$

(3) Live Load:

Table shows typical values used by the *Uniform Building Code* (*UBC*).

Minimum live Load values on slabs

Type of Use	Uniform Live Load kg/m^2
Residential	200
Residential balconies	300
Computer use	500
Offices	250
Warehouses	
▪ Light storage	600
▪ Heavy Storage	1200
Schools	
▪ Classrooms	200
Libraries	
▪ Reading rooms	300
▪ Stack rooms	600
Hospitals	200
Assembly Halls	
▪ Fixed seating	250
▪ Movable seating	500
Garages (cars)	250
Stores	
▪ Retail	400
▪ wholesale	500
Exit facilities	500
Manufacturing	
▪ Light	400
▪ Heavy	600

(4) Equivalent Partition Weight:

This load is usually taken as the weight of all walls carried by the slab divided by the floor area and treated as a dead load rather than a live load.

Loads Assigned to Beams

The beams are usually designed to carry the following loads:
- Their own weights.
- Weights of partitions applied directly on them.
- Floor loads.

The floor loads on beams supporting the slab in the shorter direction may be assumed uniformly distributed throughout their spans.

Approximate Structural Analysis

ACI Code 8.3.3 permits the use of the following approximate moments and shears for design of continuous beams and one-way slabs, provided:

1. Positive Moment:

a. End Spans:

When discontinuous end unrestrained, $M_u = w_u l_n^2/11$

When discontinuous end is integral with support, $M_u = w_u l_n^2/14$

where l_n is the corresponding clear span length

b. Interior Spans:

$$M_u = w_u l_n^2/16$$

2. Negative Moment:

a. Negative moment at exterior face of first interior support:

Two spans, $M_u = w_u l_n^2/9$

More than two spans, $M_u = w_u l_n^2/10$

where l_n is the average of adjacent clear span lengths.

b. Negative moment at other faces of interior supports:

$$M_u = w_u l_n^2/11$$

c. Negative moment at interior face of exterior support:

Support is edge beam, $M_u = w_u l_n^2/24$

Support is a column, $M_u = w_u l_n^2/16$

3. Shear:

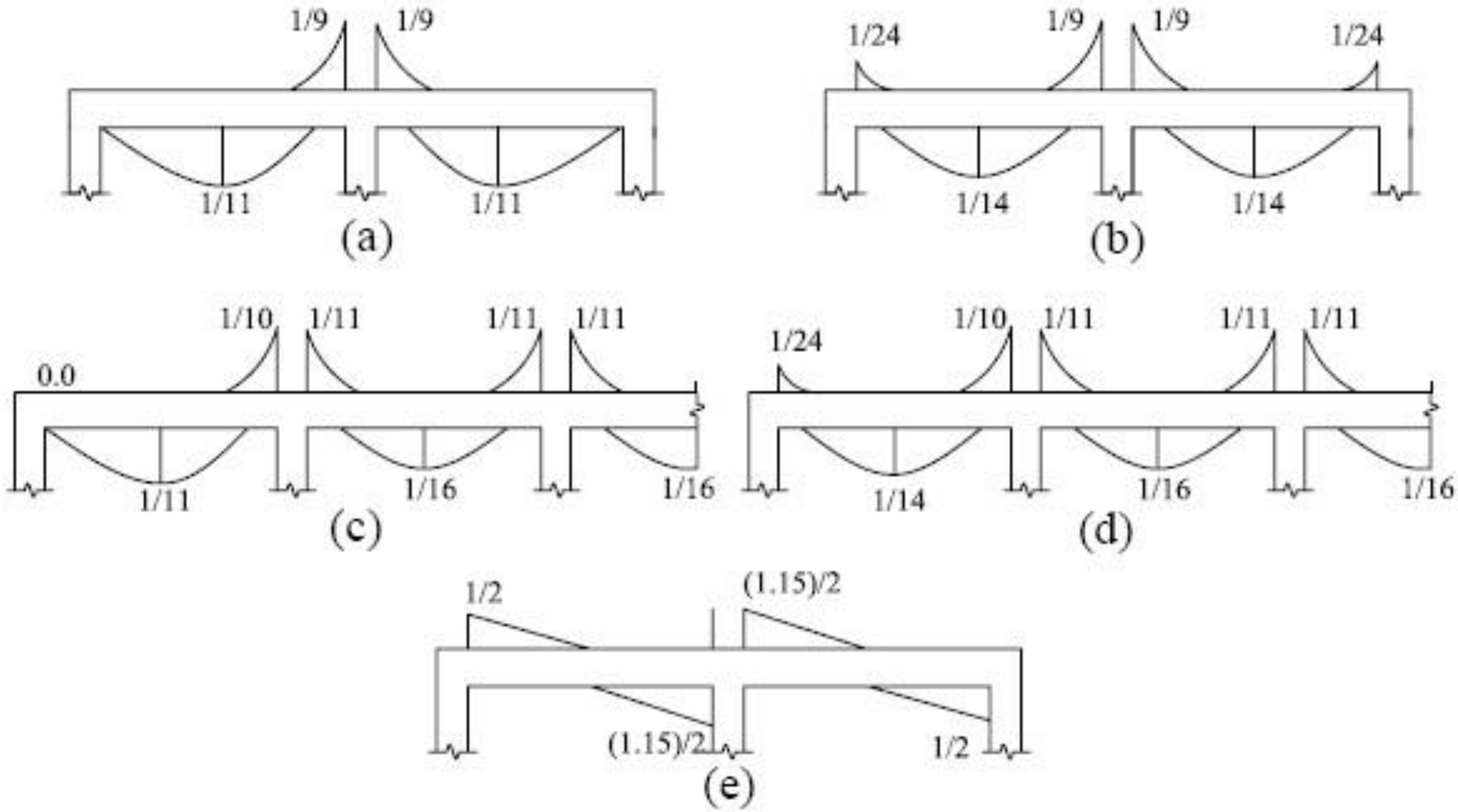
a. Shear in end members at face of first interior support:

$$V_u = 1{,}15 w_u l_n / 2$$

b. Shear at face of all other supports:

$$V_u = w_u l_n / 2$$

where l_n is the corresponding clear span length.



(a) Two spans, exterior edge unrestrained; (b) two spans, support is spandrel beam; (c) more than two spans, exterior edge unrestrained; (d) more than two spans, support is spandrel beam; (e) two spans, shearing force diagram

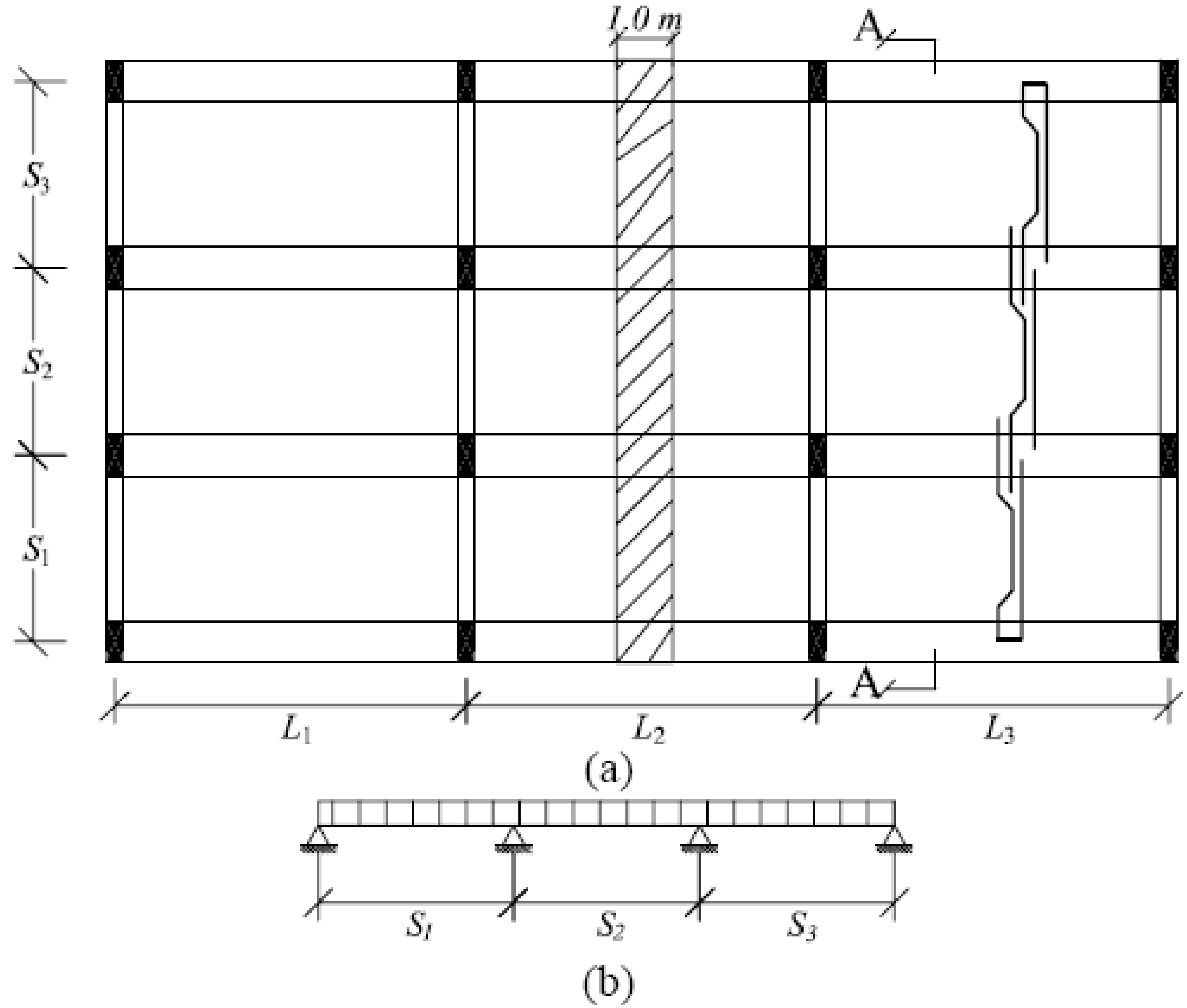
Summary of One-way Solid Slab Design Procedure

Once design compressive strength of concrete and yield stress of reinforcement are specified, the next steps are followed:

1.Select representative 1 *m* wide design strip/strips to span in the short direction.

2.Choose a slab thickness to satisfy deflection control requirements. When several numbers of slab panels exist, select the largest calculated thickness.

3. Calculate the factored load W_u by magnifying service dead and live loads according to this equation $w_u = 1.40w._d + 1.70w_l$



(a) Representative strip and reinforcement; (b) strip and loads

4. Draw the shear force and bending moment diagrams for each of the strips.
5. Check adequacy of slab thickness in terms of resisting shear by satisfying the following equation:

$$V_u \le 0.53\Phi\sqrt{f_c^{'}}bd$$

where

V_u = factored shear force

V_c = shear force resisted by concrete alone

Φ = strength reduction factor for shear is equal to 0.85.

b = width of strip = 100 cm

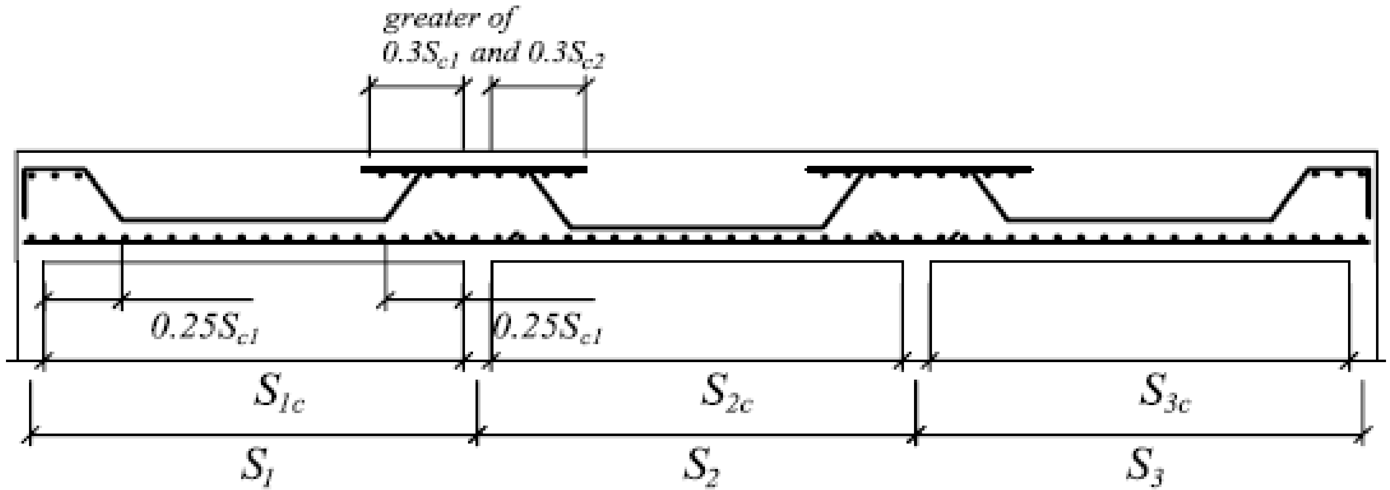
d = effective depth of slab

If the previous equation is not satisfied, go ahead and enlarge the thickness to do so.

6. Design flexural and shrinkage reinforcement:
Flexural reinforcement ratio is calculated from the following equation:

$$\rho = \frac{0.85 f_c'}{f_y}\left[1 - \sqrt{1 - \frac{2.61\times10^5 M_u}{b d^2 f_c'}}\right]$$

Make sure that the reinforcement ratio is not larger than ¾ ρ_b
Compute the area of shrinkage reinforcement, where $A_{s\,min} = 0.0018bh$ Select appropriate bar numbers and diameters for both, main and secondary reinforcement.
Check reinforcement spacing, modify your bar selection if needed.
7. Draw a plan of the slab and representative cross sections showing the dimensions and the selected reinforcement.



Section A-A

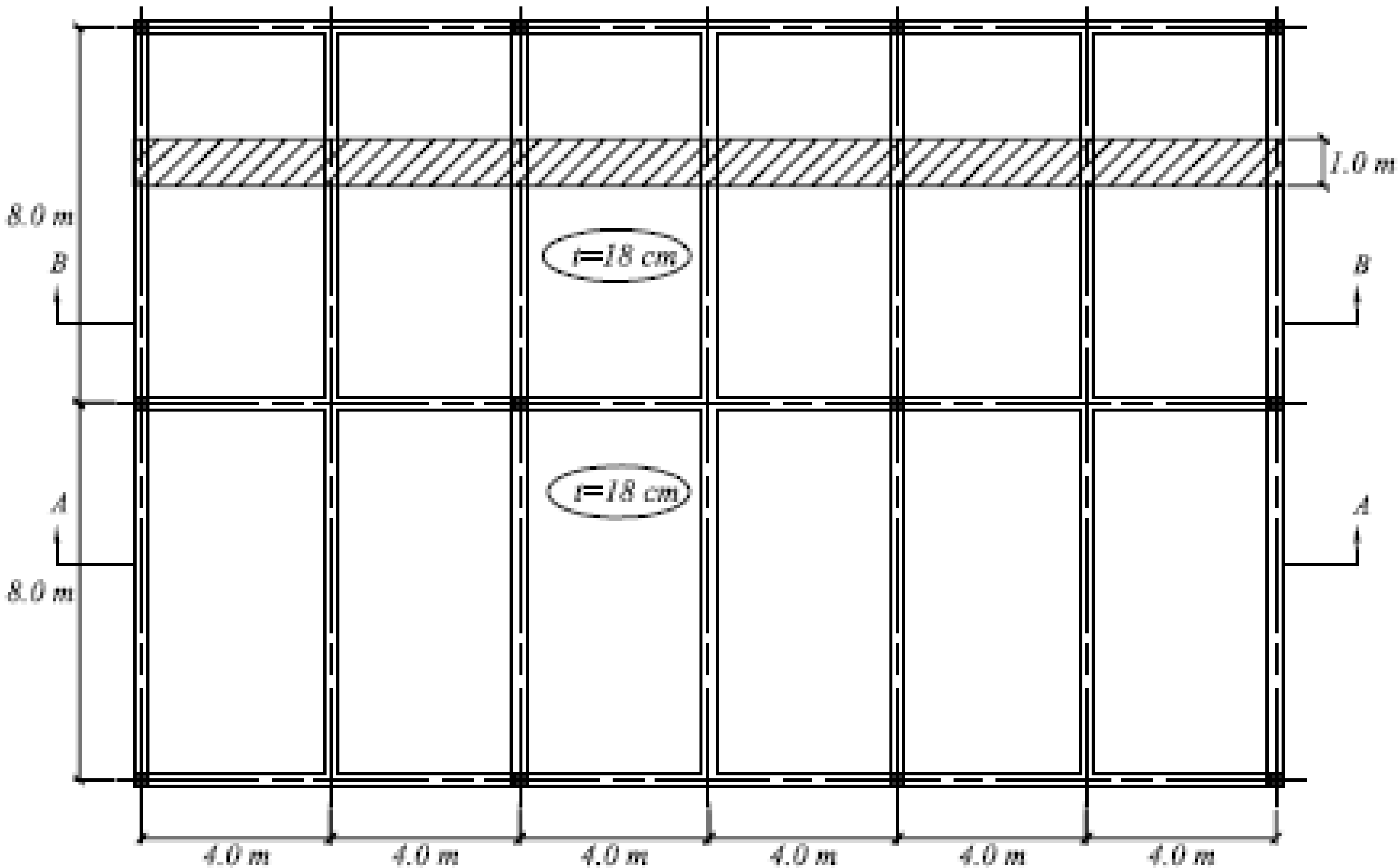
Example (8.1):

Using the ACI Code approximate structural analysis, design for a warehouse, a continuous one-way solid slab supported on beams 4.0 m apart as shown. Assume that the beam webs are 30 cm wide. The dead load is 300 kg/m² in addition to own weight of the slab, and the live load is 300 kg/m²

Use $f_c^{'} = 250\, kg/cm^2$ and $f_y = 4200\, kg/cm^2$.

Solution :

1- Select a representative 1 *m* wide slab strip:



Representative strip

2- Select slab thickness:

The clear span length $l_n = 4.0 - 0.30 = 3.70\, m$

For one-end continuous spans, $h_{\min} = l/24 = 400/24 = 16.67 cm$

Slab thickness is taken as 18 *cm*.

3- Calculate the factored load W_u per unit length of the selected strip:

Own weight of slab = $0.18 \times 2.50 = 0.45\ ton/m^2$

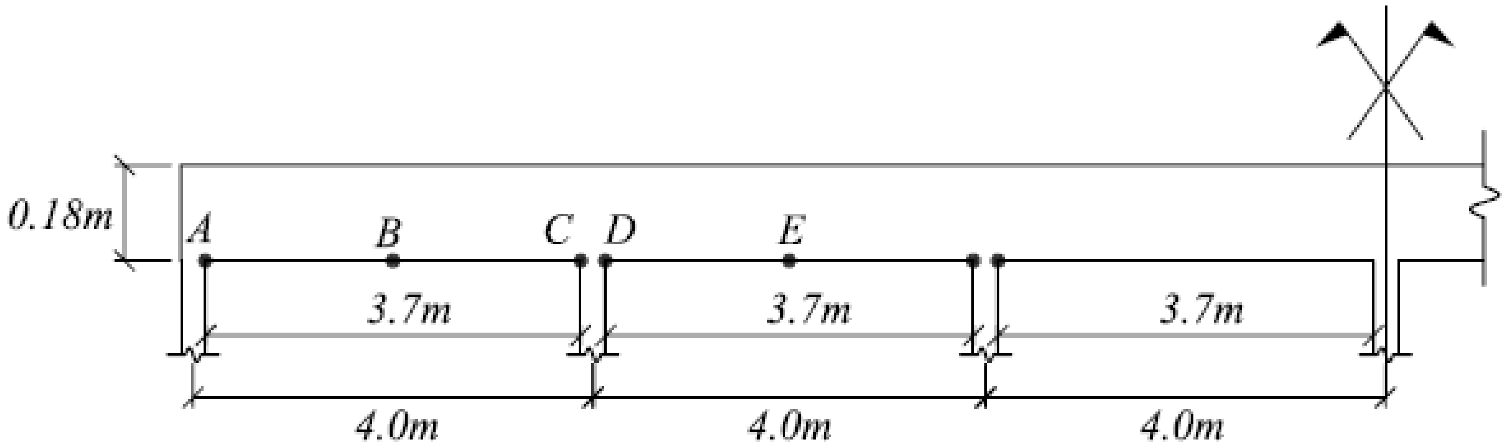
$$w_u = 1.40(0.30 + 0.45) + 1.70(0.30) = 1.56\ ton/m^2$$

For a strip 1 *m* wide, $w_u = 1.56\ ton/m$

4- Evaluate the maximum factored shear forces and bending moments in the strip:

The results are shown in the following table. Points at which moments and shear are calculated.

	A	B	C	D	E
Moment coefficient	- 1/24	1/14	- 1/10	-1 /11	1/16
Factored moment in $t.m$	- 0.890	1.525	- 2.135	- 1.941	1.335
Reinforcement ratio	0.0010	0.0017	0.0024	0.0022	0.0015
Steel reinforcement cm^2	1.54	2.62	3.70	3.39	2.31
Minimum reinforcement cm^2	3.24	3.24	3.24	3.24	3.24
Bar size mm	ϕ 10	ϕ 10	ϕ 10	ϕ 10	ϕ 10
Bar spacing cm	20	20	20	20	20



Points at which moments and shear are evaluated

5- Check slab thickness for beam shear:

Effective depth $d = 18 - 2 - 0.60 = 15.40\ cm$, assuming $\phi\, 12\ mm$ bars.

$$V_{u\max} = \frac{1.15}{2} w_u\, l_n = \frac{1.15}{2}(1.56)(3.70) = 3.32\ ton$$

$$\Phi\ V_c = 0.85(0.53)\sqrt{250}\ (100)(15.40)/1000 = 10.97\ ton$$

i.e. , slab thickness is adequate in terms of resisting beam shear.

6- Design flexural and shrinkage reinforcement:

Steel reinforcement ratios are then calculated, and be checked against minimum and maximum code specified limits, where

$$\rho = \frac{0.85\, f_c'}{f_y}\left[1-\sqrt{1-\frac{2.61\times10^5\, M_u}{b\, d^2\, f_c'}}\right]$$

Minimum reinforcement = 0.0018 (100) (18) = 3.24 cm^2/m

Maximum reinforcement ratio =¾ ρ_b. = 0.75 (0.0255)= 0.0191

Calculate the area of shrinkage reinforcement:

Area of shrinkage reinforcement = 0.0018 (100) (18) = 3.24 cm^2/m

Select reinforcement bars:

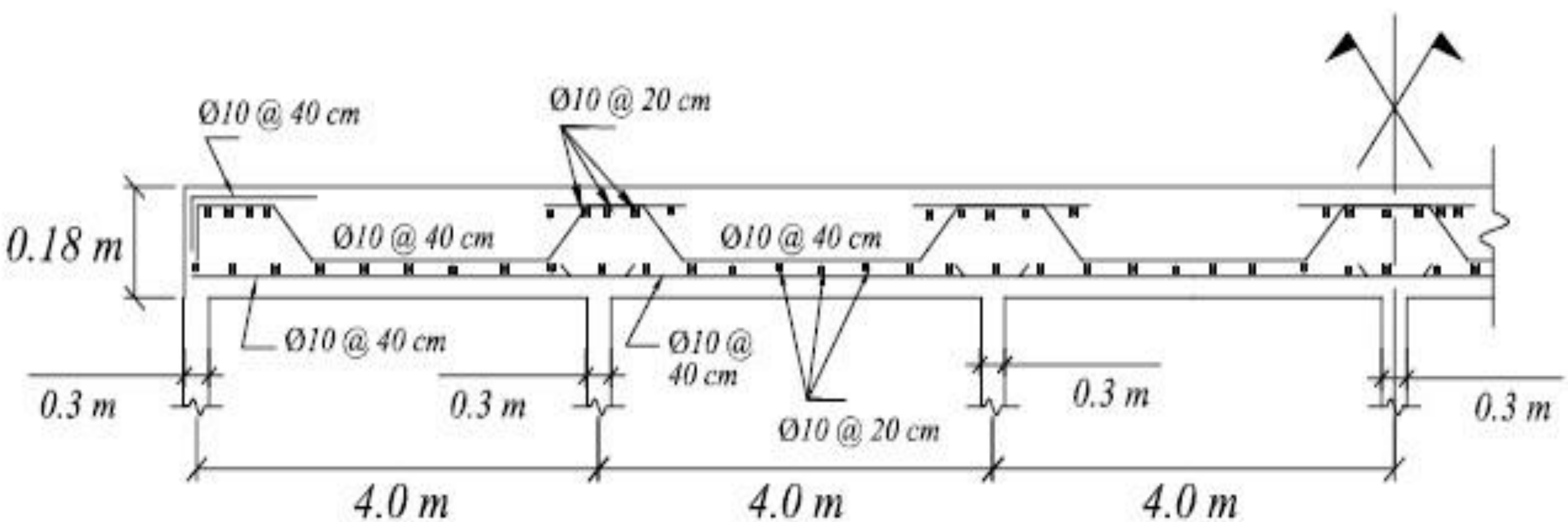
Main and secondary reinforcement bars are also shown in the table

For shrinkage reinforcement use ϕ 10 mm @ 20 cm, or 5 ϕ 10 mm @ 1 m.

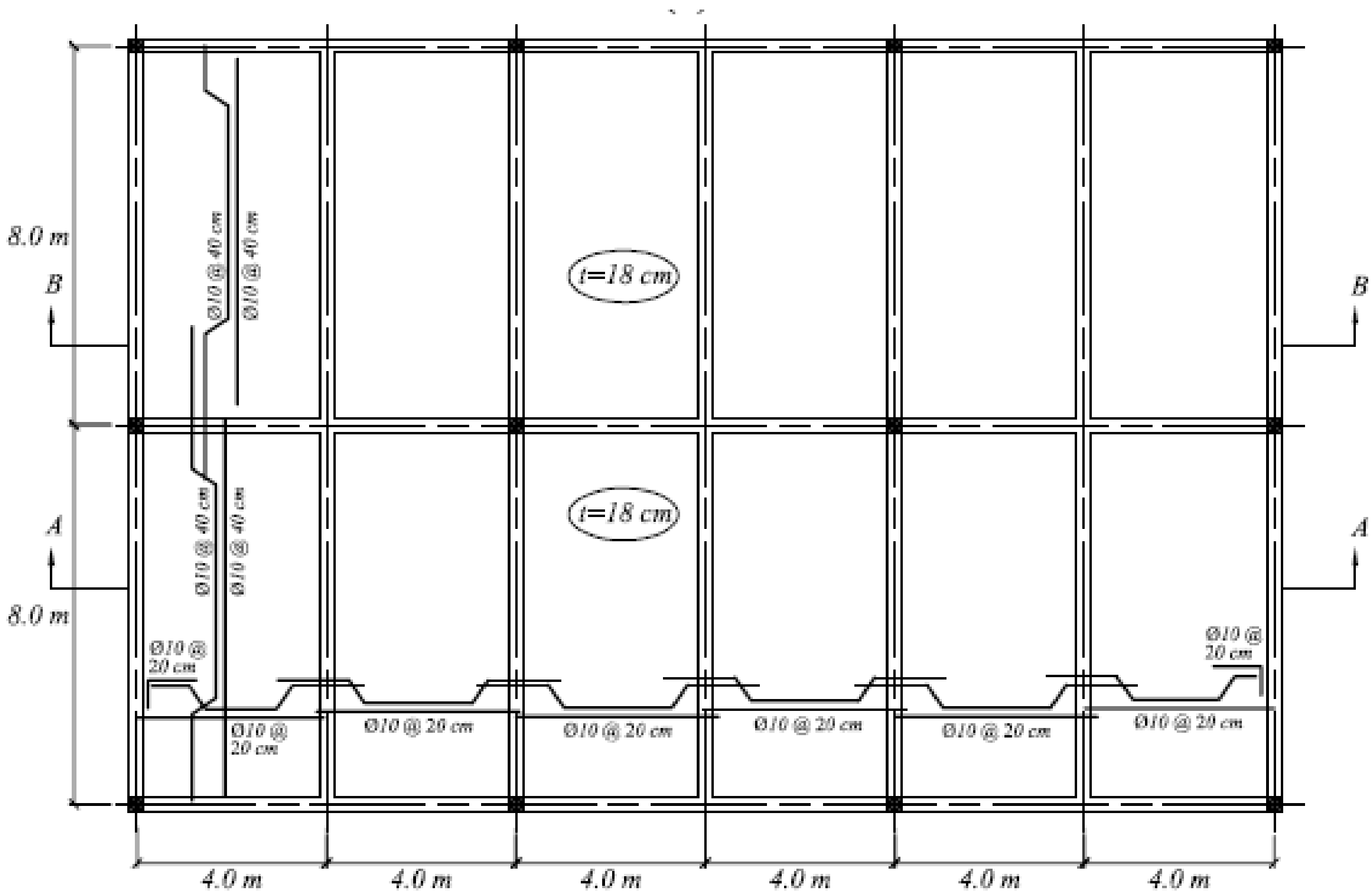
Check bar spacing against code specified values

For main reinforcement, spacing between bars is not to exceed the larger of 3 (18) = 54 *cm* and 45 *cm*, which is already satisfied as shown in the table. For shrinkage reinforcement, spacing between bars is not to exceed the larger of 5 (18) = 90 *cm* and 45 *cm*, which is also satisfied.

7- Prepare neat sketches showing the reinforcement and slab thickness:



(c)



(d)

(continued); (c) Section A-A; (d) reinforcement details

Example (8.2):

Design the slab shown in Example (8.1) using any available structural analysis software.

Solution :

1 Select a representative 1 *m* wide slab strip:

The selected representative strip is shown.

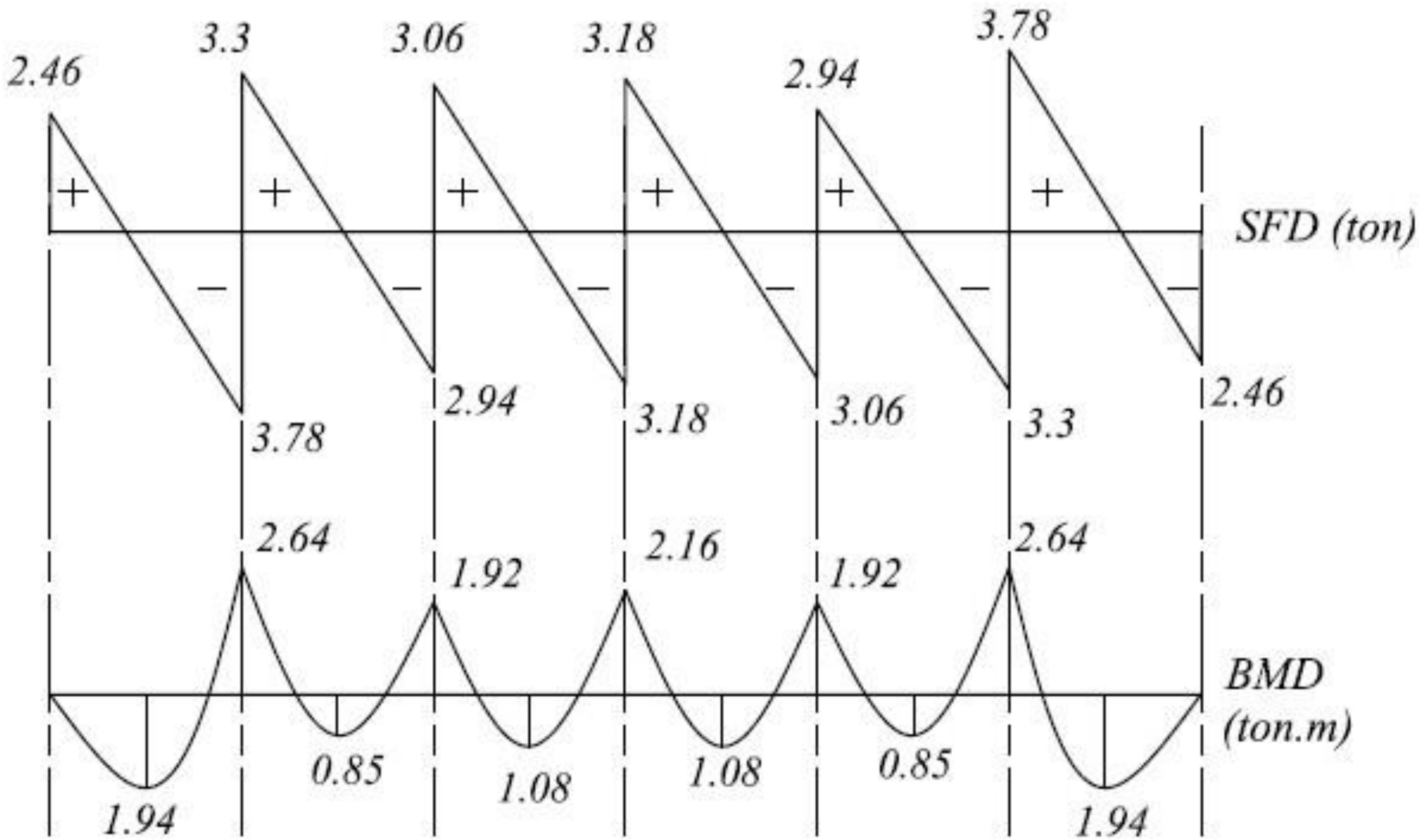
2 Select slab thickness:

Same as in Example (8.1), the thickness is taken as 18 *cm*.

3 Calculate the factored load W_u per unit length of the selected strip:

For a strip 1 *m* wide, $w_u = 1.56\, ton/m$

4- Evaluate the maximum factored shear forces and bending moments in the strip:



Shearing force and bending moment diagrams

5- Check slab thickness for beam shear:

Effective depth $d = 18 - 2 - 0.60 = 15.40\ cm$, assuming $\phi\, 12\ mm$ bars.

$$V_{u\,max} = 3.78\ ton$$

$$\Phi\ V_c = 0.85(0.53)\sqrt{250}\ (100)(15.40)/1000 = 10.97\ ton$$

i.e. , slab thickness is adequate in terms of resisting beam shear.

6- Design flexural and shrinkage reinforcement:

Steel reinforcement ratios are calculated and checked against minimum and maximum code specified limits.

For $M_u = -2.64\ t.m$

$$\rho = \frac{0.85(250)}{4200}\left[1-\sqrt{1-\frac{2.61\times10^5(2.64)}{(100)(15.4)^2(250)}}\right] = 0.00303$$

$A_s = 0.00303(100)(15.4) = 4.67\ cm^2/m$, use $\phi 10\ mm$ @ $15\ cm$.

For $M_u = -2.16\ t.m$

$$\rho = \frac{0.85(250)}{4200}\left[1-\sqrt{1-\frac{2.61\times10^5(2.16)}{(100)(15.4)^2(250)}}\right] = 0.00246$$

$A_s = 0.00246(100)(15.4) = 3.79\ cm^2/m$, use $\phi 10\ mm$ @ $20\ cm$.

For $M_u = -1.92\ t.m$

$$\rho = \frac{0.85(250)}{4200}\left[1-\sqrt{1-\frac{2.61\times10^5(1.92)}{(100)(15.4)^2(250)}}\right] = 0.00218$$

$$A_s = 0.00218(100)(15.4) = 3.36\ cm^2/m, \text{ use } \phi\, 10\ mm\ @\ 20\ cm.$$

For $M_u = 1.94\ t.m$

$$\rho = \frac{0.85(250)}{4200}\left[1 - \sqrt{1 - \frac{2.61\times10^5\,(1.94)}{(100)(15.4)^2(250)}}\right] = 0.0022$$

$$A_s = 0.0022(100)(15.4) = 3.39\ cm^2/m, \text{ use } \phi\, 10\ mm\ @\ 20\ cm.$$

For $M_u = 1.08\ t.m$

$$\rho = \frac{0.85(250)}{4200}\left[1 - \sqrt{1 - \frac{2.61\times10^5\,(1.08)}{(100)(15.4)^2(250)}}\right] = 0.0012$$

$$A_s = 0.0018(100)(18.0) = 3.24\ cm^2/m, \text{ use } \phi\, 10\ mm\ @\ 20\ cm.$$

For $M_u = 0.85\ t.m$

$A_s = 0.0018(100)(18.0) = 3.24\ cm^2/m$, use $\phi\,10\ mm\ @\ 20\ cm$.

Calculate the area of shrinkage reinforcement:

Area of shrinkage reinforcement = 0.0018 (100) (18) = 3.24 cm^2/m, use $\phi\,10\ mm\ @\ 20\ cm$.

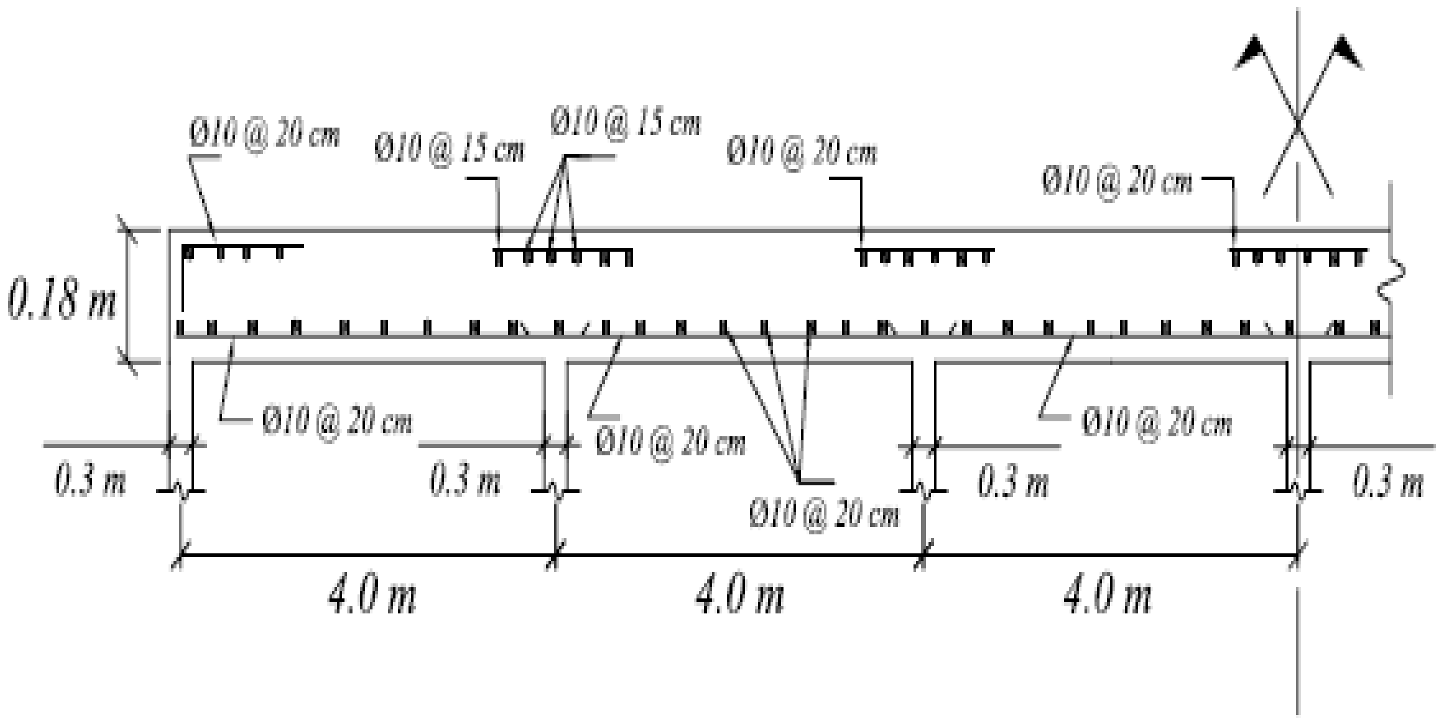
Select reinforcement bars:

It is already done in step 6.

Check bar spacing:

Same as in Example (8.1).

7- Prepare neat sketches showing the reinforcement and slab thickness:


Ø10 @ 20 cm
Ø10 @ 15 cm
Ø10 @ 15 cm
Ø10 @ 20 cm
Ø10 @ 20 cm
0.18 m
Ø10 @ 20 cm
Ø10 @ 20 cm
Ø10 @ 20 cm
0.3 m
0.3 m
0.3 m
0.3 m
Ø10 @ 20 cm
4.0 m
4.0 m
4.0 m

Section A-A

Design of Two Way Slabs

Moments in Two way Restrained Slabs with Corners Held Down

Analysis by Coefficients

- ***Restrained slabs are defined in codes as those slabs which are cast integral with RC frames and which are not free to lift up at the corners.***
- ***These slabs may be continuous or discontinuous at the edges. Those which are discontinuous at edges are also referred to be simply supported.***
- ***Coefficients specified in IS:456-2000, Table 26 in Annexure D can be used for analysis of such slabs.***

Moments in Two way Restrained Slabs with Corners Held Down

Analysis by Coefficients

- *Conditions to be satisfied for use of these coefficients are:*

  1. Loading on adjacent spans should be the same

  2. Span in each direction should be approximately equal.

- *The span moment per unit width and the negative moments at continuous edges for the slabs are calculated from the equations in terms of "l_x",*

$$M_x = \alpha_x w l_x^2$$ *for short span*

$$M_y = \alpha_y w l_x^2$$ *for longer span*

Design of Two-way Slabs

- ***Restrained two way slab is divided into middle strip and edge strips. Middle strip is forming three-fourth of slab width in width directions.***

- ***Torsion steel must be provided at discontinuous edges as specified in code.***

- ***Coefficients are given in Table 26 of IS: 456-2000. These coefficients are derived from the yield line theory of slabs.***

Design of Two-way Slabs

- ***Table given for coefficients is applicable for slabs carrying uniformly distributed loads not for concentrated loads, for which analysis should be done, separately.***
- ***Span moments/edge moments per unit width are calculated by determining ratio of "l_y" and "l_x" and for different edge conditions.***

Coefficients for Moments

- **Slabs restrained against corners lifted up**

$$\propto_y^+ = \frac{\left[24 + 2n_d + 1.5n_d^2\right]}{1000}$$

- **n_d = Number of discontinuous edges = 0,1,2,3 and 4.**

$$\propto_x^+ = \frac{2}{9}\left[\frac{3 - \sqrt{18 \propto_y^+ \left(C_{s1} + C_{s2}\right)}}{r} \frac{1}{\left(C_{l1} + C_{l2}\right)^2}\right]$$

- **C = 1.0 for a discontinuous edge**

  = $\sqrt{\frac{7}{3}}$ for continuous edge

- **Subscripts "s" and "l" denote "short edge" and "long edge"**
- **Subscripts "1" and "2" represent two edges in short and longer direction**

Important Design Issues from Table 26

1. ***Edge moments of continuous supports are 1.33 times the span moments.***

2. ***Long span moment coefficient"α_y" is a constant for given end conditions of slab, irrespective of the span ratios.***

3. ***Short span coefficient varies sharply with variation of the ratio of spans***

Important Design Issues from Table 26

- ***While using Table 26 for a series of slabs, moments calculated at an interior support will sometimes be different on two sides of that support because of the differences in continuity condition of slabs on opposite sides of support.***

Arrangement of Reinforcements

- ***While using design of two-way slabs with the help of coefficients, restrained slabs are considered to be divided into middle and edge strips.***

- ***Moments given in Table 26 apply only to middle strips, and no further redistribution is allowed for these moments.***

- ***Edge strips have to be reinforced only with nominal minimum steel for crack control.***

Arrangement of Reinforcements

- ***Middle strip should have steel (+ve and –ve) calculated for various sections. In edge strips , steel is placed as positive steel at the bottom of slab.***
- ***Negative moments may be experienced at discontinuous edges since, in practice, they are not supported on rollers but partially restrained at their ends.***
- ***The magnitude of this moment depends on the degree of fixity at the edge of the slab and is intermediate.***

Arrangement of Reinforcement

- ***Usual practice is to provide at these edges top reinforcement for negative moment equal to***

$$\frac{wl^2}{24}$$

- ***As per IS:456.2000, 50 percent of the steel provided at mid span should be extended along these edges, and the negative steel has to extend into the span 0.1 times the span length from the face of the beam.***

Design of Two-way Slabs

- ***Slab thickness should be calculated based on the greater value of the negative B.M on the short span.***

$$M_u = Kf_{ck}bd^2$$

Hence

- ***Total thickness = d (short) + 0.5ø + cover***
- ***Total thickness = d (long) + 0.5ø + ø + cover***

- ***The slab should satisfy span/effective depth ratio to control deflection.***
  - ***Simply supported = 28***
  - ***Continuous = 32***
- ***Depth of slab selected from deflection criterion will be generally greater than the minimum required from strength criterion.***
- ***Short span steel will be placed in the lower layer.***

- ***Restrained moments are obtained for the middle strips only. The reinforcement is distributed uniformly in the middle strips.***

- ***Each direction is to be provided only with the minimum reinforcement placed at the bottom of the slabs.***

- ***In addition, corner steel reinforcement should be provided at the discontinuous edges.***
- ***Corner reinforcement consists of two mats, are placed on the top and the other at the bottom of the slab, with each mat having steel in both x and y directions.***
- ***Where the slab is discontinuous on both sides of a corner, fall torsion steel has to be provided.***

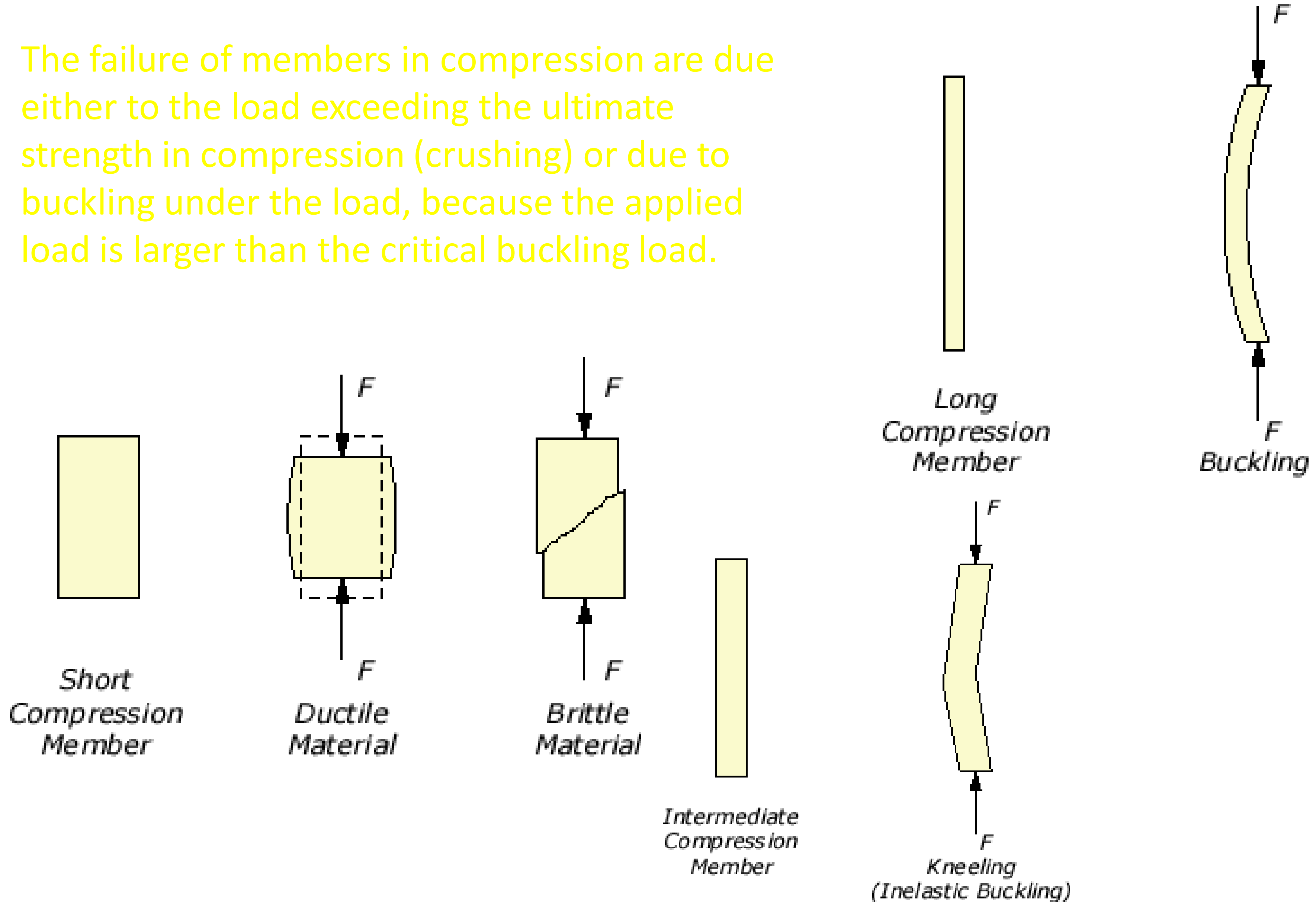
- ***The area of the full torsion reinforcement per unit width in each of the four layer should be as follows.***
- ***(Area of full corner steel) = [3/4][Area required for the maximum span per unit width in each of four layers]***
- ***These steels are to be provided for a distance of one-fifth the short span .***

UNIT-IV

COLUMNS

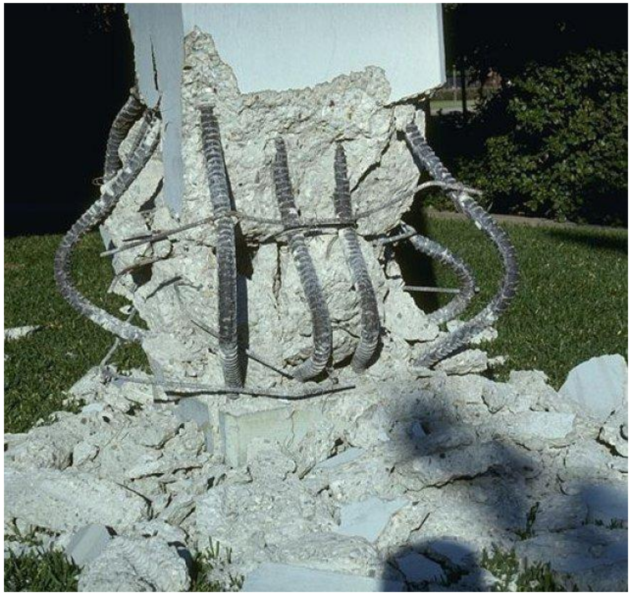
Compression Members

The failure of members in compression are due either to the load exceeding the ultimate strength in compression (crushing) or due to buckling under the load, because the applied load is larger than the critical buckling load.



Long members are referred to as columns. Columns, normally fail in buckling.

Compression Members

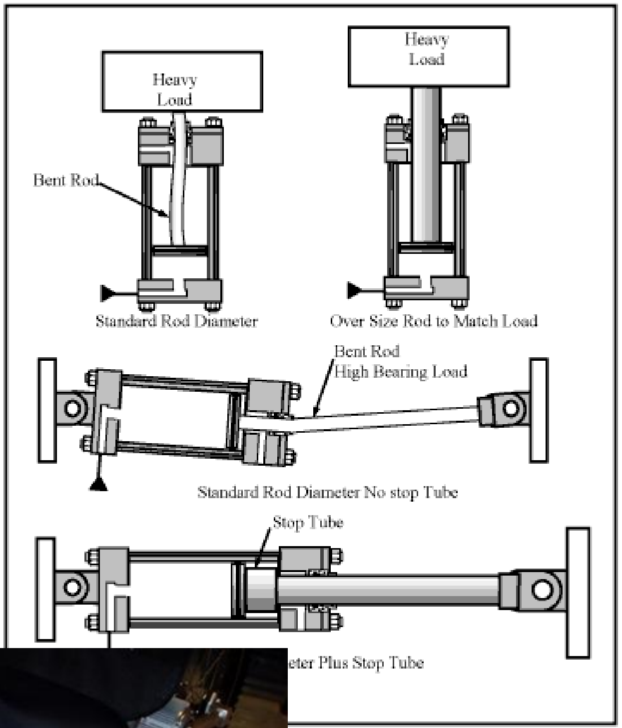


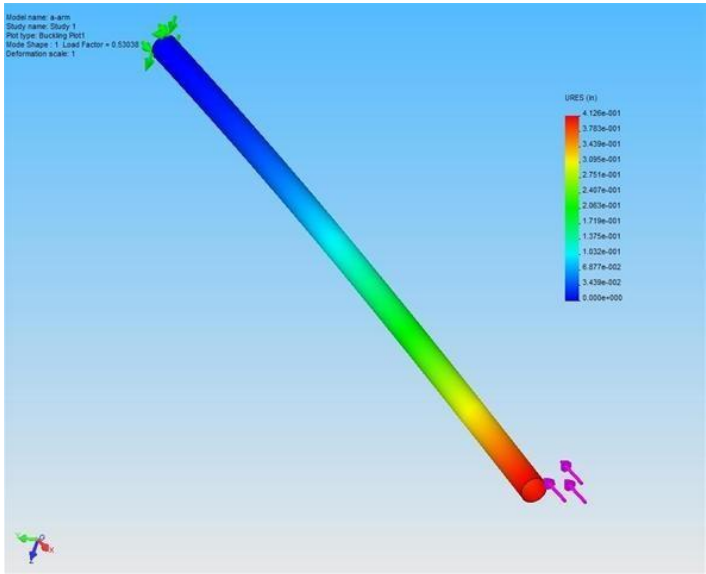
Crushing failure – 1985 Mexico earthquake.

Buckling failure

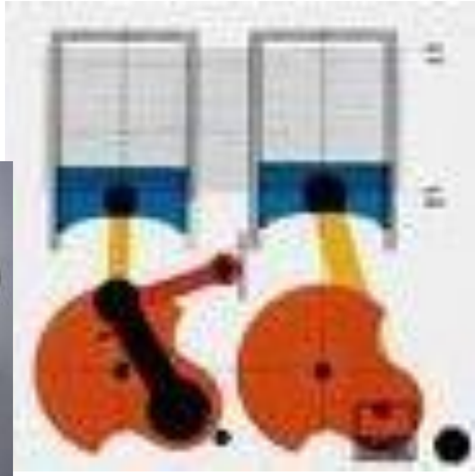
Compression Members

Actuators





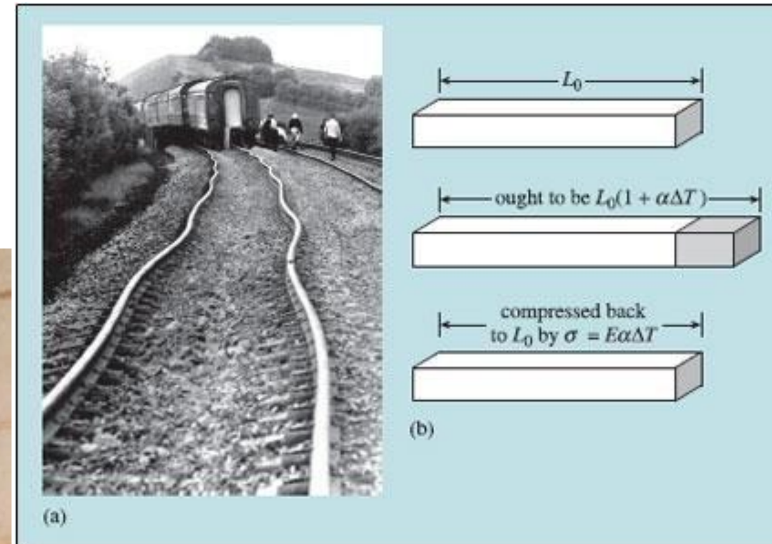
Members in compression



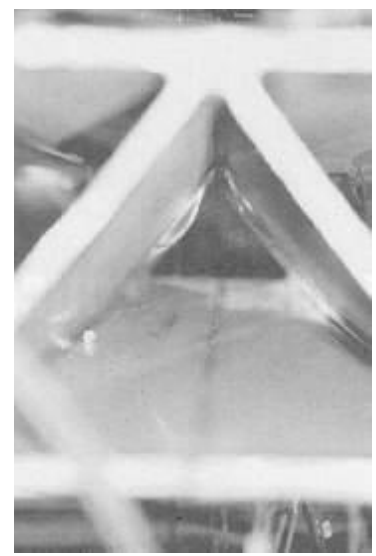
Connecting rods

Compression Members

Trusses



Structures

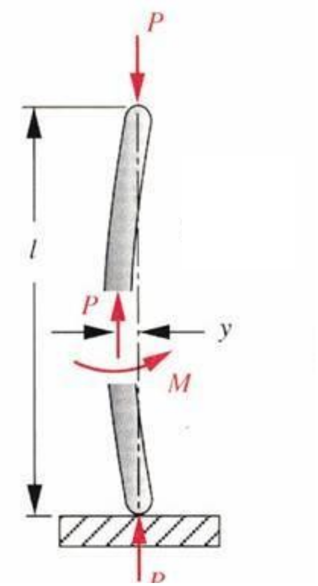


Column Design – Euler Column

Euler formula

Euler column – both ends are pinned or rounded

$$M = -Py$$



From deflection of beam, relating curvature to the moment, we have:

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

Rearranging the terms:

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = 0$$

Second order, linear, and homogeneous differential equation

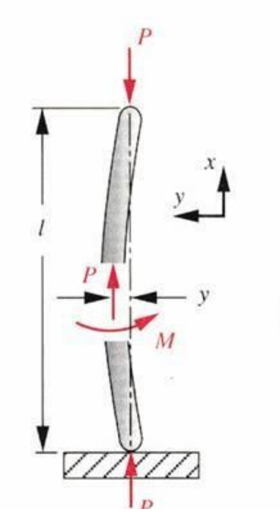
$$y = A\sin\left(\sqrt{\frac{P}{EI}}x\right) + B\cos\left(\sqrt{\frac{P}{EI}}x\right)$$

Column Design – Euler Column

Boundary conditions

No deflections at the ends

$$\begin{cases} y = 0 \; at \; x = 0 \\ y = 0 \; at \; x = l \end{cases}$$



$$y = A \sin\left(\sqrt{\frac{P}{EI}}x\right) + B\cos\left(\sqrt{\frac{P}{EI}}\,x\right)$$

Applying the first boundary condition:

$$0 = (0) + B(1) \longrightarrow B = 0$$

Applying the second boundary condition:

$$0 = A \sin\left(\sqrt{\frac{P}{EI}}l\right)$$

For a nontrivial solution, *A* cannot be zero. Therefore:

$$0 = \sin\left(\sqrt{\frac{P}{EI}}\;l\right) \longrightarrow \sqrt{\frac{P}{EI}}\;l = n\,\pi,$$

Where n is an integer, n =1, 2, 3, ...

Column Design – Euler Column

The smallest load occurs when n = 1, therefore,

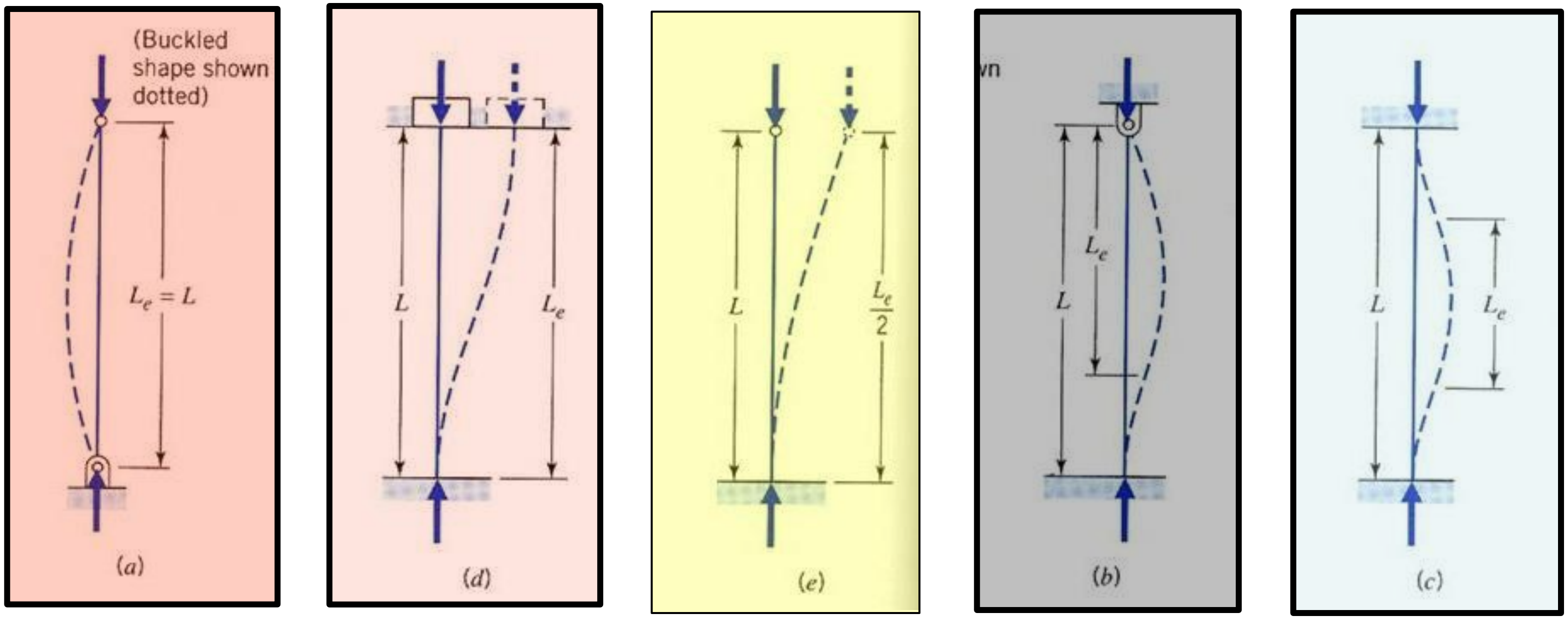
$$\sqrt{\frac{P}{EI}}\, l = \pi$$

$$\boxed{P_{\text{critical}} = \frac{\pi^2 E\, I}{l^2}}$$

Note: the strength of a material has no influence on the critical load, only the *modulus of elasticity* effects the critical load

Euler Column – End Conditions and Effective Length

Euler's
end
condition



Pinned-Pinned, or rounded-rounded | Fixed-sliding | Fixed-Free | Fixed-pinned | Fixed-Fixed

A I
S C

End Conditions	Theoretical Value	AISC* Recommended
Rounded-Rounded	$l_{eff} = l$	$l_{eff} = l$
Pinned-Pinned	$l_{eff} = l$	$l_{eff} = l$

[redacted]

$$P_{cr} = \frac{\pi^2 E I}{(l_{eff})^2}$$

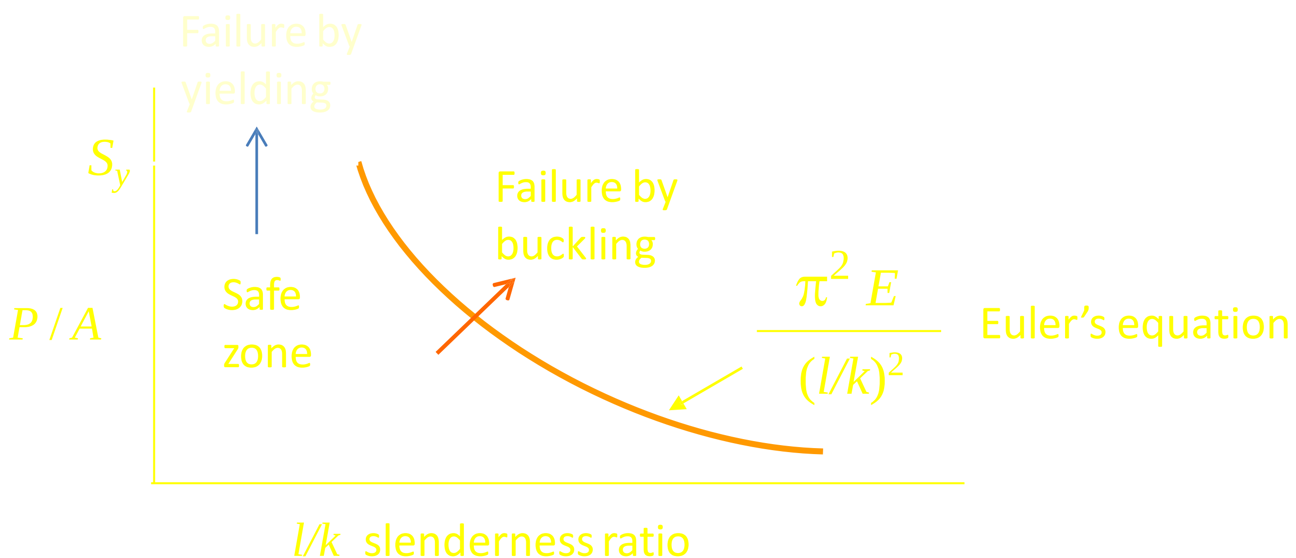
Euler Column – Slenderness Ratio, S_r

$I = A\,k^2$, where k = radius of gyration

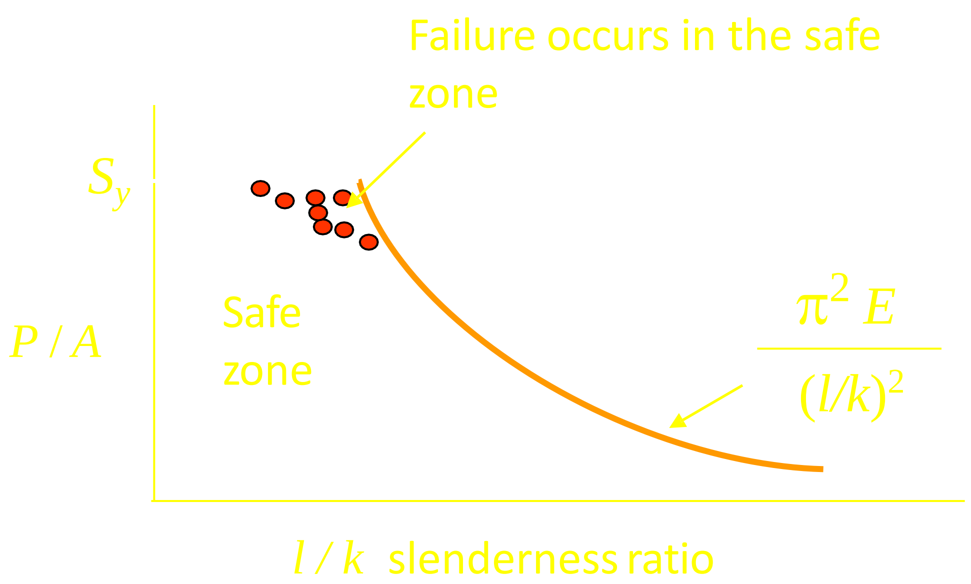
$$P_{\text{critical}} = \frac{\pi^2 E\, I}{l^2}$$

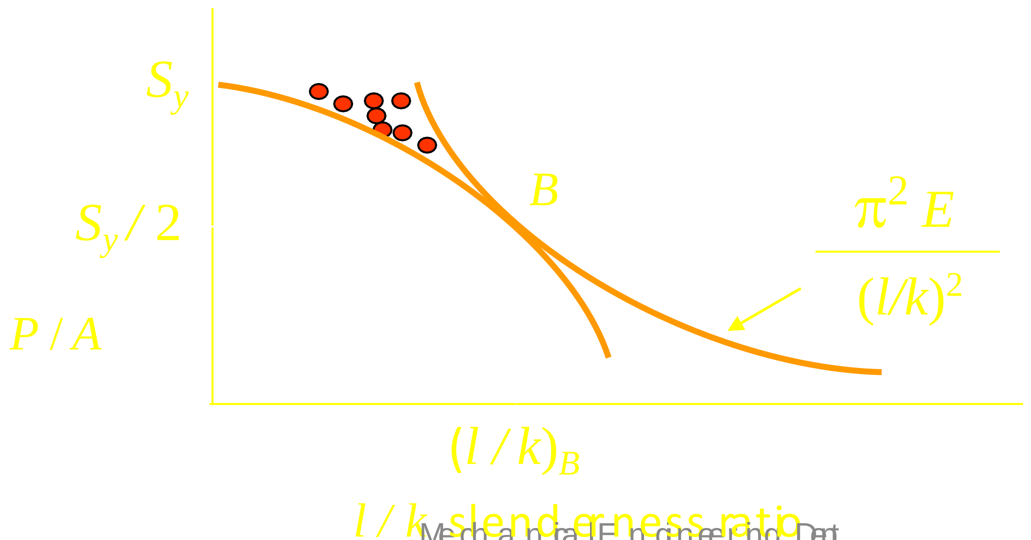
$$(P_{\text{critical}} / A) = \frac{\pi^2 E}{(l/k)^2}$$

Design graph



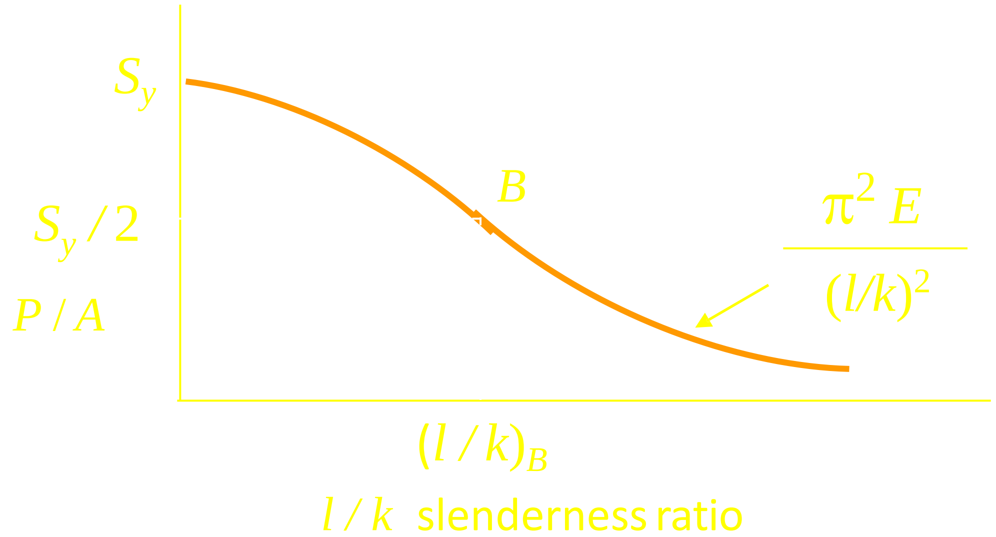
Euler Column – Design Curve





Design Curve – Johnson's Equation

Point B is also on the Euler's equation

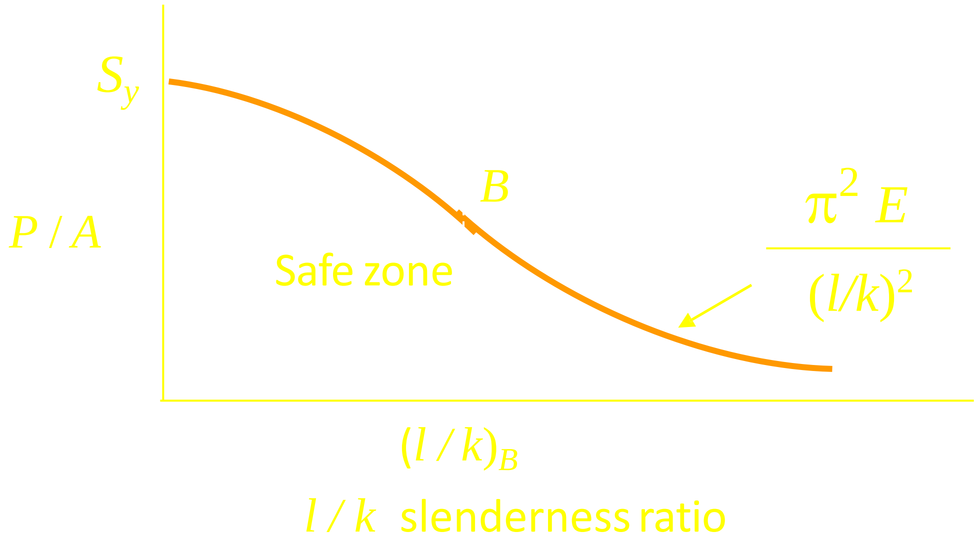


$$(P_{cr}/A) = \frac{\pi^2 E}{(l/k)^2}, \quad (S_y/2) = \frac{\pi^2 E}{(l/k)_B^{\,2}}$$

$$\boxed{(l/k)_B = \left(\frac{2K\pi^2 E}{S_y}\right)^{1/2}}$$

K depends on the end condition

Design Curve – Johnson's Equation



Johnson equation for short columns

$$(P_{\text{critical}} / A) = a - b\,(l/k)^2$$

Boundary conditions,

$$\begin{cases} P / A = S_y \quad at \quad l / k = 0 \\ P / A = S_y / 2 \quad at \quad l / k = (l / k)_B \end{cases}$$

Design Curve – Johnson's Equation

Applying the boundary condition,

$$(P_{critical} / A) = a - b\,(l/k)^2$$

$$a = S_y \quad and \quad b = (S_y / 2) / (l / k)_B$$

$$(l/k)_B = \left(\frac{2K\pi^2 E}{S_y}\right)^{1/2}$$

$$\boxed{(P_{critical} / A) = S_y - (S_y / 2\pi)^2 (1/KE)\,(l/k)^2}$$

K depends on the end condition

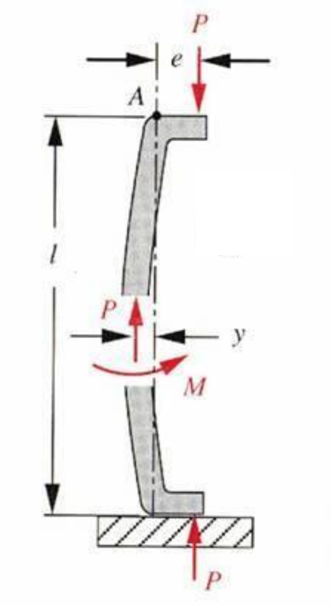
Column Design – Eccentric Loading

The Secant Formula

$$M + Py + Pe = 0$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = -\frac{Pe}{EI}$$

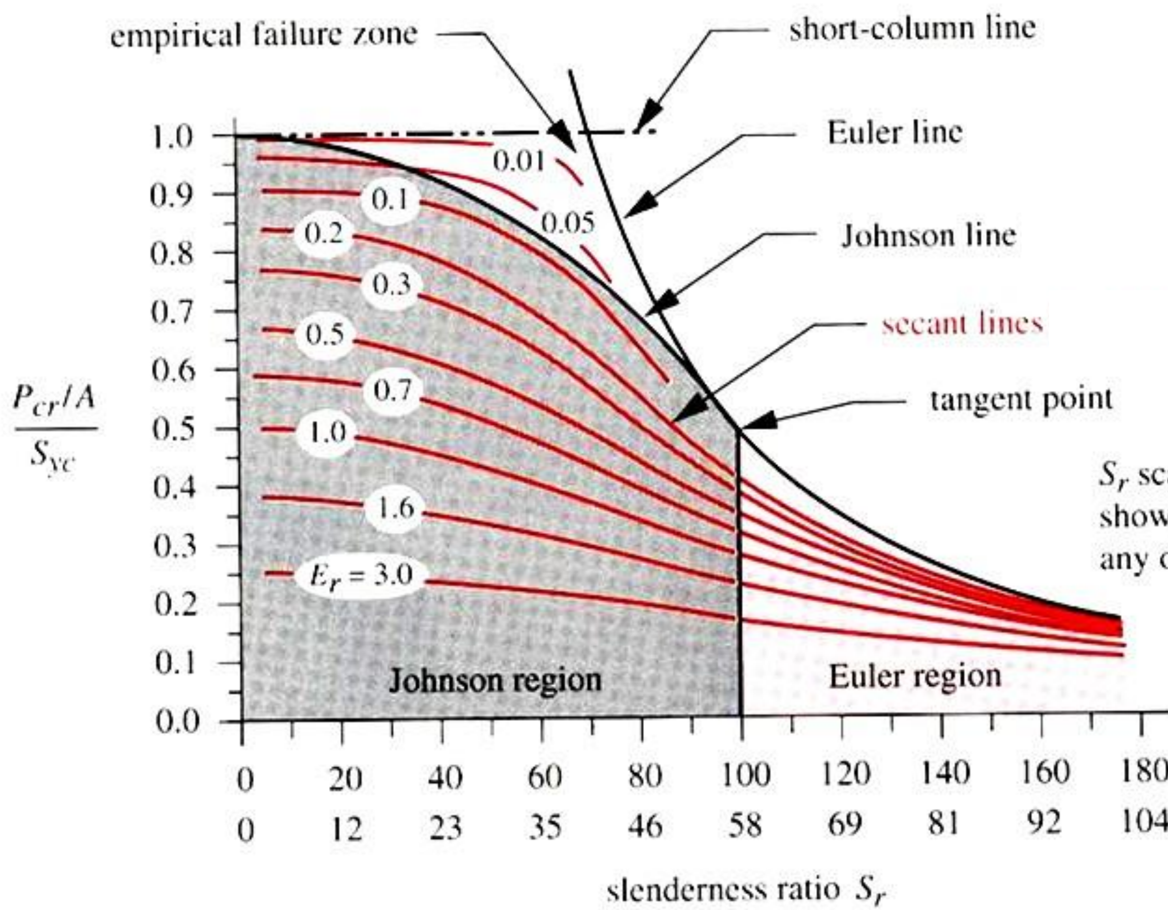
$$\boxed{(P_{cr} / A) = \frac{S_y}{1 + (eK / k^2)\ \mathrm{Sec}\left[(l/k)\,(P / 4AE)^{1/2}\right]}}$$



The secant column formula

Where (eK / k^2) is called the *eccentricity ratio*

Column Design Curve



S_r scales for steel and aluminum are shown. To convert the horizontal scale for any other material's E, apply the ratio

$$\frac{S_{r_1}}{S_{r_2}} = \sqrt{\frac{E_1}{E_2}}$$

E = 30 Mpsi = 207 GPa

E =10.4 Mpsi = 71.7 GPa

Example – Column Design

Design a column to carry a central load of 3600 lb. The column has to be 15” long. Due to space limitation the largest dimension cannot exceed 1.0 inch. The column will be welded at both ends.

Select material → 1035 CD steel → $E = 30\text{x}10^6$ psi, and $S_y = 67{,}000$ psi

Select cross section → tube with outside diameter not to exceed 1.0”

Choose a safety factor → $n = 4$

Select thickness and calculate the outside diameter to obtain safety factor of 4.

Johnson equation

$$\boxed{(P_{critical} / A) = S_y - (S_y / 2\pi)^2 (1/KE) (l/k)^2}$$

$$(P_{cr} / A) = 67000 - (67000/ 2\pi)^2 (1/30\text{x}10^6) (l/k)^2$$

$$(P_{cr} / A) = 67000 - 3.79 (l/k)^2$$

Example – Column Design

$$(P_{cr} / A) = \frac{\pi^2 E}{(l/k)^2} = \frac{2.96\text{x}10^8}{(l/k)^2}$$

$$(l/k)_B = \left(\frac{2K\pi^2 E}{S_y}\right)^{1/2} = \left(\frac{2 \text{ x } 1 \text{ x } \pi^2 \text{ x } 30\text{x}10^6}{67000}\right)^{1/2} = 94$$

$$A = \pi/4\,(d_o^2 - d_i^2)$$

$$I = \pi/64\,(d_o^4 - d_i^4)$$

$$k = (I/A)^{1/2} = [\,(d_o^2 + d_i)^2 /\ 16]^{1/2}$$

$$d_i = d_o - 2t$$

Example – Column Design

Select thickness $t = 3/16$

Use Euler eq.

d_o	d_i	A	k	l/k	P_{cr}	$n = (P_{cr} / 3600) = 4$
0.5	0.125	0.185	0.1288	$116 > 94$	4032	$1.12 < 4$
1.0	0.625	0.479	0.295	$50 < 94$	27400	$7.4 > 4$
0.75	0.375	0.337	0.210	$71 < 94$	16060	$4.46 \approx 4$

Use Johnson eq.

Specify, 1035 CD steel tube with outside diameter of 3/4” and thickness of 3/16”

Example – Column Design

Consider a solid bar

Johnson equation

$$(P_{cr} / A) = 67000 - 3.79\,(l/k)^2$$

$$\frac{16060}{\pi/4\,(d)^2} = 67000 - 3.79\left(\frac{15}{d/4}\right)^2$$

$$\boxed{d = .713,\ \ \text{select } d = .75}$$

$$\text{Weight ratio} = \frac{d^2}{d_o^2 - d_i^2} = \frac{(.713)^2}{(.75)^2 - (.375)^2} = 1.2$$

Solid bar is 20% heavier

Biaxial Bending

Biaxial bending in columns

As the position of live load on a floor varies, building columns may be subject to loading patterns that produce biaxial bending, i.e., bending about both principal axes of the cross section. Nevertheless, to simplify design, code provisions specify loading patterns that produce uniaxial bending in most building columns. Corner columns, routinely designed for biaxial bending, are the exception.

If the moments in the weak direction (y axis here) are rather small compared to bending in the strong direction (x axis), it is rather common to neglect the smaller moment. This practice is probably reasonable as long as e_y is less than about 20% of e_x,

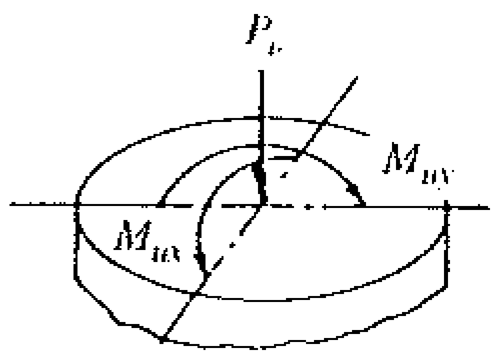
Biaxial bending in columns

Circular columns have polar symmetry and thus the same ultimate capacity in all directions. The design process is the same, therefore, regardless of the directions of the moments. If there is bending about both the x and y axes, the biaxial moment can be computed by combining the two moments or their eccentricities as follows:

$$M_u = \sqrt{(M_{ux})^2 + (M_{uy})^2}$$

or

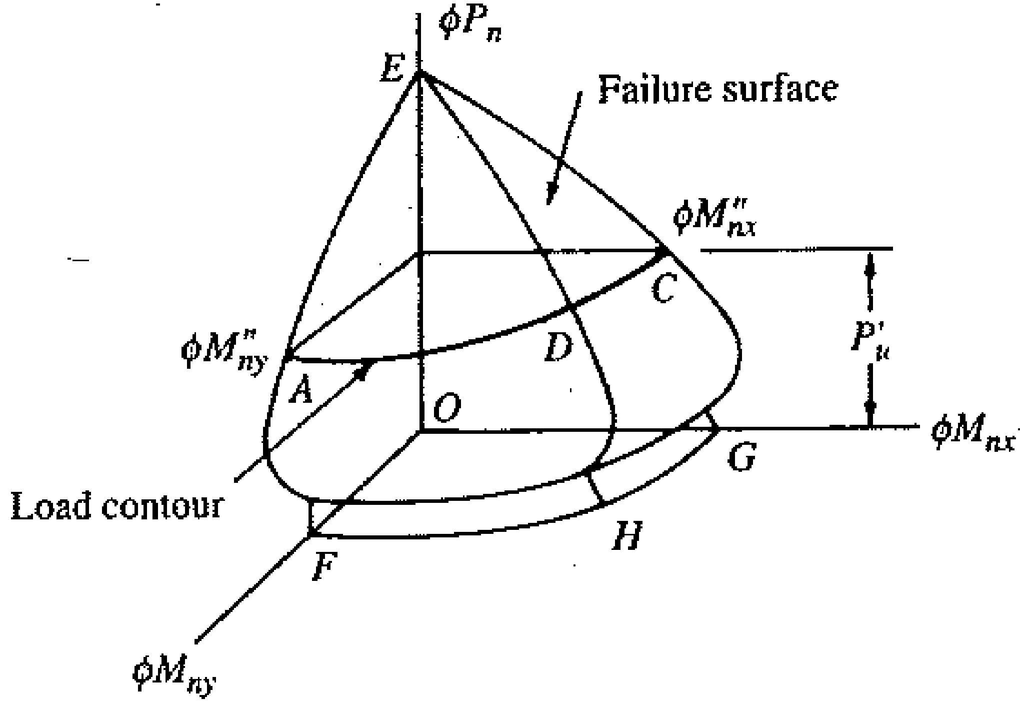
$$e = \sqrt{(e_x)^2 + (e_y)^2}$$

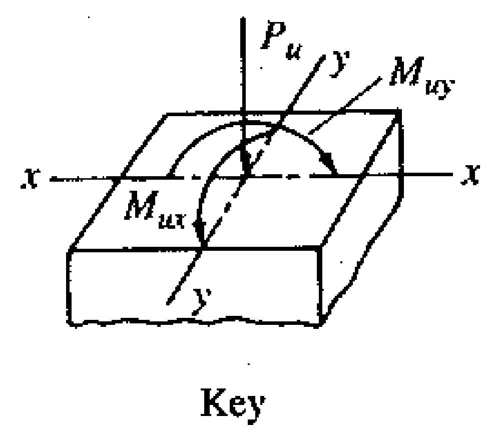


Biaxial bending in columns

For shapes other than circular ones, it is necessary to consider the three-dimensional interaction effects. Whenever possible, it is desirable to make columns subject to biaxial bending circular in shape. Should it be necessary to use square or rectangular columns for such cases, the reinforcing should be placed uniformly around the perimeters.

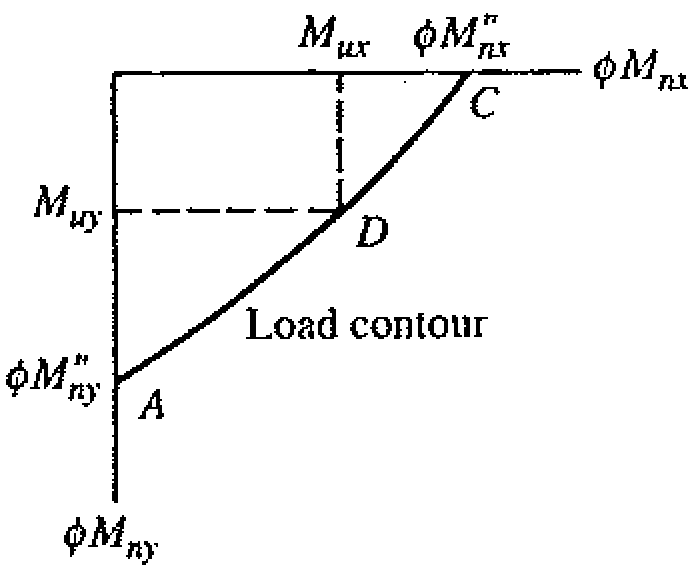
The design procedure presented in this section is based on expressing the required uniaxial bending strength about each principal axis in terms of the design moments about both principal axes. The relationship can be developed by using the three-dimensional failure surface generated by interaction curves for various ratios of biaxial eccentricity. In Fig. 7.67 ECG, the

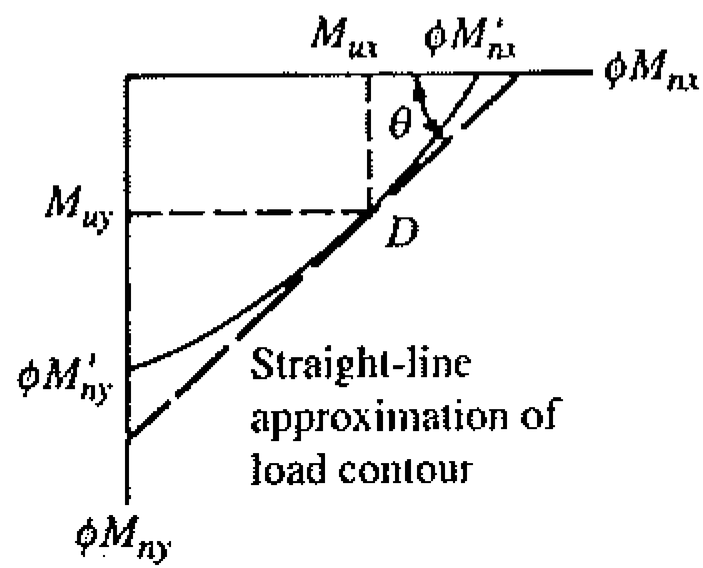




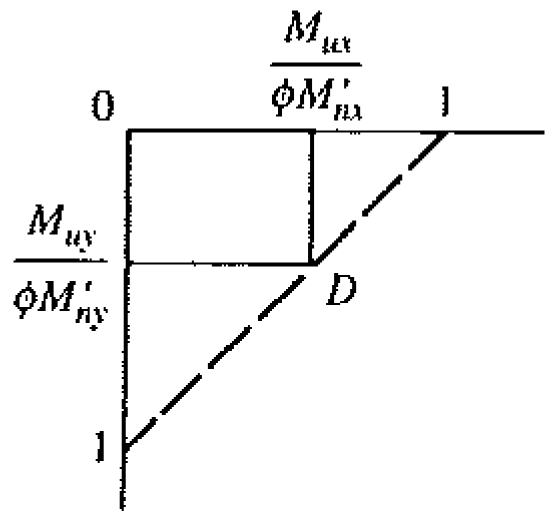
Biaxial bending in columns

A design procedure[8] using the failure surface can be developed by considering the variation of the biaxial moment capacity at a particular value of axial load. For example, curve *ADC* in Fig. 7.67, cut by a horizontal plane that lies a vertical distance P'_u above the plane of the horizontal reference axes, represents combinations of moment M_{ux} and M_{uy} that produce failure when the cross section is loaded by the axial force P'_u. A plan view of curve *ADC*, referred to as a *load contour*,





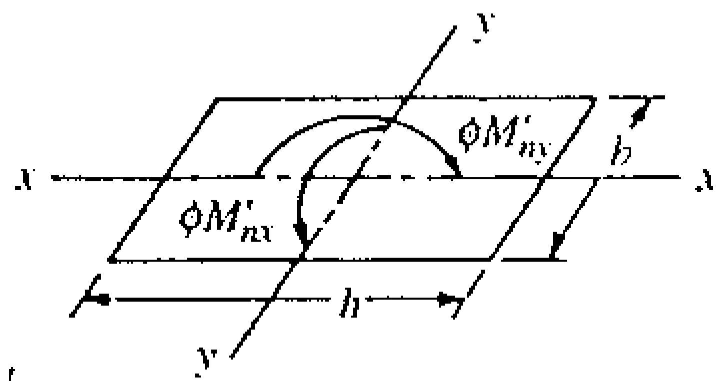
Biaxial bending in columns



Nondimensional plot of linearized load contour.

$$\frac{M_{ux}}{\phi M'_{nx}} + \frac{M_{uy}}{\phi M'_{ny}} = 1$$

Biaxial bending in columns



$$\frac{M_{ux}}{\phi M'_{nx}} + \frac{M_{uy}}{\phi M'_{ny}} = 1$$

$$M_{ux} + M_{uy}\frac{\phi M'_{nx}}{\phi M'_{ny}} = \phi M'_{nx}$$

If the restriction is imposed that the rectangular cross section be reinforced with the same area of steel along each of the four sides, the moment capacities about each principal axis will be approximately proportional to the dimensions of the sides.

$$\frac{\phi M'_{nx}}{\phi M'_{ny}} = \frac{b}{h} \tag{7.43}$$

$$\boxed{\phi M'_{nx} = M_{ux} + M_{uy}\frac{b}{h}} \tag{7.44}$$

If a square column is designed, $b = h$

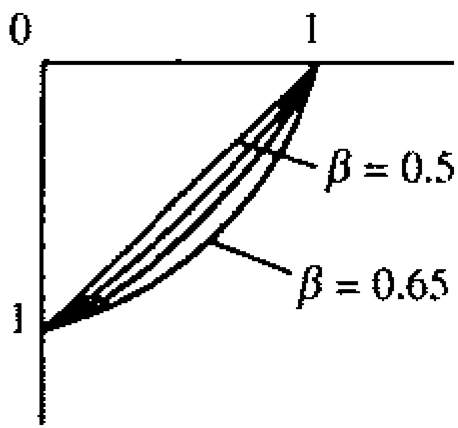
$$\boxed{\phi M'_{nx} = M_{ux} + M_{uy}} \tag{7.45}$$

Biaxial bending in columns

A more accurate estimate of $\phi M'_{nx}$ is possible if the assumption is made that the load contour is a curve. The result is given by

$$\boxed{\phi M'_{nx} = M_{ux} + M_{uy}\frac{b}{h}\frac{1-\beta}{\beta}} \qquad (7.46)$$

where β is related to the curvature of the nondimensionalized load contour (see Fig. 7.71). Charts[6] for evaluating β give values between 0.55 and 0.65 for most columns. For a value of $\beta = 0.5$, Eq. (7.46) reduces to Eq. (7.44).



Biaxial bending in columns

The Bresler Equation

Since the design of columns by the procedure discussed in Sec. 7.14 is approximate, the capacity of the column should be verified to determine if the steel area is adequate. The axial capacity of columns can be checked by the use of the Bresler equation.[11]

$$\boxed{\frac{1}{P_n} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_0}} \tag{7.47}$$

where P_n = theoretical axial capacity of column under biaxial bending

P_{nx} = theoretical axial capacity if bending occurs about x-x axis only; $e_x = 0$ (see Fig. 7.72)

P_{ny} = theoretical axial capacity if bending occurs about the y-y axis only; $e_y = 0$ (see Fig. 7.72)

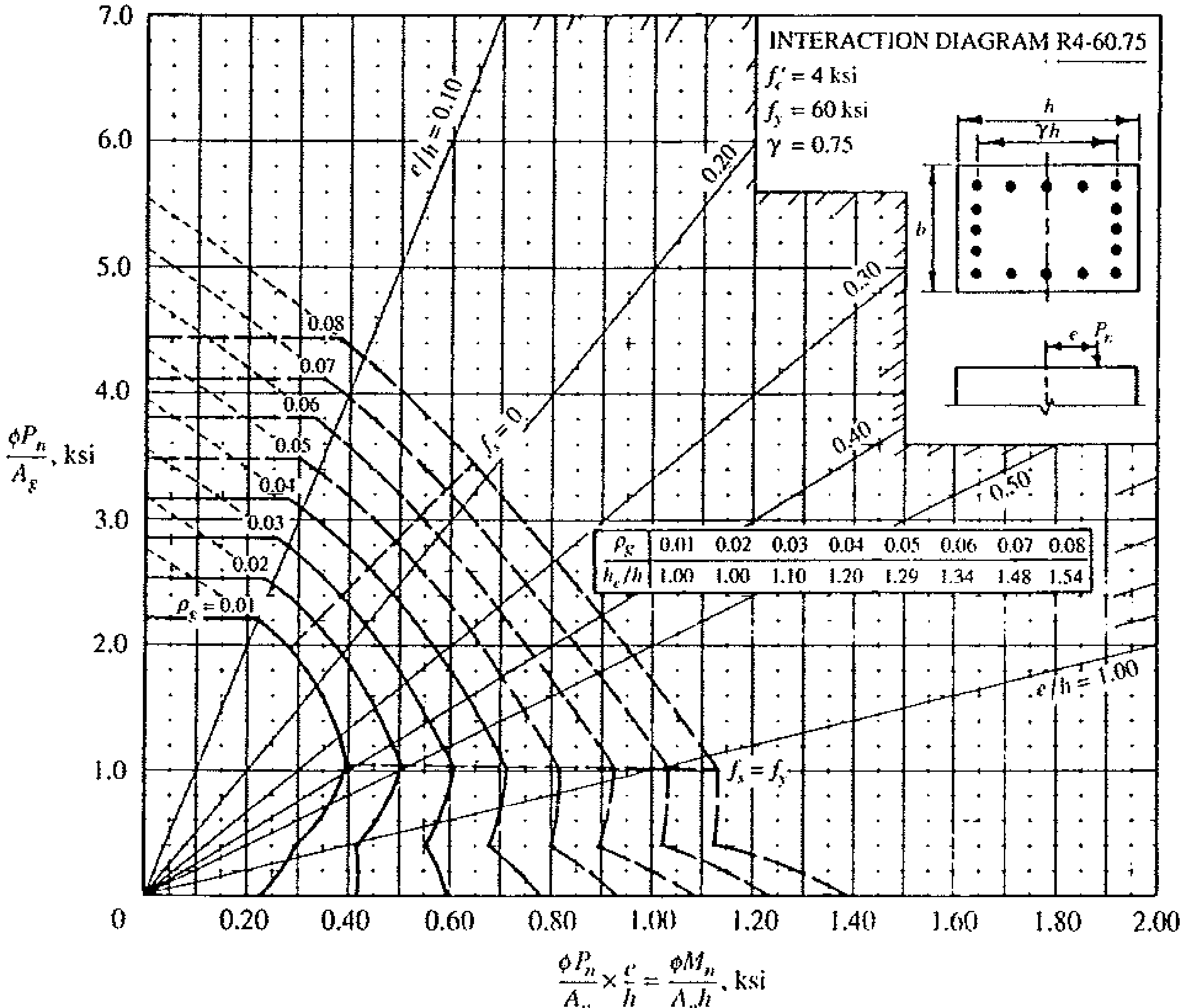
P_0 = theoretical axial capacity of column carrying pure axial load only; use Eq. (7.24)

Tests have shown that Eq. (7.47) predicts the strength of columns under biaxial bending with excellent accuracy. The equation is limited to cases where P_n is equal to or greater than $0.1P_0$.

$$P_0 = A_{st} f_y + (A_g - A_{st})(0.85 f_c')$$

Biaxial bending in columns

Interaction diagrams for biaxial bending



Biaxial bending in columns

Design Procedure for biaxial bending:

A. Determine reinforcement based on the biaxial bending capacity:

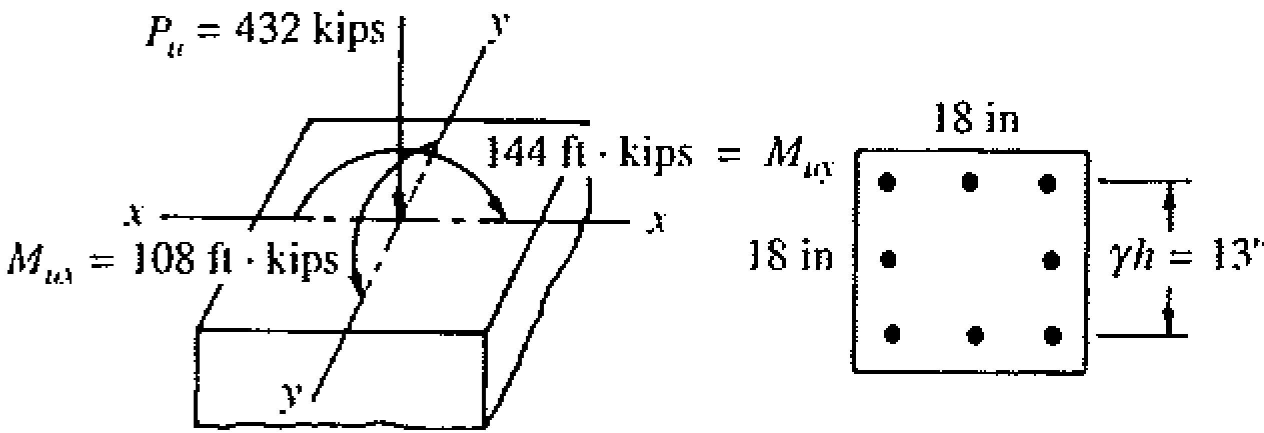
1Determine the dimensions based on a reasonable stress in the column.
2Determine γ in the week axis direction (any direction for square sections.
3- Calculate the biaxial bending moment in the week axis direction.
4- Use an interaction diagram to design the reinforcement for the section.

B- Use the Bresler equation to check the axial capacity of the section

1- Calculate P_{nx} from the interaction diagram assuming only M_{nx} is applied.
2- Calculate P_{ny} from the interaction diagram assuming only M_{ny} is applied.
3- Calculate P_0.
4- Calculate P_n and check $\phi P_n \geq P_u$

Biaxial bending in columns

EXAMPLE 7.23. (*a*) Using the straight-line approximation of the load contour given by Eq. (7.45), design a short square column to support design loads of $P_u = 432$ kips, $M_{ux} = 108$ ft · kips, and $M_{uy} = 144$ ft · kips. (*b*) Check results with Eq. (7.47). $f_c' = 4$ kips/in^2, and $f_y = 60$ kips/in^2. Use equal areas of steel on all four sides (see Fig. 7.73).



Biaxial bending in columns

Solution. (*a*) Estimate a trial area. Assume the average compressive stress to be $0.3f_c'$.

$$A_g = \frac{P_u}{0.3f_c'} = \frac{432 \text{ kips}}{1.2} = 360 \text{ in}^2$$

Try 18 by 18 in, giving $A_g = 324 \text{ in}^2$. Then assuming $\gamma h = 13$ in,

$$\gamma = \frac{13}{18} = 0.722$$

Using Eq. (7.45), select the steel based on the required bending strength about the y-y axis.

$$\phi M'_{nx} = M_{ux} + M_{uy} = 108 + 144 = 252 \text{ ft} \cdot \text{kips}$$

Using Fig. 7.43, select ρ_g; set $\phi P_n = P_u$.

$$\frac{\phi P_n}{A_g} = \frac{432}{324} = 1.333$$

$$e = \frac{\phi M'_{nx}}{P_u} = \frac{252(12)}{432} = 7 \text{ in} \qquad \frac{e}{h} = \frac{7}{18} = 0.389$$

Read $\rho_g = 0.025$; then

$$A_{st} = \rho_g A_g = 0.025(324) = 8.1 \text{ in}^2$$

Use eight no. 9s; $A_{st,\text{sup}} = 8 \text{ in}^2$.

Biaxial bending in columns

(*b*) Use Eq. (7.47) to check the capacity of the cross section.

$$\frac{1}{P_n} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_0}$$

For P_{ny} consider bending about y-y axis, $M_{uy} = 144$ ft · kips.

$$e = \frac{144(12)}{432} = 4 \text{ in} \qquad \frac{e}{h} = \frac{4}{18} = 0.222$$

For $\rho_g = 0.025$ and $e/h = 0.222$, read from Fig. 7.43

$$\frac{\phi P_{ny}}{A_g} = 2 \qquad \text{then} \qquad P_{ny} = \frac{2(324)}{0.7} = 925.7 \text{ kips}$$

For P_{nx}, consider bending about the x-x axis, $M_{ux} = 108$ ft · kips

$$e = \frac{108(12)}{432} = 3 \text{ in} \qquad \frac{e}{h} = \frac{3}{18} = 0.167$$

For $\rho_g = 0.025$ and $e/h = 0.167$, read from Fig. 7.43

$$\frac{\phi P_{nx}}{A_g} = 2.22 \qquad \text{then} \qquad P_{nx} = \frac{2.22(324)}{0.7} = 1027.5 \text{ kips}$$

$$P_0 = A_{st} f_y + (A_g - A_{st})(0.85 f_c') = (8 \text{ in}^2)(60 \text{ kips/in}^2) + (324 - 8)(3.4 \text{ kips/in}^2)$$
$$= 1554.4 \text{ kips}$$

Biaxial bending in columns

Substituting into the Bresler equation gives

$$\frac{1}{P_n} = \frac{1}{925.7} + \frac{1}{1027.5} - \frac{1}{1554.4} \tag{7.47}$$

$$P_n = 709 \text{ kips} \qquad P_n > 0.1 P_0$$

$$\phi P_n = 0.7(709 \text{ kips}) = 496.3 \text{ kips} > 432 \text{ kips}$$

The design is OK, but A_{st} can be reduced.

Biaxial bending in columns

EXAMPLE 7.24. (*a*) Repeat the design of the column in Example 7.23 using Eq. (7.46) and a value of $\beta = 0.65$ (the value recommended by Ref. 6). (*b*) Check results with Eq. (7.47).

Solution. (*a*) Compute the required moment capacity $\phi M'_{nx}$:

$$\phi M'_{nx} = M_{ux} + \frac{b}{h}\frac{1-\beta}{\beta}M_{uy}$$

For a square column $h = b$ and

$$\phi M'_{nx} = 108 + \frac{1-0.65}{0.65}144 = 185.5 \text{ ft} \cdot \text{kips}$$

Using Fig. 7.43 select ρ_g. Setting $\phi P_n = P_u$

$$\frac{\phi P_n}{A_g} = \frac{432}{324} = 1.333 \text{ kips/in}^2 \qquad \gamma = \frac{13}{18} = 0.722$$

$$e = \frac{\phi M'_{nx}}{P_u} = \frac{185.5(12)}{432} = 5.15 \text{ in} \qquad \frac{e}{h} = \frac{5.15 \text{ in}}{18 \text{ in}} = 0.29$$

Read $\rho_g = 0.018$; then

$$A_{st} = \rho_g A_g = 5.83 \text{ in}^2$$

Use eight no. 8 bars:

$$A_{st,\text{ sup}} = 6.28 \text{ in}^2 \qquad \rho_g = \frac{A_{st}}{A_g} = 0.0193$$

Biaxial bending in columns

(*b*) Check using the Bresler equation. Compute P_{nx}.

$$e = \frac{108(12)}{432} = 3 \text{ in} \qquad \text{and} \qquad \frac{e}{h} = 0.167$$

with $\rho_g = 0.0193$ and $e/h = 0.167$, enter Fig. 7.43. Read $\phi P_{nx}/A_g = 2.1$; then $P_{nx} = 972$ kips. Similarly $P_{ny} = 833.1$ kips.

$$P_0 = A_{st} f_y + (A_g - A_{st})(0.85 f_c') = 1457 \text{ kips}$$

$$\frac{1}{P_n} = \frac{1}{972} + \frac{1}{833.1} - \frac{1}{1457} \tag{7.47}$$

$$P_n = 648 \text{ kips}$$

$$\phi P_n = 453.7 \text{ kips} > 432 \text{ kips} \qquad \text{OK}$$

Biaxial bending in columns

EXAMPLE 7.25. (*a*) Using Eq. (7.46) with a trial value of $\beta = 0.65$, redesign the column of Example 7.23 with a 16- by 20-in cross section. (*b*) Verify capacity with Eq. (7.47).

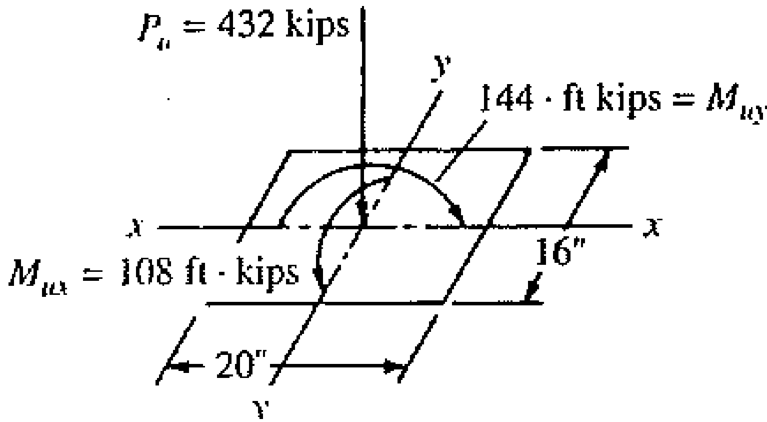


FIGURE 7.74

Solution. (*a*) Compute the bending capacity about the minor principal axis (Fig. 7.74).

$$\phi M'_{nx} = M_{ux} + \frac{b}{h}\frac{1-\beta}{\beta}M_{uy} = 108 + \frac{16}{20}\frac{1-0.65}{0.65}144 = 170\text{ ft}\cdot\text{kips}$$

Using ACI design aids, compute $\rho_g = 0.016$. Then $A_{st} = 5.12\text{ in}^2$. Use four no. 8 and four no. 7; $A_{st,\text{sup}} = 5.55\text{ in}^2$.
(*b*) Equation (7.47) gives $P_n = 634$ kips,

$$\phi P_n = 444\text{ kips} > 432\text{ kips} \qquad \text{OK}$$

UNIT-V
FOOTINGS

Lecture Goals

- Footing Classification
- Footing Design

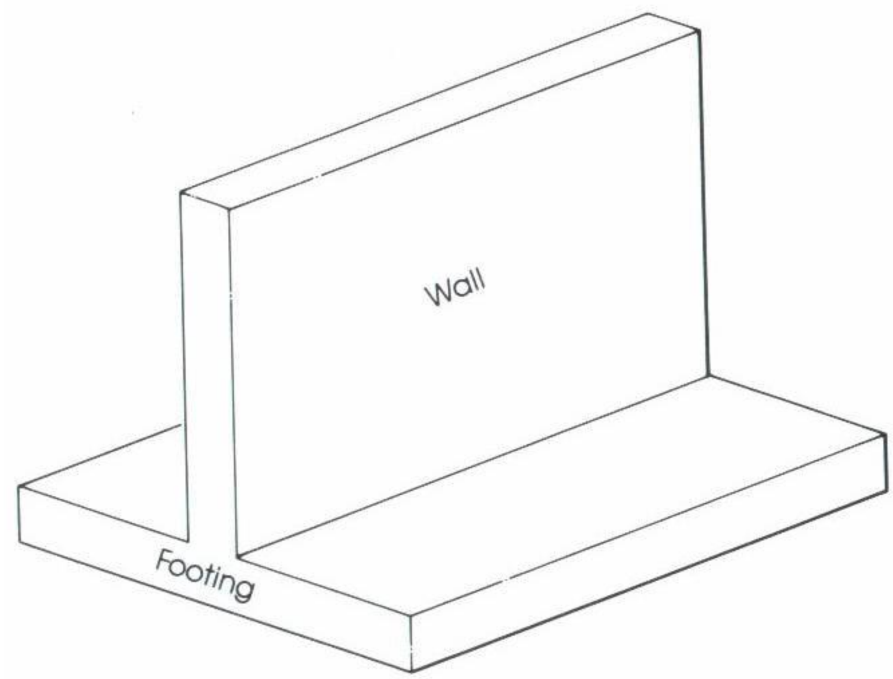
Footing

Definition

Footings are structural members used to support columns and walls and to transmit and distribute their loads to the soil in such a way that the load bearing capacity of the soil is not exceeded, excessive settlement, differential settlement,or rotation are prevented and adequate safety against overturning or sliding is maintained.

Types of Footing

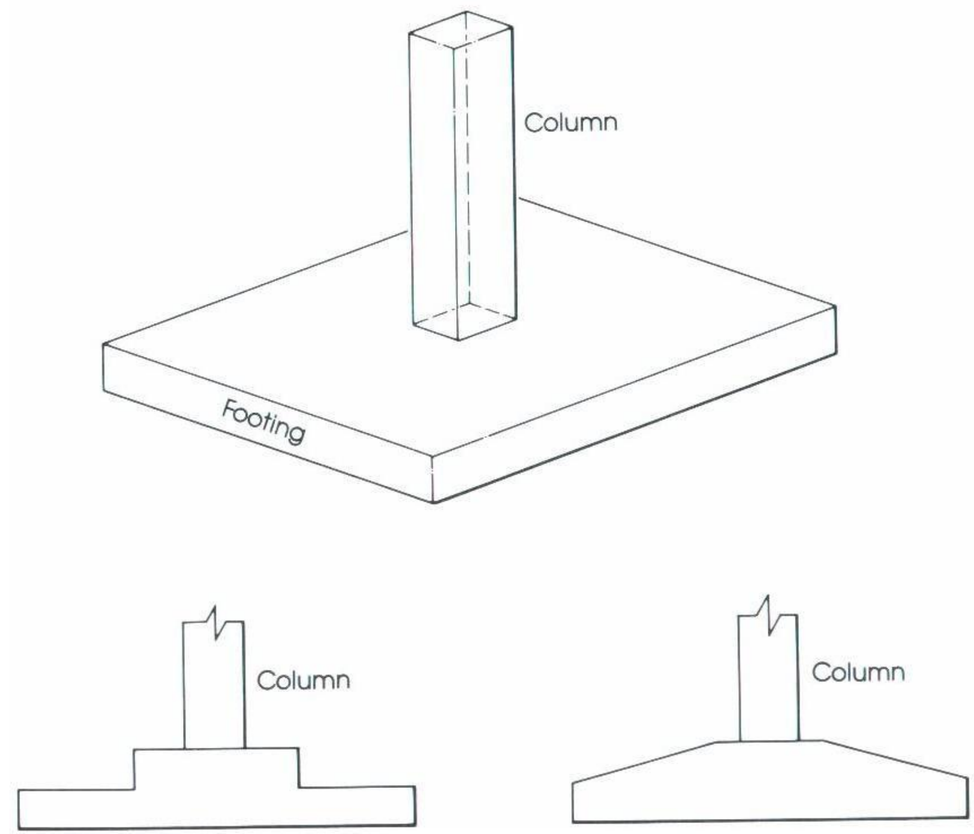
Wall footings are used to support structural walls that carry loads for other floors or to support nonstructural walls.



Wall footing.

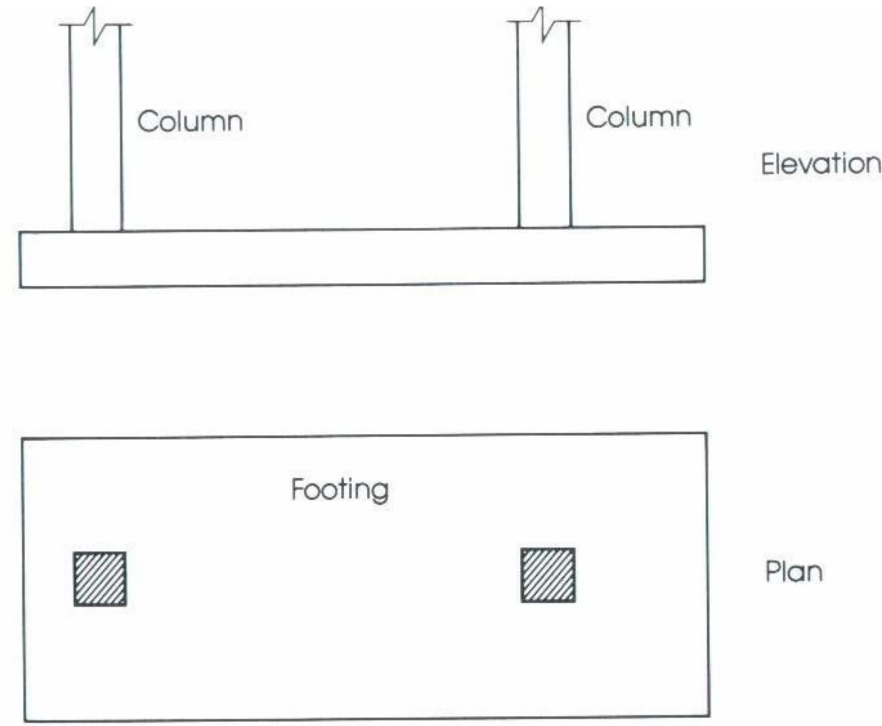
Types of Footing

Isolated or single footings are used to support single columns. This is one of the most economical types of footings and is used when columns are spaced at relatively long distances.



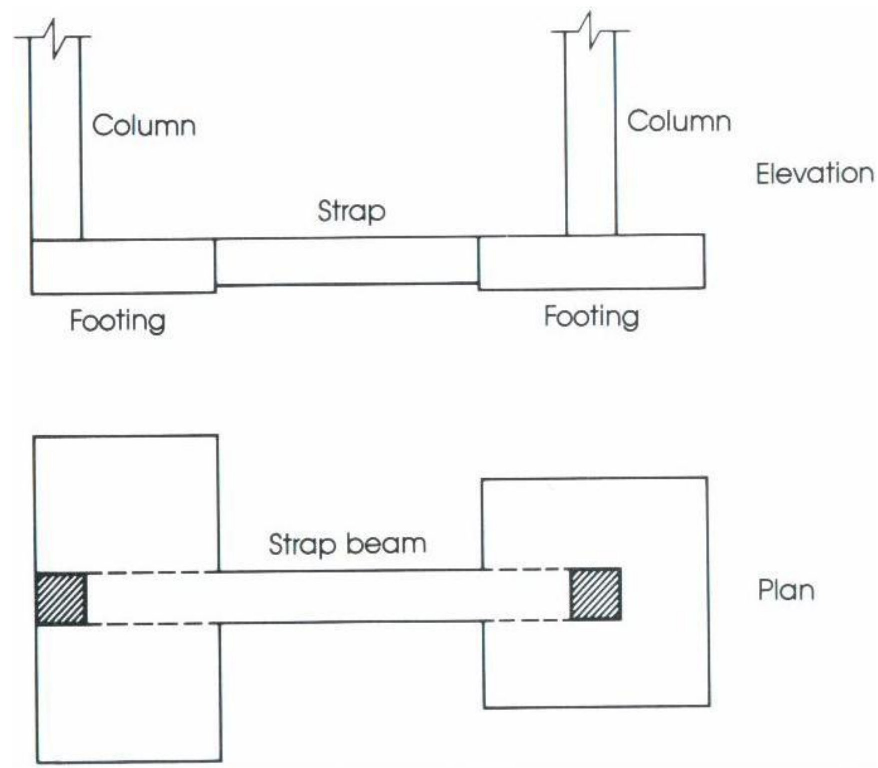
Types of Footing

Combined footings usually support two columns, or three columns not in a row. Combined footings are used when two columns are so close that single footings cannot be used or when one column is located at or near a property line.



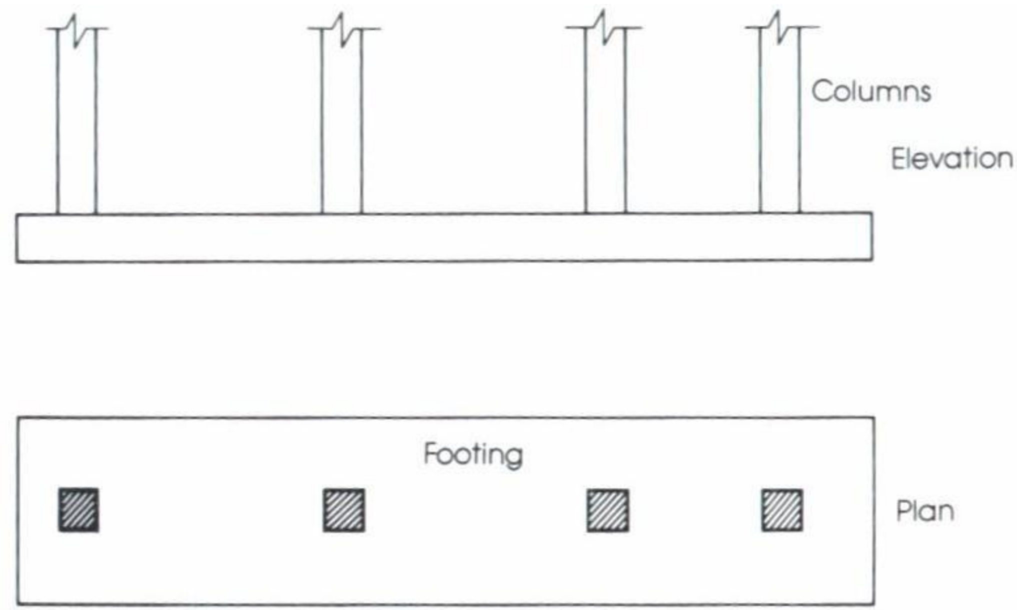
Types of Footing

Cantilever or strap footings consist of two single footings connected with a beam or a strap and support two single columns. This type replaces a combined footing and is more economical.



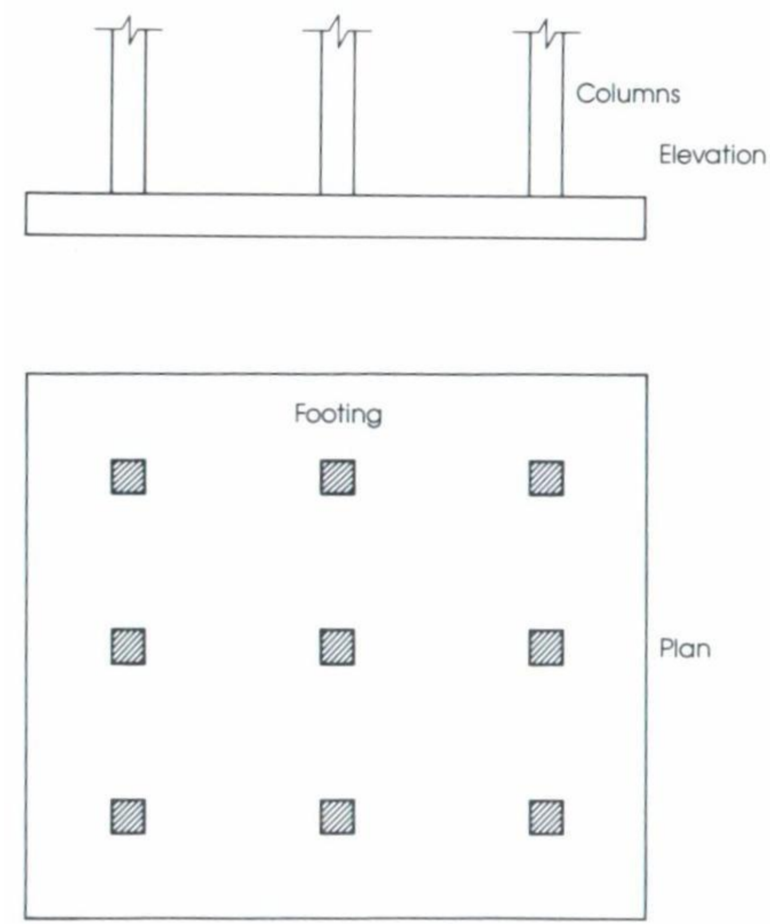
Types of Footing

Continuous footings support a row of three or more columns. They have limited width and continue under all columns.



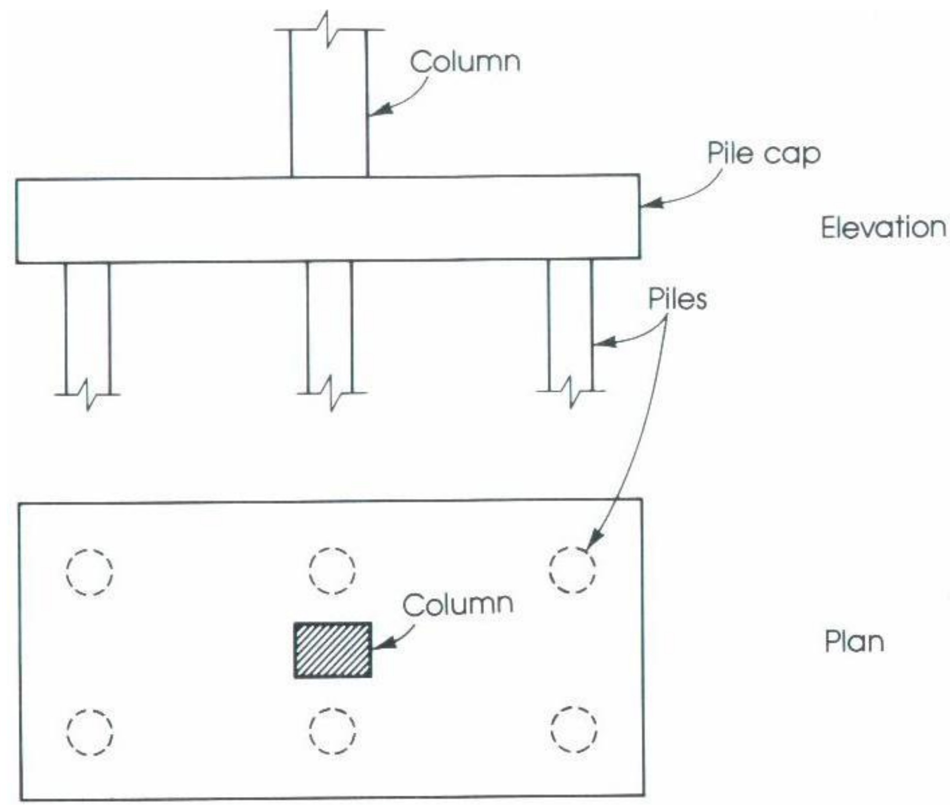
Types of Footing

Rafted or mat foundation consists of one footing usually placed under the entire building area. They are used, when soil bearing capacity is low, column loads are heavy single footings cannot be used, piles are not used and differential settlement must be reduced.



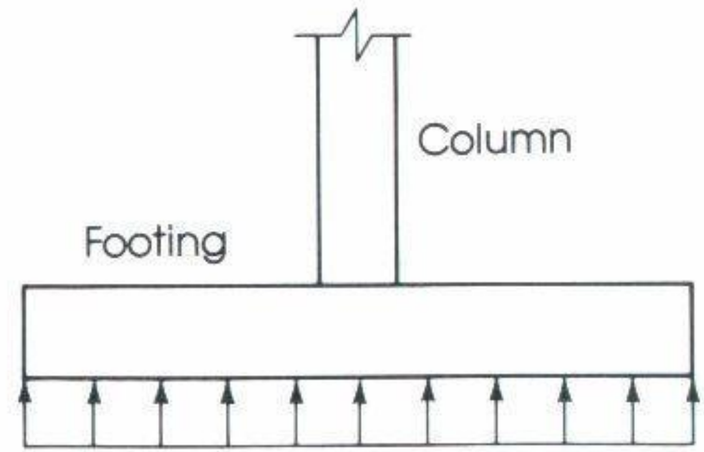
Types of Footing

Pile caps are thick slabs used to tie a group of piles together to support and transmit column loads to the piles.



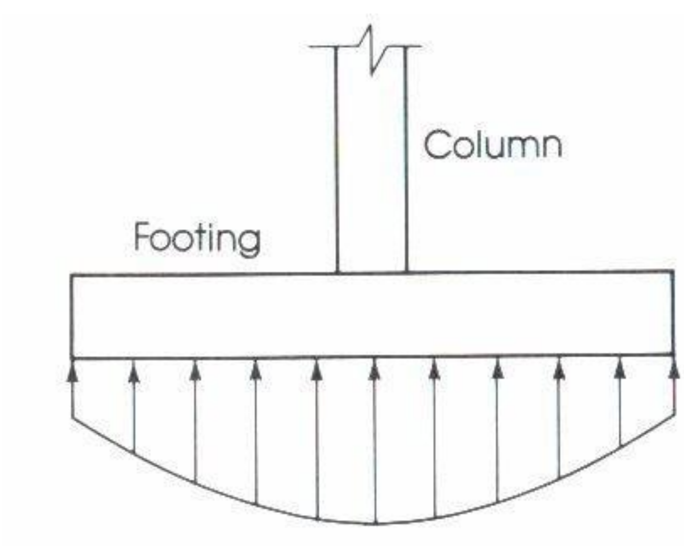
Distribution of Soil Pressure

When the column load P is applied on the centroid of the footing, a uniform pressure is assumed to develop on the soil surface below the footing area.

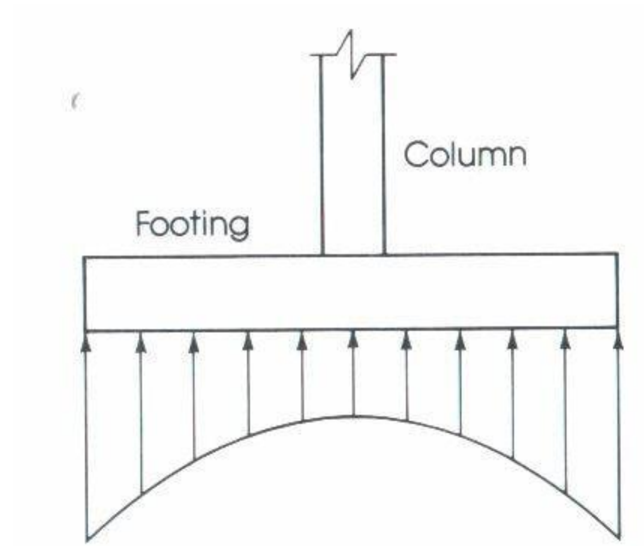


However the actual distribution of the soil is not uniform, but depends on may factors especially the composition of the soil and degree of flexibility of the footing.

Distribution of Soil Pressure



Soil pressure distribution in cohesionless soil.



Soil pressure distribution in cohesive soil.

Design Considerations

Footings must be designed to carry the column loads and transmit them to the soil safely while satisfying code limitations.

1. The area of the footing based on the allowable bearing soil capacity
2. Two-way shear or punch out shear.
3. One-way bearing
4. Bending moment and steel reinforcement required

Design Considerations

Footings must be designed to carry the column loads and transmit them to the soil safely while satisfying code limitations.

1. Bearing capacity of columns at their base
2. Dowel requirements
3. Development length of bars
4. Differential settlement

Size of Footing

The area of footing can be determined from the actual external loads such that the allowable soil pressure is not exceeded.

$$\text{Area of footing} = \frac{\text{Totalload}\left(\text{including self - weight}\right)}{\text{allowable soil pressure}}$$

Strength design requirements

$$q_{\mathrm{u}} = \frac{P_{\mathrm{u}}}{\text{area of footing}}$$

Two-Way Shear (Punching Shear)

For two-way shear in slabs (& footings) V_c is smallest of

$$V_c = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f_c}\, b_0 d$$

ACI 11-35

where, β_c = long side/short side of column concentrated load or reaction area < 2

b_0 = length of critical perimeter around the column

When β > 2 the allowable V_c is reduced.

Design of two-way shear

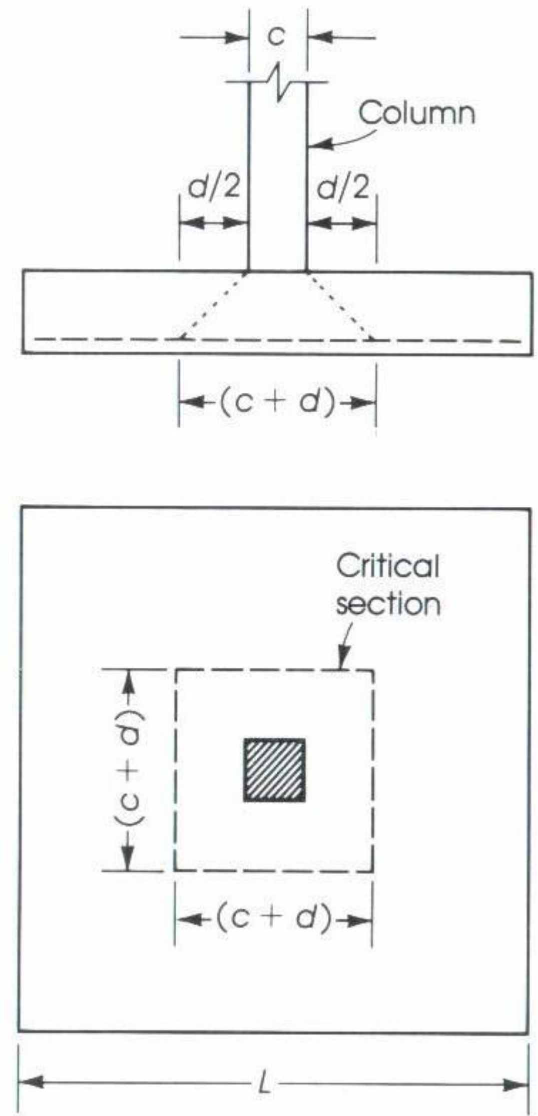
1. Assume d.
2. Determine b_0.

   $b_0 = 4(c+d)$

   for square columns where one side = c

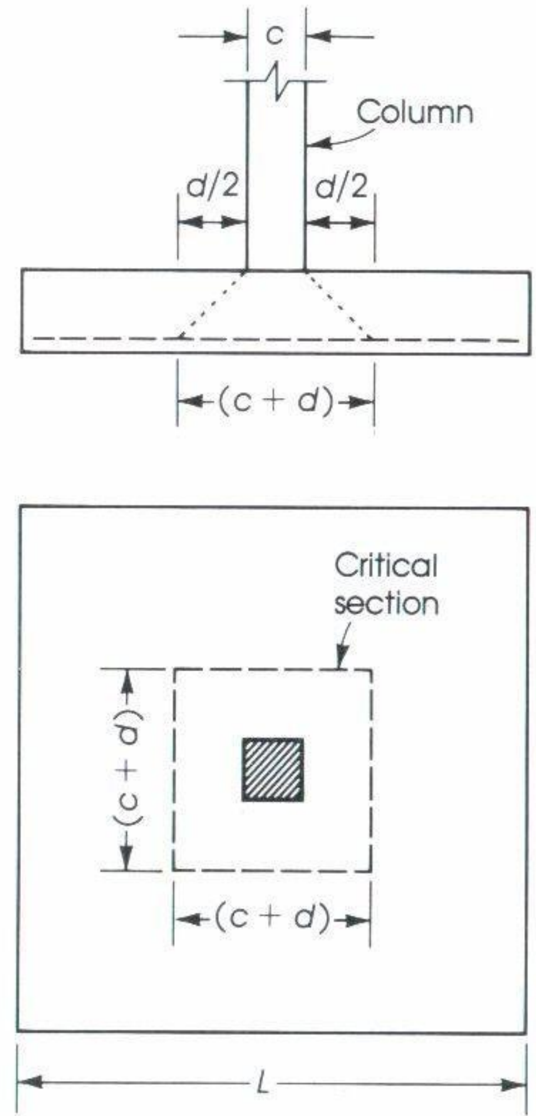
   $b_0 = 2(c_1+d) + 2(c_2+d)$

   for rectangular columns of sides c_1 and c_2.



Design of two-way shear

3. The shear force V_u acts at a section that has a length $b_0 = 4(c+d)$ or $2(c_1+d) + 2(c_2+d)$ and a depth d; the section is subjected to a vertical downward load P_u and vertical upward pressure q_u.



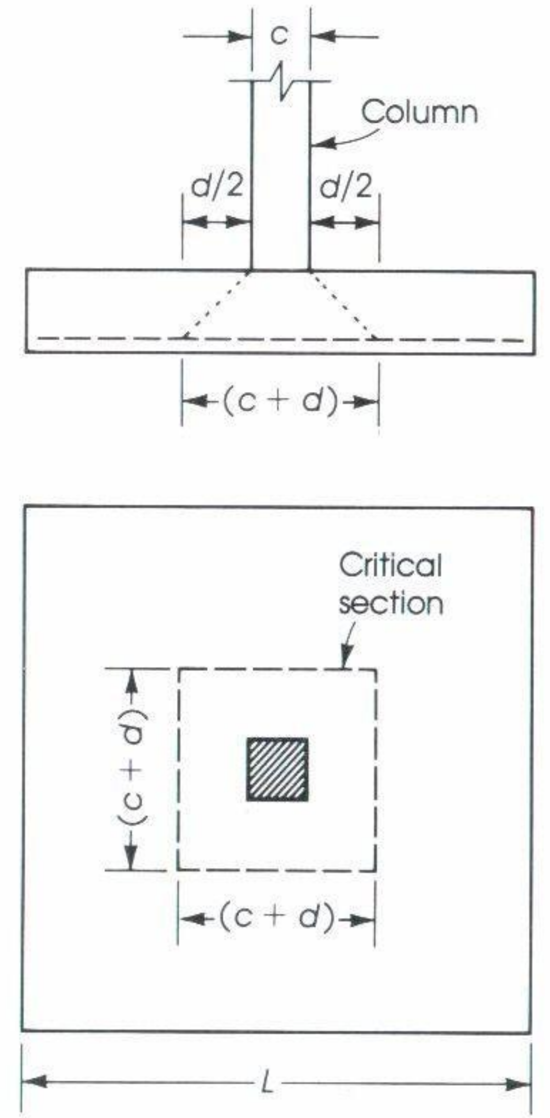
Design of two-way shear

4. Allowable $$\phi V_c = 4\phi\sqrt{f_c}\, b_0 d$$

Let $V_u = \phi V_c$

$$d = \frac{V_u}{4\phi\sqrt{f_c}\, b_0}$$

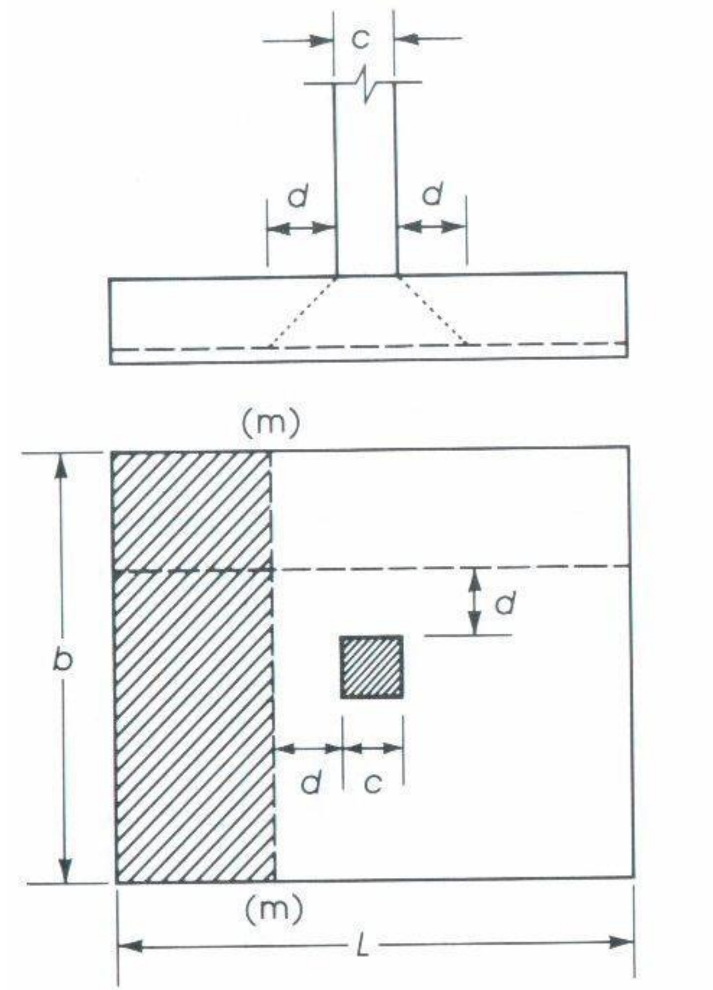
If d is not close to the assumed d, revise your assumptions



Design of one-way shear

For footings with bending action in one direction the critical section is located a distance d from face of column

$$\phi V_c = 2\phi\sqrt{f_c}\, b_0 d$$

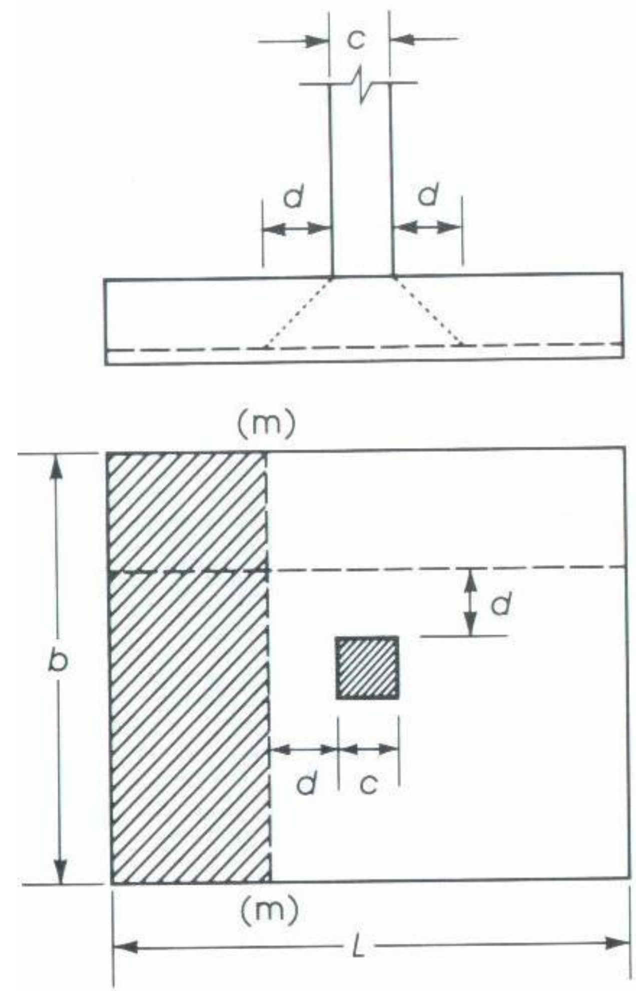


Design of one-way shear

The ultimate shearing force at section m-m can be calculated

$$V_u = q_u \, b\left(\frac{L}{2} - \frac{c}{2} - d\right)$$

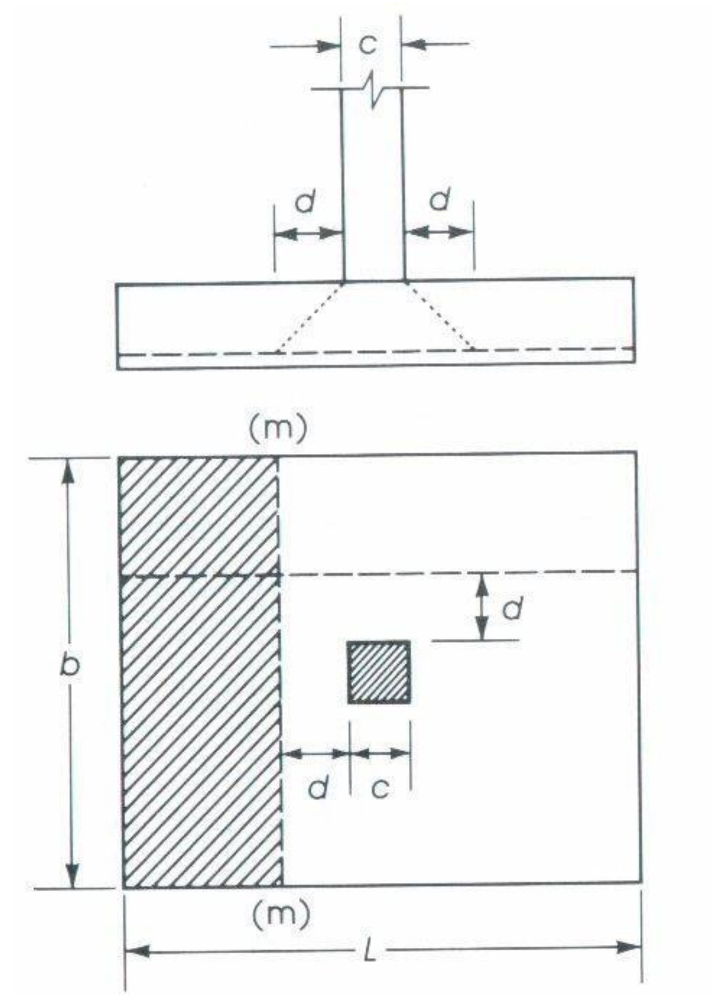
If no shear reinforcement is to be used, then d can be checked



Design of one-way shear

If no shear reinforcement is to be used, then d can be checked, assuming $V_u = \phi V_c$

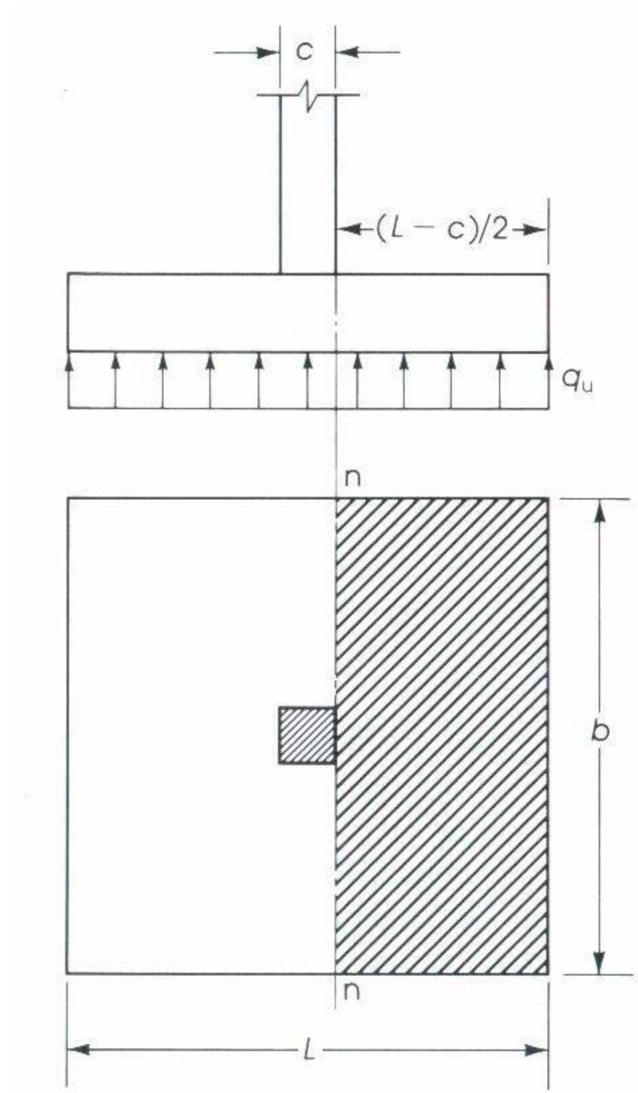
$$d = \frac{V_u}{2\phi\sqrt{f_c}\, b}$$



Flexural Strength and Footing reinforcement

The bending moment in each direction of the footing must be checked and the appropriate reinforcement must be provided.

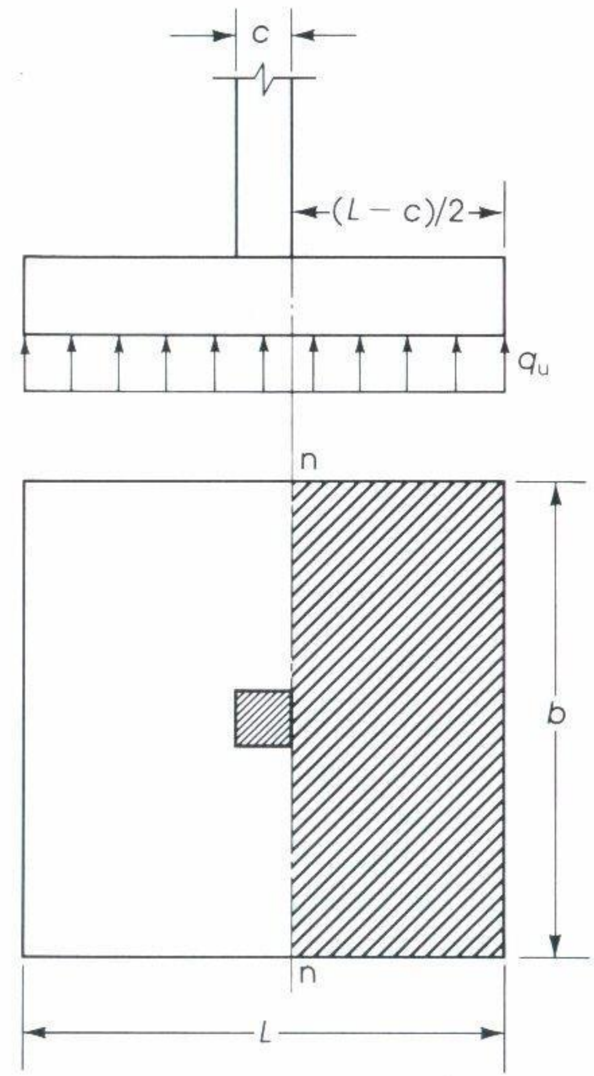
$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2} \right)}$$



Flexural Strength and Footing reinforcement

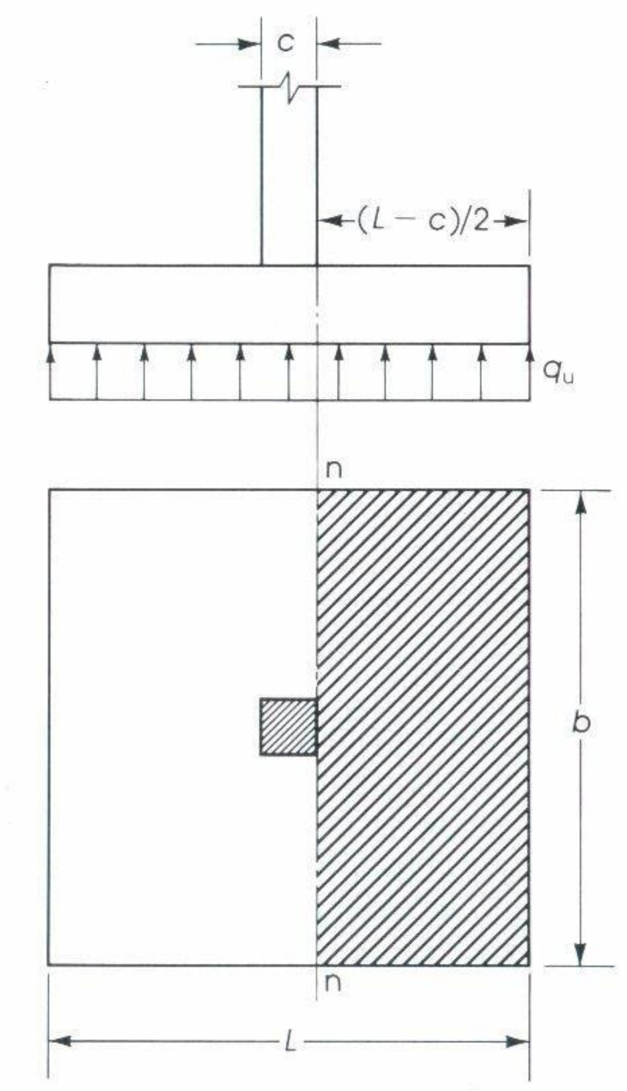
Another approach is to calculated $R_u = M_u / bd^2$ and determine the steel percentage required ρ . Determine A_s then check if assumed a is close to calculated a

$$a = \frac{f_y A_s}{0.85\, f_c\, b}$$



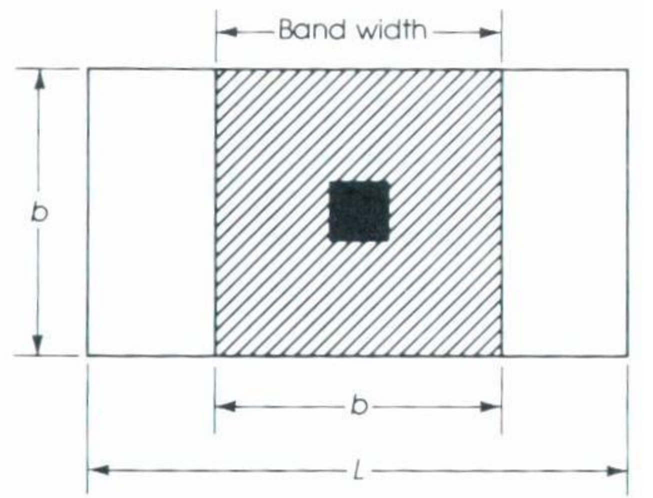
Flexural Strength and Footing reinforcement

The minimum steel percentage required in flexural members is $200/f_y$ with minimum area and maximum spacing of steel bars in the direction of bending shall be as required for shrinkage temperature reinforcement.



Flexural Strength and Footing reinforcement

The reinforcement in one-way footings and two-way footings must be distributed across the entire width of the footing.



$$\frac{\text{Reinforcement in band width}}{\text{Total reinforcement in short direction}} = \frac{2}{\beta + 1}$$

where

$$\beta = \frac{\text{long side of footing}}{\text{short side of footing}}$$

Bearing Capacity of Column at Base

The loads from the column act on the footing at the base of the column, on an area equal to area of the column cross-section. Compressive forces are transferred to the footing directly by bearing on the concrete. Tensile forces must be resisted by reinforcement, neglecting any contribution by concrete.

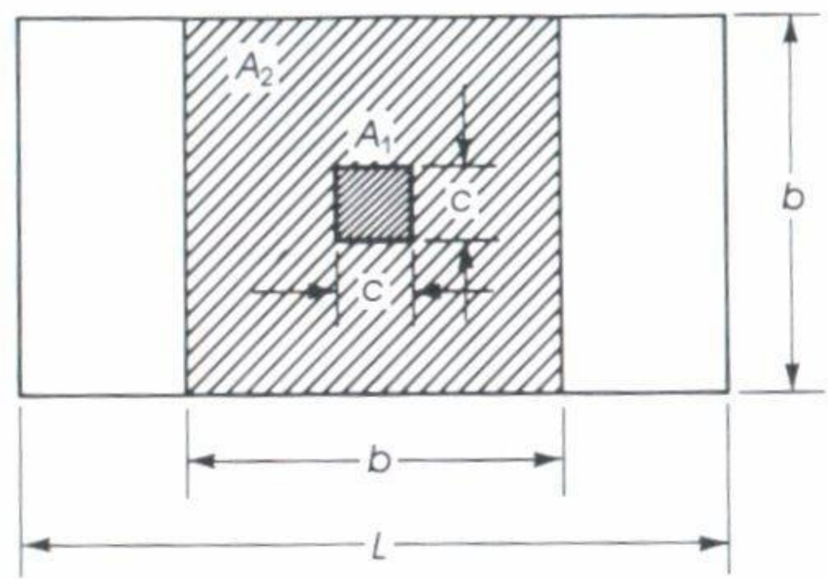
Bearing Capacity of Column at Base

Force acting on the concrete at the base of the column must not exceed the bearing strength of the concrete

$$N_1 = \phi\left(0.85 f_c A_1\right)$$

where ϕ = 0.7 and

A_1 =bearing area of column



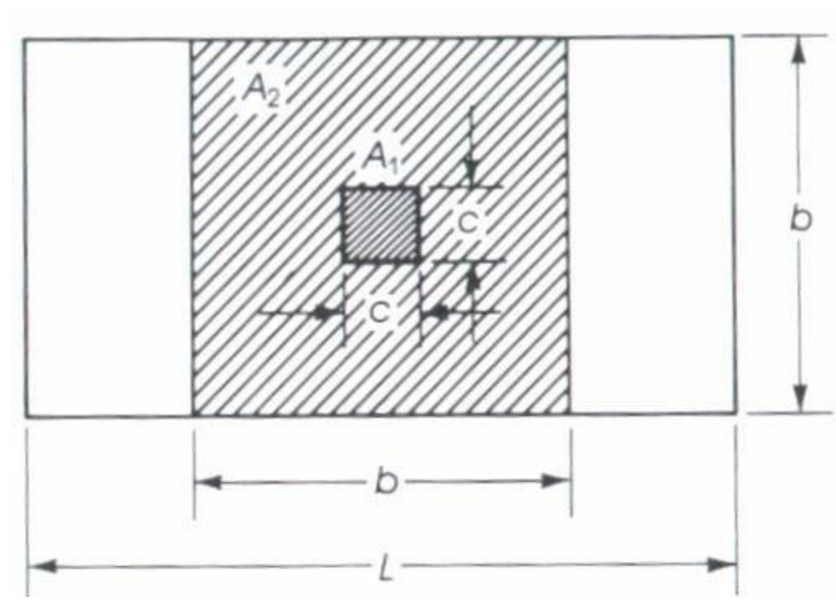
Bearing Capacity of Column at Base

The value of the bearing strength may be multiplied by a factor for bearing on footing when the supporting surface is wider on all sides than the loaded area. $\sqrt{A_2 / A_1} \le 2.0$

The modified bearing strength

$$N_2 \le \phi\left(0.85 f_c A_1\right)\sqrt{A_2 / A_1}$$

$$N_2 \le 2\phi\left(0.85 f_c A_1\right)$$



Dowels in Footings

A minimum steel ratio $\rho = 0.005$ of the column section as compared to $\rho = 0.01$ as minimum reinforcement for the column itself. The number of dowel bars needed is four these may be placed at the four corners of the column. The dowel bars are usually extended into the footing, bent at the ends, and tied to the main footing reinforcement. The dowel diameter shall not exceed the diameter of the longitudinal bars in the column by more than 0.15 in.

Development length of the Reinforcing Bars

The development length for compression bars was given

$$l_d = 0.02 f_y d_b / \sqrt{f_c}$$

but not less than

$$0.003 f_y d_b \geq 8 \text{ in.}$$

Dowel bars must be checked for proper development length.

Differential Settlement

Footing usually support the following loads:

1. Dead loads from the substructure and superstructure
2. Live load resulting from material or occupancy
3. Weight of material used in back filling
4. Wind loads

General Requirements for Footing Design

1. A site investigation is required to determine the chemical and physical properties of the soil.
2. Determine the magnitude and distribution of loads form the superstructure.
3. Establish the criteria and the tolerance for the total and differential settlements of the structure.

General Requirements for Footing Design

4. Determine the most suitable and economic type of foundation.
5. Determine the depth of the footings below the ground level and the method of excavation.
6. Establish the allowable bearing pressure to be used in design.

General Requirements for Footing Design

7. Determine the pressure distribution beneath the footing based on its width
8. Perform a settlement analysis.

THANKS