INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

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DEPARTMENT OF MECHANICAL ENGINEERING

Course: ROBOTICS

Course Code: A70355

IV B.Tech I Sem

JNTUH- R15

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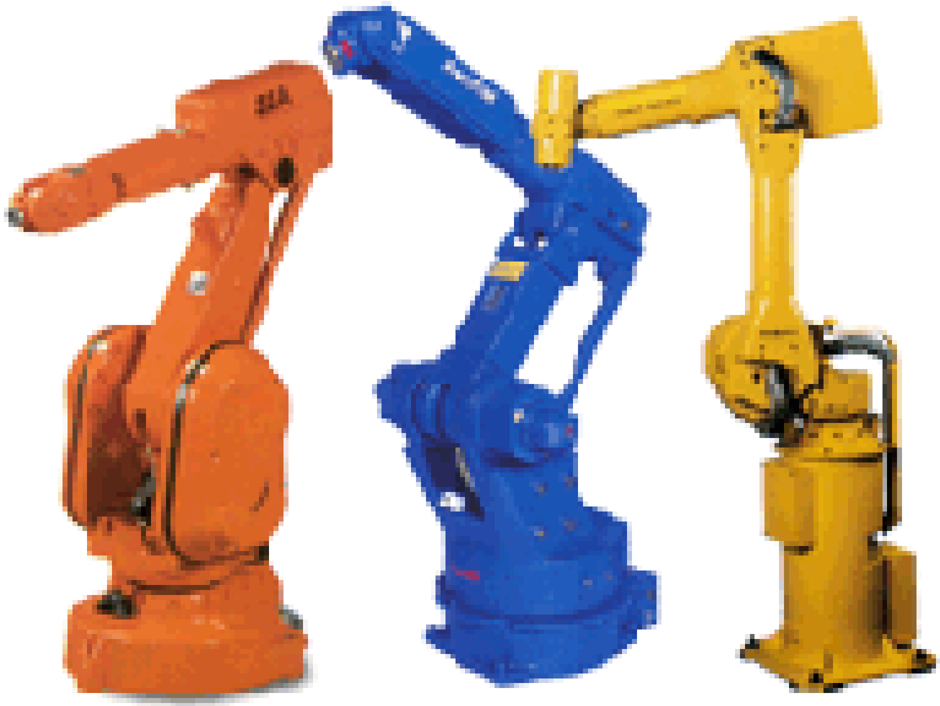
UNIT-I

Introduction to Robotics



We will be studying Industrial manipulator type Robots.

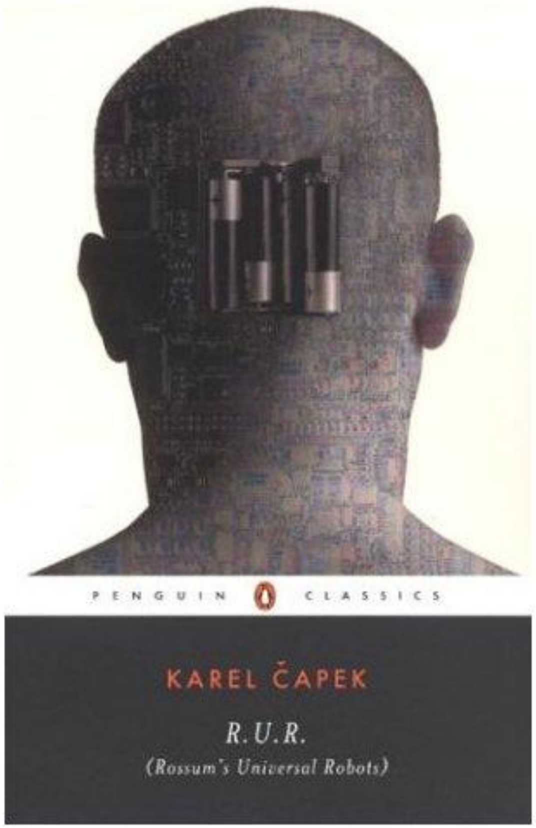
- Introduction to Robotics
- Classification of Robots
- Robot coordinates
- Work volumes and Reference Frames
- Robot Applications.

Automation vs. robots

- Automation –Machinery designed to carry out a specific task
  - Bottling machine
  - Dishwasher
  - Paint sprayer

(These are always better than robots, because they can be optimally designed for a particular task).

- Robots – machinery designed to carry out a variety of tasks
  - Pick and place arms
  - Mobile robots
  - Computer Numerical Control machines



"Robot" coined by Karel Capek in a 1921 science-fiction Czech play

Robotics Terminology

- **Robot** - Mechanical device that performs human tasks, either automatically or by remote control.
- **Robotics** - Study and application of robot technology.
- **Telerobotics** - Robot that is operated remotely.

Laws of Robotics

- Asimov proposed three “Laws of Robotics”

- Law 1: A robot may not injure a human being or through inaction, allow a human being to come to harm.
- Law 2: A robot must obey orders given to it by human beings, except where such orders would conflict with the first law.
- Law 3: A robot must protect its own existence

Ideal Tasks

Tasks which are:

- Dangerous
  - Space exploration
  - chemical spill cleanup
  - disarming bombs
  - disaster cleanup
- Boring and/or repetitive
  - Welding car frames
  - part pick and place
  - manufacturing parts.
- High precision or high speed
  - Electronics testing
  - Surgery
  - precision machining.

Robotics Timeline

- 1922 Czech author Karel Capek wrote a story called Rossum’s Universal Robots and introduced the word “Rabota”(meaning worker)
- 1954 George Devol developed the first programmable Robot.
- 1955 Denavit and Hartenberg developed the homogenous transformation matrices
- 1962 Unimation was formed, first industrial Robots appeared.
- 1973 Cincinnati Milacron introduced the T3 model robot, which became very popular in industry.
- 1990 Cincinnati Milacron was acquired by ABB

ROBOT

- Defined by Robotics Industry Association (RIA) as
  – a re-programmable, multifunctional manipulator designed to move material, parts, tools or specialized devices through variable programmed motion for a variety of tasks
- possess certain anthropomorphic characteristics
  – mechanical arm
  – sensors to respond to input
  – Intelligence to make decisions

Accessories

- **Acutators : Actuators are the muscles of the manipulators. Common types of actuators are servomotors, stepper motors, pneumatic cylinders etc.**

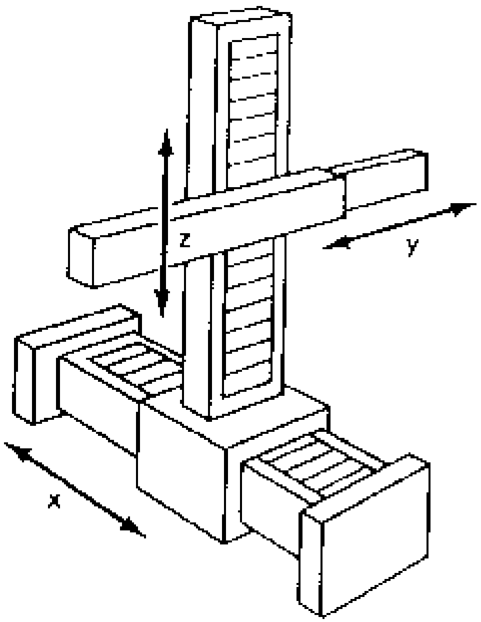
- **Sensors : Sensors are used to collect information about the internal state of the robot or to communicate with the outside environment. Robots are often equipped with external sensory devices such as a vision system, touch and tactile sensors etc which help to communicate with the environment**

- **Controller : The controller receives data from the computer, controls the motions of the actuator and coordinates these motions with the sensory feedback information**.

Robot Configurations

Some of the commonly used configurations in Robotics are

- **Cartesian/Rectangular Gantry(3P) :** These Robots are made of 3 Linear joints that orient the end effector, which are usually followed by additional revolute joints.



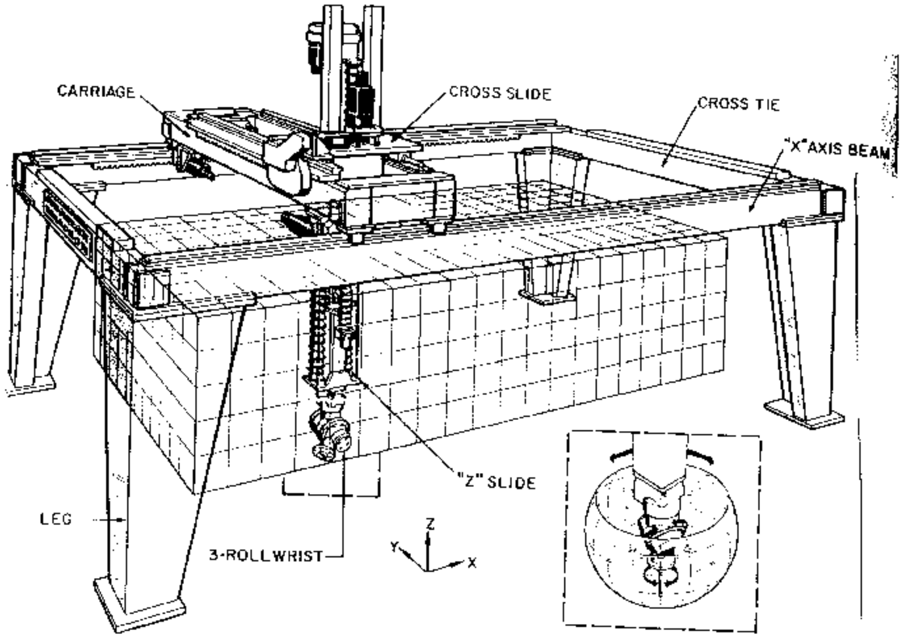
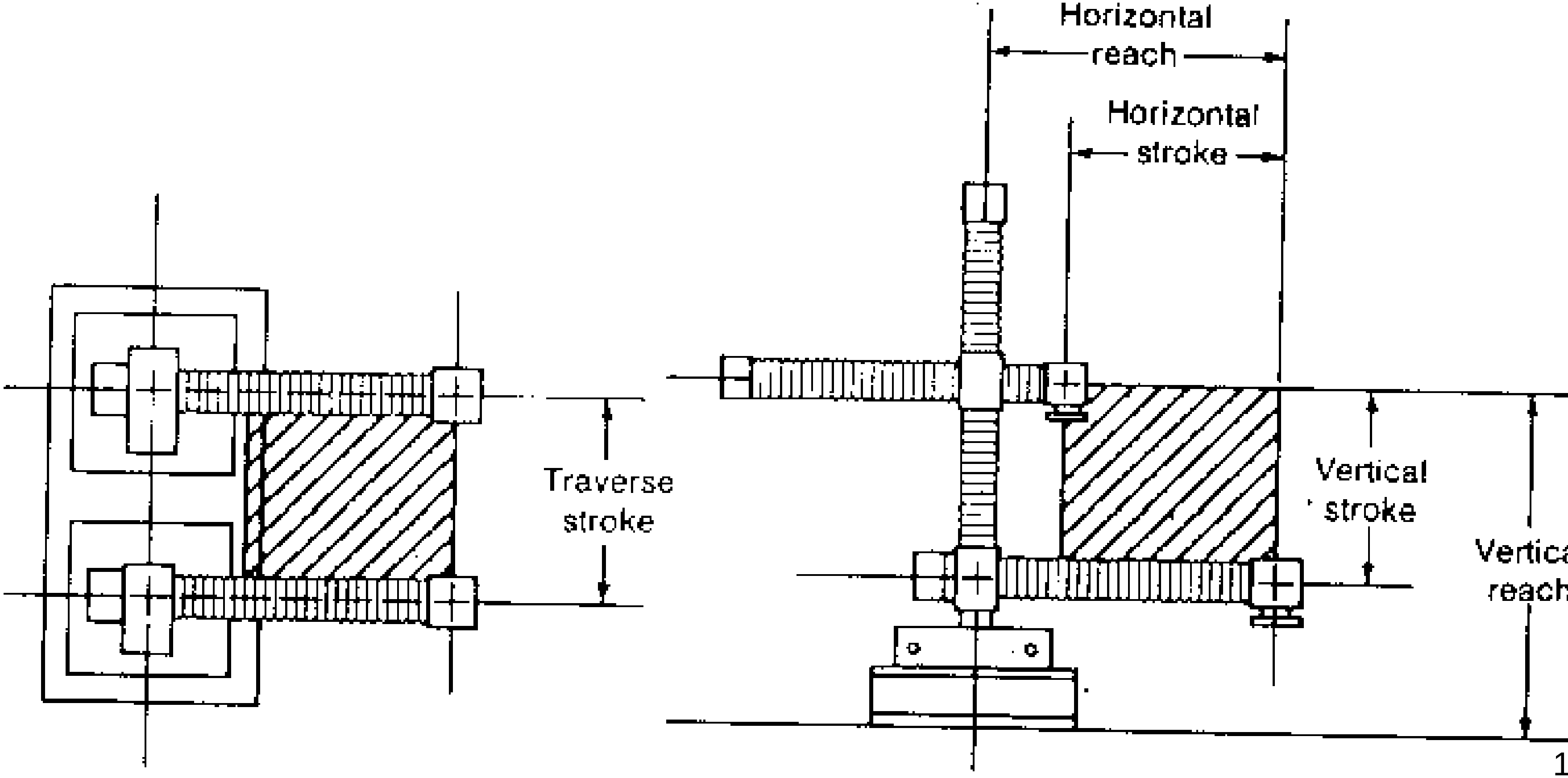


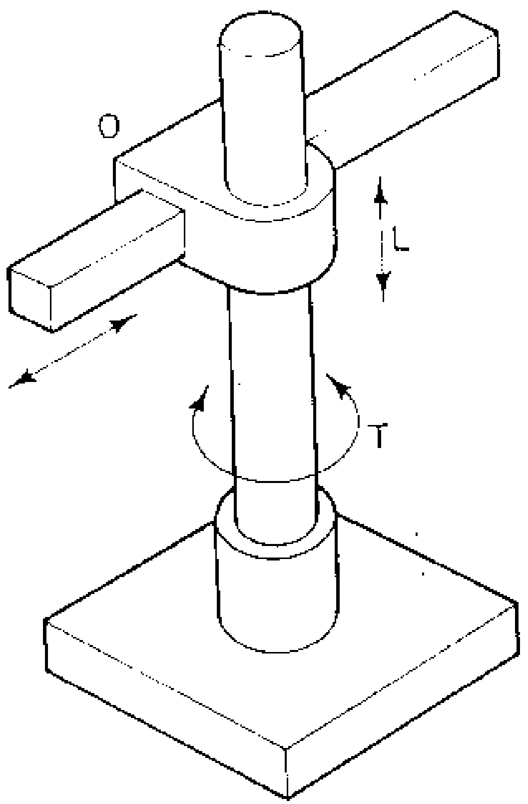
Figure 9.3. Gantry configuration robot. (Courtesy of Cincinnati Milacron.)

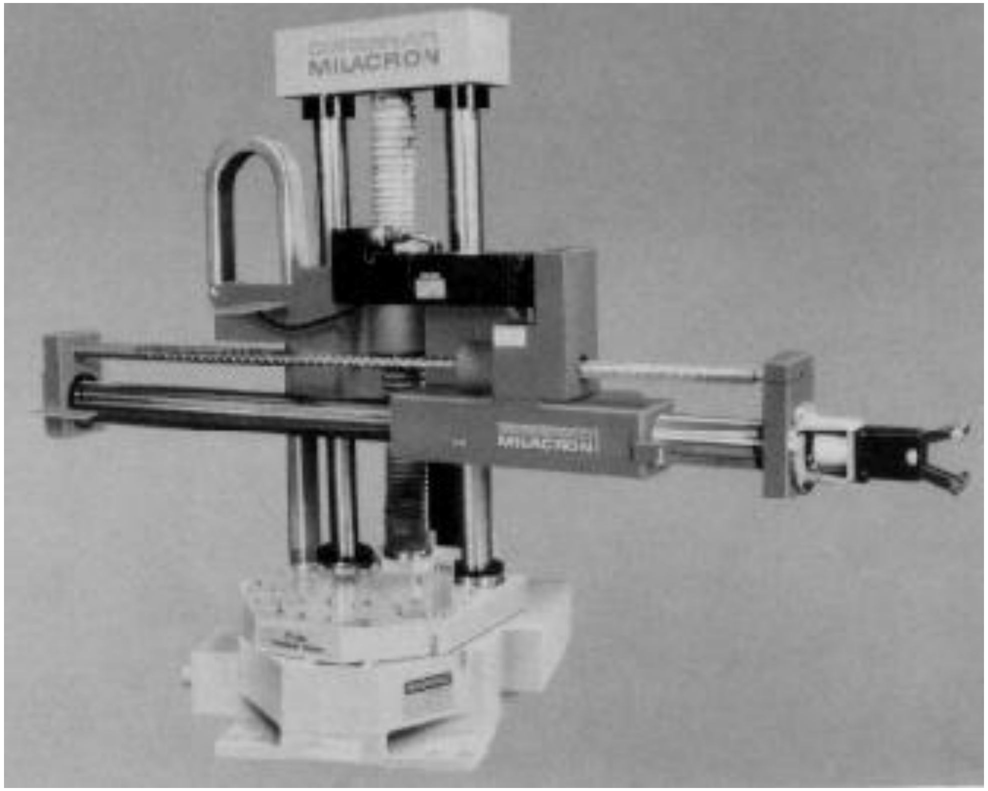
Cartesian Robot - Work Envelope



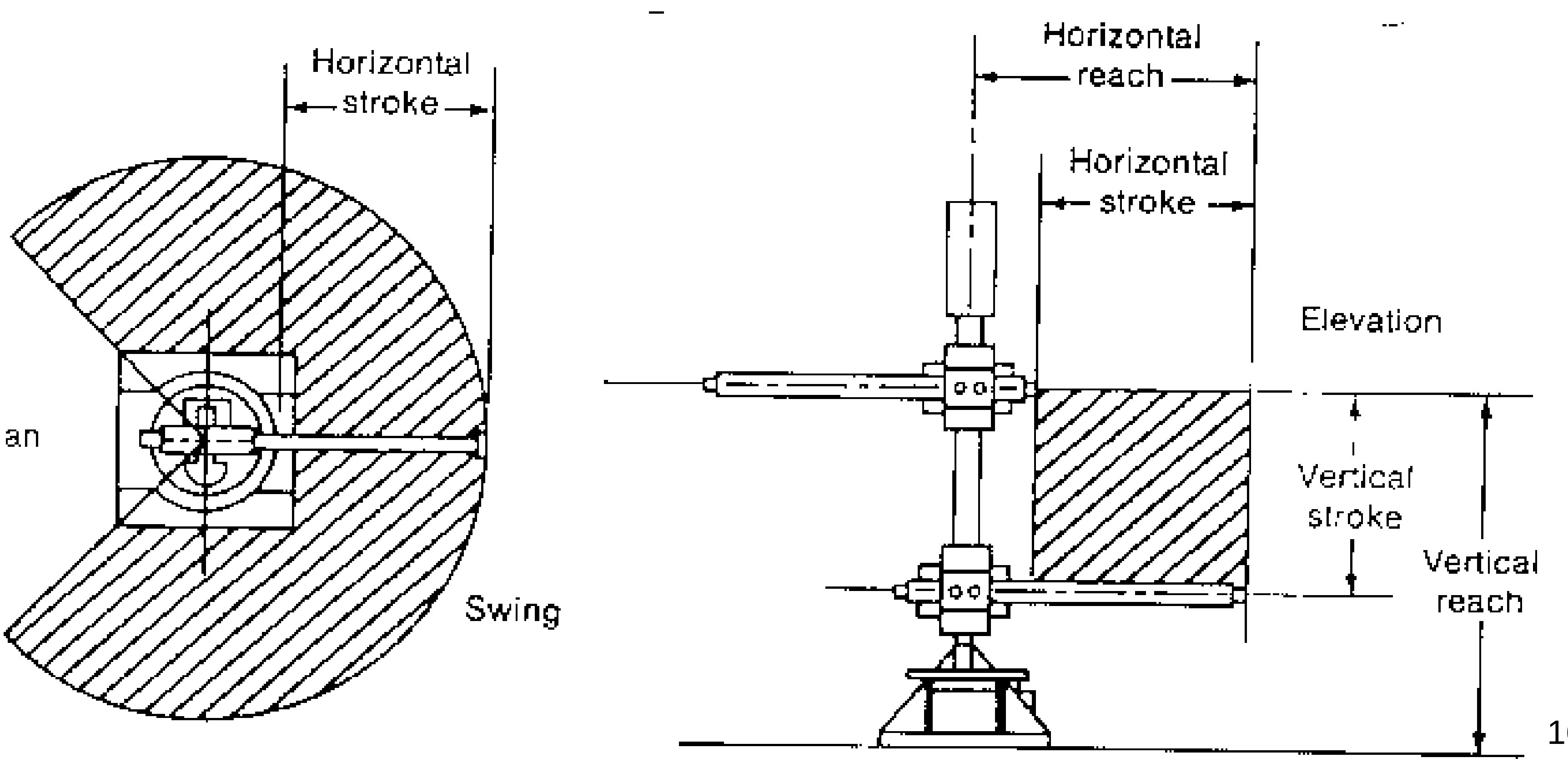
Robot Configurations

- **Cylindrical (R2P):** Cylindrical coordinate Robots have 2 prismatic joints and one revolute joint.



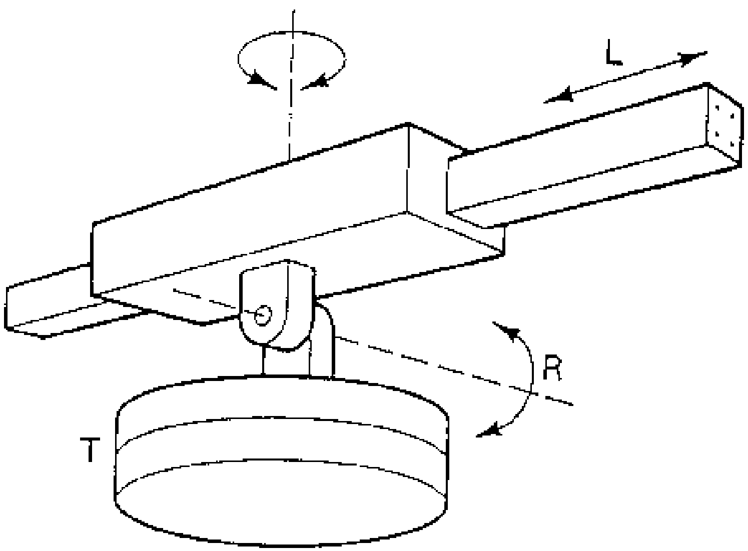


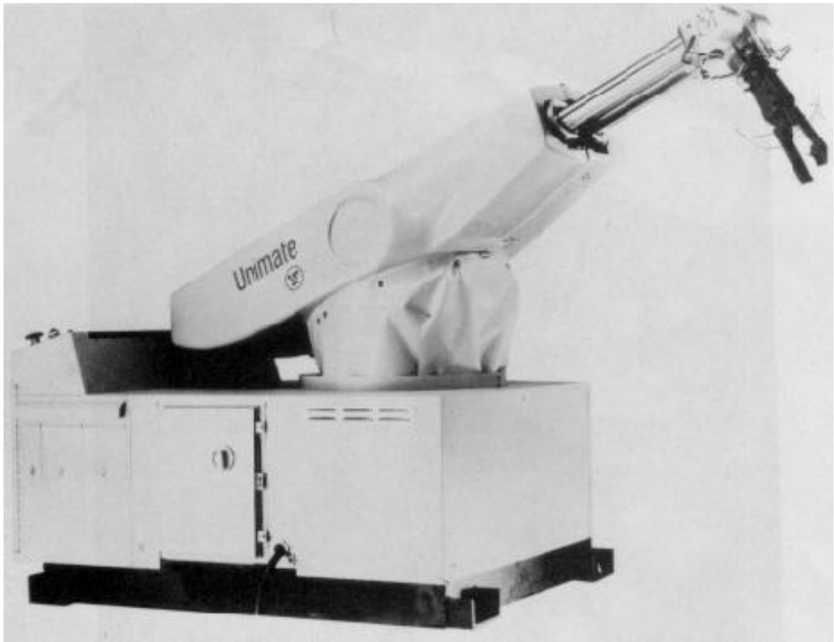
Cylindrical Robot - Work Envelope



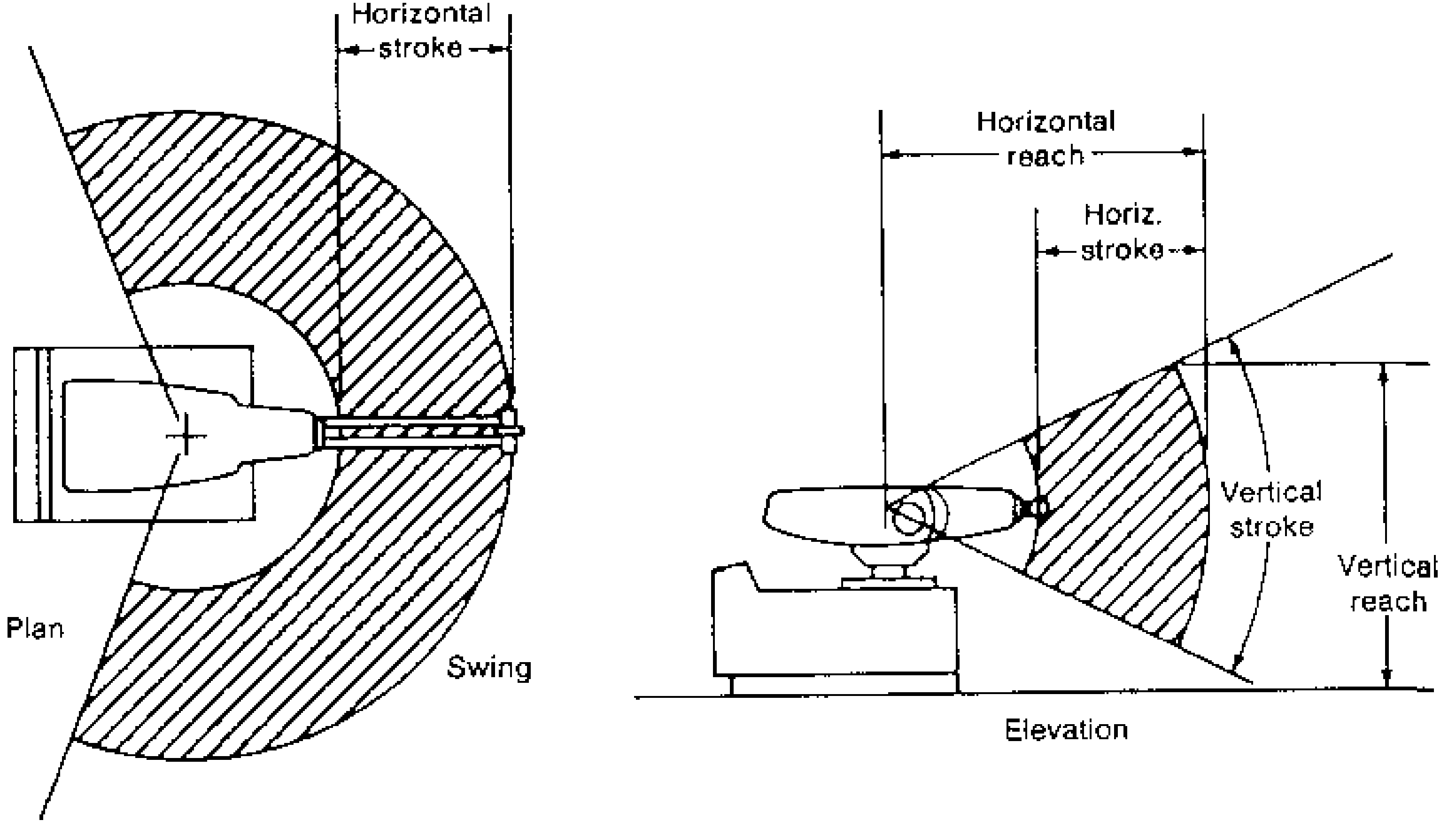
Robot Configurations

- **Spherical joint (2RP):** They follow a spherical coordinate system, which has one



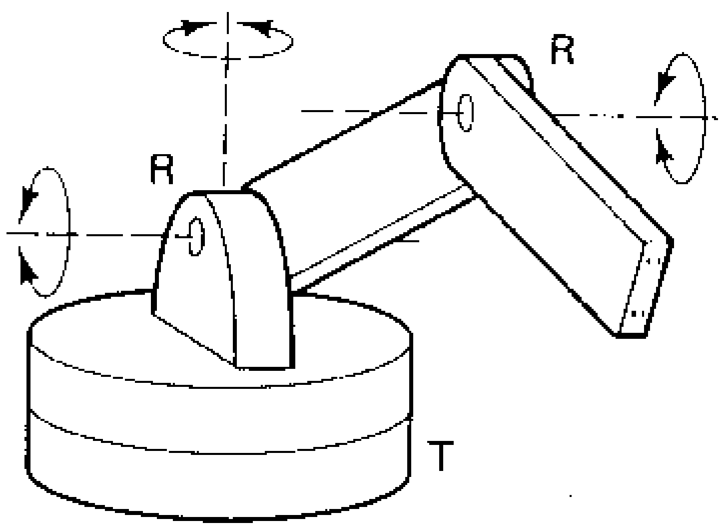


Spherical Robot - Work Envelope



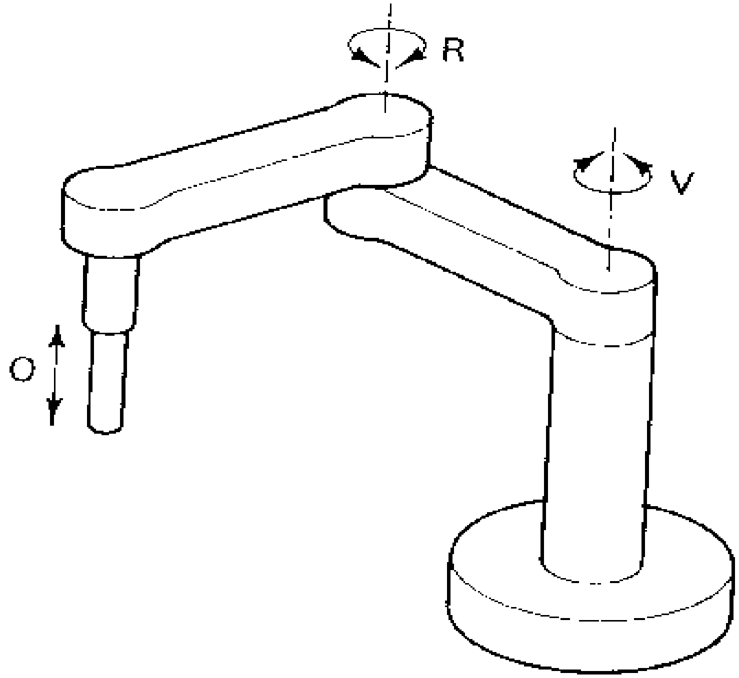
Robot Configurations

- **Articulated/anthropomorphic(3R) :**An articulated robot's joints are all revolute, similar to a human's arm.

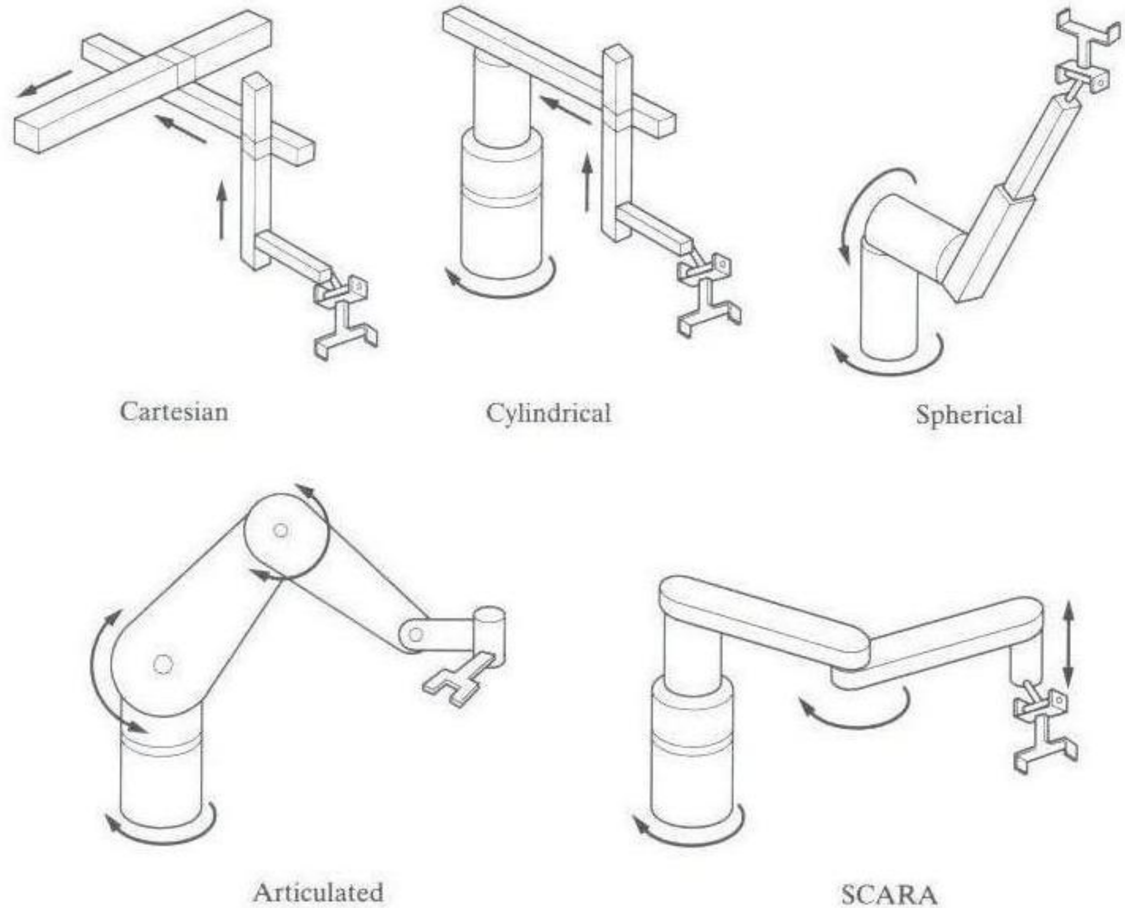


Robot Configurations

- **Selective Compliance Assembly Robot Arm (SCARA) (2R1P):** They have two revolute joints that are parallel and allow the Robot to move in a horizontal plane, plus an additional prismatic joint that moves vertically



Robot Configurations

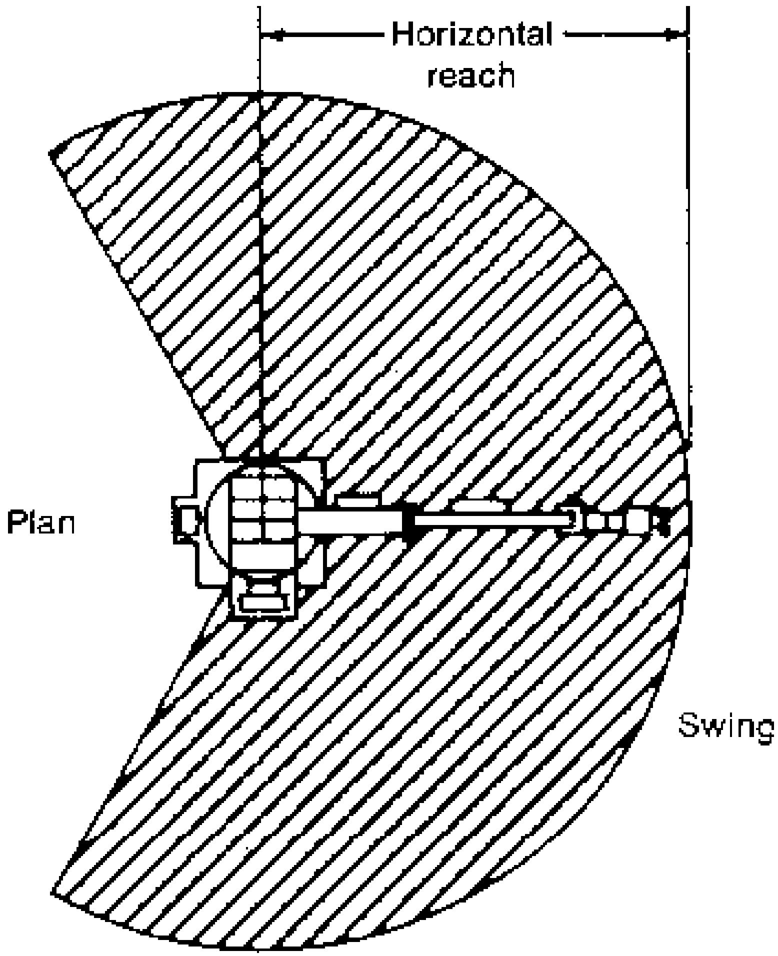


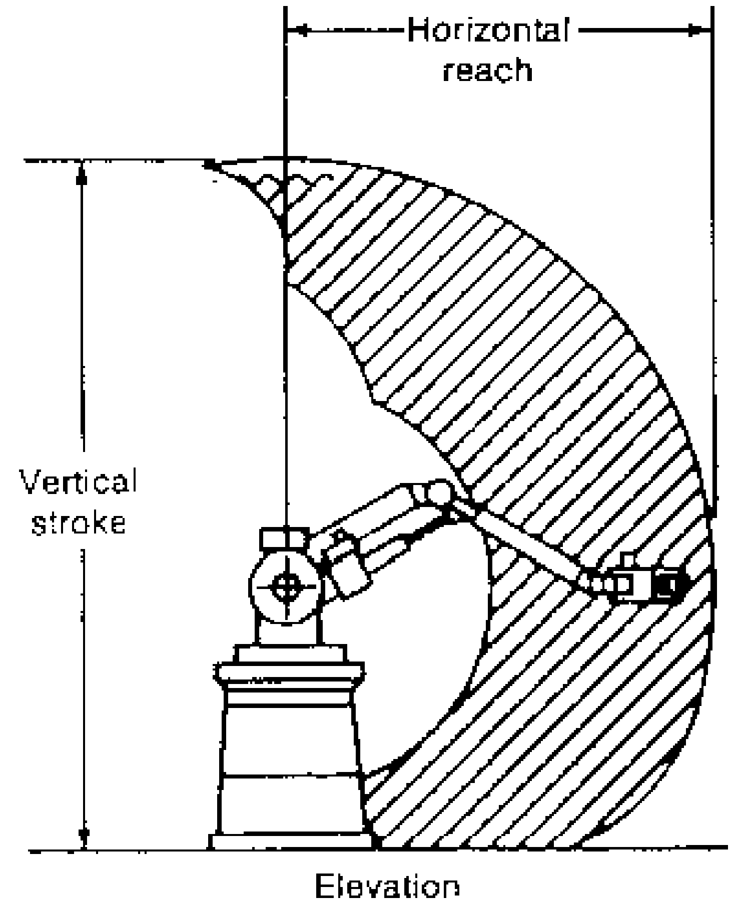
Work Envelope concept

- Depending on the configuration and size of the links and wrist joints, robots can reach a collection of points called a **Workspace**.

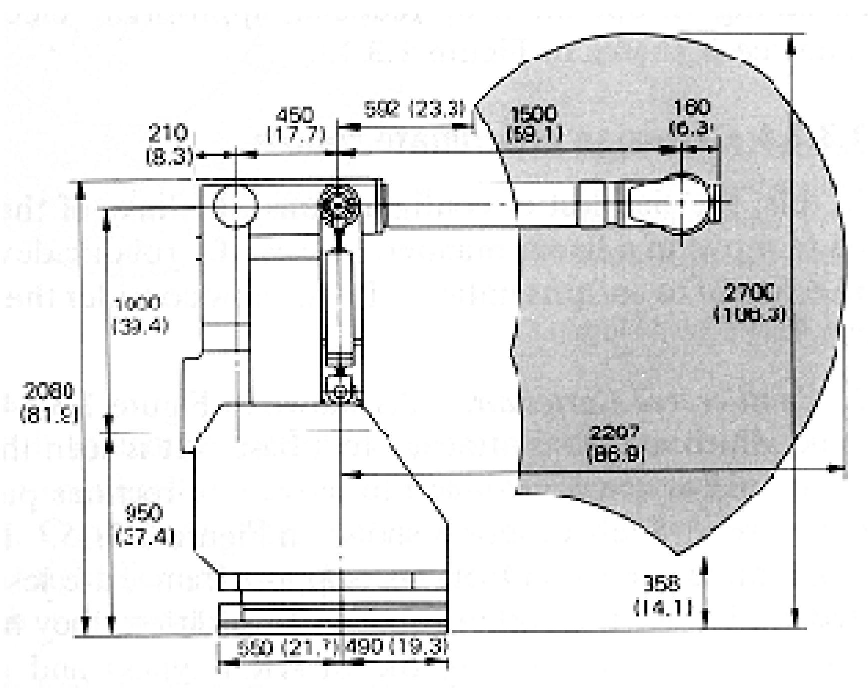
- Alternately Workspace may be found empirically, by moving each joint through its range of motions and combining all space it can reach and subtracting what space it cannot reach

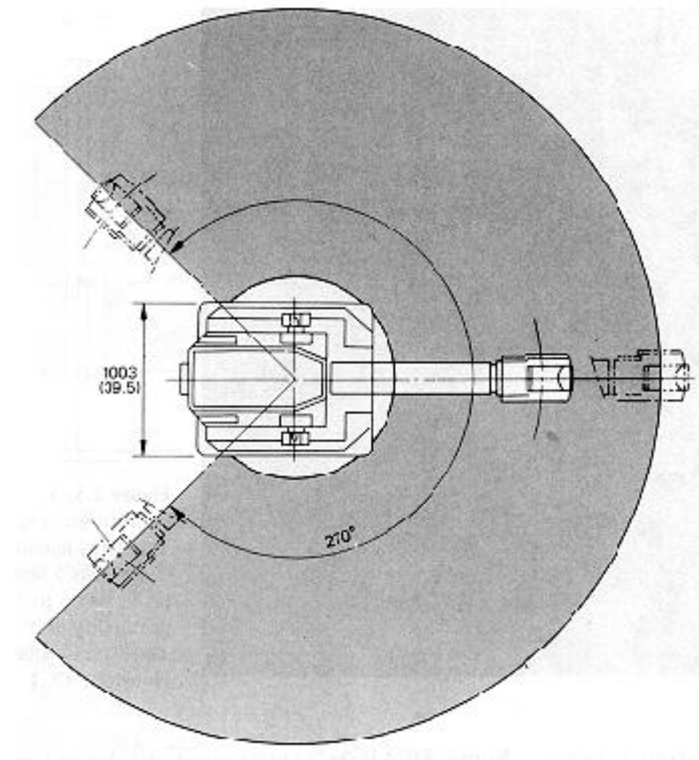
Pure Spherical Jointed Arm - Work envelope





2) Parallelogram Jointed

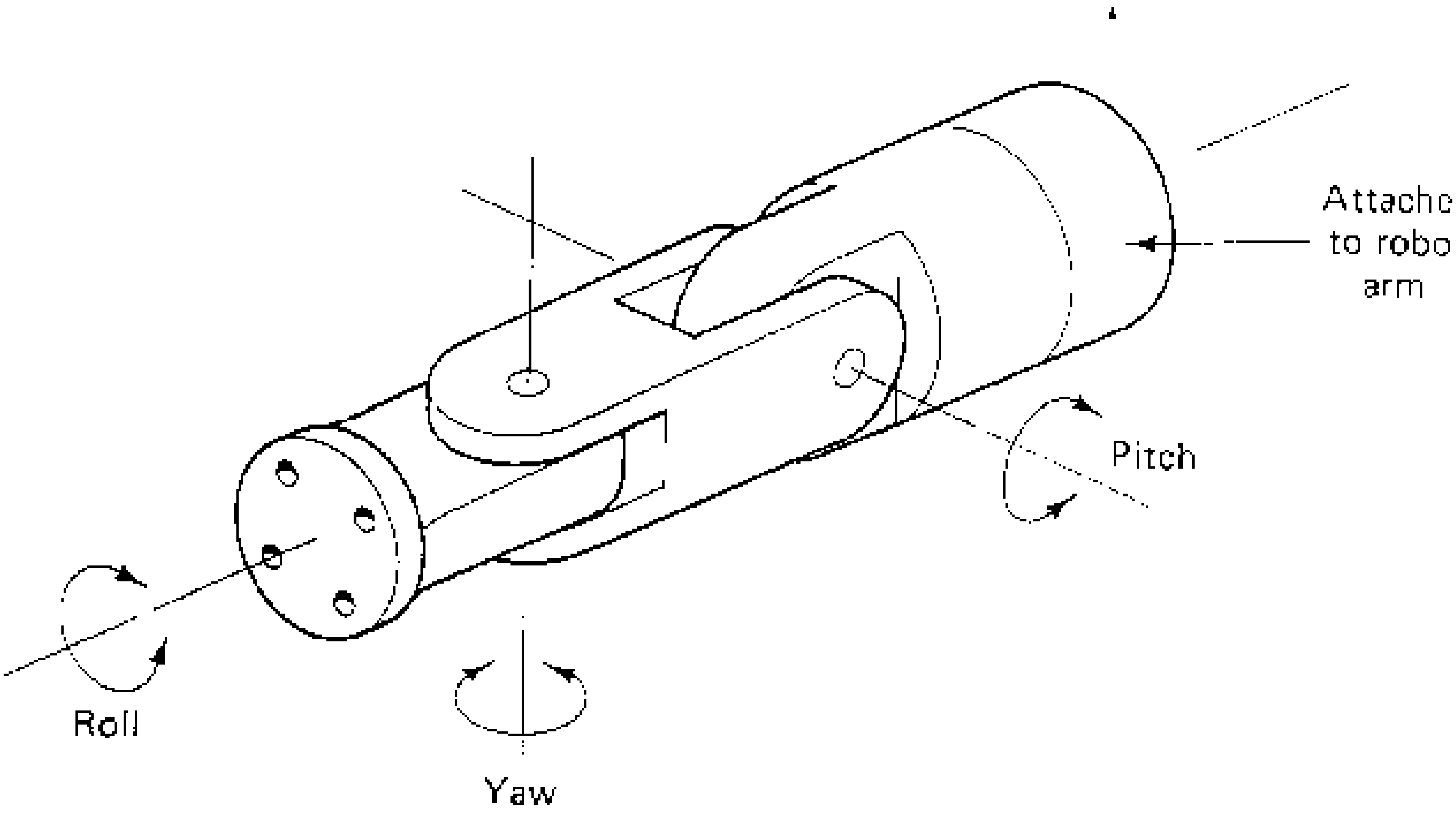




WRIST

- typically has 3 degrees of freedom
  - ***Roll*** involves rotating the wrist about the arm axis
  - ***Pitch*** up-down rotation of the wrist
  - ***Yaw*** left-right rotation of the wrist
- End effector is mounted on the wrist

WRIST MOTIONS



CONTROL METHODS

- ***Non Servo Control***
  - implemented by setting limits or mechanical stops for each joint and sequencing the actuation of each joint to accomplish the cycle
  - end point robot, limited sequence robot, bang-bang robot
  - No control over the motion at the intermediate points, only end points are known

- Programming accomplished by
  - setting desired sequence of moves
  - adjusting end stops for each axis accordingly
  - the sequence of moves is controlled by a “squencer”, which uses feedback received from the end stops to index to next step in the program
- Low cost and easy to maintain, reliable
- relatively high speed
- repeatability of up to 0.01 inch
- limited flexibility
- typically hydraulic, pneumatic drives

- ***Servo Control***
  - Point to point Control
  - Continuous Path Control
- Closed Loop control used to monitor position, velocity (other variables) of each joint

Point-to-Point Control

- Only the end points are programmed, the path used to connect the end points are computed by the controller
- user can control velocity, and may permit linear or piece wise linear motion
- Feedback control is used during motion to ascertain that individual joints have achieved desired location

- Often used hydraulic drives, recent trend towards servomotors
- loads up to 500lb and large reach
- Applications
  - pick and place type operations
  - palletizing
  - machine loading

Continuous Path Controlled

- in addition to the control over the endpoints, the path taken by the end effector can be controlled
- Path is controlled by manipulating the joints throughout the entire motion, via closed loop control
- Applications:
  - spray painting, polishing, grinding, arc welding

ROBOT PROGRAMMING

- Typically performed using one of the following
  - On line
    - teach pendant
    - lead through programming
  - Off line
    - robot programming languages
    - task level programming

Use of Teach Pendant

- hand held device with switches used to control the robot motions
- End points are recorded in controller memory
- sequentially played back to execute robot actions
- trajectory determined by robot controller
- suited for point to point control applications

- Easy to use, no special programming skills required
- Useful when programming robots for wide range of repetitive tasks for long production runs
- RAPID

Lead Through Programming

- lead the robot physically through the required sequence of motions
- trajectory and endpoints are recorded, using a sampling routine which records points at 60-80 times a second
- when played back results in a smooth continuous motion
- large memory requirements

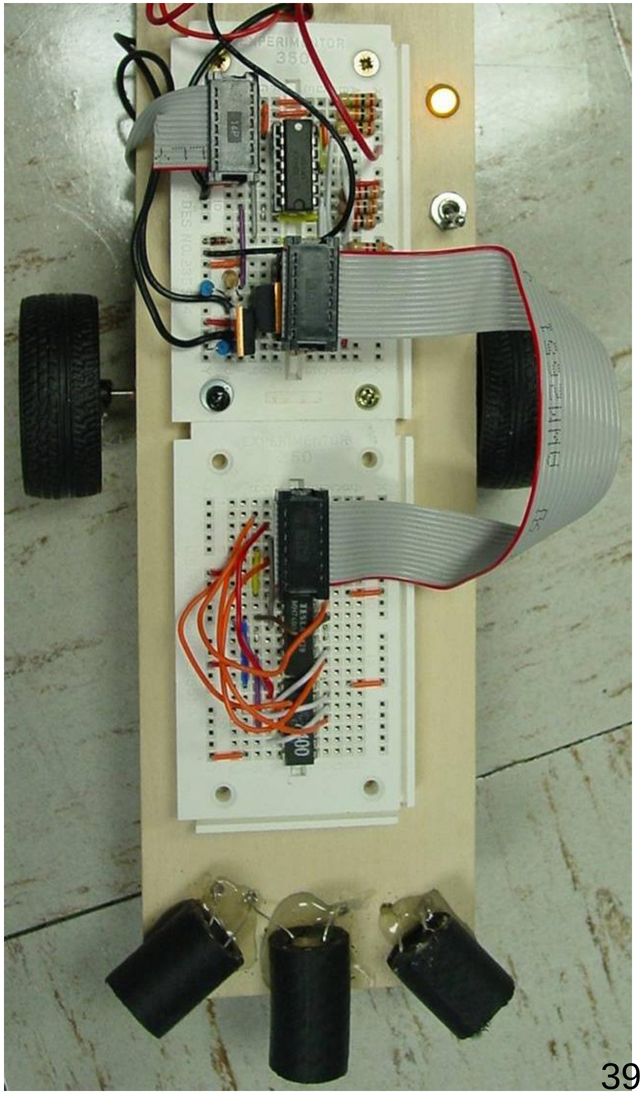
Programming Languages

- Motivation
  - need to interface robot control system to external sensors, to provide “real time” changes based on sensory equipment
  - computing based on geometry of environment
  - ability to interface with CAD/CAM systems
  - meaningful task descriptions
  - off-line programming capability

- Large number of robot languages available
  - AML, VAL, AL, RAIL, RobotStudio, etc. (200+)
- Each robot manufacturer has their own robot programming language
- No standards exist
- Portability of programs virtually non-existent

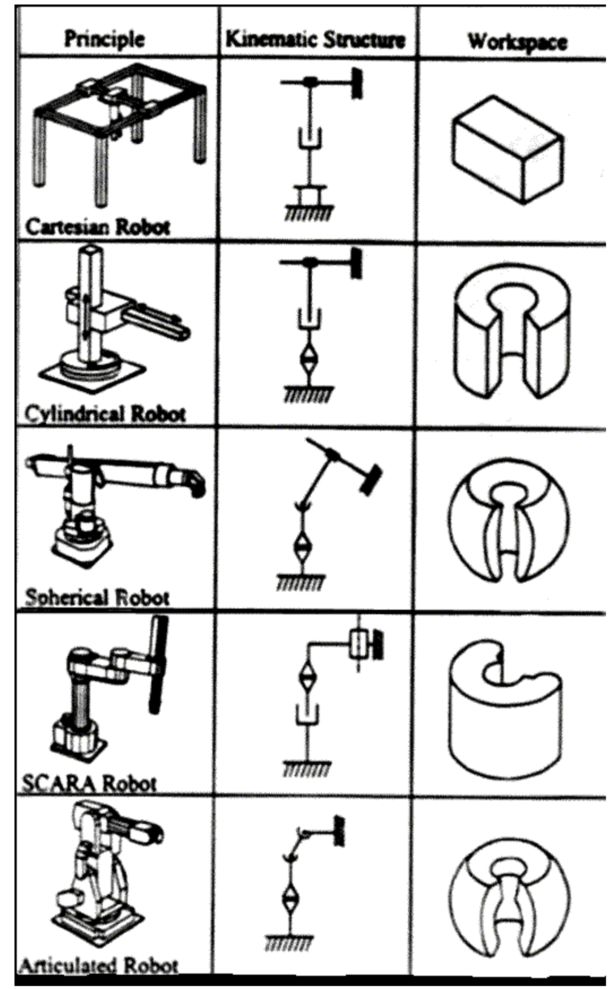
Sensory

- Uses sensors for feedback.
- Closed-loop robots use sensors in conjunction with actuators to gain higher accuracy – servo motors.
- Uses include mobile robotics, telepresence, search and rescue, pick and place with machine vision.



Measures of performance

- Working volume
  - The space within which the robot operates.
  - Larger volume costs more but can increase the capabilities of a robot
- Speed and acceleration
  - Faster speed often reduces resolution or increases cost
  - Varies depending on position, load.
  - Speed can be limited by the task the robot performs (welding, cutting)
- Resolution
  - Often a speed tradeoff
  - The smallest step the robot can take



Performance

- Accuracy

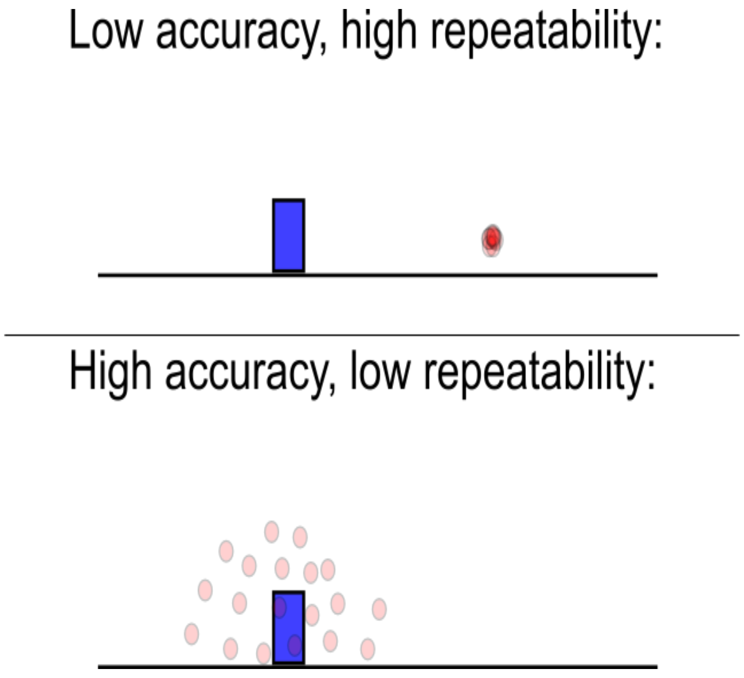
  –The difference between the actual position of the robot and the programmed position

- Repeatability

  Will the robot always return to the same point under the same control conditions?

  Increased cost

  Varies depending on position, load



Control

- Open loop, i.e., no feedback, deterministic
- Closed loop, i.e., feedback, maybe a sense of touch and/or vision

Actuators

- Actuator is the term used for the *mechanism that drives* the robotic arm.
- There are 3 main types of Actuators

1. Electric motors
2. Hydraulic
3. Pneumatic cylinder

- *Hydraulic* and *pneumatic* actuators are generally suited to driving *prismatic joints* since they produce *linear* motion directly
- Hydraulic and pneumatic actuators are also known as *linear actuators.*
- *Electric motors* are more suited to driving *revolute joints* as they produce rotation

Hydraulic Actuators

- A car makes use of a hydraulic system. If we look at the braking system of the car we see that only moderate force applied to the brake pedal is sufficient to produce force large enough to stop the car.
- The underlying principle of all hydraulic systems was first discovered by the French scientist Blaise Pascal in 1653. He stated that "if external pressure is applied to a confined fluid, then the pressure is transferred without loss to all surfaces in contact with the fluid"
- The word fluid can mean both a gas or a liquid
- Where large forces are required we can expect to find hydraulic devices (mechanical diggers on building sites, pit props in coal mines and jacks for lifting cars all use the principle of hydraulics.

Hydraulic Actuators

- Each hydraulic actuator contains the following parts:

1. Pistons
2. Spring return piston
3. Double acting cylinder
4. Hydraulic transfer value
5. And in some cases a hydraulic accumulator

- Advantages of the hydraulic mechanism

1. A hydraulic device can produce an enormous range of forces without the need for gears, simply by controlling the flow of fluid
2. Movement of the piston can be smooth and fast
3. Position of the piston can be controlled precisely by a low-current electrically operated value
4. There are no sparks to worry about as there are with electrical motor, so the system is safe to use in explosive atmospheres such as in paint spraying or near inflammable materials

Pneumatic Actuators

- A pneumatic actuator uses air instead of fluid
- The relationship between force and area is the same in a pneumatic system compared to a hydraulic system
- We know that air is compressible, so in order to build up the pressure required to operate the piston, extra work has to be done by the pump to compress the air. This means that pneumatic devices are less efficient
- If you have ever used a bicycle pump you may have noticed that it becomes hot as it is used. The heat produced by the mechanical work done in compressing the air. Heat represents wasted energy.

Pneumatic Actuators

- Advantages of the Pneumatic system:

1. Generally less expensive than an equivalent hydraulic system. Many factories have compresses air available and one large compressor pump can serve several robots
2. Small amount of air leakage is ok, but in a hydraulic system it will require prompt attention
3. The compressibility of air can also be an advantage in some applications. Think about a set of automatic doors which are operated pneumatically. If a person is caught in the doors they will not be crushed.
4. A pressure relief valve can be incorporated to release pressure when a force is exceeded, for example the gripper of a robot will incorporate a relief value to ensure it does not damage itself or what it is gripping
5. Pneumatic devices are faster to respond compared to a hydraulic system as air is lighter than fluid.

- A pneumatic system has its downfalls and the main one is that it can produce the enormous forces a hydraulic system can. Another is concerned with the location of the pistons. As air is compressible heavy loads on the robot arm may cause the pistons to move even when all the valves on the cylinder are closed. It is for this reason that pneumatic robots are best suited for pick and place robots.

Electric Motors

- Not all electric motors are suited for use as actuators in robots
- There are three basic characteristics of a motor, when combined will determine the suitability of a motor for a particular job. The 3 characteristics are power, torque and speed. Each of these characteristics are interdependent, that means that you can not alter one without affecting the others.

Electric Motors

- Two types of power: electrical and mechanical, both are measured in watts.
- Torque is how strong a motor is or how much turning force it is able to produce and is measured in newton-metres.
- The speed is measured in revolutions per minute and is rotation of the motor
- There are 3 different types of motors

1. AC motor which operates by alternating current electricity
2. DC motor which operates by direct current electricity
3. Stepper motors which operates by pulses of electricity

- Any type of electric motor could be used for a robot as long as it is possible to electronically control the speed and power so that it behaves the way we want.
- DC motors and Stepper motors are commonly used in robotics

Robot End effectors

- Introduction
- Types of End effectors
- Mechanical gripper
- Types of gripper mechanism
- Gripper force analysis
- Other types of gripper
- Special purpose grippers

Consider Typical Robots

What could a robot do without "end effectors"?

End effector

Device that attaches to the wrist of the robot arm and enables the general-purpose robot to perform a specific task.

Two types:

Grippers – to grasp and manipulate objects (e.g., parts) during work cycle

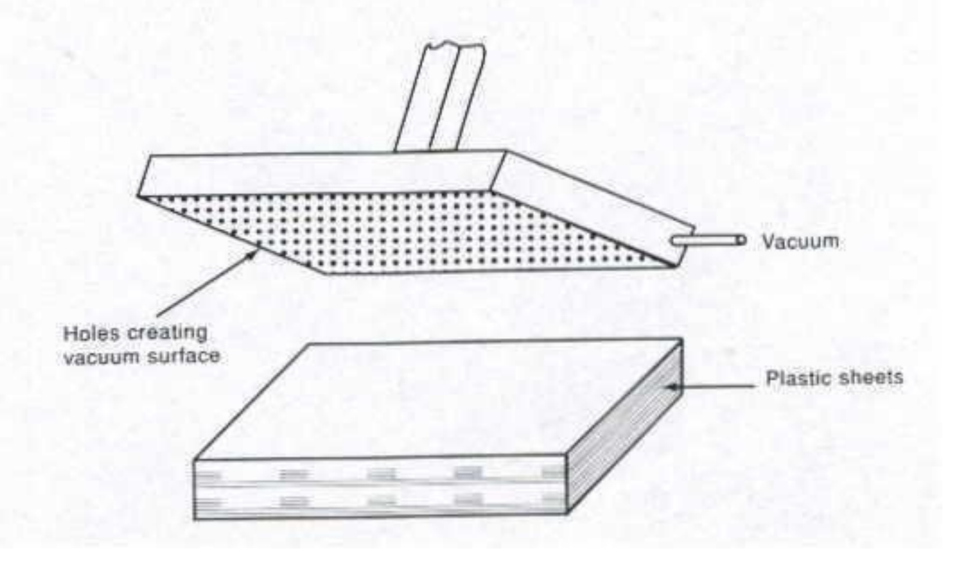
Tools – to perform a process, e.g., spot welding, spray painting

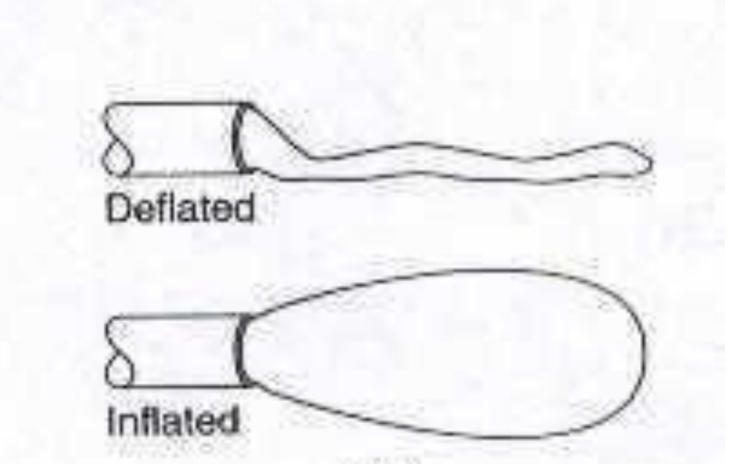
Unilateral vs Multilateral Gripper

Unilateral– only one point or surface is touching the object to be handled. (fig 1)

Example : vacuum pad gripper & Electro magnetic gripper

Multilateral – more than two points or surfaces touching the components to be handled (fig.a)





(a)

Gripper

End-effector that holds or grasp an object (in assembly, pick and place operation and material handling) to perform some task.

Four Major Types of gripper

1. Mechanical
2. Suction or vaccum cups 3. Magnetised gripper
4. Adhesives

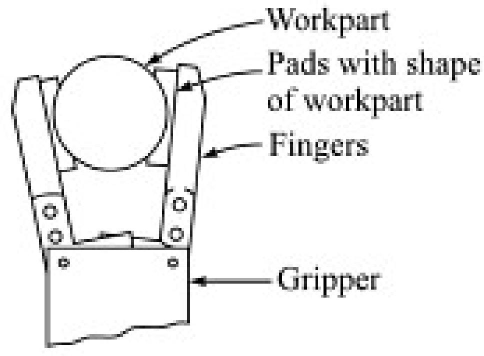
Mechanical Gripper

It is an end effector that uses mechanical fingers actuated by a mechanism to grasp an object.

- Two ways of constraining part in gripper

1. Physical construction of parts within finger. Finger encloses the part to some extent and thereby designing the contact surface of finger to be in approximate shape of part geometry.

2. Holding the part is by friction between fingers and workpart. Finger must apply force that is sufficient for friction to retain the part against gravity.



'ig. 5.2 *Physical constriction method of finger design.*

Mechanical Gripper

To resist the slippage, the gripper must be designed to exert a force that depends on the weight of the part, coeff of friction and acceleration of part.

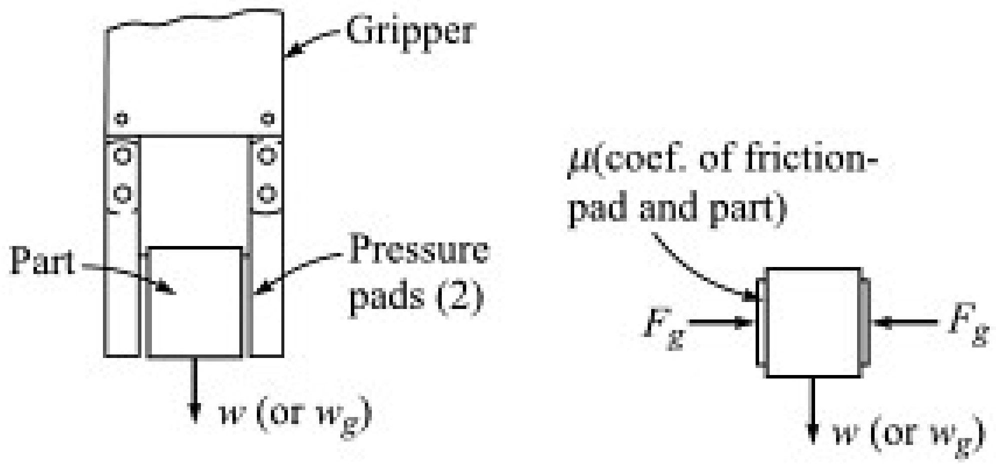
$$\mu n_f F_g = w \tag{5.1}$$

where μ = coefficient of friction of the finger contact surface against the part surface

n_f = number of contacting fingers

F_g = gripper force

w = weight of the part or object being gripped



Mechanical Gripper

$$\mu n_f F_g = wg$$

where g = the g factor. The g factor is supposed to take account of the combined effect of gravity and acceleration. If the acceleration force is applied in the same direction as the gravity force, then the g value = 3.0. If the acceleration is applied in the opposite direction, then the g value = 1.0 (2 × the weight of the part due to acceleration minus 1 × the weight of the part due to gravity). If the acceleration is applied in a horizontal direction, then use g = 2.0.

Mechanical Gripper Mechanism

Two ways of gripper mechanism based on finger movement

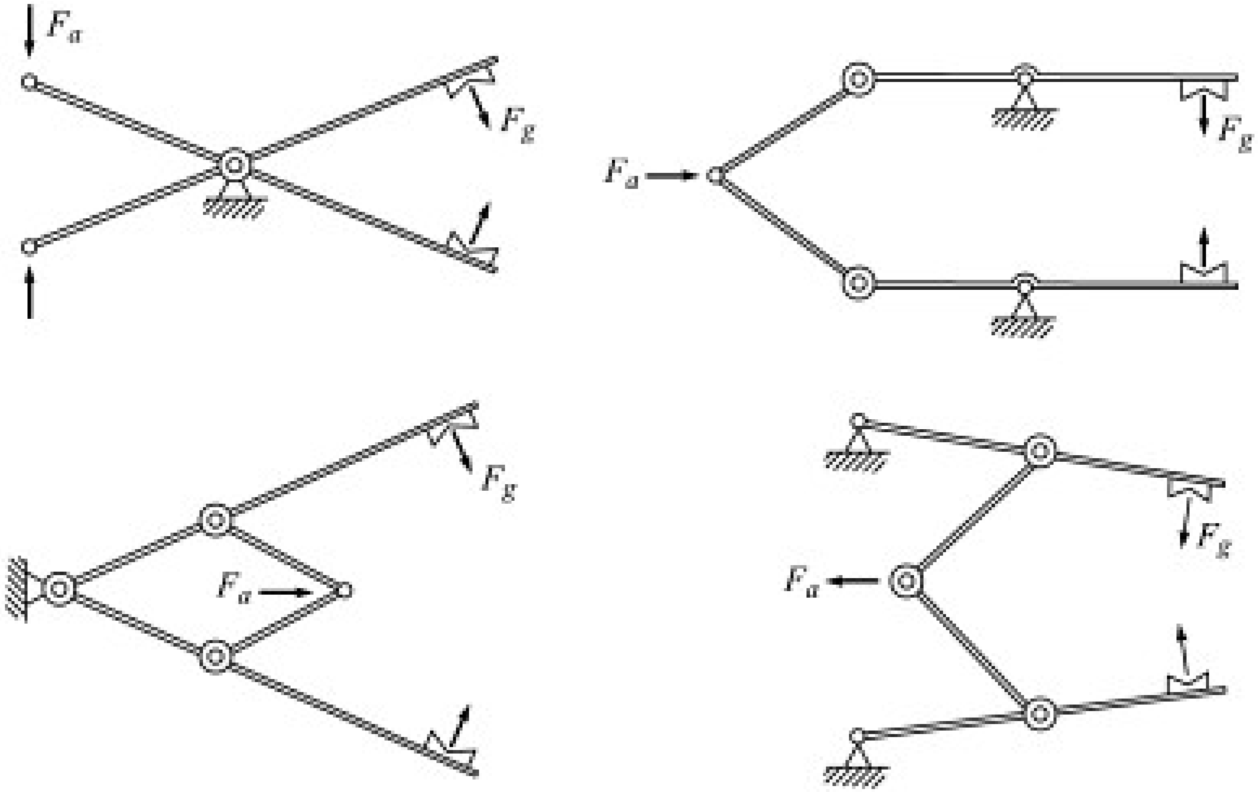
1. Pivoting movement – Eg. Link actuation
2. Linear or translational movement – Eg. Screw and cylinder

Four ways of gripper mechanism based on kinematic devices

1. Linkage actuation
2. Gear and rack actuation 3. Cam actuation
4. Screw actuation

Mechanical Gripper Mechanism

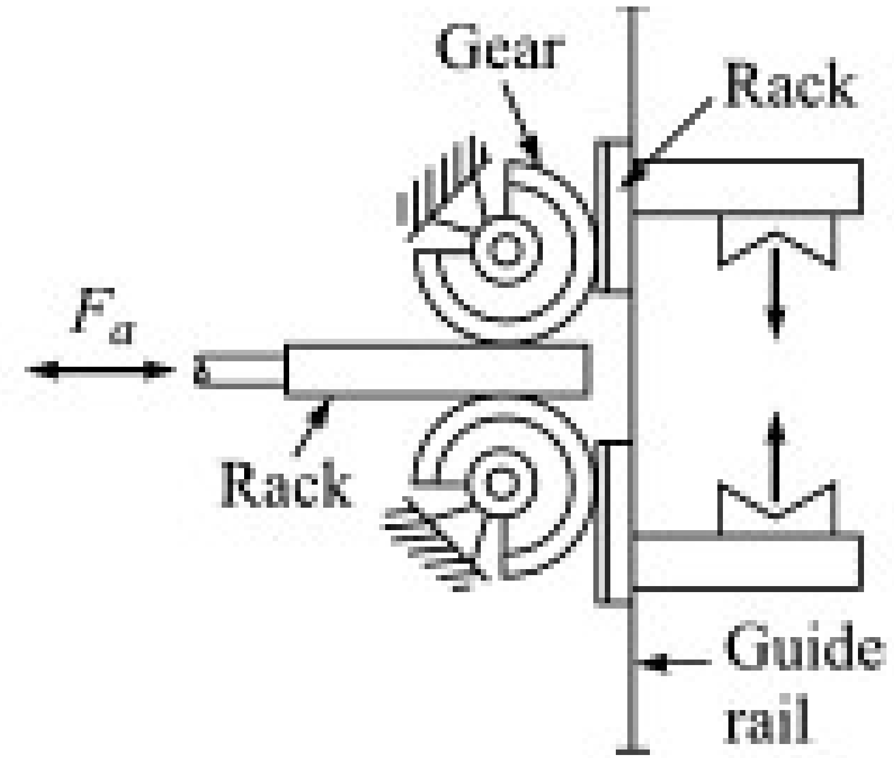
1. Linkage actuation



Some possible linkages for robot grippers.

Mechanical Gripper Mechanism

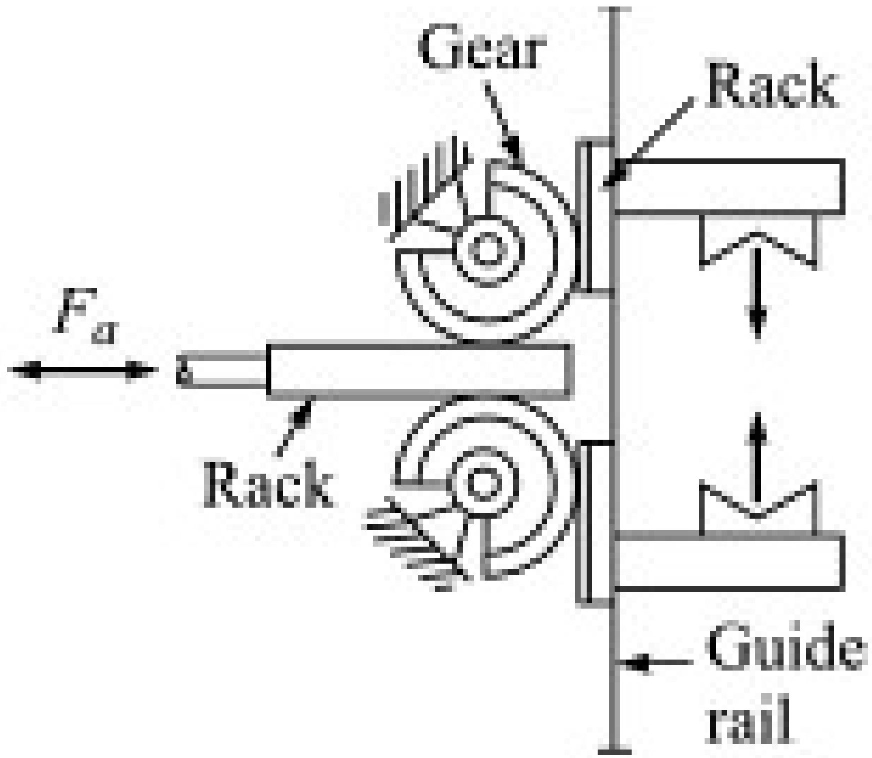
2. Gear and rack actuation



Gear-and-rack method of actuating the gripper.

Mechanical Gripper Mechanism

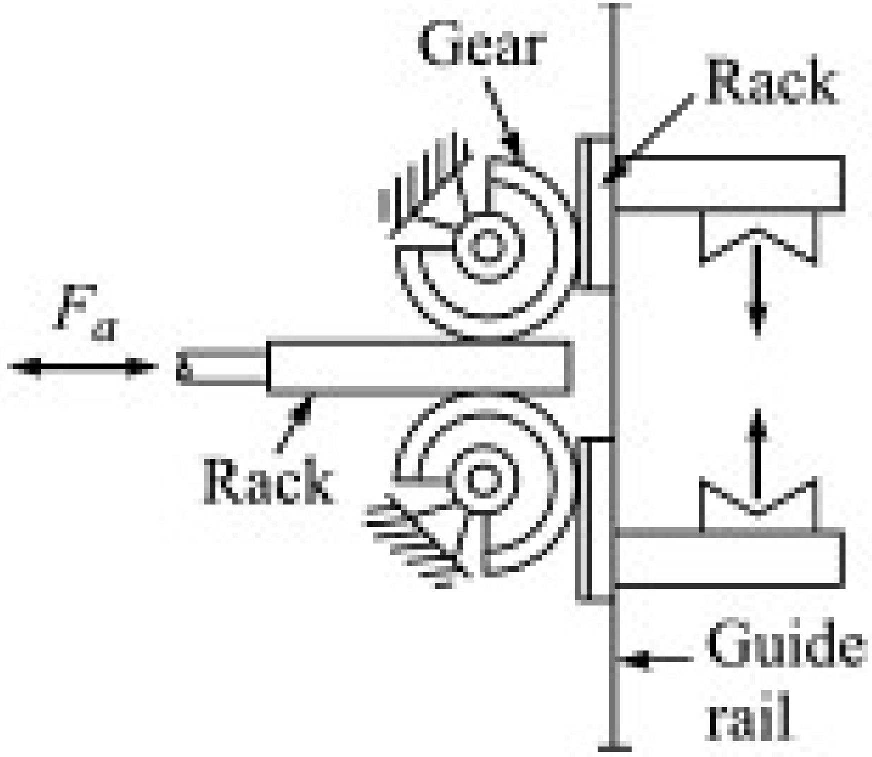
2. Gear and rack actuation



Gear-and-rack method of actuating the gripper.

Mechanical Gripper Mechanism

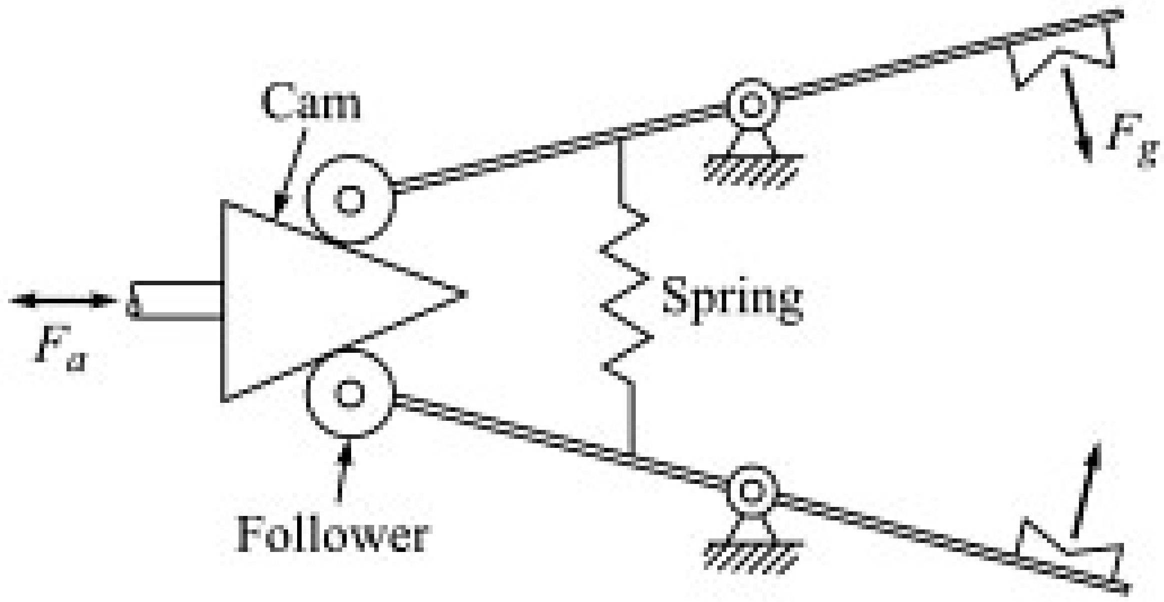
2. Gear and rack actuation



Gear-and-rack method of actuating the gripper.

Mechanical Gripper Mechanism

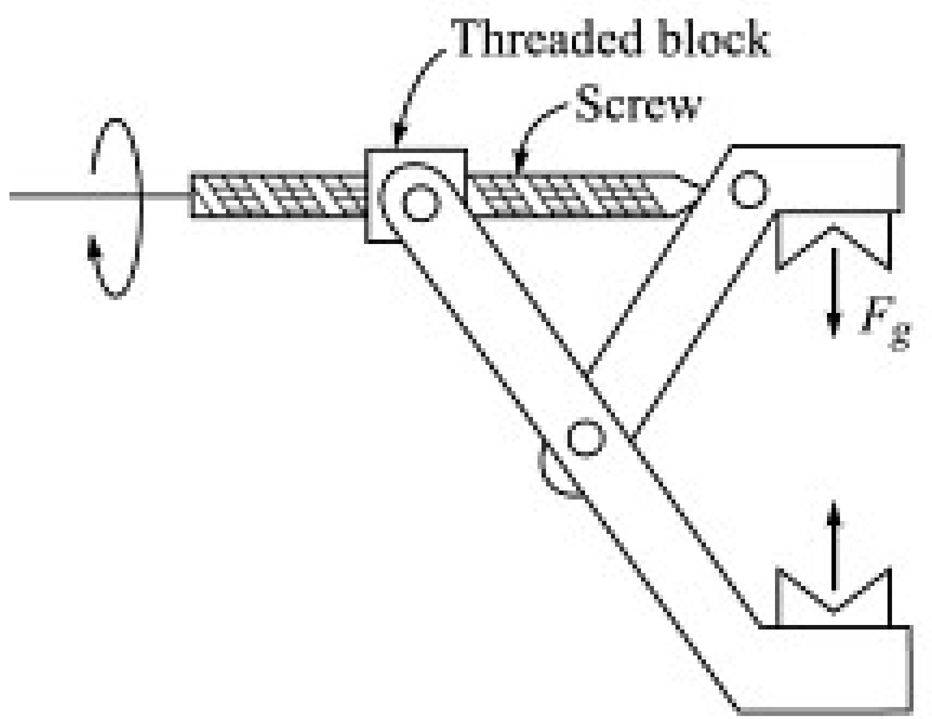
3. Cam actuation



Cam-actuated gripper.

Mechanical Gripper Mechanism

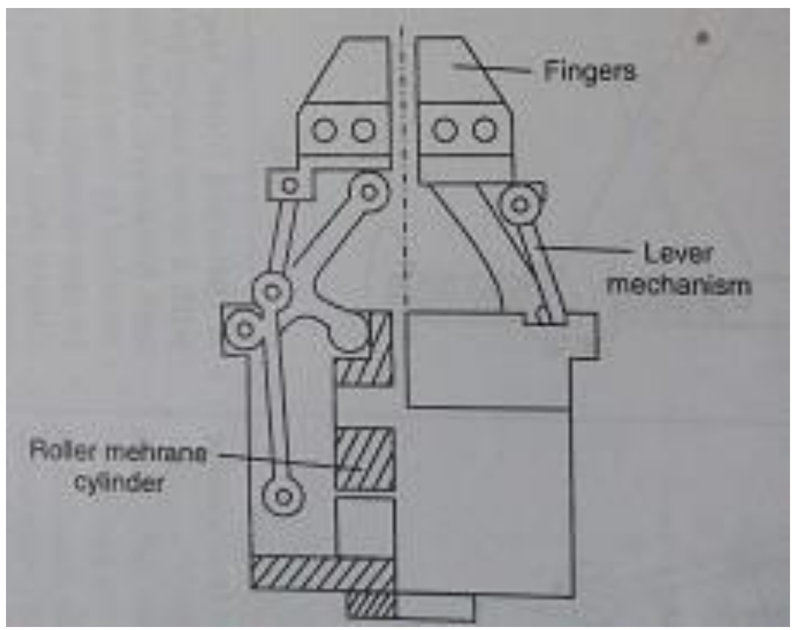
4. Screw actuation



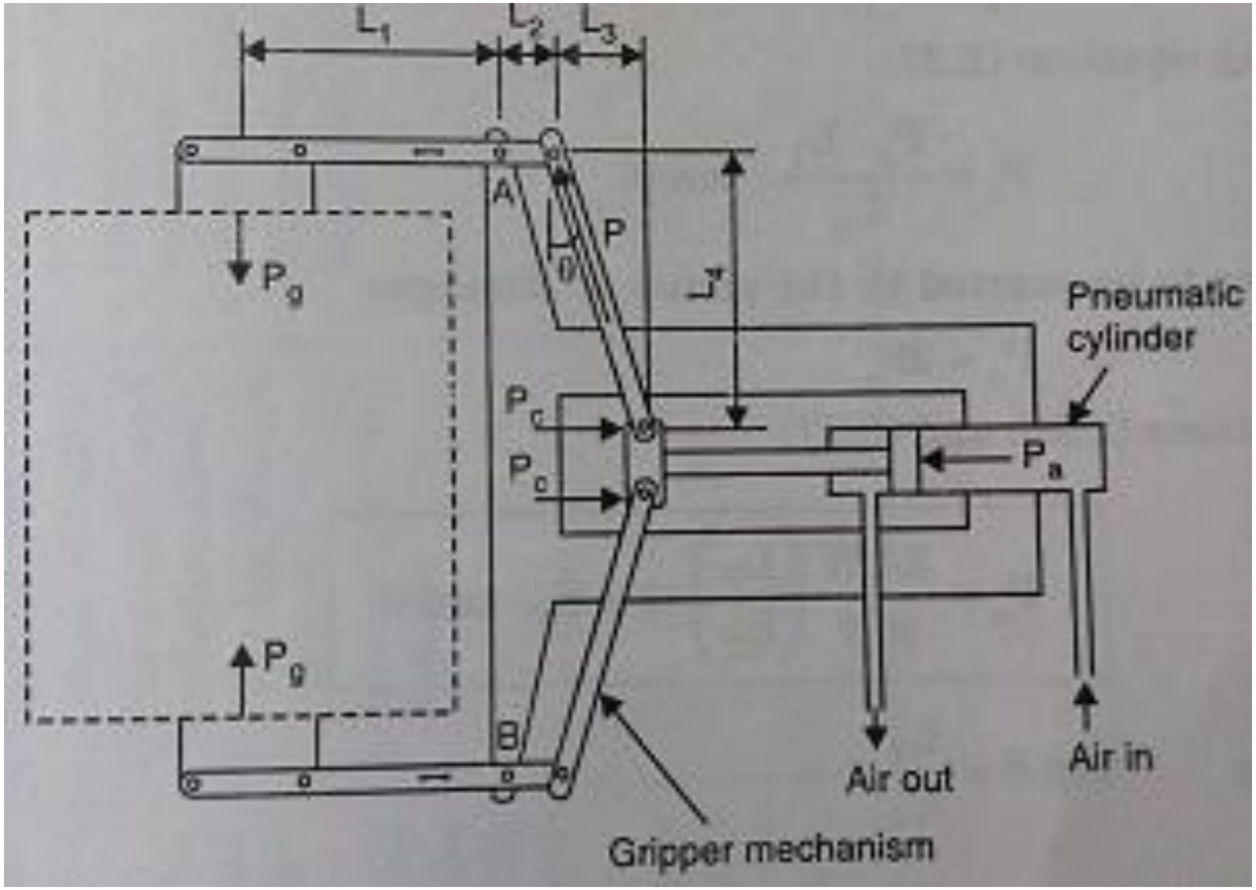
Screw-type gripper actuation.

Pneumatic or air operated Gripper

- Equipped with roller membrane cylinder with a rolling motion replacing conventional piston cylinder.
- This motion is transmitted to fingers by means of lever mechanism.
- The grippers are actuated by switching valves in the circuit.
- The finger stroke is limited by end stops or workpiece to be gripped.

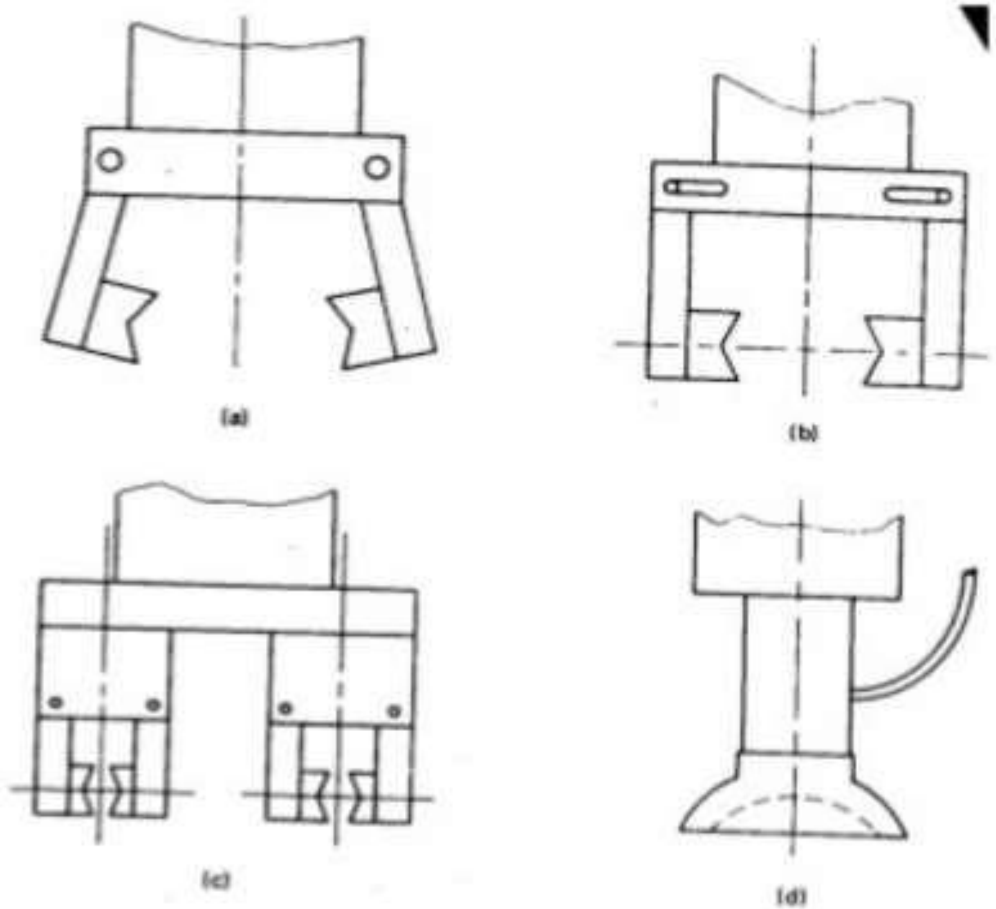


Gripper force analysis



2) Hooks and Scoops

- Hooks and scoops are the simplest type of end effectors that can be classes as grippers.
- A scoop or ladle is commonly used to scoop up molten metal and transfer it to the mould
- A hook may be all that is needed to lift a part especially if precise positioning in not required and if it is only to be dipped into a liquid.
- Hook are used to load and unload parts hanging from the overhead conveyors. The parts to be handled by a hook must have some sort of handle, eyebolt or ring to enable the hook to hold it.
- Scoops are used for handling the materials in liquid or power from, the limitation of scoop is, it is difficult to control the amount of martial being handled by the scoop. In addition, spilling of the material during handling is another problem.



Hooks and Scoops design

3) Magnetic Grippers

- Magnetic grippers obviously only work on magnetic objects and therefore are limited in working with
certain metals.
- For maximum effect the magnet needs to have complete contact with the surface of the metal to be gripped. Any air gaps will reduce the strength of the magnetic force, therefore flat sheets of metal are best suited to magnetic grippers.
- If the magnet is strong enough, a magnetic gripper can pick up an irregular shaped object. In some cases the shape of the magnet matches the shape of the object
- A disadvantage of using magnetic grippers is the temperature. Permanent magnets tend to become demagnetized when heated and so there is the danger that prolonged contact with a hot work piece will weaken them to the point where they can no longer be used. The effect of heat will depend on the time the magnet spends in contact with the hot part. Most magnetic materials are relatively unaffected by temperatures up to around 100 degrees.
- Electromagnets can be used instead and are operated by a DC electric current and lose nearly all of
their magnetism when the power is turned off.
- Permanent magnets are also used in situations where there is an explosive atmosphere and sparks from electrical equipment would cause a hazard

Magnetic Grippers design

4) Suction Grippers

- There are two types of suction grippers:

1. Devices operated by a vacuum – the vacuum may be provided by a vacuum pump or by compressed air

2. Devices with a flexible suction cup – this cup presses on the work piece. Compressed air is blown into the suction cup to release the work piece. The advantage of the suction cup is that if there is a power failure it will still work as the work piece will not fall down. The disadvantage of the suction cup is that they only work on clean, smooth surfaces.

- There are many more advantages for using a suction cup rather than a mechanical grip including: there is no danger of crushing fragile objects, the exact shape and size does not matter and the suction cup does not have to be precisely positioned on the object

- The downfalls of suction cups as an end effector include: the robot system must include a form of pump for air and the level of noise can cause annoyance in some circumstances

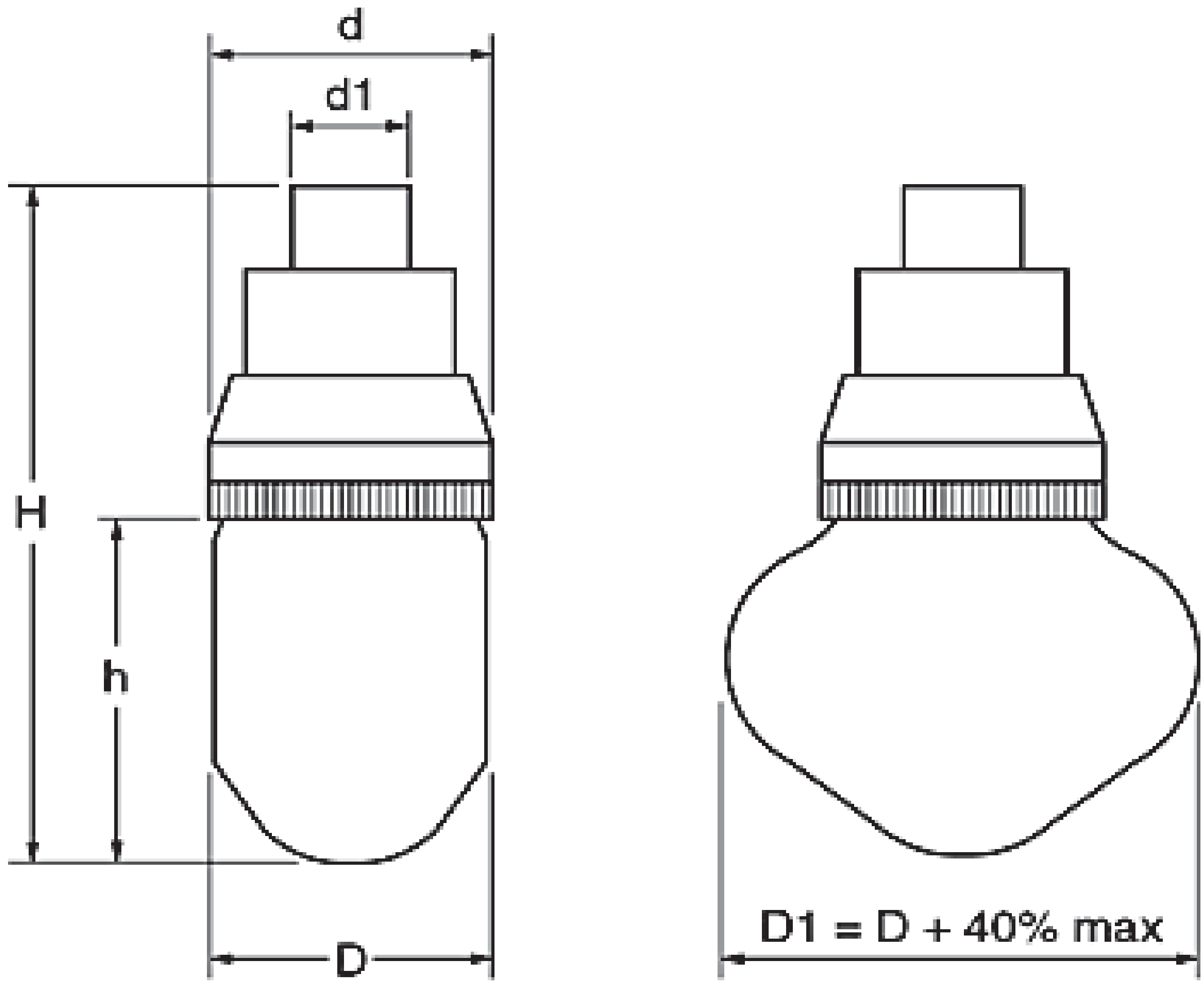
Vacuum gripper design

5) Expandable Bladder Type Grippers

- A **bladder gripper** or **bladder hand** is a specialized **robotic end effector** that can be used to grasp, pick up, and move rod-shaped or cylindrical objects.
- The main element of the gripper is an inflatable, donut-shaped or cylindrical sleeve that resembles the cuff commonly used in blood pressure measuring apparatus.
- The sleeve is positioned so it surrounds the object to be gripped, and then the sleeve

is inflated until it is tight enough to accomplish the desired task.

- The pressure exerted by the sleeve can be measured and regulated using force sensors.
- ***Bladder grippers are useful in handling fragile objects.*** However, they do not operate fast, and they can function only with objects within a rather narrow range of physical sizes.



Expandable Bladder Type Grippers

6) Adhesive Grippers

- Adhesive Substance can be used for grasping action in adhesive grippers.
- In adhesive grippers, the adhesive substance losses its tackiness due to repeated usage. This reduces the reliability of the gripper. In order to overcome this difficulty, the adhesive material is continuously fed to the gripper in the form of ribbon by feeding mechanism.
- A major asset of the adhesive gripper is the fact that it is simple. As long as the adhesive keep its stickiness it will continue to function without maintenance, however, there are certain limitations, the most significant is the fact that the adhesive cannot readily be disabled in order to release the grasp on an object. Some other means, such as devices that lock the gripped object into place, must be used.
- The adhesive grippers are used for handling fabrics and other lightweight materials.

Adhesive Grippers Design

Types of Tools

- A common tool used as an end effector is the welding tool. Welding is the process of joining two pieces of metal by melting them at the join and there are 3 main welding tools: a welding torch, spot welding gun and a stud welding tool
- Other common tools are paints praying, deburring tools, pneumatic tools such as a nut runner to tighten nuts.

Issues in choosing actuators

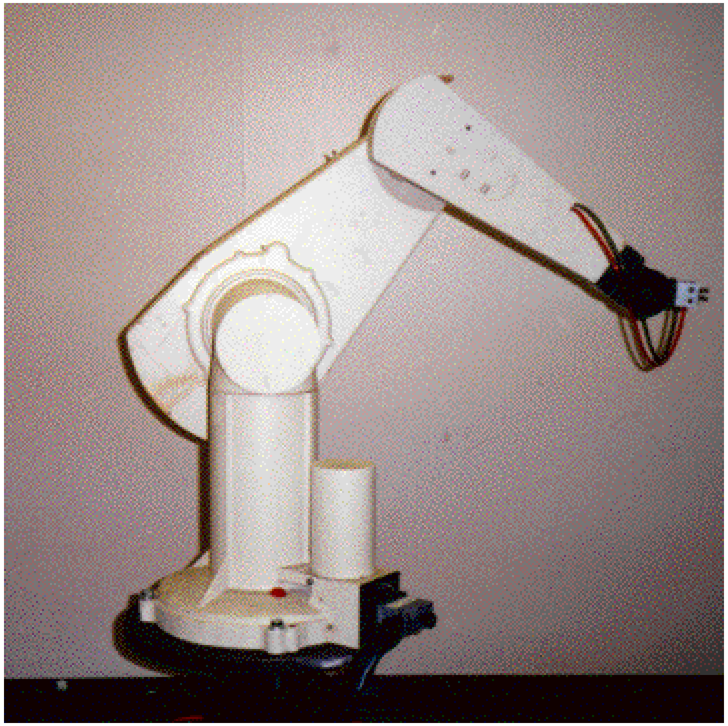
- Load (e.g. torque to overcome own inertia)
- Speed (fast enough but not too fast)
- Accuracy (will it move to where you want?)
- Resolution (can you specify exactly where?)
- Repeatability (will it do this every time?)
- Reliability (mean time between failures)
- Power consumption (how to feed it)
- Energy supply & its weight
- Also have many possible trade-offs between physical design and ability to *control*

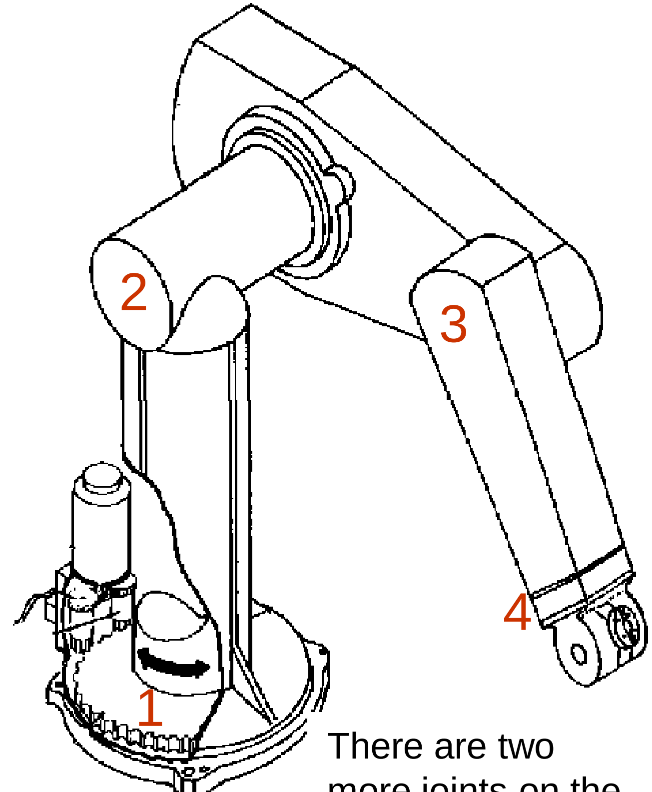
UNIT-II

An Introduction to Robot Kinematics

Kinematics studies the motion of bodies

An Example - The PUMA 560



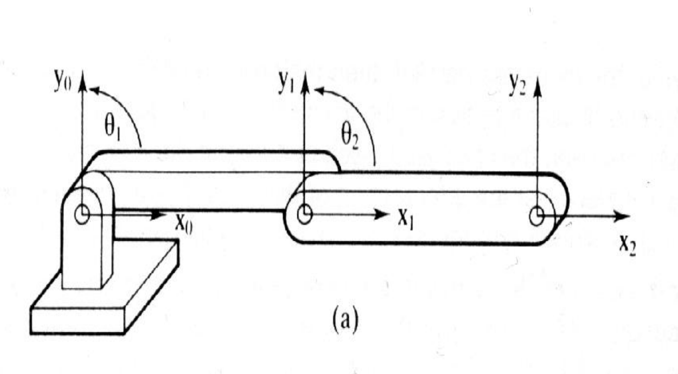


There are two more joints on the end effector (the gripper)

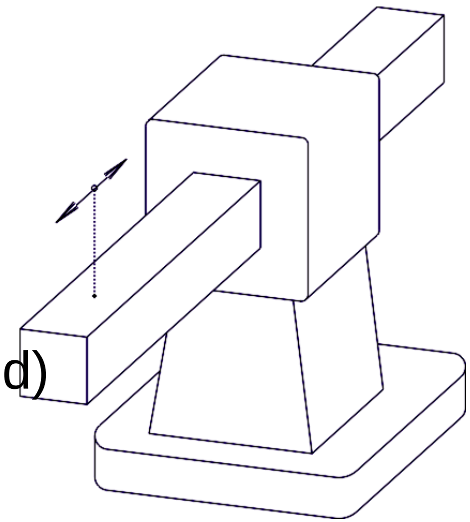
The PUMA 560 has SIX revolute joints

A revolute joint has ONE degree of freedom (1 DOF) that is defined by its angle

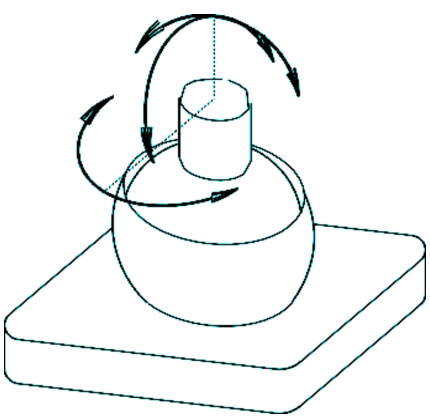
Other basic joints



Revolute Joint
1 DOF (Variable - Y)



Prismatic Joint
1 DOF (linear) (Variables - d)



Spherical Joint
3 DOF (Variables - Y_1, Y_2, Y_3)

We are interested in two kinematics topics

Forward Kinematics (angles to position)

What you are given: The length of each link
The angle of each joint

What you can find: The position of any point
(i.e. it's (x, y, z) coordinates

Inverse Kinematics (position to angles)

What you are given: The length of each link
The position of some point on the robot

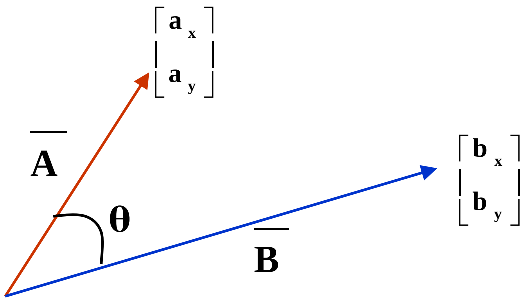
What you can find: The angles of each joint needed to obtain
that position

Quick Math Review

Dot Product:

Geometric Representation:

$$\overline{\mathbf{A}} \bullet \overline{\mathbf{B}} = \|\mathbf{A}\|\|\mathbf{B}\|\cos\theta$$



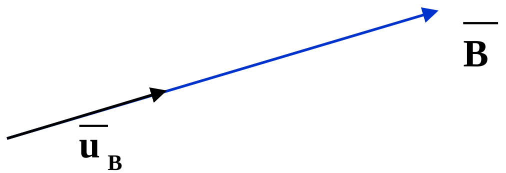
Matrix Representation:

$$\overline{\mathbf{A}} \bullet \overline{\mathbf{B}} = \begin{bmatrix} \mathbf{a}_x \\ \mathbf{a}_y \end{bmatrix} \bullet \begin{bmatrix} \mathbf{b}_x \\ \mathbf{b}_y \end{bmatrix} = \mathbf{a}_x \mathbf{b}_x + \mathbf{a}_y \mathbf{b}_y$$

Unit Vector

Vector in the direction of a chosen vector but whose magnitude is 1.

$$\overline{\mathbf{u}}_B = \frac{\overline{\mathbf{B}}}{\|\mathbf{B}\|}$$



Quick Matrix Review

Matrix Multiplication:

An (m x n) matrix A and an (n x p) matrix B, can be multiplied since the number of columns of A is equal to the number of rows of B.

<u>Non-Commutative Multiplication</u>

AB is NOT equal to BA

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} (ae + bg) & (af + bh) \\ (ce + dg) & (cf + dh) \end{bmatrix}$$

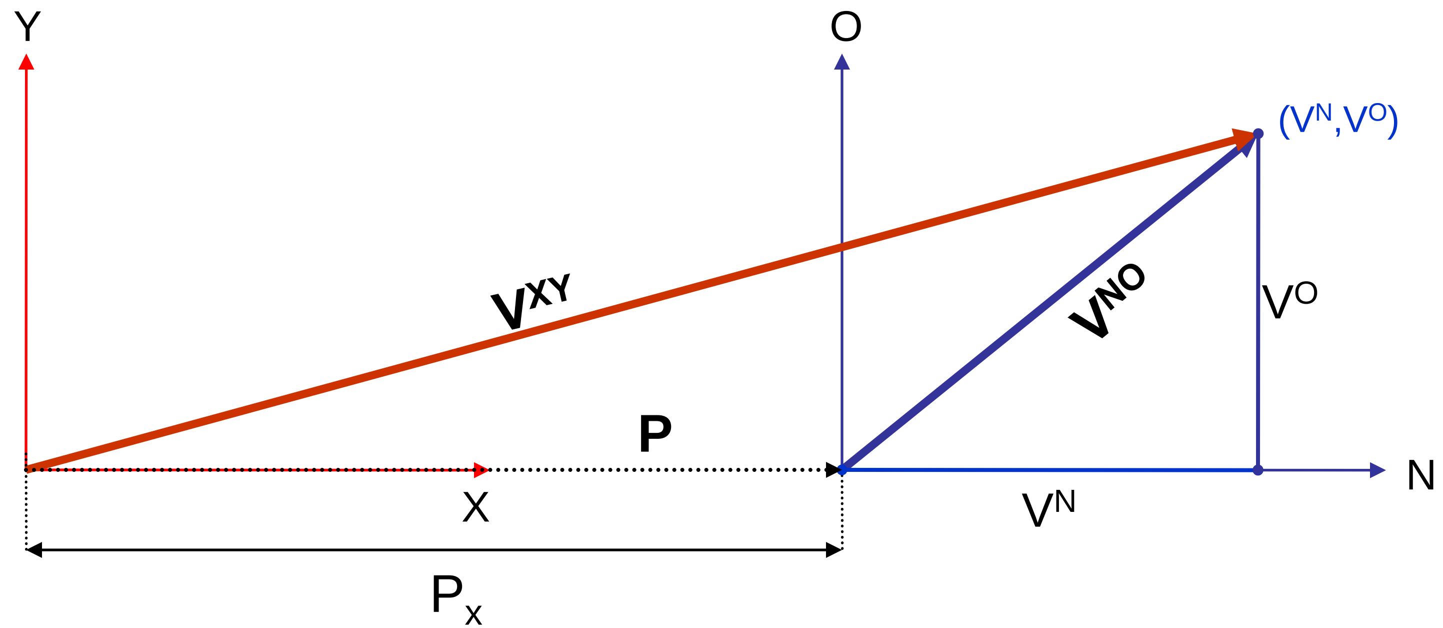
Matrix Addition:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} (a + e) & (b + f) \\ (c + g) & (d + h) \end{bmatrix}$$

Basic Transformations

Moving Between Coordinate Frames

Translation Along the X-Axis

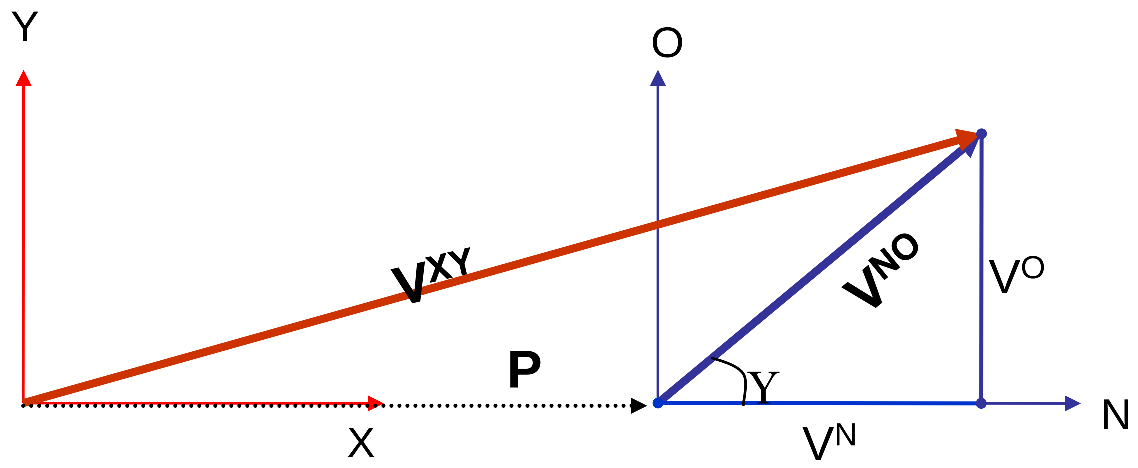


P_x = distance between the XY and NO coordinate planes

Notation:

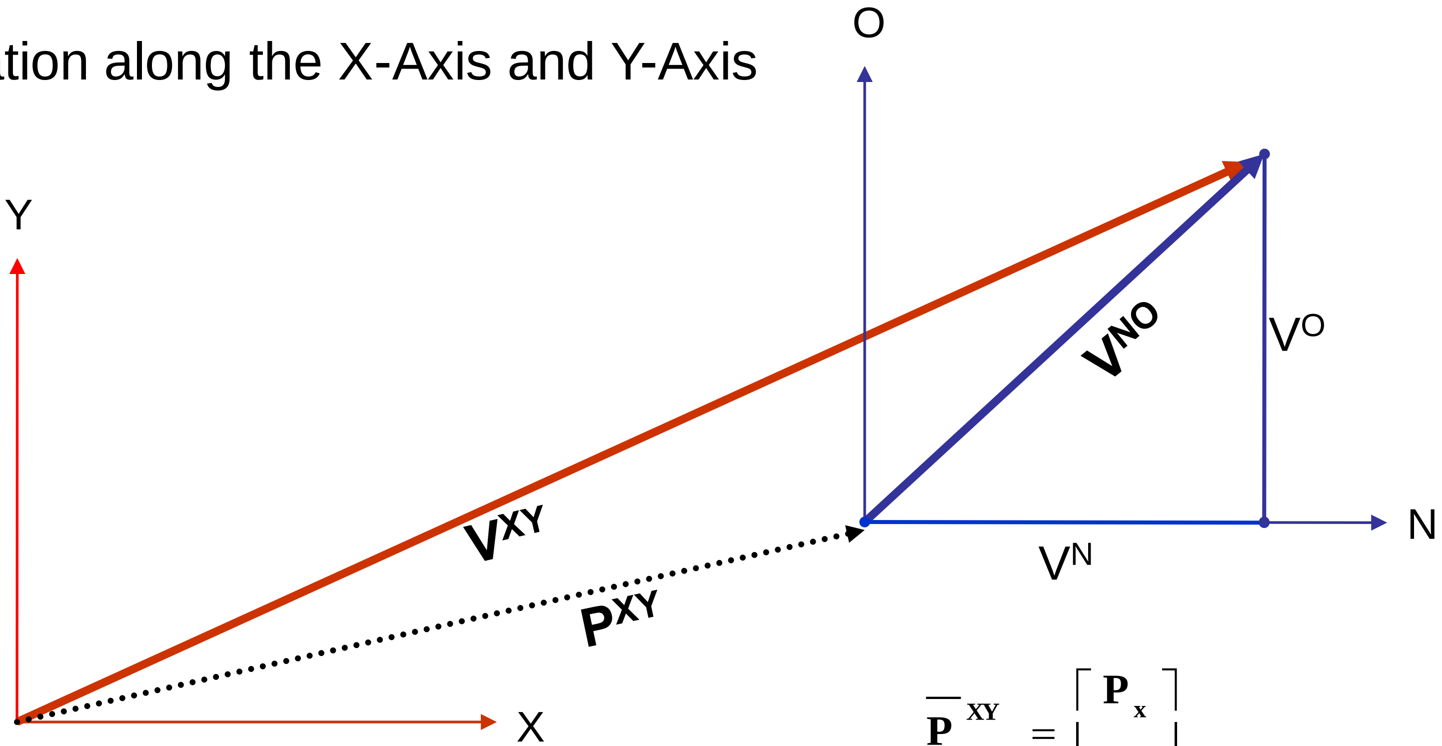
$$\overline{\mathbf{V}}^{\mathbf{XY}} = \begin{bmatrix} \mathbf{V}^{\mathbf{X}} \\ \mathbf{V}^{\mathbf{Y}} \end{bmatrix} \qquad \overline{\mathbf{V}}^{\mathbf{NO}} = \begin{bmatrix} \mathbf{V}^{\mathbf{N}} \\ \mathbf{V}^{\mathbf{O}} \end{bmatrix} \qquad \overline{\mathbf{P}} = \begin{bmatrix} \mathbf{P}_{\mathbf{x}} \\ \mathbf{0} \end{bmatrix}$$

Writing $\overline{V}^{XY}$ in terms of $\overline{V}^{NO}$



$$\overline{V}^{XY} = \begin{bmatrix} P_X + V^N \\ V^O \end{bmatrix} = \overline{P} + \overline{V}^{NO}$$

Translation along the X-Axis and Y-Axis



$$\overline{\mathbf{P}}^{XY} = \begin{bmatrix} \mathbf{P}_x \\ \mathbf{P}_Y \end{bmatrix}$$

$$\overline{\mathbf{V}}^{XY} = \overline{\mathbf{P}} + \overline{\mathbf{V}}^{NO} = \begin{bmatrix} \mathbf{P}_X + \mathbf{V}^N \\ \mathbf{P}_Y + \mathbf{V}^O \end{bmatrix}$$

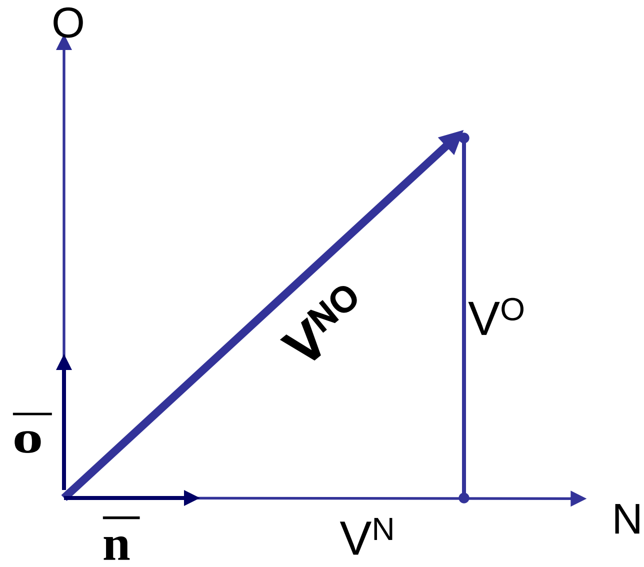
Using Basis Vectors

Basis vectors are unit vectors that point along a coordinate axis

$\overline{\mathbf{n}}$ Unit vector along the N-Axis

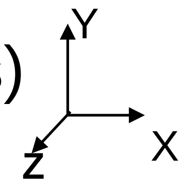
$\overline{\mathbf{o}}$ Unit vector along the N-Axis

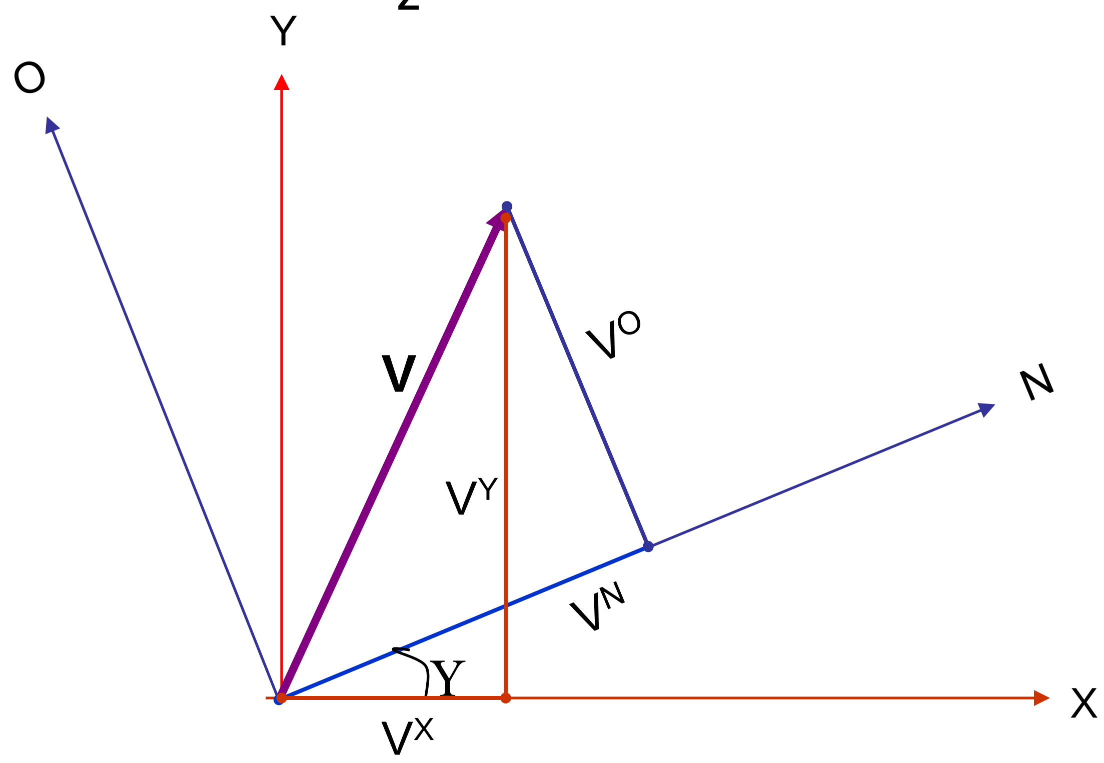
$\left\|\mathbf{V}^{NO}\right\|$ Magnitude of the V^{NO} vector



$$\overline{\mathbf{V}}^{NO} = \begin{bmatrix} \mathbf{V}^{N} \\ \mathbf{V}^{O} \end{bmatrix} = \begin{bmatrix} \left\|\mathbf{V}^{NO}\right\| \cos\theta \\ \left\|\mathbf{V}^{NO}\right\| \sin\theta \end{bmatrix} = \begin{bmatrix} \left\|\mathbf{V}^{NO}\right\| \cos\theta \\ \left\|\mathbf{V}^{NO}\right\| \cos(90-\theta) \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{V}}^{NO} \bullet \overline{\mathbf{n}} \\ \overline{\mathbf{V}}^{NO} \bullet \overline{\mathbf{o}} \end{bmatrix}$$

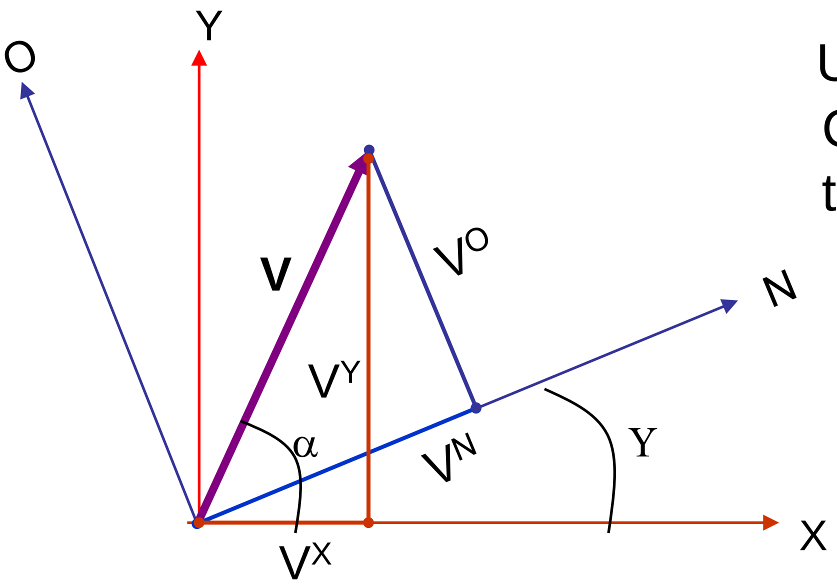
Rotation (around the Z-Axis)





Y = Angle of rotation between the XY and NO coordinate axis

$$\overline{\mathbf{V}}^{\mathbf{XY}} = \begin{bmatrix} \mathbf{V}^{\mathbf{X}} \\ \mathbf{V}^{\mathbf{Y}} \end{bmatrix} \qquad \overline{\mathbf{V}}^{\mathbf{NO}} = \begin{bmatrix} \mathbf{V}^{\mathbf{N}} \\ \mathbf{V}^{\mathbf{O}} \end{bmatrix}$$



Unit vector along X-Axis
Can be considered with respect to
the XY coordinates or NO coordinates

$$\left\|\overline{\mathbf{V}}^{\mathbf{XY}}\right\| = \left\|\overline{\mathbf{V}}^{\mathbf{NO}}\right\|$$

$$\mathbf{V}^{\mathbf{X}} = \left\|\overline{\mathbf{V}}^{\mathbf{XY}}\right\| \cos \alpha = \left\|\overline{\mathbf{V}}^{\mathbf{NO}}\right\| \cos \alpha = \overline{\mathbf{V}}^{\mathbf{NO}} \bullet \overline{\mathbf{x}}$$

$$\mathbf{V}^{\mathbf{X}} = (\mathbf{V}^{\mathbf{N}} * \overline{\mathbf{n}} + \mathbf{V}^{\mathbf{O}} * \overline{\mathbf{o}}) \bullet \overline{\mathbf{x}}$$

(Substituting for V^{NO} using the N and O components of the vector)

$$\mathbf{V}^{\mathbf{X}} = \mathbf{V}^{\mathbf{N}}(\overline{\mathbf{x}} \bullet \overline{\mathbf{n}}) + \mathbf{V}^{\mathbf{O}}(\overline{\mathbf{x}} \bullet \overline{\mathbf{o}})$$

$$= \mathbf{V}^{\mathbf{N}}(\cos \theta) + \mathbf{V}^{\mathbf{O}}(\cos(\theta + 90))$$

$$= \mathbf{V}^{\mathbf{N}}(\cos \theta) - \mathbf{V}^{\mathbf{O}}(\sin \theta)$$

Similarly….

$$V^{Y} = \left\| \overline{V}^{NO} \right\| \sin\alpha = \left\| \overline{V}^{NO} \right\| \cos(90 - \alpha) = \overline{V}^{NO} \bullet \overline{y}$$

$$V^{Y} = (V^{N} * \overline{n} + V^{O} * \overline{o}) \bullet \overline{y}$$

$$V^{Y} = V^{N}(\overline{y} \bullet \overline{n}) + V^{O}(\overline{y} \bullet \overline{o})$$

$$= V^{N}(\cos(90 - \theta)) + V^{O}(\cos\theta)$$

$$= V^{N}(\sin\theta) + V^{O}(\cos\theta)$$

So….

$$V^{X} = V^{N}(\cos\theta) - V^{O}(\sin\theta)$$

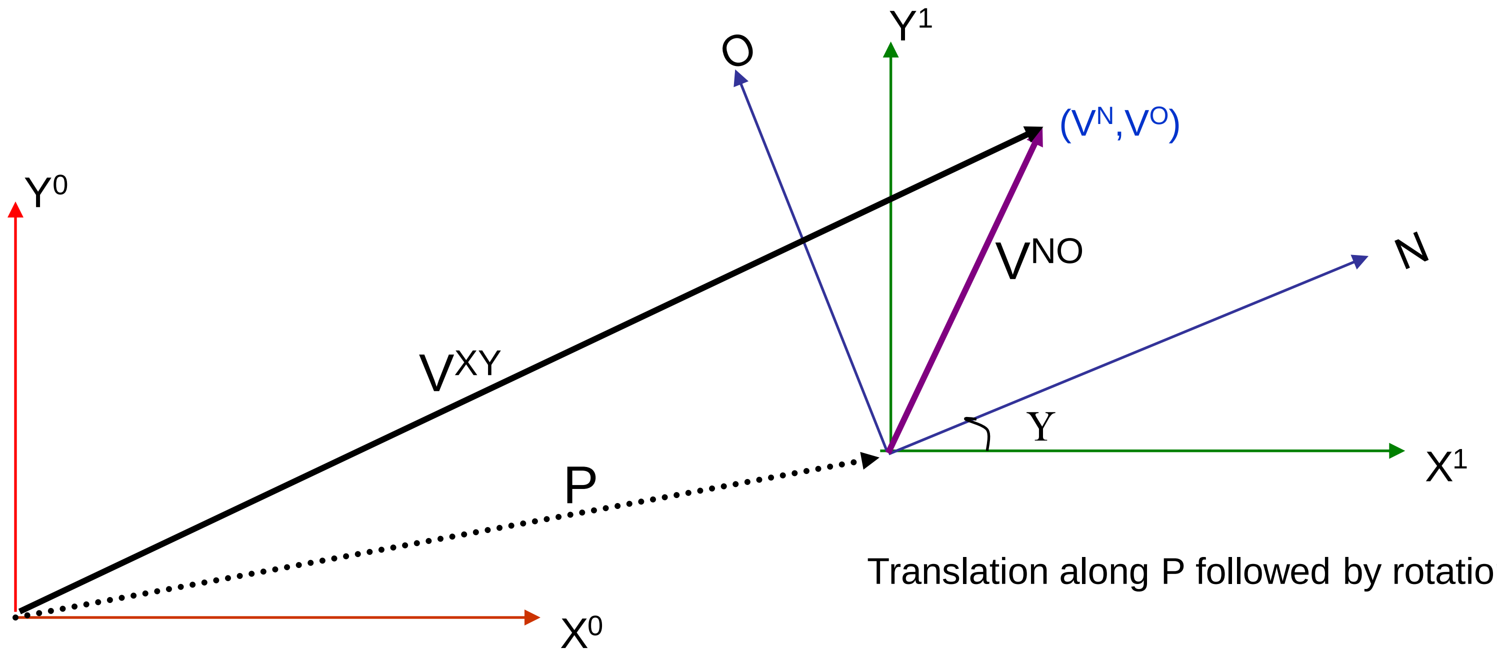
$$V^{Y} = V^{N}(\sin\theta) + V^{O}(\cos\theta)$$

$$\overline{V}^{XY} = \begin{bmatrix} V^{X} \\ V^{Y} \end{bmatrix}$$

Written in Matrix Form

$$\overline{V}^{XY} = \begin{bmatrix} V^{X} \\ V^{Y} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V^{N} \\ V^{O} \end{bmatrix}$$

Rotation Matrix about the z-axis



Translation along P followed by rotation by θ

$$\mathbf{V}^{XY} = \begin{bmatrix} \mathbf{V}^{X} \\ \mathbf{V}^{Y} \end{bmatrix} = \begin{bmatrix} \mathbf{P}_x \\ \mathbf{P}_y \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \mathbf{V}^{N} \\ \mathbf{V}^{O} \end{bmatrix}$$

(Note : P_x, P_y are relative to the original coordinate frame. Translation followed by rotation is different than rotation followed by translation.)

In other words, knowing the coordinates of a point (V^N,V^O) in some coordinate frame (NO) you can find the position of that point relative to your original coordinate frame (X^0Y^0).

HOMOGENEOUS REPRESENTATION

Putting it all into a Matrix

$$\mathbf{V}^{XY} = \begin{bmatrix} \mathbf{V}^{X} \\ \mathbf{V}^{Y} \end{bmatrix} = \begin{bmatrix} \mathbf{P}_x \\ \mathbf{P}_y \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \mathbf{V}^{N} \\ \mathbf{V}^{O} \end{bmatrix}$$

What we found by doing a translation and a rotation

$$= \begin{bmatrix} \mathbf{V}^{X} \\ \mathbf{V}^{Y} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{P}_x \\ \mathbf{P}_y \\ 1 \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{V}^{N} \\ \mathbf{V}^{O} \\ 1 \end{bmatrix}$$

Padding with 0’s and 1’s

$$= \begin{bmatrix} \mathbf{V}^{X} \\ \mathbf{V}^{Y} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & \mathbf{P}_x \\ \sin\theta & \cos\theta & \mathbf{P}_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{V}^{N} \\ \mathbf{V}^{O} \\ 1 \end{bmatrix}$$

Simplifying into a matrix form

$$\mathbf{H} = \begin{bmatrix} \cos\theta & -\sin\theta & \mathbf{P}_x \\ \sin\theta & \cos\theta & \mathbf{P}_y \\ 0 & 0 & 1 \end{bmatrix}$$

Homogenous Matrix for a Translation in XY plane, followed by a Rotation around the z-axis

Rotation Matrices in 3D – OK,lets return from homogenous repn

$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation around the Z-Axis

$$R_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

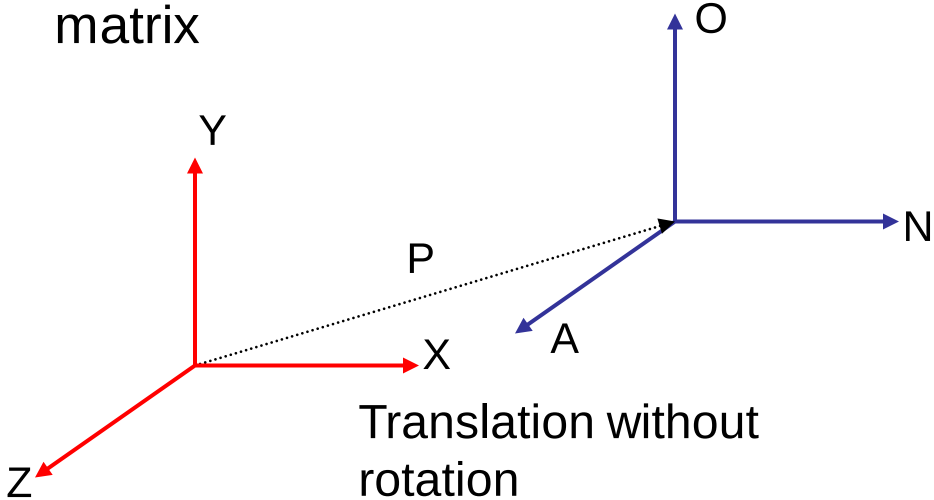
Rotation around the Y-Axis

$$R_z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

Rotation around the X-Axis

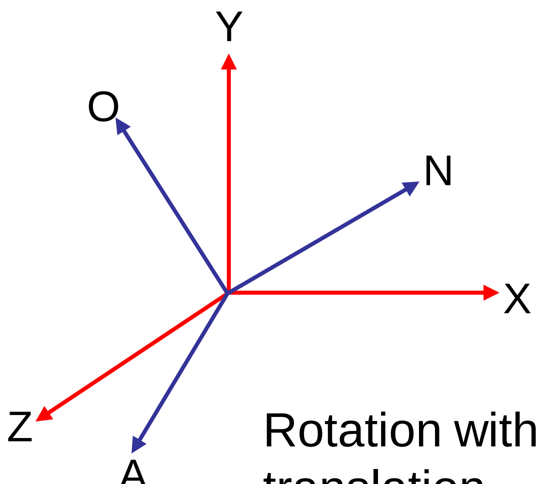
Homogeneous Matrices in 3D

H is a 4x4 matrix that can describe a translation, rotation, or both in one matrix



Translation without rotation

$$H = \begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Rotation without translation

$$H = \begin{bmatrix} n_x & o_x & a_x & 0 \\ n_y & o_y & a_y & 0 \\ n_z & o_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation part:

Could be rotation around z-axis, x-axis, y-axis or a combination of the three.

Homogeneous Continued….

$$V^{XY} = H \begin{bmatrix} V^N \\ V^O \\ V^A \\ 1 \end{bmatrix}$$

The (n,o,a) position of a point relative to the current coordinate frame you are in.

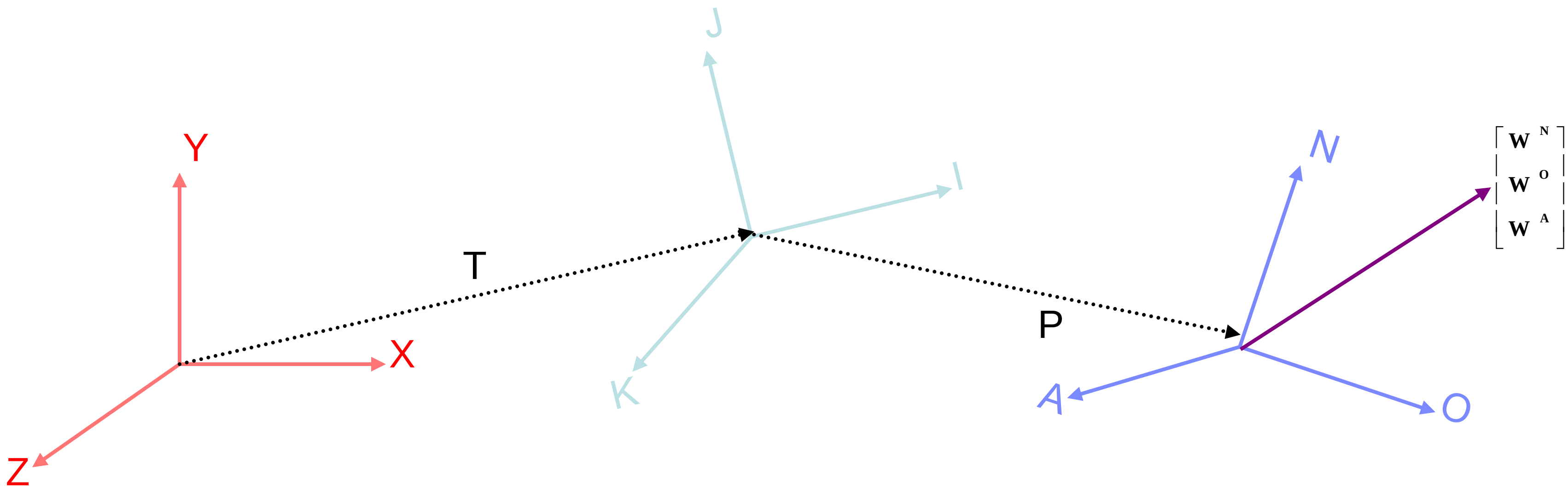
$$V^{XY} = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V^N \\ V^O \\ V^A \\ 1 \end{bmatrix}$$

$$V^X = n_x V^N + o_x V^O + a_x V^A + P_x$$

The rotation and translation part can be combined into a single homogeneous matrix IF and ONLY IF both are relative to the same coordinate frame.

Finding the Homogeneous Matrix

EX.



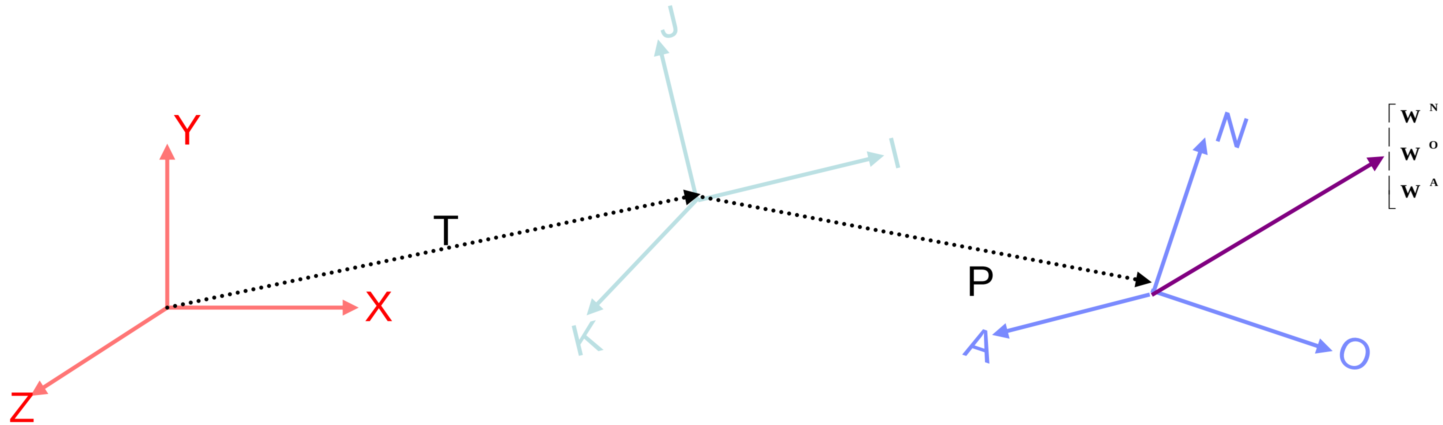
$\begin{bmatrix} w^X \\ w^Y \\ w^Z \end{bmatrix}$ Point relative to the X-Y-Z frame

$\begin{bmatrix} w^I \\ w^J \\ w^K \end{bmatrix}$ Point relative to the I-J-K frame

$\begin{bmatrix} w^N \\ w^O \\ w^A \end{bmatrix}$ Point relative to the N-O-A frame

$$\begin{bmatrix} W^I \\ W^J \\ W^K \end{bmatrix} = \begin{bmatrix} P_i \\ P_j \\ P_k \end{bmatrix} + \begin{bmatrix} n_i & o_i & a_i \\ n_j & o_j & a_j \\ n_k & o_k & a_k \end{bmatrix} \begin{bmatrix} W^N \\ W^O \\ W^A \end{bmatrix}$$

$$\begin{bmatrix} W^I \\ W^J \\ W^K \\ 1 \end{bmatrix} = \begin{bmatrix} n_i & o_i & a_i & P_i \\ n_j & o_j & a_j & P_j \\ n_k & o_k & a_k & P_k \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} W^N \\ W^O \\ W^A \\ 1 \end{bmatrix}$$



$$\begin{bmatrix} W^X \\ W^Y \\ W^Z \end{bmatrix} = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} + \begin{bmatrix} i_x & j_x & k_x \\ i_y & j_y & k_y \\ i_z & j_z & k_z \end{bmatrix} \begin{bmatrix} W^I \\ W^J \\ W^k \end{bmatrix} \longrightarrow \begin{bmatrix} W^X \\ W^Y \\ W^Z \\ 1 \end{bmatrix} = \begin{bmatrix} i_x & j_x & k_x & T_x \\ i_y & j_y & k_y & T_y \\ i_z & j_z & k_z & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} W^I \\ W^J \\ W^K \\ 1 \end{bmatrix}$$

Substituting for $\begin{bmatrix} W^I \\ W^J \\ W^K \end{bmatrix}$

$$\begin{bmatrix} W^X \\ W^Y \\ W^Z \\ 1 \end{bmatrix} = \begin{bmatrix} i_x & j_x & k_x & T_x \\ i_y & j_y & k_y & T_y \\ i_z & j_z & k_z & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_i & o_i & a_i & P_i \\ n_j & o_j & a_j & P_j \\ n_k & o_k & a_k & P_k \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} W^N \\ W^O \\ W^A \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} W^X \\ W^Y \\ W^Z \\ 1 \end{bmatrix} = H \begin{bmatrix} W^N \\ W^O \\ W^A \\ 1 \end{bmatrix}$$

$$H = \begin{bmatrix} i_x & j_x & k_x & T_x \\ i_y & j_y & k_y & T_y \\ i_z & j_z & k_z & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_i & o_i & a_i & P_i \\ n_j & o_j & a_j & P_j \\ n_k & o_k & a_k & P_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Product of the two matrices

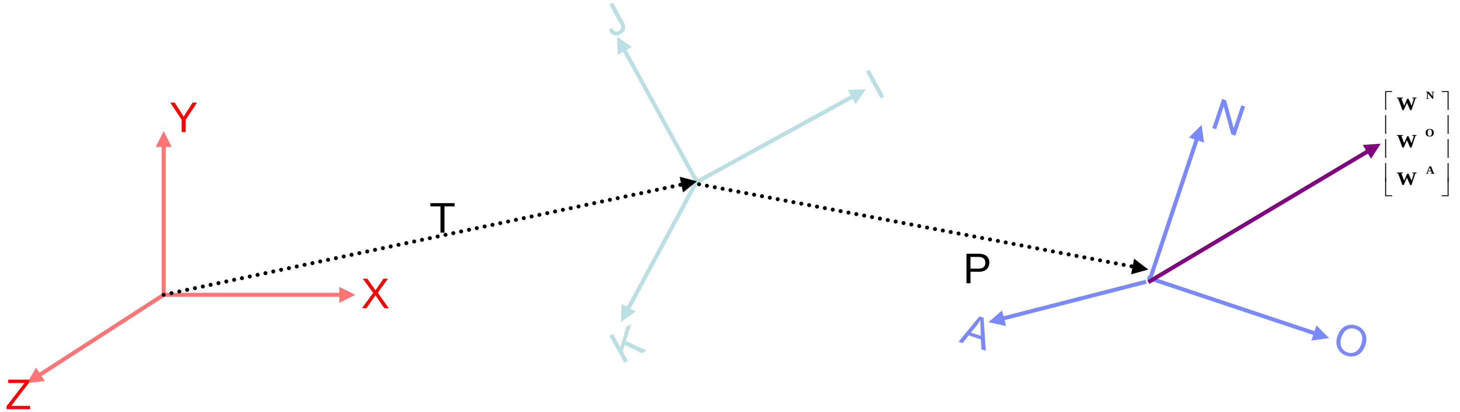
Notice that H can also be written as:

$$H = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_x & j_x & k_x & 0 \\ i_y & j_y & k_y & 0 \\ i_z & j_z & k_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & P_i \\ 0 & 1 & 0 & P_j \\ 0 & 0 & 1 & P_k \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_i & o_i & a_i & 0 \\ n_j & o_j & a_j & 0 \\ n_k & o_k & a_k & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

H = (Translation relative to the XYZ frame) * (Rotation relative to the XYZ frame)

* (Translation relative to the IJK frame) * (Rotation relative to the IJK frame)

The Homogeneous Matrix is a concatenation of numerous translations and rotations



One more variation on finding H:

H = (Rotate so that the X-axis is aligned with T)

* (Translate along the new t-axis by || T || (magnitude of T))

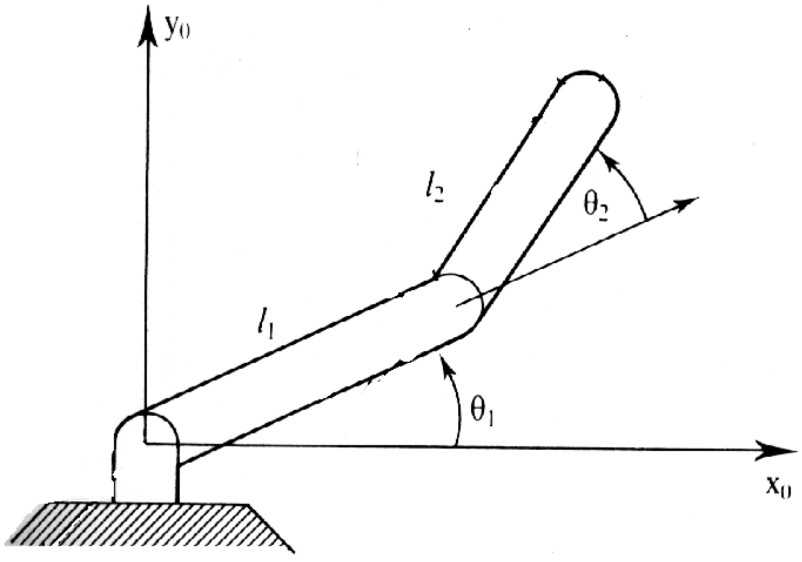
* (Rotate so that the t-axis is aligned with P)

* (Translate along the p-axis by || P ||)

* (Rotate so that the p-axis is aligned with the O-axis)

This method might seem a bit confusing, but it's actually an easier way to solve our problem given the information we have. Here is an example…

FORWARD KINEMATICS



The Situation:

You have a robotic arm that starts out aligned with the x_o-axis.
You tell the first link to move by Υ_1 and the second link to move by Υ_2.

The Quest:

What is the position of the end of the robotic arm?

Solution:

1. Geometric Approach

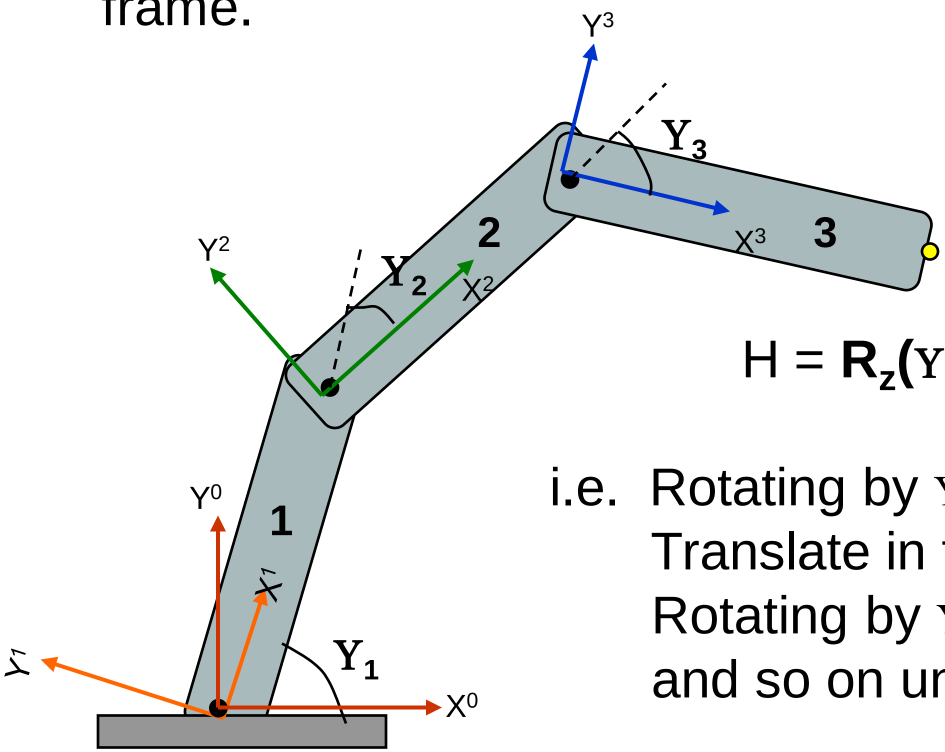
This might be the easiest solution for the simple situation. However, notice that the angles are measured relative to the direction of the previous link. (The first link is the exception. The angle is measured relative to it's initial position.) For robots with more links and whose arm extends into 3 dimensions the geometry gets much more tedious.

2. Algebraic Approach

Involves coordinate transformations.

Example Problem:

You are have a three link arm that starts out aligned in the x-axis. Each link has lengths l_1, l_2, l_3, respectively. You tell the first one to move by γ_1 , and so on as the diagram suggests. Find the Homogeneous matrix to get the position of the yellow dot in the X^0Y^0 frame.



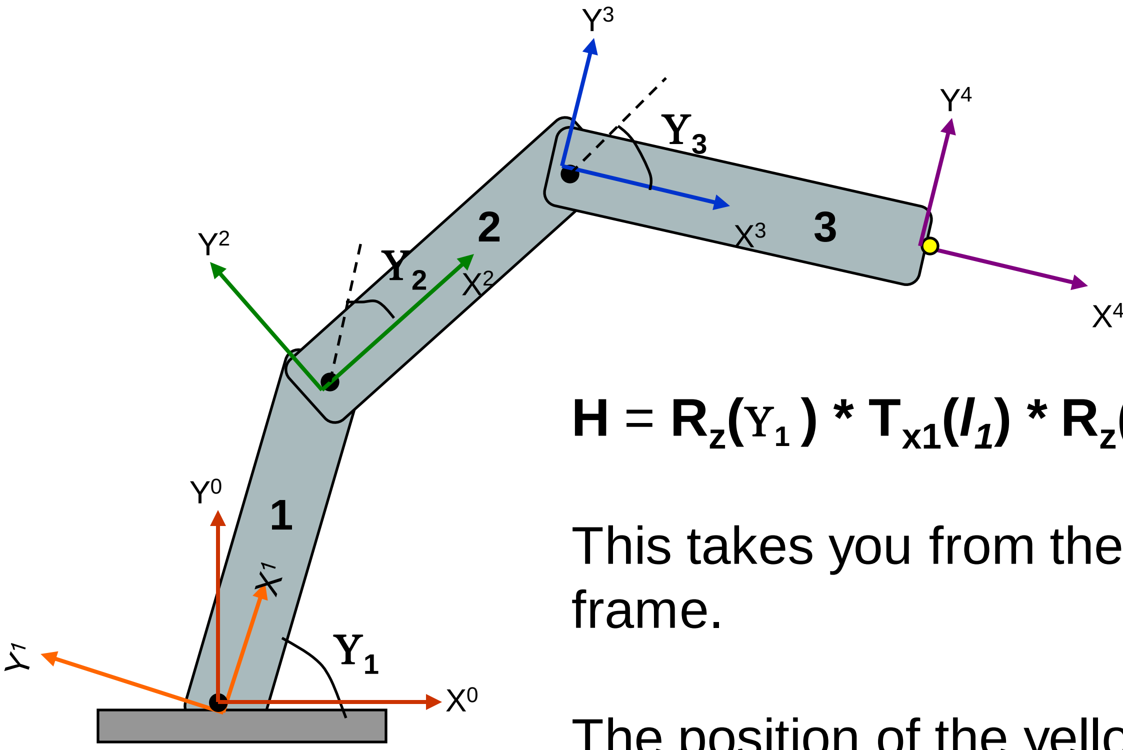
$$H = \mathbf{R_z}(\gamma_1) * \mathbf{T_{x1}}(\mathbf{l_1}) * \mathbf{R_z}(\gamma_2) * \mathbf{T_{x2}}(\mathbf{l_2}) * \mathbf{R_z}(\gamma_3)$$

i.e. Rotating by γ_1 will put you in the X^1Y^1 frame.
Translate in the along the X^1 axis by l_1.
Rotating by γ_2 will put you in the X^2Y^2 frame.
and so on until you are in the X^3Y^3 frame.

The position of the yellow dot relative to the X^3Y^3 frame is (l_1, 0). Multiplying H by that position vector will give you th coordinates of the yellow point relative the the X^0Y^0 frame

Slight variation on the last solution:

Make the yellow dot the origin of a new coordinate X^4Y^4 frame



$$H = R_z(Y_1) * T_{x1}(l_1) * R_z(Y_2) * T_{x2}(l_2) * R_z(Y_3) * T_{x3}(l_3)$$

This takes you from the X^0Y^0 frame to the X^4Y^4 frame.

The position of the yellow dot relative to the X^4Y^4 frame is (0,0).

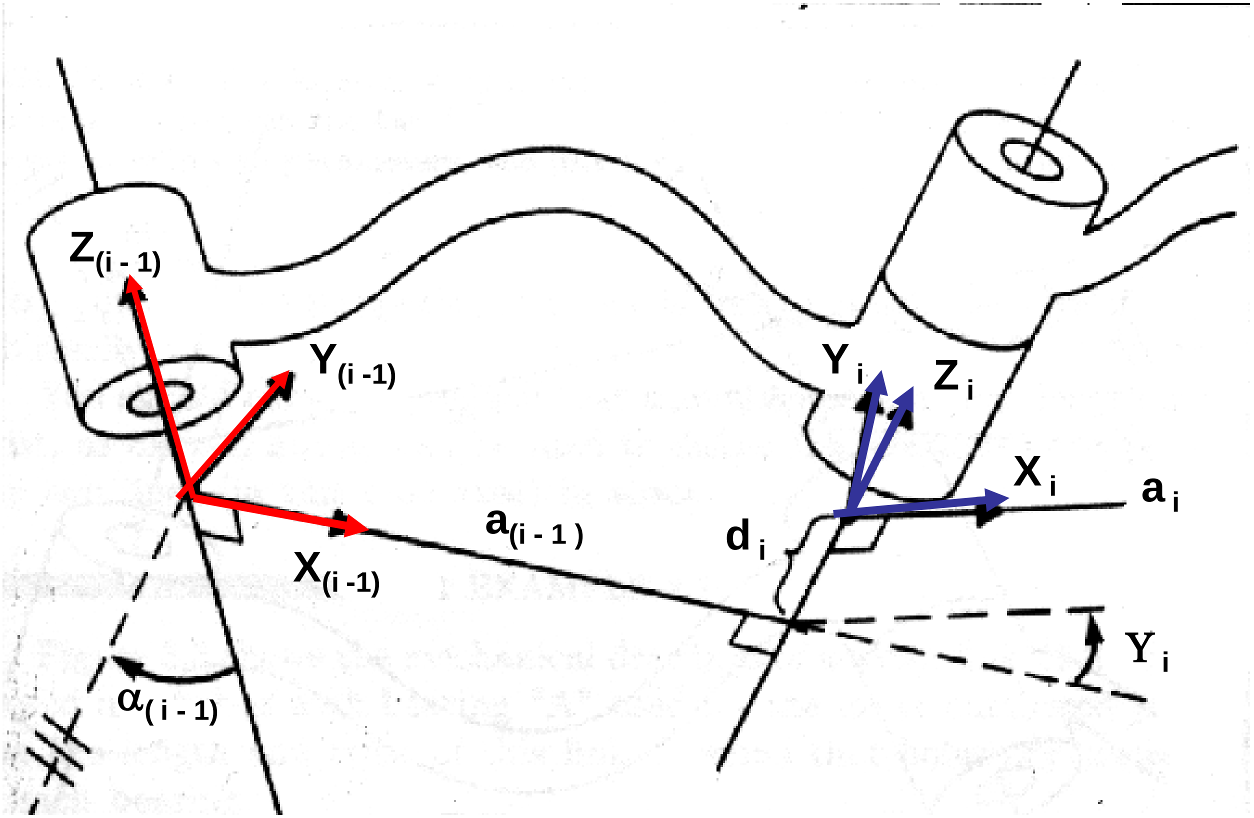
$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = H \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Notice that multiplying by the (0,0,0,1) vector will equal the last column of the H matrix.

More on Forward Kinematics…

Denavit - Hartenberg Parameters

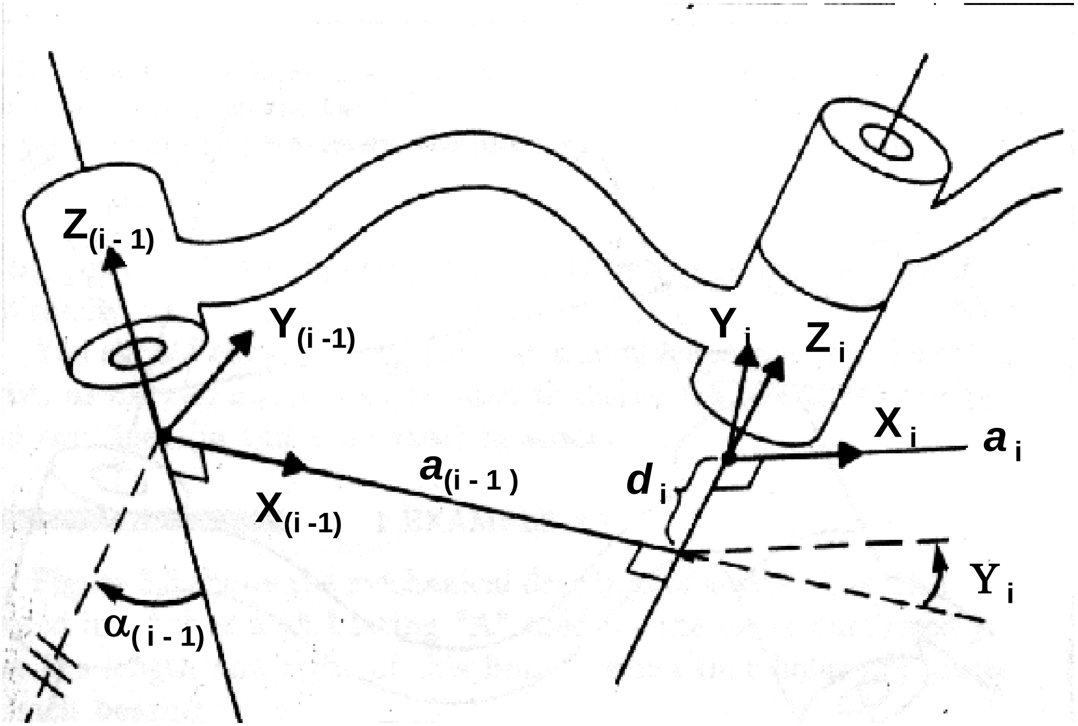
Denavit-Hartenberg Notation



IDEA: Each joint is assigned a coordinate frame. Using the Denavit-Hartenberg notation, you need 4 parameters to describe how a frame (*i*) relates to a previous frame (*i -1*).

THE PARAMETERS/VARIABLES: α, a , d, Y

The Parameters



You can align the two axis just using the 4 parameter s

1) $a_{(i-1)}$

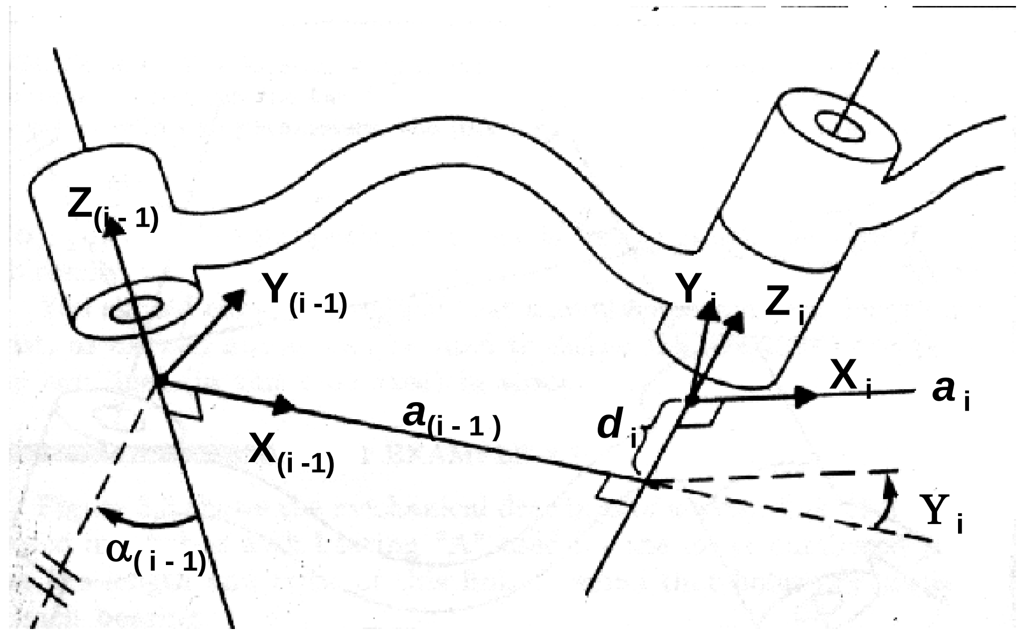
<u>Technical Definition:</u> $a_{(i-1)}$ is the length of the perpendicular between the joint axes. The joint axes is the axes around which revolution takes place which are the $Z_{(i-1)}$ and $Z_{(i)}$ axes. These two axes can be viewed as lines in space. The common perpendicular is the shortest line between the two axis-lines and is perpendicular to both axis-lines.

$a_{(i-1)}$ *cont...*

Visual Approach - "A way to visualize the link parameter $a_{(i-1)}$ is to imagine an expanding cylinder whose axis is the $Z_{(i-1)}$ axis - when the cylinder just touches the joint axis *i* the radius of the cylinder is equal to $a_{(i-1)}$." (Manipulator Kinematics)

It's Usually on the Diagram Approach - If the diagram already specifies the various coordinate frames, then the common perpendicular is usually the $X_{(i-1)}$ axis. So $a_{(i-1)}$ is just the displacement along the $X_{(i-1)}$ to move from the *(i-1)* frame to the *i* frame.

If the link is prismatic, then $a_{(i-1)}$ is a variable, not a parameter.



2) $\alpha_{(i-1)}$

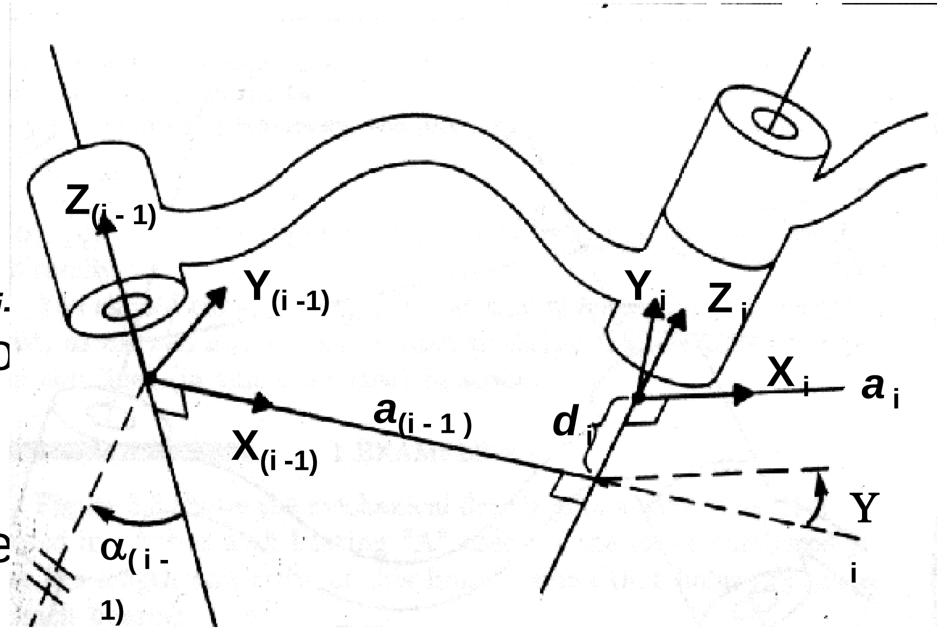
Technical Definition: Amount of rotation around the common perpendicular so that the joint axes are parallel.

i.e. How much you have to rotate around the $X_{(i-1)}$ axis so that the $Z_{(i-1)}$ is pointing in the same direction as the Z_i axis. Positive rotation follows the right hand rule.

3) $d_{(i-1)}$

Technical Definition: The displacement along the Z_i axis needed to align the $a_{(i-1)}$ common perpendicular to the a_i common perpendicular.

In other words, displacement along the Z_i to align the $X_{(i-1)}$ and X_i axes.



4) Υ_i

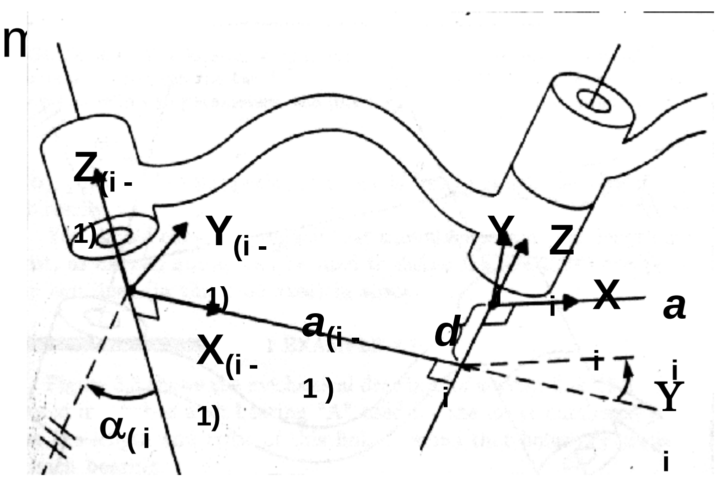
Amount of rotation around the Z_i axis needed to align the $X_{(i-1)}$ axis with the X_i axis.

The Denavit-Hartenberg Matrix

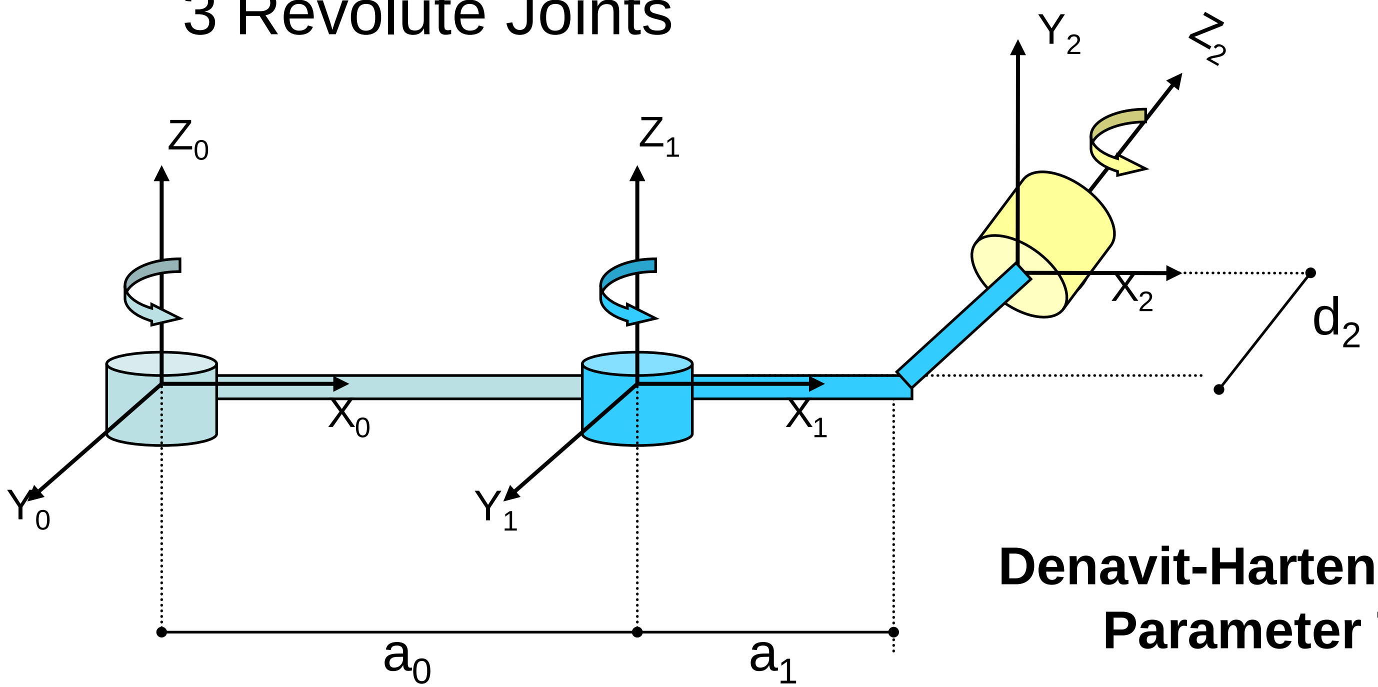
$$\begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{(i-1)} \\ \sin\theta_i \cos\alpha_{(i-1)} & \cos\theta_i \cos\alpha_{(i-1)} & -\sin\alpha_{(i-1)} & -\sin\alpha_{(i-1)} d_i \\ \sin\theta_i \sin\alpha_{(i-1)} & \cos\theta_i \sin\alpha_{(i-1)} & \cos\alpha_{(i-1)} & \cos\alpha_{(i-1)} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Just like the Homogeneous Matrix, the Denavit-Hartenberg Matrix is a transformation matrix from one coordinate frame to the next. Using a series of D-H Matrix multiplications and the D-H Parameter table, the final result is a transformation matrix from frame.

Put the transformation here



3 Revolute Joints



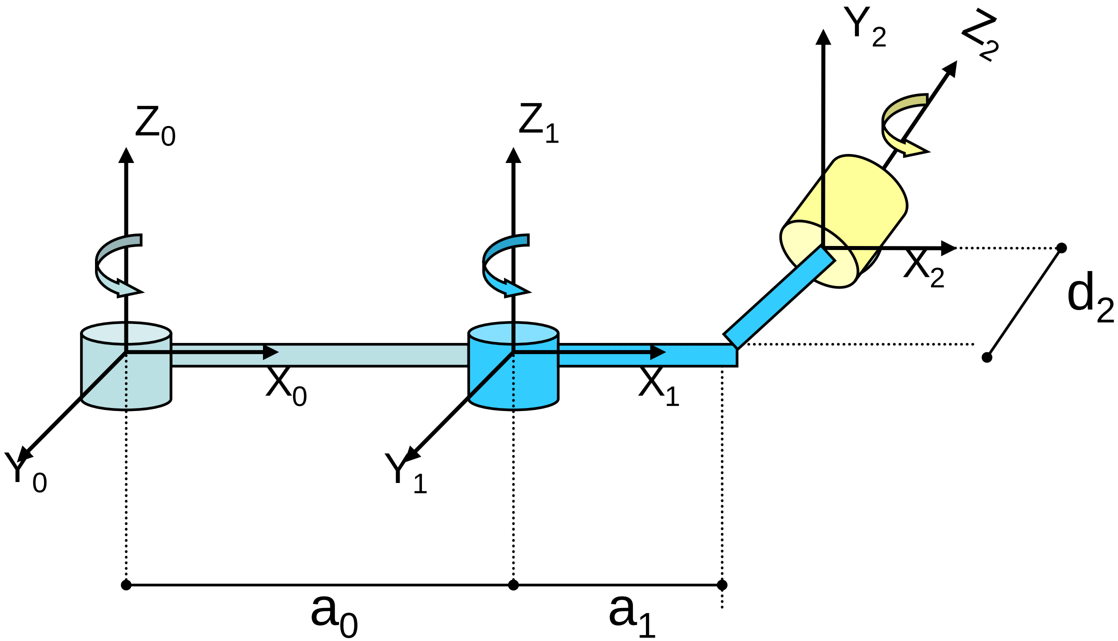
Denavit-Hartenberg Link Parameter Table

Notice that the table has two uses:

1) To describe the robot with its variables and parameters.

2) To describe some state of the robot by having a numerical values for the variables.

i	$\alpha_{(i-1)}$	$a_{(i-1)}$	d_i	θ_i
0	0	0	0	θ_0
1	0	a_0	0	θ_1
2	-90	a_1	d_2	θ_2



i	$\alpha_{(i-1)}$	$a_{(i-1)}$	d_i	θ_i
0	0	0	0	θ_0
1	0	a_0	0	θ_1
2	-90	a_1	d_2	θ_2

$$\mathbf{V}^{X_0 Y_0 Z_0} = \mathbf{T}\begin{bmatrix} \mathbf{V}^{X_2} \\ \mathbf{V}^{Y_2} \\ \mathbf{V}^{Z_2} \\ \mathbf{1} \end{bmatrix}$$

$$\mathbf{T} = ({}_{0}\mathbf{T})({}^{0}_{1}\mathbf{T})({}^{1}_{2}\mathbf{T})$$

Note: T is the D-H matrix with *(i-1)* = 0 and *i* = 1.

i	$\alpha_{(i-1)}$	$a_{(i-1)}$	d_i	θ_i
0	0	0	0	θ_0
1	0	a_0	0	θ_1
2	-90	a_1	d_2	θ_2

$$
{}_0T = \begin{bmatrix} \cos\theta_0 & -\sin\theta_0 & 0 & 0 \\ \sin\theta_0 & \cos\theta_0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

This is just a rotation around the Z_0 axis

$$
{}^0_1T = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & a_0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

This is a translation by a_0 followed by a rotation around the Z_1 axis

$$
{}^1_2T = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_1 \\ 0 & 0 & 1 & d_2 \\ -\sin\theta_2 & -\cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

This is a translation by a_1 and then d_2 followed by a rotation around the X_2 and Z_2 axis

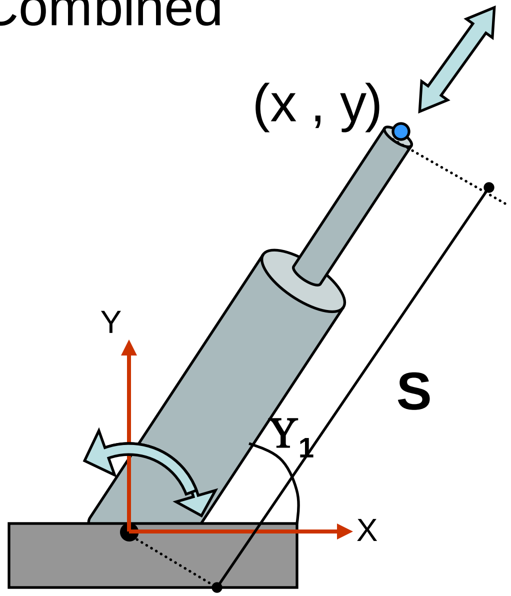
$$
T = ({}_0T)({}^0_1T)({}^1_2T)
$$

Inverse Kinematics

From Position to Angles

A Simple Example

Revolute and Prismatic Joints Combined



Finding γ:

$$\theta = \arctan\left(\frac{y}{x}\right)$$

More Specifically:

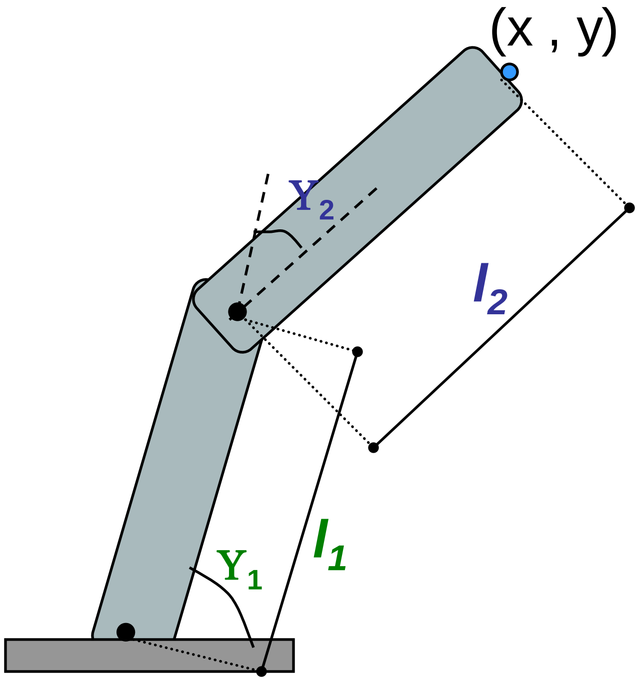
$$\theta = \arctan 2\left(\frac{y}{x}\right)$$

arctan2() specifies that it’s in the first quadrant

Finding **S**:

$$S = \sqrt{(x^2 + y^2)}$$

Inverse Kinematics of a Two Link Manipulator



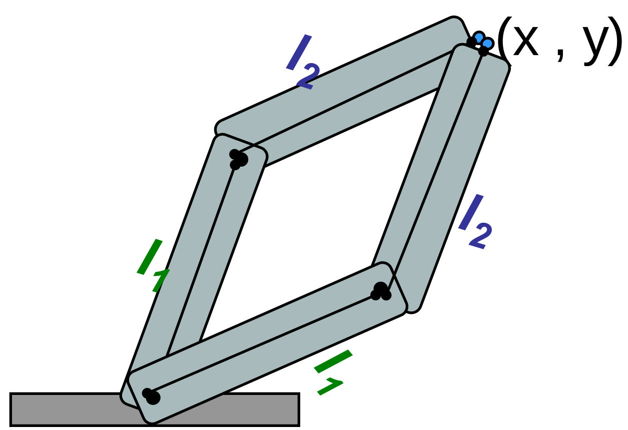
Given: l_1, l_2 , x , y

Find: Y_1, Y_2

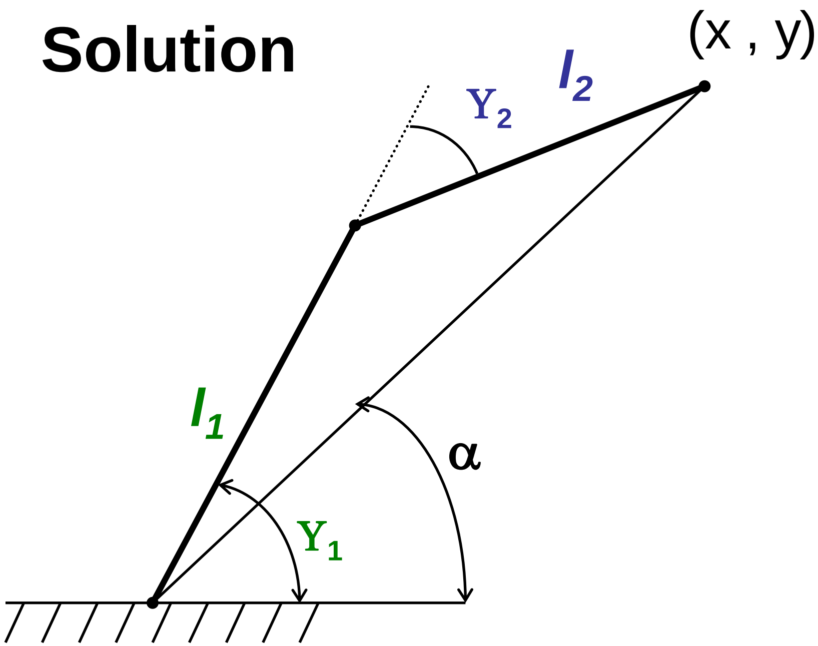
Redundancy:

A unique solution to this problem does not exist. Notice, that using the "givens" two solutions are possible.

Sometimes no solution is possible.



The Geometric Solution



Using the Law of Cosines:

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin \bar{\theta}_1}{l_2} = \frac{\sin(180 - \theta_2)}{\sqrt{x^2 + y^2}} = \frac{\sin(\theta_2)}{\sqrt{x^2 + y^2}}$$

$$\theta_1 = \bar{\theta}_1 + \alpha$$

$$\alpha = \arctan 2\left(\frac{y}{x}\right)$$

Using the Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$(x^2 + y^2) = l_1^2 + l_2^2 - 2l_1l_2\cos(180 - \theta_2)$$

$$\cos(180 - \theta_2) = -\cos(\theta_2)$$

$$\cos(\theta_2) = \frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1l_2}$$

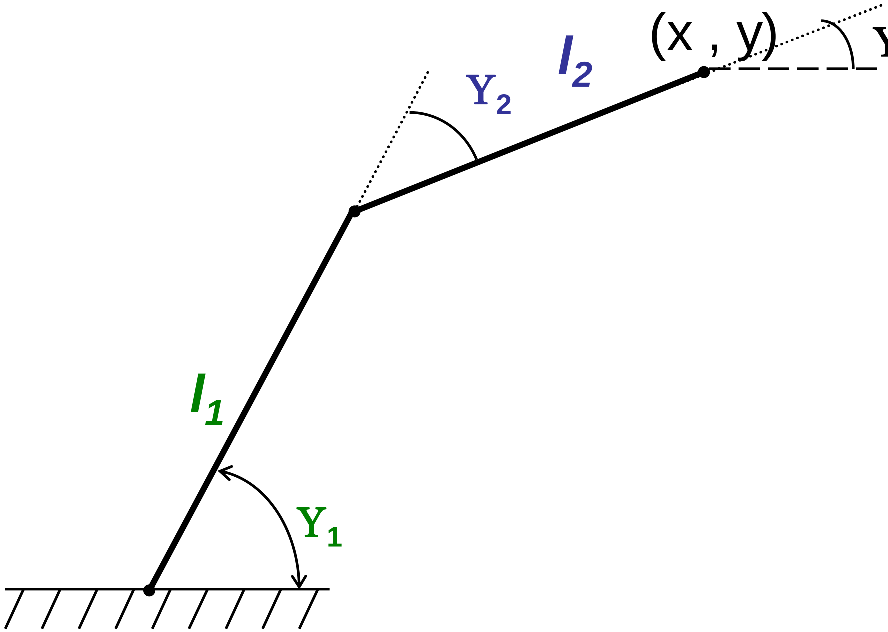
$$\theta_2 = \arccos\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1l_2}\right)$$

Redundant since θ_2 could be in the first or fourth quadrant.

Redundancy caused since θ_2 has two possible values

$$\theta_1 = \arcsin\left(\frac{l_2\sin(\theta_2)}{\sqrt{x^2 + y^2}}\right) + \arctan 2\left(\frac{y}{x}\right)$$

The Algebraic Solution



$$c_1 = \cos\theta_1$$

$$c_{1+2} = \cos(\theta_2 + \theta_1)$$

$$(1)\quad x = l_1 c_1 + l_2 c_{1+2}$$

$$(2)\quad y = l_1 s_1 + l_2 \sin_{1+2}$$

$$(3)\quad \theta = \theta_1 + \theta_2$$

$$(1)^2 + (2)^2 = x^2 + y^2 =$$

$$= \left(l_1^2 c_1^2 + l_2^2 (c_{1+2})^2 + 2 l_1 l_2 c_1 (c_{1+2})\right) + \left(l_1^2 s_1^2 + l_2^2 (\sin_{1+2})^2 + 2 l_1 l_2 s_1 (\sin_{1+2})\right)$$

$$= l_1^2 + l_2^2 + 2 l_1 l_2 \left(c_1 (c_{1+2}) + s_1 (\sin_{1+2})\right)$$

$$= l_1^2 + l_2^2 + 2 l_1 l_2 c_2$$

← Only Unknown

$$\therefore\ \theta_2 = \arccos\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

Note :

$$\cos(a \pm b) = (\cos a)(\cos b) \mp (\sin a)(\sin b)$$

$$\sin(a \pm b) = (\cos a)(\sin b) \pm (\cos b)(\sin a)$$

$$x = l_1 c_1 + l_2 c_{1+2}$$
$$= l_1 c_1 + l_2 c_1 c_2 - l_2 s_1 s_2$$
$$= c_1 (l_1 + l_2 c_2) - s_1 (l_2 s_2)$$

Note :

$$\cos(a \pm b) = (\cos a)(\cos b) \mp (\sin a)(\sin b)$$
$$\sin(a \pm b) = (\cos a)(\sin b) \pm (\cos b)(\sin a)$$

$$y = l_1 s_1 + l_2 \sin_{1+2}$$
$$= l_1 s_1 + l_2 s_1 c_2 + l_2 s_2 c_1$$
$$= c_1 (l_2 s_2) + s_1 (l_1 + l_2 c_2)$$

We know what θ_2 is from the previous slide. We need to solve for θ_1 . Now we have two equations and two unknowns (sin θ_1 and cos θ_1)

$$c_1 = \frac{x + s_1 (l_2 s_2)}{(l_1 + l_2 c_2)}$$

Substituting for c_1 and simplifying many times

$$y = \frac{x + s_1 (l_2 s_2)}{(l_1 + l_2 c_2)} (l_2 s_2) + s_1 (l_1 + l_2 c_2)$$
$$= \frac{1}{(l_1 + l_2 c_2)} \left(x\, l_2 s_2 + s_1 (l_1^2 + l_2^2 + 2 l_1 l_2 c_2) \right)$$

Notice this is the law of cosines and can be replaced by $x^2 + y^2$

$$s_1 = \frac{y (l_1 + l_2 c_2) - x\, l_2 s_2}{x^2 + y^2}$$

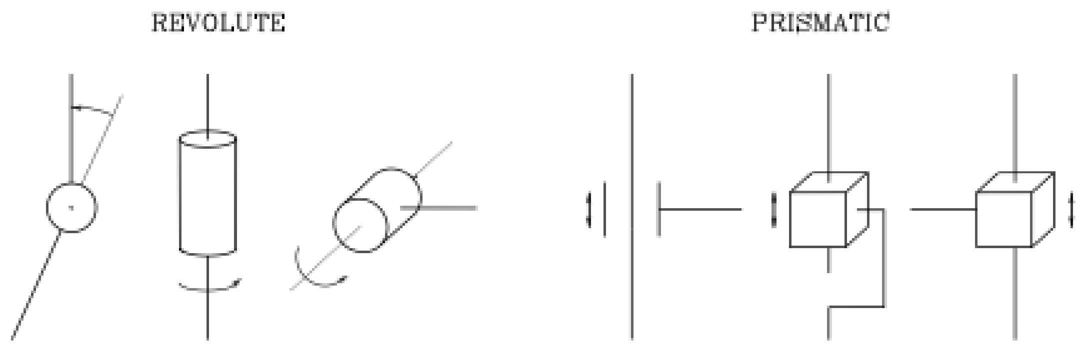
$$\theta_1 = \arcsin \left(\frac{y (l_1 + l_2 c_2) - x\, l_2 s_2}{x^2 + y^2} \right)$$

UNIT-III

DIRECT KINEMATICS

• Manipulator

series of links connected by means of joints

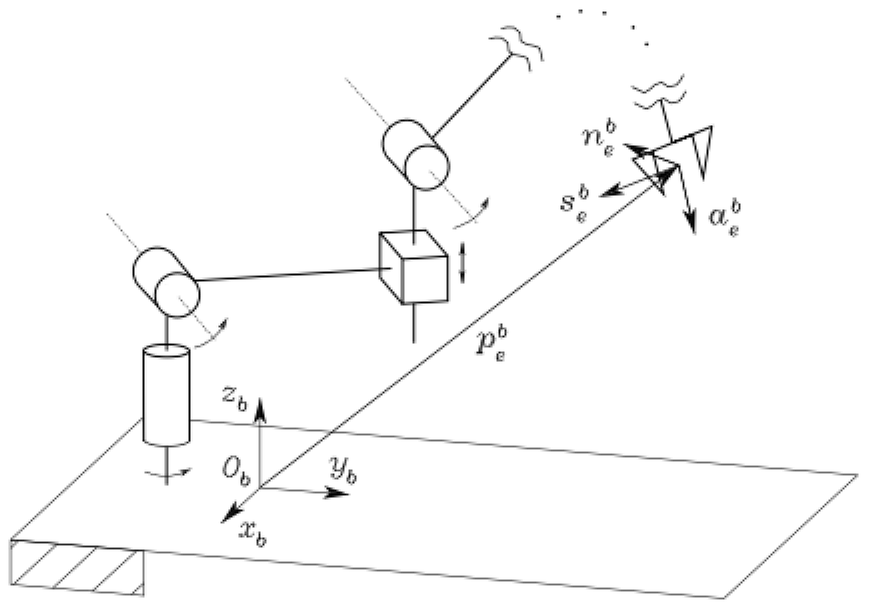


Kinematic chain (from base to end-effector)
open (only one sequence)
closed (loop)

Degree of freedom

associated with a joint articulation = joint variable

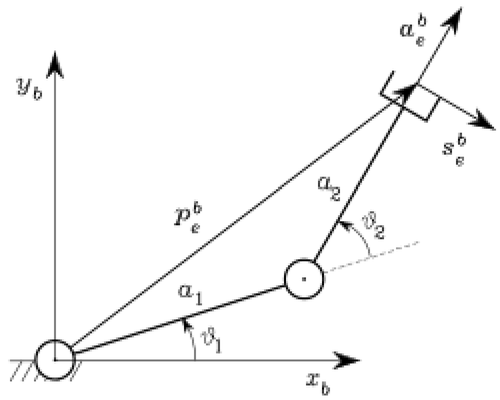
Base frame and end-effector frame



Direct kinematics equation

$$T_e^b(q) = \begin{bmatrix} n_e^b(q) & s_e^b(q) & a_e^b(q) & p_e^b(q) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

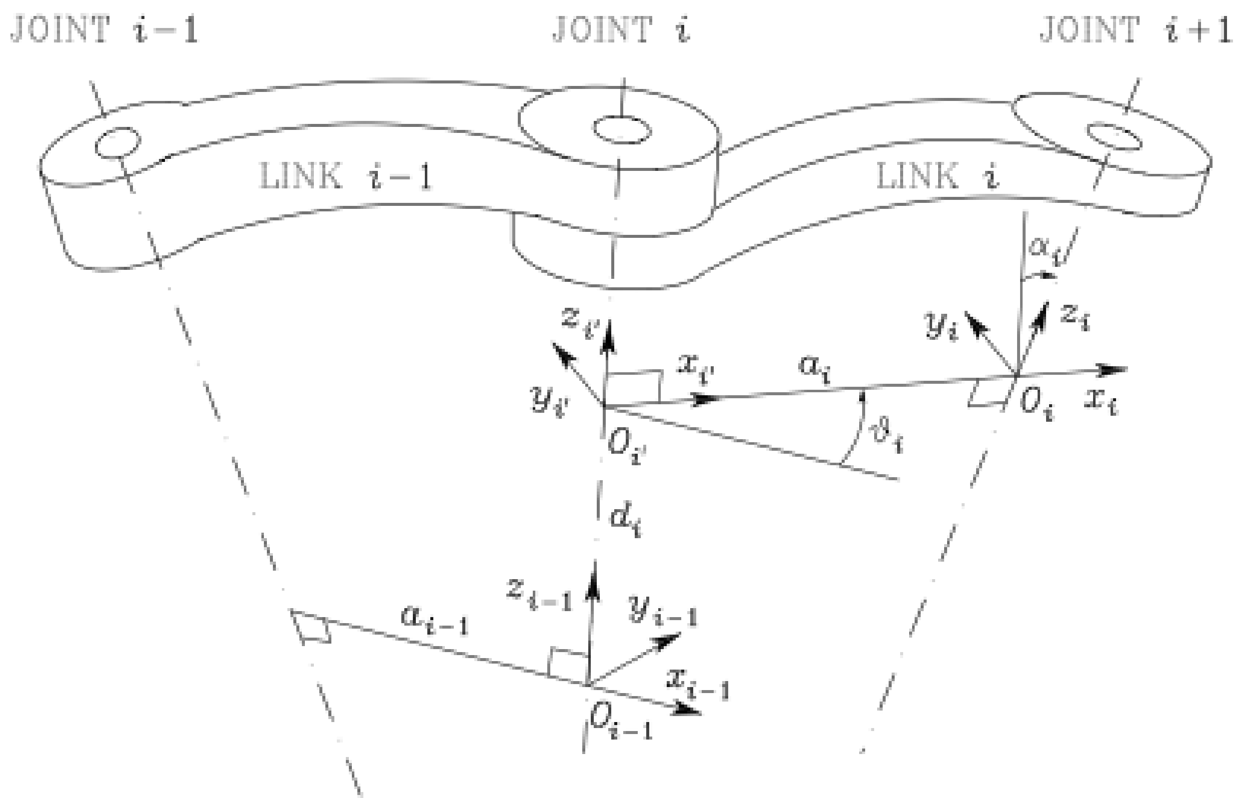
Two-link planar arm



$$
\boldsymbol{T}_e^b(\boldsymbol{q}) = \begin{bmatrix} \boldsymbol{n}_e^b & \boldsymbol{s}_e^b & \boldsymbol{a}_e^b & \boldsymbol{p}_e^b \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} 0 & s_{12} & c_{12} & a_1 c_1 + a_2 c_{12} \\ 0 & -c_{12} & s_{12} & a_1 s_1 + a_2 s_{12} \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Denavit–Hartenberg convention



choose axis z_i along axis of Joint i + 1

• locate O_i at the intersection of axis z_i with the common normal to axes z_{i-1} and z_i, and O'_i at intersection of common normal with axis z_{i-1}

• choose axis x_i along common the normal to axes z_{i-1} and z_i with positive direction from Joint i to Joint i + 1

• choose axis y_i so as to complete right-handed frame

• Nonunique definition of link frame:

For Frame 0, only the direction of axis z_0 is specified: then O_0 and and X_0 can be chosen arbitrarily.

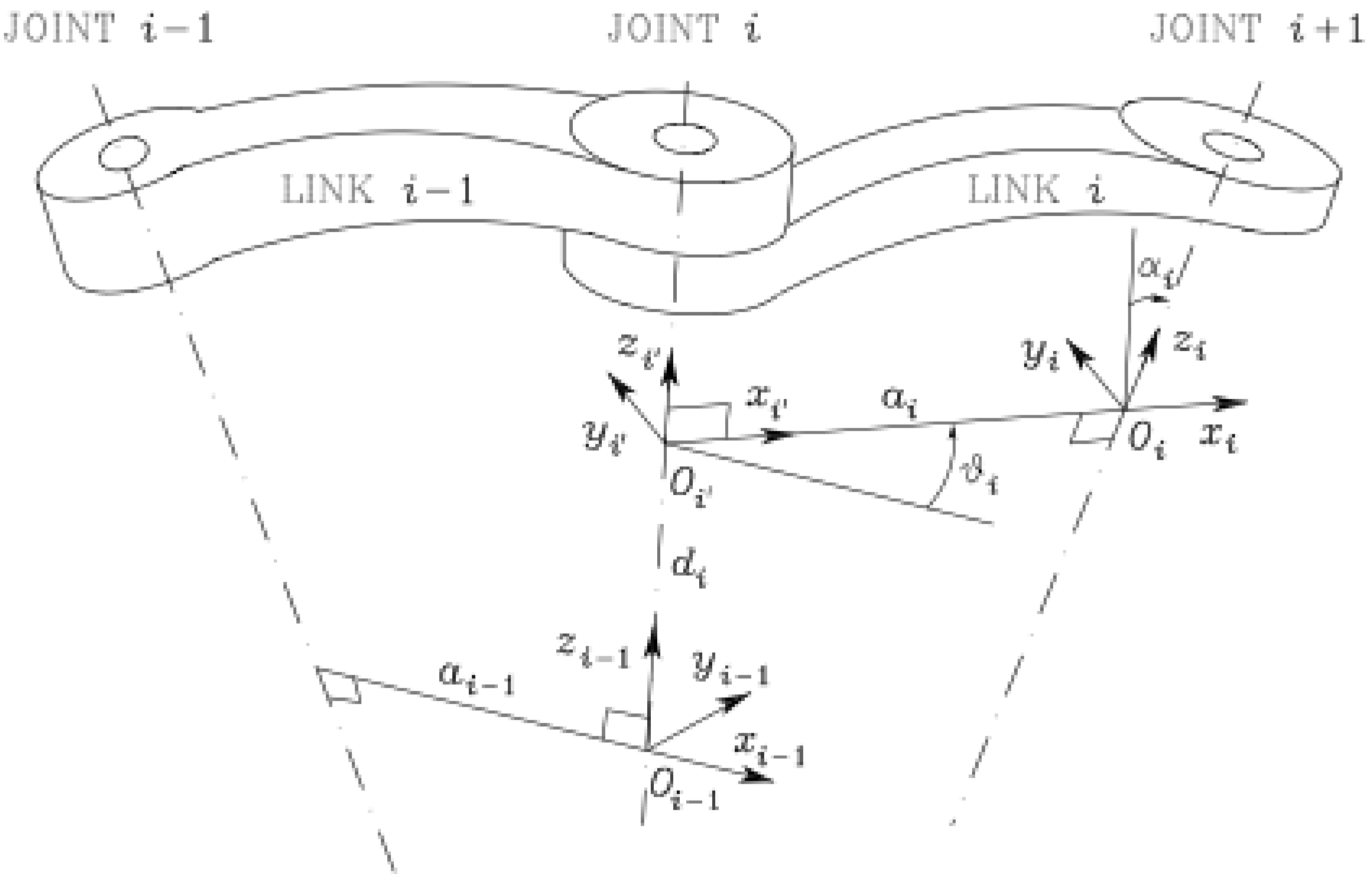
For Frame n, since there is no Joint n + 1, z_n is not uniquely defined while x_n has to be normal to axis z_{n-1}; typically Joint n is revolute and thus z_n can be aligned with z_{n-1} .

when two consecutive axes are parallel, the common normal between them is not uniquely defined.

when two consecutive axes intersect, the positive direction of x_i is arbitrary.

When Joint i is prismatic, only the direction of z_{i-1} is specified.

Denavit–Hartenberg parameters



a_i distance between O_i and O_i';

d_i coordinate of O_i' and $z_{i-1;}$

α_i angle between axes z_{i-1} and z_i about axis x_i to be taken positive when rotation is made counter-clockwise

υ_i angle between axes x_{i-1} and x_i about axis z_{i-1} to be taken positive when rotation is made counter-clockwise

a_i and α_i are always constant

if Joint i is revolute the variable is υ_i

if Joint i is prismatic the variable is di

- Coordinate transformation

$$\boldsymbol{A}_{i'}^{i-1} = \begin{bmatrix} c_{\vartheta_i} & -s_{\vartheta_i} & 0 & 0 \\ s_{\vartheta_i} & c_{\vartheta_i} & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{A}_{i}^{i'} = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \end{bmatrix}$$

$$\boldsymbol{A}_{i}^{i-1}(q_i) = \boldsymbol{A}_{i'}^{i-1}\boldsymbol{A}_{i}^{i'} = \begin{bmatrix} c_{\vartheta_i} & -s_{\vartheta_i}c_{\alpha_i} & s_{\vartheta_i}s_{\alpha_i} & a_i c_{\vartheta_i} \\ s_{\vartheta_i} & c_{\vartheta_i}c_{\alpha_i} & -c_{\vartheta_i}s_{\alpha_i} & a_i s_{\vartheta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Procedure

Find and number consecutively the joint axes; set the directions of axes $z_{0,}$….., $z_{n-1.}$

Choose Frame 0 by locating the origin on axis $z_{0;}$ axes x_0 and y_0 are chosen so as to obtain a righthanded frame. If feasible, it is worth choosing Frame 0 to coincide with the base frame.

Execute steps from 3 to 5 for i = 1, . . . , n − 1:Find and number consecutively the joint axes; set the directions of axes $z_{0,}$….., $z_{n-1.}$
Choose Frame 0 by locating the origin on axis $z_{0;}$ axes x_0 and y_0 are chosen so as to obtain a righthanded frame. If feasible, it is worth choosing Frame 0 to coincide with the base frame.
Execute steps from 3 to 5 for i = 1, . . . , n − 1:
Locate the origin O_i at the intersection of z_i with the common normal to axes z_{i-1} and $z_{i\,.}$ If axes $z_{\,i-1}$ and z_i are parallel and Joint i is revolute, then locate O_i so that d_i=0; if

Joint i is prismatic, locate O_i at a reference position for the joint range, e.g., a mechanical limit.

Choose axis x_i along the common normal to axes z_{i-1} and z_i with direction from Joint i to Joint i + 1 .

Choose axis y_i so as to obtain a right-handed frame to complete.

Choose Frame n; if Joint n is revolute, then align z_n with $z_{n-1,}$ otherwise, if Joint n is prismatic, then choose z_n arbitrarily. Axis x_n is set according to step 4.

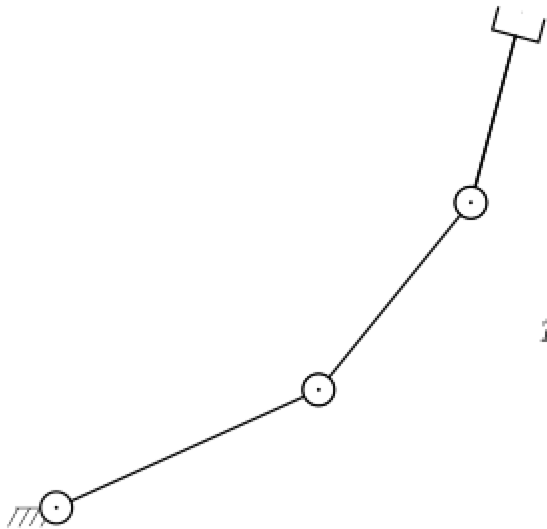
For i = 1, . . . , n, form the table of parameters $a_{i,}$ $d_{i,}$ $\alpha_{i,}$ $\upsilon_{i.}$

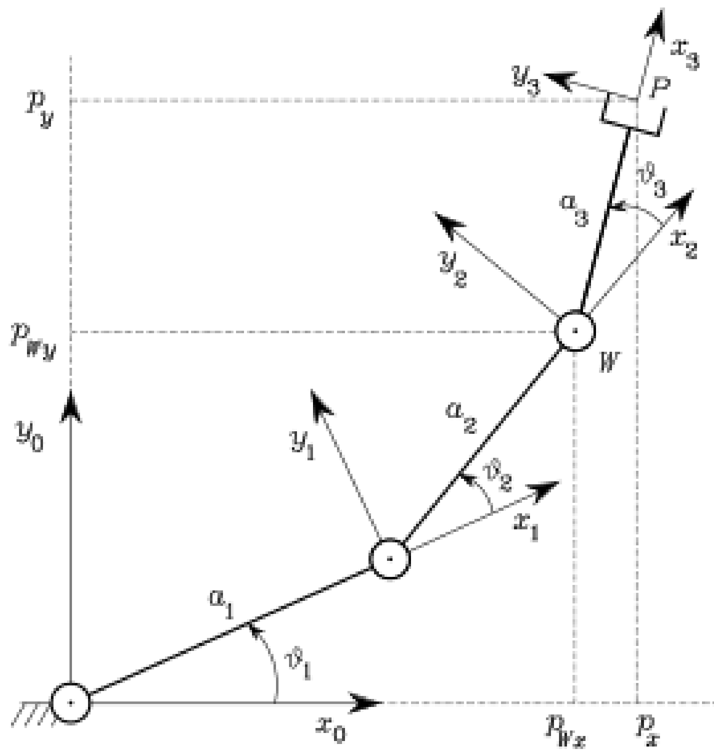
On the basis of the parameters in 7, compute the homogeneous transformation matrices $A_i^{i-1}(q_i)$ for $i_=1$, . . . , n.

Compute the homogeneous transformation $T_n^0(q)=A_1^0 \ldots A_n^{n-1}$ they yields the position and orientation of Frame n with respect to Frame 0.

Given T_0^b and T_e^n , compute the direct kinematics function as $T_e^b(q)= T_0^b T_n^0 T_e^n$ that yields the position and orientation of the end-effector frame with respect to the base frame.

Three–link planar arm





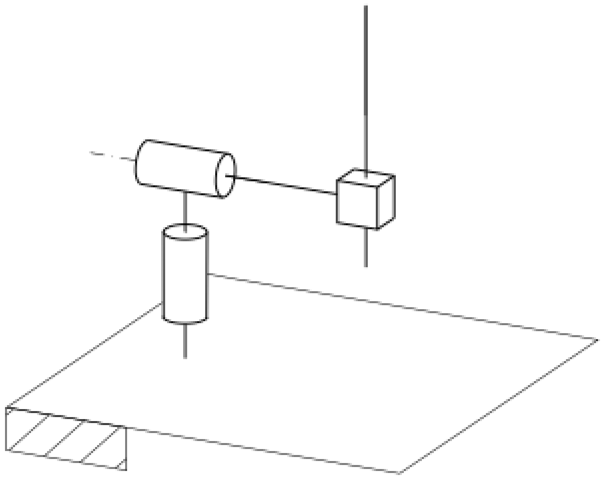
Link	a_i	α_i	d_i	ϑ_i
1	a_1	0	0	ϑ_1
2	a_2	0	0	ϑ_2
3	a_3	0	0	ϑ_3

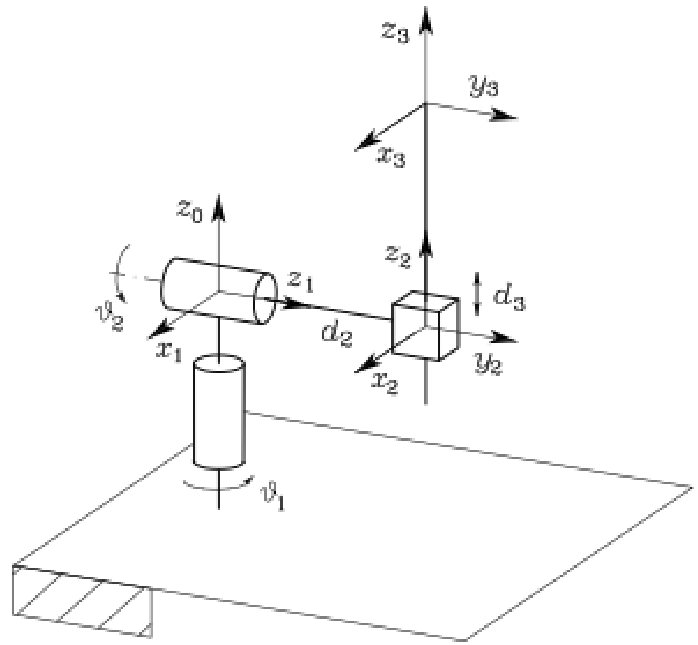
$$\boldsymbol{A}_i^{i-1} = \begin{bmatrix} c_i & -s_i & 0 & a_i c_i \\ s_i & c_i & 0 & a_i s_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad i = 1, 2, 3$$

$$\boldsymbol{T}_3^0 = \boldsymbol{A}_1^0 \boldsymbol{A}_2^1 \boldsymbol{A}_3^2$$

$$= \begin{bmatrix} c_{123} & -s_{123} & 0 & a_1 c_1 + a_2 c_{12} + a_3 c_{123} \\ s_{123} & c_{123} & 0 & a_1 s_1 + a_2 s_{12} + a_3 s_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Spherical arm





Link	a_i	α_i	d_i	ϑ_i
1	0	$-\pi/2$	0	ϑ_1
2	0	$\pi/2$	d_2	ϑ_2
3	0	0	d_3	0

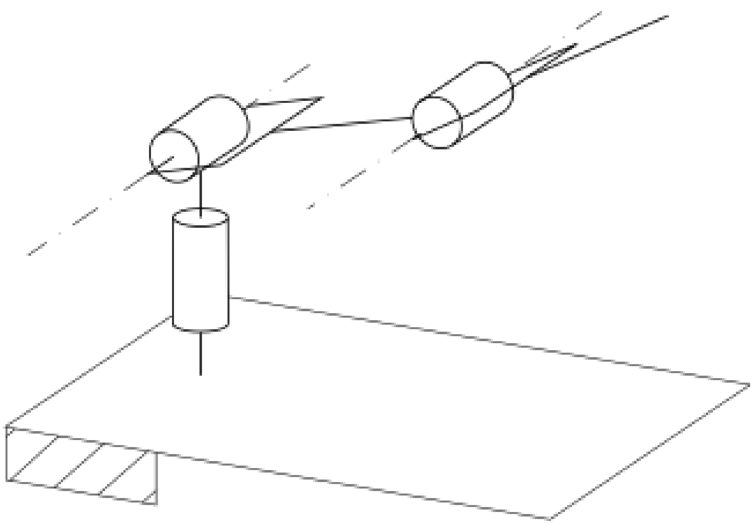
$$A_1^0 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2^1 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

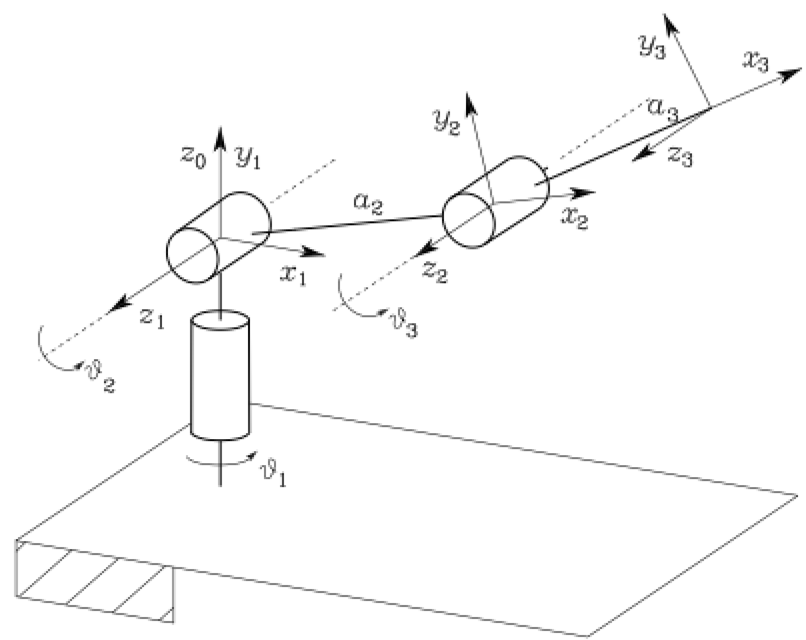
$$A_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1^0 A_2^1 A_3^2$$

$$= \begin{bmatrix} c_1 c_2 & -s_1 & c_1 s_2 & c_1 s_2 d_3 - s_1 d_2 \\ s_1 c_2 & c_1 & s_1 s_2 & s_1 s_2 d_3 + c_1 d_2 \\ -s_2 & 0 & c_2 & c_2 d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Anthropomorphic arm





Link	a_i	α_i	d_i	ϑ_i
1	0	$\pi/2$	0	ϑ_1
2	a_2	0	0	ϑ_2
3	a_3	0	0	ϑ_3

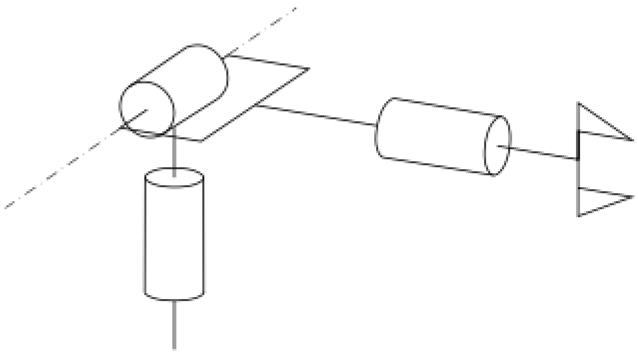
$$\boldsymbol{A}_1^0 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \boldsymbol{A}_i^{i-1} = \begin{bmatrix} c_i & -s_i & 0 & a_i c_i \\ s_i & c_i & 0 & a_i s_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

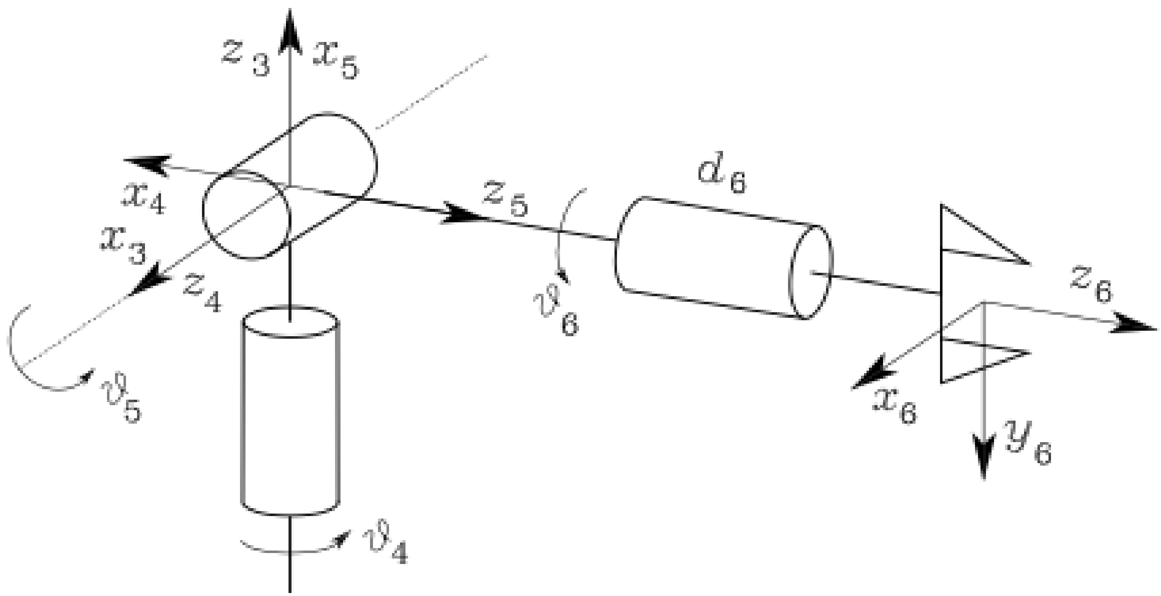
$$i = 2, 3$$

$$\boldsymbol{T}_3^0 = \boldsymbol{A}_1^0 \boldsymbol{A}_2^1 \boldsymbol{A}_3^2$$

$$= \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & s_1 & c_1(a_2 c_2 + a_3 c_{23}) \\ s_1 c_{23} & -s_1 s_{23} & -c_1 & s_1(a_2 c_2 + a_3 c_{23}) \\ s_{23} & c_{23} & 0 & a_2 s_2 + a_3 s_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Spherical wrist





Link	a_i	α_i	d_i	ϑ_i
4	0	$-\pi/2$	0	ϑ_4
5	0	$\pi/2$	0	ϑ_5
6	0	0	d_6	ϑ_6

$$\boldsymbol{A}_4^3 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \boldsymbol{A}_5^4 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{A}_6^5 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{T}_6^3 = \boldsymbol{A}_4^3 \boldsymbol{A}_5^4 \boldsymbol{A}_6^5$$

$$= \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

JOINT SPACE AND OPERATIONAL SPACE

Joint space

$$\boldsymbol{q} = \begin{bmatrix} q_1 \\ \vdots \\ q_n \end{bmatrix}$$

$q_i = \upsilon_i$ (revolute joint)
$q_i = d_i$ (prismatic joint)

Operational space

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{p} \\ \boldsymbol{\phi} \end{bmatrix}$$

P (position)
Φ (orientation)

Direct kinematics equation

x = k(q)

UNIT-IV

INTRODUCTION

- Path and trajectory planning means the way that a robot is moved from one location to another in a controlled manner.
- The sequence of movements for a controlled movement between motion segment, in straight-line motion or in sequential motions.
- It requires the use of both kinematics and dynamics of robots.

PATH VS. TRAJECTORY

- **Path:** A sequence of robot configurations in a particular order without regard to the timing of these configurations.
- **Trajectory:** It concerned about when each part of the path must be attained, thus specifying timing.

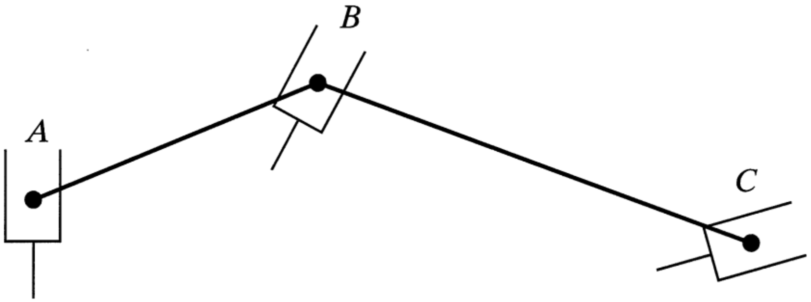


Fig. Sequential robot movements in a path.

JOINT-SPACE VS. CARTESIAN-SPACE DESCRIPTIONS

- Joint-space description:
  - The description of the motion to be made by the robot by its joint values.
  - The motion between the two points is unpredictable.
- Cartesian space description:
  - The motion between the two points is known at all times and controllable.
  - It is easy to visualize the trajectory, but is is difficult to ensure that singularity.

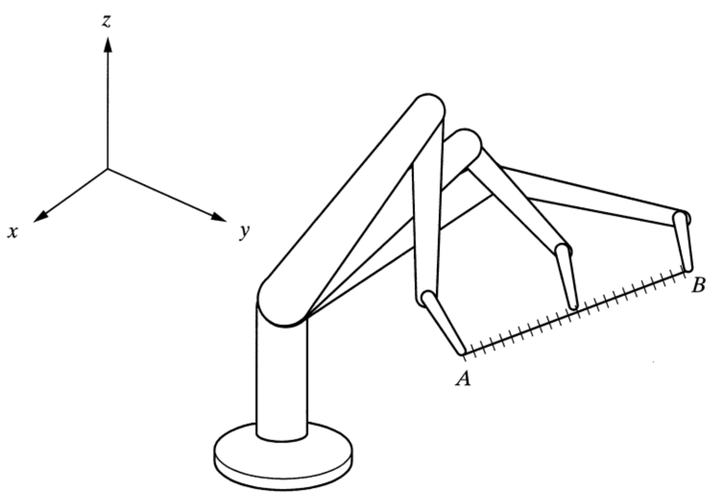


Fig. Sequential motions of a robot to follow a straight line.

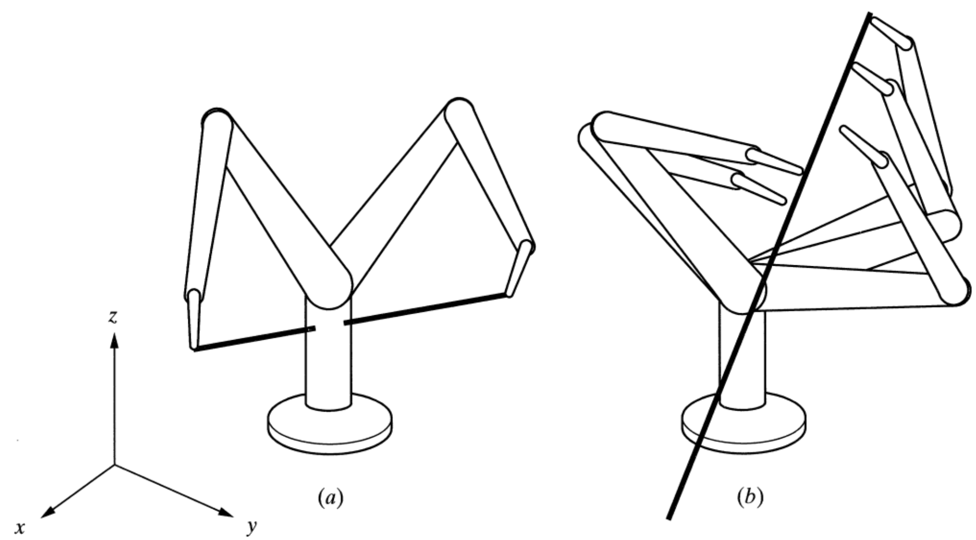
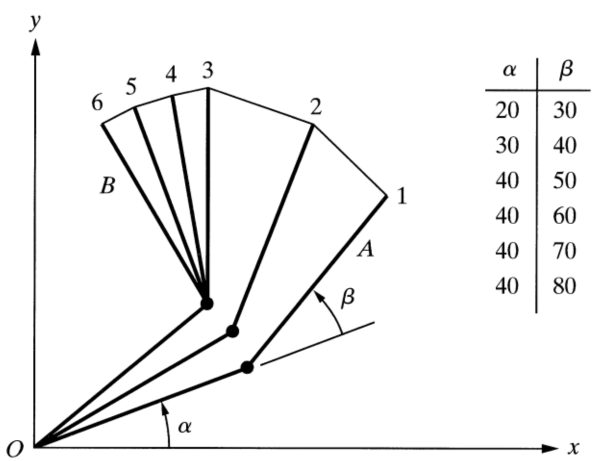


Fig. Cartesian-space trajectory (a) The trajectory specified in Cartesian coordinates may force the robot to run into itself, and (b) the trajectory may requires a sudden change in the joint angles.

BASICS OF TRAJECTORY PLANNING

- Let's consider a simple 2 degree of freedom robot.
- We desire to move the robot from Point A to Point B.
- Let's assume that both joints of the robot can move at the maximum rate of 10 degree/sec.
- Let's assume that both joints of the robot can move at the maximum rate of 10 degree/sec.



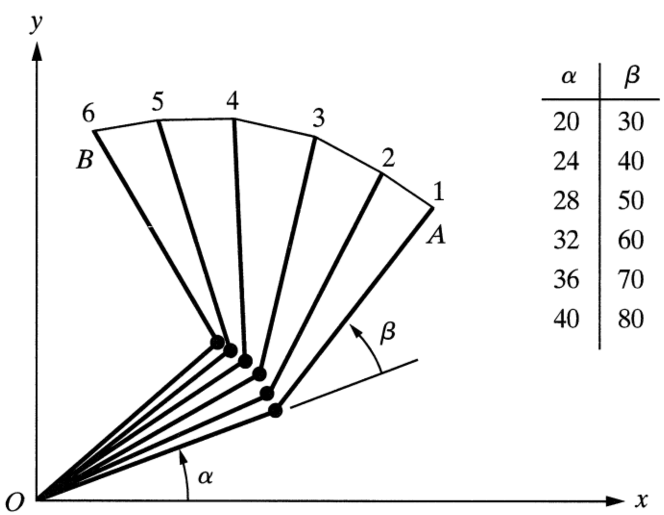
α	β
20	30
30	40
40	50
40	60
40	70
40	80

Fig. 5.4 Joint-space nonnormalized movements of a robot with two degrees of freedom.

- **Move the robot from A to B, to run both joints at their maximum angular velocities.**
- **After 2 [sec], the lower link will have finished its motion, while the upper link continues for another 3 [sec].**
- **The path is irregular and the distances traveled by the robot's end are not uniform.**

BASICS OF TRAJECTORY PLANNING

- Let's assume that the motions of both joints are normalized by a common factor such that the joint with smaller motion will move proportionally slower and the both joints will start and stop their motion simultaneously.



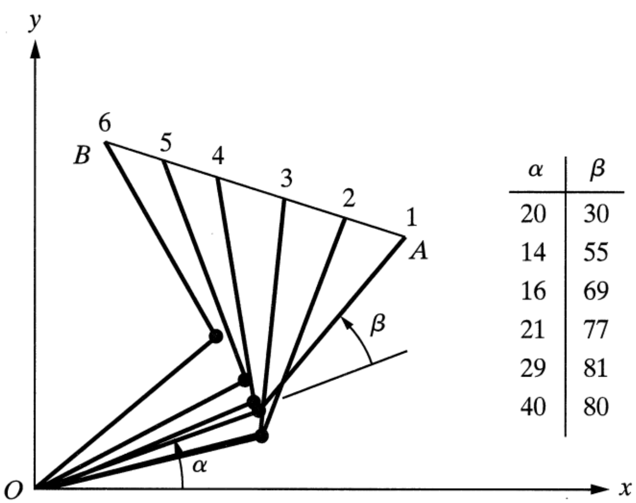
α	β
20	30
24	40
28	50
32	60
36	70
40	80

Fig. Joint-space, normalized movements of a robot with two degrees of freedom.

- **Both joints move at different speeds, but move continuously together.**
- **The resulting trajectory will be different.**

BASICS OF TRAJECTORY PLANNING

- Let's assume that the robot's hand follow a known path between point A to B with straight line.
- The simplest solution would be to draw a line between points A and B, so called interpolation.



α	β
20	30
14	55
16	69
21	77
29	81
40	80

Fig. Cartesian-space movements of a two-degree-of-freedom robot.

- **Divide the line into five segments and solve for necessary angles α and β at each point.**
- **The joint angles are not uniformly changing.**

BASICS OF TRAJECTORY PLANNING

- Let's assume that the robot's hand follow a known path between point A to B with straight line.
- The simplest solution would be to draw a line between points A and B, so called interpolation.

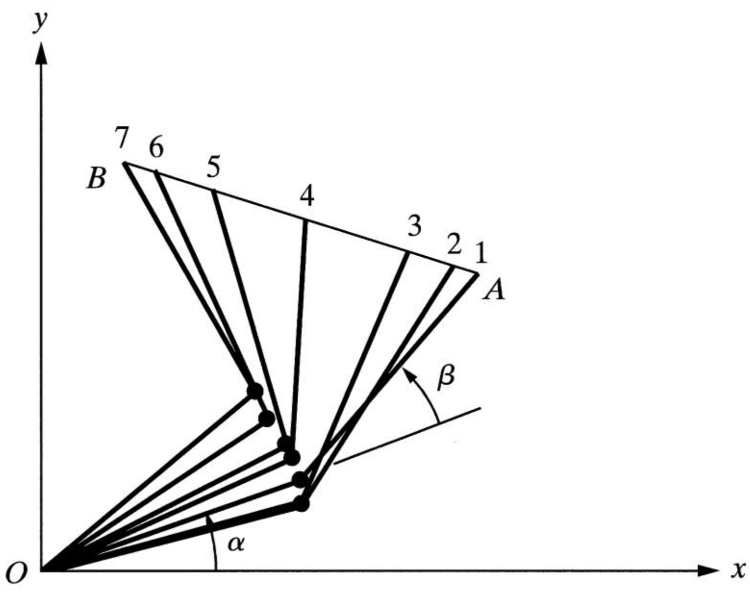


Fig. Trajectory planning with an acceleration-deceleration regiment.

- **It is assumed that the robot's actuators are strong enough to provide large forces necessary to accelerate and decelerate the joints as needed.**
- **Divide the segments differently.**
  - **The arm move at smaller segments as we speed up at the beginning.**
  - Go at a constant cruising rate.
  - Decelerate with smaller segments as approaching point B.

BASICS OF TRAJECTORY PLANNING

- Next level of trajectory planning is between multiple points for continuous movements.
- Stop-and-go motion create jerky motions with unnecessary stops.
- Blend the two portions of the motion at point B.
- Specify two via point D and E before and after point B

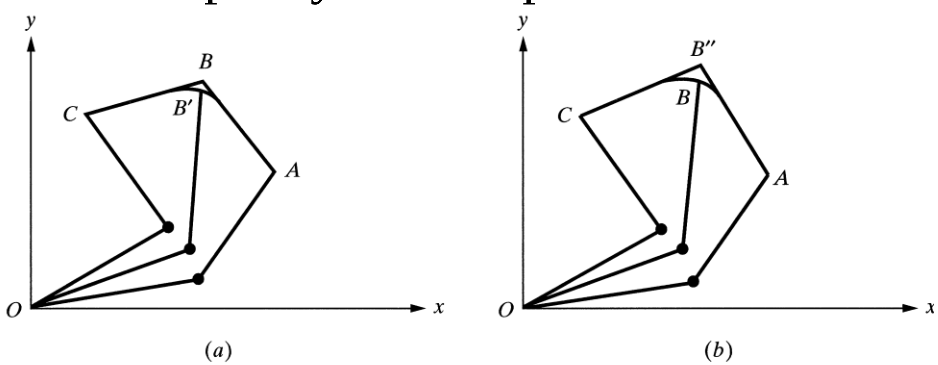


Fig. Blending of different motion segments in a path.

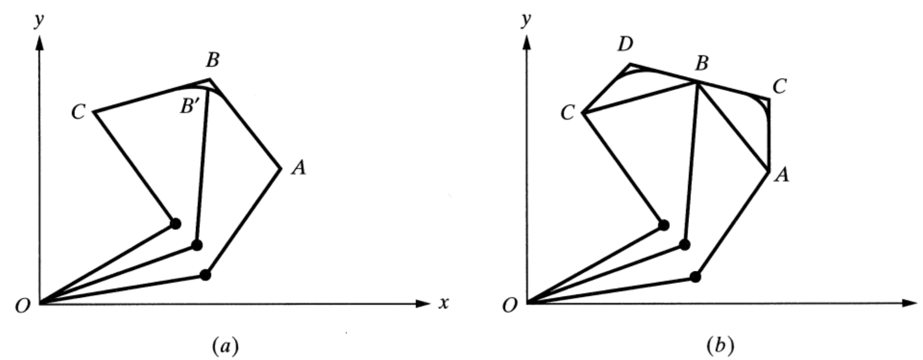


Fig. 5.9 An alternative scheme for ensuring that the robot will go through a specified point during blending of motion segments. Two via points D and E are picked such that point B will fall on the straight-line section of the segment ensuring that the robot will pass through point B.

JOINT-SPACE TRAJECTORY PLANNING

Third-Order Polynomial Trajectory Planning

- How the motions of a robot can be planned in joint-space with controlled characteristics.
- Polynomials of different orders
- Linear functions with parabolic blends

- **The initial location and orientation of the robot is known, and using the inverse kinematic equations, we find the final joint angles for the desired position and orientation.**

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$$

$$\theta(t_i) = \theta_i$$
$$\theta(t_f) = \theta_f$$
$$\dot{\theta}(t_i) = 0$$
$$\dot{\theta}(t_f) = 0$$

- **Initial Condition**

$$\dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2$$

- **First derivative of the polynomial of equation**

$$\theta(t_i) = c_0 = \theta_i$$
$$\theta(t_f) = c_0 + c_1 t_f + c_2 t_f^2 + c_3 t_f^3$$
$$\dot{\theta}(t_i) = c_1 = 0$$
$$\dot{\theta}(t_f) = c_1 + 2c_2 t_f + 3c_3 t_f^2 = 0$$

- **Substituting the initial and final conditions**

- It is desired to have the first joint of a six-axis robot go from initial angle of 30° to a final angle of 75° in 5 seconds. Using a third-order polynomial, calculate the joint angle at 1, 2 3, and 4 seconds.

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$$

$$\theta(0) = c_0 = 30$$

$$\dot{\theta}(0) = c_1 = 0$$

JOINT-SPACE TRAJECTORY PLANNING

Fifth-Order Polynomial Trajectory Planning

- Specify the initial and ending accelerations for a segment.
- To use a fifth-order polynomial for planning a trajectory, the total number of boundary conditions is 6.
- Calculation of the coefficients of a fifth-order polynomial with position, velocity and a acceleration boundary conditions can be possible with below equations.

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + c_5 t^5$$

$$\dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2$$

$$\ddot{\theta}(t) = 2c_2 + 6c_3 t + 12c_4 t^2 + 20c_5 t^3$$

JOINT-SPACE TRAJECTORY PLANNING

Linear Segments with Parabolic Blends

- Linear segment can be blended with parabolic sections at the beginning and the end of the motion segment, creating continuous position and velocity.
- Acceleration is constant for the parabolic sections, yielding a continuous velocity at the common points A and B.

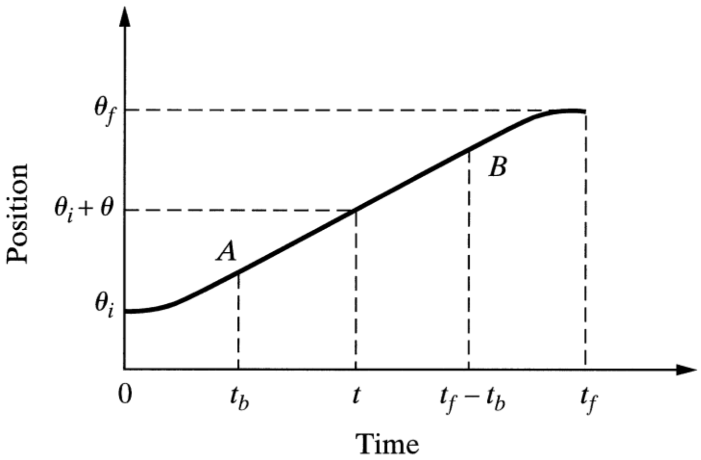


Fig. Scheme for linear segments with parabolic blends.

$$\theta(t) = c_0 + c_1 t + \frac{1}{2} c_2 t^2$$
$$\dot{\theta}(t) = c_1 + c_2 t$$
$$\ddot{\theta}(t) = c_2$$

$\Rightarrow$

$$\theta(t) = \theta_i + \frac{1}{2} c_2 t^2$$
$$\dot{\theta}(t) = c_2 t$$
$$\ddot{\theta}(t) = c_2$$

JOINT-SPACE TRAJECTORY PLANNING

Linear Segments with Parabolic Blends and Via Points

- The position of the robot at time t_0 is known and using the inverse kinematic equations of the robot, the joint angles at via points and at the end of the motion can be found.

- To blend the motion segments together, the boundary conditions of each point to calculate the coefficients of the parabolic segments is used.
- Maximum allowable accelerations should not be exceeded.

JOINT-SPACE TRAJECTORY PLANNING

Higher Order Trajectories

- Incorporating the initial and final boundary conditions together with this information enables us to use higher order polynomials in the below form, so that the trajectory will pass through all specified points.

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + \cdots \cdots + c_{n-1} t^{n-1} + c_n t^n$$

- It requires extensive calculation for each joint and higher order polynomials.
- Combinations of lower order polynomials for different segments of the trajectory and blending together to satisfy all required boundary conditions is required.

CARTESIAN-SPACE TRAJECTORIES

- Cartesian-space trajectories relate to the motions of a robot relative to the Cartesian reference frame.

- In Cartesian-space, the joint values must be repeatedly calculated through the inverse kinematic equations of the robot.

- Computer Loop Algorithm

(1) Calculate the position and orientation of the hand based on the selected function for the trajectory.

(2) Calculate the joint values for the position and orientation through the inverse kinematic equations of the robot.

(3) Send the joint information to the controller.

(4) Go to the beginning of the loop

UNIT-V

APPLICATION OF ROBOT'S

Robot applications can be studied under present and future applications.

Under present applications they can be classified into three major headings. They are

1.Material Transfer, Machine Loading and Unloading.

2.Processing operations.

3.Assembly and inspection.

In future applications category the list is exhaustive and ever increasing like

1.Medical
2. Military (Artillery, Loading, Surveillance)
3.Home applications.
4.Electronic industry.
5.Fully automated machine shop etc.,

MATERIAL HANDLING APPLICATIONS:

The material handling applications can be divided into two specific categories

1. Material transfer applications.
2. Machine loading/ unloading applications

GENERAL CONSIDERATIONS IN ROBOT MATERIAL HANDLING:

If a robot has to transfer parts or load a machine, then the following points are to be considered.

1. Part Positioning and Orientation
2. Gripper design
3. Minimum distances moved
4. Robot work volume
5. Robot weight capacity
6. Accuracy and repeatability
7. Robot configuration
8. Machine Utilization Problems

MATERIAL TRANSFER APPLICATIONS

1. Pick and place operations.
2. Palletizing and related operations.
3. Machine loading and unloading.

In these applications the robot is used to serve a production machine by transferring parts to and/or from the machine. This application can be dealt under the following three headings.

MACHINE LOADING:

The robot loads the raw material into the machine but the part/material is ejected by some other means.

MACHINE UNLOADING:

In this case the loading of raw material into the machine is done automatically but after completing the process the finished component is removed by robot.

Robots are being successfully used to in the loading and unloading function in the following production operations. They are

1. Die casting.
2. Plastic molding.
3. Forging and related operations.
4. Machining operations.
5. Stamping press operations.

PROCESSING OPERATIONS: The processing operations that are performed by a robot can be categorized into the following four types. They are

1. Spot welding.
2. Continuous arc welding.
3. Spray coating.
4. Other processing operations.