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Department of Civil Engineering

STRENGTH OF MATERIALS II

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COURSE OUTLINE

UNIT	TITLE	CONTENTS
I	**Torsion of Circular Shafts & Springs**	Theory of pure torsion , Derivation of Torsion equations : T/J = q/r - /L, - Torsional moment of resistance –Polar section modulus –Power transmitted by shafts –Combined bending and torsion and end thrust - Design of shafts according to theories of failure. Introduction - Types of springs – deflection of close and open coiled helical springs under axial pull and axial couple – springs in series and parallel – Carriage or leaf springs.
II	**Columns and Struts & Beams Curved in Plan**	Types of columns - Short, medium and long columns – Axially loaded compression members – Crushing load - Euler's theorem for long columns - assumptions - derivation of Euler's critical load formulae for various end conditions – Equivalent length of a column - slenderness ratio –Euler's critical stress - Limitations of Euler's theory - Rankine - Gordon formula - Long columns subjected to eccentric loading - Secant formula - Empirical formulae – Straight line formula - Prof. Perry's formula. Introduction - circular beams loaded uniformly and supported on symmetrically place Columns – Semi-circular beam simply-supported on three equally spaced supports.

UNIT	TITLE	CONTENTS
III	**Beam Columns & Direct and Bending Stresses**	Laterally loaded struts - subjected to uniformly distributed and concentrated loads –Maximum B.M. and stress due to transverse and lateral loading. Stresses under the combined action of direct loading and bending moment, core of a section – Determination of stresses in the case of chimneys, retaining walls and dams - conditions for stability stresses due to direct loading and bending moment about both axis
IV	**Unsymmetrical Bending & Shear Centre**	Introduction - Centroid principle axes of section – Graphical method for locating principal axes - Moments of inertia referred to any set of Rectangular axes –Stresses in beams subject to unsymmetrical bending –Principal axes - Resolution of bending moment into two rectangular axes through the centroid - Location of neutral axis – Deflection of beams under unsymmetrical bending. Shear center for symmetrical and unsymmetrical (channel, I, T and L) sections

UNIT	TITLE	CONTENTS
v	Thin Cylinders & **Thick Cylinders**	Thin seamless cylindrical shells –Derivation of formula for longitudinal and circumferential stresses –hoop, longitudinal and volumetric strains –changes in dia and volume of thin cylinders -Thin spherical shells. Introduction Lame's theory for thick cylinders –Derivation of Lame's formulae –distribution of hoop and radial stresses across thickness –design of thick cylinders - compound cylinders –Necessary difference of radii for shrinkage –Thick spherical shells

UNIT1

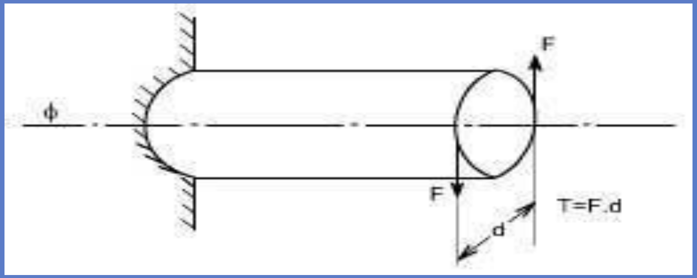
TORSION OF CIRCULAR SHAFTS

SPRINGS

Torsion of circular shafts

- **Definition of Torsion**: Consider a shaft rigidly clamped at one end and twisted at the other end by a torque T = F.d applied in a plane perpendicular to the axis of the bar such a shaft is said to be in torsion.

- **Effects of Torsion**: The effects of a torsional load applied to a bar are



  – To impart an angular displacement of one end cross 1 section with respect to the other end.

-To setup shear stresses on any cross section of the bar perpendicular to its axis.

$$G = \frac{\text{shear stress}(\tau)}{\text{shear strain}(\gamma)}$$

where γ is the shear stress set up at radius R.

Then $\frac{\tau}{G} = \gamma$

Equating the equations (1) and (2) we get $\frac{R\theta}{L} = \frac{\tau}{G}$

$\frac{\tau}{R} = \frac{G\theta}{L}\left(= \frac{\tau'}{r}\right)$ where τ' is the shear stress at any radius r.

GENERATION OF SHEAR STRESSES

- The physical understanding of the phenomena of setting up of shear stresses in a shaft subjected to a torsion may be understood from the figure 1-3.

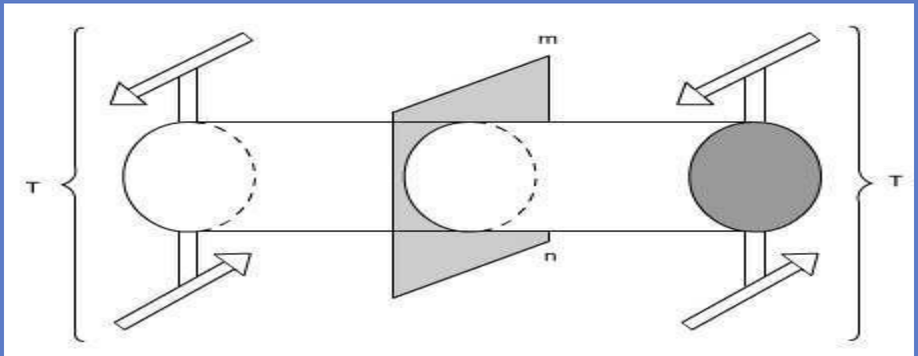


Fig 1: Here the cylindrical member or a shaft is in static equilibrium where T is the resultant external torque acting on the member. Let the member be imagined to be cut by some imaginary plane 1mn'.

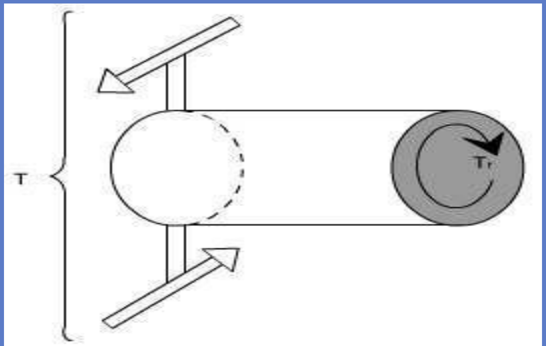
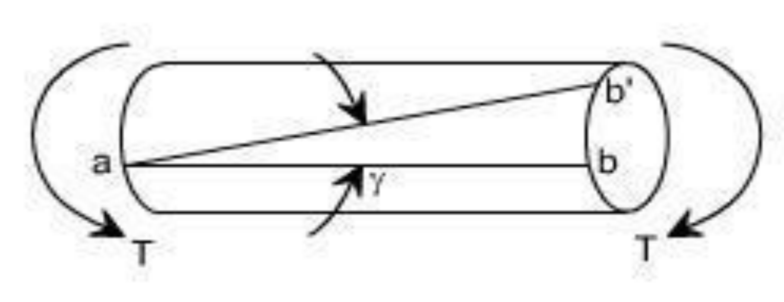


Fig 2: When the plane 1mn' cuts remove the portion on R.H.S. and we get a fig 2. Now since the entire member is in equilibrium, therefore, each portion must be in equilibrium. Thus, the member is in equilibrium under the action of resultant external torque T and developed resisting Torque Tr .

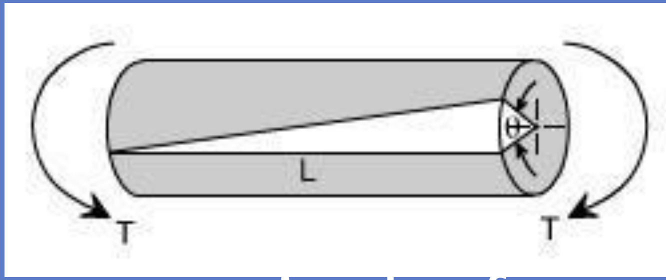
- **Twisting Moment:** The twisting moment for any section along the bar / shaft is defined to be the algebraic sum of the moments of the applied couples that lie to one side of the section under consideration. The choice of the side in any case is of course arbitrary.
- **Shearing Strain:** If a generator a 1 b is marked on the surface of the unloaded bar, then after the twisting moment 'T' has been applied this li ne moves to a b'. The angle 1γ' measured in radians, between the final and original positions of the generators is defined as the shearing strain at the surface of the bar or shaft. The same definition will hold at any interior point of the bar.



Modulus of Elasticity in shear: T he ratio of the shear stress to the shear strain is called the modulus of elasticity in shear OR Modulus of Rigidity and in represented by the symbol

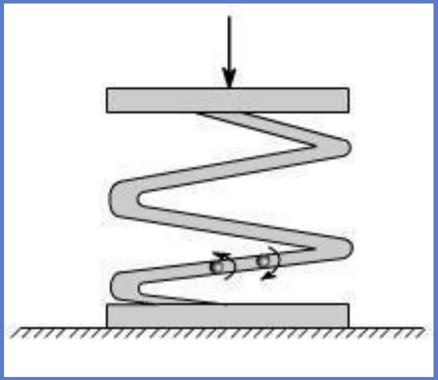
Angle of Twist: If a shaft of length L is subjected to a constant twisting moment T along its l ngth, than the angle θ

through which one end of t he bar will twist relative to the other is known is the angle of twist.



Despite the difference s in the forms of loading, we see that there are number of similarities between bending and torsion, including for example, a linear variation of stresses and strain with position.

Not all torsion problems, involve rotating machinery, however, for example some types of vehicle suspension system employ torsion al springs. Indeed, even coil springs are really curved members in torsion as shown in figure.



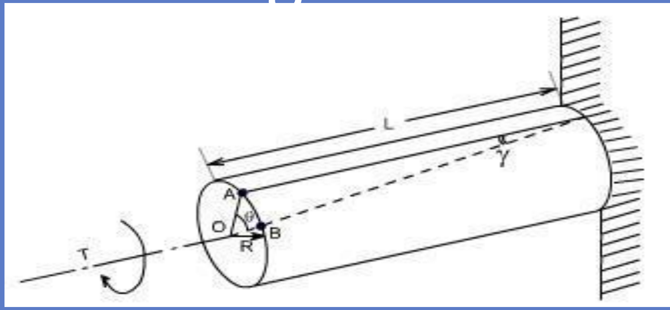
Many torque carrying engineering members are cylindrical in shape. Examples are drive shafts, bolts and screw drivers.

Simple Torsion Theory or Development of Torsion Formula :
Here we are basically interested to derive an equation between the relevant parameters

- **Relationship in Torsion:**
- **1st Term:** It refers to applied loading ad a property of section, which in the instance is the polar second moment of area.
- **2nd Term:** This refers to stress, and the stress increases as the distance from the axis increases.
- **3rd Term:** it refers to the deformation and contains the terms modulus of rigidity & combined term (θ / l) which is equivalent to strain for the purpose of designing a circular shaft to with stand a given torque we must develop an equation giving the relation between Twisting moments max m shear stain produced and a quantity representing the size and shape of the cross 1 sectional area of the shaft.

Assumption:

- The materiel is homogenous i.e of uniform elastic properties exists throughout the material.
- The material is elastic, follows Hook's law, with shear stress proportional to shear strain.
- The stress does not exceed the elastic limit.
- The circular section remains circular
- Cross section remain plane.
- Cross section rotate as if rigid i.e. every diameter rotates through the same angle.



Consider now the solid circular shaft of radius R subjected to a torque T at one end, the other end being fixed Under the action of this torque a radial line at the free end of the shaft twists through an angle θ , point A moves to B, and AB subtends an angle 1 γ ' at the fixed end. This is then the angle of distortion of the shaft i.e the shear strain.

Since angle in radius = arc / Radius arc AB = $R\theta$

= L γ [since L and γ also constitute the arc AB] Thus, $\gamma = R\theta / L$ (1)

From the definition of Modulus of rigidity or Modulus of elasticity in shear

$$G = \frac{\text{shear stress}(\tau)}{\text{shear strain}(\gamma)}$$

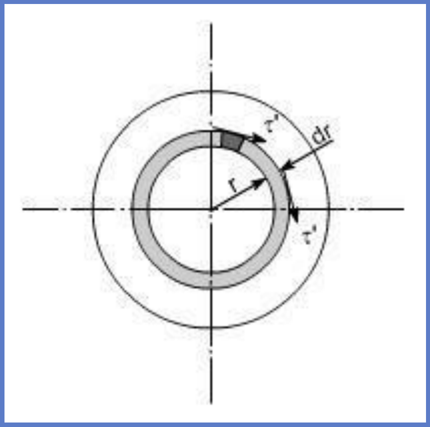
where γ is the shear stress set up at radius R.

Then $\frac{\tau}{G} = \gamma$

Equating the equations (1) and (2) we get $\frac{R\theta}{L} = \frac{\tau}{G}$

$\frac{\tau}{R} = \frac{G\theta}{L}\left(= \frac{\tau'}{r}\right)$ where τ' is the shear stress at any radius r.

Stresses: Let us consider a small strip of radius r and thickness dr which is subjected to shear stress



The force set up on each element

= stress x area

= τ' x 2π r dr (approximately)

This force will produce a moment or torque about the center axis of the shaft.

= τ' . 2 π r dr . r

= 2 π τ' . r2 dr The total torque T on the section, will be the sum of all the contributions.

Since τ' is a function of r, because it varies with radius so writing down_ ' in terms of r from th e equation (1).

i.e $\tau' = \frac{G\theta . r}{L}$

we get $T = \int_0^R 2\pi \frac{G\theta}{L} . r^3 dr$

$$T = \frac{2\pi G\theta}{L} \int_0^R r^3 dr$$

$$= \frac{2\pi G\theta}{L} \left[\frac{R^4}{4} \right]_0^R$$

$$= \frac{G\theta}{L} . \frac{2\pi R^4}{4}$$

$$= \frac{G\theta}{L} . \frac{\pi R^4}{2}$$

$$= \frac{G\theta}{L} . \left[\frac{\pi d^4}{32} \right]$$ now substituting $R = d/2$

$$= \frac{G\theta}{L} . J$$

since $\frac{\pi d^4}{32} = J$ the polar moment of inertia

or $\frac{T}{J} = \frac{G\theta}{L}$ ......(2)

if we combine the equation no.(1) and (2) we get $\boxed{\frac{T}{J} = \frac{\tau'}{r} = \frac{G.\theta}{L}}$

Where

T = applied external Torque, which is constant over Length L;

J = Polar moment of Inertia

[D = Outside diameter ; d = inside diameter]

G = Modules of rigidity (or Modulus of elasticity in shear)

θ = It is the angle of twist in radians on a length L.

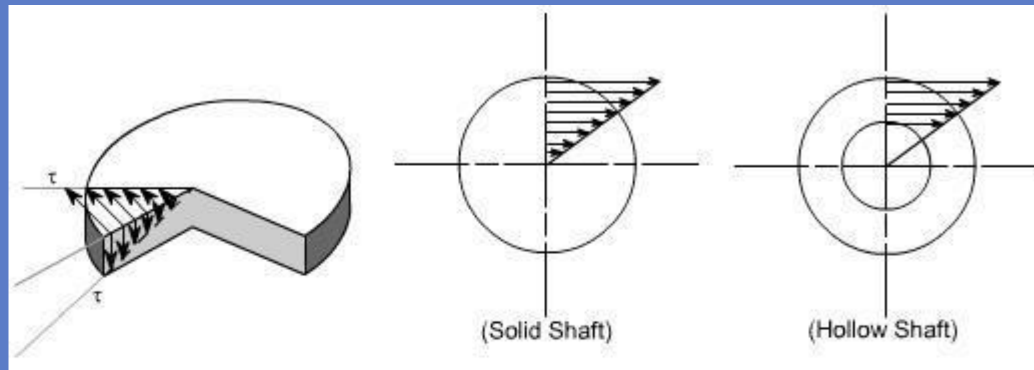
Tensional Stiffness: The tensional stiffness k is defined as the torque per radius twist i.e, k = T /_ _= GJ / L

Power Transmitted by a shaft : If T is the applied Torque and ω is the angular velocity of the shaft, then the power transmitted by the shaft is

Distribution of shear stresses in circular Shafts subjected to torsion :

The simple torsion equation is writt en as

This states that the shearing stress varies directly as the distance 1r' from the axis of the sha ft and the following is the stress distribution in the plane of cross section and also the complementary shearing stresses in an axial plane.



Hence the maximum strear stress o ccurs on the outer surface of the shaft where r = R
The value of maximum shearing stress in the solid circular shaft can be determined as

$$\frac{\tau}{r} = \frac{T}{J}$$

$$\tau_{max}\Big|_{r=d/2} = \frac{T.R}{J} = \frac{T}{\frac{\pi d^4}{32}} \cdot d/2$$

where d=diameter of solid shaft

$$\text{or} \quad \tau_{max^m} = \frac{16T}{\pi d^3}$$

Power Transmitted by a shaft:

- In practical application, the diamete r of the shaft must sometimes be calculated from the power which it is required to transmit.
- Given the power required to be tran smitted, speed in rpm 1N' Torque T, the formula connecting These quantities can be derived as follows
- **Torsional stiffness:** The torsional stiffness k is defined as the torque per radian twist .

- For a ductile material, the plastic flo w begins first in the outer surface. For a material which is weaker in shear longitudinally than transverse ly 1 for instance a wooden shaft, with the fibres parallel t o axis the first cracks will be produced by the she aring stresses acting in the axial section and they will uppe r on the surface of the shaft in the longitudin al direction.

- **Definition:** A spring may be defined as an elastic member whose primary function is to deflect or distort under the action of applied load; it recovers its original shape when load is released.
- **Important types of springs are:**

  There are various types of springs such as
- **helical spring:** They are made of wire coiled into a helical form, the load being applied along the axis of the helix. In these type of springs the major stresses is torsional shear stress due to twisting. They are both used in tension and compression.
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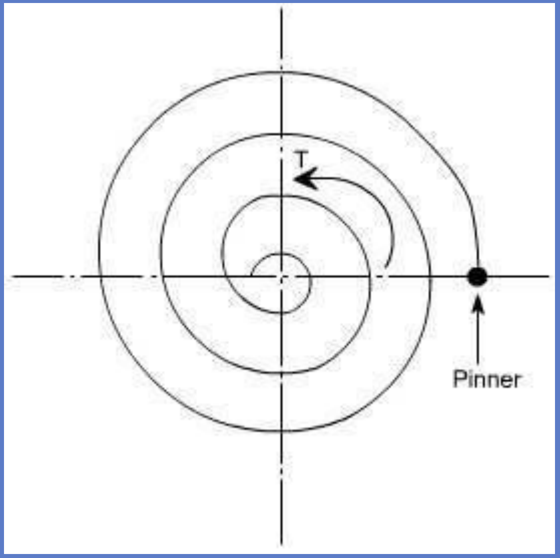
(i) **Spiral springs:** They are made of flat strip of metal wound in the form of spiral and loaded in torsion.

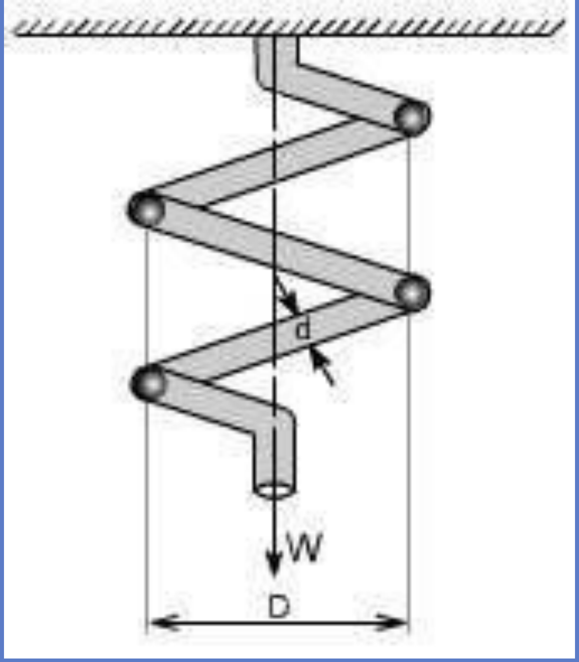
In this the major stresses are tensile and compression due to bending.



Uses of springs :

- To apply forces and to control motions as in brakes and clutches.
- To measure forces as in spring balance.
- To store energy as in clock springs.
- To reduce the effect of shock or impact loading as in carriage springs.
- To change the vibrating characteristics of a member as inflexible mounting of motors.

- **Derivation of the Formula :**
- In order to derive a necessary formula which governs the behaviour of springs, consider a closed coiled spring subjected to an axial load W.



Let

W = axial load

D = mean coil diameter d = diameter of spring wire n = number of active coils

C = spring index = D / d For circular wires

l = length of spring wire G = modulus of rigidity

x = deflection of spring q = Angle of twist

when the spring is being subjected to an axial load to the wire of the spring gets be twisted like a shaft.

If q is the total angle of twist along the wire and x is the deflection of spring under the action of load W along the axis of the coil, so that

$x = D / 2 . \theta$

again $l = \pi D n$ [consider ,one half turn of a close coiled helical spring]

UNIT-2

COLUMNS AND STRUTS
AND BEAMS CURVED IN PLAN

Introduction

- Structural members which carry co mpressive loads may be divided into two broad categories depending on their relative lengths and cross-sectional dimensions.

Columns:

- Short, thick members are generally termed columns and these usually fail by crushing when the yield stress of the material in compression is exceeded.

Struts

- Long, slender columns are generally termed as struts, they fail by buckling some time before the yield stress in compression is reached. The buckling occurs owing to one the following reasons.
- the strut may not be perfectly straight initially.
- the load may not be applied exactly along the axis of the Strut.
- one part of the material may yield in compression more readily than others owing to some lack of uniformity in the material properties through out the strut.

Euler's Theory

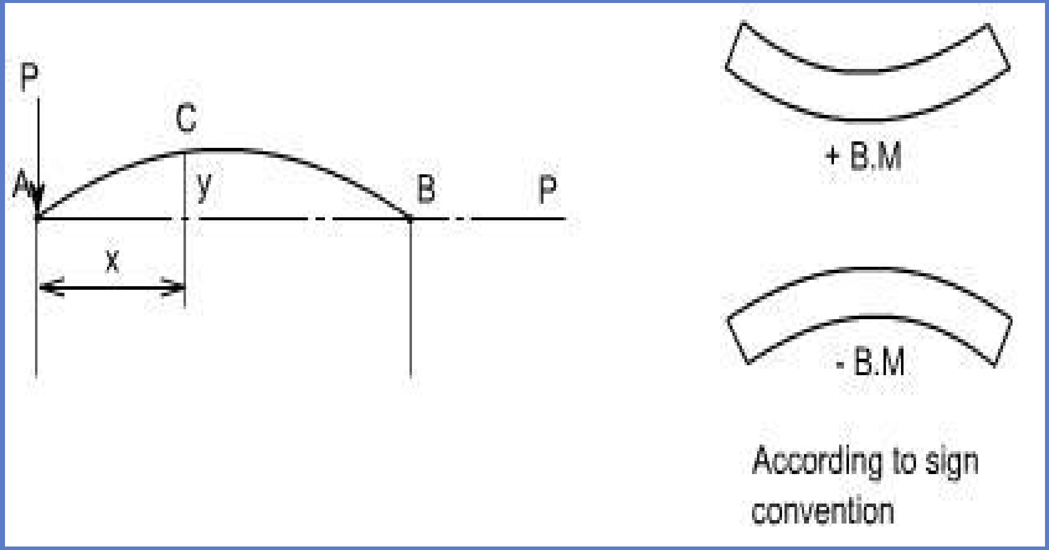
The struts which fail by buckling can be analyzed by Euler's theory. In the following sections, different cases of the struts have been analyzed.

Case A

Strut with pinned ends

- Consider an axially loaded strut, shown below, and is subjected to an axial load 1P' this load 1P' produces a deflection 1y' at a distance 1x' fr om one end.
- Assume that the ends are either pin jointed or rounded so that there is no moment at either end.

- **Assumption:**
- The strut is assumed to be initially straight, the end load being applied axially through centroid.



Let us define a operator

D = d/dx

(D2 +n 2) y =0 where n2 = P/EI

This is a second order differential equation which has a solution of the form consisting of com plimentary function and particular integral but for the time being we are interested in the complementary solution only[in this P.I = 0; since the R.H.S of Diff. equation = 0]

Thus y = A cos (nx) + B sin (nx) Where A and B are some constantss.

UNIT-3

BEAM COLUMNS

DIRECT AND BENDING STRESSES

Analysis of stress and strain :

Concept of stress : Let us introduce the concept of stress as we know that the main problem of engineering mechanics of material is the investigation of the internal resistance of the body, i.e. the nature of forces set up within a body to balance the effect of the externally applied forces.

The externally applied forces are termed as loads. These externally applied forces may be due to any one of the reason.

- due to service conditions
- due to environment in which the component works
- through contact with other mem bers
- due to fluid pressures
- due to gravity or inertia forces.

As we know that in mechanics of d eformable solids, externally applied forces acts on a body and body suffers a deformation. From equilibrium point of view, this action should be opposed or react d by internal forces which are set up within the particles of material due to cohesion.

- These internal forces give rise to a concept of stress. Therefore, let us define a stress Therefore, let us define a term stress
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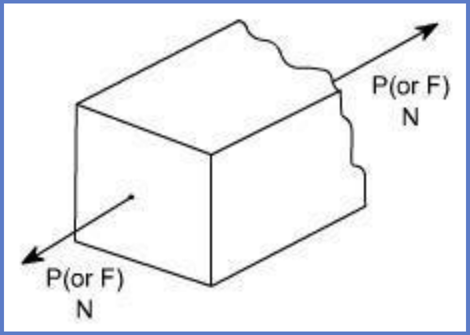
- **Stress:**



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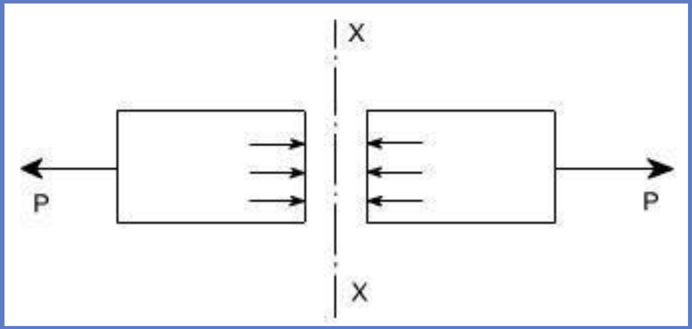
- Let us consider a rectangular bar of some cross 1 sectional area and subjected to some load or force (in Newtons)

- Let us imagine that the same recta ngular bar is assumed to be cut into two halves at section XX. The each portion of this rectangular bar is in equilibrium under the action of load P and the internal forces acting at the section XX has been shown
- 

- Now stress is defined as the force intensity or force per unit area. Here we use a symbol σ to represent the stress.

- Where A is the area of the X 1 section
- Here we are using an assumption that the total force or total load carried by the rectangular b ar is uniformly distributed over its cross 1 section.
- But the stress distributions may be for from uniform, with local regions of high stress known as stress concentrations.
- If the force carried by a component is not uniformly distributed over its cross 1 sectional area, A, we must consider a small area, 1δA' which c arries a small load δP, of the total force 1P', Then definition of stress is

- **TYPES OF STRESSES :**
- only two basic stresses exists : (1) normal stress and (2) shear shear stress. Other stresses either are similar to these basic stresses or ar e a combination of these e.g. bending stress is a combina tion tensile, compressive and shear stresses. Torsional stress, as encountered in twisting of a shaft is a shearing stress.
- Let us define the normal stresses and shear stresses in the following sections.
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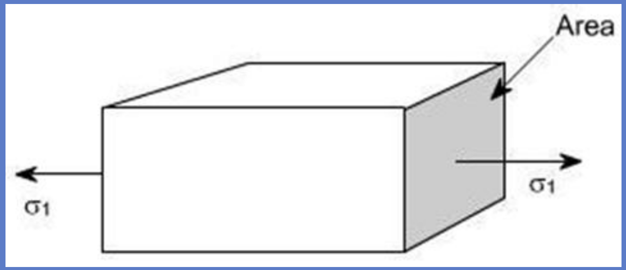
- **Normal stresses :** We have defined stress as force per unit area. If the stresses are normal to the areas concerned, then these are termed as normal stresses. The normal stresses are generally de noted by a Greek letter (σ)
- 

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- This is also known as uniaxial state of stress, because the stresses acts only in one direction however, such a state rarely exists, therefore we have biaxial and triaxial state of stresses where either the two mutually perpendicular normal stresses acts or three mutually perpendicular normal stresses acts as shown in the figures below :
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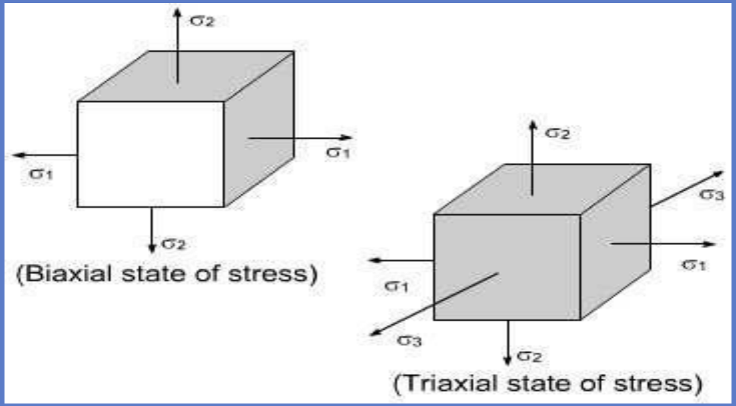
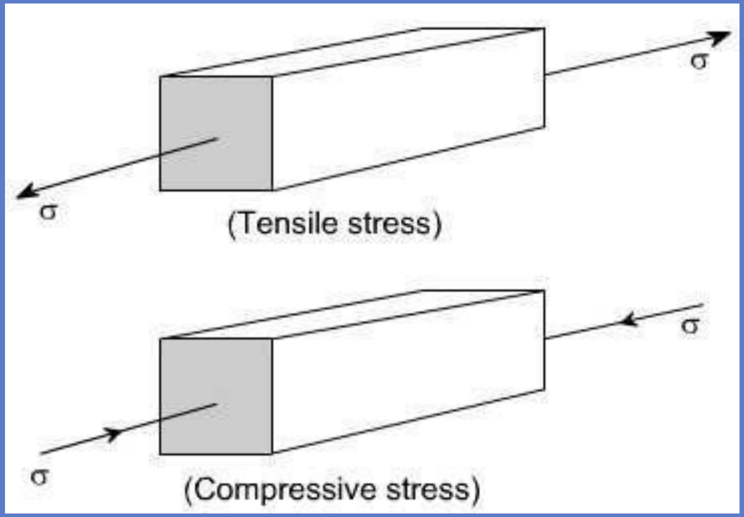
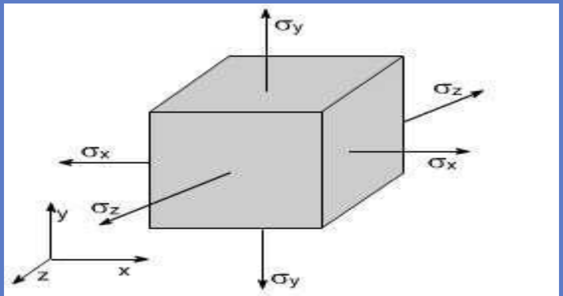


- **Tensile or compressive stresses :**
- The normal stresses can be either tensile or compressive whether the stresses acts out of the area or into the area

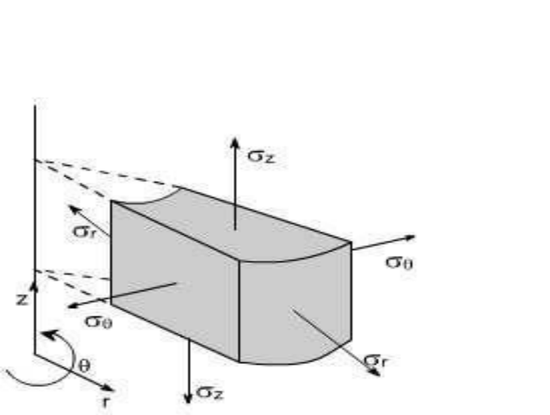


- **<u>Cartesian - co-ordinate system</u>**
- In the Cartesian co-ordinates system, we make use of the axes, X, Y and Z
- Let us consider the small element of the material and show the various normal stresses acting the faces



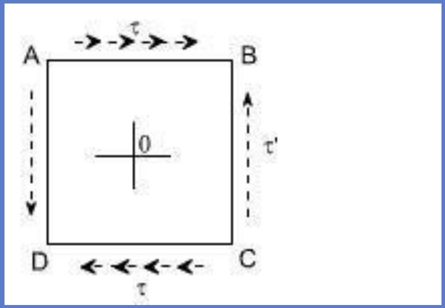
Thus, in the Cartesian co-ordinates system the normal stresses have been represented by σx, σyand σz.

- **<u>Cylindrical - co-ordinate system</u>**
- In the Cylindrical - co-ordinate system we make use of co-ordinates r, θ and Z.



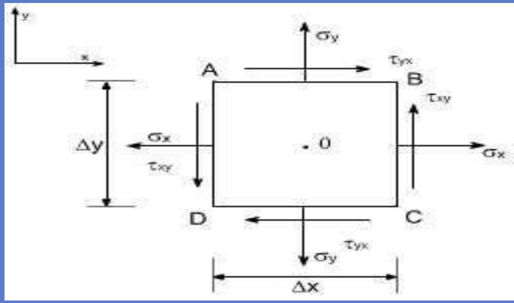
- Thus, in the Cylindrical co-ordinates system, the normal stresses i.e components acting over a element is being denoted by σ_r, $\sigma_\theta$and σ_z.

- **Complementary shear stresses:**
- The existence of shear stresses on any two sides of the element induces complementary shear stresses on the other two sides of the element to maintain equilibrium.



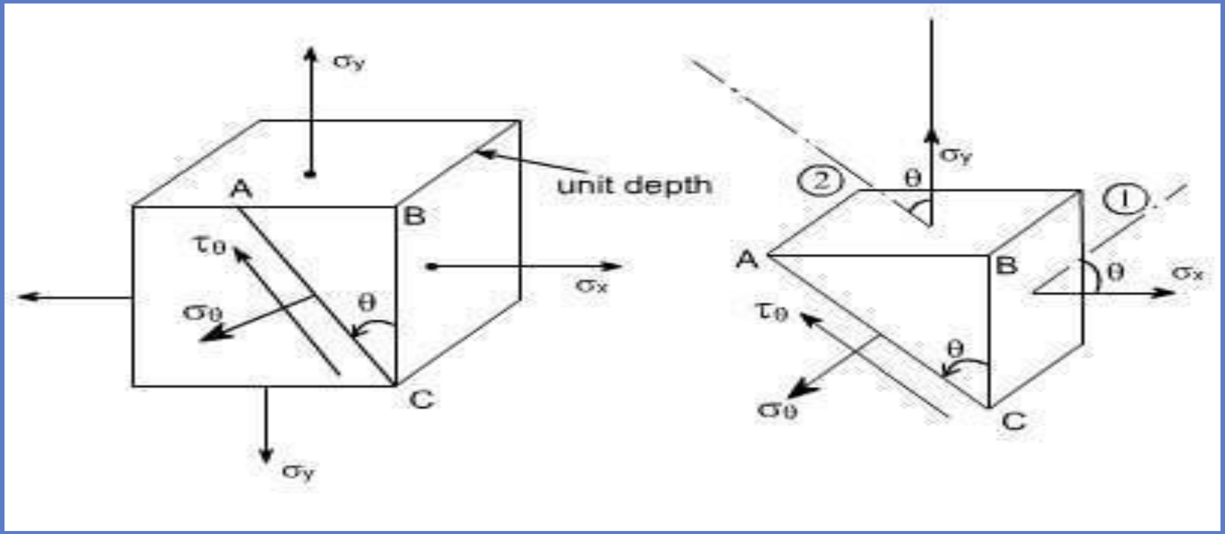
- on planes AB and CD, the shear stress τ acts. To maintain the static equilibrium of this element, on planes AD and BC, τ' should act, we shall see that τ'

- which is known as the complementary shear stress would come out to equal and opposite to the_ _. Let us prove this thing for a general case as discussed below:



- The figure shows a small rectangular element with sides of length x, y parallel to x and y directions. Its thickness normal to the plane of paper is z in z 1 direction. All nine normal and shear stress components may act on the element, only those in x and y directions are shown.

- **<u>Material subjected to two mutually perpendicular direct stresses:</u>**
- Now consider a rectangular element of unit depth, subjected to a system of two direct stresses both tensile, σx and σyacting right angles to each other.



- for equilibrium of the portion ABC, resolving perpendicular to AC

σθ . AC.1 = σy sin θ . AB.1 + σx cos θ . BC.1

converting AB and BC in terms of A C so that AC cancels out from the sides

σθ = σy si 2 + σxcos2 n θ θ

Futher, recalling that cos2 _sin θ = cos2θ or (1 − cos2θ)/2 = sin2 q

Similarly (1 + cos2θ)/2 = cos2

Hence by these transformations the expression for σθ reduces to

$= 1/2\sigma y\ (1 - \cos 2\theta) + 1/2\sigma x\ (1 + \cos 2\theta)$

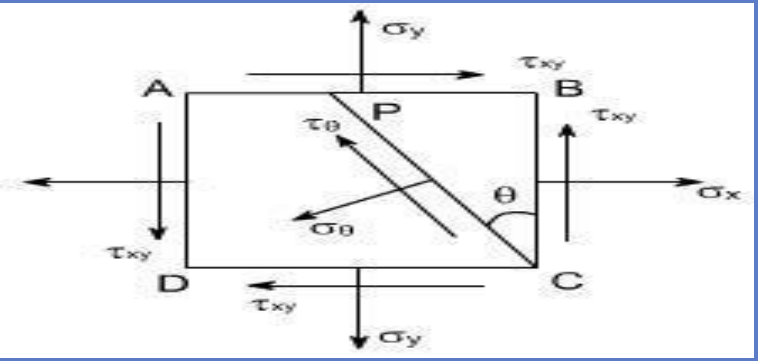
On rearranging the various terms w e get

Now resolving parallal to AC

sq.AC.1= −τxy..cosθ.AB.1+_ xy.BC.s inθ.1

The 1 ve sign appears because this component is in the same direction as that of AC.

Again converting the various quantities in terms of AC so that the AC cancels out from the two sides.

- **Material subjected to combined direct and shear stresses:**
- Now consider a complex stress system shown below, acting on an element of material.
- The stresses σx and σy may be com pressive or tensile and may be the result of direct forces or as a result of bending.The shear stresses may b e as shown or completely reversed and occur as a result o f either shear force or torsion as shown in the figu re below:
- 

- As per the double subscript notatio n the shear stress on the face BC should be notified as τ_{yx} , however, we have already seen that for a pair of shear stresses there is a set of complementary shear stre sses generated such that $\tau_{yx} = \tau_{xy}$
- By looking at this state of stress, it may be observed that this state of stress is combination of two different cases:
- Material subjected to pure stae o f stress shear. In this case the various formulas deserved are as follows
- $\sigma_\theta = \tau_{yx} \sin 2\_$
- $\tau_\theta = - \tau_{yx} \cos 2\_$

- Material subjected to two mutually perpendicular direct stresses. In this case the various formula's derived are as follows
- To get the required equations for the case under consideration,let us add the respective equ ations for the above two cases such that
- These are the equilibrium equation s for stresses at a point. They do not depend on material proportions and are equally valid for elastic and inelastic behaviour
- This eqn gives two values of 2θ that differ by 1800 .Hence the planes on which maximum and minimum normal stresses occurate 900 apart.

For σ_θ to be a maximum or minimum $\frac{d\sigma_\theta}{d\theta} = 0$

Now

$$\sigma_\theta = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\frac{d\sigma_\theta}{d\theta} = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta .2 + \tau_{xy}\cos 2\theta .2$$

$$= 0$$

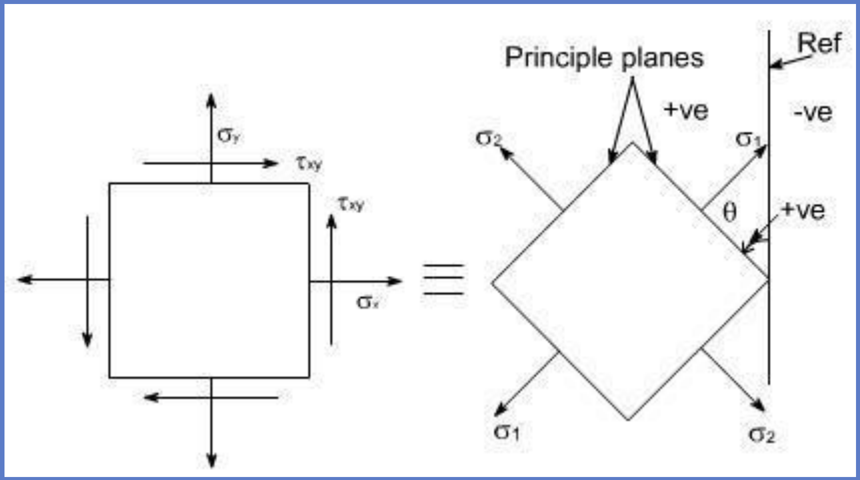
i.e. $-(\sigma_x - \sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta .2 = 0$

$\tau_{xy}\cos 2\theta .2 = (\sigma_x - \sigma_y)\sin 2\theta$

Thus,

$$\boxed{\tan 2\theta = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}}$$

- Hence the maximum and minimum values of normal stresses occur on planes of zero sheari ng stress. The maximum and minimum normal str esses are called the principal stresses, and the planes on which they act are called principal plane the soluti on of equation
-
-
- will yield two values of 2θ separate d by 1800 i.e. two values of θ separated by 900 .Thus the two principal stresses occur on mutually perpend icular planes termed principal planes.
- Therefore the two 1 dimensional c omplex stress system can now be reduced to the equivalent system of principal stresses.



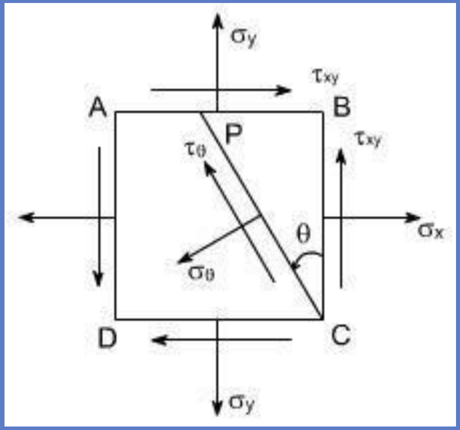
σy
τxy
τxy
σx
Principle planes
Ref
+ve
-ve
σ2
σ1
θ
+ve
σ1
σ2

UNIT-4

UNSYMMETRICAL BENDING AND SHEAR CENTRE

- **GRAPHICAL SOLUTION MOHR’S STRESS CIRCLE**

- The transformation equations for plane stress can be represented in a graphical form known as Mohr's circle. This grapical representation is very useful in depending the relationships between nor mal and shear stresses acting on any inclined plan e at a point in a stresses body.
- To draw a Mohr's stress circle consider a complex stress system as shown in the figure
- The above system represents a co mplete stress system for any condition of applied load in t wo dimensions

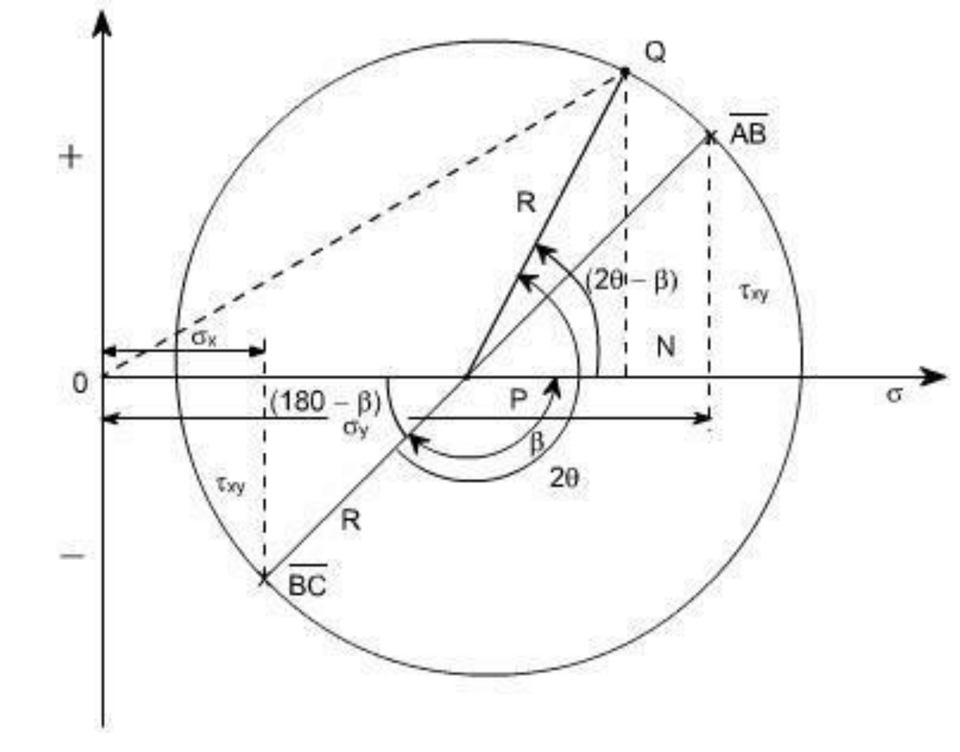


The above system represents a co mplete stress system for any condition of applied load in t wo dimensions

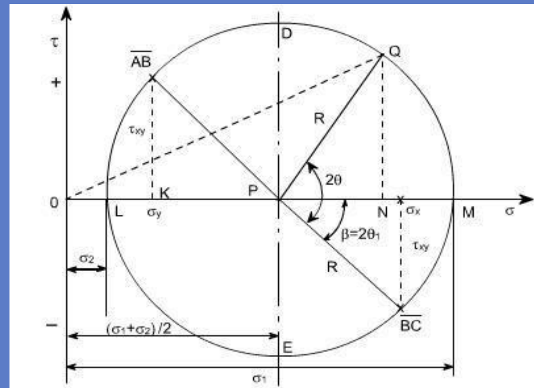
The Mohr's stress circle is used to find out graphically the direct stress σ and sheer stress_ on any plane inclined at θ to the plane on which σxacts.The direction of θ here is taken in anticlockwise dire ction from the BC.

-
- **STEPS:**
-
- In order to do achieve the desired o bjective we proceed in the following manner
-
- Label the BlockABCD.
-
- Set up axes for the direct stress (as abscissa) and shear stress (asordinate)
-
- Plot the stresses on two adjace nt faces e.g. AB and BC, using thefollowing sign convention.
-
- Direct stresses_tensile positive; compressive, negative Shear stresses 1 tending to turn block clockwise, positive 1 tending to turn block counter clockwise, negative
- [i.e shearing stresses are +ve when its movement about the centre of the element is clockwise]
-
- This gives two points on the graph which may than be labeled as respectively to denote stresses on theseplanes.
-
-
- The point P where this line cuts the s axis is than the centre of Mohr's stress circle and the line joining isdiameter. Therefore the circle can now bedrawn.

Now every point on the circle then r epresents a state of stress on some plane through C.



- **Proof:**



- Consider any point Q on the circum ference of the circle, such that PQ makes an angle 2 _with BC, and drop a perpendicular from Q to meet the s axis at N.Then OQ represents the resultant stress on the plane an angle θ to BC. Here we have assu med that $\sigma x > \sigma y$
- Now let us find out the coordinates of point Q. These are ON and QN. From the figure drawn earlier
- ON = OP + PN OP = OK + KP
- $OP = \sigma y + 1/2\ (\sigma x - \sigma y)$

$= \sigma y / 2 + \sigma y / 2 + \sigma x / 2 + \sigma y / 2$

$= (\sigma x + \sigma y) / 2$

PN = $R\cos(2\theta - \beta)$ hence ON = OP + PN
$= (\sigma x + \sigma y) / 2 + R\cos(2\theta - \_)$

$= (\_x + \sigma y) / 2 + R\cos 2\theta \cos\beta + R\sin 2\theta \sin\beta$ now make the substitutions for $R\cos\beta$ and $R\sin\beta$.

If we examine the equation (1) and (2), we see that this is the same equation which we have already derived analytically

Thus the co-ordinates of Q are the normal and shear stresses on the plane inclined at θ to B C in the original stress system.

N.B: Since anglePQis2θon Mohr'scircleandnotθitbecomesobviousthatanglesaredo ubledon Mohr's circle. This is the only differ ence, however, as They are measured in the same directi on andfrom

the same plane in both figures.

Further points to be noted are :

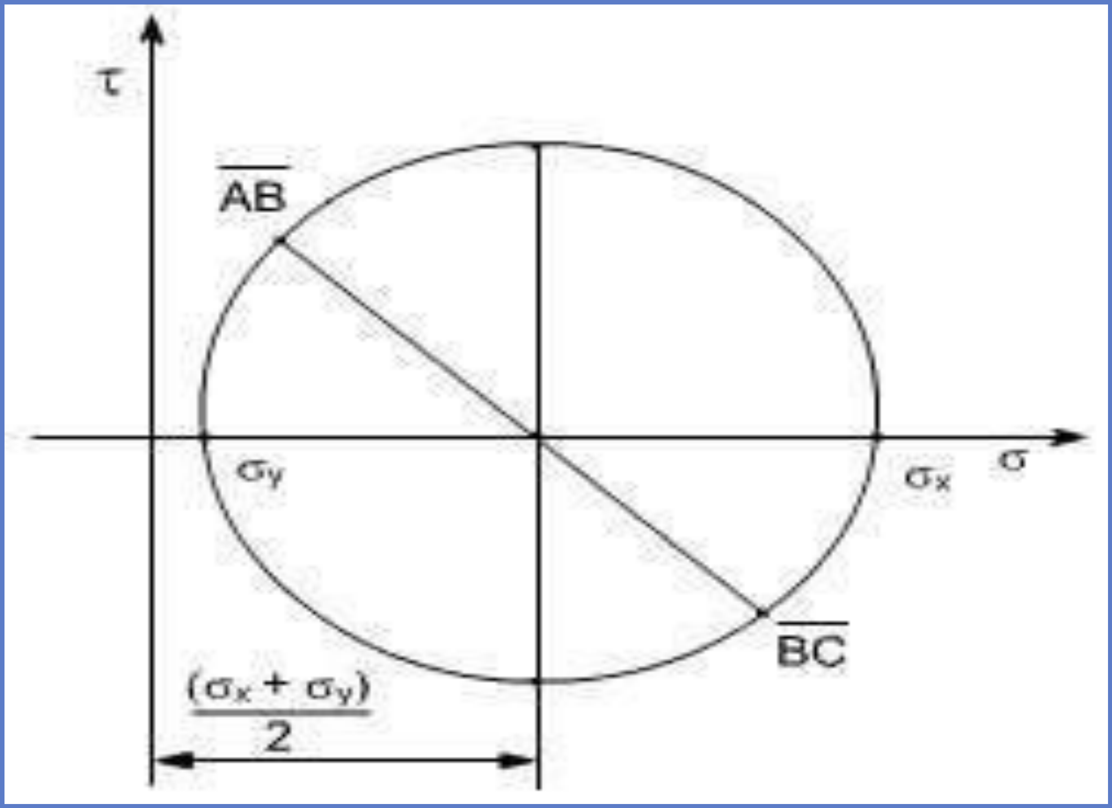
The direct stress is maximum when Q is at M and at this point obviously the sheer stress is zero, hence by definition OM is the length representing the maximum principal stresses $\sigma 1$ and $2\theta 1$ gives the angle of the plane $\theta 1$ fromBC. Similar OL is the other principal stress and is represented by$\sigma 2$

The maximum shear stress is given by the highest point on the circle and is represented y the radius of thecircle.

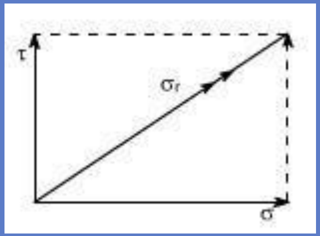
This follows that since shear stresses and complimentary sheer stresses have the same value; therefore the centre of the circle will always lie o n the s axis midway between σx and σy .[since $+\tau xy$ &$-\tau xy$ are shear stress & complimentary shear stress so they are same in magnitude but different in sign.]

From the above point the maxim um sheer stress i.e. the Radius of the Mohr's stress circle wouldbe

While the direct stress on the plane of maximum shear must be mid 1 may between σx andσ y i.e


τ
AB
σy
σx
σ
BC
(σx + σy)
2

-
- As already defined the principal planes are the planes on which the shear components are zero. Therefore are conclude that on principal plane the sheer stress iszero.
- Sincetheresultantoftwostressat900canbefoundfromtheparallogram ofvectorsasshownin
- the diagram.Thus, the resultant stress on the plane at q to BC is given by OQ on Mohr's Circle.
-



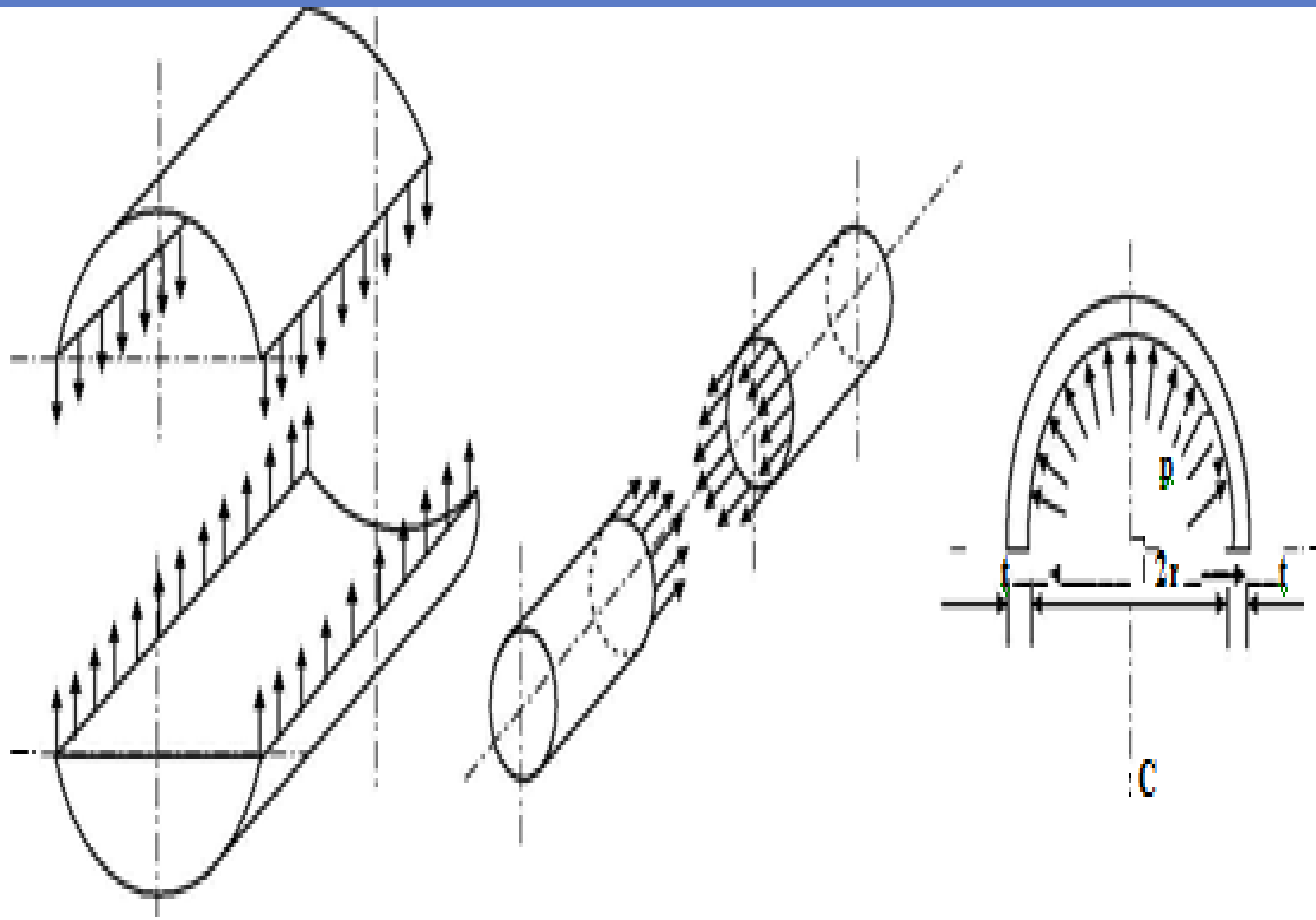
-
- The graphical method of solution for a complex stress problems using Mohr's circle is a very powerful technique, since all the information relating to any plane within the stressed element is contained in the single construction. It thus, provides a convenient and rapid means of solution. Which is less prone to arithmetical errors and is highlyrecommended.

UNIT-5

THIN CYLINDERS AND THICK CYLINDERS

Stresses in thin cylinders

- If the wall thickness is less than about 7% of the inner diameter then the cylinder may be treated as a thin one. Thin walled cylinders are used as boiler shells, pressure tanks, pipes and in other low pressure processing equipments. In general three types of stresses are developed in pressure cylinders viz. circumferential or hoop stress, longitudinal stress in closed end cylinders and radial stresses. These stresses are demonstrated in **figure**



(a) (b) (c)

Circumferential stress (b) Longitudinal stress and (c) Radial stress developed in thincylinders.

In a thin walled cylinder the circumferential stresses may be assumed to be constant over the wall thickness and stress in the radial direction may be neglected for the analysis. Considering the equilibrium of a cut out section the circumferential stress $\sigma\theta$ and

longitudinal stress σz can be found. Consider a section of thin cylinder of radius r, wall thickness t and length L and subjected to an internal pressure p as shown in **figure- 9.1.1.2(a)**. Consider now an element of included angle $d\theta$ at an angle of θ from vertical.

For equilibrium we may write

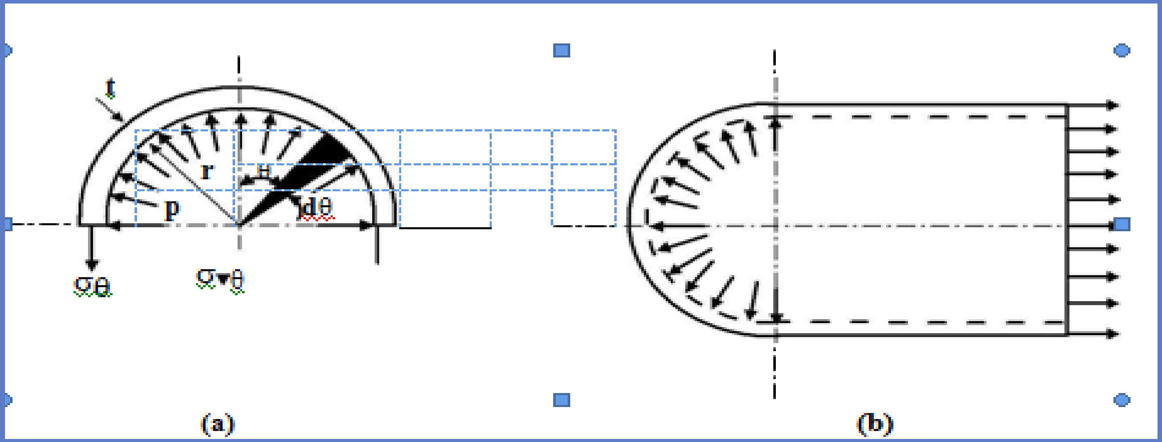
$2\int 2prd\theta L\cos\theta=2\sigma\theta tL$

0

pr

This gives $\sigma\theta$ **=t**

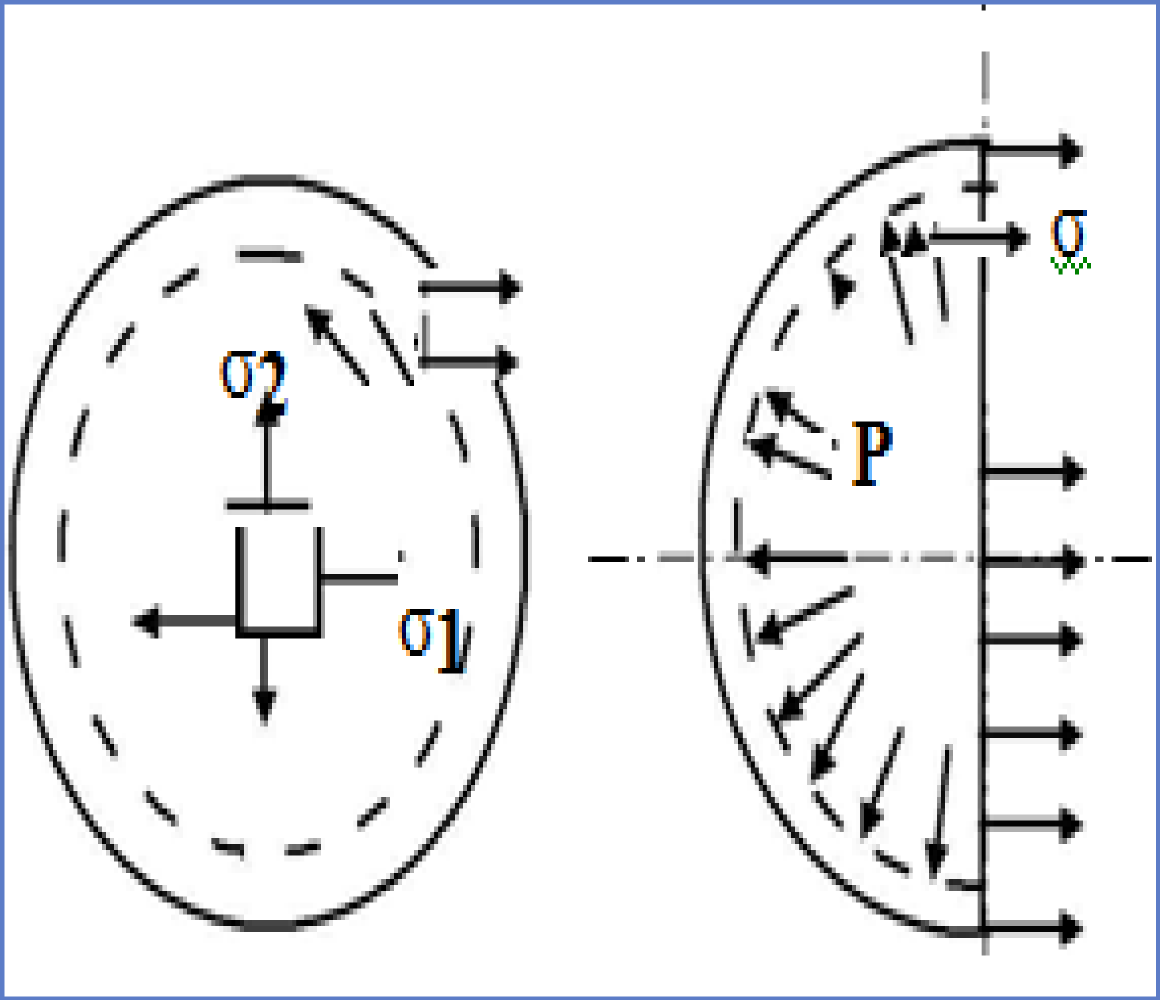
- Considering a section along the longitudinal axis as shown in **figure-9.1.1.2 (b)** we may
- 2 2
- write pπr2 = σz π (ro -ri)
- **where ri and ro are internal and external radii of the vessel and since ri≈ ro = r (say)and ro – ri = t we have σz= _____**



– ***Circumferential stress in a thin cylinder (b) Longitudinalstress in a thincylinder***

- Thin walled spheres are also sometimes used. Consider a sphere of internal radius r subjected to an internal pressure p as shown in **figure-9.1.1.3**. The circumferential and longitudinal stresses developed on an element of the surface of the sphere are equal in magnitude and in the absence of any shear stress due to symmetry both the stresses are principal stresses. From the equilibrium condition in a cut section we have
- **$\sigma_1 = \sigma_2 =$**

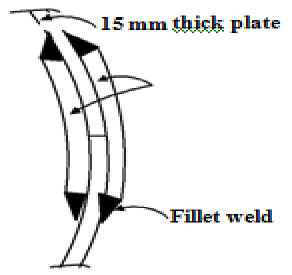
F- Stresses in a ***sphericalshell***



- DesignPrinciples

- **Pressure vessels are generally manufactured from curved sheets joined by welding. Mostly V– butt welded joints are used. The riveted joints may also be used but since the plates are weakened at the joint due to the rivet holes the plate thickness should be enhanced by taking into account the joint efficiency. It is probably more instructive to follow the design procedure of a pressure vessel. We consider a mild steel vessel of 1m diameter comprising a 2.5 m long cylindrical section with hemispherical ends to sustain an internal pressure of (say) 2MPa.**

- The plate thickness is given by
- t ≥ pr where σyt is the tensile yield stress. The
- σ
- minimum plate thickness should conform to the “Boiler code” .



-Longitudinal welded joint with coverplates.

- **Riveted Joint**
-
- The joints may also be riveted in some situations but the design must be checked for safety. The required plate thickness must take account the joint efficiency η.

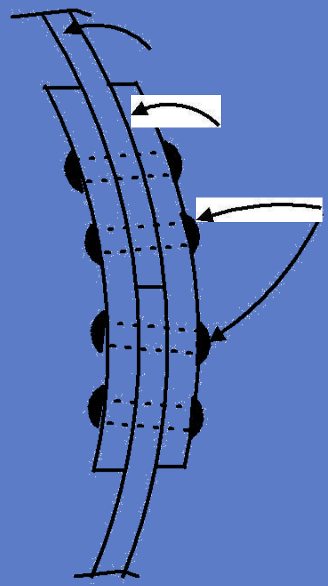
- This gives tc = pr/ ησ Substituting
- p = 2MPa, r = 0.5 m, η = 70 % and σty = (385/5)

- MPa we have tc= 18.5 mm. Let us use mild steel plate of 20 mm thickness for the cylinder body and 10mm thick plate for the hemispherical end cover. The cover plate thickness may be taken as 0.625tc i.e. 12.5 mm. The hoop stress is
- now given byσθ= pr = 50MPa and therefore the rivets must withstand σθtc i.e. 1 tc

- MN per meter.
- We may begin with 20mm diameter rivets with the allowable shear and bearing stresses of 100 MPa and 300 MPa respectively. This gives bearing load ona
-
- single rivet Fb = 300x106x0.02x0.02 = 120 kN. Assuming double shear
- the shearing load on a single rivet FS = 100x106x2x π/4(0.02) 2= 62.8kN.

- The rivet pitch based on bearing load is therefore (120 kN/ 1MN per meter) i.e.
- 0.12m and based on shearing load is (62.8 kN/ 1MN per meter) i.e. 0.063m. We may therefore consider a minimum allowable pitch of 60mm. This gives approximately 17 rivets of 20 mm diameter per meter. If two rows are used the pitch is doubled to 120mm. For the hemispherical shaped end cover the bearing load is 60 kN and therefore the rivet pitch is again approximately 60 mm.

- The maximum tensile stress developed in the plate section is σt = 1x106/[(1-
- 17x0.02)x0.02] = 75.76 MPa which is a safe value considering the allowable tensile stress of 385 MPa with a factor of safety of 5. A longitudinal riveted joint with cover plates is shown in **figure–** and the whole riveting arrangement is shown in **figure-**



20 mm thick plate
12.5 mm thick plates
20 mm diameter rivets at 120 mm pitch

9.1.2.3F- A longitudinal joint with two cover plates

Thin Cylinders Subjected to Inter nal Pressure:

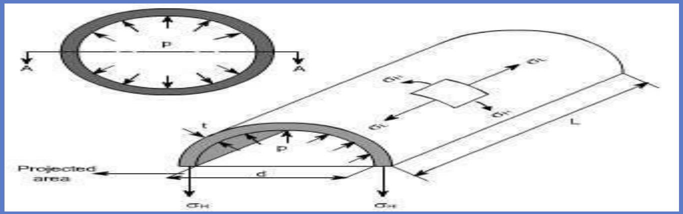
When a thin 1 walled cylinder is subjected to internal pressure, three mutually perpendicular principal stresses will be set up in the cylind er materials, namely

Circumferential or hoopstress
The radialstress
Longitudinalstress

now let us define these stresses and determine the expressions for them

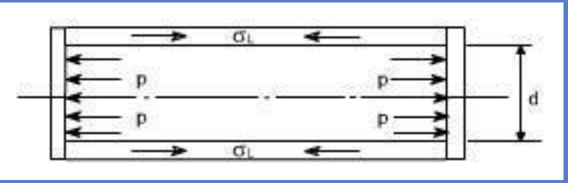
Hoop or circumferential stress:

This is the stress which is set up in resisting the bursting effect of the applied pressure and c an be most conveniently treated by considering the equilibrium of the cylinder.



In the figure we have shown a one half of the cylinder. This cylinder is subjected to an internal pressure p.

- **Longitudinal Stress:**
- Consider now again the same figure and the vessel could be considered to have closed ends and contains a fluid under a gage pressure p.Then the walls of the cylinder will have a longitudinal stress as well as a ciccumferential stress.
-



-
- Total force on the end of the cylind er owing to internal pressure