Hall Ticket No | | | | | | | | | | | Question Paper Code: AHSB05

INSTITUTE OF AERONAUTICAL ENGINEERING
(Autonomous)

B.Tech III Semester End Examinations (Regular), February – 2021

Regulation: IARE–R18

COMPLEX ANALYSIS AND SPECIAL FUNCTIONS

Time: 3 Hours **(ECE)** **Max Marks: 70**

Answer any Four Questions from Part A
Answer any Five Questions from Part B

PART – A

1. Verify whether the function u(x,y)=e^x (x siny-y cosy) is harmonic. **[5M]**
2. Prove that $\int_c \frac{e^z}{(z^2+\pi^2)^2}dz = \frac{i}{\pi}$ where C is $|z| = 4$. **[5M]**
3. Explain the types of evaluation of integrals by Cauchy's Residue theorem. **[5M]**
4. Solve the integral $\int_0^a x^4\sqrt{a^2-x^2}dx$ using Beta-Gamma functions **[5M]**
5. State the most general solution of Bessel differential equation. **[5M]**
6. Find the analytic function whose imaginary part is (2 sinx siny)/(cosh2y - cos2x) . **[5M]**
7. Prove that $\int_0^{\frac{\pi}{2}} \sin^m\theta\cos^n\theta d\theta = \frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$. **[5M]**
8. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$. **[5M]**

PART – B

9. If f(z) is analytic, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. **[10M]**
10. Find the bilinear transformation which maps the points z=1,i,-1 on to the points w =i,0,-i . Hence find the invariant points of this transformation. **[10M]**
11. Show that $\int_c \frac{\cos\pi z^2}{(z-1)(z-2)}dz = 4\pi i$ where C is |z|=3 using Cauchys integral formule. **[10M]**
12. Using Cauchy's integral formula, evaluate $\int_c \frac{z+4}{z^2+2z+5}dz$ where C is the circle |z+1-i|=2. **[10M]**
13. Expand f(z) = $\int_c \frac{(7z-2)}{z(z-2)(z+1)}$ as a series valid in 1<|z+1|<3. **[10M]**
14. Using contour integration, show that $\int_{-\infty}^{\infty} \frac{x^2dx}{(x^2+1)(x^2+4)} = \frac{\pi}{3}$ **[10M]**
15. Express the following integral as Beta function and then in terms of Gamma function $\int_0^{\infty} \frac{y}{(1+y^3)^2}dy$. **[10M]**
16. Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **[10M]**
17. Make use of generating function show that $\sin(x\sin\theta) = 2(J_1\sin\theta + J_3\sin 3\theta + J_5\sin 5\theta.....)$. **[10M]**
18. Prove the relation $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$. **[10M]**

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