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INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal-500043, Hyderabad

B.Tech IV SEMESTER END EXAMINATIONS (REGULAR / SUPPLEMENTARY) - AUGUST 2023

Regulation: UG-20

COMPLEX ANALYSIS AND PROBABILITY DISTRIBUTIONS

Time: 3 Hours (ELECTRICAL AND ELECTRONICS ENGINEERING)

Max Marks: 70

Answer ALL questions in Module I and II

Answer ONE out of two questions in Modules III, IV and V

All Questions Carry Equal Marks

All parts of the question must be answered in one place only

MODULE – I

1. (a) Prove that $u = x^2 - y^2$ and $v = \frac{-y}{x^2 + y^2}$ are harmonic functions of (x,y)

[BL: Apply| CO: 1|Marks: 7]

- (b) Find the bilinear transform which maps the points $z = 0, i, \infty$ onto $w = 1, -i, -1$

[BL: Apply| CO: 1|Marks: 7]

MODULE – II

2. (a) State Cauchy's integral theorem and use it to show that $\int_C \frac{4-3z}{z(z-1)(z-2)} dz = 2\pi i$ where C is the circle $|z| = \frac{3}{2}$

[BL: Apply| CO: 2|Marks: 7]

- (b) Evaluate $\int_0^{2+i} Z^2 dz$ along the real axis from $z = 0$ to $z = 2$ and then along a line parallel to y-axis from $z = 2$ to $z = 2 + i$

[BL: Apply| CO: 2|Marks: 7]

MODULE – III

3. (a) Expand $f(z) = \frac{z-1}{z+1}$ in Taylor's series about the point $z=0$

[BL: Apply| CO: 3|Marks: 7]

- (b) Expand $f(z) = \frac{z-1}{(z-2)(z-3)^2}$ as Laurent series valid for $|z| > 3$.

[BL: Apply| CO: 3|Marks: 7]

4. (a) Determine the poles and residues for the function $f(z) = \frac{z^2}{(z-1)(z-2)(z-3)}$

[BL: Apply| CO: 4|Marks: 7]

- (b) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{z(z-1)(z-2)} dz$ where C is the circle $|z|=3$.

[BL: Apply| CO: 4|Marks: 7]

MODULE – IV

5. (a) The probability mass function of a discrete random variable X is given in the following Table 1. Find the value of K, $P(X < 2)$ and $E(X)$

[BL: Apply| CO: 5|Marks: 7]

Table 1

X	-2	-1	0	1	2	3
P(x)	0.1	K	0.2	2K	0.3	K

- (b) Consider a random variable X having probability density function $\begin{cases} 4x^3 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$
Determine E(X) and V(x) . [BL: Apply| CO: 5|Marks: 7]

6. (a) Determine the central moments for the following frequency distribution given in Table 2:

Table 2

8	9	10	11	12	13	14	15	16
1	8	28	56	70	56	28	8	1

[BL: Apply| CO: 5|Marks: 7]

- (b) A petrol pump owner sells on an average Rs. 80,000 worth of petrol on rainy days and an average of Rs. 95,000 on clear days. Statistics from the Meteorological department show that the probability is 0.76 for clear weather and 0.24 for rainy weather on coming Monday. Find the expected value of petrol sale on coming Monday.

[BL: Apply| CO: 5|Marks: 7]

MODULE – V

7. (a) List out the basic properties of Normal distribution. Determine mean and variance of Poisson distribution [BL: Apply| CO: 6|Marks: 7]
(b) The weight of ball bearings proved in a certain factory are normally distributed with mean 0.5 Newton's and S.D 0.002 Newton's. Find the percentage of ball bearings produced with weights
i) Less than 0.498 Newton's
ii) More than 0.501 Newton's [Given $P(0 < Z < 1) = 0.3413$ and $P(0 < Z < 0.5) = 0.1915$].
[BL: Apply| CO: 6|Marks: 7]
8. (a) Ten coins are tossed simultaneously. Find the probability of getting
i) Atleast seven heads
ii) Exactly seven heads [BL: Apply| CO: 6|Marks: 7]
(b) If 2% of electric bulbs manufactured by a certain company are defective. Find the probability that in a sample of 200 bulbs i) Less than 2 bulbs ii) More than 3 bulbs are defective [$e^{-4} = 0.0183$]
[BL: Apply| CO: 6|Marks: 7]

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