

(Autonomous) Dundigal-500043, Hyderabad

B.Tech IV SEMESTER END EXAMINATIONS (REGULAR / SUPPLEMENTARY) - AUGUST 2023

Regulation: UG-20

COMPLEX ANALYSIS AND SPECIAL FUNCTIONS

Time: 3 Hours (ELECTRONICS AND COMMUNICATION ENGINEERING) Max Marks: 70

Answer ALL questions in Module I and II Answer ONE out of two questions in Modules III, IV and V All Questions Carry Equal Marks All parts of the question must be answered in one place only

$\mathbf{MODULE}-\mathbf{I}$

1. (a) Prove that $u = x^2 - y^2$ and $v = \frac{-y}{x^2 + y^2}$ are harmonic functions of (x,y)

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[BL: Apply] CO: 1|Marks: 7]

(b) Find the bilinear transform which maps the points $z = 0, i, \infty$ onto w = 1, -i, -1[BL: Apply] CO: 1|Marks: 7]

$\mathbf{MODULE}-\mathbf{II}$

2. (a) State Cauchy's integral theorem and use it to show that $\int_C \frac{4-3z}{z(z-1)(z-2)} dz = 2\pi i$ where C is the circle $|z| = \frac{3}{2}$ [BL: Apply| CO: 2|Marks: 7]

(b) Evaluate $\int_0^{2+i} Z^2 dz$ along the real axis from z = 0 to z = 2 and then along a line parallel to yaxis from z = 2 to z = 2 + i [BL: Apply] CO: 2|Marks: 7]

$\mathbf{MODULE}-\mathbf{III}$

3. (a) Expand
$$f(z) = \frac{z-1}{z+1}$$
 in Taylor's series about the point z=0

[BL: Apply] CO: 3|Marks: 7]

(b) Expand
$$f(z) = \frac{z-1}{(z-2)(z-3)^2}$$
 as Laurent series valid for $|z| > 3$.

[BL: Apply] CO: 3|Marks: 7]

4. (a) Determine the poles and residues for the function $f(z) = \frac{z^2}{(z-1)(z-2)(z-3)}$ [BL: Apply] CO: 4|Marks: 7]

(b) Evaluate
$$\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{z(z-1)(z-2)} dz$$
 where C is the circle $|z|=3$. [BL: Apply| CO: 4|Marks: 7]

MODULE - IV

5. (a) Prove that
$$\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}} dx$$
 [BL: Apply] CO: 5|Marks: 7]

- (b) Evaluate the following integrals: i) $\int_0^\infty x^4 e^{-x^2} dx$ ii) $\int_0^{\pi/2} \sqrt{\tan \theta} d\theta$ [BL: Apply] CO: 5|Marks: 7]
- 6. (a) State and prove the relationship between Beta and Gamma functions.

[BL: Understand] CO: 5|Marks: 7]

(b) Evaluate
$$\int_0^2 x(8-x^3)^{\frac{1}{3}} dx$$
 [BL: Apply| CO: 5|Marks: 7]

MODULE-V

7. (a) Show that $J_{-n}(x) = (-1)^n J_n(x)$ where n is a positive integer. [BL: Apply] CO: 6|Marks: 7] (b) Prove that $\frac{\mathrm{d}}{\mathrm{d}x}[x^n J_n(x)] = x^n J_{n-1}(x)$ [BL: Apply] CO: 6|Marks: 7]

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8. (a) Show the recurrence relation
$$xJ'_n = nJ_n - xJ_{n+1}$$

(b) Prove that $J_{1/2}(x) = \sqrt{(\frac{2}{\pi x})} \sin x$

- [BL: Apply] CO: 6|Marks: 7]
- [BL: Apply] CO: 6|Marks: 7]