Time: 3 Hours (ELECTRONICS AND COMMUNICATION ENGINEERING) Max Marks: 70

## Answer ALL questions in Module I and II

Answer ONE out of two questions in Modules III, IV and V
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

## MODULE - I

1. (a) Prove that $u=x^{2}-y^{2}$ and $v=\frac{-y}{x^{2}+y^{2}}$ are harmonic functions of ( $\mathrm{x}, \mathrm{y}$ )
[BL: Apply| CO: 1|Marks: 7]
(b) Find the bilinear transform which maps the points $z=0, i, \infty$ onto $w=1,-i,-1$
[BL: Apply| CO: 1|Marks: 7]

## MODULE - II

2. (a) State Cauchy's integral theorem and use it to show that $\int_{C} \frac{4-3 z}{z(z-1)(z-2)} d z=2 \pi i$ where C is the circle $|z|=\frac{3}{2}$
[BL: Apply| CO: 2|Marks: 7]
(b) Evaluate $\int_{0}^{2+i} Z^{2} d z$ along the real axis from $z=0$ to $z=2$ and then along a line parallel to y axis from $z=2$ to $z=2+i$
[BL: Apply| CO: 2|Marks: 7]

## MODULE - III

3. (a) Expand $f(z)=\frac{z-1}{z+1}$ in Taylor's series about the point $\mathrm{z}=0$
[BL: Apply| CO: 3|Marks: 7]
(b) Expand $f(z)=\frac{z-1}{(z-2)(z-3)^{2}}$ as Laurent series valid for $|z|>3$.
[BL: Apply| CO: 3|Marks: 7]
4. (a) Determine the poles and residues for the function $f(z)=\frac{z^{2}}{(z-1)(z-2)(z-3)}$
[BL: Apply| CO: 4|Marks: 7]
(b) Evaluate $\int_{C} \frac{\sin \pi z^{2}+\cos \pi z^{2}}{z(z-1)(z-2)} d z$ where C is the circle $|\mathrm{z}|=3$.
[BL: Apply| CO: 4|Marks: 7]
MODULE - IV
5. (a) Prove that $\beta(m, n)=\int_{0}^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} d x=\int_{0}^{\infty} \frac{x^{n-1}}{(1+x)^{m+n}} d x \quad$ [BL: Apply| CO: $5 \mid$ Marks: 7$]$
(b) Evaluate the following integrals:
i) $\int_{0}^{\infty} x^{4} e^{-x^{2}} d x$
ii) $\int_{0}^{\pi / 2} \sqrt{\tan \theta} d \theta$
[BL: Apply| CO: 5|Marks: 7]
6. (a) State and prove the relationship between Beta and Gamma functions.
[BL: Understand| CO: 5|Marks: 7]
(b) Evaluate $\int_{0}^{2} x\left(8-x^{3}\right)^{\frac{1}{3}} d x$
[BL: Apply| CO: 5|Marks: 7]

## MODULE - V

7. (a) Show that $J_{-n}(x)=(-1)^{n} J_{n}(x)$ where n is a positive integer.
[BL: Apply| CO: 6|Marks: 7]
(b) Prove that $\frac{\mathrm{d}}{\mathrm{d} x}\left[x^{n} J_{n}(x)\right]=x^{n} J_{n-1}(x)$
[BL: Apply| CO: 6|Marks: 7]
8. (a) Show the recurrence relation $x J_{n}^{\prime}=n J_{n}-x J_{n+1}$
[BL: Apply| CO: 6|Marks: 7]
(b) Prove that $J_{1 / 2}(x)=\sqrt{\left(\frac{2}{\pi x}\right)} \sin x$
[BL: Apply| CO: 6|Marks: 7]
