Hall Ticket No | | | | | | | | | | | Question Paper Code:ACAC01

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

(Dundigal-500043, Hyderabad)

B.Tech III SEMESTER END EXAMINATIONS (REGULAR) - FEBRUARY 2022

Regulation:UG-20

PROBABILISTIC MODELLING AND REASONING

Time: 3 Hours **(AIML)** **Max Marks: 70**

Answer ALL questions in Module I and II

Answer ONE out of two questions in Modules III, IV and V

NOTE: Provision is given to answer TWO questions from among one of the Modules III / IV / V

All Questions Carry Equal Marks

All parts of the question must be answered in one place only

MODULE – I

1. (a) Explain i) Gram Schmidt process ii) Pseudo inverse of a matrix. **[7M]**

   (b) Determine the QR decomposition of the following matrix. **[7M]**

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

MODULE – II

2. (a) Explain i) Skewness and kurtosis of a probability distribution ii) Memory less property. **[7M]**

   (b) Suppose the weighs of 800 male students are normally distributed with mean=140 pounds and S.D= 10 pounds. Determine number of students whose weights are

   i) Between 138 and 148 pounds

   ii) More than 150 pounds. **[7M]**

MODULE – III

3. (a) State Neyman-Pearson lemma. Describe the minimum variance unbiased estimators. **[7M]**

   (b) A coin is flipped 100 times. Given that there were 55 heads, find the maximum likelihood estimate for the probability p of heads on a single toss. **[7M]**

4. (a) Derive the likelihood function of Normal distribution and find the maximum likelihood estimates. **[7M]**

   (b) Suppose that the lifetime of Badger brand light bulbs is modeled by an exponential distribution with (unknown) parameter λ. We test 5 bulbs and find they have lifetimes of 2, 3, 1, 3, and 4 years, respectively. What is the maximum likelihood estimate for λ? **[7M]**

MODULE – IV

5. (a) Illustrate the basic concepts of decision functions

   i) Loss function

   ii) Minimax

   iii) Expected utility principle **[7M]**

(b) Suppose that the distribution of lifetimes of TV tubes can be adequately modelled by an exponential distribution with mean so $f(x|\theta) = \frac{1}{\theta}e^{\frac{-x}{\theta}}$ for $x \geq 0$ and 0 otherwise. Under usual production conditions, the mean lifetime is 2000 hours but if a fault occurs in the process, the mean lifetime drops to 1000 hours. A random sample of 20 tube lifetimes is to taken in order to test the hypotheses $H_0 : \; = 2000$ versus $H_1 : \; = 1000$. Use the Neman-Pearson lemma to find the most powerful test with significance level **[7M]**.

6. (a) State the Cramer-Rao inequality. Explain Expectation-Maximization (EM) algorithm with flow chart. **[7M]**

(b) The mean life time of a sample of 169 light bulbs manufactured by a company is found to be 1350 hours with a standard deviation of 100 hours. Establish 90% confidence limits within which the mean life time of light bulbs is expected to lie. **[7M]**

MODULE – V

7. (a) Describe the Kullback-Leibler divergence for discrete and continuous random variables and illustrate with simple examples. **[7M]**

(b) An engineering company advertises a job in three newspapers, A, B and C. It is known that these papers attract undergraduate engineering readerships in the proportions 2:3:1. The probabilities that an engineering undergraduate sees and replies to the job advertisement in these papers are 0.002, 0.001 and 0.005 respectively. Assume that the undergraduate sees only one job advertisement. If the engineering company receives only one reply to it advertisements, calculate the probability that the applicant has seen the job advertised in place i) A, ii) B, iii) C. **[7M]**

8. (a) State the two different measures of entropy based on logarithms. Describe the statistical tests and Bayesian model comparison. **[7M]**

(b) Calculate the entropy in bits for each of the following random variables:

i) Pixel values in an image whose possible grey values are all the integers from 0 to 255 with uniform probability.

ii) Humans classified according to whether they are, or are not, mammals.

iii) Gender in a tri-sexed insect population whose three genders occur with probabilities 1/4, 1/4, and 1/2.

iv) A population of persons classified by whether they are older, or not older, than the population's median age. **[7M]**

– ○ ○ ◯ ○ ○ –