INSTITUTE OF AERONAUTICAL ENGINEERING
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# B.Tech III SEMESTER END EXAMINATIONS (REGULAR) - FEBRUARY 2022 <br> Regulation:UG-20 <br> PROBABILITY THEORY AND STOCHASTIC PROCESS 

Time: 3 Hours
(ECE)
Max Marks: 70

## Answer ALL questions in Module I and II <br> Answer ONE out of two questions in Modules III, IV and V

NOTE: Provision is given to answer TWO questions from among one of the Modules III / IV / V
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

## MODULE - I

1. (a) Give the classification of random variables. Derive expressions for mean and variance for Binomial random variable.
[7M]
(b) A man wins in a gambling game if he gets two heads in five flips of a biased coin. The probability of getting a head with the coin is 0.7 .
i) Find the probability that man will win. Should he play this game?
ii) What is the probability of winning if he wins by getting at least four heads in five flips?

Should he play this new game.
[7M]

## MODULE - II

2. (a) Explain joint distribution function and joint density function of two random variables X and Y . List any five properties of joint distribution function of two random variables.
(b) Find the probability density function of the random variable Y obtained by the transformation $Y=3 X^{3}-3 X^{2}+2$ of the discrete random variable X whose density function is given in Table 1 .
[7M]
Table 1

| X | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{X})$ | 0.2 | 0.15 | 0.3 | 0.15 | 0.2 |

## MODULE - III

3. (a) State and prove the properties of correlation between two random variables. Write about the covariance between two random variables.
[7M]
(b) Consider two correlated random variables X and Y with variances 4 and 9 respectively, which are transformed to uncorrelated variables $X_{1}$ and $Y_{1}$ by angle of rotation $\theta=\pi / 8$. Compute the correlation coefficient between the variables.
[7M]
4. (a) Explain the Gaussian density function for $N$ random variables. State the properties of jointly Gaussian random variables.
(b) Find variance and covariance of $\mathrm{X}-2 \mathrm{Y}$. If $\mathrm{E}[\mathrm{X}]=2, \mathrm{E}[\mathrm{Y}]=3, \mathrm{E}[\mathrm{XY}]=10, \mathrm{E}\left[X^{2}\right]=9$, and $\mathrm{E}\left[Y^{2}\right]=16$.

## MODULE - IV

5. (a) Briefly explain the distribution and density function in the context of stationary and independent random process. State wide sense stationary random process.
(b) $\mathrm{X}(\mathrm{t})$ is a stationary random process with a mean of 3 and an auto correlation function of $9+2 e^{-|\tau|}$.Find the variance of the random process.
6. (a) Analyze the output of an LTI system driven by a random process and develop the expression for the auto correlation value of output process.
(b) A random process is defined as $\mathrm{X}(\mathrm{t})=\mathrm{A} \cos \left(\omega_{c} t+\theta\right)$ where $\theta$ is a uniform random variable over $(0,2 \pi)$. Verify the process is ergodic in the mean sense and auto correlation sense.

## MODULE - V

7. (a) Explain the cross-power density spectrum of two random processes and derive the expression for it.
(b) Find the average power in a random process defined by $\mathrm{X}(\mathrm{t})=\mathrm{A} \cos (\omega \mathrm{t}+\theta)$ where A and $\omega$ are constants and $\theta$ is a random variable uniformly distributed on the interval $(0, \pi / 2)$.
8. (a) Distinguish between white and colored noises. Where these noises are observed? Explain.
(b) Let the auto correlation function of a certain random process $\mathrm{X}(\mathrm{t})$ be given by
$R_{X X}(\tau)=\frac{A^{2}}{2} \cos (\omega \tau)$.Find its power spectral density $S_{X X}(\omega)$.
