Hall Ticket No | | | | | | | | | | | Course Code: ACACO1

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad - 500 043

B.Tech III SEMESTER END EXAMINATIONS (REGULAR / SUPPLEMENTARY) - FEBRUARY 2023

Regulation: UG20

PROBABILISTIC MODELLING AND REASONING

Time: 3 Hours **CSE (ARTIFICIAL INTELLIGENCE & MACHINE LEARNING)** **Max Marks: 70**

Answer ALL questions in Module I and II
Answer ONE out of two questions in Modules III, IV and V
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

MODULE – I

1. (a) Determine the variance-covariance matrix for the following data matrix

[BL: Apply| CO: 1|Marks: 7]

$$\mathbf{X} = \begin{bmatrix} 4.0 & 2.0 & 0.60 \\ 4.2 & 2.1 & 0.59 \\ 3.9 & 2.0 & 0.58 \\ 4.3 & 2.1 & 0.62 \\ 4.1 & 2.2 & 0.63 \end{bmatrix}$$

(b) Compute the singular value decomposition of the following matrix:

$$A=\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

[BL: Apply| CO: 1|Marks: 7]

MODULE – II

2. (a) Determine the moment generating function of exponential distribution and hence obtain its mean, variance and kurtosis. [BL: Understand| CO: 2|Marks: 7]

(b) Most graduate schools of business require applicants for admission to take the graduate management admission council's GMAT examination. Scores on the GMAT are roughly normally distributed with a mean of 527 and a standard deviation of 112.

i) What is the probability of an individual scoring above 500 on the GMAT?

Ii) How high must an individual score on the GMAT in order to score in the highest 5%?

[BL: Apply| CO: 2|Marks: 7]

MODULE – III

3. (a) A random sample of 100 students gave a mean weight of 58 kg and standard deviation of 4kg. Find the 95% and 99% confidence limits of mean of the population.[BL: Apply| CO: 3|Marks: 7]

(b) Suppose X is a single observation from a population with probability density function given by: $f(x) = \theta x^{\theta-1}$ for $0 < x < 1$. Find the test with the best critical region that is, find the most powerful test, with significance level $\alpha = 0.05$, for testing the simple null hypothesis $\mathrm{H}_0 : \theta = 3$ against the simple alternative hypothesis $\mathrm{H}_\mathrm{A} : \theta = 2$ with 5% significance level.

[BL: Apply| CO: 3|Marks: 7]

4. (a) The mean life time of a sample of 169 light bulbs manufactured by a company is found to be 1350 hours with a standard deviation of 100 hours. Establish 90confidence limits within which the mean life time of light bulbs is expected to lie. [BL: Apply| CO: 3|Marks: 7]

   (b) Consider tirst a particular θ_1 where $\theta_1 > 2$ and apply the Neyman-Pearson lemma to the hypotheses $H_0 : \theta = 2$vs $H_1 : \theta = \theta_1$ where $f\left(\frac{x}{\theta}\right) = \frac{1}{\theta} e^{\frac{-x}{\theta}}$ for x ≥ 0 and 0 otherwise. Calculate C^* and k^*. with 5% significance level. [BL: Apply| CO: 3|Marks: 7]

MODULE – IV

5. (a) State and prove the invariance property of MLE with an example. Give any two applications of MLE in real life situations. [BL: Understand| CO: 4|Marks: 7]

   (b) Find the Cramer-Rao lower bound for any unbiased estimator of the parameter μ for the normal density $f\left(x/\mu, \sigma^2\right) = ke^{-\frac{1}{2}\left(\left(\frac{x-\mu}{\sigma}\right)^2\right)}$ where k $=\frac{1}{\sqrt{2\pi\sigma}}$, based on the single observation sample. [BL: Apply| CO: 4|Marks: 7]

6. (a) Obtain C.R lower bound for parameter 'λ' when random samples are drawn from Poisson distribution with parameter 'λ'. [BL: Apply| CO: 4|Marks: 7]

   (b) Write about Maximum likelihood estimation (MLE). Obtain the MLE of the parameter θ when random observations are drawn from uniform distribution with density given as $f(x, \theta) = \frac{1}{\theta}$: $0 < x < \theta$ $\theta > 0$ [BL: Apply| CO: 4|Marks: 7]

MODULE – V

7. (a) There are two bags in a game. First bag contains 5 red, 6 white balls and the second bag contains 3 red, 4 white balls. One bag is selected at random and a ball is drawn from it. If it is found to be red, what is the probability of

   i) Selecting the first bag

   ii) Selecting the second bag (Use Bayes' Rule). [BL: Apply| CO: 6|Marks: 7]

   (b) Consider a binary symmetric communication channel, whose input source is the alphabet X = {0, 1} with probabilities {0.5, 0.5}; output alphabet Y = {0, 1} and with channel matrix:

   Where ϵ is the probability of transmission error.

   i) What is the entropy of the source, H(X)?

   ii) What is the probability distribution of the outputs, p(Y), and what is the entropy of this output distribution, H(Y)?

   iii) What is the joint probability distribution for the source and the output, p(X, Y), and what is the joint entropy, H(X, Y)? [BL: Apply| CO: 6|Marks: 7]

8. (a) If the probability that a communication system will have high fidelity is 0.81 and the probability that it will have selectivity and fidelity is 0.18. what is the probability that a system with high fidelity will also have high selectivity? [BL: Apply| CO: 6|Marks: 7]

   (b) Three machines A, B, C with capacities proportional to 2:3:4 is producing bullets. The probabilities that the machines produce defectives are 0.1, 0.2 and 0.1 respectively. A bullet is taken from a day's production and found to be defective. What is the probability that it came from

   i) Machine A ii) Machine C. [BL: Apply| CO: 6|Marks: 7]

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