INSTITUTE OF AERONAUTICAL ENGINEERING
(Autonomous)
Dundigal, Hyderabad - 500043
B.Tech III SEMESTER END EXAMINATIONS (REGULAR/SUPPLEMENTARY) - FEBRUARY 2023

Regulation: UG20
DISCRETE MATHEMATICAL STRUCTURES
Time: 3 Hours (Common to CSE \| IT \| CSIT) Max Marks: 70
Answer ALL questions in Module I and II
Answer ONE out of two questions in Modules III, IV and V
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

## MODULE - I

1. (a) Identify whether the following inference is valid or invalid? If invalid, state the fallacy
$C \vee D$
$(\mathrm{C} \vee \mathrm{D}) \rightarrow \sim \mathrm{H}$
$\sim \mathrm{H} \rightarrow(\mathrm{A} \wedge \sim \mathrm{B})$
$(\mathrm{A} \wedge \sim \mathrm{B}) \rightarrow(\mathrm{R} \vee \mathrm{S})$
$R \vee S$
[BL: Apply| CO: $1 \mid$ Marks: 7$]$
(b) Construct the truth table for $\{\sim(p \vee q) \vee[(\sim p) \wedge q] \vee p\}$ and check whether it is tautology or not.
[BL: Apply| CO: 1|Marks: 7]

## MODULE - II

2. (a) List out any two set operations with an example. Determine the elements of the sets A and B , where $\mathrm{A}-\mathrm{B}=\{1,2,4\}, \mathrm{B}-\mathrm{A}=\{7,8\}$ and $\mathrm{AUB}=\{1,2,3,4,5,7,8,9\}$.
[BL: Apply| CO: $2 \mid$ Marks: 7$]$
(b) Let f be function from $\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\}$ to $\{1,2,3,4\}$ with $\mathrm{f}(\mathrm{a})=4, \mathrm{f}(\mathrm{b})=2, \mathrm{f}(\mathrm{c})=1$ and $\mathrm{f}(\mathrm{d})=3$. Investigate whether f is one-one, into and onto function. Give reasons.
[BL: Apply| CO: 2|Marks: 7]

## MODULE - III

3. (a) If G is a group and H is a subgroup of index 2 in G , then prove that H is a normal subgroup of G. Write in detail about subgroup.
[BL: Apply| CO: 3|Marks: 7]
(b) There are 30 females and 35 males in the junior class while there are 25 females and 20 males in the senior class. In how many ways can a committee of 10 be chosen, so that there are exactly 5 females and 3 males from juniors?
[BL: Apply| CO: 3|Marks: 7]
4. (a) Suppose that $N$ and $M$ are two normal subgroups of $G$ and that $N \cap M=\{e\}$. Show that for any $\mathrm{n} \epsilon \mathrm{N}, \mathrm{m} \epsilon \mathrm{M}, \mathrm{nm}=\mathrm{mn}$. Write about semigroup in detail.
[BL: Understand| CO: 4|Marks: 7]
(b) Let R denote the group of real numbers with addition and $\mathrm{R} *$ denote the group of non-zero real numbers with multiplication. For $\mathrm{x} \epsilon \mathrm{R},|\mathrm{x}|$ denotes the absolute value of x . For each part, determine whether the mapping given is a group homomorphism. Justify your answers briefly. Define $\phi: \mathrm{R} \rightarrow \mathrm{R}$ by $\phi(\mathrm{x})=3 \mathrm{x}$ for all $\mathrm{x} \epsilon \mathrm{R}$. Explain about homomorphism with an example.
[BL: Apply| CO: 4|Marks: 7]

## MODULE - IV

5. (a) Using generating functions method solve the following recurrence relation
$a_{n}-6 a_{n-2}-8 a_{n-3}=0$ for $\mathrm{n} \leq 3 \quad$ [BL: Understand| CO: 5|Marks: 7]
(b) Solve the following recurrence relation $a_{n}-9 a_{n-1}+26 a_{n-2}-24 a_{n-3}=0$ for $\mathrm{n}=3$ with the initial conditions $a_{0}=0, a_{1}=1, a_{2}=10$ using any one method.
[BL: Apply| CO: 5|Marks: 7]
6. (a) Determine the following recurrence relation use characteristic roots method.
$a_{n}-7 a_{n-1}+16 a n-2-12 a_{n-3}=0$ for $\mathrm{n} \leq 3$ and $a_{0}=1, a_{1}=0$ and $a_{2}=8$
[BL: Understand| CO: 5|Marks: 7]
(b) A vending machine dispensing books of stamps accepts only dollar coins, $\$ 1$ bills and $\$ 5$ bills.
i) Find the recurrence relation for the number of ways to deposit n dollars, where the order in which the coins and bills are deposited matters.
ii) Find initial conditions and solve the recurrence relation.
[BL: Apply| CO: 5|Marks: 7]

## MODULE - V

7. (a) How many edge-disjoint Hamiltonian cycles exists in the complete graph with seven vertices, also draw the graph to show this Hamiltonian cycles.
[BL: Understand| CO: 6|Marks: 7]
(b) Discuss about chromatic number and write the use of it. Find the chromatic number for the following graph shown in Figure 1.
[BL: Apply| CO: 6|Marks: 7]


Figure 1
8. (a) Write about in-degree and out-degree of a graph. Explain the procedure for Kruskal's algorithm with suitable examples.
[BL: Understand| CO: 6|Marks: 7]
(b) Using Prim's algorithm, determine a minimal spanning tree for the following graph shown in Figure 2. Find its minimum cost.
[BL: Apply| CO: 6|Marks: 7]


Figure 2

