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INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous) Dundigal, Hyderabad - 500 043

B.Tech III SEMESTER END EXAMINATIONS (REGULAR/SUPPLEMENTARY) - FEBRUARY 2023 Regulation: UG20

DISCRETE MATHEMATICAL STRUCTURES

Time: 3 Hours

 $({\rm Common \ to \ CSE} \mid {\rm IT} \mid {\rm CSIT})$

Max Marks: 70

Answer ALL questions in Module I and II Answer ONE out of two questions in Modules III, IV and V All Questions Carry Equal Marks All parts of the question must be answered in one place only

MODULE - I

1. (a) Identify whether the following inference is valid or invalid? If invalid, state the fallacy

 $\begin{array}{l} C \lor D \\ (C \lor D) \rightarrow \sim H \\ \sim H \rightarrow (A \land \sim B) \\ (A \land \sim B) \rightarrow (R \lor S) \\ \hline \\ R \lor S \end{array}$

[BL: Apply] CO: 1|Marks: 7]

(b) Construct the truth table for $\{\sim (p \lor q) \lor [(\sim p) \land q] \lor p\}$ and check whether it is tautology or not. [BL: Apply] CO: 1|Marks: 7]

$\mathbf{MODULE}-\mathbf{II}$

2. (a) List out any two set operations with an example. Determine the elements of the sets A and B, where A-B = $\{1,2,4\}$, B-A = $\{7,8\}$ and AUB = $\{1,2,3,4,5,7,8,9\}$.

[BL: Apply] CO: 2|Marks: 7]

(b) Let f be function from $\{a,b,c,d\}$ to $\{1,2,3,4\}$ with f(a) = 4, f(b) = 2, f(c) = 1 and f(d) = 3. Investigate whether f is one-one, into and onto function. Give reasons.

[BL: Apply] CO: 2|Marks: 7]

$\mathbf{MODULE}-\mathbf{III}$

- 3. (a) If G is a group and H is a subgroup of index 2 in G, then prove that H is a normal subgroup of G. Write in detail about subgroup. [BL: Apply] CO: 3|Marks: 7]
 - (b) There are 30 females and 35 males in the junior class while there are 25 females and 20 males in the senior class. In how many ways can a committee of 10 be chosen, so that there are exactly 5 females and 3 males from juniors?
 [BL: Apply] CO: 3|Marks: 7]
- 4. (a) Suppose that N and M are two normal subgroups of G and that $N \cap M = \{e\}$. Show that for any n ϵ N, m ϵ M, nm = mn. Write about semigroup in detail.

[BL: Understand] CO: 4|Marks: 7]

(b) Let R denote the group of real numbers with addition and R * denote the group of non-zero real numbers with multiplication. For x ϵ R, $|\mathbf{x}|$ denotes the absolute value of x. For each part, determine whether the mapping given is a group homomorphism. Justify your answers briefly. Define $\phi : \mathbf{R} \to \mathbf{R}$ by $\phi(\mathbf{x}) = 3\mathbf{x}$ for all x ϵ R. Explain about homomorphism with an example. [BL: Apply] CO: 4[Marks: 7]

$\mathbf{MODULE}-\mathbf{IV}$

- 5. (a) Using generating functions method solve the following recurrence relation $a_n - 6a_{n-2} - 8a_{n-3} = 0$ for $n \le 3$ [BL: Understand CO: 5|Marks: 7]
 - (b) Solve the following recurrence relation $a_n 9a_{n-1} + 26a_{n-2} 24a_{n-3} = 0$ for n=3 with the initial conditions $a_0 = 0$, $a_1 = 1$, $a_2 = 10$ using any one method. [BL: Apply] CO: 5|Marks: 7]
- 6. (a) Determine the following recurrence relation use characteristic roots method. $a_n - 7a_{n-1} + 16an - 2 - 12a_{n-3} = 0$ for $n \le 3$ and $a_0 = 1$, $a_1 = 0$ and $a_2 = 8$ [BL: Understand] CO: 5|Marks: 7]
 - (b) A vending machine dispensing books of stamps accepts only dollar coins, \$1 bills and \$5 bills.i) Find the recurrence relation for the number of ways to deposit n dollars, where the order in which the coins and bills are deposited matters.

ii) Find initial conditions and solve the recurrence relation. [BL: Apply] CO: 5[Marks: 7]

$\mathbf{MODULE}-\mathbf{V}$

- 7. (a) How many edge-disjoint Hamiltonian cycles exists in the complete graph with seven vertices, also draw the graph to show this Hamiltonian cycles. [BL: Understand] CO: 6|Marks: 7]
 - (b) Discuss about chromatic number and write the use of it. Find the chromatic number for the following graph shown in Figure 1. [BL: Apply] CO: 6|Marks: 7]



Figure 1

- 8. (a) Write about in-degree and out-degree of a graph. Explain the procedure for Kruskal's algorithm with suitable examples. [BL: Understand] CO: 6|Marks: 7]
 - (b) Using Prim's algorithm, determine a minimal spanning tree for the following graph shown in Figure 2. Find its minimum cost. [BL: Apply] CO: 6|Marks: 7]



Figure 2