Hall Ticket No | | | | | | | | | | | Question Paper Code: ACAC01

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal-500043, Hyderabad

B.Tech III SEMESTER END EXAMINATIONS (REGULAR/ SUPPLEMENTARY) - FEBRUARY 2024

Regulation: UG20

PROBABILISTIC MODELLING AND REASONING

Time: 3 Hours **CSE(AI&ML)** **Max Marks: 70**

Answer ALL questions in Module I and II

Answer ONE out of two questions in Modules III, IV and V

All Questions Carry Equal Marks

All parts of the question must be answered in one place only

MODULE – I

1. (a) Outline the concept of Dimensionality Reduction and how it is achieved through Principal Component Analysis (PCA) using population principal components and sample principal coefficients.

   [BL: Understand| CO: 1|Marks: 7]

   (b) Given B = {u1; u2; u3}, where u1 = (1, 2, 1), u2 = (1, 1, 3) and u3 = (2, 1, 1), use the Gram-Schmidt procedure to find a corresponding orthonormal basis

   [BL: Apply| CO: 1|Marks: 7]

MODULE – II

2. (a) Discuss the properties of the normal distribution, including the moment generating function (MGF), cumulant generating function, skewness, and kurtosis. Mention the significance of these properties in Applying the behavior of the normal distribution.

   [BL: Understand| CO: 2|Marks: 7]

   (b) Let X be a random variable with probability density function $f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & otherwise \end{cases}$

   X is exponentially distributed with mean value of 3. Determine $P(X > 3)$ and the moment generating function. [BL: Apply| CO: 2|Marks: 7]

MODULE – III

3. (a) Summarize Maximum Likelihood Estimate (MLE) and explain its relationship with the log-likelihood function. [BL: Apply| CO: 3|Marks: 7]

   (b) Let $X_1, X_2, X_3, ...X_n$ be Poisson distributed. Calculate an estimate using the Poisson estimator $f(\frac{x}{\lambda}) = \frac{\lambda^x e^{-\lambda}}{x!}$, x=0,1,2....,$\lambda > 0$, when $x_1 = 1, x_2 = 2, x_3 = 4, x_4 = 2$ and hence find the maximum likelihood estimator of $X \hat{=} 4$ [BL: Apply| CO: 3|Marks: 7]

4. (a) Analyze the application of CRLB in estimating the rate parameter of a Poisson distribution.

   [BL: Apply| CO: 4|Marks: 7]

(b) A coin is flipped 100 times. Given that there are 55 heads, determine the maximum likelihood estimate for the probability p of heads on a single toss using log likelihood.

[BL: Apply| CO: 4|Marks: 7]

MODULE – IV

5. (a) Elaborate the basic concepts of decision theory, including decision functions, the loss function, minimax, and the expected utility principle. Discuss how these concepts are applied in making decisions under uncertainty or risk. [BL: Understand| CO: 5|Marks: 7]

(b) In a sample of 400 textile workers, 184 expressed dissatisfaction regarding a prospective plan to modify working conditions. The management felt that this is a strong negative reaction. So they want to know the proportion of total workers who have this feeling of dissatisfaction. Obtain an unbiased estimate of the population proportion. [BL: Apply| CO: 5|Marks: 7].

6. (a) Interpret point estimation and interval estimation in the context of decision theory. Explain the importance of these estimation techniques in making informed decisions and managing uncertainty. [BL: Understand| CO: 5|Marks: 7]

(b) Consider a test of simple hypotheses $H_0 : \theta = \theta_0$ versus $H_1 : \theta = \theta_1$ based on a random sample from a distribution with probability mass function $f(x/\theta)$, for x = 1,2, ... ,7. The values of the likelihood function at θ and θ_1 are given in the Table 1:

Table 1

x	1	2	3	4	5	6	7
L(θ_0)	0.01	0.01	0.01	0.01	0.01	0.01	0.94
L(θ_1)	0.06	0.05	0.04	0.03	0.02	0.01	0.79

Use the Neyman-Pearson Lemma to find the most powerful test for H0 versus H1 with significance level $\alpha = 0.04$. Compute the probability of Type II error for this test.

[BL: Apply| CO: 5|Marks: 7]

MODULE – V

7. (a) Discuss the concepts of bit, surprisal, and entropy in the context of information theory and Bayesian inference. Explain the relevance of these concepts in quantifying uncertainty and information content. [BL: Apply| CO: 6|Marks: 7]

(b) A binary channel matrix is given by $\begin{bmatrix} 2/3 & 1/3 \\ 1/10 & 9/10 \end{bmatrix}$ The probabilities of the two symbols being transmitted are 1/3 and 2/3 respectively.

i) Determine the probabilities of the two symbols received at the destination.

ii) Determine H(x), H(x y) and I(x:y). [BL: Apply| CO: 6|Marks: 7]

8. (a) Summarize Kullback Leibler divergence between p and q and prove that D(p // q)$\geq$0 and equality holds if p = q. [BL: Apply| CO: 6|Marks: 7]

(b) A businessman goes to hotels X, Y, Z, 20%, 50% and 30% of the time respectively. It is known that 5%, 4%, 8% of the rooms in X, Y, Z hotels have faulty plumbing. Calculate the probability that business man's room having faulty pluming is assigned to hotel Z?

[BL: Apply| CO: 6|Marks: 7]