Hall Ticket No | | | | | | | | | | | Question Paper Code: AECC04

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal-500043, Hyderabad

B.Tech III SEMESTER END EXAMINATIONS (REGULAR/ SUPPLEMENTARY) - FEBRUARY 2024

Regulation: UG20

PROBABILITY THEORY AND STOCHASTIC PROCESS

Time: 3 Hours (ELECTRONICS AND COMMUNICATION ENGINEERING) **Max Marks: 70**

Answer ALL questions in Module I and II
Answer ONE out of two questions in Modules III, IV and V
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

MODULE – I

1. (a) State and prove total probability theorem.Discuss about conditional probability and mention their properties. [BL: Understand| CO: 1|Marks: 7]

   (b) The number of monthly breakdown of a computer is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month

   i) Without a breakdown

   ii) With only one breakdown

   iii) With at least one breakdown. [BL: Apply| CO: 1|Marks: 7]

MODULE – II

2. (a) Summarize joint probability distribution functions and central limit theorem. Discuss their properties. [BL: Understand| CO: 2|Marks: 7]

   (b) A coin is tossed 300 times. Using central limit theorem, find the probability that heads will appear more than 140 times and less than 150 times.

   [BL: Apply| CO: 2|Marks: 7]

MODULE – III

3. (a) Outline about transformation of random variables. Show that the linear transformation of vector Gaussian random variable is another vector Gaussian random variable.

   [BL: Understand| CO: 3|Marks: 7]

   (b) Two random variables X and Y have the joint density function

   $$f(x,y) = \begin{cases} 2-x-y & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & otherwise \end{cases}$$

   Find the covariance between X and Y. [BL: Apply| CO: 3|Marks: 7]

4. (a) Discuss any four properties of the covariance between two random variables with suitable examples. [BL: Understand| CO: 4|Marks: 7]

   (b) The joint probability density function of X and Y is given by $f(x,y) = e^{-(x+y)}$.

   Find the probability density function of $U = \dfrac{X+Y}{2}$ [BL: Apply| CO: 4|Marks: 7]

MODULE – IV

5. (a) Write short notes on strict sense and wide sense stationary process. State and prove any two properties of cross correlation function. [BL: Understand| CO: 5|Marks: 7]

   (b) Prove that the random process $X(t) = Acos(\omega t + \theta)$ is correlation Ergodic where A and ω are constants and θ is uniformly distributed random variable in $(0, 2\pi)$.
   [BL: Apply| CO: 5|Marks: 7].

6. (a) Obtain an expression for cross correlation function of input and output processes of a linear time invariant (LTI) system. [BL: Understand| CO: 5|Marks: 7]

   (b) If $X(t)$ is a Gaussian process with $\mu(t) = 10$ and $c(t_1, t_2) = 16e^{|t_1-t_2|}$, find the probability that i) $X(10) \leq 8$ ii) $|X(2) - X(6)| \leq 4$. [BL: Apply| CO: 5|Marks: 7]

MODULE – V

7. (a) Discuss about power density spectrum. Determine the cross power spectral density of input and output random processes of a linear time invariant system.
   [BL: Understand| CO: 6|Marks: 7]

   (b) The Cross power spectrum of a real random process $X(t)$ and $Y(t)$ is given by
   $S_{x,y}(\omega) = \begin{cases} a + bi\omega & |\omega| < 1 \\ 0 & otherwise \end{cases}$. Find the cross correlation function.
   [BL: Apply| CO: 6|Marks: 7]

8. (a) Write short notes on noise bandwith. Distinguish between white and colored noises. Where these noises are observed? Explain. [BL: Understand| CO: 6|Marks: 7]

   (b) If $N(t)$ is band limited Gaussian white noise centered at a carrier frequency ω_0 with a power spectral density $S_{NN}(\omega) = \begin{cases} \frac{N_0}{2} & |\omega - \omega_0| \leq \omega_B \\ 0 & otherwise \end{cases}$.
   Find the autocorrelation of $N(t)$. [BL: Apply| CO: 6|Marks: 7]

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