Hall Ticket No | | | | | | | | | | | Question Paper Code:AHSC12

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal-500043, Hyderabad

B.Tech IV SEMESTER END EXAMINATIONS (REGULAR) - JULY 2022

Regulation:UG20

COMPLEX ANALYSIS AND SPECIAL FUNCTIONS

Time: 3 Hours (ELECTRONICS AND COMMUNICATION ENGINEERING) **Max Marks: 70**

Answer ALL questions in Module I and II

Answer ONE out of two questions in Modules III, IV and V

(NOTE: Provision is given to answer TWO questions from among one of the Modules III / IV / V

All Questions Carry Equal Marks

All parts of the question must be answered in one place only

MODULE – I

1. (a) Find the bilinear transform which maps the points $z = 0, -i, -1$ into the points $w = i, 1, 0$.
[BL: Understand| CO: 1|Marks: 7]

(b) Show that the function $f(z) = \begin{cases} \frac{(\overline{z})^2}{z} & z \neq 0 \\ 0 & z = 0 \end{cases}$ is not differentiable at z=0 even though Cauchy-Riemann equations are satisfied there. [BL: Apply| CO: 1|Marks: 7]

MODULE – II

2. (a) State Cauchy integral theorem and use it to evaluate the integral $\int_C \frac{e^z}{(z^2+\pi^2)^2}$ dz where $C\ is\ |z| = 4$. [BL: Understand| CO: 2|Marks: 7]

(b) Evaluate $\oint_C (x+y)dx + (y-2x)dy$ along:
i) The parabola $y = 2x^2$ from (1, 2) to (2, 8)
ii) The straight lines from (1, 1) to (1, 8) and then from (1, 8) to (2, 8)
iii) The straight line from (1, 1) to (2, 8). [BL: Apply| CO: 2|Marks: 7]

MODULE – III

3. (a) State Cauchy's residue theorem. Hence evaluate $\int_C \frac{e^z dz}{\sin z}$ where C is the circle |z|=1.
[BL: Apply| CO: 3|Marks: 7]

(b) Show that $\int_0^{2\pi} \frac{\cos 2\theta \ d\theta}{5 - 4\cos\theta} = \frac{\pi}{6}$ using residue theorem. [BL: Apply| CO: 4|Marks: 7]

4. (a) Find the nature of singularities of the following functions [BL: Understand| CO: 3|Marks: 7]
i) $f_1(z) = \frac{z - \sin z}{z^4}$
ii) $f_2(z) = \frac{1 - e^z}{1 + e^z}$

(b) Give two Laurent series expansions in the power of z for the function $f(z) = \frac{1}{z^2(1-z)}$ and specify the regions in which those expansions are valid. [BL: Understand| CO: 4|Marks: 7]

MODULE – IV

5. (a) Prove that $\dfrac{\beta(p,q+1)}{q} = \dfrac{\beta(p+1,q)}{q} = \dfrac{\beta(p,q)}{p+q}$ [BL: Understand| CO: 5|Marks: 7]

   (b) Evaluate $\int_0^1 x^3\sqrt{1-x}$ dx using Beta Gamma functions [BL: Apply| CO: 5|Marks: 7]

6. (a) Show that $\Gamma(1/2)\Gamma(2n) = 2^{2n-1}\Gamma(n)\Gamma(n+\frac{1}{2})$ [BL: Understand| CO: 5|Marks: 7]

   (b) By using techniques involving the Gamma function, find the exact value of $\int_0^\infty x^6 e^{-4x^2}$ dx. Give the answer in the form k $\sqrt{\pi}$, where k is a rational constant [BL: Apply| CO: 5|Marks: 7]

MODULE – V

7. (a) State and prove generating function of Bessel's function. [BL: Understand| CO: 6|Marks: 7]

   (b) Show that $[J_{1/2}(x)]^2 + [J_{-1/2}(x)]^2 = \dfrac{2}{\pi x}$. [BL: Apply| CO: 6|Marks: 7]

8. (a) Show that $\dfrac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$ [BL: Understand| CO: 6|Marks: 7]

   (b) Prove that $\dfrac{d}{dx}[J_n^2(x) + J_{n+1}^2(x)] = 2[\dfrac{n}{x}J_n^2(x) - \dfrac{n+1}{x}J_{n+1}^2(x)]$ [BL: Apply| CO: 6|Marks: 7]

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