

**INSTITUTE OF AERONAUTICAL ENGINEERING**

(Autonomous)

Dundigal-500043, Hyderabad

B.Tech II SEMESTER END EXAMINATIONS (REGULAR) - SEPTEMBER 2022

Regulation:UG20

MATHEMATICAL TRANSFORM TECHNIQUES**Time: 3 Hours**

(Common to AE | ECE | EEE | ME | CE)

Max Marks: 70

Answer ALL questions in Module I and II

Answer ONE out of two questions in Modules III, IV and V

All Questions Carry Equal Marks

All parts of the question must be answered in one place only

MODULE – I

1. (a) Using Laplace transform, solve $\frac{d^2y}{dt^2} + \frac{dy}{dt} = t^2 + 2t$ given that $y(0)=4, y'(0) = -2$.
[BL: Apply| CO: 1|Marks: 7]
- (b) Using convolution theorem find the inverse Laplace transform of $\frac{4}{(s^2 + 2s + 5)^2}$
[BL: Apply| CO: 2|Marks: 7]

MODULE – II

2. (a) If $F(S)$ and $G(S)$ are the Fourier transforms of $f(x)$ and $g(x)$ respectively then prove the following
i) $F(af(x) + bg(x)) = aF(S) + bG(S)$
ii) $F_s(f(x) \cos ax) = 1/2(F_s(S + a) + F_s(S - a))$ [BL: Apply| CO: 3|Marks: 7]
- (b) Express the function $f(x) = \begin{cases} 1 & |x| \leq 1 \\ 0 & x > 1 \end{cases}$ as a Fourier integral. Hence, evaluate
i) $\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda$
ii) $\int_0^\infty \frac{\sin \lambda}{\lambda} d\lambda$ [BL: Apply| CO: 3|Marks: 7]

MODULE – III

3. (a) Using double integrals show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$
[BL: Apply| CO: 4|Marks: 7]
- (b) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing to polar coordinates and hence evaluate $\int_0^\infty e^{-x^2} dx$.
[BL: Apply| CO: 4|Marks: 7]
4. (a) Solve $\int_0^4 \int_0^{2\sqrt{z}} \int_0^{\sqrt{4z-x^2}} dx dy dz$ [BL: Apply| CO: 4|Marks: 7]
- (b) Evaluate $\int_C \vec{A} \cdot d\vec{r}$ if $\vec{A} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$ and C is the curve $x = t, y = t^2, z = t^3$ from $(0, 0, 0)$ to $(1, 1, 1)$.
[BL: Apply| CO: 4|Marks: 7]

MODULE – IV

5. (a) Find the value of 'a' if the vector field $(ax^2y + yz)\vec{i} + (xy^2 - xz^2)\vec{j} + (2xy - 2x^2y^2)\vec{k}$ has zero divergence. Find the curl of the vector field when it has zero divergence. [BL: Apply| CO: 5|Marks: 7]
- (b) Evaluate by Green's theorem $\int_C e^{-x}(\sin y dx + \cos y dy)$ where C is the rectangle with vertices $(0, 0), (\pi, 0), (\pi, \frac{\pi}{2}), (0, \frac{\pi}{2})$ [BL: Apply| CO: 5|Marks: 7]
6. (a) Using the Gauss divergence theorem, evaluate $\int \int_S \vec{F} \cdot \vec{n} dS$, where $\vec{F} = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$ and S is the sphere $x^2 + y^2 + z^2 = a^2$. [BL: Apply| CO: 5|Marks: 7]
- (b) Evaluate $\int_C (xy dx + xy^2 dy)$ by Stoke's theorem, C is the square in the XY-plane with vertices $(1, 0), (-1, 0), (0, 1), (0, -1)$. [BL: Apply| CO: 5|Marks: 7]

MODULE – V

7. (a) Find the singular solution of $z = px + qy$. [BL: Apply| CO: 6|Marks: 7]
- (b) Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$ [BL: Apply| CO: 6|Marks: 7]
8. (a) Form partial differential equation by eliminating the arbitrary constant a and b from the equations
- i) $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$
- ii) $z = (x - a)^2 + (y - b)^2 + 1$ [BL: Apply| CO: 6|Marks: 7]
- (b) Find the partial differential equation $px + qy = pq$ by Charpit method. [BL: Apply| CO: 6|Marks: 7]

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