

All parts of the question must be answered in one place only

#### $\mathbf{Unit} - \mathbf{I}$

- 1. (a) For what value of K such that the matrix  $\begin{bmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3 \end{bmatrix}$  has rank 3 [7M] (b) Find Inverse by elementary row operations of  $\begin{bmatrix} -1 & -3 & 3 & 1 \\ 1 & 1 & -1 & 0 \\ 2 & 5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$  [7M]
- 2. (a) Express the matrix A as sum of symmetric and skew symmetric matrices.

	3	-2	6	
Where $A =$	2	7	-1	[7M]
	5	4	0	

(b) Solve the system of equations x + y + z = 1, 3x + y - 3z = 5, x - 2y - 5z = 10 by using Method of factorization. [7M]

#### Unit - II

- 3. (a) Prove that the sum of the Eigen Values of a matrix is equal to its trace and Product of the Eigen Values is equal to determinant. [7M]
  - (b) Diagonalize the matrix  $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ -1 & 2 & 2 \end{bmatrix}$  [7M]
- 4. (a) Prove that the Eigen values of a skew- Hermitian matrix are either zero or purely imaginary. [7M]

(b) Show that 
$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$
 satisfies its characteristic equation and hence find  $A^{-1}$  [7M]

## $\mathbf{Unit}-\mathbf{III}$

5. (a) Solve 
$$\left[x\frac{dy}{dx} + y = x^3y^6\right]$$
 [7M]

(b) A copper ball is heated to a temperature of  $80^{\circ}c$  and time t = 0, then it is placed in water which is maintained at  $30^{\circ}c$ . If at t = 3 minutes, the temperature of the ball is reduced to  $50^{\circ}c$  find the time at which the temperature of the ball is  $40^{\circ}c$ . [7M]

6. (a) Solve 
$$(1 + e^{\frac{x}{y}})dx + e^{\frac{x}{y}}(1 - \frac{x}{y})dy = 0$$
 [7M]

(b) The number N of bacteria in a culture grew at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What was the value of N after  $1\frac{1}{2}$  hours? [7M]

# $\mathbf{Unit} - \mathbf{IV}$

7. (a) Solve  $(D^3 - 4D^2 - D + 4)y = e^{3x}cos2x.$  [7M]

(b) Solve 
$$y^{11} + 6y^1 + 9y = 0$$
,  $y(0) = -4$ ,  $y^1(0) = 14$ . [7M]

- 8. (a) Solve  $(D^2 + 3D + 2) y = 2\cos(2x + 3) + 2e^x + x^2$ . [7M]
  - (b) Solve  $(D^2 + 1)y = \sin x \sin 2x + e^x x^2$ . [7M]

### $\mathbf{Unit} - \mathbf{V}$

- 9. (a) Verify mean value Theorem for  $f(x) = e^x$ . and  $g(x) = e^{-x}$  in [a, b]. [7M]
  - (b) Find the maximum and minimum values of  $f(x, y) = x^3 + 3xy^2 3x^2 3y^2 + 4.$  [7M]

10. (a) If 
$$u = Tan^{-1} \left[ \frac{2xy}{x^2 - y^2} \right]$$
, prove that  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$  [7M]

(b) Show that the functions u = x + y + z,  $v = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$  and  $w = x^3 + y^3 + z^3 - 3xyz$ are functionally related. [7M]