INSTITUTE OF AERONAUTICAL ENGINEERING

(AUTONOMOUS)

Code No: **BCC002**

MODEL QUESTION PAPER-II

I M.Tech I Semester Regular Examinations, February 2017

NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS

(CAD/CAM)

Time: 3 hours Max. Marks: 70

Answer ONE Question from each unit

All Questions Carry Equal Marks

All parts of the question must be answered in one place only

UNIT-I

1 (a) Summarize the advantages and disadvantages of finite difference and finite element method. [7M]

(b) Solve by Crank Nicolson method the partial differential equation [7M]

$\frac{\partial u}{\partial t} = x\frac{\partial^2 u}{\partial^2 x}; 0 < x < 1, t > 0$ Subject to the conditions $u = 0; x = 0, t > 0$,

$\frac{\partial u}{\partial x} = \frac{-1}{2}u; x = 1, t > 0$,

$u = x(1-x); t = 0 \& 0 \le x \le 1$ by taking h=0..

2. (a) Distinguish between the explicit finite difference approximations to one dimensional equation to implicit finite difference method. [7M]

(b) Solve the parabolic partial differential equation by numerical method [7M]

$$\frac{\partial u}{\partial y} - \frac{\partial^2 u}{\partial^2 x} = 0; 0 < x < 4, 0 < t$$

Subject to the conditions $u(0, y) = 10, u(x, 0) = 0, u(4, y) = y$ taking h=k=1.

UNIT-II

3. (a) Explain alternate direction implicit method and also stability analysis by matrix method. [7M]

(b) Summarize about Von Newmann fully implicit stability analysis partial differential equation. [7M]

4. (a) Explain the meanings of the concepts of consistency, stability, and convergence of numerical methods. [7M]

(b) Explain Stability analysis of implicit methods and describe the types of errors. [7M]

UNIT-III

5. (a) Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; 0 \le x \le 1, t \ge 0$ using implicit method given that [7M]

$$u(0,t) = 0; u(1,t) = 0; u(x,0) = \sin \pi x; \frac{\partial u(x,0)}{\partial t} = 0 \quad h = 0.2, k = 0.1$$

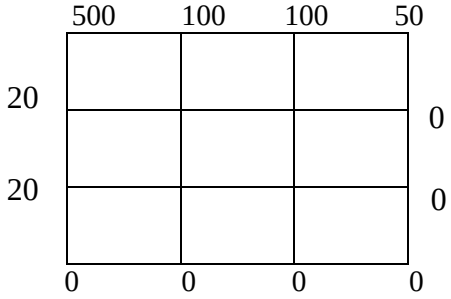
(b) Summarize explicit method for solving hyperbolic partial differential equation. [7M]

6. (a) Prove there are no explicit, unconditionally stable, consistent finite difference schemes for hyperbolic systems of partial differential equations. [7M]

(b) Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}; 0 \le x \le 1, t \ge 0$ subject to the following conditions $u = x^2 + xt^2$, along the initial line t=0 by using the method of characteristics find the solution between the grid points x =0.1 and x =0.2 [7M]

UNIT-IV

7. (a) Solve the elliptic equation at the nodal points of the following square grid using the boundary values indicated [7M]

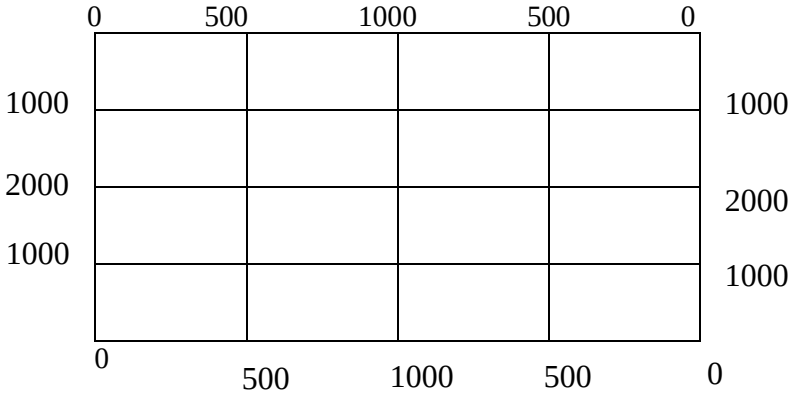


(b) Solve $u_{xx} + u_{yy} = -81xy, 0 < x < 1, 0 < y < 1$ given that [7M]

$$u(0,y) = 0, u(x,0) = 0, u(1,y) = 100, u(x,1) = 0 \text{ and } \boldsymbol{h} = \frac{1}{3}.$$

8. (a) Solve the Laplace equation $u_{xx} + u_{yy} = 0$. [7M]

for the following square region having the boundary conditions



(b) Solve $u_{xx}+u_{yy}=8x^2y^2$ for the square mesh with $u(x,y)=0$ on the boundary and mesh length=1. [7M]

UNIT-V

9. (a) Discuss different steps involved in finite difference approach. [7M]

(b) Solve the boundary value problem $y''+2=0, 0<x<1, y(0)=y(1)=0$ by stones implicit method. [7M]

10 (a) Explain weighted residual method with an example. [7M]

(b) Solve the boundary value problem $y''-y+x=0, 0<x<1, y(0)=0, y(1)=1$ by Galerkin method. [7M]