

**LECTURE NOTES**  
**ON**  
**REINFORCED CONCRETE STRUCTURES**  
**DESIGN AND DRAWING**  
**(ACE009)**

**III B. Tech I semester**  
**(Regulation- R16)**

Mr. Gude Ramakrishna  
Associate Professor



**DEPARTMENT OF CIVIL ENGINEERING**  
**INSTITUTE OF AERONAUTICAL ENGINEERING**  
**(Autonomous)**

DUNDIGAL, HYDERABAD - 500 043

## **UNIT-1**

### **DESIGN OF BEAMS**

#### **1.1 Introduction**

A structure refers to a system of connected parts used to support forces (loads). Buildings, bridges and towers are examples for structures in civil engineering. In buildings, structure consists of walls floors, roofs and foundation. In bridges, the structure consists of deck, supporting systems and foundations. In towers the structure consists of vertical, horizontal and diagonal members along with foundation.

A structure can be broadly classified as (i) sub structure and (ii) super structure. The portion of building below ground level is known as sub-structure and portion above the ground is called as super structure. Foundation is sub structure and plinth, walls, columns, floor slabs with or without beams, stairs, roof slabs with or without beams etc are super-structure. Many naturally occurring substances, such as clay, sand, wood, rocks natural fibers are used to construct buildings. Apart from this many manmade products are in use for building construction. Bricks, tiles, cement concrete, concrete blocks, plastic, steel & glass etc are manmade building materials.

#### **1.2 Objectives**

1. To understand various design philosophies.
2. To understand the necessity of reinforcement in RC structure.
3. To understand the stress block parameter of RC beam section.
4. To understand the necessity of partial safety in design of RC member.

#### **1.3 Advantages Disadvantages of RC members**

##### **Advantages**

- It has high tensile and compressive strength.
- It is more durable and may long up to 100 years.
- It imparts ductility.
- Raw materials used for construction of RC buildings are easily available and can be transported.
- Overall cost for constructing a building using RC proves to be economical compared to steel and pre-stressed structures.
- RC components can be moulded to any desired shape , if formwork is designed properly.

- If RC structures are properly designed then it can resist the earthquake forces.

### Disadvantage

- Tensile strength of RC member is about  $1/10^{\text{th}}$  of its compressive strength **1.4**

### Materials required for RC member

#### a. Concrete

Concrete is a product obtained artificially by hardening of the mixture of cement, sand, gravel and water in predetermined proportions. Depending on the quality and proportions of the ingredients used in the mix the properties of concrete vary almost as widely as different kinds of stones. Concrete has enough strength in compression, but has little strength in tension. Due to this, concrete is weak in bending, shear and torsion. Hence the use of plain concrete is limited applications where great compressive strength and weight are the principal requirements and where tensile stresses are either totally absent or are extremely low.

### Properties of Concrete

#### 1. Grade of concrete

|             |     |
|-------------|-----|
| Mild        | M20 |
| Moderate    | M25 |
| Severe      | M30 |
| Very Severe | M35 |
| Extreme     | M40 |

#### 2. Tensile strength $F_{cr}$

$$= 0.7 * \sqrt{f_{ck}}$$

#### 3. Modulus of elasticity $E_c =$

$$5000 * \sqrt{f_{ck}}$$

#### 4. Shrinkage of

**concrete:** Depends on

- Constituents of concrete
- Size of the member
- Environmental conditions

**5. Creep of concrete:** Depends on

- Strength of the concrete
- Stress in concrete
- Duration of loading

**6. Durability:** Mainly depends on

- Type of Environment
- Cement content
- Water cement ratio
- Workmanship
- Cover to the reinforcement

**7. Cover to the reinforcement**

Nominal cover is essential

- Resist corrosion
- Bonding between steel and concrete

**b) Reinforcements**

- ❑ Bamboo, natural fibers (jute, coir etc) and steel are some of the types of reinforcements

**Roles of reinforcement in RCC**

- To resist Bending moment in case of flexural members
- To reduce the shrinkage of concrete
- To improve the load carrying capacity of the compression member
- To resist the effect of secondary stresses like temperature etc.
- To prevent the development of wider cracks formed due to tensile stress

**Advantages of Steel Reinforcement**

- It has high tensile and compressive stress
- It is ductile in nature
- It has longer life
- It allows easy fabrication ( easy to cut, bend or weld)
- It is easily available
- It has low co-efficient of thermal expansion same as that of concrete

## Disadvantages of Steel Reinforcement

- More prone to corrosion
- Loses its strength when exposed to high temperature.



## Classification of Steel bars

### 1. Mild Steel plain bars

- Cold worked steel bars
- Hot rolled mild steel bars

Eg: Fe250

### 2. High Yield Strength Deformed (HYSD) Bars

Eg: Fe415 & Fe500

### 3. Steel wire Fabric 4. Structural Steel 5. CRS and TMT



**8mm-10mm** size bars are used in Slabs and Stair ups, which serves as a load bearing member in slab homes.

**12mm-25mm** size bars are used in Beams & Columns, to make them withstand external loads.

**32mm-36mm** size bars are used in the construction of very complex projects like dams, bridges.

Based on the designs also, we go for the sizes. Sometimes, we use different sizes according to the project specifications.

### Stress-strain curves for reinforcement

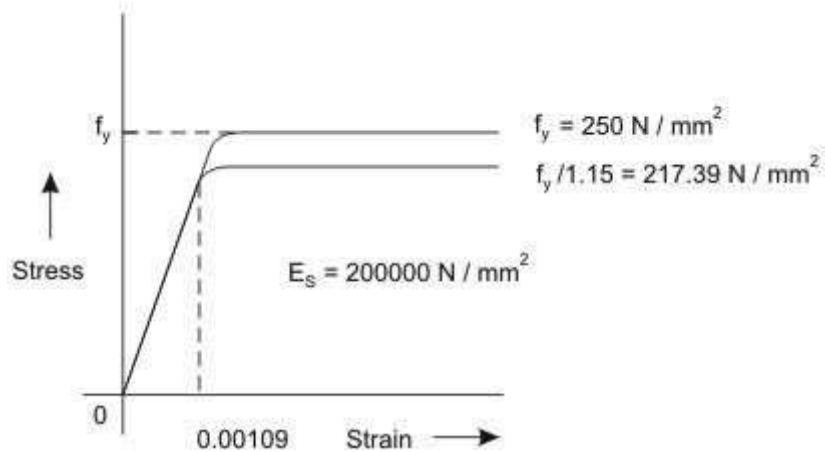
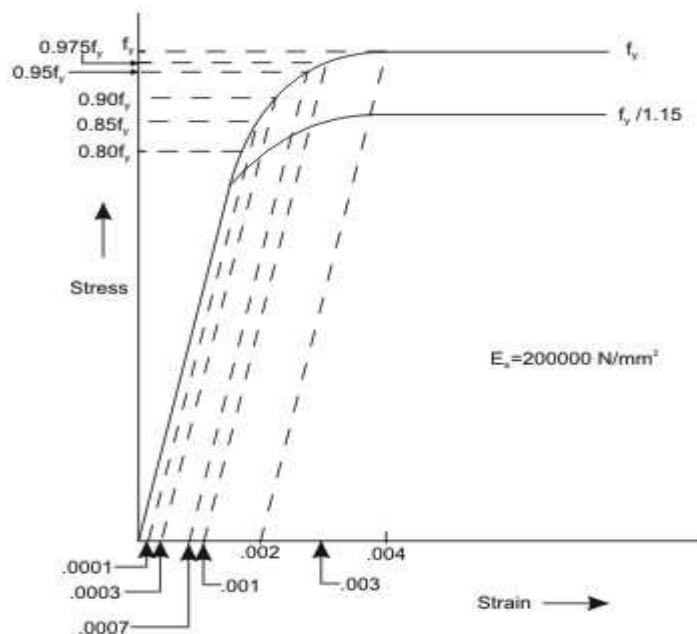


Figure 1.2: Stress-strain curve for Mild steel (idealised) (Fe 250) with definite yield point

Figure 1.3: Stress-strain curve for cold worked deform bar

Figures 1.2 and 1.3 show the representative stress-strain curves for steel having definite yield point and not having definite yield point, respectively. The characteristic yield strength  $f_y$  of steel is assumed as the minimum yield stress or 0.2 per cent of proof stress for steel having no



2

definite yield point. The modulus of elasticity of steel is taken to be 200000 N/mm .

For mild steel, the stress is proportional to the strain up to the yield point. Thereafter, post yield strain increases faster while the stress is assumed to remain at constant value of  $f_y$ .

For cold-worked bars (Fig. 1.3), the stress is proportional to the strain up to a stress of  $0.8 f_y$ .

Thereafter, the inelastic curve is defined as given below:

| Stress      | Inelastic strain |
|-------------|------------------|
| $0.80 f_y$  | Nil              |
| $0.85 f_y$  | 0.0001           |
| $0.90 f_y$  | 0.0003           |
| $0.95 f_y$  | 0.0007           |
| $0.975 f_y$ | 0.0010           |
| $1.00 f_y$  | 0.0020           |

Linear interpolation is to be done for intermediate values. The two grades of cold-worked bars used as steel reinforcement are Fe 415 and Fe 500 with the values of  $f_y$  as 415 N/mm<sup>2</sup> and 500 N/mm<sup>2</sup>, respectively.

### 1.5 Introduction to RCC Design

#### Objective:-

1. Structure should perform satisfactorily during its life span
2. Structure should take up the forces which are likely and deform within the limit
3. The structure should resist misuse or fire.

#### Design of RC member involves

1. Deciding the size or dimension of the structural element and amount of reinforcement required.
2. To check whether the adopted size perform satisfactorily during its life span.

### 1.6 Methods of Design or Design philosophy

1. Working stress method
2. Ultimate or load factor method
3. Limit state method

#### Working Stress Method – Based on Elastic theory

##### Assumptions:-

- Plane section remains plane before and after deformation takes place
- Stress –strain relation under working load, is linear for both steel and concrete
- Tensile stress is taken care by reinforcement and none of them by concrete ➤ Modular Ratio between steel and concrete remains constant.

#### Modular ratio

$$m = \frac{E_s}{E_c} = \frac{280}{\sigma_{cbc}}$$

Where  $\sigma_{cbc}$  = is permissible stress

**Advantages:**

1. Method is simple
2. Method is reliable
3. Stress is very low under working condition , therefore serviceability is automatically satisfied

**Limitations:-**

1. Stress strain relation for concrete is not linear for concrete
2. It gives an idea that failure load = working load \* factor of safety, but it is not true
3. This method gives uneconomical section

**. Ultimate load method or Load factor method**

- 
- This method uses design load = ultimate load \* load factor
  - Load factor =  $\frac{\text{Collapse Load}}{\text{Working Load}}$
  -

This method gives slender and thin section which results in excessive deflection and cracks

- This method does not take care of shrinkage of concrete
- This method does not take of serviceability

**Limit State Method**

Limit state is an acceptable limit for both safety and serviceability before which failure occurs

1. Limit state of collapse
2. Limit state of serviceability
3. Other limit state

**Limit state of Collapse The structure may get collapse because of**

- Rupture at one or more cross-sections
- Buckling
- Overturning

While designing the structure following ultimate stresses should be considered

1. Flexure
2. Shear
3. Torsion
4. Tension
5. Compression



### **Limit state of Serviceability a) Limit state of deflection**

- Lack of safety
- Appearance
- Ponding of water
- Misalignment in machines
- Door, window frames, flooring materials undergoes crack **Methods for**

#### **controlling deflecting**

- Empirical formula – span/depth
- Theoretical - dimension

### **b) Limit state of cracking**

- Appearance
- Lack of safety
- Leakage
- Creation of maintenance problem
- Reduction in stiffness with increase in deflection
- corrosion

### **Other Limit states**

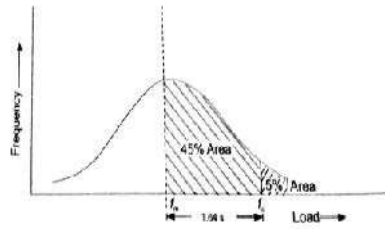
- a) Vibration
- b) Fire resistance
- c) Chemical and environmental actions
- d) Accidental loads

### **1.7 Types of Loads on RCC Structures**

1. Dead Load IS 875 (Part 1 )1987
2. Live Load IS 875 (Part 2 )1987
3. Wind Load IS 875 (Part 3 )1987
4. Snow Load IS 875 (Part 4 )1987
5. Earthquake Load IS 1893 2002
  - Low intensity Zone (IV or less) – Zone II
  - Moderate intensity Zone (VII) – Zone III
  - Severe intensity Zone (VIII) – Zone IV
  - Very Severe intensity Zone (IX and above ) – Zone V

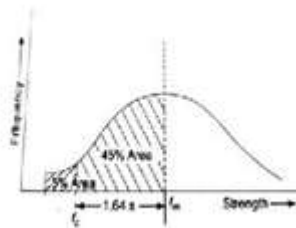
### 1.8 Characteristic load

Characteristic load = Mean Load + 1.64S



### Characteristic Strength

Characteristic Strength = Mean Strength - 1.64S



### 1.9 Partial Safety factor

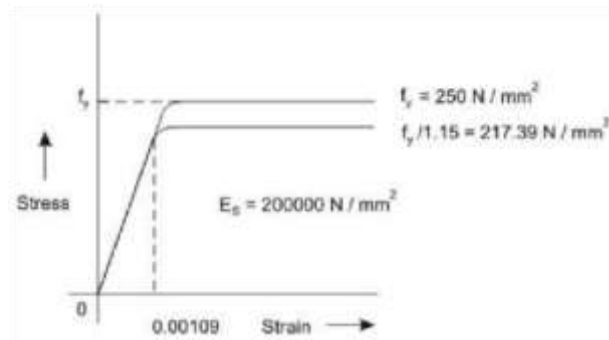
2. For material

$$\gamma_m = \frac{F}{F_{m,k}}$$

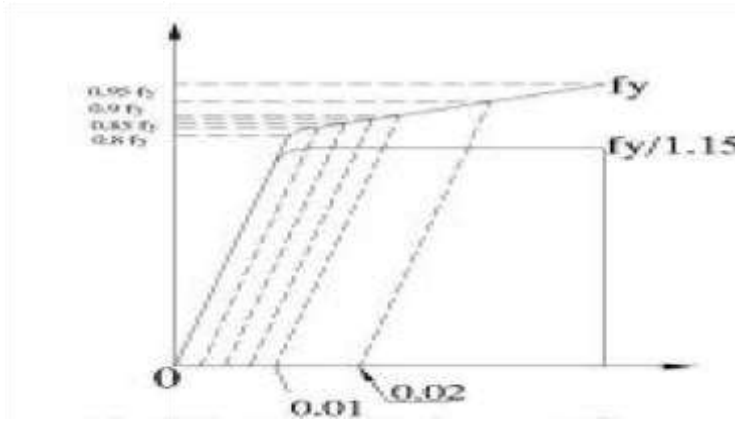
3. For load

$$F_d = F * \gamma_f$$

### 1.10 Stress-strain curves for reinforcement



Stress-strain curve for Mild steel (idealized) (Fe 250) with definite yield point



**Stress-strain curve for cold worked deform bar**

Figures show the representative stress-strain curves for steel having definite yield point and not having definite yield point, respectively. The characteristic yield strength  $f_y$  of steel is assumed as the  $m_i$  Method of RCC design

A reinforced concrete structure should be designed to satisfy the following criteria-

- i) Adequate safety, in items stiffness and durability
- iii) Reasonable economy.

The following design methods are used for the design of RCC Structures.

- a) The working stress method (WSM)
- b) The ultimate load method (ULM)
- c) The limit state method (LSM)

### **Working Stress Method (WSM)**

This method is based on linear elastic theory or the classical elastic theory. This method ensured adequate safety by suitably restricting the stress in the materials (i.e. concrete and steel) induced by the expected working loads on the structures. The assumption of linear elastic behaviour considered justifiable since the specified permissible stresses are kept well below the ultimate strength of the material. The ratio of yield stress of the steel reinforcement or the cube strength of the concrete to the corresponding permissible or working stress is usually called factor of safety.

The WSM uses a factor of safety of about 3 with respect to the cube strength of concrete and a factor of safety of about 1.8 with respect to the yield strength of steel.

### **Ultimate load method (ULM)**

The method is based on the ultimate strength of reinforced concrete at ultimate load is obtained by enhancing the service load by some factor called as load factor for giving a desired margin of safety .Hence the method is also referred to as the load factor method or the ultimate strength method.

In the ULM, stress condition at the state of in pending collapse of the structure is analysed, thus using, the non-linear stress – strain curves of concrete and steel. The safely measure in the design is obtained by the use of proper load factor. The satisfactory strength performance at ultimate loads does not guarantee satisfactory strength performance at ultimate loads does not guarantee satisfactory serviceability performance at normal service loads.

### **Limit state method (LSM)**

Limit states are the acceptable limits for the safety and serviceability requirements of the structure before failure occurs. The design of structures by this method will thus ensure that they will not reach limit states and will not become unfit for the use for which they are intended. It is worth mentioning that structures will not just fail or collapse by violating (exceeding) the limit states. Failure, therefore, implies that clearly defined limit states of structural usefulness has been exceeded.

Limit state are two types

- i) Limit state of collapse
- ii) Limit state of serviceability.

### **Limit states of collapse**

The limit state of collapse of the structure or part of the structure could be assessed from rupture of one or more critical sections and from buckling due to elastic bending, shear, torsion and axial loads at every section shall not be less than the appropriate value at that section produced by the probable most unfavourable combination of loads on the structure using the appropriate factor of safely.

### **Limit state of serviceability**

Limit state of serviceability deals with deflection and crocking of structures under service loads, durability under working environment during their anticipated exposure conditions during service, stability of structures as a whole, fire resistance etc.

## Characteristic and design values and partial safety factor

### 1. Characteristic strength of materials.

The term 'characteristic strength' means that value of the strength of material below which not more than minimum acceptable percentage of test results are expected to fall. IS 456:2000 have accepted the minimum acceptable percentage as 5% for reinforced concrete structures. This means that there is 5% probability or chance of the actual strength being less than the characteristic strength.

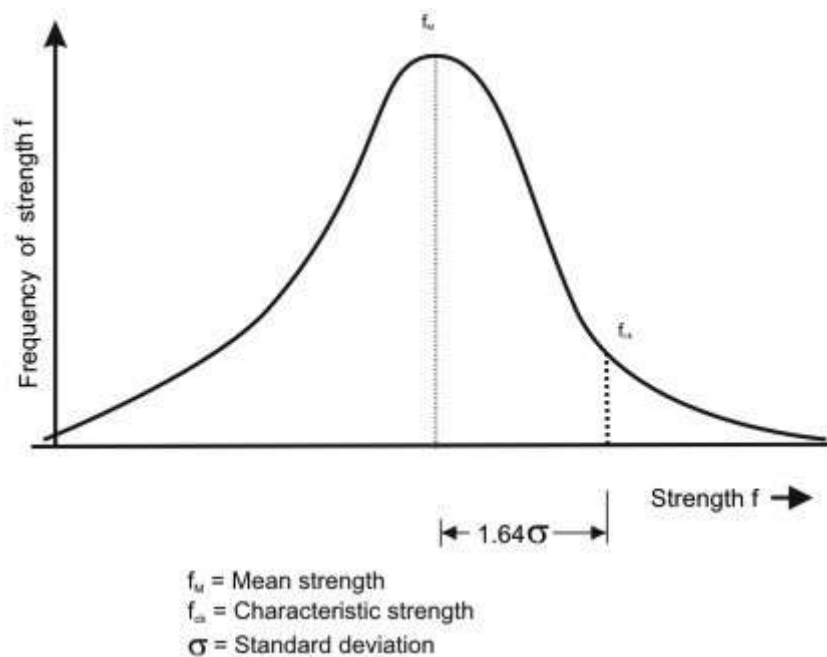


Figure 1.4: Frequency distribution curve for strength

Figure shows frequency distribution curve of strength material (concrete or steel). The value of K corresponding to 5% area of the curve is 1.65.

The design strength should be lower than the mean strength ( $f_m$ )  
Characteristic strength = Mean strength - K x standard deviation or  
 $f_k = f_m - K S_d$

Where,  $f_k$  = characteristic strength of the material

$f_m$  = mean strength

K = constant = 1.65

$S_d$  = standard deviation for a set of test results.

The value of standard deviation ( $S_d$ ) is given by

$$S_d = \sqrt{\frac{\sum \delta^2}{n-1}}$$

Where,  $\delta$  = deviation of the individual test strength from the average or mean strength of  $n$  samples.

$n$  = number of test results.

IS 456:2000 has recommended minimum value of  $n=30$ .

### Characteristic strength of concrete

Characteristic strength of concrete is denoted by  $f_{ck}$  ( $N/mm^2$ ) and its value is different for different grades of concrete e.g. M 15, M25 etc. In the symbol 'M' used for designation of concrete mix, refers to the mix and the number refers to the specified characteristic compressive strength of 150 mm size cube at 28 days expressed in  $N/mm^2$

### Characteristic strength of steel

Until the relevant Indian Standard specification for reinforcing steel are modified to include the concept of characteristic strength, the characteristic value shall be assumed as the minimum yield stress or 0.2% proof stress specified in the relevant Indian Standard specification. The characteristic strength of steel designated by symbol  $f_y$  ( $N/mm^2$ )

### Characteristic loads

The term 'Characteristic load' means that values of load which has a 95% probability of not being exceeded during that life of the structure.

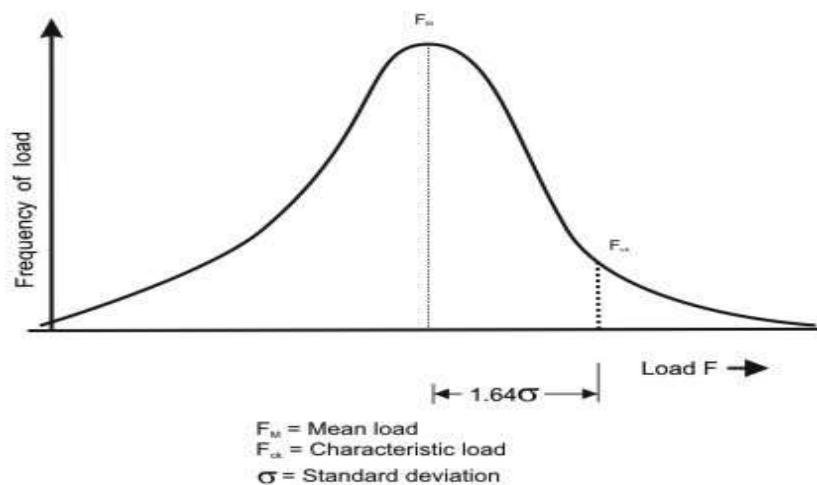


Figure 1.5: Frequency distribution curve for load

The design load should be more than average load obtained from statistic, we have

$$F_k = F_m + K S_d$$

Where,  $F_k$  = characteristic load;

$F_m$  = mean load

$K$  = constant = 2.65;

$S_d$  = standard deviation for the load.

Since data are not available to express loads in statistical terms, for the purpose of this standard, dead loads given in IS 875(Part-1), imposed loads given in IS 875(Part-2), wind loads. Given in IS 875 (Part-3), snow load as given in IS 875(Part-4) and seismic forces given in IS 1893 shall be assumed as the characteristic loads.

### **Design strength of materials**

The design strength of materials ( $f_d$ ) is given by

$$f_d = \frac{f_k}{\gamma_m}$$

Where,  $f_k$  = characteristic strength of material.

$\gamma_m$  = partial safety factor appropriate to the material and the limit state being considered

### **Design loads**

The design load ( $F_d$ ) is given by.

$$F_d = F_k \cdot \gamma_f$$

$\gamma_f$  = partial safety factor appropriate to the nature of loading and the limit state being considered.

The design load obtained by multi plying the characteristic load by the partial safety factor for load is also known as factored load.

### **Partial safety factor ( $\gamma_m$ ) for materials**

When assessing the strength of a structure or structural member for the limit state of collapse, the values of partial safety factor,  $\gamma_m$  should be taken as 1.15 for steel.

Thus, in the limit state method , the design stress for steel reinforcement is given by  $f_y / \gamma_{ms} = f_y / 1.15 = 0.87f_y$ .

According to IS 456:2000 for design purpose the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength of concrete in cube and partial safety factor  $\gamma_{mc} = 1.5$  shall be applied in addition to this. Thus, the design stress in concrete is given by

$$0.67 f_{ck} / \gamma_{mc} = 0.67 f_{ck} / 1.5 = 0.446 f_{ck}$$

### Partial safety factor for loads

The partial safety factors for loads, as per IS 456:2000 are given in table below

| Load combination | Limit State of collapse |     |       | Limit State of Serviceability |     |       |
|------------------|-------------------------|-----|-------|-------------------------------|-----|-------|
|                  | DL                      | LL  | WL/EL | DL                            | LL  | WL/EL |
| DL+IL            | 1.5                     | 1.5 | -     | 1.0                           | 1.0 | -     |
| DL+WL            | 1.5 or 0.9*             | -   | 1.5   | 1.0                           | -   | 1.0   |
| DL+IL+WL         | 1.2                     | 1.2 | 1.2   | 1.0                           | 0.8 | 0.8   |

(\* This value is to be considered when stability against overturning or stress reversal is critical)

### Limit state of collapse in flexure

The behaviour of reinforced concrete beam sections at ultimate loads has been explained in detail in previous section. The basic assumptions involved in the analysis at the ultimate limit state of flexure (Cl. 38.1 of the Code) are listed here.

- Plane sections normal to the beam axis remain plane after bending, i.e., in an initially straight beam, strain varies linearly over the depth of the section.
- The maximum compressive strain in concrete (at the outermost fibre)  $\epsilon_{cu}$  shall be taken as 0.0035 in bending.
- The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test. An acceptable stress-strain curve is given below in figure 1.6. For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor  $\gamma_c = 1.5$  shall be applied in addition to this.



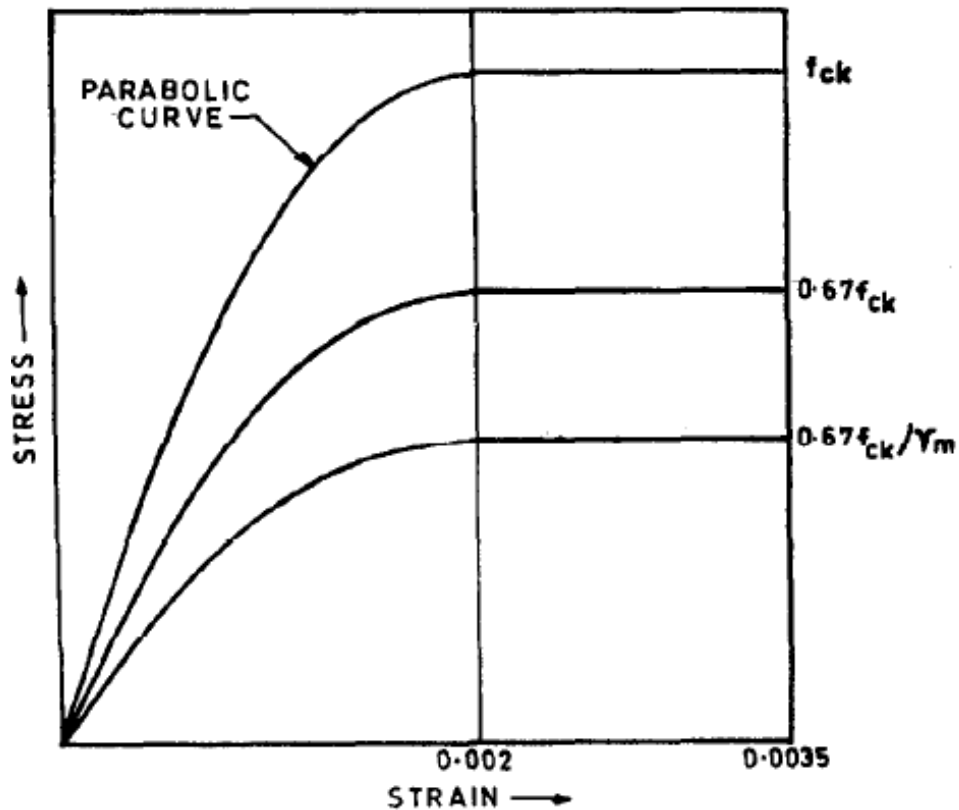


Figure 1.6 Stress-strain curve for concrete

d) The tensile strength of the concrete is ignored.

e) The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. Typical curves are given in figure 1.3. For design purposes the partial safety factor  $\gamma_m$  equal to 1.15 shall be applied.

f) The minimum yield stress or 0.2 per cent of proof stress for steel having no definite yield point. The modulus of elasticity of steel is taken to be  $200000 \text{ N/mm}^2$

g) For mild steel, the stress is proportional to the strain up to the yield point. Thereafter, post yield strain increases faster while the stress is assumed to remain at constant value of  $f_y$

a) The maximum strain in the tension reinforcement in the section at failure shall not be less

Than : 
$$\frac{f_y}{1.15E_s} + 0.002$$

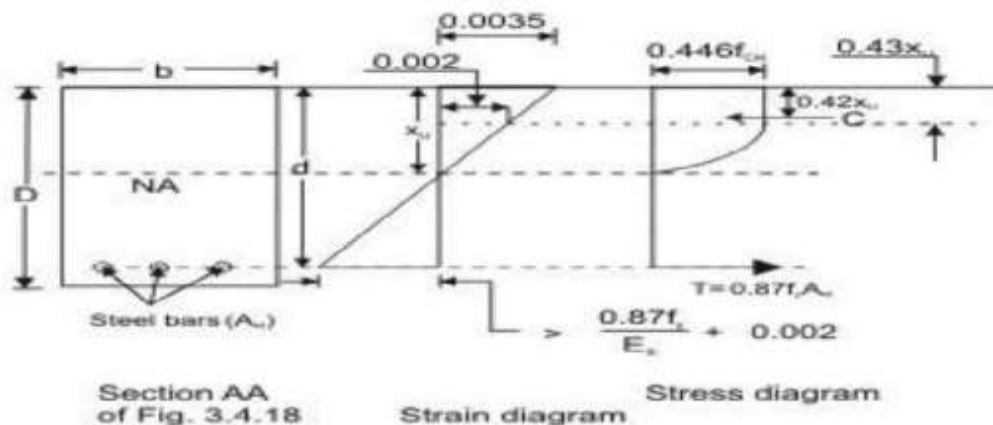
### 1.11 Limit state of collapse in flexure

The behaviour of reinforced concrete beam sections at ultimate loads has been explained in detail in previous section. The basic assumptions involved in the analysis at the ultimate limit state of flexure (Cl. 38.1 of the Code) are listed here.

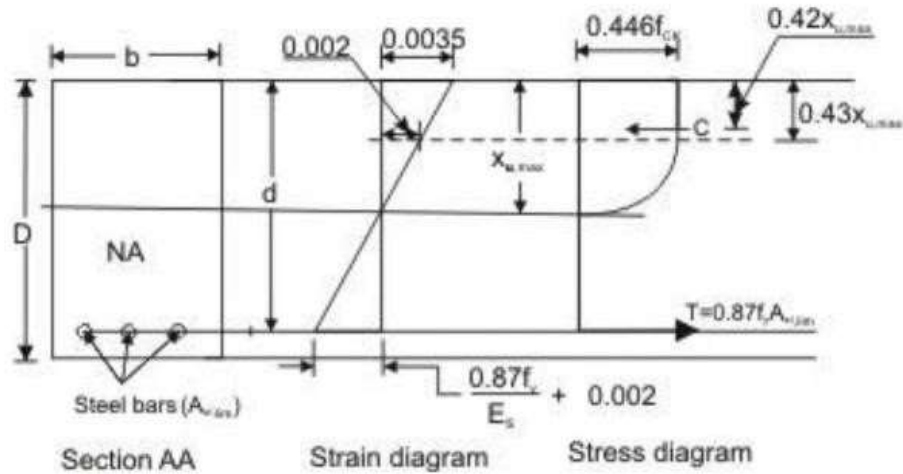
- Plane sections normal to the beam axis remain plane after bending, i.e., in an initially straight beam, strain varies linearly over the depth of the section.
- The maximum compressive strain in concrete (at the outermost fibre)  $\epsilon_{cu}$  shall be taken as 0.0035 in bending.
- The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test. An acceptable stress-strain curve is given below in figure 1.6. For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor  $\gamma_c = 1.5$  shall be applied in addition to this.
- The tensile strength of the concrete is ignored.
- The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. Typical curves are given in figure 1.3. For design purposes the partial safety factor equal to 1.15 shall be applied.
- The maximum strain in the tension reinforcement in the section at failure shall not be

less than 
$$\frac{f_y}{1.15f_{yk}} + 0.002$$

### 1.12 Limiting Depth of Neutral Axis



**Rectangular beam under flexure  $x_u < x_{u,max}$**



### Rectangular beam under flexure $x_u = x_{u,max}$

Based on the assumption given above, an expression for the depth of the neutral axis at the ultimate limit state,  $x_u$ , can be easily obtained from the strain diagram in Fig

Considering similar triangles,

$$\frac{x_u}{d} = \frac{0.0035}{0.0035 + \frac{0.87f_y}{E_s} + 0.002} \quad (1)$$

According to IS 456:2000 cl no 38.1 (f), when the maximum strain in tension reinforcement is equal to  $\frac{0.87f_y}{E_s} + 0.002$ , then the value of neutral axis will be  $x_{u,max}$ .

$$\text{Therefore, } \frac{x_{u,max}}{d} = \frac{0.0035}{0.0035 + \frac{0.87f_y}{E_s} + 0.002} \quad (2)$$

The values of  $x_{u,max}$  for different grades of steel, obtained by applying Eq. (2), are listed in table.

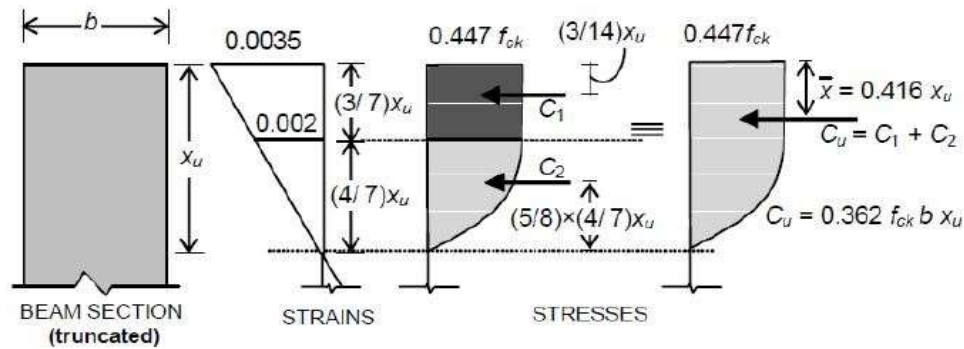
#### Limiting depth of neutral axis for different grades of steel

| Steel Grade     | Fe 250 | Fe 415 | Fe 500 |
|-----------------|--------|--------|--------|
| $x_{u,max} / d$ | 0.5313 | 0.4791 | 0.4791 |

The limiting depth of neutral axis  $x_{u,max}$  corresponds to the so-called balanced section, i.e., a section that is expected to result in a balanced failure at the ultimate limit state in flexure. If the neutral axis depth  $x_u$  is less than  $x_{u,max}$ , then the section is under-reinforced (resulting in a tension failure); whereas if  $x_u$  exceeds  $x_{u,max}$ , it is over-reinforced (resulting in a compression failure).

### 1.13 Analysis of Singly Reinforced Rectangular Sections

Analysis of a given reinforced concrete section at the ultimate limit state of flexure implies the determination of the *ultimate moment MR of resistance* of the section. This is easily obtained from the couple resulting from the flexural stresses



#### Concrete stress-block parameters in compression

$$M_R \approx C * z \approx T * z$$

where  $C$  and  $T$  are the resultant (ultimate) forces in compression and tension respectively and  $z$  is the lever arm.

$$T \approx 0.87 f_y A_{st}$$

#### Concrete Stress Block in Compression

In order to determine the magnitude of  $C_u$  and its line of action, it is necessary to analyze the concrete stress block in compression. As ultimate failure of a reinforced concrete beam in flexure occurs by the crushing of concrete, for both under- and over-reinforced beams, the shape of the compressive stress distribution ('stress block') at failure will be, in both cases, as shown in Fig. The value of  $C_u$  can be computed knowing that the compressive stress in concrete is uniform at  $0.447 f_{ck}$  for a depth of  $3x_u/7$ , and below this it varies parabolically over a depth of  $4x_u/7$  to zero at the neutral axis.

For a rectangular section of width  $b$ ,

$$C_u = 0.447 f_{ck} b \left[ \frac{3x_u}{7} + \left( \frac{2}{3} x \frac{4x_u}{7} \right) \right]$$

Therefore,  $C_u = 0.36 * f_{ck} * b * x_u$

Also, the line of action of  $C_u$  is determined by the centroid of the stress block, located at a distance  $x$  from the concrete fibres subjected to the maximum compressive strain.

Accordingly, considering moments of compressive forces  $C_u$ ,  $C_1$  and  $C_2$  about the maximum compressive strain location,

$$(0.362 f_{ck} b x_u) \bar{x} - (0.117 f_{ck} b x_u) \left[ \left( \frac{3}{7} \right) \left( \frac{1.5 x_u}{7} \right) + \left( \frac{2}{3} \times \frac{4}{7} \right) \left( x_u - \frac{5}{8} \times \frac{4 x_u}{7} \right) \right]$$

Solving  $\bar{x} = 0.416 x_u$

### Depth of Neutral Axis

For any given section, the depth of the neutral axis should be such that  $C_u = T$ , satisfying equilibrium of forces.

Equating  $C = T$ ,

$$x_u - \frac{0.87 f_y A_{st}}{0.361 f_{ck} b}, \text{ valid only if resulting } x_u \leq x_{u,\max}$$

### 1.14 Ultimate Moment of Resistance

The ultimate moment of resistance MR of a given beam section is

Accordingly, in terms of the concrete compressive strength,

$$M_{uR} = 0.361 f_{ck} b x_u (d - 0.416 x_u) \text{ for all } x_u$$

Alternatively, in terms of the steel tensile stress,

$$M_{uR} = f_{st} A_{st} (d - 0.416 x_u) \text{ for all } x_u$$

With  $f_{st} = 0.87 f_y$  for  $x_u \leq x_{u,\max}$

#### Limiting Moment of Resistance

The *limiting moment of resistance* of a given (singly reinforced, rectangular) section, according to the Code (Cl. G-1.1), corresponds to the condition, defined by Eq. (2). From Eq. (9), it follows that:

$$M_{u,\lim} = 0.361 f_{ck} b x_{u,\max} (d - 0.416 x_{u,\max}) \quad (11)$$

$$M_{u,\lim} = 0.361 f_{ck} \left( \frac{x_{u,\max}}{d} \right) \left( 1 - \frac{0.416 x_{u,\max}}{d} \right) b d^2 \quad (11a)$$

### 1.15 Modes of failure: Types of section

A reinforced concrete member is considered to have failed when the strain of concrete in extreme compression fibre reaches its ultimate value of 0.0035. At this stage, the actual strain in steel can have the following values:

- Equal to failure strain of steel
- More than failure strain, corresponding to under reinforced section.
- Less than failure strain corresponding to over reinforced section.

Thus for a given section, the actual value of  $x_u / d$  can be determined from Eq. (7). Three cases arise.

Case-1:  $x_u / d$  equal to the limiting value  $x_{u,\max} / d$  : Balanced section.

Case-2:  $x_u / d$  less than limiting value: under-reinforced section.

Case-3:  $x_u/d$  more than limiting value: over-reinforced section.

### Balanced Section

In balanced section, the strain in steel and strain in concrete reach their maximum values simultaneously. The percentage of steel in this section is known as critical or limiting steel percentage. The depth of neutral axis (NA) is  $x_u = x_{u,max}$ .

### Under-reinforced section

An under-reinforced section is the one in which steel percentage ( $p_t$ ) is less than critical or limiting percentage ( $p_{t,lim}$ ). Due to this the actual NA is above the balanced NA and  $x_u < x_{u,max}$ .

### Over-reinforced section

In the over reinforced section the steel percentage is more than limiting percentage due to which NA falls below the balanced NA and  $x_u > x_{u,max}$ . Because of higher percentage of steel, yield does not take place in steel and failure occurs when the strain in extreme fibres in concrete reaches its ultimate value.

### 1.16 General Aspects of Serviceability:

The members are designed to withstand safely all loads liable to act on it throughout its life using the limit state of collapse. These members designed should also satisfy the serviceability limit states. To satisfy the serviceability requirements the deflections and cracking in the member should not be excessive and shall be less than the permissible values. Apart from this the other limit states are that of the durability and vibrations. Excessive values beyond this limit state spoil the appearance of the structure and affect the partition walls, flooring etc. This will cause the user discomfort and the structure is said to be unfit for use.

The different load combinations and the corresponding partial safety factors to be used for the limit state of serviceability are given in Table 18 of IS 456:2000.

| Load combination | Limit State of Collapse |    |     | Limit state of serviceability |     |     |
|------------------|-------------------------|----|-----|-------------------------------|-----|-----|
|                  | DL                      | IL | WL  | DL                            | IL  | WL  |
| DL + IL          | 1.5                     |    | 1.0 | 1.0                           | 1.0 | -   |
| DL + WL          | 1.5 or 0.9              | -  | 1.5 | 1.0                           | -   | 1.0 |
| DL + IL + WL     | 1.2                     |    |     | 1.0                           | 0.8 | 0.8 |

### Limit state of serviceability for flexural members:

#### Deflection

The check for deflection is done through the following two methods specified by IS 456:2000 (Refer clause 42.1)

## 1 Empirical Method

In this method, the deflection criteria of the member is said to be satisfied when the actual value of span to depth ratio of the member is less than the permissible values. The IS code procedure for calculating the permissible values are as given below

a. Choosing the basic values of span to effective depth ratios ( $l/d$ ) from the following, depending on the type of beam.

1. Cantilever = 8

2. Simply supported = 20

3. Continuous = 26

b. Modify the value of basic span to depth ratio to get the allowable span to depth ratio.

Allowable  $l/d$  = Basic  $l/d$  x  $M_t$  x  $M_c$  x  $M_f$

Where,  $M_t$  = Modification factor obtained from fig 4 IS 456:2000. It depends on the area of tension reinforcement provided and the type of steel.

$M_c$  = Modification factor obtained from fig 5 IS 456:2000. This depends on the area of compression steel used.

$M_f$  = Reduction factor got from fig 6 of IS 456:2000

Note: The basic values of  $l/d$  mentioned above is valid upto spans of 10m. The basic values are multiplied by  $10 / \text{span}$  in meters except for cantilever. For cantilevers whose span exceeds 10 m the theoretical method shall be used.

## 2 Theoretical method of checking deflection

The actual deflections of the members are calculated as per procedure given in annexure 'C' of IS 456:2000. This deflection value shall be limited to the following

- i. The final deflection due to all loads including the effects of temperature, creep and shrinkage shall not exceed  $\text{span} / 250$ .
- ii. The deflection including the effects of temperature, creep and shrinkage occurring after erection of partitions and the application of finishes shall not exceed  $\text{span}/350$  or 20 mm whichever is less.

## Cracking in structural members

Cracking of concrete occurs whenever the tensile stress developed is greater than the tensile strength of concrete. This happens due to large values of the following:

1. Flexural tensile stress because of excessive bending under the applied load
2. Diagonal tension due to shear and torsion.

3. Direct tensile stress under applied loads (for example hoop tension in a circular tank)
4. Lateral tensile strains accompanying high axis compressive strains due to Poisson's effect (as in a compression test)
5. Settlement of supports.

In addition to the above reasons, cracking also occurs because of

1. Restraint against volume changes due to shrinkage, temperature creep and chemical effects.
2. Bond and anchorage failures.

Cracking spoils the aesthetics of the structure and also adversely affect the durability of the structure. Presence of wide cracks exposes the reinforcement to the atmosphere due to which the reinforcements get corroded causing the deterioration of concrete. In some cases, such as liquid retaining structures and pressure vessels cracks affects the basic functional requirement itself (such as water tightness in water tank).

Permissible crack width

The permissible crack width in structural concrete members depends on the type of structure and the exposure conditions. The permissible values are prescribed in clause 35.3.2

IS 456:2000 and are shown in table below

Table: Permissible values of crack width as per IS 456:2000

| No. | Types of Exposure  | Permissible widths of crack at surface (mm) |
|-----|--|---|
| 1   | Protected and not exposed to aggressive environmental conditions | 0.3   |
| 2   | Moderate environmental conditions                                | 0.2   |

### **Control of cracking**

The check for cracking in beams are done through the following 2 methods specified in IS 456:2000 clause 43.1

#### **1. By empirical method:**

In this method, the cracking is said to be in control if proper detailing (i.e. spacing) of reinforcements as specified in clause 26.3.2 of IS 456:2000 is followed. These specifications regarding the spacing have been already discussed under heading general specifications. In addition, the following specifications shall also be considered

- i. In the beams where the depth of the web exceeds 750 mm, side face reinforcement shall be provided along the two faces. The total area of such reinforcement shall not be less



than 0.1% of the web area and shall be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness whichever is less. (Refer clause 25.5.1.3 IS456:2000)

ii. The minimum tension reinforcement in beams to prevent failure in the tension zone by cracking of concrete is given by the following

$A_s = 0.85 f_y / 0.87 f_y$  (Refer clause 26.5.1.1 IS 456:2000)

iii. Provide large number of smaller diameter bars rather than large diameter bars of the same area. This will make the bars well distributed in the tension zone and will reduce the width of the cracks.

2. By crack width computations In the case of special structures and in aggressive environmental conditions, it is preferred to compute the width of cracks and compare them with the permissible crack width to ensure the safety of the structure at the limit state of serviceability.

IS 456-2000 has specified an analytical method for the estimation of surface crack width in Annexure-F which is based on the British Code (BS : 8110) specifications where the surface crack width is less than the permissible width, the crack control is said to be satisfied.

### **Outcome**

1. Able to know various design philosophies.
2. Able to know the necessity of reinforcement in RC structure.
3. Able to know the stress block parameter of RC beam section.
4. Able to know the necessity of partial safety in design of RC member.

### **1.18 Assignment questions**

1. What are the modes of failure of singly reinforced beam?
2. What are the methods of design philosophies?
3. What is moment of resistance?
4. What are the loads that are likely to act on the structure?
5. What is singly reinforced beam?

## UNIT-2

### SHEAR, BOND AND TORSION

#### **Beam:**

Beam is a structural member which is normally placed horizontally. It provides resistance to bending when loads are applied on it. Most commonly used material for beam is RCC (Reinforced Cement Concrete). RCC beam can be various types depending on different criteria.

RCC beam can be various types depending on different criteria. Such as depending on shape, beam can be rectangular, T-beam etc. Depending on reinforcement placement, beam can be double reinforced beam, single reinforced beam, etc.

#### **Objective**

1. To design singly and doubly reinforced beam

#### **Types of RCC Beams**

RCC beams are 4 types depending on their supporting systems.

- i. Simply supported beam
- ii. Semi-continuous beam
- iii. Continuous beam, and
- iv. Cantilever beam.

#### **Simply Supported Beam**

This type of beam has a single span. It is supported by two supports at both ends. This beam is also called simple beam.

#### **Semi-Continuous Beam**

This beam doesn't have more than two spans. And supports are not more than three.

Technically this beam is a continuous beam.

#### **Continuous Beam**

This type of beam has more than two spans and has more than three supports along its length.

The supports are in one straight line thus the spans are also in a straight line.

#### **Cantilever Beam**

It has only one support in one end, another end is open.

There is another type of beam we can see in the civil engineering world which is called over-hanging beam. This beam extends beyond its supports. Actually this beam is a combination of simply supported and cantilever beam

In this chapter, it is intended to learn the method of designing the beams using the principles developed in previous chapters. Design consists of selecting proper materials, shape and size of the structural member keeping in view the economy, stability and aesthetics. The design of beams are done for the limit state of collapse and checked for the other limit states. Normally the beam is designed for flexure and checked for shear, deflection, cracking and bond.

## **Design procedure**

The procedure for the design of beam may be summarized as follows:

- i. Estimation of loads
- ii. Analysis
- iii. Design

### **i. Estimation of loads**

The loads that get realized on the beams consist of the following:

- a. Self weight of the beam.
- b. Weight of the wall constructed on the beam
- c. The portion of the slab loads which gets transferred to the beams. These slab loads are due to live loads that are acting on the slab dead loads such as self weight of the slab, floor finishes, partitions, false ceiling and some special fixed loads. The economy and safety of the beams achieved depends on the accuracy with which the loads are estimated.

The dead loads are calculated based on the density whereas the live loads are taken from IS:875 depending on the functional use of the building.

## **2. Analysis**

For the loads that are acting on the beams, the analysis is done by any standard method to obtain the shear forces and bending moments.

## **3. Design**

- a. Selection of width and depth of the beam.

The width of the beam selected shall satisfy the slenderness limits specified in IS 456 : 2000 clause 23.3 to ensure the lateral stability.

- b. Calculation of effective span ( $l_e$ ) (Refer clause 22.2, IS 456:2000)
- c. Calculation of loads ( $w$ )
- d. Calculation of critical moments and shears.
- e. The moment and shear that exists at the critical sections are considered for the design.
- f. Check for the depth based on maximum bending moment.

Considering the section to be nearly balanced section and using the equation

Annexure G, IS 456-2000 obtain the value of the required depth  $d_{required}$ . If the assumed depth " $d$ " is greater than the " $d_{required}$ ", it satisfies the depth criteria based on flexure. If the assumed section is less than the " $d_{required}$ ", revise the section.

- g. Calculation of steel.

As the section is under reinforced, use the equation G.1.1.(b) to obtain the steel.

- h. Check for shear.
- i. Check for developmental length.
- j. Check for deflection.

k. Check for  $A_{st \text{ min}}$ ,  $A_{st \text{ max}}$  and distance between the two bars.

### **Anchorage of bars or check for development length**

In accordance with clause 26.2 IS 456: 2000, the bars shall be extended (or anchored) for a certain distance on either side of the point of maximum bending moment where there is maximum stress (Tension or Compression). This distance is known as the development length and is required in order to prevent the bar from pulling out under tension or pushing in under compression. The development length ( $L_d$ ) is given by

$$L_d = \frac{\phi \sigma_s}{4 Z_{bd}}$$

where,  $\phi$  = Nominal diameter of the bar

$\sigma_s$  – Stress in bar at the section considered at design load

$Z_{bd}$  – Design bond stress given in table 26.2.1.1 (IS 456 : 2000)

Table 26.2.1.1: Design bond stress in limit state method for plain bars in tension shall be as below:

| Grade of concrete                                   | M 20 | M 25 | M 30 | M 35 | M 40 and above |
|---|------|------|------|------|----------------|
| Design bond stress<br>$\tau_{bd}$ N/mm <sup>2</sup> | 1.2  | 1.4  | 1.5  | 1.7  | 1.9            |

Note: Due to the above requirement it can be concluded that no bar can be bent up or curtailed upto a distance of development length from the point of maximum moment.

Due to practical difficulties if it is not possible to provide the required embedment or development length, bends hooks and mechanical anchorages are used.

Flexural reinforcement shall not be terminated in a tension zone unless any one of the following condition is satisfied:

- The shear at the cut-off points does not exceed two-thirds that permitted, including the shear strength of web reinforcement provided.
- Stirrup area in excess of that required for shear and torsion is provided along each terminated bar over a distance from the cut-off point equal to three-fourths the effective depth of the member. The excess stirrup area shall be not less than  $0.4bs/f_y$ , where  $b$  is the breadth of the beam,  $s$  is the spacing and  $f_y$  is the characteristic strength of reinforcement in N/mm<sup>2</sup>. The resulting spacing shall not exceed  $d/8$  where is the ratio of the area of bars cut-off to the total area of bars at the section, and  $d$  is the effective depth.
- For 36 mm and smaller bars, the continuing bars provide double the area required for flexure at the cut-off point and the shear does not exceed three-fourths that permitted.

**Positive moment reinforcement:**

- a. At least one-third the positive moment reinforcement in simple members and one-fourth the positive moment reinforcement in continuous members shall extend along the same face of the member into the support, to a length equal to  $L_d/3$ .
- b. When a flexural member is part of the primary lateral load resisting system, the positive reinforcement required to be extended into the support as described in (a) shall be anchored to develop its design stress in tension at the face of the support.
- c. At simple supports and at points of inflection, positive moment tension reinforcement shall be limited to a diameter such that  $L_d$  computed for  $f_d$  by 26.2.1 IS 456:2000 does not exceed.

$$\frac{M_1}{V} + L_0$$

Where,  $M_1$  = moment of resistance of the section assuming all reinforcement at the section to be stressed to  $f_d$ ;

$f_d = 0.87f_y$  in the case of limit state design and the permissible stress in the case of working stress design;

$V$  = shear force at the section due to the design loads;

$L_0$  = sum of the anchorage beyond the centre of the support and the equivalent anchorage value of any hook or mechanical anchorage at simple support; and at a point of inflection,  $L_0$  is limited to the effective depth of the members or  $12\phi$ , whichever is greater; and

$\phi$  = diameter of bar.

The value of  $M_1/V$  in the above expression may be increased by 30 percent when the ends of the reinforcement are confined by a compressive reaction.

**Negative moment reinforcement:**

At least one third of the total reinforcement provided for negative moment at the support shall extend beyond the point of inflection for a distance not less than the effective depth of the member or  $12\phi$  or one-sixteenth of the clear span whichever is greater. Anchorage of bars

Anchoring of bars is done to provide the development length and maintain the integrity of the structure.

Anchoring bars in tension:

- a. Deformed bars may be used without end anchorages provided development length requirement is satisfied. Hooks should normally be provided for plain bars in tension.
- b. Bends and hooks – shall conform to IS 2502
  1. Bends – The anchorage value of bend shall be taken as 4 times the diameter of the bar for each 45° bend subject to a maximum of 16 times the diameter of the bar.
  2. Hooks – The anchorage value of a standard U-type hook shall be equal to 16 times the diameter of the bar.

**Anchoring bars in compression:**

The anchorage length of straight bar in compression shall be equal to the development length of bars in compression as specified in clause 26.2.1 of IS 456:2000. The projected length of hooks, bends and straight lengths beyond bends if provided for a bar in compression, shall only be considered for development length.

**Mechanical devices for anchorage:**

Any mechanical or other device capable of developing the strength of the bar without damage to concrete may be used as anchorage with the approval of the engineer-in-charge.

Anchoring shear reinforcement:

a. Inclined bars – The development length shall be as for bars in tension; this length shall be measured as under:

1. In tension zone, from the end of the sloping or inclined portion of the bar, and
2. In the compression zone, from the mid depth of the beam.

b. Stirrups – Notwithstanding any of the provisions of this standard, in case of secondary reinforcement, such as stirrups and transverse ties, complete development lengths and anchorages shall be deemed to have been provided when the bar is bent through an angle of at least 90° round a bar of at least its own diameter and is continued beyond the end of the curve for a length of at least four bar diameters.

**Reinforcement requirements**

1. Minimum reinforcement:

The minimum area of tension reinforcement shall be not less than that given by the following:

Where,

$$\frac{A_s}{bd} = \frac{0.85}{f_y}$$

Where,  $A_s$  = minimum area of tension reinforcement.

$b$  = breadth of beam or the breadth of the web of T-beam,  $d$  = effective depth, and

$f_y$  = characteristic strength of reinforcement in  $\text{N/mm}^2$

2. Maximum reinforcement – The maximum area of tension reinforcement shall not exceed  $0.04Bd$ .

**Compression reinforcement:**

The maximum area of compression reinforcement shall not exceed  $0.04bD$ .

Compression reinforcement in beams shall be enclosed by stirrups for effective lateral restraint.

Pitch and diameter of lateral ties:

The pitch of shear reinforcement shall be not more than the least of the following distances:

1. The least lateral dimension of the compression members;
2. Sixteen times the smallest diameter of the longitudinal reinforcement bar to be tied; and
3. 300 mm.

The diameter of the polygonal links or lateral ties shall be not less than one-fourth of the diameter of the largest longitudinal bar, and in no case less than 16 mm.

Slenderness limits of beams to ensure lateral stability.

A beam is usually a vertical load carrying member. However, if the length of the beam is very large it may bend laterally. To ensure lateral stability of a beam the following specifications have been given in the code.

A simply supported or continuous beam shall be so proportioned that the clear distance between the lateral restraints does not exceed  $60b$  or whichever is less, where  $d$  is the effective depth of the beam and  $b$  the breadth of the compression face midway between the lateral restraints.

For a cantilever, the clear distance from the free end of the cantilever to the lateral restraint shall not exceed  $25b$  or whichever is less.

## UNIT -3

### DESIGN OF SLABS

#### Introduction to Slabs

A slab is a flat two dimensional planar structural element having thickness small compared to its other two dimensions. It provides a working flat surface or a covering shelter in buildings. It primarily transfer the load by bending in one or two directions. Reinforced concrete slabs are used in floors, roofs and walls of buildings and as the decks of bridges. The floor system of a structure can take many forms such as in situ solid slab, ribbed slab or pre-cast units. Slabs may be supported on monolithic concrete beam, steel beams, walls or directly over the columns. Concrete slab behave primarily as flexural members and the design is similar to that of beams.

#### Objective

1. To design one-way and two-way slabs

#### CLASSIFICATION OF SLABS

Slabs are classified based on many aspects

- 1) **Based of shape:** Square, rectangular, circular and polygonal in shape.
- 2) **Based on type of support:** Slab supported on walls, Slab supported on beams, Slabsupported on columns (Flat slabs).
- 3) **Based on support or boundary condition:** Simply supported, Cantilever slab,Overhanging slab, Fixed or Continues slab.
- 4) **Based on use:** Roof slab, Floor slab, Foundation slab, Water tank slab.
- 5) **Basis of cross section or sectional configuration:** Ribbed slab /Grid slab, Solid slab,Filler slab, Folded plate
- 6) **Basis of spanning directions:**

One way slab – Spanning in one direction

Two way slab - Spanning in two direction

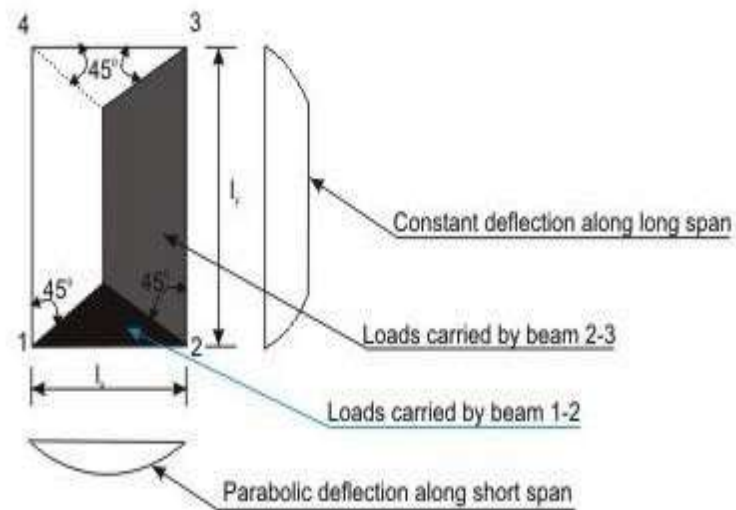
#### METHODS OF ANALYSIS

The analysis of slabs is extremely complicated because of the influence of number of factors stated above. Thus the exact (close form) solutions are not easily available. The various methods are:

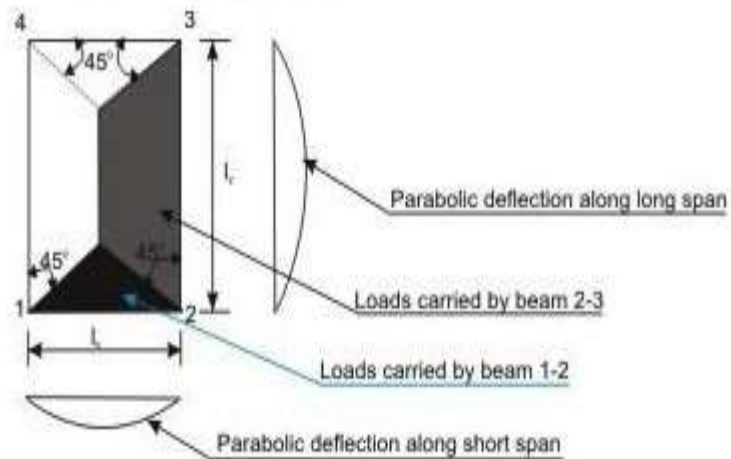


- 1) Classical methods – Levy and Naviers solutions (Plate analysis)
- 2) Yield line analysis – Used for ultimate /limit analysis
- 3) Numerical techniques – Finite element and Finite difference method.
- 4) Semi empirical – Prescribed by codes for practical design which uses coefficients.

### One-way and Two-way Slabs



One-way slab ( $l_y/l_x > 2$ )



Two-way slab ( $l_y/l_x \leq 2$ )

#### a. Effective span of slab :

Effective span of slab shall be lesser of the two

$l = \text{clear span} + d$  (effective depth )

$l = \text{Center to center distance between the support.}$

**b. Depth of slab:**

The depth of slab depends on bending moment and deflection criterion. the trail depth can be obtained using:

- Effective depth  $d = \text{Span} / ((l/d)_{\text{Basic}} \times \text{modification factor})$
- For obtaining modification factor, the percentage of steel for slab can be assumed from 0.2 to 0.5%
- The effective depth  $d$  of two way slabs can also be assumed using cl.24.1, IS 456

| Type of support  | Fe-250 | Fe-415 |
|------------------|--------|--------|
| Simply supported | 1/35   | 1/28   |
| continuous       | 1/40   | 1/32   |

OR

The following thumb rules can be used

- One way slab  $d = (1/22)$  to  $(1/28)$ .
- Two way simply supported slab  $d = (1/20)$  to  $(1/30)$
- Two way restrained slab  $d = (1/30)$  to  $(1/32)$

**c. Load on slab:**

The load on slab comprises of Dead load, floor finish and live load. The loads are calculated per unit area (load/m<sup>2</sup>).

Dead load =  $D \times 25$  kN/m<sup>2</sup> (Where  $D$  is thickness of slab in m)

Floor finish (Assumed as) = 1 to 2 kN/m<sup>2</sup>

Live load (Assumed as) = 3 to 5 kN/m<sup>2</sup> (depending on the occupancy of the building)

**DETAILING REQUIREMENTS AS PER IS 456: 2000**

**a. Nominal Cover:**

For Mild exposure – 20 mm

For Moderate exposure – 30 mm

However, if the diameter of bar do not exceed 12 mm, or cover may be reduced by 5 mm. Thus for main reinforcement up to 12 mm diameter bar and for mild exposure, the nominal cover is 15 mm.

**b. Minimum reinforcement :** The reinforcement in either direction in slab shall not be less than

- 0.15% of the total cross sectional area for Fe-250 steel
- 0.12% of the total cross sectional area for Fe-415 & Fe-500 steel.

c. **Spacing of bars:** The maximum spacing of bars shall not exceed  $\square\square$ Main Steel – 3d or 300 mm whichever is smaller

- Distribution steel – 5d or 450 mm whichever is smaller

Note: The minimum clear spacing of bars is not kept less than 75 mm (Preferably 100 mm) though code do not recommend any value.

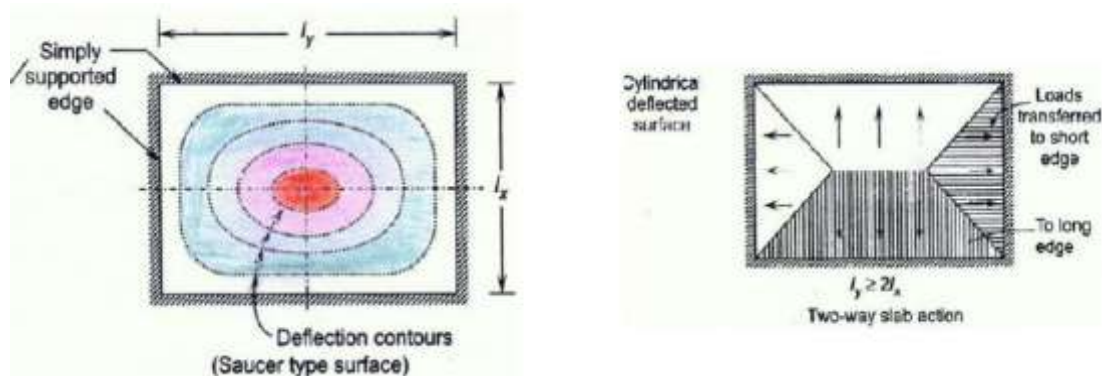
d. **Maximum diameter of bar:** The maximum diameter of bar in slab, shall not exceed  $D/8$ , where D is the total thickness of slab.

### BEHAVIOR OF ONE WAY SLAB

When a slab is supported only on two parallel opposite edges, it spans only in the direction perpendicular to two supporting edges. Such a slab is called one way slab. Also, if the slab is supported on all four edges and the ratio of longer span ( $l_y$ ) to shorter span ( $l_x$ ) i.e.  $l_y/l_x > 2$ , practically the slab spans across the shorter span. Such a slabs are also designed as one way slabs. In this case, the main reinforcement is provided along the spanning direction to resist one way bending.

### BEHAVIOR OF TWO WAY SLABS

A rectangular slab supported on four edge supports, which bends in two orthogonal directions and deflects in the form of dish or a saucer is called two way slabs. For a two way slab the ratio of  $l_y/l_x$  shall be  $\leq 2.0$ .



Since, the slab rest freely on all sides, due to transverse load the corners tend to curl up and lift up. The slab loses the contact over some region. This is known as lifting of corner. These slabs are called two way simply supported slabs. If the slabs are cast monolithic with the beams, the corners of the slab are restrained from lifting. These slabs are called restrained slabs. At corner, the rotation occurs in both the direction and causes the corners to lift. If the corners of slab are restrained from lifting, downward reaction results at corner & the end strips gets restrained against rotation. However, when the ends are restrained and the rotation of central strip still occurs and causing rotation at corner (slab is acting as unit) the end strip is subjected to torsion.

## Types of Two Way Slab

Two way slabs are classified into two types based on the support conditions:

- a) Simply supported slab
- b) Restrained slabs

### Two way simply supported slabs

The bending moments  $M_x$  and  $M_y$  for a rectangular slabs simply supported on all four edges with corners free to lift or the slabs do not having adequate provisions to prevent lifting of corners are obtained using

$$M_x = \alpha_x W l_x$$

$$M_y = \alpha_y W l_x$$

Where,  $\alpha_x$  and  $\alpha_y$  are coefficients given in Table 1 (Table 27, IS 456-2000)

$W$ - Total load /unit area

$l_x$  &  $l_y$  – lengths of shorter and longer span.

### Two way restrained slabs

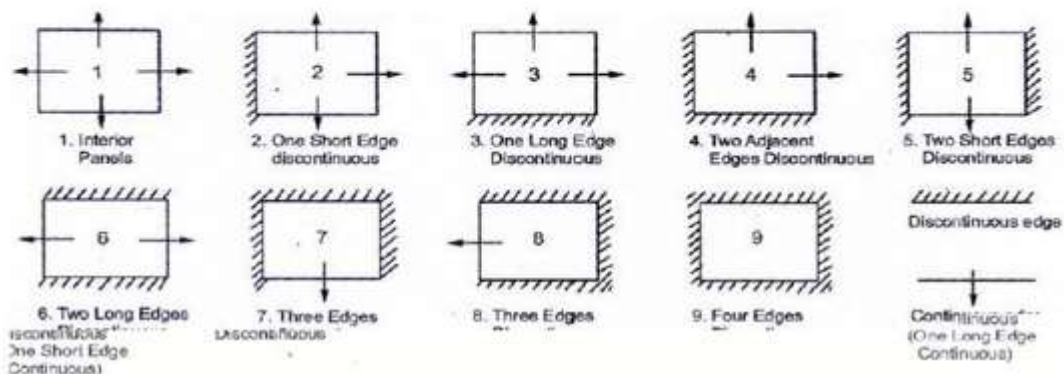
When the two way slabs are supported on beam or when the corners of the slabs are prevented from lifting the bending moment coefficients are obtained from Table 2 (Table 26, IS456-2000) depending on the type of panel shown in Fig. 3. These coefficients are obtained using yield line theory. Since, the slabs are restrained; negative moment arises near the supports. The bending moments are obtained using;

$$M_x \text{ (Negative)} = \alpha_x^{(-)} W l_x^2$$

$$M_x \text{ (Positive)} = \alpha_x^{(+)} W l_x^2$$

$$M_y \text{ (Negative)} = \alpha_y^{(-)} W l_x^2$$

$$M_y \text{ (Positive)} = \alpha_y^{(+)} W l_x^2$$



## **ONE WAY CONTINUOUS SLAB**

The slabs spanning in one direction and continuous over supports are called one way continuous slabs. These are idealised as continuous beam of unit width. For slabs of uniform section which support substantially UDL over three or more spans which do not differ by more than 15% of the longest, the B.M and S.F are obtained using the coefficients available in Table 12 and Table 13 of IS 456-2000. For moments at supports where two unequal spans meet or in case where the slabs are not equally loaded, the average of the two values for the negative moments at supports may be taken.

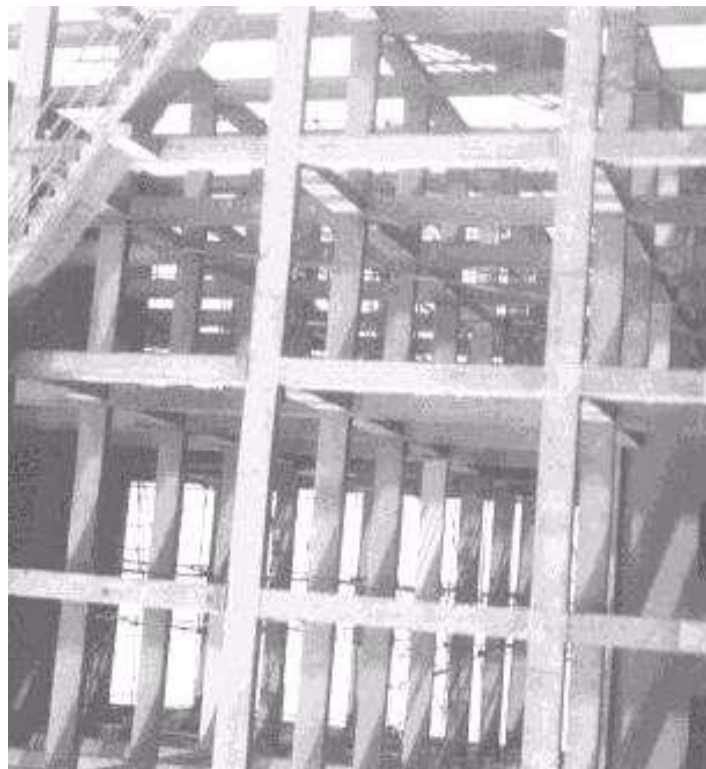
## UNIT-4

### DESIGN OF COLUMNS

#### Design of axially loaded columns

As we already know, the columns are the members predominantly subjected to direct compressive force. There can be cases when column sections can be subjected to large flexural stresses (in frames for instance, as shown in the photograph in Figure 7.19), but such cases are not being considered here.

A column is said to be subjected to axial load when the line of the resultant of loads supported by the column coincides with the center of gravity of the column section. Such loading generates only compressive stresses in the column section and in view of concrete's strength in compression, reinforced concrete is a very useful material that can withstand axial compressive loads. The other positive factor is based on the architectural requirements or the amount of the load to be supported. Reinforced concrete columns can be cast in various shapes like square, rectangular, hexagonal, circular, etc. Columns of L-shape or T-shape are also sometimes used in various buildings.



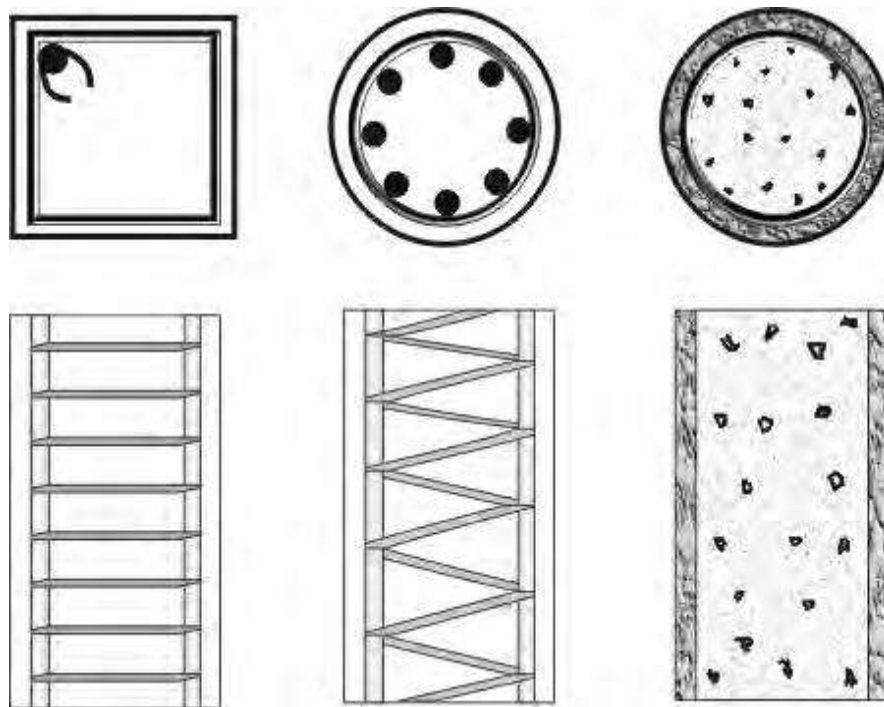
**Figure 7.19**

*RCC frame: combination of beams-columns*

As concrete itself is quite strong in compression, the longitudinal steel bars in columns supplement this bearing capacity of the column. In column section, the steel bars are

uniformly spaced along the perimeter of the column *as near to the surface as permissible*. The longitudinal bars are held in position by transverse reinforcement, termed *lateral binders*. The binders firstly prevent the displacement of the longitudinal bars during concrete pouring operation and further prevent the tendency of their buckling outwards under loads.

Transverse reinforcement in RCC columns can be of two types. The first type consists of individual binders made of small diameter bars, bent around the longitudinal bars and spaced evenly. The diameter, spacing and the profile of such binders depend upon the number and diameter of longitudinal bars and the overall cross sectional size of the column. In the second type, reinforcing bars are wound around the longitudinal bars in the form of a closely spaced continuous helix and are termed as spiral or helical reinforcements. Apart from rendering support to longitudinal bars against buckling, the helical reinforcement, in addition, also acts to confine the concrete within it. In effect, it forms a core of concrete that increases the load carrying capacity of the column. Some of the common arrangements of binders and helical reinforcements are shown in Figure 7.20.



The behavior of RCC section in columns is quite different from its behavior in beams, as we will see later. The following points may be useful to help design an economical and cost effective column section:

- Columns with separate lateral ties work out more economical than columns with spiral reinforcement.
- While in conformity with all other design norms, the axially loaded column with a low percentage of steel works out more economical for each ton of load supported, compared to the column with a higher percentage of steel as the main reinforcement.
- It is better to avoid lean concrete for compression members. The use of rich concrete mix normally results into economical design.

## Types of columns

- On the basis of response to loading, reinforced concrete columns can be broadly divided into the following categories:
- Columns with longitudinal steel with individual lateral ties or binders
- Columns with longitudinal steel with helical transverse reinforcement
- Composite columns with a steel structural member encased within the concrete

Of all the above types, columns reinforced with longitudinal steel and provided with lateral ties/spirals are most commonly used. Encased columns are recommended if the load to be carried is very high and the overall dimension of the column section is required to be restricted as a result of space or aesthetic consideration.

## Effective length

A column is a compression member and, therefore, is prone to buckling under load. In the case of reinforced concrete, the actual length ( $L$ ) of a column from practical considerations is taken as the clear distance from the floor and the underside of the beam of the least depth, at the next floor level that frames into the column from any direction. As we have seen in the section on 'Analysis of Columns', the effective length ( $L_{eff}$ ) of the column would be dependent on the end restraints.

The theoretically derived effective length for a column and the values adopted for RCC columns are shown in the Table below.

| Nature of End Restraint   | Effective Length ( $L_{eff}$ ) |
|---|--------------------------------|
| Effectively held in position and restrained against rotation at both ends   | 0.5 L                          |
| Effectively held in position at both ends, restrained against rotation at one end   | 0.7L                           |
| Effectively held in position at both ends but not restrained against rotation   | L                              |
| Effectively held in position and restrained against rotation at one end and at the other restrained against rotation but not held in position.        | L                              |
| Effectively held in position at one end but not restrained against rotation and at the other end restrained against rotation but not held in position | 2L                             |
| Effectively held in position and restrained against rotation as one end but not held in position nor restrained against rotation at the other end.    | 2L                             |



### Permissible stresses in RCC columns

Based on research, the design codes recommend somewhat lower permissible stresses in concrete and steel in direct compression, compared to the corresponding value in the bending. The reason for it is that unlike in the case of bending, the value of stress remains uniform throughout the section and also remains unchanged from section to section. Consequently, no section gets relief anytime. Hence, there has been no scope to transfer the stress from the higher stresses fiber to the adjoining lower stresses one.

The permissible stresses for various grades of concrete and for various type of steel reinforcement, to be considered in the design of column, have been indicated below. The corresponding value of stress in concrete in bending has also been shown for comparison.

### Permissible compressive stress in concrete

| Grade of Concrete | In Bending Compression | In Direct Compression |
|-------------------|------------------------|-----------------------|
| M15               | 5.0 N/mm <sup>2</sup>  | 4.0 N/mm <sup>2</sup> |
| M 20              | 7.0 N/mm <sup>2</sup>  | 5.0 N/mm <sup>2</sup> |
| M25               | 8.5 N/mm <sup>2</sup>  | 6.0 N/mm <sup>2</sup> |
| M30               | 10.0 N/mm <sup>2</sup> | 8.0 N/mm <sup>2</sup> |

### Permissible compressive stresses in steel reinforcement

For steel bars, the values of permissible stress in direct compression are as follows:

- For ordinary Mild Steel = 130 N/mm<sup>2</sup>
- For High Yield Strength Steel = 190 N/mm<sup>2</sup>

### Capacity of columns

While determining the load carrying capacity of the reinforcement concrete column, the capacity of concrete and steel is determined separately and added to determine the combined capacity. This is a deviation from the principle followed for the composite section, where the strain in each of the material remains same and the load distribution is determined accordingly. The reason for this deviation is, the special deformation characteristic of concrete under compression. Concrete undergoes the following three types of strain:

- Elastic strain – The strain due to application of load on the member. This is observed immediately on the application of load.
- Shrinkage strain – Due to chemical reaction, which results into hardening, the concrete undergoes reduction in its volume. This phenomenon is known as *shrinkage*. Shrinkage is not dependent on load and its effect is observed in the medium term.
- Creep – *Creep* is a phenomenon special to loaded concrete. As per this, concrete under compression for a substantial period undergoes slow deformation, which is time dependent. The effect of creep can be observed in the long term only.

Hence, in the RCC column section, concrete observes strain due to shrinkage and creep, in addition to, the usual elastic strain. The effect of these two, on the composite behavior of materials, is quite complex. However, research has proved that it is safe to consider independent load carrying capacity of both materials in the RCC section.

The load carrying capacity of various types of columns can be determined as follow:

### Short columns

RCC columns are classified as either long or short, based on the ratio of their length and sectional dimensions. A column is considered to be short when the ratio of its effective length, to its least lateral dimensions does not exceed 12. If this ratio exceeds 12, the column is considered to be a long column. On account of its tendency to buckle, a long column has less load carrying capacity than a short column of the same sectional area.

The permissible axial load on a short column reinforced with longitudinal bars and lateral ties is given by the relation,

$$P = \sigma_{cc} \times A_c + \sigma_{sc} \times A_s$$

Here,  $\sigma_{cc}$  = permissible stress in concrete in direct compression

$A_c$  = net cross-sectional area of concrete

$\sigma_{sc}$  = permissible compressive stress for reinforcement steel bars

$A_s$  = cross-sectional area of longitudinal steel.

The permissible load for columns with helical reinforcement shall be 1.05 times, the permissible load for similar members with lateral ties or rings. This provision can be made applicable only if, the ratio of volume of helical reinforcement, to the volume of core is not less than, the value given by expression,

$$0.36 \times (A_g / A_c - 1) \times (f_{ck} / f_y).$$

Here,  $A_g$  = gross area of the section

$A_c$  = area of the core of the helically reinforced column measured to the outside diameter of the helix.

## Long columns

The principle for the determination of load carrying capacity of a long column is same as for a short column. However, in the long columns, the maximum permissible values of stresses in concrete and steel can be brought down by multiplying the respective stresses by a reduction coefficient  $C_r$ , given by the following formula:

$$C_r = 1.25 - L_{\text{eff}}/48b$$

Where,

$$L_{\text{eff}} = \text{effective length of column}$$
$$b = \text{least lateral dimension of column}$$

For more accurate calculation, the following formula for reduction coefficient can also be used:

$$C_r = 1.25 - l_{\text{ef}}/160 i_{\text{min}}$$

Where,  $i_{\text{min}} = \text{least radius of gyration for the column section}$

## Column design – common norms

Some of the commonly accepted norms for RCC column design and detailing have been described below:

### Longitudinal reinforcement

- The minimum area of cross-section of longitudinal bars must be at least 0.8% of the gross-sectional area of the column.
- In any column that has a larger cross-sectional area than that required to support the load, the minimum percentage steel must be derived based on, the area of concrete required to resist direct stress and not on the actual area provided.
- The maximum area of cross-section of longitudinal bars must not exceed 6% of the gross cross-sectional area of the concrete. Although, it is recommended that the maximum area of steel should preferably be kept within 4% to avoid difficulties in placing and compacting of concrete.
- The bar should not be less than 12 mm in diameter, so that it is sufficiently rigid, to stand up straight in the column form during fixing and concreting.
- The minimum number of longitudinal bars provided in a column must be four, in rectangular columns and six in circular columns.
- Spacing of longitudinal bars measured along the periphery of a column should not exceed 300 mm.

## **Transverse reinforcement**

Transverse reinforcement may be in the form of lateral ties or spirals. They may also be in the form of polygonal links with internal angles not exceeding  $135^\circ$ . All of the transverse reinforcement should be properly anchored. The reinforcement should satisfy the following requirements:

- The minimum diameter of the lateral ties or helical reinforcement shall not be less than  $\frac{1}{4}$  of the diameter of the largest sized longitudinal bars with a lower limit of 5mm.
- The maximum diameter of the ties should preferably be not more than 12mm as it requires bending to the desired shape and size..
- The pitch of the ties, which is the longitudinal distance between two adjoining ties, should not exceed the smallest of the following.
  1. The least lateral dimension of the column
  2. 16 times the smallest diameter of the longitudinal reinforcement bar to be tied
  3. 48 times the diameter of lateral tie or transverse reinforcement.
- The pitch of the helical turns should not be more than the  $\frac{1}{6}$  of the core diameter up to the centre of helix or 75 mm, whichever is lower.

The illustration below would demonstrate the process of RCC column design in accordance with all the stated norms:

**Illustration** : A 7.5 m long column is hinged at one of its ends and fixed on the other end. The total vertical load acting on the column is 600000 N. It is required to design the reinforced section for this column.

The design data is as follow.

Characteristic strength of concrete = 20 N/mm<sup>2</sup>

Permissible direct compressive strength in concrete = 5 N/mm<sup>2</sup>

Permissible direct compressive stress in steel = 190 N/mm<sup>2</sup>

For these end condition, the effective length of column would be 0.7 times the actual length.

Hence,  $L_{\text{eff}} = 0.7 \times 7.5 = 5.25 \text{ m}$

We make a trial with the section of 300mm  $\times$  300mm for this column.

Gross area of the section =  $300 \times 300 = 90000 \text{ mm}^2$

For this section, the ratio  $L_{\text{eff}} / b$  is as follows:

$L_{\text{eff}} / b = 5.25 \times 1000 / 300 = 17.5$

Since the value of  $L_{eff}/b$  is more than 12, the column would be classified as a *long* one.

The stress reduction factor for the section,

$$\begin{aligned}Cr &= 1.25 - L_{eff}/48b \\ &= 1.25 - 5.25 \times 1000 / (48 \times 300) \\ &= 0.885\end{aligned}$$

Reduced Permissible Stress in concrete =  $0.885 \times 5 = 4.425 \text{ N/mm}^2$   
Reduced Permissible Stress in steel =  $0.885 \times 190 = 168.15 \text{ N/mm}^2$

We need to determine the area of steel required for the section. If this is assumed as  $A_s$ , the load capacity of the column would be,

$$P = \frac{4.425 \times (300 \times 300 - A_s) + 168.15 \times A_s}{A_s}$$

Equating this to the actual load on the column,

$$\begin{aligned}600000 &= \frac{4.425 \times (300 \times 300 - A_s) + 168.15 \times A_s}{A_s} \\ A_s &= 1233 \text{ mm}^2\end{aligned}$$

We provide 20mm diameter bars (area 314 mm<sup>2</sup> for each) for the section.

$$\begin{aligned}\text{Number of bars} &= \frac{1233}{314} \\ &= 3.92\end{aligned}$$

Hence we provide four numbers 20 mm diameter bars, which meet the criteria for the minimum number of bars for column section.

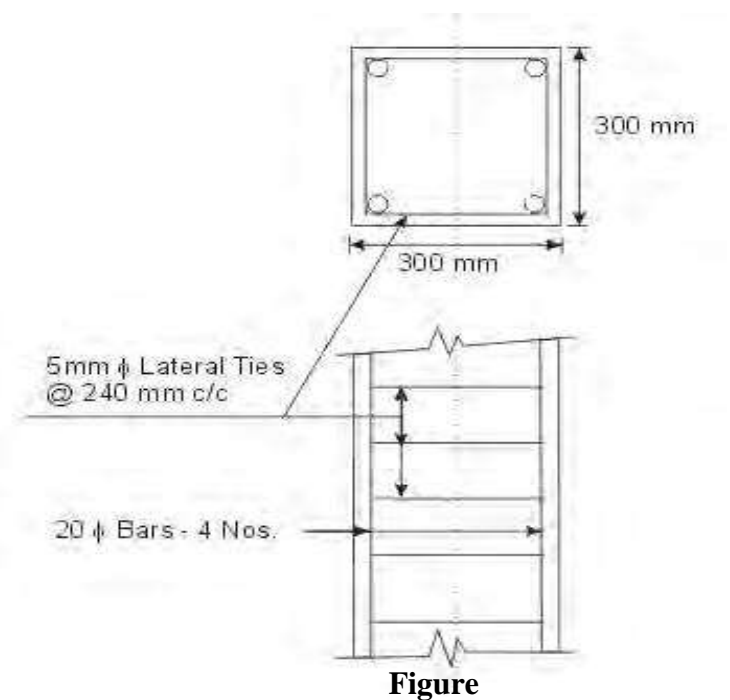
The next step is to design the transverse reinforcement. The diameter of the tie bar has to be larger than  $\frac{1}{4}$  of the main bar's diameter or 5mm. For 20 mm diameter, the value derived from both the criteria is 5mm. Hence we provide ties of 5 mm diameter.

To decide the distance between the ties, we know that the pitch should be, the least of the following three:

- The least lateral dimension of the column : *300mm in this case*
- 16 times the smallest diameter of the longitudinal reinforcement bar :  $16 \times 20 = 320\text{mm}$  *for this case*
- 48 times the diameter of a lateral tie or transverse reinforcement:  $48 \times 5 = 240\text{mm}$  *for this case.*

Since the least value of the three is 240 mm, we provide 5 mm diameter lateral ties @ 240 mm center to center along the longitudinal axis of the beam.

The sectional details of the column have been shown in Figure 7.21 below:



**Figure**

### *Column reinforcement details – illustration*

Compression members are structural elements primarily subjected to axial compressive forces and hence, their design is guided by considerations of strength and buckling. Examples of compression member pedestal, column, wall and strut

#### Definitions

(a) Effective length: The vertical distance between the points of inflection of the compression member in the buckled configuration in a plane is termed as effective length  $l_e$  of that compression member in that plane. The effective length is different from the unsupported length  $l$  of the member, though it depends on the unsupported length and the type of end restraints. The relation between the effective and unsupported lengths of any compression member is

$$l_e = k l (1)$$

Where  $k$  is the ratio of effective to the unsupported lengths. Clause 25.2 of IS 456 stipulates the effective lengths of compression members (vide Annex E of IS 456). This parameter is needed in classifying and designing the compression members.

(b) Pedestal: Pedestal is a vertical compression member whose effective length  $l_e$  does not exceed three times of its least horizontal dimension  $b$  (cl. 26.5.3.1h, Note). The other horizontal dimension  $D$  shall not exceed four times of  $b$ .

(c) Column: Column is a vertical compression member whose unsupported length  $l$  shall not exceed sixty times of  $b$  (least lateral dimension), if restrained at the two ends. Further, its unsupported length of a cantilever column shall not exceed  $100b^2/D$ , where  $D$  is the larger lateral dimension which is also restricted up to four times of  $b$  (vide cl. 25.3 of IS 456).

(d) Wall: Wall is a vertical compression member whose effective height  $H_w$  to thickness  $t$  (least lateral dimension) shall not exceed 30 (cl. 32.2.3 of IS 456). The larger horizontal dimension i.e., the length of the wall  $L$  is more than  $4t$ .

### Classification of Columns Based on Types of Reinforcement

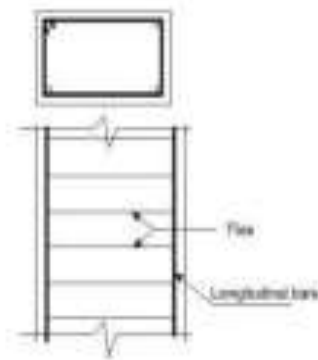


Figure 3.1(a) Tied Column

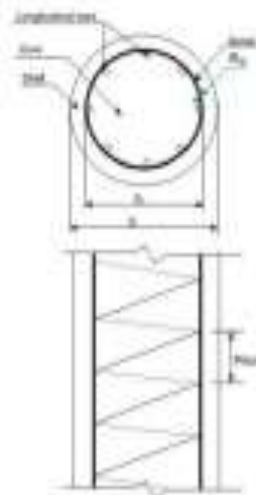


Figure 3.1(b) Column with helical reinforcement

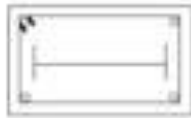


Figure 3.1(c) Composite column (steel section)

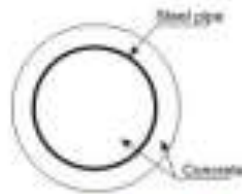


Figure 3.1(d) Composite column (steel pipe)

Figure 3.1 Tied, helically bound and composite columns

Based on the types of reinforcement, the reinforced concrete columns are classified into three groups:  
 Based on the types of reinforcement, the reinforced concrete columns are classified into three groups:

- (i) Tied columns: The main longitudinal reinforcement bars are enclosed within closely spaced lateral ties (Fig.3.1a).
- (ii) Columns with helical reinforcement: The main longitudinal reinforcement bars are enclosed within closely spaced and continuously wound spiral reinforcement. Circular and octagonal columns are mostly of this type (Fig. 3.1b).
- (iii) Composite columns: The main longitudinal reinforcement of the composite columns consists of structural steel sections or pipes with or without longitudinal bars (Fig. 3.1c and d).

Out of the three types of columns, the tied columns are mostly common with different shapes of the cross-sections viz. square, rectangular etc. Helically bound columns are also used for circular or octagonal shapes of cross-sections.

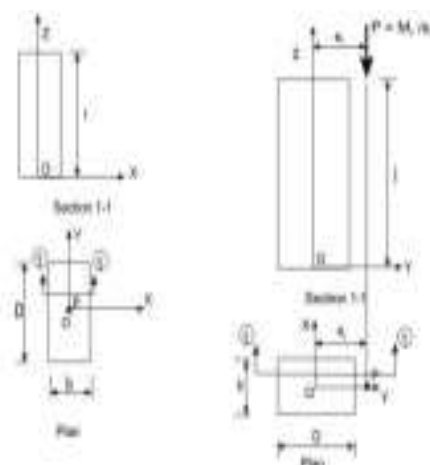


Figure 3.2(a) Axial loading (concentric)    Figure 3.2(b) Axial loading with uniaxial bending



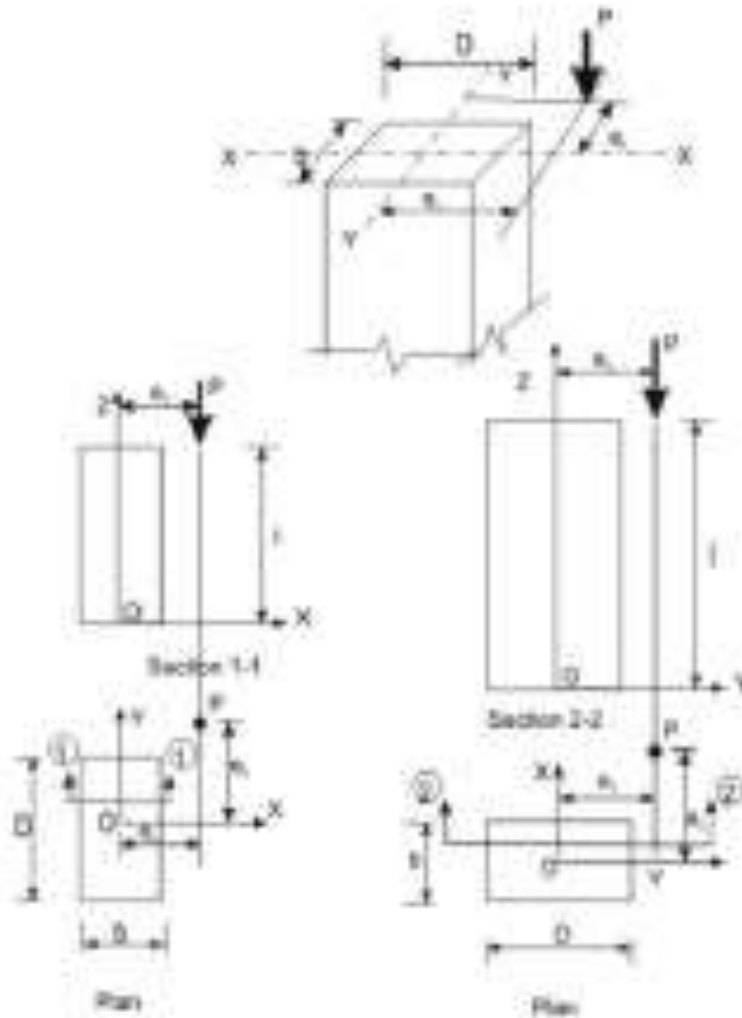


Figure 3.2(c) Axial loading with biaxial bending

Columns are classified into the three following types based on the loadings:

- (i) Columns subjected to axial loads only (concentric), as shown in Fig. 3.2a.
- (ii) Columns subjected to combined axial load and uniaxial bending, as shown in Fig. 3.2b.
- (iii) Columns subjected to combined axial load and bi-axial bending, as shown in Fig. 3.2c.

### Classification of Columns Based on Slenderness Ratios

Columns are classified into the following two types based on the slenderness ratios:

- (i) Short columns
- (ii) Slender or long columns

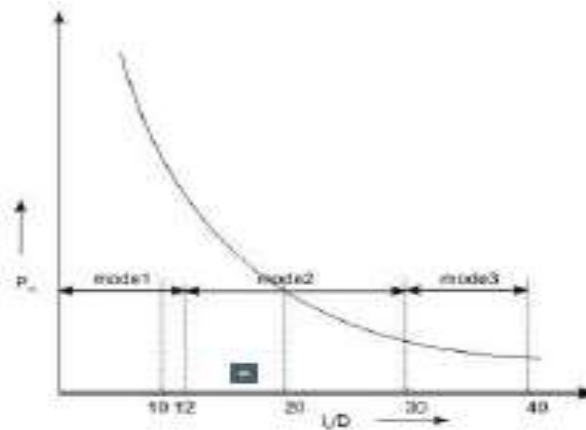


Figure 3.3 Modes of failure of columns

Figure 3.3 presents the three modes of failure of columns with different slenderness ratios when loaded axially. In the mode 1, column does not undergo any lateral deformation and collapses due to material failure. This is known as compression failure. Due to the combined effects of axial load and moment a short column may have material failure of mode 2. On the other hand, a slender column subjected to axial load only undergoes deflection due to beam-column effect and may have material failure under the combined action of direct load and bending moment. Such failure is called combined compression and bending failure of mode 2. Mode 3 failure is by elastic instability of very long column even under small load much before the material reaches the yield stresses. This type of failure is known as elastic buckling.

The slenderness ratio of steel column is the ratio of its effective length  $le$  to its least radius of gyration  $r$ . In case of reinforced concrete column, however, IS 456 stipulates the slenderness ratio as the ratio of its effective length  $le$  to its least lateral dimension. As mentioned earlier in sec. 3.1(a), the effective length  $le$  is different from the unsupported length, the rectangular reinforced concrete column of cross-sectional dimensions  $b$  and  $D$  shall have two effective lengths in the two directions of  $b$  and  $D$ . Accordingly, the column may have the possibility of buckling depending on the two values of slenderness ratios as given below:

Slenderness ratio about the major axis =  $lex/D$

Slenderness ratio about the minor axis =  $ley/b$

Based on the discussion above, cl. 25.1.2 of IS 456 stipulates the following:

A compression member may be considered as short when both the slenderness ratios  $lex/D$  and  $ley/b$  are less than 12 where  $lex$  = effective length in respect of the major axis,  $D$  = depth in respect of the major axis,  $ley$  = effective length in respect of the minor axis, and  $b$  = width of the member. It shall otherwise be considered as a slender compression member.

Further, it is essential to avoid the mode 3 type of failure of columns so that all columns should have material failure (modes 1 and 2) only. Accordingly, cl. 25.3.1 of IS 456 stipulates the maximum unsupported length between two restraints of a column to sixty times its least lateral dimension. For cantilever columns, when one end of the column is unrestrained, the unsupported length is restricted to  $100b^2/D$  where  $b$  and  $D$  are as defined earlier.

### **Longitudinal Reinforcement**

The longitudinal reinforcing bars carry the compressive loads along with the concrete. Clause 26.5.3.1 stipulates the guidelines regarding the minimum and maximum amount, number of bars, minimum diameter of bars, spacing of bars etc. The following are the salient points:

- (a) The minimum amount of steel should be at least 0.8 per cent of the gross cross-sectional area of the column required if for any reason the provided area is more than the required area.
- (b) The maximum amount of steel should be 4 per cent of the gross cross-sectional area of the column so that it does not exceed 6 per cent when bars from column below have to be lapped with those in the column under consideration.
- (c) Four and six are the minimum number of longitudinal bars in rectangular and circular columns, respectively.
- (d) The diameter of the longitudinal bars should be at least 12 mm.
- (e) Columns having helical reinforcement shall have at least six longitudinal bars within and in contact with the helical reinforcement. The bars shall be placed equidistant around its inner circumference.
- (f) The bars shall be spaced not exceeding 300 mm along the periphery of the column.
- (g) The amount of reinforcement for pedestal shall be at least 0.15 per cent of the cross-sectional area provided.

### **Transverse Reinforcement**

Transverse reinforcing bars are provided in forms of circular rings, polygonal links (lateral ties) with internal angles not exceeding 135° or helical reinforcement. The transverse reinforcing bars are provided to ensure that every longitudinal bar nearest to the compression face has effective lateral support against buckling. Clause 26.5.3.2 stipulates the guidelines of the arrangement of transverse reinforcement. The salient points are:

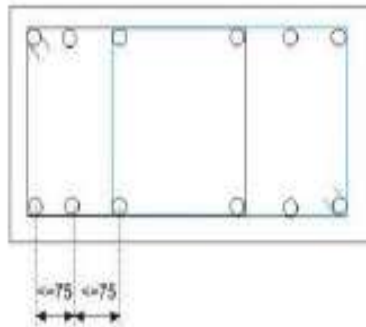


Figure 3.4 Lateral tie (Arrangement 1)

- (a) Transverse reinforcement shall only go round corner and alternate bars if the longitudinal bars are not spaced more than 75 mm on either side (Fig.3.4).

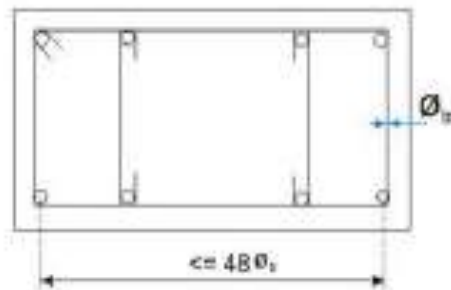


Figure 3.5 Lateral tie (Arrangement 2)

- (b) Longitudinal bars spaced at a maximum distance of 48 times the diameter of the tie shall be tied by single tie and additional open ties for in between longitudinal bars (Fig.3.5)

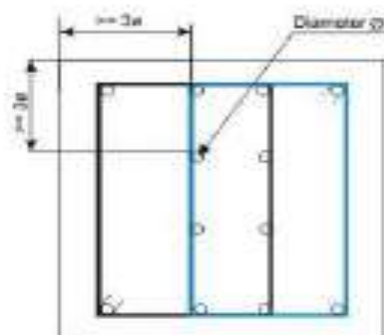
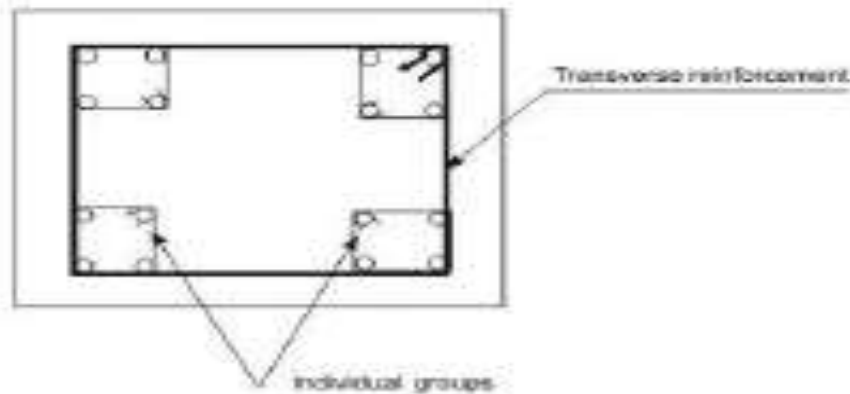


Figure 3.6 Lateral tie (Arrangement 3)

- (c) For longitudinal bars placed in more than one row (Fig.10.21.9): (i) transverse reinforcement is provided for the outer-most row in accordance with (a) above, and (ii) no bar of the inner row is closer to the nearest compression face than three times the diameter of the largest bar in the inner row.



**Figure 3.7 Lateral tie (Arrangement 4)**

#### **Pitch and Diameter of Lateral Ties**

- (a) Pitch: The maximum pitch of transverse reinforcement shall be the least of the following:
- (i) the least lateral dimension of the compression members;
  - (ii) sixteen times the smallest diameter of the longitudinal reinforcement bar to be tied; and
  - (iii) 300 mm.
- (b) Diameter: The diameter of the polygonal links or lateral ties shall be not less than one-fourth of the diameter of the largest longitudinal bar, and in no case less than 6 mm.

#### **Assumptions in the Design of Compression Members by Limit State of Collapse**

The following are the assumptions in addition to given in 38.1 (a) to (e) for flexure for the design of compression members (cl. 39.1 of IS 456).

- (i) The maximum compressive strain in concrete in axial compression is taken as 0.002.
- (ii) The maximum compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending and when there is no tension on the section shall be 0.0035 minus 0.75 times the strain at the least compressed extreme fibre.

### Minimum Eccentricity

In practical construction, columns are rarely truly concentric. Even a theoretical column loaded axially will have accidental eccentricity due to inaccuracy in construction or variation of materials etc. Accordingly, all axially loaded columns should be designed considering the minimum eccentricity as stipulated in cl. 25.4 of IS 456 and given below (Fig.3.2c)

$$e_x \min \geq \text{greater of } (l/500 + D/30) \text{ or } 20 \text{ mm}$$

$$e_y \min \geq \text{greater of } (l/500 + b/30) \text{ or } 20 \text{ mm}$$

where  $l$ ,  $D$  and  $b$  are the unsupported length, larger lateral dimension and least lateral dimension, respectively.

### Governing Equation for Short Axially Loaded Tied Columns

Factored concentric load applied on short tied columns is resisted by concrete of area  $A_c$  and longitudinal steel of areas  $A_{sc}$  effectively held by lateral ties at intervals. Assuming the design strengths of concrete and steel are  $0.4f_{ck}$  and  $0.67f_y$ , respectively, we can write

$$P_u = 0.4f_{ck} A_c + 0.67f_y A_{sc} \quad (1)$$

Where  $P_u$  = factored axial load on the member,

$f_{ck}$  = characteristic compressive strength of the concrete,

$A_c$  = area of concrete,

$f_y$  = characteristic strength of the compression reinforcement, and

$A_{sc}$  = area of longitudinal reinforcement for columns.

The above equation, given in cl. 39.3 of IS 456, has two unknowns  $A_c$  and  $A_{sc}$  to be determined from one equation. The equation is recast in terms of  $A_g$ , the gross area of concrete and  $p$ , the percentage of compression reinforcement employing

$$A_{sc} = pA_g/100 \quad (2)$$

$$A_c = A_g(1 - p/100) \quad (3)$$

Accordingly, we can write

$$P_u/A_g = 0.4f_{ck} + (p/100)(0.67f_y - 0.4f_{ck}) \quad (4)$$

Equation 4 can be used for direct computation of  $A_g$  when  $P_u$ ,  $f_{ck}$  and  $f_y$  are known by assuming  $p$  ranging from 0.8 to 4 as the minimum and maximum percentages of longitudinal reinforcement.

Equation 10.4 also can be employed to determine  $A_g$  and  $p$  in a similar manner by assuming  $p$ .

## UNIT-V

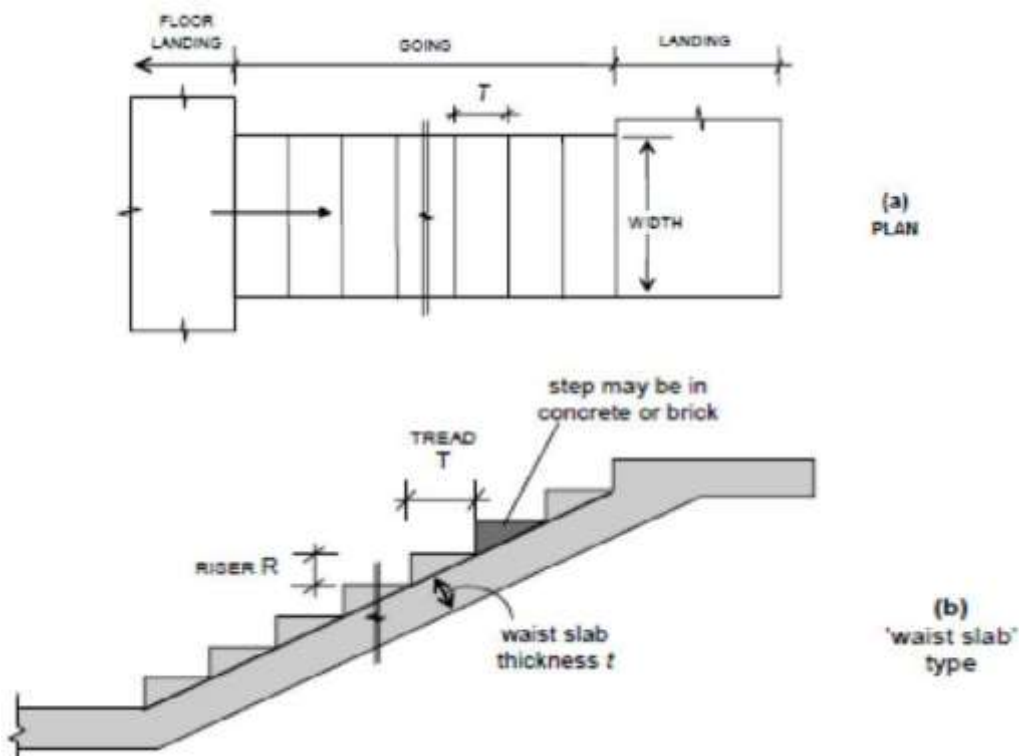
### DESIGN OF FOOTINGS AND STAIRCASE

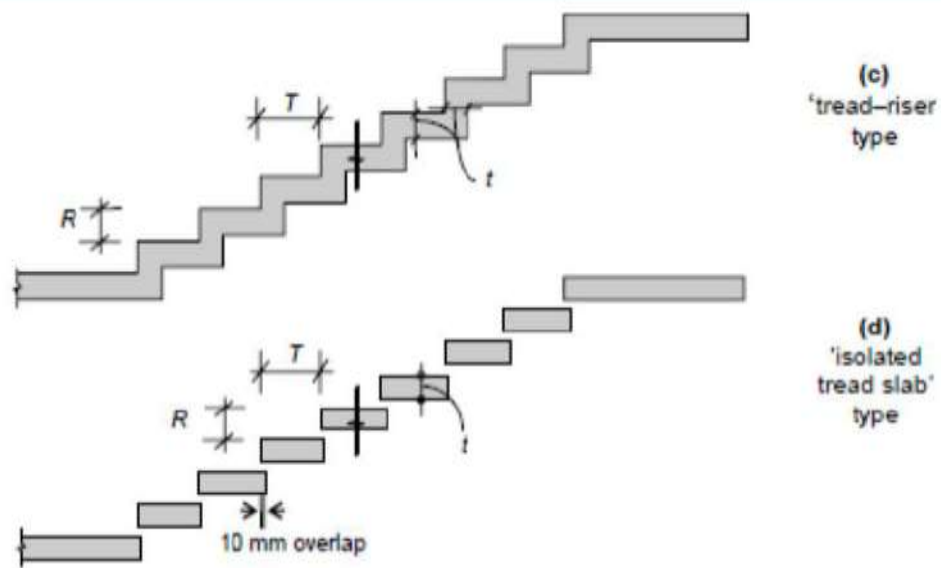
#### Introduction

The staircase is an important component of a building, and often the only means of access between the various floors in the building. It consists of a *flight* of steps, usually with one or more intermediate *landings* (horizontal slab platforms) provided between the floor levels. The horizontal top portion of a step (where the foot rests) is termed *tread* and the vertical projection of the step (i.e., the vertical distance between two neighbouring steps) is called *riser*. Values of 300 mm and 150 mm are ideally assigned to the tread and riser respectively particularly in public buildings. However, lower values of tread (up to 250 mm) combined with higher values of riser (up to 190 mm) are resorted to in residential and factory buildings. The *width* of the stair is generally around 1.1 – 1.6m, and in any case, should normally not be less than 850 mm; large stair widths are encountered in entrances to public buildings. The horizontal projection (plan) of an inclined flight of steps, between the first and last risers, is termed *going*. A typical flight of steps consists of two landings and one going, as depicted in Fig. Generally, risers in a flight should not exceed about 12 in number. The steps in the flight can be designed in a number of ways: with *waist slab*, with *tread-riser* arrangement (without waist slab) or with *isolated tread slabs* — as shown in Fig respectively.

#### Objectives

1. To design a dog-legged and open newel staircases





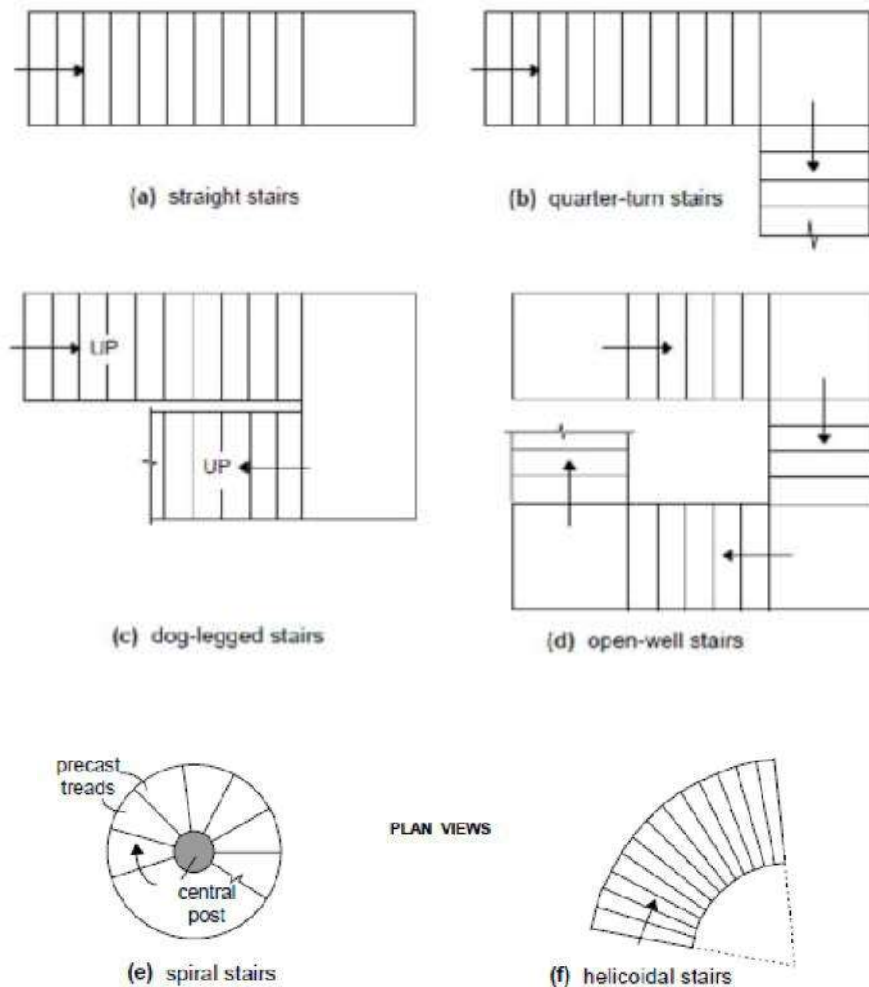
## TYPES OF STAIRCASES

### Geometrical Configurations

A wide variety of staircases are met with in practice. Some of the more common geometrical configurations are depicted in Fig. These include:

1. Straight stairs (with or without intermediate landing)
2. Quarter-turn stairs
3. Dog-legged stairs
4. Open well stairs
5. Spiral stairs
6. Helicoidal stairs





### Structural Classification

Structurally, staircases may be classified largely into two categories, depending on the predominant direction in which the slab component of the stair undergoes flexure:

1. Stair slab spanning transversely (stair widthwise);
2. Stair slab spanning longitudinally (along the incline).

### Stair Slab Spanning Transversely

The slab component of the stair (whether comprising an isolated tread slab, a tread-riser unit or a waist slab) is supported on its side(s) or cantilevers laterally from a central support. The slab supports gravity loads by bending essentially in a *transverse vertical plane*, with the span along the *width* of the stair.

In the case of the cantilevered slabs, it is economical to provide isolated treads (without risers). However, the tread-riser type of arrangement and the waist slab type are also sometimes employed in practice, as cantilevers. The spandrel beam is subjected to torsion (equilibrium torsion'), in addition to flexure and shear.

When the slab is supported at the two sides by means of 'stringer beams' or masonry walls, it may be designed as simply supported, but reinforcement at the top should be provided near the supports to resist the 'negative' moments that may arise on account of possible partial fixity.

### **Stair Slab Spanning Longitudinally**

In this case, the supports to the stair slab are provided parallel to the riser at two or more locations, causing the slab to bend longitudinally between the supports. It may be noted that longitudinal bending can occur in configurations other than the straight stair configuration, such as quarter-turn stairs, dog-legged stairs, open well stairs and helicoidally stairs.

The slab arrangement may either be the conventional waist slab type or the tread-riser type. The slab thickness depends on the 'effective span', which should be taken as the centre-to-centre distance between the beam/wall supports, according to the Code (Cl. 33.1a, c). In certain situations, beam or wall supports may not be available parallel to the riser at the landing. Instead, the flight is supported between the landings, which span transversely, the flight (spanning longitudinally) should be taken as the going of the stairs plus at each end either half the width of the landing or one metre, whichever is smaller.

### **Numerical Problem**

Design a (waist slab type) dog-legged staircase for an office building, given the following data:

1. Height between floor = 3.2 m;
2. Riser = 160 mm, tread = 270 mm;
3. Width of flight = landing width = 1.25 m
4. Live load = 5.0 kN/m
5. Finishes load = 0.6 kN/m

Assume the stairs to be supported on 230 mm thick masonry walls at the outer edges of the landing, parallel to the risers [Fig. 12.13(a)]. Use M 20 concrete and Fe 415 steel. Assume mild exposure conditions.

### **DESIGN OF FOOTING AND STAIR CASE**

The superstructure is placed on the top of the foundation structure, designated as substructure as they are placed below the ground level. The elements of the superstructure transfer the loads and moments to its adjacent element below it and finally all loads and moments come to the foundation structure, which in turn, transfers them to the underlying soil or rock. Thus, the foundation structure effectively supports the superstructure. However, all types of soil get compressed significantly and cause the structure to settle. Accordingly, the major requirements of the design of foundation structures are the two as given below (see cl.34.1 of IS 456)

:

1. Foundation structures should be able to sustain the applied loads, moments, forces and induced reactions without exceeding the safe bearing capacity of the soil.
2. The settlement of the structure should be as uniform as possible and it should be within the tolerable limits. It is well known from the structural analysis that differential settlement of supports causes additional moments in statically indeterminate structures. Therefore, avoiding the differential settlement is considered as more important than maintaining uniform overall settlement of the structure.

### **Types of Foundation Structures**

#### **1. Shallow Foundation**

Shallow foundations are used when the soil has sufficient strength within a short depth below the ground level. They need sufficient plan area to transfer the heavy loads to the base soil. These heavy loads are sustained by the reinforced concrete columns or walls (either of bricks or reinforced concrete) of much less areas of cross-section due to high strength of bricks or reinforced concrete when compared to that of soil. The strength of the soil, expressed as the safe bearing capacity of the soil is normally supplied by the geotechnical experts to the structural engineer. Shallow foundations are also designated as footings. The different types of shallow foundations or footings are discussed below.

- (i) Plain concrete pedestal footings
- (ii) Isolated footings
- (iii) Combined footings
- (iv) Strap footings

(v) Strip foundation or wall footings

(vi) Raft or mat foundation

## **2. Deep foundations**

As mentioned earlier, the shallow foundations need more plan areas due to the low strength of soil compared to that of masonry or reinforced concrete. However, shallow foundations are selected when the soil has moderately good strength, except the raft foundation which is good in poor condition of soil also. Raft foundations are under the category of shallow foundation as they have comparatively shallow depth than that of deep foundation. It is worth mentioning that the depth of raft foundation is much larger than those of other types of shallow foundations.

However, for poor condition of soil near to the surface, the bearing capacity is very less and foundation needed in such situation is the pile foundation. Piles are, in fact, small diameter columns which are driven or cast into the ground by suitable means. Precast piles are driven and cast-in-situ are cast. These piles support the structure by the skin friction between the pile surface and the surrounding soil and end bearing force, if such resistance is available to provide the bearing force. Accordingly, they are designated as frictional and end bearing piles. They are normally provided in a group with a pile cap at the top through which the loads of the superstructure are transferred to the piles.

Piles are very useful in marshy land where other types of foundation are impossible to construct. The length of the pile which is driven into the ground depends on the availability of hard soil/rock or the actual load test. Another advantage of the pile foundations is that they can resist uplift also in the same manner as they take the compression forces just by the skin friction in the opposite direction.

However, driving of pile is not an easy job and needs equipment and specially trained persons or agencies. Moreover, one has to select pile foundation in such a situation where the adjacent buildings are not likely to be damaged due to the driving of piles. The choice of driven or bored piles, in this regard, is critical.

Exhaustive designs of all types of foundations mentioned above are beyond the scope of this course. Accordingly, this module is restricted to the design of some of the shallow footings, frequently used for normal low rise buildings only.

## Isolated Footing

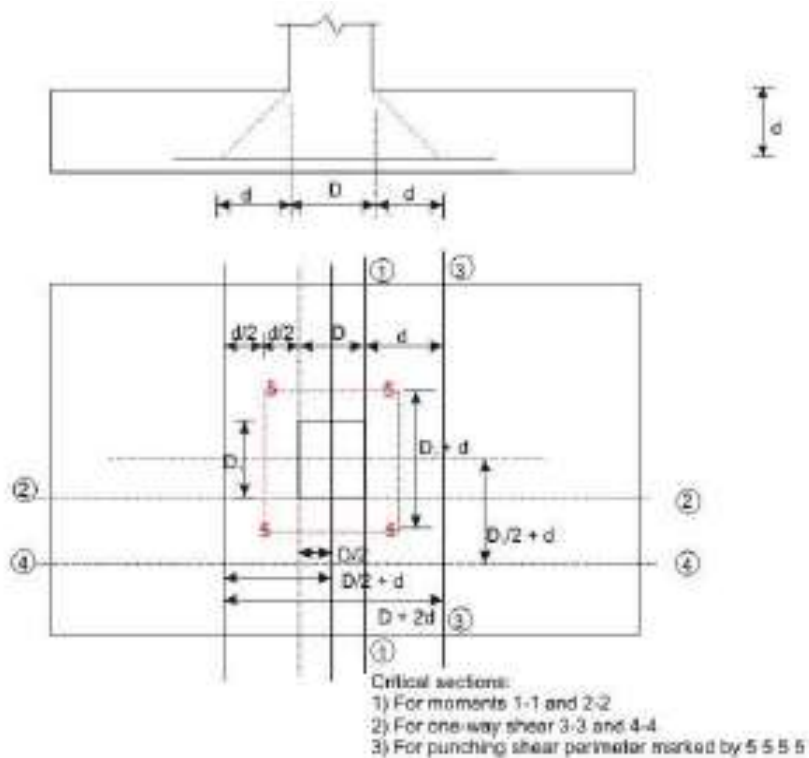


Figure 3.8: Uniform and rectangular footing

## Design Considerations

### (a) Minimum nominal cover (cl. 26.4.2.2 of IS 456)

The minimum nominal cover for the footings should be more than that of other structural elements of the superstructure as the footings are in direct contact with the soil. Clause 26.4.2.2 of IS 456 prescribes a minimum cover of 50 mm for footings. However, the actual cover may be even more depending on the presence of harmful chemicals or minerals, water table etc.

### (b) Thickness at the edge of footings (cls. 34.1.2 and 34.1.3 of IS 456)

The minimum thickness at the edge of reinforced and plain concrete footings shall be at least 150 mm for footings on soils and at least 300 mm above the top of piles for footings on piles, as per the stipulation in cl.34.1.2 of IS 456.

For plain concrete pedestals, the angle  $\alpha$  (see Fig.11.28.1) between the plane passing through the bottom edge of the pedestal and the corresponding junction edge of the column with pedestal and the horizontal plane shall be determined from the following expression (cl.34.1.3 of IS 456)

$$0.5 \tan 0.9 \left\{ \frac{100}{\alpha} \right\} \leq \frac{q_a}{f_{ck}} \leq 1$$

where  $q_a$  = calculated maximum bearing pressure at the base of pedestal in N/mm<sup>2</sup>, and  
 $f_{ck}$  = characteristic strength of concrete at 28 days in N/mm<sup>2</sup>.

**(c) Bending moments (cl. 34.2 of IS 456)**

1. It may be necessary to compute the bending moment at several sections of the footing depending on the type of footing, nature of loads and the distribution of pressure at the base of the footing. However, bending moment at any section shall be determined taking all forces acting over the entire area on one side of the section of the footing, which is obtained by passing a vertical plane at that section extending across the footing (cl.34.2.3.1 of IS 456).

2. The critical section of maximum bending moment for the purpose of designing an isolated concrete footing which supports a column, pedestal or wall shall be:

- (i) at the face of the column, pedestal or wall for footing supporting a concrete column, pedestal or reinforced concrete wall, and
- (ii) halfway between the centre-line and the edge of the wall, for footing under masonry wall. This is stipulated in cl.34.2.3.2 of IS 456.

The maximum moment at the critical section shall be determined as mentioned in 1 above.

For round or octagonal concrete column or pedestal, the face of the column or pedestal shall be taken as the side of a square inscribed within the perimeter of the round or octagonal column or pedestal (see cl.34.2.2 of IS 456 and Figs.11.28.13a and b).

**(d) Shear force (cl. 31.6 and 34.2.4 of IS 456)**

Footing slabs shall be checked in one-way or two-way shears depending on the nature of bending. If the slab bends primarily in one-way, the footing slab shall be checked in one-way vertical shear. On the other hand, when the bending is primarily two-way, the footing slab shall be checked in two-way shear or punching shear. The respective critical sections and design shear strengths are given below:

### **1. One-way shear (cl. 34.2.4 of IS 456)**

One-way shear has to be checked across the full width of the base slab on a vertical section located from the face of the column, pedestal or wall at a distance equal to

- (i) effective depth of the footing slab in case of footing slab on soil, and
- (ii) half the effective depth of the footing slab if the footing slab is on piles.

The design shear strength of concrete without shear reinforcement is given in Table 19 of cl.40.2 of IS 456.

### **2. Two-way or punching shear (cls.31.6 and 34.2.4)**

Two-way or punching shear shall be checked around the column on a perimeter half the effective depth of the footing slab away from the face of the column or pedestal.

The permissible shear stress, when shear reinforcement is not provided, shall not exceed  $\tau_c$ , where  $\tau_c = (0.5 + c\beta) \tau_{cs}$ , but not greater than one,  $c\beta$  being the ratio of short side to long side of the column, and  $\tau_{cs} = 0.25(f_{ck})^{1/2}$  in limit state method of design, as stipulated in cl.31.6.3 of IS 456.  $\tau_c \leq \tau_{cs}$

Normally, the thickness of the base slab is governed by shear. Hence, the necessary thickness of the slab has to be provided to avoid shear reinforcement.

### **(e) Bond (cl.34.2.4.3 of IS 456)**

The critical section for checking the development length in a footing slab shall be the same planes as those of bending moments in part (c) of this section. Moreover, development length shall be checked at all other sections where they change abruptly. The critical sections for checking the development length are given in cl.34.2.4.3 of IS 456, which further recommends to check the anchorage requirements if the reinforcement is curtailed, which shall be done in accordance with cl.26.2.3 of IS 456

### **(f) Tensile reinforcement (cl.34.3 of IS 456)**

The distribution of the total tensile reinforcement, calculated in accordance with the moment at critical sections, as specified in part (c) of this section, shall be done as given below for one-way and two-way footing slabs separately.

- (i) In one-way reinforced footing slabs like wall footings, the reinforcement shall be distributed uniformly across the full width of the footing i.e., perpendicular to the direction of wall. Nominal distribution reinforcement shall be provided as per cl. 34.5 of IS 456 along the length of the wall to take care of the secondary moment, differential settlement, shrinkage and temperature effects.
- (ii) In two-way reinforced square footing slabs, the reinforcement extending in each direction shall be distributed uniformly across the full width/length of the footing.

iii) In two-way reinforced rectangular footing slabs, the reinforcement in the long direction shall be distributed uniformly across the full width of the footing slab. In the short direction, a central band equal to the width of the footing shall be marked along the length of the footing, where the portion of the reinforcement shall be determined as given in the equation below. This portion of the reinforcement shall be distributed across the central band

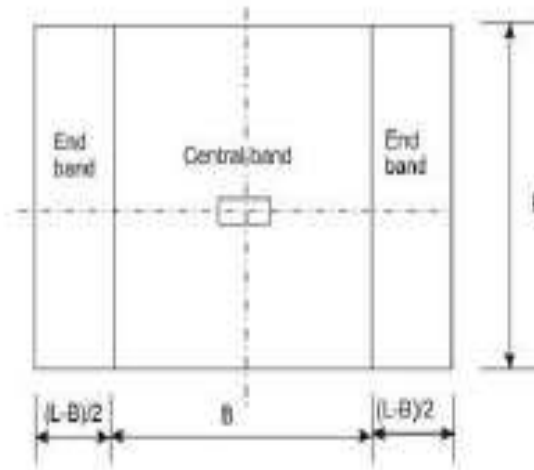


Figure 3.10 Bands for reinforcement in rectangular footing

Reinforcement in the central band =  $\frac{2}{\beta+1}$  (Total reinforcement in the short direction)

Where  $\beta$  is the ratio of longer dimension to shorter dimension of the footing slab (Fig.3.10).

Each of the two end bands shall be provided with half of the remaining reinforcement, distributed uniformly across the respective end band.

**(g) Transfer of load at the base of column (cl.34.4 of IS 456)**

All forces and moments acting at the base of the column must be transferred to the pedestal, if any, and then from the base of the pedestal to the footing, (or directly from the base of the column to the footing if there is no pedestal) by compression in concrete and steel and tension in steel. Compression forces are transferred through direct bearing while tension forces are transferred through developed reinforcement. The permissible bearing stresses on full area of concrete shall be taken as given below from cl.34.4 of IS 456:



□  $br = 0.25f_{ck}$  , in working stress method, and

□  $br = 0.45f_{ck}$  , in limit state method

The stress of concrete is taken as  $0.45f_{ck}$

while designing the column. Since the area of

footing is much larger, this bearing stress of concrete in column may be increased

considering the dispersion of the concentrated load of column to footing. Accordingly, the

permissible bearing stress of concrete in footing is given by (cl.34.4 of IS 456):

□  $br = 0.45f_{ck}(A_1/A_2)^{1/2}$

with a condition that

$(A_1/A_2)^{1/2} \leq 2$

where  $A_1$  = maximum supporting area of footing for bearing which is geometrically similar to and concentric with the loaded area  $A_2$

$A_2$  = loaded area at the base of the column.

The above clause further stipulates that in sloped or stepped footings,  $A_1$  may be taken as the area of the lower base of the largest frustum of a pyramid or cone contained wholly within the footing and having for its upper base, the area actually loaded and having side slope of one vertical to two horizontal.

If the permissible bearing stress on concrete in column or in footing is exceeded, reinforcement shall be provided for developing the excess force (cl.34.4.1 of IS 456), either by extending the longitudinal bars of columns into the footing (cl.34.4.2 of IS 456) or by providing dowels as stipulated in cl.34.4.3 of IS 456 and given below:

(i) Sufficient development length of the reinforcement shall be provided to transfer the compression or tension to the supporting member in accordance with cl.26.2 of IS 456, when transfer of force is accomplished by reinforcement of column (cl.34.4.2 of IS 456).

(ii) Minimum area of extended longitudinal bars or dowels shall be 0.5 per cent of the cross-sectional area of the supported column or pedestal (cl.34.4.3 of IS 456).

(iii) A minimum of four bars shall be provided (cl.34.4.3 of IS 456).

(iv) The diameter of dowels shall not exceed the diameter of column bars by more than 3 mm.

(v) Column bars of diameter larger than 36 mm, in compression only can be doweled at the footings with bars of smaller size of the necessary area. The dowel shall extend into the column, a distance equal to the development length of the column bar and into the footing, a distance equal to the development length of the dowel, as stipulated in cl.34.4.4 of IS 456.

#### **(h) Nominal reinforcement (cl. 34.5 of IS 456)**

Clause 34.5.1 of IS 456 stipulates the minimum reinforcement and spacing of the bars in footing slabs as per the requirements of solid slab (cls.26.5.2.1 and 26.3.3b(2) of IS 456, respectively).

#### **Design of Staircase**

The staircase is an important component of a building, and often the only means of access between the various floors in the building. It consists of a *flight* of steps, usually with one or more intermediate *landings* (horizontal slab platforms) provided between the floor levels. The horizontal top portion of a step (where the foot rests) is termed *tread* and the vertical projection of the step (i.e., the vertical distance between two neighbouring steps) is called *riser* [Fig. 2.10]. Values of 300 mm and 150 mm are ideally assigned to the tread and riser respectively — particularly in public buildings. However, lower values of tread (up to 250 mm) combined with higher values of riser (up to 190 mm) are resorted to in residential and factory buildings. The *width* of the stair is generally around 1.1 – 1.6m, and in any case, should normally not be less than 850 mm; large stair widths are encountered in entrances to public buildings. The horizontal projection (plan) of an inclined flight of steps, between the first and last risers, is termed *going*. A typical flight of steps consists of two landings and one

going, as depicted in Fig. 2.10(a). Generally, risers in a flight should not exceed about 12 in number. The steps in the flight can be designed in a number of ways: with *waist slab*, with *tread-riser* arrangement (without waist slab) or with *isolated tread slabs* — as shown in Fig. 2.10(b), (c), (d) respectively.

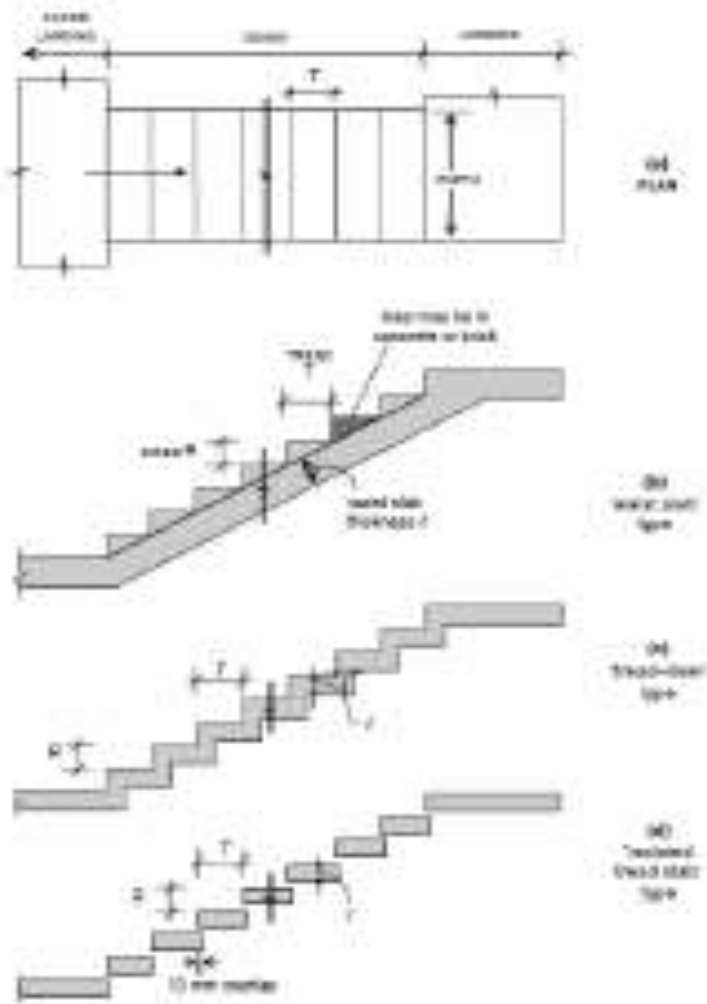


Figure 2.10 A typical flight in a staircase

## TYPES OF STAIRCASES

### Geometrical Configurations

A wide variety of staircases are met with in practice. Some of the more common geometrical configurations are depicted in Fig. 2.11. These include:

- straight stairs (with or without intermediate landing) [Fig. 2.11 (a)]
- quarter-turn stairs [Fig. 2.11 (b)]
- dog-legged stairs [Fig. 2.11 (c)]
- open well stairs [Fig. 2.11 (d)]
- spiral stairs [Fig. 2.11 (e)]
- helicoidal stairs [Fig. 2.11 (f)]

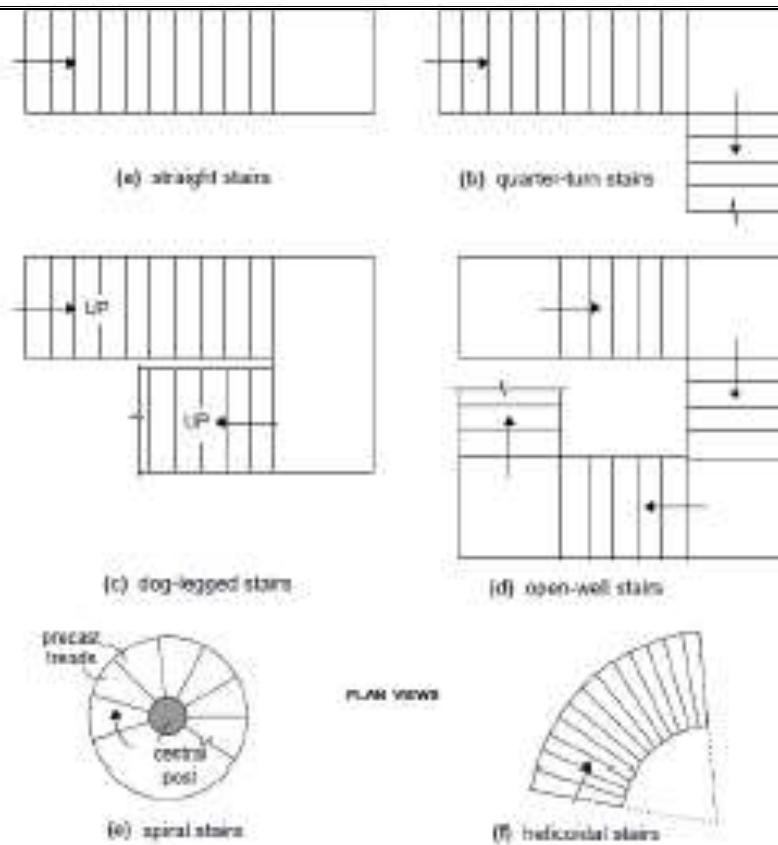


Fig. 2.11 Common geometrical configurations of stairs

### Structural Classification

Structurally, staircases may be classified largely into two categories, depending on the predominant direction in which the slab component of the stair undergoes flexure:

1. Stair slab spanning transversely (stair widthwise);
2. Stair slab spanning longitudinally (along the incline).

#### Stair Slab Spanning Transversely

The slab component of the stair (whether comprising an isolated tread slab, a tread-riser unit or a waist slab) is supported on its side(s) or cantilevers laterally from a central support. The slab supports gravity loads by bending essentially in a *transverse vertical plane*, with the span along the *width* of the stair.

In the case of the cantilevered slabs, it is economical to provide isolated treads (without risers). However, the tread-riser type of arrangement and the waist slab type are also sometimes employed in practice, as cantilevers. The spandrel beam is subjected to torsion ('\_equilibrium torsion'), in addition to flexure and shear.

When the slab is supported at the two sides by means of ‘stringer beams’ or masonry walls, it may be designed as simply supported, but reinforcement at the top should be provided near the supports to resist the ‘negative’ moments that may arise on account of possible partial fixity.

### Stair Slab Spanning Longitudinally

In this case, the supports to the stair slab are provided parallel to the riser at two or more locations, causing the slab to bend longitudinally between the supports. It may be noted that longitudinal bending can occur in configurations other than the straight stair configuration, such as quarter-turn stairs, dog-legged stairs, open well stairs and helicoidal stairs .

The slab arrangement may either be the conventional ‘waist slab’ type or the ‘tread-riser’ type. The slab thickness depends on the ‘effective span’, which should be taken as the centre-to-centre distance between the beam/wall supports, according to the Code (Cl. 33.1a, c). In certain situations, beam or wall supports may not be available parallel to the riser at the landing. Instead, the flight is supported between the landings, which span transversely, parallel to the risers. In such cases, the Code (Cl. 33.1b) specifies that the effective span for the flight (spanning longitudinally) should be taken as *the going of the stairs plus at each end either half the width of the landing or one metre, whichever is smaller.*

### Numerical Problem

Design a ‘waist slab’ type dog-legged staircase for an office building, given the following data:

- Height between floor = 3.2 m;
- Riser = 160 mm, tread = 270 mm;
- Width of flight = landing width = 1.25 m
- Live load = 5.0 kN/m<sup>2</sup>
- Finishes load = 0.6 kN/m<sup>2</sup>

Assume the stairs to be supported on 230 mm thick masonry walls at the outer edges of the landing, parallel to the risers [Fig. 12.13(a)]. Use M 20 concrete and Fe 415 steel. Assume *mild* exposure conditions.

### Solution

Given:  $R = 160 \text{ mm}$ ,  $T = 270 \text{ mm} \Rightarrow +RT22$

$= 314 \text{ mm}$  Effective span = c/c distance between supports = 5.16 m [Fig below].

- Assume a waist slab thickness  $\approx l/20 = 5160/20 = 258 \rightarrow 260 \text{ mm}$ .

Assuming 20 mm clear cover (*mild* exposure) and 12  $\theta$  main bars,

effective depth  $d = 260 - 20 - 12/2 = 234 \text{ mm}$ .

The slab thickness in the landing regions may be taken as 200 mm, as the bending moments are relatively low here.

*Loads on going* [fig. below] on projected plan area:

- (1) self-weight of waist slab @  $25 \times 0.26 \times 314/270 = 7.56 \text{ kN/m}^2$
- (2) self-weight of steps @  $25 \times (0.5 \times 0.16) = 2.00 \text{ kN/m}^2$

(3) finishes (given) = 0.60 kN/m<sup>2</sup>

(4) live load (given) = 5.00 kN/m<sup>2</sup>

Total = 15.16 kN/m<sup>2</sup>

⇒ Factored load = 15.16 × 1.5 = 22.74 kN/m<sup>2</sup>

• *Loads on landing*

(1) self-weight of slab @ 25 × 0.20 = 5.00 kN/m<sup>2</sup>

(2) finishes @ 0.6 kN/m<sup>2</sup>

(3) live loads @ 5.0 kN/m<sup>2</sup>

Total = 10.60 kN/m<sup>2</sup>

⇒ Factored load = 10.60 × 1.5 = 15.90 kN/m<sup>2</sup>

• *Design Moment* [Fig. below]

Reaction  $R = (15.90 \times 1.365) + (22.74 \times 2.43) / 2 = 49.33$  kN/m

Maximum moment at midspan:

$$\begin{aligned} M_u &= (49.33 \times 2.58) - (15.90 \times 1.365) \times (2.58 - 1.365/2) \\ &\quad - (22.74) \times (2.58 - 1.365)^2 / 2 \\ &= 69.30 \text{ Kn-m} \end{aligned}$$

• *Main reinforcement*

$$= 1.265 \text{ MPa } R_{bd} \square$$

Assuming  $f_{ck} = 20$  MPa,  $f_y = 415$  MPa,

$$2.0381 \times 10 \times 100 \times 100 \text{ t stp } A_x \square \square \square$$

$$\Rightarrow 2.32 \times (0.381 \times 10) \times 10 \times 234 \times 892 / \text{ stre } q A_{xxx} \text{ mm } m \square \square \square$$

Required spacing of 12  $\theta$  bars = 127 mm

Required spacing of 16  $\theta$  bars = 225 mm

**Provide 16  $\theta$  @ 220c/c**

• *Distributors*

$$2 \times (0.0012312 / \text{ stre } q A_{bt} \text{ mm } m \square \square$$

spacing 10  $\theta$  bars = 251 mm

**Provide 10  $\theta$  @ 250c/c as distributors.**

## **Introduction**

Whenever two or more columns in a straight line are carried on a single spread footing, it is called a combined footing. Isolated footings for each column are generally the economical. Combined footings are provided only when it is absolutely necessary, as

- i) When two columns are close together, causing overlap of adjacent isolated footings
- ii) Where soil bearing capacity is low, causing overlap of adjacent isolated footings
- iii) Proximity of building line or existing building or sewer, adjacent to a building column.

The combined footing may be rectangular, trapezoidal or Tee-shaped in plan. The geometric proportions and shape are so fixed that the centroid of the footing area coincides with the resultant of the column loads. This results in uniform pressure below the entire area of footing.

Trapezoidal footing is provided when one column load is much more than the other. As a result, the both projections of footing beyond the faces of the columns will be restricted. Rectangular footing is provided when one of the projections of the footing is restricted or the width of the footing is restricted.

### **Rectangular combined footing**

Longitudinally, the footing acts as an upward loaded beam spanning between columns and cantilevering beyond. Using statics, the shear force and bending moment diagrams in the longitudinal direction are drawn. Moment is checked at the faces of the column. Shear force is critical at distance 'd' from the faces of columns or at the point of contra flexure. Two-way shear is checked under the heavier column.

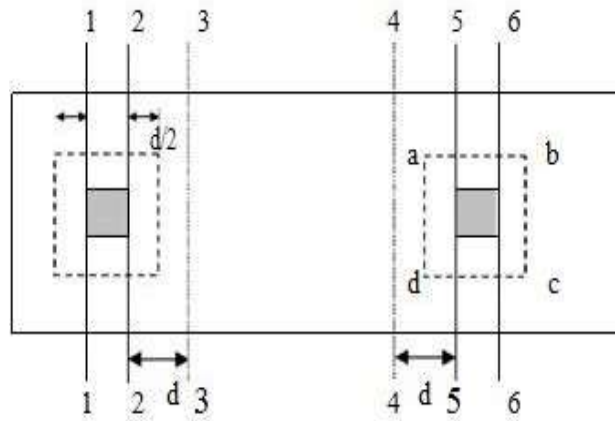
The footing is also subjected to transverse bending and this bending is spread over a transverse strip near the column.

Combined footing may be of slab type or slab and beam type or slab and strap beam type

Design:

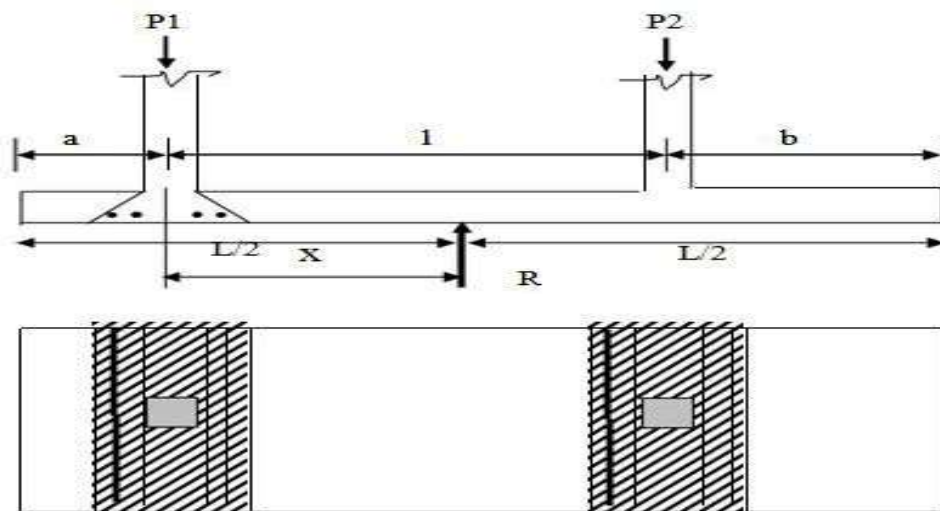
1. Locate the point of application of the column loads on the footing.
2. Proportion the footing such that the resultant of loads passes through the centre of footing
3. Compute the area of footing such that the allowable soil pressure is not exceeded.
4. Calculate the shear forces and bending moments at the salient points and hence draw SFD and BMD.
5. Fix the depth of footing from the maximum bending moment.
6. Calculate the transverse bending moment and design the transverse section for depth and reinforcement. Check for anchorage and shear.
7. Check the footing for longitudinal shear and hence design the longitudinal steel.

8. Design the reinforcement for the longitudinal moment and place them in the appropriate positions.
9. Check the development length for longitudinal steel
10. Curtail the longitudinal bars for economy
11. Draw and detail the reinforcement Prepare the bar bending schedule.



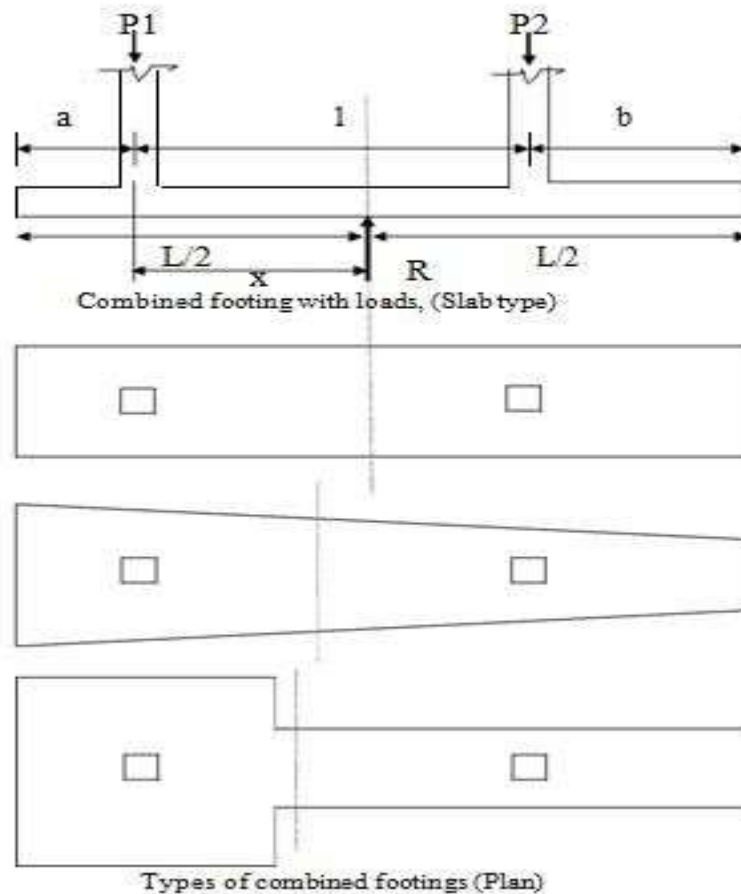
Section 1-1, 2-2, 5-5, and 6-6 are sections for critical moments Section 3-3, 4-4 are sections for critical shear (one

#### CRITICAL SECTIONS FOR MOMENTS



TRANSVERSE BEAM BELOW





### Objective

1. To provide basic knowledge in the areas of limit state method and concept of design of RC and Steel structures
2. To identify, formulate and solve engineering problems in RC and Steel Structures.

### Step by Step Procedure of Isolated Footing Design:

#### Step -1: Determining size of footing:

Loads on footing consists of load from column, self-weight of footing and weight of soil above footing. For simplicity, self-weight of footing and weight of soil on footing is considered as 10 to 15% of the vertical load.

Load on column = 1000 kN

Extra load at 10% of load due to self-weight of soil =  $1000 \times 10\% = 100\text{kN}$

Therefore, total load  $P = 1100 \text{ kN}$ .

Size of footing to be designed can be square, rectangular or circular in plan. Here we will consider square isolated footing.

Therefore, length of footing (L) = Width of footing (B)

Therefore area of footing required =  $\frac{P}{\text{SBC}}$

$$= 1100/300 = 3.67 \text{ m}^2$$

Provide Length and width of footing = 2m

$$\text{Area of footing} = 2 \times 2 = 4\text{m}^2$$

Now the pressure on isolated footing is calculated as

$$\frac{P}{A} \pm \frac{M_y}{Z_y} \pm \frac{M_z}{Z_z}$$

When calculated,  $P_{\max} = 325 \text{ kN/m}^2$

$$p_{\min} = 175 \text{ kN/m}^2$$

But  $p_{\max}$  is greater than SBC of soil, so we need to revise the size of footing so that  $P_{\max}$  is below  $300 \text{ kN/m}^2$ .

Consider width and length of footing =  $L = B = 2.25\text{m}$

Now,  $p_{\max} = 250.21 \text{ kN/m}^2$  ( $< 300 \text{ kN/m}^2 \rightarrow \text{OK}$ )

and  $p_{\min} = 144.86 \text{ kN/m}^2 > 0$  (OK)

Hence, factored upward pressure of soil =  $p_{u\max} = 375.315 \text{ kN/m}^2$

$$p_{u\min} = 217.29 \text{ kN/m}^2$$

Further, average pressure at the center of the footing is given by  $P_{u,\text{avg}} = 296.3 \text{ kN/m}^2$

and, factored load,  $P_u = 1500 \text{ kN}$ , factored uniaxial moment,  $M_u = 150 \text{ kN-m}$ .

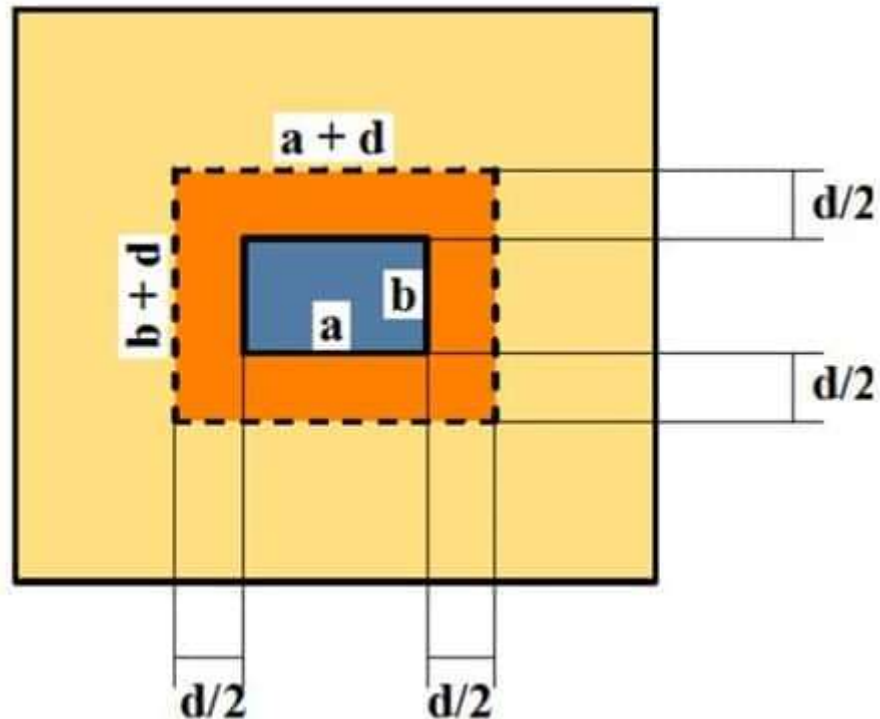
### Step 2: Two way shear

Assume an uniform overall thickness of footing,  $D = 500 \text{ mm}$

Assuming 16 mm diameter bars for main steel, effective depth of footing 'd' is

$$d = 500 - 50 - 8 = 452 \text{ mm}$$

The critical section for the two way shear or punching shear occurs at a distance of  $d/2$  from the face of the column (Fig. 1), where  $a$  and  $b$  are the dimensions of the column.



**Fig 1: Critical section for Two Way Shear (Punching Shear)**

Hence, punching area of footing =  $(a + d)^2 = (0.45 + 0.442)^2 = 0.796 \text{ m}^2$   
 where  $a = b =$  side of column

Punching shear force = Factored load – (Factored average pressure x punching area of footing)

$$= 1500 - (296.3 \times 0.796)$$

$$= 1264.245 \text{ kN}$$

Perimeter along the critical section =  $4(a + d) = 4(450 + 442) = 3568 \text{ mm}$

Therefore, nominal shear stress in punching or punching shear stress  $\zeta_v$  is calculated as below:

$$\zeta_v = \frac{\text{Punching shear force}}{\text{perimeter} \times \text{effective thickness}}$$

$$= 1264.245 \times 1000 / (3568 \times 442) = 0.802 \text{ N/mm}^2$$

$$\text{Allowable shear stress} = k_s \cdot \zeta_c$$

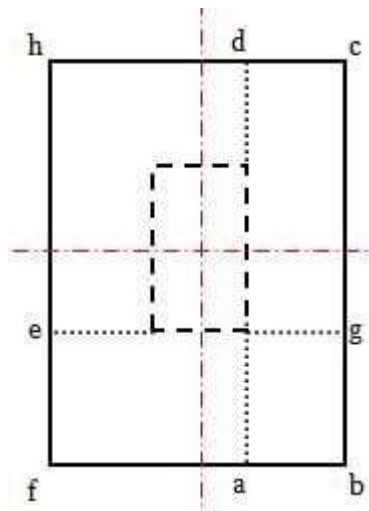
$$\text{where } \zeta_c = 0.25 \sqrt{f_{ck}} = 1.369 \text{ N/mm}^2$$

$$k_s = (0.5 + \beta_c) \frac{0.5 + \frac{0.45}{0.45}}{0.45} = 1$$

therefore, allowable shear stress =  $1 \times 1.369 = 1.369 \text{ N/mm}^2$   
 Since the punching shear stress ( $0.802 \text{ N/mm}^2$ ) is less than the allowable shear stress ( $1.369 \text{ N/mm}^2$ ), the assumed thickness is sufficient to resist the punching shear force. Hence, the assumed thickness of footing  $D = 500 \text{ mm}$  is sufficient. Please note, there is much difference between allowable and actual values of shear stress, so depth of footing can be revised and reduced. For our example, we will continue to use  $D = 500 \text{ mm}$ .

### Step 3: Design for flexure:

The critical section for flexure occurs at the face of the column (Fig. 2).



**Fig. 2 Critical section for flexure**

The projection of footing beyond the column face is treated as a cantilever slab subjected to factored upward pressure of soil.

Factored maximum upward pressure of soil,  $p_{u,max} = 375.315 \text{ kN/m}^2$

Factored upward pressure of soil at critical section,  $p_u = 312.1 \text{ kN/m}^2$

Projection of footing beyond the column face,  $l = (2250 - 450)/2 = 900 \text{ mm}$

Bending moment at the critical section in the footing is given by:

$M_u = \text{Total force} \times \text{Distance from the critical section}$

Considering uniform soil pressure of  $375.315$ ,  $M_u = 180 \text{ kN/m}^2$

$$\frac{M_u}{bd^2} = 0.92$$

from SP 16, percentage of reinforcement can be found for M30 concrete, fe415 steel for

above  $\frac{M_u}{bd^2} =$  pt = 0.265%

$$A_{st} = p_t \times b \times d$$

considering 1m wide footing,  $A_{st}$  required = 1171.1 mm<sup>2</sup>/ m width  
Provide 16 diabar @ 140mm c/c

Repeat this exercise for other direction as well. Since, uniform base pressure is assumed, and it is a square footing,  $M_u$  and  $A_{st}$  for other direction will be same.

#### **Step 4: Check for One-Way Shear:**

The critical section for one way shear occurs at a distance of 'd' from the face of the column.

Factored maximum upward pressure of soil,  $p_{u,max}$  = 375.315 kN/m<sup>2</sup>

Factored upward pressure of soil at critical section,  $p_u$  = 375.315 kN/m<sup>2</sup>

For the cantilever slab, total Shear Force along critical section considering the entire width B is

$$\begin{aligned} V_u &= \text{Total Force} \times (1 - d) \times B \\ &= 375.315 \times (0.9 - 0.442) \times 2 = 343.8 \text{ kN} \end{aligned}$$

$$\text{Nominal shear stress} = V_u / (B \times d) = 0.346 \text{ N/mm}^2$$

For,  $p_t = 0.265$ , and M30, allowable shear force from Table – 19, IS456 is greater than 0.346 N/mm<sup>2</sup>

Therefore, the foundation is safe in one-way shear.

#### **Step 5: Check for development length**

Sufficient development length should be available for the reinforcement from the critical section.

Here, the critical section considered for  $L_d$  is that of flexure.

The development length for 16 mm diameter bars is given by

$$L_d = 47 \times \text{diameter of bar} = 47 \times 16 = 752 \text{ mm.}$$

Providing 60 mm side cover, the total length available from the critical section is

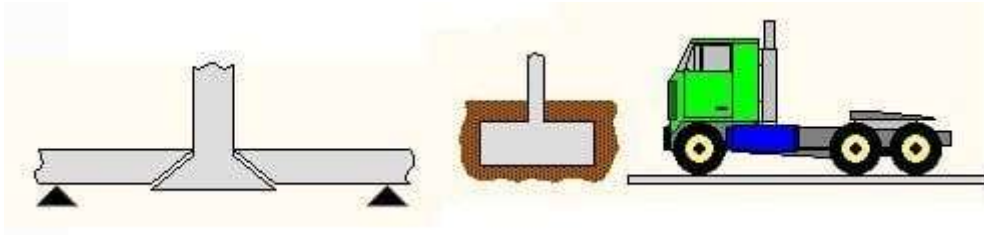
$$0.5 \times (L - a) - 60 = 0.5 \times (2250 - 450) - 60 = 840 > L_d, \text{ Hence O.K.}$$

#### **What is Punching Shear?**

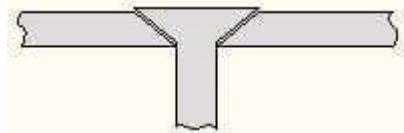
The punching shear is a failure mechanism in structural members like slabs and foundation by shear under the action of concentrated loads.

The action of concentrated loads is on a smaller area in the structural members. In most cases, this reaction is the one from the column acting against the slab. Eventually the slab will fail. One possible method of failure is that the load punches through the slab.

Some examples of the occurrence of concentrated loads on a slab are a column, particularly on a pad foundation, and wheel loads. This same type of failure could also happen in another way. Turning the structure upside down we get a flat slab supported by a column, where there is a high concentration of shear force around the column head.



When the total shear force exceeds the shear resistance of the slab, the slab will be pushed down around the column, or this can be viewed as the column being punched through the slab.



Punching shear failure mechanism is observed in normal floor slabs, flat slabs, and in the foundation slabs below the column. In pad foundations, where weight and depth are not so critical, it's effects are satisfied by providing sufficient depth.

### **Punching Shear in Reinforced Concrete Slabs**

The Punching shear in reinforced concrete slabs can be considered as a 2D analog of the shear observed in beams. This kind of failure occurs as a sudden rupture. This rupture cannot be restrained by the help of main reinforcement.

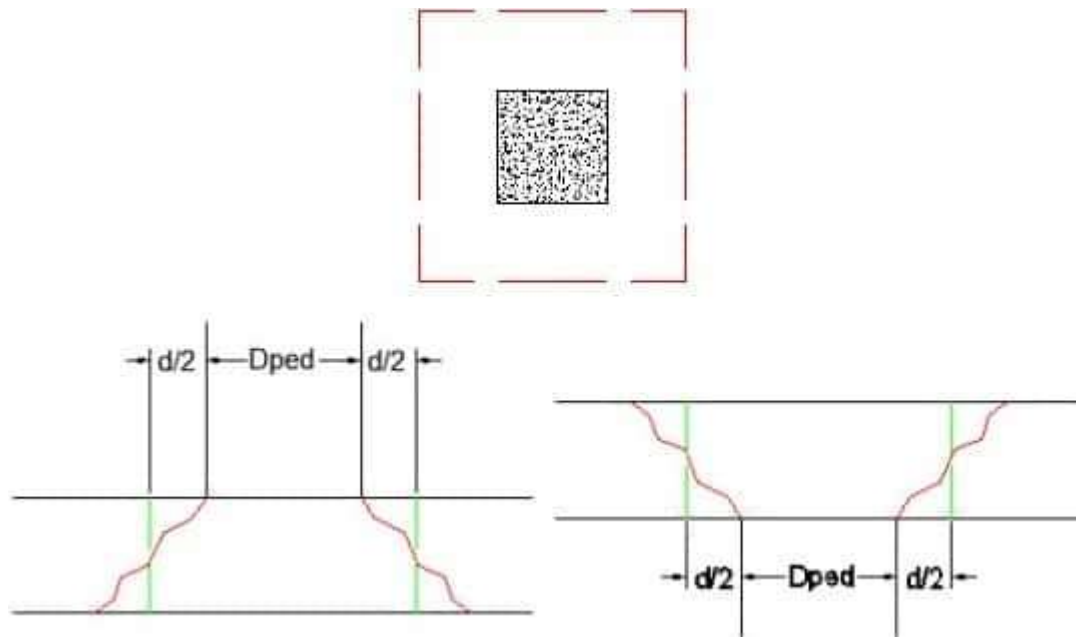
Hence, there is a reduction of ultimate load capacity of the structure below the flexural capacity by the action of shear. But when compared to beam shear, the punching shear is less critical.

In slabs, the punching shear occurs when they are subjected to high values of concentrated loads. These load categories includes the wheel loads on the bridge slabs or the loads from above floors on the columns that support the slab floors.

### **Calculations in Punching Shear**

The calculations of punching shear dealt in the design is based on the punching shear force that is punching against the thickness of the slab or foundation. This can happen only if there exists shear alone in the system. This is not possible if there exist any form of moment in the pedestal or the column.

The **Maximum Punching Shear Stress** is determined based on the punching shear failure cone and the applied values of shear and moments.



**Fig.1. Punching Shear Failure Zone for Slabs above and below the column**

The 'd' is the effective depth of the slab. The punching shear perimeter is formed at a distance of  $d/2$  from the edges of the column or the pedestal. In the figure-1,  $D_{ped}$  is the depth of the pedestal.

### **Design Considerations for Punching Shear in Slabs**

The Punching Failure in the structure can be prevented by taking the following control measures:

1. Undergo proper checking to make sure that the concrete itself is strong enough.
2. If the concrete lacks adequate strength, check whether the amount of reinforcement that is provided is reasonable.
3. If it is not reasonable, it is recommended to change the form of the structure.

### **The methods involved in changing the form of the structure are:**

1. Increasing the depth of the slab
2. The column size can be made larger
3. Incorporation of drop panels
4. Introduction of Flared column heads
5. Other foreign codes can be referred to practice other liberal designs

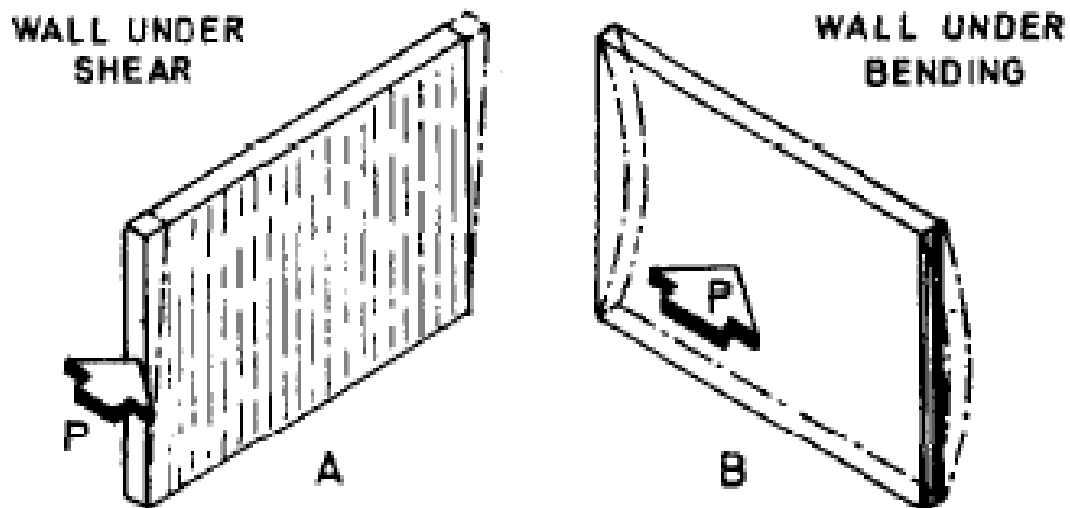
Different failure zones are mentioned below in figure-2 based along with the area where the reinforcement have to be provided. Usually vertical and traverse failure lines are possible. We do not actually know where the failure can occur. So, each possible failure plane must be reinforced.

Masonry is one of the main items of construction in a building and needs careful consideration. It comprises masonry units such as brick, stone, concrete block laid in mortar. There is a large variety of units and a number of different types and grades of mortars that are used in masonry. Architects and Engineers should have good knowledge of properties of units and mortars so as to be able to choose an appropriate combination of the two, to meet the requirements for a particular situation.

## STRUCTURAL DESIGN

### General

- i) Some general guidance on the design concept of load bearing masonry structures is given in the following paragraphs.
- ii) A building is basically subjected to two types of loads, namely:
  - a) Vertical loads on account of dead loads of materials used in construction, plus live loads due to occupancy; and
  - b) Lateral loads due to wind and seismic forces. While all walls in general can take vertical loads, ability of a wall to take lateral loads depends on its disposition in relation to the direction of lateral load. This could be best explained with the help of an illustration. In Fig. 4.1, the wall **A** has good resistance against a lateral load, while wall **B** offers very little resistance to such load. The lateral loads acting on the face of a building are transmitted through floors (which act as horizontal beams) to cross walls which act as shear walls. From cross walls, loads are transmitted to the foundation. This action is illustrated in Fig. 4.2. Stress pattern in cross walls due to lateral loads is illustrated in Fig. 4.3.



Resistance of brick wall to take lateral loads is greater in case of wall **A** than that in case of wall **B**.

FIG. 4.1 ABILITY OF A WALL TO TAKE LATERAL LOADS



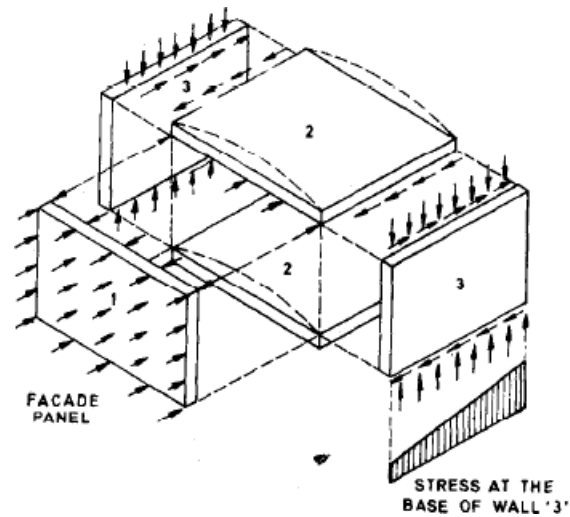


FIG. 4.2 Function of Lateral Support to Wall

iii) As a result of lateral load, in the cross walls there will be an increase of compressive stress on the leeward side, and decrease of compressive stress on the windward side. These walls should be designed for no tension and permissible compressive stress. It will be of interest to note that a wall which is carrying-greater vertical loads, will be in a better position to resist lateral loads than the one which is lightly loaded in the vertical

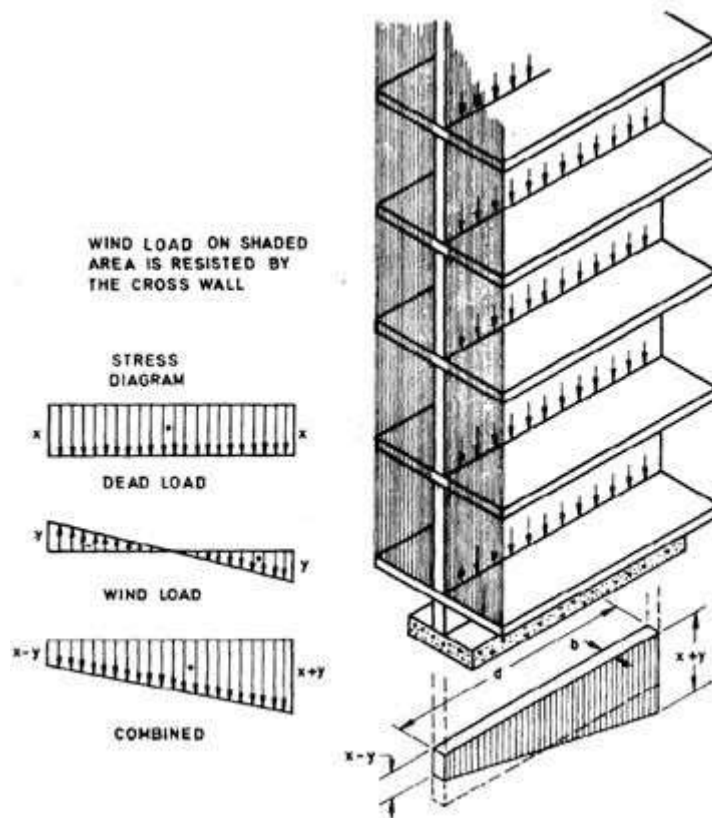


FIG. 4.3 Stress Pattern in Cross Wall Acting as Shear Wall

direction.

iv) A structure should have adequate stability in the direction of both the principal axes. The so called 'cross wall' construction may not have much lateral resistance in the longitudinal direction. In multi-storeyed buildings, it is desirable to adopt 'cellular' or 'box type' construction from consideration of stability and economy as illustrated in Fig. E-19.

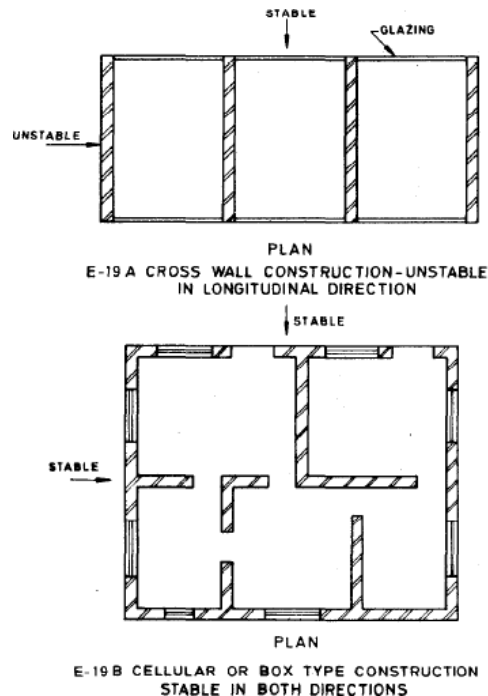


Fig. 4.4 Stability Of Cross Wall and Cellular (Box Type) Construction

v) Size, shape and location of openings in the external walls have considerable influence on stability and magnitude of stresses due to lateral loads. vi) If openings in longitudinal walls are so located that portions of these walls act as flanges to cross walls, the strength of the cross walls get considerably increased and structure becomes much more stable.

vii) Ordinarily a load-bearing masonry structure is designed for permissible compressive and shear stresses (with no tension) as a vertical cantilever by accepted principles of engineering mechanics. No moment transfer is allowed for, at floor to wall connections and lateral forces are assumed to be resisted by diaphragm action of floor, roof slabs, which acting as horizontal beams, transmit lateral forces to cross walls in proportion to their relative stiffness (moment of inertia).

#### **Design of isolated column footing.**

The superstructure is placed on the top of the foundation structure, designated as substructure as they are placed below the ground level. The elements of the superstructure transfer the loads and moments to its adjacent element below it and finally all loads and moments come to the foundation structure, which in turn, transfers them to the underlying soil or rock. Thus, the foundation structure effectively supports the superstructure. However, all types of soil get compressed significantly and cause the structure to settle. Accordingly, the major requirements of the design of foundation structures are the two as given below (see cl.34.1 of IS 456):

1. Foundation structures should be able to sustain the applied loads, moments, forces and induced reactions without exceeding the safe bearing capacity of the soil.
2. The settlement of the structure should be as uniform as possible and it should be within the tolerable limits. It is well known from the structural analysis that differential settlement of supports causes additional moments in statically indeterminate structures. Therefore, avoiding the differential settlement is considered as more important than maintaining uniform overall settlement of the structure.

## **Types of Foundation Structures**

### **1. Shallow Foundation**

Shallow foundations are used when the soil has sufficient strength within a short depth below the ground level. They need sufficient plan area to transfer the heavy loads to the base soil. These heavy loads are sustained by the reinforced concrete columns or walls (either of bricks or reinforced concrete) of much less areas of cross-section due to high strength of bricks or reinforced concrete when compared to that of soil. The strength of the soil, expressed as the safe bearing capacity of the soil is normally supplied by the geotechnical experts to the structural engineer. Shallow foundations are also designated as footings. The different types of shallow foundations or footings are discussed below.

- (i) Plain concrete pedestal footings
- (ii) Isolated footings
- (iii) Combined footings
- (iv) Strap footings
- (v) Strip foundation or wall footings
- (vi) Raft or mat foundation

### **2. Deep Foundation:**

As mentioned earlier, the shallow foundations need more plan areas due to the low strength of soil compared to that of masonry or reinforced concrete. However, shallow foundations are selected when the soil has moderately good strength, except the raft foundation which is good in poor condition of soil also. Raft foundations are under the category of shallow foundation as they have comparatively shallow depth than that of deep foundation. It is worth mentioning that the depth of raft foundation is much larger than those of other types of shallow foundations.

However, for poor condition of soil near to the surface, the bearing capacity is very less and foundation needed in such situation is the pile foundation. Piles are, in fact, small diameter columns which are driven or cast into the ground by suitable means. Precast piles are driven and cast-in-situ are cast. These piles support the structure by the skin friction between the pile surface and the surrounding soil and end bearing force, if such resistance is available to provide the bearing force. Accordingly, they are designated as frictional and end bearing piles. They are normally provided in a group with a pile cap at the top through which the loads of the superstructure are transferred to the piles.

Piles are very useful in marshy land where other types of foundation are impossible to construct. The length of the pile which is driven into the ground depends on the availability of hard soil/rock or the actual load test. Another advantage of the pile foundations is that they can resist uplift also in the same manner as they take the compression forces just by the skin friction in the opposite direction.

However, driving of pile is not an easy job and needs equipment and specially trained persons or agencies. Moreover, one has to select pile foundation in such a situation where the adjacent buildings are not likely to be damaged due to the driving of piles. The choice of driven or bored piles, in this regard, is critical.

Exhaustive designs of all types of foundations mentioned above are beyond the scope of this course. Accordingly, this module is restricted to the design of some of the shallow footings, frequently used for normal low rise buildings only.