Hall Ticket No | | | | | | | | | | | Question Paper Code: AHS004

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

B.Tech IV Semester End Examinations (Regular) - May, 2018

Regulation: IARE – R16

COMPLEX ANALYSIS AND PROBABILITY DISTRIBUTION

Time: 3 Hours **(Common to AE | EEE)** **Max Marks: 70**

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

UNIT – I

1. (a) If f(z) is an analytic function with constant modulus, show that f(z)is constant. **[7M]**

   (b) Show that the function $f(z) = \bar{Z}$is continuous at every point but not differentiable at any point. **[7M]**

2. (a) Find the points at which the function $f(z) = e^{|z|^2}$ is analytic. **[7M]**

   (b) Show that$u\,(x, y) = \cos x \, \cosh \, y$ is harmonic and find its conjugate harmonic function. **[7M]**

UNIT – II

3. (a) Evaluate $\int\limits_C \text{Re}z \, dz$, where C is the unit circle $x^2 + y^2 = 1$. **[7M]**

   (b) Evaluate $\int\limits_c \frac{e^{z^2+1}}{z} \, dz$, where $c: \; z = x + iy = 5\cos(t) - 3i\sin(t) \, , \; 0 \le t \le 2\pi$. **[7M]**

4. (a) Evaluate $\int\limits_c Z^{-2}dz$, C is a $|z - 1| = 1$. **[7M]**

   (b) Evaluate $\oint\limits_C \frac{z-\sin z}{z\sin z} \, dz$, where $C : |z - 3| = 1$. **[7M]**

UNIT – III

5. (a) Find the Taylor's series expansion of $f(z) = \log z$, about $z_0 = i$. Also find the radius of convergence **[7M]**

   (b) Evaluate $\int_0^{2\pi} \frac{d\theta}{2-\cos\theta}$ using contour integration. **[7M]**

6. (a) By using Cauchy's residue theorem, evaluate the integral $\int\limits_c \frac{z^2}{(z-1)^2(z+2)}dz$ where C is the circle |z|=5/2. **[7M]**

   (b) Find the bilinear transformation that maps the points z_1=0, z_2=-i, z_3=-1 on the points w_1=i, w_2=1, w_3=0. **[7M]**

UNIT – IV

7. (a) A box contains 12 items of which 4 are defective. A sample of 3 items is selected from the box. Let X denote the number of defective items in the sample. Find the probability distribution of X. Determine the mean and variance. **[7M]**

   (b) A petrol pump is supplied with petrol once a day. If its daily volume X of sales in thousands of litre is distributed by f(x)=$5(1-x)^4$, 0≤X≤1. What must be the capacity of its tank in order that the probability that its supply will be exhausted in a given day shall be 0.01? **[7M]**

8. (a) A continuous random variable X has p.d.f f(x)=$3x^2$,0≤x≤1. Find 'a' and 'b' such that
   i. P(x≤a)=P(x≥a)
   ii. P(X<b)=0.05.

   [7M]

   (b) If the moments of a variable X are defined by $E(x^r)$=0.6, r=1,2,3. Show that P(X=0)=0.4, P(X=1)=0.6,$P(X \geq 2) = 0$.

   [7M]

UNIT – V

9. (a) If X is a normal distribution with mean 5 and variance 2 then find P $\{|X-1| \leq 5\}$. **[7M]**

   (b) 30% of items produced from a factory are defective. Find the probability that in a sample of 5 chosen at random
   i. None is defective
   ii. One is defective
   iii. At least 3 are defective.

   [7M]

10. (a) If the probability that an individual suffers a bad reaction from a certain injection is 0.001. Determine the probability that out of 2000 individuals
    i. Exactly 3 suffers a bad reaction
    ii. More than 2 individuals suffers a bad reaction
    iii. None suffers a bad reaction.

    [7M]

    (b) If the probability density function of a random variable is then $f_X(x) = \frac{1}{\sqrt{10\pi}} e^{-\frac{(x-2)^2}{10}}$ find mean, variance and $P\{-1 < X \leq 3\}$

    [7M]

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INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

B.Tech IV Semester End Examinations (Supplementary) - July, 2018

Regulation: IARE – R16

COMPLEX ANALYSIS AND PROBABILITY DISTRIBUTION

Time: 3 Hours **(Common to AE | EEE)** **Max Marks: 70**

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

UNIT – I

1. (a) Show that an analytic function with constant absolute value is constant. **[7M]**

   (b) Find the derivative of an analytic function whose real part is $e^x \sin y$. **[7M]**

2. (a) Show that $e^{(\overline{z})}$ is everywhere analytic. **[7M]**

   (b) Find the analytic function, whose real part is sin2x/(cosh2y-cos2x). **[7M]**

UNIT – II

3. (a) Evaluate $I = \int_{(z=0)}^{(2+i)} (\bar{z})^2 dz$ along the straight line y=x/2. **[7M]**

   (b) Verify Cauchy's theorem for the integral of f(z)=1/z taken along the triangle formed by the points (1, 2), (3, 2), (1, 4). **[7M]**

4. (a) Expand f(z)=(7z-2)/(z+1)z(z-2) in power series in the region 1<|z+1|<3. **[7M]**

   (b) Evaluate $\int\limits_c e^{2z}/(z+1)^4 dz$, where C is the circle |z-1|=3. **[7M]**

UNIT – III

5. (a) Find the Laurent's series expansion of $f(z) = \frac{1-\cos z}{z^3}$ about $z = 0$ and hence find the type of isolated singularity and also residue at z=0. **[7M]**

   (b) Evaluate $\int_{-\pi}^{\pi} \frac{\cos\theta d\theta}{1+a^2-2a\cos\theta}$ using contour integration. **[7M]**

6. (a) Evaluate $\int\limits_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$ using contour integration. **[7M]**

   (b) Find the Taylor's expansion of f(z)=(2z^3+1)/(z^2+z) about the point z=i. **[7M]**

UNIT – IV

7. (a) If the distribution of a random variable X is $F(x) = \begin{cases} 1 - e^{-(x-1)}, & \text{if } x \geq 1 \\ 0, & x < 1 \end{cases}$ then find $f_X(x)$, $P\{2 \leq X \leq 3\}$, $P\{X \leq 2\}$ **[7M]**

(b) Check whether the function $f(x) = \begin{cases} 0, & x < 2 \\ \frac{1}{18}(2x+3), & 2 \leq x \leq 4 \\ 0, & x > 4 \end{cases}$ is a valid density. If, so find $P\{2 \leq X \leq 3\}$. **[7M]**

8. (a) Determine whether the $G_X(x) = \begin{cases} 1 - e^{-\frac{x}{2}}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ is a valid distribution function. If it is a distribution function then find $P\{-2 < X \leq 3\}$. **[7M]**

(b) If the probability density function of a random variable X is $f_X(x) = e^{-x}$ then find its mean, variance and $P\{|X| \leq 1\}$. **[7M]**

UNIT – V

9. (a) The mean and varience of a variable X with parameters n and p are 16 and 8. Find P(X≤ 1) and P(X>2). **[7M]**

(b) The Probability of a man hitting a target is 1/3

i.If he fires 5 times, What is he probability of his hitting the target twice?

ii. How many times must he fire so that the probability of his hitting the target least once is more than 90%? **[7M]**

10. (a) Fit a Poisson distribution to the data shown in Table 1 which gives the number of doddens in a sample of clover seeds. **[7M]**

Table 1

No.of doddens:	0	1	2	3	4	5	6	7	8
Observed freq:	56	156	132	92	37	22	4	0	1

(b) Of a large group of men, 5% are under 60 inches in height and 40% are between 60 and 65 inches. Assuming a normal distribution, find the mean height and standard deviation. **[7M]**

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INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

B.Tech IV Semester End Examinations (Regular / Supplementary) - May, 2019

Regulation: IARE – R16

COMPLEX ANALYSIS AND PROBABILITY DISTRIBUTION

Time: 3 Hours **(Common to AE | EEE)** **Max Marks: 70**

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

UNIT – I

1. (a) Define the term Continuity of a complex variable function f(z). Justify whether every differentiable function is continuous or not. Give a valid example. **[7M]**

   (b) If $f(z) = u + iv = \frac{1}{z}$, then show that u(x,y)=$c_1$ and v(x,y)=c_2 the curves are intersects orthogonally. **[7M]**

2. (a) Define the term Analyticity and Differentiability of a complex variable function f (z). Prove that an analytic function f (z) with constant real part is always constant **[7M]**

   (b) Show that the function u= x^3-3xy^2 is harmonic and find the corresponding analytic function. **[7M]**

UNIT – II

3. (a) Define the term Power series expansions of complex functions. Write the Cauchy's integral formula and Cauchy's integral formula for multiple connected region. **[7M]**

   (b) Verify Cauchy's theorem for the function f(z)=z+1 in the region of c with vertices z=0, z=1, z=1+i, z=i. **[7M]**

4. (a) Define the term line integral. Evaluate $\int_0^{2+i} z^2 dz$ along the real axis to 2 and then vertically to 2+i. **[7M]**

   (b) Evaluate $\int_c \left(3x^2 + 4\,x\,y + i\,x^2\right) dz$ along the parabola $y = x^2$ from (0,0) to (1,1). **[7M]**

UNIT – III

5. (a) State Cauchy's Residue theorem of an analytic function f(z) within and on the closed curve, Taylor's theorem and Laurent theorem of complex power series. **[7M]**

   (b) Represent the function f(z)= $\frac{4z+3}{z(z-3)(z-2)}$ as Laurent series **[7M]**

   (i) With in |z|=1

   (ii) In the annulus region |z|=2 and |z|=3

   (iii) Exterior to |z|=3.

6. (a) Define **[7M]**
   i. The Isolated singularity of an analytic function f(z).
   ii. Pole of order m of an analytic function f(z).
   iii. Essential and removable singularity of an analytic function f(z).

   (b) Prove that $\int_0^{\pi} \frac{\cos 2\theta}{1-2a\ \cos\theta+a^2}\, d\theta = \frac{\pi a^2}{1-a^2},\ \left(a^2 < 1\right)$ using Residue theorem. **[7M]**

UNIT – IV

7. (a) Express the relation between the probability mass and cumulative mass function of a random variable. List the important properties of probability mass function **[7M]**

   (b) A random variable X has the following probability distribution as shown in Table 1. **[7M]**
   Determine (i) k (ii) Mean (iii) Variance (iv) $P\,(X < 6)$, (v) $P\,(0 < X < 5)$

Table 1

x	0	1	2	3	4	5	6	7
p(x)	0	K	2k	2k	3k	k^2	$2k^2$	$7k^2$+k

8. (a) Define the term probability density function. Explain mean and variance of a probability density function. Obtain the first 4 moments for the set of numbers 2, 4, 6 and 8. **[7M]**

   (b) Let X denote the maximum of the two numbers that appear when a pair of fair dice is thrown once. Find (i) Discrete probability distribution (ii) Expectation and (iii) Variance **[7M]**

UNIT – V

9. (a) Explain in detail about mean and variance of Binomial distribution. Draft the recurrence relation for the Binomial distribution. **[7M]**

   (b) Assume that 50% of all engineering students are good in mathematics. Determine the probabilities that among 18 engineering students (i)exactly10 (ii) At least 10 (iii) At most 8 (iv) At most 9 are good in mathematics. **[7M]**

10. (a) Explain the median and variance of a Normal distribution. **[7M]**

    (b) The marks obtained in mathematics by 1000 students is normally distributed with mean 78% and standard deviation 11%. Determine (i) How many students got marks above 90%. (ii) What was the highest mark obtained by the lowest 10% of the students. **[7M]**

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