INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad - 500 043

ELECTRICAL AND ELECTRONICS ENGINEERING

ASSIGNMENT

Course Name	:	**Engineering Mathematics - III**
Course Code	:	**A30007**
Class	:	**II B. Tech I Sem**
Branch	:	**EEE**
Year	:	**2016 - 2017**
Course Faculty	:	Mr. G. Nagendra Kumar, Assistant Professor

OBJECTIVES

To meet the challenge of ensuring excellence in engineering education, the issue of quality needs to be addressed, debated and taken forward in a systematic manner. Accreditation is the principal means of quality assurance in higher education. The major emphasis of accreditation process is to measure the outcomes of the program that is being accredited.

In line with this, Faculty of Institute of Aeronautical Engineering, Hyderabad has taken a lead in incorporating philosophy of outcome based education in the process of problem solving and career development. So, all students of the institute should understand the depth and approach of course to be taught through this question bank, which will enhance learner's learning process.

S. No	Question	Blooms Taxonomy Level	Course Outcome
	UNIT - I **LINEAR ODE WITH VARIABLE COEFFICIENTS AND SERIES SOLUTION (SECOND ORDER ONLY)**		
	Part – A (Short Answer Questions)		
1	Solve $(x^2D^2 - 4xD + 6)y = x^2$	Evaluate	1
2	Solve$(x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y) = logx$	Analyse	1
3	Solve $(x^2D^2 + 4xD + 2)y = e^x$	Evaluate	1
4	Solve $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 12y = x^3\log x$	Analyse	1
5	Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x.$	Evaluate	1
6	Solve in series the equation $x(1-x)y'' - (1+3x)\ y' - y = 0$	Analyse	3
7	Solve in series the equation $(x - x^2)y'' + (1-5x)y' - 4y = 0$	Evaluate	3
8	Solve$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12logx}{x^2}$	Evaluate	1
9	Solve$(x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y) = x^4$	Evaluate	1
10	Find the power series solution of the equation $y'' + (x-3)y' + y = 0$ in powers of (x-2) (i.e,about x=2)	Analyse	3
11	Solve in series the equation $\frac{d^2y}{dx^2} - y = 0$ about x=0	Evaluate	3
12	Solve in series the equation $y'' + y = 0$ about x=0	Evaluate	3
13	Solve in series the equation $\frac{d^2y}{dx^2} + xy = 0$	Evaluate	3

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14	Solve in series the equation $2x^2\, y'' + (x^2 - x)\, y' + y = 0$	Evaluate	3
15	Solve in series the equation $y'' + x^2 y = 0$ about x=0	Evaluate	3
	Part – B (Long Answer Questions)		
1	Solve $(2x-1)^3 \frac{d^3y}{dx^3} + (2x-1)\frac{dy}{dx}$ - 2y = x.	Evaluate	1
2	Solve $(x^2D^2 - 3xD + 1)y = \log x \left(\frac{\sin(\log x)+1}{x}\right)$	Evaluate	1
3	Solve in series the equation $4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$	Understand	3
4	Solve in series the equation $9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$	Evaluate	3
5	Solve in series the equation $x(1-x)y'' - 3x\ y' - y = 0$	Evaluate	3
6	$Solve (x+1)^2\frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} + y = \sin 2(\log(1+x))$	Evaluate	1
7	Solve in series the equation $2x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + 3y = 0$	Analyse	3
8	Solve in series the equation $(x-x^2)y'' + (1-x)y' - y = 0$	Understand	3
9	Solve in series the equation $x(1-x)y'' - (1+3x)\ y' - y = 0$	Evaluate	3
10	Solve $(x^2D^2 - 4xD + 6)y = (\log x)^2$	Analyse	1
11	Solve$(x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y) = (1+x)^2$	Evaluate	1
12	Solve $(x+1)^2\frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 4y = x^2$ + x+1	Evaluate	1
13	Solve$\left(x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 8\right)y = 65\cos(\log x)$	Understand	1
14	Solve in series the equation $xy'' + (1+x)\ y' + 2y = 0$	Understand	3
15	Solve$(x+1)^2\frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx}$ = (2x+1)(2x+4)	Evaluate	1
	UNIT - II **SPECIAL FUNCTIONS**		
	Part – A (Short Answer Questions)		
1	Express f(x) = $2x + 10x^3$ in terms of Legendre polynomials	Analyse	4
2	Show that $x^3 = \frac{2}{5}P_3(x) + \frac{3}{5}P_1(x)$	Analyse	4
3	Evaluate the value of $J_{\frac{1}{2}}(x)$ is	Analyse	4
4	$Show\ that \frac{d}{dx}[x^{-n}J_n(x)] = -x^nJ_{n+1}$(x)	Analyse	4
5	Prove that $J_{-n}(x) = (-1)^n J_n(x)$ n is a positive integer	Create	4
6	Show that $J_3(x) + 3J_0'(x) + 4J_0'''(x) = 0$	Analyse	4
7	Prove that $\frac{d}{dx}[x^nJ_n(x)] = x^nJ_{n-1}$(x)	Analyse	4
8	Prove that a) $\int_0^r x\ J_0(ax) = \frac{r}{a}J_1(ar)$	Analyse	4
9	show that $J_n(x)$ is an even function function if 'n' is even and odd function when 'n' is odd	remember	4
10	Prove that $\left[J_{\frac{1}{2}}\right]^2 + \left[J_{-\frac{1}{2}}\right]^2 = \frac{2}{\pi x}$	Analyse	4
11	Show that $\int_0^x x^nJ_{n-1}(x)dx = x^nJ_n(x)$	Analyse	4
12	show $\int_0^x x^{n+1}J_n(x)dx = x^{n+1}J_{n+1}(x)$	Analyse	4

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13	Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$	Analyse	4
14	Show that $J_0(x) = \frac{1}{\pi}\int_0^{\pi} cos(xsin\theta)d\theta$ satisfies Bessel's equation of order zero.	Analyse	4
15	Express $J_2(x)$ in terms of $J_0(x)$ and $J_1(x)$	Create	4
Part – B (Long Answer Questions)			
1	Prove that $J_n(x)$ =0 has no repeated roots except at x=0	Evaluate	4
2	Show that $J_3(x) + 3J_0'(x) + 4J_0'''(x) = 0$	Understand	4
3	Prove that $J_0^2 + 2(J_1^2 + J_2^2 + J_3^2 + \cdots) = 1$	Evaluate	4
4	show that $J_{n-1}(x) = \frac{2}{x}[nJ_n - (n+2)J_{n+2} + (n+5)J_{n+5....}]$	Evaluate	4
5	Show that $\int_0^1 x^2 P_{n+1}(x)P_{n-1}(x)dx = \frac{n(n+1)}{(4n^2-1)(2n+3)}$	Evaluate	4
6	If f(x) = 0 if -1 < x < 0 =1 if 0<x<1 Then show that f(x) = $\frac{1}{2}P_0(x) + \frac{3}{4}P_1(x) - \frac{7}{16}P_3(x) + \cdots$	Evaluate	4
7	Show that P_n(x) is the coefficient of t^nin the expansion of $(1-2xt+t^2)^{\frac{-1}{2}}$	Remember	4
8	Prove (2n+1) x$P_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$	Understand	4
9	Prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = \begin{cases} 0, if \alpha \neq \beta \\ \frac{1}{2}[J_{n+1}(\alpha)]^2 \ if \ \alpha = \beta \end{cases}$	Evaluate	4
10	Show that a) $J_n(x) = \frac{1}{\pi}\int_0^{\pi} cos(n\theta - xsin\theta)d\theta$ b) $J_0(x) = \frac{1}{\pi}\int_0^{\pi} cos(xsin\theta)d\theta = \frac{1}{\pi}\int_0^{\pi} cos(xcos\theta)d\theta$	Remember	4
11	State and prove Legendre's Rodrigue's formula.	Analyse	4
12	Show that $x^4 = \frac{8}{35}P_4(x) + \frac{4}{7}P_2(x) + \frac{1}{5}P_0(x)$	Understand	4
13	Express P(x) = $x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre Polynomials.	Evaluate	4
14	Using Rodrigue's formula prove that $\int_{-1}^{1} x^m p_n(x)dx = 0$ if m<n.	Evaluate	4
15	State and prove orthogonality of Legendre polynomials	Analyse	4
UNIT - III **COMPLEX FUNCTIONS-DIFFERENTIATION AND INTEGRATION**			
Part – A (Short Answer Questions)			
1	Show that f(z) = z^3 is analytic for all z	Analyse	5
2	Show that the function f(z) = $\sqrt{\lvert xy \rvert}$ is not analytic at the origin although Cauchy – Riemann equations are satisfied at the point.	understand	5
3	Show that f(z) = $\lvert z \rvert^2$ is not analytic .	understand	5
4	Find whether f(z) = $\frac{x-iy}{x^2+y^2}$ is analytic or not.	understand	5
5	Find whether f(z) = sinxsiny – icosxcosy is analytic or not	understand	5
6	Find k such that f(x,y) = $x^3 + 3kxy^2$ may be harmonic and find its conjugate.	Analyse	5
7	Find the most general analytic function whose real part is u = $x^2 - y^2 - x$	Analyse	5
8	Find an analytic function whose imaginary part is v = $e^x(xsiny + ycosy)$	understand	5
9	If f(z) is an analytic function of z and if u - v = $\frac{cosx+sinx-e^{-y}}{2cosx-e^y-e^{-y}}$find f(z) subject to the	Analyse	5

S. No	Question	Blooms Taxonomy Level	Course Outcome
	condition f($\frac{\pi}{2}$) = 0		
10	If f(z) is an analytic function of z and if u + v = $\frac{sin2x}{2cosh2y-cos2x}$find f(z) in terms of z.	remember	5
11	Let w = f(z) = z^2 find the values of w which correspond to (i) z = 2+i (ii) z = 1+3i	Analyse	5
12	Show that f(z) = $\|z\|^2$ is a function which is continous at all z but not differentiable at any z $\neq$ 0.	understand	5
13	Find all values of k such that f(x) = $e^x(cosky + isinky)$ is analytic	understand	5
14	Show that u = $e^{-x}(xsiny - ycosy)$ is harmonic	understand	5
15	Verify that u = $x^2 - y^2 - y$ is harmonic in the whole complex plane and find a conjugate harmonic function v of u .	understand	5
	Part – B (Long Answer Questions)		
1	Show that the function u = $e^{-2xy}\sin(x^2 - y^2)$ is harmonic , find the conjugate function 'v' and express u + iv as an analytic function of z.	Apply	5
2	Find whether the function u = log$\|z\|^2$ is harmonic . If so find the analytic function whose real part is u .	Apply	5
3	Find the imaginary part of an analytic function whose real part is $e^x(xcosy - ysiny)$	Apply	5
4	Find the regular function whose imaginary part is $\frac{x-y}{x^2+y^2}$	Apply	5
5	If f(z) = u+iv is an analytic function of z find f(z) if 2u + v = e^{2x}[(2x+y)cos2y + (x-2y)sin2y]	Analyse	5
6	Evaluate $\int_0^{1+i}(x - y + ix^2)$ dz (i) along the straight from z = 0 to z = 1+i . (ii) along the real axis from z = 0 to z = 1 and then along a line parallel to real axis from z = 1 to z = 1+i along the imaginary axis from z = 0 to z = I and then along a line parallel to real axis z = i to z = 1+ i	Apply	6
7	Verify Cauchy's theorem for the integral of z^3 taken over the boundary of the rectangle with vertices -1 ,1,1+i ,-1+i	Apply	6
8	Evaluate $\int_c \frac{e^{2z}}{(z-1)(z-2)}$ dz where c is the circle $\|z\|$ =3 using Cauchy's integral formula.	Apply	6
9	Evaluate $\int_c \frac{z^3e^{-z}}{(z-1)^3}$ dz where c is $\|z-1\| = \frac{1}{2}$ using Cauchy's integral formula.	Evaluate	6
10	Evaluate $\int_c \frac{5z^2 - 3z + 2}{(z-1)^3}dz$ where c is any simple closed curve enclosing z = 1 using Cauchy's integral formula.	Evaluate	6
11	Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \|Realf(z)\|^2 = 2\|f'(z)\|^2$ where w = f(z) is analytic.	Apply	5
12	Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) log\|f'(z)\|$ where w = f(z) is analytic	Apply	5
13	If f(z) is a regular function of z prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \|f(z)\|^2 = 4\|f'(z)\|^2$	Apply	5
14	Show that the function defined by f(z)= $\begin{cases} \frac{xy^2\ (x+iy}{x^2+y^4}, z \neq 0 \\ 0 \quad if\ z = 0 \end{cases}$ Is not analytic although Cauchy Riemann equations are satisfied at origin.	Apply	5
15	Show that u = $x^3 - 3xy^2$ and find a conjugate harmonic function v and the analytic function	Analyse	5

S. No	Question	Blooms Taxonomy Level	Course Outcome
	UNIT - IV **POWER SERIES EXPANSIONS OF COMPLEX FUNCTIONS AND CONTOUR INTEGRATION**		
	Part – A (Short Answer Questions)		
1	What circle does the maclaurin's series for the function tanhz coverage to the function.	Analyse	7
2	Expand f(z)= $\frac{1}{z^2}$ in powers of z+1	Analyse	7
3	Expand e^z as taylor's series about z=1	Analyse	7
4	Expand e^z as taylor's series about z=3	Evaluate	7
5	Find the residue of $\frac{z^2}{(z-a)(z-b)(z-c)}$ $at\ z=\infty$	Evaluate	9
6	Determine the poles and the residue of the function $f(z)=\frac{ze^z}{(z+2)^4(z-1)}$	Remember	9
7	Evaluate the Taylor's series expansion of $\left(\frac{1}{z-2}-\frac{1}{z-1}\right)$ in the region $\|z\|<1$	Analyse	7
8	Obtain the Taylor series expansion of f(z) = $\frac{1}{z}$ about the point z = 1	Analyse	7
9	Obtain the Taylor series expansion of f(z) = e^z about the point z = 1	Evaluate	7
10	Find the poles and residues of $\frac{1}{z^2-1}$	Analyse	9
11	Find zeros and poles of $\left(\frac{z+1}{z^2+1}\right)^2$	Analyse	9
12	Find the poles of the function $f(z)=\frac{1}{(z+1)(z+3)}$ and residues at these poles	Analyse	9
13	Find the residue of the function $f(z)=\frac{z^3}{(z^2-1)}$ $at\ z=\infty$	Evaluate	9
14	Find the residue of $\frac{z^2}{(z-a)(z-b)(z-c)}$ $at\ z=\infty$	Evaluate	9
15	Define residue at pole of order m	remember	9
	Part – B (Long Answer Questions)		
1	Evaluate $\int_c \frac{2z-1}{z(2z+1)(z+2)}dz$ where c is the circle $\|z\|$ = 1	Evaluate	9
2	Evaluate $\int_{-\infty}^{\infty} \frac{x^2dx}{(x^2+1)(x^2+4)}$	Evaluate	9
3	Evaluate $\oint_c \tan z dz$ where c is circle $\|z\|$ = 2.	Evaluate	9
4	Evaluate $\oint_c \frac{dz}{(z^2+4)^2}$ where c is $\|z-i\|$ = 2.	Evaluate	9

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5	Evaluate $\oint_c \frac{\coth z}{z-i}dz$ where c is $\lvert z\rvert = 2$	Evaluate	9
6	Determine the poles and the residue of the function $f(z)=\frac{ze^z}{(z+2)^4(z-1)}$	Evaluate	9
7	Evaluate $\oint_c \frac{4-3z}{(z-2)(z-1)z}$ dz where c is the circle $\lvert z\rvert = 1.5$ using residue theorem	Apply	9
8	Show that $\int_0^{2\pi} \frac{1+4\cos\theta}{17+8\cos\theta}d\theta = 0$	Apply	10
9	Evaluate $\int_0^{\infty} \frac{dx}{x^6+1}$	Apply	10
10	Show that $\int_0^{2\pi} \frac{d\theta}{4\cos^2\theta+\sin^2\theta} = \pi$	Analyse	10
11	Expand f(z) = $\frac{z-1}{z+1}$ in Taylor's series about the point (i) z = 0 (ii) z = 1	Apply	7
12	Expand f(z) = $\frac{z-1}{z^2}$ in Taylor's series in powers of z -1 and determine the region of convergence.	Evaluate	7
13	Obtain Laurent's series expansion of f(z) = $\frac{z^2-4}{z^2+5z+4}$ valid in 1< z < 2	Analyse	7
14	Expand $f(z)=\frac{e^{2z}}{(z-1)^3}$ about z = 1 as Laurent's series also find the region of convergence.	apply	7
15	Expand f(z) = $\frac{7z-2}{z(z+1)(z-2)}$ about z=-1 in the region$1<\lvert z+1\rvert<3$ as Laurent's series	apply	7

UNIT - V
CONFORMAL MAPPING

Part – A (Short Answer Questions)

1	Find the map of the circle $\lvert z\rvert$ = c under the transformation w = Z-2+4i	Analyse	11
2	Determine the bilinear transformation whose fixed points are i,-i.	Analyse	12
3	Find the fixed points of the transformation $w = \frac{2i-6z}{iz-3}$	Evaluate	11
4	Find the points at which w = coshz is not conformal	Evaluate	11
5	Find the image of $\lvert z\rvert$ = 2 under the transformation w = 3z	Analyse	11
6	Find the Bi-linear transformation which carries the points from (-i,0,i) to (-1,i,1)	Evaluate	112
7	Determine the bilinear transformation whose fixed points are 1,-1	Apply	12
8	Find the bilinear transformation which maps z = -1 , i , 1 into the points w = -i , 0 , i	Evaluate	12
9	Find the bilinear transformation which maps the points (-1,0,1) into the points (0,i,3i)	Evaluate	12

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10	Find the fixed points of the transformation $w = \frac{6z-9}{z}$ Type equation here.	Evaluate	11
11	Find the fixed points of the transformation $w = \frac{z-1+i}{z+2}$	Evaluate	11
12	Find the fixed points of the transformation $w = \frac{1}{z-2i}$	Evaluate	11
13	Find the bilinear transformation which maps the points (-2,1,0) into w=1,0,i respectively.	Evaluate	12
14	Find the bilinear transformation which maps the points (2,i,-2) into the points (1,i,-1) .	Evaluate	12
15	Find the bilinear transformation which maps the points (0,-i,-1) into the points (i,1, 0) .	Evaluate	12
Part – A (Long Answer Questions)			
1	Find the Bi-linear transformation which carries the points from $(0,1,\infty) to (-5,-1,3)$	Evaluate	12
2	Find the image of the triangle with vertices 1,1+I,1-i in the z-plane under the transformation w=3z+4-2i.	Evaluate	11
3	Find the image of the triangle with vertices i,1+i,1-i in the z-plane under the transformation $e^{\frac{5\pi i}{3}}(z-2+4i)$	Remember	11
4	Sketch the transformation $w = e^z$	Understand	11
5	Sketch the transformation $w = \log z$	Understand	11
6	Find the Bi-linear transformation which carries the points from $(1, i, -1) to (0,1,\infty)$	Apply	12
7	Show that transformation w = z^2 maps the circle $\|z-1\| = 1$ into the cardioid r = 2(1+cosθ) where $w = re^{i\theta}$ in the w-plane.	Evaluate	11
8	Determine the bilinear transformation that maps the points (1-2i,2+i,2+3i) into the points (2+i,1+3i,4)	Apply	12
9	Find the image under the transformation $w = \frac{z-i}{1-iz}$ find the image of $\|w\| =$ 1 $(ii)\|z\|$=1 in the w-plane	Apply	11
10	Find the image of the region in the z-plane between the lines y = 0 and y = $\frac{\pi}{2}$ under the transformation w = e^z	Evaluate	11
11	Show that the relation $w = \frac{5-4z}{4z-2}$ transforms the circle $\|z\|$ = 1 into a circle of radius unity in the w-plane	Analyse	11
12	Show that the transformation $w = \frac{i(1-z)}{(1+z)}$ transforms the circle $\|z\|$ = 1 into the real axis in the w-plane and the interior of circle into upper half of the w-plane the w-plane	Analyse	11
13	Show that the transformation $w = \frac{3-z}{z-2}$ transforms the circle $\left\|z - \frac{5}{2}\right\| = \frac{1}{2}$ in the z-plane into the imaginary axis in the w-plane	Analyse	11

S. No	Question	Blooms Taxonomy Level	Course Outcome
14	Show that the transformation w = cosz maps the half of the z-plane to the right of the imaginary axis into the entire w-plane	Analyse	11
15	Show that the transformation $w = \frac{2z+3}{z-4}$ changes the circle $x^2 + y^2$-4x = 0 into the straight line 4u+3 = 0	Analyse	11

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