Hall Ticket No | | | | | | | | | | | Question Paper Code: BCC002

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

M.Tech I Semester End Examinations (Regular) - January/February, 2018

Regulation: IARE–R16

NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS (CAD/CAM)

Time: 3 Hours **Max Marks: 70**

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

UNIT – I

1. (a) Summarize the discrete grid point method for finite difference approximation. **[7M]**

   (b) Solve by Crank-Nicolson method, $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, 0 < x < 1, t > 0, u(x,0) = 100(x - x^2), u(o,t) = 0, u(1,t) = 0$. Compute u for one time step with h $=\frac{1}{4}$ **[7M]**

2. (a) Define finite difference method. Summarize the advantages and disadvantages of finite difference method. **[7M]**

   (b) Compute u for one step by Crank-Nicolson method if $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < 5$ u(x,0)=20, u(0,t)=0 and u(5,t)=100. **[7M]**

UNIT – II

3. (a) Explain the meanings of the concepts of consistency, stability, and convergence of numerical methods. **[7M]**

   (b) "Prove that sum of consistency analysis and stability analysis is consistency convergence analysis". Justify your answer. **[7M]**

4. (a) Explain five point formula for finite difference by alternate direction implicit method. **[7M]**

   (b) Explain the stability analysis of implicit methods and describe the types of errors. **[7M]**

UNIT – III

5. (a) Explain the method of characteristics for the hyperbolic partial differential equation. **[7M]**

   (b) Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, 0 < x < 1; t > 0, using explicit method given that u(x, 0) = 0; u_t(x, 0) = 0, u(0, t) = 0 and u(1, t) = 100 sin(πt). Compute u for four time steps with h = 0.25. **[7M]**

6. (a) Prove that there is no explicit, unconditionally stable system for hyperbolic partial differential equation **[7M]**

   (b) Explain the Lax-Wendroff for the partial differential equation with an example in multiple dimensional. **[7M]**

UNIT – IV

7. (a) Given the values of u(x, y) on the boundary of the square in the figure 1, evaluate the function u(x, y) satisfying the Laplace equation $u_{xx} + u_{yy} = 0$ at the pivotal points. **[7M]**

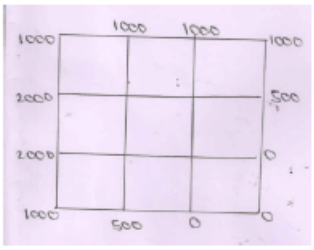

Figure 1

(b) Solve $u_{xx} + u_{yy} = -(x + y)^2$ over the square region bounded by lines x =0, y = 0, x = 3, y = 3 given that u = 0 throughout the boundaries taking h = 1. **[7M]**

8. (a) Explain solution of Laplace's equation **[7M]**

(b) Solve the Poisson equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides x = 0; y = 0; x = 3; y = 3 with u = 0 on the boundary and mesh length=1. **[7M]**

UNIT – V

9. (a) Explain the convergence of iteration methods to solve large linear systems. **[7M]**

(b) Using Galerkin's method to solve the boundary value problem y''-y+x = 0; 0 < x < 1; y(0) = 0, y(1) = 0. **[7M]**

10. (a) Explain weighted residual method with an example **[7M]**

(b) Using finite element method to solve y"+1 = 0; 0 < x < 1; y(0) = 0; y(1) = 0. **[7M]**

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