

Time: 3 Hours

Max Marks: 70

Answer ONE Question from each Unit All Questions Carry Equal Marks All parts of the question must be answered in one place only

$\mathbf{UNIT} - \mathbf{I}$

- 1. (a) Prove that the matrix $A = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$ is Unitary and find A^{-1} . [7M]
 - (b) Solve the following equations by LU decomposition method x 3y + 4z = 12, -x + 5y 3z = -12, 4x-8y+23z=58. [7M]
- 2. (a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & -1 & 2 & -1 \\ 3 & 2 & 3 & 4 \\ 1 & -2 & -3 & 2 \end{bmatrix}$ by reducing to Echelon form. [7M] (b) Express the matrix $A = \begin{bmatrix} 1 & 5 & 7 \\ -1 & -2 & -4 \\ 8 & 2 & 12 \end{bmatrix}$ as sum of Symmetric and Skew Symmetric matrix.

[7M]

$\mathbf{UNIT} - \mathbf{II}$

3. (a) Show that Eigen values of a matrix and its transpose are same. [7M]

(b) Diagonalize the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$. [7M]

- 4. (a) Examine whether $X_1 = [1, 2, 3]^T$, $X_2 = [3, -2, 1]^T$, $X_3 = [1, -6, 5]^T$ are linearly independent or dependent. [7M]
 - (b) Find all the Eigen Values and Eigen Vectors of $\begin{vmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{vmatrix}$ [7M]

$\mathbf{UNIT} - \mathbf{III}$

5. (a) Solve
$$\frac{dy}{dx} = -\frac{(x+y\cos x)}{1+\sin x}$$
. [7M]

(b) Solve
$$\frac{dy}{dx} = y \tan x - y^2 \sec x.$$
 [7M]

- 6. (a) Find the orthogonal trajectories of $\frac{x^2}{a^2} + \frac{y^2}{a-b} = 1$, where a and b are parameters. [7M]
 - (b) If the temperature of the air is 40°C and the substance cools from 80°C to 60°C in 20 minutes, what will be the temperature of the substance after 40 minutes? [7M]

UNIT - IV

- 7. (a) Solve $(D^3 3D^2 + 4D 2)y = e^x + \cos x$.
 - (b) Solve $(D^2 2D + 2) y = e^x \tan x$ by the method of variation of parameters. [7M]
- 8. (a) If $D^4x = a^4t$ then show that $x = C_1 \cos at + C_2 \sin at + C_3 \cosh at + C_4 \sinh at$.
 - [7M] (b) Solve y"+5y'-6y=sin4x sinx. [7M]

$\mathbf{UNIT}-\mathbf{V}$

- 9. (a) Verify Cauchy mean value theorem for $\log x$ and 1/x in [1,e]. [7M]
 - (b) A rectangular box open at the top is to have a volume of 32 cc. Find the dimensions of the box requiring least material for its construction.
- 10. (a) If $U = \tan^{-1} \left[\frac{x^3 + y^3}{x y} \right]$ then show that i. $xU_x + yU_y = \sin 2U$ ii. $x^2U_{xx} + y^2U_{yy} + 2xyU_{xy} = \sin 4U - \sin 2U$ [7M]
 - (b) Find the minimum and maximum values of the function $x^3 + 3xy^2 15x^2 15y^2 + 72x$. [7M]

[7M]