INSTITUTEOFAERONAUTICALENGINEERING
(Autonomous)
Dundigal, Hyderabad-500043
AERONAUTICAL ENGINEERING
TUTORIAL QUESTION BANK

| Course Title | FINITE ELEMENT METHODS |  |  |  |  |
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| Course Code | AAE009 |  |  |  |  |
| Programme | B.Tech |  |  |  |  |
| Semester | V AE |  |  |  |  |
| Course Type | Core |  |  |  |  |
| Regulation | IARE - R16 |  |  |  |  |
| Course Structure | Theory |  |  | Practical |  |
|  | Lectures | Tutorials | Credits | Laboratory | Credits |
|  | 3 | 1 | 4 | - | - |
| Chief Coordinator | Mr. S Devaraj, Assistant Professor |  |  |  |  |
| Course Faculty | Mr. S Devaraj, Assistant Professor Ms. Ch RaghaLeena, Assistant Professor |  |  |  |  |

## COURSE OBJECTIVES:

| The course should enable the students to: |  |
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| I | Introduce basic concepts of finite element methods including domain discretization, polynomial <br> interpolation and application of boundary conditions. |
| II | Understand the theoretical basics of governing equations and convergence criteria of finite element <br> method. |
| III | Develop of mathematical model for physical problems and concept of discretization of continuum. |
| IV | Discuss the accurate Finite Element Solutions for the various field problems. |
| V | Use the commercial Finite Element packages to build Finite Element models and solve a selected <br> range of engineering problems |

## COURSE OUTCOMES (COs):

| CO 1 | Describe the concept of FEM and difference between the FEM with other methods and problems <br> based on 1-D bar elements and shape functions. |
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| CO 2 | Derive elemental properties and shape functions for truss and beam elements and related problems. |
| CO 3 | Understand the concept deriving the elemental matrix and solving the basic problems of CST and axi- <br> symmetric solids. |
| CO 4 | Explore the concept of steady state heat transfer in fin and composite slab. |
| CO 5 | Understand the concept of consistent and lumped mass models and slove the dynamic analysis of all <br> types of elements. |

COURSE LEARNING OUTCOMES (CLOs):

| AAE009.01 | Describe the basic concepts of FEM and steps involved in it. |
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| AAE009.02 | Understand the difference between the FEM and Other methods. |
| AAE009.03 | Understand the stress-strain relation for 2-D and their field problem. |
| AAE009.04 | Understand the concepts of shape functions for one dimensional and quadratic elements, <br> stiffness matrix and boundary conditions. |
| AAE009.05 | Apply numerical methods for solving one dimensional bar problems. |
| AAE009.06 | Derive the elemental property matrix for beam and bar elements. |
| AAE009.07 | Solve the equations of truss and beam elements. |
| AAE009.08 | Understand the concepts of shape functions for beam element. |
| AAE009.09 | Apply the numerical methods for solving truss and beam problems. |
| AAE009.10 | Derive the element stiffness matrices for triangular elements and axi- symmetric solids and <br> estimate the load vector and stresses. |
| AAE009.11 | Formulate simple and complex problems into finite elements and solve structural and thermal <br> problems. |
| AAE009.12 | Understand the concept of CST and LST and their shape functions. |
| AAE009.13 | Understand the concepts of steady state heat transfer analysis for one dimensional slab, fin and <br> thin plate. |
| AAE009.14 | Derive the stiffness matrix for for fin element. <br> AAE009.15Solve the steady state heat transfer problems for fin and composite slab. <br> AAE009.16Understand the concepts of mass and spring system and derive the equations for various <br> structural problems |
| AAE009.17 | Understand the concept of dynamic analysis for all types of elements. |
| AAE009.18 | Calculate the mass matrices, Eigen values, Eigen vectors, natural frequency and mode shapes <br> for dynamic problems. |

## TUTORIAL QUESTION BANK

| UNIT- I |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| INTRODUCTION |  |  |  |  |
| Part - A (Short Answer Questions) |  |  |  |  |
| $\begin{aligned} & \text { S } \\ & \text { No } \end{aligned}$ | QUESTIONS | $\begin{gathered} \text { Blooms } \\ \text { Taxonomy } \\ \text { Level } \end{gathered}$ | Course Outcomes | Course <br> Learning <br> Outcomes <br> (CLOs) |
| 1 | Explain any two characteristics of shape functions. | Remember | CO 1 | AAE009.04 |
| 2 | What is degree of freedom and boundary conditions? | Understand | CO 1 | AAE009.05 |
| 3 | Give the expression for shape functions of a linear element. | Remember | CO 1 | AAE009.04 |
| 4 | Specify some applications of finite element methods | Remember | CO 1 | AAE009.02 |
| 5 | Name the different methods used for solving problems in FEM | Remember | CO 1 | AAE009.02 |
| 6 | Draw the shape functions of quadratic element and linear element | Remember | CO 1 | AAE009.04 |
| 7 | What is the element stiffness matrix for a quadratic element | Remember | CO 1 | AAE009.04 |
| 8 | Write the expressions for stress strain relationship for 2D elastic problems | Remember | CO 1 | AAE009.03 |
| 9 | What is the stiffness matrix for one dimensional element? | Remember | CO 1 | AAE009.03 |
| 10 | Discuss different types of elements | Remember | CO 1 | AAE009.05 |
| Part - B (Long Answer Questions) |  |  |  |  |
| 1 | Explain the concept of FEM briefly and outline the steps involved in FEM. | Remember | CO 1 | AAE009.01 |
| 2 | What is the difference between the plane stress and plane strain condition? | Remember | CO 1 | AAE009.03 |
| 3 | Define principle of virtual work. Describe the FEM formulation for 1D bar element. | Understand | CO 1 | AAE009.05 |
| 4 | Derive element stiffness matrix and load vector for linear element using potential energy approach. | Understand | CO 1 | AAE009.04 |
| 5 | Compare finite element method with finite difference method. | Remember | CO 1 | AAE009.02 |
| 6 | Explain the criteria for nodal selection for structural elements. | Remember | CO 1 | AAE009.01 |


| 7 | Briefly discuss the discretization process and types of elements used for discretization. | Remember | CO 1 | AAE009.01 |
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| 8 | Explain the equilibrium state of the system, when the system is subjected to different types of loads and explain the stress and equilibrium relations | Remember | CO 1 | AAE009.03 |
| 9 | Derive stress strain relationships for 2D elastic problems. | Understand | CO 1 | AAE009.03 |
| 10 | What are the advantages disadvantages and applications of FEM | Understand | CO 1 | AAE009.01 |
| Part - C (Problem Solving and Critical Thinking Questions) |  |  |  |  |
| 1 | Consider the following figure. An axial load $\mathrm{P}=200 \mathrm{KN}$ is applied as shown <br> a) Determine the nodal displacements. <br> b) Determine the stress in each material. <br> c) Determine the reaction forces. | Understand | CO 1 | AAE009.05 |
| 2 | In the figure given below, a load $\mathrm{P}=60 \mathrm{KN}$ is applied as shown. Determine the displacement field, stress and support reactions in the body. Take E as 20 GPa <br> (a) <br> (b) | Understand | CO 1 | AAE009.05 |
| 3 | Consider the thin (steel) plate in figure. The plate has a uniform thickness t $=10 \mathrm{~mm}$, Young's modulus $\mathrm{E}=100 \mathrm{Gpa}$, and weight density $=78500 \mathrm{~N} / \mathrm{m}^{3}$. In addition to its self-weight, the plate is subjected to a point load $P=60 \mathrm{~N}$ at its midpoint. <br> a) Write down expressions for the element stiffness matrices and element body force vectors <br> b) Determine the stresses in each element and reaction force at the support. <br> Consider $1 \mathrm{in}=1 \mathrm{~cm}$ for SI units <br> (a) <br> (b) | Understand | CO 1 | AAE009.05 |
| 4 | Consider the bar shown in figure Determine the <br> a) nodal displacements <br> b) Flement ctrecsec and cunnort reartinnc $F=90 \cap \mathrm{~F}_{\mathrm{a}}$ | Understand | CO 1 | AAE009.05 |
| 5 | A bar is subjected to an axial force is divided into a number of quadratic elements. For a particular element the nodes $1,3,2$ are located at 15 mm , | Understand | CO 1 | AAE009.05 |


|  | 18 mmand 21 mm respectivelly from origin. If the axial displacements of the three nodes are given by $\mathrm{u} 1=0.00015 \mathrm{~mm}, \mathrm{u} 3=0.0033 \mathrm{and} \mathrm{u} 2=0.00024 \mathrm{~mm}$. Determine the following <br> a) shape function <br> b) variation of the displacement $\mathrm{u}(\mathrm{x})$ in the element <br> c) axial stain in the element |  |  |  |
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| 6 | Consider the following fig. An axial load $\mathrm{P}=200 \mathrm{KN}$ is applied as shown. Using an elimination approach, do the following <br> a) Determine the nodal displacements. <br> b) Determine the stress in each material. | Understand | CO 1 | AAE009.05 |
| 7 | A stepped bar is subjected to an axial (vertical) force $\mathrm{P}=108 \mathrm{~N}$ at node 2 as shown in figure. If the areas of the cross section of the steps are given by $\mathrm{A} 1=0.1 \mathrm{~m} 2$ and $\mathrm{A} 2=0.05 \mathrm{~m} 2$ and Young's moduli E1 $=200 \mathrm{GPa}$ and E2 $=70 \mathrm{GPa}$, determine the following <br> a) The displacements of node 3 <br> b) The displacements of nodes and the stresses in two steps | Understand | CO 1 | AAE009.05 |
| 8 | An axial load $\mathrm{P}=300 \mathrm{X} 103 \mathrm{~N}$ is applied at 200 C to the rod as shown in Figure below. The temperature is the raised to $60^{\circ} \mathrm{C}$. <br> a) Assemble the K and F matrices. <br> b) Determine the nodal displacements and stresses. | Understand | CO 1 | AAE009.05 |
| 9 <br>  <br>  <br> 10 | An axial load $\mathrm{P}=200 \times 103 \mathrm{~N}$ is applied on a bar as shown in figure. Determine nodal displacements, stress in each material and reactions. | Understand | CO 1 | AAE009.05 |
| 10 | Determine the nodal displacement, Element stresses for axially loaded bar as shown in the figure below | Understand | CO 1 | AAE009.05 |

UNIT-II
ANALYSIS OF TRUSSES AND BEAMS
Part - A (Short Answer Questions)

| 1 | Represent the truss in local coordinate system and global coordinate system. | Remember | CO 2 | AAE009.09 |
| :---: | :---: | :---: | :---: | :---: |
| 2 | Write the transformation matrix of a truss. | Remember | CO 2 | AAE009.07 |
| 3 | Write the stress equation for truss elements | Remember | CO 2 | AAE009.07 |
| 4 | Write the stiffness matrix for a plane truss. | Remember | CO 2 | AAE009.07 |
| 5 | Write the stiffness matrix for a space truss. | Remember | CO 2 | AAE009.08 |
| 6 | Write the expression for element stiffness matrix of a beam | Remember | CO 2 | AAE009.06 |
| 7 | What is the load vector expression for a cantilever beam carrying UDL over its entire span? | Remember | CO 2 | AAE009.09 |
| 8 | What is the expression for UDL load vector of simply supported beam | Remember | CO 2 | AAE009.05 |
| 9 | What is the load vector expression for a cantilever beam carrying point load at its end? | Remember | CO 2 | AAE009.09 |
| 10 | Write the stiffness matrix for a beam. | Remember | CO 2 | AAE009.06 |
| Part - B (Long Answer Questions) |  |  |  |  |
| 1 | What is a beam? Derive the shape functions for beams and draw the shape functions. | Understand | CO 2 | AAE009.08 |
| 2 | Derive the stiffness matrix for beam elements. | Understand | CO 2 | AAE009.06 |
| 3 | Derive the stiffness matrix for two dimensional plane truss elements. | Understand | CO 2 | AAE009.07 |
| 4 | Assemble the global stiffness matrix and nodal displacement-for the fig. shown below Understand the problem by using SI units only. Take $1 \mathrm{lb}=$ $4.44 \mathrm{~N} \quad 1 \mathrm{in} 2=645.16 \mathrm{~mm} 21 \mathrm{psi}=6.89 \mathrm{KP} \quad 1 \mathrm{in}=25.4 \mathrm{~mm}$ | Understand | CO 2 | AAE009.09 |
| 5 | Find the deflection at the load and the slopes at the ends for the steel shaft shown in figure. Consider the shaft to be simply supported at bearings A and B. Solve by FEM technique. Take E $=200 \mathrm{GPa}$. | Understand | CO 2 | AAE009.09 |
| 6 | For the truss shown in figure. Determine the displacements and stresses in the bars | Understand | CO 2 | AAE009.09 |


| 7 | For the beam shown in Figure below, determine the following: <br> a) <br> Slopes at nodes 2 and 3. <br> b) <br> deflection at the mid-point of the distributed load. <br> Consider all the elementshave $\mathrm{E}=200 \mathrm{GPa}, \mathrm{I}=5 \mathrm{X} 106 \mathrm{~mm} 4$. | Understand | CO 2 | AAE009.09 |
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| 8 | A beam fixed at one end and supported by a roller at the other end, has a 20 kN concentrated load applied at the centre of the span (Figure below). Calculate the deflection under the load and construct the shear force and bending moment diagrams for the beam. <br> Figure beam with a point load | Understand | CO 2 | AAE009.09 |
| 9 | Calculate nodal displacement and element stresses for the truss shown in figure. Take $\mathrm{E}=70 \mathrm{GPa}$ and cross sectional area $\mathrm{A}=2 \mathrm{~cm} 2$ for all truss members. | Understand | CO 2 | AAE009.09 |
| 10 | Determine Nodal displacements and Element stresses in the truss shown in figure. $\mathrm{E}=80 \mathrm{GPa}$. | Understand | CO 2 | AAE009.09 |
| Part - C (Problem Solving and Critical Thinking Questions) |  |  |  |  |
| 1 | For the truss given below, a horizontal load of $\mathrm{P}=4000 \mathrm{lb}$ is applied in the x -direction at node 2 . <br> a) Write down the element stiffness matrix k for each element. <br> b) Assemble the K matrix <br> c) Using elimination approach, Understand for Q | Understand | CO 2 | AAE009.09 |


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| 2 | Apply the beam shown in Figure below by finite element method and determine the end reactions. Also determine the deflections at mid spans given $\mathrm{E}=2 \mathrm{X} 105 \mathrm{~N} / \mathrm{mm} 2$, and $\mathrm{I}=5 \mathrm{X} 106 \mathrm{~mm}^{4}$. | Understand | CO 2 | AAE009.09 |
| 3 | For the truss shown in fig, Understand for the horizontal and vertical components of displacement at node 1 and determine the stress in each element. All elements have $\mathrm{A}=500 \mathrm{~mm} 2$ and $\mathrm{E}=70 \mathrm{GPa}$. | Understand | CO 2 | AAE009.09 |
| 4 | For the truss element shown below, determine the following: <br> a) The K matrix <br> b) The nodal vector q <br> c) The stress in the element | Understand | CO 2 | AAE009.09 |
| 5 | Determine the nodal displacements and slopes for the beam shown in fig. find the moment at the midpoint of element. <br> Take $\mathrm{E}=200 \mathrm{GPa}, \mathrm{I}=5 \times 104 \mathrm{~mm} 4, \mathrm{M}=6 \mathrm{KN}-\mathrm{M}$. | Understand | CO 2 | AAE009.09 |
| 6 | Determine the deflection and slope under the point load for the beam shown in fig given. $\mathrm{E}=200 \mathrm{GPa}, \mathrm{I}=4 \times 10-6 \mathrm{~m} 4, \mathrm{I} 2=2 \times 10-6 \mathrm{~m} 4 .$ | Understand | CO 2 | AAE009.09 |


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| 7 | A beam fixed at one end and supported by a roller at the other end, has a 20 kN concentrated load applied at the centre of the span (Figure below). Calculate the deflection under the load and construct the shear force and bending moment diagrams for the beam. | Understand | CO 2 | AAE009.09 |
| 8 | For a three bar truss shown in figure, determine the displacements in node 1 and the stress in element 3 . Take $\mathrm{E}=250 \mathrm{~mm} 2, \mathrm{E}=200 \mathrm{GPa}$. | Understand | CO 2 | AAE009.09 |
| 9 | Determine the nodal displacements and slopes at the position of one-fourth distance from the support of shaft: <br> Take $\mathrm{E}=200 \mathrm{GPa}, \mathrm{I}=6 \times 104 \mathrm{~mm} 4$. The shaft is simply supported at A and B. | Understand | CO 2 | AAE009.09 |
| 10 | Estimate the displacement vector, stresses for the truss structure as shown below figure. Take $\mathrm{E}=2 \times 105 \mathrm{~N} / \mathrm{mm} 2$. | Understand | CO 2 | AAE009.09 |
| 11 | Calculate the deflection under load, shear force and bending moment at mid span and reactions at supports for the beam shown in figure. Take $\mathrm{E}=200$ GPa and $\mathrm{I}=24 \times 10-6 \mathrm{~m}^{4}$. | Understand | CO 2 | AAE009.09 |
| 12 | A beam fixed at one end and supported by a roller at the end, has a 20 KN concentrated load applied at the centre of the span, as shown in fig. calculate the deflection under the load and construct shear force and bending moment diagram for the beam. <br> Take E $=20 \times 106 \mathrm{~N} / \mathrm{c}, 2, \mathrm{I}=2500 \mathrm{~cm}^{4}$. | Understand | CO 2 | AAE009.09 |


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| UNIT -III |  |  |  |  |
| CONTINUUM ELEMENTS |  |  |  |  |
| Part - A (Short Answer Questions) |  |  |  |  |
| 1 | What is a CST element with example? | Remember | CO 3 | AAE009.12 |
| 2 | List any four two dimensional elements | Remember | CO 3 | AAE009.11 |
| 3 | Represent the node numbering of Constant strain triangle element. | Remember | CO 3 | AAE009.12 |
| 4 | What is LST element with example? | Remember | CO 3 | AAE009.12 |
| 5 | What is the condition for number of unknown polynomial coefficients of a 2-D element | Remember | CO 3 | AAE009.13 |
| 6 | Define plane stress and plane strain | Remember | CO 3 | AAE009.10 |
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| 7 | Write the expression of traction force for four node quadrilateral element. | Remember | CO 3 | AAE009.12 |
| 8 | Represent the node numbering of Linear strain triangle element. | Remember | CO 3 | AAE009.13 |
| 9 | What is meant by axi symmetric solid? | Remember | CO 3 | AAE009.10 |
| 10 | Differentiate between linear and nonlinear elements. | Remember | CO 3 | AAE009.11 |
| 11 | What is isoparametric representation | Remember | CO 3 | AAE009.11 |
| 12 | What are the conditions for a problem to be axisymmetric? | Remember | CO 3 | AAE009.10 |
| Part - B (Long Answer Questions) |  |  |  |  |
| 1 | Derive the strain displacement matrix for triangular element | Understand | CO 3 | AAE009.10 |
| 2 | Derive the Jacobian of transformation | Understand | CO 3 | AAE009.11 |
| 3 | Derive force terms for constant strain triangle. | Understand | CO 3 | AAE009.10 |
| 4 | Differentiate CST and LST elements. | Understand | CO 3 | AAE009.12 |
| 5 | Define Iso-parametric, Super Parametric and Sub-Parametric elements | Understand | CO 3 | AAE009.10 |
| 6 | Apply the element stiffness matrix for the triangular element shown in figure under plane strain condition. Assume the following values. $\mathrm{E}=200 \mathrm{GPa}, \mu=0.25, \mathrm{t}=1 \mathrm{~mm}$. | Understand | CO 3 | AAE009.12 |
| 7 | Determine the nodal displacements and element stresses for the two dimensional loaded plate as shown in figure. Assume planestress conditions. Body force may be neglected in comparison to the external forces. Take $\mathrm{E}=210 \mathrm{GPa}, \mu=0.25$; thickness $=10 \mathrm{~mm}$. | Understand | CO 3 | AAE009.12 |
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| 8 | Derive the shape function for four noded Quadrilateral Element. | Understand | CO 3 | AAE009.12 |
| 9 | Derive the shape function and strain displacement matrices for triangular element of revolving body | Understand | CO 3 | AAE009.10 |
| 10 | Derive the element stiffness matrix for four noded quadrilateral element | Understand | CO 3 | AAE009.10 |
| 11 | Derive the stiffness matrix for axisymmetric element. | Understand | CO 3 | AAE009.11 |
| 12 | Evaluate the axisymmetric stiffness matrix K of the triangular element shown in the figure. Consider the coordinates of nodes as $1(2,1), 2(4,0)$, and $3(3,2)$. Also assume $\mathrm{E}=2.6 \mathrm{GPa}$ and $v=0.2$. | Understand | CO 3 | AAE009.11 |


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| 13 | Define Iso-parametric, Super Parametric and Sub-Parametric elements? | Understand | CO 3 | AAE009.10 |
| Part - C (Problem Solving and Critical Thinking) |  |  |  |  |
| 1 | For the point P located inside the triangle, the shape functions N1 and N2 are 0.15 and 0.25 , respectively. Determine the x and y coordinate of P . | Understand | CO 3 | AAE009.10 |
| 2 | For the triangular element shown in figure, obtain strain-displacement relation matrix B and determine the strains $\varepsilon_{\mathrm{x}}$, $\varepsilon_{\mathrm{y}} \mathrm{and} \gamma_{\mathrm{xy}}$. | Understand | CO 3 | AAE009.10 |
| 3 | Determine the jacobian for the $(x, y)-(\xi, \eta)$ transformation for the element shown in fig, also find the area of the triangle. | Understand | CO 3 | AAE009.11 |
| 4 | For the plane stress element shown in figure, determine the stiffness matrix. Assume $\mathrm{E}=200 \mathrm{GPa}$ and $\mu=0.3$. Thickness $=10 \mathrm{~mm}$ | Understand | CO 3 | AAE009.10 |
| 5 | For the two dimensional plate shown in figure, determine the deflection at the point of load application. | Understand | CO 3 | AAE009.11 |


| 6 | Calculate the element stiffness matrix and thermal force vector for the plane stress element shown in figure. The element experiences a rise of $10^{\circ} \mathrm{C} . \mathrm{E}=15 \times 104 \mathrm{~N} / \mathrm{mm}^{2}, \mathrm{t}=5 \mathrm{~mm}, \mu=0.25, \alpha=6 \times 10-6 /{ }^{\circ} \mathrm{C}$ | Understand | CO 3 | AAE009.11 |
| :---: | :---: | :---: | :---: | :---: |
| 7 | For the axisymmetric element shown in figure. Determine the stiffness matrix. Let $\mathrm{E}=2.1 \times 10^{5} \mathrm{MN} / \mathrm{m}^{2}$ and $\mu=0.25$. The coordinates are in mm . | Understand | CO 3 | AAE009.10 |
| 8 | The ( $\mathrm{x}, \mathrm{y}$ ) co-ordinates of nodes $\mathrm{i}, \mathrm{j}$ and k of an axisymmetric triangular element are given by $(3.4),(6,5)$ and $(5,8) \mathrm{cm}$ respectively. The element displacement (in cm ) vector is given as $\mathrm{q}=[0.002,0.001,0.004,-0.003$, $0.007]^{\mathrm{T}}$. Determine the element strains. | Understand | CO 3 | AAE009.11 |
| 9 | A long cylinder of inside diameter 80 mm and outside diameter 120 mm snugly fits in a hole over its full length. The cylinder is then subjected to an internal pressure of 2 MPa . Using two elements on the 10 mm length shown, find the displacements at the inner radius. | Understand | CO 3 | AAE009.11 |
| 10 | A four noded rectangular element is shown in figure. Determine Jacobian matrix, Strain displacement matrix and element stresses. Take $\mathrm{E}=2 \times 10^{5}$ $\mathrm{N} / \mathrm{mm}^{2}, \mu=0.5, \mathrm{u}=[0,0,0.005,0.008,0.008,0,0]^{\mathrm{T}}, \varepsilon=0, \eta=0$ | Understand | CO 3 | AAE009.12 |
| 11 | The Cartesian global coordinates of the corner nodes of an isoparametric quadrilateral element are given by $(1,0),(2,0),(2.5,1.5)$ and $(1.5,1)$. Find its Jacobian matrix. | Understand | CO 3 | AAE009.12 |
| UNIT -IV |  |  |  |  |
| STEADY STATE HEAT TRANSFER ANALYSIS |  |  |  |  |
| Part - A (Short Answer Questions) |  |  |  |  |
| 1 | Define steady state heat transfer. | Remember | CO 4 | AAE009.13 |
| 2 | Define fins or extended surfaces | Remember | CO 4 | AAE009.14 |
| 3 | Write the basic equation of heat transfer. | Remember | CO 4 | AAE009.13 |
| 4 | Specify the applications of heat transfer problems. | Remember | CO 4 | AAE009.13 |


| 5 | What is conduction and convetion heat transfer? | Understand | CO 4 | AAE009.13 |
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| 6 | Write thermal conductivity matrix for two dimensional heat transfer problems. | Remember | CO 4 | AAE009.14 |
| 7 | Differentiate between convection and radiation heat transfer. | Understand | CO 4 | AAE009.13 |
| 8 | Formulate the equation of one dimensional criteria of composite wall. | Understand | CO 4 | AAE009.15 |
| 9 | Define the terms thermal conductivity and convection heat transfer coefficient. | Understand | CO 4 | AAE009.13 |
| 10 | Analyse the heat transfer characteristics of fins. | Understand | CO 4 | AAE009.13 |
| 11 | Write the finite element equation for 1dimensional heat conduction element. | Remember | CO 4 | AAE009.14 |
| Part - B (Long Answer Questions) |  |  |  |  |
| 1 | Derive thermal stiffness matrix for one dimensional heat conduction with lateral surface convection and with internal heat generation. | Understand | CO 4 | AAE009.13 |
| 2 | Describe heat transfer analysis for composite wall. | Understand | CO 4 | AAE009.15 |
| 3 | What are different types of boundary conditions for 1D heat conduction problems? | Understand | CO 4 | AAE009.13 |
| 4 | Consider a brick wall of thickness $\mathrm{L}=30 \mathrm{~cm}, \mathrm{k}=0.7 \mathrm{~W} / \mathrm{m}^{0} \mathrm{C}$. The inner surface is at $28^{\circ} \mathrm{C}$ and the outer surface is exposed to cold air at $-15^{\circ} \mathrm{C}$. The heat transfer coefficient associated with the outside surface is $\mathrm{h}=40$ $\mathrm{W} / \mathrm{m}^{20} \mathrm{C}$. Determine the steady state temperature distribution within the wall and also the heat flux through the wall. | Understand | CO 4 | AAE009.15 |
| 5 | Derive the finite element equation for straight fin. | Understand | CO 4 | AAE009.14 |
| 6 | Derive the conductivity matrix for two dimensional triangular element subjected to convection on one face of the element. | Understand | CO 4 | AAE009.13 |
| 7 | Calculate the temperature distribution in the stainless steel fin of circular cross section length 10 cm shown in figure. The cross section of the fin is circular with diameter of 2 cm . Discretize the fin into 5 elements. <br> $h=0.0225 \mathrm{~W} / \mathrm{m}^{2} \mathrm{C}, \mathrm{T}_{\mathrm{a}^{2}}=25^{\circ} \mathrm{C}$ <br> $\mathrm{I}_{0}=10^{\circ} \mathrm{C}$ | Understand | CO 4 | AAE009.14 |
| 8 | A furnace wall is made up of three layers, inside layer with thermal conductivity $8.5 \mathrm{~W} / \mathrm{mK}$, the middle layer with conductivity $0.25 \mathrm{~W} / \mathrm{mK}$ the outer layer with conductivity $0.08 \mathrm{~W} / \mathrm{mK}$. The respective thicknesses of the inner, middle and outer layer are $25 \mathrm{~cm}, 5 \mathrm{~cm}$ and 3 cm respectively. The inside temperature of the wall is $600^{\circ} \mathrm{C}$ and the outside of the wall is exposed to atmospheric air at $30^{\circ} \mathrm{C}$ with heat transfer coefficient of 45 $\mathrm{W} / \mathrm{m}^{2} \mathrm{~K}$. Determine the nodal temperatures. | Understand | CO 4 | AAE009.15 |
| 9 | Explain the methodology for the treatment of all three boundary conditions in a 1-D heat transfer element? | Understand | CO 4 | AAE009.13 |
| 10 | Derive one dimensional steady state heat conduction equation and apply to one dimensional fin problem | Understand | CO 4 | AAE009.14 |
| 11 | Derive stiffness matrix for 1-D heat conduction problem. | Understand | CO 4 | AAE009.13 |
| 12 | Derive the conductivity matrix and thermal load vector for the one dimensional finite element for the three boundary conditions. | Understand | CO 4 | AAE009.13 |
| 13 | Derive angle of twist for a uniform shaft subjected to torsion. | Understand | CO 4 | AAE009.13 |
| Part - C (Problem Solving and Critical Thinking) |  |  |  |  |
| 1 | Determine the temperature distribution through the composite wall shown in figure, when convection heat loss occurs on the left surface. Assume unit area. Assume wall thickness <br> $\mathrm{t}_{1}=4 \mathrm{~cm}, \mathrm{t}_{2}=2 \mathrm{~cm}, \mathrm{k}_{1}=0.5 \mathrm{w} / \mathrm{cm}^{0} \mathrm{c}, \mathrm{k}_{2}=0.05 \mathrm{w} / \mathrm{cm}^{0} \mathrm{c}, \mathrm{h}=0.1 \mathrm{w} / \mathrm{cm}^{20} \mathrm{c}$ and $\mathrm{T}_{\alpha}=-5^{0} \mathrm{c}$. | Understand | CO 4 | AAE009.15 |
| 2 | Determine the nodal temperature in a composite wall, the wall is maintained at 100 deg c at the left face and convection mode of heat transfer occurs between the right face and existing fluid .take $\mathrm{k}_{1}=0.06 \mathrm{w} / \mathrm{cm}$ | Understand | CO 4 | AAE009.15 |


|  | deg c and $\mathrm{k}_{2}=0.2 \mathrm{w} / \mathrm{cm}$ deg c , convection co efficient of heat transfer between walls and fluid $\mathrm{h}=0.1 \mathrm{w} / \mathrm{cm}^{20} \mathrm{C}$ and $\mathrm{T} \infty=25^{\circ} \mathrm{C}$. Consider unit area $=1$ $\mathrm{cm}^{2}$ perpendicular to the direction of heat flow. |  |  |  |
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| 3 | A metallic fin with thermal conductivity $\mathrm{K}=360 \mathrm{~W} / \mathrm{m}^{0} \mathrm{c}$, 1 mm thick and 100 mm long extends from a plane wall whose temperature is $235^{\circ} \mathrm{c}$. Determine the distribution and amount of heat transferred from the fin to air at $20^{\circ} \mathrm{c}$ with $\mathrm{h}=9 \mathrm{~W} / \mathrm{m}^{20} \mathrm{c}$ take width of the fin is 1000 mm . Assume tip is insulted. | Understand | CO 4 | AAE009.14 |
| 4 | Calculate the temperature distribution in a one dimensional fin with the physical properties $\mathrm{k}=3 \mathrm{~W} / \mathrm{cm}^{0} \mathrm{C}, \mathrm{h}=0.1 \mathrm{~W} / \mathrm{cm}^{20} \mathrm{C}, \mathrm{T}_{\infty}=20^{\circ} \mathrm{C}$. the fin is rectangular in shape and is 8 cm long, 4 cm wide and 1 cm thick. Assume that convection heat loss occurs from the end of the fin. | Understand | CO 4 | AAE009.14 |
| 5 | A metallic fin 0.15 cm thick and 12 cm long is attached to a furnace whose wall temperature is 2200 C . If the thermal conductivity of the material of the fin is $350 \mathrm{~W} / \mathrm{m}^{0} \mathrm{C}$ and convection coefficient is $9 \mathrm{~W} / \mathrm{m}^{20} \mathrm{C}$, determine the temperature distribution if the width of the fin is 2 cm . Assume that the tip of the fin is open to the atmosphere and that the ambient temperature is $25^{0} \mathrm{C}$. | Understand | CO 4 | AAE009.14 |
| 6 | Compute the element matrices and vectors for the element shown in figure when the edges jk and ki experience convection heat loss. | Understand | CO 4 | AAE009.13 |
| 7 | Heat is entering into a large plate at the rate of $\mathrm{q}_{0}=-300 \mathrm{~W} / \mathrm{m}^{2}$. The plate is 25 mm thick. The outside surface of the plate is maintained at a temperature of $10^{\circ} \mathrm{C}$. Using two finite elements solve for the vector of nodal temperatures T . Thermal conductivity $\mathrm{k}=1.0 \mathrm{~W} / \mathrm{m}^{0} \mathrm{C}$. | Understand | CO 4 | AAE009.13 |
| 8 | A composite wall consists of three materials as shown in figure. The outer temperature is $\mathrm{T}_{0}=200 \mathrm{C}$. Convection heat transfer takes place on the inner surface of the wall with $\mathrm{T}_{\infty}=8000 \mathrm{C}$ and $\mathrm{h}=25 \mathrm{~W} / \mathrm{m}^{2 \circ} \mathrm{C}$. Determine the temperature distribution in the wall. | Understand | CO 4 | AAE009.15 |
| 9 | A bar of length 10 cm with rectangular section of width 3 cm and depth 2 cm is experiencing a temperature of $90^{\circ} \mathrm{C}$ at its left end. Assuming convection over the length of bar obtain the temperature distribution along the length. Use two 1 D elements with nodes at its ends and the following data $\mathrm{k}=5 \mathrm{~W} / \mathrm{cm}^{0} \mathrm{C}, \mathrm{h}=0.2 \mathrm{~W} / \mathrm{cm}^{20} \mathrm{C}, \mathrm{T}_{\infty}=25^{\circ} \mathrm{C}$ | Understand | CO 4 | AAE009.13 |
| 10 | Consider the shaft with rectangular cross section shown in figure. Determine in terms of M and G , the angle of twist per unit length. | Understand | CO 4 | AAE009.13 |


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| Part - C (Problem Solving and Critical Thinking) |  |  |  |  |
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| 1 | Consider axial vibration of the steel bar shown in figure below. Develop the global stiffness mass matrix and determine the natural frequencies and mode shapes using the characteristic polynomial technique. | Understand | CO 5 | AAE009.18 |
| 2 | Evaluate the lowest Eigenvalue and the corresponding Eigenmode for the beam shown in Figure below $\begin{aligned} E & =200 \mathrm{GPa} \\ \rho & =7840 \mathrm{~kg} / \mathrm{m}^{3} \\ I & =2000 \mathrm{~mm}^{4} \\ A & =240 \mathrm{~mm}^{2} \end{aligned}$ | Understand | CO 5 | AAE009.18 |
| 3 | Determine the Eigen values and Eigen vectors for the stepped bar shown in Figure below. $\begin{aligned} & E=30 \times 10^{6} \mathrm{psi} \\ & \text { Specific weight } f=0.283 \mathrm{lb} / \mathrm{in}^{3} \end{aligned}$ | Understand | CO 5 | AAE009.18 |
| 4 | Consider axial vibration of the Aluminium bar shown in Figure below, develop the global stiffness and determine the nodal displacements and stresses using elimination approach and with help of linear and quadratic shape function concept. Assume Young's Modulus E = 70Gpa. | Understand | CO 5 | AAE009.18 |
| 5 | Find the natural frequencies in the vibration of two element simply supported beam having the parameters as length $\mathrm{L}=2 \mathrm{~m}$, area of cross section $\mathrm{A}=30 \mathrm{~cm}^{2}$, moment of inertia $\mathrm{I}=400 \mathrm{~mm}^{4}$ density $\rho=7800 \mathrm{~kg} / \mathrm{m}^{3}$ and Young's modulus $\mathrm{E}=200 \mathrm{GPa}$. | Understand | CO 5 | AAE009.18 |
| 6 | Consider the axial vibrations of a steel bar shown in the figure <br> a) Develop global stiffness and mass matrices <br> b) Determine the natural frequencies? | Understand | CO 5 | AAE009.18 |
| 7 | Determine the first two natural frequencies of longitudinal vibration of the stepped bar shown in figure and plot the mode shapes. All the dimensions are in $\mathrm{mm} \mathrm{E}=200 \mathrm{GPa}$ and $\rho=0.78 \mathrm{~kg} / \mathrm{cc}$. what will be the effect on natural frequencies if a concentrated mass of 100 kg is added to the tip of bar? | Understand | CO 5 | AAE009.18 |


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| 8 | Consider a simply supported beam which is discretized into 2 elements as shown in figure. Obtain the natural frequencies. The following data for beam is length $=2 \mathrm{~m}$, area of cross section $\mathrm{A}=30 \mathrm{~cm}^{2}$, moment of inertia $\mathrm{I}=$ $400 \mathrm{~mm}^{4}$, mass density $\rho=7800 \mathrm{~kg} / \mathrm{m}^{3}$. | Understand | CO 5 | AAE009.18 |
| 9 | Determine the natural frequencies of the system in figure. Consider at least two elements. | Understand | CO 5 | AAE009.18 |
| 10 | Find the natural frequencies of longitudinal vibration of the unconstrained stepped bar shown in figure | Understand | CO 5 | AAE009.18 |
| 11 | Determine all natural frequencies of the simply supported beam as shown in figure? | Understand | CO 5 | AAE009.18 |
| 12 | Determine the Eigen values and Eigen Vectors for the stepped bar as shown in figure? | Understand | CO 5 | AAE009.18 |

Prepared by:
Ms. Ch RaghaLeena, AssistantProfessor

