

ELECTRO-MAGNETIC FIELD THEORY

B.Tech III semester

LECTURE NOTES

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Prepared By

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SYLLABUS

UNIT – I
ELECTROSTATICS

Electrostatic fields: Coulomb's law, electric field intensity due to line and surface charges, work done in moving a point charge in an electrostatic field, electric potential, properties of potential function, potential gradient, Gauss's law, application of Gauss's law, Maxwell's first law, Laplace's and Poisson's equations, solution of Laplace's equation in one variable.

UNIT – II
CONDUCTORS AND DIELECTRICS

Electric dipole: Dipole moment, potential and electric field intensity due to an electric dipole, torque on an electric dipole in an electric field, behavior of conductors in an electric field, electric field inside a dielectric material, polarization, conductor and dielectric, dielectric boundary conditions, capacitance of parallel plate and spherical and coaxial capacitors with composite dielectrics, energy stored and energy density in a static electric field, current density, conduction and convection current densities, Ohm's law in point form, equation of continuity.

UNIT – III
MAGNETOSTATICS

Static magnetic fields: Biot-Savart's law, magnetic field intensity, magnetic field intensity due to a straight current carrying filament, magnetic field intensity due to circular, square and solenoid current carrying wire, relation between magnetic flux, magnetic flux density and magnetic field intensity, Maxwell's second equation, $\text{div}(\mathbf{B})=0$.

Ampere's circuital law and its applications: Magnetic field intensity due to an infinite sheet of current and a long current carrying filament, point form of Ampere's circuital law, Maxwell's third equation, $\text{Curl}(\mathbf{H})=\mathbf{J}_c$, field due to a circular loop, rectangular and square loops.

UNIT – IV
FORCE IN MAGNETIC FIELD AND MAGNETIC POTENTIAL

Magnetic force: Moving charges in a magnetic field, Lorentz force equation, force on a current element in a magnetic field, force on a straight and a long current carrying conductor in a magnetic field, force between two straight long and parallel current carrying conductors, magnetic dipole and dipole moment, a differential current loop as a magnetic dipole, torque on a current loop placed in a magnetic field;

Scalar magnetic potential and its limitations: Vector magnetic potential and its properties, vector magnetic potential due to simple configurations, Poisson's equations, self and mutual inductance, Neumann's formula, determination of self-inductance of a solenoid, toroid and determination of mutual inductance between a straight long wire and a square loop of wire in the same plane, energy stored and density in a magnetic field, characteristics and applications of permanent magnets.

UNIT – V
TIME VARYING FIELDS AND FINITE ELEMENT METHOD

Time varying fields: Faraday's laws of electromagnetic induction, integral and point forms, Maxwell's fourth equation, $\text{curl}(\mathbf{E})=-\partial\mathbf{B}/\partial t$, statically and dynamically induced EMFs, modification of Maxwell's equations for time varying fields, displacement current; Numerical methods: Finite difference method (FDM), finite element method (FEM), charge simulation method (CSM), boundary element method, application of finite element method to calculate electrostatic and magneto static fields.

UNIT-1
ELECTRO-STATICS

1.1 Introduction

The most known particles are photons, electrons and neutrons with different masses. Their masses are

$$m_e = 9.10 \times 10^{-31} \text{ kilograms}$$

$$m_p = 1.67 \times 10^{-27} \text{ kilograms}$$

these masses leads to gravitational force between them, given as

$$F = G m_e m_p / r^2$$

The force between two opposite charges placed 1cm apart likely to be 5.5×10^{-67} and force between two like charges placed 1cm apart likely to be 2.3×10^{-24} . this force between them is called as electric force .

Electric force is larger than gravitational force.

- ➔ Gravitational force is due to their masses.
- ➔ Electric force is due to their properties.
- ➔ Neutron has only mass but no electric force.

ELECTROSTATICS:

Electrostatics is the study of charge at rest. The study of electric and magnetic field can be done using MAXWELL'S equations. Electrostatic field is developed between static charges. Electrostatics got wide variety applications like X-rays, lightning protections etc.

Let us study the behavior of electric field using COLOUMB's and GAUSS laws.

1.2 Point Charge

A charge with smallest dimensions on the body compare to other charges is called as point charge. A group of charges concentrated on any pin head may be also called as point charge.

1.3 Columb's Law

Coloumb stated that the force between two point charges is

- ➔ Directly proportional to product of charges.
- ➔ Inversely proportional square of distance between the.

$$F \propto Q_1 Q_2 / r^2$$

$F = K Q_1 Q_2 / r^2$, where K is the proportionality constant.

$K = 1 / 4\pi\epsilon$, where ϵ is the permittivity of the medium.

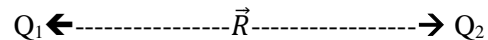
$E = \epsilon_0 \epsilon_r$, $\epsilon_0 = \text{absolute permittivity} = 8.854 \times 10^{-12}$
 $\epsilon_r = \text{relative permittivity}$

most common medium is air or vacuum whose relative permittivity is 1, hence permittivity of air or vacuum is

$$\epsilon = 9 \times 10^9 \text{ m/F}$$

Force between two point charges using vector analysis:

Let us consider two point charges separated by some distance given as \vec{R} .



According to coloumb's law force between them is given as

$$F = (K Q_1 Q_2 / r^2) \times \hat{a} , \text{ where } \hat{a} \text{ is the unit vector direction of force.}$$

Let F_2 is the force experienced by Q_2 due to Q_1 and F_1 is force experienced by Q_1 due to Q_2 . The direction of forces opposes each other , hence we can write in vector form as

$$\vec{F}_1 = - \vec{F}_2$$

Hence unit vector can be \hat{a}_{12} or \hat{a}_{21} , from the vector analysis we can write

$$\hat{a}_{12} = \vec{R}_{12} / R_{12} = \vec{R} / R \text{ and}$$

$$\hat{a}_{21} = \vec{R}_{21} / R_{21} = -\vec{R} / R$$

Therefore the magnitude of force between them can be written as

$$F_1 = F_2 = (K Q_1 Q_2 / R^2) \times \widehat{R}$$

1.4 Electric Field

It is the region around the point and group charges in which another charge experiences force is called as electric field.

The force between two charges can be studied in terms of electric field as :

- 1) A charge can develop field surrounding it in space only.
- 2) The field of one charge leads to force on the other charge .

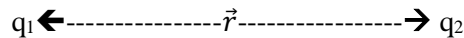
1.5 Electric Field Intensity

If an point charge q experiences the force F , then the electric field intensity of charge is defines as

$$E = F/q$$

Here charge q is called as test charge because the force experienced by it is due field of other charge.

The units of electric field intensity are N/C or V/m.


$$q_1 \leftarrow \text{-----} \vec{r} \text{-----} \rightarrow q_2$$

the force experienced by q_2 because of field of q_1 is

$$\text{vector, } F_2 = (K q_1 q_2 / r^2) \times \hat{a}$$

Therefore electric field intensity on q_2 charge is

$$\text{Vector, } E = F_2/q_2 = (K q_1 / r^2) \times \hat{a}$$

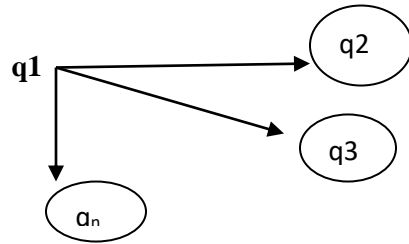
the force experienced by q_1 because of field of q_2 is

$$\text{vector, } F_1 = (K q_1 q_2 / r^2) \times \hat{a}$$

Therefore electric field intensity on q_2 charge is

$$\text{Vector, } E = F_1/q_1 = (K q_2 / r^2) \times \hat{a}$$

1.5.1 Electric Field Intensity due to n point charges



let the point charges q_2, q_3, \dots, q_n are placed at a distance of r_2, r_3, \dots, r_n from q_1 . Hence total electric field intensity on q_1 due remaining point charges is

$$\text{force due to } q_2 \text{ on } q_1, F_2 = (K q_1 q_2 / r^2) \times \hat{a}_2$$

$$\text{force due to } q_3 \text{ on } q_1, F_3 = (K q_1 q_3 / r^2) \times \hat{a}_3$$

$$\text{force due to } q_n \text{ on } q_1, F_n = (K q_1 q_n / r^2) \times \hat{a}_n$$

therefore total electric field intensity is , $\vec{E} = (F_2 + F_3 + \dots + F_n) / q_1$

$$= (K q_2 / r^2) \times \hat{a}_2 + (K q_3 / r^2) \times \hat{a}_3 + \dots + (K q_n / r^2) \times \hat{a}_n$$

1.6 Charge Distribution

Charge distribution is of three types- line charge, surface and volume charge distribution.

Line charge: Here charge is distributed through out some length . the total charge distributed through a wire of length l is

$$Q = \int \rho_l dl$$

Where, ρ_l ----- line charge density

Hence electric field intensity due to line charge is ,

$$E = \int (K \int \rho_l dl / r^2) \times \hat{a}$$

Surface charge: Here charge is distributed through given area . the total charge distributed in an surface area is

$$Q = \int \rho_s ds$$

Where, ρ_s ----- surface charge density

Hence electric field intensity due to surface charge is ,

$$E = \int (K \int \rho_s ds / r^2) \times \hat{a}$$

Volume charge: Here charge is distributed through given volume . the total charge distributed in an volume is

$$Q = \int \rho_v dv$$

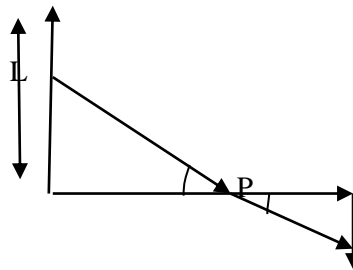
Where, ρ_v ----- volume charge density

Hence electric field intensity due to volume charge is ,

$$E = \int (K \int \rho_v dv / r^2) \times \hat{a}$$

1.7 Electric field intensity due line charge

Let us consider a straight wire of length l is symmetrically placed in X-Y axis as shown in below figure



For a small length of dl on y-axis the charge is dq , the electric field intensity due dq at test point p is

$$dE = Kdq / (y^2 + a^2)$$

then,

$$dE_x = Kdq \cdot \cos\theta / (y^2 + a^2) \text{-----} 0$$

$$\cos\theta = a / \sqrt{y^2 + a^2} \text{-----} 1$$

we can write charge per unit length as $dq/dl = Q/l$,

$$dq = Q \cdot dl / l \quad (dl = dy)$$

$$dq = Q \cdot dy/l \text{ -----}2$$

substituting equation 1 and 2 in 0

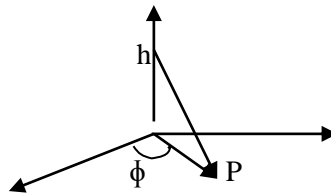
$$dE_x = KQ \, dy \cdot a / l \cdot (y^2 + a^2)^{3/2}$$

integrating on both sides with limits $-l/2$ and $l/2$, $E_x = \int KQ \, dy \cdot a / l \cdot (y^2 + a^2)^{3/2}$

and when l tends to ∞ , $E_x = KQ / a^2$.

1.8 Electric field intensity due surface charge

Let us consider an infinite sheet placed uniformly in xy plane as shown in figure.



Let us consider a small area ds in xy plane, $ds = \rho \cdot d\rho \cdot d\phi$.

Which is located at distance of ρ from origin making an angle of ϕ .

P be the point on z axis given as $(0,0,h)$.

Distance from P to ds is \vec{R} .

$$\vec{R} = -\rho \mathbf{a}_\rho + h \mathbf{a}_z$$

$$R = \sqrt{(\rho^2 + h^2)}$$

$$\mathbf{a}_R = (-\rho \mathbf{a}_\rho + h \mathbf{a}_z) / \sqrt{(\rho^2 + h^2)}$$

hence the electric field intensity at ds is given as ,

$$\vec{E} = \int \rho s \, ds \cdot \mathbf{a}_R / 4\pi\epsilon \cdot R^2$$

$$\vec{E} = \int \rho s \, \rho \cdot d\rho \cdot d\phi \cdot (-\rho \mathbf{a}_\rho + h \mathbf{a}_z) / \sqrt{(\rho^2 + h^2)} \cdot 4\pi\epsilon \cdot R^2$$

The limits of ρ from 0 to ∞ and ϕ from 0 to 2π .

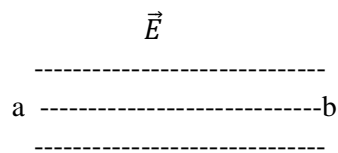
$$\vec{E} = \iint \rho_s \rho \cdot d\rho \cdot d\phi \cdot (-\rho \mathbf{a}_\rho + h \mathbf{a}_z) / \sqrt{(\rho^2 + h^2)} \cdot 4\pi\epsilon \cdot R^2$$

By simplifying above equation, the electric field intensity

$$\vec{E} = \rho_s \mathbf{a}_z / 2\epsilon.$$

1.9 Work done in moving point charge

Let us consider a charge q is placed in the existing electric field. The charge q experiences force \vec{F} .



Here charge q is made to move from a to b of length l through electric field intensity \vec{E} .

$$dw = -\vec{F} \cdot d\mathbf{l} = -q\vec{E} \cdot d\mathbf{l}$$

integrating on both sides, $w = -q \int \vec{E} \cdot d\mathbf{l}$ with limits a to b.

1.10 Electric Potential

From the above discussion work done to move point charge through the existing electric field is

$$w = -q \int \vec{E} \cdot d\mathbf{l}$$

but we know that electric potential is defined work done to move unit charge

$$V = w/q$$

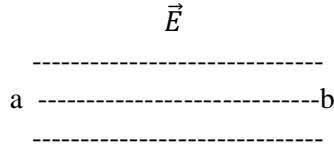
Therefore, $V = w/q = - \int \vec{E} \cdot d\mathbf{l}$ with limits a to b

$$V = -\vec{E} \cdot \mathbf{l} \text{ with limits a to b}$$

Hence, electric potential $V = V_a - V_b$

1.11 Electric potential due to point charge

P (point charge)



Let the charge q is moved from a to b and at point charge is Q from r_a and r_b ,

We know that electric potential, $V = q/(4\pi\epsilon r)$

Electric field intensity at point p due to charge at a is, $V_a = Q/(4\pi\epsilon r_a)$

Electric field intensity at point p due to charge at b is, $V_b = Q/(4\pi\epsilon r_b)$

Hence potential difference or electric potential from a to b is,

$$V_{ab} = V_a - V_b$$

$$V_{ab} = Q/(4\pi\epsilon r_a) - Q/(4\pi\epsilon r_b)$$

$$V_{ab} = Q(r_b - r_a)/(4\pi\epsilon r_a r_b).$$

1.12 Electric Flux

Micheal faraday has conducted experiment on two concentric spheres, inner layer is positively charges and outer layer negatively charge, then he observe that their some sort of displacement from inner layer to outer layer , this displacement is pronounced as electric flux between spheres.

1.13 Electric Flux Density

We know that electric field intensity is , $E = (K q / r^2)$

$$= q/ (4\pi\epsilon r^2)$$

$$D=\epsilon E = q/ (4\pi r^2)\text{---electric flux density}$$

Electric flux density is defined as charge per unit area.

Potential gradient: potential gradient is defined as electric change in electric potential due to change in the distance or length.

$$E = - \nabla V$$

Properties of Potential:

- ➔ Potential is the energy acquired by the charge.
- ➔ When charge travel from one end to other end in any element there is potential change from high to low.
- ➔ Potential acquired by point charge leads to electric field.

1.14 Gauss Law

Gauss law states that the total flux in the given surface is equal to charge enclosed in it.

$$\phi = Q.$$

the total flux enlaced in given surface is

$$\begin{aligned}\phi &= \int E \, ds \\ &= \int Q / (4\pi\epsilon r^2) \, ds \\ &= Q / (4\pi\epsilon r^2) \cdot \int ds \\ &= Q4\pi r^2 / (4\pi\epsilon r^2). \\ &= Q / \epsilon.\end{aligned}$$

Applications of Gauss law

- ➔ To apply gauss law first assume Gaussian surface.
- ➔ The electric field intensity must be normal to the Gaussian surface.
- ➔ Gaussian surface must be symmetry.

1.15 Maxwell First Equation

We know that electric flux passing through the surface is equal to $1/\epsilon$ times the net charge enclosed.

$$\phi = \int_s E ds = Q/\epsilon$$

$$\phi = \int_s \epsilon E ds = Q \epsilon / \epsilon$$

$$\phi = \int_s D ds = Q$$

from the Stokes theorem we can say that surface integral function is volume integral of divergence of same function.

$$Q = \int_s D ds = \int_v (\nabla \cdot D) dv \text{ -----3}$$

from the Gauss law we can write, $Q = \int_v \rho_v dv \text{ ----- 4}$

by comparing equation 3 and 4

$$(\nabla \cdot D) = \rho_v \text{ -----5}$$

Equation 3 and 5 are said to be Maxwell's first and second equation.

1.16 Poisson and Laplace Equations

From the Maxwell's equation we know that,

$$(\nabla \cdot D) = \rho_v \text{ -----6}$$

$$D = \epsilon E \text{ ----- 7}$$

Equation 7 in 6,

$$\nabla \cdot \epsilon E = \rho_v$$

but we know that,

$$E = -\nabla V$$

$$\nabla \cdot \epsilon(-\nabla V) = \rho_v$$

$$\nabla^2 V = -\rho_v / \epsilon \text{ -----8}$$

Equation 8 is called Poisson's equation.

In the uniform Gaussian surface,

$$\rho_v = 0$$

Then equation 8 can be rewritten as,

$$\nabla^2 V = 0 \text{ -----9}$$

Equation 9 is called as Laplace equation.

UNIT-II
ELECTRIC DIPOLE AND CAPACITANCE

2.1 Electric Dipole and Dipole Moment

Two opposite charges $+q$ and $-q$ separated by some distance d forms the electric dipole.

$$+q \text{ ----- } d \text{ ----- } -q$$

The distance travelled by the point charge is defined as dipole moment (or) the product of charge and distance travelled by it is called as electric dipole.

$$P = qd \text{ ----- } 1$$

Here, $P \rightarrow$ electric dipole moment

$d \rightarrow$ distance between opposite charges

the line between two charges is called as axis of dipole. Potential

2.2 Electric Dipole Potential

Let us assume two charges separated by distance d as shown in the figure

$$+q \text{ ----- } d \text{ ----- } -q$$

Here, $O \rightarrow$ center of the axis between charges

$P \rightarrow$ be the test point where potential is required.

$OP \rightarrow$ with length of r .

$AA^1 \rightarrow$ perpendicular from A to OP .

$BB^1 \rightarrow$ perpendicular from A to OP .

$$\angle POB = \theta$$

$$r \gg d$$

the line

$$AP = A^1P = OP + OA^1 \text{ ----- } 2$$

from the right angle triangle AA^1O , $OA^1 = OA \cos \theta$

hence equation 2 can be written as, $AP = A^1P = r + OA \cos \Theta$

but, $OA = d/2$

$$AP = A^1P = r + d/2 \cos \Theta$$

Hence the potential at P due negative charge at A is ,

$$V_A = -Kq/ AP = -Kq/ r + d/2 \cos \Theta$$

Similarly from the right angle triangle BB^1O , $BP = B^1P = r - d/2 \cos \Theta$

Hence the potential at P due negative charge at A is ,

$$V_B = Kq/ BP = Kq/ r - d/2 \cos \Theta$$

Therefore the total potential acting on P is , $V = V_A + V_B$

$$V = Kq[(1/ r - d/2 \cos \Theta) - (1/ r + d/2 \cos \Theta)]$$

$$= Kqd.\cos \Theta/ (r^2 - d^2/4 \cos^2 \Theta)$$

But we know that,

$$r \gg d$$

$$V = Kqd.\cos \Theta/ r^2$$

$$V = KP.\cos \Theta/ r^2 , (P = qd) \text{ ----- 3}$$

2.3 Electric Field Intensity due to Dipole

We know that electric field intensity in terms of electric potential is given as ,

$$E = - \nabla V$$

From equation 3 we can say that potential due dipole is in spherical co-ordinates, therefore find electric field intensity we shall use spherical co-ordinates.

$$\nabla V = -[dv/dr + (1/r)dv/d\Theta]$$

Simplifying ∇V ,

$$dv/dr = -2KP.\cos \Theta/ r^3$$

$$(1/r)dv/d\theta = -KP.\sin \theta / r^3$$

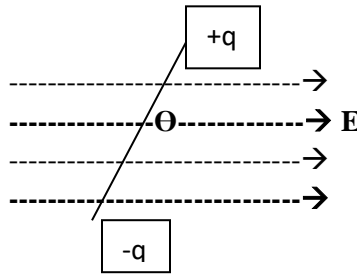
Substituting above two equations in E, $E = -[(-2KP.\cos \theta / r^3) + (- KP.\sin \theta / r^3)]$

$$= [(2KP.\cos \theta / r^3) + (KP.\sin \theta / r^3)]$$

$$= KP/ r^3 [(2\cos \theta) + (\sin \theta)] \text{----- 4}$$

2.4 Torque due to Electric Dipole

Let us consider two opposite charges are placed in the uniform electric field with their line of axis of $2r$.



The experienced by +q is , $F_1 = E.q$

The experienced by -q is , $F_2 = -E.q$

The total experienced by the dipole is , $F = F_1 + F_2$

$$F = 0$$

But the due to force experienced by +q it tends to oscillate in the direction of E and -q in the direction opposite to E, which leads torque of dipole.

T = magnitude of F x perpendicular distance
Between their line of action

$$T = E.q \times 2r \sin\theta$$

$$T = PE.\sin\theta. \quad (q.2r = P) \text{-----5}$$

2.5 Polarization

If an piece if dielectric or insulator placed between the charges plates of condenser, then center of gravity of negative charges is concentrated towards positive plate and center of gravity of positives charges concentrated towards negative plate, this process of separation opposite charges is called a polarization.

Polarization is also defined as electric dipole moment per unit volume.

Let A be the area of cross section of dielectric,
 l be the distance by with opposite charges are separated,
 q total charge in the volume of dielectric

then polarization,

$$P = \text{dipole moment} / \text{volume}$$

$$= q.l / A.l$$

$$= q / A \text{ ----- } 6$$

i.e the polarization numerically equal to surface charge density.

2.6 Dielectric Constant and Electric Susceptibility

Dielectric constant is defined as ratio capacitance of capacitor with dielectric to the capacitance of capacitor without dielectric .

Capacitance of capacitor with dielectric has low potential(V_d) than the capacitance of capacitor without dielectric(V) .

$$K = V / V_d \text{----- } 7$$

The polarization is directly proportional to the electric field intensity created between charges.

$$P \propto E$$

$$P = K_e E$$

$$K_e = P / E = \text{electric susceptibility} \text{----- } 8$$

2.7 Capacitor and its Capacitance

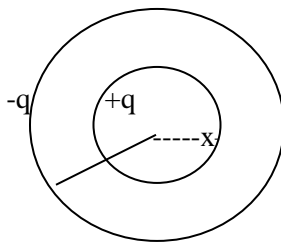
The basic capacitor element is formed by separated two parallel plates with some dielectric medium.

When some voltage is applied to such an element charge is formed between the plates, their by capacitance of capacitor is defined as charge Q developed between the plates when voltage V is applied.

$$C = Q / V \text{ ----- 9}$$

The units of capacitance are Farads (F).

2.8 Capacitance of the Isolated Sphere



Let us consider an isolated sphere which is positively charges with radius x and negatively charges plate placed at infinite distance.

The electric flux density due to positive charge, $D = Kq / x^2$

Electric field intensity due to positive charge, $\epsilon E = Kq / x^2$

$$E = Kq / . x^2$$

Work done,

$$w = - q \int E dl.$$

$$W = -q \int E dx \text{ with limits } \infty \text{ to } x$$

$$V = - \int E dx \text{ with limits } \infty \text{ to } x$$

$$V = - \int Kq / . x^2 dx \text{ with limits } \infty \text{ to } x$$

$$= -K.q / (-. x) \text{ with limits } \infty \text{ to } x$$

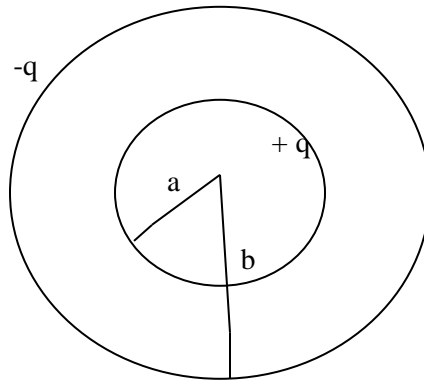
$$= K.q / (. x)$$

But the capacitance is given charge per voltage,

$$C = q / V$$

$$C = (x) / K \text{ ----- } 10$$

2.9 Capacitance of the spherical Sphere



Let us consider an isolated sphere which is positively charges with radius a and negatively charges plate placed at b distance.

The electric flux density due to positive charge, $D = Kq / x^2$

Electric field intensity due to positive charge, $\epsilon E = Kq / x^2$

$$E = Kq / . x^2$$

Work done,

$$w = - q \int E dl.$$

$$W = -q \int E dx \text{ with limits } b \text{ to } a$$

$$V = - \int E dx \text{ with limits } b \text{ to } a$$

$$V = - \int Kq / . x^2 dx \text{ with limits } b \text{ to } a$$

$$= -K \cdot q / (-x) \text{ with limits } b \text{ to } a$$

$$= \frac{Kq}{ab} \cdot [(1/a) - (1/b)]$$

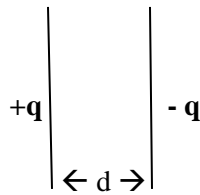
$$= \frac{Kq(b-a)}{ab}$$

But the capacitance is given charge per voltage,

$$C = q / V$$

$$C = ab / K(b-a) \text{ ----- } 11$$

2.10 Capacitance of the Parallel Plates



Let potential applied to these parallel plates is V their by forming charge q between them.

Electric flux density between plates,

$$D = q / A$$

$$\epsilon E = q / A$$

$$E = q / \epsilon \cdot A,$$

$$V = E \cdot d$$

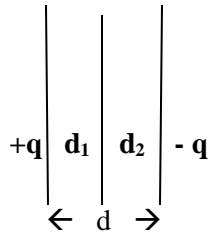
$$V = q d / \epsilon \cdot A$$

But the capacitance is given charge per voltage,

$$C = q / V$$

$$C = \epsilon \cdot A / d \text{ ----- } 12$$

2.11 Capacitance of the Parallel Plates with two dielectric mediums



Let potential applied to first part is V_1 their by forming charge q between them.

Electric flux density between plates,

$$D = q / A$$

$$\epsilon E_1 = q / A$$

$$E_1 = q / \epsilon_1.A,$$

$$V_1 = E.d_1$$

$$V_1 = q d_1 / \epsilon_1.A$$

But the capacitance is given charge per voltage,

$$C = q / V$$

$$C_1 = \epsilon_1.A / d_1 \text{ ----- } 13$$

Let potential applied to first part is V_2 their by forming charge q between them.

Electric flux density between plates,

$$D = q / A$$

$$\epsilon E_2 = q / A$$

$$E_2 = q / \epsilon_2.A,$$

$$V_2 = E.d_2$$

$$V_2 = q d_2 / \epsilon_2.A$$

But the capacitance is given charge per voltage,

$$C = q / V$$

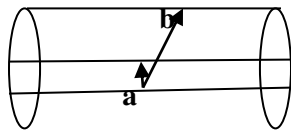
$$C_2 = \epsilon_2 \cdot A / d_2 \text{ ----- 14}$$

Hence total capacitance between plates with multiple dielectric mediums is ,

$$\begin{aligned} C &= C_1 + C_2 \\ &= (\epsilon_1 \cdot A / d_1) + (\epsilon_2 \cdot A / d_2) \\ &= A / [(d_1/ \epsilon_1) + (d_2/ \epsilon_2) \text{ ----- 15.} \end{aligned}$$

2.12 Capacitance of the Co-axial Cable

Let us consider co-axial cable two isolated sphere with radius a and b from center of axis



the length of cable is , then line charge distribution $\rho_l = q / l$

the electric flux density generally in cable is , $D = \rho_l / 2\pi r$

therefore electric field intensity , $E = \rho_l / 2\pi r \epsilon$

the electric potential of the cable is , $V = -\int E dr$, with limits b to a

$$= -\int (\rho_l / 2\pi r \epsilon) dr$$

$$= -(\rho_l / 2\pi \epsilon) \int dr/r$$

$$= -(\rho_l / 2\pi \epsilon) \cdot \ln(r)$$

By applying limits,

$$V = -(\rho_l / 2\pi \epsilon) \cdot [\ln(a) - \ln(b)]$$

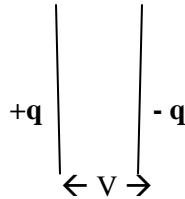
$$V = (\rho_l / 2\pi \epsilon) \cdot \ln(b/a)$$

The capacitance of co-axial cable,

$$C = \rho_l / V$$

$$C = 2\pi\epsilon / \ln(b/a) \text{ ----- 21}$$

2.13 Energy Stored in the Capacitor



By the definition capacitance between plates is , $C = q / V$

Electric potential, $V = dw / dq$

$$dw = V dq$$

$$dw = (q / C) dq$$

integrating on both sides,

$$w = \int (q / C) dq$$

$$w = q^2 / 2C \quad (\text{or}) \text{ ----- 16}$$

$$w = (CV)^2 / 2C$$

$$w = CV^2 / 2 \quad (\text{or}) \text{ ----- 17}$$

$$w = q^2 / 2C$$

$$w = Vq / 2 \quad \text{-----18}$$

2.14 Energy Density in the Static Electric Field

Energy density of capacitor is defined energy stored per unit volume.

$$W_d = \text{energy stored} / \text{volume}$$

$$W_d = CV^2 / 2 / Ad$$

$$W_d = \epsilon A V^2 / d / 2. Ad$$

$$W_d = \epsilon V^2 / 2d^2$$

$$W_d = \epsilon E^2 / 2$$

$$W_d = DE / 2 \text{ ----- 19}$$

From equation 19 we can write,

$$dW / dV = DE / 2$$

$$dW = (DE / 2) dV$$

integrating on both sides, energy stored $W = \int_v (DE / 2) dV \text{ ----- 20}$

2.15 current

The flow of electrons from one end to other end constitutes current. The rate of change of

Charge is also defined as current.

$$i = q / t = dq / dt \text{ ----- 22}$$

the units of current is ampere.

Current Density

If charge is distributed in the given area, then current density is defined as current constituted

In given area.

$$J = i / A \text{ (A/mt}^2\text{) ----- 23}$$

$$J = di / ds$$

$$di = J .ds$$

integrating on both sides, $i = \int J .ds$

Convection Current Density

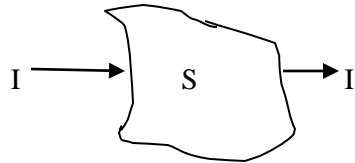
Let us consider a material with volume of charges (ρ_v) moving with drift velocity (V_d), then

Convection Current density is defined as product volume of charges moving with drift velocity.

$$\mathbf{J} = \rho_v \times \mathbf{V}_d \text{-----} 24.$$

2.16 Equation of Continuity

Let us an surface area through charges are moving in and out as shown in the figure



Let the charge q is moving through an area of S .

According law of conservation of charge,

$$[I]_s = - dq / dt$$

But current passing through area is ,

$$[I]_s = \int \mathbf{J} \, ds$$

Total charge in the given volume is,

$$q = \int_v \rho_v \, ds$$

From above three equations we can write,

$$\int \mathbf{J} \, ds = -(d/dt). \int_v \rho_v \, dv \text{-----} 25$$

from the stokes theorem we can write,

$$\int \mathbf{J} \, ds = \int_v \nabla \cdot \mathbf{J} \, dv \text{-----} 26$$

by comparing equation 25 and 26,

$$\int_v \nabla \cdot \mathbf{J} \, dv = -(d/dt). \int_v \rho_v \, dv$$

$$\int_v \nabla \cdot \mathbf{J} \, dv + (d/dt). \int_v \rho_v \, dv = 0$$

$$\int_v [\nabla \cdot \mathbf{J} + d\rho_v / dt] \, dv = 0 \text{-----} 27$$

equation 27 is called as equation of continuity or maxwell's fifth equation.

UNIT-III
MAGNETO-STATICS

3.1 Introduction

Magneto-statics is the study of magnetic field developed by the constant current through the coil

Or due to permanent magnets.

The behavior of constant magnetic field is studied by using two basic laws, they are

- Bi-Savart's law
- Ampere's circuital law.

3.2 Magnetic Field



Let us consider a constant current I is passing through coil shown above which develops constant

Flux surrounding the coil thereby forming north and south poles. This formation magnetic from

North pole to south pole is called as magnetic field. The direction of magnetic flux in an coil is

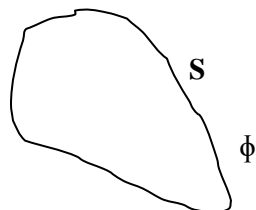
Given by right hand thumb rule.

Right hand thumb rule says that if four fingers of hand folded such that they show direction of winding

Then thumb indicates direction of flux.

3.3 Definitions of Magnetic Field

Magnetic flux density



magnetic flux density is defined as flux per unit area, $B = d\phi / ds$ (Wb/m² or Tesla)

$$d\phi = B ds$$

by integrating on both sides we can determine total magnetic flux in area,

$$\phi = \int B ds \text{ -----} 28$$

Magnetic Field Intensity

The force experienced by coil when some current passes through it is magnetic field

Intensity. Mathematically magnetic field intensity is given as,

$$H = \text{magnetic force} / \text{length}$$

$$\text{Magnetic force} = NI$$

$$\text{Length} = l$$

$$\text{Therefore magnetic field intensity, } H = NI / l \text{ (AT/mt) -----} 29$$

Magnetic Permeability

Permeability is the inherent property of core which helps in sustaining flux in the core.

$$\text{Mathematically permeability is given as, } \mu = B / H \text{ -----} 30$$

From equation 30 the relation between flux density and intensity is ,

$$B = \mu H \text{ -----} 31$$

Where

$$\mu = \mu_0 \mu_r$$

$$\mu_0 = \text{absolute permeability} = 4\pi \times 10^{-7} \text{ H/mt}$$

$$\mu_r = \text{relative permeability}$$

varies from core to core

Intensity of Magnetization

When a magnetic substance is placed in a magnetic field it experiences magnetic momentum.

The magnetic momentum per unit volume of substance is intensity of magnetization.

$$I = M / V$$

$$M = m.l \quad (m - \text{pole strength of bar, } l - \text{length})$$

$$V = A.l$$

intensity of magnetization, $I = m.l / A.l$

$$I = m / A$$

Magnetic Susceptibility

The ratio intensity of magnetization to the magnetic field intensity is called as Magnetic Susceptibility.

$$K = I / H.$$

Total flux density, $B = B$ due to magnetic field + B due to intensity of magnetization of bar

$$B = \mu_0 H + I$$

But we know that, $\mu = B / H$

$$= (\mu_0 H + I) / H$$

$$= \mu_0 + (I/H)$$

$$\mu_0 \mu_r = \mu_0 + K$$

$$\mu_r = 1 + K / \mu_0 \quad \text{----- 31}$$

$\mu_r > 1$, paramagnetic materials

$\mu_r < 1$, diamagnetic materials

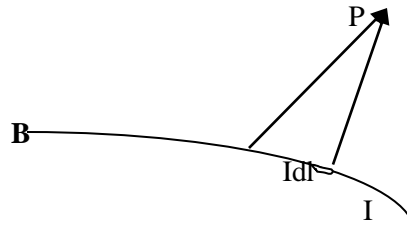
$\mu_r = 0$, non-magnetic materials

3.4 Biot-Savart's Law

Bio and savart are two scientists who conducted experiments on current carrying conductor

To determine magnetic flux density(B) at any point surrounding that conductor. Their

Conclusion is named as “Biot-Savart's Law”.



Let us consider an conductor carrying current I, which develops magnetic flux density B surrounding

It. Here Idl is called as current element. To find total electric field intensity conductor is divided into

Number of current elements.

The magnetic field intensity due to current element Idl is dH at point P. According Bio-Savart's law

$$dH \propto Idl \text{ (current element)}$$

$$dH \propto \sin\theta \text{ (angle between current element and length joining point)}$$

$$dH \propto 1 / r^2 \text{ (square of distance between current element and point)}$$

by combining above three,

$$dH \propto Idl \cdot \sin\theta / r^2$$

by removing proportionality,

$$dH = Idl \cdot \sin\theta / 4\pi r^2$$

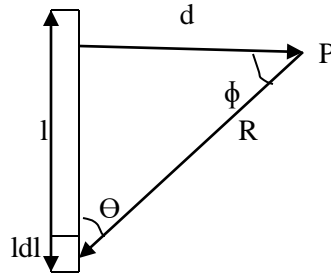
total magnetic field intensity at point P,

$$H = \int Idl \cdot \sin\theta / 4\pi r^2$$

therefore total flux density at point P, $B = \mu H$

$$B = \mu \int Idl \cdot \sin\Theta / 4\pi r^2 \text{ ----- 32}$$

3.5 Magnetic Field Intensity due to a finite length of current carrying filament



Let us consider a straight conductor of length l, a test point P at which electric field intensity is to be Determined at a distance of d from conductor. Assume current element with a distance of R to P.

From Bio-Savart's law magnetic field intensity at test point P due to current element ldl is ,

$$dH = Idl \cdot \sin\Theta \cdot \hat{a} / 4\pi R^2 \text{ ----- a}$$

from above right angle triangle, $\Theta + \phi = 90^\circ \text{ ----- b}$

using equation a and b, $dH = Idl \cdot \cos \phi \cdot \hat{a} / 4\pi R^2 \text{ ----- c}$

the unit vector \hat{a} , indicates the direction H at point P.

$$\hat{a} = \hat{R} / R \text{ ----- d}$$

from above right angle triangle, $R = \sqrt{l^2 + d^2} \text{ ----- e}$

$$\cos \phi = d / \sqrt{l^2 + d^2} \text{ ----- f}$$

$$\tan\phi = l / d \text{ ----- g}$$

$$l = d \cdot \tan\phi$$

$$dl = d \sec^2 \phi \, d\phi \text{ ----- h}$$

substituting d,e,f in c,

$$dH = Idl \cdot \cos \phi \cdot d \cdot \hat{R} / 4\pi (l^2 + d^2)^2$$

integrating on both sides

$$H = \int Idl \cdot \cos \phi \cdot d \cdot \hat{R} / 4\pi (l^2 + d^2)^2$$

$$H = \frac{I}{4\pi d^2} \int dl / (l^2 / d^2 + 1)^{3/2}$$

$$H = \frac{I}{4\pi d^2} \int dl / (\tan^2 \phi + 1)^{3/2}$$

Substituting equation h in above equation is ,

$$H = \frac{I}{4\pi d^2} \int d \sec^2 \phi \, d\phi / (\sec^2 \phi)^{3/2}$$

$$H = \frac{I}{4\pi d^2} \int d \sec^2 \phi \, d\phi / (\sec^3 \phi)$$

$$H = \frac{I}{4\pi d} \int \cos \phi \, d\phi$$

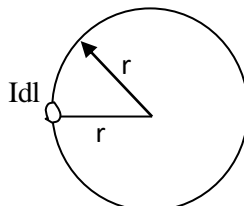
$$H = \frac{I}{4\pi d} \sin \phi \text{ -----33}$$

For straight line of infinite length, ϕ varies between $-\pi/2$ to $\pi/2$

$$\text{Substituting above limits in equation 33, } H = \frac{I}{2\pi d} \text{ ----- 34}$$

3.5 Magnetic Field Intensity due to a circular current carrying filament

Let us consider circular conductor with radius r,



magnetic field intensity at the center of circular conductor is,

from above figure we can say that idl and center are at 90°

using Bio-Savart's law magnetic field intensity at center point P due to current element idl is,

$$dH = idl \sin 90 / 4\pi r^2$$

$$dH = idl / 4\pi r^2$$

integrating on both sides,

$$H = \int idl / 4\pi r^2$$

$$H = i \int dl / 4\pi r^2 \quad (\int dl = 2\pi r)$$

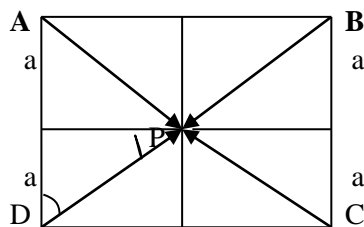
$$H = i 2\pi r / 4\pi r^2$$

$$H = i / 2r \text{ ----- 34}$$

Magnetic field intensity at the center of circular conductor with N number of turns is,

$$H = Ni / 2r \text{ ----- 35}$$

3.6 Magnetic Field Intensity due to a Square current carrying filament



From the above figure we can say that each side AB, BC, CD, DA has magnetic field intensity at the center of square conductor.

In every right angle triangle angle between current element and center is 45° .

The total magnetic field intensity at the center of square due to all corners using Bio-Savart's law

Because of any one side,

$$H = (I / 4\pi a) \times [\sin 45^\circ + \sin 45^\circ]$$

Using all sides,

$$H = 4(I / 4\pi a) \times [\sin 45^\circ + \sin 45^\circ]$$

$$H = (I / \pi a) \times [2 / \sqrt{2}]$$

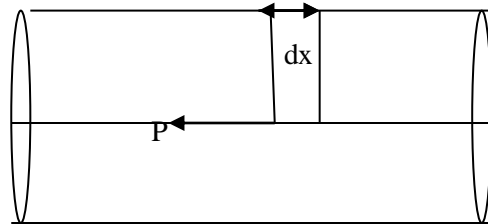
$$H = (\sqrt{2} \cdot I / \pi a) \text{ -----36}$$

3.7 Magnetic Field Intensity due to a Solenoid current carrying filament

The construction of solenoid is same as coil wound on a cylinder , let us take take cylinder

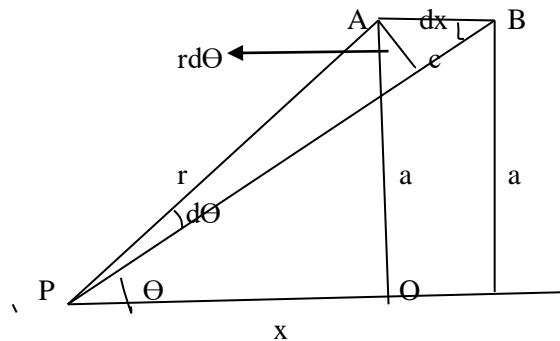
As reference and derive expression for H due to solenoid. The solenoid with length l, number of turns

N allowing an current of I is shown in below figure,



Assume a small length dx , with total turns ndx in it , let us derive what is the magnetic field intensity

Due to dx on P, their by total H at P.



total number of turns = N

total length = l

number of turns per unit length, $n = N / l$

x be the distance of the point,

the magnetic field intensity due to length dx on P is ,

$$dH = (Ia^2 / 2r^3) ndx$$

from figure ,

$$r = \sqrt{a^2 + x^2} \text{ , substituting } r \text{ in } dH.$$

$$dH = (Ia^2 / 2 (a^2 + x^2)^{3/2}) ndx$$

from above right angle triangles, $d\theta \ll \theta$, hence $\sin d\theta = d\theta$

$$\sin \theta = r d\theta / dx$$

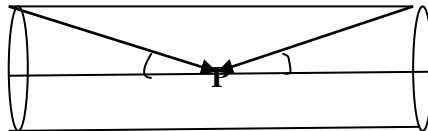
$$\sin \theta = a / r$$

substituting above deduction in dH ,

$$dH = (Ia^2 r \cdot d\theta / \sin \theta / 2r^3) n$$

$$dH = I \cdot n \cdot \sin \theta \cdot d\theta / 2 \text{ ----- a}$$

if seen from end points of solenoid the magnetic field intensity at P is



Here from one end to other end angle varies from 0 to 2π , substituting above and integrating equation a

$$\int dH = \int I \cdot n \cdot \sin \theta \cdot d\theta / 2$$

$$H = - I \cdot n \cdot \cos \theta / 2, \text{ substituting above limits -----b}$$

$$H = -(I \cdot n / 2) [\cos 2\pi - \cos 0]$$

$$H = I \cdot n = NI / l$$

if seen from end point of solenoid the magnetic field intensity at P at same end point,

then the limits varies between 0 to $\pi/2$

substituting above limits in b

$$H = -(I.n/2) [\cos\pi/2 - \cos 0]$$

$$H = n.I/2 = N.I/ 2l \text{ ----- 37}$$

3.8 Maxwell's Second Equation

From the gauss law we can write magnetic flux in the given surface is surface integral of

Magnetic flux density.

$$\Psi = \int B.ds$$

But total flux density in closed surface is always zero,

$$\Psi = \int B.ds = 0$$

By applying divergence theorem we can write,

$$\int B.ds = \int_v \nabla \cdot B.dv = 0$$

hence we can write , $\nabla \cdot B = 0$, is Maxwell's second equation----- 38

3.9 Ampere Circuital Law

The ampere circuital law states line integral magnetic field intensity around any closed path

Is equal to total current enclosed in that path.

$$\int H dl = I \text{ -----39}$$

Ampere's law is analogous to Gauss law electro-statics.

Applications of Ampere's law :

- ➔ The magnetic field intensity in the surrounding closed path is always tangential at each and every point on it.
- ➔ At each every point on the closed path magnetic field intensity has the same value.

3.10 Maxwell's Third Equation

From the ampere circuital law we know that,

$$\oint H \, dl = I$$

but current can be written as,

$$\int J \, ds = I$$

equating above two equations,

$$\oint H \, dl = \int J \, ds \text{ -----a}$$

from Stokes theorem,

$$\oint H \, dl = \int \nabla \times H \, ds \text{ ----- b}$$

by combining equation a and b,

$$\int \nabla \times H \, ds = \int J \, ds$$

by comparing on both sides,

$$\nabla \times H = J, \quad \nabla \times H = \text{curl of } H \text{ -----40}$$

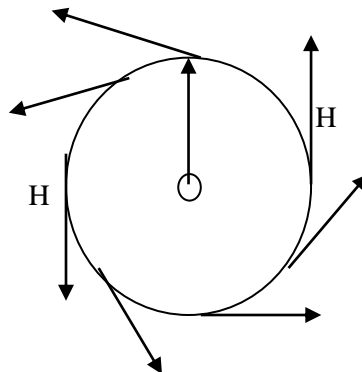
equation 40 is called as differential, integral or point form of Ampere's law and also called

as **Maxwell's Third Equation**

3.11 Magnetic field intensity due to long straight conductor using Ampere's law

Let us consider a straight conductor as shown in figure with closed path of magnetic field

Intensity surrounding it with radius of r.



From ampere's circuital law we can write magnetic field intensity in closed path,

$$\int H \, dl = I \text{ -----a}$$

but we can write,

$$\int H \, dl = H \int dl$$

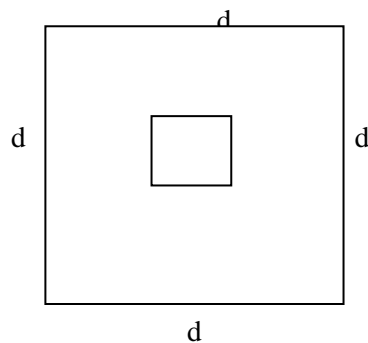
$$= H 2\pi r \text{ ----- b}$$

Equating a and b,

$$H 2\pi r = I$$

$$H = I / 2\pi r \text{ ----- 41}$$

3.12 Magnetic field intensity due to infinite sheet conductor using ampere's law



let us consider a square sheet as shown above with surrounding current path of side d.

according to Ampere's law ,

$$\int H \, dl = I$$

where $\int dl$ indicates the mean length closed path,

$$\int dl = 4d$$

there by ,

$$H \int dl = I$$

$$H.4d = I$$

$$H = I/4d. \text{-----42}$$

UNIT-IV
FORCE IN MAGNETIC FIELD AND MAGNETIC POTENTIAL

4.1 Force on moving charge.

When an charge Q is with velocity \vec{V} is placed in the magnetic field of density \vec{B} , then it

Experiences force called as magnetic force.

$$\vec{F}_m = Q(\vec{V} \times \vec{B}) \text{-----} 43$$

$$= QVB \sin\theta \text{ a}_r$$

\vec{V} is parallel to \vec{B} then $\theta = 0$, therefore $\sin\theta = 0$, hence always velocity direction and flux density

Direction must be normal to each other.

4.2 Moving charges in the magnetic field

The limitations of moving charge in the existing magnetic field,

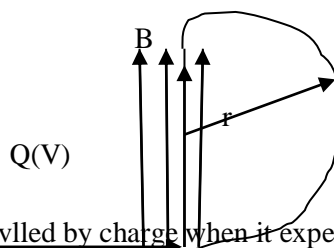
- ➔ If the velocity of charge in the magnetic field is zero then force experienced also zero.
- ➔ If the velocity direction and magnetic field direction are parallel to each other then force Experienced is zero.
- ➔ To say that moving charge in the magnetic field experiences force velocity and field must be normal to each other.

From the above discussion the force experienced by moving charge is ,

$$F_m = QVB.$$

Similarly we can also write force experienced by moving charge due to its mass is ,

$$F_m = ma$$



r is the radius made by path travllled by charge when it experiences force.

$$F_m = mV^2/r$$

By equating both forces,

$$QVB = mV^2/r$$

$$r = mV / QB$$

time taken to complete one revolution in field is ,

$$T = 2\pi r / V$$

$$= 2\pi m / QB$$

Hence frequency of charge in field is ,

$$F = 1/ T$$

$$= QB / 2\pi m, \text{ as this expression of frequency is independent}$$

Of velocity it is called as cyclotron.

4.3 Lorentz force equation

We know that the force acquire by point charge when kept in the static electric field is,

$$\vec{F}_e = Q \vec{E}$$

The force experienced by moving charge in the magnetic field is ,

$$\vec{F}_m = Q(\vec{V} \times \vec{B})$$

The total force on the charge in the presence of both field is,

$$\begin{aligned} \vec{F} &= \vec{F}_e + \vec{F}_m \\ &= Q \vec{E} + Q(\vec{V} \times \vec{B}) \end{aligned}$$

$$= Q(\vec{E} + (\vec{V} \times \vec{B})) \text{-----} 44$$

Equation 44 is called as **Lorentz force equation**.

4.4 Force on current element due to magnetic field

Let us a long conductor of length l which is partitioned into number parts allowing current

Of I. each part of conductor is of length dl, therefore individual part is represented with Idl called

As current element.

Force due to current element at any point

We know that convection current density is ,

$$\vec{J} = \rho_v \vec{V}$$

The current elements are ,

$$\vec{J} dv = K ds = \vec{I} dl$$

Using above two equations,

$$\vec{I} dl = \rho_v \vec{V} dv = Q\vec{V}$$

Also current element,

$$\vec{I} dl = (dQ/ dt).dl$$

$$= dQ. \vec{V}$$

The force experienced by moving charge we know as ,

$$\overrightarrow{dFm} = Q(\vec{V} \times \vec{B})$$

$$= \vec{I} dl \times \vec{B}$$

Integrating on both sides we can determine force due current element,

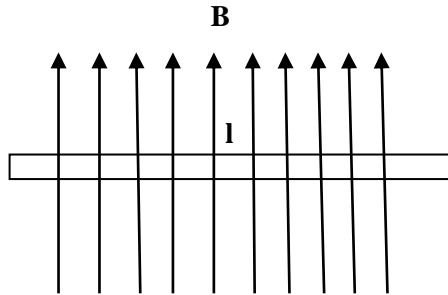
$$\overrightarrow{Fm} = \int \vec{I} dl \times \vec{B} \text{-----} 45$$

Similarly,

$$\overrightarrow{Fm} = \int_s \vec{K} ds \times \vec{B}$$

$$\vec{F}_m = \int_V \vec{J} \, dv \times \vec{B}$$

4.5 Force on a straight long current carrying conductor placed in the magnetic field



Let us consider a straight conductor placed in the magnetic field as shown in the figure,

Of length l , allowing current of I , hence current element if Idl ,

The velocity of charges in the given length of conductor is \vec{V} .

The force experienced by current element is ,

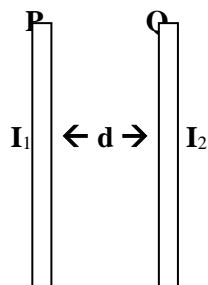
$$\begin{aligned} d\vec{F}_m &= dQ(\vec{V} \times \vec{B}) \\ &= dQ(dl/dt \times \vec{B}) \\ &= I(d\vec{l} \times \vec{B}) \end{aligned}$$

Their by integrating on both sides,

$$\vec{F}_m = I(\vec{l} \times \vec{B})$$

$$F_m = BIl \sin\theta \text{ ----- 46}$$

4.6 Force on a straight parallel long current carrying conductors placed in the magnetic field



Let us consider two straight parallel current carrying conductors of length l separated by distance d

As shown above,

The magnetic field intensity due conductor P on Q is,

$$H = I_1 / 2\pi d$$

The magnetic flux density due conductor P on Q is,

$$B = \mu_0 I_1 / 2\pi d$$

Hence force experienced by conductor Q due to field of P is,

$$\begin{aligned} F_1 &= B I_2 l \\ &= \mu_0 I_1 I_2 l / 2\pi d \end{aligned}$$

Similarly force experienced by P due to conductor Q is ,

$$F_2 = \mu_0 I_1 I_2 l / 2\pi d$$

Hence force per unit length of conductor is ,

$$(F / l) = \mu_0 I_1 I_2 / 2\pi d \text{ ----- 47}$$

4.7 Magnetic dipole and dipole moment

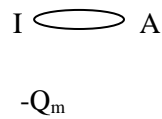
Magnetic dipole is formed when two opposite magnetic charges are separated by distance l .

$$-Q_m \text{ ----- } l \text{ ----- } +Q_m$$

The line joining two charges is termed as axis of dipole. Direction magnetic dipole is from $-Q_m$ to $+Q_m$

In other words a bar magnet with pole strength Q_m and l has , magnetic dipole moment, $m = Q_m l$.





Let us consider a bar conductor allowing current I their forming loop of area A, magnet poles formed As shown in the figure.

Magnetic dipole moment , $m = IA$

Numerically both dipole moment must be same, $Q_m l = IA$

Magnetization

If their exist an conductor consisting of number of dipoles in its volume , then magnet dipole Moment per unit volume is called as magnetization.

$$\begin{aligned} M &= m / V \\ &= Q_m \cdot l / A \cdot l \\ &= Q_m / A \end{aligned}$$

Magnetic susceptibility

When the magnetic field is applied to an material the ,

Total magnetic field intensity is ,

$$\begin{aligned} \vec{B} &= \mu_0 \vec{H} + \mu_0 \vec{M} \\ &= \mu_0 \mu_r H \end{aligned}$$

Therefore,

$$\mu_0 \mu_r \vec{H} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

$$\vec{M} = (\mu_r - 1) \vec{H}$$

$$\vec{M} = X_m \vec{H}$$

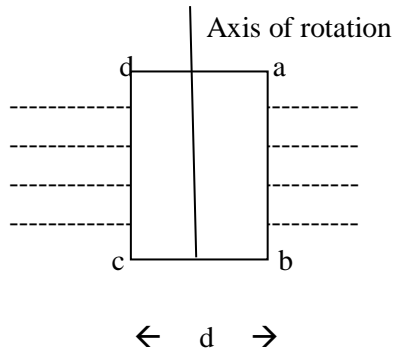
$X_m = (\mu_r - 1)$, is called as magnetic susceptibility

$$= \vec{M} / \vec{H} \text{ ----- 48}$$

4.7 Torque due to Magnetic dipole

Let us a sheet of side abcd placed in the magnetic field , the side ab experiences the force into

The page and side cd out of the page. Angles made by sheet with magnetic field are α and β .



the total torque experienced by sheet due to dipole is ,

$$\begin{aligned}
 T &= 2 \times \text{torque on each side} \\
 &= 2 \times \text{force} \times \text{distance from axis of rotation} \\
 &= 2 \times F \times d/2 \\
 &= 2 \times BIl \cos \beta \times d/2 \\
 &= BIA \cos \beta \\
 &= mB \cos \beta \quad \text{or} \quad mB \sin \alpha
 \end{aligned}$$

Therefore torque vector , $\vec{T} = \vec{m} \times \vec{B} \text{ ----- 49}$

4.8 Scalar magnetic potential

From the electro-statics we know that, $E = -\nabla V$

Similarly in the magneto-statics , $H = -\nabla V_m$

V_m – vector magnetic potential

Applying curl on both sides of H, $\nabla \times H = -\nabla \times (\nabla V_m)$

But curl of divergence of any vector is zero, $\nabla \times H = 0$

We can also write , $\nabla \times H = J$

From the above two equations we can write , $J = 0$.

This is possible only in the case constant magnetic field.

from the electro-statics we know that, $\int E \, dl = V$

Similarly in the magneto-statics , $\int H \, dl = V_m$

Ampere circuital law says that, $\int H \, dl = I$

Comparing last two equations, $V_m = I$ -----50

Hence the units of scalar magnetic potential is Amperes.

4.9 Vector magnetic potential

We know that divergence magnetic flux density over uniform closed surface is always zero.

$$\nabla \cdot B = 0$$

Also divergence of curl of vector is always zero.

$$\nabla \cdot (\nabla \times A) = 0$$

By comparing above two equations,

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mu \mathbf{H} = \nabla \times \mathbf{A}$$

$$\mathbf{H} = (\nabla \times \mathbf{A}) / \mu$$

Applying curl on both sides,

$$\nabla \times \mathbf{H} = \nabla \times (\nabla \times \mathbf{A}) / \mu = \mathbf{J}$$

But,

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla \cdot (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} = \mu \mathbf{J}$$

For time invariant fields divergence of vector is zero, hence above can be written as

$$-\nabla^2 \mathbf{A} = \mu \mathbf{J}$$

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}$$

Form the electro-statics we know that,

$$dv = dq / 4\pi\epsilon$$

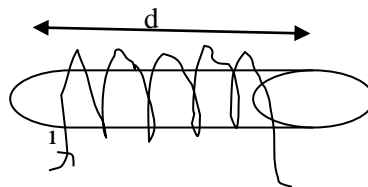
Similarly in the magneto-statics ,

$$dA = \mu idl / 4\pi r$$

Integrating on both sides,

$$A = \int \mu idl / 4\pi r, \text{ A- vector magnetic potential -----}51$$

4.10 Self inductance of a solenoid



let us consider a solenoid as shown in figure with length l allowing an current of i A.

N – total turns of solenoid coil

N – number of turns per unit length

magnetic field density inside solenoid is , $B = \mu_0 n.i.$

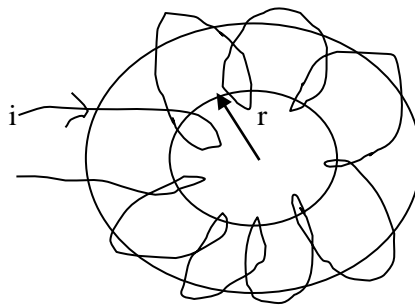
total flux linking with coil is

$$\begin{aligned}\phi &= N B A \\ &= \mu_0 n l.i.A .n \\ &= \mu_0 n^2.i.A .l\end{aligned}$$

Self inductance is the property of coil which is responsible for emf induced in it,

$$\begin{aligned}L &= N \phi / i \\ &= \mu_0 n^2.i.A .l / i \\ &= \mu_0 N^2A / l \text{ H} \text{-----}52\end{aligned}$$

4.11 Self inductance of a Toroid



Let us a toroid on which a coil N turns is wound allowing an current of i A.

Let r be the mean radius of the toroid.

Magnetic flux density in the toroid, $B = \mu_0 Ni / l$

Where , $l = 2\pi r$

$$B = \mu_0 Ni / 2\pi r$$

Total flux linkage with toroid is , $\phi = NBA$

$$= (N \mu_0 Ni / 2\pi r) . A$$

But, area $A = \pi R^2$

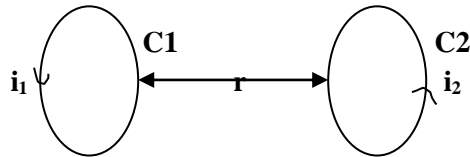
$$\begin{aligned}\phi &= (N \mu_0 Ni / 2\pi r). \pi R^2 \\ &= (N^2 \mu_0 i R^2 / 2r).\end{aligned}$$

Therefore self inductance of toroid is ,

$$L = \phi / i$$

$$= (N^2 \mu_0 R^2 / 2r). H \text{ ----- } 53$$

4.11 Neumann's formulae



let us consider two circular coils brought as near as possible allowing i_1 and i_2 currents, with separation of r , of an areas S_1 and S_2 .

the magnetic flux density due to current i_1 is ,

$$B_1 = \nabla \times A_1.$$

Vector magnetic potential ,

$$A_1 = \int \mu i_1 dl_1 / 4\pi r$$

Hence flux with second coil due to i_1 ,

$$\phi_{21} = B_1 dS_2$$

hence total flux linking with second coil is ,

$$\Psi_{21} = \int B_1 dS_2$$

$$= \int (\nabla \times A_1) dS_2$$

From stokes theorem,

$$\int (\nabla \times A_1) dS_2 = \int A_1 dl_2$$

Substituting this in above equation ,

$$\begin{aligned}\Psi_{21} &= \int A_1 dl_2 \\ &= \int \int \mu i_1 dl_1 dl_2 / 4\pi r\end{aligned}$$

Therefore mutual inductance between two coils is ,

$$M_{21} = \Psi_{21} / i_1$$

Mutual inductance is the imaginary concept which says that there is flux linkage with second Coil because of current flowing through first coil.

$$M_{21} = \int \int \mu i_1 dl_1 dl_2 / 4\pi r / i_1$$

$$M_{21} = \int \int \mu dl_1 dl_2 / 4\pi r \text{ ----- } 54$$

This M_{21} is called as **Neumann's formulae.**

4.11 Energy stored in the magnetic field

Let the work done to increase the current by di is dw , by law of conservation of energy

Work done is equal to energy stored .

$$\begin{aligned}dw &= vi dt \\ &= L \cdot di \cdot dt/dt\end{aligned}$$

$$dw = L di$$

integrating on both sides ,

$$\int dw = \int L di$$

$$w = Li^2 / 2$$

but we know that,

$$L = N\Phi / i = \Psi / i$$

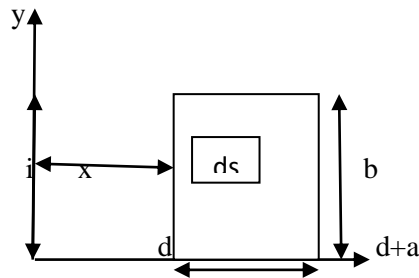
using above expressions we can write energy stored in the magnetic field also as,

$$w = \Psi i / 2$$

$$= \Psi^2 / 2 L. \text{-----} 55$$

4.12 Mutual inductance between straight long and square conductors

Let us consider a straight and square conductor placed in the xy plane as shown.



here straight conductor is placed on y-axis and square loop as shown is xy plane.

The magnetic flux density due to straight wire o square loop is ,

$$B = \mu_0.i / 2\pi x$$

The flux linking with square loop because current in straight wire is,

$$M = \Psi / I$$

From the gauss law we know that,

$$\Psi = \int_s B ds$$

$$= \int \int (\mu_0.i / 2\pi x) dx dy, \text{ with limits}$$

$$x = d \text{ to } d+a, y = 0 \text{ to } b$$

$$= \int (\mu_0.i / 2\pi x).y dx$$

Substituting limits of y,
$$= \int (\mu_0 i / 2\pi x) \cdot b \, dx$$

Then,
$$= (\mu_0 i b / 2\pi) \cdot \ln(x)$$

Substituting limits of x,
$$= (\mu_0 i b \ln(d+a) / 2\pi \ln(d)).$$

Therefore mutual inductance between two conductors is ,

$$M = \Psi / i$$

$$= (\mu_0 \cdot B \cdot \ln(d+a) / 2\pi \ln(d)). \text{----- } 56$$

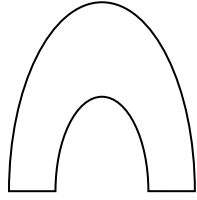
4.13 Characteristics and applications of permanent magnets

Characteristics :

- ➔ Permanent magnets are the one which readily available in nature in the form of Bar and horse shoe shapes etc.
- ➔ Permanent magnets irrespective of supply always exhibits magnetic properties.
- ➔ Permanent magnets always develops a constant magnetic field.
- ➔ The strength of the permanent magnets measured in terms of their cohesive force.
- ➔ An permanent magnet with high cohesive force will have long life.
- ➔ Permanent magnet got the disadvantage of ageing effect i.e in long run they may get rusted.

Applications:

- ➔ Permanent magnets are used in the applications where ever it is required to develop Constant magnetic field . Eg- Dc generator, Dc motor.



Horse shoe magnet



Bar magnet

UNIT-V

TIME VARYING FIELDS AND NUMERICAL METHODS

5.1 introduction

Time varying fields are produced due to accelerated charges or time varying currents.

Here we shall study how time varying current affects electric and magnet fields.

5.2 Faraday's law of electro-magnetic induction

Micheal faraday has stated two laws

- i) If any coil experiences change in flux or variable flux then emf is induced in it.
- ii) The emf induced in the coil is directly proportional to rate of change of flux linking

With the coil.

$$E \propto - d\phi / dt$$

E- electro-motive force.

- Sign indicates that magnetic flux developed in coil opposes the current through it.

From Lenz law.

For an coil with N turns emf induced in it ,

$$E = - N.d\phi / dt$$

5.3 Maxwell's Fourth equation or vector form of faraday's law

We know from the gauss law,

$$\Phi = \int_s B ds$$

hence emf induced due to above flux is ,

$$e = - d\phi / dt$$

$$= -d(\int_s B ds) / dt \text{ -----a}$$

Electric potential is given as ,

$$e = \int E dl \text{ -----b}$$

equating above two equations,

$$\int E dl = - (\int_s dB ds) / dt \text{ ----- c}$$

by applying stokes theorem,

$$\int E dl = \int_s (\nabla \times E) ds$$

substituting above equation in c,

$$\int_s (\nabla \times E) ds = - (\int_s dB ds) / dt$$

comparing on both sides,

$$\nabla \times E = -dB/dt \text{ ----- 57}$$

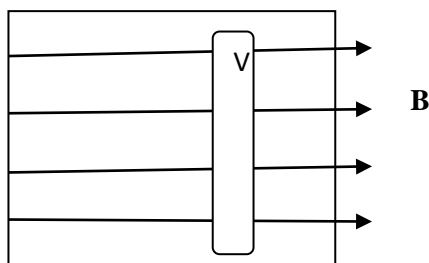
Equation 57 is called as Maxwell's fourth equation of vector form of faraday's law.

5.4 Types of induced emf

The emf induced in the coil according faraday's law is mainly of two types. They are

- i) Dynamically induced emf
- ii) Statically induced emf.

Dynamically induced emf



Let us consider a straight conductor with charge velocity of v moving against the existing Magnetic field. Force experienced by conductor is ,

$$\vec{F} = Q ((\vec{V} \times \vec{B}))$$

$$\vec{F}/Q = ((\vec{V} \times \vec{B}))$$

$$\vec{E} = (\vec{V} \times \vec{B})$$

Hence electric potential induced in the conductor is ,

$$e = \int \vec{E} \cdot d\vec{l}$$

$$= \int (\vec{V} \times \vec{B}) \cdot d\vec{l}$$

therefore potential induced can be written as,

$$e = Bvl \sin\theta$$

the maximum value of potential induced is,

$$e = Bvl \text{ ----- } 58$$

Statically induced emf

If an conductor experiences variable flux then emf induced in it is called as statically induced

Emf.

$$e = \int \vec{E} \cdot d\vec{l} = -Nd\phi / dt$$

since the flux is alternating,

$$\phi = \phi_m \sin\omega t$$

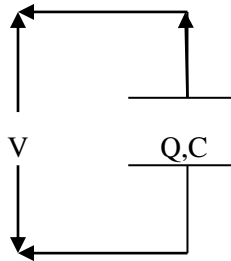
then the emf induced is ,

$$e = -Nd (\Phi_m \sin wt) / dt$$

$$= N \dot{\Phi}_m w \cos wt \text{ ----- } 59$$

5.5 Displacement current

Let us consider a capacitor is connected to Ac source as shown in figure



The current flowing through capacitor is ,

$$i_c = C dV / dt$$

the capacitance of capacitor,

$$C = \epsilon A / d$$

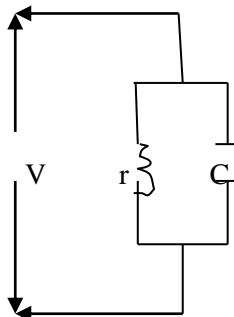
Then,

$$i_c = (\epsilon A / d). dV / dt$$

$$i_c / A = \epsilon dE / dt$$

$$J_c = dD / dt$$

J_c is called as displacement current.



Above is the figure of actual capacitor with internal resistance,

Then the total current is ,

$$i = i_r + i_c$$

where ,

I – total current

i_r – current through resistance

i_c – current through capacitance

dividing above KCL on both sides by area A ,

$$i / A = i_r / A + i_c / A$$

$$J = J_r + J_c$$

J_r – conducting current

J_c – displacement current

5.6 Maxwell's equations in time varying fields

In the time varying fields we can write,

$$E = E_0 \cos \omega t$$

$$= E_0 e^{j\omega t}$$

Similarly,

$$D = D_0 e^{j\omega t}$$
$$dD / dt = D_0 \omega e^{j\omega t} = J \omega D_0$$

likely,

$$dB / dt = J \omega B$$

we know that,

$$\nabla \times E = - dB / dt$$

$$= J_w B$$

Also,

$$\nabla \times E = -J_w \mu H$$

$$\nabla \times H = J + dD/dt$$

$$= \sigma E + J_w D_0$$

$$= \sigma E + J_w \epsilon E$$

$$= E (\sigma + J_w \epsilon)$$

Integak form,,

$$\int D \, ds = \int \rho_v \, dv$$

$$\int B \, ds = 0$$

$$\int E \, dl = -J_w \int B \, ds$$

$$\int H \, dl = (\sigma + J_w \epsilon) \int E \, ds$$

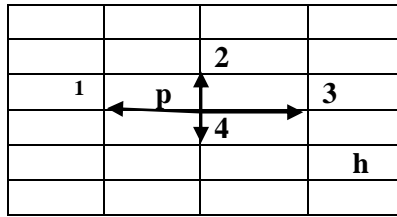
5.7 Numerical methods.

There are various numerical methods to calculate the electric field intensity on a uniform and non-uniform fields. They are

- a) Finite difference method
- b) Finite element method
- c) Boundary element method
- d) Charge simulation method.

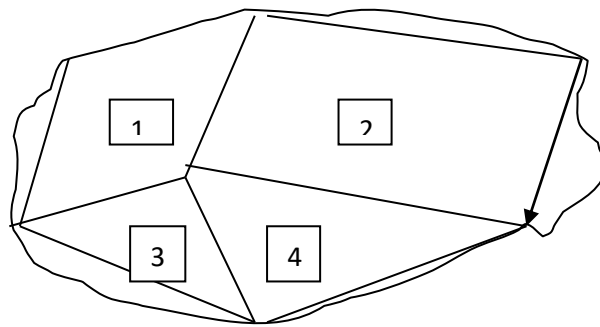
Let us check short discussion of each method to calculate electric field intensity.

a) Finite difference method



- Here let us consider an Uniform field whose electric field intensity is to be determined
- The test area is portioned into number meshes as shown in the figure.
- Each mesh is of step size h .
- Each mesh can be studied with the knowledge of their four nodes.
- With the knowledge of mesh field we can determine any unknown potential in the given field.
- If P is the test point at field is to be determined, then the field function at P depends on Fields at neighboring nodes.

b) Finite element method



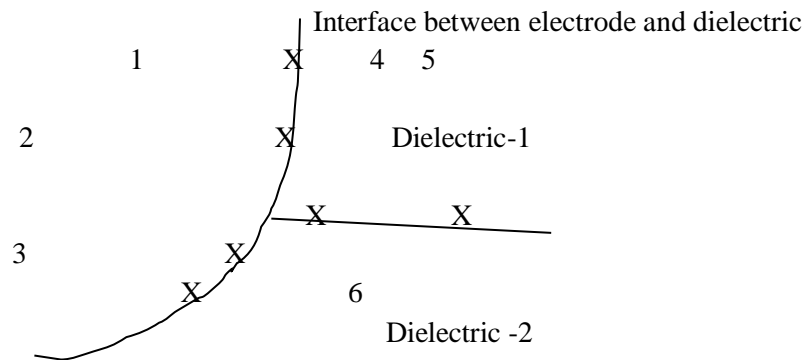
- This method is applicable to both uniform and non-uniform test fields.
- Let us consider a non-uniform field whose total field is to be determined.
- Here total area of field is portioned into number known shapes like triangle, trapezoidal etc.
- Each shape is pronounced as element.
- Each element is described with corner nodes.
- If the all node voltages of individual element is known then total potential of element

Can be determined.

→ Their by with all the element voltages we can calculate total electric field intensity of given

Medium.

c) Charge simulation method



→ In this method we can find field due to point charges.

→ Here 1,2,3 -----6 are the point charges.

→ The x marks indicates counter points at which field is to be calculated.

→ Let us consider counter point 1 between electrode and dielectric -1 , hence the total potential

At this counter point is due to fields of point charges 1,2,4,5.

→ Similarly we can field at each counter point, their by total potential of interface is sum of

All counter point field.

→ This method has good accuracy and speed.

→ Can not be applicible for electrodes of irregular shapes

→ Accuracy in 2D is 1% and in 3D is 2%.

→ As point to point field is calculated , this can be easily simulated in the PC.

→ This method can be used for unbounded fields.

*** Boundary method is almost similar to charge simulation method , but here instead of point

Charges surface charges are considered. This method required large time and complexity increases.