## LECTURE NOTES

## ON

# ELECTRICAL TECHNOLOGY 2018-2019 <br> III Semester (IARE-R16) 

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## UNIT - I <br> DC TRANSIENT ANALYSIS

## Introduction:

For higher order differential equation, the number of arbitrary constants equals the order of the equation. If these unknowns are to be evaluated for particular solution, other conditions in network must be known. A set of simultaneous equations must be formed containing general solution and some other equations to match number of unknown with equations.
We assume that at reference time $t=0$, network condition is changed by switching action. Assume that switch operates in zero time. The network conditions at this instant are called initial conditions in network.

## 1. Resistor:



Eq. 1 is linear and also time dependent. This indicates that current through resistor changes if applied voltage changes instantaneously. Thus in resistor, change in current is instantaneous as there is no storage of energy in it.

## 2. Inductor:

If dc current flows through inductor, dil/dt becomes zero as dc current is constant with respect to time. Hence voltage across inductor, VL becomes zero. Thus, as for as dc quantities are considered, in steady stake, inductor acts as short circuit


$$
i L=\frac{1}{L} \int V_{L} d t
$$

In above eqn. The limits of integration is from $-\infty$ to $t$
Assuming that switching takes place at $t=0$, we can split limits into two intervals as $-\infty$ tc $t$

$$
\begin{gathered}
i_{L}=\frac{1}{L} \int_{-}^{t} V_{L} d t \\
i_{L}=\frac{1}{L} \int_{-\infty}^{0} V_{L} d t+\frac{1}{L} \int_{0}^{1} V_{L} d t \\
i_{L}=i_{L}+\frac{1}{L} \int_{0}^{1} V_{L} d t \\
\text { at } \mathrm{t}=0^{t} \text { we can write } i_{L}\left(0^{+}\right)= \\
i_{L}\left(0^{+}\right)=i_{L}\left(0^{+}\right)
\end{gathered}
$$

## 3. Capacitor:



If dc voltage is applied to capacitor, $\mathrm{dVC} / \mathrm{dt}$ becomes zero as dc voltage is constant with respect to time.
Hence the current through capacitor iC becomes zero, Thus as far as dc quantities are considered capacitor acts as open circuit.

$$
\begin{gathered}
V_{C}=\frac{1}{C} \int i_{C} d t \\
V_{C}=\frac{1}{C} \int_{-\infty}^{t} i_{C} d t
\end{gathered}
$$

Splitting limits of integration

$$
V_{C}=\frac{1}{C} \int_{-\infty}^{0-} i_{C} d t+\frac{1}{C} \int_{0}^{t} i_{C} d t
$$

At $\mathrm{t}\left(0^{+}\right)$, equation is given by

$$
\begin{aligned}
& V_{C} \mathbf{C}^{+}=V_{C}+\frac{1}{C} \int_{0-}^{0+} i_{C} d t \\
& V_{C} \mathbf{l}^{+}=V_{C}
\end{aligned}
$$

Thus voltage across capacitor cannot change instantaneously.

## Initial Condition for (DC steady state solution)

- Initial condition: response of a circuit before a switch is first activated.
- Since power equals energy per unit time, finite power requires continuous change in energy.
- Primary variables: capacitor voltages and inductor currents-> energy storage elements

$$
W_{L}(t)=\frac{1}{2} L i_{L}^{2}(t) \quad W_{C}(t)=\frac{1}{2} C v_{C}^{2}(t)
$$

- Capacitor voltages and inductor currents cannot change instantaneously but

Should be continuous -> Continuity of capacitor voltages and inductor currents

- The value of an inductor current or a capacitor voltage just prior to the closing (or opening) of a switch is equal to the value just after the switch has been closed (or opened).
$v_{C}\left(t=0^{-}\right)=v_{C}\left(t=0^{+}\right)$
$i_{L}\left(t=0^{-}\right)=i_{L}\left(t=0^{+}\right)$


## 1. TRANSIENT RESPONSE OF RL CIRCUITS:

Consider the following series RL circuit given below


In the above circuit, the switch was kept open up to $t=0$ and it was closed at $t=0$. So, the DC voltage source having $V$ volts is not connected to the series RL circuit up to this instant. Therefore, there is no initial current flows through inductor.
The circuit diagram, when the switch is in closed position is shown in the following figure.


Now, the current $;$ flows in the entire circuit, since the DC voltage source having $\boldsymbol{V}$ volts is connected to the series RL circuit.

Now, apply KVL around the loop.

$$
\begin{array}{r}
V=R i+L \frac{d i}{d t} \\
\frac{d i}{d t}+\left(\frac{R}{L}\right) i=\frac{V}{L} \quad \text { Equation } \mathbf{1}
\end{array}
$$

The above equation is a first order differential equation and it is in the form of

$$
\frac{d y}{d t}+P y=Q \quad \text { Equation } 2
$$

By comparing Equation 1 and Equation 2, we will get the following relations.

$$
\begin{aligned}
x & =t \\
y & =i \\
P & =\frac{R}{L} \\
Q & =\frac{V}{L}
\end{aligned}
$$

The solution of Equation 2 will be

$$
y e^{\int p d x}=\int Q e^{\int p d x} d x+k \quad \text { Equation } 3
$$

Where, $\mathbf{k}$ is the constant.
Substitute, the values of $x, y, P \& Q$ in Equation 3.

$$
\begin{aligned}
& i e^{\int\left(\frac{R}{L}\right) d t}=\int\left(\frac{V}{L}\right)\left(e^{\int\left(\frac{R}{L}\right) d t}\right) d t+k \\
& \Rightarrow i e^{\left(\frac{R}{L}\right) t}=\frac{V}{L} \int e^{\left(\frac{R}{L}\right) t} d t+k \\
& \Rightarrow i e^{\left(\frac{R}{L}\right) t}=\frac{V}{L}\left\{\frac{e^{\left(\frac{R}{L}\right)} t}{\frac{R}{L}}\right\}+k \\
& \Rightarrow i=\frac{V}{R}+k e^{-\left(\frac{R}{L}\right)} t
\end{aligned}
$$

Equation 4
We know that there is no initial current in the circuit. Hence, substitute, $t=0$ and $z=0$ in Equation 4 in order to find the value of the constant $\boldsymbol{k}$.

$$
\begin{gathered}
0=\frac{V}{R}+k e^{-\left(\frac{R}{L}\right)(0)} \\
0=\frac{V}{R}+k(1) \\
k=-\frac{V}{R}
\end{gathered}
$$

Substitute, the value of $k$ in Equation 4.

$$
\begin{gathered}
i=\frac{V}{R}+\left(-\frac{V}{R}\right) e^{-\left(\frac{R}{L}\right) t} \\
i=\frac{V}{R}-\frac{V}{R} e^{-\left(\frac{R}{L}\right) t}
\end{gathered}
$$

Therefore, the current flowing through the circuit is

$$
i=-\frac{V}{R} e^{-\left(\frac{R}{L}\right) t}+\frac{V}{R}
$$

So, the response of the series RL circuit, when it is excited by a DC voltage source, has the following two terms.


- The second term $\frac{V}{R}$ corresponds with the steady state response. These two responses are shown in the following figure.


We can re-write the Equation 5 as follows -

$$
\begin{aligned}
& i=\frac{V}{R}\left(\mathbf{1}-e^{-\left(\frac{R}{L}\right) t}\right) \\
& \Rightarrow i=\frac{V}{R}\left(\mathbf{1}-e^{-\left(\frac{t}{\tau}\right)}\right)
\end{aligned}
$$

## Equation 6

Where, $\boldsymbol{T}$ is the time constant and its value is equal to $\frac{L}{R}$.

Both Equation 5 and Equation 6 are same. But, we can easily understand the above waveform of current flowing through the circuit from Equation 6 by substituting a few values of $\mathbf{t}$ like $0, \tau, 2 \tau, 5 \tau$, etc.

In the above waveform of current flowing through the circuit, the transient response will present up to five time constants from zero, whereas the steady state response will present from five time constants onwards.

| Actual time $(t)$ in sec | Growth of current in inductor |
| :---: | :---: |
| (Eq.10.15) |  |
| $i(0)=0$ |  |
| $t=0$ | $i(\tau)=0.632 \times \frac{V_{s}}{R}$ |
| $t=\tau\left(=\frac{L}{R}\right)$ | $i(2 \tau)=0.865 \times \frac{V_{s}}{R}$ |
| $t=2 \tau$ | $i(3 \tau)=0.950 \times \frac{V_{s}}{R}$ |
| $t=3 \tau$ | $i(4 \tau)=0.982 \times \frac{V_{s}}{R}$ |
| $t=4 \tau$ | $i(5 \tau)=0.993 \times \frac{V_{s}}{R}$ |
| $t=5 \tau$ |  |

## 2. TRANSIENT RESPONSE OF RC CIRCUITS:

Ideal and real capacitors: An ideal capacitor has an infinite dielectric resistance and plates (made of metals) that have zero resistance. However, an ideal capacitor does not exist as all dielectrics have some leakage current and all capacitor plates have some resistance. A capacitor's of how much charge (current) it will allow to leak through the dielectric medium. Ideally, a charged capacitor is not supposed to allow
leaking any current through the dielectric medium and also assumed not to dissipate any power loss in capacitor plate's resistance. Under this situation, the model as shown in fig. 10.16(a) represents the ideal capacitor. However, all real or practical capacitor leaks current to some extend due to leakage resistance of dielectric medium. This leakage resistance can be visualized as a resistance connected in parallel with the capacitor and power loss in capacitor plates can be realized with a resistance connected in series with capacitor. The model of a real capacitor is shown in fig.

Let us consider a simple series RC-circuit shown in fig. 10.17(a) is connected through a switch ' S ' to a constant voltage source .


The switch ' S ' is closed at time $\mathrm{t}=0$ ' It is assumed that the capacitor is initially charged with a voltage and the current flowing through the circuit at any instant of time ' ' after closing the switch is

## 3. Current decay in source free series RL circuit:


$\mathrm{t}=0-$, , switch k is kept at position 'a' for very long time. Thus, the network is in steady state. Initial current through inductor is given as,

$$
i_{L} \mathbf{i}^{-}=I_{0}=\frac{V}{R}=i_{L} \mathbf{U}^{+} \text {, }
$$

$\qquad$
Because current through inductor cannot change instantaneously
Assume that at $\mathrm{t}=0$ switch k is moved to position ' b '.
Applying KVL,

$$
\begin{aligned}
& L \frac{d i}{d t}+i R=0 \\
& \therefore L \frac{d i}{d t}=-i R
\end{aligned}
$$

Rearranging the terms in above equation by separating variables

$$
\frac{d i}{i}=-\frac{R}{L} d t
$$

Integrating both sides with respect to corresponding variables

$$
\therefore \ln \left\lvert\,=-\frac{R}{L}\right.
$$

Where $k$ ' is constant of integration.
To find- $k$ ':
Form equation 1 , at $\mathrm{t}=0, \mathrm{i}=\mathrm{I} 0$
Substituting the values in equation 3
Where $k$ ' is constant of integration.

To find- $\mathrm{k}^{\prime}$ : Form equation 1 , at $\mathrm{t}=0, \mathrm{i}=\mathrm{I} 0$
Substituting the values in eq

$$
\begin{aligned}
& \ln \boldsymbol{\|}_{-}^{-}=-\frac{R}{L} t+\ln \boldsymbol{\|}_{0_{-}}^{-} \\
& \ln |-\ln |_{0}^{-}=-\frac{R}{L} t \\
& \frac{i}{i_{0}}=e^{-\frac{R}{L} t} \\
& \therefore i=I_{0} \cdot e^{-\frac{R}{L} t}
\end{aligned}
$$

From the graph, H is clear that current is exponentially decaying. At point P on graph. The current value is (0.363) times its maximum value. The characteristics of decay are determined by values R and L which are two parameters of network.

The voltage across inductor is given by

$$
\begin{gathered}
V_{L}=L \frac{d i}{d t}=L \frac{d}{d t}\left[I_{0} \cdot e^{-\frac{R}{L} t}\right]=L \cdot I_{0}\left(-\frac{R}{L}\right) \cdot e^{-\frac{R}{L} t} \\
\therefore V_{L}=-I_{0} \cdot \mathrm{Re}^{-\frac{R}{L} t} \\
\text { But } \quad I_{0} \cdot R=V \\
\therefore V_{L}=-V \cdot e^{-\frac{R}{L} t} \quad \text { Volts }
\end{gathered}
$$

## 4. TRANSIENT RESPONSE OF RLC CIRCUITS

In the preceding lesson, our discussion focused extensively on dc circuits having resistances with either inductor or capacitor (i.e., single storage element) but not both. Dynamic response of such first order system has been studied and discussed in detail. The presence of resistance, inductance, and capacitance in the dc circuit introduces at least a second order differential equation or by two simultaneous coupled linear first order differential equations. We shall see in next section that the complexity of analysis of second order circuits increases significantly when compared with that encountered with first order circuits. Initial conditions for the circuit variables and their derivatives play an important role and this is very crucial to analyze a second order dynamic system.
5. Response of a series R-L-C circuit:

Consider a series RL circuit as shown in fig.11.1, and it is excited with a dc voltage source $\mathrm{C}--\mathrm{sV}$. Applying around the closed path for,

$$
L \frac{d i(t)}{d t}+R i(t)+v_{c}(t)=V_{s}
$$



The current through the capacitor can be written as Substituting the current ''expression in eq.(11.1) and rearranging the terms,

$$
\begin{gathered}
i(t)=C \frac{d v_{c}(t)}{d t} \\
L C \frac{d^{2} v_{c}(t)}{d t^{2}}+R C \frac{d v_{c}(t)}{d t}+v_{c}(t)=V_{s}
\end{gathered}
$$

The above equation is a 2 nd-order linear differential equation and the parameters associated with the differential equation are constant with time. The complete solution of the above differential equation has two components; the transient response and the steady state response. Mathematically, one can write the complete solution as

$$
\begin{gathered}
v_{c}(t)=v_{c n}(t)+v_{c f}(t)=\left(A_{1} e^{\alpha_{1} t}+A_{2} e^{\alpha_{2} t}\right)+A \\
L C \frac{d^{2} v_{c}(t)}{d t^{2}}+R C \frac{d v_{c}(t)}{d t}+v_{c}(t)=0 \Rightarrow \frac{d^{2} v_{c}(t)}{d t^{2}}+\frac{R}{L} \frac{d v_{c}(t)}{d t}+\frac{1}{L C} v_{c}(t)=0 \\
a \frac{d^{2} v_{c}(t)}{d t^{2}}+b \frac{d v_{c}(t)}{d t}+c v_{c}(t)=0 \text { (where } a=1, b=\frac{R}{L} \text { and } c=\frac{1}{L C} \text { ) }
\end{gathered}
$$

Since the system is linear, the nature of steady state response is same as that of forcing function (input voltage) and it is given by a constant value. Now, the first part of the total response is completely dies out with time while and it is defined as a transient or natural response of the system. The natural or transient response (see Appendix in Lesson-10) of second order differential equation can be obtained from the homogeneous equation (i.e., from force free system) that is expressed by

$$
\alpha^{2}+\frac{R}{L} \alpha+\frac{1}{L C}=0 \Rightarrow a \alpha^{2}+b \alpha+c=0\left(\text { where } a=1, b=\frac{R}{L} \text { and } c=\frac{1}{L C}\right)
$$

And solving the roots of this equation (11.5) on that associated with transient part of the complete solution (eq.11.3) and they are given below.

$$
\begin{aligned}
& \alpha_{1}=\left(-\frac{R}{2 L}+\sqrt{\left(\frac{R}{2 L}\right)^{2}-\frac{1}{L C}}\right)=\left(-\frac{b}{2 a}+\frac{1}{a} \sqrt{\left(\frac{b}{2}\right)^{2}-a c}\right) \\
& \alpha_{2}=\left(-\frac{R}{2 L}-\sqrt{\left(\frac{R}{2 L}\right)^{2}-\frac{1}{L C}}\right)=\left(-\frac{b}{2 a}-\frac{1}{a} \sqrt{\left(\frac{b}{2}\right)^{2}-a c}\right)
\end{aligned}
$$

where, $b=\frac{R}{L}$ and $c=\frac{1}{L C}$.
The roots of the characteristic equation are classified in three groups depending upon the values of the parameters, Rand of the circuit Case-A (over damped response): That the roots are distinct with negative real parts. Under this situation, the natural or transient part of the complete solution is written as

$$
\dot{v}_{c n}(t)=A_{1} e^{\alpha_{1} t}+A_{2} e^{\alpha_{2} t}
$$

and each term of the above expression decays exponentially and ultimately reduces to zero as and it is termed as over damped response of input free system. A system that is over damped responds slowly to any change in excitation. It may be noted that the exponential term $t \rightarrow \infty 11$ tAeatakes longer time to decay its value to zero than the term21tAe $\alpha$. One can introduce a factor $\xi$ that provides information about the speed of system response and it is defined by damping ratio

$$
(\xi)=\frac{\text { Actual damping }}{\text { critical damping }}=\frac{b}{2 \sqrt{a c}}=\frac{R / L}{2 / \sqrt{L C}}>1
$$

## UNIT II <br> TWO PORT NETWORKS

## Introduction:

A pair of terminals through which a current may enter or leave a network is known as a port. Two-terminal devices or elements (such as resistors, capacitors, and inductors) result in one-port networks. Most of the circuits we have dealt with so far are two-terminal or one-port circuits, represented in Figure 2(a). We have considered the voltage across or current through a single pair of terminals-such as the two terminals of a resistor, a capacitor, or an inductor. We have also studied four-terminal or two-port circuits involving op amps, transistors, and transformers, as shown in Figure 2(b). In general, a network may have $n$ ports. A port is an access to the network and consists of a pair of terminals; the current entering one terminal leaves through the other terminal so that the net current entering the port equals zero.

(a)

(b)

Figure 2: (a) One-port network, (b) two-port network.
A pair of terminals through which a current may enter or leave a network is known as a port. A port is an access to the network and consists of a pair of terminals; the current entering one terminal leaves through the other terminal so that the net current entering the port equals zero. There are several reasons why we should study two-ports and the parameters that describe them. For example, most circuits have two ports. We may apply an input signal in one port and obtain an output signal from the other port. The parameters of a two-port network completely describe its behavior in terms of the voltage and current at each port. Thus, knowing the parameters of a two port network permits us to describe its operation when it is connected into a larger network. Two-port networks are also important in modeling electronic devices and system components. For example, in electronics, two-port networks are employed to model transistors and Op-amps. Other examples of electrical components modeled by two-ports are transformers and transmission lines.

Four popular types of two-port parameters are examined here: impedance, admittance, hybrid, and transmission. We show the usefulness of each set of parameters, demonstrate how they are related to each other

## IMPEDANCE PARAMETERS:



Impedance and admittance parameters are commonly used in the synthesis of filters. They are also useful in the design and analysis of impedance-matching networks and power distribution networks. We discuss impedance parameters in this section and admittance parameters in the next section.
A two-port network may be voltage-driven as in Figure 3 (a) or current-driven as in Figure 3(b). From either Figure 3(a) or (b), the terminal voltages can be related to the terminal currents as

$$
\begin{aligned}
& \mathrm{V} 1=\mathrm{Z}_{11} \mathrm{I}_{1}+\mathrm{Z}_{12} \mathrm{I}_{2} \\
& \mathrm{~V} 2=\mathrm{Z}_{21} \mathrm{I}_{1}+\mathrm{Z}_{22} \mathrm{I}_{2}
\end{aligned}
$$

Where the z terms are called the impedance parameters, or simply z parameters, and have units of ohms.

(a)

(b)

The values of the parameters can be evaluated by setting $\mathbf{I}_{1}=0$ (input port open-circuited) or $\mathbf{I}_{2}=0$ (output port open-circuited).

$$
\begin{array}{ll}
\mathbf{z}_{11}=\left.\frac{\mathbf{V}_{1}}{\mathbf{I}_{1}}\right|_{\mathbf{I}_{2}=0}, & \mathbf{z}_{12}=\left.\frac{\mathbf{V}_{1}}{\mathbf{I}_{2}}\right|_{\mathbf{I}_{1}=0} \\
\mathbf{z}_{21}=\left.\frac{\mathbf{V}_{2}}{\mathbf{I}_{1}}\right|_{\mathbf{I}_{2}=0}, & \mathbf{z}_{22}=\left.\frac{\mathbf{V}_{2}}{\mathbf{I}_{2}}\right|_{\mathbf{I}_{1}=0}
\end{array}
$$

Since the z parameters are obtained by open-circuiting the input or output port, they are also called the opencircuit impedance parameters. Specifically,
$\mathbf{z}_{11}=$ Open-circuit input impedance
$\mathbf{z}_{12}=$ Open-circuit transfer impedance from port 1 to port $2 \mathbf{z}_{21}$
$=$ Open-circuit transfer impedance from port 2 to port $1 \mathbf{z}_{22}=$
Open-circuit output impedance
We obtain $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$ by connecting a voltage $\mathbf{V}_{1}$ (or a current source $\mathbf{I}_{1}$ ) to port 1 with port 2 open-circuited as in Figure 4 and finding $\mathbf{I}_{1}$ and $\mathbf{V}_{2}$; we then get


Determination of the $z$ parameters: (a) finding $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$ (b) finding $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$.
$\mathrm{Z}_{11}=\mathrm{V}_{1} / \mathrm{I}_{1}, \mathrm{Z}_{21}=\mathrm{V}_{2} / \mathrm{I}_{1}$
We obtain $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$ by connecting a voltage $\mathbf{V}_{2}$ (or a current source $\mathbf{I}_{2}$ ) to port 2 with port 1 open-circuited as in Figure 4) and finding $\mathbf{I}_{2}$ and $\mathbf{V}_{1}$; we then get
$\mathrm{Z}_{12}=\mathrm{V}_{1} / \mathrm{I}_{2}, \mathrm{Z}_{21}=\mathrm{V}_{2} / \mathrm{I}_{2}$
The above procedure provides us with a means of calculating or measuring the $z$ parameters. Sometimes $\mathbf{z}_{11}$ and $\mathbf{z}_{22}$ are called driving-point impedances, while $\mathbf{z}_{21}$ and $\mathbf{z}_{12}$ are called transfer impedances. A driving-point impedance is the input impedance of a two-terminal (one-port) device. Thus, $\mathbf{z}_{11}$ is the input driving-point impedance with the output port open-circuited, while $\mathbf{z}_{22}$ is the output driving-point impedance with the input port open circuited.

When $\mathbf{z}_{11}=\mathbf{z}_{22}$, the two-port network is said to be symmetrical. This implies that the network has mirror like symmetry about some center line; that is, a line can be found that divides the network into two similar halves.

When the two-port network is linear and has no dependent sources, the transfer impedances are equal ( $\mathbf{z}_{12}=\mathbf{z}_{21}$ ), and the two-port is said to be reciprocal. This means that if the points of excitation and response are interchanged, the transfer impedances remain the same. A two-port is reciprocal if interchanging an ideal voltage source at one port with an ideal ammeter at the other port gives the same ammeter reading.

## ADMITTANCE PARAMETERS:

In the previous section we saw that impedance parameters may not exist for a two-port network. So there is a need for an alternative means of describing such a network. This need is met by the second set of parameters, which we obtain by expressing the terminal currents in terms of the terminal voltages. In either Figure 5(a) or (b), the terminal currents can be expressed in terms of the terminal voltages as


Determination of the $y$ parameters: (a) finding $y_{11}$ and $y_{21}$, (b) finding $y_{12}$ and $y_{22}$.

$$
\begin{aligned}
& \mathrm{I}_{1}=\mathrm{Y}_{11} \mathrm{~V}_{1}+\mathrm{Y}_{12} \mathrm{~V}_{2} \\
& \mathrm{I}_{2}=\mathrm{Y}_{21} \mathrm{~V}_{1}+\mathrm{Y}_{22} \mathrm{~V}_{2}
\end{aligned}
$$

The $\mathbf{Y}$ terms are known as the admittance parameters (or, simply, y parameters) and have units of Siemens The values of the parameters can be determined by setting $\mathbf{V} 1=0$ (input port short-circuited) or $\mathbf{V} 2=0$ (output port short-circuited). Thus,

$$
\begin{array}{ll}
\mathbf{y}_{11}=\left.\frac{\mathbf{I}_{1}}{\mathbf{V}_{1}}\right|_{\mathbf{V}_{2}=0}, & \mathbf{y}_{12}=\left.\frac{\mathbf{I}_{1}}{\mathbf{V}_{2}}\right|_{\mathbf{V}_{1}=0} \\
\mathbf{y}_{21}=\left.\frac{\mathbf{I}_{2}}{\mathbf{V}_{1}}\right|_{\mathbf{V}_{2}=0}, & \mathbf{y}_{22}=\left.\frac{\mathbf{I}_{2}}{\mathbf{V}_{2}}\right|_{\mathbf{V}_{1}=0}
\end{array}
$$

Since the $y$ parameters are obtained by short-circuiting the input or output port, they are also called the shortcircuit admittance parameters. Specifically,
$\mathbf{y}_{11}=$ Short-circuit input admittance
$\mathbf{y}_{12}=$ Short-circuit transfer admittance from port 2 to port 1
$\mathbf{y}_{21}=$ Short-circuit transfer admittance from port 1 to port 2
$\mathbf{y}_{22}=$ Short-circuit output admittance
We obtain $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ by connecting a current $\mathbf{I}_{1}$ to port 1 and short-circuiting port 2 and finding V1And $\mathbf{I}_{2}$.
Similarly, we obtain $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$ by connecting a current source $\mathbf{I}_{2}$ to port 2 and short-circuiting port 1 and
finding $\mathbf{I}_{1}$ and $\mathbf{V}_{2}$, and then getting
This procedure provides us with a means of calculating or measuring the $y$ parameters. The impedance and admittance parameters are collectively referred to as admittance parameters

## HYBRID PARAMETERS:

The $z$ and $y$ parameters of a two-port network do not always exist. So there is a need for developing another set of parameters. This third set of parameters is based on making V1 and I2 the dependent variables. Thus, we obtain

$$
\begin{aligned}
\mathbf{V}_{1} & =\mathbf{h}_{11} \mathbf{I}_{1}+\mathbf{h}_{12} \mathbf{V}_{2} \\
\mathbf{I}_{2} & =\mathbf{h}_{21} \mathbf{I}_{1}+\mathbf{h}_{22} \mathbf{V}_{2}
\end{aligned}
$$

Or in matrix form,

$$
\left[\begin{array}{l}
\mathbf{V}_{1} \\
\mathbf{I}_{2}
\end{array}\right]=\left[\begin{array}{ll}
\mathbf{h}_{11} & \mathbf{h}_{12} \\
\mathbf{h}_{21} & \mathbf{h}_{22}
\end{array}\right]\left[\begin{array}{l}
\mathbf{I}_{1} \\
\mathbf{V}_{2}
\end{array}\right]=[\mathbf{h}]\left[\begin{array}{l}
\mathbf{I}_{1} \\
\mathbf{V}_{2}
\end{array}\right]
$$

The $\mathbf{h}$ terms are known as the hybrid parameters (or, simply, h parameters) because they are a hybrid Combination of ratios. They are very useful for describing electronic devices such as transistors; it is much easier to measure experimentally the $h$ parameters of such devices than to measure their $z$ or $y$ parameters. The hybrid parameters are as follows.

It is evident that the parameters $\mathbf{h}_{11}, \mathbf{h}_{12}, \mathbf{h}_{21}$, and $\mathbf{h}_{22}$ represent impedance, a voltage gain, a current gain, and admittance, respectively. This is why they are called the hybrid parameters. To be specific,
$\mathbf{h}_{11}=$ Short-circuit input impedance
$\mathbf{h}_{12}=$ Open-circuit reverse voltage gain
$\mathbf{h}_{21}=$ Short-circuit forward current gain
$\mathbf{h}_{22}=$ Open-circuit output admittance

The procedure for calculating the $h$ parameters is similar to that used for the $z$ or $y$ parameters. We apply a voltage or current source to the appropriate port, short-circuit or open-circuit the other port, depending on the parameter of interest, and perform regular circuit analysis.

## TRANSMISSION PARAMETERS:

$$
\left[\begin{array}{c}
\mathbf{V}_{1} \\
\mathbf{I}_{1}
\end{array}\right]=\left[\begin{array}{cc}
\mathbf{A} & \mathbf{B} \\
\mathbf{C} & \mathbf{D}
\end{array}\right]\left[\begin{array}{c}
\mathbf{V}_{2} \\
-\mathbf{I}_{2}
\end{array}\right]
$$

Since there are no restrictions on which terminal voltages and currents should be considered independent and which should be dependent variables, we expect to be able to generate many sets of parameters. Another set of parameters relates the variables at the input port to those at the output port.

$$
\begin{aligned}
\mathrm{V}_{1} & =\mathrm{AV}_{2}-\mathrm{BI}_{2} \\
\mathrm{I}_{1} & =\mathrm{CV}_{2}-\mathrm{DI}_{2}
\end{aligned}
$$

Thus,

$$
\begin{array}{ll}
\mathbf{h}_{11}=\left.\frac{\mathbf{V}_{1}}{\mathbf{I}_{1}}\right|_{\mathbf{V}_{2}=0}, & \mathbf{h}_{12}=\left.\frac{\mathbf{V}_{1}}{\mid \mathbf{V}_{2}}\right|_{\mathbf{I}_{1}=0} \\
\mathbf{h}_{21}=\left.\frac{\mathbf{I}_{2}}{\mathbf{I}_{1}}\right|_{\mathbf{V}_{2}=0}, & \mathbf{h}_{22}=\left.\frac{\mathbf{I}_{2}}{\mathbf{V}_{2}}\right|_{\mathbf{I}_{1}=0}
\end{array}
$$

The above Equations are relating the input variables $\left(\mathbf{V}_{1}\right.$ and $\left.\mathbf{I}_{1}\right)$ to the output variables $\left(\mathbf{V}_{2}\right.$ and $\left.-\mathbf{I}_{2}\right)$. Notice that in computing the transmission parameters, $-\mathbf{I}_{2}$ is used rather than $\mathbf{I}_{2}$, because the current is considered to be
leaving the network, as shown in Figure 6. This is done merely for conventional reasons; when you cascade twoports (output to input), it is most logical to think of $\mathbf{I}_{2}$ as leaving the two-port. It is also customary in the power industry to consider $\mathbf{I}_{2}$ as leaving the two-port.


Terminal variables used to define the $\mathbf{A B C D}$ parameters.
The two-port parameters in above equations provide a measure of how a circuit transmits voltage and current from a source to a load. They are useful in the analysis of transmission lines (such as cable and fiber) because they express sending-end variables $\left(\mathbf{V}_{1}\right.$ and $\left.\mathbf{I}_{1}\right)$ in terms of the receiving-end variables $\left(\mathbf{V}_{2}\right.$ and $\left.-\mathbf{I}_{2}\right)$. For this reason, they are called transmission parameters. They are also known as $\mathbf{A B C D}$ parameters. They are used in the design of telephone systems, microwave networks, and radars.
The transmission parameters are determined as

$$
\begin{array}{ll}
\mathbf{A}=\left.\frac{\mathbf{V}_{1}}{\mathbf{V}_{2}}\right|_{\mathbf{I}_{2}=0}, & \mathbf{B}=-\left.\frac{\mathbf{V}_{1}}{\mathbf{I}_{2}}\right|_{\mathbf{V}_{2}=0} \\
\mathbf{C}=\left.\frac{\mathbf{I}_{1}}{\mathbf{V}_{2}}\right|_{\mathbf{I}_{2}=0}, & \mathbf{D}=-\left.\frac{\mathbf{I}_{1}}{\mathbf{I}_{2}}\right|_{\mathrm{V}_{2}=0}
\end{array}
$$

Thus, the transmission parameters are called, specifically,
A = Open-circuit voltage ratio
$\mathbf{B}=$ Negative short-circuit transfer impedance
C = Open-circuit transfer admittance
$\mathbf{D}=$ Negative short-circuit current ratio
$\mathbf{A}$ and $\mathbf{D}$ are dimensionless, $\mathbf{B}$ is in ohms, and $\mathbf{C}$ is in Siemens. Since the transmission parameters provide a direct relationship between input and output variables, they are very useful in cascaded networks.

## Condition of symmetry:

A two port network is said to be symmetrical if the ports can be interchanged without port voltages and currents

## Condition of reciprocity:

A two port network is said to be reciprocal, if the rate of excitation to response is invariant to an interchange of the position of the excitation and response in the network. Network containing resistors, capacitors and inductors are generally reciprocal

## Condition for reciprocity and symmetry in two port parameters:

In Z parameters a network is termed to be reciprocal if the ratio of the response to the excitation remains unchanged even if the positions of the response as well as the excitation are interchanged.
A two port network is said to be symmetrical it the input and the output port can be interchanged without altering the port voltages or currents.

| Parameter | Condition for reciprocity | Condition for symmetry |
| :---: | :---: | :---: |
| Z | ${ }_{12}{ }_{12}{ }_{21}$ | ${ }_{22}{ }_{12}{ }_{11}$ |
| Y | ${ }_{12}{ }_{21}$ | ${ }_{11}{ }_{22}$ |
| h | ${ }^{11}{ }_{11}{ }^{11} 1$ | ${ }^{n}{ }_{11}{ }^{\text {n }}$ 21 |
| ABCD | $\mathrm{AD}-\mathrm{BC}=1$ | A = D |

## Interconnecting Two-Port Networks:

Two-port networks may be interconnected in various configurations, such as series, parallel, or cascade connection, resulting in new two-port networks. For each configuration, certain set of parameters may be more useful than others to describe the network. A series connection of two two-port networks a and b with opencircuit impedance parameters Za and Zb , respectively. In this configuration, we use the Z -parameters since they are combined as a series connection of two impedances.


The Z-parameters of the series connection are Z 11= Z11A +Z 11 B
Or in the matrix form $[\mathrm{Z}]=[\mathrm{ZA}]+[\mathrm{ZB}]$

## Parallel Connection

$[\mathrm{Y}]=[\mathrm{YA}]+[\mathrm{YB}]$

## Cascade Connection



## RELATIONSHIPS BETWEEN PARAMETERS:

Since the six sets of parameters relate the same input and output terminal variables of the same two-port network, they should be interrelated. If two sets of parameters exist, we can relate one set to the other set. Let us demonstrate the process with two examples.

Given the $z$ parameters, let us obtain the $y$ parameters.

$$
\left[\begin{array}{l}
\mathbf{V}_{1} \\
\mathbf{V}_{2}
\end{array}\right]=\left[\begin{array}{ll}
\mathbf{z}_{11} & \mathbf{z}_{12} \\
\mathbf{z}_{21} & \mathbf{z}_{22}
\end{array}\right]\left[\begin{array}{l}
\mathbf{I}_{1} \\
\mathbf{I}_{2}
\end{array}\right]=[\mathbf{z}]\left[\begin{array}{l}
\mathbf{I}_{1} \\
\mathbf{I}_{2}
\end{array}\right]
$$

$$
\left[\begin{array}{l}
\mathbf{I}_{1} \\
\mathbf{I}_{2}
\end{array}\right]=[\mathbf{z}]^{-1}\left[\begin{array}{l}
\mathbf{V}_{1} \\
\mathbf{V}_{2}
\end{array}\right]
$$

$$
\left[\begin{array}{l}
\mathbf{I}_{1} \\
\mathbf{I}_{2}
\end{array}\right]=\left[\begin{array}{ll}
\mathbf{y}_{11} & \mathbf{y}_{12} \\
\mathbf{y}_{21} & \mathbf{y}_{22}
\end{array}\right]\left[\begin{array}{l}
\mathbf{V}_{1} \\
\mathbf{V}_{2}
\end{array}\right]=[\mathbf{y}]\left[\begin{array}{l}
\mathbf{V}_{1} \\
\mathbf{V}_{2}
\end{array}\right]
$$


$\mathbf{z}_{21} \quad \mathbf{z}_{22} \quad-\frac{\mathbf{y}_{21}}{\Delta_{\mathrm{y}}} \quad \frac{\mathbf{y}_{11}}{\Delta_{y}} \quad-\frac{\mathbf{h}_{21}}{\mathbf{h}_{22}} \quad \frac{1}{\mathbf{h}_{22}} \quad \frac{\mathbf{g}_{21}}{\mathbf{g}_{11}} \quad \frac{\Delta_{g}}{\mathbf{g}_{11}}$
$\begin{array}{lllllllll}\mathbf{y} & \frac{\mathbf{z}_{22}}{\Delta_{z}} & -\frac{\mathbf{z}_{12}}{\Delta_{z}} & \mathbf{y}_{11} & \mathbf{y}_{12} & \frac{1}{\mathbf{h}_{11}} & -\frac{\mathbf{h}_{12}}{\mathbf{h}_{11}} \quad \frac{\Delta_{g}}{\mathbf{g}_{22}} \quad \frac{\mathbf{g}_{12}}{\mathbf{g}_{22}}\end{array}$
$-\frac{\mathbf{z}_{21}}{\Delta_{z}} \quad \frac{\mathbf{z}_{11}}{\Delta_{z}} \quad \mathbf{y}_{21} \quad \mathbf{y}_{22} \quad \frac{\mathbf{h}_{21}}{\mathbf{h}_{11}} \quad \frac{\Delta_{h}}{\mathbf{h}_{11}} \quad-\frac{\mathrm{g}_{21}}{\mathrm{~g}_{22}} \quad \frac{1}{\mathbf{g}_{22}}$
$\mathbf{h} \quad \frac{\Delta_{z}}{\mathbf{z}_{22}} \quad \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}} \quad \frac{1}{\mathrm{y}_{11}} \quad-\frac{\mathrm{y}_{12}}{\mathrm{y}_{11}} \quad \mathbf{h}_{11} \quad \mathbf{h}_{12} \quad \frac{\mathrm{~g}_{22}}{\Delta_{g}} \quad-\frac{\mathrm{g}_{12}}{\Delta_{g}}$
$-\frac{\mathbf{z}_{21}}{\mathbf{z}_{22}} \quad \frac{1}{\mathbf{z}_{22}} \quad \frac{\mathrm{y}_{21}}{\mathbf{y}_{11}} \quad \frac{\Delta_{y}}{\mathrm{y}_{11}} \quad \mathbf{h}_{21} \quad \mathbf{h}_{22} \quad-\frac{\mathbf{g}_{21}}{\Delta_{s}} \quad \frac{\mathrm{~g}_{11}}{\Delta_{s}}$
g $\frac{1}{\mathbf{z}_{11}}-\frac{\mathbf{z}_{12}}{\mathbf{z}_{11}} \quad \frac{\Delta_{y}}{\mathbf{y}_{22}} \quad \frac{\mathbf{y}_{12}}{\mathbf{y}_{22}} \quad \frac{\mathbf{h}_{22}}{\Delta_{h}} \quad-\frac{\mathbf{h}_{12}}{\Delta_{h}} \quad \mathbf{g}_{11} \quad \mathbf{g}_{12}$
$\begin{array}{llllllll}\frac{\mathbf{z}_{21}}{\mathbf{z}_{11}} & \frac{\Delta z}{\mathbf{z}_{11}} & -\frac{\mathbf{y}_{21}}{\mathbf{y}_{22}} \quad \frac{1}{\mathbf{y}_{22}} \quad-\frac{\mathbf{h}_{21}}{\Delta_{h}} & \frac{\mathbf{h}_{11}}{\Delta_{h}} & \mathbf{g}_{21} & \mathbf{g}_{22}\end{array}$

## UNIT III FILTERS AND SYMMETRICAL ATTENUATORS

## PASSIVE FILTERS:

Frequency-selective or filter circuits pass to the output only those input signals that are in a desired range of frequencies (called pass band). The amplitude of signals outside this range of frequencies (called stop band) is reduced (ideally reduced to zero). Typically in these circuits, the input and output currents are kept to a small value and as such, the current transfer function is not an important parameter. The main parameter is the voltage transfer function in the frequency domain, $\mathrm{Hv}(\mathrm{j} \omega)=\mathrm{Vo} / \mathrm{Vi}$. As $\mathrm{Hv}(\mathrm{j} \omega)$ is complex number, it has both a magnitude and a phase, filters in general introduce a phase difference between input and output signals. To minimize the number of subscripts, hereafter, we will drop subscript v of Hv. Furthermore, we concentrate on the \|open-loop\| transfer functions, Hvo, and denote this simply by $\mathrm{H}(\mathrm{j} \omega)$.

## Low-Pass Filters:

An ideal low-pass filter's transfer function is shown. The frequency between the pass- and-stop bands is called the cut-off frequency $(\omega c)$. All of the signals with frequencies below $\omega c$ are transmitted and all other signals are stopped.
In practical filters, pass and stop bands are not clearly defined, $|\mathrm{H}(\mathrm{j} \omega)|$ varies continuously from its maximum toward zero. The cut-off frequency is, therefore, defined as the frequency at which $|\mathrm{H}(\mathrm{j} \omega)|$ is reduced to $1 / \sqrt{2}=0.7$ of its maximum value. This corresponds to signal power being reduced by $1 / 2$ as $P \propto V 2$.

## Band-pass filters:

A band pass filter allows signals with a range of frequencies (pass band) to pass through and attenuates signals with frequencies outside this range.

## Constant - K Low Pass Filter:

A network, either $T$ or $\backslash[\mid$ pi $\backslash]$, is said to be of the constant- $k$ type if $\mathrm{Z}_{1}$ and $\mathrm{Z}_{2}$ of the network satisfy the relation

$$
Z_{1} Z_{2}=k^{2}
$$

Where $Z_{1}$ and $Z_{2}$ are impedance in the $T$ and [pi] sections as shown in Fig. Equation 17.20 states that $Z_{1}$ and $Z_{2}$ are inverse if their product is a constant, independent of frequency. $k$ is a real constant, that is the resistance. $k$ is often termed as design impedance or nominal impedance of the constant $k$-filter.

The constant $k, T$ or $\backslash[\mid$ pil] type filter is also known as the prototype because other more complex networks can be derived from it. Where $Z_{1}=j \omega_{L}$ and $Z_{2}=1 / j \omega_{C}$. Hence $Z_{1} Z_{2}=\backslash\left[\{L\right.$ lover $\left.C\}=\left\{k^{\wedge} 2\right\} \backslash\right]$ which is independent of frequency

The pass band can be determined graphically. The reactances of $Z_{1}$ and $4 Z_{2}$ will vary with frequency as drawn in Fig.30.2. The cut-off frequency at the intersection of the curves $\mathrm{Z}_{1}$ and $4 \mathrm{Z}_{2}$ is indicated as $f_{\mathrm{c}}$. On the X -axis as
$Z_{1}=-4 Z_{2}$ at cut-off frequency, the pass band lies between the frequencies at which $Z_{1}=0$, and $Z_{1}=-4 Z_{2}$.


All the frequencies above $f_{\mathrm{c}}$ lie in a stop or attenuation band

$$
\left.\begin{array}{l}
\text { The characteristic impedance of a } \backslash \backslash \text { pil } \backslash \text {-network is given by } \\
Z_{0 \pi}=\frac{Z_{1} Z_{2}}{Z_{0 T}}=\frac{k}{\sqrt{1-\left(\frac{f}{f_{c}}\right)^{2}}} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots
\end{array}\right)
$$

## Constant K-High Pass Filter:

Constant K-high pass filter can be obtained by changing the positions of series and shunt arms of the networks shown in Fig.30.1. The prototype high pass filters are shown in Fig.30.5, where $Z_{1}=-j / \omega_{\mathrm{C}}$ and $Z_{2}=j \omega L$.

(a)

(b)

Again, it can be observed that the product of $\mathrm{Z}_{1}$ and $\mathrm{Z}_{2}$ is independent of frequency, and the filter design obtained will be of the constant $k$ type. The plot of characteristic impedance with respect to frequency is shown


## m-Derived T-Section:

It is clear from previous chapter Figs $30.3 \& 30.7$ that the attenuation is not sharp in the stop band for k-type filters. The characteristic impedance, $Z_{0}$ is a function of frequency and varies widely in the transmission band. Attenuation can be increased in the stop band by using ladder section, i.e. by connecting two or more identical sections. In order to join the filter sections, it would be necessary that their characteristic impedance be equal to each other at all frequencies. If their characteristic impedances match at all frequencies, they would also have the same pass band. However, cascading is not a proper solution from a practical point of view. This is because practical elements have a certain resistance, which gives rise to attenuation in the pass band also. Therefore, any attempt to increase attenuation in stop band by cascading also results in an increase of ${ }^{\circ}{ }^{\text {a }}$ in the pass band. If the constant k section is regarded as the prototype, it is possible to design a filter to have rapid attenuation in the stop band, and the same characteristic impedance as the prototype at all frequencies. Such a filter is called m-derived filter. Suppose a prototype T-network shown in Fig.31.1 (a) has the series arm modified as shown in Fig.31.1 (b), where m is a constant. Equating the characteristic impedance of the networks in us has

(a)

(b)

$$
Z_{0 \mathrm{~T}}=\mathrm{Z}_{0 \mathrm{~T}}
$$

Where $\mathrm{Z}_{0 \text { T }}$ is the characteristic impedance of the modified (m-derived) T-network.
Thus m-derived section can be obtained from the prototype by modifying its series and shunt arms. The same technique can be applied to $\backslash[$ pil] section network. Suppose a prototype p-network shown in Fig. 31.3 (a) has the shunt arm modified as shown in Fig. 31.3 (b).


The characteristic impedances of the prototype and its modified sections have to be equal for matching.


The characteristic impedance of the modified (m-derived) $\backslash[\mid$ pil $\rceil$-network

$$
\therefore \quad \sqrt{\frac{Z_{1} Z_{2}}{1+\frac{Z_{1}}{4 Z_{2}}}}=\sqrt{\frac{Z_{1}^{\prime} \frac{Z_{2}}{m}}{1+\frac{Z_{1}^{\prime}}{4 \cdot Z_{2} / m}}}
$$

$$
\text { Or } \quad Z_{1}^{\prime}=\frac{Z_{1} Z_{2}}{\frac{Z_{1}}{4 m}+\frac{Z_{2}}{m}-\frac{m Z_{1}}{4}}
$$

$$
=\frac{Z_{1} Z_{2}}{\frac{Z_{2}}{m}+\frac{Z_{1}}{4 m}\left(1-m^{2}\right)}
$$

$$
\begin{equation*}
Z_{1}^{\prime}=\frac{Z_{1} Z_{2} \frac{4 m^{2}}{\left(1-m^{2}\right)}}{\frac{Z_{2} 4 m_{2}}{m\left(1-m_{2}\right)}+Z_{1} m}=\frac{m Z_{1} \frac{Z_{2} 4 m}{\left(1-m^{2}\right)}}{m Z_{1}+\frac{Z_{2} 4 m}{\left(1-m^{2}\right)}} . \tag{31.2}
\end{equation*}
$$

$$
m \mathrm{Z}_{1}
$$

The series arm of the m -derived $\backslash\left\lceil\lfloor\mathrm{pi} \backslash]\right.$ section is a parallel combination of $\mathrm{mZ} \mathrm{Z}_{1}$ and $4 \mathrm{mZ} \mathrm{Z}_{2} / 1-\mathrm{m}^{2}$

## m-Derived Low Pass Filter:

In Fig.31.5, both m-derived low pass T and $\backslash[$ pi 1$\rfloor$ filter sections are shown. For the T -section shown Fig. 31.5
(a) The shunt arm is to be chosen so that it is resonant at some frequency $f_{\mathrm{x}}$ above cut-off frequency $f_{\mathrm{c}}$ its impedance will be minimum or zero. Therefore, the output is zero and will correspond to infinite attenuation at
this particular frequency.

$m \omega_{r} L=\frac{1}{\left(\frac{1-m^{2}}{4 M}\right) \omega_{r} C}$
$\omega_{r}^{2}=\frac{4}{L C\left(1-m^{2}\right)}$
$f_{r}=\frac{1}{\pi \sqrt{L C\left(1-m^{2}\right)}}$
$\therefore \quad \alpha=2 \cosh ^{-1} \frac{m \frac{f}{f_{c}}}{\sqrt{1-\left(\frac{f}{f_{\alpha}}\right)^{2}}}$

And

$$
\beta=2 \sin ^{-1} \sqrt{\left|\frac{Z_{1}}{4 Z_{1}}\right|}=2 \sin ^{-1} \frac{m \frac{f}{f_{c}}}{\sqrt{1-\left(\frac{f}{f_{c}}\right)^{2}(1=m)^{2}}}
$$

The variation of attentuationforalow pass m-derived section cantze verified

(a)

(b)

(c)

## m-derived High Pass Filter:

If the shunt arm in T-section is series resonant, it offers minimum or zero impedance. Therefore, the output is zero and, thus, at resonance frequency or the frequency corresponds to infinite attenuation.

$$
\omega_{r} \frac{L}{m}=\frac{1}{\omega_{r} \frac{4 m}{1-m^{2}} C}
$$


the m-derived $\backslash[$ pi $1 \backslash$-section, the resonant circuit is constituted by the series arm inductance and capacitance

$$
\begin{aligned}
\frac{4 m}{1-m^{2}} \omega_{2} L & =\frac{1}{\frac{\omega_{q} C}{m}} \\
\omega_{q}^{2} & =\omega_{\alpha}^{2}=\frac{1-m^{2}}{4 L C} \\
\omega_{g} & =\frac{\sqrt{1-m^{2}}}{2 \sqrt{L C}} \text { or } f_{\alpha}=\frac{\sqrt{1-m^{2}}}{4 \pi \sqrt{L C}}
\end{aligned}
$$



## Classification of Attenuators

i. Symmetrical attenuators
ii. Asymmetrical attenuators
iii. Balanced attenuator
iv. Unbalanced attenuator
v. Symmetrical T-type attenuator
${ }^{\text {vi. }}$ Design equations of Symmetrical T-type attenuators

$$
\begin{array}{r}
\frac{R_{1}}{2}=R_{0}\left[\frac{N-1}{N+1}\right] \\
2 N \\
R_{2}=R_{0}\left[\begin{array}{ll}
N^{2} & 1
\end{array}\right]
\end{array}
$$

Series and Shunt arm impedances
From this lecture student can able to design Symmetrical T-type attenuator Symmetrical $\pi$-type attenuator
. Design equations of the Symmetrical $\pi$-type attenuator

$$
\begin{aligned}
& R_{1}=R_{0}\left[\frac{N^{2}-1}{2 N}\right] \\
& 2 R_{2}=R_{0}\left[\frac{N}{N} 1\right. \\
& \hline
\end{aligned}
$$

Series and shunt arm impedances

- From this lecture student can able to design Symmetrical $\pi$-type attenuator Bridged-T type Attenuator

Design equations of Bridged-T attenuator

$$
\begin{gathered}
R_{A}=R_{0}(N-1) \\
R_{B}=\frac{R_{0}}{\left(\begin{array}{ll}
N \quad 1
\end{array}\right)} \\
-+
\end{gathered}
$$

- From this lecture student can able to design a Bridged T-type attenuator

$$
\begin{aligned}
& R_{A}=R_{0}\left[\frac{N}{N-1}\right] \\
& R_{B}=R_{0}\left[\frac{N^{+} 1}{N} 1\right]
\end{aligned}
$$

## UNIT-IV

## DC MACHINES

## DC generator:

## Introduction:

The electrical machines deals with the energy transfer either from mechanical to electrical form or from electrical to mechanical form, this process is called electromechanical energy conversion. An electrical machine which converts mechanical energy into electrical energy is called an electric generator while an electrical machine which converts electrical energy into the mechanical energy is called an electric motor. A DC generator is built utilizing the basic principle that EMF is induced in a conductor when it cuts magnetic lines of force. A DC motor works on the basic principle that a current carrying conductor placed in a magnetic field experiences a force.

## Working principle:

All the generators work on the principle of dynamically induced EMF. The change in flux associated with the conductor can exist only when there exists a relative motion between the conductor and the flux. The relative motion can be achieved by rotating the conductor w.r.t flux or by rotating flux w.r.t conductor. So, a voltage gets generated in a conductor as long as there exists a relative motion between conductor and the flux. Such an induced EMF which is due to physical movement of coil or conductor w.r.t flux or movement of flux w.r.t coil or conductor is called dynamically induced EMF. Whenever a conductor cuts magnetic flux, dynamically induced EMF is produced in it according to
Faraday's laws of Electromagnetic Induction. This EMF causes a current to flow if the conductor circuit is closed.In a practical generator, the conductors are rotated to cut the magnetic flux, keeping flux stationary. To have a large voltage as output, a number of conductors are connected together in a specific manner to form a winding. The winding is called armature winding of a dc machine and the part on which this winding is kept is called armature of the dc machine. The magnetic field is produced by a current carrying winding which is called field winding.The conductors placed on the armature are rotated with the help of some external device. Such an external device is called a prime mover. The commonly used prime movers are diesel engines, steam engines, steam turbines, water turbines etc. The purpose of the prime mover is to rotate the electrical conductor as required by Faraday's laws The direction of induced EMF can be obtained by using Flemings right hand rule. The magnitude of induced EMF $=\mathrm{e}=$

BLV $\sin =E m \sin$. The nature of the induced EMF for a conductor rotating in the magnetic field is alternating. As conductor rotates in a magnetic field, the voltage component at various positions is different. Hence the basic nature of induced EMF in the armature winding in case of dc generator is alternating. To get dc output which is unidirectional, it is necessary to rectify the alternating induced EMF. A device which is used in dc generator to convert alternating induced EMF to unidirectional dc EMF is called commutator.

## Construction of DC machines:

D. C. machine consists of two main parts

Stationary part: It is designed mainly for producing a magnetic flux.

Rotating part: It is called the armature, where mechanical energy is converted into electrical (electrical generate) or conversely electrical energy into mechanical (electric into)


## Parts of a Dc Generator:

The stationary parts and rotating parts are separated from each other by an air gap. The stationary part of a D. C. machine consists of main poles, designed to create the magnetic flux, commutating poles interposed between the main poles and designed to ensure spark less operation of the brushes at the commutator and a frame / yoke. The armature is a cylindrical body rotating in the space between the poles and comprising a slotted armature core, a winding inserted in the armature core slots, a commutator and brush

Yoke:
It saves the purpose of outermost cover of the dc machine so that the insulating materials get protected from harmful atmospheric elements like moisture, dust and various gases like $\mathrm{SO}_{2}$, acidic fumes etc. It provides mechanical support to the poles. It forms a part of the magnetic circuit. It provides a path of low reluctance for magnetic flux. Choice of material: To provide low reluctance path, it must be made up of some magnetic material. It is prepared by using cast iron because it is the cheapest. For large machines rolled steel or cast steel, is used which provides high permeability i.e., low reluctance and gives good mechanical strength.

Poles: Each pole is divided into two parts
a) pole core
b) pole shoe


Pole core basically carries a field winding which is necessary to produce the flux. It directs the flux produced through air gap to armature core to the next pole. Pole shoe enlarges the area of armature core to come across the flux, which is necessary to produce larger induced EMF. To achieve this, pole core has been given a particular shape. Choice of material: It is made up of magnetic material like cast iron or cast steel. As it requires a definite shape and size, laminated construction is used. The laminations of required size and shape are stamped together to get a pole which is then bolted to yoke.

Armature: It is further divided into two parts namely,
(1) Armature core
(2) Armature winding.

Armature core is cylindrical in shape mounted on the shaft. It consists of slots on its periphery and the air ducts to permit the air flow through armature which serves cooling purpose


Armature core provides house for armature winding i.e., armature conductors. To provide a path of low reluctance path to the flux it is made up of magnetic material like cast iron or cast steel. Choice of material: As it has to provide a low reluctance path to the flux, it is made up of magnetic material like cast iron or cast steel. It is made up of laminated construction to keep eddy current loss as low as possible. A single circular lamination used for the construction of the armature core is shown below.

## Armature winding:

Armature winding is nothing but the inter connection of the armature conductors, placed in the slots provided on the armature core. When the armature is rotated, in case of generator magnetic flux gets cut by armature conductors and EMF gets induced in them. Generation of EMF takes place in the armature winding in case of generators. To carry the current supplied in case of dc motors. To do the useful work it the external circuit.
Choice of material: As armature winding carries entire current which depends on external load. It has to be made up of conducting material, which is copper

## Field winding:

The field winding is wound on the pole core with a definite direction.
Functions: To carry current due to which pole core on which the winding is placed behaves as an electromagnet, producing necessary flux. As it helps in producing the magnetic field i.e. exciting the pole as electromagnet it is called 'Field winding' or _Exciting winding'.

Choice of material: As it has to carry current it should be made up of some conducting material like the aluminum or copper. But field coils should take any type of shape should bend easily, so copper is the proper choice. Field winding is divided into various coils called as field coils. These are connected in series with each other and wound in such a direction around pole cores such that alternate N and S poles are formed.

Commutator: The rectification in case of dc generator is done by device called as commutator.
Functions: To facilitate the collection of current from the armature conductors. To convert internally developed alternating EMF to in directional (Dc) EMF .To produce unidirectional torque in case of motor. Choice of material: As it collects current from armature, it is also made up of copper segments. It is cylindrical in shape and is made up of wedge shaped segments which are insulated from each other by thin layer of mica.

Brushes and brush gear: Brushes are stationary and rest on the surface of the Commutator. Brushes are rectangular in shape. They are housed in brush holders, which are usually of box type. The brushes are made to press on the commutator surface by means of a spring, whose tension can be adjusted with the help of lever. A flexible copper conductor called pigtail is used to connect the brush to the external circuit.

Functions: To collect current from commutator and make it available to the stationary external circuit.
Choice of material: Brushes are normally made up of soft material like carbon.
Bearings: Ball-bearings are usually used as they are more reliable. For heavy duty machines, roller bearings are preferred.

## Types of armature winding

Armature conductors are connected in a specific manner called as armature winding and according to the way of connecting the conductors; armature winding is divided into two types.

Lap winding: In this case, if connection is started from conductor in slot 1 then the connections overlap each other as winding proceeds, till starting point is reached again.

There is overlapping of coils while proceeding. Due to such connection, the total number of conductors get divided into $P$ number of parallel paths, where
$\mathrm{P}=$ number of poles in the machine.

Large number of parallel paths indicates high current capacity of machine hence lap winding is pertained for high current rating generators.

Wave winding: In this type, winding always travels ahead avoiding over lapping. It travels like a progressive wave hence called wave winding. Both coils starting from slot 1 and slot 2 areprogressing in wave fashion. Due to this type of connection, the total number of conductors get divided into two number of parallel paths always, irrespective of number of poles of machine.
EMF equation
EMF generated/path $=\mathrm{PN} / 60(\mathrm{Z} / \mathrm{P})=\mathrm{ZN} / 60$
$\mathrm{Z}=$ total number of armature conductors.
$=$ number of slots x number of conductors/slot
$\mathrm{N}=$ armature rotation in revolutions (speed for armature) per minute (rpm) $\mathrm{A}=$ No.of parallel paths into which the $Z^{\text {‘ }}$ no. of conductors are divided.
$\mathrm{E}=\mathrm{EMF}$ induced in any parallel path
$\mathrm{E}_{\mathrm{g}}=\mathrm{EMF}$ generated in any parallel path
$\mathrm{A}=2$ for simplex - wave winding
$\mathrm{A}=\mathrm{P}$ for simplex lap-winding

## DC MOTOR:

A dc motor is similar in construction to a dc generator. As a matter of fact a dc generator will run as a motor when its field \& armature windings are connected to a source of direct current.

The basic construction is same whether it is generator or a motor.

## Working principle:

The principle of operation of a dc motor can be stated as when a current carrying conductor is placed in a magnetic field; it experiences a mechanical force. In a practical dc motor, the field winding produces the required magnetic held while armature conductor play the role of current carrying conductor and hence the armature conductors experience a force. As conductors are placed in the slots which are on the periphery, the individual force experienced by the conductive acts as a twisting or turning force on the armature which is called a torque. The torque is the product of force and the radius at which this force acts, so overall armature experiences a torque and starts rotating. Consider a single conductor placed in a magnetic field, the magnetic field is produced by a permanent magnet but in practical dc motor it is produced by the field winding when it carries a current. Now this conductor is excited by a separate supply so that it carries a current in a particular direction. Consider that it
carries a current away from an current. Any current carrying conductor produces its own magnetic field around it, hence this conductor also produces its own flux, around. The direction of this flux can be determined by right hand thumb rule. For direction of current considered the direction of flux around a conductor is clock-wise. Now, there are two fluxes present

- Flux produced by permanent magnet called main flux
- Flux produced by the current carrying conductor

From the figure shown below, it is clear that on one side of the conductor, both the fluxes are in the same direction in this case, on the left of the conductor there gathering of the flux lines as two fluxes help each other. A too against this, on the right of the conductor, the two fluxes are in opposite direction and hence try to cancel each other. Due to this, the density of the flux lines in this area gets weakened.

So on the left, there exists high flux density area while on the right of the conductor then exists low flux density area The flux distribution around the conductor arts like a stretched ribbed bond under tension. The exerts a mechanical force on the conductor which acts from high flux density area towards low flux density area, i.e. from left to right from the case considered as shown above.

In the practical dc motor, the permanent magnet is replaced by the field winding which produces the required flux winding which produces the required flux called main flux and all the armature conductors, would on the periphery of the armature gram, get subjected total he mechanical force. Due to this, overall armature experiences a twisting force called torque and armature of the motor status rotating.

## Direction of rotation of motor

The magnitude of the force experienced by the conductor in a motor is given by $\mathrm{F}=$ BIL newtons. The direction of the main field can be revoked y changing the direction of current passing through the field winding, which is possible by interchanging the polarities of supply which is given to the field winding. The direction of current through armature can be reversed by changing supply polarities of dc supplying current to the armature.
It directions of both the currents are changed then the direction of rotation of the motor remains undamaged. In a dc motor both the field and armature is connected to a source of direct current. The current through the armature winding establish its own magnetic flux the interaction both the main field and the armature current produces the torque, there by sensing the motor to rotate, once the motor starts rotating, already existing magnetic flux there wire be an induced EMF in the armature conductors due to generator action. This EMF acts in a direction apposite to supplied voltage. Therefore it is called Black EMF.

## Significance of Back EMF

In the generating action, when a conductor cuts the lines of flux, EMF gets induced in the conductor in a motor, after a motoring action, armature starts rotating and armature conductors cut the main flux. After a motoring action, there exists a generating action there is induced EMF in the rotating armature conductors according to Faraday's law of electromagnetic induction. This induced EMF in the armature always acts in the opposite direction of the supply voltage. This is according to the lenz's law which states that the direction of the induced EMF is always so as to oppose the case producing it. In a dc motor, electrical input i.e., the supply voltage is the cause and hence this induced EMF opposes the supply voltage. The EMF tries to set $u$ a current throughout he armature which is in the opposite direction to that which supply voltage is forcing through the conductor so, as this EMF always opposes the supply voltage, it is called back EMF and devoted as Eb. Through it is denoted as Eb , basically it gets generated by the generating action which we have seen

$$
\frac{Z N P}{60 A}
$$

Voltage equation of a Motor
The voltage v applied across the motor armature has to (1) over core the back EMF Eb and supply the armature ohmic drop Ia Ra

$$
\mathrm{V}=\mathrm{Eb}+\mathrm{Ia} \mathrm{R}_{\mathrm{a}}
$$

This is known as voltage equation of a motor

Torque: The turning or twisting movement of a body is called Torque

$$
\begin{aligned}
& \mathrm{T}_{\mathrm{sh}}=\text { output } /((2 \Pi \mathrm{~N}) / 60) \\
& \mathrm{T}_{\mathrm{sh}}=9.55(\text { output }) / \mathrm{N}
\end{aligned}
$$

## TRANSFORMERS AND THEIR PERFORMANCE

## INTRODUCTION

Transformer is a static device which transfers electrical energy from one electrical circuit to another electrical circuit without change in frequency through magnetic medium. The winding which receives energy is called primary winding and the winding which delivers energy to the load is called secondary winding.

Based on the voltage levels transformers are classified into two types
i. Step down transformer ii. Step up transformer.

## CONSTRUCTION

## CORE-TYPE AND SHELL-TYPE CONSTRUCTION

Depending upon the manner in which the primary and secondary windings are placed on the core, and the shape of the core, there are two types of transformers, called (a) core type, and (b) shell type. In core type transformers, the windings are placed in the form of concentric cylindrical coils placed around the vertical limbs of the core. The low-voltage (LV) as well as the high-voltage (HV) winding are made in two halves, and placed on the two limbs of core. The LV winding is placed next to the core for economy in insulation cost. Figure a shows the cross-section of the arrangement. In the shell type transformer, the primary and secondary windings are wound over the central limb of a three-limb core as shown in Figure b. The HV and LV windings are split into a number of sections, and the sections are interleaved or sandwiched i.e. the sections of the HV and LV windings are placed alternately.
H.V. winding

Core

(a) core type

(b) Shell Type

CORE
The core is built-up of thin steel laminations insulated from each other. This helps in reducing the eddy current losses in the core, and also helps in construction of the transformer. The steel used for core is of high silicon content, sometimes heat treated to produce a high permeability and low hysteresis loss. The material commonly used for core is CRGO (Cold Rolled Grain Oriented) steel. Conductor material used for windings is mostly copper. However, for small distribution transformer aluminum is also sometimes used. The conductors, core and whole windings are insulated using various insulating materials depending upon the voltage.

## INSULATING OIL

In oil-immersed transformer, the iron core together with windings is immersed in insulating oil. The insulating oil provides better insulation, protects insulation from moisture and transfers the heat produced in core and windings to the atmosphere. The transformer oil should posses the following quantities:
(a) High dielectric strength,
(b) Low viscosity and high purity,
(c) High flash point, and
(d) Free from sludge.

Transformer oil is generally a mineral oil obtained by fractional distillation of crude oil.

## TANK AND CONSERVATOR

The transformer tank contains core wound with windings and the insulating oil. In large transformers small expansion tank is also connected with main tank is known as conservator. Conservator provides space when insulating oil expands due to heating. The transformer tank is provided with tubes on the outside, to permits circulation of oil, which aides in cooling. Some additional devices like breather and Buchholz relay are connected with main tank.

Buchholz relay is placed between main tank and conservator. It protect the transformer under extreme heating of transformer winding. Breather protects the insulating oil from moisture when the cool transformer sucks air inside. The silica gel filled breather absorbs moisture when air enters the tank. Some other necessary parts are connected with main tank like, Bushings, Cable Boxes, Temperature gauge, Oil gauge, Tapings, etc.

## WORKING PRINCIPLE

In its simplest form a single-phase transformer consists of two windings, wound on an iron core one of the windings is connected to an ac source of supply f. The source supplies a current to this winding (called primary winding) which in turn produces a flux in the iron core. This flux is alternating in nature If the supplied voltage has a frequency $f$, the flux in the core also alternates at a frequency $f$. the alternating flux linking with the second winding, induces a voltage $\mathrm{E}_{2}$ in the second winding (according to faraday's law). [Note that this alternating flux linking with primary winding will also induce a voltage in the primary winding, denoted as $E_{1}$. Applied voltage $V_{1}$ is very nearly equal to $E_{1}$ ]. If the number of turns in the primary and secondary windings is $\mathrm{N}_{1}$ and $\mathrm{N}_{2}$ respectively, we shall see later in this unit that $\mathrm{E} 1 / \mathrm{N} 1=\mathrm{E} 2 / \mathrm{N} 2$. The load is connected across the secondary winding, between the terminals $\mathrm{a}_{1}, \mathrm{a}_{2}$. Thus, the load can be supplied at a voltage higher or lower than the supply voltage, depending upon the ratio $\mathrm{N}_{1} / \mathrm{N}_{2}$


## IDEAL TRANSFORMER

Under certain conditions, the transformer can be treated as an ideal transformer. The assumptions necessary to treat it as an ideal transformer are:

Primary and secondary windings have zero resistance. This means that ohmic loss ( $\left.I^{2} R \operatorname{loss}\right)$, and resistive voltage drops in windings are zero.

There is no leakage flux, i.e. the entire flux is mutual flux that links both the primary and secondary windings.

Permeability of the core is infinite this means that the magnetizing current needed for establishing the flux is zero.

Core loss (hysteresis as well as eddy current losses) are zero.

## IDEAL TRANSFORMER ON NO LOAD


(a) Phasor Diagram at No Load

(b) Equivalent Circuit at No Load

## IDEAL TRANSFORMER ON LOAD



$$
\mathrm{V}_{1} / \mathrm{V}_{2}=\mathrm{N}_{1} / \mathrm{N}_{2}=\mathrm{I}_{1} / \mathrm{I}_{2}
$$

## EQUIVALENT CIRCUIT OF REAL TRANSFORMER



## REGULATION OF TRANSFORMER

Voltage regulation of a transformer is defined as the drop in the magnitude of load voltage (or secondary terminal voltage) when load current changes from zero to full load value. This is expressed as a fraction of secondary rated voltage
$(\%)$ Regulation $=($ Secondary terminal voltage at no load - Secondary terminal voltage at any load $) /$ secondary rated voltage.

Percentage voltage regulation $=\left(\mathrm{V}-\mathrm{E}_{0}\right) * 100 / \mathrm{V}$

## LOSSES AND EFFICIENCY OF TRANSFORMER

A transformer doesn't contains any rotating part so it is free from friction and windage losses.
In transformer the losses occur in iron parts as well as in copper coils. In iron core the losses are sum of hysteresis and eddy current losses. The hysteresis losses are
$P_{h} \alpha \mathrm{fB}^{\mathrm{X}}{ }_{\text {max }}$ and eddy current loss is equal to $\mathrm{P}_{\mathrm{e}} \alpha \mathrm{f}^{2} B_{\text {max }}$.

Where $-f \|$ is frequency $-B_{\text {max }} \|$ is maximum flux density.

## IRON LOSSES OR CORE LOSSES

To minimize hysteresis loss in transformer, we use Cold Rolled Grain Oriented (CRGO) silicon steel to build up the iron core.

## EDDY CURRENT LOSS

When the primary winding variable flux links with iron core then it induces some EMF on the surface of core. The magnitude of EMF is different at various points in core. So, there is current between different points in Iron Core having unequal potential.

These currents are known at eddy currents. $I^{2} R$ loss in iron core is known as eddy current loss. These losses depend on thickness of core. To minimize the eddy current losses we use the Iron Core which is made of laminated sheet stampings. The thickness of stamping is around 0.5 mm .

## COPPER LOSSES

In a transformer the primary and secondary winding currents increase with increases in load. Due to these currents there is some $I^{2} R$ losses. These are known as copper losses or ohmic losses. The total $I^{2} R$ loss in both windings at rated or full load current is equal to $I_{1}{ }^{2} R_{1}=I_{2}^{2} R_{2}$.

## EFFICIENCY OF SINGLE PHASE TRANSFORMER

Efficiency $(\eta)=$ output power/input power

$$
=(\text { input power }- \text { total losses }) / \text { input power }
$$

Alternatively ${ }_{\eta}=$ output power/(output power + total losses)
In a transformer, if $P_{i}$ is the iron loss, and $P_{c}$ is the copper loss at full load (when the load current is equal to the rated current of the transformer, the total losses in the transformer are $P_{i}+P_{c}$. In any transformer, copper losses are variable and iron losses are fixed.

When the load on transformer is x times full load then

$$
\begin{aligned}
\eta=x V_{2} & I_{2} \cos \mathrm{v} /\left(x V_{2} I_{2} \cos \mathrm{v}+\mathrm{P}_{\mathrm{i}}+\mathrm{x}^{2 *} \mathrm{P}_{\mathrm{c}}\right) \\
\eta & =x K V A \cos \mathrm{v} /\left(x K V A \cos \mathrm{v}+\mathrm{P}_{\mathrm{i}}+\mathrm{x}^{2 *} \mathrm{P}_{\mathrm{c}}\right)
\end{aligned}
$$

## OPEN CIRCUIT TEST

Practically we can determine the iron losses by performing the open circuit test and also the core loss components of equivalent circuit.

We perform open circuit test in low voltage winding in transformer keeping the high voltage winding open. The circuit is connected as shown in Figure. The instruments are connected on the LV side. The advantage of performing the test from LV side is that the test can be performed at rated voltage.

When we apply rated voltage then watt meter shows iron losses [There is some copper loss but this is negligible when compared to iron loss]. The ammeter shows no load current $I_{0}$ which is very small [2-5 \% of rated current]. Thus, the drops in $R_{1}$ and $X_{l 1}$ can be neglected.


We have

$$
\begin{aligned}
& \quad W_{0}=\text { iron loss } \\
& \quad I_{0}=\text { no load current } \\
& \cos v \quad \frac{W_{0}}{=} \\
& =\quad V_{i} I_{0}
\end{aligned}
$$

Then

$$
I_{e}=I_{0} \cos v
$$

So
$I_{m}=I_{0} \sin \mathrm{v}$.
And
$\mathrm{R}_{0}=\mathrm{V}_{\mathrm{i}} / \mathrm{I}_{\mathrm{e}}$
$\mathrm{X}_{0}=\mathrm{V}_{\mathrm{i}} / \mathrm{I}_{\mathrm{m}}$


## SHORT CIRCUIT TEST:



From short circuit test we can determine copper losses and also the winding components of equivalent circuit. It's an indirect method to find out the copper losses. To perform this test, we apply a reduced voltage to the primary winding through instruments keeping LV winding short circuited. The connections are shown in Figure. We need to apply only $5-10 \%$ of rated voltage to primary to circulated rated current in the primary and secondary winding. The applied voltage is adjusted so that the ammeter shows rated current of the winding. Under this condition, the watt-meter reading shows the copper losses of the transformer. Because of low value of applied voltage, iron losses, are very small and can be neglected.

Connection diagram for short circuit test


Equivalent circuit under shot circuit

At a rated current watt meter shows full load copper loss. We have and equivalent impedance

$$
\begin{aligned}
& \mathrm{W}_{\mathrm{sc}}=\text { copper loss } \\
& \mathrm{I}_{\mathrm{sc}}=\text { full } \\
& \text { load current } \\
& \mathrm{V}_{\mathrm{sc}}=\text { supply }
\end{aligned}
$$

$$
\begin{aligned}
& \text { voltage } \\
& \mathrm{R}_{\mathrm{eq}}=\mathrm{W}_{\mathrm{sc}} / \mathrm{I}_{\mathrm{sc}}{ }^{2} \\
& \mathrm{eq}=\mathrm{V}_{\mathrm{sc}} / \mathrm{I}_{\mathrm{sc}} \\
& \mathrm{X}_{\mathrm{eq}}=\mathrm{V}\left(\mathrm{Zeq}^{2}-{ }^{\text {Req }}{ }^{2}\right)
\end{aligned}
$$

So we calculate equivalent reactance. These $R_{\text {eq }}$ and $X_{\text {eq }}$ are equivalent resistance and reactance of both windings referred in HV side. These are known as equivalent circuit resistance and reactance.

