LECTURE NOTES

ON

COMPUTER METHODS IN POWER SYSTEMS

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UNIT-I

POWER SYSTEM NETWORK MATRICES

[CONTENTS: Definitions of important terms, Incidence matrices: Element node incidence matrix and Bus incidence matrix, Primitive networks and matrices, Performance of primitive networks, Frames of reference, Singular transformation analysis, Formation of bus admittance matrix, examples]

INTRODUCTION

The solution of a given linear network problem requires the formation of a set of equations describing the response of the network. The mathematical model so derived, must describe the characteristics of the individual network components, as well as the relationship which governs the interconnection of the individual components. In the bus frame of reference the variables are the node voltages and node currents.

The independent variables in any reference frame can be either currents or voltages. Correspondingly, the coefficient matrix relating the dependent variables and the independent variables will be either an impedance or admittance matrix. The formulation of the appropriate relationships between the independent and dependent variables is an integral part of a digital computer program for the solution of power system problems. The formulation of the network equations in different frames of reference requires the knowledge of graph theory. Elementary graph theory concepts are presented here, followed by development of network equations in the bus frame of reference.

ELEMENTARY LINEAR GRAPH THEORY: IMPORTANT TERMS

The geometrical interconnection of the various branches of a network is called the *topology* of the network. The connection of the network topology, shown by replacing all its elements by lines is called a *graph.* A *linear graph* consists of a set of objects called *nodes* and another set called *elements* such that each element is identified with an ordered pair of nodes. An *element* is defined as any line segment of the graph respective of the characteristics of the components involved. A graph in which a direction is assigned to each element is called an *oriented graph* or a *directed graph*. It is to be noted that the directions of currents in various elements are arbitrarily assigned and the network equations are derived, consistent with the assigned directions. Elements are indicated by numbers and the nodes by encircled numbers. The ground node is taken as the reference node. In electric networks the convention is to use associated directions for the voltage drops. This means the voltage drop in a branch is taken to be in the direction of the current through the branch. Hence, we need not mark the voltage polarities in the oriented graph.

Connected Graph : This is a graph where at least one path (disregarding orientation) exists between any two nodes of the graph. A representative power system and its oriented graph are as shown in Fig 1, with:

e = number of elements = 6	l = number of links = e-b = 3
n = number of nodes = 4	Tree = T(1,2,3) and
b = number of branches = n-1 = 3	Co-tree = T(4,5,6)

Sub-graph : sG is a sub-graph of G if the following conditions are satisfied:

- sG is itself a graph
- Every node of sG is also a node of G
- Every branch of sG is a branch of G

For eg., sG(1,2,3), sG(1,4,6), sG(2), sG(4,5,6), sG(3,4),.. are all valid sub-graphs of the oriented graph of Fig.1c.

Loop : A sub-graph L of a graph G is a loop if

- L is a connected sub-graph of G
- Precisely two and not more/less than two branches are incident on each node in L

In Fig 1c, the set{1,2,4} forms a loop, while the set{1,2,3,4,5} is not a valid, although the set(1,3,4,5) is a valid loop. The KVL (Kirchhoff's Voltage Law) for the loop is stated as follows: *In any lumped network, the algebraic sum of the branch voltages around any of the loops is zero.*

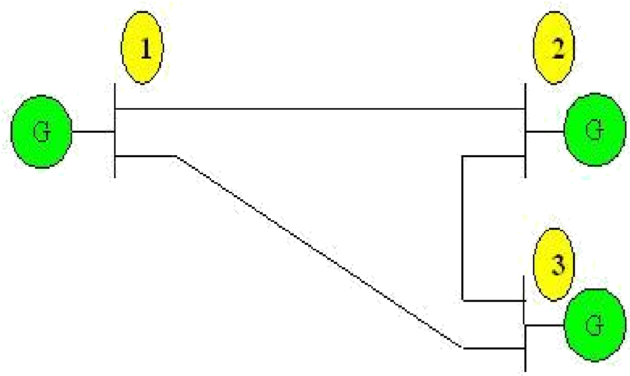


Fig 1a. Single line diagram of a power system

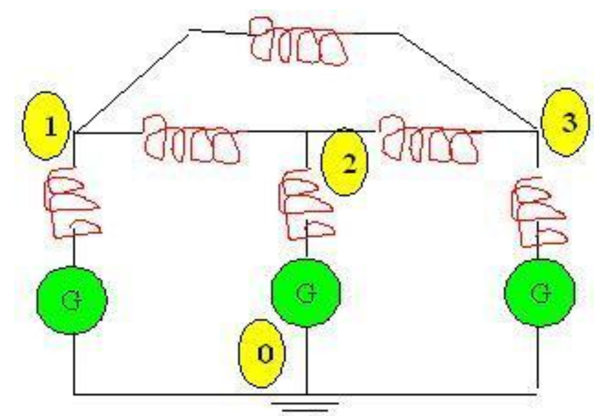


Fig 1b. Reactance diagram

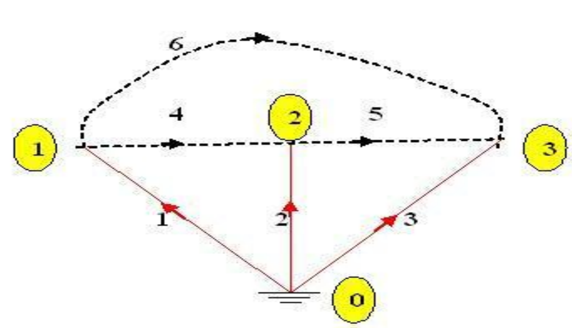


Fig 1c. Oriented Graph

Cutset : It is a set of branches of a connected graph G which satisfies the following conditions :

- The removal of all branches of the cutset causes the remaining graph to have two separate unconnected sub-graphs.
- The removal of all but one of the branches of the set, leaves the remaining graph connected.

Referring to Fig 1c, the set {3,5,6} constitutes a cutset since removal of them isolates node 3 from rest of the network, thus dividing the graph into two unconnected sub-graphs. However, the set(2,4,6) is not a valid cutset! The KCL (Kirchhoff's Current Law) for the cutset is stated as follows: *In any lumped network, the algebraic sum of all the branch currents traversing through the given cutset branches is zero.*

Tree: It is a connected sub-graph containing all the nodes of the graph G, but without any closed paths (loops). There is one and only one path between every pair of nodes in a tree. The elements of the tree are called twigs or branches. In a graph with *n* nodes,

The number of branches: b = n-1 **(1)**

For the graph of Fig 1c, some of the possible trees could be T(1,2,3), T(1,4,6), T(2,4,5), T(2,5,6), etc.

Co-Tree : The set of branches of the original graph G, not included in the tree is called the *co-tree*. The co-tree could be connected or non-connected, closed or open. The branches of the co-tree are called *links*. By convention, the tree elements are shown as solid lines while the co-tree elements are shown by dotted lines as shown in Fig.1c for tree T(1,2,3). With e as the total number of elements,

The number of links: $\boldsymbol{l}$ = e – b = e – n + 1 **(2)**

For the graph of Fig 1c, the co-tree graphs corresponding to the various tree graphs are as shown in the table below:

Tree	T(1,2,3)	T(1,4,6)	T(2,4,5)	T(2,5,6)
Co-Tree	T(4,5,6)	T(2,3,5)	T(1,3,6)	T(1,3,4)

Basic loops: When a link is added to a tree it forms a closed path or a loop. Addition of each subsequent link forms the corresponding loop. A loop containing only one link and remaining branches is called a *basic loop* or a *fundamental loop*. These loops are defined for a particular tree. Since each link is associated with a basic loop, the number of basic loops is equal to the number of links.

Basic cut-sets: Cut-sets which contain only one branch and remaining links are called *basic cutsets* or fundamental cut-sets. The basic cut-sets are defined for a particular tree. Since each branch is associated with a basic cut-set, the number of basic cut-sets is equal to the number of branches.

<u>Examples on Basics of LG Theory:</u>

Example-1: *Obtain the oriented graph for the system shown in Fig. E1. Select any four possible trees. For a selected tree show the basic loops and basic cut-sets.*

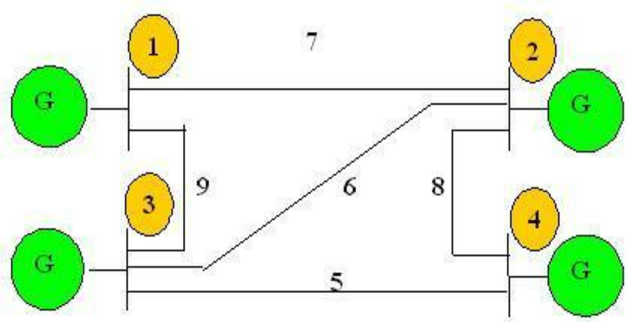


Fig. E1a. Single line diagram of Example System

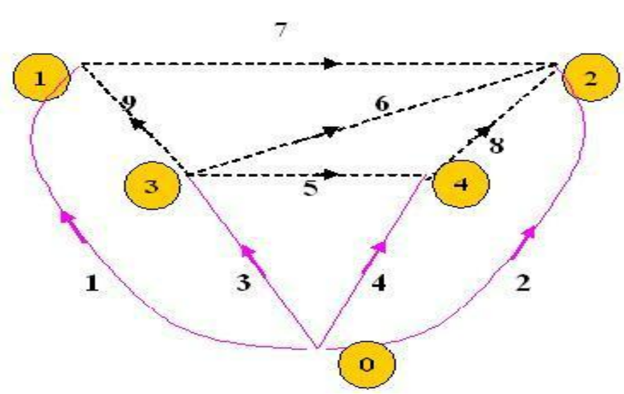


Fig. E1b. Oriented Graph of Fig. E1a.

For the system given, the oriented graph is as shown in figure E1b. some of the valid Tree graphs could be T(1,2,3,4), T(3,4,8,9), T(1,2,5,6), T(4,5,6,7), etc. The basic cut-sets (A,B,C,D) and basic loops (E,F,G,H,I) corresponding to the oriented graph of Fig.E1a and tree, T(1,2,3,4) are as shown in Figure E1c and Fig.E1d respectively.

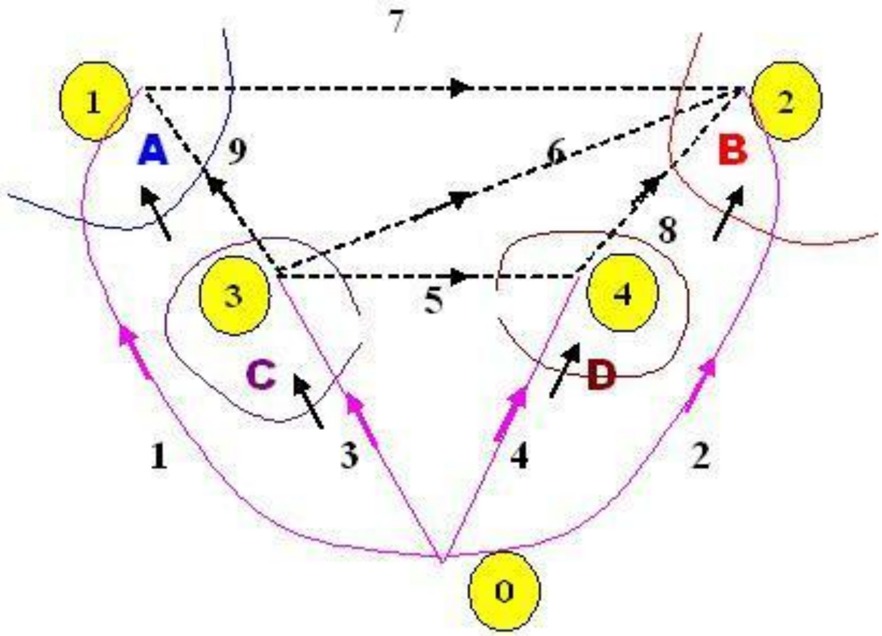


Fig. E1c. Basic Cutsets of Fig. E1a.

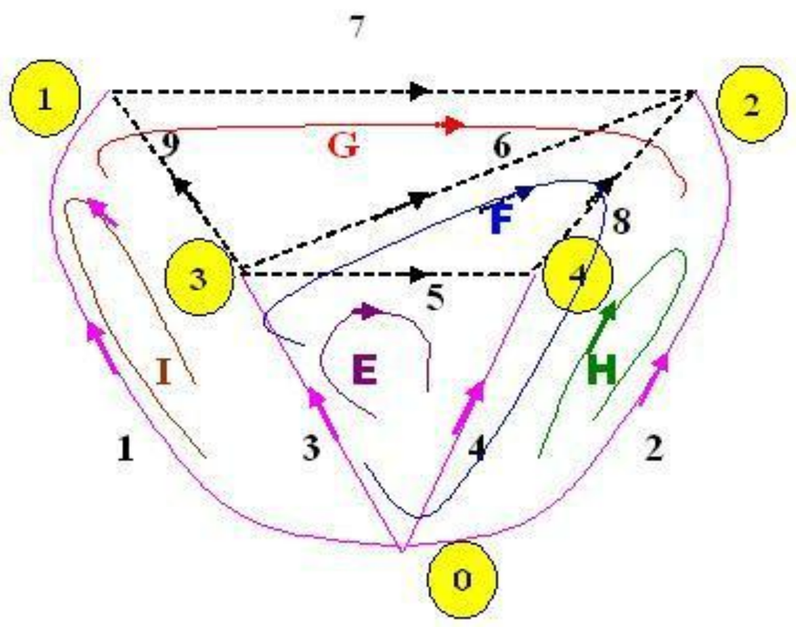


Fig. E1d. Basic Loops of Fig. E1a.

INCIDENCE MATRICES

Element–node incidence matrix: $\hat{A}$

The incidence of branches to nodes in a connected graph is given by the element-node incidence matrix, $\hat{A}$. An element a_{ij} of $\hat{A}$ is defined as under:

a_{ij} = 1 if the branch-i is incident to and oriented away from the node-j.

= -1 if the branch-i is incident to and oriented towards the node-j.

= 0 if the branch-i is not at all incident on the node-j.

Thus the dimension of $\hat{A}$ is e × *n*, where e is the number of elements and *n* is the number of nodes in the network. For example, consider again the sample system with its oriented graph as in fig. 1c. the corresponding element-node incidence matrix, is obtained as under:

$\hat{A}$ =

Elements \ Nodes	0	1	2	3
1	1	-1		
2	1		-1	
3	1			-1
4		1	-1	
5			1	-1
6		1		-1

It is to be noted that the first column and first row are not part of the actual matrix and they only indicate the element number node number respectively as shown. Further, the sum of every row is found to be equal to zero always. Hence, the rank of the matrix is less than *n*. Thus in general, the matrix $\hat{A}$ satisfies the identity:

$$\sum_{j=1}^{n} \mathbf{a_{ij} = 0} \quad \forall\ i = 1,2,\ldots..e. \qquad \textbf{(3)}$$

Bus incidence matrix: A

By selecting any one of the nodes of the connected graph as the reference node, the corresponding column is deleted from $\hat{A}$ to obtain the bus incidence matrix, A. The dimensions of A are e × *(n-1)* and the rank is *n-1*. In the above example, selecting node-0 as reference node, the matrix A is obtained by deleting the column corresponding to node-0, as under:

A =

Elements \ Buses	1	2	3
1	-1		
2		-1	
3			-1
4	1	-1	
5		1	-1
6	1		-1

=

A_b	Branches
A_l	Links

It may be observed that for a selected tree, say, T(1,2,3), the bus incidence matrix can be so arranged that the branch elements occupy the top portion of the A-matrix followed by the link elements. Then, the matrix-A can be partitioned into two sub matrices A_b and A_l as shown, where,

(i) A_b is of dimension (bxb) corresponding to the branches and

(ii) A_l is of dimension (lxb) corresponding to links.

A is a rectangular matrix, hence it is singular. A_b is a non-singular square matrix of dimension-b. Since A gives the incidence of various elements on the nodes with their direction of incidence, the KCL for the nodes can be written as

$$A^T \bar{i} = 0 \qquad \textbf{(4)}$$

where A^T is the transpose of matrix A and $\bar{i}$ is the vector of branch currents. Similarly for the branch voltages we can write,

$$\bar{v} = A \bar{E}_{bus} \qquad \textbf{(5)}$$

<u>Examples on Bus Incidence Matrix:</u>

Example-2: *For the sample network-oriented graph shown in Fig. E2, by selecting a tree, T(1,2,3,4), obtain the incidence matrices A and* $\hat{A}$. *Also show the partitioned form of the matrix-A.*

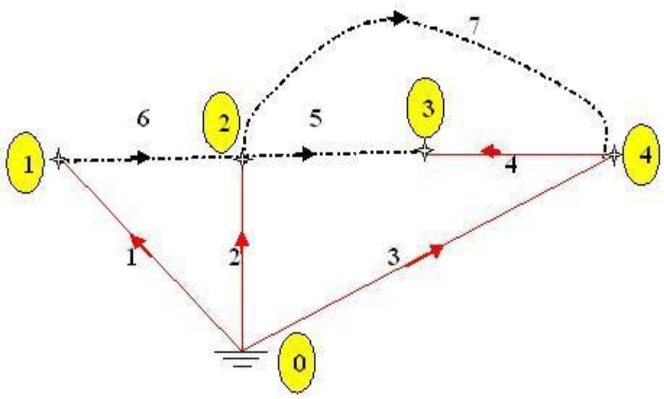


Fig. E2. Sample Network-Oriented Graph

$$\hat{A} = \text{Elements} \overset{\text{nodes}}{\begin{bmatrix} e \backslash n & 0 & 1 & 2 & 3 & 4 \\ 1 & 1 & -1 & 0 & 0 & 0 \\ 2 & 1 & 0 & -1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & 0 & -1 & 1 \\ 5 & 0 & 0 & 1 & -1 & 0 \\ 6 & 0 & 1 & -1 & 0 & 0 \\ 7 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}}$$

$$\mathbf{A} = \text{Elements} \overset{\text{buses}}{\begin{bmatrix} e \backslash b & 1 & 2 & 3 & 4 \\ 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & -1 & 1 \\ 5 & 0 & 1 & -1 & 0 \\ 6 & 1 & -1 & 0 & 0 \\ 7 & 0 & 1 & 0 & -1 \end{bmatrix}}$$

Corresponding to the Tree, T(1,2,3,4), matrix-A can be partitioned into two sub-matrices as under:

$$A_b = \text{branches} \overset{\text{buses}}{\begin{bmatrix} b \backslash b & 1 & 2 & 3 & 4 \\ 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & -1 & 1 \end{bmatrix}}$$

$$A_l = \text{links} \overset{\text{buses}}{\begin{bmatrix} l \backslash b & 1 & 2 & 3 & 4 \\ 5 & 0 & 1 & -1 & 0 \\ 6 & 1 & -1 & 0 & 0 \\ 7 & 0 & 1 & 0 & -1 \end{bmatrix}}$$

Example-3: *For the sample-system shown in Fig. E3, obtain an oriented graph. By selecting a tree, T(1,2,3,4), obtain the incidence matrices A and* $\hat{A}$*. Also show the partitioned form of the matrix-A.*

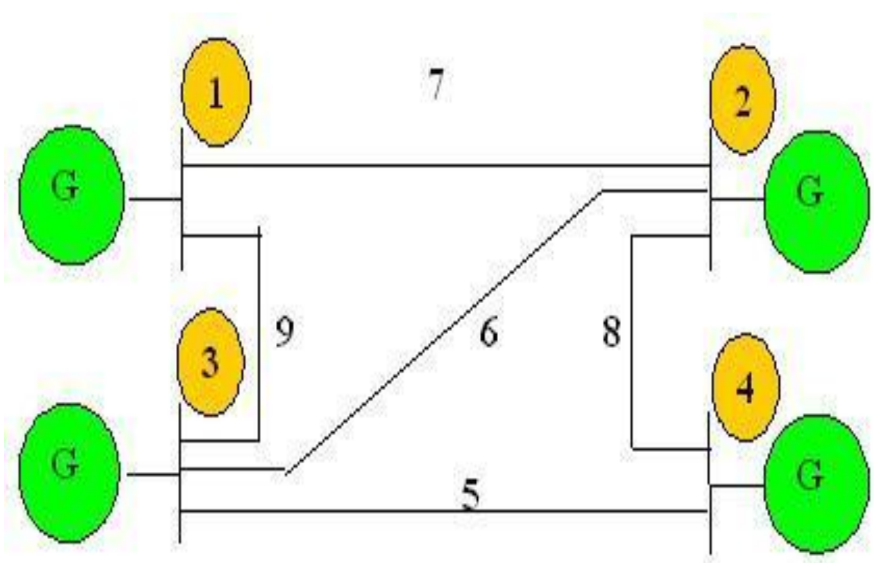


Fig. E3a. Sample Example network

Consider the oriented graph of the given system as shown in figure E3b, below.

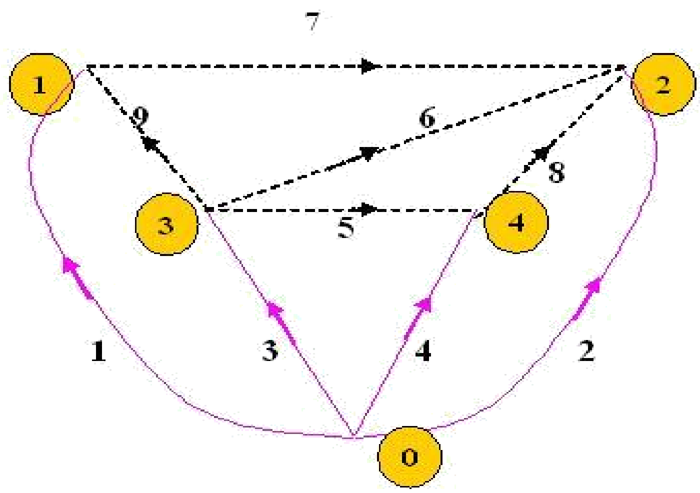


Fig. E3b. Oriented Graph of system of Fig-E3a.

Corresponding to the oriented graph above and a Tree, T(1,2,3,4), the incidence matrices Ậand A can be obtained as follows:

$\hat{A}$ =

e\n	0	1	2	3	4
1	1	-1			
2	1		-1		
3	1			-1	
4	1				-1
5				1	-1
6			-1	1	
7		1	-1		
8			-1		1
9		-1		1	

A =

e\b	1	2	3	4
1	-1			
2		-1		
3			-1	
4				-1
5			1	-1
6		-1	1	
7	1	-1		
8		-1		1
9	-1		1	

Corresponding to the Tree, T(1,2,3,4), matrix-A can be partitioned into two sub-matrices as under:

A_b =

e\b	1	2	3	4
1	-1			
2		-1		
3			-1	
4				-1

A_l =

e\b	1	2	3	4
5			1	-1
6		-1	1	
7	1	-1		
8		-1		1
9	-1		1	

PRIMITIVE NETWORKS

So far, the matrices of the interconnected network have been defined. These matrices contain complete information about the network connectivity, the orientation of current, the loops and cutsets. However, these matrices contain no information on the nature of the elements which form the interconnected network. The complete behaviour of the network can be obtained from the knowledge of the behaviour of the individual elements which make the network, along with the incidence matrices. An element in an electrical network is completely characterized by the relationship between the current through the element and the voltage across it.

General representation of a network element: In general, a network element may contain active or passive components. Figure 2 represents the alternative impedance and admittance forms of representation of a general network component.

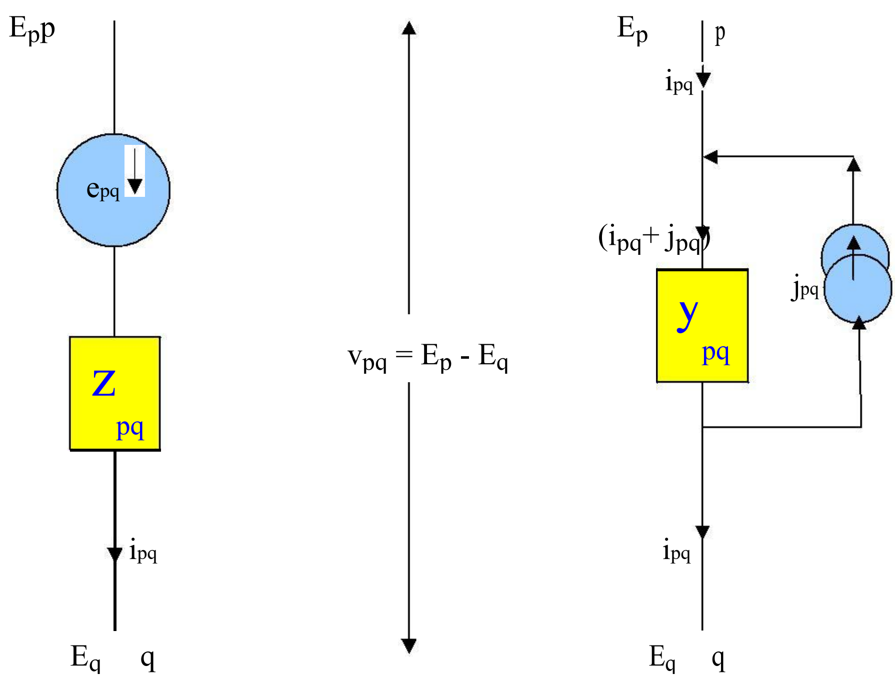


Fig.2 Representation of a primitive network element

(a) Impedance form (b) Admittance form

The network performance can be represented by using either the impedance or the admittance form of representation. With respect to the element, p-q, let,

v_{pq} = voltage across the element p-q,

e_{pq} = source voltage in series with the element p-q, i_{pq}= current through the element p-q,

j_{pq}= source current in shunt with the element p-q, z_{pq}= self impedance of the element p-q and

y_{pq}= self admittance of the element p-q.

Performance equation: Each element p-q has two variables, v_{pq} and i_{pq}. The performance of the given element p-q can be expressed by the performance equations as under:

$v_{pq} + e_{pq} = z_{pq}i_{pq}$ (in its impedance form)

$i_{pq} + j_{pq} = y_{pq}v_{pq}$ (in its admittance form) **(6)**

Thus the parallel source current jpq in admittance form can be related to the series source voltage, epq in impedance form as per the identity:

$j_{pq} = - y_{pq} e_{pq}$ **(7)**

A set of non-connected elements of a given system is defined as a *primitive Network and a*n element in it is a fundamental element that is not connected to any other element. In the equations above, if the variables and parameters are replaced by the corresponding vectors and matrices, referring to the complete set of elements present in a given system, then, we get the performance equations of the primitive network in the form as under:

$v + e = [z]\ i$

$i + j = [y]\ v$ **(8)**

Primitive network matrices:

A diagonal element in the matrices, [z] or [y] is the self impedance z_{pq-pq} or self admittance, y_{pq-pq}. An off-diagonal element is the mutual impedance, z_{pq-rs} or mutual admittance, y_{pq-rs}, the value present as a mutual coupling between the elements p-q and r-s. The primitive network admittance matrix, [y] can be obtained also by inverting the primitive impedance matrix, [z]. Further, if there are no mutually coupled elements in the given system, then both the matrices, [z] and [y] are diagonal. In such cases, the self impedances are just equal to the reciprocal of the corresponding values of self admittances, and vice-versa.

Examples on Primitive Networks:

Example-4: *Given that the self impedances of the elements of a network referred by the bus incidence matrix given below are equal to:* $Z_1=Z_2=0.2$, $Z_3=0.25$, $Z_4=Z_5=0.1$ *and* $Z_6=0.4$ *units, draw the corresponding oriented graph, and find the primitive network matrices. Neglect mutual values between the elements.*

A =

-1	0	0
0	-1	0
0	0	-1
1	-1	0
0	1	-1
1	0	-1

Solution:

The element node incidence matrix, $\hat{A}$ can be obtained from the given A matrix, by pre-augmenting to it an extra column corresponding to the reference node, as under.

$\hat{A}$ =

1	-1	0	0
1	0	-1	0
1	0	0	-1
0	1	-1	0
0	0	1	-1
0	1	0	-1

Based on the conventional definitions of the elements of A , the oriented graph can be formed as under:

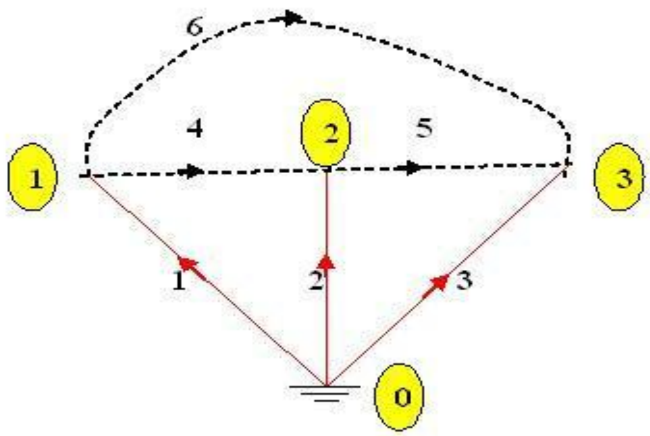


Fig. E4 Oriented Graph

Thus the primitive network matrices are square, symmetric and diagonal matrices of order e=no. of elements = 6. They are obtained as follows.

[z] =

0.2	0	0	0	0	0
0	**0.2**	0	0	0	0
0	0	**0.25**	0	0	0
0	0	0	**0.1**	0	0
0	0	0	0	**0.1**	0
0	0	0	0	0	**0.4**

And

[y] =

5.0	0	0	0	0	0
0	**5.0**	0	0	0	0
0	0	**4.0**	0	0	0
0	0	0	**10**	0	0
0	0	0	0	**10**	0
0	0	0	0	0	**2.5**

Example-5: *Consider three passive elements whose data is given in Table E5 below. Form the primitive network impedance matrix.*

Table E5

Element number	**Self impedance** (z_{pq-pq})		**Mutual impedance,** (z_{pq-rs})	
	Bus-code, (p-q)	**Impedance in p.u.**	**Bus-code, (r-s)**	**Impedance in p.u.**
1	1-2	j 0.452		
2	2-3	j 0.387	1-2	j 0.165
3	1-3	j 0.619	1-2	j 0.234

Solution:

[z] =

	1-2	2-3	1-3
1-2	j 0.452	j 0.165	j 0.234
2-3	j 0.165	j 0.387	0
1-3	j 0.234	0	j 0.619

Note:

- The size of [z] is e × e, where e= number of elements,
- The diagonal elements are the self impedances of the elements
- The off-diagonal elements are mutual impedances between the corresponding elements.
- Matrices [z] and [y] are inter-invertible.

FORMATION OF Y_{BUS} AND Z_{BUS}

The bus admittance matrix, Y_{BUS} plays a very important role in computer aided power system analysis. It can be formed in practice by either of the methods as under:

1. Rule of Inspection
2. Singular Transformation
3. Non-Singular Transformation
4. Z_{BUS} Building Algorithms, etc.

The performance equations of a given power system can be considered in three different frames of reference as discussed below:

Frames of Reference:

Bus Frame of Reference: There are b independent equations (b = no. of buses) relating the bus vectors of currents and voltages through the bus impedance matrix and bus admittance matrix:

$$E_{BUS} = Z_{BUS} I_{BUS}$$

$$I_{BUS} = Y_{BUS} E_{BUS} \quad \textbf{(9)}$$

Branch Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

$$E_{BR} = Z_{BR} I_{BR}$$

$$I_{BR} = Y_{BR} E_{BR} \quad \textbf{(10)}$$

Loop Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

$$E_{LOOP} = Z_{LOOP} I_{LOOP}$$

$$I_{LOOP} = Y_{LOOP} E_{LOOP} \quad \textbf{(11)}$$

Of the various network matrices refered above, the bus admittance matrix (Y_{BUS}) and the bus impedance matrix (Z_{BUS}) are determined for a given power system by the rule of inspection as explained next.

Rule of Inspection

Consider the 3-node admittance network as shown in figure5. Using the basic branch relation: I = (YV), for all the elemental currents and applying Kirchhoff's Current Law principle at the nodal points, we get the relations as under:

At node 1: $I_1 = Y_1V_1 + Y_3 (V_1-V_3) + Y_6 (V_1 - V_2)$

At node 2: $I_2 = Y_2V_2 + Y_5 (V_2-V_3) + Y_6 (V_2 - V_1)$

At node 3: $0 = Y_3 (V_3-V_1) + Y_4V_3 + Y_5 (V_3 - V_2)$ **(12)**

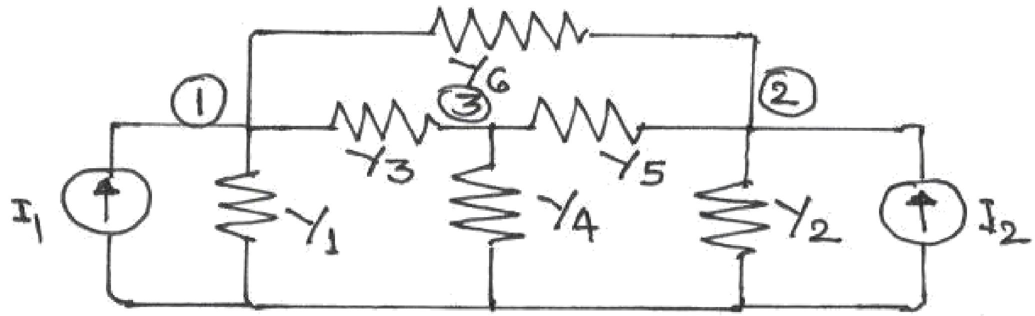


Fig. 3 Example System for finding Y_{BUS}

These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

$$\begin{vmatrix} I_1 \\ I_2 \\ 0 \end{vmatrix} = \begin{vmatrix} (Y_1+Y_3+Y_6) & -Y_6 & -Y_3 \\ -Y_6 & (Y_2+Y_5+Y_6) & -Y_5 \\ -Y_3 & -Y_5 & (Y_3+Y_4+Y_5) \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix} \qquad \textbf{(13)}$$

In other words, the relation of equation (9) can be represented in the form

$$I_{BUS} = Y_{BUS}\, E_{BUS} \qquad \textbf{(14)}$$

Where, Y_{BUS} is the bus admittance matrix, I_{BUS} & E_{BUS} are the bus current and bus voltage vectors respectively.

By observing the elements of the bus admittance matrix, Y_{BUS} of equation (13), it is observed that the matrix elements can as well be obtained by a simple inspection of the given system diagram:

Diagonal elements: A diagonal element (Y_{ii}) of the bus admittance matrix, Y_{BUS}, is equal to the sum total of the admittance values of all the elements incident at the bus/node i,

Off Diagonal elements: An off-diagonal element (Y_{ij}) of the bus admittance matrix, Y_{BUS}, is equal to the negative of the admittance value of the connecting element present between the buses I and j, if any.

This is the principle of the rule of inspection. Thus the algorithmic equations for the rule of inspection are obtained as:

$$Y_{ii} = \Sigma\, y_{ij} \quad (j = 1,2,\ldots\ldots n)$$

$$Y_{ij} = -\,y_{ij} \quad (j = 1,2,\ldots\ldots n) \qquad \textbf{(15)}$$

For i = 1,2,….n, n = no. of buses of the given system, y_{ij} is the admittance of element connected between buses i and j and y_{ii} is the admittance of element connected between bus i and ground (reference bus).

Bus impedance matrix

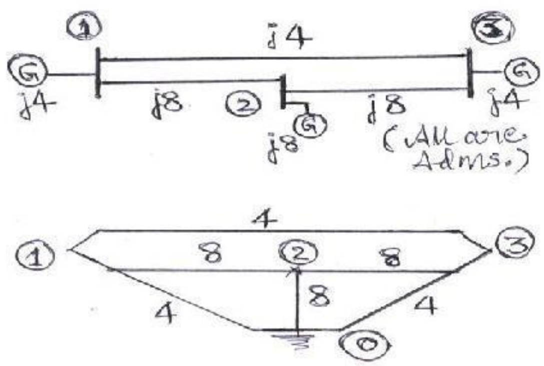
In cases where, the bus impedance matrix is also required, it cannot be formed by direct inspection of the given system diagram. However, the bus admittance matrix determined by the rule of inspection following the steps explained above, can be inverted to obtain the bus impedance matrix, since the two matrices are inter-invertible.

Note: It is to be noted that the rule of inspection can be applied only to those power systems that do not have any mutually coupled elements.

<u>Examples on Rule of Inspection:</u>

Example 6: *Obtain the bus admittance matrix for the admittance network shown aside by the rule of inspection*

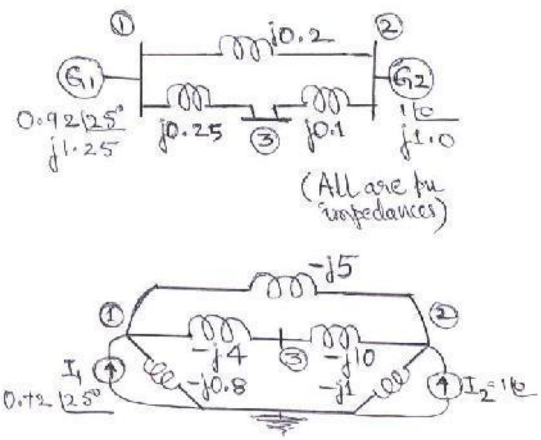
16 -8 -4 Y_{BUS} = j
-8 24 -8 -4 -8 16

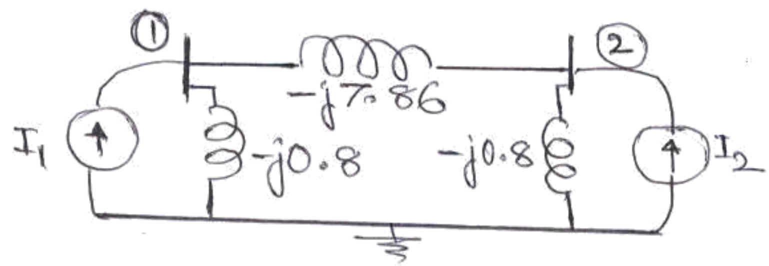


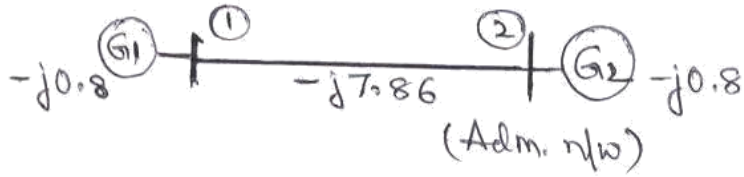
Example 7: *Obtain Y_{BUS} for the impedance network shown aside by the rule of inspection. Also, determine Y_{BUS} for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.*

$$Y_{BUS} = j\begin{vmatrix} -9.8 & 5 & 4 \\ 5 & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1}$$







$$Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C$$

$$Y_{BUS} = j\begin{vmatrix} -8.66 & 7.86 \\ 7.86 & -8.66 \end{vmatrix}$$

SINGULAR TRANSFORMATIONS

The primitive network matrices are the most basic matrices and depend purely on the impedance or admittance of the individual elements. However, they do not contain any information about the behaviour of the interconnected network variables. Hence, it is necessary to transform the primitive matrices into more meaningful matrices which can relate variables of the interconnected network.

Bus admittance matrix, Y_{BUS} and Bus impedance matrix, Z_{BUS}

In the bus frame of reference, the performance of the interconnected network is described by *n* independent nodal equations, where *n* is the total number of buses (n+1 nodes are present, out of which one of them is designated as the reference node). For example a 5-bus system will have 5 external buses and 1 ground/ ref. bus). The performance equation relating the bus voltages to bus current injections in bus frame of reference in admittance form is given by

$$I_{BUS} = Y_{BUS} E_{BUS} \quad \textbf{(17)}$$

Where E_{BUS} = vector of bus voltages measured with respect to reference bus

I_{BUS} = Vector of currents injected into the bus

Y_{BUS} = bus admittance matrix

The performance equation of the primitive network in admittance form is given by

$$i + j = [y]\ v$$

Pre-multiplying by A^t (transpose of A), we obtain

$$A^t i + A^t j = A^t [y]\ v \quad \textbf{(18)}$$

However, as per equation (4),

$$A^t i = 0,$$

since it indicates a vector whose elements are the algebraic sum of element currents incident at a bus, which by Kirchhoff's law is zero. Similarly, $A^t j$ gives the algebraic sum of all source currents incident at each bus and this is nothing but the total current injected at the bus. Hence,

$$A^t j = I_{BUS} \quad \textbf{(19)}$$

Thus from (18) we have, $I_{BUS} = A^t [y]\ v$ **(20)**

However, from (5), we have

$$v = A\ E_{BUS}$$

And hence substituting in (20) we get,

$$I_{BUS} = A^t [y]\ A\ E_{BUS} \quad \textbf{(21)}$$

Comparing (21) with (17) we obtain,

$$Y_{BUS} = A^t [y]\ A \quad \textbf{(22)}$$

The bus incidence matrix is rectangular and hence singular. Hence, (22) gives a singular transformation of the primitive admittance matrix [y]. The bus impedance matrix is given by ,

$$Z_{BUS} = Y_{BUS}^{-1} \quad \textbf{(23)}$$

Note: This transformation can be derived using the concept of power invariance, however, since the transformations are based purely on KCL and KVL, the transformation will obviously be power invariant.

Examples on Singular Transformation:

Example 8: *For the network of Fig E8, form the primitive matrices [z] & [y] and obtain the bus admittance matrix by singular transformation. Choose a Tree T(1,2,3). The data is given in Table E8.*

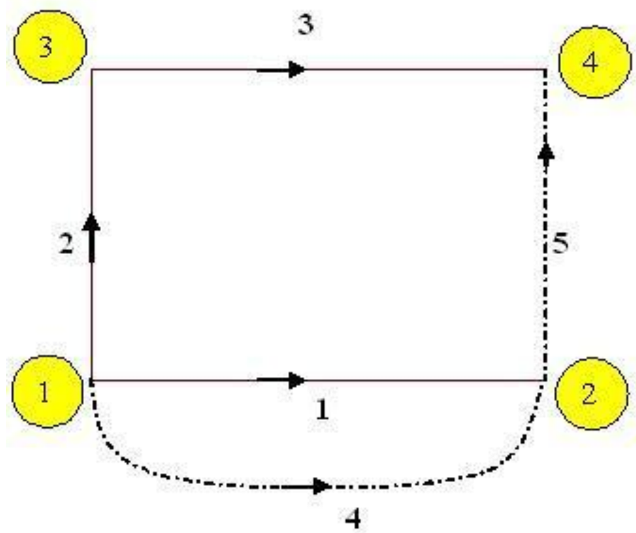


Fig E8 System for Example-8

Table E8: Data for Example-8

Elements	Self impedance	Mutual impedance
1	j 0.6	-
2	j 0.5	j 0.1(with element 1)
3	j 0.5	-
4	j 0.4	j 0.2 (with element 1)
5	j 0.2	-

Solution:

The bus incidence matrix is formed taking node 1 as the reference bus

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

The primitive incidence matrix is given by,

$$[z] = \begin{bmatrix} j0.6 & j0.1 & 0.0 & j0.2 & 0.0 \\ j0.1 & j0.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & j0.5 & 0.0 & 0.0 \\ j0.2 & 0.0 & 0.0 & j0.4 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & j0.2 \end{bmatrix}$$

The primitive admittance matrix $[y] = [z]^{-1}$ and given by,

$$[y] = \begin{bmatrix} -j2.0833 & j0.4167 & 0.0 & j1.0417 & 0.0 \\ j0.4167 & -j2.0833 & 0.0 & -j0.2083 & 0.0 \\ 0.0 & 0.0 & -j2.0 & 0.0 & 0.0 \\ j1.0417 & -j0.2083 & 0.0 & -j3.0208 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -j5.0 \end{bmatrix}$$

The bus admittance matrix by singular transformation is obtained as

$$Y_{BUS} = A^t [y] A = \begin{bmatrix} -j8.0208 & j0.2083 & j5.0 \\ j0.2083 & -j4.0833 & j2.0 \\ j5.0 & j2.0 & -j7.0 \end{bmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1} = \begin{bmatrix} j0.2713 & j0.1264 & j0.2299 \\ j0.1264 & j0.3437 & j0.1885 \\ j0.2299 & j0.1885 & j0.3609 \end{bmatrix}$$

SUMMARY

The formulation of the mathematical model is the first step in obtaining the solution of any electrical network. The independent variables can be either currents or voltages. Correspondingly, the elements of the coefficient matrix will be impedances or admittances.

Network equations can be formulated for solution of the network using graph theory, independent of the nature of elements. In the graph of a network, the tree-branches and links are distinctly identified. The complete information about the interconnection of the network, with the directions of the currents is contained in the bus incidence matrix.

The information on the nature of the elements which form the interconnected network is contained in the primitive impedance matrix. A primitive element can be represented in impedance form or admittance form. In the bus frame of reference, the performance of the interconnected system is described by *(n-1)* nodal equations, where *n* is the number of nodes. The bus admittance matrix and the bus impedance matrix relate the bus voltages and currents. These matrices can be obtained from the primitive impedance and admittance matrices.

FORMATION OF BUS IMPEDANCE MATRIX

[**CONTENTS:** *Node elimination by matrix algebra, generalized algorithms for* Z_{BUS} *building, addition of BRANCH, addition of LINK, special cases of analysis, removal of elements, changing the impedance value of an element, examples*]

NODE ELIMINATION BY MATRIX ALGEBRA

Nodes can be eliminated by the matrix manipulation of the standard node equations. However, *only those nodes at which current does not enter or leave the network can be considered for such elimination*. Such nodes can be eliminated either in one group or by taking the eligible nodes one after the other for elimination, as discussed next.

CASE-A: Simultaneous Elimination of Nodes:

Consider the performance equation of the given network in bus frame of reference in admittance form for a n-bus system, given by:

$$\mathbf{I}_{BUS} = \mathbf{Y}_{BUS}\,\mathbf{E}_{BUS} \qquad \textbf{(1)}$$

Where I_{BUS} and E_{BUS} are n-vectors of injected bus current and bus voltages and Y_{BUS} is the square, symmetric, coefficient bus admittance matrix of order n.

Now, of the n buses present in the system, let p buses be considered for node-elimination so that the reduced system after elimination of p nodes would be retained with m (= n-p) nodes only. Hence the corresponding performance equation would be similar to (1) except that the coefficient matrix would be of order m now, i.e.,

$$\mathbf{I}_{BUS_{new}} = \mathbf{Y}_{BUS}^{new}\,\mathbf{E}_{BUS} \qquad \textbf{(2)}$$

Where Y_{BUS} is the bus admittance matrix of the reduced network and the vectors I_{BUS} and E_{BUS} are of order m. It is assumed in (1) that I_{BUS} and E_{BUS} are obtained with their elements arranged such that the elements associated with p nodes to be eliminated are in the lower portion of the vectors. Then the elements of Y_{BUS} also get located accordingly so that (1) after matrix partitioning yields,

$$
\begin{matrix} \\ \begin{bmatrix} I_{BUS-m} \\ I_{BUS-p} \end{bmatrix} \end{matrix} = \begin{matrix} & m & p \\ m & Y_A & Y_B \\ p & Y_C & Y_D \end{matrix} \begin{matrix} \\ \begin{bmatrix} E_{BUS-m} \\ E_{BUS-p} \end{bmatrix} \end{matrix} \qquad (3)
$$

Where the self and mutual values of Y_A and Y_D are those identified only with the nodes to be retained and removed respectively and $Y_C = Y_B^t$ is composed of only the corresponding mutual admittance values, that are common to the nodes m and p.

Now, for the p nodes to be eliminated, it is necessary that, each element of the vector I_{BUS-p} should be zero. Thus we have from (3):

$$
\begin{aligned} I_{BUS-m} &= Y_A E_{BUS-m} + Y_B E_{BUS-p} \\ I_{BUS-p} &= Y_C E_{BUS-m} + Y_D E_{BUS-p} = 0 \end{aligned} \qquad (4)
$$

Solving, $$\mathbf{E_{BUS-p} = - Y_D^{-1} Y_C E_{BUS-m}} \qquad (5)$$

Thus, by simplification, we obtain an expression similar to (2) as,

$$\mathbf{I_{BUS-m}} = \{Y_A - Y_B Y_D^{-1} Y_C\} \mathbf{E_{BUS-m}} \qquad (6)$$

Thus by comparing (2) and (6), we get an expression for the new bus admittance matrix in terms of the sub-matrices of the original bus admittance matrix as:

$$\mathbf{Y_{BUS}^{new} = \{Y_A - Y_B Y_D^{-1} Y_C\}} \qquad (7)$$

This expression enables us to construct the given network with only the necessary nodes retained and all the unwanted nodes/buses eliminated. However, it can be observed from (7) that the expression involves finding the inverse of the sub-matrix Y_D (of order p). This would be computationally very tedious if p, the nodes to be eliminated is very large, especially for real practical systems. In such cases, it is more advantageous to eliminate the unwanted nodes from the given network by considering one node only at a time for elimination, as discussed next.

CASE-B: Separate Elimination of Nodes:

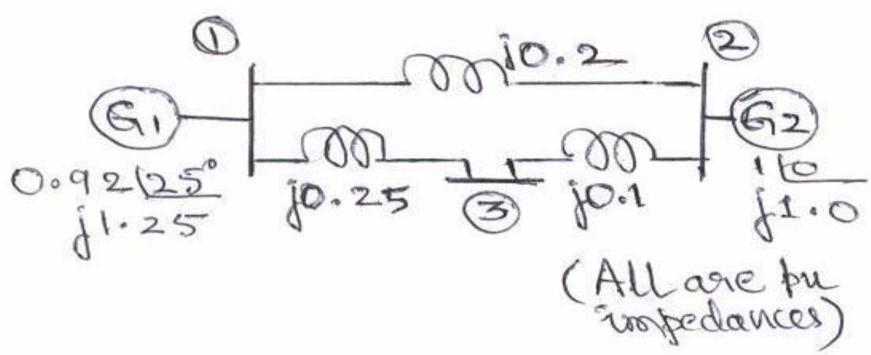
Here again, the system buses are to be renumbered, if necessary, such that the node to be removed always happens to be the last numbered one. The sub-matrix Y_D then would be a single element matrix and hence it inverse would be just equal to its own reciprocal value. Thus the generalized algorithmic equation for finding the elements of the new bus admittance matrix can be obtained from (6) as,

$$\mathbf{Y_{ij}^{new} = Y_{ij}^{old} - Y_{in}\, Y_{nj} \,/\, Y_{nn}} \;\forall\; i,j = 1,2,\ldots\ldots n. \qquad \mathbf{(8)}$$

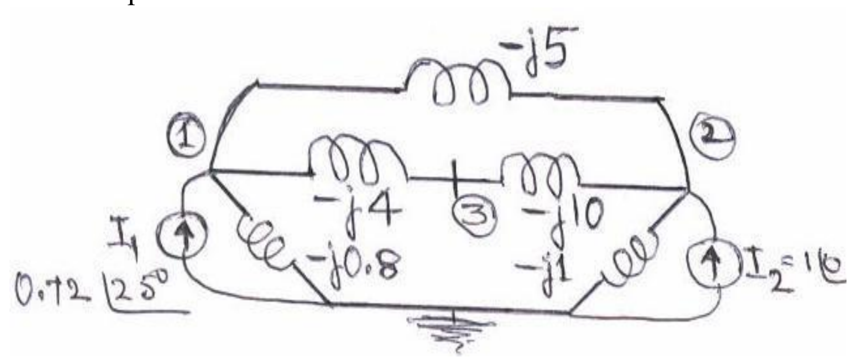
Each element of the original matrix must therefore be modified as per (7). Further, this procedure of eliminating the last numbered node from the given system of n nodes is to be iteratively repeated p times, so as to eliminate all the unnecessary p nodes from the original system.

<u>**Examples on Node elimination:**</u>

Example-1: *Obtain Y_{BUS} for the impedance network shown below by the rule of inspection. Also, determine Y_{BUS} for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.*



The admittance equivalent network is as follows:



The bus admittance matrix is obtained by RoI as:

$$Y_{BUS} = j\begin{vmatrix} -9.8 & 5 & 4 \\ 5 & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$

The reduced matrix after elimination of node 3 from the given system is determined as per the equation:

$$Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C$$

$Y_{BUS}^{new} =$

n/n	1	2
1	-j8.66	j7.86
2	j7.86	-j8.66

Alternatively,

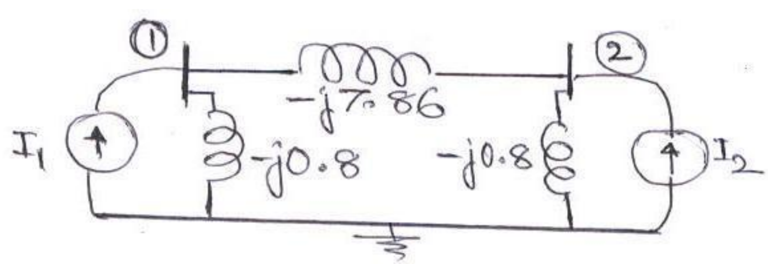
$$Y_{ij}^{new} = Y_{ij}^{old} - Y_{i3}\, Y_{3j} / Y_{33} \quad \forall\ i,j = 1,2.$$

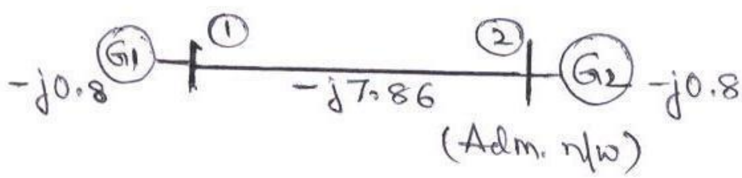
$Y_{11} = Y_{11} - Y_{13}Y_{31}/\ Y_{33} = -j8.66$

$Y_{22} = Y_{22} - Y_{23}Y_{32}/\ Y_{33} = -j8.66$

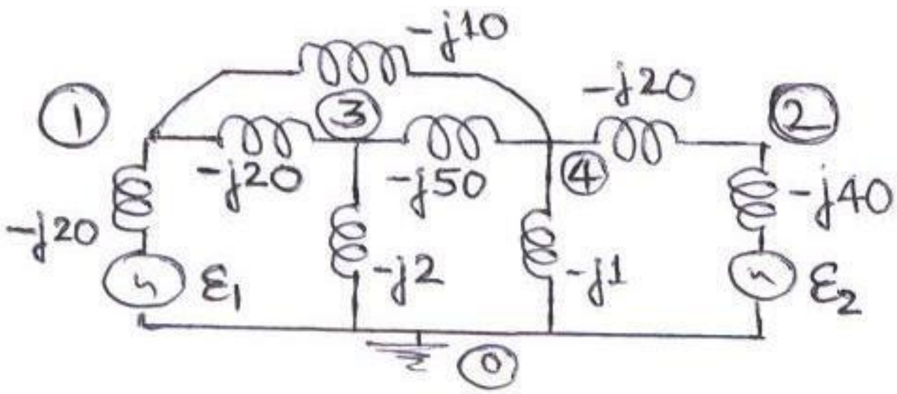
$Y_{12} = Y_{21} = Y_{12} - Y_{13}Y_{32}/Y_{33} = j7.86$

Thus the reduced network can be obtained again by the rule of inspection as shown be low.





Example-2: *Obtain Y_{BUS} for the admittance network shown below by the rule of inspection. Also, determine Y_{BUS} for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.*



Y_{BUS} =

n/n	1	2	3	4
1	-j50	0	j20	j10
2	0	-j60	0	j72
3	j20	0	-j72	j50
4	j10	j72	j50	-j81

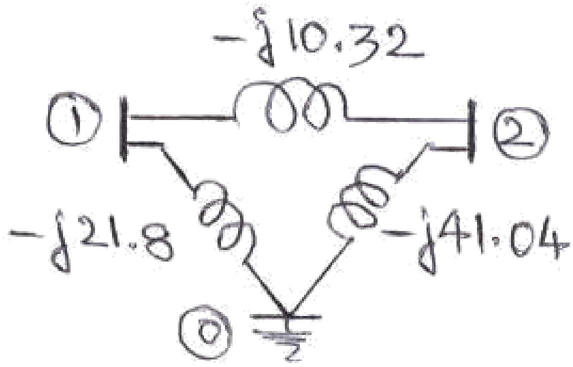
$$= \begin{vmatrix} Y_A & Y_B \\ Y_C & Y_D \end{vmatrix}$$

$$Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C$$

I_{BUS}^{new} =

n/n	1	2
1	-j32.12	j10.32
2	j10.32	-j51.36

Thus the reduced system of two nodes can be drawn by the rule of inspection as under:



Z_{BUS} building

FORMATION OF BUS IMPEDANCE MATRIX

The bus impedance matrix is the inverse of the bus admittance matrix. An alternative method is possible, based on an algorithm to form the bus impedance matrix directly from system parameters and the coded bus numbers. The bus impedance matrix is formed adding one element at a time to a partial network of the given system. The performance equation of the network in bus frame of reference in impedance form using the currents as independent variables is given in matrix form by

$$\bar{E}_{bus} = [Z_{bus}]\bar{I}_{bus} \tag{9}$$

When expanded so as to refer to a n bus system, (9) will be of the form

$$E_1 = Z_{11} I_1 + Z_{12} I_2 + ... + .Z_{1k} I_k ... + Z_{1n} I_n$$

$$E_k = Z_{k1} I_1 + Z_{k2} I_2 + ... + Z_{kk} I_k + ... + Z_{kn} I_n$$

$$E_n = Z_{n1} I_1 + Z_{n2} I_2 + ... + Z_{nk} I_k + ... + Z_{nn} I_n \tag{10}$$

Now assume that the bus impedance matrix Z_{bus} is known for a partial network of m buses and a known reference bus. Thus, Z_{bus} of the partial network is of dimension $m \times m$. If now a new element is added between buses p and q we have the following two possibilities:

(i) *p* is an existing bus in the partial network and *q* is a new bus; in this case *p-q* is a **branch** added to the p-network as shown in Fig 1a, and

(ii) both *p* and *q* are buses existing in the partial network; in this case *p-q* is a **link** added to the p-network as shown in Fig 1b.

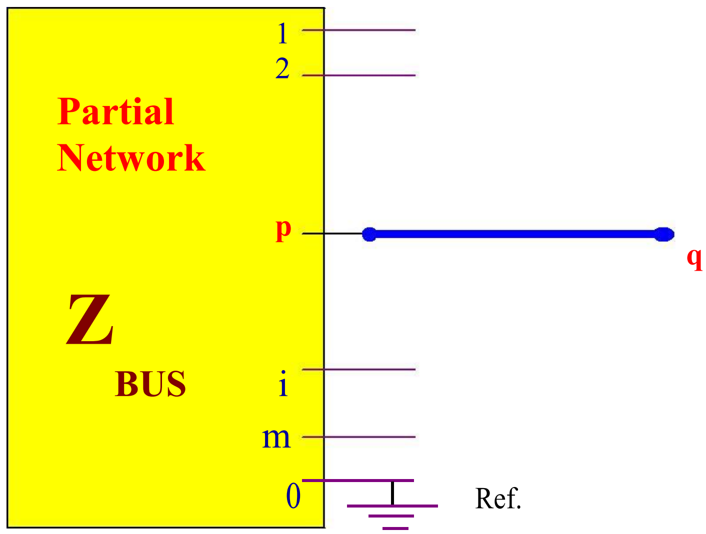


Fig 1a. Addition of branch p-q

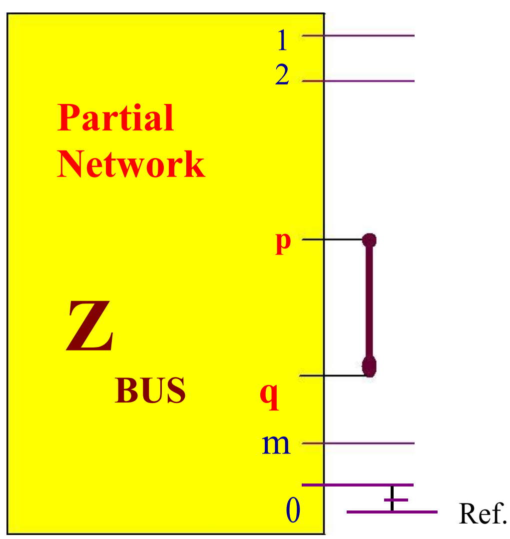


Fig 1b. Addition of link p-q

If the added element ia a branch, p-q, then the new bus impedance matrix would be of order m+1, and the analysis is confined to finding only the elements of the new row and column (corresponding to bus-q) introduced into the original matrix.

If the added element ia a link, p-q, then the new bus impedance matrix will remain unaltered with regard to its order. However, all the elements of the original matrix are updated to take account of the effect of the link added.

ADDITION OF A BRANCH

Consider now the performance equation of the network in impedance form with the added branch p-q, given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{1p} & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & Z_{2p} & Z_{2m} & Z_{2q} \\ & & & & \\ Z_{p1} & Z_{p2} & Z_{pp} & Z_{pm} & Z_{pq} \\ & & & & \\ Z_{m1} & Z_{m2} & Z_{mp} & Z_{mm} & Z_{mq} \\ Z_{q1} & Z_{q2} & Z_{qp} & Z_{qm} & Z_{qq} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_q \end{bmatrix} \qquad (11)$$

It is assumed that the added branch *p-q* is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have

Vector y_{pq-rs} is not equal to zero and $\mathbf{Z_{ij}= Z_{ji}}$ $\forall$ i,j=1,2,…m,q (12)

<u>To find Z_{qi}:</u>

The elements of last row-q and last column-q are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.2. Since all other bus currents are zero, we have from (11) that

$E_k = Z_{ki} I_i = Z_{ki}$ $\quad \forall$ k = 1, 2,…i…....p,….m, q (13)

Hence, $E_q = Z_{qi}$; $E_p = Z_{pi}$ ………

Also, $E_q=E_p - v_{pq}$; so that $Z_{qi} = Z_{pi} - v_{pq}$ $\forall$ i =1, 2,…i…....p,….m, $\neq$q (14)

<u>To find v_{pq}:</u>

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pq} \\ \bar{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pq,pq} & \bar{y}_{pq,rs} \\ \bar{y}_{rs,pq} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pq} \\ \bar{v}_{rs} \end{bmatrix} \tag{15}$$

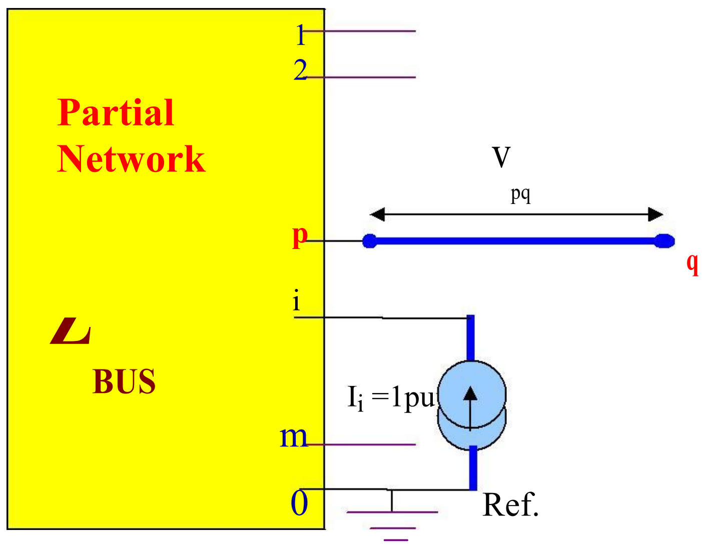


Fig.2 Calculation for Z_{qi}

where i_{pq} is current through element *p-q*

$\bar{i}_{rs}$ is vector of currents through elements of the partial network

v_{pq} is voltage across element *p-q*

$y_{pq,pq}$ is self – admittance of the added element

$\bar{y}_{pq,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

$\bar{v}_{rs}$ is vector of voltage across elements of partial network.

$\bar{y}_{rs,pq}$ is transpose of $\bar{y}_{pq,rs}$.

$\bar{y}_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-q, is zero, $i_{pq} = 0$. We thus have from (15),

$$i_{pq} = y_{pq,pq} v_{pq} + \bar{y}_{pq,rs} \bar{v}_{rs} = 0 \tag{16}$$

Solving, $v_{pq} = -\frac{\bar{y}_{pq,rs}\,\bar{v}_{rs}}{y_{pq,pq}}$ or

$$v_{pq} = -\frac{\bar{y}_{pq,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pq,pq}} \qquad (17)$$

Using (13) and (17) in (14), we get

$$Z_{qi} = Z_{pi} + \frac{\bar{y}_{pq,rs}\left(\bar{Z}_{ri} - \bar{Z}_{si}\right)}{y_{pq,pq}} \qquad i = 1,2......m;\ i \neq q \qquad (18)$$

To find zqq:

The element Z_{qq} can be computed by injecting a current of 1pu at bus-q, I_q = 1.0 pu. As before, we have the relations as under:

$E_k = Z_{kq} I_q = Z_{kq} \quad \forall$ k = 1, 2,…i…....p,….m, q (19)

Hence, $E_q = Z_{qq}$; $E_p = Z_{pq}$; Also, $E_q = E_p - v_{pq}$; so that $Z_{qq} = Z_{pq} - v_{pq}$ (20)

Since now the current in the added element is $i_{pq} = -I_q = -1.0$, we have from (15)

$$i_{pq} = y_{pq,pq}\, v_{pq} + \bar{y}_{pq,rs}\bar{v}_{rs} = -1$$

Solving, $v_{pq} = -1 + \frac{\bar{y}_{pq,rs}\,\bar{v}_{rs}}{y_{pq,pq}}$

$$v_{pq} = -1 + \frac{\bar{y}_{pq,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pq,pq}} \qquad (21)$$

Using (19) and (21) in (20), we get

$$Z_{qq} = Z_{pq} + \frac{1 + \bar{y}_{pq,rs}\left(\bar{Z}_{rq} - \bar{Z}_{sq}\right)}{y_{pq,pq}} \qquad (22)$$

Special Cases

The following special cases of analysis concerning Z_{BUS} building can be considered with respect to the addition of branch to a p-network.

Case (a): If there is no mutual coupling then elements of $\bar{y}_{pq,rs}$ are zero. Further, if p is the reference node, then E_p=0. thus,

$Z_{pi} = 0 \qquad i = 1,2......m : i \neq q$

And $Z_{pq} = 0$.

Hence, from (18) (22) $Z_{qi} = 0 \qquad i = 1,2.......m;\ i \neq q$

And $Z_{qq} = z_{pq,pq}$ \ (23)

Case (b): If there is no mutual coupling and if p is not the ref. bus, then, from (18) and (22), we again have,

$$Z_{qi} = Z_{pi}\,, \quad i = 1,2....m;\ i \neq q$$

$$Z_{qq} = Z_{pq} + z_{pq,pq} \qquad (24)$$

ADDITION OF A LINK

Consider now the performance equation of the network in impedance form with the added link p-l, (p-l being a fictitious branch and l being a fictitious node) given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_l \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{1p} & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & Z_{2p} & Z_{2m} & Z_{2q} \\ & & & & \\ Z_{p1} & Z_{p2} & Z_{pp} & Z_{pm} & Z_{pq} \\ Z_{m1} & Z_{m2} & Z_{mp} & Z_{mm} & Z_{mq} \\ Z_{l1} & Z_{l2} & Z_{li} & Z_{lm} & Z_{ll} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_l \end{bmatrix} \qquad (25)$$

It is assumed that the added branch *p-q* is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have

Vector y_{pq-rs} is not equal to zero and **$Z_{ij} = Z_{ji}$** $\forall$ i,j=1,2,…m,l. (26)

<u>To find Z_{li}:</u>

The elements of last row-l and last column-l are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.3. Further, the current in the added element is made zero by connecting a voltage source, e_l in series with element p-q, as shown. Since all other bus currents are zero, we have from (25) that

$E_k = Z_{ki} I_i = Z_{ki}$ $\quad \forall$ k = 1, 2,…i…...p,….m, l (27)

Hence, $e_l = E_l = Z_{li}$; $E_p = Z_{pi}$; $E_p = Z_{pi}$ ………

Also, $e_l = E_p - E_q - v_{pq}$;

So that $Z_{li} = Z_{pi} - Z_{qi} - v_{pq}$ $\forall$ i=1,2,…i…p,...q,….m, ≠l (28)

To find v_{pq}:

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pl} \\ \bar{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pl,pl} & \bar{y}_{pl,rs} \\ \bar{y}_{rs,pl} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pl} \\ \bar{v}_{rs} \end{bmatrix} \qquad (29)$$

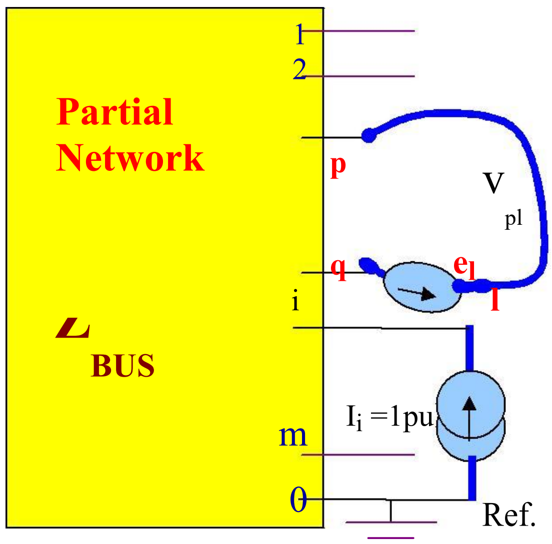


Fig.3 Calculation for Z_{li}

where i_{pl} is current through element *p-q*

$\bar{i}_{rs}$ is vector of currents through elements of the partial network

v_{pl} is voltage across element *p-q*

$y_{pl,pl}$ is self – admittance of the added element

$\bar{y}_{pl,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

$\bar{v}_{rs}$ is vector of voltage across elements of partial network.

$\bar{y}_{rs,pl}$ is transpose of $\bar{y}_{pl,rs}$.

$\bar{y}_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-l, is zero, $i_{pl} = 0$. We thus have from (29),

$$i_{pl} = y_{pl,pl} v_{pl} + \bar{y}_{pl,rs} \bar{v}_{rs} = 0 \quad (30)$$

Solving, $v_{pl} = -\frac{\bar{y}_{pl,rs} \bar{v}_{rs}}{y_{pl,pl}}$ or

$$v_{pl} = -\frac{\bar{y}_{pl,rs} \left(\bar{E}_r - \bar{E}_s \right)}{y_{pl,pl}} \quad (31)$$

However,

$$\bar{y}_{pl,rs} = \bar{y}_{pq,rs}$$

And
$$y_{pl,pl} = y_{pq,pq} \quad (32)$$

Using (27), (31) and (32) in (28), we get

$$Z_{li} = Z_{pi} - Z_{qi} + \frac{\bar{y}_{pq,rs} \left(\bar{Z}_{ri} - \bar{Z}_{si} \right)}{y_{pq,pq}} \qquad i = 1,2......m; i \neq l \quad (33)$$

To find Z_{ll}:

The element Z_{ll} can be computed by injecting a current of 1pu at bus-l, $I_l = 1.0$ pu.

As before, we have the relations as under:

$E_k = Z_{kl} I_l = Z_{kl} \quad \forall$ k = 1, 2,…i…....p,…q,….m, l (34)

Hence, $e_l = E_l = Z_{ll}$; $E_p = Z_{pl}$;

Also, $e_l = E_p - E_q - v_{pl}$;

So that $Z_{ll} = Z_{pl} - Z_{ql} - v_{pl} \quad \forall$ i=1,2,…i….p,...q,….m, ≠l (35)

Since now the current in the added element is $i_{pl} = -I_l = -1.0$, we have from (29)

$$i_{pl} = y_{pl,pl} v_{pl} + \bar{y}_{pl,rs} \bar{v}_{rs} = -1$$

Solving, $v_{pl} = -1 + \frac{\bar{y}_{pl,rs} \bar{v}_{rs}}{y_{pl,pl}}$

$$v_{pl} = -1 + \frac{\bar{y}_{pl,rs} \left(\bar{E}_r - \bar{E}_s \right)}{y_{pl,pl}} \quad (36)$$

However,

$$\bar{y}_{pl,rs} = \bar{y}_{pq,rs}$$

And
$$y_{pl,pl} = y_{pq,pq} \quad (37)$$

Using (34), (36) and (37) in (35), we get

$$Z_{ll} = Z_{pl} - Z_{ql} + \frac{1 + \bar{y}^{pq,rs}\left(\bar{Z}_{rl} - \bar{Z}_{sl}\right)}{y_{pq,pq}} \tag{38}$$

Special Cases Contd….

The following special cases of analysis concerning Z_{BUS} building can be considered with respect to the addition of link to a p-network.

Case (c): If there is no mutual coupling, then elements of $\bar{y}_{pq,rs}$ are zero. Further, if *p* is the reference node, then E_p=0. thus,

$$Z_{li} = -Z_{qi}\,, \quad i = 1,2....m; i \neq l$$

$$Z_{ll} = -Z_{ql} + z_{pq,pq} \tag{39}$$

From (39), it is thus observed that, when a link is added to a ref. bus, then the situation is similar to adding a branch to a fictitious bus and hence the following steps are followed:

1. The element is added similar to addition of a branch (case-b) to obtain the new matrix of order m+1.
2. The extra fictitious node, l is eliminated using the node elimination algorithm.

Case (d): If there is no mutual coupling, then elements of $\bar{y}_{pq,rs}$ are zero. Further, if *p* is not the reference node, then

$Z_{li} = Z_{pi} - Z_{qi}$

$Z_{ll} = Z_{pl} – Z_{ql} – z_{pq,pq}$
$= Z_{pp} + Z_{qq} – 2\, Z_{pq} + z_{pq,pq}$ (40)

UNIT-II
POWER FLOW STUDIES

MODIFICATION OF Z_{BUS} FOR NETWORK CHANGES

An element which is not coupled to any other element can be removed easily. The Z_{bus} is modified as explained in sections above, by adding in parallel with the element (to be removed), a link whose impedance is equal to the negative of the impedance of the element to be removed. Similarly, the impedance value of an element which is not coupled to any other element can be changed easily. The Z_{bus} is modified again as explained in sections above, by adding in parallel with the element (whose impedance is to be changed), a link element of impedance value chosen such that the parallel equivalent impedance is equal to the desired value of impedance. When mutually coupled elements are removed, the Z_{bus} is modified by introducing appropriate changes in the bus currents of the original network to reflect the changes introduced due to the removal of the elements.

Examples on Z_{BUS} building

Example 1: *For the positive sequence network data shown in table below, obtain Z_{BUS} by building procedure.*

Sl. No.	p-q (nodes)	Pos. seq. reactance in pu
1	0-1	0.25
2	0-3	0.20
3	1-2	0.08
4	2-3	0.06

Solution:
The given network is as shown below with the data marked on it. Assume the elements to be added as per the given sequence: 0-1, 0-3, 1-2, and 2-3.

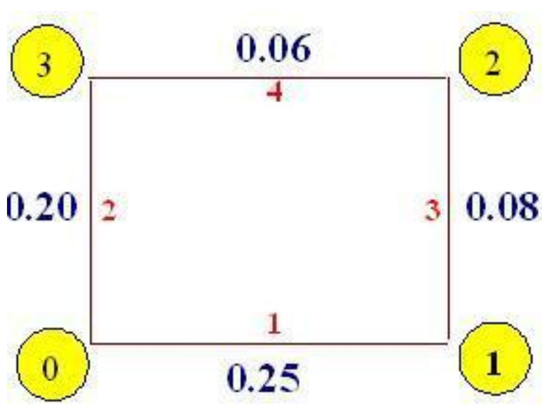
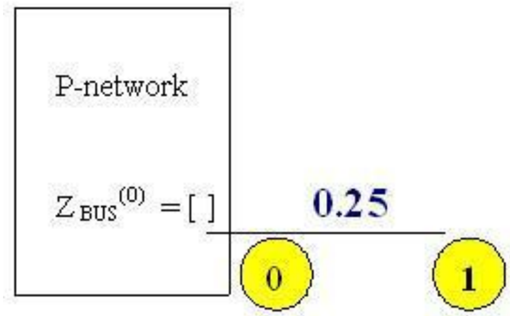


Fig. E1: Example System

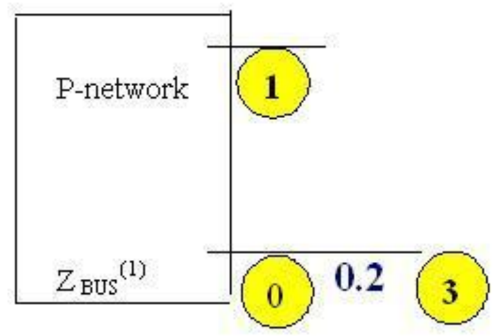
Consider building Z_{BUS} as per the various stages of building through the consideration of the corresponding partial networks as under:

Step-1: Add element–1 of impedance 0.25 pu from the external node-1 (q=1) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;



$$Z_{BUS}^{(1)} = \begin{array}{c|c|} & 1 \\ \hline 1 & 0.25 \\ \hline \end{array}$$

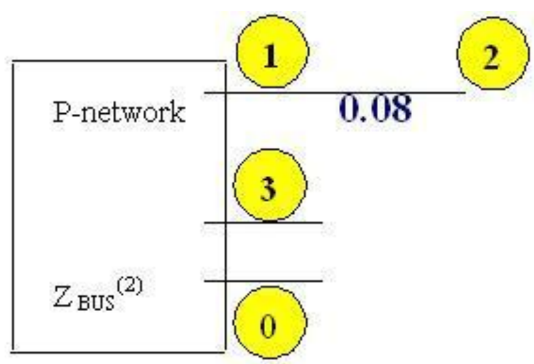
Step-2: Add element–2 of impedance 0.2 pu from the external node-3 (q=3) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;



$Z_{BUS}^{(2)}$ =

	1	3
1	0.25	0
3	0	0.2

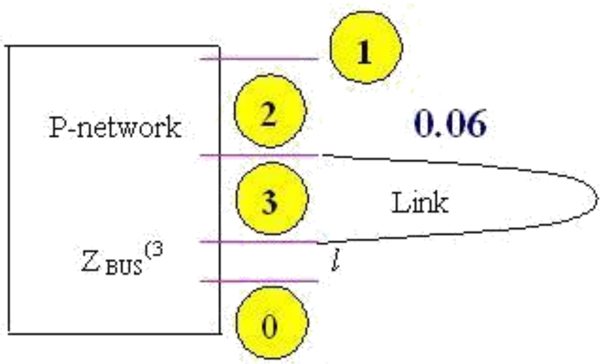
Step-3: Add element–3 of impedance 0.08 pu from the external node-2 (q=2) to internal node-1 (p=1). (Case-b), as shown in the partial network;



$Z_{BUS}^{(3)}$ =

	1	3	2
1	0.25	0	0.25
3	0	0.2	0
2	0.25	0	0.33

Step-4: Add element–4 of impedance 0.06 pu between the two internal nodes, node-2 (p=2) to node-3 (q=3). (Case-d), as shown in the partial network;



$Z_{BUS}^{(4)}$ =

	1	3	2	l
1	0.25	0	0.25	0.25
3	0	0.2	0	-0.2
2	0.25	0	0.33	0.33
l	0.25	-0.2	0.33	0.59

The fictitious node l is eliminated further to arrive at the final impedance matrix as under:

$Z_{BUS}^{(final)}$ =

	1	3	2
1	0.1441	0.0847	0.1100
3	0.0847	0.1322	0.1120
2	0.1100	0.1120	0.1454

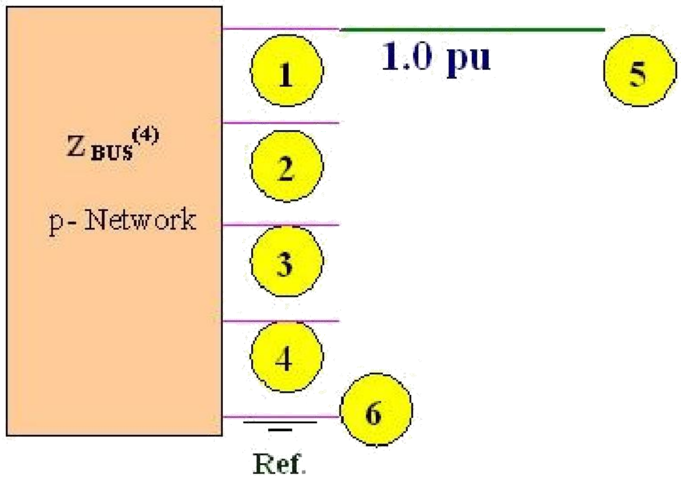
Example 2: *The Z_{BUS} for a 6-node network with bus-6 as ref. is as given below. Assuming the values as pu reactances, find the topology of the network and the parameter values of the elements involved. Assume that there is no mutual coupling of any pair of elements.*

Z_{BUS} =

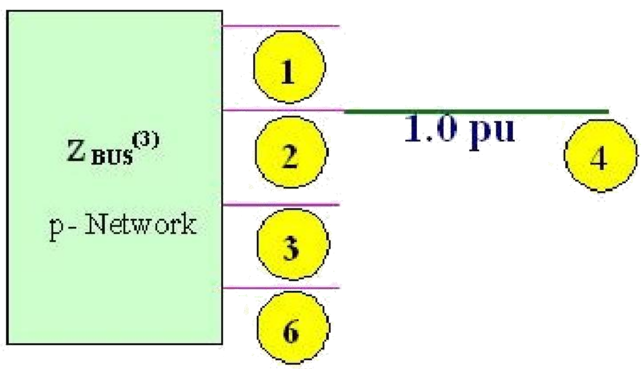
	1	2	3	4	5
1	2	0	0	0	2
2	0	2	0	2	0
3	0	0	2	0	0
4	0	2	0	3	0
5	2	0	0	0	3

Solution:

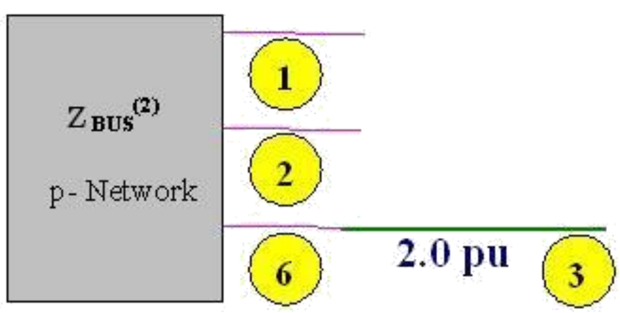
The specified matrix is so structured that by its inspection, we can obtain the network by backward analysis through the various stages of Z_{BUS} building and p-networks as under:



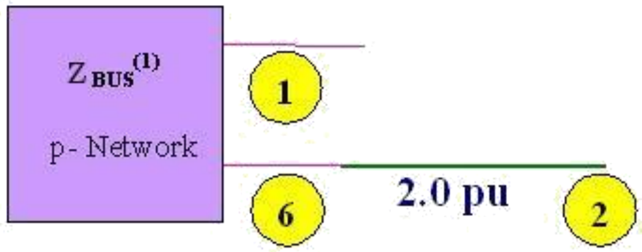
Z BUS(4)
p- Network
1.0 pu
1
2
3
4
5
6
Ref.

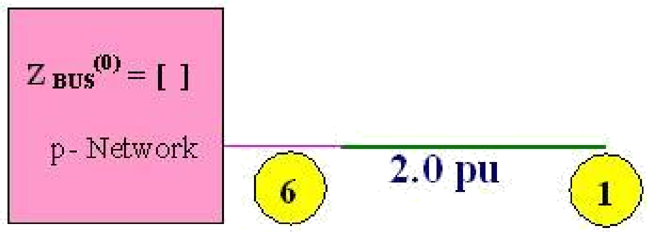


Z BUS(3)
p- Network
1.0 pu
1
2
3
4
6



Z BUS(2)
p- Network
2.0 pu
1
2
6
3





Thus the final network is with 6 nodes and 5 elements connected as follows with the impedance values of elements as indicated.

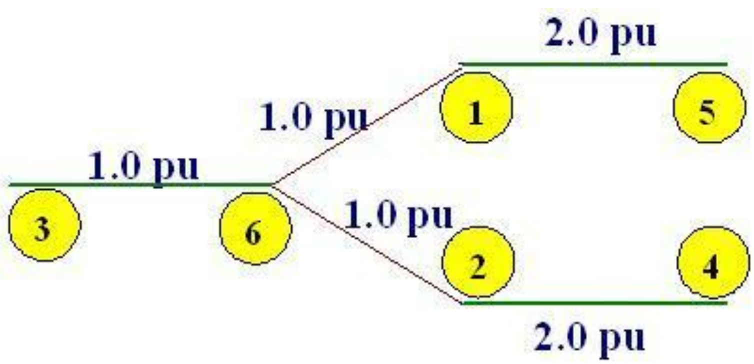
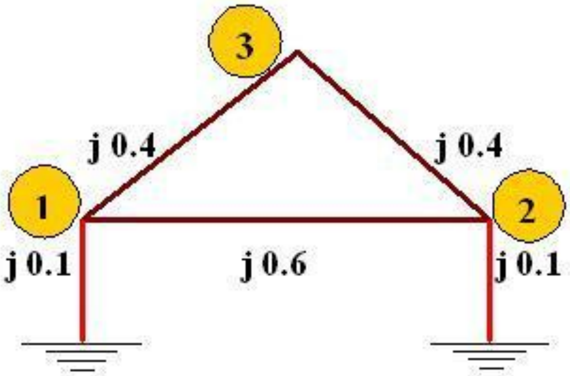


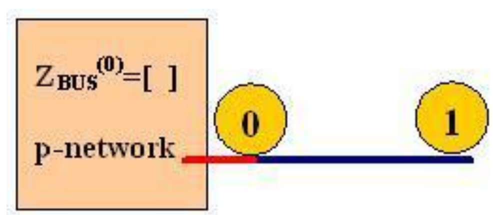
Fig. E2: Resultant network of example-2

Example 3: *Construct the bus impedance matrix for the system shown in the figure below by building procedure. Show the partial networks at each stage of building the matrix. Hence arrive at the bus admittance matrix of the system. How can this result be verified in practice?*



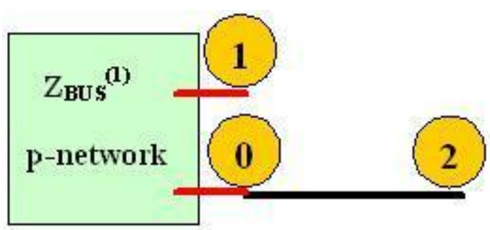
Solution: The specified system is considered with the reference node denoted by node-0. By its inspection, we can obtain the bus impedance matrix by building procedure by following the steps through the p-networks as under:

Step1: Add branch 1 between node 1 and reference node. (q =1, p = 0)



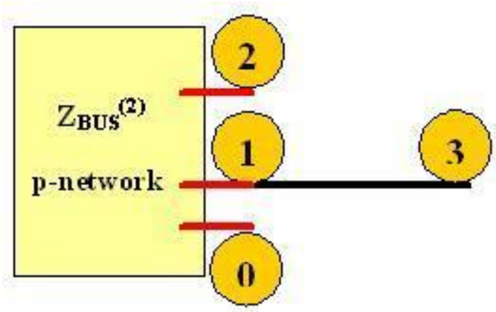
$$Z_{bus}^{(1)} = \begin{matrix} & 1 \\ 1 & [j0.1] \end{matrix}$$

Step2: Add branch 2, between node 2 and reference node. (q = 2, p = 0).



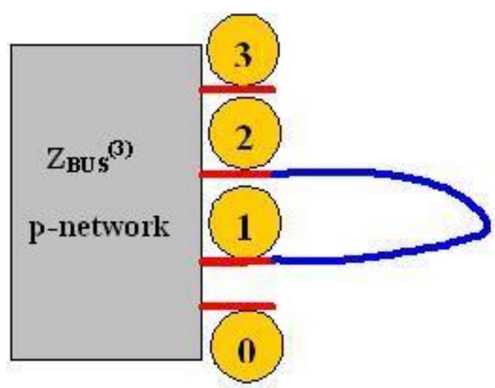
$$Z_{Bus} = \begin{array}{c} \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} j0.1 & 0 \\ 0 & j0.15 \end{bmatrix} \end{array}$$

Step3: Add branch 3, between node 1 and node 3 (p = 1, q = 3)



$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j0.1 & 0 & j0.1 \\ 0 & j0.15 & 0 \\ j0.1 & 0 & j0.5 \end{bmatrix} \end{array}$$

Step 4: Add element 4, which is a link between node 1 and node 2. (p = 1, q = 2)



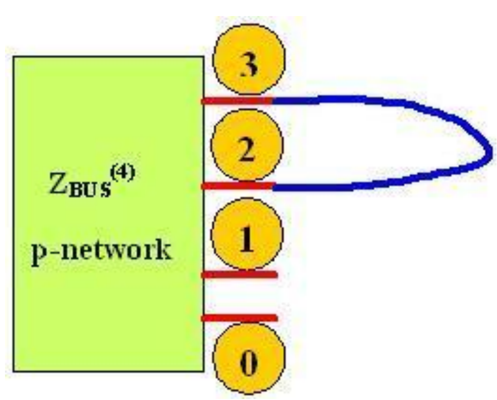
$$
Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array}
\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\
\begin{bmatrix} j0.1 & 0 & j0.1 & j0.1 \\ 0 & j0.15 & 0 & -j0.15 \\ j0.1 & 0 & j0.5 & j0.1 \\ j0.1 & -j0.15 & j0.1 & j0.85 \end{bmatrix} \end{array}
$$

Now the extra node-*l* has to be eliminated to obtain the new matrix of step-4, using the algorithmic relation:

$$\mathbf{Y_{ij}^{new} = Y_{ij}^{old} - Y_{in}\,Y_{nj} / Y_{nn}} \quad \forall\ i,j = 1,2,3.$$

$$
Z_{bus} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\
\begin{bmatrix} j0.08823 & j0.01765 & j0.08823 \\ j0.01765 & j0.12353 & j0.01765 \\ j0.08823 & j0.01765 & j0.48823 \end{bmatrix} \end{array}
$$

Step 5: Add link between node 2 and node 3 (p = 2, q=3)



$$Z_{l1} = Z_{21} - Z_{31} = j0.01765 - j0.08823 = -j0.07058$$

$$Z_{l2} = Z_{22} - Z_{32} = j0.12353 - j0.01765 = j0.10588$$

$$Z_{l3} = Z_{23} - Z_{33} = j0.01765 - j0.48823 = -j0.47058$$

$$Z_{ll} = Z_{2l} - Z_{3l} + z_{23,23}$$

$$= j0.10588 - (-j0.47058) + j0.4 = j0.97646$$

Thus, the new matrix is as under:

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\ \begin{bmatrix} j0.08823 & j0.01765 & j0.08823 & -j0.07058 \\ j0.01765 & j0.12353 & j0.01765 & j0.10588 \\ j0.08823 & j0.01765 & j0.48823 & -j0.47058 \\ -j0.07058 & j0.10588 & -j0.47058 & j0.97646 \end{bmatrix} \end{array}$$

Node *l* is eliminated as shown in the previous step:

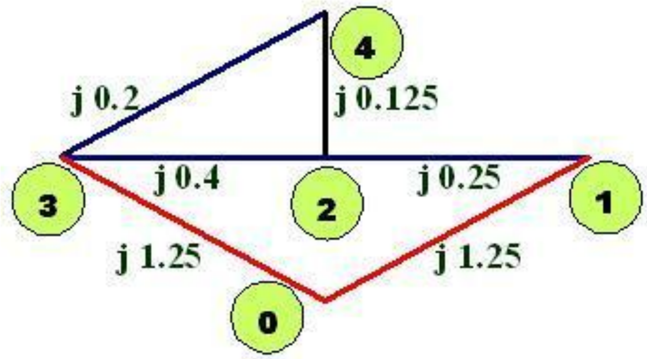
$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j0.08313 & j0.02530 & j0.05421 \\ j0.02530 & j0.11205 & j0.06868 \\ j0.05421 & j0.06868 & j0.26145 \end{bmatrix} \end{array}$$

Further, the bus admittance matrix can be obtained by inverting the bus impedance matrix as under:

$$Y_{bus} = [Z_{bus}]^{-1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} -j14.1667 & j1.6667 & j2.5 \\ j1.6667 & -j10.8334 & j2.5 \\ j2.5 & j2.5 & -j5.0 \end{bmatrix} \end{array}$$

As a check, it can be observed that the bus admittance matrix, Y_{BUS} can also be obtained by the rule of inspection to arrive at the same answer.

Example 4: Form the bus impedance matrix for the network shown below.



Solution:

Add the elements in the sequence, 0-1, 1-2, 2-3, 0-3, 3-4, 2-4, as per the various steps of building the matrix as under:

Step1: Add element 1, which is a branch between node-1 and reference node.

$$Z_{bus} = \begin{array}{c} \\ 1 \end{array}\begin{array}{c} 1 \\ [j1.25] \end{array}$$

Step2: Add element 2, which is a branch between nodes 1 and 2.

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} j1.25 & j1.25 \\ j1.25 & j1.5 \end{bmatrix} \end{array}$$

Step3: Add element 3, which is a branch between nodes 2 and 3

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j1.25 & j1.25 & j1.25 \\ j1.25 & j1.5 & j1.5 \\ j1.25 & j1.5 & j1.9 \end{bmatrix} \end{array}$$

Step4: Add element 4, which is a link from node 3 to reference node.

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\ \begin{bmatrix} j1.25 & j1.25 & j1.25 & j1.25 \\ j1.25 & j1.5 & j1.5 & j1.5 \\ j1.25 & j1.5 & j1.9 & j1.9 \\ j1.25 & j1.5 & j1.9 & j3.15 \end{bmatrix} \end{array}$$

Eliminating node *l*,

$$Z_{bus} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j0.75397 & j0.65476 & j0.49603 \\ j0.65476 & j0.78571 & j0.59524 \\ j0.49603 & j0.59524 & j0.75397 \end{bmatrix} \end{array}$$

Step5: Add element 5, a branch between nodes 3 and 4.

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} j0.75397 & j0.65476 & j0.49603 & j0.49603 \\ j0.65476 & j0.65476 & j0.59524 & j0.59524 \\ j0.49603 & j0.59524 & j0.75397 & j0.75397 \\ j0.49603 & j0.59524 & j0.75397 & j0.95397 \end{bmatrix} \end{array}$$

Step 6: Add element 6, a link between nodes 2 & 4.

$$Z_{bus} = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ l \end{array} \begin{bmatrix} j0.75397 & j0.65476 & j0.49603 & j0.49603 & j0.15873 \\ j0.65476 & j0.65476 & j0.59524 & j0.59524 & j0.19047 \\ j0.49603 & j0.59524 & j0.75397 & j0.75397 & -j0.15873 \\ j0.49603 & j0.59524 & j0.75397 & j0.95397 & -j0.35873 \\ j0.15873 & j0.19047 & -j0.15873 & -j0.35873 & j0.67421 \end{bmatrix}$$

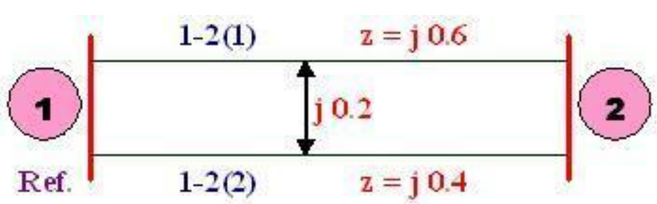
Eliminating node *l* we get the required bus impedance , matrix

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} j0.7166 & j0.6099 & j0.5334 & j0.5805 \\ j0.6099 & j0.7319 & j0.6401 & j0.6966 \\ j0.5334 & j0.6401 & j0.7166 & j0.6695 \\ j0.5805 & j0.6966 & j0.6695 & j0.7631 \end{bmatrix} \end{array}$$

<u>Example 5:</u> Form the bus impedance matrix for the network data given below.

Element	Self Impedance		Mutual Impedance	
	Bus p-q	$Z_{pq,\,pq}$ (pu)	Bus r-s	$Z_{pq,\,rs}$ (pu)
1	1 – 2(1)	j0.6		
2	1 – 2(2)	j0.4	1 – 2(1)	j0.2

Solution:
Let bus-1 be the reference. Add the elements in the sequence 1-2(1), 1-2(2). Here, in the step-2, there is mutual coupling between the pair of elements involved.



Step1: Add element 1 from bus 1 to 2, element 1-2(1). (p=1, q=2, p is the reference node)

$$Z_{bus} = \begin{array}{c} \\ 2 \end{array} \begin{array}{c} 2 \\ [j0.6] \end{array}$$

Step2: Add element 2, element 1-2(2), which is a link from bus1 to 2, mutually coupled with element 1, 1-2(1).

$$Z_{bus} = \begin{array}{c} \\ 2 \\ l \end{array} \begin{array}{c} \begin{array}{cc} 2 & l \end{array} \\ \begin{bmatrix} j0.6 & Z_{2l} \\ Z_{l2} & Z_{ll} \end{bmatrix} \end{array}$$

Where,

$$Z_{2l} = Z_{l2} = -Z_{22} + \frac{y_{12(2),12(1)}(Z_{12} - Z_{22})}{y_{12(2),12(2)}}$$

$Z_{12} = Z_{1l} = 0$ (as bus 1 is reference)

Consider the primitive impedance matrix for the two elements given by

$$[z] = \begin{array}{c} \\ 1-2(1) \\ 1-2(2) \end{array} \begin{array}{c} \begin{array}{cc} \text{1-2(1)} & \text{1-2(2)} \end{array} \\ \begin{bmatrix} j0.6 & j0.2 \\ j0.2 & j0.4 \end{bmatrix} \end{array}$$

Thus the primitive admittance matrix is obtained by taking the inverse of [z] as

$$[y] = \begin{array}{c} \\ 1-2(1) \\ 1-2(2) \end{array} \begin{array}{c} \begin{array}{cc} \text{1-2(1)} & \text{1-2(2)} \end{array} \\ \begin{bmatrix} -j2.0 & j1.0 \\ j1.0 & -j3.0 \end{bmatrix} \end{array}$$

Thus,

$y_{12(1),12(2)} = j1.0; \quad y_{12(2),12(2)} = -j3.0$

So that we have,

$$Z_{2l} = Z_{l2} = -j0.6 + \frac{(j1.0)(-j0.6)}{-j3.0} = -j0.4$$

$$Z_{ll} = -Z_{2l} + \frac{1 + y_{12(2),12(1)}(Z_{1l} - Z_{2l})}{y_{12(2),12(2)}} = j0.4 + \frac{1 + (j1.0)(j0.4)}{-j3.0} = j0.6$$

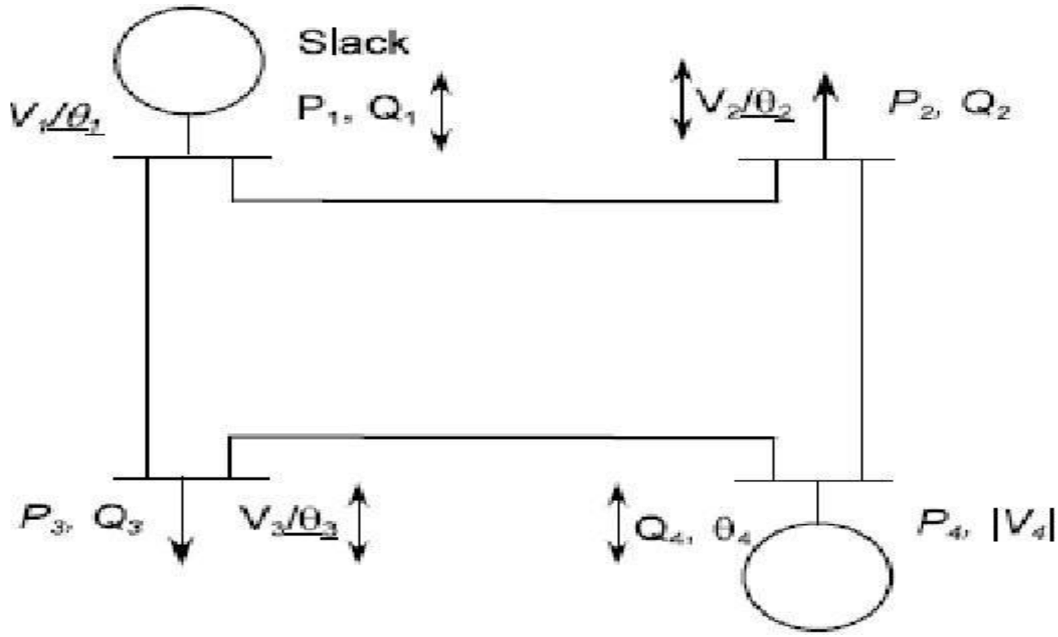
$$Z_{bus} = \begin{array}{c} \\ 2 \\ l \end{array} \begin{array}{c} \begin{array}{cc} 2 & l \end{array} \\ \begin{bmatrix} j0.6 & -j0.4 \\ -j0.4 & j0.6 \end{bmatrix} \end{array}$$

Thus, the network matrix corresponding to the 2-node, 1-bus network given, is obtained after eliminating the extra node-l as a single element matrix, as under:

$$Z_{bus} = 2\overset{2}{[j0.3333]}$$

INTRODUCTION

In a three phase ac power system active and reactive power flows from the generating station to the load through different networks buses and branches. The flow of active and reactive power is called power flow or load flow. Power flow studies provide asystematic mathematical approach for determination of various bus voltages, there phase angle active and reactive power flows through different branches, generators and loads under steady state condition. Power flow analysis is used to determine the steady state operating condition of a power system. Power flow analysis is widely used by power distribution professional during the planning and operation of power distribution system.



There three methods for load flow studies mainly

1. Gauss siedel method
2. Newton raphson method
3. Fast decoupled method.

a. OBJECTIVE OF LOAD FLOW STUDY

i. Power flow analysis is very important in planning stages of new networks or addition to existing ones like adding new generator sites, meeting increase load demand and locating new transmission sites.
ii. The load flow solution gives the nodal voltages and phase angles and hence the power injection at all the buses and power flows through interconnecting power channels.
iii. It is helpful in determining the best location as well as optimal capacity of proposed generating station, substation and new lines.
iv. It determines the voltage of the buses. The voltage level at the certain buses must be kept within the closed tolerances.
v. System transmission loss minimizes.
vi. Economic system operation with respect to fuel cost to generate all the power needed
vii. The line flows can be known. The line should not be

overloaded, it means, we should not operate the close to their stability or thermal limits.

BUS CLASSIFICATION

A bus is a node at which one or many lines, one or many loads and generators are connected. In a power system each node or bus is associated with 4 quantities, such as magnitude of voltage, phage angle of voltage, active or true power and reactive power in load flow problem two out of these 4 quantities are specified and remaining 2 are required to be determined through the solution of equation. Depending on the quantities that have been specified, the buses are classified into 3 categories.

VARIABLES AND BUS CLASSIFICATION

Buses are classified according to which two out of the four variables are specified

- **Load bus**: No generator is connected to the bus. At this bus the real and reactive power are specified.it is desired to find out the volatage magnitude and phase angle through load flow solutions.It is required to specify only Pd and Qd at such bus as at a load bus voltage can be allowed to vary within the permissible values.

- **Generator bus or voltage controlled bus**: Here the voltage magnitude corresponding to the generator voltage and real power Pg corresponds to its rating are specified.It is required to find out the reactive power generation Qg and phase angle of the bus voltage.

- **Slack (swing) bus**: For the Slack Bus, it is assumed that the voltage magnitude $|V|$ and voltage phase Θ are known,whereas real and reactive powers Pg and Qg are obtained through the load flow solution.

UNIT-III
SHORT CIRCUIT ANALYSIS

Power System Fault Analysis

Introduction

The fault analysis of a power system is required in order to provide information for the selection of switchgear, setting of relays and stability of system operation. A power system is not static but changes during operation (switching on or off of generators and transmission lines) and during planning (addition of generators and transmission lines). Thus fault studies need to be routinely performed by utility engineers (such as in the CEB).

Faults usually occur in a power system due to either insulation failure, flashover, physical damage or human error. These faults, may either be three phase in nature involving all three phases in a symmetrical manner, or may be asymmetrical where usually only one or two phases may be involved. Faults may also be caused by either short-circuits to earth or between live conductors, or may be caused by broken conductors in one or more phases. Sometimes simultaneous faults may occur involving both short-circuit and broken- conductor faults (also known as open-circuit faults).

Balanced three phase faults may be analysed using an equivalent single phase circuit. With asymmetrical three phase faults, the use of symmetrical components help to reduce the complexity of the calculations as transmission lines and components are by and large symmetrical, although the fault may be asymmetrical.

Fault analysis is usually carried out in per-unit quantities (similar to percentage quantities) as they give solutions which are somewhat consistent over different voltage and power ratings, and operate on values of the order of unity.

In the ensuing sections, we will derive expressions that may be used in computer simulations by the utility engineers.

Equivalent Circuits - Single phase and Equivalent Single Phase Circuits

In a balanced three phase circuit, since the information relating to one single phase gives the information relating to the other two phases as well, it is sufficient to do calculations in a single phase circuit. There are two common forms used. These are (i) to take any one single phase of the three phase circuit and (ii) to take an equivalent single phase circuit to represent the full three phase circuit.

Single Phase Circuit

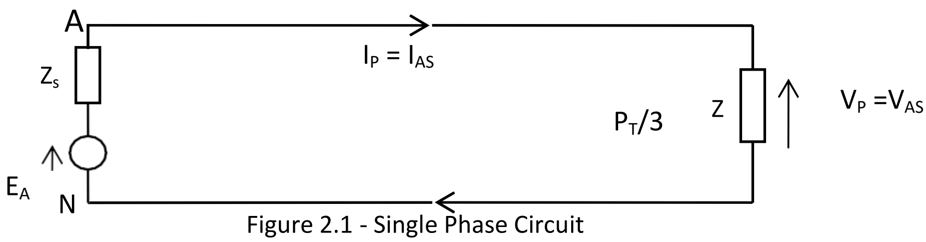


Figure 2.1 - Single Phase Circuit

S

Figure 2.1 shows one single phase "***AN***" of the three phase circuit "***ABC N***". Since the system is balanced, there is no current in the neutral, and there is no potential drop across the neutral wire. Thus the star point **"S"** of the system would be at the same potential as the neutral point **"N"**. Also, the line current is the same as the phase current, the line voltage is √3 times the phase voltage, and the total power is 3 times the power in a single phase.

$$I = I_P = I_L,\ V = V_P = V_L/\sqrt{3} \text{ and } S = S_P = S_T/3$$

Working with the single phase circuit would yield single phase quantities, which can then be converted to three phase quantities using the above conversions.

Equivalent Single Phase Circuit

Of the parameters in the single phase circuit shown in figure 2.1, the Line Voltage and the Total Power (rather than the Phase Voltage and one-third the Power) are the most important quantities. It would be useful to have these quantities obtained directly from the circuit rather than having conversion factors of √3 and 3 respectively. This is achieved in the Equivalent Single Phase circuit, shown in figure 2.2, by multiplying the voltage by a factor of √3 to give Line Voltage directly.

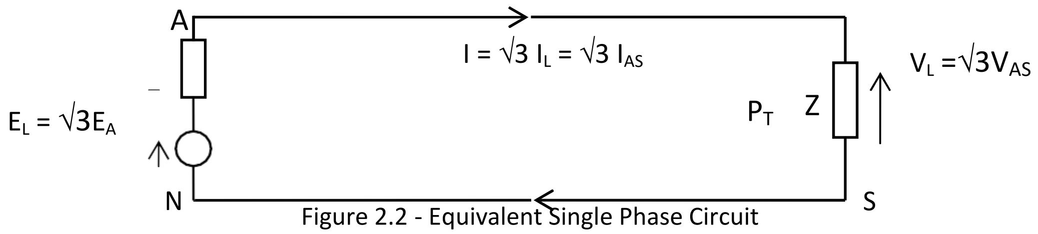


Figure 2.2 - Equivalent Single Phase Circuit

The Impedance remains as the per-phase impedance. However, the Line Current gets artificially amplified by a factor of √3. This also increases the power by a factor of $(\sqrt{3})^2$, which is the required correction to get the total power.

Thus, working with the Equivalent single phase circuit would yield the required three phase quantities directly, other than the current which would be $\sqrt{3}\, I_L$.

Revision of Per Unit Quantities

Per unit quantities, like percentage quantities, are actually fractional quantities of a reference quantity. These have a lot of importance as per unit quantities of parameters tend to have similar values even when the system voltage and rating change drastically. The per unit system permits multiplication and division in addition to addition and subtraction without the requirement of a correction factor (when percentage quantities are multiplied or divided additional factors of 0.01 or100 must be brought in, which are not in the original equations, to restore the percentage values). Per-unit values are written with "pu" after the value.

For power, voltage, current and impedance, the per unit quantity may be obtained by dividing by the respective base of that quantity.

$$S_{pu} = \frac{S}{S_{base}} \qquad V_{pu} = \frac{V}{V_{base}} \qquad I_{pu} = \frac{I}{I_{base}} \qquad Z_{pu} = \frac{Z}{Z_{base}}$$

Expressions such as Ohm's Law can be applied for per unit quantities as well. Since Voltage, Current, Impedance and Power are related, only two Base or reference quantities can be independently defined. The Base quantities for the other two can be derived there from. Since Power and Voltage are the most often specified, they are usually chosen to define the independent base quantities.

Calculation for Single Phase Systems

If VA_{base} and V_{base} are the selected base quantities of *power* (complex, active or reactive) and *voltage* respectively, then

Base current $$I_{base} = \frac{V_{base} I_{base}}{V_{base}} = \frac{VA_{base}}{V_{base}}$$

Base Impedance $$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V^2{}_{base}}{I_{base} V_{base}} = \frac{V^2{}_{base}}{VA_{base}}$$

In a power system, voltages and power are usually expressed in *kV* and *MVA*, thus it is usual to select an MVA_{base} and a kV_{base} and to express them as

$$\text{Base current} \qquad I_{base} = \frac{MVA_{base}}{kV_{base}} \quad \text{in } kA, \qquad [\because 10^6/10^3 = 10^3]$$

$$\text{Base Impedance} \qquad Z_{base} = \frac{kV_{base}^2}{MVA_{base}} \quad \text{in } \Omega, \qquad [\because (10^3)^2/10^6 = 1]$$

In these expressions, all the quantities are single phase quantities.

Calculations for Three Phase Systems

In three phase systems the line voltage and the total power are usually used rather than the single phase quantities. It is thus usual to express base quantities in terms of these.

If $VA_{3\phi base}$ and V_{LLbase} are the base three-phase power and line-to-line voltage respectively,

$$\text{Base current} \qquad I_{base} = \frac{VA_{base}}{V_{base}} = \frac{3VA_{base}}{3V_{base}} = \frac{VA_{3\phi base}}{\sqrt{3}V_{LLbase}}$$

$$\text{Base Impedance} \qquad Z_{base} = \frac{V^2{}_{base}}{VA_{base}} = \frac{\left(\sqrt{3}\right)^2 V^2{}_{base}}{3VA_{base}} = \frac{V^2{}_{LLbase}}{VA_{3\phi base}}$$

$$\text{Base current} \qquad I_{base} = \frac{MVA_{3\phi base}}{\sqrt{3}kV_{LLbase}} \quad \text{in } kA$$

$$\text{Base Impedance} \qquad Z_{base} = \frac{kV_{LLbase}^2}{MVA_{3\phi base}} \quad \text{in } \Omega$$

It is to be noted that while the base impedance for the three phase can be obtained directly from the $VA_{3\phi base}$ and V_{LLbase} (or $MVA_{3\phi base}$ and kV_{LLbase}) without the need of any additional factors, the calculation of base current needs an additional factor of √3. However this is not usually a problem as the value of current is rarely required as a final answer in power systems calculations, and intermediate calculations can be done with a variable $\sqrt{3}I_{base}$.

Thus in three phase, the calculations of per unit quantities becomes

$$S_{pu} = \frac{S_{actual}(MVA)}{MVA_{3\phi base}},$$

$$V_{pu} = \frac{V_{actual}(kV)}{kV_{LLbase}},$$

$$I_{pu} = I_{actual}(kA).\frac{\sqrt{3}kV_{LLbase}}{MVA_{3\phi base}} \quad \text{and}$$

$$Z_{pu} = Z_{actual}(\Omega).\frac{MVA_{3\phi base}}{kV^2_{LLbase}}$$

P and Q have the same base as S, so that

$$P_{pu} = \frac{P_{actual}(MW)}{MVA_{3\phi base}}, \quad Q_{pu} = \frac{Q_{actual}(\text{Mvar})}{MVA_{3\phi base}}$$

Similarly, R and X have the same base as Z, so that

$$R_{pu} = R_{actual}(\Omega).\frac{MVA_{3\phi base}}{kV^2_{LLbase}}, \quad X_{pu} = X_{actual}(\Omega).\frac{MVA_{3\phi base}}{kV^2_{LLbase}}$$

The *power factor* remains unchanged in *per unit*.

Conversions from one Base to another

It is usual to give data in per unit to its own rating [ex: The manufacturer of a certain piece of equipment, such as a transformer, would not know the exact rating of the power system in which the equipment is to be used. However, he would know the rating of his equipment]. As different components can have different ratings, and different from the system rating, it is necessary to convert all quantities to a common base to do arithmetic or algebraic operations. Additions, subtractions, multiplications and divisions will give meaningful results only if they are to the same base. This can be done for three phase systems as follows.

$$S_{puNew} = S_{puGiven}.\frac{MVA_{3\phi baseGiven}}{MVA_{3\phi baseNew}}, \quad V_{puNew} = V_{puGiven}.\frac{kV_{LLbaseGiven}}{kV_{LLbaseNew}}, \quad \text{and}$$

$$Z_{pu} = Z_{puGiven}.\frac{MVA_{3\phi baseNew}}{MVA_{3\phi baseGiven}}.\frac{kV^2_{LLbaseGiven}}{kV^2_{LLbaseNew}}$$

Example:

A 200 MVA, 13.8 kV generator has a reactance of 0.85 p.u. and is generating 1.15 pu voltage. Determine (a) the actual values of the line voltage, phase voltage and reactance, and (b) the corresponding quantities to a new base of 500 MVA, 13.5 kV.

(a) Line voltage = 1.15 * 13.8 = 15.87 kV

Phase voltage = 1.15 * 13.8/√3 = 9.16 kV

Reactance = 0.85 * 13.8^2/200 = 0.809 Ω

(b) Line voltage = 1.15 * 13.8/13.5 = 1.176 pu

Phase voltage = 1.15 * (13.8/√3)/(13.5/√3) = 1.176 pu

Reactance = 0.85 * $(13.8/13.5)^2$/(500/200) = 0.355 pu

Per Unit Quantities across Transformers

When a transformer is present in a power system, although the power rating on either side of a transformer remains the same, the voltage rating changes, and so does the base voltage across a transformer. [This is like saying that full or 100% (or 1 pu) voltage on the primary of a 220kV/33 kV transformer corresponds to 220 kV while on the secondary it corresponds to 33 kV.] Since the power rating remains unchanged, the impedance and current ratings also change accordingly.

While a common $MVA_{3\phi base}$ can and must be selected for a power system to do analysis, a common V_{LLbase} must be chosen corresponding to a particular location (or side of transformer) and changes in proportion to the nominal voltage ratio whenever a transformer is encountered. Thus the current base changes inversely as the ratio. Hence the impedance base changes as the square of the ratio.

For a transformer with turns ratio N_P:N_S, base quantities change as follows.

Quantity	**Primary Base**	**Secondary Base**
Power (S, P and Q)	S_{base}	S_{base}
Voltage (V)	V_{1base}	$V_{1base} . N_S/N_P = V_{2base}$
Current (I)	$S_{base}/\sqrt{3}V_{1base}$	$S_{base}/\sqrt{3}V_{1base} . N_P/N_S = S_{base}/\sqrt{3}V_{2base}$
Impedance (Z, R and X)	V_{1base}^2/S_{base}	$V_{1base}^2/S_{base} . (N_S/N_P)^2 = V_{2base}^2/S_{base}$

Example :

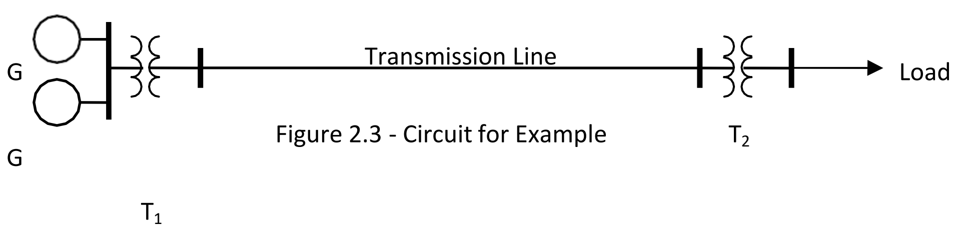


Figure 2.3 - Circuit for Example

In the single line diagram shown in figure 2.3, each three phase generator G is rated at 200 MVA, 13.8 kV and has reactances of 0.85 pu and are generating 1.15 pu. Transformer T_1 is rated at 500 MVA, 13.5 kV/220 kV and has a reactance of 8%. The transmission line has a reactance of 7.8 Ω. Transformer T_2 has a rating of 400 MVA, 220 kV/33 kV and a reactance of 11%. The load is 250 MVA at a power factor of 0.85 lag. Convert all quantities to a common base of 500 MVA, and 220 kV on the line and draw the circuit diagram with values expressed in pu.

Solution:

The base voltage at the generator is (220*13.5/220) 13.5 kV, and on the load side is (220*33/220) 33 kV. [Since we have selected the voltage base as that corresponding to the voltage on that side of the transformer, we automatically get the voltage on the other side of the transformer as the base on that side of the transformer and the above calculation is in fact unnecessary.

Generators G

Reactance of 0.85 pu corresponds 0.355 pu on 500 MVA, 13.5 kV base (see earlier example)

Generator voltage of 1.15 corresponds to 1.176 on 500 MVA, 13.5 kV base

Transformer T_1

Reactance of 8% (or 0.08 pu) remains unchanged as the given base is the same as the new chosen base.

Transmission Line

Reactance of 7.8 Ω corresponds to $7.8 * 500/220^2 = 0.081$ pu

Transformer T_2

Reactance of 11% (0.11 pu) corresponds to 0.11 * 500/400 =

0.1375 pu (voltage base is unchanged and does not come into the

calculations) *Load*

Load of 250 MVA at a power factor of 0.85 corresponds to 250/500 = 0.5 pu at a power factor of 0.85 lag (power factor angle = 31.79°)

$\therefore$ resistance of load = 0.5 * 0.85 = 0.425 pu

and reactance of load = 0.5 * sin 31.79° = 0.263 pu

The circuit may be expressed in per unit as shown in figure 2.4.

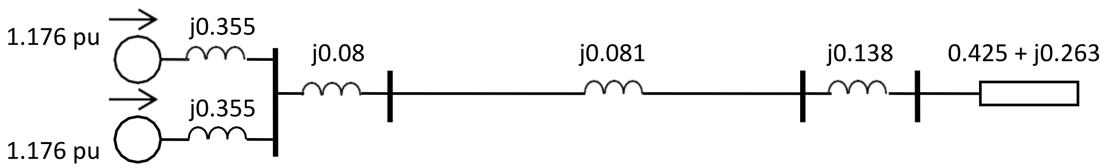
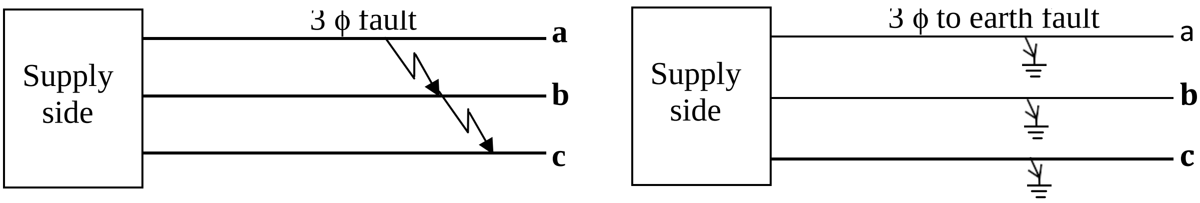


Figure 2.4 - Circuit with per unit values

Symmetrical Three Phase Fault Analysis

A three phase fault is a condition where either (a) all three phases of the system are short-circuited to each other, or (b) all three phase of the system are earthed.



This is in general a balanced condition, and we need to only know the positive-sequence network to analyse faults. Further, the single line diagram can be used, as all three phases carry equal currents displaced by 120^o.

Typically, only 5% of the initial faults in a power system, are three phase faults with or without earth. Of the unbalanced faults, 80 % are line-earth and 15% are double line faults with or without earth and which can often deteriorate to 3 phase fault. Broken conductor faults account for the rest.

Fault Level Calculations

In a power system, the maximum the fault current (or fault MVA) that can flow into a zero impedance fault is necessary to be known for switch gear solution. This can either be the balanced three phase value or the value at an asymmetrical condition. The Fault Level defines the value for the symmetrical condition. The fault level is usually expressed in MVA (or corresponding per-unit value), with the maximum fault current value being converted using the nominal voltage rating.

$$MVA_{base} = \sqrt{3} \ . \ Nominal\ Voltage(kV) \ . \ I_{base}\ (kA)$$

$$MVA_{Fault} = \sqrt{3} \ . \ Nominal\ Voltage(kV) \ . \ I_{sc}\ (kA)$$

where

MVA_{Fault} – Fault Level at a given point in MVA

I_{base} – Rated or base line current

I_{sc} – Short circuit line current flowing in to a

fault The per unit value of the Fault Level may thus be

written as

$$MVA_{base} = \sqrt{3}\ .\ Nominal\ Voltage(kV)\ .\ I_{base}\ (kA)$$

$$MVA_{Fault} = \sqrt{3}\ .\ Nominal\ Voltage(kV)\ .\ I_{sc}\ (kA)$$

where

MVA_{Fault} – Fault Level at a given point in MVA

I_{base} – Rated or base line current

I_{sc} – Short circuit line current flowing in to a fault

The per unit value of the Fault Level may thus be written as

$$Fault\ Level = \frac{\sqrt{3}\ .\ \text{Nominal Voltage}\ .\ I_{sc}}{\sqrt{3}\ .\ \text{Nominal Voltage}\ .\ I_{base}} = \frac{\sqrt{3} I_{sc}}{\sqrt{3} I_{base}} = I_{sc,pu} = \frac{V_{Nominal,pu}}{Z_{pu}}$$

The per unit voltage for nominal value is unity, so that

$$Fault\ Level\ (pu) = \frac{1}{Z_{pu}},$$

$$Fault\ MVA = Fault\ Level\ (pu) \times MVA_{base} = \frac{MVA_{base}}{Z_{pu}}$$

The Short circuit capacity (SCC) of a busbar is the fault level of the busbar. The strength of a busbar (or the ability to maintain its voltage) is directly proportional to its SCC. An infinitely strong bus (or Infinite bus bar) has an infinite SCC, with a zero equivalent impedance and will maintain its voltage under all conditions.

Magnitude of short circuit current is time dependant due to synchronous generators. It is initially at its largest value and decreasing to steady value. These higher fault levels tax Circuit Breakers adversely so that current limiting reactors are sometimes used.

The Short circuit MVA is a better indicator of the stress on CBs than the short circuit current as CB has to withstand recovery voltage across breaker following arc interruption.

The currents flowing during a fault is determined by the internal emfs of machines in the network, by the impedances of the machines, and by the impedances between the machines and the fault.

Figure 2.6 shows a part of a power system, where the rest of the system at two points of coupling have been represented by their Thevenin's equivalent circuit (or by a voltage source of 1 pu together its fault level which corresponds to the per unit value of the effective Thevenin's impedance).

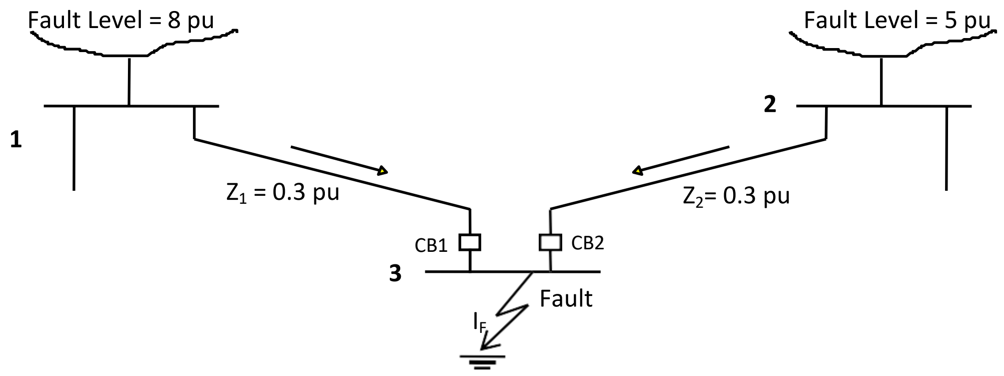


Figure 2.6 – Circuit for Fault Level Calculation

With CB1 and CB2 open, short circuit capacities are

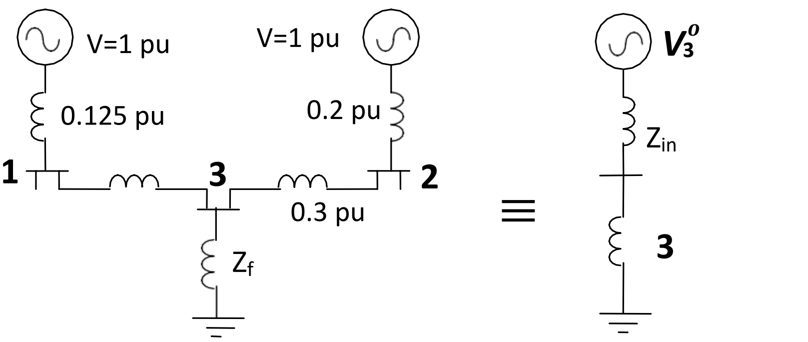
SCC at bus 1 = 8 p.u. gives Z_{g1} = 1/8 = 0.125 pu

SCC at bus 2 = 5 p.u. gives Z_{g2} = 1/5 = 0.20 pu

Each of the lines are given to have a per unit impedance of 0.3 pu.

$Z_1 = Z_2 = 0.3$ p.u.

With CB1 and CB2 closed, what would be the SCCs (or Fault Levels) of the busbars in the system ?



System Equivalent Circuit Thevenin's Equivalent at 3

Figure 2.7a Determination of Short circuit capacities

This circuit can be reduced and analysed as in figure 2.7b.

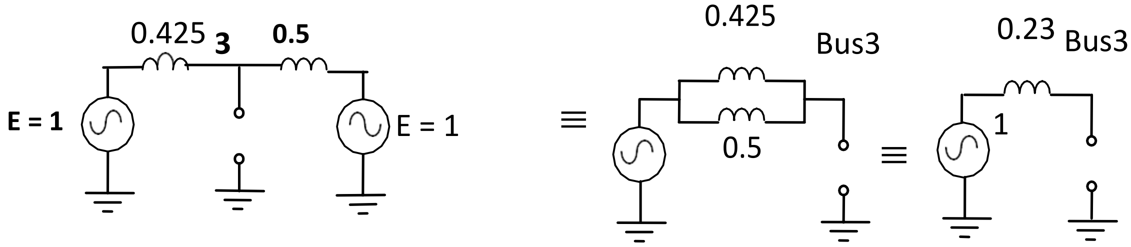


Figure 2.7b Determination of Short circuit capacity at Bus 3

Thus, the equivalent input impedance is given by to give Z_{in} as 0.23 pu at bus 3, so that the short circuit capacity at busbar 3 is given as

| SCC3 |= 1/0.23 = 4.35 p.u

The network may also be reduced keeping the identity of Bus 1 as in figure 2.7c.

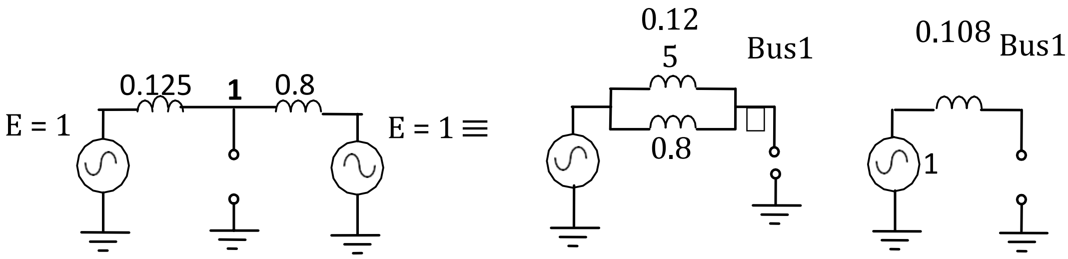


Figure 2.7c Determination of Short circuit capacity at Bus 1

UNIT - IV
POWER SYSTEM STEADY STATE STABILITY ANALYSIS

Stability of power system is its ability to return to normal or stable operating condition after been subjected to some of disturbance. Instability means a condition representing loss of synchronism or fall out of step.

The instability of power system is divided into two parts

1. Steady state stability
2. Transient stability

Increase in load is a kind of disturbance to power system. If the increase in load takes place gradually and slowly in small steps and the system withstand this change in load and operates satisfactorily then this system phenomena is said to be STEADY STATE STABILITY.

Cause of transient disturbances

1. Sudden change of load.
2. Switching operation.
3. Loss of generation.
4. Fault.

Due to the following sudden disturbances in the power system, rotor angular difference, rotor speed and power transfer undergo fast changes whose magnitude are dependent upon the severity of disturbances.

If the disturbance is so large that the angular difference increases so much which can cause the machine out of synchronism. This kind of instability is denoted as transient instability. It is a very fast phenomenon it occurs within one second for the generating unit closer to the disturbance.

Dynamics Of A Synchronous Machine

The kinetic energy of the rotor at synchronous machine is

$$KE = \frac{1}{2} J\omega_{sm}^{2} \times 10^{-6} MJ \tag{1}$$

J =rotor moment of inertia in kg-m^2

$$KE = \frac{1}{2}\left(J\left(\frac{2}{P}\right)^{2} \omega_s \times 10^{-6} \right)\omega_s = \frac{1}{2} M\omega_s$$

Where

$$M = J\left(\frac{2}{P}\right)^{2} \omega_s \times 10^{-6} \text{=Moment of inertia MJ-s/elect. rad}$$

now inertia constant h be written as

$$GH = KE = \frac{1}{2} M\omega_s \text{ mj}$$

g =machine rating(base)in mva(3-phase)

h =inertia constant in mj/mva or mw-s/mva

so,

$$M = \frac{2GH}{\omega_s} = \frac{GH}{\pi f} \text{MJ-s/elect.rad} \tag{4}$$

$$= \frac{GH}{180 f} \text{ MJ-s/elect.rad} \tag{5}$$

Taking G as base, the inertia constant in pu is

$$M = \frac{H}{\pi f} \text{ s}^2\text{/elect.rad} \tag{6}$$

$$M = \frac{H}{180 f} \text{ s}^2\text{/elect.degree} \tag{7}$$

Swing Equation

The differential equation that relates the angular momentum *M*, acceleration power P_a and the rotor angle δ is known as swing equation. Solution of swing equation shows how the rotor angle changes with respect time following a disturbance. The plot δ Vs *t* is known as swing curve. The differential equation governing the rotor dynamics can then be written as.

$$J\frac{d^2\theta_m}{dt^2} = T_m - T_e$$

where,

J = rotor moment of inertia in kg-m^2, θ_m = angle in radian (mech.)

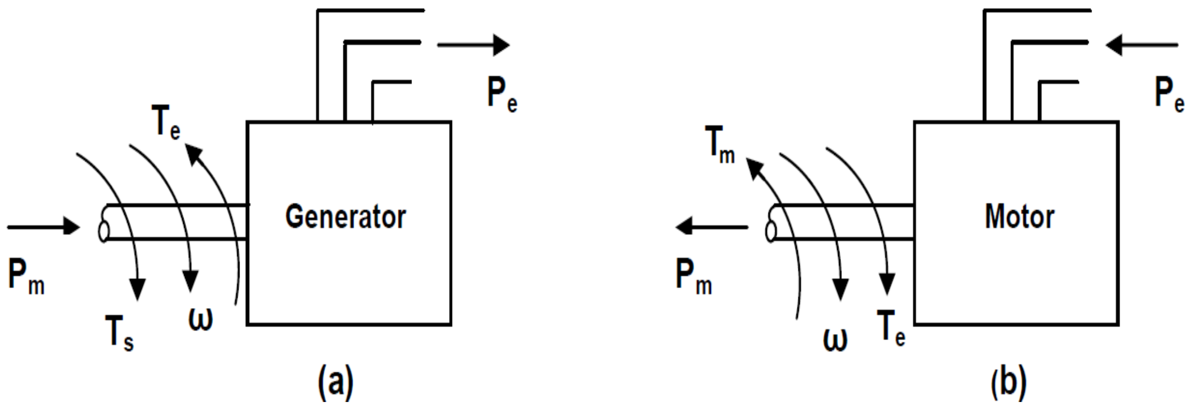


Fig. 1 Electrical and mechanical power flow in motor

While the rotor undergoes dynamics as per Equation (9), the rotor speed changes by insignificant magnitude for the time period of interest (1s)

Equation (8) can therefore be converted into its more convenient power form by assuming the rotor .speed (ω_{sm}). Multiplying both sides of Equation (8) by ω_{sm} we can write

$$J\omega_{sm}\frac{d^2\theta_m}{dt^2}\times 10^{-6} = P_m - P_e \text{MW} \qquad (9)$$

Where,

P_m= mechanical power input in MW

P_e=electrical power output in MW; stator copper loss is assumed neglected.

Rewriting Equation (9)

$$\left(J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}\right)\frac{d^2\theta_e}{dt^2} = (P_m - P_e)MW$$

$$M\frac{d^2\theta_e}{dt^2} = P_m - P_e\,MW$$

Where

θ_e =angle in rad.(elect.)

As it is more convenient to measure the angular position of the rotor with respect to a synchronously rotating frame of reference.

Let us assume,

$$\delta = \theta_e - \omega_s t \qquad (13)$$

δ *is* rotor angular displacement from synchronously rotating reference frame, called ***Torque Angle/Power Angle.***

From Equation (9)

$$\frac{d^2\theta_e}{dt^2} = \frac{d^2\delta}{dt^2} \qquad (14)$$

Hence Equation (11) can be written in terms of δ as

$$M\frac{d^2\delta}{dt^2} = P_m - P_e MW \qquad (15)$$

Using Equation (11) we can also write

$$\left(\frac{GH}{\pi f}\right)\frac{d^2\delta}{dt^2}$$

Dividing throught by *G*, the MVA rating of the machine

$$M\,(pu)\frac{d^2\delta}{dt^2} = P_m - P_e \qquad (17)$$

Where

$$M\,(pu) = \frac{H}{\pi f}, \quad \frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \text{ pu}$$

Equation (17) is called as swing equation and it describes the rotor dynamics for a synchronous machine (generating/motoring). It is a second-order differential equation where the damping term (proportional to $d\delta/dt$) is absent because of the assumption of a loss less machine and the fact that the torque of damper winding has been ignored. Since the electrical power P_e depends upon the sine of angle δ the swing equation is a non-linear second-order differential equation.

Multi-Machine System

In a multi-machine system a common system base must be
chosen Let
G_{mach}=machine rating
(base) G_{system}=system base
Equation(18) can then be written as

$$\frac{G_{mach}}{G_{system}}\left(\frac{H_{mach}}{f}\frac{d^2\delta}{dt^2}\right)=(P_m-P_e)\frac{G_{mach}}{G_{system}}$$

Or $$\frac{H_{system}}{\pi f}\frac{d^2\delta}{dt^2}=P_m-P_e \text{ pu in system base.}$$

Where

$$H_{system}=H_{mach}\left(\frac{G_{mach}}{G_{system}}\right)$$

Consider the swing equations of two machines or a common system base.

$$\frac{H_1}{\pi f}\frac{d^2\delta_1}{dt^2}=P_{m1}-P_{e1} \text{ pu}$$

$$\frac{H_2}{\pi f}\frac{d^2\delta_2}{dt^2}=P_{m2}-P_{e2} \text{ pu}$$

Since the machine rotors swings together (coherently or in unison)

$\delta_1=\delta_2=\delta$

Adding Equation (20) and (21)

$$\frac{H_{eq}}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \qquad (22)$$

Where

$$P_m = P_{m1} + P_{m2}$$

$$P_e = P_{e1} + P_{e2}$$

$$H_{eq} = H_1 + H_2$$

The two machines swinging coherently are thus reduced to a single machine as in Equation (22), the equivalent inertia in (22) can be written as

$$H_{eq} = H_{1mach} \, {}^{G_{1mach}}\!\big/_{G_{system}} + H_{2mach} \, {}^{G_{2mach}}\!\big/_{G_{system}} \qquad (23)$$

The above results are easily extendable to any number of machines swinging coherently. To solving the swing equation (Equation (23), certain simplifying assumptions are usually made. These are:

1. Mechanical power input to the machine (P_m) remains constant during the period of electromechanical transient of interest. In other words, it means that the effect of the turbine governing loop is ignored being much slower than the speed of the transient. This assumption leads to pessimistic result-governing loop helps to stabilize the system.
2. Rotor speed changes are insignificant-these have already been ignored in formulating the swing equation.
3. Effect of voltage regulating loop during the transient is ignored, as a consequence the generated machine emf remains constant. This assumption also leads to pessimistic results- voltage regulator helps to stabilize the system.

Before the swing equation can be solved, it is necessary to determine the dependence of the electrical power output (P_e) upon the rotor angle.

Simplified Machine Model

For a non-salient pole machine, the per-phase induced emf-terminal voltage equation under steady conditions is.

$$E = V + jX_d I_d + jX_q I_q ; X_d > X_q \qquad (24)$$

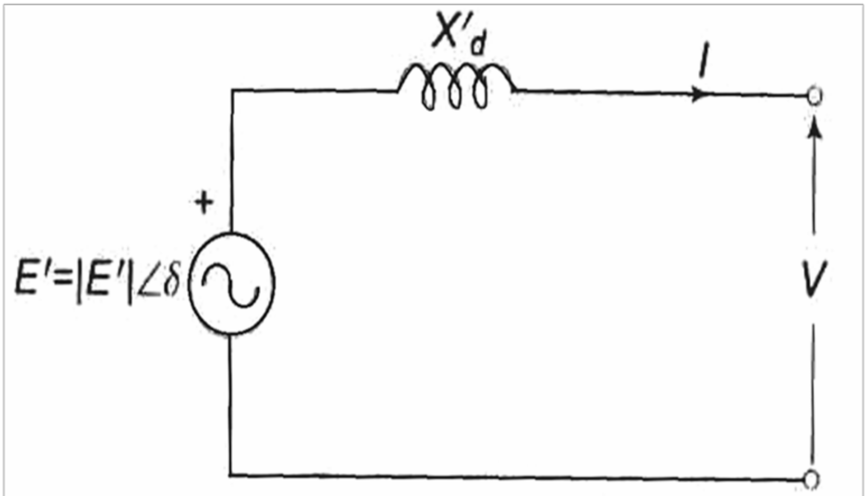


Fig. 3 Simplified machine model.

The machine model corresponding to Eq. (26) is drawn in Fig. (3) which also applies to a cylindrical rotor machine where $X_d' = X_q' = X_s'$ (transient synchronous reactance).

Power Angle Curve

For the purposes of stability studies $|E'|$, transient emf of generator motor remains constant or is the independent variable determined by the voltage regulating loop but V, the generator determined terminal voltage is a dependent variable. Therefore, the nodes (buses) of the stability study network to the ernf terminal in the machine model as shown in Fig. 4, while the machine reactance (X_d') is absorbed in the system network as different from a load flow study. Further, the loads (other than large synchronous motor) will be replaced by equivalent static admittances (connected in shunt between transmission network buses and the reference bus).

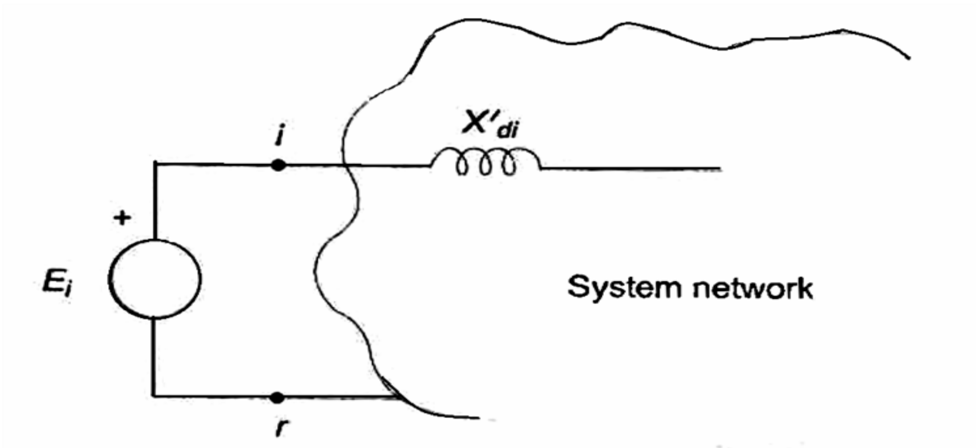


Fig. 4 Simplified Machine studied Network

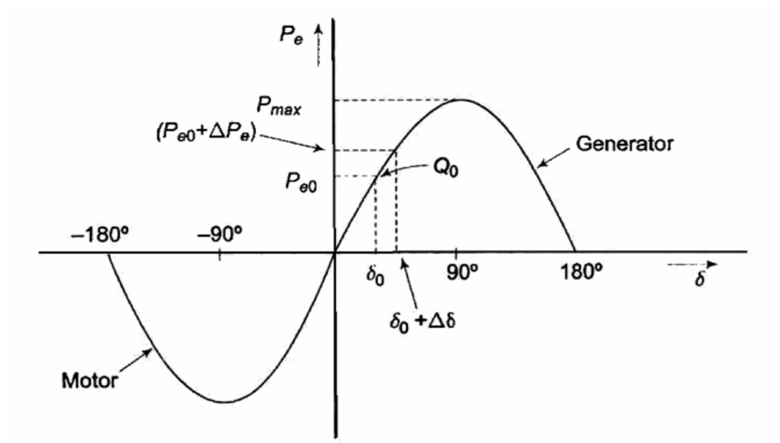


Fig 5 Power Angle Curve

This is so because load voltages vary during a stability study (in a load flow study, these remain constant within a narrow band). The simplified power angle equation is

$$P_e = P_{\max} \sin\delta \qquad (27)$$

Wher
e

$$P_{\max} = \frac{E_1' \; E_2'}{X} \qquad (28)$$

The graphical representation of power angle equation (28) is shown in Fig. 5. The swing equation (27) can now be written as

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \quad \text{pu} \qquad (29)$$

It is a non linear second-order differential equation with no damping.

Machine Connected to Infinite Bus

Figure 6 is the circuit model of a single machine connected to infinite bus through a line of reactance X_e. In this simple case

$$X'_{transfer} = X'_d + X_e$$

From Eq (30) we get

$$P_e = \frac{|E'||V|}{X_{transfer}} \sin\delta = P_{max} \sin\delta \qquad (30)$$

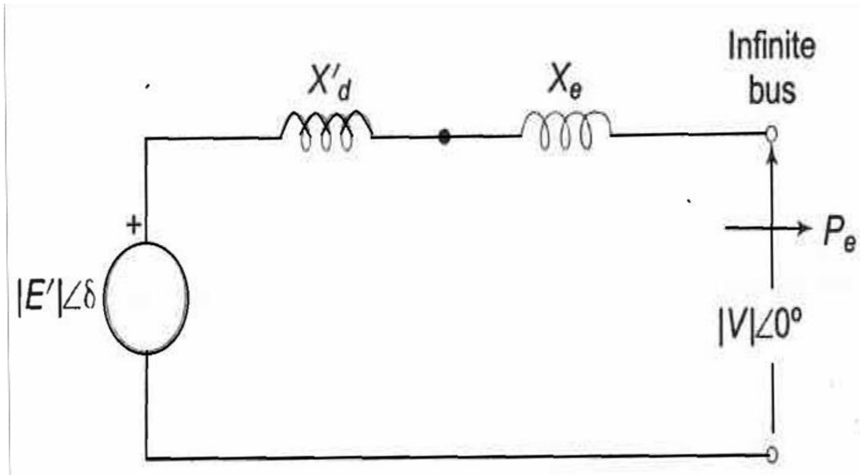


Fig. 6 Machine connected to infinite bus bar

The dynamics of this system are described in Eq. (15) as

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \quad \text{pu} \qquad (31)$$

Two Machine Systems

The case of two finite machines connected through a line (X_e) is illustrated in Fig. 5 where one of the machines must be generating and the other must be motoring. Under steady condition, before the system goes into dynamics and the mechanical input/output of the two machines is assumed to remain constant at these values throughout the dynamics (governor action assumed slow).During steady state or in dynamic condition, the electrical power output of the generator must be absorbed by the motor (network being lossless).

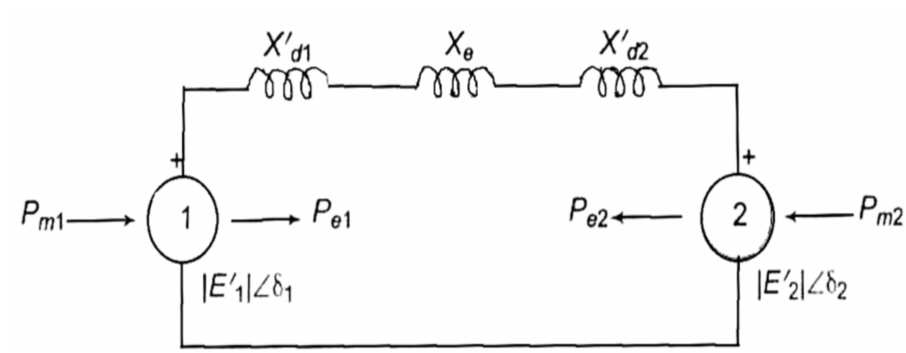


Fig. 7 Two machine system

Thus at all time

$$P_{m1} = -P_{m2} = P_m \qquad (32)$$

$$P_{e1} = -P_{em\,2} = P_e \qquad (33)$$

$= \quad -$

Steady State Stability

The steady state stability limit of a particular circuit of a power system is defined as the maximum power that can be transmitted to the receiving end without loss of synchronism.

Consider the simple system of Fig. 7 whose dynamics is described by equations

$$M\frac{d^2\delta_e}{dt^2} = P_m - P_e MW \tag{40}$$

$$M = \frac{H}{\pi f} \text{ in pu system} \tag{41}$$

And,

$$P_e = \frac{|E||V|}{X_d}\sin\delta = P_{max}\sin\delta \tag{42}$$

For determination of steady state stability, the direct axis reactance (X_d) and, voltage behind X_d

are used in the above equations. Let the system be operating with steady power transfer of $P_{e0}=P_m$ with torque angle δ_0 as indicated in the figure. Assume a small increment ΔP in the electric power with the input from the prime mover remaining fixed at P_m (governor response is slow compared to the speed of energy dynamics), causing the torque angle to change to $(\delta_0 + \Delta\delta)$.

$$M\frac{d^2\Delta\delta}{dt^2} = P_m - (P_{e0} + \Delta P_e) = -\Delta P_e$$

or

$$M\frac{d^2\Delta\delta}{dt^2} + \left[\frac{\partial P_e}{\partial\delta}\right]_0 \Delta\delta = 0 \qquad (43)$$

or

$$Mp^2 + \left(\frac{\partial P_e}{\partial\delta}\right)_0 \Delta\delta = 0$$

Where

$$p = \frac{d}{dt}$$

The system stability to small change is determined from the characteristic equation.

$$Mp^2 + \left[\frac{\partial p_e}{\partial\delta}\right]_0 = 0$$

Its two roots are

$$p = \pm\left[\frac{-\left(\partial p_e / \partial\delta\right)}{M}\right]^{\frac{1}{2}}$$

As long as $\left(\partial p_e / \partial\delta\right)_0$ it positive, the roots are purely imaginary and conjugate and the system

behaviour is oscillatory about δ_0. Line resistance and damper windings of machine, which have been ignored in the above modelling, cause the system oscillations to decay. The system is therefore stable for a small increment in power so long as

$$\left(\frac{\partial p_e}{\partial \delta} \right)_0 > 0 \tag{44}$$

When $\left(\partial p_e/\partial \delta\right)_0$, is negative, the roots are real, one positive and the other negative but of equal

magnitude. The torque angle therefore increases without bound upon occurrence of a small power increment (disturbance) and the synchronism is soon lost. The system is therefore unstable for

$$\left(\partial p_e / \partial \delta \right)_0 < 0 \quad (45)$$

$\left(\partial p_e / \partial \delta \right)_0$ is known as synchronizing coefficient. This is also called stiffness (electrical) of synchronous machine.

Assuming $|E|$ and $|V|$ to remain constant, the system is unstable, if

$$\frac{|E||V|}{X} \cos \delta_0 < 0$$

$$\delta_0 > 90^\circ \quad (46)$$

The maximum power that can be transmitted without loss of stability (steady state) occurs for

$$\delta_0 = 90^\circ \quad (47)$$

$$P_{\max} = \frac{|E||V|}{X} \quad (48)$$

If the system is operating below the limit of steady stability condition (Eq. 48), it may continue to oscillate for a long time if the damping is low. Persistent oscillations are a threat to system security. The study of system damping is the study of dynamical stability.

The above procedure is also applicable for complex systems wherein governor action and excitation control are also accounted for. The describing differential equation is linerized about the operating point. Condition for steady state stability is then determined from the corresponding characteristic equation (which now is of order higher than two).

It was assumed in the above account that the internal machine voltage $|E|$ remains constant (i.e., excitation is held constant). The result is that as loading increases, the terminal voltage $|V_t|$ dips heavily which cannot be tolerated in practice. Therefore, we must consider the steady state stability limit by assuming that excitation is adjusted for every load increase to keep
$|V_t|$ constant. This is how the system will be operated practically. It may be understood that we are still not considering the effect of automatic excitation control.

Some Comment on Steady State Stability

Knowledge of steady state stability limit is important for various reasons. A system can be operated above its transient stability limit but not above its steady state limit. Now, with increased fault clearing speeds, it is possible to make the transient limit closely approach the steady state limit.

As is clear from Eq. (50), the methods of improving steady state stability limit of a system are to reduce X and increase either or both $|E|$ and $|V|$. If the transmission lines are of sufficiently high reactance, the stability limit can be raised by using two parallel lines which incidentally also increases the reliability of the system. Series capacitors are sometimes employed in lines to get better voltage regulation and to raise the stability limit by decreasing the line reactance. Higher excitation voltages and quick excitation system are also employed to improve the stability limit.

UNIT-V

POWER SYSTEM TRANSIENT STATE STABILITY ANALYSIS

Transient Stability

The dynamics of a single synchronous machine connected to infinite bus bars is governed by the nonlinear differential equation

$$M\frac{d^2\delta}{dt^2} = P_m - P_e$$

where

$$P_e = P_{\max}\sin\delta \tag{49}$$

or

$$M\frac{d^2\delta}{dt^2} = P_m - P_{\max}\sin\delta$$

As said earlier, this equation is known as the swing equation. No closed form solution exists for swing equation except for the simple case $P_m = 0$ (not a practical case) which involves elliptical integrals. For small disturbance (say, gradual loading), the equation can be linearised leading to the concept of steady state stability where a unique criterion of stability $\left(\partial p_e / \partial\delta > 0\right)$ could be established. No generalized criteria are available for determining system stability with large disturbances (called transient stability). The practical approach to the transient stability problem is therefore to list all important severe disturbances along with their possible locations to which the system is likely to be subjected according to the experience and judgement of the power system analyst. Numerical solution of the swing equation (or equations for a multi- machine case) is then obtained in the presence of such disturbances giving a plot of δ *Vs* t called the swing curve. If δ starts to decrease after reaching a maximum value, it is normally assumed that the system is stable and the oscillation of δ around the equilibrium point will decay and finally die out. As already pointed out in the introduction, important severe disturbances are a short circuit or a sudden loss of load.

For ease of analysis certain assumptions and simplifications are always made (some of these have already been made in arriving at the swing equation (Eq. 49). All the assumptions are listed, below along with their justification and consequences upon accuracy of results.

1. Transmission line as well as synchronous machine resistance is ignored. This leads to pessimistic result as resistance introduces damping term in the swing equation which helps stability.
2. Damping term contributed by synchronous machine damper windings is ignored. This also leads to pessimistic results for the transient stability limit.
3. Rotor speed is assumed to be synchronous. In fact it varies insignificantly during the course of the stability transient.
4. Mechanical input to machine is assumed to remain constant during the transient, i.e., regulating action of the generator loop is ignored. This leads to pessimistic results.
5. Voltage behind transient reactance is assumed to remain constant, i.e., action of voltage regulating loop is ignored. It also leads to pessimistic results.
6. Shunt capacitances are not difficult to account for in a stability study. Where ignored, no greatly significant error is caused.
7. Loads are modelled as constant admittances. This is a reasonably accurate representation. *Note:* Since rotor speed and hence frequency vary insignificantly, the network parameters remain fixed during a stability study.

A digital computer programme to compute the transient following sudden disturbance can be suitably modified to include the effect of governor action and excitation control.

Preset day power system are so large that even after lumping of machines (Eq.(24)), the system remains a multi-machine one. Even then, a simple two machine system greatly aids the

understanding of the transient stability problem. It has been shown in that an equivalent single machine infinite bus system can be found for a two- machine system (Eq. 45) to (Eq. 49)

Upon occurrence of a severe disturbance, say a short circuit, the power transfer between machines is greatly reduced, causing the machine torque angles to swing relatively. The circuit breakers near the fault disconnect the unhealthy part of the system so that power transfer can be partly restored, improving the chances of the system remain stable. The shorter the time to breaker operating, called *clearing time*, the higher is the probability of the system being stable. Most of the line faults are transient in nature and get cleared on opening the line. Therefore, it is common practice now to employ *auto-reclose breakers* which automatically close rapidly after each of the two sequential openings. If the fault still persists, the circuit breakers open and lock permanently till cleared manually. Since in the majority of faults the first *reclosure* will be successful, the chances of system stability are greatly enhanced by using *autoreclose* breakers.

The procedure of determining the stability of a system upon occurrence of a disturbance followed by various switching off and switching on action called a *stability study*. Steps to be followed in stability study are outlined below for single- machine infinite bus bar system shown in fig. 6. The fault is assumed to be transient one which is cleared by the time of first reclosure. In the case of a permanent fault, this system completely falls apart. This will not be the case in a multi-machine system. The steps listed, in fact, apply to a system of any size.

1. From prefault loading, determine the voltage behind transient reactance and the torque angle δ_0 of the machine with reference to the infinite bus.
2. For the specified fault, determine the power transfer $P_e(\delta)$ during fault. In this equation system $P_e = 0$ for a three-phase fault.
3. From the swing equation starting with δ_0 as obtained in step 1, calculate δ as a function of time using a numerical technique of solving the nonlinear differential equation.
4. After clearance of the fault, once again determine $P_e(\delta)$

Equal Area Criteria for Stability

In a system where one machine is swinging with respect to an infinite bus, it is possible to study transient stability by means of a simple criterion, without resorting to the numerical solution of a swing equation.

Consider the equation

$$M \frac{d^2\delta}{dt^2} = P_m - P_e = P_a \tag{50}$$

P_a =accelerating power

If the system is unstable δ continues to increase indefinitely with time and the machine loses synchronism. On the other hand, if the system is stable, $\delta(t)$ performs oscillations (nonsinusoidal) whose amplitude decreases in actual practice because of damping terms (not included in the swing equation).These two situations are shown in fig. 6. Since the system is no- linear, the nature of its response1 [$\delta(t)$] is not unique and it may exhibit instability in a fashion different from that indicated in Fig. 6, depending upon the nature and severity of disturbance.

However, experience indicates that the response $\delta(t)$ in a power system generally falls in the two broad categories as shown in the figure. It can easily be visualized now (this has also been stated earlier) that for a stables system, indication of stability will be given by observation of the first swing where δ will go to a maximum and will start to reduce.

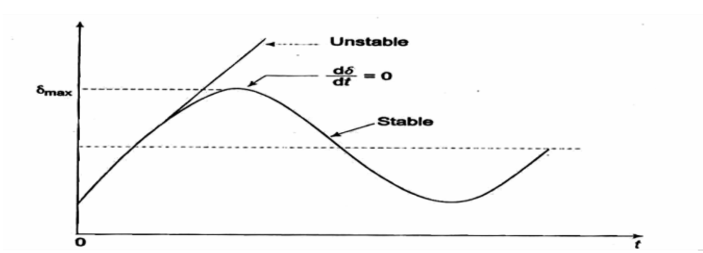


Fig. 8 Plot of δ vs t for stable and unstable system.

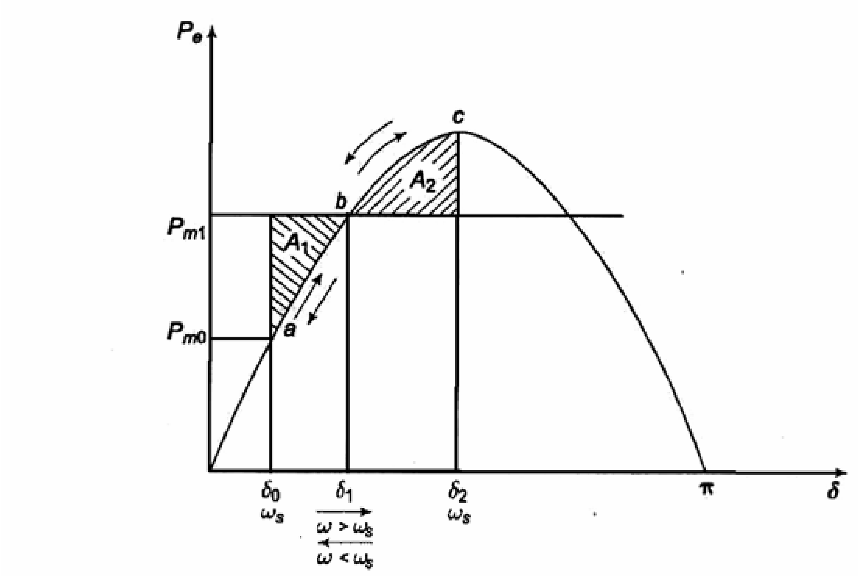


Fig. 9 P_e- δ diagram for sudden increase in mechanical input

The condition of stability can therefore be stated as: the system is stable if the area under P_a (accelerating power) - δ curve reduces to zero at some value of δ. In other words, the positive (accelerating) area under P_a - δ curve must equal the negative (decelerating) area and hence the name „equal area" criterion of stability. To illustrate the equal area criterion of stability, we now consider several types of disturbances that may occur in a single machine infinite bus bar system. Figure 9 shows the transient model of a single machine tied to infinite bus-bar. The electrical power transmitted is given by

$$P_e = \frac{|E'||V|}{X_d} \sin\delta = P_{max} \sin\delta$$

Under steady operating condition

$$P_{m0} = P_{e0} = P_{max} \sin\delta_0$$

This is indicated by the point a in the P_e - δ diagram of Fig. 8.

Let the mechanical input to the rotor be suddenly increased to P_{m1} (by opening the steam valve). The accelerating power $P_a = P_{m1} - P_e$ causes the rotor speed to increase $(\omega > \omega_s)$ and begins to reduce but the angle continues to increase till at angle δ_2, ($\omega > \omega_s$) once again (state point at *c*. At *c*), the-decelerating area A_2 equals the accelerating area A_1, (areas are shaded), i.e,

$$\int_{\delta_0}^{\delta} P_a \, d\delta = 0$$

Since the rotor is decelerating, the speed reduces below ω_s and the rotor angle begins to reduce. The state point now traverses $P_e \, Vs \delta$ curve in the opposite direction as indicated by the arrows in Fig. 8.It is easily seen that the system oscillates about the new steady state point *b* ($\delta = \delta_1$) with angle excursion up to δ_0 and δ_2 on the two sides. These oscillations are similar to the simple harmonic motion of an inertia-spring system except that these are not sinusoidal.

As the oscillations decay out because of inherent system damping (not modelled), the system settles to the new steady state where

$$P_{m1} = P_e = P_{\max} \sin \delta_1$$

From Fig. 12.20, areas A_1=A_2 are given by

$$A_1 = \int_{\delta_0}^{\delta_0} (P_{m1} - P_e) d\delta$$

or

$$A_1 = \int_{\delta_0}^{\delta_0} (P_e - P_{m1}) d\delta$$

For the system to be stable, it should be possible to find angle δ_2 such that A_1=A_2. As P_{m1} is increased, a limiting condition is finally reached when A_1 equals the area above the P_{m1} line as shown in Fig 10.Under this condition, δ_2 acquires the maximum value such that

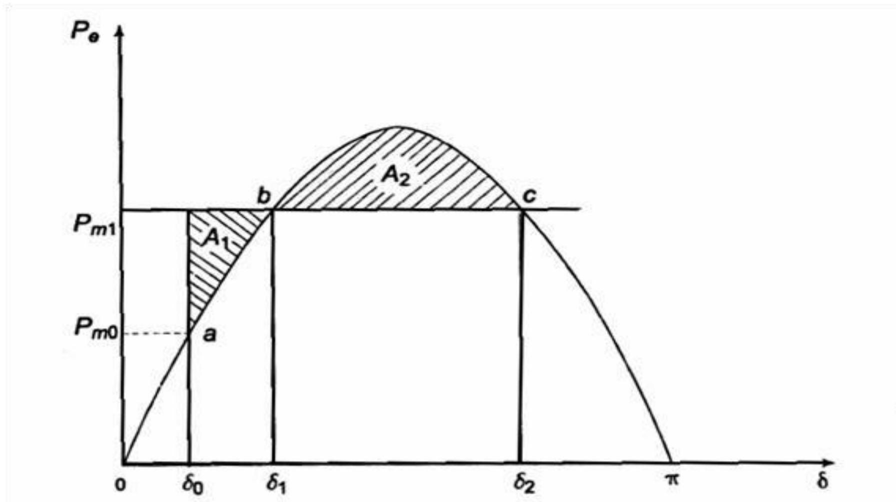


Fig. 10 Limiting case of transient stability with mechanical input suddenly increased

It has thus been shown by use of the equal area criterion that there-is an upper limit to sudden increase in mechanical input ($P_{m1} - P_{m0}$), for the system in question to remain stable'

It may be noted from Fig. 9 that the system will remain stable even though the rotor may oscillate beyond $\delta = 90^\circ$, so long as the equal area criteria is met. The condition of $\delta = 90^\circ$ is meant for use in steady state stability only and does not apply to the transient stability case.

Effect of Clearing Time on Stability

Let the system of Fig. 9 be operating with mechanical input P_m at a steady angle of δ (P_m=P_e) as shown by the point *a* on the P_e *Vs* δ diagram of Fig. 10. If a 3-phase fault occurs at the point *P* of the outgoing radial line, the electrical output of the generator instantly reduces to zero, i.e., $P_e = 0$ and the state point drops to *b*. The acceleration area A_1 begins to increase and so does the rotor angle while the state point moves along *bc*. At time t_c corresponding to angle δ_c,

the faulted line is cleared by the opening of the line circuit breaker. The values of t_c and δ_c are respectively known as *clearing time* and, *clearing angle*. The system once again becomes healthy and transmits $P_e = P_{\max} \sin \delta$ i.e. the state point shifts to *d* on the original P_e *Vs* δ curve.

The rotor now decelerates and the decelerating area A_2, begins while the state point moves

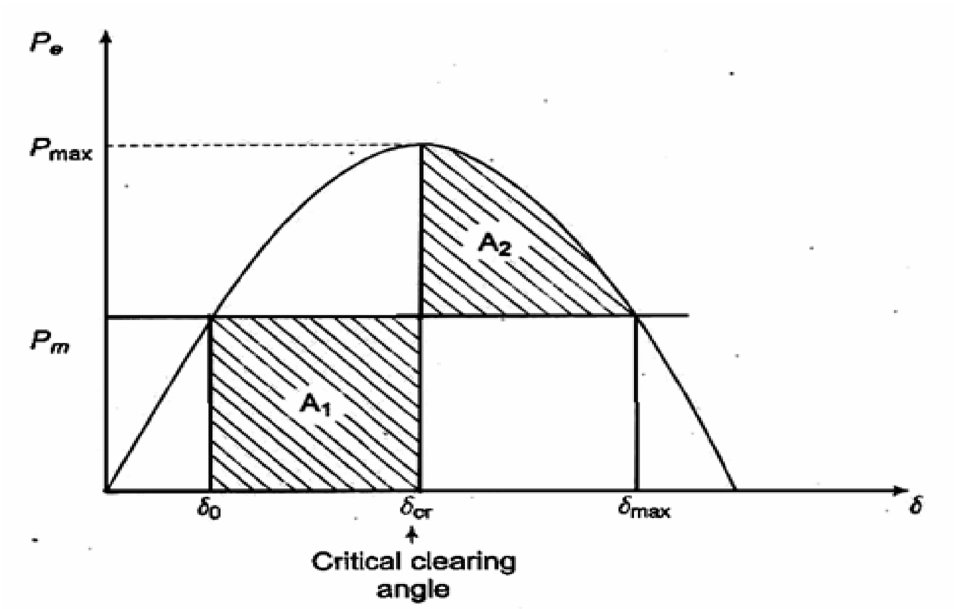


Fig. 10 Limiting case of transient stability with critical angle

The value of clearing time corresponding to a clearing angle can be established only by numerical integration except in this simple case. The equal area criterion therefore gives only qualitative answer to system stability as the time when the breaker should be opened is hard to establish.

As the clearing of the faulty line is delayed, A_1 increases and so does δ_1, to find A_2=A_1 till $\delta_1 = \delta_{\max}$ as shown in Fig. 10. For a clearing time (or angle) larger than this value, the system would be unstable as A_2<A. The maximum allowable value of the clearing time and angle for the system to remain stable are known respectively as *critical clearing time and angle.*

For this simple case (P_e=0 during fault), explicit relationships for δ_c (critical) and t_c (critical) are established below. All angles are in radians.

It is easily seen from Fig. 10

$$\delta_{\max} = \pi - \delta_0 \quad (56)$$

and

$$P_m = P_{\max} \sin \delta_0 \quad (57)$$

Now

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - 0)d\delta = P_m(\delta_{cr} - \delta_0)$$

and

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{max}\sin\delta - P_m)d\delta$$

$$= P_{max}(\cos\delta_{cr} - \cos\delta_m) - P_m(\delta_{max} - \delta_{cr})$$

For the system to be stable, $A_2 = A_1$ which gives

$$\cos\delta_{cr} = \frac{P_m}{P_{max}}(\delta_{max} - \delta_0) + \cos\delta_{max} \qquad (58)$$

Where

δ_{cr} =critical clearing angle.

Substituting Eq. (58) and (59) in Eq.(60), we get

$$\delta_{cr} = \cos^{-1}[(\pi - 2\delta_0)\sin\delta_0 - \cos\delta] \qquad (59)$$

During the period the fault is persisting, the swing equation is

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H} P_m; \quad \text{where } P_e = 0 \qquad (60)$$

From Eq. (61)

$$\delta_{cr} = \sqrt{\frac{2H\,(\delta_{cr} - \delta_0)}{\pi\ f\ P}} \tag{62}$$

Where δ_{cr}, is given by the expression of Eq. (62)

An explicit relationship for determining t_{cr} is possible in this case as during the faulted condition P_e =0 and so the swing equation can be integrated in closed form. This will not be the case in most other situations.

Consider now a single machine tied to infinite bus through two parallel lines as in Fig. 11a circuit model of the system is given in Fig. 11b.

Let us study the transient stability of the system when one of the lines is suddenly switched off with the system operating at a steady road. Before switching off, power angle curve is given by

$$P_{eI} = \frac{|E'||V|}{X'_d \,\square\, X_1 X_2} \sin\delta = P_{\max I} \sin\delta$$

Immediately on switching off line 2, power angle curve is given by

$$P_{eII} = \frac{|E'||V|}{X'_d \,\square\, X_1 X_2} \sin\delta = P_{\max II} \sin\delta$$

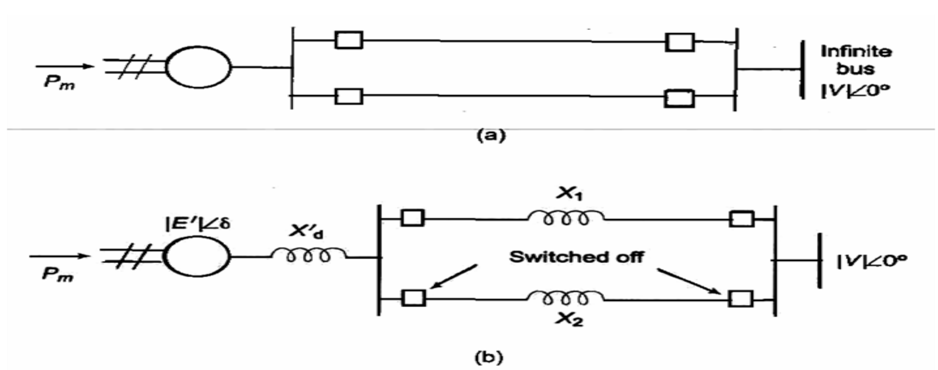


Fig. 11 Single machine tied to infinite bus through two parallel lines

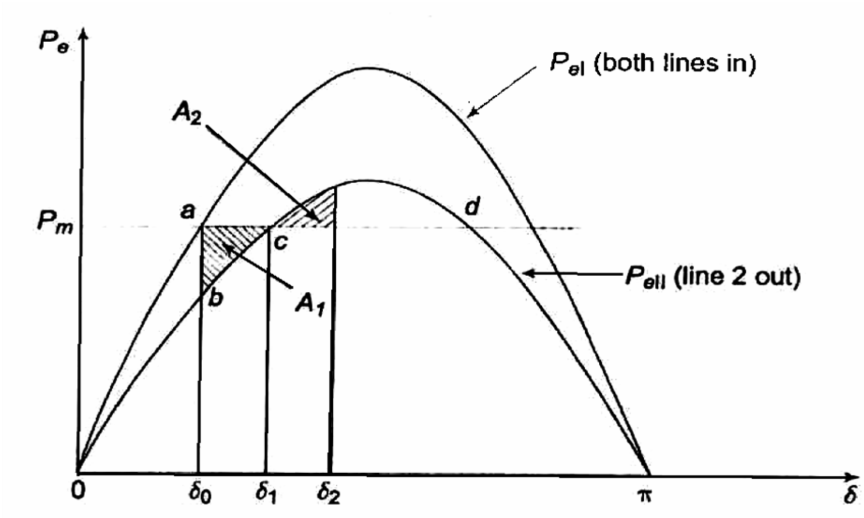


Fig. 12 Equal area criterion applied to the opening of one of the two lines in parallel

Both these curves are plotted in Fig. 12, wherein $P_{maxII} < P_{maxI}$ as $(X'_d + X_1) > (X'_d + X_1 || X_2)$

.The system is operating initially with a steady power transfer P_e=P_m at a torque angle δ_0 on curve I. Immediately on switching off line 2, the electrical operating point shifts to curve II (point b). Accelerating energy corresponding to area A_1 is put into rotor followed by decelerating energy for $\delta_1 > \delta_0$. Assuming that an area A_2 corresponding to decelerating energy (energy out of rotor) can be found such that A_1 = A_2, the system will be stable and will finally operate at c

corresponding to a new, rotor angle $\delta_1 > \delta_1$. This is so because a single line offers larger

reactance and larger rotor angle is needed to transfer the same steady power.

It is also easy to see that if the steady load is increased (line P_m is shifted upward in Fig. 12, a limit is finally reached beyond which decelerating area equal to A_1 cannot be found and therefore, the system behaves as an unstable one, For the limiting case of stability, δ_1 has maximum value given by

$$\delta_1 = \delta_{\max} = \pi - \delta_c$$

This is the same condition as in the previous example.

We shall assume the fault to be a three-phase one. Before the occurrence of a fault, the power angle curve is given by

Upon occurrence of a three-phase fault at the generator end of line 2 (see Fig. 15a), the generator gets isolated from the power system for purposes of power flow as shown by Fig. 15b. Thus during the period the fault lasts,

$P_{eII}=0$

The rotor therefore accelerates and angle δ increases. Synchronism will be lost unless the fault is cleared in time.

The circuit breakers at the two ends of the faulted line open at time t_c (corresponding to angle δ_c), the clearing time, disconnecting the faulted line.

The power flow is now restored via the healthy line (through higher line reactance X_2 in place of $X_l \,//\, X_2$), with power angle curve

$$P_{eII} = \frac{|E'||V|}{X'_d \; X_1 X_2} \sin\delta = P_{\max II} \sin\delta$$

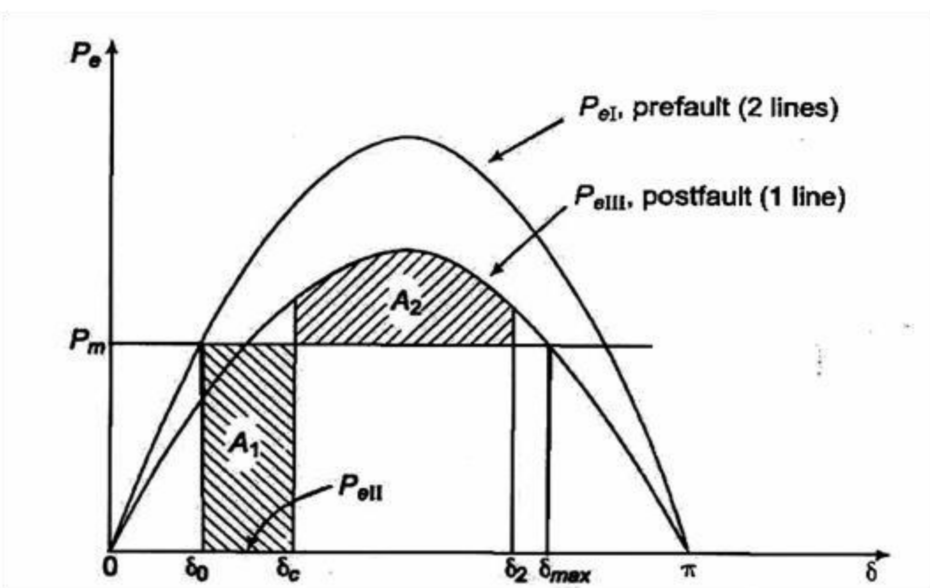


Fig. 13 Equal area criteria applied to the system, I system is normal, II fault applied, III faulted line isolated.

Obviously, $P_{maxII} < P_{maxI}$. The rotor now starts to decelerate as shown in Fig. 13. The system will be stable if a decelerating area A_2 can be found equal to accelerating area A_1 before δ reaches the maximum allowable value δ_{max} .As area A_1 depends upon clearing time t_c

(corresponding to clearing angle δ_c), clearing time must be less than a certain value (critical clearing time) for the system to be stable. It is to be observed that the equal area criterion helps to determine critical clearing angle and not critical clearing time. Critical clearing time can be obtained by numerical solution of the swing equation

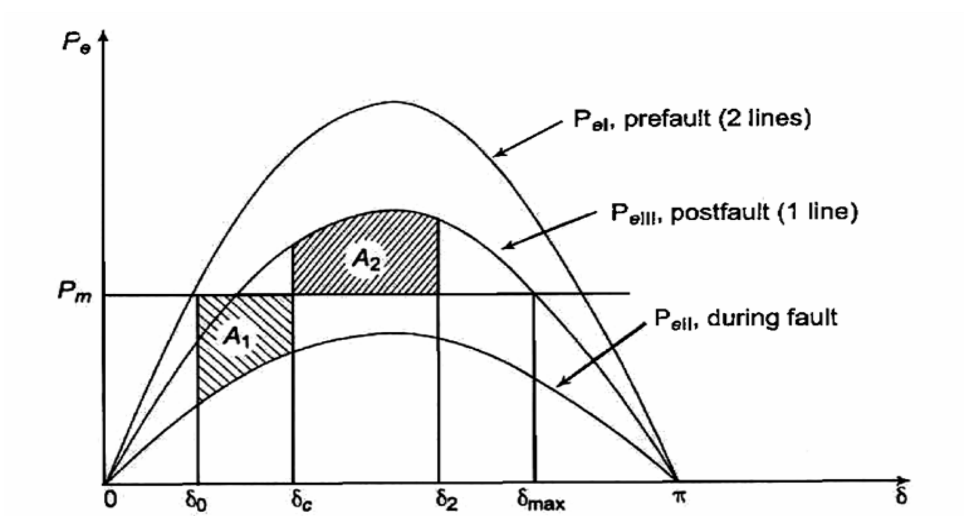
It also easily follows that larger initial loading (P_m.) increases A_1 for a given clearing angle (and time) and therefore quicker fault clearing would be needed to maintain stable operation. The power angle curve during fault is therefore given by

$$P_{eII} = \frac{|E'||V|}{X_{II}} \sin\delta = P_{maxII} \sin\delta$$

P_{eI} , P_{eIII} and P_{eII} as obtained above are all plotted in Fig. 1 Accelerating area A_1 corresponding to a given clearing angle δ is less in this case then in case *a* giving a better chance for stable operation. Stable system operation is shown in Fig. 14, wherein it is possible

to find an area A_2 equal to A_1 for $\delta_2 < \delta_{max}$. As the clearing angle δ_c is increased, area A_1

increases and to find $A_2 = A_1$, δ_2 increases till it has a value δ_{max}, the maximum allowable for stability This case of critical clearing angle is shown in Fig. 15



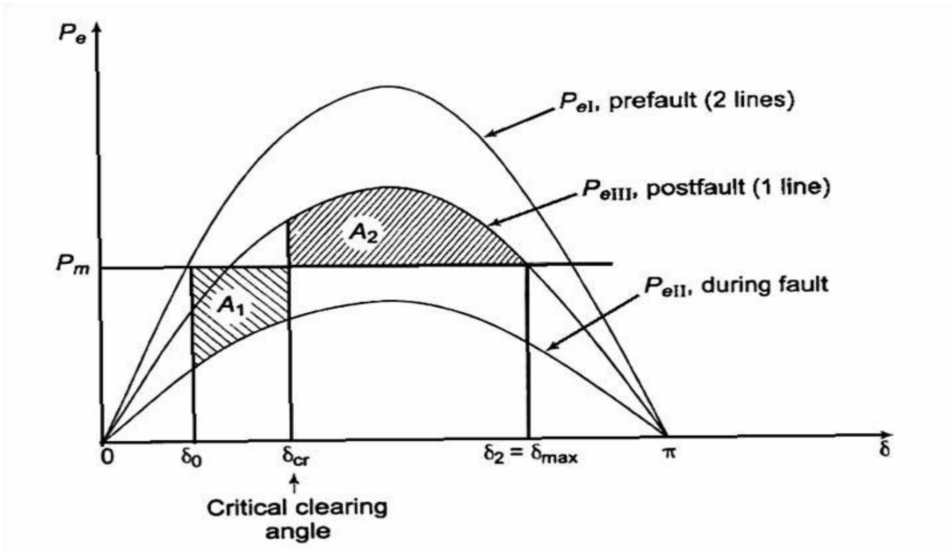


Fig. 15 Fault on middle of one line of the system of, case of critical clearing angle