LECTURE NOTES
ON
DIGITAL COMMUNICATIONS

III B.Tech II semester (JNTUH-R15)

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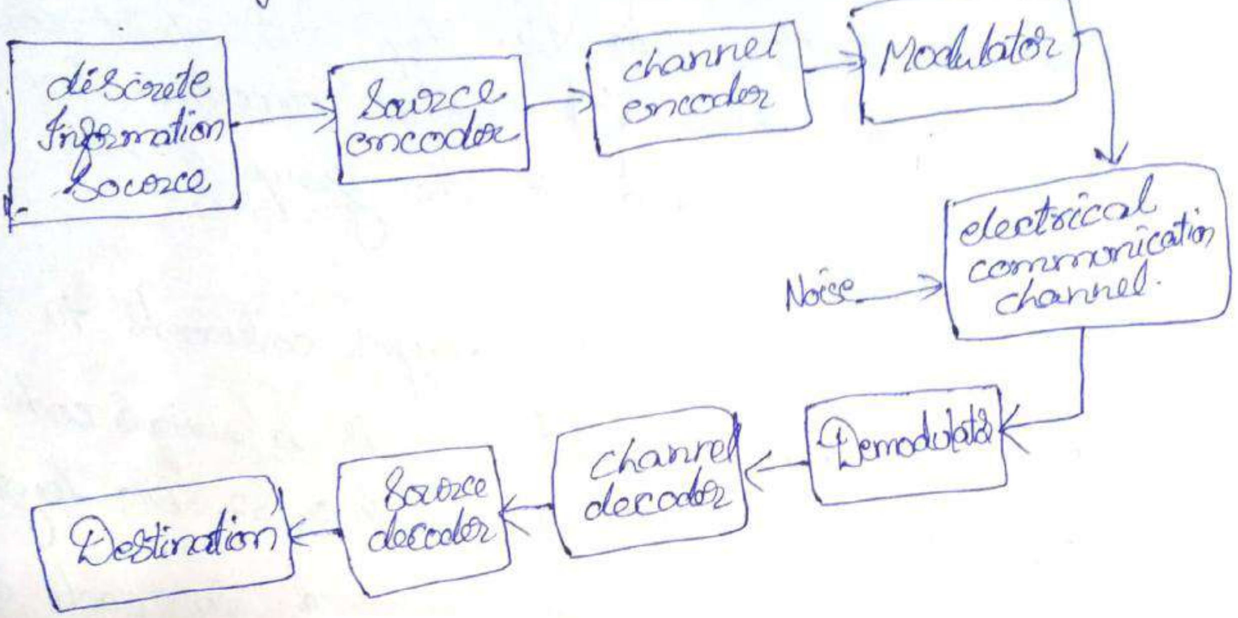
UNIT-I Elements of Digital communication Systems.

1. Model of digital Communication Systems



Information Source

The information source generates the message signal to be transmitted. In case of analog communication, the information source is analog. In case of digital communication, the information source produce a message signal which is not continously varying with time.

→ The examples of discrete information sources are data from computers, teletype etc.

→ The analog signal can be converted to discrete signal by sampling & quantization.

Digital modulators and demodulators

whenever the modulating signal is discrete, then digital modulation techniques are used. The carrier signal used by digital modulators signal is $S_1(t)$ to transmit binary '0' & $S_2(t)$ to transmit binary '1', & always continous sinusoidal wave of high frequency.

→ FSK, PSK, ASK, DPSK, MSK are the examples of various digital modulators. Since these modulations use continous carrier wave, they are also called digital CW modulators.

→ In the reciever, the digital modulator converts the i/p modulated signal to the sequence of binary bits. The most important parameter for the demodulator is method of demodulation.

Communication channel:-

→ The connection between transmitter and reciever is established through communication channel. We have seen that the communication can takes place through wirelines, wireless or fibre optic channels.

→ The other media such as optical disks, magnetic tapes & disks etc can also be called a communication channel, because they can also carry data through them.

Resources available with communication channels

1, channel bandwidth.

2, power in the transmitted signal.

Certain issues in digital transmission:-

The issues in digital transmission are

1, Line coding

2, Scrambling

3, T_1 digital System

4, Multiplexing T_1 lines - the T_2, T_3 & T_4 lines.

Line coding

→ We know how to represent analog data in binary form. But the binary digits 0&1 can't be transmitted directly.

→ ∴ Assigning electrical voltages to the binary digits 0's & 1's that suits communication over an electrical line is called as "line coding".

→ The issues in line coding are

(i) power and bandwidth required for transmission

(ii) ability to extract timing information

(iii) presence of low frequency & dc component which is unsuitable for a.c coupled circuits.

(iv) error monitoring abilities etc.

→ In line coding the i/p data is continously processed to generate o/p code.

→ Different line codes are:-

1, Unipolar NRZ
2, Unipolar RZ
3, Bipolar NRZ
4, Bipolar RZ
5, Split phase (Manchester)
6, Gray coding
7, M-ary coding

Advantages of Digital communication System

→ Because of the advances in digital IC technologies and high speed computers, digital communication systems are simpler & cheaper compared to analog systems.

→ Using data Encryption, only permitted recievers can be allowed to detect the transmitted data. This is very useful in military applications.

→ Wide dynamic range is possible since the data is converted to the digital form.

→ Using multiplexing, the speech, video & other data can be merged and transmitted over common channel.

→ Since the transmitted signal is digital, a large amount of noise interference can be tolerated.

→ Since channel coding is used, the errors can be detected and corrected in recievers.

→ Digital communication is adaptive to other advanced branches of data processing such as DSP, image processing, data compression etc.

Disadvantages:-

→ Because of analog to digital conversion, the data rate becomes high. Hence more transmission bandwidth is required for digital communication.

→ Digital communication needs synchronization in case of synchronous modulation.

Shannon Hartley Theorem for Gaussian channels.

(continous channel capacity theorem)

when shannon's theorem of channel capacity is applied specifically to a channel in which the noise is gaussian is known as Shannon-Hartley theorem. It is called information capacity theorem

$$C = B\log_2\left(1+\frac{S}{N}\right) \text{ bits/sec}$$

Here B is the channel bandwidth,

S is the signal power.

N is the total noise power within channel bandwidth

We know that signal noise power is given as,

$$P = \int_{-B}^{B} \text{power spectral density.}$$

Here B is bandwidth. And power spectral density of white noise is $\frac{N_0}{2}$. Hence noise power N becomes,

$$\text{Noise power } N = \int_{-B}^{B} \frac{N_0}{2}\, df$$

$$\therefore \quad N = N_0 B$$

Tradeoff between Bandwidth and signal to noise Ratio

Channel capacity of the gaussian channel is given as,

$$C = B\log_2\left[1+\frac{S}{N}\right]$$

Above eqn shows that the channel capacity depends on two factors:

1, Bandwidth of the channel

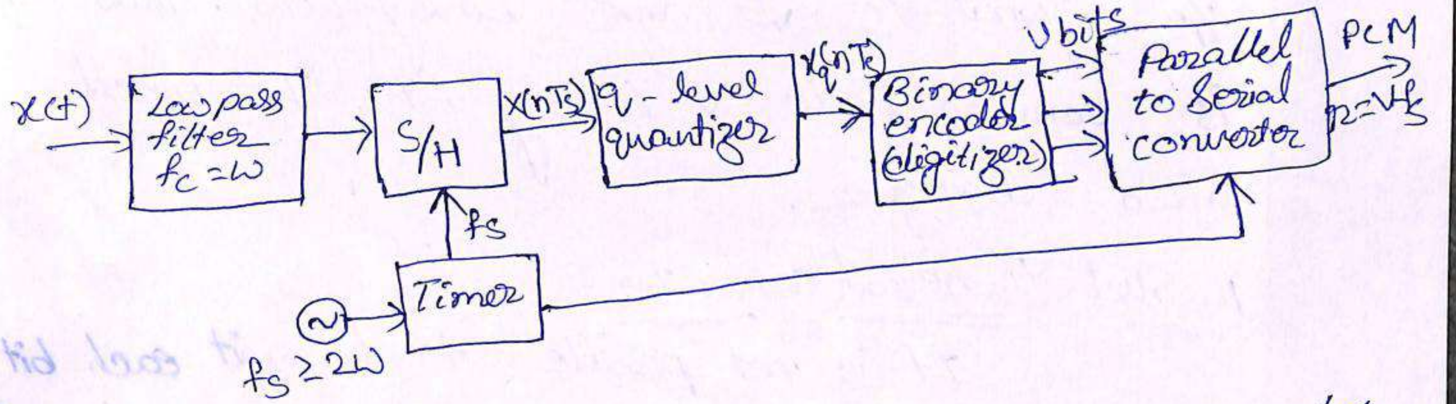
2, Signal to noise ratio $\left[\frac{S}{N}\right]$

Scanned by CamScanner

UNIT-II Pulse digital modulation

Pulse code Modulation

PCM generator



Principle: The pulse code modulator technique samples the input signal $x(t)$ at frequency $f_s \geq 2W$. This sampled variable amplitude pulse is then digitised by the analog to digital converter. The parallel bits then obtained are converted to a serial bit stream.

Low pass filter:- The signal $x(t)$ is first passed through the LPF of cut off frequency W Hz. This LPF blocks all the frequencies components above W Hz. Thus $x(t)$ is band limited to W Hz.

Sample & hold circuit:- The S&H ckt then samples this signal at rate of f_s. f_s is selected sufficiently above Nyquist rate. The o/p of S&H is $x(nT_s)$. This is discrete in time & continous in amplitude.

quantizer:- A q level quantizer compares i/p $x(nT_s)$ with its fixed digital levels. It then assigns anyone of the digital level to $x(nT_s)$ which results in minimum distortion or error. This error is called quantization error. Thus o/p of quantizer is a digital level called $x_q(nT_s)$.

Encoder:- The quantized signal ~~cabbeah~~ level $x_q(nT_s)$ is given to binary encoder. This encoder converts i/p signal to 'v' digits binary word. Thus $x_q(nT_s)$ is converted to 'v' binary bits. The encoder is also called digitizer.

Parallel to serial converter

It is not possible to transmit each bit of the binary word seperately on transmission line. Therefore 'v' binary bits are converted to serial bit stream to generate single baseband signal. In a parallel to serial converter normally shift register does this job. The o/p of PCM generator is thus a single base band signal of binary bits.

Oscillator & timer:- ~~An~~ oscillator generates the clocks for sample and hold and parallel to serial converter.

Transmission Bandwidth in PCM

Let the quantizer use 'v' number of binary digits to represent each level. Then the number of levels that can be represented by 'v' digits will be,

$$q = 2^v$$

Here q represents total no. of digital levels of q level quantizer.

=> Each sample is converted to 'v' binary bits. i.e Number of bits per sample = v. we know that number of samples per second = f_s

∴ number of bits per second is given by, = number of bits per sample × no. of samples per second.

$= v \times f_s$

Signalling rate:- The no. of bits per second is also called signalling rate of PCM.

$$r = v f_s$$

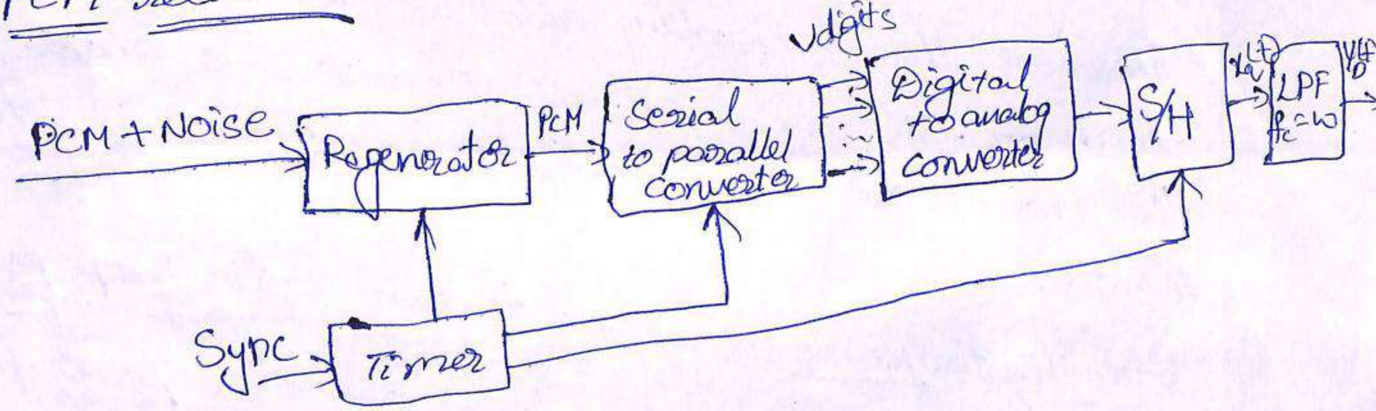
Bandwidth of PCM :- Bandwidth needed for PCM transmission will be given by half of the signalling rate.

i.e

Transmission B.W of PCM

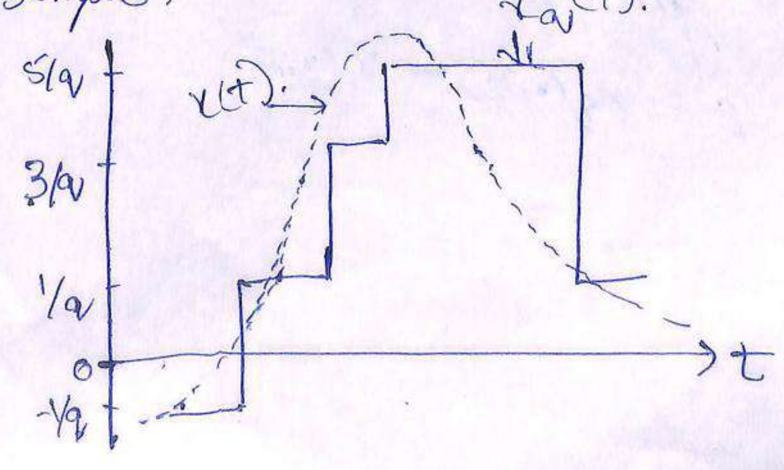
$$\left\{\begin{array}{l} B_T \geq \frac{1}{2} r \\ B_T \geq \frac{1}{2} v f_s \quad \text{since } f_s \geq 2W \\ B_T \geq vW \end{array}\right.$$

PCM reciever



Regenerator & Serial to parallel converter:-

The regenerator at the start of PCM reciever reshapes the pulses & removes the noise. This signal is then converted to parallel digital words for each sample.



D/A converter & S/H

The digital word is converted to its analog Value $x_q(t)$ along with sample & hold

Reconstruction filter :- The o/p of S/H is passed through Low pass reconstruction filter to get $y_D(t)$. As shown in reconstructed signal it is impossible to reconstruct exact original signal $x(t)$ because of permanent quantization error introduced during quantization at the transmitter.

The quantization error can be reduced by increasing the binary levels. This is equivalent to increasing binary digits per sample. But increasing bits 'v' increases the signalling rate as well as transmission bandwidth. The choice of these parameters is made, such that the noise due to quantization error is in tollerable limits.

Quantization Noise & Signal to Noise ratio in PCM

Derivation of quantization Error/Noise or noise power for uniform (Linear) Quantization.

Quantization error

Because of quantization, inherent errors are introduced in the signal. This error is called quantization error.

$$\varepsilon = x_q(nT_s) - x(nT_s).$$

Step Size

→ Let an i/p $x(nT_s)$ be of continous amplitude in the range $-x_{max}$ to $+x_{max}$.

→ The total excursion of i/p $x(nT_s)$ is mapped into 'q' levels on vertical axis.

→ Total amplitude range $= x_{max} - (-x_{max})$
$= 2x_{max}$.

If the amplitude range is divided into q levels then the step size

$$\delta = \frac{x_{max} - (-x_{max})}{q}$$

$$= \frac{2x_{max}}{q}$$

If signal $x(t)$ is normalised to minimum & maximum values equal to 1, then $x_{max} = 1, -x_{max} = -1$

$$\therefore \quad \delta = \frac{2}{q}$$

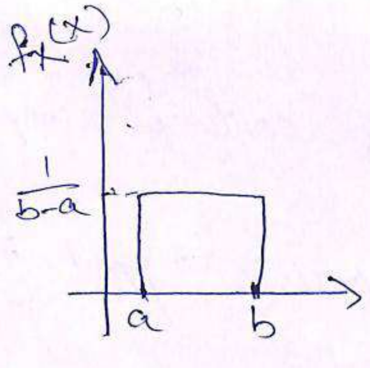
PDF of quantization error

If step size 'δ' is sufficiently small, then it is possible reasonable to assume that the quantization error 'ε' will be uniformly distributed random variable. The maximum quantization error is given by.

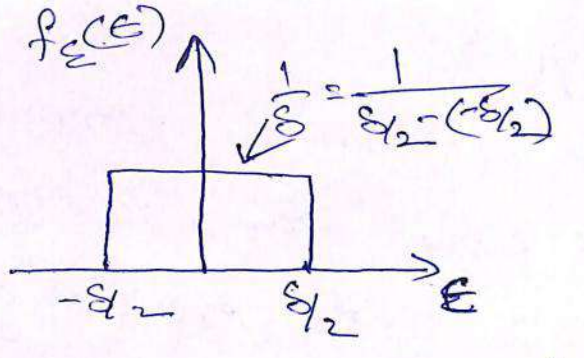
$$\varepsilon_{max} = \left|\frac{\delta}{2}\right|$$

$$\text{i.e} \quad -\frac{\delta}{2} \geq \varepsilon_{max} \geq \frac{\delta}{2}$$

Thus over the interval $\left[-\frac{\delta}{2}, \frac{\delta}{2}\right]$ quantization error is uniformly distributed random variable.



(a) uniform distribution



(b) uniform distribution for quantization error.

In above fig. (a) random variable is said to be uniformly distributed over an interval (a, b). Then PDF is given by,

$$f_X(x) = \begin{cases} 0 & \text{for } x \le a \\ \frac{1}{b-a} & \text{for } a < x \le b \\ 0 & \text{for } x > b \end{cases}$$

Thus with the help of above eqn we can define the PDF for quantization error 'ϵ' as

$$f_\epsilon(\varepsilon) = \begin{cases} 0 & \text{for } \epsilon \le -\frac{\delta}{2} \\ \frac{1}{\delta} & \text{for } -\frac{\delta}{2} \le \epsilon \le \frac{\delta}{2} \\ 0 & \text{for } \epsilon > \frac{\delta}{2} \end{cases} \quad \text{— (1)}$$

Noise power

The signal to quantization noise ratio of the quantizer is defined as

$$\frac{S}{N} = \frac{\text{Signal power (normalized)}}{\text{noise power (normalized)}} \rightarrow \text{divided max voltage.}$$

of. 5V 10V 100V

$\frac{5}{100}$ 0.05 $\frac{10}{100}$ 0.1 $\frac{100}{100}$ 1

If type of signal at i/p i.e. $x(t)$ is known, then it is possible to calculate signal power.

The noise power is given as.

$$\text{Noise power} = \frac{V_{noise}^2}{R}$$

Here V_{noise}^2 is the mean Square value of noise voltage (

Since noise is defined by random variable 'ε' & PDF $f_\varepsilon(\varepsilon)$, its mean Square is given as.

mean square value = $E[\varepsilon^2] = \bar{\varepsilon}^2$

The mean Square value of a random variable 'x' is given as,

$$\bar{X}^2 = E[X^2] = \int_{-\infty}^{\infty} x^2 f_x(x)\,dx$$

Here $E[\varepsilon^2] = \int_{-\infty}^{\infty} \varepsilon^2 f_\varepsilon(\varepsilon)\,d\varepsilon$

from eqn (1) we can write.

$$E[\varepsilon^2] = \int_{-\delta/2}^{\delta/2} \varepsilon^2 \times \frac{1}{\delta}\,d\varepsilon = \frac{1}{\delta}\left[\frac{\varepsilon^3}{3}\right]_{\delta/2}^{\delta/2}$$

$$= \frac{1}{\delta}\left[\frac{(\delta/2)^3}{3} + \frac{(\delta/2)^3}{3}\right]$$

$$= \frac{1}{3\delta}\left[\frac{\cancel{2}\delta^3}{\cancel{8}4}\right]$$

$$E[\varepsilon^2] = \frac{\delta^2}{12}$$

∴ V_{noise}^2 = mean Square value $= \frac{\delta^2}{12}$

when load resistance R=1ohm. then the noise pow

is normalised i.e

Noise powe $= \frac{V_{noise}^2}{R} = \frac{\frac{\delta^2}{12}}{1} = \frac{\delta^2}{12}$.

Derivation of maximum Signal to quantization Noise ratio for linear quantization

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{\text{Normalized noise power}}$$

$$= \frac{\text{Normalized Signal power}}{\delta^2/12}$$

The number of bits 'V' & quantization levels 'q' are related as

$$q = 2^V$$

$$\therefore \delta = \frac{2x_{max}}{2^V}$$

$$\therefore \frac{S}{N} = \frac{\text{Normalized Signal power}}{\left(\frac{2x_{max}}{2^V}\right)^2 / 12}$$

$$\frac{S}{N} = \frac{P}{\frac{4x^2_{max}}{2^{2V}} / 12} = \frac{3P}{x^2_{max}} \times 2^{2V}$$

Max. Signal to quantization noise ratio : $\frac{S}{N} = \frac{3P}{x^2_{max}} 2^{2V}$

Signal to noise ratio increases with increase in bits per sample.

If we assumed that input $x(t)$ is normalized, i.e

$$x_{max} = 1$$

then $\frac{S}{N} = \frac{3P \times 2^{2V}}{\cancel{x^2_{max}}}$.

If the destination signal power P is normalised, i.e,

$$P \leq 1$$

then S/N is given as $\frac{S}{N} \leq 3 \times 2^{2V}$

Expressing $\frac{S}{N}$ in decibles.

$$\left[\frac{S}{N}\right] dB \leq 10 \log_{10} \left[\frac{S}{N}\right] dB$$

$$\leq 10 \log_{10} \left[3 \times 2^{2V}\right]$$

$$\leq (4.8 + 6V)\, dB.$$

Non uniform quantization & companding

⇒ In non uniform quantization, the step size is not fixed. It varies according to certain law or as per i/p signal amplitude. $\varepsilon_{max} = |\delta/2|$ $\delta = \frac{2x_{max}}{qV} = 2/q_v = 1/8$ $= 2/16$ $\varepsilon_{max} = (1/16)$ of fullrange voltage 16V. Let $\varepsilon = 1V$.

Necessity of nonuniform quantization for Speech Signal

We know that speech & music signals are characterized by large crest factor. That is for such signals the ratio of peak to rms value is very high.

$$\text{Crest factor} = \frac{\text{Peak value}}{\text{rms value}}$$

$$= \text{very high for speech \& music}.$$

If we normalize the signal power i.e if $P=1$, then

$$\left[\frac{S}{N}\right]dB \geq (4.8+6v)\,dB.$$

Here power P is defined as, $P = \frac{\overline{V^2_{signal}}}{R} = \frac{\overline{x^2(t)}}{R}$

$$\overline{V^2_{signal}} = \text{mean square value of signal voltage}$$

$$= \overline{x^2(t)}$$

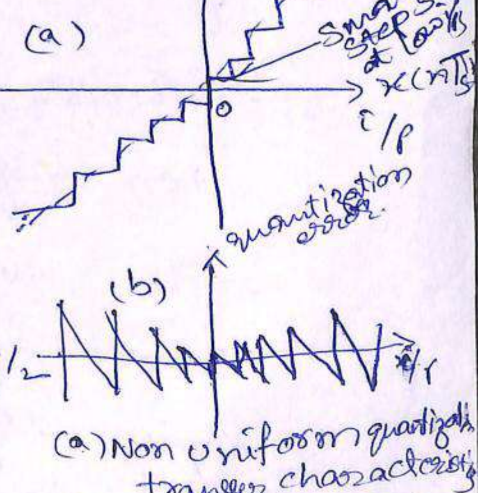
∴ Normalized power will be,

$$P = \frac{\overline{x^2(t)}}{1} \; [\text{with } R=1]$$

$$P = \overline{x^2(t)}$$

$$\therefore \text{Crest factor} = \frac{\text{Peak value}}{\text{RMS Value}} = \frac{x_{max}}{[\overline{x^2(t)}]^{1/2}}$$

$$= \frac{x_{max}}{\sqrt{P}}$$



(a) Non uniform quantization transfer characteristics
(b) Quantization error.

when we normalize the signal $x(t)$, then

$$x_{max} = 1$$

Putting above value of x_{max} in crest factor

$$\text{Crest factor} = \frac{1}{\sqrt{P}}$$

For a large crest factor of voice & music signals P should be very very less then in above eqn.

$$P << 1$$

Therefore actual signal to noise ratio will be significantly less than the value that is given by the eqn

$$\left[\frac{S}{N}\right]dB \geq (4.8 + 6v)dB.$$

-> Since in this eqn $P = 1$.

-> consider eqn $\left[\frac{S}{N}\right] = 3 \times 2^{2v} \times P$

$$(3 \times 2^{2v} \times P)\big|_{P<<1} << (3 \times 2^{2v} \times P)\big|_{P=1}$$

This eqn shows that the signal to noise ratio for larg crest factor signal (P<<1) will be very less than tha of theoretical value.

-> Therefore such large crest factor signals should use non uniform quantization.

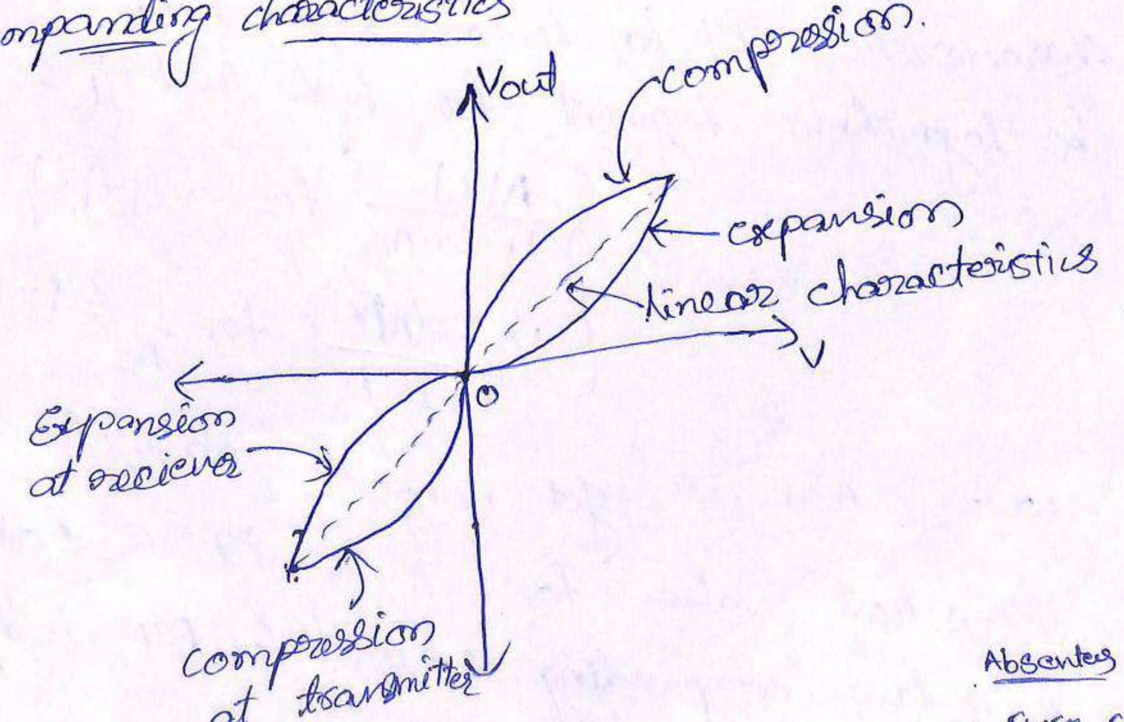
-> The quantization noise is directly related to step size Therefore at low signal levels (P<<1) noise can be kept low keeping step size low. This means that at low signal le S/N ratio can be increased by decreasing step size. This means that step size δ should be varied acco to the signal level to keep S/N ratio at required value. This is nothing but non uniform quantizatio

Companding in PCM

Definition

Normally we don't know how the signal level will vary in advance. Therefore the non uniform quantization becomes difficult to implement. Therefore the signal is amplified at low signal levels & attenuated at high signal levels. After this process, uniform quantization is used. This is equivalent to more step size at low signal levels & small step size at high signal levels. At the reciever a reverse process is done. That is signal is attenuated at low signal levels and amplified at high signal levels to get original signal. Thus compression of signal at transmitter & expansion at reciever is called combinely as companding.

Companding characteristics



Absentees III-B.

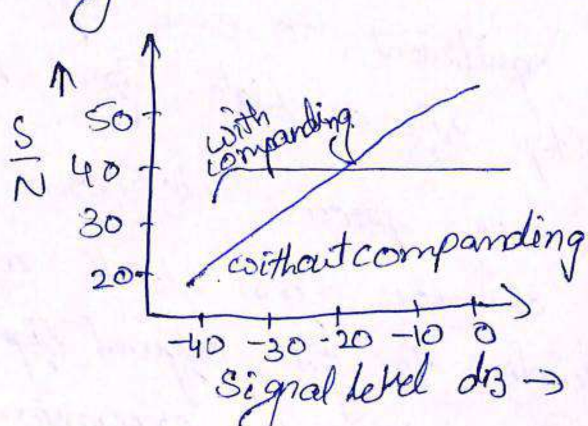
53, 54, 55, 56, 58, 59,
62, 63, 64, 65, 67, 68, 69,
73, 78, 80, 85, 87,
89, 90, 93, 96, 98, 99,
A1, A3, LE 8, 9, 10

μ-law companding for speech signals

Normally for speech & music signals a μ-law compression is used. This is defined as

$$Z(x) = (\mathrm{Sgn}\, x)\, \frac{\ln(1+\mu|x|)}{\ln(1+\mu)} \quad |x| \le 1$$

fig shows the variation of signal to noise ratio w.r.t signal level without companding & with compandi



A-law for companding

The A-law provides piecewise compressor characteristic. It has linear segment for low level i/p's & logarithmic segment for high level i/p's. It is defined as

$$Z(x) = \begin{cases} \dfrac{A|x|}{1+\ln A} & \text{for } 0 \le |x| \le \dfrac{1}{A} \\[2ex] \dfrac{1+\ln(A|x|)}{1+\ln A} & \text{for } \dfrac{1}{A} \le |x| \le 1 \end{cases}$$

when A = 1 we get uniform quantization. The practical value for A is 87.56. Both A-law & μ-law companding is used for PCM telephone systems.

Advantages of PCM

1, Effect of channel noise & interference is reduced.

2, PCM permits regeneration of pulses along the transmission path. This reduces the noise interference.

3, The bandwidth & S/N ratio are related by exponential law.

4, Multiplexing of various PCM signal is easily possible.

5, Encryption or decryption can be easily incorporated for Security purpose. ✓

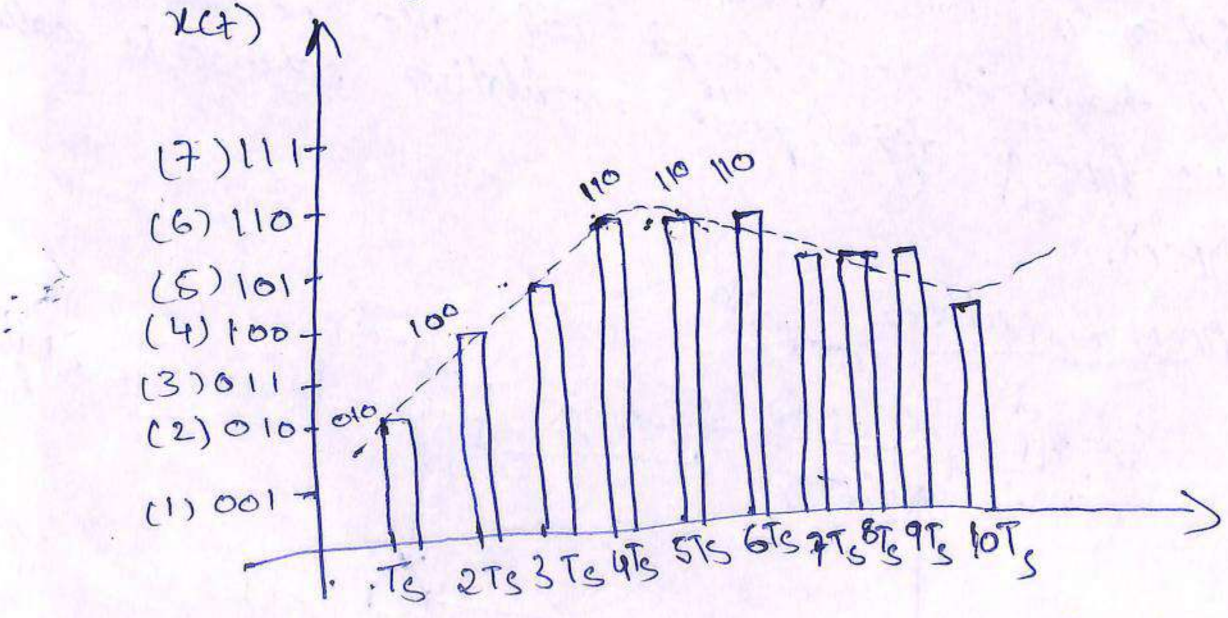
Limitations of PCM

1, PCM Systems are complex compared to analog Pulse modulation methods.

2, The channel bandwidth is also increased because of digital coding of analog pulses.

Differential pulse code modulation

Redundant information in PCM



Redundant information in PCM.

fig Shows a continous time Signal $x(t)$ by dotted line. This Signal is Sampled by flat top Sampling at intervals T_S, $2T_S$, $3T_S$ ... nT_S. The Sampling frequency is Selected to be higher than nyquist rate. The Samples are encoded by using 3 bit PCM. The Sample is quantized to the nearest digital level. ~~as to~~ The encoded value of each Sample is written on the top of the Samples. we can See from the fig that the Samples taken at $4T_S$, $5T_S$ & $6T_S$ are encoded to Same value of (110). This information can be carried only by one Sample. But the three Samples are carrying the Same information means it is redundant.

Principle of DPCM.

If this redundancy is reduced, then overall bit rate will decrease & number of bits required to transmit one Sample will also be reduced. This type of digital pulse modulation Scheme is called DPCM.

DPCM transmitter

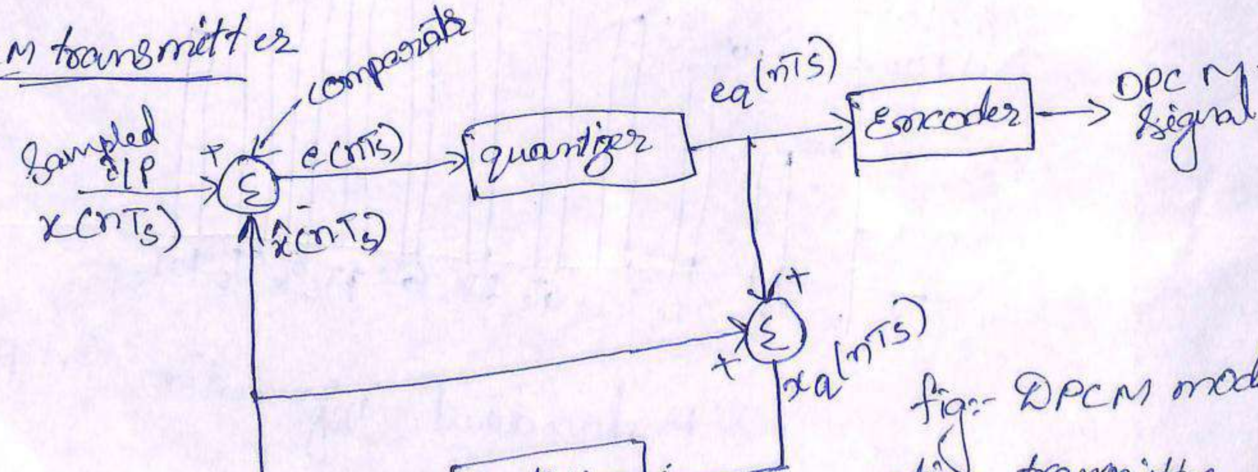


fig:- DPCM modulation transmitter.

→ The DPCM works on the principle of prediction. The value of the present sample is predicted from past samples. The prediction may not be exact but it is very close to actual sample value.

→ fig shows transmitter of DPCM. The sampled signal is denoted by $x(nT_s)$ & predicted signal is $\hat{x}(nT_s)$. The comparator finds out difference b/w sample value & actual value. This is called error & denoted by $e(nT_s)$.

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

Thus error is the difference between input sample $x(nT_s)$ and prediction of it $\hat{x}(nT_s)$. The predicted value is produced by using a prediction filter. The quantizer o/p signal $e_q(nT_s)$ & previous prediction is added and given as i/p to the prediction filter. This signal is called $x_q(nT_s)$. This makes the prediction more & more close to the actual sampled signal. We can see that the $e_q(nT_s)$ is very small & can be ed encoded by using small no. of bits. Thus no. of bits per sample are reduced in DPCM.

→ The quantizer o/p can be written as,

$$e_q(nT_s) = e(nT_s) + q(nT_s)$$

Here $q(nT_s)$ is the quantizer o/p error. The prediction filter i/p $x_q(nT_s) = \hat{x}(nT_s) + e_q(nT_s)$

Substituting $e_q(nT_s)$ is

$$x_q(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s)$$

we know $e(nT_s) = x(nT_s) - \hat{x}(nT_s)$

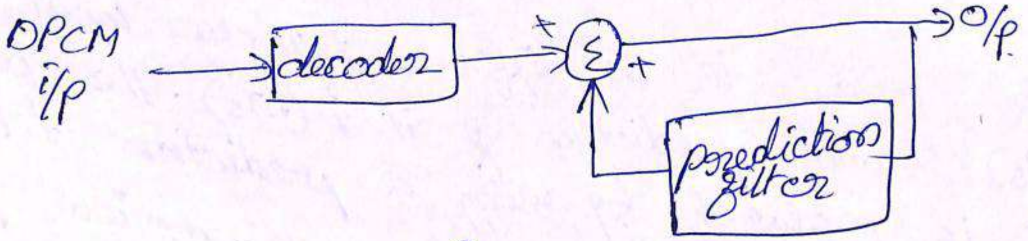
$$e(nT_s) + \hat{x}(nT_s) = x(nT_s)$$

Putting the value of $e(nT_S) + \hat{x}(nT_S)$

$$x_q(nT_S) = x(nT_S) + q(nT_S).$$

Thus the quantized version of signal $x_q(nT_S)$ is sum of original sample value & quantization error $q(nT_S)$. The quantization error can be 'positive or negative.

Reconstruction of DPCM ~~sample~~ signal.



DPCM reciever.

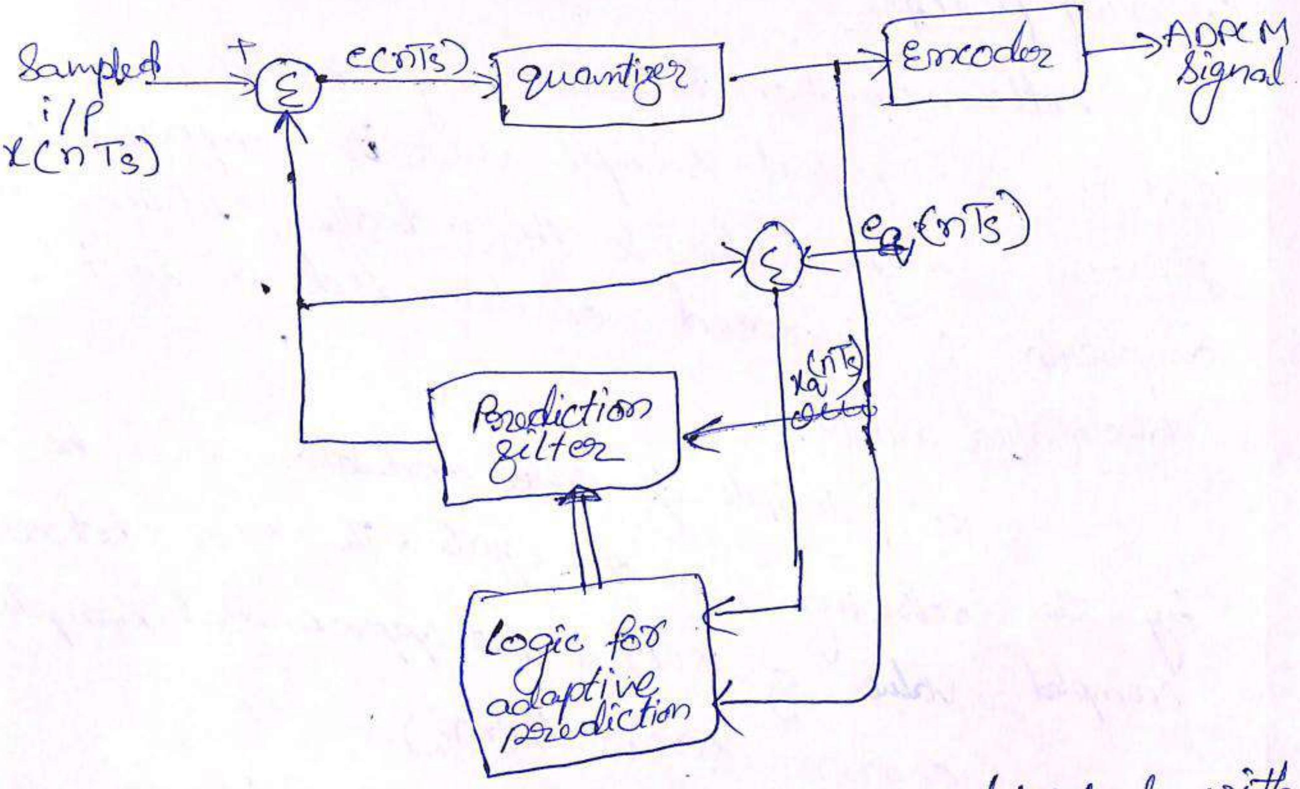
The decoder first reconstructs the quantized error signal from incoming binary signal. The prediction filter o/p & quantized error signals are summed up to give the quantized version of original signal.

→ Thus the signal at the reciever differs from actual signal by quantization error $q(nT_S)$. which is introduced permanently in the reconstructed signal

ADPCM

Principle:- The ADPCM uses adaptive quantizer that has time varying step size $\delta(n)$. This step size depends upon amplitude level & spectrum of speech signal.

Block diagram & implementation



→ The adaptive quantization can be implemented with forward estimation or backward estimation.

→ In forward estimation, unquantized sample of the input signal are used to obtain step size.

→ In backward estimation samples of quantizer o/p are used to obtain step size.

→ The adaptive predictor generates step size from $x_q(nT_s)$ & $e_q(nT_s)$. The prediction filter accordingly generates an estimate of i/p signal named as $\hat{x}(nT_s)$.

Advantages:-

1, The bit rate required by PCM is reduced to half of its earlier value.

2, Because of backward estimation the problems of level estimation, delay & requirement of buffer are eliminated.

3, efficient for speech coding at low bit rates.

Delta Modulation

operating principle

Delta modulation transmits only one bit per sample. That is the present sample value is compared with previous sample value & the indication, whether the amplitude is increased or decreased is sent.

Mathematical analysis

The principle of delta modulation can be explained by the following set of eqns. The error between the sampled value of $x(t)$ & last approximated sample is given as

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

Here $e(nT_s)$ = Error at present sample
$x(nT_s)$ = sample signal of $x(t)$
$\hat{x}(nT_s)$ = Last sample approximation of staircase o/p waveform.

We can call $u(nT_s)$ as the present sample approximation of stair case o/p.

Then $u[(n-1)T_s] = \hat{x}(nT_s)$
= Last sample approximation of staircase o/p waveform.

Let the quantity $b(nT_s)$ be defined as,

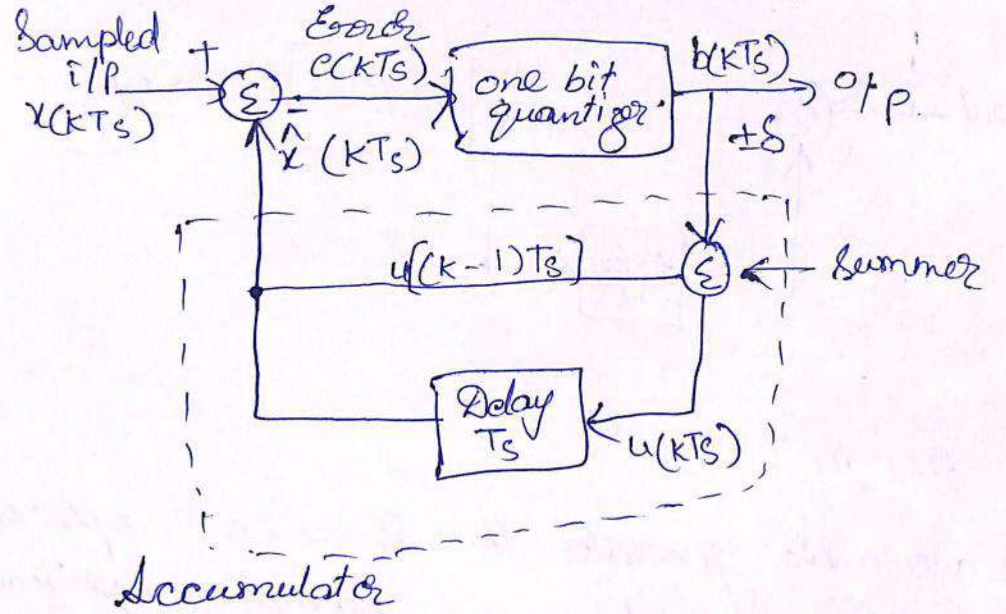
$$b(nT_s) = \delta \, \mathrm{sgn}[e(nT_s)]$$

That is depending on sign of error $e(nT_s)$ the sign of step size δ will be decided. In otherwords

$$b(nT_s) = +\delta \quad \text{if } x(nT_s) \geq \hat{x}(nT_s)$$
$$= -\delta \quad \text{if } x(nT_s) < \hat{x}(nT_s)$$

If $b(nT_s) = +\delta$; binary 1 is transmitted.
& if $b(nT_s) = -\delta$; " 0 " "
T_s → Sampling interval.

DM transmitter



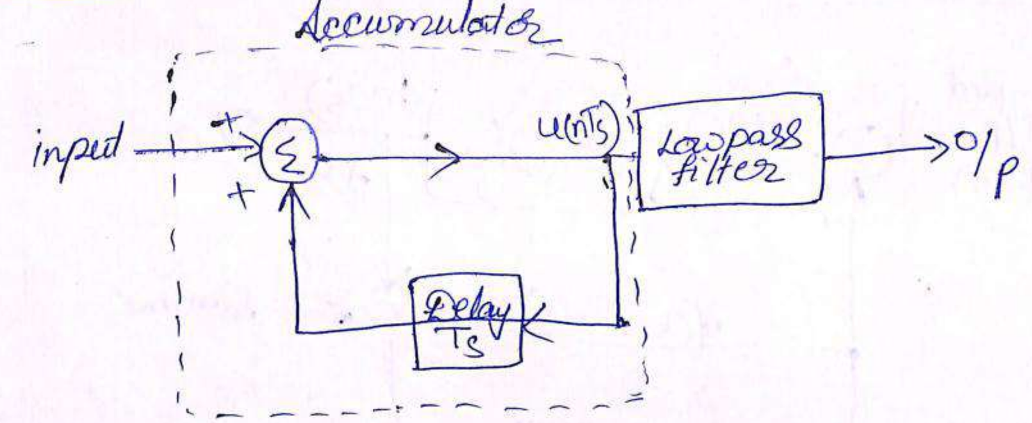
The Summer in the accumulator adds quantizer o/p (±δ) with the previous sample approximation. This gives the present sample approximation, i.e.

$$u(nT_s) = u(nT_s - T_s) + (\pm\delta) \text{ or}$$
$$= u[(n-1)T_s] + b(nT_s)$$

→ The previous sample approximation $u[(n-1)T_s]$ is restored by delaying on sample period T_s. The sampled i/p signal $x(nT_s)$ & Stair case approximated signal $\hat{x}(nT_s)$ are substracted to get error signal $e(nT_s)$.

→ Depending on the sign of $e(nT_s)$ one bit quantizer produces an o/p step of +δ or −δ. If the step size is +δ, then binary '1' is transmitted & if it is −δ, then binary '0' is transmitted.

DM receiver



DM rx

→ The accumulator generates the staircase approximated signal o/p & is delayed by one sampling period Ts. It is then added to i/p signal.

→ If i/p is binary '1' then it adds +δ to the previous o/p.
" " " " '0' then one step δ is subtracted from the delayed signal.

→ The low pass filter has cut off frequency equal to highest frequency in x(t). This filter smoothens the staircase signal to reconstruct x(t).

Advantages

The DM has following advantages over PCM.

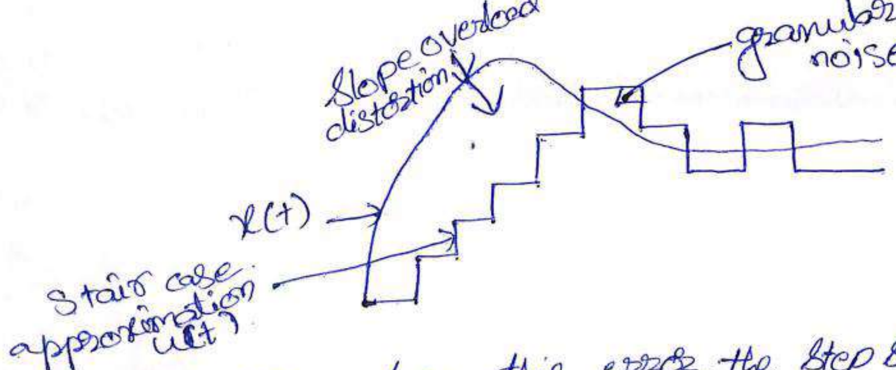
1) DM transmits only one bit for one sample. Thus signaling rate & transmission channel bandwidth is quite small for DM.

2) The tx & rx implementation is very much simple for DM. There is no analog to digital converter involved in DM,

Disadvantages

1, Slope overload distortion

This distortion arises because of the large dynamic range of i/p signal. The rate of rise of i/p systems signal x(t) is so high that the staircase signal cannot approximate it, the step size δ becomes too small for stair case signal u(t) to follow the steep segment of x(t). Thus there is a large error b/w the staircase approximated signal & the original i/p x(t). This error is called slope overload distortion.



Remedy:- To reduce this error, the step size should be increased when slope of signal of x(t) is high.

Granular noise

It occurs when the step size is too large compared to small ~~S~~variations in i/p signal, That is ~~stair~~ case for very small variations in i/p signal, the staircase signal is changed by large amount (δ) because of large step size. fig shows when i/p signal is almost flat, the stair case u(t) keeps on oscillating by ±δ around the signal. The error b/w i/p & approximated signal is called granular noise.

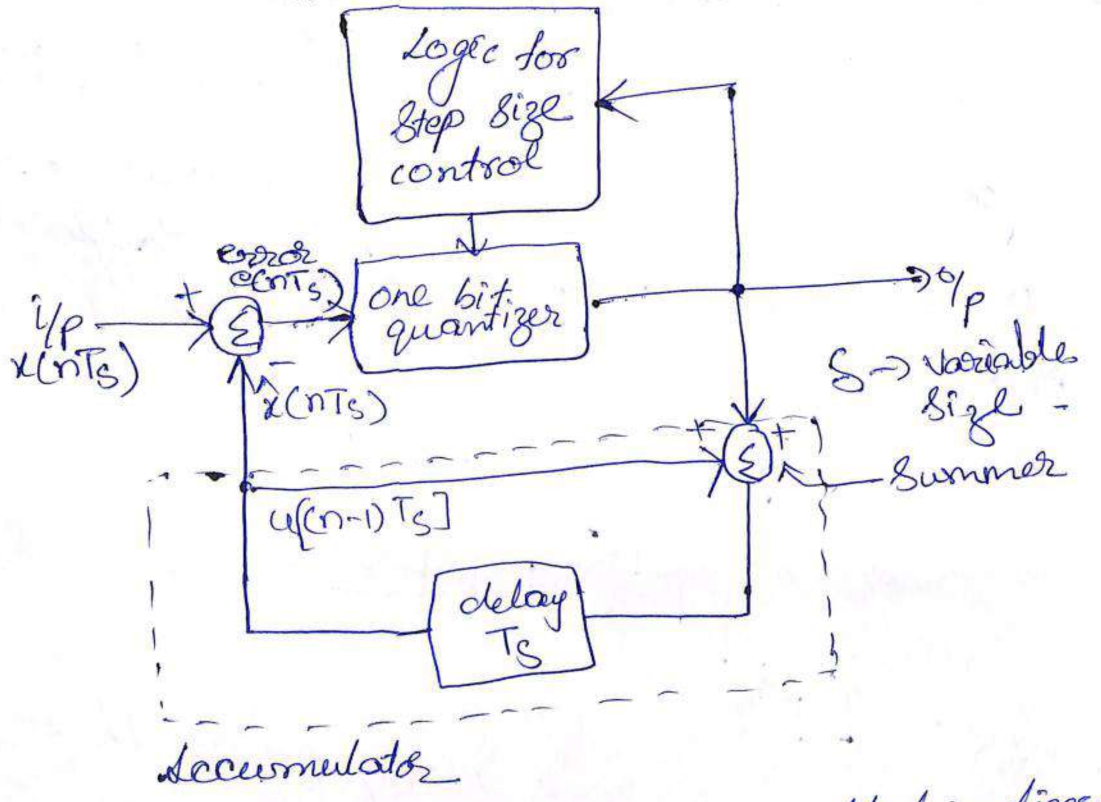
The solution is to make step size small

Adaptive Delta modulation

⇒ To overcome the quantization errors due to slope overload & granular noise. The step size is made adaptive.

→ The adaptive delta modulators can take continous changes in step size or discrete changes in step size.

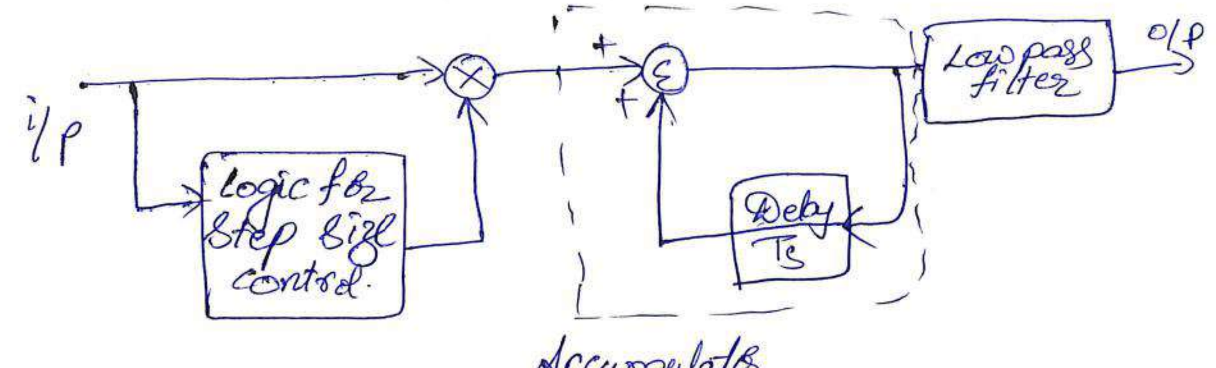
Transmitter



⇒ The logic step size control is added in diagram.

⇒ The step size increases or decreases according to certain rule depending on one bit quantizer o/p. For example if one bit quantizer o/p is high then step size may be doubled for next sample. If one bit quantizer o/p is low, then step size may be reduced by one step.

Reciever



Accumulator .

Reciever
In reciever of ADM the first part generates the Step Size from each incoming bit. Exactly the Same process is followed as that in transmitter. The previous i/p & present i/p decides the Step Size. It is then given to an accumulator which builds up Stair case waveform. The LPF than Smoothens out the Stair case waveform to reconstruct the Smooth Signal

Advantages

1, The S/N ratio is better than ordinary D.M because of the reduction in Slope overload distortion & granular noise

2, Because of variable Step Size, the dynamic range of ADM is wide.

3, Utilization of bandwidth is better than delta modulation.

⇒ Consider a sine wave of frequency f_m & amplitude A_m applied to a delta modulator of step size δ. Show that the slope overload distortion will occur if

$A_m > \frac{\delta}{2\pi f_m T_s}$ where T_s is sampling period.

Sol:- Let sine wave be represented by.

$$x(t) = A_m \sin(2\pi f_m t)$$

Slope of $x(t)$ will be maximum when derivation of $x(t)$ w.r.t 't' will me max. The max slope of delta modulator is .

$$\text{max slope} = \frac{\text{Stepsize}}{\text{Sampling period}}$$

$$= \frac{\delta}{T_s}$$

Slope overload distortion will take place if slope of sine wave is greater than slope of delta modulator

i.e

$$\max\left|\frac{d}{dt}x(t)\right| \geq \frac{\delta}{T_s}$$

$$\max\left|\frac{d}{dt}A_m\sin(2\pi f_m t)\right| > \frac{\delta}{T_s}$$

$$\max\left|A_m \overset{2\pi f_m}{\wedge} \cos(2\pi f_m t)\right| > \frac{\delta}{T_s}$$

$$A_m 2\pi f_m > \frac{\delta}{T_s}.$$

$$\boxed{A_m = \frac{\delta}{2\pi f_m T_s}}$$

2, Derive an expression for signal to quantization noise power ratio for D.M. Assume that no slope overload distortion exists

w.k.t $A_m \le \frac{\delta}{2\pi f_m T_s}$.

From above eqn, the max signal amplitude.

$$A_m = \frac{\delta}{2\pi f_m T_s}$$

Signal power $P = \frac{V^2}{R}$

Here V is r.m.s value of signal. Here $V = \frac{A_m}{\sqrt{2}}$.

$$\text{So } P = \frac{\left(\frac{A_m}{\sqrt{2}}\right)^2}{R}$$

Normalise signal power

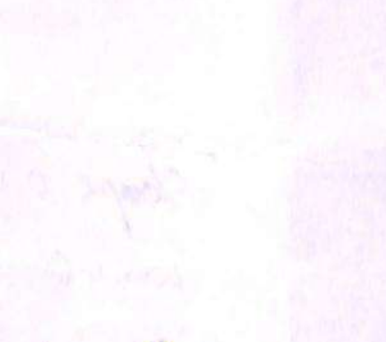
$$P = \left(\frac{A_m}{\sqrt{2}}\right)^2 = \frac{A_m^2}{2}$$

$$P = \frac{\delta^2}{8\pi^2 f_m^2 T_s^2}$$

(ii) To obtain noise power

w.k.t the max quantization error in D.M is δ.
Let the error be uniformly distributed over $[-\delta, \delta]$.

From the fig the PDF of quantization error is



$$f_\epsilon(\epsilon) = \begin{cases} 0 & \text{for } \epsilon < -\delta \\ \frac{1}{2\delta} & \text{for } -\delta < \epsilon < \delta \\ 0 & \text{for } \epsilon > \delta. \end{cases}$$

$$\text{Noise power} = \frac{V_{noise}^2}{R}$$

mean square value is given as

$$E[\epsilon^2] = \int_{-\infty}^{\infty} \epsilon^2 \cdot \cancel{\frac{1}{2\delta}} f_\epsilon(\epsilon) d\epsilon$$

$$E[\varepsilon^2] = \int_{-\alpha}^{\alpha} \varepsilon^2 \frac{1}{2\delta} d\varepsilon$$

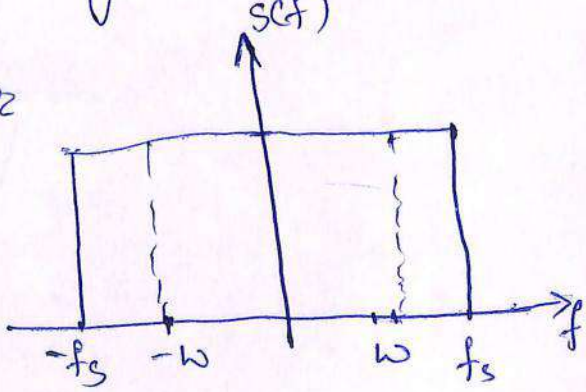
$$= \frac{1}{2\delta}\left[\frac{\varepsilon^3}{3}\right]_{-\delta}^{\delta}$$

$$= \frac{1}{2\delta}\left[\frac{\delta^3}{3} + \frac{\delta^3}{3}\right]$$

$$= \underline{\underline{\frac{\delta^2}{3}}}$$

Normalised noise power $= \frac{\delta^2}{3}$ $\because R = 1$.

The's noise power is uniformly distributed over $-f_s$ to f_s range. At the o/p of DM receiver there is low pass reconstruction filter. whose cut off frequency is 'W'. This cutoff frequency is equal to highest signal frequency. The reconstruction filter passes part of noise power at o/p so from fig ~~o/p noise~~



$$\text{o/p noise power} = \frac{W}{f_s} \times \text{noise power}$$

$$= \frac{W}{f_s} \times \frac{\delta^2}{3}$$

w.k.t $f_s = \frac{1}{T_s}$

$$\text{o/p noise power} = \frac{W T_s \delta^2}{3}$$

$$\frac{S}{N} = \frac{\frac{\delta^2}{8\pi^2 f_m^2 T_s^2}}{\frac{W T_s \delta^2}{3}} = \frac{3}{8\pi^2 W f_m^2 T_s^3}$$

→ Consider the speech signal with $f_m = 3.4\,KHz$ & max amplitude of 1V. This speech signal is applied to a DM. whose bit rate is set at 20 kbps. Discuss the choice of appropriate step size for modulator.

Given, $A_m = 1V$ & $f_m = 3.4\,KHz$

$$f_s = \frac{1}{T_s} = 20\times10^3$$

The max rate of change of i/p signal occurs at highest frequency component & max amplitude.

Hence $A_m > \frac{\delta}{2\pi f_m T_s}$

$$\delta < A_m 2\pi f_m T_s.$$

$$\delta < 1 \times 2\pi \times 3.4\times10^3 \times \frac{1}{20\times10^3}$$

$$\boxed{\delta < 1.068 \text{ Volts}}$$

Tutorial

1, A television Signal with a Bandwidth of 4.2MHz is transmitted using Binary PCM. The number of quantization level is 512.

Calculate, (i) code word length (ii) Transmission bandwidth. (iii) Final bit rate (iv) o/p Signal to quantization noise ratio.

(i) $q = 2^v$

$512 = 2^v$

$v = \log_2 512$

$\underline{V = 9}$

(ii) $B_T \geq vW$

$= B_T \geq 9 \times 4.2 \times 10^6$

$B_T \geq 37.8MHz$

(iii) $r = vf_s$

$= 9 \times 2 \times 4.2 \times 10^6$

$= \underline{75.6 \times 10^6}$

(iv) $\left[\frac{S}{N}\right]_{dB} \leq 4.8 + 6v\, dB$

$\leq 4.8 + 6 \times 9\, dB$

$= 58.8dB$

2, The B.W of Signal i/p to the PCM is restricted to 4KHz. The i/p varies from -3.8V to +3.8V & has avg power of 30mW. The required S/N is 20dB. The modulator produces binary o/p. Assume uniform quantization.

(i) Calculate the number of bits required per Sample.

(ii) o/p's of 30 Such PCM coders are time multiplexed. what is the minimum required transmission bandwidth for the multiplexed Signal?

(i) $\frac{S}{N} = 100$.

$\frac{S}{N} = \frac{3P.2^{2V}}{x_{max}^2}$

$100 = \frac{3 \times 30 \times 10^{-3} \times 2^{2V}}{(3.8)^2}$

$\underline{V = 7 \text{ bits}}$

(ii) $W = 4KHz$

Since there are 30 PCM coders which are time multiplexed.

$B_T \geq 30 \times 4 \times V$

$\geq 30 \times 7 \times 4\, KHz$

$\geq 840KHz$

$r = 2 \times B_T = 1680$ bits/sec.

UNIT II Digital modulation Technique

Syllabus:- Introduction, ASK, ASK modulator, coherent ASK detector, Non-coherent ASK detector, FSK, Bandwidth & frequency Spectrum of FSK, Non coherent FSK Detector, Coherent FSK detector, FSK detection using PLL, BPSK, coherent PSK detection, QPSK, Differential PSK.

Introduction

There are basically two types of transmission of digital signals.

- Baseband data transmission:- The digital data is transmitted over the channel directly. There is no carrier or any modulation. This is suitable for transmission over short distance

Passband data transmission:- The digital data modulates high frequency sinusoidal carrier. Hence it is also called digital CW modulation. It is suitable for transmission over long distances.

- Types of passband ~~transmission~~ Modulation:-

1, PSK:- In this technique, digital data modulates phase of the carrier.

2, FSK:- In this technique, digital data modulates frequency of the carrier.

3, ASK:- In this technique, the digital data modulates amplitude of the carrier.

2/1/16

Absentees - III b

57, 64, 66, 67, 66, 69, 70, 76, 79, 80,
91, 96, A2, A3, LE'S 7, 8, 10

Types of Reception for passband transmission Scheme

Coherent (Synchronous) detection:- In this method, the local carrier generated at the receiver is phase locked with the carrier at the transmitter. Hence it is also called Synchronous detection.

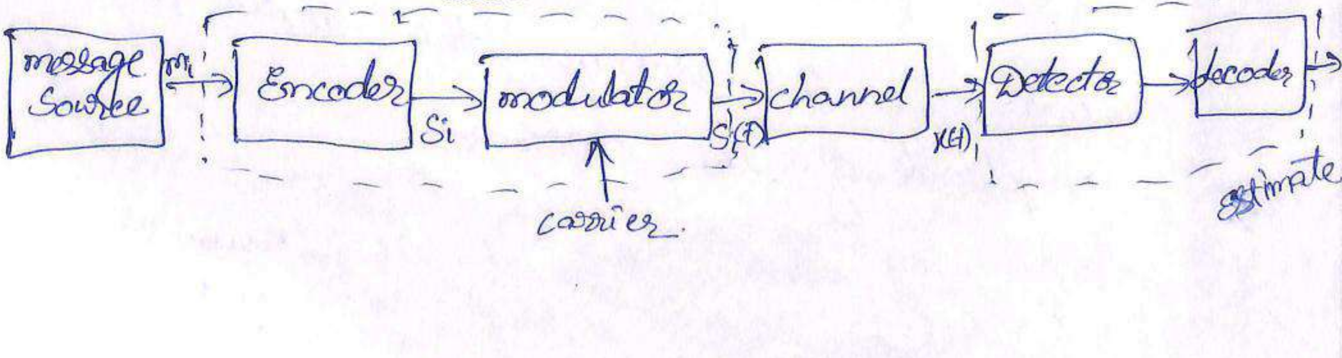
Non coherent detection:- In this method, the receiver carrier need not be phase locked with transmitter carrier. Hence it is also called envelope detection.

Non coherent detection is simple but it has higher probability of error.

Advantages of passband transmission over Baseband transmission

1, Long distance transmission
2, analog channels, can be used for ~~bandwidth~~ transmission conservation
3, Multiplexing techniques can be used for bandwidth conservation.
x(4) minimum channel bandwidth.
5, Maximum resistance to interfering signals.
6, Minimum circuit complexity]x
4) Problems such as ISI & crosstalk are absent.
5, passband transmission can talk place over wireless channels also.
6, Large number of modulation techniques are available.

Passband transmission model



Amplitude Shift Keying or ON-off keying

ASK or ON-off keying is the simplest digital modulation technique.

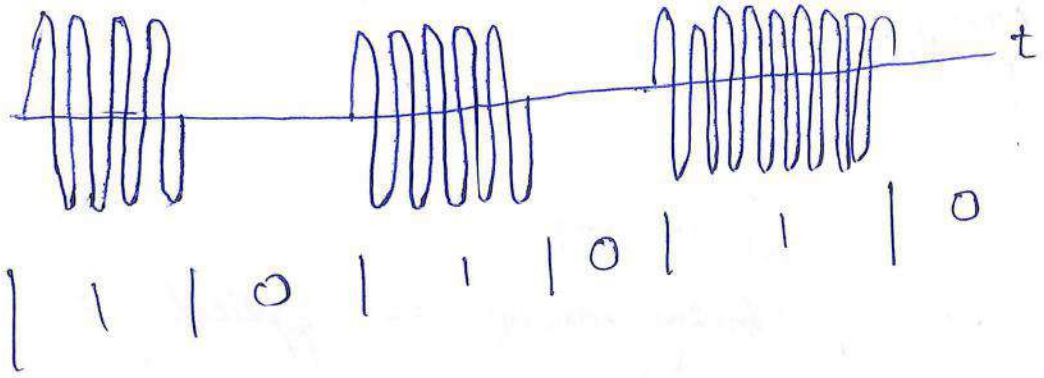
→ In this method, there is only one unit energy carrier and is switched on or off depending upon the input binary sequence. The ASK waveform can be represented as,

$$S(t) = \sqrt{2P_s}\cos(2\pi f_0 t) \quad (\text{To transmit '1'})$$

To transmit symbol '0', the signal $S(t) = 0$. That is no signal is transmitted. $S(t)$ contains some complete cycles of carrier frequency 'f'. Thus

Symbol '1' ⇒ pulse is transmitted,
Symbol '0' ⇒ no pulse is transmitted

Thus ASK waveform looks like an ON-off of the signal. Hence it is also called ON-off keying.



$$P_s = \frac{A_m^2}{2}$$

$$A_m = \sqrt{2P_s}$$

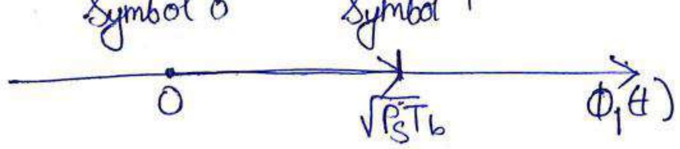
Signal space diagram of ASK

The ASK waveform of eqn 3.7 $S(t) = \sqrt{2P_S}\cos(2\pi f_0 t)$ for symbol '1' can be represented as,

$$S(t) = \sqrt{P_S T_b}\sqrt{2/T_b}\cos(2\pi f_0 t)$$

$$= \sqrt{P_S T_b}\,\phi_1(t)$$

Thus there is only one carrier function $\phi_1(t)$. The signal space diagram will have two points on $\phi_1(t)$. One will be at zero & other will be at $\sqrt{P_S T_b}$.

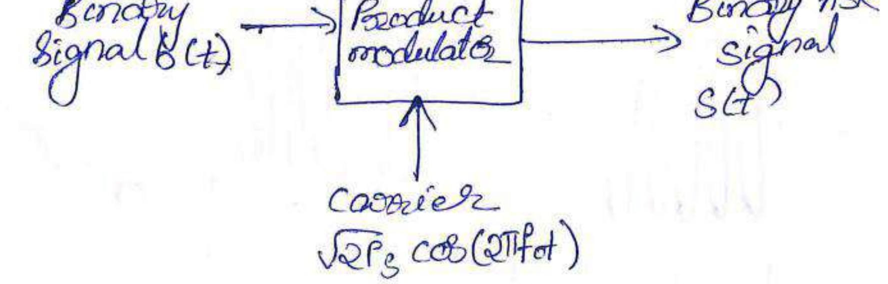


Signal space diagram of ASK.

Therefore the distance b/w the two signal points will be,

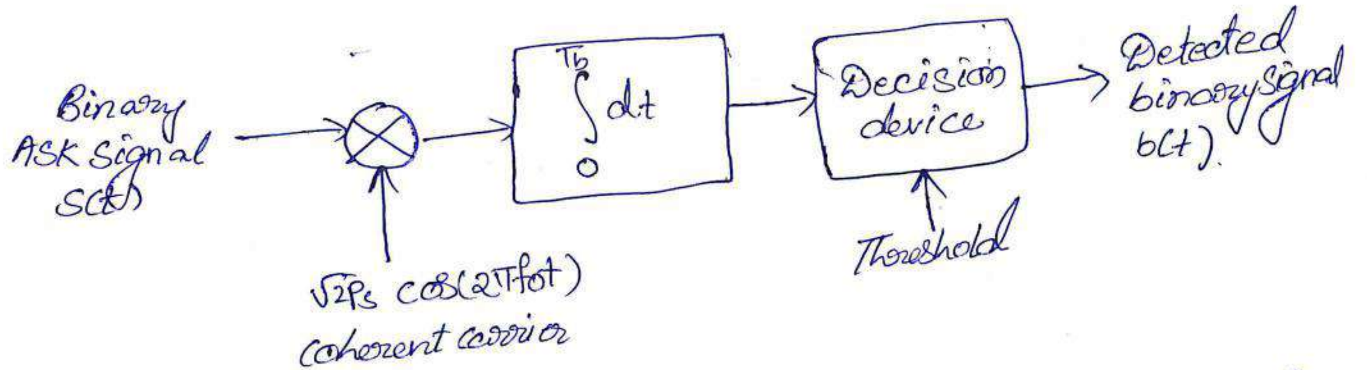
$$d = \sqrt{P_S T_b} = \sqrt{E_b}$$

ASK generator



The i/p binary sequence is applied to the product modulator. The product modulator amplitude modulates the sinusoidal carrier. It passes the carrier when i/p bit is '1'. It blocks the carrier when i/p bit is '0'.

Coherent ASK detector



The ASK signal is applied to the correlator consisting of multiplier & integrator. The locally generated coherent carrier is applied to the multiplier. The o/p of multiplier is integrated over bit period. The decision device takes the decision at end of every bit period. It compares o/p of integrator with threshold. Decision is taken in favour of '1' when threshold is exceeded. Decision is taken as '0' if threshold is not exceeded.

Non coherent ASK reception

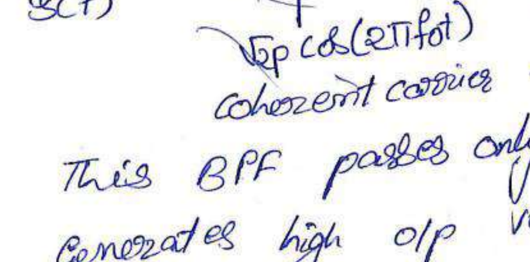
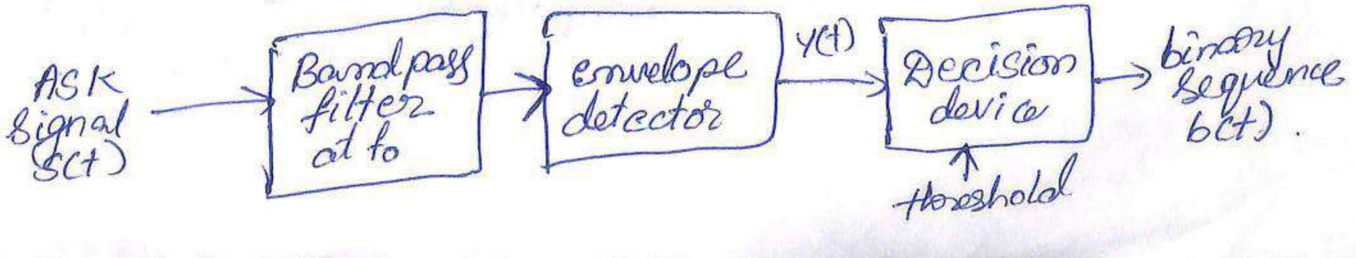


fig shows the block diagram of non-coherent ASK receiver. In this fig observe that the received ASK signal is given to BPF. This BPF passes only carrier frequency f_0. The envelope detector generates high o/p voltage when carrier is present. When carrier (f_0) is absent, there is only noise at the i/p of envelope detector. Hence it produces low o/p. The decision device is basically a regenerator. It generates the binary sequence b(t). Threshold is provided to the decision device to overcome effects due to noise. when $Y(t) >$ threshold, $b(t) = 1$
" " < " , $b(t) = 0$.



Binary frequency Shift Keying (BFSK)

In BFSK, the frequency of the carrier is shifted according to the binary symbol. The phase of the carrier is unaffected. That is we have two different frequency signals according to binary symbols. Let there be a frequency shift by Ω. Then we can write the following eqns.

If $b(t) = '1'$; $S_H(t) = \sqrt{2P_s}\cos(2\pi f_0 + \Omega)t$

If $b(t) = '0'$; $S_L(t) = \sqrt{2P_s}\cos(2\pi f_0 - \Omega)t$

Thus there is increase or decrease in frequency by Ω.
We can rewrite the eqn as $S(t) = \sqrt{2P}\cos[(2\pi f_0 + d(t)\Omega)t]$

→ Thus when symbol '1' is to be transmitted, the carrier frequency will be $f_0 + \left(\frac{\Omega}{2\pi}\right]$

$d(t) = 1V$ if $b(t) = 1$
$= -1V$ if $b(t) = 0$

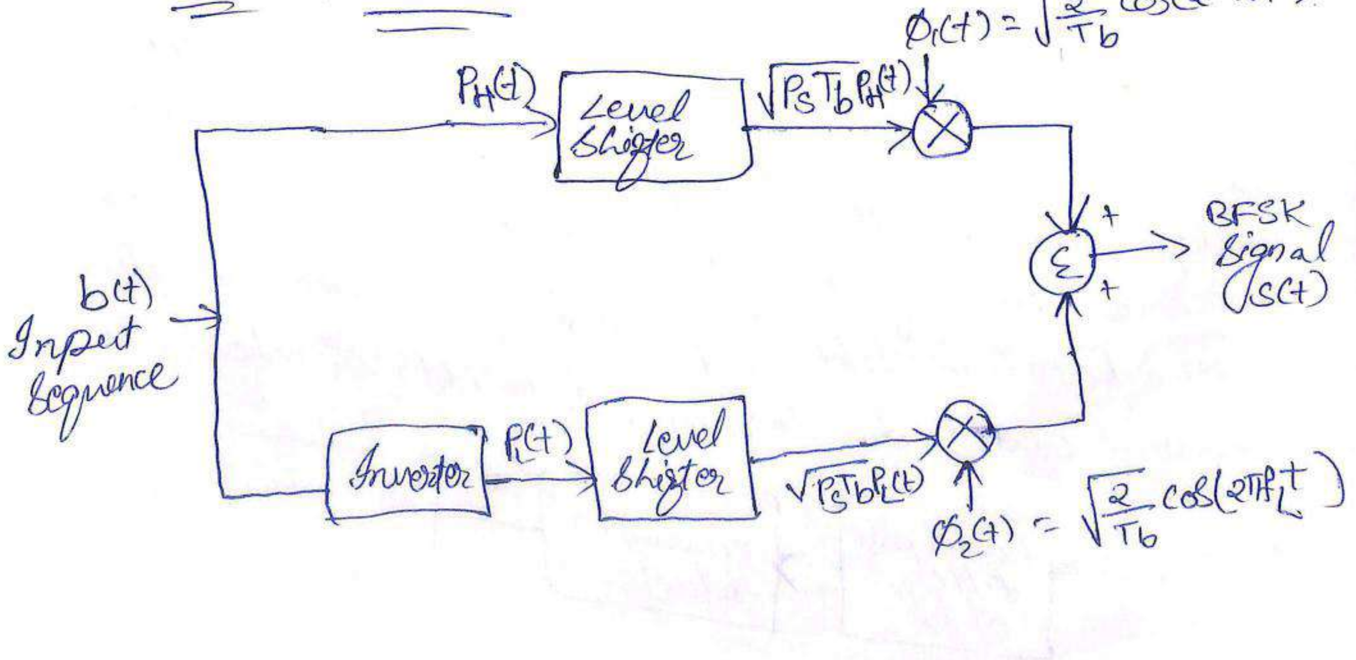
→ If symbol '0' is to be transmitted, the carrier frequency will be $f_0 - \left(\frac{\Omega}{2\pi}\right]$. i.e,

$f_H = f_0 + \frac{\Omega}{2\pi}$ for symbol '1'

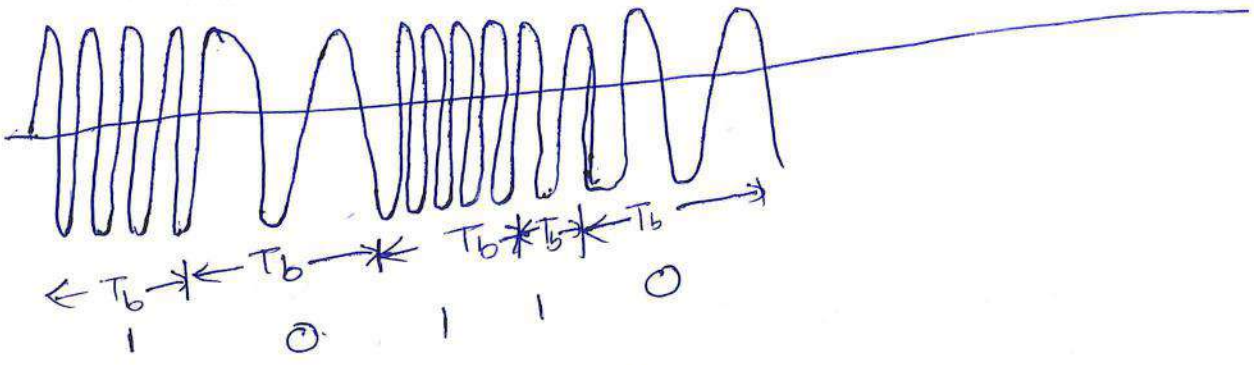
$f_L = f_0 - \frac{\Omega}{2\pi}$ for symbol '0'.

~~P(H)~~
for $b(t) = 1$ $P_H(t) = 1$
for $b(t) = 0$ $P_L(t) = 1$

BFSK Transmitter



→We Know that i/p sequence b(t) is same as $P_H(t)$. An inverter is added after b(t) to get $P_L(t)$. $P_H(t)$ & $P_L(t)$ are unipolar signals. The level shifter converts the '+1' level to $\sqrt{P_S T_b}$. Zero level is unaffected. Thus the o/p of the level shifter will be either $\sqrt{P_S T_b}$ if (+1) or zero (if i/p is zero). Further there are product modulators after level shifter. The two carrier signals $\phi_1(t)$ & $\phi_2(t)$ are used. $\phi_1(t)$ & $\phi_2(t)$ are orthogonal to each other. In one bit period of i/p signal (i.e T_b), $\phi_1(t)$ or $\phi_2(t)$ have integral no. of cycles. Therefore the modulated signal has continous phase. Such BF-SK signal fig. as shown.



Spectrum & Bandwidth of BFSK

We can write BFSK signal S(t) as

$$S(t) = \sqrt{2P_s}\, P_H(t) \cos(2\pi f_H t) + \sqrt{2P_s}\, P_L(t) \cos(2\pi f_L t)$$

In BFSK the coefficients $P_H(t)$ or $P_L(t)$ are unipolar. Therefore let us convert those coefficients in bipolar as follows

$$P_H(t) = \frac{1}{2} + \frac{1}{2} P_H'(t)$$

$$P_L(t) = \frac{1}{2} + \frac{1}{2} P_L'(t)$$

When $P_H'(t)$ & $P_L'(t)$ will be bipolar (i.e +1 or -1). Putting those values in eqn.

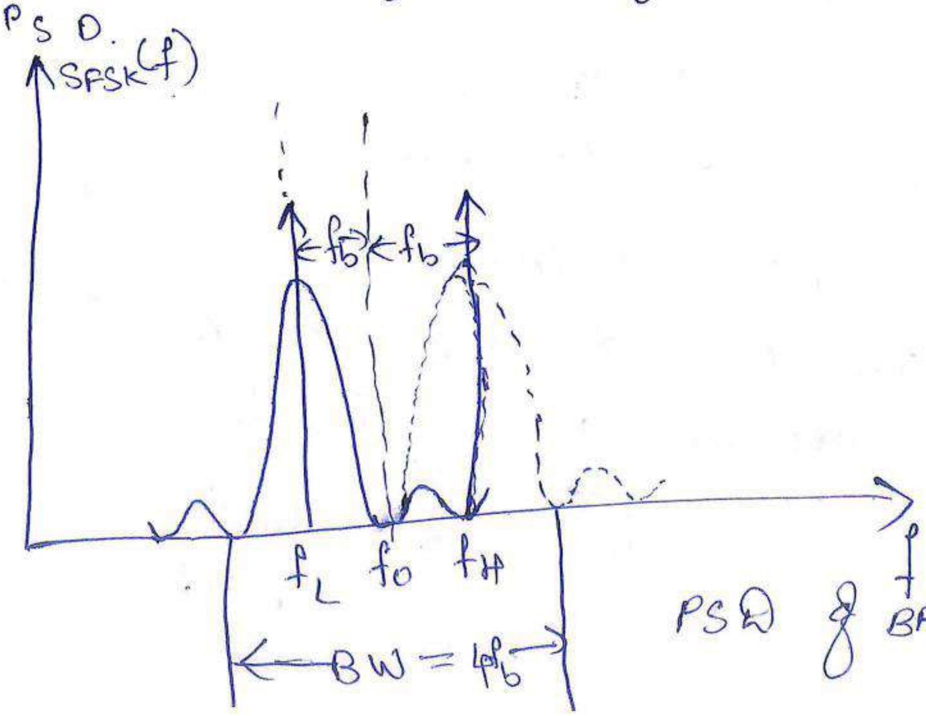
$$S(t) = \sqrt{2P_s}\left[\frac{1}{2}+\frac{1}{2}P_H'(t)\right]\cos(2\pi f_H(t) + \sqrt{2P_s}\left[\frac{1}{2}+\frac{1}{2}P_L'(t)\right]\cos(2\pi f_L t)$$

$$= \underbrace{\sqrt{\frac{P_s}{2}}\cos(2\pi f_H t)}_{\text{single frequency impulse at } f_H} + \sqrt{\frac{P_s}{2}}\cos(2\pi f_L t) + \left.\begin{array}{l} \sqrt{\frac{P_s}{2}}\,P_H'(t)\cos(2\pi f_H t) \\ + \sqrt{\frac{P_s}{2}}\,P_L'(t)\cos(2\pi f_L t) \end{array}\right\} \begin{array}{l}\cancel{\text{single freq}} \\ \text{BPSK eqn.}\end{array}$$

Once Spectrum is located at f_H & other at f_L. Therefore we can write the power spectral density of BFSK as,

$$S(f) = \sqrt{\frac{P_s}{2}}\left\{\delta(f-f_H)+\delta f+f_L) + \frac{P_sT_b}{2}\left\{\frac{\sin(\pi f_H T_b)}{\pi f_H T_b}\right\}^2 + \frac{P_sT_b}{2}\left\{\frac{\sin(\pi f_L T_b)}{\pi f_L T_b}\right\}^2\right\}$$

fig shows the PSD of BFSK Signal



PSD of BFSK Signal.

Bandwidth of BFSK Signal

It is clear that the width of one lobe is $2f_b$. The two main lobes due to f_H & f_L are placed such-that the total width due to both main lobes is $4f_b$. i.e.

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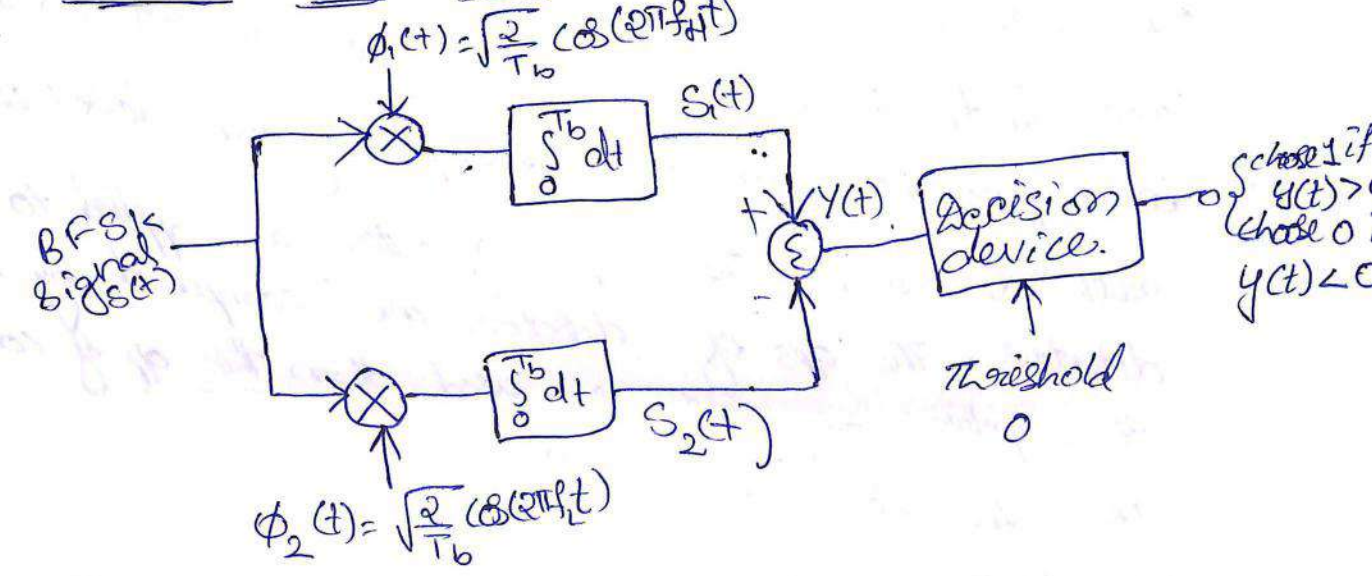
$$B.W \text{ of } BFSK = 2f_b + 2f_b$$
$$= 4f_b$$

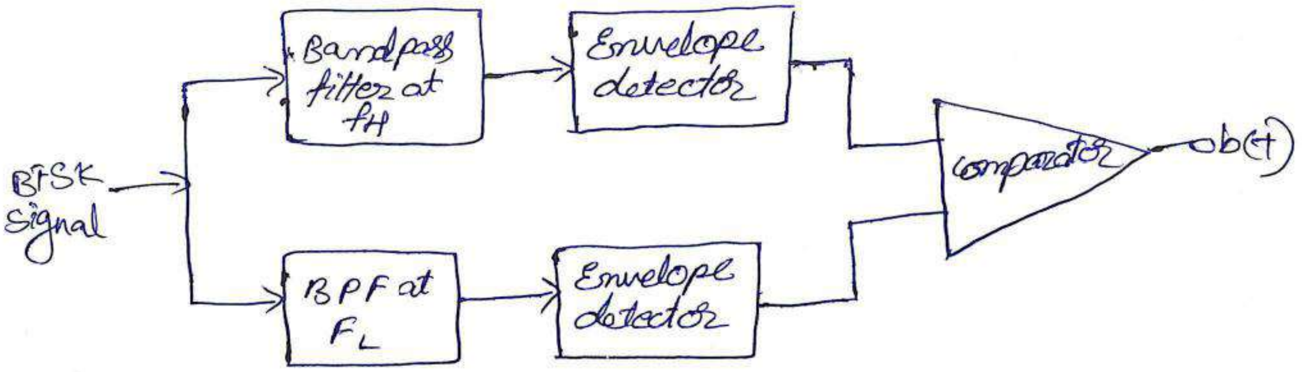
Coherent BFSK receiver



There are two correlators for two frequencies of FSK signal. These correlators are supplied with locally generated carriers $\phi_1(t)$ & $\phi_2(t)$. If the transmitted frequency is f_H, then the o/p $S_1(t)$ will be higher than $S_2(t)$. Hence $y(t)$ will be greater than zero.

The decision device then decides in favour of binary 1. If $S_2(t) > S_1(t)$ then $y(t) < 0$ & decision device decides in favour of 0. The coherent carriers are generated using similar method.

Non Coherent BFSK receiver

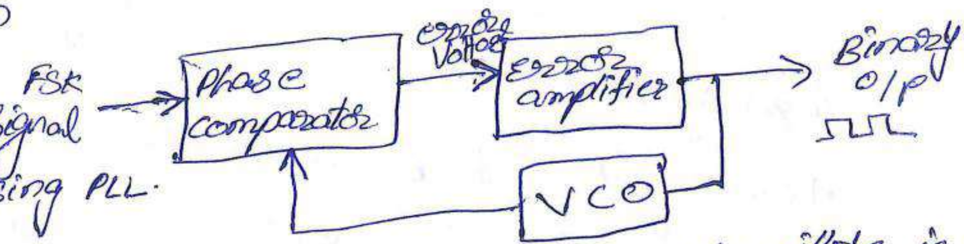


The receiver consists of two band pass filters; one with centre frequency f_H & other with centre frequency f_L. Since $f_H - f_L = 2f_b$, the o/p's of filters do not overlap. The band pass filters pass their corresponding main lobes without much distortion. The o/p's of filters are applied to envelope detectors. The o/p's of detectors are compared by the comparator. If unipolar comparator is used, then the o/p of comparator is the bit sequence b(t).

FSK detection using PLL

Principle:- The error voltage in PLL is proportional to difference b/w the phase or frequency of the i/p signal & VCO frequency.

Block diagram



FSK detection using PLL.

-> The free running frequency of Voltage control oscillator is kept in b/w F_H & F_L.

-> When F_H is transmitted, the error voltage becomes +ve & hence binary o/p goes high. It remains high as long as f_H is transmitted for a bit period.

when F_L is transmitted the error voltage becomes negative and hence binary goes low. It remains low as long as f_L is transmitted for a bit period.

→ The VCO generates free running frequency b/w F_H & F_L. Hence phase comparator. Hence phase comparator detects the difference b/w VCO frequency & f_H or f_L.

→ PLL detector is a non coherent type of detector.

Geometrical representation of Orthogonal BFSK or Signal Space representation of orthogonal BFSK.

The different Signal points are represented geometric in $\phi_1 \phi_2$ plane. For geometrical representation of BFSK such orthogonal carriers are required. We know that two carriers $\phi_1(t)$ & $\phi_2(t)$ of two different frequencies f_H & f_L are used for modulation. to make $\phi_1(t)$ & $\phi_2(t)$ orthogonal, the frequencies f_H & f_L should be some integer multiple of base band frequen f_b.

i.e

$$f_H = mf_b$$
$$f_L = nf_b$$

then carriers will be

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi m f_b t)$$
$$\phi_2(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi n f_b t)$$

The carrier $\phi_1(t)$ & $\phi_2(t)$ are orthogonal over time period T_b

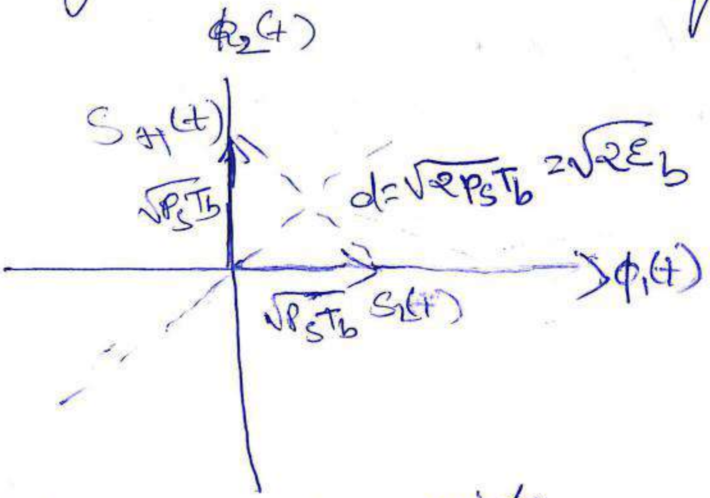
we can write

$$S_H(t) = \sqrt{P_S T_b} \sqrt{\frac{2}{T_b}} \cos(2\pi f_H t)$$
$$S_L(t) = \sqrt{P_S T_b} \sqrt{\frac{2}{T_b}} \cos(2\pi f_L t)$$

$\Rightarrow S_H(t) = \sqrt{P_S T_b}\,\phi_1(t)$

$\Rightarrow S_2(t) = \sqrt{P_S T_b}\,\phi_2(t)$

Base on the above 2 eqn we can draw signal space diagram as shown in fig.



Distance b/w two points.

$$d^2 = (\sqrt{P_S T_b})^2 + (\sqrt{P_S T_b})^2$$

$$d^2 = 2P_S T_b$$

$$d = \sqrt{2P_S T_b}$$

$$d = \sqrt{2E_b}$$

Binary phase shift keying (BPSK)

Principle

In BPSK binary symbol 1 & 0 modulate the phase of the carrier.

Let the carrier be,

$$S(t) = A\cos(2\pi f_0 t)$$

'A' represents peak value of sinusoidal carrier. In the standard 1Ω load resistor, the power dissipated will be.

$$P = \frac{1}{2}A^2$$

$$A = \sqrt{2P}$$

$$\therefore \; S(t) = \sqrt{2P}\cos(2\pi f_0 t)$$

→ when the symbol is changed, then the phase of the carrier is changed by 180°

→ consider for example,

Symbol 1 ⇒ $S_1(t) = \sqrt{2P}\cos(2\pi f_0 t)$

If next symbol is '0' then,

Symbol 0 ⇒ $S_2(t) = \sqrt{2P}\cos(2\pi f_0 t + \pi)$

$\therefore \; S_2(t) = -\sqrt{2P}\cos 2\pi f_0 t$.

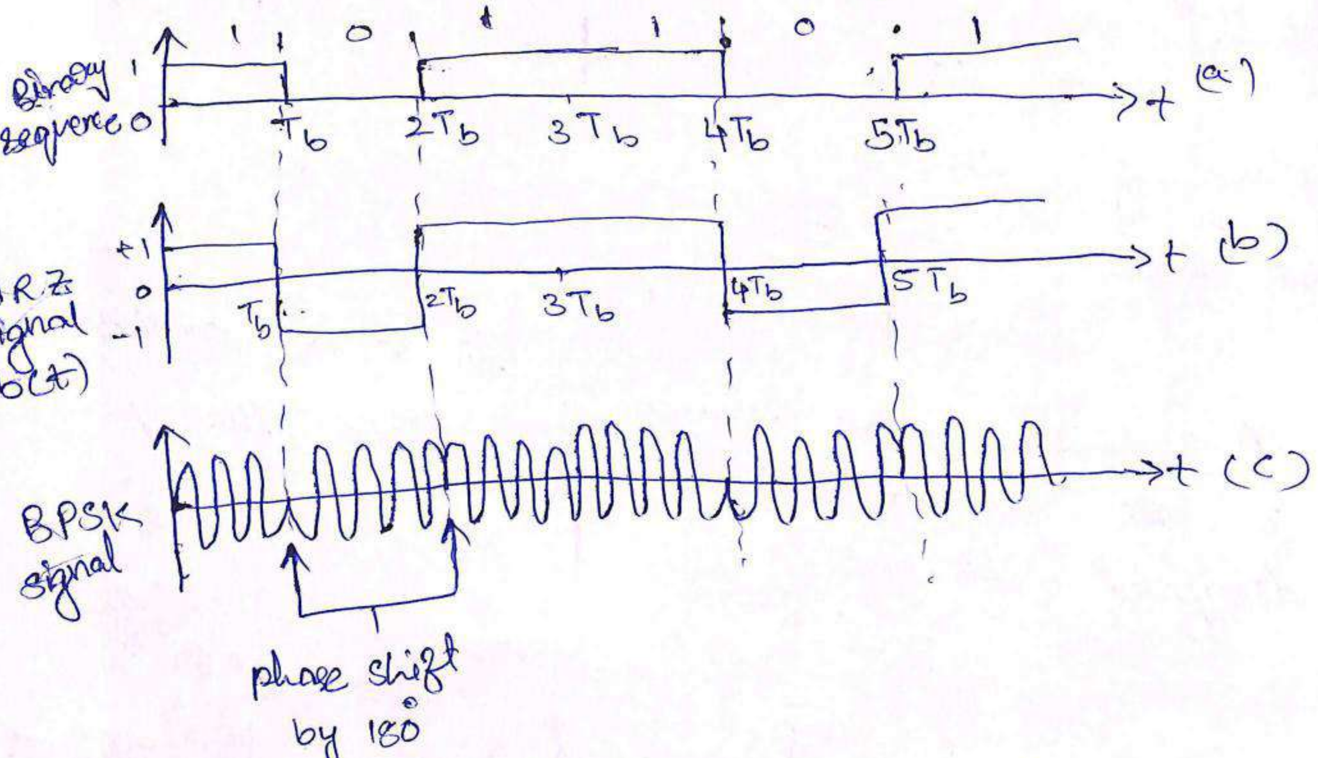
with the above eqn we can define.

$$S(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)$$

Here $b(t) = +1$ when binary '1' is to be transmitted

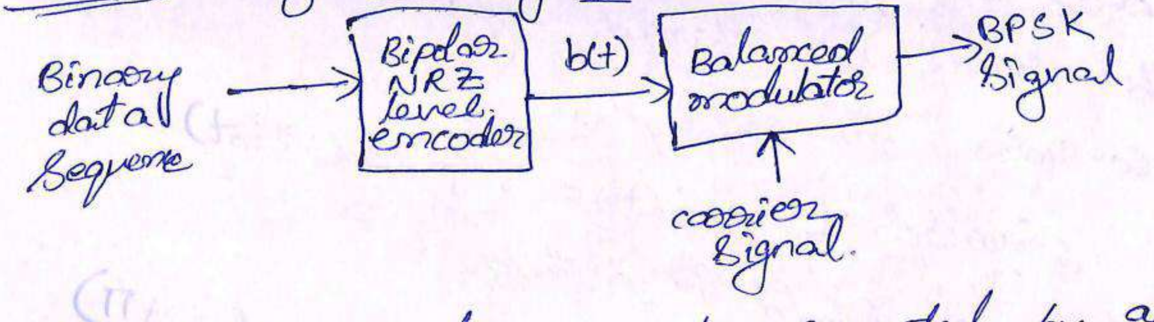
$= -1$ " " '0' " " "

Graphical representation of BPSK Signal



Generation & reception of BPSK Signal

Generator of BPSK Signal

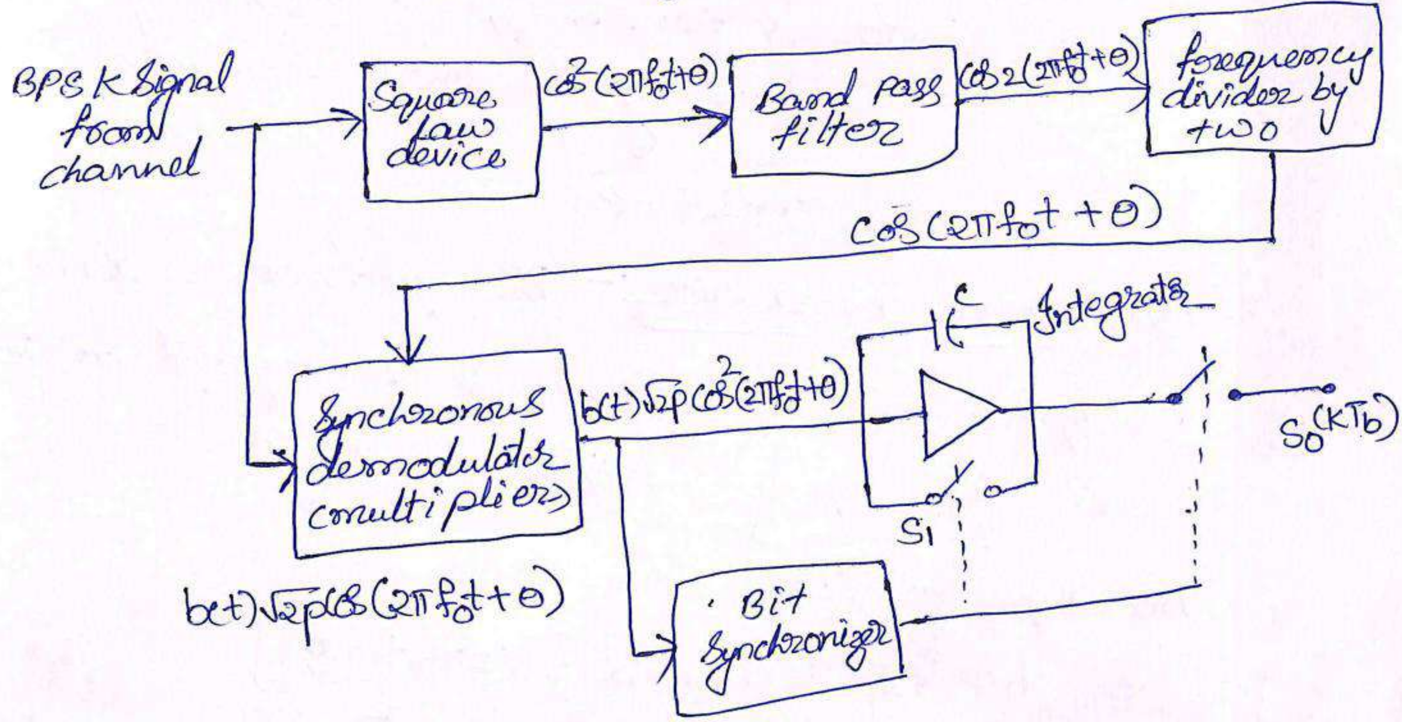


→ The BPSK Signal can be generated by applying carrier Signal to the balanced modulator.

→ The base band Signal b(t) is applied as a modulating Signal to the balanced modulator.

→ The NRZ level encoder converts the binary data Sequence into bipolar NRZ Signal.

Coherent Reception of BPSK Signal



Operation

1, **Phase Shift in received signal** :- This signal undergoes the phase change depending upon the time delay from transmitter to receiver. This phase change is normally fixed phase shift in transmitted signal. Let the phase shift be θ ∴ The signal at i/p of receiver is $s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t+\theta)$.

2, **Square law device** :- The received signal is passed through a square law device. At the o/p of square law device. The signal is $\cos^2(2\pi f_0 t+\theta)$

$$= \frac{1}{2} + \frac{1}{2}\cos 2(2\pi f_0 t+\theta)$$ Here $\frac{1}{2}$ represents DC level.

3, **Band pass filter** :- The signal is then passed through a B.P.F whose pass band is centred around $2f_0$. B.P.F removes the DC level of $\frac{1}{2}$ & its o/p we get

$$\cos 2(2\pi f_0 t+\theta).$$

4. Frequency divider :- The above signal is passed through a frequency divider by two. ∴ o/p of frequency divided we get a carrier signal whose frequency is f_0.

i.e $\cos(2\pi f_0 t + \theta)$

5. Synchronous demodulator :- The synchronous demodulator multiplies the i/p signal & recovered carrier.
∴ At the o/p of multiplier we get.

$$b(t)\sqrt{2p}\ \cos^2(2\pi f_0 t + \theta)$$

$$= b(t)\sqrt{2p}\ \frac{1}{2}\left[1 + \cos 2(2\pi f_0 t + \theta)\right]$$

$$= b(t)\sqrt{\frac{P}{2}}\left[1 + \cos 2(2\pi f_0 t + \theta)\right]$$

6. Bit Synchronizer & integrator :- The integrator integrates over one bit period. The bit synchronizer take care of starting & ending times of a bit.

→ At the end of bit duration T_b, the bit synchronizer closes switch S_2 temporarily. This connects the o/p of integrator to the decision device. It is equivalent to sampling the o/p of integrator.

→ The synchronizer then opens switch S_2 & S_1 is closed temporarily. This resets the integrator voltage to zero. The integrator then integrates next bit.

To Show the o/p of integrator depends upon transmitted bit.

In the K^{th} bit interval we can write o/p signal as

$$S_o(KT_b) = b(KT_b)\sqrt{\frac{P}{2}}\int_{(K-1)T_b}^{KT_b}[1+\cos 2(2\pi f_0 t+\theta)]dt$$

Here T_b is one bit period.

$$S_o(KT_b) = b(KT_b)\sqrt{\frac{P}{2}}\int_{(K-1)T_b}^{KT_b} 1\,dt + \int_{(K-1)T_b}^{KT_b}\cos 2(2\pi f_0 t+\theta)]\,dt$$

Here $\int_{(K-1)T_b}^{KT_b}\cos 2(2\pi f_0 t+\theta)\,dt = 0$; becoz avg value of Sinusoidal waveform is '0' if integration is performed over full cycles. we get.

$$S_o(KT_b) = b(KT_b)\sqrt{\frac{P}{2}}\,[t]_{(K-1)T_b}^{KT_b}$$

$$\therefore\ S_o(KT_b) = b'(KT_b)\sqrt{\frac{P}{2}}\,T_b.$$

This eqn Shows that o/p of receiver depends on i/p.

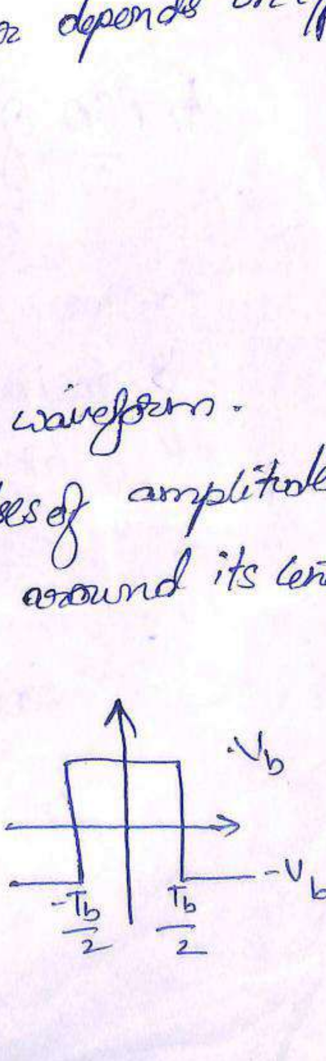
i.e $S_o(KT_b) \propto b(KT_b)$

Spectrum of BPSK Signal

1) Fourier transform of basic NRZ pulse

w.r.t waveform of b(t) is NRZ bipolar waveform. In this waveform there are rectangular pulses of amplitude $\pm V_b$. If we say that each pulse is $\pm\frac{T_b}{2}$ around its center. Then F.T of this type of pulse is

$$X(f) = V_b T_b\frac{\sin(\pi f T_b)}{\pi f T_b}$$



2, **PSD of NRZ pulse**

For large no. of such +ve & -ve pulses the PSD S(f) is given as

$$S(f) = \frac{|\overline{X(f)}|^2}{T_S}$$

Here $\overline{x(f)}$ denotes avg value of x(f) due to all pulse in b(t) & T_s is symbol duration. ~~by~~ Substituting x(f)

$$S(f) = \frac{V_b^2 T_b^2}{T_S}\left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$

3, **PSD of baseband signal b(t)**

For BPSK ~~sign.~~ since only one bit is transmitted at a time, symbol & bit durations are same $T_b = T_s$.

$$S(f) = V_b^2 T_b \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2.$$

4, **PSD of BPSK Signal**

The BPSK signal is generated by modulating a carrier by base band signal b(t). Because of modulatn of carrier of frequency f_0, the spectral components ~~is divided~~ ~~by~~ are translated from f to f_0+f & f_0-f. The magnitude of those components is divided by half.

$\therefore$ we can write PSD of BPSK as

$$S_{BPSK}(f) = V_b^2 T_b\left\{\frac{1}{2}\left[\frac{\sin\pi(f_0-f)T_b}{\pi(f_0-f)T_b}\right]^2 + \frac{1}{2}\left[\frac{\sin\pi(f_0+f)T_b}{\pi(f_0+f)T_b}\right]^2\right\}$$

Let us assume that value of $\pm V_b = \pm\sqrt{P}$.

i.e NRZ having amplitudes of $+\sqrt{P}$ & $-\sqrt{P}$.

$$S_{BPSK}(f) = \frac{PT_b}{2}\left\{\left[\frac{\sin\pi(f_0-f)T_b}{\pi(f_0-f)T_b}\right]^2 + \left[\frac{\sin\pi(f_0+f)T_b}{\pi(f_0+f)T_b}\right]^2\right.$$

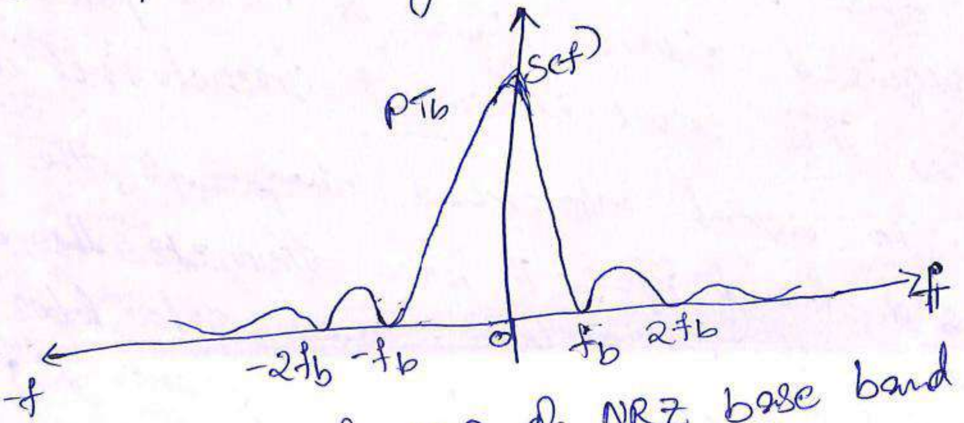
The above eqn giving PSD of BPSK signal for modulating signal b(t) having amplitude of ± $\sqrt{P}$, we know that modulating signal is given by $S(t) = \pm\sqrt{2P}\cos(2\pi f_0 t)$ since $A=\sqrt{2P}$

If $b(t) = \pm\sqrt{P}$, then the carrier becomes,

$$\phi(t) = \sqrt{2P}\cos(2\pi f_0 t)$$

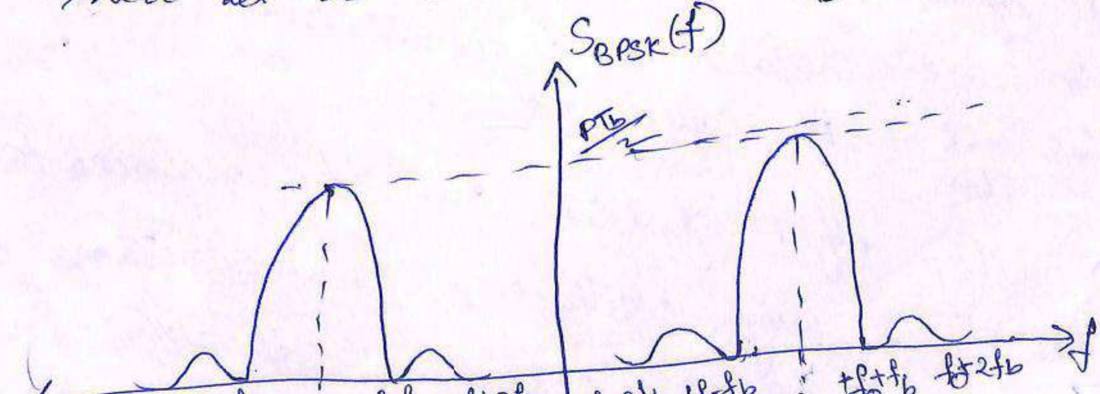
Plot of PSD

→ eqn For one rectangular pulse, the shape of S(f) will be a sinc pulse. as fig shows the plot of magnitude of S(f).



Plot of PSD of NRZ base band signal

→ Now let us consider the PSD of BPSK signal :



The fig thus clearly shows that there are two lobes, one at f_0 & other at $-f_0$. But the amplitude of main lobes are = $\frac{PT_b}{2}$.

Absentees 9/1/11

53, 58, 59, 62, 64, 67, 68, 70, 73, 80, 83, 85, 93, 96, A2, A3, LE, 7, 8, 9, 10

Interchannel Interference & Inter Symbol interference

Let us assume that BPSK signals are assumed multiplexed with help of different carrier frequencies for different baseband signals. Then at any frequency, the spectral components due to all multiplexed channels will be present. This is because $S(f)$ as well as $S_{BPSK}(f)$ of every channel extends over all the frequency range.

Therefore a BPSK receiver tuned to a particular carrier frequency will also receive frequency components due to other channel. This will make interference with the required channel signals & error probability will increase. This result is called Interchannel interference.

To avoid interchannel interference, the BPSK signal is passed through filter. The filter attenuates the side lobes & passes only main lobe. Since side lobes are attenuated to high level, the interference is reduced. Because of the filtering the phase distortion takes place in bipolar NRZ, i.e $b(t)$. Therefore the individual bits mix with adjacent bits (symbols) in the Samechannel. This effect is called ISI.

→ The effect of ISI can be reduced to some extent by using equilizers at receiver. These equilizers have the reverse effect to the filter's adverse effects. Normally equilizers are also filter structures

Geometrical representation of BPSK Signals

We know that BPSK signal carries the information about two symbols. These are symbol '1' & symbol '0'. We can represent BPSK signal geometrically to show those two symbols.

(i) w.k.t BPSK signal is given as.

$$S(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)$$

(ii) Let's rearrange the above eqn as

$$S(t) = b(t)\sqrt{PT_b}\sqrt{\frac{2}{T_b}}\cos(2\pi f_0 t)$$

(iii) Let $\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_0 t)$ represents an orthonormal carrier signal.

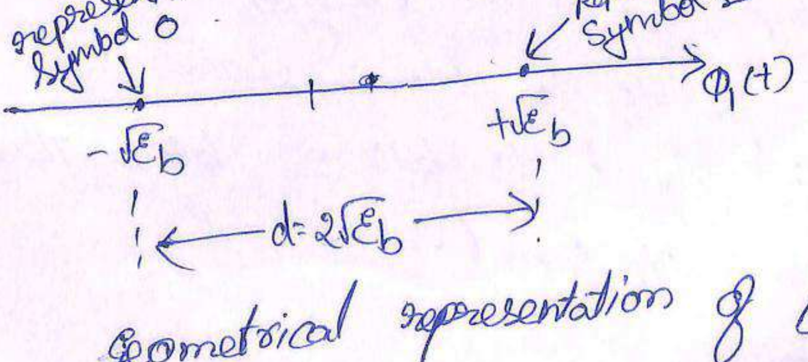
$$\therefore \quad S(t) = b(t)\sqrt{PT_b}\,\phi_1(t).$$

(iv) The bit energy $\mathcal{E}_b$ is defined interms of power P & bit duration T_b as.

$$\mathcal{E}_b = PT_b.$$

$$\therefore \quad S(t) = \cancel{b(t)}\pm\sqrt{\mathcal{E}_b}\,\phi_1(t).$$

(v) Thus on the single axis of $\phi_1(t)$ there will be two points. One point will be located at $+\sqrt{E_b}$ & other at $-\sqrt{E_b}$. This is shown in fig.



Geometrical representation of BPSK Signal.

The Seperation b/w the two points $+\sqrt{E_b}$ & $-\sqrt{E_b}$ represent the isolation in Symbol '1' & '0' in BPSK Signal. This seperation is normally called distance d.

As this distance 'd' increases, the isolation b/w the Symbols in BPSK Signal is more. Therefore probability of error reduces.

Bandwidth of BPSK Signal

The Spectrum of the BPSK Signal is centred around the carrier frequency f_0 & extends from $f_0 - f_b$ to $f_0 + f_b$.

∴ B.W of BPSK Signal = Highest frequency - lowest frequency in main lobe.

$$= f_0 + f_b - (f_0 - f_b)$$

$$\boxed{B.W = 2f_b}$$

Thus minimum B.W in BPSK Signal is equal to twice of the highest frequency contained in baseband Signal.

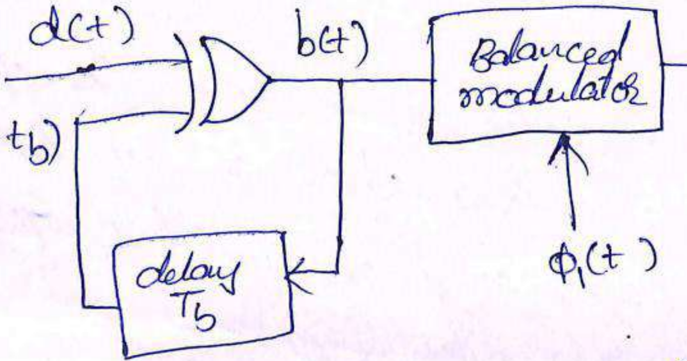
Drawbacks of BPSK: Ambiguity in o/p Signal.

To generate the carrier in receiver, we start by Squaring $b(t)\sqrt{2P}\cos(2\pi f_0 t + \theta)$. If the received Signal is $-b(t)\sqrt{2P}\cos(2\pi f_0 t + \theta)$ then the Squared Signal remains same as before. Therefore carrier is unchanged even if the i/p Signal has changed its Sign. Therefore it is not possible to determine whether the received Signal is equal to $b(t)$ or $-b(t)$. This result in ambiguity in o/p signal.

Differential phase Shift Keying

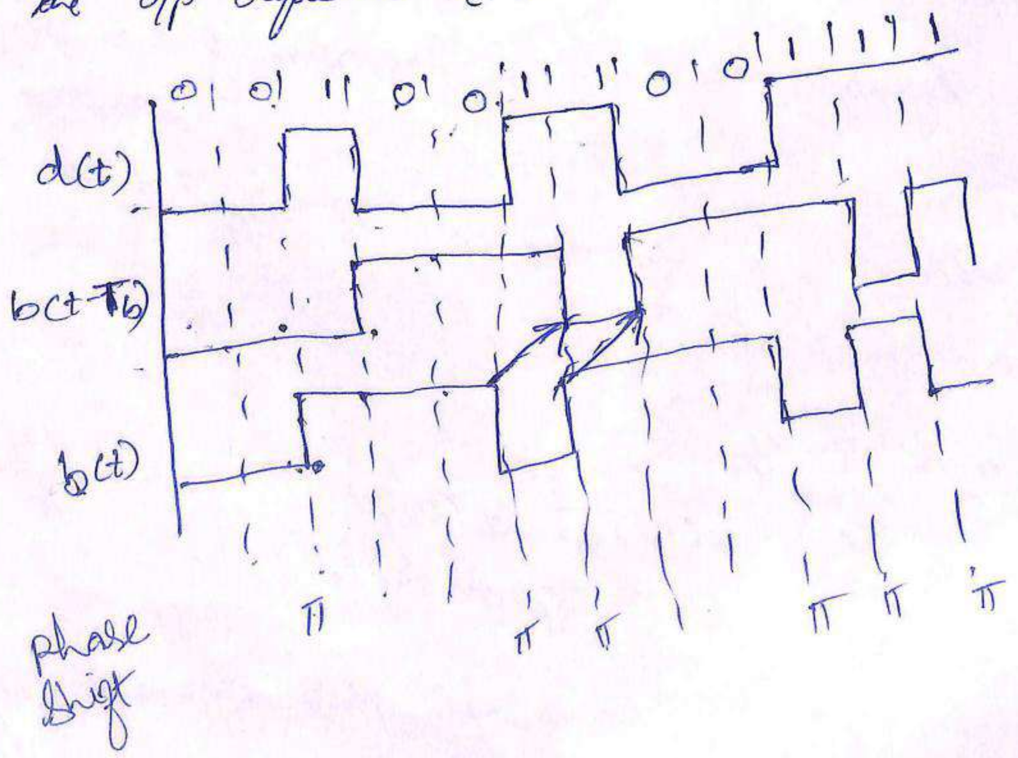
Principle:- DPSK is differentially coherent modulation method:- DPSK does not need a Synchronous (coherent) carrier at the demodulator. The i/p sequence of binary bits is modified Such that the next bit depends upon the previous bit. Therefore in the receiver the previous ~~b~~ received bits are used to detect the present bit.

DPSK tx



$S(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)$
$= \pm\sqrt{2P}\cos(2\pi f_0 t)$
DPSK Signal.

The i/p Sequence is d(t). o/p Sequence is b(t) & b(t-Tb) is the previous o/p delayed by one bit period. Depending upon values of d(t) & b(t-Tb), exclusive OR gate generates the o/p sequence b(t).



From fig we can observe.

when $d(t) = 0$; $b(t) = b(t-T_b)$

& $d(t) = 1$; $b(t) = \overline{b(t-T_b)}$

-> The sequence $b(t)$ modulates phase of the carrier.

-> when $b(t)$ changes the level, phase of the carrier is changed. Since $b(t)$ changes its level only if $d(t)=1$ It shows that phase of the carrier is changed only if $d(t) = 1$.

-> In BPSK the phase of the carrier ~~is~~ changes on both the symbol 1 & 0. where as in DPSK phase of the carrier changes only on symbol 1. This is the main difference b/w BPSK & DPSK.

-> Always two successive bits of $d(t)$ are checked for any change of level. Hence ~~f~~.one symbol has two bits.

$$\text{Symbol duration } (T) = \text{Duration of two bits } (2T_b)$$

$$T = 2T_b.$$

As shown in fig the $b(t)$ is applied to a balanced modulator.

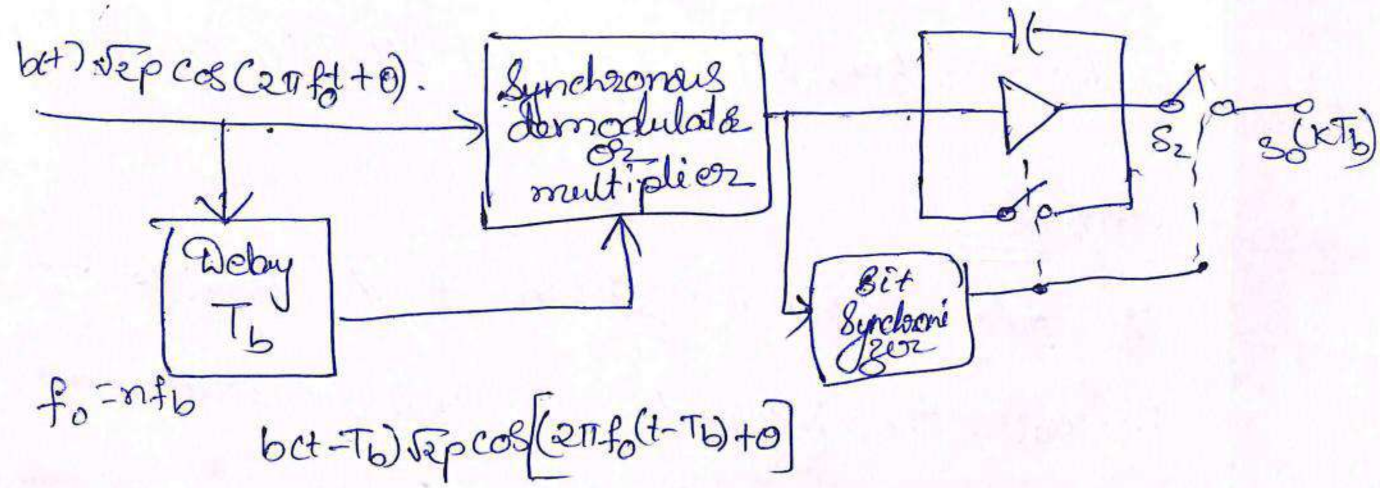
The modulator o/p is,

$$S(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)$$

$$= \pm\sqrt{2P}\cos(2\pi f_0 t).$$

As shown in waveforms the phase changes only when $d(t)=1$.

DPSK receiver



- phase shift in rx Signal

rx signal $= b(t)\sqrt{2P}\cos(2\pi f_0 t+\theta)$

$\cos(A-B)+\cos(A+B) = 2\cos A\cos B$

Multiplier o/p

$= b(t)b(t-T_b)(2P)\cos(2\pi f_0 t+\theta)\cos[2\pi f_0(t-T_b)+\theta]$

Multiplier o/p $= b(t)\, b(t-T_b)\, P \left\{\cos 2\pi f_0 T_b + \cos\left[4\pi f_0\left[t-\frac{T_b}{2}\right]+2\theta\right]\right\}$

- f_0 is the carrier frequency & T_b is one bit period.

T_b contains integral no. of cycles of f_0.

w.r.t $f_b = \frac{1}{T_b}$

if T_b contains 'n' cycles of f_0.

$$f_0 = nf_b \Rightarrow f_0 = \frac{n}{T_b}$$

$$\therefore f_0 T_b = n$$

Absentees 3B 11/1/16

53, 56, 59, 65, 66, 69, 70,
77, 79, 82, 85, 88, 90,
93, 95 97, 99, A1,
LE's 7, 8, 9, 10

$$\text{Multiplier o/p} = b(t)\, b(t-T_b)\, P \left\{ \cos 2\pi n + \cos\left[4\pi f_0 \left(t - \frac{T_b}{2}\right] + 2\theta\right] \right\}$$

Since $\cos 2\pi n = 1$

$$\text{Multiplier o/p} = b(t)\, b(t-T_b)\, P + b(t)\, b(t-T_b)\, P \cos\left[4\pi f_0 \left(t-\frac{T_b}{2}\right) + 2\theta\right]$$

Integrator

The above signal is given to the integrator. In K^{th} bit interval, the integrator o/p can be written as.

$$S_0(KT_b) = b(KT_b)\, b[(k-1)T_b]\, P \int_{(K-1)T_b}^{KT_b} dt$$

$$+ b(KT_b)\, b[(k-1)T_b]\, P \int_{(K-1)T_b}^{KT_b} \cos\left[4\pi f_0 \left(t - \frac{T_b}{2}\right] + 2\theta\right] dt$$

$$S_0(KT_b) = b(KT_b)\, b[(k-1)T_b]\, P [KT_b - (k-1)T_b]$$

$$= b(KT_b)\, b[(k-1)T_b]\, PT_b .$$

$\Rightarrow$ If $b(t)\, b(t-T_b) = 1V$ then $d(t) = 0$

$\Rightarrow$ If $b(t)\, b(t-T_b) = -1V$ then $d(t) = 1$

decision device.

$$S_0(KT_b) = -PT_b \quad \text{then } d(t) = 1$$
$$\qquad\qquad +PT_b \quad \text{then } d(t) = 0.$$

B.W of DPSK.

Symbol duration $T = 2T_b$.

$$B.W = \frac{2}{T}$$

$$= \frac{1}{T_b}$$

$$= f_b$$

Since one previous bit is always used to define the phase shift in next bit the symbol can said to have 2 bits'

Adv & disadv.

adv. 1, DPSK does not need carrier at Rx. Hence the complicated ckt for generation of local carrier is avoided.

2, B.W is reduced than BPSK

disadv 1, The probability of error is higher than BPSK

2, DPSK uses two successive bits for Rx. Error in first bit creates error in 2nd bit

3, Noise interference is more.

Quadrature phase Shift keying

Principle: -

In communication Systems w.k.t there are two main resources i.e transmission power & channel Bandwidth. The channel B.W depends upon bit ~~power~~ rate ~~& transmi~~ or Signalling rate f_b.

-> If two or more bits are combined in some Symbols then Signalling rate is reduced. Therefore the frequency reqd of the carrier required is also reduced. This reduces the transmission channel Bandwidth.

-> In QPSK, two successive bits in the data sequence are grouped together. This reduces the Signal rate & hence reduces the bandwidth of the channel.

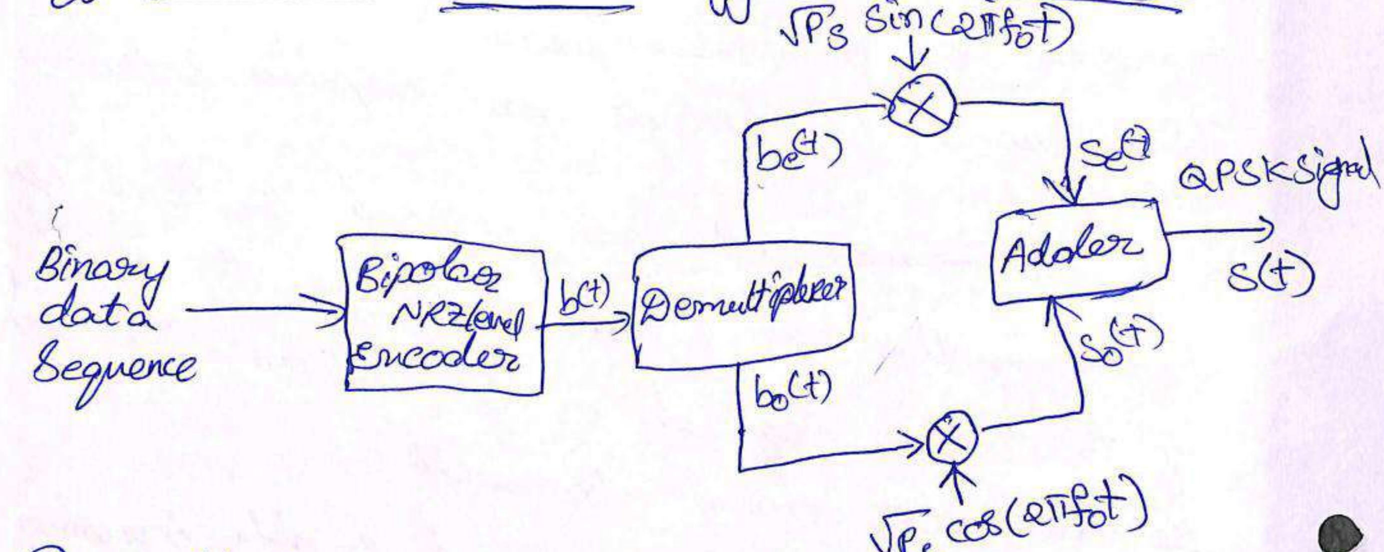
-> This combination of two bits forms four distinct Symbols when the Symbol is changed to next Symbol, the phase of the carrier is changed by 45°.

Input successive bits		Symbol	phase shift in carrier
1(1V)	0(-1V)	S_1	$\pi/4$
0(-1V)	0(-1V)	S_2	$3\pi/4$
0(-1V)	1(1V)	S_3	$5\pi/4$
1(1V)	1(1V)	S_4	$7\pi/4$

Qpsk transmitter & receiver

~~Offset (QPSK) or~~

Offset QPSK (OQPSK) or Staggered QPSK transmitter



Demultiplexer

The demultiplexer divides $b(t)$ into two ~~Sequence~~ seperate bit streams of the odd numbered & even numbered bits. $b_e(t)$ represents even numbered sequence & $b_o(t)$ represents odd numbered sequence. The symbol duration of both of these odd & even numbered sequence is $2T_b$.

We Observe that first even bit occurs after the first odd bit. Therefore even numbered bit sequence $b_e(t)$ starts with delay of one bit period due to first odd bit. Thus first symbol of $b_e(t)$ is delayed by one bit period 'T_b' w.r.t first symbol of $b_o(t)$. This delay T_b is called offset. Hence the name offset QPSK is given. This shows that change in levels of $b_e(t)$ & $b_o(t)$ cannot occur at same time because of offset or staggering.

Modulation of quadrature carriers

The bit stream $b_e(t)$ modulates $\sqrt{P_s}\cos(2\pi f_0 t)$ & $b_o(t)$ modulates $\sqrt{P_s}\sin(2\pi f_0 t)$. These modulators are balanced modulator. The carriers are also called quadrature carriers. The two modulated signals are.

$$S_e(t) = b_e(t)\sqrt{P_s}\sin(2\pi f_0 t)$$

$$S_o(t) = b_o(t)\sqrt{P_s}\cos(2\pi f_0 t)$$

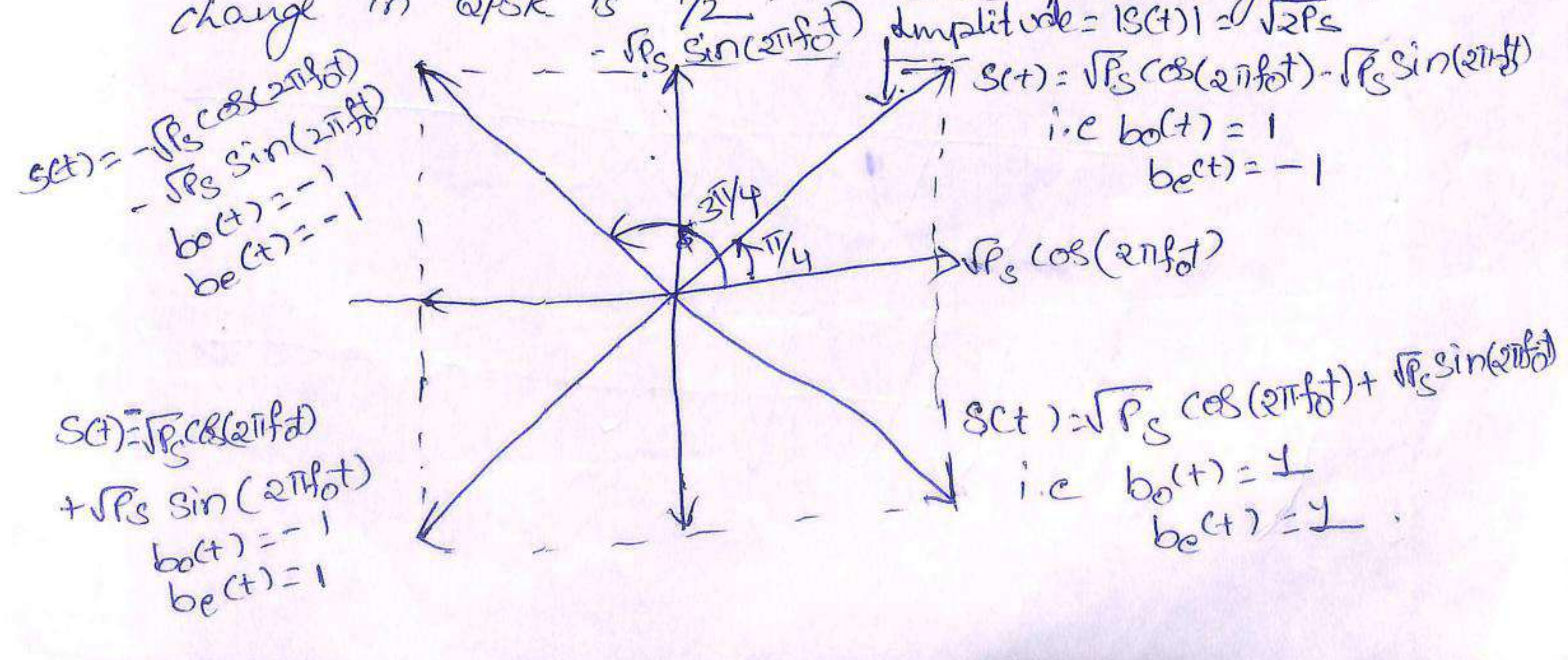
Addition of modulated carriers

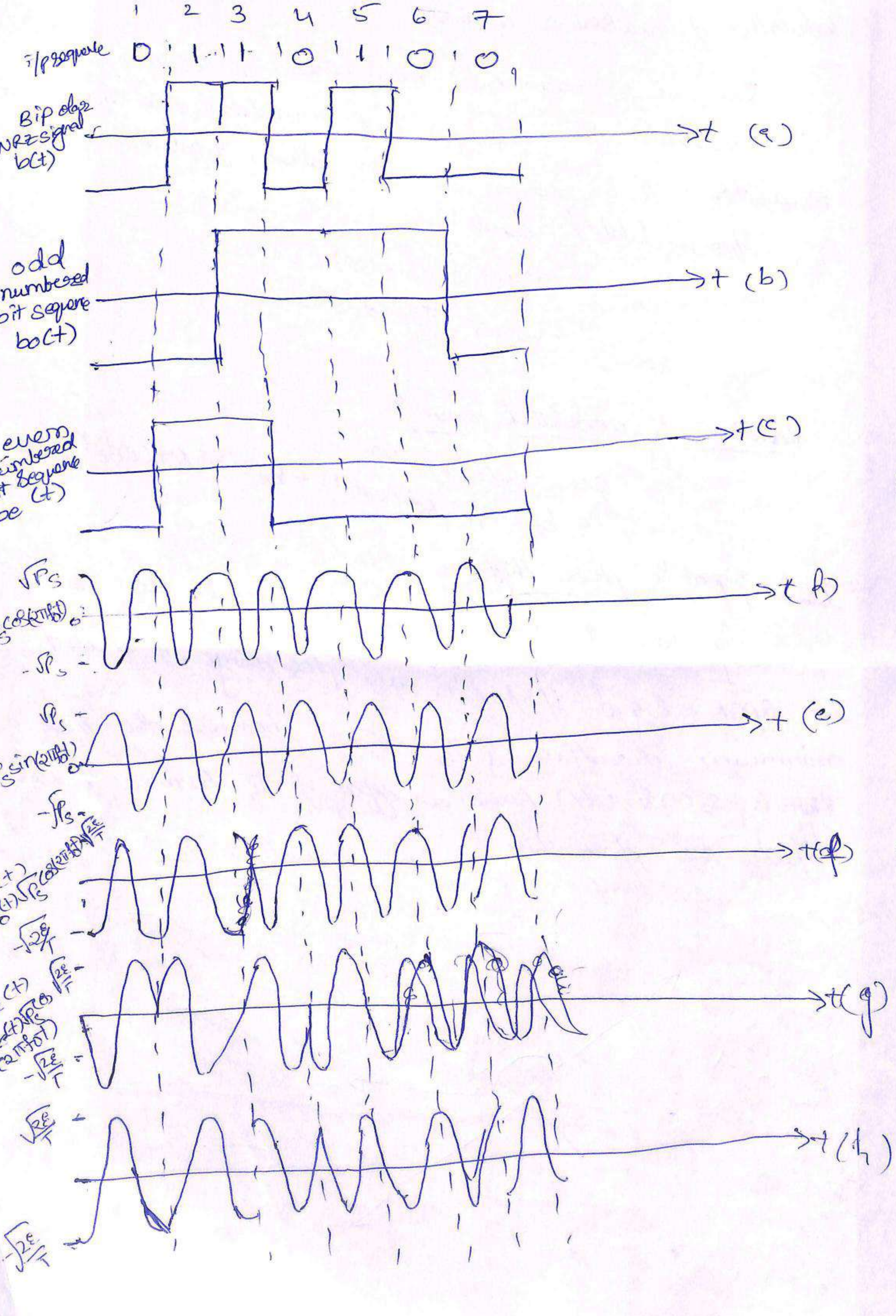
$$S(t)' = S_o(t) + S_e(t)$$

$$= b_o(t)\sqrt{P_s}\cos(2\pi f_0 t) + b_e(t)\sqrt{P_s}\sin(2\pi f_0 t)$$

QPSK Signal & phase Shift:-

fig shows the QPSK signal represented by above eqn. In QPSK signal if there is any phase change, it occurs at minimum duration of T_b. This is because the two signals $S_e(t)$ & $S_o(t)$ have an offset of T_b. Because of this offset, the phase shift in QPSK signal is $\pi/2$. Since $b_o(t)$ & $b_e(t)$ cannot change at same time, ~~because of~~ The phase change in QPSK is $\pi/2$. fig shows phasor diagram.




1 2 3 4 5 6 7
I/p sequence 0 1 1 0 1 0 0
Bipolar NRZ signal b(t)
t (a)
odd numbered bit sequence bo(t)
t (b)
even numbered bit sequence be(t)
t (c)
√Ps
√Ps cos(2πf0t)
0
-√Ps
t (d)
√Ps
√Ps sin(2πf0t)
0
-√Ps
t (e)
So(t) = bo(t)√Ps cos(2πf0t)
√(2E/T)
-√(2E/T)
t (f)
Se(t) = be(t)√Ps cos(2πf0t)
√(2E/T)
-√(2E/T)
t (g)
√(2E/T)
-√(2E/T)
t (h)

To Show that o/p of integrator depends upon respective bit sequence

Let's consider the product signal at the o/p of upper multiplier

$$S(t)\sin(2\pi f_0 t) = b_0(t)\sqrt{P_s}\cos(2\pi f_0 t)\sin(2\pi f_0 t) + b_e(t)\sqrt{P_s}\sin^2(2\pi f_0 t)$$

The signal is integrated by upper integrator.

$$\therefore \int_{(2k-1)T_b}^{(2k+1)T_b} S(t)\sin(2\pi f_0 t)\,dt = b_0(t)\sqrt{P_s}\int_{(2k-1)T_b}^{(2k+1)T_b}\cos(2\pi f_0 t)\sin(2\pi f_0 t)\,dt + b_e(t)\sqrt{P_s}\int_{(2k-1)T_b}^{(2k+1)T_b}\sin^2(2\pi f_0 t)\,dt$$

$$\Rightarrow \int_{(2k-1)T_b}^{(2k+1)T_b} S(t)\sin(2\pi f_0 t)\,dt = \frac{b_0(t)\sqrt{P_s}}{2}\int_{(2k-1)T_b}^{(2k+1)T_b}\sin 4\pi f_0 t\,dt + \frac{b_e(t)\sqrt{P_s}}{2}\int_{(2k-1)T_b}^{(2k+1)T_b} 1\,dt - \frac{b_e(t)\sqrt{P_s}}{2}\int_{(2k-1)T_b}^{(2k+1)T_b}\cos 4\pi f_0 t\,dt$$

⇒ for above eqn 1st & 3rd terms have full cycles of over two bit period & hence integration will be zero.

$$\int_{(2k-1)T_b}^{(2k+1)T_b} S(t)\sin(2\pi f_0 t)\,dt = \frac{b_e(t)\sqrt{P_s}}{2}\Big[t\Big]_{(2k-1)T_b}^{(2k+1)T_b}$$

$$= \frac{b_e(t)\sqrt{P_s}}{2}\left[(2k+1)T_b - (2k-1)T_b\right]$$

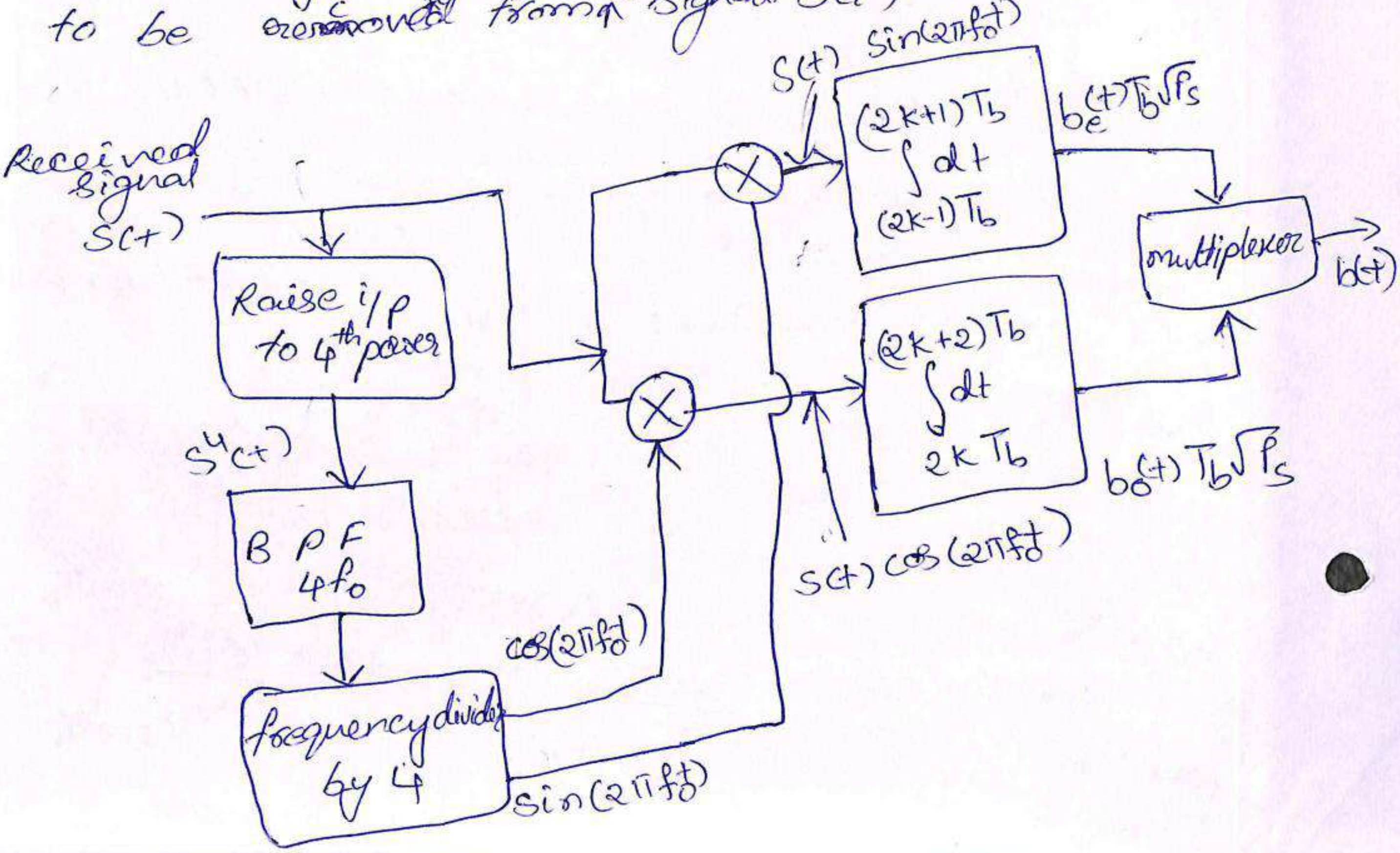
$$= b_e(t)\sqrt{P_s}\,T_b$$

→ Thus the upper integrator responds to even sequence only.

||ly we can obtain the o/p of lower integrator as $b_0(t)\sqrt{P_s}\,T_b$

The QPSK receiver

This is Synchronous reception. Therefore coherent carrier is to be recovered from rx Signal S(t)



Isolation of carrier

The rx Signal is first raised to its 4th power, i.e $S^4(t)$.

The it passed through BPF centred around $4f_0$.

The o/p of BPF is a coherent carrier of frequency $4f_0$.

this is divided by 4 & it gives two coherent quadrature carriers $\cos(2\pi f_0 t)$ & $\sin(2\pi f_0 t)$

Synchronous demodulators.

It consists of multiplier & integrator

Integrator :- It integrates the product signal over two bit interval

Sampling & multiplexing odd & even bit Sequences

~~At the end of this period~~ The o/p of integrator is sampled at offset of onebit period T_b : Hence the o/p of multiplexer is b(t).

Signal space representation of QPSK Signal

(1) The QPSK signal ~~cab~~ can be written as.

$$S(t) = \sqrt{2P_s}\cos\left[2\pi f_0 t + (2m+1)\pi/4\right]\ m = 0,1,2,3$$

→ The above eqn can be expanded as,

$$S(t) = \sqrt{2P_s}\cos(2\pi f_0 t)\cos\left[(2m+1)\pi/4\right] - \sqrt{2P_s}\sin(2\pi f_0 t)\sin\left[(2m+1)\pi/4\right]$$

$$S(t) = \left\{\sqrt{P_s T_b}\cos\left[(2m+1)\pi/4\right]\right\}\sqrt{\frac{2}{T_s}}\cos(2\pi f_0 t)$$
$$- \left\{\sqrt{P_s T_b}\sin\left[(2m+1)\pi/4\right]\right\}\sqrt{\frac{2}{T_s}}\sin(2\pi f_0 t)$$

→ Let $\phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_0 t)$

& $\phi_2(t) = \sqrt{\frac{2}{T_s}}\sin(2\pi f_0 t)$

→ Let $b_o(t) = \sqrt{2}\cos\left[(2m+1)\pi/4\right]$

& $b_e(t) = -\sqrt{2}\sin\left[(2m+1)\pi/4\right]$

$$\therefore S(t) = \sqrt{P_s T_s}\cdot\frac{1}{\sqrt{2}}b_o(t)\phi_1(t) + \sqrt{P_s T_s}\frac{1}{\sqrt{2}}b_e(t)\phi_2(t)$$

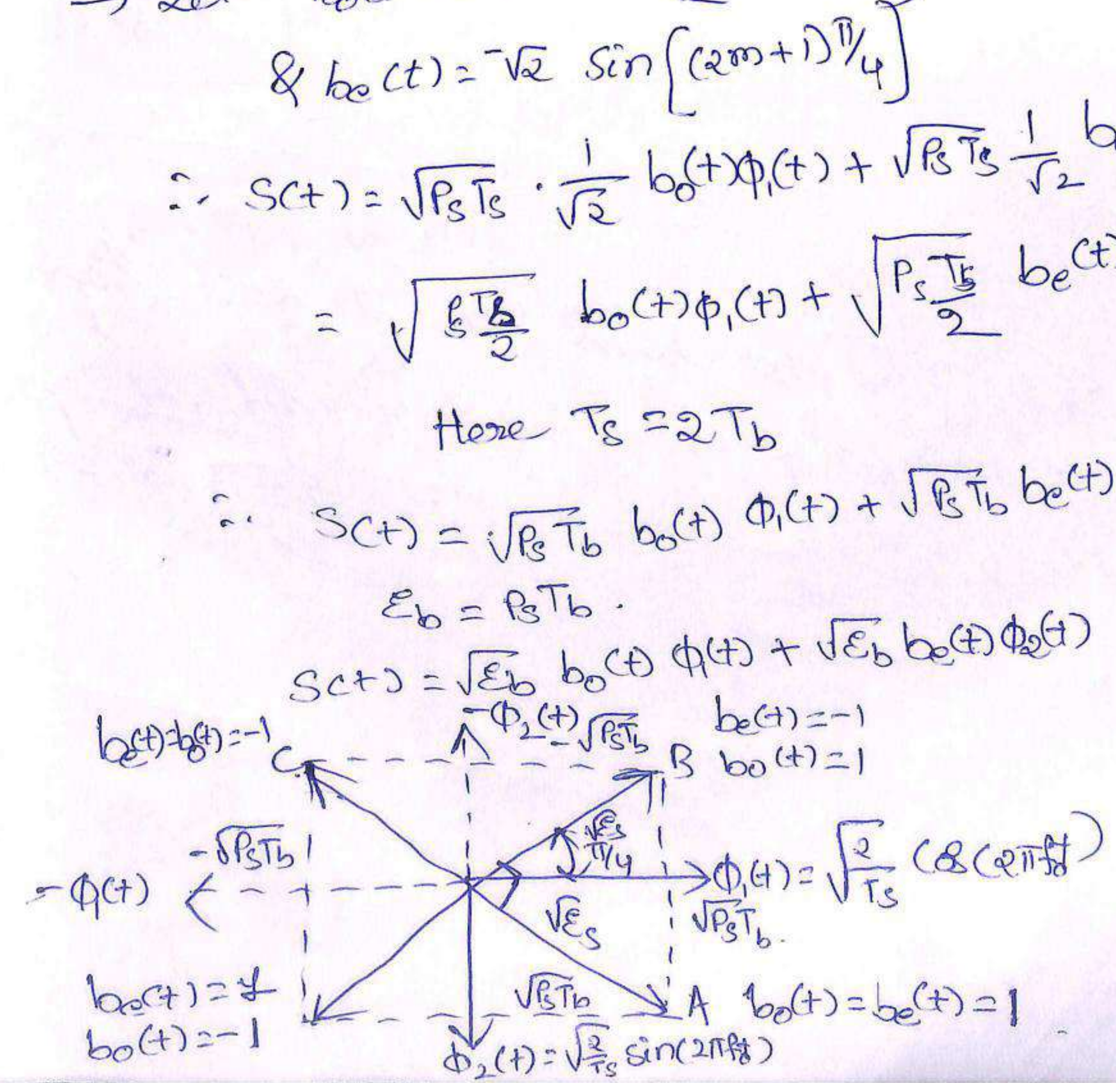
$$= \sqrt{\frac{P_s T_b}{2}}\ b_o(t)\phi_1(t) + \sqrt{\frac{P_s T_s}{2}}\ b_e(t)\phi_2(t)$$

Here $T_s = 2T_b$

$$\therefore S(t) = \sqrt{P_s T_b}\ b_o(t)\ \phi_1(t) + \sqrt{P_s T_b}\ b_e(t)\phi_2(t)$$

$$\mathcal{E}_b = P_s T_b.$$

$$S(t) = \sqrt{\mathcal{E}_b}\ b_o(t)\ \phi_1(t) + \sqrt{\mathcal{E}_b}\ b_e(t)\phi_2(t)$$



Spectrum of QPSK Signal

1, PSD of NRZ waveform

$$S(f) = V_b^2 T_b \left[\frac{\sin(\pi f T_b)}{(\pi f T_b)}\right]^2$$

& $V_b = \sqrt{P_s}$

PSD of b(t) $\rightarrow$ $S(f) = P_s T_b \left[\frac{\sin(\pi f T_b)}{(\pi f T_b)}\right]^2$

2, PSD of even & odd number sequence.

$$S_e(f) = P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$$

$$\& \; S_o(f) = P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$$

3, PSD of QPSK Signal =

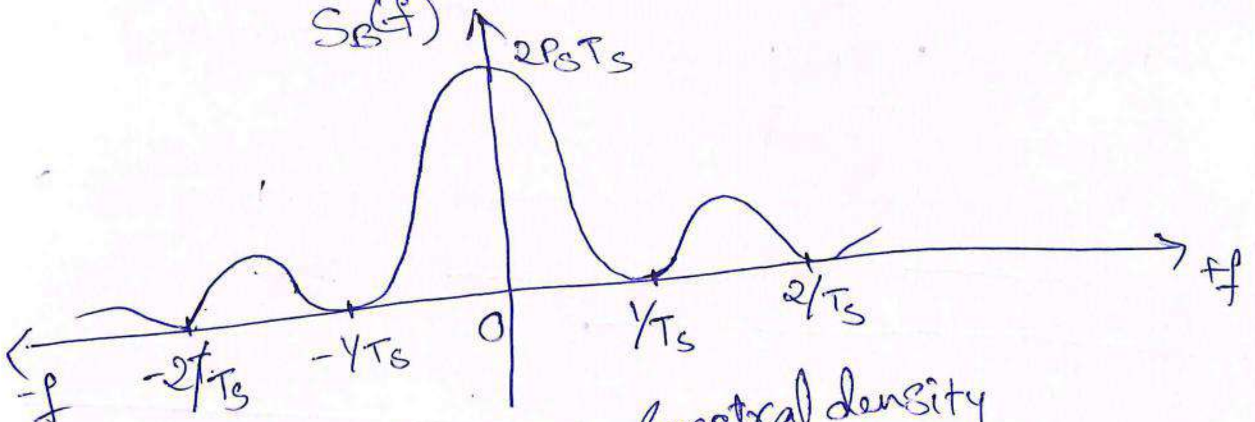
$$S_B(f) = S_e(f) + S_o(f)$$

$$= P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2 + P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$$

$$= 2 P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$$

This eqn gives baseband PSD of QPSK Signal. upon modulation of carrier of frequency f_0, the spectral density given by above eqn is shifted at $\pm f_0$.

Band width of QPSK Signal



plot of Power Spectral density of QPSK Signal.

$$B.W = \frac{1}{T_S} - \left[\frac{-1}{T_S}\right]$$

$$= \frac{2}{T_S}$$

$$T_S = 2T_b$$

$$B.W = \frac{2}{2T_b} = f_b.$$

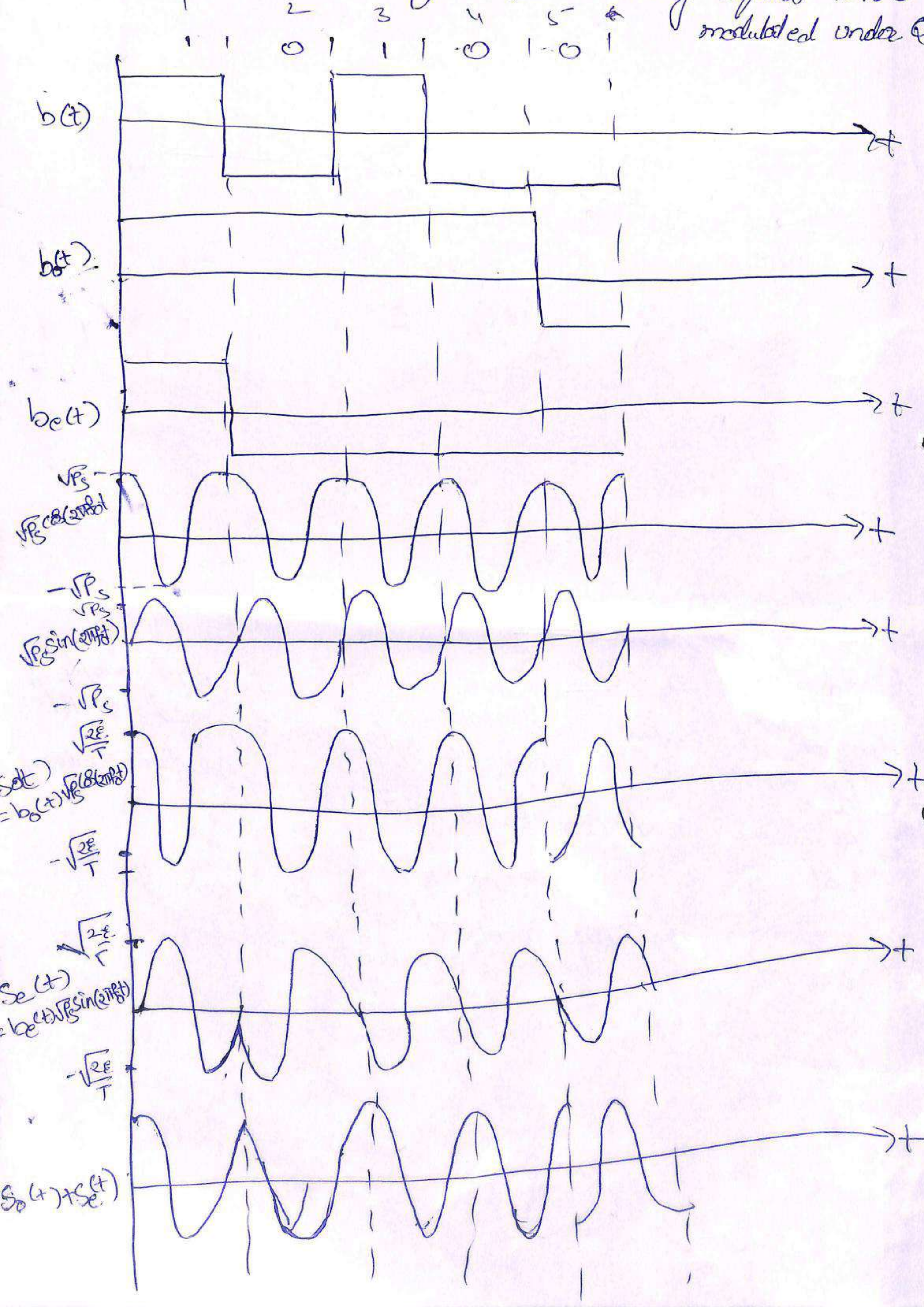
Advantages of QPSK

1) For the same bit error rate, the B.W required by QPSK is reduced to half as compared to BPSK.

2) Because of reduced band width, the information transmission rate of QPSK is higher.

3) Variation in OQPSK amplitude is not much. Hence carrier power almost remains constant.

-Q write the waveforms for a binary Sequence 10100 modulated under QPSK.



Basebandtransmission

def: -

problems occured: - The presence of o/p due to the other bits interfere with the o/p of reqd bit.

1) I S I : This ~~pa~~ interference takes place due to dispersive nature of channel.

corrective measures to minimise errors

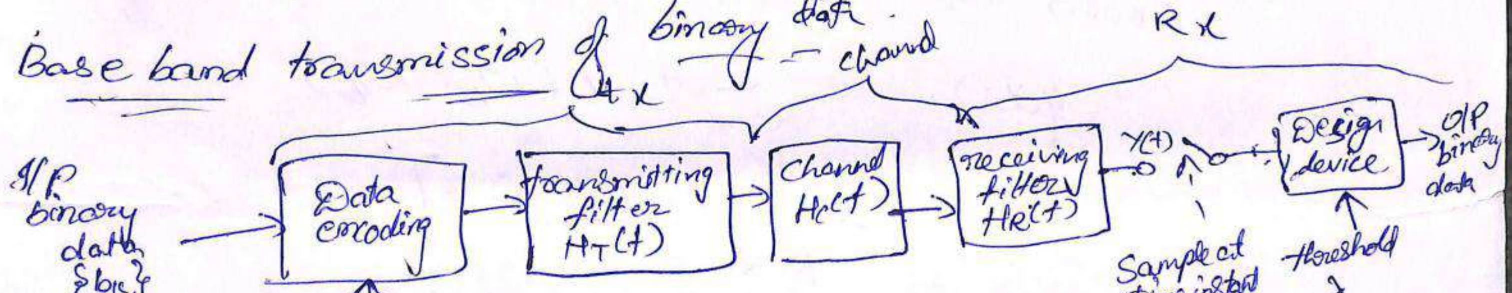
1) Nyquist criterion gives a condition of distortionless base band transmission

=> It is possible to reduce the effect of ISI by with the help of raised cosine spectrum.

-> Correlative level coding is also used to minimize effect of ISI.

-> Equalizers are used to compensate for distortion.

Base band transmission of binary data



Baseband binary data transmission.

The binary data b_K is applied to the data encoder. The encoder generates the pulse waveform $x(t)$

$$x(t) = \sum_{K=-\infty}^{\infty} A_K g(t - KT_b)$$

Here T_b is the duration of each i/p binary bit

$g(t)$ is shaping pulse

$$\& \quad A_K = \begin{cases} +a & \text{if } b_K = 1 \\ -a & \text{"} \ b_K = 0 \end{cases}$$

The Signal $Y(t)$ is sampled synchronously. The sampling instants are $t = iT_b$. These sampling instants are synchronous to clk pulses at tx. The sampled signal $Y(t_i)$ is then given to decision device

$Y(t_i) > \lambda$ Select symbol 1,

$y(t_i) < \lambda$ " " 0

Intersymbol Interference

→ consider the o/p $y(t)$ of receiving filter $y(t)$ can be given interms of A_K as,

$$y(t) = \mu \sum_{K=\alpha}^{\alpha} A_K P(t-KT_b) \quad -(1)$$

Here μ is the scaling factor
$p(t)$ is the shape different from that $g(t)$

Here $A_K g(t)$ is the signal applied to i/p of cascade of transmitting filter, channel & receiving filter. The o/p of cascade & connection is $\mu A_K P(t)$.

From eqn (1) at $t = iT_b$ we can write.

$$y(t_i) = \mu \sum_{K=-\alpha}^{\alpha} A_K P(iT_b - KT_b)$$

$$= \mu \sum_{K=\alpha}^{\alpha} A_K P[(i-K)T_b]$$

Let us rearrange the above eqn as

$$y(t_i) = \mu A_i P(0) + \mu \sum_{K=-\alpha}^{\alpha} A_K P[(i-K)T_b]$$

The first term represents the value of $y(t_i)$ when $i=k$. $p(t)$ is normalized such that $p(0) = 1$. Hence above eqn becomes,

$$y(t_i) = \mu A_i + \mu \sum_{K=-\lambda}^{\alpha} A_K P[(i-K)T_b]$$

(if the ISI is absent 2nd term will not be present)

& $i = 0, \pm 1, \pm 2, \pm 3,$

→ The first term in above eqn is μA_i. It is the contribution of the i^{th} transmitted bit.

→ The second term represent the residual effect of all other bits transmitted before & after the sampling instant t_i.

Baseband transmission & optimal Reception of digital Signal

Topics :- pulse Shaping for optimum transmissions, & baseband Signal Rx, probability of error, optimum Receiver, optimal of coherent Reception, Signal Space representation & probability of error, eye diagrams, cross talk.

I. Pulse Shaping for optimum transmission:-

Optimum transmission is nothing but nyquist criterion for distortionless baseband binary transmission.

Nyquist pulse Shaping criterion:-

(i) In time domain:- If the received pulse p(t) satisfies the below condition then, we get a Siagnal which is free from ISI.

$$P[(i-k)T_b] = \begin{cases} 1 \text{ for } i=k \\ 0 \text{ for } i \neq k \end{cases}$$

→ The above eqn is the condition in time domain.

2) In frequency domain:-

→ The frequency domain condition for zero ISI is

where T_b → bit period.
f_b → bit rate. $f_b = \frac{1}{T_b}$

$$\boxed{\sum_{n=-\infty}^{\infty} P(f-nf_b) = T_b}$$

The above eqn is called nyquist pulse Shaping for base band transmission.

Nyquist bandwidth - (B_0)

The nyquist bandwidth is the minimum transmission bandwidth for zero ISI.

→ nyquist bandwidth B_0 is related to bit period T_b as

$$T_b = \frac{1}{2B_0}$$

$$B_0 = \frac{1}{2T_B} = \frac{f_b}{2}$$

$$\boxed{B_0 = \frac{\text{bit rate}}{2}}$$

Raised cosine channel: -

In freq domain: -

In the raised cosine spectrum, the frequency response P(f) decreases towards zero.

The raised cosine Spectrum is given as follows

$$P(f) = \begin{cases} \frac{1}{2B_0} & \text{for } -f_1 < f < f_1 \\ \frac{1}{4B_0}\left\{1+\cos\left[\frac{\pi(|f|-f_1)}{2B_0-2f_1}\right]\right\} & \text{for } f_1 < |f| < 2B_0-f_1 \\ 0 & \text{elsewhere.} \end{cases}$$

where $f_1 = B_0 - B_0\alpha$

B_0 = Nyquist B.W $= \frac{f_b}{2}$

α = roll-off factor

$$\alpha = 1 - \frac{f_1}{B_0}$$

In time domain: -

The inverse Fourier transform of raised cosine Spectrum gives the time domain pulse p(t)

$$p(t) = \text{Sinc}(2B_0 T)\,\frac{\cos(2\pi\alpha B_0 t)}{1-16\alpha^2 - B_0^2 t^2}$$

B.W required for raised cosine channel

→ The B.W required for raised cosine ~~chan~~ Spectrum will be 'B'

$$B = 2B_0 - f_1 \quad \text{— (1)}$$

w.k.t $\alpha = 1 - \frac{f_1}{B_0}$

$$\Rightarrow f_1 = B_0 - B_0\alpha$$

Substitute in eqn (1)

$$B = 2B_0 - B_0 + B_0\alpha$$

$$\boxed{B = B_0(1+\alpha)}$$

Prob:- The o/p of a digital computer is a rate of 64kbps. If the roll off factors (i) $\alpha = 1$ (ii) $\alpha = 0.5$ (iii) $\alpha = 0.25$ (iv) $\alpha = 0$. Find the b.w req'd to transmit data in each case.

Sol:- $f_b = 64$ kbps.

$B_0 = \frac{f_b}{2} = 32$ kbps.

(i) $B = 64$ KHz
(ii) $B = 48$ KHz
(iii) $B = 40$ KHz
(iv) $B = B_0 = 32$ KHz

→ If $\alpha = 0$, $B = B_0$ i.e raised cosine B.W = nyquist B.W

→ $\alpha\uparrow$, $B\uparrow$ above nyquist B.W B_0.

→ If $\alpha = 1$, $B = 2B_0$.

A base band signal receiver:-

→ The ckt diagram of a simple base band signal receiver for the detection of digital signal is as shown in fig.

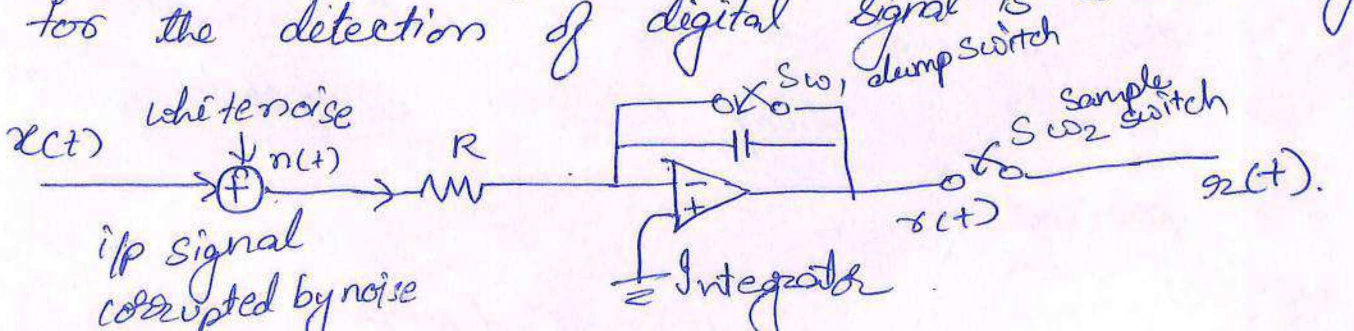


fig:- Base band signal rx.

→ The digital signal $x(t)$ is corrupted by white noise $n(t)$ during transmission over the channel.

→ Such noise signal $[x(t)+n(t)]$ is given to the i/p of Integrate & dump filter.

→ The capacitor is discharged fully at the begining of the bit interval. This is acheived by temporarily closing switch SW, at the begining of bit interval.

→ The integrator then integrates noisy i/p signal over one bit period. we get $r(t)$

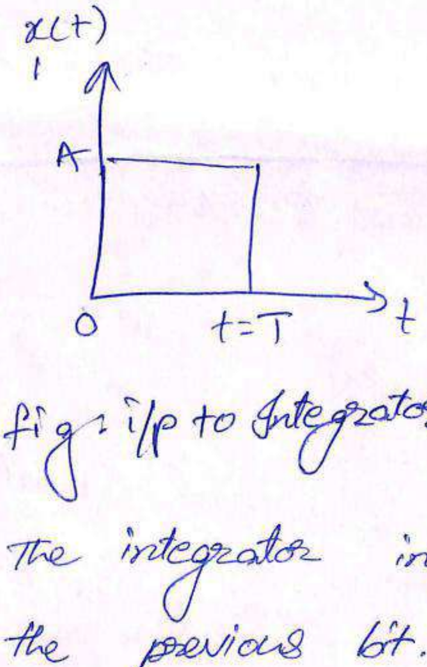


fig: i/p to Integrator

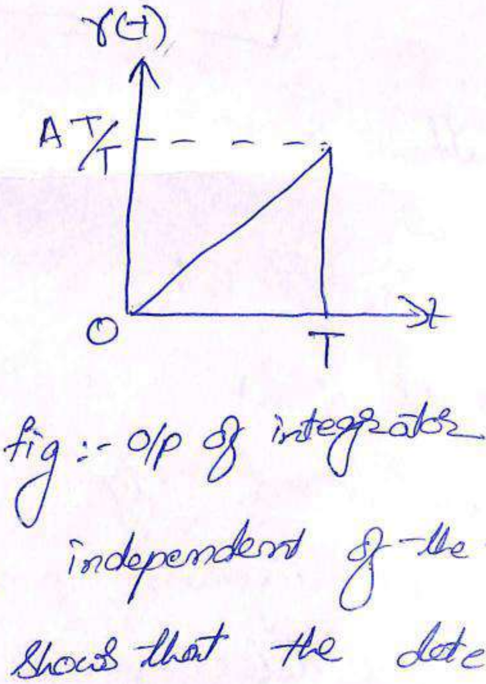


fig:- o/p of integrator

→ The integrator integrates independent of the the value of the previous bit. This shows that the detection in integrate & dump filter is unaffected by values of previous bit.

→ The o/p of integrator will decrease after $t>T$.

calculation of signal power

o/p of integrator: $r(t) = \int_0^T x(t)+n(t)dt$

$= \int_0^T x(t) + \int_0^T n(t)dt$

$= x_0(t) + n_0(t)$. consider the o/p signal voltage

$x_0(t) = \frac{1}{RC}\int_0^T x(t)dt$ since $x(t) = A$

$= \frac{1}{RC}\int_0^T A dt = \frac{AT}{RC} = \frac{AT}{\tau}$

o/p signal power $= \frac{x_0^2(t)}{1\Omega} = \frac{A^2 T^2}{\tau^2}$

Signal to noise ratio of baseband Rx :- (figure of merit).

A — tutorial present
52, LE5, LE4, LE10, LE17

→ S/N ratio, $\rho = \frac{\text{Signal Power}}{\text{noise power}}$

$$\rho = \frac{A^2T^2}{\tau^2} \Big/ \frac{N_0 T}{2\tau^2}$$

$$\boxed{\rho = \frac{2A^2T}{N_0} \text{ or } \frac{A^2T}{N_0/2}}$$

→ It is also called figure of merit.

III Probability of error of integrate & dump Rxr :-

→ An i/p pulse applied is $x(t)$ whose amplitude is ±A.

→ The o/p of integrator is,

$$r(t) = x_0(t) + n_0(t)$$

$$x_0(t) = \frac{AT}{\tau} \quad \text{for } x(t) = A$$

$$x_0(t) = -\frac{AT}{\tau} \quad \text{for } x(t) = -A.$$

→ Since the occurence of $-A$ or $+A$ are mutually exclusive, the probability of error is given by these two are also equal.

$$\therefore P_e = p\left[n_0(t) > \frac{AT}{\tau}\right) = p\left(n_0(t) < -\frac{AT}{\tau}\right]$$

i/p $x(t)$	Value of $n_0(t)$ for error in the o/p	Probability of error P_e
$-A$	Error introduced if $n_0(t) > \frac{AT}{\tau}$	Probability of error can be obtained by evaluating the probability that $n_0(t) > \frac{AT}{\tau}$
A	Error introduced if $n_0(t) < -\frac{AT}{\tau}$	Probability can be obtained by evaluating the probability that $n_0(t) < \frac{AT}{\tau}$

PDF of white Gaussian distributed noise of zero mean, $\Rightarrow f_x(n_0(t)) = \frac{\tau}{\sqrt{\pi N_0 T}} e^{-[n_0(t)]^2 / \left[\frac{N_0 T}{\tau^2}\right]}$

$$\therefore \; P_e = P\left(n_0(t) > \frac{AT}{\tau}\right) = P\left(n_0(t) < -\frac{AT}{\tau}\right)$$

$$P_e = P\left(n_0(t) > \frac{AT}{\tau}\right) = \int_{\frac{AT}{\tau}}^{\infty} f_x(n_0(t)) \, d[n_0(t)]$$

PDF of gaussian distributed noise

$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-m)^2/2\sigma^2}$

Here $x = n_0(t)$

& $m = 0$

$\sigma = (\overline{x^2} - m_x^2)^{1/2}$

$= \sqrt{\frac{N_0 T}{2\tau^2}}$

$$f_x(n_0(t)) = \frac{T}{\sqrt{\pi N_0 T}} e^{-\left[(n_0(t))^2 / \frac{N_0 T}{\tau^2}\right]}$$

$$P_e = \int_{AT/\tau}^{\infty} \frac{\tau}{\sqrt{\pi N_0 T}} e^{-\left[(n_0(t))^2 / \frac{N_0 T}{\tau^2}\right]} d[n_0(t)]$$

Let $\frac{n_0(t)^2}{\frac{N_0 T}{\tau^2}} = y^2 \Rightarrow \frac{n_0(t)}{\sqrt{N_0 T}/\tau} = y$

$$\Rightarrow n_0(t) = \frac{y}{\tau}\sqrt{N_0 T}$$

$$d[n_0(t)] = \sqrt{\frac{N_0 T}{\tau}} \, dy.$$

where $n_0(t) \rightarrow \alpha$, $y \rightarrow \alpha$

$n_0(t) \rightarrow \frac{AT}{\tau}$, $y = \sqrt{\frac{A^2T}{N_0}}$

$$\therefore \; P_e = \int_{\sqrt{\frac{A^2T}{N_0}}}^{\alpha} \frac{\tau}{\sqrt{\pi N_0 T}} e^{-y^2} \sqrt{\frac{N_0 T}{\tau}} \, dy$$

$$P_e = \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{A^2T}{N_0}}}^{\alpha} e^{-y^2} dy$$

$$P_e = \frac{1}{2} \left\{ \frac{2}{\sqrt{\pi}} \int_{\sqrt{\frac{A^2 \cancel{T}}{N_0}}}^{\alpha} e^{-y^2} dy \right.$$

$$P_e = \frac{1}{2} \left\{ \frac{2}{\sqrt{\pi}} \int_{\sqrt{\frac{A^2T}{N_0}}}^{\alpha} e^{-y^2} dy \right\}$$

$$\boxed{P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{A^2T}{N_0}}}$$

let $A^2T = E$

$P_E = \frac{1}{2} \operatorname{erfc} \sqrt{E/N_0}$

w.k.t complementary error function

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\alpha} e^{-x^2} dx$$

→ P_e falls rapidly as the ratio E/N_0 increases.

→ max P_e is $1/2$ when E/N_0 very small.

→ that mean if signal is lost entirely in noise P_e will be $1/2$

→ that mean the receiver will make incorrect decisions half number of time.

Optimum receiver

It is used to minimise the probability of error. Here we considers generalised gaussion noise of zero mean &

→ The block diagram of optimum receiver is following.

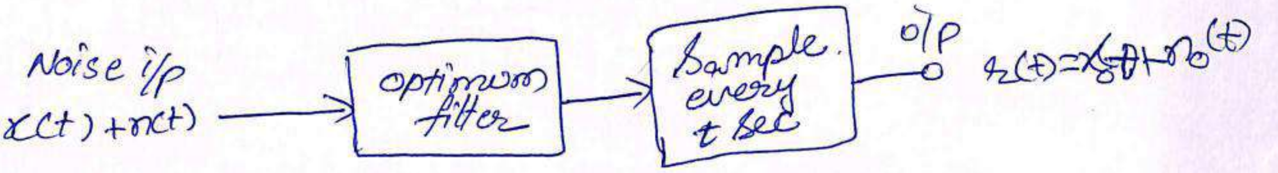


fig optimum receiver.

The i/p $x(t) = x_1(t) = A$ for binary '1'

$= x_2(t) = -A$ for binary '0'

→ i/p to the Rxer $= x(t)+n(t) = x_1(t)+n(t)$ or $x_2(t)+n(t)$

o/p from the Rxer $= x_0(T)+n_0(T) = x_{01}(T)+n_0(T)$

or $x_{02}(T)+n_0(T)$

→ In the absence of noise i.e $n_0(T)=0$.

$$Y(T) = \underset{x_1(t)}{x_{01}(T)} \text{ or } x_{02}(T).$$

In the presence of noise select $x_1(t)$ if $x(t)$ is close to $x_{01}(T)$ than $x_{02}(T)$ vie verse

→ ∴ Decision boundary $= \left[\frac{x_{01}(T)+x_{02}(T)}{2}\right]$

Probability of error of optimum filter

→ Error will be generated if

$$n_0(T) \geq \frac{x_{01}(T)-x_{02}(T)}{2}$$

→ Probibility of error, Pe ie

$$P_e = P\left[n_0(T) \geq \frac{x_{01}(t)-x_{02}(t)}{2}\right]$$

$$= \int_{\frac{x_{01}(T)-x_{02}(T)}{2}}^{\infty} f_x(n_0(t))\, d(n_0(t))$$

Suppose $x_2(t)$ was t x but $x_{01}(t) > x_{02}(t)$ If $n_0(T)$ is +ve & larger in magnitude then voltage difference $\frac{1}{2}[x_{01}(T)+x_{02}(T)]-x_{02}$ then error is generated.

where $f_x(n_0(t)) \Rightarrow$ P.d.f of $n_0(t)$ i.e Gaussian noise

$$f_x(n_0(t)) = \frac{1}{\sigma\sqrt{2\pi}} e^{-[(n_0(t)]^2/2\sigma^2}$$

$$\therefore \quad P_e = \int_{\frac{x_{01}(t)-x_{02}(T)}{2}}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-[\cancel{2}n_0(t)]^2/2\sigma^2} \cdot d[n_0(t)]$$

Let $\frac{n_0(t)^2}{2\sigma^2} = y^2$

$$\Rightarrow n_0(t) = \sigma\sqrt{2}\,y$$

$$d(n_0(t)) = \sigma\sqrt{2}\,dy.$$

$$n_0(t) \to \infty, \; y \to \infty$$

$$n_0(t) \Rightarrow \frac{x_{01}(T)-x_{02}(T)}{2}, \; y \cancel{\to \infty} = \frac{x_0(T)-x_{02}(T)}{2\sqrt{2}\sigma}$$

~~$n_0(t/2) =$~~

$$\therefore P_e = \int_{\frac{x_{01}(t)-x_{02}(T)}{2\sqrt{2}\sigma}}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-y^2} \sigma\sqrt{2}\, dy$$

$$= \frac{1}{\sqrt{\pi}} \int_{\frac{x_0(T)-x_{02}(T)}{2\sqrt{2}\sigma}}^{\infty} e^{-y^2} dy$$

$$= \frac{1}{2} \left[\frac{2}{\sqrt{\pi}} \int_{\frac{x_{01}(T)-x_0(T)}{2\sqrt{2}\sigma}}^{\infty} e^{-y^2} dy \right.$$

$$\boxed{P_e = \frac{1}{2} \operatorname{erfc}\left[\frac{x_{01}(T)-x_{02}(T)}{2\sqrt{2}\sigma}\right]}$$

The transfer function of optimum filter is

$$H(f) = K \cdot \frac{X^*(f)}{S_{ni}(f)} e^{-j2\pi fT}$$ PSD of noise → $S_{ni}(f)$

The (S/N)max of optimum Rxer is.

$$(S/N)_{max} = P_{max} = \left[\frac{x_o^2(T)}{\sigma^2}\right]_{max} = \left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max}$$

$$= \int_{-\infty}^{\infty} \frac{|x(t)|^2}{S_{ni}(f)} df .$$

II Matched filter :-

when the noise is white gaussian noise, than the optimum filter is called "matched filter".

Impulse response of the Matched filter :-

w.r.t the transfer function of the optimum filter is

$$H(f) = K \cdot \frac{X^*(f)}{S_{ni}(f)} \cdot e^{j2\pi fT} \quad \text{——} (1)$$

For the matched filter, the noise is white noise & its PSD is given by :

$$S_{ni}(f) = \frac{N_o}{2}$$

put in (1)

$$\therefore H(f) = K \cdot \frac{X^*(f)}{N_o/2} e^{j2\pi fT}$$

$$\boxed{\therefore H(f) = \frac{2K}{N_o} X^*(f) e^{j2\pi fT}}$$

The above eqn is transfer function of matched filter from the property of F.T,

$$x^*(f) = X(-f)$$

$$\therefore \quad H(f) = \frac{2k}{N_0} X(-f) e^{-j2\pi f T}$$

→ The impulse response of a matched filter is obtaine

by taking Inverse F.T of H(f)

$$\therefore \quad h(t) = \text{IFT}\{H(f)\}$$

$$= \text{IFT}\left[\frac{2k}{N_0} X(-f) e^{-j2\pi f T}\right\}$$

$$h(t) = \frac{2k}{N_0} \text{IFT}\{X(-f) e^{-j2\pi f T}\}$$

→ we K.T

$$FT\,[x(-t)] = X(-f)$$

$$F.T\,[x(T-t)\} = X(-f) e^{-j2\pi f T}$$

$$\therefore \quad h(t) = \frac{2k}{N_0} x(T-t)$$

$$\text{if} \quad x(t) = x_1(t) - x_2(t)$$

$$\rightarrow \boxed{h(t) = \frac{2k}{N_0}\left[x_1(T-t) - x_2(T-t)\right]}$$

Probability of error of matched filter

→ w.k.T the probility of error of optimum filter

is $$P_e = \frac{1}{2}\,\text{erfc}\left\{\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\,\sigma}\right\} \quad \text{—(1)}$$

w.k.T max (S/N) i.e P_{max} of optimum filter is

$$\left[\frac{x_{o1}(T)-x_{o2}(T)}{\sigma}\right]^2_{max} = \int_{-\alpha}^{\alpha} \frac{|X(f)|^2}{S_{ni}(f)}\, df$$

→ for matched filter, $S_{ni}(f) = \frac{N_o}{2}$ (∵ white noise PSD)

$$\therefore \left[\frac{x_{o1}(T)-x_{o2}(T)}{\sigma}\right]_{max}^2 = \int_{-\alpha}^{\alpha} \frac{|X(f)|^2}{N_0/2} . df$$

$$= \frac{2}{N_0} \int_{-\alpha}^{\alpha} |X(f)|^2 df \quad \text{—— (2)}$$

→ From parseval's theorem

$$\text{Energy} = \int_{-\alpha}^{\alpha} |X(f)|^2 df = \int_{-\alpha}^{\alpha} x^2(t)\, dt = \int_0^T x^2(t)\, dt$$

$$\therefore \int_{-\alpha}^{\alpha} |X(f)|^2 df = \int_0^T [x_1(t) - x_2(t)]^2 dt$$

$$= \int_0^T x_1^2(t)\, dt + \int_0^T x_2^2(t)\, dt - 2\int_0^T x_1(t)\, x_2(t)\, dt$$

If we select ~~w.k.t~~ $x_1(t) = -x_2(t)$. then the energies are equal

$$= \int_0^T x_1^2(t)\, dt + \int_0^T x_2^2(t)\, dt + 2\int_0^T x_1^2(t)\, dt$$

$$E + E + 2E = 4E$$

$$\therefore \int_{-\alpha}^{\alpha} |X(f)|^2 df = 4E .$$

∴ From eqn (1)

$$\therefore \left[\frac{x_{o1}(T)-x_{o2}(T)}{\sigma}\right]^2_{max} = \frac{2}{N_0} 4E = \frac{8E}{N_0}$$

$$\frac{x_{01}-(T) - x_{02}(T)}{\sigma} = \sqrt{\frac{8E}{N_0}} = 2\sqrt{2}\frac{E}{N_0}$$

from eqn-(1)

$$P_E = \frac{1}{2}\,\mathrm{erfc}\left[\frac{2\sqrt{2}\,E/N_0}{2\sqrt{2}}\right]$$

$$\boxed{P_E = \frac{1}{2}\,\mathrm{erfc}\left[\sqrt{\frac{E}{N_0}}\right]}$$

VI Optimal of coherent reception:-

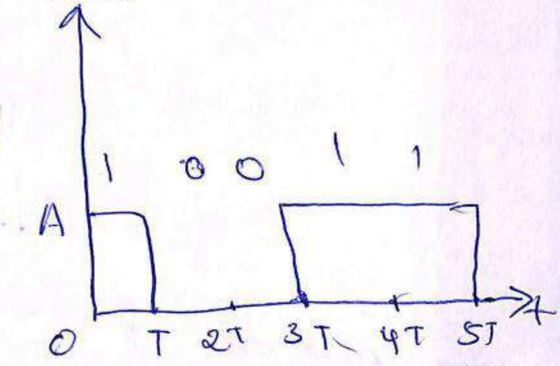
When the signal is transmitted ove a base band or band pass channel, the noise interfers it. Therefore the signal loses its shape & it becomes difficult to detect a particular value of a digit.

→ The base band signal can be detected by the techniques like integrator, optimum filter, matched filter & corelator. All these techniques maximise the signal to noise ratio of received ~~noise~~ signal.

→ Normally the S/N is maximum at end of symbol period 'T'. The o/p is then sampled at $t=T$ & decision is taken. The ~~o/p is then sa~~ probability of error in o/p is minimum when $\frac{S}{N}$ is max.

→ In bandpass transmission, the noise interference depend upon the type of modulation technique used. Hence the probability of error in the o/p also depends on particular type of digital modulation.

Detection of PCM Signal:-



Consider the binary PCM Signal is as shown in fig.

→ This is a baseband signal.

→ Let us calculate the probability of error at the receiver in the presence of white gaussian noise.

→ Let this signal be received by matched filter receiver

→ The received signal shape will be distorted because of presence of noise.

binary '1' ⇒ $x_1(t) = A$, for $0 \leq t < T$

'0' ⇒ $x_2(t) = 0$, for $0 \leq t \leq T$.

→ w.k.t. the probability of the optimum filter is

$$P_e = \frac{1}{2} \operatorname{erfc}\left\{\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\,\sigma}\right\} \quad \text{— (1)}$$

→ & the P_{max} (i.e S/N) is $\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max}$

$$= \int_{-\infty}^{\infty} \frac{|X(f)|^2}{S_{ni}(f)} df .$$

→ For a matched filter, we consider the white gaussian noise & its PSD is $S_{ni}(f) = \frac{N_0}{2}$.

$$\therefore \left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \int_{-\alpha}^{\alpha} \frac{|X(f)|^2}{N_0/2} df$$

$$= \frac{2}{N_0} \int_{-\alpha}^{\alpha} |X(f)|^2 df \quad \text{—— (2)}$$

→ From parseval's energy theorem.

$$E = \int_{-\alpha}^{\alpha} |X(f)|^2 df = \int_{-\alpha}^{\alpha} x^2(t) dt$$

$x(t) = x_1(t) - x_2(t)$. But $x_2(t) = 0$.

$\therefore$ for $0 \leq t \leq T$, $x(t) = x_1(t) = A$.

$$\therefore \int_{-\alpha}^{\alpha} |X(f)|^2 df = \int_0^T A^2 dt$$

$$= A^2 [t]_0^T$$

$$\int_{-\alpha}^{\alpha} |X(f)|^2 df = A^2 T.$$

Subs in eqn (2)

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \frac{2}{N_0} A^2 T = \frac{2A^2T}{N_0}$$

$$\rightarrow \frac{x_{01}(T) - x_{02}(T)}{\sigma} = \sqrt{\frac{2A^2T}{N_0}}$$

Sub in eqn (1)

$$P_e = \frac{1}{2} \operatorname{erfc} \left[\frac{\sqrt{2A^2T/N_0}}{2\sqrt{2}} \right]$$

$$\boxed{P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{A^2T}{4N_0}}}$$

∴ This is the expression for Pe of matched filter detection of binary PCM signal.

$$\rightarrow \boxed{P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E}{4N_0}}}$$

VII Signal space representation to calculate error probability

→ If d is the distance b/w nearest signal points, then probability of error is given as.

$$P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{d^2}{4N_0}} \quad \text{---} (1)$$

→ If there are more than one point in neighbourhood, then

$$P_e < \frac{1}{M} \sum_{j=1}^{M} \sum_{k=1}^{M} \frac{1}{2} \operatorname{erfc} \sqrt{\frac{d_{kj}^2}{4N_0}} \quad \text{---} (2)$$

It is max. value of Pe.

Error probability of BPSK :- (QPSK)

→ w.k.t the distance b/w two signal points in BPSK & QPSK is

$$d = 2\sqrt{E_b} = 2\sqrt{P_s T_b}$$

Subs in eq-(1)

$$P_e = \frac{1}{2}\operatorname{erfc}\sqrt{\frac{d^2}{4N_0}}$$

$$= \frac{1}{2}\operatorname{erfc}\sqrt{\frac{4E_b}{4N_0}}$$

$$\boxed{P_e = \frac{1}{2}\operatorname{erfc}\sqrt{\frac{E_b}{N_0}}}$$

→ w.k.t this $P_e < \frac{1}{M}\sum_{j=1}^{M}\sum_{\substack{k=1\\k\neq j}}^{M}\frac{1}{2}\operatorname{erfc}\sqrt{\frac{d_{kj}^2}{4N_0}}$ i.e from eqn(2)

$$P_e < \frac{1}{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\frac{1}{2}\operatorname{erfc}\sqrt{\frac{d^2_{kj}}{4N_0}}$$

$$< \frac{1}{2}\left[\sum_{\substack{k=1\\k\neq 1}}^{2}\frac{1}{2}\operatorname{erfc}\sqrt{\frac{d^2_{k1}}{4N_0}} + \sum_{\substack{k=1\\k\neq 2}}^{2}\frac{1}{2}\operatorname{erfc}\sqrt{\frac{d^2_{k2}}{4N_0}}\right]$$

$$< \frac{1}{2}\left[\frac{1}{2}\operatorname{erfc}\sqrt{\frac{d^2_{21}}{4N_0}} + \frac{1}{2}\operatorname{erfc}\sqrt{\frac{d^2_{12}}{4N_0}}\right]$$

$$< \frac{1}{2}\left[\frac{1}{2}\operatorname{erfc}\sqrt{\frac{4E_b}{N_0}} + \frac{1}{2}\operatorname{erfc}\sqrt{\frac{4E_b}{4N_0}}\right]$$

$$< \frac{1}{2}\left[\operatorname{erfc}\sqrt{\frac{E_b}{N_0}}\right]$$

∴ for M=2, it is equal.

Probability of error of BFSK :-

→ w.k.T, the distance b/w two signal points in BFSK is $d = \sqrt{2E_b}$

$$\therefore P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{2E_b}{4N_0}}$$

$$P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{2N_0}}$$

$$\boxed{P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{0.5 E_b}{N_0}}}$$

Probability of error of BASK :-

→ w.k.t the distance b/w two signal points in BAK is $d = \sqrt{E_b}$

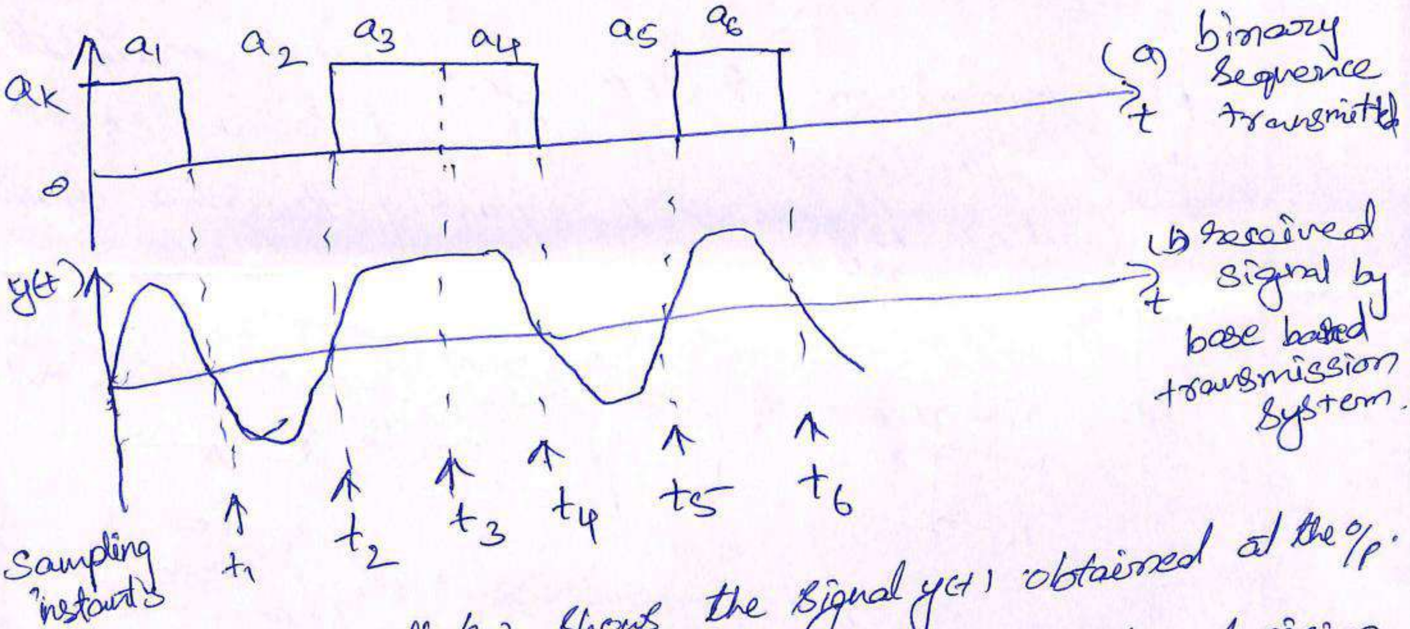
$$\therefore P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{d^2}{4N_0}}$$

$$\boxed{P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{4N_0}}}$$

Eye patterns

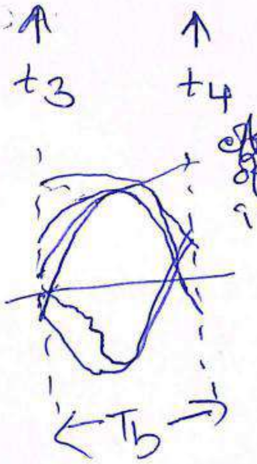
→ The eyepattern is used to Study of the effect of ISI.

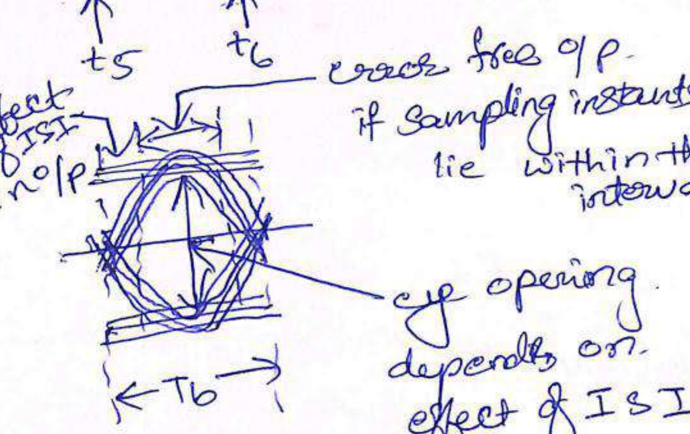
→ when the Sequence is transmitted over a baseband binary transmission System, the Signal obtained at o/p i.e y(t) is a continous time depending on Symbol. that was transmitted. But because of the nature of transmission channel, the Signal becomes continous with increasing & decreasing amplitude.



Here fig b) Shows the Signal y(t) obtained at the o/p. fig (b) also Shows various Sampling instants, decision is ~~taken~~ by the decision device- t_1, t_2, t_3 ---etc. Thus based on the Signal obtained over the period T_b b/w two Sampling instants, decision is taken by decision device. If we cut the signal y(t) as shown in (b) in each interval (T_b) & place it over one another, then we obtain diagram as Shown in (c). This diagram is called eyepattern.

(c) eye pattern of signal in (b) ~~for large~~ ~~no of bits~~ in y(t)

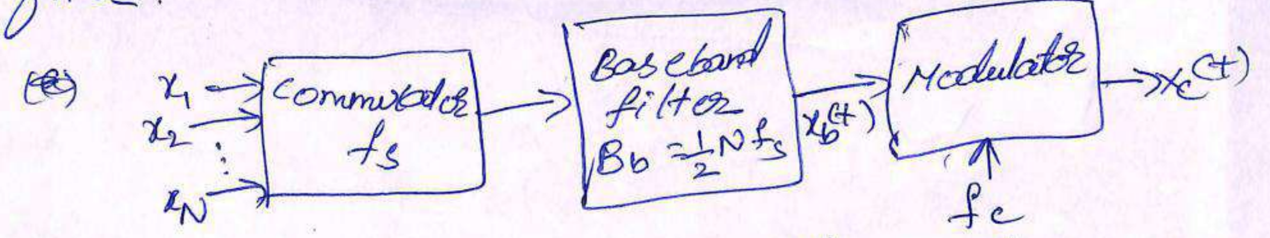




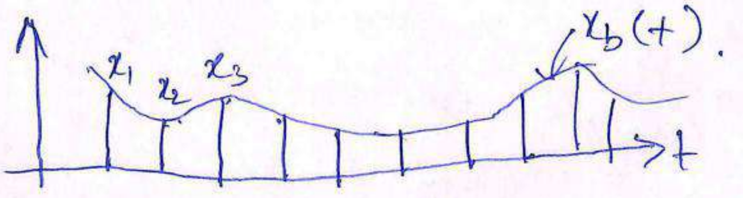
(d) eye pattern for large no. of bits in y(t).

cross talk & guard times

RF transmission of TDM signal needs modulation. Hence TDM signal is converted to a smooth modulating waveform by passing through a baseband filter.



(a) TDM transmission with base band filtering



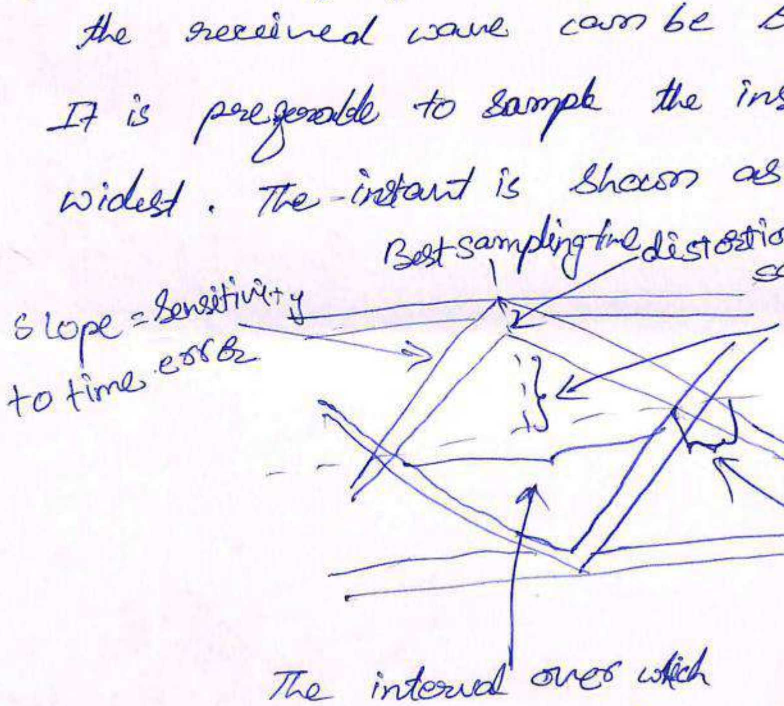
(b) Base band form

definition:- Thus the baseband wave form passes through the values of all the individual samples. The baseband filtering gives rise to interchannel crosstalk from one sample value to the next. In other words crosstalk means the individual sample amplitudes. interfere with each other. This interference can be reduced by increasing the distance b/w the ~~8~~ individual signal samples to avoid crosstalk is called guard time.

Performance of data transmission system using Eye pattern

-> Various important conclusions can be derived from eye pattern.

i) The width of eye opening defines the interval over which the received wave can be sampled without error from ISI. It is preferable to sample the instant at which eye is open widest. The instant is shown as best sampling time in fig.



ii) The sensitivity of the system to timing errors is determined by rate of closure of the eye as the sampling time is varied.

iii) The height of the eye opening at the specified sampling time, is called margin over noise.

-> As the effect of ISI increases the eye opening reduces. If eye is closed completely, then it is not possible to avoid errors in o/p.

-> If there are are M-levels (M-ary system) then eye pattern contains (M-1) eye openings.

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Information theory

Syllabus:-

Information and entropy, condition entropy and redundancy. Shannon Fano coding. Mutual information, Information loss due to noise. Source codings, Huffman code, variable length coding, Source coding to increase average information per bit. Lossy Source coding.

→ Information theory is ~~related~~ to used for mathematical modelling & analysis of communication Systems.

Mathematical representation of Source

Consider the Source which emits the discrete Symbols randomly from set of fixed alphabet i.e

$$X = \{x_0, x_1, x_2 --- x_{K-1}\}$$

The various ~~prob~~ Symbols in 'X' have probabilities of occur $P_0, P_1, P_2, ---$ etc, which can be written as.

$$P(X = x_K) = P_K \quad K = 0, 1, 2, --- K-1$$

This set of probabilities satisfy the following condition,-

$$\sum_{K=0}^{K-1} P_K = 1.$$

Such information Source is called discrete Information Source.

Definition of ~~of~~ information

Let us consider the communication System which transmits messages $m_1, m_2, m_3, ---$ with probabilities of occurrence $P_1, P_2, P_3 ---$

The amount of information transmitted through the message m_K with probability P_K is given as.

$$\text{Amount of information } I_K = \log_2\left[\frac{1}{P_K}\right]$$

Properties of Information.

1) If there is more uncertainty about the message, I_k carried is also more.

2) If receiver knows the message being transmitted, the amount of information carried is zero.

3) If I_1 is the information carried by message m_1 & I_2 is the information carried by m_2, The amount of information carried combinely due to m_1 & m_2 is I_1+I_2

4) If there are $M=2^N$ equally likely messages, then the amount of information carried by each message will be N bits,

Problems:-

1) Calculate the amount of information if binary digits occur with equal likelihood in binary PCM.

Sol → In PCM there are only two binary levels i.e 1 or 0
Since they occur with equal likelihood, their probabilities of occurre.

$$P_1 = P_2 = \frac{1}{2}$$

$$I_1 = \log_2\left[\frac{1}{P_1}\right] \quad \& \quad I_2 = \log_2\left[\frac{1}{P_2}\right]$$

$$I_1 = I_2 = \log_2 2 = 1 \text{ bit}$$

→ If there are M equally likely & independent messages, then prove that amount of information carried by each message will be. $I = N$ bits. where $M = 2^N$ & N is an integer.

Sol:- Since all the M messages are equally likely & independent probability of occurence of each message will be $\frac{1}{M}$.

$$I = \log_2\left[\frac{1}{P_K}\right]$$

$$= \log_2 M$$

$$= \log_2 2^N$$

$$= N \text{ bits}$$

Probl- If I_1 is the information carried by message m_1 & I_2 is the information carried by message m_2, then prove the amount of information carried compositely due to m_1 & m_2 is $I_{1,2} = I_1 + I_2$.

Sol:- The individual amounts carried by messages m_1 & m_2 are

$$I_1 = \log_2\left[\frac{1}{P_1}\right] \qquad I_2 = \log_2\left[\frac{1}{P_2}\right]$$

Since m_1 & m_2 are independent, the probability of composite message is P_1P_2.

$$\therefore \quad I_{1,2} = \log_2\left[\frac{1}{P_1P_2}\right]$$

$$= \log_2\frac{1}{P_1} + \log_2\frac{1}{P_2}$$

$$I_{12} = \underline{I_1 + I_2}$$

Average information content of Symbols in long independent Sequences (Entropy)

Consider that we have M different messages. Let the messages be $m_1, m_2, m_3 --- m_M$ & they have probabilities of occurrence as $P_1, P_2, P_3, --- P_m$. Suppose that sequence of L messages are transmitted. We say that.

P_1L messages of m_1 are transmitted
P_2L " " m_2 " "
⋮
P_ML " " m_M " "

Here $I_1 = \log_2\left[\frac{1}{P_1}\right]$

Since there are P_1L no. of messages of m_1, the total information due to all messages of m_1 will be

$$I_1 = P_1L \log_2\left[\frac{1}{P_1}\right]$$

$''^{ly}$

$$I_{2(total)} = P_2 L \log_2 \left[\frac{1}{P_2}\right]$$

Thus the total information caried due to Sequence of L messags will be

$$I_{(total)} = I_{1(total)} + I_{2(total)} + \cdots + I_{M(total)}$$

$$\therefore I_{(total)} = P_1 L \log_2 \left[\frac{1}{P_1}\right] + P_2 L \log_2 \left[\frac{1}{P_2}\right] + \cdots + P_M L \log_2 \left[\frac{1}{P_M}\right]$$

Avg information = $\dfrac{\text{Total information}}{\text{no. of messags}}$

$$\text{Entropy } (H) = \frac{I_{(total)}}{L}$$

$$H = P_1 \log_2 \left[\frac{1}{P_1}\right] + P_2 \log_2 \left[\frac{1}{P_2}\right] + \cdots + P_M \log_2 \left[\frac{1}{P_M}\right]$$

$$\boxed{H = \sum_{K=1}^{M} P_K \log_2 \left[\frac{1}{P_K}\right]}$$

Properties of Entropy

1) Entropy is zero if the even is sure or it is impossible i.e

$$H = 0 \quad \text{if } P_K = 0 \text{ or } 1.$$

2) when $P_K = \frac{1}{M}$ for all the M symbols, then the symbols are equally likely ∴ for such source entropy is givens

$$H = \log_2 M$$

3) upper boundary on entropy is given as

$$H_{max} = \log_2 M$$

lower bounday on Entropy is zero.

Information Rate :-

Information Rate : $R = rH$

Here R is information rate.

r is rate at which messages are generated

H is entropy or avg information.

$R = r$ messages/sec $\times H$ information bits/message

= Information bits/second.

→ An analog signal is bandlimited to B Hz & sampled at Nyquist rate. The samples are quantized into 4 levels. Each level represents one message. Thus there are 4 messages. The probabilities of occurrence of these 4 levels are $P_1 = P_4 = \frac{1}{8}$ & $P_2 = P_3 = \frac{3}{8}$. Find out information rate of source.

Sol :- $H = P_1 \log_2\left[\frac{1}{P_1}\right] + P_2 \log_2\left[\frac{1}{P_2}\right] + P_3 \log_2\left[\frac{1}{P_3}\right] + P_4 \log_2\left[\frac{1}{P_4}\right]$

$= \frac{1}{8}\log_2[8] + \frac{3}{8}\log_2\frac{8}{3} + \frac{3}{8}\log_2 8/3 + \frac{1}{8}\log_2 8$

$H = 1.8$ bits/message.

Or

W.k.T the signal is sampled at nyquist rate.

Nyquist rate = 2B samples/sec.

Since every sample generates one message signal

Messages per second

= 2B messages/sec.

$R = r \times H$

$= 2B \times 1.8 = 3.6B$ bit/sec

→ A black & white TV picture consists of 525 lines of picture informations. Assuming each line consists of 525 picture elements, each having 256 brightness levels & the pictures are repeated at rate of 30 per sec. Calculate the average information conveyed by a TV set to a viewer.

Sol:- r :-

$= 525 \times 30$

$= 8.26875 \times 10^6$ picture elements/sec

H:-
Here the picture element can take any one of the 256 brightness levels. All the brightness levels are equally probable. For M number of equally likely messages the

$H = \log_2 M$

$= \log_2 256 = 8$ bits.

$R = r \times H = 8.26875 \times 10^6 \times 8$

$= 66.15$ Mbps

→ Show that for M number of equally likely messages then entropy of the source is $\log_2 M$.

$$P = \frac{1}{M}$$

$$P_1 = P_2 = P_3 = P_4 = \dots P_M = \frac{1}{M}$$

$$H = \sum_{K=1}^{M} P_K \log_2\left[\frac{1}{P_K}\right]$$

$$= P_1 \log_2\left[\frac{1}{P_1}\right] + P_2 \log_2\left(\frac{1}{P_2}\right] + \dots + P_M \log_2\left[\frac{1}{P_M}\right]$$

$$H = \frac{1}{M}\log_2(M) + \frac{1}{M}\log_2\left[M\right] + \dots + \frac{1}{M}\log_2\left[M\right)$$

$$\boxed{H = \log_2 M}$$

Prob:- A code is composed of dots & dashes. Assume that the dash is 3 times as long as dots & it has one third the probability of occurence. calculate

(a) Information in a dot & a dash.

(b) Avg information in dot-dash code.

(c) Assume that a dot lasts for 10ms & this same interval is allowed b/w symbols. calculate average rate of information.

Sol:- $P(dash) = \frac{1}{3}(P dot)$

$P(dot) = 3 P(dash)$

$P(dash) + P(dot) = 1$

$P(dash) + 3 (p dash) = 1.$

$\Rightarrow P(dash) = \frac{1}{4}$ & $P(dot) = \frac{3}{4}$

a) $I(dot) = \log_2 \frac{1}{P dot}$

$= \log_2 \frac{4}{3} = 0.415$ bits

$I(dash) = \log_2 \frac{1}{P(dash)} = \log_2 4 = \underline{\underline{2 \text{ bits}}}$

(b) $H = P(dot) \log_2 \frac{1}{P(dot)} + P(dash) \log_2 \frac{1}{P(dash)}$

$= \frac{3}{4} \log_2 \frac{4}{3} + \frac{1}{4} \log_2 4$

$= \underline{\underline{0.811 \text{ bits/sec}}}$

(c) Avg rate of information.

$T_{dot} = 10ms$, $T_{dash} = 3 \cdot T_{dot} = 30ms$, $T_{symbol} = 10ms$.

$T = P(dot) \times T_{dot} + P(dash) \times T_{dash} + T_{symbols}$

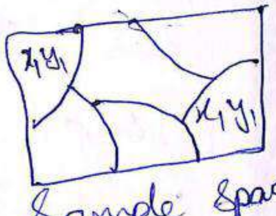
$= 0.025$ sec/symbol.

$r = \frac{1}{T} = 40$ symbol/sec

$R = r \times H = \underline{\underline{32.44 \text{ bits/sec}}}$

Marginal & conditional Entropies & Redundancy

fig shows the sample space that consists of joint occurrence of two events x_i & y_j. In such situation we have four sets of probability schemes:



Sample space of joint probabilities

i) Probability of X i.e $p(x_i)$

(ii) " " Y i.e $p(y_j)$

(3) joint probability of XY i.e $P(x_i, y_j)$

(4) Transition probability. $P(x_i/y_j)$ & $P(y_j/x_i)$

→ If we add all joint probabilities for fixed y_j we get $p(y_j)$ i.e.

$$P(Y_j) = \sum_{i=1}^{M} (x_i, y_j) \quad — (1)$$

→ lly we add all joint probabilities for fixed x_i we ge $p(x_i)$ i.e

$$p(x_i) = \sum_{j=1}^{m} (x_i, y_j) \quad — (2)$$

Joint Entropy

It represents entropy of joint occurrence of two or more events

The joint entropy is given as

$$H(X,Y) = H(Y,X) = \sum_{i=1}^{M} \sum_{j=1}^{M} P(x_i, y_j) \log_2 \frac{1}{p(x_i, y_j)}$$

⇒ Here $H(X,Y)$ represents entropy of joint occurrence of X & Y.

Marginal Entropy

when the entropy of individual even is evaluated from joint probabilities of the events, it is called marginal entropy

In the joint occurence of x_i & Y_j, eqn (1) & eqn (2) give the probabilities of occurrence of $P(x_i)$ & $P(y_j)$

Then the entropies of x_i & y_j can be calculated.

i.e
$$H(X) = \sum_{i=1}^{M} P(x_i)$$
$$= \sum_{i=1}^{M} \left\{ \sum_{j=1}^{M} P(x_i, y_j) \right\}$$

Illy
$$H(Y) = \sum_{j=1}^{M} P(y_j)$$
$$H(Y) = \sum_{j=1}^{M} \left\{ \sum_{i=1}^{M} P(x_i, y_j) \right\}.$$

H(X) & H(Y) are called marginal entropy.

Conditional Entropy

It is also called equivocation. The conditional entropy. H(X/Y) represents uncertainity of X, on average, when Y is known.

Illy H(Y/X) " " " Y, " " , when X is ~~known~~ transmitted.

→ This conditional entropy indicates the information lost accross noisy channel.

i.e
$$H(X/Y) = \sum_{i=1}^{M} \sum_{j=1}^{M} P(x_i, y_j) \log_2 \frac{1}{P(x_i/y_j)}$$

&
$$H(Y/X) = \sum_{i=1}^{M} \sum_{j=1}^{M} P(x_i, Y_j) \log_2 \frac{1}{P(y_j/x_i)}$$

Source coding theorem (Shannon's first theorem)

→ If the messages have different probabilities & they are assigned same no. of binary digits then information carrying capability of binary PCM is not completely utilized. That is actual information rate is less than maximum achievable rate.

→ But if all the messages have same probabilities, then maximum information rate is possible.

→ The PCM method used to code the discrete messages from source is one of source coding methods. The device which performs source coding is called source encoder (like PCM, DM, ADM).

→ The efficient source encoder should satisfy following requirements.

1, The codewords generated by the encoder should be binary in nature

2, The source code should be unique in nature. That is every code word should represent unique message.

Average no. of bits in codeword ($\bar{N}$)

$$\bar{N} = \sum_{K=1}^{L} P_K n_K.$$

Here P_K is probability of K^{th} message
& n_K are bits in codeword of K^{th} message.

→ Let N_{min} be the minimum value of $\bar{N}$. Then the ~~codeword~~ coding efficiency of the source encoder is defined as,

$$\eta = \frac{N_{min}}{\bar{N}}$$

The source encoder is efficient if η approaches unity

Statement of Source coding Theorem

Given a discrete memoryless source of entropy H, the average codeword length $\bar{N}$ for any distortionless source encoding is bounded as,

$$\bar{N} \geq H.$$

Here the entropy H represents the fundamental limit on avg number of bits per symbol i.e $\bar{N}$. This limit says that the avg number of bits per symbol cannot be made smaller than entropy H. Hence $N_{min} = H$.

coding Efficiency.

$$\eta = \frac{\text{Entropy}}{\text{Avg no. of bits}} = \frac{H}{\bar{N}}$$

code redundancy

It is measure of redundancy of bits in the encoded message sequence. It is given as,

$$\text{redundancy } (\gamma) = 1 - \text{code efficiency}$$
$$= 1 - \eta$$

Redundancy Should be as low as possible.

code variance

$$\sigma^2 = \sum_{K=0}^{L-1} P_K (n_K - \bar{N})^2$$

Here σ^2 is variance of the code.

M is the no. of Symbols.

P_K is probability of K^{th} Symbol.

n_K is the no. of bits assigned to K^{th} Symbol.

$\bar{N}$ is average code word length.

Variance is the measure of variability in codeword lengths. Variance should be as small as possible.

Data compaction: variable length source coding algorithms

(Entropy coding)

Data compaction: — It is also called loss less data compression. It is basically a source coding technique that has two aspect.

1, It is efficient in terms of average no. of bits per symbol.

2, Original data can be reconstructed without loss of any information.

3, Thus the data compaction is used to remove the redund any from signal priorities.

→ Three techniques are used for data compaction.

1, prefix coding or instantaneous coding.
2, Shannon - fano coding.
3, Huffman coding.

Prefix coding

This is the variable length coding algorithm. It assigns binary digits to the messages as per their probability of occurrence. Prefix of the codeword means any sequence which is initial part of the codeword. In prefix code, no code word is the prefix of any other codeword.

Source Symbol	Probability of occurrence	Prefix code
S_0	0.5	0 ← codeword.
S_1	0.25	$\underbrace{1}_{\text{prefix}}0$ ← ”
S_2	0.125	$\underbrace{11}_{\text{prefix}}0$ ← ”
S_3	0.125	$\underbrace{11}_{\text{prefix}}1$ ← codeword

Shannon-Fano Algorithm

If the probabilities of all the messages are not equally likely, then average information is reduced. This intern reduces the information rate (R). This problem is solved by coding the messages with different no. of bits. As the probability of message is increased less no. of bits are used to code it.

→ Apply Shannon-Fano coding procedure for the following message ensemble. Also determine its efficiency.

x	x_1	x_2	x_3	x_4	x_5	x_6	x_7
p	0.4	0.2	0.12	0.08	0.08	0.08	0.04

Take M=2

Sol: $H = 0.4 \log_2 \frac{1}{0.4} + 0.2 \log_2 \frac{1}{0.2} + 0.12 \log_2 \frac{1}{0.12} + 0.08 \log_2 \frac{1}{0.08} + 0.08 \log_2 \frac{1}{0.08} + 0.08 \log_2 \frac{1}{0.08} + 0.04 \log_2 \frac{1}{0.04}$

$= 2.42$ bits/message.

$\bar{N} = (0.4)1 + (0.2)3 + (0.12)3 + (0.08)4 + (0.08)4 + (0.08)4 + (0.04)4$
$= 2.48$ bits/message.

$\eta = \frac{H}{\bar{N}} = 0.978$
$= 97.5\%$

message x_k	probability of P_k						
x_1	0.4	0				0	1
x_2	0.2	1	0	0		100	3
x_3	0.12	1	0	1		101	3
x_4	0.08	1	1	0	0	1100	4
x_5	0.08	1	1	0	1	1101	4
x_6	0.08	1	1	1	0	1110	4
x_7	0.04	1	1	1	1	1111	4

prob:- Apply Shannon-fano coding procedure to find the coding efficiency for following message ensemble

x_K	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
P_K	$1/4$	$1/8$	$1/16$	$1/16$	$1/16$	$1/4$	$1/16$	$1/8$

sol:- $H = 2.42$ bits/message.

$$= \frac{1}{4}\log_2 4 + \frac{1}{8}\log_2 8 + \frac{1}{16}\log_2 16 + \frac{1}{16}\log_2 16 + \frac{1}{16}\log_2 16$$
$$+ \frac{1}{4}\log_2 4 + \frac{1}{16}\log_2 16 + \frac{1}{8}\log_2 8$$

To obtain Shannon fano coding.

x_K	P_K	I	x_K	P_K	I	II	III	IV	code	
x_1	$1/4$	0	x_1	$1/4$	0	0			00	2
x_2	$1/8$	1	x_6	$1/4$	0	1			01	2
x_3	$1/16$	1	x_2	$1/8$	1	0	0		100	3
x_4	$1/16$	1	x_8	$1/8$	1	0	1		101	3
x_5	$1/16$	1	x_3	$1/16$	1	1	0	0	1100	4
x_6	$1/4$	1	x_4	$1/16$	1	1	0	1	1101	4
x_7	$1/16$	1	x_5	$1/16$	1	1	1	0	1110	4
x_8	$1/8$	1	x_7	$1/16$	1	1	1	1	1111	4

$$\bar{N} = \left(\frac{1}{4}\right)2 + \left(\frac{1}{4}\right)2 + \left(\frac{1}{8}\right)3 + \left(\frac{1}{8}\right)3 + \left(\frac{1}{16}\right)4 + \left(\frac{1}{16}\right)4 + \left(\frac{1}{16}\right)4$$

$= 2.75$ bits/message.

$$\eta = \frac{H}{\bar{N}} = \frac{2.75}{2.75} = 1 \text{ or } 100\%$$

→ consider 5 messages given by the probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{16}$

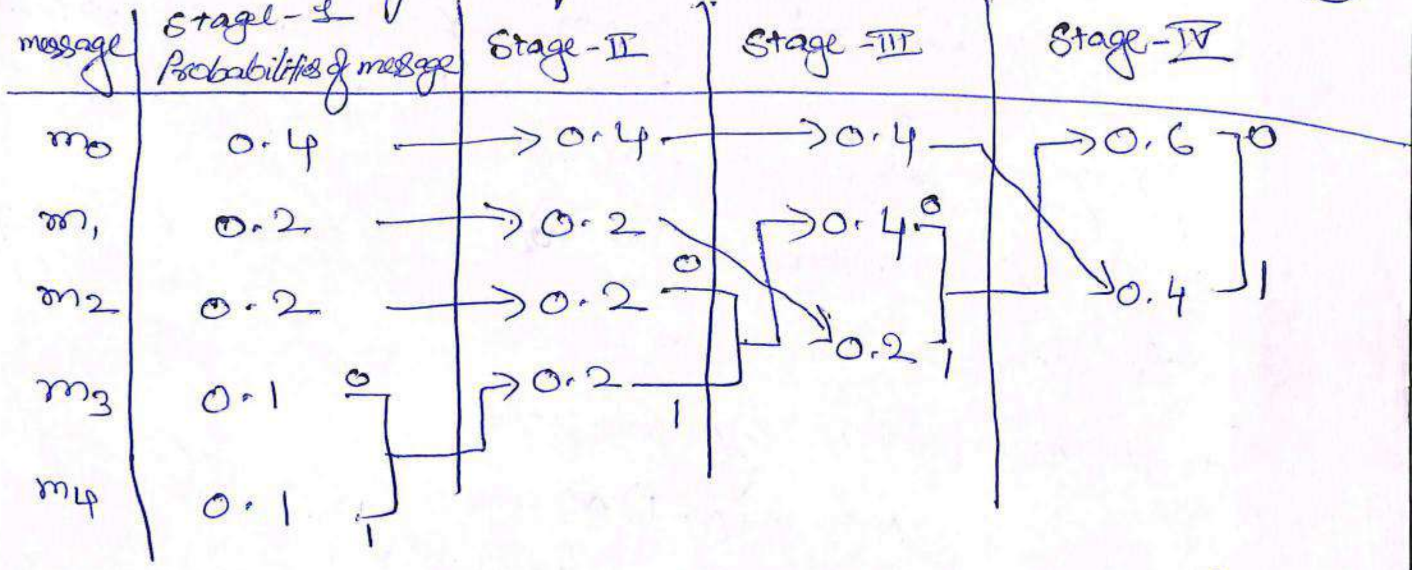
(a) calculate H, (b) $\bar{N}$

sol :- $H = 1.875$ bits/message

$\bar{N} = 1.875$ bits/message

$\eta = 1$ or 100%

Huffman Binaary coding



message	Probability	Digits obtained by tracing	code word obtained by reading digits from LSB side	No. of digits
m_0	0.4	1	1	1
m_1	0.2	10	01	2
m_2	0.2	000	000	3
m_3	0.1	0100	0010	4
m_4	0.1	1100	0011	4.

$$H = 0.4 \log\frac{1}{0.4} + 0.2 \log\frac{1}{0.2} + 0.2 \log\frac{1}{0.2} + 0.1 \log\frac{1}{0.1} + 0.1 \log\frac{1}{0.1}$$

$= \underline{2.12193}$ bits of information/message.

$$\bar{N} = (0.4)1 + (0.2)2 + (0.2)3 + (0.1)4 + (0.1)4.$$

$= \underline{2.2 \text{ binary digits/message}}$

$$\eta = \frac{H}{\bar{N}} = \frac{2.12193}{2.2} =$$

Huffman Ternary Coding

→ It combines three Symbols having lowest probabilities. are combined. The combined probability is placed at appropriate level in next Stage.

→ The three Symbols which are combined are assigned the codes 0, 1 & 2.

→ If there are 'M' Symbols, then

$$M = r + (r-1)\alpha.$$

Here for ternary coding $r = 3$.

$$M = 3 + 2\alpha$$

$$\alpha = \frac{M-3}{2}$$

The value of α Should be integer, 'M' must be 5, 7, 9, 11 - - & so on.

→ If α is not an integer, then necessary no. of Symbols must be appended with zero probability. These are dummy symbols.

Huffman Quarternary coding

→ It combines four Symbols having lowest probabilities.

→ The four Symbols which are combined are assigned the codes 0, [illegible] 3.

→ Let $r = 4$.

$$M = r + (r-1)\alpha$$

$$= 4 + 3\alpha$$

$$\alpha = \frac{M-4}{3}.$$

The value α Should be integer So 'M' must be 7, 10, 13, 16 - - -

→ If α is not an integer, the necessary number of Symbols must be appended with zero probabality.

→ To obtain Huffman ternary codes.

Symbol S_K	Probability P_K I	II	III	IV	digits obtained by tracing	Huffman code	number of digits
S_1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}^0$	$\frac{1}{3}$	0	0	1
S_3	$1/3$	$\frac{1}{3}$	$1/3^1$		1	1	1
S_4	$1/9$	$1/9^0$	$\frac{1}{3}_2$		02	20	2
S_5	$1/9$	$\frac{1}{9}^1$			12	21	2
S_2	$1/27^0$	$1/9_2$			022	220	3
S_6	$1/27^1$				122	221	3
S_7	$1/27^2$				222	222	3

→ $H = 2.2893903$ bits / symbol

To obtain entropy of the Source for ternary coding.

The entropy in 'r' units per Symbol is obtained as

$$H_r = \frac{H}{\log_2 r} = \frac{2.2893903}{\log_2 3} = \underline{\underline{1.4444}}\ \text{ternary units / Symbol}$$

$$\bar{N}_3 = \frac{1}{3}\times 1 + \frac{1}{3}\times 1 + \frac{1}{9}\times 2 + \frac{1}{9}\times 2 + \frac{1}{27}\times 3 + \frac{1}{27}\times 3 + \frac{1}{27}\times 3$$

$$= \underline{\underline{1.4444}} \qquad \eta = \frac{H_3}{N_3} = \underline{\underline{1 \text{ or } 100\%}}$$

→ To obtain Huffman quarternary code.

For $r = 4$.

$$H_r = \frac{H}{\log_2 4} = \frac{2.2893903}{2} = \underline{1.144695}\ \text{quarternary units / symbol.}$$

S_1	$\frac{1}{3}$	$\frac{1}{3}^0$	0	0	1
S_3	$1/3$	$1/3^1$	1	1	1
S_4	$1/9$	$2/9^2$	1	1	1
S_5	$1/9^0$	$1/9^3$	02	20	2
S_2	$1/27^1$		12	21	2
S_6	$1/27^2$		22	22	2
S_7	$1/27^3$		32	23	2

$$\bar{N}_4 = \frac{1}{3}\times 1 + \frac{1}{3}\times 1 + \frac{1}{9}\times 1 + \frac{1}{9}\times 2 + \frac{1}{27}\times 2 + \frac{1}{27}\times 2 + \frac{1}{27}\times 2$$

$= 1.22229$ quarternary units / Symbol

$$\eta_4 = \frac{H_4}{N_4} = 0.9365 = \underline{\underline{93.65\%}}$$

→ A DMS has 4 Symbols x_1, x_2, x_3, x_4 with $P(x_1) = 1/2$, $P(x_2) = 1/4$, $P(x_3) = 1/8 = P(x_4)$.

(a) Construct Shannon fano code

(b) Repeat for Huffman code & compare the result.

Shannon fano code.

Symbols	P_k	I	II	III	code	no. of bits
x_1	$1/2$	0			0	1
x_2	$1/4$	1	0		10	2
x_3	$1/8$	1	1	0	110	3
x_4	$1/8$	1	1	1	111	4

$H = 1.75$ bits/symbol

$\overline{N} = 1.75$ bits/Symbol

$\eta = 1$ or 100%.

Huffman code.

Symbol	$P(x_i)$	I	II	III	digits tracing	huffman code	Code word length
x_1	$1/2$	$1/2$	$1/2$ 0	1	0	0	1
x_2	$1/4$	$1/4$ 0	$1/2$ 1		01	10	2
x_3	$1/8$ 0	$1/4$ 1			011	110	3
x_4	$1/8$ 1				111	111	3

$H = 1.75$ bits/ ~~symbol~~ message

$\overline{N} = 1.75$ bits/message

$\eta = 1$ or 100%

Mutual information

It is defined as the amount of information transferred when x_i is transmitted & y_i is received. It is represented by $I(x_i, y_j)$ & given as,

$$I(x_i, y_j) = \log_2 \frac{P(x_i/y_j)}{P(x_i)} \text{ bits}$$

Here $I(x_i, y_j)$ is the mutual information.

$P(x_i/y_j)$ is the conditional probability that x_i was transmitted & y_j is received.

$P(x_i)$ is the probability of symbol x_i for transmission.

- The average mutual information is represented by $I(X;Y)$. It is calculated in bits/Symbol. The average mutual information is defined as the amount of source information gained per received symbol. It is different from entropy.

It is given as $I(X;Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) I(x_i, y_j)$

Thus $I(x_i, y_j)$ is weighted by joint probabilities $P(x_i, y_j)$ overall the possible joint events.

- $$I(X;Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) \log_2 \frac{P(x_i/y_j)}{P(x_i)}$$

Properties of Mutual information

1, The mutual Information of the channel is Symmetric i.e

$$I(X;Y) = I(Y, X)$$

2, The mutual information can be expressed in terms of entropies of channel i/p or o/p & conditional entropies

that is

$$I(X;Y) = H(X) - H(X/Y)$$
$$= H(Y) - H(Y/X)$$

Here $H(X/Y)$ & $H(Y/X)$ are conditional entropies.

(iii) The mutual information is always positive, i.e

$$I(X;Y) \geq 0$$

(iv) The mutual information is related to the joint entropy H(X,Y) by following relation.

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

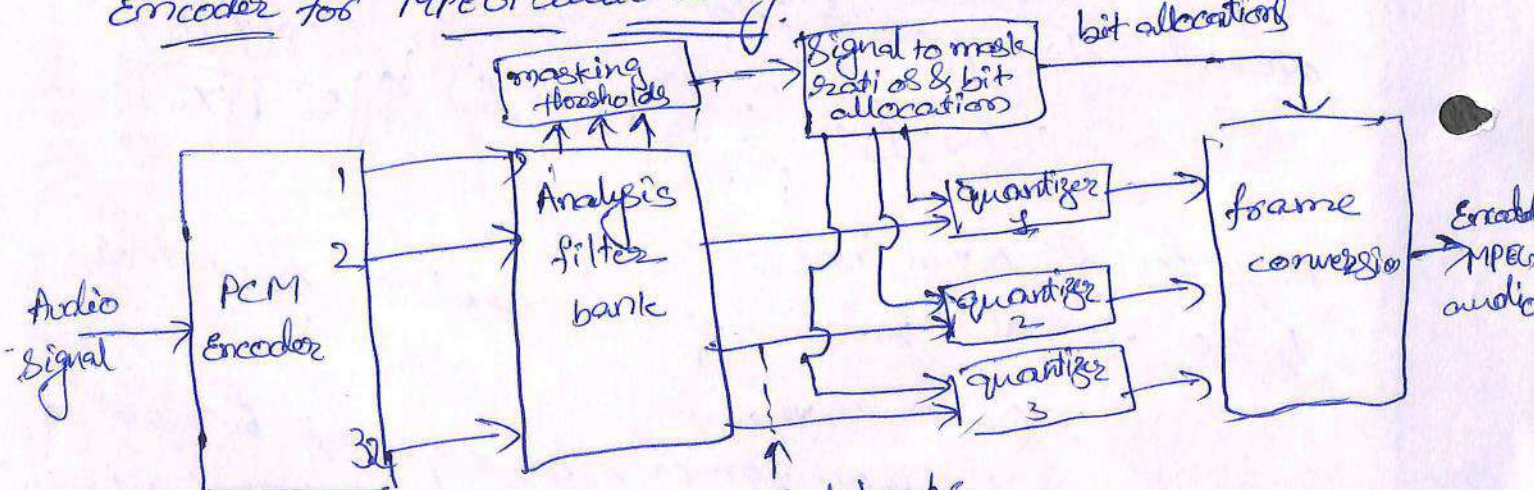
Lossy Source coding.

In lossy source coding some of the information is lost during coding & that is never recovered. These technique are MPEG & JPEG used for video & images.

MPEG coding for audio signals:-

MPEG stands for Motion pictures Expert group (MPEG). ~~It was formed by ISO~~. MPEG has developed the standards for compression of video with audio. MPEG audio codes are used for compression of audio. This compression mainly uses perceptual coding.

Encoder for MPEG audio coding.



MPEG audio encoder

PCM Encoder

It Generates 32 PCM Samples of i/p audio Signal. The time deration of 32 PCM Samples depends upon Sampling frequency.

Analysis filter bank:-

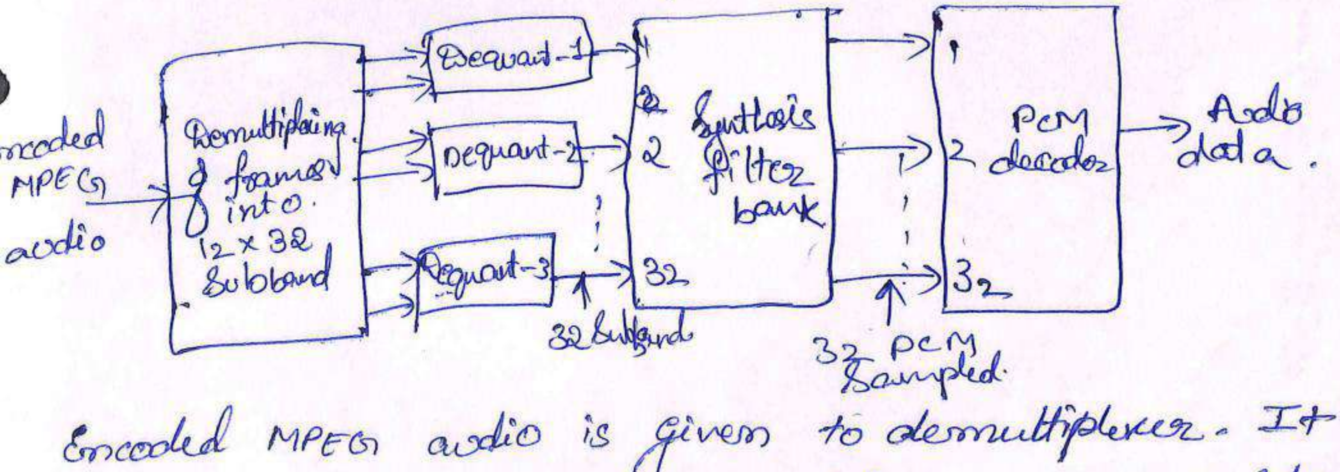
These 32 PCM Samples of audio Segment are given to analysis filter bank. It calculates 32 point DFT of i/p 32 PCM Samples. This conversion Generats 32 frequency components are treateated as 32 Sub bands by the analysis filter bank. The analysis filter bank accumulates 12 Sets of 32 PCM Samples.

Quantizer blocks:-

The quantizer masks the frequency components that are below masking threshold. For each Subband there is Seperate quantizer.

The quantizer frequency samples are carried in specific frame

Decoder for MPEG audio decoding.



Encoded MPEG audio is given to demultiplexer. It demultiplexes the incomming encoded frames into 12 Sets of 32 Subbands. The information is given to the dequantizers. The dequantized 32 Subband Samples are given to Synthesis filter bank. The Synthesis filter bank produces 2 PCM Samples for each Set. These PCM Samples are then decoded by PCM decoder to generate audio data.

JPEG coding for images

JPEG stands for Joint photographic Experts group.

Types of JPEG There are two types of JPEG algorithms.

1) Baseline JPEG:-
During decoding, this JPEG algorithm draws line after line until complete image is shown.

2) Progressive JPEG:- During decoding this JPEG algorithm draws whole ~~line~~ image at a time but in poor quality. Progressive JPEG is used for image on web. The users can make out image before it is fully downloaded.

Absentees III ECE 2(b)
70, 74, 75,
77, 79, 80, 84,
99, LE'S 7, 8, 9, 10

UNIT-IV. Linear block codes

Matrix description of linear block codes, Error detection & error correction capabilities of linear block codes. Cyclic codes :- Algebraic structure, Encoding, Syndrome calculation, Decoding.

Introduction

→ Errors are introduced in the data when it passes through the channel. The channel noise interferes the signal. The signal power is also ~~introd~~ reduced.

→ ~~Hence~~ ~~er~~ The transmission of the data over the channel depends upon two parameters. They are transmitted power and channel bandwidth. The power spectral density of channel noise & the two parameters determine S/N ratio.

→ The S/N ratio determines the probability of error of the modulation scheme. For given S/N ratio, the error probability can be reduced further by using coding techniques. The coding techniques also reduce S/N ratio for fixed probability of error.

Types of codes

The code are mainly classified as block codes & convolution codes

(i) Block codes :- These codes consists of 'n' no. of bits in one block or codeword. This codeword consists of 'k' message bits & (n-k) redundant bits. Such blocks are (n, k) block codes.

(ii) Convolutional codes :- The coding operation is discrete time convolution of I/P sequence with the impulse response of encoder. The convolution encoder accepts the message bits continously & generates the encoded sequence continously.

- The codes can also be classified as.

(i) Linear codes :- If the two codewords of the linear code are added by modulo - 2 arithmetic, then it produces third codeword in code.

(ii) Non linear code :- Addition of non linear codewords doesnot necessarily produce third code word.

Methods of controlling errors

There are two main methods used for error control coding.

Forward acting error correction

In this method, the errors are detected & corrected by proper coding techniques at the receiver. The check bits or redundant bits are used by the receiver to detect & correct errors. The error detection & correction capability of the receiver depends upon no. of redundant bits in the transmitted message.

(ii) Error detection with retransmission.

In this method, the decoder checks the i/p sequence. When it detects any error, it discards that part of the sequence in which error was detected & requests the transmitter for retransmission. The transmitter then again retransmits part of sequence in which error was detected.

Types of errors

1) random errors :- These errors are created due to white gaussian noise in the channel in particular interval donot affect the performance of the system in subsequent intervals. In other words these errors are totally uncorrelated. Hence they are called random errors

2) Burst error :- These errors are regenerated due to impulsive noise in the channel. These impulsive noise are generated due to lightening & switching transients. These noise burst affect several successive symbols. Such errors are called burst errors.

Some of the important terms used in Error control coding.

codeword:- The encoded block of 'n' bits is called codeword. It contains message bits & redundant bits.

Block length:- The number of bits 'n' after coding is called the block length of the code.

code rate:- The ratio of message bits (k) & the encoder o/p bits (n) is called code rate. Code rate is defined by 'r' ~~is called~~ i.e

$$r = \frac{k}{n}$$

we find that $0 < r < 1$

channel data rate:- It is the bit rate at the o/p of encoder. If the bit rate at the i/p of encoder is R_S, then channel data rate will be,

$$\text{channel data rate } (R_O) = \frac{n}{k} R_S$$

Hamming distance:- The hamming distance b/w the two code vectors is equal to the no. of elements in which they differ.

ex, The hamming distance b/y $x = (101)$ & $y = (110)$ is 2

demin minimum distance :- It is the smallest hamming distance b/w the valid code vectors.

For (n, k) block code, the $d_{min} \leq n-k+1$.

code effeciency:- It is the ratio of message bits in a block to the transmitted bits for the block by encoder i.e,

$$\text{code efficiency} = \frac{\text{message bits in a block}}{\text{transmitted bits for the block.}} = \frac{k}{n}$$

For (n, k) block code there are k message bits & 'n' transmitted bits.

weight of the code:- The no. of non zero elements in the transmitted code vector is called vector weight.

for example if $X = 0110101$

Then weight of code vector $w(X) = 5$

Linear block codes

Principle of block coding.

For block of K message bits, n-k parity bits or check bits are added. Hence the total bits at the o/p of channel encoder are n. Such codes are called (n,k) blockcode.

Systematic codes:- In the Systematic block code, the message bits appear at the beginning of code word. The message bits appear first & then check bits are transmitted in a block.

non Systematic codes:- In this it is not possible to identify message bits & check bits. They are mixed in block.

~~linear codes~~ -

Consider that the particular code vector consists of $m_1, m_2, m_3 --- m_k$ message bits & $c_1, c_2, c_3 -- c_q$ check bits.

Then code vector

$$X = (m_1, m_2, -- m_k \; c_1, c_2, -- c_q).$$

$$X = (M|C)$$

~~where~~

$M = K$ bit message vector &

$c = q$ -bit check vector.

The check bits play the role of error detection & correction

Matrix Description of Linear block codes

The code vector can be represented as,

$X = MG$.

Here x = code vector of $1 \times n$ size or n bits

M = message vector of $1 \times k$ size or k-bits.

G = generator matrix of $k \times n$ size.

$$[X]_{1 \times n} = [M]_{1 \times k} [G]_{k \times n}.$$

The generator matrix depends upon linear block code used. Generally it is represented as,

$$G = [I_k \mid P_{k \times q}]_{k \times n}.$$

Here I_k = $k \times k$ identity matrix &

P = $k \times q$ submatrix.

The check vector can be obtained as,

$C = MP$.

Thus in the expanded form we can write above eqns,

$$[c_1, c_2, --- c_q]_{1 \times q} = [m_1, m_2 --- m_k]_{1 \times k} \begin{bmatrix} P_{11} & P_{12} & --- & P_{1q} \\ P_{21} & P_{22} & --- & P_{2q} \\ \vdots \\ P_{k1} & P_{k2} & --- & P_{kq} \end{bmatrix}_{k \times q}.$$

By solving the above matrix eqn. check vector can be obtained i.e.

$$c_1 = m_1 P_{11} \oplus m_2 P_{21} \oplus m_3 P_{31} \oplus --- \oplus m_k P_{k1}$$

$$c_2 = m_1 P_{12} \oplus m_2 P_{22} \oplus m_3 P_{32} \oplus -- \oplus m_k P_{k2}$$

$$c_3 = m_1 P_{13} \oplus m_2 P_{23} \oplus m_3 P_{33} \oplus - \oplus m_k P_{k3}$$

& so on.

Here all additions are mod-2 addition.

Prob:- The generator matrix for a (6, 3) block code is given below. Find all code vectors of this code.

$$G = \begin{bmatrix} 1 & 0 & 0 & : & 0 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 1 & ; & 1 & 1 & 0 \end{bmatrix}$$

→ The code vectors can be obtained through following steps.

i) determine the P submatrix from generator matrix.

(ii) Obtain equations for check bits using $C = MP$.

(iii) Determine check bits for every message vector.

(i) $G = [I_K : P_K]$

$$I_k = I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_{k\times q} = P_{3\times3} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

(ii) Here $k=3$, $q=3$ & $n=6$.

That is, block size of message vector is 3 bits. Hence there will be total 8 possible message vector.

Sr.No	m_1	m_2	m_3
1	0	0	0
2	0	0	1
3	0	1	0
4	0	1	1
5	1	0	0
6	1	0	1
7	1	1	0
8	1	1	1

$$\begin{array}{ccc} c_1 & c_2 & c_3 \end{array}$$

$$[c_1\, c_2\, c_3] = [m_1\, m_2\, m_3] \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$c_1 = m_2 \oplus m_3$

$c_2 = m_1 \oplus m_3$

$c_3 = m_1 \oplus m_2$

(iii) for first block $(m_1, m_2, m_3) = 000$

$(c_1, c_2, c_3) = 000$.

for second block 001

$(c_1, c_2, c_3) = 110$

001110 ✓ code vec

Parity check matrix

For every block code there is $q \times n$ parity check matrix H. It is defined as.

$$H = [P^T : I_q]_{q \times n}.$$

P^T is the transpose of P sub matrix.

Hamming codes

These are (n,k) linear block codes.

These codes satisfy the following conditions,

(a) number of check bits $q \geq 3$.

2) block length $n = 2^q - 1$

3) number of message bits $k = n - q$.

4) minimum distance dmin $= 3$.

w.r.t code rate $r = \frac{k}{n}$.

$$r = \frac{n-q}{n}$$

$$= 1 - \frac{q}{n}.$$

putting value of $n = 2^q - 1$ $\therefore r = 1 - \frac{q}{2^q - 1} = \underline{1 - \frac{q}{2^q - 1}}$

→ Prove that $GH^T = HG^T = 0$ for systematic linear block codes.

$$[G]_{k \times n} = [I_{k \times k} | P_{k \times q}]_{k \times n}$$

$$\& \ [H]_{q \times n} = [P^T_{q \times k} | I_{q \times q}]_{q \times n}.$$

$$HG^T = [P^T_q \times k | I_{q \times q}]_{q \times n} \begin{bmatrix} I_{k \times k} \\ \hline P^T_{q \times k} \end{bmatrix}$$

$$= [P^T \oplus P^T]_{q \times k} = 0$$

lly $GH^T = [I_{k \times k} | P_{k \times q}] \begin{bmatrix} P^T_{q \times k} \\ \hline I_{q \times q} \end{bmatrix}$

$$= [P \oplus P] = 0$$

Error detection & correction capabilities of Hamming codes

S.No	Name of errors detected/corrected	distance requirement.
1	Detect upto 'S' errors per word	$d_{min} \geq S+1$
2	correct upto 't' errors per word	$d_{min} \geq 2t+1$
3	correct upto 't' errors & detect S>t errors per word.	$d_{min} \geq t+S+1$

$3 \geq 2+1$

$S=2$

Error control capabilities.

Since the min distance (d_{min}) of Hamming code is 3. It can be used to detect double errors or correct single error.

→ The parity check matrix of a particular (7,4) linear block codes is given by

$$[H] = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(1) find the generator matrix (G)

(2) List all the code vectors

(3) what is the minimum distance b/w code vectors?

(4) How many errors can be detected? How many errors can be corrected?

Sol: Here $n=7,\ k=4$.
$q = n-k = 3$.

$$H = [P^T : I_q]_{q \times n}.$$

$$P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$G = [I_k : P_{k \times q}]_{q \times n} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}_{4 \times 7}$$

To find all the code vectowords:

$C = M P$.

$M = 4$.

$$[C_1\ C_2\ C_3]_{1\times3} = [m_1\ m_2\ m_3\ m_4]_{1\times4}\,[P]_{4\times3}$$

$$= [m_1\ m_2\ m_3\ m_4]_{1\times4}\begin{bmatrix}1&1&1\\1&1&0\\1&0&1\\0&1&1\end{bmatrix}$$

$$c_1 = m_1 \oplus m_2 \oplus m_3$$

$$c_2 = m_1 \oplus m_2 \oplus m_4$$

$$c_3 = m_1 \oplus m_3 \oplus m_4$$

Since K=4 there total possible 16 message vectors from.0000 to 1111 → So total code vectors are

→ 1,

				code vectors			
m_1	m_2	m_3	m_4	c_1	c_2	c_3	weight of code
0	0	0	0	0	0	0	0
0	0	0	1	0	1	1	3
0	0	1	0	1	0	1	3
0	0	1	1	1	1	0	4
0	1	0	0	1	1	0	3
0	1	0	1	1	0	1	4
0	1	1	0	0	1	1	4
0	1	1	1	0	0	0	3
1	0	0	0	1	1	1	4
1	0	0	1	1	0	0	3
1	0	1	0	0	1	0	3
1	0	1	1	0	0	1	4
1	1	0	0	0	0	1	3
1	1	0	1	0	1	0	4
1	1	1	0	1	0	0	4
1	1	1	1	1	1	1	7

(iii) minimum distance b/w code vectors.

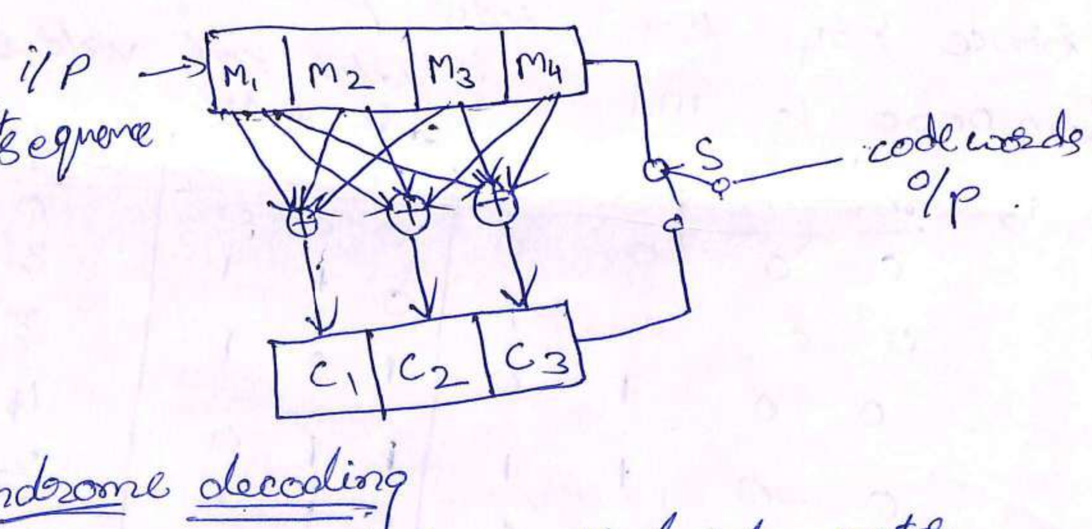
$d_{min} = 3$.

∴ we can write, the minimum distance of linear block code is equal to the minimum weight of a non-zero code vector i.e.

$\therefore d_{min} = [w(X)]_{min}$.

(iv) Error detection & correction.
2 1

Encoder for (7,4) Hamming code.



Syndrome decoding

If X is the transmitted code vector.
If Y is the receiver " "
X = Y no transmission error.
X ≠ Y if the errors created during transmission.

The decoder detects or corrects those errors in Y by using the stored bit pattern in decoder about the code. For larger block lengths more & more bits are required to be stored in decoder. This increase the memory requirement & adds to the complexity & cost of the System. To avoid this problems Syndrome decoding is used in LBC.

w.k.t for (n,k) linear block; there exists a parity check matrix H of size q×n.

$$H = [P^T : I_q]_{q\times n}.$$

$$H^T = \begin{bmatrix} P \\ \hdashline I_q \end{bmatrix}_{n\times q}.$$

The H^T has very important property.

$$XH^T = (000---0).$$

$$\text{or } [H]_{1\times n}[H^T]_{n\times q} = (000---0)_{1\times q}$$

This is true for all code vectors.

→ Consider the previous example.

$$H^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{7\times 3}.$$

Consider the code vectors.

$$X = (0\,0\,1\;\;0\,1\,0\,1).$$

$$XH^T = (0010101)\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= (0\,0\,0)$$

Thus X belongs to the valid code vector at transmitter. At the receiver, the received code vector is Y. Then we can write,

$YH^T = (00--0)$ if $X=Y$ i.e no errors. & Y is valid code vector.

YH^T = non zero if $X\neq Y$ i.e some errors

Definition of Syndrome (S)

- when Some errors are present in recei code vecto Y. then YH^T is non zero. The nonzero o/ of the product YH^T is is called Syndrome & it i used to detect errors in Y.

i.e $S = YH^T$.

or $[S]_{1\times q} = [Y]_{1\times n} [H^T]_{n\times q}$

Detecting error with the help of Syndrome & error vecto

The non zero elements of 'S' represent errors in o/p. when all elements of 'S' are zero, the two cases are p

1, no error in the o/p & $Y = X$.

2, Y is Some other valid code vector other than X. This means the transmission errors are undetectable.

→ Lets consider an n-bit error vector E. Let this vector represent the position of transmission errors in Y.

$X = (1\ 0\ \underset{\uparrow}{1}\ 1\ \underset{\uparrow}{0})$ be a transmitted vector.

$Y = (1\ 0\ 0\ 1\ 1)$ " " received vector

then $E = (0\ 0\ \overset{\uparrow}{1}\ 0\ \overset{\uparrow}{1})$ represents the error vector.

The non zero entries represents errors in Y.

using mod-2 addition rules we can write

$$Y = X \oplus E.$$

$$= (1\oplus 0 \quad 0\oplus 0 \quad 1\oplus 1 \quad 1\oplus 0 \quad 0\oplus 1)$$

$$= (1\ 0\ 0\ 1\ 1).$$

or $X = Y \oplus E$

$$= (1\oplus 0 \quad 0\oplus 0 \quad 1\oplus 0 \quad 1\oplus 0 \quad 1\oplus 1)$$

$$= (1\ 0\ 1\ 1\ 0).$$

Relation ship b/w Syndrome vector(S) & Error Vector (E)

$$S = YH^T$$

$$S = (X \oplus E)H^T$$

$$= XH^T \oplus EH^T$$

we know $XH^T = 0$

$$\boxed{S = EH^T}$$

This relation shows that Syndrome depends on Error pattern only. Syndrome vector 'S' is of size $1 \times q$. Thus q bits of Syndrome can only represent 2^q Syndrome vectors. Each Syndrome vector corresponds to particular error pattern.

→ The parity check matrix of a(7,4) Hamming code is given as follows.

$$H = \left[\begin{array}{cccc:ccc} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]_{3\times7}$$

calculate the Syndrome vector for Single bit errors.

Sol:- $n = 7$, $K = 4$ $q = n-k = 3$.

w.k.t Syndrome vector is a q bit vector. For this example Syndrome will be 3 bit vector. Therefore there will be $2^3 - 1 = 7$ non-zero Syndromes. These shows that '7' single bit error patterns will be represented by these 7 non-zero Syndromes. Error vector is a n bit vector representing error pattern. for this example E is '7' bit vector.

Sr.no	bit in error	Bits of vector (E), Non zero bits show error
1	1st	1000000
2	2nd	0100000
3	3rd	0010000
4	4th	0001000
5	5th	0000100
6	6th	0000010
7	7th	0000001

calculation of Syndromes

$$S = EH^T$$

$$[S]_{1\times 3} = [E]_{1\times 7}[H^T]_{7\times 3}$$

syndrome for 1st bit in error

$$S = [1000000]\begin{bmatrix}1&0&1\\1&1&1\\1&1&0\\0&1&1\\1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$= [101]$$

syndrome for 2nd bit in error

$$S = [0100000]\begin{bmatrix}1&0&1\\1&1&1\\1&1&0\\0&1&1\\1&0&0\\0&1&0\\0&0&1\end{bmatrix} = \underline{[111]}$$

- Syndrome vectors are rows of H^T.

Error correction using Syndrome vector

Let us see how single bit errors can be corrected using Syndrome decoding.

Let the transmitted code vector be,

$$x = (1\ 0\ 0\ 1\ 1\ 1\ 0)$$

Let there be error be created in 3rd bit in received vector Y.

$Y = (1\ 0\ \textcircled{1}\ 1\ 1\ 1\ 0)$ encircled bit shows it is in error.

Now error can be corrected by following steps.

1) calculate $S = YH^T$.

2) check the row of H^T same as 'S'

3) for p^{th} bit row in H^T p^{th} bit in error. Hence corresponding error vector E.

4) obtain correct vector by $X = Y \oplus E$

(i) $S = YH^T$

$$S = [1\ 0\ 1\ 1\ 1\ 1\ 0] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [1\ 1\ 0]$$

$$S = YH^T = EH^T = [1\ 1\ 0]$$

(ii) So the on comparing syndrome with H^T the syndrome is 3rd row of H^T. So the error pattern of syndrome is $E = (0\ 0\ 1\ 0\ 0\ 0\ 0)$

(iii) $X = Y \oplus E$

$$= [1\ 0\ 1\ 1\ 1\ 1\ 0] \oplus [0\ 0\ 1\ 0\ 0\ 0\ 0]$$

$$= [1\ 0\ 0\ 1\ 1\ 1\ 0].$$

what happens if double error occurs in Y?

Let $X = 1001110$

& $Y = 10\textcircled{1}\textcircled{0}110$
error in 3rd & 4th bit

$$S = [1010110]\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= [1\,0\,1]$$

The Syndrome corresponds to $\mathcal{E} = 1000000$.
This shows error in first bit. Thus error detection & correction goes wrong.

~~So~~
→ For a linear block code, prove with examples that:
(i) The syndrome depends only on error pattern & not on transmitted codeword.
(ii) All error patterns that differ by a code word have same syndrome.

Sol: (i) $\cdot\ S = \mathcal{E}H^T$
Above eqn shows the syndrome (S) depend upon the ~~codeword (X)~~ error pattern. It does not depend on codeword (X)

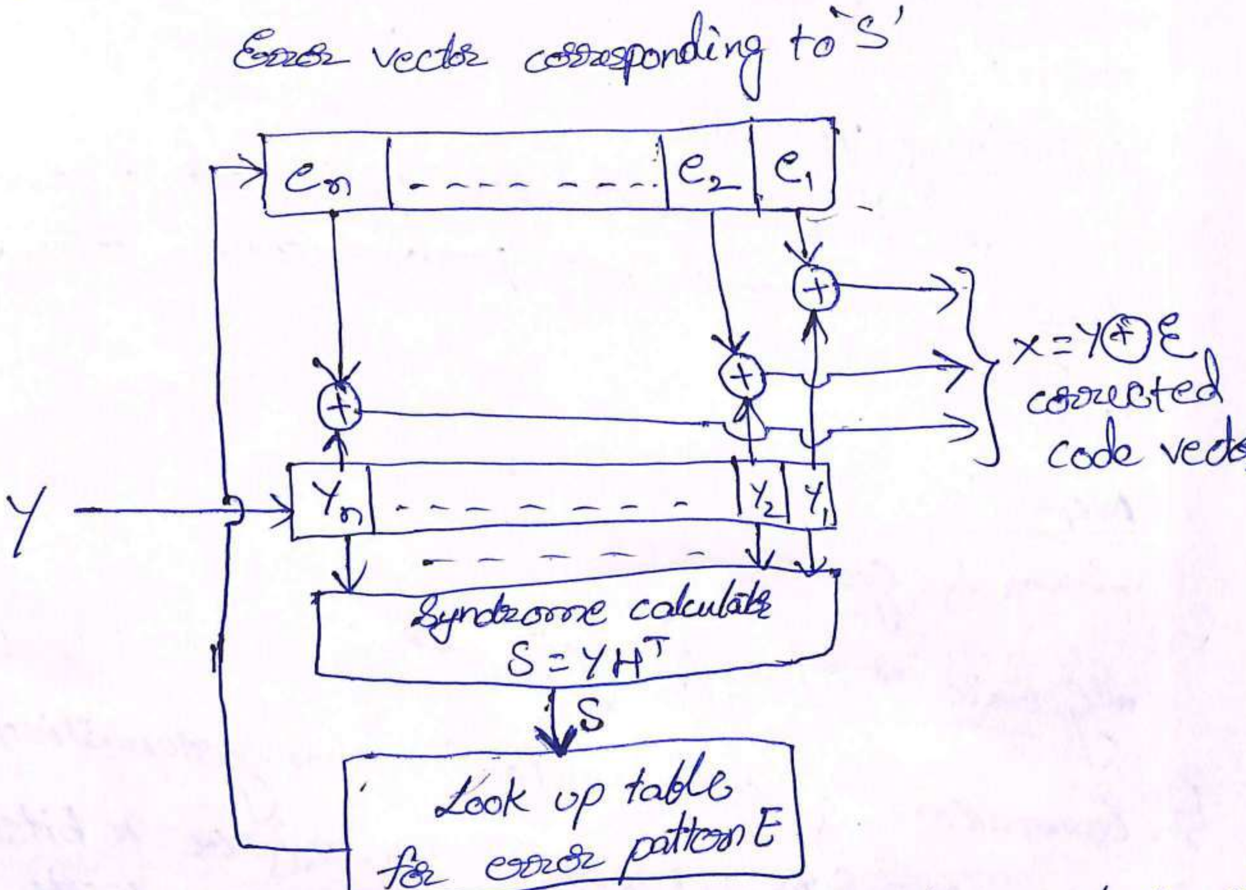
(ii) Let us consider the two code vectors X_1 & X_2. Let error be introduced in the first bit. Then error pattern for both of these codeword will be same. i.e

$\mathcal{E} = (1000\ 0000)$ for 8bit code vector

$S_1 = Y_1 H^T$
$= (X_1 \oplus \mathcal{E}) H^T = \mathcal{E}H^T \quad (\because X_1 H^T = 0)$

$S_2 = Y_2 H^T \qquad \because X_2 H^T = 0$
$= (X_2 \oplus \mathcal{E}) H^T$
$= X_2 H^T \oplus \mathcal{E}H^T = \underline{\mathcal{E}H^T}$

Syndrome decoder for (n, k) Block code:-



Syndrome decoder for Linear block code to correct errors.

→ Here the received n bit vector 'Y' is stored in a n bit register. From this vector Syndrome is calculated using.

$$S = YH^T.$$

→ Thus H^T is stored in the Syndrome calculator. The q-bit Syndrome vector is then applied to a Look up table of error patterns. Depending on particular syndrome an error pattern is selected. This error pattern is added to vector Y.

The o/p is thus $Y \oplus E = X$.

→ The block diagram shown above correct only single errors.

→ Syndrome decoding is called maximum likelihood decoding.

Binary cyclic codes

Cyclic codes can be in Systematic or non Systematic form.

Definition of cyclic code

A linear code is called cyclic code if every cyclic shift of the code vector produces some other code vector.

Properties.

Linear & cyclic.

Algebraic Structure of cyclic codes.

1) Generation of code vector in Non Systematic form.

Let $M = \{m_{k-1}, m_{k-2}, \cdots m_1, m_0\}$ be k bits of message vector. Then it can be represented by the polynomial as

$$M(P) = M_{k-1}P^{k-1} + m_{k-2}P^{k-2} + \cdots + m_1(P) + m_0.$$

Let $X(P)$ represent the codeword polynomial. it is given as

$$X(P) = M(P)G(P)$$

Here $G(p)$ is the generating polynomial of degree 'q'. for (n,k) cyclic code $q = n-k$.

$$G(p) = p^q + g_{q-1}p^{q-1} + \cdots + g_1 P + 1$$

2) The codewords are represented by polynomial. for example consider a n-bit codeword.

$$X = (x_{n-1}, x_{n-2}, \cdots x_1, x_0)$$

This codeword can be represented by polynomial of degree less than or equal to $n-1$

$$X(P) = x_{n-1}P^{n-1} + x_{n-2}P^{n-2} + \cdots + x_1 P + d_0$$

Here $x(p)$ is a polynomial of degree $n-1$

→ The generator polynomial of a (7,4) cyclic code is ⑩.
$G(P) = P^3 + P + 1$. Find all code vectors for code in non systematic form.

Sol:- Here $n = 7$ $K = 4$.

So there $2^4 = 16$ message vectors of

consider a message $(0101) = (m_3\ m_2\ m_1\ m_0) = M$.

$$M(P) = m_3 P^3 + m_2 P^2 + m_1 P + m_0$$

$$M(P) = P^2 + 1$$

$$X(P) = G(P)\,M(P)$$

$$= (P^3+P+1)(P^2+1)$$

$$= P^5 + P^3 + P^3 + P + P^2 + 1$$

$$= P^5 + (1 \oplus 1)P^3 + P^2 + P + 1$$

$$= 0P^6 + P^5 + 0P^4 + 0P^3 + P^2 + P + 1$$

Note :- the degree of above polynomial $n-1 = 6$.

$$X = (0100111)$$

	message bits				Non Systematic code vector						
	m_2	m_2	m_1	m_0	x_6	x_5	x_4	x_3	x_2	x_1	x_0
8)	0	1	1	1	0	1	1	0	0	0	1
9)	1	0	0	0	1	0	1	1	0	0	0

The cyclic shift of x_9 produces x_8

Generation of code vectors in systematic form.

$$X = (k \text{ message bits} : (n-k) \text{ check bits})$$

$$= (m_{k-1}\, m_{k-2} \cdots m_1, m_0 : c_{q-1}\, c_{q-2} \cdots c_1 c_0)$$

Here the check bits form a polynomial as,

$$c(p) = c_{q-1}p^{q-1} + c_{q-2}p^{q-2} + \cdots c_1 p + c_0.$$

The check bit polynomial is obtained by.

$$c(p) = \text{rem}\left[\frac{p^q M(p)}{G(p)}\right]$$

→ The generator polynomial of (7,4) cyclic code is $G(p) = p^3+p+1$ find all code vectors for the code in systematic form.

Sol: Here $n=7$, $k=4$ & $q = n-k = 3$.

$2^4 = 16$ message vectors.

$M = (0101) = M(p) = p^2+1$, $G(p) = p^3+p+1$.

$$p^q(M(p)) = p^3[p^2+1] = p^5+p^3.$$

$$c(p) = \text{rem}\left[\frac{p^3 M(p)}{G(p)}\right]$$

$$= p^3+0p^2+p+1 \overset{p^2 \cancel{+0}}{\overline{\big) p^5+0p^4+p^3+0p^2+0p+0}}$$

$$\quad \oplus p^5 \oplus 0p^4 \oplus p^3 \oplus p^2$$

$$\quad 0 \quad 0 \quad 0 \quad \underline{p^2+0p+0}$$

$$c(p) = p^2+0p+0.$$

$$= c_2p^2 + c_1p + c_0.$$

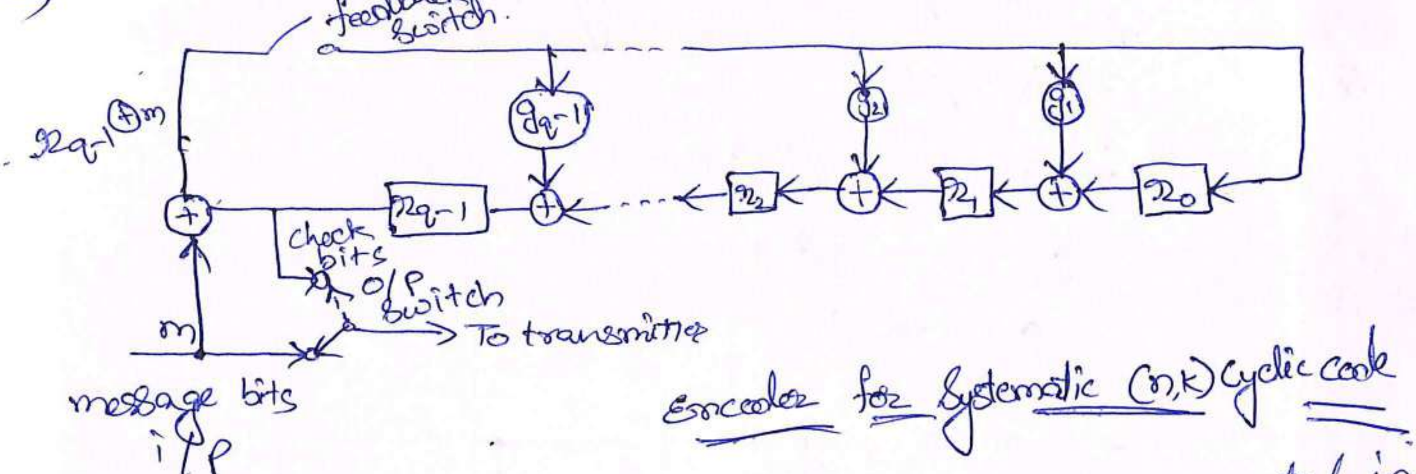
$$C = (c_2\; c_1\; c_0) = (1\;0\;0).$$

$$X = (0\;1\;0\;1\;1\;0\;0).$$

lly remaining also

Encoding using (n-k) bit Shift register.

→.



Encoder for Systematic (n,k) Cyclic code

→[r_1]→ } there are flipflops. They are connected in
→[r_2]→ } sequential order to make a shift register. The contents of the shift register are shifted from i/p to o/p when clock pulse is applied.

g_1 g_2 They represent closed path if g=1 & open path if g=0.

→(+)→ These symbols represent mod-2 addition.

Operation:- The feedback switch is first closed. The o/p switch is connected to message i/p. All the shift registers are initialized to all zero state. The K message bits are shifted to tx as well as shifted into the registers. After the shift of 'k' message bits the registers contain 'q' check bits. The feed back switch is now opened & o/p switch is connected to check bits 'are then shifted to transmitter.

→ Design the encoder for (7,4) cyclic code generated by $G(p) = p^3+p+1$ & verify its operation for any message vector.

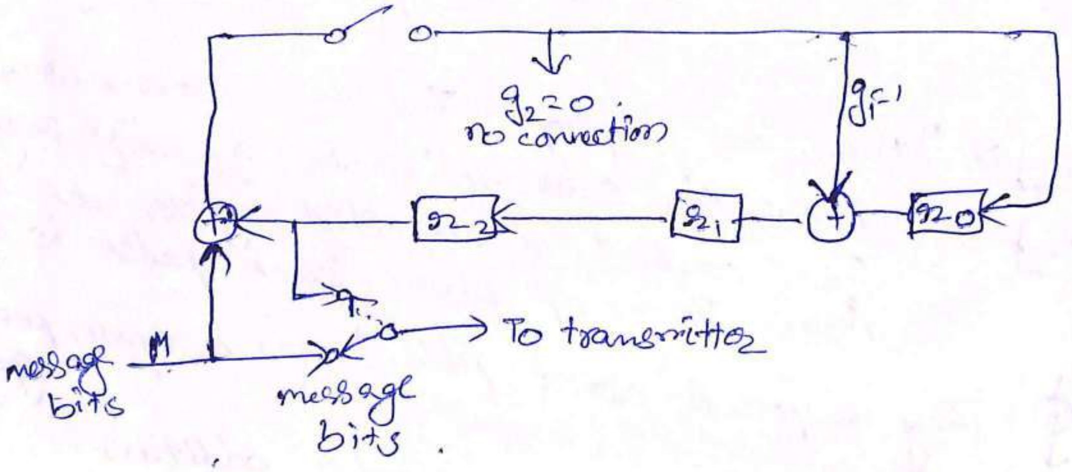
Sol:

$$G(p) = p^3+p+1.$$

$$G(p) = p^3+g_2p^2+g_1p+1$$

$$g_2 = 0 \quad g_1 = 1.$$

$$q = (n-k) = 3.$$



I/p message bit m	register bit i/p before shift			register bit o/p after shift		
	$r_2=r_2'$	$r_1=r_1'$	$r_0=r_0'$	$r_2'=r_1$	$r_1'=r_0\oplus r_2\oplus m$	$r_0'=r_2\oplus m$
–	0	0	0	0	0	0
1	0	0	0	0	$0\oplus0\oplus1=1$	$0\oplus1=1$.
1	0	~~0~~1	1	~~$1\oplus0\oplus1=0$~~	$1\oplus0\oplus1=0$	$0\oplus1=1$
0	1	0	1	0	$1\oplus1\oplus0=0$	$1\oplus0=1$
0	0	0	1	0	$1\oplus0\oplus0=1$	$0\oplus0=0$.

$$x = (m_3 m_2 m_1 m_0 c_2 c_1 c_0) = (1\,1\,0\,0\ 0\,1\,0).$$

Syndrome decoding, Error detection & error correction ⑫

Syndrome decoding can be used to correct errors.

Let we we know $X = Y \oplus E$

or $Y = X \oplus E$

In the polynomial form we can write the above eqns.

$$Y(p) = X(p) + E(p).$$

Since $X(p) = M(p)G(p)$

$$Y(p) = M(p)G(p) + E(p). \quad \text{——— (1)}$$

Let $Y(p)$ be divided by $G(p)$.

$$\frac{Y(p)}{G(p)} = M(p) + \frac{E(p)}{G(p)}$$

$$= \text{Quotient} + \frac{\text{remainder}}{G(p)}.$$

In the above eqn $Y(p)=X(p)$ i.e if no error.

$$\frac{X(p)}{G(p)} = M(p) \cancel{+ \frac{E(p)}{G(p)}} \text{ Quotient} + \frac{\text{remainder}}{G(p)}$$

Since $X(p) = M(p)G(p)$ quotient will be equal to $M(p)$ & remainder will be zero. This shows that if there is no error. then remainder will be zero. Here $G(p)$ is the factor of code vector polynomial. Let's represent quotient by $Q(p)$. & remainder by $R(p)$ then

$$\frac{Y(p)}{G(p)} = Q(p) + \frac{R(p)}{G(p)}$$

$R(p)$ will be the polynomial of degree less than or equal to $q-1$. Multiply both sides by $G(p)$

$$Y(p) = Q(p)G(p) + R(p). \quad \text{— (2)}$$

on comparing eqn (1) & (2)

$$M(p)G(p) + E(p) = Q(p)G(p) + R(p)$$

$$E(p) : M(p)G(p) \oplus Q(p)G(p) \oplus R(p)$$

$$E(p) = [M(p) \oplus Q(p)]G(p) \oplus R(p)$$

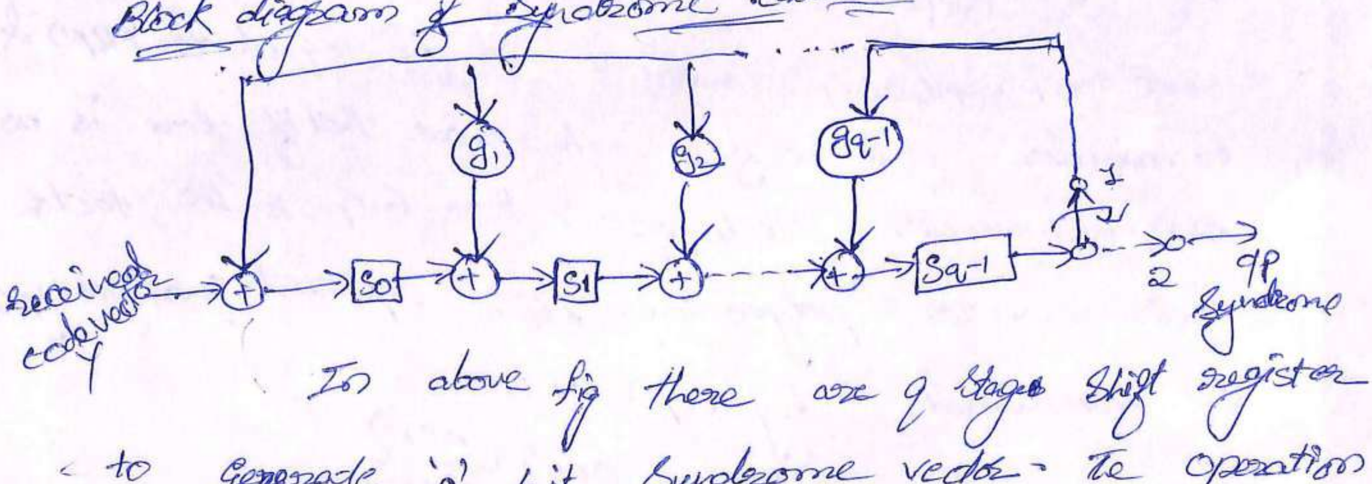
This eqn shows that for fixed message vector & Generator polynomial, an error pattern or error vector E depends on remainder R. For every remainder R there will be specific error vector. Therefore we can call the remainder vector R' as Syndrome vector 'S'. or $R(p) = S(p)$.

$$\therefore \frac{Y(p)}{G(p)} = Q(p) + \frac{S(p)}{G(p)}$$

Thus the Syndrome vector is obtained by dividing received vector Y(p) by G(p) i.e

$$S(p) = \text{rem}\left[\frac{Y(p)}{G(p)}\right]$$

Block diagram of Syndrome Calculator



In above fig there are q stages shift register to generate 'q' bit Syndrome vector. The operation as follows. Initially all the shift registers contents are zero and the switch is closed in position 1. The received vector Y is shifted bit by bit into the shift register. The contents of flipflops keep on changing according to the i/p bits of Y & value of $g_1 g_2$ etc.

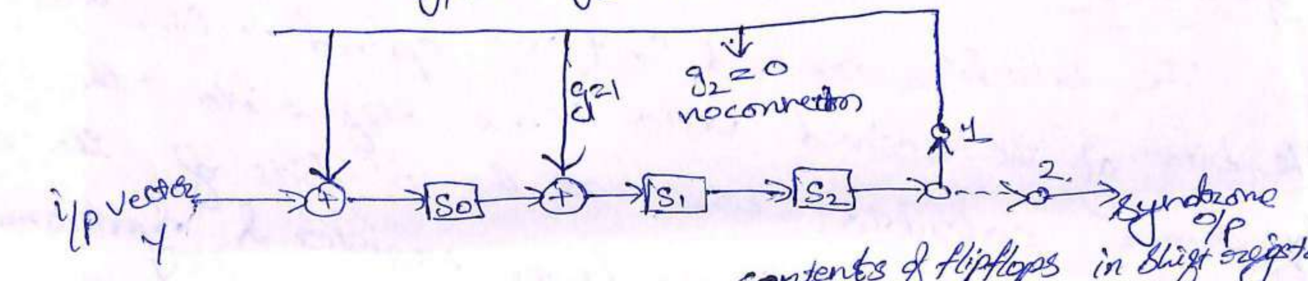
After all the bits are shifted, the 'q' flipflops of shift register contains the q bit syndrome vector. The switch is then closed to position 2 & clocks are applied to the shift register. The o/p is a syndrome vector.

→ Design a syndrome calculator for a (7,4) cyclic hamming code generated by the polynomial $G(p) = p^3 + p + 1$. Calculate the syndrome for $Y = (1001101)$.

Sol: for a given code $n=7$, $K=4$, $q = n-k = 3$.

$G(p) = p^3 + 0p^2 + p + 1$.

$g_1 = 1 \quad g_2 = 0$.

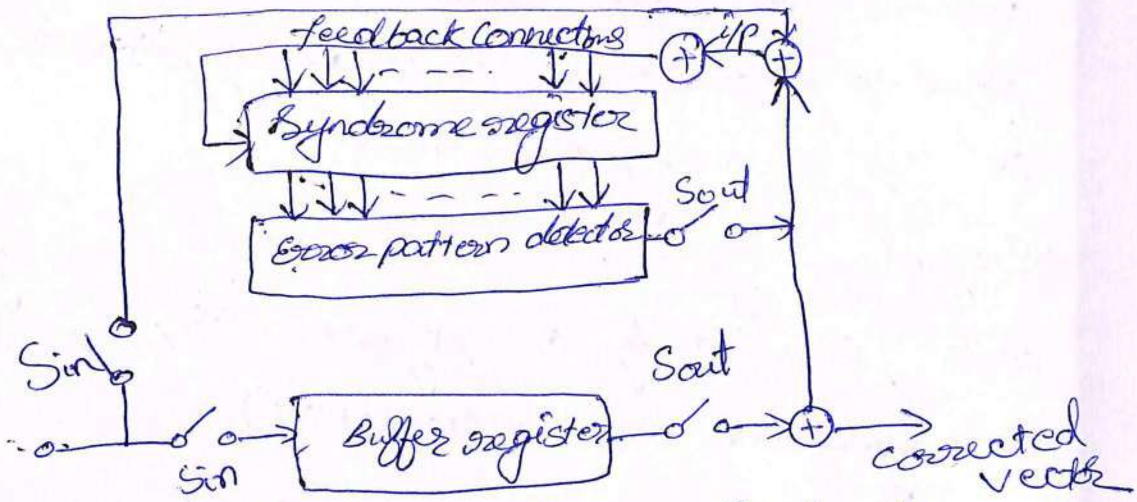


contents of flipflops in shift registers

Shift	received vector i.e bits of Y	$S_0 = Y \oplus S_2$	$S_1 = S_0 \oplus S_2$	$S_2 = S_1$
-	—	0	0	0
1	1	$1 \oplus 0 = 1$	0	0
2	0	$0 \oplus 0 = 0$	$1 \oplus 0 = 1$	0
3	0	$0 \oplus 0 = 0$	$0 \oplus 0 = 0$	1
4	1	$1 \oplus 1 = 0$	1	0
5	1	1	0	1
6	0	1	0	0
7	1 Syndrome →	1	1	0

$S = (0\ 1\ 1) = (S_2\ S_1\ S_0)$

Decoder for cyclic codes



Generalized block diagram of decoder for cyclic codes.

operation

The switches named Sout are opened & Sin are closed. The bits of the received vector Y are shifted into the Syndrome calculator. when all the 'n' bits of the received vector Y are shifted into buffer register & Syndrome calculator the Syndrome register holds a Syndrome vector. The Syndrome vector is given to the error pattern detector. A particular Syndrom detects a specific error pattern. The switches Sin are opened & Sout are closed. The Shifts are then applied to the flipflops of buffer ~~st~~ register, error register & Syndrome register. The error pattern is then added bit by bit to received vector. The o/p is the corrected error free vector.

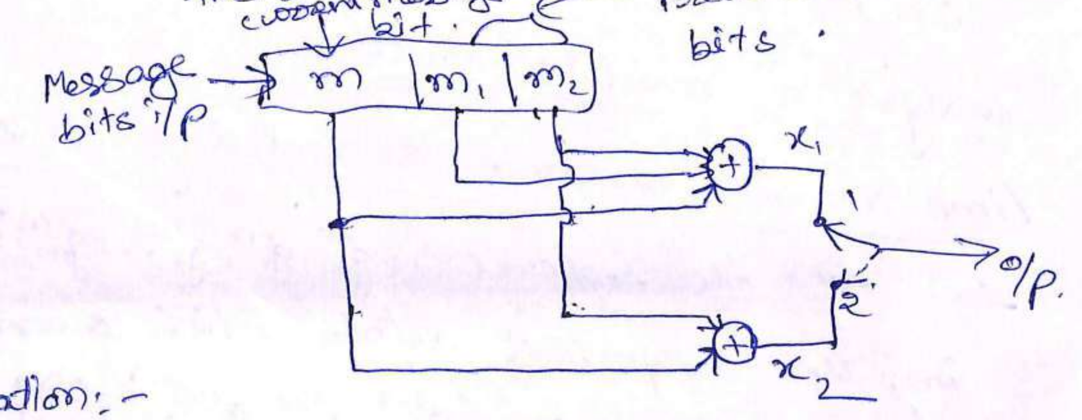
UNIT – 7

Convolution codes

Encoding, Decoding using State, Tree & trellis diagrams, Decoding using viterbi algorithm, Comparision of error rates in coded & uncoded transmission.

Definition of Convolution coding.

· convolution coding is done by combining the fixed number of i/p bits. The i/p bits are stored in fixed length shift register & they are combined with mod-2 adder. This operation is called convolution code.



operation:-

$$x_1 = m \oplus m_1 \oplus m_2$$

$$x_2 = m \oplus m_2$$

The o/p then switch then samples x_1 & x_2. Thus o/p stream.

$$X = x_1 x_2 x_1 x_2 \ldots \text{ \& so on.}$$

→ Here the for every i/p message bit, two encoded o/p bit.

no. of message bits, $k = 1$

no. of Encoded o/p bits for on message bit, $n = 2$

code rate of the convolution encoder $r = \frac{k}{n} = \frac{1}{2}$.

Constraint length (K)

It is defined as number of shifts over which a single message bit can influence the encoder o/p.

first shift → message bit is entered in position 'm'

Second shift → messag bit is shifted in position 'm_1'

Third shift → " " " " " 'm_2'

∴ So constraint length $k = 3$.

Dimension of the code

$(n, k) = (2, 1)$.

Analysis of convolution encoder

Time domain approach to analysis of convolution Encoder.

→ Let the sequence $\{g_0^{(1)}, g_1^{(1)}, g_2^{(1)} \cdots g_m^{(1)}\}$ denote the impulse response of adder which generates x_1'

Let the sequence $\{g_0^{(2)}, g_1^{(2)}, g_2^{(2)} \cdots g_m^{(2)}\}$ denote the impulse response of adder which generates x_2'

This impulse responses are also called generator sequences of the code.

→ Let incoming message sequence be $\{m_0, m_1, m_2 \cdots\}$.

→ The encoder generates the two o/p sequences x_1 & x_2.

$$x_1 = x_i^{(1)} = \sum_{l=0}^{M} g_l^{(1)} m_{i-l} \qquad i = 0, 1, 2 \cdots$$

Here $m_{i-l} = 0$ for $l \geq i$.

$$\text{||ly} \quad x_2 = x_i^{(2)} = \sum_{l=0}^{M} g_l^{(2)} m_{i-l}$$

Next x_1 & x_2 are multiplexed by the switch. Hence o/p sequence

$$\{x_i\} = \{x_0^{(1)}, x_0^{(2)}, x_1^{(1)}, x_1^{(2)}, x_2^{(1)}, x_2^{(2)} \cdots\}.$$

Here $V_1 = x_i^{(1)} = \{x_0^{(1)}, x_1^{(1)}, x_2^{(1)} \cdots\}$.

$V_2 = x_i^{(2)} = \{x_0^{(2)}, x_1^{(2)} x_2^{(2)} x_3^{(2)} \cdots\}$

②

→ For the convolution encoder of fig determine the following:

1, dimension of the code.

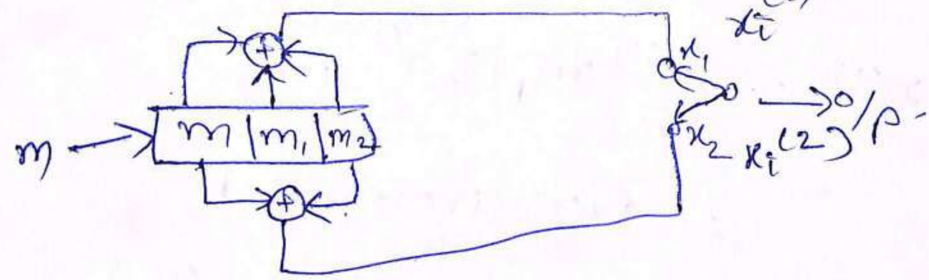
2, code rate.

3, constraint length

4, generating sequences

5, o/p sequence for $m = \{10011\}$

→ Sol:



1, $(2,1) = (n,K)$

2, code rate $= \frac{K}{n} = \frac{1}{2}$.

3, constraint length $K = 3$ bits.

4, Generating Sequences.

$x_i^{(1)}$ is generated by adding all the three bits.

$g_i^{(1)} = \{1\ 1\ 1\}$.

$g_0^{(1)} = 1$ represents connection of bit m.

$g_1^{(1)} = 1$ " " " " m_1

$g_2^{(1)} = 1$ " " " " m_2.

$x_i^{(2)}$ is generated by addition of first & last bits.

$g_i^{(2)} = \{1\ 0\ 1\}$.

$g_0^{(2)} = 1$, $g_1^{(2)} = 0$, $g_2^{(2)} = 1$.

5, To obtain o/p Sequence.

for $m = (\underset{m_0}{1}\ \underset{m_1}{0}\ \underset{m_2}{0}\ \underset{m_3}{1}\ \underset{m_4}{1})$.

o/p due to adder 1.

$$x_i^{(1)} = \sum_{l=0}^{M} g_l^{(1)} m_{i-l}$$

$i=0 \rightarrow x_0^{(1)} = g_0^{(1)} m_0.$

$= 1 \times 1 = 1.$

$i=1 \rightarrow x_1^{(1)} = g_0^{(1)} m_1 \oplus g_1^{(1)} m_0.$

$= (1 \times 0) \oplus (1 \times 1) = 1.$

$i=2 \rightarrow x_2^{(1)} = g_0^{(1)} m_2 \oplus g_1^{(1)} m_1 + g_2^{(1)} m_0$

$= (1 \times 0) \oplus (1 \times 0) \oplus (1 \times 1) = 1$

$i-3\ x_3^{(1)} = g_0^{(1)} m_3 \oplus g_1^{(1)} m_2 \oplus g_2^{(1)} m_1$

$= (1 \times 1) \oplus (1 \times 0) \oplus (1 \times 0)$

$= 1.$

$i=4\ \ x_4^{(1)} = g_0^{(1)} m_4 \oplus g_1^{(1)} m_3 \oplus g_2^{(1)} m_2$

$= (1 \times 1) \oplus (1 \times 1) \oplus (1 \times 0) = 0.$

$i=5\ \ x_5^{(1)} = g_0^{(1)} m_5 \oplus g_1^{(1)} m_4 \oplus g_2^{(1)} m_3$

$= g_1^{(1)} m_4 \oplus g_2^{(1)} m_3$

$= 1 \times 1 \oplus (1 \times 1) = \underline{0}$

$x = \{1\,1, 10, 11, 11, 01\,01, 11\}$

$i=6\ \ x_6^{(1)} = g_0^{(1)} m_6 + g_1^{(1)} m_5 \oplus g_2^{(1)} m_4$

$= g_2^{(1)} m_4 = 1$

$x_1 = \{1\,1\,1\,1\,0\,0\},\ x_2 = \{1\,0\,1\,1\,1\,1\,$[illegible]$\}$

$i=0 \quad = \quad x_0^{(2)} = \sum_{l=0}^{M} g_0^{(2)} m_0 \oplus$

$= \quad 1 \times 1 = \underline{\underline{1}}$

$i=1 \quad x_1^{(2)} = \sum_{l=0}^{M} g_l^{(2)} m_{i-l}$

$= g_0^{(2)} m_1 \oplus g_1^{(2)} m_0$

$= (1 \times 0) \oplus (0 \times 1)$

$= \underline{\underline{0}}$

$i=2 \quad x_2(2) = g_0^{(2)} m_2 \oplus g_1^{(2)} m_1 \oplus g_2^{(2)} m_0$

$= \quad 1 \times 0 \oplus 0 \times 0 \oplus 1 \times 1$

$= \underline{\underline{1}}$

$i=3 \quad x_3^{(2)} = g_0^{(2)} m_3 \oplus g_1^{(2)} m_2 \oplus g_2^{(2)} m_1$

$= 1 \times 1 \oplus 0 \times 0 \oplus 1 \times 0$

$= \underline{\underline{1}}$

with $i=4 \quad x_4^{(2)} = g_0^{(2)} m_4 \oplus g_1^{(2)} m_3 \oplus g_2^{(2)} m_2 \oplus g_3^{(2)} m_1 \oplus g_4^{(2)} m_0$

$= \quad 1 \times 1 \oplus 0 \times 1 \oplus 1 \times 0 + 0 + 0$

$= \underline{\underline{1}}$

with $i=5 \quad x_5^{(2)} = g_1^{(2)} m_4 \oplus g_2^{(2)} m_3$

$= 0 \times 1 \oplus (1 \times 1)$

$= \underline{\underline{1}}$

with $i=6 \quad x_6^{(2)} = \sum_{l=} g_2^{(2)} m_4 =$

$= \quad 1 \times 1 \qquad x_2 = x_i^{(2)} = \{1\ 0 1 \cdot 1\ 1\ 1\}$

$= \underline{\underline{1}}$

Transform Domain approach to analysis of Convolution Encoders

Let the impulse responses be represented by polynomials.

i.e $g^{(1)}(p) = g_0^{(1)} + g_1^{(1)} p + g_2^{(1)} p^2 + \dots + g_M^{(1)} p^M$.

||ly $g^{(2)}(p) = g_0^{(2)} + g_1^{(2)} p + g_2^{(2)} p^2 + \dots + g_M^{(2)} p^M$.

The variable 'p' is unity delay operator in above eqn. It represents time delay of the bits in impulse response.

||ly $M(p) = m_0 + m_1 p + m_2 p^2 + \dots + m_{L-1} p^{L-1}$

Here L is the length of the message sequence.

→ The convolution sums are converted to polynomial multiplica in transform domain i.e.

$$x^{(1)}(p) = g^{(1)}(p) \cdot m(p)$$

$$x^{(2)}(p) = g^{(2)}(p) \cdot m(p)$$

→ consider previous examp.

$$g_i^{(1)} = \{1\ 1\ 1\}$$

$$g^{(1)}(p) = 1 + 1 \times p + p^2$$

$$= p^2 + p + 1$$

$$g_i^{(2)} = \{1\ 0\ 1\}$$

$$= 1 + 0 \cdot p + p^2$$

$$= 1 + p^2$$

$$m = \{1\ 0\ 0\ 1\ 1\} \Rightarrow m(p) = 1 + 0 \cdot p + 0 \cdot p^2 + 1 \cdot p^3 + 1 \cdot p^4$$

$$= 1 + p^3 + p^4$$

$$x^{(1)}(p) = (p^2 + p + 1)(1 + p^3 + p^4)$$

$$= p^2 + p^5 + p^6 + p + p^3 + p^5 + 1 + p^3 + p^4$$

$$= 1 + p + p^2 + p^3 + 0 \cdot p^4 + 0 \cdot p^5 + p^6$$

$$= \underline{\{1\,1\,1\,1\,0\,0\,1\}}$$

(4).

$$x^{(2)}(p) = g^{(2)}p \cdot m(p)$$

$$= (1+p^2)(1+p^3+p^4)$$

$$= 1+p^2+p^3+p^5+p^4+p^6$$

$$= 1+p^2+p^3+p^4+p^5+p^6$$

$$x^{(2)}(p) = \{1 0 1 1 1 1 1\}$$

$$x_i = \{11, 10, 11, 11, 01, 01, 11\}$$

code tree, Trellis & State diagram for a convolution Encoder

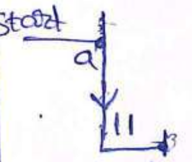
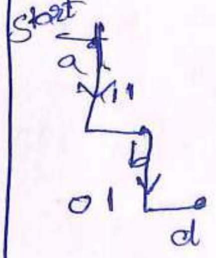
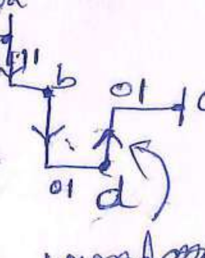
Let us consider the development of code tree for m=110.

where $x_2 = m \oplus m_1 \oplus m_2$

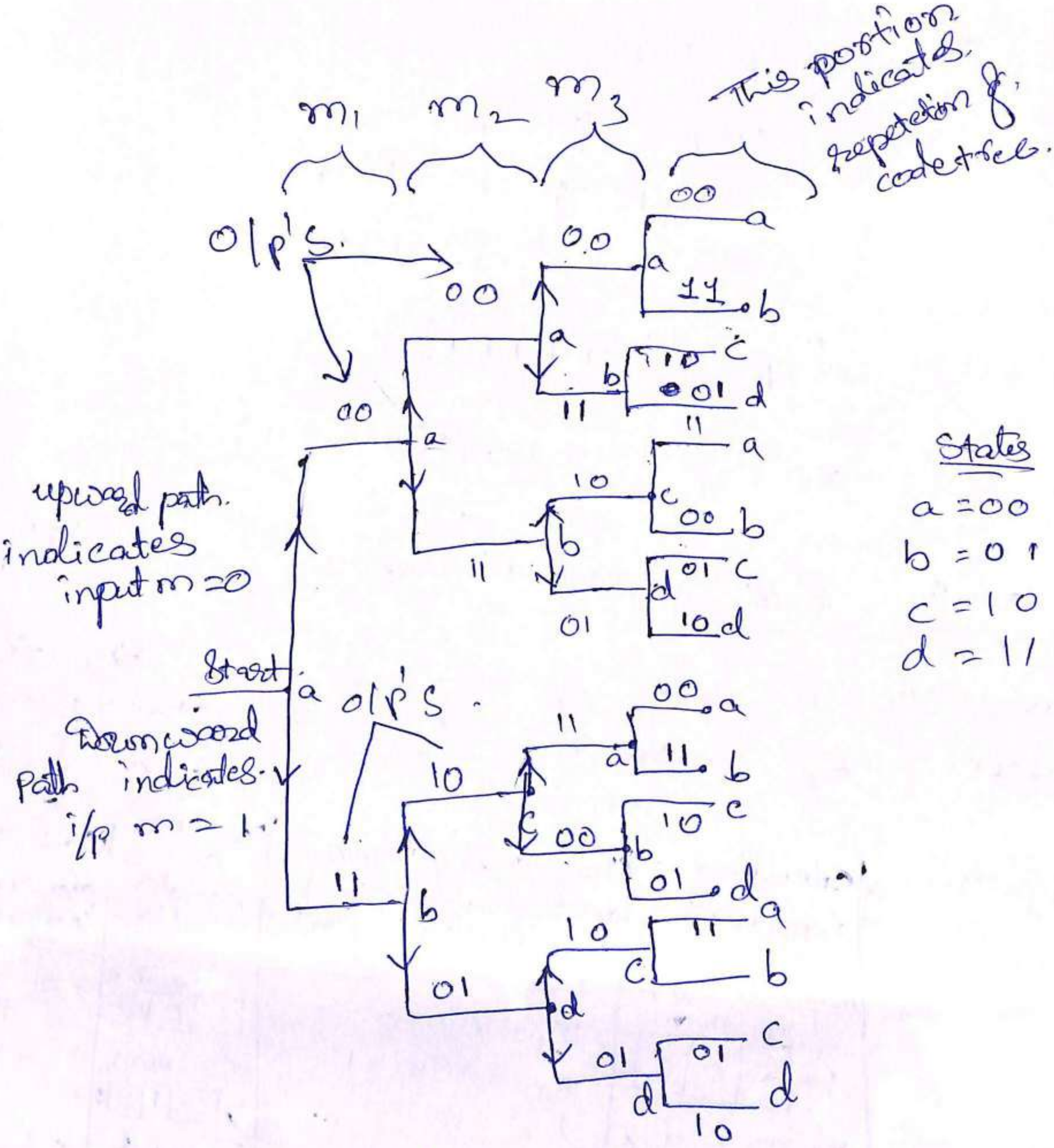
$x_2 = m \oplus m_2$

states of Encoder

m_2	m_1	st
0	0	a
0	1	b
1	0	c
1	1	d

S.no	Input message bit m	calculating o/p's x_1 & x_2 & status of shift register after entering of m	status of shift register after transmission of o/p & shift right by one bit	new state of encoder $m_2 m_1$	code tree diagram downward arrow indicates i/p is 1
1	1	$m\ m_1\ m_2$ 1 → |0|0| $m_2 m_1 \Rightarrow a$ 0 0	$x_1 = 1 \oplus 0 \oplus 0 = 1$ $x_2 = 1 \oplus 0 = 1$	|1|0|0| i.e $m_1 m_2$ | |1|0| $m_2 m_1 \rightarrow 01 \rightarrow b$	 Code tree for m=1
2	1	$m\ m_1\ m_2$ 1 → |1|0| → b $x_1 = 0$ $x_2 = 1$	$m\ m_1\ m_2$ |1|1|0| $m_1 m_2 \Rightarrow 11 \Rightarrow$ i.e d $m_1\ m_2$ | |1|1|	11 i.e d	
3	0	$m\ m_1\ m_2$ 0 → | |1|1| → d $m\ m_1\ m_2$ |0|1|1| $x_1 = 0$ $x_2 = 1$	$m\ m_1\ m_2$ |0|1|1| i.e $m_1\ m_2$ | |0|1| 1	10 i.e c	 upward arrow means message bit is '0' code tree for m=110

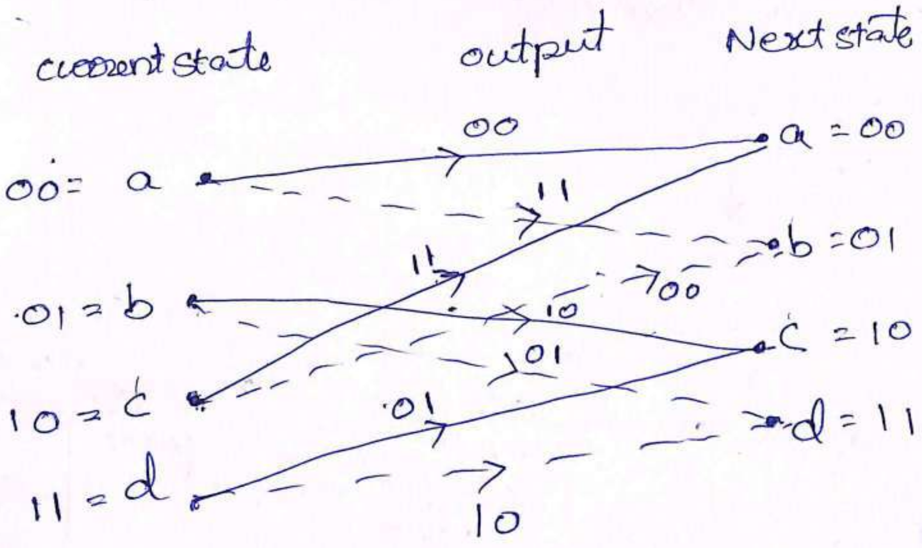
Analysis of convolution encoder



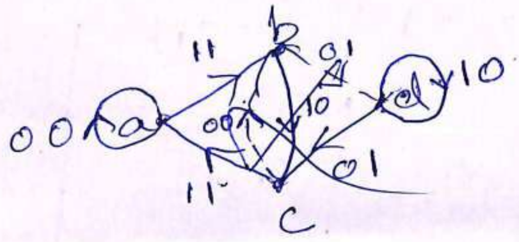
-> code tree for convolution encoder.

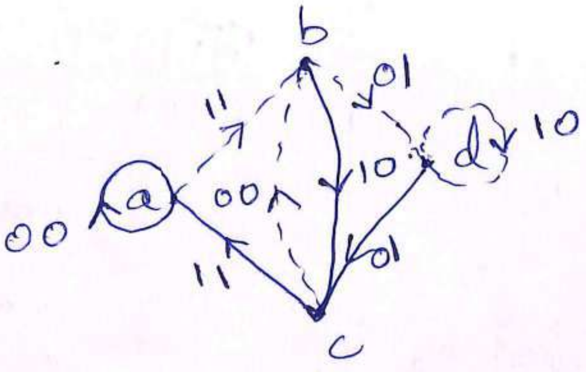
we can observe that the branch pattern begins to repeat after third bit. The repetition starts after 3rd bit since particular message bit is stored in shift registers ~~of~~ of encoder for three shifts. If the length of the shift register is increased by one bit, then the pattern of code tree will repeat after fourth message bit.

Code trellis

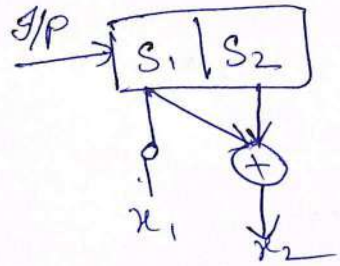


State diagram



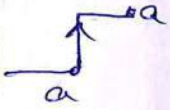
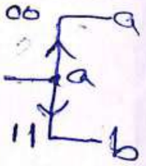
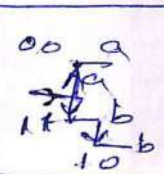
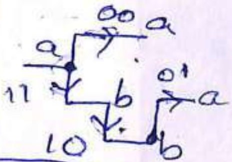


→ For given convolutional encoder sketch...
code tree, code trellis, State diagram. for m = 0 1 1 0

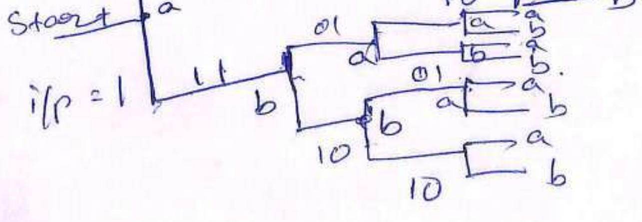


S_2	S_1	state.
0	0	a
0	1	b
1	0	c
1	1	d

Sol:- $x_1 = S_1$ $x_2 = S_1 \oplus S_2$

Sr no	I/P message bit	Status of Shift register after entry of m	calculation x_1 & x_2	Status of Shift register after ex. transmission of o/p. & Shift it by one bit	new status of encoder $S_2 S_1$	code tree diagram.
y	0	0 0 a	$x_1 = 0$. $x_2 = 0$	0 0 i.e 0 0	a	
2	1	S_1 S_2 1 0 State : b	$x_1 = 1$ $x_2 = 1$	1 0 i.e 1	b	
3	1	1 1 State d	$x_1 = 1$ $x_2 = 0$	1 1	b	
4	0	S_1 S_2 0 → 1 S_1 S_2 i.e 0 1 b	b $x_1 = 0$ $x_2 = 1$	0 1 0 1	a	

To obtain code tree.



Code trellis.

State diagram

Comparision of error rates in coded & uncoded transmission

→ Let the coded or uncoded words have the duration of T_ω. The coded word has more digits than does the uncoded word. Hence bit duration of coded word is less compared to uncoded word.

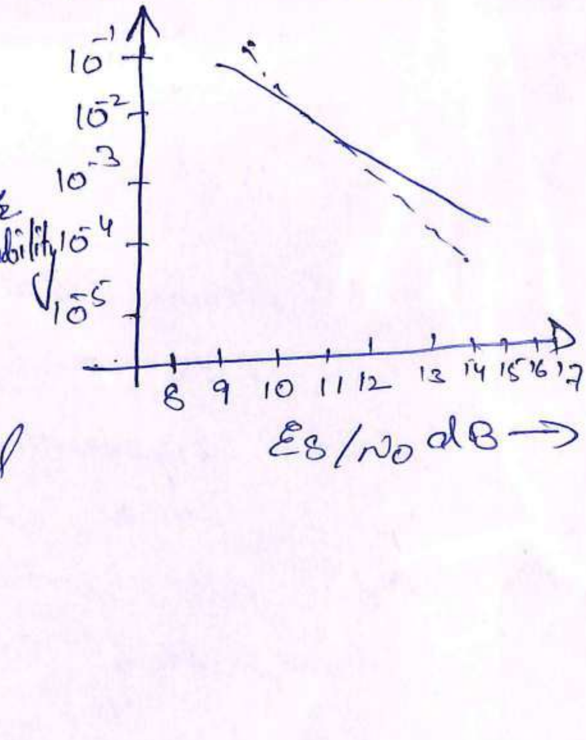
Let P_e represent probability of bit error in uncoded word & $P_e^{(c)}$ represent probability of bit error in coded word. They are given as follows.

$$\text{uncoded}: \; P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{P_s T_\omega}{K N_0}}$$

$$\text{coded}: \; P_e^{(c)} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{P_s T_\omega}{n N_0}}$$

fig shows the plot of these error probabilities with respect to word energy to noise ratio.

→ observe that error probability is less in coded case for $\frac{E_s}{N_0}$ ratios higher than 10dB.



→ coding gain is defined as

$$A = \frac{\left[\frac{E_b}{N_0}\right] \text{uncoded}}{\left[\frac{E_b}{N_0}\right] \text{coded}}.$$

Viterbi algorithm for decoding of convolutional codes

metric:- It is the discrepancy between the received signal Y & the decoded signal at particular node. This metric can be added over few nodes for a particular path.

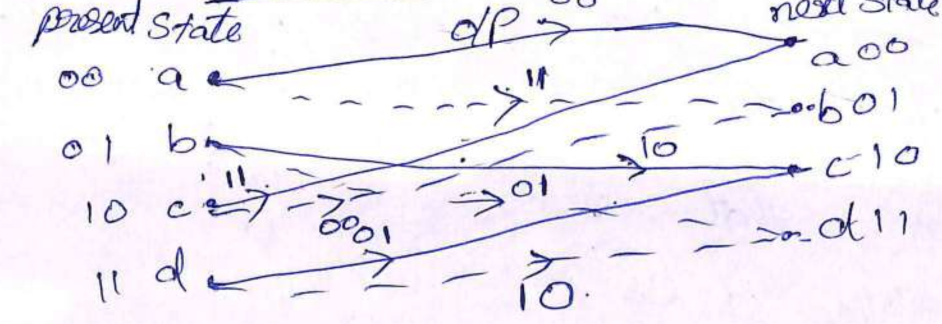
Surviving path:- This is the path of the decoded signal with minimum metric.

Let the first six received bits be.

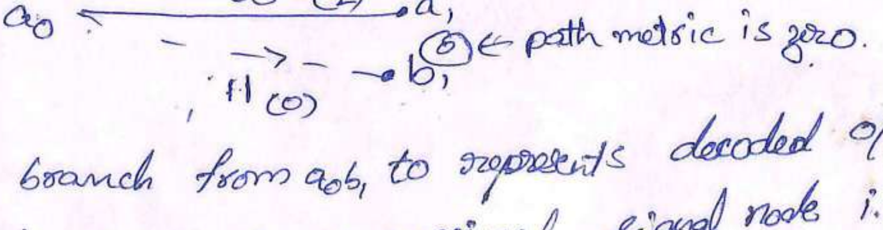
$Y = 11\ 01\ 11$

a) Decoding of first message bit for Y = 11

consider the code trellis.

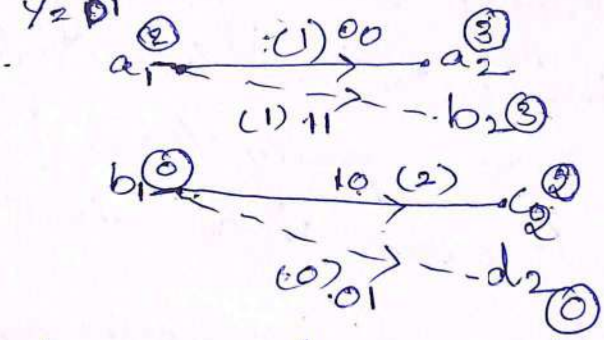


→ we know that for single bit i/p the encoder transmits two bits (x_1, x_2) o/p. These o/p's are received at decoder and represented by Y. Thus Y given represents the o/p for three successive message bits. Assume decoder at state a_0. Now look at code trellis It shows that if current state is a → next state will be a & b.

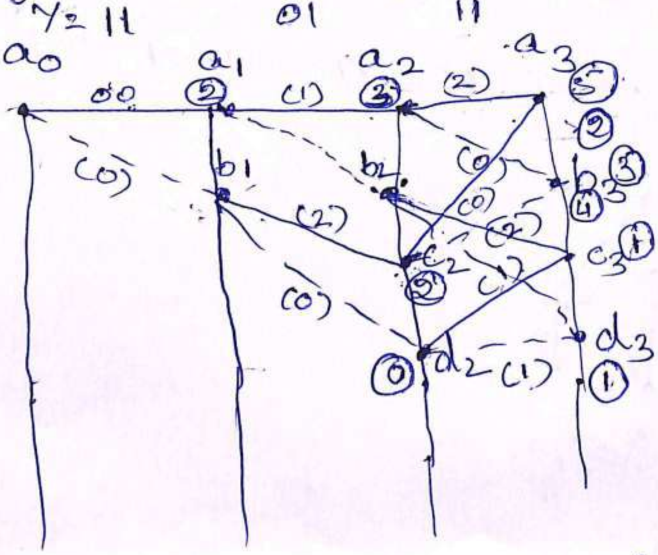


The branch from $a_0 b_1$ to represents decoded o/p as 11 which is same as a received signal node i.e 11. Hence metric of that branch is zero. The metric of branch from a_0 to a_1 is 2.

Decoding of second message bit



Decoding of 3rd message bit for Y = 11



This method ~~let~~ as apply for 12 message bits.

Y = 11 01 11 00 01 10 00 11 11 10 11 00

The method of decoding is called maximum liklihood decoding.

Surviving paths

$= 2^{(K-1)k}$

K → constraint length

k → message bits

If K = 3 & k = 1

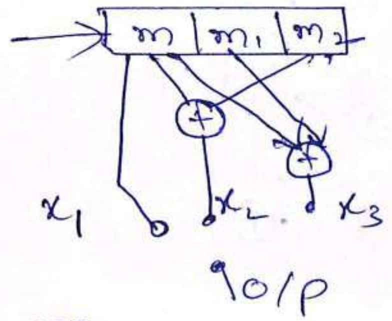
Surviving paths $= 2^{(3-1)\times 1} = 4$.

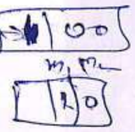
~~Thus~~ During decoding we can see that a viterbi decoder has to store four surviving paths for four nodes. If the no. of message bits to be decoded are very large, then the storage requirement is also large. To avoid this problem metric diversion effect is used.

Metric diversion effect

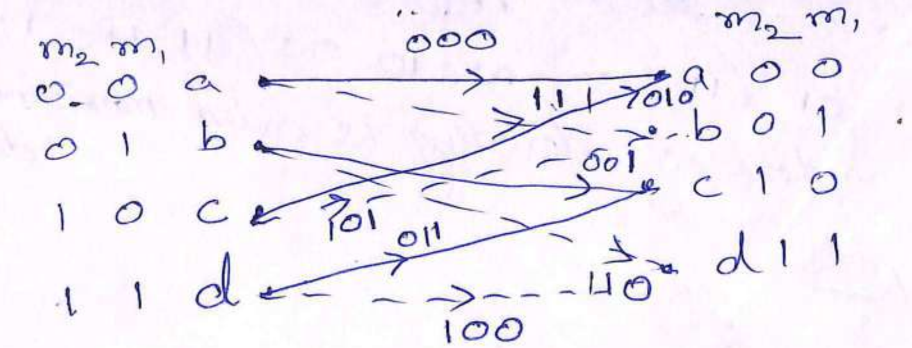
For the two surviving paths originating from same node; the running metric of less likely path tends to increase more rapidly than the metric of other path. This is called metric diversion effect

→ For the convolutional encoder arrangement shown in fig. Draw the trellis & use viterbi's algorithm to decode the sequence, 100 110 111 101 001 101 001 010



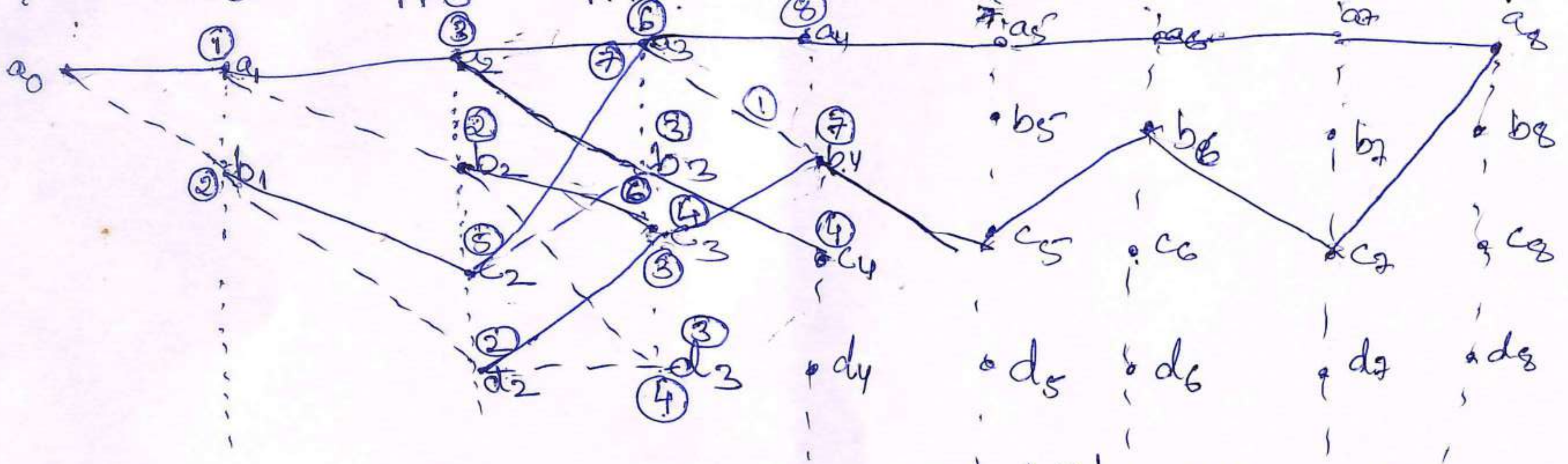


$x_1 = m.$
$x_2 = m \oplus m_2$
$x_3 = m \oplus m_1$



Viterbi. algorithm to decode given sequence

Y 100 110 111 101 001 101 001 010



Y+E 111 110 011 101 001 101 ~~010~~ 001 010

Decoded message sequence 1 1 0 1 0 1 0 0

Surviving path = $a_0 b_1 d_2 c_3 b_4 c_5 b_6 c_7 a_8$
lowest running metric ③.

UNIT-IV

Baseband transmission & optimal Reception of digital Signal

Topics :- pulse Shaping for optimum transmissions, & baseband Signal Rx, probability of error, optimum Receiver, optimal of coherent Reception, Signal Space representation & probability of error, eye diagrams, cross talk.

I. Pulse Shaping for optimum transmission:-

Optimum transmission is nothing but nyquist criterion for distortionless baseband binary transmission.

Nyquist pulse Shaping Criterion:-

(i) In time domain:- If the received pulse p(t) satisfies the below condition then, we get a Siegnal which is free from ISI.

$$P\left[(i-k)T_b\right] = \begin{cases} 1 \text{ for } \overset{i}{k}=k \\ 0 \text{ for } \overset{i}{k}\neq k \end{cases}$$

→ The above eqn is the condition in time domain.

2) In frequency domain:-

→ The frequency domain condition for zero ISI is

where $T_b \rightarrow$ bit period

$f_b \Rightarrow$ bit rate, $f_b = \frac{1}{T_b}$

$$\boxed{\sum_{n=-\infty}^{\infty} P(f-nf_b) = T_b}$$

The above eqn is called nyquist pulse shaping for base band transmission.

Nyquist bandwidth - (B_0)

The nyquist bandwidth is the minimum transmission bandwidth for zero ISI.

→ nyquist bandwidth B_0 is related to bit period T_b as

$$T_b = \frac{1}{2B_0}$$

$$B_0 = \frac{1}{2T_B} = \frac{f_b}{2}$$

$$\boxed{B_0 = \frac{\text{bit rate}}{2}}$$

Raised cosine channel: -

In freq domain: -

In the raised cosine spectrum, the frequency response P(f) decreases towards zero.

The raised cosine Spectrum is given as follows

$$P(f) = \begin{cases} \frac{1}{2B_0} & \rightarrow \text{for } -f_1 < f < f_1 \\ \frac{1}{4B_0}\left\{1 + \cos\left[\frac{\pi(|f| - f_1)}{2B_0 - 2f_1}\right]\right\} & \text{for } f_1 < |f| < 2B_0 - f_1 \\ 0 & \text{elsewhere.} \end{cases}$$

where $f_1 = B_0 - B_0\alpha$

B_0 = Nyquist B.w $= \frac{f_b}{2}$

α = roll-off factor

$$\alpha = 1 - \frac{f_1}{B_0}$$

In time domain: -

The inverse Fourier transform of raised cosine Spectrum gives the time domain pulse $p(t)$

$$p(t) = \text{sinc}(2B_0 T) \frac{\cos(2\pi\alpha B_0 t)}{1 - 16\alpha^2 - B_0^2 t^2}$$

B.W required for raised cosine channel

→ The B.W required for raised cosine ~~chan~~ Spectrum will be 'B'

$$B = 2B_0 - f_1 \quad —(1)$$

w.k.t $\alpha = 1 - \frac{f_1}{B_0}$

$\Rightarrow f_1 = B_0 - B_0\alpha$

Substitute in eqn (1)

$B = 2B_0 - B_0 + B_0\alpha$

$$\boxed{B = B_0(1+\alpha)}$$

Prob:- The o/p of a digital computer is a rate of 64kbps. If the roll off factors (i) $\alpha=1$ (ii) $\alpha=0.5$ (iii) $\alpha=0.25$ (iv) $\alpha=0$. Find the b.w req'd to transmit data in each case.

Sol:- $f_b = 64$ kbps.

$B_0 = \frac{f_b}{2} = 32$ kbps.

(i) $B = 64$ KHz

(ii) $B = 48$ KHz

(iii) $B = 40$ KHz

(iv) $B = B_0 = 32$ KHz

→ If $\alpha=0$, $B=B_0$ i.e raised cosine B.W = nyquist B.W

→ α↑, B↑ above nyquist B.W B_0

→ If $\alpha=1$, $B=2B_0$.

A base band Signal receiver:-

→ The ckt diagram of a simple base band signal receiver for the detection of digital signal is as shown in fig.

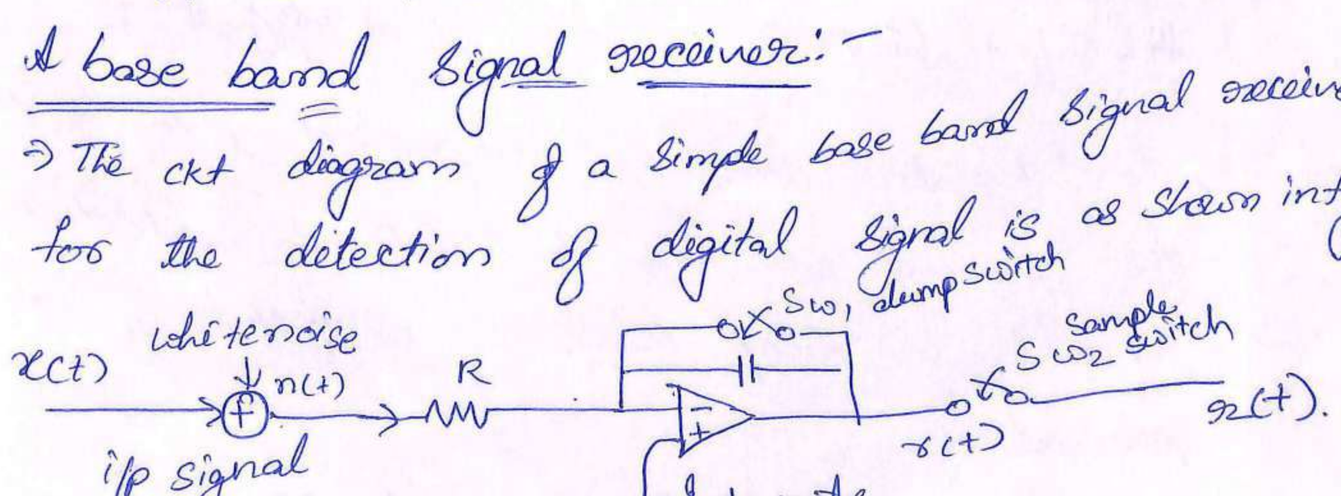


fig:- Base band Signal Rx.

→ The digital signal $x(t)$ is corrupted by white noise $n(t)$ during transmission over the channel.

→ Such noise signal $[x(t)+n(t)]$ is given to the i/p of Integrate & dump filter.

→ The capacitor is discharged fully at the begining of the bit interval. This is acheived by temporarily closing switch sw, at the begining of bit interval.

→ The integrator then integrates noisy i/p signal over one bit period. we get $r(t)$

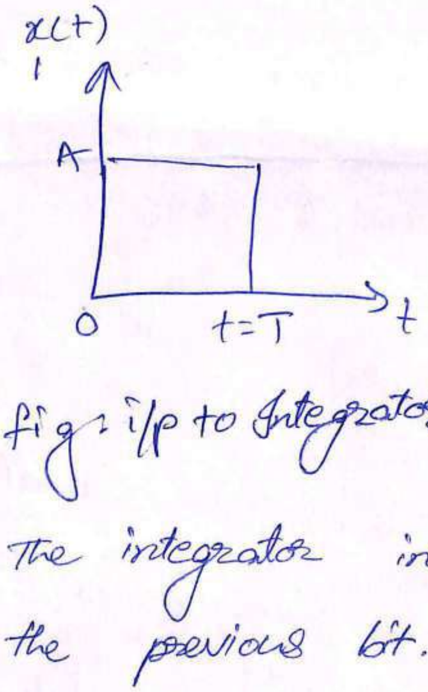


fig: i/p to Integrator

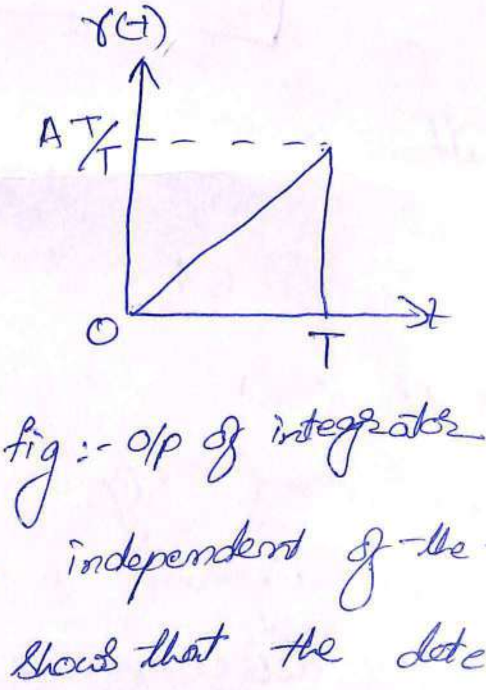


fig:- o/p of integrator

→ The integrator integrates independent of the the value of the previous bit. This shows that the detection in integrate & dump filter is unaffected by values of previous bit.

→ The o/p of integrator will decrease after $t>T$.

calculation of signal power

o/p of integrator:- $r(t) = \int_0^T x(t)+n(t)dt$

$= \int_0^T x(t) + \int_0^T n(t)dt$

$= x_0(t)+n_0(t)$. consider the o/p signal voltage

$x_0(t) = \frac{1}{RC}\int_0^T x(t)dt$ since $x(t)=A$

$= \frac{1}{RC}\int_0^T A dt = \frac{AT}{RC} = \frac{AT}{\tau}$

o/p signal power $= \frac{x_0^2(t)}{1\Omega} = \frac{A^2 T^2}{\tau^2}$

UNIT - V

Spread Spectrum modulation

→ Earlier we have been communication concerned ab

a, transmission bandwidth efficient utilization.

b, transmitted power.

→ There are some other applications where it is necessary t resist external interference & to make it difficult for unauthorized receivers to receive the messages being transmitted. This type of communication is called Sec communication.

→ Such communication is very important in military wh techniques called Spread Spectrum modulation is use

→ The Spread Spectrum modulation can be defined in two parts as follows:-

1, The transmitted data sequence occupies a much more bandwidth than the minimum required bandwidth &

2, The spectrum spreading (i.e increase of signal bandwidth) at the transmitter and desp at receiver is obtained by special code which is independent of data sequence (message signal)

Applications of Spread Spectrum modulation

1, Antijam capability - particularly for narrow band jammin

2, Interference rejection.

3, multiple access capability.

4, multipath protection.

5, low probability of intercept

6, Secure communication.

7, improved Spectral efficiency.

Classification of Spread Spectrum Systems

→ They are two types depending upon the operating concept

1, averaging type

2, avoidance type.

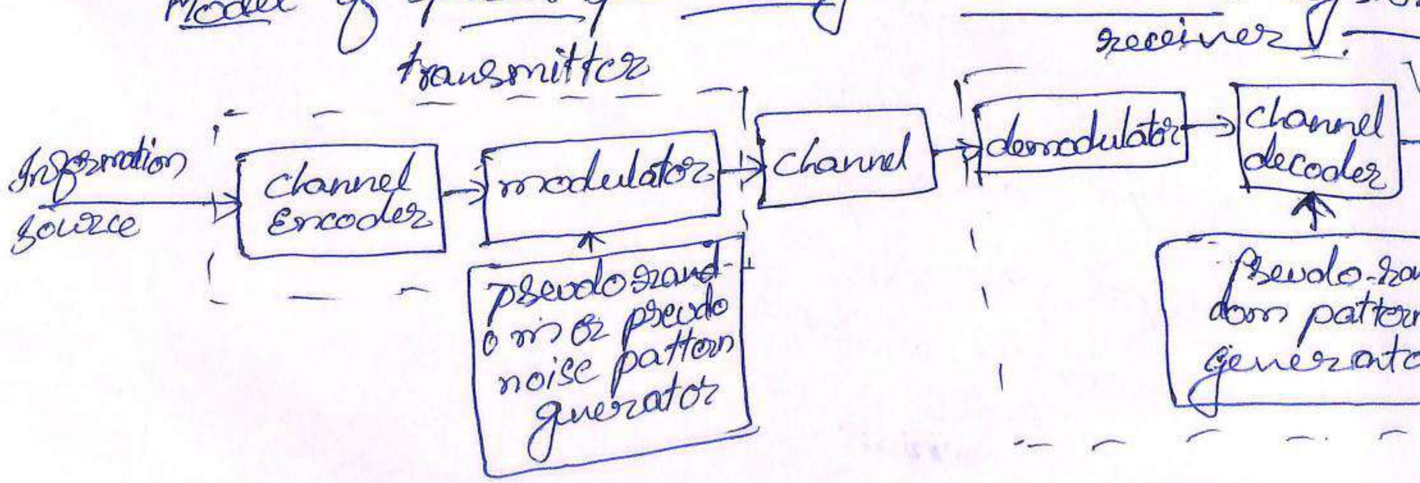
Averaging Systems :- In this Systems the interference is reduced by averaging it over a long period.

Avoidance Systems :- In these systems the interference is reduced by making the signal to avoid the interference a large fraction of time.

The Spread Spectrum techniques are also classified based on modulation techniques employed.

1, direct sequence → averaging
2, frequency hopping.
3, Time hopping.
4, chirp.
5, hybrid methods.
} avoidance.

Model of Spread Spectrum Digital communication System



Types of modulation.

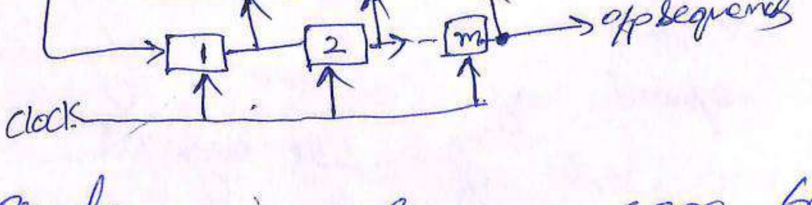
The pseudo noise sequence at the modulator is used with the PSK modulation to shift the phase of the PSK signal pseudo randomly. Such technique is called direct sequence spread spectrum (DSSS) modulation. & also called pseudonoise (PN) spread spectrum modulation.

when the pseudo noise sequence in modulate is used in conjuction with M-ary FSK to shift the frequency of FSK signal pseudo randomly. The technique is called frequency hopped (FH) spread spectrum method.

Generation of Pseudo-noise (PN) Sequences.

Definition :- The pseudo noise sequence is a noise like high frequency signal. This signal is binary in nature. Thus it looks like pulses. The sequence is not completely random but it is generated by a well defined logic. The same logic is used in tx & rx. Since the sequence is generated by well defined logic. It is called pseudo random sequence.

generation



The Pseudo-noise sequence can be generated by feedback shift register & combinational logic.

1, The Shift register consists of 'm' flipflops.

2, The o/p of flipflop are given to the logic ckt. Depending upon the o/p of F/f, the o/p of logic ckt is calculated & given as an i/p to first flip flop of Shift register.

3, The PN sequence is generated at o/p of last flip-flop in Shift register. At each pulse of clk, the state of flipflop is Shifted to the next flipflop & logic ckt o/p is Shifted in the first flipflop.

Period of o/p Sequence

→ The Pseudo-noise sequence generated at the o/p of the flipflop repeated after 2^m digit. This is bec the shift register will have 2^m states

→ Normally the logic ckt is mod-2 adders.

If Shift register enters in zero state, if it will not come out of it & o/p Sequence will ~~not repeat of~~ be zeros only. To prevent this the zero state of shift register is not allowed. ∴ the total of no. of states of m state Shift register will be 2^m-1. ∴ the o/p Pseu noise Sequence will have a period of 2^m-1 b

Maximum length Sequence

when the Pseudo-noise sequence generated by linear feedback shift register has length of 2^m-1 is called maximum length sequence.

Properties

1) **Balance property** :- The number of 1's is always one more than the number of zeros in each period of maximum length sequence.

2) **Run property** : The run means subsequence of identical symbols i.e 1's & 0's within one period of the sequence. The length of the run is equal to length sequence is 2^{m-1}.

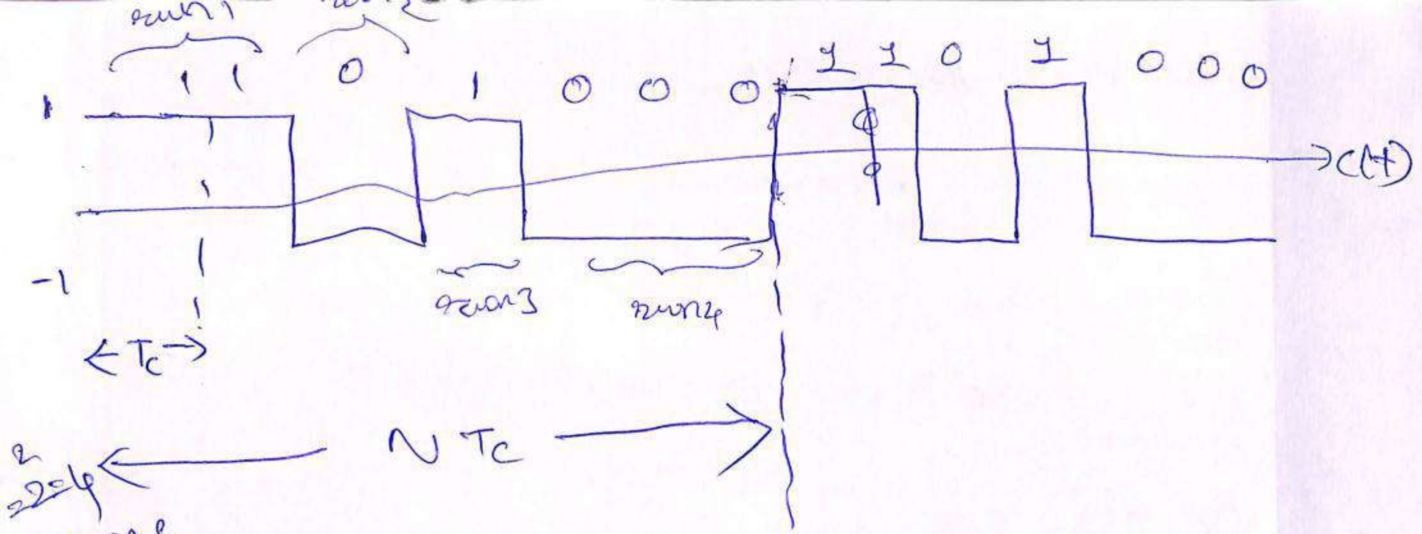
3) **Correlation property** :- The autocorrelation function of maximum length sequence is periodic & it is binary valued.

Chip rate :- Let the bits of maximum length sequence occur at rate R_c. This is also called chip rate in chips per second. Then duration of every ~~chi~~ bit is

$$T_c = 1/R_c$$

→ Let N represents the period of maximum length sequence.

$$N = 2^m - 1$$



$2^{3-1} = 2^2 = 4$

total 4 runs.

Hence the period of the waveform c(t) is given as

$$\boxed{T_b = NT_c}$$

T_b is the period of max length of sequence.

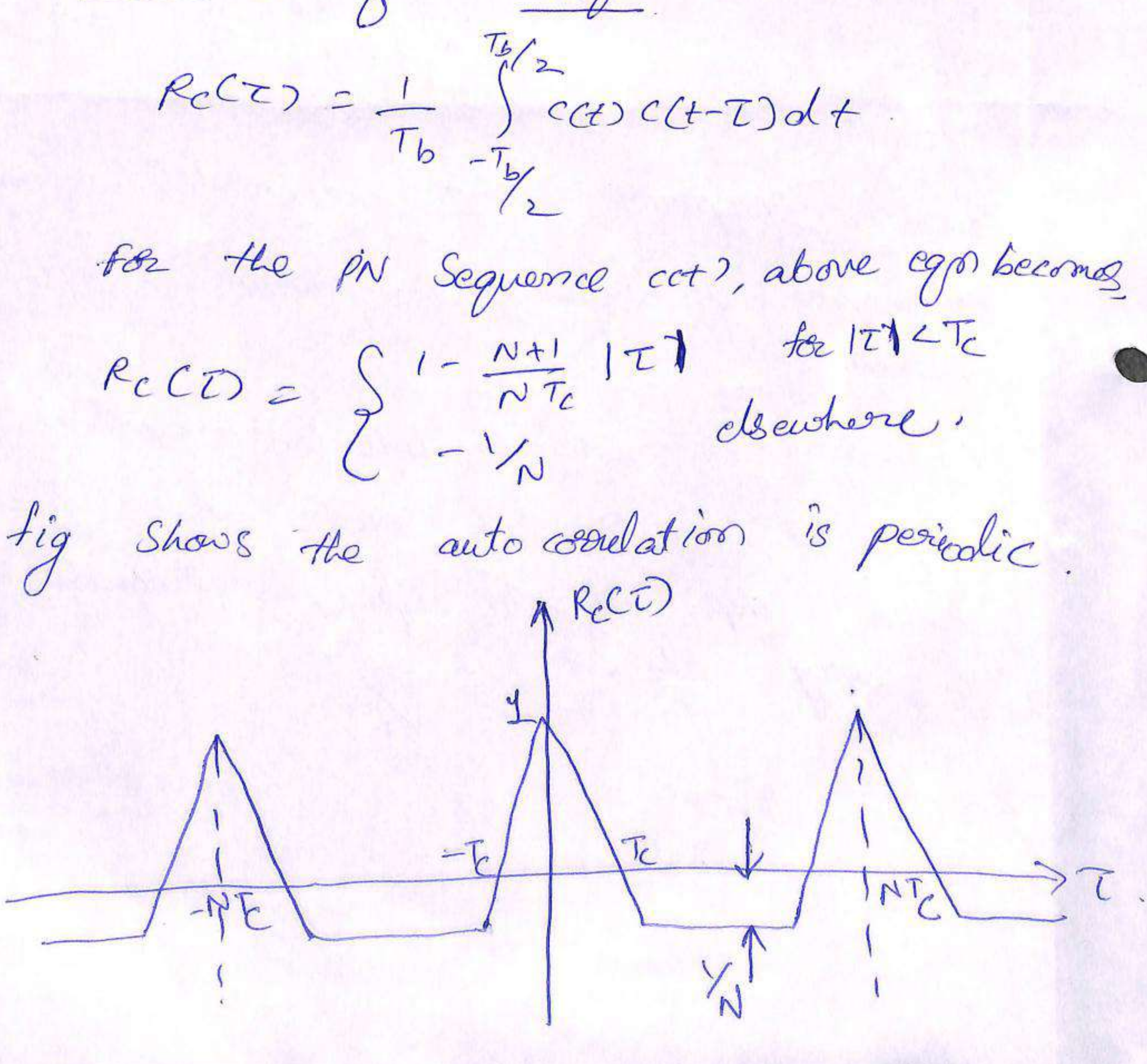
Autocorrelation of PN Sequence

$$R_c(\tau) = \frac{1}{T_b}\int_{-T_b/2}^{T_b/2} c(t)\,c(t-\tau)\,dt.$$

for the PN Sequence c(t), above eqn becomes,

$$R_c(\tau) = \begin{cases} 1 - \frac{N+1}{NT_c}|\tau| & \text{for } |\tau| < T_c \\ -1/N & \text{elsewhere.} \end{cases}$$

fig Shows the autocorrelation is periodic.

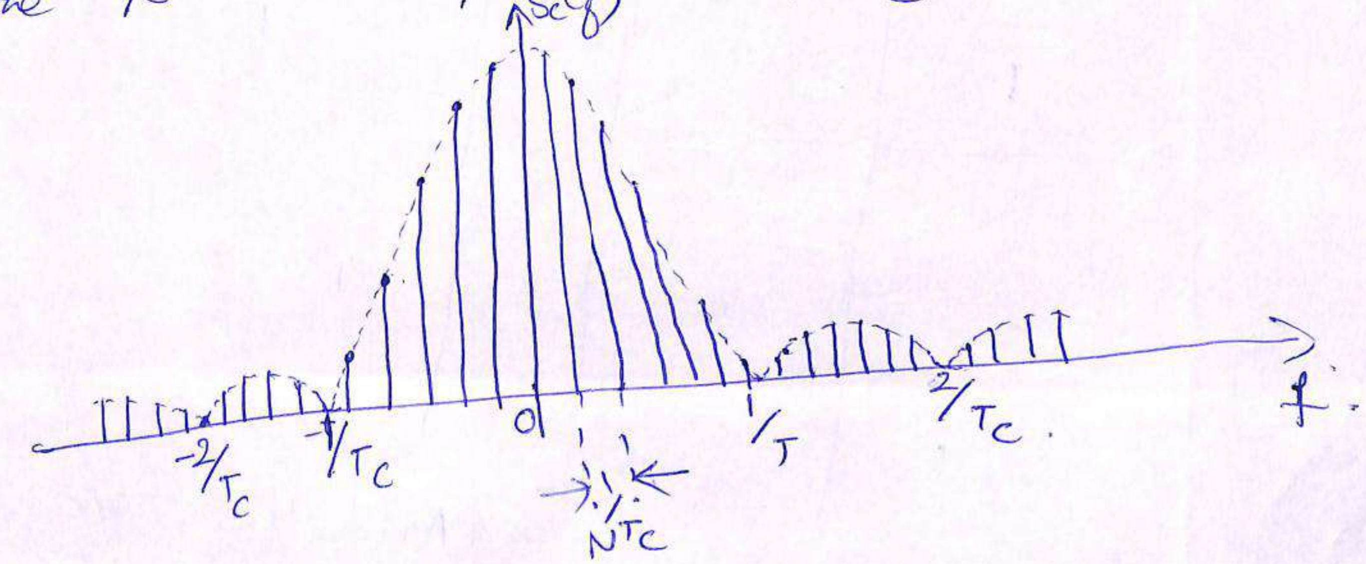


PSD of PN Sequence

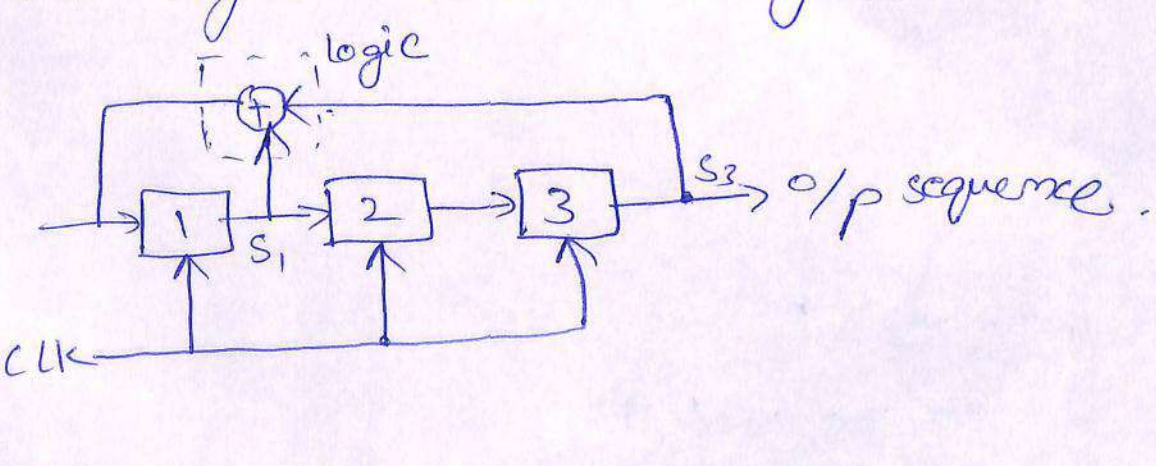
$$S_c(f) = F.T\left[R_c(\tau)\right]$$

$$= \frac{1}{N^2}\delta(f) + \frac{1+N}{N^2}\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \text{sinc}^2\left(\frac{n}{N}\right)\delta\left(f-\frac{n}{NT_c}\right)$$

since autocorrelation function is periodic in nature. its psd is discrete. The impulse functions in the psd are spaced at $1/NT_c$.



Explain the pseudo-noise sequence generated by feed back register shown in fig.



State of flip-flop			O/P PN Sequence equal to S_3
$S_1 = S_1 \oplus S_3$	S_2	S_3	S_3
1	0	0	0
$1 \oplus 0 = 1$	1	0	0
$1 \oplus 0 = 1$	1	1	1
0	[illegible]	1	1
1	0	1	1
0	1	0	0
0	0	1	1
1	0	0	0

And the sequence repeats.

31/3 Absentees III - 2b)

67, 70, 88, LE9

Direct Sequence Spread Spectrum with coherent BPSK

DS-SS BPSK Transmitter

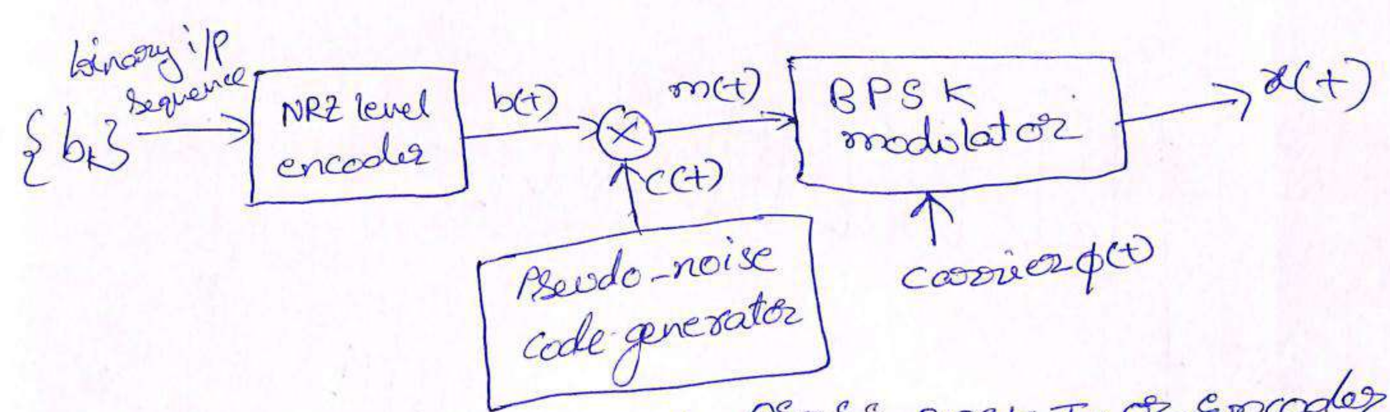


fig:- DS-SS BPSK Tx or Encoder

Operation

1) The binary data sequence is given to NRZ level encoder. This encoder converts b_K into bipolar NRZ waveform.

2) The Pseudo noise sequence generator generates & encodes this sequence in bipolar NRZ signal.

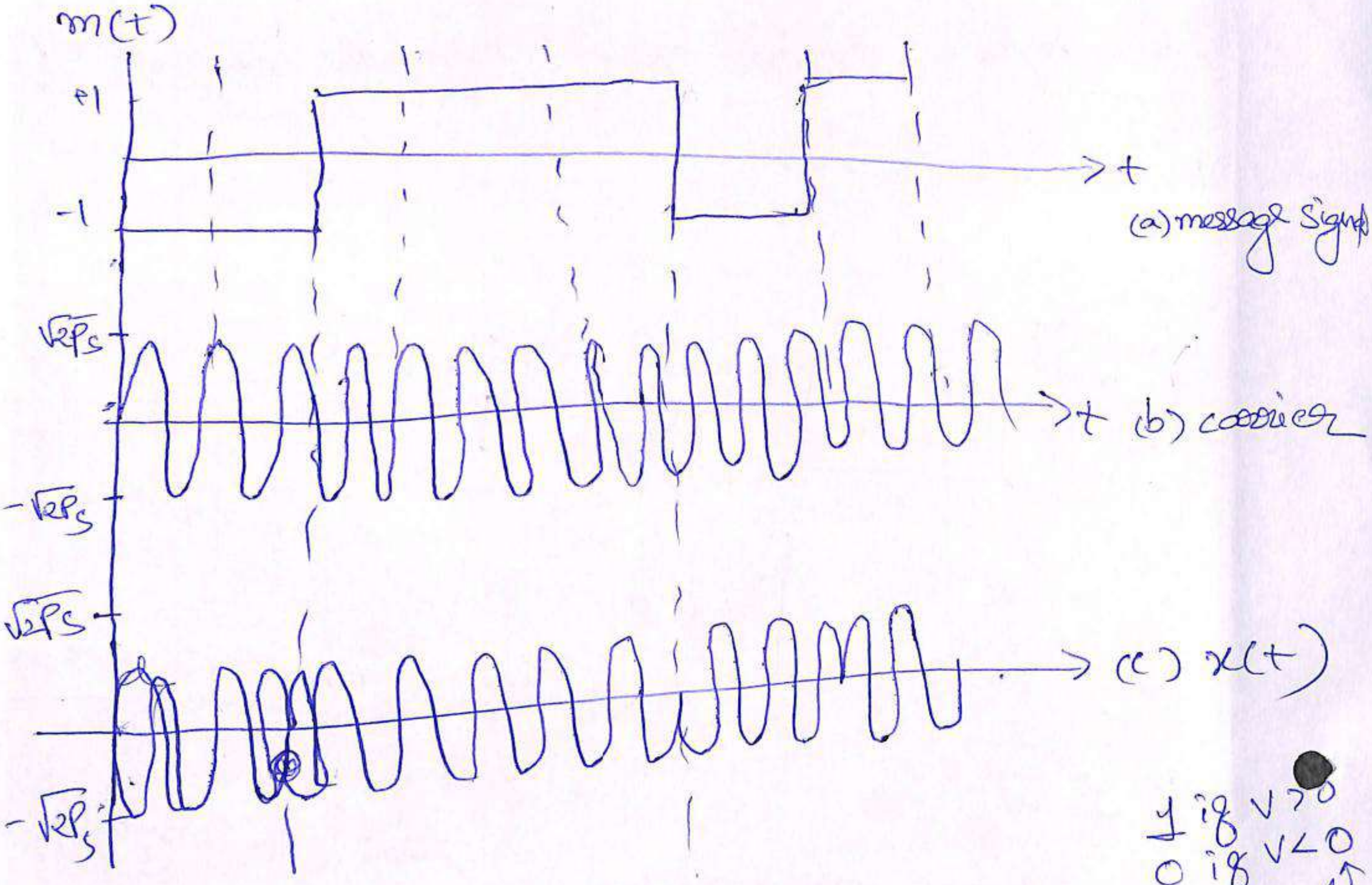
3) The multiplier multiplies $b(t)$ & $c(t)$. The o/p of multiplier is direct sequence spread signal $m(t)$.

4) This signal is given as modulating signal to BPSK transmitter. The direct sequence BPSK or (DS/BPSK) signal is generated as o/p i.e $x(t)$

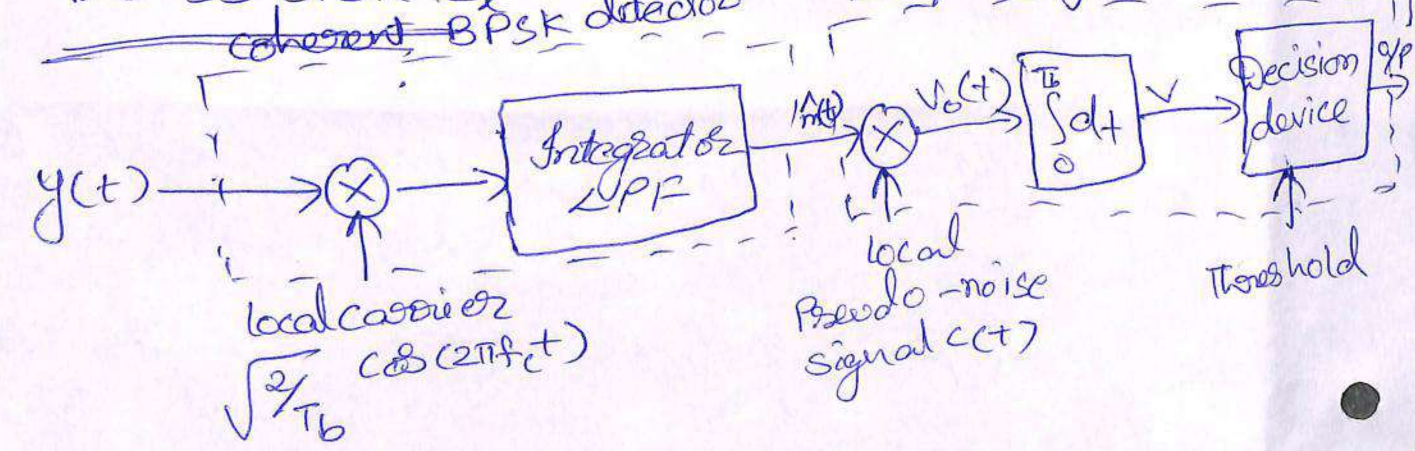
$$\phi(t) = \sqrt{2P_S}\,\sin(2\pi f_c t)$$

then $$x(t) = \sqrt{2P_S}\, m(t)\,\sin(2\pi f_c t)$$

Thus when $m(t)$ is positive, there is phase shift of '0' & if it is -ve, there is phase shift of 180.



DS-SS BPSK Rx ~~coherent BPSK detector~~



1) The received signal y(t) is applied to the multiplier which is also supplied with locally generated coherent carrier.

2) The o/p of multiplier is then applied to LPF
the B.W of LPF is equal to m(t).

3) The o/p signal is then applied to second demodulator which despreads the signal. The local pseudo noise is exact replica of that used in tx.

4) The integrator integrates the product of detected message Signal & Pseudonoise signal over a bit period T_b.

5) the decision is then taken depending upon the polarity of o/p V of integrator.

Performance of Direct Sequence spread Spectrum System

It can be evaluated ~~depe~~ on the basis of processing gain & probability of error.

1) Processing gain.

It is defined as ratio of the bandwidth of spread spectrum message signal to the bandwidth of unspreaded data signal.

i.e processing Gain = $\frac{BW(\text{spread signal})}{BW(\text{unspreaded signal})}$

B.W of unspreaded or data Signal.

$$B.W(\text{Data Signal}) = \frac{1}{\text{one bit period}} = \frac{1}{T_b}$$

B.W of spreaded signal

The spreading pseudo noise Signal $c(t)$ is multiplied by data Signal & spreaded message Signal $m(t)$ is produced. ~~Ob~~ The one bit period in message signal $m(t)$ is same as that in spreading Pseudo-noise Signal $c(t)$. This period is represented by T_c.

$$BW(\text{Spreaded message Signal}) = \frac{1}{\text{one bit period}} = \frac{1}{T_c}$$

Processing gain $= \frac{1/T_c}{1/T_b} = T_b/T_c$.

we know $T_b = NT_c$.

$\therefore$ Processing gain = N (Some times)

Probability of error of DS/BPSK System

wkt the P_E of BPSK

$$P(E) = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}$$

Here $N_0/2$ is noise spectraldensity

E_b is energy bit.

For Direct Sequence spread Spectrum modulation the noise spectral density is given as

$$\frac{N_0}{2} = \frac{JT_c}{2}$$

$$N_0 = JT_c$$

there J is the average interference power.

$$P(E) = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{JT_c}}$$

Jamming margin

w.k.T $\frac{E_b}{N_0} = \frac{P_S T_b}{N_0}$. putting $N_0 = JT_c$.

$$\frac{E_b}{N_0} = \frac{P_S T_b}{JT_c}$$

$$\frac{J}{P_S} = \frac{T_b}{T_c} \cdot 1/E_b/N_0$$

$$\boxed{\frac{J}{P_S} = \frac{PG}{E_b/N_0}} \quad \because \frac{T_b}{T_c} = PG$$

CDMA with direct sequence SS (SSMA)

In this application, many users transmit their signals on the same channel bandwidth. Each transmitter reciever pair has Pseudo-noise sequence. Thus signals of a particular transmitter are recieved by its intended reciever only, even if many users are transmitting at the same time. This method is also called Spread Spectrum multiple access. The signals from other users appear as additive interference which are rejected by spread spectrum decoder. The level of interference depends upon number of users transmitting at any time. The main advantage of CDMA is that the number of user sharing the same channel can be increased & decreased very easily. Large number of users can transmit on the same channel if their messages are for short periods of time. For this method it is desirable that the PN sequences be mutually orthogonal.

Frequency hop Spread Spectrum (FH-SS) Signals.

→ In the direct Sequence spread Spectrum modulation, the Pseudo-noise Sequence of large bandwidth is multiplied with the narrowband data signal. ∴ the o/p signal is spreaded over the complete o/p bandwidth at every instant.

→ But there is a limitation of physical devices which generates Pseudo-noise Sequence. Hence very large bandwidth are not possible with DSSS. To overcome this FHSS is used.

→ FH means to transmit the data bits in different frequency Slots. The total band width of the o/p Signal is equal to Sum of all these frequency Slots or "hops".

→ Types of ~~Fr~~ Frequency hopping.

1) Slow frequency hopping.
2) fast frequency hopping.

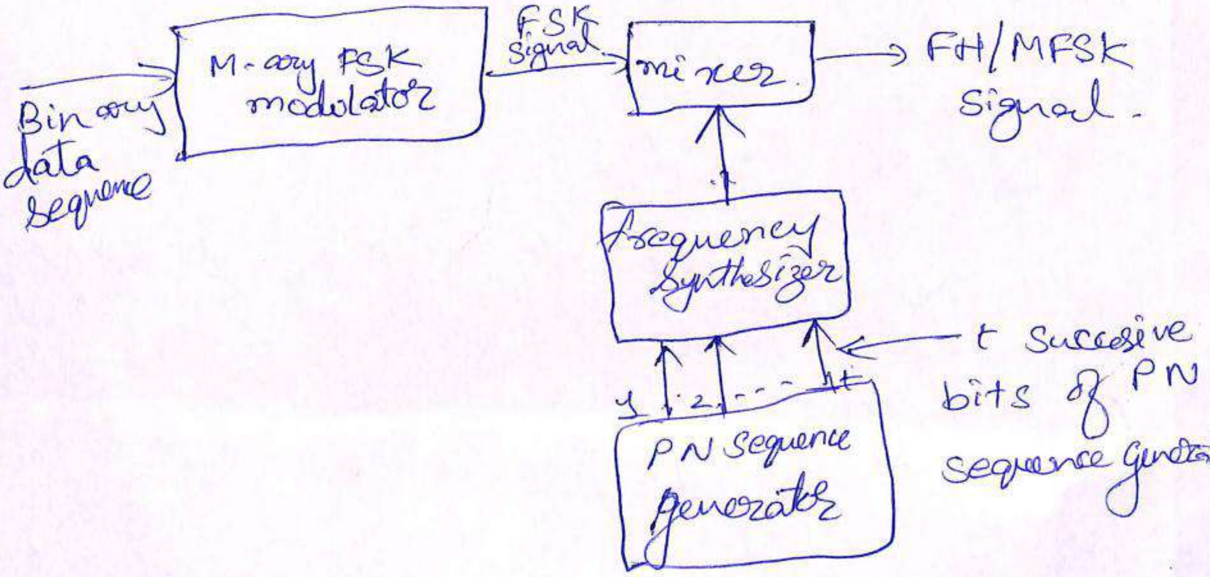
hop rate (R_H) :- The rate of change of frequency 'hops'

Symbol rate (R_S) :- The rate at which K-bit Symbols of data i/p Sequence are generated.

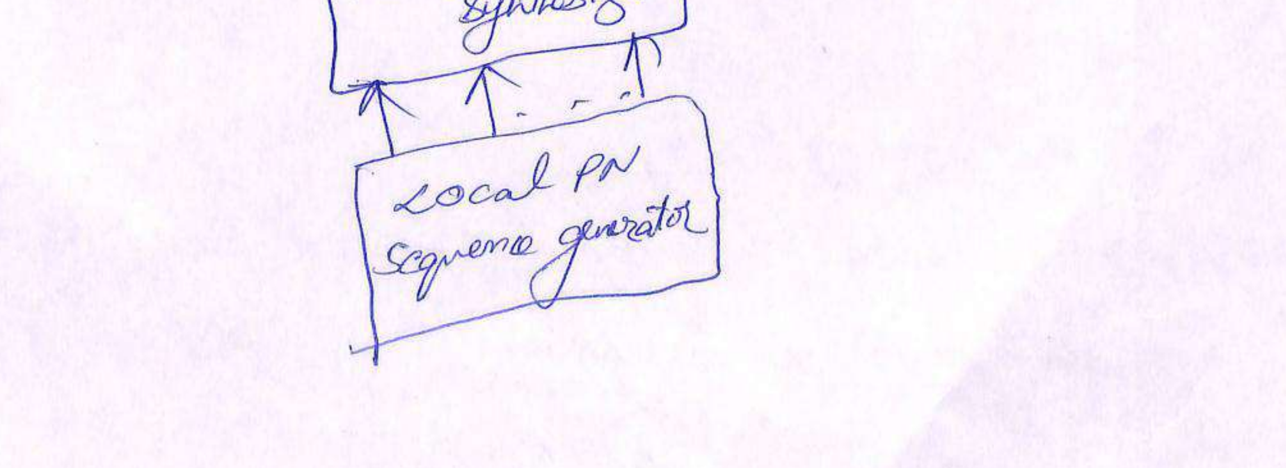
Slow frequency hopping

Definition:- when several symbols of data are transmitted in one frequency hop (slot) then it is slow frequency hopping. This means symbol rate is higher than hop rate. Here hop rate is slower, hence it is called slow f.h.

Tx of FH/MFSK



Reciever



Chip rate for Slow frequency hopping

→ In FH/MFSK the individual frequency of smallest duration is called 'chip'. In slow frequency hopping multiple Symbols are transmitted per hop.

→ With in the single hop very symbol will have independent frequency. Therefore this frequency is chip for slow frequency hopping.

→ In other words

$$R_c = R_s = \frac{R_b}{K}$$

Here R_b is i/p bit rate & 'K' no. of bits/symbol

Processing gain :-

→ Let f_s be Symbol frequency.

→ w.k.t 2^t frequency hops generated because of t bits of PN sequence

$$\therefore \; PG = \frac{B.\omega \text{ of Spreaded Signal}}{B.\omega \text{ of un Spreaded Signal}} = \frac{2^t f_s}{f_s} = 2^t$$

$$\boxed{\text{Here } f_s = R_s}$$

Fast frequency hopping.

Several frequency hops ⇒ one symbol transmitted

Chip rate for fast hopping

$$R_c = R_h$$

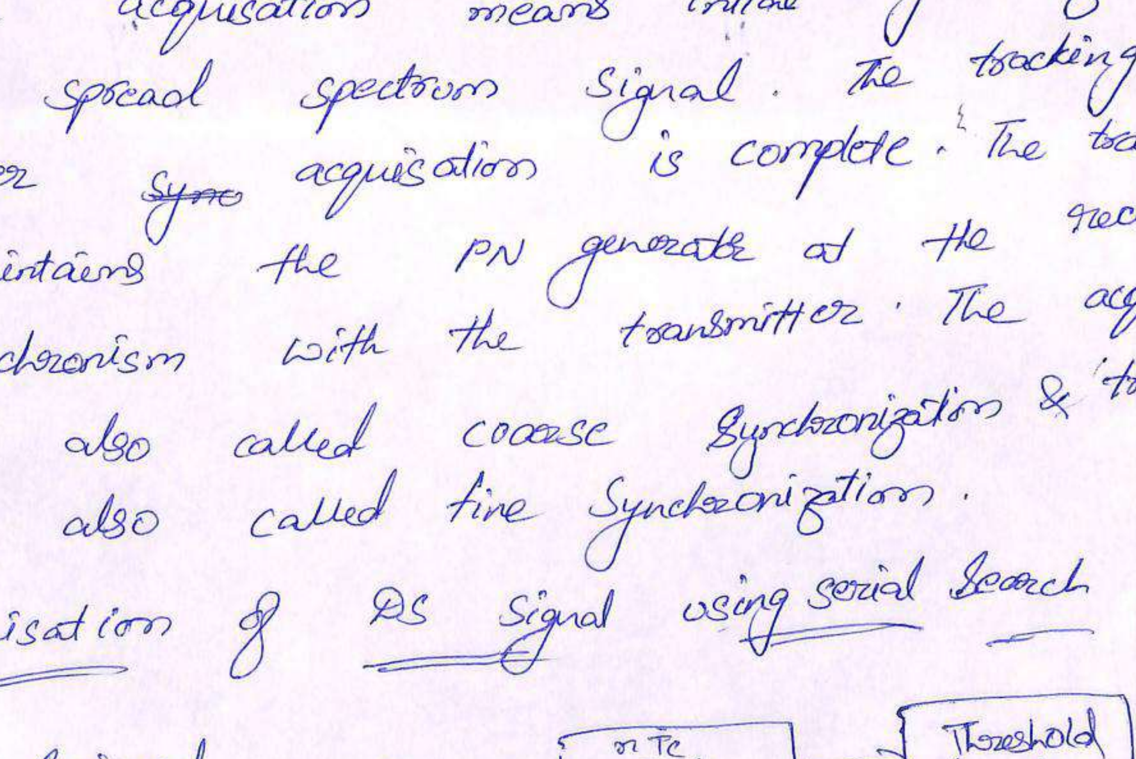
Synchronization in Spread Spectrum Systems

→ Spread spectrum systems are essentially synchronous. The PN sequences generated at rx & the tx must be same and locked to each other so that ~~so~~ the transmitted signal can be extracted.

→ The ~~Spr~~ Synchronization of the spread spectrum system can be considered in two parts Acquisation & tracking.

→ The acquisation means initial synchronization of spread spectrum signal. The tracking starts after ~~sync~~ acquisation is complete. The tracking maintains the PN generator at the reciever in synchronism with the transmitter. The acquisation is also called coarse synchronization & tracking is also called fine synchronization.

- Acquisation of DS signal using serial search



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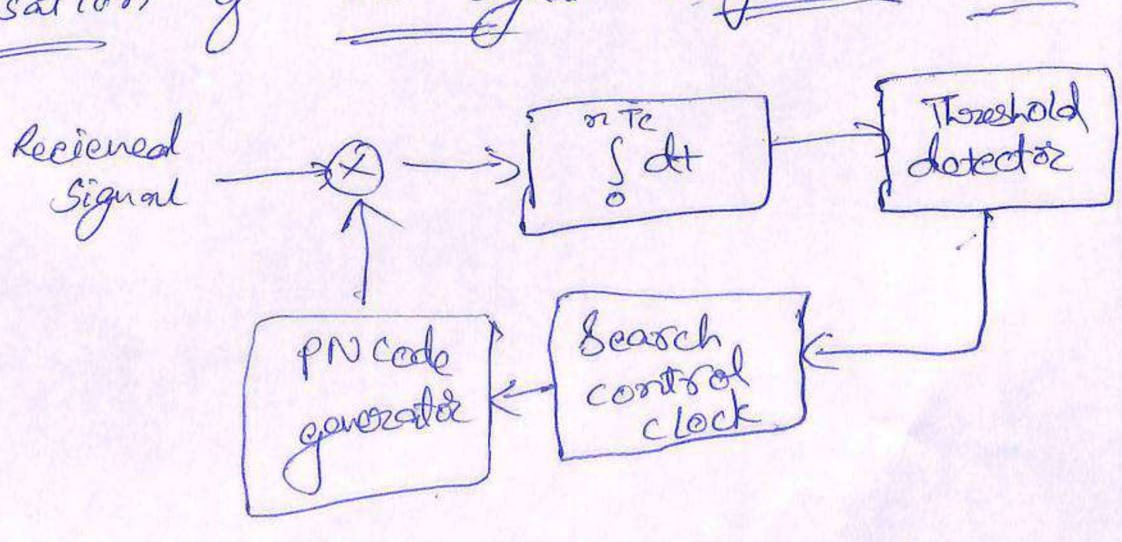

Acquisation of FH Signal using Serial search