LECTURE NOTES

ON

ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

III B.Tech II Sem (JNTU –R15)

Prepared by

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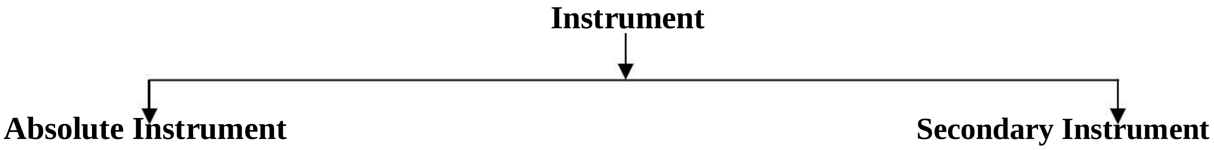
UNIT – I
INTRODUCTION TO MEASURING INSTRUMENTS

1.1 Definition of instruments

An instrument is a device in which we can determine the magnitude or value of the quantity to be measured. The measuring quantity can be voltage, current, power and energy etc. Generally instruments are classified in to two categories.



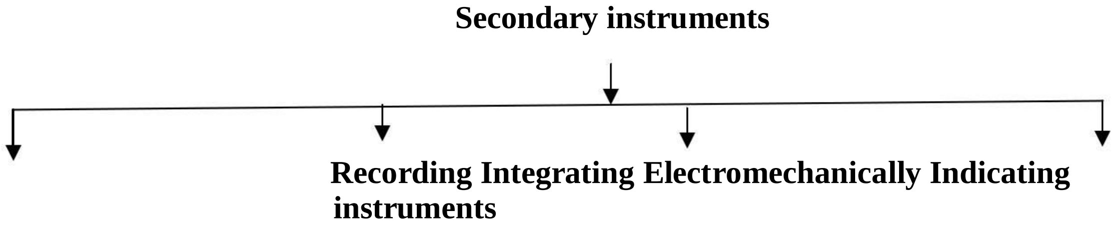
1.2 Absolute instrument

An absolute instrument determines the magnitude of the quantity to be measured in terms of the instrument parameter. This instrument is really used, because each time the value of the measuring quantities varies. So we have to calculate the magnitude of the measuring quantity, analytically which is time consuming. These types of instruments are suitable for laboratory use. Example: Tangent galvanometer.

1.3 Secondary instrument

This instrument determines the value of the quantity to be measured directly. Generally these instruments are calibrated by comparing with another standard secondary instrument.

Examples of such instruments are voltmeter, ammeter and wattmeter etc. Practically secondary instruments are suitable for measurement.



1.3.1 Indicating instrument

This instrument uses a dial and pointer to determine the value of measuring quantity. The pointer indication gives the magnitude of measuring quantity.

1.3.2 Recording instrument

This type of instruments records the magnitude of the quantity to be measured continuously over a specified period of time.

1.3.3 Integrating instrument

This type of instrument gives the total amount of the quantity to be measured over a specified period of time.

1.3.4 Electromechanical indicating instrument

For satisfactory operation electromechanical indicating instrument, three forces are necessary. They are

(a) Deflecting force

(b) Controlling force

(c)Damping force

1.4 Deflecting force

When there is no input signal to the instrument, the pointer will be at its zero position. To deflect the pointer from its zero position, a force is necessary which is known as deflecting force. A system which produces the deflecting force is known as a deflecting system. Generally a deflecting system converts an electrical signal to a mechanical force.

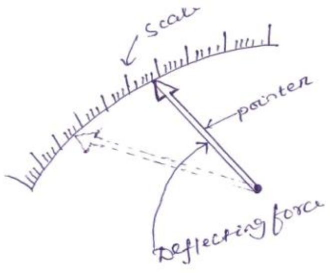


Fig. 1.1 Pointer scale

1.4.1 Magnitude effect

When a current passes through the coil (Fig.1.2), it produces a imaginary bar magnet. When a soft-iron piece is brought near this coil it is magnetized. Depending upon the current direction the poles are produced in such a way that there will be a force of attraction between the coil and the soft iron piece. This principle is used in moving iron attraction type instrument.

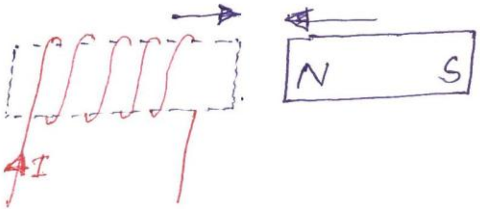


Fig. 1.2

If two soft iron pieces are place near a current carrying coil there will be a force of repulsion between the two soft iron pieces. This principle is utilized in the moving iron repulsion type instrument.

1.4.2 Force between a permanent magnet and a current carrying coil

When a current carrying coil is placed under the influence of magnetic field produced by a permanent magnet and a force is produced between them. This principle is utilized in the moving coil type instrument.

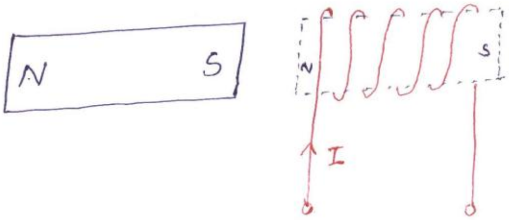


Fig. 1.3

1.4.3 Force between two current carrying coil

When two current carrying coils are placed closer to each other there will be a force of repulsion between them. If one coil is movable and other is fixed, the movable coil will move away from the fixed one. This principle is utilized in electrodynamometer type instrument.

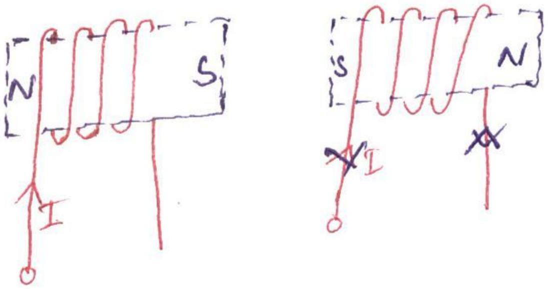


Fig. 1.4

1.5 Controlling force

To make the measurement indicated by the pointer definite (constant) a force is necessary which will be acting in the opposite direction to the deflecting force. This force is known as controlling force. A system which produces this force is known as a controlled system. When the external signal to be measured by the instrument is removed, the pointer should return back to the zero position. This is possibly due to the controlling force and the pointer will be indicating a steady value when the deflecting torque is equal to controlling torque.

$$T_d = T_c \tag{1.1}$$

1.5.1 Spring control

Two springs are attached on either end of spindle (Fig. 1.5).The spindle is placed in jewelled bearing, so that the frictional force between the pivot and spindle will be minimum. Two springs are provided in opposite direction to compensate the temperature error. The spring is made of phosphorous bronze.

When a current is supply, the pointer deflects due to rotation of the spindle. While spindle is rotate, the spring attached with the spindle will oppose the movements of the pointer. The torque produced by the spring is directly proportional to the pointer deflectionθ .

$$T_C \propto \theta \tag{1.2}$$

The deflecting torque produced T_d proportional to ‘I’. When $T_C = T_d$, the pointer will come to a steady position. Therefore

$$\theta \propto I \tag{1.3}$$

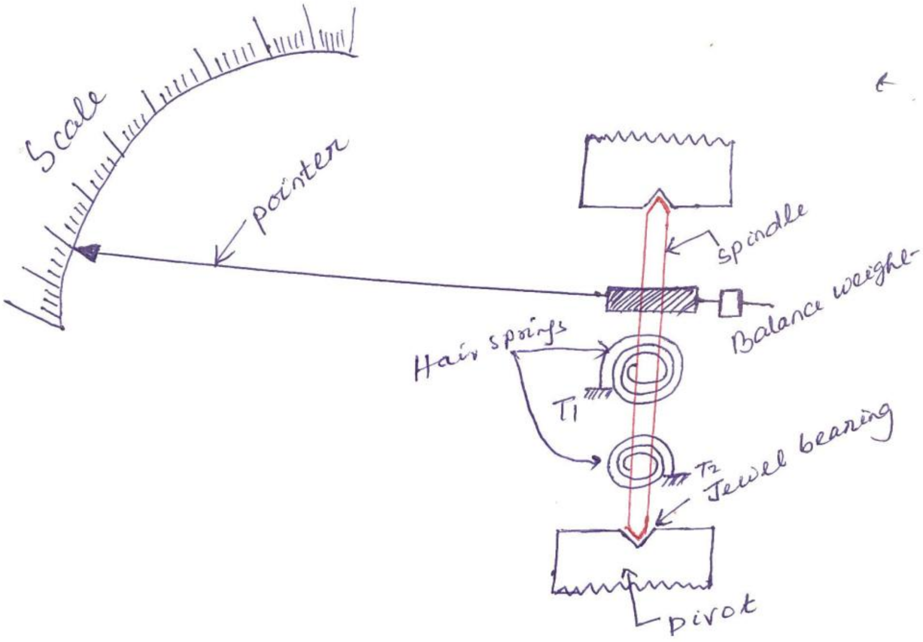


Fig. 1.5

Since, θ and I are directly proportional to the scale of such instrument which uses spring controlled is uniform.

1.6 Damping force

The deflection torque and controlling torque produced by systems are electro mechanical. Due to inertia produced by this system, the pointer oscillates about it final steady position before coming to rest. The time required to take the measurement is more. To damp out the oscillation is quickly, a damping force is necessary. This force is produced by different systems.

(a) Air friction damping
(b) Fluid friction damping
(c) Eddy current damping

1.6.1 Air friction damping

The piston is mechanically connected to a spindle through the connecting rod (Fig. 1.6). The pointer is fixed to the spindle moves over a calibrated dial. When the pointer oscillates in clockwise direction, the piston goes inside and the cylinder gets compressed. The air pushes the piston upwards and the pointer tends to move in anticlockwise direction.

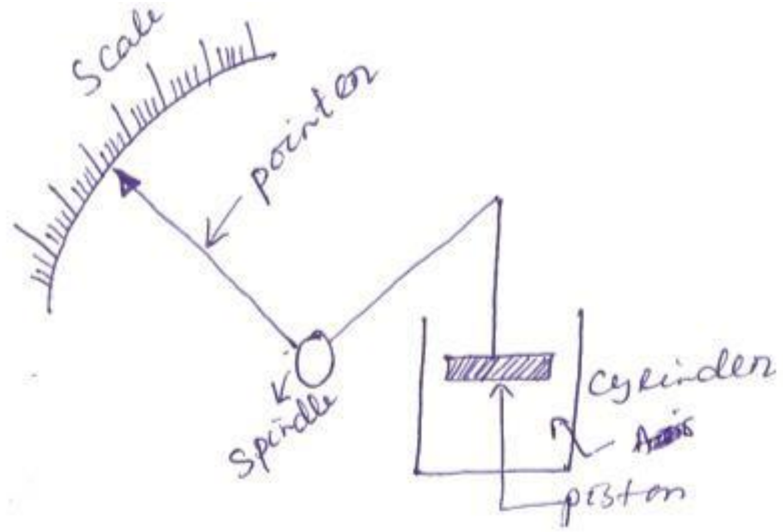


Fig. 1.6

If the pointer oscillates in anticlockwise direction the piston moves away and the pressure of the air inside cylinder gets reduced. The external pressure is more than that of the internal pressure. Therefore the piston moves down wards. The pointer tends to move in clock wise direction.

1.6.2 Eddy current damping

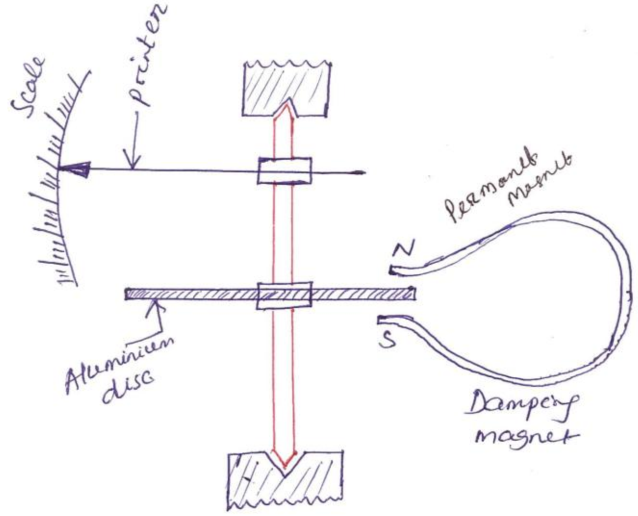


Fig. 1.6 Disc type

An aluminum circular disc is fixed to the spindle (Fig. 1.6). This disc is made to move in the magnetic field produced by a permanent magnet.

When the disc oscillates it cuts the magnetic flux produced by damping magnet. An emf is induced in the circular disc by faradays law. Eddy currents are established in the disc since it has several closed paths. By Lenz's law, the current carrying disc produced a force in a direction opposite to oscillating force. The damping force can be varied by varying the projection of the magnet over the circular disc.

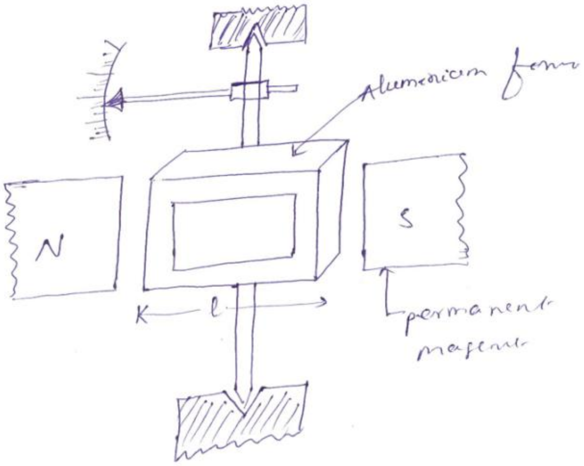


Fig. 1.6 Rectangular type

1.7 Permanent Magnet Moving Coil (PMMC) instrument

One of the most accurate type of instrument used for D.C. measurements is PMMC instrument.

Construction: A permanent magnet is used in this type instrument. Aluminum former is provided in the cylindrical in between two poles of the permanent magnet (Fig. 1.7). Coils are wound on the aluminum former which is connected with the spindle. This spindle is supported with jeweled bearing. Two springs are attached on either end of the spindle. The terminals of the moving coils are connected to the spring. Therefore the current flows through spring 1, moving coil and spring 2.

Damping: Eddy current damping is used. This is produced by aluminum former.

Control: Spring control is used.

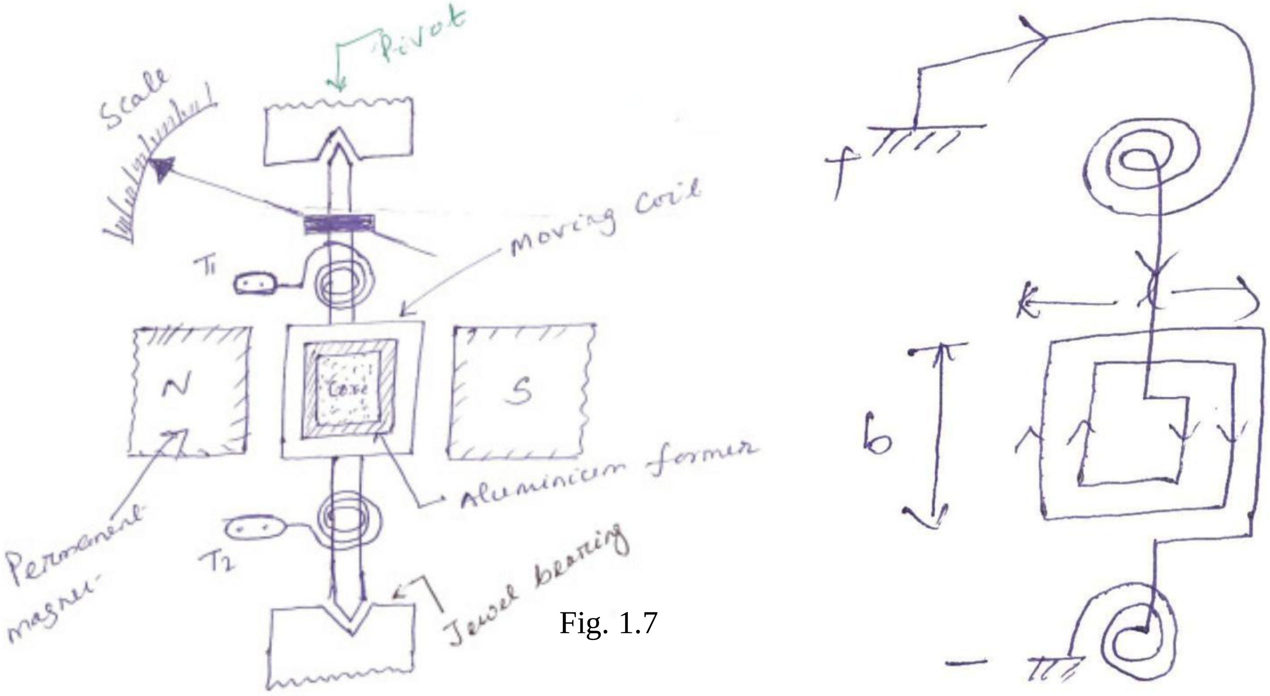


Fig. 1.7

Principle of operation

When D.C. supply is given to the moving coil, D.C. current flows through it. When the current carrying coil is kept in the magnetic field, it experiences a force. This force produces a torque and the former rotates. The pointer is attached with the spindle. When the former rotates, the pointer moves over the calibrated scale. When the polarity is reversed a torque is produced in the opposite direction. The mechanical stopper does not allow the deflection in the opposite direction. Therefore the polarity should be maintained with PMMC instrument.

If A.C. is supplied, a reversing torque is produced. This cannot produce a continuous deflection. Therefore this instrument cannot be used in A.C.

Torque developed by PMMC

Let T_d =deflecting torque

T_C = controlling torque

θ = angle of deflection

K=spring constant

b=width of the coil

l=height of the coil or length of coil

N=No. of turns

I=current

B=Flux density

A=area of the coil

The force produced in the coil is given by

$F = BIL \sin\theta$ (1.4)

When $\theta = 90^\circ$

For N turns, $F = NBIL$ (1.5)

Torque produced $T_d = F \times \perp r$ distance (1.6)

$T_d = NBIL \times b = BINA$ (1.7)

$T_d = BANI$ (1.8)

$T_d \propto I$ (1.9)

Advantages

- Torque/weight is high
- Power consumption is less
- Scale is uniform
- Damping is very effective
- Since operating field is very strong, the effect of stray field is negligible
- Range of instrument can be extended

Disadvantages

- Use only for D.C.
- Cost is high
- Error is produced due to ageing effect of PMMC
- Friction and temperature error is present

1.7.1 Extension of range of PMMC instrument

Case-I: Shunt

A low shunt resistance connected in parrel with the ammeter to extent the range of current. Large current can be measured using low current rated ammeter by using a shunt.

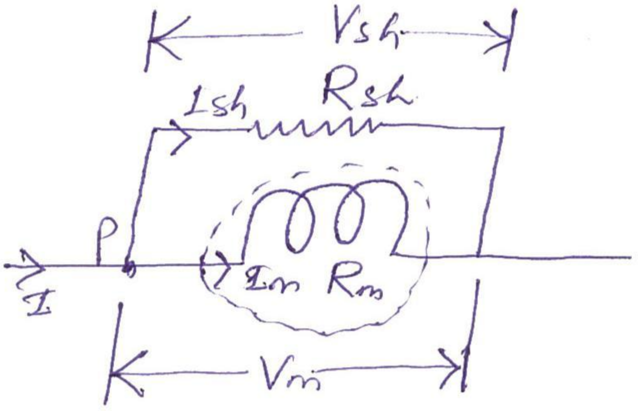


Fig. 1.8

Let R_m =Resistance of meter

R_{sh} =Resistance of shunt

I_m = Current through meter

I_{sh} =current through shunt

I= current to be measure

$$\therefore V_m = V_{sh} \tag{1.10}$$

$$I_m R_m = I_{sh} R_{sh}$$

$$\frac{I_m}{I_{sh}} = \frac{R_{sh}}{R_m} \tag{1.11}$$

Apply KCL at ‘P’ $I = I_m + I_{sh}$ (1.12)

Eq^n (1.12) ÷ by I_m

$$\frac{I}{I_m} = 1 + \frac{I_{sh}}{I_m} \tag{1.13}$$

$$\frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}} \tag{1.14}$$

$$\therefore I = I_m\left(1 + \frac{R_m}{R_{sh}}\right) \tag{1.15}$$

$1 + \frac{R_m}{R_{sh}}$ is called multiplication factor

Shunt resistance is made of manganin. This has least thermoelectric emf. The change is resistance, due to change in temperature is negligible.

Case (II): Multiplier

A large resistance is connected in series with voltmeter is called multiplier (Fig. 1.9). A large voltage can be measured using a voltmeter of small rating with a multiplier.

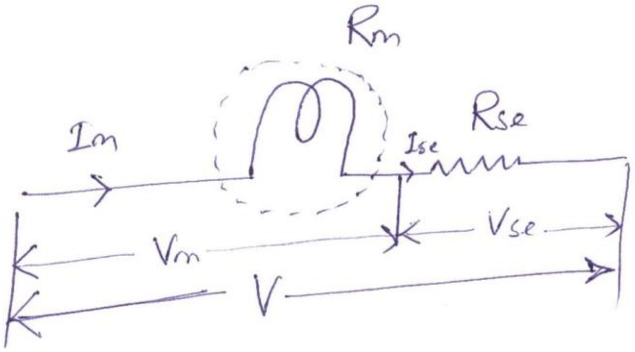


Fig. 1.9

Let R_m =resistance of meter

R_{se} =resistance of multiplier

V_m =Voltage across meter

V_{se} = Voltage across series resistance

V= voltage to be measured

$$I_m = I_{se} \tag{1.16}$$

$$\frac{V_m}{R_m} = \frac{V_{se}}{R_{se}} \tag{1.17}$$

$$\therefore \frac{V_{se}}{V_m} = \frac{R_{se}}{R_m} \tag{1.18}$$

Apply KVL, $V = V_m + V_{se}$ (1.19)

Eq^n (1.19) $\div V_m$

$$\frac{V}{V_m} = 1 + \frac{V_{se}}{V_m} = 1 + \frac{R_{se}}{R_m} \quad (1.20)$$

$$\therefore V = V_m \left(1 + \frac{R_{se}}{R_m}\right) \quad (1.21)$$

$1 + \frac{R_{se}}{R_m}$ → Multiplication factor

1.8 Moving Iron (MI) instruments

One of the most accurate instrument used for both AC and DC measurement is moving iron instrument. There are two types of moving iron instrument.

- Attraction type
- Repulsion type

1.8.1 Attraction type M.I. instrument

Construction: The moving iron fixed to the spindle is kept near the hollow fixed coil (Fig. 1.10). The pointer and balance weight are attached to the spindle, which is supported with jeweled bearing. Here air friction damping is used.

Principle of operation

The current to be measured is passed through the fixed coil. As the current is flow through the fixed coil, a magnetic field is produced. By magnetic induction the moving iron gets magnetized. The north pole of moving coil is attracted by the south pole of fixed coil. Thus the deflecting force is produced due to force of attraction. Since the moving iron is attached with the spindle, the spindle rotates and the pointer moves over the calibrated scale. But the force of attraction depends on the current flowing through the coil.

Torque developed by M.I

Let 'θ' be the deflection corresponding to a current of 'i' amp

Let the current increases by di, the corresponding deflection is '$\theta + d\theta$'

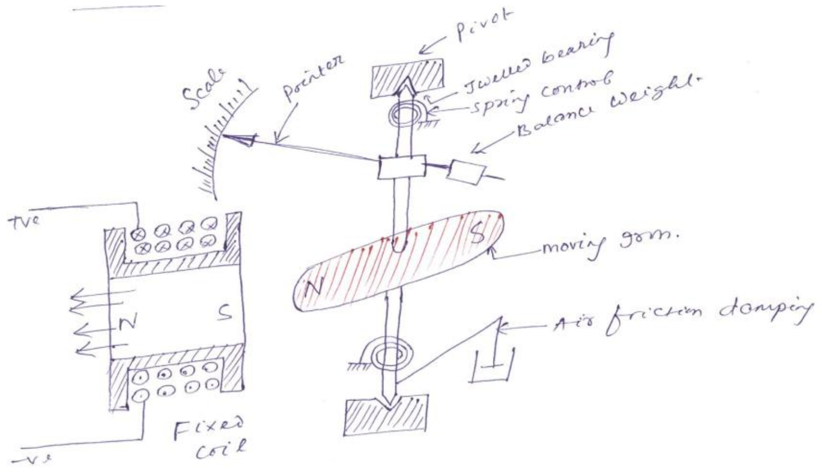


Fig. 1.10

There is change in inductance since the position of moving iron change w.r.t the fixed electromagnets.

Let the new inductance value be ‘L+dL’. The current change by ‘di’ is dt seconds.

Let the emf induced in the coil be ‘e’ volt.

$$e = \frac{d}{dt}(Li) = L\frac{di}{dt} + i\frac{dL}{dt} \tag{1.22}$$

Multiplying by ‘idt’ in equation (1.22)

$$e \times idt = L\frac{di}{dt} \times idt + i\frac{dL}{dt} \times idt \tag{1.23}$$

$$e \underset{n}{\times} idt = Lidi + i^{2}dL \tag{1.24}$$

Eq (1.24) gives the energy is used in to two forms. Part of energy is stored in the inductance.

Fig. 1.11

Change in energy stored=Final energy-initial energy stored

$$= \frac{1}{2}(L + dL)(i + di)^2 - \frac{1}{2} Li^2$$

$$= \frac{1}{2}\{(L + dL)(i^2 + di^2 + 2idi) - Li^2\}$$

$$= \frac{1}{2}\{(L + dL)(i^2 + 2idi) - Li^2\}$$

$$= \frac{1}{2}\{Li^2 + 2Lidi + i^2 dL + 2ididL - Li^2\}$$

$$= \frac{1}{2}\{2Lidi + i^2 dL\}$$

$$= Lidi + \frac{1}{2} i^2 dL \qquad (1.25)$$

Mechanical work to move the pointer by $d\theta$

$$= T_d \, d\theta \qquad (1.26)$$

By law of conservation of energy,

Electrical energy supplied=Increase in stored energy+ mechanical work done.

Input energy= Energy stored + Mechanical energy

$$Lidi + i^2 dL = Lidi + \frac{1}{2} i^2 dL + T_d \, d\theta \qquad (1.27)$$

$$\frac{1}{2} i^2 dL = T_d \, d\theta \qquad (1.28)$$

$$\frac{1}{2} i^2 \frac{dL}{d\theta} = T_d \qquad (1.29)$$

At steady state condition $T_d = T_C$

$$\frac{1}{2} i^2 \frac{dL}{d\theta} = K\theta \qquad (1.30)$$

$$\theta = \frac{1}{2K} i^2 \frac{dL}{d\theta} \qquad (1.31)$$

$$\theta \propto i^2 \qquad (1.32)$$

When the instruments measure AC, $\theta \propto i^2_{rms}$

Scale of the instrument is non uniform.

Advantages

MI can be used in AC and DC
It is cheap

Supply is given to a fixed coil, not in moving coil. Simple construction

Less friction error.

Disadvantages

It suffers from eddy current and hysteresis error Scale is not uniform

It consumed more power

Calibration is different for AC and DC operation

1.8.2 Repulsion type moving iron instrument

Construction: The repulsion type instrument has a hollow fixed iron attached to it (Fig. 1.12). The moving iron is connected to the spindle. The pointer is also attached to the spindle in supported with jeweled bearing.

Principle of operation: When the current flows through the coil, a magnetic field is produced by it. So both fixed iron and moving iron are magnetized with the same polarity, since they are kept in the same magnetic field. Similar poles of fixed and moving iron get repelled. Thus the deflecting torque is produced due to magnetic repulsion. Since moving iron is attached to spindle, the spindle will move. So that pointer moves over the calibrated scale. Damping: Air friction damping is used to reduce the oscillation.

Control: Spring control is used.

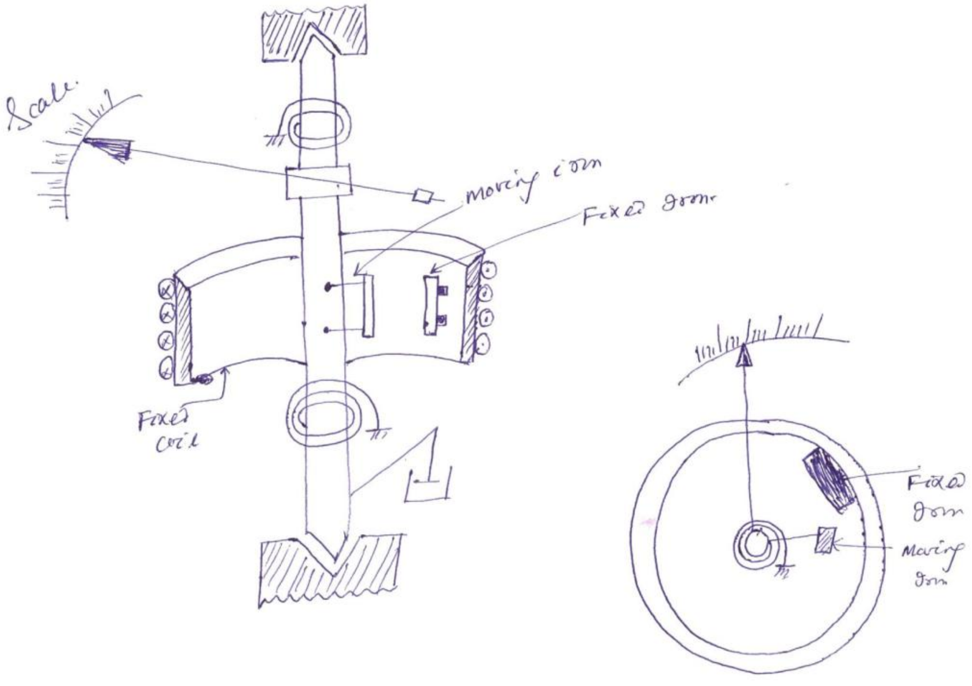


Fig. 1.12

1.9 Dynamometer (or) Electromagnetic moving coil instrument (EMMC)

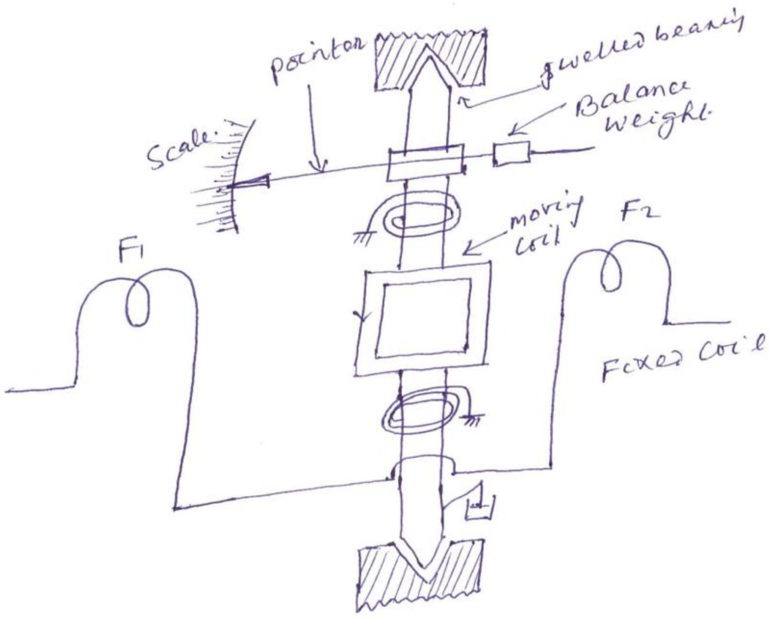


Fig. 1.13

This instrument can be used for the measurement of voltage, current and power. The difference between the PMMC and dynamometer type instrument is that the permanent magnet is replaced by an electromagnet.

Construction: A fixed coil is divided in to two equal half. The moving coil is placed between the two half of the fixed coil. Both the fixed and moving coils are air cored. So that the hysteresis effect will be zero. The pointer is attached with the spindle. In a non metallic former the moving coil is wounded.

Control: Spring control is used.

Damping: Air friction damping is used.

Principle of operation:

When the current flows through the fixed coil, it produced a magnetic field, whose flux density is proportional to the current through the fixed coil. The moving coil is kept in between the fixed coil. When the current passes through the moving coil, a magnetic field is produced by this coil.

The magnetic poles are produced in such a way that the torque produced on the moving coil deflects the pointer over the calibrated scale. This instrument works on AC and DC. When AC voltage is applied, alternating current flows through the fixed coil and moving coil. When the current in the fixed coil reverses, the current in the moving coil also reverses. Torque remains in the same direction. Since the current i_1 and i_2 reverse simultaneously. This is because the fixed and moving coils are either connected in series or parallel.

Torque developed by EMMC

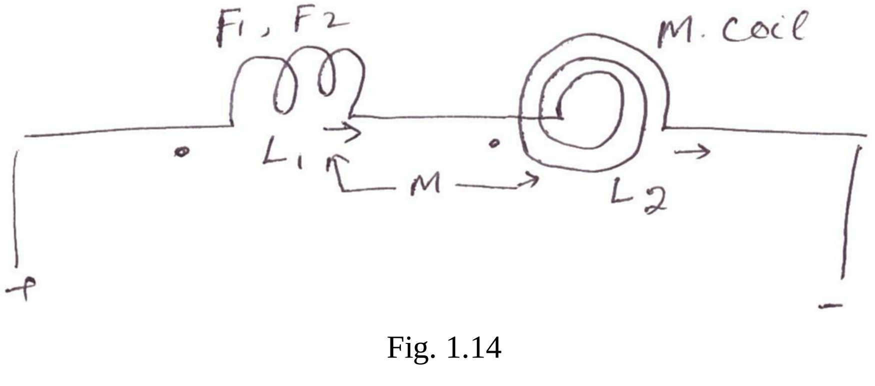


Fig. 1.14

Let

L_1=Self inductance of fixed coil

L_2= Self inductance of moving coil

M=mutual inductance between fixed coil and moving coil i_1=current through fixed coil

i_2=current through moving coil

Total inductance of system,

$$L_{total} = L_1 + L_2 + 2M \tag{1.33}$$

But we know that in case of M.I

$$T_d = \frac{1}{2} i^2 \frac{d(L)}{d\theta} \tag{1.34}$$

$$T_d = \frac{1}{2} i^2 \frac{d}{d\theta}(L_1 + L_2 + 2M) \tag{1.35}$$

The value of L_1 and L_2 are independent of 'θ' but 'M' varies with θ

$$T_d = \frac{1}{2} i^2 \times 2 \frac{dM}{d\theta} \tag{1.36}$$

$$T_d = i^2 \frac{dM}{d\theta} \tag{1.37}$$

If the coils are not connected in series $i_1 \neq i_2$

$$\therefore T_d = i_1 i_2 \frac{dM}{d\theta} \tag{1.38}$$

$$T_C = T_d \tag{1.39}$$

$$\therefore \theta = \frac{i_1 i_2}{K} \frac{dM}{d\theta} \tag{1.40}$$

Hence the deflection of pointer is proportional to the current passing through fixed coil and moving coil.

1.9.1 Extension of EMMC instrument

Case-I Ammeter connection

Fixed coil and moving coil are connected in parallel for ammeter connection. The coils are designed such that the resistance of each branch is same. Therefore

$I_1 = I_2 = I$

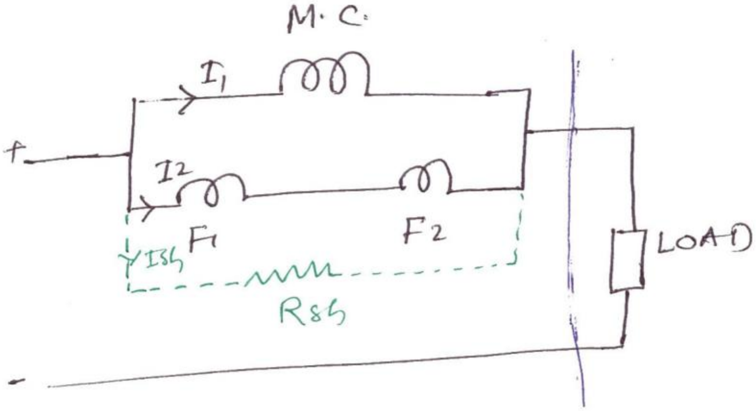


Fig. 1.15

To extend the range of current a shunt may be connected in parallel with the meter. The value R_{sh} is designed such that equal current flows through moving coil and fixed coil.

$$\therefore T_d = I_1 I_2 \frac{dM}{d\theta} \tag{1.41}$$

$$\text{Or } \therefore T_d = I^2 \frac{dM}{d\theta} \tag{1.42}$$

$$T_C = K\theta \tag{1.43}$$

$$\theta = \frac{I^2}{K} \frac{dM}{d\theta} \tag{1.44}$$

$$\therefore \theta \propto I^2 \text{ (Scale is not uniform)} \tag{1.45}$$

Case-II Voltmeter connection

Fixed coil and moving coil are connected in series for voltmeter connection. A multiplier may be connected in series to extent the range of voltmeter.

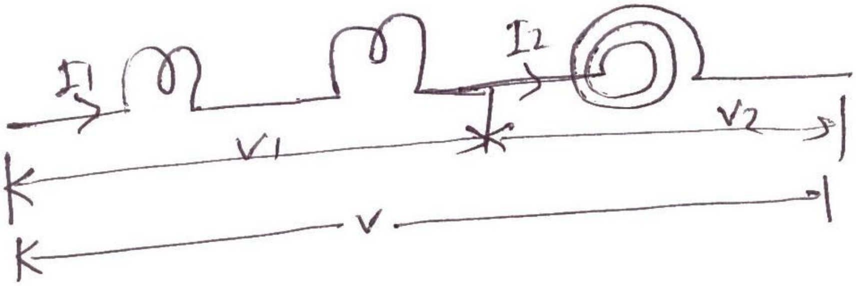


Fig. 1.16

$$I_1 = \frac{V_1}{Z_1}, I_2 = \frac{V_2}{Z_2} \tag{1.46}$$

$$T_d = \frac{V_1}{Z_1} \times \frac{V_2}{Z_2} \times \frac{dM}{d\theta} \tag{1.47}$$

$$T_d = \frac{K_1 V}{Z_1} \times \frac{K_2 V}{Z_2} \times \frac{dM}{d\theta} \tag{1.48}$$

$$T_d = \frac{KV^2}{Z_1 Z_2} \times \frac{dM}{d\theta} \tag{1.49}$$

$$T_d \propto V^2 \tag{1.50}$$

$$\therefore \theta \propto V^2 \quad \text{(Scale in not uniform)} \tag{1.51}$$

Case-III As w**attmeter**

When the two coils are connected to parallel, the instrument can be used as a wattmeter. Fixed coil is connected in series with the load. Moving coil is connected in parallel with the load. The moving coil is known as voltage coil or pressure coil and fixed coil is known as current coil.

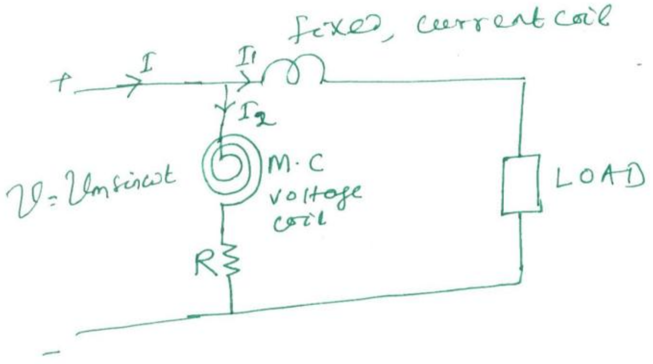


Fig. 1.17

Assume that the supply voltage is sinusoidal. If the impedance of the coil is neglected in comparison with the resistance 'R'. The current,

$$I_2 = \frac{V_m \sin wt}{R} \tag{1.52}$$

Let the phase difference between the currents I$_1$ and I$_2$ is φ

$$I_1 = I_m \sin(wt - \varphi) \tag{1.53}$$

$$T_d = I_1 I_2 \frac{dM}{d\theta} \tag{1.54}$$

$$T_d = I_m \sin(wt - \varphi) \times \frac{V_m \sin wt}{R} \frac{dM}{d\theta} \tag{1.55}$$

$$T_d = \frac{1}{R}(I_m V_m \sin wt \sin(wt - \varphi))\frac{dM}{d\theta} \tag{1.56}$$

$$T_d = \frac{1}{R} I_m V_m \sin wt.\sin(wt - \varphi)\frac{dM}{d\theta} \tag{1.57}$$

The average deflecting torque

$$(T_d)_{avg} = \frac{1}{2\Pi}\int_0^{2\Pi} T_d \times d(wt) \tag{1.58}$$

$$(T_d)_{avg} = \frac{1}{2\Pi}\int_0^{2\Pi} \frac{1}{R} \times I_m V_m \sin wt.\sin(wt - \varphi)\frac{dM}{d\theta} \times d(wt) \tag{1.59}$$

$$(T_d)_{avg} = \frac{V_m I_m}{2 \times 2\Pi} \times \frac{1}{R} \times \frac{dM}{d\theta}\int \{\cos\varphi - \cos(2wt - \varphi)\}dwt \tag{1.60}$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\Pi R} \times \frac{dM}{d\theta}\int_0^{2\Pi}\cos\varphi.dwt - \int_0^{2\Pi}\cos(2wt - \varphi).dwt \tag{1.61}$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\Pi R} \times \frac{dM}{d\theta}\left[\cos\varphi\,[wt]_0^{2\Pi}\right] \tag{1.62}$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\Pi R}\frac{dM}{d\theta}[\cos\varphi\,(2\Pi - 0)] \tag{1.63}$$

$$(T_d)_{avg} = \frac{V_m I_m}{2} \times \frac{1}{R} \times \frac{dM}{d\theta} \times \cos\varphi \tag{1.64}$$

$$(T_d)_{avg} = V_{rms} \times I_{rms} \times \cos\varphi \times \frac{1}{R} \times \frac{dM}{d\theta} \tag{1.65}$$

$(T_d)_{avg} \propto KVI \cos\varphi$ (1.66)

$T_C \propto \theta$ (1.67)

$\theta \propto KVI \cos\varphi$ (1.68)

$\theta \propto VI \cos\varphi$ (1.69)

Advantages

It can be used for voltmeter, ammeter and wattmeter

Hysteresis error is nill

Eddy current error is nill

Damping is effective

It can be measure correctively and accurately the rms value of the voltage

Disadvantages

Scale is not uniform

Power consumption is high(because of high resistance)

Cost is more

Error is produced due to frequency, temperature and stray field.

Torque/weight is low.(Because field strength is very low)

Errors in PMMC

The permanent magnet produced error due to ageing effect. By heat treatment, this error can be eliminated.

The spring produces error due to ageing effect. By heat treating the spring the error can be eliminated.

When the temperature changes, the resistance of the coil vary and the spring also produces error in deflection. This error can be minimized by using a spring whose temperature co-efficient is very low.

1.10 Difference between attraction and repulsion type instrument

An attraction type instrument will usually have a lower inductance, compare to repulsion type instrument. But in other hand, repulsion type instruments are more suitable for economical production in manufacture and nearly uniform scale is more easily obtained. They are therefore much more common than attraction type.

1.11 Characteristics of meter

1.11.1 Full scale deflection current(I_{FSD})

The current required to bring the pointer to full-scale or extreme right side of the instrument is called full scale deflection current. It must be as small as possible. Typical value is between 2 $\propto$ A to 30mA.

1.11.2 Resistance of the coil(R_m)

This is ohmic resistance of the moving coil. It is due to ρ , L and A. For an ammeter this should be as small as possible.

1.11.3 Sensitivity of the meter(S)

$$S = \frac{1}{I_{FSD}} (\Omega / volt), \uparrow S = \frac{Z \uparrow}{V}$$

It is also called ohms/volt rating of the instrument. Larger the sensitivity of an instrument, more accurate is the instrument. It is measured in Ω/volt. When the sensitivity is high, the impedance of meter is high. Hence it draws less current and loading affect is negligible. It is also defend as one over full scale deflection current.

1.12 Error in M.I instrument

1.12.1 Temperature error

Due to temperature variation, the resistance of the coil varies. This affects the deflection of the instrument. The coil should be made of manganin, so that the resistance is almost constant.

1.12.2 Hysteresis error

Due to hysteresis affect the reading of the instrument will not be correct. When the current is decreasing, the flux produced will not decrease suddenly. Due to this the meter reads a higher value of current. Similarly when the current increases the meter reads a lower value of current. This produces error in deflection. This error can be eliminated using small iron parts with narrow hysteresis loop so that the demagnetization takes place very quickly.

1.12.3 Eddy current error

The eddy currents induced in the moving iron affect the deflection. This error can be reduced by increasing the resistance of the iron.

1.12.4 Stray field error

Since the operating field is weak, the effect of stray field is more. Due to this, error is produced in deflection. This can be eliminated by shielding the parts of the instrument.

1.12.5 Frequency error

When the frequency changes the reactance of the coil changes.

$$Z = \sqrt{(R_m + R_S)^2 + X_L^2} \qquad (1.70)$$

$$I = \frac{V}{Z} = \frac{V}{\sqrt{(R_m + R_S)^2 + X_L^2}} \qquad (1.71)$$

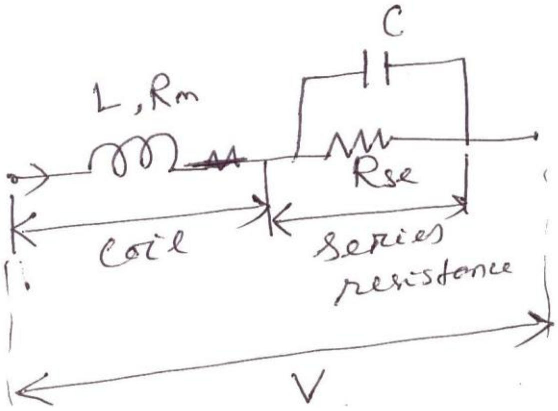


Fig. 1.18

Deflection of moving iron voltmeter depends upon the current through the coil. Therefore, deflection for a given voltage will be less at higher frequency than at low frequency. A capacitor is connected in parallel with multiplier resistance. The net reactance, ($X_L - X_C$) is very small, when compared to the series resistance. Thus the circuit impedance is made independent of frequency. This is because of the circuit is almost resistive.

$$C = 0.41 \frac{L}{(R_S)^2} \qquad (1.72)$$

1.13 Electrostatic instrument

In multi cellular construction several vans and quadrants are provided. The voltage is to be measured is applied between the vanes and quadrant. The force of attraction between the vanes and quadrant produces a deflecting torque. Controlling torque is produced by spring control. Air friction damping is used.

The instrument is generally used for measuring medium and high voltage. The voltage is reduced to low value by using capacitor potential divider. The force of attraction is proportional to the square of the voltage.

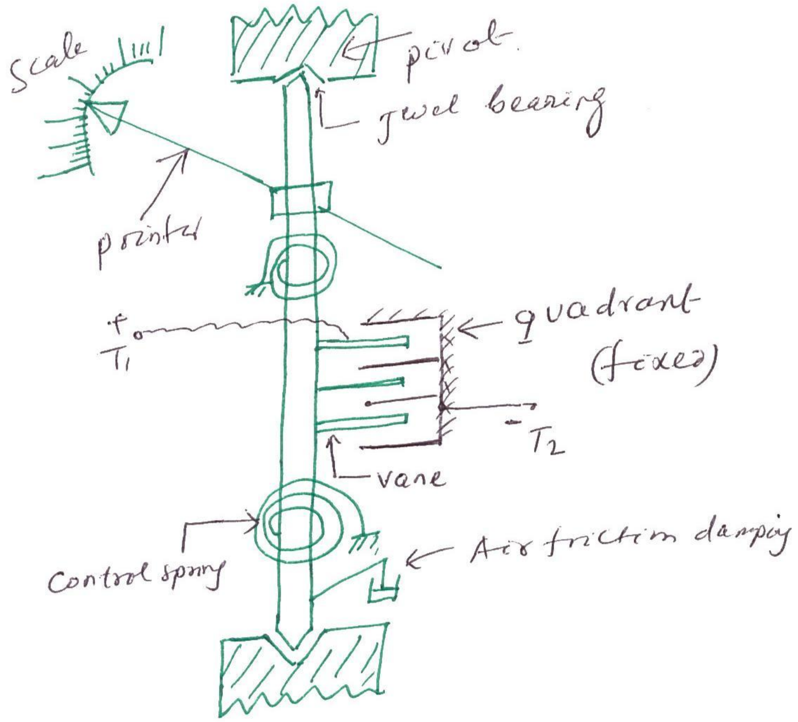


Fig. 1.19

<u>Torque develop by electrostatic instrument</u>

V=Voltage applied between vane and quadrant

C=capacitance between vane and quadrant
Energy stored=$-CV$
Let ‘θ’ be the deflection corresponding to a voltage V. (1.73)

Let the voltage increases by dv, the corresponding deflection is’$\theta + d\theta$’
When the voltage is being increased, a capacitive current flows

$$i = \frac{dq}{dt} = \frac{d(CV)}{dt} = \frac{dC}{dt} V + C\frac{dV}{dt} \quad (1.74)$$

$V \times dt$ multiply on both side of equation (1.74)

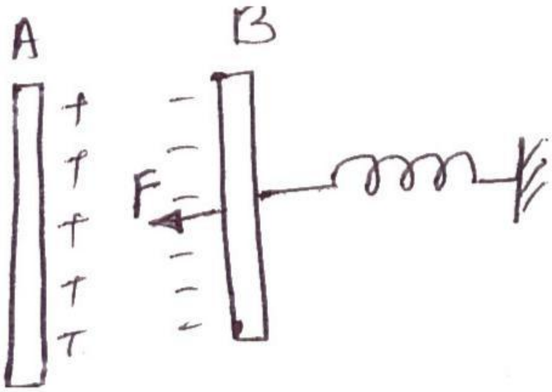


Fig. 1.20

$$Vidt = \frac{dC}{dt} V^2 dt + CV\frac{dV}{dt} dt \tag{1.75}$$

$$Vidt = V^2 dC + CVdV \tag{1.76}$$

Change in stored energy= $\frac{1}{2}(C + dC)(V + dV)^2 - \frac{1}{2}CV^2$ (1.77)

$$= \frac{1}{2}\left[(C + dC)V^2 + dV^2 + 2VdV\right] - \frac{1}{2}CV^2$$

$$= \frac{1}{2}\left[CV^2 + CdV^2 + 2CVdV + V^2 dC + dCdV^2 + 2VdVdC\right] - \frac{1}{2}CV^2$$

$$= \frac{1}{2}V^2 dC + CVdV$$

$$V^2 dC + CVdV = \frac{1}{2}V^2 dC + CVdV + F \times rd\theta \tag{1.78}$$

$$T_d \times d\theta = \frac{1}{2}V^2 dC \tag{1.79}$$

$$T_d = \frac{1}{2}V^2 \frac{dC}{d\theta} \tag{1.80}$$

At steady state condition, $T_d = T_C$

$$K\theta = \frac{1}{2}V^2 \frac{dC}{d\theta} \tag{1.81}$$

$$\theta = \frac{1}{2K}V^2 \frac{dC}{d\theta} \tag{1.82}$$

Advantages

It is used in both AC and DC.

There is no frequency error.

There is no hysteresis error.

There is no stray magnetic field error. Because the instrument works on electrostatic principle.

It is used for high voltage

Power consumption is negligible.

Disadvantages

Scale is not uniform

Large in size

Cost is more

1.14 Multi range Ammeter

When the switch is connected to position (1), the supplied current I_1

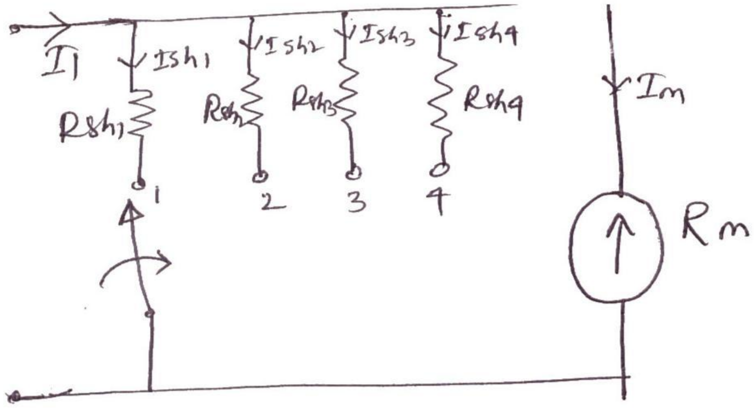


Fig. 1.21

$$I_{sh1} R_{sh1} = I_m R_m \tag{1.83}$$

$$R_{sh1} = \frac{I_m R_m}{I_{sh1}} = \frac{I_m R_m}{I_1 - I_m} \tag{1.84}$$

$$R_{sh1} = \frac{R_m}{\frac{I_1}{I_m} - 1}, \quad R_{sh1} = \frac{R_m}{m_1 - 1}, \quad m_1 = \frac{I_1}{I_m} = \text{Multiplying power of shunt}$$

$$R_{sh2} = \frac{R_m}{m_2 - 1}, \quad m_2 = \frac{I_2}{I_m} \tag{1.85}$$

$$R_{sh3} = \frac{R_m}{m_3 - 1}, \quad m_3 = \frac{I_3}{I_m} \tag{1.86}$$

$$R_{sh4} = \frac{R_m}{m_4 - 1}, \quad m_4 = \frac{I_4}{I_m} \tag{1.87}$$

1.15 Ayrton shunt

$$R_1 = R_{sh1} - R_{sh2} \tag{1.88}$$

$$R_2 = R_{sh2} - R_{sh3} \tag{1.89}$$

$$R_3 = R_{sh3} - R_{sh4} \tag{1.90}$$

$$R_4 = R_{sh4} \tag{1.91}$$

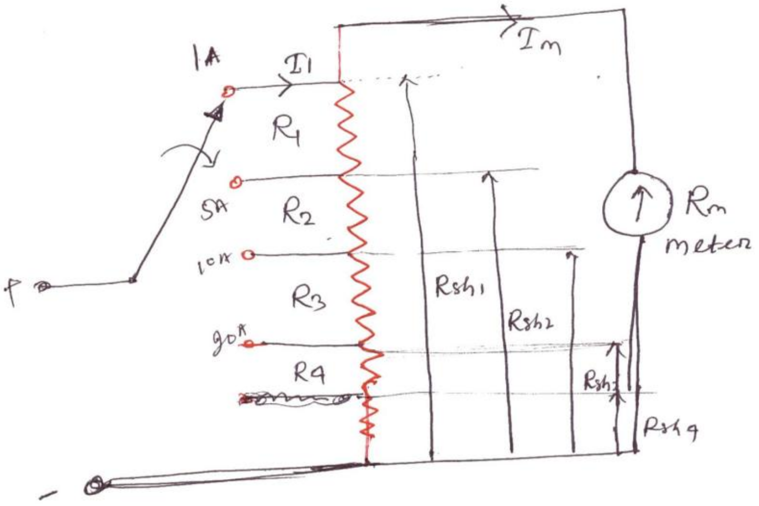


Fig. 1.22

Ayrton shunt is also called universal shunt. Ayrton shunt has more sections of resistance. Taps are brought out from various points of the resistor. The variable points in the o/p can be connected to any position. Various meters require different types of shunts. The Aryton shunt is used in the lab, so that any value of resistance between minimum and maximum specified can be used. It eliminates the possibility of having the meter in the circuit without a shunt.

1.16 Multi range D.C. voltmeter

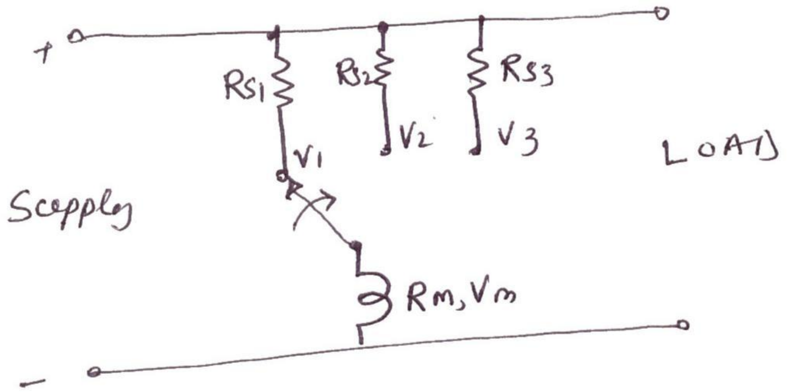


Fig. 1.23

$R_{s1} = R_m\,(m_1 - 1)$

$$R_{s2} = R_m\,(m_2 - 1) \qquad (1.92)$$

$R_{s3} = R_m\,(m_3 - 1)$

$$m_1 = \frac{V_1}{V_m},\; m_2 = \frac{V_2}{V_m},\; m_3 = \frac{V_3}{V_m} \qquad (1.93)$$

We can obtain different Voltage ranges by connecting different value of multiplier resistor in series with the meter. The number of these resistors is equal to the number of ranges required.

1.17 Potential divider arrangement

The resistance R_1, R_2 , R_3 and R_4 is connected in series to obtained the ranges V_1,V_2 ,V_3 and V_4

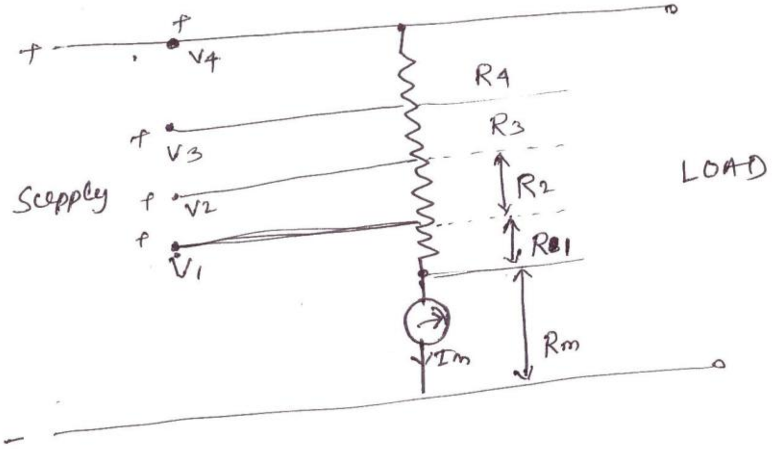


Fig. 1.24

Consider for voltage V1, $(R_1 + R_m)I_m = V_1$

$$\therefore R_1 = \frac{V_1}{I_m} - R_m = \frac{V_1}{\left(\frac{V_m}{R_m}\right)} - R_m - \frac{V_1}{V_m} R_m - R_m \qquad (1.94)$$

$$R_1 = (m_1 - 1)R_m \qquad (1.95)$$

For V2, $(R_2 + R_1 + R_m)I_m = V_2 \Rightarrow R_2 = \frac{V_2}{I_m} - R_1 - R_m$ (1.96)

$$R_2 = \frac{V_2}{\frac{V_m}{R_m}} - (m_1 - 1)R_m - R_m \qquad (1.97)$$

$$R_2 = m_2 R_m - R_m - (m_1 - 1)R_m$$

$$= R_m (m_2 - 1 - m_1 + 1) \qquad (1.98)$$

$$R_2 = (m_2 - m_1)R_m \qquad (1.99)$$

For V3 $(R_3 + R_2 + R_1 + R_m)I_m = V_3$

$$R_3 = \frac{V_3}{I_m} - R_2 - R_1 - R_m$$

$$= \frac{V_3}{V_m} R_m - (m_2 - m_1)R_m - (m_1 - 1)R_m - R_m$$

$$= m_3 R_m - (m_2 - m_1)R_m - (m_1 - 1)R_m - R_m$$

$$R_3 = (m_3 - m_2)R_m$$

For V_4 $(R_4 + R_3 + R_2 + R_1 + R_m) I_m = V_4$

$$R_4 = \frac{V_4}{I_m} - R_3 - R_2 - R_1 - R_m$$

$$= \frac{V_4}{v_m} R_m - (m_3 - m_2)R_m - (m_2 - m_1)R_m - (m_1 - 1)R_m - R_m$$

$$R_4 = R_m\left[m_4 - m_3 + m_2 - m_2 + m_1 - m_1 + 1 - 1\right]$$

$$R_4 = (m_4 - m_3)R_m$$

Example: 1.1

A PMMC ammeter has the following specification
Coil dimension are 1cm × 1cm. Spring constant is $0.15 \times 10^{-6}\ N - m / rad$, Flux density is $1.5 \times 10^{-3}\ wb / m^2$ .Determine the no. of turns required to produce a deflection of 90^0 when a current 2mA flows through the coil.

Solution:

At steady state **condition** $T_d = T_C$

$BANI = K\theta$

$$\Rightarrow N = \frac{K\theta}{BAI}$$

$A = 1 \times 10^{-4}\ m^2$

$K = 0.15 \times 10^{-6}\ \frac{N - m}{rad}$

$B = 1.5 \times 10^{-3}\ wb / m^2$

$I = 2 \times 10^{-3}\ A$

$\theta = 90^\circ = \frac{\Pi}{2}\ rad$

N=785 ans.

The pointer of a moving coil instrument gives full scale deflection of 20mA. The potential difference across the meter when carrying 20mA is 400mV.The instrument to be used is 200A for full scale deflection. Find the shunt resistance required to achieve this, if the instrument to be used as a voltmeter for full scale reading with 1000V. Find the series resistance to be connected it?

Solution:

Case-1

V_m =400mV

$I_m = 20mA$

I=200A

$$R_m = \frac{V_m}{I_m} = \frac{400}{20} = 20\Omega$$

$$I = I_m \left(1 + \frac{R_m}{R_{sh}}\right)$$

$$200 = 20 \times 10^{-3} \left(1 + \frac{20}{R_{sh}}\right)$$

$$R_{sh} = 2 \times 10^{-3}\ \Omega$$

Case-II

V=1000V

$$V = V_m \left(1 + \frac{R_{se}}{R_m}\right)$$

$$4000 = 400 \times 10^{-3} \left(1 + \frac{R_{se}}{20}\right)$$

$$R_{se} = 49.98k\Omega$$

Example: 1.2

The coil of a 600V M.I meter has an inductance of 1 henery. It gives correct reading at 50HZ and requires 100mA. For its full scale deflection, what is % error in the meter when connected to 200V D.C. by comparing with 200V A.C?

Solution:

$V_m = 600V, I_m = 100mA$

Case-I A.C.

$$Z_m = \frac{V_m}{I_m} = \frac{600}{0.1} = 6000\Omega$$

$$X_L = 2\Pi fL = 314\Omega$$

$$R_m = \sqrt{Z_m^2 - X_L^2} = \sqrt{(6000)^2 - (314)^2} = 5990\Omega$$

$$I_{AC} = \frac{V_{AC}}{Z} = \frac{200}{6000} = 33.33mA$$

Case-II D.C

$$I_{DC} = \frac{V_{DC}}{R_m} = \frac{200}{5990} = 33.39mA$$

$$\text{Error} = \frac{I_{DC} - I_{AC}}{I_{AC}} \times 100 = \frac{33.39 - 33.33}{33.33} \times 100 = 0.18\%$$

Example: 1.3

The relationship between inductance of moving iron ammeter, the current and the position of pointer is as follows:

Reading (A)	1.2	1.4	1.6	1.8
Deflection (degree)	36.5	49.5	61.5	74.5
Inductance ($\propto H$)	575.2	576.5	577.8	578.8

Calculate the deflecting torque and the spring constant when the current is 1.5A?

Solution:

For current I=1.5A, θ =55.5 degree=0.96865 rad

$$\frac{dL}{d\theta} = \frac{577.65 - 576.5}{60 - 49.5} = 0.11\mu H / \text{degree} = 6.3\mu H / rad$$

Deflecting torque , $T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} = \frac{1}{2}(1.5)^2 \times 6.3\times10^{-6} = 7.09\times10^{-6} N-m$

Spring constant, $K = \frac{T_d}{\theta} = \frac{7.09\times10^{-6}}{0.968} = 7.319\times10^{-6} \frac{N-m}{rad}$

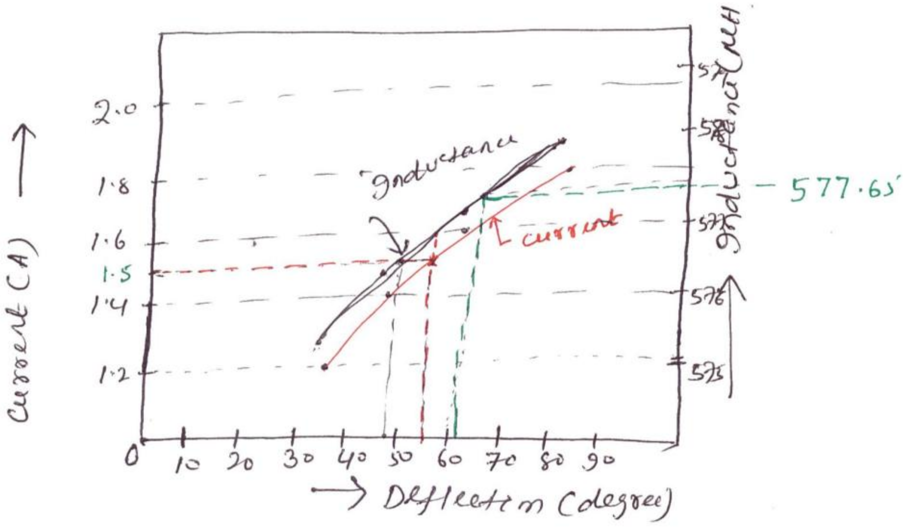


Fig. 1.25

Example: 1.4

For a certain dynamometer ammeter the mutual inductance 'M' varies with deflection θ as $M = -6\cos(\theta + 30^\circ)mH$ .Find the deflecting torque produced by a direct current of 50mA corresponding to a deflection of 60^0.

Solution:

$$T_d = I_1 I_2 \frac{dM}{d\theta} = I^2 \frac{dM}{d\theta}$$

$$M = -6\cos(\theta + 30^\circ)$$

$$\frac{dM}{d\theta} = 6\sin(\theta + 30) mH$$

$$\frac{dM}{d\theta}\Big|_{\theta=60} = 6\sin 90 = 6 mH/\deg$$

$$T_d = I^2 \frac{dM}{d\theta} = (50\times10^{-3})^2 \times 6\times10^{-3} = 15\times10^{-6} N-m$$

Example: 1.5

The inductance of a moving iron ammeter with a full scale deflection of 90^0 at 1.5A, is given by the expression $L = 200 + 40\theta - 4\theta^2 - \theta^3 \mu H$, where θ is deflection in radian from the zero position. Estimate the angular deflection of the pointer for a current of 1.0A.

Solution:

$$L = 200 + 40\theta - 4\theta^2 - \theta^3 \mu H$$

$$\left.\frac{dL}{d\theta}\right|_{\theta=90^\circ} = 40 - 8\theta - 3\theta^2 \mu H / rad$$

$$\left.\frac{dL}{d\theta}\right|_{\theta=90^\circ} = 40 - 8\times\frac{\Pi}{2} - 3\left(\frac{\Pi}{2}\right)^2 \mu H / rad = 20\mu H / rad$$

$$\therefore \theta = \frac{1}{2K} I^2 \frac{dL}{d\theta}$$

$$\Pi = \frac{1}{2}\frac{(1.5)^2}{2K} \times 20 \times 10^{-6}$$

K=Spring constant=$14.32\times 10^{-6}\ N - m / rad$

For I=1A, $\therefore \theta = \frac{1}{2K} I^2 \frac{dL}{d\theta}$

$$\therefore \theta = \frac{1}{2}\times\frac{(1)^2}{14.32\times 10^{-6}}\left(40 - 8\theta - 3\theta^2\right)$$

$$3\theta + 36.64\theta^2 - 40 = 0$$

$$\theta = 1.008 rad, 57.8^\circ$$

Example: 1.6

The inductance of a moving iron instrument is given by $L = 10 + 5\theta - \theta^2 - \theta^3 \mu H$, where θ is the deflection in radian from zero position. The spring constant is $12\times 10^{-6}\ N - m / rad$.
Estimate the deflection for a current of 5A.

Solution:

$$\frac{dL}{d\theta} = (5 - 2\theta)\frac{\mu H}{rad}$$

$$\therefore \theta = \frac{1}{2K} I^2 \frac{dL}{d\theta}$$

$$\therefore \theta = \frac{1}{2}\times\frac{(5)^2}{12\times 10^{-6}}(5 - 2\theta)\times 10^{-6}$$

$$\therefore \theta = 1.69 rad, 96.8^\circ$$

Example: 1.7

The following figure gives the relation between deflection and inductance of a moving iron instrument.

Deflection (degree)	20	30	40	50	60	70	80	90
Inductance (μH)	335	345	355.5	366.5	376.5	385	391.2	396.5

Find the current and the torque to give a deflection of (a) 30^0 (b) 80^0 . Given that control spring constant is $0.4 \times 10^{-6}\ N-m/\deg ree$

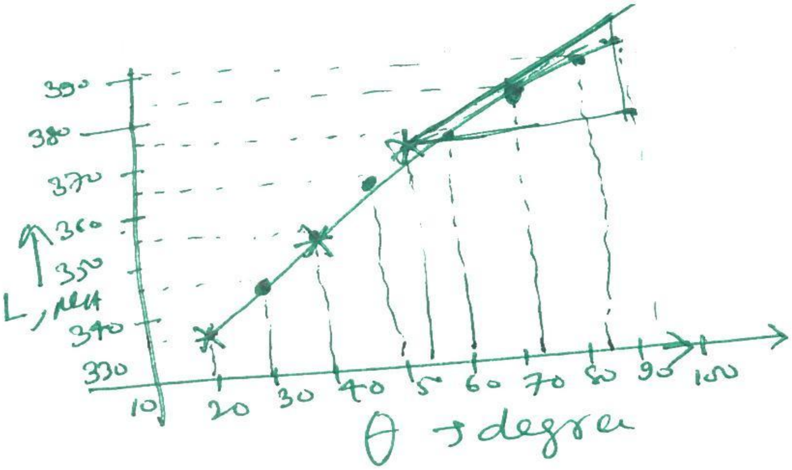
Solution:

$$\theta = \frac{1}{2K} I^2 \frac{dL}{d\theta}$$

(a) For $\theta = 30^\circ$

The curve is linear

$$\therefore \left.\frac{dL}{d\theta}\right|_{\theta=30} = \frac{355.5-335}{40-20} = 1.075\mu H/\deg ree = 58.7\mu H/rad$$



Example: 1.8

In an electrostatic voltmeter the full scale deflection is obtained when the moving plate turns through 90^0. The torsional constant is $10 \times 10^{-6}\ N - m / rad$. The relation between the angle of deflection and capacitance between the fixed and moving plates is given by

Deflection (degree)	0	10	20	30	40	50	60	70	80	90
Capacitance (PF)	81.4	121	156	189.2	220	246	272	294	316	334

Find the voltage applied to the instrument when the deflection is 90^0?

Solution:

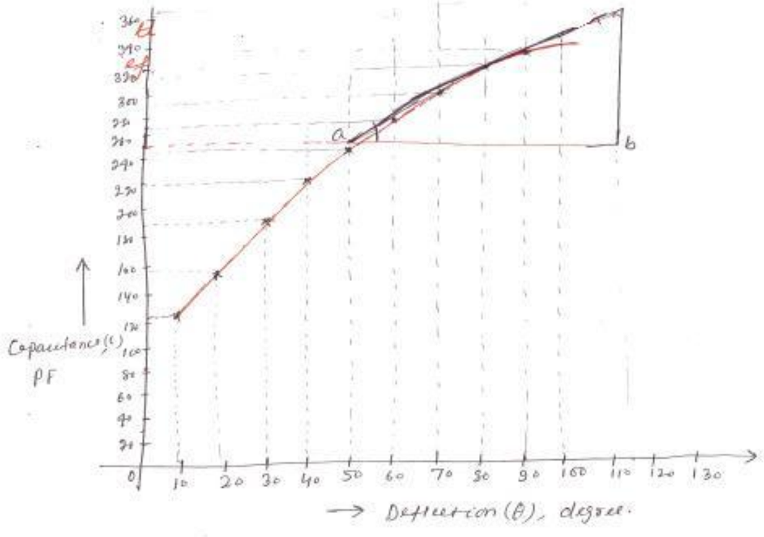


Fig. 1.27

$$\frac{dC}{d\theta} = \tan\theta = \frac{bc}{ab} = \frac{370 - 250}{110 - 44} = 1.82 PF / \deg ree = 104.2 PF / rad$$

$$\text{Spring constant } K = 10 \times 10^{-6} \frac{N - m}{rad} = 0.1745 \times 10^{-6} N - m / \deg ree$$

$$\theta = \frac{1}{2K} V^2 \frac{dC}{d\theta} \Rightarrow V = \sqrt{\frac{2K\theta}{\frac{dC}{d\theta}}}$$

$$V = \sqrt{\frac{2 \times 0.1745 \times 10^{-6} \times 90}{104.2 \times 10^{-12}}} = 549 volt$$

Example: 1.9

Design a multi range d.c. mille ammeter using a basic movement with an internal resistance R_m = 50Ω and a full scale deflection current $I_m = 1mA$. The ranges required are 0-10mA; 0-50mA; 0-100mA and 0-500mA.

Solution:

Case-I 0-10mA

Multiplying power $m = \frac{I}{I_m} = \frac{10}{1} = 10$

$\therefore$ Shunt resistance $R_{sh1} = \frac{R_m}{m-1} = \frac{50}{10-1} = 5.55\Omega$

Case-II 0-50mA

$m = \frac{50}{1} = 50$

$R_{sh2} = \frac{R_m}{m-1} = \frac{50}{50-1} = 1.03\Omega$

Case-III 0-100mA, $m = \frac{100}{1} = 100\Omega$

$R_{sh3} = \frac{R_m}{m-1} = \frac{50}{100-1} = 0.506\Omega$

Case-IV 0-500mA, $m = \frac{500}{1} = 500\Omega$

$R_{sh4} = \frac{R_m}{m-1} = \frac{50}{500-1} = 0.1\Omega$

Example: 1.10

A moving coil voltmeter with a resistance of 20Ω gives a full scale deflection of 120^0, when a potential difference of 100mV is applied across it. The moving coil has dimension of 30mm*25mm and is wounded with 100 turns. The control spring constant is $0.375 \times 10^{-6}\ N-m$ / deg ree. Find the flux density, in the air gap. Find also the diameter of copper wire of coil winding if 30% of instrument resistance is due to coil winding. The specific resistance for copper=$1.7 \times 10^{-8}\ \Omega m$.

Solution:

Data given

$V_m = 100mV$

$R_m = 20\Omega$

$\theta = 120^\circ$

N=100

$K = 0.375 \times 10^{-6} N - m / \deg ree$

$R_C = 30\% of R_m$

$\rho = 1.7 \times 10^{-8} \Omega m$

$I_m = \frac{V_m}{R_m} = 5 \times 10^{-3} A$

$T_d = BANI, T_C = K\theta = 0.375 \times 10^{-6} \times 120 = 45 \times 10^{-6} N - m$

$B = \frac{T_d}{ANI} = \frac{45 \times 10^{-6}}{30 \times 25 \times 10^{-6} \times 100 \times 5 \times 10^{-3}} = 0.12 wb / m^2$

$R_C = 0.3 \times 20 = 6\Omega$

Length of mean turn path =2(a+b) =2(55)=110mm

$R_C = N\frac{\rho l}{A}$

$A = \frac{N \times \rho \times (l_t)}{R_C} = \frac{100 \times 1.7 \times 10^{-8} \times 110 \times 10^{-3}}{6}$

$= 3.116 \times 10^{-8} m^2$

$= 31.16 \times 10^{-3} mm^2$

$A = \frac{\Pi}{\ } d^2 \Rightarrow d = 0.2mm$

Example: 1.11

A moving coil instrument gives a full scale deflection of 10mA, when the potential difference across its terminal is 100mV. Calculate

(1) The shunt resistance for a full scale deflection corresponding to 100A

(2) The resistance for full scale reading with 1000V.
Calculate the power dissipation in each case?

Solution:

Data given

$I_m = 10mA$

$V_m = 100mV$

$I = 100A$

$$I = I_m \, 1 + \frac{R_m}{R_{sh}}$$

$$100 = 10 \times 10^{-3} \, 1 + \frac{10}{R_{sh}}$$

$$R_{sh} = 1.001 \times 10^{-3}\ \Omega$$

$R_{se} = ??, V = 1000V$

$$R_m = \frac{V_m}{I_m} = \frac{100}{10} = 10\Omega$$

$$V = V_m \, 1 + \frac{R_{se}}{R_m}$$

$$1000 = 100 \times 10^{-3} \, 1 + \frac{R_{se}}{10}$$

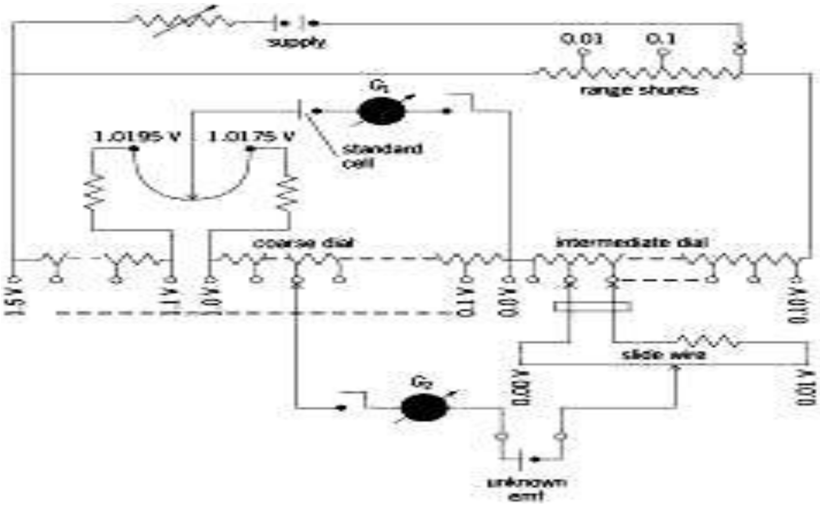
$$\therefore R_{se} = 99.99K\Omega$$

UNIT – II
POTENTIOMETERS AND INSTRUMENT TRANSFORMERS

D.C & A.C Potentiometers

An instrument that precisely measures an electromotive force (emf) or a voltage by opposing to it a known potential drop established by passing a definite current through a resistor of known characteristics. (A three-terminal resistive voltage divider is sometimes also called a potentiometer.) There are two ways of accomplishing this balance: (1) the current *I* may be held at a fixed value and the resistance *R* across which the *IR* drop is opposed to the unknown may be varied; (2) current may be varied across a fixed resistance to achieve the needed *IR* drop.

The essential features of a general-purpose constant-current instrument are shown in the illustration. The value of the current is first fixed to match an *IR* drop to the emf of a reference standard cell. With the standard-cell dial set to read the emf of the reference cell, and the galvanometer (balance detector) in position *G*1, the resistance of the supply branch of the circuit is adjusted until the *IR* drop in 10 steps of the coarse dial plus the set portion of the standard-cell dial balances the known reference emf, indicated by a null reading of the galvanometer. This adjustment permits the potentiometer to be read directly in volts. Then, with the galvanometer in position *G*2, the coarse, intermediate, and slide-wire dials are adjusted until the galvanometer again reads null. If the potentiometer current has not changed, the emf of the unknown can be read directly from the dial settings. There is usually a switching arrangement so that the galvanometer can be quickly shifted between positions 1 and 2 to check that the current has not drifted from its set value.



Circuit diagram of a general-purpose constant-current potentiometer, showing essential features

Potentiometer techniques may also be used for current measurement, the unknown current being sent through a known resistance and the *IR* drop opposed by balancing it at the voltage terminals of the potentiometer. Here, of course, internal heating and consequent resistance change of the current-carrying resistor (shunt) may be a critical factor in measurement accuracy; and the shunt design may require attention to dissipation of heat resulting from its I^2R power consumption. Potentiometer techniques have been extended to alternating-voltage measurements, but generally at a reduced accuracy level (usually 0.1% or so). Current is set on an ammeter which must have the same response on ac as on dc, where it may be calibrated with a potentiometer and shunt combination. Balance in opposing an unknown voltage is achieved in one of two ways: (1) a slide-wire and phase-adjustable supply; (2) separate in-phase and quadrature adjustments on slide wires supplied from sources that have a 90° phase difference. Such potentiometers have limited use in magnetic testing.

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An electrical measuring device used in determining the electromotive force (emf) or voltage by means of the compensation method. When used with calibrated standard resistors, a potentiometer can be employed to measure current, power, and other electrical quantites; when used with the appropriate measuring transducer, it can be used to gauge various non-electrical quantities, such as temperature, pressure, and the composition of gases.

Distinction is made between DC and AC potentiometers. In DC potentiometers, the voltage being measured is compared to the emf of a standard cell. Since at the instant of compensation the current in the circuit of the voltage being measured equals zero, measurements can be made without reductions in this voltage. For this type of potentiometer, accuracy can exceed 0.01 percent. DC potentiometers are categorized as either high-resistance, with a slide-wire resistance ranging from The higher resistance class can measure up to 2 volts (V) and is used in testing highly accurate apparatus. The low-resistance class is used in measuring voltage up to 100 mV. To measure higher voltages, up to 600 V, and to test voltmeters, voltage dividers are connected to potentiometers. Here the voltage drop across one of the resistances of the voltage divider is compensated; this constitutes a known fraction of the total voltage being measured.

In AC potentiometers, the unknown voltage is compared with the voltage drop produced by a current of the same frequency across a known resistance. The voltage being measured is then adjusted both for amplitude and phase. The accuracy of AC potentiometers is of the order of 0.2 percent. In electronic automatic DC and AC potentiometers, the measurements of voltage are carried out automatically. In this case, the compensation of the unknown voltage is achieved with the aid of a servomechanism that moves the slide along the resistor, or rheostat. The servomechanism is actuated by the imbalance of the two voltages, that is, by the difference between the compensating voltage and the voltage that is being compensated. In electronic automatic potentiometers, the results of measurements are read on dial indicators, traced on recorder charts or received as numerical data. The last method makes it possible to input the data directly into a computer. In addition to measurement, electronic automatic potentiometers are also capable of regulating various parameters of industrial processes. In this case, the slide of the rheostat is set in a position that predetermines, for instance, the temperature of the object to be regulated. The voltage imbalance of the potentiometer drives the servomechanism, which then increases or decreases the electric heating or regulates the fuel supply.

A voltage divider with a uniform variation of resistance, a device that allows some fraction of a given voltage to be applied to an electric circuit. In the simplest case, the device consists of a conductor of high resistance equipped with a sliding contact. Such dividers are used in electrical engineering, radio engineering, and measurement technology. They can also be utilized in analog computers and in automation systems, where, for example, they function as sensors for linear or angular displacement

Instrument Transformers Basics

Why instrument transformers?

In power systems, currents and voltages handled are very large.
Direct measurements are not possible with the existing equipment's. Hence it is required to step down currents and voltages with the help of instrument transformers so that they can be measured with instruments of moderate sizes

Instrument Transformers

Transformers used in conjunction with measuring instruments for measurement purposes are called "Instrument Transformers".

The instrument used for the measurement of current is called a "Current Transformer" or simply "CT".

The transformers used for the measurement of voltage are called "Voltage transformer" or "Potential transformer" or simply "PT".

Instrument Transformers:

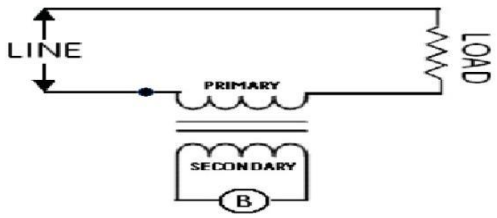


Fig 1.Current Transformer

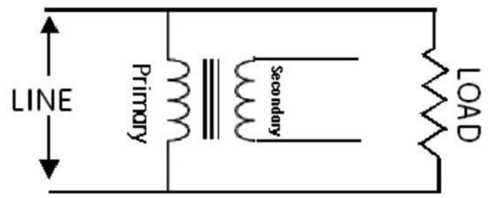


Fig 2. Potential Transformer

Fig 1. Indicates the current measurement by a C.T. The current being measured passes through the primary winding and the secondary winding is connected to an ammeter. The C.T. steps down the current to the level of ammeter.

Fig 2. Shows the connection of P.T. for voltage measurement. The primary winding is connected to the voltage being measured and the secondary winding to a voltmeter. The P.T. steps down the voltage to the level of voltmeter.

Merits of Instrument Transformers:

1. Instruments of moderate size are used for metering i.e. 5A for current and 100 to 120 volts for voltage measurements

2. Instrument and meters can be standardized so that there is saving in costs. Replacement of damaged instruments is easy.

3. Single range instruments can be used to cover large current or voltage ranges, when used with suitable multi range instrument transformers.

4. The metering circuit is isolated from the high voltage power circuits. Hence isolation is not a problem and the safety is assured for the operators

5. There is low power consumption in metering circuit.

6. Several instruments can be operated from a single instrument transformer.

Ratios of Instrument Transformer:

Some definitions are:

1. Transformation ratio: It is the ratio of the magnitude if the primary phasor to secondary phasor.

Transformation ratio:

$$R = \frac{|\text{Primary phasor}|}{|\text{secondary phasor}|}$$

R= Primary winding Current/ secondary winding Current for a C.T

R= Primary Winding Voltage/ Secondary winding Voltage for P.T

Nominal Ratio: It is the ratio of rated primary winding current (voltage) to the ratedsecondary winding current (voltage).

$$K_n = \frac{\text{rated primary winding current}}{\text{rated secondary winding current}}$$

$$= \frac{\text{rated primary winding voltage}}{\text{rated secondary winding voltage}}$$ for a P.T.

Turns ratio: This is defined as below

$$n = \frac{\text{number of turns of secondary winding}}{\text{number of turns of primary winding}} \quad \text{for a C.T.}$$

$$= \frac{\text{number of turns of primary winding}}{\text{number of turns of secondary winding}} \quad \text{for a P.T.}$$

Burden of an Instrument Transformer:

The rated burden is the volt ampere loading which is permissible without errors exceeding the particular class of accuracy.

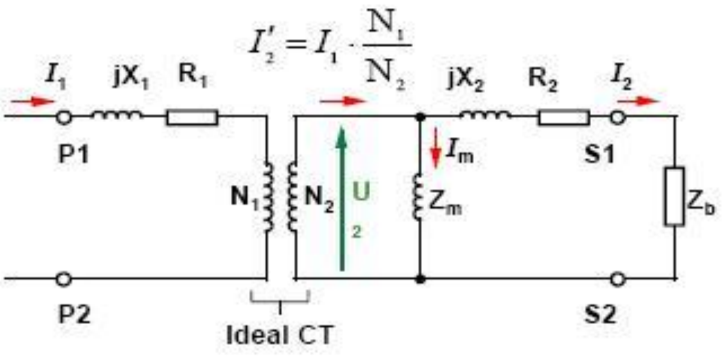
$$\text{Total secondary winding burden} = \frac{(\text{secondary winding induced voltage})^2}{(\text{impedance of secondary winding circuit including impedance of secondary winding})}$$

$$= (\text{secondary winding current})^2 \times (\text{impedance of secondary winding circuit including secondary winding})$$

$$\text{secondary winding burden due to load} = \frac{(\text{secondary winding terminal voltage})^2}{(\text{impendance of load on secondary winding})}$$

$$= (\text{secondary winding current})^2 \times (\text{impedance of load in the secondary winding circuit})$$

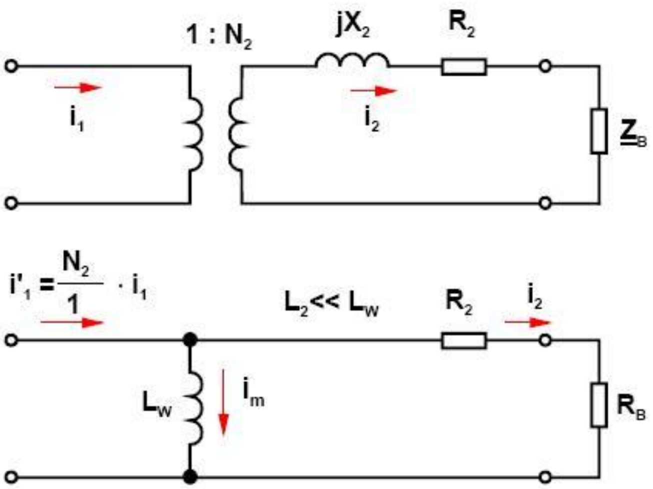
Current Transformer equivalent circuit:



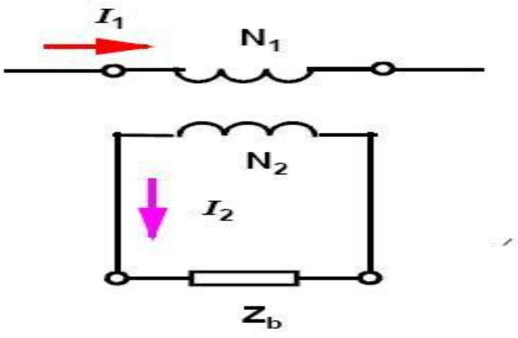
X1 = Primary leakage reactance
R1 = Primary winding resistance
X2 = Secondary leakage reactance

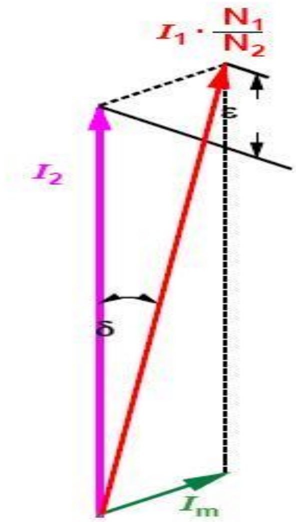
Z0 = Magnetizing impedance
R2 = Secondary winding resistance
Zb = Secondary load
Note: Normally the leakage fluxes X1 and X2 can be neglected

Current transformer, simplified equivalent circuit:



Current transformer: Phase displacement (δ) and current ratio error (ε):





Current Transformer Basics:

Current Transformers (CT's) can be used for monitoring current or for transforming primary current into reduced secondary current used for meters, relays, control equipment and other instruments. CT's that transform current isolate the high voltage primary, permit grounding of the secondary, and step-down the magnitude of the measured current to a standard value that can be safely handled by the instrument.

Ratio :The CT ratio is the ratio of primary current input to secondary current output at full load. For example, a CT with a ratio of 300:5 is rated for 300 primary amps at full load and will produce 5 amps of secondary current when 300 amps flow through the primary. If the primary current changes the secondary current output will change accordingly. For example, if 150 amps flow through the 300 amp rated primary the secondary current output will be 2.5 amps (150:300 = 2.5:5).

Current Transformer: Cautions:

Inspect the physical and mechanical condition of the CT before installation.

Check the connection of the transformer requirements for the instrument or the system requirements before connecting the CT.

Inspect the space between the CT phases, ground and secondary conductor for adequate clearance between the primary and secondary circuitry wiring.

Verify that the shorting device on the CT is properly connected until the CT is ready to be installed. The secondary of the CT must always have a burden (load) connected when not in use. NOTE: A dangerously high secondary voltage can develop with an open-circuited secondary.

Construction of Current Transformer:

Current transformers are constructed in various ways. In one method there are two separate windings on a magnetic steel core. The primary winding consists of a few turns of heavy wire capable of carrying the full load current while the secondary winding consist of many turns of smaller wire with a current carrying capacity of between 5/20 amperes, dependent on the design. This is called the wound type due to its wound primary coil.

Another very common type of construction is the so-called “window,” “through” or donut type current transformer in which the core has an opening through which the conductor carrying the primary load current is passed. This primary conductor constitutes the primary winding of the CT (one pass through the “window” represents a one turn primary), and must be large enough in cross section to carry the maximum current of the load.

Window-type

Construction of Current Transformer:

Another distinguishing feature is the difference between indoor and outdoor construction.

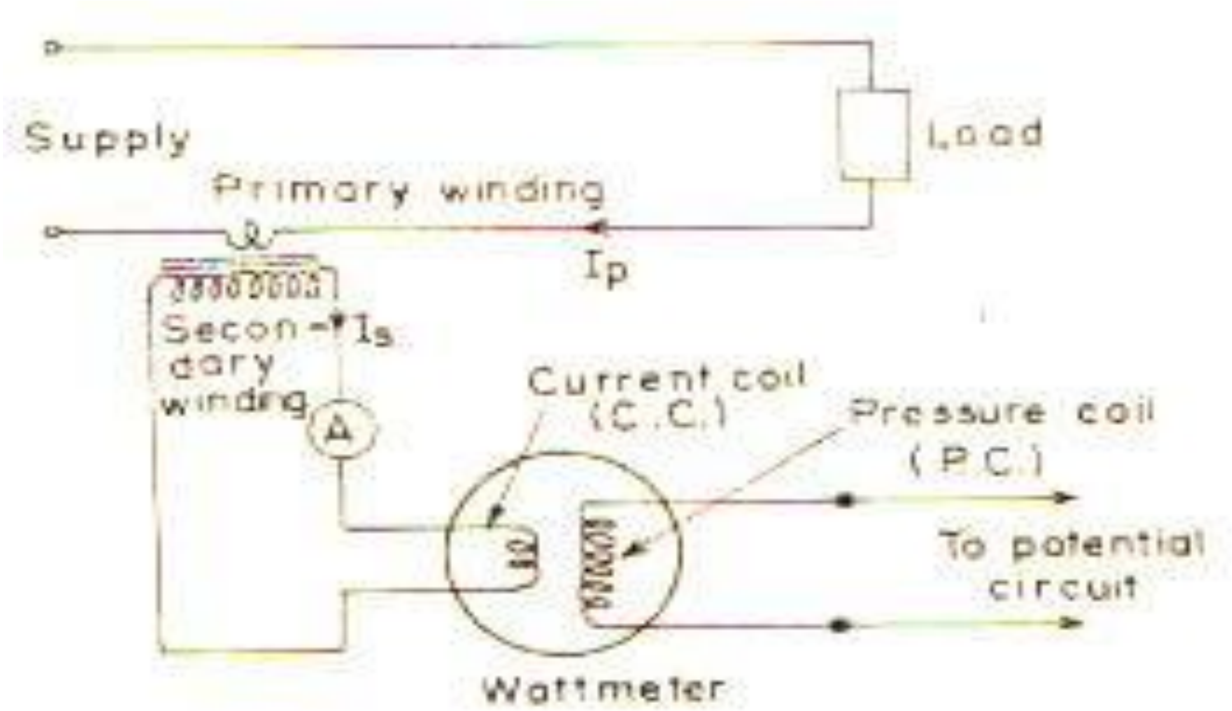
15kV Outdoor CT

15kV Indoor CT

Construction of Current Transformer

Indoor Type	Outdoor Type
indoor units are protected due to their being mounted in an enclosure of some kind	The outdoor unit must be protected for possible contaminated environments
Not Required	outdoor units will have larger spacing between line and ground, which is achieved by the addition of skirts on the Design.
Not Required	For outdoor types the hardware must be of the non-corrosive type and the insulation must be of the non-arc-Tracking type.
The indoor types must be compatible for connection to bus type electrical construction	outdoor types are normally on the pole-top installations.

Circuit connection for current and power measurement using C.T.



Equivalent Circuit of C.T.

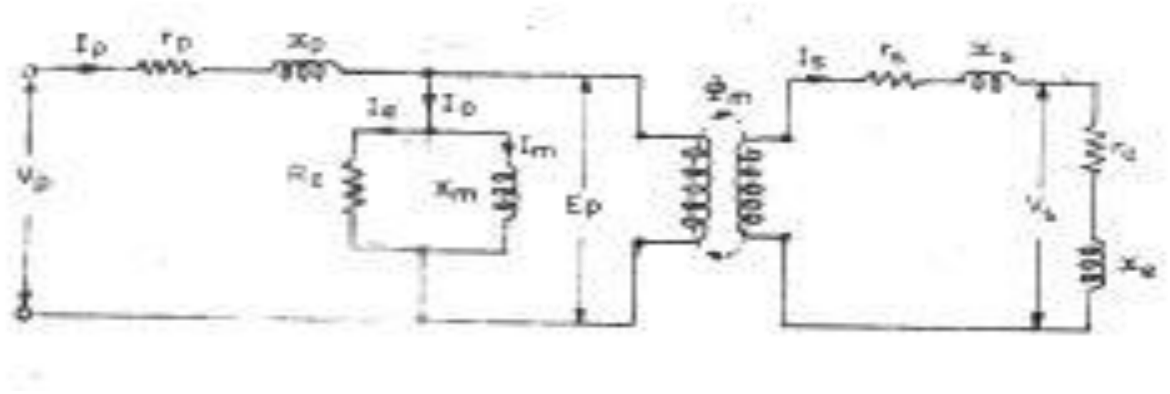


Fig. 1 Equivalent circuit of C.T.

Phasor diagram

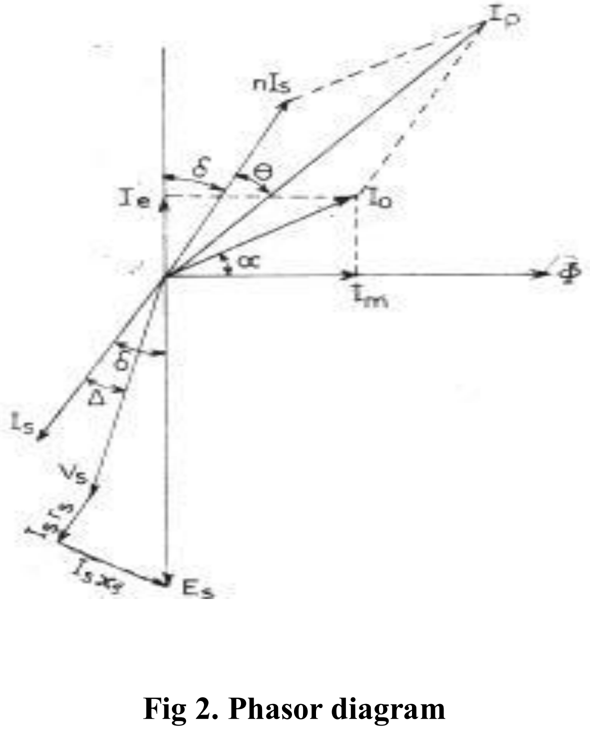


Fig 2. Phasor diagram

A section of Phasor diagram

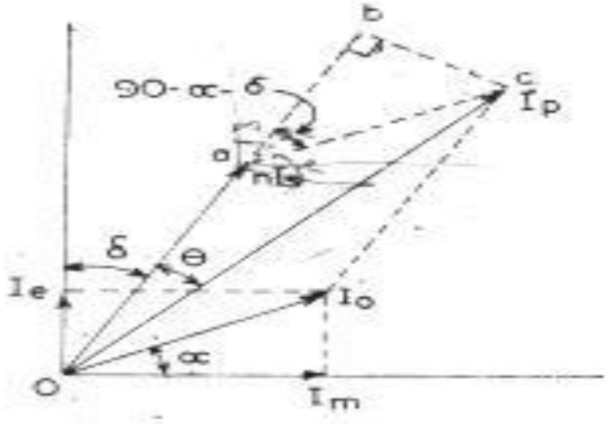


Fig 3. A section of Phasor diagram

Relationship in current transformer:

Fig 1 represents the equivalent circuit and Fig 2 the phasor diagram of a current transformer.

n= turns ratio = (No. of secondary winding turns)/(No. of primary winding turns)
rs = resistance of the secondary winding;
xs = reactance of the secondary winding;
re = resistance of external burden i.e. resistance of meters, current coils etc. including leads;
xe = reactance of external burden i.e. reactance of meters, current coils etc. including leads;
Ep = primary winding induced voltage Es
= secondary winding induced voltage Np
= No. of primary winding turns;
Ns = No. of secondary winding turns;
Vs = Voltage at the secondary winding
terminals; Is = secondary winding current;
Ip = primary winding current;
Θ = phase angle of transformer;
secondary winding current = phase angle of total burden including impedance of secondarywinding

$$\tan^{-1}\left(\frac{x_s+x_e}{r_s+r_e}\right)$$

= phase angle of secondary winding load circuit i.e. of external burden $\tan^{-1}\left(\frac{x_e}{r_e}\right)$

Io = exiting current;

Im = magnetizing component of exciting current, Ie = loss component of exciting current,

α = angle between exciting current Io and working flux ϕ

Consider a small section of the phasor as shown in Fig. 3. e have ∠bac= 90° -δ-α, ac = Io , oa = nIs and oc = Ip.

bc= Io sin (90°-δ-α) = Io cos (δ+α), ab = Io cos (90°-δ-α) = Io sin (δ+α).

Now $(Oc)^2 = (oa + ab)^2 + (bc)^2$

or

$$Ip^2 = [nIs + Io\sin(\delta+\alpha)]^2 + [Io\cos(\delta+\alpha)]^2$$
$$= n^2Is^2 + I_0^2\sin^2(\delta+\alpha) + 2\,nIsIo\sin(\delta+\alpha) + Io^2\cos^2(\delta+\alpha).$$
$$= n^2I_s^2 + 2nIsIo\sin(\delta+\alpha) + Io^2$$
$$\therefore Ip = [n^2I_s^2 + 2nIsIo\sin(\delta+\alpha) + Io^2]^{½} \qquad (1)$$

Transformation ratio

$$R = Ip/Is = [n2\,IsIo\sin(\delta+\alpha) + Io2]^{½}/Is \qquad (2)$$

Now in a well-designed current transformer Io <<nIs. Usually Io is less than 1 percent of IpandIp is, therefore, very nearly equal to nIs.

Eqn. (2) can be written as

$$R = \frac{[n^2I_s^2 + 2\,nI_sI_0\sin(\delta+\alpha) + I_0^2\sin^2(\delta+\alpha)]^{\frac{1}{2}}}{I_s}$$

$$\frac{nI_s + I_0\sin(\delta+\alpha)}{I_s} = n + \frac{I_0}{I}\sin(\delta+\alpha)(3)$$

The above theory is applicable to case when the secondary burden has a lagging power factor i.e., when the burden is inductive.

Eqn. (3) can be further expanded as:

$$R \approx n + \frac{I_0}{I_s}(\sin\delta\cos\alpha + \cos\delta\sin\alpha)$$
$$\approx n + \frac{I_m\sin\delta + I_e\cos\delta}{I_s}$$

As

$$I_m = I_0 \cos\alpha \; and \qquad I_e = I_0 \sin\alpha$$

Phase angle:

The angle by which the secondary current phasor, when reversed, differs in phase from primary current, is known as the "phase angle" of the transformer.
+ve if secondary reversed current leads the primary current
-ve if secondary reversed current lags behind the primary current.
The angle between Is and Ip is θ. Therefore, the phase angle is θ.
From the phasor diagram,

$$\tan\theta = \frac{bc}{ob} = \frac{bc}{oa+ab} = \frac{I_0\cos(\delta+\alpha)}{nI_s + I_0\sin(\delta+\alpha)}$$

As ϑ is very small, we can write

$$\theta = \frac{I_0\cos(\delta+\alpha)}{nI_s + I_0\sin(\delta+\alpha)} \; rad \; .(5)$$

Now Io is very small as compared to nIs, and, therefore we can neglect the term Io sin($\delta+\alpha$)

$$\therefore \theta = \frac{I_0\cos(\delta+\alpha)}{nI_s} \, rad \qquad (6)$$

$$\approx \frac{I_0\cos\delta\cos\alpha - I_0\sin\delta\sin\alpha}{nI_s} \approx \frac{I_m\cos\delta - I_e\sin\delta}{nI_s} \, rad \qquad (7)$$

$$(8)$$

$$\approx \frac{180}{\Pi}\left(\frac{I_m\cos\delta - I_e\sin\delta}{nI_s}\right) degree$$

Errors in current transformers:

- Turns ratio and transformation ratios are not equal.
- The value of transformation ratio is not constant.

It depends upon:

1. Magnetizing and loss components of exciting current,
2. The secondary winding load current and its power

This introduces considerable errors into current measurement

In power measurement it is necessary that the phase of secondary winding current shall be displaced by exactly 180° from that of the Primary current. Here, phase difference is different from 180° by an angle θ. Hence due to C.T. two types of errors are introduced in power measurements.

Due to actual transformation ratio being different from the turn's ratio.

Due to secondary winding current not being 180° out of phase with the primary winding current.

Ratio error and phase angle error

Ratio Error is defined as:

Percentage ratio error = ((nominal ratio – actual ratio)/(actual ratio))x100

$$= \frac{K_n - R}{R} \cdot 100 \qquad (9)$$

Phase angle

$$\approx \frac{180}{3.14} \left(\frac{I_m \cos\delta - I_e \sin\delta}{n I_s} \right) \text{degree}$$

Problem No.1

Two current transformers of the same nominal ratio 500/5 A, are tested by Silsbee's method. With the current in the secondary of the transformer adjusted at its rated value, the content in the middle conductor I = 0.05e-j126.9° A expressed with respect to current in the secondary of standard transformer as the reference. It is known that standard transformer has a ratio correction factor (RCF) of 1.0015 and phase error +8'. Find RCF and phase angle error of transformer under test.

Problems on CT

1. A current transformer has a single turn primary and 200 turns secondary winding. The secondary winding supplies a current of 5 A to a non-inductive burden of 1 resistance. The requisite flux is set up in the core by an mmf of 80 A. The frequency is 50 Hz and the net cross section of the core is 1000 sq. mm. Calculate the ratio and phase angle of the transformer. Also find the flux density in the core. Neglect the effects of magnetic leakage, iron losses and copper losses.

A current transformer with a bar primary has 300 turns in its secondary winding. The resistance and reactance of secondary circuits are 1.5 and 1.0 respectively including the transformer winding. With 5 A flowing in the secondary winding, the magnetizing mmf is 100 A and the iron losses is 1.2W. Determine the ratio and phase angle error.

Solution: Primary winding turns Np= 1;

Secondary winding turns Ns = 300;
Turns ratio = Ns/Np = 300/1 = 300.

Secondary circuit burden impedance = $\sqrt{(1.5)^2+(1.0)^2}$ = 1.8

For secondary winding circuit:

$\cos\delta = 1.5/1.8 = 0.833$ and $\sin\delta = 1.0/1.8 = 0.555$.
Secondary induced voltage Es = 5 × 1.8 = 9.0 V.
Primary induced voltage Ep = Es/n = 9.0/300 = 0.03 V.
Loss component of current referred to primary winding

Ie = iron loss/(Ep) = 1.2/0.03 = 40

A. Magnetizing current

Im = (magnetizing mmf)/(primary winding turns)
= 100/1 = 100 A

Actual ratio R =

$$n+\frac{I_m \sin\delta + I_e \cos\delta}{I_s}$$

= 300+(100 ×0.555+40 × 0.833)/5 = 317.6

In the absence of any information to the contrary we can take nominal ratio to be equal to the turns ratio, or
Kn = n = 300

Percentage ratio error $=\frac{K_n - R}{R}\cdot 100$

= (300-317.6)/317.6 = -5.54%.

Phase angle ϑ =

$$\frac{180}{\Pi}\left(\frac{I_m \cos\delta - I_e \sin\delta}{nI_s}\right) \deg ree$$

= 180/π ((100 × 0.833 - 40 × 0.555)/(300 × 5))
= 2.34°.

Problem No. 3

A 100/5 A, 50 Hz CT has a bar primary and a rated secondary burden of 12.5 VA. The secondary winding has 196 turns and a leakage inductance of 0.96 mH. With a purely resistive burden at rated full load, the magnetization mmf is 16 A and the loss excitation requires 12 A. Find the ratio and phase angle errors.

Solution: Secondary burden = 12.5 VA.

Secondary winding current = 5 A

Secondary circuit impedance = $12.5/5^2 = 0.5$.

Secondary circuit reactance = $2\pi \times 50 \times 96 \times 10^{-3} = 0.3$

Phase angle of secondary circuit $\delta = \sin^{-1} 0.3/0.5 = \sin^{-1} 0.6$

Therefore, $\sin \delta = 0.6$ and $\cos\delta = \sqrt{(1)^2 + (0.6)^2}$

$= 0.8$.

Primary winding turns Np = 1. Secondary winding turns Ns = 196.

Turns ratio n = Ns/Np = 196.

Nominal ratio = Kn = 1000/5 =

200 Magnetizing current

Im = (magnetizing mmf)/(primary winding turns)

= 16/1 =16 A

Loss component Ie = (excitation for loss/primary winding turns)

= 12/1 = 12 A.

Actual ratio R =

$$n + \frac{I_m \sin\delta + I_e \cos\delta}{I_s}$$

$= 196 + ((12\times0.8+16\times0.6)/5) = 199.84$

Ratio error = ((nominal ratio – actual ratio)/(actual ratio)) ×100

= ((200-199.84) / 199.84) ×100

= +0.08%

Phase angle ϑ =

$$\frac{180}{\Pi}\left(\frac{I_m \cos\delta - I_e \sin\delta}{nI_s}\right) \deg ree$$

$180/\pi\ ((16\times0.8 - 12\times0.6)/(196\times5)) = 0.327°$

= 19.6'.

Potential Transformer Basics

Potential transformers are normally connected across two lines of the circuit in which the voltage is to be measured. Normally they will be connected L-L (line-to-line) or L-G (line-to-ground). A typical connection is as follows:

Relationships in a Potential Transformer:

The theory of a potential transformer is the same as that of a power transformer. The main difference is that the power loading of a P.T. is very small and consequently the exciting current is of the same order as the secondary winding current while in a power transformer the exciting current is a very small fraction of secondary winding load current.

Fig 3.and Fig 4. shows the equivalent circuit and phasor diagram of a potential transformer respectively.

I_s = secondary winding current,
r_s = resistance of secondary winding Φ =
working flux in wb.
I_m = magnetizing component of no load (exciting) current in
A, I_e = iron loss component of no load (exciting) current in A,
I_0 = no load (exciting) current in A,
E_s = secondary winding induced voltage
V_s = secondary winding terminal
voltage, N_p = primary winding turns
N_s = secondary winding turns
x_s = reactance of secondary winding
r_e =resistance of secondary load circuit
x_e = reactance of secondary load circuit
= phase angle of secondary load circuit = $\tan^{-1} x_e$
$/r_e E_p$ = primary winding induced voltage,
I_p = primary winding current
r_p = resistance of primary winding
x_p = reactance of primary winding
Turns ratio n = $N_p / N_s = E_p / E_s$

Secondary voltages when referred to primary side are to be multiplied by n. When secondary currents are referred to primary side, they must be divided by n

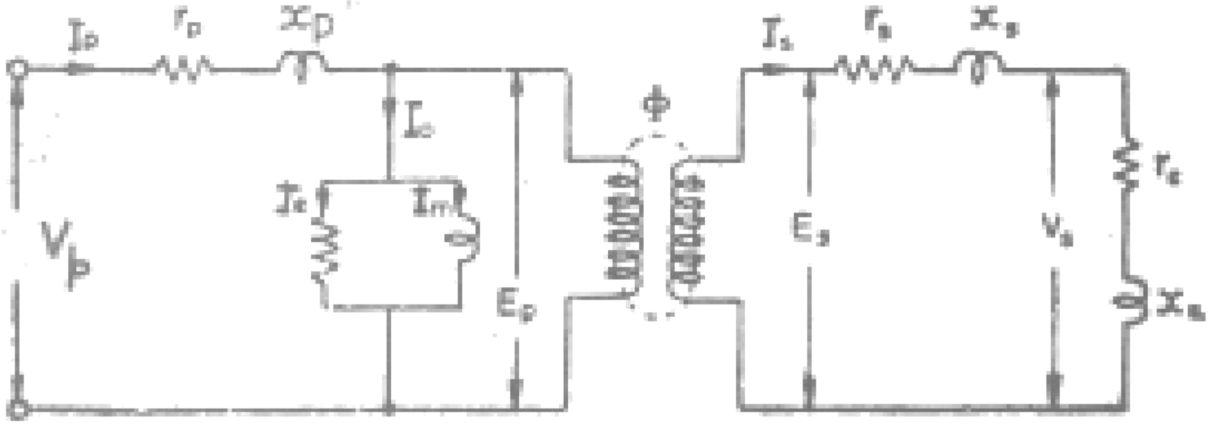


Fig. 3 Equivalent circuit of a P.T.

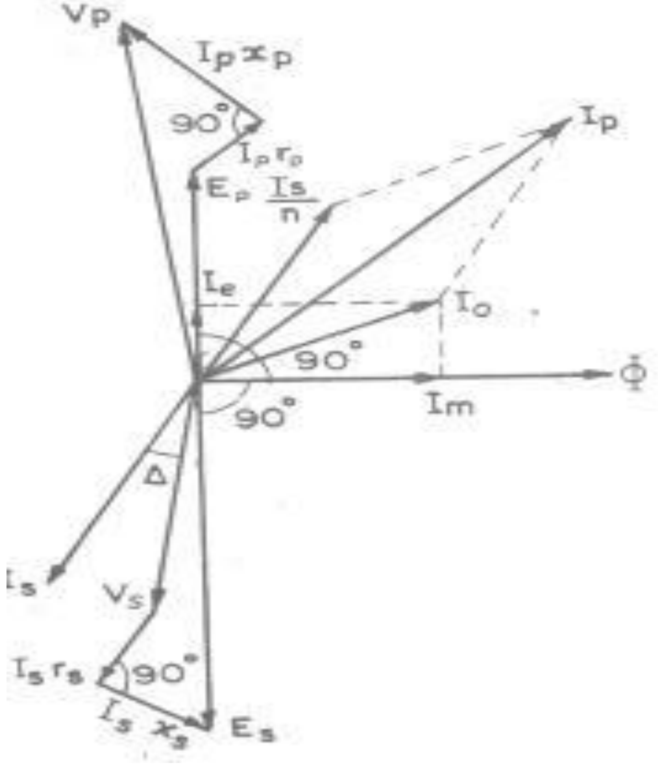


Fig. 4 Phasor diagram of P.T.

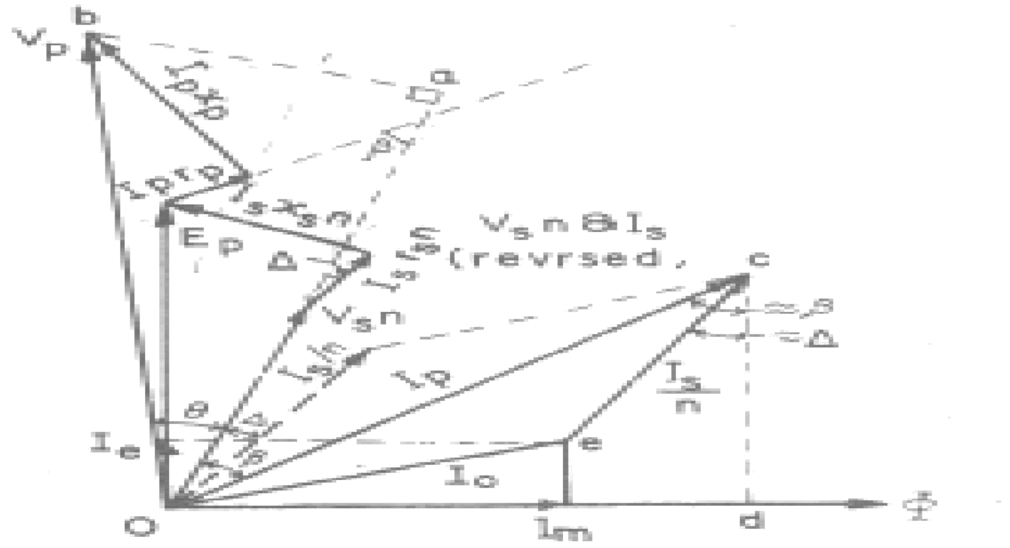


Fig. 5 Enlarged and concise phasor diagram of a P.T

Actual Transformation ratio

An enlarged concise phasor diagram is shown in Fig. 5.

ϑ = phase angle of the transformer

= angle between V_P and V_S reversed

= phase angle of secondary load circuit

= phase angle between I_P and V_S reversed. Now

$oa = V_P \cos\vartheta$

From Phasor diagram

$oa = n V_S + nI_S r_S \cos \quad + nI_S x_S \sin \quad + I_P r_P \cos\beta + I_P x_P \sin\beta$

Or

$V_P \cos\vartheta = n V_S + nI_S r_S \cos \quad + nI_S x_S \sin \quad + I_P r_P \cos\beta + I_P x_P \sin\beta$

$= n V_S + nI_S (r_S \cos \quad + x_S \sin \quad) + I_P r_P \cos\beta + I_P x_P \sin\beta$ ……..(i)

Phase angle ϑ is very small and, therefore, both V_P and V_S reversed can be taken perpendicular to Φ and, hence

$\angle ocd = \beta$ (approximately) and $\angle ecd =$ (approximately).

Thus $I_P \cos\beta = I_e + (I_S / n) \cos$

$I_P \sin\beta = I_m + (I_S / n) \sin$

Now ϑ is very small usually less than 1° and therefore, $\cos\vartheta = 1$ and hence we can write: $V_P \cos\vartheta = V_P$

Substituting the above values in (i), we have:

$$V_P = n V_S + nI_S (r_S \cos \quad + x_S \sin \quad) + (I_e + (I_S / n) \cos \quad) r_P + (I_m + (I_S / n) \sin \quad) x_P$$

$$= n\,V_S + I_S\cos\ \ (n\,r_S + r_P/n) + I_S\sin\ \ (n\,x_S + x_P/n) + I_e r_P + I_m x_P$$

……..(ii)

$$= n\,V_S + (I_S/n)\cos\ \ (n^2 r_S + r_P) + (I_S/n)\sin\ \ (n^2 x_S + x_P) + I_e r_P + I_m x_P$$

$$= n\,V_S + (I_S/n)\cos\ \ R_P + (I_S/n)\sin\ \ X_P + I_e r_P + I_m x_P$$

$= n\,V_S + (I_S/n)(R_P\cos + X_P\sin\ ) + I_e r_P + I_m x_P$ …..(iii) Here

R_P = equivalent resistance of the transformer referred to the primary side = $n^2 r_S + r_P$

and X_P = equivalent reactance of the transformer referred to the primary side = $n^2 x_S + x_P$

Actual transformation (voltage) ratio $R = V_P / V_S$

$$= n + \frac{((I_S/n)(R_P\cos\ + X_P\sin\ \ ) + I_e r_P + I_m x_P)}{V} \quad \text{……(iv)}$$

$_S$Eqn (ii) may be written as:

$$V_P = n\,V_S + nI_S\cos(r_S + r_P/n^2) + nI_S\sin(x_S + x_P/n^2) + I_e r_P + I_m x_P$$

$$V_P = n\,V_S + nI_S\cos\ \ R_S + nI_S\sin\ \ X_S + I_e r_P + I_m x_P$$

$= n\,V_S + nI_S(R_S\cos\ + X_S\sin\ ) + I_e r_P + I_m x_P$ ……(v)

Where

R_S = equivalent resistance of transformer referred to secondary side = $r_S + r_P/n^2$

X_S = equivalent reactance of transformer referred to secondary side = $x_S + x_P/n^2$.

Actual transformation (voltage) ratio $R = V_P / V_S$

$$= n + \frac{nI_S(R_S\cos\ \ + X_S\sin\ ) + I_e r_P + I_m x_P}{V_S} \quad \text{……(vi)}$$

Using eqns. (iii) and (v), the difference between actual transformation ratio and turns ratio is:

$R - n =$ $((I_S/n)(R_P\cos + X_P\sin\ ) + I_e r_P + I_m x_P)$ …….(vii)

V_S

$$= \frac{nI_S(R_S\cos\ \ \ + X_S\sin\ ) + I_e r_P + I_m x_P}{V_S} \quad \text{…….(viii)}$$

$$\tan\theta = \frac{ab}{oa} = \frac{I_P x_P\cos\beta - I_P r_P\sin\beta + nI_s x_s\cos - nI_s r_s\sin}{nV_s + nI_s r_s\cos + nI_s x_P\sin + I_P r_P\cos\beta + I_P x_P\sin\beta}$$

The terms in the denominator involving I_P and I_S are small and, therefore, they can be neglected as compared with nV_S .

$$= \frac{I_P x_P \cos\beta - I_P r_P \sin\beta + nI_s x_s \cos \ - nI_s r_s \sin}{nV_s}$$

$$= \frac{x_P I_e + \frac{I_s}{n}\cos \ - r_P I_m + \frac{I_s}{n}\sin \ + nI_s x_s \cos - nI_s r_s \sin}{nV_s}$$

$$= \frac{I_s \cos \ \frac{x_P}{n} + nx_s - I_s \sin \ \frac{r_P}{n} + nr_s + I_e x_p - I_m r_s}{nV_s}$$

$$= \frac{\frac{I_s \cos}{n}\left(x_P + n^2 x_s\right) - \frac{I_s \sin}{n}\left(r_P + n^2 r_s\right) + I_e x_P - I_m r_P}{nV_S}$$

Since ϑ is small, ϑ = tan ϑ

$$\therefore \theta = \frac{\frac{I_s}{n}\left(X_P \cos \ - R_P \sin\right) + I_e x_P - I_m r_P}{nV_s} \, rad .$$

$$= \frac{I_s}{V_s}\left(X_s \cos - R_s \ \sin \ \right) + \frac{I_e x_P - I_m r_P}{nV_s} \, rad$$

Errors in potential transformers

Ratio error (Voltage Error):

The actual ratio of transformation varies with operating condition and the error in secondary voltage may be defined as,

$$\% \text{ Ratio Error} = \frac{K_n - R}{R} \times 100$$

Phase angle error:

In an ideal voltage transformer, there should not be any phase difference between primary winding voltage and secondary winding voltage reversed. However, in an actual transformer there exists a phase difference between V_P and V_S reversed.

$$\therefore \theta = \frac{\frac{I_s}{n}\left(X_P \cos \ - R_P \sin\right) + I_e x_P - I_m r_P}{nV_s} \, rad .$$

$$= \frac{I_s}{V_s}\left(X_s \cos - R_s \ \sin \ \right) + \frac{I_e x_P - I_m r_P}{nV_s} \, rad$$

Problem No. 1

A potential transformer, ratio 1000/100 volt, has the following constants:
Primary resistance = 94.5 , secondary resistance=0.86
Primary reactance = 66.2 , total equivalent reactance = 110 No
load current = 0.02 A at 0’4 p.f.

Calculate: (i) Phase angle error at no load
(ii) burden in VA at UPF at which phase angle will be Zero.

Problem No.2

A potential transformer rated 6900/115 Volts, has 22500 turns in the primary winding and 375 turns in the secondary winding. With 6900 volts applied to the primary and the secondary circuit open circuited, the primary winding current is 0.005A lagging the voltage by 73.7°. With a particular burden connected to the secondary, the primary winding current is 0.0125A lagging the voltage by 53.1°. Primary winding resistance = 1200 , Primary winding reactance = 2000 , secondary winding resistance = 0.4 , secondary winding reactance = 0.7 .

(i) Find the secondary current and terminal voltage using the applied primary voltage $V_P = 6900 + j0$ as reference. Find the load burden also.
(ii) Find the actual transformation ratio and also the phase angle.

If the actual ratio = the nominal ratio under above conditions, what change should be made in the primary turns?

Testing of Instrument Transformers

Methods for finding ratio and phase angle errors experimentally are broadly classified into two groups:

1. **Absolute method**: In these methods the transformer errors are determined interms of constants i.e., resistance, inductance and capacitance of the testing circuit.

2. **Comparison method**: In these methods, the errors of the transformer under testare compared with those of a standard current transformer whose errors are known.

Each of the two methods can be classified, according to measurement technique employed as

1. Deflection Method: These methods use the deflections of suitable instruments for measuring quantities related to the phasor under consideration or to their deflection. The required ratio and phase angles are then found out from the magnitudes of deflection. These methods may be made direct reading in some cases.
2. Null Methods: These methods make use of a network in which the appropriate phasor quantities are balanced against one another. The ratio and phase angle errors are then found out from the impedance elements of the network. The method may be made direct reading in terms of calibrated scales on the adjustable elements in the network.

Testing of Current Transformer

There are three methods:

1. **Mutual Inductance method**: This is an absolute method using null deflection.
2. **Silsbee's Method**: This is a comparison method. There are two types; deflectionand null.
3. **Arnold's Method**: This is a comparison method involving null techniques.

Silsbee's Method:

The arrangement for Silsbee's deflection method is shown in Fig.1. Here the ratio and phase angle of the test transformer 'X' are determined in terms of that of a standard transformer 'S' having the same nominal ratio.

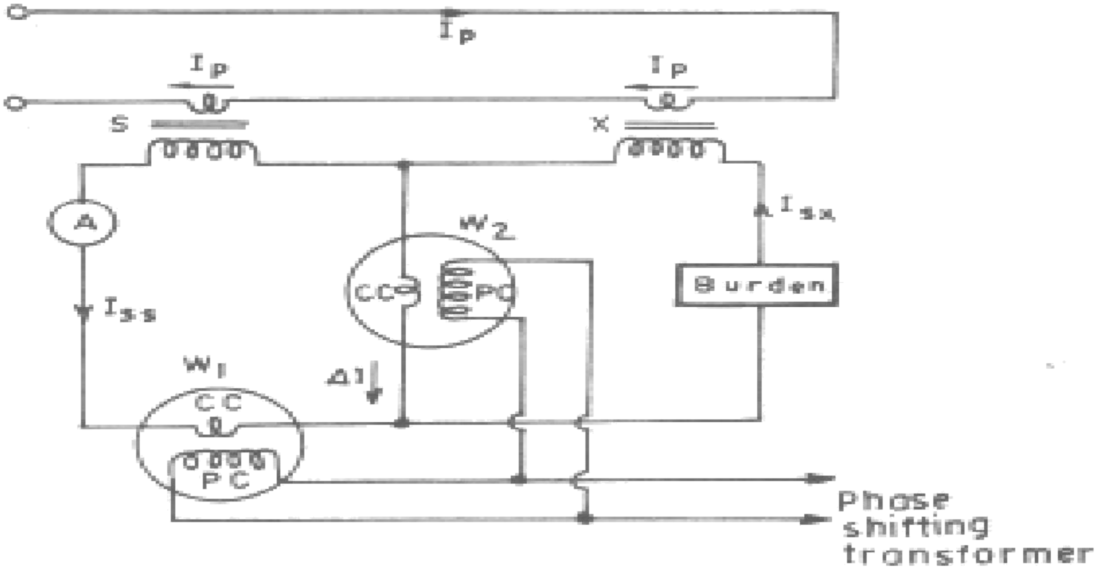


Fig. 1 Silsbee’s deflection method

Procedure:

The two transformers are connected with their primaries in series. An adjustable burden is put in the secondary circuit of the transformer under test.

An ammeter is included in the secondary circuit of the standard transformer so that the current may be set to desired value. W_1 is a wattmeter whose current coil is connected to carry the secondary current of the standard transformer. The current coil of wattmeter W_2 carries a current I which is the difference between the secondary currents of the standard and test transformer. The voltage circuits of wattmeters are supplied in parallel from a phase shifting transformer at a constant voltage V.

The phasor diagram is shown in Fig. 2

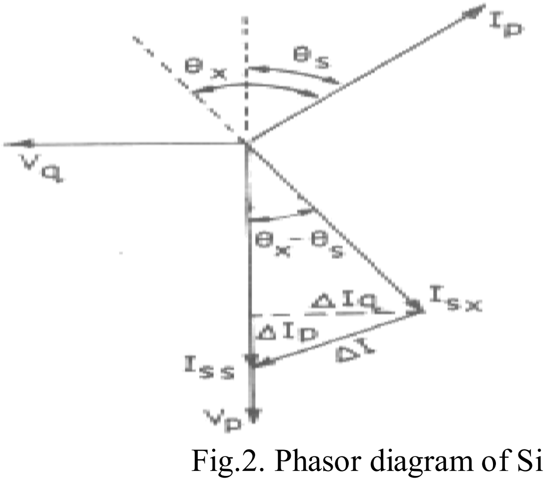


Fig.2. Phasor diagram of Silsbee’s method

1. The phase of the voltage is so adjusted that wattmeter W_1 reads zero. Under these conditions voltage V is in quadrature with current I_{ss} . The position of voltage phasor for this case is shown as V_q .

Reading of wattmeter, W_1 $W_{1q} = V_q I_{ss} \cos 90° = 0$.
Reading of wattmeter, W_2
$W_{2q} = V_q \times$ component of current I in phase with $V_q = V_q I_q =$
$V_q I_{sx} \sin(\theta_x - \theta_s)$
Where θ_x = phase angle of C.T. under test, θ_s =
phase angle of standard C.T.
$= W_{1p} - VI_{sx}\cos(\theta_x - \theta_s) \approx W_{1p} - VI_{sx}$
As $(\theta_x - \theta_s)$ is very small and, therefore, $\cos(\theta_x - \theta_s) = 1$ For
above, $VI_{sx} = W_{1p} - W_{2p}$.
Actual ratio of transformer under test $R_x = I_p / I_{sx}$.
Actual ratio of standard transformer $R_s = I_p / I_{ss}$.

$$\frac{R_x}{R_S} = \frac{I_{ss}}{I_{sx}} = \frac{VI_{ss}}{VI_{sx}} = \frac{W_{1p}}{W_{1p} - W_{2p}} = \frac{1}{1 - \left({W_{2p}}/{W_{1p}} \right)} \approx 1 + \frac{W_{2p}}{W_{1p}}$$

The phase of voltage V is shifted through 90° so that it occupies a position V_P and is in phase with I_{ss} .
Reading of wattmeter W_1, $W_{1p} = V_p I_{ss} \cos\theta = V_p I_{ss}$.

Reading of wattmeter W_2, $W_{2p} = V_p \times$component of current I_p

I in phase with $V_p = V_p [I_{ss} - I_{sx}\cos(\vartheta_x - \vartheta_s)]$
If the voltage is kept same for both sets of readings, then $V = V_p = V_q$.

We have $W_{2q} = VI_{sx}\sin(\theta_x - \theta_s)$, $W_{1p} = VI_{ss}$
$W_{2p} = V[I_{ss} - I_{sx}\cos(\theta_x - \theta_s)] = VI_{ss} - VI_{sx}\cos(\theta_x - \theta_s)]$

$$R_x = R_s \left(1 + \frac{W_{2p}}{W_{1p}}\right)$$

$$\sin(\theta_x - \theta_s) = \frac{W_{2q}}{VI_{sx}} \qquad \cos(\theta_x - \theta_s) = \frac{VI_{ss} - W_{2p}}{VI_{sx}} = \frac{W_{1p} - W_{2p}}{VI_{sx}}$$

$$\therefore \tan(\theta_x - \theta_s) = \frac{W_{2q}}{W_{1p} - W_{2p}} \; or \; (\theta_x - \theta_s) = \frac{W_{2q}}{W_{1p} - W_{2p}} \; rad$$

Or phase angle of test transformer,

$$\theta = \frac{W_{2q}}{W_{1p} - W_{2p}} + \theta_s \, rad \approx \frac{W_{2q}}{W_{1p}} + \theta_s \, rad$$

as W_{2p} is very small. Hence if the ratio and phase angle errors of standard transformer are known, we can compute the errors of the test transformer. W_2 must be a sensitive instrument. Its current coil may be designed for small values. It is normally designed to carry about 0.25A for testing CTs having a secondary current of 5A.

Problem No.1

Two current transformers of the same nominal ratio 500/5 A, are tested by Silsbee's method. With the current in the secondary of the transformer adjusted at its rated value, the content in the middle conductor $I = 0.05e^{-j126.9°}$ A expressed with respect to current in the secondary of standard transformer as the reference. It is known that standard transformer has a ratio correction factor (RCF) of 1.0015 and phase error +8'. Find RCF and phase angle error of transformer under test.

UNIT – III
MEASUREMENT OF POWER AND ENERGY

Single And Three Phase Wattmeters And Energy Meters

Single Phase Induction Type Meters

The construction and principle of operation of Single Phase Energy Meters is explained below

Construction of Induction Type Energy Meters

There are four main parts of the operating mechanism

[1]. Driving system
[2]. Moving system
[3]. Braking system
[4]. Registering system

Driving system

The driving system of the meter consists of two electro-magnets.

The core of these electromagnets is made up of silicon steel laminations. The coil of one of the electromagnets is excited by the load current. This coil is called the current coil.

The coil of second electromagnet is connected across the supply and, therefore, carries a current proportional to the supply voltage. This coil is called the pressure coil.

Consequently the two electromagnets are known as series and shunt magnets respectively.

Copper shading bands are provided on the central limb. The position of these bands is adjustable.

The function of these bands is to bring the flux produced by the shunt magnet exactly in quadrature with the applied voltage.

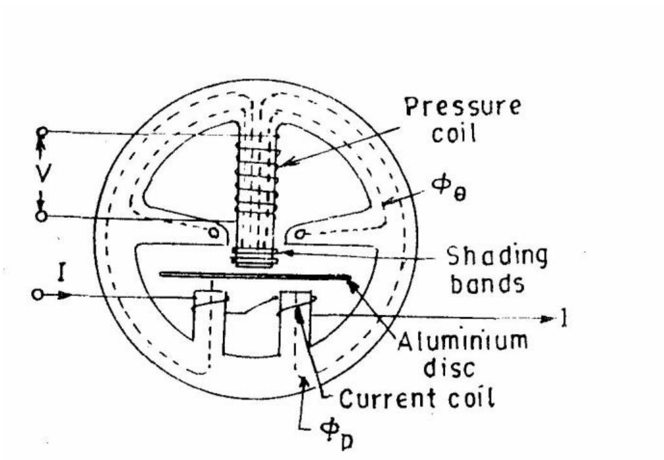
Moving System

This consists of an aluminum disc mounted on a light alloy shaft.

This disc is positioned in the air gap between series and shunt magnets. The upper bearing of the rotor (moving system) is a steel pin located in a hole in the bearing cap fixed to the top of the shaft.

The rotor runs on a hardened steel pivot, screwed to the foot of the shaft. The pivot is supported by a jewel bearing.

A pinion engages the shaft with the counting or registering mechanism.

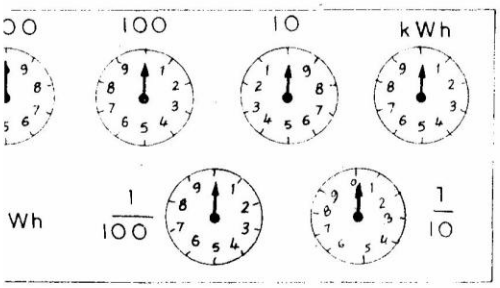


(Fig) single phase energy meter

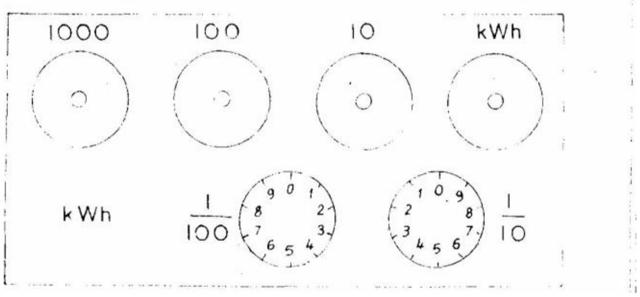
Braking System

A permanent magnet positioned near the edge of the aluminium disc forms the braking system. The aluminium disc moves in the field of this magnet and thus provides a braking torque.

The position of the permanent magnet is adjustable, and therefore braking torque can be adjusted by shifting the permanent magnet to different radial positions as explained earlier.



(fig) Pointer type



(fig) cyclometer register

Registering (counting) Mechanism

The function of a registering or counting mechanism is to record continuously a number which is proportional to the revolutions made by the moving system. By a suitable system, a train of reduction gears the pinion on the rotor shaft drives a series of five or six pointers.

These rotate on round dials which are marked with ten equal divisions.

The pointer type of register is shown in Fig. Cyclo-meter register as shown in Fig can also he used.

Errors in Single Phase Energy Meters

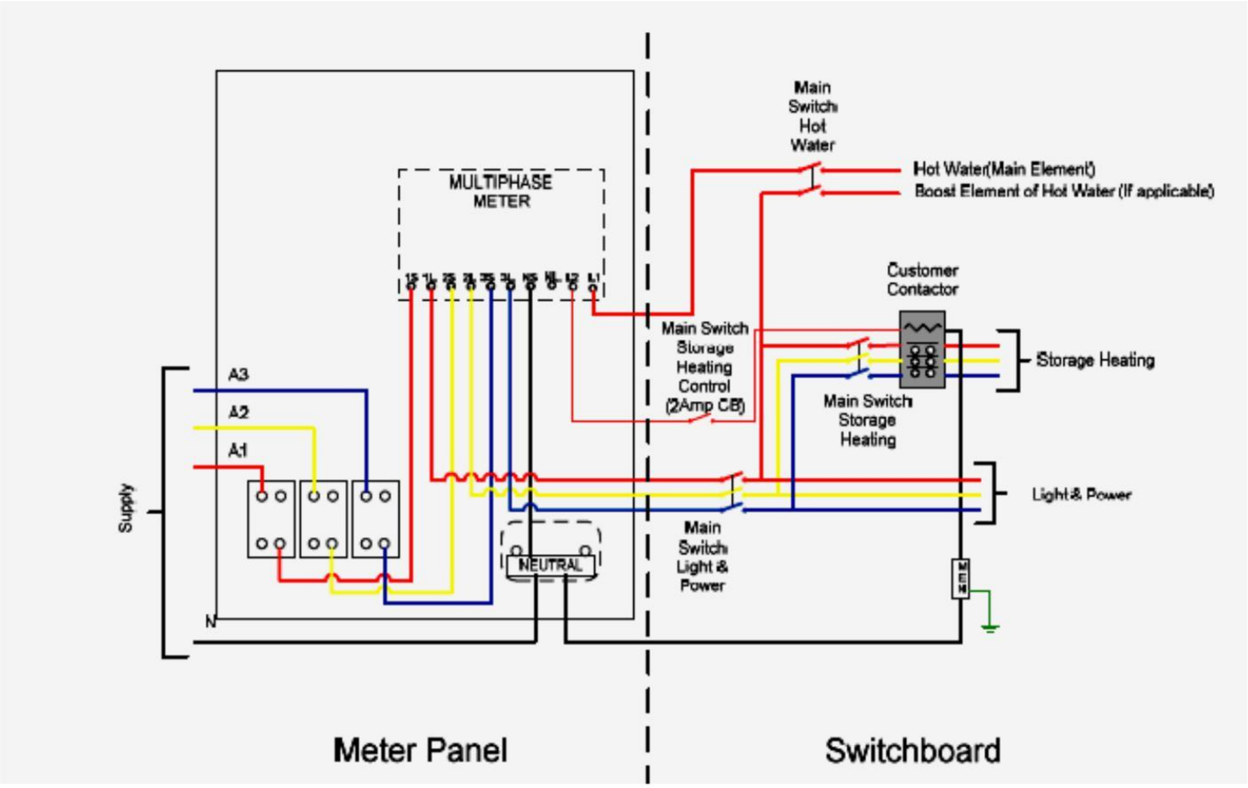
The errors caused by the driving system are

[5]. Incorrect magnitude of fluxes.
[6]. Incorrect phase angles.

[7]. Lack of Symmetry in magnetic circuit.

The errors caused by the braking system are
changes in strength of brake magnet
changes in disc resistance
[8]. abnormal friction
[9]. self braking effect

Three Phase General Supply with Controlled Load



- L1 – 30A Load Control (Hot Water)
- L2 – Maximum 2A Load Control (Storage Heating)
- 2.5mm² with 7 strands for conductors to control customer contactor
- Load carrying conductors not less than 4mm² or greater than 35mm²
- All metering neutrals to be black colour 4mm² or 6 mm² with minimum 7 stranded conductors.
- Not less than 18 strand for 25 & 35mm²
- conductors Refer to SIR’s for metering obligations
- Comply with Electrical Safety (Installations) Regulations 2009 and AS/NZS 3000
- Customer needs to provide 2A circuit breaker as a Main Switch and their load control contactor
- Within customer’s switchboard
- Meter panel fuse not required for an overhead supply.
- Off Peak controlled load only includes single phase hot water & single or multi-phase storage heating
- Wiring diagram applicable for Solar

- Metering diagram is applicable for 2 or 3 phase load.
  For 2 phase loads – Red and Blue phase is preferred.

WATTMETER

Electrodynamometer Wattmeters

These instruments are similar in design and construction to electrodynamometer type ammeters and voltmeters.

The two coils are connected in different circuits for measurement of power.

The fixed coils or " field coils" arc connected in series with the load and so carry the current in the circuit.

The fixed coils, therefore, form the current coil or simply C.C. of the wattmeter.

The moving coil is connected across the voltage and, therefore, carries a current proportional to the voltage.

A high non-inductive resistance is connected in series with the moving coil to limit the current to a small value.

Since the moving coil carries a current proportional to the voltage, it is called the ' ' pressure coil' ' or " voltage coil" or simply called P.C. of the wattmeter.

Construction of Electrodynamometer Wattmeter

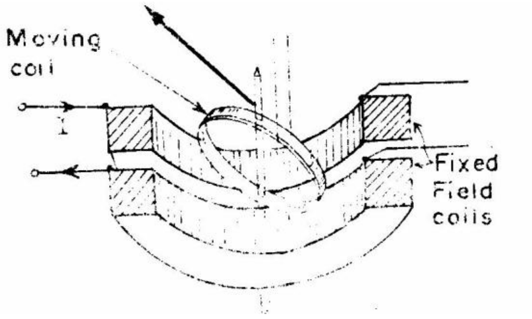
Fixed Coils

The fixed coils carry the current of the circuit.

They are divided into two halves.

The reason for using fixed coils as current coils is that they can be made more massive and can be easily constructed to carry considerable current since they present no problem of leading the current in or out.

The fixed coils are wound with heavy wire. This wire is stranded or laminated especially when carrying heavy currents in order to avoid eddy current losses in conductors. The fixed coils of earlier wattmeters were designed to carry a current of 100 A but modem designs usually limit the maximum current ranges of wattmeters to about 20 A. For power measurements involving large load currents, it is usually better to use a 5 A wattmeter in conjunction with a current transformer of suitable range.



(Fig) Dynamometer wattmeter

Damping

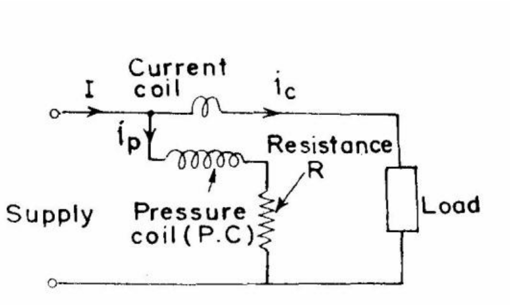
Air friction damping is used.

The moving system carries a light aluminium vane which moves in a sector shaped box. Electromagnetic or eddy current damping is not used as introduction of a permanent magnet (for damping purposes) will greatly distort the weak operating magnetic field.

Scales and Pointers

They are equipped with mirror type scales and knife edge pointers to remove reading errors due to parallax.

Theory of Electrodynamometer Watt-meters



(Fig) circuit of electrodynamometer

It is clear from above that there is a component of power which varies as twice the frequency of current and voltage (mark the term containing 2 At).

Average deflecting torque

$$T_d = \frac{1}{T}\int_0^T T_i \, d(\omega t) = \frac{1}{T}\int_0^T I_p I \left[\cos\phi - \cos(2\omega t - \phi)\right] \frac{dM}{d\theta} \, . \, d(\omega t)$$

$$= I_p I \cos\phi \, . \, dM/d\theta$$

$$= (VI/R_p) \cos\phi \, . \, dM/d\theta$$

Controlling torque exerted by springs Tc= Kș

Where, K = spring constant; ș= final steady deflection.

Errors in electrodynamometer

[10]. Errors due to inductance effects ii) Stray magnetic field errors

[11]. Eddy current errors

[12]. Temperature error

Ferrodynamic Wattmeters

The operating torque can be considerably increased by using iron cores for the coils.

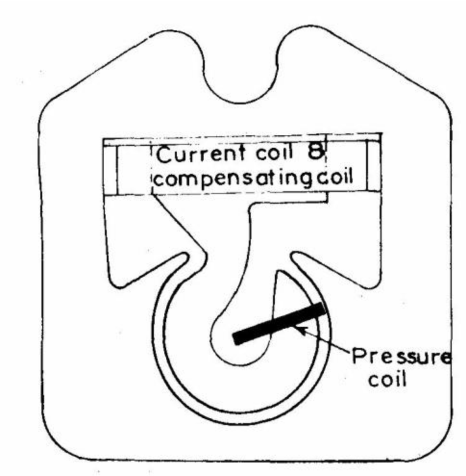
Ferrodynamic wattmeters employ cores of low loss iron so that there is a large increase in the flux density and consequently an increase in operating torque with little loss in accuracy.

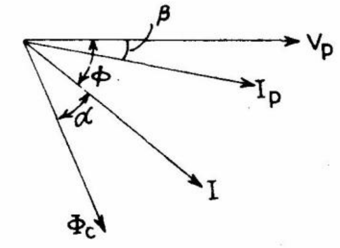
The fixed coil is wound on a laminated core having pole pieces designed to give a uniform radial field throughout the air gap.

The moving coil is asymmetrically pivoted and is placed over a hook shaped pole piece.

This type of construction permits the use of a long scale up to about 270° and gives a deflecting torque which is almost proportional to the average power. With this construction there is a tendency on the part of the pressure coil to creep (move further on the hook) when only the pressure coil is energized.

This is due to the fact that a coil tries to take up a position where it links with maximum flux. The creep causes errors and a compensating coil is put to compensate for this voltage creep.





The use of ferromagnetic core makes it possible to employ a robust construction for the moving element.

Also the Instrument is less sensitive to external magnetic fields. On the other hand, this construction introduces non-linearity of magnetization curve and introduction of large eddy current & hysteresis losses in the core.

Three Phase Wattmeters

Dynamometer type three-phase wattmeter consists of two separate wattmeter movements mounted together in one case with the two moving coils mounted on the same spindle.

The arrangement is shown in Fig.

There are two current coils and two pressure coils.

A current coil together with its pressure coil is known as an element.

Therefore, a three phase wattmeter has two elements.

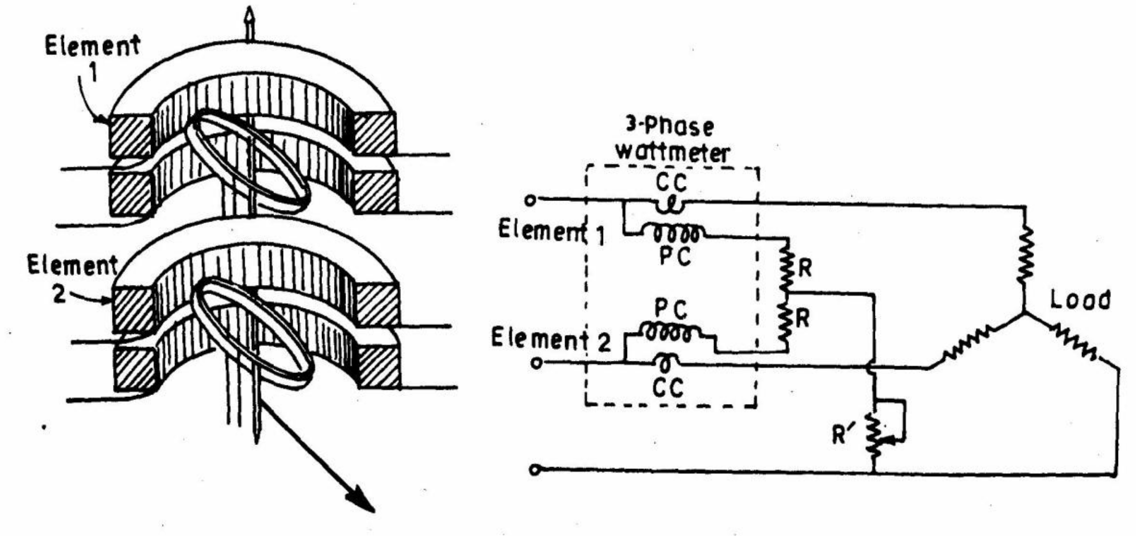
The connections of two elements of a 3 phase wattmeter are the same as that for two wattmeter method using two single phase wattmeter.

The torque on each element is proportional to the power being measured by it. The total torque deflecting the moving system is the sum of the deflecting torque of’ the two elements.

Hence the total deflecting torque on the moving system is proportional to the total Power.

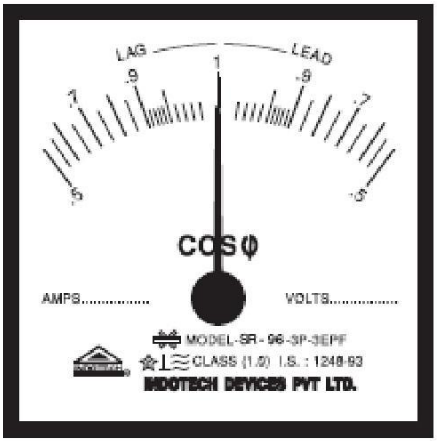
In order that a 3 phase wattmeter read correctly, there should not be any mutual interference between the two elements.

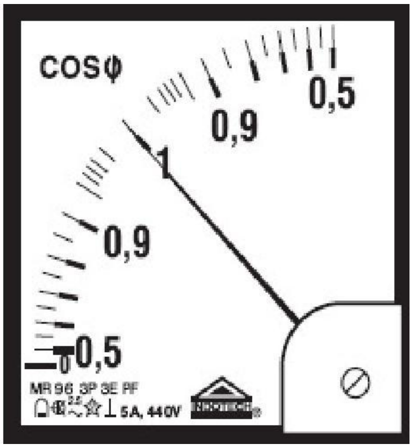
A laminated iron shield may be placed between the two elements to eliminate the mutual effects.



(fig) three phase wattmeter

Power Factor Meter





Power factor of a single phase circuit is given by
cos Φ = P/VI.
By measuring power, current and voltage power factor can be calculated using the above equation. This method is not accurate.

It is desirable to have instantaneous indication of power factor.

Power factor meter indicate directly the power factor of the circuit.

Power factor meters have: 1. Current Coil
2. Pressure Coil.

1. The current circuit carries the current whose PF is to be measured.
2. The pressure circuit is connected across the circuit whose PF is to be measured and is usually split into two paths.
3. The deflection of the instrument depends upon the phase difference between the main current and currents in two paths of pressure coil i.e., the power factor of the circuit.
4. The deflection is indicated by a pointer.

UNIT – IV
DC AND AC BRIDGES

4.1 General form of A.C. Bridge

AC bridge are similar to D.C. bridge in topology(way of connecting).It consists of four arm AB,BC,CD and DA .Generally the impedance to be measured is connected between ‘A’ and ‘B’. A detector is connected between ‘B’ and ’D’. The detector is used as null deflection instrument. Some of the arms are variable element. By varying these elements, the potential values at ‘B’ and ‘D’ can be made equal. This is called balancing of the bridge.

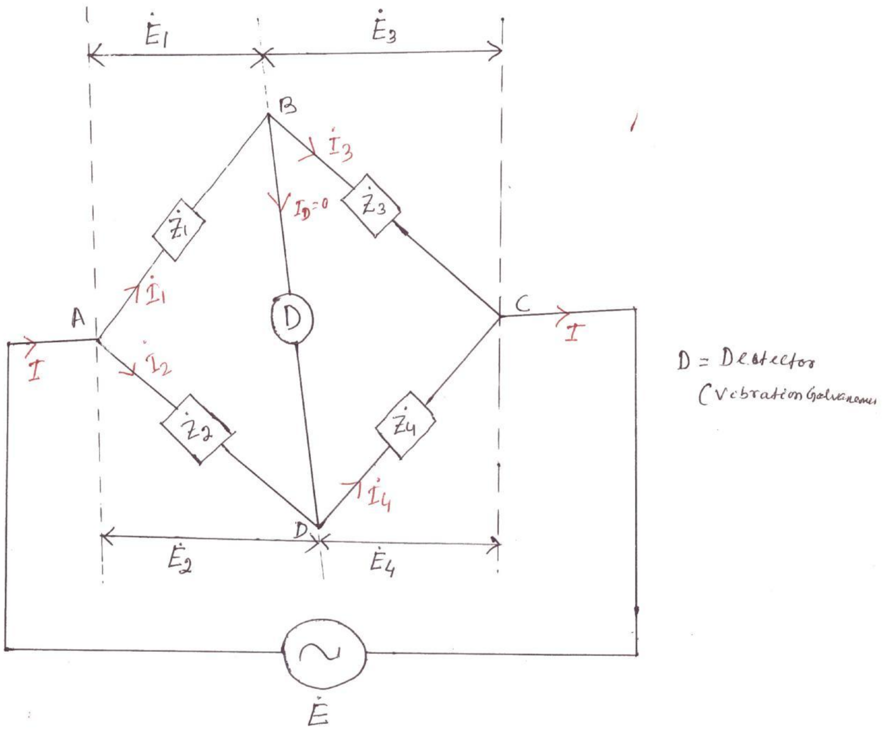


Fig. 4.1 General form of A.C. bridge

At the balance condition, the current through detector is zero.

$$\therefore \dot{I}_1 = \dot{I}_3$$

$$\dot{I}_2 = \dot{I}_4$$

$$\therefore \frac{\dot{I}_1}{\dot{I}_2} = \frac{\dot{I}_3}{\dot{I}_4} \qquad (2.1)$$

At balance condition,

Voltage drop across 'AB'=voltage drop across 'AD'.

$$\dot{E}_1 = \dot{E}_2$$

$$\therefore \dot{I}_1 \dot{Z}_1 = \dot{I}_2 \dot{Z}_2 \tag{2.2}$$

Similarly, Voltage drop across 'BC'=voltage drop across 'DC'

$$\dot{E}_3 = \dot{E}_4$$

$$\therefore \dot{I}_3 \dot{Z}_3 = \dot{I}_4 \dot{Z}_4 \tag{2.3}$$

From Eqn. (2.2), we have $\therefore \dfrac{\dot{I}_1}{\dot{I}_2} = \dfrac{\dot{Z}_2}{\dot{Z}_1}$ (2.4)

From Eqn. (2.3), we have $\therefore \dfrac{\dot{I}_3}{\dot{I}_4} = \dfrac{\dot{Z}_4}{\dot{Z}_3}$ (2.5)

From equation -2.1, it can be seen that, equation -2.4 and equation-2.5 are equal.

$$\therefore \frac{\dot{Z}_2}{\dot{Z}_1} = \frac{\dot{Z}_4}{\dot{Z}_3}$$

$$\therefore \dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$$

Products of impedances of opposite arms are equal.

$$\therefore |Z_1| \angle\theta_1 |Z_4| \angle\theta_4 = |Z_2| \angle\theta_2 |Z_3| \angle\theta_3$$

$$\Rightarrow |Z_1||Z_4| \angle\theta_1 + \theta_4 = |Z_2||Z_3| \angle\theta_2 + \theta_3$$

$$|Z_1||Z_4| = |Z_2||Z_3|$$

$$\theta_1 + \theta_4 = \theta_2 + \theta_3$$

- For balance condition, magnitude on either side must be equal.
- Angle on either side must be equal.

Summary

For balance condition,

- $\dot{I}_1 = \dot{I}_3$, $\dot{I}_2 = \dot{I}_4$
- $|Z_1||Z_4| = |Z_2||Z_3|$
- $\theta_1 + \theta_4 = \theta_2 + \theta_3$
- $\dot{E}_1 = \dot{E}_2$ & $\dot{E}_3 = \dot{E}_4$

4.2 Types of detector

The following types of instruments are used as detector in A.C. Bridge.

- Vibration galvanometer
- Head phones (speaker)
- Tuned amplifier

4.2.1 Vibration galvanometer

Between the point ‘B’ and ‘D’ a vibration galvanometer is connected to indicate the bridge balance condition. This A.C. galvanometer which works on the principle of resonance. The A.C. galvanometer shows a dot, if the bridge is unbalanced.

4.2.2 Head phones

Two speakers are connected in parallel in this system. If the bridge is unbalanced, the speaker produced more sound energy. If the bridge is balanced, the speaker do not produced any sound energy.

4.2.3 Tuned amplifier

If the bridge is unbalanced the output of tuned amplifier is high. If the bridge is balanced, output of amplifier is zero.

4.3 Measurements of inductance

4.3.1 Maxwell’s inductance bridge

The choke for which R_1 and L_1 have to measure connected between the points ‘A’ and ‘B’. In this method the unknown inductance is measured by comparing it with the standard inductance.

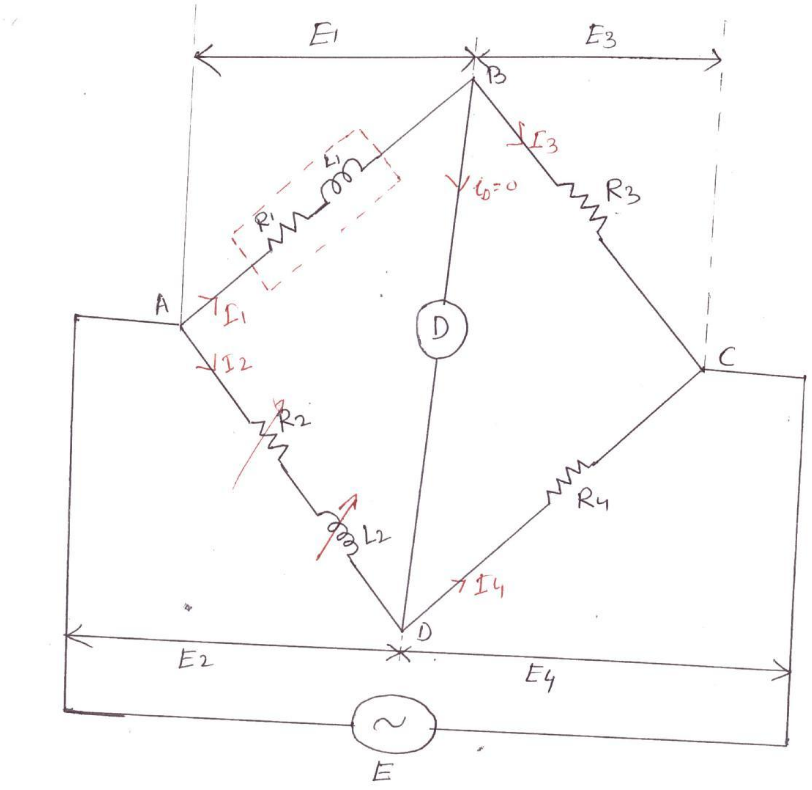


Fig. 4.2 Maxwell’s inductance bridge

L_2 is adjusted, until the detector indicates zero current.

Let R_1= unknown resistance

L_1= unknown inductance of the choke.

L_2= known standard inductance

R_1,R_2,R_4= known resistances.

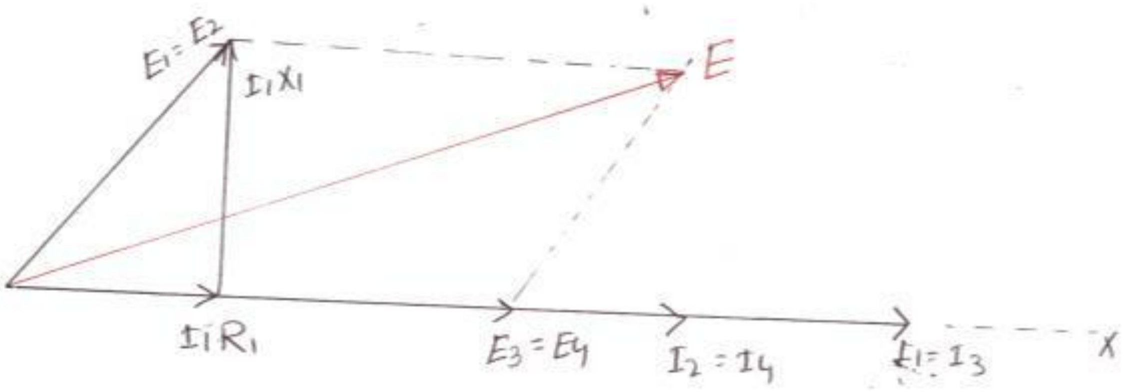


Fig 2.3 Phasor diagram of Maxwell's inductance bridge

At balance condition, $\dot{Z}_1\dot{Z}_4 = \dot{Z}_2\dot{Z}_3$

$$(R_1 + jXL_1)R_4 = (R_2 + jXL_2)R_3$$

$$(R_1 + jwL_1)R_4 = (R_2 + jwL_2)R_3$$

$$R_1R_4 + jwL_1R_4 = R_2R_3 + jwL_2R_3$$

Comparing real part,

$$R_1R_4 = R_2R_3$$

$$\therefore R_1 = \frac{R_2R_3}{R_4} \qquad (2.6)$$

Comparing the imaginary parts,

$$wL_1R_4 = wL_2R_3$$

$$L_1 = \frac{L_2R_3}{R_4} \qquad (2.7)$$

Q-factor of choke, $Q = \dfrac{WL_1}{R_1} = \dfrac{WL_2R_3R_4}{R_4R_2R_3}$

$$Q = \frac{WL_2}{R_2} \qquad (2.8)$$

Advantages

Expression for R_1 and L_1 are simple. Equations area simple

They do not depend on the frequency (as w is cancelled) R_1 and L_1 are independent of each other.

Disadvantages

Variable inductor is costly.

Variable inductor is bulky.

4.3.2 Maxwell's inductance capacitance bridge

Unknown inductance is measured by comparing it with standard capacitance. In this bridge, balance condition is achieved by varying 'C_4'.

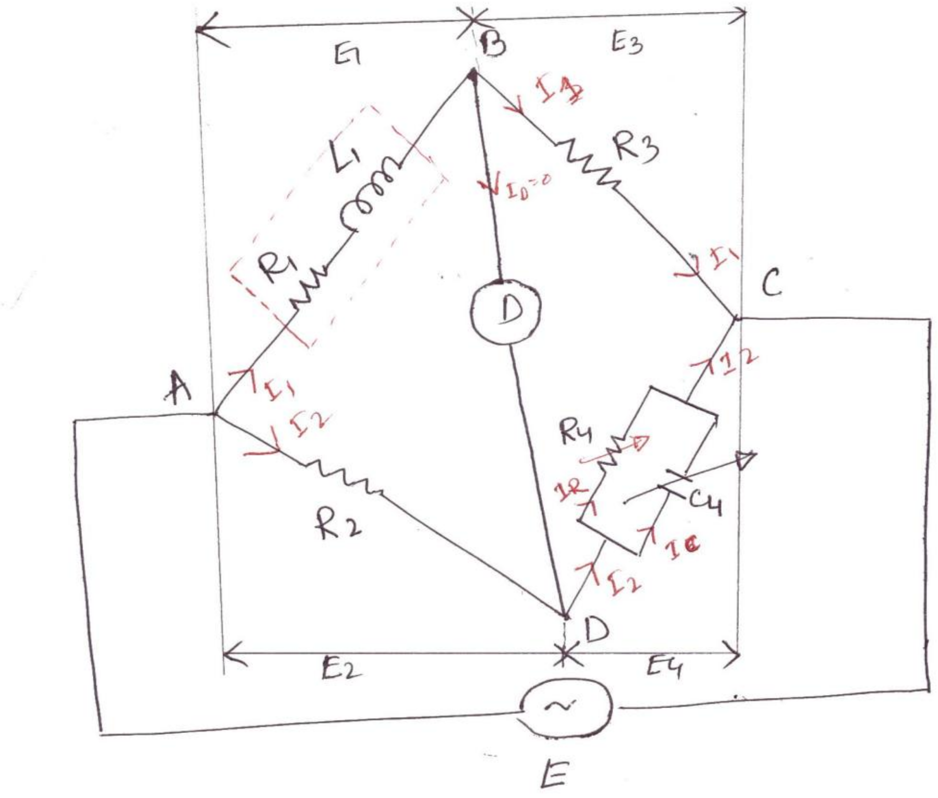


Fig 4.4 Maxwell's inductance capacitance bridge

At balance condition, $Z_1Z_4=Z_3Z_2$

$$Z_4 = R_4 \parallel \frac{1}{jwC_4} = \frac{R_4 \times \frac{1}{jwC_4}}{R_4 + \frac{1}{jwC_4}} \tag{2.9}$$

$$Z_4 = \frac{R_4}{jwR_4C_4 + 1} = \frac{R_4}{1 + jwR_4C_4}$$

∴ Substituting the value of Z_4 from eqn. (2.10) in eqn. (2.9) we get (2.10)

$$(R_1 + jwL_1) \times \frac{R_4}{1 + jwR_4C_4} = R_2 R_3$$

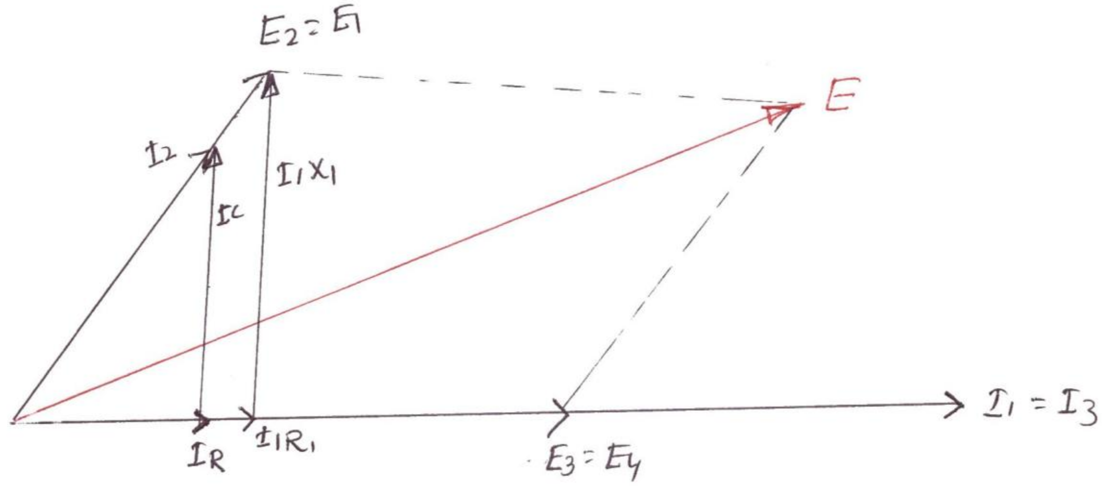


Fig 2.5 Phasor diagram of Maxwell's inductance capacitance bridge

$$(R_1 + jwL_1)R_4 = R_2 R_3 (1 + jwR_4C_4)$$

$$R_1R_4 + jwL_1R_4 = R_2 R_3 + jwC_4 R_4 R_2 R_3$$

Comparing real parts,

$$R_1R_4 = R_2 R_3$$

$$\Rightarrow R_1 = \frac{R_2 R_3}{R_4} \qquad (2.11)$$

Comparing imaginary part,

$$wL_1 R_4 = wC_4 R_4 R_2 R_3$$

$$L_1 = C_4 R_2 R_3 \qquad (2.12)$$

Q-factor of choke,

$$Q = \frac{WL_1}{R_1} = w \times C_4 R_2 R_3 \times \frac{R_4}{R_2 R_3}$$

$$Q = wC_4 R_4 \qquad (2.13)$$

Advantages

Equation of L_1 and R_1 are simple.

They are independent of frequency.

They are independent of each other.

Standard capacitor is much smaller in size than standard inductor.

Disadvantages

Standard variable capacitance is costly.

It can be used for measurements of Q-factor in the ranges of 1 to 10.

It cannot be used for measurements of choke with Q-factors more than 10.

We know that $Q = wC_4R_4$

For measuring chokes with higher value of Q-factor, the value of C_4 and R_4 should be higher. Higher values of standard resistance are very expensive. Therefore this bridge cannot be used for higher value of Q-factor measurements.

4.3.3 Hay’s bridge

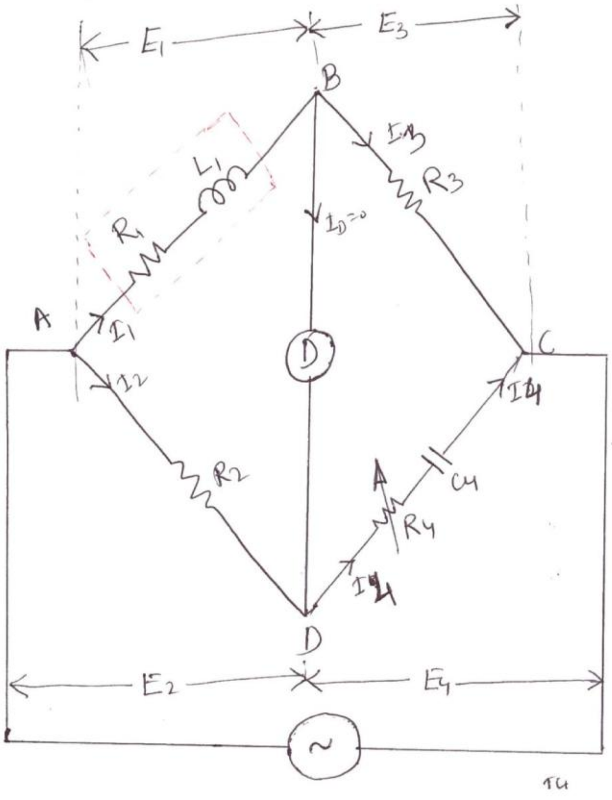


Fig 4.6 Hay’s bridge

$$\dot{E}_1 = I_1R_1 + jI_1X_1$$

$$\overset{\cdots}{E} = E_1 + E_3$$

$$\dot{E}_4 = \dot{I}_4 R_4 + \frac{I_4}{jwC_4}$$

$$\dot{E}_3 = I_3R_3$$

$$Z_4 = R_4 + \frac{1}{jwC_4} = \frac{1 + jwR_4C_4}{jwC_4}$$

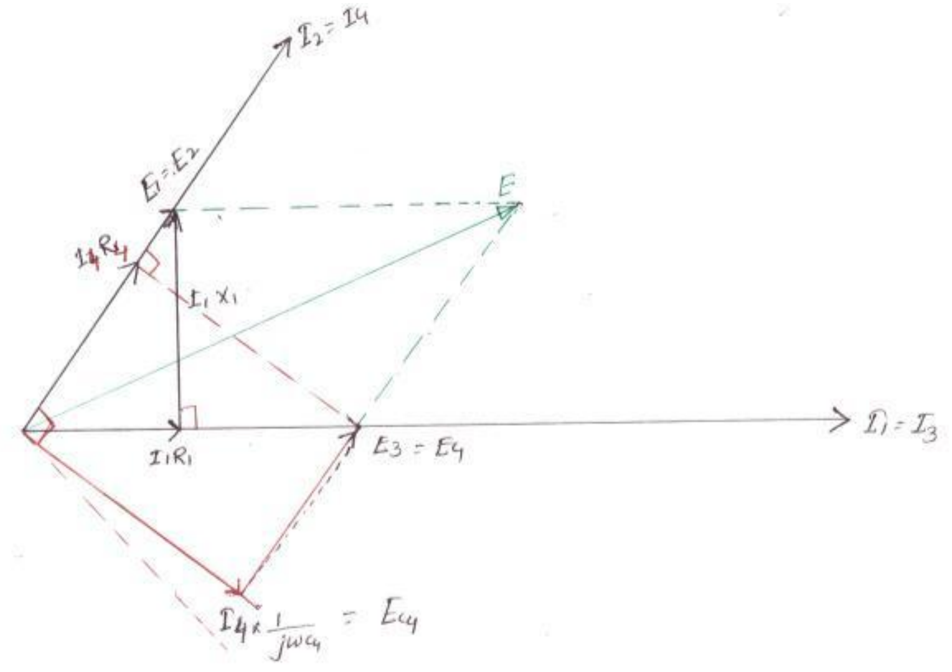


Fig 4.7 Phasor diagram of Hay's bridge

At balance condition, $Z_1Z_4=Z_3Z_2$

$$(R_1 + jwL_1)\left(\frac{1 + jwR_4C_4}{jwC_4}\right) = R_2 R_3$$

$$(R_1 + jwL_1)(1+ jwR_4C_4) = jwR_2C_4 R_3$$

$$R_1 + jwC_4R_4 R_1 + jwL_1 + j^2 w^2 L_1C_4R_4 = jwC_4 R_2 R_3$$

$$(R_1 - w^2 L_1C_4R_4) + j(wC_4 R_4R_1 + wL_1) = jwC_4 R_2R_3$$

Comparing the real term,

$$R_1 - w^2 L_1C_4 R_4 = 0$$

$$R_1 = w^2 L_1 C_4 R_4 \qquad (2.14)$$

Comparing the imaginary terms,

$$wC_4R_4R_1 + wL_1 = wC_4R_2R_3$$

$$C_4 R_4 R_1 + L_1 = C_4 R_2 R_3$$

$$L_1 = C_4 R_2 R_3 - C_4 R_4 R_1 \qquad (2.15)$$

Substituting the value of R$_1$ fro eqn. 2.14 into eqn. 2.15, we have,

$$L_1 = C_4 R_2 R_3 - C_4 R_4 \times w^2 L_1C_4 R_4$$

$$L_1 = C_4 R_2 R_3 - w^2 L_1C_4^2 R_4^2$$

$$L_1(1+ w^2 L_1C_4^2 R_4^2) = C_4 R_2 R_3$$

$$L_1 = \frac{C_4 R_2 R_3}{1+ w^2 L_1 C_4^2 R_4^2} \qquad (2.16)$$

Substituting the value of L$_1$ in eqn. 2.14 , we have

$$R \quad \underline{\qquad\qquad\qquad} \quad = w^2 C_4^2 R_2 R_3 R_4 \qquad (2.17)$$

$$Q = \frac{wL_1}{R_1} = \frac{w \times C_4 R_2 R_3}{1+ w^2 C_4^2 R_4^2} \times \frac{1+ w^2 C_4^2 R_4^2}{w^2 C_4^2 R_4 R_2 R_3}$$

$$Q = \frac{1}{wC_4 R_4} \qquad (2.18)$$

Advantages

Fixed capacitor is cheaper than variable capacitor.

This bridge is best suitable for measuring high value of Q-factor.

Disadvantages

Equations of L1and R1 are complicated.

Measurements of R_1 and L_1 require the value of frequency.

This bridge cannot be used for measuring low Q- factor.

4.3.4 Owen's bridge

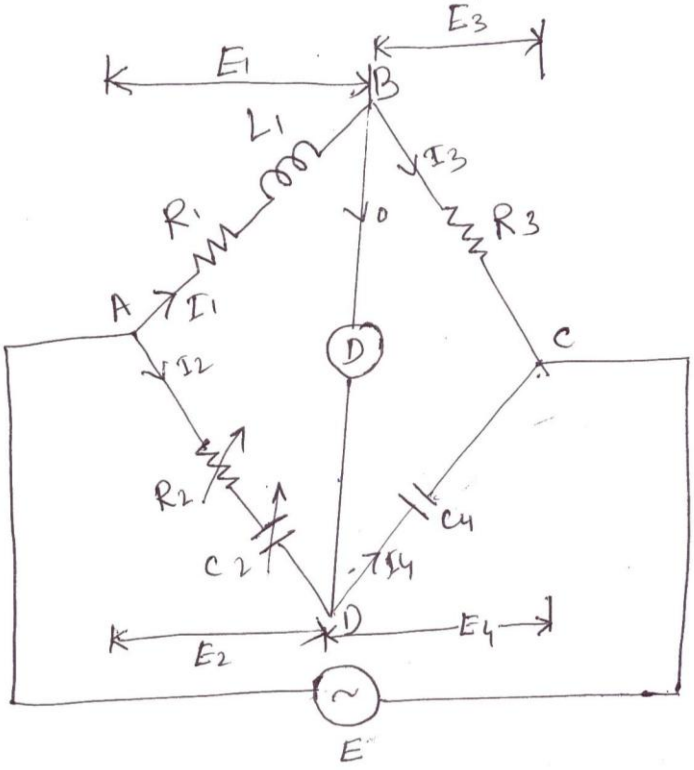


Fig 4.8 Owen's bridge

$E_1 = I_1R_1 + jI_1X_1$

I_4 leads E_4 by 90^0

$$E = \dot{E}_1 + \dot{E}_3$$

$$\dot{E}_2 = I_2 R_2 + \frac{I_2}{jwC_2}$$

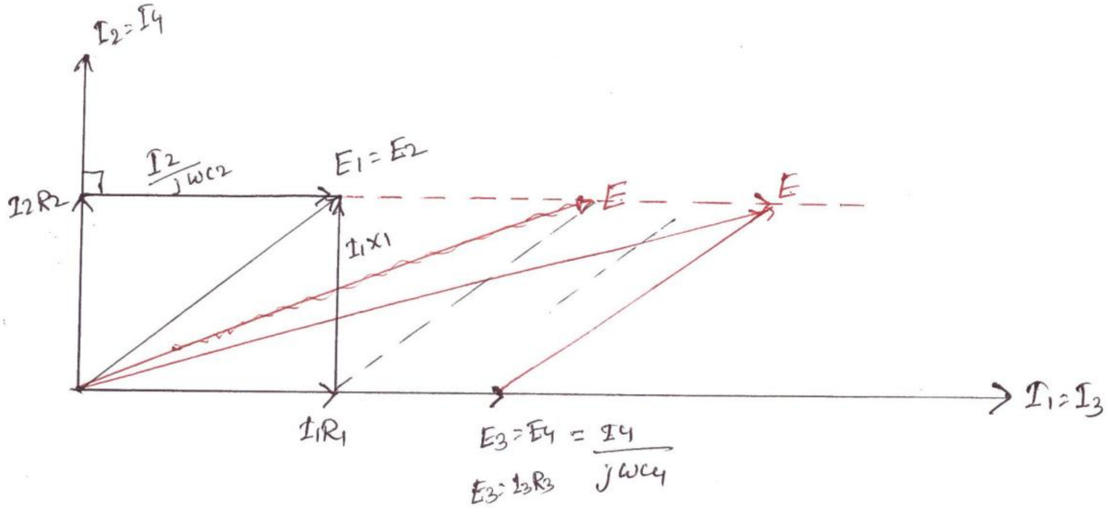


Fig 4.9 Phasor diagram of Owen’s bridge

Balance condition, $\dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$

$$Z_2 = R_2 + \frac{1}{jwC_2} = \frac{jwC_2 R_2 + 1}{jwC_2}$$

$$\therefore (R_1 + jwL_1) \times \frac{1}{jwC_4} = \frac{(1 + jwR_2C_2) \times R_3}{jwC_2}$$

$$C_2 (R_1 + jwL_1) = R_3C_4 (1 + jwR_2C_2)$$

$$R_1C_2 + jwL_1C_2 = R_3C_4 + jwR_2C_2R_3C_4$$

Comparing real terms,

$$R_1C_2 = R_3C_4$$

$$R_1 = \frac{R_3 C_4}{C_2}$$

Comparing imaginary terms,

$$wL_1C_2 = wR_2C_2\, R_3C_4$$

$$L_1 = R_2\, R_3C_4$$

$$\text{Q- factor} = \frac{WL_1}{R_1} = \frac{wR_2\, R_3C_4C_2}{R_3C_4}$$

$$Q = wR_2C_2$$

Advantages

Expression for R_1 and L_1 are simple.

R_1 and L_1 are independent of Frequency.

Disadvantages

The Circuits used two capacitors.

Variable capacitor is costly.

Q-factor range is restricted.

4.3.5 Anderson’s bridge

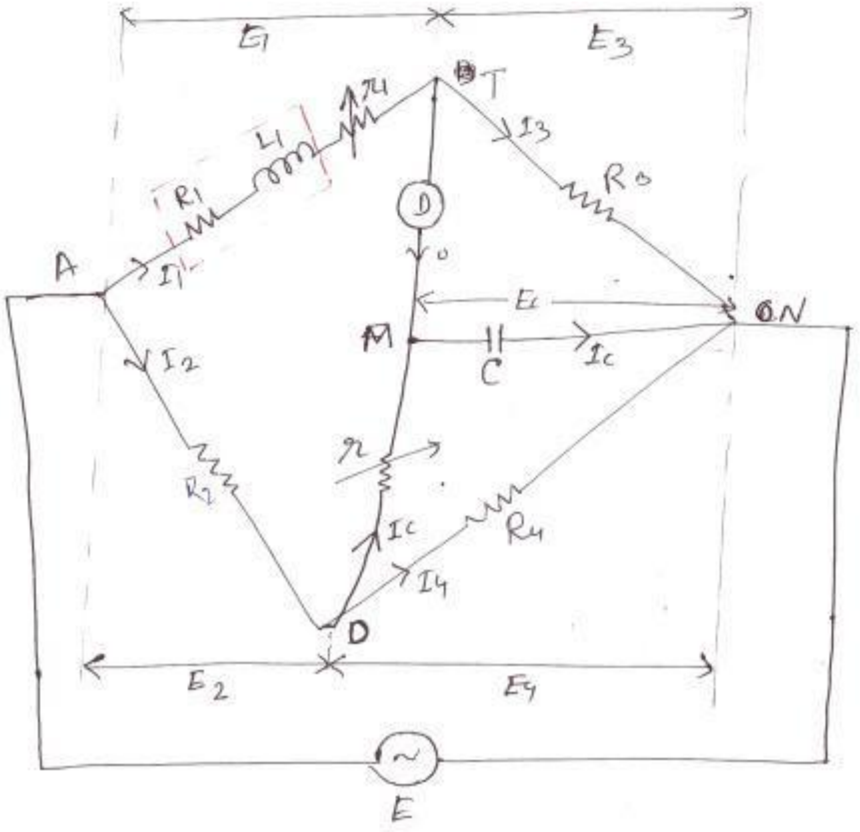


Fig 4.10 Anderson’s bridge

$E_1 = I_1(R_1 + r_1) +$

jI_1X_1 $E_3 = E_C$

$E_4 = I_C r + E_C$

$I_2 = I_4 + I_C$

$E_2 + E_4 = E$

$E_1 + E_3 = E$

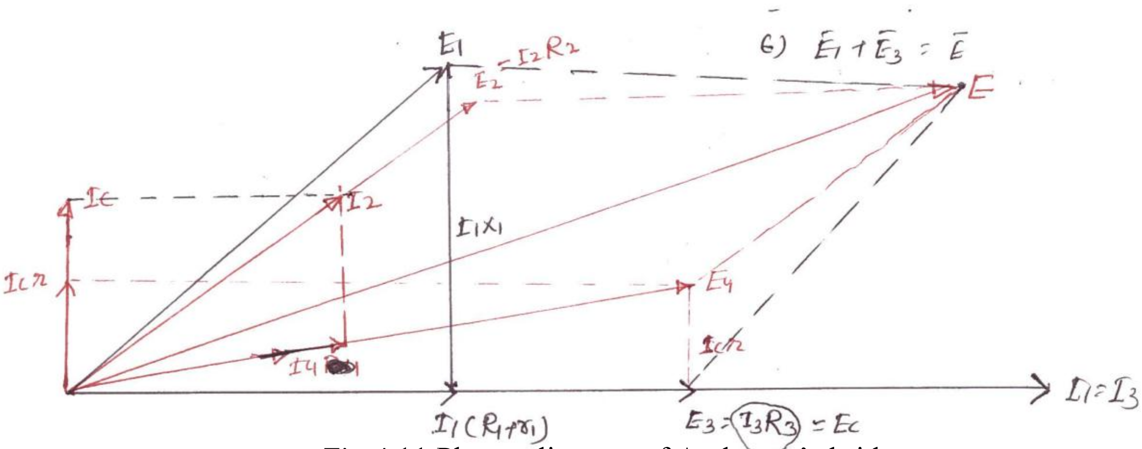


Fig 4.11 Phasor diagram of Anderson's bridge

Step-1 Take I_1 as references vector .Draw $I_1R_1^1$ in phase with I_1

$R_1^1 = (R_1 + r_1)$, I_1X_1 is ⊥r to $I_1R_1^1$

$E_1 = I_1R_1^1 + jI_1X_1$

Step-2 $I_1 = I_3$,E_3 is in phase with I_3 , From the circuit ,

$E_3 = E_C$, I_C leads E_C by 90^0

Step-3 $E_4 = I_C r + E_C$

Step-4 Draw I_4 in phase with E_4 , By KCL , $\bar{I}_2 = \bar{I}_4 + \bar{I}_C$

Step-5 Draw E_2 in phase with I_2

Step-6 By KVL , $\bar{E}_1 + \bar{E}_3 = \bar{E}$ or $\bar{E}_2 + \bar{E}_4 = \bar{E}$

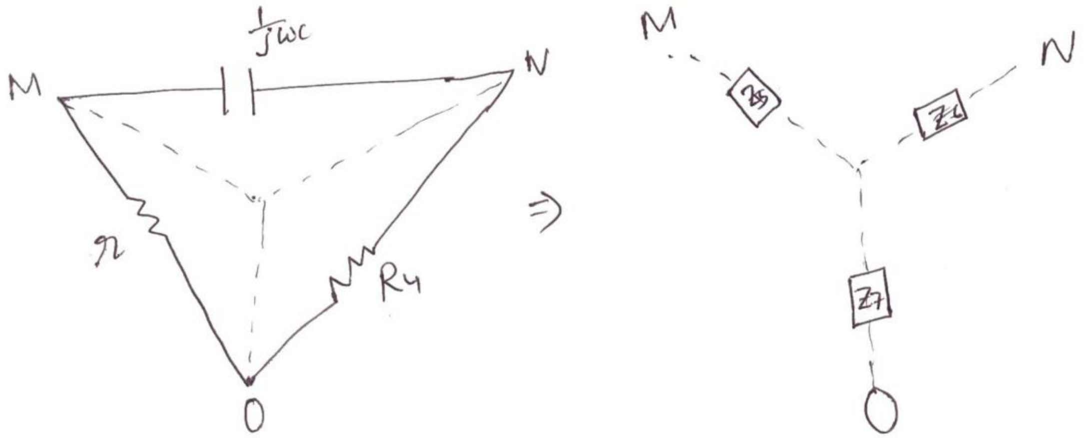


Fig 4.12 Equivalent delta to star conversion for the loop MON

$$Z_7 = \frac{R_4 \times r}{R_4 + r + \frac{1}{jwc}} = \frac{jwCR_4 r}{1 + jwC(R_4 + r)}$$

$$Z_6 = \frac{R_4 \times \frac{1}{jwC}}{R_4 + r + \frac{1}{jwc}} = \frac{R_4}{1 + jwC(R_4 + r)}$$

$$(R_1^1 + jwL_1) \times \frac{R_4}{1 + jwC(R_4 + r)} = R_3 \left(R_2 + \frac{jwCR_4 r}{1 + jwC(R_4 + r)}\right)$$

$$\Rightarrow \frac{(R_1^1 + jwL_1)R_4}{1 + jwC(R_4 + r)} = R_3 \frac{R_2(1 + jwC(R_4 + r)) + jwCrR_4}{1 + jwC(R_4 + r)}$$

$$\Rightarrow R_1^1 R_4 + jwL_1 R_4 = R_2 R_3 + jCwR_2 R_3 (r + R_4) + jwCrR_4 R_3$$

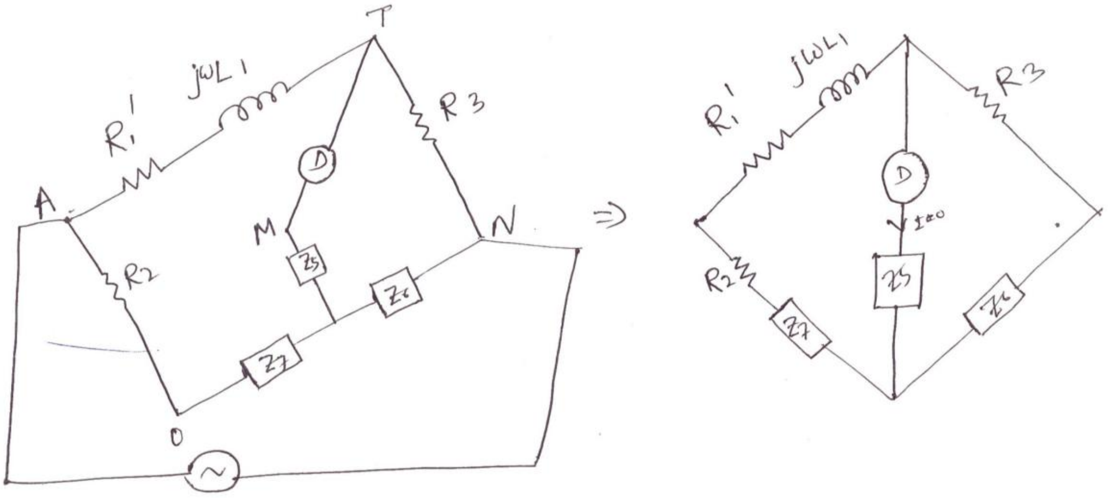


Fig 4.13 Simplified diagram of Anderson’s bridge

Comparing real term,

$$R_1^{1}R_4 = R_2R_3$$

$$(R_1 + r_1)R_4 = R_2 R_3$$

$$R_1 = \frac{R_2 R_3}{R_4} - r_1$$

Comparing the imaginary term,

$$wL_1R_4 = wCR_2 R_3 (r + R_4) + wcrR_3R_4$$

$$L_1 = \frac{R_2 R_3C}{R_4}(r + R_4) + R_3rC$$

$$L_1 = R_3 C \frac{R_2}{R_4}(r + R_4) + r$$

Advantages

Variable capacitor is not required.

Inductance can be measured accurately.

R_1 and L_1 are independent of frequency.

Accuracy is better than other bridges.

Disadvantages

Expression for R_1 and L_1 are complicated.

This is not in the standard form A.C. bridge.

4.4 Measurement of capacitance and loss angle. (Dissipation factor)

4.4.1 Dissipation factors (D)

A practical capacitor is represented as the series combination of small resistance and ideal capacitance.

From the vector diagram, it can be seen that the angle between voltage and current is slightly less than 90^0. The angle ‘ δ ’ is called loss angle.

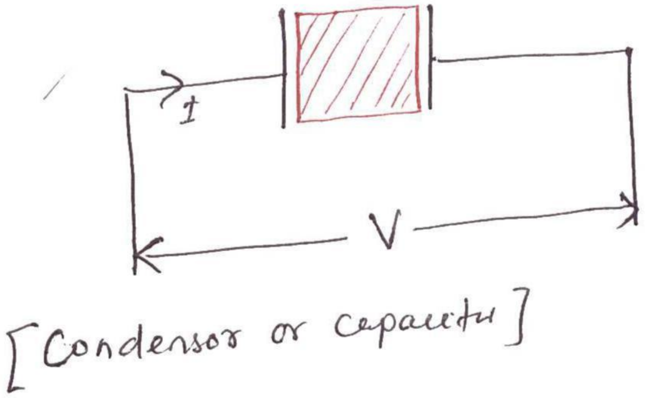


Fig 4.14 Condensor or capacitor

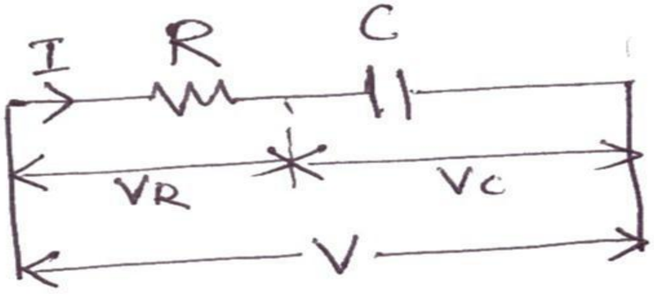


Fig 4.15 Representation of a practical capacitor

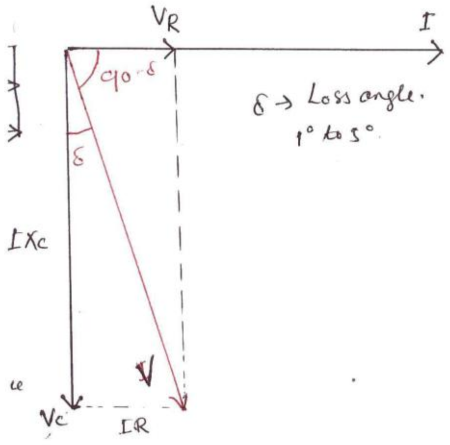


Fig 4.16 Vector diagram for a practical capacitor

A dissipation factor is defined as 'tan δ'.

$$\therefore \tan\delta = \frac{IR}{IX_C} = \frac{R}{X_C} = wCR$$

$$D = wCR$$

$$D = \frac{1}{Q}$$

$D = \tan\delta = \dfrac{\sin\delta}{\cos\delta} \cong \dfrac{\delta}{1}$ For small value of 'δ' in radians

$D \cong \delta \cong$ Loss Angle ('δ' must be in radian)

4.4.2 Desauty's Bridge

C_1= Unknown capacitance
At balance condition,

$$\frac{1}{jwC_1} \times R_4 = \frac{1}{jwC_2} \times R_3$$

$$\frac{R_4}{C_1} = \frac{R_3}{C_2}$$

$$\Rightarrow C_1 = \frac{R_4 C_2}{R_3}$$

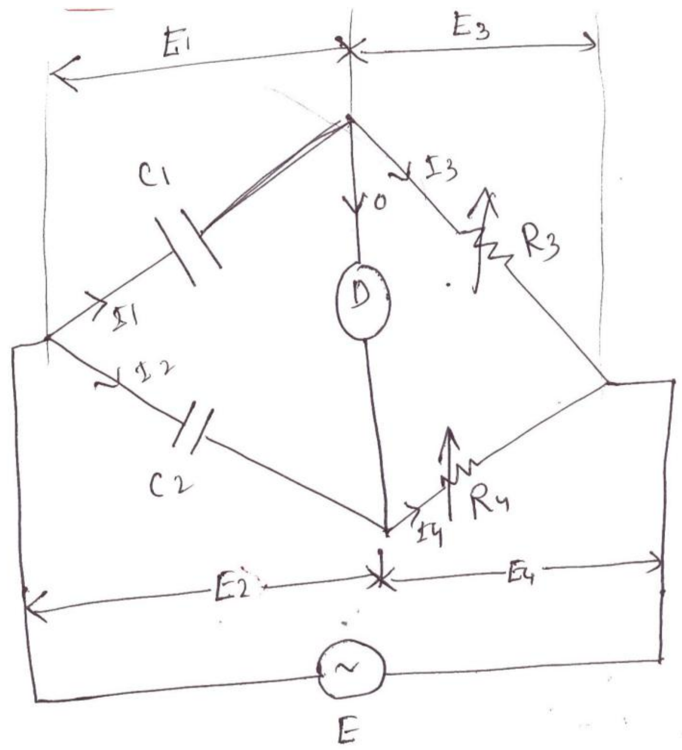


Fig 4.17 Desauty's bridge

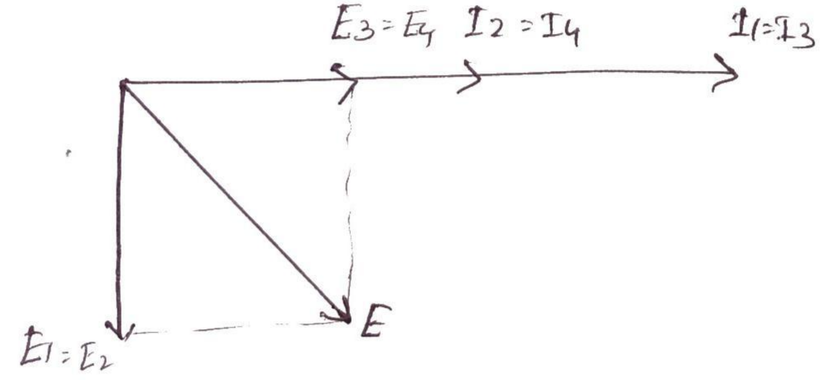


Fig 4.18 Phasor diagram of Desauty's bridge

4.4.3 Modified desauty's bridge

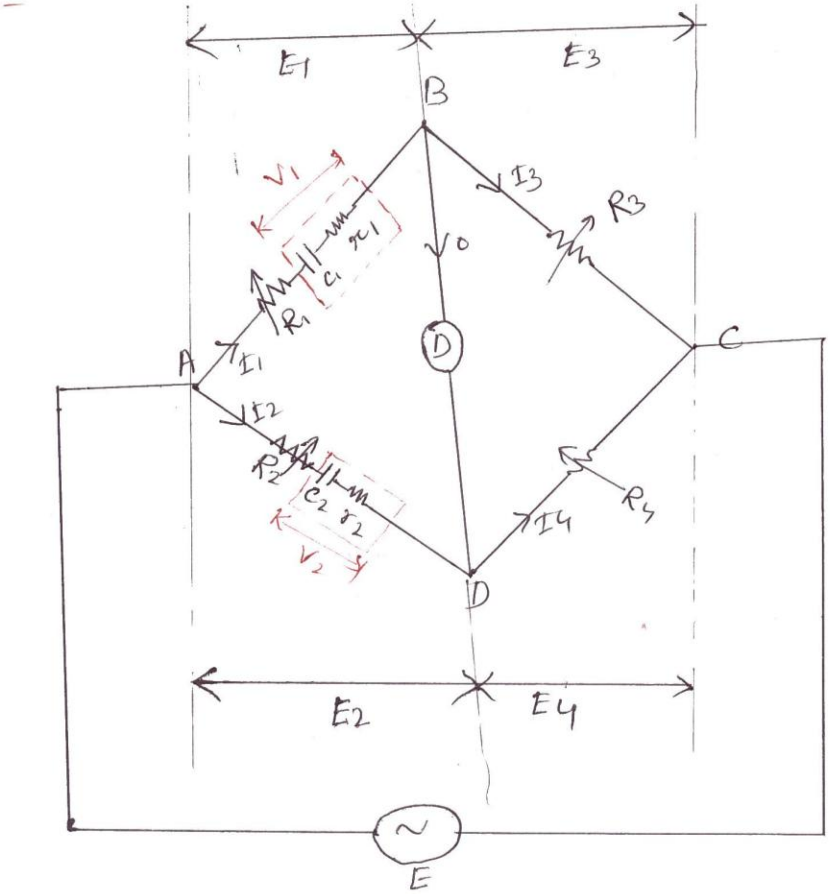


Fig 4.19 Modified Desauty's bridge

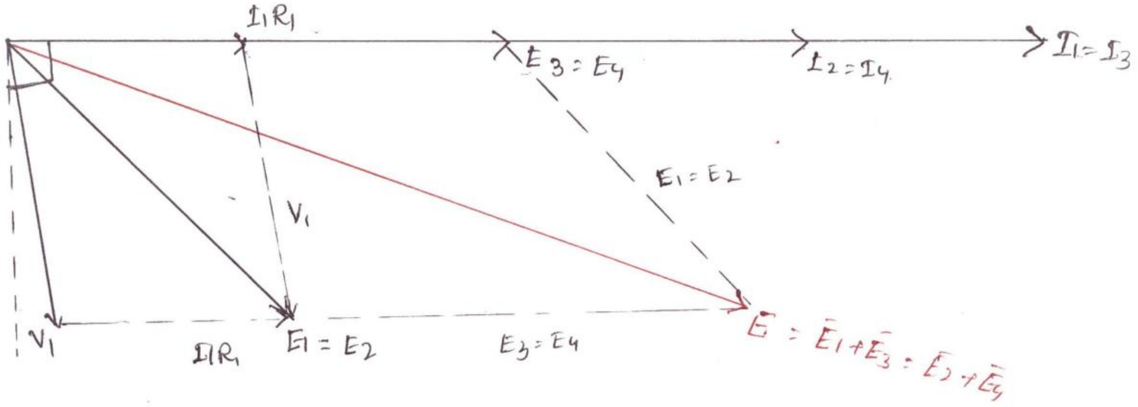


Fig 4.20 Phasor diagram of Modified Desauty's bridge

$$R_1^1 = (R_1 + r_1)$$

$$R_2^1 = (R_2 + r_2)$$

At balance condition, $(R_1^1 + \frac{1}{jwC_1})R_4 = R_3(R_2^1 + \frac{1}{jwC_2})$

$$R_1^1 R_4 + \frac{R_4}{jwC_1} = R_3 R_2^1 + \frac{R_3}{jwC_2})$$

Comparing the real term, $R_1^1 R_4 = R_3 R_2^1$

$$R_1^1 = \frac{R_3 R_2^1}{R_4}$$

$$R + r_1 = \frac{(R_2 + r_2)R_3}{R_4}$$

Comparing imaginary term,

$$\frac{R_4}{wC_1} = \frac{R_3}{wC_2}$$

$$C_1 = \frac{R_4 C_2}{R_3}$$

Dissipation factor D=wC_1r_1

Advantages

- r_1 and c_1 are independent of frequency.
- They are independent of each other.
- Source need not be pure sine wave.

4.4.4 Schering bridge

$$E_1 = I_1 r_1 - jI_1 X_4$$

C_2 = C_4= Standard capacitor (Internal resistance=0)

C_4= Variable capacitance.

C_1= Unknown capacitance.

r_1= Unknown series equivalent resistance of the capacitor.

R_3=R_4= Known resistor.

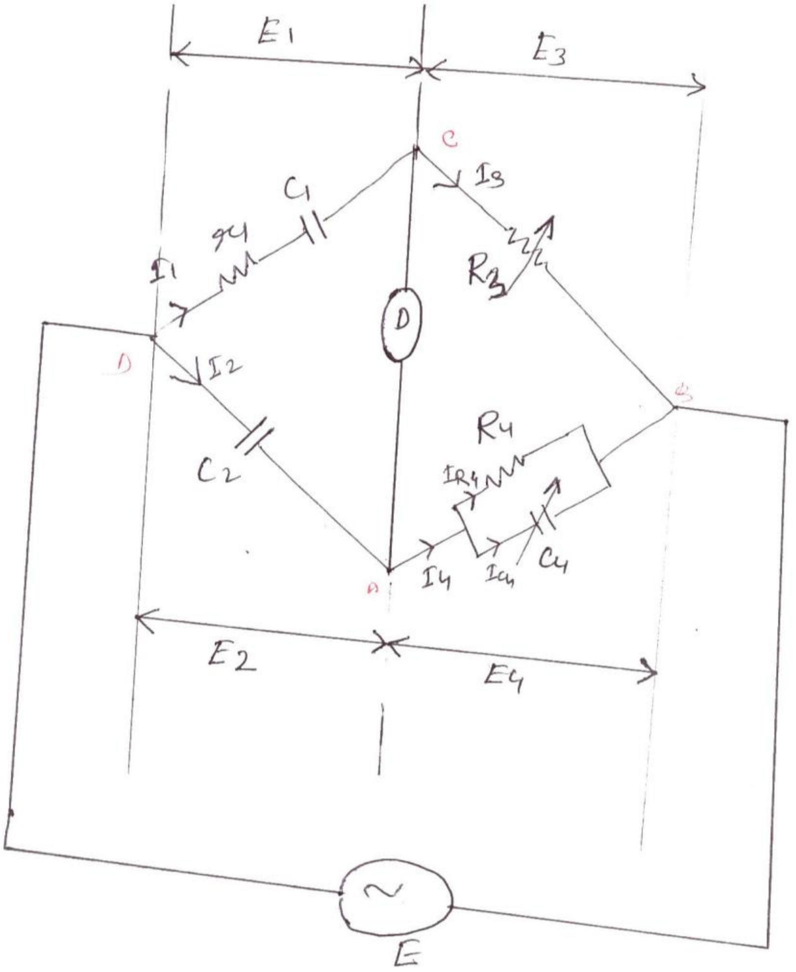


Fig 4.21 Schering bridge

$$Z_1 = r_1 + \frac{1}{jwC_1} = \frac{jwC_1 r_1 + 1}{jwC_1}$$

$$Z_4 = \frac{R_4 \times \frac{1}{jwC_4}}{R_4 + \frac{1}{jwC_4}} = \frac{R_4}{1 + jwC_4 R_4}$$

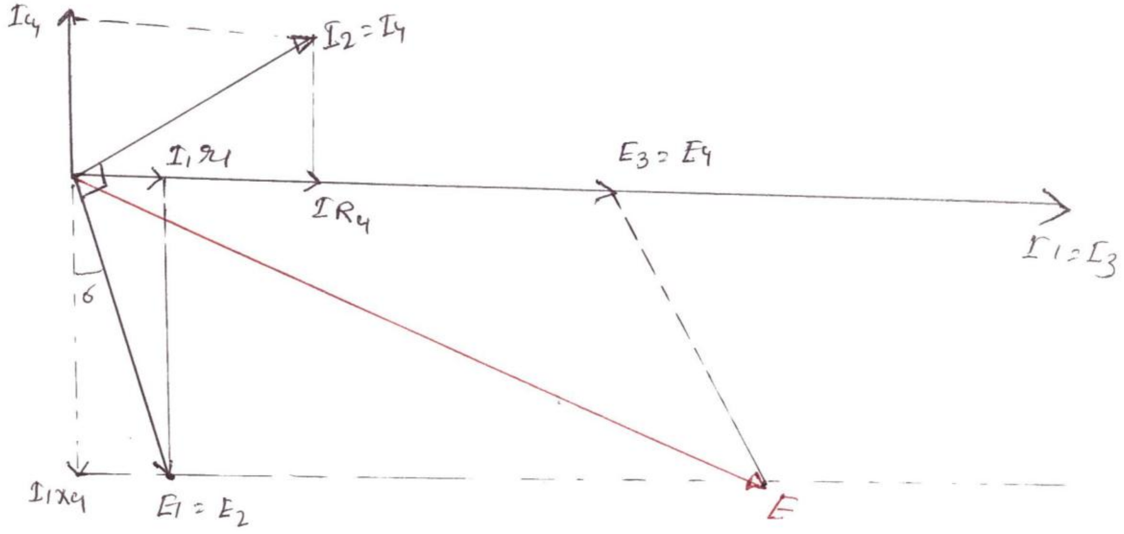


Fig 4.22 Phasor diagram of Schering bridge

At balance condition, $\dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$

$$\frac{1+jwC_1r_1}{jwC_1} \times \frac{R_4}{1+jwC_4R_4} = \frac{R_3}{jwC_2}$$

$$(1+jwC_1r_1)R_4C_2 = R_3C_1(1+jwC_4r_4)$$

$$R_2C_2 + jwC_1r_1R_4C_2 = R_3C_1 + jwC_4R_4R_3C_1$$

Comparing the real part,

$$\therefore C_1 = \frac{R_4C_2}{R_3}$$

Comparing the imaginary part,

$$wC_1r_1R_4C_2 = wC_4R_3R_4C_1$$

$$r_1 = \frac{C_4R_3}{C_2}$$

Dissipation factor of capacitor,

$$D = wC_1 r_1 = w \times \frac{R_4 C_2 \times C_4 R_3}{R_3 C_2}$$

$$\therefore D = wC_4 R_4$$

Advantages

In this type of bridge, the value of capacitance can be measured accurately. It can measure capacitance value over a wide range.

It can measure dissipation factor accurately.

Disadvantages

It requires two capacitors.

Variable standard capacitor is costly.

4.5 Measurements of frequency

24.5.1 Wein's bridge

Wein's bridge is popularly used for measurements of frequency of frequency. In this bridge, the value of all parameters are known. The source whose frequency has to measure is connected as shown in the figure.

$$Z_1 = r_1 + \frac{1}{jwC_1} = \frac{jwC_1 r_1 + 1}{jwC_1}$$

$$Z_2 = \frac{R_2}{1 + jwC_2 R_2}$$

At balance condition, $\dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$

$$\frac{jwC_1 r_1 + 1}{jwC_1} \times R_4 = \frac{R_2}{1 + jwC_2 R_2} \times R_3$$

$$(1 + jwC_1 r_1)(1 + jwC_2 R_2) R_4 = R_2 R_3 \times jwC_1$$

$$\left[1 + jwC_2 R_2 + jwC_1 r_1 - w^2 C_1 C_2 r_1 R_2\right] = jwC_1 \frac{R_2 R_3}{R_4}$$

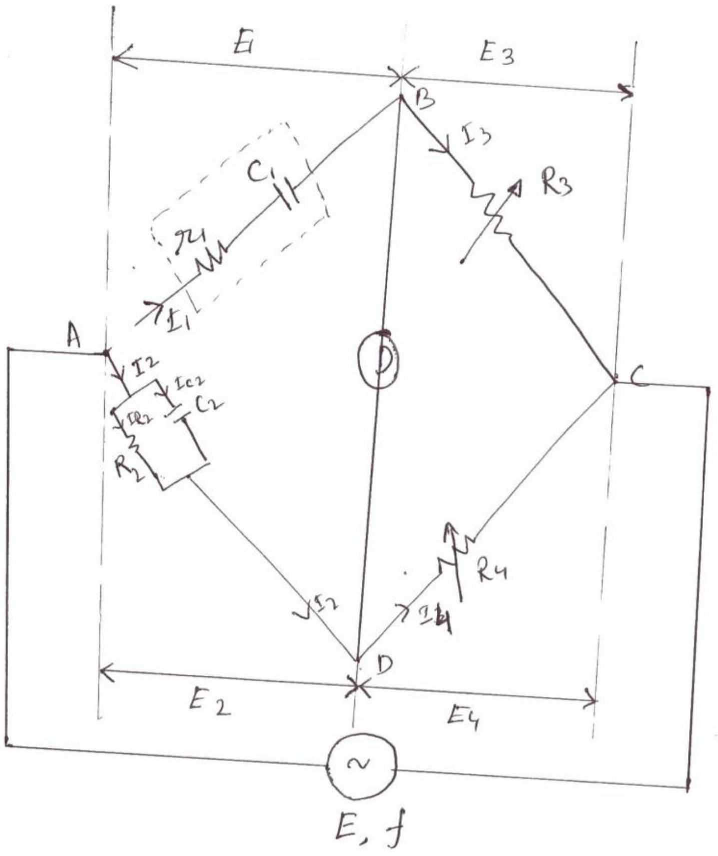


Fig 4.23 Wein's bridge

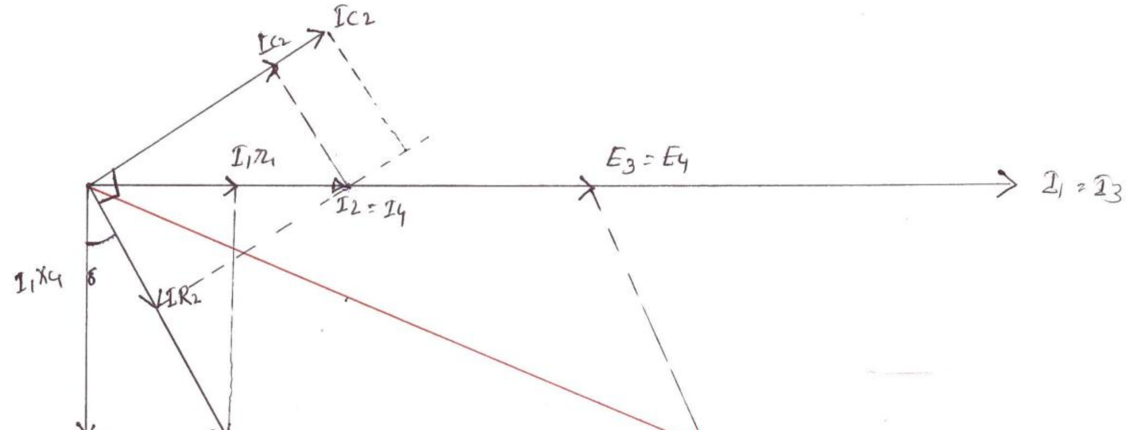


Fig 4.24 Phasor diagram of Wein's bridge

Comparing real term,

$$1 - w^2 C_1 C_2 r_1 R_2 = 0$$

$$w^2 C_1 C_2 r_1 R_2 = 1$$

$$w^2 = \frac{1}{C_1 C_2 r_1 R_2}$$

$$w = \frac{1}{\sqrt{C_1 C_2 r_1 R_2}}, \quad f = \frac{1}{2\Pi \sqrt{C_1 C_2 r_1 R_2}}$$

NOTE

The above bridge can be used for measurements of capacitance. In such case, r_1 and C_1 are unknown and frequency is known. By equating real terms, we will get R_1 and C_1. Similarly by equating imaginary term, we will get another equation in terms of r_1 and C_1. It is only used for measurements of Audio frequency.

A.F=20 HZ to 20 KHZ

R.F=>> 20 KHZ

Comparing imaginary term,

$$wC_2 R_2 + wC_1 r_1 = wC_1 \frac{R_2 R_3}{R_4}$$

$$C_2 R_2 + C_1 r_1 = \frac{C_1 R_2 R_3}{R_4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2.19)$$

$$C_1 = \frac{1}{w^2 C_2 r_1 R_2}$$

Substituting in eqn. (2.19), we have

$$C_2 R_2 + \frac{r_1}{w^2 C_2 r_1 R_2} = \frac{R_2 R_3}{R_4} C_1$$

Multiplying $\frac{R_4}{R_2 R_3}$ in both sides, we have

$$C_2 R_2 \times \frac{R_4}{R_2 R_3} + \frac{1}{w^2 C_2 R_2} \times \frac{R_4}{R_2 R_3} = C_1$$

$$C_1 = \frac{C_2 R_4}{R_3} + \frac{R_4}{w^2 C_2 R_2^2 R_3}$$

$$w^2 C_1 r_1 C_2 R_2 = 1$$

$$r_1 = \frac{1}{w^2 C_2 R_2 C_1} = \frac{1}{w^2 C_2 R_2 \left(\frac{C_2 R_4}{R_3} + \frac{R_4}{w^2 C_2 R_2^2 R_3}\right)}$$

$$= \frac{1}{\frac{w^2 C_2^2 R_2 R_4}{R_3} + \frac{R_4}{R_2 R_3}}$$

$$\therefore r_1 = \frac{1}{\frac{R_3}{R_4}\left(w^2 C_2^2 R_2 + \frac{1}{R_2}\right)}$$

$$\therefore r_1 = \frac{R_3}{R_4} \frac{1}{\left(w^2 C_2^2 R_2 + \frac{1}{R_2}\right)}$$

4.5.2 High Voltage Schering Bridge

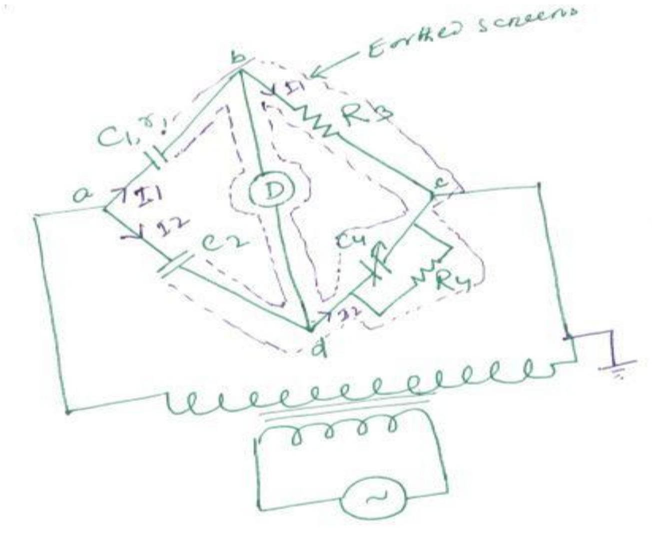


Fig 4.25 High Voltage Schering bridge

(1) The high voltage supply is obtained from a transformer usually at 50 HZ.

4.6 Wagner earthing device:

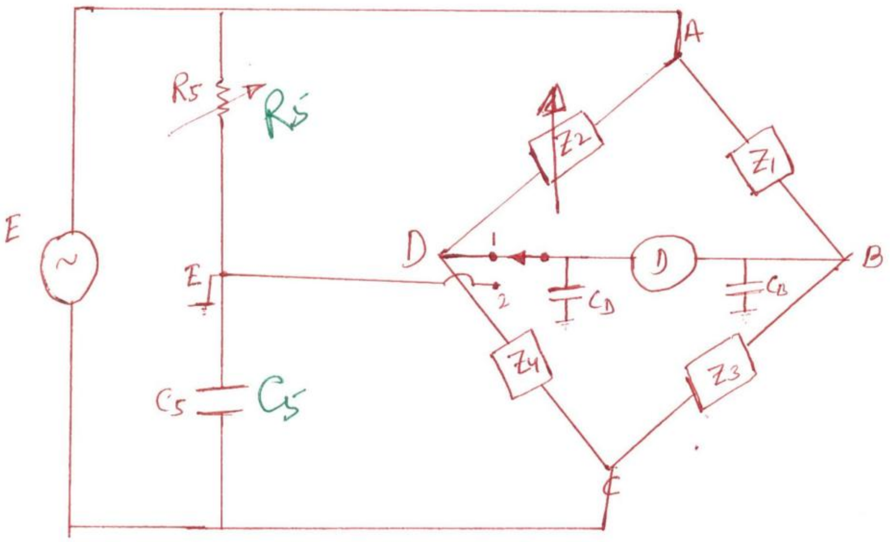


Fig 4.26 Wagner Earthing device

Wagner earthing consists of ‘R’ and ‘C’ in series. The stray capacitance at node ‘B’ and ‘D’ are C_B, C_D respectively. These Stray capacitances produced error in the measurements of ‘L’ and ‘C’. These error will predominant at high frequency. The error due to this capacitance can be eliminated using wagner earthing arm.

Close the change over switch to the position (1) and obtained balanced. Now change the switch to position (2) and obtained balance. This process has to repeat until balance is achieved in both the position. In this condition the potential difference across each capacitor is zero. Current drawn by this is zero. Therefore they do not have any effect on the measurements.

What are the sources of error in the bridge measurements?

Error due to stray capacitance and inductance.
Due to external field.

Leakage error: poor insulation between various parts of bridge can produced this error.
Eddy current error.

Frequency error.

Waveform error (due to harmonics)

Residual error: small inductance and small capacitance of the resistor produce this error.

Precaution

The load inductance is eliminated by twisting the connecting the connecting lead.

In the case of capacitive bridge, the connecting lead are kept apart.($QC = \frac{A \in 0 \in r}{)\, d}$

In the case of inductive bridge, the various arm are magnetically screen.

In the case of capacitive bridge, the various arm are electro statically screen to reduced the stray capacitance between various arm.

To avoid the problem of spike, an inter bridge transformer is used in between the source and bridge.

The stray capacitance between the ends of detector to the ground, cause difficulty in balancing as well as error in measurements. To avoid this problem, we use wagner earthing device.

4.7 Ballastic galvanometer

This is a sophisticated instrument. This works on the principle of PMMC meter. The only difference is the type of suspension is used for this meter. Lamp and glass scale method is used to obtain the deflection. A small mirror is attached to the moving system. Phosphorous bronze wire is used for suspension.

When the D.C. voltage is applied to the terminals of moving coil, current flows through it. When a current carrying coil kept in the magnetic field, produced by permanent magnet, it experiences a force. The coil deflects and mirror deflects. The light spot on the glass scale also move. This deflection is proportional to the current through the coil.

$i = \frac{Q}{t}$, $Q = it = +idt$

$\theta \propto Q$, deflection ∝ Charge

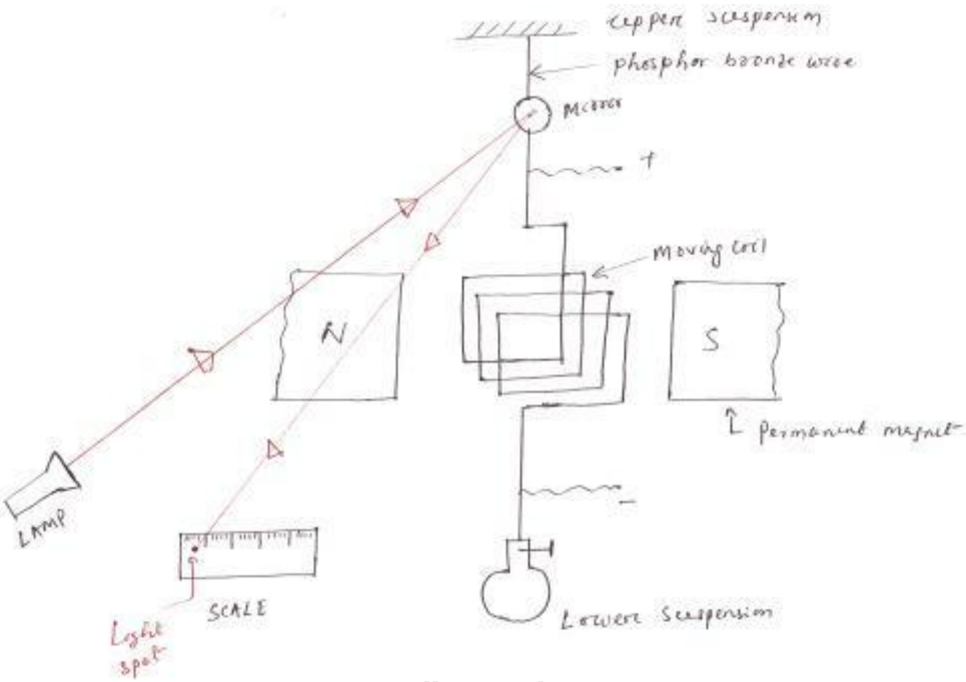


Fig 4.27 Ballastic galvanometer

4.8 Measurements of flux and flux density (Method of reversal)

D.C. voltage is applied to the electromagnet through a variable resistance R_1 and a reversing switch. The voltage applied to the toroid can be reversed by changing the switch from position 2 to position ‘1’. Let the switch be in position ‘2’ initially. A constant current flows through the toroid and a constant flux is established in the core of the magnet.

A search coil of few turns is provided on the toroid. The B.G. is connected to the search coil through a current limiting resistance. When it is required to measure the flux, the switch is changed from position ‘2’ to position ‘1’. Hence the flux reduced to zero and it starts increasing in the reverse direction. The flux goes from + φ to - φ , in time ‘t’ second. An emf is induced in the search coil, science the flux changes with time. This emf circulates a current through R_2 and B.G. The meter deflects. The switch is normally closed. It is opened when it is required to take the reading.

4.8.1 Plotting the BH curve

The curve drawn with the current on the X-axis and the flux on the Y-axis, is called magnetization characteristics. The shape of B-H curve is similar to shape of magnetization characteristics. The residual magnetism present in the specimen can be removed as follows.

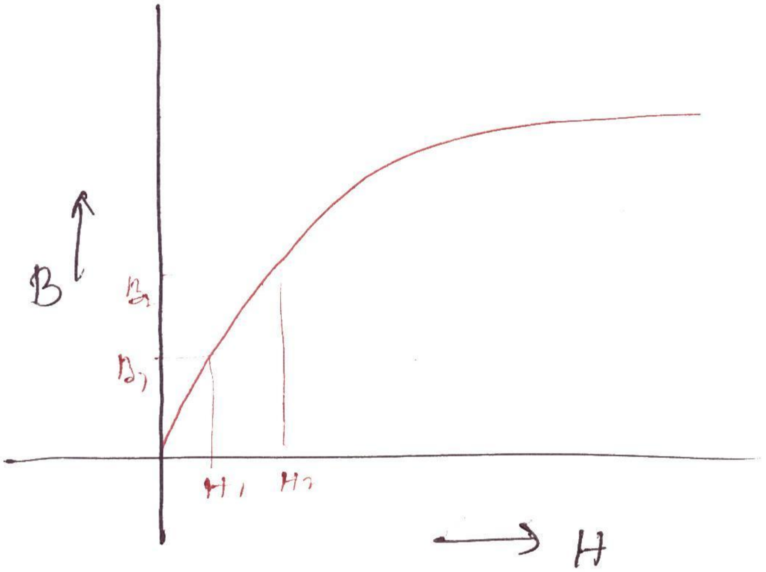


Fig 4.28 BH curve

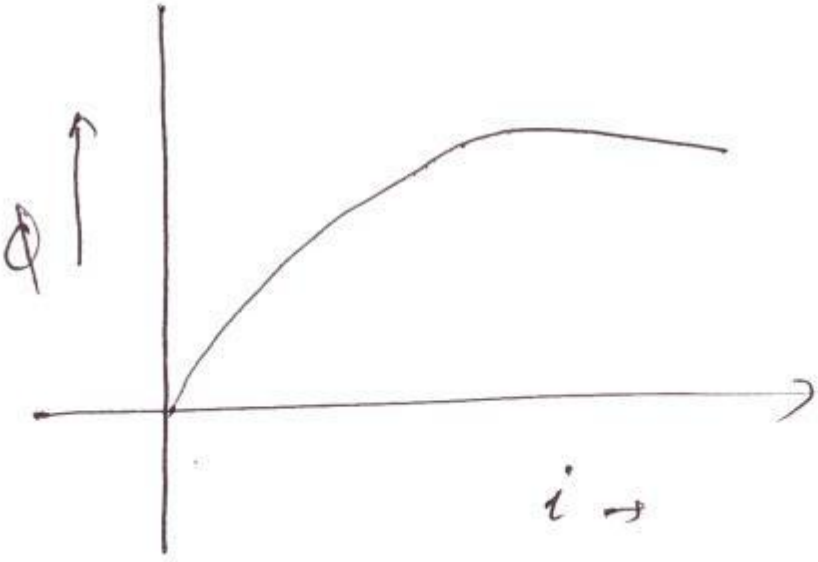


Fig 4.29 Magnetization characteristics

Close the switch 'S_2' to protect the galvanometer, from high current. Change the switch S1 from position '1' to '2' and vice versa for several times.

To start with the resistance 'R_1' is kept at maximum resistance position. For a particular value of current, the deflection of B.G. is noted. This process is repeated for various value of current. For each deflection flux can be calculated.($B = \frac{\varphi}{A}$)

Magnetic field intensity value for various current can be calculated.().The B-H curve can be plotted by using the value of 'B' and 'H'.

4.8.2 Measurements of iron loss:

Let R_P= pressure coil resistance

R_S = resistance of coil S1

E= voltage reading= Voltage induced in S_2

I= current in the pressure coil

V_P= Voltage applied to wattmeter pressure coil.

W= reading of wattmeter corresponding voltage V

W_1= reading of wattmeter corresponding voltage E

$$W \rightarrow V \qquad \frac{W_1}{W} = \frac{E}{V} \Rightarrow W_1 = \frac{E \times W}{V}$$
$$W_1 \rightarrow E_P$$

W1=Total loss=Iron loss+ Cupper loss.

The above circuit is similar to no load test of transformer.

In the case of no load test the reading of wattmeter is approximately equal to iron loss. Iron loss depends on the emf induced in the winding. Science emf is directly proportional to flux. The voltage applied to the pressure coil is V. The corresponding of wattmeter is 'W'. The iron loss corresponding E is $E = \frac{WE}{V}$. The reading of the wattmeter includes the losses in the pressure coil and copper loss of the winding S1. These loses have to be subtracted to get the actual iron loss.

4.9 Galvanometers

D-Arsonval Galvanometer

Vibration Galvanometer

Ballistic C

4.9.1 D-arsonval galvanometer (d.c. galvanometer)

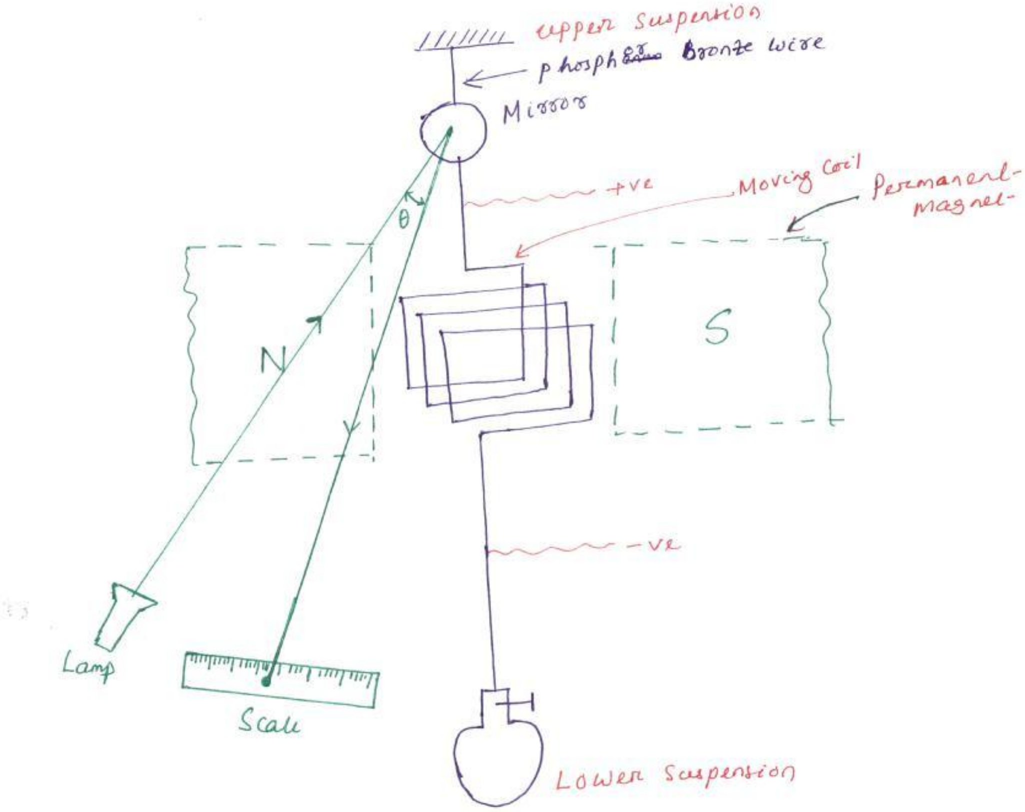


Fig 4.30 D-Arsonval Galvanometer

Galvanometer is a special type of ammeter used for measuring □ A or mA. This is a sophisticated instrument. This works on the principle of PMMC meter. The only difference is the type of suspension used for this meter. It uses a sophisticated suspension called taut suspension, so that moving system has negligible weight.

Lamp and glass scale method is used to obtain the deflection. A small mirror is attached to the moving system. Phosphors bronze is used for suspension.

When D.C. voltage is applied to the terminal of moving coil, current flows through it. When current carrying coil is kept in the magnetic field produced by P.M. , it experiences a force. The light spot on the glass scale also move. This deflection is proportional to the current through the coil. This instrument can be used only with D.C. like PMMC meter.

The deflecting Torque,

T_D=BINA

T_D=GI, Where G=BAN

$T_C=K_S\theta=S\theta$

At balance, $T_C=T_D \Rightarrow S\theta=GI$

$$\therefore \theta = \frac{GI}{S}$$

Where G= Displacements constant of Galvanometer

S=Spring constant

4.9.2 Vibration Galvanometer (A.C. Galvanometer)

The construction of this galvanometer is similar to the PMMC instrument except for the moving system. The moving coil is suspended using two ivory bridge pieces. The tension of the system can be varied by rotating the screw provided at the top suspension. The natural frequency can be varied by varying the tension wire of the screw or varying the distance between ivory bridge piece.

When A.C. current is passed through coil an alternating torque or vibration is produced. This vibration is maximum if the natural frequency of moving system coincide with supply frequency. Vibration is maximum, science resonance takes place. When the coil is vibrating , the mirror oscillates and the dot moves back and front. This appears as a line on the glass scale. Vibration galvanometer is used for null deflection of a dot appears on the scale. If the bridge is unbalanced, a line appears on the scale

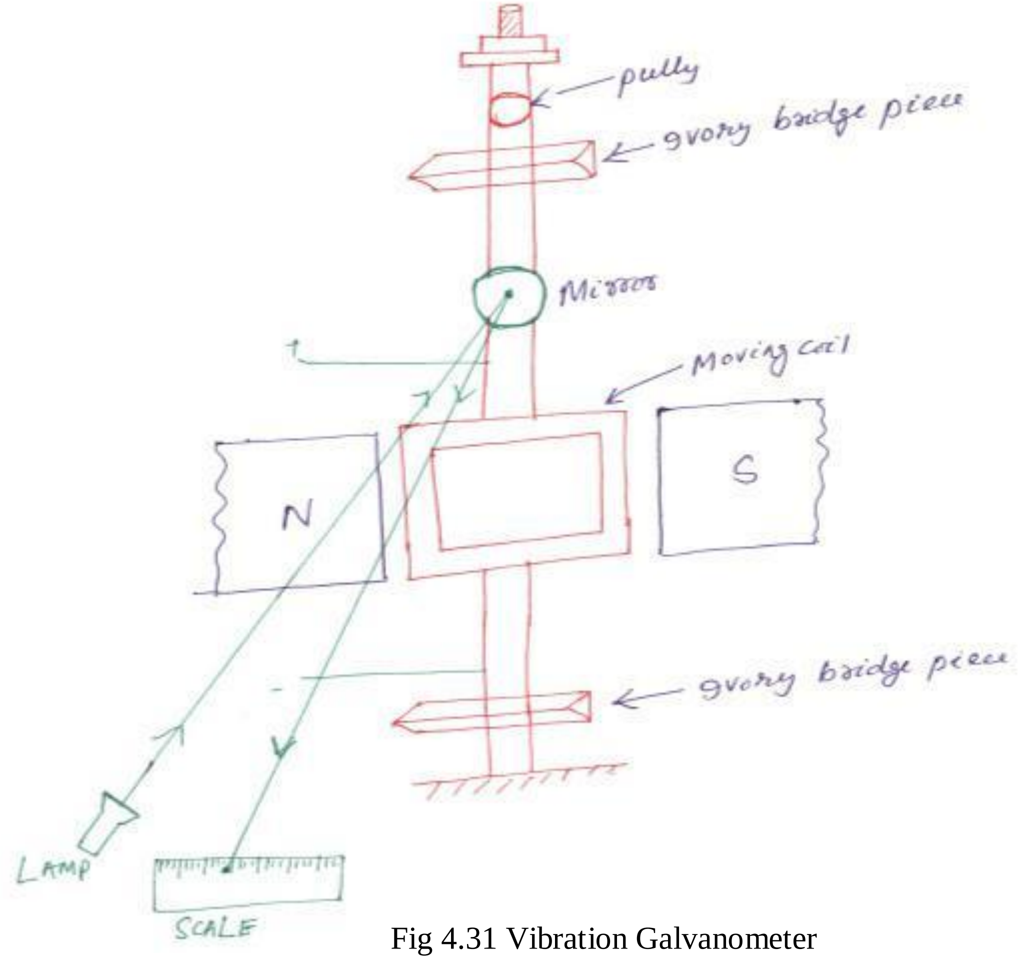


Fig 4.31 Vibration Galvanometer

Example 2.2-In a low- Voltage Schering bridge designed for the measurement of permittivity, the branch 'ab' consists of two electrodes between which the specimen under test may be inserted, arm 'bc' is a non-reactive resistor R_3 in parallel with a standard capacitor C_3, arm CD is a non-reactive resistor R_4 in parallel with a standard capacitor C_4, arm 'da' is a standard air capacitor of capacitance C_2. Without the specimen between the electrode, balance is obtained with following values , $C_3=C_4=120$ pF, $C_2=150$ pF, $R_3=R_4=5000\Omega$.With the specimen inserted, these values become $C_3=200$ pF,$C_4=1000$ pF,$C_2=900$ pF and $R_3=R_4=5000\Omega$. In such test w=5000 rad/sec. Find the relative permittivity of the specimen?

Sol: Relative permittivity (εr) $= \dfrac{\text{capacitance measured with given medium}}{\text{capacitance measured with air medium}}$

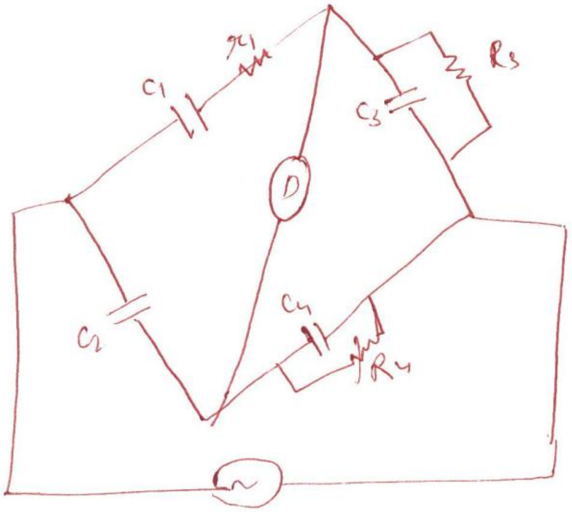


Fig 4.32 Schering Bridge

$$C_1 = C_2 \left(\frac{R_4}{R_3}\right)$$

Let capacitance value C_0, when without specimen dielectric.

Let the capacitance value C_S when with the specimen dielectric.

$$C_0 = C_2 \left(\frac{R_4}{R_3}\right) = 150 \times \frac{5000}{5000} = 150\ pF$$

$$C_S = C_2 \left(\frac{R_4}{R_3}\right) = 900 \times \frac{5000}{5000} = 900\ pF$$

$$\varepsilon r = \frac{C_S}{C_0} = \frac{900}{150} = 6$$

Example 4.3- A specimen of iron stamping weighting 10 kg and having a area of 16.8 cm^2 is tested by an episten square. Each of the two winding S_1 and S_2 have 515 turns. A.C. voltage of 50 HZ frequency is given to the primary. The current in the primary is 0.35 A. A voltmeter connected to S_2 indicates 250 V. Resistance of S_1 and S_2 each equal to 40 Ω. Resistance of pressure coil is 80 kΩ. Calculate maximum flux density in the specimen and iron loss/kg if the wattmeter indicates 80 watt?

Sol^n- $E = 4.44 f\varphi_m N$

$$B_m = \frac{E}{4.44 fAN} = 1.3 wb/m^2$$

Iron loss= $W(1+\frac{R_S}{R_P}) - \frac{E^2}{(R_S + R_P)}$

$$= 80(1 + \frac{40}{80\times10^3}) - \frac{250^2}{(40 + 80\times10^3)} = 79.26 watt$$

Iron loss/ kg=79.26/10=7.926 w/kg.

UNIT – V
TRANSDUCERS AND OSCILLOSCOPES

TRANSDUCERS

[1]. The input quantity for most instrumentation systems is nonelectrical. In order to use electrical methods and techniques for measurement, the nonelectrical quantity is converted into a proportional electrical signal by a device called transducer.

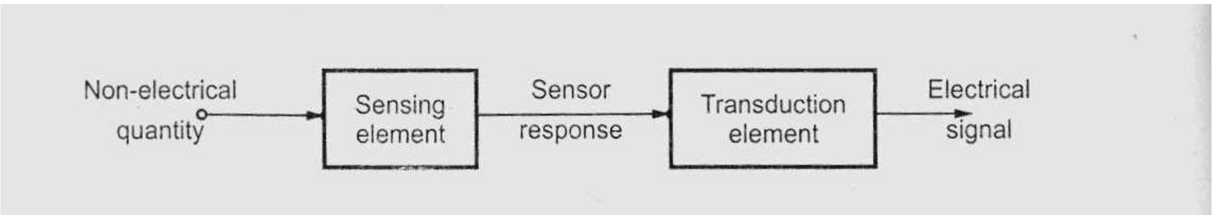
[2]. Another definition states that transducer is a device which when actuated by energy in one system, supplies energy in the same form or in another form to a second system.

[3]. When transducer gives output in electrical form it is known as electrical transducer. Actually, electrical transducer consists of two parts which are very closely related to Each other.

[4]. These two parts are sensing or detecting element and transduction element. The sensing or detecting element is commonly known as sensor.

[5]. Definition states that sensor is a device that produces a measurable response to a

[6]. The transduction element transforms the output of the sensor to an electrical output, as shown in the Fig.



(Fig)Transducer elements in cascade

5.1 Classification of Electrical Transducers

Transducers may be classified according to their structure, method of energy conversion and application. Thus we can say that transducers are classified

[1]. As active and passive transducer
[2]. According to transduction principle
[3]. As analog and digital transducer
[4]. As primary and secondary transducer
[5]. As transducer and inverse transducer

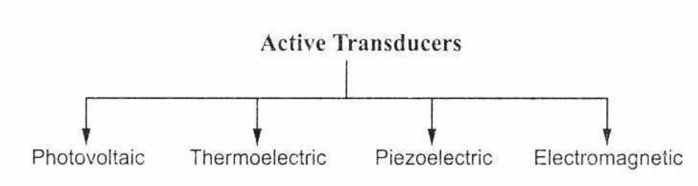
Active and Passive Transducer
Active Transducers

[1]. Active transducers are self-generating type of transducers.

[2]. These transducers develop an electrical parameter (i.e. voltage or current) which is proportional to the quantity under measurement.

[3]. These transducers do not require any external source or power for their operation.

[4]. They can be subdivided into the following commonly used types



Passive Transducers

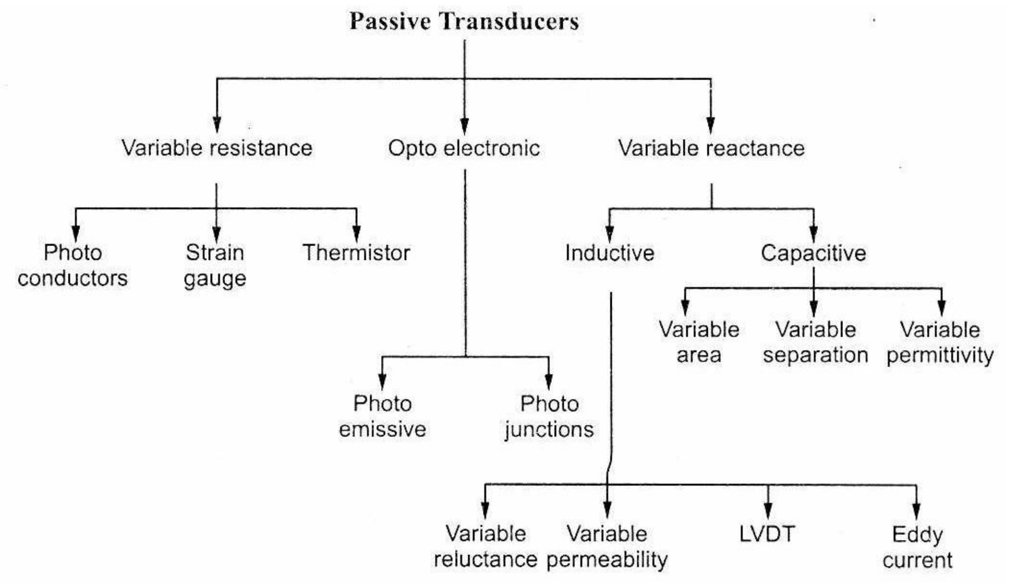
[1]. Passive transducers do not generate any electrical signal by themselves.

Ø To obtain an electrical signal from such transducers, an external source of power is essential.

[2]. Passive transducers depend upon the change in an electrical parameter (R, L, or C).

[3]. They are also known as externally power driven transducers.

[4]. They can be subdivided into the following commonly used types.



According to Transduction Principle

The transducers can be classified according to principle used in transduction.

[1]. Capacitive transduction

[2]. Electromagnetic transduction

[3]. Inductive transduction

[4]. Piezoelectric transduction

[5]. Photovoltaic transduction

[6]. Photoconductive transduction

Analog and Digital Transducers

The transducers can be classified on the basis of the output which may be a continuous function of time or the output may be in discrete steps.

Analog Transducers

Ø These transducers convert the input quantity into an analog output which is a continuous function of time.

[1]. A strain gauge, LVDT, thermocouples or thermistors are called analog transducers as they produce an outp

which is a continuous function of time.

Digital Transducers

Ø Digital transducers produce an electrical output in the form of pulses which forms an unique code.

[1]. Unique code is generated for each discrete value sensed.

Primary or Secondary

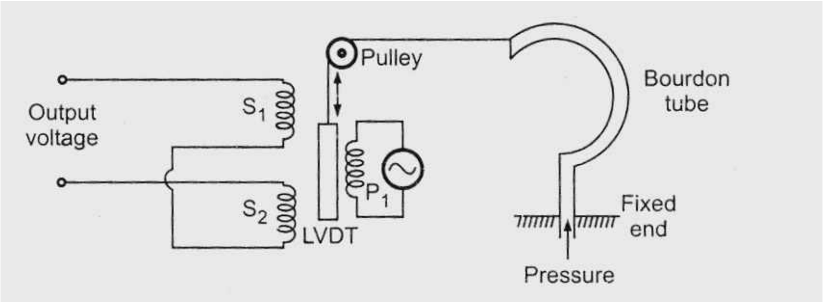
[1]. Some transducers consist of mechanical device along with the electrical device.

In such transducers mechanical device acts as a primary transducer and converts physical quantity into mechanical signal.

[2]. The electrical device then converts mechanical signal produced by primary transducer into an electrical signal.

[3]. Therefore, electrical device acts as a secondary transducer.

[4]. For an example, in pressure measurement Bourdons tube acts as a primary transducer which converts a pressure into displacement and LVDT acts as a secondary transducer which converts this displacement into an equivalent electrical signal.



(Fig) pressure Measurement

Transducer and Inverse Transducer

[1]. Transducers convert non-electrical quantity into electrical quantity whereas inverse transducer converts electrical quantity into non-electrical quantity.

[2]. For example, microphone is a transducer which converts sound signal into an Electrical signal whereas loudspeaker is an inverse transducer which converts electrical signal into sound signal.

Advantages of Electrical Transducers

[1]. Electrical signal obtained from electrical transducer can be easily processed (mainly amplified) and brought to a level suitable for output device which may be an indicator or recorder.

[2]. The electrical systems can be controlled with a very small level of power

[3]. The electrical output can be easily used, transmitted, and processed for the purpose of measurement.

[4]. With the advent of IC technology, the electronic systems have become extremely small in size, requiring small space for their operation.

[5]. No moving mechanical parts are involved in the electrical systems. Therefore there is no question of mechanical wear and tear and no possibility of mechanical failure.

Electrical transducer is almost a must in this modem world. Apart from the merits described above, some disadvantages do exist in electrical sensors.

Disadvantages of Electrical Transducers

[1]. The electrical transducer is sometimes less reliable than mechanical type because of the ageing and drift of the active components.

Ø Also, the sensing elements and the associated signal processing circuitry are comparatively expensive.

With the use of better materials, improved technology and circuitry, the range of accuracy and stability have been increased for electrical transducers.

[2]. Using negative feedback technique, the accuracy of measurement and the stability of the system are improved, but all at the expense of increased circuit complexity, more space, and obviously, more cost.

[3]. **Accuracy**: It is defined as the closeness with which the reading approaches an accepted standard value or ideal value or true value, of the variable being measured.

[4]. **Ruggedness**: The transducer should be mechanically rugged to withstand overloads. It should have overload protection.

[5]. **Linearity**: The output of the transducer should be linearly proportional to the input quantity under measurement. It should have linear input - output characteristic. -

[6]. **Repeatability:** The output of the transducer must be exactly the same, under same environmental conditions, when the same quantity is applied at the input repeatedly.

[7]. **High output**: The transducer should give reasonably high output signal so that it can be easily processed and measured. The output must be much larger than noise. Now-a-days, digital output is preferred in many applications;

[8]. **High Stability and Reliability:** The output of the transducer should be highly stable and reliable so that there will be minimum error in measurement. The output must remain unaffected by environmental conditions such as change in temperature, pressure, etc.

[9]. **Sensitivity**: The sensitivity of the electrical transducer is defined as the electrical output obtained per unit change in the physical parameter of the input quantity. For example, for a transducer used for temperature measurement, sensitivity will be expressed in mV/' C. A high sensitivity is always desirable for a given transducer.

[10]. **Dynamic Range:** For a transducer, the operating range should be wide, so that it can be used over a wide range of measurement conditions.

9**. Size:** The transducer should have smallest possible size and shape with minimal weight and volume. This will make the measurement system very compact.

[11]. **Speed of Response:** It is the rapidity with which the transducer responds to changes in the measured quantity. The speed of response of the transducer should be as high as practicable.

5.2 Transducer Selection Factors

[1]. Nature of measurement
[2]. Loading effect
[3]. Environmental considerations
[4]. Measuring system
[5]. Cost & Availability

5.3 Resistance Transducers

Temperature Sensors

Temperature is one of the fundamental parameters indicating the physical condition of matter, i.e. expressing its degree of hotness or coldness. Whenever a body is heat' various effects are observed. They include

[1]. Change in the physical or chemical state, (freezing, melting, boiling etc.)
[2]. Change in physical dimensions,
[3]. Changes in electrical properties, mainly the change in resistance,
[4]. Generation of an emf at the junction of two dissimilar metals.

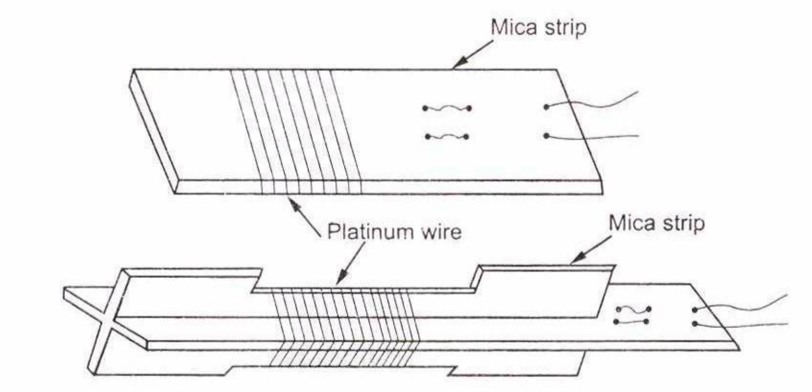
One of these effects can be employed for temperature measurement purposes. Electrical methods are the most convenient and accurate methods of temperature measurement. These methods are based on change in resistance with temperature and generation of thermal e.m.f. The change in resistance with temperature may be positive or negative. According to that there are two types

[1]. Resistance Thermometers —Positive temperature coefficient
[2]. Thermistors —Negative temperature coefficient

Construction of Resistance Thermometers

[1]. The wire resistance thermometer usually consists of a coil wound on a mica or ceramic former, as shown in the Fig.

[2]. The coil is wound in bifilar form so as to make it no inductive. Such coils are available in different sizes and with different resistance values ranging from 10 ohms to 25,000 ohms.



(Fig) Resistance Thermometer

Advantages of Resistance Thermometers

[1]. The measurement is accurate.
[2]. Indicators, recorders can be directly operated.
[3]. The temperature sensor can be easily installed and replaced.
[4]. Measurement of differential temperature is possible.
[5]. Resistance thermometers can work over a wide range of temperature from -20' C to + 650° C.
[6]. They are suitable for remote indication.
[7]. They are smaller in size
[8]. They have stability over long periods of time.

Limitations of Resistance Thermometers

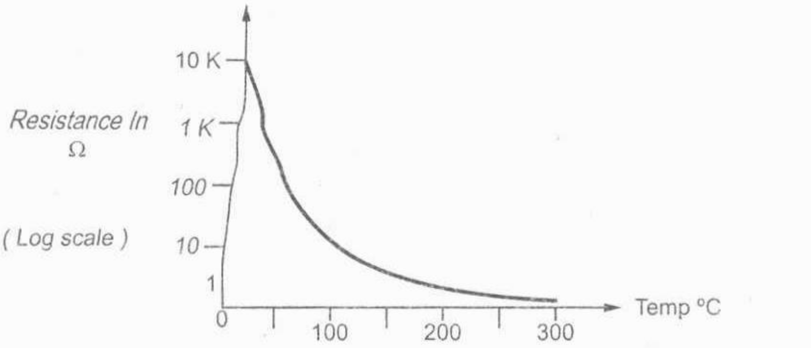
[1]. A bridge circuit with external power source is necessary for their operation.
[2]. They are comparatively costly.

Thermistors

[1]. Thermistor is a contraction of a term ‘ thermal-resistors’ .

[2]. Thermistors are semiconductor device which behave as thermal resistors having negative temperature coefficient [i.e. their resistance decreases as temperature increases.

[3]. The below Fig. shows this characteristic.

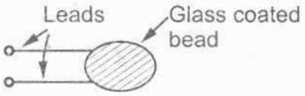


Construction of Thermistor

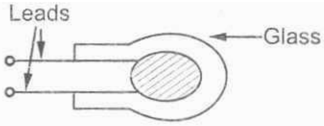
[1]. Thermistors are composed of a sintered mixture of metallic oxides, manganese, nickel, cobalt, copper, iron, and uranium.

Ø Their resistances at temperature may range from 100 to 100k .

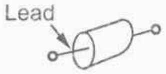
[2]. Thermistors are available in variety of shapes and sizes as shown in the Fig.



(a) Bead



(b) Probe



(c) Disc

(c) Rod

[3]. Smallest in size are the beads with a diameter of 0.15 mm to 1.25 mm.

[4]. Beads may be sealed in the tips of solid glass rods to form probes.

[5]. Disks and washers are made by pressing thermistor material under high pressure into flat cylindrical shapes.

[6]. Washers can be placed in series or in parallel to increase power dissipation rating.

[7]. Thermistors are well suited for precision temperature measurement, temperature control, and temperature compensation, because of their very large change in resistance with temperature.

Ø They are widely used for measurements in the temperature range -100 C to +100 C

[8]. Comparatively large change in resistance for a given change in temperature
[9]. Fast response over a narrow temperature range.

Limitations of Thermistor

[1]. The resistance versus temperature characteristic is highly non-linear.
[2]. Not suitable over a wide temperature range.
[3]. Because of high resistance of thermistor, shielded cables have to be used to minimize interference.

Applications of Thermistor

[1]. The thermistors relatively large resistance change per degree change in temperature [known as sensitivity] makes it useful as temperature transducer.

[2]. The high sensitivity, together with the relatively high thermistor resistance that May be selected [e.g. 100k.], makes the thermistor ideal for remote measurement or control. Thermistor control systems are inherently sensitive, stable, and fast acting, and they require relatively simple circuitry.

[3]. Because thermistors have a negative temperature coefficient of resistance, thermistors are widely used to compensate for the effects of temperature on circuit performance.

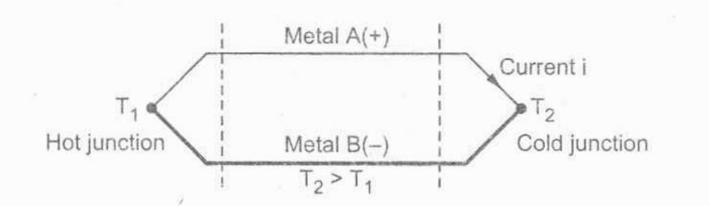
[4]. Measurement of conductivity.

Temperature Transducers

They are also called thermo-electric transducers. Two commonly used temperature transducers are

[1]. Resistance Temperature Detectors
[2]. Thermocouples.

Thermocouples



(Fig) Basic circuit

[3]. The thermocouple is one of the simplest and most commonly used methods of measuring process temperatures.

5.4 Capacitive Transducers

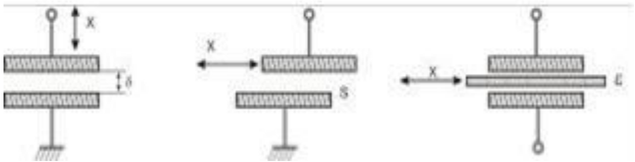
Capacitive transducers are capacitors that change their capacity under the influence of the input magnitude, which can be linear or angular movement. The capacity of a flat capacitor, composed of two electrodes with sizes **a´b**, with area of overlapping **s**, located at a distance **δ** from each other (in **d** << **a**/10 and **d** << **b**/10) is defined by the formula

$$C=\varepsilon_0 \varepsilon s/d$$

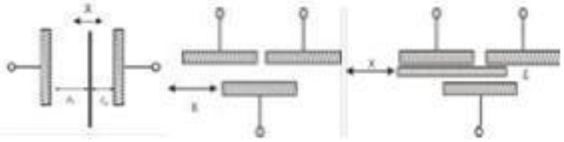
where: $\varepsilon_0=8,854.10^{-12}$ F/m is the dielectric permittivity of vacuum;

[4]. - permittivity of the area between the electrodes (for air **e**= 1,0005);

S=a.b – overlapping cross-sectional area of the electrodes. The Capacity can be influenced by changing the air gap **d**, the active area of overlapping of the electrodes **s** and the dielectric properties of the environment



Single capacitive transducers



Differential capacitive transducers

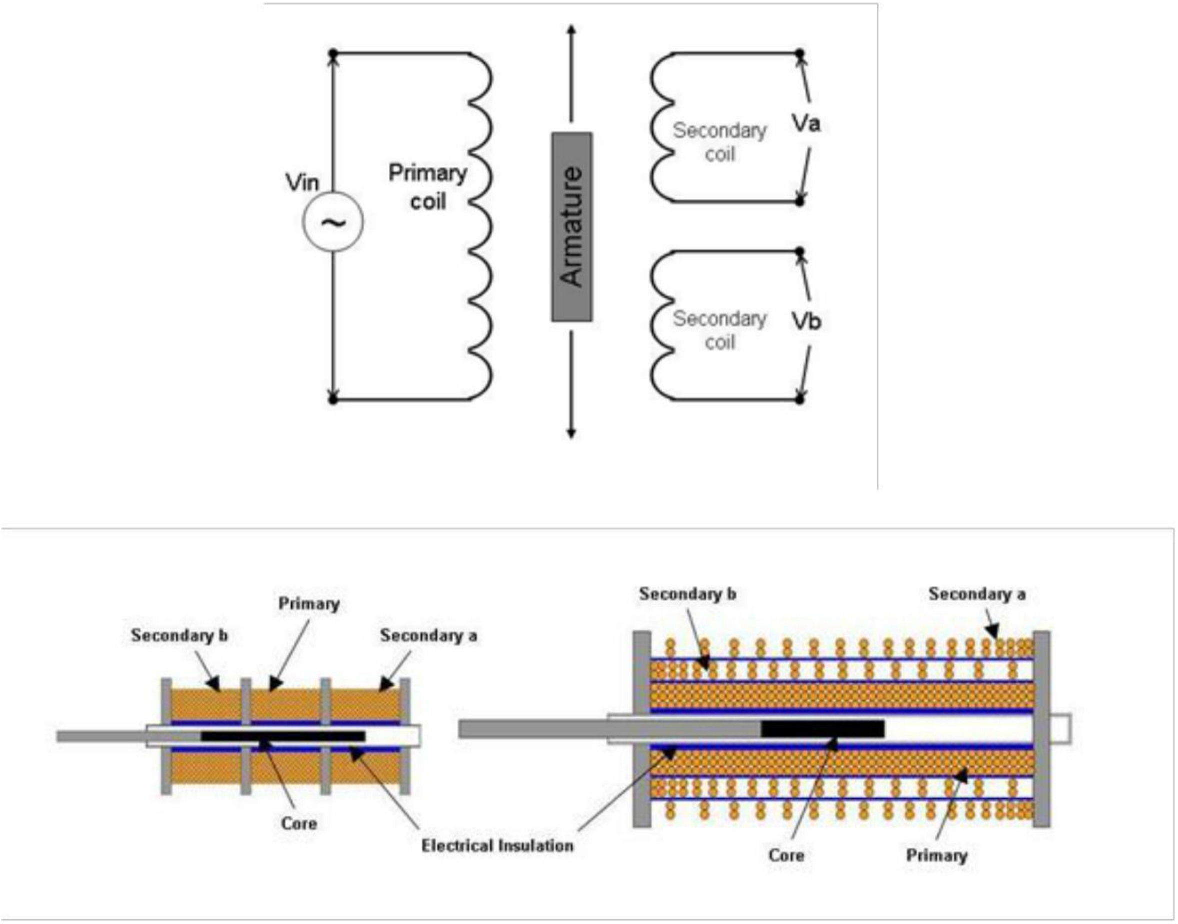
Application of capacitive transducers

Capacitive sensors have found wide application in automated systems that require precise determination of the placement of theobjects, processes in microelectronics, assembly of precise equipment associated with spindles for high speed drilling machines, ultrasonic welding machines and in equipment for Vibration measurement. They can be used not only to measure displacements (large and small), but also the level of fluids, fuel bulk materials, humidity environment, concentration of substances and others Capacitive sensors are often used for non-contact measurement of the thickness of various materials, such as silicon wafers, brake discs and plates of hard discs. Among the possibilities of the capacitive sensors is the measurement of density, thickness and location of Dielectrics.

5.5 Inductive Transducers

An LVDT, or Linear Variable Differential Transformer, is a transducer that converts a linear displacement or position from a mechanical reference (or zero) into a proportional electrical signal containing phase (for direction) and amplitude information (for distance). The LVDT operation does not require electrical contact between the moving part (probe or core rod assembly) and the transformer, but rather relies on electromagnetic coupling; this and the fact that they operate without any built-in electronic circuitry are the primary reasons why LVDTs have been widely used in applications where long life and high reliability under severe environments are a required, such Military/Aerospace applications.

The LVDT consists of a primary coil (of magnet wire) wound over the whole length of a non-ferromagnetic bore liner (or spool tube) or a cylindrical coil form. Two secondary coils are wound on top of the primary coil for “long stroke” LVDTs (i.e. for actuator main RAM) or each side of the primary coil for “Short stroke” LVDTs (i.e. for electro-hydraulic servo-valve or EHSV). The two secondary windings are typically connected in “opposite series” (or wound in opposite rotational directions). A ferromagnetic core, which length is a fraction of the bore liner length, magnetically couples the primary to the secondary winding turns that are located above the length of the core.



The LVDT: construction and principle of operation

When the primary coil is excited with a sine wave voltage (Vin), it generate a variable magnetic field which, concentrated by the core, induces the secondary voltages (also sine waves). While the secondary windings are designed so that the differential output voltage (Va-Vb) is proportional to the core position from null, the phase angle (close to 0 degree or close to 180 degrees depending of direction) determines the direction away from the mechanical zero. The zero is defined as the core position where the phase angle of the (Va-Vb) differential output is 90 degrees.

The differential output between the two secondary outputs (Va-Vb) when the core is at the mechanical zero (or “Null Position”) is called the Null Voltage; as the phase angle at null position is 90 degrees, the Null Voltage is a “quadrature” voltage. This residual voltage is due to the complex nature of the LVDT electrical model, which includes the parasitic capacitances of the windings.

5.6 Digital Transducers

A transducer measures physical quantities and transmits the information as coded digital signals rather than as continuously varying currents or voltages. Any transducer that presents information as discrete samples and that does not introduce a quantization error when the reading is represented in the digital form may be classified as a digital transducer. Most transducers used in digital systems are primarily analogue in nature and incorporate some form of conversion to provide the digital output. Many special techniques have been developed to avoid the necessity to use a conventional analogue - to-digital conversion technique to produce the digital signal. This article describes some of the direct methods which are in current use of producing digital outputs from transducers.

Some of the techniques used in transducers which are particularly adaptable for use in digital systems are introduced. The uses of encoder discs for absolute and incremental position measurement and to provide measurement of angul ar speed are outlined. The application of linear gratings for measurement of translational displacement is compared with the use of Moire fringe techniques used for similar purposes. Synchro devices are briefly explained and the various techniques used to produce a digital output from synchro resolvers are described. Brief descriptions of devices which develop a digital output from the natural frequency of vibration of some part of the transducer are presented. Digital techniques including vortex flow meters and instruments using laser beams are also briefly dealt with. Some of them are as follows:

[1]. Shaft Encoders
[2]. Digital Resolvers
[3]. Digital Tachometers
[4]. Hall Effect Sensors
[5]. Limit Switches

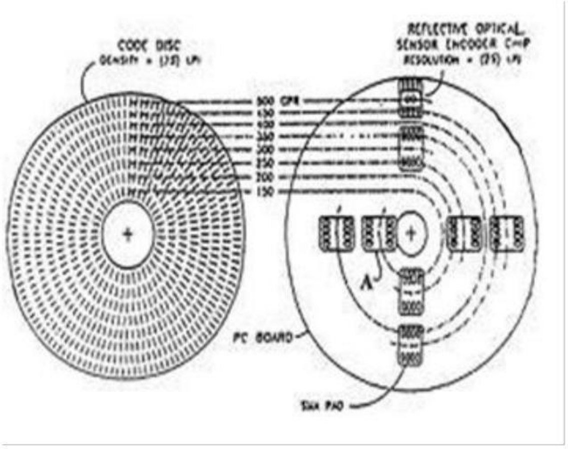
Shaft Encoders:

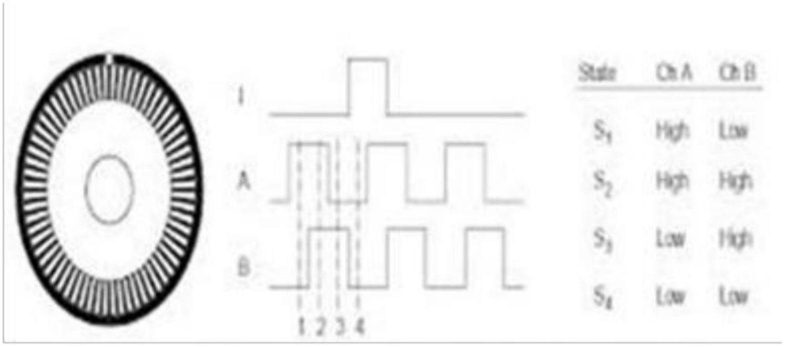
An encoder is a device that provides a coded reading of a measurement. A Shaft encoders can be one of the encoder that provide digital output measurements of angular position and velocity. This shaft encoders are excessively applicable in robotics, machine tools, mirror positioning systems, rotating machinery controls (fluid and electric), etc. Shaft encoders are basically of two types-Absolute and Incremental encoders.

An "absolute" encoder maintains position information when power is removed from the system. The position of the encoder is available immediately on applying power. The relationship between the encoder value and the physical position of the controlled machinery is set at assembly; the system does not need to return to a calibration point to maintain position accuracy. An "incremental" encoder accurately records changes in position, but does not power up with a fixed relation between encoder state and physical position. Devices controlled by incremental encoders may have to "go home" to a fixed reference point to initialize the position measurement. A multi-turn absolute rotary encoder includes additional code wheels and gears. A high-resolution wheel measures the fractional rotation, and lower-resolution geared code wheels record the number of whole revolutions of the shaft.

An absolute encoder has multiple code rings with various binary weightings which provide a data word representing the absolute position of the encoder within one

An incremental encoder works differently by providing an A and a B pulse output that provide no usable count information in their own right. Rather, the counting is done in the external electronics. The point where the counting begins depends on the counter in the external electronics and not on the position of the encoder. To provide useful position information, the encoder position must be referenced to the device to which it is attached, generally using an index pulse. The distinguishing feature of the incremental encoder is that it reports an incremental change in position of the encoder to the counting electronics.





State	Ch A	Ch B
S_1	High	Low
S_2	High	High
S_3	Low	High
S_4	Low	Low

5.7 Piezoelectric Transducers

Piezoelectric transducers produce an output voltage when a force is applied to them. They are frequently used as ultrasonic receivers and also as displacement transducers, particularly as part of devices measuring acceleration, force and pressure. In ultra- sonic receivers, the sinusoidal amplitude variations in the ultrasound wave received are translated into sinusoidal changes in the amplitude of the force applied to the piezoelectric transducer. In a similar way, the translational movement in a displacement transducer is caused by mechanical means to apply a force to the piezoelectric transducer. Piezoelectric transducers are made from piezoelectric materials. These have an asymmetrical lattice of molecules that distorts when a mechanical force is applied to it. This distortion causes a reorientation of electric charges within the material, resulting in a relative displacement of positive and negative charges. The charge displacement induces surface charges on the material of opposite polarity between the two sides. By implanting electrodes into the surface of the material, these surface charges can be measured as an output voltage. For a rectangular block of material, the induced voltage is given by:

$$V = \frac{kFd}{A}$$

Where F is the applied force in g, A is the area of the material in mm, d is the thickness of the material and k is the piezoelectric constant. The polarity of the induced voltage depends on whether the material is compressed or stretched.

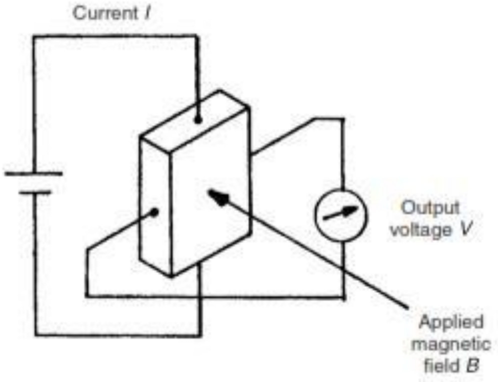
Where F is the applied force in g, A is the area of the material in mm, d is the thickness of the material and k is the piezoelectric constant. The polarity of the induced voltage depends on whether the material is compressed or stretched.

Materials exhibiting piezoelectric behaviour include natural ones such as quartz, synthetic ones such as lithiumsulphate andferroelectric ceramics such as barium titanate. The piezoelectric constant varies widely between different materials. Typical values of k are 2.3 for quartz and 140 for barium titanate. Applying equation (13.1) for a force of 1 g applied to a crystal of area 100 mm^2 and thickness 1 mm gives an output of 23 µV for quartz and 1.4 mV for barium titanate.

The piezoelectric principle is invertible, and therefore distortion in a piezoelectric material can be caused by applying a voltage to it. This is commonly used in ultrasonic transmitters, where the application of a sinusoidal voltage at a frequency in the ultra- sound range causes a sinusoidal variation in the thickness of the material and results in a sound wave being emitted at the chosen frequency. This is considered further in the section below on ultrasonic

5.8 Hall-effect transducers

Basically, a Hall-effect sensor is a device that is used to measure the magnitude of a magnetic field. It consists of a conductor carrying a current that is aligned orthogonally with the magnetic field, as shown in Figure 13.4. This produces a transverse voltage difference across the device that is directly proportional to the magnetic field strength. For an excitation current I and magnetic field strength B, the output voltage is given by V D KIB, where K is known as the Hall constant

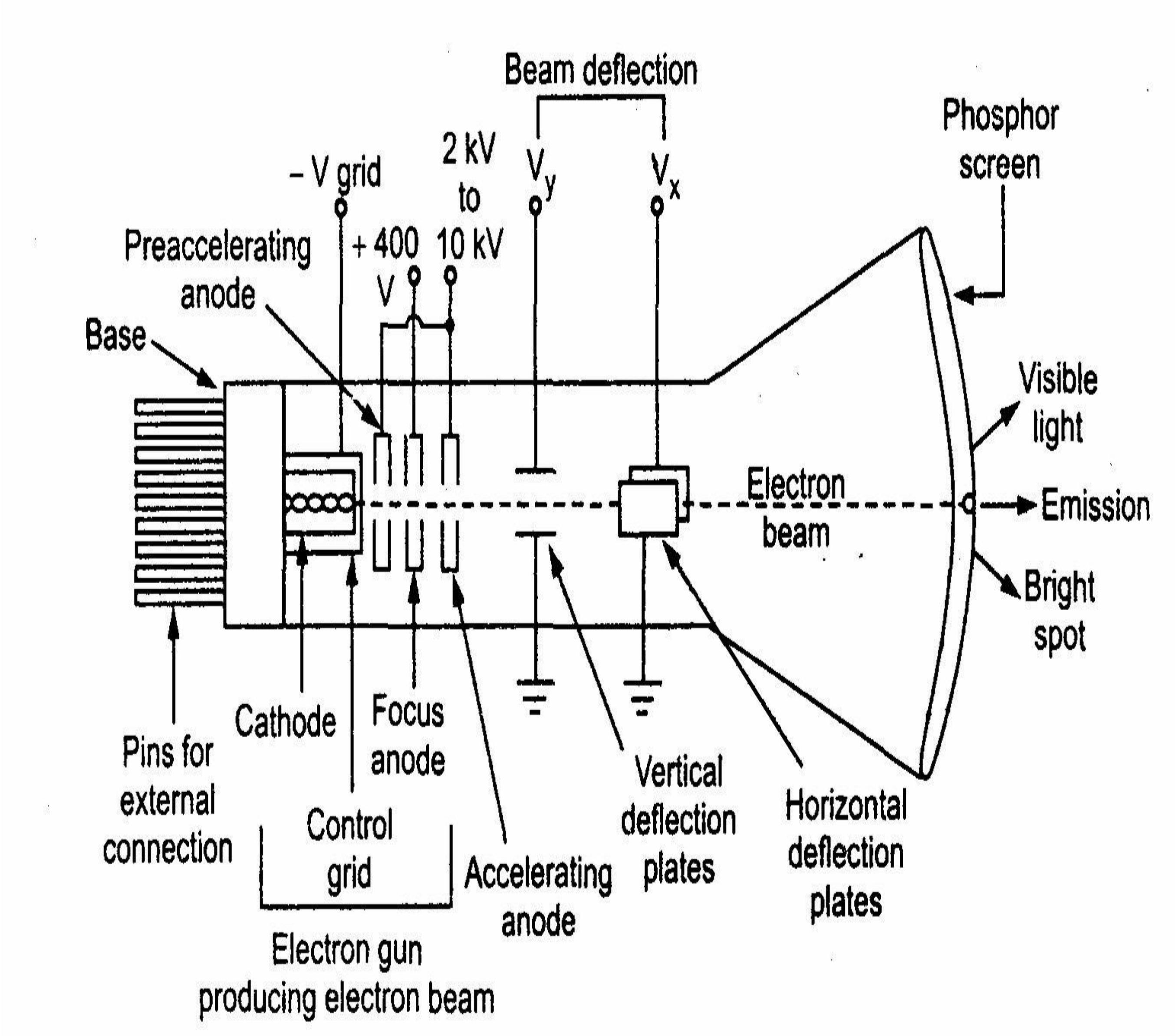


The conductor in Hall-effect sensors is usually made from a semiconductor material as opposed to a metal, because a larger voltage output is produced for a magnetic field of a given size. In one common use of the device as a proximity sensor, the magnetic field is provided by a permanent magnet that is built into the device. The magnitude of this field changes when the device becomes close to any ferrous metal object or boundary. The Hall Effect is also commonly used in keyboard pushbuttons, in which a magnet is attached underneath the button. When the button is depressed, the magnet moves past a Hall-effect sensor. The induced voltage is then converted by a trigger circuit into a digital output. Such pushbutton switches can operate at high frequencies without contact bounce.

CRT Display

The device which allows, the amplitude of such signals, to be displayed primarily as a function of time, is called cathode ray oscilloscope. The cathode ray tube (CRT) is the heart of the C.R.O. The CRT generates the electron beam, accelerates the beam, deflects the beam and also has a screen where beam becomes visible as a spot. The main parts of the CRT are

[1]. Electron gun
[2]. Deflection system
[3]. Fluorescent screen
Glass tube or envelope
Base



Electron gun

[1]. The electron gun section of the cathode ray tube provides a sharply focused, electron beam directed towards the fluorescent-coated screen.

[2]. This section starts from thermally heated cathode, emitting the electrons.

[3]. The control grid is given negative potential with respect to cathode.

[4]. This grid controls the number of electrons in t beam, going to the screen.

[5]. The momentum of the electrons (their number x their speed) determines the intensity, or brightness, of the light emitted from the fluorescent screen due to the electron bombardment.

[6]. The light emitted is usually of the green colour.

Deflection System

[1]. When the electron beam is accelerated it passes through the deflection system, with which beam can be positioned anywhere on the screen.

Fluorescent Screen

[1]. The light produced by the screen does not disappear immediately when bombardment by electrons ceases, i.e., when the signal becomes zero.

[2]. The time period for which the trace remains on the screen after the signal becomes zero is known as “persistence or fluorescence” .

[3]. The persistence may be as short as a few microsecond, or as long as tens of seconds or even minutes.

[4]. Medium persistence traces are mostly used for general purpose applications. Ø Long persistence traces are used in the study of transients.

[5]. Long persistence helps in the study of transients since the trace is still seen on the screen after the transient has disappeared.

Glass Tube

[1]. All the components of a CRT are enclosed in an evacuated glass tube called envelope.

[2]. This allows the emitted electrons to move about freely from one end of the tube to the other end.

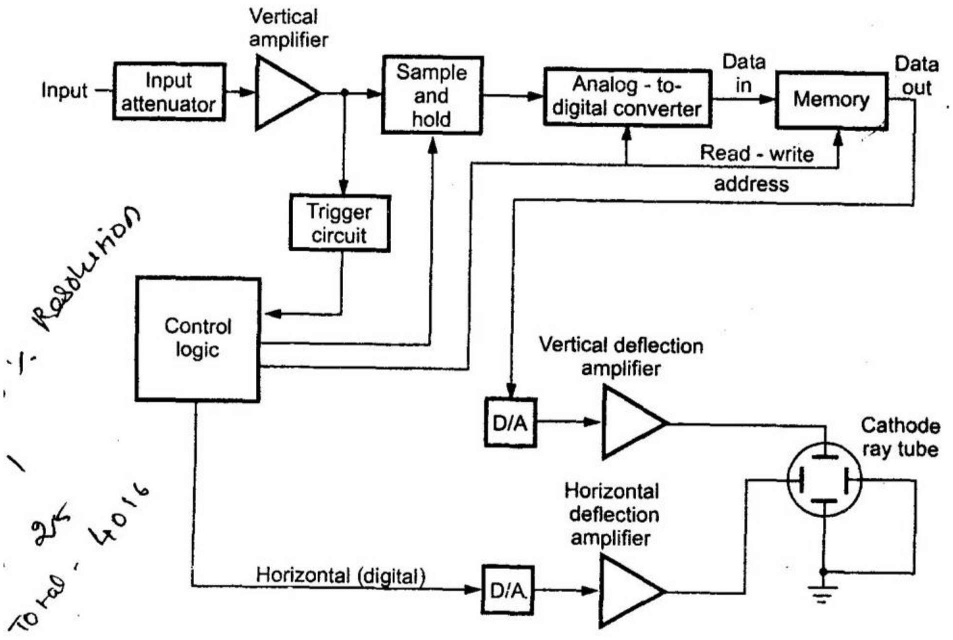
Base

[1]. The base is provided to the CRT through which the connections are made to the various parts.

Digital Storage Oscilloscope

Block Diagram

The block diagram of digital storage oscilloscope is shown in the Fig.



The input signal is applied to the amplifier and attenuator section.

The oscilloscope uses same type of amplifier and attenuator circuitry as used in the conventional oscilloscopes.

[1]. The attenuated signal is then applied to the vertical amplifier.

[2]. To digitize the analog signal, analog to digital (A/D) converter is used.

[3]. The output of the vertical amplifier is applied to the A/D converter section.

[4]. The successive approximation type of A/D converter is most oftenly used in the digital storage oscilloscopes.

[5]. The sampling rate and memory size are selected depending upon the duration & the waveform to be recorded.

Once the input signal is sampled, the A/D converter digitizes it.

The signal is then captured in the memory.

[6]. Once it is stored in the memory, many manipulations are possible as memory can be readout without being erased.

The digital storage oscilloscope has three modes:

[7]. Roll mode

[8]. Store mode

[9]. Hold or save mode.

Advantages

[1]. It is easier to operate and has more capability. ii) The storage time is infinite.

[2]. The display flexibility is available. The number of traces that can be stored and recalled depends on the size of the memory.

[3].The cursor measurement is possible.

v) The characters can be displayed on screen along with the waveform which can indicate waveform information such as minimum, maximum, frequency, amplitude etc.

vi) The X-Y plots, B-H curve, P-V diagrams can be displayed.

[4]. The pretrigger viewing feature allows to display the waveform before trigger pulse.

[5]. Keeping the records is possible by transmitting the data to computer system where the further processing is possible

Signal processing is possible which includes translating the raw data into finished information e.g. computing parameters of a captured signal like r.m.s. value, energy stored etc.