LECTURE NOTES

ON

FUNDAMENTALS OF ELECTRICAL AND ELECTRONICS ENGINEERING

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ELECTRICAL AND ELECTRONICS ENGINEERING

LECTURE NOTES

Course Title	FUNDAMENTALS OF ELECTRICAL AND ELECTRONICS ENGINEERING			
Course Code	AEE001			
Course Structure	**Lectures**	**Tutorials**	**Practical's**	**Credits**
	4	-	-	4
Course Coordinator	Mr. K Lingaswamy Reddy, Assistant Professor.			
Team of Instructor	Dr. V Chandra Jagan Mohan, Professor, EEE Mr. A Naresh Kumar, Assistant Professor, EEE			

Syllabus:

UNIT-I

ELECTRIC CIRCUIT ELEMENTS

Electrical circuit elements: Voltage and current sources, linear, non linear, active and passive elements, inductor current and capacitor voltage continuity, Kirchhoff's laws, elements in series and parallel, superposition in linear circuits, controlled sources, energy and power in elements, energy in mutual inductor and constraint on mutual inductance.

UNIT-II

NETWORK ANALYSIS AND THEOREMS

Network analysis: Nodal analysis with independent and dependant sources, modified nodal analysis, mesh analysis, notion of network graph, nodes, trees, twigs, links, co-tree, independent sets of branch currents and voltages; Network theorems: Voltage shift theorem, zero current theorem, Tellegen's theorem, reciprocity, substitution theorem, Thevenin's and Norton's theorems, pushing a voltage source through a node, splitting a current source, compensation theorem, maximum power transfer theorem.

UNIT-III

AC CIRCUITS

RLC circuits: Natural, step and sinusoidal steady state responses, series and parallel RLC circuits.

AC signal measurement: Complex, apparent, active and reactive power, power factor; Introduction to three phase supply: Three phase circuits, star-delta transformations, balance and unbalanced three phase load, power measurement, two wattmeter method.

UNIT-IV

SEMICONDUCTOR DIODE AND APPLICATIONS

P-N diode, symbol, V-I characteristics, half wave rectifier, full wave rectifier, bridge rectifier and filters, diode as a switch, Zener diode as a voltage regulator.

UNIT-V

BIPOLAR JUNCTION TRANSISTOR AND APPLICATIONS

DC characteristics, CE, CB, CC configurations, biasing, load line, Transistor as an amplifier.

Text Books:

1. A. Chakrabarti, "Circuit Theory", Dhanpat Rai Publications, 6th Edition, 2004.
2. K. S. Suresh Kumar, "Electric Circuit Analysis", Pearson Education, 1st Edition, 2013.
3. William Hayt, Jack E. Kemmerly S. M. Durbin, "Engineering Circuit Analysis", Tata Mc Graw Hill, 7th Edition, 2010.

4. J. P. J. Millman, C. C. Halkias, Satyabrata Jit, “Millman’s Electronic Devices and Circuits”, Tata Mc Graw Hill, 2nd Edition, 1998.
5. R. L. Boylestad, Louis Nashelsky, “Electronic Devices and Circuits”, PEI/PHI, 9th Edition, 2006

Reference Books:

1. David A. Bell, “Electronic Devices and Circuits”, Oxford University Press, 5th Edition, 2005.
2. M. Arshad, “Network Analysis and Circuits”, Infinity Science Press, 9th Edition, 2016.
3. A. Bruce Carlson, “Circuits”, Cengage Learning, 1st Edition, 2008.
4. S. Salivahanan, N. Suresh Kumar, A. Vallavaraj, “Electronic Devices and Circuits”, Tata Mc Graw Hill, 2nd Edition, 2011.

Web References:

1. http:// www.powerlab.ee.ncku.edu.tw
2. http:// www.textofvideo.nptel.iitm.ac.in
3. http:// www.textofvideo.nptel.iitm.ac.in

E-Text Books:

1. http://www.textbooksonline.tn.nic.in
2. http://www.bookboon.com
3. http://www.ktustudents.in

UNIT – I

ELECTRIC CIRCUIT ELEMENTS

INTRODUCTION

Given an electrical network, the network analysis involves various methods. The process of finding the network variables namely the voltage and currents in various parts of the circuit is known as network analysis. Before we carry out actual analysis it is very much essential to thoroughly understand the various terms associated with the network. In this chapter we shall begin with the definition and understanding in detail some of the commonly used terms. The basic laws such as Ohm's law, KCL and KVL, those can be used to analyse a given network Analysis becomes easier if we can simplify the given network. We will be discussing various techniques, which involve combining series and parallel connections of R, L and C elements.

SYSTEMS OF UNITS

As engineers, we deal with measurable quantities. Our measurement must be communicated in standard language that virtually all professionals can understand irrespective of the country. Such an international measurement language is the International System of Units (SI). In this system, there are six principal units from which the units of all other physical quantities can be derived.

Quantity	Basic Unit	Symbol
Length	Meter	M
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Temperature	Kelvin	K
Luminous intensity	candela	Cd

One great advantage of SI unit is that it uses prefixes based on the power of 10 to relate larger and smaller units to the basic unit.

Multiplier	Prefix	Symbol
10^{12}	Tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	K
10^{-3}	milli	m
10^{-6}	micro	
10^{-9}	nano	n
10^{-12}	pico	p

BASIC CONCEPTS AND DEFINITIONS

CHARGE

The most basic quantity in an electric circuit is the *electric charge.* We all experience the effect of electric charge when we try to remove our wool sweater and have it stick to our body or walk across a carpet and receive a shock.

Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C). Charge, positive or negative, is denoted by the letter q or Q.

We know from elementary physics that all matter is made of fundamental building blocks known as atoms and that each atom consists of electrons, protons, and neutrons. We also know that the charge 'e' on an electron is negative and equal in magnitude to 1.602×10^{-19} C, while a proton carries a positive charge of the same magnitude as the electron and the neutron has no charge. The presence of equal numbers of protons and electrons leaves an atom neutrally charged.

CURRENT

Current can be defined as the motion of charge through a conducting material, measured in Ampere (A). Electric current, is denoted by the letter i or I.

The unit of current is the ampere abbreviated as (A) and corresponds to the quantity of total charge that passes through an arbitrary cross section of a conducting material per unit second. Mathematically,

Where is the symbol of charge measured in Coulombs (C), I is the current in amperes (A) and t is the time in second (s).

The current can also be defined as the rate of charge passing through a point in an electric circuit.
Mathematically,

The charge transferred between time t_1 and t_2 is obtained as

A constant current (also known as a direct current or DC) is denoted by symbol I whereas a time-varying current (also known as alternating current or AC) is represented by the symbol or ( ). Figure 1.1 shows direct current and alternating current.

Current is always measured through a circuit element as shown in Fig. 1.1

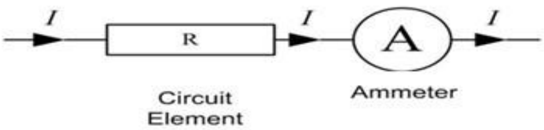


Fig. 1.1 Current through Resistor (R)

Two types of currents:
1) A direct current (DC) is a current that remains constant with time.
2) An alternating current (AC) is a current that varies with time.

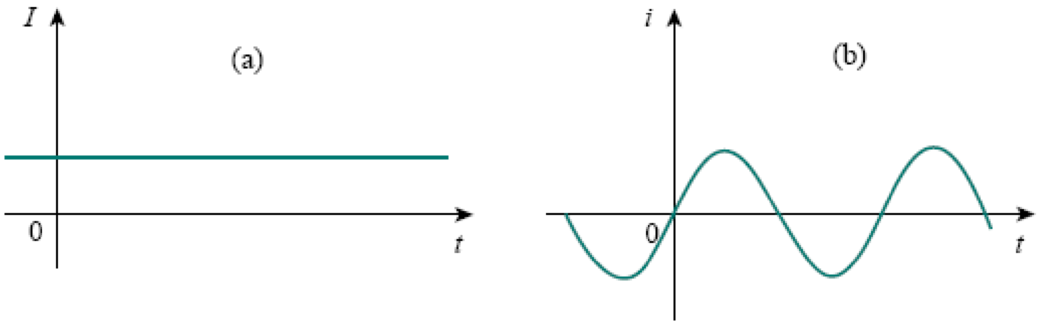


Fig. 1.2Two common types of current: (a) direct current (DC), (b) alternative current (AC)

Example 1.1
Determine the current in a circuit if a charge of 80 coulombs passes a given point in 20 seconds (s).

Solution:

Example 1.2
How much charge is represented by 4,600 electrons?

Solution:
Each electron has - 1.602x10^{-19} C. Hence 4,600 electrons will have:

-1.602x10^{-19}x4600 = -7.369x10^{-16} C

Example 1.3
The total charge entering a terminal is given by =5 sin4 . Calculate the current at =0.5 .

Solution:

At =0.5 .
= 31.42

Example 1.4
Determine the total charge entering a terminal between =1 and =2 if the current passing the terminal is = (3 ^{2}−) .
Solution:

VOLTAGE (or) POTENTIAL DIFFERENCE

To move the electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf), typically represented by the battery in Fig. 1.3. This emf is also known as voltage or potential difference. The voltage $_{ab}$between two points aand b in an electric circuit is the energy (or work) needed to move a unit charge from a to b.

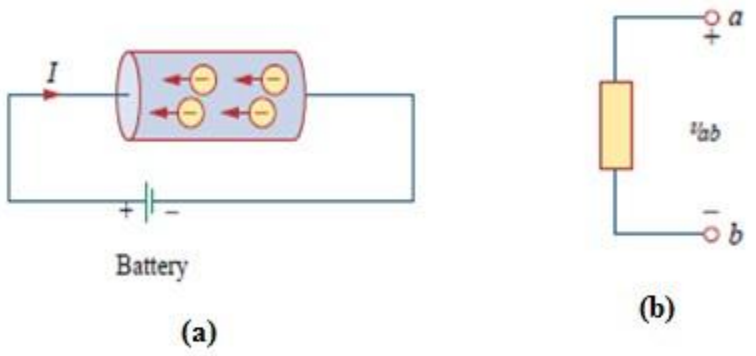


Fig. 1.3(a) Electric Current in a conductor, (b)Polarity of voltage $_{ab}$

Voltage (or potential difference) is the energy required to move charge from one point to the other, measured in volts (V). Voltage is denoted by the letter v or V.

Mathematically,

where w is energy in joules (J) and q is charge in coulombs (C). The voltage $_{ab}$ or simply V is measured in volts (V).

1 volt = 1 joule/coulomb = 1 newton-meter/coulomb

Fig. 1.3 shows the voltage across an element (represented by a rectangular block) connected to points a and b. The plus (+) and minus (-) signs are used to define reference direction or voltage polarity. The $_{ab}$ can be interpreted in two ways: (1) point a is at a potential of $_{ab}$ volts higher than point b, or (2) the potential at point a with respect to point b is $_{ab}$. It follows logically that in general

Voltage is always measured across a circuit element as shown in Fig. 1.4

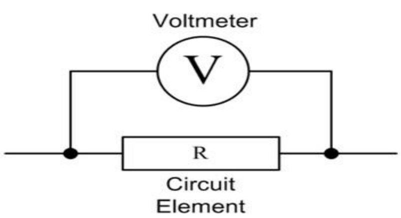


Fig. 1.4 Voltage across Resistor (R)

Example 1.5
An energy source forces a constant current of 2 A for 10 s to flow through a lightbulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

Solution:
Total charge dq= i*dt = 2*10 = 20 C

The voltage drop is

4 POWER

Power is the time rate of expending or absorbing energy, measured in watts (W). Power, is denoted by the letter p or P.
Mathematically,

Where p is power in watts (W), w is energy in joules (J), and t is time in seconds (s).

From voltage and current equations, it follows that;

Thus, ifthe magnitude of current I and voltage are given, then power can be evaluated as the product of the two quantities and is measured in watts (W).

Sign of power:

Plus sign: Power is absorbed by the element. (Resistor, Inductor)

Minus sign: Power is supplied by the element. (Battery, Generator)

Passive sign convention:

If the current enters through the positive polarity of the voltage, $p = +vi$
If the current enters through the negative polarity of the voltage, $p = -vi$

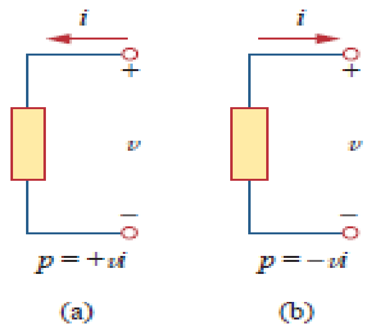


Fig 1.5 Polarities for Power using passive sign convention
(a) Absorbing Power (b) Supplying Power

1.3.5 ENERGY
Energy is the capacity to do work, and is measured in joules (J).
The energy absorbed or supplied by an element from time 0 to t is given by,

The electric power utility companies measure energy in watt-hours (WH) or Kilo watt-hours (KWH)

$$1 \text{ WH} = 3600 \text{ J}$$

Example 1.6

A source e.m.f. of 5 V supplies a current of 3A for 10 minutes. How much energy is provided in this time?

Solution:

$$= \quad = 5 \times 3 \times 10 \times 60 = 9$$

Example 1.7

An electric heater consumes 1.8Mj when connected to a 250 V supply for 30 minutes. Find the power rating of the heater and the current taken from the supply.

Solution:

$= \ / \ = (1.8\times10^6)/(30\times60) = 1000$

Power rating of heater = 1kW

$=$

Thus

$= \ / \ =1000/250=4$

Hence the current taken from the supply is 4A.

Example 1.7

Find the power delivered to an element at $=3$ if the current entering its positive terminals is $=5\cos60$ and the voltage is: (a) $=3$, (b) $=3$didt.

Solution:

(a) The voltage is $=3 =15\cos60$; hence, the power is: $= \ =75\cos260$

At $=3$,

$=75\cos260 \ \times3\times10^{-3} =53.48$

(b) We find the voltage and the power as

$=3 \quad =3 -60 \ \ 5\sin60 \ \ =-900 \ \ \sin60$

$= \ =-4500 \ \sin60 \ \ \cos60$

At $=3$,

$=-4500 \ \sin0.18 \ \cos0.18 \ \ =-6.396$

OHM'S LAW

Georg Simon Ohm (1787–1854), a German physicist, is credited with finding the relationship between current and voltage for a resistor. This relationship is known as Ohm's law.

Ohm's law states that at constant temperature, the voltage (V) across a conducting material is directly proportional to the current (I) flowing through the material.

Mathematically,

V I

V=RI

Wherethe constant of proportionality R is called the resistance of the material. The V-I relation for resistor according to Ohm's law is depicted in Fig.1.6

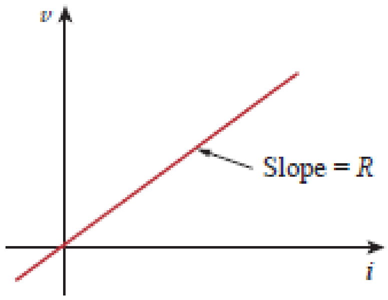


Fig. 1.6 V-I Characteristics for resistor

Limitations of Ohm's Law:

1. Ohm's law is not applicable to non-linear elements like diode, transistor etc.
2. Ohm's law is not applicable for non-metallic conductors like silicon carbide.

CIRCUIT ELEMENTS

An element is the basic building block of a circuit. An electric circuit is simply an interconnection of the elements. Circuit analysis is the process of determining voltages across (or the currents through) the elements of the circuit.

There are 2 types of elements found in electrical circuits.

a) Active elements (Energy sources): The elements which are capable of generating or delivering the energy are called active elements.
E.g., Generators, Batteries

b) Passive element (Loads): The elements which are capable of receiving the energy are called passive elements.
E.g., Resistors, Capacitors and Inductors

ACTIVE ELEMENTS (ENERGY SOURCES)

The energy sources which are having the capacity of generating the energy are called active elements. The most important active elements are voltage or current sources that generally deliver power/energy to the circuit connected to them.

There are two kinds of sources

a) Independent sources
b) Dependent sources

INDEPENDENT SOURCES:

An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit elements.

Ideal Independent Voltage Source:
An ideal independent voltage source is an active element that gives a constant voltage across its terminals irrespective of the current drawn through its terminals. In other words, an ideal independent voltage source delivers to the circuit whatever current is necessary to maintain its terminal voltage. The symbol of idea independent voltage source and its V-I characteristics are shown in Fig. 1.7

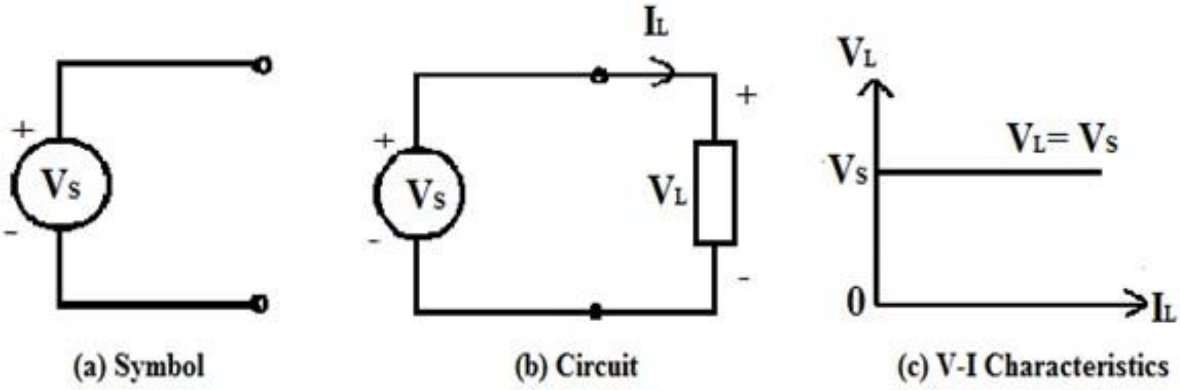


Fig. 1.7 Ideal Independent Voltage Source

Practical Independent Voltage Source:
Practically, every voltage source has some series resistance across its terminals known as internal resistance, and is represented by Rse. For ideal voltage source Rse = 0. But in practical voltage source Rse is not zero but may have small value. Because of this Rse voltage across the terminals decreases with increase in current as shown in Fig. 1.8

Terminal voltage of practical voltage source is given by

$$V_L = V_S - I_L\,Rse$$

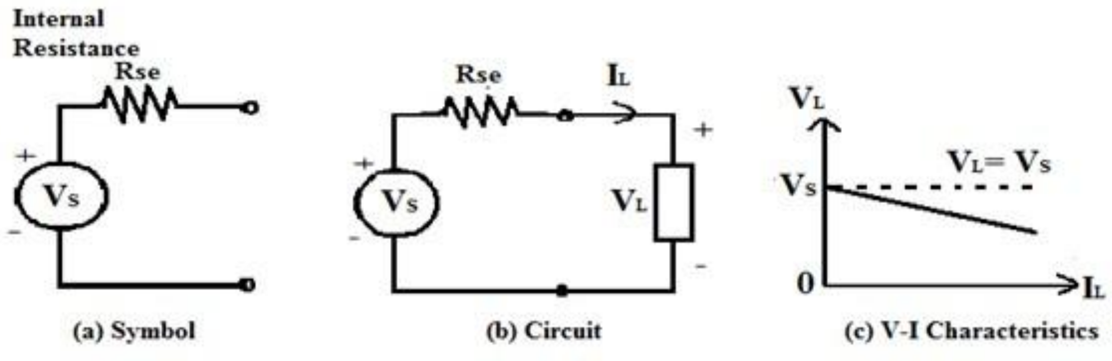


Fig. 1.8 Practical Independent Voltage Source

Ideal Independent Current Source:
An ideal independent Current source is an active element that gives a constant current through its terminals irrespective of the voltage appearing across its terminals. That is, the

current source delivers to the circuit whatever voltage is necessary to maintain the designated current. The symbol of idea independent current source and its V-I characteristics are shown in Fig. 1.9

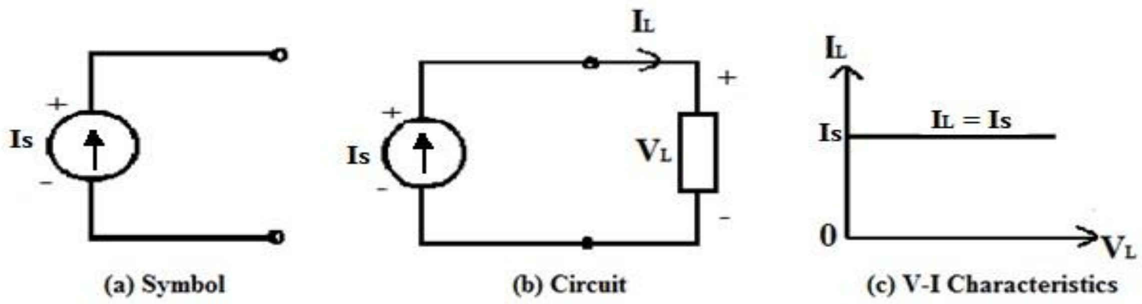


Fig. 1.9 Ideal Independent Current Source

Practical Independent Current Source:

Practically, every current source has some parallel/shunt resistance across its terminals known as internal resistance, and is represented by Rsh. For ideal current source Rsh = ∞ (infinity). But in practical voltage source Rsh is not infinity but may have a large value. Because of this Rsh current through the terminals slightly decreases with increase in voltage across its terminals as shown in Fig. 1.10.

Terminal current of practical current source is given by

$$I_L = Is - Ish$$

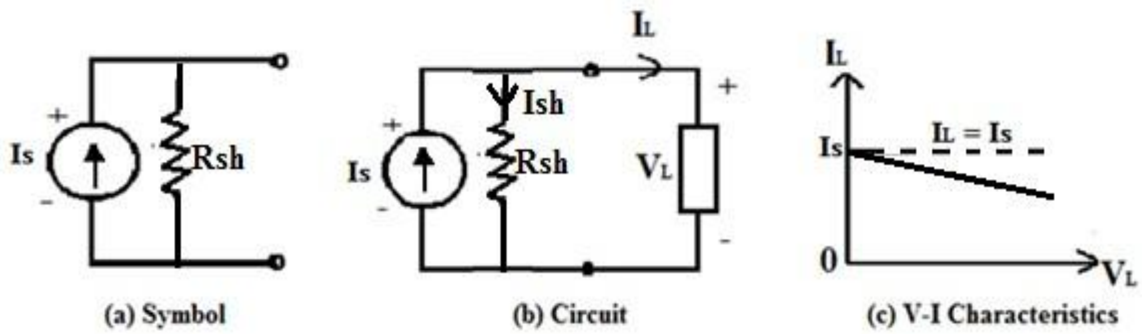


Fig. 1.10 Practical Independent Current Source

DEPENDENT (CONTROLLED) SOURCES

An ideal dependent (or controlled) source is an active element in which the source quantity is controlled by another voltage or current.

Dependent sources are usually designated by diamond-shaped symbols, as shown in Fig. Since the control of the dependent source is achieved by a voltage or current of some other element in the circuit, and the source can be voltage or current, it follows that there are four possible types of dependent sources, namely:

1. A voltage-controlled voltage source (VCVS)
2. A current-controlled voltage source (CCVS)
3. A voltage-controlled current source (VCCS)
4. A current-controlled current source (CCCS)

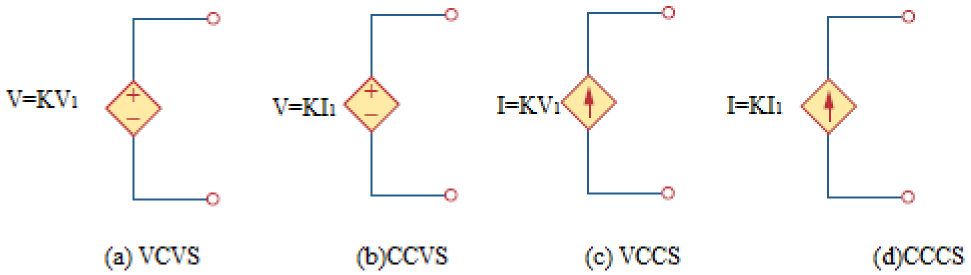


Fig. 1.11 Symbols for Dependent voltage source and Dependent current source

Dependent sources are useful in modeling elements such as transistors, operational amplifiers, and integrated circuits. An example of a current-controlled voltage source is shown on the right-hand side of Fig. 1.12, where the voltage 10i of the voltage source depends on the current i through element C. Students might be surprised that the value of the dependent voltage source is 10i V (and not 10i A) because it is a voltage source. The key idea to keep in mind is that a voltage source comes with polarities (+ -) in its symbol, while a current source comes with an arrow, irrespective of what it depends on.

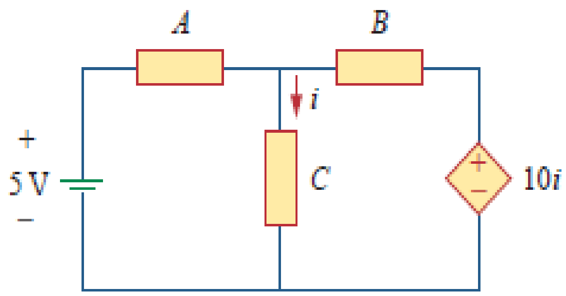


Fig. 1.12 The source in right hand side is current-controlled voltage source

PASSIVE ELEMENTS (LOADS)

Passive elements are those elements which are capable of receiving the energy. Some passive elements like inductors and capacitors are capable of storing a finite amount of energy, and return it later to an external element. More specifically, a passive element is defined as one that cannot supply average power that is greater than zero over an infinite time interval. Resistors, capacitors, Inductors fall in this category.

RESISTOR

Materials in general have a characteristic behavior of resisting the flow of electric charge. This physical property, or ability to resist the flow of current, is known as resistance and is represented by the symbol R.The Resistance is measured in ohms ( ). The circuit element used to model the current-resisting behavior of a material is called the resistor.

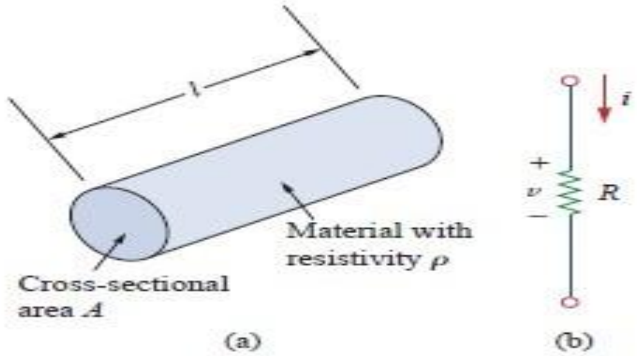


Fig. 1.13 (a) Typical Resistor, (b) Circuit Symbol for Resistor

The resistance of a resistor depends on the material of which the conductor is made and geometrical shape of the conductor. The resistance of a conductor is proportional to the its length (and inversely proportional to its cross sectional area (A). Therefore the resistance of a conductor can be written as,

The proportionality constant is called the specific resistance o resistivity of the conductor and its value depends on the material of which the conductor is made.

The inverse of the resistance is called the conductance and inverse of resistivity is called specific conductance or conductivity. The symbol used to represent the conductance is G and conductivity is . Thus conductivity and its units are Siemens per meter

By using Ohm's Law, The power dissipated in a resistor can be expressed in terms of R as below

The power dissipated by a resistor may also be expressed in terms of G as

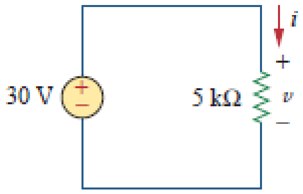
The energy lost in the resistor from time 0 to t is expressed as

Or

Where V is in volts, I is in amperes, R is in ohms, and energy W is in joules

Example 1.8

In the circuit shown in Fig. below, calculate the current i, the conductance G, the power p and energy lost in the resistor W in 2hours.



Solution:

The voltage across the resistor is the same as the source voltage (30 V) because the resistor and the voltage source are connected to the same pair of terminals. Hence, the current is

The conductance is

We can calculate the power in various ways

or

or

Energy lost in the resistor is

INDUCTOR

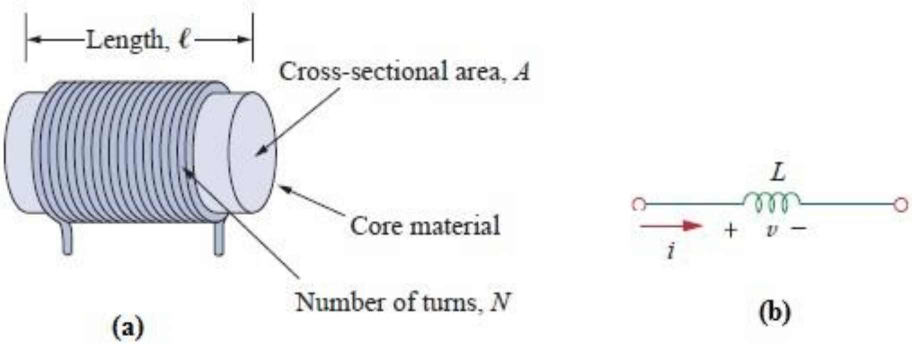


Fig. 1.14 (a) Typical Inductor, (b) Circuit symbol of Inductor

A wire of certain length, when twisted into a coil becomes a basic inductor. The symbol for inductor is shown in Fig.1.14 (b). If current is made to pass through an inductor, an electromagnetic field is formed. A change in the magnitude of the current changes the electromagnetic field. Increase in current expands the fields, and decrease in current reduces it. Therefore, a change in current produces change in the electromagnetic field, which induces a voltage across the coil according to Faraday's law of electromagnetic induction. i.e., the voltage across the inductor is directly proportional to the time rate of change of current.
Mathematically,

Where L is the constant of proportionality called the *inductance* of an inductor. The unit of inductance is Henry (H).we can rewrite the above equation as

Integrating both sides from time 0 to t, we get

From the above equation we note that the current in an inductor is dependent upon the integral of the voltage across its terminal and the initial current in the coil .
The power absorbed by the inductor is

The energy stored by the inductor is

From the above discussion, we can conclude the following.

1. The induced voltage across an inductor is zero if the current through it is constant. That means an inductor acts as short circuit to DC.
2. A small change in current within zero time through an inductor gives an infinite voltage across the inductor, which is physically impossible. In a fixed inductor the current cannot change abruptly i.e., the inductor opposes the sudden changes in currents.

3. The inductor can store finite amount of energy. Even if the voltage across the inductor is zero
4. A pure inductor never dissipates energy, only stores it. That is why it is also called a non-dissipative passive element. However, physical inductors dissipate power due to internal resistance.

Example 1.9

Find the current through a 5-H inductor if the voltage across it is

Also, find the energy stored at t = 5 s. assume initial conditions to be zero.

Solution:

The power

Then the energy stored is

1.5.2.2 CAPACITOR

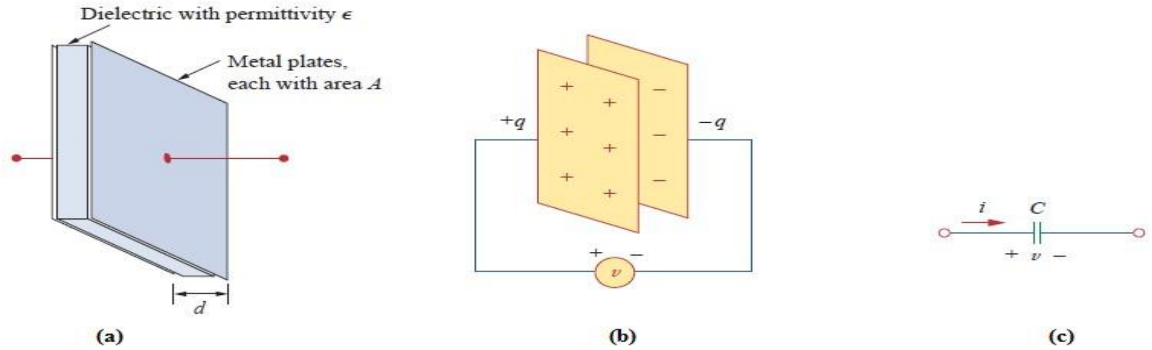


Fig. 1.15 (a) Typical Capacitor, (b) Capacitor connected to a voltage source, (c) Circuit Symbol of capacitor

Any two conducting surfaces separated by an insulating medium exhibit the property of a capacitor. The conducting surfaces are called electrodes, and the insulating medium is called dielectric. A capacitor stores energy in the form of an electric field that is established by the opposite charges on the two electrodes. The electric field is represented by lines of force between the positive and negative charges, and is concentrated within the dielectric.

When a voltage source v is connected to the capacitor, as in Fig 1.15 (c), the source deposits a positive charge q on one plate and a negative charge — q on the other. The capacitor is said to store the electric charge. The amount of charge stored, represented by q, is directly pro-proportional to the applied voltage v so that

Where C, the constant of proportionality, is known as the capacitance of the capacitor. The unit of capacitance is the farad (F).

Although the capacitance C of a capacitor is the ratio of the charge q per plate to the applied voltage v, it does not depend on q or v. It depends on the physical dimensions of the capacitor. For example, for the parallel-plate capacitor shown in Fig.1.15 (a), the capacitance is given by

Where A is the surface area of each plate, d is the distance between the plates, and is the permittivity of the dielectric material between the plates.
The current flowing through the capacitor is given by

We can rewrite the above equation as

Integrating both sides from time 0 to t, we get

From the above equation we note that the voltage across the terminals of a capacitor is dependent upon the integral of the current through it and the initial voltage .
The power absorbed by the capacitor is

The energy stored by the capacitor is

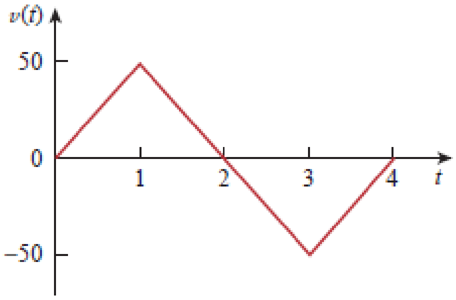
From the above discussion we can conclude the following,

1. The current in a capacitor is zero if the voltage across it is constant; that means, the capacitor acts as an open circuit to DC.

2. A small change in voltage across a capacitance within zero time gives an infinite current through the capacitor, which is physically impossible. In a fixed capacitance the voltage cannot change abruptly. i.e., A capacitor will oppose the sudden changes in voltages.
3. The capacitor can store a finite amount of energy, even if the current through it is zero.
4. A pure capacitor never dissipates energy, but only stores it; that is why it is called non-dissipative passive element. However, physical capacitors dissipate power due to internal resistance.

Example 1.10

Determine the current through a 200 capacitor whose voltage is shown in Fig. below

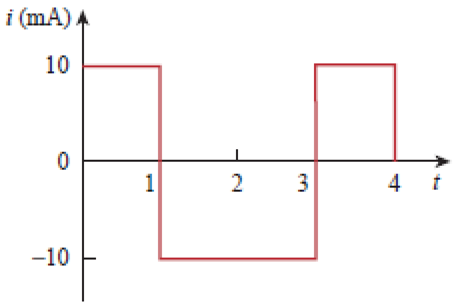


Solution:

The voltage waveform can be described mathematically as

Since — we take the derivative of to obtain the i

Hence, the current wave form is as shown in the fig. below



NETWORK/CIRCUIT TERMINOLOGY

In the following section various definitions and terminologies frequently used in electrical circuit analysis are outlined.

- **Network Elements:** The individual components such as a resistor, inductor, capacitor, diode, voltage source, current source etc. that are used in circuit are known as network elements.
- **Network:** The interconnection of network elements is called a network**.**
- **Circuit:** A network with at least one closed path is called a circuit. So, all the circuits are networks but all networks are not circuits.
- **Branch:** A branch is an element of a network having only two terminals.
- **Node:** A node is the point of connection between two or more branches. It is usually indicated by a dot in a circuit**.**
- **Loop:** A loop is any closed path in a circuit. A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once.
- **Mesh or Independent Loop:** Mesh is a loop which does not contain any other loops in it.

KIRCHHOFF'S LAWS

The most common and useful set of laws for solving electric circuits are the Kirchhoff's voltage and current laws. Several other useful relationships can be derived based on these laws. These laws are formally known as Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL).

KIRCHHOFF'S CURRENT LAW (KCL)

This is also called as Kirchhoff's first law or Kirchhoff's nodal law. Kirchhoff's first law is based on the law of conservation of charge, which requires that the algebraic sum of charges within a system cannot change.

Statement: Algebraic sum of the currents meeting at any junction or node is zero. The term 'algebraic' means the value of the quantity along with its sign, positive or negative.

Mathematically, KCL implies that

Where N is the number of branches connected to the node and is the nth current entering (or leaving) the node. By this law, currents entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa.

Alternate Statement: Sum of the currents flowing towards a junction is equal to the sum of the currents flowing away from the junction.

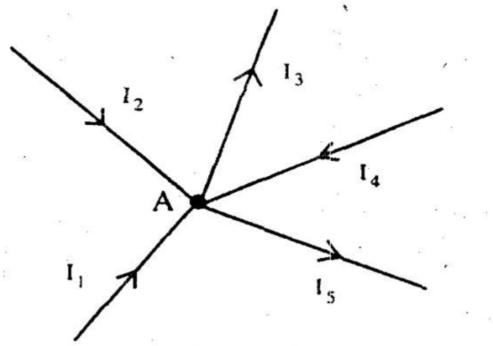


Fig 1.16 Currents meeting in a junction

Consider Fig. 1.16 where five branches of a circuit are connected together at the junction or node A. Currents I_1, I_2 and I_4 are flowing towards the junction whereas currents I_3 and I_5 are flowing away from junction A. If a positive sign is assigned to the currents I_2 and I_4 that are flowing into the junction then the currents I_3 and I_4 flowing away from the junction should be assigned with the opposite sign i.e. the negative sign.

Applying Kirchhoff's current law to the junction A

$$I_1 + I_2 - I_3 + I_4 - I_5 = 0 \text{ (algebraic sum is zero)}$$

The above equation can be modified as $I_1 + I_2 + I_4 = I_3 + I_5$ (sum of currents towards the junction = sum of currents flowing away from the junction).

KIRCHHOFF'S VOLTAGE LAW (KVL)

This is also called as Kirchhoff's second law or Kirchhoff's loop or mesh law. Kirchhoff's second law is based on the principle of conservation of energy.

Statement: Algebraic sum of all the voltages around a closed path or closed loop at any instant is zero. Algebraic sum of the voltages means the magnitude and direction of the voltages; care should be taken in assigning proper signs or polarities for voltages in different sections of the circuit.

Mathematically, KVL implies that

Where N is the number of voltages in the loop (or the number of branches in the loop) and is the nth voltage in a loop.

The polarity of the voltages across active elements is fixed on its terminals. The polarity of the voltage drop across the passive elements (Resistance in DC circuits) should be assigned with reference to the direction of the current through the elements with the concept that the current flows from a higher potential to lower potential. Hence, the entry point of the current through the passive elements should be marked as the positive polarity of voltage drop across the element and the exit point of the current as the negative polarity. The direction of currents in different branches of the circuits is initially marked either with the known direction or assumed direction.

After assigning the polarities for the voltage drops across the different passive elements, algebraic sum is accounted around a closed loop, either clockwise or anticlockwise, by assigning a particular sign, say the positive sign for all rising potentials along the path of tracing and the negative sign for all decreasing potentials. For example consider the circuit shown in Fig. 1.17

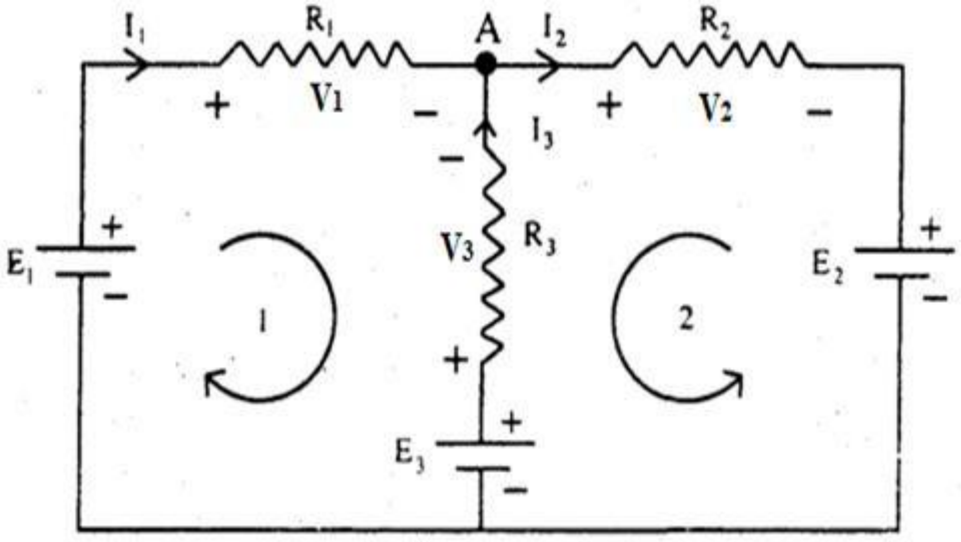


Fig. 1.17 Circuit for KVL

The circuit has three active elements with voltages E_1, E_2 and E_3. The polarity of each of them is fixed. R_1, R_2, R_3 are three passive elements present in the circuit. Currents I_1 and I_3 are marked flowing into the junction A and current I_2 marked away from the junction A with known information or assumed directions. With reference to the direction of these currents, the polarity of voltage drops V_1, V_2 and V_3 are marked.

For loop1 it is considered around clockwise

$$+ E_1 - V_1 + V_3 - E_3 = 0$$

$$+ E_1 - I_1 R_1 + I_3 R_3 - E_3 = 0$$

$$E_1 - E_3 = I_1 R_1 - I_3 R_3$$

For loop2 it is considered anticlockwise

$$+ E_2 + V_2 + V_3 - E_3 = 0$$

$$+ E_2 + I_2 R_2 + I_3 R_3 - E_3 = 0$$

$$E_2 - E_3 = - I_2 R_2 - I_3 R_3$$

Two equations are obtained following Kirchhoff's voltage law. The third equation can be written based on Kirchhoff's current law as

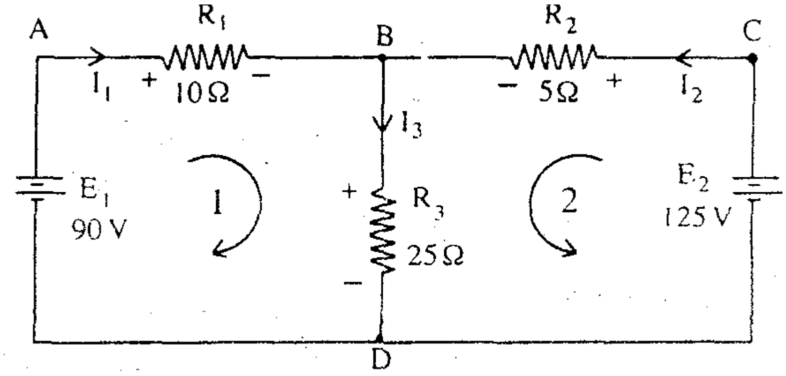
$$I_1 - I_2 + I_3 = 0$$

With the three equations, one can solve for the three currents I_1, I_2, and I_3.

If the results obtained for I_1, I_2, and I_3 are all positive, then the assumed direction of the currents are said to be along the actual directions. A negative result for one or more currents will indicate that the assumed direction of the respective current is opposite to the actual direction.

Example 1.11

Calculate the current supplied by two batteries in the circuit given below



Solution:

The four junctions are marked as A, B, C and D. The current through R_1 is assumed to flow from A to B and through R_2, from C to B and finally through R_3 from B to D. With reference to current directions, polarities of the voltage drop in R_1, R_2 and R_3 are then marked as shown in the figure. Applying KCL to junction B

$$I_3 = I_1 + I_2 \qquad \text{.....(1)}$$

Applying KVL to loop 1

$$E_1 - I_1R_1 - I_3R_3 = 0$$

$$I_1R_1 + I_3R_3 = E_1$$

$$10I_1 + 25I_3 = 90 \qquad \text{...... (2)}$$

Substituting Eq. (1) in Eq. (2)

$$10I_1 + 25(I_1 + I_2) = 90$$

$$35I_1 + 25I_2 = 90 \qquad \text{....... (3)}$$

Applying KVL to loop 2

$$E_2 - I_2R_2 - I_3R_3 = 0$$

$$I_2R_2 + I_3R_3 = E_2$$

$$5I_2 + 25I_3 = 125 \qquad \text{...... (4)}$$

Substituting Eq. (1) in Eq. (4)

$$5I_2 + 25(I_1 + I_2) = 125$$

$$25I_1 + 30I_2 = 125 \qquad \text{....... (5)}$$

Multiplying Eq. (3) by 6/5 we get

$$42I_1 + 30I_2 = 108 \qquad \text{...... (6)}$$

Subtracting Eq. (6) from Eq. (5)

$$-17I_1 = 17$$

$$I_1 = -1\text{ A}$$

Substituting the value of I_1 in Eq. (5) we get

$$I_2 = 5\text{ A}$$

As the sign of the current I_1 is found to be negative from the solution, the actual direction of I_1 is from B to A to D i.e. 90 V battery gets a charging current of 1 A.

1.8 RESISTIVE NETWORKS

1.8.1 SERIES RESISTORS AND VOLTAGE DIVISION

Two or more resistors are said to be in series if the same current flows through all of them. The process of combining the resistors is facilitated by combining two of them at a time. With this in mind, consider the single-loop circuit of Fig. 1.18.

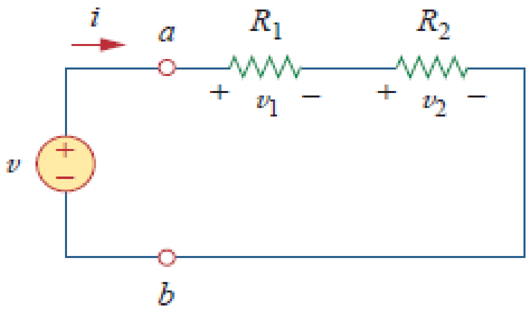


Fig.1.18 A single loop circuit with two resistors in series

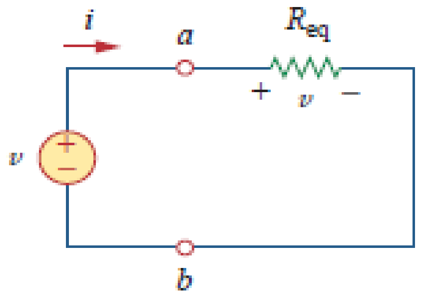


Fig. 1.19 Equivalent Circuit of series resistors

The two resistors are in series, since the same current i flow in both of them. Applying Ohm's law to each of the resistors, we obtain

………. (1)

If we apply KVL to the loop (moving in the clockwise direction), we have

……….. (2)

Combining equations (1) and (2), we get

……….. (3)

Or

……….. (4)

Equation (3) can be written as

………. (5)

implying that the two resistors can be replaced by an equivalent resistor ;that is

…………… (6)

Thus, Fig. 1.18 can be replaced by the equivalent circuit in Fig. 1.19. The two circuits in Fig 1.18 and 1.19 are the equivalent because they because they exhibit the same voltage-current relationships at the terminals a-b. An equivalent circuit such as the one in Fig. 1.19 is useful in simplifying the analysis of a circuit.

In general, the equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

For N resistors in series then,

……… (7)

VOLTAGE DIVISION:

To determine the voltage across each resistor in Fig. 1.18, we substitute Eq. (4) into Eq. (1) and obtain

……… (8)

Notice that the source voltage is divided among the resistors in direct proportion to their resistances; the larger the resistance, the larger the voltage drop. This is called the principle of voltage division, and the circuit in Fig. 1.18 is called a voltage divider. In general, if a voltage divider has N resistors ( ) in series with the source voltage , the nth resistor ( ) will have a voltage drop of

…….. (9)

1.8.2 PARALLEL RESISTORS AND CURRENT DIVISION

Two or more resistors are said to be in parallel if the same voltage appears across each element.Consider the circuit in Fig. 1.20, where two resistors are connected in parallel and therefore have the same voltage across them.

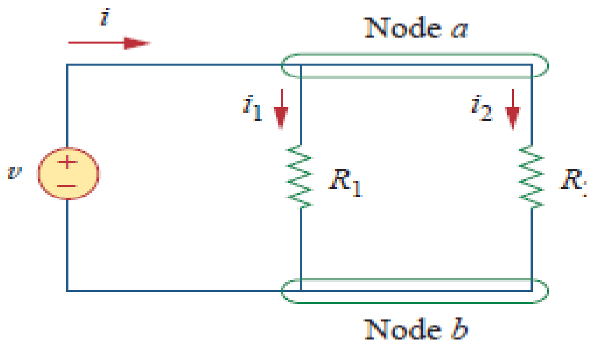


Fig. 1.20 Two resistors in parallel

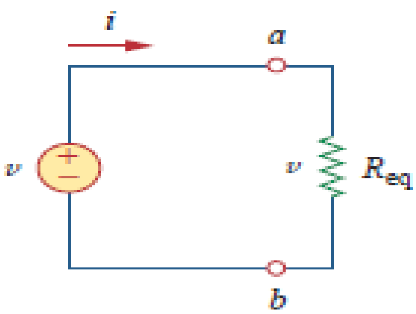


Fig. 1.21Equivalent circuit of Fig. 1.20

……… (1)

………. (2)

Applying KCL at node gives the total current as

…….. (3)

Substituting Eq. (2) into Eq. (3), we get

……… (4)

where is the equivalent resistance of the resistors in parallel.

………. (5)

Or

……….. (6)

Thus,

The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

It must be emphasized that this applies only to two resistors in parallel. From Eq. (6), if

We can extend the result in Eq. (5) to the general case of a circuit with N resistors in parallel. The equivalent resistance is

……… (7)

Thus,

The equivalent Resistance of parallel-connected resistors is the reciprocal of the sum of the reciprocals of the individual resistances.

Note that is always smaller than the resistance of the smallest resistor in the parallel combination.

Current Division:

Given the total current i entering node in Fig. 1.20, then how do we obtain currents ?
We know that the equivalent resistor has the same voltage, or

……. (8)

Substitute (8) in (2), we get

……….. (9)

This shows that the total current is shared by the resistors in inverse proportion to their resistances. This is known as the principle of current division, and the circuit in Fig.1.20 is known as a current divider. Notice that the larger current flows through the smaller resistance.

INDUCTIVE NETWORKS

Now that the inductor has been added to our list of passive elements, it is Necessary to extend the powerful tool of series-parallel combination. We need to know how to find the equivalent inductance of a series-connected or parallel-connected set of inductors found in practical circuits.

SERIES INDUCTORS

Two or more inductors are said to be in series, if the same current flows through all of them. Consider a series connection of N inductors, as shown in Fig. 1.22(a), with the equivalent circuit shown in Fig. 1.22(b). The inductors have the same current through them.

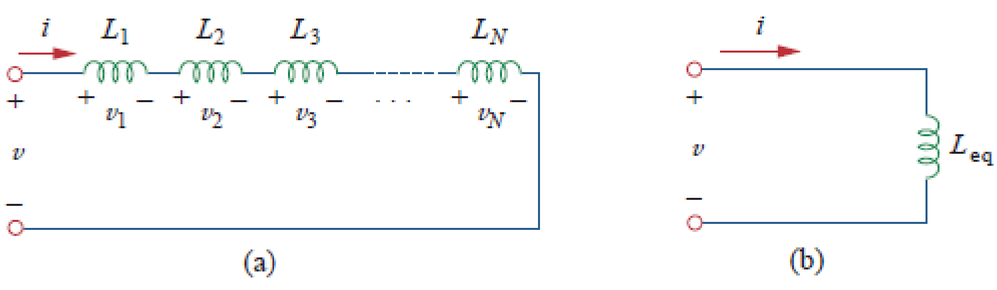


Fig. 1.22 (a) series connection of N inductors (b) Equivalent circuit for the series inductors

Applying KVL to the loop,

We know that the voltage across an inductor is

Therefore, Eq. (1) becomes

Where,

Thus

The equivalent inductance of series-connected inductors is the sum of the individual inductances.

* Inductors in series are combined in exactly the same way as resistors in series.

INDUCTORS IN PARALLEL

Two or more inductors are said to be in parallel, if the same voltage appears across each element. We now consider a parallel connection of N inductors, as shown in Fig. 1.23(a), with the equivalent circuit in Fig. 1.23(b). The inductors have the same voltage across them.

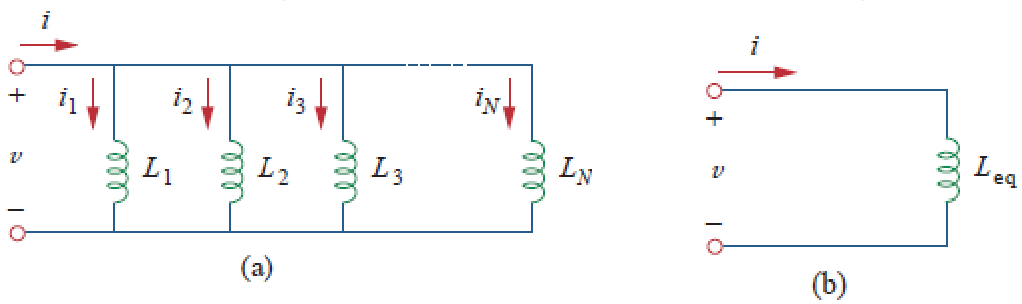


Fig. 1.23 (a) Parallel connection of N inductors (b) Equivalent circuit for parallel inductors

Using KCL,

But the current through the inductor is

If we neglect the initial value of current i.e, then current through inductor becomes

Hence,

Where,

Thus,

The equivalent inductance of parallel inductors is the reciprocal of the sum of the reciprocals of the individual inductances.

* Note that the inductors in parallel are combined in the same way as resistors in parallel.

CAPACITIVE NETWORKS

We know from resistive circuits and inductive circuits that the series-parallel combination is a powerful tool for reducing circuits. This technique can be extended to series-parallel connections of capacitors, which are sometimes encountered. We desire to replace these capacitors by a single equivalent capacitor $C_{eq.}$

SERIES CAPACITORS

Two or more capacitors are said to be in series, if the same current flows through all of them. Consider a series connection of N capacitors, as shown in Fig. 1.24(a), with the equivalent circuit shown in Fig. 1.24(b). The capacitors have the same current through them.

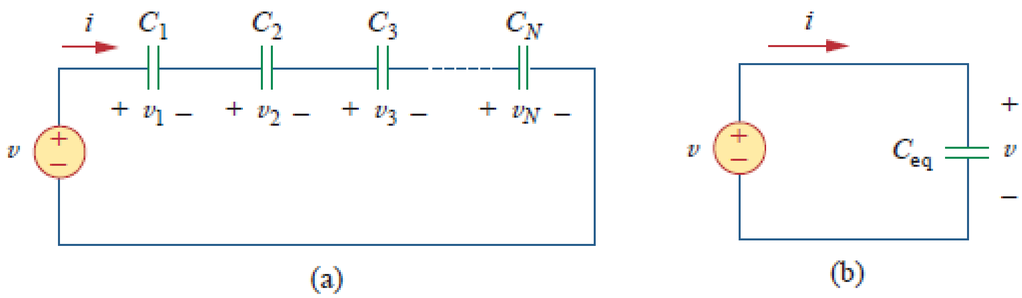


Fig. 1.24 (a) series connection of N capacitors (b) Equivalent circuit for the series capacitors

Applying KVL to the loop,

We know that the voltage across a capacitor is

If we neglect the initial value of voltage i.e, then voltage across the capacitor becomes

Hence, Eq. (1) becomes

Where,

Thus,

The equivalent capacitance of series-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.

* Note that the capacitors in series are combined in the same way as resistors in parallel.

For N=2,

PARALLEL CAPACITORS

Two or more capacitors are said to be in parallel, if the same voltage appears across each element. Consider a parallel connection of N capacitors, as shown in Fig. 1.25(a), with the equivalent circuit in Fig. 1.25(b). The capacitors have the same voltage across them.

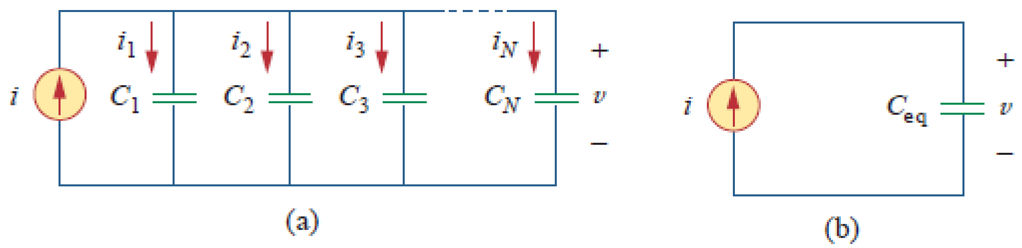


Fig. 1.25 (a) Parallel connection of N capacitors (b) Equivalent circuit for parallel capacitors

Applying KCL to Fig. 1.25(a)

We know that the current through capacitor is

Therefore, Eq. (1) becomes

Where,

Thus

The equivalent capacitance of parallel-connected capacitors is the sum of the individual capacitances.

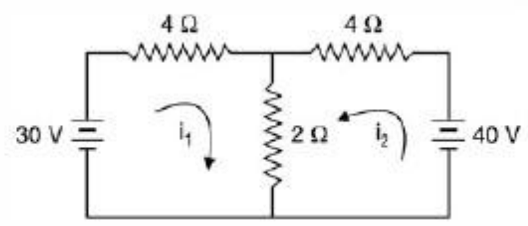
* Capacitors in parallel are combined in exactly the same way as resistors in series.

UNIT II

NETWORK ANALYSIS AND THEOREMS

MESH ANALYSIS:

Mesh analysis is a technique used to solve the complex networks consisting of more number of meshes. Mesh analysis is nothing but applying KVL to each and every loop in circuit and solving for mesh currents. By finding the mesh currents we can solve any require data of the network.



Let V1 = 30v, V2 = 40v, R1 =4 ohms, R2 = 2 ohms and R3 = 4 ohms

i1, i2 are the mesh currents, here positive direction of currents is assumed, but in general we can assume current directions in any fashion and can solve.

Applying KVL to first loop we get, V1 = i1R1 – (i1+i2) R2.

V1 = (R1+R2) i1 + i2.R2. ------- 1

Applying KVL to second loop we get, V2 =i2R3 – (i1+i2) R2.

V2 = (R1+R2) i1 + i2.R3. ------ 2

Hence by solving eq. 1 and 2 we can get mesh currents i1 and i2.

Mesh analysis by inspection method:

Inspection method is direct method using mesh currents can be find directly without applying KVL. Let us take same network as above , representing eq 1 and 2 in matrix form.

V1		R1	R1+R2	i1
V2	=	R1+R2	R3	i2

Genereally we can write in matrix form as,

V1		R11	R12	i1
V2	=	R21	R22	i2

Where, V1 – sum of all the voltage sources in 1st mesh according current flow.

V2 – sum of all the voltage sources in 2nd mesh according current flow.

R11- self resistance of first loop, adding total resistance in 1st loop

R21= R12- mutual resistance between 1st and 2nd loop,

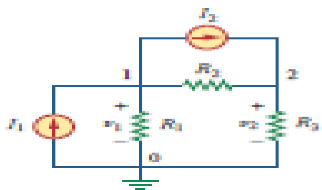
+ve if mesh currents are in same direction

-ve if mesh currents are in opposite direction

R22- self resistance of second loop, adding total resistance in 2nd loop

NODAL ANALYSIS:

Nodal analysis is a technique used to solve the complex networks consisting of more number of nodes. Node analysis is nothing but applying KCL to each and every node in circuit and solving for node voltages. By finding the node voltages we can solve any require data of the network.



For the above circuit current division takes place at two nodes 1 and 2.

Let, I3, I4, I5 are the currents flowing through R1,R2 and R3.

Applying KCL at node 1, $I1 = I3+I4$

$$= Va / R1 + (Va-Vb) / R2$$

$$= Va(1/R1 + 1/R2) - Vb / R2 \text{ ----1}$$

Applying KCL at node 2, $I2+I4 = I5$

$$I2+(Va-Vb) / R2 = Vb / R3$$

$$I2 = - Va / R2 + Vb (1/R3 + 1/R2) \text{ ----2}$$

By solving above eq.1 and 2 we can get node voltages V1 and V2.

Nodal analysis by inspection method:

Inspection method is direct method using node voltages can be find directly without applying KCL. Let us take same network as above, representing eq 1 and 2 in matrix form.

I1		(1/R1 +1/R2)	-1/ R2	V1
I2	=	-1/ R2	(1/R3 +1/R2)	V2

Generally we can write in matrix form as,

I1		G11	G12	V1
I2	=	G21	G22	V2

Where, I1 – sum of all the current sources in 1st node according current flow.

I2 – sum of all the voltage sources in 2^{nd} node according current flow.

G11- self conductance of first node, adding total conductance in 1^{st} node

G21= G12- mutual conductance between 1^{st} and 2^{nd} loop , Always -ve

G22- self conductance of second node, adding total conductance in 2^{nd} node

CONVENTIONS:

- Power delivered = Vs. Is
- Power lost = I^2 . R
- voltage is positive if flow is from negative to positive called as poetntial rise.(Vs)
- voltage is negative if flow is from positive to negative called as poetntial drop.(I.R)

NETWORK TOPOLOGY:

Network topology is the one of the technique to solve electrical networks consisting of number og meshes or number of nodes, where it is difficult to apply mesh and nodal analysis.

Graph theory is the technique where all the elements of the network are Represented by straight lines irrespective of their behavior. Here matrix methods are used to solve complex networks. Before seeing the actual matrices, the knowledge of some of the definitions is very important.

DEFINITONS:

Node: A node is junction where two or more than two elements are connected.

Degree of the node: number of elements connected to the node is defined as degree of the node.

Branch: A branch is a element(s) connected between pair of nodes.

Path: It is traversal of signal between pair of nodes.

Loop: It is the path started from an node and ends at the same node.

Graph: A graph is formed when all the elements of the network are replaced by straight line irrespective of their behaviour.

Oriented and non-oriented graph: If the graph of the network is represented with directions in each and every branch then it is oriented, if atleast one branch of graph has no direction then iti is non-oriented graph.

Planar and non-planar graph: if a graph can be on plane surface without cross over then system is planar and vice-versa is non-linear.

Tree: Tree is the sub-graph of the graph, which consists of same number of nodes as that of original graph without any closed path.

Branches of the tree are called as twigs. Numbers of possible twigs in a tree are (n - 1).

n – Number of nodes

Co-tree: The set of braches which are removed to form tree are called as co-tree.

Links: Branches removed to form tree are called as links.

$$l = b – (n-1)$$

INCIDENCE MATRIX (A):

Incidence matrix is formed between number of nodes and number of branches. This matrix is useful easy understanding of network and any complex network can be easily feed into system for coding. Order of incidence matrix is (n*b)

Procedure to incidence matrix:

aij = 1, if jth branch is incidence to ith node and direction is away from node.

aij = -1, if jth branch is incidence to ith node and direction is towards from node.

aij = 0, if jth branch is not incidence to ith node .

Properties of incidence matrix:

1. Number of non zero entries of row indicates degree of the node.
2. The non zero entries of the coloumn represents branch connections.
3. If two coloumns has same entries then they are in parallel.

Reduced incidence matrix:

Reduced incidence matrix is formed by eliminating one of the row from incidence matrix generally ground node row is eliminating, which is representing by A!. Number of possible trees of for any graph are det(A! A!T).

Once we form the incidence matrix , we can write KCL equations any complex without appling KCL as follows,

A!.Ib = 0

Where, A! - reduced incidence matrix

Ib - branch current matrix(an coloumn matrix)

TIE-SET MATRIX:

Tie-set matrix is formed between link currents and branch currents. The order of tie-set matrix is (Il*Ib). it is represented by B. Tie-set is defined as a loop which consists of one link any number of branches. Hence total number of tie-set possible with above definition are number of links.

Procedure to form tie-set matrix:

Bij = 1, if current direction of jth branch coincides with direction of ith link.

Bij = -1, if current direction of jth branch opposes with direction of ith link.

Bij = 0, if jth branch has no relation with ith link.

Once we form tie-set matrix KVL equations of any complex network can be written as B. $V_b = 0$

Where, B = tie-set matrix

V_b = branch voltage matrix (coloumn matrix)

From the knowledge of tie-set matrix we can calculate all the branch currents in terms of link currents, which is given as

Ib = BT . Il

Where, Ib - branch current matrix(an coloumn matrix)

Il - link current matrix(an coloumn matrix)

BT- transpose of tie-set matrix

CUT-SET MATRIX:

Cut-set matrix is formed between twig voltages and branch voltages. The order of cut-set matrix is ((n-1)*b). it is represented by C. Cut-set is defined as minimum set of branches by removing which graph is divided into two sub-graphs, where one of the part is an isolated node. Hence total number of cut-set possible with above definition is number of twigs. Generally cut-set direction is assumed in direction of branch, as cut-set consists of one branch any number of links.

Procedure to form tie-set matrix:

cij = 1, if direction of jth branch coincides with direction of ith cut-set.

cij = -1, if direction of jth branch opposes with direction of ith cut-set.

cij = 0, if direction of jth branch is not in the ith cut-set.

Once we form cut-set matrix KCL equations of any complex network can be written as C.Ib = 0

Where, C= cut-set matrix

Ib = branch current matrix(coloumn matrix)

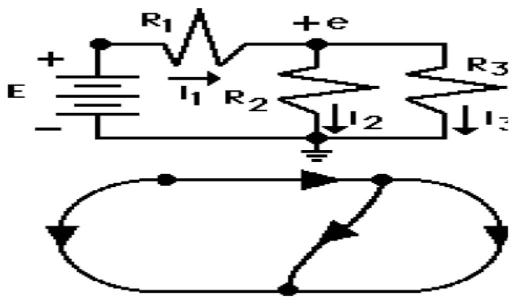
From the knowledge of cut-set matrix we can calculate all the branch voltages in terms of twig voltages, which is given as

Vt = CT . Vb

Where, Vb - branch voltage matrix(an coloumn matrix)

Vt – twig voltage matrix(an coloumn matrix)

CT- transpose of cut-set matrix



Eg: Determine all studied above for given circuit with its graph.

Nodes,n   = 3

Branches = 4

Twigs   = 3-1 = 2

Links   = 4-3+1 = 2

Incidence matrix:

A	**=**		**1**	**2**	**3**	**4**
		a	**1**	**1**	**0**	**0**
		b	**0**	**-1**	**1**	**1**
		c	**-1**	**0**	**-1**	**-1**

Reduced Incidence matrix:

A!	**=**		**1**	**2**	**3**	**4**
		a	**1**	**1**	**0**	**0**
		b	**0**	**-1**	**1**	**1**

Tie-set matrix:

As tree can be formed with two branches remaining two are links, let 2,4 arethe links.

B	**=**		**i1**	**i2**	**i3**	**i4**
		i2	**-1**	**1**	**1**	**0**
		i4	**0**	**0**	**-1**	**1**

We can write all branch currents as, Ib = BT . Il

i1	=	-1	0	i2
i2		1	0	
i3		1	-1	i4
i4		0	1	

Hence , i1 = -i2

i2 = i2

i3 = i2-i4

i4 = i4

2.6.1 Cut-set matrix:

Let the two cut-set are (1,2) and (2,3,4) as possible cut-sets are (n-1) = (3-1) =2.

Here 1 and 3 are branches or twigs hence cutsets direction is same as these branch directions.

			1	**2**	**3**	**4**
C	**=**	**C(1,2)**	**1**	**-1**	**0**	**0**
		C(2,3,4)	**0**	**-1**	**1**	**1**

We can write all branch voltages as, Vb = CT . Vt

V1		1	0	V1
V2	=	-1	-1	
V3		0	1	V3
V4		0	1	

Hence, V1 = V1

V2 = -V1-V3

V3= V3

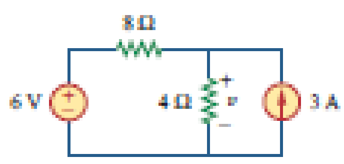
V4 = V3

2.7 **INTRODUCTION of** Network theorems

Network theorems are also can be termed as network reduction techniques. Each and every theorem got its importance of solving network. Let us see some important theorems with DC and AC excitation with detailed procedures.

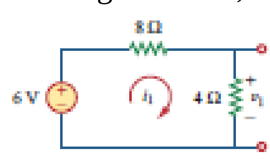
2.7.2 SUPER-POSITION THEOREM:

DC: "In an any linear, bi-lateral network consisting number of sources, response in any element(resistor) is given as sum of the individual responses due to individual sources, while other sources are non-operative**"**



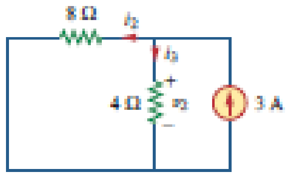
Eg: Let V = 6v, I = 3A, R1 = 8 ohms and R2 = 4 ohms

Let us find current through 4 ohms using V source, while I is zero. Then equivalent circuit is

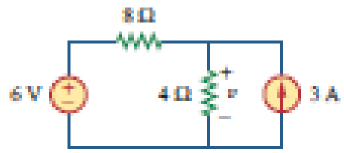


Let i1 is the current through 4 ohms, i1 = V / (R1+R2)

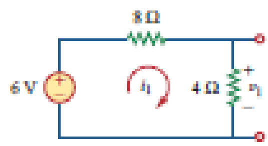
Let us find current through 4 ohms using I source, while V is zero. Then equivalent circuit is



Let i2 is the current through 4 ohms, i2 = I. R1 / (R1+R2)
Hence total current through 4 ohms is = I1+I1 (as both currents are in same direction or otherwise I1-I2)

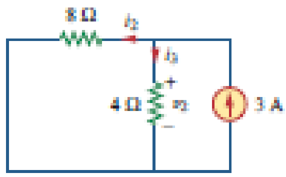


Let V = 6v, I = 3A, Z1 = 8 ohms and Z2 = 4 ohms
Let us find current through 4 ohms using V source, while I is zero. then equivalent circuit is



Let i1 is the current through 4 ohms, i1 = V / (Z1+Z2)

Let us find current through 4 ohms using I source, while V is zero.then equivalent circuit is

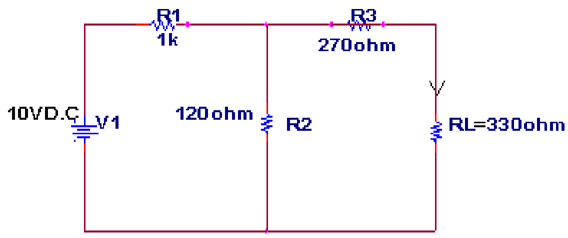


Let i2 is the current through 4 ohms, i2 = I. Z1 / (Z1+Z2)

Hence total current through 4 ohms is = I1+I1(as both currents are in same direction or otherwise I1-I2).

4.7.3 RECIPROCITY THEOREM:

DC: “In any linear bi-lateral network ratio of voltage in one mesh to current in other mesh is same even if their positions are inter-changed”.



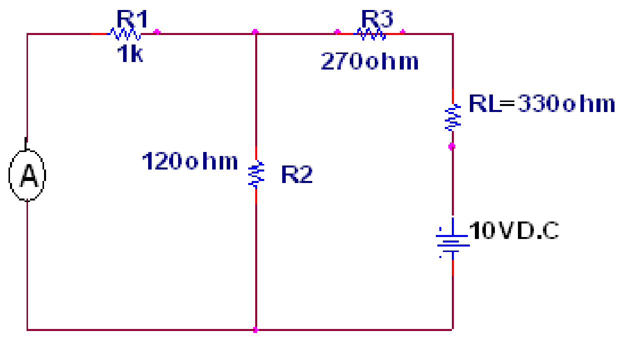
Eg: Find the total resistance of the circuit, Rt = R1+ [R2(R3+Rl)] / R2+R3+RL.

Hence source current, I = V1 / Rt.

Current through RL is I1 = I. R2 / (R2+R3+RL)

Take the ratio of, V1 / I1

Draw the circuit by inter changing position of V1 and I1



Eg: Find the total resistance of the circuit, Rt = (R3+RL) + [R2(Rl)] / R2+R1.

Hence source current, I = V1 / Rt.

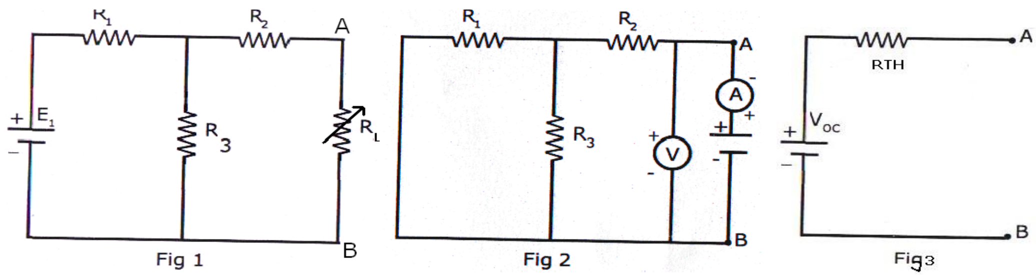
Current through RL is I1 = I. R2 / (R2+R1)

Take the ratio of , V1 / I1 ---2

If ratio 1 = ratio 2, then circuit is said to be satisfy reciprocity.

THEVENIN'S THEOREM:

DC: " An complex network consisting of number voltage and current sources cand be replaced by simple series circuit consisting of equivalent voltage source in series with equivalent resistance, where equivalen voltage is called as open circuit voltage and equivalent resistance is called as thevenin's resistance calculated across open circuit terminals while all energy sources are non-operative"



Eg: Here we need to find current through RL using thevenin's theorem.

Open circuit the AB terminals to find the Thevenin's voltage.

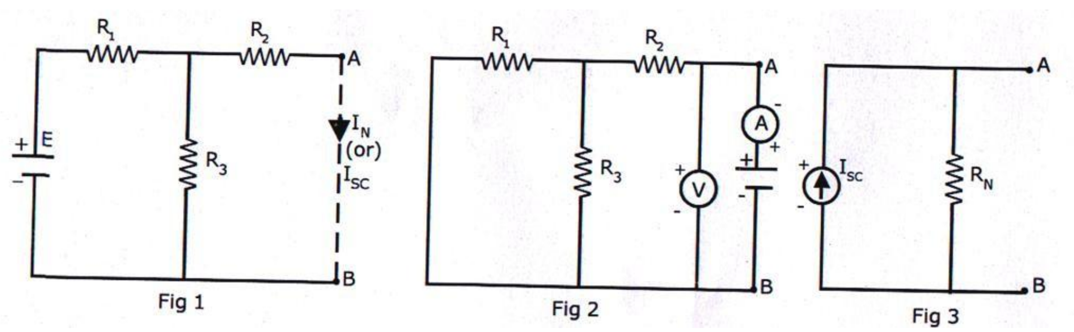
Thevenin's voltage , Vth = E1. R3 / (R1+R3) ----1 from figure .1

Thevenin's resistance, Rth = (R1.R3)/ (R1+R3) + R2 ----2 from figure 2.

Now draw the thevenin's equivalent circuit as shown in figure 3 with calculated values.

NORTON'S THEOREM:

DC: "A complex network consisting of number voltage and current sources cand be replaced by simple parallel circuit consisting of equivalent current source in parallel with equivalent resistance, where equivalent current source is called as short circuit current and equivalent resistance is called as Norton's resistance calculated across open circuit terminals while all energy sources are non-operative"



Here we need to find current through RL using Norton's theorem.

Short circuit the AB terminals to find the Norton are current.

Total resistance of circuit is, Rt = (R2.R3) / (R2+R3) + R1

Source current, I = E / Rt

Norton's current, IN = I. R3 / (R2+R3) ----1 from figure .1

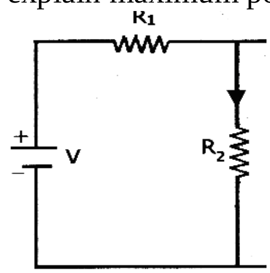
Norton's resistance, RN = (R1.R3)/ (R1+R3) + R2 ----2 from figure 2.

Now draw the Norton's equivalent circuit as shown in figure 3 with calculated values.

MAXIMUM POWER TRANSFER THEOREM:

DC: "In linear bi-lateral network maximum power can be transferred from source to load if load resistance is equal to source or thevenin's or internal resistances".

Eg: For the below circuit explain maximum power transfer theorem.



Let I be the source current, I = V / (R1+R2)

Power absorbed by load resistor is, $PL = I^2.R2$

$$= [V / (R1+R2)]^2.R2.$$

To say that load resistor absorbed maximum power , dPL / dR2 = 0.

When we solve above condition we get, R2 = R1.

Hence maximum power absorbed by load resistor is, $PLmax = V^2 / 4R2$.

MILLIMAN'S THEOREM:

DC: " An complex network consisting of number of parallel branches , where each parallel branch consists of voltage source with series resistance, can be replaced with equivalent circuit consisting of one voltage source in series with equivalent resistance"

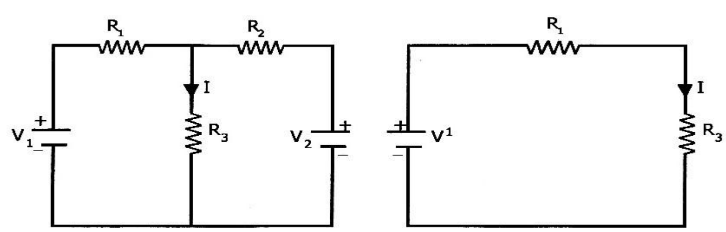


Fig 1 Fig 2

Where equivalent voltage source value is ,

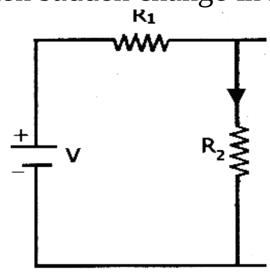
$$V' = \frac{(V1G1+V2G2+\text{------}+VnGn)}{G1+G2+\text{----------------}Gn}$$

Equivalent resistance is ,

R' = 1 / (G1+G2+-------------------Gn)

COMPENSATION THEOREM:

DC: "compensation theorem states that any element in the network can be replaced with Voltage source whose value is product of current through that element and its value" It is useful in finding change in current when sudden change in resistance value.



For the above circuit source current is given as, I = V / (R1+R2)

Element R2 can be replaced with voltage source of, V' = I.R2

Let us assume there is change in R2 by ΔR, now source current is I'= V / (R1+R2+ ΔR)

Hence actual change in current from original circuit to present circuit is = I – I'.

This can be find using compensation theorem as, making voltage source non-operative and replacing ΔR with voltage source of I'. ΔR.

Then change in current is given as = I'. ΔR / (R1+R2)

Tellegens theotem

This theorem has been introduced in the year of 1952 by Dutch Electrical Engineer Bernard D.H. Tellegen. This is a very useful theorem in network analysis. According to Tellegen theorem, the summation of instantaneous powers for the n number of branches in an electrical network is zero. Are you confused? Let's explain. Suppose n number of branches in an electrical network have i_1, i_2, i_3, ...i_n respective instantaneous currents through them. These currents satisfy Kirchhoff's Current Law. Again, suppose these branches have instantaneous voltages across them are v_1, v_2, v_3, ... v_n respectively. If these

$$\sum_{k=1}^{n} v_k \cdot i_k = 0$$

Voltages across these elements satisfy Kirchhoff Voltage Law then, v_k is the instantaneous voltage across the k^{th} branch and i_k is the instantaneous current flowing through this branch.

Tellegen theorem is applicable mainly in general class of lumped network s that consist of linear, non-linear, active, passive,

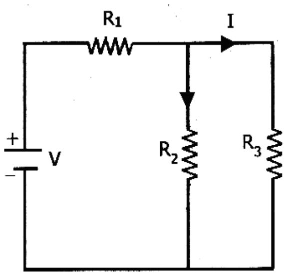


Fig 1

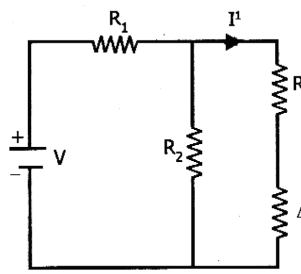


Fig 2

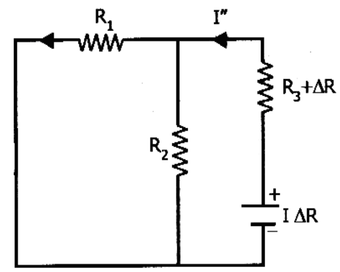
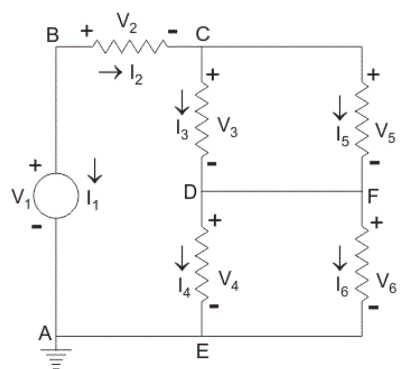


Fig 3

time variant and time variant elements. This theorem can easily be explained by the following example.



In the network shown, arbitrary reference directions have been selected for all of the branch currents, and the corresponding branch voltages have been indicated, with positive reference direction at the tail of the current arrow.

For this network, we will assume a set of branch voltages satisfy the Kirchhoff voltage law and a set of branch current satisfy Kirchhoff current law at each node. We will then show that these arbitrary assumed

$$\sum_{k=1}^{n} v_k \cdot i_k = 0$$

Voltages and currents satisfy the equation. And it is the condition of Tellegen theorem. In the network shown in the figure, let v_1, v_2 and v_3 be 7, 2 and 3 volts respectively. Applying Kirchhoff voltage law around loop ABCDEA. We see that $v_4 = 2$ volt is required. Around loop CDFC, v_5 is required to be 3 volt and around loop DFED, v_6 is required to be 2. We next apply Kirchhoff current law successively to nodes B, C and D. At node B let $i_i = 5$ A, then it is required that $i_2 = -5$ A. At node C let $i_3 = 3$ A and then i_5 is required to be -8. At node D assume i_4 to be 4 then

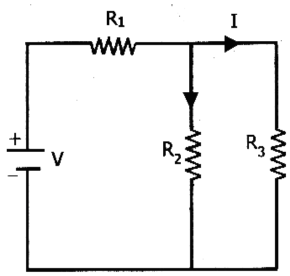


Fig 1

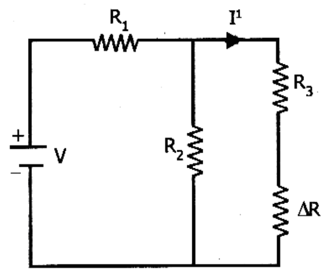


Fig 2

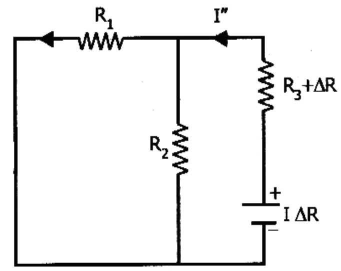


Fig 3

$k=1$

i_6 is required to be − 9. Carrying out the operation of equation, we get,

$$7 \times 5 + 2 \times (-5) + 3 \times 3 + 2 \times 4 + 3 \times (-8) + 2 \times (-9) = 0$$

Hence Tellegen theorem is verified.

Statement

Substitution theorem states that "if an element in a network is replaced by a voltage source whose voltage at any instant of time is equals to the voltage across the element in the previous network then the initial condition in the rest of the network will be unaltered". Or alternately "if an element in a network is replaced by a current source whose current at any instant of time is equal to the current through the element in the previous network then the initial condition in the rest of the network will be unaltered".

Explanation

Let us take a circuit as shown in fig - a, Let, V is supplied voltage and Z_1, Z_2 and Z_3 is different circuit impedances. V_1, V_2 and V_3 are the voltages across Z_1, Z_2 and Z_3 impedance respectively and I is the supplied current whose I_1 part is flowing through the Z_1 impedance whereas I_2 part is flowing through the Z_2 and Z_3 impedance.

Now if we replace Z_3 impedance with V_3 voltage source as shown in fig-b or with I_2 current source as shown in fig-c then according to Substitution Theorem all initial condition through other impedances and source will remain unchanged.

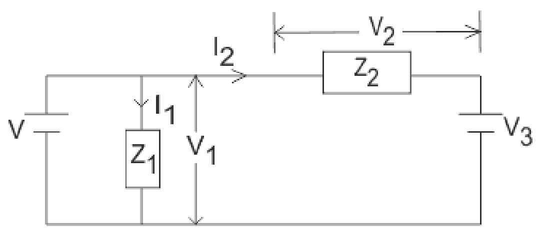


fig-b

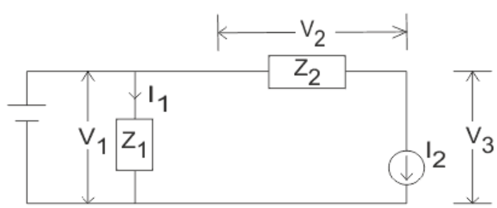


fig - c

i.e. - current through source will be I, voltage across Z_1 impedance will be V_1, current through Z_2 will be I_2 etc.

Current-Source Splitting Theorem

Figure E.4(a) shows an ideal current source *I* connected between two networks N_1 and N_2. This source can be either independent or dependent. The branch with a single ideal current source can be replaced with a branch with any number of identical ideal current sources *I* connected in series. Figure E.4(b) shows two identical current sources. In the simplest case, an ideal current source can be replaced by two ideal current sources connected in series. The point between two current sources can be connected to any other point in the circuit. The current through the conductor between these two points is zero, as shown in Fig. E.4(c). The single conductor can be replaced by two conductors, as shown in Fig. E.4(d). The conductor at the bottom can be cut
and removed from the circuit, as shown in Fig. E.4(f).

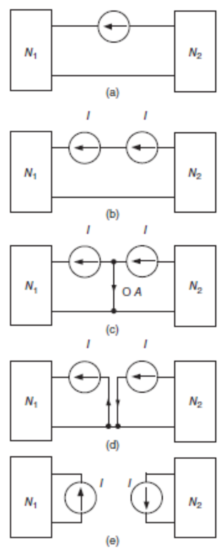


Fig: current source shifting theorem

Voltage Source Splitting Theorem

Figure E.5(a) shows an ideal voltage source *V* connected to two different points in the network *N*. A single conductor can be replaced by two conductors, as shown in Fig. E.5(b). The single ideal voltage source *V* can be replaced by any number of identical ideal voltage sources *V* connected in parallel, as shown in Fig. E.5(c). The current flowing in the conductor between two identical voltage sources is zero. Therefore, the conductor connecting the two voltage sources can be removed from the circuit, as shown in Fig. E.5(d).

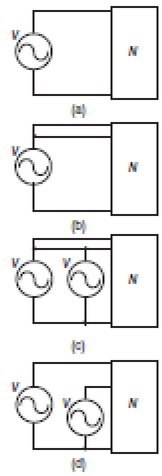


Fig: voltage source splitting theorem

UNIT III

AC CIRCUITS

INTRODUCTION:

The alternating quantity is one whose value varies with time. This alternating quantity may be periodic and non-periodic. Periodic quantity is one whose value will be repeated for every specified interval. Generally to represent alternating voltage or current we prefer sinusoidal wave form, because below listed properties

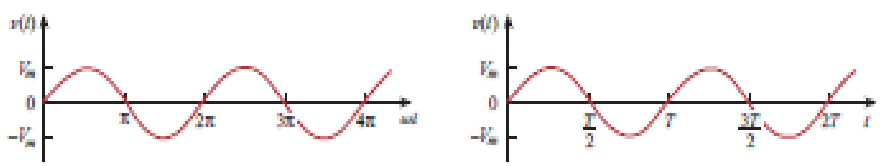
1. Derivative of sine is an sine function only.
2. Integral of sine is an sine function only.
3. It is easy to generate sine function using generators.
4. Most of the 2^{nd} order system response is always sinusoidal.

Alternating quantity:

As said above an alternating voltage or current can be represented with sine wave. Sine wave can be defined with degree or radians as reference.

At, 0 degrees --- 0
90 degrees --- maximum
180 degrees --- 0
270 degrees --- maximum
360 degrees --- 0

i.e value of sine function varies with time, firstly increases from zero and reaches maximum and again falls to zero, there after tends to increase in opposite direction and reaches maximum value and falls to zero. This the variation of sine in 1^{st} cycle is called as positive half cycle and other negative half cycle.(i.e during +ve half cycle direction is required one and during 2^{nd} half cycle direction actual required direction.). Therefore one positive and negative cycle combinely forms one complete cycle.



Sine equation , voltage , V(t) = Vm sin wt.

Where, Vm = peak value or maximum value
W = angular frequency.

Definitions:

Peak to peak value: It is total value from positive peak to the negative peak.(2Vm)

Instantaneous value: It is the magnitude of wave form at any specified time. V(t)

Average value : It is ratio of area covered by wave form to its length.(Vd)

$$Vd = (1/T) \int V\ (t)\ dwt.$$

$$Vd = (1\ /\ 2\Pi) \int Vm \sin wt.dwt$$

$$= -\ Vm\ /\ 2\Pi.\ \cos wt \text{---with limits of } 2\Pi \text{ and } 0$$

$= 0.$ (i.e. average value of sine wave over a full cycle is zero)

Hence it is defined for half cycle.

$$Vd = (1\ /\ \Pi) \int Vm \sin wt.dwt$$

$$= -\ Vm\ /\ \Pi\ .\ \cos wt \text{ with limits of } \Pi \text{ and } 0$$

$$= 2Vm\ /\ \Pi$$

RMS value:

It is the root mean square value of the function, which given as

$$Vrms = \quad (1/T) \int V(t)]2\ dwt.$$

$$= \quad (1/2\Pi) \int Vm2[\ (1-\cos 2wt)/2]dwt.$$

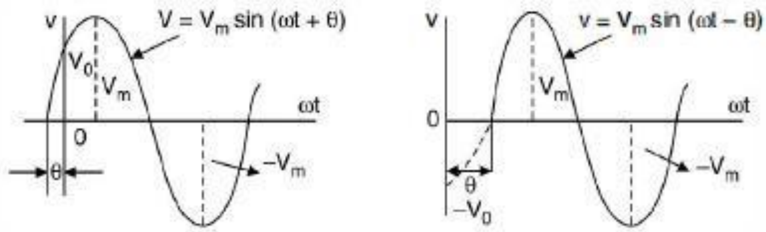
$$= \quad (1/2\Pi)\ .Vm2[\ (wt-\sin 2wt\ /\ 2wt)/2]$$

$= Vm\ /\ \overline{}$ = effective value.

Peak factor:
It is the ratio of peak value to the rms value.
$Pp = Vp\ /\ Vrms = \overline{}$

Form factor:
It is the ratio of average value to the rms value.
$Fp = Vd\ /\ Vrms = 2\ \overline{}\ /\ \Pi = 1.11$
Eg: Find the peak, peak to peak, average, rms, peak factor and form factor of given current function , $i(t) = 5 \sin wt$.

Phase and phase difference:
Phase of the sine indicates staring phase of the sine wave. i.e
Let , $V(t) = Vm \sin wt$, here we can say that phase is zero as function starts from origin.
$V(t)= Vm \sin(wt-\theta)$, here we can say that phase of function is θ degrees to right shift.
$V(t)= Vm \sin(wt+\theta)$, here we can say that phase of function is θ degrees to left shift.



Phase difference is the difference of phase between two wave forms taking one as reference.
Eg: If wave form A is Vm sin (wt+15), B is Vm sin (wt – 30) and C is Vm sin (wt + 45).
Determine the phase difference between every pair if wave forms.
When A and B are compared , phase difference is 45 degrees.
When C and B are compared , phase difference is 75 degrees.
When A and C are compared , phase difference is 30 degrees.

Phasor diagram:

Phasor diagram is the pictorial representation of sine wave. Here magnitude and phase of the wave function are represented in four quadrant axis. We assume positive phases in anti-clock wise direction and negative phases in clock wise direction. From the phasor diagram we can esily identify the phase difference between different wave forms. We can also identify whether function is right shift or left shift.

Phase relations of network parameters:

Resistor:

Let us consider resistor allowing alternating current i(t). Then the voltage drop across resistor is given as,

If, V(t) = Vm sin wt
V(t) = i(t).R
i(t) = V(t) / R
= Vm sin wt / R
i(t) = Im sin wt.

The ratio of V(t) / i(t) = Z = impedance offered by resistor.(ohms).

Z = Vm sin wt / Im sin wt.
= Vm / Im

Hence we can say that V(t) and i(t) in resistor element are in phase.

Inductor:

Let us consider an coil of N turns allowing current i(t).(Im sin wt)
Hence emf induced in the coil is ,

V(t) = L di(t) / dt
= L d(Im sin wt) / dt
= L w Im coswt
= Vm coswt = Vm sin(wt + 90).

Where, $V_m = L w I_m = I_m .X_L$

X_L = reactance offered by coil.

Impedance offered by coil is , $Z = V(t) / i(t)$

$= V_m \sin(wt + 90) / I_m \sin wt$

The function $V_m \sin wt = V_m$.

$Z = V_m \quad / I_m$

$Z = V_m \quad / I_m$

$= j\, wL = j\, X_L \quad (j = 1 \quad)$

As there is left shift in V (t), we can say that i(t) lags V(t) by 90 degrees.

Capacitor:

Let us consider an capacitor allowing current i(t).(Im sin wt)

Hence voltage across it is ,

$V(t) = 1 / C \int i(t)\, dt$

$= 1 / C \int I_m \sin wt\, dt$

$= - \cos wt .I_m / wC$

$= V_m \sin(wt - 90).$

Where, $V_m = I_m/ wC = I_m .X_C$

X_C = reactance offered by capacitor

Impedance offered by capacitor is , $Z = V(t) / i(t)$

$= V_m \sin(wt -90) / I_m \sin wt$

The function $V_m \sin wt = V_m$.

$Z = V_m \quad / I_m$

$Z = V_m \quad / I_m$

$= -j\, wL =- j\, X_L \quad (j = 1 \quad)$

As there is right shift in V(t), we can say that i(t) leads V(t) by 90 degrees.

Power in Ac circuits

In the case of DC circuits power is given as product of voltage and current in that element.

$P = V.I$ (W)

Let $V(t) = V_m \sin wt$

$i(t) = I_m \sin(wt +90)$

Instantaneous power, $P (t) = V (t).i (t)$

$= V_m \sin wt.\ I_m \sin(wt + \Phi)$

$= V_m.I_m \sin wt \sin (wt + \Phi).$

Average power, $P_{av} = 1/ 2\pi \int p(t)\, dwt$.

$= 1/ 2\pi \int V_m.I_m [\cos \Phi - \cos(2wt + \Phi)]\ dwt$

As average value over full cycle is equal to zero, hence second term can be neglected.

$P_{av} = 1/ 2\pi \int V_m.I_m [\cos \Phi]\ dwt$

$= V_{rms} I_{rms} \cos \Phi[wt]$ ----------- with limits 2π and 2π

$P_{av} = V_{rms}. I_{rms} \cos \Phi.(W)$ = true power= active power.

$\cos \Phi = P_{av} / V_{rms}. I_{rms}$.= defined as power factor of the circuit.

$\cos \Phi = P_{av} / P_a$

= true power / apparent power

= actual power utilized by load / total generated power.

P_a = apparent power = $V_{rms}. I_{rms}$ = V-A

Let us consider commercial inductor,

$Z = R + jX_L$

Where, Z = impedance of the coil

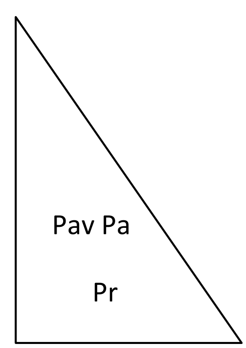
R = internal resistance of the coil

XL = reactance offered by the coil.

$I(t) Z = I(t) R + I(t) jX_L$

$I^2 Z = I^2R + j I^2X_L$

$P_a = P_{av} + j P_r$



power triangle with phase Φ

Pav = Pa cos Φ = Vrms.Irms cos Φ = active power= W

Pr = Pa sin Φ = Vrms.Irms sin Φ= reactive power = VAR

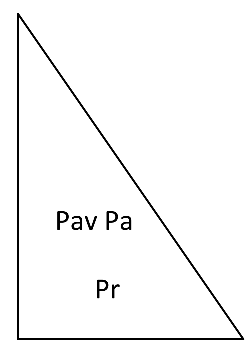
Let us consider commercial capacitor,

$$Z = R - jXC$$

Where, Z = impedance of the capacitor

R = internal resistance of the capacitor

XC = reactance offered by the capacitor.



power triangle with phase Φ

$$I(t)\,Z = I(t)\,R - I(t)\,jXC$$

$$I^2 Z = I^2R - j\,I^2XC$$

$$Pa = Pav - j\,Pr$$

Pav = Pa cos Φ = Vrms.Irms cos Φ = active power= W

Pr = Pa sin Φ = Vrms.Irms sin Φ= reactive power = VAR

Complex power:

Complex power is represented with S.

S = V(t).i(t)*

= P+jQ or P-jQ

Where, P = active power

Q = reactive power

Here only useful power is true power where as net reactive power over an cycle will be zero.

Complex numbers:

Complex numbers can be represented in two ways, either rectangle form or polar form

Rectangular form = a + j b

Polar form = $\overline{\qquad\qquad}$ tan-1 (b/a)

Here j operator plays major role in complex number, which is define unit vector rotating in anti-clock wise direction with phase 90.

j = 1 =

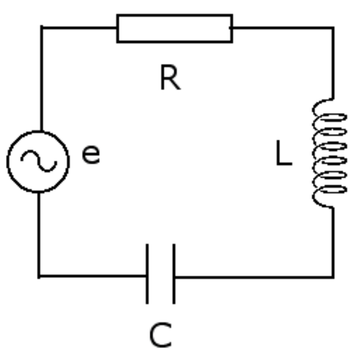
j2 = -1

j3 = -

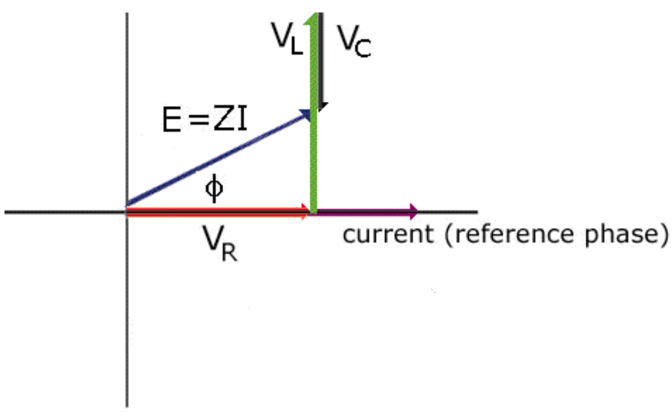
RLC series A.C. circuits

The e.m.f. that is supplied to the circuit is distributed between the resistor, the inductor, and the capacitor. Since the elements are in series the common current is taken to have the reference phase.

$$\underline{I} = I_m \exp\left[j\omega t\right]$$



On a phasor diagram this is:



Adding the potentials around the circuit:

$$\underline{E} = \underline{V}_R + \underline{V}_L + \underline{V}_C$$

$$= R\underline{I} + j\omega L\underline{I} - \frac{j}{\omega C}\underline{I}$$

$$= \left(R + j\left\{\omega L - \frac{1}{\omega C}\right\}\right) I_m \exp\left[j\omega t\right]$$

$$= \sqrt{R^2 + \left\{\omega L - \frac{1}{\omega C}\right\}^2} \exp\left[+j\tan^{-1}\left(\frac{\omega L - \dfrac{1}{\omega C}}{R}\right)\right] I_m \exp\left[j\omega t\right]$$

$$\underline{E} = ZI_m \exp\left[j\left(\omega t + \phi\right)\right], \quad \text{where}$$

$$Z = \sqrt{R^2 + \left\{\omega L - \frac{1}{\omega C}\right\}^2}, \quad \text{and} \quad \tan\phi = \frac{\omega L - \dfrac{1}{\omega C}}{R}$$

The physical current and potentials are:

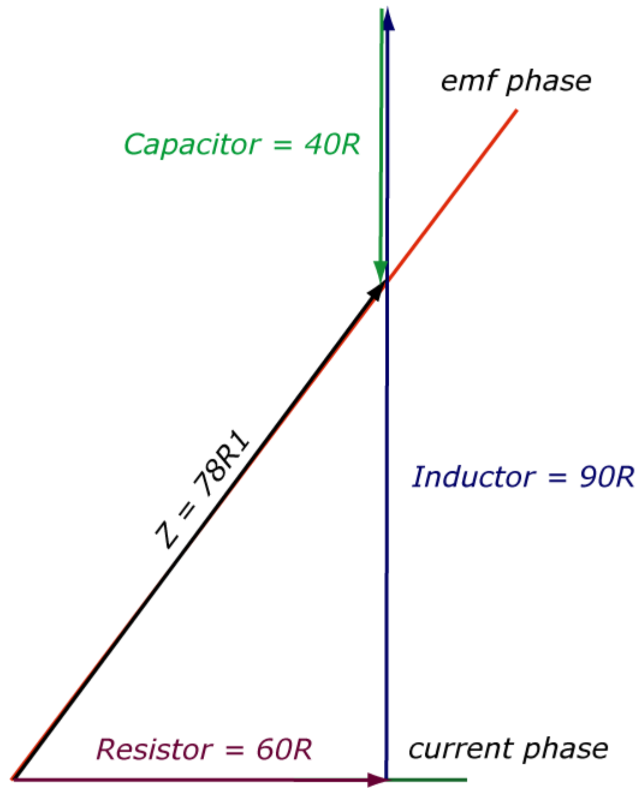
$$i = \mathrm{Im}\left\{I_m \exp\left[j\omega t\right]\right\} = I_m \sin\omega t$$

$$v_R = \mathrm{Im}\left\{RI_m \exp\left[j\omega t\right]\right\} = RI_m \sin\omega t$$

$$v_L = \mathrm{Im}\left\{j\omega LI_m \exp\left[j\omega t\right]\right\} = \omega LI_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$v_C = \mathrm{Im}\left\{\frac{I_m}{j\omega C} \exp\left[j\omega t\right]\right\} = \frac{I_m}{\omega C} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$e = \mathrm{Im}\left\{ZI_m \exp\left[j\left(\omega t + \phi\right)\right]\right\} = ZI_m \sin\left(\omega t + \phi\right)$$



Example

A 240V, 250/π Hz supply is connected in series with 60R, 180mH and 50μF. Take the emf as the reference phase and find:

(a) the complex impedance of the circuit

(b) the complex, real (i.e. physical) and rms currents, and

(c) the complex, real (i.e. physical) and rms potential differences across each element.

$$\omega = 2\pi\left(\frac{250}{\pi}\right) = 500 rad.s^{-1}$$

$$\underline{Z} = R + j\omega L - \frac{j}{\omega C}$$

$$= 60 + j\,500 \times \left(180 \times 10^{-3}\right) - \frac{j}{500 \times \left(50 \times 10^{-6}\right)}$$

$$= 60 + j\left(90 - 40\right)$$

$$= \sqrt{60^2 + \left(90 - 40\right)^2}\exp\left[j\tan^{-1}\frac{50}{60}\right]$$

$$= 78.1\ \exp\left[0.69j\right] \qquad \left(\phi = 39.8^o\right)$$

The complex impedance for the circuit is 78.1 Ω, and the phase angle between current and applied emf is 0.69 radians (or 39.8^0).

$$E = 240\ V \quad \therefore E_m = 240\sqrt{2} = 339\,V$$

$$\underline{E} = 339\ \exp\left[j\left(500t\right)\right]$$

$$\underline{I} = \frac{\underline{E}}{\underline{Z}} = \frac{339}{78.1}\exp\left[j\left(500t - 0.69\right)\right]$$

$$\underline{I} = 4.3\ \exp\left[j\left(500t - 0.69\right)\right]\ A$$

$$i = 4.3\ \sin\left(500t - 0.69\right)\ A$$

$$I = \frac{E}{Z} = \frac{240}{78.1} \quad or = \frac{4.3}{\sqrt{2}} = 3\ A$$

$$\underline{V}_R = R\underline{I} = 60 \times 4.3\ \exp\left[j\left(500t - 0.69\right)\right]\ V$$

$$\underline{V}_R = 255\ \times \exp\left[j\left(500t - 0.69\right)\right]\ V$$

$$v_R = 255\ \times\ \sin\left(500t - 0.69\right) V$$

$$V_R = \frac{255}{\sqrt{2}} \quad or\ = 60 \times 3\ = 180\ V$$

$$\underline{V}_L = j\omega L\,\underline{I} = 90j \times 4.3\exp\left[j\left(500t - 0.69\right)\right]$$

$$= 90\exp\left[\frac{\pi}{2}j\right] \times 4.3\exp\left[j\left(500t - 0.69\right)\right]$$

$$= 382\exp\left[j\left(500t - 0.69 + \frac{\pi}{2}\right)\right]$$

$$\underline{V}_L = 382\exp\left[j\left(500t + 0.88\right)\right]\ V$$

$$v_L = 382\sin\left(500t + 0.88\right)\ V$$

$$V_L = \frac{382}{\sqrt{2}} \quad or \quad = 90 \times 3 = 270\ V$$

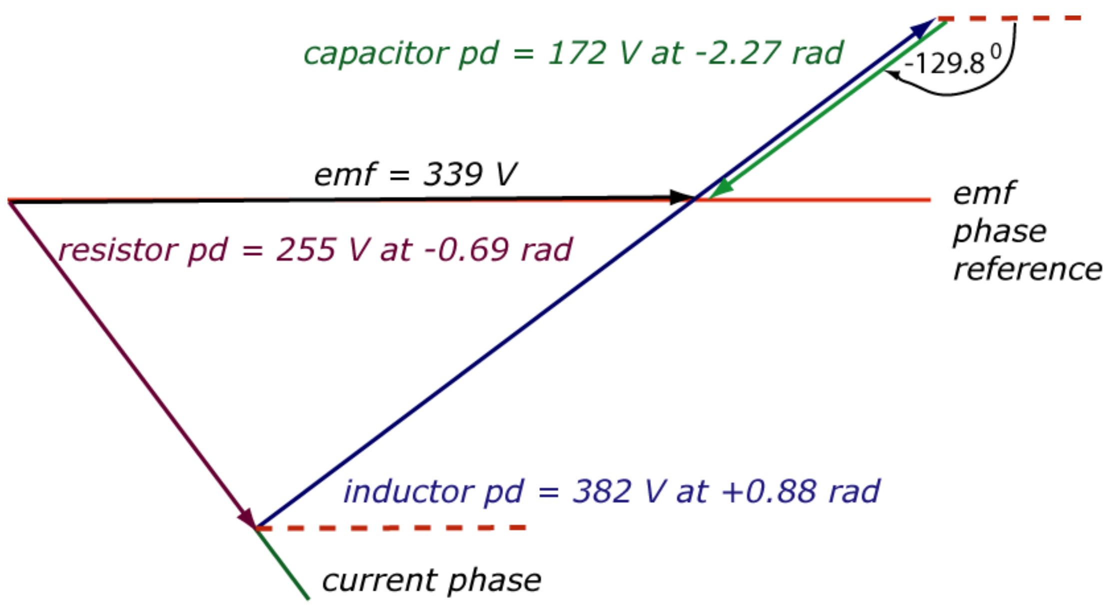
$$\underline{V}_C = -\frac{j}{\omega C}\underline{I} = 40\exp\left[-\frac{\pi}{2}j\right] \times 4.3\exp\left[j\left(500t - 0.69\right)\right]$$

$$= 172\exp\left[j\left(500t - \frac{\pi}{2} - 0.69\right)\right]$$

$$\underline{V}_C = 172\exp\left[j\left(500t - 2.27\right)\right] \qquad \left(\phi = -129.8^{\circ}\right)$$

$$v_C = 172\sin\left(500t - 2.27\right)\ V$$

$$V_C = \frac{172}{\sqrt{2}} \quad or \quad = 40 \times 3 = 120\ V$$



Complex potentials and currents hold both magnitude and phase information. Resistor/Capacitor and Resistor/Inductor circuits can form filters to block high or low frequency signals.

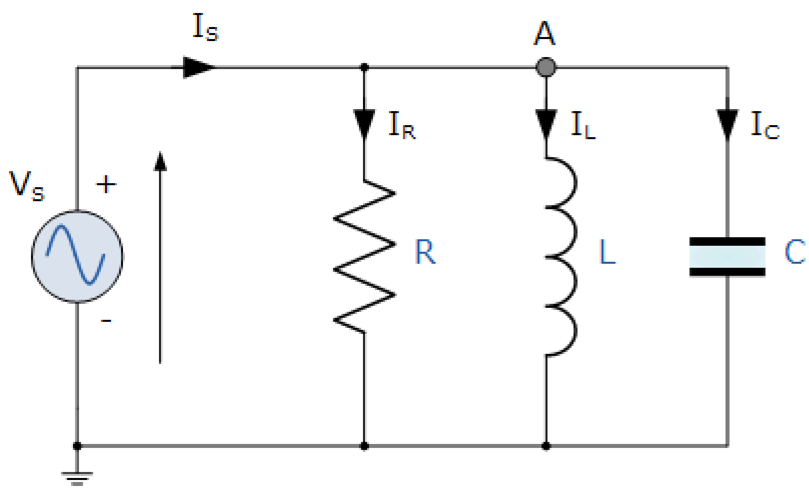
Average Power is calculated with rms quanties.

Apparent Power is the product of applied emf and current.
Real Power is the product of applied emf, current and cos(the phase angle between emf and current) (in Watt). cos(phase angle between emf and current) is called the power factor.
Apparent and Complex power are given the unit VA (Volt Amp).
Reactive Power is given the unit VAR (Volt Amp Reactive).

However, the analysis of a *parallel RLC circuits* can be a little more mathematically difficult than for series RLC circuits so in this tutorial about parallel RLC circuits only pure components are assumed in this tutorial to keep things simple.

This time instead of the current being common to the circuit components, the applied voltage is now common to all so we need to find the individual branch currents through each element. The total impedance, Z of a parallel RLC circuit is calculated using the current of the circuit similar to that for a DC parallel circuit, the difference this time is that admittance is used instead of impedance. Consider the parallel RLC circuit below.

Parallel RLC Circuit



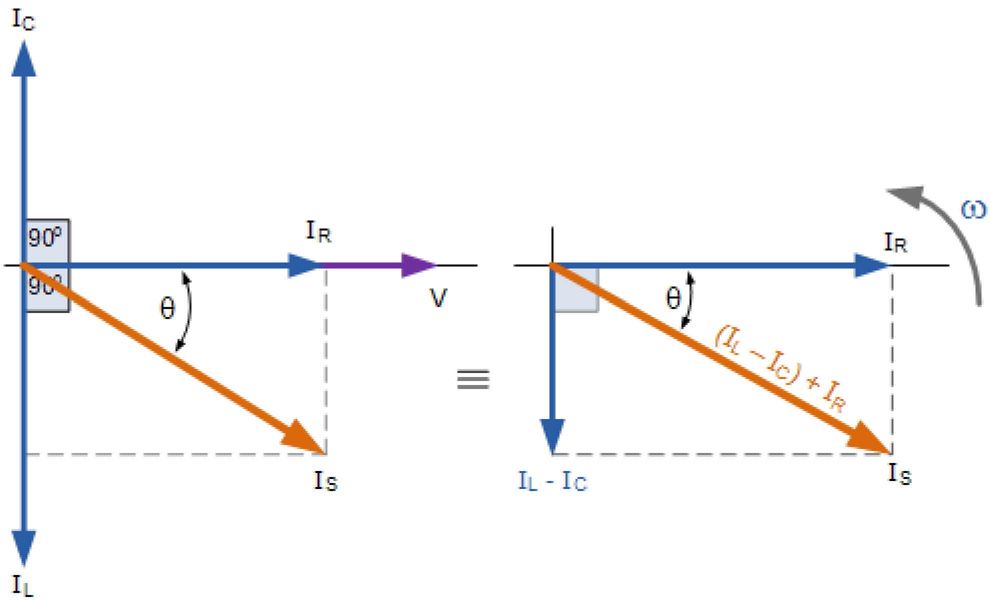
In the above parallel RLC circuit, we can see that the supply voltage, V_S is common to all three components whilst the supply current I_S consists of three parts. The current flowing through the resistor, I_R, the current flowing through the inductor, I_L and the current through the capacitor, I_C.

But the current flowing through each branch and therefore each component will be different to each other and to the supply current, I_S. The total current drawn from the supply will not be the mathematical sum of the three individual branch currents but their vector sum.

Like the series RLC circuit, we can solve this circuit using the phasor or vector method but this time the vector diagram will have the voltage as its reference with the three current vectors plotted with respect to the voltage. The phasor diagram for a parallel RLC circuit is produced by combining together the three individual phasors for each component and adding the currents vectorially.

Since the voltage across the circuit is common to all three circuit elements we can use this as the reference vector with the three current vectors drawn relative to this at their corresponding angles. The resulting vector I_S is obtained by adding together two of the vectors, I_L and I_C and then adding this sum to the remaining vector I_R. The resulting angle obtained between V and I_S will be the circuits phase angle as shown below.

Phasor Diagram for a Parallel RLC Circuit



We can see from the phasor diagram on the right hand side above that the current vectors produce a rectangular triangle, comprising of hypotenuse I_S, horizontal axis I_R and vertical axis $I_L – I_C$ Hopefully you will notice then, that this forms a **Current Triangle** and we can therefore use Pythagoras's theorem on this current triangle to mathematically obtain the magnitude of the branch currents along the x-axis and y-axis and then determine the total current I_S of these components as shown.

Current Triangle for a Parallel RLC Circuit

$$I_S^2 = I_R^2 + (I_L - I_C)^2$$

$$I_S = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$\therefore \; I_S = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L} - \frac{V}{X_C}\right)^2} = \frac{V}{Z}$$

$$\text{where:} \quad I_R = \frac{V}{R}, \quad I_L = \frac{V}{X_L}, \quad I_C = \frac{V}{X_C}$$

Since the voltage across the circuit is common to all three circuit elements, the current through each branch can be found using Kirchoff's Current Law, (KCL). Kirchoff's current law or junction law states that "the total current entering a junction or node is exactly equal to the current leaving that node", so the currents entering and leaving node "A" above are given as:

$$\text{KCL: } I_S - I_R - I_L - I_C = 0$$

$$I_S - \frac{V}{R} - \frac{1}{L}\int v\,dt - C\frac{dv}{dt} = 0$$

Taking the derivative, dividing through the above equation by C and rearranging gives us the following Second-order equation for the circuit current. It becomes a second-order equation because there are two reactive elements in the circuit, the inductor and the capacitor.

$$I_S - \frac{d^2V}{dt^2} - \frac{dV}{RCdt} - \frac{V}{LC} = 0$$

$$\therefore\ I_{S(t)} = \frac{d^2V}{dt^2} + \frac{dV}{dt}\frac{1}{RC} + \frac{1}{LC}V$$

The opposition to current flow in this type of AC circuit is made up of three components: X_L X_C and R and the combination of these three gives the circuit impedance, Z. We know from above that the voltage has the same amplitude and phase in all the components of a parallel RLC circuit. Then the impedance across each component can also be described mathematically according to the current flowing through and the voltage across each element as.

Impedance of a Parallel RLC Circuit

$$R = \frac{V}{I_R} \qquad X_L = \frac{V}{I_L} \qquad X_C = \frac{V}{I_C}$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}}$$

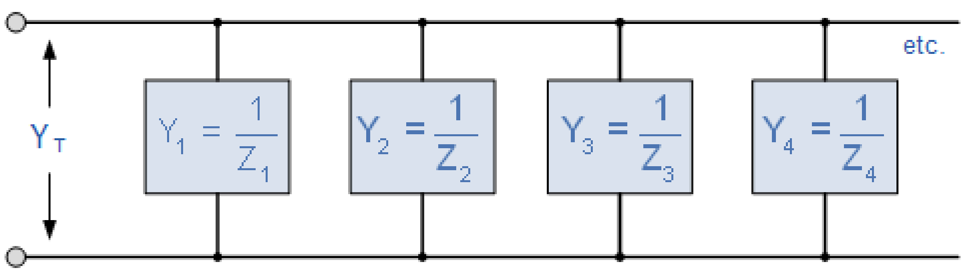
$$\therefore \frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

You will notice that the final equation for a parallel RLC circuit produces complex impedance's for each parallel branch as each element becomes the reciprocal of impedance, (1/Z) with the reciprocal of impedance being called **Admittance**.

In parallel AC circuits it is more convenient to use **admittance**, symbol (Y) to solve complex branch impedance's especially when two or more parallel branch impedance's are involved (helps with the math's). The total admittance of the circuit can simply be found by the addition of the parallel admittances. Then the total impedance, Z_T of the circuit will therefore be $1/Y_T$ Siemens as shown.

3.11.1 Admittance of a Parallel RLC Circuit

$$\frac{1}{Z_T} = Y_T = Y_1 + Y_2 + Y_3 + Y_4 + \ldots\ldots \text{etc}$$



The new unit for admittance is the *Siemens*, abbreviated as S, (old unit mho's ℧, ohm's in reverse). Admittances are added together in parallel branches, whereas impedance's are added together in series branches. But if we can have a reciprocal of impedance, we can also have a reciprocal of resistance and reactance as impedance consists of two components, R and X. Then the reciprocal of resistance is called **Conductance** and the reciprocal of reactance is called **Susceptance**.

Three Phase AC Circuits

Introduction

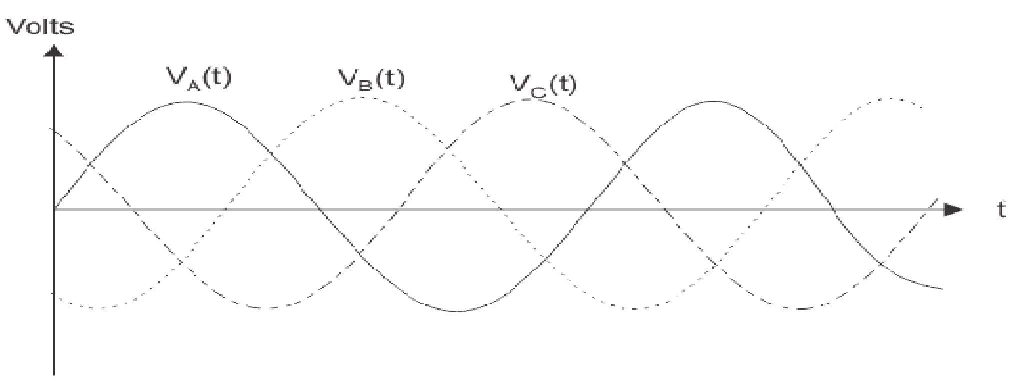
Three-phase systems are commonly used in generation, transmission and distribution of electric power. Power in a three-phase system is constant rather than pulsating and three-phase motors start and run much better than single-phase motors. A three-phase system is a generator-load pair in which the generator produces three sinusoidal voltages of equal amplitude and frequency but differing in phase by 120 from each other.

There are two types of system available in electric circuit, single phase and **three phase system**. In single phase circuit, there will be only one phase, i.e the current will flow through only one wire and there will be one return path called neutral line to complete the circuit. So in single phase minimum amount of power can be transported. Here the generating station and load station will also be single phase. This is an old system using from previous time.

In polyphase system, that more than one phase can be used for generating, transmitting and for load system. **Three phase circuit** is the polyphase system where three phases are send together from the generator to the load. Each phase are having a phase difference of 120°, i.e 120° angle electrically. So from the total of 360°, three phases are equally divided into 120° each. The power in **three phase system** is continuous as all the three phases are involved in generating the total power. The sinusoidal waves for 3 phase system is shown below The three phases can be used as single phase each. So if the load is single phase, then one phase can be taken from the **three phase circuit** and the neutral can be used as ground to complete the circuit..

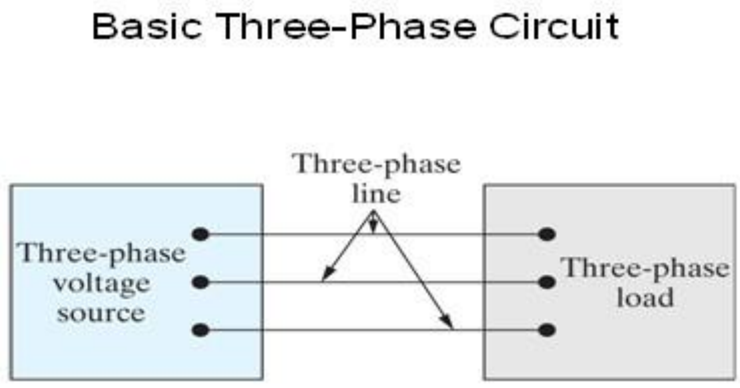
The phase voltages $v_a(t)$, $v_b(t)$ and $v_c(t)$ are as follows

$$v_a = V_m \cos \omega t$$
$$v_b = V_m \cos\left(\omega t - 120^\circ\right)$$
$$v_c = V_m \cos\left(\omega t - 240^\circ\right),$$



the corresponding phasors are

$$V_a = V_m$$
$$V_b = V_m e^{-j120^\circ}$$
$$V_c = V_m e^{-j240^\circ}.$$



Basic three Phase Circuit

Advantages of Three Phase is preferred Over Single Phase

The three phase system can be used as three single phase line so it can act as three single phase system. The three phase generation and single phase generation is same in the generator except the arrangement of coil in the generator to get 120° phase difference. The conductor needed in three phase circuit is 75% that of conductor needed in single phase circuit. And also the instantaneous power in single phase system falls down to zero as in single phase we can see from the sinusoidal curve but in three phase system the net power from all the phases gives a continuous power to the load. the will have better and higher efficiency compared to the single phase system.

In three phase circuit, connections can be given in two types:

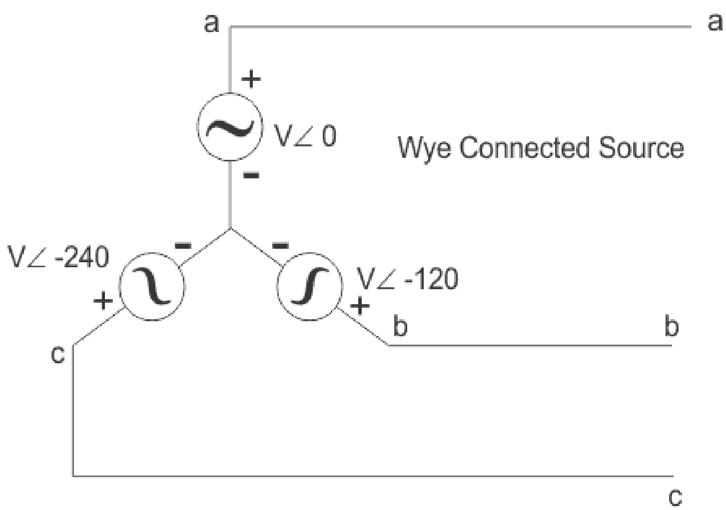
1. Star connection
2. Delta connection

STAR CONNECTION

In star connection, there is four wire, three wires are phase wire and fourth is neutral which is taken from the star point. Star connection is preferred for long distance power transmission because it is having the neutral point. In this we need to come to the concept of balanced and unbalanced current in power system.

When equal current will flow through all the three phases, then it is called as balanced current. And when the current will not be equal in any of the phase, then it is unbalanced current. In this case, during balanced condition there will be no current flowing through the neutral line and hence there is no use of the neutral terminal. But when there will be unbalanced current flowing in the three phase circuit, neutral is having a vital role. It will take the unbalanced current through to the ground and protect the transformer. Unbalanced current affects transformer and it may also cause damage to the transformer and for this star connection is preferred for long distance transmission.

The Star Connection



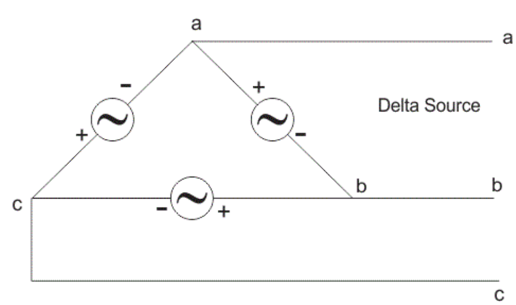
In star connection, the line voltage is √3 times of phase voltage. Line voltage is the voltage between two phases in three phase circuit and phase voltage is the voltage between one phase to the neutral line. And the current is same for both line and phase. It is shown as expression below

$$E_{Line} = \sqrt{3}E_{phase} \; and \; I_{Line} = I_{Phase}$$

Delta Connection

In delta connection, there are three wires alone and no neutral terminal is taken. Normally delta connection is preferred for short distance due to the problem of unbalanced current in the circuit. The figure is shown below for delta connection. In the load station, ground can be used as neutral path if required. In delta connection, the line voltage is same with that of phase voltage. And the line current is √3 times of phase current. It is shown as expression below,

$$E_{Line} = E_{phase} \; and \; I_{Line} = \sqrt{3}I_{Phase}$$



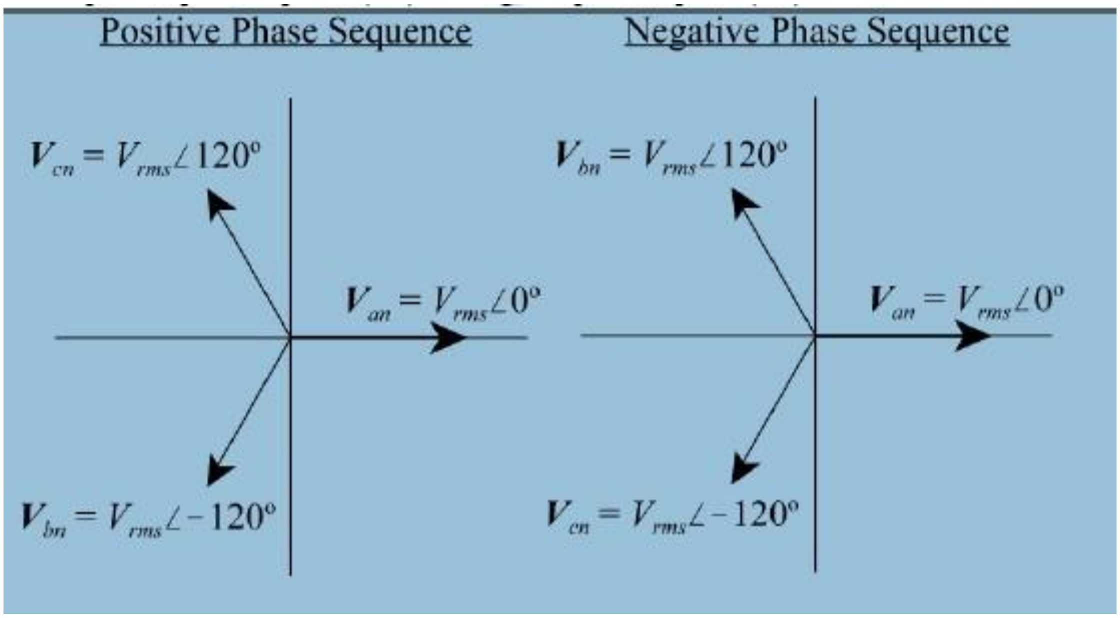
If we compare the line-to-neutral voltages with the line-to-line voltages, we find the following relationships,

Line-to-neutral voltages	Line-to-line voltages
$V_{an} = V_{rms}\angle 0°$	$V_{ab} = \sqrt{3}V_{rms}\angle 30°$
$V_{bn} = V_{rms}\angle -120°$	$V_{bc} = \sqrt{3}V_{rms}\angle -90°$
$V_{cn} = V_{rms}\angle 120°$	$V_{ca} = \sqrt{3}V_{rms}\angle 150°$

In three phase circuit, star and delta connection can be arranged in four different ways-

1. Star-Star connection
2. Star-Delta connection
3. Delta-Star connection
4. Delta-Delta connection

Phase Sequence



But the power is independent of the circuit arrangement of the three phase system. The net power in the circuit will be same in both star and delta connection. The power in three phase circuit can be calculated from the equation below,

$$P_{Total} = 3 \times E_{phase} \times I_{phase} \times PF$$

Since there is three phases, so the multiple of 3 is made in the normal power equation and the PF is power factor. Power factor is a very important factor in three phase system and sometimes due to certain error, it is corrected by using capacitors.

ANALYSIS OF BALANCED THREE PHASE CIRCUITS

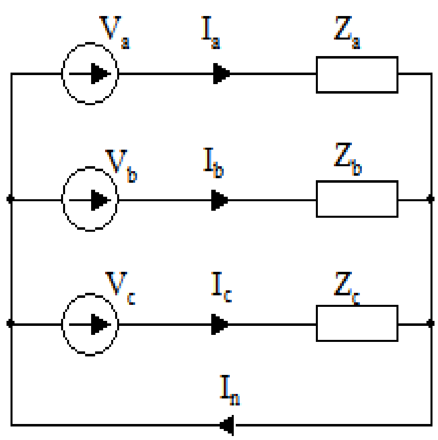
In a balanced system, each of the three instantaneous voltages has equal amplitudes, but is separated from the other voltages by a phase angle of 120. The three voltages (or phases) are typically labeled a, b and c. The common reference point for the three phase voltages is designated as the neutral connection and is labeled as n.

A three-phase system is shown in Fig. In a special case all impedances are identical

$$Z_a = Z_b = Z_c = Z$$

Such a load is called a balanced load and is described by equations

$$I_a = \frac{V_a}{Z} \qquad I_b = \frac{V_b}{Z} \qquad I_c = \frac{V_c}{Z}\ .$$



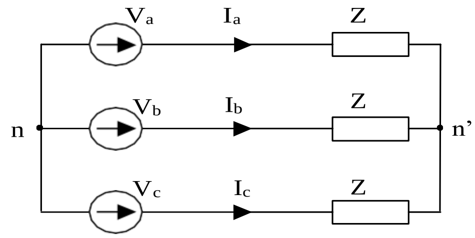
Using KCL, we have,

$$V_a + V_b + V_c = V_m\left(1 + e^{-j120^\circ} + e^{-j240^\circ}\right) =$$
$$= V_m\left(1 + \cos 120^\circ - j\sin 120^\circ + \cos 240^\circ - j\sin 240^\circ\right) = V_m\left(1 - \frac{1}{2} - j\frac{\sqrt{3}}{2} - \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = 0\ .$$

$$I_n = I_a + I_b + I_c = \frac{1}{Z}\left(V_a + V_b + V_c\right) \quad ,(4)$$

From the above result, we obtain $I_n = 0$.

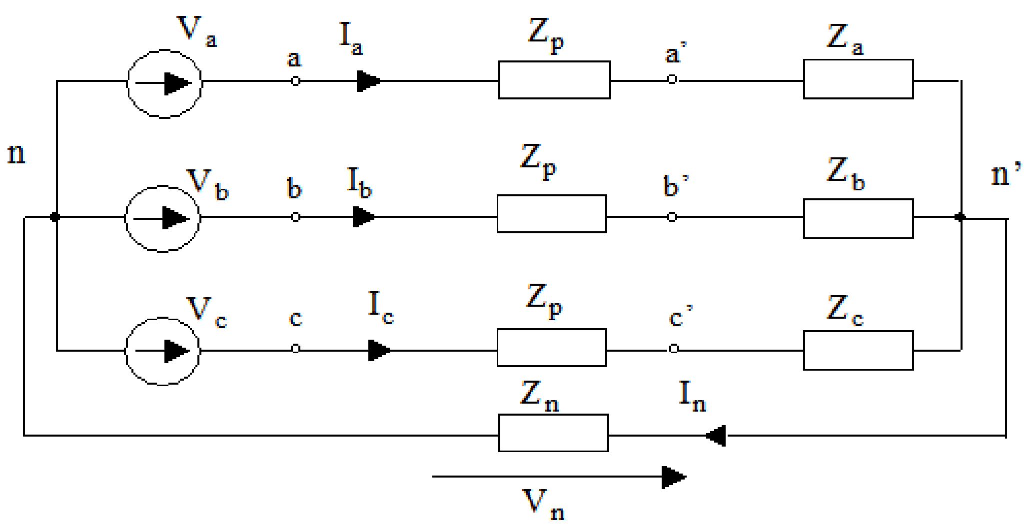
Since the current flowing though the fourth wire is zero, the wire can be removed



ANALYSIS OF UNBALANCED LOADS

Three-phase systems deliver power in enormous amounts to single-phase loads such as lamps, heaters, air-conditioners, and small motors. It is the responsibility of the power systems engineer to distribute these loads equally among the three-phases to maintain the demand for power fairly balanced at all times. While good balance can be achieved on large power systems, individual loads on smaller systems are generally unbalanced and must be analyzed as unbalanced three phase systems.

When the three phases of the load are not identical, an unbalanced system is produced. An unbalanced Y-connected system is shown in Fig.1. The system of Fig.1 contains perfectly conducting wires connecting the source to the load. Now we consider a more realistic case where the wires are represented by impedances Z_p and the neutral wire connecting n and n' is represented by impedance Z_n



the node n as the datum, we express the currents I_a, I_b, I_c and I_n in terms of the node voltage V_n

$$I_a = \frac{V_a - V_n}{Z_a + Z_p}$$

$$I_b = \frac{V_b - V_n}{Z_b + Z_p}$$

$$I_c = \frac{V_c - V_n}{Z_c + Z_p}$$

$$I_n = \frac{V_n}{Z_n}$$

The node equation is

$$\frac{V_n}{Z_n} - \frac{V_a - V_n}{Z_a + Z_p} - \frac{V_b - V_n}{Z_b + Z_p} - \frac{V_c - V_n}{Z_c + Z_p} = 0$$

And $$V_n = \frac{\dfrac{V_a}{Z_a + Z_p} + \dfrac{V_b}{Z_b + Z_p} + \dfrac{V_c}{Z_c + Z_p}}{\dfrac{1}{Z_n} + \dfrac{1}{Z_a + Z_p} + \dfrac{1}{Z_b + Z_p} + \dfrac{1}{Z_c + Z_p}}$$

Power in three-phase circuits

In the balanced systems, the average power consumed by each load branch is the same and given by

$$\tilde{P}_{av} = V_{eff} I_{eff} \cos\phi$$

where V_{eff} is the effective value of the phase voltage, I_{eff} is the effective value of the phase current and ϕ is the angle of the impedance. The total average power consumed by the load is the sum of those consumed by each branch, hence, we have

$$P_{av} = 3\tilde{P}_{av} = 3V_{eff} I_{eff} \cos\phi$$

In the balanced Y systems, the phase current has the same amplitude as the line current $I_{eff} = (I_{eff})_L$, whereas the line voltage has the effective value $(V_{eff})_L$ which is $\sqrt{3}$ times greater than the effective value of the phase voltage, $(V_{eff})_L = \sqrt{3} V_{eff}$. Hence, using (22), we obtain

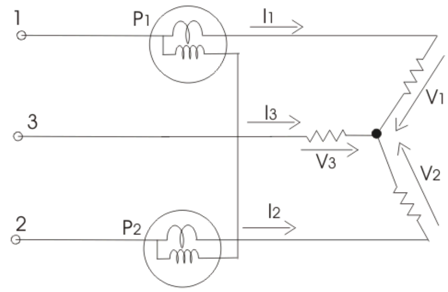
Measurement of Three Phase Power by Two Wattmeters Method

In this method we have two types of connections

(a) Star connection of loads

(b) Delta connection of loads.

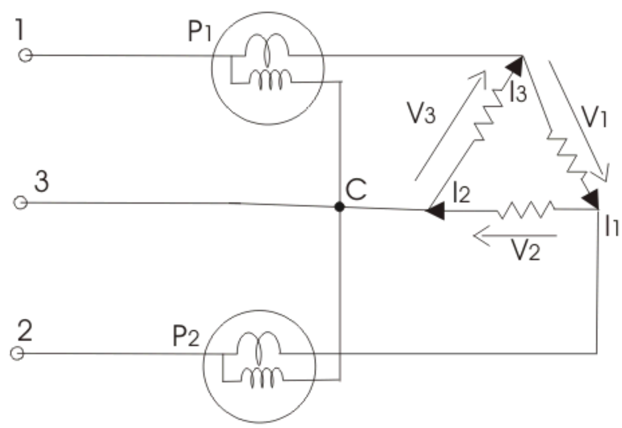
When the star connected load, the diagram is shown in below-



For star connected load clearly the reading of wattmeter one is product phase current and voltage difference (V_2-V_3). Similarly the reading of wattmeter two is the product of phase current and the voltage difference (V_2-V_3). Thus the total power of the circuit is sum of the reading of both the wattmeters. Mathematically we can write

$$P = P_1 + + P_2 = I_1(V_1 + V_2) + I_2(V_2 - V_3)$$

But we have I1+I2+I3=0, hence putting the value of I1+I2=-I3. We get total power as V1I1+V2I2+V3I3.
For delta connected load, the diagram is shown in below



The reading of wattmeter one can be written as

$$P_1 = -V_3(I_1 - I_3)$$

And reading of wattmeter two is

$$P_2 = -V_2(I_2 - I_1)$$

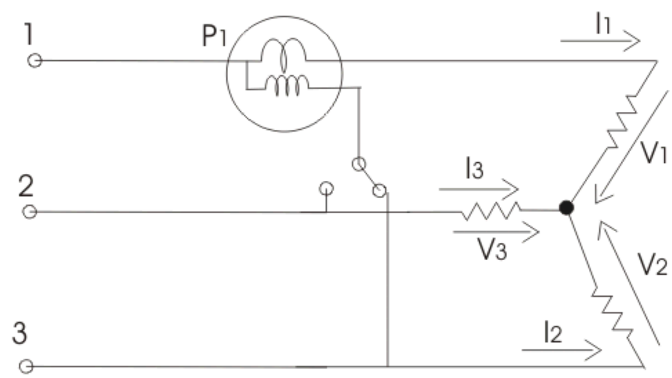
$$Total\ power\ is\ P = P_1 + P_2 = V_2 I_2 + V_3 I_3 - I_1(V_2 + V_3)$$

But $V_1+V_2+V_3=0$, hence expression for total power will reduce to $V_1I1+V_2I_2+V_3I_3$.

13.16 Measurement of Three Phase Power by One Wattmeter Method

Limitation of this method is that it cannot be applied on unbalanced load. So under this condition we have $I_1=I_2=I_3=I$ and $V_1=V_2=V_3=V$.

Diagram is shown below:



Two switches are given which are marked as 1-3 and 1-2, by closing the switch 1-3 we get reading of wattmeter as

$$P_1 = V_{13} I_1 \cos(30 - \phi) = \sqrt{3} \times VI\cos(30 - \phi)$$

Similarly the reading of wattmeter when switch 1-2 is closed is

$$P_2 = V_{12} I_1 \cos(30 + \phi) = \sqrt{3} \times VI\cos(30 + \phi)$$

$$Total\ power\ is\ P_1 + P_2 = 3VI\cos\phi$$

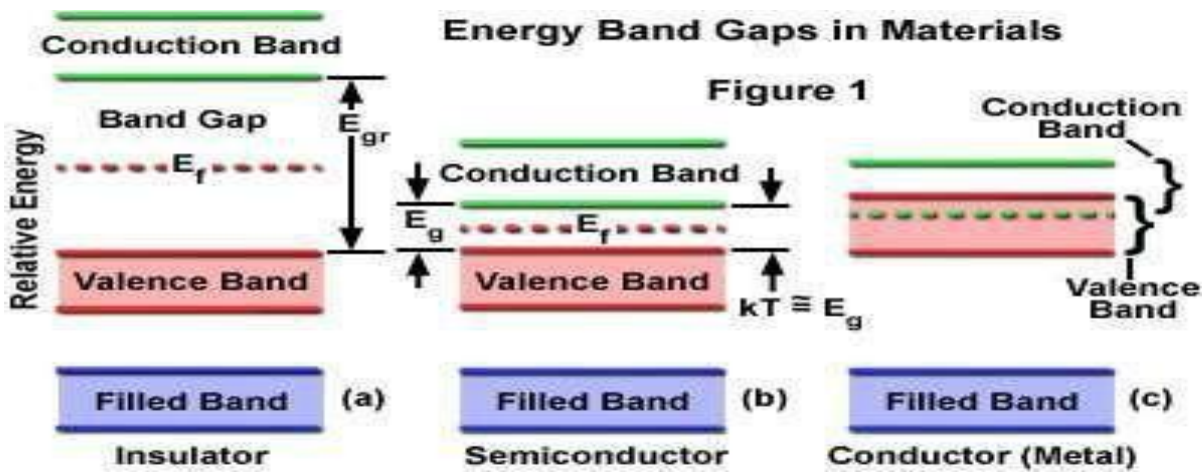
UNIT IV

SEMICONDUCTOR DIODE AND APPLICATIONS

INTRODUCTION:

We start our study of nonlinear circuit elements. These elements (diodes and transistors) are made of semiconductors. A brief description of how semiconductor devices work is rst given to understand their i v characteristics. You will see a rigorous analysis of semiconductors in the breadth courses.

Energy Bands in Solids:

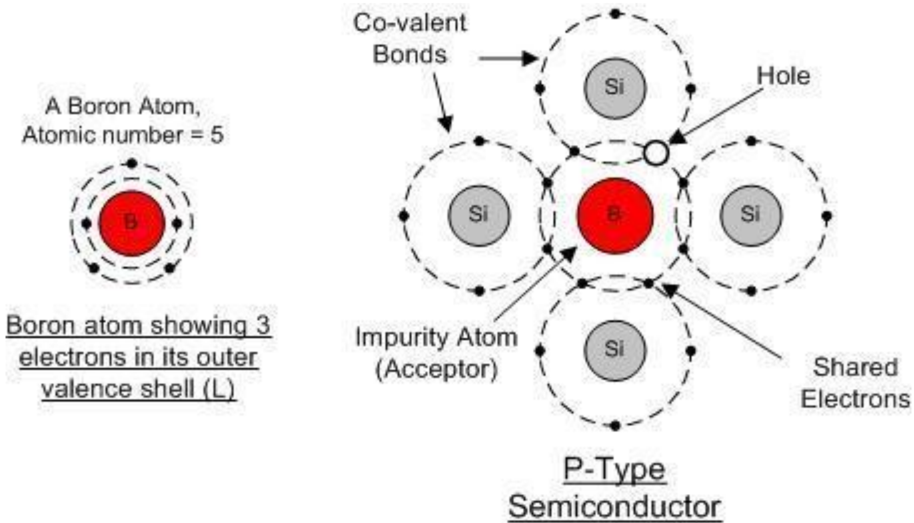


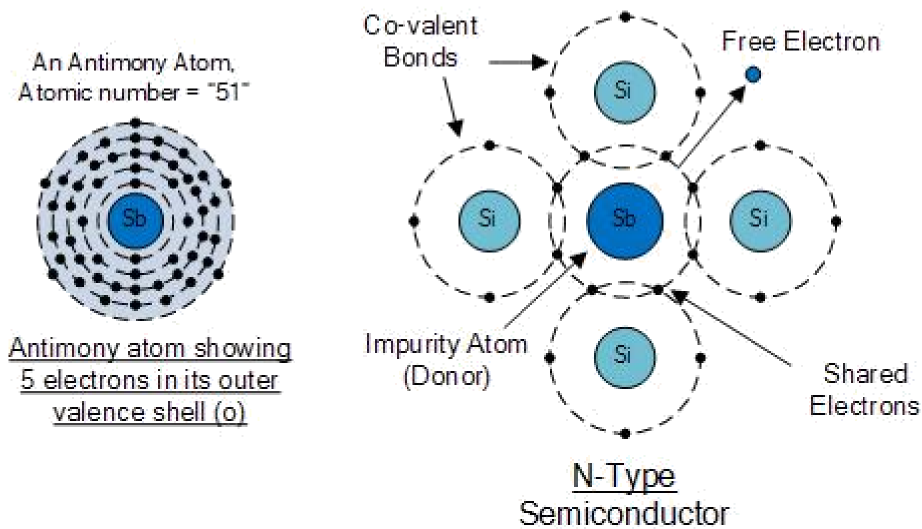
Semiconductors :

Semiconductor material are mainly made of elements from group IVB of the periodic table like C (diamond), Si, Ge, SiC. These material have 4 electrons in their outer most electronic shell. Each atom can form a \covalent" bond with four of its neighbors sharing one electron with that atom. In this manner, each atom \sees" eight electrons in its outer most electronic shell (4 of its own, and one from each neighbor), completely lling that shell. It is also possible to form this type of covalent bond by combining elements from group IIIB (sharing three electrons) with element from group VB (sharing ve electrons). Examples of these semiconductors are GaAs or AlGaAs and are usually called \3-5" semiconductors. We focus mostly on Si semiconductors in this class. Figure below shows this covalent

bond structure for Si. A pair of electrons and holes are slow shown. Note that Si form a tetrahedron structure and an atom in the center of the tetrahedron share electrons with atoms on the each vertex. Figure below is a two-dimensional representation of such a structure. The leftgure is for a pure Si semiconductor and an electron-hole pair is depicted. Both electrons and holes are called \mobile" carriers as they are responsible for carrying electric current.

If we add a small amount of an element from group VB, such as P, to the semiconductor, we create a n-type semiconductor and the impurity dopant is called a n-type dopant. Each of these new atoms also form a covalent bond with four of its neighbors. However, as a n-type dopant has 5 valance electron, the extra electron will be located in the \empty" energy band. As can be seen, there is no hole associated with this electron. In addition to electrons from the n-type dopant, there are electron-hair pair in the solid from the base semiconductor (Si in he above gure) which are generated due to temperature e ects. In a n-type semiconductor, the number of free electrons from the dopant is much larger than the number of electrons from electron-hole pairs. As such, a n-type semiconductor is considerably more conductive than the base semiconductor (in this respect, a n-type semiconductor is more like a \resistive" metal than a semiconductor).





In summary, in a n-type semiconductor there are two charge carriers: \holes" from the base semiconductor (called the \minority" carriers) and electrons from both the n-type dopant and electron-hole pairs (called the \majority" carrier).

Similarly, we can create a p-type semiconductor by adding an element from group IIIB, such as B, to the semiconductor. In this case, the p-type dopant generate holes. We will have two charge carriers: majority carriers are \holes" from the p-type dopant and electron-hole pairs and minority carriers are electrons from the base semiconductor (from electron-hole pairs).

The charge carriers (electrons and holes) move in a semiconductor through two mechanisms: First, charge carriers would move from regions of higher concentration to lower concentration in order to achieve a uniform distribution throughout the semiconductor. This process is called **Diff usion**" and is characterized by the diffusion coefficient, D. Second, charge carriers move under the influence of an electricfield. This motion is called the drift and is characterized by the mobility.

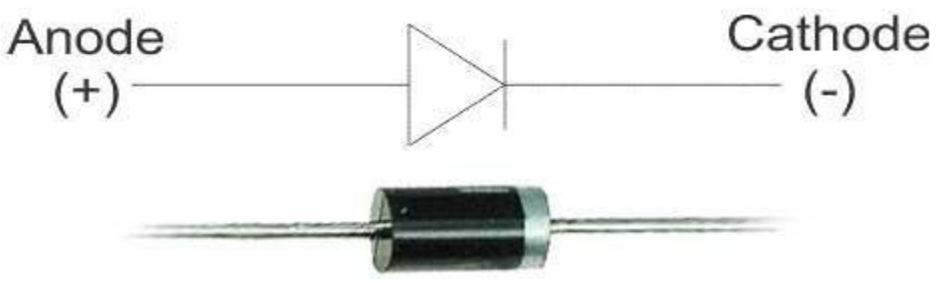
4.2 DIODE | WORKING PRINCIPLE AND TYPES OF DIODE

What is a Diode?

A diode is a device which only allows unidirectional flow of current if operated within a rated specified voltage level. A diode only blocks current in the reverse direction while the reverse voltage is within a limited range otherwise reverse barrier breaks and the voltage at which this breakdown occurs is called reverse breakdown voltage. The diode acts as a valve in the electronic and electrical circuit. A P-N junction is the simplest form of the diode which behaves as ideally short circuit when it is in forward biased and behaves as ideally open circuit when it is in the reverse biased. Beside simple PN junction diodes, there are different types of diodes although the fundamental principle is more or less same. So a particular arrangement of diodes can convert AC to pulsating DC, and hence, it is sometimes also called as a rectifier. The name diode is derived from "di-ode" which means a device having two electrodes.

4.2.1 Symbol of Diode

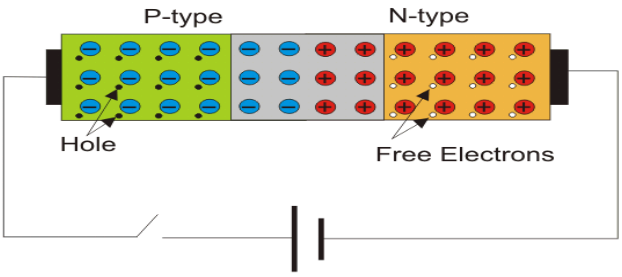
The symbol of a diode is shown below, the arrowhead points in the direction of conventional current flow.



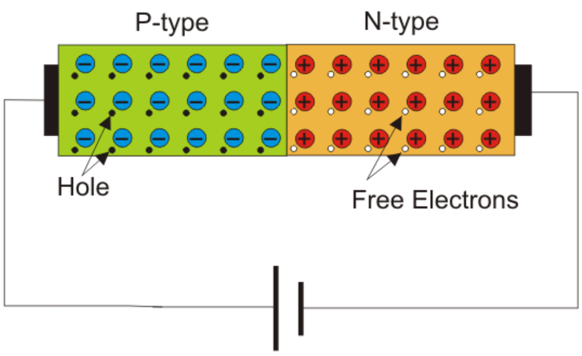
A simple **PN junction diode** can be created by doping donor impurity in one portion and acceptor impurity in other portion of a silicon or germanium crystal block. These make a p n junction at the middle portion of the block beside which one portion is p type (which is doped by trivalent or acceptor impurity) and other portion is n type (which is doped by pentavalent or donor impurity). It can also be formed by joining a p-type (intrinsic semiconductor doped with a trivalent impurity) and n-type semiconductor (intrinsic semiconductor doped with a pentavalent impurity) together with a special fabrication technique such that a p-n junction is formed. Hence, it is a device with two elements, the p-type forms anode and the n-type forms the cathode. These terminals are brought out to make the external connections.

4.2.3 Working Principle of Diode

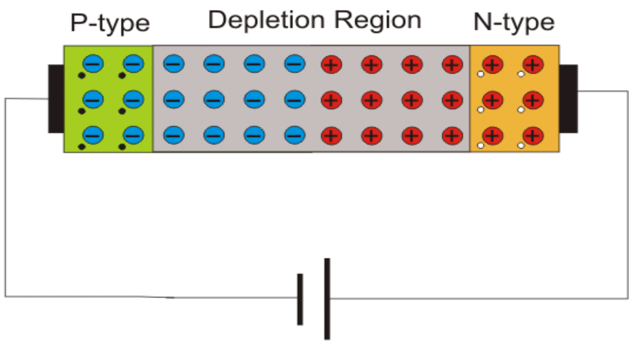
The n side will have a large number of electrons and very few holes (due to thermal excitation) whereas the p side will have a high concentration of holes and very few electrons. Due to this, a process called diffusion takes place. In this process free electrons from the n side will diffuse (spread) into the p side and combine with holes present there, leaving a positive immobile (not moveable) ion in the n side. Hence, few atoms on the p side are converted into negative ions. Similarly, few atoms on the n-side will get converted to positive ions. Due to this large number of positive ions and negative ions will accumulate on the n-side and p-side respectively. This region so formed is called as depletion region. Due to the presence of these positive and negative ions a static electric field called as "barrier potential" is created across the p-n junction of the diode. It is called as "barrier potential" because it acts as a barrier and opposes the further migration of holes and electrons across the junction.



In a PN junction diode when the forward voltage is applied i.e. positive terminal of a source is connected to the p-type side, and the negative terminal of the source is connected to the n-type side, the diode is said to be in forward biased condition. We know that there is a barrier potential across the junction. This barrier potential is directed in the opposite of the forward applied voltage. So a diode can only allow current to flow in the forward direction when forward applied voltage is more than barrier potential of the junction. This voltage is called forward biased voltage. For silicon diode, it is 0.7 volts. For germanium diode, it is 0.3 volts. When forward applied voltage is more than this forward biased voltage, there will be forward current in the diode, and the diode will become short circuited. Hence, there will be no more voltage drop across the diode beyond this forward biased voltage, and forward current is only limited by the external resistance">resistance connected in series with the diode. Thus, if forward applied voltage increases from zero, the diode will start conducting only after this voltage reaches just above the barrier potential or forward biased voltage of the junction. The time taken by this input voltage to reach that value or in other words the time taken by this input voltage to overcome the forward biased voltage is called recovery time.



Now if the diode is reverse biased i.e. positive terminal of the source is connected to the n-type end, and the negative terminal of the source is connected to the p-type end of the diode, there will be no current through the diode except reverse saturation current. This is because at the reverse biased condition the depilation layer of the junction becomes wider with increasing reverse biased voltage. Although there is a tiny current flowing from n-type end to p-type end in the diode due to minority carriers. This tiny current is called reverse saturation current. Minority carriers are mainly thermally generated electrons and holes in p-type semiconductor and n-type semiconductor respectively. Now if reverse applied voltage across the diode is continually increased, then after certain applied voltage the depletion layer will destroy which will cause a huge reverse current to flow through the diode. If this current is not externally limited and it reaches beyond the safe value, the **diode** may be permanently destroyed. This is because, as the magnitude of the reverse voltage increases, the kinetic energy of the minority charge carriers also increase. These fast moving electrons collide with the other atoms in the device to knock-off some more electrons from them. The electrons so released further release much more electrons from the atoms by breaking the covalent bonds. This process is termed as carrier multiplication and leads to a considerable increase in the flow of current through the p-n junction. The associated phenomenon is called Avalanche Breakdown.



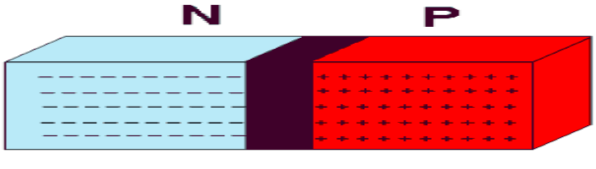
Types of Diode

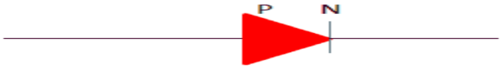
The **types of diode** are as follow-

1) Zener diode
2) P-N junction diode
3) Tunnel diode
4) Varractor diode
5) Schottky diode
6) Photo diode
7) PIN diode
8) Laser diode
9) Avalanche diode
10) Light emitting diode

DIODE CHARACTERISTICS

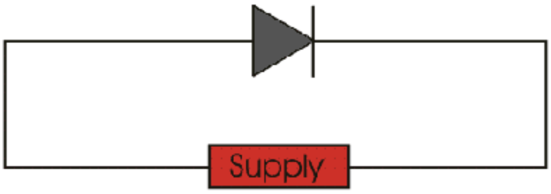
Semiconductor materials (Si, Ge) are used to form variety of electronic devices. The most basic device is diode. Diode is a two terminal P-N junction device. P-N junction is formed by bringing a P type material in contact with N type material.When a P-type material is brought in contact with N- type material electrons and holes start recombining near the junction. This result in lack of charge carriers at the junction and thus the junction is called depletion region. Symbol of P-N junction is given as:





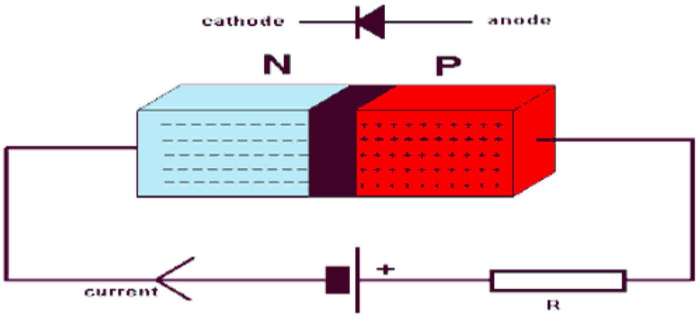
Biased i.e. when voltage is applied across the terminals of P-N junction, it is called diode.

Diode is unidirectional device that allows the flow of current in one direction only depending on the biasing.



Forward Biasing Characteristic of Diode

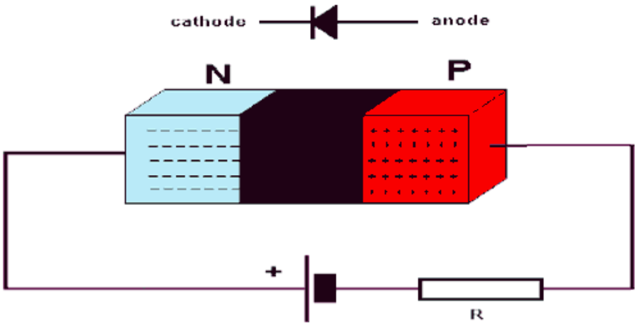
When, P terminal is more positive as compared to N terminal i.e. P- terminal connected to positive terminal of battery and N-terminal connected to negative terminal of battery, it is said to be forward biased.



Positive terminal of the battery repels majority carriers, holes, in P-region and negative terminal repels electrons in the N-region and push them towards the junction. This result in increase in concentration of charge carriers near junction, recombination takes place and width of depletion region decreases. As forward bias voltage is raised depletion region continues to reduce in width, and more and more carriers recombine. This results in exponential rise of current.

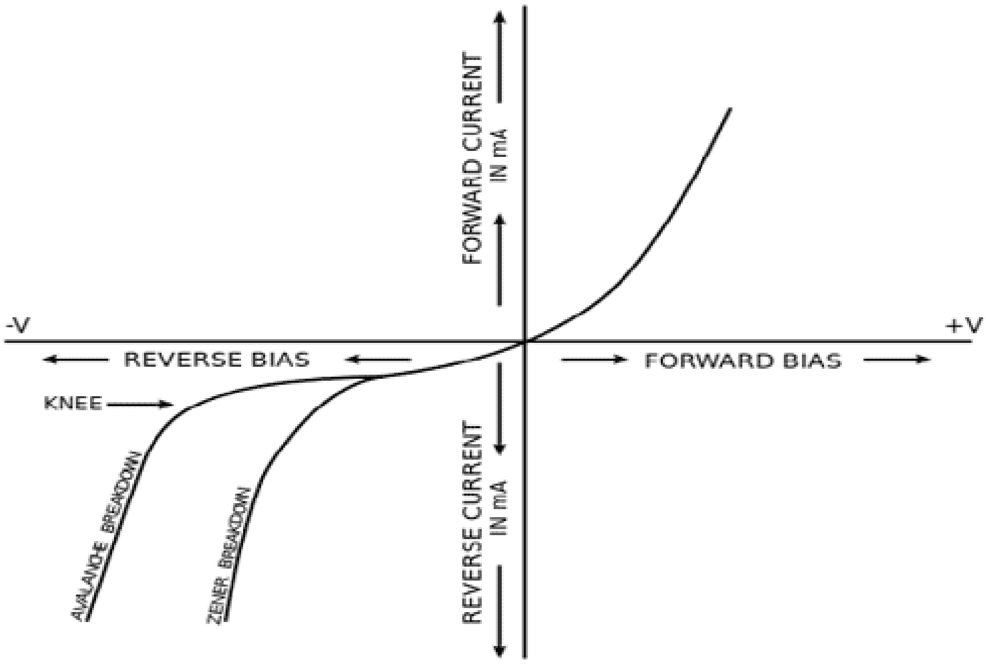
Reverse Biasing Characteristic of Diode

In reverse biasing P- terminal is connected to negative terminal of the battery and N- terminal to positive terminal of battery. Thus applied voltage makes N-side more positive than P-side.



Negative terminal of the battery attracts majority carriers, holes, in P-region and positive terminal attracts electrons in the N-region and pull them away from the junction. This result in decrease in concentration of charge carriers near junction and width of depletion region increases. A small amount of current flow due to minority carriers, called as reverse bias current or leakage current. As reverse bias voltage is raised depletion region continues to increase in width and no current flows. It can be concluded that diode acts only when forward biased. Operation of diode can be summarized in form of I-V **diode characteristics** graph. For reverse bias diode, $V<0$, $I_D = I_S$ Where, V = supply voltage I_D = diode current I_S = reverse saturation current For forward bias, $V > 0$, $I_D = I_S(e^{V/NV_T} - 1)$

Where,
V_T = volt's equivalent of temperature = KT/Q = T/11600
Q = electronic charge = 1.632×10^{-19} C
K = Boltzmann's constant = 1.38×10^{-23}
N = 1, for Ge
= 2, for Si



As reverse bias voltage is further raised, depletion region width increases and a point comes when junction breaks down. This results in large flow of current. Breakdown is the knee of **diode characteristics** curve. Junction breakdown takes place due to two phenomena:

Avalanche Breakdown(for V> 5V)

Under very high reverse bias voltage kinetic energy of minority carriers become so large that they knock out electrons from covalent bonds, which in turn knock more electrons and this cycle continues until and unless junction breakdowns.

Zener Effect (for V<5V)

Under reverse bias voltage junction barrier tends to increase with increase in bias voltage. This results in very high static electric field at the junction. This static electric field breaks covalent bond and set minority carriers free which contributes to reverse current. Current increases abruptly and junction breaks down.

P-N JUNCTION DIODE AND CHARACTERISTICS OF P-N JUNCTION

The volt-ampere characteristics of a diode explained by the following equations:

$$I = I_S(e^{V_D/(\eta V_T)} - 1)$$

where

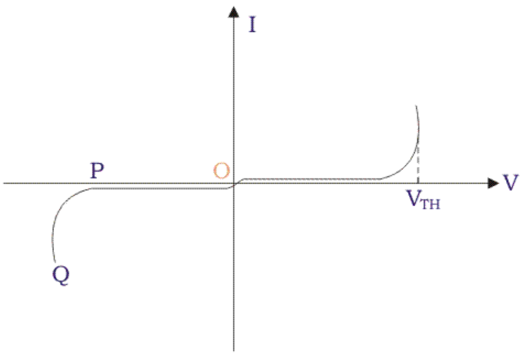
I = current flowing in the diode, I0 = reverse saturation current

VD = Voltage applied to the diode

VT = volt- equivalent of temperature = k T/q = T/ 11,600 = 26mV (@ room temp)

η = 1(for Ge) and 2 (for Si)

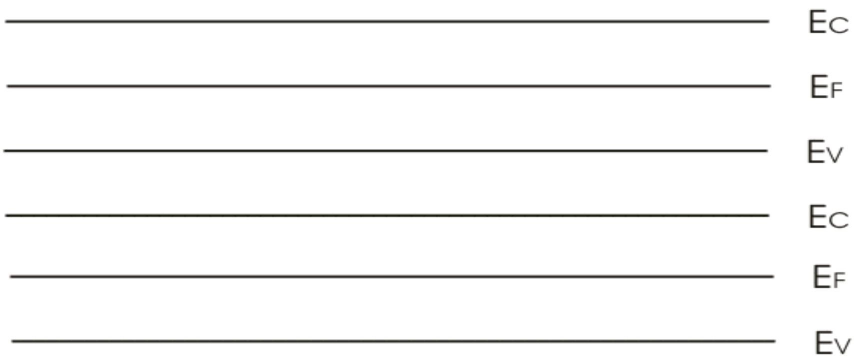
It is observed that **Ge** diodes has smaller cut-in-voltage when compared to **Si** diode. The reverse saturation current in **Ge** diode is larger in magnitude when compared to silicon diode.



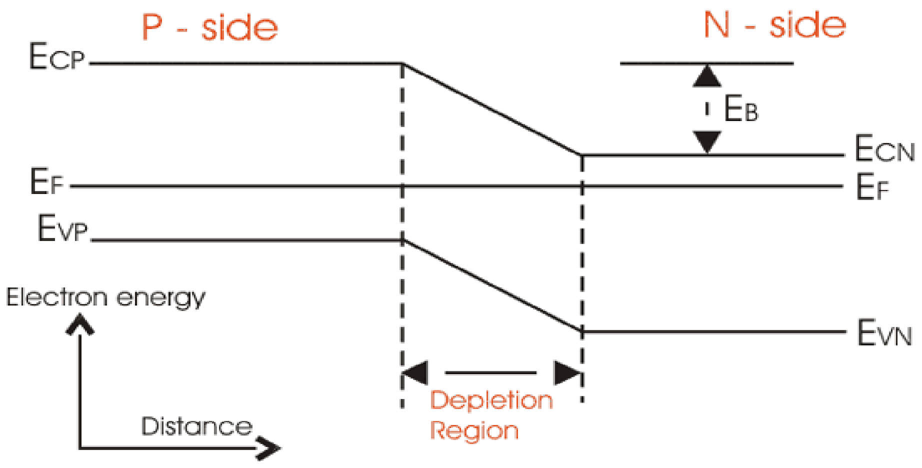
When, V is positive the junction is forward biased and when V is negative, the junction is reversing biased. When V is negative and less than VTH, the current is very small. But when V exceeds VTH, the current suddenly becomes very high. The voltage VTH is known as threshold or cut in voltage. For Silicon diode VTH = 0.6 V. At a reverse voltage corresponding to the point P, there is abrupt increment in reverse current. The PQ portion of the characteristics is known as breakdown region.

P-N Junction Band Diagram

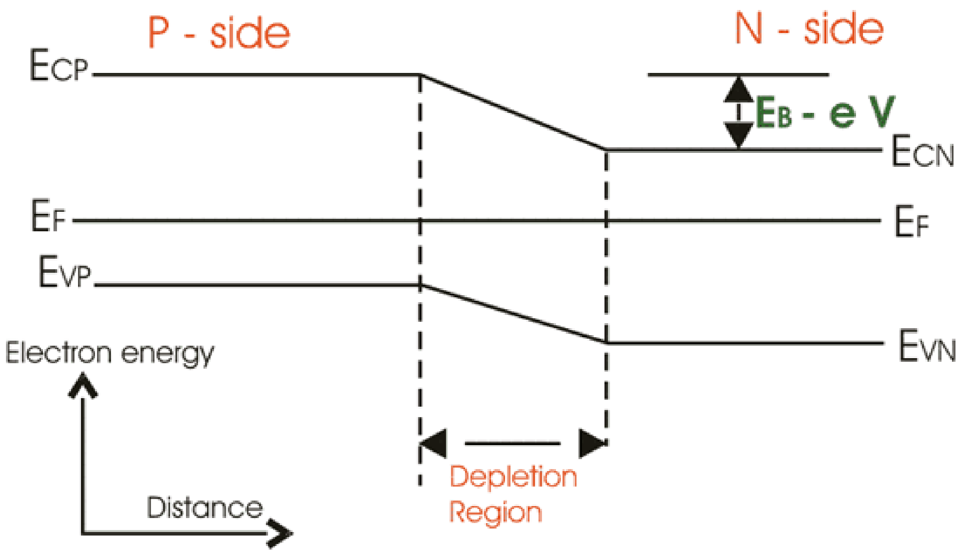
For an n-type semiconductor, the Fermi level E_F lies near the conduction band edge. E_C but for an p - type semiconductor, E_F lies near the valance band edge E_V.

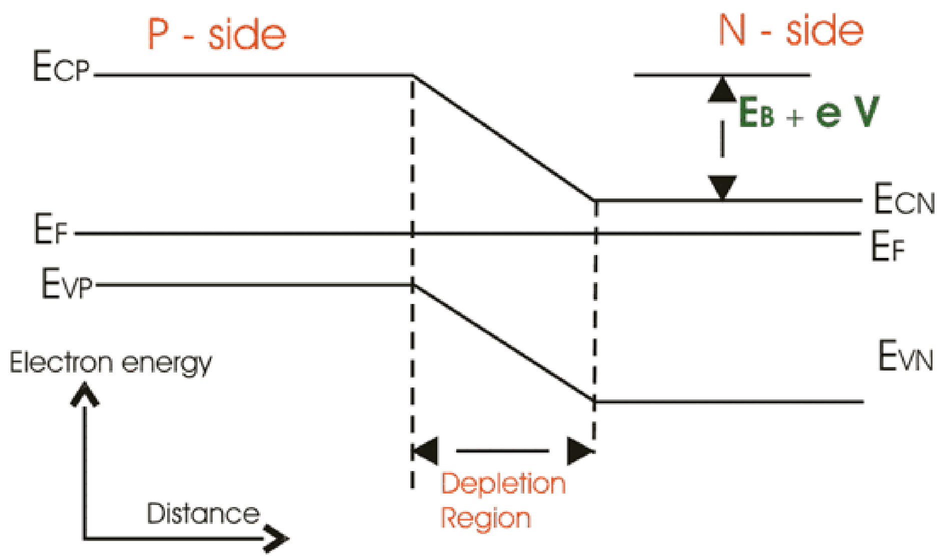


Now, when a p-n junction is built, the Fermi energy E_F attains a constant value. In this scenario the p-sides conduction band edge. Similarly n–side valance band edge will be at higher level than E_{cn}, n-sides conduction band edge of p - side. This energy difference is known as barrier energy. The barrier energy is $E_B = E_{cp} - E_{cn} = E_{vp} - E_{vn}$



If we apply forward bias voltage V, across junction then the barrier energy decreases by an amount of eV and if V is reverse bias is applied the barrier energy increases by eV.





P-N Junction Diode Equation

The p-n junction diode equation for an ideal diode is given below

$I = I_S[\exp(eV/K_BT) - 1]$

Here,

I_S = reverse saturation current

e = charge of electron

K_B = Boltzmann constant

T = temperature For a normal p-n junction diode, the equation becomes

$I = I_S[\exp(eV/\eta K_BT) - 1]$

Here,

η = emission co-efficient, which is a number between 1 and 2, which typically increases as the current increases.

APPLICATIONS OF DIODES

- Rectifying a voltage, such as turning AC into DC voltages
- Isolating signals from a supply
- Voltage Reference
- Controlling the size of a signal
- Mixing signals
- Detection signals
- Lighting
- Lasers diodes

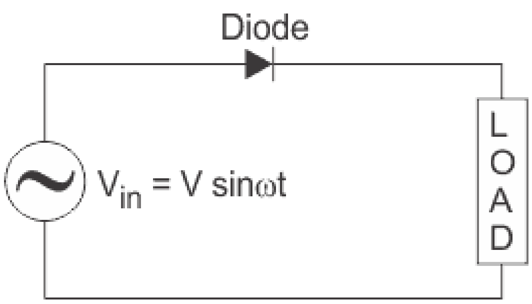
HALF WAVE DIODE RECTIFIER

Electric current flows through a p - n junction diode when it is forward biased and we get output current through the load. Let, we supply a sinusoidal voltage $V_{in} = V\sin\omega t$ as a source voltage. Now, if the input voltage is positive, the diode is forward biased and when that is negative, the diode is in reverse bias condition. When the input voltage is positive, i.e, for the positive cycle of the input voltage, the current flows through the diode.

So, the current will flow through the load also and we obtain output voltage across the load. But for the negative half cycle of the input, the p-n junction get reverse biased and no current flows through the diode as a result we obtain zero current and zero voltage across the load.

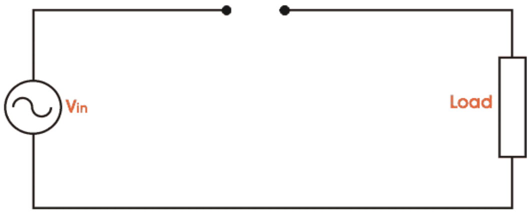
Circuit Diagram of Half Wave Rectifier

The basic diagram of **half wave diode rectifier** is given below,

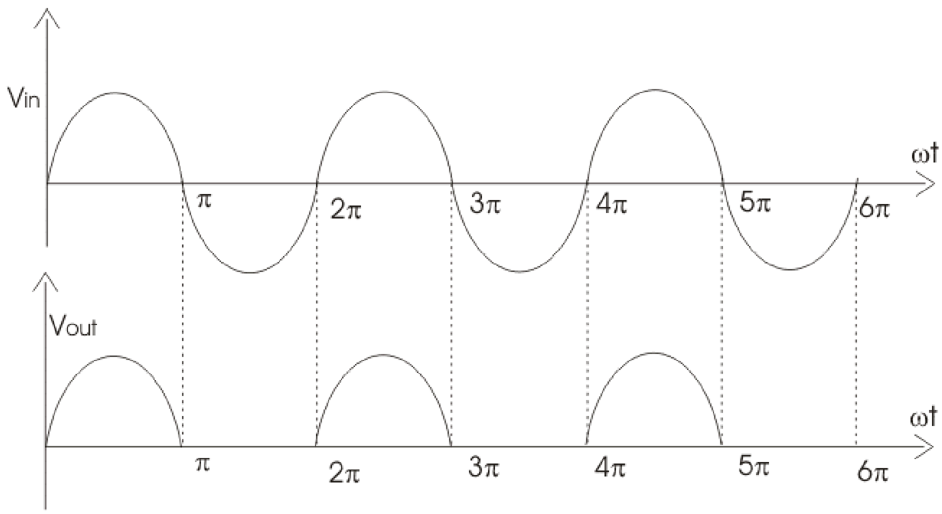


For positive half cycle

For negative half cycle



Input voltage and Output Voltage Waveforms



Now, different parameters for **half wave rectifier** is given below

The average of load current (I_{DC}) :
Let, the load current be $i_L = I_m \sin\omega t$,

$$I_{dc} = \frac{1}{2\pi}\int_0^{\pi} I_m\, sin\omega t = \frac{I_m}{\pi}$$

Ripple factor of half wave rectifier,

$$Ripple\ factor(r) = \frac{(I_{rms}^2 - I_{dc}^2)}{I_{dc}} = 1.21$$

The rms value of the load current (I_{rms}),

$$I_{rms} = \frac{I_m}{4}$$

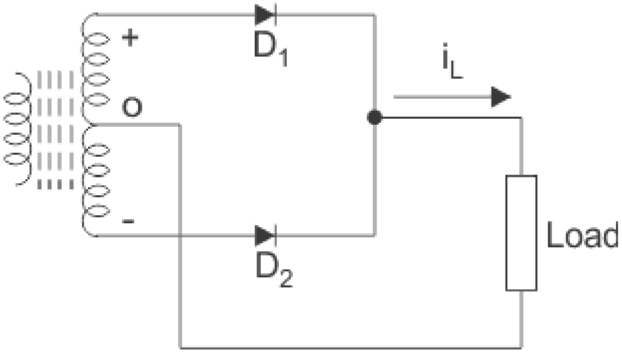
4.8 FULL WAVE DIODE RECTIFIER

The diode works only when it is in forward bias, only the current flows through p-n junction diode and output current across the load is found. If two diodes are connected in such a way that one diode conducts during one half of the input voltage and the other one conducts during the next half of the cycle, in a unidirectional can flow through the load during the full cycle of the impact voltage. This is known as **full wave rectifier.**

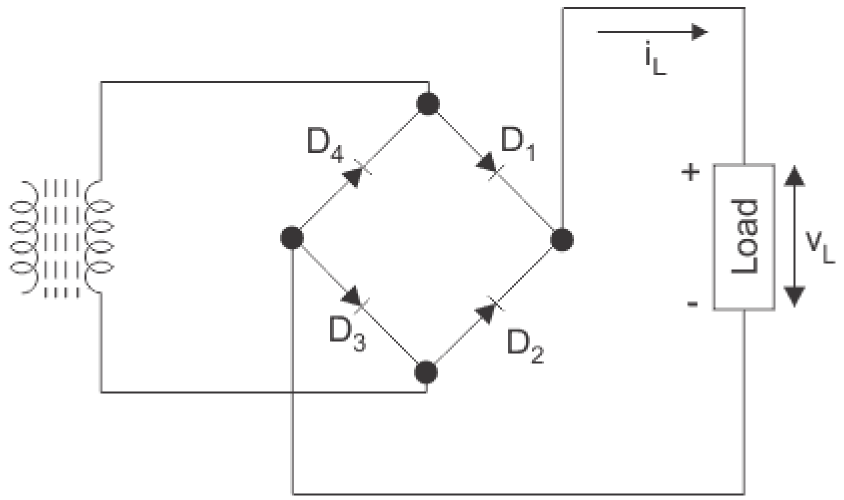
According to the diagram given below a center tapped transformer D_1, and D_2 are two p-n junction diodes with similar characteristics D_1 conducts for negative half of the output voltage. Thus we get output voltage and the output current for the entire input cycle.

4.8.1 Circuit Diagram of Full Wave Diode Rectifier

The circuit diagram of the **full wave diode rectifier** given below,

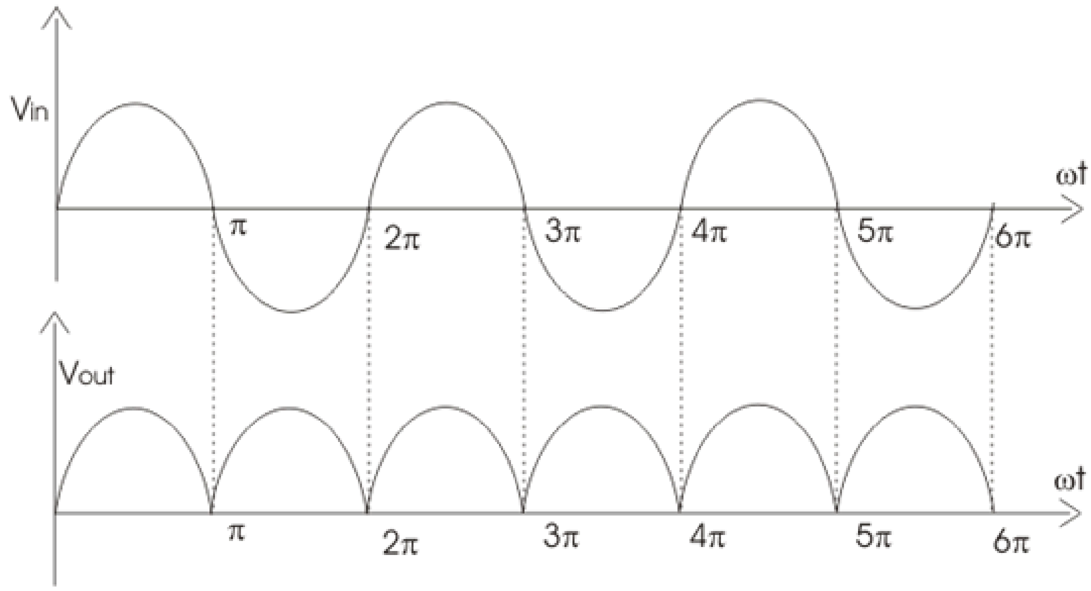


Full wave rectification can also be achieved using a bridge rectifier which is made of four diodes.



According to the figure, when D_1 and D_3 are forward biased, they conduct but D_2 and D_4 and on D_1 and D_3 are reverse biased in both cases load current in the same direction.

Bridge rectifier has several advantages over simple full wave rectifier. It performance and efficiency is better than that of the simple full time rectifier.



Now, different parameters for **half wave rectifier** is given below
The average of load current (I_{dc}) : Let, the load current be $i_L = I_m \sin\omega t$

$$I_{dc} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\omega t = \frac{2I_m}{\pi}$$

Ripple factor of half wave rectifier,

$$Ripple\ factor(r) = \frac{(I_{rms}^2 - I_{dc}^2)^{\frac{1}{2}}}{I_{dc}} = 0.482$$

$$Here,\ I_{rms} = \frac{I_m}{\sqrt{2}}$$

HALF WAVE RECTIFIERS

Rectifiers are the circuits used to convert alternating current (AC) into direct current (DC). Half-Wave Rectifiers are designed using a diode (D) and a load resistor (R_L) as shown in

Figure 1. In these rectifiers, only one-half of the input waveform is obtained at the output i.e. the output will comprise of either positive pulses or the negative pulses only. The polarity of the output voltage so obtained (across R_L) depends on the direction of the diode used in the circuit of half-wave rectifier. This is evident from the figure as Figure 1a shows the output waveform consisting of only positive pulses while the Figure 1b has only negative pulses in its output waveform.

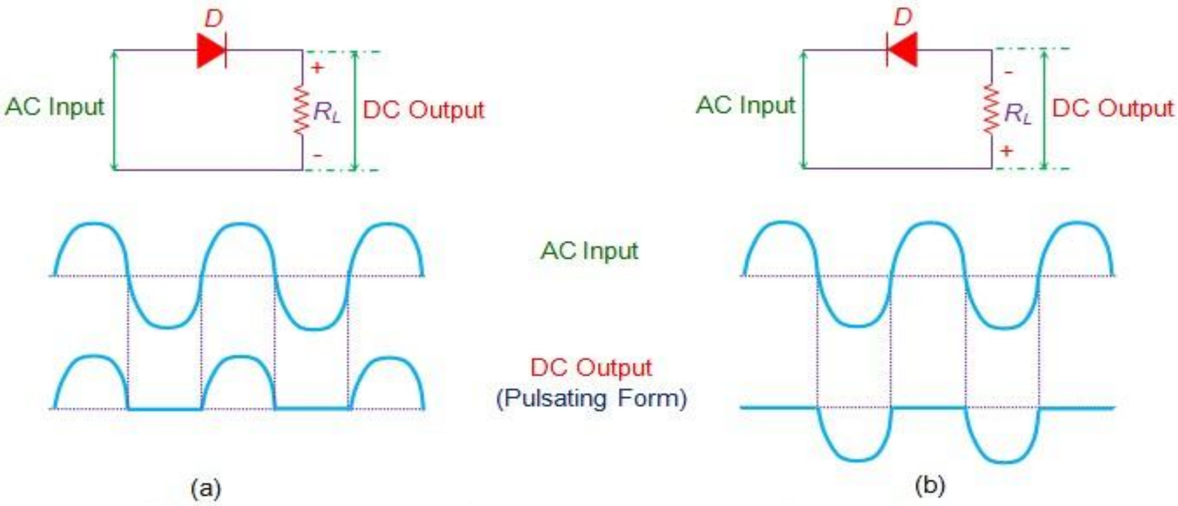


Figure 1 Half Wave Rectifier with Input and Output Waveforms

This is because, in Figure 1a the diode gets forward biased only during the positive pulse of the input which causes the current to flow across R_L, producing the output voltage.

Further for the same case, if the input pulse becomes negative, then the diode will be reverse biased and hence there will be no current flow and no output voltage. Similarly for the circuit shown in Figure 1b, the diode will be forward biased only when the input pulse is negative, and thus the output voltage will contain only the negative pulses. Further it is to be noted that the input to the half-wave rectifier can be supplied even via the transformer. This is advantageous as the transformer provides isolation from the power line as well as helps in obtaining the desired level of DC voltage. Next, one can connect a capacitor across the resistor in the circuit of half wave rectifier to obtain a smoother DC output (Figure 2). Here the capacitor charges through the diode D during the positive pulse of the input while it discharges through the load resistor R_L when the input pulse will be negative. Thus the output waveform of such a rectifier will have ripples in it as shown in the figure.

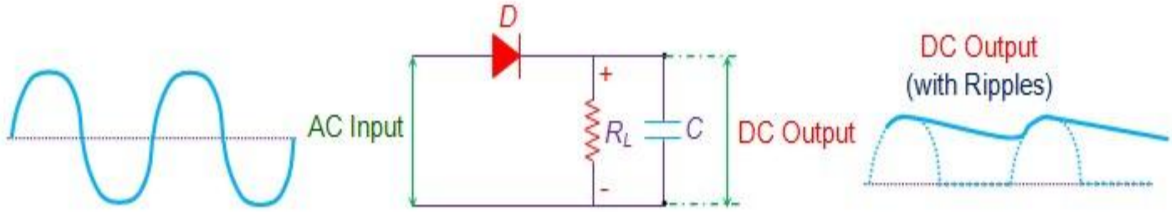


Figure 2 Half-Wave Rectifier with a RC Filter

Different parameters associated with the half wave rectifiers are

1. **Peak Inverse Voltage (PIV):** This is the maximum voltage which should be withstood by the diode under reverse biased condition and is equal to the peak of the input voltage, Vm.

2. **Average Voltage:** This is the DC content of the voltage across the load and is given by Vm/π. Similarly DC current is given as Im/π, where Im is the maximum value of the current.
3. **Ripple Factor (r):** It is the ratio of root mean square (rms) value of AC component to the DC component in the output and is given by

   $$r = \sqrt{\left(\frac{V_{rms}}{V_{DC}}\right)^2 - 1}$$

   Further, for half-wave rectifier, rms voltage is given as Vm/2 which results in the ripple factor of 1.21.
4. **Efficiency:** It is the ratio of DC output power to the AC input power and is equal to 40.6 %.
5. **Transformer Utilization Factor:** It is the ratio of DC power delivered to the load to the AC rating of the transformer secondary and is equal to 0.287.
6. **Form Factor:** This is the ratio of rms value to the average value and is thus equal to 1.57 for half-wave rectifier.
7. **Peak Factor:** It is the ratio of peak value to the rms value and is equal to 2.

Half wave rectifiers are advantageous as they are cheap, simple and easy to construct. These are quite rarely used as they have high ripple content in their output. However they can be used in non-critical applications like those of charging the battery. They are also less preferred when compared to other rectifiers as they have low output power, low rectification efficiency and low transformer utilization factor. In addition, if AC input is fed via the transformer, then it might get saturated which inturn results in magnetizing current, hysteresis loss and/or result in the generation of harmonics. Lastly it is important to note that the explanation provided here applies only for the case where the diode is ideal. Although for a practical diode, the basic working remains the same, one will have to consider the voltage drop across the diode as well as its reverse saturation current into consideration during the analysis.

FULL WAVE RECTIFIERS

The circuits which convert the input alternating current (AC) into direct current (DC) are referred to as rectifiers. If such rectifiers rectify both the positive as well as negative pulses of the input waveform, then they are called Full-Wave Rectifiers. Figure 1 shows such a rectifier designed using a multiple winding transformer whose secondary winding is equally divided into two parts with a provision for the connection at its central point (and thus referred to as the centre-tapped transformer), two diodes (D_1 and D_2) and a load resistor (R_L). Here the AC input is fed to the primary winding of the transformer while an arrangement of diodes and the load resistor which yields the DC output, is made across its secondary terminals.

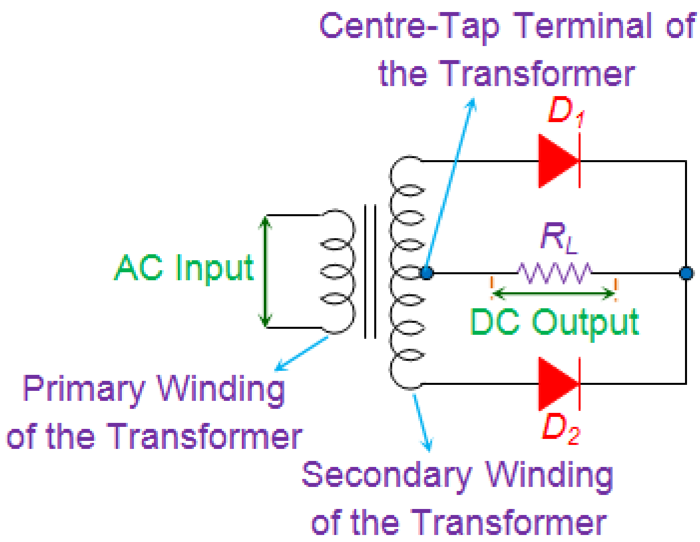


Figure 1 Full Wave Rectifier

The circuit can be analyzed by considering its working during the positive and the negative input pulses separately.

Figure 2a shows the case where the AC pulse is positive in nature i.e. the polarity at the top of the primary winding is positive while its bottom will be negative in polarity. This causes the top part of the secondary winding to acquire a positive charge while the common centre-tap terminal of the transformer will become negative.

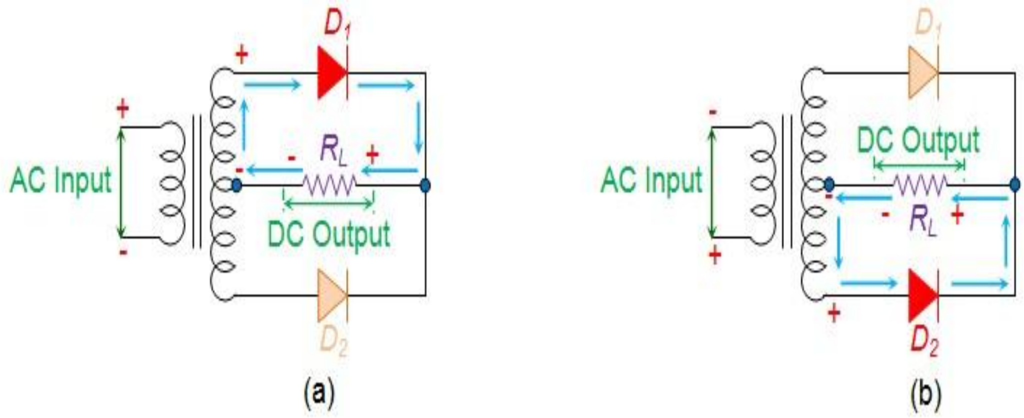


Figure 2 Conduction Path of Full Wave Rectifier for (a) Positive Input Pulse (b) Negative Input Pulse

This causes the diode D_1 to be forward biased which inturn causes the flow of current through R_L along the direction shown in Figure 2a. However at the same time, diode D_2 will be reverse biased and hence acts like an open circuit. This causes the appearance of positive pulse across the R_L, which will be the DC output. Next, if the input pulse becomes negative in nature, then the top and the bottom of the primary winding will acquire the negative and the positive polarities respectively. This causes the bottom of the secondary winding to become positive while its centre-tapped terminal will become negative. Thus the diode D_2 gets forward biased while the D_1 will get reverse biased which allows the flow of current as shown in the Figure 2b. Here the most important thing to note is the fact that the direction in which the current flows via R_L will be identical in either case (both for positive as well as for negative input pulses). Thus we get the positive output pulse even for the case of negative input pulse (Figure 3), which indicates that both the half cycles of the input AC are rectified.

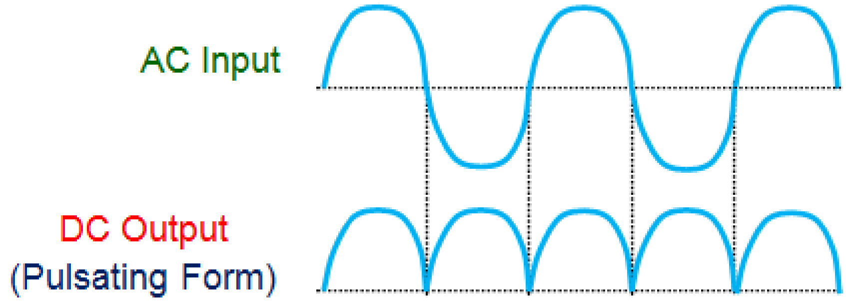


Figure 3 Input and Output Waveforms of Full Wave Rectifier

Such circuits are referred to as (i) Centre-Tapped **Full Wave Rectifiers** as they use a centre-tapped transformer, (ii) Two-Diode Full-Wave Rectifiers because of the use of two diodes and/or (iii) Bi-Phase Circuits due to the fact that in these circuits, the output voltage will be the phasor addition of the voltages developed across the load resistor due to two individual diodes, where each of them conducts only for a particular half-cycle. However as evident from Figure 3, the output of the rectifier is not pure DC but pulsating in nature, where the frequency of the output waveform is seen to be double of that at the input. In order to smoothen this, one can connect a capacitor across the load resistor as shown by the Figure 4. This causes the capacitor to charge via the diode D_1 as long as the input positive pulse increases in its magnitude. By the time the input pulse reaches the positive maxima, the capacitor would have charged to the same magnitude. Next, as long as the input positive pulse keeps on decreasing, the capacitor tries to hold the charge acquired (being an energy-storage element).

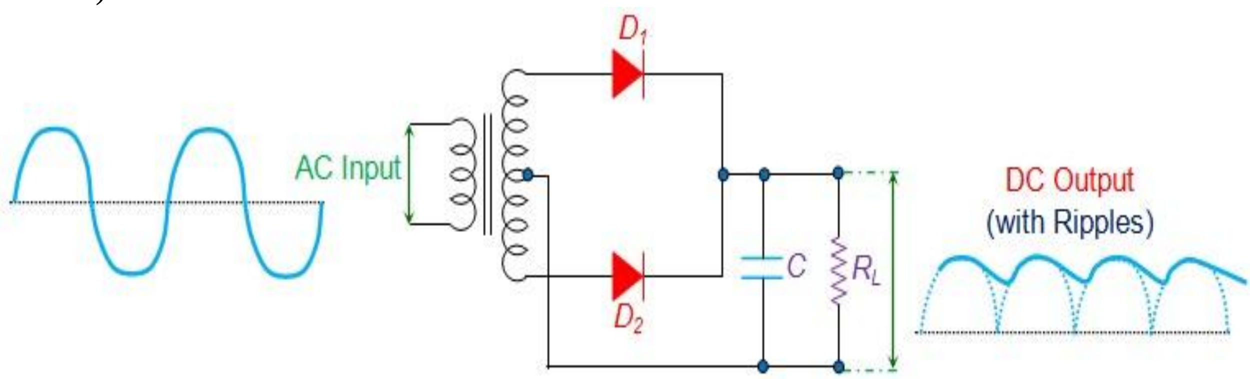


Figure 4 Full-Wave Rectifier with a RC Filter

However there will be voltage-loss as some amount of charge gets lost through the path provided by the load resistor (nothing but discharging phenomenon). Further, as the input pulse starts to go low to reach the negative maxima, the capacitor again starts to charge via the path provided by the diode D_2 and acquires an almost equal voltage but with opposite polarity. Next, as the input voltage starts to move towards 0V, the capacitor slightly discharges via R_L. This charge-discharge cycle of the capacitor causes the ripples to appear in the output waveform of the full-wave rectifier with RC filter as shown in Figure 4.

Different parameters and their values for the centre-tapped full-wave rectifiers are

1. **Peak Inverse Voltage (PIV):** This is the maximum voltage which occurs across the diodes when they are reverse biased. Here it will be equal to twice the peak of the input voltage, $2V_m$.

2. **Average Voltage:** It is the DC voltage available across the load and is equal to $2V_m/\pi$. The corresponding DC current will be $2I_m/\pi$, where Im is the maximum value of the current.
3. **Ripple Factor (r):** This is the ratio of the root mean square (rms) value of AC component to the dc component at the output. It is given by

$$r = \sqrt{\left(\frac{V_{rms}}{V_{DC}}\right)^2 - 1}$$

and will be equal to 0.482 as the rms voltage for a full-wave rectifier is given as

$$V_m/\sqrt{2}$$

4. **Efficiency:** This is the ratio of DC output power to the AC input power and is equal to 81.2 %.
5. **Transformer Utilization Factor (TUF):** This factor is expressed as the ratio of DC power delivered to the load to the AC rating of the transformer secondary. For the full-wave rectifier this will be 0.693.
6. **Form Factor:** This is the ratio of rms value to the average value and is equal to 1.11.
7. **Peak Factor:** It is the ratio of peak value to the rms value and is equal to $\sqrt{2}$ for the full-wave rectifiers.

Further it is to be noted that the two-diode full-wave rectifier shown in Figure 1 is costly and bulky in size as it uses the complex centre-tapped transformer in its design. Thus one may resort to another type of full-wave rectifier called Full-Wave Bridge Rectifier (identical to Bridge Rectifier) which might or might not involve the transformer (even if used, will not be as complicated as a centre-tap one). It also offers higher TUF and higher PIV which makes it ideal for high power applications. However it is to be noted that the full wave bridge rectifier uses four diodes instead of two, which in turn increases the magnitude of voltage drop across the diodes, increasing the heating loss. **Full wave rectifiers** are used in general power supplies, to charge a battery and to provide power to the devices like motors, LEDs, etc. However due to the ripple content in the output waveform, they are not preferred for audio applications. Further these are advantageous when compared to half-wave rectifiers as they have higher DC output power, higher transformer utilization factor and lower ripple content, which can be made more smoother by using π-filters. All these merits mask-up its demerit of being costly in comparison to the half-wave rectifiers due to the use of increased circuit elements. At last, it is to be noted that the explanation provided here considers the diodes to be ideal in nature. So, incase of practical diodes, one will have to consider the voltage drop across the diode, its reverse saturation current and other diode characteristics into account and reanalyze the circuit. Nevertheless the basic working remains the same.

4.10 BRIDGE RECTIFIERS

Bridge Rectifiers are the circuits which convert alternating current (AC) into direct current (DC) using the diodes arranged in the bridge circuit configuration. They usually comprise of four or more number of diodes which cause the output generated to be of the same polarity irrespective of the polarity at the input. Figure 1 shows such a bridge rectifier composed of four diodes D_1, D_2, D_3 and D_4 in which the input is supplied across two terminals A and B in the figure while the output is collected across the load resistor R_L connected between the terminals C and D.

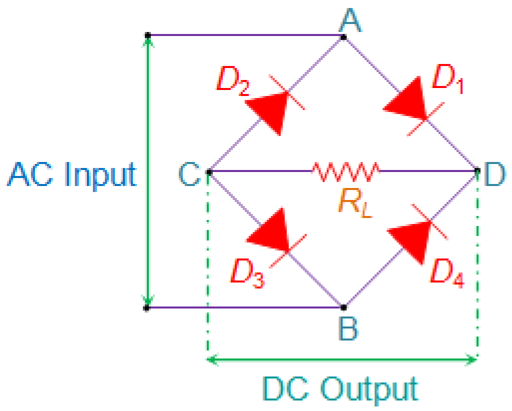


Figure 1 Bridge Rectifier

Now consider the case wherein the positive pulse appears at the AC input i.e. the terminal A is positive while the terminal B is negative. This causes the diodes D_1 and D_3 to get forward biased and at the same time, the diodes D_2 and D_4 will be reverse biased.

As a result, the current flows along the short-circuited path created by the diodes D_1 and D_3 (considering the diodes to be ideal), as shown by Figure 2a. Thus the voltage developed across the load resistor R_L will be positive towards the end connected to terminal D and negative at the end connected to the terminal C.

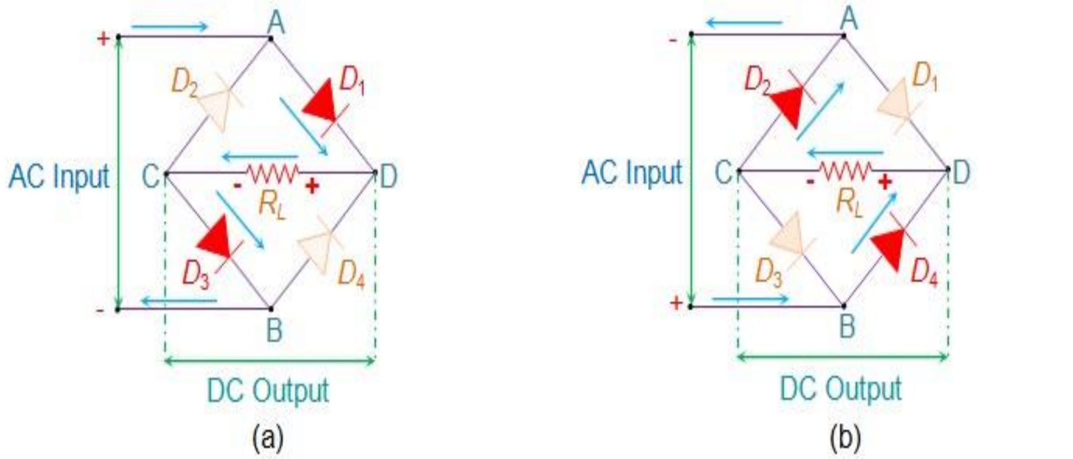


Figure 2 Current Path Through the Bridge Rectifier for (a) Positive half-cycle (b) Negative Half-Cycle

Next if the negative pulse appears at the AC input, then the terminals A and B are negative and positive respectively. This forward biases the diodes D_2 and D_4, while reverse biasing D_1 and D_3 which causes the current to flow in the direction shown by Figure 2b. At this instant, one has to note that the polarity of the voltage developed across R_L is identical to that produced when the incoming AC pulse was positive in nature. This means that for both positive and negative pulse, the output of the bridge rectifier will be identical in polarity as shown by the wave forms in Figure 3.

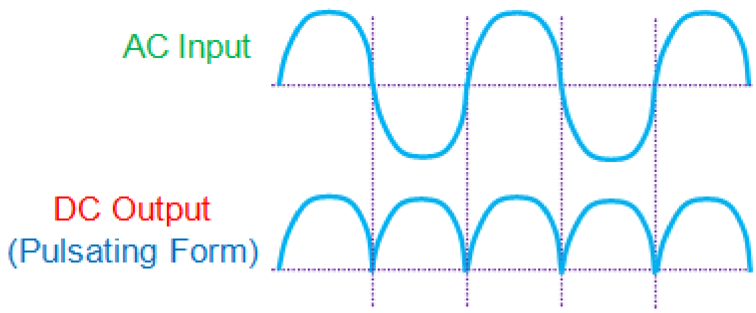


Figure 3 Input-Output Waveforms of a Bridge Rectifier

However it is to be noted that the bridge rectifier's DC will be pulsating in nature. In order to obtain pure form of DC, one has to use capacitor in conjunction with the bridge circuit (Figure 4).

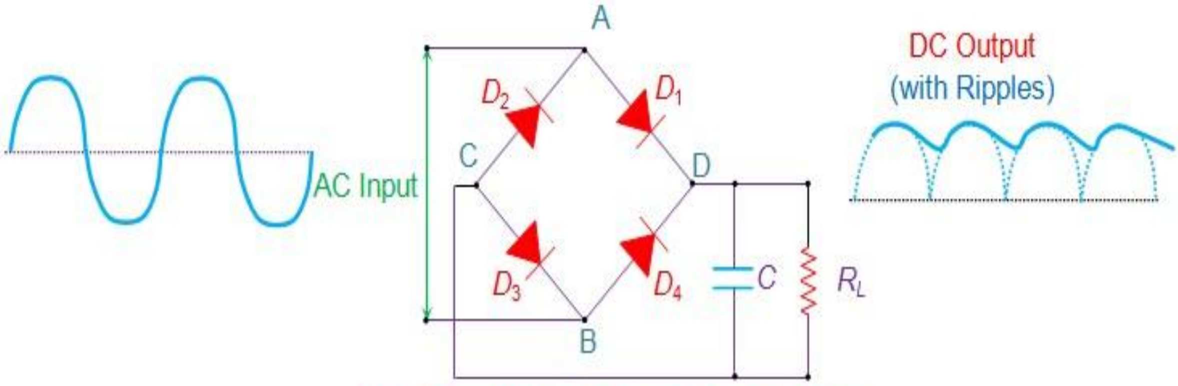


Figure 4 Bridge Rectifier with a RC Filter

In this design, the positive pulse at the input causes the capacitor to charge through the diodes D_1 and D_3. However as the negative pulse arrives at the input, the charging action of the capacitor ceases and it starts to discharge via R_L. This results in the generation of DC output which will have ripples in it as shown in the figure. This ripple factor is defined as the ratio of AC component to the DC component in the output voltage. In addition, the mathematical expression for the ripple voltage is given by the equation

$$V_r = \frac{I_l}{fC}$$

Where, V_r represents the ripple voltage.

I_l represents the load current.

f represents the frequency of the ripple which will be twice the input frequency.

C is the Capacitance.

Further, the **bridge rectifiers** can be majorly of two types, viz., Single-Phase Rectifiers and Three-Phase Rectifiers. In addition, each of these can be either Uncontrolled or Half-Controlled or Full-Controlled. Bridge rectifiers for a particular application are selected by considering the load current requirements. These bridge rectifiers are quite advantageous as they can be constructed with or without a transformer and are suitable for high voltage applications. However here two diodes will be conducting for every half-cycle and thus the voltage drop across the diodes will be higher. Lastly one has to note that apart from converting AC to DC, **bridge rectifiers** are also used to detect the amplitude of modulated radio signals and to supply polarized voltage for welding applications.

UNIT V

BIPOLAR JUNCTION TRANSISTOR AND APPLICATIONS

Introduction

The transistor was invented in 1947 by John Bardeen, Walter Brattain and William Shockley at Bell Laboratory in America. A transistor is a semiconductor device, commonly used as an Amplifier or an electrically Controlled Switch. There are two types of transistors:

1) Unipolar Junction Transistor

2) Bipolar Junction Transistor

In Unipolar transistor, the current conduction is only due to one type of carriers i.e., majority charge carriers. The current conduction in bipolar transistor is because of both the types of charge carriers i.e., holes and electrons. Hence it is called as Bipolar Junction Transistor and it is referred to as BJT.

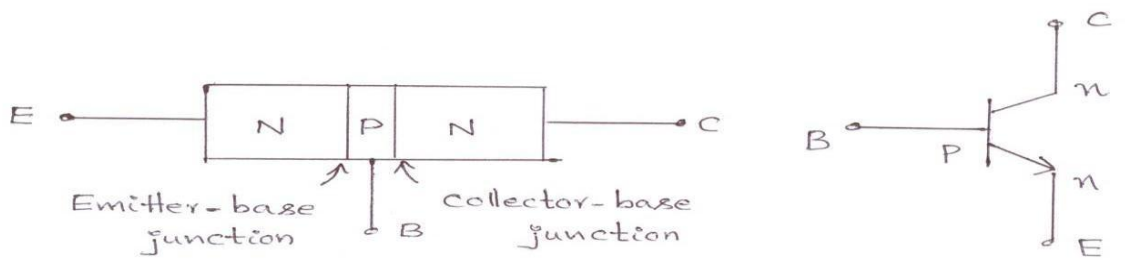
BJT is a semiconductor device in which one type of semiconductor material is sand witched between two opposite types of semiconductor i.e., an n-type semiconductor is sandwiched between two p-type semiconductors or a p-type semiconductor is sandwiched between two n-type semiconductor. Hence the BJTs are of two types.

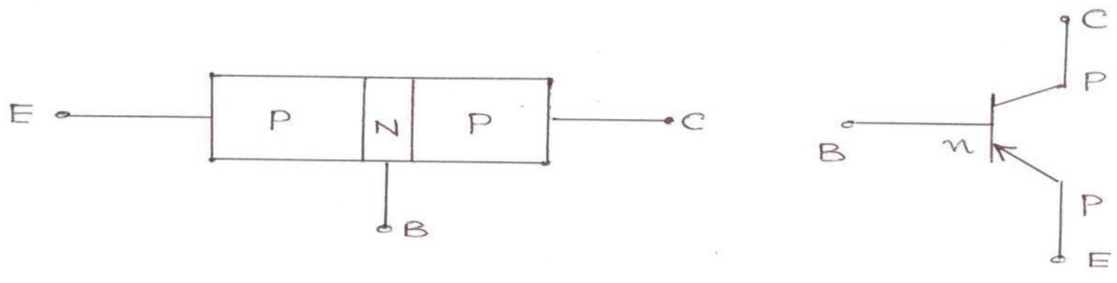
They are:

1) n-p-n Transistor

2) p-n-p Transistor

The two types of BJTs are shown in the figure below.





The arrow head represents the conventional current direction from p to n. Transistor has three terminals.

1) Emitter 2) Base 3) Collector

Transistor has two p-n junctions. They are:

1) Emitter-Base Junction

2) Collector-Base Junction

Emitter: Emitter is heavily doped because it is to emit the charge carriers.

Base: The charge carriers emitted by the emitter should reach collector passing through the base. Hence base should be very thin and to avoid recombination, and to provide more collector current base is lightly doped.

Collector: Collector has to collect the most of charge carriers emitted by the emitter. Hence the area of cross section of collector is more compared to emitter and it is moderately doped.

Transistor can be operated in three regions.

1) Active region.

2) Saturation region.

3) Cut-Off region.

Active Region: For the transistor to operate in active region base to emitter junction is forward biased and collector to base junction is reverse biased.

Saturation Region: Transistor to be operated in saturation region if both the junctions i.e., collector to base junction and base to emitter junction are forward biased.

Cut-Off Region: For the transistor to operate in cut-off region both the junctions i.e., base to emitter junction and collector to base junction are reverse biased.

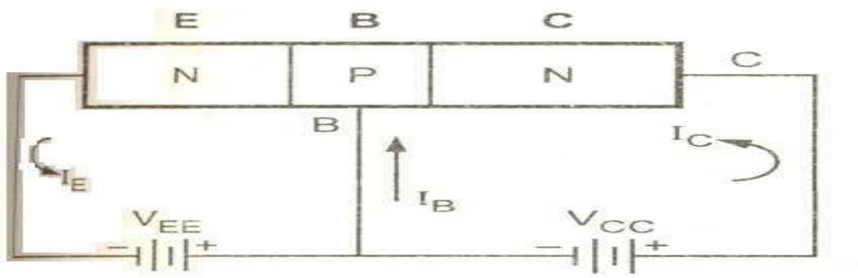
Transistor can be used as

1) Amplifier 2) Switch

For the transistor to act as an amplifier, it should be operated in active region. For the transistor to act as a switch, it should be operated in saturation region for ON state, and cut-off region for OFF state.

Transistor Operation:

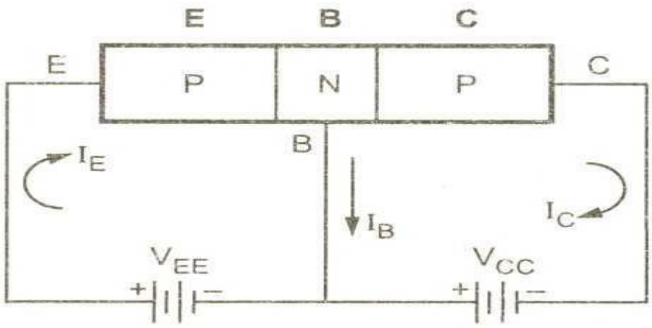
Working of a n-p-n transistor:



The n-p-n transistor with base to emitter junction forward biased and collector base junction reverse biased is as shown in figure.

As the base to emitter junction is forward biased the majority carriers emitted by the n type emitter i.e., electrons have a tendency to flow towards the base which constitutes the emitter Current IE As the base is p-type there is chance of recombination of electrons emitted by the emitter with the holes in the p-type base. But as the base is very thin and lightly doped only few electrons emitted by the n-type emitter less than 5% combines with the holes in the p-type base, the remaining more than 95% electrons emitted by the n-type emitter cross over into the collector region constitute the collector current. The current distributions are as shown in fig **IE = IB + IC**

Working of a p-n-p transistor:



The p-n-p transistor with base to emitter junction is forward biased and collector to base junction reverse biased is as show in figure. As the base to emitter junction is forward biased the majority carriers emitted by the type emitter i.e., holes have a tendency to flow towards the base which constitutes the emitter current IE. As the base is n-type there is a chance of recombination of holes emitted by the emitter with the electrons in the n-type base. But as the base us very thin and lightly doped only few electrons less than 5% combine with the holes emitted by the p-type emitter, the remaining 95% charge carriers cross over into the collector region to constitute the collector current. The current distributions are shown in figure.

IE = IB + IC

Current components in a transistor:

The figure below shows the various current components which flow across the forward biased emitter junction and reverse-biased collector junction in P-N-P transistor

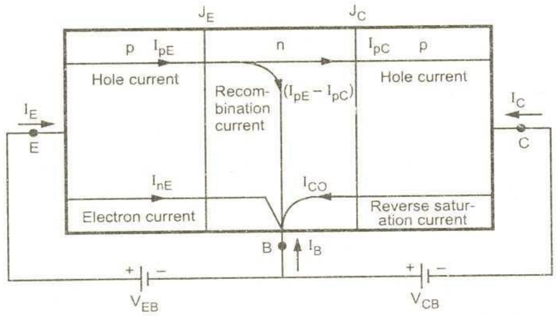


Figure. Current components in a transistor with forward-biased emitter and reverse-biased Collector junctions. The emitter current consists of the following two parts:

1) Hole current IpE constituted by holes (holes crossing from emitter into base).

2) Electron current InE constituted by electrons (electrons crossing from base into the emitter).

Therefore, Total emitter current IE = IpE (majority)+ InE (Minority)

The holes crossing the emitter base junction JE and reaching the collector base junction JC constitutes collector current IpC. Not all the holes crossing the emitter base junction JE reach collector base junction JC because some of them combine with the electrons in the n-type base Since base width is very small, most of the holes cross the collector base junction JC and very few recombine, constituting the base current (IpE – IpC).

When the emitter is open-circuited, IE=0, and hence IpC=0. Under this condition, the base and collector together current IC equals the reverse saturation current ICO, which consists of the following two parts: IPCO caused by holes moving across IC from N-region to P-region. InCO caused by electrons moving across IC from P-region to N-region. ICO = InCO + IpCO

In general,

$$IC = InC + IpC$$

Thus for a P-N-P transistor,

$$IE = IB + IC$$

Transistor circuit configurations:

Following are the three types of transistor circuit configurations:

1) Common-Base (CB)

2) Common-Emitter (CE)

3) Common-Collector (CC)

Here the term 'Common' is used to denote the transistor lead which is common to the input and output circuits. The common terminal is generally grounded.

It should be remembered that regardless the circuit configuration, the emitter is always forward-biased while the collector is always reverse-biased.

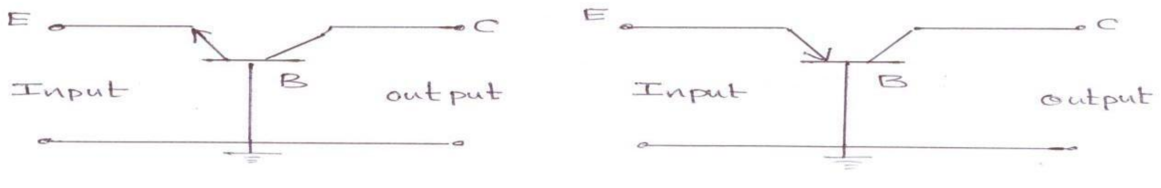


Fig. Common – Base configuration

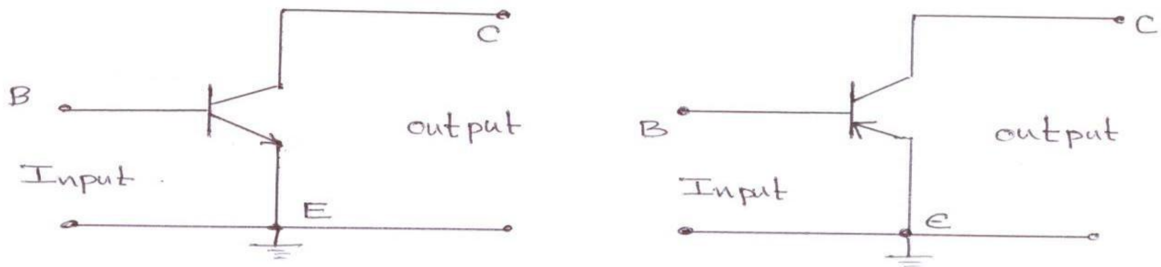


Fig. Common – emitter configuration

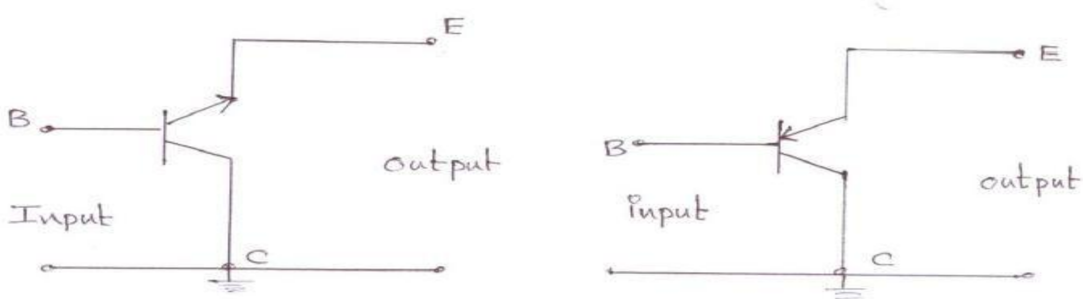


Fig. Common – Collector configuration

Common – Base (CB) configurations:

In this configuration, the input signal is applied between emitter and base while the output is taken from collector and base. As base is common to input and output circuits, hence the name

common-base configuration. Figure show the common-base P-N-P transistor circuit.

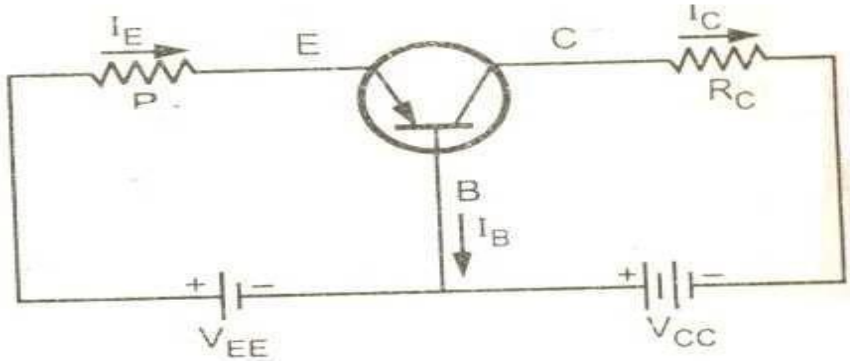


Fig. Common – base PNP transistor amplifier.

Current Amplification Factor (α) **:**

When no signal is applied, then the ratio of the collector current to the emitter current is called dc alpha (α dc) of a transistor.

$$\alpha_{dc} = \frac{-I_C}{I_E} \quad ,............... \text{(1)}$$

(Negative sign signifies that IE flows into transistor while IC flows out of it). ‘ α’ of a transistor is a measure of the quality of a transistor. Higher is the value of ‘α ’, better is the transistor in the sense that collector current approaches the emitter current. By considering only magnitudes of the currents, IC = a IE and hence IB = IE - IC

Therefore,

$$IB = IE - a\,IE = IE(1-a) \ldots\ldots\ldots\ldots (2)$$

When signal is applied, the ratio of change in collector current to the change in emitter current at constant collector-base voltage is defined as current amplification factor,

$$\alpha_{dc} = -\frac{\Box I_C}{\Box I_E} \ldots\ldots\ldots\ldots\ldots (3)$$

For all practical purposes, *dc* a =a*ac* =a and practical values in commercial transistors range from 0.9 to 0.99.

Total Collector Current:

The total collector current consists of the following two parts

i) a IE , current due to majority carriers

ii) ICBO, current due to minority carriers

\ Total collector current IC = a IE + ICBO ………… (4)

The collector current can also be expressed as IC = a (IB+IC) + ICBO (Q IE = IB + IC)

$$\Rightarrow I_C(1-\alpha) = \alpha I_B + I_{CBO}$$

$$\Rightarrow I_C = \left(\frac{\alpha}{1-\alpha}\right) I_B + \left(\frac{1}{1-\alpha}\right) I_{CBO} \quad \ldots (5)$$

Common-Emitter (CE) configuration:

In this configuration, the input signal is applied between base and emitter and the output is taken from collector and emitter. As emitter is common to input and output circuits, hence the name common emitter configuration.

Figure shows the common-emitter P-N-P transistor circuit.

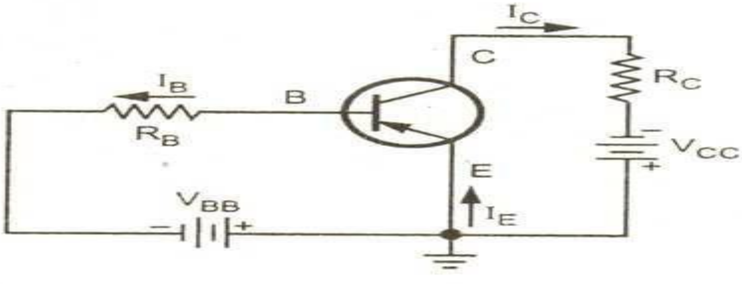


Fig. Common-Emitter PNP transistor amplifier.

Current Amplification Factor (β):

When no signal is applied, then the ratio of collector current to the base current is called dc beta (βdc) of a transistor.

$$\beta_{dc} = \beta = \frac{I_C}{I_B} \quad \text{.................. (1)}$$

When signal is applied, the ratio of change in collector current to the change in base current is defined as base current amplification factor. Thus,

$$\beta_{dc} = \beta = \frac{\Delta I_C}{\Delta I_B} \quad \text{.............(2)}$$

From equation (1), $I_C = \beta I_B$

Almost in all transistors, the base current is less than 5% of the emitter current. Due to this fact, 'β' ranges from 20 to 500. Hence this configuration is frequently used when appreciable current gain as well as voltage gain is required.

Total Collector Current:

The Total collector current IC = β IB + ICEO ………… (3)

Where ICEO is the leakage current.

But, we have, $I_C = \left(\frac{\alpha}{1-\alpha}\right) I_B + \left(\frac{1}{1-\alpha}\right) I_{CBO}$ ....(4)

Comparing equations (3) and (4), we get

$$\beta = \frac{\alpha}{1-\alpha} \text{ and } I_{CEO} = \frac{1}{1-\alpha} I_{CBO} \quad \text{...........(5)}$$

Relation between α and β:

We know that $\alpha = \frac{I_C}{I_E}$ and $\beta = \frac{I_C}{I_B}$

$I_E = I_B + I_C$ (or) $I_B = I_E - I_C$

Now
$$\beta = \frac{I_C}{I_E - I_C} = \frac{\frac{I_C}{I_E}}{1 - \frac{I_C}{I_E}} = \frac{\alpha}{1-\alpha} \quad \text{................ (6)}$$

$$\Rightarrow \beta(1-\alpha) = \alpha \text{ (or) } \beta = \alpha(1+\beta)$$

$$\Rightarrow \alpha = \frac{\beta}{1+\beta} \quad \text{................ (7)}$$

It can be seen that $1-\alpha = \frac{1}{1+\beta}$ .... (8)

Common – Collector (CC) Configuration:

In this configuration, the input signal is applied between base and collector and the output is taken from the emitter. As collector is common to input and output circuits, hence the name common collector configuration. Figure shows the common collector PNP transistor circuit.

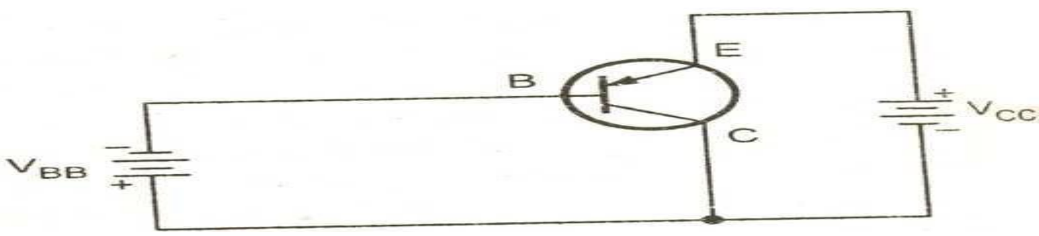


Fig. Common collector PNP transistor amplifier.

Current Amplification Factor γ):

When no signal is applied, then the ratio of emitter current to the base current is called as dc gamma (γdc) of the transistor.

$$\gamma_{dc} = \gamma = \frac{I_E}{I_B} \quad \text{............... (1)}$$

When signal is applied, then the ratio of change in emitter current to the change in base current is known as current amplification factor 'γ'.

$$\gamma_{ac} = \gamma = \frac{\Delta I_E}{\Delta I_B} \quad \text{................(2)}$$

This configuration provides the same current gain as common emitter circuit as $\Delta I_E = \Delta I_C$ but the voltage gain is always less than one.

Total Emitter Current:

We know that $I_E = I_B + I_C$ Also $I_C = \alpha I_E + I_{CBO}$

$$I_E = I_B + (\alpha I_E + I_{CBO})$$

$$\Rightarrow I_E(1-\alpha) = I_B + I_{CBO}$$

$$\Rightarrow I_E = \frac{I_B}{1-\alpha} + \frac{I_{CBO}}{1-\alpha}$$

(or) $\Rightarrow I_E = (1+\beta)I_B + (1+\beta)I_{CBO}$ .... (3) $\left(\because \frac{1}{1-\alpha} = 1+\beta\right)$

Relation between γ and α :

We know that $\gamma = \frac{I_E}{I_B}$ and $\alpha = \frac{I_C}{I_B}$

Also $I_B = I_E - I_C$

Now $\gamma = \frac{I_E}{I_E - I_C} = \frac{1}{1 - \frac{I_C}{I_E}} = \frac{1}{1-\alpha}$

$$\because \gamma = \frac{1}{1-\alpha} \quad \text{......................... (4)}$$

Relation between γ and β :

We know that $\frac{1}{1-\alpha} = 1+\beta$

$\because$ From equation (4), $\gamma = \frac{1}{1-\alpha} = 1+\beta$ .... (5)

Characteristics of Common-Base Circuit:

The circuit diagram for determining the static characteristic curves of an NPN transistor in the common base configuration is shown in fig. below.

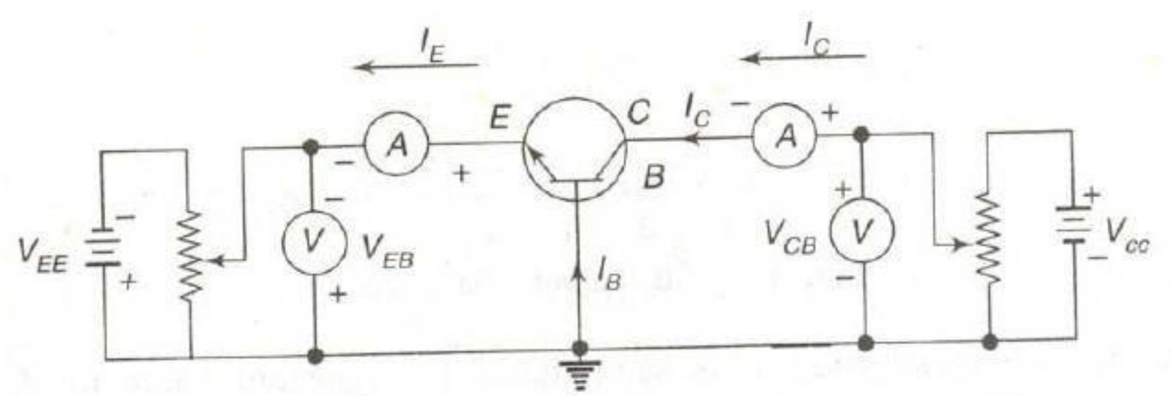


Fig. Circuit to determine CB static characteristics.

Input Characteristics:

To determine the input characteristics, the collector-base voltage VCB is kept constant at zero volts and the emitter current IE is increased from zero in suitable equal steps by increasing VEB. This is repeated for higher fixed values of VCB. A curve is drawn between emitter current IE and emitter-base voltage VEB at constant collector-base voltage VCB.

The input characteristics thus obtained are shown in figure below.

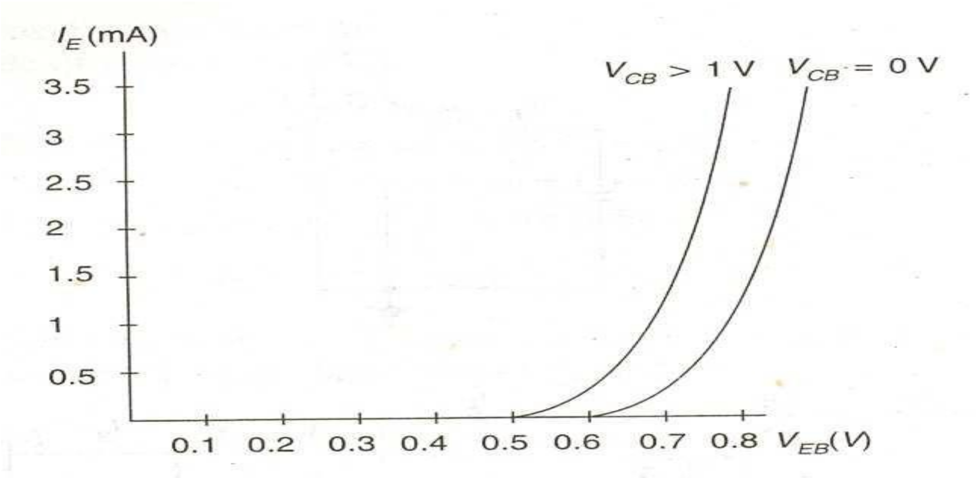
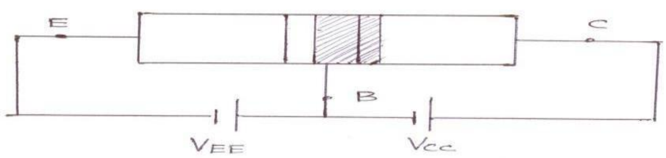


Fig. CB Input characteristics.

Early effect (or) Base – Width modulation:

As the collector voltage VCC is made to increase the reverse bias, the space charge width between collector and base tends to increase, with the result that the effective width of the base

decreases. This dependency of base-width on collector-to-emitter voltage is known as Early effect (or) Base-Width modulation.



Thus decrease in effective base width has following consequences:

i. Due to Early effect, the base width reduces, there is a less chance of recombination of holes with electrons in base region and hence base current IB decreases.

ii. As IB decreases, the collector current IC increases.

iii. As base width reduces the emitter current IE increases for small emitter to base voltage.

iv. As collector current increases, common base current gain (a) increases.

Punch Through (or) Reach Through:

When reverse bias voltage increases more, the depletion region moves towards emitter junction and effective base width reduces to zero. This causes breakdown in the transistor. This condition is called “Punch Through” condition.

Output Characteristics:

To determine the output characteristics, the emitter current IE is kept constant at a suitable value by adjusting the emitter-base voltage VEB. Then VCB is increased in suitable equal steps and the collector current IC is noted for each value of IE. Now the curves of IC versus VCB are plotted for constant values of IE and the output characteristics thus obtained is shown in figure below.

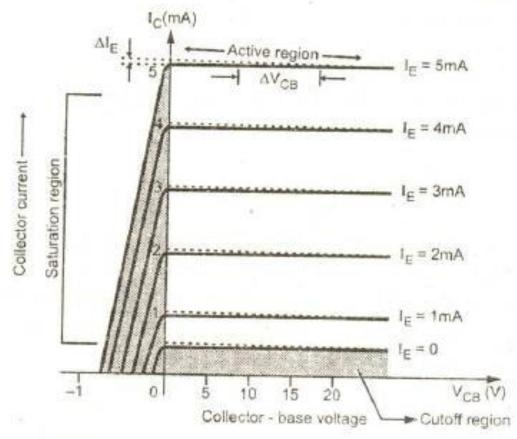


Fig. CB Output characteristics

From the characteristics, it is seen that for a constant value of IE, IC is independent of VCB and the curves are parallel to the axis of VCB. Further, IC flows even when VCB is equal to zero. As the emitter-base junction is forward biased, the majority carriers, i.e., electrons, from the emitter are injected into the base region. Due to the action of the internal potential barrier at the reverse

biased collector-base junction, they flow to the collector region and give rise to IC even when VCB is equal to zero.

Transistor Parameters:

The slope of the CB characteristics will give the following four transistor parameters. Since these parameters have different dimensions, they are commonly known as common base hybrid parameters (or) h-parameters.

i) Input Impedance (hib):

It is defined as the ratio of change in (input) emitter to base voltage to the change in (input) emitter current with the (output) collector to base voltage kept constant. Therefore,

$$h_{ib} = \frac{\Delta V_{EB}}{\Delta I_E} \text{ , } V_{CB} \text{ constant}$$

It is the slope of CB input characteristics curve.

The typical value of hib ranges from 20I to 50I.

ii) Output Admittance (hob):

It is defined as the ratio of change in the (output) collector current to the corresponding change in the (output) collector-base voltage, keeping the (input) emitter current IE constant. Therefore,

$$h_{ob} = \frac{\Delta I_C}{\Delta V_{CB}} \text{ , } I_E \text{ constant}$$

It is the slope of CB output characteristics IC versus VCB.

The typical value of this parameter is of the order of 0.1 to 10µmhos

iii) Forward Current Gain (hfb):

It is defined as a ratio of the change in the (output) collector current to the corresponding change in the (input) emitter current keeping the (output) collector voltage VCB constant. Hence,

$$h_{fb} = \frac{\Delta I_C}{\Delta I_E} \text{ , } V_{CB} \text{ constant}$$

It is the slope of IC versus IE curve. Its typical value varies from 0.9 to 1.0.

iv) Reverse Voltage Gain (hrb):

It is defined as a ratio of the change in the (input) emitter voltage and the corresponding change in (output) collector voltage with constant (input) emitter current, IE. Hence,

$$h_{rb} = \frac{\Delta V_{EB}}{\Delta V_{CB}} \text{ , } I_E \text{ constant.}$$

It is the slope of VEB versus VCB curve. Its typical value is of the order of 10^{-5} to 10^{-4}

Characteristics of Common-Emitter Circuit:

The circuit diagram for determining the static characteristic curves of the an N-P-N transistor in the common emitter configuration is shown in figure below.

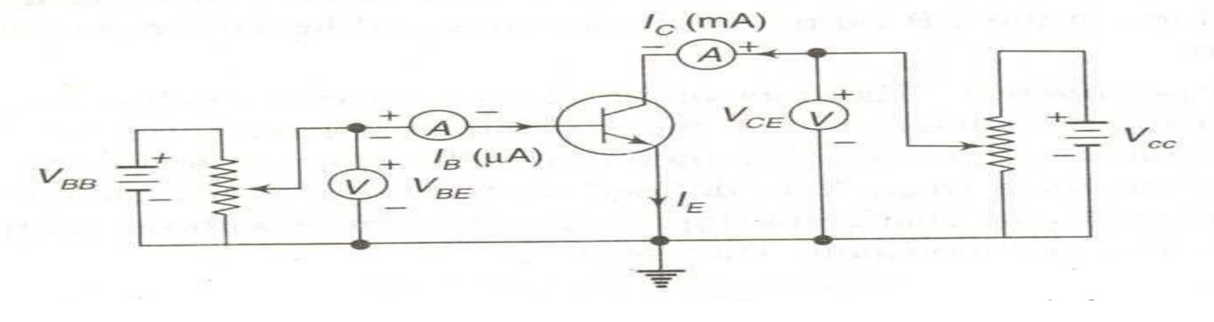


Fig. Circuit to determine CE Static characteristics.

Input Characteristics:

To determine the input characteristics, the collector to emitter voltage is kept constant at zero volts and base current is increased from zero in equal steps by increasing VBE in the circuit. The value of VBE is noted for each setting of IB. This procedure is repeated for higher fixed values

of VCE, and the curves of IB versus VBE are drawn.

The input characteristics thus obtained are shown in figure below.

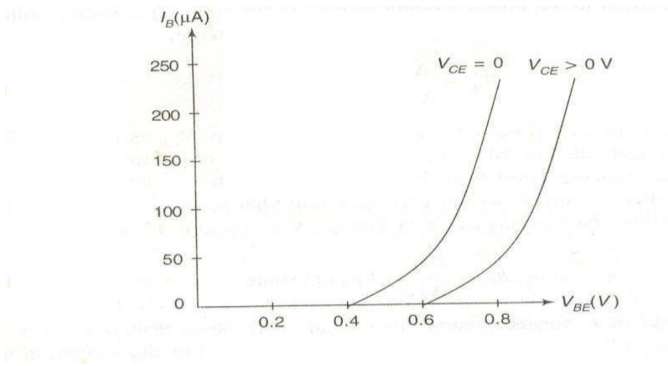


Fig. CE Input Characteristics.

When VCE=0 , the emitter-base junction is forward biased and he junction behaves as a forward biased diode. When VCE is increased, the width of the depletion region at the reverse biased collector-base junction will increase. Hence he effective width of the base will decrease. This effect causes a decrease in the base current IB. Hence, to get the same value of IB as that for VCE=0, VBE should be increased. Therefore, the curve shifts to the right as VCE increases.

Output Characteristics:

To determine the output characteristics, the base current IB is kept constant at a suitable value by adjusting base-emitter voltage, VBE. The magnitude of collector-emitter voltage VCE is increased in suitable equal steps from zero and the collector current IC is noted for each setting of VCE. Now the curves of IC versus VCE are plotted for different constant values of IB. The output characteristics thus obtained are shown in figure below.

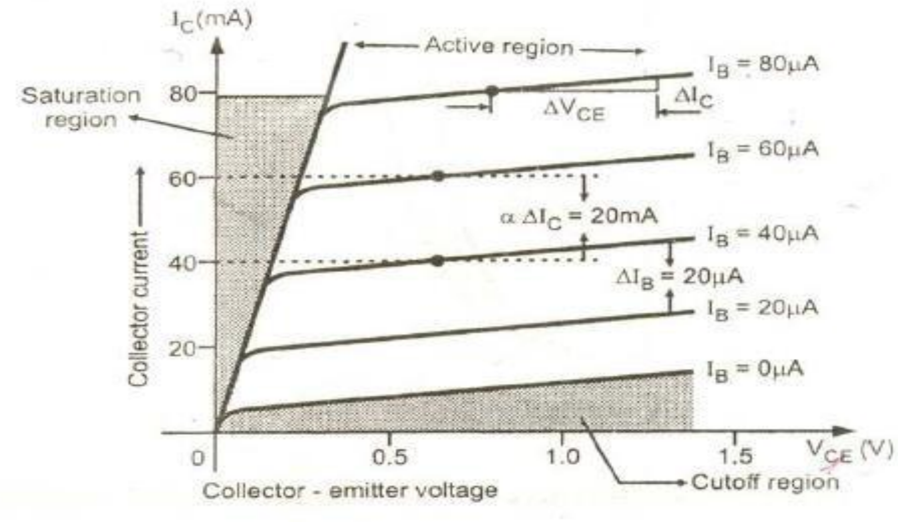


Fig. CE Output characteristics

The output characteristics of common emitter configuration consist of three regions: Active, Saturation and Cut-off regions.

Active Region: The region where the curves are approximately horizontal is the "Active" region of the CE configuration. In the active region, the collector junction is reverse biased. As VCE is increased, reverse bias increase. This causes depletion region to spread more in base than in collector, reducing the changes of recombination in the base. This increase the value of a *dc* . This Early effect causes collector current to rise more sharply with increasing VCE in the active region of output characteristics of CE transistor.

Saturation Region: If VCE is reduced to a small value such as 0.2V, then collector-base junction becomes forward biased, since the emitter-base junction is already forward biased by 0.7V. The input junction in CE configuration is base to emitter junction, which is always forward biased to operate transistor in active region. Thus input characteristics of CE configuration are similar to forward characteristics of p-n junction diode. When both the junctions are forwards

biased, the transistor operates in the saturation region, which is indicated on the output characteristics. The saturation value of VCE, designated *VCE*(*Sat*) , usually ranges between 0.1V to 0.3V.

Cut-Off Region: When the input base current is made equal to zero, the collector current is the reverse leakage current ICEO. Accordingly, in order to cut off the transistor, it is not enough to reduce IB=0. Instead, it is necessary to reverse bias the emitter junction slightly. We shall define cut off as the condition where the collector current is equal to the reverse saturation current ICO and the emitter current is zero.

Transistor Parameters:

The slope of the CE characteristics will give the following four transistor parameters. Since these parameters have different dimensions, they are commonly known as Common emitter hybrid parameters (or) h-parameters.

i) Input Impedance (hib):

It is defined as the ratio of change in (input) base voltage to the change in (input) base current with the (output) collector voltage (VCE), kept constant. Therefore,

$$h_{ie} = \frac{\Delta V_{BE}}{\Delta I_B} \text{ , } \Delta V_{CE} \text{ constant}$$

It is the slope of CB input characteristics IB versus VBE.

The typical value of hie ranges from 500Ωto 2000Ω.

ii) Output Admittance (hoe):

It is defined as the ratio of change in the (output) collector current to the corresponding change in the (output) collector voltage. With the (input) base current IB kept constant. Therefore,

$$h_{oe} = \frac{\Delta I_C}{\Delta V_{CE}} \text{ , } I_B \text{ constant}$$

It is the slope of CE output characteristics IC versus VCE.

The typical value of this parameter is of the order of 0.1 to 10μmhos.

iii) Forward Current Gain (hfe):

It is defined as a ratio of the change in the (output) collector current to the corresponding change in the (input) base current keeping the (output) collector voltage VCE constant. Hence,

$$h_{fe} = \frac{\Delta I_C}{\Delta I_B} \text{ , } V_{CE} \text{ constant}$$

It is the slope of IC versus IB curve.

It's typical value varies from 20 to 200.

iv) Reverse Voltage Gain (hre):

It is defined as a ratio of the change in the (input) base voltage and the corresponding change in (output) collector voltage with constant (input) base current, IB. Hence,

$$h_{re} = \frac{\Delta V_{BE}}{\Delta V_{CE}}, \ I_E \text{ constant.}$$

It is the slope of VBE versus VCE curve.

It's typical value is of the order of 10-5 to 10-4

Characteristics of common collector circuit:

The circuit diagram for determining the static characteristics of an N-P-N transistor in the common collector configuration is shown in fig. below.

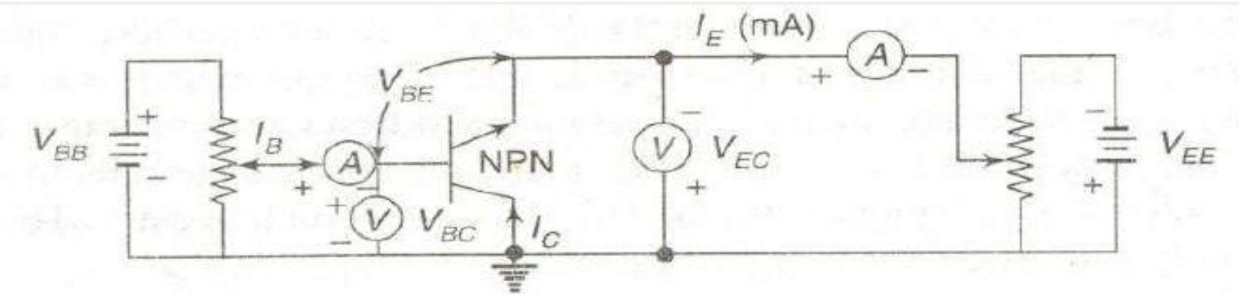


Fig. Circuit to determine CC static characteristics.

Input Characteristics:

To determine the input characteristic, VEC is kept at a suitable fixed value. The base collector voltage VBC is increased in equal steps and the corresponding increase in IB is noted. This is repeated for different fixed values of VEC. Plots of VBC versus IB for different values of VEC shown in figure are the input characteristics.

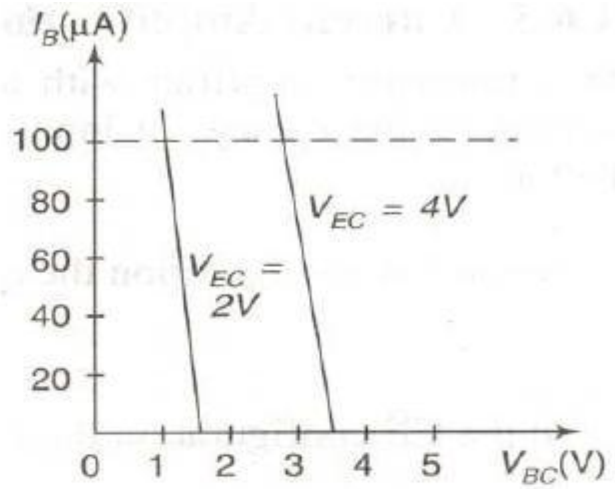


Fig. CC Input Characteristics.

Output Characteristics:

The output characteristics shown in figure below are the same as those of the common emitter configuration.

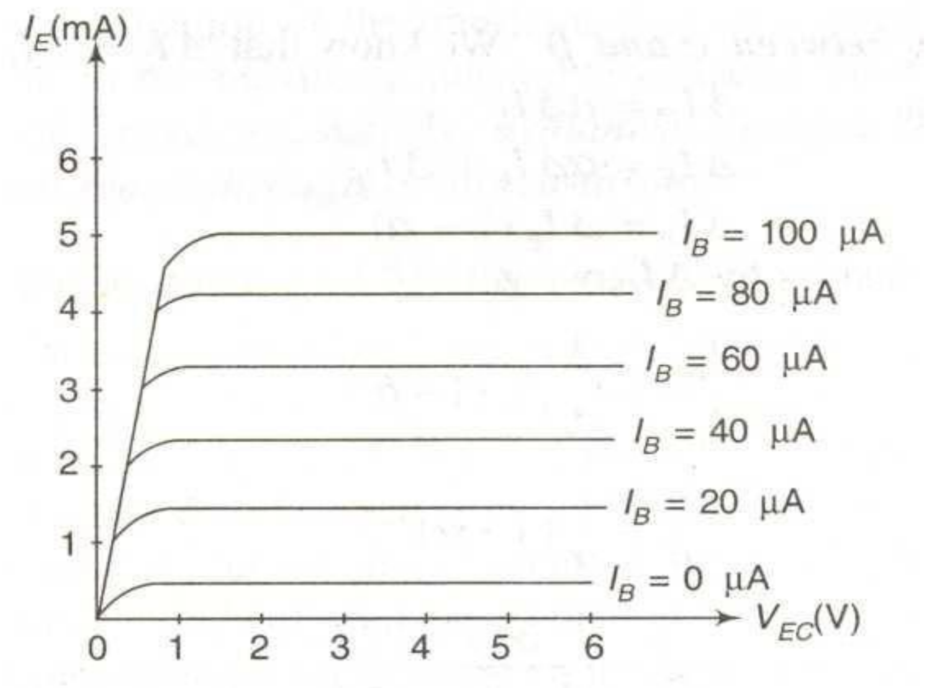


Fig. CC output characteristics.

Comparison:

Table: A comparison of CB, CE and CC configurations

Property	CB	CE	CC
Input Resistance	Low (About 100Ω)	Moderate (About 750Ω)	High (About 750kΩ)
Output Resistance	High (About 450kΩ)	Moderate (About 45kΩ)	Low (About 25Ω)
Current Gain	1	High	High
Voltage Gain	About 150	About 500	Less than 1
Phase Shift between input and output voltages	0° (or) 360°	180°	0° (or) 360°
Applications	For high frequency circuits	For Audio frequency circuits	For impedance matching

Problem:

1 A Germanium transistor used in a complementary symmetry amplifier has ICBO=10μA at 27oC and hfe=50. (a) find IC when IB=0.25mA and (b) Assuming hfe does not increase with temperature; find the value of new collector current, if the transistor's temperature rises to 50oC.

Solution: Given data: ICBO = 10μA and hfe (=β) = 50

a) IC = βIB+(1+β)ICBO

= 50x(0.25x10-3)+(1+50)x(10x10-6)A

=**13.01mA**

b) I’CBO (β=50) = ICBO x 2(T2-T1)/10

= 10 X 2(50-27)/10

= 10 x 22.3 μA

= 49.2μA

IC at 50oC is

IC= βIB+(1+β)I’CBO

= 50x(0.25x10-3)+(1+50)x(49.2x10-6)

=15.01 mA.